

VCE VM Numeracy Units 3 & 4

Chapter 7 — Dimension and direction

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Contents

Section	Page
7A Location, landmarks and space	2
7B Maps, compass directions and bearings	17
7C Giving and following directions	31
7D Angles in degrees	48
Chapter 07 — Test A (multiple choice)	60
Test A — Answers	63
Test A — Fully-worked solutions	64
Chapter 07 — Test B (written response)	65
Test B — Answers	69
Test B — Fully-worked solutions	70

Chapter 7 Dimension and direction — 7A Location, landmarks and space

Theory

The study design describes what this section builds (Unit 4, Area of Study 5): working out where an object is in one, two or three dimensions — along a line, on a flat surface or in space — and using the right words to say where it is and how it moves.

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-07.

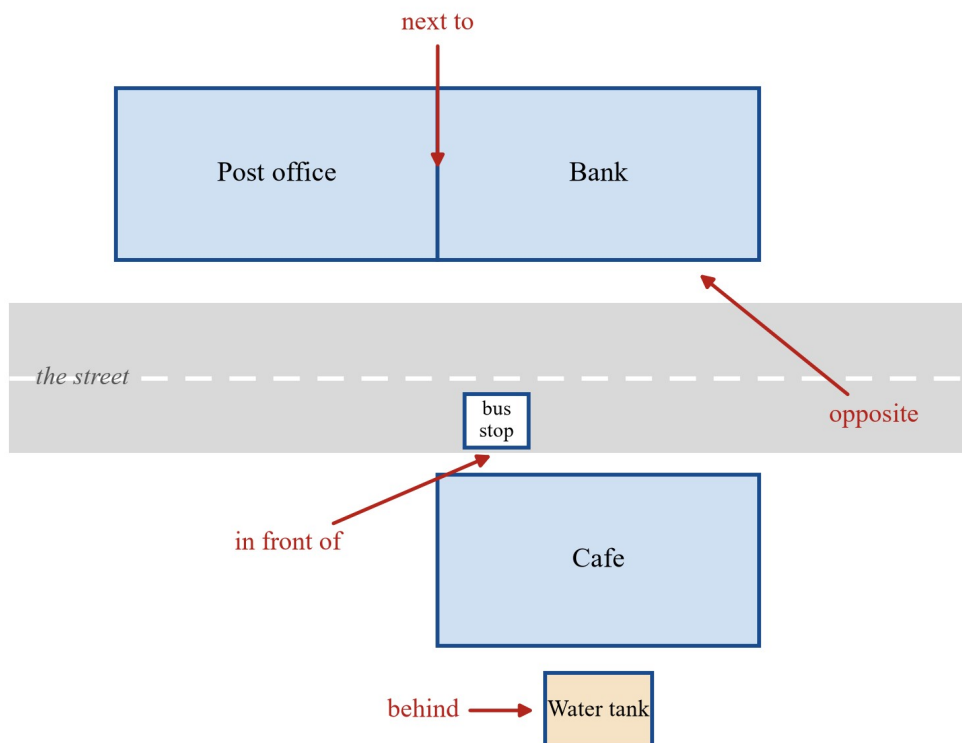
About the places in this section. The street, the town of Aldervale, the rail lines, the warehouse, the farm, the hall and the fete ground in this section are **made up** (not real). They are drawn to be easy to read, so you can practise the ideas instead of memorising a real map.

Where things are: location and landmarks.

A **landmark** is anything fixed and recognisable that you can place other things against: a station, a shop, a water tank, a gate. To describe a **location** means to say where something is *in relation to* a landmark. A small set of position words does most of that work:

Word	What it means
next to	side by side with, sharing a side
opposite	across from, facing over a space such as a street
in front of	on the side the thing faces
behind	on the side the thing faces away from
between	in the space separating two things

Describing location with landmarks (plan view)



Read against that plan, each of these is a complete location:

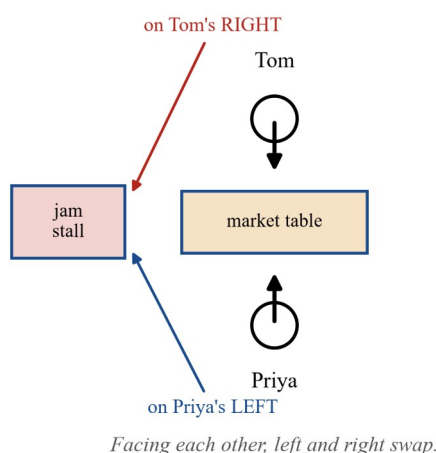
- the bank is **next to** the post office;
- the cafe is **opposite** the bank, across the street;
- the bus stop is **in front of** the cafe, at the street edge;
- the water tank is **behind** the cafe.

Notice that every sentence names a landmark. “Behind” on its own tells nobody anything; “behind the cafe” tells somebody exactly where to walk.

Left and right depend on which way you face.

Left and right are the trickiest position words because they belong to a *facing*, not to a place. Two people facing each other do not share a left.

Which left? Which right?

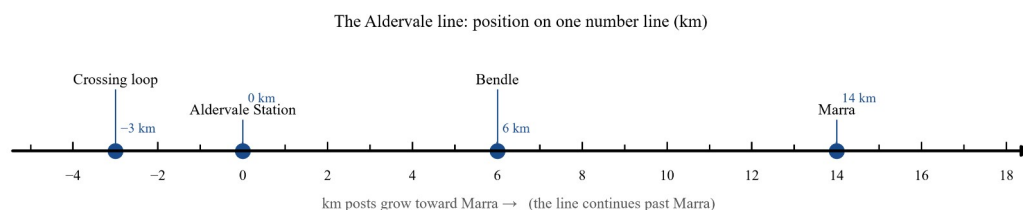


Priya and Tom face each other across the market table. The jam stall sits on Priya's **left** — and on Tom's **right**. Both are telling the truth. If they turn to face the same way, they suddenly agree: standing side by side looking at the cake stall, the jam stall at the left-hand end of the stall front is on the left of *both* of them. And if you stand facing a gate with an office on your right, then turn around to face away from the gate, the office has moved to your left without moving at all.

On a map or a grid the frame is the page: **right** means the across-number grows, **up** means the up-number grows, and left and down mean they shrink. Compass directions (north, south, east, west) come in §7B.

One dimension: position on a number line.

One number fixes a position on a line. The Aldervale rail line keeps a **km post** for every stopping place, counted from Aldervale Station:



Positions can be negative: the crossing loop sits at -3 km, on the other side of the station from Marra. Two facts carry the whole line:

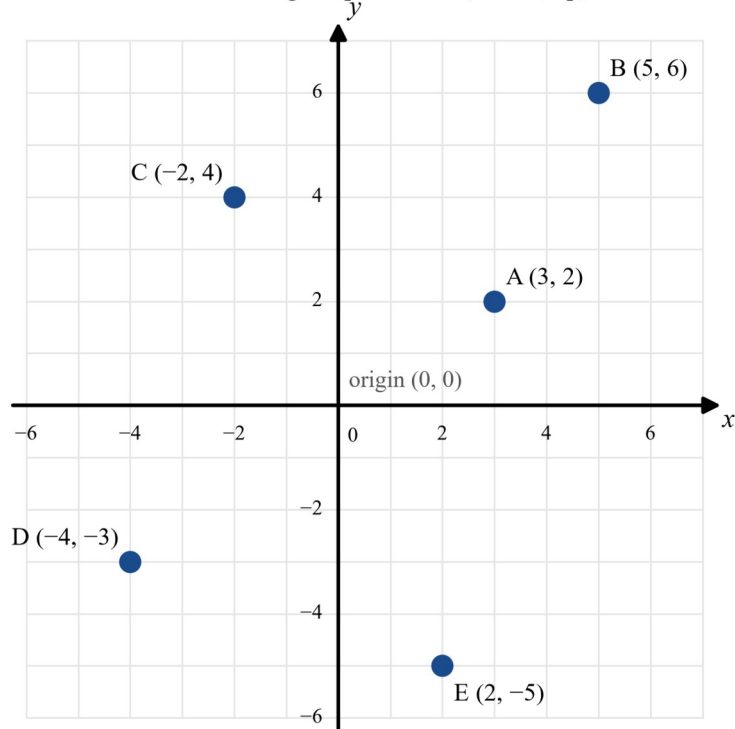
- **Distance between two positions** is the bigger position minus the smaller one. From the crossing loop to Marra is $14 - (-3) = 17$ km. Taking away a negative position adds, which is why the 17 km is longer than the 14 km from the station to Marra.
- **Movement** adds or takes away from a position. A train leaving Bendle (post 6) that runs 5 km toward Marra reaches post $6 + 5 = 11$; running 3 km back toward the station puts it at $11 - 3 = 8$.

The **halfway** point between two positions is their sum halved: halfway between posts 4 and 9 is $(4 + 9) \div 2 = 6.5$.

Two dimensions: position on a grid.

On a flat map one number is not enough, because a place can sit across *and* up. A **coordinate grid** fixes a position with an **ordered pair** (across, up), written in brackets: $(4, 3)$ means 4 across and 3 up from the origin $(0, 0)$. The order matters — $(3, 5)$ and $(5, 3)$ are different corners of the grid.

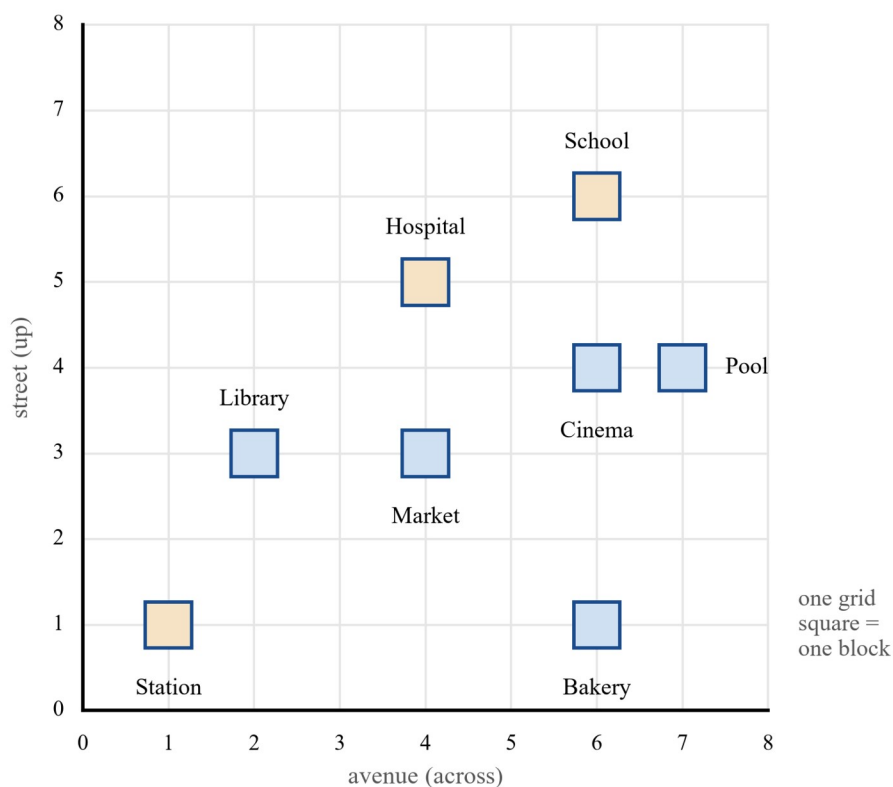
A coordinate grid: position = (across, up)



Read off the grid: A is $(3, 2)$, B is $(5, 6)$, C is $(-2, 4)$, D is $(-4, -3)$ and E is $(2, -5)$. Negative coordinates work exactly as they do on the number line: left of the origin is negative across, below the origin is negative up.

A real town map is the same idea with streets instead of grid lines. Each building in Aldervale sits at a grid corner:

Aldervale town map

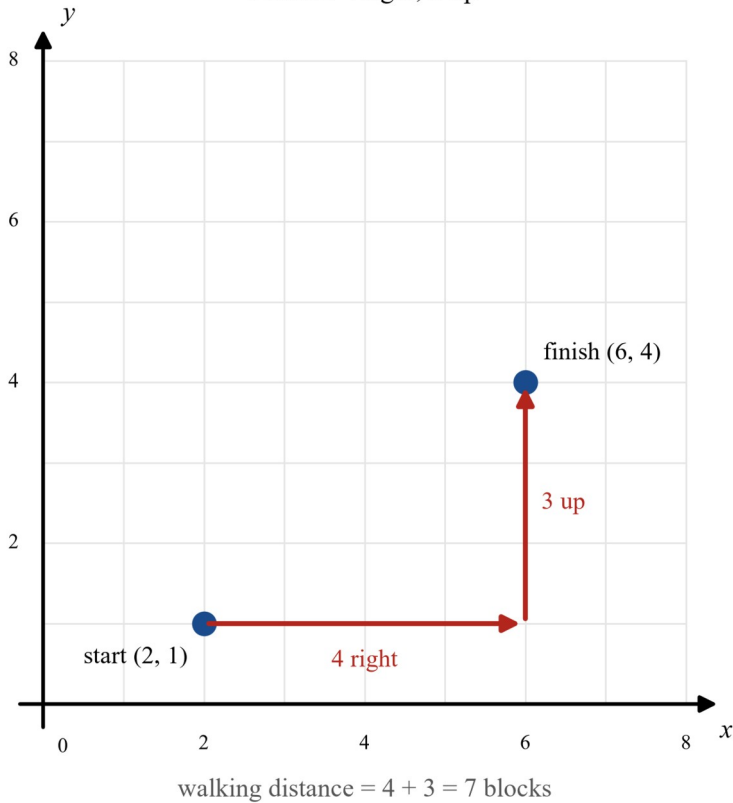


Aldervale is a made-up town; this map exists only for this section.

Movement on a grid: slides and walking distance.

A **slide** moves a position across and up without turning. Sliding 4 right and 3 up adds 4 to the across-number and 3 to the up-number:

A slide: 4 right, 3 up



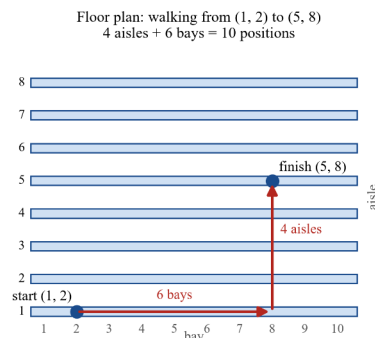
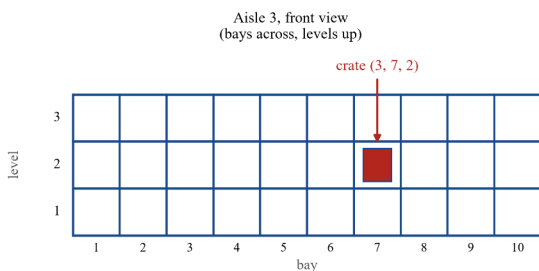
Start at $(2, 1)$: across $2 + 4 = 6$, up $1 + 3 = 4$, so the slide ends at $(6, 4)$.

One grid square of a map is called a **block**. The **walking distance** between two corners counts the blocks you actually walk — across blocks plus up-and-down blocks — because you cannot cut diagonally through a block of buildings. The slide above walks $4 + 3 = 7$ blocks even though the straight line from start to finish looks shorter. Halfway between two corners halves each number: halfway between $(2, 1)$ and $(6, 7)$ is $(4, 4)$.

Three dimensions: position with three numbers.

Add height and a position needs three numbers. The Aldervale supply warehouse records every crate as **(aisle, bay, level)**: which aisle, how far along that aisle, and how many levels up.

Warehouse positions: (aisle, bay, level)



The marked crate sits at $(3, 7, 2)$: aisle 3, bay 7, level 2. One bay along and one level up from there is $(3, 8, 3)$. The order is part of the meaning, just as on the grid: $(3, 7, 2)$ and $(2, 7, 3)$ are different crates.

Words for movement.

Position says where something is; movement says how it gets somewhere. The language this section uses:

- **slide or move** — across and up or down, as on a grid;
- **left / right / up / down** — always inside a frame: a person's facing, or the page;
- **forward / back** — along the way something is facing or running;
- **up / down a level** — the third dimension, as in a car park or warehouse;
- **walking distance** — blocks walked, counted across plus up-and-down.

Dimensions	What fixes a position	Example from this section
one	a single number on a line	km post 6, Bendle
two	an ordered pair (across, up)	(4, 3), the market
three	an ordered triple (aisle, bay, level)	(3, 7, 2), a crate

Precision.

Positions and distances in this section are exact: a crate is at bay 7, not at “about bay 7”. Where an answer is not a whole number, the question states the precision it wants — for example “to 1 decimal place” — and the answer is given to that precision and no finer. Halfway points are the usual place this comes up: halfway between 35 cm and 118 cm is exactly 76.5 cm, which is already correct to 1 decimal place.

Technology.

A GPS unit or a map app can check a position or a route, and drivers and pickers use exactly that idea in real workplaces. Nothing in this section needs one: every position here can be read off a diagram or counted on a grid.

Worked examples

Example 1 — scaffolded

Use the Aldervale rail line number line in the Theory.

- How far is it from Bendle to Marra?
- How far is it from the crossing loop to Aldervale Station?
- A maintenance vehicle starts at Bendle and runs 11 km toward Marra. The crew realise they have left a tool behind and run 4 km back toward Bendle. What km post are they at now, and which two stopping places are they between?

Working	Comment
(a) Bendle is post 6 and Marra is post 14: $14 - 6 = 8$. The distance is 8 km.	Distance on a line = bigger position – smaller position.
(b) The crossing loop is post -3 and the station is post 0: $0 - (-3) = 3$. The distance is 3 km.	Taking away a negative position adds: minus -3 is plus 3.
(c) From Bendle toward Marra: $6 + 11 = 17$. Post 17 is past Marra (14), so the vehicle overshoots. Running 4 km back: $17 - 4 = 13$. The vehicle is at post 13.	Moving toward Marra adds to the km post. Running back subtracts.
Check: 13 sits between 6 and 14, so the vehicle is between Bendle and Marra — 7 km past Bendle and 1 km this side of Marra.	A position between two stops lies between their posts.

Example 2 — scaffolded

Use the Aldervale town map in the Theory.

- Write the coordinates of the library and the bakery.
- What is the walking distance, in blocks, from the library to the bakery?
- A walker starts at the bakery and moves 2 left and 4 up. Which building do they reach?

Working	Comment
(a) Read across then up: the library is $(2, 3)$ and the bakery is $(6, 1)$.	The ordered pair is (avenue, street) = (across, up).
(b) Across: $6 - 2 = 4$ blocks. Up and down: $3 - 1 = 2$ blocks. Add the blocks: $4 + 2 = 6$ blocks of walking.	Walking counts across blocks and up-down blocks separately. No cutting diagonally through a block.
(c) From the bakery $(6, 1)$: 2 left is $6 - 2 = 4$ across; 4 up is $1 + 4 = 5$ up. The position is $(4, 5)$. $(4, 5)$ on the map is the hospital. Check: from the hospital $(4, 5)$, 2 right and 4 down returns to $(6, 1)$, the bakery.	Left subtracts from across; up adds to up. Read the building standing at that corner. Reversing the move lands back where it started.

Example 3 — unscaffolded

Positions in the Aldervale supply warehouse are ordered triples (aisle, bay, level), as in the Theory diagram. A crate is recorded at $(3, 7, 2)$.

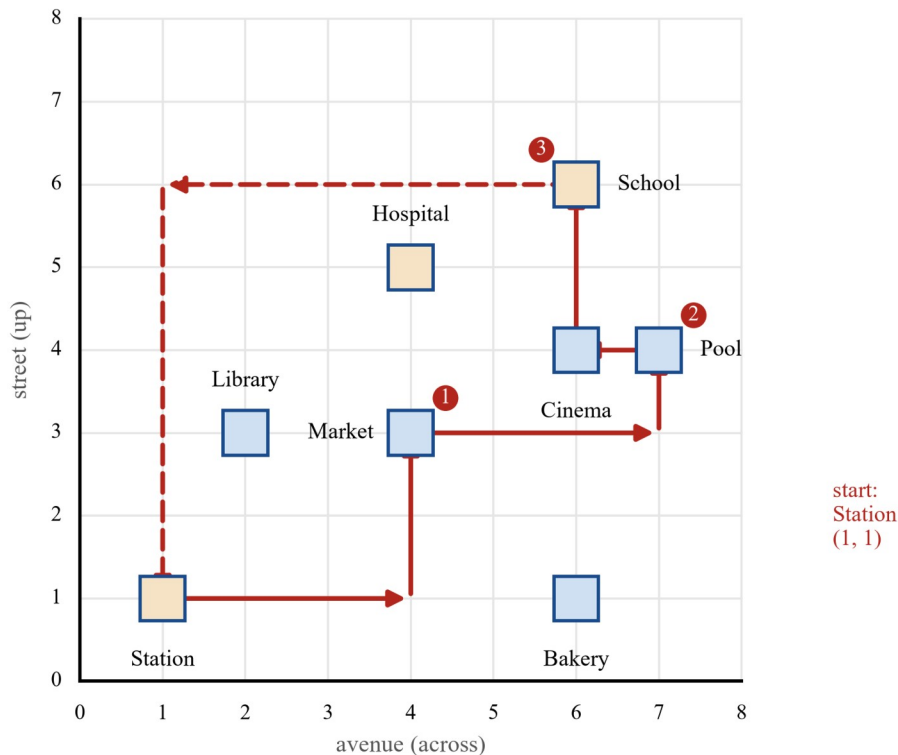
- (a) Explain in words where this crate is.
- (b) A forklift takes the crate 2 bays further along the same aisle and down 1 level. Write its new position.
- (c) On the ground level, a picker walks from $(1, 2)$ to $(5, 8)$, moving only along aisles and along bays. How many positions does she move through in all?
- (d) The crate is in aisle 3, at bay 7 along that aisle, on level 2 — the second level up from the floor.
- (e) Two bays along: $7 + 2 = 9$. Down 1 level: $2 - 1 = 1$. The aisle does not change, so the new position is $(3, 9, 1)$.
- (f) Aisles: $5 - 1 = 4$. Bays: $8 - 2 = 6$. Walking distance adds them: $4 + 6 = 10$ positions.

$\therefore$ the crate finishes at $(3, 9, 1)$, and the picker moves through 10 positions.

Example 4 — unscaffolded

Use the delivery route diagram below, which runs on the Aldervale town map from the Theory.

Delivery route: Station → stop 1 → stop 2 → stop 3 → back



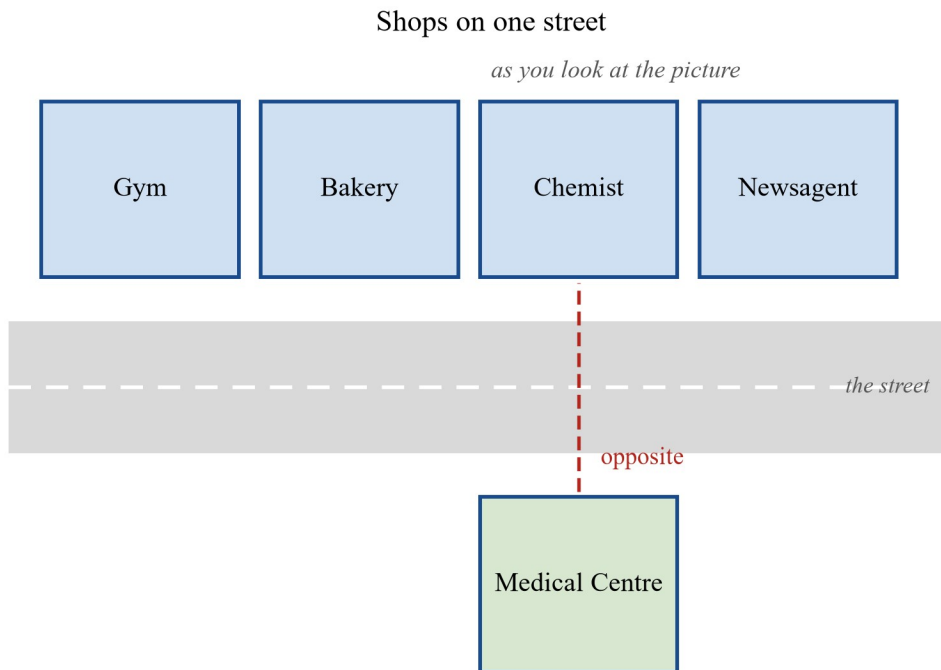
- Write each numbered stop as coordinates and name it, and give the walking distance in blocks of each leg from the stop before it.
- What is the walking distance from the school back to the station?
- How many blocks does the whole round trip cover?
- Stop 1 is $(4, 3)$, the market: from the station $(1, 1)$ that is 3 across and 2 up, $3 + 2 = 5$ blocks. Stop 2 is $(7, 4)$, the pool: 3 across and 1 up from the market, $3 + 1 = 4$ blocks. Stop 3 is $(6, 6)$, the school: 1 across and 2 up from the pool, $1 + 2 = 3$ blocks.
- School $(6, 6)$ to station $(1, 1)$: across $6 - 1 = 5$, down $6 - 1 = 5$, so $5 + 5 = 10$ blocks.
- Add the four legs: $5 + 4 + 3 + 10 = 22$ blocks.

∴ the legs run 5, 4 and 3 blocks out and 10 blocks home, 22 blocks in all.

Practice questions

Scaffolded

Q1. Use the shop strip diagram.



- (a) Which shop is between the gym and the chemist?
- (b) Which shop is next to the newsagent?
- (c) Which building is opposite the chemist, across the street?

Q2. On the Kester rail line, km posts are counted from Kester station: Redgum is at -5 , Tantarra at 9 and Wirra Hill at 17 .

- (a) How far is it from Kester to Tantarra?
- (b) How far is it from Redgum to Wirra Hill?
- (c) A train leaves Tantarra and runs 6 km toward Redgum, then 2 km back toward Tantarra. What km post is it at now?

Q3. Use the Aldervale town map in the Theory.

- (a) Write the coordinates of the pool.
- (b) Write the coordinates of the school.
- (c) Which building stands at $(6, 4)$?

Q4. Slides on a coordinate grid.

- (a) A counter at $(2, 1)$ slides 3 right and 4 up. Where is it now?
- (b) A counter at $(6, 7)$ slides 4 left and 2 down. Where is it now?
- (c) Describe the slide that takes $(1, 2)$ to $(4, 6)$, using right or left and up or down.

Q5. In the Aldervale library, a box position is an ordered triple (level, shelf, position), with positions counted from the left end of the shelf.

- (a) Explain in words where a box at $(2, 5, 12)$ is.
- (b) A box is moved from $(1, 3, 8)$ to $(1, 3, 14)$. How many positions along the shelf did it move?
- (c) A box is moved from $(2, 7, 3)$ to $(2, 2, 3)$. Which of the three numbers changed, and what did it change to?

Q6. Use the Aldervale town map in the Theory. Give each answer as a walking distance in blocks.

- (a) From the station to the hospital.
- (b) From the hospital to the pool.
- (c) From the market to the school.

Unscaffolded

Q7. A shelf is 240 cm long. Measured from the left end, a vase stands at 35 cm, a clock at 118 cm and a photo frame at 190 cm.

- (a) How far apart are the vase and the clock?
- (b) How far apart are the clock and the photo frame?
- (c) A bracket is fixed halfway between the vase mark and the clock mark. How far is the bracket from the left end of the shelf, to 1 decimal place?

Q8. A city car park numbers its levels -3 , -2 , -1 , 0 , 1 , 2 , 3 , 4 and 5 , where 0 is the street level and negative levels are underground.

- (a) How many levels is it from level -2 to level 3 ?
- (b) A car parks on level -1 . The driver takes a lift up 6 levels to an exit. What level is the exit on?
- (c) A service lift starts on level 4 and goes down 7 levels. What level does it stop on?

Q9. Use the left-and-right diagram in the Theory to help.

- (a) Priya and Tom face each other across the market table, and the jam stall sits off to one side. Priya says the jam stall is on her left. Whose right is it on?
- (b) Priya and Tom stand side by side, both facing the cake stall. The jam stall sits at the left-hand end of the cake stall front. Tom says the jam stall is on his left. Is Priya's answer the same as Tom's? Explain why.
- (c) You stand facing the front gate of a yard, and the site office is on your right. You turn around to face away from the gate. Which side of you is the site office on now?

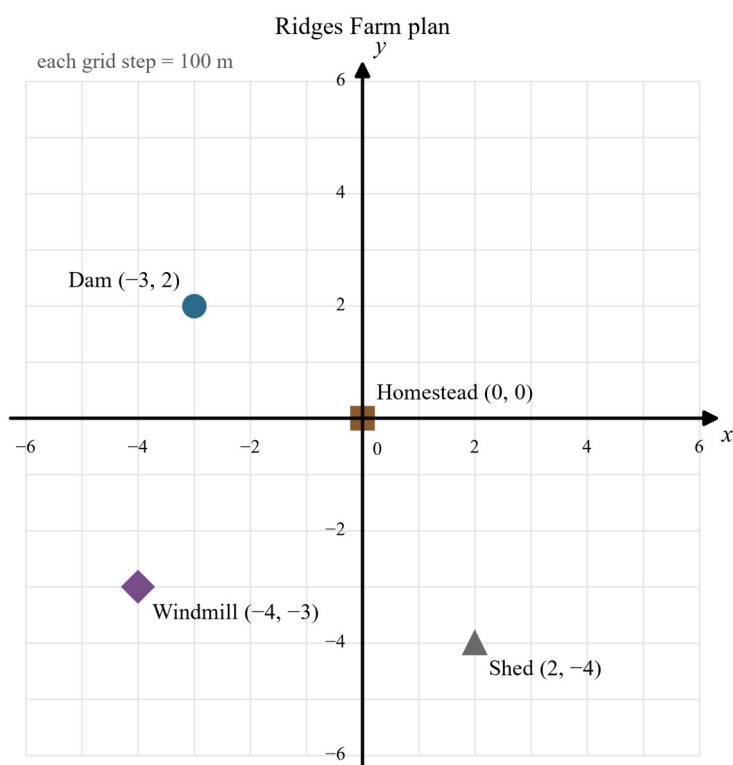
Q10. Use the Aldervale town map in the Theory.

- (a) What is the walking distance, in blocks, from the pool to the school?
- (b) Start at the station and move 5 right and 3 up. Which building do you reach? Give its coordinates too.
- (c) Start at the cinema and move 2 left and 1 up. Which building do you reach? Give its coordinates too.

Q11. In the Aldervale station car park, a space is recorded as an ordered triple (level, row, space).

- (a) Write the position of a car on level 2, row 3, space 11.
- (b) A car at $(5, 1, 4)$ is moved down 2 levels, keeping its row and space. Write its new position.
- (c) A car at $(1, 6, 20)$ is moved 8 spaces toward space 1 in the same row, then up 3 levels. Write its new position.

Q12. Use the Ridges Farm plan. Each grid step is 100 m of walking.



- Write the coordinates of the dam.
- Start at the shed and walk 5 steps left and 6 steps up. Where are you? Which farm feature is there?
- What is the walking distance from the windmill to the homestead, in steps and in metres?

Q13. Slides on a coordinate grid, with negatives.

- Describe the slide from $(2, 7)$ to $(6, 3)$ using right or left and up or down, and give its walking distance.
- Describe the slide from $(-1, 2)$ to $(3, 5)$ the same way, and give its walking distance.
- A point at $(4, 1)$ slides 2 left and 5 down. Where is it now?

Q14. Use the Aldervale Community Hall seating chart. Seats are numbered 1 to 10 along each row, rows are numbered 1 to 6 with row 1 nearest the stage, and a seat is written as the ordered pair (seat, row).

Aldervale Community Hall seating chart

	seat number									
	1	2	3	4	5	6	7	8	9	10
Row 6	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
Row 5	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
Row 4	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
Row 3	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
Row 2	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
Row 1	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

row 1 is nearest the stage

STAGE

- Write row 3, seat 8 as an ordered pair.
- In one row, how many seats apart are seat 2 and seat 9?
- In one row, how many seats are there from seat 3 to seat 8, counting both ends?

Q15. On one side of Mere Street the buildings are numbered in order: the flower shop is number 18, the hardware store is number 42 and the bookshop is number 66.

- How many numbers apart are the flower shop and the hardware store?
- How many numbers apart are the hardware store and the bookshop?
- Show that the number halfway between the flower shop and the bookshop is the hardware store's number. Write one step per line.

Q16. Positions in the Aldervale supply warehouse are ordered triples (aisle, bay, level), as in the Theory diagram.

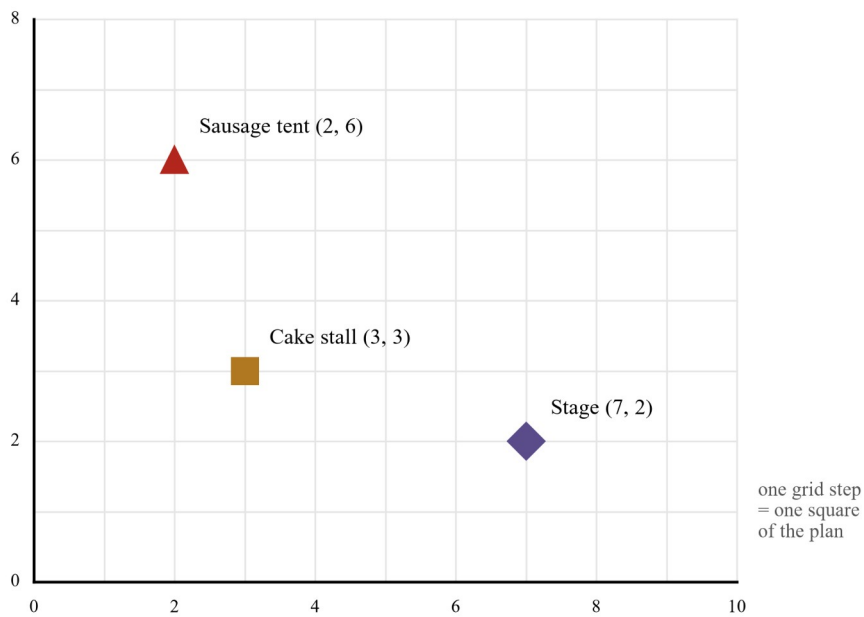
- A crate is lowered straight down from $(2, 9, 3)$ to $(2, 9, 1)$. How many levels did it move?
- On the ground level, a picker walks from $(2, 9, 1)$ to $(6, 4, 1)$, moving only along aisles and along bays. How many positions does she move through in all?
- A pick slot is 2 aisles less and 3 bays more than $(6, 4, 1)$, on the same level. Write its position.

Q17. Use the Aldervale town map in the Theory.

- Start at the station and move 1 right and 2 up. Which building do you reach? Give its coordinates too.
- From the library, move 2 right. Which building do you reach? Give its coordinates too.
- Describe the move from the bakery to the cinema using right or left and up or down.

Q18. Use the Aldervale School fete plan.

Aldervale School fete plan



- The drinks stall is 4 right and 2 down from the sausage tent. Write its coordinates.
 - What is the walking distance, in steps, from the drinks stall to the stage?
 - A volunteer walks from the cake stall to the sausage tent, then to the drinks stall. How many steps is that walk in all?
 - A first-aid point is placed halfway between the cake stall and the stage. Give its coordinates, to 1 decimal place.
-

Answers

Q1. (a) The bakery. (b) The chemist. (c) The medical centre. **Q2.** (a) 9 km. (b) 22 km. (c) Km post 5. **Q3.** (a) (7, 4). (b) (6, 6). (c) The cinema. **Q4.** (a) (5, 5). (b) (2, 5). (c) 3 right and 4 up. **Q5.** (a) Level 2, shelf 5 on that level, position 12 along that shelf from the left end. (b) 6 positions. (c) The middle number, the shelf: 7 changed to 2. **Q6.** (a) 7 blocks. (b) 4 blocks. (c) 5 blocks. **Q7.** (a) 83 cm. (b) 72 cm. (c) 76.5 cm, correct to 1 decimal place. **Q8.** (a) 5 levels. (b) Level 5. (c) Level - 3. **Q9.** (a) Tom's right. (b) Yes, the same — left — because they face the same way. (c) The left. **Q10.** (a) 3 blocks. (b) The cinema, (6, 4). (c) The hospital, (4, 5). **Q11.** (a) (2, 3, 11). (b) (3, 1, 4). (c) (4, 6, 12). **Q12.** (a) (-3, 2). (b) (-3, 2), which is the dam. (c) 7 steps, 700 m. **Q13.** (a) 4 right and 4 down; walking distance 8. (b) 4 right and 3 up; walking distance 7. (c) (2, -4). **Q14.** (a) (8, 3). (b) 7 seats. (c) 6 seats. **Q15.** (a) 24. (b) 24. (c) 42, the hardware store's number. **Q16.** (a) 2 levels. (b) 9 positions. (c) (4, 7, 1). **Q17.** (a) The library, (2, 3). (b) The market, (4, 3). (c) 0 across and 3 up — straight up the same avenue. **Q18.** (a) (6, 4). (b) 3 steps. (c) 10 steps. (d) (5.0, 2.5), correct to 1 decimal place.

Fully-worked solutions

Q1. (a) The row runs gym, bakery, chemist, newsagent, so the shop between the gym and the chemist is the bakery. (b) Next to the newsagent, on the side toward the gym, is the chemist. (c) The dashed line on the diagram crosses the street from the chemist to the medical centre, so the medical centre is opposite the chemist.

Q2. (a) $9 - 0 = 9$ km. (b) $17 - (-5) = 22$ km; taking away -5 adds 5. (c) Toward Redgum the posts shrink: $9 - 6 = 3$. Back toward Tantarra they grow: $3 + 2 = 5$. The train is at km post 5.

Q3. (a) The pool sits 7 across and 4 up: (7, 4). (b) The school sits 6 across and 6 up: (6, 6). (c) Across 6, up 4 on the map is the cinema.

Q4. (a) Across $2 + 3 = 5$, up $1 + 4 = 5$: (5, 5). (b) Across $6 - 4 = 2$, up $7 - 2 = 5$: (2, 5). (c) Across: $4 - 1 = 3$ right. Up: $6 - 2 = 4$ up. The slide is 3 right and 4 up.

Q5. (a) Level 2 of the shelves, shelf 5 on that level, and position 12 counting from the left end of that shelf. (b) The level and shelf are unchanged, so only the position moved: $14 - 8 = 6$ positions. (c) Compare (2, 7, 3) with (2, 2, 3): the first and third numbers match, so the middle number changed — the shelf, from 7 to 2. The box moved to shelf 2 on the same level and the same place along it.

Q6. (a) Station (1, 1) to hospital (4, 5): across $4 - 1 = 3$, up $5 - 1 = 4$, so $3 + 4 = 7$ blocks. (b) Hospital (4, 5) to pool (7, 4): across $7 - 4 = 3$, down $5 - 4 = 1$, so $3 + 1 = 4$ blocks. (c) Market (4, 3) to school (6, 6): across $6 - 4 = 2$, up $6 - 3 = 3$, so $2 + 3 = 5$ blocks.

Q7. (a) $118 - 35 = 83$ cm. (b) $190 - 118 = 72$ cm. (c) Halfway means the sum halved: $(35 + 118) \div 2 = 153 \div 2 = 76.5$ cm, which is already correct to 1 decimal place.

Q8. (a) $3 - (-2) = 5$ levels. (b) $-1 + 6 = 5$: level 5. (c) $4 - 7 = -3$: level -3, three levels underground.

Q9. (a) Priya and Tom face each other, so their lefts are opposite each other: the jam stall that is on Priya's left is on Tom's right. (b) Yes, Priya also says left. Left and right follow the facing, and here the two face the same way, so they agree: the jam stall at the left-hand end of the stall front is ahead-left of both of them. They only disagreed in part (a) because they faced each other. (c) Turning around swaps your left and right without moving anything else, so the site office that was on your right is now on your left.

Q10. (a) Pool (7, 4) to school (6, 6): across $7 - 6 = 1$, up $6 - 4 = 2$, so $1 + 2 = 3$ blocks. (b) Station (1, 1): across $1 + 5 = 6$, up $1 + 3 = 4$, giving (6, 4), the cinema. (c) Cinema (6, 4): across $6 - 2 = 4$, up $4 + 1 = 5$, giving (4, 5), the hospital.

Q11. (a) In the order (level, row, space): (2, 3, 11). (b) Down 2 levels: $5 - 2 = 3$, row and space unchanged: (3, 1, 4). (c) Toward space 1 the space number shrinks: $20 - 8 = 12$. Up 3 levels: $1 + 3 = 4$. The row stays 6: (4, 6, 12).

Q12. (a) The dam is 3 left and 2 up from the homestead: $(-3, 2)$. (b) Shed $(2, -4)$: across $2 - 5 = -3$, up $-4 + 6 = 2$, giving $(-3, 2)$ — which is exactly the dam. (c) Windmill $(-4, -3)$ to homestead $(0, 0)$: across $0 - (-4) = 4$, up $0 - (-3) = 3$, so $4 + 3 = 7$ steps; at 100 m a step that is $7 \times 100 = 700$ m.

Q13. (a) Across: $6 - 2 = 4$ right. Up: $3 - 7 = -4$, so 4 down. Walking distance $4 + 4 = 8$. (b) Across: $3 - (-1) = 4$ right. Up: $5 - 2 = 3$ up. Walking distance $4 + 3 = 7$. (c) Across $4 - 2 = 2$, up $1 - 5 = -4$: $(2, -4)$.

Q14. (a) The pair is (seat, row), so row 3 seat 8 is $(8, 3)$. (b) Same row: $9 - 2 = 7$ seats apart. (c) Counting both ends adds one to the gap: $8 - 3 + 1 = 6$ seats.

Q15. (a) $42 - 18 = 24$. (b) $66 - 42 = 24$. (c) One step per line:

Halfway between 18 and 66 is $(18 + 66) \div 2$.

$$18 + 66 = 84.$$

$$84 \div 2 = 42.$$

42 is the hardware store's number, as required.

Q16. (a) $3 - 1 = 2$ levels. (b) Aisles: $6 - 2 = 4$. Bays: $9 - 4 = 5$. Walking distance $4 + 5 = 9$ positions. (c) 2 aisles less: $6 - 2 = 4$. 3 bays more: $4 + 3 = 7$. Same level, 1: $(4, 7, 1)$.

Q17. (a) Station $(1, 1)$: across $1 + 1 = 2$, up $1 + 2 = 3$, giving $(2, 3)$, the library. (b) Library $(2, 3)$: across $2 + 2 = 4$, up unchanged at 3, giving $(4, 3)$, the market. (c) Bakery $(6, 1)$ to cinema $(6, 4)$: across $6 - 6 = 0$, up $4 - 1 = 3$, so no move right or left and 3 up — straight up the same avenue.

Q18. (a) Sausage tent $(2, 6)$: across $2 + 4 = 6$, up $6 - 2 = 4$, so the drinks stall is $(6, 4)$. (b) Drinks $(6, 4)$ to stage $(7, 2)$: across $7 - 6 = 1$, down $4 - 2 = 2$, so $1 + 2 = 3$ steps. (c) Cake stall $(3, 3)$ to sausage tent $(2, 6)$: $1 + 3 = 4$ steps; sausage tent $(2, 6)$ to drinks $(6, 4)$: $4 + 2 = 6$ steps; in all $4 + 6 = 10$ steps. (d) Halve each coordinate: across $(3 + 7) \div 2 = 5.0$, up $(3 + 2) \div 2 = 2.5$, so $(5.0, 2.5)$, correct to 1 decimal place.

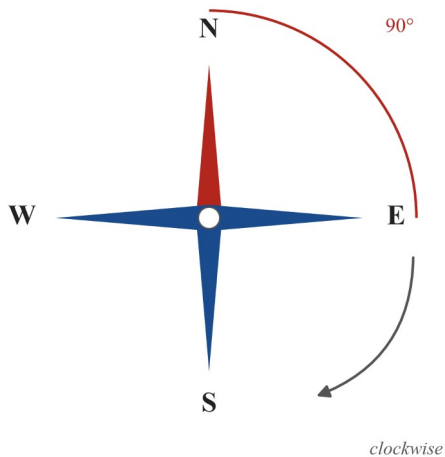
Theory

Maps and north.

A map is a picture of a place seen from above. Nearly every map is drawn with **north at the top**, and it shows this with a small arrow or star marked **N**, called the **north arrow** or **compass rose**. Before you use a paper map outside, you **set** it: turn the map round in your hands until the north arrow on the map points the same way as north on the ground. Then everything on the map lines up with everything around you, and you can read directions straight off it. A phone compass or a map app can do the setting for you, but the map itself always works with nothing but your eyes (§10D looks at the digital tools; everything in this section can be done with pen, paper and a printed map).

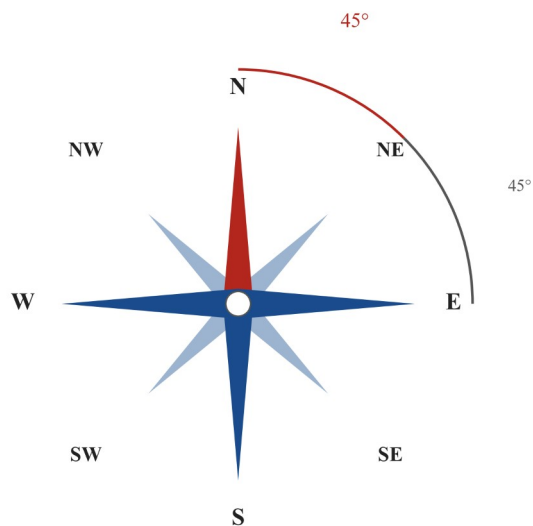
The four main directions.

Direction on a map is described with the **compass points**. The four main points are called the **cardinal directions**: **north** (N), **east** (E), **south** (S) and **west** (W). They always go in this clockwise order — N, E, S, W and back to N — and each turn from one to the next is a quarter of a full turn, which is 90° . (Angles in degrees are studied further in §7D.) An everyday check: in Australia the sun rises in the east and sets in the west.



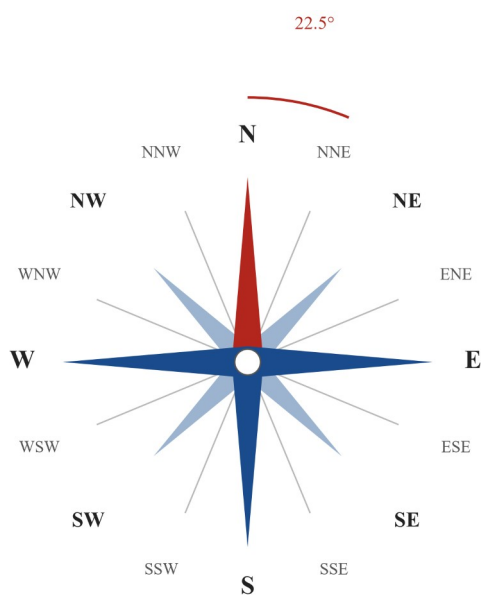
The four halfway points.

Halfway between each pair of cardinal directions is another point, named from the two cardinals it sits between: **north-east** (NE), **south-east** (SE), **south-west** (SW) and **north-west** (NW). Each of these is halfway through a 90° turn, so it is 45° from the cardinals either side of it. With the cardinals, these eight points are the **eight-point compass**, and they are enough for most everyday directions: “the shed is south-west of the house”.



The 16-point compass.

Sailors, pilots, farmers and bushwalkers often need more than eight points, so each 45° gap is split in half again. The eight new points have three-letter names: the first letter is the cardinal the point is *closer to*. For example **NNE** (north-north-east) is halfway between N and NE, and **SSW** (south-south-west) is halfway between S and SW. That gives the **16-point compass**, where neighbouring points are $360^\circ \div 16 = 22.5^\circ$ apart.

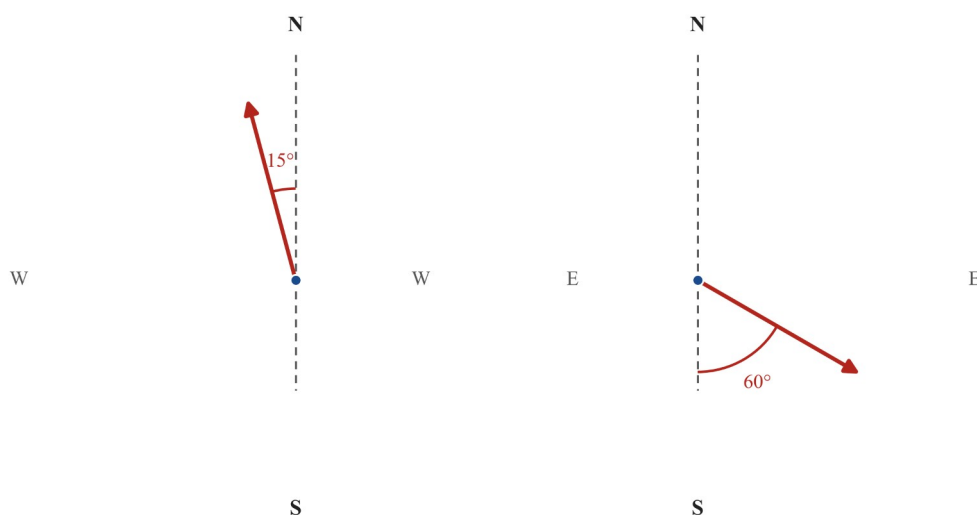


Compass point	Name	Turn clockwise from north
N	north	0°
NNE	north-north-east	22.5°
NE	north-east	45°
ENE	east-north-east	67.5°
E	east	90°
ESE	east-south-east	112.5°
SE	south-east	135°

Compass point	Name	Turn clockwise from north
SSE	south-south-east	157.5°
S	south	180°
SSW	south-south-west	202.5°
SW	south-west	225°
WSW	west-south-west	247.5°
W	west	270°
WNW	west-north-west	292.5°
NW	north-west	315°
NNW	north-north-west	337.5°

Bearings: writing a direction with degrees.

A **bearing** is a direction written with an angle. The form used in this section starts from **north or south**, then turns toward **east or west**. So $N 15^\circ W$ means “face north, then turn 15° toward the west”, and $S 60^\circ E$ means “face south, then turn 60° toward the east”. The first letter is where you start facing, the number is how far you turn, and the last letter is which way you turn.



$N 15^\circ W$ — face north, turn 15° toward the west $S 60^\circ E$ — face south, turn 60° toward the east

Bearing	What it means
$N 15^\circ W$	from north, turn 15° toward the west
$N 45^\circ E$	from north, turn 45° toward the east — the same direction as NE
$S 60^\circ E$	from south, turn 60° toward the east
$S 22.5^\circ W$	from south, turn 22.5° toward the west — the same direction as SSW

Compass points and bearings are two ways of saying the same thing. Because NE is halfway between N and E it is $N 45^\circ E$; because SSW is one 16-point step (22.5°) west of south it is $S 22.5^\circ W$. A bearing that lands between two named points, like $N 15^\circ W$, is simply finer than the 16-point compass: it sits between N and NNW, closer to NNW.

Worked model. Write NE as a bearing. NE is 45° around from north toward the east, so NE is $N 45^\circ E$.

Worked model. Which way is $N 30^\circ E$? NNE is 22.5° from north and NE is 45° from north. 30° is between those two, so $N 30^\circ E$ lies between NNE and NE — and, because $30 - 22.5 = 7.5$ while $45 - 30 = 15$, the nearest named point is NNE.

Reverse directions.

Every direction has a **reverse**: the way back. On the compass the reverse is the point straight across the rose — NE reverses to SW, and NNE reverses to SSW. For a bearing, keep the same angle but swap north for south and east for west: the reverse of $N 20^\circ E$ is $S 20^\circ W$.

Worked model. A delivery van drives out of town to the NE. The reverse direction, back to town, is SW.

Reading direction on a map.

To say where one place is compared with another, stand at the place you are measuring *from* and look along the map toward the other place. “The hospital is NE **of** the station” and “the hospital is NE **from** the station” both mean: start at the station, and the hospital lies to the north-east. Reading the other way round gives the reverse direction: the station is SW of the hospital. Map apps and GPS units read these directions for you and usually say them as words (NE, SSW) or as a bearing like $N 15^\circ W$; on paper you read them off the north arrow, and a map with a scale also lets you work out real distances the same way as in §5C. Giving and following whole routes in words is the next section (§7C).

When an answer needs rounding.

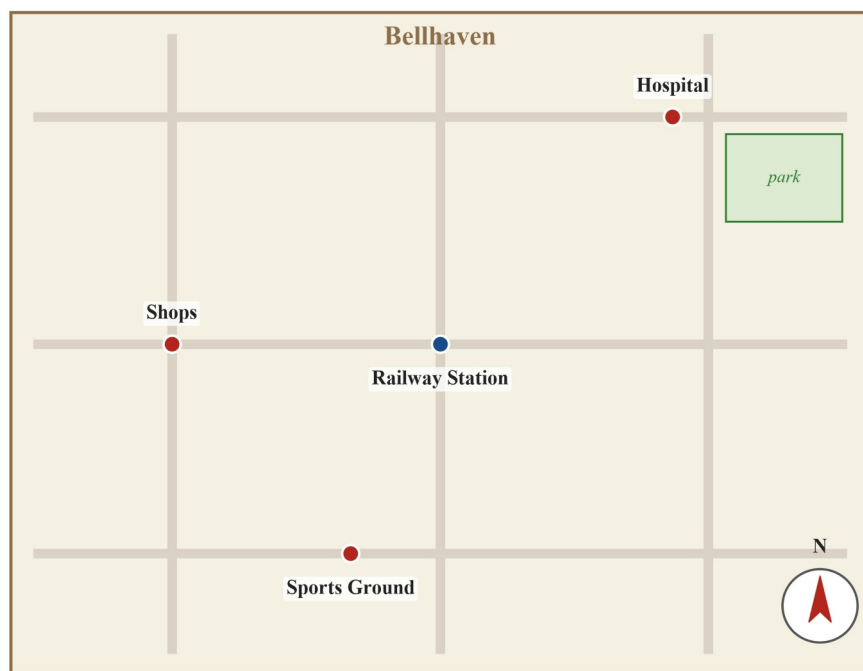
Most compass and bearing numbers in this section are exact — 90° , 45° , 22.5° and the bearings built from them. When a number does need rounding, the question says how: for example “to the nearest degree”. Keep the full number in your working and round only at the very last step.

About the maps and places in this section. The towns, reserves, harbours, farms and properties in the maps here are **made up** (not real): pretend places and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real town, park, business or person.

Worked examples

Example 1 — scaffolded

This is a street map of the made-up town of Bellhaven.



- (a) What compass direction is the Hospital from the Railway Station?
- (b) What compass direction is the Railway Station from the Hospital?
- (c) What compass direction is the Sports Ground from the Railway Station?

Working

Comment

(a) Start at the Railway Station. The Hospital is up and to the right on the map — halfway between N and E.

Up is north, right is east, because the north arrow points up.

So the Hospital is **NE** of the Railway Station.

NE is the halfway point between N and E.

(b) Now start at the Hospital and look back at the station: down and to the left.

Measuring *from* the other place gives the reverse direction.

So the Railway Station is **SW** of the Hospital.

SW is straight across the rose from NE.

(c) From the station, the Sports Ground is down and slightly left — between S and SW.

One 16-point step (22.5°) west of south.

So the Sports Ground is **SSW** of the Railway Station.

Three-letter names start with the closer cardinal: S.

Example 2 — scaffolded

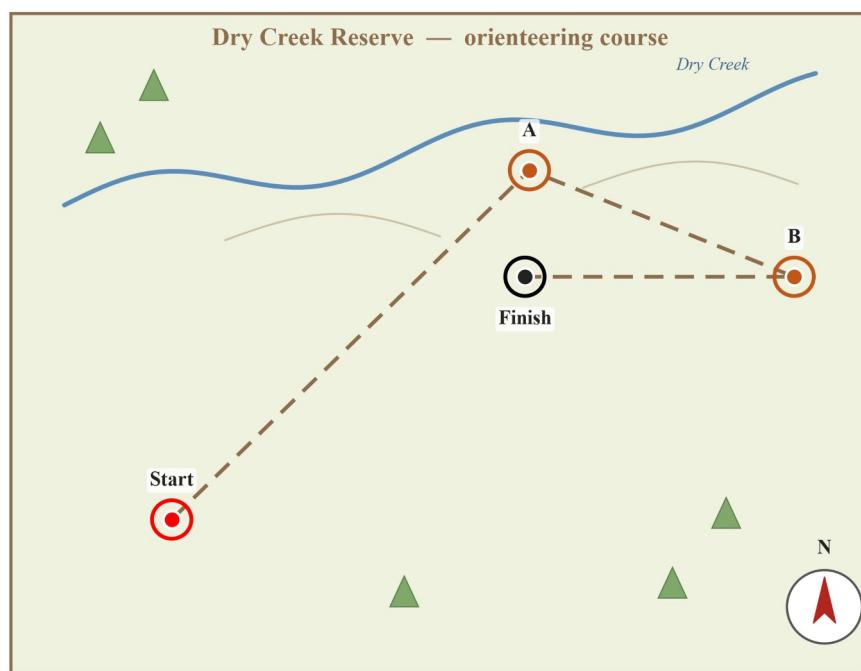
A walking track leaves a camp on a bearing of $N 30^\circ E$.

- (a) Between which two compass points does the track head?
- (b) Which single compass point is closest to the track's direction?
- (c) At the far end the walkers turn around and head back to camp. Write the bearing of the return leg.

Working	Comment
(a) $N 30^\circ E$ is 30° around from north toward the east. NNE is 22.5° from north and NE is 45° from north. $22.5 < 30 < 45$. So the track heads between NNE and NE.	First letter N: start facing north; last letter E: turn east. Read the turns from the 16-point table. The bearing sits inside that 22.5° gap.
(b) Gap down to NNE: $30 - 22.5 = 7.5$. Gap up to NE: $45 - 30 = 15$. $7.5 < 15$, so the closest point is NNE.	Compare the two gaps. The smaller gap wins.
(c) Reverse of $N 30^\circ E$: keep the 30°, swap $N \rightarrow S$ and $E \rightarrow W$. The return bearing is $S 30^\circ W$.	The way back is straight across the rose. Same angle, opposite starting face and turn.

Example 3 — unscaffolded

Runners in the made-up Dry Creek Reserve follow the orienteering course $\text{Start} \rightarrow A \rightarrow B \rightarrow \text{Finish}$ shown on the map.



Give the compass direction of each leg: Start to A, A to B, and B to Finish.

Start to A: from Start, control A is up and to the right at 45° , the halfway point between N and E, so this leg runs **NE**.

A to B: from A, control B is down and to the right — 112.5° clockwise from north as seen from A, one 16-point step south of due east — so this leg runs **ESE**.

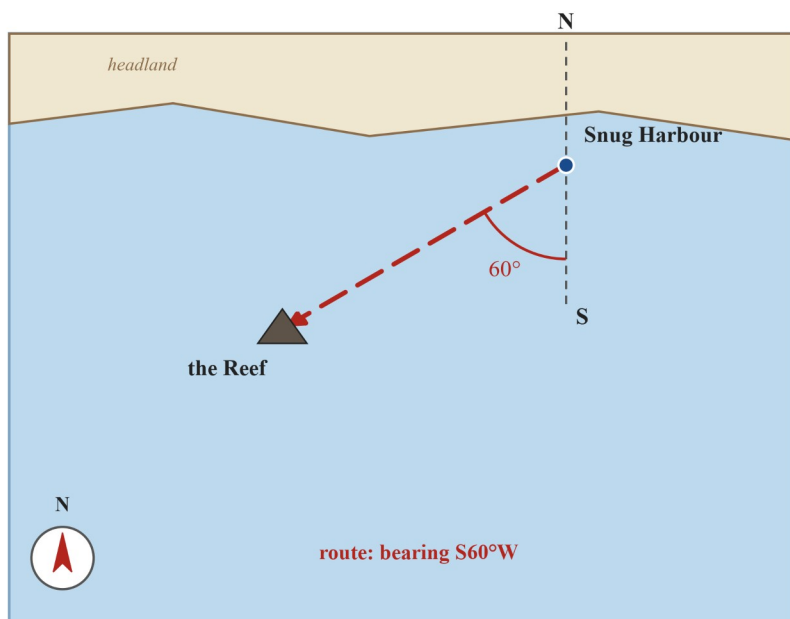
B to Finish: from B, the Finish is straight left along the map, so this leg runs due **W**.

Check: the reverse of the first leg, A back to Start, is SW — straight across the rose from NE, as a reverse must be.

$\therefore$ the legs of the course run NE, then ESE, then W.

Example 4 — unscaffolded

A fishing boat leaves Snug Harbour and travels to a reef on a bearing of $S 60^\circ W$.



- (a) Between which two compass points does the boat travel?
- (b) Which single compass point is closest to the boat's course?
- (c) After fishing, the boat returns straight to the harbour. Write the bearing of the return trip.
- (d) $S 60^\circ W$ is 60° around from south toward the west. SW is 45° from south and WSW is 67.5° from south, and $45 < 60 < 67.5$, so the boat travels between **SW and WSW**.
- (e) Gap down to SW: $60 - 45 = 15$. Gap up to WSW: $67.5 - 60 = 7.5$. The smaller gap is up to WSW, so the closest compass point is **WSW**.
- (f) The return trip is the reverse of $S 60^\circ W$: keep the 60° and swap $S \rightarrow N$ and $W \rightarrow E$, giving $N 60^\circ E$.

Check: 60° east of north is between NE (45° from north) and ENE (67.5° from north), the mirror image of the outward leg between SW and WSW — as a reverse should be.

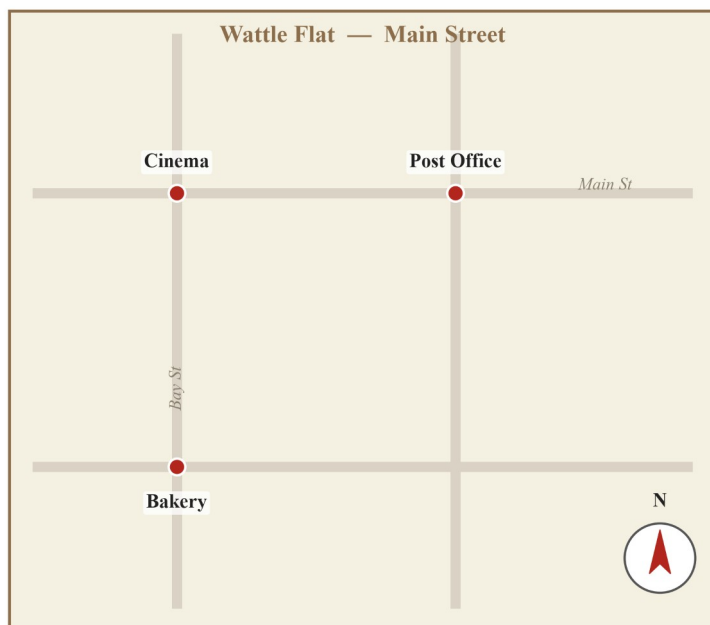
$\therefore$ the boat travels between SW and WSW (closest point WSW), and returns on a bearing of $N 60^\circ E$.

Practice questions

Scaffolded

- Q1.** Compass point names. (a) Write the abbreviation for north-east. (b) Which compass point is the reverse (straight opposite) of west? (c) Which compass point is halfway between north and west?
- Q2.** Turns between compass points. (a) How many degrees is the clockwise turn from N to E? (b) How many degrees is the clockwise turn from N to NE? (c) How many degrees is the clockwise turn from E to S?
- Q3.** The 16-point compass. (a) Name the compass point halfway between N and NE. (b) What three words does the abbreviation SSW stand for? (c) Name the compass point halfway between SW and W.
- Q4.** Bearings and compass points. (a) Write SE as a bearing in the form $S \theta^\circ E$. (b) In words: what does the bearing $N 15^\circ W$ tell you to do? (c) Write NW as a bearing in the form $N \theta^\circ W$.

Q5. This is a street map of part of the made-up town of Wattle Flat.

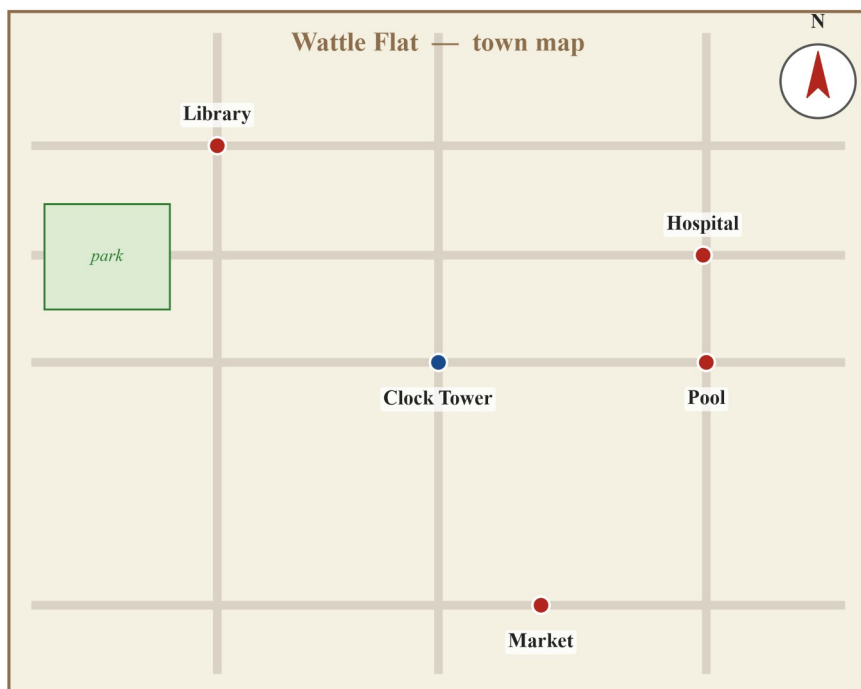


- What compass direction is the Post Office from the Bakery?
- What compass direction is the Bakery from the Post Office?
- What compass direction is the Cinema from the Post Office?

Q6. Reverse directions. (a) What is the reverse compass point of NW? (b) What is the reverse bearing of $N 20^\circ E$? (c) What is the reverse compass point of NNW?

Unscaffolded

Q7. This is the town map of made-up Wattle Flat.



- What compass direction is the Library from the Clock Tower?
- What compass direction is the Clock Tower from the Library?
- What compass direction is the Hospital from the Clock Tower?

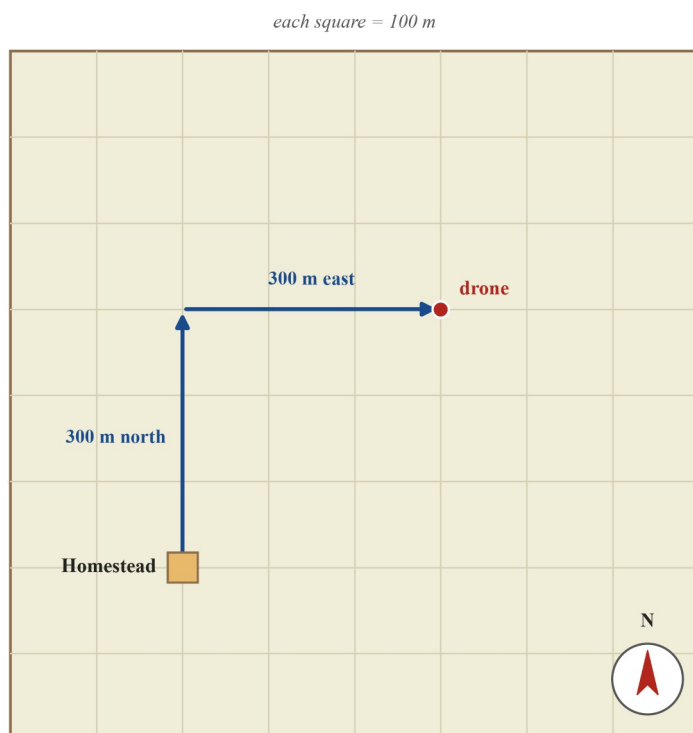
Q8. Walkers in the made-up Fern Range use this map, which has a scale of 1 : 40 000. The walking track from the Camp to the Hut measures 4.5 cm on the map.



- What is the real length of the track, in km?
- What compass direction is the Hut from the Camp?
- What compass direction is the Camp from the Hut?

Q9. Two ferries leave the same wharf. (a) Ferry 1 sails on a bearing of $S 22.5^\circ W$. Name this compass point. (b) Ferry 1 then turns around and returns to the wharf. Write the bearing of the return trip, and name that compass point. (c) Ferry 2 sails on a bearing of $N 45^\circ E$. Name this compass point.

Q10. A farm drone takes off from the homestead, flies 300 m due north, then 300 m due east, and hovers. Each square on the plan below is 100 m.

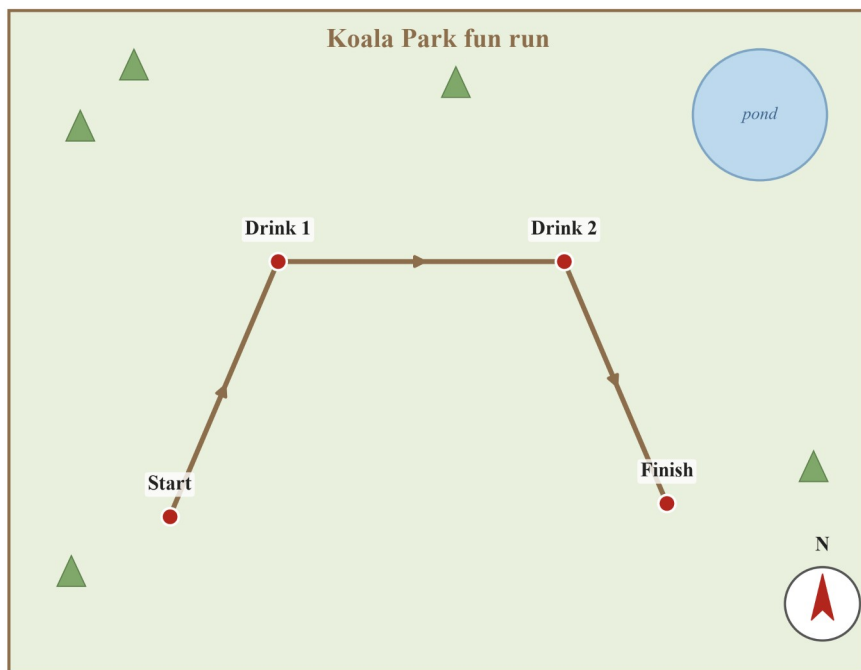


- (a) What compass direction is the drone from the homestead?
- (b) The drone flies straight back to the homestead. What compass direction is that flight?
- (c) A second flight goes 300 m due north but then only 150 m due east. Is the drone now exactly north-east of the homestead? Give a reason.

Q11. Winds are named after the direction they blow *from*: a southerly blows from the south. (a) A southerly blows from which compass point, and toward which compass point? (b) A wind vane points to the direction the wind is coming from. A vane on a shed roof is pointing NW. What wind is this, in words? (c) Smoke from a chimney drifts toward ESE. From which compass point is the wind blowing, and what is the wind called in words?

Q12. On a construction site the supervisor radios: “the steel stockpile is $S\ 60^\circ\ E$ of the site gate”. (a) Between which two compass points does this bearing lie? (b) Which single compass point is closest to it? (c) Describe the bearing $S\ 60^\circ\ E$ in words, starting at the gate.

Q13. Runners in the made-up Koala Park fun run follow the route Start → Drink 1 → Drink 2 → Finish shown on the map.

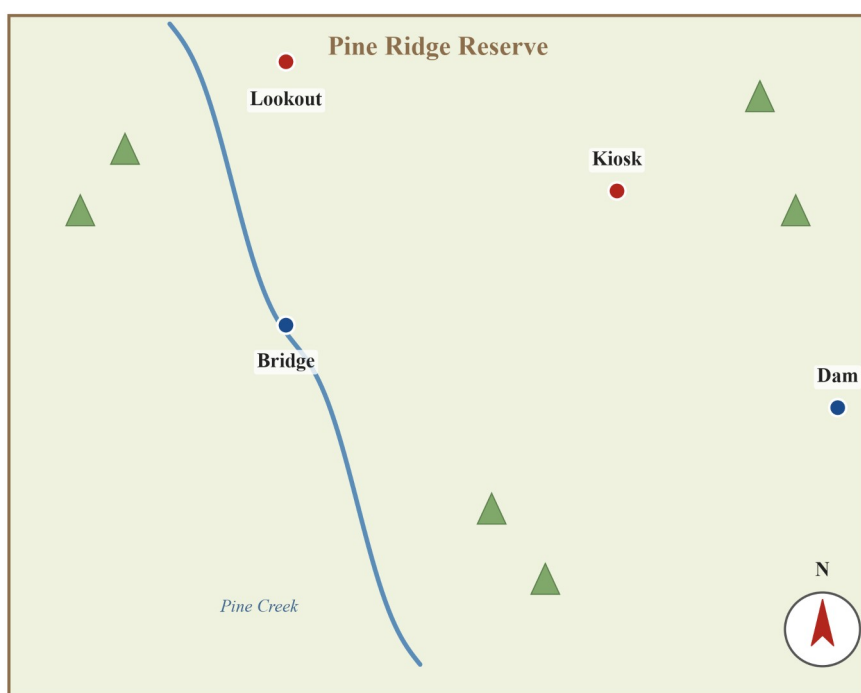


Give the compass direction of each leg of the run. (a) Start to Drink 1. (b) Drink 1 to Drink 2. (c) Drink 2 to Finish.

Q14. A phone map app tells a walker: “head 800 m on a bearing of $N 15^\circ W$ ”. (a) In words: what does $N 15^\circ W$ tell the walker to do? (b) Between which two compass points does this bearing lie? (c) Which single compass point is closest to it?

Q15. Show that the compass point SSW is the same direction as the bearing $S 22.5^\circ W$. Write one step per line.

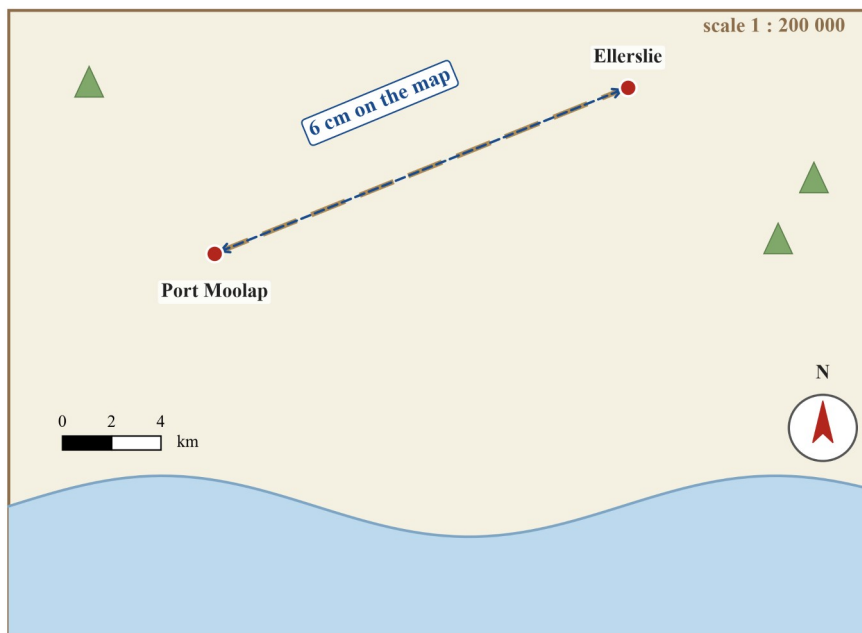
Q16. This is a map of made-up Pine Ridge Reserve.



- What compass direction is the Bridge from the Kiosk?
- What compass direction is the Lookout from the Bridge?
- What compass direction is the Kiosk from the Dam?

Q17. An SES rescue helicopter searches for a bushwalker. (a) The walker was last seen due west of the lookout. What compass direction does the helicopter fly from the lookout? (b) A report now says the walker is SW of the lookout. Write SW as a bearing in the form $S \theta^\circ W$. (c) The helicopter finds the walker and flies straight back to the lookout. What compass direction is that flight?

Q18. This map of a made-up coastal district has a scale of 1 : 200 000. The coast road from Port Moolap to Ellerslie measures 6 cm on the map.



- What is the real length of the coast road, in km?
- What compass direction is Ellerslie from Port Moolap?
- A ferry crossing back from Ellerslie shows a heading of 67.4° west of south on its GPS. Write this as a bearing in the form $S \theta^\circ W$, to the nearest degree.

Answers

Q1. (a) NE. (b) E. (c) NW. **Q2.** (a) 90° . (b) 45° . (c) 90° . **Q3.** (a) NNE. (b) south-south-west. (c) WSW. **Q4.** (a) $S 45^\circ E$. (b) face north, then turn 15° toward the west. (c) $N 45^\circ W$. **Q5.** (a) NE. (b) SW. (c) W. **Q6.** (a) SE. (b) $S 20^\circ W$. (c) SSE. **Q7.** (a) NW. (b) SE. (c) ENE. **Q8.** (a) 1.8 km. (b) NE. (c) SW. **Q9.** (a) SSW. (b) $N 22.5^\circ E$, which is NNE. (c) NE. **Q10.** (a) NE. (b) SW. (c) no — NE needs equal amounts of north and east; this flight went twice as far north as east. **Q11.** (a) from S, toward N. (b) a north-westerly (a north-west wind). (c) from WNW; a west-north-westerly. **Q12.** (a) between SE and ESE. (b) ESE. (c) face south at the gate, then turn 60° toward the east. **Q13.** (a) NNE. (b) E. (c) SSE. **Q14.** (a) face north, then turn 15° toward the west and walk. (b) between N and NNW. (c) NNW. **Q15.** $360^\circ \div 16 = 22.5^\circ$; SSW is the first point west of south, so SSW = $S 22.5^\circ W$. **Q16.** (a) WSW. (b) N. (c) NW. **Q17.** (a) W. (b) $S 45^\circ W$. (c) NE. **Q18.** (a) 12 km. (b) ENE. (c) $S 67^\circ W$, to the nearest degree.

Fully-worked solutions

Q1. (a) North-east is written **NE**. (b) Straight across the rose from W is **E**. (c) Halfway between N and W is **NW**.

Q2. (a) N to E is one quarter of a full turn: $360^\circ \div 4 = 90^\circ$. (b) NE is halfway between N and E: $90^\circ \div 2 = 45^\circ$. (c) E to S is one quarter turn clockwise: 90° .

Q3. (a) Halfway between N and NE is **NNE** (north-north-east). (b) SSW stands for **south-south-west**, the point halfway between S and SW. (c) Halfway between SW and W is **WSW** (west-south-west).

Q4. (a) SE is 45° around from south toward the east: $S 45^\circ E$. (b) $N 15^\circ W$ means: face **north**, then turn 15° toward the **west**. (c) NW is 45° around from north toward the west: $N 45^\circ W$.

Q5. (a) From the Bakery, the Post Office is up and to the right at the halfway angle: **NE**. (b) The way back is the reverse: **SW**. (c) From the Post Office, the Cinema is straight left along Main Street: due **W**.

Q6. (a) Across the rose from NW is **SE**. (b) Keep the 20° , swap $N \rightarrow S$ and $E \rightarrow W$: $S 20^\circ W$. (c) Across the rose from NNW is **SSE**.

Q7. (a) From the Clock Tower, the Library is up and to the left at the halfway angle: **NW**. (b) The reverse direction: **SE**. (c) From the Clock Tower, the Hospital is up and to the right but closer to east than to NE — one 16-point step north of due east: **ENE**.

Q8. (a) Real length = map length $\times$ scale factor (§5C): $4.5 \times 40\,000 = 180\,000$ cm. Then $180\,000$ cm = 1800 m = 1.8 km. (b) From the Camp, the Hut is up and to the right at 45° : **NE**. (c) The reverse direction: **SW**.

Q9. (a) $S 22.5^\circ W$ is 22.5° west of south — exactly one 16-point step — which is **SSW**. (b) The reverse keeps the 22.5° and swaps $S \rightarrow N$, $W \rightarrow E$: $N 22.5^\circ E$, one step east of north, which is **NNE**. (c) $N 45^\circ E$ is halfway between N and E: **NE**.

Q10. (a) 300 m north then 300 m east puts the drone equal amounts north and east of the homestead — the halfway line: **NE**. (b) The flight home is the reverse: **SW**. (c) **No**. NE means equal amounts of north and east (a 45° line). This flight went 300 m north but only 150 m east — twice as much north as east — so the drone lies between N and NE, not on the NE line. (Measured as an angle it is about 27° from north, to the nearest degree, not 45° .)

Q11. (a) A southerly blows **from S**, so it blows **toward N** — the reverse point. (b) The vane points NW, so the wind comes from NW: a **north-westerly** (a north-west wind). (c) Smoke drifting toward ESE means the wind pushes from the reverse point, **WNW**: a **west-north-westerly** (a west-north-west wind).

Q12. (a) Measured from south toward the east: SE is 45° and ESE is 67.5° . $45 < 60 < 67.5$, so the bearing lies **between SE and ESE**. (b) Gap down to SE: $60 - 45 = 15$; gap up to ESE: $67.5 - 60 = 7.5$. The smaller gap wins: **ESE**. (c) Stand at the gate, face **south**, then turn 60° toward the **east**; the stockpile is straight ahead.

Q13. (a) From Start, Drink 1 is up and slightly right — one 16-point step (22.5°) east of north: **NNE**. (b) From Drink 1, Drink 2 is straight right along the map: due **E**. (c) From Drink 2, the Finish is down and slightly right — one 16-point step (22.5°) east of south: **SSE**.

Q14. (a) Face **north**, turn 15° toward the **west**, and walk 800 m that way. (b) NNW is 22.5° west of north, and $15 < 22.5$, so the bearing lies **between N and NNW**. (c) Gap to N: 15; gap to NNW: $22.5 - 15 = 7.5$. The smaller gap wins: **NNW**.

Q15. A full turn is 360° . The 16-point compass splits a full turn into 16 equal gaps: $360^\circ \div 16 = 22.5^\circ$. Going from south toward the west, the first point reached is SSW. So SSW is 22.5° west of south. A turn of 22.5° west of south is written S 22.5° W. So SSW is the same direction as S 22.5° W.

Q16. (a) From the Kiosk, the Bridge is down and to the left, one 16-point step past SW toward W (247.5° clockwise from north): **WSW**. (b) From the Bridge, the Lookout is straight up along the map: due **N**. (c) From the Dam, the Kiosk is up and to the left at the halfway angle: **NW**.

Q17. (a) Due west of the lookout means the helicopter flies **W**. (b) SW is halfway between S and W, which is 45° from south toward the west: S 45° W. (c) The flight back to the lookout is the reverse of SW, which is **NE**.

Q18. (a) Real length = map length $\times$ scale factor (§5C): $6 \times 200\,000 = 1\,200\,000$ cm. Then $1\,200\,000$ cm = 12 000 m = 12 km. (b) From Port Moolap, Ellerslie is up and to the right, one 16-point step (22.5°) north of due east: **ENE**. (c) Rounding 67.4 to the nearest degree gives 67° , so the bearing is S 67° W, to the nearest degree. (Check: this is very close to WSW, 67.5° west of south — exactly what a return trip from an ENE outward leg should show.)

Chapter 7 Dimension and direction — 7C Giving and following directions

Theory

What this section is about.

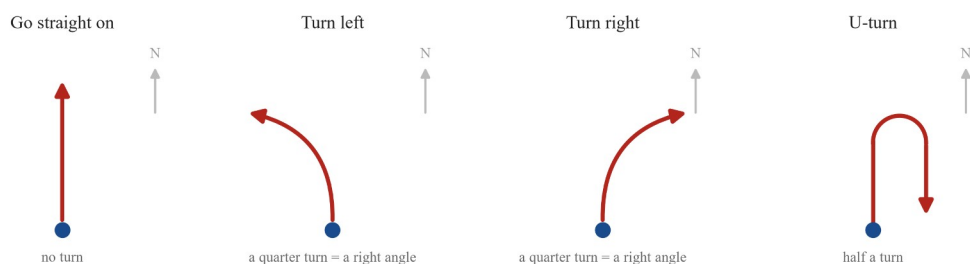
Giving and following directions is maths you use with your feet. On the **following** side, you take instructions — spoken out loud or written down — and turn them into a walk to a specified location. On the **giving** side, you write or say instructions clear enough that someone who has never stood where you are standing can follow them without asking you anything. This section covers both sides: oral and written instructions to specified locations, and directions between several destinations, including unfamiliar places, using maps and technology.

Every route in this section is built from the same three ingredients: a **starting place and facing**, a list of **legs** (the straight stretches between turns), and a **turn** at the end of each leg. Get those three right and any route can be described, followed, measured and checked.

The turn language: quarter turns, left and right.

Directions use turns, not angles, but the two are the same idea. A **quarter turn** is a turn of 90° — the angle between one street and the next on a square grid. Turn a quarter turn to the right and north becomes east; turn a quarter turn to the left and north becomes west. A **U-turn** is two quarter turns, 180° , and points you back the way you came.

Straight on means no turn at all.



Whose left? Two walkers on the same street, facing opposite ways



Left and right depend on the way the walker is facing — directions always say which way you start facing.

Reading the panels from the left: facing north, **straight on** leaves you facing north; a **left** turn leaves you facing west; a **right** turn leaves you facing east; and a **U-turn** leaves you facing south. The last panel is the one people get wrong most often — *whose* left and *whose* right.

Whose left? Whose right?

Left and right are not fixed on a map the way north and south are. They move with the walker. Facing north, your right hand points east. Facing west, your right hand points north and your left hand points south. So “the shop is on your left” means nothing until you know which way the walker is facing.

When you follow directions on a paper map, put yourself on the map: imagine standing at the start, facing the way the first leg goes, and only then decide what is on your left and your right. Turn the page if it helps — there is no rule that says north must be at the top of your desk.

Five checks for directions that can be followed.

Before you call a set of directions finished — yours or anyone else’s — check it against these five:



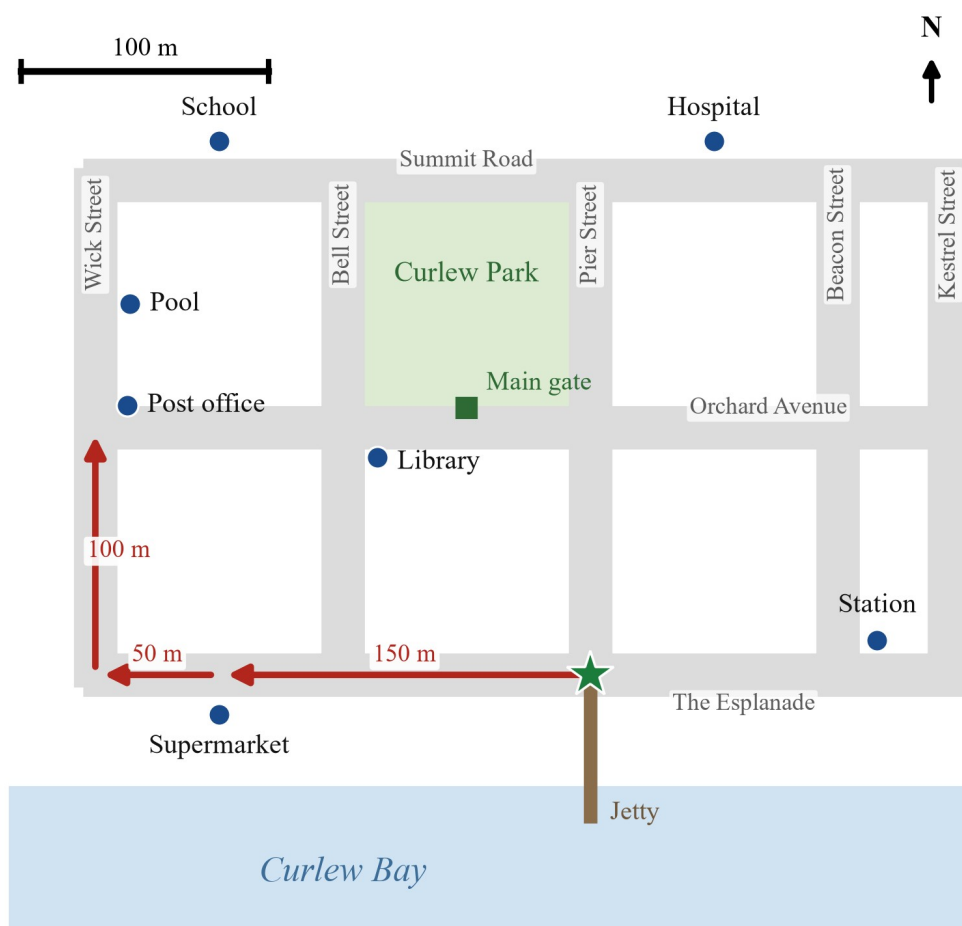
1. A **start and a facing**: “Start at the jetty, facing west along The Esplanade.”
2. A **turn word at every change of direction**: left, right, straight on or U-turn.
3. A **distance or a landmark for every leg**: “walk 150 m” or “walk to the next corner”.
4. A **side cue for landmarks you pass**: “the supermarket is on your left”.
5. An **arrival cue at the end**: “the post office is on your right at the corner — you are there.”

Directions that fail check 1 leave the walker facing the wrong way at step one. Directions that fail checks 2 and 3 leave the walker guessing at every corner. A set of directions that passes all five can be followed by a stranger, which is the whole point.

Following directions on a map — a model walk.

Here is a set of written directions around the made-up bayside town of Curlew Bay. One grid block on the map is 100 m, and north is at the top.

1. Start at the jetty on The Esplanade, facing west.
2. Walk 150 m west along The Esplanade. The supermarket is on your left.
3. Keep going 50 m west to the corner of Wick Street.
4. Turn right and walk 100 m north up Wick Street. The post office is on your right.
5. You are at the corner of Wick Street and Orchard Avenue.



Made-up data: Curlew Bay, its streets and every place on this map are made up for this book.

Trace the red route on the map as you read each step. Step 2 passes the supermarket, which sits on the south side of The Esplanade — south is on your left while you face west, so the side cue checks out. At Wick Street you are facing west and the next leg goes north, so the turn is a **right** turn, a quarter turn clockwise. Step 4 passes the post office on the east side of Wick Street — east is on your right while you face north, so that cue checks out too.

The legs add to the total walk: $150 + 50 + 100 = 300$ m. Adding leg distances is how you cost a route, and it works the same whether the legs came from a map, from an app or from someone’s voice.

Giving directions other people can follow.

Giving directions is the same five checks in reverse. Plan the route first — on a map if you have one — then write the legs in order with a turn word at every corner. Name streets where you can (“turn right into Pier Street”) because a street sign is a landmark the walker can confirm for free, and add a side cue or two so the walker can tell halfway along a leg whether they are still on your route. Finish with an arrival cue so the walker knows when to stop.

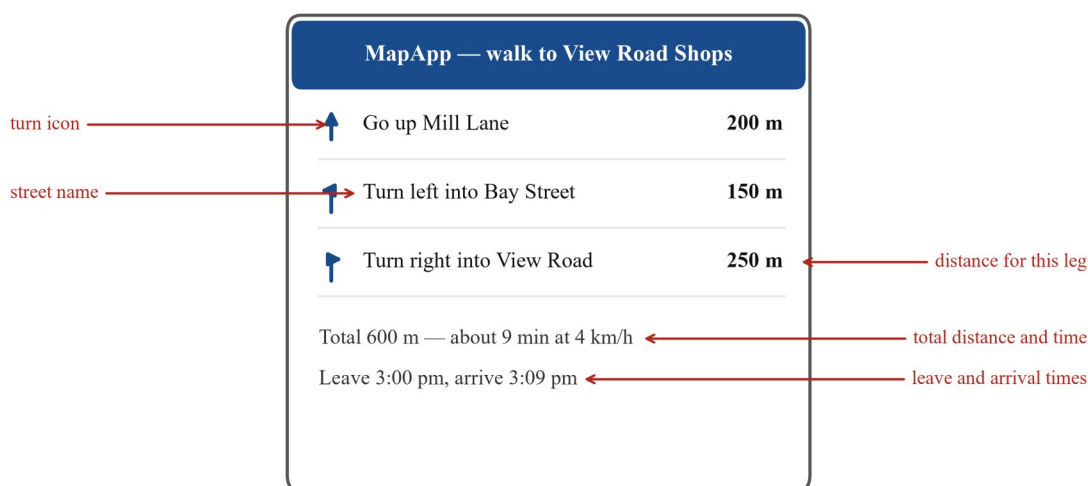
If you are giving directions out loud to someone standing next to you, face the same way they will walk when you say left and right. Pointing north while saying “left” to a walker who is facing east is how good friends get lost.

More than one destination.

Plenty of real trips have several stops: a delivery run, a guided tour, a morning of errands. Treat each pair of stops as its own little route — a leg list with a turn at every corner — then join them in the order you will visit. Keeping a running total of the legs tells you the whole trip’s distance, and comparing two different stop orders tells you which is shorter. Questions 9, 10 and 18 practise exactly this.

Maps, apps and other technology.

A directions app writes the same five checks on a screen for you. The card below is a typical walking card, with the parts labelled.



Made-up data: the app screen, its street names and every number on it are made up for this book.

Read a card top to bottom: each row is one leg, with a **turn icon**, the **street name** and the **distance for that leg**; the footer gives the **total distance and time** and the **leave and arrival times**. The total is just the legs added — here $200 + 150 + 250 = 600$ m — and the time is that distance at an assumed walking speed, here 4 km/h, which gives 9 min. Leave at 3:00 pm and the app expects you at 3:09 pm.

Apps and GPS are one way to get directions, and it helps to know what a card is telling you, but they are a convenience, not a requirement: every skill in this section works with a paper map, a pencil and the five checks. If a device’s route ever disagrees with the street signs in front of you, trust the signs and re-check your starting facing first.

When the place is unfamiliar.

Directions to somewhere you have never been lean on the map more heavily. Orient the map first: find the **north arrow** and line it up with the way the page is held. Locate yourself second — the last corner or landmark you are sure of — because every leg is measured from somewhere. Use the **scale bar** to turn map lengths into metres so you know how far a leg should feel. Confirm each leg as you walk it with street names and the side cues you were given. And if a landmark fails to appear where the directions promise it, stop: walk back to the last point you were sure of and re-check your facing, rather than pushing on in a direction that has already gone wrong.

Skills this section leans on — a quick look back.

Three older skills do quiet work in this section.

- **Scale (§5C).** A map scale of 1 : 20 000 means 1 cm on the map is 20 000 cm, that is 200 m, in real life. So 3.5 cm on that map is $3.5 \times 200 = 700$ m on the ground.
- **Speed (§5B).** At a walking speed of 4 km/h, a 1.2 km walk takes $1.2 \div 4 = 0.3$ h, which is 18 min.
- **Time (§4D).** Leaving at 9:05 am for an 18-minute walk, you arrive at 9:23 am.

Compass points and bearings (§7B) sit underneath all of this too: this section keeps to left, right and street names, but “north” on a map arrow is a bearing doing its everyday job.

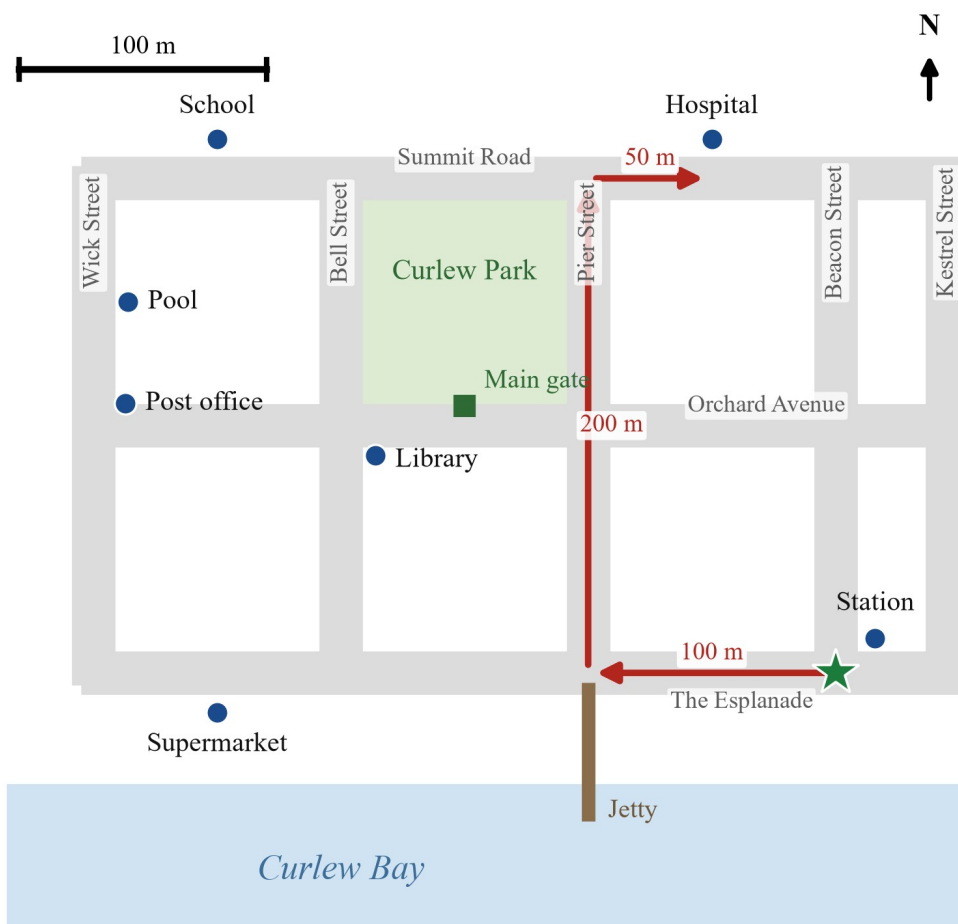
Most distances and times in this section work out exactly. When one does not, the question says how to round — “to 1 decimal place”, “to the nearest minute”. Keep the full number in your working and round only at the very last step.

About the data in this section. The towns, maps, walking tracks, holiday parks and app screens here are **made up** (not real): pretend places, pretend names and pretend numbers, built for this book so you can practise giving and following directions. Nothing here is real data about a real town, business or person.

Worked examples

Example 1 — scaffolded

Erin steps out of the station and walks to the hospital along the red route on the Curlew Bay map.



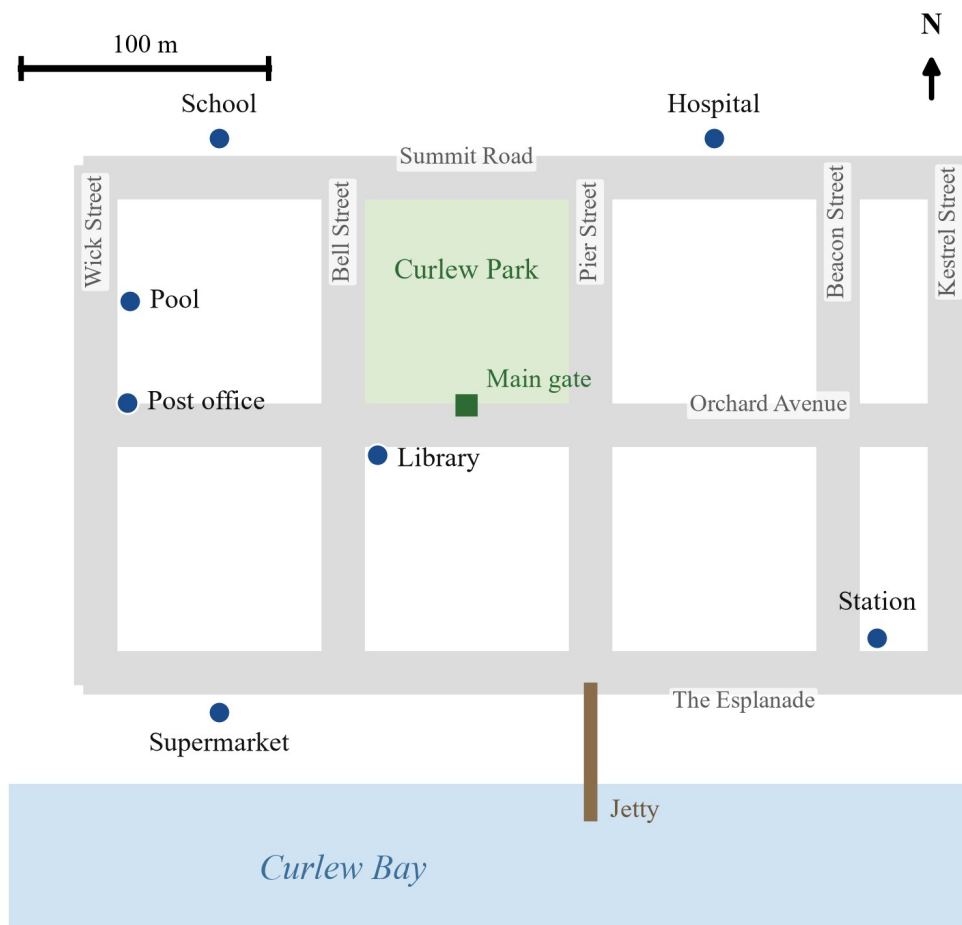
Made-up data: Curlew Bay and every place and distance on this map are made up.

- Describe each turn in the route as left, right, straight on or U-turn.
- How far does Erin walk altogether?
- On the last leg, which side of the street is the hospital on?

Working	Comment
Leg 1: 100 m west along The Esplanade, from the station corner to Pier Street.	Read the labelled legs off the red route, in order.
Turn 1 at Pier Street: Erin is facing west and the next leg goes north, so the turn is a right turn.	West to north is a quarter turn clockwise.
Leg 2: 200 m north up Pier Street to Summit Road.	One grid block is 100 m; this leg is two blocks.
Turn 2 at Summit Road: facing north, the next leg goes east, so this is a right turn too.	North to east is again a quarter turn clockwise.
Leg 3: 50 m east along Summit Road to the hospital.	Half a block is 50 m.
Total: $100 + 200 + 50 = 350$ m.	The legs add to the whole walk.
On the last leg Erin faces east; the hospital stands on the north side of Summit Road, so it is on her left .	Facing east, your left hand points north.

Example 2 — scaffolded

Noah walks from the supermarket to the library on the Curlew Bay map below. Fill in the four blanks so his directions pass all five checks.



Made-up data: Curlew Bay and every place on this map are made up for this book.

“Start at the supermarket on The Esplanade, facing east. Walk _____ m east along The Esplanade to Bell Street. Turn _____ and walk _____ m north along Bell Street to Orchard Avenue. The library is on your _____, at the corner.”

Working	Comment
The supermarket sits halfway along the block between Wick Street and Bell Street, so the first leg is half a block: 50 m.	One grid block is 100 m.
Facing east, Noah needs to head north up Bell Street:	Right would send him south, into the bay.

east to north is a quarter turn anticlockwise, a **left** turn.

Bell Street from The Esplanade to Orchard Avenue is one whole block: 100 m.

Walking north, Noah's right hand points east, and the library stands on the east side of Bell Street at the corner: on his **right**.

Check the total: $50 + 100 = 150$ m.

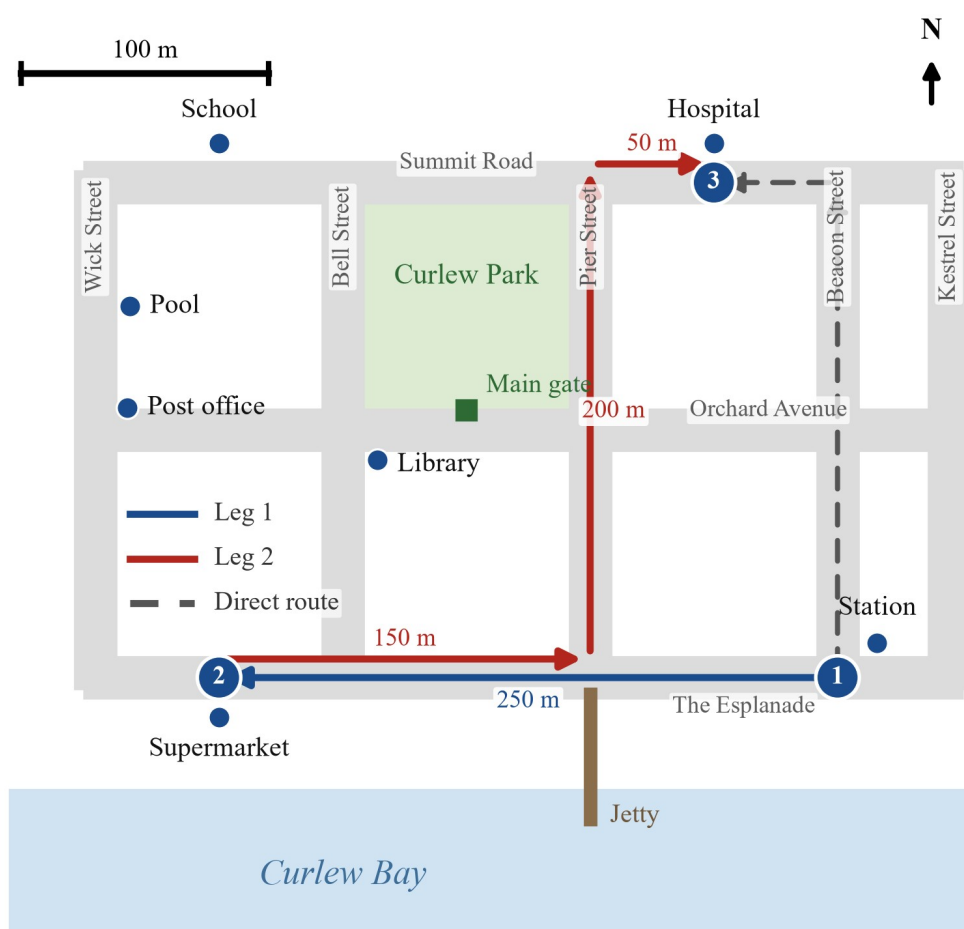
Count blocks, then multiply by 100 m.

Side cues depend on the facing.

A total gives the walker a way to catch a wrong turn early.

Example 3 — unscaffolded

Beth drives the meals van in Curlew Bay: first the station, then the supermarket, then the hospital, along the red route shown.



Made-up data: Curlew Bay and every place and distance on this map are made up.

- How far does Beth drive altogether?
- The grey dashed route straight from the station to the hospital is 250 m. How much longer is Beth's two-stop run than that direct route?
- Leg 1, station to supermarket: 250 m west along The Esplanade. Leg 2, supermarket to hospital: 150 m east along The Esplanade to Pier Street, then 200 m north up Pier Street, then 50 m east along Summit Road, which is $150 + 200 + 50 = 400$ m. Altogether Beth drives $250 + 400 = 650$ m.
- The direct route is 250 m, so the two-stop run is $650 - 250 = 400$ m longer. That is the price of the supermarket stop: the van doubles back along The Esplanade before heading north.

$\therefore$ Beth drives 650 m in all, which is 400 m more than the direct 250 m route.

Example 4 — unscaffolded

Tomas's phone shows this walking card from his front door to the Seaview Centre.

MapApp — walk to Seaview Centre		
	Go up Grand Parade	300 m
	Turn right into Tower Street	250 m
	Turn left into Marine Drive	450 m
MapApp estimate: 12 min		

Made-up data: the app screen, its street names and every number on it are made up.

- (a) What is the total walk, in metres and in kilometres?
- (b) At a walking speed of 4 km/h, how many minutes should the walk take Tomas?
- (c) If Tomas leaves at 10:42 am, what time should he arrive?
- (d) The app's own estimate is 12 min. What walking speed, in km/h, is the app assuming?
- (e) The legs add: $300 + 250 + 450 = 1\,000$ m, and 1 000 m is 1.0 km.
- (f) At 4 km/h, 1.0 km takes $1.0 \div 4 = 0.25$ h, and $0.25 \times 60 = 15$ min.
- (g) 10:42 am plus 15 min: $10:42 \rightarrow 10:57$ am, so Tomas should arrive at 10:57 am.
- (h) The app fits 1.0 km into 12 min, which is $12 \div 60 = 0.2$ h. Speed is distance over time: $1.0 \div 0.2 = 5$ km/h. The app assumes a 5 km/h walker, brisker than the 4 km/h used in part (b) — which is why its estimate is shorter.

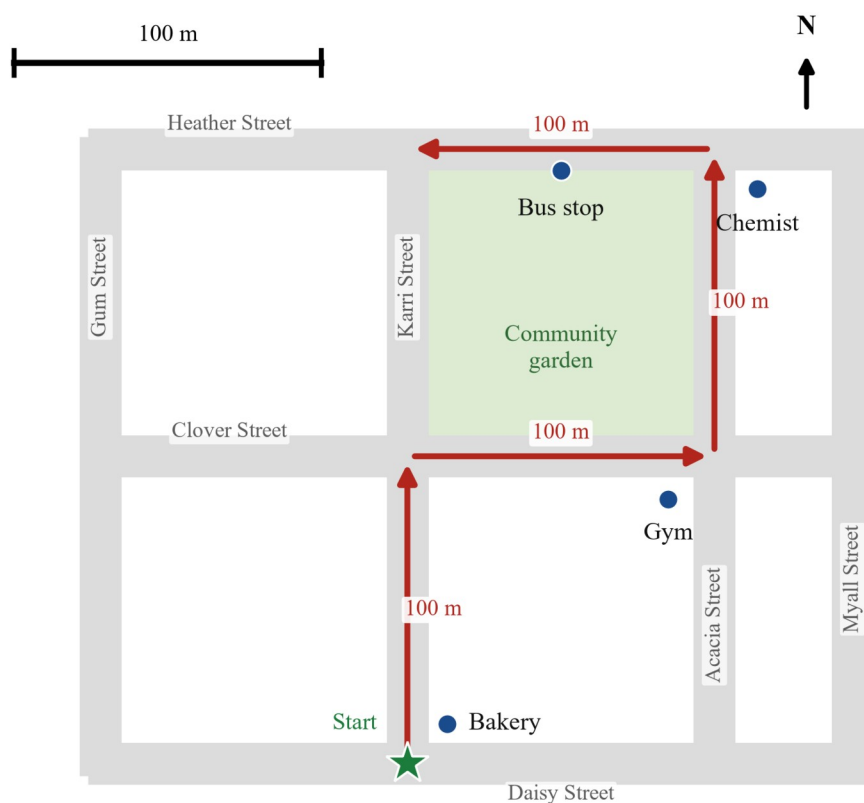
$\therefore$ the walk is 1 000 m (1.0 km), 15 min at 4 km/h, arriving 10:57 am, and the app's 12-minute estimate assumes 5 km/h.

Practice questions

Scaffolded

Q1. Turn vocabulary. (a) How many quarter turns make a U-turn? (b) What angle, in degrees, is a quarter turn? (c) You are facing north and turn a quarter turn to the right. Which way do you face now?

Q2. Ollie follows these spoken directions in the neighbourhood below.



Made-up data: this neighbourhood, its streets and places are made up for this book.

“Start at the corner of Karri Street and Daisy Street, facing north. Walk 100 m north along Karri Street to Clover Street. Turn right and walk 100 m east along Clover Street to Acacia Street — the gym is on your right. Turn left and walk 100 m north along Acacia Street to Heather Street — the chemist is on your right. Turn left and walk 100 m west along Heather Street — the bus stop is on your left, in front of the community garden.”

- Where does Ollie end up?
- How far does Ollie walk altogether?
- Count the turns: how many right turns and how many left turns did Ollie make?

Q3. A note left in the staff room at the Curlew Bay school reads: “Walk from the school to Wick Street. Turn at Wick Street and walk to the pool. The pool is on your right.” Use the Curlew Bay map in Example 2. (a) Give two reasons why these directions are hard to follow. (b) Walking from Summit Road down Wick Street towards the pool, which side of Wick Street is the pool really on? (c) Rewrite the directions so a stranger could follow them, with a distance on every leg, the correct turn word, one side cue and a total distance.

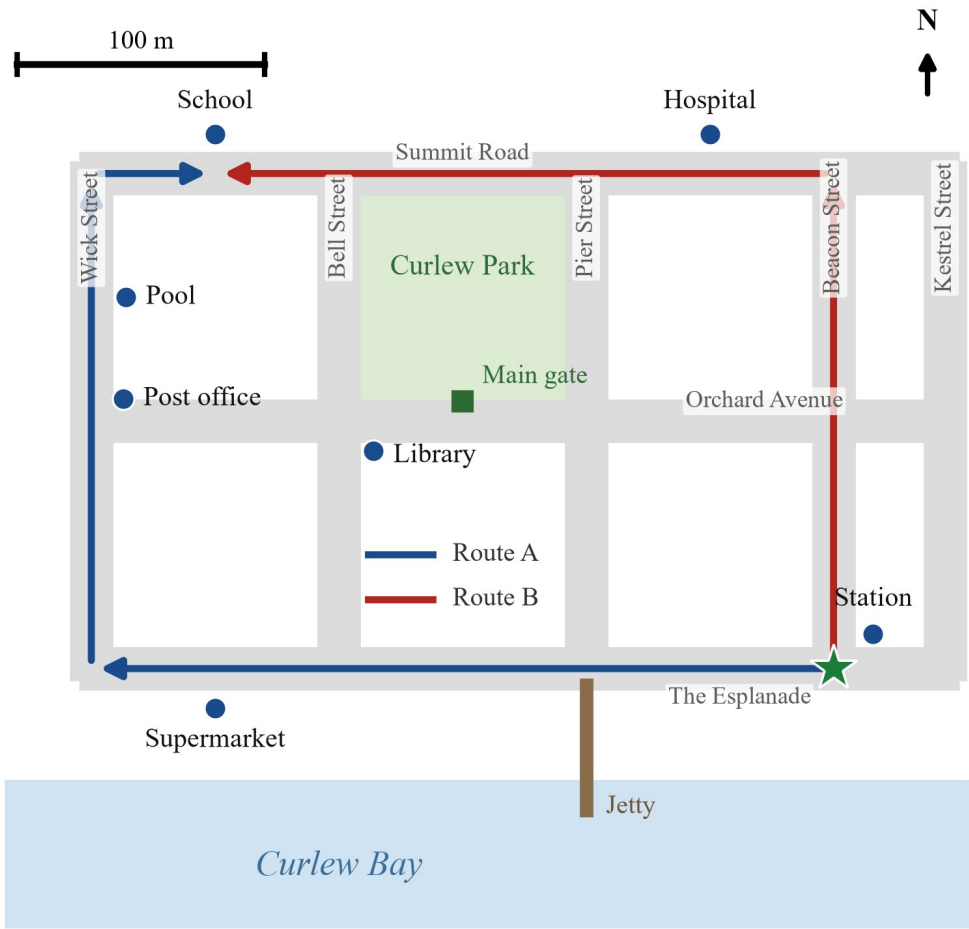
Q4. A bushwalking map has a scale of 1 : 25 000. (a) How many metres in real life does 1 cm on the map stand for? (b) A track measures 3.5 cm on the map. How long is it in real life, in metres? (c) A fire trail is 2 km long in real life. How many centimetres long is it on the map? (d) A river walk measures 4.8 cm on the map. How long is it in real life, in metres and in kilometres?

Q5. Riley’s phone shows these three walking legs to the lookout.

Step	Distance
Go up Bay Road	250 m
Turn left into Cliff Steps	200 m
Turn right into Lighthouse Track	100 m

Made-up data: the streets and distances in this table are made up for this book.

- (a) What is the total walk, in metres?
 - (b) Give the total in kilometres, to 1 decimal place.
 - (c) Riley walks at 50 m per minute. How many minutes does the walk take?
- Q6.** Two walking routes from the station to the school are drawn on the Curlew Bay map: route A in blue and route B in red.



Made-up data: Curlew Bay and every place on this map are made up for this book.

- (a) How long is each route, in metres?
- (b) How much shorter is route B than route A?
- (c) Walking at 50 m per minute, how long does each route take? How many minutes does route B save?

Unscaffolded

Questions 7, 8 and 13 use the Curlew Bay map below (the same map as Example 2).



Made-up data: Curlew Bay and every place on this map are made up for this book.

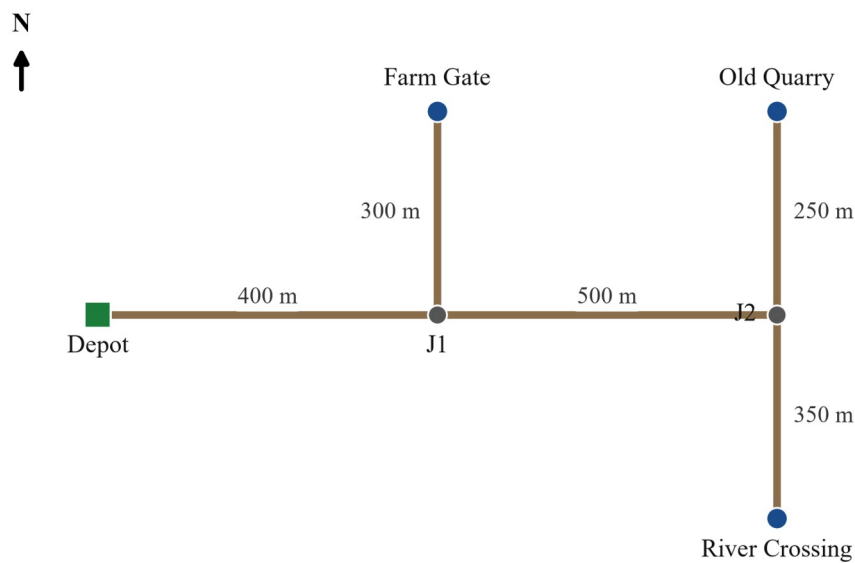
Q7. Fill in the four blanks so these directions walk from the library to the station.

“Start at the library, at the corner of Bell Street and Orchard Avenue, facing east. Walk _____ m east along Orchard Avenue to Beacon Street. Turn _____. Walk _____ m south along Beacon Street to The Esplanade. The station is on your _____, at the corner.”

- (a) Complete the directions.
- (b) What is the total walk, in metres?

Q8. You are at the pool and a new neighbour asks you to walk her to the station. (a) Write a set of directions she can follow, with a distance on every leg, a turn word at every corner and at least one side cue. (b) What is the total walk, in metres? (c) Explain why *any* route from the pool to the station that stays on the streets of this grid is the same total distance, and say what that distance must cover in each compass direction.

Q9. Nick drives a delivery loop on the sealed tracks below, starting and finishing at the Depot, in the order Depot → Farm Gate → Old Quarry → River Crossing → Depot.



Made-up data: the tracks, junctions and distances in this diagram are made up.

- (a) How far is it from the Depot to the Farm Gate?
 - (b) How far is it from the Farm Gate to the Old Quarry, via J1 and J2?
 - (c) How long is Nick’s whole loop, in metres and in kilometres?
- Q10.** A colleague drives the same four places in a different order: Depot → Old Quarry → Farm Gate → River Crossing → Depot. (a) How long is this loop, in metres and in kilometres? (b) A friend claims both loops must be the same length because they visit the same places. Is the claim correct? How much longer is one loop than the other?
- Q11.** A walking card on a visitor’s phone reads as below.

MapApp — walk to the Water Tower

↑

Go up Gannet Road

600 m

↘

Turn right into Gully Way

500 m

↑

Continue on Pelican Path

500 m

MapApp estimate: 20 min

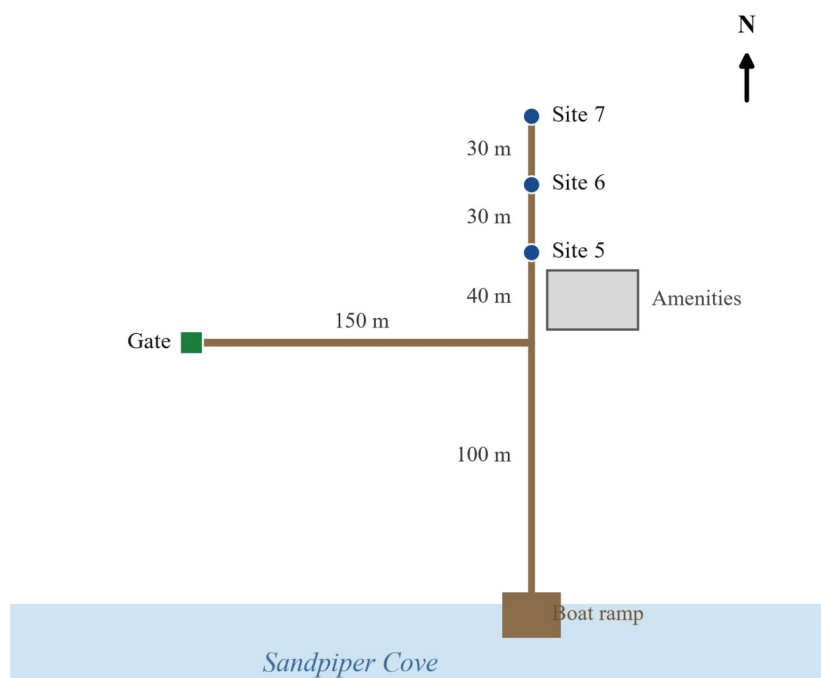
Made-up data: the app screen, its street names and every number on it are made up.

- (a) What is the total walk, in kilometres?
- (b) At a walking speed of 4 km/h, how many minutes should the walk take?
- (c) If the visitor leaves at 9:38 am, what time should they arrive at the Water Tower?
- (d) The walk actually takes the visitor 20 min. What was their average speed, in km/h, to 1 decimal place?

Q12. On a topographic map with a scale of 1 : 20 000, a walking track measures 4.5 cm. (a) How long is the track in real life, in metres? (b) A group walks the track and back the same way. How far is the return trip, in kilometres? (c) At a walking speed of 3 km/h, how many minutes does the return trip take?

Q13. Leo writes directions from the library to the jetty for his cousin: “Leave the library and walk along Orchard Avenue to Pier Street. Turn and walk to the jetty.” (a) Rewrite Leo’s directions so they pass all five checks: a facing at the start, a turn word at the corner, a distance on every leg, and an arrival cue. (b) What is the total walk, in metres?

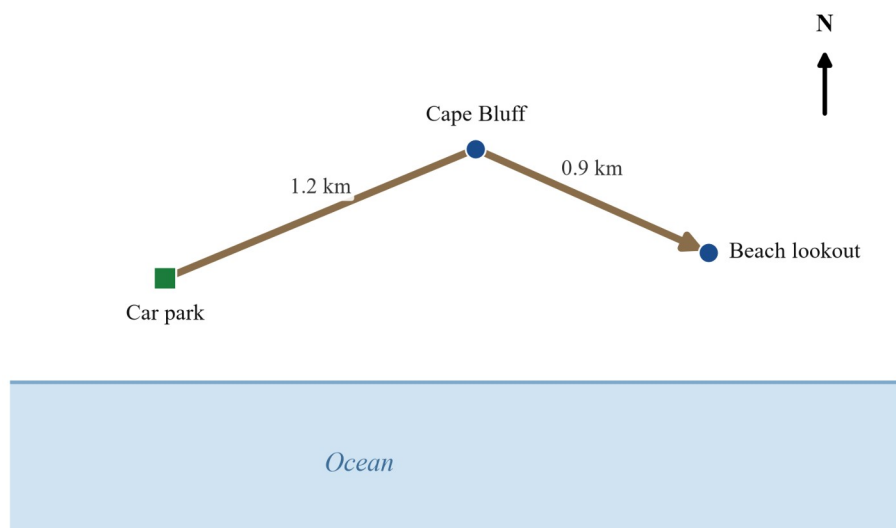
Q14. The Carter family arrives at the Sandpiper Cove holiday park. The track from the gate runs east to the amenities block at the fork; one arm continues north past sites 5, 6 and 7, and the other arm runs south to the boat ramp.



Made-up data: the park, its sites and every distance on this map are made up.

- How far is it from the gate to the amenities block?
- The family is booked into site 6. How far is the walk from the gate to site 6?
- From site 6, how far is it down to the boat ramp?

Q15. Pete walks the Cape Bluff track: 1.2 km from the carpark to the bluff, then 0.9 km from the bluff to the lookout.



Made-up data: the track, its points and the leg distances are made up.

- How far is the lookout from the carpark, one way?
- Pete walks back to the carpark the same way. How far has he walked altogether?
- At a walking speed of 3 km/h, how long does the return walk in part (b) take? Give your answer in hours and minutes.

Q16. Wes rides his bike from home to the shops, then on to work, then home along the direct road. The distances are in the table.

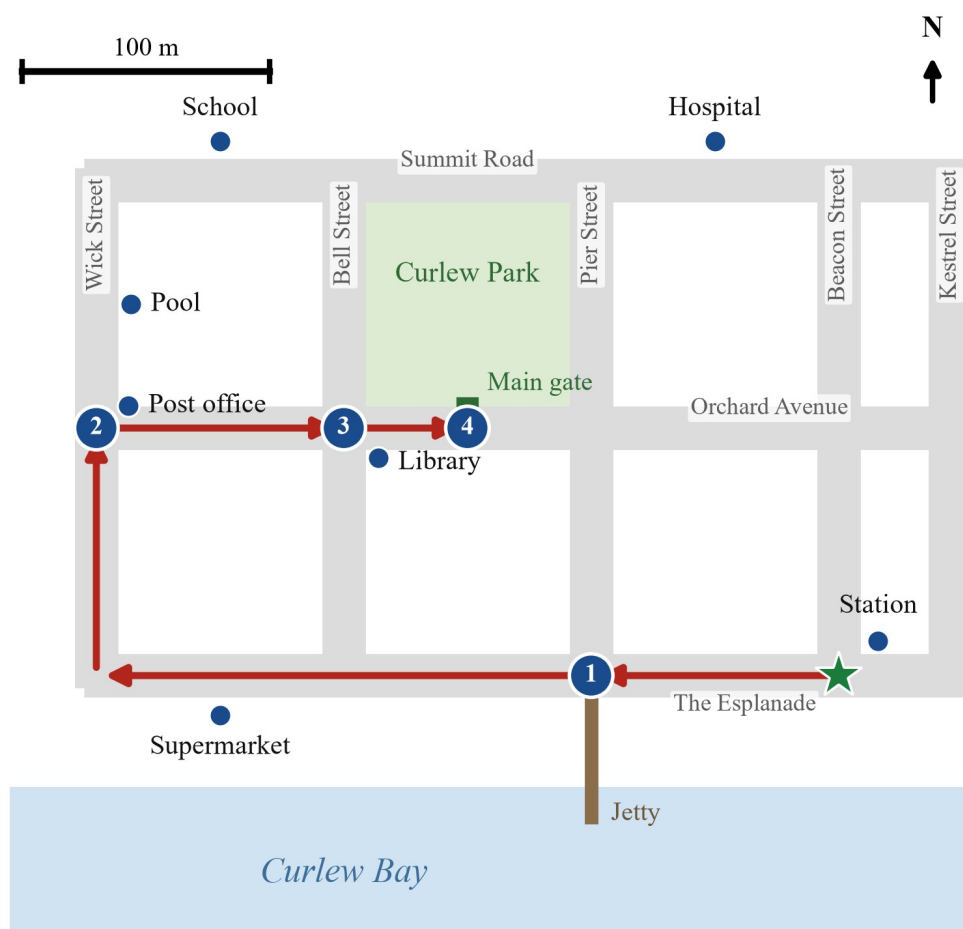
Stretch	Distance
Home to the shops	1.8 km
Shops to work	2.6 km
Work to home (direct road)	3.1 km

Made-up data: the distances in this table are made up for this book.

- How far does Wes ride altogether?
- How much longer is the way home → shops → work than riding straight from home to work?
- Riding at 12 km/h, how long does the whole trip take, to the nearest minute?

Q17. Carl compares two ways to get to the showgrounds, walking at 50 m per minute. Option A: walk 450 m to the station, ride the train for 8 min, then walk 200 m from the station to the showgrounds. Option B: walk straight there, 1150 m on foot the whole way. (a) How many minutes does each option take, door to door? (b) How much walking, in metres, does each option involve? (c) Compare the options: what does Option B cost Carl in extra time and extra walking?

Q18. Ollie guides a walking tour of Curlew Bay, starting at the station and stopping in the order 1, 2, 3, 4 marked on the map: the jetty, the post office, the library and the Curlew Park main gate.



Made-up data: Curlew Bay and every place on this map are made up for this book.

- Write the tour as numbered directions, with a distance on every leg and one side cue on each leg.
- How far does the tour walk altogether?
- Ollie gives a 15-minute talk at each of the four stops, and the group walks at 50 m per minute between stops. If the tour starts at 10:15 am, what time does it finish at the main gate?
- After the tour, Ollie walks from the pool to the park's main gate, via Wick Street and Orchard Avenue. How far is that walk?

Answers

Q1. (a) 2 quarter turns. (b) 90° . (c) East. **Q2.** (a) The bus stop, in front of the community garden. (b) 400 m. (c) 1 right turn and 2 left turns. **Q3.** (a) Any two of: no starting facing; no turn word at the corner; no distances; the side cue is wrong; no arrival cue. (b) The left. (c) Model: start at the school on Summit Road facing west; walk 50 m west to Wick Street; turn left; walk 50 m south down Wick Street; the pool is on your left — 100 m in all. **Q4.** (a) 250 m. (b) 875 m. (c) 8 cm. (d) 1 200 m, that is 1.2 km. **Q5.** (a) 550 m. (b) 0.6 km, to 1 decimal place. (c) 11 min. **Q6.** (a) Route A 550 m; route B 450 m. (b) 100 m. (c) A 11 min, B 9 min; route B saves 2 min. **Q7.** (a) 200 m; right; 100 m; left. (b) 300 m. **Q8.** (a) Model: start at the pool facing south; walk 50 m south down Wick Street to Orchard Avenue; turn left; walk 300 m east along Orchard Avenue to Beacon Street; turn right; walk 100 m south; the station is on your left at the corner. (b) 450 m. (c) Every street route must cover 300 m east and 150 m south in some order, so all of them total 450 m. **Q9.** (a) 700 m. (b) 1 050 m. (c) 3 600 m, that is 3.6 km. **Q10.** (a) 4 600 m, that is 4.6 km. (b) The claim is wrong; this loop is 1 000 m (1.0 km) longer than Q9's loop. **Q11.** (a) 1.6 km. (b) 24 min. (c) 10:02 am. (d) 4.8 km/h. **Q12.** (a) 900 m. (b) 1.8 km. (c) 36 min. **Q13.** (a) Model: start at the library facing east; walk 100 m east along Orchard Avenue to Pier Street; turn right; walk 100 m south down Pier Street to The Esplanade; the jetty is straight ahead, over The Esplanade. (b) 200 m. **Q14.** (a) 150 m. (b) 220 m. (c) 170 m. **Q15.** (a) 2.1 km. (b) 4.2 km. (c) 84 min, that is 1 h 24 min. **Q16.** (a) 7.5 km. (b) 1.3 km. (c) 38 min, to the nearest minute. **Q17.** (a) Option A 21 min; Option B 23 min. (b) Option A 650 m; Option B 1 150 m. (c) Option B costs 2 min more time and 500 m more walking. **Q18.** (a) Model: 1 — start at the station facing west and walk 100 m west along The Esplanade to Pier Street (stop 1, the jetty, on your left, out over the water); 2 — keep going 200 m west to Wick Street, turn right and walk 100 m north, the post office on your right (stop 2); 3 — turn right and walk 100 m east along Orchard Avenue, the library on your right at Bell Street (stop 3); 4 — walk 50 m east to the park's main gate on your left (stop 4). (b) 550 m. (c) 11:26 am. (d) 200 m.

Fully-worked solutions

Q1. (a) A U-turn faces you back the way you came, 180° , and $180 \div 90 = 2$ quarter turns. (b) A full turn is 360° , so a quarter turn is $360 \div 4 = 90^\circ$. (c) From north, a quarter turn to the right runs clockwise to the next compass point: east.

Q2. (a) Trace the four 100 m legs on the map: north on Karri Street, east on Clover Street, north on Acacia Street, then west on Heather Street. The last leg runs along the front of the community garden, where the bus stop sits, so Ollie ends at the **bus stop**. (b) Four legs of 100 m: $100 \times 4 = 400$ m. (c) Leg 1 to leg 2: north to east is a right turn. Leg 2 to leg 3: east to north is a left turn. Leg 3 to leg 4: north to west is a left turn. So 1 right turn and 2 left turns. (The side cues confirm the trace: gym on the right heading east, chemist on the right heading north, bus stop on the left heading west.)

Q3. (a) The note never says which way to face at the school, never says left or right at Wick Street, gives no distances, and its side cue is wrong — so a stranger could turn the wrong way at the very first corner and never notice. (b) The pool stands on the east side of Wick Street. Walking south down Wick Street, your left hand points east, so the pool is on the **left**, not the right. (c) Model: "Start at the school on Summit Road, facing west. Walk 50 m west along Summit Road to Wick Street. Turn left and walk 50 m south down Wick Street. The pool is on your left, halfway along the block." Total: $50 + 50 = 100$ m.

Q4. (a) 1 : 25 000 means 1 cm on the map is 25 000 cm in real life, and $25\,000 \div 100 = 250$ m. (b) $3.5 \times 250 = 875$ m. (c) 2 km is 2 000 m, and $2\,000 \div 250 = 8$ cm on the map. (d) $4.8 \times 250 = 1\,200$ m, which is 1.2 km.

Q5. (a) The legs add: $250 + 200 + 100 = 550$ m. (b) $550 \div 1\,000 = 0.55$ km, which rounds to 0.6 km to 1 decimal place. (c) At 50 m per minute, $550 \div 50 = 11$ min.

Q6. (a) Route A: 300 m west along The Esplanade, 200 m north up Wick Street, 50 m east along Summit Road: $300 + 200 + 50 = 550$ m. Route B: 200 m north up Beacon Street, 250 m west along Summit Road: $200 + 250 = 450$ m. (b) $550 - 450 = 100$ m shorter. (c) At 50 m per minute: route A $550 \div 50 = 11$ min; route B $450 \div 50 = 9$ min. Route B saves $11 - 9 = 2$ min.

Q7. (a) The library corner to Beacon Street is two blocks east along Orchard Avenue: 200 m. Facing east, the next leg goes south down Beacon Street: east to south is a **right** turn. Beacon Street to The Esplanade is one block: 100 m.

Walking south, your left hand points east and the station stands on the east side of Beacon Street: on your **left**. (b) $200 + 100 = 300$ m.

Q8. (a) Model: “Start at the pool, facing south. Walk 50 m south down Wick Street to Orchard Avenue. Turn left and walk 300 m east along Orchard Avenue to Beacon Street. Turn right and walk 100 m south along Beacon Street. The station is on your left, at the corner of The Esplanade.” (b) $50 + 300 + 100 = 450$ m. (c) The pool sits 300 m west and 150 m north of the station on this grid. Streets only run north–south and east–west, so every route must clock up exactly 300 m of eastward walking and 150 m of southward walking, in whatever order — $300 + 150 = 450$ m every time. (Extra back-tracking would only add distance, so 450 m is also the shortest any street route can be.)

Q9. (a) Depot to J1 is 400 m and J1 to the Farm Gate is 300 m: $400 + 300 = 700$ m. (b) Farm Gate to J1 (300 m), J1 to J2 (500 m), J2 to Old Quarry (250 m): $300 + 500 + 250 = 1\,050$ m. (c) The four stretches of the loop: Depot → Farm Gate 700 m; Farm Gate → Old Quarry 1 050 m; Old Quarry → River Crossing $250 + 350 = 600$ m; River Crossing → Depot $350 + 500 + 400 = 1\,250$ m. Total $700 + 1\,050 + 600 + 1\,250 = 3\,600$ m, that is 3.6 km.

Q10. (a) Depot → Old Quarry: $400 + 500 + 250 = 1\,150$ m. Old Quarry → Farm Gate: $250 + 500 + 300 = 1\,050$ m. Farm Gate → River Crossing: $300 + 500 + 350 = 1\,150$ m. River Crossing → Depot: $350 + 500 + 400 = 1\,250$ m. Total $1\,150 + 1\,050 + 1\,150 + 1\,250 = 4\,600$ m, that is 4.6 km. (b) The claim is wrong. Visiting the same places does not fix the distance, because the legs between them change with the order: $4\,600 - 3\,600 = 1\,000$ m, so this loop is 1.0 km longer than Q9’s loop.

Q11. (a) $600 + 500 + 500 = 1\,600$ m, which is 1.6 km. (b) At 4 km/h: $1.6 \div 4 = 0.4$ h, and $0.4 \times 60 = 24$ min. (c) 9:38 am plus 24 min: 9:38 → 9:62, that is 10:02 am. (d) In 20 min the visitor covers 1.6 km, so each minute covers $1.6 \div 20 = 0.08$ km, and an hour of that pace is $0.08 \times 60 = 4.8$ km/h — already exact at 1 decimal place.

Q12. (a) 1 : 20 000 gives 1 cm = 200 m, so $4.5 \times 200 = 900$ m. (b) Out and back: $900 \times 2 = 1\,800$ m, which is 1.8 km. (c) At 3 km/h: $1.8 \div 3 = 0.6$ h, and $0.6 \times 60 = 36$ min.

Q13. (a) Model: “Start at the library, at the corner of Bell Street and Orchard Avenue, facing east. Walk 100 m east along Orchard Avenue to Pier Street. Turn right and walk 100 m south down Pier Street to The Esplanade. The jetty is straight ahead, over The Esplanade — you are there.” Leo’s draft had the legs but no facing, no turn word and no distances. (b) $100 + 100 = 200$ m.

Q14. (a) The gate-to-fork track is labelled 150 m, and the amenities block sits at the fork: 150 m. (b) Gate to fork 150 m, then north up the sites arm: fork to site 5 is 40 m and site 5 to site 6 is 30 m, so $150 + 40 + 30 = 220$ m. (c) Site 6 back down to the fork is $30 + 40 = 70$ m, then fork to boat ramp 100 m: $70 + 100 = 170$ m, all of it straight south.

Q15. (a) $1.2 + 0.9 = 2.1$ km one way. (b) Out and back: $2.1 \times 2 = 4.2$ km. (c) At 3 km/h: $4.2 \div 3 = 1.4$ h. The 0.4 h is $0.4 \times 60 = 24$ min, so the return walk takes 84 min, that is 1 h 24 min.

Q16. (a) $1.8 + 2.6 + 3.1 = 7.5$ km. (b) Home → shops → work is $1.8 + 2.6 = 4.4$ km against 3.1 km direct: $4.4 - 3.1 = 1.3$ km longer. (c) At 12 km/h: $7.5 \div 12 = 0.625$ h, and $0.625 \times 60 = 37.5$ min, which is 38 min to the nearest minute.

Q17. (a) Option A: $450 \div 50 = 9$ min walking, plus 8 min on the train, plus $200 \div 50 = 4$ min walking: $9 + 8 + 4 = 21$ min. Option B: $1\,150 \div 50 = 23$ min. (b) Option A walking: $450 + 200 = 650$ m. Option B walking: 1 150 m. (c) Option B takes $23 - 21 = 2$ min more and $1\,150 - 650 = 500$ m more walking — a small time price for no train, which is exactly the trade-off Carl has to weigh.

Q18. (a) Model directions:

1. Start at the station, facing west. Walk 100 m west along The Esplanade to Pier Street — stop 1, the jetty, runs out into the bay on your left.
2. Keep going 200 m west along The Esplanade to Wick Street, turn right and walk 100 m north — stop 2, the post office, is on your right at the corner.
3. Turn right and walk 100 m east along Orchard Avenue — stop 3, the library, is on your right at the corner of Bell Street.
4. Walk 50 m further east — stop 4, the Curlew Park main gate, is on your left.

(b) The legs add: $100 + 300 + 100 + 50 = 550$ m.

(c) Walking time: $550 \div 50 = 11$ min. Talk time: $4 \times 15 = 60$ min. Total $11 + 60 = 71$ min. From 10:15 am, 71 min is 1 h 11 min, so the tour finishes at 11:26 am.

(d) Pool to Wick Street is 50 m south, then Wick Street to the main gate is 150 m east along Orchard Avenue: $50 + 150 = 200$ m.

Chapter 7 Dimension and direction — 7D Angles in degrees

Theory

In §3B you used angle facts inside shapes to work out unknown angles without measuring anything. This section is the other half of the story: the angles themselves — how big a turn is, how to measure one with a protractor, how to draw one, and how to talk about angles in everyday situations such as clocks, wheels, roofs and ladders. (§7B uses degrees to describe compass directions; here the turn itself is the subject.)

An angle is an amount of turn.

Two straight arms meeting at a point make an **angle**, and the meeting point is called the **vertex**. The size of an angle is how much turn you need to swing one arm onto the other. Turn is measured in **degrees**: a full turn around a point is split into 360 equal steps, and one step is one degree, written 1° . A turn also has a direction — **clockwise** (the way clock hands move) or **anticlockwise** — but the size of an angle does not depend on which way you turn.

Turns and degrees.

The most useful turns have names you already know from everyday language:

Turn	Degrees
quarter turn	90°
half turn	180°
three-quarter turn	270°
full turn	360°
two full turns	720°



quarter turn
 90° clockwise



half turn
 180° clockwise



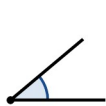
three-quarter turn
 270° clockwise



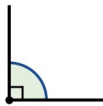
full turn
 360° clockwise

Turns bigger than one full turn just keep counting: two full turns are $2 \times 360^\circ = 720^\circ$. Skateboarders and BMX riders name their tricks by the total turn — a “360” is one full turn, and a “540” is one full turn plus a half turn (Example 4 works this one out).

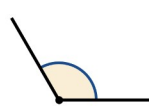
The six angle types — quick review.



acute
40°



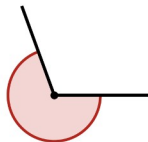
right angle
90°



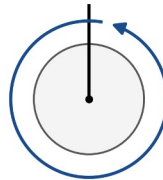
obtuse
120°



straight angle
180°



reflex
250°



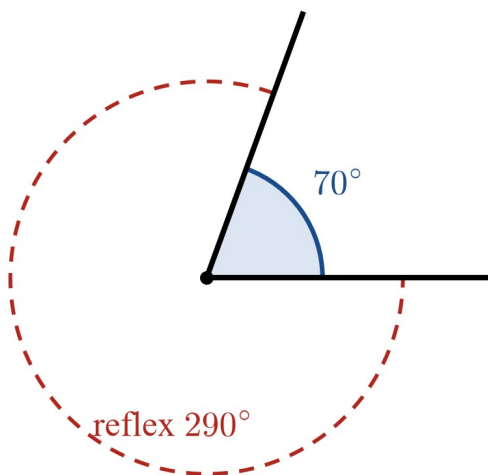
full turn
360°

Each angle in the picture is drawn true-size:

- **Acute angle:** between 0° and 90° .
- **Right angle:** exactly 90° — a quarter turn, marked with a small square.
- **Obtuse angle:** between 90° and 180° .
- **Straight angle:** exactly 180° — a half turn, which looks like a straight line.
- **Reflex angle:** between 180° and 360° .
- **Full turn:** exactly 360° — all the way around, back to the start.

Two angles at every corner.

Two arms meeting at a point split the full turn into **two** angles — the smaller one and the bigger one that goes the other way around.



$$70^\circ + 290^\circ = 360^\circ$$

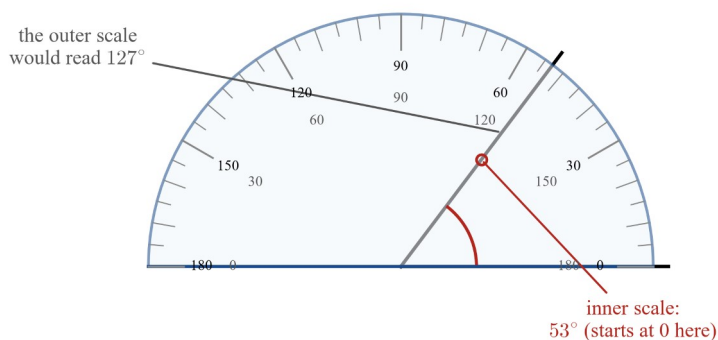
Unless a question says otherwise, “the angle” means the **smaller** of the two. The two angles always add to a full turn, so the small angle and the reflex angle at the same vertex are partners: in the picture, $70^\circ + 290^\circ = 360^\circ$. That gives a handy rule:

$$\text{reflex angle} = 360^\circ - \text{smaller angle}$$

Measuring an angle with a protractor.

A **protractor** is a clear half-circle tool for measuring angles. It has two scales running in opposite directions, so you can measure angles that open either way. To measure an angle:

1. Put the protractor's centre mark (the small hole or cross) on the **vertex**.
2. Turn the protractor so its straight baseline (the 0° – 180° line) lies along one arm.
3. Find where the other arm crosses the scale. Read the scale that starts at 0° on the arm you lined up.
4. Sanity-check against the shape: an acute angle reads under 90° , an obtuse angle between 90° and 180° . For a reflex angle, measure the smaller angle and take it from 360° .



The angle opens from the 0 of the inner (black) scale, so the inner scale gives the true size: 53° (acute).

In the picture, the arm crosses the inner scale at 53° and the outer scale at 127° . The angle opens from the arm lying on the baseline, and the inner scale is the one starting at 0° on that arm, so the angle is 53° . Notice that the two readings add to 180° : $53^\circ + 127^\circ = 180^\circ$. That always happens, and it is how you repair the most common protractor mistake — reading the wrong scale (Example 2).

Drawing an angle with a protractor.

To draw an angle of 135° :

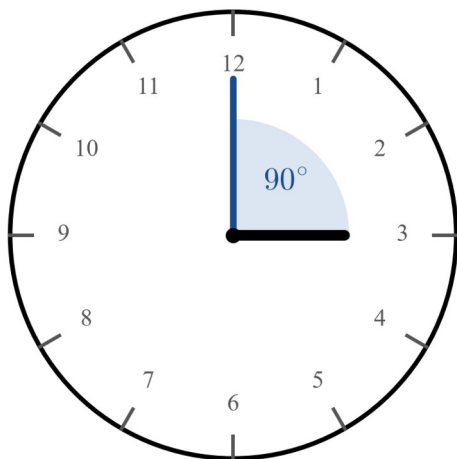
1. Draw a straight baseline with a ruler and mark the vertex at one end.
2. Put the protractor's centre mark on the vertex and its baseline along the line.
3. Find 135° on the scale that starts at 0° on your baseline, and make a small dot there.
4. Take the protractor away and use the ruler to join the vertex to the dot.
5. Check the result: 135° is obtuse (between 90° and 180°), so the drawn angle should open wider than a right angle but not as far as a straight line.

Estimate first, measure second.

Before measuring, compare the angle with ones you know: 30° is a thin sliver of a turn, 45° is half a right angle, 90° is the corner of a page or a doorway, and 180° is flat. An angle that looks like a wide-open right angle is probably obtuse — somewhere between 90° and 180° — not acute. Estimate first, then measure, and if the number disagrees wildly with your eye, measure again.

Angles on a clock face.

A clock face is a full turn split into 12 equal steps by the hour marks, so the marks are $360^\circ \div 12 = 30^\circ$ apart at the centre. The minute hand turns a full 360° in 60 minutes, so it moves $360^\circ \div 60 = 6^\circ$ per minute. The hour hand turns a full 360° in 12 hours, so it is much slower: it moves 30° per hour, which is 0.5° per minute.



3:00 — the hands are a quarter turn apart:
 $3 \times 30^\circ = 90^\circ$

At exactly 3:00 the minute hand points at the 12 and the hour hand points at the 3, so the hands are three hour-mark steps apart: $3 \times 30^\circ = 90^\circ$ — a right angle, as the picture shows. At times between the hours the hour hand creeps past its mark, so hand angles take two steps of working (Example 3 and Q9).

Why small differences matter.

At work and around the home, angles are rarely just numbers on a page. Roof pitch, ramp slope, ladder setup, solar-panel tilt and machine settings are all described in degrees, and a few degrees can be the difference between safe and unsafe, or between catching the sun and missing it. Safety instructions often give an allowed **range** of angles — a ladder safety card, for instance, might require an angle at the foot between 70° and 75° (Q12 uses a made-up card like this). Phone apps with tilt meters can measure these angles too, but a protractor is all you need in this section.

About the data in this section. The chairs, roofs, ladders, wheels, cakes and rides here are **made up** (not real): pretend projects and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real place, person or business.

Worked examples

Example 1 — scaffolded

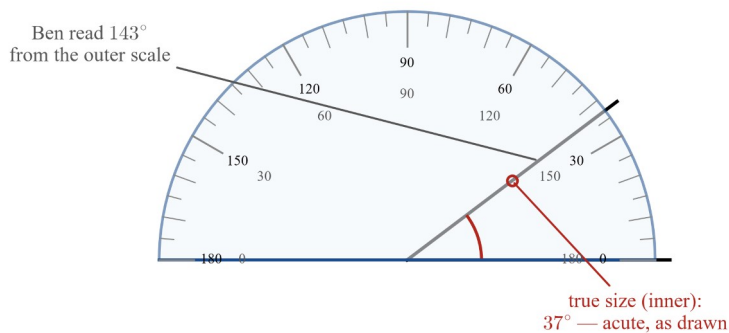
Kai sits on a swivel chair whose seat spins freely.

- He turns the seat three-quarters of a full turn, clockwise. How many degrees has the seat turned?
- By the end of the day he has spun a total of two and a quarter turns. How many degrees is that altogether?

Working	Comment
A full turn is 360°, so a quarter turn is $360^\circ \div 4 = 90^\circ$.	Every turn calculation starts from the turns table.
(a) Three quarters of a turn: $3 \times 90^\circ = 270^\circ$.	Three of the four quarter-turns.
(b) Two and a quarter turns: $2 \times 360^\circ = 720^\circ$, then $720^\circ + 90^\circ = 810^\circ$.	Count the whole turns first, then the extra quarter.
Check (b): $810^\circ = 360^\circ + 360^\circ + 90^\circ \checkmark$.	Two full turns and a quarter turn, exactly as the question said.

Example 2 — scaffolded

Ben measures an acute angle on a technical drawing with his protractor. He lines the protractor up correctly but reads the wrong scale, and records 143° . Find the true size of the angle.



Ben read the scale that did NOT start at 0 on the arm the angle opens from. $180^\circ - 143^\circ = 37^\circ$.

Working

Comment

The drawing shows a sharp, narrow angle.

An acute angle must measure under 90° , so 143° cannot be right.

The two protractor scales always add to 180° .

That is what makes a wrong-scale reading fixable.

True size: $180^\circ - 143^\circ = 37^\circ$.

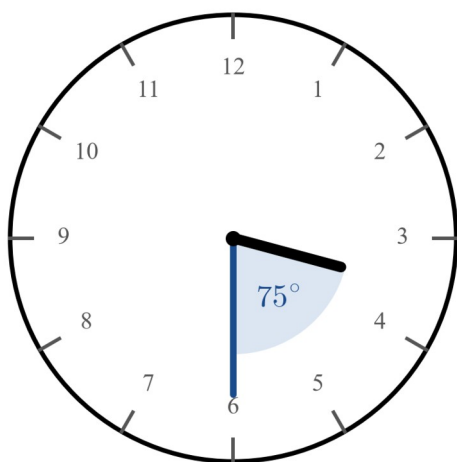
Take the wrong reading away from 180° .

Check: $143^\circ + 37^\circ = 180^\circ$ ✓, and 37° is acute.

The fixed answer now matches the shape of the drawing.

Example 3 — unscaffolded

A wall clock shows 3:30. Find the smaller angle between the hour hand and the minute hand.



3:30 — minute hand at 180° , hour hand at 105° : the smaller angle is 75°

The minute hand points at the 6: it is 30 minutes past the hour, so it has turned $30 \times 6^\circ = 180^\circ$ clockwise from the 12.

The hour hand has left the 3 and is halfway to the 4: it turns 30° for each hour plus 0.5° for each minute, so it is $3 \times 30^\circ + 30 \times 0.5^\circ = 90^\circ + 15^\circ = 105^\circ$ clockwise from the 12.

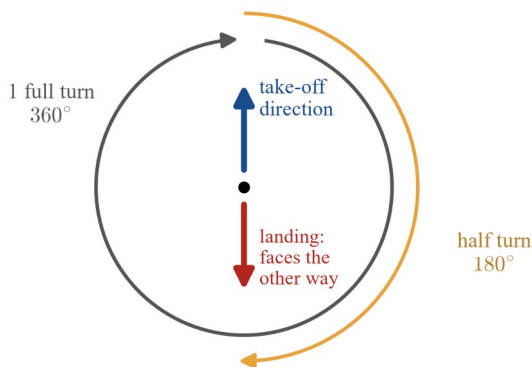
The angle between the hands is $180^\circ - 105^\circ = 75^\circ$. This is under 180° , so it is already the smaller angle (the other side is $360^\circ - 75^\circ = 285^\circ$, a reflex angle). It is acute, which matches the picture.

$\therefore$ at 3:30 the smaller angle between the hands is 75° .

Example 4 — unscaffolded

In skateboarding and BMX, tricks are named after the total amount of turn the rider spins in the air. A “540” is a spin of 540° ; a “720” is a spin of 720° .

- (a) How many full turns, plus how much extra, is a 540?
- (b) How many degrees short of a 720 is a 540?



$$\text{a “540” spin: } 540^\circ = 360^\circ + 180^\circ$$

Take out whole turns first: $540^\circ - 360^\circ = 180^\circ$, so a 540 is 1 full turn and a half turn — the rider lands facing the opposite way to the take-off.

A 720 is two full turns, since $2 \times 360^\circ = 720^\circ$. The gap between the two tricks is $720^\circ - 540^\circ = 180^\circ$ — exactly a half turn. Check: $360^\circ + 180^\circ = 540^\circ$ ✓.

$\therefore$ a 540 is 1 full turn plus 180° , and it is 180° short of a 720.

Practice questions

Scaffolded

Q1. Name the type of each angle. (a) 15° , 84° , 90° , 138° . (b) 180° , 275° , 360° . (c) By how much does a 275° angle go past a straight angle?

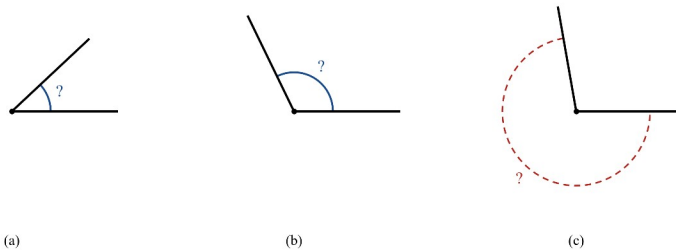
Q2. Convert each turn into degrees. (a) A quarter turn, and a half turn. (b) 5 full turns. (c) One and three-quarter turns.

Q3. A clock shows an exact hour time, so the minute hand points at the 12. The hour marks are 30° apart at the centre. Find the smaller angle between the hands at: (a) 2:00. (b) 5:00. (c) 9:00.

Q4. Mia lines her protractor up correctly but reads the wrong scale each time. (a) Her angle looks acute, and she records 128° . Find its true size. (b) Her angle looks obtuse, and she records 61° . Find its true size. (c) A wrong reading and the true reading always add to what?

Q5. (a) Find the reflex angle at the same vertex as a 70° angle. (b) Find the reflex angle at the same vertex as a 135° angle. (c) What do an angle and its reflex angle always add to? Use your answer to check (a) and (b).

Q6. The three angles below are drawn true-size. Measure each one with a protractor and record it to the nearest degree.

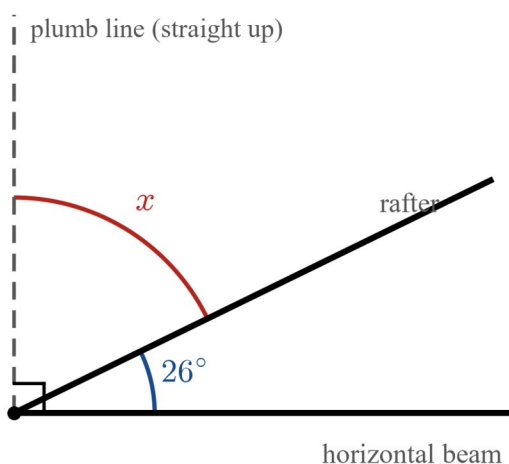


Drawn true-size — measure each angle with a protractor and record it to the nearest degree.

- (a) Angle (a).
- (b) Angle (b).
- (c) Angle (c): the two arms make a small angle and a large reflex angle (the dashed arc). Measure the small angle, then use it to find the reflex angle.
- (d) Now draw an angle of 110° with your protractor and ruler, and classify it.

Unscaffolded

Q7. A roof rafter makes an angle of 26° with the horizontal beam — this is the **pitch** angle of the roof. The dashed plumb line (a weighted string that hangs straight down) shows the vertical.



- (a) Find x , the angle between the rafter and the plumb line.
- (b) Find the reflex angle at the same vertex as the 26° pitch angle.
- (c) Classify both the 26° angle and x .

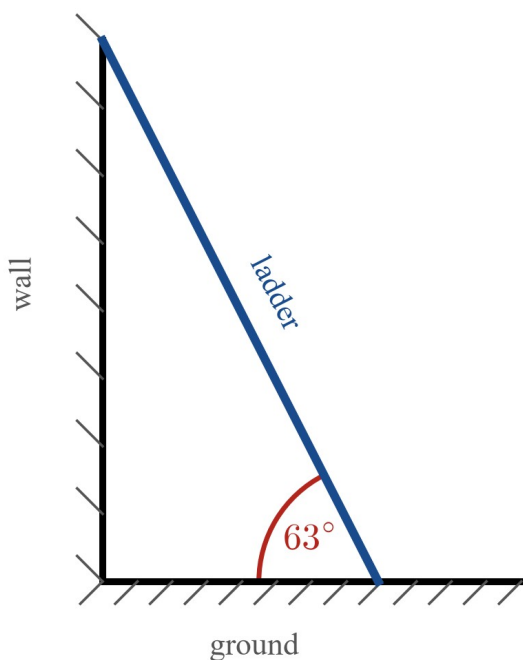
Q8. Show that the minute hand of a clock turns 6° per minute, and that the hour hand turns 0.5° per minute. Write one step per line.

Q9. Use the two rates from Q8 to find the smaller angle between the hands of a clock at: (a) 6:30. (b) 9:45.

Q10. A snowboarder lands a “1080” — a spin of 1080° in the air. (a) How many full turns is 1080° ? (b) How many more degrees is a 1080 than the 540 of Example 4? (c) After landing a 1080, is the rider facing the same way as at take-off? Explain.

Q11. A pottery wheel spins at 12 rpm — that is, 12 full turns (revolutions) every minute. (a) How many degrees does it turn in 1 minute? (b) How many degrees does it turn in 5 minutes? (c) How many full turns does it make in 5 minutes?

Q12. A ladder safety card says the angle between the ladder and the ground must be between 70° and 75° . A worker sets a ladder at 63° to the ground, as shown.

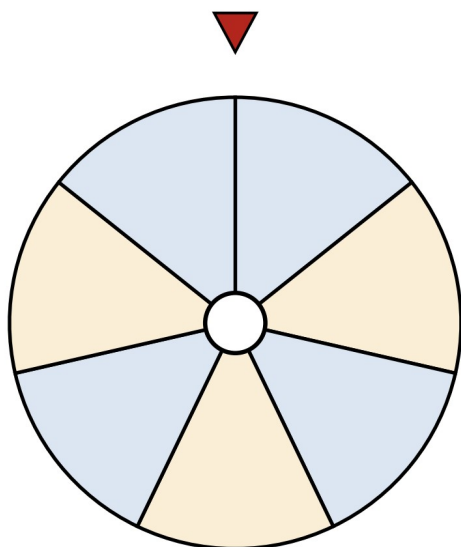


- (a) Find the angle between the ladder and the wall.
- (b) Is the ladder set inside the safe range? What should the worker do?
- (c) How many more degrees does the angle at the foot need, to reach the bottom of the safe range?

Q13. A solar panel is mounted tilted 32° from the horizontal. (a) Find the angle between the panel and the vertical. (b) The installer raises the tilt by 9° . What is the new tilt from the horizontal? (c) Find the new angle with the vertical.

Q14. A deck chair backrest sits at 97° to the seat. It is reclined until the angle is 132° . (a) Through how many degrees did the backrest turn? (b) Classify the 132° angle. (c) Flat is 180° . How many more degrees would recline the backrest to flat?

Q15. A fundraising prize wheel is divided into 7 equal sectors that meet at the centre. (A **sector** is a slice of the disc, like a cut piece of pizza.) A pointer is fixed at the top.



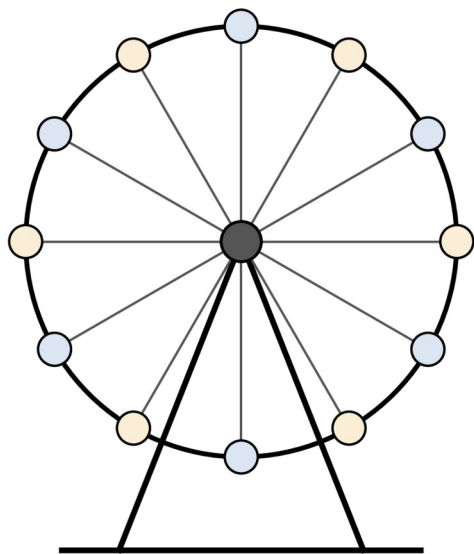
7 equal sectors — the angle of one sector
is the answer to Q15(a).

- (a) Find the angle of one sector, correct to 1 decimal place.
- (b) Multiply your rounded answer by 7. Does it come to exactly 360° ? Explain why or why not.
- (c) The wheel is spun and stops after exactly 4 full turns. How many degrees did it turn?

Q16. A round cake is cut into equal slices from the centre. (a) What is the angle of each of 10 equal slices? (b) What is the angle of each of 8 equal slices? (c) Could a whole cake be cut into slices that are each exactly 50° ? Explain with a calculation.

Q17. A forklift's forks start flat, at a 0° tilt. The operator tilts them back 15° to slide under a pallet, then tilts further until the total tilt is 62° , to steady the load. (a) Through how many degrees did the forks turn in the second tilt? (b) Classify the 62° tilt angle. (c) The mast of the forklift is vertical. At the 62° tilt, find the angle between the forks and the mast.

Q18. A Ferris wheel has 12 cabins equally spaced around the rim, and it turns at 2 rpm — 2 full turns every minute.



12 equally spaced cabins

- (a) How many degrees apart, at the centre, are neighbouring cabins?
 - (b) How many degrees does the wheel turn in 1 minute?
 - (c) A rider stays on for 3.5 minutes. How many degrees does the wheel turn in that time?
 - (d) How many full turns is that? Does a cabin that started at the top finish at the top?
-

Answers

Q1. (a) acute; acute; right angle; obtuse. (b) straight angle; reflex angle; full turn. (c) $275^\circ - 180^\circ = 95^\circ$. **Q2.** (a) 90° ; 180° . (b) 1800° . (c) 630° . **Q3.** (a) 60° . (b) 150° . (c) 90° (the long way round is 270° ; the smaller angle is 90°). **Q4.** (a) $180^\circ - 128^\circ = 52^\circ$, acute. (b) $180^\circ - 61^\circ = 119^\circ$, obtuse. (c) 180° . **Q5.** (a) 290° . (b) 225° . (c) 360° ; check $70^\circ + 290^\circ = 360^\circ$ ✓ and $135^\circ + 225^\circ = 360^\circ$ ✓. **Q6.** (a) 43° . (b) 116° . (c) small angle 100° , so the reflex angle is 260° . Accept any readings within 2° of these values. (d) drawing: the finished angle measures 110° (within 2°) and is obtuse. **Q7.** (a) $x = 90^\circ - 26^\circ = 64^\circ$. (b) $360^\circ - 26^\circ = 334^\circ$. (c) both acute. **Q8.** Minute hand: $360^\circ \div 60 = 6^\circ$ per minute. Hour hand: $12 \times 60 = 720$ minutes in a full turn, so $360^\circ \div 720 = 0.5^\circ$ per minute. **Q9.** (a) 15° . (b) 22.5° (exact — no rounding needed). **Q10.** (a) $1080 \div 360 = 3$ full turns. (b) $1080^\circ - 540^\circ = 540^\circ$ more. (c) yes — 3 whole turns with nothing left over, and each full turn ends facing the way it started. **Q11.** (a) $12 \times 360^\circ = 4320^\circ$. (b) $21\,600^\circ$. (c) $12 \times 5 = 60$ turns. **Q12.** (a) $90^\circ - 63^\circ = 27^\circ$. (b) no — 63° is below 70° , so the ladder is too flat and could slide out; move the foot closer to the wall. (c) $70^\circ - 63^\circ = 7^\circ$. **Q13.** (a) $90^\circ - 32^\circ = 58^\circ$. (b) $32^\circ + 9^\circ = 41^\circ$. (c) $90^\circ - 41^\circ = 49^\circ$. **Q14.** (a) $132^\circ - 97^\circ = 35^\circ$. (b) obtuse. (c) $180^\circ - 132^\circ = 48^\circ$. **Q15.** (a) 51.4 to 1 decimal place. (b) $7 \times 51.4^\circ = 359.8^\circ$, not exactly 360° — the small rounding shortfalls add up. (c) $4 \times 360^\circ = 1440^\circ$. **Q16.** (a) 36° . (b) 45° . (c) no — $360 \div 50 = 7.2$ is not a whole number; 7 slices use 350° and leave 10° over. **Q17.** (a) $62^\circ - 15^\circ = 47^\circ$. (b) acute. (c) $90^\circ - 62^\circ = 28^\circ$. **Q18.** (a) 30° . (b) 720° . (c) 2520° . (d) 7 full turns; yes — with no turn left over, a cabin that starts at the top finishes at the top.

Fully-worked solutions

Q1. (a) Compare each angle with 90° : 15° and 84° are under 90° , so both are acute; 90° is a right angle; 138° is between 90° and 180° , so it is obtuse. (b) 180° is a straight angle; 275° is between 180° and 360° , so it is a reflex angle; 360° is a full turn. (c) A straight angle is 180° , so the 275° angle goes past it by $275^\circ - 180^\circ = 95^\circ$.

Q2. (a) A quarter turn is $360^\circ \div 4 = 90^\circ$; a half turn is $360^\circ \div 2 = 180^\circ$. (b) $5 \times 360^\circ = 1800^\circ$. (c) One and three-quarter turns is one full turn plus a three-quarter turn: $360^\circ + 270^\circ = 630^\circ$.

Q3. With the minute hand on the 12, the angle is just the number of hour-mark steps times 30° . (a) 2:00 — 2 steps: $2 \times 30^\circ = 60^\circ$. (b) 5:00 — 5 steps: $5 \times 30^\circ = 150^\circ$. (c) 9:00 — 9 steps one way is $9 \times 30^\circ = 270^\circ$, and the other way is $360^\circ - 270^\circ = 90^\circ$. The smaller angle is 90° — a right angle.

Q4. (a) The two scales add to 180° , so the true size is $180^\circ - 128^\circ = 52^\circ$. That is under 90° , so it is acute, which matches the drawing ✓. (b) The true size is $180^\circ - 61^\circ = 119^\circ$. That is between 90° and 180° , so it is obtuse, which matches the drawing ✓. (c) They always add to 180° — the two protractor scales run in opposite directions across the same half turn.

Q5. (a) The two angles at a vertex make a full turn: $360^\circ - 70^\circ = 290^\circ$. (b) $360^\circ - 135^\circ = 225^\circ$. (c) An angle and its reflex angle always add to 360° , a full turn. Check: $70^\circ + 290^\circ = 360^\circ$ ✓ and $135^\circ + 225^\circ = 360^\circ$ ✓.

Q6. (a) Line the protractor up on the vertex and read the inner scale: angle (a) is 43° , acute. (b) Angle (b) reads 116° , obtuse — matching its wide-open look. (c) The small angle at (c) measures 100° , so the dashed reflex angle is $360^\circ - 100^\circ = 260^\circ$. Measurements within 2° of 43° , 116° , 100° and 260° are fine — hand measuring is never exact. (d) Drawing: baseline, protractor centre on the vertex, dot at 110° , join with the ruler. The finished angle should measure 110° (within 2°) and look obtuse.

Q7. (a) The plumb line is vertical and the beam is horizontal, and vertical and horizontal meet at 90° (marked in the picture). So $x = 90^\circ - 26^\circ = 64^\circ$. (b) The reflex angle and the 26° angle make a full turn together: $360^\circ - 26^\circ = 334^\circ$. (c) 26° is under 90° , so it is acute; $x = 64^\circ$ is also under 90° , so it is acute too. A shallow roof pitch leaves both angles acute.

Q8. Step 1: the minute hand makes one full turn — 360° — in 1 hour, and 1 hour is 60 minutes. Step 2: so each minute it turns $360^\circ \div 60 = 6^\circ$. The minute hand turns 6° per minute. Step 3: the hour hand makes one full turn — 360° — in 12 hours, and $12 \times 60 = 720$ minutes. Step 4: so each minute it turns $360^\circ \div 720 = 0.5^\circ$. The hour hand turns 0.5° per minute. $\therefore$ the minute hand turns 6° per minute and the hour hand turns 0.5° per minute, as required.

Q9. (a) At 6:30 the minute hand is $30 \times 6^\circ = 180^\circ$ clockwise from the 12. The hour hand is $6 \times 30^\circ + 30 \times 0.5^\circ = 180^\circ + 15^\circ = 195^\circ$ clockwise from the 12 — it has crept 15° past the 6. The smaller angle between the hands is $195^\circ - 180^\circ = 15^\circ$. (b) At 9:45 the minute hand is $45 \times 6^\circ = 270^\circ$ clockwise from the 12. The hour hand is $9 \times 30^\circ + 45 \times 0.5^\circ = 270^\circ + 22.5^\circ = 292.5^\circ$ clockwise from the 12. The smaller angle between the hands is $292.5^\circ - 270^\circ = 22.5^\circ$. This value is exact, so no rounding is needed.

Q10. (a) $1080 \div 360 = 3$, so a 1080 is exactly 3 full turns. (b) $1080^\circ - 540^\circ = 540^\circ$ more — one and a half turns more. (c) Yes. 1080° is 3 whole turns with nothing left over, and every full turn brings the rider back to the facing it started from, so the rider lands facing the take-off direction.

Q11. (a) Each turn is 360° , so 12 turns is $12 \times 360^\circ = 4320^\circ$ in 1 minute. (b) $4320^\circ \times 5 = 21600^\circ$ in 5 minutes. (c) $12 \times 5 = 60$ full turns in 5 minutes.

Q12. (a) The wall is vertical and the ground is horizontal, so they meet at 90° . The angle between the ladder and the wall is $90^\circ - 63^\circ = 27^\circ$. (b) No. The safe range is 70° to 75° and 63° is below 70° , so the ladder is too flat — it could slide out at the foot. The worker should move the foot of the ladder closer to the wall until the angle at the foot is inside the range. (The card rule here is made up for this book — with real equipment, always follow the instructions on the ladder itself.) (c) The bottom of the safe range is 70° , so the angle needs $70^\circ - 63^\circ = 7^\circ$ more.

Q13. (a) Horizontal and vertical meet at 90° , so the angle with the vertical is $90^\circ - 32^\circ = 58^\circ$. (b) $32^\circ + 9^\circ = 41^\circ$ from the horizontal. (c) $90^\circ - 41^\circ = 49^\circ$ with the vertical. Raising the tilt makes the angle with the vertical smaller by the same 9° : $58^\circ - 9^\circ = 49^\circ$ ✓.

Q14. (a) The backrest turned through $132^\circ - 97^\circ = 35^\circ$. (b) 132° is between 90° and 180° , so it is obtuse. (c) Flat is 180° , so it needs $180^\circ - 132^\circ = 48^\circ$ more.

Q15. (a) The 7 equal sectors share one full turn: $360 \div 7 = 51.428571\dots$, so one sector is 51.4° correct to 1 decimal place. (b) $7 \times 51.4^\circ = 359.8^\circ$ — not exactly 360° . Rounding cuts a tiny amount off each sector, and the seven small cuts add up to a total shortfall of $360^\circ - 359.8^\circ = 0.2^\circ$. The exact total is still 360° : round only for reporting, not in the middle of a calculation. (c) $4 \times 360^\circ = 1440^\circ$.

Q16. (a) The 10 slices share a full turn: $360^\circ \div 10 = 36^\circ$ each. (b) $360^\circ \div 8 = 45^\circ$ each. (c) No. $360 \div 50 = 7.2$, which is not a whole number, so whole 50° slices cannot use up the full turn exactly: 7 slices would use $7 \times 50^\circ = 350^\circ$, leaving $360^\circ - 350^\circ = 10^\circ$ of cake that is not a full slice.

Q17. (a) The forks went from a 15° tilt to a 62° tilt, so the second tilt turned them through $62^\circ - 15^\circ = 47^\circ$. (b) 62° is under 90° , so the tilt angle is acute. (c) The mast is vertical, and vertical and horizontal meet at 90° , so the angle between the forks and the mast is $90^\circ - 62^\circ = 28^\circ$.

Q18. (a) The 12 cabins share a full turn: $360^\circ \div 12 = 30^\circ$ apart at the centre. (b) 2 rpm is 2 full turns per minute: $2 \times 360^\circ = 720^\circ$ per minute. (c) In 3.5 minutes: $720^\circ \times 3.5 = 2520^\circ$. (d) $2520^\circ \div 360^\circ = 7$ — exactly 7 full turns with nothing left over. So yes: every full turn returns a cabin to where it started, and a cabin that began at the top finishes at the top.

Chapter 07 — Test A (multiple choice)

Chapter 07 (Dimension and direction). Materials: pen, paper, ruler, protractor and a basic scientific calculator.

Section A: 12 questions (12 marks). Approximate time: 25 minutes. Answer all questions. In each question, choose the one best answer.

About the data in this test. The rail lines, towns, maps, walks and numbers in this test are **made up** (not real): pretend places and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real place, business or person.

Question 1

The Fernleigh rail line numbers its kilometre posts from Fernleigh Station, which is post 0. Posts toward Pine Hill are positive and posts toward Willow Bend are negative. Willow Bend is at post -4 and Pine Hill is at post 12. How far is it by rail from Willow Bend to Pine Hill?

- A. 8 km
- B. 12 km
- C. 16 km
- D. -16 km

Question 2

On a city grid, a walker goes from the corner at $(2, 5)$ to the corner at $(6, 2)$, staying on the grid lines (no diagonal shortcuts). How many blocks does the walker travel?

- A. 5
- B. 7
- C. 12
- D. 1

Question 3

In a storage room, each crate's position is written as an ordered triple (aisle, bay, level). A crate sits at $(4, 8, 3)$. A worker moves it 2 bays further along its aisle, where bay numbers go up, and 1 level up. What is the crate's new ordered triple?

- A. $(4, 10, 4)$
- B. $(6, 8, 4)$
- C. $(4, 6, 2)$
- D. $(2, 10, 4)$

Question 4

On a 16-point compass rose, which point lies exactly halfway between south and south-west?

- A. SSE
- B. WSW
- C. SW
- D. SSW

Question 5

A walking app gives the bearing $N 20^\circ W$. Which instruction matches this bearing?

- A. Face west, then turn 20° toward the north
- B. Face north, then turn 20° toward the west
- C. Face north, then turn 20° toward the east
- D. Face north-west, then turn 20° toward the west

Question 6

What is the reverse of the bearing $N 35^\circ E$?

- A. $N 35^\circ W$
- B. $S 35^\circ E$
- C. $S 35^\circ W$
- D. $S 55^\circ W$

Question 7

A walking map has a scale of $1:50\,000$. A track measures 3 cm on the map. How long is the track on the ground?

- A. 1.5 km
- B. 15 km
- C. 0.15 km
- D. 150 km

Question 8

A walker sets off facing north and walks 200 m, turns right and walks 100 m, then turns right again and walks 200 m. Where is the walker now, compared with the starting point?

- A. Back at the starting point
- B. 100 m west of the start
- C. 200 m south of the start
- D. 100 m east of the start

Question 9

Mia walks 900 m to school at a steady speed of 60 m per minute. She leaves home at 8:45 am. At what time does she arrive?

- A. 8:55 am
- B. 9:00 am
- C. 9:15 am
- D. 8:30 am

Question 10

How is an angle of 215° classified?

- A. Obtuse
- B. Straight
- C. Reflex
- D. Acute

Question 11

At exactly 4:00, what is the smaller angle between the hour hand and the minute hand of a clock?

- A. 120°
- B. 90°
- C. 150°
- D. 240°

Question 12

The drum of a cement mixer turns at 15 revolutions per minute. Through how many degrees does a mark on the drum turn in 2 minutes?

- A. 5400°
- B. 2700°
- C. 30°
- D. $10\,800^\circ$

End of Test A

Test A — Answers

Que stio n	1	2	3	4	5	6	7	8	9	10	11	12
Ans wer	C	B	A	D	B	C	A	D	B	C	A	D

Test A — Fully-worked solutions

Question	Answer	Working
1	C	Posts run from -4 up to 12 : $12 - (-4) = 16$ km.
2	B	Across $6 - 2 = 4$ blocks and up–down $5 - 2 = 3$ blocks: $4 + 3 = 7$ blocks, with no diagonal shortcut.
3	A	The aisle stays 4 ; the bay goes $8 + 2 = 10$; the level goes $3 + 1 = 4$: $(4, 10, 4)$.
4	D	South is 180° and south-west is 225° ; halfway is 202.5° , the point SSW.
5	B	N 20° W means start facing north, then turn 20° toward the west.
6	C	The reverse bearing swaps N for S and E for W: S 35° W.
7	A	$3 \times 50\,000 = 150\,000$ cm = 1500 m = 1.5 km.
8	D	North 200 m, then east 100 m, then south 200 m: the 200 m north and the 200 m south cancel, leaving 100 m east of the start.
9	B	$900 \div 60 = 15$ min of walking; $8:45$ am plus 15 min is $9:00$ am.
10	C	215° sits between 180° and 360° , so it is a reflex angle.
11	A	At $4:00$ the hour hand is $4 \times 30 = 120^\circ$ from the 12 and the minute hand is at the 12 : the smaller angle is 120° .
12	D	$15 \times 2 = 30$ turns; $30 \times 360 = 10\,800$, so $10\,800^\circ$.

Chapter 07 — Test B (written response)

Chapter 07 (Dimension and direction). Materials: pen, paper, ruler, protractor and a basic scientific calculator.

Section B: 8 questions (39 marks). Approximate time: 45 minutes. Answer all questions in the spaces provided. In all questions where a numerical answer is required, you should only round when instructed to do so. Where more than one mark is available, appropriate working must be shown.

About the data in this test. The rail lines, towns, maps, walks and numbers in this test are **made up** (not real): pretend places and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real place, business or person.

Question 1 (4 marks)

The Tanmara rail line numbers its kilometre posts from Tanmara Station, which is post 0. Posts toward Bellview are positive and posts toward Cobble Hill are negative. Cobble Hill is at post -6 and Bellview is at post 7 .

a. How far is it by rail from Cobble Hill to Bellview? 1 mark

b. A work train leaves Cobble Hill and runs 9 km toward Bellview. Which kilometre post is it at now? 1 mark

c. A new crossing loop is built halfway between Cobble Hill and Bellview. Which post number is it at? Give your answer to 1 decimal place. 2 marks

Question 2 (4 marks)

a. A counter sits at $(3, 7)$ on a game board. It slides 4 squares right and 2 squares down. Write the ordered pair of its new square. 1 mark

b. On a city grid, how many blocks is it from the corner at $(1, 2)$ to the corner at $(5, 8)$, walking along the grid lines only (no diagonal shortcuts)? 1 mark

c. In a storage room, each box is labelled with an ordered triple (row, shelf, level). A box at $(2, 9, 4)$ is moved 3 shelves toward shelf 1, staying in its row, and 2 levels down. Write its new ordered triple. 2 marks

Question 3 (5 marks)

a. Which 16-point compass direction lies exactly halfway between north and north-east? 1 mark

b. Write the bearing $S 45^\circ W$ as a point of the 8-point compass. 1 mark

c. Write the compass point ENE as a bearing in the form $N \theta^\circ E$. 1 mark

d. A fishing boat travels out from the pier on a bearing of $N 25^\circ E$. Write the bearing for its return trip to the pier, and name the two 16-point compass directions this return bearing lies between. 2 marks

Question 4 (4 marks)

A walking map of the Koora Range has a scale of 1:10 000, with north at the top of the map.

a. How many metres on the ground does 1 cm on this map stand for? 1 mark

b. A fire track measures 5.5 cm on the map. How long is it on the ground, in metres? 1 mark

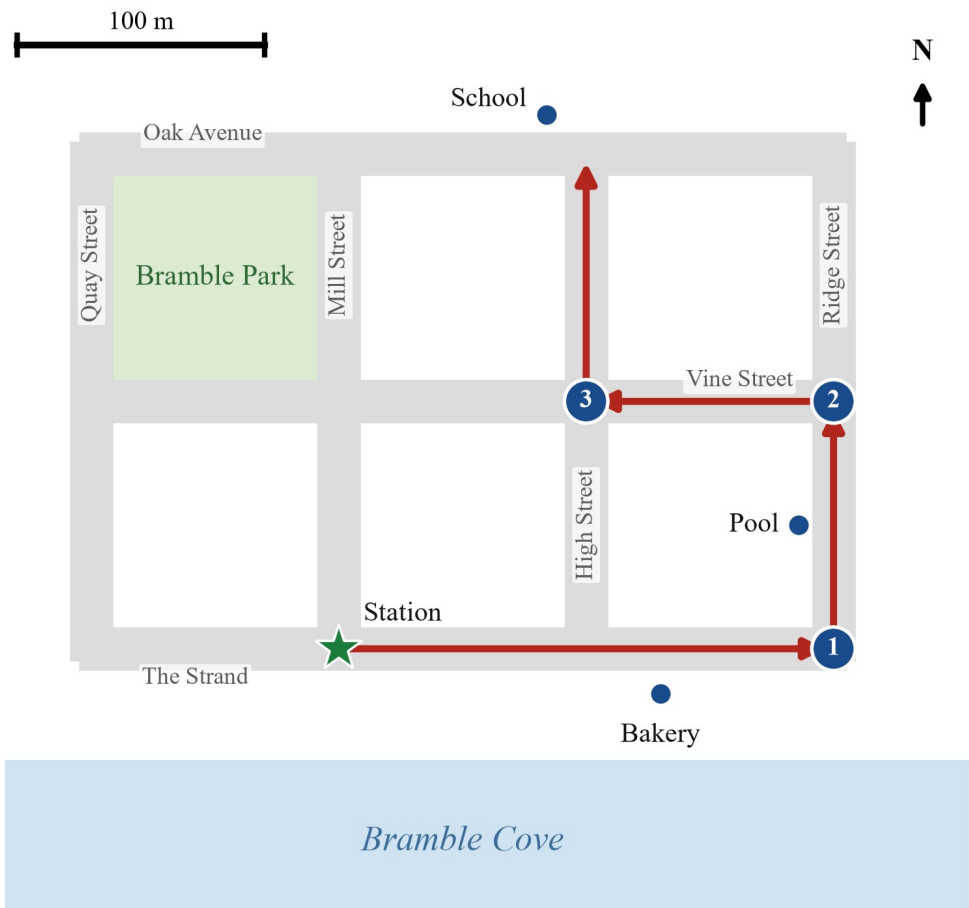
c. From the camp, the lookout lies in a direction turned 45° from north toward the west. Which 8-point compass direction is the lookout from the camp? 1 mark

d. Seen from the lookout, which 8-point compass direction points back to the camp? 1 mark

Question 5 (7 marks)

The map below shows Bramble Cove. Each block is 100 m and north is at the top of the map. A delivery rider starts at the station, on the corner of Mill Street and The Strand, facing east along The Strand, and follows these directions.

- 1. Go 200 m east along The Strand to Ridge Street.
- 2. Turn left into Ridge Street and go 100 m north to Vine Street.
- 3. Turn left into Vine Street and go 100 m west to High Street.
- 4. Turn right into High Street and go 100 m north to Oak Avenue. The school is on this corner.



Made-up data: Bramble Cove, its streets and every place on this map are made up for this book.

- a.** How far does the rider travel in total, from the station to the school? 2 marks

- b.** Name the three turns in the directions, in order (left, right, U-turn or straight on). 3 marks

- c.** The pool is drawn on the west side of Ridge Street. As the rider travels north along Ridge Street in direction 2, is the pool on her left or her right? Explain how you know. 2 marks

Question 6 (5 marks)

A charity walk has three legs: 460 m, then 340 m, then 650 m.

a. How long is the whole walk, in metres and in kilometres?

2 marks

b. A walker averages 4 km/h over the walk. How long does the walk take this walker? Give your answer in minutes, to the nearest minute.

2 marks

c. The walker sets off at 10:26 am. Using your rounded answer to part b, what time does the walker finish?

1 mark

Question 7 (5 marks)

a. Classify each angle as acute, right, obtuse, straight or reflex: 62° , 152° , 271° .

3 marks

b. Two angles around a point make a full turn. One of them is 65° . What is the size of the other (reflex) angle?

1 mark

c. A student measures an acute angle with a protractor, lined up correctly but read on the wrong scale, and records 137° . What is the true size of the angle?

1 mark

Question 8 (5 marks)

a. Through how many degrees does the minute hand of a clock turn in 10 minutes?

1 mark

b. What is the smaller angle between the hour hand and the minute hand at exactly 2:30? Show your working.

2 marks

c. The Ferris wheel at the showground turns at 8 revolutions per minute. Through how many degrees does one cabin turn in 3 minutes?

2 marks

End of Test B

Test B — Answers

Question 1. (a) 13 km (b) post 3 (c) post 0.5

Question 2. (a) (7, 5) (b) 10 blocks (c) (2, 6, 2)

Question 3. (a) NNE (b) SW (c) N 67.5° E (d) S 25° W, between SSW and SW

Question 4. (a) 100 m (b) 550 m (c) NW (d) SE

Question 5. (a) 500 m (b) left, left, right (c) left — facing north, her left hand points west and the pool is on the west side

Question 6. (a) 1450 m, that is 1.45 km (b) 22 minutes (c) 10:48 am

Question 7. (a) acute, obtuse, reflex (b) 295° (c) 43°

Question 8. (a) 60° (b) 105° (c) 8640°

Test B — Fully-worked solutions

Question 1. **a.** Cobble Hill is post -6 and Bellview is post 7 , so the distance is $7 - (-6) = 13$ km. **b.** Starting at -6 and running 9 km toward Bellview adds 9 : $-6 + 9 = 3$, so the train is at post 3 . **c.** Halfway between -6 and 7 is $(-6 + 7) \div 2 = 1 \div 2 = 0.5$. To 1 decimal place, the crossing loop is at post 0.5 .

Question 2. **a.** Across: $3 + 4 = 7$; down: $7 - 2 = 5$. The new square is $(7, 5)$. **b.** Across $5 - 1 = 4$ blocks and up $8 - 2 = 6$ blocks: $4 + 6 = 10$ blocks. **c.** The row stays 2 ; shelves toward shelf 1 : $9 - 3 = 6$; levels: $4 - 2 = 2$. The new triple is $(2, 6, 2)$.

Question 3. **a.** North is 0° and north-east is 45° ; halfway is 22.5° , the point NNE. **b.** $S 45^\circ W$ turns 45° from south toward west, which is exactly the point SW. **c.** ENE sits 67.5° clockwise from north, toward the east: $N 67.5^\circ E$. **d.** The return bearing swaps N for S and E for W: $S 25^\circ W$. It sits at 205° clockwise from north; SSW is 202.5° and SW is 225° , so the return bearing lies between SSW and SW.

Question 4. **a.** 1 cm on the map is $10\,000$ cm on the ground, and $10\,000$ cm = 100 m. **b.** $5.5 \times 100 = 550$ m. **c.** A direction turned 45° from north toward the west is the 8-point direction NW. **d.** The direction back to the camp is straight across the compass rose from NW: SE.

Question 5. **a.** $200 + 100 + 100 + 100 = 500$ m in total. **b.** Direction 2 turns left (east to north), direction 3 turns left (north to west) and direction 4 turns right (west to north): left, left, right. **c.** Facing north, her left hand points west. The pool is on the west side of Ridge Street, so the pool is on her left.

Question 6. **a.** $460 + 340 + 650 = 1450$ m, that is $1\,450$ m or 1.45 km. **b.** Time = distance $\div$ speed = $1.45 \div 4 = 0.3625$ h, and $0.3625 \times 60 = 21.75$ min. To the nearest minute, the walk takes 22 minutes. **c.** $10:26$ am plus 22 minutes is $10:48$ am.

Question 7. **a.** 62° is between 0° and 90° : acute. 152° is between 90° and 180° : obtuse. 271° is between 180° and 360° : reflex. **b.** A full turn minus 65° : $360 - 65 = 295$, so the reflex angle is 295° . **c.** Read on the wrong scale, the two readings add to 180° : $180 - 137 = 43$, so the true angle is 43° — acute, as drawn.

Question 8. **a.** The minute hand turns 6° each minute: $10 \times 6 = 60$, so 60° . **b.** At $2:30$ the minute hand is $30 \times 6 = 180^\circ$ from the 12, and the hour hand is $2 \times 30 + 30 \times 0.5 = 75^\circ$ from the 12. The gap is $180 - 75 = 105^\circ$, which is already the smaller angle. **c.** In 3 minutes the wheel turns $8 \times 3 = 24$ times; $24 \times 360 = 8640$, so a cabin turns through 8640° .