

VCE VM Numeracy Units 3 & 4

Chapter 5 — Relationships

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Chapter 5 Relationships — 5A Variables and formulas in real life

Theory

Formulas in everyday life.

A plant nursery charges a \$40 delivery fee plus \$6 for each tree it supplies. Suppose an order is for 15 trees. The cost is the fee plus 15 lots of \$6:

$$\$40 + 15 \times \$6 = \$40 + \$90 = \$130$$

Now write that rule once, for any number of trees. Let n stand for the number of trees and C for the total cost in dollars. Then:

$$C = 40 + 6 \times n$$

A rule written like this, in maths language, is a **formula**. A formula shows how one quantity depends on another. The letters in it are **variables**: letters that stand for numbers, where the number can change from case to case. Here n changes with the order, and C changes with n . The 40 and the 6 never change, so they are called **constants**.

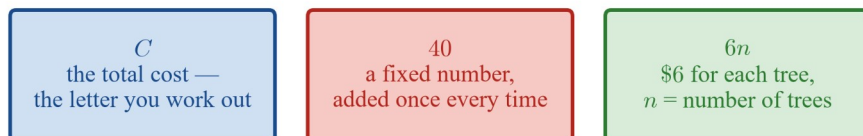
Most formulas are written with the multiplication sign left out, so $6 \times n$ becomes $6n$:

$$C = 40 + 6n$$

(§5E covers these shorter ways of writing algebra in full. This section uses formulas the way they appear in real life.)

one real-life formula

$$C = 40 + 6n$$



n is a variable: a letter that stands for a number that can change

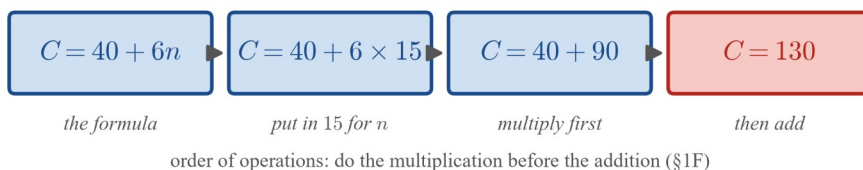
Substituting: putting a number into a formula.

To use a formula, replace each letter with its number, then work the arithmetic out. That is called **substituting**. For 15 trees:

$$C = 40 + 6 \times 15 = 40 + 90 = 130$$

so the order costs \$130. Notice the order of operations: the multiplication 6×15 is done before the 40 is added (§1F).

substitute $n = 15$ into $C = 40 + 6n$

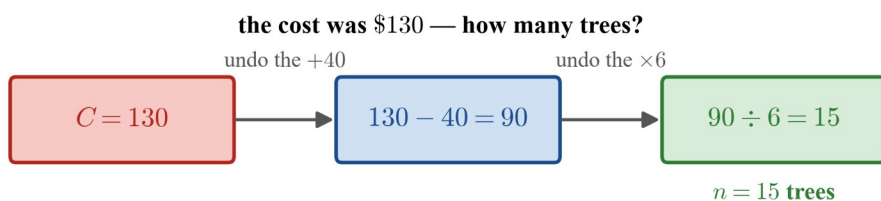


Working backwards: back-tracking.

Sometimes the result is known and the input is what you want. Suppose the nursery's invoice says \$130 and you want to check how many trees were supplied. Run the formula backwards, undoing each step in reverse order with the **inverse operation**:

$130 - 40 = 90$ (subtraction undoes the adding of 40), then $90 \div 6 = 15$ (division undoes the multiplying by 6).

So 15 trees were supplied. Checking forwards agrees: $C = 40 + 6 \times 15 = 40 + 90 = 130$, the amount on the invoice. This backwards move — undo the steps, last one first — is called **back-tracking**, and it works on any formula.



inverse operations run the formula backwards: subtraction undoes addition,

division undoes multiplication

check forwards: $C = 40 + 6 \times 15 = 40 + 90 = 130$, which matches

A gallery of everyday formulas.

These formulas turn up again and again in real life and in this book.

Situation	Formula	What the letters mean
Area of a rectangle	$A = l w$	l = length, w = width
Perimeter of a rectangle	$P = 2l + 2w$	l and w as above
Volume of a rectangular box	$V = l w h$	h = height, with l and w as above
Celsius to Fahrenheit	$F = 1.8 C + 32$	C = temperature in $^{\circ}\text{C}$, F = temperature in $^{\circ}\text{F}$
A job with a fixed fee	$C = f + r n$	f = fixed fee paid once, r = charge per unit, n = number of units

Try three rows of the gallery on easy numbers. A rectangle 3 m by 2 m has area $A = 3 \times 2 = 6 \text{ m}^2$. A rectangle 5 m by 3 m has perimeter $P = 2 \times 5 + 2 \times 3 = 10 + 6 = 16 \text{ m}$. A box 4 m by 3 m by 2 m has volume $V = 4 \times 3 \times 2 = 24 \text{ m}^3$. And the temperature row: freezing point, 0°C , gives $F = 1.8 \times 0 + 32 = 32^{\circ}\text{F}$; a mild 20°C day gives $F = 68^{\circ}\text{F}$; boiling water, 100°C , gives $F = 212^{\circ}\text{F}$.

Describing a relationship in words.

A formula is also a story about how two quantities move together, and being able to tell that story in words is half of what this section is about. Look at each part of the formula and say what it does in the situation.

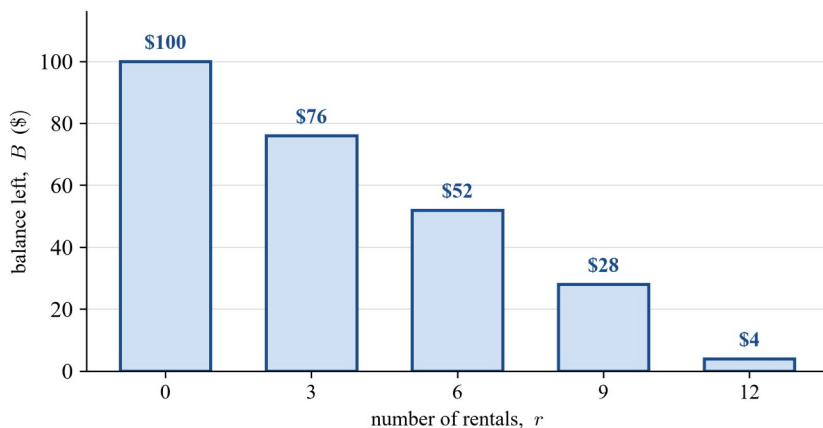
A gift card starts with \$100 on it, and each rental paid for with the card costs \$8. With r = number of rentals and B = balance left in dollars:

$$B = 100 - 8r$$

- The 100 is the **starting value**: put $r = 0$ in and $B = 100$, the balance before any rentals.
- The -8 says the balance **falls** by \$8 for every rental. As r goes up, B goes down: this is a **decreasing** relationship.

After 6 rentals the balance is $B = 100 - 8 \times 6 = 100 - 48 = 52$, so \$52 left. The bars below show the balance dropping by \$8 for each rental. The situation also sets sensible limits on the letters: after 13 rentals the formula would give a negative balance, which a real card cannot have, so r only makes sense up to 12 here.

gift card $B = 100 - 8r$ — as r goes up, B goes down



An **increasing** relationship works the same way with a plus sign. A caterer charges a \$20 booking fee plus \$9 per person, so with p = number of people and T = total cost in dollars, $T = 20 + 9p$. The 20 is paid once whatever the head count, and the +9 says the cost **rises** by \$9 for each extra person.

Building a formula from words.

Most real-life formulas are not handed to you: you build them from a description. The caterer above is a good model. Follow these steps.

1. **Name the variables.** p = number of people, T = total cost in dollars.
2. **Find the fixed part.** The \$20 booking fee is paid once, whatever p is.
3. **Find the changing part.** \$9 per person means 9 lots of p , that is $9p$.
4. **Join the parts.** Total = fixed part + changing part: $T = 20 + 9p$.

Checking the formula on a case you can work out by hand is always worth it. For 30 people:
 $T = 20 + 9 \times 30 = 20 + 270 = 290$, so \$290 — which matches $30 \times \$9 = \$270$ plus the \$20 fee.

When an answer needs rounding.

Substituting sometimes lands on a tidy number, and sometimes it does not. Three friends splitting a \$100 bill equally each owe $\$100 \div 3 = \$33.333\dots$, which is \$33.33 correct to the nearest cent. Whenever an answer in this book needs rounding, the question says so — “correct to the nearest cent”, “to 1 decimal place”. Sometimes the situation chooses the rounding instead: a job needing 4.2 bags of sand must buy 5 bags, because part bags are not sold, so that one rounds *up* (see §1C).

Where formulas lead.

The same skills return throughout Chapter 5: §5B uses formulas for rates (pay per hour, speed per km), §5C meets ratio and proportion, §5D shows relationships as tables and graphs, and §5E sets out the shorter ways of writing algebra. This section is the starting point: read a formula, substitute into it, back-track through it, and say in words what the relationship means.

About the data in this section. The prices, fees, card balances and quantities here are **made up** (not real): pretend nurseries, pretend cards and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

Use substitution to find the value asked for in each case. (a) $A = lw$, with $l = 3.2$ m and $w = 1.5$ m (b) $F = 1.8C + 32$, with $C = 25$ (c) $C = 45 + 12n$, with $n = 8$

Working	Comment
(a) $A = lw = 3.2 \times 1.5 = 4.8$, so $A = 4.8 \text{ m}^2$.	Put 3.2 in for l and 1.5 in for w , then multiply.
(b) $F = 1.8 \times 25 + 32 = 45 + 32 = 77$, so $F = 77^\circ\text{F}$.	Multiply before you add. 25°C is a warm day, and 77°F sounds right for one.
(c) $C = 45 + 12 \times 8 = 45 + 96 = 141$, so $C = \$141$.	Multiply first: $12 \times 8 = 96$. Then add the 45.

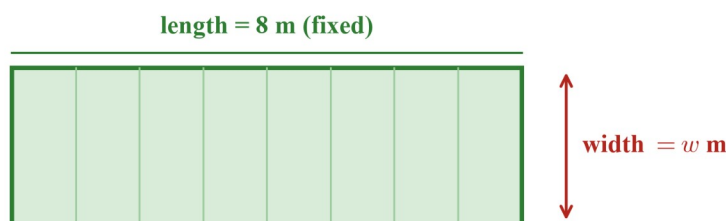
Example 2 — scaffolded

A plumber charges an \$80 call-out fee plus \$45 for each hour of work. The cost of a job is $C = 80 + 45t$, with t = time in hours and C = cost in dollars. (a) Find the cost of a 4-hour job. (b) One job cost \$305. How many hours did it take? (c) What do the 80 and the 45 mean in this situation?

Working	Comment
(a) $C = 80 + 45 \times 4 = 80 + 180 = 260$, so the job costs \$260.	Substitute $t = 4$; multiply before adding.
(b) Back-track: $305 - 80 = 225$, then $225 \div 45 = 5$, so $t = 5$ hours. Check: $80 + 45 \times 5 = 80 + 225 = 305$.	Undo the $+80$ with subtraction, then the $\times 45$ with division — last step first. The check forwards matches the bill.
(c) The 80 is the call-out fee: paid once on every job, whatever the time. The 45 is the charge per hour: the cost rises \$45 for each extra hour.	Every formula of the form fixed part $+$ rate $\times$ number tells you both stories at once.

Example 3 — unscaffolded

A rectangular veggie patch has a length of 8 m, fixed by the fence line it sits against, and a width of w m.



perimeter: $P = 2 \times 8 + 2w = 16 + 2w$ area: $A = 8 \times w = 8w$

the length never changes; only the width, w , is free to vary

- Write a formula for the perimeter P (in m) and a formula for the area A (in m^2) of the patch. Then find P and A when $w = 3$.
- Find w when the perimeter is 30 m.
- Find the area of the patch at that width.

The length never changes, so only the width, w , is free to vary.

The perimeter is two lengths plus two widths: $P = 2 \times 8 + 2w = 16 + 2w$. The area is length times width: $A = 8w$.

- With $w = 3$: $P = 16 + 2 \times 3 = 16 + 6 = 22$ m, and $A = 8 \times 3 = 24 \text{ m}^2$.
- Back-track $P = 30$: undo the $+16$ with $30 - 16 = 14$, then undo the $\times 2$ with $14 \div 2 = 7$, so $w = 7$ m. Check: $2 \times 8 + 2 \times 7 = 16 + 14 = 30$ m, the perimeter given.
- At $w = 7$: $A = 8 \times 7 = 56 \text{ m}^2$.

$\therefore$ the patch has $P = 16 + 2w$ and $A = 8w$; at $w = 3$ m that is $P = 22$ m and $A = 24 \text{ m}^2$, and a perimeter of 30 m means $w = 7$ m with $A = 56 \text{ m}^2$.

Example 4 — unscaffolded

A landscaper quotes a fixed labour fee of \$75 plus \$22 for each square metre of turf laid.

landscaper's quote: $C = 75 + 22n$



n = number of square metres of turf laid

- (a) Write a formula for the cost C (in dollars) in terms of n , the number of square metres of turf laid.
- (b) Find the cost of laying 30 m² of turf.
- (c) One quote came to \$1 241. How many square metres of turf does that quote cover?
- (d) Check your answer to (c) by substituting forwards.
- (e) The cost is the fixed fee plus \$22 for each square metre:

$$C = 75 + 22n$$

- (b) With $n = 30$: $C = 75 + 22 \times 30 = 75 + 660 = 735$, so the job costs \$735.
- (c) Back-track from $C = 1\,241$: undo the $+ 75$ with $1\,241 - 75 = 1\,166$, then undo the $\times 22$ with $1\,166 \div 22 = 53$, so $n = 53$ m².
- (d) Check forwards: $C = 75 + 22 \times 53 = 75 + 1\,166 = 1\,241$, the quoted amount, so the back-tracking is confirmed.

$\therefore$ the quote follows $C = 75 + 22n$; laying 30 m² costs \$735, and a \$1 241 quote covers 53 m² of turf.

Practice questions

Scaffolded

Q1. Find the value of each expression when $x = 5$. (a) $4x$ (b) $x + 13$ (c) $3x - 7$

Q2. Substitute the given numbers into each formula. (a) $A = lw$, with $l = 6$ m and $w = 2.5$ m (b) $P = 2l + 2w$, with $l = 9$ cm and $w = 4$ cm (c) $F = 1.8C + 32$, with $C = 30$

Q3. A print shop charges a \$30 set-up fee plus \$8 per poster. The cost of an order is $C = 30 + 8p$, with p = number of posters and C in dollars. (a) Find the cost of an order of 15 posters. (b) Find the cost of an order of 25 posters. (c) What do the 30 and the 8 mean in this situation?

Q4. A phone plan costs $M = 19 + 4g$ dollars a month, with g = gigabytes of data used that month. (a) Find M when $g = 3$. (b) Find M when $g = 6$. (c) Describe in words how the monthly cost changes as the data used goes up.

Q5. A job cost \$90 under the formula $C = 55 + 7n$. Work backwards to find n . (a) Undo the $+ 55$: what is $90 - 55$? (b) Undo the $\times 7$: what is your answer to (a) divided by 7? (c) Check your n by substituting it forwards into $C = 55 + 7n$.

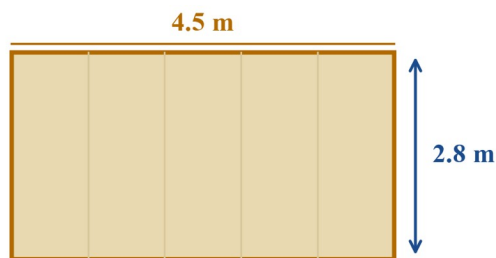
Q6. A square has side length s cm and perimeter $P = 4s$ cm. (a) Find P when $s = 7.5$. (b) Find s when $P = 44$. Check by substituting forwards. (c) Describe in words the relationship between P and s .

Unscaffolded

Q7. A cupcake order costs $C = 35 + 12b$ dollars, with b = number of boxes of cupcakes. (a) What do the 35 and the 12 mean in this situation? (b) Find the cost of an order of 9 boxes. (c) Find the cost of an order of 14 boxes.

Q8. A dog groomer charges $G = 25 + 28d$ dollars, with d = number of dogs groomed. One day's takings were \$137.
 (a) How much of the \$137 comes from the per-dog charges, once the fixed part of the formula is taken off? (b) Hence find how many dogs were groomed that day. (c) Check your answer by substituting forwards.

Q9. A rectangular garden bed measures 4.5 m by 2.8 m.



a rectangular garden bed

- Find the area of the garden bed.
- Find the perimeter of the garden bed.
- Edging strip is sold in 15 m rolls. Is one roll enough to go all the way around the bed? How much of the roll is left over?

Q10. Temperatures in degrees Celsius (C) and degrees Fahrenheit (F) are related by $F = 1.8C + 32$.

temperature conversion

$$F = 1.8C + 32$$

C = degrees Celsius ($^{\circ}\text{C}$) F = degrees Fahrenheit ($^{\circ}\text{F}$)

- Find F when $C = 15$.
- Find F when $C = -5$.
- A reading of 100°F is what temperature in degrees Celsius, correct to 1 decimal place?

Q11. Two cleaning firms quote for a job lasting h hours. Sparkle Cleaning: $C = 90 + 30h$ Shine Cleaning: $C = 50 + 35h$
 (a) What does Sparkle Cleaning charge for a 4-hour job? (b) What does Shine Cleaning charge for a 4-hour job? (c) Which firm is cheaper for a 4-hour job, and by how much?

Q12. A planter box is a rectangular box 1.2 m long, 0.8 m wide and 0.5 m deep. (a) Find the volume of the box. (b) Potting mix is sold in bags that each hold 0.1 m^3 . How many whole bags are needed to fill the box? (c) Explain why you rounded the way you did in (b).

Q13. A gym charges a joining fee plus \$12 a month. The total paid after m months is $T = f + 12m$ dollars, where f is the joining fee. After 7 months one member has paid \$149 altogether. (a) How much of the \$149 is the 7 months of monthly charges? (b) Hence find the joining fee, f . Check by substituting forwards. (c) What will that member have paid altogether after 10 months?

Q14. A community hall costs $H = 110 + 35n$ dollars to hire for n days. A group has a budget of \$400 for the hire. (a) Show that hiring the hall for 4 days costs \$250. Write one step per line. (b) Show that hiring the hall for 9 days is over the group's budget. Write one step per line. (c) What is the greatest whole number of days the group can hire the hall for? Explain your reasoning.

Q15. Gia's savings account follows $S = 210 + 65w$, with S = amount in the account in dollars and w = number of weeks since she started saving. (a) How much is in the account after 4 weeks? (b) After how many weeks does the

account first hold \$860? Check by substituting forwards. (c) Describe in words what the 210 and the 65 tell you about Gia's saving.

Q16. A car park charges \$3 for the first hour and \$2 for each extra hour, with part hours charged as whole hours.



(a) Copy and complete the table for stays of 1, 2, 3 and 4 hours.

Hours parked, h	1	2	3	4
Cost, C (\$)				

Made-up data: the prices in this table are made up for this section.

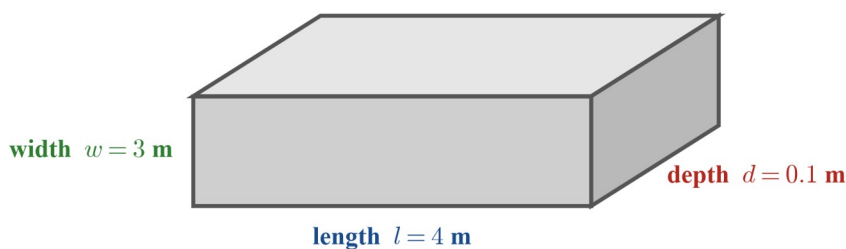
(b) Find a formula for C in terms of h that fits your table.

(c) Use your formula to find the cost of parking for 8 hours, and check it against the sign's prices.

Q17. Fatima's group hires a bus for a trip. The hire costs $C = 85 + 1.25k$ dollars, with k = kilometres travelled. (a) What do the 85 and the 1.25 mean in this situation? (b) The trip is 76 km. Find the cost of the hire. (c) The cost is split equally between the 14 people in the group. How much does each person pay, correct to the nearest cent?

Q18. A concrete slab is a rectangular box 4 m long, 3 m wide and 0.1 m deep.

volume of a rectangular slab: $V = l \times w \times d$



(a) Find the volume V of concrete in the slab, using $V = l \times w \times d$.

(b) The job costs $C = 175V + 90$ dollars. Find C for this slab.

(c) Another job was quoted at \$527.50. What volume of concrete does that quote cover? Check by substituting forwards.

(d) Describe in words how the cost C changes as the volume V goes up, and say what the 90 means.

Answers

Q1. (a) 20. (b) 18. (c) 8. **Q2.** (a) 15 m^2 . (b) 26 cm. (c) 86°F . **Q3.** (a) \$150. (b) \$230. (c) the 30 is the set-up fee, charged once per order; the 8 is the price per poster. **Q4.** (a) \$31. (b) \$43. (c) the cost starts at \$19 and rises \$4 for each extra gigabyte used. **Q5.** (a) 35. (b) $35 \div 7 = 5$, so $n = 5$. (c) $55 + 7 \times 5 = 55 + 35 = 90$. **Q6.** (a) 30 cm. (b) 11 cm; check $4 \times 11 = 44$. (c) P is always 4 times s , so P grows whenever s grows. **Q7.** (a) the 35 is a fixed fee charged on every order; the 12 is the price per box. (b) \$143. (c) \$203. **Q8.** (a) \$112. (b) 4 dogs. (c) $25 + 28 \times 4 = 25 + 112 = 137$. **Q9.** (a) 12.6 m^2 . (b) 14.6 m. (c) yes, one roll is enough, with 0.4 m left over. **Q10.** (a) 59°F . (b) 23°F . (c) 37.8°C , correct to 1 decimal place. **Q11.** (a) \$210. (b) \$190. (c) Shine Cleaning, by \$20. **Q12.** (a) 0.48 m^3 . (b) 5 bags. (c) 4 bags hold only 0.4 m^3 , which would not fill the box, and part bags cannot be bought — so round up. **Q13.** (a) \$84. (b) \$65; check $65 + 84 = 149$. (c) \$185. **Q14.** (a) \$250. (b) \$425, which is more than \$400. (c) 8 days: 8 days costs \$390, inside the budget, while 9 days costs \$425, over it. **Q15.** (a) \$470. (b) 10 weeks; check $210 + 65 \times 10 = 860$. (c) the account starts at \$210 and grows by \$65 each week. **Q16.** (a) \$3, \$5, \$7, \$9. (b) $C = 2h + 1$. (c) \$17; check $3 + 2 \times 7 = 17$. **Q17.** (a) the 85 is the fixed hire fee, paid whatever the distance; the 1.25 is the charge per kilometre. (b) \$180. (c) \$12.86 each, correct to the nearest cent. **Q18.** (a) 1.2 m^3 . (b) \$300. (c) 2.5 m^3 ; check $175 \times 2.5 + 90 = 527.50$. (d) C rises by \$175 for each extra m^3 of concrete; the \$90 is a fixed delivery fee charged on every job.

Fully-worked solutions

Q1. (a) $4x = 4 \times 5 = 20$. (b) $x + 13 = 5 + 13 = 18$. (c) $3x - 7 = 3 \times 5 - 7 = 15 - 7 = 8$.

Q2. (a) $A = lw = 6 \times 2.5 = 15 \text{ m}^2$. (b) $P = 2l + 2w = 2 \times 9 + 2 \times 4 = 18 + 8 = 26 \text{ cm}$. (c) $F = 1.8 \times 30 + 32 = 54 + 32 = 86^\circ\text{F}$.

Q3. (a) $C = 30 + 8 \times 15 = 30 + 120 = 150$, so \$150. (b) $C = 30 + 8 \times 25 = 30 + 200 = 230$, so \$230. (c) The 30 is the set-up fee: it is charged once per order, whatever the number of posters. The 8 is the price per poster: the cost rises \$8 for each extra poster.

Q4. (a) $M = 19 + 4 \times 3 = 19 + 12 = 31$, so \$31. (b) $M = 19 + 4 \times 6 = 19 + 24 = 43$, so \$43. (c) The plan costs \$19 even with no data used, and the cost rises \$4 for each extra gigabyte — an increasing relationship.

Q5. (a) $90 - 55 = 35$. (b) $35 \div 7 = 5$, so $n = 5$. (c) Check forwards: $55 + 7 \times 5 = 55 + 35 = 90$, the cost given.

Q6. (a) $P = 4 \times 7.5 = 30 \text{ cm}$. (b) $44 \div 4 = 11$, so $s = 11 \text{ cm}$. Check: $4 \times 11 = 44$, the perimeter given. (c) The perimeter is always 4 times the side length, so P grows whenever s grows; double s and P doubles.

Q7. (a) The 35 is a fixed fee charged on every order, and the 12 is the price per box of cupcakes. (b) $C = 35 + 12 \times 9 = 35 + 108 = 143$, so \$143. (c) $C = 35 + 12 \times 14 = 35 + 168 = 203$, so \$203.

Q8. (a) $137 - 25 = 112$, so \$112 of the takings comes from the per-dog charges. (b) $112 \div 28 = 4$, so $d = 4$ dogs were groomed. (c) Check forwards: $25 + 28 \times 4 = 25 + 112 = 137$, the day's takings.

Q9. (a) $A = lw = 4.5 \times 2.8 = 12.6 \text{ m}^2$. (b) $P = 2l + 2w = 2 \times 4.5 + 2 \times 2.8 = 9 + 5.6 = 14.6 \text{ m}$. (c) The roll holds 15 m and the perimeter is 14.6 m, so yes, one roll is enough. Left over: $15 - 14.6 = 0.4 \text{ m}$.

Q10. (a) $F = 1.8 \times 15 + 32 = 27 + 32 = 59^\circ\text{F}$. (b) $F = 1.8 \times (-5) + 32 = -9 + 32 = 23^\circ\text{F}$. (c) Back-track: $100 - 32 = 68$, then $68 \div 1.8 = 37.777\dots$, which is 37.8°C correct to 1 decimal place. Check: putting 37.8 back into the formula returns a touch over 100°F , exactly as expected after rounding to 1 decimal place.

Q11. (a) Sparkle: $C = 90 + 30 \times 4 = 90 + 120 = 210$, so \$210. (b) Shine: $C = 50 + 35 \times 4 = 50 + 140 = 190$, so \$190. (c) Shine Cleaning is cheaper for a 4-hour job, by $210 - 190 = 20$, that is \$20.

Q12. (a) $V = lwh = 1.2 \times 0.8 \times 0.5 = 0.48 \text{ m}^3$. (b) $0.48 \div 0.1 = 4.8$ bags. Bags are sold whole: 4 bags hold only $4 \times 0.1 = 0.4 \text{ m}^3$, which is not enough, so round up to 5 bags (which hold 0.5 m^3 , enough to fill the box). (c) Part bags cannot be bought, and 4 bags would leave the box unfilled, so the situation forces rounding *up* rather than to the nearest whole number (see §1C).

Q13. (a) $12 \times 7 = 84$, so \$84 of the \$149 is monthly charges. (b) $f = 149 - 84 = 65$, so the joining fee is \$65. Check: $65 + 84 = 149$. (c) $T = 65 + 12 \times 10 = 65 + 120 = 185$, so \$185 altogether after 10 months.

Q14. (a) $35 \times 4 = 140$ $110 + 140 = 250$ So hiring the hall for 4 days costs \$250. (b) $35 \times 9 = 315$ $110 + 315 = 425$ \$425 is more than the \$400 budget, so 9 days is over the group's budget. (c) Money left for daily charges after the fixed part: $400 - 110 = 290$. Days that buys: $290 \div 35 = 8\frac{2}{7}$, which sits between 8 and 9, so test both. 8 days:

$H = 110 + 35 \times 8 = 110 + 280 = 390$, inside the budget. 9 days: $H = 110 + 35 \times 9 = 425$, over the budget. The greatest whole number of days is 8.

Q15. (a) $S = 210 + 65 \times 4 = 210 + 260 = 470$, so \$470. (b) $860 - 210 = 650$, then $650 \div 65 = 10$, so $w = 10$ weeks. Check forwards: $210 + 65 \times 10 = 210 + 650 = 860$. (c) The 210 is what was in the account when Gia started (week 0), and the 65 is what she puts in each week — the account grows by \$65 every week.

Q16. (a) The first hour costs \$3 and each hour after that adds \$2: 2 hours: $3 + 2 = 5$; 3 hours: $5 + 2 = 7$; 4 hours: $7 + 2 = 9$ (that is, $3 + 2 + 2 + 2$). The table row is \$3, \$5, \$7, \$9. (b) The first hour costs \$3 and each of the remaining $h - 1$ hours costs \$2, so $C = 3 + 2(h - 1) = 2h + 1$. Testing against the table: $h = 1$ gives 3, $h = 2$ gives 5, $h = 3$ gives 7, $h = 4$ gives 9 — all four match. (c) $C = 2 \times 8 + 1 = 17$, so \$17. Check against the sign: the first hour at \$3 plus 7 extra hours at \$2 is $3 + 2 \times 7 = 3 + 14 = 17$, the same amount.

Q17. (a) The 85 is the fixed hire fee, paid whatever the distance travelled, and the 1.25 is the charge per kilometre. (b) $C = 85 + 1.25 \times 76 = 85 + 95 = 180$, so \$180. (c) $\$180 \div 14 = \$12.8571\dots$ per person, which is \$12.86 correct to the nearest cent.

Q18. (a) $V = l \times w \times d = 4 \times 3 \times 0.1 = 1.2 \text{ m}^3$. (b) $C = 175 \times 1.2 + 90 = 210 + 90 = 300$, so \$300. (c) $527.50 - 90 = 437.50$, then $437.50 \div 175 = 2.5$, so $V = 2.5 \text{ m}^3$. Check forwards: $175 \times 2.5 + 90 = 437.50 + 90 = 527.50$, the quote given. (d) The cost rises by \$175 for each extra cubic metre of concrete — the 175 is the price per m^3 — and the 90 is a fixed delivery fee, charged on every job whatever the volume.

Chapter 5 Relationships — 5B Rates of change

Theory

What a rate is.

Apples at \$4 a kilogram. Pay at \$25 an hour. A car doing 80 kilometres an hour. Each of these is a **rate**: a comparison of two quantities measured in *different* units, saying how much of one there is **per** one unit of the other. The word *per* is the signal — and *per* means **divide**.

A rate is a cousin of the ratio from §2G, with one difference. A ratio compares quantities of the **same** kind (girls to boys, cups of flour to eggs). A rate compares quantities of **different** kinds (kilometres to hours, dollars to hours), so its units never cancel: a rate always carries its units with it.

Writing and reading a rate.

A rate is written with a slash, and the slash reads as “per”:

- 80 km/h reads as “80 kilometres **per hour**” — every 1 hour of travel covers 80 km.
- \$25/h reads as “25 dollars **per hour**” — every 1 hour of work earns \$25.
- 12 L/min reads as “12 litres **per minute**” — every 1 minute delivers 12 L.

Here are the rates this section uses most:

Rate	Reads as	Where you meet it
km/h	kilometres per hour	speed signs, car speedometers
m/s	metres per second	sprints, the speed of a thrown ball
rev/min (RPM)	revolutions per minute — one revolution is one full turn	drills, wheels, washing-machine spins
L/min, L/h	litres per minute, litres per hour	hoses, taps, fuel pumps
\$/h	dollars per hour	job ads, payslips
\$/m ³	dollars per cubic metre	water bills, loads of mulch or concrete
beats/min	beats per minute	pulse checks, fitness trackers
\$/kg	dollars per kilogram	unit prices on the shelf (§2G)

Most rates are “per **one** unit”, but a few are per some other fixed amount. Car fuel use is written in **litres per 100 km** (L/100 km), because one kilometre uses less than a litre. The idea is the same; only the block size changes.

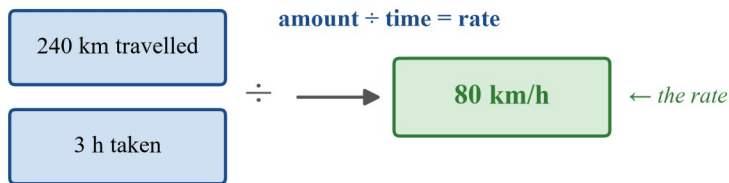
Finding a rate: divide.

To find a rate, divide the amount by the number of units it took:

$$\text{rate} = \text{amount} \div \text{time (or other unit)}$$

A family drives 240 km in 3 hours at a steady speed. The rate is

$240 \div 3 = 80$, and the units come along for the ride: 80 km/h.

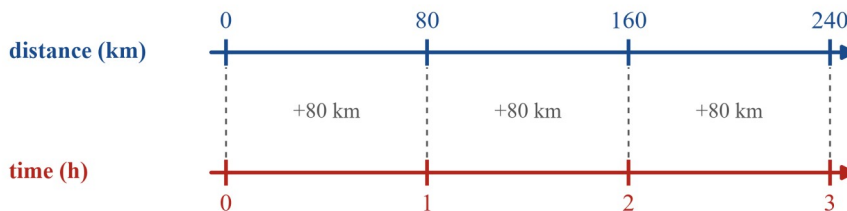


80 km/h reads as “80 kilometres per 1 hour” — the slash / means “per”

Always write the units in a rate. “80” alone is not a rate; “80 km/h” is.

Steady rates.

Real speeds and flows change from moment to moment — a car slows for corners, a hose splutters. In this section every rate is a **steady rate**: one that stays the same the whole time, so the same division works anywhere in the job. A steady rate of 80 km/h adds 80 km for every extra hour, exactly as the double number line shows:



a steady rate of 80 km/h: every extra hour adds another 80 km

A quick direction check from §1C still applies: more hours must mean more kilometres, and a bigger rate must mean a bigger amount in the same time.

Using a rate: three jobs.

Every rate question is one of three jobs. The same rate, 80 km/h, serves all three:

Know the RATE and the TIME — find the AMOUNT: multiply

$$80 \text{ km/h} \times 2.5 \text{ h} = 200 \text{ km}$$

Know the AMOUNT and the TIME — find the RATE: divide

$$240 \text{ km} \div 3 \text{ h} = 80 \text{ km/h}$$

Know the AMOUNT and the RATE — find the TIME: divide

$$200 \text{ km} \div 80 \text{ km/h} = 2.5 \text{ h}$$

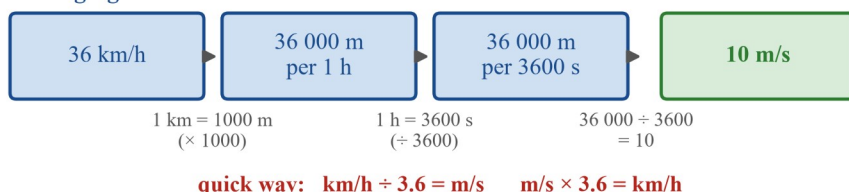
- **Multiply** when you know the rate and want the amount.
- **Divide (amount $\div$ time)** when you know the amount and the time and want the rate.
- **Divide (amount $\div$ rate)** when you know the amount and the rate and want the time.

Pick the job first, then the operation follows.

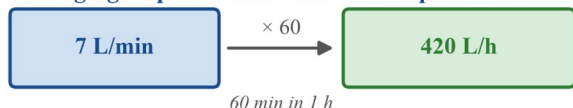
Changing the units of a rate.

Sometimes a rate arrives in one pair of units and is needed in another — a speed in km/h wanted in m/s, or a “per minute” flow wanted “per hour”. Change the top unit and the bottom unit, one at a time, using the conversions from §4A and §4D:

Changing 36 km/h into m/s



Changing a “per minute” rate into a “per hour” rate



The same trick turns any “per minute” rate into a “per hour” rate: there are 60 minutes in 1 hour, so multiply by 60. A tap dripping at 7 L/min wastes $7 \times 60 = 420$ L in an hour.

When a rate needs rounding.

Rates come from division, and division often refuses to finish. A job that pays \$100 for 3 hours of work has an hourly rate of $\$100 \div 3 = \$33.3333\dots$, which is \$33.33 correct to the nearest cent. Whenever an answer in this book needs rounding, the question says so — “correct to the nearest cent”, “to 1 decimal place”. Keep the full unrounded value in your calculator for any later steps, and round only where the question tells you to.

About the data in this section. The payslips, prices, timers, tanks and machine counts here are **made up** (not real): pretend people, pretend workplaces and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

Tia’s payslip shows 25 hours worked and total pay of \$937.50 for the month.

PAYSIP — Tia	
Hours worked	25
Total pay	\$937.50
Pay period: one month	

- Find Tia’s hourly rate of pay.
- How much would she earn for a 32-hour week at this rate?
- How many hours must she work to earn \$1 500 at this rate?

Working

- (a) rate = pay $\div$ hours: $\$937.50 \div 25 = \$37.50$ per hour.
- (b) pay = rate $\times$ hours: $\$37.50 \times 32 = \$1\,200.00$.
- (c) hours = pay $\div$ rate: $\$1\,500 \div \$37.50 = 40$ hours.
- Check:** $40 \times \$37.50 = \$1\,500$, the amount asked for.

Comment

A rate is one quantity divided by the other. The units travel with it: dollars per hour.

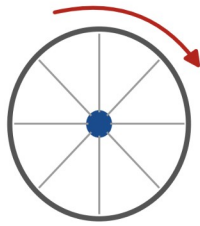
Rate and time known, amount wanted — multiply.

Amount and rate known, time wanted — divide.

Undo the division with a multiplication.

Example 2 — scaffolded

A bicycle wheel spinning on a test rig makes 2 250 turns in 30 minutes, at a steady rate.



1 revolution = 1 full turn

Test rig readout
turns counted: 2 250
time: 30 min

- (a) Find the rate in revolutions per minute (RPM).
- (b) How many turns does the wheel make in 12 minutes?
- (c) How many turns does it make in 1 hour?

Working

- (a) $2\,250 \div 30 = 75$. The rate is 75 revolutions per minute, written 75 RPM.
- (b) $75 \times 12 = 900$ turns.
- (c) 1 hour is 60 minutes, so $75 \times 60 = 4\,500$ turns.

Comment

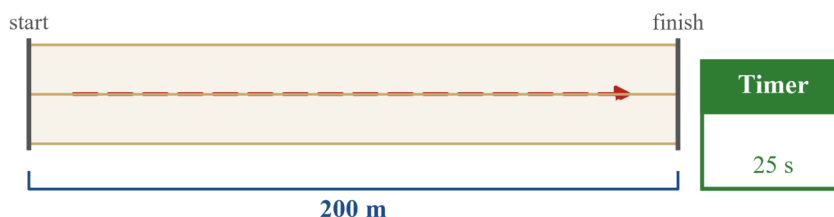
“Per minute” tells you to divide by the minutes.

Rate and time known — multiply.

Put the time in the **same unit** as the rate before multiplying (§4D).

Example 3 — unscaffolded

Jordan runs 200 m in 25 s at a steady speed.



- (a) Find Jordan’s speed in m/s.
- (b) Write this speed in km/h.
- (c) A cyclist rides past at a steady 30 km/h. Who is travelling faster, and by how much?
- (d) Speed is a rate: distance $\div$ time.

$200 \div 25 = 8$, so Jordan’s speed is 8 m/s.

- (b) In 1 hour, which is 3600 s, Jordan would cover $8 \times 3600 = 28\,800$ m, and 28 800 m is 28.8 km. So the speed is 28.8 km/h. (The quick way agrees: $8 \times 3.6 = 28.8$.)

(c) Compare like with like: 30 km/h against 28.8 km/h. The cyclist is faster, by $30 - 28.8 = 1.2$ km/h.

∴ Jordan runs at 8 m/s, which is 28.8 km/h, and the cyclist is travelling faster by 1.2 km/h.

Example 4 — unscaffolded

Two garden yards sell the same mulch by the truckload. Yard 1 charges \$351 for a load of 13 m³. Yard 2 charges \$312 for a load of 12 m³. Which yard is cheaper per cubic metre, and how much would you save by buying 15 m³ from the cheaper yard?

Yard 1 — one truck load	Yard 2 — one truck load
mulch delivered: 13 m ³	mulch delivered: 12 m ³
price: \$351	price: \$312

Find each yard's rate in dollars per cubic metre by dividing price by volume.

Yard 1: $\$351 \div 13 = \$27$ per m³. Yard 2: $\$312 \div 12 = \$26$ per m³.

Yard 2 is cheaper, by \$1 per m³. For 15 m³:

Yard 1: $15 \times \$27 = \$405$. Yard 2: $15 \times \$26 = \$390$.

The saving is $\$405 - \$390 = \$15$. A direction check from §1C agrees: the cheaper rate must give the cheaper total for the same volume, and it does.

∴ Yard 2 is the cheaper mulch at \$26 per m³, and buying 15 m³ there saves \$15.

Practice questions

Scaffolded

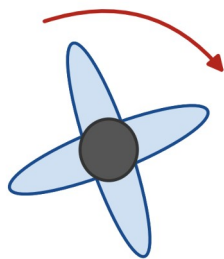
Q1. Write each of these as a rate with “per” in it. (a) 255 km travelled in 3 h (b) \$187.50 earned in 15 h of work (c) 900 revolutions in 15 min

Q2. Use each rate to find the missing amount. (a) A car travels at a steady 90 km/h for 4 h. How far does it go? (b) A casual worker is paid \$32.40 per hour. What is the pay for 7 h of work? (c) A hose delivers 12 L per minute. How many litres does it deliver in 25 min?

Q3. Use each rate to find the missing time. (a) A walker moves at a steady 5 km/h. How long does 12.5 km take? (b) A tap fills a 56 L bucket at 8 L per minute. How many minutes does the bucket take to fill? (c) A job pays \$30 per hour. How many hours of work earn \$480?

Q4. Change each rate into the new units. (a) 3 L/min into L/h (b) 1 200 rev/min into rev/h (c) 2 m/s into km/h

Q5. A ceiling fan makes 1 050 turns in 10 minutes, at a steady rate.



1 turn = 1 revolution

Counted by hand
1 050 turns in 10 minutes

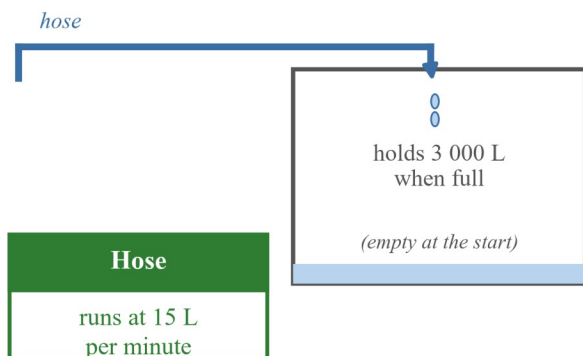
- Find the fan's rate in revolutions per minute.
- How many turns does the fan make in 4 minutes?
- How many turns does it make in 1 hour?

Q6. Find each rate, rounded as stated. (a) A worker is paid \$499.40 for 34 h of work. Find the hourly rate of pay, correct to the nearest cent. (b) A swimmer covers 200 m in 31 s. Find the speed in m/s, to 1 decimal place. (c) A load of topsoil costs \$112 for 4.5 m³. Find the price per cubic metre, correct to the nearest cent.

Unscaffolded

Q7. Ruby runs 400 m in 62.5 s at a steady speed. (a) Find her speed in m/s. (b) Write this speed in km/h. (c) At this speed, how far does she run in 90 s?

Q8. A water tank holds 3 000 L when full, and is empty at the start. A hose runs into it at a steady 15 L per minute.



- How many minutes does the tank take to fill?
- Write this time in hours and minutes.
- A second hose, running at 10 L per minute, is turned on as well. With both hoses running, how long does the tank take to fill? Give your answer in hours.

Q9. Two part-time jobs are advertised. Job A pays \$627.90 for 23 h. Job B pays \$561.60 for 20 h. (a) Find the hourly rate of pay for each job. (b) Which job pays more per hour, and by how much? (c) Find the pay for a 30-hour week in each job, and the difference between the two weekly pays.

Q10. A delivery van travels 480 km on 38.4 L of diesel. (a) Find the van's fuel use in litres per 100 km. (b) How much diesel does a 700 km trip use? (c) Diesel costs \$1.94 a litre. Find the fuel cost of the 700 km trip.

Q11. The wheel of a bench grinder makes 3 150 turns in 45 s, at a steady rate. (a) Find the rate in revolutions per second. (b) Write this rate in revolutions per minute (RPM). (c) How many turns does the wheel make in 5 min?

Q12. A concrete truck delivers 9 m³ of concrete, and the bill is \$1 660.50. (a) Find the price per cubic metre. (b) Find the cost of 6.5 m³ of the same concrete. (c) How many cubic metres of this concrete can a builder buy for \$3 690?

Q13. Speeds in different units. (a) Write a speed of 25 m/s in km/h. (b) Write a speed of 27 km/h in m/s. (c) In a 60 km/h speed zone, a car is timed over 300 m and takes 17.5 s. Find its speed in m/s, to 2 decimal places, and then in km/h, to 1 decimal place. Was the car travelling over the speed limit? If so, by how much, to 1 decimal place?

Q14. A bakery oven bakes loaves at a steady rate. It bakes 240 loaves in 4 h.

Time (h)	0	2	4	6	8
Loaves baked	0	120	240		

Made-up data: the numbers in this table are made up for this section.

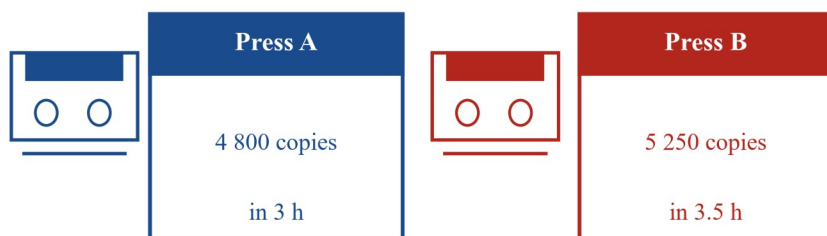
- Find the baking rate in loaves per hour.
- Fill in the two empty cells of the table.
- How many hours does the oven take to bake 900 loaves?

Q15. A ferry crosses 4.8 km of harbour in 6 min, at a steady speed. (a) Show that the ferry's speed is 800 m per minute. Write one step per line. (b) Find the ferry's speed in m/s, to 1 decimal place. (c) Find the ferry's speed in km/h.

Q16. Just after a training run, Mei counts 135 heartbeats in 90 s. (a) Find her heart rate in beats per minute. (b) Her resting heart rate is 72 beats per minute. How many more beats per minute is her rate just after training? (c) At the after-training rate, how many beats does her heart make in 24 h?

Q17. A tap drips at a steady 35 mL per minute. (a) How many mL of water does the tap lose in 1 hour? (b) How many litres does it lose in 1 day of 24 h? (c) Water costs \$3.10 per kilolitre (kL), where 1 kL = 1 000 L. Find the cost of the water this tap wastes in one year of 365 days, correct to the nearest cent.

Q18. Two printing presses run at steady rates. Press A prints 4 800 copies in 3 h. Press B prints 5 250 copies in 3.5 h.



- Find the rate of each press in copies per hour.
- Which press is faster?
- In an 8-hour day, how many more copies does the faster press print than the slower one?
- Press A runs on its own. How long does it take to print 10 000 copies? Give your answer in hours and minutes.

Answers

Q1. (a) 85 km/h. (b) \$12.50 per hour. (c) 60 rev/min (60 RPM). **Q2.** (a) 360 km. (b) \$226.80. (c) 300 L. **Q3.** (a) 2.5 h. (b) 7 min. (c) 16 h. **Q4.** (a) 180 L/h. (b) 72 000 rev/h. (c) 7.2 km/h. **Q5.** (a) 105 RPM. (b) 420 turns. (c) 6 300 turns. **Q6.** (a) \$14.69 per hour, correct to the nearest cent. (b) 6.5 m/s, to 1 decimal place. (c) \$24.89 per m³, correct to the nearest cent. **Q7.** (a) 6.4 m/s. (b) 23.04 km/h. (c) 576 m. **Q8.** (a) 200 min. (b) 3 h 20 min. (c) 2 h. **Q9.** (a) Job A \$27.30 per hour; Job B \$28.08 per hour. (b) Job B, by \$0.78 per hour. (c) Job A \$819.00, Job B \$842.40; difference \$23.40. **Q10.** (a) 8 L per 100 km. (b) 56 L. (c) \$108.64. **Q11.** (a) 70 rev/s. (b) 4 200 RPM. (c) 21 000 turns. **Q12.** (a) \$184.50 per m³. (b) \$1 199.25. (c) 20 m³. **Q13.** (a) 90 km/h. (b) 7.5 m/s. (c) 17.14 m/s and 61.7 km/h; yes, over the limit by 1.7 km/h. **Q14.** (a) 60 loaves per hour. (b) 360 and 480. (c) 15 h. **Q15.** (a) $4\,800 \div 6 = 800$, so 800 m per minute. (b) 13.3 m/s, to 1 decimal place. (c) 48 km/h. **Q16.** (a) 90 beats per minute. (b) 18 beats per minute more. (c) 129 600 beats. **Q17.** (a) 2 100 mL. (b) 50.4 L. (c) \$57.03, correct to the nearest cent. **Q18.** (a) Press A 1 600 copies per hour; Press B 1 500 copies per hour. (b) Press A. (c) 800 copies. (d) 6 h 15 min.

Fully-worked solutions

Q1. (a) $255 \div 3 = 85$, so the rate is 85 km/h. (b) $\$187.50 \div 15 = \$12.50$, so the rate is \$12.50 per hour. (c) $900 \div 15 = 60$, so the rate is 60 revolutions per minute (60 RPM).

Q2. (a) Amount = rate $\times$ time: $90 \times 4 = 360$ km. (b) $\$32.40 \times 7 = \$226.80$. (c) $12 \times 25 = 300$ L.

Q3. (a) Time = amount $\div$ rate: $12.5 \div 5 = 2.5$ h. (b) $56 \div 8 = 7$ min. (c) $\$480 \div \$30 = 16$ h.

Q4. (a) There are 60 min in 1 h, so $3 \times 60 = 180$ L/h. (b) $1\,200 \times 60 = 72\,000$ rev/h. (c) In 1 hour (3600 s) the object covers $2 \times 3600 = 7\,200$ m = 7.2 km, so the rate is 7.2 km/h. (Quick way: $2 \times 3.6 = 7.2$.)

Q5. (a) $1\,050 \div 10 = 105$, so the rate is 105 revolutions per minute. (b) $105 \times 4 = 420$ turns. (c) 1 hour is 60 min: $105 \times 60 = 6\,300$ turns.

Q6. (a) $\$499.40 \div 34 = \$14.6882\dots$, which is \$14.69 correct to the nearest cent. (b) $200 \div 31 = 6.4516\dots$ m/s, which is 6.5 m/s to 1 decimal place. (c) $\$112 \div 4.5 = \$24.888\dots$, which is \$24.89 correct to the nearest cent.

Q7. (a) $400 \div 62.5 = 6.4$, so her speed is 6.4 m/s. (b) $6.4 \times 3.6 = 23.04$, so the speed is 23.04 km/h. (c) $6.4 \times 90 = 576$ m.

Q8. (a) Time = amount $\div$ rate: $3\,000 \div 15 = 200$ min. (b) 200 min is 3 h (180 min) plus 20 min, so 3 h 20 min. (c) Both hoses together deliver $15 + 10 = 25$ L per minute. Then $3\,000 \div 25 = 120$ min, and 120 min is 2 h.

Q9. (a) Job A: $\$627.90 \div 23 = \$27.30$ per hour. Job B: $\$561.60 \div 20 = \$28.08$ per hour. (b) Job B pays more, by $\$28.08 - \$27.30 = \$0.78$ per hour. (c) Job A: $\$27.30 \times 30 = \$819.00$. Job B: $\$28.08 \times 30 = \$842.40$. The difference is $\$842.40 - \$819.00 = \$23.40$.

Q10. (a) 480 km is 4.8 lots of 100 km, so the fuel per 100 km is $38.4 \div 4.8 = 8$ L. The van uses 8 L per 100 km. (b) 700 km is 7 lots of 100 km: $7 \times 8 = 56$ L. (c) $56 \times \$1.94 = \$108.64$.

Q11. (a) $3\,150 \div 45 = 70$, so the rate is 70 revolutions per second. (b) There are 60 s in 1 min: $70 \times 60 = 4\,200$ RPM. (c) $4\,200 \times 5 = 21\,000$ turns.

Q12. (a) $\$1\,660.50 \div 9 = \$184.50$ per m³. (b) $\$184.50 \times 6.5 = \$1\,199.25$. (c) $\$3\,690 \div \$184.50 = 20$, so 20 m³.

Q13. (a) $25 \times 3.6 = 90$, so the speed is 90 km/h. (b) $27 \div 3.6 = 7.5$, so the speed is 7.5 m/s. (c) Speed = $300 \div 17.5 = 17.142857\dots$ m/s, which is 17.14 m/s to 2 decimal places. In km/h: $17.142857\dots \times 3.6 = 61.714\dots$, which is 61.7 km/h to 1 decimal place. The limit is 60 km/h, so yes — the car was over the limit, by $61.714\dots - 60 = 1.714\dots$, that is by 1.7 km/h to 1 decimal place.

Q14. (a) $240 \div 4 = 60$, so the oven bakes 60 loaves per hour. (b) 6 h: $60 \times 6 = 360$ loaves. 8 h: $60 \times 8 = 480$ loaves. (c) $900 \div 60 = 15$ h.

Q15. (a) 4.8 km = 4 800 m, because 1 km is 1 000 m. $4\,800 \div 6 = 800$. So the ferry travels 800 m in each minute: its speed is 800 m per minute. (b) 1 min is 60 s, so $800 \div 60 = 13.333\dots$ m/s, which is 13.3 m/s to 1 decimal place. (c) One hour holds ten 6-minute blocks ($60 \div 6 = 10$), and each block covers 4.8 km: $4.8 \times 10 = 48$ km/h.

Q16. (a) 90 s is 1.5 min, so the rate is $135 \div 1.5 = 90$ beats per minute. (b) $90 - 72 = 18$ beats per minute more. (c) In 1 hour: $90 \times 60 = 5\,400$ beats. In 24 h: $5\,400 \times 24 = 129\,600$ beats.

Q17. (a) $35 \times 60 = 2\,100$ mL in one hour. (b) $2\,100 \times 24 = 50\,400$ mL in one day, and $50\,400 \text{ mL} = 50.4 \text{ L}$. (c) One year: $50.4 \times 365 = 18\,396 \text{ L}$, which is 18.396 kL. The cost is $18.396 \times \$3.10 = \$57.0276$, which is \$57.03 correct to the nearest cent.

Q18. (a) Press A: $4\,800 \div 3 = 1\,600$ copies per hour. Press B: $5\,250 \div 3.5 = 1\,500$ copies per hour. (b) $1\,600 > 1\,500$, so Press A is faster. (c) The gap is $1\,600 - 1\,500 = 100$ copies each hour, so in 8 h: $100 \times 8 = 800$ copies. (d) $10\,000 \div 1\,600 = 6.25$ h. The 0.25 h is a quarter of an hour, 15 min, so the time is 6 h 15 min.

Chapter 5 Relationships — 5C Ratio and proportion in context

Theory

A quick look back at ratio and proportion.

A **ratio** compares two or more quantities measured in the same units (§2G). The order of the terms matters: 1 : 4 is not the same ratio as 4 : 1, because it compares the other way round. A **proportion** is a statement that two ratios are equal, and two quantities are in **direct proportion** when one is always the same multiple of the other — double one and the other doubles.

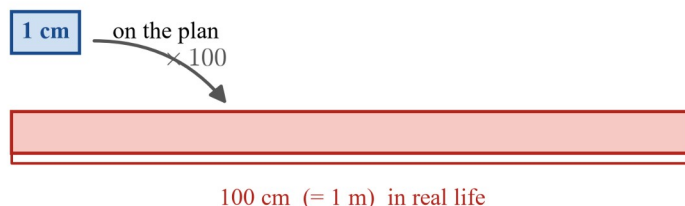
This section puts ratio and proportion to work in three everyday situations: the scales on maps and plans, the ratios used when things are mixed, and magnification factors.

Scale drawings, maps and plans.

Some things are too big to draw at their real size — a house, a park, a whole walking area. A **scale drawing** shows the thing smaller (or, for tiny parts, bigger) than it really is. Every length on the drawing is the real length multiplied by the same number, which is why the drawing keeps the right shape.

The **scale** is written as a ratio, for example 1 : 100. The first number is a length on the drawing and the second number is the matching length in real life, measured in the *same* units. So 1 : 100 means 1 cm on the drawing stands for 100 cm in real life. The second number of the scale is called the **scale factor** — here, 100.

- drawing length → real length: **multiply** by the scale factor
- real length → drawing length: **divide** by the scale factor



*scale 1 : 100 — the second number is the scale factor:
multiply any plan length by 100 to get the real length*

Working with map scales.

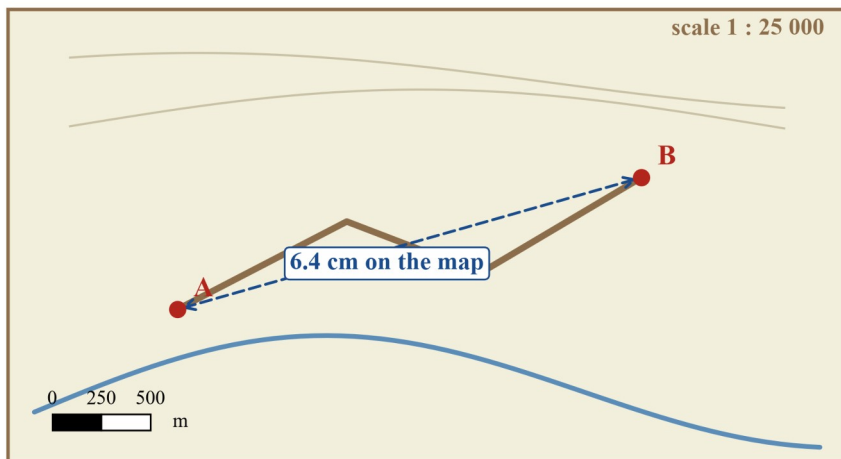
Walking maps often use scales like 1 : 25 000 or 1 : 50 000. Both sides of the scale are the same unit (centimetres), so a map distance in centimetres usually has to be changed into metres or kilometres (§4A): 100 cm = 1 m and 100 000 cm = 1 km. These three scales are worth knowing:

Map scale	1 cm on the map represents
1 : 25 000	250 m
1 : 50 000	500 m
1 : 100 000	1 km

Worked model. On a 1 : 25 000 map a walking track measures 6.4 cm. The real distance is

$$6.4 \times 25\,000 = 160\,000 \text{ cm} = 1600 \text{ m} = 1.6 \text{ km}$$

Going the other way, a 3 km track is 300 000 cm on the ground, so on the map it measures $300\,000 \div 25\,000 = 12$ cm.



$$6.4 \times 25\,000 = 160\,000 \text{ cm} = 1600 \text{ m} = 1.6 \text{ km in real life}$$

A **plan** is just a scale drawing of a building or a piece of land, seen from above. House and shed plans often use 1:50 or 1:100; a **site plan** shows a whole block of land, often at 1:200 or smaller. The method is exactly the same as for a map. Map apps and GPS units work these distances out for you, but knowing how a scale works lets you check what they tell you (§1C, §1F).

Mixing in a given ratio.

Many products are sold strong and must be mixed with water (or with another product) before use: cordial, cleaning chemicals, two-stroke fuel, concrete. The label gives the mix as a ratio, for example 1:4. The order matters: cordial mixed 1:4 means 1 part cordial to 4 parts water — not 4 parts cordial to 1 part water. A **part** can be any unit you like (mL, L, a cup, a shovel) as long as every part of the ratio uses the same unit. The undiluted product is called the **concentrate**.

Mixing problems come in two shapes:

1. **One amount is known.** Find one part first, then multiply.
2. **The total is known.** Add the terms of the ratio to count the parts, divide the total by that count to find one part, then multiply — exactly the sharing method of §2G.

Worked model. Cordial is mixed 1:4 (cordial : water). How much cordial and how much water make 300 mL of drink? The drink is $1 + 4 = 5$ parts, so one part is $300 \div 5 = 60$ mL. That gives $1 \times 60 = 60$ mL of cordial and $4 \times 60 = 240$ mL of water.



5 parts altogether = 300 mL of drink

60 mL

240 mL

one part = $300 \text{ mL} \div 5 = 60 \text{ mL}$: 60 mL of cordial + 240 mL of water

Recipes are the same idea. To cook for a different number of people, multiply *every* ingredient by the same number — the number you multiply by is $n \div m$ when a recipe for m people is changed to serve n people.

With chemicals the ratio is not a suggestion. A mix that is too strong can be dangerous or damage surfaces, and a mix that is too weak will not do its job, so the label or safety sheet is followed exactly.

Magnification factors.

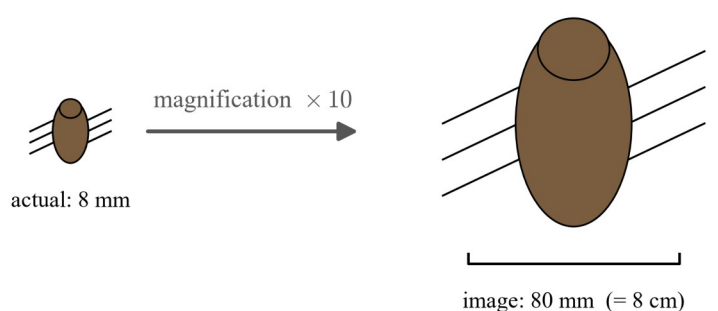
Photos, microscopes and technical drawings make small things look bigger. The **magnification factor** says how many times bigger:

$$\text{image size} = \text{actual size} \times \text{magnification factor}$$

From that one line you also get the other two facts you need:

- $\text{actual size} = \text{image size} \div \text{magnification factor}$
- $\text{magnification factor} = \text{image size} \div \text{actual size}$

Worked model. A beetle 8 mm long is photographed at magnification $\times 10$. In the photo the beetle measures $8 \times 10 = 80$ mm, which is 8 cm.



$$\text{image size} = \text{actual size} \times \text{magnification factor}; \quad 8 \text{ mm} \times 10 = 80 \text{ mm}$$

Choosing the operation.

What you know	What you want	What to do
drawing length, scale $1:n$	real length	multiply by n
real length, scale $1:n$	drawing length	divide by n
one part of a mix, ratio $a:b$	another part	find one part, then multiply
total of a mix, ratio $a:b$	each part	$\text{total} \div (a + b)$, then multiply
recipe for m people	amounts for n people	multiply every amount by $n \div m$
actual size, magnification factor	image size	multiply by the factor
image size, magnification factor	actual size	divide by the factor

When an answer needs rounding.

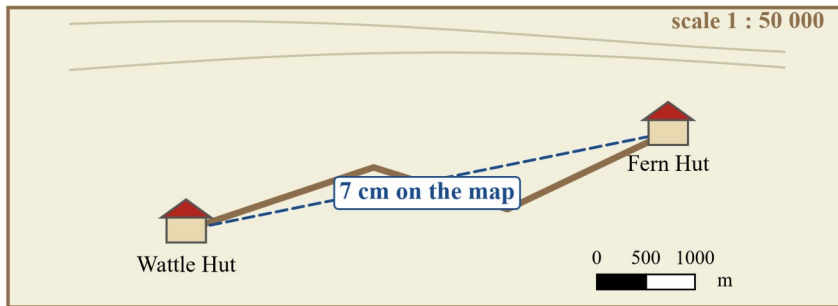
Most scale, mixing and magnification numbers in this section work out exactly. When one does not, the question says how to round — “to 1 decimal place”, “to 2 decimal places”, “to the nearest whole number”. Keep the full number in your calculator or working and round only at the very last step.

About the data in this section. The maps, plans, products, recipes and photographs here are **made up** (not real): pretend places, pretend products and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

A bushwalking map has a scale of 1 : 50 000.



- (a) Wattle Hut and Fern Hut are 7 cm apart on the map. What is the real distance between them, in km?
 (b) A fire trail is 45 km long in real life. What length is it on this map, in cm?

Working

Comment

(a) 1 cm on the map is 50 000 cm on the ground, and
 $50\,000\text{ cm} = 500\text{ m} = 0.5\text{ km}$.

Change what 1 cm means into handy units first.

So 7 cm on the map is $7 \times 0.5 = 3.5\text{ km}$ in real life.

Drawing $\rightarrow$ real: multiply.

(b) $45\text{ km} = 45\,000\text{ m} = 4\,500\,000\text{ cm}$.

Real $\rightarrow$ drawing needs the same units, cm.

$4\,500\,000 \div 50\,000 = 90$, so the trail is 90 cm on the map.

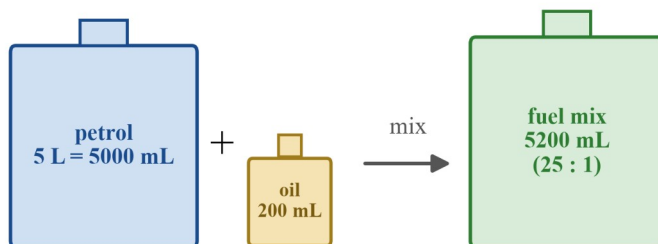
Real $\rightarrow$ drawing: divide.

Check: $90\text{ cm} \times 50\,000 = 4\,500\,000\text{ cm} = 45\text{ km}$.

Multiplying back returns the real length.

Example 2 — scaffolded

A chainsaw runs on two-stroke fuel mixed in the ratio 25 : 1 (petrol : oil).



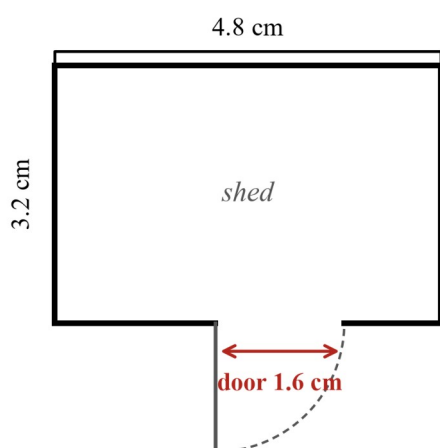
$5000\text{ mL} \div 25 = 200\text{ mL of oil}$ — the ratio must be exact or the engine is damaged

- (a) How much oil, in mL, is mixed with 5 L of petrol?
 (b) What is the total volume of the fuel mix, in L?
 (c) Check that your mix really is in the ratio 25 : 1.

Working	Comment
(a) 5 L = 5000 mL of petrol. The ratio is 25 parts petrol to 1 part oil, so one part is $5000 \div 25 = 200$ mL. Oil needed: $1 \times 200 = 200$ mL.	Work in one unit — mL here. The oil is the 1-part side of the ratio.
(b) Total: $5000 + 200 = 5200$ mL = 5.2 L.	The mix is petrol <i>plus</i> oil.
(c) 5000 : 200; dividing both terms by 200 gives 25 : 1.	The check returns the ratio on the label.

Example 3 — unscaffolded

The plan of a garden shed is drawn at a scale of 1 : 50. On the plan the shed is 4.8 cm long and 3.2 cm wide, and the door is drawn 1.6 cm wide.



plan scale 1 : 50

Find the real length and width of the shed in metres, and the real width of the door in centimetres.

The scale factor is 50, so every plan length is multiplied by 50 to get the real length.

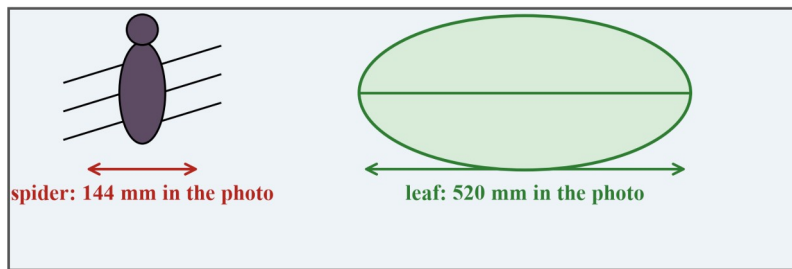
Length: $4.8 \times 50 = 240$ cm, and 240 cm = 2.4 m. Width: $3.2 \times 50 = 160$ cm, and 160 cm = 1.6 m. Door: $1.6 \times 50 = 80$ cm.

A direction check from §1C: the real shed must be much bigger than the drawing, and 2.4 m against 4.8 cm is 50 times bigger — as it should be.

$\therefore$ in real life the shed is 2.4 m long and 1.6 m wide, and the door is 80 cm wide.

Example 4 — unscaffolded

A science poster shows one photograph taken through the same lens at the same magnification. In the photo a spider measures 144 mm long and a leaf measures 520 mm long. The spider is 18 mm long in real life.



same photo, same magnification: spider actual 18 mm $\rightarrow 144 \div 18 = \times 8$

- Find the magnification factor of the photo.
- Find the real length of the leaf, in mm.
- magnification factor = image size $\div$ actual size = $144 \div 18 = 8$, so the photo magnifies $\times 8$.
- The leaf is in the same photo, so the same factor applies: actual size = image size $\div$ factor = $520 \div 8 = 65$ mm.

Check: $65 \times 8 = 520$ mm, the leaf length seen in the photo.

$\therefore$ the photo magnifies $\times 8$, and the leaf is 65 mm (6.5 cm) long in real life.

Practice questions

Scaffolded

Q1. Write each ratio in simplest form. (a) 20 mm : 60 mm (b) 5 cm : 2 m (c) 300 mL : 1.5 L

Q2. A street map has a scale of 1 : 20 000. (a) What real distance, in m, does 1 cm on the map represent? (b) Two streets are 5 cm apart on the map. What is the real distance, in m? (c) A creek is 1.6 km long in real life. What length is it on the map, in cm?

Q3. A cordial label says to mix 1 : 4 (cordial : water). (a) How many parts are there in the finished drink? (b) How much water, in mL, is mixed with 60 mL of cordial? (c) What volume of drink does that make, in mL?

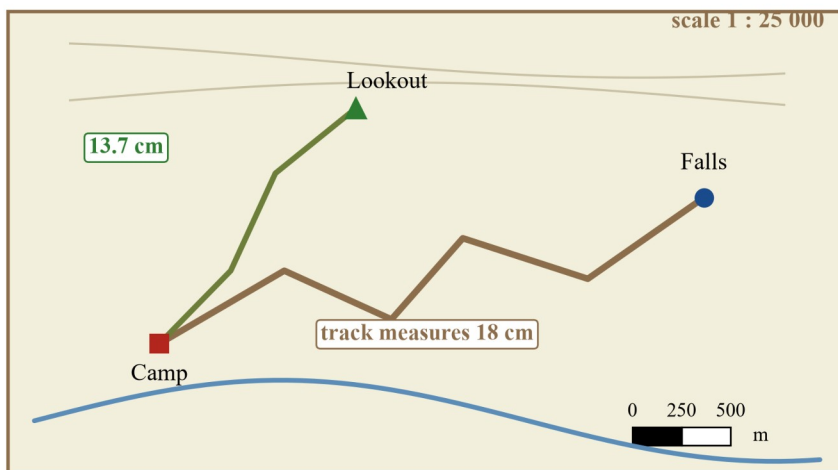
Q4. A concrete mix is 1 : 2 : 3 (cement : sand : gravel), measured in shovels. (a) How many parts are there in the mix? (b) How many shovels of sand and of gravel go with 4 shovels of cement? (c) How many shovels of mix does that make altogether?

Q5. A photo is enlarged by magnification $\times 3$. The original photo is 8 cm wide and 12 cm high. (a) How wide is the enlargement, in cm? (b) How high is the enlargement, in cm? (c) An insect measures 15 mm across on the original photo. How wide is it on the enlargement, in mm?

Q6. A classroom floor plan is drawn at a scale of 1 : 100. (a) The room is 6 m long in real life. What length is it on the plan, in cm? (b) A bench is drawn 2.5 cm long on the plan. What is its real length, in m? (c) A doorway is drawn 9 mm wide on the plan. What is its real width, in cm?

Unscaffolded

Q7. Bushwalkers use a map with a scale of 1:25 000.



- (a) The track from the Camp to the Falls measures 18 cm on the map. What is the real distance, in km?
- (b) A second track is 6.25 km long in real life. What length is it on the map, in cm?
- (c) The track from the Camp to the Lookout measures 13.7 cm on the map. What is the real distance, in km, to 1 decimal place?

Q8. A model railway is built to a scale of 1:87. (a) A real carriage is 21.75 m long. How long is the model carriage, in cm? (b) A model platform is 40 cm long. How long is the real platform, in m?

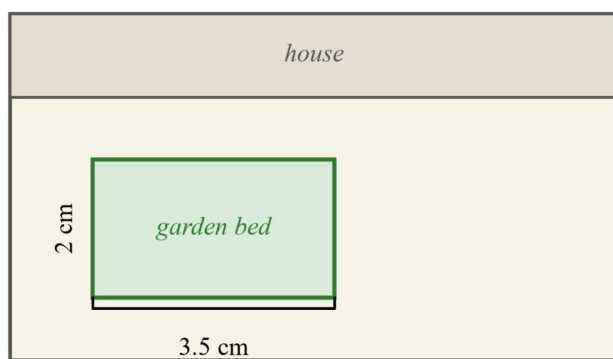
Q9. A mower uses two-stroke fuel mixed 50:1 (petrol : oil). (a) How much oil, in mL, is mixed with 4 L of petrol? (b) What is the total volume of that fuel mix, in mL? (c) The mixing jug holds at most 400 mL of oil. What is the largest volume of petrol, in L, that can be mixed with 400 mL of oil at this ratio?

Q10. A cleaning product is diluted 1:9 (product : water). (a) How much product and how much water, in mL, make 5 L of cleaning solution? (b) A cleaner uses 200 mL of product. How much water is added, and what volume of solution does that make, in L? (c) Explain in one sentence why the ratio on the label should be followed exactly.

Q11. A lamington recipe for 6 people uses 180 g of flour, 90 g of sugar and 120 g of butter. (a) Write the ratio of flour to sugar in simplest form. (b) Find the amount of each ingredient needed for 15 people. (c) Find the amount of each ingredient needed for 4 people.

Q12. A microscope magnifies $\times 250$. (a) The image of a cell is 30 mm wide. What is the real width of the cell, in mm? (b) The image of a second cell is 18 mm wide. What is its real width, in mm? (c) A third object is 0.09 mm wide in real life. How wide is its image, in mm?

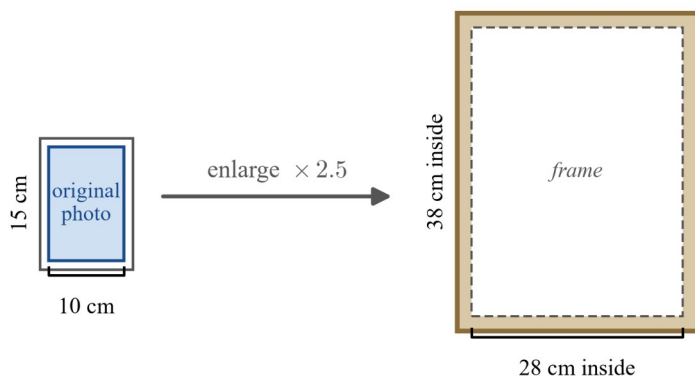
Q13. A site plan of a backyard is drawn at a scale of 1 : 200. On the plan a rectangular garden bed measures 3.5 cm by 2 cm.



site plan, scale 1 : 200

- Find the real length and width of the garden bed, in m.
- Find the real perimeter of the garden bed, in m.
- Find the real area of the garden bed, in m².

Q14. A 10 cm by 15 cm photo is printed at magnification $\times 2.5$. A frame for it has inside measurements of 28 cm by 38 cm.



will the $\times 2.5$ print fit inside the frame?

- Find the width and height of the enlarged print, in cm.
- Will the enlarged print fit inside the frame? Show the comparisons you make.
- Find the largest magnification factor, to 2 decimal places, at which the photo still fits inside the frame.

Q15. On a map, 6 cm represents a real distance of 4.8 km. (a) Write the scale of the map in the form 1 : n . (b) Two towns are 14 cm apart on this map. What is the real distance, in km? (c) Write the scale in words: “1 cm on the map represents ... m in real life”.

Q16. Green paint is mixed from blue and yellow paint in the ratio 3 : 5 (blue : yellow). (a) How much blue and how much yellow paint, in L, make 4 L of green paint? (b) A painter uses all of a 900 mL tin of blue paint. How much yellow paint is needed, and what volume of green paint does that make, in mL? (c) The painter has 900 mL of blue paint and only 1.2 L of yellow paint. What is the largest volume of green paint that can be mixed, in mL, and how much blue paint is left over?

Q17. Show that on a 1 : 25 000 map a distance of 7.2 cm represents a real distance of 1.8 km. Write one step per line.

Q18. A walking group uses two different maps of the same area. (a) On the first map, scale $1:50\,000$, their walk measures 23 cm. What is the real length of the walk, in km? (b) How long, in cm, does a 16 km walk measure on the first map? (c) On the second map the walk from part (a) measures 46 cm. Find the scale of the second map, in the form $1:n$.

Answers

Q1. (a) 1:3. (b) 1:40. (c) 1:5. **Q2.** (a) 200 m. (b) 1000 m (1 km). (c) 8 cm. **Q3.** (a) 5 parts. (b) 240 mL. (c) 300 mL. **Q4.** (a) 6 parts. (b) 8 shovels of sand, 12 of gravel. (c) 24 shovels. **Q5.** (a) 24 cm. (b) 36 cm. (c) 45 mm. **Q6.** (a) 6 cm. (b) 2.5 m. (c) 90 cm. **Q7.** (a) 4.5 km. (b) 25 cm. (c) 3.4 km, to 1 decimal place. **Q8.** (a) 25 cm. (b) 34.8 m. **Q9.** (a) 80 mL. (b) 4080 mL. (c) 20 L. **Q10.** (a) 500 mL of product, 4500 mL of water. (b) 1800 mL of water; 2 L of solution. (c) any correct reason — too strong is dangerous or damaging, too weak does not clean. **Q11.** (a) 2:1. (b) 450 g flour, 225 g sugar, 300 g butter. (c) 120 g flour, 60 g sugar, 80 g butter. **Q12.** (a) 0.12 mm. (b) 0.072 mm. (c) 22.5 mm. **Q13.** (a) 7 m by 4 m. (b) 22 m. (c) 28 m². **Q14.** (a) 25 cm by 37.5 cm. (b) yes — $25 < 28$ and $37.5 < 38$. (c) $\times 2.53$, to 2 decimal places. **Q15.** (a) 1:80 000. (b) 11.2 km. (c) 1 cm represents 800 m. **Q16.** (a) 1.5 L blue, 2.5 L yellow. (b) 1500 mL yellow; 2400 mL green. (c) 1920 mL green; 180 mL of blue left over. **Q17.** $7.2 \times 25\,000 = 180\,000$ cm; $180\,000$ cm = 1800 m; 1800 m = 1.8 km. **Q18.** (a) 11.5 km. (b) 32 cm. (c) 1:25 000.

Fully-worked solutions

Q1. (a) Divide both terms by 20: $20:60 = 1:3$. (b) Same units first: 2 m is 200 cm, so the ratio is 5:200; divide by 5: 1:40. (c) Same units first: 1.5 L is 1500 mL, so the ratio is 300:1500; divide by 300: 1:5.

Q2. (a) 1 cm on the map is 20 000 cm on the ground, and $20\,000$ cm = 200 m. (b) $5 \times 200 = 1000$ m, which is 1 km. (c) 1.6 km = 160 000 cm on the ground; $160\,000 \div 20\,000 = 8$, so 8 cm on the map.

Q3. (a) $1 + 4 = 5$ parts. (b) The cordial is the 1-part side, so one part is 60 mL; water is $4 \times 60 = 240$ mL. (c) $60 + 240 = 300$ mL of drink. (Check: this is the 5 parts, $5 \times 60 = 300$.)

Q4. (a) $1 + 2 + 3 = 6$ parts. (b) One part is 4 shovels, so sand is $2 \times 4 = 8$ shovels and gravel is $3 \times 4 = 12$ shovels. (c) $4 + 8 + 12 = 24$ shovels of mix. (Check: 6 parts $\times 4 = 24$.)

Q5. (a) $8 \times 3 = 24$ cm. (b) $12 \times 3 = 36$ cm. (c) $15 \times 3 = 45$ mm — the magnification applies to every length in the photo.

Q6. (a) Real $\rightarrow$ plan: divide by 100. 6 m = 600 cm; $600 \div 100 = 6$ cm. (b) Plan $\rightarrow$ real: multiply by 100. $2.5 \times 100 = 250$ cm = 2.5 m. (c) $9 \times 100 = 900$ mm = 90 cm.

Q7. (a) $18 \times 25\,000 = 450\,000$ cm = 4500 m = 4.5 km. (b) 6.25 km = 625 000 cm; $625\,000 \div 25\,000 = 25$ cm on the map. (c) $13.7 \times 25\,000 = 342\,500$ cm = 3425 m = 3.425 km, which is 3.4 km to 1 decimal place.

Q8. (a) 21.75 m = 2175 cm; real $\rightarrow$ model: divide by 87: $2175 \div 87 = 25$ cm. (b) Model $\rightarrow$ real: multiply by 87: $40 \times 87 = 3480$ cm = 34.8 m.

Q9. (a) 4 L = 4000 mL of petrol; one part is $4000 \div 50 = 80$ mL, and the oil is that 1 part: 80 mL. (b) $4000 + 80 = 4080$ mL. (c) Oil is the 1-part side, so the petrol is 50 parts: $400 \times 50 = 20\,000$ mL = 20 L.

Q10. (a) The solution is $1 + 9 = 10$ parts. 5 L = 5000 mL, so one part is $5000 \div 10 = 500$ mL: 500 mL of product and $9 \times 500 = 4500$ mL of water. (b) Water is $9 \times 200 = 1800$ mL; the solution is $200 + 1800 = 2000$ mL = 2 L. (c) For example: a mix stronger than the label can be dangerous or damage surfaces, while a weaker mix will not clean or disinfect properly.

Q11. (a) 180:90; divide by 90: 2:1. (b) $15 \div 6 = 2.5$, so multiply every amount by 2.5: flour $180 \times 2.5 = 450$ g, sugar $90 \times 2.5 = 225$ g, butter $120 \times 2.5 = 300$ g. (c) $4 \div 6 = \frac{2}{3}$, so multiply every amount by $\frac{2}{3}$: flour $180 \times \frac{2}{3} = 120$ g, sugar $90 \times \frac{2}{3} = 60$ g, butter $120 \times \frac{2}{3} = 80$ g.

Q12. (a) $\text{actual} = \text{image} \div \text{factor} = 30 \div 250 = 0.12 \text{ mm}$. (b) $18 \div 250 = 0.072 \text{ mm}$. (c) $\text{image} = \text{actual} \times \text{factor} = 0.09 \times 250 = 22.5 \text{ mm}$.

Q13. (a) Length: $3.5 \times 200 = 700 \text{ cm} = 7 \text{ m}$. Width: $2 \times 200 = 400 \text{ cm} = 4 \text{ m}$. (b) Perimeter $= 2 \times (7 + 4) = 22 \text{ m}$. (c) Area $= 7 \times 4 = 28 \text{ m}^2$. (Both side lengths are real lengths before the area is found — the scale is never applied to an area.)

Q14. (a) Width $10 \times 2.5 = 25 \text{ cm}$; height $15 \times 2.5 = 37.5 \text{ cm}$. (b) Compare each side with the frame: $25 \text{ cm} < 28 \text{ cm}$ and $37.5 \text{ cm} < 38 \text{ cm}$. Both fit, so yes, the print fits inside the frame. (c) Width allows $28 \div 10 = 2.8$; height allows $38 \div 15 = 2.5333 \dots$. The smaller of the two is the limit, because the height runs out first: to 2 decimal places the largest factor is $\times 2.53$. (Check: $15 \times 2.53 = 37.95 \text{ cm}$, which still fits; $15 \times 2.54 = 38.1 \text{ cm}$, which does not.)

Q15. (a) $4.8 \text{ km} = 480\,000 \text{ cm}$, so the scale is $6 : 480\,000$; dividing both terms by 6 gives $1 : 80\,000$. (b) $14 \times 80\,000 = 1\,120\,000 \text{ cm} = 11.2 \text{ km}$. (c) $80\,000 \text{ cm} = 800 \text{ m}$, so 1 cm on the map represents 800 m in real life.

Q16. (a) $3 + 5 = 8$ parts; one part is $4 \div 8 = 0.5 \text{ L}$. Blue: $3 \times 0.5 = 1.5 \text{ L}$. Yellow: $5 \times 0.5 = 2.5 \text{ L}$. (b) Blue is 3 parts, so one part is $900 \div 3 = 300 \text{ mL}$; yellow is $5 \times 300 = 1500 \text{ mL}$. Green: $900 + 1500 = 2400 \text{ mL}$. (c) One part from the blue would be $900 \div 3 = 300 \text{ mL}$; one part from the yellow would be $1200 \div 5 = 240 \text{ mL}$. The smaller one-part wins, because the yellow runs out first, so one part is 240 mL: blue used $3 \times 240 = 720 \text{ mL}$, yellow used 1200 mL, green $720 + 1200 = 1920 \text{ mL}$. Blue left over: $900 - 720 = 180 \text{ mL}$.

Q17. $7.2 \times 25\,000 = 180\,000$, so the distance is 180 000 cm on the ground. $180\,000 \text{ cm} = 1800 \text{ m}$, because $100 \text{ cm} = 1 \text{ m}$. $1800 \text{ m} = 1.8 \text{ km}$, because $1000 \text{ m} = 1 \text{ km}$. So 7.2 cm on a $1 : 25\,000$ map does represent a real distance of 1.8 km.

Q18. (a) $23 \times 50\,000 = 1\,150\,000 \text{ cm} = 11.5 \text{ km}$. (b) $16 \text{ km} = 1\,600\,000 \text{ cm}$; $1\,600\,000 \div 50\,000 = 32 \text{ cm}$ on the first map. (c) The walk is 1 150 000 cm on the ground and 46 cm on the second map, so the scale is $46 : 1\,150\,000$; dividing both terms by 46 gives $1 : 25\,000$. (Check: $46 \times 25\,000 = 1\,150\,000 \text{ cm} = 11.5 \text{ km}$.)

Chapter 5 Relationships — 5D Tables, conversion charts and graphs

Theory

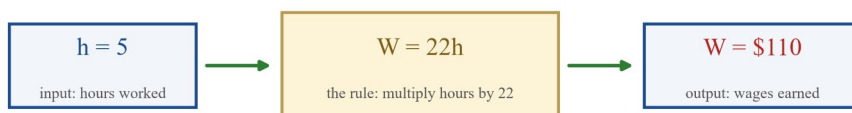
In work and in everyday life, one quantity often depends on another. Pay depends on hours worked. Fuel left in a tank depends on hours of driving. The cost of a job depends on how many items are made. When one quantity depends on another like this, there is a **relationship** between the two. Section 5A writes relationships as formulas and section 5B writes them as rates. This section shows the same relationships three other ways — as a **table of values**, as a **graph**, and as a **conversion chart** — and builds the skill of moving between the three and choosing the best one for the job.

To **represent** a relationship means to show it in one of these forms. The relationship does not change when the form changes; only the picture does.

About the data in this section. Every business, person, price, plan and workplace in this section is **made up** (not real). They are built for this book so you can practise reading and drawing tables, graphs and conversion charts. Nothing here is real data about a real person, business or place.

Three ways to show one relationship.

Suppose a casual worker is paid \$22 per hour, so the wage rule is $W = 22 \times h$, with h the hours worked and W the wage in dollars. The rule can be pictured as a machine that takes hours in and gives dollars out:



Made-up data: the wage and every number in this picture are made up for this section.

Read the machine left to right: 5 hours in gives $W = 22 \times 5 = \$110$ out. The machine is the **rule**. A table of values and a graph are the other two ways to show exactly this relationship, and both are built from the rule.

Tables of values.

A **table of values** is a list of matching pairs: each input written beside the output the rule gives it. To build one, choose inputs, **substitute** each into the rule — substitute means put the number in place of the letter — and calculate the output. A screen-printing shop charges a \$60 set-up fee plus \$4 per shirt, so $C = 60 + 4n$, with n shirts and C dollars:

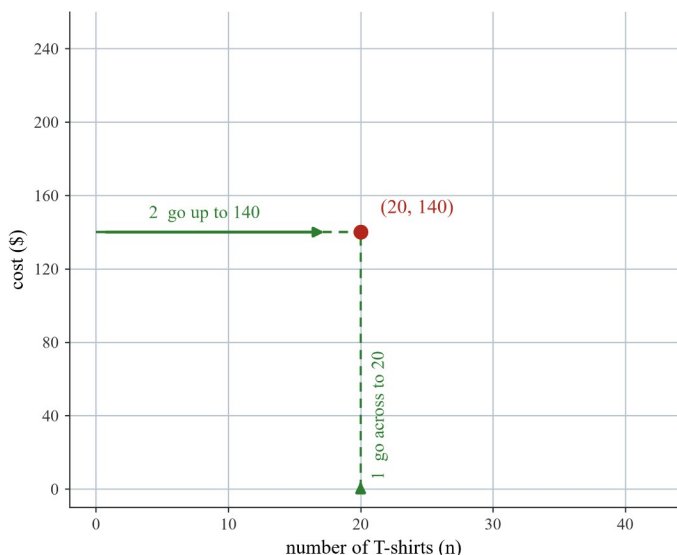
Shirts, n	Cost, C (\$)
0	60
10	100
20	140
30	180
40	220

Made-up data: the shop and its prices are made up for this section.

One row in full: for $n = 20$, $C = 60 + 4 \times 20 = 60 + 80 = 140$, so the pair is 20 shirts, \$140.

Drawing a graph from a table.

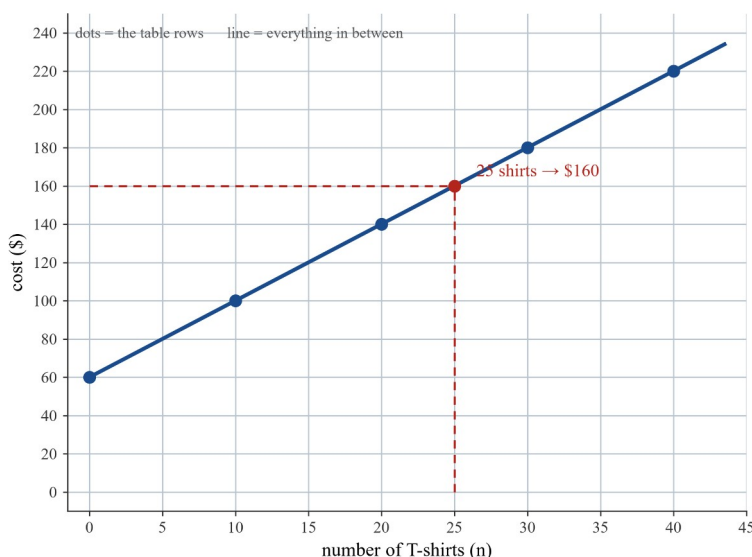
A **graph** draws each pair as a dot on two axes. The **horizontal axis** (across the page) carries the input — the quantity you choose. The **vertical axis** (up the page) carries the output — the quantity the rule produces. Each pair becomes one dot: go across to the input, then up to the output.



Made-up data: the shirt prices and every number in this picture are made up for this section.

The dot above is the pair (20 shirts, \$140): step 1 go across to 20, step 2 go up to 140. Plot every row of the table the same way.

Once the dots are plotted, joining them with a straight line fills in the pairs the table skipped. The line is not new information — it is the in-between values the rule would give:



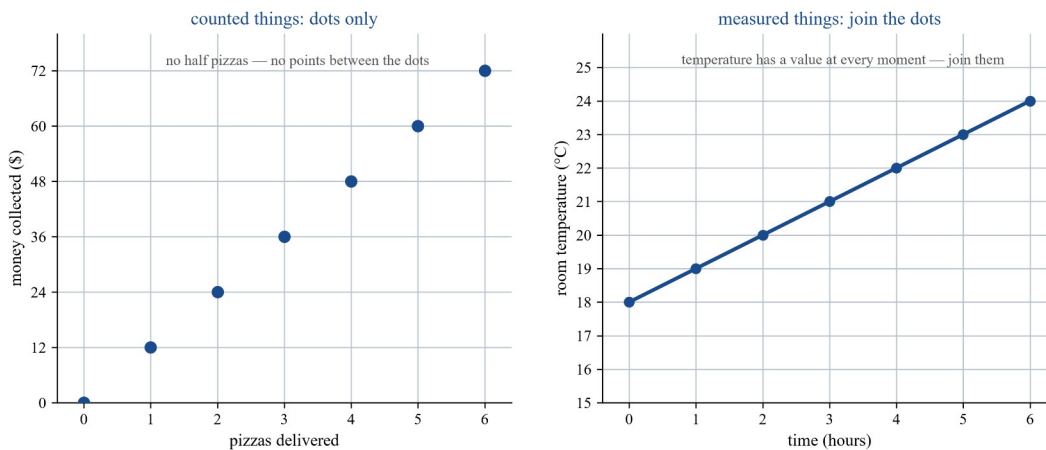
Made-up data: the shirt prices and every number in this picture are made up for this section.

The dots are the table's rows. The line between them lets you read any in-between pair: go up from 25 shirts to the line, then across, and the cost reads as \$160 — which matches the rule, $60 + 4 \times 25 = 160$.

Join the dots, or leave them as dots?

Joining is only sensible when in-between values make sense. Ask: **can the input be counted, or is it measured?**

- **Counted inputs** come in whole numbers only — pizzas, muffins, people, shirts. There is no such thing as 2.5 pizzas ordered, so the in-between dots would mean nothing: leave the graph as **separate dots**.
- **Measured inputs** can take any in-between value — time, distance, temperature, volume. A shift can last 7.5 hours, so the in-between values are real: **join the dots with a line**.



Made-up data: the pizza prices, the room and every number in this picture are made up for this section.

On the left, pizzas at \$12 each ($P = 12n$) give dots only: 0, 12, 24, 36, 48, 60 and 72 dollars for 0 to 6 pizzas. On the right, a room warming by 1 °C per hour ($T = 18 + h$) gives a joined line, because time and temperature both flow through the in-between values.

Reading a graph: telling the story of the change.

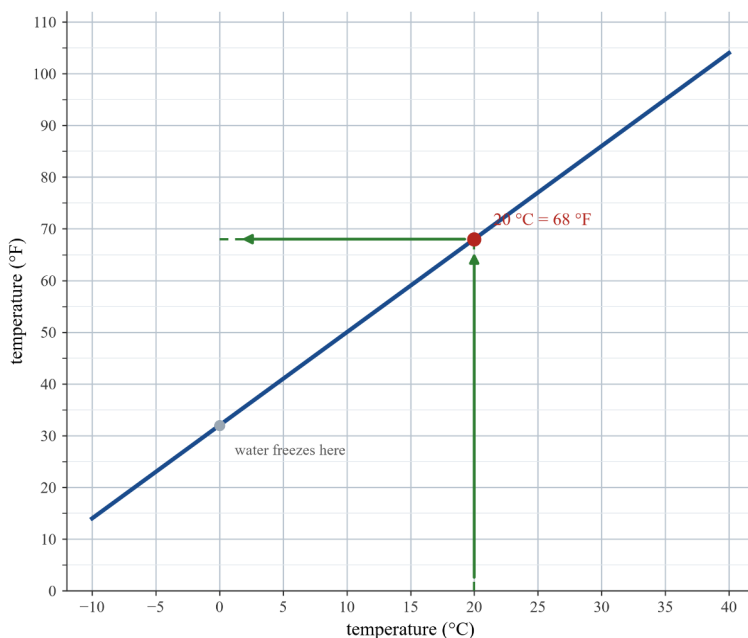
A graph's job is to show **change** — how the output moves as the input grows. Reading left to right along the line:

What the line does	What it tells you
Rises	The output is increasing — getting bigger
Rises steeply	Increasing quickly — a lot of change per step of the input
Flat (horizontal)	No change — the output stays the same while the input grows
Falls	The output is decreasing — getting smaller

Steepness is the eye's speedometer: the steeper the line, the faster the change. A flat stretch is always worth a second look, because it means the story paused — a stop, a wait, a hold at one temperature.

Conversion charts.

A **conversion chart** is a graph with one unit on the horizontal axis and a different unit on the vertical axis, so a single dot states the same measurement in both units. The chart below converts between °C and °F, the two temperature scales from section 4A:



To read 20 °C in °F: start at 20 on the horizontal axis, go up to the line (green arrow up), then go across to the vertical axis (green arrow across) and read 68 °F. The grey dot marks where water freezes: 0 °C on the horizontal axis lines up with 32 °F on the vertical axis.

A chart gives a quick **estimate**; the two-step rule from section 4A gives the exact value. Check the reading above with the rule: $20 \times 1.8 = 36$, then $36 + 32 = 68$ °F — the chart and the rule agree. Use the chart to see where you are, and the rule (section 4A's conversion table) when the answer must be exact.

Which representation is best?

All three forms show the same relationship, but each earns its keep in a different situation:

Representation	Best at	Weak spot
Rule (formula)	An exact answer for any input	You must calculate every time
Table of values	Instant look-up of the pairs listed	Silent about values between rows
Graph	The whole story at a glance — rise, fall, flat stretches, where two lines cross	Values are read off, so they are estimates

A pay clerk wants the rule. A shop counter wants a table of prices on the wall. A nurse watching a patient's temperature over three days wants the graph. Choosing the right form is part of representing a relationship well.

Tools that help.

Spreadsheets and phone apps will draw a graph from a table in seconds, and at work you will often use them. But every graph in this section can be drawn and read with pen, paper, a ruler and a basic calculator — and the tool cannot tell you whether the story it drew is sensible. That stays your job: check a few pairs against the rule, and check the shape against what the situation should do.

Worked examples

Example 1 — scaffolded

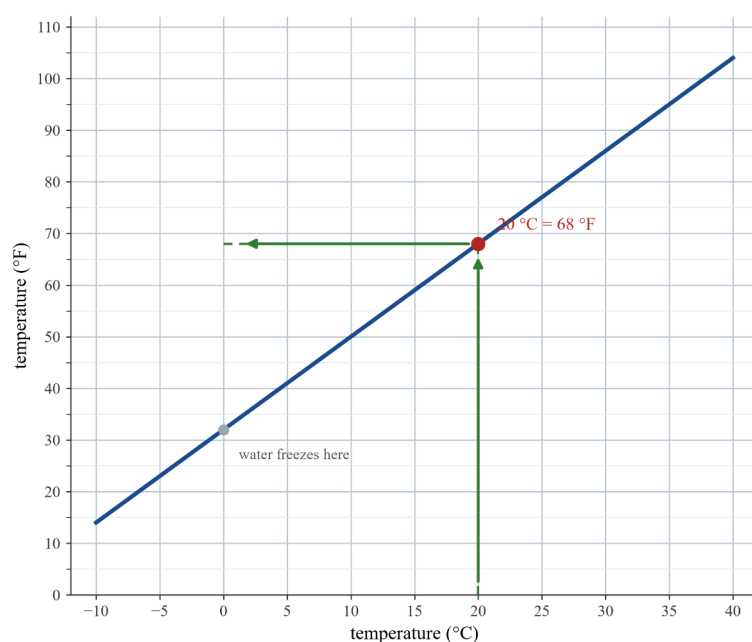
Rosa sells pies at a market stall for \$6 each. Her takings follow the rule $T = 6 \times p$, with p pies sold and T dollars taken. Build the table, then use it to answer: how much does Rosa take for 45 pies, and how many pies had she sold once the takings reached \$300?

Pies, p	Takings, T (\$)
0	0
10	60
20	120
30	180
40	240
50	300

Working	Comment
The rule is $T = 6 \times p$.	Write the rule before anything else.
Table rows: $p = 0, 10, 20, 30, 40, 50$ give $T = 0, 60, 120, 180, 240, 300$.	Substitute each input: $6 \times 10 = 60$, and so on.
45 pies is not a table row, so use the rule: $T = 6 \times 45 = 270$.	A table lists only some pairs; the rule covers all of them.
Takings of \$300: undo the multiply — $300 \div 6 = 50$ pies.	Working backwards means doing the opposite operation.
Rosa takes \$270 for 45 pies, and \$300 of takings means 50 pies sold.	Answer both questions asked, in words.

Example 2 — scaffolded

Ben is in the United States and his weather app shows 77 °F. He wants the same temperature in °C. Read it off the conversion chart from Theory, then check the reading with section 4A's two-step rule.



Working	Comment
Start at 77 on the vertical (°F) axis, a little below halfway from 75 to 80.	Find the unit you already have, first.
Go across to the line, then down to the horizontal axis: it lands on 25 °C, a gridline on the °C side.	Chart readings are estimates — landing on a gridline makes this one clean.
Check, step 1: $77 - 32 = 45$.	°F to °C subtracts 32 first (section 4A).
Check, step 2: $45 \div 1.8 = 25$.	Then divides by 1.8.
The chart and the rule agree on 25 °C, so the reading is	Two methods agreeing is the strongest check there is.

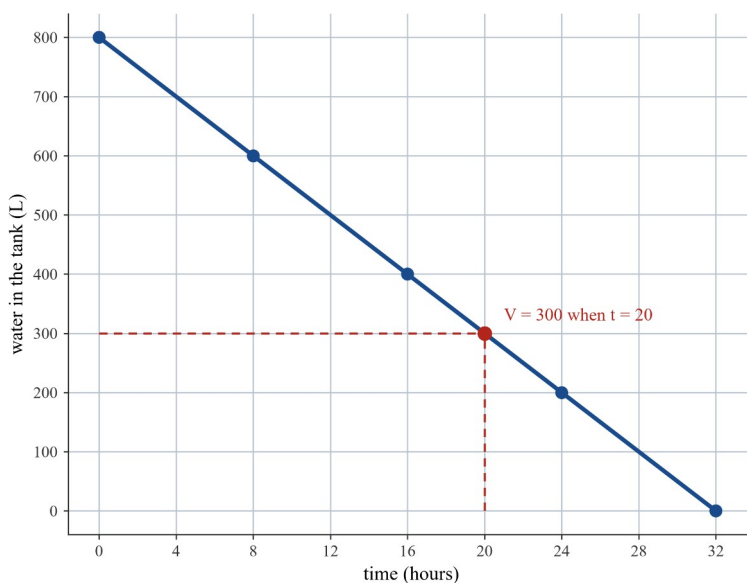
trusted.

Example 3 — unscaffolded

A farm tank holds 800 L of water. While the tank feeds a dripper, the volume left follows $V = 800 - 25t$, with t the hours since the dripper started and V the litres left. Build a table using $t = 0, 8, 16, 24, 32$, draw the graph, and use it to find how long the dripper has been running when 300 L are left.

Hours, t	Water left, V (L)
0	800
8	600
16	400
24	200
32	0

Plotting those pairs and joining them (time is measured, so joining is correct) gives a straight line falling from (0, 800) to (32, 0). Reading across from 300 L on the vertical axis to the line, then down to the horizontal axis, gives $t = 20$:



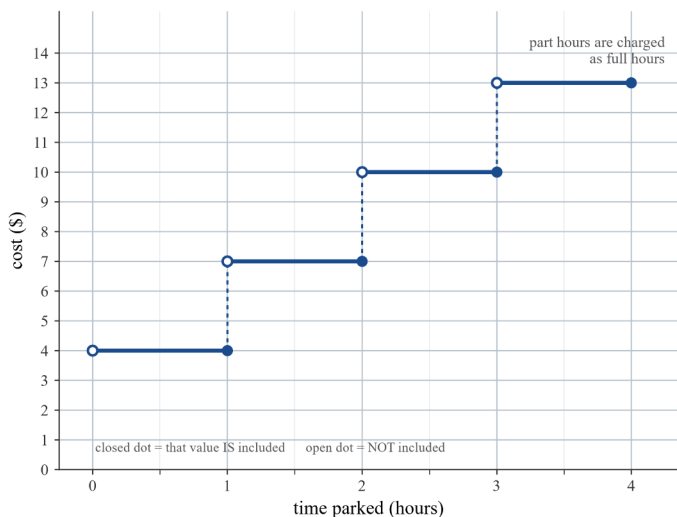
Made-up data: the tank, the dripper and every number in the table and graph are made up for this section.

Check with the rule: $800 - 25 \times 20 = 800 - 500 = 300$ ✓.

∴ the dripper has been running 20 hours when 300 L are left.

Example 4 — unscaffolded

A shopping centre charges \$4 for parking up to and including 1 hour, then \$3 for each extra hour **or part hour**. The step graph below shows the cost. Each flat **step** is one price, and the price jumps at every whole hour. A **closed dot** means that time is included in the step; an **open dot** means it is not — so exactly 2 hours costs \$7, but 2 hours and one minute jumps to \$10.



Made-up data: the shopping centre and its parking fees are made up for this section.

- (a) Parking 2.5 hours: 2.5 sits in the step between 2 and 3 hours, which costs \$10.
- (b) A \$7 ticket: the \$7 step runs from just after 1 hour up to and including 2 hours, so \$7 means up to 2 hours parked.
- (c) Comparing 3.1 hours and 3.9 hours: both sit in the step between 3 and 4 hours, so both cost \$13 — part hours are charged as full hours.

$\therefore$ 2.5 hours costs \$10; a \$7 ticket means up to 2 hours parked; and 3.1 and 3.9 hours cost the same, \$13.

Practice questions

Scaffolded

Q1. A forklift loads crates onto a truck at 5 crates per minute, so $b = 5 \times m$, with m the minutes and b the crates loaded.

Minutes,

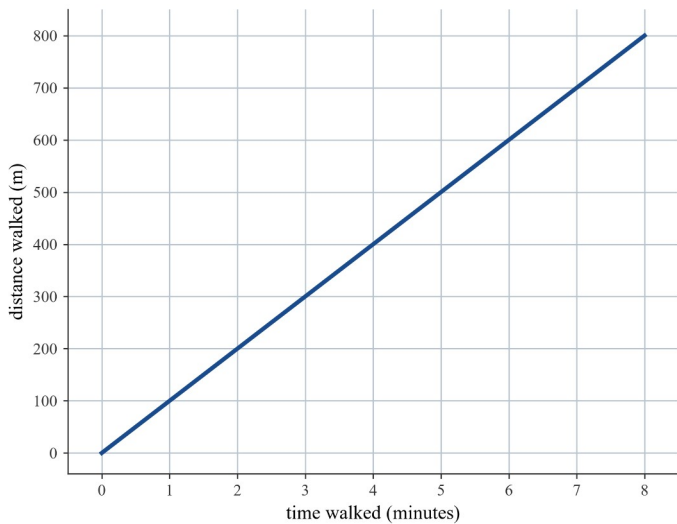
m 0 2 4 6 8 10

Crates, b

- (a) Complete the table.
- (b) Use the rule to find b when $m = 9$.
- (c) Find m when $b = 55$.

Q2. A car uses 0.08 L of fuel per kilometre, so $F = 0.08 \times d$, with d the kilometres driven and F the litres of fuel used. (a) Make a table of values for $d = 0, 50, 100, 150, 200, 250, 300$, and plot the pairs. (b) Should the dots be joined with a line? Give a reason. (c) Use the rule to find the fuel used for 220 km, to the nearest litre.

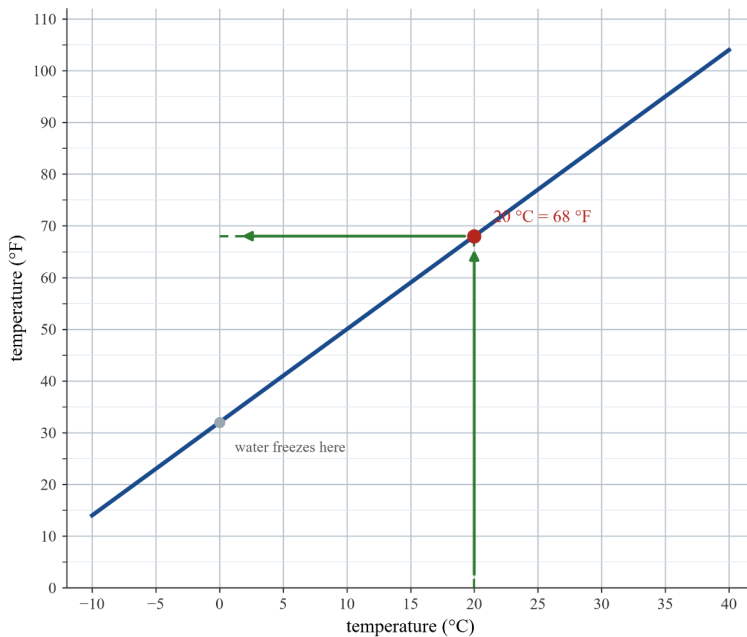
Q3. The graph below shows Hana’s walk home from school at a steady pace.



Made-up data: Hana and every number on the graph are made up for this section.

- Read the distance walked at $t = 6$ minutes.
- Use the graph to find how long Hana had been walking when she reached 500 m.
- Describe the story of the change in one sentence: what does the line’s shape say about the walk?

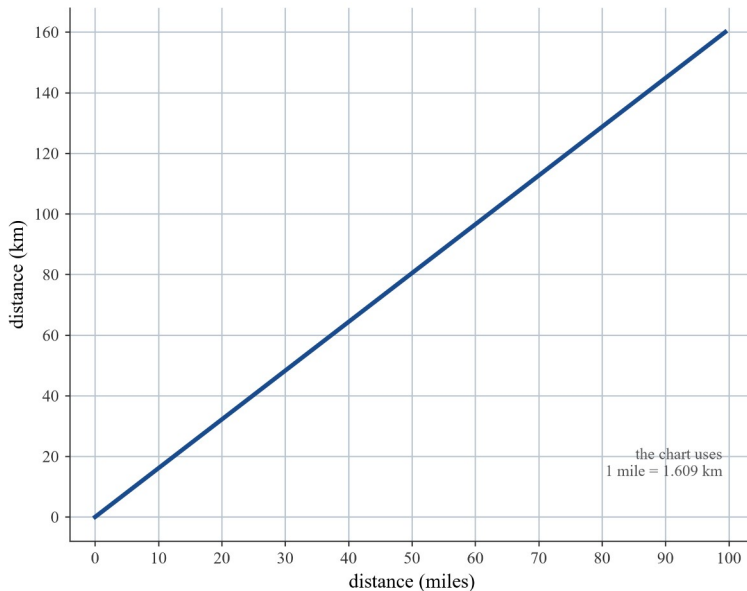
Q4. Use the °C/°F conversion chart from Theory.



- Read 10 °C in °F.
- Read 95 °F in °C.
- The grey dot marks where water freezes. Read its temperature in °F.

Q5. Tessa puts \$35 of her wage into savings each week, so her savings follow $S = 35 \times w$, with w the weeks and S the dollars saved. (a) Make a table of values for $w = 0, 2, 4, 6, 8, 10$. (b) Show that Tessa has saved \$245 after 7 weeks. (c) Find how many weeks it takes for her savings to reach \$280.

Q6. The chart below converts between miles and kilometres, using section 4A's factor 1 mile = 1.609 km.



- (a) Read 50 miles in kilometres, to the nearest 5 km.
- (b) Read 100 km in miles, to the nearest 5 miles.
- (c) Convert 45 miles to kilometres using the factor 1.609, to the nearest km.

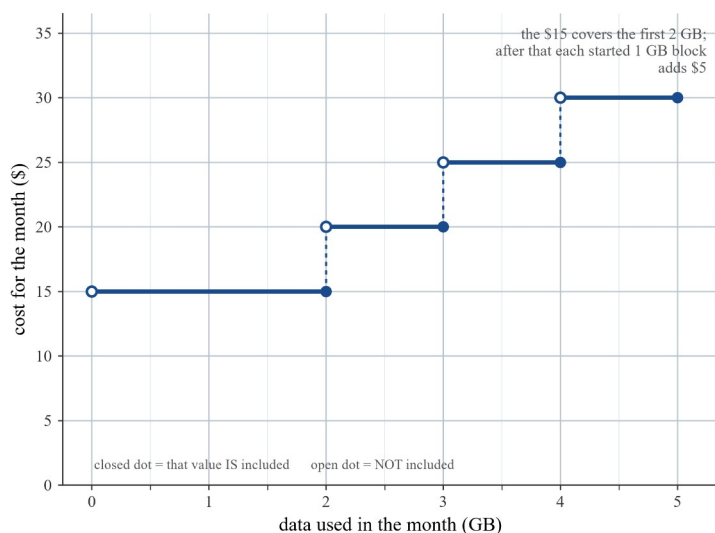
Un scaffolded

Q7. Kai is paid \$24 per hour for weekend work, so his pay follows $P = 24 \times h$, with h the hours worked and P the dollars paid. (a) Make a table of values for $h = 0, 5, 10, 15, 20$. (b) Draw the graph from your table. Join the dots or leave them as dots? Give a reason. (c) Read the pay for 12 hours off your graph, then check it with the rule.

Q8. Ella buys muffins at \$4.50 each for her team, so the cost follows $C = 4.5 \times n$, with n the muffins and C the dollars. (a) Ella plots the pairs for $n = 0$ to 10. Explain why her graph must stay as separate dots. (b) Find the cost of 7 muffins. (c) How many muffins can Ella buy with \$40? Show the check that decides it.

Q9. A tractor's tank holds 60 L of diesel. While it pumps water, the fuel left follows $F = 60 - 7.5t$, with t the hours of pumping and F the litres left. (a) Make a table of values for $t = 0, 2, 4, 6, 8$. (b) After how many hours of pumping is the tank empty? (c) Find the fuel left after 3 hours, and describe in one sentence how the fuel changes over the whole 8 hours.

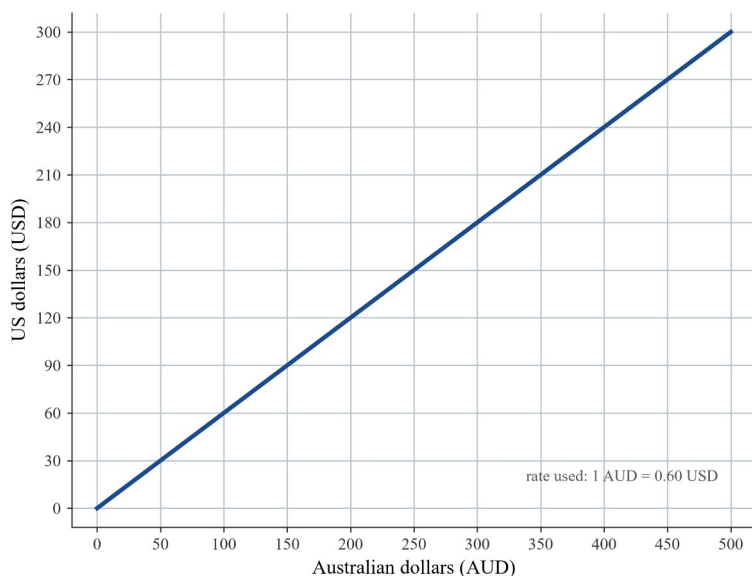
Q10. The step graph below is a mobile data plan: \$15 covers up to 2 GB of data, then each extra 1 GB block **or part block** adds \$5.



Made-up data: the plan and its prices are made up for this section.

- Find the monthly cost for 1.8 GB of data.
- Find the monthly cost for 4.5 GB of data.
- Jae used 3.2 GB and was charged \$25. Jae expected \$20, reasoning “just over 3 GB should be the 3 GB price”. Use the graph to explain why \$25 is the correct charge.

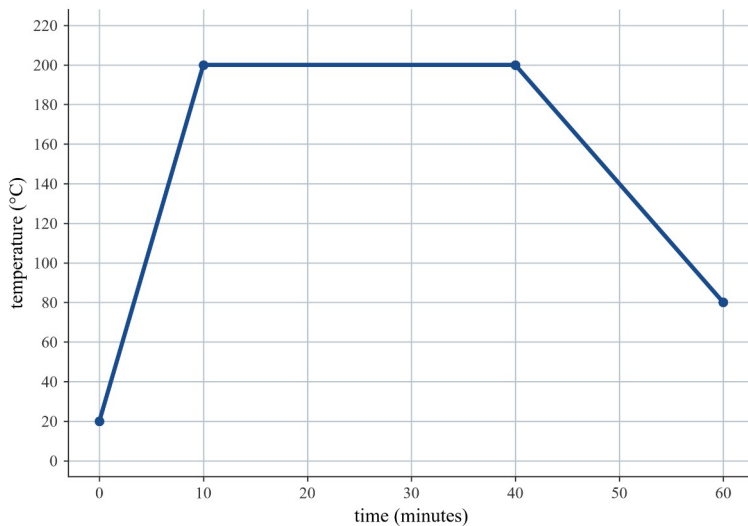
Q11. A travel-money desk displays the conversion chart below. The rate on it is 1 Australian dollar = 0.60 US dollars.



Made-up data: the travel-money desk and its rate are made up for this section. Real rates change every day and are rarely this neat.

- Read 200 Australian dollars in US dollars.
- Read 210 US dollars in Australian dollars.
- Read the rate back off the chart: pick one point on the line and divide its US-dollar value by its Australian-dollar value. What do you get?

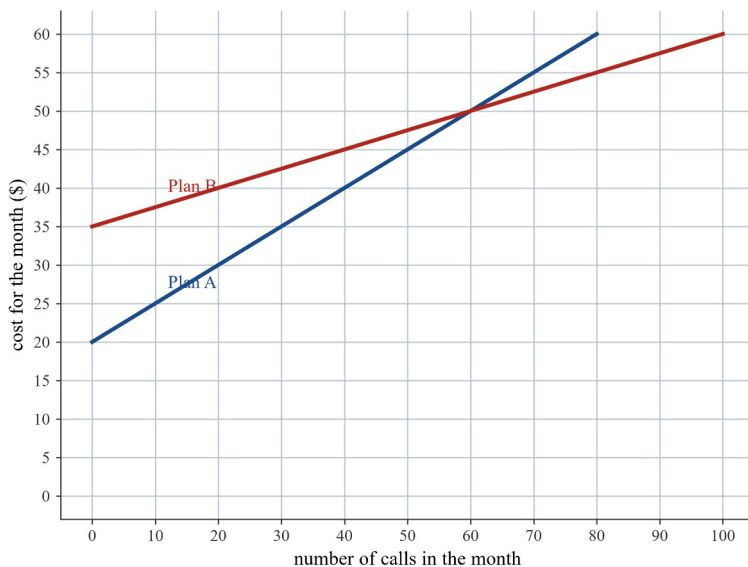
Q12. The graph below shows an oven's temperature from switch-on through cooling.



Made-up data: the oven and every temperature on the graph are made up for this section.

- Read the oven's temperature at $t = 25$ minutes.
- For how many minutes does the oven hold steady at 200°C ?
- Between 40 and 60 minutes the oven cools. How many $^{\circ}\text{C}$ per minute does it fall over that stretch?

Q13. Zac is choosing between two phone plans. Plan A costs \$20 per month plus \$0.50 per call; Plan B costs \$35 per month plus \$0.25 per call. The graph below shows both, with n the calls in a month and the monthly cost in dollars.



Made-up data: both plans and their prices are made up for this section.

- Find each plan's cost for 20 calls, and say which is cheaper there.
- The lines cross at one point. Read the number of calls and the cost at that point, then check both with the rules.
- Which plan is cheaper for 80 calls a month? Show the pair of calculations that decides it.

Q14. A painter keeps a record of how much paint wall jobs of different sizes use:

Walls, w	2	4	6	8	10
Paint, L (L)	0.5	1	1.5	2	2.5

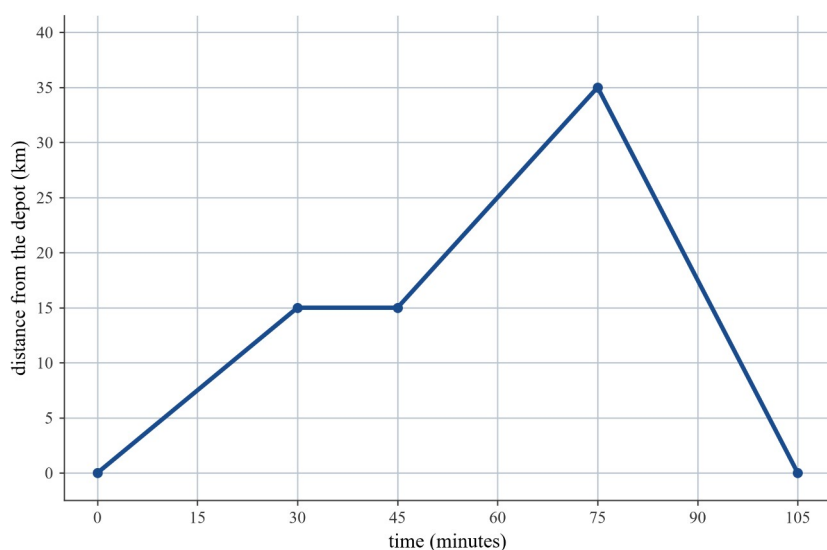
Made-up data: the painter's record and every number in it are made up for this section.

- How many extra litres does each 2 extra walls take?
- Write the rule in words: how many litres does one single wall take?
- Use your rule to find the paint needed for 12 walls.

Q15. Choose the best representation — rule, table of values, or graph — for each job, and give a one-sentence reason.

- A nurse records a patient's temperature every 4 hours for 3 days and wants to see at a glance whether it is going up or going down.
- A print shop's rule is "cost = 40 cents $\times$ number of sheets". A customer wants the exact cost of 37 sheets.
- A café wants to show customers the price of each of its five milkshake sizes on a sign at the counter.

Q16. The graph below shows a delivery van's distance from its depot over one trip.



Made-up data: the van's trip and every number on the graph are made up for this section.

- For how many minutes was the van stopped? How can you tell from the graph?
- The trip has three driving legs: out to 15 km, on to 35 km, and back to the depot. Which leg was fastest? Check by working each leg's speed in km/h.
- How many kilometres did the van travel in total over the whole trip?

Q17. Ava's class grows seedlings. Her seedling starts at 8 cm and grows about 2.5 cm per week, so its height follows $H = 8 + 2.5 \times w$, with w the weeks and H the height in centimetres. (a) Make a table of values for $w = 0, 2, 4, 6, 8$, and draw the graph — joined line or separate dots? Say which. (b) Use the rule to find the height the model predicts at week 12. (c) Give one reason why the real seedling might not be that height at week 12, even though the rule says so.

Q18. A school swim-a-thon raises money with a flat \$40 of sponsorships plus \$18 per lap swum, so the money raised follows $T = 40 + 18 \times L$, with L the laps and T the dollars. The school's goal is \$500. (a) Make a table of values for $L = 0, 5, 10, 15, 20, 25$. (b) Half the goal is \$250. How many laps must be swum to reach at least \$250? Give your answer to the nearest lap, then check it against whole laps. (c) Does 30 laps reach the \$500 goal? If so, by how much does it beat it?

Answers

Q1. (a) $b = 0, 10, 20, 30, 40, 50$. (b) 45 crates. (c) 11 minutes. **Q2.** (a) $F = 0, 4, 8, 12, 16, 20, 24$ L. (b) yes — distance is measured, so the in-between values are real. (c) 18 L (to the nearest litre). **Q3.** (a) 600 m. (b) 5 minutes. (c) a steady rise — one pace, no stops. **Q4.** (a) 50°F . (b) 35°C . (c) 32°F . **Q5.** (a) $S = 0, 70, 140, 210, 280, 350$. (b) shown — $S = 245$. (c) 8 weeks. **Q6.** (a) 80 km (to the nearest 5 km). (b) 60 miles (to the nearest 5 miles). (c) 72 km (to the nearest km). **Q7.** (a) $P = 0, 120, 240, 360, 480$. (b) join — hours are measured. (c) \$288. **Q8.** (a) muffins are counted, so part values would mean nothing. (b) \$31.50. (c) 8 muffins. **Q9.** (a) $F = 60, 45, 30, 15, 0$ L. (b) 8 hours. (c) 37.5 L; a steady fall to empty. **Q10.** (a) \$15. (b) \$30. (c) 3.2 GB sits in the 3–4 GB step, which costs \$25. **Q11.** (a) 120 US dollars. (b) 350 Australian dollars. (c) 0.60. **Q12.** (a) 200°C . (b) 30 minutes. (c) 6°C per minute. **Q13.** (a) A \$30, B \$40 — Plan A. (b) 60 calls, both \$50. (c) Plan B (\$55 against \$60). **Q14.** (a) 0.5 L. (b) 0.25 L per wall. (c) 3 L. **Q15.** (a) graph — the rise and fall over time is the point. (b) rule — one exact value is wanted. (c) table — a few fixed prices to look up. **Q16.** (a) 15 minutes (flat from 30 to 45 min). (b) the return leg, 70 km/h against 30 and 40 km/h. (c) 70 km. **Q17.** (a) $H = 8, 13, 18, 23, 28$ cm; joined line (weeks and height are measured). (b) 38 cm. (c) real plants slow down — the rule is a model. **Q18.** (a) $T = 40, 130, 220, 310, 400, 490$. (b) 12 laps (to the nearest lap). (c) yes — \$580, beating the goal by \$80.

Fully-worked solutions

Q1. (a) Substitute each minute value: $5 \times 0 = 0$, $5 \times 2 = 10$, $5 \times 4 = 20$, $5 \times 6 = 30$, $5 \times 8 = 40$, $5 \times 10 = 50$. (b) $b = 5 \times 9 = 45$ crates. (c) Undo the multiply: $55 \div 5 = 11$ minutes.

Q2. (a) $0.08 \times 0 = 0$; $0.08 \times 50 = 4$; $0.08 \times 100 = 8$; $0.08 \times 150 = 12$; $0.08 \times 200 = 16$; $0.08 \times 250 = 20$; $0.08 \times 300 = 24$. (b) Yes, join the dots: distance is measured, so 127 km and every other in-between value is a real trip with a real fuel use. (c) $F = 0.08 \times 220 = 17.6$ L before rounding, so 18 L to the nearest litre.

Q3. (a) Up from 6 minutes to the line, then across: 600 m. (b) Across from 500 m to the line, then down: 5 minutes. (c) The line rises steadily the whole way with no flat or steep stretches, so Hana walked at one steady pace without stopping.

Q4. (a) Up from 10°C to the line, then across: 50°F . (b) Across from 95°F to the line, then down: 35°C . (c) The grey dot sits at 0°C on the horizontal axis and 32°F on the vertical axis, so water freezes at 32°F .

Q5. (a) $35 \times 0 = 0$; $35 \times 2 = 70$; $35 \times 4 = 140$; $35 \times 6 = 210$; $35 \times 8 = 280$; $35 \times 10 = 350$. (b) Substitute $w = 7$ into $S = 35 \times w$, one step per line:

$$S = 35 \times 7$$

$$S = 245$$

which is the \$245 required. (c) Undo the multiply: $280 \div 35 = 8$ weeks.

Q6. (a) Up from 50 miles to the line, then across: the reading sits a touch above 80 km — the exact value is $50 \times 1.609 = 80.45$ km — so 80 km to the nearest 5 km. (b) Across from 100 km to the line, then down: a little past 60 miles — to 4 decimal places, $100 \div 1.609 = 62.1504$ miles — so 60 miles to the nearest 5 miles. (c) $45 \times 1.609 = 72.405$ km before rounding, so 72 km to the nearest km.

Q7. (a) $24 \times 0 = 0$; $24 \times 5 = 120$; $24 \times 10 = 240$; $24 \times 15 = 360$; $24 \times 20 = 480$. (b) Join the dots: hours worked are measured, so a 7.5-hour shift is real and its pay is real. (c) The graph reads \$288 at 12 hours; the rule checks it: $24 \times 12 = 288$ ✓.

Q8. (a) Muffins are counted in whole numbers — nobody buys 2.5 muffins — so the in-between dots would mean nothing: separate dots. (b) $C = 4.5 \times 7 = 31.5$, so \$31.50. (c) $40 \div 4.5 = 8.89$ before rounding, which is not a whole number of muffins; check the whole numbers either side: 8 muffins cost $4.5 \times 8 = \$36$ (fits in \$40) and 9 muffins cost $4.5 \times 9 = \$40.50$ (too dear), so Ella can buy 8 muffins.

Q9. (a) $60 - 7.5 \times 0 = 60$; at $t = 2$: $60 - 15 = 45$; at $t = 4$: $60 - 30 = 30$; at $t = 6$: $60 - 45 = 15$; at $t = 8$: $60 - 60 = 0$. (b) The table and the graph both reach 0 L at $t = 8$, so the tank is empty after 8 hours of pumping. (c) $F = 60 - 7.5 \times 3 = 60 - 22.5 = 37.5$ L. The line falls steadily from 60 L to 0 L, so the fuel decreases at the same rate the whole way — 7.5 L every hour.

Q10. (a) 1.8 GB is inside the first step (up to and including 2 GB), so \$15. (b) 4.5 GB is inside the step from 4 to 5 GB, so \$30. (c) On the graph, the \$20 step ends with a closed dot at exactly 3 GB; past 3 GB the price jumps to the \$25 step, which runs from just after 3 GB up to and including 4 GB. 3.2 GB sits in that step, so the charge is \$25: the part block from 3 to 3.2 GB is charged as a full 1 GB block.

Q11. (a) Up from 200 Australian dollars to the line, then across: 120 US dollars. (b) Across from 210 US dollars to the line, then down: 350 Australian dollars. (c) Take the point (200 Australian, 120 US): $120 \div 200 = 0.60$ — the rate the chart was built from. Pick any other point on the line and the same division returns the same rate.

Q12. (a) 25 minutes sits inside the flat stretch, so the temperature reads 200 °C. (b) The flat stretch runs from 10 to 40 minutes: $40 - 10 = 30$ minutes held at a steady 200 °C. (c) The fall is $200 - 80 = 120$ °C over $60 - 40 = 20$ minutes, so $120 \div 20 = 6$ °C per minute.

Q13. (a) Plan A: $20 + 0.5 \times 20 = 20 + 10 = \$30$. Plan B: $35 + 0.25 \times 20 = 35 + 5 = \$40$. Plan A is cheaper at 20 calls. (b) The lines cross at 60 calls and \$50. Check: Plan A gives $20 + 0.5 \times 60 = 20 + 30 = 50$ and Plan B gives $35 + 0.25 \times 60 = 35 + 15 = 50$ ✓. (c) Plan A: $20 + 0.5 \times 80 = 20 + 40 = \$60$. Plan B: $35 + 0.25 \times 80 = 35 + 20 = \$55$. Plan B is cheaper at 80 calls — past the crossing at 60 calls, the plan with the smaller per-call price wins.

Q14. (a) Each step of 2 walls adds 0.5 L: $1 - 0.5 = 0.5$, $1.5 - 1 = 0.5$, and so on down the table. (b) 0.5 L per 2 walls halves to 0.25 L for one single wall — a quarter of a litre per wall. (c) $0.25 \times 12 = 3$ L.

Q15. (a) A graph: the nurse wants the story of the change — rising, falling or steady over the three days — and a graph shows that at a glance. (b) The rule: the customer wants one exact value, and a graph only ever gives an estimate, while 37 sheets is a quick calculation. (c) A table of values: there are only five fixed sizes, and a sign people look up is exactly what a table is for.

Q16. (a) The line is flat from 30 to 45 minutes — the distance from the depot is not changing — so the van was stopped $45 - 30 = 15$ minutes. (b) Leg 1: 15 km in 30 minutes = 30 km/h. Leg 2: $35 - 15 = 20$ km in $75 - 45 = 30$ minutes = 40 km/h. Return: 35 km in $105 - 75 = 30$ minutes = 70 km/h. The return leg was fastest, which is also the steepest stretch of the graph. (c) $15 + 20 + 35 = 70$ km travelled in total.

Q17. (a) $8 + 2.5 \times 0 = 8$; at $w = 2$: $8 + 5 = 13$; at $w = 4$: $8 + 10 = 18$; at $w = 6$: $8 + 15 = 23$; at $w = 8$: $8 + 20 = 28$. Join the dots: weeks and height are both measured, so the in-between values are real. (b) $H = 8 + 2.5 \times 12 = 8 + 30 = 38$ cm. (c) Real seedlings do not grow at 2.5 cm per week forever — growth slows as a plant matures, runs out of room in its pot, or meets a cold week — so the rule is a model that fits the first weeks well and week 12 only roughly. Section 1C's reasonableness habit applies: a model's answer is a prediction, not a promise.

Q18. (a) $40 + 18 \times 0 = 40$; at $L = 5$: $40 + 90 = 130$; at $L = 10$: $40 + 180 = 220$; at $L = 15$: $40 + 270 = 310$; at $L = 20$: $40 + 360 = 400$; at $L = 25$: $40 + 450 = 490$. (b) Solve $40 + 18 \times L = 250$: first $250 - 40 = 210$, then $210 \div 18 = 11.67$ laps before rounding, so 12 laps to the nearest lap. Check with whole laps: 11 laps raises $40 + 18 \times 11 = \$238$ (short of \$250) and 12 laps raises $40 + 18 \times 12 = \$256$ (past it), so 12 laps is the first whole number of laps that reaches half the goal. (c) $T = 40 + 18 \times 30 = 40 + 540 = \$580$, which beats \$500 by $580 - 500 = \$80$ — yes, the goal is reached.

Chapter 5 Relationships — 5E Conventions for writing algebra

Theory

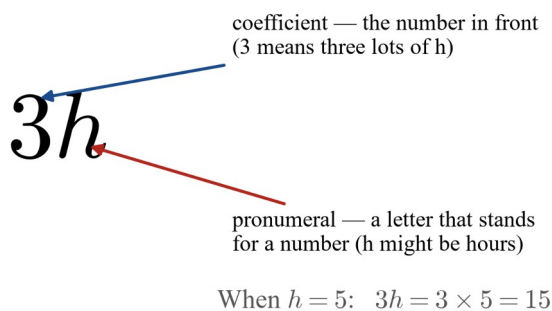
About the data in this section. The job sheets, delivery rounds, candle stalls, flat-pack kits, garden beds, smoothie stalls and prices here are **made up** (not real): pretend jobs and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Algebra is a shorthand for “a number I do not know yet, or a number that changes”. A letter stands in for that number, and the conventions — the agreed rules for writing — are what let two people read the same line of algebra and get the same answer. In the study design (Unit 3, Area of Study 4), this is key knowledge: the standard rules people follow when they build, use and write algebraic expressions.

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-07.

The parts of a term.

A **pronumeral** is a letter that stands for a number: h might be hours, d might be deliveries. A **term** is one piece of an expression — a plain number, a pronumeral, or a number times a pronumeral. In the term $3h$ the 3 is the **coefficient**, the number in front: $3h$ means three lots of h . An **expression** is one or more terms joined by $+$ and $-$ signs.



Two coefficients are usually left invisible: $1 \times h$ is written h (not $1h$), and $-1 \times h$ is written $-h$.

The multiplication sign disappears.

- Two plain numbers keep the sign: $3 \times 4 = 12$. Dropping it there would read as thirty-four, a different number altogether.
- A number and a letter drop the sign, with the number first: $3 \times h = 3h$. Never write $h3$.
- Two letters drop the sign: $a \times b = ab$.
- A number and a bracket drop the sign: $4 \times (x + 2) = 4(x + 2)$. At $x = 5$ that is $4 \times 7 = 28$.

KEEP	$3 \times 4 = 12$		two plain numbers — keep the $\times$ sign
DROP	$3 \times h$	$\rightarrow 3h$	number and letter — drop the $\times$, number first
DROP	$a \times b$	$\rightarrow ab$	two letters — drop the $\times$
DROP	$4 \times (x + 2)$	$\rightarrow 4(x + 2)$	number and bracket — drop the $\times$

Never write $h3$ for $3h$ — the number goes first. When $x = 5$: $4(x + 2) = 4 \times 7 = 28$.

Division is written as a fraction.

$h \div 4$ is written $\frac{h}{4}$. At $h=12$: $\frac{12}{4}=3$. The letter's place matters: $\frac{4}{h}$ at $h=12$ is $\frac{4}{12}=\frac{1}{3}$, a completely different number.

Top and bottom are not interchangeable.

Powers: a letter times itself.

$h \times h$ is written h^2 ("h squared") and $h \times h \times h$ is written h^3 . As with numbers (Section 2E), the index sits on the letter only: $2h^2$ means $2 \times h \times h$, so at $h=3$ it is $2 \times 9=18$. Brackets move the index: $(2h)^2$ squares everything inside the brackets, so at $h=3$ it gives 36, not 18.

Adding a letter to itself and multiplying a letter by itself are different jobs:

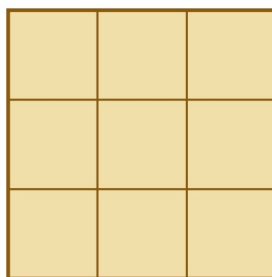
- $2x = x + x$: at $x=3$ that is $3+3=6$;
- $x^2 = x \times x$: at $x=3$ that is $3 \times 3=9$.

At $x=5$ the gap widens: $x+x=10$ but $x^2=25$.

$$2x = x + x = 3 + 3 = 6$$



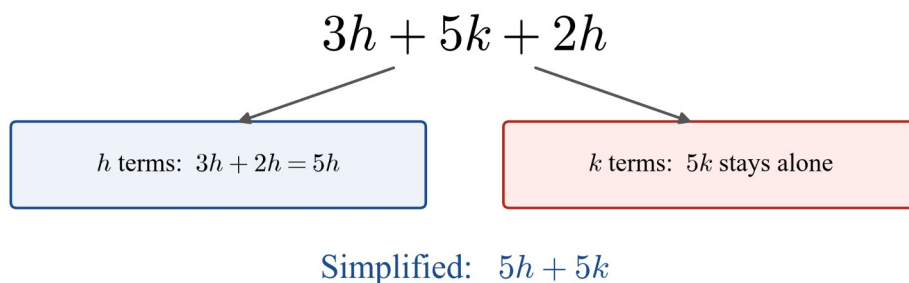
$$x^2 = x \times x = 3 \times 3 = 9$$



With $x=3$: $2x=6$ but $x^2=9$ — adding is not the same as multiplying.

Like terms.

Like terms are terms whose pronumerals and indices match exactly: $3h$ and $2h$ are like terms; $3h$ and $3h^2$ are not; $3h$ and $5k$ are not. Only like terms can be collected by adding or subtracting: $3h+5h=8h$. Unlike terms stay separate, so $3h+5k+2h$ simplifies to $5h+5k$ — the h terms collect ($3h+2h=5h$) and the $5k$ stays alone. Check at $h=2$ and $k=1$: unsimplified, $6+5+4=15$; simplified, $10+5=15$. Same total, so the simplifying is safe.

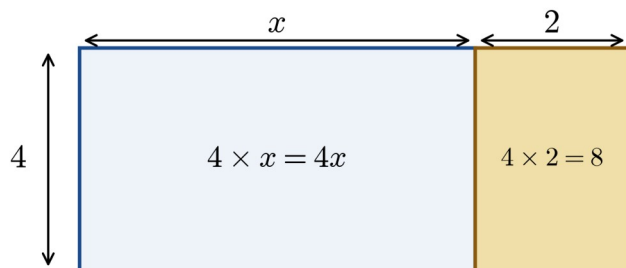


Check with $h=2$ and $k=1$: $3h+5k+2h=6+5+4=15$
and $5h+5k=10+5=15$ — same total, so the simplifying is safe.

Expanding one bracket.

$4(x+2)$ means four lots of the whole bracket, so the 4 multiplies every term inside: $4 \times x + 4 \times 2 = 4x + 8$. Writing the bracket out this way is called **expanding** (or multiplying out). At $x=5$: bracket side, $4 \times 7 = 28$; expanded side, $4 \times 5 + 8 = 20 + 8 = 28$. Same area, same answer — expanding changes the look, not the value.

$$4(x+2) = 4x + 8$$



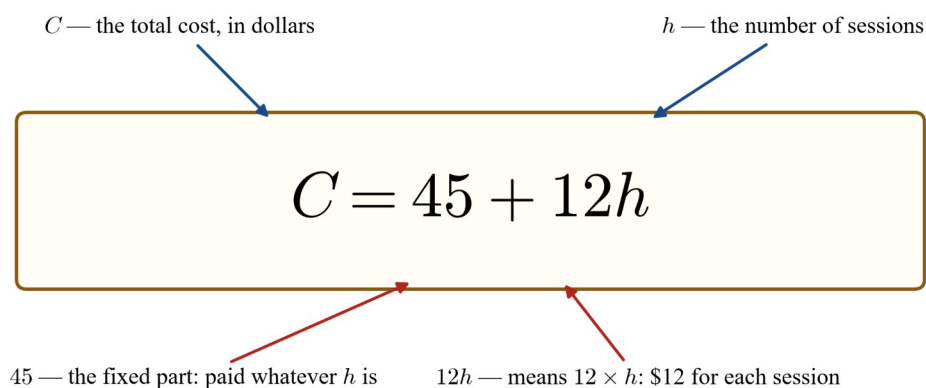
When $x=5$: $4 \times 7 = 28$ and $4 \times 5 + 8 = 28$ — same area.

Reading a formula.

A **formula** is a rule that connects quantities, written with pronumerals. Read each part of $C = 45 + 12h$, a call-out price where C is the total cost in dollars and h is the number of sessions:

- 45 is the fixed part — paid whatever h is;
- $12h$ means $12 \times h$: \$12 for each session;
- C is what the formula hands you at the end.

At $h=2$: $C = 45 + 24 = 69$, so \$69.00. At $h=3$: $C = 45 + 36 = 81$, so \$81.00.



$$h=2: C = 45 + 24 = 69 \text{ dollars} \quad h=3: C = 45 + 36 = 81 \text{ dollars}$$

Substituting: keep the order.

Substituting means putting a number in for a pronumeral. Then evaluate in the standard order: brackets, then indices (powers), then $\times$ and $\div$, then $+$ and $-$, working top to bottom as you meet them. For $3 + 2h^2$ at $h=4$: the index first ($4^2=16$), then the multiply ($2 \times 16=32$), then the add ($3+32=35$). Adding first would give a different, wrong answer.

Evaluate $3 + 2h^2$ when $h = 4$

1 substitute	$3 + 2 \times 4^2$	put 4 in for h
2 index	$3 + 2 \times 16$	$4^2 = 16$ first
3 multiply	$3 + 32$	$2 \times 16 = 32$
4 add	35	the last step

The order

- 1 brackets
- 2 indices (powers)
- 3 $\times$ and $\div$
- 4 $+$ and $-$
*top to bottom,
as you meet them*

Words into algebra.

Words	Algebra
k plus 7	$k + 7$
3 times m	$3m$
y divided by 5	$\frac{y}{5}$
twice w , then take away 4	$2w - 4$

Reading the last row back at $w = 6$: $2 \times 6 - 4 = 12 - 4 = 8$.

Three mistakes worth meeting now.

Mistake	Why it fails	Correct reading
$4d = 4 + d$	$4d$ means $4 \times d$: at $d = 3$, $4d = 12$, while $4 + d = 7$	$4d = 4 \times d$
$x + x = x^2$	adding is not multiplying: at $x = 5$, $x + x = 10$ but $x^2 = 25$	$x + x = 2x$
$5d + 3 = 8d$	$5d$ and 3 are not like terms: at $d = 2$, $5d + 3 = 13$ but $8d = 16$	$5d + 3$ stays as it is

Worked examples

Example 1 — scaffolded

A bakery job sheet uses d for the number of deliveries one driver does in a day. The sheet has three lines: the total deliveries from the 2 morning drivers and the 1 afternoon driver (each driver does d); the cake deliveries, which run at one fifth of d ; and the loaves loaded, which is 6 more than 4 times d . Write each line as an algebraic expression, then find its value when $d = 10$.

Made-up data: the bakery job sheet and its numbers are made up for this section.

Working	Comment
Purpose and question(s): turn three job-sheet lines into algebra, then evaluate each at $d = 10$.	Three expressions and three values — list them before calculating.
Mathematics in the context: 2 morning drivers and 1 afternoon driver each do d deliveries; cakes run at $d \div 5$; loaves are 6 more than $4 \times d$.	The words carry the operations; the conventions carry the writing.

Working	Comment
Step 1 — total deliveries: $2 \times d + 1 \times d = 3d$; at $d = 10$: $3 \times 10 = 30$.	Like terms: $2d + 1d = 3d$. Check: $2 \times 10 + 1 \times 10 = 30$.
Step 2 — cake deliveries: $d \div 5 = \frac{d}{5}$; at $d = 10$: $\frac{10}{5} = 2$.	Division is written as a fraction, letter on top.
Step 3 — loaves loaded: $4 \times d + 6 = 4d + 6$; at $d = 10$: $40 + 6 = 46$.	Number first, then the added term: “6 more than” comes last.

At $d = 10$ the job sheet reads: 30 deliveries, 2 cake deliveries and 46 loaves loaded.

Example 2 — scaffolded

Dana runs a candle stall. Her stock sheet counts candles in boxes of c : 5 boxes of scented, 3 boxes of plain, plus 8 loose candles on the table. Write the total number of candles as an expression, simplify it, then find the total when $c = 6$.

Made-up data: Dana’s stall and its stock counts are made up for this section.

Working	Comment
Purpose and question(s): count all the candles. Questions: what is the simplified expression? What is the total at $c = 6$?	Simplify first — the substitution then costs one step.
Mathematics in the context: 5 boxes of c plus 3 boxes of c plus 8 loose candles: $5c + 3c + 8$.	The boxes of c are like terms; the loose 8 is not.
Step 1 — collect like terms: $5c + 3c + 8 = 8c + 8$.	$5c + 3c = 8c$; the $+8$ stays alone.
Step 2 — substitute: at $c = 6$: $8 \times 6 + 8 = 48 + 8 = 56$ candles.	Multiply before add, as the order says.
Check: unsimplified at $c = 6$: $5 \times 6 + 3 \times 6 + 8 = 56$ — the same total.	Two routes to one answer: that is a check.

The simplified expression is $8c + 8$, and at $c = 6$ the stall holds 56 candles.

Example 3 — unscaffolded

A flat-pack kit holds x legs and 5 bolts. A store stacks 6 unopened kits. Write an expression for the total number of pieces (legs and bolts together) in the 6 kits, expand it, and find the total when $x = 4$.

Made-up data: the kits and their contents are made up for this section.

Identify first: the purpose is to count pieces across 6 identical kits, and the mathematics is one bracket (the pieces per kit), expanded, then a substitution. Pieces per kit: $x + 5$. Six kits: $6(x + 5)$. Expanding, the 6 multiplies both terms inside the bracket: $6 \times x + 6 \times 5 = 6x + 30$. At $x = 4$: $6 \times 4 + 30 = 24 + 30 = 54$ pieces. Check by counting parts: legs $6 \times 4 = 24$, bolts $6 \times 5 = 30$, and $24 + 30 = 54$; check again straight from the bracket: $6 \times (4 + 5) = 54$.

$\therefore$ the 6 kits hold $6x + 30$ pieces, which is 54 pieces when $x = 4$.

Example 4 — unscaffolded

A trainee’s job card shows four lines of algebra. Exactly one line is correct; the other three break a writing convention.

Job card — trainee's lines

- (i) $p + p = p^2$
- (ii) $7 + 2t = 9t$
- (iii) $3 \times 4k = 12k$
- (iv) $d \times 4 = d4$

Made-up data: the job card and its lines are made up for this section.

Find the three broken lines, rewrite each one correctly, and check every rewrite by substituting a value.

Identify first: the purpose is to audit four lines against the conventions, and the mathematics is collecting like terms, the order of operations, and the number-first rule. Line (i): $p + p$ is two like terms, so $p + p = 2p$, not p^2 — at $p = 3$: $3 + 3 = 6$, while p^2 would give 9. Line (ii): 7 and $2t$ are not like terms, so $7 + 2t$ cannot become $9t$ — at $t = 2$: $7 + 2 \times 2 = 11$, while $9t$ would give 18. Line (iii): $3 \times 4k = 12k$ is correct — at $k = 2$ both sides give 24. Line (iv): $d \times 4$ is right as a value but wrong as writing, because the number goes first: $4d$ — at $d = 3$ that is 12.

$\therefore$ lines (i), (ii) and (iv) break conventions — corrected they read $2p$, $7 + 2t$ and $4d$ — and line (iii) is the one correct line.

Practice questions

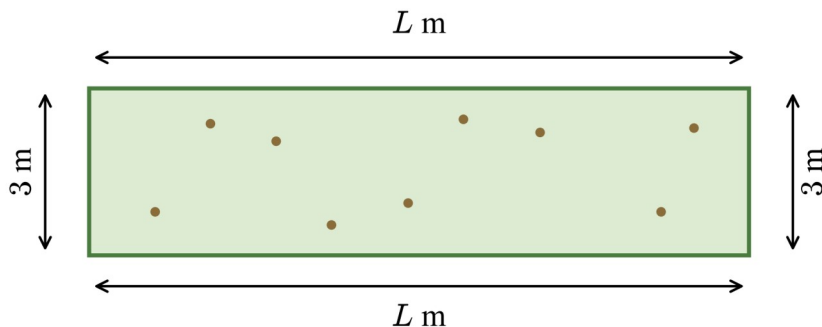
Scaffolded

- Q1.** Write each product without multiplication signs. (a) $7 \times c$ (b) $m \times 6$ (c) $p \times q$
- Q2.** Write each phrase as an algebraic expression. (a) 9 more than k (b) t divided by 4 (c) 5 times h , then take away 2; find its value when $h = 4$
- Q3.** Collect like terms to simplify each expression. (a) $4d + 3d$ (b) $6y + 2 - 3y$ (c) $8n - 3n + n$
- Q4.** A campsite charges a booking fee of \$35 plus \$18 per night. The cost in dollars for n nights is $R = 35 + 18n$. (a) Find R when $n = 2$. (b) Find R when $n = 0$. What does the 35 represent? (c) Find R when $n = 5$.
- Q5.** True or false? If false, give the correct version. (a) $1 \times h = h$ (b) $3h + 5h = 8h$ (c) $x + x = x^2$
- Q6.** Expand each bracket. (a) $3(x + 2)$ (b) $5(m + 4)$ (c) $2(y + 7)$; check your expansion at $y = 3$ by evaluating both versions

Unscaffolded

- Q7.** A poster shop charges a \$25 setup fee plus \$3 per poster. Ana writes the cost of p posters as $25 + 3p$; Ben writes $25 + 3 \times p$. (a) Explain in one sentence why Ana's and Ben's expressions say the same thing. (b) Find the cost when $p = 8$ using each expression. (c) Jo adds before multiplying and writes $(25 + 3)p = 28p$. Find Jo's value at $p = 8$ and explain in one sentence why it cannot be right.

Q8. A rectangular garden bed is L m long and 3 m wide, as shown.



- Write the perimeter as a sum of its four sides.
- Collect like terms to write the perimeter as $2L + 6$.
- Find the perimeter when $L = 5$, and check it by adding the four side lengths.

Q9. Sam simplifies $4a + 3b$ to $7ab$. (a) Explain in one sentence what Sam has done wrong. (b) Write the correct simplification of $4a + 3b$. (c) Show, using $a = 2$ and $b = 1$, that Sam's version gives a different value from the original expression.

Q10. A bench is built from a centre section of length $5(x + 2)$ cm and an end section of length $4x$ cm, so its total length is $5(x + 2) + 4x$ cm. (a) Expand the bracket. (b) Collect like terms to simplify the total length. (c) Find the total length when $x = 3$, and check it against the unexpanded expression.

Q11. A screen printer charges a \$30 setup fee plus \$6.75 per shirt. The cost in dollars for s shirts is $C = 30 + 6.75s$. (a) Find C when $s = 9$. (b) Find C when $s = 7$. (c) Hence find the cost per shirt for the 7-shirt order, correct to the nearest cent.

Q12. A hardware order lists 6 bolts, 2 hooks, then 3 more bolts and 5 more hooks. A bolt costs b dollars and a hook costs h dollars. (a) Write the total cost as a sum of four terms. (b) Collect like terms to simplify the total. (c) A classmate adds all four coefficients and multiplies the letters, getting $16bh$. Show, using $b = 2$ and $h = 1$, that this is wrong.

Q13. An expression is $A = 2x^2 + 5$. (a) Find A when $x = 3$. (b) Write in words what the term $2x^2$ tells you to do. (c) Show that $2x^2$ and $(2x)^2$ give different values when $x = 3$.

Q14. A smoothie stall counts cups sold with $P = 5h + 18$, where h is the number of hours since opening.

h (hours)	0	1	2	3
P (cups)				

Made-up data: the stall's hourly cup counts are made up for this section.

- Complete the table.
- Describe in one sentence how P changes from one hour to the next.
- Find P when $h = 8$.

Q15. A strip of turf is 4 m wide and $(x + 3)$ m long. (a) Write the area of the strip as a number times a bracket. (b) Expand your bracket. (c) Find the area in m^2 when $x = 6$, and check it against the bracket version.

Q16. Kai writes a job total as $40 + 15h$; Rai writes the same job as $15h + 40$. (a) Evaluate both expressions at $h = 2$. (b) Evaluate both expressions again at $h = 5$. Are the two expressions always equal? (c) Explain in one sentence why the order of the two terms does not change the total.

Q17. A tile pattern uses $T = 5n + 12$ tiles for n rows. (a) Find T when $n = 7$, one step per line. (b) Find T when $n = 20$. (c) A shelf holds at most 500 tiles. Does a 100-row pattern fit on the shelf? Use a value to justify your answer.

Q18. Four phrase cards and four expression cards are shown below.

1. 7 added to 3 times n

2. the sum of n and 7, multiplied by 3

3. n divided by 3, then add 7

4. 3 subtracted from 7 times n

A. $3(n + 7)$

B. $7n - 3$

C. $\frac{n}{3} + 7$

D. $3n + 7$

- (a) Match each phrase card (1–4) to its expression card (A–D).
(b) Show, using $n = 2$, that phrase 1 and phrase 2 give different values.
(c) Evaluate expression card C at $n = 6$ and expression card B at $n = 2$.
-

Answers

Q1. (a) $7c$. (b) $6m$. (c) pq . **Q2.** (a) $k+9$. (b) $\frac{t}{4}$. (c) $5h-2$; at $h=4$: 18. **Q3.** (a) $7d$. (b) $3y+2$. (c) $6n$. **Q4.** (a) \$71. (b) \$35 — the booking fee, charged even for zero nights. (c) \$125. **Q5.** (a) true. (b) true. (c) false: $x+x=2x$ (at $x=5$: 10, not 25). **Q6.** (a) $3x+6$. (b) $5m+20$. (c) $2y+14$; check at $y=3$: $2 \times (3+7)=20$ and $2 \times 3+14=20$. **Q7.** (a) $3p$ means $3 \times p$, so the two expressions are the same rule written two ways. (b) \$49 by each expression. (c) \$224; the \$25 setup is charged once, not once per poster, so Jo's version overcharges. **Q8.** (a) $L+3+L+3$. (b) $2L+6$. (c) 16 m; check $5+3+5+3=16$. **Q9.** (a) $4a$ and $3b$ are not like terms, so they cannot be merged into one term. (b) $4a+3b$ — it is already simplified. (c) original 11, Sam's 14 — different. **Q10.** (a) $5x+10$. (b) $9x+10$. (c) 37 cm; check $5 \times (3+2)+4 \times 3=37$. **Q11.** (a) \$90.75. (b) \$77.25. (c) \$11.04 per shirt, correct to the nearest cent. **Q12.** (a) $6b+2h+3b+5h$. (b) $9b+7h$. (c) the true total is \$25; $16bh$ gives \$32 — a different number, so it is wrong. **Q13.** (a) 23. (b) square x , then double it: $2 \times x \times x$. (c) at $x=3$: $2x^2=18$ but $(2x)^2=36$. **Q14.** (a) 18, 23, 28, 33. (b) P grows by 5 cups each hour. (c) 58. **Q15.** (a) $4(x+3)$. (b) $4x+12$. (c) 36 m^2 ; check $4 \times (6+3)=36$. **Q16.** (a) 70 and 70. (b) 115 and 115; yes, always equal. (c) adding in either order gives the same total, and the two terms are not like terms, so neither absorbs the other. **Q17.** (a) 47. (b) 112. (c) no: $T=512$ tiles, which is more than 500. **Q18.** (a) $1 \rightarrow D$, $2 \rightarrow A$, $3 \rightarrow C$, $4 \rightarrow B$. (b) 13 and 27 — different. (c) 9 and 11.

Fully-worked solutions

Q1. (a) Number and letter: drop the sign, number first: $7c$. (b) Number first: $6m$. (c) Two letters: drop the sign: pq .

Q2. (a) "9 more than" means add 9: $k+9$. (b) Division is written as a fraction: $\frac{t}{4}$. (c) $5 \times h$ then take away 2: $5h-2$.
At $h=4$: $5 \times 4 - 2 = 20 - 2 = 18$.

Q3. (a) $4d+3d=7d$. (b) $6y-3y=3y$, and the plain 2 stays: $3y+2$. (c) $8n-3n+n=6n$ (the lone n is $1n$, so it counts in the collecting).

Q4. (a) $R=35+18 \times 2=35+36=71$, so \$71. (b) $R=35+18 \times 0=35+0=35$, so \$35. The 35 is the fixed booking fee — it is charged even for zero nights. (c) $R=35+18 \times 5=35+90=125$, so \$125.

Q5. (a) True: a coefficient of 1 is left invisible. (b) True: $3h+5h=8h$ collects like terms. (c) False: adding is not multiplying. At $x=5$: $x+x=10$ but $x^2=25$. The correct version is $x+x=2x$.

Q6. (a) $3(x+2)=3x+6$. (b) $5(m+4)=5m+20$. (c) $2(y+7)=2y+14$. Check at $y=3$: bracket side $2 \times (3+7)=20$; expanded side $2 \times 3+14=20$ — the same.

Q7. (a) $3p$ is the convention for $3 \times p$, so Ana's and Ben's expressions are one rule written two ways. (b) Ana: $25+3 \times 8=25+24=49$. Ben: $25+3 \times 8=49$. Both give \$49. (c) Jo: $28 \times 8=224$, so \$224. It cannot be right because the \$25 setup fee is charged once per order, not once per poster — Jo charges it 8 times.

Q8. (a) Perimeter = $L+3+L+3$ (the four sides in order). (b) Collect the L terms and the number terms: $L+L=2L$ and $3+3=6$, so $2L+6$. (c) At $L=5$: $2 \times 5+6=10+6=16$ m. Check by adding the sides: $5+3+5+3=16$ m — the same.

Q9. (a) Sam has merged unlike terms: $4a$ and $3b$ have different pronumerals, so they cannot be added into a single term. (b) $4a+3b$ is already simplified — it stays as two terms. (c) At $a=2$, $b=1$: original $4 \times 2+3 \times 1=8+3=11$; Sam's $7 \times 2 \times 1=14$. Different values, so the simplifying was not allowed.

Q10. (a) $5(x+2)=5x+10$. (b) $5x+10+4x=9x+10$. (c) At $x=3$: $9 \times 3+10=27+10=37$ cm. Check from the unexpanded form: $5 \times (3+2)+4 \times 3=25+12=37$ cm — the same.

Q11. (a) $C=30+6.75 \times 9=30+60.75=90.75$, so \$90.75. (b) $C=30+6.75 \times 7=30+47.25=77.25$, so \$77.25. (c) Cost per shirt: $\$77.25 \div 7 = \$11.0357\dots$, which is \$11.04 correct to the nearest cent.

Q12. (a) Total = $6b + 2h + 3b + 5h$. (b) Bolts: $6b + 3b = 9b$; hooks: $2h + 5h = 7h$; total = $9b + 7h$. (c) At $b = 2$, $h = 1$: true total $9 \times 2 + 7 \times 1 = 18 + 7 = 25$, so \$25. The classmate's $16 \times 2 \times 1 = 32$, so \$32. Different numbers, so $16bh$ is wrong — unlike terms add their coefficients only, and never multiply the letters.

Q13. (a) At $x = 3$: $x^2 = 9$, then $2 \times 9 = 18$, then $18 + 5 = 23$. (b) $2x^2$ means: square x first, then multiply by 2 — that is $2 \times x \times x$. (c) At $x = 3$: $2x^2 = 2 \times 9 = 18$, while $(2x)^2 = (2 \times 3)^2 = 36$. The index sits on the letter only unless brackets move it.

Q14. (a) $h = 0$: $5 \times 0 + 18 = 18$. $h = 1$: $5 + 18 = 23$. $h = 2$: $10 + 18 = 28$. $h = 3$: $15 + 18 = 33$.

h (hours)	0	1	2	3
P (cups)	18	23	28	33

(b) P grows by 5 cups for every extra hour — the $5h$ term adds 5 each time.

(c) At $h = 8$: $5 \times 8 + 18 = 40 + 18 = 58$ cups.

Q15. (a) Area = $4(x + 3)$ m². (b) $4(x + 3) = 4x + 12$. (c) At $x = 6$: $4 \times 6 + 12 = 24 + 12 = 36$ m². Check from the bracket: $4 \times (6 + 3) = 4 \times 9 = 36$ m² — the same.

Q16. (a) Kai: $40 + 15 \times 2 = 40 + 30 = 70$. Rai: $15 \times 2 + 40 = 30 + 40 = 70$. (b) Kai: $40 + 15 \times 5 = 40 + 75 = 115$. Rai: $15 \times 5 + 40 = 75 + 40 = 115$. Yes — the two expressions are always equal. (c) Addition gives the same total in either order, and $15h$ and 40 are not like terms, so writing them in a different order changes nothing but the look.

Q17. (a) One step per line: $T = 5n + 12$

$$T = 5 \times 7 + 12$$

$$T = 35 + 12$$

$T = 47$ (b) $T = 5 \times 20 + 12 = 100 + 12 = 112$ tiles. (c) At $n = 100$: $T = 5 \times 100 + 12 = 500 + 12 = 512$ tiles. $512 > 500$, so no — the 100-row pattern does not fit on the shelf.

Q18. (a) Phrase 1 (“7 added to 3 times n ”) is card D, $3n + 7$. Phrase 2 (“the sum of n and 7, multiplied by 3”) is card A, $3(n + 7)$. Phrase 3 (“ n divided by 3, then add 7”) is card C, $\frac{n}{3} + 7$. Phrase 4 (“3 subtracted from 7 times n ”) is card B, $7n - 3$. (b) At $n = 2$: card D gives $3 \times 2 + 7 = 6 + 7 = 13$; card A gives $3 \times (2 + 7) = 3 \times 9 = 27$. Different values, so the two phrases are different rules — the brackets in phrase 2 put the adding first. (c) Card C at $n = 6$: $\frac{6}{3} + 7 = 2 + 7 = 9$. Card B at $n = 2$: $7 \times 2 - 3 = 14 - 3 = 11$.

Chapter 05 — Test A (multiple choice)

Chapter 5 (Relationships). Materials: one scientific calculator and paper. There are 12 multiple-choice questions, each worth 1 mark (12 marks). Approximate time: 25 minutes. Choose the one correct response for each question. Every business, price and measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1

A plumber charges a fixed fee of \$70 plus \$45 for each hour of work. The cost, in dollars, for h hours of work is $C = 70 + 45h$. What is the cost when $h = 3$?

- A. \$115
- B. \$135
- C. \$345
- D. \$205

Question 2

The cost, in dollars, of hiring a machine for n days is $C = 75 + 18n$. One hire cost \$201. What is the value of n ?

- A. 8
- B. 7
- C. 11
- D. 126

Question 3

A train travels 368 km in 4 hours at a steady speed. What is its speed, in km/h?

- A. 23 km/h
- B. 364 km/h
- C. 92 km/h
- D. 1 472 km/h

Question 4

A packing machine fills boxes at a steady rate of 9 boxes per minute. How many boxes does it fill in 15 minutes?

- A. 135 boxes
- B. 24 boxes
- C. 6 boxes
- D. 0.6 boxes

Question 5

A sprinter runs at 5 m/s. What is this speed in km/h?

- A. 3.6 km/h
- B. 30 km/h
- C. 50 km/h
- D. 18 km/h

Question 6

On a map of scale 1 : 25 000, two campsites are 8 cm apart. What is the real distance between them, in km?

- A. 0.2 km
- B. 2 km
- C. 20 km
- D. 200 km

Question 7

A cordial is mixed with water in the ratio 1 : 4 (cordial : water). How much cordial, in mL, is needed to make 450 mL of the drink?

- A. 112.5 mL
- B. 360 mL
- C. 90 mL
- D. 225 mL

Question 8

Under a microscope of magnification $\times 40$, the image of a cell is 12 mm wide. What is the actual width of the cell?

- A. 0.3 mm
- B. 3 mm
- C. 30 mm
- D. 480 mm

Question 9

A machine starts at 0 and, at a steady rate, makes 18 parts in 3 minutes and 36 parts in 6 minutes. Which rule gives the number of parts y made in x minutes?

- A. $y = x + 6$
- B. $y = 3x$
- C. $y = 18x$
- D. $y = 6x$

Question 10

A car park charges \$5 for up to and including 1 hour of parking, then \$4 for each extra hour or part hour. What is the cost of parking for 3.5 hours?

A. \$13

B. \$15

C. \$17

D. \$19

Question 11

Expand $8(n+2)$.

A. $8n+2$

B. $8n+10$

C. $n+16$

D. $8n+16$

Question 12

Simplify $7m+3m+5$.

A. $10m$

B. $10m+5$

C. $15m$

D. $7m+8$

End of Test A

Test A — Answers

Que stio n	1	2	3	4	5	6	7	8	9	10	11	12
Ans wer	D	B	C	A	D	B	C	A	D	C	D	B

Test A — Fully-worked solutions

Question 1. Substitute $h=3$ into the formula: $C=70+45\times 3=70+135=205$, so the cost is \$205. (\$135 leaves out the fixed fee, \$115 is the fee plus only one hour, and \$345 wrongly multiplies $70+45$ by 3.) **Answer D.**

Question 2. Back-track, undoing each step in reverse order with the inverse operation: $201-75=126$ (subtract the fixed fee), then $126\div 18=7$ (divide by the daily rate). Check: $75+18\times 7=75+126=201$ ✓. (126 stops after the first step, and 11 comes from ignoring the fee and dividing $201\div 18\approx 11.2$.) **Answer B.**

Question 3. Speed = distance $\div$ time = $368\div 4=92$ km/h. (1 472 multiplies instead of dividing, 364 subtracts, and 23 divides by 4 twice.) **Answer C.**

Question 4. Amount = rate $\times$ time = $9\times 15=135$ boxes. (24 adds the two numbers, 6 subtracts them, and 0.6 divides the wrong way round.) **Answer A.**

Question 5. To change m/s into km/h, multiply by 3.6: $5\times 3.6=18$ km/h. (30 multiplies by 6, 50 multiplies by 10, and 3.6 is the conversion factor itself, not the answer.) **Answer D.**

Question 6. Drawing $\rightarrow$ real: multiply by the scale factor. $8\times 25\,000=200\,000$ cm, and $200\,000$ cm = 2 000 m = 2 km. (The other options each slip a power of 10 in the unit change.) **Answer B.**

Question 7. The mix has $1+4=5$ equal parts and the total is known, so one part is $450\div 5=90$ mL. The cordial is the 1-part side: 90 mL. (112.5 mL divides by 4 instead of 5, and 360 mL is the water, not the cordial.) **Answer C.**

Question 8. Actual size = image size $\div$ magnification factor = $12\div 40=0.3$ mm. (480 mm multiplies instead of dividing — that would make the cell bigger than its image.) **Answer A.**

Question 9. Test each rule on the pairs given. $y=6x$ fits all three: $6\times 0=0$, $6\times 3=18$ and $6\times 6=36$. $y=x+6$ fails at $x=0$ (it gives 6, not 0), $y=3x$ gives 9 at $x=3$, and $y=18x$ gives 54 at $x=3$. **Answer D.**

Question 10. The first hour costs \$5. The time left is $3.5-1=2.5$ hours, and each extra hour **or part hour** costs \$4, so the extra time is charged as 3 lots of \$4 (the part hour counts as a full hour): $5+3\times 4=5+12=17$. (\$15 charges the half hour at only half price, and \$13 ignores it altogether.) **Answer C.**

Question 11. Multiply everything inside the bracket by 8: $8(n+2)=8\times n+8\times 2=8n+16$. Check at $n=1$: $8\times (1+2)=24$ and $8\times 1+16=24$ ✓. ($8n+2$ forgets to multiply the 2, and $8n+10$ adds $8+2$ instead of multiplying.) **Answer D.**

Question 12. $7m$ and $3m$ are like terms: $7m+3m=10m$. The 5 is a plain number, not an m term, so it stays separate: $10m+5$. Check at $m=2$: $14+6+5=25$ and $10\times 2+5=25$ ✓. ($15m$ wrongly adds the 5 into the m terms; $10m$ drops the 5.) **Answer B.**

Chapter 05 — Test B (written response)

Chapter 5 (Relationships). Materials: one scientific calculator and paper. There are 8 written questions, worth 43 marks in total. Approximate time: 40 minutes. Answer every question in the spaces provided and show your working. Where an answer needs rounding, the question states the precision — round only as instructed. Every business, price and measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1 (6 marks)

A café charges a \$45 booking fee plus \$12 per person for a group booking. The cost, in dollars, for n people is $C = 45 + 12n$.

a. Find the cost of a booking for 20 people.

2 marks

b. One booking cost \$297. Find n .

2 marks

c. Explain what the 45 and the 12 represent in this situation.

2 marks

Question 2 (6 marks)

A hose fills a tank at a steady rate of 14 litres per minute.

a. How many litres does the hose deliver in 22 minutes?

1 mark

b. The tank holds 3920 L when it is full. How many minutes does it take to fill the empty tank at this rate?

2 marks

c. Write your time from (b) in hours and minutes.

1 mark

d. A second hose delivers 275 L in 25 minutes. Find its rate in litres per minute, and state which hose is faster.

2 marks

Question 3 (5 marks)

A road map has a scale of 1 : 100 000.

- a.** What real distance, in km, does 1 cm on the map represent? 1 mark

- b.** A route measures 9 cm on the map. Find the real distance of the route, in km. 2 marks

- c.** A highway is 32 km long in real life. What length is it on the map, in cm? 2 marks

Question 4 (5 marks)

A science magazine prints a photograph of a butterfly at a magnification of $\times 6$.

- a.** In the photograph, the image of the butterfly is 54 mm long. Find the actual length of the butterfly, in mm. 2 marks

- b.** A leaf in the same photograph is 22 mm long in real life. Find the length of its image, in mm. 1 mark

- c.** Another photograph in the magazine shows a spider. The image of the spider is 56 mm wide and its actual width is 7 mm. Find the magnification factor of that photograph. 2 marks

Question 5 (5 marks)

A school printer prints at a steady rate of 7 sheets per minute. The number of sheets p printed in m minutes is given by $p = 7 \times m$.

- a.** Make a table of values of p for $m = 0, 5, 10, 15$ and 20 . 2 marks

- b.** On a graph of p against m , should the points be joined by a line or left as separate dots? Give a reason. 2 marks

- c.** Use the rule to find the number of sheets printed in 12 minutes. 1 mark

Question 6 (6 marks)

A rectangular sign is w metres wide and 4 metres tall. Its perimeter is P metres.

- a.** Write an expression for P as the sum of the four sides of the sign. 1 mark

- b.** Collect like terms to simplify your expression for P . 1 mark

- c.** Find P when $w = 7$, and check your answer by adding the four side lengths. 2 marks

- d.** Expand $6(t + 9)$. 1 mark

- e.** Joel simplifies $6k + 4$ to $10k$. Explain why Joel is wrong. 1 mark

Question 7 (5 marks)

A cyclist travels 45 km in 1.5 hours at a steady speed.

- a.** Find the speed of the cyclist, in km/h. 1 mark

- b.** Convert this speed into m/s, correct to 1 decimal place. 2 marks

- c.** A runner has a speed of 4 m/s. Convert this speed into km/h. 1 mark

- d.** Who is travelling faster, the cyclist or the runner? Use your speeds from (a) and (c) to explain your answer. 1 mark

Question 8 (5 marks)

The conversion factor between miles and kilometres is 1 mile = 1.609 km. A conversion chart is drawn with miles on the horizontal axis and kilometres on the vertical axis.

a. Convert 30 miles into km, to the nearest km.

1 mark

b. Convert 80 km into miles, correct to 1 decimal place.

2 marks

c. The point (40 miles, 64.36 km) lies on the chart. Divide the km value by the miles value and say what this number represents.

2 marks

End of Test B

Test B — Answers

Q1. (a) \$285 (b) $n = 21$ (c) the 45 is the fixed booking fee, charged once whatever the number of people; the 12 is the extra cost for each person. **Q2.** (a) 308 L (b) 280 min (c) 4 h 40 min (d) 11 L/min; the first hose is faster. **Q3.** (a) 1 km (b) 9 km (c) 32 cm. **Q4.** (a) 9 mm (b) 132 mm (c) $\times 8$. **Q5.** (a) $p = 0, 35, 70, 105, 140$ (b) joined — time is measured, so the in-between values are real (c) 84 sheets. **Q6.** (a) $P = w + 4 + w + 4$ (b) $P = 2w + 8$ (c) 22 m; check $7 + 4 + 7 + 4 = 22$ (d) $6t + 54$ (e) $6k$ and 4 are not like terms, so they cannot be joined into one term — $6k + 4$ is already simplest. **Q7.** (a) 30 km/h (b) 8.3 m/s (c) 14.4 km/h (d) the cyclist — 30 km/h against 14.4 km/h. **Q8.** (a) 48 km (b) 49.7 miles (c) 1.609 — the conversion factor: the number of km in 1 mile.

Test B — Fully-worked solutions

Question 1.

- a. Substitute $n = 20$: $C = 45 + 12 \times 20 = 45 + 240 = 285$. The cost is \$285.
- b. Undo the steps in reverse order: $297 - 45 = 252$ (subtract the fixed fee), then $252 \div 12 = 21$ (divide by the cost per person). So $n = 21$. Check: $45 + 12 \times 21 = 45 + 252 = 297$ ✓.
- c. The 45 is the fixed booking fee — it is charged once per booking, whatever the number of people (it is the cost when $n = 0$). The 12 is the cost per person — the cost rises by \$12 for each extra person.

Question 2.

- a. Amount = rate $\times$ time = $14 \times 22 = 308$ L.
- b. Time = amount $\div$ rate = $3920 \div 14 = 280$ minutes.
- c. $280 \div 60 = 4$ remainder 40, so the time is 4 h 40 min. Check: $4 \times 60 + 40 = 280$ ✓.
- d. Rate = amount $\div$ time = $275 \div 25 = 11$ L/min. The first hose delivers 14 L/min and the second delivers 11 L/min, and $14 > 11$, so the first hose is faster.

Question 3.

- a. 1 cm represents 100 000 cm, and $100\,000\text{ cm} = 1\,000\text{ m} = 1\text{ km}$.
- b. Drawing $\rightarrow$ real: multiply. $9 \times 1 = 9$, so the route is 9 km. (Or by first principles: $9 \times 100\,000 = 900\,000\text{ cm} = 9\,000\text{ m} = 9\text{ km}$.)
- c. Real $\rightarrow$ drawing: divide. $32\text{ km} = 32\,000\text{ m} = 3\,200\,000\text{ cm}$, and $3\,200\,000 \div 100\,000 = 32$, so the highway is 32 cm on the map. Check: $32 \times 100\,000 = 3\,200\,000\text{ cm} = 32\text{ km}$ ✓.

Question 4.

- a. Actual size = image size $\div$ magnification factor = $54 \div 6 = 9$ mm.
- b. Image size = actual size $\times$ magnification factor = $22 \times 6 = 132$ mm.
- c. Magnification factor = image size $\div$ actual size = $56 \div 7 = 8$, so that photograph is at a magnification of $\times 8$.

Question 5.

- a. Substitute each value of m into $p = 7 \times m$:

m (minutes)	0	5	10	15	20
p (sheets)	0	35	70	105	140

- b. The points should be joined by a line. Time is measured, so in-between values (like 2.5 minutes) are real, and the number of sheets rises steadily between the plotted points.
- c. $p = 7 \times 12 = 84$ sheets.

Question 6.

- a. $P = w + 4 + w + 4$.
- b. $w + w = 2w$ and $4 + 4 = 8$, so $P = 2w + 8$.
- c. $P = 2 \times 7 + 8 = 14 + 8 = 22$ m. Check by adding the sides: $7 + 4 + 7 + 4 = 22$ ✓.
- d. $6(t + 9) = 6 \times t + 6 \times 9 = 6t + 54$. Check at $t = 1$: $6 \times (1 + 9) = 60$ and $6 \times 1 + 54 = 60$ ✓.
- e. Joel added the 4 onto the 6, but the 4 is a plain number, not a k term. $6k$ and 4 are not like terms, so they cannot be joined into one term: $6k + 4$ is already in its simplest form. Check at $k = 2$: $6 \times 2 + 4 = 16$, but Joel's $10k$ gives $10 \times 2 = 20$ — the two expressions give different values, so the simplification is wrong.

Question 7.

- a. Speed = distance $\div$ time = $45 \div 1.5 = 30$ km/h.
- b. km/h $\rightarrow$ m/s: divide by 3.6. $30 \div 3.6 = 8.333\dots$, so the speed is 8.3 m/s, correct to 1 decimal place.
- c. m/s $\rightarrow$ km/h: multiply by 3.6. $4 \times 3.6 = 14.4$ km/h.
- d. In the same units, the cyclist travels at 30 km/h and the runner at 14.4 km/h. $30 > 14.4$, so the cyclist is travelling faster. (Equivalently, 8.3 m/s against 4 m/s.)

Question 8.

- a. Miles $\rightarrow$ km: multiply by 1.609. $30 \times 1.609 = 48.27$, which is 48 km to the nearest km.
- b. Km $\rightarrow$ miles: divide by 1.609. $80 \div 1.609 = 49.720\dots$, which is 49.7 miles, correct to 1 decimal place.
- c. $64.36 \div 40 = 1.609$. This is the conversion factor — the number of kilometres in 1 mile. On the chart, the km value rises by 1.609 for each extra mile.