

VCE VM Numeracy Units 3 & 4

Chapter 4 — Quantity and measures

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Chapter 4 Quantity and measures — 4A Metric and non-metric units; conversion

Theory

Every measurement is a number **and** a unit. “188” on its own tells you nothing; “188 cm” tells you a person’s height. Australia measures in the **metric system**: millimetres, centimetres, metres and kilometres for length; grams and kilograms for mass; millilitres and litres for capacity; and degrees Celsius (°C) for temperature. But most of the world does not. The United States still measures in **inches, feet, pounds and degrees Fahrenheit**, and the United Kingdom mixes the two systems.

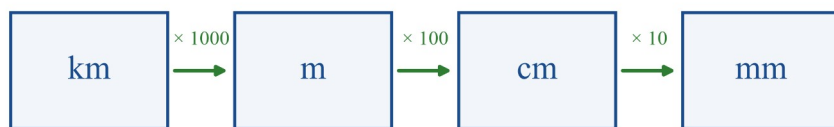
Even in Australia you meet **non-metric units** every week: phone and TV screens sold in inches, imported food labels in ounces and pounds, spanners and timber in inches, overseas weather reports in °F, and old family recipes in pints and cups. This section gives you the units, and the conversions between them, that everyday life and work actually use.

About the data in this section. Every product, recipe, sign, forecast, person and workplace in this section is **made up** (not real). They are built for this book so you can practise converting units. Nothing here is real data about a real person, business or place.

Metric units refresher.

Metric units are built on tens, and the name tells you the size. **Milli-** means one thousandth, **centi-** means one hundredth, and **kilo-** means one thousand. So:

- 1 cm = 10 mm
- 1 m = 100 cm = 1000 mm
- 1 km = 1000 m
- 1 kg = 1000 g
- 1 L = 1000 mL



each step to the right multiplies by that number; to go back left, divide by it

Step down to a smaller unit and the number gets **bigger** — multiply by the number on the arrow. Step up to a bigger unit and the number gets **smaller** — divide by that same number. Two quick examples: $2.5 \text{ km} = 2.5 \times 1000 = 2500 \text{ m}$, and $85 \text{ mm} = 85 \div 10 = 8.5 \text{ cm}$.

The non-metric units you will meet.

The non-metric units in this book come from the old British **imperial** system and the US system that grew from it. Inside each family the units link together like metric ones do — just not in tens:

What is measured	Units (small to large)	How they link
Length	inch (in), foot (ft), yard (yd), mile	$12 \text{ in} = 1 \text{ ft}$; $3 \text{ ft} = 1 \text{ yd}$; $1760 \text{ yd} = 1 \text{ mile}$
Mass	ounce (oz), pound (lb), stone (st)	$16 \text{ oz} = 1 \text{ lb}$; $14 \text{ lb} = 1 \text{ st}$
Capacity	fluid ounce (fl oz), pint (pt), gallon (gal)	$8 \text{ pt} = 1 \text{ gal}$ — but see the warning below

Watch out: US and UK pints and gallons are different sizes. A US pint is about 473 mL while a UK pint is about 568 mL. Always check which one a recipe or label means before you convert.

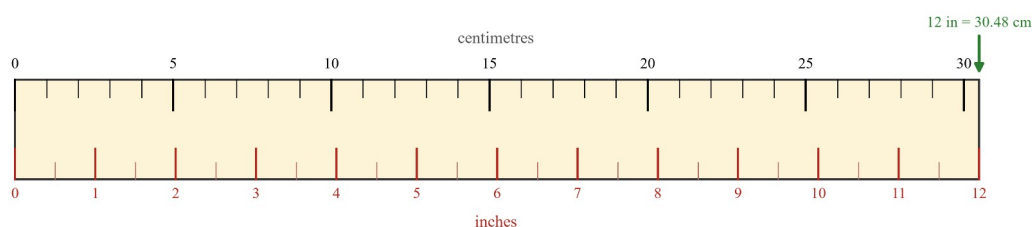
The conversion table — your toolkit for this section.

Between metric and non-metric units there is no neat pattern of tens. Each pair of units has its own **conversion factor**: the number you multiply or divide by to move between them. This table is the one used everywhere in this book. (Phone apps and online converters keep more decimal places than this table, so their answers can differ from yours in the last digit. That is fine — use this book’s factors.)

To convert	Use
inches → centimetres	$\times 2.54$ (1 in = 2.54 cm, exact)
centimetres → inches	$\div 2.54$
feet → centimetres	$\times 30.48$ (1 ft = 30.48 cm)
miles → kilometres	$\times 1.609$
kilometres → miles	$\div 1.609$
ounces → grams	$\times 28.35$
kilograms → pounds	$\times 2.205$ (1 kg = 2.205 lb)
pounds → kilograms	$\div 2.205$
US fluid ounces → millilitres	$\times 29.6$
US pints → millilitres	$\times 473$
UK pints → millilitres	$\times 568$
US gallons → litres	$\times 3.79$
UK gallons → litres	$\times 4.55$
°C → °F	$\times 1.8$, then $+ 32$
°F → °C	$- 32$, then $\div 1.8$

Multiply or divide? Choosing the right one.

Each row of the table reads one way. To travel the other way, do the opposite operation: inches to centimetres is $\times 2.54$, so centimetres to inches is $\div 2.54$.



The ruler above carries both scales. The 12 in mark lines up with 30.48 cm, because $12 \times 2.54 = 30.48$.

A check that always works is to ask: **should the new number be bigger or smaller?** Centimetres are *smaller* than inches, so a length written in centimetres needs *more* of them — the number grows, so you multiply. Going from centimetres back to inches, the number must shrink, so you divide. Worked both ways:

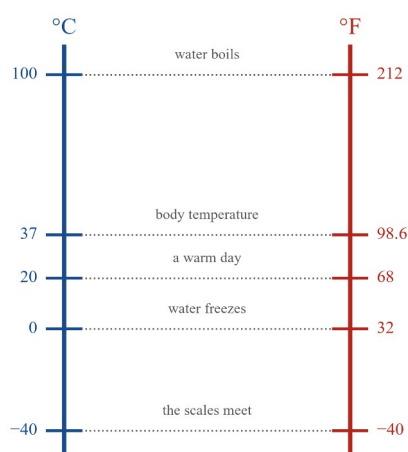
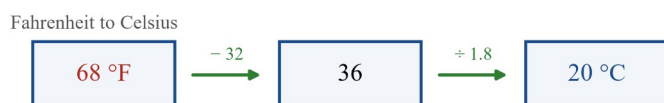
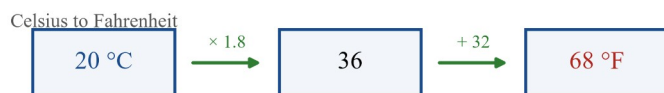
- $12 \text{ in} = 12 \times 2.54 = 30.48 \text{ cm}$ (number grew — correct, cm are smaller).
- $90 \text{ cm} = 90 \div 2.54 = 35.4331 \text{ in}$ before rounding, so 35.4 in to 1 decimal place (number shrank — correct, inches are bigger).

The most common mistake in this section is multiplying when you should divide, and ending up with a person “381 in” tall. Make the bigger-or-smaller check first, every time.

Temperature converts differently.

Length, mass and capacity conversions are one multiply or one divide. Temperature is not, because the two scales do not start at the same place and their degrees are not the same size. Water freezes at 0 °C but at 32 °F, and a step of 1 °C is as big as a step of 1.8 °F. So a temperature conversion takes **two steps**: scale by 1.8, then shift by 32.

- **°C to °F:** multiply by 1.8, then add 32.
- **°F to °C:** subtract 32, then divide by 1.8.



Four benchmarks are worth remembering, because they let you check any answer: water freezes at $0\text{ }^{\circ}\text{C} = 32\text{ }^{\circ}\text{F}$; body temperature is $37\text{ }^{\circ}\text{C} = 98.6\text{ }^{\circ}\text{F}$; water boils at $100\text{ }^{\circ}\text{C} = 212\text{ }^{\circ}\text{F}$; and the two scales show the same number at -40 , since $-40\text{ }^{\circ}\text{C} = -40\text{ }^{\circ}\text{F}$.

Quick estimates for checking.

Rough anchor	Why it sticks
1 in $\approx$ 2.5 cm	about the width of your thumb
1 ft $\approx$ 30 cm	about a school ruler
1 mile $\approx$ 1.6 km	a bit more than one and a half kilometres
1 kg $\approx$ 2.2 lb	a bit more than double
1 lb $\approx$ 450 g	a bit under half a kilogram
$^{\circ}\text{C} \rightarrow ^{\circ}\text{F}$ roughly: double it and add 30	$20\text{ }^{\circ}\text{C}$ gives about $70\text{ }^{\circ}\text{F}$ (true value $68\text{ }^{\circ}\text{F}$)

Rounding and precision.

Conversions rarely land on a whole number. 5 in is exactly 12.7 cm, but 5 cm is 1.9685 in before rounding — 1.97 in to 2 decimal places. So every question in this section tells you how precise the answer must be: “to 1 decimal place”, “to the nearest whole number”, and so on. Keep full precision while you work, round **only at the final step**, and write the precision with your answer. Outside this book, if nobody states a precision, choose a sensible one (1 decimal place or the nearest whole unit is usual for everyday work) and say which one you used.

Tools that help.

Phones, calculators and online converters will do these conversions for you, and knowing which tool to trust is part of your mathematical toolkit. But a converter cannot tell you whether its answer is sensible — that stays your job. Pick the factor, predict bigger or smaller, convert, then check against a quick anchor. If an app says a 2 kg bag of rice weighs 0.9 lb, your anchor ($1\text{ kg} \approx 2.2\text{ lb}$) tells you straight away that something is wrong.

Worked examples

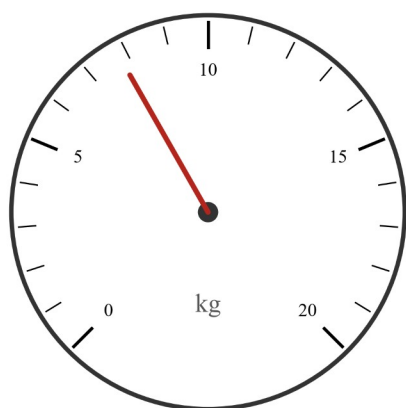
Example 1 — scaffolded

Mei is ordering a laptop stand from a US website. The listing says the stand is 12.6 in wide. The nook on Mei's desk is 34 cm wide. Will the stand fit? Convert to 1 decimal place.

Working	Comment
The factor is 1 in = 2.54 cm.	Read the factor from the conversion table.
Centimetres are the smaller unit, so the number must grow: multiply.	Decide multiply-or-divide before calculating.
$12.6 \times 2.54 = 32.004$ cm.	Keep full precision for now.
Round to 1 decimal place: 32.0 cm.	Round only at the step the question asks for.
$32.0 < 34$, so the stand fits, with about 2 cm to spare.	Answer the question asked, not just the number.

Example 2 — scaffolded

At the clinic, the scale below shows baby Anh's mass. His grandmother overseas thinks in pounds. Convert the reading to pounds, to 1 decimal place.

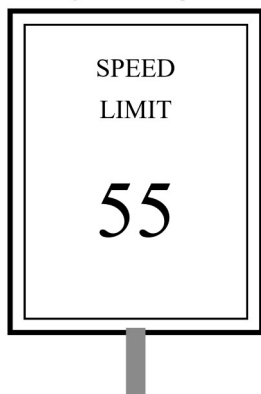


Working	Comment
The dial reads 7.8 kg (a little before the 8).	Read the tool first, then convert.
The factor is 1 kg = 2.205 lb. Pounds are the smaller unit, so the number grows: multiply.	Bigger-or-smaller check first.
$7.8 \times 2.205 = 17.199$ lb.	Keep full precision for now.
Round to 1 decimal place: 17.2 lb.	State the precision with the answer.
Check: $8 \text{ kg} \times 2.2 = 17.6 \text{ lb}$ — close to 17.2, so the answer is sensible.	A quick anchor catches big slips.

Example 3 — unscaffolded

A driver in the US passes the sign below. Speed limits there are in miles per hour (mph), and the next town is 96 miles further on.

US signs are in miles per hour



The “per hour” carries straight through, so the speed converts with the miles-to-km factor:

$$55 \times 1.609 = 88.495,$$

which rounds to 88 km/h to the nearest whole number. The distance converts the same way:

$$96 \times 1.609 = 154.464,$$

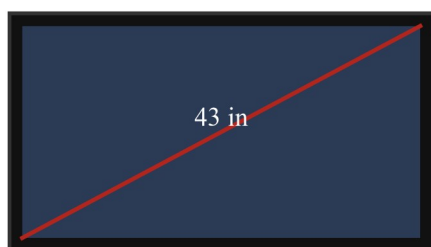
which rounds to 154 km to the nearest whole number.

$\therefore$ the limit is about 88 km/h and the next town is about 154 km away.

Example 4 — unscaffolded

Jay wants to move his TV in the car, lying the screen flat across the gap between the seat backs. The gap is 105 cm wide. The TV is advertised as a “43 in” screen, and a screen size is its diagonal — its longest measurement, as shown below. Will it fit? Convert to 1 decimal place.

the screen size is the diagonal, corner to corner



The diagonal in centimetres is

$$43 \times 2.54 = 109.22,$$

so the diagonal is 109.2 cm to 1 decimal place. The screen’s longest measurement (109.2 cm) is longer than the 105 cm gap, so lying flat will not work.

$\therefore$ the TV does not fit flat across the gap; Jay will need to angle it or fold a seat.

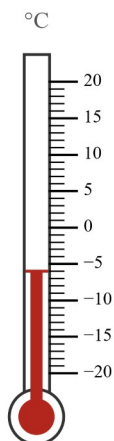
Practice questions

Scaffolded

Q1. Convert using the metric ladder. (a) 2.5 km to metres. (b) 85 mm to centimetres. (c) 1.45 kg to grams.

Q2. Use 1 in = 2.54 cm. (a) Convert 9 in to centimetres. (b) Convert 25 cm to inches, to 1 decimal place. (c) Which is longer: 9 in or 24 cm? Show the comparison that decides it.

Q3. The outdoor thermometer below shows the temperature on a frosty morning.



- (a) Write down the temperature in $^{\circ}\text{C}$.
- (b) Convert it to $^{\circ}\text{F}$.
- (c) Water freezes at 0°C . Is this morning above or below freezing?

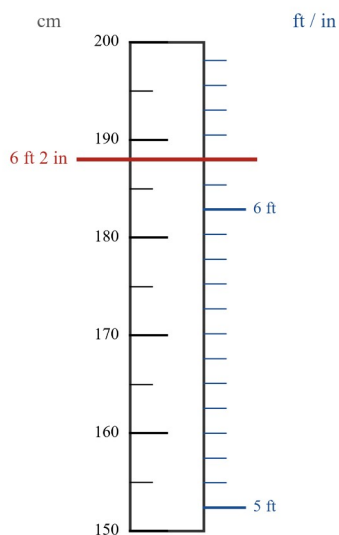
Q4. The US recipe card below is for banana loaf.

Banana loaf — a US recipe	
Butter	6 oz
Milk	8 fl oz
Oven temperature	350 $^{\circ}\text{F}$

- (a) Convert the butter amount to grams, to the nearest gram.
- (b) Convert the oven temperature to $^{\circ}\text{C}$, to the nearest whole degree.
- (c) Convert the milk amount to millilitres, to the nearest mL. (The card means US fluid ounces.)

Q5. A weather app shows 31°C in Cairns. Mei wants to tell her US penpal the same temperature in $^{\circ}\text{F}$, to 1 decimal place. (a) Step 1: multiply 31 by 1.8. (b) Step 2: add 32 to your step-1 answer. What is the temperature in $^{\circ}\text{F}$? (c) Body temperature is 98.6°F . Is the Cairns day hotter or cooler than body temperature?

Q6. The height chart below marks a basketball player listed at 6 ft 2 in.

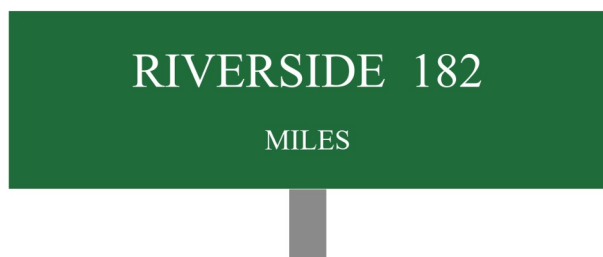


- Write the player's height in inches only.
- Convert it to centimetres, to the nearest cm.
- A rival is listed at 185 cm. Who is taller, and by how many centimetres (using your rounded answer to (b))?

Unscaffolded

Q7. Dana is driving in the US. The sign below shows the distance to the next town. Later she fills the tank with 12.5 US gallons of petrol; her tank holds 50 L.

US distance signs are in miles



- Convert the distance on the sign to kilometres, to the nearest km.
- Convert 12.5 US gallons to litres, to 1 decimal place.
- Does that fill-up fit in her 50 L tank?

Q8. The packet below is an imported lolly bag.



- (a) Convert the net weight to grams, to the nearest gram.
- (b) An Australian bag of the same lollies is 175 g. Using your answer to (a), which bag is heavier, and by how many grams?
- (c) Check your answer to (a) with the anchor $1 \text{ oz} \approx 28 \text{ g}$. Is it sensible?

Q9. A UK recipe asks for 1.5 UK pints of milk. (a) Convert this to millilitres. (b) Write the same amount in litres. (c) Explain why using US pints instead would spoil the recipe, and say how many mL of milk the recipe would then be short.

Q10. The forecast below is for the made-up town of Lakeview.

Lakeview 3-day forecast	
Monday	50 °F
Wednesday	68 °F
Friday	86 °F
temperature shown in °F	

Made-up data: the town name and every number in the forecast are made up for this section.

- (a) Convert each day's temperature to °C.
- (b) Which day will feel like a hot summer day (30 °C or more)?
- (c) On which day would you take a coat (below 12 °C)?

Q11. A mechanic's socket set includes a socket stamped $\frac{1}{2}$ in and one stamped $\frac{3}{4}$ in. A bolt head measures 19 mm across its flats. (a) Convert $\frac{1}{2}$ in to millimetres. (b) Convert $\frac{3}{4}$ in to millimetres. (c) Which socket fits the 19 mm bolt head? Explain your choice.

Q12. A newborn baby weighs 7 lb 5 oz. (a) Write the baby's mass in ounces only. (b) Convert it to grams, to the nearest gram. (c) Convert it to kilograms, to 2 decimal places.

Q13. Serena's uncle in England says he weighs 12 stone 6 lb. (Remember 1 stone = 14 lb.) (a) Write his mass in pounds only. (b) Convert it to kilograms, to 1 decimal place. (c) Check your answer with the anchor $1 \text{ kg} \approx 2.2 \text{ lb}$. Is it sensible?

Q14. A US car advertisement claims 35 miles per US gallon. An Australian car advertisement claims 6.8 L per 100 km. (a) Convert 35 miles to kilometres, to 1 decimal place. (b) One US gallon is 3.79 L. How many kilometres does the US car travel on one litre, to 1 decimal place? (c) The Australian car's claim works out as $100 \div 6.8$ km per litre. Decide, to 1 decimal place, which car travels further on one litre of fuel.

Q15. A garden centre sells an imported bag of potting mix marked 20 lb beside an Australian bag marked 10 kg. (a) Convert 20 lb to kilograms, to 1 decimal place. (b) Which bag is heavier? (c) How much heavier is it, to 1 decimal place? (Keep full precision until the last step.)

Q16. Imported framing timber is stamped "2 in $\times$ 4 in" and comes in 8 ft lengths. Australian framing timber is sold as 90 mm $\times$ 45 mm. (a) Convert the 2 in and 4 in sizes to centimetres, to 1 decimal place. (b) Convert the 8 ft length to metres, to 2 decimal places. (c) Convert the Australian 90 mm and 45 mm sizes to inches, to 1 decimal place, and say which timber is the wider of the two.

Q17. (a) Tom is 150 cm tall and converts his height to inches for a US form by calculating $150 \times 2.54 = 381$. Explain how you can tell he is wrong without a calculator, then find the correct height in inches, to 1 decimal place. (b) Show that -40°C and -40°F are the same temperature. (c) A patient's thermometer reads 100.4°F . Convert it to $^\circ\text{C}$, to 1 decimal place. A temperature of 38°C or more counts as a fever — does this patient have one?

Q18. Zoe is planning a US road trip. One day's driving is 265 miles; her car's tank holds 13.2 US gallons; and the morning forecast for the start town is 59°F . (a) Convert the day's driving distance to kilometres, to the nearest km. (b) Convert the tank capacity to litres, to 1 decimal place. (c) Convert the morning temperature to $^\circ\text{C}$, to the nearest whole degree.

Answers

Q1. (a) 2500 m. (b) 8.5 cm. (c) 1450 g. **Q2.** (a) 22.86 cm. (b) 9.8 in (to 1 d.p.). (c) 24 cm — it beats 22.86 cm. **Q3.** (a) -6°C . (b) 21.2°F . (c) below freezing. **Q4.** (a) 170 g (nearest g). (b) 177°C (nearest degree). (c) 237 mL (nearest mL). **Q5.** (a) 55.8. (b) 87.8°F (to 1 d.p.). (c) cooler than body temperature. **Q6.** (a) 74 in. (b) 188 cm (nearest cm). (c) the player, by 3 cm. **Q7.** (a) 293 km (nearest km). (b) 47.4 L (to 1 d.p.). (c) yes, 47.4 L fits in 50 L. **Q8.** (a) 150 g (nearest g). (b) the Australian bag, by 25 g. (c) yes — the estimate is about 148.4 g. **Q9.** (a) 852 mL. (b) 0.852 L. (c) US pints are smaller; the recipe would be 142.5 mL short. **Q10.** (a) Mon 10°C , Wed 20°C , Fri 30°C . (b) Friday. (c) Monday. **Q11.** (a) 12.7 mm. (b) 19.05 mm. (c) the $\frac{3}{4}$ in socket. **Q12.** (a) 117 oz. (b) 3317 g (nearest g). (c) 3.32 kg (to 2 d.p.). **Q13.** (a) 174 lb. (b) 78.9 kg (to 1 d.p.). (c) yes — the estimate is about 79.1 kg. **Q14.** (a) 56.3 km (to 1 d.p.). (b) 14.9 km/L (to 1 d.p.). (c) the US car (14.9 vs 14.7 km/L). **Q15.** (a) 9.1 kg (to 1 d.p.). (b) the 10 kg bag. (c) 0.9 kg (to 1 d.p.). **Q16.** (a) 5.1 cm and 10.2 cm (to 1 d.p.). (b) 2.44 m (to 2 d.p.). (c) 3.5 in and 1.8 in (to 1 d.p.); the US timber is wider. **Q17.** (a) inches are bigger than cm, so the number must shrink, not grow; 59.1 in (to 1 d.p.). (b) $-40 \times 1.8 = -72$, then $-72 + 32 = -40$. (c) 38.0°C (to 1 d.p.) — yes, a fever. **Q18.** (a) 426 km (nearest km). (b) 50.0 L (to 1 d.p.). (c) 15°C (nearest degree).

Fully-worked solutions

Q1. (a) km $\rightarrow$ m steps down the ladder, so multiply: $2.5 \times 1000 = 2500$ m. (b) mm $\rightarrow$ cm steps up, so divide: $85 \div 10 = 8.5$ cm. (c) kg $\rightarrow$ g steps down, so multiply: $1.45 \times 1000 = 1450$ g.

Q2. (a) Inches to centimetres makes the number grow: $9 \times 2.54 = 22.86$ cm. (b) Centimetres to inches makes the number shrink: $25 \div 2.54 = 9.8425$ in before rounding, so 9.8 in to 1 decimal place. (c) 9 in = 22.86 cm from (a), and $22.86 < 24$, so 24 cm is the longer length.

Q3. (a) The column ends one mark below -5°C , one mark being 1°C : the reading is -6°C . (b) $^{\circ}\text{C}$ to $^{\circ}\text{F}$: $-6 \times 1.8 = -10.8$, then $-10.8 + 32 = 21.2$, so 21.2°F . (c) -6°C is below 0°C , so the morning is below freezing — which matches a $^{\circ}\text{F}$ reading (21.2°F) below the freezing mark of 32°F .

Q4. (a) $6 \times 28.35 = 170.1$ g before rounding, so 170 g to the nearest gram. (b) $^{\circ}\text{F}$ to $^{\circ}\text{C}$: $350 - 32 = 318$, then $318 \div 1.8 = 176.6667^{\circ}\text{C}$ before rounding, so 177°C to the nearest whole degree. (c) $8 \times 29.6 = 236.8$ mL before rounding, so 237 mL to the nearest mL.

Q5. (a) $31 \times 1.8 = 55.8$. (b) $55.8 + 32 = 87.8$, so 87.8°F to 1 decimal place. (c) 87.8°F is less than 98.6°F , so the Cairns day is cooler than body temperature (hot, but not that hot).

Q6. (a) $6\text{ ft} = 6 \times 12 = 72$ in, plus 2 in: 74 in. (b) $74 \times 2.54 = 187.96$ cm before rounding, so 188 cm to the nearest cm. (c) $188 - 185 = 3$, so the player is taller by 3 cm.

Q7. (a) $182 \times 1.609 = 292.838$ km before rounding, so 293 km to the nearest km. (b) $12.5 \times 3.79 = 47.375$ L before rounding, so 47.4 L to 1 decimal place. (c) 47.4 L is less than the 50 L the tank holds, so yes — the fill-up fits.

Q8. (a) $5.3 \times 28.35 = 150.255$ g before rounding, so 150 g to the nearest gram. (b) $175 - 150 = 25$, so the Australian bag is heavier by 25 g. (c) The anchor gives $5.3 \times 28 = 148.4$ g, which is close to 150 g, so the answer in (a) is sensible.

Q9. (a) $1.5 \times 568 = 852$ mL. (b) $852 \div 1000 = 0.852$ L. (c) A US pint is only 473 mL, so 1.5 US pints is $1.5 \times 473 = 709.5$ mL — the recipe would be $852 - 709.5 = 142.5$ mL short of milk, enough to change how the dish turns out. That is why the pint's country matters.

Q10. (a) $^{\circ}\text{F}$ to $^{\circ}\text{C}$, subtract 32 then divide by 1.8: Monday $(50 - 32) \div 1.8 = 18 \div 1.8 = 10^{\circ}\text{C}$; Wednesday $(68 - 32) \div 1.8 = 36 \div 1.8 = 20^{\circ}\text{C}$; Friday $(86 - 32) \div 1.8 = 54 \div 1.8 = 30^{\circ}\text{C}$. (b) Friday, at 30°C . (c) Monday, at 10°C , which is below 12°C .

Q11. (a) $\frac{1}{2}$ in = 0.5 in, and $0.5 \times 2.54 = 1.27$ cm = 12.7 mm. (b) $\frac{3}{4}$ in = 0.75 in, and $0.75 \times 2.54 = 1.905$ cm = 19.05 mm. (c) The $\frac{3}{4}$ in socket: its 19.05 mm opening is almost exactly the 19 mm bolt head, while the $\frac{1}{2}$ in socket (12.7 mm) is far too small to go over it.

Q12. (a) $7 \text{ lb} = 7 \times 16 = 112$ oz, plus 5 oz: 117 oz. (b) $117 \times 28.35 = 3316.95$ g before rounding, so 3317 g to the nearest gram. (c) $3316.95 \div 1000 = 3.31695$ kg before rounding, so 3.32 kg to 2 decimal places.

Q13. (a) $12 \text{ stone} = 12 \times 14 = 168$ lb, plus 6 lb: 174 lb. (b) Pounds to kilograms makes the number shrink: $174 \div 2.205 = 78.9116$ kg before rounding, so 78.9 kg to 1 decimal place. (c) The anchor gives $174 \div 2.2 = 79.1$ kg to 1 decimal place, close to 78.9 kg, so the answer is sensible.

Q14. (a) $35 \times 1.609 = 56.315$ km before rounding, so 56.3 km to 1 decimal place. (b) One US gallon is 3.79 L, so the car travels $56.315 \div 3.79 = 14.8588$ km per litre before rounding — 14.9 km/L to 1 decimal place. (c) The Australian car travels $100 \div 6.8 = 14.7059$ km per litre before rounding — 14.7 km/L to 1 decimal place. Since $14.9 > 14.7$, the US car travels further on one litre of fuel.

Q15. (a) Pounds to kilograms makes the number shrink: $20 \div 2.205 = 9.0703$ kg before rounding, so 9.1 kg to 1 decimal place. (b) The Australian bag: 10 kg beats 9.0703 kg. (c) Carrying the unrounded value from (a) — 9.0703 kg to 4 decimal places — $10 - 9.0703 = 0.9297$ kg, so the Australian bag is heavier by 0.9 kg to 1 decimal place.

Q16. (a) $2 \times 2.54 = 5.08$ cm, which is 5.1 cm to 1 decimal place; $4 \times 2.54 = 10.16$ cm, which is 10.2 cm to 1 decimal place. (b) $8 \text{ ft} = 8 \times 12 = 96$ in, and $96 \times 2.54 = 243.84$ cm = 2.4384 m before rounding, so 2.44 m to 2 decimal places. (c) $90 \text{ mm} = 9$ cm, and $9 \div 2.54 = 3.5433$ in before rounding — 3.5 in to 1 decimal place; $45 \text{ mm} = 4.5$ cm, and $4.5 \div 2.54 = 1.7717$ in before rounding — 1.8 in to 1 decimal place. The US timber's 4 in face beats the Australian 3.5 in face, so the US timber is the wider of the two.

Q17. (a) Inches are *bigger* than centimetres, so the same height must be a *smaller* number of inches — 381 in (about 9.7 m of person) is impossible, and Tom multiplied where he should have divided. Correctly: $150 \div 2.54 = 59.0551$ in before rounding, so 59.1 in to 1 decimal place.

(b) Start at -40°C and apply the $^\circ\text{C}$ -to- $^\circ\text{F}$ rule, one step per line.

$$-40 \times 1.8 = -72$$

$$-72 + 32 = -40$$

The steps end at -40°F — the same number we started with in $^\circ\text{C}$. So -40°C and -40°F are the same temperature.

(c) $^\circ\text{F}$ to $^\circ\text{C}$: $100.4 - 32 = 68.4$, then $68.4 \div 1.8 = 38$, so 38.0°C to 1 decimal place. That is on the 38°C fever line, so yes — this patient has a fever.

Q18. (a) $265 \times 1.609 = 426.385$ km before rounding, so 426 km to the nearest km. (b) $13.2 \times 3.79 = 50.028$ L before rounding, so 50.0 L to 1 decimal place. (c) $^\circ\text{F}$ to $^\circ\text{C}$: $59 - 32 = 27$, then $27 \div 1.8 = 15$, so 15°C to the nearest whole degree — a cool morning, worth a jacket.

Chapter 4 Quantity and measures — 4B Perimeter and area (incl. circles and compound shapes)

Theory

Every flat shape — a garden bed, a floor, a paddock, a metal plate — can be measured in two different ways. You can measure **around** it, or you can measure **over** it. This section is about both: the **perimeter** (the distance around the outside) and the **area** (the amount of flat surface inside), for simple shapes, for circles, and for shapes built out of several pieces.

About the numbers in this section. Every garden, room, track, plan and price in this section is **made up** (not real). They are built for this book so you can practise perimeter and area. Nothing here is real data about a real person, business or place.

Perimeter and area: the two measurements of a flat shape.

The **perimeter** of a shape is the total distance around its outside edge. Perimeter is a length, so it is measured in length units: millimetres (mm), centimetres (cm), metres (m) or kilometres (km).

The **area** of a shape is the amount of flat surface it covers. Area is measured in square units: square centimetres (cm²), square metres (m²), hectares (ha) or square kilometres (km²). One square centimetre is the area of a square with sides of 1 cm.

Choosing the right measurement is a job skill in itself:

Job	Measurement needed
Fencing around a paddock	perimeter
Edging trim around a round table	perimeter (circumference)
Turf laid on a lawn	area
Tiles laid on a floor	area
Paint on a wall	area

Perimeter of straight-sided shapes.

To find the perimeter of any shape with straight sides, add the lengths of **all** the sides. For the two most common shapes there are shortcuts. A rectangle with length l and width w has

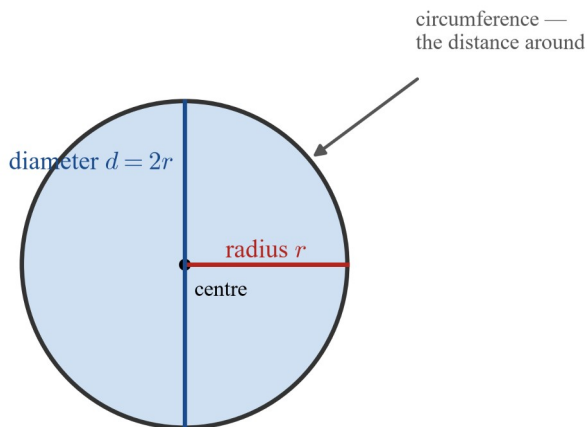
$$P = 2l + 2w = 2(l + w),$$

and a square with side s has $P = 4s$. For a triangle or any other straight-sided shape, simply add every side. Always check that all sides are in the same unit before you add.

Worked model. A rectangular sign is 8 m long and 3 m wide. Its perimeter is $P = 2 \times (8 + 3) = 2 \times 11 = 22$ m.

Circles: centre, radius, diameter and pi.

A circle is named by three parts, shown below. The **centre** is the middle point. The **radius** (r) is the distance from the centre to any point on the circle. The **diameter** (d) is the distance straight across the circle through the centre, so $d = 2r$. The perimeter of a circle has its own name: the **circumference**.



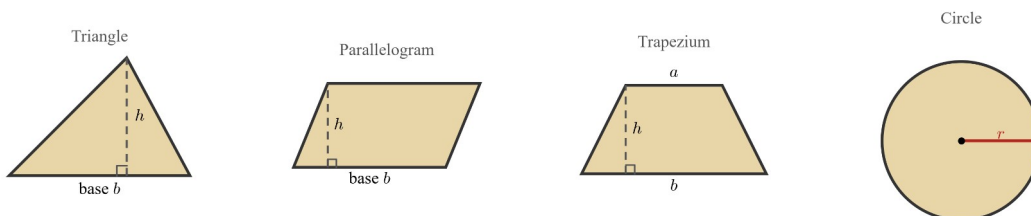
For **every** circle, the circumference divided by the diameter gives the same number. That number is called **pi**, written π . Its value never ends and never repeats: $\pi = 3.14159\dots$ (to 2 decimal places, $\pi \approx 3.14$). Your scientific calculator has a π key that stores many more digits than 3.14 — use it, and round only at the end, at the precision the question asks for. Because $\text{circumference} \div \text{diameter} = \pi$,

$$C = \pi d = 2\pi r.$$

Worked model. A circle has radius 5 cm. Its circumference is $C = 2 \times \pi \times 5 = 31.41592\dots$, so $C = 31.42$ cm to 2 decimal places.

Area formulas for common shapes.

The area formulas used in this section are collected below the figure. In every formula, h is the **height measured perpendicular to the base** — perpendicular means meeting at right angles (90°), shown by the small square in each picture. In a trapezium, a and b are the two **parallel sides** (the pair of sides that stay the same distance apart and never meet).



Shape	Area formula	In words
Square (side s)	$A = s^2$	side $\times$ side
Rectangle (length l , width w)	$A = lw$	length $\times$ width
Triangle (base b , height h)	$A = \frac{1}{2}bh$	half of base $\times$ height
Parallelogram (base b , height h)	$A = bh$	base $\times$ height
Trapezium (parallel sides a , b , height h)	$A = \frac{1}{2}(a+b)h$	half of (sum of parallel sides) $\times$ height
Circle (radius r)	$A = \pi r^2$	pi $\times$ radius $\times$ radius

Worked models. A trapezium with parallel sides 3 cm and 5 cm and height 4 cm has

$A = \frac{1}{2} \times (3+5) \times 4 = \frac{1}{2} \times 8 \times 4 = 16 \text{ cm}^2$. A circle of radius 5 cm has $A = \pi \times 5^2 = 78.53981\dots$, so $A = 78.54 \text{ cm}^2$ to 2 decimal places.

Parts of circles: semicircles and quarter-circles.

The curved edge of a piece of a circle is called an **arc**. An arc is a fraction of the full circumference, so its length is the same fraction of $2\pi r$:

- a **semicircle** (half a circle) has arc length $\frac{1}{2} \times 2\pi r = \pi r$;
- a **quarter-circle** has arc length $\frac{1}{4} \times 2\pi r = \frac{\pi r}{2}$.

The perimeter of a piece of a circle is **not just the arc**: you must add the straight edges too. A semicircular shape has perimeter = arc + diameter, and a quarter-circle shape has perimeter = arc + two radii. Forgetting the straight edges is the most common mistake with these shapes.

Big units of area: hectares and square kilometres.

Land is usually too big for square metres, so two bigger units are used:

- $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ (a square metre holds 100×100 square centimetres);
- 1 hectare (ha) = $10\,000 \text{ m}^2$ — the area of a square 100 m by 100 m;
- $1 \text{ km}^2 = 1\,000\,000 \text{ m}^2 = 100 \text{ ha}$.

Farms, paddocks, parks and suburbs are measured in hectares or square kilometres. To change m^2 to ha, divide by 10 000; to change ha to m^2 , multiply by 10 000.

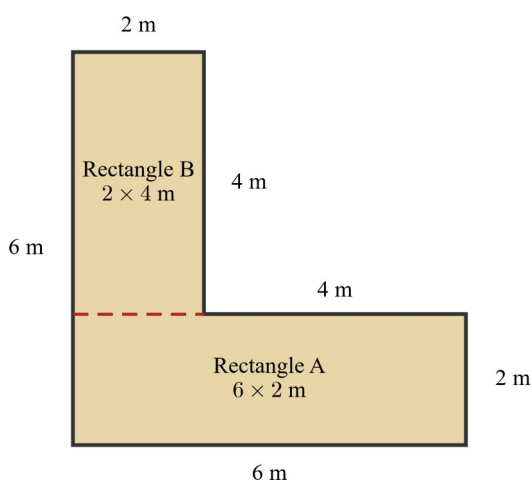
Compound shapes.

A **compound shape** is a shape built by joining simple shapes together, or by cutting a simple shape out of another one. An L-shaped room, a house block with a corner cut away, and a lawn with a path around it are all compound shapes.

Area. Split the shape into rectangles, triangles or circles you already know, find each piece, and **add**. Or, if the shape is a big shape with a piece missing, find the big shape and **subtract** the missing piece. Both ways must give the same answer — a good check.

Perimeter. The perimeter is the **outside edge only**. Lines you draw inside the shape to split it are not part of the perimeter. If a side length is not given, find it by comparing with the parallel sides opposite it.

Worked model. The L-shape below is split along the dashed line into two rectangles.



Perimeter: add the six outside sides, $6 + 2 + 4 + 4 + 2 + 6 = 24 \text{ m}$. (The dashed line is not counted — it is inside the shape.) Area: rectangle A is $6 \times 2 = 12 \text{ m}^2$, rectangle B is $2 \times 4 = 8 \text{ m}^2$, so the total area is $12 + 8 = 20 \text{ m}^2$.

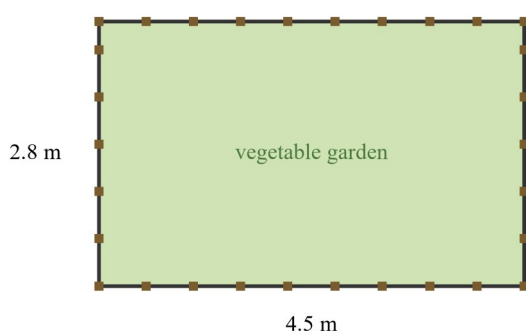
Rounding, units and checking.

Carry every digit your calculator shows through the working, and round **only** where the question tells you to — “to 2 decimal places”, “to the nearest cent”, “to the nearest whole number”. Answers that use π almost always need rounding, so the question will say how fine to be. Always write the unit with your answer: a length in m or cm, an area in m^2 or cm^2 . And always ask whether the answer is reasonable: a fencing length should be bigger than any single side, and a part of a shape should have a smaller area than the whole shape.

Worked examples

Example 1 — scaffolded

Dani digs a rectangular vegetable garden 4.5 m long and 2.8 m wide. She puts a low fence all the way around it to keep pets out. The fencing panel costs \$12.50 per metre.



Working

$$\text{Perimeter} = 2 \times (4.5 + 2.8)$$

$$= 2 \times 7.3 = 14.6 \text{ m}$$

$$\text{Cost} = 14.6 \times 12.50 = \$182.50$$

The fence needs 14.6 m of panel and costs \$182.50.

Comment

A rectangle's perimeter is $P = 2(l + w)$: two lengths plus two widths.

Add inside the brackets first.

Fencing is sold by the metre, so multiply the metres by the price per metre.

Record the unit, and money to the cent. This one lands exactly on a cent.

Example 2 — scaffolded

A round table has a radius of 1.5 m. The maker fits a metal trim right around the edge of the top, and a glass sheet over the whole top.

(a) Find the length of trim needed (the circumference), to 2 decimal places.

(b) Find the area of the glass sheet, to 2 decimal places.

Working

$$\text{Diameter} = 2 \times 1.5 = 3 \text{ m}$$

$$C = \pi d = \pi \times 3 = 9.424777 \dots$$

$$C = 9.42 \text{ m, to 2 decimal places}$$

$$A = \pi r^2 = \pi \times 1.5^2 = \pi \times 2.25$$

$$= 7.068583 \dots = 7.07 \text{ m}^2, \text{ to 2 decimal places}$$

Comment

The diameter is twice the radius: $d = 2r$.

Use the π key. Keep all the digits on the screen for now.

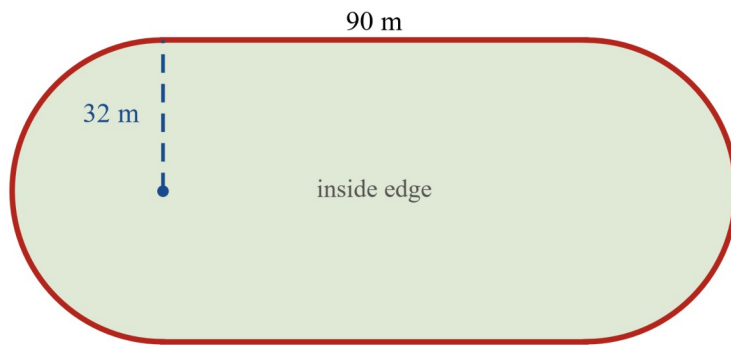
Round only at the end, at the precision the question asks for.

Area uses the **radius**, not the diameter. Square the radius first.

Area answers carry square units.

Example 3 — unscaffolded

The running track below has two straight sections, each 90 m long, joined by two semicircular ends. The radius of the inside edge of each end is 32 m. Find the distance of one lap of the track along the inside edge, to the nearest metre.



The two semicircular ends together make exactly one full circle of radius 32 m, so the curved part of one lap is

$$2 \times \pi \times 32 = 201.061929 \dots \text{m}.$$

The two straights add

$$2 \times 90 = 180 \text{ m},$$

so one lap along the inside edge is

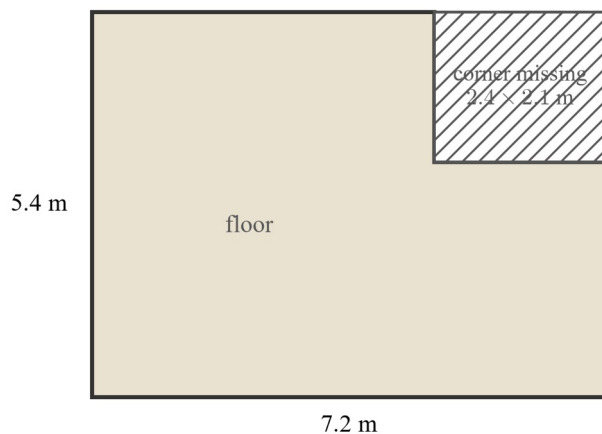
$$180 + 201.061929 \dots = 381.061929 \dots \text{m}.$$

Rounded to the nearest metre, this is 381 m.

$\therefore$ one lap of the track along the inside edge is 381 m, to the nearest metre.

Example 4 — unscaffolded

The floor of a small office is a rectangle 7.2 m by 5.4 m with a rectangular corner 2.4 m by 2.1 m cut out for a stair, as shown below. New carpet costs \$46 per square metre. Find the area to be carpeted, and the cost of the carpet correct to the nearest cent.



The floor is a big rectangle with a corner missing, so complete the rectangle and subtract. The full rectangle is

$$7.2 \times 5.4 = 38.88 \text{ m}^2,$$

and the missing corner is

$$2.4 \times 2.1 = 5.04 \text{ m}^2,$$

so the floor area is

$$38.88 - 5.04 = 33.84 \text{ m}^2.$$

The carpet cost is

$$33.84 \times 46 = 1556.64,$$

that is, \$1 556.64.

$\therefore$ the area to be carpeted is 33.84 m^2 and the carpet costs \$1 556.64, correct to the nearest cent.

Practice questions

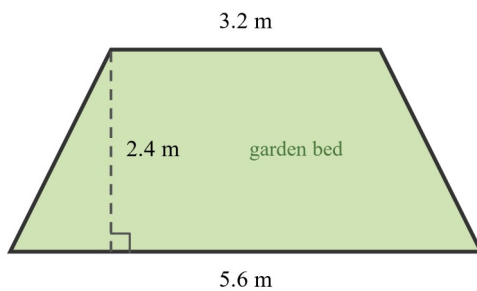
Scaffolded

Q1. A rectangle is 12 cm long and 7 cm wide. (a) Find its perimeter. (b) Find its area. (c) A square has sides of 6.4 cm. Find its perimeter.

Q2. A circle has a radius of 7 cm. Use the π key on your calculator. (a) Write down its diameter. (b) Find its circumference, to 2 decimal places. (c) Find its area, to 2 decimal places.

Q3. A boat sail is a triangle with base 3.6 m and height 2.5 m. A wall panel in the same boat is a parallelogram with base 4.2 m and height 1.8 m. (a) Find the area of the sail. (b) Find the area of the wall panel. (c) Both faces are painted on one side only. One tin of paint covers 8 m^2 . How many tins are needed?

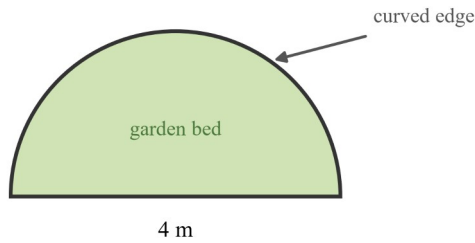
Q4. A garden bed is a trapezium with parallel sides 3.2 m and 5.6 m and height 2.4 m, as shown below.



- (a) Find the sum of the two parallel sides.
- (b) Find the area of the garden bed.
- (c) Mulch (bark spread over the soil) is sold in bags that each cover 2 m^2 , at \$7.50 a bag. How many bags are needed, and what do they cost?

Q5. A bedroom floor is a rectangle 4.2 m by 3.6 m. (a) Find the area of the floor. (b) Carpet costs \$38.50 per square metre. Find the cost of the carpet, correct to the nearest cent. (c) Underlay (the soft layer laid under carpet) costs \$6.20 per square metre. Find the cost of the underlay, correct to the nearest cent.

Q6. A garden bed is a semicircle (half a circle) with a straight side of 4 m, as shown below. Use the π key on your calculator.



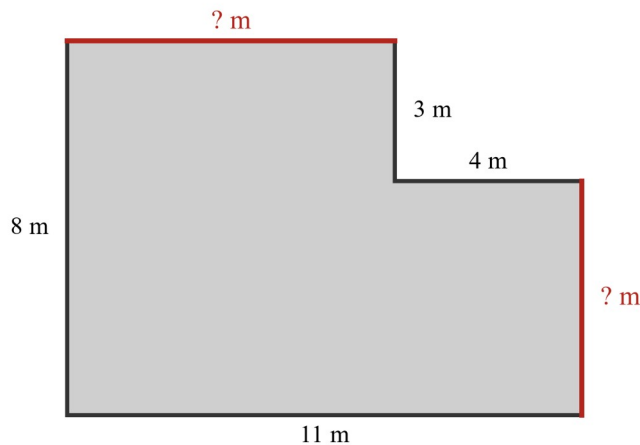
- Find the circumference of the **full** circle with diameter 4 m, to 2 decimal places.
- Find the length of the curved edge of the bed, to 2 decimal places.
- Find the full perimeter of the bed (curved edge plus the straight side), to 2 decimal places.

Unscaffolded

Q7. A rectangular paddock measures 38 m by 24 m. The fence around it includes a gate 4 m wide, and no fencing panel is needed across the gate. (a) Find the perimeter of the paddock. (b) Find the length of fencing panel needed. (c) Fencing panel costs \$9.80 per metre. Find the cost of the panel, correct to the nearest cent.

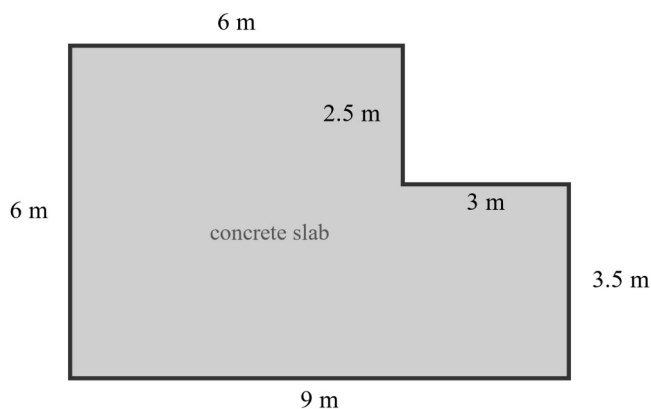
Q8. A lawn sprinkler waters a circle of radius 2.5 m. Use the π key on your calculator. (a) Find the circumference of the watered circle, to 2 decimal places. (b) Find the area of the watered circle, to 2 decimal places. (c) Fertiliser is spread at 60 g per square metre over the watered area. Find the mass of fertiliser needed, to the nearest gram.

Q9. A concrete pad is the L-shape shown below. Two side lengths are missing.



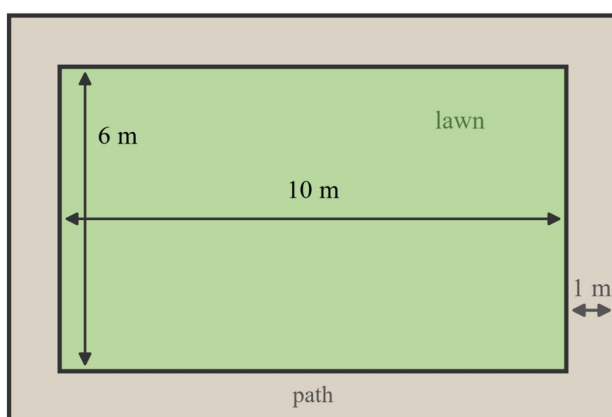
- Find the two missing side lengths marked “? m”.
- Find the perimeter of the pad.
- The pad fits exactly inside a full rectangle 11 m by 8 m. Explain why the perimeter of the pad is the same as the perimeter of that full rectangle.

Q10. A concrete slab has the compound shape shown below.



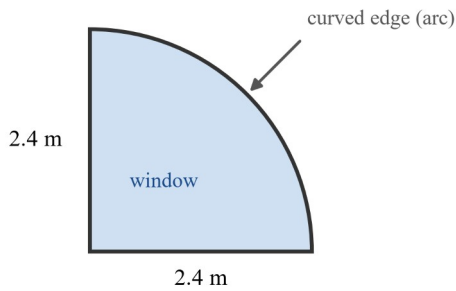
- (a) Split the shape into two rectangles with a horizontal line, and write down the dimensions of each rectangle.
- (b) Find the total area of the slab.
- (c) Concrete for the slab costs \$118 per square metre. Find the cost of the concrete, correct to the nearest cent.

Q11. A rectangular lawn 10 m by 6 m has a paved path 1 m wide all the way around it, as shown below.



- (a) Write down the length and the width of the outer rectangle (lawn plus path).
- (b) Find the area of the path.
- (c) The path is laid with pavers (flat paving blocks), 8 pavers per square metre. How many pavers are needed?

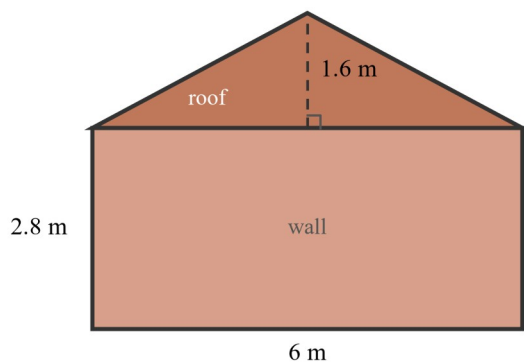
Q12. A window is a quarter-circle with radius 2.4 m, as shown below. Use the π key on your calculator.



- (a) Find the circumference of the full circle with radius 2.4 m, to 2 decimal places.
- (b) Find the length of the curved edge (the arc) of the window, to 2 decimal places.
- (c) Using your answer to (b), find the full perimeter of the window (arc plus the two straight edges), to 2 decimal places.

Q13. A square metal plate has sides of 20 cm. A circular hole of diameter 10 cm is cut out of its centre. Use the π key on your calculator. (a) Find the area of the full square plate. (b) Find the area of the circular hole, to 2 decimal places. (c) Find the area of metal left in the plate, to 2 decimal places.

Q14. The front of a garden shed is a rectangle 6 m wide and 2.8 m high, topped by a triangular roof section with base 6 m and height 1.6 m, as shown below.



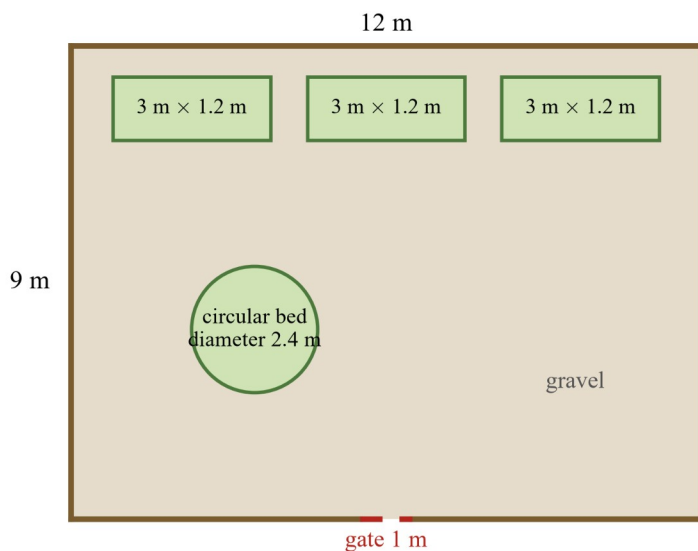
- (a) Find the area of the rectangular wall.
- (b) Find the area of the triangular roof section.
- (c) The whole front face (wall plus roof section) is painted with two coats of paint. One tin of paint covers 12 m^2 . How many tins are needed?

Q15. A rectangular paddock measures 850 m by 600 m. (a) Find its area in square metres. (b) Find its area in hectares. (c) The farm runs one cow per 1.5 ha on this paddock. How many cows can it carry?

Q16. Square tiles measure 25 cm by 25 cm. They are laid on a rectangular floor 5 m by 3 m. (a) Find the area of one tile in square centimetres. (b) Find the area of one tile in square metres. (c) Find the number of tiles needed to cover the floor exactly.

Q17. Two garden beds hold the same plants. One bed is a square with sides of 4.2 m. The other bed is a circle with diameter 5 m. Use the π key on your calculator. (a) Find the area of the square bed. (b) Find the area of the circular bed, to 2 decimal places. (c) Which bed needs more mulch, and by how much, to 2 decimal places?

Q18. A community group builds a garden on a rectangular plot 12 m by 9 m, shown below. A fence goes right around the plot, with a gate 1 m wide in the front. Inside are three rectangular garden beds, each 3 m by 1.2 m, and one circular garden bed with diameter 2.4 m. The rest of the plot is gravel path. Use the π key on your calculator.



- Find the length of fence needed, and its cost at \$17.20 per metre, correct to the nearest cent.
- Find the total area of the four garden beds (three rectangles plus the circle), to 2 decimal places.
- Using your answer to (b), find the area covered by gravel, to 2 decimal places, and the cost of the gravel at \$14.50 per square metre, correct to the nearest cent.

Answers

Q1. (a) 38 cm. (b) 84 cm². (c) 25.6 cm. **Q2.** (a) 14 cm. (b) 43.98 cm (2 d.p.). (c) 153.94 cm² (2 d.p.). **Q3.** (a) 4.5 m². (b) 7.56 m². (c) 2 tins. **Q4.** (a) 8.8 m. (b) 10.56 m². (c) 6 bags, \$45.00. **Q5.** (a) 15.12 m². (b) \$582.12. (c) \$93.74. **Q6.** (a) 12.57 m (2 d.p.). (b) 6.28 m (2 d.p.). (c) 10.28 m (2 d.p.). **Q7.** (a) 124 m. (b) 120 m. (c) \$1 176.00. **Q8.** (a) 15.71 m (2 d.p.). (b) 19.63 m² (2 d.p.). (c) 1178 g. **Q9.** (a) 5 m and 7 m. (b) 38 m. (c) sliding the 4 m and 3 m notch sides out to the edge of the full rectangle pairs the six sides into the rectangle's four sides, so the totals are the same. **Q10.** (a) bottom 9 m × 3.5 m; top 6 m × 2.5 m. (b) 46.5 m². (c) \$5 487.00. **Q11.** (a) 12 m and 8 m. (b) 36 m². (c) 288 pavers. **Q12.** (a) 15.08 m (2 d.p.). (b) 3.77 m (2 d.p.). (c) 8.57 m (2 d.p.). **Q13.** (a) 400 cm². (b) 78.54 cm² (2 d.p.). (c) 321.46 cm² (2 d.p.). **Q14.** (a) 16.8 m². (b) 4.8 m². (c) 4 tins. **Q15.** (a) 510 000 m². (b) 51 ha. (c) 34 cows. **Q16.** (a) 625 cm². (b) 0.0625 m². (c) 240 tiles. **Q17.** (a) 17.64 m². (b) 19.63 m² (2 d.p.). (c) the circular bed, by 1.99 m² (2 d.p.). **Q18.** (a) 41 m of fence, \$705.20. (b) 15.32 m². (c) 92.68 m² of gravel, \$1 343.86.

Fully-worked solutions

Q1. (a) $P = 2 \times (12 + 7) = 2 \times 19 = 38$ cm. (b) $A = 12 \times 7 = 84$ cm². (c) $P = 4 \times 6.4 = 25.6$ cm.

Q2. (a) $d = 2r = 2 \times 7 = 14$ cm. (b) $C = \pi d = \pi \times 14 = 43.98229\dots$, so $C = 43.98$ cm to 2 decimal places. (c) $A = \pi r^2 = \pi \times 7^2 = \pi \times 49 = 153.93804\dots$, so $A = 153.94$ cm² to 2 decimal places.

Q3. (a) $A = \frac{1}{2}bh = \frac{1}{2} \times 3.6 \times 2.5 = 4.5$ m². (b) $A = bh = 4.2 \times 1.8 = 7.56$ m². (c) Total area = $4.5 + 7.56 = 12.06$ m². $12.06 \div 8 = 1.5075$, and tins cannot be bought in parts, so 2 tins are needed (one tin would leave 4.06 m² unpainted).

Q4. (a) $3.2 + 5.6 = 8.8$ m. (b) $A = \frac{1}{2}(a+b)h = \frac{1}{2} \times 8.8 \times 2.4 = 10.56$ m². (c) $10.56 \div 2 = 5.28$, and bags cannot be bought in parts, so 6 bags are needed. Cost = $6 \times 7.50 = \$45.00$.

Q5. (a) $A = 4.2 \times 3.6 = 15.12$ m². (b) $15.12 \times 38.50 = 582.12$, so the carpet costs \$582.12, correct to the nearest cent. (c) $15.12 \times 6.20 = 93.744$, so the underlay costs \$93.74, correct to the nearest cent (93.744 rounds down to 93.74).

Q6. (a) $C = \pi d = \pi \times 4 = 12.56637\dots$, so 12.57 m to 2 decimal places. (b) The curved edge is half of that: $12.56637\dots \div 2 = 6.28318\dots$, so 6.28 m to 2 decimal places. (c) Perimeter = curved edge + straight side = $6.28318\dots + 4 = 10.28318\dots$, so 10.28 m to 2 decimal places.

Q7. (a) $P = 2 \times (38 + 24) = 2 \times 62 = 124$ m. (b) The gate takes the place of 4 m of panel: $124 - 4 = 120$ m. (c) $120 \times 9.80 = 1176.00$, so the panel costs \$1 176.00, correct to the nearest cent.

Q8. (a) $C = 2\pi r = 2 \times \pi \times 2.5 = 15.70796\dots$, so 15.71 m to 2 decimal places. (b) $A = \pi r^2 = \pi \times 2.5^2 = \pi \times 6.25 = 19.63495\dots$, so 19.63 m² to 2 decimal places. (c) $19.63495\dots \times 60 = 1178.097\dots$ g, so 1178 g to the nearest gram. (Using the rounded area from (b): $19.63 \times 60 = 1177.8$ g, which also rounds to 1178 g.)

Q9. (a) The right-hand side and the 3 m side together match the 8 m left side, so the right side is $8 - 3 = 5$ m. The top side and the 4 m side together match the 11 m bottom side, so the top side is $11 - 4 = 7$ m. $P = 11 + 5 + 4 + 3 + 7 + 8 = 38$ m. (c) Slide the 4 m side straight up to the top edge: it joins the 7 m top side to make the full 11 m top of the rectangle. Slide the 3 m side straight across to the right-hand edge: it joins the 5 m side to make the full 8 m right-hand side. The six sides of the pad therefore pair up into the four sides of the rectangle, so both perimeters are $2 \times (11 + 8) = 38$ m.

Q10. (a) A horizontal line through the shape gives a bottom rectangle 9 m by 3.5 m (since $6 - 2.5 = 3.5$) and a top rectangle 6 m by 2.5 m (since $9 - 3 = 6$). (b) Bottom: $9 \times 3.5 = 31.5$ m². Top: $6 \times 2.5 = 15$ m². Total: $31.5 + 15 = 46.5$ m². Check by subtracting: $9 \times 6 - 3 \times 2.5 = 54 - 7.5 = 46.5$ m² — the same. (c) $46.5 \times 118 = 5487.00$, so the concrete costs \$5 487.00, correct to the nearest cent.

Q11. (a) The path adds 1 m at each end: length = $10 + 1 + 1 = 12$ m, width = $6 + 1 + 1 = 8$ m. (b) Outer rectangle: $12 \times 8 = 96 \text{ m}^2$. Lawn: $10 \times 6 = 60 \text{ m}^2$. Path: $96 - 60 = 36 \text{ m}^2$. (c) $36 \times 8 = 288$ pavers.

Q12. (a) $C = 2\pi r = 2 \times \pi \times 2.4 = 15.07964\dots$, so 15.08 m to 2 decimal places. (b) The arc is a quarter of the circumference: $15.07964\dots \div 4 = 3.76991\dots$, so 3.77 m to 2 decimal places. (c) Perimeter = arc + two radii = $3.77 + 2.4 + 2.4 = 8.57$ m, to 2 decimal places.

Q13. (a) $A = 20 \times 20 = 400 \text{ cm}^2$. (b) The hole has radius $10 \div 2 = 5$ cm, so $A = \pi \times 5^2 = 78.53981\dots$, that is 78.54 cm^2 to 2 decimal places. (c) $400 - 78.53981\dots = 321.46018\dots$, so the metal left is 321.46 cm^2 to 2 decimal places.

Q14. (a) $A = 6 \times 2.8 = 16.8 \text{ m}^2$. (b) $A = \frac{1}{2}bh = \frac{1}{2} \times 6 \times 1.6 = 4.8 \text{ m}^2$. (c) Front face = $16.8 + 4.8 = 21.6 \text{ m}^2$. Two coats cover $21.6 \times 2 = 43.2 \text{ m}^2$ of paint. $43.2 \div 12 = 3.6$, and tins cannot be bought in parts, so 4 tins are needed.

Q15. (a) $A = 850 \times 600 = 510\,000 \text{ m}^2$. (b) $510\,000 \div 10\,000 = 51$ ha. (c) $51 \div 1.5 = 34$ cows.

Q16. (a) $25 \times 25 = 625 \text{ cm}^2$. (b) $625 \div 10\,000 = 0.0625 \text{ m}^2$. (c) Floor area = $5 \times 3 = 15 \text{ m}^2 = 150\,000 \text{ cm}^2$. Tiles: $150\,000 \div 625 = 240$ tiles. (Same answer in metres: $15 \div 0.0625 = 240$.)

Q17. (a) $A = 4.2 \times 4.2 = 17.64 \text{ m}^2$. (b) The circle has radius $5 \div 2 = 2.5$ m, so $A = \pi \times 2.5^2 = 19.63495\dots$, that is 19.63 m^2 to 2 decimal places. (c) $19.63495\dots - 17.64 = 1.99495\dots$, so the circular bed needs more mulch, by 1.99 m^2 to 2 decimal places.

Q18. (a) Perimeter of the plot = $2 \times (12 + 9) = 42$ m. The gate replaces 1 m of fence: $42 - 1 = 41$ m. Cost = $41 \times 17.20 = 705.20$, that is \$705.20, correct to the nearest cent. (b) Rectangular beds: $3 \times (3 \times 1.2) = 3 \times 3.6 = 10.8 \text{ m}^2$. Circular bed: radius $2.4 \div 2 = 1.2$ m, so $A = \pi \times 1.2^2 = 4.52389\dots = 4.52 \text{ m}^2$ to 2 decimal places. Total garden beds: $10.8 + 4.52 = 15.32 \text{ m}^2$. (c) Plot area = $12 \times 9 = 108 \text{ m}^2$. Gravel = $108 - 15.32 = 92.68 \text{ m}^2$ (using the answer to (b)). Cost = $92.68 \times 14.50 = 1343.86$, that is \$1 343.86, correct to the nearest cent.

Chapter 4 Quantity and measures — 4C Volume and capacity

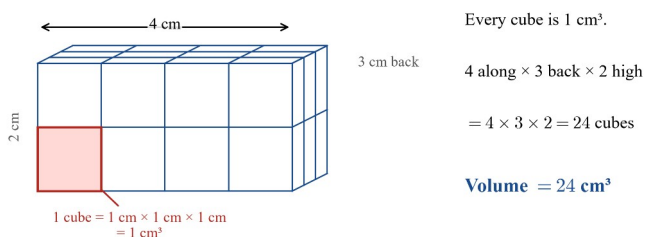
About the data in this section. Every name, price, size, time and other number in this section is **made up** (not real): pretend people, pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Theory

Volume is the amount of space a solid takes up. **Capacity** is the amount a container can hold — how much water, soil, grain or air fits inside it. Volume is measured in cubic units (cm^3 , m^3); capacity is measured in litres (mL, L, kL). This section connects the two, and gives you a formula for each common solid.

Volume starts with cubes.

A cube with sides of 1 cm has a volume of one **cubic centimetre**, written cm^3 . A cube with sides of 1 m has a volume of one **cubic metre**, written m^3 . For a box shape, the volume is just the number of unit cubes that fill it.



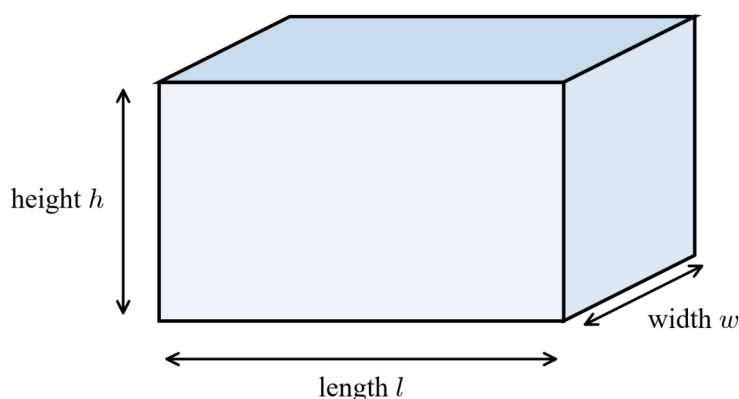
Worked model 1. Count the cubes in the box above by layers. Each layer is a 4 by 3 rectangle, so one layer holds $4 \times 3 = 12$ cubes. There are 2 layers, so the box holds $12 \times 2 = 24$ cubes: its volume is 24 cm^3 .

Because $1 \text{ m} = 100 \text{ cm}$, a 1 m cube holds $100 \times 100 \times 100 = 1\,000\,000 \text{ cm}^3$, that is $1\,000\,000 \text{ cm}^3$. Keep that number handy: it is the bridge between cm^3 and m^3 .

Cuboids (rectangular boxes).

A **cuboid** is a box shape where every corner is square. Counting cubes in layers gives the formula: length times width times height.

A cuboid (rectangular box): every corner is square.



$$V = l \times w \times h$$

Worked model 2. A storage box measures 30 cm by 20 cm by 15 cm. Its volume is $V = 30 \times 20 \times 15 = 9000 \text{ cm}^3$, that is 9000 cm^3 . Filled with water it would hold 9 L, because (as the bridge below shows) 1000 cm^3 is exactly 1 L.

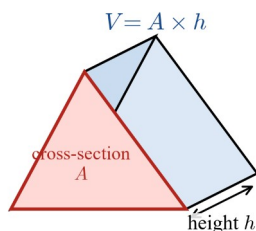
Prisms and cylinders: the same cross-section all the way through.

Slice a solid straight across and look at the cut face: that shape is its **cross-section**. A **prism** is a solid whose cross-section is the same shape and size all the way through. A **cylinder** is a prism whose cross-section is a circle. For any prism, stacking copies of the cross-section gives:

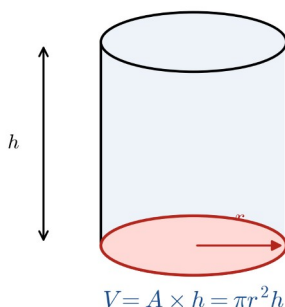
$$V = A \times h$$

where A is the area of the cross-section and h is the height (the length of the prism). For a cylinder the cross-section is a circle of area $A = \pi r^2$, so $V = \pi r^2 h$.

Triangular prism — the same cross-section all the way through



Cylinder — the cross-section is a circle

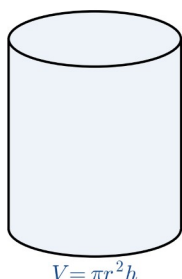


Worked model 3. A cylinder has radius 10 cm and height 25 cm. Its cross-section area is $A = \pi \times 10^2 = 314.1592\dots$, about 314.2 cm² to 1 decimal place (d.p.). Its volume is $V = \pi \times 100 \times 25 = 7853.9816\dots$, about 7854 cm³ to the nearest whole number. Carry the full calculator value into the next step; round only the answer you report.

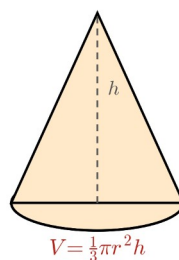
Cones and pyramids: one third of the matching prism.

Fill a cone with water and pour it into a cylinder with the same radius and the same height: it takes exactly three cone-fuls to fill the cylinder. So a cone is one third of its cylinder, and a pyramid is one third of a prism with the same base and height.

cylinder



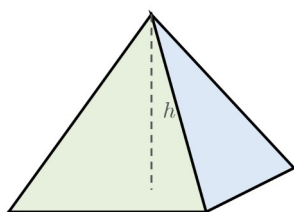
cone (same r and h)



$\times 3$
←
3 cones fill
1 cylinder

square-based pyramid

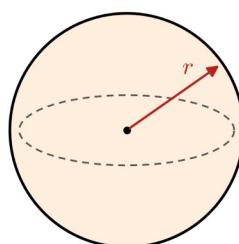
$$V = \frac{1}{3} \times A \times h$$



square base, area A

sphere

$$V = \frac{4}{3} \pi r^3$$



a hemisphere is half a sphere

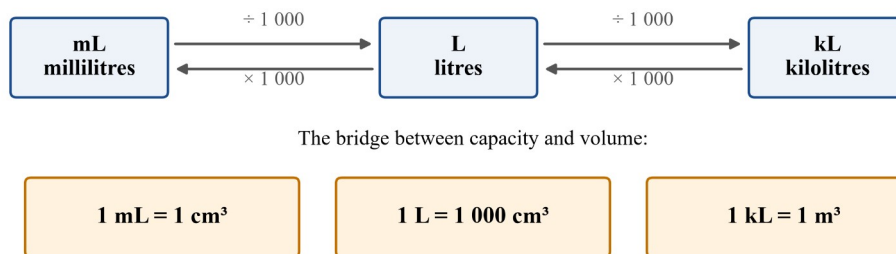
For a cone, $A = \pi r^2$, so $V = \frac{1}{3} \pi r^2 h$. For a pyramid with base area A , $V = \frac{1}{3} \times A \times h$. A **sphere** (a perfect ball) of radius r has $V = \frac{4}{3} \pi r^3$, and a **hemisphere** — half a sphere — is half of that.

Worked model 4. The cone with the same radius 10 cm and height 25 cm as the cylinder in model 3 holds $7853.9816... \div 3 = 2617.9938...$, about 2618 cm³ to the nearest whole number: exactly one third of 7854 cm³ before rounding.

Worked model 5. A sphere of radius 10 cm holds $\frac{4}{3} \times \pi \times 10^3 = 4188.7902...$, about 4189 cm³ to the nearest whole number.

Capacity: mL, L and kL, and the bridge to volume.

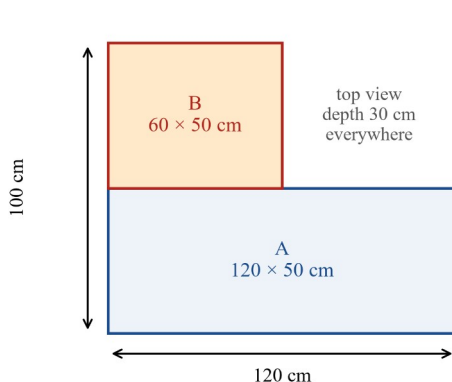
Capacity units sit on a ladder of thousands: 1000 mL make 1 L, and 1000 L make 1 kL (kilolitre). Move up the ladder by dividing by 1000; move down by multiplying by 1000. The bridge between capacity and volume is the cubic centimetre and the cubic metre:



Worked model 6. A farm dam holds 2.4 kL. Down the ladder: $2.4 \times 1000 = 2400$ L. A skip bin holds 3 m³ of rubble. Across the bridge: $3 \text{ m}^3 = 3000 \text{ L}$ (which is also 3 kL).

Compound shapes: split, work out, add.

A **compound** solid is one you can split into simpler solids. Find the volume of each piece, then add. If a solid has a hole or a gap inside (like the pipe in Q10), subtract the missing piece instead.



Split, work out, add:

$$A: 120 \times 50 \times 30 = 180\,000 \text{ cm}^3$$

$$B: 60 \times 50 \times 30 = 90\,000 \text{ cm}^3$$

$$\text{Total} = 270\,000 \text{ cm}^3$$

$$270\,000 \div 1000 = 270 \text{ L of soil}$$

The L-shaped garden bed above splits into two cuboids, both 30 cm deep: piece A is $120 \times 50 \times 30 = 180\,000 \text{ cm}^3$ and piece B is $60 \times 50 \times 30 = 90\,000 \text{ cm}^3$. The total is $180\,000 + 90\,000 = 270\,000 \text{ cm}^3$, that is 270 000 cm³, so the bed needs 270 L of soil.

Rounding in volume work.

The rule from section 2C carries straight over: carry full precision through the working, round only the answer you report, say what precision you rounded to, and remember that exactly halfway rounds up. Rounding too early can change a real answer. The cylinder in model 3 has volume 0.0078539... m³ — think instead of the hot water system in

Example 2: its volume is $0.235619... \text{ m}^3$. Rounded early to 0.24 m^3 it converts to 240 L; converted first and rounded once it is 236 L. Four litres of difference, from one early rounding.

The formula set.

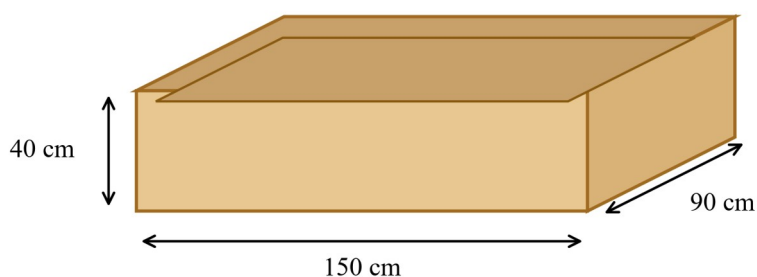
Solid	Volume formula
Cuboid	$V = l \times w \times h$
Prism	$V = A \times h$ (A = cross-section area)
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{1}{3} \pi r^2 h$
Pyramid	$V = \frac{1}{3} \times A \times h$ (A = base area)
Sphere	$V = \frac{4}{3} \pi r^3$

All of these are also on the formula sheet at the back of the book.

Worked examples

Example 1 — scaffolded

Raised garden bed — soil fills it to the top.



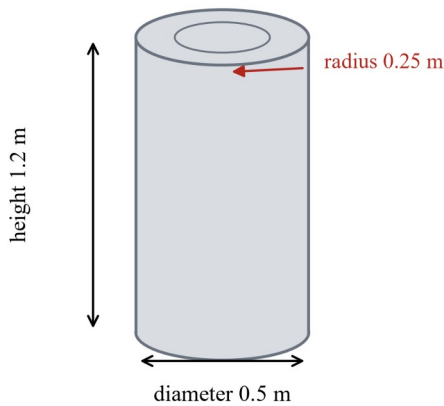
A raised garden bed is a cuboid 150 cm long, 90 cm wide and 40 cm high. Soil fills it to the top.

- Find the volume of the soil in cm^3 .
- Find the capacity of the bed in litres.
- Soil is sold in 30 L bags. How many bags are needed to fill the bed?

Working	Comment
(a) $V = 150 \times 90 \times 40 = 540000$	Length $\times$ width $\times$ height: the volume is $540\,000 \text{ cm}^3$.
(b) $540000 \div 1000 = 540$	$1000 \text{ cm}^3 = 1 \text{ L}$, so divide by 1000: the capacity is 540 L.
(c) $540 \div 30 = 18$	The division is exact: 18 bags, with no part bag left over.

Example 2 — scaffolded

Cylindrical hot water system



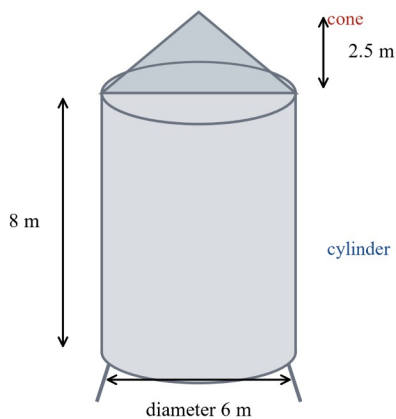
A cylindrical hot water system stands 1.2 m high, and its circular base has a diameter of 0.5 m.

- (a) Write down the radius of the base.
- (b) Find the volume of the cylinder in m^3 , to 2 decimal places.
- (c) Find its capacity in litres, to the nearest whole litre.

Working	Comment
(a) $r = 0.5 \div 2 = 0.25$	The radius is half the diameter: 0.25 m.
(b) $V = \pi \times 0.25^2 \times 1.2 = 0.235619... \approx 0.24$	Round to 2 d.p. here, but keep the full calculator value for part (c).
(c) $0.235619... \times 1000 = 235.6194... \approx 236$	$1 \text{ m}^3 = 1000 \text{ L}$. From the full value this is 236 L; starting from 0.24 m^3 would give 240 L, so round only once, at the end.

Example 3 — unscaffolded

Grain silo = cylinder + cone roof



A grain silo (a tall farm bin for grain) is a cylinder with a cone roof on top. The cylinder part is 8 m high, the cone roof is 2.5 m high, and both share a circular base of diameter 6 m.

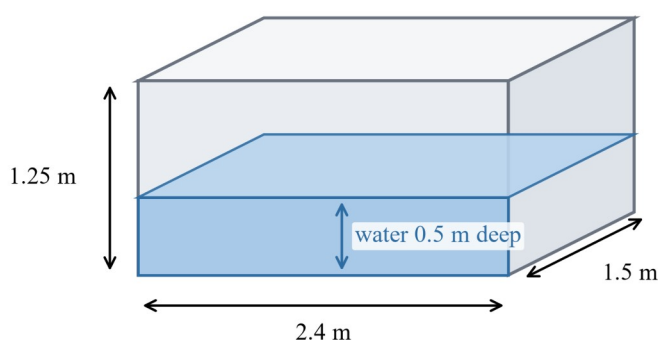
- (a) Show that the cylinder part has a volume of about 226.19 m^3 , to 2 decimal places.
- (b) Find the volume of the space inside the cone roof, to 2 decimal places.

- (c) Hence find the total volume of the silo, to the nearest cubic metre.
- (d) The shared radius is $6 \div 2 = 3$ m. The cylinder part is $V = \pi \times 3^2 \times 8 = 72\pi = 226.19467 \dots \text{ m}^3$. To 2 d.p. the deciding digit is the 4 (thousandths), which is less than 5, so round down: 226.19 m^3 , as required.
- (e) The cone roof space is $V = \frac{1}{3} \times \pi \times 3^2 \times 2.5 = 7.5\pi = 23.561944 \dots \text{ m}^3$. To 2 d.p. the deciding digit is the 1, so round down: 23.56 m^3 .
- (f) At full precision the total is $72\pi + 7.5\pi = 79.5\pi = 249.7566 \dots \text{ m}^3$. To the nearest whole number the deciding digit is the 7 (tenths), so round up: 250 m^3 . (Adding the rounded parts, $226.19 + 23.56 = 249.75$, also rounds to 250 m^3 .)

$\therefore$ the silo holds about 250 m^3 of grain: 226.19 m^3 in the cylinder part plus 23.56 m^3 under the cone roof.

Example 4 — unscaffolded

Rectangular farm tank — part full



A rectangular farm tank measures 2.4 m long, 1.5 m wide and 1.25 m high. The water in the tank is 0.5 m deep.

- Find the capacity of the tank when full, in litres.
- Find how many litres of water are in the tank now.
- Find how many more litres of water are needed to fill the tank.
- Full, the water is a cuboid 2.4 m by 1.5 m by 1.25 m: $V = 2.4 \times 1.5 \times 1.25 = 4.5 \text{ m}^3$. Across the bridge, $4.5 \times 1000 = 4500 \text{ L}$.
- Now, the water is a cuboid 2.4 m by 1.5 m by only 0.5 m deep: $V = 2.4 \times 1.5 \times 0.5 = 1.8 \text{ m}^3$, that is $1.8 \times 1000 = 1800 \text{ L}$.
- The difference is $4500 - 1800 = 2700 \text{ L}$. Check: the empty space above the water is $1.25 - 0.5 = 0.75 \text{ m}$ deep, and $2.4 \times 1.5 \times 0.75 = 2.7 \text{ m}^3 = 2700 \text{ L}$, which agrees.

$\therefore$ the tank holds 4500 L when full, it holds 1800 L now, and 2700 L more would fill it.

Practice questions

Scaffolded

Q1. Find the volume of: (a) a cuboid 8 cm long, 5 cm wide and 3 cm high (b) a cuboid 2.4 m long, 1.5 m wide and 0.6 m high (c) a cube with side length 7 cm

Q2. Convert: (a) 4.2 L to mL (b) 3550 mL to L (c) 6.5 m^3 to L

Q3. A cylindrical food can has a radius of 5 cm and a height of 12 cm. (a) Find the area of its circular base, to 1 decimal place. (b) Find its volume, to 1 decimal place. (c) Find its capacity in litres, to 2 decimal places.

Q4. A concrete ramp is a triangular prism. Its cross-section is a right-angled triangle with base 40 cm and height 15 cm, and the ramp is 90 cm long. (a) Find the area of the triangular cross-section. (b) Find the volume of the ramp in cm^3 . (c) Write this volume in m^3 .

Q5. A cone has radius 6 cm and height 10 cm. A cylinder has the same radius and the same height. (a) Find the volume of the cone, to 1 decimal place. (b) Find the volume of the cylinder, to 1 decimal place. (c) How many times larger is the volume of the cylinder than the volume of the cone?

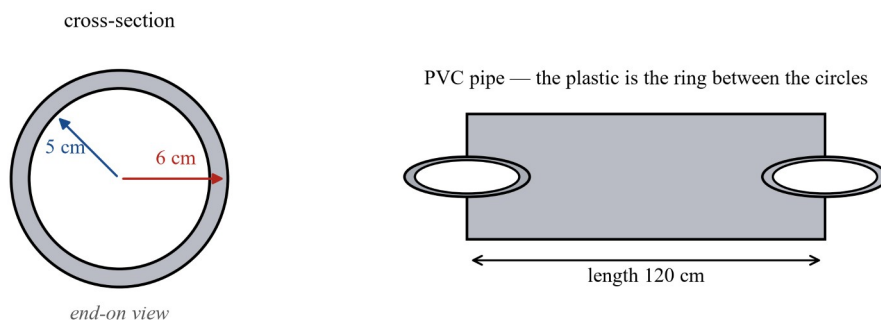
Q6. A basketball is a sphere of radius 12 cm. (a) Show that its volume is exactly $2304\pi \text{ cm}^3$. (b) Find the volume of the basketball, to the nearest cm^3 . (c) A display stand is a hemisphere (half a sphere) of radius 12 cm. Find its volume, to the nearest cm^3 .

Unscaffolded

Q7. An aquarium holds 96 L of water when it is full. Its base is 60 cm by 40 cm. (a) Write 96 L in cm^3 . (b) Find the area of the base in cm^2 . (c) Hence find the depth of the aquarium in cm.

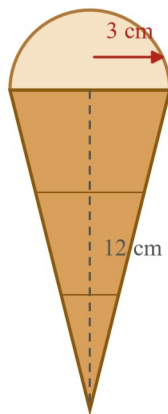
Q8. A rectangular water tank measures 3.2 m by 2.1 m by 1.4 m. (a) Find its volume in m^3 . (b) Find its capacity in litres. (c) Write this capacity in kL, to 1 decimal place.

Q9. A cylindrical stockpot has an inside diameter of 22 cm and an inside height of 15 cm. (a) Write down the inside radius. (b) Find the volume inside the pot, to the nearest cm^3 . (c) Find the capacity of the pot in litres, to 2 decimal places.



Q10. A length of PVC pipe is 120 cm long. Seen end-on, the plastic is a ring: the outer circle has radius 6 cm and the inner circle (the hole through the pipe) has radius 5 cm. (a) Find the volume of the outer cylinder (radius 6 cm, length 120 cm), to the nearest cm^3 . (b) Find the volume of the hole (radius 5 cm, length 120 cm), to the nearest cm^3 . (c) Hence find the volume of plastic in the pipe, to the nearest cm^3 .

cone + hemisphere scoop



scoop = half a sphere, same radius as the cone top

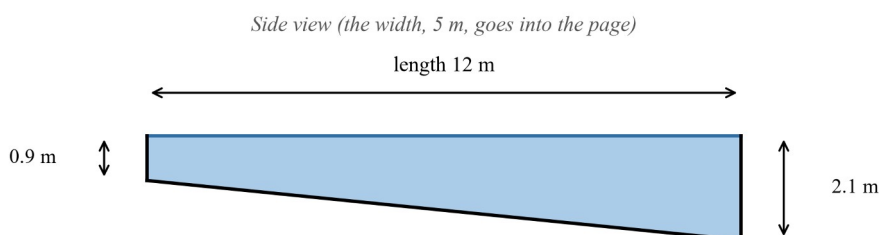
Q11. An ice cream is a cone of radius 3 cm and height 12 cm, topped by a hemisphere (half a sphere) of radius 3 cm. (a) Find the volume of the cone part, to 1 decimal place. (b) Find the volume of the hemisphere scoop, to 1 decimal place. (c) Find the total volume of the ice cream, to 1 decimal place.

Q12. A drink dispenser holds 15 L. The cups next to it hold 330 mL each. (a) Write 15 L in mL. (b) Calculate $15000 \div 330$, to 1 decimal place. (c) How many full cups can be poured from a full dispenser, and how many mL are left in the dispenser afterwards?

Q13. A rectangular container has a base 12 cm by 8 cm. Water is poured in to a depth of 6 cm. A mango is then lowered into the water until it is completely under, and the water level rises by 1.5 cm. (a) Find the volume of water in the container before the mango, in cm^3 . (b) The mango takes up space, so the water level rises by exactly the volume of the mango. Find the volume of the mango in cm^3 . (c) Find the total volume of water plus mango in mL, and write it in litres to 2 decimal places.

Q14. A concrete slab is 8 m long, 5 m wide and 12 cm thick. Concrete costs \$180 per m^3 , and a mixer truck carries 2.5 m^3 per load. (a) Find the volume of the slab in m^3 . (b) Find the cost of the concrete for the slab, to the nearest cent. (c) Find the number of truck loads needed.

Q15. A cylindrical can has radius 3.4 cm and height 12 cm. A carton is a cuboid 7 cm by 7 cm by 11.5 cm. (a) Find the volume of the can, to 1 decimal place. (b) Find the volume of the carton. (c) Which one holds more, and by how much (to 1 decimal place)?



Q16. A swimming pool is 12 m long and 5 m wide. Seen from the side, the water is 0.9 m deep at the shallow end and 2.1 m deep at the deep end, with the floor sloping in a straight line between them, so the side view is a trapezium (a four-sided shape with one pair of parallel sides — the two depths). (a) Find the area of the side-view cross-section in m^2 . (b) Find the volume of water in the pool in m^3 . (c) Write this capacity in kL and in litres.

Q17. The roof space of a house is a square-based pyramid: the base is 3 m by 3 m and the height is 1.5 m. (a) Find the area of the base. (b) Find the volume of the roof space in m^3 . (c) A cone with the same base area and the same height has the same volume as the pyramid. State that volume, and explain why no new calculation is needed.

Q18. A cylindrical rainwater tank has a diameter of 2.2 m and a height of 1.6 m. A pump fills the tank at 15 L per minute, and water costs \$2.25 per kL. (a) Find the volume of the tank in m^3 , to 2 decimal places, and in litres to the nearest whole litre. (b) Find how long the pump takes to fill the tank from empty. Give your answer in hours, to 1 decimal place. (c) Find the cost of that water, to the nearest cent.

Answers

Q1. (a) 120 cm^3 ; (b) 2.16 m^3 ; (c) 343 cm^3 . **Q2.** (a) 4200 mL ; (b) 3.55 L ; (c) 6500 L . **Q3.** (a) 78.5 cm^2 ; (b) 942.5 cm^3 ; (c) 0.94 L . **Q4.** (a) 300 cm^2 ; (b) $27\,000 \text{ cm}^3$; (c) 0.027 m^3 . **Q5.** (a) 377.0 cm^3 ; (b) 1131.0 cm^3 ; (c) 3 times. **Q6.** (a) $\frac{4}{3} \times 12^3 = \frac{4}{3} \times 1728 = 2304$, so $V = 2304\pi \text{ cm}^3$; (b) 7238 cm^3 ; (c) 3619 cm^3 . **Q7.** (a) $96\,000 \text{ cm}^3$; (b) 2400 cm^2 ; (c) 40 cm . **Q8.** (a) 9.408 m^3 ; (b) 9408 L ; (c) 9.4 kL . **Q9.** (a) 11 cm ; (b) 5702 cm^3 ; (c) 5.70 L . **Q10.** (a) $13\,572 \text{ cm}^3$; (b) 9425 cm^3 ; (c) 4147 cm^3 . **Q11.** (a) 113.1 cm^3 ; (b) 56.5 cm^3 ; (c) 169.6 cm^3 . **Q12.** (a) $15\,000 \text{ mL}$; (b) 45.5 ; (c) 45 full cups, with 150 mL left in the dispenser. **Q13.** (a) 576 cm^3 ; (b) 144 cm^3 ; (c) $720 \text{ mL} = 0.72 \text{ L}$. **Q14.** (a) 4.8 m^3 ; (b) $\$864.00$; (c) 2 loads. **Q15.** (a) 435.8 cm^3 ; (b) 563.5 cm^3 ; (c) the carton, by 127.7 cm^3 . **Q16.** (a) 18 m^2 ; (b) 90 m^3 ; (c) $90 \text{ kL} = 90\,000 \text{ L}$. **Q17.** (a) 9 m^2 ; (b) 4.5 m^3 ; (c) 4.5 m^3 — a cone and a pyramid are both $\frac{1}{3} \times A \times h$. **Q18.** (a) 6.08 m^3 , 6082 L ; (b) 6.8 h ; (c) $\$13.68$.

Fully-worked solutions

Q1. (a) $V = 8 \times 5 \times 3 = 120 \text{ cm}^3$. (b) $V = 2.4 \times 1.5 \times 0.6 = 2.16 \text{ m}^3$. (c) A cube is a cuboid with equal sides:
 $V = 7 \times 7 \times 7 = 7^3 = 343 \text{ cm}^3$.

Q2. (a) $1 \text{ L} = 1000 \text{ mL}$, so $4.2 \times 1000 = 4200 \text{ mL}$. (b) $3550 \div 1000 = 3.55 \text{ L}$. (c) $1 \text{ m}^3 = 1000 \text{ L}$, so $6.5 \times 1000 = 6500 \text{ L}$.

Q3. (a) $A = \pi r^2 = \pi \times 5^2 = 78.5398 \dots \approx 78.5 \text{ cm}^2$ to 1 d.p. (b) $V = A \times h = 78.5398 \dots \times 12 = 942.4777 \dots \approx 942.5 \text{ cm}^3$ to 1 d.p. (c) $942.4777 \dots \div 1000 = 0.9424 \dots \approx 0.94 \text{ L}$ to 2 d.p.

Q4. (a) $A = \frac{1}{2} \times 40 \times 15 = 300 \text{ cm}^2$. (b) $V = A \times h = 300 \times 90 = 27000 \text{ cm}^3$, that is $27\,000 \text{ cm}^3$. (c) $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$, so $27000 \div 1000000 = 0.027 \text{ m}^3$.

Q5. (a) $V = \frac{1}{3} \times \pi \times 6^2 \times 10 = 120\pi = 376.9911 \dots \approx 377.0 \text{ cm}^3$ to 1 d.p. (b)

$V = \pi \times 6^2 \times 10 = 360\pi = 1130.9733 \dots \approx 1131.0 \text{ cm}^3$ to 1 d.p. (c) $1130.9733 \dots \div 376.9911 \dots = 3$: the cylinder is 3 times the cone, exactly as the cone figure promises.

Q6. (a) $V = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times 12^3 = \frac{4}{3} \times 1728 \times \pi = 2304\pi \text{ cm}^3$ exactly. (b) $2304\pi = 7238.2294 \dots \approx 7238 \text{ cm}^3$ to the nearest cm^3 . (c) Half of the sphere: $7238.2294 \dots \div 2 = 3619.1147 \dots \approx 3619 \text{ cm}^3$ to the nearest cm^3 .

Q7. (a) $1 \text{ L} = 1000 \text{ cm}^3$, so $96 \text{ L} = 96\,000 \text{ cm}^3$. (b) $60 \times 40 = 2400 \text{ cm}^2$. (c) The water fills a cuboid whose volume is base area times depth, so the depth is $96000 \div 2400 = 40 \text{ cm}$.

Q8. (a) $V = 3.2 \times 2.1 \times 1.4 = 9.408 \text{ m}^3$. (b) $9.408 \times 1000 = 9408 \text{ L}$. (c) $9408 \div 1000 = 9.408 \approx 9.4 \text{ kL}$ to 1 d.p.

Q9. (a) $r = 22 \div 2 = 11 \text{ cm}$. (b) $V = \pi \times 11^2 \times 15 = 1815\pi = 5701.9906 \dots \approx 5702 \text{ cm}^3$ to the nearest cm^3 . (c) $5701.9906 \dots \div 1000 = 5.7019 \dots \approx 5.70 \text{ L}$ to 2 d.p. (keep the final zero to show 2 d.p.).

Q10. (a) $\pi \times 6^2 \times 120 = 4320\pi = 13571.6802 \dots \approx 13572 \text{ cm}^3$, that is $13\,572 \text{ cm}^3$ to the nearest cm^3 . (b) $\pi \times 5^2 \times 120 = 3000\pi = 9424.7779 \dots \approx 9425 \text{ cm}^3$ to the nearest cm^3 . (c) The plastic is the ring:
 $13571.6802 \dots - 9424.7779 \dots = 4146.9023 \dots \approx 4147 \text{ cm}^3$ to the nearest cm^3 . (The rounded parts agree: $13\,572 - 9425 = 4147$.)

Q11. (a) $V = \frac{1}{3} \times \pi \times 3^2 \times 12 = 36\pi = 113.0973 \dots \approx 113.1 \text{ cm}^3$ to 1 d.p. (b) A hemisphere is half a sphere:

$V = \frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3 = 18\pi = 56.5486 \dots \approx 56.5 \text{ cm}^3$ to 1 d.p. (c) $36\pi + 18\pi = 54\pi = 169.6460 \dots \approx 169.6 \text{ cm}^3$ to 1 d.p. (and $113.1 + 56.5 = 169.6$ agrees).

Q12. (a) $15 \times 1000 = 15000$ mL, that is 15 000 mL. (b) $15000 \div 330 = 45.4545 \dots \approx 45.5$ to 1 d.p. (c) Only whole cups count, so round **down**: 45 full cups. They use $45 \times 330 = 14850$ mL, leaving $15000 - 14850 = 150$ mL in the dispenser (not enough for a 46th cup, which would need 330 mL).

Q13. (a) $12 \times 8 \times 6 = 576$ cm³. (b) The rise is 1.5 cm across the same 12 cm by 8 cm base, so the mango's volume is $12 \times 8 \times 1.5 = 144$ cm³. (c) $576 + 144 = 720$ mL (since 1 cm³ = 1 mL); in litres, $720 \div 1000 = 0.72$ L to 2 d.p.

Q14. (a) The thickness is 12 cm = 0.12 m, so $V = 8 \times 5 \times 0.12 = 4.8$ m³. (b) $4.8 \times \$180 = \$864.00$ to the nearest cent (the product is exact, so no rounding is needed). (c) $4.8 \div 2.5 = 1.92$ loads; one load is not enough, so round **up** to 2 loads.

Q15. (a) $V = \pi \times 3.4^2 \times 12 = 435.8017 \dots \approx 435.8$ cm³ to 1 d.p. (b) $V = 7 \times 7 \times 11.5 = 563.5$ cm³. (c) The carton holds more: $563.5 - 435.8017 \dots = 127.6982 \dots \approx 127.7$ cm³ more, to 1 d.p.

Q16. (a) The parallel sides of the trapezium are the two depths, 0.9 m and 2.1 m, and the distance between them is the 12 m length (the trapezium area formula from section 4B): $A = \frac{1}{2} \times (0.9 + 2.1) \times 12 = 18$ m². (b) $V = A \times h = 18 \times 5 = 90$ m³. (c) 90 m³ = 90 kL, and $90 \times 1000 = 90000$ L, that is 90 000 L.

Q17. (a) $3 \times 3 = 9$ m². (b) $V = \frac{1}{3} \times A \times h = \frac{1}{3} \times 9 \times 1.5 = 4.5$ m³. (c) 4.5 m³. The cone formula $\frac{1}{3} \pi r^2 h$ is just $\frac{1}{3} \times A \times h$ with $A = \pi r^2$, so any cone and any pyramid with the same base area and the same height have the same volume — nothing new to calculate. (A cylinder with that base and height would hold $9 \times 1.5 = 13.5$ m³, three times as much.)

Q18. (a) $r = 2.2 \div 2 = 1.1$ m, so $V = \pi \times 1.1^2 \times 1.6 = 1.936 \pi = 6.0821 \dots \approx 6.08$ m³ to 2 d.p. In litres, $6.0821 \dots \times 1000 = 6082.1233 \dots \approx 6082$ L to the nearest whole litre. (b) Time in minutes is $6082.1233 \dots \div 15 = 405.4748 \dots$ minutes; in hours, $405.4748 \dots \div 60 = 6.7579 \dots \approx 6.8$ h to 1 d.p. (c) $6082.1233 \dots$ L is 6.0821... kL, so the cost is $6.0821 \dots \times \$2.25 = \$13.6847 \dots$, that is \$13.68 to the nearest cent.

Chapter 4 Quantity and measures — 4D Time: analogue and digital, 24-hour, time zones, durations

Theory

Units of time.

Time is measured in units that fit inside each other. These are the facts the whole section builds on:

Unit	Made of
1 minute	60 seconds
1 hour	60 minutes (that is, 3600 seconds)
1 day	24 hours
1 week	7 days
1 common year	365 days
1 leap year	366 days

Because an hour is 60 minutes and not 100, time is not a decimal system. Half an hour is 30 minutes, a quarter of an hour is 15 minutes, and three-quarters of an hour is 45 minutes. This is why adding and subtracting times takes care.

Reading an analogue clock.

An **analogue** clock shows the time with hands moving around a round face. (Analogue means “shown with moving parts”, like the hands of a clock or the needle of a scale.) The face has two main hands:

- the **hour hand** — short and thick. It points near a number 1 to 12, and moves slowly from one number to the next across a whole hour;
- the **minute hand** — long. In one hour it travels right around the face.

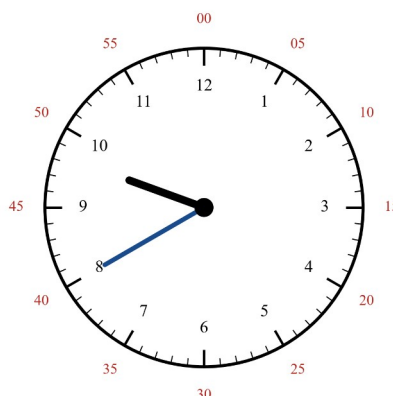
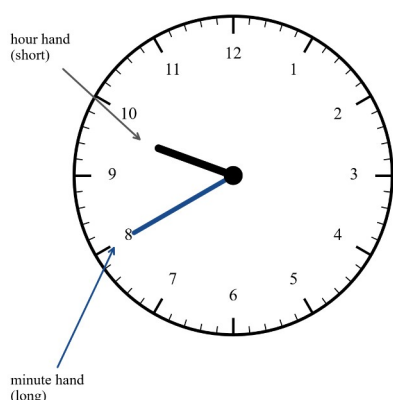
Some clocks also have a thin **second hand** that ticks around once a minute.

The minute hand reads in fives. When it points at the 1 it means 5 minutes, at the 2 it means 10 minutes, and so on — each number stands for that number times 5 minutes. So a minute hand on the 8 means $8 \times 5 = 40$ minutes.

The hour hand tells the hour, but between o'clock times it sits *between* two numbers. At 9:40 the hour hand is nearly at the 10, because most of the hour from 9 to 10 has gone by. Read the number it has *passed*, not the one it is nearing: 9:40, not 10:40.

The clock shows 9:40

Minute hand on the 8: count 5s — $8 \times 5 = 40$ minutes



An analogue clock cannot show whether a time is morning or evening — the face looks the same at 9:40 in the morning and at 9:40 at night. The situation tells you which one is meant.

Digital time and the 12-hour clock.

A **digital** clock shows the time as numbers, such as 9:40. Most digital clocks use the **12-hour clock**: the day is split into two halves of 12 hours each, tagged **am** and **pm**.

- **am** runs from midnight to just before midday (morning times).
- **pm** runs from midday to just before midnight (afternoon, evening and night times).

Midday itself is written 12:00 pm (also called **noon**), and midnight itself is written 12:00 am. Those two are the easy ones to get wrong: midday is *pm*, midnight is *am*.

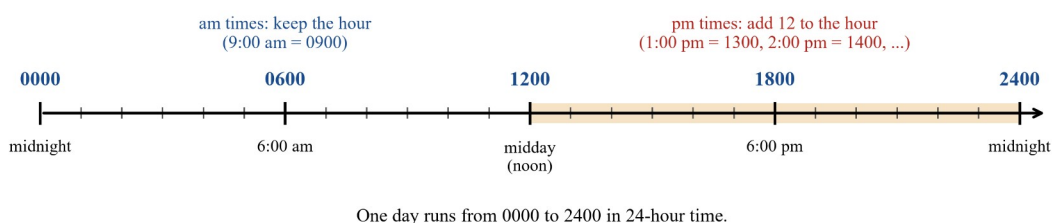
24-hour time.

Timetables, hospitals, the defence forces and most workplaces use **24-hour time**, which needs no am or pm. The day is numbered from midnight as 0000 through to 2359, written as four digits with no colon. Midnight is 0000 (or 2400 at the very end of a day), and midday is 1200.

To change a 12-hour time to 24-hour time:

- an **am** time keeps its hour, written with a leading zero when the hour is less than 10: 7:20 am becomes 0720. Midnight times change 12 to 00: 12:15 am becomes 0015;
- a **pm** time adds 12 to the hour: 3:35 pm becomes 1535, because $3 + 12 = 15$. Midday times keep the 12: 12:15 pm is 1215.

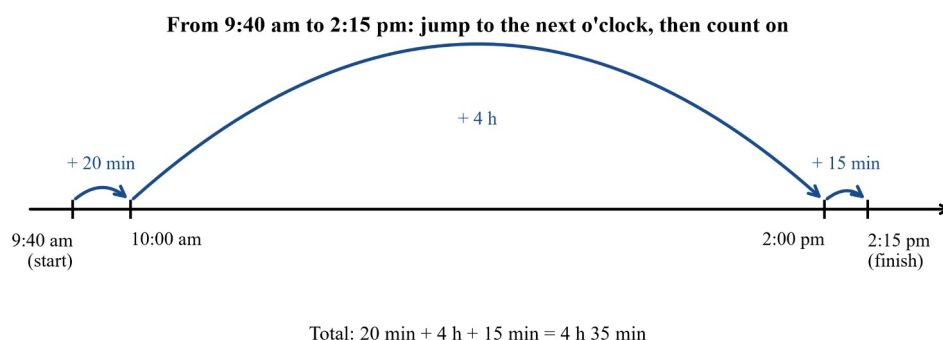
To change back, reverse the rules: 1535 has an hour over 12, so take 12 away ($15 - 12 = 3$) and tag it pm — 3:35 pm. An hour under 12 is an am time: 0720 is 7:20 am.



Durations.

A **duration** (also called **elapsed time**) is how much time passes between a start and a finish. A shift, a trip, a film and a phone call all have durations.

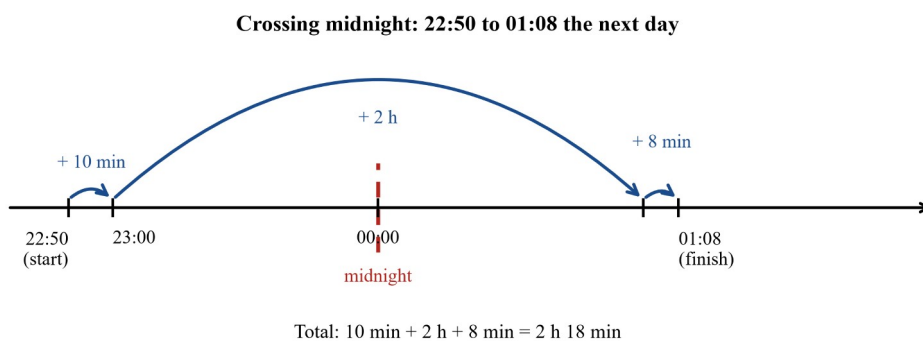
The safest way to find a duration is to **jump** along a number line of the day: jump from the start to the next o'clock, then jump whole hours, then jump the leftover minutes, and add the jumps up.



From 9:40 am to 2:15 pm the jumps are 20 minutes, then 4 hours, then 15 minutes, giving a duration of 4 h 35 min.

Subtracting times directly also works, but remember an hour holds 60 minutes, not 100. To take 9:40 from 14:15, borrow one hour as 60 minutes: 14:15 is the same as 13:75, and $13:75 - 9:40 = 4:35$.

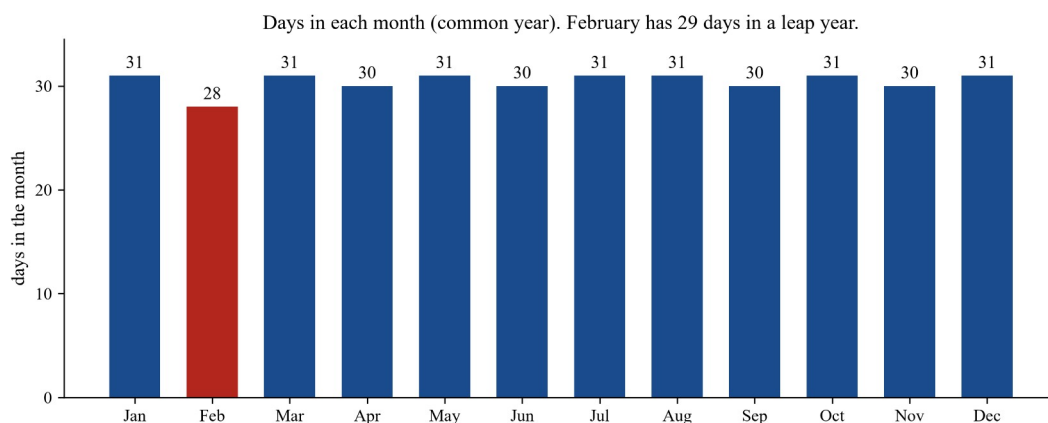
When a duration **crosses midnight**, split it at midnight. From 22:50 to 01:08 the next day: 22:50 to midnight is 1 h 10 min, and midnight to 01:08 is 1 h 8 min, giving 1 h 10 min + 1 h 8 min = 2 h 18 min. Jumping to the next hour first works just as well, as below.



Times are also written as **decimal hours** in rosters and pay records: 8.75 hours means 8 hours plus 0.75 of an hour, and $0.75 \times 60 = 45$ minutes, so 8.75 h is 8 h 45 min. The other way, 90 minutes is $90 \div 60 = 1.5$ h.

Calendar time.

The months are not all the same length. The chart gives the days in each month of a common year.



A **leap year** comes every 4 years (2024, 2028, 2032, ...) and gives February a 29th day, making a 366-day year.

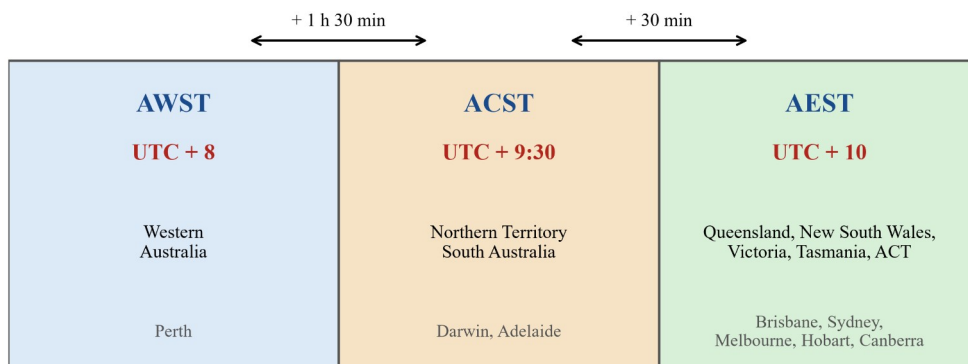
To count the days between two dates in different months, count in three parts: to the end of the first month, through the whole months in between, then into the last month. From 12 March to 5 April: 12 to 31 March is 19 days, then 5 more days into April, giving $19 + 5 = 24$ days.

Day-of-week questions use the 7-day week. Because $90 = 12 \times 7 + 6$, a 90-day job that starts on a Monday finishes 6 week-days past Monday — a Sunday.

Time zones.

The sun rises earlier in the east of a country than in the west, so the whole world cannot share one clock time. The world is split into **time zones**, vertical strips of the Earth that keep the same clock time. Every zone is described by how far its clocks sit ahead of **UTC** (Coordinated Universal Time), the base time the world counts from. A zone written UTC + 10 runs 10 hours ahead of UTC.

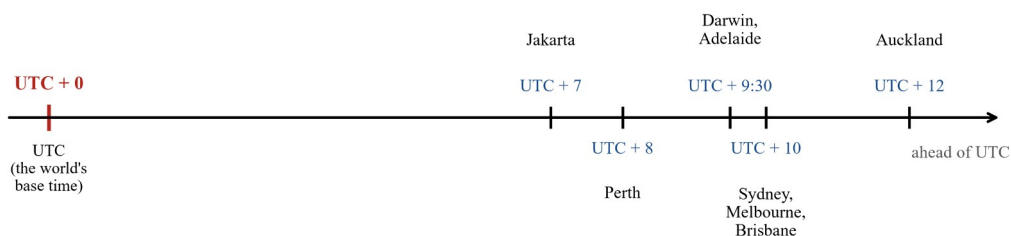
Australia keeps three standard time zones:



Australia's three standard time zones (schematic — not a map to scale). East is ahead of west.

Going **east** across zones, clocks go **forward**; going **west**, they go **back**. Perth (UTC + 8) is 2 hours behind Sydney (UTC + 10), so when Sydney reads 18:30, Perth reads 16:30. Adelaide (UTC + 9:30) sits only 30 minutes behind Sydney — zones can differ by half an hour.

Local time = UTC + the offset. London in winter (GMT) sits on UTC + 0.



If adding or subtracting hours carries a time past midnight, the **date changes** as well: 23:20 UTC plus 10 hours is 09:20 the *next* day in Sydney.

Daylight saving. In summer, some places (New South Wales, Victoria, South Australia, Tasmania, the ACT and New Zealand) put their clocks forward one hour; Queensland, Western Australia and the Northern Territory do not. Every calculation in this section uses **standard time** (the offsets above), so no daylight-saving step is ever needed here.

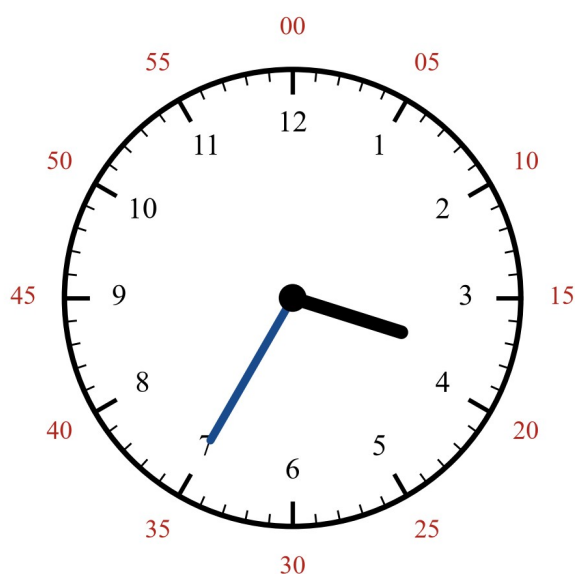
About the data in this section. The timetables, rosters, shifts and flight times here are **made up** (not real): pretend names and pretend numbers, built for this book so you can practise the maths. What is real is the clock maths itself — 60 minutes in an hour, the days in each month, and the standard time-zone offsets of the cities named. Nothing else here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

The clock on the staff-room wall shows the start of the afternoon shift.

Example 1: the clock shows 3:35



(a) Read the clock and write the time in 12-hour digital form.

(b) Write the same time in 24-hour time.

Working	Comment
The minute hand points at the 7. Count in fives: $7 \times 5 = 35$ minutes.	Each face number is 5 minutes for the minute hand.
The hour hand has passed the 3 but not reached the 4, so the hour is 3. The clock reads 3:35.	Read the number the hour hand has passed, not the one it is nearing.
The shift is an afternoon one, so the time is pm: 3:35 pm.	An analogue face cannot show am or pm — the situation tells you.
24-hour time: a pm hour adds 12, so $3 + 12 = 15$. The time is 1535.	Minutes never change in the conversion.

Example 2 — scaffolded

Maree's Saturday shift runs from 8:45 am to 3:30 pm, with a 45-minute unpaid meal break. (a) How long is she at work? (b) How long is she paid for?

Working	Comment
Change the finish to 24-hour time: 3:30 pm is 15:30.	pm: add 12 to the hour.
Jump 8:45 $\rightarrow$ 9:00 (+15 min), 9:00 $\rightarrow$ 15:00 (+6 h), 15:00 $\rightarrow$ 15:30 (+30 min).	Jump to the next o'clock, then whole hours, then the leftover minutes.
Add the jumps: 15 min + 6 h + 30 min = 6 h 45 min at work.	The jumps add to the duration.
Take off the unpaid break: 6 h 45 min - 45 min = 6 h paid.	Only paid time counts for part (b).
Check: 8:45 + 6 h = 14:45, and 14:45 + 45 min = 15:30 — the stated finish.	A backwards check lands exactly on 3:30 pm.

Example 3 — unscaffolded

A community garden project starts on 22 September and runs for 35 days. On what date does it finish?

September has 30 days. From 22 September to the end of September is $30 - 22 = 8$ days. That leaves $35 - 8 = 27$ days, which fall in October, so the finish date is 27 October.

Check by counting: 8 days to 30 September, plus 27 days of October, is $8 + 27 = 35$ days in all.

∴ the project finishes on 27 October.

Example 4 — unscaffolded

(a) A team in Sydney (AEST, UTC + 10) starts a video call at 18:30. What time is it in Perth (AWST, UTC + 8)?

(b) A weather station at sea sends a report at 23:20 UTC on 3 June. Give the date and time in Sydney (AEST).

(c) Perth is $10 - 8 = 2$ hours behind Sydney. Going west, take 2 hours off: $18:30 - 2 \text{ h} = 16:30$.

(d) Sydney is 10 hours ahead of UTC: $23:20 + 10 \text{ h} = 33:20$. That is past midnight, so take away 24 hours and move on one day: $33:20 - 24 \text{ h} = 09:20$ on 4 June.

∴ Perth joins the call at 16:30, and the Sydney team reads the report at 09:20 on 4 June.

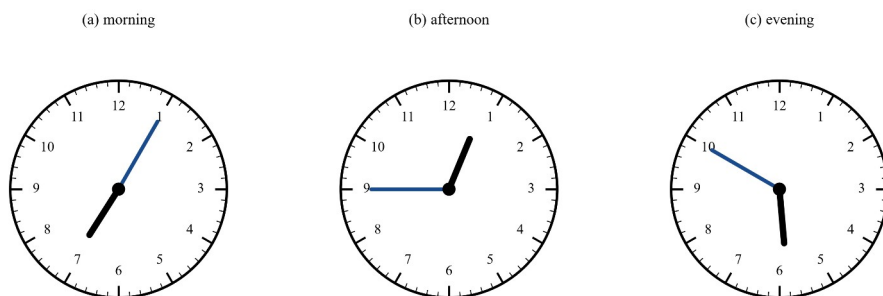
Practice questions

Scaffolded

Q1. Write each time in the units asked. (a) 3 hours, in minutes (b) 4 minutes 15 seconds, in seconds (c) 2.5 hours, in hours and minutes

Q2. Change between 12-hour and 24-hour time. (a) 0815, in 12-hour time (b) 1730, in 12-hour time (c) 12:40 pm, in 24-hour time

Q3. Each clock below shows a time. Clock (a) is a morning time, clock (b) an afternoon time and clock (c) an evening time.



(a) Write clock (a) in 12-hour digital time, then in 24-hour time.

(b) Write clock (b) in 12-hour digital time, then in 24-hour time.

(c) Write clock (c) in 12-hour digital time, then in 24-hour time.

Q4. Find the duration between each pair of times, all on the same day. (a) 9:10 am to 11:35 am (b) 7:45 pm to 9:05 pm (c) 1420 to 1705

Q5. Use the time-zone diagram in the Theory to help. (a) How many hours ahead of Perth (AWST) is Sydney (AEST)? How many minutes ahead of Adelaide (ACST) is Sydney? (b) It is 10:00 am in Sydney (AEST). What time is it in Adelaide (ACST)? (c) It is 06:00 UTC. What time is it in Brisbane (AEST)?

Q6. Calendar facts. (a) How many days are in August? How many in November? (b) How many days are there from 12 April to 26 April? (c) 2028 is a leap year. How many days does February 2028 have?

Un scaffolded

Q7. Write each time in the form asked. (a) 0047, in 12-hour time (b) 11:58 pm, in 24-hour time (c) midday, in 24-hour time (d) 12:05 am, in 24-hour time

Q8. A clock in a hospital ward has its hour hand between the 10 and the 11, and its minute hand on the 8. It is the evening visiting time. Write the time (a) in 12-hour digital form and (b) in 24-hour time.

Q9. (a) A late film session starts at 2250 and runs for 2 hours 18 minutes. What time does it finish? (b) A baker starts work at 22:35 and finishes at 06:10 the next morning. How long is the shift?

Q10. A nurse's roster shows a shift from 1430 to 2215, with a 30-minute unpaid meal break and a 15-minute unpaid tea break. (a) How long is the shift from start to finish? (b) How many hours and minutes is the nurse paid for? (c) The drive home takes 40 minutes. What time does the nurse get home?

Q11. (a) Write 8.75 hours in hours and minutes. (b) Write 345 minutes in hours and minutes. (c) Write 500 minutes in hours, correct to 2 decimal places.

Q12. (a) How many days are there from 27 January to 9 February? (b) How many days are there from 18 June to 5 August? (c) A seed catalogue says one pumpkin type takes 280 days from planting to picking. Seeds go in on 4 March. On what date are the pumpkins picked?

Q13. A road-repair job starts on Monday 3 August and takes 90 days. (a) On what date does it finish? (b) What day of the week is that? (c) Write 90 days in weeks and days.

Q14. The timetable below shows the weekday trains running in both directions between Wallaby Creek and Fern Hill. Asha lives in Wallaby Creek.

Wallaby Creek to Fern Hill line — weekday timetable (both directions)

Train	Runs	Departs	Arrives
A	Wallaby Creek to Fern Hill	06:12	08:37
B	Fern Hill to Wallaby Creek	09:48	11:41
C	Wallaby Creek to Fern Hill	13:05	15:52
D	Fern Hill to Wallaby Creek	17:30	19:44

Made-up data: the train names and every time in this timetable are made up for this section.

- (a) How long does Train B take?
- (b) Which train is the quickest, and how long does it take?
- (c) Asha leaves home on Train A and comes back home on Train D, which arrives in Wallaby Creek at 19:44. How long is she away from home?

Q15. (a) It is 4:30 pm in Brisbane (AEST). What time is it in Perth (AWST)? (b) A match in Adelaide (ACST) starts at 19:40. What time is that in Sydney (AEST)? (c) A satellite picture is marked 22:30 UTC on 12 March. Give the date and time in Melbourne (AEST).

Q16. A plane leaves Darwin (ACST) at 23:50 on a Friday and flies for 6 hours 40 minutes to Jakarta (UTC + 7). On what day, and at what Jakarta local time, does it land?

Q17. Priya, in Perth (AWST), starts a video call at 16:15 on a Thursday. (a) What time is it in Auckland (NZST, UTC + 12) when the call starts? (b) The call runs for 1 hour 25 minutes. What time does it end in Perth, and in Auckland? (c) Priya's cousin in London (UTC + 0 in winter) joins at the start of the call. What time is it in London then?

Q18. Jayden works a fly-in fly-out roster: 2 weeks on site, then 1 week at home, over and over. His first on-site block starts on Monday 6 April. (a) On what date does his first week at home start? (b) After three full repeats of the 3-week pattern, on what date — and what day of the week — does his next on-site block start? (c) Each day on site he works an 11-hour shift starting at 0600. What time does the shift finish? (d) How many hours does he work in one 14-day on-site block?

Answers

Q1. (a) 180 minutes. (b) 255 seconds. (c) 2 h 30 min. **Q2.** (a) 8:15 am. (b) 5:30 pm. (c) 1240. **Q3.** (a) 7:05 am, 0705. (b) 12:45 pm, 1245. (c) 5:50 pm, 1750. **Q4.** (a) 2 h 25 min. (b) 1 h 20 min. (c) 2 h 45 min. **Q5.** (a) 2 hours ahead of Perth; 30 minutes ahead of Adelaide. (b) 9:30 am. (c) 16:00. **Q6.** (a) 31 days; 30 days. (b) 14 days. (c) 29 days. **Q7.** (a) 12:47 am. (b) 2358. (c) 1200. (d) 0005. **Q8.** (a) 10:40 pm. (b) 2240. **Q9.** (a) 01:08 the next day. (b) 7 h 35 min. **Q10.** (a) 7 h 45 min. (b) 7 h. (c) 2255. **Q11.** (a) 8 h 45 min. (b) 5 h 45 min. (c) 8.33 h, correct to 2 decimal places. **Q12.** (a) 13 days. (b) 48 days. (c) 9 December. **Q13.** (a) 1 November. (b) Sunday. (c) 12 weeks and 6 days. **Q14.** (a) 1 h 53 min. (b) Train B, 1 h 53 min. (c) 13 h 32 min. **Q15.** (a) 2:30 pm (14:30). (b) 20:10. (c) 08:30 on 13 March. **Q16.** 04:00 on the Saturday (Jakarta local time). **Q17.** (a) 20:15 on the Thursday. (b) Perth 17:40; Auckland 21:40. (c) 08:15. **Q18.** (a) Monday 20 April. (b) Monday 8 June. (c) 1700. (d) 154 hours.

Fully-worked solutions

Q1. (a) $3 \times 60 = 180$ minutes. (b) $4 \times 60 = 240$ seconds, plus 15 gives 255 seconds. (c) The 2 is 2 hours; the 0.5 is half an hour, $0.5 \times 60 = 30$ minutes. So 2 h 30 min.

Q2. (a) The hour 08 is under 12, so it is am: 8:15 am. (b) The hour 17 is over 12, so take 12 away: $17 - 12 = 5$, pm: 5:30 pm. (c) A midday time keeps its hour: 12:40 pm is 1240.

Q3. (a) The minute hand is on the 1, so 5 minutes; the hour hand has just passed the 7. Morning, so 7:05 am, which is 0705. (b) The minute hand is on the 9, so $9 \times 5 = 45$ minutes; the hour hand is near the 1 but has passed the 12. Afternoon, so 12:45 pm, which is 1245. (c) The minute hand is on the 10, so 50 minutes; the hour hand is almost at the 6 but has passed the 5. Evening, so 5:50 pm; $5 + 12 = 17$, which is 1750.

Q4. (a) 9:10 to 11:10 is 2 hours, then 11:10 to 11:35 is 25 minutes: 2 h 25 min. (b) 7:45 pm to 8:45 pm is 1 hour, then to 9:05 pm is 20 minutes: 1 h 20 min. (c) 14:20 to 17:20 would be 3 hours, which is 15 minutes too many, so 2 h 45 min. (Or borrow: $16:65 - 14:20 = 2:45$.)

Q5. (a) Sydney is UTC + 10 and Perth is UTC + 8: $10 - 8 = 2$ hours ahead. Adelaide is UTC + 9:30, so Sydney is 30 minutes ahead of Adelaide. (b) Adelaide is 30 minutes behind Sydney: $10:00 \text{ am} - 30 \text{ min} = 9:30 \text{ am}$. (c) Brisbane is 10 hours ahead of UTC: $06:00 + 10 \text{ h} = 16:00$.

Q6. (a) From the chart: August has 31 days and November has 30 days. (b) Same month, so subtract the dates: $26 - 12 = 14$ days. (c) A leap year gives February a 29th day: 29 days.

Q7. (a) The hour 00 is a midnight hour: change 00 to 12 and tag am: 12:47 am. (b) $11 + 12 = 23$: 2358. (c) Midday keeps its hour: 1200. (d) A midnight hour changes 12 to 00: 0005.

Q8. (a) A minute hand on the 8 means $8 \times 5 = 40$ minutes. The hour hand has passed the 10, so the clock reads 10:40; the evening makes it 10:40 pm. (b) $10 + 12 = 22$: 2240.

Q9. (a) Jump $22:50 \rightarrow 23:00$ (+10 min), $23:00 \rightarrow 01:00$ (+2 h), $01:00 \rightarrow 01:08$ (+8 min): $10 + 120 + 8 = 138$ minutes = 2 h 18 min, and the clock reads 01:08 on the next day. (b) Split at midnight: 22:35 to midnight is 1 h 25 min, midnight to 06:10 is 6 h 10 min. Total 1 h 25 min + 6 h 10 min = 7 h 35 min.

Q10. (a) 14:30 to 22:30 would be 8 hours, which is 15 minutes too many, so 7 h 45 min. (Or borrow: $21:75 - 14:30 = 7:45$.) (b) Breaks total $30 + 15 = 45$ minutes. $7 \text{ h } 45 \text{ min} - 45 \text{ min} = 7 \text{ h paid}$. (c) $22:15 + 40 \text{ min}$: $22:15 + 45 \text{ min}$ would be 23:00, so 40 min lands at 22:55.

Q11. (a) $0.75 \times 60 = 45$ minutes, so 8 h 45 min. (b) $345 \div 60 = 5$ remainder 45, so 5 h 45 min. (c) $500 \div 60 = 8.3333\dots$, which is 8.33 h correct to 2 decimal places.

Q12. (a) 27 to 31 January is 4 days, plus 9 days of February: $4 + 9 = 13$ days. (b) 18 to 30 June is 12 days, July is 31 days, then 5 days of August: $12 + 31 + 5 = 48$ days. (c) March: 4 to 31 March is 27 days (running total 27). April 30 (57), May 31 (88), June 30 (118), July 31 (149), August 31 (180), September 30 (210), October 31 (241), November 30 (271). That leaves $280 - 271 = 9$ days of December: the pumpkins are picked on 9 December.

Q13. (a) August: 3 to 31 August is 28 days (running total 28). September 30 days (58), October 31 days (89). That leaves $90 - 89 = 1$ day of November: the job finishes on 1 November. (b) $90 = 12 \times 7 + 6$, so the finish is 6 week-days past Monday: Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday — a Sunday. (c) 12 weeks and 6 days.

Q14. (a) 09:48 to 11:48 would be 2 hours, which is 7 minutes too many, so 1 h 53 min. (Or borrow: $10:101 - 09:48 = 1:53$.) (b) Train A: 06:12 $\rightarrow$ 08:37 is 2 h 25 min. Train B is 1 h 53 min. Train C: 13:05 $\rightarrow$ 15:52 is 2 h 47 min. Train D: 17:30 $\rightarrow$ 19:44 is 2 h 14 min. The quickest is Train B at 1 h 53 min. (c) Train A leaves Wallaby Creek at 06:12, and Train D brings her home at 19:44. Away from 06:12 to 19:44: 06:12 to 19:12 is 13 hours, then to 19:44 is 32 minutes: 13 h 32 min.

Q15. (a) Perth is 2 hours behind Brisbane: $4:30 \text{ pm} - 2 \text{ h} = 2:30 \text{ pm}$ (14:30). (b) Sydney is 30 minutes ahead of Adelaide: $19:40 + 30 \text{ min} = 20:10$. (c) Melbourne is 10 hours ahead of UTC: $22:30 + 10 \text{ h} = 32:30$; take away 24 hours and move on one day: 08:30 on 13 March.

Q16. Darwin is UTC + 9:30, so the take-off in UTC is $23:50 - 9 \text{ h } 30 \text{ min} = 14:20$ UTC on the Friday. Add the flight: $14:20 + 6 \text{ h } 40 \text{ min} = 21:00$ UTC on the Friday. Jakarta is UTC + 7: $21:00 + 7 \text{ h} = 28:00$; take away 24 hours and move on one day: the plane lands at 04:00 Jakarta time on the Saturday.

Q17. Perth is UTC + 8, so the call starts at $16:15 - 8 \text{ h} = 08:15$ UTC on the Thursday. (a) Auckland is UTC + 12: $08:15 + 12 \text{ h} = 20:15$ on the Thursday. (b) Perth: $16:15 + 1 \text{ h } 25 \text{ min} = 17:40$. Auckland: $20:15 + 1 \text{ h } 25 \text{ min} = 21:40$. (c) London sits on UTC + 0 in winter, so it is 08:15 there.

Q18. (a) Two weeks on site is 14 days: 6 April + 14 days = 20 April, and 14 days is exactly 2 weeks, so it is again a Monday. (b) One full repeat of the pattern is $14 + 7 = 21$ days; three repeats is $3 \times 21 = 63$ days. 6 to 30 April is 24 days, May is 31 days (55), leaving $63 - 55 = 8$ days of June: 8 June. 63 days is exactly 9 weeks, so it is a Monday. (c) $0600 + 11 \text{ h} = 1700$. (d) $14 \times 11 = 154$ hours.

Chapter 4 Quantity and measures — 4E Temperature

Theory

Temperature tells us how hot or how cold something is. We read it off thermometers, dials and digital displays. We watch it change on graphs and in logs. And we calculate with it: how much warmer, how much colder, how fast it climbs, what the average was. This section builds those skills with the units we meet every day and at work.

Three units for temperature.

The **degree Celsius** ($^{\circ}\text{C}$) is the unit Australia uses. On this scale water freezes at 0°C and boils at 100°C . The **degree Fahrenheit** ($^{\circ}\text{F}$) is used in a few countries, including the United States, and on older cooking charts: water freezes at 32°F and boils at 212°F . Scientists use the **kelvin** (K), which carries no degree sign: water freezes at about 273 K and boils at about 373 K. Converting between Celsius and Fahrenheit is a calculation of its own — Section 4A teaches it. Here we only read conversions off a chart (Q14).

Unit	Water freezes	Water boils	Where you meet it
Celsius ($^{\circ}\text{C}$)	0°C	100°C	weather, fridges, ovens, health
Fahrenheit ($^{\circ}\text{F}$)	32°F	212°F	some countries, older cooking charts
kelvin (K)	about 273 K	about 373 K	science and food-safety standards

Three units you will meet for temperature

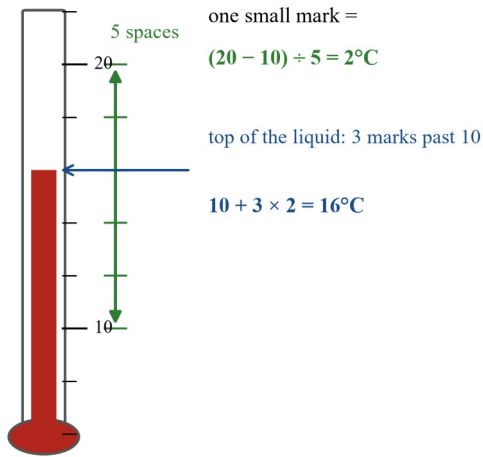
Celsius ($^{\circ}\text{C}$)	Fahrenheit ($^{\circ}\text{F}$)	kelvin (K)
the everyday unit in Australia	still used in the USA and a	used mostly in science
weather, cooking, body,	few other countries	no degree symbol
work and fridge temperatures	converting between $^{\circ}\text{C}$ and $^{\circ}\text{F}$	0 K is absolute zero —
water freezes at 0°C	is done in section 4A	as cold as anything can get
water boils at 100°C	water freezes at 32°F	water freezes at about 273 K

Reading a scale: find the step first.

An analogue thermometer (one with a liquid column and a printed scale) does not label every mark. To read it, first find the **step**, the size of one space between marks: pick two labelled marks, subtract the smaller from the bigger, and divide by the number of spaces between them. Then count marks from a label to the top of the liquid.

In the figure the labels 10 and 20 sit five spaces apart, so the step is $(20 - 10) \div 5 = 2$ degrees per space. The liquid tops out 3 marks above the 10 label: $10 + 3 \times 2 = 16$, so the reading is 16°C . Below zero, count from a label the same way: 3 marks below -10 with a step of 2 is $-10 - 3 \times 2 = -16^{\circ}\text{C}$.

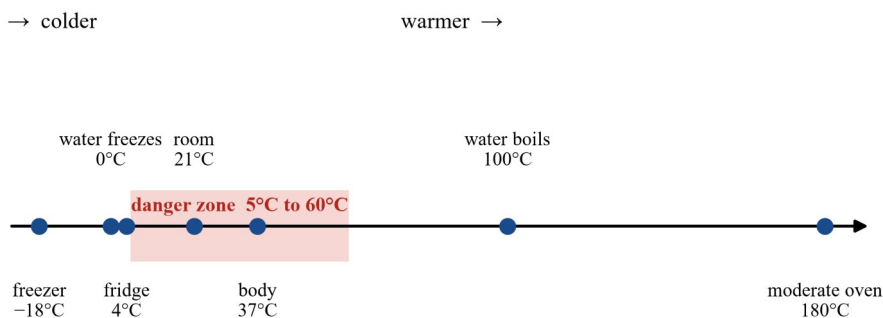
labels every 10°



Digital thermometers skip all of this: the number appears on a screen, often to one decimal place.

A ladder of landmark temperatures.

Some temperatures are worth knowing by heart, because everything else is judged against them. The ladder below runs from a hard-freeze freezer at -18°C to a hot oven at 180°C . The shaded band from 5°C to 60°C is the **danger zone** for food: the band where bacteria grow fastest. Cold food must stay below 5°C and hot food must stay at 60°C or above, so the danger zone is where food should spend as little time as possible. Notice that a fridge at 4°C sits just under the band, while room temperature (21°C) and body temperature (37°C) sit inside it.



bacteria in food multiply fastest in the danger zone

Differences, rises and falls.

The **difference** between two temperatures is the higher minus the lower. An afternoon of 14°C and an overnight low of -3°C differ by $14 - (-3) = 14 + 3 = 17$ degrees. A **fall** subtracts: 22°C falling 7 degrees lands on $22 - 7 = 15^{\circ}\text{C}$. A **rise** adds: -2°C rising 5 degrees lands on $-2 + 5 = 3^{\circ}\text{C}$. The negative-number rules are the ones from Section 2D — taking away a negative is the same as adding a positive.

Averages of temperatures.

The **mean** (the average) of a set of readings is their sum divided by how many there are: 12, 16 and 20 sum to 48, and $48 \div 3 = 16$, so the mean is 16°C . The **range** is the warmest minus the coldest; it says how spread out the readings are. When a mean does not come out exactly, carry full precision through the working, round only at the end, and say how far you rounded — the items here ask for one decimal place (1 d.p.).

About the data in this section. The weather logs, town temperatures, food-safety readings, oven charts and compost heaps in this section are **made up** (not real): pretend places, pretend appliances and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

Three thermometers sit in a food shop overnight: thermometer A on the salad bench, thermometer B in the freezer chest, and probe C, a digital probe (a temperature sensor with a number on a screen) in the fridge tray. The figure shows the two analogue scales and the digital display. On A the labels 20 and 30 sit five spaces apart; on B the labels -20 and -10 sit five spaces apart. Read each thermometer, then find which is warmest and how much warmer it is than the coldest.

Working

Step on A: $(30 - 20) \div 5 = 2$, so one space is 2°C .

Reading A: $30 + 3 \times 2 = 36$, so 36°C .

Step on B: $(-10) - (-20) = 10$, and $10 \div 5 = 2$, so one space is 2°C here too.

Reading B: $-10 - 3 \times 2 = -16$, so -16°C .

Reading C: the screen shows -3.8°C .

Warmest: A at 36°C , since $36 > -3.8 > -16$.

Gap, A to B: $36 - (-16) = 36 + 16 = 52$, so 52°C .

Comment

Five spaces between the labels 20 and 30.

The liquid tops out 3 marks above the 30 label.

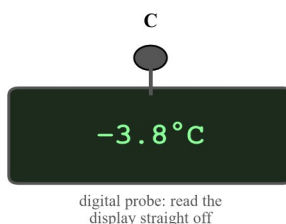
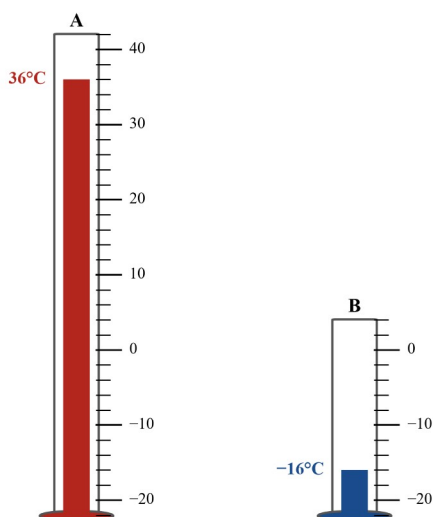
Bigger label minus smaller label, then divide by the spaces.

The liquid sits 3 marks below the -10 label.

A digital probe needs no scale reading; it is already to 1 d.p.

Put the three readings in order first.

Higher minus lower; subtracting a negative adds.



Example 2 — scaffolded

A weather log (a written record of readings) keeps the temperature six times in one day, every three hours from 6 am. The graph below shows the six readings. Find the 9 am and 6 pm readings, the warmest reading and its time, the rise from 6 am to noon, and the range for the day.

9 am: 2°C . **6 pm:** 4°C .

Read straight off the marked points.

Warmest: 9°C , at 3 pm.

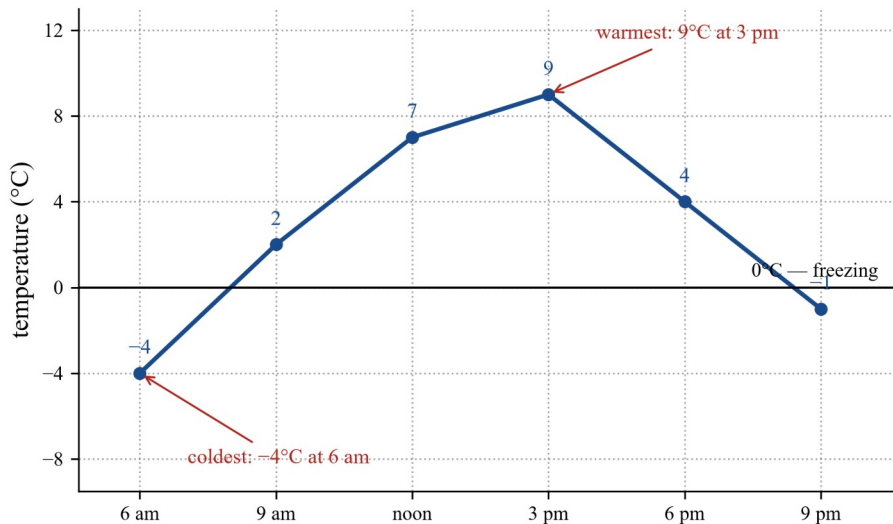
The highest point on the graph.

Rise, 6 am to noon: $7 - (-4) = 7 + 4 = 11$, so a rise of 11°C .

Later reading minus earlier reading.

Range: $9 - (-4) = 9 + 4 = 13$, so 13°C .

Warmest (9 at 3 pm) minus coldest (-4 at 6 am).



Made-up data: these six readings are invented for this book.

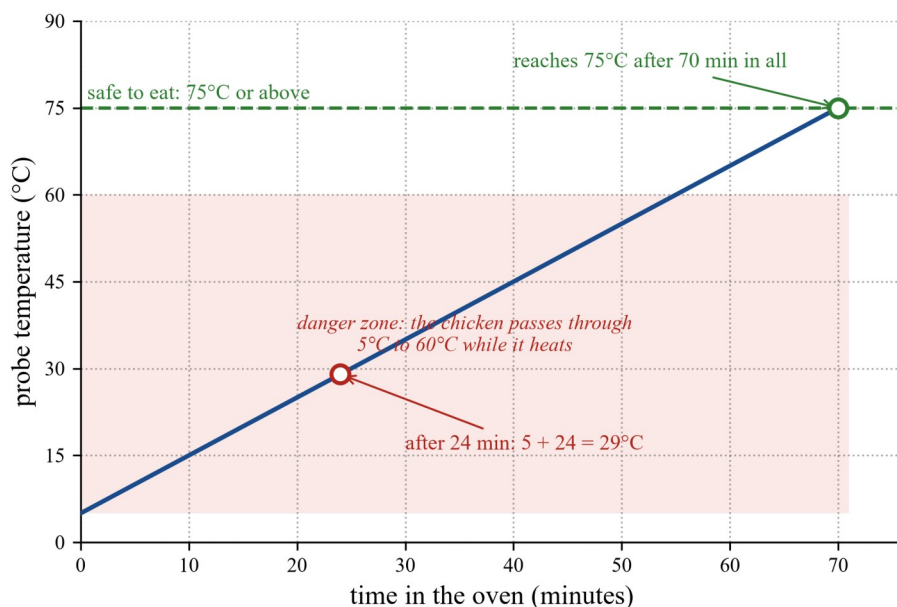
Example 3 — unscaffolded

A chicken comes out of the fridge at 5°C and goes straight into the oven. A probe shows the temperature inside the meat, and it climbs 1°C every minute. After 24 minutes the probe reads 29°C . Poultry is safe to eat at 75°C or above. Check the 24-minute reading, decide whether the chicken is safe then, and find how many more minutes it needs and the total time in the oven.

Start at 5°C and add 1°C for each of the 24 minutes: $5 + 24 = 29$, which agrees with the probe.

29°C is not safe, because safe means 75°C or above and $29 < 75$. The chicken still has to climb $75 - 29 = 46$ degrees, and at 1°C a minute that takes $46 \div 1 = 46$ more minutes.

The total time in the oven is $24 + 46 = 70$ minutes. Note where those minutes are spent: the chicken starts inside the danger zone (5°C to 60°C) and passes right through it on the way up, which is why cooking should not be stopped and started.



∴ after 24 minutes the probe reads 29°C , which is not yet safe; the chicken needs 46 more minutes, 70 minutes in all, to reach 75°C .

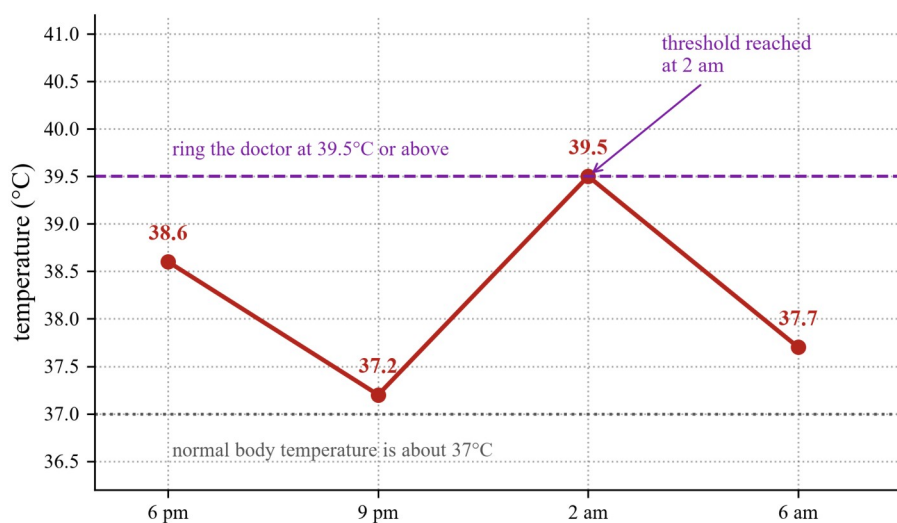
Example 4 — unscaffolded

A night nurse records a patient's temperature four times. At 6 pm it is 38.6°C . By 9 pm it has fallen 1.4°C . By 2 am it has risen 2.3°C . By 6 am it has fallen 1.8°C . The ward rule is to ring the doctor at 39.5°C or above. Find the last three readings, say whether the rule was triggered, and find the range and the mean of the four readings, the mean to 1 d.p.

Work along the night one change at a time: 9 pm is $38.6 - 1.4 = 37.2$; 2 am is $37.2 + 2.3 = 39.5$; 6 am is $39.5 - 1.8 = 37.7$. So the readings are 38.6°C , 37.2°C , 39.5°C and 37.7°C .

The rule was triggered: at 2 am the temperature reached 39.5°C , and the rule says ring at 39.5°C or above.

The range is the warmest minus the coldest: $39.5 - 37.2 = 2.3$, so 2.3°C . The sum of the four readings is $38.6 + 37.2 + 39.5 + 37.7 = 153.0$, and the mean is $153.0 \div 4 = 38.25$ exactly. Rounded half-up to 1 d.p. that is 38.3°C .



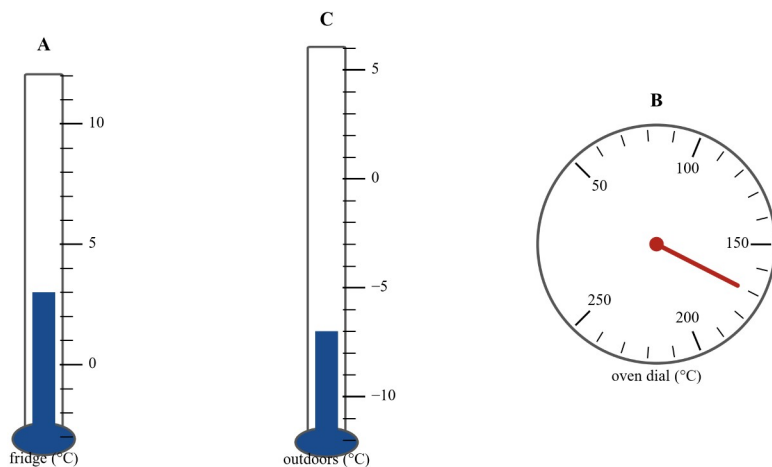
Made-up data: these four readings are invented for this book.

∴ the later readings are 37.2°C , 39.5°C and 37.7°C ; the call-the-doctor rule is triggered at 2 am; the range is 2.3°C and the mean is 38.3°C to 1 d.p.

Practice questions

Scaffolded

Q1. The figure shows three scales: A a fridge thermometer, B an oven dial, and C an outdoor thermometer. Each analogue scale has five spaces between its numbered labels.



- (a) Write down the temperature shown on fridge thermometer A.
- (b) Write down the temperature shown on oven dial B.
- (c) Write down the temperature shown on outdoor thermometer C.

Q2. Overnight minimum temperatures in five towns are listed below.

Town	Overnight minimum ($^{\circ}\text{C}$)
Ridgeway	-6
Flatton	2
Coastvale	9
Lake Bend	-2
Hilltop	-11

Made-up data: the towns and temperatures are invented for this book.

- (a) Write the five temperatures from coldest to warmest.
- (b) Which towns were below freezing (below 0°C)?
- (c) How much warmer was the warmest town than the coldest town?

Q3. A cool-room log starts at -5°C at 9 am. A fault in the cooling unit then lets the temperature fall 6°C by 11 am. The fault is fixed, and by 3 pm the temperature has risen 8°C . (a) What is the temperature at 11 am? (b) What is the temperature at 3 pm? (c) Find the overall (net) change from 9 am to 3 pm, and say what the sign of your answer tells you.

Q4. A hot cabinet in a café must hold milk between 60°C and 65°C . A probe in the milk reads 62.4°C . (a) Is the milk inside the required band? (b) A cooling fault then drops the temperature 3.7°C . What does the probe read now, and is the milk still inside the band? (c) How many degrees must the cabinet warm the milk to get back to the bottom of the band, 60°C ?

Q5. A food-safety audit reads six temperatures in a kitchen. Remember the danger zone is 5°C to 60°C .

Item	Temperature ($^{\circ}\text{C}$)
Cold display	4
Hot holding unit	68
Soup on the bench	22
Ham on the bench	18
Freezer	-20
Cooked roast in the oven	76

Made-up data: these audit readings are invented for this book.

- (a) Which two items sit inside the danger zone?
- (b) The cold display is below the danger zone. How many degrees below 5°C is it?
- (c) The hot holding unit is above the danger zone. How many degrees above 60°C is it?

Q6. Four daily maximum temperatures for one week are listed below.

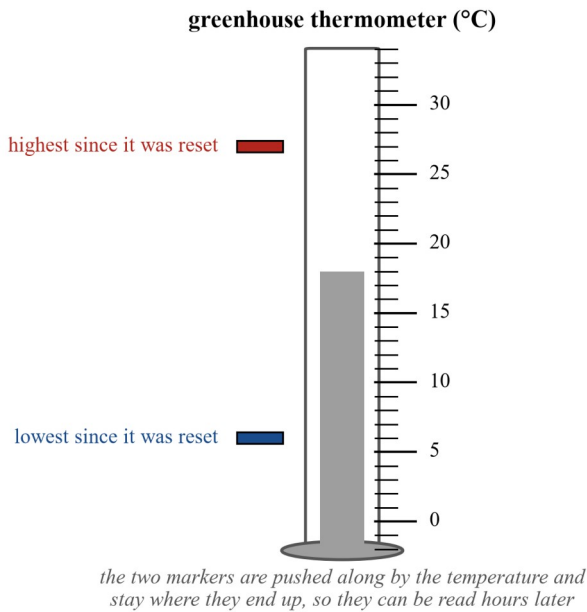
Day	Mon	Tue	Wed	Thu
Maximum ($^{\circ}\text{C}$)	18.5	21.2	24.8	19.5

Made-up data: these maxima are invented for this book.

- (a) Find the sum of the four maxima.
- (b) Find the mean of the four maxima, to 1 d.p.
- (c) How many of the four days reached 20°C or above?

Un scaffolded

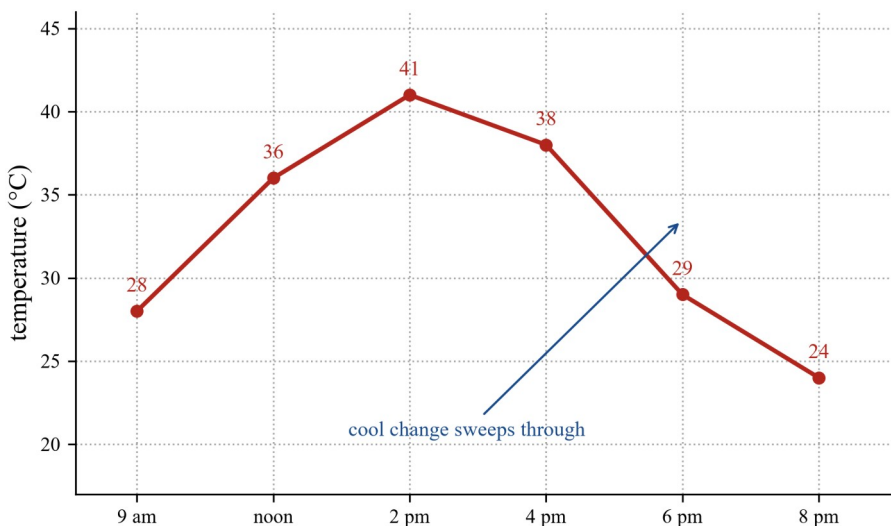
Q7. A greenhouse thermometer keeps two sliding markers: one pushed down to the lowest temperature since it was reset, one pushed up to the highest. The figure shows the markers after day 1. The next day the markers sit at 11°C and 24°C . Tomato plants need the greenhouse to reach at least 20°C at the warmest point of the day.



Made-up data: the marker readings are invented for this book.

- (a) Find the range (highest minus lowest) for day 1.
- (b) Find the range for day 2, and say which day had the bigger range.
- (c) Did the greenhouse reach 20°C on each of the two days? Justify.

Q8. A weather station graphs one day of temperatures, with readings at 9 am, noon, and then every two hours until 8 pm. A cool change (a sudden switch to cooler air) sweeps through in the afternoon.



Made-up data: these readings are invented for this book.

- (a) What was the peak temperature, and at what time?
- (b) How much did the temperature fall between 2 pm and 8 pm?
- (c) Find the biggest fall between two readings next to each other in time, and say between which two readings it happened.
- (d) Find the mean of the six readings, to 1 d.p.

Q9. A clinic logs a patient's temperature five times in one day. A reading of 38.0°C or above counts as a fever.

Time	6 am	10 am	2 pm	6 pm	10 pm
Temperature ($^{\circ}\text{C}$)	37.8	38.5	39.1	38.0	37.4

Made-up data: these readings are invented for this book.

- Which readings count as a fever?
- Find the change in temperature from 6 am to 2 pm.
- Find the mean of the five readings, to 1 d.p.
- Find the range of the five readings.

Q10. A pie warmer starts at 18°C and climbs 2°C every 5 minutes. (a) What is the temperature after 20 minutes? (b) How long does the pie warmer take to reach 70°C from the start? Give your answer in hours and minutes. (c) Hot food must be held at 60°C or above to stay out of the danger zone. Is the temperature from part (a) a safe holding temperature? Explain.

Q11. A quality check reads four cooling units in a shop and compares each reading with the range it must stay in.

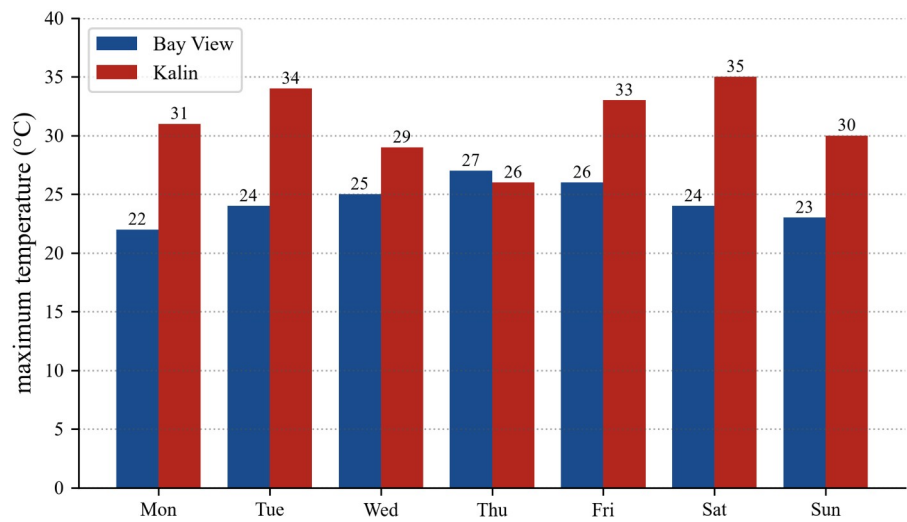
Unit	Reading ($^{\circ}\text{C}$)	Must stay between ($^{\circ}\text{C}$)
Fridge A	4	1 and 5
Fridge B	7	1 and 5
Freezer	-13	-22 and -15
Cool room	9	10 or below

Made-up data: these readings and limits are invented for this book.

- Which two units are outside their range?
- For each of those two units, how many degrees past its limit is the reading?
- A supervisor claims both faulty units are within 3°C of their limit, so a setting fix (not a repair) is enough. Is the claim true? Justify.

Q12. A student's homework asks: what is the difference between 6°C and -7°C ? The student writes $6 - 7 = -1$. (a) Write the correct calculation and its answer. (b) Explain in words the mistake the student most likely made, then check your answer by counting steps on a number line from -7 up to 6 .

Q13. Two towns, Bay View and Kalin, record their maximum temperature every day for a week.



Made-up data: the towns and maxima are invented for this book.

- (a) On Friday, which town was hotter, and by how much?
- (b) Find the range of the week’s maxima for each town.
- (c) Find the mean of the week’s maxima for each town, to 1 d.p., and find the difference between the two means, to 1 d.p.

Q14. A recipe card prints a small conversion chart for ovens.

Oven label (Fahrenheit)	350 °F	400 °F	450 °F
Chart Celsius	180 °C	200 °C	230 °C

Made-up data: this chart is invented for this book.

A fan-forced oven runs hot, so its settings sit 20 °C below the chart temperature. (a) A recipe says 200 °C. What is the fan-forced setting? (b) What is the fan-forced setting for 220 °C? (c) A recipe card from the United States says 350 °F. Use the chart to find the Celsius temperature, then find the fan-forced setting.

Q15. A compost heap starts at 25 °C and heats up 5 °C per day for 6 days. (a) What is the temperature after 6 days? (b) Hot compost needs 60 °C or above to kill weed seeds. Has the heap reached that? If not, how many more days does it need at the same rate? (c) Turning the heap (forking it over) drops the temperature 12 °C. After turning, the heap heats at 4 °C per day. How many days does it take to climb back to 60 °C?

Q16. A yogurt maker starts at 19 °C and rises 1.5 °C per hour. (a) What is the temperature after 6 hours? (b) How many hours from the start does it take to reach 31 °C? (c) The yogurt maker is switched on at 7:20 am. At what time does it reach 31 °C?

Q17. A weather log lists the minimum and maximum temperature for each day of a week.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minimum (°C)	3	-1	0	5	8	11	9
Maximum (°C)	17	16	14	19	23	27	21

Made-up data: these minima and maxima are invented for this book.

- (a) Which day had the biggest gap between its minimum and its maximum?
- (b) Frost forms at 0 °C or below. On which mornings was there frost?
- (c) Find the mean of the seven maxima, to 1 d.p.
- (d) On which days was the maximum 20 °C or above?

Q18. A pizza oven starts at 30 °C and climbs 6 °C per minute until it reaches its setting of 210 °C. (a) How many minutes does the oven take to reach its setting? (b) The oven is lit at 4:00 pm. A pizza goes in once the oven reaches its setting, and cooks for 18 minutes. At what time does the pizza come out? (c) Opening the door drops the oven 35 °C. With the door shut again it climbs 5 °C per minute. How many minutes does it take to get back to 210 °C? (d) The manager's log claims the oven was back to 200 °C or above within 5 minutes of the door shutting. Is the claim true? Justify with two readings.

Answers

Q1. (a) 3°C . (b) 170°C . (c) -7°C . **Q2.** (a) $-11, -6, -2, 2, 9$. (b) Hilltop, Lake Bend and Ridgeway. (c) 20°C . **Q3.** (a) -11°C . (b) -3°C . (c) $+2^{\circ}\text{C}$: the positive sign means the cool room ended 2°C warmer than at 9 am. **Q4.** (a) yes. (b) 58.7°C , no. (c) 1.3°C . **Q5.** (a) the soup on the bench and the ham on the bench. (b) 1°C . (c) 8°C . **Q6.** (a) 84.0°C . (b) 21.0°C to 1 d.p. (c) 2 days. **Q7.** (a) 21°C . (b) 13°C ; day 1 had the bigger range. (c) yes — 27°C and 24°C , both at least 20°C . **Q8.** (a) 41°C at 2 pm. (b) 17°C . (c) a fall of 9°C , from 4 pm to 6 pm. (d) 32.7°C to 1 d.p. **Q9.** (a) 10 am, 2 pm and 6 pm. (b) a rise of 1.3°C . (c) 38.2°C to 1 d.p. (d) 1.7°C . **Q10.** (a) 26°C . (b) 130 minutes, that is 2 h 10 min. (c) no — 26°C sits inside the danger zone, well below 60°C . **Q11.** (a) fridge B and the freezer. (b) 2°C each. (c) true — both are 2°C past their limit, and $2 \leq 3$. **Q12.** (a) $6 - (-7) = 13$, so 13°C . (b) the student subtracted 7 instead of -7 ; counting gives 7 steps up to 0 plus 6 steps up to 6, 13 in all. **Q13.** (a) Kalin, by 7°C . (b) Bay View 5°C ; Kalin 9°C . (c) Bay View 24.4°C , Kalin 31.1°C ; difference 6.7°C , all to 1 d.p. **Q14.** (a) 180°C . (b) 200°C . (c) 180°C , then 160°C . **Q15.** (a) 55°C . (b) no; 1 more day. (c) 48°C ; 3 days. **Q16.** (a) 28°C . (b) 8 hours. (c) 3:20 pm. **Q17.** (a) Tuesday (a gap of 17°C). (b) Tuesday and Wednesday. (c) 19.6°C to 1 d.p. (d) Friday, Saturday and Sunday. **Q18.** (a) 30 minutes. (b) 4:48 pm. (c) 7 minutes. (d) true — 195°C at 4 minutes (still below), 200°C at exactly 5 minutes.

Fully-worked solutions

Q1. (a) Labels 0 and 5, five spaces: step $(5 - 0) \div 5 = 1$. The liquid tops out 3 marks above 0: $0 + 3 \times 1 = 3$, so 3°C . (b) Labels 50 and 100, five spaces: step $(100 - 50) \div 5 = 10$. The needle sits 2 marks past 150: $150 + 2 \times 10 = 170$, so 170°C . (c) Labels -10 and -5 , five spaces: step $(-5) - (-10) = 5$, $5 \div 5 = 1$. The liquid tops out 3 marks above -10 : $-10 + 3 \times 1 = -7$, so -7°C .

Q2. (a) Coldest to warmest: $-11, -6, -2, 2, 9$ (Hilltop, Ridgeway, Lake Bend, Flatton, Coastvale). (b) Below freezing means below 0: Hilltop (-11), Lake Bend (-2) and Ridgeway (-6). Flatton and Coastvale are above 0. (c) Warmest minus coldest: $9 - (-11) = 9 + 11 = 20$, so 20°C warmer.

Q3. (a) A fall subtracts: $-5 - 6 = -11$, so -11°C at 11 am. (b) A rise adds: $-11 + 8 = -3$, so -3°C at 3 pm. (c) Net change: $-3 - (-5) = -3 + 5 = +2$. Check on the changes alone: $-6 + 8 = +2$ ✓. The positive sign means the room finished 2°C warmer than it started.

Q4. (a) The band is 60 to 65, and $60 \leq 62.4 \leq 65$, so yes. (b) $62.4 - 3.7 = 58.7$, so 58.7°C . That is below 60, so no, the milk is now outside the band. (c) From 58.7 up to 60: $60 - 58.7 = 1.3$, so a rise of 1.3°C .

Q5. (a) Inside the danger zone means between 5 and 60: the soup on the bench (22) and the ham on the bench (18). The cold display (4) and freezer (-20) sit below it; the hot holding unit (68) and the roast (76) sit above it. (b) $5 - 4 = 1$, so 1°C below the bottom of the zone. (c) $68 - 60 = 8$, so 8°C above the top of the zone.

Q6. (a) $18.5 + 21.2 + 24.8 + 19.5 = 84.0$, so the sum is 84.0°C . (b) Mean $= 84.0 \div 4 = 21$ exactly, which is 21.0°C to 1 d.p. (c) 21.2 and 24.8 are at or above 20; 18.5 and 19.5 are not. So 2 days.

Q7. (a) Day 1: highest 27, lowest 6, so the range is $27 - 6 = 21$, that is 21°C . (b) Day 2: $24 - 11 = 13$, that is 13°C . $21 > 13$, so day 1 had the bigger range. (c) Yes. The warmest point was 27°C on day 1 and 24°C on day 2, and both are at least 20°C , so the tomatoes' requirement was met on each day.

Q8. (a) The highest point is 41°C , at 2 pm. (b) $41 - 24 = 17$, so a fall of 17°C . (c) The changes between neighbouring readings are $+8, +5, -3, -9, -5$. The biggest fall is 9°C , between 4 pm (38) and 6 pm (29) — when the cool change sweeps through. (d) Sum: $28 + 36 + 41 + 38 + 29 + 24 = 196$. Mean: $196 \div 6 = 32.666\dots$, which is 32.7°C to 1 d.p.

Q9. (a) Fever means 38.0 or above: 38.5 (10 am), 39.1 (2 pm) and 38.0 (6 pm). The 37.8 and 37.4 readings do not count. (b) $39.1 - 37.8 = 1.3$, a rise of 1.3°C . (c) Sum: $37.8 + 38.5 + 39.1 + 38.0 + 37.4 = 190.8$. Mean: $190.8 \div 5 = 38.16$, which is 38.2°C to 1 d.p. (d) $39.1 - 37.4 = 1.7$, so a range of 1.7°C .

Q10. (a) 20 minutes is 4 steps of 5 minutes, so 4 rises of 2°C : $18 + 2 \times 4 = 26$, so 26°C . (b) The rise needed is $70 - 18 = 52$ degrees, which is $52 \div 2 = 26$ steps of 5 minutes: $26 \times 5 = 130$ minutes. 130 minutes is 2 h 10 min. (c) No. Safe hot holding means 60°C or above, and 26°C sits inside the danger zone (5 to 60), so pies held at that temperature are not safe.

Q11. (a) Fridge B reads 7 against a limit of 1 to 5: out. The freezer reads -13 against -22 to -15 : out (-13 is warmer than -15). Fridge A (4 in 1 to 5) and the cool room (9, at or below 10) are fine. (b) Fridge B: $7 - 5 = 2$ degrees above its top limit. Freezer: $-13 - (-15) = -13 + 15 = 2$ degrees above its top limit. So 2°C each. (c) True. Both faults are 2°C past their limit and $2 \leq 3$, so both sit within 3°C of the limit, as claimed.

Q12. (a) Difference means higher minus lower: $6 - (-7) = 6 + 7 = 13$, so 13°C . (b) The student subtracted 7 instead of -7 — dropping the minus sign on the second number ($6 - 7 = -1$). Check on the number line: from -7 up to 0 is 7 steps, and from 0 up to 6 is 6 more, $7 + 6 = 13$ steps in all, which matches part (a).

Q13. (a) Friday: Bay View 26, Kalin 33. Kalin was hotter by $33 - 26 = 7$, so 7°C . (b) Bay View: highest 27, lowest 22, range $27 - 22 = 5$. Kalin: highest 35, lowest 26, range $35 - 26 = 9$. So 5°C and 9°C . (c) Bay View sum $22 + 24 + 25 + 27 + 26 + 24 + 23 = 171$; mean $171 \div 7 = 24.428\dots$, so 24.4°C to 1 d.p. Kalin sum $31 + 34 + 29 + 26 + 33 + 35 + 30 = 218$; mean $218 \div 7 = 31.142\dots$, so 31.1°C to 1 d.p. Difference of the exact means: $218/7 - 171/7 = 47/7 = 6.714\dots$, so 6.7°C to 1 d.p. (The rounded means give the same difference: $31.1 - 24.4 = 6.7$.)

Q14. (a) $200 - 20 = 180$, so 180°C . (b) $220 - 20 = 200$, so 200°C . (c) The chart prints 350°F as 180°C ; the fan-forced setting is then $180 - 20 = 160$, so 160°C .

Q15. (a) $25 + 5 \times 6 = 25 + 30 = 55$, so 55°C . (b) No: $55 < 60$. The heap still needs $60 - 55 = 5$ degrees, and at 5°C a day that is 1 more day (day 7 reaches 60°C). (c) After turning: $60 - 12 = 48$, so 48°C . The climb back needs $60 - 48 = 12$ degrees at 4°C a day: $12 \div 4 = 3$ days. Check: $48 + 3 \times 4 = 60$ ✓.

Q16. (a) $19 + 1.5 \times 6 = 19 + 9 = 28$, so 28°C . (b) The rise needed is $31 - 19 = 12$ degrees; at 1.5 degrees an hour that is $12 \div 1.5 = 8$ hours. (c) 7:20 am plus 8 hours is 3:20 pm.

Q17. (a) Gaps (maximum minus minimum): Mon $17 - 3 = 14$; Tue $16 - (-1) = 17$; Wed $14 - 0 = 14$; Thu $19 - 5 = 14$; Fri $23 - 8 = 15$; Sat $27 - 11 = 16$; Sun $21 - 9 = 12$. The biggest is 17, on Tuesday. (b) Frost needs a minimum of 0 or below: Tuesday (-1) and Wednesday (0). (c) Sum of maxima: $17 + 16 + 14 + 19 + 23 + 27 + 21 = 137$. Mean: $137 \div 7 = 19.571\dots$, which is 19.6°C to 1 d.p. (d) Maxima at or above 20: Friday (23), Saturday (27), Sunday (21).

Q18. (a) The rise needed is $210 - 30 = 180$ degrees; at 6 degrees a minute that is $180 \div 6 = 30$ minutes. (b) Lit at 4:00 pm, at setting by 4:30 pm, cooked 18 minutes: the pizza comes out at 4:48 pm. (c) After the door: $210 - 35 = 175$, so 175°C . The climb back needs $210 - 175 = 35$ degrees at 5 degrees a minute: $35 \div 5 = 7$ minutes. (d) True. From 175°C at 5°C a minute: after 4 minutes $175 + 20 = 195$ (still below 200), after 5 minutes $175 + 25 = 200$, which is 200°C or above. So the oven was back to 200°C or above within 5 minutes, exactly at the 5-minute mark.

Chapter 4 Quantity and measures — 4F Estimation, measurement tools, accuracy and tolerance

Theory

Every measuring job runs the same three steps, whether it is a shelf at home or a part in a workshop: **estimate** first, **measure** second with a tool that suits the job, then **compare** the two. This section builds those three habits. It also adds the idea that makes measuring trustworthy: every reading comes with a small band of possible true values, and every job comes with a **tolerance**, the amount of difference that is still acceptable. Good measuring is knowing both bands, and checking that they line up.

About the measurements and jobs in this section. Every measurement, person, job and workplace in this section is **made up** (not real). They are built for this book so you can practise estimating, measuring and checking tolerances. Nothing here is real data about a real person, business or place.

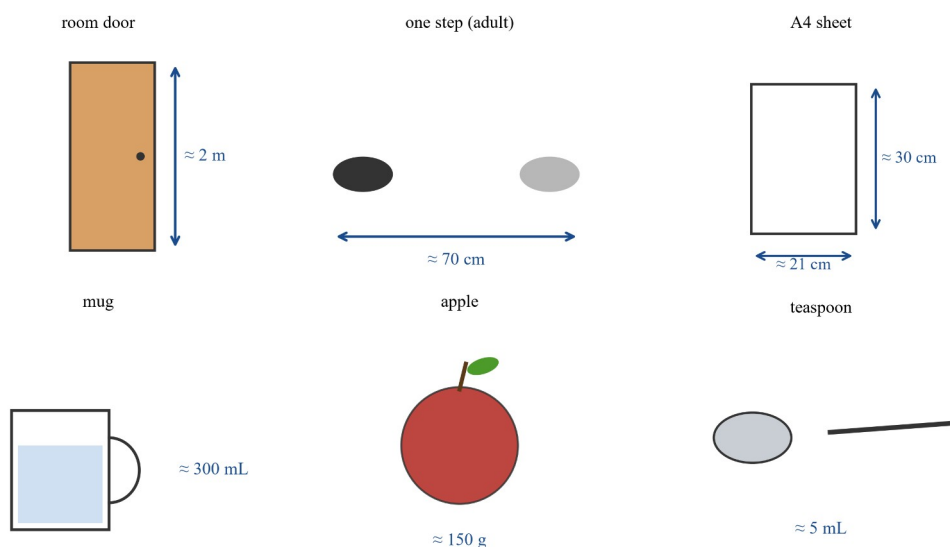
Estimate first, then measure.

An estimate is a sensible guess at a size, made without a tool. It costs a few seconds and it catches big mistakes: if your measurement lands nowhere near your estimate, something went wrong — the wrong unit, the wrong end of the tape, the wrong mark on the scale. Stop and read the tool again before trusting a number that disagrees with your estimate. Estimates also choose the tool: you would not take a 30 cm ruler to measure a hallway. Section 1C uses estimates the same way, to check calculations.

Strategy 1: benchmarks.

A **benchmark** is a size you already know, carried in your head, that new sizes can be measured against. Six useful benchmarks:

Benchmark	Useful size
Height of an internal door	about 2 m
One adult step	about 70 cm (0.7 m)
A4 sheet of paper	about 21 cm by 30 cm
Mug of drink	about 300 mL
Apple	about 150 g
Teaspoon of liquid	about 5 mL



Once you carry these, other sizes fall out of them. A workbench about half a door-height is about 1 m tall. Walk a hallway in 12 steps and the hallway is about $12 \times 0.7 = 8.4\text{ m}$, which is 8 m to the nearest metre.

Strategy 2: chunking.

Chunking means measure one chunk properly, then count the chunks. A storehand counts the bolts in one jar: 250. The shelf holds 8 identical jars, so the shelf holds about $8 \times 250 = 2000$ bolts — one careful count instead of two thousand.

Strategy 3: bracketing.

Bracketing means trap the size between two values you are sure of, then sit your estimate between them. A parcel is heavier than 1 kg and lighter than 2 kg, so halfway, about 1.5 kg, is a fair estimate.

Strategy 4: counting squares.

For a flat shape drawn on a square grid, count the whole squares inside the shape, then add the squares that are half covered or more, and skip the squares less than half covered. Each counted square adds one square of area. Question 9 practises this on a 1 cm grid.

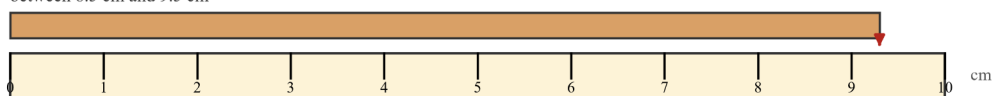
Measurement tools and their smallest division.

The **smallest division** of a tool is the finest mark on its scale, or the last digit its display shows. A tool only ever gives you the nearest mark, so the smallest division tells you how finely you can read. The steps for reading any scale are the same on every tool (section 1E), and changing between units such as mm and cm is section 4A.

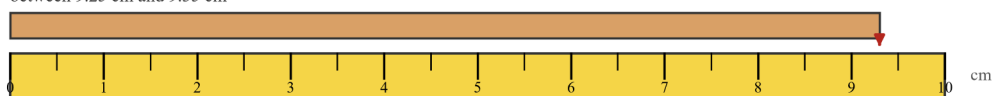
Tool	Measures	Typical smallest division
Ruler (30 cm)	length	1 mm (0.1 cm)
Tape measure	length	1 mm
Trundle wheel with counter	long distances	0.1 m on the counter
Calipers	small lengths	0.1 mm or 0.01 mm
Digital kitchen scale	mass	1 g
Digital bathroom scale	mass	0.1 kg
Dial scale	mass	depends on the dial (5 g, 25 g, 1 kg)
Measuring jug	volume	25 mL or 50 mL
Thermometer	temperature	1 °C or 0.1 °C

Calipers are a hand tool like a pair of curved jaws that closes onto a small object and shows its size on a fine scale; they are the tool of choice when a ruler is not fine enough.

marks every 1 cm:
read ≈ 9 cm, so the true length is
between 8.5 cm and 9.5 cm



marks every 1 mm:
read 9.3 cm, so the true length is
between 9.25 cm and 9.35 cm



The same wooden bar, measured on two rulers. On the ruler marked every 1 cm the far end sits nearest the 9 cm mark, so you read about 9 cm and the true length is somewhere between 8.5 cm and 9.5 cm. On the ruler marked every 1

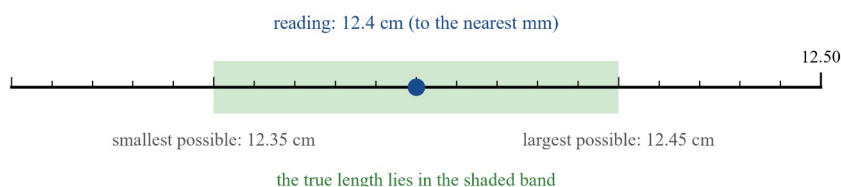
mm the same end reads 9.3 cm, so the true length is between 9.25 cm and 9.35 cm. Same bar, same reader — the finer tool gives a much narrower band of possible true lengths. That band is what accuracy means.

Accuracy: the limits of a reading.

The **accuracy** of a reading is how close it can be to the true value. Because a reading is taken to the nearest division, the true value lies within half a division either side of it. For a reading r on a tool whose smallest division is d :

$$\text{true value lies between } r - (d \div 2) \text{ and } r + (d \div 2)$$

These two ends are the **limits of accuracy** of the reading.

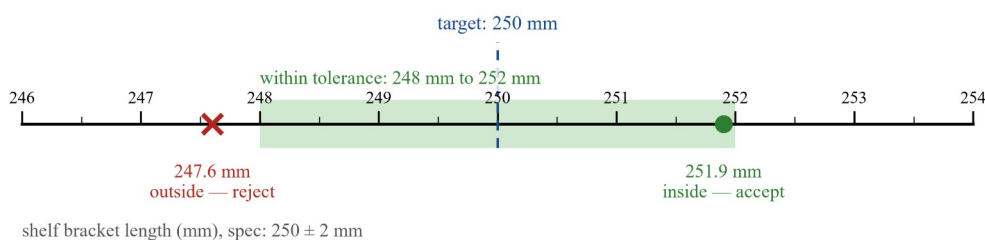


A ruler marked in millimetres reads 12.4 cm. The smallest division is 1 mm = 0.1 cm, and half of it is 0.05 cm, so the true length lies between $12.4 - 0.05 = 12.35$ cm and $12.4 + 0.05 = 12.45$ cm — the shaded band. Always write a reading with its fineness, “12.4 cm, to the nearest millimetre”, so the band can be rebuilt any time.

Tolerances — how much difference is acceptable.

A **tolerance** is the amount a measurement may differ from the target and still be acceptable. Jobs state tolerances in three written forms.

- **The $\pm$ form.** A shelf bracket specified as 250 ± 2 mm is accepted from $250 - 2 = 248$ mm up to $250 + 2 = 252$ mm. A bracket exactly 248 mm is accepted: the limits themselves count as within tolerance.
- **The range form.** A tyre placard (the label in a car’s door frame) may say 200–240 kPa. The midpoint is 220 kPa and the half-width is 20 kPa, so this is the same as 220 ± 20 kPa.
- **The one-sided form.** Some jobs allow difference on one side only. A door opening specified as 820 mm (+5 mm, –0 mm) may be up to 5 mm over but not any under: accepted widths run from 820 mm to 825 mm.



On the number line above, the green band is the tolerance for a 250 ± 2 mm bracket. A bracket measuring 247.6 mm sits 0.4 mm below the lower limit, so it is rejected; a bracket measuring 251.9 mm sits inside the band, so it is accepted.

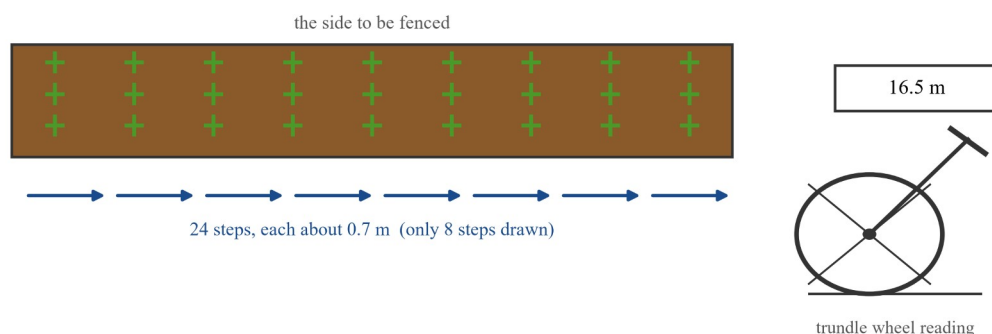
Matching the tool to the tolerance.

Now put the two bands side by side: the band of possible true values from your reading, and the band of accepted values from the tolerance. If the whole reading band sits inside the tolerance, the job definitely passes. If the whole reading band sits outside, it definitely fails. If the reading band crosses a tolerance limit, this tool cannot settle the question — the honest verdict is **unsure**, and a finer tool is needed (Example 3 and Questions 14, 17 and 18). As a rule of thumb, choose a tool whose smallest division is many times finer than the tolerance, so the reading band sits comfortably inside the accepted band (Question 15).

Worked examples

Example 1 — scaffolded

Dan fences one side of the community garden bed below. He paces it first: 24 steps, each about 0.7 m. He then measures it with a trundle wheel whose counter reads to 0.1 m and shows 16.5 m. Find Dan's estimate to the nearest metre, and compare it with the measured length.



Working

Pacing: $24 \times 0.7 = 16.8$ m.

16.8 m is 17 m to the nearest metre.

The trundle wheel reads 16.5 m.

$16.8 - 16.5 = 0.3$ m.

Comment

Estimate = number of steps $\times$ length of one step.

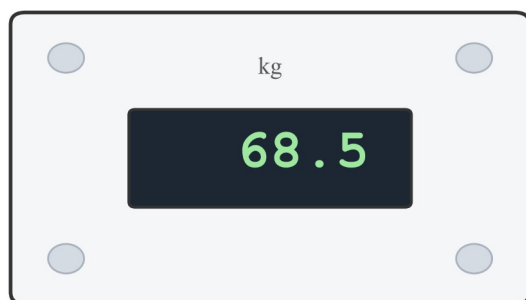
Give the estimate to the precision the question asks for.

Measure after estimating, with a tool fine enough for the job.

Compare: estimate and measurement agree to within 0.3 m, so both can be trusted.

Example 2 — scaffolded

A digital bathroom scale shows 68.5 kg, as below. The scale reads to the nearest 0.1 kg. Find the limits of accuracy of this reading: the smallest and largest masses it could be.



reads to the nearest 0.1 kg

Working

Smallest division = 0.1 kg.

Half a division = $0.1 \div 2 = 0.05$ kg.

Lower limit: $68.5 - 0.05 = 68.45$ kg.

Upper limit: $68.5 + 0.05 = 68.55$ kg.

The mass is between 68.45 kg and 68.55 kg.

Comment

The last digit on the display is tenths of a kg.

The band of possible true values is half a division each way.

A mass this low still displays as 68.5 kg.

And so does a mass this high.

Give both limits, with the unit.

Example 3 — unscaffolded

A workshop cuts shelf brackets to 250 ± 2 mm. The quality checker measures them with calipers that read to 0.1 mm. Bracket A reads 247.6 mm and bracket B reads 251.9 mm. For each bracket, say whether it is definitely within tolerance, definitely outside tolerance, or unsure.

The accepted band is $250 - 2 = 248$ mm to $250 + 2 = 252$ mm. The calipers' smallest division is 0.1 mm, so half a division is 0.05 mm. Bracket A's possible true lengths run from

$$247.6 - 0.05 = 247.55 \text{ mm to } 247.6 + 0.05 = 247.65 \text{ mm ,}$$

and every value in that band is below 248 mm, so bracket A is definitely outside tolerance. Bracket B's possible true lengths run from

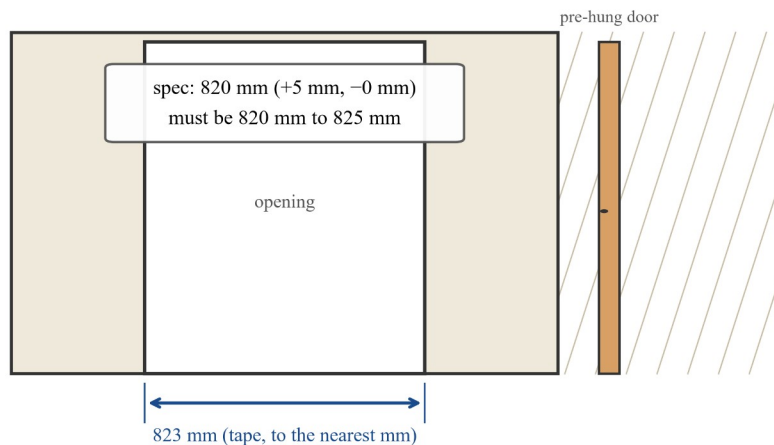
$$251.9 - 0.05 = 251.85 \text{ mm to } 251.9 + 0.05 = 251.95 \text{ mm ,}$$

and every value in that band lies inside 248–252 mm, so bracket B is definitely within tolerance. Neither band crosses a limit, so neither verdict is “unsure”.

∴ bracket A is definitely outside tolerance (reject), and bracket B is definitely within tolerance (accept).

Example 4 — unscaffolded

A builder fits a pre-hung door into the opening below. The specification says the opening must be 820 mm (+5 mm, –0 mm). She estimates first with hand spans: 8 spans, each about 10 cm. Her tape reads to the nearest mm and shows 823 mm. Does the opening definitely meet the specification?



The estimate is $8 \times 100 = 800$ mm, which says the opening is in the right ballpark and the tape reading is believable. The one-sided specification accepts widths from $820 + 0 = 820$ mm up to $820 + 5 = 825$ mm. The tape's smallest division is 1 mm, so the reading's band runs from $823 - 0.5 = 822.5$ mm to $823 + 0.5 = 823.5$ mm. The whole band lies inside 820–825 mm.

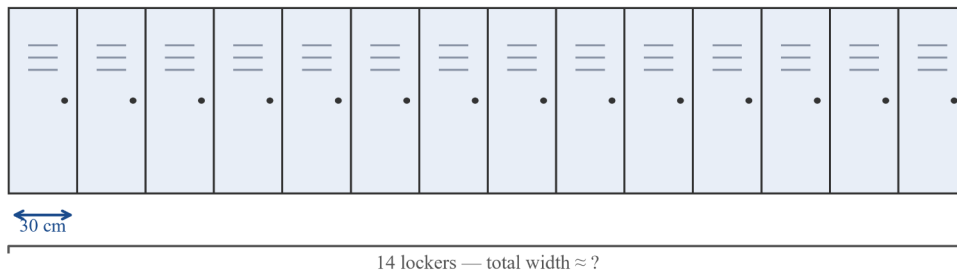
∴ yes — every possible true width (822.5–823.5 mm) is within 820–825 mm, so the opening definitely meets the specification.

Practice questions

Scaffolded

Q1. Use the benchmarks from Theory (door about 2 m, step about 0.7 m, mug about 300 mL). (a) A workbench stands about half as high as an internal door. Estimate its height. (b) A car is about 6 adult steps long. Estimate its length in metres. (c) A soup pot holds about 4 mugfuls. Estimate its capacity in mL, then in L.

Q2. A row of identical lockers runs along a hallway wall, as below. One locker is 30 cm wide.



- Estimate the total width of the row in centimetres.
- Write this estimate in metres.
- Write this estimate in metres and centimetres.

Q3. Smallest divisions. (a) A ruler is marked in millimetres. What is its smallest division, in cm? (b) A jug is marked every 50 mL. Juice is poured to about 430 mL. Between which two marks does the surface sit? (c) A dial scale is marked every 5 g. What is half of one division? Could this scale reliably show a 2 g difference? Explain.

Q4. A shelf is measured as 45 cm, to the nearest cm. (a) Could the true length be 45.2 cm? Could it be 45.6 cm? Explain. (b) Write the smallest possible true length. (c) Write the largest possible true length.

Q5. Dowels are cut to 10 ± 0.5 mm. (a) Write the smallest accepted length. (b) Write the largest accepted length. (c) A dowel measures 10.6 mm. Is it accepted?

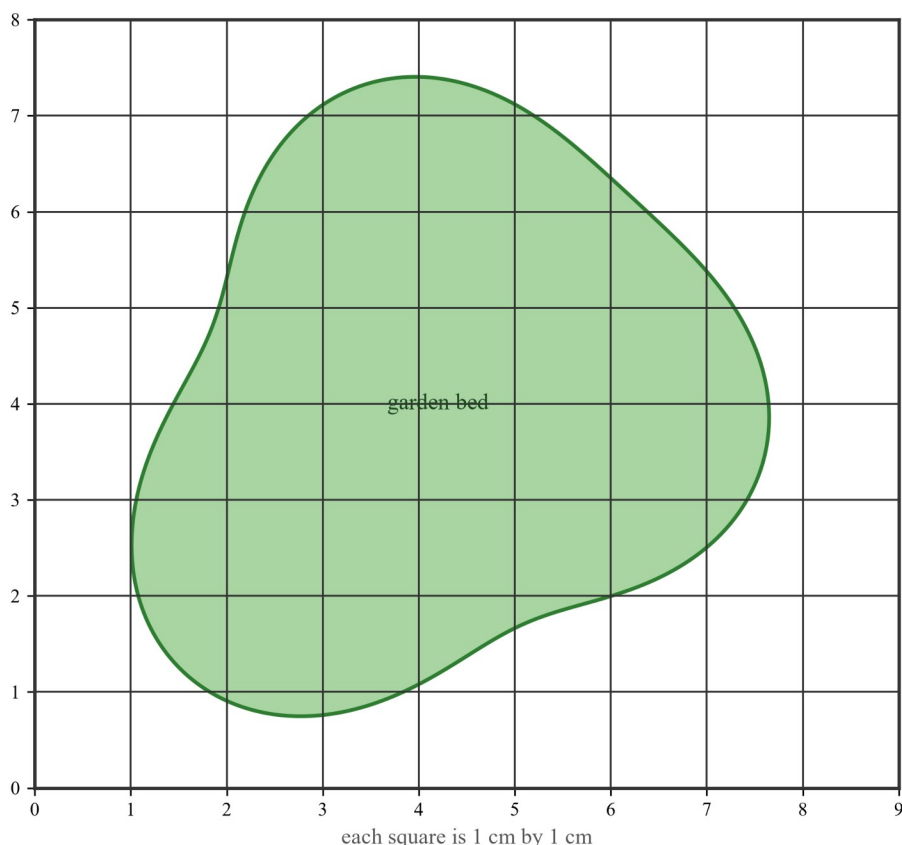
Q6. The tyre placard in a car's door frame says the tyres must be inflated to between 200 and 240 kPa. (a) Find the midpoint of this range. (b) Write the range in $\pm$ form about the midpoint. (c) A gauge shows 245 kPa. Is the tyre within the placard range?

Un scaffolded

Q7. Kim calibrates her step on a straight 9 m length marked out with a tape: it takes her exactly 12 steps. (**Calibrating** means measuring your own benchmark so it is yours, not a book's.) (a) Find the length of one of Kim's steps, in metres. (b) Kim paces a path in 36 steps. Estimate the length of the path. (c) The tape measures the path as 26.6 m. How far out was the pace estimate?

Q8. Square floor tiles are 30 cm by 30 cm. A row of 18 tiles runs exactly across a hallway. (a) Estimate the length of the hallway in centimetres. (b) Write this estimate in metres. (c) The hallway is 3 tiles wide. Estimate its width in metres.

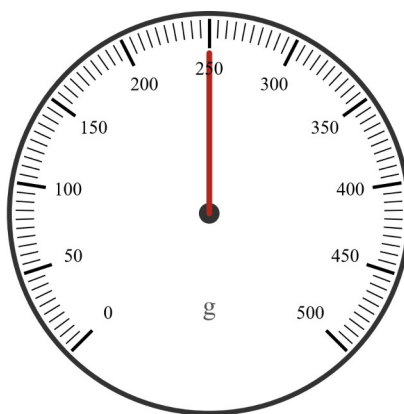
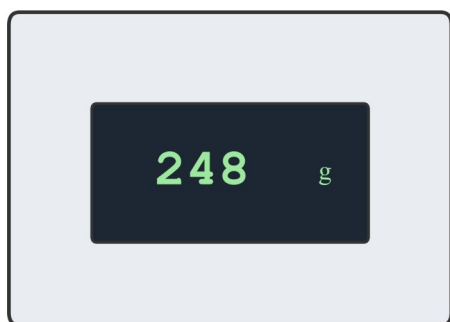
Q9. A garden bed is drawn on a 1 cm grid below. Estimate its area by counting squares: whole squares, plus squares that are half covered or more.



Q10. A postage class accepts letters up to 250 g. The post office weighs a letter on two scales, shown below: a digital scale reading to the nearest 1 g shows 248 g, and a dial scale marked every 5 g shows 250 g.

digital — nearest 1 g

dial — marks every 5 g



- Using the digital reading and its limits of accuracy, is the letter definitely under the 250 g limit?
- Using the dial reading and its limits of accuracy, is the letter definitely under the 250 g limit?
- Which scale should the post office trust for this decision, and why?

Q11. A measuring jug is marked every 50 mL. A smoothie reads 450 mL on it. (a) What is half of the jug's smallest division? (b) Write the smallest possible true volume. (c) Write the largest possible true volume.

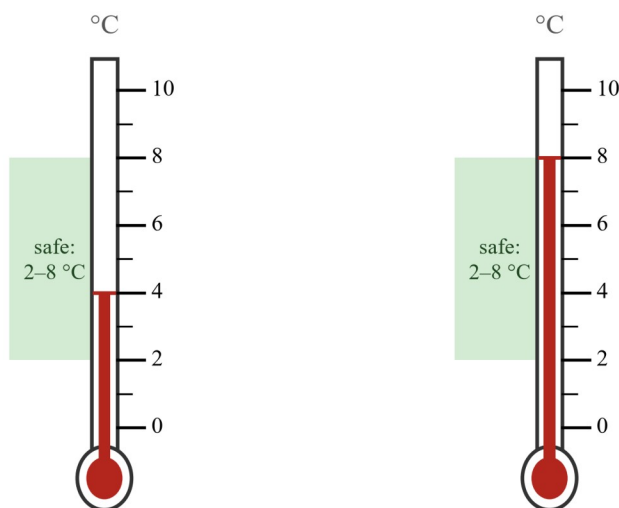
Q12. Steel rods are cut to 1200 ± 3 mm. A checker measures each rod with a tape that reads to the nearest mm. Three rods read 1196 mm, 1199 mm and 1204 mm. (a) Write the accepted range of lengths. (b) For each rod, say whether it is definitely within tolerance or definitely outside (and if outside, which side). (c) For the 1204 mm rod: how much must be cut off to reach exactly 1200 mm? What is the least that must be cut off to bring it within tolerance?

Q13. A furniture maker cuts tenons — the tongues on the ends of rails that slot into a frame — to 25 mm (+ 0.5 mm, – 0.2 mm). She checks them with calipers reading to 0.1 mm. (a) Write the largest accepted tenon thickness. (b) Write the smallest accepted tenon thickness. (c) A tenon reads 24.7 mm. Is it definitely within tolerance, definitely outside, or unsure? Show your answer with limits of accuracy.

Q14. Vaccines must be stored between 2 °C and 8 °C. The fridge thermometer has 1 °C divisions. The morning reading is 4 °C and the evening reading is 8 °C, as below.

morning: reads 4 °C

evening: reads 8 °C



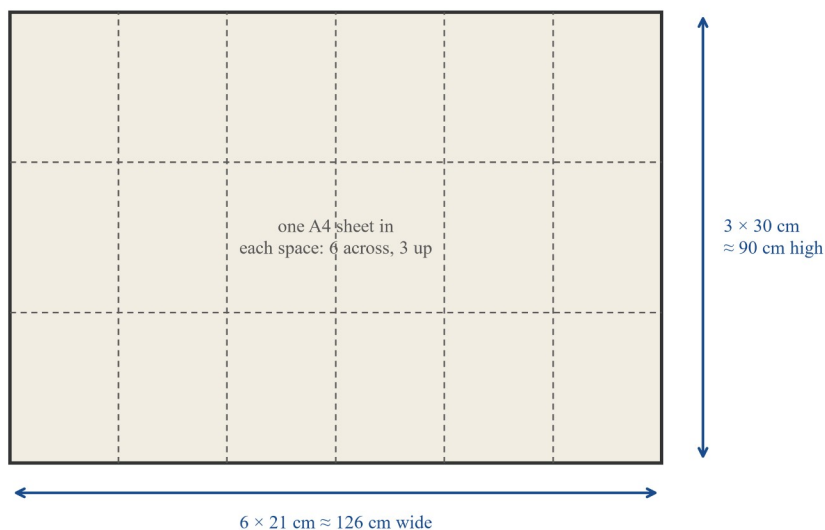
- Is the morning reading definitely inside the safe range? Show your answer with limits of accuracy.
- Is the evening reading definitely inside the safe range? Show your answer with limits of accuracy.
- What should the worker do about the evening reading?

Q15. Choose the tool for each job, and explain your choice with limits of accuracy. (a) A small pin must be made to a tolerance of ± 0.1 mm. Which of a mm ruler and calipers reading to 0.01 mm can check it? (b) A bench is about 900 mm long and must be checked to a tolerance of ± 10 mm. Which of a 30 cm ruler and a 5 m tape with mm marks can measure it? (c) Flour bags are filled to 500 ± 5 g. Which of a kitchen scale reading to 1 g and a bathroom scale reading to 100 g can check them?

Q16. Ned paints a mural on a wall about as long as three door-heights laid end to end. The job sheet comes back reading “612” with the unit written as m. (a) Write Ned’s estimate for the wall, in metres. (b) The wall was later measured as 612 cm. Write this in metres and compare it with the estimate. (c) Taken as written, 612 m is how many times Ned’s estimate? About how many 100 m football fields is it? What should Ned conclude about the job sheet?

Q17. A concrete slab must be 100 ± 5 mm thick. A worker checks it with a ruler that reads to the nearest mm. (a) Write the accepted range of thicknesses. (b) Four readings are 96 mm, 101 mm, 104 mm and 94 mm. For each, say whether the slab is definitely within tolerance or definitely outside. (c) One reading is 95 mm. Is the slab within tolerance, outside tolerance, or unsure? Explain with limits of accuracy.

Q18. A class makes a noticeboard from a board they estimate with A4 sheets: 6 sheets across and 3 sheets up fit the board exactly, as below.



- Estimate the size of the board in centimetres, then write each dimension in metres.
- The screw holes must be spaced 140 ± 3 mm apart. A tape reading to the nearest mm shows 138 mm between two holes. Is the spacing definitely within tolerance?
- The holes are 8.0–8.4 mm across. A screw shaft reads 8 mm on a mm ruler. Can the class be sure the screw fits through a hole? What tool would settle the question?

Answers

Q1. (a) about 1 m. (b) about 4.2 m. (c) about 1200 mL = 1.2 L. **Q2.** (a) 420 cm. (b) 4.2 m. (c) 4 m 20 cm. **Q3.** (a) 0.1 cm. (b) between the 400 mL and 450 mL marks. (c) 2.5 g; no — a 2 g difference is less than half a division, so the needle may not reach the next mark. **Q4.** (a) 45.2 cm yes (it lies in 44.5–45.5 cm); 45.6 cm no (it lies outside). (b) 44.5 cm. (c) 45.5 cm. **Q5.** (a) 9.5 mm. (b) 10.5 mm. (c) no — 10.6 mm is above 10.5 mm. **Q6.** (a) 220 kPa. (b) 220 ± 20 kPa. (c) no — 245 kPa is above 240 kPa. **Q7.** (a) 0.75 m. (b) 27 m. (c) 0.4 m (the estimate was over). **Q8.** (a) 540 cm. (b) 5.4 m. (c) 0.9 m. **Q9.** about 32 cm² (any count from 29 to 33 cm² is a good estimate). **Q10.** (a) yes — the band 247.5–248.5 g lies wholly under 250 g. (b) unsure — the band 247.5–252.5 g crosses 250 g. (c) the digital scale — its whole band sits under the limit, so its verdict is certain. **Q11.** (a) 25 mL. (b) 425 mL. (c) 475 mL. **Q12.** (a) 1197–1203 mm. (b) 1196 mm outside (too short), 1199 mm within, 1204 mm outside (too long). (c) 4 mm; 1 mm. **Q13.** (a) 25.5 mm. (b) 24.8 mm. (c) definitely outside (too thin) — the band 24.65–24.75 mm lies wholly below 24.8 mm. **Q14.** (a) yes — the band 3.5–4.5 °C lies inside 2–8 °C. (b) unsure — the band 7.5–8.5 °C crosses 8 °C. (c) measure again with a finer thermometer before acting; “unsure” is not a fail. **Q15.** (a) the calipers — the ruler’s band is ± 0.5 mm, wider than the ± 0.1 mm tolerance; the calipers’ band is ± 0.005 mm. (b) the 5 m tape — a 30 cm ruler cannot span 900 mm at all, and the tape’s band of ± 0.5 mm sits inside ± 10 mm. (c) the kitchen scale — its band of ± 0.5 g sits inside ± 5 g; the bathroom scale’s band of ± 50 g is far wider. **Q16.** (a) about 6 m. (b) 6.12 m — close to the estimate, so the measurement is believable. (c) 102 times; about 6 football fields of 100 m; the unit on the job sheet is a slip — it should read cm, not m. **Q17.** (a) 95–105 mm. (b) 96 mm within, 101 mm within, 104 mm within, 94 mm outside. (c) unsure — the band 94.5–95.5 mm crosses the 95 mm limit. **Q18.** (a) 126 cm by 90 cm, that is 1.26 m by 0.9 m. (b) yes — the band 137.5–138.5 mm lies inside 137–143 mm. (c) no — the band 7.5–8.5 mm crosses 8.0–8.4 mm; calipers would settle it.

Fully-worked solutions

Q1. (a) Half of a 2 m door-height is $2 \div 2 = 1$ m. (b) Six steps of about 0.7 m give $6 \times 0.7 = 4.2$ m. (c) Four mugfuls of about 300 mL give $4 \times 300 = 1200$ mL, and $1200 \div 1000 = 1.2$ L.

Q2. (a) The picture shows 14 lockers, each a 30 cm chunk: $14 \times 30 = 420$ cm. (b) $420 \div 100 = 4.2$ m. (c) 420 cm is 4 m and 20 cm.

Q3. (a) One millimetre is 0.1 cm, so the smallest division is 0.1 cm. (b) The marks run 400 mL, 450 mL, 500 mL; 430 mL sits between the 400 mL and 450 mL marks. (c) Half of a 5 g division is $5 \div 2 = 2.5$ g. A 2 g difference is less than half a division, so the true mass can change by 2 g without the needle reaching the next mark — the scale cannot reliably show it.

Q4. The reading is to the nearest cm, so the band is half a cm each way: 44.5–45.5 cm. (a) 45.2 cm lies inside that band, so yes, it is possible; 45.6 cm lies outside it, so no. (b) The smallest possible true length is $45 - 0.5 = 44.5$ cm. (c) The largest is $45 + 0.5 = 45.5$ cm.

Q5. (a) $10 - 0.5 = 9.5$ mm. (b) $10 + 0.5 = 10.5$ mm. (c) No: 10.6 mm is above the largest accepted length of 10.5 mm.

Q6. (a) The midpoint is $(200 + 240) \div 2 = 220$ kPa. (b) The half-width is $240 - 220 = 20$ kPa, so the range is 220 ± 20 kPa. (c) No: 245 kPa is above the top of the range, 240 kPa.

Q7. (a) Twelve steps cover 9 m, so one step is $9 \div 12 = 0.75$ m. (b) $36 \times 0.75 = 27$ m. (c) $27 - 26.6 = 0.4$ m; the pace estimate was 0.4 m over the tape.

Q8. (a) $18 \times 30 = 540$ cm. (b) $540 \div 100 = 5.4$ m. (c) Three tiles of 30 cm: $3 \times 30 = 90$ cm = 0.9 m.

Q9. Counting the whole squares inside the bed, then adding the squares that are half covered or more, gives about 32 squares; each square is 1 cm by 1 cm = 1 cm², so the area is about 32 cm². Any count from 29 to 33 cm² is a good estimate: measured exactly, the shape’s area is 30.7 cm² to 1 decimal place, so every count in that band is within 2.5 cm² of it.

Q10. (a) The digital scale's smallest division is 1 g, so half a division is 0.5 g and the band is $248 - 0.5 = 247.5$ g to $248 + 0.5 = 248.5$ g. Every value in the band is under 250 g, so yes, the letter is definitely under the limit. (b) The dial's smallest division is 5 g, so half a division is 2.5 g and the band is $250 - 2.5 = 247.5$ g to $250 + 2.5 = 252.5$ g. That band crosses 250 g, so the letter could be over the limit: the verdict is unsure. (c) The digital scale: its whole band sits under the limit, so an "under the limit" verdict from it is certain, while the dial can never settle this letter.

Q11. (a) $50 \div 2 = 25$ mL. (b) $450 - 25 = 425$ mL. (c) $450 + 25 = 475$ mL.

Q12. (a) $1200 - 3 = 1197$ mm and $1200 + 3 = 1203$ mm, so accepted lengths run 1197–1203 mm. (b) The tape's band is half a mm each way. The 1196 mm rod: 1195.5–1196.5 mm, wholly below 1197 mm, so definitely outside (too short). The 1199 mm rod: 1198.5–1199.5 mm, wholly inside, so definitely within. The 1204 mm rod: 1203.5–1204.5 mm, wholly above 1203 mm, so definitely outside (too long). (c) To reach the target: $1204 - 1200 = 4$ mm. To reach the top of the accepted range: $1204 - 1203 = 1$ mm, so 1 mm is the least cut that brings it within tolerance.

Q13. (a) $25 + 0.5 = 25.5$ mm. (b) $25 - 0.2 = 24.8$ mm. (c) The calipers' band is $24.7 - 0.05 = 24.65$ mm to $24.7 + 0.05 = 24.75$ mm. Every value in that band is below the smallest accepted thickness of 24.8 mm, so the tenon is definitely outside tolerance — too thin.

Q14. (a) The thermometer's smallest division is 1°C , so the band is $4 - 0.5 = 3.5^\circ\text{C}$ to $4 + 0.5 = 4.5^\circ\text{C}$. The whole band lies inside $2\text{--}8^\circ\text{C}$, so yes, the morning reading is definitely inside the safe range. (b) The evening band is $8 - 0.5 = 7.5^\circ\text{C}$ to $8 + 0.5 = 8.5^\circ\text{C}$. That band crosses the 8°C limit, so the true temperature could be above it: the verdict is unsure. (c) Measure again with a finer thermometer (say 0.1°C divisions) before moving the vaccines. "Unsure" does not mean the fridge failed — it means this tool cannot settle the question.

Q15. (a) The mm ruler's band is ± 0.5 mm, which is wider than the whole ± 0.1 mm tolerance, so it cannot check the pin. Calipers reading to 0.01 mm have a band of ± 0.005 mm, which sits comfortably inside the tolerance, so they can. (b) A 30 cm ruler cannot span 900 mm at all, so it is out regardless of fineness. The 5 m tape reads to 1 mm, a band of ± 0.5 mm, which sits well inside the ± 10 mm tolerance, so the tape can. (c) The kitchen scale reads to 1 g, a band of ± 0.5 g, inside the ± 5 g tolerance, so it can check the bags. The bathroom scale reads to 100 g, a band of ± 50 g, ten times the tolerance, so it cannot.

Q16. (a) Three door-heights: $3 \times 2 = 6$ m. (b) $612 \text{ cm} = 612 \div 100 = 6.12$ m, which sits close to the 6 m estimate, so the measurement is believable. (c) $612 \div 6 = 102$, so as written the sheet claims a wall 102 times the estimate — about 6 football fields of 100 m each. No school wall is that long, so the unit on the sheet is a slip: the measurement was 612 cm, keyed in as m. The estimate is what caught it.

Q17. (a) $100 - 5 = 95$ mm and $100 + 5 = 105$ mm, so accepted thicknesses run 95–105 mm. (b) The ruler's band is half a mm each way. 96 mm: 95.5–96.5 mm, wholly inside — within. 101 mm: 100.5–101.5 mm, wholly inside — within. 104 mm: 103.5–104.5 mm, wholly inside — within. 94 mm: 93.5–94.5 mm, wholly below 95 mm — outside. (c) The band for 95 mm is 94.5–95.5 mm. It crosses the 95 mm lower limit, so the slab could be just under or just over: the verdict is unsure, and a finer tool (or a second reading) is needed.

Q18. (a) Across: $6 \times 21 = 126$ cm. Up: $3 \times 30 = 90$ cm. In metres: $126 \div 100 = 1.26$ m and $90 \div 100 = 0.9$ m. (b) The accepted spacing is $140 - 3 = 137$ mm to $140 + 3 = 143$ mm. The tape's band is $138 - 0.5 = 137.5$ mm to $138 + 0.5 = 138.5$ mm, wholly inside 137–143 mm, so yes, the spacing is definitely within tolerance. (c) The ruler's band for the shaft is $8 - 0.5 = 7.5$ mm to $8 + 0.5 = 8.5$ mm. That band crosses both hole limits (8.0 and 8.4 mm), so the shaft could be too thick for the smallest hole: the class cannot be sure. Calipers, with a far narrower band, would settle the question.

Chapter 04 — Test A (multiple choice)

Chapter 4 Quantity and measures (Sections 4A–4F). Allowed materials: one scientific calculator. Section A: 12 questions (12 marks). Time: about 25 minutes.

About the data in this test. Every name, place, price, time and measurement in this test is **made up** (not real). They are built for this book so you can practise the mathematics from Chapter 4. Nothing here is real data about a real person, business or place.

Section A — Multiple-choice questions

Choose the response that is correct for the question.

Question 1

A parcel weighs 5.3 kg. Using $1 \text{ kg} = 2.205 \text{ lb}$, the weight of the parcel in pounds (lb), correct to one decimal place, is

- A. 2.4 lb
- B. 10.6 lb
- C. 11.6 lb
- D. 11.7 lb

Question 2

A patient's thermometer reads 98°F . To convert from $^\circ\text{F}$ to $^\circ\text{C}$, subtract 32 and then divide by 1.8. The reading in $^\circ\text{C}$, correct to the nearest whole degree, is

- A. 37°C
- B. 54°C
- C. 66°C
- D. 208°C

Question 3

A circular water tank has a radius of 3 m. Using the π key on your calculator, the circumference of the tank, correct to two decimal places, is

- A. 9.42 m
- B. 18.84 m
- C. 18.85 m
- D. 28.27 m

Question 4

A garden bed is shaped like a trapezium. Its two parallel sides are 5 m and 9 m long, and its height is 4 m. The area of the garden bed is

- A. 18 m^2
- B. 28 m^2
- C. 56 m^2
- D. 180 m^2

Question 5

A rectangular water tank measures 2 m by 1.5 m by 0.8 m. Using $1 \text{ m}^3 = 1000 \text{ L}$, the capacity of the full tank is

- A. 2400 L
- B. 240 L
- C. 24 L
- D. 2.4 L

Question 6

A cylinder has a radius of 2 cm and a height of 10 cm. Using the π key on your calculator, its volume, correct to the nearest cm^3 , is

- A. 63 cm^3
- B. 503 cm^3
- C. 125 cm^3
- D. 126 cm^3

Question 7

A night shift starts at 21:40 and finishes at 06:25 the next day. The length of the shift is

- A. 8 hours 15 minutes
- B. 8 hours 45 minutes
- C. 9 hours 15 minutes
- D. 9 hours 25 minutes

Question 8

Brisbane time is UTC + 10 and Adelaide time is UTC + 9:30 (standard time). When it is 18:30 in Brisbane, the time in Adelaide is

- A. 17:30
- B. 18:30
- C. 18:00
- D. 19:00

Question 9

At midday the temperature in a town was 2°C . By midnight it had fallen by 9°C . The temperature at midnight was

- A. -11°C
- B. -7°C
- C. 7°C
- D. 11°C

Question 10

Cold food must be kept below 5 °C, and hot food must be kept at 60 °C or above. The danger zone is the band of temperatures between these two limits. Which one of the following temperatures is in the danger zone?

- A. 4 °C
- B. – 18 °C
- C. 75 °C
- D. 22 °C

Question 11

A ruler is marked in millimetres. A pencil is measured as 8.6 cm long. The limits of accuracy of this reading are

- A. 8.55 cm to 8.65 cm
- B. 8.5 cm to 8.7 cm
- C. 8.1 cm to 9.1 cm
- D. 8.6 cm to 8.65 cm

Question 12

A building specification is written as 150 ± 10 mm. Written as a range, this specification is

- A. 145 mm to 155 mm
- B. 140 mm to 150 mm
- C. 140 mm to 160 mm
- D. 150 mm to 160 mm

End of Section A

Test A — Answers

Que stio n	1	2	3	4	5	6	7	8	9	10	11	12
Ans wer	D	A	C	B	A	D	B	C	B	D	A	C

Test A — Fully-worked solutions

Question	Answer	Working
1	D	$5.3 \times 2.205 = 11.6865$. Correct to one decimal place (the second decimal is 8, so round up): 11.7 lb. Option A divides instead of multiplying ($5.3 \div 2.205 = 2.4$); option B uses the rough factor 2; option C truncates instead of rounding.
2	A	Subtract 32: $98 - 32 = 66$. Divide by 1.8: $66 \div 1.8 = 36.666\dots$. To the nearest whole degree: 37°C . Option B skips the subtraction; option C skips the division; option D applies the $^\circ\text{C}$ -to- $^\circ\text{F}$ rule in reverse.
3	C	$C = 2\pi r = 2 \times \pi \times 3 = 18.84955\dots$. Correct to two decimal places: 18.85 m. Option A is πr (half the circumference); option B rounds π to 3.14 too early; option D is the area, $\pi r^2 = 28.27 \text{ m}^2$.
4	B	$A = \frac{1}{2}(a+b)h = \frac{1}{2} \times (5+9) \times 4 = \frac{1}{2} \times \text{m}^2$. Option A adds the three numbers; option C forgets the $\frac{1}{2}$; option D multiplies all three.
5	A	$V = lwh = 2 \times 1.5 \times 0.8 = 2.4 \text{ m}^3$. Convert: $2.4 \times 1000 = 2400 \text{ L}$. Options B, C and D use the wrong power of ten at the m^3 -to-L step.
6	D	$V = \pi r^2 h = \pi \times 2^2 \times 10 = \pi \times 4 \times 10 =$ To the nearest cm^3 : 126 cm^3 . Option A misses the square ($\pi \times 2 \times 10 = 63$); option B uses the diameter 4 cm as the radius; option C truncates.
7	B	From 21:40 to 24:00 is 2 hours 20 minutes. From 00:00 to 06:25 is 6 hours 25 minutes. Total: 8 hours 45 minutes. Option A subtracts the minutes backwards ($40 - 25$); option C subtracts column by column without borrowing; option D reads 8.85 hours as 8 hours 85 minutes.
8	C	Adelaide (UTC +9:30) is 30 minutes behind Brisbane (UTC +10): $18:30 - 0:30 = 18:00$. Option A goes back a full hour; option B ignores the offset; option D goes the

Question	Answer	Working
9	B	wrong way. Falling means subtracting: $2 - 9 = -7$, so the temperature at midnight was -7°C . Option A starts from -2 ; option C drops the negative sign; option D adds instead of subtracting.
10	D	The danger zone sits between 5°C and 60°C . Only 22°C lies inside it: 4°C is cold and safe, -18°C is frozen, and 75°C is hot and safe.
11	A	The smallest division is $1\text{ mm} = 0.1\text{ cm}$, so the true length is within half of that (0.05 cm) of the reading: $8.6 - 0.05 = 8.55$ and $8.6 + 0.05 = 8.65$. Option B uses a whole division each side; option C uses half a centimetre each side; option D only allows error above the reading.
12	C	$150 - 10 = 140$ and $150 + 10 = 160$, so the range is 140 mm to 160 mm . Option A halves the tolerance; options B and D apply the tolerance in only one direction.

Chapter 04 — Test B (written response)

Chapter 4 Quantity and measures (Sections 4A–4F). Allowed materials: one scientific calculator; one ruler. Section B: 7 questions (37 marks). Time: about 40 minutes. Show your working for every part. Only round where the question tells you to, and give each rounded answer at the precision asked for.

About the data in this test. Every name, place, price, time and measurement in this test is **made up** (not real). They are built for this book so you can practise the mathematics from Chapter 4. Nothing here is real data about a real person, business or place.

Section B — Written-response questions

Question 1 (4 marks)

Nadia is checking some measurements on overseas websites.

- a.** A bag of rice is labelled 9.7 lb. Using $1 \text{ kg} = 2.205 \text{ lb}$, convert this weight to kilograms, correct to two decimal places. 1 mark

- b.** A computer monitor is listed as 27 in wide. Using $1 \text{ in} = 2.54 \text{ cm}$, convert this width to centimetres, correct to one decimal place. 1 mark

- c.** Nadia's pen friend in Seattle says it is 19°C there. Convert 19°C to $^\circ\text{F}$, correct to one decimal place. (Multiply by 1.8, then add 32.) 2 marks

Question 2 (6 marks)

A circular garden bed has a radius of 2.5 m. A fence is built around the edge of the garden bed, with a 1 m section left open as a gate. Use the π key on your calculator, and only round where you are told to.

- a.** Find the circumference of the garden bed, correct to two decimal places. 1 mark

- b.** Hence find the length of fencing needed, correct to two decimal places. 1 mark

- c.** Find the area of the garden bed, correct to two decimal places. 2 marks

- d.** Compost is spread over the whole garden bed at a rate of 8 kg per square metre. Find the mass of compost needed, correct to the nearest kilogram. 2 marks

Question 3 (5 marks)

A cylindrical water tank has a diameter of 1.8 m and a height of 1.5 m.

a. Write down the radius of the tank.

1 mark

b. Show that the volume of the tank is 3.82 m^3 , correct to two decimal places.

2 marks

c. Using $1 \text{ m}^3 = 1000 \text{ L}$, find the capacity of the full tank in litres, correct to the nearest litre. Use the full value of the volume from your calculator, not the rounded value from part b.

2 marks

Question 4 (6 marks)

Marcus is a security guard. One night his shift starts at 22:45 and finishes at 07:15 the next day. He takes two unpaid 15-minute breaks during the shift.

a. Find the length of Marcus's shift, in hours and minutes.

2 marks

b. Find the number of hours Marcus is paid for on this shift.

1 mark

c. Marcus calls his sister in Perth (UTC + 8) at 23:30 Melbourne time (UTC + 10). What is the time in Perth when he calls? All times are standard time.

1 mark

d. Marcus works a 12-day roster. His first day is Thursday 5 March. Find the date of the last day of the roster, and the day of the week it falls on.

2 marks

Question 5 (5 marks)

The freezer in a delivery van develops a fault. The driver's log shows the temperature inside the freezer at five times one morning.

Time	6 am	8 am	10 am	12 pm	2 pm
Temperature (°C)	-5	-2	2	6	9

Made-up data: the times and temperatures in this log are invented for this test.

a. Find the rise in temperature from 6 am to 12 pm.

1 mark

b. Find the mean of the five temperatures in the log, correct to one decimal place. 2 marks

c. Find the range of the five temperatures. 1 mark

d. To be safe, cold food must be kept below 5 °C. List the times in the log when the food in the freezer was not being kept safe. 1 mark

Question 6 (5 marks)

Sam the carpenter is checking whether a new door will fit its frame. The specification for the width of the frame opening is 760 mm, with a tolerance of + 4 mm, – 0 mm. Sam measures the opening with a tape measure marked in millimetres and gets a reading of 763 mm.

a. Before measuring, Sam estimates the width as about one step. Using the benchmark that one step is about 0.7 m, convert this estimate to millimetres. 1 mark

b. Write down the accepted range of widths for the frame opening, using the + 4 mm, – 0 mm tolerance. 1 mark

c. Sam's tape measure is marked in millimetres. Write the limits of accuracy of the 763 mm reading as a range. 1 mark

d. Using your answers to parts **b** and **c**, explain whether Sam can be sure that the door will fit the opening. 2 marks

Question 7 (6 marks)

Rosa runs a food stall at the Saturday market.

- a.** Rosa buys a packet of spice labelled 8 oz. Using $1 \text{ oz} = 28.35 \text{ g}$, convert this mass to grams, correct to the nearest gram, and then to kilograms, correct to two decimal places. 2 marks

- b.** Rosa's largest frying pan has a diameter of 30 cm. Find the area of the base of the pan, correct to the nearest cm^2 . 2 marks

- c.** The oil in the pan starts at 24°C and heats up at a steady rate of 12°C per minute. Rosa needs the oil at 180°C . How many minutes does she have to wait? 1 mark

- d.** Rosa puts a basket of chips in the fryer at 18:47. They take 25 minutes to cook. At what time does she take them out? 1 mark

End of Section B

Test B — Answers

Q1. (a) 4.40 kg. (b) 68.6 cm. (c) 66.2 °F.

Q2. (a) 15.71 m. (b) 14.71 m. (c) 19.63 m². (d) 157 kg.

Q3. (a) 0.9 m. (b) 3.82 m³. (c) 3817 L.

Q4. (a) 8 hours 30 minutes. (b) 8 hours. (c) 21:30. (d) Monday 16 March.

Q5. (a) 11 °C. (b) 2.0 °C. (c) 14 °C. (d) 12 pm and 2 pm.

Q6. (a) 700 mm. (b) 760 mm to 764 mm. (c) 762.5 mm to 763.5 mm. (d) Yes — the door definitely fits.

Q7. (a) 227 g; 0.23 kg. (b) 707 cm². (c) 13 minutes. (d) 19:12.

Test B — Fully-worked solutions

Question 1.

- a. Divide by the conversion factor: $9.7 \div 2.205 = 4.39909\dots$ Correct to two decimal places (the third decimal is 9, so round up): 4.40 kg.
- b. Multiply by the conversion factor: $27 \times 2.54 = 68.58$. Correct to one decimal place (the second decimal is 8, so round up): 68.6 cm.
- c. Multiply by 1.8: $19 \times 1.8 = 34.2$. Add 32: $34.2 + 32 = 66.2$. This is already at one decimal place: 66.2 °F.

Question 2.

- a. $C = 2\pi r = 2 \times \pi \times 2.5 = 15.70796\dots$ Correct to two decimal places: 15.71 m.
- b. The fence goes all the way around the circle except for the 1 m gate: $15.70796\dots - 1 = 14.70796\dots$ Correct to two decimal places: 14.71 m of fencing.
- c. $A = \pi r^2 = \pi \times 2.5^2 = \pi \times 6.25 = 19.63495\dots$ Correct to two decimal places: 19.63 m².
- d. Mass = area $\times$ rate = $19.63495\dots \times 8 = 157.0796\dots$ Correct to the nearest kilogram: 157 kg. (Using the rounded area gives $19.63 \times 8 = 157.04$, the same 157 kg.)

Question 3.

- a. The radius is half the diameter: $1.8 \div 2 = 0.9$ m.

b. $V = \pi r^2 h$

$$V = \pi \times 0.9^2 \times 1.5$$

$$0.9^2 = 0.81$$

$$\pi \times 0.81 \times 1.5 = 3.81703\dots$$

The third decimal is 7, so round the second decimal up: $V = 3.82$ m³ correct to two decimal places, as required.

- c. $3.81703\dots \times 1000 = 3817.03\dots$ Correct to the nearest litre: 3817 L. (The rounded volume 3.82 would give 3820 L — this is why the question asks you to use the full calculator value.)

Question 4.

- a. From 22:45 to 24:00 is 1 hour 15 minutes. From 00:00 to 07:15 is 7 hours 15 minutes. Total: 1 h 15 min + 7 h 15 min = 8 hours 30 minutes.
- b. The two breaks take $2 \times 15 = 30$ minutes and are not paid: 8 h 30 min – 30 min = 8 hours paid.
- c. Perth (UTC + 8) is 2 hours behind Melbourne (UTC + 10): $23:30 - 2:00 = 21:30$. The time in Perth is 21:30.
- d. The first day is day 1, so the last day is 11 days after Thursday 5 March. The date is $5 + 11 = 16$ March. Eleven days is 1 week and 4 days, so the day of the week is Thursday plus 4 days, which is Monday. The roster ends on Monday 16 March.

Question 5.

a. Rise = $6 - (-5) = 6 + 5 = 11$ °C.

b. Mean = $\frac{-5 + (-2) + 2 + 6 + 9}{5} = \frac{10}{5} = 2.0$ °C (correct to one decimal place).

c. Range = highest – lowest = $9 - (-5) = 14$ °C.

d. Cold food is safe below 5 °C. The readings at 12 pm (6 °C) and 2 pm (9 °C) are not below 5 °C, so the food was not being kept safe at 12 pm and 2 pm.

Question 6.

a. $0.7 \times 1000 = 700$ mm.

b. Lower limit: $760 - 0 = 760$ mm. Upper limit: $760 + 4 = 764$ mm. The accepted range is 760 mm to 764 mm.

c. The smallest division on the tape is 1 mm, so the true width could be up to half a millimetre below or above the reading: 762.5 mm to 763.5 mm.

d. Yes — Sam can be sure. Every possible true width, from 762.5 mm to 763.5 mm, sits inside the accepted range of 760 mm to 764 mm. Even at its widest and narrowest possible values the opening is definitely within tolerance, so the door will fit.

Question 7.

a. $8 \times 28.35 = 226.8$ g; correct to the nearest gram, 227 g. In kilograms: $226.8 \div 1000 = 0.2268$ kg; correct to two decimal places, 0.23 kg.

b. Radius = $30 \div 2 = 15$ cm. $A = \pi r^2 = \pi \times 15^2 = \pi \times 225 = 706.858 \dots$ Correct to the nearest cm²: 707 cm².

c. The rise needed is $180 - 24 = 156$ °C. At 12 °C per minute: $156 \div 12 = 13$ minutes.

d. $18:47 + 13$ minutes = 19:00, and $19:00 + 12$ more minutes = 19:12. Rosa takes the chips out at 19:12.