

VCE VM Numeracy Units 3 & 4

Formula Sheet — every formula used in Units 3 & 4

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Formula Sheet — every formula used in this book

This sheet collects every formula used anywhere in Units 3 and 4, with the meaning of every letter. Keep it beside you while you work through the chapters, and use it for the practice assessments in Chapters 12 to 14 wherever your teacher allows.

Three things to remember before you start.

- **Work in the right order.** Brackets first, then indices (powers and roots), then multiplication and division, then addition and subtraction, working from left to right. Substituting $h = 4$ into $3 + 2h^2$ gives $3 + 2 \times 16 = 35$, not $(3 + 2 \times 4)^2$.
- **Carry full precision, round only at the end.** Rounding too early changes the answer. Every question that needs rounding tells you the precision to use — nearest cent, one decimal place, and so on. Halfway values round up. Round in the direction that fits the real situation: a budget or a ceiling rounds **down**, a target or a floor rounds **up**.
- **Use the numbers the question gives you.** The GST of 10% and the Medicare levy of 2% are the standard rates used in this book. Superannuation rates, tax rates, award rates and insurance rates change, so they are always stated in the question — use the question's number, not a remembered one.

The mathematical toolkit (Chapter 1)

Description	Formula
Multiply two cells in a spreadsheet — B2 and C2 are cell names	=B2 * C2
Add up a block of cells	=SUM (D2 : D6)
Mean of a block of cells	=AVERAGE (D2 : D6)
Largest and smallest values in a block of cells	=MAX (D2 : D6) and =MIN (D2 : D6)
How many records are in a block of cells	=COUNT (D2 : D6)
Dragging a formula down copies it	the cell names move down one row each time
A formula and a function	a formula names the cells yourself; a function is a built-in formula word
A formula always means “whatever is in these cells now”	correcting one input recalculates every total that uses it
Sorting	move the whole rows , not the key column on its own
Filtering	hides rows, it never deletes them; a total under a filter counts the shown rows only
The symbols used in this book	= equals; $\approx$ approximately equal to (“is about equal to”); % out of 100; $\therefore$ therefore; $\frac{22}{7} \approx 3.14$ is a fraction approximation of π
The problem-solving cycle	four stages in order: identify the mathematics, act on and use mathematics, evaluate and reflect, communicate and report
Signpost words	total $\rightarrow$ add; left / change / difference $\rightarrow$ subtract; each / per $\rightarrow$ multiply or a rate; share equally $\rightarrow$ divide; how many tins or buses $\rightarrow$ divide, then round up
Three ways to estimate	round to friendly numbers ($12 \times \$3.85 \approx 12 \times \$4 = \$48$); compare with a known benchmark; think about which direction the answer must move
Six checks on an answer	estimate it; check size and units; check the direction; check the type of number; use real-world knowledge; work another way — if you multiplied, divide back
An answer exactly ten times out	the decimal point slipped one place — check every $\times 10$ and $\div 10$
Contextual rounding	whole things you must buy round up ; a budget or a limit rounds down ; a measurement rounds to the stated place
Reading any scale	one small space = (gap between two labels) $\div$ (spaces between them); reading = label below + (spaces past $\times$ value of one space)
Measuring from a start that is not zero	length = end reading – start reading

Number (Chapter 2)

Description	Formula
Place value	each place is worth 10 times the place to its right; after the decimal point, each place is one tenth of the place to its left
Multiplying and dividing by 10, 100, 1000	the digits move that many places — left when you multiply, right when you divide
Rounding to a given place	look at the deciding digit, the next one to the right: 5 or more, round up; 4 or less, round down
Halfway values	always round up, so $87.5 \rightarrow 88$ and $\$4.625 \rightarrow \$4.63$
Square of a number	$n^2 = n \times n$
Cube of a number	$n^3 = n \times n \times n$
Index of 1	$n^1 = n$
Powers of ten	$10^1 = 10$, $10^2 = 100$, $10^3 = 1000$
An index applies only to what it sits on	$2 \times 5^2 = 50$ but $(2 \times 5)^2 = 100$
A negative base needs brackets	$(-4)^2 = 16$ but $-4^2 = -16$
Square root	$\sqrt{a}$ is the positive number that multiplies by itself to give a , so $\sqrt{25} = 5$
Squaring and square-rooting undo each other	$\sqrt{a^2} = a$ and $(\sqrt{a})^2 = a$, for $a \geq 0$
Side of a square from its area	$s = \sqrt{A}$
Multiplying and dividing signed numbers	same signs give a positive answer; different signs give a negative answer
Subtracting a negative	$a - (-b) = a + b$
Swapping the order of a multiplication	never changes the product: $4 \times 7 = 7 \times 4$
Splitting a multiplication	$34 \times 27 = (34 \times 20) + (34 \times 7)$ — multiply the parts, then add
Factors of a number	the whole numbers that divide into it exactly; list them in pairs, and stop when the pairs repeat
Prime and composite	a prime has exactly two factors (1 and itself); a composite has more than two; 1 is neither
Testing for a prime	divide by primes in order and stop once the square of the test number is bigger than the number itself
Divisibility tests	2: last digit even; 3: digit sum divisible by 3; 4: last two digits divisible by 4; 5: ends in 0 or 5; 6: passes both the 2 and 3 tests; 9: digit sum divisible by 9; 10: ends in 0
Highest common factor (HCF)	the biggest number that divides into both amounts exactly — used for simplest forms
Lowest common multiple (LCM)	the smallest number both amounts divide into exactly — used for common denominators and for cycles that line up
Estimating a square root	trap it between the perfect squares either side: $6^2 = 36 < 40 < 49 = 7^2$, so $\sqrt{40}$ is between 6 and 7
Multiplying decimals	multiply as whole numbers, then put back the total number of decimal places: $0.6 \times 0.3 = 0.18$

Description	Formula
Fraction to a decimal	divide the top by the bottom
Decimal to a fraction	use place value, then simplify: $0.6 = \frac{6}{10} = \frac{3}{5}$
Decimal to a percentage	multiply by 100
Percentage to a decimal	divide by 100
Percentage to a fraction	over 100, then simplify: $20\% = \frac{20}{100} = \frac{1}{5}$
Scaling a fraction to a percentage	when the bottom divides into 100, multiply top and bottom up: $\frac{21}{25} = \frac{84}{100} = 84\%$
Benchmarks to know	$\frac{1}{2} = 50\%$, $\frac{1}{4} = 25\%$, $\frac{3}{4} = 75\%$, $\frac{1}{5} = 20\%$, $\frac{1}{10} = 10\%$, $\frac{1}{8} = 12.5\%$, $\frac{1}{3} \approx 33\%$, $\frac{2}{3} \approx 67\%$
Adding and subtracting fractions	same bottom: add the tops; different bottoms: first rename them with a common denominator (the LCM)
Mixed number to improper	$\frac{(\text{whole} \times \text{bottom}) + \text{top}}{\text{bottom}}$, so $2\frac{3}{5} = \frac{13}{5}$
“Of” means multiply	$\frac{3}{4}$ of 20 = $\frac{3}{4} \times 20 = 15$
Multiplying fractions	tops $\times$ tops, bottoms $\times$ bottoms, then simplify
Dividing by a fraction	multiply by the reciprocal (the fraction turned upside down): $3 \div \frac{1}{2} = 3 \times 2 = 6$
Percentage of an amount	$\frac{p}{100} \times \text{amount}$
Percentages without a calculator	find 10% (move the point one place left), then build: halve it for 5%, double it for 20%, so $15\% = 10\% + 5\%$
Increase an amount by $p\%$	multiply by $1 + \frac{p}{100}$
Decrease an amount by $p\%$	multiply by $1 - \frac{p}{100}$
Undo a percentage change	original amount = new amount $\div$ multiplier
Successive percentage changes	multiply the multipliers — $1.10 \times 0.90 = 0.99$, a fall of 1%, not no change
Equivalent ratios	multiply or divide both terms by the same number
Simplest form of a ratio	whole-number terms with no common factor bigger than 1 — divide both by the HCF; convert to the same units first
One part from a difference	difference in amounts $\div$ difference in parts, so 2 extra parts worth \$86 gives one part = \$43
The unitary method	find the value of one, then multiply up
Two equal ratios form a proportion:	

$$a:b=c:d \text{ means } \frac{a}{b} = \frac{c}{d}$$

To share a total in the ratio $a:b$:

$$\text{one part} = \text{total} \div (a + b)$$

Percentage change, where the change is new – old and the original amount is the value you started from:

$$\text{percentage change} = \frac{\text{change}}{\text{original amount}} \times 100 \%$$

Shape (Chapter 3)

Description	Formula
Prism with a base of n sides — n is the number of sides of the base	faces = $n + 2$, edges = $3n$, vertices = $2n$
Pyramid with a base of n sides	faces = $n + 1$, edges = $2n$, vertices = $n + 1$
Angles on a straight line	add to 180°
Angles at a point	add to 360°
Vertically opposite angles	are equal
Angles in a triangle — a, b, c are the three angles	$a + b + c = 180^\circ$
Equilateral triangle	every angle = $180^\circ \div 3 = 60^\circ$
Isosceles triangle	the two base angles are equal: base angle = $(180^\circ - \text{top angle}) \div 2$
Right-angled triangle	the two acute angles add to 90°
Exterior angle of a triangle	= the sum of the two interior angles far away from it
Angles in a quadrilateral	add to 360°
Parallelogram and rhombus	opposite angles are equal; neighbouring angles add to 180°
Kite	one pair of opposite angles is equal
Co-interior angles between parallel lines	add to 180°
Reflex angle	$360^\circ - \text{the smaller angle}$
Supplementary angle	$180^\circ - \text{the angle}$
Missing side of a compound shape with right-angled corners	the sides facing each other add up to the same total: $x = \text{whole side} - \text{known part}$

Angles of any polygon, where n is the number of sides:

$$\text{angle sum} = (n - 2) \times 180^\circ \quad \text{each angle of a regular polygon} = \frac{(n - 2) \times 180^\circ}{n}$$

Description	Formula
Tessellation test	the angles meeting at a point must add to exactly 360° ; a regular polygon tessellates on its own only when $360^\circ \div \text{its interior angle}$ is a whole number
A rule for a growing pattern	find the amount added each term: rule = $(\text{step} \times \text{term number}) + \text{adjustment}$, for example matches = $2 \times \text{term number} + 1$ — then check it on the first term

Rotational symmetry, where the order is the number of times a shape lands exactly on itself in one full turn:

$$\text{smallest turn} = 360^\circ \div \text{order} \quad \text{order} = 360^\circ \div \text{smallest turn}$$

Description	Formula
Regular polygon with n sides	n axes of symmetry and rotational order n , so its smallest turn = $360^\circ \div n$
An order of 1	means no rotational symmetry

Enlargement, where k is the scale factor:



Description	Formula
Scale factor	$k = \text{new length} \div \text{matching old length}$
Similar shapes	every matching-length ratio comes out the same, and that common ratio is k ; every angle stays the same
Lengths and perimeters	multiply by k
Going backwards	$\text{old length} = \text{new length} \div k$
Areas	multiply by k^2
Volumes	multiply by k^3

Units and conversion (Chapter 4)

Description	Formula
Metric prefixes	milli = one thousandth, centi = one hundredth, kilo = one thousand
Metric length	1 cm = 10 mm, 1 m = 100 cm = 1000 mm, 1 km = 1000 m
Metric mass	1 kg = 1000 g
Metric capacity	1 L = 1000 mL
Moving along the metric ladder	going down to a smaller unit, multiply; going up, divide
Imperial length	12 in = 1 ft, 3 ft = 1 yd, 1760 yd = 1 mile
Imperial mass	16 oz = 1 lb, 14 lb = 1 st
Imperial capacity	8 pt = 1 gal
Area units	1 m ² = 10 000 cm ² , 1 ha = 10 000 m ² , 1 km ² = 100 ha = 1 000 000 m ²
Volume units	1 m ³ = 1 000 000 cm ³
Volume and capacity	1 cm ³ = 1 mL, 1000 cm ³ = 1 L, 1 m ³ = 1000 L = 1 kL, 1 kL = 1000 L
A smaller unit gives a bigger number	check which way your answer should move before you multiply or divide
A unit slip	putting centimetres into a metres box makes the answer 100 times too big; a kilometre box makes it 1000 times too big

The conversion factors used in this book. To go the other way, divide by the same factor.

To convert	Multiply by
inches to centimetres	2.54
feet to centimetres	30.48
miles to kilometres	1.609
ounces to grams	28.35
kilograms to pounds	2.205
US fluid ounces to millilitres	29.6
US pints to millilitres	473
UK pints to millilitres	568
US gallons to litres	3.79
UK gallons to litres	4.55

Quick estimates, useful for checking whether an answer is sensible:

Description	Formula
Rough length and mass equivalents	1 in $\approx$ 2.5 cm, 1 ft $\approx$ 30 cm, 1 mile $\approx$ 1.6 km, 1 kg $\approx$ 2.2 lb, 1 lb $\approx$ 450 g, 1 oz $\approx$ 28 g
Rough Celsius to Fahrenheit	double the Celsius reading, then add 30
Everyday anchors	a door $\approx$ 2 m, an adult step $\approx$ 70 cm, a mug $\approx$ 300 mL, an apple $\approx$ 150 g, a teaspoon = 5 mL, an A4 page $\approx$ 21 cm $\times$ 30 cm
Sensible speeds	walking $\approx$ 5 km/h; ordinary town driving $\approx$ 48 km/h

Perimeter, area and volume (Chapter 4)

Description	Formula
Value of π	$\pi = 3.14159 \dots \approx 3.14$
Diameter and radius — d is the diameter, r is the radius	$d = 2r$
Perimeter of a rectangle — l is the length, w is the width	$P = 2l + 2w = 2(l + w)$
Perimeter of a square — s is the side	$P = 4s$
Circumference of a circle	$C = \pi d = 2\pi r$
Arc of a semicircle	πr
Arc of a quarter-circle	$\frac{\pi r}{2}$
Perimeter of a semicircle	arc + diameter
Perimeter of a quarter-circle	arc + two radii
Area of a square	$A = s^2$
Area of a rectangle	$A = lw$
Area of a triangle — b is the base, h is the height at 90° to that base	$A = \frac{1}{2}bh$
Area of a parallelogram — h is the perpendicular height	$A = bh$
Area of a trapezium — a and b are the parallel sides, h is the perpendicular height	$A = \frac{1}{2}(a + b)h$
Area of a circle	$A = \pi r^2$
Area of a semicircle	$A = \frac{1}{2}\pi r^2$
Volume of a cuboid — h is the height	$V = l \times w \times h$
Volume of any prism — A is the area of the cross-section, h is the height	$V = A \times h$
Volume of a cylinder	$V = \pi r^2 h$
Volume of a cone — h is the perpendicular height	$V = \frac{1}{3}\pi r^2 h$
Volume of a pyramid — A is the area of the base	$V = \frac{1}{3} \times A \times h$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a hemisphere	half the volume of the sphere
Radius from the diameter	$r = d \div 2$

Working with compound shapes and real jobs:

Description	Formula
A compound shape	split it and add, or complete it and subtract
A perimeter	count the outside edge only
A ring or a path	outer area – inner area
Fencing with a gate	perimeter – gate width
Water pushed up by an object	base area $\times$ rise in the water level
A part-filled container	water now = $l \times w \times$ depth of water
A slab or a layer	area $\times$ thickness

Description	Formula
Tins, bags or tiles needed	$\text{area} \div \text{coverage of one tin, bag or tile} \text{ — round up}$
Cost of edging or fencing	$\text{length} \times \text{price per metre}$
Volume of mulch or concrete	$\text{area} \times \text{depth}$
Fill time	$\text{volume} \div \text{pump rate}$
Cups round down, truck loads round up	round in the direction that fits the real situation

Time, temperature and accuracy (Chapter 4)

Description	Formula
Time units	1 min = 60 s, 1 h = 60 min = 3600 s, 1 day = 24 h, 1 week = 7 days
Year lengths	a common year has 365 days; a leap year has 366
Parts of an hour	half = 30 min, quarter = 15 min, three-quarters = 45 min
Minutes from an analogue clock	minute-hand number $\times 5$
Twelve-hour to twenty-four-hour	pm: add 12 to the hour; midnight 12 becomes 00; midday is 1200
Twenty-four-hour to twelve-hour	an hour above 12: subtract 12 and write pm; an hour below 12: write am
Decimal hours to hours and minutes	the fractional part $\times 60$, so 0.75 h = 45 min
Minutes to decimal hours	minutes $\div 60$, so 90 min = 1.5 h
A duration that crosses midnight	split it at midnight, then add the two parts
Subtracting times	borrow one hour as 60 minutes, so 14:15 = 13:75
Paid time	shift time – unpaid breaks
Time zones	east of home: add the difference in offsets; west: subtract
Past midnight in a time zone	subtract 24 h and move the date forward one day

Description	Formula
Celsius to Fahrenheit	multiply by 1.8, then add 32
Fahrenheit to Celsius	subtract 32, then divide by 1.8
The same rules written as formulas — C is the temperature in $^{\circ}\text{C}$, F in $^{\circ}\text{F}$	$F = 1.8C + 32$ and $C = (F - 32) \div 1.8$
Fixed points	$0^{\circ}\text{C} = 32^{\circ}\text{F}$, $37^{\circ}\text{C} = 98.6^{\circ}\text{F}$, $100^{\circ}\text{C} = 212^{\circ}\text{F}$, $-40^{\circ}\text{C} = -40^{\circ}\text{F}$
A rise adds, a fall subtracts	difference = higher reading – lower reading
Temperature changing at a steady rate — T is the temperature, t is the time	$T = \text{start} + \text{rate} \times t$
Time to reach a target temperature	$(\text{target} - \text{start}) \div \text{rate}$
Fan-forced oven	set 20°C below the chart temperature

Reading any scale, and how accurate a reading is. In the formulas below, r is the reading and d is the smallest division on the tool.

Description	Formula
Step of one space on a scale	$(\text{bigger label} - \text{smaller label}) \div \text{number of spaces between them}$
A reading on a scale	$\text{label} + (\text{marks} \times \text{step})$
Calibrate your step	$\text{step length} = \text{known distance} \div \text{number of steps}$
Estimate by pacing	$\text{distance} = \text{steps} \times \text{length of one step}$
Estimating by chunking	$\text{one count} \times \text{number of chunks}$
Counting squares on a grid	$\text{whole squares} + \text{squares that are half covered or more}$

Description	Formula
A percentage tolerance	tolerance in units = amount $\times \frac{p}{100}$

the true value lies between $r - (d \div 2)$ and $r + (d \div 2)$

Tolerances are written in three forms. The limits themselves always count as inside.

Description	Formula
The $\pm$ form	accepted from target $-t$ up to target $+t$, so 250 ± 2 mm accepts 248 mm to 252 mm
The range form	midpoint = $(\text{low} + \text{high}) \div 2$; half-width = high – midpoint
The one-sided form	820 mm $(+5 \text{ mm}, -0 \text{ mm})$ accepts 820 mm to 825 mm
Deciding pass or fail	a band wholly inside the tolerance passes; wholly outside, it fails; crossing a limit means you need a finer tool

Formulas, rates and graphs (Chapter 5)

Description	Formula
Substitution	replace each letter with its number, then use the order of operations
Back-tracking	undo the steps in reverse order, using the inverse operation each time
The fixed-plus-rate template — f is the fixed fee paid once, r is the charge per unit, n is the number of units	$C = f + rn$
A decreasing rule — r is the number of rentals, B is the balance left in dollars	$B = 100 - 8r$
A rule built from words	name the variables, then total = fixed part + changing part
The starting value of a rule	the value when the input is 0
Dropping the multiplication sign	$3 \times h = 3h$, $a \times b = ab$, $4 \times (x + 2) = 4(x + 2)$
Multiplying by 1 or by -1	$1 \times h = h$ and $-1 \times h = -h$
Division written as a fraction	$h \div 4 = \frac{h}{4}$, which is not the same as $\frac{4}{h}$
A power	$h \times h = h^2$ and $h \times h \times h = h^3$
An index belongs to the letter only	$2h^2 = 2 \times h \times h$
Brackets square everything inside	$(2h)^2 = 4h^2$
Adding is not multiplying	$x + x = 2x$ but $x \times x = x^2$
Like terms	the letters and the indices must match, so $3h + 5h = 8h$
The distributive law	$a(b + c) = ab + ac$, so $4(x + 2) = 4x + 8$
The commutative law of addition	$a + b = b + a$, so $40 + 15h = 15h + 40$
The three jobs a rate can do:	

$$\text{rate} = \text{amount} \div \text{time} \quad \text{amount} = \text{rate} \times \text{time} \quad \text{time} = \text{amount} \div \text{rate}$$

The same three jobs for a journey. For a speed in km/h the time must be in hours, so convert minutes first by dividing by 60.

$$\text{speed} = \text{distance} \div \text{time} \quad \text{distance} = \text{speed} \times \text{time} \quad \text{time} = \text{distance} \div \text{speed}$$

Description	Formula
A rate per minute to a rate per hour	$\times 60$
Metres per second to kilometres per hour	$\times 3.6$
Kilometres per hour to metres per second	$\div 3.6$
Average speed for a whole trip	total distance $\div$ total time, including stops
Speed on one leg of a travel graph	change in distance $\div$ change in time for that leg
Fuel economy in L per 100 km	litres $\div$ (km $\div$ 100)
From km per litre to L per 100 km	$100 \div$ (km per litre)
Combined rate of two machines or taps	add the individual rates
Heart rate	beats $\div$ minutes

Scale, proportion and mixing:

Description	Formula
A scale of $1:n$ — n is the scale factor	drawing to real: $\times n$; real to drawing: $\div n$
Centimetres to kilometres	$100\text{ cm} = 1\text{ m}$ and $100\,000\text{ cm} = 1\text{ km}$
Map scales	$1:25\,000 \rightarrow 1\text{ cm} = 250\text{ m}$; $1:50\,000 \rightarrow 500\text{ m}$; $1:100\,000 \rightarrow 1\text{ km}$
Plan scales	$1:50$ and $1:100$ for a house or shed; $1:200$ and $1:500$ for a site
Deriving a scale	divide both terms by the drawing term, so $6:480\,000$ becomes $1:80\,000$
Scaling a recipe	multiply every ingredient by new serves $\div$ old serves
Mixing in the ratio $a:b$ with the total known	one part = total $\div (a + b)$
Mixing with one amount known	find one part first, then multiply up
A limiting ingredient	the smaller one-part value decides how much you can make
Scale is never applied to an area	convert the real lengths first, then find the area

$$\text{image size} = \text{actual size} \times \text{magnification factor}$$

$$\text{actual size} = \text{image size} \div \text{magnification factor} \quad \text{factor} = \text{image} \div \text{actual}$$

Drawing and reading graphs:

Description	Formula
The axes	input on the horizontal axis, output on the vertical axis
Joined line or separate dots	join the dots when the input is measured; leave them separate when it is counted
Reading a graph	rising means increasing; rising steeply means increasing quickly; flat means no change; falling means decreasing
A rate from a graph	fall or rise $\div$ time taken
Stopped time on a travel graph	the length of the flat stretch
Step graphs	a part hour or part block is charged as a full one; a closed dot is included, an open dot is not
Two competing rules	set them equal and solve, or compare them at the same input; the crossing point is where they cost the same
A conversion chart	gives estimates; the rule gives the exact answer

Numeracy in context — money, work and shopping (Chapters 6 and 11)

Description	Formula
Unit price	total price $\div$ amount — the lower unit price is the better buy
Discount in dollars	$\frac{p}{100} \times \text{marked price}$
Sale price	marked price – discount
A discount as a multiplier	$p\%$ off means multiply by $1 - \frac{p}{100}$, so 15 % off is $\times 0.85$
Instalments with a deposit	(price – deposit) $\div$ number of payments
Instalment plan total	deposit + (number of payments $\times$ amount of each)
Sharing a cost	total $\div$ number of people
Fuel used on a trip	(km $\div$ 100) $\times$ rate in L per 100 km
Fuel cost	litres $\times$ price per litre
Scaling a recipe	new serves $\div$ old serves, then multiply every ingredient
Cost per unit of area	price $\div$ area $\times$ 100, giving the cost per 100 cm ²
A quote with a setup fee	(quantity $\times$ unit price) + setup fee
Payback time on a purchase	price paid $\div$ saving each year

Wages and pay, where W is the pay for those hours, R is the hourly rate and H is the number of hours worked:

$$W = R \times H$$

Description	Formula
Time-and-a-half	hourly rate $\times$ 1.5
Double time	hourly rate $\times$ 2
Average hourly rate	gross pay $\div$ total hours
Allowances	added on top of the hourly pay

Commission pay, where P is the total weekly pay, B is the retainer or base paid whatever happens, c is the commission rate as a decimal and S is the value of sales that week:

$$P = B + c \times S$$

Bills and plans, where C is the total cost, F is the fixed charge for the period, u is the charge per unit and q is the number of units used:

$$C = F + u \times q$$

Description	Formula
Unit cost on a bill	price $\div$ units, for example $\$30 \div 10 = \$3.00$ per GB
Over-inclusion plan	plan fee + (extra units $\times$ per-unit fee)

Simple interest, where I is the interest, P is the principal (the amount borrowed), r is the yearly rate as a decimal and n is the number of years. For six months, $n = 0.5$.

$$I = P \times r \times n$$

Description	Formula
Total repaid on a loan	principal + interest

Description	Formula
Each repayment	$\text{total repaid} \div \text{number of repayments}$
The longer you borrow	the more interest you pay
Credit cards	$\text{interest for the month} = \text{monthly rate} \times \text{balance}$
Long-term cost of a purchase	$(\text{each repayment} \times \text{number of repayments}) + \text{running costs}$

Tax and the standard rates used in this book:

Description	Formula
Adding the GST	$\text{price} \times 1.10$
Taking the GST out	$\text{GST inside an inclusive price} = \text{price} \div 11$
Why it is $\div 11$	an inclusive price is 110 %, and the GST is 10 of those 110 parts
Medicare levy	2 % of taxable income
Superannuation	$\text{rate} \times \text{gross pay}$, with the rate given in the question
Tax as a share of income	$\text{tax paid} \div \text{income} \times 100$

Budgets and saving:

Description	Formula
Surplus	$\text{income} - \text{total expenses}$
Deficit	a negative surplus — expenses are more than income
Months to reach a savings goal	$\text{goal} \div \text{surplus each month}$ — round a part month up
Average change per year	$\text{total change} \div \text{number of years}$

Events, venues and running something:

$$\text{total cost} = \text{fixed cost} + (\text{cost per person} \times \text{number of guests})$$

Description	Formula
Cost per person	$\text{total cost} \div \text{number of guests}$
Comparing venues	cost them both at the same guest number; the cheaper one can change as guests grow
Guests you can afford	$(\text{budget} - \text{fixed cost}) \div \text{cost per person}$ — round down
Income for an event	$(\text{tickets} \times \text{price}) + \text{sponsorship}$
Profit	$\text{income} - \text{costs}$
Entry fee that covers the cost	$\text{total cost} \div \text{number of people}$ — round up to the cent

Numeracy in context — rates, data in the news and odds (Chapters 6 and 11)

Description	Formula
A rate as a percentage	$\text{part} \div \text{whole} \times 100$
A rate per 100 000 people	$\text{count} \div \text{population} \times 100\,000$
Parts per million	$\text{count} \div \text{population} \times 1\,000\,000$
A share written as “1 in n ”	$\text{part} \div \text{whole}$, then divide both terms by the smaller one
A number from a stated share	$\text{share as a decimal} \times \text{population}$
Percentage-point change	$\text{new value} - \text{old value}$
Per cent change on a rate	$(\text{new} - \text{old}) \div \text{old} \times 100$
Percentage points are not per cent	a rise from 5.8 % to 6.4 % is 0.6 percentage points but 10.3 per cent
A percentage rise applied to a rate	$\text{the rate} \times \text{the rise}$, giving an answer in percentage points
Storage or capacity as a percentage	$\text{amount held} \div \text{capacity} \times 100$
Recycling rate	$\text{recycled} \div \text{collected} \times 100$
GDP per person	$\text{GDP} \div \text{population}$
Growth rate over a period	$(\text{end value} - \text{start value}) \div \text{number of years}$

Work and employment statistics:

Description	Formula
Labour force	$\text{employed} + \text{unemployed}$
Employed	$\text{labour force} - \text{unemployed}$
Not in the labour force	$\text{working-age population} - \text{labour force}$
Unemployment rate	$\text{unemployed} \div \text{labour force} \times 100$
Underemployment rate	$\text{underemployed} \div \text{labour force} \times 100$
Participation rate	$\text{labour force} \div \text{working-age population} \times 100$
Gender pay gap	$(\text{men's earnings} - \text{women's earnings}) \div \text{men's earnings} \times 100$
Women's pay as a share of men's	$\text{women's earnings} \div \text{men's earnings} \times 100$
Injury rate per 100 workers	$\text{injuries} \div \text{workers} \times 100$
Defect rate	$\text{defects} \div \text{items checked} \times 100\%$
Compare rates, not counts	a smaller count can still be a bigger rate if the group is smaller

Prices, index numbers and inflation:

Description	Formula
Index number	the base period is 100; an index of 118.4 means 18.4 % above the base
Inflation rate	$(\text{new index} - \text{old index}) \div \text{old index} \times 100$
A rise in index points	$\text{new index} - \text{old index}$
Restating a price with an index	$\text{old price} \times \text{new index} \div \text{old index}$
Back to base-period dollars	$\text{price} \div (\text{index} \div 100)$
Affordability	$\text{median house price} \div \text{average yearly wage}$
Interest at a stated yearly rate	$\text{rate} \div 100 \times \text{amount}$
An official rate rise	$\text{new rate} - \text{old rate}$, in percentage points

Description	Formula
Your own inflation rate	add up (each rise as a decimal $\times$ your spend on it), then divide by your total spend

Elections:

Description	Formula
Turnout	$\text{votes cast} \div \text{people enrolled} \times 100$
Formal votes	$\text{votes cast} - \text{informal votes}$
A share of the vote	$\text{part} \div \text{formal votes} \times 100$
An outright win	more than half of the formal votes
A majority of seats	$(\text{seats} \div 2) + 1$

Sport and betting, where decimal odds are given in the question. A decimal odd is the payout for each dollar staked, and the stake comes back inside it.

Description	Formula
Payout	$\text{stake} \times \text{decimal odds}$
Profit	$\text{payout} - \text{stake}$
Implied probability	$1 \div \text{decimal odds}$
Odds of exactly \$2.00	imply exactly 50%
Shorter odds	mean more likely to win and less profit
The bookmaker's margin	add up the implied probabilities of every outcome; the total is over 100%, and the extra is the margin
Long-run average return per bet	$\text{probability of winning} \times \text{return if it wins}$
Average loss per bet	$\text{stake} - \text{average return}$
Ladder points	$3 \times \text{wins} + 1 \times \text{draws}$
Win percentage	$\text{wins} \div \text{games played} \times 100$
Probability from a record	$\text{wins} \div \text{games played}$

Reading data critically — questions to ask of any chart or headline:

Description	Formula
What is the whole?	a percentage is meaningless until you know what it is a percentage of
A count or a rate?	500 cases in a big city can be a lower rate than 50 in a small town
Percentage points or per cent?	they are different answers to the same change
“Up to” a percentage	is a limit, not a promise
Does the axis start at zero?	a truncated axis makes a small change look large
The illusion of a truncated axis	$\text{bar height} = \text{value} - \text{axis start}$, so the bars overstate the ratio
Mean or median?	with skewed money data the median describes the typical case better
Small counts make big percentages	a rise from 3 to 9 is 200% but only 6 more cases
Is the period and the source stated?	check both before you repeat a claim

Numeracy in context — health, safety and everyday systems (Chapters 6 and 11)

Description	Formula
Body mass index — w is the weight in kg, h is the height in m	$BMI = \frac{w}{h^2}$
BMI categories	below 18.5 underweight, 18.5 to 24.9 healthy, 25.0 to 29.9 overweight, 30.0 and above obese
A target weight from a BMI	target weight = target BMI $\times h^2$
Maximum heart rate	220 – age
A target heart-rate zone	the stated percentage $\times$ the maximum heart rate — the moderate zone in this book is 50 % to 70 %, but always use the percentages the question gives
Percentage of the maximum	reading $\div$ maximum $\times 100$
Pulse from a short count	count $\times (60 \div \text{seconds counted})$, so a 15-second count $\times 4$
Resting adult bands	heart rate 60 to 100 beats per minute; breathing 12 to 20 breaths per minute
A risk falling by a percentage	fall = rate $\times \frac{p}{100}$, then new rate = old rate – fall
Percentage of a daily guideline	amount $\div$ guideline $\times 100$

Food labels and nutrition:

Description	Formula
Scaling a per-100 g panel to a packet	packet mass $\div 100$ = number of lots of 100 g, then multiply each panel value
Energy as a share of the daily guide	kilojoules $\div$ daily guide $\times 100$
Kilojoules left in the day	daily guide – kilojoules eaten

Medicines:

Description	Formula
Dose in mg	weight in kg $\times$ mg per kg
Volume of liquid medicine	(dose in mg $\div$ strength in mg) $\times$ the volume the strength is given in
Doses in a day from an “every n hours” label	$24 \div n$
A dilution of 1 : n	1 part concentrate to n parts water
Strength of a mixed product	part $\div$ total $\times 100\%$, so 1 : 99 gives $\frac{1}{100} = 1\%$
Health costs	gap = fee – rebate
Extras cover	amount returned = percentage returned $\times$ bill

Access to services and prevention:

Description	Formula
Number of services a population needs	population $\div$ people per service
Expected number of cases	probability $\times$ group size
Cost per case avoided	cost of the program $\div$ number of cases avoided
Expected treatment cost	expected cases $\times$ cost of treating one case

Work, safety and jobs:

person-hours = number of workers $\times$ hours each

Description	Formula
Time for a job	job size $\div$ rate of work
Combined rate of two workers	add the individual rates
Posts and gaps	posts = gaps + 1
Aisle or clear space	total width – the space taken up
A load limit	exactly on the limit is allowed; over the limit is not
Fitting whole items into a space	fits along $\times$ fits across, dividing each direction and rounding down first: $90 \div 30 = 3$ and $60 \div 20 = 3$, so $3 \times 3 = 9$ trays
Compare in one unit	convert both measurements first, then compare
Painting	area $\times$ coats $\div$ coverage per litre — round up to whole tins
Screen time as a share of a week	weekly hours $\div 168 \times 100$
A finishing time at work	start + hours worked + breaks
Good output from a defect rate	total output $\times (1 - \text{defect rate as a decimal})$

Dimension and direction (Chapter 7)

Positions, movement and grids:

Description	Formula
Distance between two positions on a number line	bigger position – smaller position — negatives count, so $14 - (-3) = 17$ km
Movement on a number line	add to move toward bigger positions, subtract to move toward smaller ones
Halfway between two positions	$(\text{first} + \text{second}) \div 2$
A corner written as a pair	(across, up) from the origin $(0, 0)$; the order matters, so $(3, 5)$ and $(5, 3)$ are different corners
A slide	add to each number of the pair, so sliding 4 right and 3 up takes $(2, 1)$ to $(6, 4)$
Walking distance between two corners	blocks across + blocks up-and-down — you cannot cut diagonally through a block
Halfway between two corners	halve each number separately, so halfway between $(2, 1)$ and $(6, 7)$ is $(4, 4)$
Counting both ends of a run	add one to the gap: seats 3 to 8 is $8 - 3 + 1 = 6$ seats
A distance from a grid	grid steps $\times$ metres per step

Compass directions and bearings:

Description	Formula
The four cardinal points	N, E, S and W, always in that clockwise order: $360^\circ \div 4 = 90^\circ$ apart
An eight-point compass	the half-points sit halfway between the cardinals, $90^\circ \div 2 = 45^\circ$
The sixteen-point compass	each step = $360^\circ \div 16 = 22.5^\circ$
Turn clockwise from north	N 0° , NE 45° , E 90° , SE 135° , S 180° , SW 225° , W 270° , NW 315° ; the half-points add 22.5°
A bearing	written as north or south, then the angle, then east or west — for example N 15° W
The reverse bearing	keep the same angle and swap north for south and east for west, so N 20° E reverses to S 20° W
The nearest compass point to an angle	compare the gaps to the two neighbouring points — the smaller gap wins
Halfway between two compass points	add their degrees and halve, so halfway between S 180° and SW 225° is 202.5° , which is SSW
A turn changes your heading	add for a turn to the right, subtract for a turn to the left
Quarter turn and U-turn	a quarter turn = 90° ; a U-turn = 180° ; straight on = 0°
Grid references and maps	read along first, then up
A real distance from a map	map distance $\times$ scale factor

Angles and turns:

Description	Formula
The angle types	acute: between 0° and 90° ; right: exactly 90° ; obtuse:

Description	Formula
	between 90° and 180° ; straight: exactly 180° ; reflex: between 180° and 360° ; full turn: exactly 360°
“The angle” at a point	means the smaller of the two unless the question says otherwise; the two always add to 360°
The two parts of a right angle	unknown part = $90^\circ - \text{known part}$
Reading a protractor	the inner and outer scales both start at 0° ; the two readings always add to 180° , so a wrong-scale reading is fixed by $180^\circ - \text{reading}$
Full turns inside a big turn	turns = degrees $\div 360^\circ$; what is left over = degrees $- (\text{turns} \times 360^\circ)$
Degrees turned by something spinning	degrees = turns $\times 360^\circ$; at a rate in rpm (revolutions per minute), degrees = rpm $\times$ minutes $\times 360^\circ$
Equal sectors of a full turn	sector angle = $360^\circ \div \text{number of sectors}$ — round only for reporting, never mid-calculation
Minute hand of a clock	turns $360^\circ \div 60 = 6^\circ$ per minute
Hour hand of a clock	turns $360^\circ \div 12 = 30^\circ$ per hour, which is $360^\circ \div 720 = 0.5^\circ$ per minute
Angle between the clock hands	minute hand = minutes $\times 6^\circ$; hour hand = hours $\times 30^\circ + \text{minutes} \times 0.5^\circ$; take the difference, then the smaller of it and $360^\circ - \text{it}$

Data (Chapter 8)

Description	Formula
Scale of one square on a graph	gap between two neighbour labels $\div$ number of squares between them
Sector angle in a pie chart	$\text{frequency} \div \text{total} \times 360^\circ$
Percentage share	$\text{frequency} \div \text{total} \times 100\%$
Pictograph key	one picture stands for the number in the key; a part picture is that fraction of it
An interval like “20 to less than 30”	20 is in, 30 is not — a value exactly on a boundary always goes into the higher interval
A missing count in a table	total – all the other counts
Mean	sum of all the values $\div$ number of values
Mean written with symbols — n is the number of values	$\bar{x} = \frac{\text{sum of the values}}{n}$
Mean from a frequency table	(each value $\times$ its frequency), added up, then $\div$ the total frequency
Working backwards from a mean	needed total = mean $\times$ count; missing value = needed total – sum of the values you have
A value joining a set	new mean = (old total + new value) $\div$ new count
Median	the middle value when they are in order; with an even count, the mean of the two middle values
Position of the median	with an odd count of n values it is value number $(n + 1) \div 2$; with an even count, the two middle values are numbers $n \div 2$ and $n \div 2 + 1$
Mode	the value that appears most often
Range	highest value – lowest value
Outlier	a value far away from the rest — look for a big gap; there is no formula for one
Mean without an outlier	$(\text{sum} - \text{outlier}) \div (\text{number of values} - 1)$
A range is never negative	range ≥ 0 , and it is 0 only when every value is the same
A smaller range	means more consistent results, even when the means are the same
Overall change over time	last value – first value
Average rise per step	total rise $\div$ number of steps
Estimating one step ahead	last value + average rise — label it an estimate, not a fact
A trend	describe the direction, the speed of change and any turning points

Uncertainty (Chapter 9)

Probability, where every outcome is equally likely:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$$

Description	Formula
Probabilities run from	0 (impossible) to 1 (certain)
The opposite of an event	$P(\text{not the event}) = 1 - P(\text{event})$
Sample space	S is the list of every possible outcome
Two things happening together	list every pair; the total is the number of outcomes of one times the number of the other
Relative frequency	times it happened $\div$ number of trials — the longer the record, the more the estimate can be trusted
Expected number of times	probability $\times$ number of trials — round a decimal count of days or people to a whole number, and say so
Working backwards	total = count $\div$ probability, so 4 green tiles at $P = \frac{1}{3}$ means $4 \times 3 = 12$ tiles in all
Comparing chances	put them all in the same form first — 1 in 8 is $1 \div 8 = 12.5\%$; when the numbers of chances differ, compare rates, never raw counts
Expected cost	probability $\times$ cost if it happens
Written as “1 in how many”	$1 \div \text{probability}$, so $0.04 = \frac{4}{100} = \frac{1}{25}$, about once every 25 times
Likelihood levels	1 rare (under 5%), 2 unlikely (5% to 20%), 3 possible (20% to 50%), 4 likely (50% to 80%), 5 almost certain (over 80%)
Consequence levels	1 minor, 2 slight (up to about \$100 or first-aid only), 3 moderate (up to about \$1000 or a doctor), 4 major (up to about \$10000 or a serious injury), 5 catastrophic (over \$10000 or life-threatening)
Risk score	likelihood level $\times$ consequence level, each from 1 to 5, so the score runs from 1 to 25 — re-score after every change
Risk zones	1 to 4 low, 5 to 9 medium, 10 to 14 high, 15 to 25 extreme
Chance language	impossible $p = 0$; very unlikely $0 < p < 0.1$; unlikely $0.1 \leq p < 0.5$; even chance $p = 0.5$; likely $0.5 < p \leq 0.9$; very likely $0.9 < p < 1$; certain $p = 1$
An expected number is an average	over many tries, not a promise about one

Systematics and planning (Chapter 10)

$$\text{finish} = \text{start} + \text{duration} \quad \text{start} = \text{finish} - \text{duration}$$

Description	Formula
Total duration of a plan	add every block — tasks, travel, breaks and changeovers
An arrival time	departure + driving time + breaks
A departure time	arrival – duration
A duration	arrival – departure
Door-to-door time	add every leg — walk, wait, ride, change, walk
Buffer in a plan	deadline – planned finish
Latest start time	deadline – total of all the task durations
A task that depends on another	it starts when the one before it finishes; with several before it, start at the latest of their finishes
Working backwards	start from the deadline and subtract each duration in reverse order
A late task	pushes its finish, and every task after it, back by the same amount
Tasks done in parallel	the whole job takes as long as the longest task
Does the plan fit?	total time needed $\leq$ time available fits, and the leftover is your buffer
Checking a plan	add the durations and compare with the time you actually have
Setting a parameter	change one input in the rule, then work out the new output; change one at a time
A threshold — how many to reach a target	solve the rule for the target, test the whole numbers either side, and take the one that works
The biggest lever first	work out what each parameter change saves, then act on the largest

End of Formula Sheet