

VCE VM Numeracy Units 3 & 4

Chapter 3 — Shape

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Chapter 3 Shape — 3A Naming and classifying 2D shapes and 3D objects

Theory

About the data in this section. The signs, templates, tents, tins, gardens, quilts and measurements here are **made up** (not real): pretend objects and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Shapes are everywhere: road signs, picture frames, boxes, cans, tents, balls, tiles and quilts. This section is about **names** and **properties**. A property is a feature of a shape you can check — how many sides it has, whether sides are equal, whether corners are square. Classifying means sorting shapes into groups using their properties, then giving each shape its most exact name.

The study design asks for exactly this (Unit 3, Area of Study 2): know the names and properties of many flat shapes and solid objects (cones and pyramids, for example), and be able to describe them and classify them.

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-07.

Flat shapes are **two-dimensional (2D)**: they have length and width, and you can draw them on paper. Solid objects are **three-dimensional (3D)**: they have length, width and height, and you can hold them. Later sections build on this one: 3B looks at angles in shapes, 3C at symmetry, 3D at same-shape and scaled shapes, and 3E at compound shapes. Here we only name and classify.

Words we use.

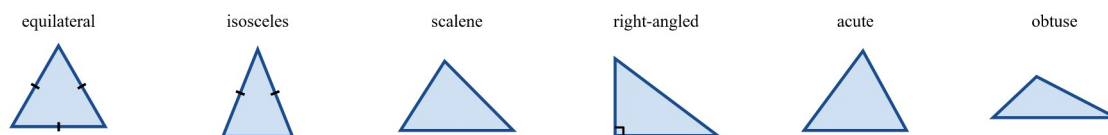
- **side** — one straight line segment making up the boundary of a flat shape.
- **vertex** (plural **vertices**) — a corner: the point where two sides meet in 2D, or where edges meet in 3D.
- **edge** — in 3D, the line segment where two faces meet.
- **face** — in 3D, one flat or curved surface on the outside of a solid object.
- **parallel** — two sides are parallel if they stay the same distance apart and never meet. We mark parallel sides with matching arrow marks.
- **right angle** — a square corner, an angle of 90° . We mark it with a small square.
- **equal sides** — sides of the same length. We mark them with matching tick marks.
- **adjacent** — next to each other: adjacent sides meet at a vertex.
- **polygon** — a closed flat shape with only straight sides.
- **quadrilateral** — a polygon with 4 sides.
- **regular** — a polygon with all sides equal and all angles equal. Any other polygon is **irregular**.
- **cross-section** — the shape of the slice you see when you cut straight through a solid object.
- **polyhedron** — a 3D object whose surfaces are all flat faces (no curved surface). The plural is polyhedra.
- **apex** — the single point where the sloping faces of a pyramid or cone meet.
- **classify** — to sort objects into groups using their properties.

Naming polygons by sides.

Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon

A **regular** polygon passes two tests: all of its sides are equal, and all of its angles are equal. A stop-sign shape passes both, so it is a regular octagon. Stretch one side of a hexagon and it fails the first test, so it is an irregular hexagon.

Triangles.



The first three are named by their sides. The last three are named by their angles.

Triangles are named by their sides, or by their angles.

Named by sides

Property

equilateral

3 equal sides

isosceles

at least 2 equal sides

scalene

no equal sides

Named by angles

Property

right-angled

one right angle (the small square mark)

acute

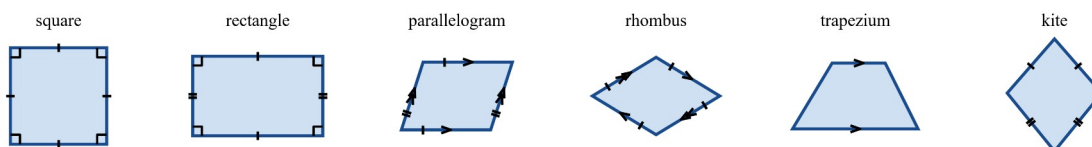
all three angles smaller than a right angle

obtuse

one angle bigger than a right angle

The two naming sets can combine: one triangle can be right-angled and isosceles at the same time. Measuring and calculating angles is Section 3B; here we only read the marks and name the triangle.

Quadrilaterals.



Marks show the properties: | equal sides > parallel sides L right angles

Shape

Properties

square

4 equal sides, 4 right angles, 2 pairs of parallel sides

rectangle

opposite sides equal, 4 right angles, 2 pairs of parallel sides

parallelogram

opposite sides equal and parallel

rhombus

4 equal sides, opposite sides parallel

trapezium

exactly one pair of parallel sides

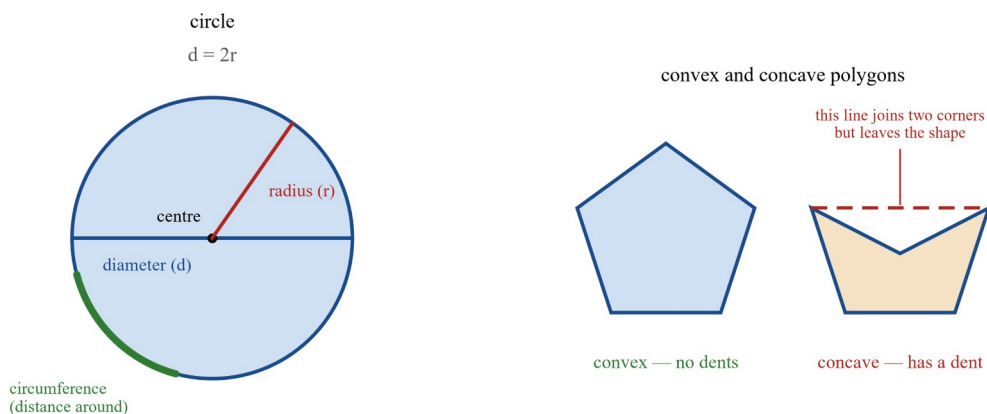
kite

2 pairs of adjacent equal sides

Notice the family links. A square is a special rectangle (a rectangle with all four sides equal) and also a special rhombus (a rhombus with four right angles). So every square is a rectangle, but not every rectangle is a square.

Book convention. In this book a trapezium has *exactly* one pair of parallel sides. A shape with two pairs of parallel sides is called a parallelogram here, not a trapezium.

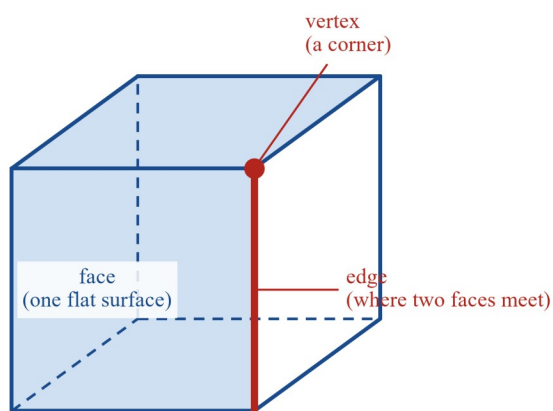
Circles, and dents.



A circle is named by four things: its **centre** (the middle point); its **radius** r (a segment from the centre to the circle — all radii of one circle are equal); its **diameter** d (a segment from one side of the circle to the other, through the centre); and its **circumference** (the distance around the circle). The diameter is always twice the radius: $d = 2r$. For example, a circle of radius 5 cm has diameter $2 \times 5 = 10$ cm. A circle is not a polygon: its boundary is curved, not straight.

A polygon is **convex** if it has no dents pointing in. It is **concave** if it has at least one dent. A quick test: if you can join two corners with a straight segment that leaves the shape (the dashed segment in the figure), the shape is concave.

Solid objects: faces, edges and vertices.

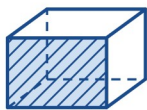


A cube has 6 faces, 12 edges and 8 vertices.

Solid objects are counted and classified with three words: **faces** (the flat or curved surfaces), **edges** (where two faces meet) and **vertices** (the corners, where edges meet). A cube has 6 faces, 12 edges and 8 vertices.

Prisms.

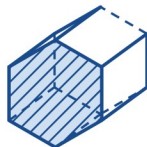
rectangular prism
(a box)



triangular prism



hexagonal prism



cylinder

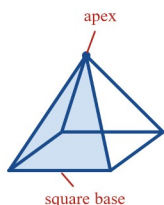


The shaded end is the cross-section. A prism has the same cross-section all the way through.

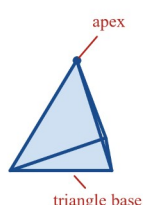
A **prism** is a polyhedron with the same cross-section all the way through. Prisms are named by that cross-section: a triangular prism, a hexagonal prism, and so on. A box is a rectangular prism, and a cube is a special rectangular prism with all edges equal. A cylinder also has the same cross-section (a circle) all the way through, but its side is curved — so a cylinder is not a polyhedron.

Pyramids, cones and spheres.

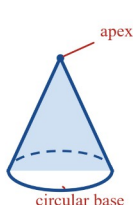
square-based
pyramid



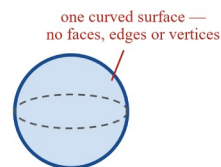
triangular-based
pyramid



cone



sphere



A **pyramid** has a polygon base and triangular faces that rise to one apex. Pyramids are named by their base: a square-based pyramid, a triangular-based pyramid. A **cone** has a circular base and one curved surface rising to an apex. A **sphere** (a ball shape) has one curved surface and no flat faces, no edges and no vertices.

Counting faces, edges and vertices.

Object	Faces	Edges	Vertices
cube	6	12	8
rectangular prism (box)	6	12	8
triangular prism	5	9	6
pentagonal prism	7	15	10
hexagonal prism	8	18	12
triangular-based pyramid	4	6	4
square-based pyramid	5	8	5
cylinder	2 flat + 1 curved	2	0
cone	1 flat + 1 curved	1	1
sphere	0 flat + 1 curved	0	0

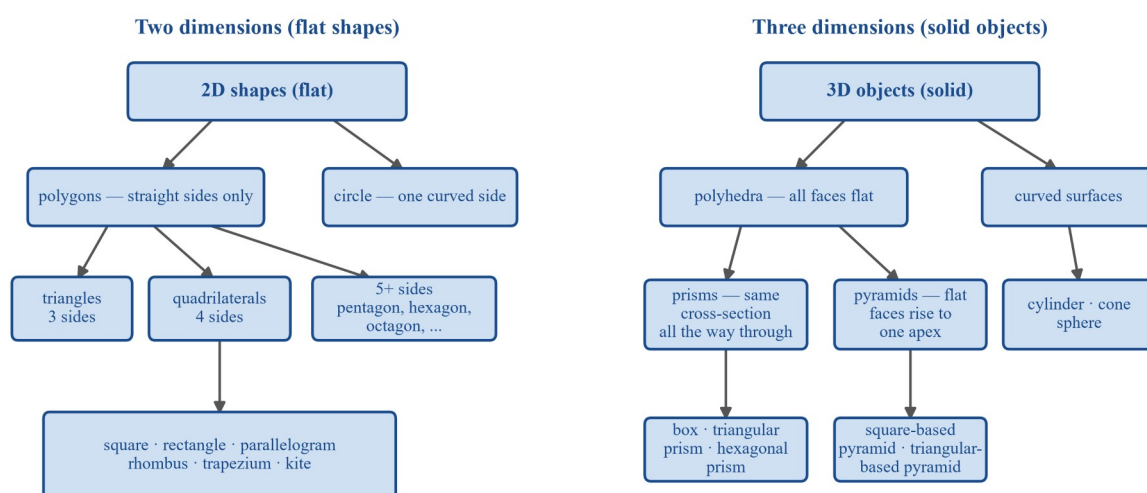
Counting tips:

- Count in an order (front then back, top then bottom) so nothing is missed or doubled.
- Hidden edges and vertices still count. In our drawings, hidden edges are dashed.

- A prism whose cross-section is an n -sided polygon has $n + 2$ faces, $3n$ edges and $2n$ vertices. Check with the hexagonal prism: $6 + 2 = 8$ faces, $3 \times 6 = 18$ edges, $2 \times 6 = 12$ vertices.
- A pyramid whose base is an n -sided polygon has $n + 1$ faces, $2n$ edges and $n + 1$ vertices. Check with the square-based pyramid: $4 + 1 = 5$ faces, $2 \times 4 = 8$ edges, $4 + 1 = 5$ vertices.

How to classify any shape.

1. Flat or solid? Flat shapes are 2D; solid objects are 3D.
2. If flat: are all sides straight? If yes, it is a polygon — count the sides for the family name, then check equal sides, equal angles and right angles for the most exact name. If the boundary is curved, it is a circle or another curved shape.
3. If solid: are all faces flat? If yes, it is a polyhedron — same cross-section all the way through means a prism (name it by the cross-section); flat faces rising to one apex means a pyramid (name it by the base). If there is a curved surface, decide between cylinder, cone and sphere.



Regular polygons have all sides equal and all angles equal.

A cylinder has a constant cross-section like a prism, but its surface is curved — it is not a polyhedron.

Everyday names and maths names.

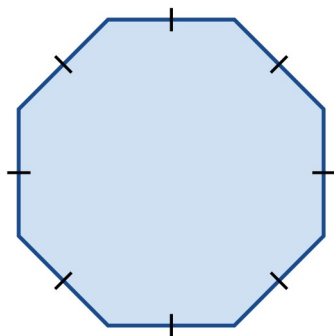
Everyday name	Mathematical name
ball	sphere
box, carton	rectangular prism
dice	cube
can, drum, pipe	cylinder
party hat, traffic cone, funnel	cone
ridged tent	triangular prism
gift box with a pointy lid	square-based pyramid

Model. Classify a dice. It is solid, so 3D. All of its faces are flat squares, so it is a polyhedron. The cross-section is the same square all the way through, so it is a prism named by that square: a rectangular prism with all edges equal — a cube. The most exact name for a dice is a cube.

Worked examples

Example 1 — scaffolded

A sign maker cuts a template for a warning sign (shown below). The template has 8 sides. Every side is the same length, and every angle is the same size. Give the most exact name for the shape of the template.



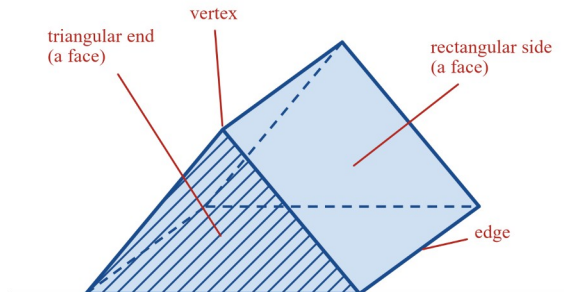
the template (all 8 sides equal)

Made-up data: the template and its measurements are made up for this section.

Working	Comment
Count the sides: 8	An 8-sided polygon is an octagon.
Check the two regular tests: all 8 sides equal, all 8 angles equal	Both tests pass, so the polygon is regular.
Most exact name: regular octagon	“Polygon” and “octagon” are true but less exact.
The template is a regular octagon.	

Example 2 — scaffolded

A camping tent (shown below) is a triangular prism: the two ends are triangles, and the two slopes and the floor are rectangles. Count the faces, edges and vertices of the tent, and classify it.



The tent is a triangular prism.

Made-up data: the tent and its measurements are made up for this section.

Working	Comment
Faces: 2 triangular ends + 3 rectangles = 5	Count the floor rectangle too — it is a face of the solid.
Edges: 3 on each triangle + 3 joining the triangles = 9	The 3 long edges join matching corners of the two ends.
Vertices: 3 on each triangle = 6	No other corners exist.
Same triangular cross-section all the way through	So it is a prism, named by its cross-section.

The tent has 5 faces, 9 edges and 6 vertices, and it is a triangular prism.

Example 3 — unscaffolded

A window panel is a quadrilateral. It has 2 pairs of parallel sides and 4 equal sides, but none of its corners are square corners. Give the most exact name for the shape of the panel.

Read the properties in order. Two pairs of parallel sides put the panel in the parallelogram family. Four equal sides narrow it to a rhombus or a square. A square needs four right angles, and the panel has none, so the square is ruled out.

∴ the window panel is a rhombus.

Example 4 — unscaffolded

Tia says: “Every square is a rectangle, but not every rectangle is a square.” Marco disagrees: he says squares and rectangles are two separate families with nothing in common. Who is correct? Explain using properties and one example.

Test Tia’s claim against the rectangle properties: 4 sides, 4 right angles, and opposite sides equal. A square has 4 right angles, and because all 4 of its sides are equal, its opposite sides are certainly equal. So every square passes the rectangle test: every square is a rectangle. Now test Marco’s claim with one example: a rectangle 5 cm by 3 cm has four right angles and equal opposite sides, so it is a rectangle — but its adjacent sides are not equal ($5 \neq 3$), so it fails the square test. That one rectangle is not a square, so the families are not identical.

∴ Tia is correct: squares sit inside the rectangle family, and a 5 cm by 3 cm rectangle shows the family is bigger than the squares.

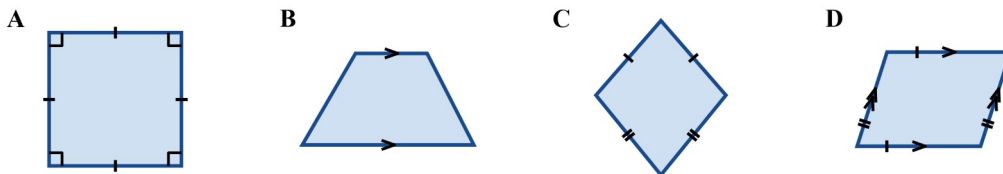
Practice questions

Scaffolded

Q1. Name each polygon from its number of sides. (a) 5 sides (b) 6 sides (c) 10 sides

Q2. A triangle has sides 7 cm, 7 cm and 12 cm. (a) Name the triangle by its sides. (b) Explain why it is not an equilateral triangle. (c) A classmate calls it a scalene triangle. Explain why they are wrong.

Q3. The four quadrilaterals below are drawn with property marks.



- (a) Name shapes A and D.
- (b) One shape has exactly one pair of parallel sides. Give its letter and its name.
- (c) One shape has two pairs of adjacent equal sides. Give its letter and its name.

Q4. A round cake tin has a radius of 12 cm. (a) Find its diameter. (b) Write down the rule that connects the diameter and the radius, in words. (c) A second tin has a diameter of 18 cm. Find its radius.

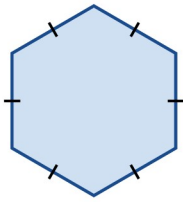
Q5. A shop display stand is a pentagonal prism: its cross-section is a pentagon (5 sides). (a) How many faces does it have? (b) How many edges does it have? (c) How many vertices does it have?

Q6. Match each object with its mathematical name: soccer ball, party hat, drain pipe, shoebox, dice. The names are: sphere, cone, cylinder, rectangular prism, cube. (a) Write the five matches. (b) Which two of these objects are polyhedra? (c) One of the objects is a special case of another object’s shape. Which one, and why?

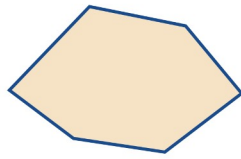
Un scaffolded

Q7. Two shapes are drawn below.

A



B

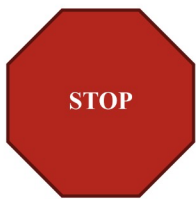


- (a) Both shapes have 6 sides. Give their polygon name.
- (b) Shape A is regular. State the two tests a polygon must pass to be regular.
- (c) Give the most exact name for shape B.

Q8. A quadrilateral has 2 pairs of parallel sides and 4 right angles. Its adjacent sides are not equal. (a) Give the most exact name for the shape. (b) Explain why it is not a square. (c) Is the shape a parallelogram? Explain.

Q9. Say whether each statement is true or false, and explain why. (a) Every rhombus is a parallelogram. (b) Every kite is a rhombus. (c) Every square is a rhombus.

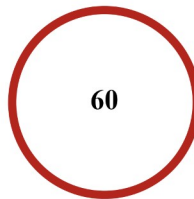
Q10. Three road signs are drawn below.



stop sign



give way sign



speed sign

- (a) Name the shape of the outline of each sign.
- (b) Which of those outlines are regular polygons? Explain.
- (c) Explain why the outline of the speed sign is not a polygon.

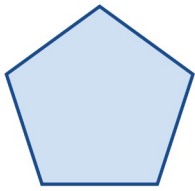
Q11. A gift box is a square-based pyramid. (a) How many faces does it have? (b) How many edges does it have? (c) How many vertices does it have? (d) How many triangular faces meet at the apex?

Q12. Consider these five objects: cylinder, triangular prism, cone, sphere, cube. (a) Which of them are polyhedra? (b) Which has exactly one curved surface and one flat face? (c) Which has no edges and no vertices?

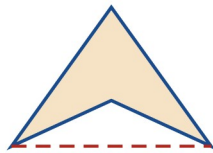
Q13. On a farm there is a grain silo (round, with straight sides and a flat top), a site hut shaped like a box, and a funnel that narrows to a point. (a) Give the mathematical name of each object. (b) A length of pipe lies nearby. Which of the three objects is the pipe in the same family as? (c) Which of the three objects is in the same family as a shoebox?

Q14. Two shapes are drawn below. In shape B, the dashed segment joins two corners.

A



B



the dashed line leaves the shape

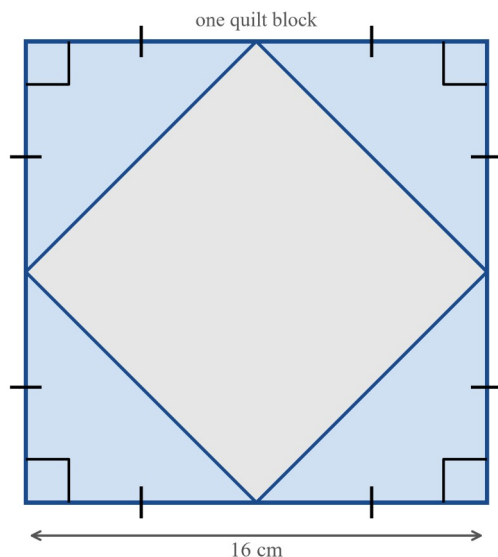
- Which shape is concave?
- Explain how the dashed segment supports your answer.
- Is a regular hexagon convex or concave? Explain.

Q15. A circular garden bed has a diameter of 2.4 m. A sprinkler standing at the centre of the bed wets a circle of radius 1.2 m. (a) Find the radius of the garden bed. (b) Does the sprinkler reach the edge of the bed? Explain. (c) Give the mathematical name for the distance around the edge of the bed.

Q16. A steel rail has the same rectangular cross-section all the way through. A pipe has the same circular cross-section all the way through. (a) Name the shape of the rail. (b) Name the shape of the pipe. (c) Explain why the pipe is not a polyhedron, even though its cross-section never changes.

Q17. Say whether each statement is true or false, and explain why. (a) A cone is a polyhedron. (b) Every rectangle is a parallelogram. (c) A cylinder has exactly 2 edges.

Q18. A quilter makes the square block below. The outer square has sides of 16 cm. The middle shape joins the midpoints of the four outer sides, and the four corners are triangles. The marks show two equal sides and a right angle in each corner triangle.



Made-up data: the quilt block and its measurements are made up for this section.

- How many corner triangles are there? Name one of them by its sides and by its angles.
- Name the middle shape and explain your answer, even though it sits tilted.
- Find the length of each short side of a corner triangle.

Answers

Q1. (a) pentagon. (b) hexagon. (c) decagon. **Q2.** (a) isosceles. (b) an equilateral triangle has 3 equal sides; here only 2 sides are equal (7 cm and 7 cm, with the third 12 cm). (c) a scalene triangle has no equal sides; this triangle has two equal sides of 7 cm. **Q3.** (a) A square, D parallelogram. (b) B, trapezium. (c) C, kite. **Q4.** (a) 24 cm. (b) the diameter is twice the radius ($d = 2r$). (c) 9 cm. **Q5.** (a) 7. (b) 15. (c) 10. **Q6.** (a) soccer ball → sphere; party hat → cone; drain pipe → cylinder; shoebox → rectangular prism; dice → cube. (b) the shoebox and the dice. (c) the dice: a cube is a rectangular prism with all edges equal. **Q7.** (a) hexagon. (b) all sides equal, and all angles equal. (c) irregular hexagon. **Q8.** (a) rectangle. (b) a square needs 4 equal sides; here adjacent sides are not equal. (c) yes — 2 pairs of parallel sides is exactly the parallelogram test. **Q9.** (a) true — a rhombus has 2 pairs of parallel sides. (b) false — a kite has two pairs of adjacent equal sides, not four equal sides. (c) true — a square has 4 equal sides and 2 pairs of parallel sides. **Q10.** (a) octagon, triangle, circle. (b) the stop sign and the give way sign: all sides equal and all angles equal. (c) its boundary is curved; a polygon has only straight sides. **Q11.** (a) 5. (b) 8. (c) 5. (d) 4. **Q12.** (a) the triangular prism and the cube. (b) the cone. (c) the sphere. **Q13.** (a) cylinder, rectangular prism, cone. (b) the silo (both are cylinders). (c) the site hut (both are rectangular prisms). **Q14.** (a) B. (b) the segment joins two corners but leaves the shape, which only happens with a dent. (c) convex — a regular polygon has no dents, so no such segment exists. **Q15.** (a) 1.2 m. (b) yes — the wet radius (1.2 m) equals the bed's radius, so the water reaches exactly the edge. (c) circumference. **Q16.** (a) rectangular prism. (b) cylinder. (c) its side is curved; a polyhedron has only flat faces. **Q17.** (a) false — a cone has a curved surface, and polyhedra have only flat faces. (b) true — a rectangle has 2 pairs of parallel sides. (c) true — the curved surface meets each flat end in one circular edge, giving 2 edges. **Q18.** (a) 4; a right-angled isosceles triangle (2 equal sides and 1 right angle). (b) a square — 4 equal sides and 4 right angles; tilting does not change the name. (c) 8 cm.

Fully-worked solutions

Q1. Count the sides and read the name from the polygon table. (a) 5 sides: **pentagon**. (b) 6 sides: **hexagon**. (c) 10 sides: **decagon**.

Q2. (a) Two sides are the same length (7 cm and 7 cm), so the triangle is **isosceles**. (b) An equilateral triangle has all 3 sides equal. Here the third side is 12 cm, not 7 cm, so only 2 sides are equal. (c) A scalene triangle has no equal sides at all. This triangle has a pair of equal sides (7 cm and 7 cm), so it cannot be scalene.

Q3. (a) Shape A has 4 equal sides and 4 right angles: a **square**. Shape D has 2 pairs of parallel sides and equal opposite sides, with no right angles: a **parallelogram**. (b) Shape B has exactly one pair of parallel sides (one arrow on each of two sides): a **trapezium**. (c) Shape C has two pairs of adjacent equal sides (single ticks on the top pair, double ticks on the bottom pair): a **kite**.

Q4. (a) The diameter is twice the radius: $2 \times 12 = 24$ cm. (b) The diameter is twice the radius; in symbols, $d = 2r$. Equally, the radius is half the diameter. (c) The radius is half the diameter: $18 \div 2 = 9$ cm.

Q5. A pentagonal prism has a 5-sided cross-section, so $n = 5$. (a) Faces: $n + 2 = 5 + 2 = 7$ (two pentagon ends and five rectangles). (b) Edges: $3n = 3 \times 5 = 15$. (c) Vertices: $2n = 2 \times 5 = 10$.

Q6. (a) Soccer ball → sphere; party hat → cone; drain pipe → cylinder; shoebox → rectangular prism; dice → cube. (b) Polyhedra have only flat faces: the **shoebox** (rectangular prism) and the **dice** (cube). The ball, hat and pipe all have curved surfaces. (c) The **dice**: a cube is a rectangular prism whose edges are all the same length, so the cube is a special case of the shoebox's shape.

Q7. (a) 6 sides means both shapes are **hexagons**. (b) The two regular tests: **all sides equal**, and **all angles equal**. The tick marks on shape A show the first test; a regular polygon must pass both. (c) Shape B has 6 sides but unequal sides, so it fails the regular tests: an **irregular hexagon**.

Q8. (a) Two pairs of parallel sides and four right angles, with adjacent sides unequal: a **rectangle**. (b) A square is a rectangle with 4 equal sides. Here the adjacent sides are not equal, so the square test fails. (c) Yes. A parallelogram needs 2 pairs of parallel sides, and this shape has them — a rectangle is a parallelogram with right angles.

Q9. (a) **True.** A rhombus has 4 equal sides and opposite sides parallel, so it has 2 pairs of parallel sides: that is the parallelogram test. (b) **False.** A kite has two pairs of *adjacent* equal sides; its four sides are not all equal, so it fails the rhombus test. (A kite with four equal sides would be a rhombus, but most kites are not.) (c) **True.** A square has 4 equal sides and 2 pairs of parallel sides, which is exactly the rhombus test. A square is a rhombus with right angles.

Q10. (a) Stop sign: 8 sides, an **octagon**. Give way sign: 3 sides, a **triangle**. Speed sign: a **circle**. (b) The **stop sign** (a regular octagon: 8 equal sides and 8 equal angles) and the **give way sign** (an equilateral triangle: 3 equal sides and 3 equal angles). Both pass the two regular tests. (c) A polygon has only straight sides. The speed sign's boundary is one curved line, so it is not a polygon at all — and therefore not a regular polygon either.

Q11. A square-based pyramid has a 4-sided base, so $n = 4$. (a) Faces: $n + 1 = 4 + 1 = 5$ (the square base and four triangles). (b) Edges: $2n = 2 \times 4 = 8$ (four base edges and four sloping edges). (c) Vertices: $n + 1 = 4 + 1 = 5$ (four base corners and the apex). (d) One triangular face sits on each base side, so 4 triangular faces meet at the apex.

Q12. (a) Polyhedra have only flat faces: the **triangular prism** and the **cube**. The cylinder, cone and sphere each have a curved surface. (b) The **cone**: one curved surface and one flat circular face. (c) The **sphere**: one curved surface, with no edges and no vertices anywhere.

Q13. (a) Silo: round with straight sides and flat ends, same circular cross-section all the way through — a **cylinder**. Site hut: box-shaped — a **rectangular prism**. Funnel: circular opening narrowing to a point — a **cone**. (b) The pipe has the same circular cross-section all the way through, like the **silo**: both are cylinders. (c) The **site hut**: like a shoebox, it is a rectangular prism.

Q14. (a) Shape **B** is concave (it has a dent pointing in); shape **A** is convex. (b) The dashed segment joins two corners of **B** but runs outside the shape. That can only happen when the shape dents in between those corners, so **B** is concave. In shape **A**, every segment joining two corners stays inside. (c) **Convex.** A regular polygon has all sides and all angles equal, so it cannot dent in; no segment joining two of its corners ever leaves the shape.

Q15. (a) The radius is half the diameter: $2.4 \div 2 = 1.2$ m. (b) Yes. The sprinkler wets a circle of radius 1.2 m from the centre, and the bed's radius is also 1.2 m, so the water reaches exactly the edge of the bed. (Check: the wet circle's diameter is $2 \times 1.2 = 2.4$ m, the same as the bed's diameter.) (c) The distance around the edge of a circle is called the **circumference**.

Q16. (a) Same rectangular cross-section all the way through, all faces flat: a **rectangular prism**. (b) Same circular cross-section all the way through: a **cylinder**. (c) A polyhedron must have only flat faces. The pipe's side is one curved surface, so the pipe is not a polyhedron — a constant cross-section alone is not enough.

Q17. (a) **False.** A cone has a curved surface (and a circular base), and polyhedra have only flat faces. (b) **True.** A rectangle has opposite sides equal and parallel, so it has 2 pairs of parallel sides: the parallelogram test. (c) **True.** An edge is where two faces meet. The curved surface of a cylinder meets the top flat face in one circular edge and the bottom flat face in another: 2 edges in total (and no vertices, since no corners exist).

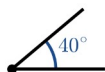
Q18. (a) There are 4 corner triangles, one at each corner of the block. The marks show two equal sides and one right angle in each, so each is a **right-angled isosceles triangle** (isosceles by sides, right-angled by angles). (b) The middle shape is a **square**. Its four sides are equal (each is drawn the same way, between the midpoints of matching sides of the block), and each of its corners is a right angle — check with the square corner of a piece of paper if needed. A square is still a square when it sits tilted like this. (c) Each short side runs from a corner of the block to the midpoint of a side, so it is half of 16 cm: $16 \div 2 = 8$ cm.

Chapter 3 Shape — 3B Angles in shapes

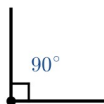
Theory

Every shape in this section is built from straight sides, and wherever two sides meet they make an **angle**. An angle is the amount of turn between the two sides, measured at the corner point where they meet. That corner point is called the **vertex** of the angle.

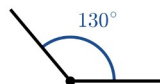
Angles are measured in **degrees**. A full turn around a point is split into 360 equal steps of turn, and one step is one degree, written 1° . So a quarter of a turn is $360^\circ \div 4 = 90^\circ$, and a half turn is 180° .



acute
(less than 90°)



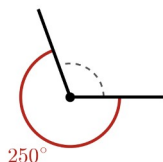
right angle
(exactly 90°)



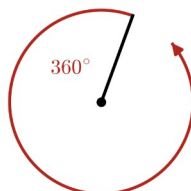
obtuse
(between 90° and 180°)



straight angle
(exactly 180°)



reflex
(between 180° and 360°)

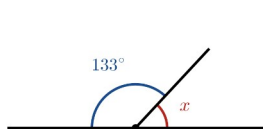


full turn
(exactly 360°)

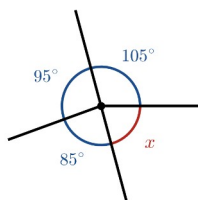
The six angle types.

- **Acute angle:** between 0° and 90° .
- **Right angle:** exactly 90° — a quarter turn, marked with a small square.
- **Obtuse angle:** between 90° and 180° .
- **Straight angle:** exactly 180° — a half turn, which looks like a straight line.
- **Reflex angle:** between 180° and 360° .
- **Full turn:** exactly 360° — all the way around, back to the start.

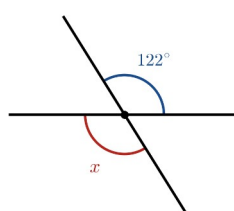
Three facts about turns.



angles on a straight line
add to 180°



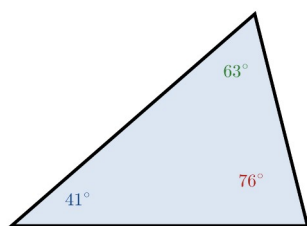
angles at a point
add to 360°



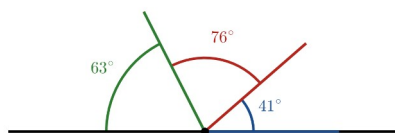
vertically opposite angles
are equal

- **Angles on a straight line add to 180° .** If one of the two angles is 133° , the other is $180^\circ - 133^\circ = 47^\circ$.
- **Angles at a point add to 360° .** In the middle picture, $105^\circ + 95^\circ + 85^\circ = 285^\circ$, so $x = 360^\circ - 285^\circ = 75^\circ$.
- **Vertically opposite angles are equal.** When two straight lines cross, the two angles facing each other across the crossing are called **vertically opposite angles**, and they are always equal: in the right-hand picture, $x = 122^\circ$. (The angle next to it is $180^\circ - 122^\circ = 58^\circ$.)

The angles of a triangle add to 180° .



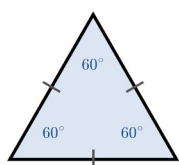
the three corners of any triangle



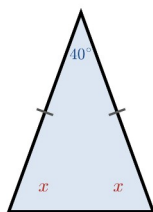
$41^\circ + 76^\circ + 63^\circ = 180^\circ$ — the three corners together make a straight line

If two angles of a triangle are 41° and 76° , they use $41^\circ + 76^\circ = 117^\circ$ of the 180° , which leaves $180^\circ - 117^\circ = 63^\circ$ for the third angle. The picture on the right shows why the rule is true: cut the three corners off any triangle, lay them together on a straight line, and they fit exactly into a half turn.

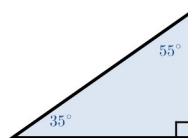
Three special triangles.



equilateral triangle:
all sides equal, all angles 60°



isosceles triangle: two equal sides
face two equal angles
 $x = (180^\circ - 40^\circ) \div 2 = 70^\circ$



right-angled triangle:
the two acute angles add to 90°

- An **equilateral triangle** has three equal sides, and its three angles are equal too: $180^\circ \div 3 = 60^\circ$ each.
- An **isosceles triangle** has two equal sides. The two angles at the base (the angles facing those equal sides) are equal as well: if the top angle is 40° , the base angles share $180^\circ - 40^\circ = 140^\circ$, so each is $140^\circ \div 2 = 70^\circ$.
- A **right-angled triangle** has one right angle. The other two angles are acute, and together they add to 90° : $35^\circ + 55^\circ = 90^\circ$.

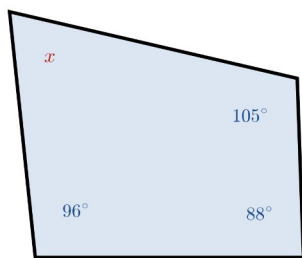
The exterior angle of a triangle.

Extend one side of a triangle past a corner. The angle outside the triangle, between the extended side and the next side, is called an **exterior angle**. An exterior angle equals the sum of the two interior angles far away from it:

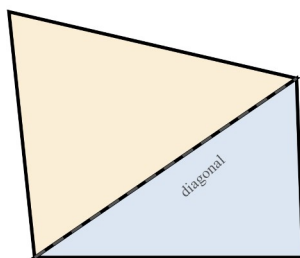
$43^\circ + 58^\circ = 101^\circ$. (Check: the interior angle next to it is $180^\circ - 101^\circ = 79^\circ$, and $43^\circ + 58^\circ + 79^\circ = 180^\circ$, as a triangle must.)

The angles of a quadrilateral add to 360° .

A **quadrilateral** is any flat shape with four straight sides. A **diagonal** is a straight line joining two opposite corners of the shape. One diagonal splits any quadrilateral into two triangles, so its four angles add to $180^\circ + 180^\circ = 360^\circ$.



a quadrilateral (any 4-sided 2D shape)



one diagonal splits it into two triangles:
 $180^\circ + 180^\circ = 360^\circ$

If three of the angles are 96° , 88° and 105° , they use $96^\circ + 88^\circ + 105^\circ = 289^\circ$, which leaves $x = 360^\circ - 289^\circ = 71^\circ$.

Special quadrilaterals and their angles.

Shape	Angle facts
Square — four equal sides and four right angles	every angle is 90°
Rectangle — opposite sides equal and parallel	every angle is 90°
Parallelogram — both pairs of opposite sides parallel	opposite angles are equal; angles next to each other add to 180°
Rhombus — a parallelogram with four equal sides	the same angle facts as a parallelogram
Trapezium — exactly one pair of parallel sides	each pair of angles sitting between the parallel sides adds to 180°
Kite — two pairs of equal sides sitting next to each other	one pair of opposite angles is equal

A plan for chasing angles.

1. Mark every angle you know on a sketch of the shape.
2. Choose the fact that fits: straight line, point, vertically opposite, triangle, quadrilateral, or one of the special shapes.
3. Write one step per line, and name the fact you used.
4. When every angle is known, add them all up as a check (180° for a triangle, 360° for a quadrilateral or a point).
5. Compare with the sketch: an angle that looks sharp and narrow should come out under 90° .

Naming triangles by their angles.

A triangle is **acute-angled** if all three of its angles are acute, **right-angled** if one angle is a right angle, and **obtuse-angled** if one angle is obtuse. The biggest angle decides the name.

You do not need a **protractor** — the clear half-circle tool used to measure angles by eye — anywhere in this section. Every angle here is worked out from the facts above instead. (Measuring and drawing angles with a protractor is a toolkit skill from §1E, and it returns in Unit 4.)

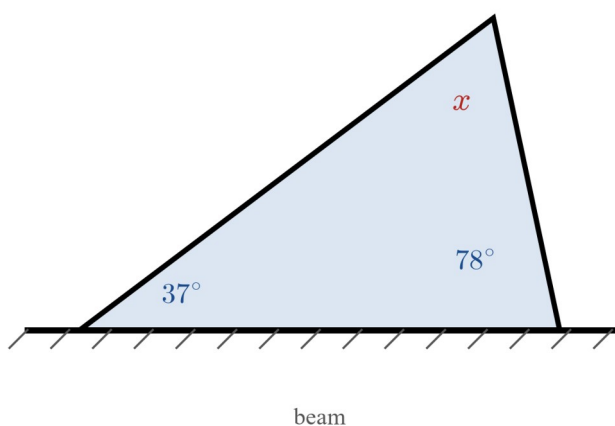
About the data in this section. The frames, gardens, paths, quilts and mosaics here are **made up** (not real): pretend projects and pretend measurements, built for this book so you can practise the maths. Nothing here is real data about a real place, person or business.

Worked examples

Example 1 — scaffolded

A shed roof frame is a triangle. Two of its angles are 37° and 78° . Find the third angle, x , and name the triangle by its angles.

Working	Comment
Add the known angles: $37^\circ + 78^\circ = 115^\circ$.	Put together what is given.
State the fact: the three angles of a triangle add to 180° .	Choose the fact that fits the shape.
Solve: $x = 180^\circ - 115^\circ = 65^\circ$.	Take the known part away from 180° .
Check: $37^\circ + 78^\circ + 65^\circ = 180^\circ$ ✓.	Put all three angles back — they fill the triangle exactly.
Name: all three angles are under 90° , so the frame is an acute-angled triangle.	The biggest angle decides the name.



Example 2 — scaffolded

A wall mural is painted in the shape of a parallelogram. One corner angle is 112° . Find the other three angles, x , y and z .

Working	Comment
State the facts: in a parallelogram, opposite angles are equal, and angles next to each other add to 180° .	Read the angle facts off the table.

Find x : $x = 180^\circ - 112^\circ = 68^\circ$.

x sits next to the 112° angle, sharing a side.

Find y : $y = 112^\circ$.

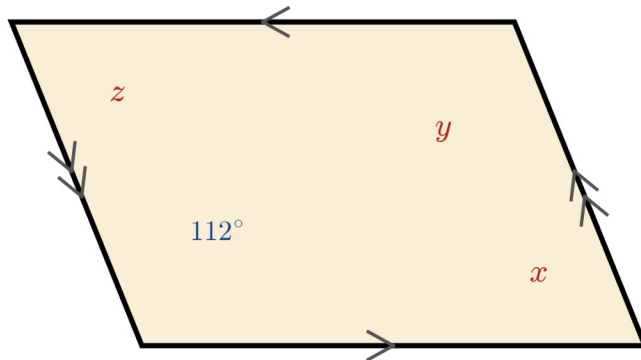
y is opposite the given angle, so it is equal to it.

Find z : $z = 68^\circ$.

z is opposite x , so it is equal to x .

Check: $112^\circ + 68^\circ + 112^\circ + 68^\circ = 360^\circ \checkmark$.

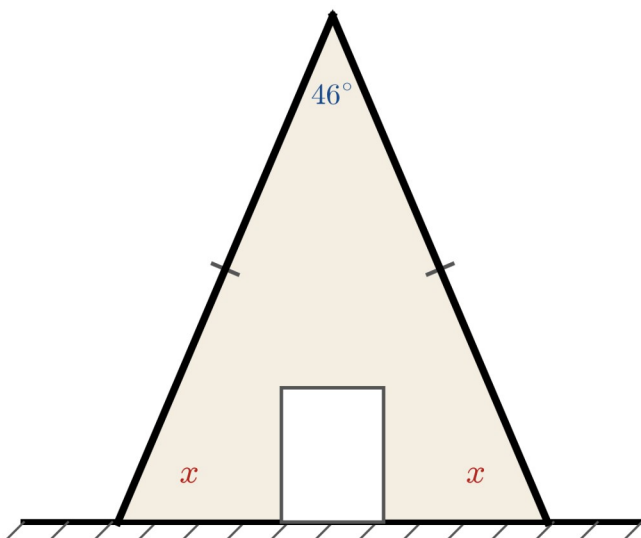
The four angles of any quadrilateral add to 360° .



Example 3 — unscaffolded

A cubby house has an A-frame front: two boards of equal length lean together at the top, making an isosceles triangle. The angle at the top is 46° . Find the angle each board makes with the ground.

The two equal boards face the two equal base angles, so those base angles must share whatever is left of the 180° once the top angle is taken out: $180^\circ - 46^\circ = 134^\circ$. Shared equally, each base angle is $134^\circ \div 2 = 67^\circ$. Check: $46^\circ + 67^\circ + 67^\circ = 180^\circ \checkmark$, and 67° is acute, which matches the steeply leaning boards in the picture.

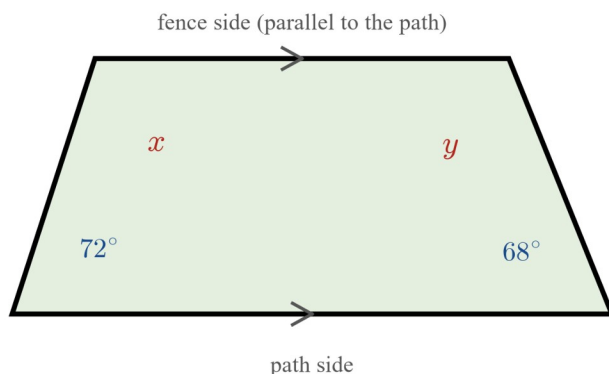


$\therefore$ each board makes an angle of 67° with the ground.

Example 4 — unscaffolded

A garden bed is a trapezium. The side along the path and the side along the fence are parallel. The two angles on the path side are 72° and 68° . Find the two angles on the fence side, x and y .

Each slant side of the trapezium joins the two parallel sides, so the two angles at its ends sit between the parallels and add to 180° . On the left, $x = 180^\circ - 72^\circ = 108^\circ$. On the right, $y = 180^\circ - 68^\circ = 112^\circ$. Check with the quadrilateral sum: $72^\circ + 68^\circ + 108^\circ + 112^\circ = 360^\circ$ ✓.



$\therefore$ the fence-side angles are $x = 108^\circ$ and $y = 112^\circ$.

Practice questions

Scaffolded

Q1. Name the type of each angle. (a) 23° , 90° , 137° , 180° , 265° . (b) How many degrees are there in a full turn? (c) Jot writes that 96° is an acute angle. Explain why Jot is wrong, and name the angle correctly.

Q2. Two angles sit together on a straight line. (a) What do the two angles add to? (b) One of the angles is 123° . Find the other. (c) A straight angle is cut into two equal angles. What size is each?

Q3. A raffle spinner is a disc divided into four sectors meeting at the centre point. (A **sector** is a slice of the disc, like a cut piece of pizza.) Three of the sector angles are 96° , 117° and 78° . (a) What do the four sector angles add to? (b) Find the fourth sector angle. (c) Is the fourth sector acute, right or obtuse?

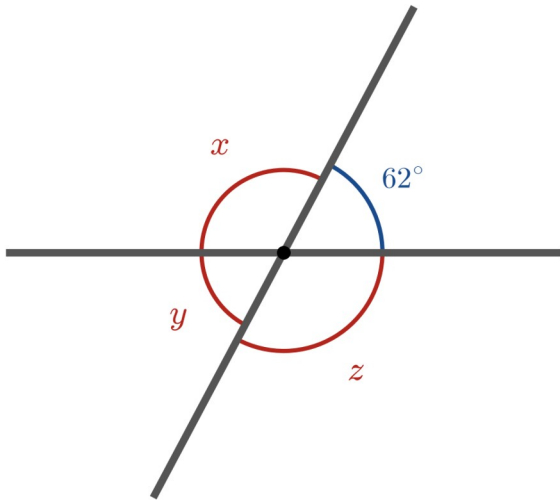
Q4. A triangular shade sail has angles of 52° and 38° . (a) Find the third angle. (b) Name the triangle by its angles. (c) What do the two acute angles add to? Which rule from the theory does this match?

Q5. A sandpit is a quadrilateral with three angles of 95° , 110° and 85° . (a) What do the four angles of a quadrilateral add to? (b) Find the fourth angle. (c) Could a quadrilateral have four angles that are all 100° ? Explain with a calculation.

Q6. A wall bracket is a right-angled triangle. Its vertical side sits against a wall and its horizontal side sits under a shelf, so the angle between those two sides is a right angle. The third, slanting side is a support brace; the angle between the brace and the wall is 33° . (a) What do the two acute angles of a right-angled triangle add to? (b) Find the angle between the brace and the shelf. (c) What is the angle between the wall and the shelf? Check that your two acute angles add to it.

Un scaffolded

Q7. Two straight paths cross in a park, making four angles at the crossing. One angle is 62° , as marked.



Find x , y and z , and check your three answers with the point rule.

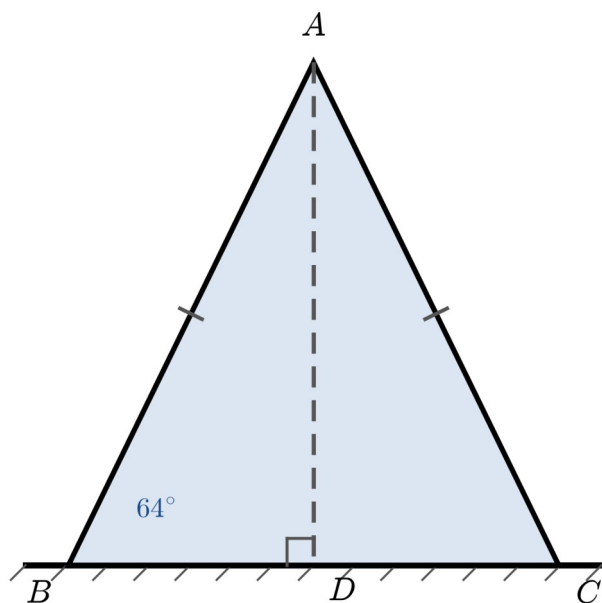
Q8. An isosceles triangle has a top angle of 34° . Show that each of the two equal base angles is 73° . Write one step per line.

Q9. A window is a rhombus. One of its angles is 104° . Find the other three angles, and check that all four add to 360° .

Q10. A planter box is an isosceles trapezium: its parallel sides are the top and the bottom, and its two slant sides are equal in length. The two bottom angles are 74° each. Find the two top angles, and check the four angles.

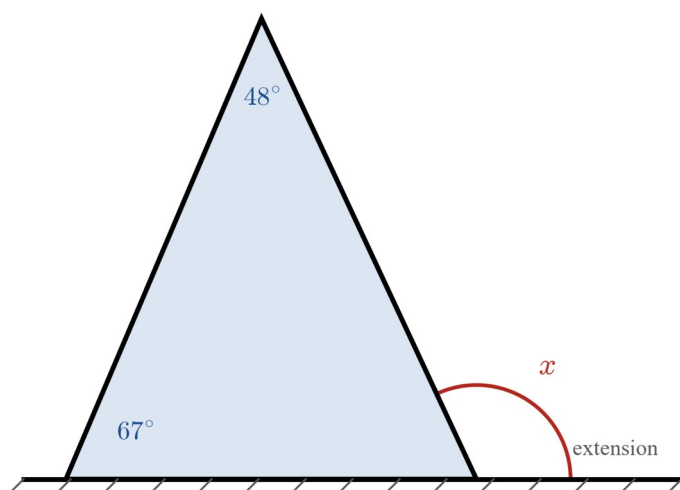
Q11. A carport roof frame is an isosceles triangle. The angle at the top is 50.5° , and the two equal slant sides (the **rafters**) make equal angles with the horizontal. (a) Find the sum of the two rafter angles. (b) Find each rafter angle, rounded to 1 decimal place. (c) Check your *exact*, unrounded rafter angles against the triangle angle sum.

Q12. A roof frame is an isosceles triangle ABC with $AB = AC$. A vertical post AD runs from the top corner A down to the point D on the horizontal beam BC . The angle ABC is 64° . (We write $\angle ABC$ for the angle at vertex B , between the sides BA and BC .)



- (a) Find $\angle ACB$.
- (b) Find $\angle BAC$.
- (c) Find $\angle ADB$.
- (d) Find $\angle BAD$.

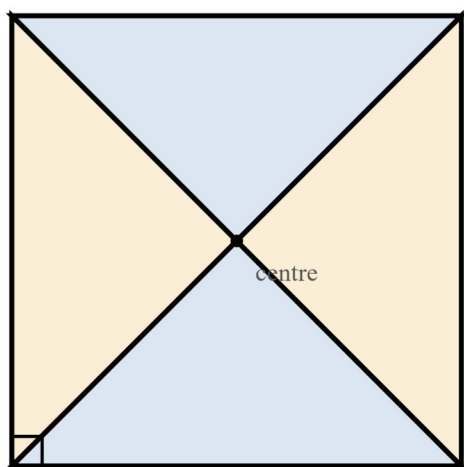
Q13. A playground climbing frame is a triangle. The ground line runs on past the front corner, as shown, and x is the exterior angle at that corner.



The top angle is 48° and the back bottom angle is 67° . (a) Use the exterior angle rule to find x . (b) Use a different method: first find the interior angle at the front corner, then use the straight ground line. (c) Do the two methods agree? Which method do you prefer, and why?

Q14. A student records the three angles of a triangle as 47° , 66° and 68° . Could these be the angles of a real triangle? Explain, and say how far out the record is.

Q15. A square quilt block has both of its diagonals sewn in, making four triangles that meet at the centre of the square.



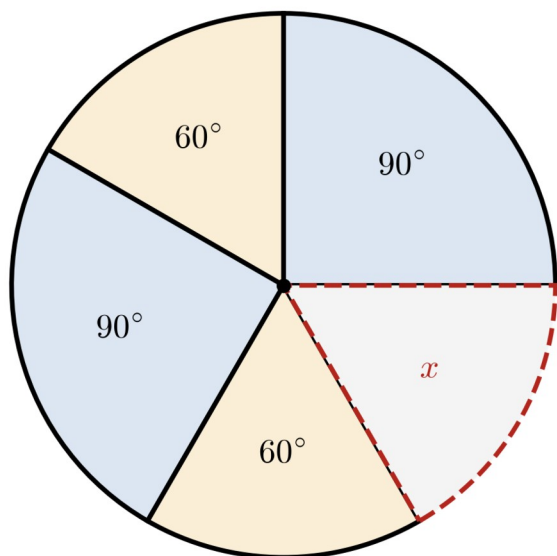
square block, both diagonals drawn

- (a) Each diagonal cuts a corner angle of the square in half. What size is each half?
- (b) Look at one of the four triangles: two of its angles are those halves. Find its third angle, at the centre.
- (c) Check your centre angle by adding the four centre angles around the middle point.

Q16. A kite has four angles. The top angle is 118° and the two side angles are 76° each. Find the bottom angle.

Q17. A picture frame is made by cutting the ends of four strips of timber at a slant of 45° (a slanted end cut like this is called a **mitre**) and joining two cuts together at each corner. (a) What corner angle do two 45° mitre cuts make when joined? (b) Check that four such corners close up into a full turn. (c) An apprentice cuts the ends at 40° instead. What corner angle do two of these cuts make, and how far is that corner from square?

Q18. A mosaic design (a pattern built from flat tiles) places, around one point: the corner of a square, the corner of an equilateral triangle, the corner of a second square, then the corner of a second equilateral triangle. A gap is left, as shown.



blue: corners of squares (90° each)
yellow: corners of equilateral triangles (60° each)

- (a) Add up the four corners already placed.
 - (b) Find the gap angle, x .
 - (c) Name a shape whose corner fits the gap exactly, and say why.
-

Answers

Q1. (a) 23° acute; 90° right angle; 137° obtuse; 180° straight angle; 265° reflex. (b) 360° . (c) 96° sits between 90° and 180° , so it is obtuse, not acute. **Q2.** (a) 180° . (b) 57° . (c) 90° each. **Q3.** (a) 360° . (b) 69° . (c) acute. **Q4.** (a) 90° . (b) right-angled. (c) 90° — the two acute angles of a right-angled triangle add to 90° . **Q5.** (a) 360° . (b) 70° . (c) no — four angles of 100° would add to $4 \times 100^\circ = 400^\circ$, but every quadrilateral adds to exactly 360° . **Q6.** (a) 90° . (b) 57° . (c) 90° ; and $33^\circ + 57^\circ = 90^\circ$ ✓. **Q7.** $x = 118^\circ$, $y = 62^\circ$, $z = 118^\circ$; check $62^\circ + 118^\circ + 62^\circ + 118^\circ = 360^\circ$ ✓. **Q8.** $180^\circ - 34^\circ = 146^\circ$ for the two base angles together; $146^\circ \div 2 = 73^\circ$ each; check $34^\circ + 73^\circ + 73^\circ = 180^\circ$ ✓. **Q9.** 76° , 104° , 76° ; check $104^\circ + 76^\circ + 104^\circ + 76^\circ = 360^\circ$ ✓. **Q10.** 106° each; check $74^\circ + 74^\circ + 106^\circ + 106^\circ = 360^\circ$ ✓. **Q11.** (a) 129.5° . (b) 64.8° to 1 decimal place (the exact value is 64.75°). (c) $64.75^\circ + 64.75^\circ + 50.5^\circ = 180^\circ$ ✓. **Q12.** (a) 64° . (b) 52° . (c) 90° . (d) 26° . **Q13.** (a) 115° . (b) interior angle 65° , so $x = 180^\circ - 65^\circ = 115^\circ$. (c) yes — both methods give 115° . **Q14.** no — $47^\circ + 66^\circ + 68^\circ = 181^\circ$, which is 1° over the 180° that a triangle allows. **Q15.** (a) 45° . (b) 90° . (c) $4 \times 90^\circ = 360^\circ$ ✓. **Q16.** 90° . **Q17.** (a) 90° . (b) $4 \times 90^\circ = 360^\circ$, a full turn ✓. (c) 80° , which is 10° off a square corner. **Q18.** (a) 300° . (b) 60° . (c) an equilateral triangle — each of its corner angles is 60° , exactly the gap.

Fully-worked solutions

Q1. (a) Compare each angle with 90° and 180° : 23° is acute, 90° is a right angle, 137° is obtuse, 180° is a straight angle, and 265° is reflex. (b) A full turn is 360° . (c) An acute angle is under 90° , but 96° is over 90° and under 180° , so 96° is obtuse. Jot's label is one type out.

Q2. (a) Angles on a straight line add to 180° . (b) $180^\circ - 123^\circ = 57^\circ$; check $123^\circ + 57^\circ = 180^\circ$ ✓. (c) $180^\circ \div 2 = 90^\circ$ each — two equal angles on a line are right angles.

Q3. (a) The four sectors meet at the centre point, so their angles add to 360° . (b) $96^\circ + 117^\circ + 78^\circ = 291^\circ$, so the fourth sector is $360^\circ - 291^\circ = 69^\circ$. (c) 69° is under 90° , so the fourth sector is acute.

Q4. (a) $52^\circ + 38^\circ = 90^\circ$, so the third angle is $180^\circ - 90^\circ = 90^\circ$. (b) One angle is a right angle, so the sail is a right-angled triangle. (c) $52^\circ + 38^\circ = 90^\circ$: this matches the rule that the two acute angles of a right-angled triangle add to 90° .

Q5. (a) The four angles of any quadrilateral add to 360° . (b) $95^\circ + 110^\circ + 85^\circ = 290^\circ$, so the fourth angle is $360^\circ - 290^\circ = 70^\circ$. (c) No. Four angles of 100° would add to $4 \times 100^\circ = 400^\circ$, but every quadrilateral adds to exactly 360° , so four obtuse angles of 100° cannot exist in one quadrilateral.

Q6. (a) The three angles add to 180° and one of them is the right angle (90°), so the two acute angles add to 90° . (b) $90^\circ - 33^\circ = 57^\circ$. (c) The wall is vertical and the shelf is horizontal, so they meet at 90° — that is the right angle of the triangle. Check: $33^\circ + 57^\circ = 90^\circ$ ✓.

Q7. x sits next to the 62° angle on one straight path, so $x = 180^\circ - 62^\circ = 118^\circ$. y is vertically opposite the 62° angle, so $y = 62^\circ$. z is vertically opposite x , so $z = 118^\circ$. Check at the point: $62^\circ + 118^\circ + 62^\circ + 118^\circ = 360^\circ$ ✓.

Q8. The two equal sides face the two base angles, so the base angles are equal to each other. Step 1: the top angle uses 34° of the 180° , leaving $180^\circ - 34^\circ = 146^\circ$ for the two base angles together. Step 2: the base angles are equal, so each is $146^\circ \div 2 = 73^\circ$. Check: $34^\circ + 73^\circ + 73^\circ = 180^\circ$ ✓. $\therefore$ each base angle is 73° , as required.

Q9. A rhombus is a parallelogram, so angles next to each other add to 180° and opposite angles are equal. Next to the 104° angle: $180^\circ - 104^\circ = 76^\circ$. Opposite the given angle: 104° . Opposite the 76° angle: 76° . Check: $104^\circ + 76^\circ + 104^\circ + 76^\circ = 360^\circ$ ✓.

Q10. The top and bottom sides are parallel, so each slant side carries a pair of angles that add to 180° : each top angle is $180^\circ - 74^\circ = 106^\circ$. Check: $74^\circ + 74^\circ + 106^\circ + 106^\circ = 360^\circ$ ✓.

Q11. (a) The top angle uses 50.5° of the 180° , leaving $180^\circ - 50.5^\circ = 129.5^\circ$ for the two rafter angles together. (b) The rafters are equal, so their angles are equal: $129.5^\circ \div 2 = 64.75^\circ$ exactly, which rounds to 64.8° to 1 decimal place. (c) Using the exact values, $64.75^\circ + 64.75^\circ + 50.5^\circ = 180^\circ$ ✓. Round only for reporting; the check uses the exact angles.

Q12. (a) $AB = AC$, so the angles facing those sides are equal: $\angle ACB = \angle ABC = 64^\circ$. (b) In triangle ABC : $\angle BAC = 180^\circ - 64^\circ - 64^\circ = 52^\circ$. (c) The post AD is vertical and the beam BC is horizontal, so they meet at a right angle: $\angle ADB = 90^\circ$. (d) In triangle ABD : $\angle BAD = 180^\circ - 90^\circ - 64^\circ = 26^\circ$. (The same answer by symmetry: the post splits the 52° top angle in half, and $52^\circ \div 2 = 26^\circ$.)

Q13. (a) The exterior angle equals the sum of the two interior angles far away from it: $x = 48^\circ + 67^\circ = 115^\circ$. (b) The interior angle at the front corner is $180^\circ - 48^\circ - 67^\circ = 65^\circ$; then the straight ground line gives $x = 180^\circ - 65^\circ = 115^\circ$. (c) Yes — both methods give 115° . The exterior angle rule is quicker; the two-step method makes a good check.

Q14. The three recorded angles add to $47^\circ + 66^\circ + 68^\circ = 181^\circ$. The angles of every triangle add to exactly 180° , so these cannot be the angles of a real triangle: the record overshoots by 1° , so at least one of the three readings is wrong.

Q15. (a) A corner of a square is 90° , and the diagonal cuts it in half: $90^\circ \div 2 = 45^\circ$. (b) Each of the four triangles has two of those 45° halves as its corner angles, so its centre angle is $180^\circ - 45^\circ - 45^\circ = 90^\circ$. (c) The four centre angles meet at the middle point: $4 \times 90^\circ = 360^\circ$ ✓ — a full turn, exactly as the point rule requires.

Q16. The four angles of the kite add to 360° . The three known angles give $118^\circ + 76^\circ + 76^\circ = 270^\circ$, so the bottom angle is $360^\circ - 270^\circ = 90^\circ$.

Q17. (a) Two 45° mitre cuts joined make $45^\circ + 45^\circ = 90^\circ$ — a square corner. (b) Four corners of 90° give $4 \times 90^\circ = 360^\circ$, exactly the full turn needed for the frame to close. (c) Two 40° cuts make $40^\circ + 40^\circ = 80^\circ$, which is $90^\circ - 80^\circ = 10^\circ$ off square. Four such corners give only $4 \times 80^\circ = 320^\circ$, so the frame cannot close flat.

Q18. (a) A square corner is 90° and an equilateral triangle corner is 60° , so the four placed corners add to $90^\circ + 60^\circ + 90^\circ + 60^\circ = 300^\circ$. (b) The angles around the point must reach 360° , so the gap is $x = 360^\circ - 300^\circ = 60^\circ$. (c) An equilateral triangle fits exactly, because each of its corner angles is 60° — the same size as the gap.

Chapter 3 Shape — 3C Reflectional and rotational symmetry

Theory

Symmetry is everywhere: bathroom tiles, company logos, road signs, quilt patches, playing cards, roof frames. A shape has **symmetry** when there is a move — a fold or a turn — that lands the shape exactly back on itself. This section looks at the two kinds of symmetry move: a **fold** (reflection) and a **turn** (rotation).

Reflectional symmetry — the fold test.

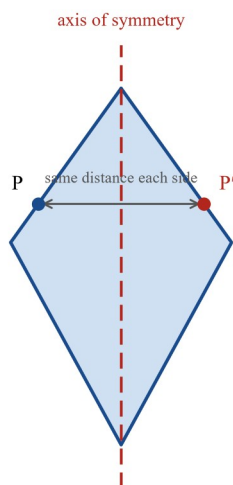
A flat shape has **reflectional symmetry** (also called **line symmetry**) if you can fold it along a straight line so that the two halves match exactly. That line is called an **axis of symmetry** or a **mirror line**.

Two quick ways to test a line:

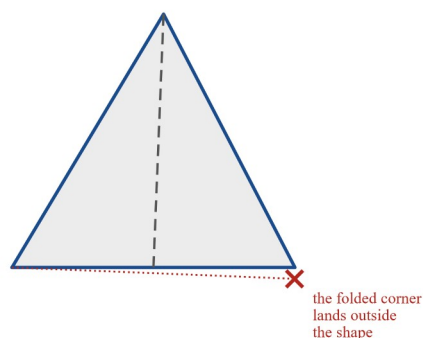
- **Fold test:** fold the shape along the line. If the two halves land exactly on top of each other, the line is an axis of symmetry.
- **Mirror test:** stand a small mirror upright on the line. If the real half plus its reflection together look like the complete shape, the line is an axis of symmetry.

Matching points (like P and P' below) sit the **same distance** from the axis, on opposite sides of it. Points that lie **on** the axis do not move at all — each one is its own image.

fold matches — 1 axis of symmetry



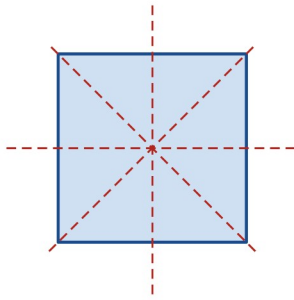
no fold matches — no axis of symmetry



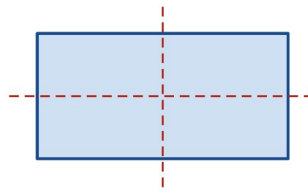
A shape can have one axis, several axes, or none. The triangle on the right above is **scalene** (all three sides different lengths), and no fold of it ever matches, so it has no axis of symmetry at all.

Axes of the familiar shapes.

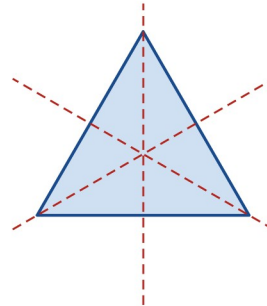
square — 4 axes



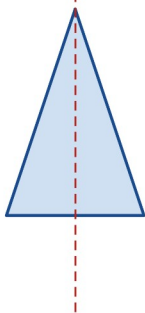
rectangle — 2 axes



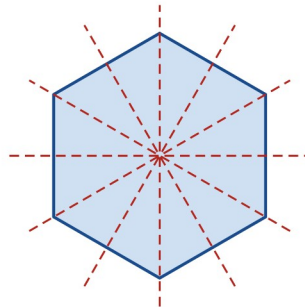
equilateral triangle — 3 axes



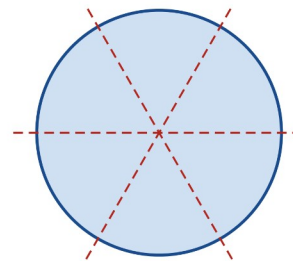
isosceles triangle — 1 axis



regular hexagon — 6 axes



circle — infinitely many axes



every line through the centre

Shape	Axes of symmetry
isosceles triangle (two equal sides)	1
rectangle	2
equilateral triangle (three equal sides)	3
square	4
regular hexagon (six equal sides)	6
circle	infinitely many (every line through the centre)

Watch out: the diagonals of a rectangle (corner to opposite corner) are **not** axes of symmetry. Fold a rectangle along a diagonal and the halves do not match — the corners stick out past each other. Only the two lines joining the midpoints of opposite sides are axes.

Rotational symmetry — the turn test.

A flat shape has **rotational symmetry** if there is a turn of less than 360° about a fixed **centre** point that lands the shape exactly back on itself. Two numbers describe it:

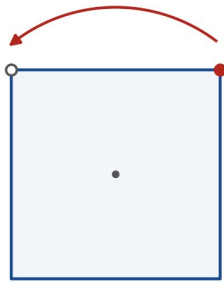
- the **order** of rotational symmetry — the number of times the shape lands on itself in one full 360° turn;
- the **smallest turn** — the smallest angle of turn that works.

The two numbers connect by a rule that works both ways:

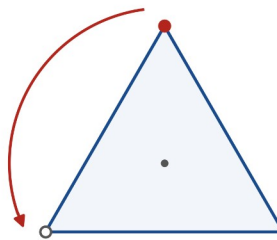
$$\text{smallest turn} = 360^\circ \div \text{order} \quad \text{order} = 360^\circ \div \text{smallest turn}$$

A shape that lands on itself only after a full 360° turn has **order 1**, and we say it has **no rotational symmetry**.

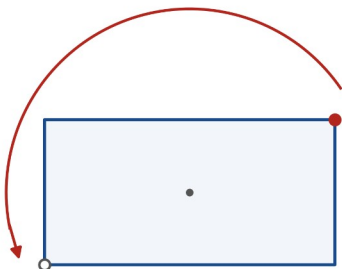
square — order 4, smallest turn 90°



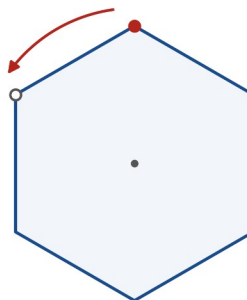
equilateral triangle — order 3, smallest turn 120°



rectangle — order 2, smallest turn 180°



regular hexagon — order 6, smallest turn 60°



In each shape above, the marked corner (red dot) moves to the open dot after one smallest turn — and the whole shape looks exactly the same as it did before the turn. That is what rotational symmetry means.

Regular polygons.

A **regular polygon** is a flat shape with straight sides where every side is the same length and every angle is the same size. Regular polygons follow one tidy rule: a regular polygon with n sides has n axes of symmetry **and** rotational order n , so its smallest turn is $360^\circ \div n$.

Worked model — a stop sign. A stop sign is a regular octagon: a regular polygon with $n=8$ sides. By the rule it has 8 axes of symmetry and rotational order 8, and its smallest turn is $360^\circ \div 8 = 45^\circ$.

Both symmetries together.

Shape	Axes of symmetry	Order	Smallest turn
square	4	4	90°
rectangle	2	2	180°
equilateral triangle	3	3	120°
regular pentagon	5	5	72°
regular hexagon	6	6	60°
isosceles triangle	1	1	none
parallelogram	0	2	180°
circle	infinitely many	every turn works	—

Different shapes sit in different camps. Some have **both** symmetries (the square). Some have reflectional symmetry **only** (the isosceles triangle, an arrow). Some have rotational symmetry **only** (the parallelogram, the letter S, a windmill). Some have **neither** (the scalene triangle, an odd L-shaped block).

Using symmetry.

Symmetry is not only something to spot — it is something to *use*:

- **Complete a half-drawn shape.** Every corner moves to the same distance on the other side of the mirror line, at the same height. Corners on the mirror line stay put. Example 2 and Questions 5 and 11 practise this on grid paper.
- **Read off measurements without measuring.** Matching lengths on the two halves are equal, so one measured half tells you the whole shape (Example 3, Question 12).
- **Designs that turn.** Repeating one piece n times evenly spaced around a centre gives order n and smallest turn $360^\circ \div n$ — bolt holes on a plate, clamps on a turntable, petals on a tile (Example 4, Question 14).

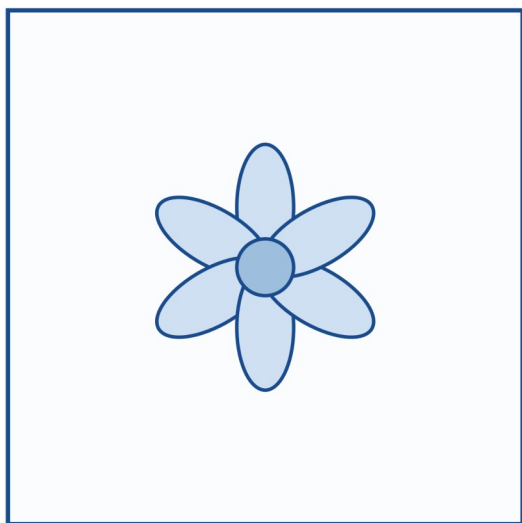
This section stays with flat (two-dimensional) shapes. Enlarging and shrinking shapes, and solid (three-dimensional) objects, come later in this chapter; repeating patterns and tilings come in section 3E.

About the data in this section. The businesses, designs, signs and measurements here are **made up** (not real): pretend companies and pretend numbers, built for this book so you can practise the mathematics. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

A tiler designs a feature tile with a six-petal flower. The six petals are identical and evenly spaced around the centre of the tile.



- How many axes of symmetry does the design have?
- What is its order of rotational symmetry?
- Find the smallest turn that maps the design onto itself.

Working

Comment

Fold through two opposite petals: the halves match. The 6 petals make 3 opposite pairs, so these folds give 3 axes.

Start the fold test with lines through the middle of the design.

Fold through two opposite gaps (the spaces between petals): the halves match again. The 6 gaps make 3

A second family of folds, halfway between the first

Working

opposite pairs, so these folds give 3 more axes.

Total: $3 + 3 = 6$ axes.

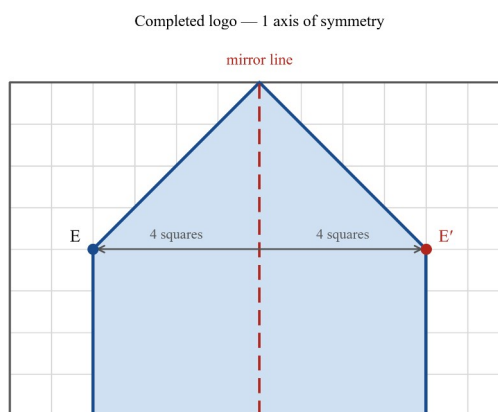
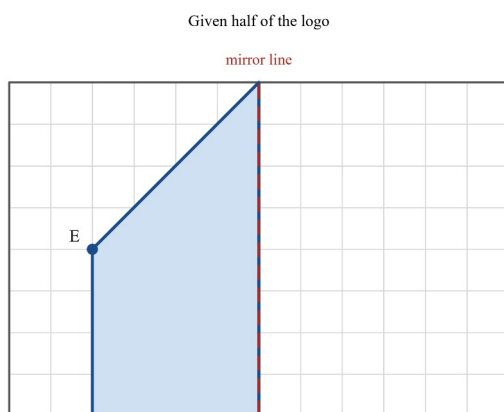
Turn test: the 6 identical petals are evenly spaced, so the design lands on itself 6 times in one full turn. Order = 6.

Smallest turn = $360^\circ \div 6 = 60^\circ$.

Check: $60^\circ \times 6 = 360^\circ$ ✓ — six turns of 60° fill the full turn exactly.

Example 2 — scaffolded

A company's logo is drawn on grid paper, but only the left half is drawn. The mirror line is the vertical grid line through the middle of the grid, and the marked corner is E .



- How many squares to the left of the mirror line is corner E ?
- Where should the designer mark the image E' of corner E ?
- Complete the logo. How many axes of symmetry does the completed logo have, and what is its order of rotational symmetry?

Working

Count the squares from E across to the mirror line: 4 squares.

E is 4 squares to the **left**, so E' is 4 squares to the **right** of the mirror line, at the **same height**.

The top corner and the bottom corner lie **on** the mirror line, so they do not move.

Join E' to the top corner and to the bottom right corner. The completed logo is a house-shaped pentagon (5 sides).

Fold test: folding along the mirror line makes the halves match, and no other fold matches. Axes: 1.

Turn test: no turn smaller than a full turn maps the house onto itself. Order = 1.

Example 3 — unscaffolded

A café is making a kite-shaped sign. (A **kite** is a four-sided shape with two pairs of equal sides sitting next to each other.) The sign is symmetrical about a vertical axis. On the left half: the distance from the axis to the left corner is 35 cm, the top-left edge is 50 cm, and the bottom-left edge is 65 cm.

Comment

family.

$\therefore$ the design has 6 axes of symmetry.

Count the matches in a full 360° turn — that count is the order.

Smallest turn = $360^\circ \div$ order.

Back-check the rule in the other direction.

Comment

Matching points are the same distance from the mirror line.

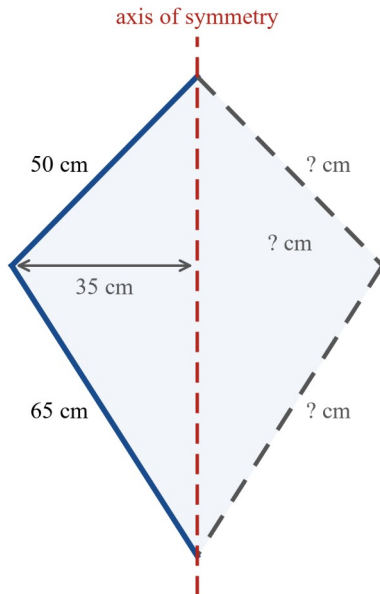
Same distance, other side, same height — that is the whole reflection rule.

Points on the mirror line are their own images.

Reflect every corner first, then join them in the same order as the original.

Test the completed shape, not just the half you were given.

$\therefore$ the completed logo has 1 axis of symmetry and order 1 — no rotational symmetry.



- Find the total width of the sign.
- Find the length of trim needed to go all the way around the sign — its **perimeter** (the distance around the edge).

Solution.

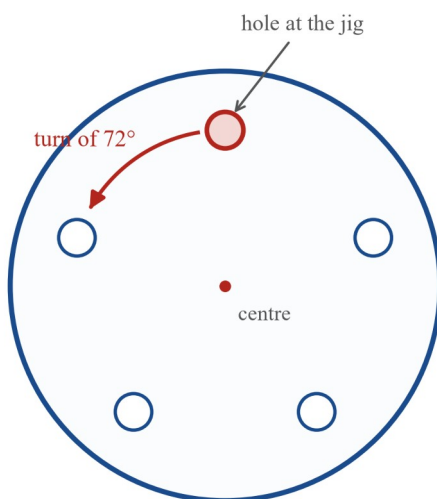
The right half is the mirror image of the left half, so matching lengths are equal: the axis is also 35 cm from the right corner, the top-right edge is also 50 cm, and the bottom-right edge is also 65 cm.

- Width = $35 + 35 = 70$ cm.
- Perimeter = $50 + 65 + 65 + 50 = 230$ cm.

$\therefore$ the sign is 70 cm wide and needs 230 cm of trim.

Example 4 — unscaffolded

A steel plate has 5 identical bolt holes, spaced evenly in a circle around the centre of the plate. A drilling jig (a guide that holds the drill in place) sits at the top of the plate. The worker turns the plate about its centre until the next hole clicks into the jig.



- What is the order of rotational symmetry of the hole pattern?

- (b) Find the smallest turn that brings the next hole to the jig.
 (c) The worker turns the plate 144° . How many holes have clicked into the jig during the turn?

Solution.

The 5 holes are identical and evenly spaced, so the pattern lands on itself 5 times in one full turn.

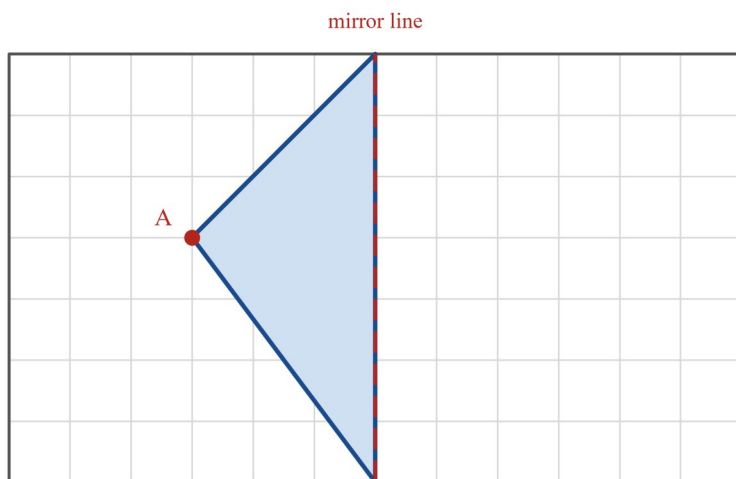
- (a) Order = 5.
 (b) Smallest turn = $360^\circ \div 5 = 72^\circ$.
 (c) Holes at the jig = $144^\circ \div 72^\circ = 2$.

$\therefore$ the order is 5, the smallest turn is 72° , and a 144° turn brings 2 holes into the jig.

Practice questions

Scaffolded

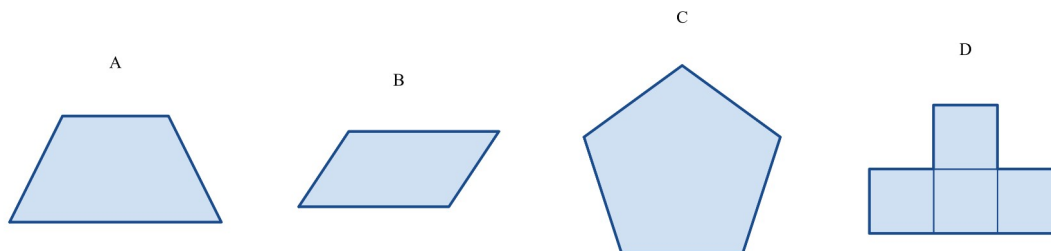
- Q1.** This question is about the words and rules of symmetry. (a) A flat shape has reflectional symmetry when it can be folded along a line so the two halves match exactly. What is that line called? (b) What does the order of rotational symmetry count? (c) Complete the rule: smallest turn = $360^\circ \div$ ____.
- Q2.** State the number of axes of symmetry of each shape. (a) a square (b) a rectangle (c) a circle
- Q3.** State the order of rotational symmetry of each shape. (a) a square (b) an equilateral triangle (c) a rectangle
- Q4.** A design has the given order of rotational symmetry. Find the smallest turn that maps the design onto itself. (a) order 4 (b) order 5 (c) order 6
- Q5.** A craft group has drawn half a kite-shaped badge on grid paper. The mirror line is the vertical grid line through the middle of the grid, and the marked corner is A.



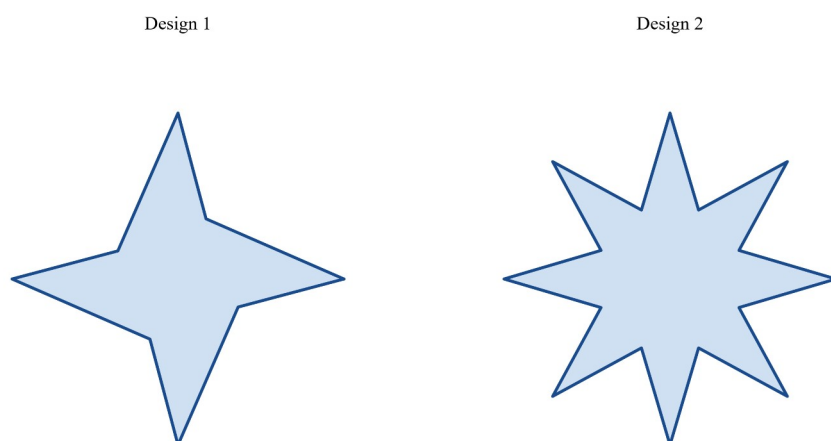
- (a) How many squares is corner A from the mirror line?
 (b) Where is the image of corner A? (Say how many squares from the mirror line, and on which side.)
 (c) Complete the badge. Name the completed shape and state how many axes of symmetry it has.
- Q6.** (a) A floor design matches itself each time it is turned 45° . What is its order of rotational symmetry? (b) A shape matches itself 3 times in one full turn. What is its smallest turn? (c) A shape matches itself only after a full 360° turn. What is its order, and does it have rotational symmetry?

Un scaffolded

Q7. For each shape A–D below, state the number of axes of symmetry and the order of rotational symmetry.

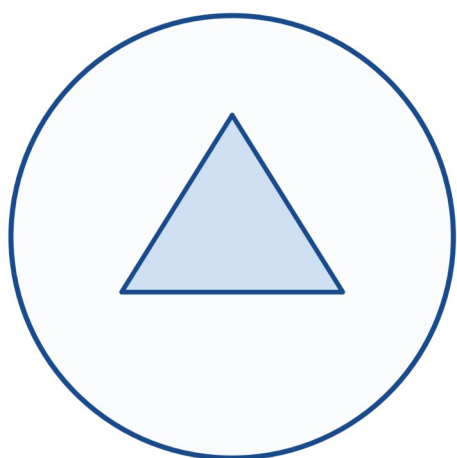


Q8. Design 1 is a four-blade windmill; Design 2 is an eight-pointed star. All blades and all points are identical and evenly spaced.



- (a) What is the order of rotational symmetry of Design 1?
- (b) Find the smallest turn that maps Design 1 onto itself.
- (c) Does Design 1 have any axes of symmetry? Give a reason.
- (d) For Design 2, state the order, the smallest turn, and the number of axes of symmetry.

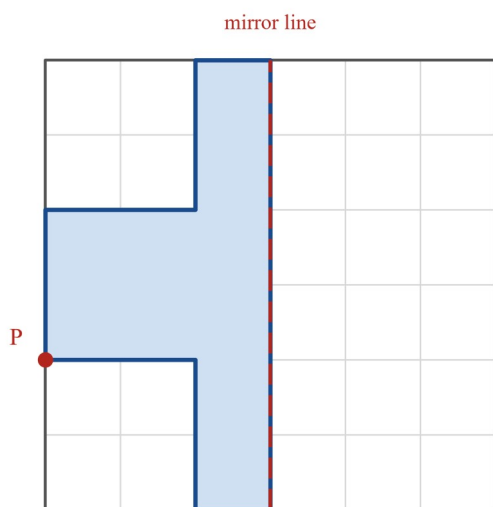
Q9. A council's logo is a circle with a symmetrical mountain — an isosceles triangle — inside it.



- (a) How many axes of symmetry does the logo have?
- (b) What is its order of rotational symmetry?
- (c) Does the logo have rotational symmetry? Explain your answer.

Q10. A hall floor has a seven-pointed star set into it, with seven identical points evenly spaced around the centre. (a) What is the order of rotational symmetry of the star? (b) Find the smallest turn that maps the star onto itself, correct to 1 decimal place. (c) How many axes of symmetry does the star have?

Q11. Half a plus sign is drawn on grid paper. The mirror line is the vertical grid line through the middle of the grid, and the marked corner is *P*.



- How many squares is corner *P* from the mirror line?
- Where is the image of corner *P*?
- Two corners of the drawn half lie on the mirror line. What happens to them when the shape is reflected?
- Complete the sign. How many axes of symmetry does the completed plus sign have, and what is its order of rotational symmetry?

Q12. A garden shed roof is symmetrical about a vertical centre line. The left rafter (the sloping beam) is 3.4 m long, and the distance from the centre line to the left edge of the roof is 2.1 m. Use symmetry to find: (a) the length of the right rafter (b) the total width of the roof (c) the total length of both rafters

Q13. Maya has a circular paper mat. She folds it exactly in half, then in half again, and opens it out flat. (a) In symmetry language, what is each crease? (b) How many axes of symmetry does the circle have altogether? (c) Maya does the same two folds with a square paper napkin, making two creases. How many axes of symmetry does the square have altogether?

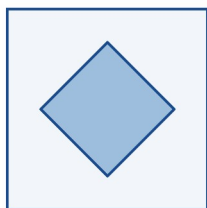
Q14. A welding turntable (a table that spins about its centre) has 6 identical clamping positions, evenly spaced around its edge. The welder turns the table until the next position clicks into place at the front. (a) What is the order of rotational symmetry of the position pattern? (b) Find the smallest turn that brings the next position to the front. (c) The table is turned 240° . How many positions have clicked into place at the front during the turn?

Q15. Tahlia says: “A rectangle is like a square, so it has 4 axes of symmetry and order 4.” Explain why each part of Tahlia’s statement is wrong, and state the correct number of axes and the correct order for a rectangle.

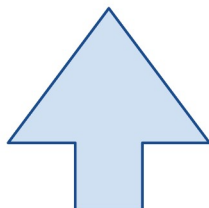
Q16. Consider the letters of the word MATHS, written in plain block capitals. (a) Which letters have a vertical axis of symmetry? (b) Which letter has rotational symmetry of order 2 but no axes of symmetry? (c) Write a three-letter word, in plain block capitals, in which every letter has a vertical axis of symmetry.

Q17. Below are four quilt blocks (decorative square fabric patches). Match each block A–D with one of these descriptions: **both** symmetries; reflectional symmetry **only**; rotational symmetry **only**; **neither** symmetry. (Remember: an order of 1 counts as no rotational symmetry.)

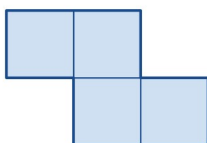
Block A



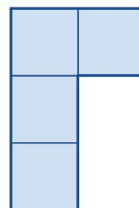
Block B



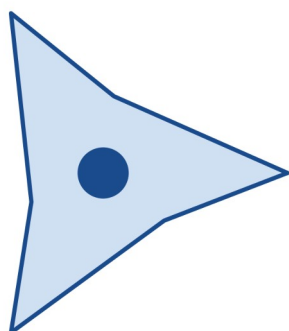
Block C



Block D



Q18. A garden pinwheel has 3 identical blades. All three blades curve the same way around the centre.



- What is the order of rotational symmetry of the pinwheel?
- Find the smallest turn that maps the pinwheel onto itself.
- Explain why the pinwheel has no axes of symmetry.

Answers

Q1. (a) An axis of symmetry (or mirror line). (b) The number of times the shape lands on itself in one full 360° turn. (c) order. **Q2.** (a) 4. (b) 2. (c) Infinitely many. **Q3.** (a) 4. (b) 3. (c) 2. **Q4.** (a) 90° . (b) 72° . (c) 60° . **Q5.** (a) 3 squares. (b) 3 squares to the right of the mirror line, at the same height. (c) A kite; 1 axis of symmetry. **Q6.** (a) Order 8. (b) 120° . (c) Order 1 — no rotational symmetry. **Q7.** A: 1 axis, order 1. B: 0 axes, order 2. C: 5 axes, order 5. D: 1 axis, order 1. **Q8.** (a) 4. (b) 90° . (c) No — a fold would make one half's blades curve the other way, so the halves never match. (d) Order 8, smallest turn 45° , 8 axes. **Q9.** (a) 1 (the vertical line through the peak). (b) 1. (c) No — it lands on itself only after a full turn, so order 1 means no rotational symmetry. **Q10.** (a) 7. (b) $360 \div 7 = 51.42857\dots$, so 51.4° correct to 1 decimal place. (c) 7. **Q11.** (a) 3 squares. (b) 3 squares to the right of the mirror line, at the same height. (c) They do not move — each is its own image. (d) 4 axes; order 4. **Q12.** (a) 3.4 m. (b) 4.2 m. (c) 6.8 m. **Q13.** (a) An axis of symmetry. (b) Infinitely many. (c) 4 — the two creases plus the two diagonals. **Q14.** (a) 6. (b) 60° . (c) $240 \div 60 = 4$ positions. **Q15.** The diagonals are not axes — the fold does not match — so a rectangle has 2 axes (the lines joining midpoints of opposite sides), not 4; and a 90° turn leaves it standing the wrong way, so the order is 2 (smallest turn 180°), not 4. **Q16.** (a) M, A, T, H. (b) S. (c) e.g. HAT (other answers are possible). **Q17.** A — both (4 axes, order 4); B — reflectional only (1 axis, order 1); C — rotational only (0 axes, order 2); D — neither (0 axes, order 1). **Q18.** (a) 3. (b) 120° . (c) Any fold line flips one half over, so its blades curve the opposite way and can never match the blades underneath.

Fully-worked solutions

Q1. (a) The line is called an **axis of symmetry** (or a **mirror line**). (b) The order counts how many times the shape lands exactly on itself during one full 360° turn. (c) $\text{smallest turn} = 360^\circ \div \text{order}$.

Q2. (a) A square folds onto itself along the vertical line, the horizontal line, and both diagonals: 4 axes. (b) A rectangle folds onto itself along the vertical and horizontal lines only (the diagonals fail the fold test): 2 axes. (c) Every line through the centre of a circle is an axis, and there is no end to those lines: **infinitely many** axes.

Q3. (a) A quarter turn (90°) maps a square onto itself; the matches come at 90° , 180° , 270° and 360° — order 4. (b) An equilateral triangle matches at 120° , 240° and 360° — order 3. (c) A rectangle matches only at the half turn, 180° (and the full turn) — order 2.

Q4. (a) $360^\circ \div 4 = 90^\circ$. (b) $360^\circ \div 5 = 72^\circ$. (c) $360^\circ \div 6 = 60^\circ$.

Q5. (a) Counting squares from A across to the mirror line gives 3 squares. (b) The image is the same distance on the other side: 3 squares to the **right** of the mirror line, at the **same height** as A. (c) Reflecting every corner and joining them in order completes a **kite**. The fold test matches only along the mirror line, so the completed badge has 1 axis of symmetry (and order 1 — any smaller turn moves the kite off itself).

Q6. (a) $\text{order} = 360^\circ \div \text{smallest turn} = 360^\circ \div 45^\circ = 8$. (b) $\text{smallest turn} = 360^\circ \div \text{order} = 360^\circ \div 3 = 120^\circ$. (c) Matching only after a full turn means 1 match in the turn: order 1, which is the way we say the shape has **no rotational symmetry**.

Q7. **A** (an isosceles trapezium — a four-sided shape with one pair of parallel sides and matching sloping sides): the vertical fold matches and no other fold does, so 1 axis; a half turn turns it upside down, so order 1. **B** (a parallelogram — opposite sides parallel and equal): no fold matches, so 0 axes; a half turn maps it onto itself, so order 2. **C** (a regular pentagon): the regular-polygon rule gives 5 axes and order 5. **D** (a T-shape made of four squares): the vertical fold matches, so 1 axis; a half turn puts the stem on top instead of the bottom, so order 1.

Q8. (a) The 4 identical blades are evenly spaced, so the design lands on itself 4 times in a full turn: order 4. (b) $\text{Smallest turn} = 360^\circ \div 4 = 90^\circ$. (c) No. All 4 blades curve the same way around the centre. A fold flips one half over, so its blades would curve the other way and could not lie on the blades underneath — no fold matches, so there are 0 axes. (d) Design 2: the 8 identical points are evenly spaced, so order 8; $\text{smallest turn} = 360^\circ \div 8 = 45^\circ$; folds through

opposite points (4 of them) and through opposite gaps (4 more) give 8 axes. $\therefore$ Design 1: order 4, smallest turn 90° , no axes; Design 2: order 8, smallest turn 45° , 8 axes.

Q9. (a) The circle on its own has infinitely many axes, but a fold only matches if the mountain halves match too. Only the vertical line through the peak does that, so the logo has 1 axis of symmetry. (b) Any turn smaller than a full turn moves the mountain off itself, so the logo matches only once in a full turn: order 1. (c) No. Order 1 means the logo lands on itself only after a full 360° turn, which is exactly what “no rotational symmetry” means.

Q10. (a) The 7 identical points are evenly spaced, so the star lands on itself 7 times in a full turn: order 7. (b) Smallest turn $= 360^\circ \div 7 = 51.42857 \dots^\circ$, which is 51.4° correct to 1 decimal place. (c) A fold through each point and the gap opposite it matches, giving 7 axes. $\therefore$ order 7, smallest turn 51.4° (to 1 decimal place), 7 axes.

Q11. (a) Counting squares from *P* across to the mirror line gives 3 squares. (b) The image is 3 squares to the **right** of the mirror line, at the **same height** as *P*. (c) Corners that lie on the mirror line do not move — each one is its own image. (d) Reflecting the six off-line corners completes the plus sign. The fold test matches along the vertical line, the horizontal line and both diagonals: 4 axes. A quarter turn maps the sign onto itself, so the order is 4 (smallest turn $360^\circ \div 4 = 90^\circ$).

Q12. (a) The right rafter is the mirror image of the left rafter, so it is the same length: 3.4 m. (b) The centre line is 2.1 m from each edge, so the width is $2 \times 2.1 = 4.2$ m. (c) Both rafters: $2 \times 3.4 = 6.8$ m. $\therefore$ the right rafter is 3.4 m, the roof is 4.2 m wide, and the two rafters total 6.8 m.

Q13. (a) Each crease is an **axis of symmetry** — the fold made the two halves match exactly. (b) A circle matches every fold through its centre, so it has **infinitely many** axes of symmetry; the two creases are just two of them. (c) The square’s two creases join the midpoints of opposite sides. The square also folds onto itself along its two diagonals, so the total is $2 + 2 = 4$ axes.

Q14. (a) The 6 identical positions are evenly spaced, so the pattern lands on itself 6 times in a full turn: order 6. (b) Smallest turn $= 360^\circ \div 6 = 60^\circ$. (c) Positions clicked into place $= 240^\circ \div 60^\circ = 4$. $\therefore$ order 6, smallest turn 60° , and 4 positions pass through the front during a 240° turn.

Q15. First part: fold a rectangle along a diagonal. The halves do **not** match — the corners stick out past each other — so the diagonals are not axes. The only axes of a rectangle are the two lines joining the midpoints of opposite sides, so a rectangle has 2 axes of symmetry, not 4. Second part: turn a rectangle 90° . The long sides end up standing upright instead of lying across, so the shape does not land on itself; the turn that works is the half turn, 180° . So the order is 2, not 4. (A square really does have 4 axes and order 4 because all four of its sides are equal — a rectangle has two long sides and two short sides, and that is what breaks Tahlia’s rule.) $\therefore$ a rectangle has 2 axes of symmetry, order 2, and smallest turn 180° .

Q16. (a) Test each letter with a vertical fold: M ✓, A ✓, T ✓, H ✓ match, and S does not. So the letters are **M, A, T, H**. (b) **S**: a half turn maps it onto itself (order 2), but no fold of S matches, so it has no axes of symmetry. (c) e.g. **HAT** — H, A and T each have a vertical axis of symmetry. (Other words such as MAM also work.)

Q17. Block A — both symmetries: the square patch with a centred diamond folds onto itself 4 ways (vertical, horizontal, both diagonals) and a quarter turn maps it onto itself: 4 axes, order 4. **Block B — reflectional symmetry only:** the vertical fold matches (1 axis), but a half turn turns the arrow upside down, so order 1. **Block C — rotational symmetry only:** no fold of the S-shape matches (0 axes), but a half turn maps it onto itself: order 2. **Block D — neither symmetry:** no fold of the L-shape matches (0 axes) and only a full turn maps it onto itself: order 1.

Q18. (a) The 3 identical blades are evenly spaced, so the pinwheel lands on itself 3 times in a full turn: order 3. (b) Smallest turn $= 360^\circ \div 3 = 120^\circ$. (c) All three blades curve the same way around the centre. Any fold line flips one half over, so the folded blades curve the opposite way from the blades lying under them — the two halves can never match, whatever line you fold along. That is why the pinwheel has no axis of symmetry. $\therefore$ the pinwheel has order 3, smallest turn 120° , and no axes of symmetry.

Chapter 3 Shape — 3D Congruence, similarity and scaling

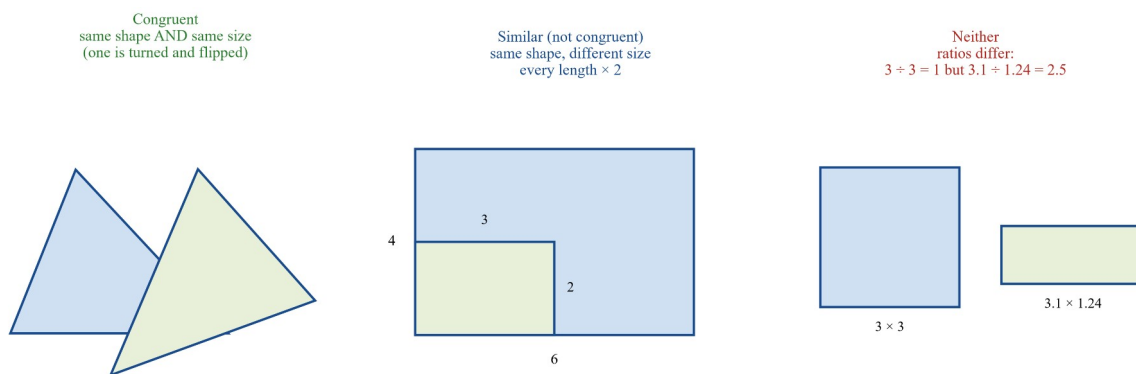
Theory

Congruent means an exact match. Two shapes are **congruent** if they have the same shape *and* the same size — one can be slid, turned or flipped onto the other and they line up exactly. Turning or flipping a shape does not change it: a left-hand glove pattern flipped over is still the same size of glove. Congruence is what you rely on when two cut pieces must match, like the two sides of a bracket, or the four corner squares of a quilt block.

Similar means same shape, any size. Two shapes are **similar** if one is an enlargement or a reduction of the other: every angle stays the same and every length is multiplied by the same number. That number is the **scale factor**,

$$k = \text{new length} \div \text{matching old length}$$

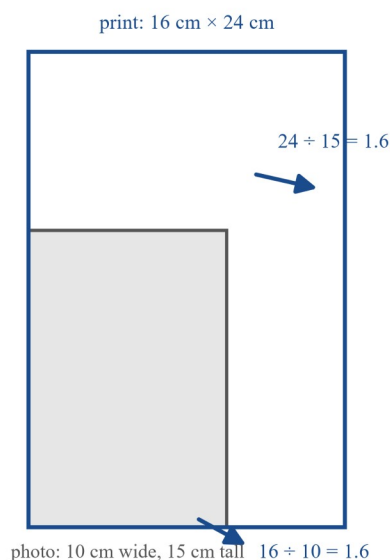
and the test for similarity is simple: *match up the lengths and divide. If every ratio comes out the same, the shapes are similar and that common ratio is k .* If $k > 1$ the change is an **enlargement**; if $k < 1$ it is a **reduction**.



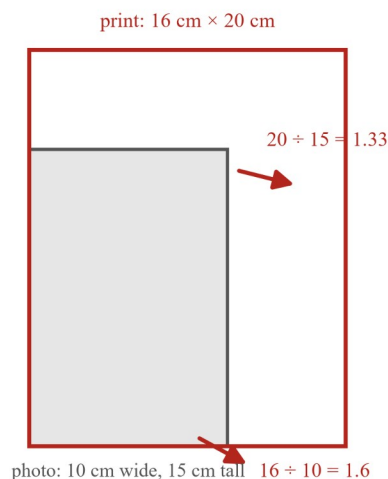
Three pairs of shapes: congruent (same shape and size, one turned and flipped), similar (same shape, every length $\times 2$), and neither (ratios differ).

The test in action. A photo is 10 cm wide and 15 cm tall. A print comes out 16 cm wide and 24 cm tall. Match and divide: widths give $16 \div 10 = 1.6$ and heights give $24 \div 15 = 1.6$. The ratios agree, so the print is similar to the photo — an enlargement with $k = 1.6$. Now suppose the print came out 16 cm wide but only 20 cm tall. Widths still give $16 \div 10 = 1.6$, but heights give $20 \div 15 = 1.33$ (to 2 decimal places). The ratios disagree, so this print is **not** similar: it has been squashed. This is exactly what people mean when they say a picture looks “stretched”.

Same ratio both ways → similar ($k = 1.6$)



Different ratios → NOT similar (stretched)



A $10\text{ cm} \times 15\text{ cm}$ photo printed at $16\text{ cm} \times 24\text{ cm}$ is similar (both ratios 1.6); printed at $16\text{ cm} \times 20\text{ cm}$ it is not (1.6 against 1.33).

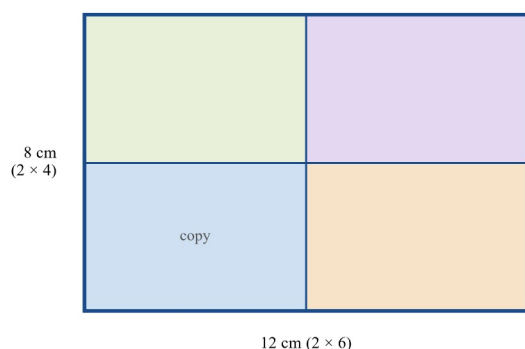
Finding a missing length. Once you know two shapes are similar with scale factor k , any length follows: new length $= k \times$ old length, and old length $=$ new length $\div k$. You only need one matched pair of lengths to find k .

Scale notation. Models and plans write the scale as a ratio. A “1:18 scale” model car means 1 cm on the model matches 18 cm on the real car — a reduction with $k = 1/18$. A plan at 1:50 means 1 cm on the plan matches 50 cm in the building. The first number belongs to the drawing, the second to the real thing.

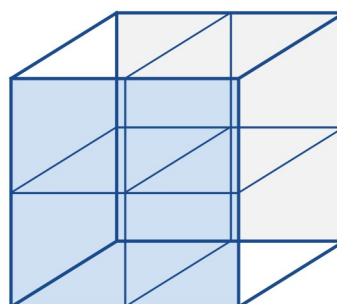
What scaling does to perimeter, area and volume.

- Perimeter is a length, so perimeter $\times k$.
- Area is measured in squares. Enlarge a flat shape with $k = 2$ and *four* copies of the original fit inside it (2 across, 2 down), so area $\times k^2 = 4$.
- Volume is measured in blocks. Enlarge a solid with $k = 2$ and *eight* copies of the original fit inside ($2 \times 2 \times 2$), so volume $\times k^3 = 8$.

Flat shape enlarged with $k = 2$:
4 copies of the original fit inside — area $\times 4$



Solid enlarged with $k = 2$:
8 copies of the original fit inside — volume $\times 8$



Left: a rectangle enlarged with $k = 2$ holds 4 copies of the original, so area $\times 4$. Right: a cube enlarged with $k = 2$ holds 8 copies, so volume $\times 8$.

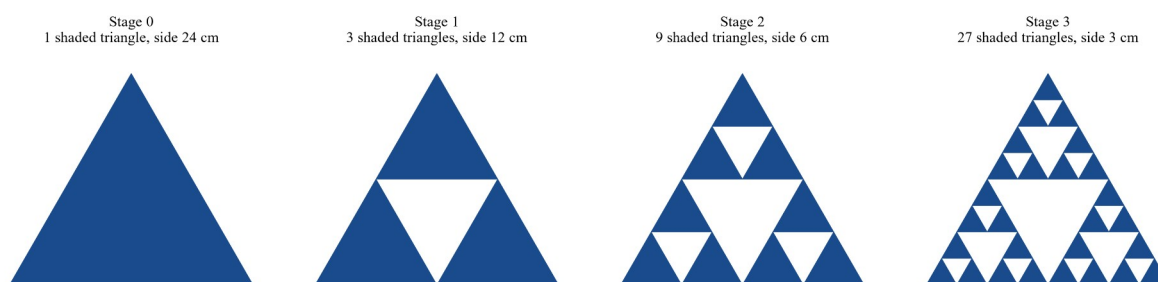
Counting copies is the whole idea — no formulas needed. The pattern holds for every scale factor:

Scale factor k	Lengths	Area factor k^2	Volume factor k^3
2	$\times 2$	4	8

Scale factor k	Lengths	Area factor k^2	Volume factor k^3
3	$\times 3$	9	27
0.5	$\times 0.5$	0.25	0.125
2.5	$\times 2.5$	6.25	15.625

Read the last row as a warning: shrink a carton to half the length, half the width and half the height and it holds only 0.125 of the volume — one eighth, not half.

Self-similarity: shapes made of copies of themselves. A shape is **self-similar** if part of it looks like a scaled-down copy of the whole. Start with a shaded triangle of side 24 cm. Replace it with 3 shaded triangles of half the side; replace each of those with 3 more, and so on. The counts go 1, 3, 9, 27 and the sides go 24, 12, 6, 3 cm — every stage is the previous stage reduced with $k = 1/2$. This figure (the Sierpinski triangle) is self-similar: zoom into a corner and you see the whole picture again.



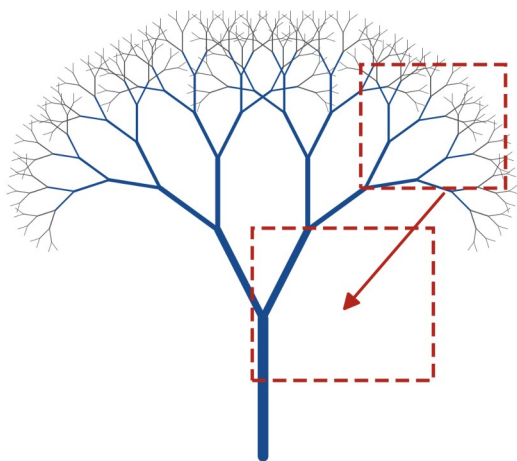
Start with a 24 cm-sided triangle. Each stage: every shaded triangle is replaced by 3 shaded triangles with half the side length.

Stages 0 to 3 of the Sierpinski triangle from a 24 cm side: 1, 3, 9, 27 shaded triangles with sides 24, 12, 6, 3 cm.

Self-similarity turns up outside mathematics too. A tree branch splits into twigs that look like little trees; a fern frond carries leaflets shaped like the frond; coastlines and clouds repeat their roughness at every zoom.

Each branch splits into two smaller copies of itself.

A twig looks like the whole tree — the shape is self-similar.



A self-similar tree: each branch splits into two smaller copies of itself, and a twig looks like the whole tree.

Scaling in everyday technology. A photocopier set to 200% makes an enlargement with $k = 2$; set to 50% it makes a reduction with $k = 0.5$. Dragging a photo corner in a phone app scales it the same way — and dragging a side edge instead is the “ratios differ” case above, which is why the picture then looks stretched. Copier settings are a quick check on the table: 200% doubles lengths and needs 4 times the paper area.

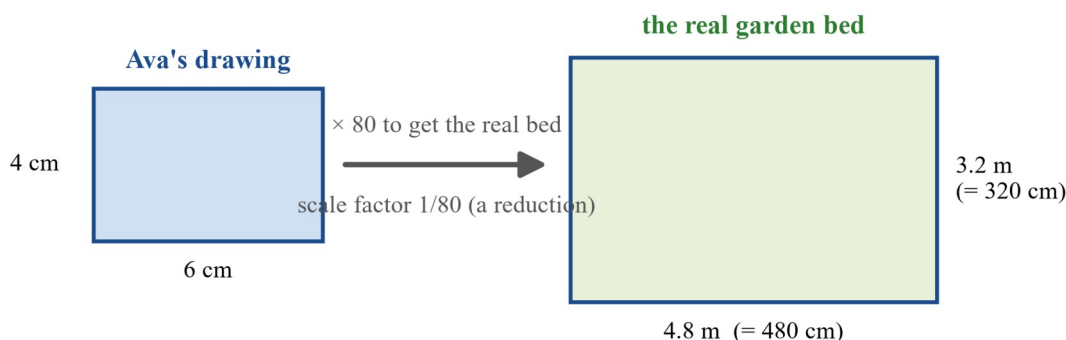
About the data in this section. The photos, flags, garden beds, quilts, cartons and mosaics here are **made up** (not real): they exist only to give you shapes with clean numbers to compare. Nothing here is real data about a real product, place or person.

Where this leads. The k^2 and k^3 ideas here are carried into the area and volume formulas of Sections 4B and 4C, and scale ratios meet maps and plans properly in Section 5C. For now the tools are just: match and divide to test similarity, multiply by k for lengths, and count copies for area and volume.

Worked examples

Example 1 — A garden-bed drawing (scaffolded)

Ava draws a rectangular garden bed at 6 cm long and 4 cm wide. The real bed is 4.8 m long and 3.2 m wide. Is the drawing similar to the real bed, and what is the scale factor?



$6 \div 480 = 1/80$ and $4 \div 320 = 1/80$ — the same ratio both ways, so the shapes are similar.

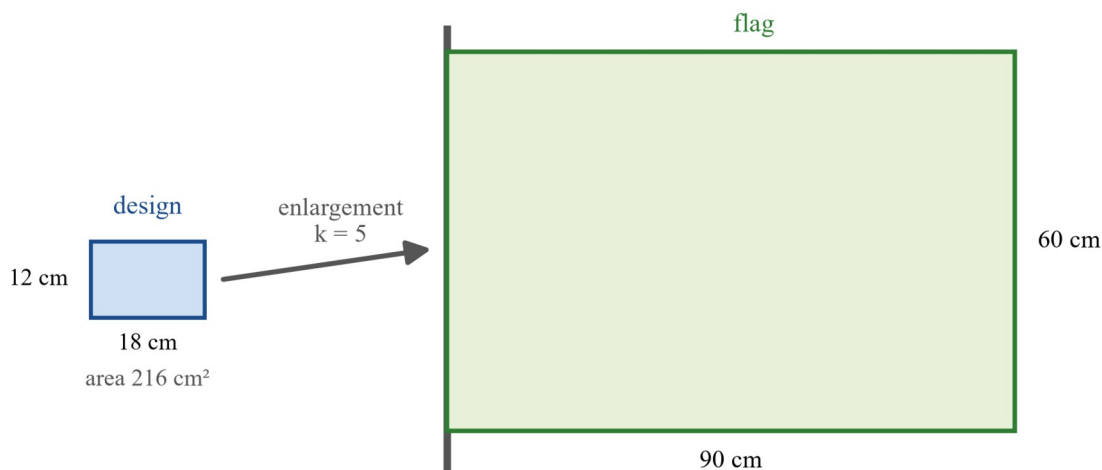
Ava's 6 cm $\times$ 4 cm drawing and the real 4.8 m $\times$ 3.2 m bed: both ratios are 1/80.

Working	Comment
4.8 m = 480 cm; 3.2 m = 320 cm	same units first (cm)
$6 \div 480 = 1/80$	length ratio, drawing : real
$4 \div 320 = 1/80$	width ratio — the same, so the shapes are similar
$k = 1/80 = 0.0125$	scale factor; $k < 1$ so the drawing is a reduction
check: $6 \times 80 = 480$ and $4 \times 80 = 320$	scaling back up returns the real bed (stage 3 check)

The drawing is similar to the bed with scale factor $1/80$: every centimetre on paper stands for 80 cm in the garden.

Example 2 — Painting a flag design (scaffolded)

A club flag is 90 cm by 60 cm. The design to be painted on it is drawn 18 cm by 12 cm and will be enlarged to cover the flag. What is the scale factor, and what area of flag will the design cover?



$90 \div 18 = 5$ and $60 \div 12 = 5$ — scale factor $k = 5$.
 Area factor $= 5^2 = 25$, so the design covers $216 \times 25 = 5400 \text{ cm}^2$ of flag.

The $18 \text{ cm} \times 12 \text{ cm}$ design enlarged with $k = 5$ onto the $90 \text{ cm} \times 60 \text{ cm}$ flag.

Working	Comment
$90 \div 18 = 5$ and $60 \div 12 = 5$	same ratio both ways $\rightarrow$ similar, $k = 5$
area factor $= 5^2 = 25$	areas scale by k^2 , not by k
$18 \times 12 = 216 \text{ cm}^2$	area of the design as drawn
$216 \times 25 = 5400 \text{ cm}^2$	area the design covers on the flag
check: $90 \times 60 = 5400 \text{ cm}^2$	the design covers the whole flag — consistent
wrong idea: $216 \times 5 = 1080 \neq 5400$	multiplying area by k is the classic error

Example 3 — Is the poster similar to the photo? (unscaffolded)

A photo is 9 cm wide and 12 cm tall. A print shop offers a poster 36 cm wide and 60 cm tall, claiming it is “the same shape, only bigger”. Test the claim, and if it fails, find the height a similar poster of that width would have.

Widths: $36 \div 9 = 4$. Heights: $60 \div 12 = 5$. The two ratios differ ($4 \neq 5$), so the offered poster is not similar to the photo — it has been stretched taller. Keeping the 36 cm width (ratio 4), a similar poster must be $12 \times 4 = 48 \text{ cm}$ tall; check $48 \div 12 = 4 = 36 \div 9$. The offered 60 cm height is $60 - 48 = 12 \text{ cm}$ too tall.

photo
9 cm $\times$ 12 cm

poster A
36 cm $\times$ 48 cm

poster B
36 cm $\times$ 60 cm



the original



$36 \div 9 = 4$ and $48 \div 12 = 4$
similar — $k = 4$



$36 \div 9 = 4$ but $60 \div 12 = 5$
NOT similar — stretched

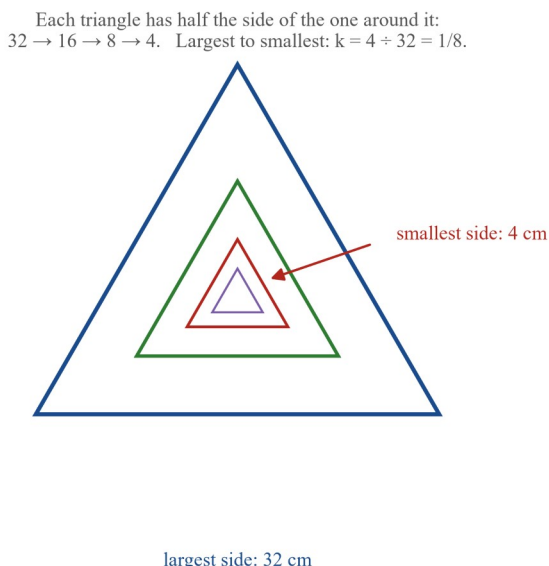
The 9 cm $\times$ 12 cm photo, a similar 36 cm $\times$ 48 cm poster ($k = 4$), and the stretched 36 cm $\times$ 60 cm offer.

∴ the offered poster is **not** similar to the photo; a similar poster 36 cm wide is 48 cm tall, so the offer is 12 cm too tall.

Example 4 — A nested-triangle logo (unscaffolded)

A logo is made of four nested triangle outlines. Each triangle has half the side length of the triangle around it, and the largest triangle has a side of 32 cm. Find the four side lengths, the scale factor from the largest triangle to the smallest, and the fraction of the largest triangle's area that the smallest triangle covers.

Halving from 32 cm: the sides are 32, 16, 8 and 4 cm, so the smallest side is 4 cm. From one triangle to the next the scale factor is $1/2$; from largest to smallest there are three halvings, $(1/2)^3 = 1/8$, which agrees with $4 \div 32 = 1/8 = 0.125$. Areas scale by k^2 , so the smallest triangle covers $(1/8)^2 = 1/64$ of the largest triangle's area.



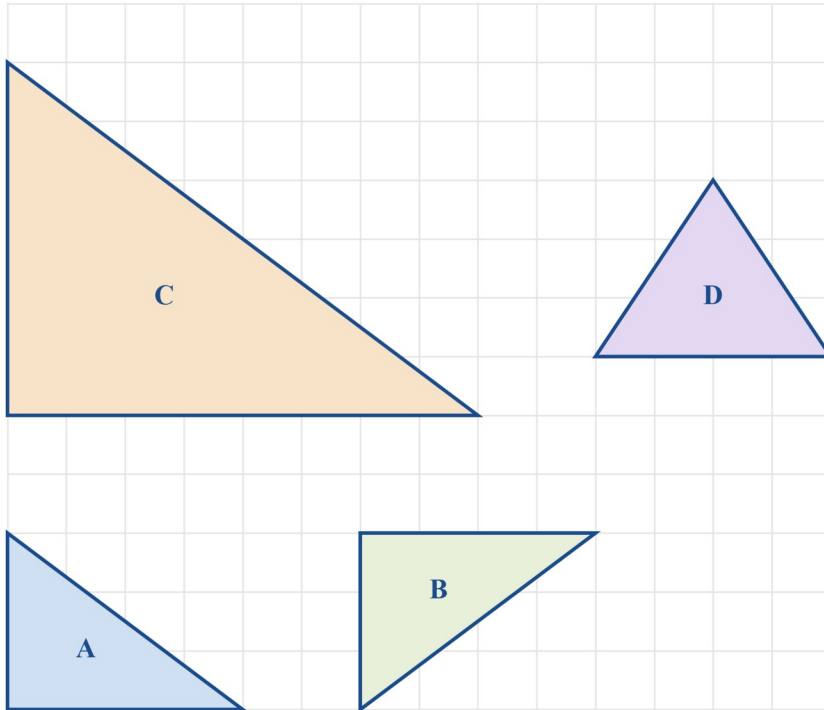
The logo: nested triangles with sides 32, 16, 8 and 4 cm; largest to smallest $k = 1/8$.

∴ the sides are 32, 16, 8, 4 cm; largest to smallest $k = 1/8 = 0.125$; the smallest triangle covers $1/64$ of the largest triangle's area.

Practice questions

Scaffolded

Q1. Four triangles are drawn on 1 cm grid paper.



Triangles A, B, C and D on 1 cm grid paper.

- (a) Which triangle is congruent to triangle A? Say how you know.
- (b) Which triangle is similar to A but *not* congruent to A? Give the scale factor from A to it.
- (c) Which triangle is neither congruent nor similar to A? Show the ratios that fail (ratios to 1 decimal place).

Q2. A photo is 8 cm wide and 12 cm tall. A print of it is 14 cm wide and 21 cm tall.

- (a) Calculate the width ratio and the height ratio (print : photo).
- (b) Is the print similar to the photo?
- (c) Is this an enlargement or a reduction? Give the scale factor.

Q3. A rectangle is 12 cm long and 5 cm wide. It is enlarged with scale factor $k = 3$.

- (a) Find the length and width of the new rectangle.
- (b) Find the perimeter of each rectangle and compare them.
- (c) Find the area of each rectangle and compare them.

Q4. A noticeboard poster is 250 cm long and 90 cm wide. It is reduced to a flyer with scale factor $k = 0.4$.

- (a) Find the length and width of the flyer.
- (b) Is this an enlargement or a reduction? How do you know from k ?
- (c) Find the area of the poster and of the flyer, and the area factor as a decimal and as a fraction.

Q5. A model car is built at 1:18 scale: 1 cm on the model matches 18 cm on the real car.

- (a) The model is 15 cm long. How long is the real car, in cm?
- (b) Write that real length in metres.
- (c) The real car is 175 cm wide. How wide is the model, to 1 decimal place?

Q6. A Sierpinski triangle starts as one shaded triangle with side 24 cm (stage 0). At each stage, every shaded triangle is replaced by 3 shaded triangles with half the side length.

- (a) How many shaded triangles are there at stages 1, 2 and 3?
- (b) What is the side length of a shaded triangle at each of stages 1, 2 and 3?
- (c) What is the scale factor from a stage 0 triangle to a stage 3 triangle?

Unscaffolded

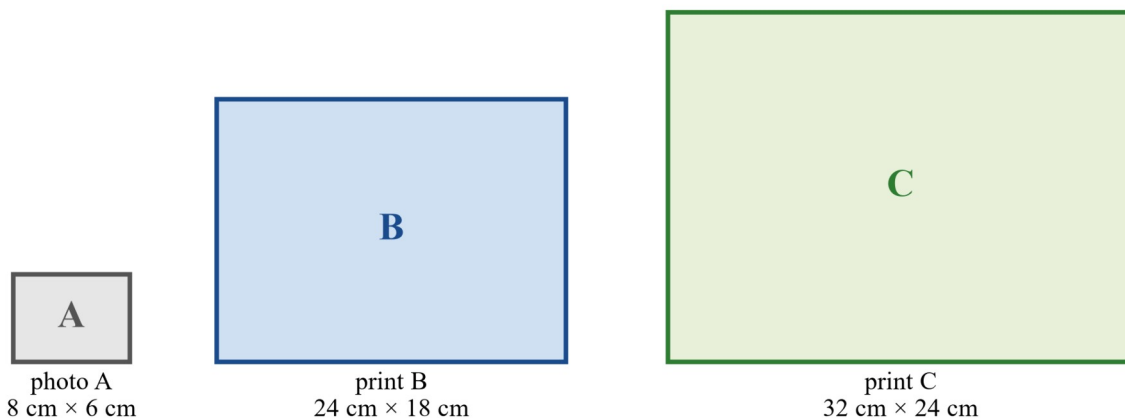
Q7. Two triangular metal plates are made for a bracket. Plate 1 has sides 30 cm, 40 cm and 50 cm. Plate 2 has sides 30 cm, 40 cm and 50 cm but is turned over and rotated. Are the plates congruent? Show the reasoning.

Q8. A photo is 10 cm wide and 14 cm tall. A shop offers a print 30 cm wide and 40 cm tall as “the same shape, bigger”. Is the offer similar to the photo? If not, what height would a similar print 30 cm wide have, and how far out is the offer? (Give any repeating ratio to 2 decimal places.)

Q9. A house plan is drawn at 1:50 scale.

- (a) A room is 6 m long and 4.5 m wide. What are its length and width on the plan, in cm?
- (b) A hallway measures 14 cm on the plan. How long is it in the house, in metres?

Q10. Photo A is 8 cm by 6 cm. Print B is 24 cm by 18 cm and print C is 32 cm by 24 cm.



Drawn to scale.

Photo A and prints B and C, drawn to scale.

- (a) Show that B and C are each similar to A, and give the scale factor from A to each.
- (b) Is B similar to C? Give the scale factor from B to C as a fraction and to 2 decimal places.

Q11. A model railway is built at 1:87 scale.

- (a) A real locomotive is 1 914 cm long. How long is the model, in cm?
- (b) The real locomotive is 348 cm tall. How tall is the model, in cm?
- (c) A model wagon is 14 cm long. How long is the real wagon, in metres?

Q12. A mural design is 20 cm by 30 cm. The mural itself is an enlargement with scale factor $k = 2.5$.

- (a) What is the area factor? (Name the wrong idea it is often confused with.)
- (b) Find the dimensions of the mural and its area.
- (c) Paint for the design sheet costs \$48. Paint costs the same amount per cm^2 . What will paint for the mural cost?

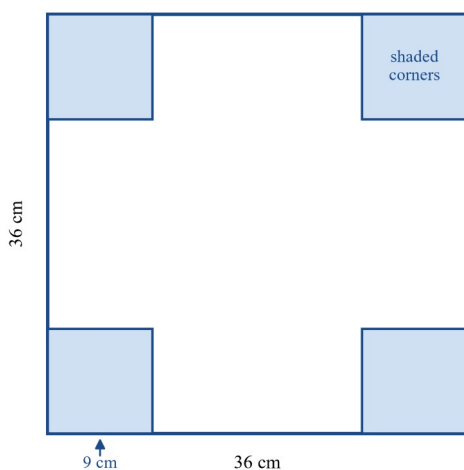
Q13. A juice carton holds 250 mL. The company makes a “family” carton with every length (height, width and depth) doubled.

- (a) What does the family carton hold? Give your answer in mL and in L.
- (b) A shopper guesses 500 mL. Explain, with a number, why that guess is wrong.

Q14. A road sign face is 60 cm by 40 cm. A model of the sign is made with scale factor $k = 0.3$.

- (a) Find the dimensions of the model sign face.
- (b) Find the area of the real sign face and of the model sign face.
- (c) Write the model area as a fraction of the real area, and check it against k^2 .

Q15. A square quilt block is 36 cm by 36 cm. Its pattern shades four corner squares, each 9 cm by 9 cm.



The 36 cm quilt block with four shaded 9 cm corner squares.

- (a) What is the scale factor from one small square to the whole block? Are the four small squares congruent to each other?
- (b) What fraction of the block is shaded? Check your answer with the area-factor shortcut $4 \times (1/4)^2$.
- (c) Fabric for a whole block costs \$100. At the same rate, what does the shaded fabric in one block cost?

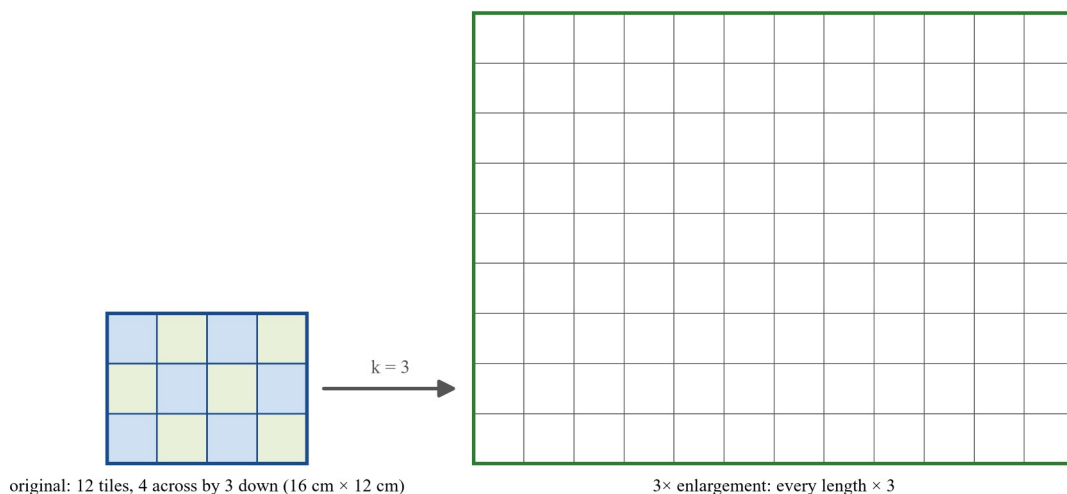
Q16. Two tablet screens are similar rectangles. The smaller screen’s diagonal is 18 cm and the larger screen’s diagonal is 27 cm.

- (a) What is the scale factor from the smaller screen to the larger one?
- (b) The smaller screen is 14 cm wide. How wide is the larger screen?

Q17. Storage box P is 10 cm long, 6 cm wide and 4 cm high. Storage box Q is 30 cm long, 18 cm wide and 12 cm high.

- Show that Q is similar to P and give the scale factor.
- Find the volume of each box and the volume factor, and check it against k^3 .
- A jug holds 2.5 L and exactly fills box P with packing beads. How many litres fill box Q?

Q18. A mosaic pattern is made from 12 square tiles, each 4 cm by 4 cm, laid 4 across and 3 down. The pattern is enlarged with scale factor 3, using more of the same 4 cm tiles.



The 12-tile original (16 cm $\times$ 12 cm) and its 3 $\times$ enlargement, drawn on a grid of the same 4 cm tiles.

- How many tiles does the enlargement need?
- What are the dimensions of the enlargement, and what is the scale factor?
- Find the area of the enlargement two different ways and the area factor, and check they agree.

Answers

Q1 (a) B — same three side lengths (3, 4, 5 cm), so congruent, $k=1$. (b) C — sides 6, 8, 10 cm, similar with $k=2$, not congruent. (c) D — matched ratios differ ($3.606 \div 3 \approx 1.2$ (1 d.p.) but $4 \div 5 = 0.8$, sides to 3 d.p.), so not similar. **Q2** (a) $14 \div 8 = 1.75$ and $21 \div 12 = 1.75$. (b) Yes, the ratios agree. (c) Enlargement, $k = 1.75$. **Q3** (a) 36 cm by 15 cm. (b) 34 cm $\rightarrow$ 102 cm, factor 3 ($= k$). (c) $60 \text{ cm}^2 \rightarrow 540 \text{ cm}^2$, factor 9 ($= k^2$). **Q4** (a) 100 cm by 36 cm. (b) Reduction, because $k = 0.4 < 1$. (c) $22\,500 \text{ cm}^2 \rightarrow 3\,600 \text{ cm}^2$; factor $0.16 = 4/25$ ($= k^2$). **Q5** (a) 270 cm. (b) 2.7 m. (c) 9.7 cm (to 1 d.p.). **Q6** (a) 3, 9, 27. (b) 12 cm, 6 cm, 3 cm. (c) $k = 1/8$. **Q7** Congruent — both plates have sides 30, 40, 50 cm (same shape and same size; turning does not change a shape). **Q8** Not similar: width ratio 3 but height ratio $40 \div 14 = 2.86$ (2 d.p.). A similar print 30 cm wide is 42 cm tall; the offer is 2 cm too short. **Q9** (a) 12 cm by 9 cm. (b) 7 m. **Q10** (a) B: $k=3$; C: $k=4$ (all matched ratios equal). (b) Yes; $k = 4/3 \approx 1.33$ (2 d.p.). **Q11** (a) 22 cm. (b) 4 cm. (c) 12.18 m. **Q12** (a) 6.25 ($= 2.5^2$), not 2.5. (b) 50 cm by 75 cm; 3 750 cm². (c) \$300. **Q13** (a) 2 000 mL = 2 L. (b) 500 mL is only 1/4 of 2 000 mL; volume scales by $2^3 = 8$, not by 2. **Q14** (a) 18 cm by 12 cm. (b) 2 400 cm² and 216 cm². (c) $9/100 = 0.09$, which equals $k^2 = 0.3^2$. **Q15** (a) $k = 1/4$; yes, the four small squares are congruent to each other. (b) 1/4 (check $4 \times (1/4)^2 = 1/4$). (c) \$25. **Q16** (a) $k = 1.5$. (b) 21 cm. **Q17** (a) Similar, $k = 3$ (all three matched ratios are 3). (b) 240 cm³ and 6 480 cm³; factor $27 = k^3$. (c) 67.5 L. **Q18** (a) 108 tiles. (b) 48 cm by 36 cm; $k = 3$. (c) 1 728 cm² both ways (48×36 and 108×16); area factor 9.

Fully-worked solutions

Q1 Read the sides off the grid. A: 3, 4, 5 cm. B: 3, 4, 5 cm (turned over). C: 6, 8, 10 cm. D: 4 cm base with two slant sides of about 3.606 cm each (3 d.p.). (a) B has exactly the same three side lengths as A, so B is congruent to A ($k=1$); it is just turned and flipped. (b) C's sides are A's sides doubled: $6 \div 3 = 2$, $8 \div 4 = 2$, $10 \div 5 = 2$, so C is similar to A with $k=2$. Because $k \neq 1$ it is not congruent. (c) For D against A, match shortest with shortest: $3.606 \div 3 \approx 1.2$ (1 d.p.); match longest with longest: $4 \div 5 = 0.8$. The ratios differ, so D is not similar to A (and not congruent, since the sides differ).

Q2 (a) Widths: $14 \div 8 = 1.75$. Heights: $21 \div 12 = 1.75$. (b) Both matched ratios equal 1.75, so the print is similar to the photo. (c) $k = 1.75 > 1$, so this is an enlargement.

Q3 (a) New length $12 \times 3 = 36$ cm; new width $5 \times 3 = 15$ cm. (b) Old perimeter $2 \times (12 + 5) = 34$ cm; new perimeter $2 \times (36 + 15) = 102$ cm; $102 \div 34 = 3$, the perimeter scales by k . (c) Old area $12 \times 5 = 60$ cm²; new area $36 \times 15 = 540$ cm²; $540 \div 60 = 9 = 3^2$, the area scales by k^2 .

Q4 (a) Flyer length $250 \times 0.4 = 100$ cm; flyer width $90 \times 0.4 = 36$ cm. (b) A reduction, because the scale factor $k = 0.4$ is less than 1. (c) Poster area $250 \times 90 = 22\,500$ cm²; flyer area $100 \times 36 = 3\,600$ cm²; factor $3\,600 \div 22\,500 = 0.16$, which is $4/25$ and equals 0.4^2 .

Q5 1:18 scale means real = model $\times$ 18, and model = real $\div$ 18. (a) $15 \times 18 = 270$ cm. (b) 270 cm = 2.7 m. (c) $175 \div 18 = 9.7222\dots$ cm, which rounds to **9.7 cm to 1 decimal place**.

Q6 (a) Each shaded triangle becomes 3, so the counts are $3^1 = 3$, $3^2 = 9$, $3^3 = 27$. (b) Sides halve each stage: 12 cm, 6 cm, 3 cm. (c) Stage 0 side 24 cm to stage 3 side 3 cm: $k = 3 \div 24 = 1/8$ (three halvings, $(1/2)^3$).

Q7 Both plates have the same three side lengths — 30 cm, 40 cm and 50 cm — so they have the same shape and the same size: they are congruent ($k=1$). Turning a plate over does not change congruence.

Q8 Widths: $30 \div 10 = 3$. Heights: $40 \div 14 = 2.857\dots$, that is 2.86 to 2 decimal places. The ratios differ, so the offered print is not similar — it is squashed. At the width ratio 3, a similar print would be $14 \times 3 = 42$ cm tall (check $42 \div 14 = 3 = 30 \div 10$). The offered 40 cm is $42 - 40 = 2$ cm too short.

Q9 1:50 means plan cm = real cm $\div$ 50, and real cm = plan cm $\times$ 50. (a) 6 m = 600 cm, so $600 \div 50 = 12$ cm on the plan; 4.5 m = 450 cm, so $450 \div 50 = 9$ cm on the plan. (b) $14 \times 50 = 700$ cm = 7 m.

Q10 (a) B against A: $24 \div 8 = 3$ and $18 \div 6 = 3$, so B is similar to A with $k = 3$. C against A: $32 \div 8 = 4$ and $24 \div 6 = 4$, so C is similar to A with $k = 4$. (b) B and C are both similar to A, so they are similar to each other: $32 \div 24 = 4/3$ and $24 \div 18 = 4/3$, so $k = 4/3 \approx 1.33$ (2 d.p.) from B to C.

Q11 1:87 means $\text{real} = \text{model} \times 87$ and $\text{model} = \text{real} \div 87$. (a) $1914 \div 87 = 22$ cm (check $22 \times 87 = 1914$). (b) $348 \div 87 = 4$ cm. (c) $14 \times 87 = 1218$ cm = 12.18 m exactly.

Q12 (a) Area factor $= k^2 = 2.5^2 = 6.25$. The wrong idea it is confused with is multiplying the area by $k = 2.5$. (b) Mural: $20 \times 2.5 = 50$ cm by $30 \times 2.5 = 75$ cm; area $50 \times 75 = 3750$ cm² (check: design area $20 \times 30 = 600$ cm² and $600 \times 6.25 = 3750$ cm²). (c) Paint rate: $\$48 \div 600 = \$0.08$ per cm²; mural paint $3750 \times 0.08 = \$300$ (same as $\$48 \times 6.25$).

Q13 (a) Every length $\times 2$, so volume $\times 2^3 = 8$: $250 \times 8 = 2000$ mL = 2 L. (b) Doubling the lengths does not double the capacity. The guess of 500 mL is $500 \div 2000 = 1/4$ of the true capacity — it comes from thinking “double” once or twice instead of in all three directions ($2^3 = 8$).

Q14 (a) Model: $60 \times 0.3 = 18$ cm by $40 \times 0.3 = 12$ cm. (b) Real area $60 \times 40 = 2400$ cm²; model area $18 \times 12 = 216$ cm². (c) $216 \div 2400 = 9/100 = 0.09$, and $k^2 = 0.3^2 = 0.09$ — they agree (check $2400 \times 0.09 = 216$).

Q15 (a) Small square to block: $9 \div 36 = 1/4$, so $k = 1/4$. The four small squares all have sides 9, 9, 9, 9 cm, so they are congruent to each other. (b) One small square $9 \times 9 = 81$ cm²; shaded $4 \times 81 = 324$ cm²; block $36 \times 36 = 1296$ cm²; fraction $324 \div 1296 = 1/4$. Shortcut: 4 squares each covering $(1/4)^2 = 1/16$ of the block gives $4 \times 1/16 = 1/4$. (c) Shaded fabric costs $1/4$ of $\$100 = \$25$.

Q16 (a) The diagonals are matching lengths, so $k = 27 \div 18 = 1.5$. (b) Larger width $= 14 \times 1.5 = 21$ cm (check $21 \div 14 = 1.5$).

Q17 (a) Matched ratios: $30 \div 10 = 3$, $18 \div 6 = 3$, $12 \div 4 = 3$ — all equal, so Q is similar to P with $k = 3$. (b) P: $10 \times 6 \times 4 = 240$ cm³; Q: $30 \times 18 \times 12 = 6480$ cm³; factor $6480 \div 240 = 27 = 3^3 = k^3$. (c) Beads scale with volume: $2.5 \times 27 = 67.5$ L.

Q18 (a) The original pattern is $4 \times 4 = 16$ cm by $3 \times 4 = 12$ cm (area $16 \times 12 = 192$ cm², 12 tiles of 16 cm² each). At $k = 3$ the enlargement holds $3 \times 3 = 9$ copies of the pattern, so it needs $12 \times 9 = 108$ of the same tiles. (b) Enlarged dimensions: $16 \times 3 = 48$ cm by $12 \times 3 = 36$ cm; scale factor $48 \div 16 = 3$. (c) Area by dimensions: $48 \times 36 = 1728$ cm². Area by tiles: $108 \times 16 = 1728$ cm². Area factor $= 3^2 = 9$, and the check $192 \times 9 = 1728$ cm² agrees with both.

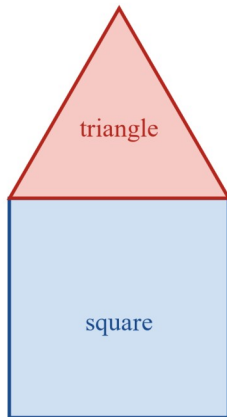
Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-07.

Theory

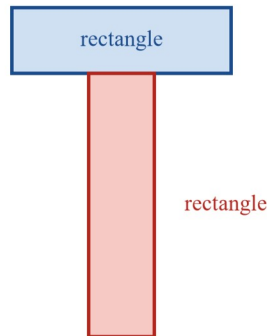
Compound 2D shapes

A **compound shape** (also called a composite shape) is a flat shape made by joining simple shapes — rectangles, triangles, circles and parts of circles — edge to edge. To describe a compound shape, name the simple parts and say how they are joined: which edges are shared, and which edges end up hidden inside the shape.

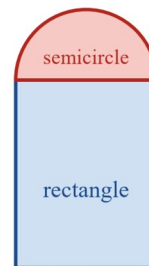
(a) square + triangle



(b) rectangle + rectangle



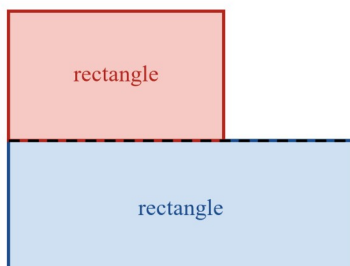
(c) rectangle + semicircle



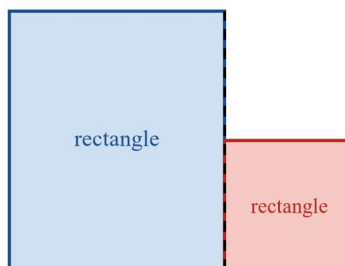
In (a) a square and a triangle share one edge to make a house outline. In (b) two rectangles meet to make a T. In (c) a semicircle sits on the top edge of a rectangle to make an arch window. Each is one shape, made of two simple pieces.

The same compound shape can be split into its simple parts in more than one way. Both splits below show the *same* L-shape made of two rectangles:

Split A — one horizontal cut



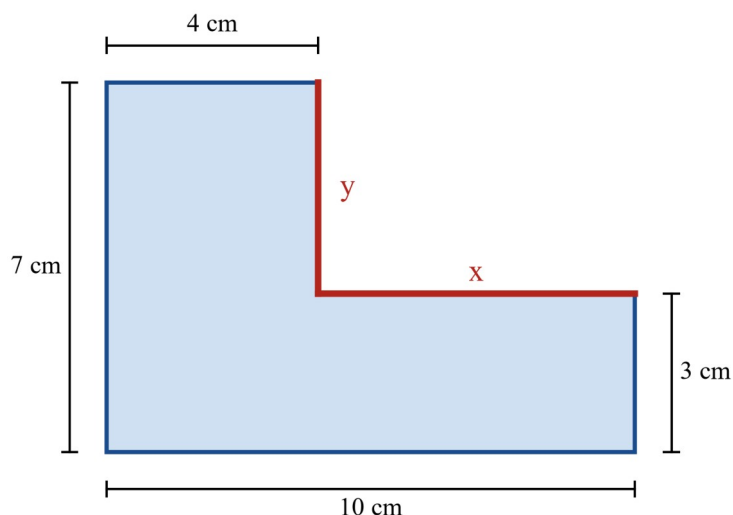
Split B — one vertical cut



Split A uses one horizontal cut; split B uses one vertical cut. Either description is correct, and both give two rectangles. (Measuring compound shapes — perimeter and area — is in sections 4B and 4C. Here we make compound shapes, describe them, and use the relationships between their side lengths.)

Missing lengths: part–whole relationships

In a compound shape whose corners are all right angles, sides on one side of the shape add up to the matching sides on the other side. In the L-shape below, the two top pieces (4 cm and x) together match the 10 cm bottom edge, and the two right-hand pieces (3 cm and y) together match the 7 cm left edge:



- $x = 10 - 4 = 6$, so $x = 6$ cm
- $y = 7 - 3 = 4$, so $y = 4$ cm
- Check: $4 + 6 = 10$ ✓ and $3 + 4 = 7$ ✓

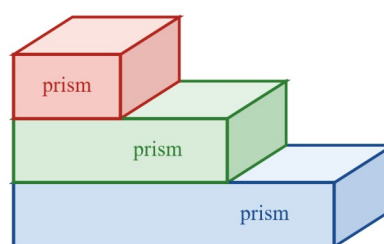
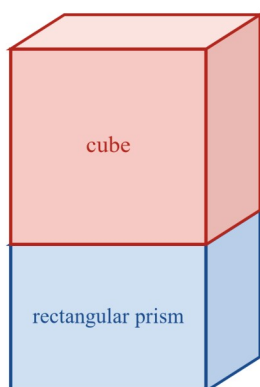
This part-whole relationship is the key tool for finding missing sides of any right-angled compound shape: label what you know, then ask which sides must add up to which.

Compound 3D objects

Solids can be joined face to face in the same way. When two solids are joined, the touching faces are hidden inside the object, so the outside of a compound solid has fewer faces than the two solids had separately. The front, side and top views of a compound solid can themselves be compound 2D shapes.

(a) a cube on a rectangular prism

(b) steps made of three rectangular prisms



In (a) a cube sits on a rectangular prism: the top face of the prism and the bottom face of the cube are hidden where they join. In (b) three rectangular prisms are stacked to make a set of steps — a compound solid you meet every day.

Nets and cross-sections

Two ideas connect 3D objects to 2D shapes:

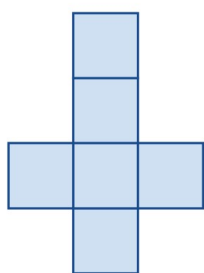
- A **net** is a flat pattern that folds up to make a solid. Every face of the solid appears once in the net.
- A **cross-section** is the flat shape you see when a solid is cut straight through. The cross-section depends on the direction of the cut.

(a) net of a cube

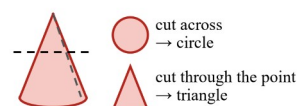
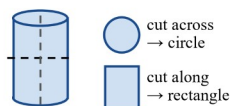
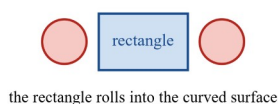
(b) net of a cylinder

(c) cross-sections of a cylinder

(d) cross-sections of a cone



6 squares fold into a cube



In (a) six squares in a cross fold into a cube. In (b) a rectangle and two circles fold into a cylinder — the rectangle rolls into the curved surface. In (c) a cylinder cut straight across gives a circle, while a cut along its length gives a rectangle. In (d) a cone cut straight across gives a circle, while a cut through its point gives a triangle. A net unfolds the *surfaces* of a solid; a cross-section slices through its *inside*. Both are 2D shapes that belong to a 3D object.

Patterns in and between shapes

Shapes make patterns too. A **repeating pattern** uses the same shape (or group of shapes) over and over, moved by slides, flips or turns. A **growing pattern** adds a fixed piece each time, so the terms keep getting bigger in a predictable way.

A **tessellation** is a repeating pattern of shapes that covers a flat surface completely, with no gaps and no overlaps. Floor tiles, pavers and honeycomb are everyday tessellations. Whether one regular polygon tessellates on its own is decided by the angles at a point: the angles around any point add to 360° , so copies of the polygon must fit around a point exactly. A **regular polygon** has all sides the same length and all angles the same size; the angle inside the shape at a corner is its **interior angle**.

Splitting a polygon into triangles from one vertex gives its interior angles: an n -sided polygon splits into $(n - 2)$ triangles, so its angles add to $(n - 2) \times 180^\circ$, and a regular polygon's interior angle is that total divided by n . For a regular hexagon: 4 triangles, $4 \times 180^\circ = 720^\circ$, and $720^\circ \div 6 = 120^\circ$.

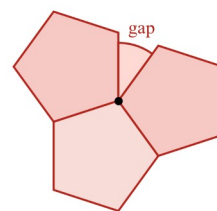
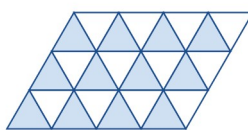
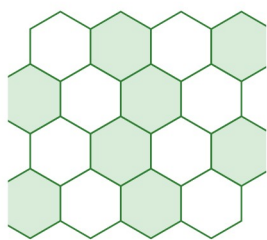
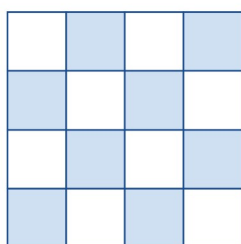
Regular polygon	Interior angle	Copies at one point	Angles at the point
Equilateral triangle	60°	6	$6 \times 60^\circ = 360^\circ$ — fits
Square	90°	4	$4 \times 90^\circ = 360^\circ$ — fits
Regular hexagon	120°	3	$3 \times 120^\circ = 360^\circ$ — fits
Regular pentagon	108°	3 leaves a gap; 4 overlaps	$3 \times 108^\circ = 324^\circ$ (gap 36°); $4 \times 108^\circ = 432^\circ$
Regular octagon	135°	2 leaves a gap; 3 overlaps	$2 \times 135^\circ = 270^\circ$ (gap 90°); $3 \times 135^\circ = 405^\circ$

(a) squares: 4 at a point

(b) hexagons: 3 at a point

(c) triangles: 6 at a point

(d) pentagons: a gap at the point



Squares, regular hexagons and equilateral triangles each fill the point exactly, so each tessellates on its own. Regular pentagons do not: three leave a 36° gap and four overlap, so pentagons alone can never cover a surface without gaps or overlaps. Growing patterns are described with a rule in words — for example, the matchstick triangles in Example 4 grow by 2 matches every term.

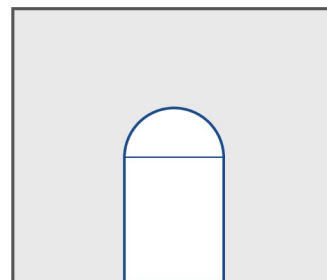
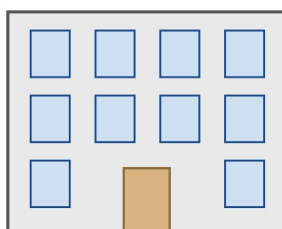
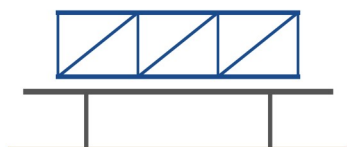
Patterns in engineering, architecture and design

Shape patterns are not only decoration — they do jobs.

(a) a bridge truss — repeating triangles

(b) a building front — a repeating grid

(c) an arch — rectangle + semicircle



- **Bridges.** A **truss** is a frame of straight members joined into triangles, like the bridge in (a). Triangles are **rigid** — stiff, they keep their shape when pushed — while a square frame collapses sideways into a slanted shape. That is why bridges, roofs and cranes repeat triangles.
- **Buildings.** The front in (b) is a repeating grid of congruent windows (congruent means identical in shape and size). Repeating one window shape keeps the design simple to draw, simple to build and simple to glaze.
- **Arches and sculptures.** The arch in (c) is a rectangle with a semicircle on top, repeated in doorways, bridges and arcades. Sculptures often stack, slide or turn one simple solid many times.

To name any shape pattern, say two things: what repeats (the unit — a triangle pair, a window, a house shape), and how it repeats (slid along a line, flipped, turned, or growing by a fixed amount each term).

Shape technologies

Technology creates and manipulates shapes far faster than hand drawing:

Technology	What it does with shapes
Geometry app	Draws shapes on screen; slides, flips and turns them to test ideas such as whether a shape tessellates
CAD (computer-aided design)	Draws exact 2D and 3D models of buildings and products, with lengths and angles stored precisely
Laser cutter	Cuts a flat net or pattern from sheet material, straight from a 2D drawing
3D printer	Builds a solid in thin flat layers — each layer is a cross-section of the object

These tools are worth seeing and worth using, but nothing in this section needs them: pen, paper and a basic calculator are enough for every example and question here.

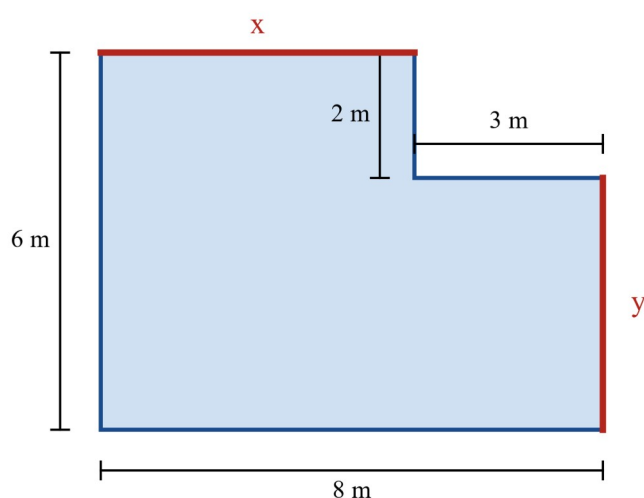
About the data in this section. The measurements and counts here — paving areas, silo heights, printer layers, brick rows, honeycomb cells, mural walls — are **made up** (not real): they are chosen to give clean numbers for practising shape ideas.

Worked examples

Example 1 — scaffolded

A paving area in a courtyard is the L-shape shown, with the four marked measurements. Find (a) x , the top edge, and (b) y , the right-hand edge. (c) Check both answers against the figure.

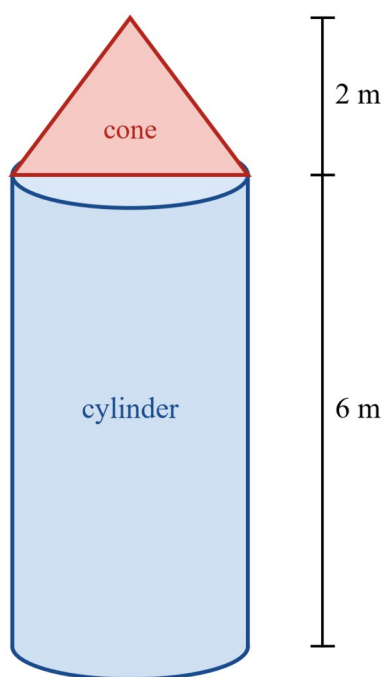
Working	Comment
(a) The top edge x and the 3 m piece together match the 8 m bottom edge: $x + 3 = 8$. So $x = 5$ m.	Part-whole relationship: the two top pieces add to the bottom edge.
(b) The right edge y and the 2 m piece together match the 6 m left edge: $y + 2 = 6$. So $y = 4$ m.	The same relationship, turned a quarter turn: the two right pieces add to the left edge.
(c) Check: $5 + 3 = 8$ ✓ and $4 + 2 = 6$ ✓.	The pieces add back to the given sides, so both answers fit the figure exactly.



Example 2 — scaffolded

A farm silo is a cylinder with a cone roof sitting on top. The cylinder body is 6 m tall and the cone roof adds another 2 m of height. (a) Name the two solids. (b) Find the total height of the silo. (c) Describe the shape of (i) the side view, (ii) the top view, and (iii) the curved surface of the cylinder unrolled flat.

Working	Comment
(a) The body is a cylinder ; the roof is a cone .	The two simple solids joined face to face make the compound solid.
(b) Total height = $6 + 2 = 8$. The silo is 8 m tall.	Heights stack: body height plus roof height.
(c)(i) A rectangle with a triangle on top — a compound 2D shape.	The side view of the cylinder is a rectangle; the cone looks like a triangle from the side.
(c)(ii) A circle.	From directly above, the cone roof hides the cylinder, and both look circular.
(c)(iii) A rectangle.	Unrolling the curved surface flattens it — the rectangle from the cylinder's net.



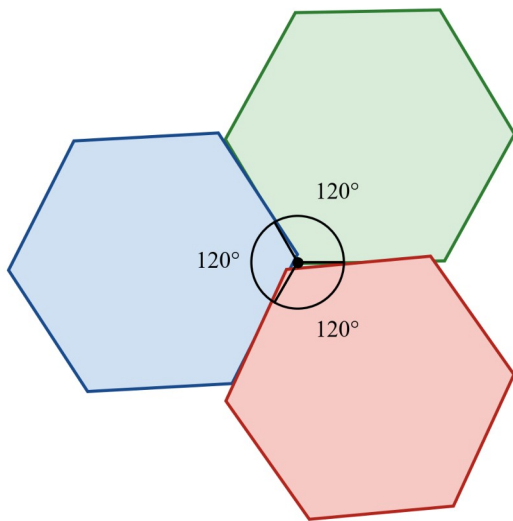
Example 3 — unscaffolded

A tiler claims that regular hexagons tessellate on their own but regular octagons do not. Show that the interior angle of a regular hexagon is 120° , then use the angles at a point to justify both halves of the claim.

Split a regular hexagon into triangles from one vertex: this gives 4 triangles. Their angles add to $4 \times 180^\circ = 720^\circ$. The hexagon's six interior angles share that total equally: $720^\circ \div 6 = 120^\circ$.

At a point in a hexagon tessellation, three hexagons meet (see figure): $3 \times 120^\circ = 360^\circ$, exactly one full turn. There is no gap and no overlap, so regular hexagons tessellate on their own.

For a regular octagon, the same split gives 6 triangles: $6 \times 180^\circ = 1080^\circ$, so one interior angle is $1080^\circ \div 8 = 135^\circ$. Three octagons at a point would give $3 \times 135^\circ = 405^\circ$, more than a full turn, so they overlap. Two give $2 \times 135^\circ = 270^\circ$, leaving a gap of $360^\circ - 270^\circ = 90^\circ$. No whole number of octagons fills the point exactly.

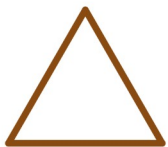


∴ Regular hexagons tessellate on their own, and regular octagons do not.

Example 4 — unscaffolded

Matchsticks are joined to make a growing strip of triangles. Term 1 uses 3 matches, term 2 uses 5, term 3 uses 7, and each new triangle adds 2 more matches. Find a rule in words for the number of matches in any term, use it to find the number of matches in term 10, and check your answer a different way.

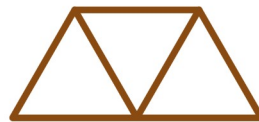
From term to term the matches grow by 2, and term 1 is $3 = 2 \times 1 + 1$. Rule in words: **matches = 2 × term number + 1** — double the term number, then add one. Term 10: $2 \times 10 + 1 = 21$ matches. Check a different way: start at term 1 (3 matches) and add 2 for each of the 9 steps up to term 10: $3 + 2 \times 9 = 21$ ✓ — the same answer.



Term 1 — 3 matches



Term 2 — 5 matches



Term 3 — 7 matches

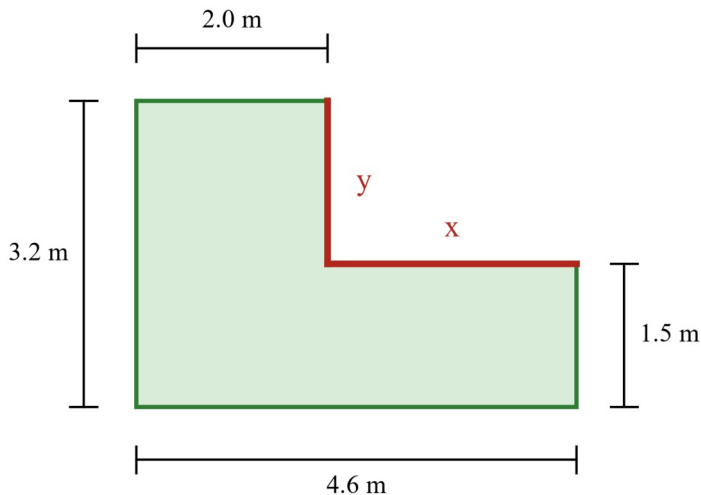
∴ Term 10 of the pattern uses 21 matches.

Practice questions

Scaffolded

Q1. A sign is made by joining an equilateral triangle to the top edge of a square sign board, so the shared edge is hidden inside the sign. (a) Sketch the sign and label the two simple shapes. (b) Count the sides of the outline of the sign. (The shared edge is not part of the outline.) (c) Give the special name for a polygon with that many sides.

Q2. A garden bed is the L-shape shown, with four sides marked.



- (a) Find x .
- (b) Find y .
- (c) Check both answers by adding the pieces back together.

Q3. A sculpture is built from identical cubes stacked in steps: the bottom row has 4 cubes, then 3, then 2, then 1 on top. (a) How many cubes are in the sculpture? (b) Sketch the front view. How many square faces does it show? (c) Is the front view a simple shape or a compound shape? Give a reason.

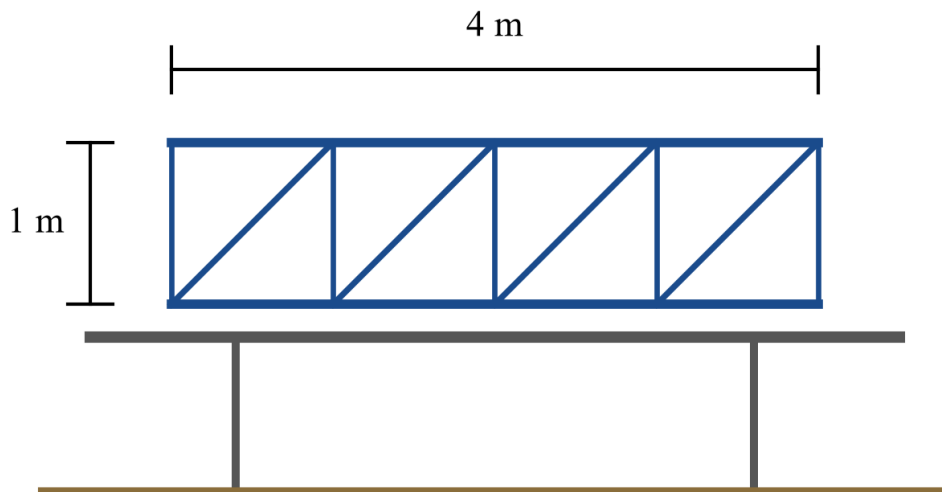
Q4. Copy and complete: a tessellation covers a flat surface with no _____ and no _____. (a) Fill in the two missing words. (b) Equilateral triangles have interior angles of 60° . How many meet at one point in a triangle tessellation? Check by multiplying. (c) Repeat part (b) for squares, whose interior angles are 90° .

Q5. A 3D printer builds a model in thin flat layers, each 0.25 mm thick. (a) How many layers does a model 18 mm tall need? (b) Each layer is a flat slice of the solid. Which 2D-shape idea from this section is each layer an example of? (c) A second model uses layers 0.35 mm thick and must be at least 18 mm tall. Find the smallest whole number of layers needed.

Q6. A brick wall is built in 9 rows. Row 1 starts with a whole brick, row 2 starts with a half brick, and the rows keep alternating so the vertical joints never line up. (a) What shape is the face of the finished wall? (b) How many of the 9 rows start with a half brick? (c) Describe the pattern of the rows in words.

Unscaffolded

Q7. A pedestrian bridge uses the truss shown: 4 m long and 1 m deep, made of four 1 m square panels, each panel cut by one diagonal (a line from corner to corner).



- (a) How many triangles make up the truss?
- (b) Explain why trusses are built from triangles rather than squares.
- (c) Describe the repeating pattern in the truss.

Q8. Squares of side 5 cm are cut from each corner of a 24 cm by 18 cm rectangle of card, and the four flaps are folded up to make an open box. (a) Sketch the cross-shaped net, marking the cut-out corners. (b) Find the length and the width of the base of the box. (c) State the height of the box and name the solid it forms (it has no top).

Q9. A birdhouse is a cube with a square-based pyramid roof; the base of the pyramid is the top face of the cube, hidden inside. (a) Name the two solids. (b) How many faces are on the outside of the birdhouse? (c) Sketch the side view and describe the compound 2D shape you see. How many sides does its outline have?

Q10. (a) Show that the interior angle of a regular pentagon is 108° . (b) Find $360 \div 108$, correct to two decimal places. (c) Use your answer to (b) to explain why regular pentagons do not tessellate on their own. (d) Circles are not polygons. Do circles tessellate? Give a reason.

Q11. A pattern of square tiles grows: term 1 is a 1×1 square of tiles, term 2 is 2×2 , term 3 is 3×3 , and so on. (a) How many tiles are in term 4 and in term 5? (b) Write a rule in words for the number of tiles in any term. (c) You have 40 tiles. What is the largest complete term you can make, and how many tiles are left over?

Q12. The front of an office building has 5 rows of windows with 6 identical windows in each row. (a) How many windows are there? (b) Describe the pattern the windows make. (c) The doorway is a rectangle with a semicircle on top. Describe this compound shape and name one other place in this section where you met it.

Q13. (a) How many squares are in any net of a cube? (b) Sketch one net of a cube. (c) A triangular prism has 2 triangular faces and 3 rectangular faces. How many faces does its net have? Sketch the net.

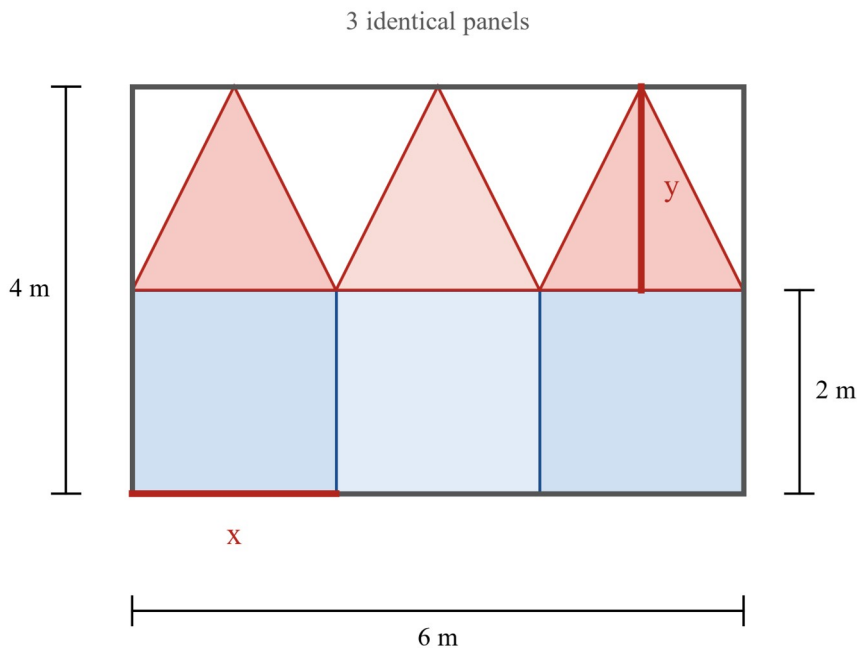
Q14. (a) A carrot is roughly a cylinder. What cross-section do you get by cutting it (i) straight across, and (ii) lengthways through the middle? (b) A party hat is roughly a cone. What cross-section do you get by cutting it (i) straight across, parallel to the base, and (ii) straight down through the point of the hat?

Q15. A border pattern is a strip of 11 equilateral triangles joined edge to edge, point-up and point-down alternately, starting point-up. (a) What single transformation takes one point-up triangle to the next point-up triangle along the strip? (b) What single transformation takes a point-up triangle to its point-down neighbour? (c) How many point-up triangles and how many point-down triangles are in the strip?

Q16. A honeycomb frame has five rows of cells: 8, 7, 8, 7, 8. (a) How many cells are in the first two rows? (b) How many cells are in the whole frame? (c) Each cell is the same regular polygon. Name it, and explain why that shape tessellates.

Q17. Match each job to the most suitable technology: geometry app, CAD, laser cutter, 3D printer. (a) Cutting a net accurately from a flat sheet of plastic. (b) Building a solid model in thin flat layers. (c) Drawing an exact 3D model of a building, with all lengths stored precisely. (d) Sliding and turning a shape on screen to test whether it tessellates.

Q18. A mural wall is 6 m wide and 4 m high. It is painted with 3 identical panels side by side. Each panel is a rectangle at the bottom with a triangle on top; the rectangle is 2 m high and the triangle reaches the top of the wall.



- Find x , the width of one panel.
- Find y , the height of the triangle part of a panel.
- The bottom part of a panel is 2 m wide and 2 m high. What special name does this shape have?
- Describe the pattern across the whole wall.

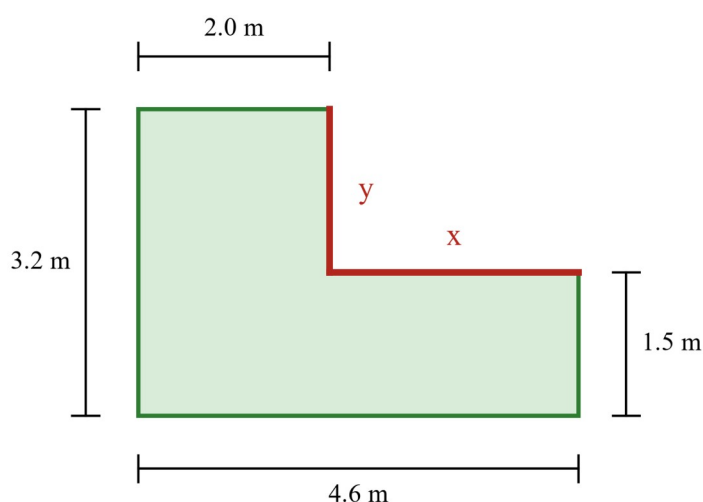
Answers

Q1. (a) Sketch: a square with an equilateral triangle on its top edge, shared edge inside. (b) 5 sides. (c) Pentagon. **Q2.** (a) $x = 2.6$ m. (b) $y = 1.7$ m. (c) $2.0 + 2.6 = 4.6$ ✓ and $1.5 + 1.7 = 3.2$ ✓. **Q3.** (a) 10 cubes. (b) 10 squares. (c) Compound — it is made of 10 squares joined edge to edge (a steps outline). **Q4.** (a) Gaps; overlaps. (b) 6; $6 \times 60^\circ = 360^\circ$ ✓. (c) 4; $4 \times 90^\circ = 360^\circ$ ✓. **Q5.** (a) 72 layers. (b) A cross-section. (c) 52 layers. **Q6.** (a) A rectangle. (b) 4 rows. (c) A two-row repeating pattern: whole-brick start, then half-brick start. **Q7.** (a) 8 triangles. (b) Triangles are rigid — a square frame collapses sideways, a triangle keeps its shape. (c) A 1 m square panel cut by a diagonal into two right triangles repeats 4 times along the 4 m span. **Q8.** (a) Sketch: a cross shape — 14 cm by 8 cm base with four 5 cm flaps, corners removed. (b) 14 cm by 8 cm. (c) 5 cm; an open rectangular prism (a box with no top). **Q9.** (a) A cube and a square-based pyramid. (b) 9 faces. (c) A square with a triangle on top (house shape); its outline has 5 sides. **Q10.** (a) $3 \times 180^\circ = 540^\circ$; $540^\circ \div 5 = 108^\circ$. (b) 3.33. (c) 3.33 is not a whole number: 3 pentagons leave a 36° gap, 4 overlap. (d) No — curved edges leave gaps between circles. **Q11.** (a) 16; 25. (b) Tiles = term number $\times$ term number. (c) Term 6 (36 tiles); 4 tiles left over. **Q12.** (a) 30 windows. (b) A repeating grid of congruent rectangles, 5 rows by 6 columns. (c) Rectangle + semicircle — the arch shape of the theory figure (the arch window). **Q13.** (a) 6. (b) Sketch: any cross, T or L arrangement of 6 squares that folds to a cube. (c) 5 faces: 2 triangles and 3 rectangles. **Q14.** (a) (i) A circle. (ii) A rectangle. (b) (i) A circle. (ii) A triangle. **Q15.** (a) A slide (translation). (b) A flip (reflection) in the shared edge, or a half-turn about the midpoint of that edge. (c) 6 point-up and 5 point-down. **Q16.** (a) 15 cells. (b) 38 cells. (c) A regular hexagon; $3 \times 120^\circ = 360^\circ$ at every point, so it fits exactly. **Q17.** (a) Laser cutter. (b) 3D printer. (c) CAD. (d) Geometry app. **Q18.** (a) $x = 2$ m. (b) $y = 2$ m. (c) A square. (d) A repeating pattern: the same house shape (square + triangle) slid along the wall 3 times.

Fully-worked solutions

Q1. (a) Sketch a square, then an equilateral triangle sitting on its top edge; the shared edge is inside the sign, so label the square and the triangle. (b) The triangle contributes $3 - 1 = 2$ outline sides and the square contributes $4 - 1 = 3$, so the outline has $2 + 3 = 5$ sides. (c) A five-sided polygon is a **pentagon**.

Q2. (a) The top pieces 2.0 m and x match the 4.6 m bottom edge: $x = 4.6 - 2.0 = 2.6$, so $x = 2.6$ m. (b) The right pieces 1.5 m and y match the 3.2 m left edge: $y = 3.2 - 1.5 = 1.7$, so $y = 1.7$ m. (c) Check: $2.0 + 2.6 = 4.6$ ✓ and $1.5 + 1.7 = 3.2$ ✓ — both answers fit the figure.



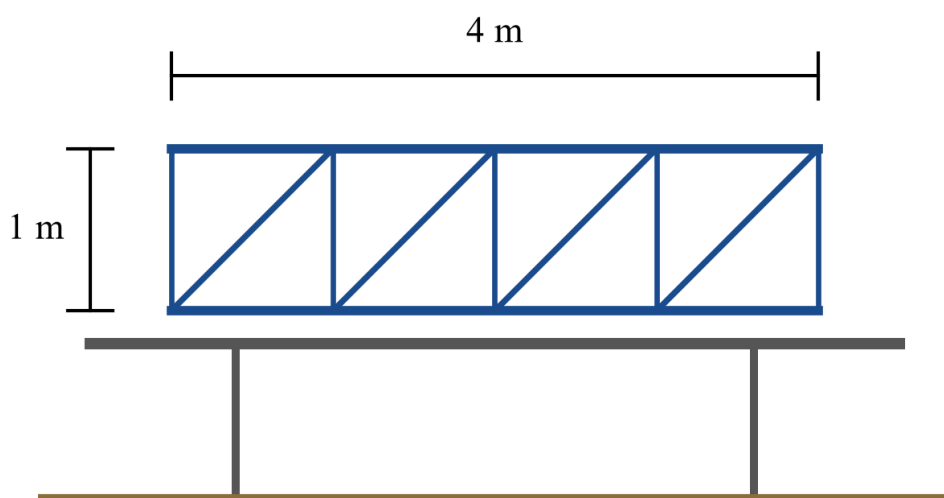
Q3. (a) $4 + 3 + 2 + 1 = 10$ cubes. (b) From the front each cube shows one square face, so the front view shows 10 squares in steps: rows of 4, 3, 2, 1. (c) The front view is a **compound shape** — it is 10 squares joined edge to edge, and its outline is a steps shape, not one simple shape.

Q4. (a) A tessellation covers a flat surface with no **gaps** and no **overlaps**. (b) 6 triangles meet at a point: $6 \times 60^\circ = 360^\circ$, exactly one full turn, so the point is filled. (c) 4 squares meet at a point: $4 \times 90^\circ = 360^\circ$ — again exactly one full turn.

Q5. (a) $18 \div 0.25 = 72$ layers. (b) Each flat layer is a **cross-section** of the solid — the same idea as cutting a cylinder or cone straight across. (c) Test 51 layers: $51 \times 0.35 = 17.85$ mm, which is $18 - 17.85 = 0.15$ mm short of 18 mm. One more layer gives $52 \times 0.35 = 18.2$ mm, which reaches 18 mm. Layers come in whole numbers, so the smallest number of layers is 52.

Q6. (a) The face of the wall is a **rectangle**. (b) Rows 2, 4, 6 and 8 start with a half brick: 4 rows. (c) The pattern repeats every two rows: a row starting with a whole brick, then a row starting with a half brick.

Q7. (a) Each of the 4 square panels is cut into 2 triangles by its diagonal: $4 \times 2 = 8$ triangles. (b) A triangle is rigid — it keeps its shape when pushed — while a square frame collapses sideways into a slanted shape. Repeating triangles keeps the bridge stiff under load. (c) The unit is one 1 m square panel cut by a diagonal into two congruent right triangles; this unit repeats 4 times, slid along the 4 m span.



Q8. (a) Sketch: a 24 cm by 18 cm rectangle with a 5 cm square marked at each corner; with the corners removed, the remaining cross shape folds up. (b) Cutting 5 cm from each end removes $2 \times 5 = 10$ cm from each measurement: base length = $24 - 10 = 14$ cm and base width = $18 - 10 = 8$ cm. (c) The flaps are as tall as the cut squares, so the height is 5 cm. The solid is a **rectangular prism with no top** (an open box).

Q9. (a) A **cube** and a **square-based pyramid**. (b) The cube shows $6 - 1 = 5$ outside faces (its top face is hidden), and the pyramid shows $5 - 1 = 4$ (its four triangles; its square base is hidden): $5 + 4 = 9$ faces. (c) The side view is a square with a triangle on top — a house shape. Its outline has 5 sides, so the outline is a pentagon (the same compound shape as Q1's sign).

Q10. (a) Split a regular pentagon into triangles from one vertex: this gives 3 triangles. Their angles add to $3 \times 180^\circ = 540^\circ$. The five interior angles share that total equally: $540^\circ \div 5 = 108^\circ$, which is what was to be shown. (b) $360 \div 108 = 3.3333\dots$; correct to two decimal places this is 3.33. (c) A whole number of pentagons must fit at a point, but 3.33 is not a whole number. With 3 pentagons: $3 \times 108^\circ = 324^\circ$, leaving a gap of $360^\circ - 324^\circ = 36^\circ$. With 4: $4 \times 108^\circ = 432^\circ$, which overlaps. So regular pentagons cannot tessellate on their own. (d) No. Circles have curved edges, so circles touching at points always leave gaps between them — a tessellation allows no gaps.

Q11. (a) Term 4: $4 \times 4 = 16$ tiles. Term 5: $5 \times 5 = 25$ tiles. (b) Rule in words: **tiles = term number $\times$ term number** (multiply the term number by itself). (c) Term 6 needs $6 \times 6 = 36$ tiles, which 40 covers; term 7 needs $7 \times 7 = 49$, which 40 does not. So the largest complete term is term 6, with $40 - 36 = 4$ tiles left over.

Q12. (a) $5 \times 6 = 30$ windows. (b) A repeating grid (an array) of congruent rectangles: 5 rows by 6 columns, each window the same shape and size. (c) The doorway is a **rectangle with a semicircle on top** — the arch shape met in the theory figure (arch window) and in the bridge/arch panel of the design figure.

Q13. (a) A cube has 6 faces, so any net of a cube has 6 squares. (b) Sketch: for example, a cross of 6 squares — four in a column with one square on each side of the second square (as in the theory figure). (c) The net shows every face once: $2 + 3 = 5$ faces — 2 triangles (the two ends) and 3 rectangles (the three side faces), with the rectangles in a strip and a triangle on each of two opposite edges.

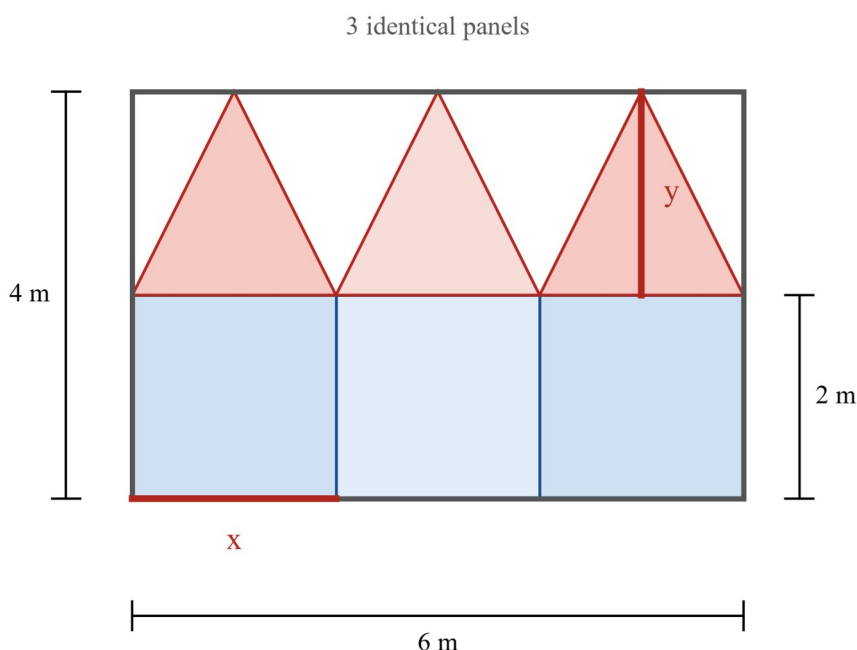
Q14. (a) (i) Straight across, the cut face is a **circle** (the cylinder's cross-section). (ii) Lengthways through the middle, the cut face is a **rectangle**. (b) (i) Straight across, parallel to the base, the cut face is a **circle** (smaller than the base). (ii) Straight down through the point, the cut face is a **triangle**.

Q15. (a) A **slide** (translation) along the strip takes one point-up triangle to the next point-up triangle. (b) A **flip** (reflection) in the shared slanted edge takes a point-up triangle to its point-down neighbour — a half-turn about the midpoint of that edge works too. (c) The strip alternates starting point-up: of the 11 triangles, 6 are point-up and 5 are point-down ($6 + 5 = 11$).

Q16. (a) $8 + 7 = 15$ cells. (b) $8 + 7 + 8 + 7 + 8 = 38$ cells. (c) Each cell is a **regular hexagon**. Its interior angle is 120° , and $3 \times 120^\circ = 360^\circ$ at every point where cells meet, so hexagons fill the plane with no gaps and no overlaps — which is exactly why honeycomb is built this way.

Q17. (a) **Laser cutter** — it cuts a flat net from sheet material straight from a 2D drawing. (b) **3D printer** — it builds a solid in thin flat layers, each a cross-section. (c) **CAD** — computer-aided design stores exact 2D and 3D models with all lengths. (d) **Geometry app** — it slides, flips and turns shapes on screen to test ideas such as tessellation.

Q18. (a) The 6 m wall holds 3 identical panels side by side: $x = 6 \div 3 = 2$, so $x = 2$ m. (b) The wall is 4 m high and the rectangle part is 2 m high: $y = 4 - 2 = 2$, so $y = 2$ m. (c) The bottom part is 2 m wide and 2 m high — all four sides equal and all corners right angles — so it is a **square**. (d) The unit is one house shape (a square with a triangle on top); it repeats 3 times, slid along the wall, filling the 6 m by 4 m rectangle exactly.



Chapter 03 — Test A (multiple choice)

Chapter 3 (Shape). Materials: one scientific calculator and paper. There are 12 multiple-choice questions, each worth 1 mark (12 marks). Approximate time: 25 minutes. Choose the one correct response for each question. Shapes described in words are not drawn to scale. Every measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1

A quadrilateral has 2 pairs of parallel sides and 4 equal sides, but none of its angles are right angles. What is its most exact name?

- A. Square
- B. Rhombus
- C. Rectangle
- D. Parallelogram

Question 2

A hexagonal prism has a cross-section that is a hexagon (a 6-sided polygon). How many edges does it have?

- A. 8
- B. 6
- C. 12
- D. 18

Question 3

Two angles sit together on a straight line. One of them is 118° . What is the size of the other angle?

- A. 62°
- B. 72°
- C. 118°
- D. 242°

Question 4

A triangle has angles of 48° and 67° . What is the size of the third angle?

- A. 115°
- B. 132°
- C. 65°
- D. 113°

Question 5

A triangle has interior angles of 37° and 46° . One side is extended past the third corner to make an exterior angle. What is the size of this exterior angle?

- A. 97°
- B. 83°
- C. 180°
- D. 277°

Question 6

How many axes of symmetry does a rectangle have?

- A. 0
- B. 1
- C. 2
- D. 4

Question 7

A design has rotational symmetry of order 8. What is the smallest turn that maps the design onto itself?

- A. 45°
- B. 60°
- C. 72°
- D. 90°

Question 8

A photo is 8 cm wide and 12 cm tall. A print of the photo is 20 cm wide and 30 cm tall. Which statement is correct?

- A. The print is similar to the photo, with scale factor 2.4.
- B. The print is similar to the photo, with scale factor 2.5.
- C. The print is not similar to the photo, because the width and height ratios differ.
- D. The print is similar to the photo, with scale factor 0.4.

Question 9

A solid object is enlarged with scale factor $k = 3$. By what factor is the volume of the solid increased?

- A. 3
- B. 9
- C. 27
- D. 6

Question 10

A model car is built at 1 : 18 scale: 1 cm on the model matches 18 cm on the real car. The model is 15 cm long. What is the length of the real car?

- A. 33 cm
- B. 27 cm
- C. about 0.83 cm
- D. 270 cm

Question 11

Which one of these regular polygons cannot tessellate on its own (cannot cover a flat surface with no gaps and no overlaps)?

- A. Equilateral triangle
- B. Square
- C. Regular hexagon
- D. Regular pentagon

Question 12

A growing pattern is built from matchsticks. Term 1 uses 4 matches, term 2 uses 7 matches and term 3 uses 10 matches, and the pattern grows by the same number of matches each term. How many matches does term 8 use?

- A. 25
- B. 24
- C. 22
- D. 28

End of Test A

Test A — Answers

Que stio n	1	2	3	4	5	6	7	8	9	10	11	12
Ans wer	B	D	A	C	B	C	A	B	C	D	D	A

Test A — Fully-worked solutions

Question 1. Two pairs of parallel sides put the shape in the parallelogram family. Four equal sides narrow it to a rhombus or a square. A square needs four right angles, and this shape has none, so it is a **rhombus**. **Answer B.**

Question 2. A prism whose cross-section is an n -sided polygon has $3n$ edges. Here $n=6$, so the number of edges is $3 \times 6 = 18$. (Its faces are $n+2=8$ and its vertices are $2n=12$.) **Answer D.**

Question 3. Angles on a straight line add to 180° . The other angle is $180^\circ - 118^\circ = 62^\circ$. **Answer A.**

Question 4. The angles of a triangle add to 180° . The third angle is $180^\circ - 48^\circ - 67^\circ = 65^\circ$. **Answer C.**

Question 5. An exterior angle of a triangle equals the sum of the two interior angles far away from it: $37^\circ + 46^\circ = 83^\circ$. **Answer B.**

Question 6. A rectangle folds onto itself only along the two lines joining the midpoints of opposite sides; its diagonals are not axes of symmetry. So a rectangle has 2 axes of symmetry. **Answer C.**

Question 7. Smallest turn $= 360^\circ \div \text{order} = 360^\circ \div 8 = 45^\circ$. **Answer A.**

Question 8. Match the widths and the heights and divide: width ratio $= 20 \div 8 = 2.5$ and height ratio $= 30 \div 12 = 2.5$. The two ratios are equal, so the print is similar to the photo, with scale factor 2.5. **Answer B.**

Question 9. Volume is measured in blocks, so it scales by $k^3 = 3^3 = 27$. (Lengths scale by $k=3$ and area by $k^2=9$.) **Answer C.**

Question 10. A 1:18 scale means the real car is 18 times the model: $15 \times 18 = 270$ cm. **Answer D.**

Question 11. A regular polygon tessellates on its own only when its interior angle divides 360° exactly. An equilateral triangle (60°), a square (90°) and a regular hexagon (120°) all do. A regular pentagon has interior angles of 108° : three leave a gap of $360^\circ - 3 \times 108^\circ = 36^\circ$ and four overlap ($4 \times 108^\circ = 432^\circ$), so pentagons cannot tessellate on their own. **Answer D.**

Question 12. The pattern grows by 3 matches each term and term 1 is $4 = 3 \times 1 + 1$, so the rule is matches $= 3 \times \text{term number} + 1$. Term 8 $= 3 \times 8 + 1 = 25$. **Answer A.**

Chapter 03 — Test B (written response)

Chapter 3 (Shape). Materials: one scientific calculator and paper. There are 7 written questions, worth 28 marks in total. Approximate time: 40 minutes. Answer every question in the spaces provided and show your working. Where an answer needs rounding, the question states the precision — round only as instructed. Every measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1 (4 marks)

a. Give the most exact name for a quadrilateral with exactly one pair of parallel sides. 1 mark

b. A prism has an octagonal cross-section (an 8-sided polygon). State the number of faces of the prism. 1 mark

c. State the number of edges of the prism. 1 mark

d. State the number of vertices of the prism. 1 mark

Question 2 (4 marks)

An isosceles triangle has a top angle of 52° . The two base angles are equal.

a. State the angle sum of any triangle. 1 mark

b. Find the sum of the two base angles. 1 mark

c. Find the size of each base angle. 1 mark

d. Name the triangle by its angles (acute-angled, right-angled or obtuse-angled) and give a reason. 1 mark

Question 3 (4 marks)

A decorative tile is a nine-pointed star, with 9 identical points evenly spaced around its centre.

a. State the order of rotational symmetry of the tile. 1 mark

b. Find the smallest turn that maps the tile onto itself. 1 mark

c. State the number of axes of symmetry of the tile. 1 mark

d. A second design has a smallest turn of 120° . State its order of rotational symmetry. 1 mark

Question 4 (4 marks)

Two rectangles are similar. Rectangle A is 8 cm wide and 12 cm tall. Rectangle B is 20 cm wide.

- a.** Find the scale factor from rectangle A to rectangle B, using the widths. 1 mark

- b.** Find the height of rectangle B. 1 mark

- c.** State the factor by which the area of rectangle B is greater than the area of rectangle A. 1 mark

- d.** The area of rectangle A is 96 cm². Find the area of rectangle B. 1 mark

Question 5 (4 marks)

A courtyard paving area is an L-shape with all corners at right angles. The two top pieces are 3.5 m and x , and together they match the 9 m bottom edge. The two right-hand pieces are 2.5 m and y , and together they match the 6 m left edge.

- a.** Find x . Show your working. 2 marks

- b.** Find y . Show your working. 2 marks

Question 6 (4 marks)

This question is about tessellations.

- a.** State the interior angle of a regular hexagon. 1 mark

- b.** Explain, using the angles at a point, why regular hexagons tessellate on their own. 2 marks

- c.** The interior angle of a regular pentagon is 108° . Explain why regular pentagons do not tessellate on their own. 1 mark

Question 7 (4 marks)

A juice carton holds 300 mL. The company makes a family carton with every length (height, width and depth) doubled.

a. State the factor by which the volume increases.

1 mark

b. Find the capacity of the family carton, in mL.

1 mark

c. State this capacity in litres.

1 mark

d. A shopper guesses that the family carton holds 600 mL (double the original). Explain, using a number, why this guess is wrong.

1 mark

End of Test B

Test B — Answers

Q1. (a) trapezium (b) 10 faces (c) 24 edges (d) 16 vertices. **Q2.** (a) 180° (b) 128° (c) 64° (d) acute-angled — all three angles (52° , 64° , 64°) are under 90° . **Q3.** (a) 9 (b) 40° (c) 9 (d) 3. **Q4.** (a) 2.5 (b) 30 cm (c) 6.25 (d) 600 cm^2 . **Q5.** (a) $x=5.5\text{ m}$ (b) $y=3.5\text{ m}$. **Q6.** (a) 120° (b) three hexagons at a point give $3 \times 120^\circ = 360^\circ$, exactly a full turn (c) $3 \times 108^\circ = 324^\circ$ leaves a 36° gap and $4 \times 108^\circ = 432^\circ$ overlaps. **Q7.** (a) 8 (b) 2 400 mL (c) 2.4 L (d) volume scales by $2^3=8$, not 2; $2\,400 \div 600 = 4$.

Test B — Fully-worked solutions

Question 1.

- a. A quadrilateral with exactly one pair of parallel sides is a **trapezium**.
- b. A prism whose cross-section is an n -sided polygon has $n + 2$ faces. With $n = 8$: $8 + 2 = 10$ faces.
- c. The prism has $3n$ edges. With $n = 8$: $3 \times 8 = 24$ edges.
- d. The prism has $2n$ vertices. With $n = 8$: $2 \times 8 = 16$ vertices.

Question 2.

- a. The angles of any triangle add to 180° .
- b. The top angle uses 52° , so the two base angles share $180^\circ - 52^\circ = 128^\circ$.
- c. The base angles are equal, so each is $128^\circ \div 2 = 64^\circ$.
- d. The three angles are 52° , 64° and 64° , and all are under 90° , so the triangle is **acute-angled**. (Check: $52^\circ + 64^\circ + 64^\circ = 180^\circ$ ✓.)

Question 3.

- a. The 9 identical points are evenly spaced, so the tile lands on itself 9 times in one full turn: order 9.
- b. Smallest turn $= 360^\circ \div \text{order} = 360^\circ \div 9 = 40^\circ$.
- c. A regular nine-point arrangement has 9 axes of symmetry (one through each point and the gap opposite it).
- d. Order $= 360^\circ \div \text{smallest turn} = 360^\circ \div 40^\circ = 9$.

Question 4.

- a. Matching the widths: scale factor $= 20 \div 8 = 2.5$.
- b. Height of B $= 12 \times 2.5 = 30$ cm.
- c. Area scales by k^2 , so the area factor is $2.5^2 = 6.25$.
- d. Area of B $= 96 \times 6.25 = 600$ cm². (Check with the dimensions: $20 \times 30 = 600$ cm² ✓.)

Question 5.

- a. The two top pieces match the bottom edge: $3.5 + x = 9$, so $x = 9 - 3.5 = 5.5$ m. (Check: $3.5 + 5.5 = 9$ ✓.)
- b. The two right-hand pieces match the left edge: $2.5 + y = 6$, so $y = 6 - 2.5 = 3.5$ m. (Check: $2.5 + 3.5 = 6$ ✓.)

Question 6.

- a. Split a regular hexagon into triangles from one vertex: this gives 4 triangles, so its angles add to $4 \times 180^\circ = 720^\circ$. The six interior angles share this equally: $720^\circ \div 6 = 120^\circ$.
- b. At a point in a hexagon tessellation, three hexagons meet: $3 \times 120^\circ = 360^\circ$, exactly one full turn. There is no gap and no overlap, so regular hexagons tessellate on their own.
- c. With pentagons, three at a point give $3 \times 108^\circ = 324^\circ$, leaving a gap of $360^\circ - 324^\circ = 36^\circ$; four give $4 \times 108^\circ = 432^\circ$, which overlaps. No whole number of pentagons fills a point exactly, so regular pentagons do not tessellate on their own.

Question 7.

- a.** Volume is measured in blocks, so doubling every length increases the volume by $2^3 = 8$.
- b.** Capacity = $300 \times 8 = 2\,400$ mL.
- c.** $2\,400 \text{ mL} = 2\,400 \div 1\,000 = 2.4$ L.
- d.** The guess of 600 mL treats the change as a doubling ($\times 2$), but the volume increases by a factor of 8. The true capacity is 2 400 mL, which is $2\,400 \div 600 = 4$ times the guess (equivalently, 600 mL is only one quarter of 2 400 mL).