

VCE VM Numeracy Units 3 & 4

Chapter 9 — Uncertainty

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Contents

Section	Page
9A Chance language and probability values	2
9B Sample spaces and simple probability	16
9C Comparing events and deciding from data	29
9D Risk and likelihood	39
Chapter 09 — Test A (multiple choice)	50
Test A — Answers	54
Test A — Fully-worked solutions	55
Chapter 09 — Test B (written response)	56
Test B — Answers	60
Test B — Fully-worked solutions	61

Chapter 9 Uncertainty — 9A Chance language and probability values

Theory

Will it rain on Saturday? Will the bus be late? Will your ticket win the raffle? We answer questions like these with words — “probably”, “not a chance”, “fifty-fifty” — but words are vague: one person’s “likely” is another person’s “maybe”. Mathematics makes chance precise by turning it into a number. This section teaches the language of chance — the words, the numbers they stand for, and how to move between the two — and then shows how to calculate a chance, estimate one from data, and use it to predict how often something will happen.

About the data in this section. Every business, team, person and number in this section is **made up** (not real). They are built for this book so you can practise turning words of chance into numbers and numbers back into words. Nothing here is real data about a real person, business or place.

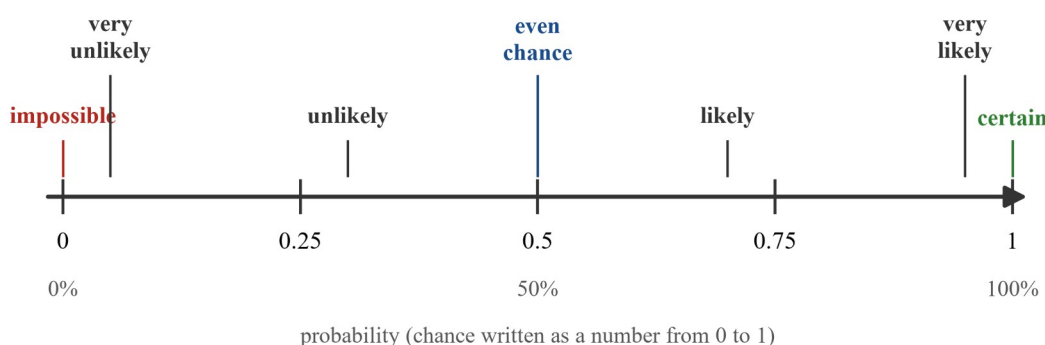
The chance scale.

Every chance, from “no hope at all” to “absolutely sure”, sits somewhere on one scale from 0 to 1. The number that says how likely an event is, is called its **probability**. An **event** is just a result we are asking about — “rain on Saturday”, “the bus is late”, “a red token is drawn”.

Probabilities are written with a P :

$$P(\text{event}) = \text{a number from 0 to 1}$$

- $P(\text{event}) = 0$ means the event is **impossible** — it can never happen.
- $P(\text{event}) = 1$ means the event is **certain** — it must happen.
- $P(\text{event}) = 0.5$ means an **even chance** — the event is exactly as likely to happen as not to happen.
- Between 0 and 1, closer to 1 means more likely and closer to 0 means less likely.



This picture shows the scale itself, not real data.

The same scale works as a percentage, because a probability and its percentage are the same number in different clothes: 0 is 0%, 0.5 is 50% and 1 is 100%. Nothing can be less likely than 0 and nothing can be more likely than 1 — if your working produces 1.3 or 120%, something has gone wrong.

The words of chance.

The words people use for chance match up with ranges of numbers. This book uses the exact bands in this table, so every question here has one clear word answer:

Word	Probability, p	Percentage
impossible	$p = 0$	0 %
very unlikely	$0 < p < 0.1$	between 0 % and 10 %
unlikely	$0.1 \leq p < 0.5$	between 10 % and 50 %

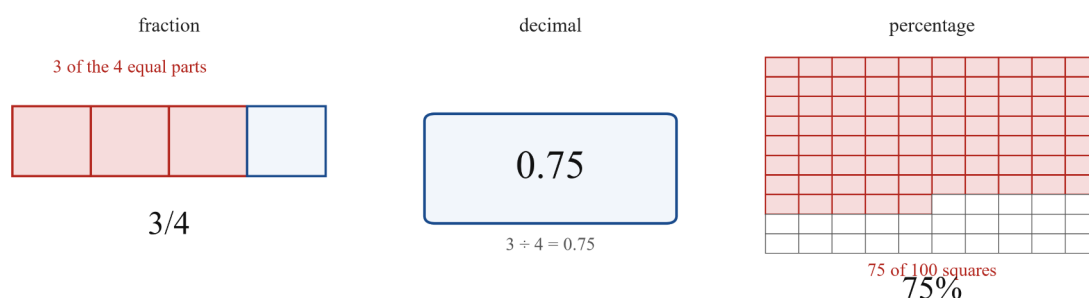
Word	Probability, p	Percentage
even chance	$p = 0.5$	exactly 50 %
likely	$0.5 < p \leq 0.9$	between 50 % and 90 %
very likely	$0.9 < p < 1$	between 90 % and 100 %
certain	$p = 1$	100 %

The edges of the middle bands are a guide — in ordinary talk, different people draw the lines a little differently — but the number itself is always exact. That is the whole point of using numbers: “72 %” ends an argument that “pretty likely” starts.

Three ways of writing one chance.

A probability can be written as a **fraction**, a **decimal** or a **percentage**, and all three are the same number in different clothes. Take the chance “three quarters”:

One chance, three ways of writing it



This picture shows the forms themselves, not real data.

The fraction $\frac{3}{4}$, the decimal 0.75 and the percentage 75 % all describe one chance. To move between the forms: divide the top by the bottom to get the decimal ($3 \div 4 = 0.75$), and multiply the decimal by 100 to get the percentage ($0.75 \times 100 = 75\%$). Section 2F practised these conversions; this section puts them to work on chance.

Some chances come up so often they are worth knowing off by heart:

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50 %
$\frac{1}{4}$	0.25	25 %
$\frac{3}{4}$	0.75	75 %
$\frac{1}{5}$	0.2	20 %
$\frac{1}{10}$	0.1	10 %
$\frac{1}{100}$	0.01	1 %
$\frac{1}{3}$	0.33 (to 2 decimal places)	33 % (to the nearest whole per cent)

The $\frac{1}{3}$ row is the odd one out: $1 \div 3 = 0.333\dots$ never ends, so the decimal and the percentage must be rounded — and whenever a value is rounded, say how far, exactly as the table does.

Chance as a ratio.

Newspapers, promotions and health reports often give a chance as a **ratio** — a count like “3 in 10”. A raffle hat holds 10 tickets and 3 of them are winners, so “3 in 10 tickets win”:

One ticket is drawn at random from a hat of 10

no. 1	WIN	no. 3	no. 4	WIN
no. 6	no. 7	WIN	no. 9	no. 10

3 winning tickets out of 10
 $P(\text{win}) = 3/10 = 0.3 = 30\%$

Made-up data: the raffle and its tickets are made up for this section.

“3 in 10” becomes a fraction straight away — 3 favourable tickets out of 10 in total — and then the other two forms follow:

$$P(\text{win}) = \frac{3}{10} = 0.3 = 30\%$$

The other $10 - 3 = 7$ tickets lose, so $P(\text{lose}) = \frac{7}{10} = 0.7 = 70\%$. Notice that a ratio like “3 in 10” is not a promise that exactly 3 of the next 10 draws win — it says how the chances stand on each single draw.

Calculating a probability.

When every possible result is **equally likely** — no result is favoured over another — a probability can be calculated by counting. Four results of a draw are worth naming:

- An **outcome** is one possible result — one ticket, one token, one face of a die.
- A **favourable outcome** is an outcome that does what we are asking about.
- Outcomes are **equally likely** when none has an advantage — a fair coin, a fair die, tickets well mixed in a hat.
- **At random** means chosen so that every outcome has the same chance of being the one picked — no looking, no picking favourites.

Count the favourable outcomes, count all the outcomes, and divide:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

A bag holds 10 tokens — 4 red and 6 blue — and one token is drawn at random:

A bag holds 10 tokens: 4 red and 6 blue. One is drawn at random.



$$P(\text{red}) = 4/10 = 2/5 = 0.4 = 40\% \quad P(\text{blue}) = 6/10 = 3/5 = 0.6 = 60\%$$

Made-up data: the bag and its tokens are made up for this section.

Every token is equally likely to be the one drawn, so counting works. There are 4 favourable outcomes for red out of 10 in total:

$$P(\text{red}) = \frac{4}{10} = \frac{2}{5} = 0.4 = 40\%$$

and for blue, $P(\text{blue}) = \frac{6}{10} = \frac{3}{5} = 0.6 = 60\%$. The token is either red or blue, so the two probabilities add to 1:

$\frac{2}{5} + \frac{3}{5} = 1$, that is $40\% + 60\% = 100\%$. When the counting is not plain to see — two dice, a pack of cards — Section 9B shows how to list every outcome first.

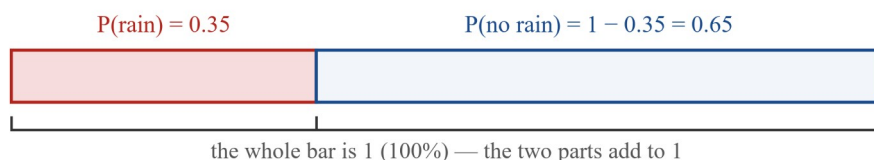
When an event does NOT happen.

Every event either happens or does not, and those two chances together cover everything, so they must add to 1:

$$P(\text{event does not happen}) = 1 - P(\text{event})$$

A weather app says $P(\text{rain}) = 0.35$. Then $P(\text{no rain}) = 1 - 0.35 = 0.65 = 65\%$:

An event and the same event NOT happening fill the whole bar



Made-up data: the rain probability in this picture is made up for this section.

One bar, two pieces: rain takes 0.35 of the bar and no-rain takes the rest. Finding “the chance it does not happen” by subtracting from 1 is usually faster than counting the not-happening outcomes — the raffle hat above has 7 losing

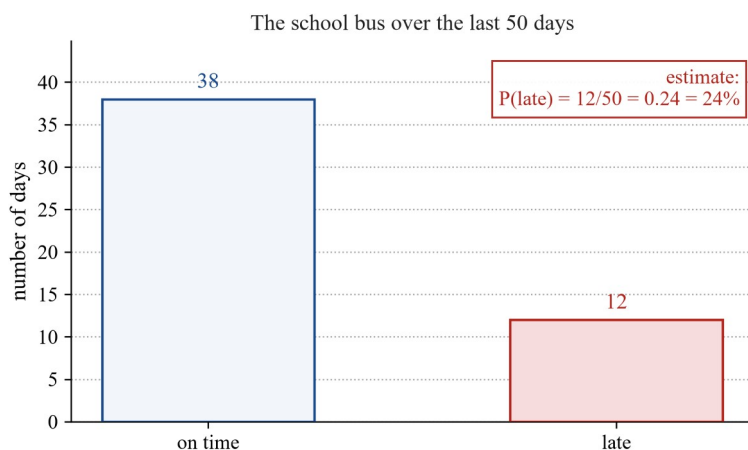
tickets, but $1 - \frac{3}{10} = \frac{7}{10}$ gets there in one step.

Estimating a probability from data.

Counting needs equally likely outcomes, and real life rarely offers them: a bus is not “a late side” and “an on-time side” like a coin. When outcomes are not equally likely, estimate the probability from records — from data about how often the event actually happened:

$$P(\text{event}) \approx \frac{\text{number of times the event happened}}{\text{total number of trials or records}}$$

The school bus was late on 12 of the last 50 school days:



Made-up data: the bus record in this picture is made up for this section.

So the estimate is

$$P(\text{late}) = \frac{12}{50} = \frac{6}{25} = 0.24 = 24\%$$

and the bus was on time on the other $50 - 12 = 38$ days, so $P(\text{on time}) = \frac{38}{50} = 0.76 = 76\%$ — check:

$24\% + 76\% = 100\%$ ✓. This is an **estimate**, not a calculation: it is built from what happened before, and it gets better as the records get longer. Twelve late days out of fifty is a firmer guide than three late days out of ten.

Predicting how often an event will happen.

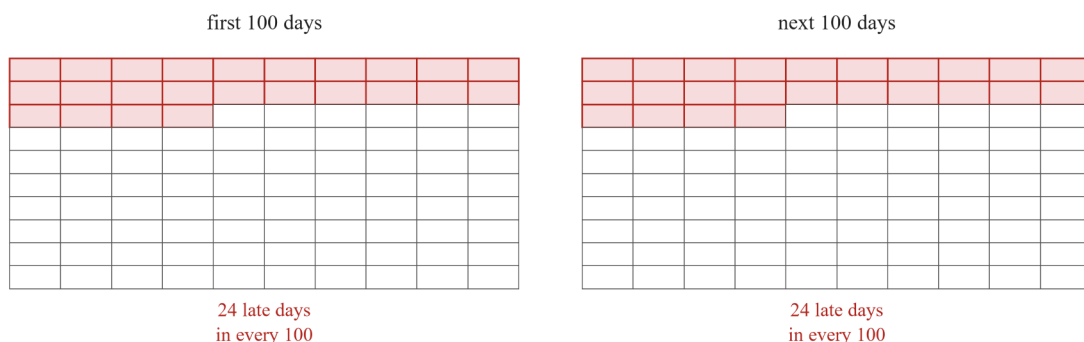
A probability of a single event can be scaled up to predict a whole run of events. Multiply the probability by the number of trials:

$$\text{expected number} = \text{probability} \times \text{number of trials}$$

Using the bus estimate, over the next 200 school days:

$$\text{expected late days} = 0.24 \times 200 = 48$$

$$P(\text{late}) = 0.24 \rightarrow \text{over 200 days expect } 0.24 \times 200 = 48 \text{ late days}$$



Made-up data: the bus record behind this picture is made up for this section.

The grids show why it works: 0.24 means about 24 late days in every 100, so two lots of 100 days carry about $24 \times 2 = 48$ late days. An expected number is the best long-run prediction, not a promise — the actual count might be 44 or 51; over many 200-day runs it settles near 48. If an expected number of days or people comes out as a decimal, round it to a whole number and say so — you cannot have 0.4 of a late day.

Checking your answers.

Three quick checks catch most mistakes with chance:

- Every probability must sit between 0 and 1 — as a percentage, between 0 % and 100 %. A probability of 1.3 or 120 % is impossible on its face.
- $P(\text{event})$ and $P(\text{event does not happen})$ must add to 1. If your two answers add to anything else, one of them is wrong.
- The word must match the size of the number: 7 % is very unlikely, not likely. Read the number back off the band table before you write the word.

Tools that help.

A calculator turns $\frac{18}{25}$ into 0.72 in a press, and a spreadsheet will count records and multiply up expected numbers over hundreds of rows. Use them for the arithmetic. What they cannot do is decide which outcomes are favourable, judge whether outcomes really are equally likely or whether the records are long enough to estimate from, or choose the word that fits the number. Those decisions stay your job.

Worked examples

Example 1 — scaffolded

A footy club sells 40 raffle tickets for its season-launch raffle. One ticket is drawn at random and wins the prize. Your friend holds 2 of the 40 tickets. (a) Find $P(\text{your friend wins})$ as a fraction in lowest terms, a decimal and a percentage. (b) Find $P(\text{your friend does not win})$ as a fraction in lowest terms, a decimal and a percentage. (c) Describe each probability in words from the chance scale.

Working	Comment
Favourable outcomes: 2 tickets. Total outcomes: 40 tickets.	The tickets are well mixed and drawn at random, so every ticket is equally likely and counting works.
$P(\text{win}) = \frac{2}{40} = \frac{1}{20}$.	Favourable over total, then simplify: divide top and bottom by 2.
$\frac{1}{20} = 1 \div 20 = 0.05 = 5\%$.	One chance, three forms: divide for the decimal, multiply by 100 for the percentage.
$P(\text{not win}) = 1 - \frac{1}{20} = \frac{19}{20} = 0.95 = 95\%$.	Subtract from 1. Check by counting: the other $40 - 2 = 38$ tickets give $\frac{38}{40} = \frac{19}{20}$ ✓
5 % is between 0 % and 10 %: very unlikely. 95 % is between 90 % and 100 %: very likely.	Match each number to its band in the words-of-chance table.

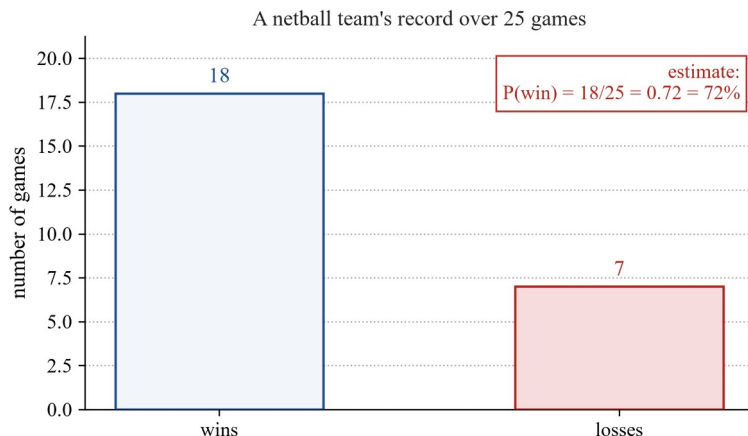
(a) $\frac{1}{20} = 0.05 = 5\%$. (b) $\frac{19}{20} = 0.95 = 95\%$. (c) winning

Answer every part, in every form asked for.

is very unlikely; not winning is very likely.

Example 2 — scaffolded

A netball team has played 25 games this season: 18 wins and 7 losses, as drawn below.



Made-up data: the team and its game record are made up for this section.

- Estimate $P(\text{the team wins its next game})$ as a decimal and a percentage.
- Predict how many of the team's next 20 games are wins, to the nearest whole number.
- Describe $P(\text{win})$ in words from the chance scale.

$$P(\text{win}) = \frac{18}{25} = 18 \div 25 = 0.72 = 72\%.$$

A game record is data, not equally likely outcomes — so this is an estimate, found from how often winning actually happened.

Expected wins: $0.72 \times 20 = 14.4$, so about 14 wins to the nearest whole number.

Expected number = probability $\times$ number of games. Nobody wins 0.4 of a game, so round to a whole number and say so.

72 % sits between 50 % and 90 %: likely.

Read the number off the band table.

$$\text{Check: } P(\text{loss}) = \frac{7}{25} = 0.28 = 28\%, \text{ and}$$

Win and not-win must add to 1.

$$0.72 + 0.28 = 1 \checkmark$$

(a) $0.72 = 72\%$. (b) 14 wins (to the nearest whole number). (c) likely.

State the rounding you used.

Example 3 — unscaffolded

A weather app gives $P(\text{rain on Saturday}) = 0.65$ and $P(\text{rain on Sunday}) = \frac{3}{5}$. (a) Write each probability as a percentage. (b) On which day is rain more likely? Show how you compared them. (c) Find $P(\text{no rain on Saturday})$ as a decimal and as a percentage.

(a) Saturday: $0.65 \times 100 = 65\%$. Sunday: $\frac{3}{5} = 3 \div 5 = 0.6$, and $0.6 \times 100 = 60\%$.

(b) Comparing needs one common form: as decimals, $0.65 > 0.6$, so rain is more likely on Saturday. The difference is $0.65 - 0.6 = 0.05$.

(c) Rain on Saturday either happens or does not, and the two chances add to 1:
 $P(\text{no rain on Saturday}) = 1 - 0.65 = 0.35 = 35\%$.

∴ Saturday is 65 % and Sunday is 60 %; rain is more likely on Saturday, and $P(\text{no rain on Saturday}) = 0.35 = 35 \%$

Example 4 — unscaffolded

A courier company checks its records and finds its van was late on 9 of the last 45 working days. (a) Estimate $P(\text{the van is late on a working day})$ as a fraction in lowest terms, a decimal and a percentage. (b) How many late days would the company expect over the next 60 working days? (c) Estimate $P(\text{the van is on time})$ as a fraction in lowest terms and a percentage, and describe it in words from the chance scale.

(a) The record is data, so estimate by dividing: $P(\text{late}) = \frac{9}{45}$. Both 9 and 45 divide by 9, so

$$\frac{9}{45} = \frac{1}{5} = 1 \div 5 = 0.2 = 20 \%$$

(b) Expected number = probability $\times$ number of days: $0.2 \times 60 = 12$ late days. This is already a whole number, so no rounding is needed.

(c) On time so far: $45 - 9 = 36$ days, so $P(\text{on time}) = \frac{36}{45}$. Both divide by 9: $\frac{36}{45} = \frac{4}{5} = 0.8 = 80 \%$. Check the

other way: $1 - \frac{1}{5} = \frac{4}{5}$ ✓. 80 % sits between 50 % and 90 %, so the van being on time is likely.

∴ $P(\text{late}) = \frac{1}{5} = 0.2 = 20 \%$, the company can expect 12 late days over the next 60 working days, and

$P(\text{on time}) = \frac{4}{5} = 80 \%$, which is likely.

Practice questions

Scaffolded

Q1. Write each chance in the form asked for. (a) $\frac{3}{10}$ as a decimal. (b) 0.45 as a percentage. (c) 80 % as a fraction in lowest terms.

Q2. Match each event below with one word from the chance scale: **certain**, **impossible**, **even chance**, **very unlikely**.

- (i) Today is Friday, and tomorrow is Saturday.
- (ii) A fair die is rolled and shows a 7.
- (iii) A fair coin is tossed and lands heads.
- (iv) You hold 1 ticket in a raffle of 1000 tickets, and one ticket is drawn at random.

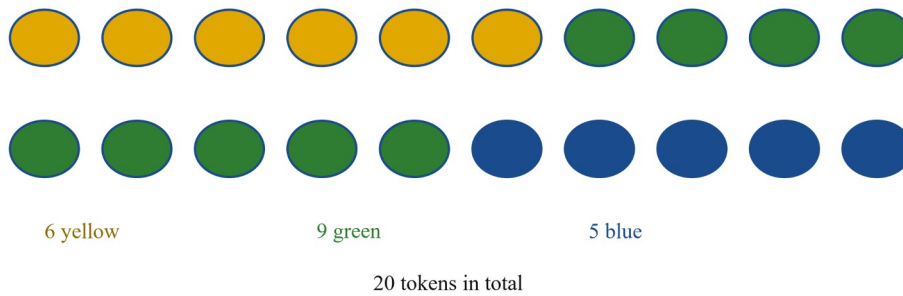
(a) Write the word that matches each event.

(b) Which event has probability 0, and which has probability 1?

(c) Write $P(\text{you win the raffle})$ for event (iv) as a decimal and as a percentage, to 1 decimal place.

Q3. A bag holds 20 tokens: 6 yellow, 9 green and 5 blue, as drawn below. One token is drawn at random.

Tokens in a bag — one token is drawn at random



Made-up data: the bag and its tokens are made up for this section.

- (a) Find $P(\text{green})$ as a decimal and as a percentage.
- (b) Find $P(\text{yellow})$ as a fraction in lowest terms, a decimal and a percentage.
- (c) Find $P(\text{not blue})$ as a fraction in lowest terms and as a percentage.

Q4. A train company estimates that $P(\text{the 7:15 train is on time}) = 0.93$. (a) Write this probability as a percentage. (b) Find $P(\text{the 7:15 train is late})$ as a decimal and as a percentage. (c) Describe each of these two probabilities in words from the chance scale.

Q5. A promotion says “1 in 6 boxes holds a prize”. The boxes look identical and one is chosen at random. (a) Write $P(\text{prize})$ as a fraction. (b) Write $P(\text{prize})$ as a percentage, to 1 decimal place. (c) Describe $P(\text{prize})$ in words from the chance scale.

Q6. A café estimates that 40 % of its customers buy a coffee. On Saturday the café serves 250 customers. (a) Write 40 % as a decimal. (b) How many customers are expected to buy a coffee? (c) What percentage of customers are expected **not** to buy a coffee, and how many customers out of the 250 is that?

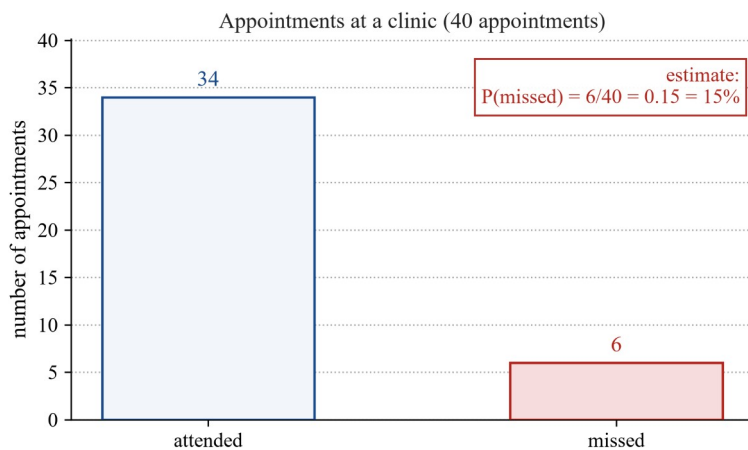
Un scaffolded

Q7. A drawer holds 15 single socks in the dark: 4 black, 6 white and 5 grey. You pull out one sock without looking. (a) Find $P(\text{black})$ as a percentage, to 1 decimal place. (b) Find $P(\text{white})$ as a fraction in lowest terms and as a percentage. (c) Find $P(\text{not grey})$ as a fraction in lowest terms and as a percentage, to 1 decimal place.

Q8. Over a practice session a golfer holes 17 of 25 putts. (a) Estimate $P(\text{the next putt is holed})$ as a percentage. (b) Predict how many of the next 40 putts are holed, to the nearest whole number. (c) Estimate $P(\text{the next putt is missed})$ as a percentage, and describe $P(\text{holed})$ in words from the chance scale.

Q9. A factory checks 120 parts off the line and finds 8 faulty. (a) Estimate $P(\text{a part is faulty})$ as a fraction in lowest terms and as a decimal, to 3 decimal places. (b) Write the estimate as a percentage, to 1 decimal place. (c) How many faulty parts would be expected in a run of 600 parts?

Q10. A health clinic records its last 40 appointments: 34 attended and 6 missed, as drawn below.



Made-up data: the clinic and its appointments are made up for this section.

- Estimate $P(\text{an appointment is missed})$ as a percentage.
- How many missed appointments would the clinic expect over its next 200 appointments?
- Estimate $P(\text{an appointment is not missed})$ as a fraction in lowest terms and as a percentage, then choose the word that best describes it: **unlikely**, **even chance** or **likely**.

Q11. Two insurance companies describe the chance that a new customer makes a claim in the first year. Company A says $P(\text{claim}) = 0.04$. Company B says “3 in 100 customers make a claim”. (a) Write each company’s probability as a percentage. (b) Which company reports the higher probability of a claim? Show the comparison. (c) Write Company A’s probability as a fraction in lowest terms, and describe it in words from the chance scale.

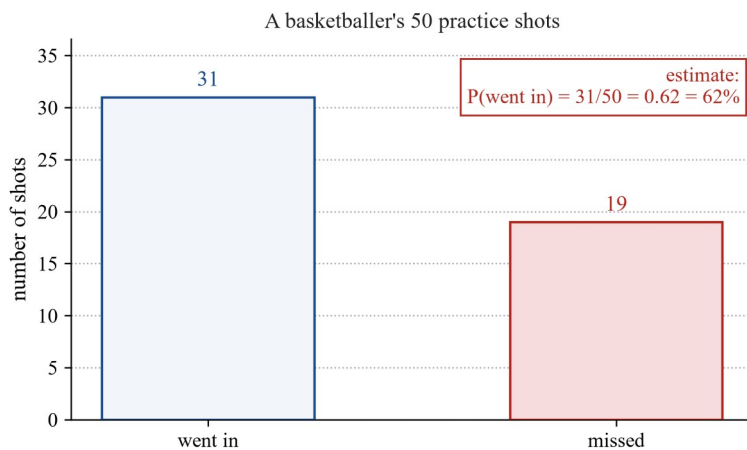
Q12. A council surveys 500 residents about a new bus service, and 45 of them rate the service excellent. One survey form is picked at random. (a) Estimate $P(\text{excellent})$ as a decimal. (b) Write the estimate as a percentage. (c) Estimate $P(\text{not excellent})$ as a decimal and as a percentage.

Q13. Each statement below is wrong. Explain what is wrong, and give the correct answer or fact where there is one. (a) “The weather app says $P(\text{rain}) = 1.3$.” (b) “ $P(\text{rain}) = 45\%$, so $P(\text{no rain}) = 45\%$.” (c) “I am sure I will win: $P(\text{win}) = 120\%$.”

Q14. For each situation below, say whether the probability is best found by **counting** equally likely outcomes or by **estimating** from data.

- (i) A fair die is rolled: $P(\text{an even number shows})$.
 - (ii) Whether it rains at the town fête on 1 December.
 - (iii) Whether a planted seed grows into a plant.
- Which situation can be answered by counting, and which must be estimated?
 - For situation (i), find $P(\text{even})$ as a fraction in lowest terms and as a percentage.
 - Explain in one sentence why situations (ii) and (iii) can only be estimated.

Q15. A basketballer counts her practice shots: 31 of 50 went in, as drawn below.



Made-up data: the basketballer and her shots are made up for this section.

- Estimate $P(\text{the next shot goes in})$ as a percentage.
- Predict how many of the next 30 shots go in, to the nearest whole number.
- Your answer to (b) is a prediction, not a promise. Explain in one sentence what that means.

Q16. A weather app gives the chance of a storm for three days: Monday 12%, Tuesday 0.4, Wednesday $\frac{2}{5}$. (a) Write all three probabilities as percentages. (b) Which day or days have the same chance of a storm? Show the comparison. (c) Find $P(\text{no storm on Tuesday})$ as a percentage, and describe the chance of a storm on Monday in words from the chance scale.

Q17. A venue's records suggest that when a casual worker is offered a shift, $P(\text{the worker says yes}) = 0.75$. The venue offers shifts to 12 casual workers for one weekend. (a) How many of the 12 workers are expected to say yes? (b) Write $P(\text{yes})$ as a fraction in lowest terms and as a percentage. (c) Find $P(\text{a worker says no})$ as a decimal and as a percentage, and describe it in words from the chance scale.

Q18. A school fête raffle sells 200 tickets, and one ticket is drawn at random to win the prize. You hold 4 of the 200 tickets. (a) Find $P(\text{you win})$ as a fraction in lowest terms, a decimal and a percentage. (b) Find $P(\text{you do not win})$ as a fraction in lowest terms and as a percentage. (c) The organiser says: "it is almost certain you will not win." Is the organiser's statement supported by your numbers? Give a reason.

Answers

Q1. (a) 0.3. (b) 45 %. (c) $\frac{4}{5}$. **Q2.** (a) (i) certain, (ii) impossible, (iii) even chance, (iv) very unlikely. (b) event (ii) has probability 0; event (i) has probability 1. (c) 0.001; 0.1 % (to 1 decimal place). **Q3.** (a) 0.45; 45 %. (b) $\frac{3}{10} = 0.3 = 30\%$. (c) $\frac{3}{4} = 75\%$. **Q4.** (a) 93 %. (b) 0.07; 7 %. (c) on time: very likely; late: very unlikely. **Q5.** (a) $\frac{1}{6}$. (b) 16.7 % (to 1 decimal place). (c) unlikely. **Q6.** (a) 0.4. (b) 100 customers. (c) 60 %; 150 customers. **Q7.** (a) 26.7 % (to 1 decimal place). (b) $\frac{2}{5} = 40\%$. (c) $\frac{2}{3}$; 66.7 % (to 1 decimal place). **Q8.** (a) 68 %. (b) 27 putts (to the nearest whole number). (c) 32 %; likely. **Q9.** (a) $\frac{1}{15}$; 0.067 (to 3 decimal places). (b) 6.7 % (to 1 decimal place). (c) 40 faulty parts. **Q10.** (a) 15 %. (b) 30 appointments. (c) $\frac{17}{20} = 85\%$; likely. **Q11.** (a) Company A: 4 %; Company B: 3 %. (b) Company A — $0.04 > 0.03$. (c) $\frac{1}{25}$; very unlikely. **Q12.** (a) 0.09. (b) 9 %. (c) 0.91; 91 %. **Q13.** (a) a probability can never be bigger than 1, and $1.3 > 1$. (b) $P(\text{no rain}) = 1 - 0.45 = 0.55 = 55\%$. (c) a probability can never be more than 100 %, and $120\% > 100\%$. **Q14.** (a) (i) counting; (ii) and (iii) estimating from data. (b) $\frac{1}{2} = 50\%$. (c) the result is not one of a short list of equally likely outcomes — it depends on past weather records and past seed trials. **Q15.** (a) 62 %. (b) 19 shots (to the nearest whole number). (c) 19 is what the record predicts on average — the actual number could be a bit higher or lower. **Q16.** (a) Monday 12 %, Tuesday 40 %, Wednesday 40 %. (b) Tuesday and Wednesday — both 0.4. (c) 60 %; unlikely. **Q17.** (a) 9 workers. (b) $\frac{3}{4} = 75\%$. (c) $0.25 = 25\%$; unlikely. **Q18.** (a) $\frac{1}{50} = 0.02 = 2\%$. (b) $\frac{49}{50} = 98\%$. (c) yes — 98 % is between 90 % and 100 %, the very likely band, so not winning is close to certain.

Fully-worked solutions

Q1. (a) Divide the top by the bottom: $\frac{3}{10} = 3 \div 10 = 0.3$. (b) Multiply by 100: $0.45 \times 100 = 45\%$. (c) $80\% = \frac{80}{100}$, and dividing top and bottom by 20 gives $\frac{4}{5}$.

Q2. (a) (i) If today is Friday then tomorrow must be Saturday — **certain**. (ii) A fair die shows 1, 2, 3, 4, 5 or 6; a 7 is not on the die, so it can never show — **impossible**. (iii) A fair coin has two sides and neither is favoured, so heads has probability $\frac{1}{2}$ — an **even chance**. (iv) One ticket out of 1000 gives a probability of $\frac{1}{1000}$, which is greater than 0 but below 0.1 — **very unlikely**. (b) Probability 0 means impossible: event (ii). Probability 1 means certain: event (i). (c) $\frac{1}{1000} = 1 \div 1000 = 0.001$; as a percentage, $0.001 \times 100 = 0.1\%$ to 1 decimal place.

Q3. (a) 9 green tokens out of 20 in total: $P(\text{green}) = \frac{9}{20} = 9 \div 20 = 0.45 = 45\%$. (b) $P(\text{yellow}) = \frac{6}{20}$; dividing top and bottom by 2 gives $\frac{3}{10} = 0.3 = 30\%$. (c) Tokens that are not blue: $20 - 5 = 15$, so $P(\text{not blue}) = \frac{15}{20} = \frac{3}{4}$ (divide top and bottom by 5) = 75 %. Check: $P(\text{blue}) = \frac{5}{20} = \frac{1}{4} = 25\%$, and $75\% + 25\% = 100\%$ ✓.

Q4. (a) $0.93 \times 100 = 93\%$. (b) The train is either on time or late, and the two chances add to 1: $P(\text{late}) = 1 - 0.93 = 0.07 = 7\%$. (c) 93 % sits between 90 % and 100 %: the train being on time is very likely. 7 % sits between 0 % and 10 %: the train being late is very unlikely.

Q5. (a) “1 in 6” means 1 favourable outcome out of 6 equally likely boxes: $P(\text{prize}) = \frac{1}{6}$. (b) $\frac{1}{6} = 1 \div 6 = 0.1666\dots$, so as a percentage it is 16.66...%, which is 16.7% to 1 decimal place. (c) 0.1666... sits between 0.1 and 0.5: finding a prize is unlikely.

Q6. (a) $40\% = 40 \div 100 = 0.4$. (b) Expected number = probability $\times$ number of customers: $0.4 \times 250 = 100$ customers. (c) Not buying a coffee: $100\% - 40\% = 60\%$ (equivalently $1 - 0.4 = 0.6$), and $0.6 \times 250 = 150$ customers. Check: $100 + 150 = 250$ ✓ — every customer either buys a coffee or does not.

Q7. (a) $P(\text{black}) = \frac{4}{15} = 4 \div 15 = 0.2666\dots$, so as a percentage 26.66...%, which is 26.7% to 1 decimal place. (b) $P(\text{white}) = \frac{6}{15}$; dividing top and bottom by 3 gives $\frac{2}{5} = 0.4 = 40\%$. (c) Socks that are not grey: $15 - 5 = 10$, so $P(\text{not grey}) = \frac{10}{15} = \frac{2}{3}$ (divide top and bottom by 5) = 0.6666..., which as a percentage is 66.66...%, that is 66.7% to 1 decimal place.

Q8. (a) The 25 putts are data, so estimate: $P(\text{holed}) = \frac{17}{25} = 17 \div 25 = 0.68 = 68\%$. (b) Expected number = $0.68 \times 40 = 27.2$, which is 27 putts to the nearest whole number. (c) Putts missed so far: $25 - 17 = 8$, so $P(\text{missed}) = \frac{8}{25} = 0.32 = 32\%$; check $68\% + 32\% = 100\%$ ✓. 68% sits between 50% and 90%: holing the next putt is likely.

Q9. (a) $P(\text{faulty}) = \frac{8}{120}$; dividing top and bottom by 8 gives $\frac{1}{15} = 1 \div 15 = 0.0666\dots$, which is 0.067 to 3 decimal places. (b) As a percentage: $0.0666\dots \times 100 = 6.66\dots\%$, which is 6.7% to 1 decimal place. (c) Expected number = $\frac{1}{15} \times 600 = 40$ faulty parts. Using the exact fraction rather than the rounded 0.067 keeps the prediction clean — $0.067 \times 600 = 40.2$ would drift away from the true 40. Keep full values in your working and round only at the end.

Q10. (a) $P(\text{missed}) = \frac{6}{40} = 6 \div 40 = 0.15 = 15\%$. (b) Expected number = $0.15 \times 200 = 30$ missed appointments. (c) Appointments attended: $40 - 6 = 34$, so $P(\text{not missed}) = \frac{34}{40} = \frac{17}{20}$ (divide top and bottom by 2) = $0.85 = 85\%$. Check: $15\% + 85\% = 100\%$ ✓. 85% sits between 50% and 90%, so of the three words offered, **likely** is the best fit.

Q11. (a) Company A: $0.04 \times 100 = 4\%$. Company B: “3 in 100” means $\frac{3}{100} = 0.03 = 3\%$. (b) $0.04 > 0.03$ — equivalently $4\% > 3\%$ — so Company A reports the higher probability of a claim. (c) $0.04 = \frac{4}{100} = \frac{1}{25}$ (divide top and bottom by 4). 4% sits between 0% and 10%: a customer making a claim is very unlikely.

Q12. (a) $P(\text{excellent}) = \frac{45}{500}$; dividing top and bottom by 5 gives $\frac{9}{100} = 0.09$. (b) $0.09 \times 100 = 9\%$. (c) Excellent and not-excellent add to 1: $P(\text{not excellent}) = 1 - 0.09 = 0.91 = 91\%$. Check: $9\% + 91\% = 100\%$ ✓.

Q13. (a) A probability is a number from 0 to 1, and $1.3 > 1$ — no event can be more likely than certain, so this reading cannot be right (it was probably meant to be 0.3). (b) Rain and no-rain must add to 1: $P(\text{no rain}) = 1 - 0.45 = 0.55 = 55\%$, not 45%. The mistake was using the rain probability again instead of subtracting it from 1. (c) A probability written as a percentage runs from 0% to 100%, and $120\% > 100\%$ — more than certain is impossible. The biggest any probability can be is $1 = 100\%$.

Q14. (a) Situation (i) is found by counting: a fair die has 6 equally likely faces, all known in advance. Situations (ii) and (iii) must be estimated from data — past weather records and past seed trials. (b) The even faces are 2, 4 and 6: 3

favourable outcomes out of 6, so $P(\text{even}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$. (c) Neither result is one of a short list of equally likely outcomes you can count in advance — each depends on how often rain followed days like this one, or how often seeds like this one grew, so only records can give a number.

Q15. (a) $P(\text{goes in}) = \frac{31}{50} = 31 \div 50 = 0.62 = 62\%$. (b) Expected number $= 0.62 \times 30 = 18.6$, which is 19 shots to the nearest whole number. (c) It means 19 is the best long-run guess from her 62% record, but in the actual next 30 shots she might make a few more or a few fewer — a prediction says what usually happens, not what must happen.

Q16. (a) Monday: 12%. Tuesday: $0.4 \times 100 = 40\%$. Wednesday: $\frac{2}{5} = 2 \div 5 = 0.4 = 40\%$. (b) Tuesday and Wednesday have the same chance: $0.4 = \frac{2}{5} = 40\%$ for both, and both are above Monday's 12%. (c) A storm on Tuesday either happens or does not: $P(\text{no storm on Tuesday}) = 1 - 0.4 = 0.6 = 60\%$. Monday's 12% sits between 10% and 50%: a storm on Monday is unlikely.

Q17. (a) Expected number $= \text{probability} \times \text{number of workers}$: $0.75 \times 12 = 9$ workers. (b) $0.75 = \frac{75}{100} = \frac{3}{4}$ (divide top and bottom by 25) $= 75\%$. (c) Saying yes and saying no add to 1: $P(\text{no}) = 1 - 0.75 = 0.25 = 25\%$. 25% sits between 10% and 50%: a worker saying no is unlikely.

Q18. (a) $P(\text{win}) = \frac{4}{200}$; dividing top and bottom by 4 gives $\frac{1}{50} = 1 \div 50 = 0.02 = 2\%$. (b)

$P(\text{not win}) = 1 - \frac{1}{50} = \frac{49}{50} = 0.98 = 98\%$; check by counting: the other $200 - 4 = 196$ tickets give $\frac{196}{200} = \frac{49}{50}$ ✓. (c)

Yes, the statement is supported: 98% sits between 90% and 100% — the very likely band, close to certain — so “almost certain you will not win” fairly describes $\frac{49}{50}$. It is still not quite certain: your 4 tickets could be the one drawn.

Chapter 9 Uncertainty — 9B Sample spaces and simple probability

Theory

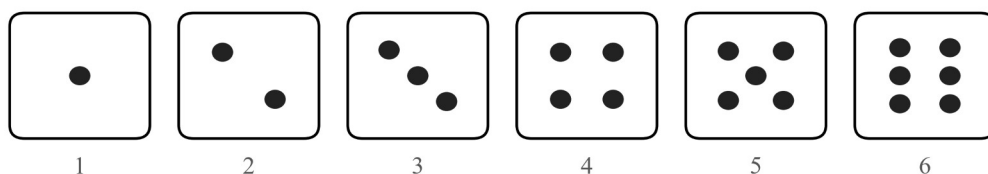
Some things in life you can predict exactly: a payslip at the agreed hourly rate, a full tank of fuel lasting its rated distance. Other things you cannot: the next face of a die, where a spinner's pointer stops, which ticket comes out of a raffle barrel. This section is about those second kinds of situations. We cannot say what *will* happen, but we can list everything that *could* happen — and once that list is written down, we can measure how likely each possibility is. That measure is called **probability**, and Section 9A puts its values on a scale from impossible to certain. Here we build the list and the pictures that go with it.

About the data in this section. Every club, business, game and person in this section is **made up** (not real). They are built for this book so you can practise listing outcomes and finding simple probabilities. Nothing here is real data about a real person, business or place.

Random outcomes.

An activity is **random** when you know every result that could happen, but you cannot say in advance which one will happen. One go of a random activity is called a **trial**, and the result of a trial is called an **outcome**. Rolling a die once is a trial; the face that lands up is the outcome.

One ordinary six-sided die: six possible outcomes



Roll one ordinary six-sided die and the outcome is one of 1, 2, 3, 4, 5, 6. Nobody can say which one before the roll — that is what random means — yet the list of possibilities is completely known, and that list is where all the maths in this section starts.

The sample space.

The **sample space** of a random activity is the complete list of every possible outcome. It is usually written inside curly brackets. For one die:

$$S = \{1, 2, 3, 4, 5, 6\}$$

which is 6 possible outcomes. For one coin toss, with H for heads and T for tails:

$$S = \{H, T\}$$

which is 2 possible outcomes. Writing the sample space first is a habit worth keeping: probabilities in this section are *counted* from the sample space, so an outcome left off the list makes every probability that follows wrong.

Equally likely outcomes and the probability fraction.

Outcomes are **equally likely** when no outcome is favoured over another — when nothing in the setup pushes the result one way. A fair die, a fair coin, a well-shuffled pile of cards and a spinner with equal-sized sectors all give equally likely outcomes.

An **event** is a description of the outcomes we are watching for, such as “roll an even number”. The outcomes that make the event happen are called the **favourable outcomes**. When all outcomes are equally likely, the probability of the event — written $P(\text{event})$, where P stands for probability — is a simple count:

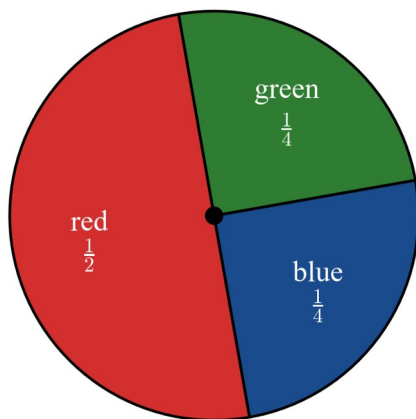
$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Roll one die. The event “roll a 4” has 1 favourable outcome out of 6, so $P(4) = \frac{1}{6}$, which as a decimal is 0.1666..., or 0.17 to two decimal places. The event “roll an even number” has 3 favourable outcomes — 2, 4 and 6 — out of 6, so $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$, the same as 50 %. Section 9A moves between fractions, decimals and percentages like these; in this section the fraction comes first and other forms follow when a question asks for them. Every probability sits between 0 (impossible) and 1 (certain).

When outcomes are NOT equally likely.

The counting fraction only works on equally likely outcomes, so check before you count. This spinner has three colours but they are not equally likely, because the sectors are different sizes:

This spinner's sectors are NOT equal

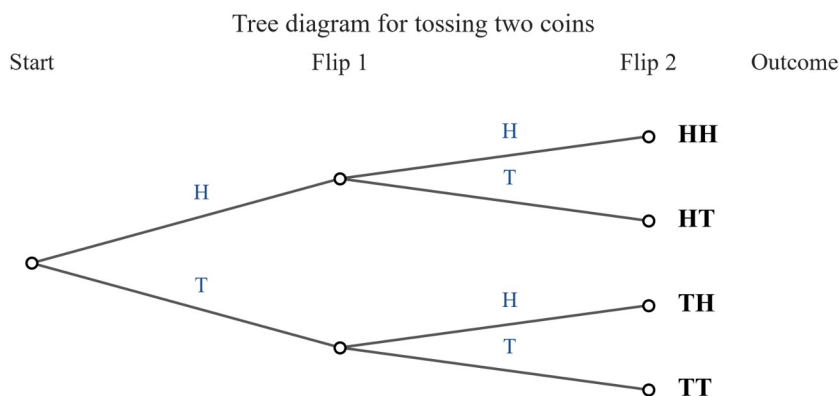


Red covers half the circle, so $P(\text{red}) = \frac{1}{2}$, while blue and green cover a quarter each: $P(\text{blue}) = \frac{1}{4}$ and $P(\text{green}) = \frac{1}{4}$. Writing “3 colours, so each is $\frac{1}{3}$ ” would be wrong.

The same trap hides in bags and barrels. A bag holding 9 red balls and 1 blue ball does not give $P(\text{red}) = \frac{1}{2}$: the equally likely outcomes are the 10 individual balls, and 9 of them are red, so $P(\text{red}) = \frac{9}{10}$. And “it will rain tomorrow or it will not” has two outcomes that are nowhere near equally likely — never write $\frac{1}{2}$ just because you can name two results. When outcomes are not equally likely, cut the activity into pieces that are: each ball, each ticket, each equal-sized sector.

Two-step activities: listing every outcome.

Toss two coins and it is tempting to write the sample space as “two heads, two tails, or one of each” — three outcomes. But those three are not equally likely: “one of each” can happen two different ways. The fix is to record the coins in order, first coin then second coin, and a **tree diagram** keeps the list honest:



Follow the branches from left to right and every path is one outcome: HH, HT, TH and TT — 4 equally likely outcomes. Note that HT and TH are *different* outcomes (head then tail, versus tail then head), which is exactly why “one of each” is twice as likely as “two heads”: $P(\text{one of each}) = \frac{2}{4} = \frac{1}{2}$ while $P(\text{HH}) = \frac{1}{4}$.

When two things happen together — a coin and a die, say — a **two-way table** does the same job. Each cell of the table is one outcome, and reading across a row and down a column pairs the two results:

Two-way table: tossing a coin and rolling a die together (12 outcomes)

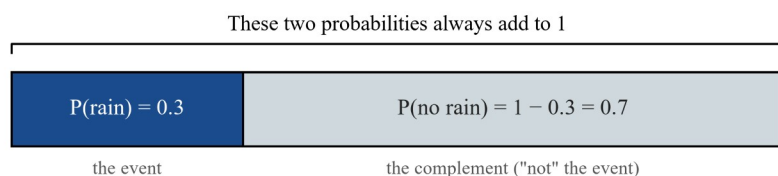
die → coin ↓	1	2	3	4	5	6
H	H 1	H 2	H 3	H 4	H 5	H 6
T	T 1	T 2	T 3	T 4	T 5	T 6

The table has 12 cells, so 12 equally likely outcomes. Trees and two-way tables are the two standard pictures of a sample space; either one earns full credit, and both stop the same mistake — missing an outcome, or counting one twice.

The other side of an event: the complement.

For any event, the **complement** is “the event does not happen”. Rain and no rain are complements; rolling a 6 and not rolling a 6 are complements. An event and its complement cover the whole sample space between them, so their probabilities always add to 1:

$$P(\text{not the event}) = 1 - P(\text{the event})$$



If the chance of rain tomorrow is 0.3, the chance of no rain is $1 - 0.3 = 0.7$. With one die, $P(\text{not a 6}) = 1 - \frac{1}{6} = \frac{5}{6}$ — and counting the five favourable outcomes directly gives the same answer, which is a handy check. When “not” is the shorter count, use the complement and subtract from 1.

Writing your answers.

Give a probability as a fraction in simplest form unless the question asks for another form; the fraction is exact, while a decimal may need rounding. When a question states a precision — “to two decimal places”, “as a percentage” — round only at the end and say so in your answer. Then sanity-check: a probability must sit between 0 and 1, and an event plus its complement must add to 1.

Tools that help.

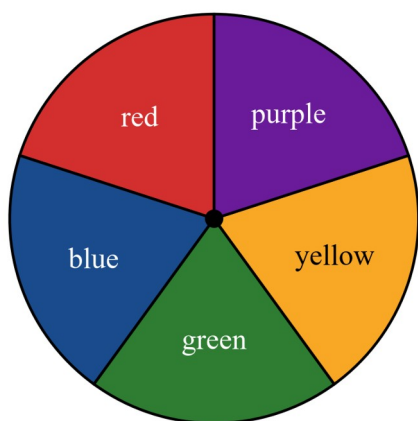
A calculator turns $\frac{1}{6}$ into a decimal in one press, and a spreadsheet will list all 36 outcomes of two dice or count the favourable cells of a big table in seconds — useful at work, where rosters and quality checks are counted this way. But no tool can decide whether outcomes are equally likely, and none will notice an outcome missing from your list. The listing and the judgement stay your job.

Worked examples

Example 1 — scaffolded

A games-night spinner has 5 equal sectors, coloured red, blue, green, yellow and purple. The spinner is spun once.

Spinner with 5 equal sectors



Made-up data: the spinner and its colours are made up for this section.

Find the sample space, then $P(\text{green})$ as a fraction, a decimal and a percentage, then $P(\text{not green})$ and $P(\text{red or blue})$.

Working

Comment

$S = \{\text{red, blue, green, yellow, purple}\}$ — 5 outcomes.

List every sector. Equal sectors mean equally likely outcomes, so the counting fraction applies.

Favourable outcomes for green: 1.

The event “green” happens on one sector only.

$$P(\text{green}) = \frac{1}{5}.$$

Favourable outcomes over possible outcomes.

$$\frac{1}{5} = 0.2 = 20\%.$$

Divide for the decimal, then multiply by 100 for the percentage. Both are exact here — no rounding needed.

$$P(\text{not green}) = 1 - \frac{1}{5} = \frac{4}{5} = 80\%.$$

The complement is the other 4 sectors; check $\frac{1}{5} + \frac{4}{5} = 1$.

Working	Comment
$P(\text{red or blue}) = \frac{2}{5} = 40\%$.	Two favourable outcomes out of five.

Example 2 — scaffolded

At a fete stall you toss a coin and roll an ordinary six-sided die together. The two-way table in the Theory above lists all 12 outcomes, from H 1 to T 6. Using that table, find:

- $P(\text{head})$
- $P(\text{tail and a 5})$
- $P(\text{head and an even number})$
- $P(\text{the number is greater than 4})$

Working	Comment
Possible outcomes: 12 (the 12 cells of the table).	Count the sample space before anything else.
(a) Heads: H 1, H 2, H 3, H 4, H 5, H 6 — 6 cells. $P(\text{head}) = \frac{6}{12} = \frac{1}{2}$.	The whole top row is the event “head”.
(b) Tail and a 5: the single cell T 5. $P = \frac{1}{12}$.	“And” means both parts at once — one cell.
(c) Head and even: H 2, H 4, H 6 — 3 cells. $P = \frac{3}{12} = \frac{1}{4}$.	Shade the cells that satisfy both conditions, then count them.
(d) Number > 4 : H 5, H 6, T 5, T 6 — 4 cells. $P = \frac{4}{12} = \frac{1}{3}$.	This event ignores the coin, so it takes one cell from each row for 5 and 6.
Answers: $\frac{1}{2}, \frac{1}{12}, \frac{1}{4}, \frac{1}{3}$.	Every fraction is in simplest form; all sit between 0 and 1.

Example 3 — unscaffolded

A football club sells 200 raffle tickets for its end-of-year draw, and every ticket has an equal chance of being drawn. Mia buys 5 tickets. One ticket is drawn for the major prize.

- Find the probability that Mia wins the major prize, as a fraction and as a percentage.
- Find the probability that Mia does not win the major prize, as a percentage.

Made-up data: the club, the raffle and the ticket counts are made up for this section.

- The equally likely outcomes are the 200 tickets, and 5 of them are Mia’s, so

$$P(\text{Mia wins}) = \frac{5}{200} = \frac{1}{40}$$

and $\frac{1}{40} = 0.025 = 2.5\%$, exactly.

- Not winning is the complement of winning:

$$P(\text{Mia does not win}) = 1 - \frac{1}{40} = \frac{39}{40} = 0.975 = 97.5\%$$

Check by counting: 195 of the 200 tickets are not Mia’s, and $\frac{195}{200} = \frac{39}{40}$ — the same answer.

$$\therefore P(\text{Mia wins}) = \frac{1}{40} = 2.5\% \text{ and } P(\text{Mia does not win}) = \frac{39}{40} = 97.5\%.$$

Example 4 — unscaffolded

Eight tiles are placed in a bag, one for each letter of the word NUMERACY. The bag is shaken and one tile is drawn at random.

Eight tiles, one for each letter of NUMERACY



Shaded tiles are the vowels: U, E and A

- (a) Find the probability of drawing a vowel, as a fraction and as a decimal correct to two decimal places.
- (b) Find the probability of drawing a consonant.
- (c) Find the probability of drawing an N or a Y.
- (d) The 8 tiles are equally likely outcomes. The vowels on the tiles are U, E and A — 3 favourable outcomes — so

$$P(\text{vowel}) = \frac{3}{8} = 0.375$$

which is 0.38 correct to two decimal places (and 37.5 % exactly).

- (b) The consonants are N, M, R, C and Y — 5 favourable outcomes — so

$$P(\text{consonant}) = \frac{5}{8}$$

Check with the complement: $1 - \frac{3}{8} = \frac{5}{8}$. ✓

- (c) Two tiles — N and Y — are favourable, so

$$P(\text{N or Y}) = \frac{2}{8} = \frac{1}{4}$$

$\therefore P(\text{vowel}) = \frac{3}{8} = 0.38$ to two decimal places, $P(\text{consonant}) = \frac{5}{8}$ and $P(\text{N or Y}) = \frac{1}{4}$.

Practice questions

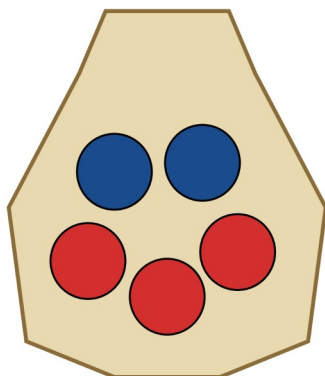
Scaffolded

Q1. An ordinary six-sided die is rolled once. (a) Write down the sample space. (b) How many possible outcomes are there? (c) Find $P(\text{roll a 4})$ as a fraction and correct to two decimal places.

Q2. An ordinary six-sided die is rolled once. (a) Find $P(\text{an even number})$. (b) Find $P(\text{a number greater than } 4)$. (c) Find $P(\text{not a } 6)$.

Q3. A bag holds 3 red balls and 2 blue balls, all the same size and weight. One ball is drawn at random.

3 red balls, 2 blue balls



Made-up data: the bag and its balls are made up for this section.

- (a) How many equally likely outcomes are there?
- (b) Find $P(\text{red})$.
- (c) Find $P(\text{blue})$, then check that your answers to (b) and (c) add to 1.

Q4. Ten cards numbered $1, 2, 3, \dots, 10$ are shuffled face down and one card is drawn at random. (a) Write down the sample space and say how many outcomes it has. (b) Find $P(\text{a prime number})$. (The primes up to 10 are 2, 3, 5, 7.) (c) Find $P(\text{an odd number})$.

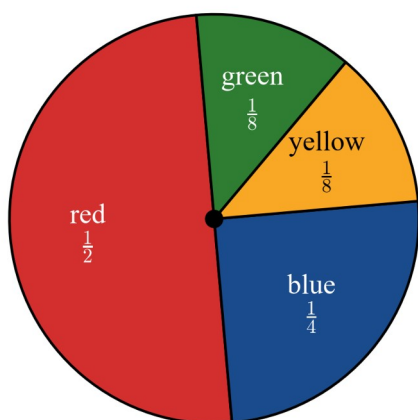
Q5. Two coins are tossed together. (a) Draw a tree diagram and list the sample space. (b) Find $P(\text{two heads})$. (c) Find $P(\text{exactly one head})$.

Q6. A raffle has 50 tickets and you hold 2 of them. One ticket is drawn at random for the prize. (a) Find $P(\text{you win})$ as a fraction in simplest form. (b) Write that probability as a decimal. (c) Find $P(\text{you do not win})$ as a decimal.

Unscaffolded

Q7. Decide whether the outcomes are equally likely in each situation below. If they are not, say which outcome is more likely and why.

The fete spinner



Made-up data: the fete spinner is made up for this section.

- (a) Spinning the fete spinner shown above, with outcomes red, blue, yellow, green. For this spinner, also write down $P(\text{red})$, $P(\text{blue})$ and $P(\text{yellow})$.
- (b) Rolling an ordinary six-sided die, with outcomes 1 to 6.
- (c) Watching tomorrow's weather, with outcomes "rain" and "no rain".
- (d) Drawing one name slip from a hat holding 12 identical, well-mixed slips.

Q8. Ten tiles are placed in a bag, one for each letter of the word APPRENTICE. One tile is drawn at random. (a) Find $P(\text{the letter C})$. (b) Find $P(\text{the letter P})$. (c) Find $P(\text{a vowel})$ as a decimal correct to two decimal places.

Q9. Two ordinary six-sided dice are rolled together and their scores are added. The grid below shows the sum for all 36 outcomes.

Rolling two dice: 36 outcomes, shown as sums (shaded cells sum to 7)

		Die 2 →					
		1	2	3	4	5	6
Die 1 ↓	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- How many equally likely outcomes are there?
- Find $P(\text{the sum is } 7)$.
- Find $P(\text{the sum is } 12)$.
- Find $P(\text{the sum is } 2 \text{ or } 12)$.

Q10. A warehouse checks phone cases from a batch of 60. Exactly 3 cases in the batch are faulty. One case is chosen at random for testing. (a) Find $P(\text{faulty})$ as a fraction and as a percentage. (b) Find $P(\text{not faulty})$ as a percentage.

Made-up data: the batch and its fault count are made up for this section.

Q11. One day of the week is chosen at random for a working bee. (a) Find $P(\text{Saturday or Sunday})$. (b) Find $P(\text{the name begins with M})$. (c) Find $P(\text{a weekday})$.

Q12. Two spinners are spun together. Each spinner has 3 equal sectors, coloured red, blue and green. (a) List the sample space of 9 outcomes (a table or a tree diagram is fine). (b) Find $P(\text{the same colour on both spinners})$. (c) Find $P(\text{red on the first spinner only})$.

Q13. A bag holds 4 green marbles and some yellow marbles, all the same size and weight. One marble is drawn at random and $P(\text{green}) = \frac{1}{3}$. (a) How many marbles are in the bag altogether? (b) How many of them are yellow? (c) Find $P(\text{yellow})$.

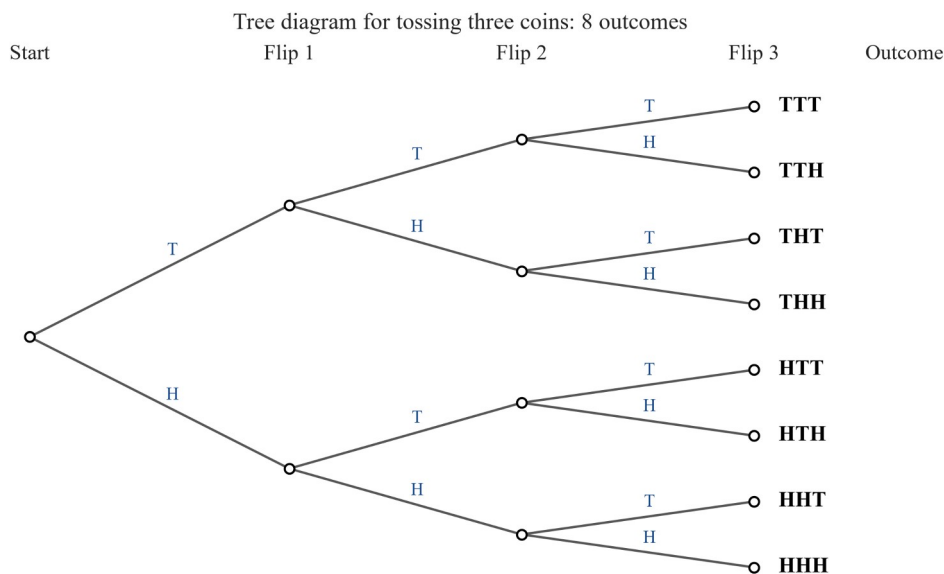
Q14. Twelve staff at a café put their names in a hat for the Saturday roster draw. One slip is drawn at random.

Ana	Ben	Cara	Dan
Eli	Farid	Gia	Hans
Ivy	Jack	Kim	Luca

Made-up data: the staff names are made up for this section.

- Find $P(\text{Jack})$.
- Find $P(\text{the name begins with a vowel})$.
- Find $P(\text{not Ana})$.

Q15. Three coins are tossed together. The tree diagram below shows every outcome.



- How many equally likely outcomes are there?
- Find $P(\text{three heads})$.
- Find $P(\text{at least one tail})$.

Q16. A school fete runs a lucky dip of 40 identical wrapped boxes, and 8 of the boxes contain a prize. One box is chosen at random. (a) Find $P(\text{a prize})$ as a fraction and as a percentage. (b) Find $P(\text{no prize})$ as a percentage.

Made-up data: the lucky dip and its box counts are made up for this section.

Q17. A spinner has 8 equal sectors numbered 1, 2, 3, ..., 8. It is spun once. (a) Find $P(\text{an odd number})$. (b) Find $P(\text{a number greater than 5})$. (c) Find $P(\text{a multiple of 4})$.

Q18. Kai and Tia play a game: a coin is tossed and an ordinary six-sided die is rolled. Kai wins the round if the result is a head **and** an even number; otherwise Tia wins the round. (a) Find $P(\text{Kai wins})$. (b) Find $P(\text{Tia wins})$. (c) Is the game fair? Justify your answer with the two probabilities.

Answers

Q1. (a) $S = \{1, 2, 3, 4, 5, 6\}$. (b) 6. (c) $\frac{1}{6}$, or 0.17 to two decimal places. **Q2.** (a) $\frac{1}{2}$. (b) $\frac{1}{3}$. (c) $\frac{5}{6}$. **Q3.** (a) 5. (b) $\frac{3}{5}$. (c) $\frac{2}{5}$; $\frac{3}{5} + \frac{2}{5} = 1$ ✓. **Q4.** (a) $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, 10 outcomes. (b) $\frac{2}{5}$. (c) $\frac{1}{2}$. **Q5.** (a) HH, HT, TH, TT — 4 outcomes. (b) $\frac{1}{4}$. (c) $\frac{1}{2}$. **Q6.** (a) $\frac{1}{25}$. (b) 0.04. (c) 0.96. **Q7.** (a) not equally likely — the sectors are different sizes; $P(\text{red}) = \frac{1}{2}$, $P(\text{blue}) = \frac{1}{4}$, $P(\text{yellow}) = \frac{1}{8}$. (b) equally likely — a fair die favours no face. (c) not equally likely — rain and no rain depend on the weather, not on equal shares. (d) equally likely — identical slips, well mixed. **Q8.** (a) $\frac{1}{10}$. (b) $\frac{1}{5}$. (c) 0.40 to two decimal places. **Q9.** (a) 36. (b) $\frac{1}{6}$. (c) $\frac{1}{36}$. (d) $\frac{1}{18}$. **Q10.** (a) $\frac{1}{20} = 5\%$. (b) 95%. **Q11.** (a) $\frac{2}{7}$. (b) $\frac{1}{7}$. (c) $\frac{5}{7}$. **Q12.** (a) RR, RB, RG, BR, BB, BG, GR, GB, GG — 9 outcomes. (b) $\frac{1}{3}$. (c) $\frac{2}{9}$. **Q13.** (a) 12. (b) 8. (c) $\frac{2}{3}$. **Q14.** (a) $\frac{1}{12}$. (b) $\frac{1}{4}$. (c) $\frac{11}{12}$. **Q15.** (a) 8. (b) $\frac{1}{8}$. (c) $\frac{7}{8}$. **Q16.** (a) $\frac{1}{5} = 20\%$. (b) 80%. **Q17.** (a) $\frac{1}{2}$. (b) $\frac{3}{8}$. (c) $\frac{1}{4}$. **Q18.** (a) $\frac{1}{4}$. (b) $\frac{3}{4}$. (c) not fair — Tia's $\frac{3}{4}$ is three times Kai's $\frac{1}{4}$.

Fully-worked solutions

Q1. (a) One die can land on any face from 1 to 6, so $S = \{1, 2, 3, 4, 5, 6\}$. (b) The list has 6 outcomes. (c) One outcome, the 4, is favourable out of 6, so $P(4) = \frac{1}{6}$. As a decimal $\frac{1}{6} = 0.1666\dots$, which is 0.17 correct to two decimal places.

Q2. (a) The even faces are 2, 4, 6 — 3 favourable outcomes out of 6 — so $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$. (b) The faces greater than 4 are 5 and 6 — 2 out of 6 — so $P(>4) = \frac{2}{6} = \frac{1}{3}$. (c) Not rolling a 6 leaves 5 favourable faces, so $P(\text{not } 6) = \frac{5}{6}$; the complement check gives $1 - \frac{1}{6} = \frac{5}{6}$. ✓

Q3. (a) The equally likely outcomes are the individual balls: $3 + 2 = 5$. (b) Three of the five balls are red, so $P(\text{red}) = \frac{3}{5}$. (c) Two of the five are blue, so $P(\text{blue}) = \frac{2}{5}$. Check: red and blue cover every ball, and $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$. ✓

Q4. (a) Any of the ten cards can be drawn, so $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with 10 equally likely outcomes. (b) The primes in the set are 2, 3, 5, 7 — 4 favourable outcomes — so $P(\text{prime}) = \frac{4}{10} = \frac{2}{5}$. (c) The odd cards are 1, 3, 5, 7, 9 — 5 out of 10 — so $P(\text{odd}) = \frac{5}{10} = \frac{1}{2}$.

Q5. (a) The tree has two branches (H, T) for the first coin and two from each of those for the second coin, giving the 4 paths HH, HT, TH, TT. (b) Only the path HH gives two heads, so $P(\text{HH}) = \frac{1}{4}$. (c) Exactly one head happens on the paths HT and TH — 2 of the 4 — so $P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2}$.

Q6. (a) The 50 tickets are equally likely and 2 are yours, so $P(\text{win}) = \frac{2}{50} = \frac{1}{25}$. (b) $\frac{1}{25} = 0.04$ exactly. (c) The complement: $P(\text{not win}) = 1 - 0.04 = 0.96$; counting gives the same, since 48 of the 50 tickets are not yours and $\frac{48}{50} = 0.96$. ✓

Q7. (a) Not equally likely: the sectors are different sizes, so the pointer is more likely to stop in a big sector. Red covers half the circle, blue a quarter and yellow one eighth, so $P(\text{red}) = \frac{1}{2}$, $P(\text{blue}) = \frac{1}{4}$ and $P(\text{yellow}) = \frac{1}{8}$ — red is the most likely colour. (b) Equally likely: an ordinary die is built so that no face is favoured, giving six outcomes each with probability $\frac{1}{6}$. (c) Not equally likely: “rain” and “no rain” are not two equal shares of anything — the weather decides, and in most of Australia a random day is more likely dry than wet. Two named outcomes alone never make $\frac{1}{2}$ each. (d) Equally likely: the slips are identical and well mixed, so no name is favoured and each has probability $\frac{1}{12}$.

Q8. The word APPRENTICE has 10 letters, so 10 equally likely tiles. (a) The letter C appears once, so $P(C) = \frac{1}{10}$. (b) The letter P appears twice, so $P(P) = \frac{2}{10} = \frac{1}{5}$. (c) The vowels are A, E, I and E — 4 tiles — so $P(\text{vowel}) = \frac{4}{10} = \frac{2}{5} = 0.4$, which written to two decimal places is 0.40.

Q9. (a) The grid has 6 rows and 6 columns, so $6 \times 6 = 36$ equally likely outcomes. (b) The shaded cells show the six ways to make 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). So $P(7) = \frac{6}{36} = \frac{1}{6}$. (c) A sum of 12 needs a 6 on both dice — 1 cell — so $P(12) = \frac{1}{36}$. (d) A sum of 2 needs two 1s — 1 cell — and a sum of 12 needs two 6s — 1 cell — so 2 cells in all and $P(2 \text{ or } 12) = \frac{2}{36} = \frac{1}{18}$.

Q10. (a) The 60 cases are equally likely and 3 are faulty, so $P(\text{faulty}) = \frac{3}{60} = \frac{1}{20}$. As a percentage, $\frac{1}{20} = 0.05 = 5\%$, exactly. (b) The complement: $P(\text{not faulty}) = 1 - \frac{1}{20} = \frac{19}{20} = 0.95 = 95\%$; counting the 57 good cases gives $\frac{57}{60} = \frac{19}{20}$. ✓

Q11. The 7 days of the week are equally likely. (a) Saturday and Sunday are 2 favourable days, so $P(\text{weekend day}) = \frac{2}{7}$. (b) Only Monday begins with M, so $P(\text{begins with M}) = \frac{1}{7}$. (c) The weekdays are Monday to Friday — 5 days — so $P(\text{weekday}) = \frac{5}{7}$; check $\frac{2}{7} + \frac{5}{7} = 1$. ✓

Q12. (a) Pair each result of the first spinner with each result of the second:

First spinner ↓, second →	red	blue	green
red	RR	RB	RG
blue	BR	BB	BG
green	GR	GB	GG

Made-up data: the spinners are made up for this section.

That is 9 equally likely outcomes. (b) The same colour on both spinners happens on RR, BB and GG — 3 cells — so $P(\text{same}) = \frac{3}{9} = \frac{1}{3}$. (c) Red on the first spinner only means red then blue, or red then green — the cells RB and RG — so $P = \frac{2}{9}$.

Q13. (a) $P(\text{green}) = \frac{1}{3}$ means the 4 green marbles are one-third of all the marbles, so the total is $4 \times 3 = 12$ marbles.

(b) Yellow marbles: $12 - 4 = 8$. (c) $P(\text{yellow}) = \frac{8}{12} = \frac{2}{3}$; check with the complement: $1 - \frac{1}{3} = \frac{2}{3}$. ✓

Q14. The 12 slips are equally likely. (a) One slip says Jack, so $P(\text{Jack}) = \frac{1}{12}$. (b) The names beginning with a vowel are Ana, Eli and Ivy — 3 slips — so $P(\text{vowel start}) = \frac{3}{12} = \frac{1}{4}$. (c) Eleven slips are not Ana, so $P(\text{not Ana}) = \frac{11}{12}$; the complement gives $1 - \frac{1}{12} = \frac{11}{12}$. ✓

Q15. (a) The tree has 8 end points: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT — 8 equally likely outcomes. (b) Only HHH gives three heads, so $P(\text{HHH}) = \frac{1}{8}$. (c) “At least one tail” is every outcome except HHH, so its complement is quicker: $P(\text{at least one tail}) = 1 - \frac{1}{8} = \frac{7}{8}$. Counting the tree agrees: 7 of the 8 paths show a T. ✓

Q16. (a) The 40 boxes are equally likely and 8 hold a prize, so $P(\text{prize}) = \frac{8}{40} = \frac{1}{5}$. As a percentage, $\frac{1}{5} = 0.2 = 20\%$, exactly. (b) The complement: $P(\text{no prize}) = 1 - \frac{1}{5} = \frac{4}{5} = 0.8 = 80\%$; counting the 32 empty boxes gives $\frac{32}{40} = \frac{4}{5}$. ✓

Q17. The 8 equal sectors give 8 equally likely outcomes. (a) The odd numbers are 1, 3, 5, 7 — 4 of the 8 — so $P(\text{odd}) = \frac{4}{8} = \frac{1}{2}$. (b) The numbers greater than 5 are 6, 7, 8 — 3 of the 8 — so $P(>5) = \frac{3}{8}$. (c) The multiples of 4 are 4 and 8 — 2 of the 8 — so $P(\text{multiple of 4}) = \frac{2}{8} = \frac{1}{4}$.

Q18. (a) The coin-and-die sample space is the 12 cells of the two-way table in the Theory. Kai wins on a head with an even number: H 2, H 4 and H 6 — 3 cells — so $P(\text{Kai}) = \frac{3}{12} = \frac{1}{4}$. (b) Tia wins on every other cell: $P(\text{Tia}) = 1 - \frac{1}{4} = \frac{3}{4}$. (c) The game is not fair. A fair game needs both players on $P = \frac{1}{2}$, but Tia wins $\frac{3}{4}$ of rounds against Kai's $\frac{1}{4}$ — three rounds in every four on average — so the game favours Tia heavily.

Chapter 9 Uncertainty — 9C Comparing events and deciding from data

Theory

Chances arrive in many outfits. A raffle ticket says “1 in 8”. A poster says “12%”. A weather app says “0.3”. Section 9A put all of these on one scale from 0 (impossible) to 1 (certain), and Section 9B showed how to count equally likely outcomes to get a chance as a fraction. This section takes the next two steps: **comparing** chances that arrive in different forms or from different amounts of data, and then **deciding** — which ticket to buy, which route to take, whether a claim is actually backed by the record behind it.

About the data in this section. Every business, team, person and number in this section is **made up** (not real). They are built for this book so you can practise comparing chances and making decisions from data. Nothing here is real data about a real person, business or place.

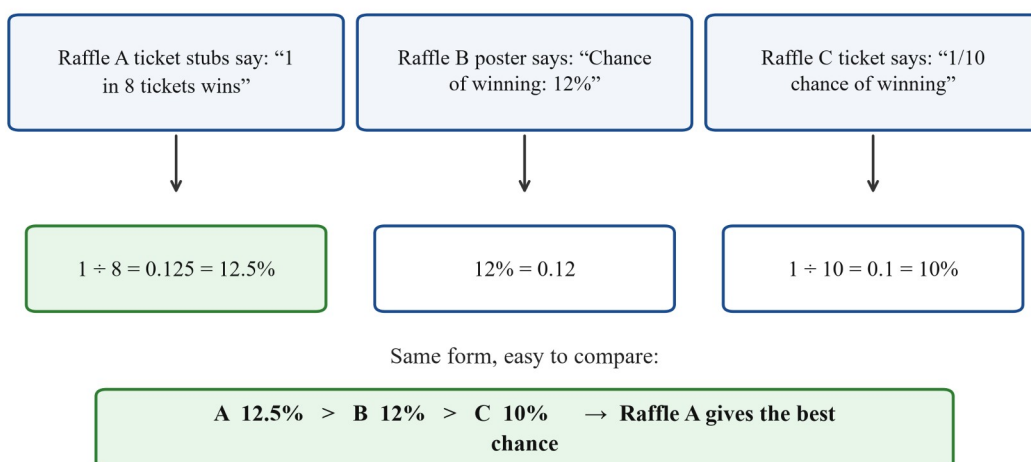
Put every chance in the same form.

You cannot compare chances until they wear the same outfit. Three raffles are running at a fete:

- Raffle A: the ticket stub says “1 in 8 tickets wins”.
- Raffle B: the poster says “Chance of winning: 12%”.
- Raffle C: the ticket says “1/10 chance of winning”.

Turn all three into percentages. Raffle A: $1 \div 8 = 0.125 = 12.5\%$. Raffle B is already a percentage, and as a decimal it is 0.12. Raffle C: $1 \div 10 = 0.1 = 10\%$. Now the comparison is easy: $10\% < 12\% < 12.5\%$, so Raffle C is the worst chance of the three and Raffle A the best — even though “1 in 8” and “1/10” look like the same kind of number and “12%” looks bigger than both at a glance.

Three raffles, three different forms — convert them all



Fractions, decimals and percentages are just different clothes for the same number. Undress them all into one form first.

Made-up data: the three raffles and every number in this picture are made up for this section.

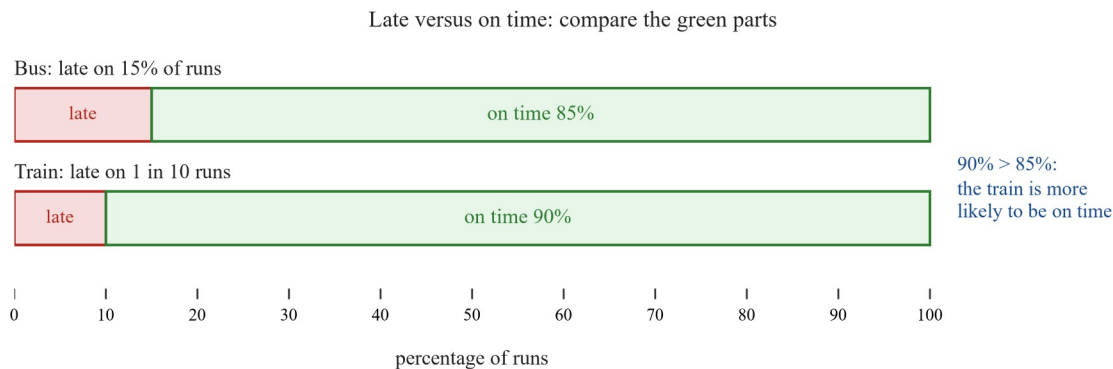
The other side of an event.

An **event** is anything that can happen or not happen: “the bus is late”, “a ticket wins”, “a shot is saved”. The chance of an event is written $P(\text{event})$, said “the probability of the event”. Every event has an other side — the event *not* happening — and the two sides fill up the whole scale between them, so they must add to 1, or to 100%:

$$P(\text{not}) = 1 - P(\text{event})$$

In percentages that is $100\% - P(\text{event})$. A town’s bus record says the bus is late on 15% of its runs, so the other side is $100\% - 15\% = 85\%$: the bus is on time on 85% of runs. The train’s record says “late on 1 in 10 runs”, which is $1 \div 10 = 10\%$, so the train is on time on $100\% - 10\% = 90\%$ of runs. Comparing the on-time sides, the train (90%)

is more likely to be on time than the bus (85 %) — a comparison you can only make once both chances are in the same form and on the same side of the event.



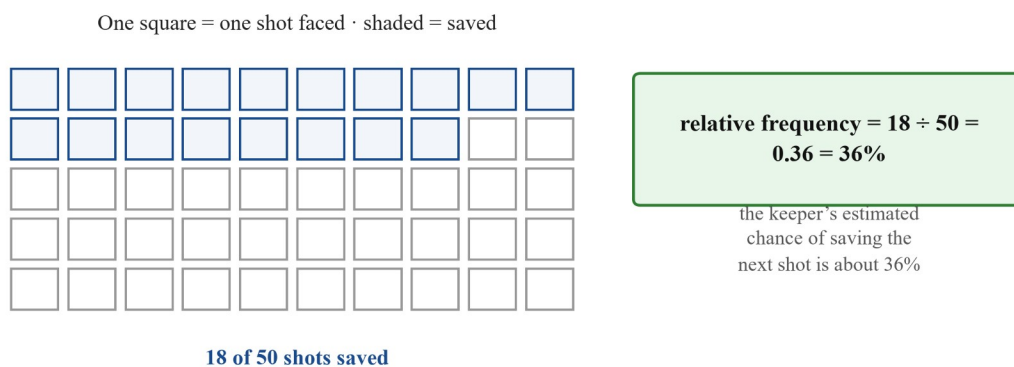
Made-up data: the bus and train records in this picture are made up for this section.

Estimating a chance from data.

Not every chance comes from counting equally likely outcomes — a saved shot, a late delivery and a faulty bottle are not equally likely in the way a die face is. When the outcomes are not equally likely, we estimate the chance from a record: count how many times the event happened, and divide by how many chances it had. That ratio has a name, the **relative frequency**:

$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{number of chances}}$$

A goalkeeper's season record shows 18 saves from 50 shots faced. The relative frequency of a save is $18 \div 50 = 0.36 = 36\%$, and that is our best estimate of the keeper's chance of saving the next shot. The longer the record, the more the estimate can be trusted: 18 saves in 50 shots says far more than 2 saves in 5 shots, because results vary and a handful of tries can land anywhere.



Made-up data: the keeper and every shot in this picture are made up for this section.

Make the comparison fair: rates, not counts.

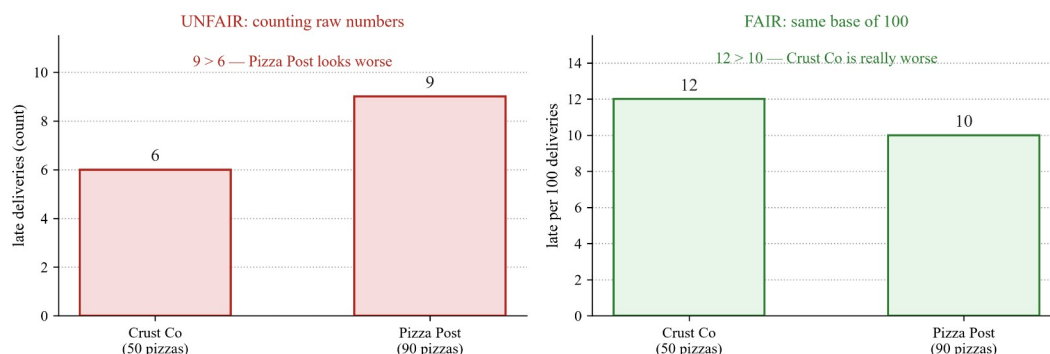
Two pizza shops deliver in the same suburb. Over a month, Crust Co had 6 late deliveries and Pizza Post had 9. Raw counts say Pizza Post is worse — but the two shops did not deliver the same number of pizzas: Crust Co delivered 50 and Pizza Post delivered 90. When the numbers of chances differ, compare **rates** — here, late deliveries per 100:

	Late deliveries	Pizzas delivered	Late per 100
Crust Co	6	50	12
Pizza Post	9	90	10

Made-up data: the two pizza shops and their delivery counts are made up for this section.

Crust Co: $6 \div 50 = 0.12 = 12\%$, so 12 late per 100. Pizza Post: $9 \div 90 = 0.1 = 10\%$, so 10 late per 100. The fair comparison flips the answer: Pizza Post is actually late **less** often, despite having more late deliveries in raw count. Whenever the numbers of chances differ, compare rates, never raw counts.

6 of 50 late = 12% · 9 of 90 late = 10% — the fair picture flips the answer



Made-up data: every number in this picture is made up for this section.

Expected numbers.

A chance also tells you what to **expect** over many tries. Multiply the chance by the number of chances:

$$\text{expected number} = \text{probability} \times \text{number of chances}$$

A bakery rejects 3% of its loaves at the quality check. In a week of 400 loaves, the expected number of rejects is $0.03 \times 400 = 12$ loaves. An expected number is a prediction about the long run, not a promise about one week: the real count wanders around 12 — one week 9, the next 15. That wandering is normal, and on its own it is not evidence that anything has changed.

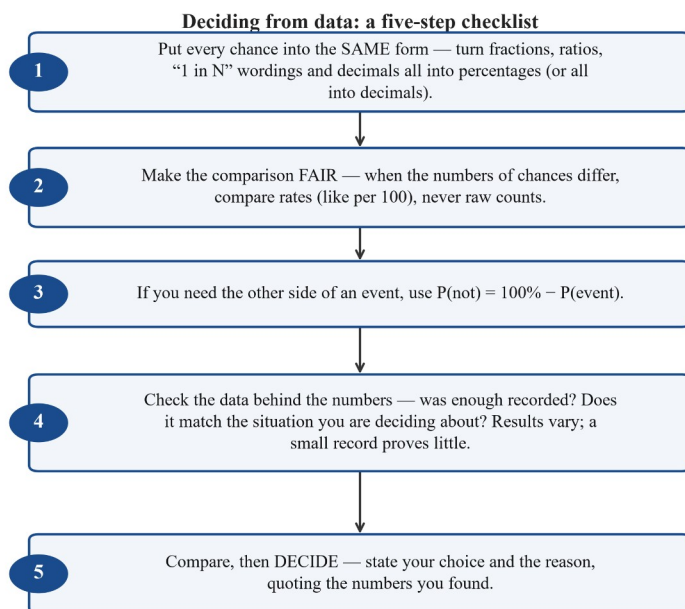
Does the data back the claim?

Chances are often handed to us as claims: “it rains on 30% of March Saturdays”. To test a claim, put the record and the claim into the same form and compare. Twenty years of March Saturdays in one town gives 90 Saturdays, of which 27 were wet. $27 \div 90 = 0.3 = 30\%$ — the record lines up with the claim exactly, so the data backs it.

Now suppose a friend calls the claim wrong because it rained on only 1 of the last 4 Saturdays — $1 \div 4 = 25\%$. That does not touch the claim at all: 4 Saturdays is far too small a record. Results vary from week to week, and a small record proves little. Trust the long record, and be just as suspicious of a counter-claim built on a handful of tries as of a claim built on one.

A checklist for deciding from data.

Everything in this section packs into five steps you can run on any decision — which ticket to buy, which route to take, which claim to believe.



Made-up data: this chart describes the method in this section, not real data.

Most mistakes happen at step 2, comparing raw counts instead of rates, and at step 4, trusting a record that is too small. If your decision survives all five steps, you can state it with the numbers behind it — which is exactly what “deciding from data” means.

Rounding and checking.

When an answer needs rounding, the question tells you how far — to one decimal place, to the nearest whole number. Keep the full value in your working and round only at the end, then say in your answer how far you rounded. Afterwards, sanity-check the number: a chance always sits between 0 and 1, or between 0 % and 100 %, and an expected number should look like the rate times the tries. If your “chance” comes out above 100 %, or your expected saves exceed the shots faced, something has gone wrong.

Tools that help.

A calculator or a spreadsheet will convert fractions and divide counts in seconds, and at work you will often let it. But the tool cannot tell you whether a comparison is fair, or whether a record is long enough to trust — it will happily compare 26 late parcels with 24 and call it a tie. Choosing the fair rate and checking the data behind it stay your job.

Worked examples

Example 1 — scaffolded

A school fete runs three prize games, each advertising its chance of winning in a different form:

- Game 1: “1 in 6 players wins a prize”
- Game 2: “Chance of winning: 0.15”
- Game 3: “1 in 5 players wins a prize”

Which game gives a player the best chance of winning a prize?

Working	Comment
Game 1: $1 \div 6 = 0.1666\dots$, so 16.7 % to one decimal place.	“1 in 6” means one win in every six plays.
Game 2: $0.15 = 15\%$.	Decimal to percentage: multiply by 100.
Game 3: $1 \div 5 = 0.2 = 20\%$.	The same conversion as Game 1.

Line them up: $15\% < 16.7\% < 20\%$.

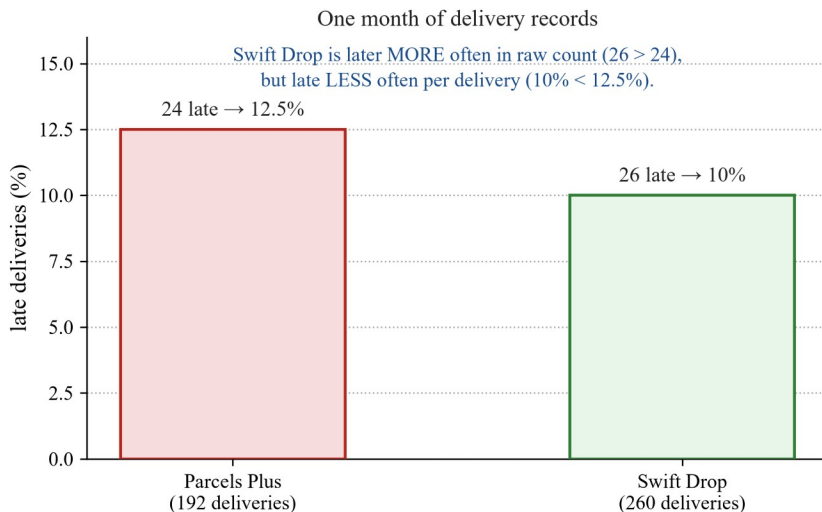
Chances can only be compared in the same form.

Game 3 gives the best chance of winning a prize.

Answer the question that was asked.

Example 2 — scaffolded

A town keeps one month of parcel-delivery records. Parcels Plus delivered 192 parcels and 24 arrived late. Swift Drop delivered 260 parcels and 26 arrived late. Which company is more reliable? Give each company's late rate and on-time rate as percentages.



Made-up data: the two companies and every delivery count in this picture are made up for this section.

Parcels Plus late: $24 \div 192 = 0.125 = 12.5\%$.

Late parcels $\div$ all parcels is the late rate.

Parcels Plus on time: $100\% - 12.5\% = 87.5\%$.

The two sides of an event add to 100%.

Swift Drop late: $26 \div 260 = 0.1 = 10\%$.

The same calculation on the second record.

Swift Drop on time: $100\% - 10\% = 90\%$.

Same again on the other side.

$10\% < 12.5\%$: Swift Drop is late less often, even though $26 > 24$ in raw count.

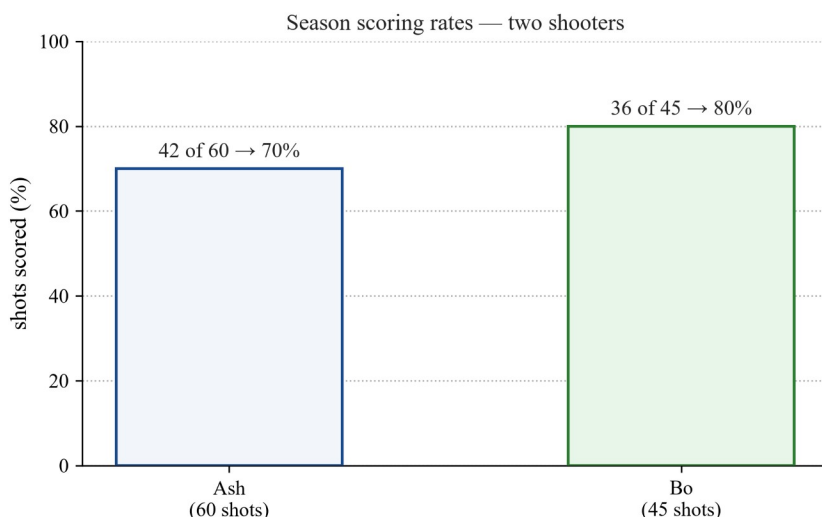
The two records hold different numbers of deliveries, so compare rates, never counts.

Swift Drop is the more reliable company: 90% on time against 87.5%.

State the decision and quote the numbers behind it.

Example 3 — unscaffolded

A netball club keeps its season record for its two shooters:



Made-up data: the two shooters and every count in this picture are made up for this section.

- (a) Turn each shooter's scoring rate into a percentage and say which shooter has the better rate.
- (b) The club picks that shooter to take 25 shots in a final. How many goals would you expect from them?
- (c) Ash: $42 \div 60 = 0.7 = 70\%$. Bo: $36 \div 45 = 0.8 = 80\%$. Bo has the better scoring rate — and notice that the raw counts would mislead here: Ash scored more goals (42 against 36) but needed more shots to do it.
- (d) Expected goals = probability $\times$ number of chances = $0.8 \times 25 = 20$ goals.

$\therefore$ Bo has the better rate (80% against 70%), and from 25 shots we would expect about 20 goals from Bo.

Example 4 — unscaffolded

A market stall runs a duck-scoop game and claims “1 in 5 scoops wins a prize”. One morning Zoe counts 150 scoops played and 18 prizes won. A scoop costs \$2.

- (a) Does the morning's record back the stall's claim? Show by calculation.
- (b) On the numbers, should Zoe pay \$2 for a scoop? Give a reason.
- (c) The claim as a percentage: $1 \div 5 = 0.2 = 20\%$. If the claim were true, the expected number of prizes in 150 scoops would be $0.2 \times 150 = 30$. The record shows 18 prizes, which is $18 \div 150 = 0.12 = 12\%$. A rate of 12% sits well below the claimed 20% — 18 prizes against an expected 30 — so the record does not back the claim.
- (d) No. On the morning's record Zoe's real chance of a prize is about 12%, roughly 1 in 8, not the 1 in 5 she is being sold: her \$2 buys much less chance than the stall claims. (Section 9D adds the other half of this kind of decision — what the prize is worth against what the play costs.)

$\therefore$ the record (12% against the claimed 20%; 18 prizes against an expected 30) does not back the claim, and on the numbers Zoe should keep her \$2.

Practice questions

Scaffolded

Q1. Each line below holds two chances in different forms. Put each pair in the same form and say which is the bigger chance. (a) 0.37 or 31% (b) $\frac{1}{4}$ or 0.24 (c) Write $\frac{2}{5}$, 45% and 0.5 in order from smallest chance to largest.

Q2. A weather app puts tomorrow's chance of rain at 30%. (a) Write tomorrow's chance of **no** rain as a decimal. (b) Write tomorrow's chance of no rain as a percentage. (c) A sports club plays its match if the chance of no rain is 60% or more. Will the match go ahead?

Q3. A bag holds 8 tokens, 3 of them gold. A fair 6-sided die is rolled. (a) Write the chance of drawing a gold token as a decimal. (b) Write the chance of rolling an even number as a decimal. (c) Which gives the better chance of a win: drawing a gold token, or rolling an even number?

Q4. A café checks 200 takeaway orders and finds 12 were made wrong. (a) Write the relative frequency of a wrong order as a decimal. (b) Write it as a percentage. (c) At the same rate, how many wrong orders would you expect in the next 500 orders?

Q5. A council times two bus routes over a month. Route 7 arrived late on 8 of its 40 runs; Route 9 arrived late on 9 of its 60 runs. (a) Write each route's late rate as a percentage. (b) Which route was on time more often? Give both on-time rates. (c) For the route you chose in (b), how many late runs would you expect per 100 runs?

Q6. Four friends state the chance of the same event in four different ways: “1 in 4”, 0.3, 35 % and $\frac{2}{5}$. (a) Write “1 in 4” and $\frac{2}{5}$ as percentages. (b) Put all four chances in order from smallest to largest. (c) Which of the four is closest to an even chance (50 %)?

Unscaffolded

Q7. Three charity raffles advertise their chance of winning: Raffle A “1 in 12”, Raffle B “9%”, Raffle C “1 in 15”. (a) Write all three chances as percentages, to one decimal place where needed. (b) Which raffle gives the best chance of winning? (c) Which gives the worst chance?

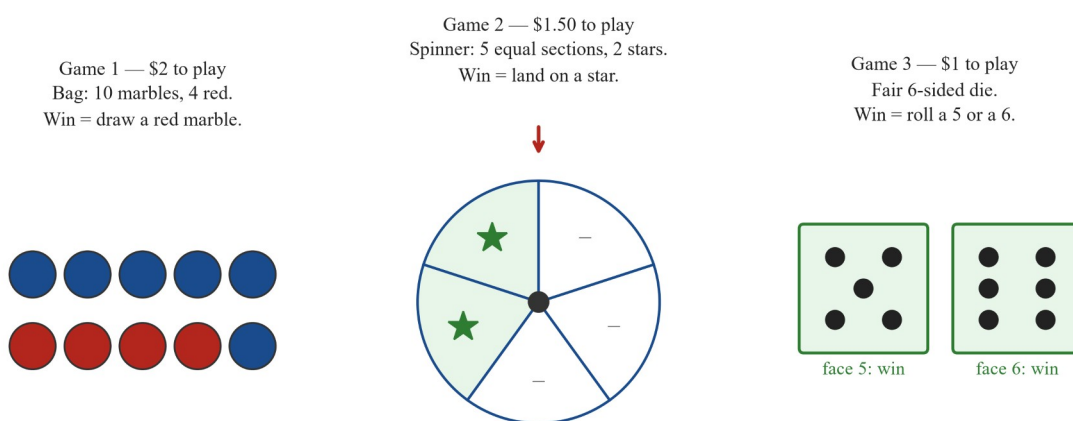
Q8. A football club compares its two goalkeepers over the season. Tau saved 84 of the 120 shots faced; Sam saved 51 of the 68 shots faced. (a) Write each keeper’s save rate as a percentage. (b) The club must pick one keeper for the finals. Who would you pick, and what else about the two records would you mention alongside your choice? (c) At your chosen keeper’s rate, how many saves would you expect from 40 shots faced?

Q9. A factory checks two bottling machines. Machine A produced 15 rejected bottles in 500; Machine B produced 9 rejected in 250. (a) Write each machine’s reject rate as a percentage. (b) Which machine is running better? Explain why the raw counts (15 against 9) give the wrong answer here. (c) At Machine A’s rate, how many rejects would you expect in 2 000 bottles?

Q10. Two weather apps forecast tomorrow differently: App A says the chance of rain is 35 %; App B says “1 in 4”. (a) Put both forecasts in the same form and say which app makes rain more likely. (b) Using App A’s forecast, how many rainy days would you expect over the next 20 days? (c) Exactly one of those 20 days is your day at the beach. On App A’s number, would you pack an umbrella? Give a reason in one sentence.

Q11. A card pack claims “1 in 6 packets contains a winning card”. A class opens 240 packets and finds 30 winners. (a) Write the claim and the class’s result as percentages, the claim to one decimal place. (b) If the claim were true, how many winning packets would you expect in 240? (c) Does the class’s record back the claim? Use your numbers from (a) and (b) in your answer.

Q12. A fete runs three games, drawn below.



Made-up data: the three games and their prices in this picture are made up for this section.

- Write the chance of winning each game as a decimal, to two decimal places.
- Which two games give exactly the same chance of winning? What is that chance as a percentage?
- A player wants the best value — the same chance for the least money. Between the two games from (b), which should they play, and why?

Q13. A mail company compares two courier services over a quarter. Swift Post had 18 problem deliveries in 240; Flash Freight had 20 in 320. (a) Write each service’s problem rate as a percentage. (b) Which service is the safer

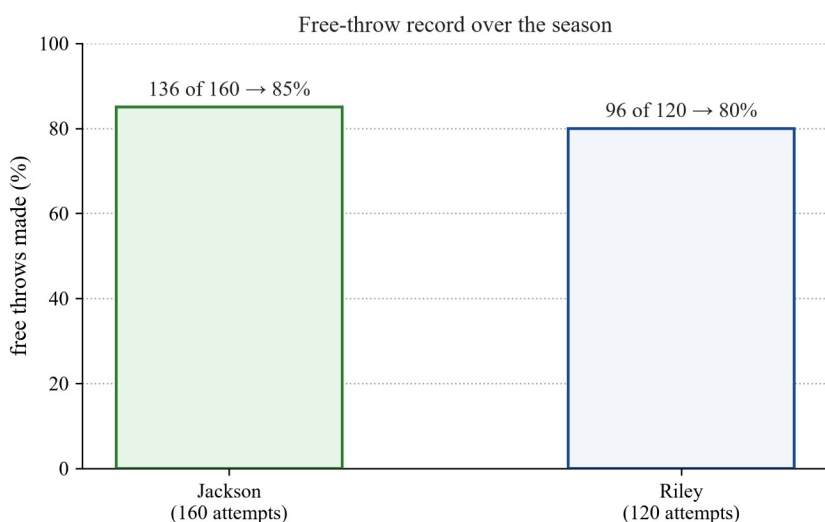
choice? (c) At the safer service's rate, how many problem deliveries would you expect in 500 parcels? Give your answer to the nearest whole parcel.

Q14. A player rolls a fair 6-sided die 120 times and counts the sixes. (a) Write down the chance of a six on one roll. (b) How many sixes would you expect over the 120 rolls? (c) The player actually rolls 26 sixes. Does this prove the die is biased? Give a reason in one sentence.

Q15. A council checks 450 recycling bins in a town and finds 63 with the wrong contents in them. The council gives a recycling grant only to towns whose wrong-bin rate is 10% or less. (a) Write the town's wrong-bin rate as a percentage. (b) Does the town get the grant? (c) At the same rate, how many wrong bins would you expect in the next 400 bins checked?

Q16. A report claims that 2 in 5 households in a city own a pet. A student surveys one street and finds 18 of the 40 households own a pet. (a) Write the report's claim as a percentage. (b) Write the street's result as a percentage, and say how far it sits from the claim. (c) The student says the street's result disproves the report. Do you agree? Give a reason.

Q17. A basketball coach compares two players' free throws over the season, drawn below.



Made-up data: the two players and every count in this picture are made up for this section.

- (a) Write each player's made rate as a percentage.
- (b) The coach picks one player to take the last free throw of a close game. Who, and why?
- (c) At the chosen player's rate, how many of 10 free throws would you expect them to make? Comment on your answer, since free throws come in whole numbers.

Q18. Two car dealers each had 12 cars come back with a fault in the first year. Dealer A sold 80 cars that year; Dealer B sold 120. (a) Write each dealer's fault rate as a percentage. (b) A friend says the dealers are equally good because both had 12 faulty cars. Use your percentages to answer the friend. (c) At Dealer B's rate, how many first-year faults would you expect from its next 150 sales?

Answers

Q1. (a) $0.37 = 37\%$, so 0.37 is the bigger chance. (b) $\frac{1}{4} = 0.25$, so $\frac{1}{4}$ is the bigger chance. (c) $\frac{2}{5} = 40\%$ and $0.5 = 50\%$, so the order is $\frac{2}{5}$, 45% , 0.5 . **Q2.** (a) 0.7 . (b) 70% . (c) yes — 70% is 60% or more, so the match goes ahead. **Q3.** (a) 0.375 . (b) 0.5 . (c) rolling an even number — $0.5 > 0.375$. **Q4.** (a) 0.06 . (b) 6% . (c) 30 wrong orders. **Q5.** (a) Route 7: 20% ; Route 9: 15% . (b) Route 9 — on time on 85% of runs against 80% . (c) 15 late runs per 100. **Q6.** (a) “1 in 4” = 25% ; $\frac{2}{5} = 40\%$. (b) 1 in 4 (25%), 0.3 (30%), 35% , $\frac{2}{5}$ (40%). (c) $\frac{2}{5}$ — 40% sits 10 percentage points from 50% , the smallest gap of the four. **Q7.** (a) A 8.3% , B 9% , C 6.7% (A and C to one decimal place). (b) Raffle B. (c) Raffle C. **Q8.** (a) Tau 70% , Sam 75% . (b) Sam on the rate — but Tau’s rate rests on 120 shots against Sam’s 68, so Sam’s higher rate comes from the smaller record. (c) 30 saves. **Q9.** (a) A 3% , B 3.6% . (b) Machine A — $3\% < 3.6\%$; the raw counts ignore that A was checked over twice as many bottles. (c) 60 rejects. **Q10.** (a) App A 35% , App B 25% — App A makes rain more likely. (b) 7 rainy days. (c) any reasoned call quoting the number, e.g. yes — 35% is more than 1 day in 3, so most people would pack one. **Q11.** (a) claim 16.7% (to one decimal place), result 12.5% . (b) 40 winning packets. (c) no — 30 winners against an expected 40, and 12.5% against the claimed 16.7% . **Q12.** (a) Game 1 0.40 , Game 2 0.40 , Game 3 0.33 (to two decimal places). (b) Games 1 and 2, both 40% . (c) Game 2 — the same 40% chance for $\$1.50$ instead of $\$2$. **Q13.** (a) Swift Post 7.5% , Flash Freight 6.25% . (b) Flash Freight. (c) 31 problem deliveries (to the nearest whole parcel; exactly 31.25). **Q14.** (a) $\frac{1}{6}$. (b) 20 sixes. (c) no — results vary from run to run, and one set of 120 rolls landing 6 above the expected number proves little. **Q15.** (a) 14% . (b) no — 14% is above the 10% cutoff. (c) 56 wrong bins. **Q16.** (a) 40% . (b) 45% , which sits 5 percentage points above the claim. (c) no — 40 households on one street is a small record and one street may not match the whole city; results vary, and a 5-point gap on 40 households is exactly the wobble a small record produces. **Q17.** (a) Jackson 85% , Riley 80% . (b) Jackson — the higher made rate, on records long enough to compare. (c) 8.5, so about 8 or 9 made throws; a rate predicts the long run, not one set of 10. **Q18.** (a) Dealer A 15% , Dealer B 10% . (b) the same count of 12 hides different rates — per 100 cars sold, A sends 15 back and B sends 10, so Dealer B is the better buy on this record. (c) 15 faults.

Fully-worked solutions

Q1. (a) $0.37 = 37\%$, and $37\% > 31\%$, so 0.37 is the bigger chance. (b) $\frac{1}{4} = 1 \div 4 = 0.25$, and $0.25 > 0.24$, so $\frac{1}{4}$ is the bigger chance. (c) In percentages: $\frac{2}{5} = 40\%$, 45% stays as it is, and $0.5 = 50\%$; from smallest to largest the order is $\frac{2}{5}$, 45% , 0.5 .

Q2. (a) The two sides of the event add to 1: $1 - 0.3 = 0.7$. (b) $0.7 = 70\%$. (c) Yes: the chance of no rain is 70% , which is 60% or more, so the match goes ahead.

Q3. (a) 3 gold tokens in 8: $3 \div 8 = 0.375$. (b) Three even faces (2, 4, 6) in six: $3 \div 6 = 0.5$. (c) Rolling an even number: $0.5 > 0.375$, so the die gives the better chance of a win.

Q4. (a) Relative frequency = times happened $\div$ chances = $12 \div 200 = 0.06$. (b) $0.06 = 6\%$. (c) Expected number = probability $\times$ chances = $0.06 \times 500 = 30$ wrong orders.

Q5. (a) Route 7: $8 \div 40 = 0.2 = 20\%$ late. Route 9: $9 \div 60 = 0.15 = 15\%$ late. (b) On-time sides: Route 7 $100\% - 20\% = 80\%$; Route 9 $100\% - 15\% = 85\%$. Route 9 was on time more often. (c) $0.15 \times 100 = 15$ late runs per 100 runs.

Q6. (a) $1 \text{ in } 4 = 1 \div 4 = 25\%$; $\frac{2}{5} = 2 \div 5 = 40\%$. (b) As percentages the four chances are 25%, 30%, 35% and 40%, so the order from smallest to largest is 1 in 4, 0.3, 35%, $\frac{2}{5}$. (c) The gaps to 50% are 25, 20, 15 and 10 percentage points, so $\frac{2}{5}$ at 40% is closest to an even chance.

Q7. (a) A: $1 \div 12 = 0.0833\dots = 8.3\%$ to one decimal place. B: 9% as given. C: $1 \div 15 = 0.0666\dots = 6.7\%$ to one decimal place. (b) Raffle B, at 9%, is the best chance. (c) Raffle C, at 6.7%, is the worst.

Q8. (a) Tau: $84 \div 120 = 0.7 = 70\%$. Sam: $51 \div 68 = 0.75 = 75\%$. (b) Sam, on the rate: $75\% > 70\%$. Worth mentioning alongside the choice: Tau's rate rests on 120 shots faced against Sam's 68, so Sam's higher rate is built from the smaller record and could move as more shots are counted. (c) $0.75 \times 40 = 30$ saves.

Q9. (a) A: $15 \div 500 = 0.03 = 3\%$. B: $9 \div 250 = 0.036 = 3.6\%$. (b) Machine A, because $3\% < 3.6\%$. The raw counts (15 against 9) ignore that A was checked over 500 bottles and B over only 250 — different numbers of chances, so the counts cannot be compared directly. (c) $0.03 \times 2000 = 60$ rejects.

Q10. (a) App B: $1 \text{ in } 4 = 1 \div 4 = 25\%$. App A's 35% is bigger than 25%, so App A makes rain more likely. (b) $0.35 \times 20 = 7$ rainy days. (c) Any reasoned call that quotes the number, e.g. yes — a 35% chance of rain is more than 1 day in 3, so leaving the umbrella behind on the one beach day is a gamble most people would not take.

Q11. (a) Claim: $1 \div 6 = 0.1666\dots = 16.7\%$ to one decimal place. Result: $30 \div 240 = 0.125 = 12.5\%$. (b) 1 in 6 of 240 packets is $240 \div 6 = 40$ winning packets. (c) No: the record shows 30 winners against the 40 the claim expects, and 12.5% against the claimed 16.7% — the record sits below the claim, so it does not back it.

Q12. (a) Game 1: $4 \div 10 = 0.40$. Game 2: $2 \div 5 = 0.40$. Game 3: $2 \div 6 = 0.333\dots = 0.33$ to two decimal places. (b) Games 1 and 2 give exactly the same chance, $0.40 = 40\%$. (c) Game 2: it sells the same 40% chance for \$1.50 that Game 1 sells for \$2, so the player loses less money per play for the same chance. (Game 3 is cheaper still at \$1, but its chance, 0.33, is worse.)

Q13. (a) Swift Post: $18 \div 240 = 0.075 = 7.5\%$. Flash Freight: $20 \div 320 = 0.0625 = 6.25\%$. (b) Flash Freight: $6.25\% < 7.5\%$, even though it had more problems in raw count (20 against 18) — it also carried more parcels, so only the rates compare fairly. (c) $0.0625 \times 500 = 31.25$, which is 31 problem deliveries to the nearest whole parcel.

Q14. (a) One fair face in six: $\frac{1}{6}$. (b) $\frac{1}{6} \times 120 = 20$ sixes. (c) No. Results vary from one run of 120 rolls to the next, and 26 is only 6 above the expected 20; a single run landing a little high does not prove bias — making that case takes a much longer record.

Q15. (a) $63 \div 450 = 0.14 = 14\%$. (b) No: the grant needs a wrong-bin rate of 10% or less, and 14% is above that cutoff. (c) $0.14 \times 400 = 56$ wrong bins.

Q16. (a) 2 in 5 = $2 \div 5 = 40\%$. (b) $18 \div 40 = 0.45 = 45\%$, which sits 5 percentage points above the report's 40%. (c) No. Forty households on one street is a small record, and one street may not match the whole city; results vary from sample to sample, and a 5-point gap on 40 households is exactly the kind of wobble a small record produces. The street neither proves nor disproves the report.

Q17. (a) Jackson: $136 \div 160 = 0.85 = 85\%$. Riley: $96 \div 120 = 0.8 = 80\%$. (b) Jackson: $85\% > 80\%$, and both records are long enough (160 and 120 attempts) for the rates to compare fairly. (c) $0.85 \times 10 = 8.5$ — but made throws come in whole numbers, so expect about 8 or 9 made throws out of 10; the 8.5 is a long-run average, not a promise for one set of 10.

Q18. (a) A: $12 \div 80 = 0.15 = 15\%$. B: $12 \div 120 = 0.1 = 10\%$. (b) The friend is comparing raw counts over different numbers of sales. Per 100 cars sold, Dealer A sends 15 back and Dealer B sends 10, so Dealer B's cars fault less often — B is the better buy on this record. (c) $0.1 \times 150 = 15$ first-year faults.

Chapter 9 Uncertainty — 9D Risk and likelihood

Theory

Risk is part of every job and every day. A ladder can slip. A phone screen can crack. A storm can flood a shed. None of these are certain — but some are more likely than others, and some would hurt a lot more than others. This section puts a number on each of those two sides, joins them into one number called a **risk score**, and then uses that score to decide what to fix first and what is worth paying for.

About the data in this section. Every business, person, price and chance in this section is **made up** (not real). They are built for this book so you can practise judging risk. Nothing here is real data about a real person, business or place.

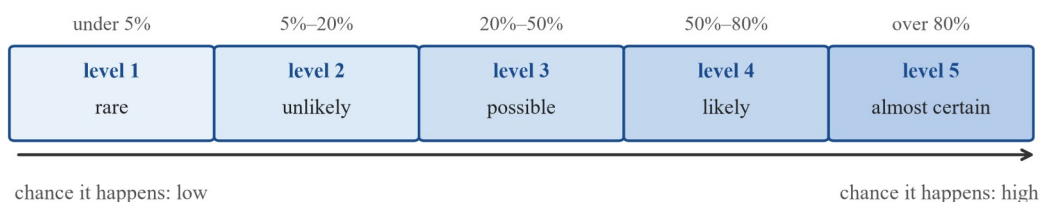
Likelihood and consequence.

Two questions sit behind every risk. The first is **likelihood**: what is the chance this happens? Section 9A gave the words and the numbers for chance — a probability between 0 and 1, or a percent. The second is **consequence**: if it does happen, how bad is that? A consequence can be dollars lost, days lost, or harm to a person. **Risk** is the two sides together: a very likely small trouble and a rare disaster are both risks, and they need different answers.

The likelihood scale.

This section scores likelihood on five levels, from rare to almost certain.

The likelihood scale used in this section



The five bands on this scale are the ones this section uses; a workplace may use its own.

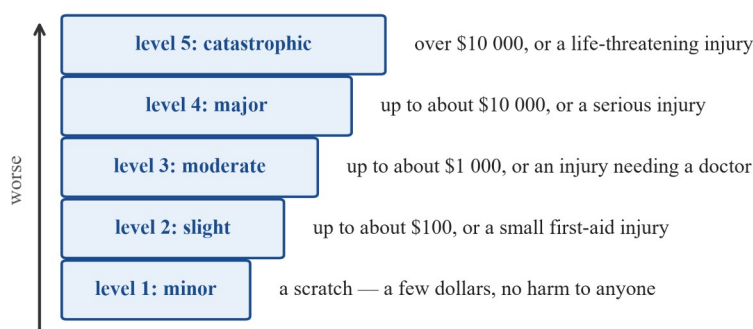
Level	Word	Chance it happens
1	rare	under 5 %
2	unlikely	5 %–20 %
3	possible	20 %–50 %
4	likely	50 %–80 %
5	almost certain	over 80 %

To place a chance on the scale, write it as a percent first if you need to: a probability of 0.3 is 30 %, which sits in the 20 %–50 % band, so it is level 3, possible.

The consequence scale.

Consequence is scored on five levels too, from a scratch to as bad as it gets. The top level is called **catastrophic**, which means the worst kind of outcome — a life-threatening injury, or a loss so big it could close a business.

The consequence scale used in this section



The five bands on this scale are the ones this section uses; a workplace may use its own.

Level	Word	What it could mean
1	minor	a few dollars, a scratch — no harm to anyone
2	slight	up to about \$100, or a small first-aid injury
3	moderate	up to about \$1 000, or an injury needing a doctor
4	major	up to about \$10 000, or a serious injury
5	catastrophic	over \$10 000, or a life-threatening injury

The risk score and the risk matrix.

One multiply turns the two levels into a single **risk score**:

One multiply turns two judgments into one score

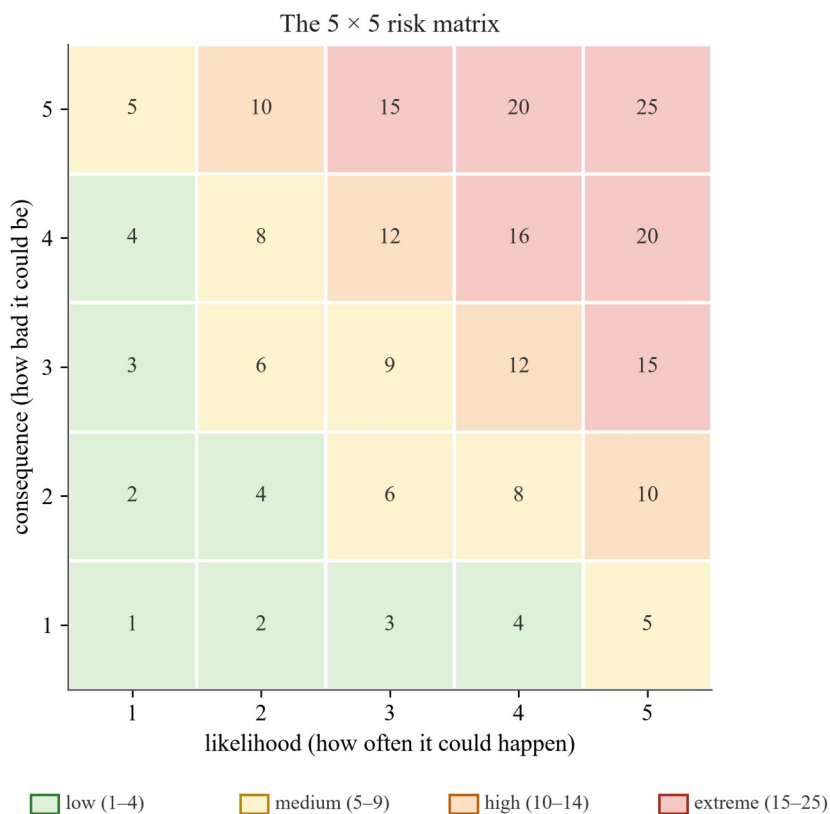


example: likelihood 4 × consequence 3 = score 12 → high risk

The score lands between 1 and 25, and the size of the score puts the risk in one of four **zones**:

Score	Zone	What it means for you
1–4	low	everyday care is enough; keep an eye on it
5–9	medium	plan a fix soon
10–14	high	fix it before most other jobs
15–25	extreme	stop the job until it is fixed

The whole picture at once is a **risk matrix** — a grid with likelihood along the bottom and consequence up the side. Each cell holds its score, and the colour of the cell is the zone.



The numbers in the cells are scores: likelihood level × consequence level.

The matrix shows why both sides matter. A forklift crossing a walking path is unlikely (level 2) but could kill (level 5): score $2 \times 5 = 10$, high. Cardboard offcuts on the floor trip people nearly every day (level 5) but only bruise them (level 1): score $5 \times 1 = 5$, medium. The forklift scores higher even though it happens less often, because its consequence is so much worse.

Using likelihood estimates to make decisions.

A likelihood estimate is not just a label — it can be multiplied by what the event would cost, to give an **expected cost**: the amount you would lose on average if the same year happened over and over.

$$\text{expected cost} = \text{probability} \times \text{cost if it happens}$$

A phone screen has a 0.05 chance of cracking in a year, and a repair costs \$240, so the expected cost is $0.05 \times 240 = \$12$ a year. A screen-protection plan costs \$29 a year. On money alone, \$12 is less than \$29, so the plan is not worth buying. (There can still be other reasons to buy it — see the questions.)

Another useful move is turning a probability into a “**1 in how many**” statement. A probability of 0.04 means 4 in 100, which is 1 in 25: about once every 25 times. That form is often the easiest one to think with.

Cutting a risk.

A risk has two handles, so there are two ways to cut it. You can lower the **likelihood** — training, checks, guards, better lighting — or you can lower the **consequence** — helmets, seatbelts, backups, spreading the cost with insurance. Each change drops one of the two levels, so you re-score the risk after every change and watch it move down the matrix. A fix that only moves the score from 15 to 10 has not finished the job: 10 is still high.

Risk is not certainty.

A small chance is not zero, and a big chance is not a promise. A 1 % yearly flood chance means about 1 house in 100 floods each year — on a street of 100 houses, that is somebody’s house, every year, and the 1 % does not say whose. And a 90 % chance still misses one time in ten. A risk score is a guide to the better choice, not a guarantee of what will happen. Judging risk well means acting on the numbers while accepting that any single year can surprise you.

Tools that help.

A calculator or a spreadsheet will do the multiplies and the expected costs in seconds, and at work you will often let it. What the tool cannot do is choose the levels: no app can tell you whether a burn is a consequence 3 or a 4, or whether a slip is likely or possible. Reading the situation and picking the two levels stays your job, and so does checking the score against common sense — a score of 25 for a paper cut means a level was misjudged.

Worked examples

Example 1 — scaffolded

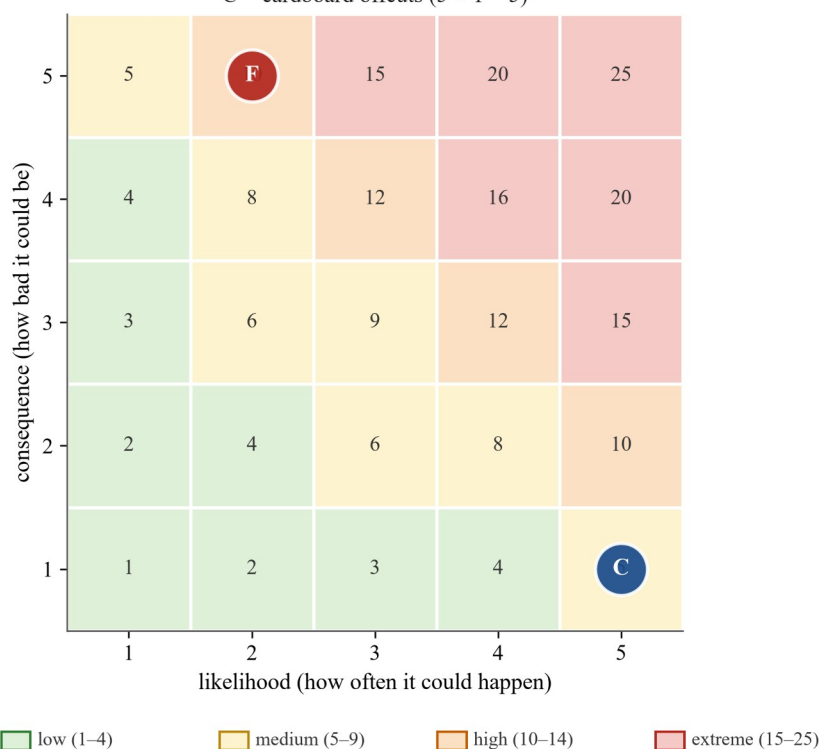
A café owner scores two hazards in the kitchen. A **hazard** is anything in a workplace that could cause harm or loss. Hazard 1: milk spilled on the tiles near the coffee machine, which happens most days; a slip means a bruise and a mop-up. Hazard 2: a hot baking tray left on the bench with no sign, which happens some weeks; a touch means a deep burn needing hospital treatment. Score both hazards and say which one the owner fixes first.

Working	Comment
Hazard 1 likelihood: most days → level 4.	“Most days” is over 50 %, so the likely band.
Hazard 1 consequence: bruise and mop-up → level 2.	A bruise is a small first-aid injury.
Hazard 1 score: $4 \times 2 = 8$.	Multiply the two levels.
Hazard 1 zone: 8 is in 5–9 → medium.	Read the zone off the score table.
Hazard 2 likelihood: some weeks → level 3.	“Some weeks” sits in the 20 %–50 % band.
Hazard 2 consequence: deep burn needing hospital → level 4.	A serious injury is level 4; a doctor-visit injury would be level 3.
Hazard 2 score: $3 \times 4 = 12$.	Likelihood 3 $\times$ consequence 4.
Hazard 2 zone: 12 is in 10–14 → high.	The higher zone wins.
Fix hazard 2 first: high (12) beats medium (8).	Always answer the question that was asked.

Example 2 — scaffolded

A warehouse supervisor plots two hazards on a risk matrix. F is a forklift path that crosses the walking path; C is cardboard offcuts left on the floor near the packing bench.

Warehouse hazards: F = forklift path ($2 \times 5 = 10$),
C = cardboard offcuts ($5 \times 1 = 5$)



Made-up data: the warehouse and both hazards in this picture are made up for this section.

Read each hazard off the matrix, score it, name its zone, and say which one the supervisor deals with first — and why the answer is not simply “the one that happens more often”.

Working

Comment

F sits at likelihood 2, consequence 5.

Read straight across and up from the point F.

F score: $2 \times 5 = 10 \rightarrow$ high.

10 is the first score in the high zone.

C sits at likelihood 5, consequence 1.

C is right at the frequent-but-tiny corner.

C score: $5 \times 1 = 5 \rightarrow$ medium.

5 is the first score in the medium zone.

Deal with F first: 10 (high) beats 5 (medium).

The higher zone gets done first.

F happens less often than C, yet scores higher: its consequence of 5 can kill, while C’s consequence of 1 only trips people.

A score weighs both sides — consequence counts as much as likelihood.

Example 3 — unscaffolded

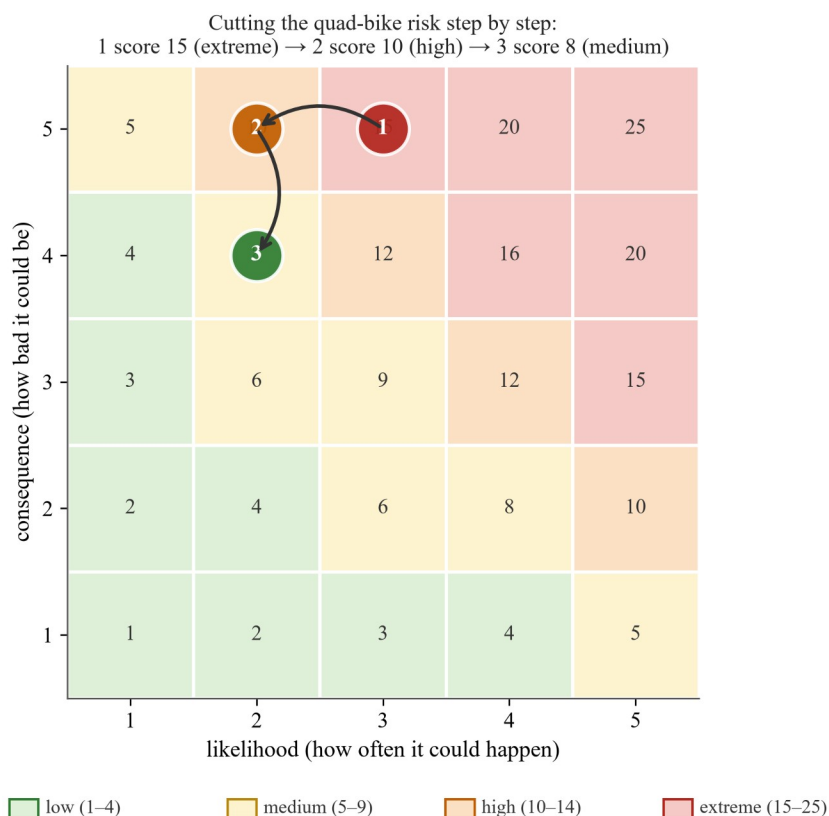
A tradie’s phone has a probability of 0.08 of a cracked screen in a year. A screen repair costs \$300. A screen-protection plan costs \$30 a year and covers every repair. On money alone, is the plan worth buying for one year?

The expected cost of cracks is the probability times the cost of a repair: $0.08 \times 300 = \$24$ a year. The plan costs \$30 a year, which is \$6 more than the \$24 the cracks are expected to cost. On money alone the plan loses you money in an average year.

$\therefore$ the expected cost is \$24 a year, below the \$30 plan, so on money alone the plan is not worth buying.

Example 4 — unscaffolded

A farm lists a quad-bike roll-over on a steep slope as likelihood 3 and consequence 5. The farm makes two changes: a rule that no steep slope is worked while the ground is wet, which drops the likelihood to level 2; then helmets and operator training for every rider, which drops the consequence to level 4. Score the risk before and after each change, and say whether one change alone would have been enough.



Made-up data: the farm and every level in this picture are made up for this section.

Before any change the score is $3 \times 5 = 15$, which is in the extreme zone — the job should stop until something changes. After the wet-ground rule the likelihood is 2, so the score is $2 \times 5 = 10$: the risk has left extreme, but 10 is still high. After helmets and training the consequence is 4, so the score is $2 \times 4 = 8$, which is medium. One change alone would not have been enough: the rule by itself leaves a high risk of 10, and helmets alone would leave $3 \times 4 = 12$, also high.

∴ the two changes together take the risk from extreme (15) to medium (8), and neither change on its own gets it out of the high zone.

Practice questions

Scaffolded

- Q1.** Rewrite each chance in the form asked for. (a) Write 30 % as a decimal. (b) Write 0.02 as a percent. (c) Write 15 % as a fraction in simplest form.
- Q2.** Place each chance on the five-level likelihood scale by writing its level number. (a) 90 % (b) 3 % (c) 65 %
- Q3.** Place each outcome on the five-level consequence scale by writing its level number. (a) a scratch on a bench top costing \$50 to polish out (b) a cut that needs stitches at the doctor (c) a fire that destroys \$15 000 of stock
- Q4.** Find the risk score, then name the zone. (a) likelihood 2, consequence 3 (b) likelihood 5, consequence 1 (c) likelihood 4, consequence 4
- Q5.** A café rates the hazard “toast burns a worker’s fingers at the toaster” as likelihood 3, consequence 2. (a) Find the risk score. (b) Name the zone. (c) The owner fits a splash guard that drops the likelihood to level 2. Find the new score and its zone.
- Q6.** A store rates two hazards: A, boxes stacked loosely on a high shelf (likelihood 4, consequence 3); B, a faded price label on a shelf edge (likelihood 2, consequence 2). (a) Find the score and zone for hazard A. (b) Find the score and zone for hazard B. (c) Which hazard does the store fix first?

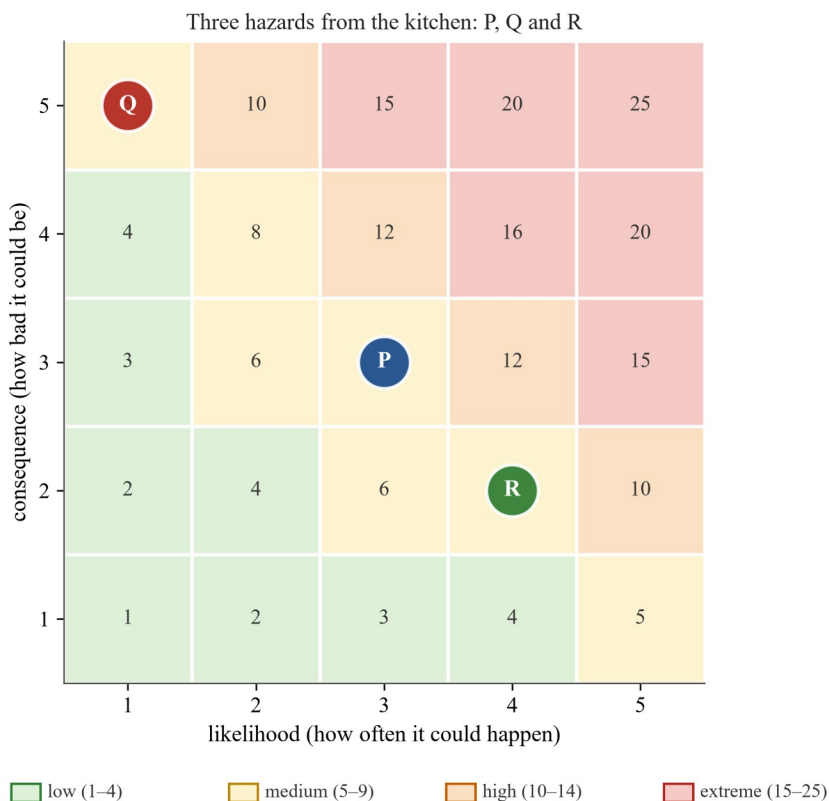
Un scaffolded

Q7. A delivery rider works in heavy rain. The chance of a slide in a year is about 70 %; a slide usually means a broken arm needing a doctor. (a) Write the likelihood level. (b) Write the consequence level. (c) Find the risk score and name the zone.

Q8. A courier firm finds that a delivery is late with probability 0.04. (a) Write 0.04 as a percent. (b) In 250 deliveries, how many late deliveries would you expect? (c) Explain in one sentence why your answer to (b) is not a promise.

Q9. A laptop has a probability of 0.03 of accidental damage in a year. A repair costs \$650. A damage plan costs \$25 a year. (a) Find the expected cost of damage for one year. (b) Compare the expected cost with the plan price, and say which is cheaper. (c) Give one reason a student might still buy the plan even though it costs more.

Q10. A head chef plots three kitchen hazards on this risk matrix: P, hot oil splashing from the fryer; Q, a gas leak catching fire; R, a wet floor beside the dishwasher.



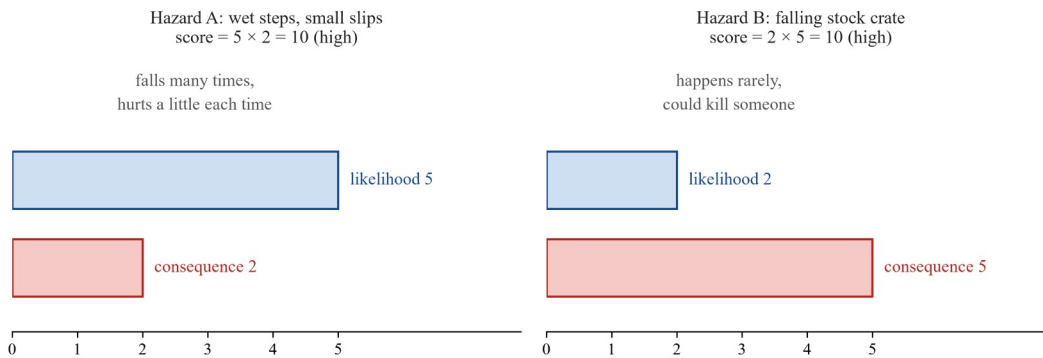
Made-up data: the kitchen and all three hazards in this picture are made up for this section.

- Read the likelihood level and the consequence level for P, Q and R off the matrix.
- Find the score and zone for each hazard.
- Which hazard does the chef deal with first? Give the score in your reason.

Q11. A workplace rates the hazard “worker falls from a ladder” as likelihood 4, consequence 3. It then brings in ladder training and a monthly ladder check, which drop the likelihood to level 2. (a) Find the score and zone before the change. (b) Find the score and zone after the change. (c) Which of the two levels changed, and by how many levels?

Q12. A bike courier rates the hazard “head injury in a crash” as likelihood 2, consequence 5. Wearing a helmet drops the consequence to level 3. (a) Find the score and zone without the helmet. (b) Find the score and zone with the helmet. (c) Which of the two levels did the helmet change, and why could it not change the other one?

Q13. A warehouse has two hazards with the same risk score.



Made-up data: the warehouse and both hazards in this picture are made up for this section.

- Show by calculation that both hazards have a score of 10.
- Name the zone that a score of 10 sits in.
- The scores are equal. Should the warehouse treat the two hazards the same? Give a reason that uses the consequence levels.

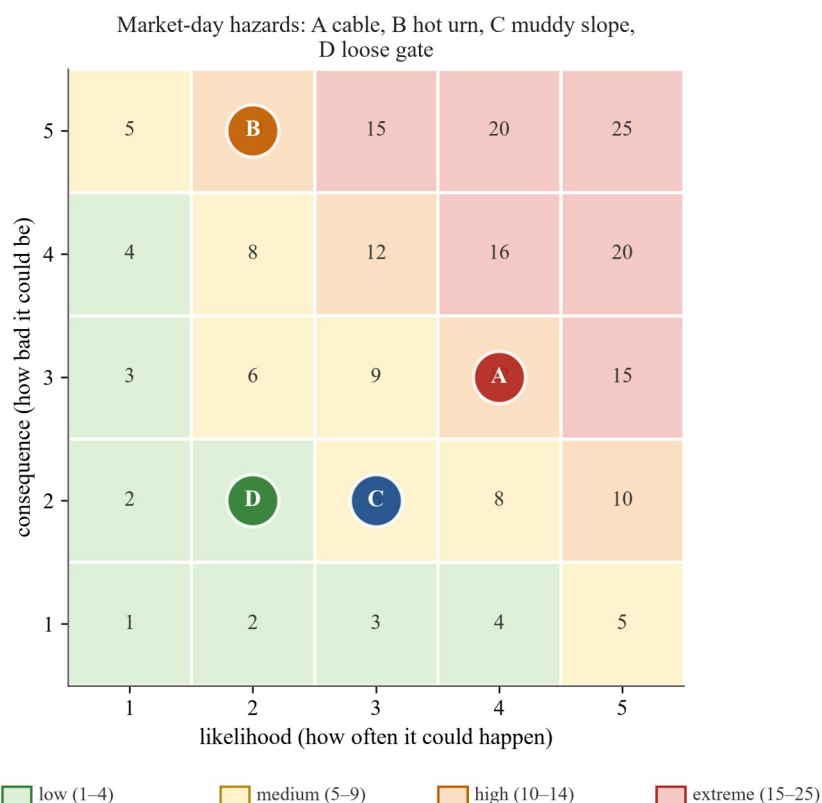
Q14. A house in a flood plain has a 1 % chance of flooding in any year. The owner says “one percent means it will never happen to us, so there is no point preparing.” (a) Write 1 % as a decimal, and as a “1 in how many” statement. (b) On a street of 100 similar houses, how many floods would you expect in one year? (c) Use your answers to say whether the owner’s claim is sound.

Q15. A farmer must decide whether to cut hay today or wait. The chance of rain tomorrow is 0.6. If rain falls on cut hay the damage is \$3 000. Cutting today means paying \$900 in overtime. (a) Find the expected cost of the rain damage if the farmer waits. (b) Compare that expected cost with the \$900 overtime. (c) State the farmer’s better choice on these numbers, with a reason.

Q16. A worker compares two job offers. Job 1, roof work: likelihood 3, consequence 4. Job 2, office filing: likelihood 2, consequence 1. (a) Find the risk score for each job. (b) Name the zone for each job. (c) Which job needs more safety controls in place before the worker starts? Give a reason using the scores.

Q17. A market garden’s greenhouse is hit by about 8 strong storms a year. In each storm the probability of wind damage is 0.15, and a damage event costs \$1 200 to fix. Bracing the frame costs \$1 500 once and would drop the probability of damage per storm to 0.05. (a) Find the expected cost of wind damage for one year without bracing. (b) Find the expected cost for one year with bracing, and the saving per year. (c) Find how many years of saving it takes to pay for the bracing, to one decimal place.

Q18. A community market plots four hazards on this risk matrix: A, a power cable run across a walking path; B, a hot water urn at the edge of a stall; C, a muddy slope to the toilet block; D, a loose gate latch at the entry.



Made-up data: the market and all four hazards in this picture are made up for this section.

- Find the risk score for each of A, B, C and D.
- How many of the four hazards sit in the high or extreme zone?
- The committee can fix two hazards before market day. Which two, and why?

Answers

Q1. (a) 0.3. (b) 2%. (c) $\frac{3}{20}$. **Q2.** (a) level 5. (b) level 1. (c) level 4. **Q3.** (a) level 2. (b) level 3. (c) level 5. **Q4.** (a) 6, medium. (b) 5, medium. (c) 16, extreme. **Q5.** (a) 6. (b) medium. (c) 4, low. **Q6.** (a) 12, high. (b) 4, low. (c) hazard A. **Q7.** (a) level 4. (b) level 3. (c) 12, high. **Q8.** (a) 4%. (b) 10 late deliveries. (c) an expected number is an average over many runs, not a count of what will happen this time. **Q9.** (a) \$19.50. (b) the expected cost of \$19.50 is cheaper than the \$25 plan. (c) for example, they could not afford a sudden \$650 repair even though it is unlikely. **Q10.** (a) P likelihood 3, consequence 3; Q likelihood 1, consequence 5; R likelihood 4, consequence 2. (b) P 9 medium, Q 5 medium, R 8 medium. (c) P, with the highest score of 9. **Q11.** (a) 12, high. (b) 6, medium. (c) the likelihood level, down 2 levels (from 4 to 2). **Q12.** (a) 10, high. (b) 6, medium. (c) the consequence level; a helmet softens the injury if a crash happens but does nothing to stop crashes happening. **Q13.** (a) A: $5 \times 2 = 10$; B: $2 \times 5 = 10$. (b) high. (c) no — B's consequence of 5 could kill someone in one event, while A's consequence of 2 only bruises, so B deserves the stricter controls even at the same score. **Q14.** (a) 0.01; 1 in 100. (b) 1 house. (c) not sound — about one house on the street floods each year, so the risk is real for somebody every year. **Q15.** (a) \$1 800. (b) \$1 800 is twice the \$900 overtime. (c) cut today — the expected loss from waiting is bigger than the certain cost of the overtime. **Q16.** (a) Job 1: 12; Job 2: 2. (b) Job 1 high, Job 2 low. (c) Job 1 — its score of 12 is in the high zone, so harnesses, edge protection and training must be in place before starting. **Q17.** (a) \$1 440. (b) \$480; saving \$960 a year. (c) 1.6 years (to one decimal place). **Q18.** (a) A 12, B 10, C 6, D 4. (b) 2 hazards (A and B). (c) A and B — they are the two in the high zone; a trip over the cable and a burn from the urn both outrank the muddy slope and the latch.

Fully-worked solutions

Q1. (a) $30\% = 30 \div 100 = 0.3$. (b) $0.02 = 0.02 \times 100 = 2\%$. (c) $15\% = \frac{15}{100} = \frac{3}{20}$ after dividing top and bottom by 5.

Q2. (a) 90% is over 80%, so level 5, almost certain. (b) 3% is under 5%, so level 1, rare. (c) 65% sits in the 50%–80% band, so level 4, likely.

Q3. (a) \$50 is under \$100 with no injury, so level 2, slight. (b) stitches at the doctor is an injury needing a doctor, so level 3, moderate. (c) \$15 000 is over \$10 000, so level 5, catastrophic.

Q4. (a) $2 \times 3 = 6$; 6 is in 5–9, so medium. (b) $5 \times 1 = 5$; 5 is in 5–9, so medium. (c) $4 \times 4 = 16$; 16 is in 15–25, so extreme.

Q5. (a) $3 \times 2 = 6$. (b) 6 is in 5–9, so medium. (c) With likelihood 2 and consequence still 2, the new score is $2 \times 2 = 4$, which is in 1–4, so low — the guard moved the hazard down one zone.

Q6. (a) Hazard A: $4 \times 3 = 12$, which is in 10–14, so high. (b) Hazard B: $2 \times 2 = 4$, which is in 1–4, so low. (c) Hazard A, because a high-zone risk is fixed before a low-zone one.

Q7. (a) 70% sits in the 50%–80% band, so likelihood level 4. (b) A broken arm needing a doctor is an injury needing a doctor, so consequence level 3. (c) $4 \times 3 = 12$, which is in 10–14, so the zone is high.

Q8. (a) $0.04 \times 100 = 4\%$. (b) Expect $0.04 \times 250 = 10$ late deliveries. (c) The 10 is an average over many blocks of 250 deliveries; any one block could have 6 late or 14 late, because each delivery is a chance event.

Q9. (a) Expected cost = $0.03 \times 650 = \$19.50$. (b) \$19.50 is less than \$25, so on money alone the expected cost is cheaper than the plan (by \$5.50 a year). (c) A student on a tight budget might not be able to meet a sudden \$650 repair at all, so paying \$25 to remove that worry can still be the better personal choice.

Q10. (a) From the matrix: P is at likelihood 3, consequence 3; Q is at likelihood 1, consequence 5; R is at likelihood 4, consequence 2. (b) P: $3 \times 3 = 9$, medium. Q: $1 \times 5 = 5$, medium. R: $4 \times 2 = 8$, medium. (c) P first: its score of 9 is the highest of the three, even though all three sit in the medium zone.

Q11. (a) Before: $4 \times 3 = 12$, which is in 10–14, so high. (b) After: $2 \times 3 = 6$, which is in 5–9, so medium. (c) The likelihood level changed, falling 2 levels from 4 to 2; the consequence of a fall stayed the same, because training does not make a fall softer.

Q12. (a) Without the helmet: $2 \times 5 = 10$, which is in 10–14, so high. (b) With the helmet: $2 \times 3 = 6$, which is in 5–9, so medium. (c) The helmet changed the consequence level, from 5 to 3. It could not change the likelihood, because a helmet does nothing about how often crashes happen — it only makes the injury less severe when one does.

Q13. (a) Hazard A: $5 \times 2 = 10$. Hazard B: $2 \times 5 = 10$. Both scores are 10. (b) 10 is in 10–14, so both sit in the high zone. (c) No. Hazard A happens many times and hurts a little each time, so guards and cleaning fix it; hazard B happens rarely but its consequence of 5 could kill someone in a single event, so the warehouse should give B the stricter controls — checks, barriers and a rule that nobody stands under a raised crate — even though the two scores match.

Q14. (a) $1\% = 1 \div 100 = 0.01$, and 0.01 is 1 in 100. (b) Expect $0.01 \times 100 = 1$ flood on the street in a year. (c) The claim is not sound. One flood a year on a street of 100 houses means the event is real and recurring, not never: each year some house takes the damage, and the 1% gives no way to know which. Preparing spreads or cuts that one house's loss, which is exactly what a small-but-real likelihood is for.

Q15. (a) Expected damage if the farmer waits $= 0.6 \times 3\,000 = \$1\,800$. (b) \$1 800 is double the \$900 the overtime costs, so waiting carries the bigger expected loss. (c) Cut today. The certain \$900 is cheaper than the \$1 800 expected loss from waiting, so on these numbers the overtime is the better buy.

Q16. (a) Job 1: $3 \times 4 = 12$. Job 2: $2 \times 1 = 2$. (b) 12 is in 10–14, so Job 1 is high; 2 is in 1–4, so Job 2 is low. (c) Job 1. A high-zone score of 12 says the work cannot start until controls — harnesses, edge protection, weather rules and training — are in place, while Job 2's low score of 2 needs only everyday care.

Q17. (a) Without bracing: $0.15 \times 8 = 1.2$ damage events expected a year, times \$1 200 each, so $0.15 \times 8 \times 1\,200 = \$1\,440$. (b) With bracing: $0.05 \times 8 \times 1\,200 = \$480$ a year, a saving of $1\,440 - 480 = \$960$ a year. (c) The bracing costs \$1 500 once, so the payback time is $1\,500 \div 960 = 1.5625$ years, which is 1.6 years to one decimal place.

Q18. (a) A: $4 \times 3 = 12$. B: $2 \times 5 = 10$. C: $3 \times 2 = 6$. D: $2 \times 2 = 4$. (b) Two hazards — A (12) and B (10) — sit in the high zone; none reach extreme. (c) Fix A and B. They are the only two hazards in the high zone, and between them they cover the two ways a risk gets big: A is likely with a moderate consequence, while B is unlikely but could scald someone badly. C (6) and D (4) can wait for mats and a new latch after market day.

Chapter 09 — Test A (multiple choice)

Chapter 9 (Uncertainty), Sections 9A–9D. Materials: pen, paper and a basic calculator. Section A: 12 multiple-choice questions (12 marks). Approximate time: 25 minutes.

About the data in this test. Every business, team, person and number in this test is **made up** (not real). They are built for this book so you can practise the maths of Chapter 9. Nothing here is real data about a real person, business or place.

Section A — Multiple-choice questions

Choose the response that is correct for the question.

Question 1

A weather app gives the probability of frost tonight as 0.07. On the chance-scale words of Section 9A, this probability is best described as

- A. very unlikely
- B. unlikely
- C. an even chance
- D. likely

Question 2

Written as a percentage, $\frac{3}{8}$ is

- A. 3.75 %
- B. 37.5 %
- C. 38 %
- D. 87.5 %

Question 3

A delivery van was late on 9 of the last 40 working days. Using this record, the probability that the van is on time on the next working day, written as a percentage, is

- A. 9 %
- B. 77.5 %
- C. 22.5 %
- D. 78 %

Question 4

Two fair coins are tossed together. The probability of getting exactly one head is

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Question 5

Two ordinary six-sided dice are rolled together and their scores are added. The probability that the sum is 9 is

A. $\frac{1}{9}$

B. $\frac{1}{6}$

C. $\frac{1}{36}$

D. $\frac{1}{4}$

Question 6

A bag holds 9 red balls and 1 blue ball, all the same size and weight. One ball is drawn at random. The probability that the ball is red is

A. $\frac{1}{2}$

B. $\frac{1}{10}$

C. $\frac{1}{9}$

D. $\frac{9}{10}$

Question 7

Four raffles advertise their chances of winning in different forms: 0.12, “1 in 8”, 12.4 % and $\frac{1}{9}$. Which of these is the biggest chance?

- A. 0.12
- B. 1 in 8
- C. 12.4 %
- D. $\frac{1}{9}$

Question 8

In one month, the Flower Pot nursery sold 80 pots and 12 of them cracked, while Garden Gear sold 150 pots and 18 of them cracked. Which statement is correct?

- A. Flower Pot is the safer buy, because its crack rate is 12 %.
- B. Flower Pot is the safer buy, because only 12 pots cracked out of 80, against 18 out of 150 at Garden Gear.
- C. Garden Gear is the safer buy, because its crack rate is 12 %, against Flower Pot’s 15 %.
- D. Garden Gear is the safer buy, because its crack rate is 15 %, against Flower Pot’s 12 %.

Question 9

A market-stall game claims “1 in 5 scoops wins a prize”. If the claim is true, the number of winning scoops expected in 200 scoops is

- A. 20
- B. 50
- C. 100
- D. 40

Question 10

A supervisor estimates the probability that a machine jams in any one week as 0.65. On the five-level likelihood scale of Section 9D, this probability sits at

- A. level 2 (unlikely)
- B. level 3 (possible)
- C. level 5 (almost certain)
- D. level 4 (likely)

Question 11

A workplace scores a hazard at likelihood level 4 and consequence level 3. The risk score and its zone are

- A. 12, high
- B. 7, medium
- C. 12, medium
- D. 12, extreme

Question 12

A machine jams with probability 0.04 on any one day. Written as a “1 in how many” statement, this probability is

- A. 1 in 4
- B. 1 in 40
- C. 1 in 25
- D. 1 in 100

End of Section A

Test A — Answers

Question	Answer
1	A
2	B
3	B
4	C
5	A
6	D
7	B
8	C
9	D
10	D
11	A
12	C

Test A — Fully-worked solutions

Question 1. 0.07 is greater than 0 and less than 0.1, so it sits in the very-unlikely band of the chance scale. The answer is **A**.

Question 2. $\frac{3}{8} = 3 \div 8 = 0.375$, and $0.375 \times 100 = 37.5\%$ — exact, so no rounding is needed. The answer is **B**.

Question 3. On-time days in the record: $40 - 9 = 31$, so the estimate is $P(\text{on time}) = \frac{31}{40} = 31 \div 40 = 0.775 = 77.5\%$, exact. Option C, 22.5%, is the late rate, which is not what was asked. The answer is **B**.

Question 4. Two coins give 4 equally likely outcomes: HH, HT, TH, TT. Exactly one head happens on HT and TH — 2 of the 4 — so $P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2}$. Option B, $\frac{1}{3}$, comes from the wrong list “two heads, two tails, one of each”, whose three results are not equally likely. The answer is **C**.

Question 5. Two dice give $6 \times 6 = 36$ equally likely outcomes. The ways to make a sum of 9 are (3, 6), (4, 5), (5, 4) and (6, 3) — 4 outcomes — so $P(\text{sum } 9) = \frac{4}{36} = \frac{1}{9}$. The answer is **A**.

Question 6. The equally likely outcomes are the 10 individual balls, and 9 of them are red, so $P(\text{red}) = \frac{9}{10}$. Two colours do not mean $\frac{1}{2}$ each — the red balls outnumber the blue ball nine to one. The answer is **D**.

Question 7. Put all four chances in the same form. As decimals: 0.12 stays as it is; 1 in 8 = $1 \div 8 = 0.125$; $12.4\% = 0.124$; $\frac{1}{9} = 0.1111\dots$. Line them up: $0.1111\dots < 0.12 < 0.124 < 0.125$, so the biggest chance is 1 in 8. The answer is **B**.

Question 8. Compare rates, not raw counts, because the two records hold different numbers of pots. Flower Pot: $12 \div 80 = 0.15 = 15\%$ cracked. Garden Gear: $18 \div 150 = 0.12 = 12\%$ cracked. Since $12\% < 15\%$, Garden Gear is the safer buy, and only option C states both rates correctly. The answer is **C**.

Question 9. 1 in 5 = $\frac{1}{5} = 0.2$. Expected number = probability $\times$ number of scoops = $0.2 \times 200 = 40$ winning scoops. The answer is **D**.

Question 10. $0.65 = 65\%$, which sits in the 50%–80% band of the likelihood scale: level 4, likely. The answer is **D**.

Question 11. Risk score = likelihood level $\times$ consequence level = $4 \times 3 = 12$. A score of 12 sits in 10–14, which is the high zone. Option B, 7, comes from adding the levels, which is not how the score works. The answer is **A**.

Question 12. $0.04 = \frac{4}{100} = \frac{1}{25}$, which reads as “1 in 25” — about one jam every 25 days. The answer is **C**.

Chapter 09 — Test B (written response)

Chapter 9 (Uncertainty), Sections 9A–9D. Materials: pen, paper and a basic calculator. Section B: 7 questions (35 marks). Approximate time: 40 minutes. Answer every question in the space provided. Round only where a question tells you to, and say how far you rounded. Show your working wherever more than one mark is available.

About the data in this test. Every business, team, person and number in this test is **made up** (not real). They are built for this book so you can practise the maths of Chapter 9. Nothing here is real data about a real person, business or place.

Section B

Question 1 (5 marks)

A hockey club sells 120 raffle tickets for its season-launch draw. You hold 6 of the 120 tickets, and one ticket is drawn at random to win the prize.

a. Find $P(\text{you win})$ as a fraction in lowest terms. 1 mark

b. Write your fraction from (a) as a decimal and as a percentage. 2 marks

c. Find $P(\text{you do not win})$ as a percentage. 1 mark

d. Use one word from the chance scale to describe $P(\text{you win})$. 1 mark

Question 2 (5 marks)

A bakery checks its records and finds its delivery van was late on 15 of the last 60 runs.

a. Estimate $P(\text{the van is late on a run})$ as a percentage. 1 mark

b. Estimate $P(\text{the van is on time})$ as a percentage. 1 mark

c. At this rate, how many late runs would you expect over the next 48 runs? 2 marks

d. Explain in one sentence why your answer to (c) is a prediction, not a promise. 1 mark

Question 3 (5 marks)

Eleven tiles are placed in a bag, one for each letter of the word APPRENTICES. The bag is shaken and one tile is drawn at random. (The vowels are A, E, I, O and U.)

a. How many equally likely outcomes are there? 1 mark

b. Find $P(\text{the letter E})$ as a fraction. 1 mark

c. Find $P(\text{a vowel})$ as a fraction, and as a decimal correct to two decimal places. 2 marks

d. Find $P(\text{not a vowel})$ as a fraction. 1 mark

Question 4 (5 marks)

Two ordinary six-sided dice are rolled together and their scores are added.

a. How many equally likely outcomes are there? 1 mark

b. Find $P(\text{the sum is 10})$ as a fraction in lowest terms. 2 marks

c. Find $P(\text{the sum is less than 5})$ as a fraction in lowest terms. 1 mark

d. Find $P(\text{the sum is not 7})$ as a fraction in lowest terms. 1 mark

Question 5 (5 marks)

Two phone-repair shops kept one month of records. FixFast repaired 200 phones and 14 came back with a problem. ScreenCare repaired 150 phones and 9 came back with a problem.

- a.** Write each shop's come-back rate as a percentage. 2 marks

- b.** Which shop is the safer choice? Use both rates in your answer. 1 mark

- c.** At ScreenCare's rate, how many come-backs would you expect from its next 400 repairs? 1 mark

- d.** FixFast advertises: "97 % of our repairs need no come-back." Does FixFast's record back this ad? Show one calculation. 1 mark

Question 6 (5 marks)

A warehouse scores the hazard "pallets stacked too high beside the packing bench" at likelihood level 4 and consequence level 3. A hazard is anything in a workplace that could cause harm or loss.

- a.** Find the risk score and name its zone. 2 marks

- b.** The warehouse brings in stacking training and a height rule, which drop the likelihood to level 2. Find the new score and name the new zone. 2 marks

- c.** Which level did the training change? Explain in one sentence why it could not change the other level. 1 mark

Question 7 (5 marks)

The probability that a laptop suffers accidental damage in any one year is 0.05. A repair costs \$560. A damage plan costs \$35 a year and covers every repair.

a. Find the expected cost of damage for one year.

2 marks

b. On money alone, is the plan worth buying for one year? Use your number from (a) in your answer.

2 marks

c. Give one reason a person might still buy the plan.

1 mark

End of Section B

Test B — Answers

Question 1. (a) $\frac{1}{20}$. (b) 0.05; 5 %. (c) 95 %. (d) very unlikely. **Question 2.** (a) 25 %. (b) 75 %. (c) 12 late runs. (d) the expected number is a long-run average — the actual count could be a little higher or lower. **Question 3.** (a) 11. (b) $\frac{2}{11}$. (c) $\frac{4}{11}$; 0.36 (to two decimal places). (d) $\frac{7}{11}$. **Question 4.** (a) 36. (b) $\frac{1}{12}$. (c) $\frac{1}{6}$. (d) $\frac{5}{6}$. **Question 5.** (a) FixFast 7 %; ScreenCare 6 %. (b) ScreenCare — $6\% < 7\%$. (c) 24 come-backs. (d) no — the record gives $100\% - 7\% = 93\%$, below the claimed 97 %. **Question 6.** (a) 12, high. (b) 6, medium. (c) the likelihood level — the training cuts how often pallets fall, but a falling pallet hurts the same, so the consequence level stands. **Question 7.** (a) \$28. (b) no — the expected cost of \$28 is below the \$35 plan. (c) any one sound reason, e.g. a sudden \$560 repair could be unaffordable, and the plan removes that worry.

Test B — Fully-worked solutions

Question 1.

a. The 120 tickets are equally likely outcomes and 6 of them are yours, so $P(\text{win}) = \frac{6}{120}$. Dividing top and bottom by 6 gives $\frac{1}{20}$.

b. $\frac{1}{20} = 1 \div 20 = 0.05$. As a percentage, $0.05 \times 100 = 5\%$. Both forms are exact, so no rounding is needed.

c. Winning and not winning add to 1: $P(\text{not win}) = 1 - \frac{1}{20} = \frac{19}{20} = 0.95 = 95\%$. Check by counting: the other $120 - 6 = 114$ tickets give $\frac{114}{120} = \frac{19}{20}$ ✓

d. 5% is between 0% and 10%, so on the chance scale winning is **very unlikely**.

Question 2.

a. The record is data, so estimate by dividing: $P(\text{late}) = \frac{15}{60} = 15 \div 60 = 0.25 = 25\%$.

b. Late and on time add to 1: $P(\text{on time}) = 1 - 0.25 = 0.75 = 75\%$. Check by counting: $60 - 15 = 45$ on-time runs, and $\frac{45}{60} = 0.75$ ✓

c. Expected number = probability $\times$ number of runs $= 0.25 \times 48 = 12$ late runs. This is already a whole number, so no rounding is needed.

d. The 12 is what the record predicts on average over many blocks of 48 runs; the actual count in any one block could be a little higher or lower, because each run is a chance event.

Question 3.

a. One tile per letter, and APPRENTICES has 11 letters, so there are 11 equally likely outcomes.

b. The letter E appears twice in APPRENTICES, so $P(E) = \frac{2}{11}$.

c. The vowel tiles are A, E, I and E — 4 tiles — so $P(\text{vowel}) = \frac{4}{11}$. As a decimal, $\frac{4}{11} = 0.3636\dots$, which is 0.36 correct to two decimal places.

d. Non-vowel tiles: $11 - 4 = 7$, so $P(\text{not a vowel}) = \frac{7}{11}$. Check: $\frac{4}{11} + \frac{7}{11} = 1$ ✓

Question 4.

- a. Each of the 6 faces of the first die pairs with each of the 6 faces of the second, so there are $6 \times 6 = 36$ equally likely outcomes.
- b. The ways to make a sum of 10 are $(4, 6)$, $(5, 5)$ and $(6, 4)$ — 3 outcomes — so $P(\text{sum } 10) = \frac{3}{36} = \frac{1}{12}$.
- c. Sums less than 5 are 2, 3 and 4. There is 1 way to make 2, 2 ways to make 3 and 3 ways to make 4, so $1 + 2 + 3 = 6$ outcomes and $P(\text{sum} < 5) = \frac{6}{36} = \frac{1}{6}$.
- d. A sum of 7 has 6 ways — $(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, $(6, 1)$ — so $P(\text{sum } 7) = \frac{6}{36} = \frac{1}{6}$. The complement gives $P(\text{not } 7) = 1 - \frac{1}{6} = \frac{5}{6}$.

Question 5.

- a. FixFast: $14 \div 200 = 0.07 = 7\%$. ScreenCare: $9 \div 150 = 0.06 = 6\%$. Both are exact.
- b. ScreenCare: its come-back rate of 6% is below FixFast's 7%. The raw counts (14 against 9) cannot decide it — the shops did different numbers of repairs, so only the rates compare fairly.
- c. Expected number = probability $\times$ number of repairs = $0.06 \times 400 = 24$ come-backs.
- d. No. FixFast's record says 7% of repairs came back, so the no-come-back side is $100\% - 7\% = 93\%$. The record's 93% sits below the claimed 97%, so the record does not back the ad.

Question 6.

- a. Risk score = likelihood level $\times$ consequence level = $4 \times 3 = 12$. A score of 12 sits in 10–14, so the zone is **high**.
- b. The likelihood is now 2 and the consequence is still 3, so the new score is $2 \times 3 = 6$. A score of 6 sits in 5–9, so the new zone is **medium**.
- c. The training changed the **likelihood** level, from 4 to 2. It could not change the consequence level: a pallet that falls hurts just as much however often falls happen — training cuts how often the accident occurs, not how bad it is.

Question 7.

- a. Expected cost = probability $\times$ cost if it happens = $0.05 \times 560 = \$28$ a year.
- b. The plan costs \$35 a year, and \$28 is less than \$35, so on money alone the plan is **not** worth buying — in an average year it costs \$7 more than the damage it covers.
- c. Any one sound reason; for example, a sudden \$560 repair bill could be unaffordable even though it is unlikely, so paying \$35 a year to remove that worry can still be the better personal choice.