

VCE VM Numeracy Units 3 & 4

Chapter 2 — Number

This work is licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-nd/4.0/> or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.

Attribution: Classbasics Pty Ltd, vwx.au

Distribution note: this document may be distributed, and whole pages may be removed from it, provided only WHOLE pages are removed (never part of a page). This allows teachers to remove tests, exams and subtopics they do not want to issue.

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

Significant effort has been made to ensure this content is legal. Please send any areas of concern to admin@vwx.au with appropriate document/legal references so errors can be verified and changes be made.

Contents

Section	Page
2A Place value: reading and writing large and small numbers	2
2B Multiplication facts, factors and multiples	11
2C Rounding whole numbers and decimals	23
2D Positive and negative numbers	33
2E Powers and square roots	44
2F Fractions, decimals and percentages: equivalence and operations	54
2G Simple ratio and proportion	66
Chapter 02 — Test A (multiple choice)	75
Test A — Answers	78
Test A — Fully-worked solutions	79
Chapter 02 — Test B (written response)	81
Test B — Answers	84
Test B — Fully-worked solutions	85

Chapter 2 Number — 2A Place value: reading and writing large and small numbers

Theory

Every number you will ever write is built from the same ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. What a digit *means* depends on where it sits. That one idea — **place value** — is what lets you read a cheque for millions of dollars, check a medicine dose of a quarter of a millilitre, and compare two swim times that differ in the last digit.

About the data in this section. The names, businesses and numbers in this section are **made up** (not real). They are here so you can practise on things that look like the slips, screens, signs and receipts you meet outside class.

Digits and place value.

In a whole number, each place is worth **10 times** the place to its right. After the decimal point the places keep going the other way: each place is worth one tenth of the place to its left.



four million, two hundred and seventy-six thousand, three hundred and fifteen point zero four eight

Place	ones	tens	hundreds	thousand s	ten thousand s	hundred thousand s	millions	ten millions	hundred millions
One digit here is worth	1	10	100	1 000	10 000	100 000	1 000 000	10 000 000	100 000 000

Place	tenths	hundredths	thousandths
One digit here is worth	0.1	0.01	0.001

The number on the chart is 4 276 315.048. The 7 sits in the ten thousands place, so it is worth 70 000. The 4 after the point sits in the hundredths place, so it is worth 0.04. Read aloud, the number is “four million, two hundred and seventy-six thousand, three hundred and fifteen point zero four eight”. In this section we read and write decimals to at most three places after the point (thousandths).

Reading and writing large numbers.

In Australia we group the digits of a large number in **threes from the right**, and we show the groups with a space: 1 000, not 1000. The groups have names:

Number	1 000	1 000 000	1 000 000 000	1 000 000 000 000
Name	thousand	million	billion	trillion

To say a large number in words, follow three rules:

- read each group as its own little number (from 1 to 999), then say the group's name — **million**, then **thousand**;
- say the last group (the ones) as just a number, with no group name after it;
- inside a group, say “and” before the last part when it is below 100 (“five hundred **and** nine”), and use “and” to join a last group below 100 (“... thousand **and** seventeen”); otherwise join the parts with commas (“six thousand, five hundred”).

Group the digits in threes from the right:



three million, four hundred and eighty-two thousand and seventeen

Say each group, then its group name. The last group is below 100, so join it with “and”.

Number	In words
6 500	six thousand, five hundred
48 210	forty-eight thousand, two hundred and ten
703 509	seven hundred and three thousand, five hundred and nine
2 500 000 000	two billion, five hundred million

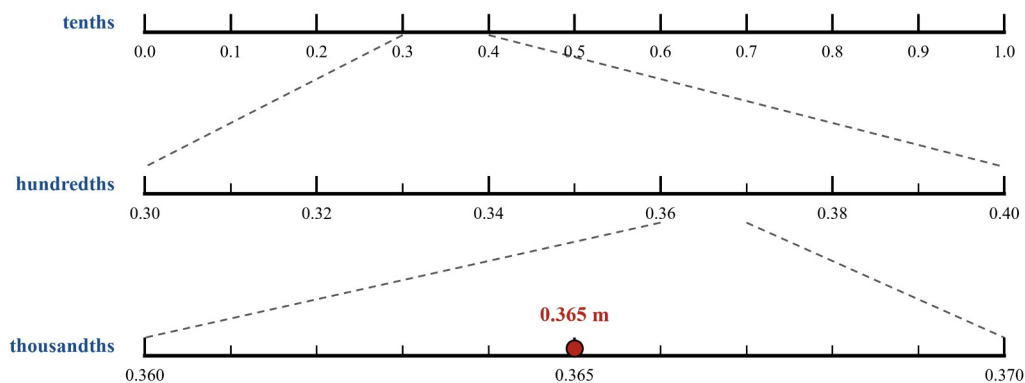
Going the other way, the words tell you where every digit sits — and every place the words skip becomes a zero. “One million **and** fifty” has a 1 in the millions group and just 50 in the ones group, with nothing said in between: 1 000 050.

Decimals and the decimal point.

A decimal is a number with a part smaller than 1, written after a **decimal point**. There are two good ways to read one aloud.

- **Point form:** say the whole-number part, say “point”, then say the digits after the point one at a time.
- **Place form:** say the digits after the point as one number, then say the name of the **last** place — tenths, hundredths or thousandths.

A metre strip zoomed in: where 0.365 m sits



Each place is 10 times smaller than the place before it: 1 m → 10 tenths → 100 hundredths → 1000 thousandths.

Number	Point form	Place form
0.7	zero point seven	seven tenths
0.34	zero point three four	thirty-four hundredths
0.365	zero point three six five	three hundred and sixty-five thousandths
0.048	zero point zero four eight	forty-eight thousandths

Zeros after the point are never “nothing”. Compare 0.5 and 0.05: the first is five tenths, the second is five hundredths, and 0.5 is 10 times as big as 0.05. The zero in 0.05 is what pushes the 5 one place further right.

Very small numbers in real life.

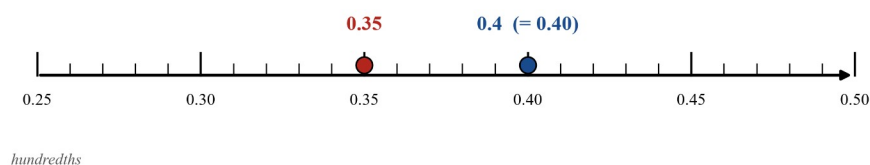
Small decimals turn up anywhere precision matters: a medicine dose of 0.25 mL (“zero point two five”, or twenty-five hundredths of a millilitre); a parcel weighed at 3.475 kg; a 50 m swim timed at 28.437 seconds; a piece of timber cut to 2.435 m. Reading these fluently means never losing track of which place each digit sits in.

Comparing and ordering.

To compare two numbers, or to put a list of them in order:

- for whole numbers, the number with **more digits** is larger — 1 000 050 beats 906 014 before you look at a single digit;
- otherwise line the numbers up **by their decimal points** and compare place by place from the left until the digits differ;
- you may add **end zeros** to a decimal without changing it, so every number shows the same number of places: $28.4 = 28.400$, which makes it easy to see against 28.437;
- beware the trap: **more digits does not mean larger** once a decimal point is involved. 0.35 has more digits than 0.4, but $0.4 = 0.40$, and 40 hundredths is bigger than 35 hundredths.

0.40 sits to the right of 0.35, so 0.4 is the larger number — even though 0.35 has more digits.



Fractions and mixed numbers in words.

A fraction names equal parts of a whole. The top number (the **numerator**) counts the parts you have; the bottom number (the **denominator**) names the size of each part. A **mixed number** is a whole number plus a fraction. In this section we only read and name fractions — the calculating with them comes in Section 2F.

Written	Said
$\frac{1}{2}$	one half
$\frac{2}{3}$	two thirds
$\frac{3}{4}$	three quarters
$\frac{5}{8}$	five eighths
$1\frac{3}{4}$	one and three quarters

Written

Said

 $2\frac{1}{2}$

two and a half

Say the numerator as a number, then the denominator as a plural (“five eighths”), and join a mixed number with “and” (“one **and** three quarters”).

Numbers in words on forms and slips.

Money slips and cheques ask for the amount **twice**: in words and in digits. The words are the safety net — a smudged or altered digit is hard to spot, but it will not match the words. Always read the two boxes against each other.

WESTHAVEN COMMUNITY CLUB	
payment slip	
Pay to:	Cedar Party Foods
For:	catering, March meeting
Amount (in words):	three hundred and twelve dollars and eighty-five cents
Amount (in digits):	\$312.85
signature	

The club, the supplier and the amount on this slip are made up.

On the slip above, the words box and the digits box say the same amount: three hundred and twelve dollars and eighty-five cents is \$312.85. The “.85” is the cents part — 85 hundredths of a dollar.

Reading numbers on screens.

Screens are not always tidy. A phone bill, a spreadsheet or an odometer may print 284317 with no grouping at all, and a spreadsheet may add its own separators. Neither changes the value: group the digits in threes **from the right** yourself, starting at the decimal point (or the end, if there is no point), and the number becomes readable. A number printed without grouping is not a different number — it is just waiting for its spaces.

Worked examples

Example 1 — scaffolded

A flood appeal closes at \$3 482 017. Write this amount in words, as it would appear on the presentation cheque.

Working

Comment

Group from the right: 3 | 482 | 017.

Spaces in threes, from the right.

Millions group is 3: “three million”.

Say the group, then its name.

Thousands group is 482: “four hundred and eighty-two thousand”.

482 says as “four hundred and eighty-two”.

Ones group is 17, below 100: “and seventeen”.

A last group below 100 joins with “and”.

Cheque words: “three million, four hundred and eighty-two thousand and seventeen dollars”.

Add “dollars” because it is money.

Example 2 — scaffolded

At the Fernhill Clinic a nurse draws up 0.25 mL of medicine, and the office writes a cheque for “two hundred and six thousand, three hundred and fifty dollars”. (a) Read 0.25 mL both ways. (b) Write the cheque amount in digits.

Working	Comment
(a) Point form: “zero point two five”.	Digits after the point, one at a time.
(a) Place form: last place is hundredths, so “twenty-five hundredths” of a millilitre.	25 hundredths = 0.25.
(b) “two hundred and six thousand” puts 206 in the thousands group.	The “and six” is inside the group: 206.
“three hundred and fifty” puts 350 in the ones group.	Last group: no group name said.
Digits: \$206 350.	Groups 206 350, space between.

Example 3 — unscaffolded

The Westhaven Swim Club runs a 50 m freestyle time trial, and the club’s timing app prints the table in the picture. Who is fastest, and what is the order from fastest to slowest?

Westhaven Swim Club
50 m freestyle time trial — times in seconds

Swimmer	Time (s)
Maya	28.437
Jin	28.374
Ella	28.4
Tom	28.473

the club’s timing app prints times without extra end zeros

The club, the swimmers and the times are made up.

In a race, the **fastest** swimmer has the **smallest** time. All four times have 28 before the point, so compare the places after it. Adding end zeros so every time shows three places: 28.437, 28.374, 28.400, 28.473. In the tenths place Jin alone has a 3, so Jin is fastest. Of the rest, the hundredths place gives 0 (Ella), then 3 (Maya), then 7 (Tom).

∴ Jin is fastest at 28.374 s, and the order from fastest to slowest is Jin 28.374 s, Ella 28.4 s, Maya 28.437 s, Tom 28.473 s.

Example 4 — unscaffolded

Sam drives the Cedar Party Foods delivery van. The odometer (the meter that counts how far the van has travelled) shows 148203 with no spaces. (a) Write the reading with grouping. (b) Write it in words. (c) Sam then drives 372 km. What does the odometer show now?

- (a) Grouping in threes from the right: 148 203 km.
- (b) “One hundred and forty-eight thousand, two hundred and three kilometres”.
- (c) Add 372 place by place: ones $3 + 2 = 5$; tens $0 + 7 = 7$; hundreds $2 + 3 = 5$; the 148 in front is untouched. The new reading is 148 575.

∴ after the trip the odometer reads 148 575 km.

Practice questions

Scaffolded

Q1. Write each number in words. (a) 4 806 (b) 37 500 (c) 906 014

Q2. Write each number in digits, grouped in threes. (a) sixty-two thousand and five (b) four hundred and seven (c) one million and fifty

Q3. Read each decimal both ways: point form and place form. (a) 0.6 (b) 0.075 (c) 2.09

Q4. A parcel weighs 3.475 kg. (a) Read 3.475 in place form, including the unit. (b) Name the place of the digit 4, and say what it is worth. (c) Which is heavier: this parcel, or one marked 3.5 kg? How do you know?

Q5. Look again at the Westhaven Community Club payment slip in the theory. (a) What amount does the slip pay, in digits? (b) The digits box ends in 85. What does that 85 mean? (c) Write \$1 024.50 in words, as it would go in the words box.

Q6. The picture shows a snapshot printed in the Fernhill Gazette.

THE FERNHILL GAZETTE	
Community snapshot	
People living in Fernhill	284 317
Cost of the new aquatic centre	\$12 650 000
Trees planted this year	8 240
numbers printed without grouping would read 284317, 12650000 and 8240	

The town, the paper and the numbers are made up.

- (a) Write the population of Fernhill in words.
- (b) The aquatic centre figure was printed as 12650000. Regroup it with spaces and say what the “12” at the front is worth.
- (c) Last year’s snapshot said 248 173 people. Which number is larger, and how do you know?

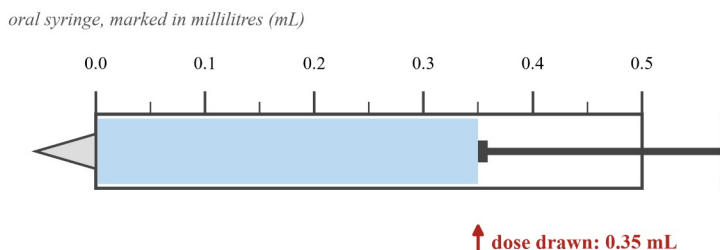
Unscaffolded

Q7. A house in Fernhill sells for \$845 750. Write the sale price in words.

Q8. A petition (a list of signatures asking council to act) is handed over with “one hundred and twenty-three thousand, four hundred and six” signatures. How many signatures is that, in digits?

Q9. Consider the number 9 074 216. (a) What digit is in the ten thousands place? (b) What is the 9 worth? (c) Write the number in words.

Q10. A nurse draws the dose shown on the 0.5 mL syringe in the picture.



- (a) Read the dose both ways.
- (b) Which is larger: the dose drawn, or 0.5 mL? How do you know?
- (c) Write 0.5 mL in thousandths.

Q11. Three offcuts of timber measure 2.435 m, 2.45 m and 2.408 m. (a) Read 2.408 m in place form, including the unit. (b) List the three lengths from shortest to longest. (c) Kai says 2.408 m must be longer than 2.45 m “because 408 is bigger than 45”. Explain Kai’s mistake.

Q12. Tia reads 0.036 aloud as “zero point thirty-six”. (a) Explain what went wrong, and why what she said sounds like a number 10 times too big. (b) Read 0.036 correctly in point form. (c) Read 0.036 in place form.

Q13. A council budget prints one line as 48000000000 with no spaces at all. (a) Rewrite the number with grouping. (b) Write it in words, as dollars. (c) Which group does the “48” sit in?

Q14. Write each of these in the other form. (a) “two and a half cups” as a mixed number (b) $\frac{5}{8}$ in words (c) “one and three quarters hours” as a mixed number

Q15. Bellwater FC lists 41 206 members and Redgum FC lists 41 602 members. (a) Which club lists more members? How do you know? (b) Write the larger number in words. (c) Show that the two clubs together list 82 808 members, adding place by place.

Q16. An ATM screen shows a balance of 4207.55. (a) Write the balance as a dollar amount with grouping. (b) Write the balance in words. (c) The digits after the point are 55. What does that 55 mean?

Q17. Three fish weigh 1.365 kg, 1.37 kg and 1.4 kg. (a) Read 1.365 kg in place form, including the unit. (b) Which fish is the heaviest? How do you know? (c) List the three weights from lightest to heaviest.

Q18. Bellwater Dam holds 1 250 000 000 L of water. After a week of rain it holds 1 250 600 000 L. (a) Write the first reading in words. (b) Exactly one digit changed. Which digit is new, what place does it sit in, and what is it worth? (c) Write the new reading in words.

Answers

Q1. (a) four thousand, eight hundred and six; (b) thirty-seven thousand, five hundred; (c) nine hundred and six thousand and fourteen. **Q2.** (a) 62 005; (b) 407; (c) 1 000 050. **Q3.** (a) zero point six, six tenths; (b) zero point zero seven five, seventy-five thousandths; (c) two point zero nine, two and nine hundredths. **Q4.** (a) three and four hundred and seventy-five thousandths kilograms; (b) tenths place, worth 0.4 (kg); (c) the 3.5 kg parcel, since $3.5 = 3.500 > 3.475$. **Q5.** (a) \$312.85; (b) 85 cents, that is eighty-five hundredths of a dollar; (c) one thousand and twenty-four dollars and fifty cents. **Q6.** (a) two hundred and eighty-four thousand, three hundred and seventeen; (b) 12 650 000 — the “12” is the millions group, worth twelve million; (c) 284 317 (hundred thousands tie at 2; ten thousands 8 beats 4). **Q7.** eight hundred and forty-five thousand, seven hundred and fifty dollars. **Q8.** 123 406. **Q9.** (a) 7; (b) 9 000 000; (c) nine million, seventy-four thousand, two hundred and sixteen. **Q10.** (a) zero point three five, thirty-five hundredths (of a millilitre); (b) 0.5 mL, since $0.5 = 0.50 > 0.35$; (c) five hundred thousandths (0.500 mL). **Q11.** (a) two and four hundred and eight thousandths metres; (b) 2.408 m, 2.435 m, 2.45 m; (c) $2.45 = 2.450$, and in the hundredths place 5 beats 0, so 2.45 m is longer — the digits sit in different places. **Q12.** (a) digits after the point are said one at a time, and she skipped the zero in the tenths place; “point thirty-six” sounds like 0.36, which is 10 times 0.036; (b) zero point zero three six; (c) thirty-six thousandths. **Q13.** (a) 48 000 000 000; (b) forty-eight billion dollars; (c) the billions group. **Q14.** (a) $2\frac{1}{2}$; (b) five eighths; (c) $1\frac{3}{4}$. **Q15.** (a) Redgum FC: $41\ 602 > 41\ 206$ (hundreds place, 6 beats 2); (b) forty-one thousand, six hundred and two; (c) ones $6 + 2 = 8$; tens $0 + 0 = 0$; hundreds $2 + 6 = 8$; thousands $1 + 1 = 2$; ten thousands $4 + 4 = 8$ — total 82 808. **Q16.** (a) \$4 207.55; (b) four thousand, two hundred and seven dollars and fifty-five cents; (c) 55 cents, that is fifty-five hundredths of a dollar. **Q17.** (a) one and three hundred and sixty-five thousandths kilograms; (b) the 1.4 kg fish, since $1.4 = 1.400$; (c) 1.365 kg, 1.37 kg, 1.4 kg. **Q18.** (a) one billion, two hundred and fifty million litres; (b) a 6 in the hundred thousands place, worth 600 000; (c) one billion, two hundred and fifty million, six hundred thousand litres.

Fully-worked solutions

Q1. Group in threes, say each group then its name. (a) 4 806: ones group 806 below 1000, thousands group 4: “four thousand, eight hundred and six”. (b) 37 500: “thirty-seven thousand” then 500 as “five hundred” (a full hundred, so no “and”): “thirty-seven thousand, five hundred”. (c) 906 014: “nine hundred and six thousand”, then 14 below 100 joins with “and”: “nine hundred and six thousand and fourteen”.

Q2. The words name each group; unsaid places become zeros. (a) “sixty-two thousand and five”: 62 in the thousands group, 5 in the ones: 62 005. (b) “four hundred and seven”: no thousands group said, so 407. (c) “one million and fifty”: 1 in the millions group, nothing in the thousands group, 50 in the ones: 1 000 050.

Q3. Point form says the digits one at a time; place form names the last place. (a) 0.6: “zero point six”; last place tenths: “six tenths”. (b) 0.075: “zero point zero seven five”; last place thousandths: “seventy-five thousandths”. (c) 2.09: “two point zero nine”; last place hundredths: “two and nine hundredths”.

Q4. (a) The last place is thousandths, so 3.475 kg reads “three and four hundred and seventy-five thousandths kilograms”. (b) The 4 is the first digit after the point: the **tenths** place, worth 0.4 (kg). (c) Write both to three places: $3.5 = 3.500$. Compare with 3.475: the tenths place gives 5 against 4, so the 3.5 kg parcel is heavier.

Q5. (a) The digits box reads \$312.85. (b) The 85 sits after the point: 85 cents, that is eighty-five hundredths of a dollar. (c) \$1 024.50: dollars group 1 024 says “one thousand and twenty-four” (the 24 joins with “and”), then the cents: “one thousand and twenty-four dollars and fifty cents”.

Q6. (a) 284 317: “two hundred and eighty-four thousand, three hundred and seventeen”. (b) 12650000 groups to 12 650 000. The “12” is the millions group, so it is worth twelve million. (c) Line up the places: both have 2 in the hundred thousands; in the ten thousands, 8 beats 4, so $284\ 317 > 248\ 173$.

Q7. Group \$845 750 as 845 | 750: “eight hundred and forty-five thousand, seven hundred and fifty dollars”. (750 is 100 or more, so the last group joins with a comma, not “and”).

Q8. “one hundred and twenty-three thousand” puts 123 in the thousands group; “four hundred and six” puts 406 in the ones group: 123 406 signatures.

Q9. (a) Counting places from the right (6 ones, 1 tens, 2 hundreds, 4 thousands), the ten thousands digit is 7. (b) The 9 sits in the millions place, so it is worth 9 000 000. (c) 9 | 074 | 216: “nine million, seventy-four thousand, two hundred and sixteen”.

Q10. (a) The plunger stops at 0.35 mL: “zero point three five”, or thirty-five hundredths (of a millilitre). (b) 0.5 mL is larger: $0.5 = 0.50$, and 50 hundredths beats 35 hundredths. (c) $0.5 \text{ mL} = 0.500 \text{ mL}$, which is five hundred thousandths (of a millilitre).

Q11. (a) The last place is thousandths: “two and four hundred and eight thousandths metres”. (b) Write all to three places: 2.408, 2.435, 2.450. From shortest: 2.408 m, 2.435 m, 2.45 m. (c) Kai compared 408 with 45 as whole numbers, but the digits sit in different places: $2.45 = 2.450$, and in the hundredths place 5 beats 0, so 2.45 m is the longer offcut.

Q12. (a) After the point, digits are said **one at a time**, and the zero in the tenths place must be said too. “Point thirty-six” is how 0.36 sounds, and 0.36 is 10 times 0.036 — so her reading names a number 10 times too big. (b) “zero point zero three six”. (c) The last place is thousandths: “thirty-six thousandths”.

Q13. (a) Group in threes from the right: 48 000 000 000. (b) 48 | 000 | 000 | 000: “forty-eight billion dollars”. (c) The “48” is the group said as “billion”, so it sits in the billions group.

Q14. (a) “two and a half” is the whole 2 plus one half: $2\frac{1}{2}$. (b) Numerator 5, denominator 8: “five eighths”. (c) “one and three quarters” is the whole 1 plus three quarters: $1\frac{3}{4}$.

Q15. (a) Line up the places: ten thousands $4 = 4$, thousands $1 = 1$, then hundreds 6 against 2 — so Redgum FC’s 41 602 is larger. (b) 41 602: “forty-one thousand, six hundred and two”. (c) Add place by place: ones: $6 + 2 = 8$; tens: $0 + 0 = 0$; hundreds: $2 + 6 = 8$; thousands: $1 + 1 = 2$; ten thousands: $4 + 4 = 8$. Reading the places back gives 82 808, so the total is 82 808 members.

Q16. (a) Group the dollars part: \$4 207.55. (b) 4 000 says “four thousand”, 7 joins with “and”: “four thousand, two hundred and seven dollars and fifty-five cents”. (c) The 55 sits after the point: 55 cents, that is fifty-five hundredths of a dollar.

Q17. (a) The last place is thousandths: “one and three hundred and sixty-five thousandths kilograms”. (b) Write all to three places: 1.365, 1.370, 1.400. The tenths place gives 3, 3, 4 — so the 1.4 kg fish is heaviest. (c) From lightest: 1.365 kg, 1.37 kg, 1.4 kg.

Q18. (a) 1 250 | 000 | 000: “one billion, two hundred and fifty million litres”. (b) The new reading adds a 6 where there was a 0: the hundred thousands place, worth 600 000 (litres). (c) 1 250 | 600 | 000: “one billion, two hundred and fifty million, six hundred thousand litres”.

Chapter 2 Number — 2B Multiplication facts, factors and multiples

Theory

About the data in this section. The prices, timetables, orders, rosters and stories here are **made up** (not real): pretend businesses, pretend people and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Section 2A was about **place value** — how the number system writes every number from its digits. This section is about how numbers **fit together**. It all starts with multiplication.

In the study design (Unit 3, Area of Study 1), the key knowledge for this topic is knowing your multiplication facts (your times tables) and understanding factors and multiples. That is exactly what we build here: multiplication you can do quickly and trust, plus the language of **factors, multiples, prime numbers**, the **highest common factor (HCF)** and the **lowest common multiple (LCM)**. These tools turn up everywhere later in the course — reducing fractions (Section 2F), working with ratio (Section 2G), and any real task with money, time or measurements.

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-07.

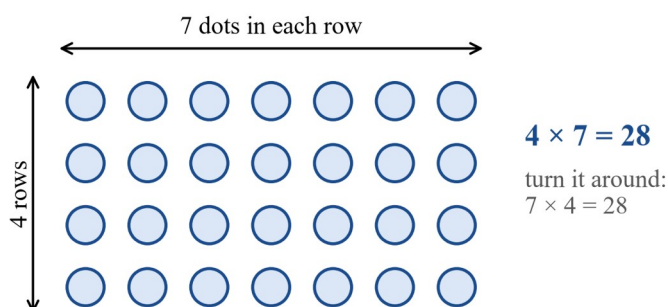
Multiplication is fast repeated addition.

To multiply means to add the same number over and over. 5×8 means “5 lots of 8”:

$$8 + 8 + 8 + 8 + 8 = 40$$

The answer to a multiplication is called the **product**. So the product of 5 and 8 is 40.

Multiplication can be drawn as an **array** — dots arranged in equal rows.



The array shows two useful facts at once: $4 \times 7 = 28$ (4 rows of 7) and $7 \times 4 = 28$ (7 rows of 4). Swapping the order of a multiplication never changes the product. So if 12×5 feels hard, try it the other way around as 5×12 .

Some facts are worth knowing off by heart — that is what “multiplication facts” means: times tables you can recall without working anything out.

- The tables up to 10×10 . For example, $6 \times 7 = 42$ and $9 \times 6 = 54$.
- Multiplying by 10, 100 and 1000 — the digits move one place to the left each time (this is place value, Section 2A). For example, $45 \times 10 = 450$ and $45 \times 100 = 4\,500$.
- Multiplying by 0 and by 1: zero times anything is 0, and one times anything is the number itself.

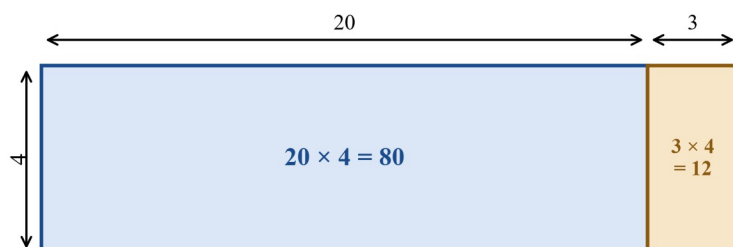
Multiplying bigger numbers without a calculator.

For numbers beyond the tables, break the job into easy pieces. Three handy strategies:

Strategy	How it works	Example
Partition (split)	Split one number into easy parts,	$23 \times 4 = (20 \times 4) + (3 \times 4) = 80 + 12 =$

Strategy	How it works	Example
Double and halve	multiply each part, then add Double one number and halve the other — the product stays the same	16×5 becomes $8 \times 10 = 80$
Build from a fact you know	Repeated doubling, or nearly-there numbers	19×8 : double 19 three times — 38, 76, 152

The partition strategy has a picture of its own. Split 23 into 20 and 3:



$$23 \times 4 = 80 + 12 = 92$$

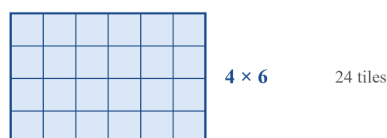
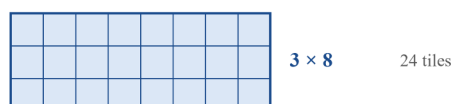
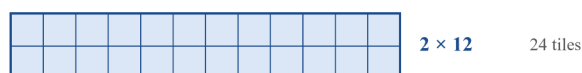
split 23 into 20 and 3 — two easy multiplications

Pen, paper and these strategies handle almost everything in this section. A basic calculator is fine for checking, but the thinking is what matters.

Factors: the numbers that divide exactly.

A **factor** of a number divides into it exactly, with nothing left over. For example, 3 is a factor of 24 because $24 \div 3 = 8$ exactly. 5 is not a factor of 24 because $24 \div 5 = 4$ remainder 4.

Factors always come in **pairs**: since $3 \times 8 = 24$, both 3 and 8 are factors of 24. Twenty-four square tiles can be arranged into rectangles in exactly four ways — one for each factor pair.



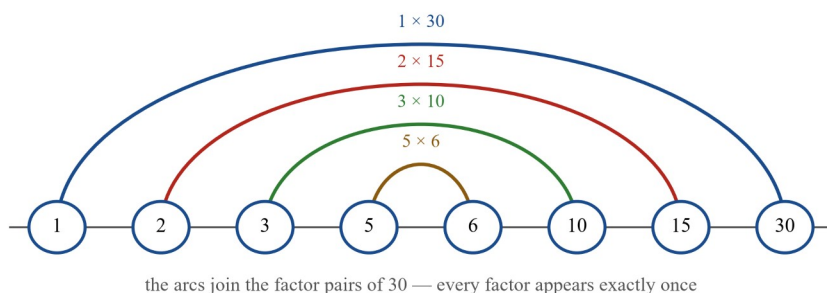
the same 24 tiles make exactly four different rectangles

So the factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24 — eight factors altogether.

Three things to remember about factors:

- 1 and the number itself are always factors.
- A factor is never bigger than the number.
- Every number has a fixed, finite list of factors.

Listing factors without missing any. Work in order. Test 1, then 2, then 3, and so on, writing each pair as you find it. Stop when the pairs would start repeating — that happens when the next number you would test already appears in a pair. The “factor rainbow” below lists the factors of 30 this way; each arc joins one pair.



The pairs are 1×30 , 2×15 , 3×10 and 5×6 , so the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

Prime and composite numbers.

A **prime number** has exactly two factors: 1 and itself. A **composite number** has more than two factors — it can be built by multiplying smaller whole numbers together.

- The primes less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29 — ten primes.
- 2 is the only even prime. Every other even number has at least the factors 1, 2 and itself, so it is composite.
- The number 1 is **neither prime nor composite** — it has only one factor.
- Ending in an odd digit does not make a number prime: $21 = 3 \times 7$, $27 = 3 \times 9$ and $91 = 7 \times 13$ are all composite.

Multiples: the times tables keep going.

The **multiples** of a number are what you get counting in steps of that number: 1 lot, 2 lots, 3 lots, and so on. The multiples of 4 are 4, 8, 12, 16, 20, 24 and onwards — the list never ends.

- Every number is a multiple of itself ($4 \times 1 = 4$), and 48 is a multiple of 24 because $24 \times 2 = 48$.
- **Membership test** — to check whether a number is a multiple, divide and look at the remainder. Is 84 a multiple of 7? $84 \div 7 = 12$ exactly, so yes. Is 74 a multiple of 12? $74 \div 12 = 6$ remainder 2, so no.

Factors and multiples run in opposite directions: the factors of 24 are all less than or equal to 24, and there are finitely many; the multiples of 24 are all greater than or equal to 24, and they never stop.

On a hundred chart, the multiples fall into neat patterns. The shaded cells below are the multiples of 4 up to 100 — there are 25 of them, and their last digits repeat the cycle 4, 8, 2, 6, 0.

the multiples of 4 (shaded) end 4, 8, 2, 6, 0, 4, 8, 2, 6, 0, ...

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

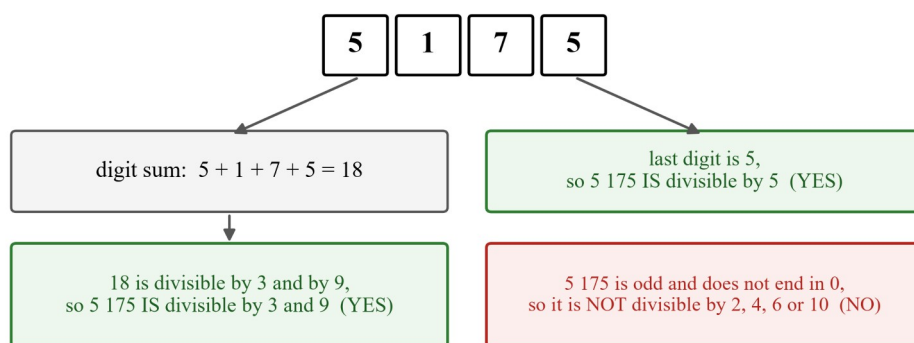
Divisibility tests: shortcuts for spotting factors.

A number is **divisible** by another when that second number is one of its factors — the division leaves nothing over. **Divisibility tests** let you spot factors without dividing.

Test for	Do this	Example
2	Is the last digit even (0, 2, 4, 6 or 8)?	238 ends in 8, so it is divisible by 2
3	Add the digits — is the sum divisible by 3?	5 175: digit sum $5 + 1 + 7 + 5 = 18$, and 18 is divisible by 3, so 5 175 is too
4	Is the number made by the last two digits divisible by 4?	144: the last two digits are 44, and $44 \div 4 = 11$, so 144 is divisible by 4
5	Does it end in 0 or 5?	5 175 ends in 5, so it is divisible by 5
6	Does it pass both the 2 test and the 3 test?	450 is even and its digit sum is 9, so it is divisible by 6
9	Add the digits — is the sum divisible by 9?	5 175: digit sum 18 is divisible by 9, so 5 175 is divisible by 9
10	Does it end in 0?	450 ends in 0, so it is divisible by 10

Here is the number 5 175 tested without a single division:

Is 5 175 divisible? Test it without dividing.



These tests are also a quick way to sanity-check calculator working: if you multiply by 27 and your answer's digits do not add up to a multiple of 3, something has gone wrong. Question 12 uses exactly this trick.

Common factors and the highest common factor (HCF).

A **common factor** of two numbers is a factor of both of them. The **highest common factor (HCF)** is the largest common factor.

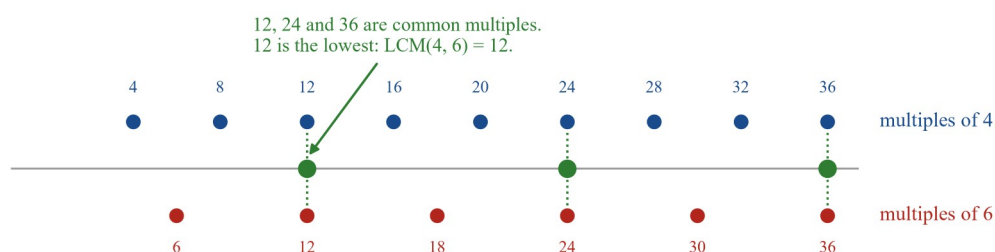
For 18 and 24:

- Factors of 18: 1, 2, 3, 6, 9, 18.
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.
- Common factors: 1, 2, 3, 6. So the HCF of 18 and 24 is 6.

The HCF answers the question: *what is the biggest size that fits into both amounts exactly?* Use it for splitting things into the largest possible equal groups, cutting materials into equal pieces with no waste, or packing two amounts into identical boxes.

Common multiples and the lowest common multiple (LCM).

A **common multiple** of two numbers is a multiple of both. The **lowest common multiple (LCM)** is the smallest common multiple.



The LCM answers the question: *when will two things that repeat on different cycles next line up?* Use it for buses or machines on different schedules, lights that blink at different rates, or people on different rotating rosters.

HCF or LCM? Reading the task.

Words in the task	Use	Because
largest, biggest, greatest; equal groups or pieces; nothing left over;	HCF	you are splitting amounts into the

Words in the task	Use	Because
no waste		biggest shared pieces
next time together; line up again; both on the same day; smallest number that ...	LCM	you are waiting for two cycles to meet again

Both ideas keep coming back: Section 2F uses the HCF to reduce fractions to their simplest form, and Section 2G uses common multiples to compare ratios.

Worked examples

Example 1 — scaffolded

Marcia has baked 60 cupcakes for a stall. She packs them into boxes so that every box holds the same number of cupcakes and no cupcakes are left over.

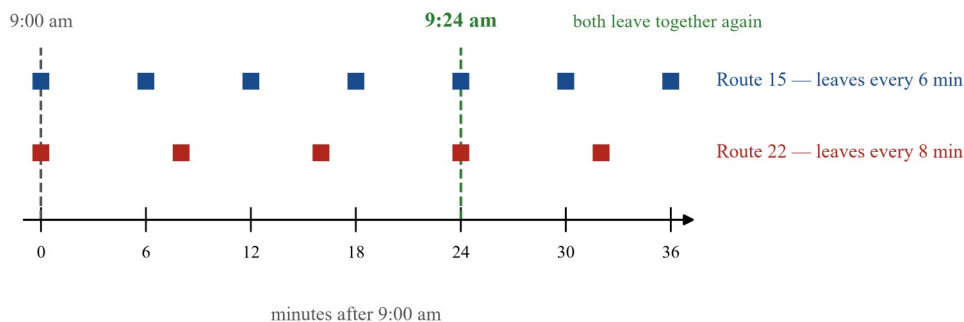
- (a) Find all of the factor pairs of 60. (b) List the factors of 60 in order from smallest to largest. (c) If each box holds 12 cupcakes, how many boxes does she fill? How does the factor list prove there are no cupcakes left over?

Working	Comment
Purpose and question(s): pack 60 cupcakes into equal boxes with none left over. Questions: (a) all factor pairs of 60; (b) the factors in order; (c) the number of boxes of 12, and why none are left over.	Equal groups with nothing left over — that points straight at factors.
Step (a) — start at 1: $60 = 1 \times 60$. $60 = 2 \times 30$ — 60 is even, so 2 is a factor. $60 = 3 \times 20$ — digit sum $6 + 0 = 6$, which is divisible by 3. $60 = 4 \times 15$ — the last two digits are 60, and $60 \div 4 = 15$. $60 = 5 \times 12$ — 60 ends in 0. $60 = 6 \times 10$ — 60 is even and its digit sum is divisible by 3. 7, 8 and 9 all fail — $60 \div 7 = 8$ remainder 4, $60 \div 8 = 7$ remainder 4, $60 \div 9 = 6$ remainder 6. The next number to test, 10, already appears in the pair 6×10 .	Every whole number starts with the pair $1 \times$ itself. Test the numbers in order: 1, 2, 3, ... Divisibility tests are quicker than long division. Test for 4: look at the last two digits. Test for 5: ends in 0 or 5. Test for 6: pass the 2 test and the 3 test.
Step (b): the factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 — twelve factors.	When the pairs would start repeating, the search is finished. Collect every number that appeared in a pair.
Step (c): $60 \div 12 = 5$, so Marcia fills 5 boxes. 12 is in the factor list, so it divides 60 exactly — no cupcakes left over.	A box size works with no leftovers exactly when it is a factor.

Marcia can use any box size from her factor list; boxes of 12 give her 5 full boxes.

Example 2 — scaffolded

At a bus interchange, Route 15 buses leave every 6 minutes and Route 22 buses leave every 8 minutes. Both routes have a bus leave at 9:00 am. The timeline below shows the departure times as minutes after 9:00 am.



Made-up data: the two routes, the interchange and every time in the timeline are made up for this section.

- (a) List the multiples of 6 up to 36 minutes. (b) List the multiples of 8 up to 36 minutes. (c) When do the two routes next leave a bus together?

Working

Comment

Purpose and question(s): find when the two routes next leave together. Questions: (a) the multiples of 6; (b) the multiples of 8; (c) the first time both lists agree.

“Next time together” — that points straight at common multiples.

Step (a): multiples of 6: 6, 12, 18, 24, 30, 36.

Route 15 leaves on these minutes.

Step (b): multiples of 8: 8, 16, 24, 32.

Route 22 leaves on these minutes.

Step (c) — compare the lists: 24 appears in both, and it is the first number that does.

The LCM of 6 and 8 is 24.

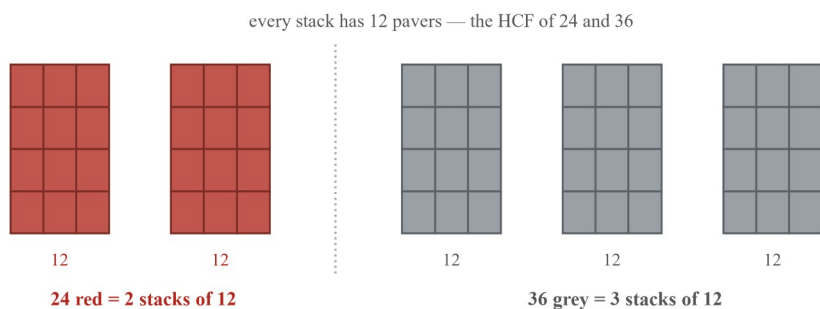
Translate back: 24 minutes after 9:00 am is 9:24 am.

The question asks for a time, so do not stop at “24”.

The two routes next leave a bus together at 9:24 am.

Example 3 — unscaffolded

A landscaping crew has 24 red pavers and 36 grey pavers left over. They want to stack the pavers so that every stack holds the same number of pavers, red and grey are not mixed, and no pavers are left over — and they want the stacks as large as possible. How many pavers go in each stack, and how many stacks are there altogether?



Made-up data: the pavers and the stacks are made up for this section.

The stack size must divide both 24 and 36 exactly, and the largest size that does this is the HCF. Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24. Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36. The common factors are 1, 2, 3, 4, 6 and 12, so the HCF is 12 — each stack holds 12 pavers. Now count the stacks: red, $24 \div 12 = 2$ stacks; grey, $36 \div 12 = 3$ stacks; altogether, $2 + 3 = 5$ stacks.

$\therefore$ each stack holds 12 pavers, and there are 5 stacks in total (2 red and 3 grey).

Example 4 — unscaffolded

A community fete orders 8 crates of soft drink, and each crate holds 24 cans.

- (a) By splitting 24 into 20 and 4, show that the order holds 192 cans.
- (b) The cans are shared equally between 6 stalls. Explain how divisibility tests show the sharing works out exactly, then find how many cans each stall gets.
- (c) Split 24 into $20 + 4$: $8 \times 24 = (8 \times 20) + (8 \times 4) = 160 + 32 = 192$, so the order holds 192 cans.
- (d) 192 ends in 2, so it is even — it passes the test for 2. Its digit sum is $1 + 9 + 2 = 12$, which is divisible by 3 — it passes the test for 3. A number that passes both the 2 test and the 3 test is divisible by 6, so 192 shares out exactly between 6 stalls with none left over. Now divide: $192 \div 6 = 32$.

$\therefore$ the order holds 192 cans and each stall receives 32 cans.

Practice questions

Scaffolded

- Q1.** (a) Without a calculator, work out: 6×7 , 8×4 , 9×6 , 7×8 , 12×5 and 9×9 . (b) Write 5×8 as repeated addition, and find the answer. (c) Sam says 6×8 and 8×6 give the same product. Is he right? Explain why.
- Q2.** (a) Here is a way to work out 45×5 : first find 45×10 , then halve it. Carry out both steps and show that the answer is 225. (b) Work out 19×8 by doubling three times: start with $19 \times 2 = 38$, then double, then double again. (c) A garden club buys 6 rose bushes at \$23 each. Without a calculator, use partitioning ($23 = 20 + 3$) to find the total cost.
- Q3.** (a) Write down all of the factor pairs of 18. (b) List the factors of 18 in order from smallest to largest. (c) One of 18 and 19 is prime and the other is composite. Which is which? Give a reason.
- Q4.** (a) Write down the first six multiples of 7. (b) Is 56 a multiple of 7? Is 84? Show how you know. (c) Eggs are sold in cartons of 12. Could you buy exactly 60 eggs? Exactly 96 eggs? Exactly 74 eggs? Explain each answer using multiples.
- Q5.** For each number below, list which of 2, 3, 4, 5, 6, 9 and 10 it is divisible by. Use the divisibility tests, not long division. (a) 450 (b) 238 (c) 144
- Q6.** A sports day has two groups: 20 juniors and 28 seniors. Each group is split into equal teams so that every team has the same number of players, the teams are as large as possible, and nobody is left out. Teams do not mix juniors with seniors. (a) List the factors of 20. (b) List the factors of 28. (c) What is the largest team size that works for both groups? How many teams are there altogether?

Unscaffolded

- Q7.** A community hall sets out 36 chairs in equal rows, with the same number of chairs in every row, using all 36 chairs. (a) List all of the factor pairs of 36. (b) If each row has 9 chairs, how many rows are there? (c) The hall wants between 4 and 10 chairs in each row. List every row size that works, and the number of rows for each.
- Q8.** A nursery has 7 trays of seedlings, and each tray holds 48 seedlings. Seedlings sell for \$2 each. (a) Use partitioning ($48 = 40 + 8$) to find how many seedlings there are altogether. (b) Farid works out 48×7 instead of 7×48 and gets the same answer. Explain why that must happen. (c) What is the total value of all the seedlings, at \$2 each?
- Q9.** A park has two sprinkler systems. System A runs every 4 hours and system B runs every 6 hours. Both systems start together at 8:00 am. (a) List the multiples of 4 and the multiples of 6 up to 24 hours. (b) After how many hours do the two systems next start together? (c) What time of day is that?
- Q10.** In a factory, machine X beeps every 9 minutes and machine Y beeps every 12 minutes. Both machines beep together at 7:15 am. (a) List the multiples of 9 and the multiples of 12 up to 36 minutes. (b) After how many minutes do the two machines next beep together? (c) What time is that?

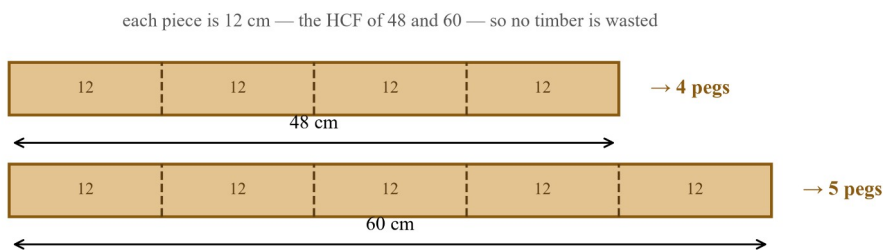
Q11. A market stall has 45 jars of jam and 30 jars of honey. The owner makes identical gift hampers using all of the jars — every hamper gets the same number of jam jars and the same number of honey jars, and the hampers are as large as possible. **(a)** List the factors of 45 and the factors of 30. **(b)** What is the largest number of hampers she can make? **(c)** How many jars of jam and how many jars of honey go in each hamper?

Q12. Tayla uses a calculator to work out 34×27 . The screen shows 818. **(a)** Without dividing, explain how a divisibility test shows that 818 cannot be right. **(b)** Work out the correct product. **(c)** Check your answer against the test for 9, and explain why the product had to pass that test.

Q13. A farm stall has 92 eggs and packs them into cartons of 12. **(a)** How many full cartons can be filled? **(b)** How many eggs are left loose? **(c)** How many more eggs would be needed to fill one more full carton?

Q14. (a) Which of 21, 23, 27 and 29 are prime? Give a reason for each of the four numbers. **(b)** Dean says 91 is prime because it does not end in an even digit and is not divisible by 3. Show that he is wrong. **(c)** Explain why 2 is the only even prime number.

Q15. A landscaping crew has two offcuts of hardwood, one 48 cm long and one 60 cm long. They cut both offcuts into garden pegs of the same whole-number length, using all of the timber with none left over.



Made-up data: the two offcuts and their measurements are made up for this section.

(a) Explain why the peg length must be a common factor of 48 and 60, and find the longest peg length that works. **(b)** How many pegs come from each offcut? **(c)** How many pegs are there altogether?

Q16. Three friends share the jobs at a community garden on rotating schedules. Sam weeds every 4 days, Anh waters every 6 days, and Dean mows every 9 days. Today all three are at the garden together. **(a)** Explain why the number of days until they are next all together must be a common multiple of 4, 6 and 9. **(b)** Find that number of days. **(c)** Over the next 108 days, on which days are all three at the garden together?

Q17. A neighbourhood group orders yoghurt for a breakfast program: 6 boxes of 20 tubs each. Each tub costs \$2.35, and delivery is a flat \$35.60. **(a)** How many tubs of yoghurt are in the order? **(b)** Find the cost of the yoghurt alone (before delivery). **(c)** The total bill (yoghurt plus delivery) is split equally between 7 families. Find each family's share, correct to the nearest cent.

Q18. Grace is catering a sausage sizzle and needs at least 100 sausages. Sausages are sold in packs of 12, and each pack costs \$4.50. She buys the packs she needs and cooks every sausage she bought. **(a)** How many packs must she buy? Explain why 8 packs are not enough. **(b)** What do the packs cost? **(c)** Grace arranges all of the sausages she bought in equal rows on trays, with between 5 and 20 sausages in each row. List every row size that works, and the number of rows for each.

Answers

Q1. (a) 42, 32, 54, 56, 60, 81. (b) $8+8+8+8+8=40$. (c) Yes — swapping the order never changes the product; both count the same array, and both equal 48. **Q2.** (a) $45 \times 10 = 450$; $450 \div 2 = 225$, and $45 \times 5 = 225$. (b) 38, then 76, then 152, so $19 \times 8 = 152$. (c) $(20 \times 6) + (3 \times 6) = 120 + 18 = 138$, so \$138.00. **Q3.** (a) 1×18 , 2×9 , 3×6 . (b) 1, 2, 3, 6, 9, 18. (c) 19 is prime (its only factors are 1 and 19); 18 is composite (for example, 2×9). **Q4.** (a) 7, 14, 21, 28, 35, 42. (b) Both yes: $56 = 7 \times 8$ and $84 = 7 \times 12$, so both divide by 7 exactly. (c) 60: yes, $12 \times 5 = 60$. 96: yes, $12 \times 8 = 96$. 74: no, $74 \div 12 = 6$ remainder 2, so 74 is not a multiple of 12. **Q5.** (a) 450: divisible by 2, 3, 5, 6, 9 and 10 (not by 4 — the last two digits are 50, and 50 is not divisible by 4). (b) 238: divisible by 2 only. (c) 144: divisible by 2, 3, 4, 6 and 9. **Q6.** (a) 1, 2, 4, 5, 10, 20. (b) 1, 2, 4, 7, 14, 28. (c) HCF of 20 and 28 is 4, so teams of 4; $20 \div 4 = 5$ junior teams and $28 \div 4 = 7$ senior teams — 12 teams altogether. **Q7.** (a) 1×36 , 2×18 , 3×12 , 4×9 , 6×6 . (b) 4 rows. (c) 4 per row (9 rows), 6 per row (6 rows) or 9 per row (4 rows). **Q8.** (a) $(7 \times 40) + (7 \times 8) = 280 + 56 = 336$ seedlings. (b) Swapping the order of a multiplication never changes the product — the two calculations count the same array turned around. (c) $336 \times \$2 = \$672.00$. **Q9.** (a) Multiples of 4: 4, 8, 12, 16, 20, 24; multiples of 6: 6, 12, 18, 24. (b) 12 hours — the LCM of 4 and 6. (c) 8:00 pm. **Q10.** (a) Multiples of 9: 9, 18, 27, 36; multiples of 12: 12, 24, 36. (b) 36 minutes — the LCM of 9 and 12. (c) 7:51 am. **Q11.** (a) 45: 1, 3, 5, 9, 15, 45; 30: 1, 2, 3, 5, 6, 10, 15, 30. (b) HCF is 15, so 15 hampers. (c) 3 jars of jam and 2 jars of honey in each hamper. **Q12.** (a) 27 is divisible by 3 (digit sum 9), so any product involving 27 must be divisible by 3; the digit sum of 818 is 17, which is not divisible by 3 — so 818 cannot be right. (b) 918. (c) Digit sum of 918 is 18, which is divisible by 9, and 27 is divisible by 9, so the product had to be divisible by 9 — it checks out. **Q13.** (a) 7 cartons ($7 \times 12 = 84$). (b) 8 eggs. (c) 4 more eggs ($12 - 8 = 4$). **Q14.** (a) 23 and 29 are prime (only 1 and themselves as factors); $21 = 3 \times 7$ and $27 = 3 \times 9$, so both are composite. (b) $91 = 7 \times 13$, so its factors are 1, 7, 13, 91 — more than two factors, so it is not prime. (c) Every even number greater than 2 has at least the factors 1, 2 and itself — three factors — so it is composite. **Q15.** (a) The peg length must divide both 48 and 60 exactly, so it must be a common factor; the longest is the HCF, which is 12 cm. (b) 48 cm gives 4 pegs; 60 cm gives 5 pegs. (c) 9 pegs. **Q16.** (a) Sam works on multiples of 4, Anh on multiples of 6 and Dean on multiples of 9; “all together” means the day number is in all three lists — a common multiple. (b) LCM of 4, 6 and 9 is 36, so 36 days. (c) Days 36, 72 and 108 — three more times within 108 days. **Q17.** (a) 120 tubs. (b) \$282.00. (c) Total $\$282.00 + \$35.60 = \$317.60$; $\$317.60 \div 7 = \$45.37$ correct to the nearest cent. **Q18.** (a) 9 packs — 8 packs give $8 \times 12 = 96$ sausages, which is under 100; 9 packs give 108. (b) $9 \times \$4.50 = \$40.50$. (c) 108 sausages; the factors of 108 between 5 and 20 are 6, 9, 12 and 18 — rows of 6 (18 rows), 9 (12 rows), 12 (9 rows) or 18 (6 rows).

Fully-worked solutions

Q1. (a) $6 \times 7 = 42$, $8 \times 4 = 32$, $9 \times 6 = 54$, $7 \times 8 = 56$, $12 \times 5 = 60$, $9 \times 9 = 81$. (b) 5 lots of 8 is $8+8+8+8+8=40$, so $5 \times 8 = 40$. (c) Sam is right. Swapping the order of a multiplication never changes the product — both calculations count the same array (one sees 6 rows of 8, the other 8 rows of 6), and both equal 48.

Q2. (a) $45 \times 10 = 450$. Halving: $450 \div 2 = 225$. Check: $45 \times 5 = 225$, so the method gives the right answer. (b) $19 \times 2 = 38$; doubling: $38 \times 2 = 76$; doubling again: $76 \times 2 = 152$. Three doublings multiply by 8, so $19 \times 8 = 152$. (c) Split 23 into $20+3$: $(20 \times 6) + (3 \times 6) = 120 + 18 = 138$. The roses cost \$138.00.

Q3. (a) Test in order: $18 = 1 \times 18$, $18 = 2 \times 9$, $18 = 3 \times 6$. The next number, 4, does not divide 18, and 6 already appears, so stop. The pairs are 1×18 , 2×9 and 3×6 . (b) Collecting every number in the pairs: 1, 2, 3, 6, 9, 18. (c) 19 is prime: its only factors are 1 and 19, exactly two. 18 is composite: it has the factors 1, 2, 3, 6, 9, 18 — more than two, since $18 = 2 \times 9$.

Q4. (a) Count in steps of 7: 7, 14, 21, 28, 35, 42. (b) $56 \div 7 = 8$ exactly ($56 = 7 \times 8$), so 56 is a multiple of 7. $84 \div 7 = 12$ exactly ($84 = 7 \times 12$), so 84 is also a multiple of 7. (c) $60 = 12 \times 5$, so yes — 5 full cartons. $96 = 12 \times 8$, so yes — 8 full cartons. $74 \div 12 = 6$ remainder 2, so no — you would have 6 cartons and 2 loose eggs, never exactly 74 in cartons of 12.

Q5. (a) 450 ends in 0: divisible by 2, 5 and 10. Digit sum $4+5+0=9$: divisible by 3 and 9. Even and divisible by 3: divisible by 6. Last two digits are 50, and $50 \div 4 = 12$ remainder 2: not divisible by 4. So 450 is divisible by

2, 3, 5, 6, 9 and 10. (b) 238 ends in 8: divisible by 2. Digit sum $2 + 3 + 8 = 13$, not divisible by 3: fails 3, and so fails 6 and 9. Last two digits 38, and $38 \div 4 = 9$ remainder 2: not divisible by 4. Does not end in 0 or 5: fails 5 and 10. So 238 is divisible by 2 only. (c) 144 ends in 4: divisible by 2. Digit sum $1 + 4 + 4 = 9$: divisible by 3 and 9. Last two digits 44, and $44 \div 4 = 11$: divisible by 4. Even and divisible by 3: divisible by 6. Does not end in 0 or 5: fails 5 and 10. So 144 is divisible by 2, 3, 4, 6 and 9.

Q6. (a) Pairs: 1×20 , 2×10 , 4×5 . Factors of 20: 1, 2, 4, 5, 10, 20. (b) Pairs: 1×28 , 2×14 , 4×7 . Factors of 28: 1, 2, 4, 7, 14, 28. (c) The team size must divide both 20 and 28, and be as large as possible — that is the HCF. Common factors: 1, 2, 4. HCF = 4, so teams of 4. Juniors: $20 \div 4 = 5$ teams. Seniors: $28 \div 4 = 7$ teams. Altogether: $5 + 7 = 12$ teams.

Q7. (a) Test in order: $36 = 1 \times 36$, 2×18 , 3×12 , 4×9 , 6×6 . The next number, 7, fails, and 9 already appears, so stop. Pairs: 1×36 , 2×18 , 3×12 , 4×9 , 6×6 . (b) $36 \div 9 = 4$, so 4 rows. (c) The row size must be a factor of 36 between 4 and 10: 4, 6 or 9. Rows of 4: $36 \div 4 = 9$ rows. Rows of 6: $36 \div 6 = 6$ rows. Rows of 9: $36 \div 9 = 4$ rows.

Q8. (a) Split 48 into $40 + 8$: $(7 \times 40) + (7 \times 8) = 280 + 56 = 336$. There are 336 seedlings. (b) Swapping the order of a multiplication never changes the product: 7×48 counts 7 rows of 48 and 48×7 counts the same array turned around, so both give 336. (c) $336 \times \$2 = \$672.00$.

Q9. (a) Multiples of 4: 4, 8, 12, 16, 20, 24. Multiples of 6: 6, 12, 18, 24. (b) The first number in both lists is 12: the LCM of 4 and 6 is 12, so they next start together after 12 hours. (c) 12 hours after 8:00 am is 8:00 pm.

Q10. (a) Multiples of 9: 9, 18, 27, 36. Multiples of 12: 12, 24, 36. (b) The first number in both lists is 36: the LCM of 9 and 12 is 36, so they next beep together after 36 minutes. (c) 36 minutes after 7:15 am is 7:51 am.

Q11. (a) Pairs of 45: 1×45 , 3×15 , 5×9 — factors 1, 3, 5, 9, 15, 45. Pairs of 30: 1×30 , 2×15 , 3×10 , 5×6 — factors 1, 2, 3, 5, 6, 10, 15, 30. (b) The number of hampers must divide both 45 and 30 and be as large as possible — the HCF. Common factors: 1, 3, 5, 15. HCF = 15, so 15 hampers. (c) Jam: $45 \div 15 = 3$ jars per hamper. Honey: $30 \div 15 = 2$ jars per hamper.

Q12. (a) 27 has digit sum $2 + 7 = 9$, so 27 is divisible by 3. Any product involving 27 must then also be divisible by 3. But 818 has digit sum $8 + 1 + 8 = 17$, and 17 is not divisible by 3 — so 818 fails the test and cannot be the product. (b) $34 \times 27 = 34 \times (20 + 7) = (34 \times 20) + (34 \times 7) = 680 + 238 = 918$. (c) 918 has digit sum $9 + 1 + 8 = 18$, which is divisible by 9, so 918 is divisible by 9. It had to be: 27 is divisible by 9 ($27 = 9 \times 3$), so any product involving 27 is divisible by 9. The answer checks out.

Q13. (a) $92 \div 12 = 7$ remainder 8 ($7 \times 12 = 84$), so 7 full cartons. (b) $92 - 84 = 8$ eggs left loose. (c) A full carton holds 12, and 8 are already there, so $12 - 8 = 4$ more eggs are needed.

Q14. (a) $21 = 3 \times 7$, so it has factors beyond 1 and itself — composite. 23: not divisible by 2, 3 or 5 (and $5 \times 5 = 25 > 23$, so no bigger test is needed) — prime. $27 = 3 \times 9$ — composite. 29: not divisible by 2, 3 or 5 (and $5 \times 5 = 25 < 29$, $6 \times 6 = 36 > 29$, so tests stop at 5) — prime. (b) $91 = 7 \times 13$, so its factors are 1, 7, 13, 91 — four factors, more than two. Dean's tests only rule out 2 and 3; they never rule out 7. 91 is composite. (c) Any even number greater than 2 has at least three factors: 1, 2 and itself. So it cannot be prime. 2 itself has exactly two factors, 1 and 2, so 2 is prime — the only even prime.

Q15. (a) The pegs all have the same whole-number length, both offcuts are used up exactly, so the length must divide 48 and must divide 60 — a common factor. The longest choice is the HCF. Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60. The largest common factor is 12, so the pegs are 12 cm long. (b) $48 \div 12 = 4$ pegs from the first offcut; $60 \div 12 = 5$ pegs from the second. (c) $4 + 5 = 9$ pegs altogether.

Q16. (a) Sam works on days 4, 8, 12, ... — the multiples of 4. Anh works on the multiples of 6, and Dean on the multiples of 9. All three together means the day number sits in all three lists, so it must be a common multiple of 4, 6 and 9. (b) LCM of 4 and 6 is 12; LCM of 12 and 9 is 36. So they are next all together in 36 days. (c) The common multiples of 4, 6 and 9 up to 108 are 36, 72 and 108 — they are all together three more times within the 108 days.

Q17. (a) $6 \times 20 = 120$ tubs. (b) $120 \times \$2.35 = \$282.00$. (c) Total bill: $\$282.00 + \$35.60 = \$317.60$. Share: $\$317.60 \div 7 = \$45.3714\dots$, so each family pays **\$45.37** correct to the nearest cent.

Q18. (a) 8 packs give $8 \times 12 = 96$ sausages, and $96 < 100$ — not enough. 9 packs give $9 \times 12 = 108$, and $108 \geq 100$, so Grace buys 9 packs. (b) $9 \times \$4.50 = \$40.50$. (c) She has 108 sausages, so the row size must be a factor of 108 between 5 and 20. The factors of 108 are 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108; in the range are 6, 9, 12 and 18. Rows of 6: $108 \div 6 = 18$ rows. Rows of 9: $108 \div 9 = 12$ rows. Rows of 12: $108 \div 12 = 9$ rows. Rows of 18: $108 \div 18 = 6$ rows.

Chapter 2 Number — 2C Rounding whole numbers and decimals

About the data in this section. Every name, price, weight, time and other number in this section is **made up** (not real): pretend people, pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Theory

Numbers in real life rarely come out neat. A scale can show 2.317 kg. A share of a bill can work out to \$28.916666... A crowd count can be 23 847. We **round** a number when we replace it with a nearby number that is simpler to read, say or use. This section shows you how to round whole numbers and decimals, and how to keep rounding under control inside a calculation.

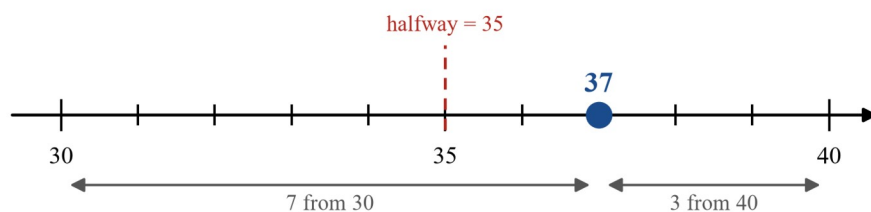
Why we round.

- **Money stops at the cent.** Bills and prices are written in dollars and cents, so an amount with part of a cent (like \$28.9166...) must be rounded to the nearest cent before anyone can pay it.
- **Measures have limits.** A kitchen scale that reads to the nearest gram cannot really tell you 2.31746 kg. Writing 2.32 kg is honest about what the scale can do.
- **Short numbers are easier to use.** A news report says “about 24 000 fans” instead of 23 847, because the rounded number is quicker to read and remember.

A rounded number is close to the exact number but not equal to it, so we use the sign $\approx$, which means “**is approximately equal to**”: for example, $2.317 \approx 2.32$ (to 2 decimal places).

Rounding whole numbers.

Rounding to the nearest ten asks: *which multiple of ten is this number closest to?* The number 37 sits between 30 and 40. It is 7 away from 30 and only 3 away from 40, so it is closer to 40, and $37 \approx 40$ to the nearest ten.



37 is past halfway and closer to 40, so 37 rounds to 40 (nearest ten)

The number exactly halfway between two tens (here, 35) is the same distance from both. The rule this book uses is that **exactly halfway rounds up**, so $35 \approx 40$ to the nearest ten. The same idea works for the nearest hundred ($6473 \approx 6500$) and the nearest thousand.

The rounding rule — it works for every place.

1. Find the digit in the place you are rounding to. Call it the **rounding digit**.
2. Look at the digit just to its right. Call it the **deciding digit**.
3. If the deciding digit is **5 or more**, add 1 to the rounding digit (**round up**). If the deciding digit is **less than 5**, leave the rounding digit as it is (**round down**).

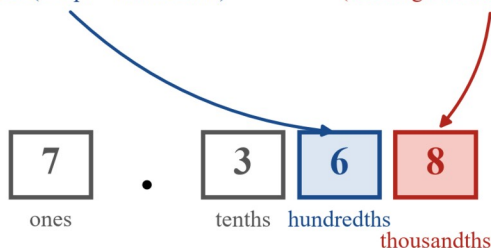
Then, for a whole number, put zeros in every place to the right of the rounding digit. For a decimal, simply drop the digits to the right of the rounding digit.

Worked model 1. Round 6 473 to the nearest hundred. The rounding digit is the 4 (hundreds); the deciding digit is the 7 (tens). 7 is 5 or more, so round up: the 4 becomes 5 and zeros fill the places after it. So $6473 \approx 6500$ to the nearest hundred.

Worked model 2. Round 7.368 to 2 decimal places. The rounding digit is the 6 (hundredths); the deciding digit is the 8 (thousandths). 8 is 5 or more, so round up: the 6 becomes 7 and the digits after it are dropped. So $7.368 \approx 7.37$ to 2 decimal places.

Step 1 find the rounding digit for the place asked (2 d.p. = hundredths)

Step 2 look at the deciding digit (next digit to the right)



Step 3 the deciding digit is 8, and $8 \geq 5$, so add 1 to the rounding digit and drop the digits after it: the 6 becomes 7.

$$7.368 \approx 7.37 \text{ (to 2 d.p.)}$$

If the deciding digit were less than 5, the rounding digit would stay the same (round down).

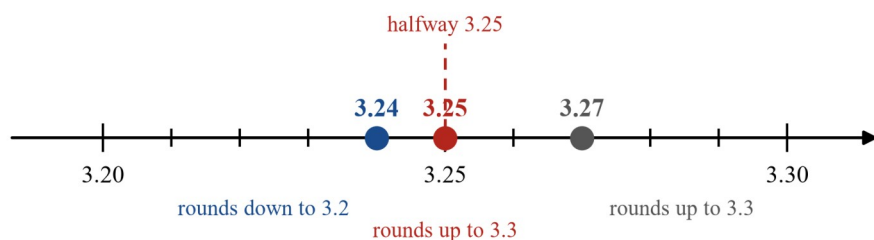
Decimal places, and the halfway rule for decimals.

The digits after the decimal point are called **decimal places** (written **d.p.**). In 7.368: the 3 is the first decimal place (tenths), the 6 is the second (hundredths) and the 8 is the third (thousandths).

ones	.	tenths (1st d.p.)	hundredths (2nd d.p.)	thousandths (3rd d.p.)
7	.	3	6	8

In this subject you round decimals to **3 decimal places at most**. Rounding to the nearest whole number means keeping no decimal places at all (the deciding digit is then the tenths digit). Calculator answers can have far more digits than three — those long answers are exactly the numbers that need rounding.

The halfway rule is the same as for whole numbers: a deciding digit of 5 or more rounds up, less than 5 rounds down. On the number line below, 3.24 is below the halfway mark and rounds down, while 3.25 (exactly halfway) and 3.27 round up.



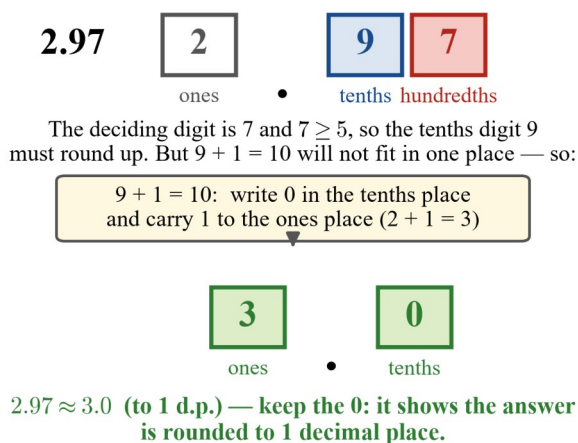
deciding digit less than 5 → round down · deciding digit 5 or more → round up (the 5 goes up)

Rounding money: the nearest cent.

“Correct to the nearest cent” means 2 decimal places, because the cent is the hundredth of a dollar. Examples: $\$2.994 \approx \$2.99$ (the deciding digit 4 rounds down) and $\$12.375 \approx \$12.38$ (the deciding digit is a half-cent, 5, so it rounds up).

When rounding carries.

Sometimes the rounding digit is a 9, and adding 1 makes 10, which will not fit in one place. Then write 0 in that place and **carry** 1 (add 1) to the place on the left. Rounding 2.97 to 1 d.p.: the deciding digit 7 rounds the tenths digit 9 up, $9 + 1 = 10$, so write 0 and carry 1 to the ones place: $2.97 \approx 3.0$.



Keep the 0 in an answer like 3.0: it tells the reader the answer was rounded to 1 d.p. (Just writing 3 would hide that.) Carries also happen in whole numbers: $198 \approx 200$ to the nearest ten.

Rounding inside a calculation.

Two rules keep a calculation honest:

- Do all of the working at **full precision** — keep every digit your calculator gives you — and round **only the final answer**, to the precision the question asks for.
- Rounding too early can change the answer, so it is an error in any task that says “round your final answer”.

Worked model 3. Six shelves each need 3.24 m of timber. The exact total is $6 \times 3.24 = 19.44$ m, which is 19 m to the nearest metre. But if you round first ($3.24 \approx 3$) you get $6 \times 3 = 18$ m — a whole metre less, and not the rounded value of the true total. Round at the end, not along the way.

Words that tell you the precision.

The question says	It means	Example
to the nearest ten	round to a multiple of 10	$37 \approx 40$
to the nearest whole number	keep no decimal places	$3.6 \approx 4$
to 1 decimal place (1 d.p.)	keep one digit after the point	$2.47 \approx 2.5$
to 2 decimal places (2 d.p.), or correct to the nearest cent	keep two digits after the point	$\$12.375 \approx \$12.38$
to 3 decimal places (3 d.p.)	keep three digits after the point	$0.84375 \approx 0.844$

How fine should the rounding be? Match the precision to the job. A bill goes to the cent. A swimming race is timed to thousandths of a second (3 d.p.). A newspaper rounds a crowd to the nearest hundred or thousand. Whatever the job, the precision is part of the answer — which is why every question in this section tells you the precision it wants, and your answer should repeat it.

Worked examples

Example 1 — scaffolded

The Fernvale community fair counted 4 387 visitors through the gate on Saturday. Round the count to:

- (a) the nearest ten
- (b) the nearest hundred
- (c) the nearest thousand

Working	Comment
(a) The rounding digit is the 8 (tens). The deciding digit is 7, which is 5 or more, so round up: $4\,387 \approx 4\,390$.	Round up: the 8 becomes 9, and a zero fills the ones place.
(b) The rounding digit is the 3 (hundreds). The deciding digit is 8, so round up: $4\,387 \approx 4\,400$.	The 3 becomes 4; zeros fill the tens and ones places.
(c) The rounding digit is the 4 (thousands). The deciding digit is 3, less than 5, so round down: $4\,387 \approx 4\,000$.	Round down: the 4 stays; zeros fill the places after it.

Example 2 — scaffolded

A post office weighs three parcels on a digital scale.

Parcel	Weight (kg)
A	1.236
B	0.875
C	2.964

Made-up data: the parcels and every weight in this table are made up for this section.

Round each weight to:

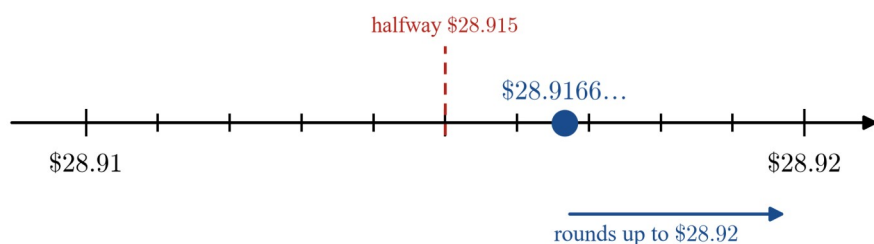
- (a) the nearest whole kilogram
- (b) 1 decimal place
- (c) 2 decimal places

Working	Comment
(a) A: $1.236 \approx 1$; B: $0.875 \approx 1$; C: $2.964 \approx 3$.	Deciding digits (tenths): 2 rounds down; 8 and 9 round up.
(b) A: $1.236 \approx 1.2$; B: $0.875 \approx 0.9$; C: $2.964 \approx 3.0$.	C carries: the 9 becomes 10, so write 0 and carry 1 — and keep the 0 to show 1 d.p.
(c) A: $1.236 \approx 1.24$; B: $0.875 \approx 0.88$; C: $2.964 \approx 2.96$.	B's deciding digit is 5, and 5 goes up. C's deciding digit 4 rounds down.

Example 3 — unscaffolded

Three friends share a restaurant bill of \$86.75 equally.

- (a) Find each person's share, correct to the nearest cent.
- (b) Three copies of your answer to (a) do not add up to \$86.75. Explain why, and give a split of the bill into three amounts (each correct to the nearest cent) that adds to exactly \$86.75.
- (c) Each share is $86.75 \div 3 = 28.916666\dots$ dollars. For the nearest cent the rounding digit is the 1 (hundredths) and the deciding digit is the 6 (thousandths), which is 5 or more, so round up: each share is \$28.92, correct to the nearest cent.



$\$86.75 \div 3 = \$28.9166\dots$ is past the halfway mark, so each person pays $\$28.92$ (nearest cent)

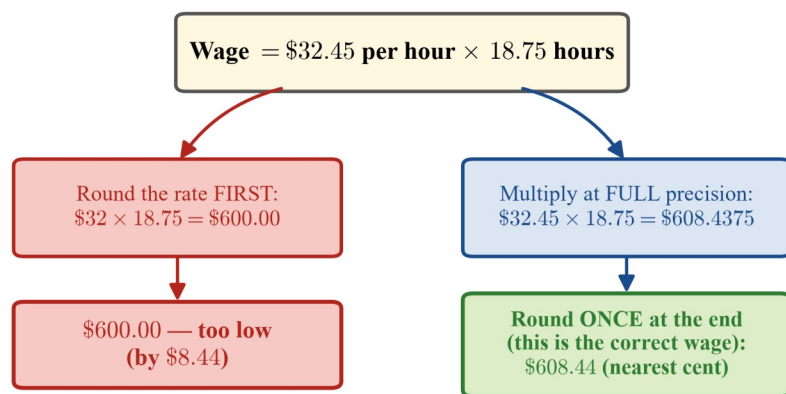
- (b) $3 \times \$28.92 = \$86.76$, one cent more than the bill, because every share was rounded up by a fraction of a cent. A split that adds exactly is $\$28.92 + \$28.92 + \$28.91 = \$86.75$: two friends pay $\$28.92$ and one pays $\$28.91$.

$\therefore$ each share is $\$28.92$ correct to the nearest cent, and a fair exact split of the bill is $\$28.92$, $\$28.92$ and $\$28.91$.

Example 4 — unscaffolded

Priya is paid $\$32.45$ per hour. In one week she works 18.75 hours.

- Find Priya's wage for the week, correct to the nearest cent.
- Priya's friend rounds the hourly rate to $\$32$ first, then multiplies by 18.75. How far is his answer from your answer to (a)?
- The wage at full precision is $32.45 \times 18.75 = 608.4375$ dollars. For the nearest cent the deciding digit is the 7 (thousandths), which is 5 or more, so round up: Priya's wage is $\$608.44$, correct to the nearest cent.
- Rounding first gives $32 \times 18.75 = 600$, that is $\$600.00$. The difference from part (a) is $\$608.44 - \$600.00 = \$8.44$.



Rounding early changes the answer and loses $\$8.44$ of Priya's pay.

Carry full precision through every step; round only the final answer, to the precision the question asks for.

$\therefore$ rounded once at the end, Priya's wage is $\$608.44$; rounding the rate first loses $\$8.44$ of her pay — so carry full precision and round only the final answer.

Practice questions

Scaffolded

Q1. Round 1 472 to: (a) the nearest ten (b) the nearest hundred (c) the nearest thousand

Q2. Round 6.375 to: (a) the nearest whole number (b) 1 decimal place (c) 2 decimal places

Q3. A service station's price computer works out these amounts before rounding to cents. Write each amount correct to the nearest cent. (a) \$1.876 (b) \$0.625 (c) \$2.994

Q4. A market stall weighs four bags of potatoes.

Bag	Weight (kg)
1	2.317
2	1.985
3	3.062
4	2.475

Made-up data: the bags and every weight in this table are made up for this section.

Round each weight to: (a) 1 decimal place (b) 2 decimal places (c) the nearest whole kilogram

Q5. Each of these numbers carries when it rounds. Round as instructed. (a) 9.96 to 1 decimal place (b) 3.997 to 2 decimal places (c) 198 to the nearest ten

Q6. Seven netball teammates share the \$450 cost of new uniforms equally. (a) Write the division that gives one person's share. (b) Work out the share and round it correct to the nearest cent. (c) Check by multiplying: is 7 times your answer to (b) exactly \$450? What do you notice?

Un scaffolded

Q7. A football club counted a crowd of 23 847 at its grand final. Round the crowd to: (a) the nearest hundred (b) the nearest thousand (c) the nearest ten thousand

Q8. A swim-timing app shows Bella's 50 m freestyle time as 31.4267 seconds. Round her time to: (a) 3 decimal places (b) 2 decimal places (c) the nearest whole second

Q9. An online store charges \$110.00 postage on an order of 6 items and splits the postage evenly across the items. (a) Find the postage share of one item, correct to the nearest cent. (b) Multiply your answer to (a) by 6. (c) Explain why your answer to (b) is not \$110.00.

Q10. A car uses 41.36 L of petrol over 512 km. (a) Show that the car uses 0.08078125 L per kilometre. (b) Hence find how many litres the car uses per 100 km, correct to 1 decimal place. (c) The driver says the car uses "about 8 L per 100 km". To what place is that value rounded?

Q11. Sam works in a café. In one week he works 24.5 hours at \$28.75 per hour, and he earns \$63.80 in tips. (a) Show that Sam's wage before tips is \$704.375. (b) Find his total earnings for the week, before rounding. (c) Write his total earnings correct to the nearest cent.

Q12. A deli's scale works out these amounts (in dollars) before rounding to cents: \$3.069, \$4.675 and \$2.328. (a) Round each amount to the nearest cent. (b) Add your three rounded amounts from (a). (c) Add the three exact amounts, then round that total to the nearest cent. (d) Should the till charge the total from (b) or the total from (c)? Give a reason.

Q13. A city's population is 2 847 391. (a) Round the population to the nearest 100 000. (b) Round it to the nearest 10 000. (c) Round it to the nearest million. (d) A newspaper writes the population as "2.8 million". Which of your answers above does this match?

Q14. A bottle of cough mixture holds 150 mL, and one dose is 5.4 mL. (a) Find how many doses the bottle holds, correct to 1 decimal place. (b) How many **full** doses can be poured from one bottle? (c) Explain why your answer to (b) is not your answer to (a) rounded to the nearest whole number.

Q15. Three students each made one rounding error. For each, explain the error and give the correct rounded value. (a) Tom rounds 2.96 to the nearest whole number and writes 2. (b) Sarah rounds 4.375 to 2 decimal places and writes 4.37. (c) Jack rounds \$6.998 to the nearest cent and writes \$6.99.

Q16. Wei works out $250 \div 7$ on his phone. The screen shows 35.714285714. (a) Round the number on the screen to the nearest whole number. (b) Round it to 2 decimal places. (c) Wei says “ $250 \div 7$ is exactly 35.714285714”. Is he correct? Explain.

Q17. A school camp costs \$10 530 in total, and 68 students are going. (a) Find the cost per student, correct to the nearest cent. (b) The school asks each family to pay \$155. To what place is that the value from (a) rounded? (c) Show that collecting \$155 from all 68 families covers the camp cost, and find how much extra is collected.

Q18. A store sells 24-packs of water bottles for \$15.38 and 6-packs for \$4.05. (a) Find the cost of one bottle from a 24-pack, correct to the nearest cent. (b) Find the cost of one bottle from a 6-pack, correct to the nearest cent. (c) Which pack is cheaper per bottle? (d) Using exact values, find how much you save by buying 24 bottles as one 24-pack instead of four 6-packs, correct to the nearest cent.

Answers

Q1. (a) 1 470; (b) 1 500; (c) 1 000. **Q2.** (a) 6; (b) 6.4; (c) 6.38. **Q3.** (a) \$1.88; (b) \$0.63; (c) \$2.99. **Q4.** (a) 2.3, 2.0, 3.1, 2.5 kg; (b) 2.32, 1.99, 3.06, 2.48 kg; (c) 2, 2, 3, 2 kg. **Q5.** (a) 10.0; (b) 4.00; (c) 200. **Q6.** (a) $450 \div 7$; (b) \$64.29; (c) no — $7 \times \$64.29 = \$450.03$, 3 cents over, because the share was rounded up. **Q7.** (a) 23 800; (b) 24 000; (c) 20 000. **Q8.** (a) 31.427 s; (b) 31.43 s; (c) 31 s. **Q9.** (a) \$18.33; (b) \$109.98; (c) each share was rounded down by a fraction of a cent, so six rounded shares fall 2 cents short of the exact total. **Q10.** (a) $41.36 \div 512 = 0.08078125$; (b) 8.1 L per 100 km; (c) the nearest whole number (of litres per 100 km). **Q11.** (a) $28.75 \times 24.5 = \$704.375$; (b) \$768.175; (c) \$768.18. **Q12.** (a) \$3.07, \$4.68, \$2.33; (b) \$10.08; (c) \$10.07; (d) (c) — round once, at the end of the calculation. **Q13.** (a) 2 800 000; (b) 2 850 000; (c) 3 000 000; (d) (a). **Q14.** (a) 27.8; (b) 27 full doses; (c) 28 doses would need 151.2 mL, more than the bottle holds — the 4.2 mL left over is not a full dose, so round down here. **Q15.** (a) the deciding digit 9 is 5 or more, so 3; (b) the deciding digit 5 goes up, so 4.38; (c) the carry makes it \$7.00. **Q16.** (a) 36; (b) 35.71; (c) no — the exact answer keeps going past the digits the screen shows, so the screen value is already rounded. **Q17.** (a) \$154.85; (b) the nearest dollar; (c) $68 \times \$155 = \$10\,540$, which covers \$10 530 with \$10 extra. **Q18.** (a) \$0.64; (b) \$0.68; (c) the 24-pack; (d) \$0.82.

Fully-worked solutions

Q1. (a) The rounding digit is the 7 (tens); the deciding digit 2 is less than 5, so round down: $1\,472 \approx 1\,470$ to the nearest ten. (b) The rounding digit is the 4 (hundreds); the deciding digit 7 rounds up: $1\,472 \approx 1\,500$ to the nearest hundred. (c) The rounding digit is the 1 (thousands); the deciding digit 4 rounds down: $1\,472 \approx 1\,000$ to the nearest thousand.

Q2. (a) The deciding digit (tenths) is 3, less than 5: $6.375 \approx 6$ to the nearest whole number. (b) The deciding digit (hundredths) is 7, so round up: $6.375 \approx 6.4$ to 1 d.p. (c) The deciding digit (thousandths) is 5, and 5 goes up: $6.375 \approx 6.38$ to 2 d.p.

Q3. (a) Deciding digit 6 rounds up: $\$1.876 \approx \$1.88$ to the nearest cent. (b) Deciding digit 5 rounds up: $\$0.625 \approx \$0.63$ to the nearest cent. (c) Deciding digit 4 rounds down: $\$2.994 \approx \$2.99$ to the nearest cent.

Q4. (a) To 1 d.p. the deciding digit is the hundredths digit: $2.317 \approx 2.3$; $1.985 \approx 2.0$ (the 8 rounds up and carries); $3.062 \approx 3.1$; $2.475 \approx 2.5$. (b) To 2 d.p. the deciding digit is the thousandths digit: $2.317 \approx 2.32$; $1.985 \approx 1.99$ (the 5 goes up); $3.062 \approx 3.06$; $2.475 \approx 2.48$ (the 5 goes up). (c) To the nearest kg the deciding digit is the tenths digit: $2.317 \approx 2$; $1.985 \approx 2$; $3.062 \approx 3$; $2.475 \approx 2$.

Q5. (a) 9.96 to 1 d.p.: the deciding digit 6 rounds the 9 up to 10, so write 0 and carry: $9.96 \approx 10.0$. (b) 3.997 to 2 d.p.: the deciding digit 7 rounds the second 9 up, which carries twice: $3.997 \approx 4.00$ (keep both zeros to show 2 d.p.). (c) 198 to the nearest ten: the deciding digit 8 rounds the 9 up to 10, so carry: $198 \approx 200$.

Q6. (a) One share is $450 \div 7$. (b) $450 \div 7 = 64.285714\dots$; the deciding digit for the cent is the 5 (thousandths) and the digits after it make the value more than half, so round up: each person pays \$64.29, correct to the nearest cent. (c) $7 \times \$64.29 = \$450.03$, which is 3 cents **more** than \$450. Each share was rounded up by a fraction of a cent, so seven rounded shares overshoot the exact total — the same idea as Example 3, where the rounded shares overshoot the bill.

Q7. (a) Rounding digit 8 (hundreds), deciding digit 4 rounds down: $23\,847 \approx 23\,800$ to the nearest hundred. (b) Rounding digit 3 (thousands), deciding digit 8 rounds up: $23\,847 \approx 24\,000$ to the nearest thousand. (c) Rounding digit 2 (ten thousands), deciding digit 3 rounds down: $23\,847 \approx 20\,000$ to the nearest ten thousand.

Q8. (a) Rounding digit 6 (thousandths), deciding digit 7 rounds up: $31.4267 \approx 31.427$ s to 3 d.p. (b) Rounding digit 2 (hundredths), deciding digit 6 rounds up: $31.4267 \approx 31.43$ s to 2 d.p. (c) Rounding digit 1 (ones), deciding digit 4 rounds down: $31.4267 \approx 31$ s to the nearest whole second.

Q9. (a) $110.00 \div 6 = 18.3333\dots$; the deciding digit for the cent is 3, less than 5, so round down: each item's share is \$18.33, correct to the nearest cent. (b) $6 \times \$18.33 = \$109.98$. (c) Each share was rounded **down** by a third of a cent, so six rounded shares come to 2 cents less than the exact \$110.00. Rounded pieces do not always add back to the exact total.

Q10. (a) Litres per kilometre:

$$41.36 \div 512 = 0.08078125$$

so the car uses 0.08078125 L per km, as required. (b) Per 100 km, multiply by 100: $0.08078125 \times 100 = 8.078125$ L. The deciding digit (hundredths) is 7, so round up: the car uses 8.1 L per 100 km, correct to 1 d.p. (c) “About 8 L per 100 km” is 8.078125 rounded to the nearest whole number.

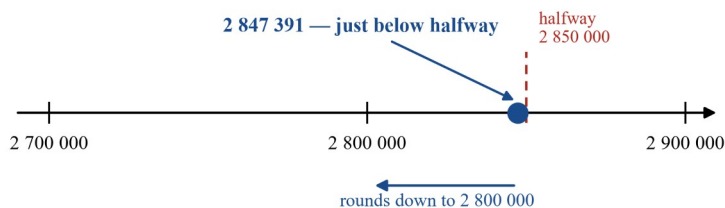
Q11. (a) Wage before tips:

$$28.75 \times 24.5 = 704.375$$

so Sam’s wage before tips is \$704.375, as required. (b) Total before rounding: $\$704.375 + \$63.80 = \$768.175$. (c) The deciding digit for the cent is the 5 (thousandths), and 5 goes up: Sam’s total earnings are \$768.18, correct to the nearest cent. (Note the full precision was kept from (a) into (c) — nothing was rounded early.)

Q12. (a) $\$3.069 \approx \$3.07$ (deciding digit 9); $\$4.675 \approx \$4.68$ (deciding digit 5 goes up); $\$2.328 \approx \$2.33$ (deciding digit 8). (b) $\$3.07 + \$4.68 + \$2.33 = \$10.08$. (c) Exact total: $\$3.069 + \$4.675 + \$2.328 = \$10.072$; the deciding digit 2 rounds down, so the exact total is \$10.07 to the nearest cent. (d) The till should charge the total from (c), \$10.07: the rule is to carry full precision and round **once**, at the end. Rounding each item first added an extra cent.

Q13. (a) Rounding digit 8 (hundred thousands), deciding digit 4 rounds down: $2\,847\,391 \approx 2\,800\,000$ to the nearest 100 000. (b) Rounding digit 4 (ten thousands), deciding digit 7 rounds up: $2\,847\,391 \approx 2\,850\,000$ to the nearest 10 000. (c) Rounding digit 2 (millions), deciding digit 8 rounds up: $2\,847\,391 \approx 3\,000\,000$ to the nearest million. (d) “2.8 million” is 2 800 000, which matches answer (a).



2 847 391 is below the halfway mark, so to the nearest 100 000 the population is 2 800 000 — the paper’s “2.8 million”.

Q14. (a) $150 \div 5.4 = 27.7777\dots$; the deciding digit for 1 d.p. is 7, so round up: the bottle holds 27.8 doses, correct to 1 d.p. (b) Full doses only: 27 doses use $27 \times 5.4 = 145.8$ mL, leaving $150 - 145.8 = 4.2$ mL — not enough for another 5.4 mL dose. So 27 full doses. (c) Rounding $27.7777\dots$ to the nearest whole number gives 28, but 28 doses would need $28 \times 5.4 = 151.2$ mL, more than the 150 mL in the bottle. Here the context forces rounding **down**: you cannot pour a dose the bottle does not contain.

Q15. (a) Tom dropped the decimals instead of rounding: the deciding digit 9 is 5 or more, so $2.96 \approx 3$ to the nearest whole number. (b) Sarah rounded the 5 down: a deciding digit of 5 goes up, so $4.375 \approx 4.38$ to 2 d.p. (c) Jack forgot the carry: the deciding digit 8 rounds the hundredths 9 up to 10, which carries again, so $\$6.998 \approx \$7.00$ to the nearest cent.

Q16. (a) Rounding digit 5 (ones), deciding digit 7 rounds up: $35.714285714 \approx 36$ to the nearest whole number. (b) Rounding digit 1 (hundredths), deciding digit 4 rounds down: $35.714285714 \approx 35.71$ to 2 d.p. (c) No. $250 \div 7$ does not end: the exact value keeps going ($35.714285714285\dots$), so the phone’s display is already a rounded version of the answer. (For parts (a) and (b) the extra hidden digits make no difference, because they sit after the deciding digit.)

Q17. (a) $10530 \div 68 = 154.852941\dots$; the deciding digit for the cent is 2, less than 5, so round down: the cost per student is \$154.85, correct to the nearest cent. (b) \$155 is $\$154.852941\dots$ rounded to the nearest dollar. (c) Collecting \$155 from each family gives $68 \times 155 = 10540$, that is \$10 540, which covers the \$10 530 cost. The extra collected is $\$10\,540 - \$10\,530 = \$10$. (Rounding the share **up** to a whole dollar here is deliberate: it makes sure the money collected is enough.)

Q18. (a) $15.38 \div 24 = 0.640833\dots$; the deciding digit for the cent is 0, less than 5: one bottle from a 24-pack costs \$0.64, correct to the nearest cent. (b) $4.05 \div 6 = 0.675$ exactly; the deciding digit is 5, and 5 goes up: one bottle from a 6-pack costs \$0.68, correct to the nearest cent. (c) Comparing the exact values, $0.640833\dots < 0.675$, so the **24-pack** is cheaper per bottle. (d) Four 6-packs cost $4 \times \$4.05 = \$16.20$ for 24 bottles; the 24-pack costs \$15.38. The saving is $\$16.20 - \$15.38 = \$0.82$, correct to the nearest cent.

Chapter 2 Number — 2D Positive and negative numbers

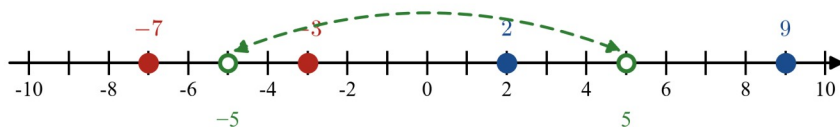
Theory

Many everyday numbers sit **below zero**. The freezer at the shop is colder than 0°C . A bank account can owe money instead of holding it. A mine level sits below sea level. Numbers below zero are written with a minus sign in front, and they change how we add, subtract, multiply and divide. This section builds the rules and then uses them on real-life tasks.

Positive, negative and zero.

A **positive number** is above zero. It can be written with a plus sign, $+5$, or with no sign at all, 5 . A **negative number** is below zero and always carries a minus sign: -5 . Read -5 as “negative five”. Zero is neither positive nor negative. It is the starting point that the other numbers are measured from.

opposites: same distance from 0, different sides



numbers get bigger as you move right: $-7 < -3 < 0 < 2 < 9$

Ordering: bigger means further right.

On a number line, numbers get bigger as you move right. So -7 is **less than** -3 , written $-7 < -3$, even though 7 looks “bigger” than 3. Reading the number line above from left to right gives the order $-7 < -3 < 0 < 2 < 9$. With two negative numbers, the one further from zero is the smaller one: a freezer at -7°C is colder than a freezer at -3°C .

Opposites.

The **opposite** of a number is the same distance from zero, on the other side. The opposite of 7 is -7 . The opposite of -12 is 12. The opposite of 0 is 0, because zero has no other side. Taking the opposite twice brings you back: the opposite of -7 is 7 again.

Adding and subtracting.

Think of movement along the number line. Adding a positive moves right. Adding a negative moves left. Subtracting moves the other way: subtracting a positive moves left, and subtracting a negative moves **right**.

- $5 + (-3) = 2$ — start at 5, move 3 left.
- $-5 + (-3) = -8$ — start at -5 , move 3 further left.
- $-5 + 8 = 3$ — start at -5 , move 8 right.
- $3 - 7 = -4$ — start at 3, move 7 left.
- $-4 - 3 = -7$ — start at -4 , move 3 left.
- $-4 - (-3) = -1$ — subtracting a negative is adding its opposite: $-4 - (-3) = -4 + 3 = -1$.

That last rule is the one people forget: **taking away a negative is the same as adding a positive**. Wiping out a debt makes you better off, not worse off.

Multiplying and dividing: the sign rules.

Signs of the two numbers	Answer sign	Example
same (both positive, or both negative)	positive	$-6 \times (-5) = 30$ and $-24 \div (-6) = 4$

Signs of the two numbers	Answer sign	Example
different (one of each)	negative	$-7 \times 4 = -28$ and $-24 \div 6 = -4$

same signs $\rightarrow$ positive

different signs $\rightarrow$ negative

$+\times+=+$	$+\times-= -$
$-\times+= -$	$-\times-= +$

The same rules work for division ($\div$) as for multiplication ($\times$).

Why a negative times a negative is positive.

The rule is not arbitrary. Watch the pattern as the first number steps down by 1 each line:

Calculation	Answer
$3 \times (-2)$	-6
$2 \times (-2)$	-4
$1 \times (-2)$	-2
$0 \times (-2)$	0
$-1 \times (-2)$	2
$-2 \times (-2)$	4

Each answer is 2 more than the one above it. That steady step does not stop at zero, so once the first number goes negative the answers must be positive. Any other rule would break the pattern.

A first money picture.

A bank account **balance** is the money in the account. Say the balance is \$85 and a bill of \$120 comes out: $\$85 - \$120 = -\$35$. A negative balance means the account is **overdrawn** — the bank is owed money. Here the account is overdrawn by \$35. Money in is added; money out is subtracted; the sign of the balance tells you which side of zero you are on.

About the data in this section. The balances, temperatures, match scores and transaction lists here are **made up** (not real): pretend people, pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

A shop cool room is at -6°C . A settings change makes the temperature fall 7°C . Later a door is left open and the temperature rises 9°C . Find the temperature after the fall, the final temperature, and the overall (net) change.

Working

After the fall: $-6 - 7 = -13$, so -13°C .

After the rise: $-13 + 9 = -4$, so -4°C .

Net change: $-7 + 9 = +2$, so 2°C warmer overall.

Check: $-6 + 2 = -4$, which matches the final temperature.

Comment

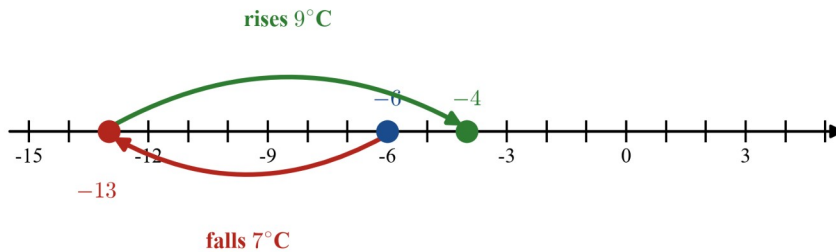
A fall means subtract. Move 7 left from -6 .

A rise means add. Move 9 right from -13 .

Add the two changes on their own: down 7, up 9.

The net change must land on the same place.

net change: $-7 + 9 = +2$



Example 2 — scaffolded

Hana's bank account has a balance of \$85.00 on Monday. During the week a phone bill of \$120.00 comes out, her wage of \$200.00 goes in, and an account fee of \$12.00 comes out. Find the balance after each of the three transactions, and state the most overdrawn the account gets.

Working

Start: \$85.00.

Phone bill: $\$85.00 - \$120.00 = -\$35.00$.

Wage: $-\$35.00 + \$200.00 = \$165.00$.

Fee: $\$165.00 - \$12.00 = \$153.00$.

Most overdrawn: $-\$35.00$, straight after the phone bill.

Comment

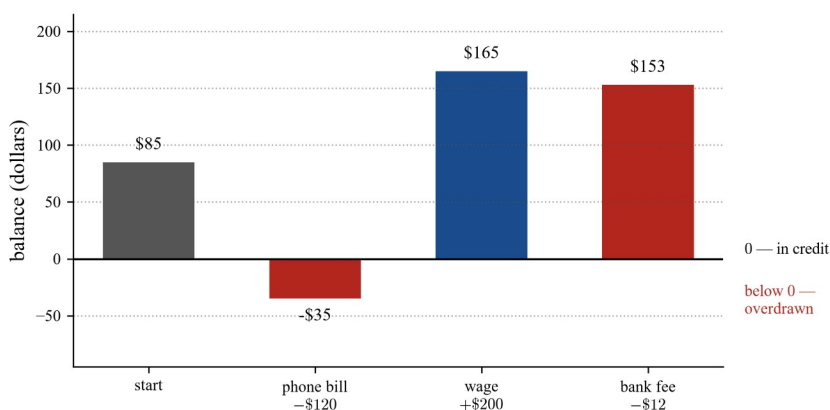
Money in the account.

More went out than was in, so the balance goes negative: overdrawn.

Money in: add. The wage clears the debt first, then builds credit.

Money out: subtract.

The lowest balance of the week.



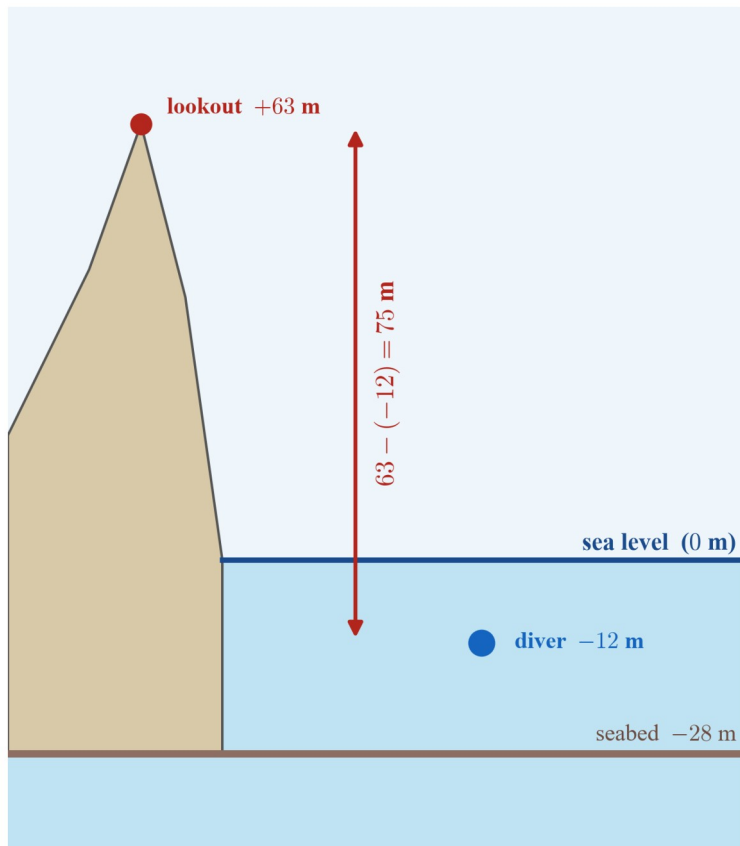
Example 3 — unscaffolded

A cliff-top lookout is 63 m above sea level. A diver hovers 12 m below sea level, and the seabed under the diver is 28 m below sea level. Find the vertical distance from the lookout to the diver, the distance the diver descends to reach the seabed, and the vertical distance from the lookout to the seabed.

Write each height as a signed number, measured from sea level: lookout $+63$, diver -12 , seabed -28 . The gap between two heights is the higher minus the lower, and subtracting a negative turns into adding:

- lookout to diver: $63 - (-12) = 63 + 12 = 75$ m;
- diver to seabed: $(-12) - (-28) = -12 + 28 = 16$ m;
- lookout to seabed: $63 - (-28) = 63 + 28 = 91$ m.

Check the third answer against the first two: $75 + 16 = 91$ m, which agrees.



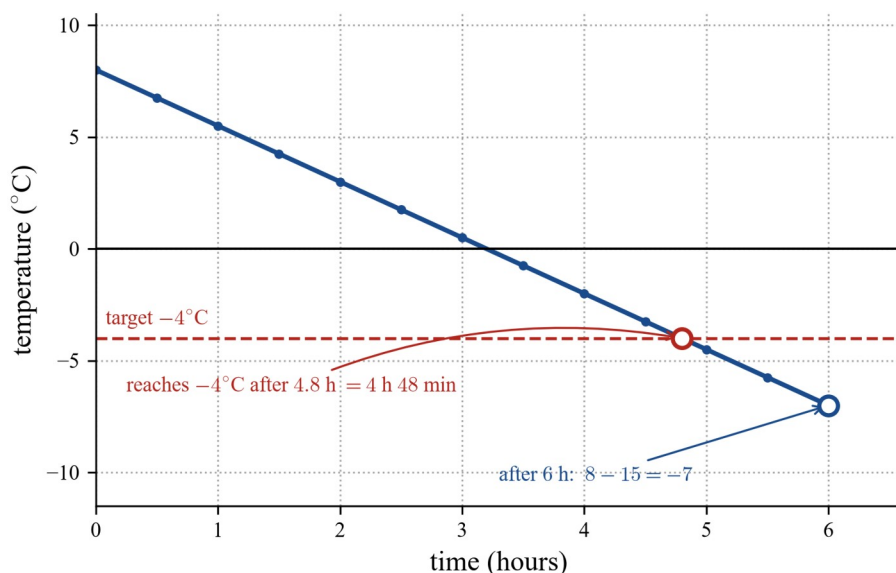
$\therefore$ the lookout is 75 m above the diver, the diver descends 16 m to the seabed, and the lookout is 91 m above the seabed.

Example 4 — unscaffolded

A chill room is switched on at 8°C to cool a delivery of fruit. Once running, the temperature falls 2.5°C every hour. Find the temperature after 6 hours, and find how long the room takes to reach -4°C .

After 6 hours the total fall is $2.5 \times 6 = 15$ degrees, so the temperature is $8 - 15 = -7^{\circ}\text{C}$.

To reach -4°C the temperature must fall from 8 down to -4 , a fall of $8 - (-4) = 8 + 4 = 12$ degrees. At 2.5 degrees an hour that takes $12 \div 2.5 = 4.8$ hours. The 0.8 of an hour is $0.8 \times 60 = 48$ minutes, so 4.8 h is 4 h 48 min. Check on the graph: the line crosses the -4°C dashed line just before the 5-hour mark.



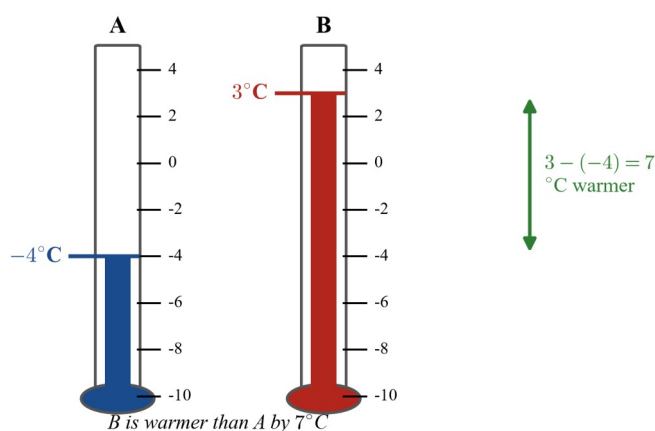
∴ after 6 hours the chill room is at -7°C , and it reaches -4°C after 4.8 hours, that is 4 h 48 min.

Practice questions

Scaffolded

Q1. (a) Write these numbers from smallest to largest: -9 , 4 , -2 , 0 , 7 , -13 . (b) Write the opposite of each: 6 , -11 , -4.5 . (c) For each pair below, write $<$ or $>$ between the two numbers: -6 and -2 ; 0 and -8 ; -15 and -10 .

Q2. The two thermometers below show overnight temperatures in two towns.



- Write down the temperature shown on thermometer A and on thermometer B.
- How much warmer is town B than town A?
- Overnight, town A's temperature falls another 5°C . What temperature does thermometer A show now?

Q3. Sam's bank account balance is \$40.00. (a) Sam withdraws \$75.00 at an ATM (a **withdrawal** takes money out). What is the new balance? (b) Sam then deposits \$150.00 (a **deposit** puts money in). What is the balance now? (c) The bank then charges a \$9.00 fee. What is the balance now?

Q4. Calculate each of these. (a) $-8 + 5$ (b) $-6 - 7$ (c) $-2 - (-9)$

Q5. Calculate each of these. (a) -7×6 (b) $-8 \times (-5)$ (c) $-36 \div 4$

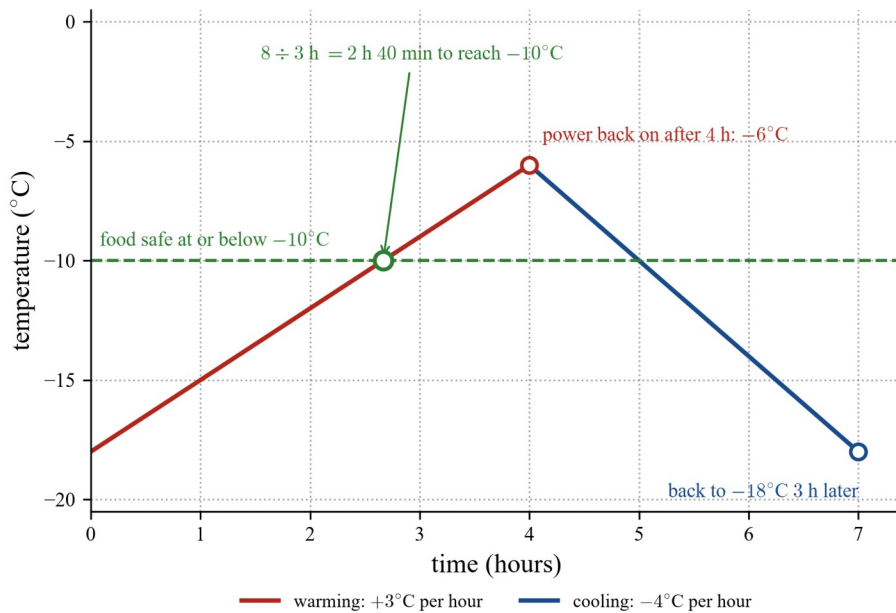
Q6. In an office building, floors above ground are numbered $1, 2, 3, \dots$ and floors below ground are numbered $-1, -2, -3, \dots$ (also called B1, B2, B3). The ground floor is 0. (a) A lift starts at floor 7 and travels down 10 floors. Which floor is it on now? (b) A lift at floor -2 travels up 9 floors. Which floor is it on now? (c) A worker starts at the

ground floor, goes up 5 floors, down 11 floors, then up 4 floors. Which floor is she on now? Give each answer as a signed number and as a floor name.

Un scaffolded

Q7. A hill top is 310 m above sea level. A town in a deep valley nearby is 28 m below sea level, and the town's water pump sits 46 m below sea level. (a) Find the vertical distance from the hill top to the town. (b) Find the vertical distance from the town down to the water pump. (c) Write the four heights (hill top, sea level, town, water pump) from lowest to highest.

Q8. A shop freezer holds food at -18°C . The power fails and the temperature rises 3°C per hour for 4 hours until the power returns. The cooling unit then brings the temperature down 4°C per hour until it is back at -18°C . Food stays safe while the temperature is -10°C or below.



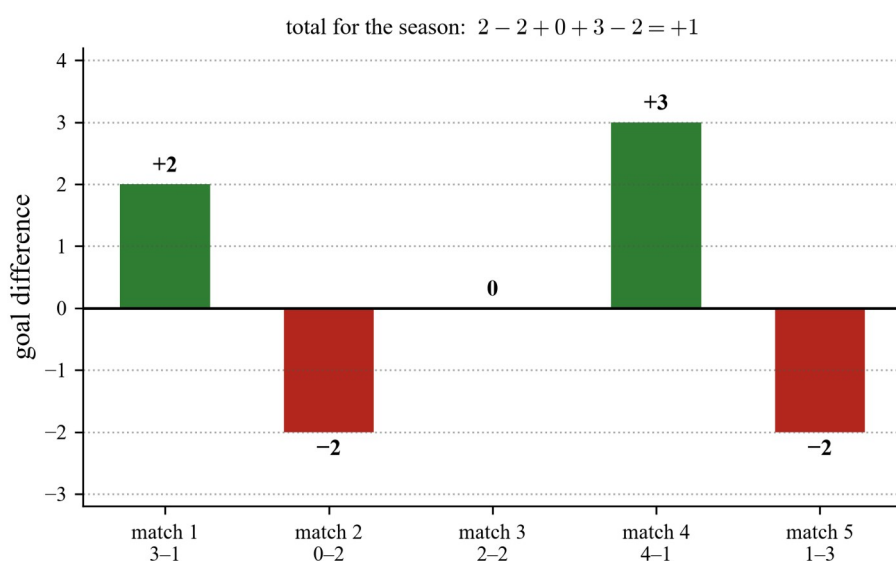
- What is the temperature when the power returns?
- For how long after the outage begins is the food still safe? Give your answer in hours and minutes.
- Once the power returns, how many hours does the cooling unit take to get back to -18°C ?

Q9. A soccer team's **goal difference** for a match is its goals for, minus its goals against. A positive goal difference means it scored more than it conceded.

Match	Goals for	Goals against
1	3	1
2	0	2
3	2	2
4	4	1
5	1	3

Made-up data: these match scores are invented for this book.

- Find the team's goal difference for each of the five matches.
- Add the five goal differences to find the total for the season so far.
- Check your total a different way: add all the goals for, add all the goals against, and subtract. Do the two totals agree?



Q10. A market stall records its profit or loss for each day of a five-day week. A negative number is a loss (the day cost more than it earned).

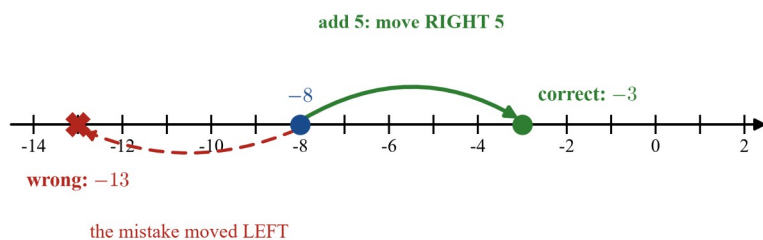
Day	Profit or loss
1	\$265.00
2	– \$78.00
3	\$345.00
4	– \$110.00
5	\$470.00

Made-up data: these daily figures are invented for this book.

- Find the total profit or loss for the week.
- Find the mean profit or loss per day, to the nearest dollar.
- Find the total of the loss days only, and the total of the profit days only. Check that the profit-day total minus the loss-day total equals your answer to (a).

Q11. Elena's bank balance is – \$45.00 and Marco's bank balance is – \$60.00. (a) Who owes the bank more? Explain your answer using the number line. (b) Find the combined balance of the two accounts. (c) Each of them pays \$25.00 into their account. Find both new balances. (d) Marco then pays in another \$40.00. Find his balance now, and say what the sign of the answer tells you.

Q12. A student's homework page shows these three answers, and all three are wrong.



- (a) $-8 + 5 = -13$
 (b) $-6 - (-4) = -10$
 (c) $-3 \times (-7) = -21$

For each part, give the correct answer and explain in words the mistake the student most likely made.

Q13. A weather station logs the temperature seven times in one day.

Time	6 am	9 am	noon	3 pm	6 pm	9 pm	midnight
Temperature (°C)	-3	1	5	4	-1	-5	-7

Made-up data: these temperatures are invented for this book.

- (a) Which reading is the coldest, and at what time?
 (b) Find the change in temperature between each pair of readings. Between which two readings is the biggest drop, and how big is it?
 (c) Find the mean of the seven readings, to one decimal place.
 (d) How much warmer is the warmest reading than the coldest reading?

Q14. For each calculation, first predict whether the answer will be positive or negative by counting the negative signs. Then calculate to check your prediction. (a) $-14 \times 9 \times (-2)$ (b) $-100 \div 5 \div (-4)$ (c) $-6 \times (-5) \times (-3)$ (d) $-250 \div (-5) \div (-5)$

Q15. A diver starts at the surface of a dam, which we call 0. She swims down 8 m, rises 3 m, sinks 11 m, then rises 16 m back to the surface. (a) Write her position after each of the four moves as a signed number, starting from 0. (b) Find the total distance she travelled, counting every move up and down. (c) What was her deepest position? Give it as a signed number and in words.

Q16. Caitlin’s account statement for one week is below. The opening balance is negative because the account started overdrawn.

Transaction	Amount	Running balance
Opening balance	– \$ 32.00	– \$ 32.00
Wages	+ \$ 780.00	
Rent	– \$ 315.00	
Groceries	– \$ 96.40	
Phone plan	– \$ 45.00	
Overdraft interest	– \$ 8.55	

Made-up data: these transactions are invented for this book.

- Complete the running balance column, one balance after each transaction.
- State the closing balance at the end of the week.
- Find the total of all the money that went out (the negative amounts).
- Check your closing balance a different way: opening balance, plus wages, plus the total money out from (c).

Q17. In a quiz of 25 questions, each correct answer scores 3 points, each wrong answer loses 2 points, and a blank question scores 0. Zoe answers 14 correct, 6 wrong and leaves 5 blank. Wei answers 10 correct, 3 wrong and leaves the rest blank. (a) Find Zoe’s score. (b) Find Wei’s score. (c) Wei goes back and answers more of the blank questions, getting every one of them correct and making no new wrong answers. How many more correct answers does Wei need to beat Zoe’s score? (Beating means strictly more, not equal.)

Q18. A chilled truck carries frozen goods at -21°C . At a stop the door is left open and the temperature rises 8°C before it is shut again. With the door shut, the cooling unit drops the temperature 2°C every 10 minutes. The cargo is safe at -18°C or below. (a) Find the temperature when the door is shut again. (b) How many minutes does the cooling unit take to get the cargo back to a safe temperature? (c) At the next delivery the door opening raises the temperature 11°C from where it was at the end of part (b). With the door shut again, how many minutes does the unit take to reach -18°C or below? (d) The driver’s log claims the cargo bay “never went above -10°C ” across the whole day. Is the claim true? Justify with the temperatures from parts (a) to (c).

Answers

Q1. (a) $-13, -9, -2, 0, 4, 7$. (b) $-6, 11, 4.5$. (c) $-6 < -2$; $0 > -8$; $-15 < -10$. **Q2.** (a) A: -4°C ; B: 3°C . (b) 7°C warmer. (c) -9°C . **Q3.** (a) $-\$35.00$. (b) $\$115.00$. (c) $\$106.00$. **Q4.** (a) -3 . (b) -13 . (c) 7 . **Q5.** (a) -42 . (b) 40 . (c) -9 . **Q6.** (a) -3 , floor B3. (b) 7 . (c) -2 , floor B2. **Q7.** (a) 338 m. (b) 18 m. (c) $-46, -28, 0, 310$. **Q8.** (a) -6°C . (b) 2 h 40 min. (c) 3 h. **Q9.** (a) $+2, -2, 0, +3, -2$. (b) $+1$. (c) yes — goals for 10 , against 9 , and $10 - 9 = +1$. **Q10.** (a) $\$892.00$. (b) $\$178$ to the nearest dollar (the exact mean is $\$178.40$). (c) loss days total $-\$188.00$; profit days total $\$1080.00$; and $\$1080.00 - \$188.00 = \$892.00$ ✓. **Q11.** (a) Marco — $-\$60.00$ is less than $-\$45.00$, so it sits further left on the number line. (b) $-\$105.00$. (c) Elena $-\$20.00$, Marco $-\$35.00$. (d) $+\$5.00$: the positive sign means the account is now in credit. **Q12.** (a) -3 . (b) -2 . (c) 21 . **Q13.** (a) -7°C , at midnight. (b) changes $+4, +4, -1, -5, -4, -2$; biggest drop 5°C , from 3 pm to 6 pm. (c) -0.9°C to one decimal place. (d) 12°C . **Q14.** (a) positive, 252 . (b) positive, 5 . (c) negative, -90 . (d) negative, -10 . **Q15.** (a) $0, -8, -5, -16, 0$. (b) 38 m. (c) -16 , that is 16 m below the surface. **Q16.** (a) $\$748.00, \$433.00, \$336.60, \$291.60, \$283.05$. (b) $\$283.05$. (c) $-\$464.95$, so $\$464.95$ went out. (d) $-\$32.00 + \$780.00 - \$464.95 = \$283.05$ ✓. **Q17.** (a) 30 . (b) 24 . (c) 3 more correct — 2 more only ties Zoe on 30 , while 3 more gives 33 . **Q18.** (a) -13°C . (b) 30 minutes. (c) 50 minutes. (d) false — the warmest the bay got was -8°C , which is above -10°C .

Fully-worked solutions

Q1. (a) Place each number on a number line and read left to right: $-13, -9, -2, 0, 4, 7$. (b) Flip each number to the other side of zero: the opposite of 6 is -6 ; of -11 is 11 ; of -4.5 is 4.5 . (c) The smaller number sits further left: $-6 < -2$ (both negative, -6 further from zero); $0 > -8$ (zero is above any negative); $-15 < -10$ (-15 further from zero).

Q2. (a) Thermometer A shows -4°C and thermometer B shows 3°C . (b) The gap is $3 - (-4) = 3 + 4 = 7$, so town B is 7°C warmer. (c) A fall of 5 from -4 : $-4 - 5 = -9$, so -9°C .

Q3. (a) Money out: $\$40.00 - \$75.00 = -\$35.00$. The account is overdrawn. (b) Money in: $-\$35.00 + \$150.00 = \$115.00$. (c) Fee out: $\$115.00 - \$9.00 = \$106.00$.

Q4. (a) $-8 + 5$: start at -8 , move 5 right, land on -3 . (b) $-6 - 7$: start at -6 , move 7 left, land on -13 . (c) $-2 - (-9) = -2 + 9 = 7$: subtracting a negative adds its opposite.

Q5. (a) Different signs: $-7 \times 6 = -42$. (b) Same signs: $-8 \times (-5) = 40$. (c) Different signs: $-36 \div 4 = -9$.

Q6. (a) $7 - 10 = -3$: floor -3 , called B3. (b) $-2 + 9 = 7$: floor 7 . (c) $0 + 5 - 11 + 4 = -2$: floor -2 , called B2.

Q7. (a) Hill top to town: $310 - (-28) = 310 + 28 = 338$ m. (b) Town to pump: $(-28) - (-46) = -28 + 46 = 18$ m. (c) Lowest to highest: $-46, -28, 0, 310$ (pump, town, sea level, hill top). Check: $338 + 18 = 356$ m and $310 - (-46) = 356$ m, the hill top to the pump, which agrees.

Q8. (a) The temperature rises 3°C for 4 h: $-18 + 3 \times 4 = -18 + 12 = -6$, so -6°C . (b) The food is safe while the temperature is -10°C or below. From -18 the temperature may rise $-10 - (-18) = 8$ degrees before it passes -10 . At 3 degrees an hour that is $8 \div 3 = 2\frac{2}{3}$ h, and $\frac{2}{3}$ h is 40 minutes: 2 h 40 min. Check the whole hours: after 2 h the freezer is at $-18 + 6 = -12$, still safe; after 3 h it is at $-18 + 9 = -9$, no longer safe. So the safe window ends between 2 h and 3 h, at 2 h 40 min. (c) From -6°C back to -18°C is a fall of $-6 - (-18) = 12$ degrees. At 4 degrees an hour: $12 \div 4 = 3$ h. Check: $-6 - 4 \times 3 = -18$ ✓.

Q9. (a) Goals for minus goals against, match by match: $3 - 1 = +2$; $0 - 2 = -2$; $2 - 2 = 0$; $4 - 1 = +3$; $1 - 3 = -2$. (b) Total: $2 - 2 + 0 + 3 - 2 = +1$. (c) Goals for: $3 + 0 + 2 + 4 + 1 = 10$. Goals against: $1 + 2 + 2 + 1 + 3 = 9$. Then $10 - 9 = +1$, the same total. The two methods agree, as they must: adding the per-match differences is the same as differencing the totals.

Q10. (a) Add the five days: $265.00 - 78.00 + 345.00 - 110.00 + 470.00 = 892.00$, a profit of \$ 892.00 for the week. (b) Mean per day: $\$ 892.00 \div 5 = \$ 178.40$, which is \$ 178 to the nearest dollar. (c) Loss days: $- 78.00 - 110.00 = - 188.00$. Profit days: $265.00 + 345.00 + 470.00 = 1\,080.00$. Check: $\$ 1\,080.00 - \$ 188.00 = \$ 892.00$, the answer to (a) ✓.

Q11. (a) Marco. On the number line $- 60$ sits to the left of $- 45$, so $- \$ 60.00 < - \$ 45.00$: a bigger debt is a smaller number. (b) Combined: $- 45.00 + (- 60.00) = - 105.00$, so $- \$ 105.00$. (c) Elena: $- 45.00 + 25.00 = - 20.00$. Marco: $- 60.00 + 25.00 = - 35.00$. Both still overdrawn. (d) Marco: $- 35.00 + 40.00 = + 5.00$. The positive sign means the debt is cleared and the account now holds \$5.00 — it is in credit.

Q12. (a) Correct: $- 8 + 5 = - 3$. The student moved left instead of right (adding a positive moves right), or added the sizes $8 + 5 = 13$ and kept the minus sign. (b) Correct: $- 6 - (- 4) = - 6 + 4 = - 2$. The student computed $- 6 - 4 = - 10$, dropping the double negative: subtracting $- 4$ means adding 4. (c) Correct: $- 3 \times (- 7) = 21$. The student used the “different signs give a negative” rule on two negatives; same signs give a positive.

Q13. (a) The smallest reading is $- 7^{\circ}\text{C}$, at midnight. (b) Each change is the next reading minus the one before: $1 - (- 3) = + 4$; $5 - 1 = + 4$; $4 - 5 = - 1$; $- 1 - 4 = - 5$; $- 5 - (- 1) = - 4$; $- 7 - (- 5) = - 2$. The biggest drop is $- 5$, a fall of 5°C between 3 pm and 6 pm. (c) Sum of the seven readings: $- 3 + 1 + 5 + 4 - 1 - 5 - 7 = - 6$. Mean: $- 6 \div 7 = - 0.857\dots$, which is $- 0.9^{\circ}\text{C}$ to one decimal place. (d) Warmest 5°C , coldest $- 7^{\circ}\text{C}$: $5 - (- 7) = 12$, so 12°C warmer.

Q14. Count the negative signs: an even count gives a positive answer, an odd count gives a negative one. (a) Two negatives (even) $\rightarrow$ positive. $- 14 \times 9 = - 126$, then $- 126 \times (- 2) = 252$ ✓. (b) Two negatives (even) $\rightarrow$ positive. $- 100 \div 5 = - 20$, then $- 20 \div (- 4) = 5$ ✓. (c) Three negatives (odd) $\rightarrow$ negative. $- 6 \times (- 5) = 30$, then $30 \times (- 3) = - 90$ ✓. (d) Three negatives (odd) $\rightarrow$ negative. $- 250 \div (- 5) = 50$, then $50 \div (- 5) = - 10$ ✓.

Q15. (a) Start 0; down 8: $- 8$; up 3: $- 8 + 3 = - 5$; down 11: $- 5 - 11 = - 16$; up 16: $- 16 + 16 = 0$. Positions: 0, $- 8$, $- 5$, $- 16$, 0. (b) Distance counts every metre moved, up or down: $8 + 3 + 11 + 16 = 38$ m. (Distance is never negative, unlike position.) (c) Deepest position: $- 16$, that is 16 m below the surface.

Q16. (a) Add each amount to the balance before it: $- 32.00 + 780.00 = 748.00$; $748.00 - 315.00 = 433.00$; $433.00 - 96.40 = 336.60$; $336.60 - 45.00 = 291.60$; $291.60 - 8.55 = 283.05$. (b) Closing balance: \$ 283.05. (c) Money out: $- 315.00 - 96.40 - 45.00 - 8.55 = - 464.95$, so \$464.95 went out. (d) Check: $- \$ 32.00 + \$ 780.00 - \$ 464.95 = \$ 283.05$, the same closing balance ✓. Note the account was only overdrawn at the start: after the wages arrived, every running balance was positive.

Q17. (a) Zoe: $14 \times 3 - 6 \times 2 + 5 \times 0 = 42 - 12 + 0 = 30$. (b) Wei: $10 \times 3 - 3 \times 2 = 30 - 6 = 24$. (c) Each extra correct answer adds 3 points (a blank was worth 0). With 2 more correct Wei has $24 + 6 = 30$, only a tie with Zoe. With 3 more Wei has $24 + 9 = 33$, which beats 30. So Wei needs 3 more correct answers.

Q18. (a) $- 21 + 8 = - 13$, so $- 13^{\circ}\text{C}$ when the door shuts. (b) Safe means $- 18^{\circ}\text{C}$ or below. Stepping down 2°C per 10 min from $- 13$: after 10 min $- 15$, after 20 min $- 17$ (still not safe), after 30 min $- 19$ (safe). So 3 steps, 30 minutes. (c) Door opening: $- 19 + 11 = - 8$, so $- 8^{\circ}\text{C}$. The fall needed to reach $- 18$ is $- 8 - (- 18) = 10$ degrees, which is $10 \div 2 = 5$ steps of 10 minutes: 50 minutes. (d) The claim is false. The temperatures reached were $- 13^{\circ}\text{C}$ in (a) and $- 8^{\circ}\text{C}$ in (c), and $- 8$ is above $- 10$. The bay went above $- 10^{\circ}\text{C}$ twice, so “never above $- 10^{\circ}\text{C}$ ” cannot be true.

Chapter 2 Number — 2E Powers and square roots

Theory

About the data in this section. The crate stacks, dance floors, banners, sandpits, patios, shade sails and prices here are **made up** (not real): pretend jobs and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

This section is about two ideas that turn up everywhere in trade and everyday maths: **powers** (repeated multiplication) and **square roots** (undoing a square). In the study design (Unit 3, Area of Study 1), you need to know powers with an index no higher than 3, and square roots, and you need to use both of them to solve problems.

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-07.

What a power means.

A **power** is a shortcut for repeated multiplication of the same number. In 5^3 the 5 is the **base** — the number being multiplied — and the 3 is the **index**, which counts how many factors there are. The plural of index is **indices**. So $5^3 = 5 \times 5 \times 5 = 125$, read as “five cubed”.

Two indices have their own names:

- index 2 is **squared**: $7^2 = 7 \times 7 = 49$, read “seven squared”;
- index 3 is **cubed**: $2^3 = 2 \times 2 \times 2 = 8$, read “two cubed”;
- index 1 just hands the number back: $8^1 = 8$.

In this section indices go up to 3 and no further. Squares, cubes and powers of 10 are the tools you need.

index — how many factors
(3 means three 5s)

5^3

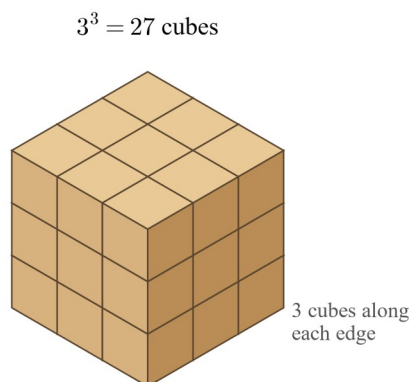
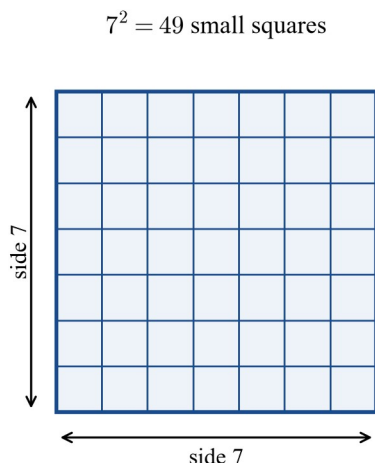
base — the number
being multiplied

$= 5 \times 5 \times 5 = 125$

read: “five cubed”

Squares and cubes you can picture.

The names come from shapes. 7^2 counts the small squares in a 7 by 7 square grid; 3^3 counts the small cubes in a 3 by 3 by 3 cube stack.



A **perfect square** is a whole number that is some whole number squared — 49 is a perfect square because $7^2 = 49$. Square roots (below) land cleanly on perfect squares.

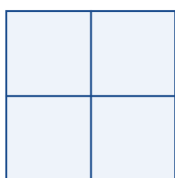
Squares and cubes worth knowing.

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

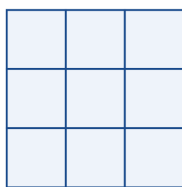
Squares keep going past 10, and the next five are worth knowing too: $11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$, $15^2 = 225$.



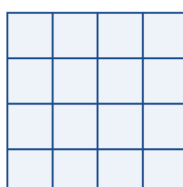
$$1^2 = 1$$



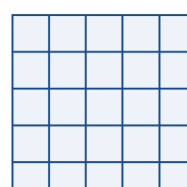
$$2^2 = 4$$



$$3^2 = 9$$



$$4^2 = 16$$



$$5^2 = 25$$

Powers of 10.

Three powers of 10 cover most of everyday numeracy: $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$. Unit labels are powers in disguise: m^2 means “square metres” (a side in metres, squared) and cm^3 means “cubic centimetres” (an edge in centimetres, cubed). When you see m^2 or cm^3 on a job sheet, an index is doing quiet work.

Three power traps.

- **The power beats the multiply.** In 2×5^2 the index sits on the 5 only: $5^2 = 25$ first, then $2 \times 25 = 50$. With brackets, $(2 \times 5)^2$, the brackets go first: $10^2 = 100$. Same numbers, different answers — the brackets decide.
- **Where the minus sign sits.** $(-4)^2 = (-4) \times (-4) = 16$, because the minus is inside the brackets and gets squared too. But -4^2 means “the negative of 4^2 ”: $4^2 = 16$ first, then the minus, giving -16 .
- **Index 1 is not 1.** $8^1 = 8$. An index of 1 means one factor of 8, not the number 1.

Square roots.

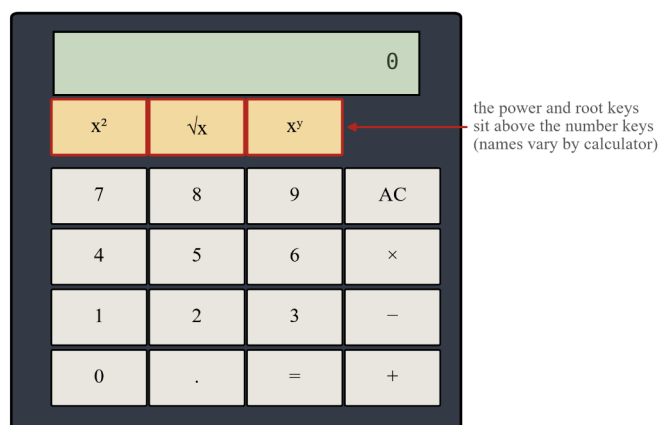
The square-root sign ($\sqrt{\quad}$) asks one question: *what number, times itself, gives this?* So $\sqrt{25} = 5$, because $5 \times 5 = 25$. The sign means the **positive** square root — the one you can actually measure.

Squaring and square-rooting undo each other:

- $\sqrt{6^2} = \sqrt{36} = 6$ — the root undoes the square;
- $(\sqrt{9})^2 = 3^2 = 9$ — the square undoes the root.

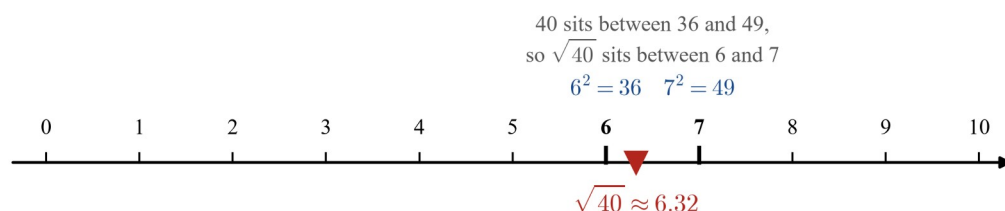
Roots of decimals work the same way: $\sqrt{0.25} = 0.5$, because $0.5 \times 0.5 = 0.25$.

On a calculator, the keys for this section are x^2 (square), x^y (any power) and $\sqrt{x}$ (square root).



Estimate first, then calculate.

Before pressing $\sqrt{}$, trap the answer between two whole numbers. For $\sqrt{40}$: 40 sits between $6^2 = 36$ and $7^2 = 49$, so $\sqrt{40}$ sits between 6 and 7, nearer 6. Only then calculate: $\sqrt{40} = 6.32$ to 2 decimal places. If the calculator ever shows something outside your trap, a key went wrong.



closer to 6 than to 7, because 40 is closer to 36 than to 49

This is the estimate-then-calculate habit from the problem-solving cycle: estimate, calculate, compare. And round only where the question tells you to — carry the full calculator value through the working.

Squares and cubes in real jobs.

What you know

a square with side s

a cube with edge s

What it gives you

area = s^2 ; going backwards, side = $\sqrt{\text{area}}$

volume = s^3

Volume is the space inside a solid, measured in cubed units (cm^3 , m^3). This section finds volume from a known edge. Going the other way — edge from volume — needs cube roots, which are outside this section, so every volume question here hands you the edge.

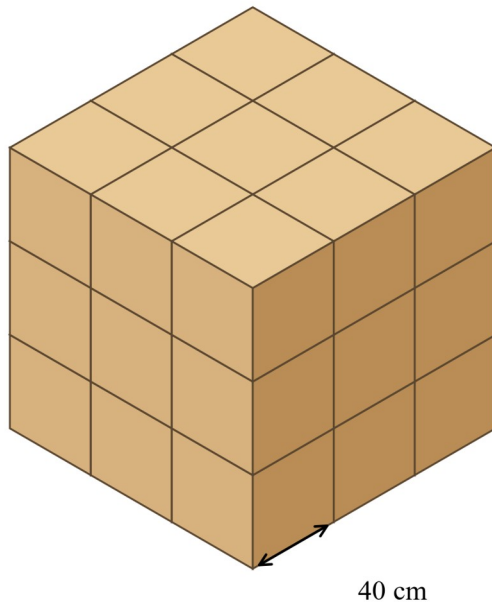
Worked examples

Example 1 — scaffolded

A store builds a display by stacking cube-shaped crates. Each crate is a cube with edges of 40 cm. The display is 3 crates wide, 3 crates deep and 3 crates high. How many crates are in the display, and what is the volume of the whole display in cm^3 ?

3 crates wide, 3 deep, 3 high

$$3 \times 3 \times 3 = 3^3 = 27 \text{ crates}$$



Made-up data: the crate display and its measurements are made up for this section.

Working

Comment

Purpose and question(s): size up the crate display.
Questions: how many crates? What is the volume of the whole display in cm^3 ?

Two questions — list both before calculating.

Mathematics in the context: each crate is a 40 cm cube; the stack is 3 by 3 by 3.

The picture carries the numbers here, not the words.

Step 1 — crates: $3 \times 3 \times 3 = 3^3 = 27$ crates.

Three rows of three, in three layers: a cube number.

Step 2 — one crate: volume
 $= 40^3 = 40 \times 40 \times 40 = 64\,000 \text{ cm}^3$.

A cube's volume is its edge cubed.

Step 3 — the display: $27 \times 64\,000 = 1\,728\,000 \text{ cm}^3$.

Twenty-seven crates, each holding $64\,000 \text{ cm}^3$.

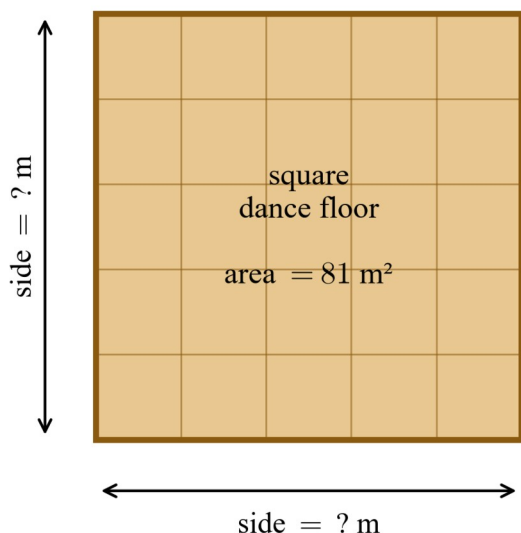
Check another way: the display is itself a cube of side $3 \times 40 = 120 \text{ cm}$, and $120^3 = 1\,728\,000 \text{ cm}^3$ — the same answer.

Two routes to one answer: that is a check.

The display uses 27 crates, and its volume is $1\,728\,000 \text{ cm}^3$.

Example 2 — scaffolded

A club marks out a square dance floor and ropes it off around the edge. The floor's area is 81 m^2 . How long is each side of the floor, and how much rope goes around it?



Made-up data: the dance floor and its area are made up for this section.

Working

Comment

Purpose and question(s): buy the right length of rope.
Questions: what is the side length? What is the perimeter?

Area in, side out — then the perimeter.

Mathematics in the context: a square; area = 81 m^2 ; rope runs around all four sides.

A square has four equal sides.

Step 1 — side: area = side^2 , so $\text{side} = \sqrt{81} = 9 \text{ m}$, because $9 \times 9 = 81$.

The square root undoes the squaring.

Step 2 — rope: perimeter = $4 \times 9 = 36 \text{ m}$.

Four equal sides.

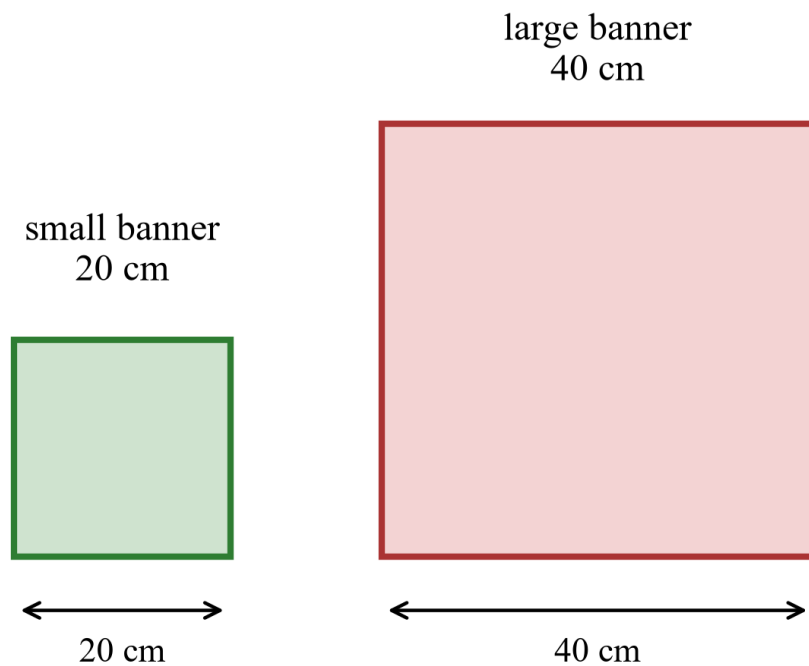
Check: $9^2 = 81 \text{ m}^2$, the area we started with.

Undo the step backwards to check it.

Each side of the floor is 9 m, and the rope around it is 36 m long.

Example 3 — unscaffolded

Tam prints square banners. A small banner with sides of 20 cm uses \$2.50 of fabric. A customer orders a large banner with sides of 40 cm and reasons: “Double the side, so double the price — \$5.00, right?” Fabric cost follows area. Is \$5.00 correct, and if not, what should the fabric cost?



drawn to scale: each side doubles, and the area grows 4 times

Made-up data: the banners, their sizes and the fabric prices are made up for this section.

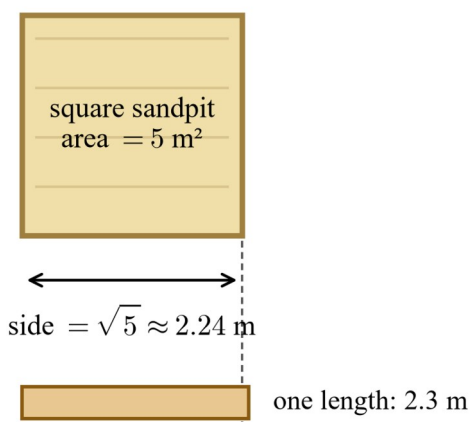
Identify first: the purpose is to price the large banner's fabric, and the mathematics is area, then a comparison of areas, then a cost. Small banner: $20^2 = 400 \text{ cm}^2$ of fabric. Large banner: $40^2 = 1600 \text{ cm}^2$. Comparing, $1600 \div 400 = 4$, so the large banner uses **four** times the fabric, not two. The fabric cost is $4 \times \$2.50 = \$10.00$, so the customer's \$5.00 is wrong. Doubling each side multiplies the area by $2 \times 2 = 4$ — that is the trap in “double the side”.

$\therefore$ the large banner needs 4 times the fabric, so its fabric costs \$10.00, not \$5.00.

Example 4 — unscaffolded

A centre builds a square sandpit with an area of 5 m^2 . Edging planks are sold in 2.3 m lengths. How many whole lengths must the centre buy to edge the sandpit?

*the length is a little longer
than one side of the pit*



Made-up data: the sandpit and the plank length are made up for this section.

Identify first: the purpose is to buy whole plank lengths to go around a square; the mathematics is a square root for the side, a perimeter, then division with a round-up. Trap the side first: $2^2 = 4$ and $3^2 = 9$, and $4 < 5 < 9$, so the side is

between 2 m and 3 m. The calculator gives $\sqrt{5} = 2.23607\dots$, so the side is 2.24 m to 2 decimal places. The edging runs around all four sides: $4 \times \sqrt{5} = 8.94$ m to 2 decimal places. Keep the full calculator value for the next step: $(4 \times \sqrt{5}) \div 2.3 = 3.889\dots$, and lengths are sold whole, so round up to 4. Check: $4 \times 2.3 = 9.2$ m covers the 8.94 m needed, while $3 \times 2.3 = 6.9$ m falls short.

$\therefore$ the centre must buy 4 lengths of edging plank.

Practice questions

Scaffolded

Q1. Write each power as repeated multiplication, then find its value. (a) 6^2 (b) 5^3 (c) 15^2

Q2. Find each square root. (a) $\sqrt{49}$ (b) $\sqrt{144}$ (c) $\sqrt{0.25}$

Q3. (a) Find $4^3 - 3^3$. (b) Which is bigger, 2^3 or 3^2 ? Show both values. (c) Find $(-4)^2$.

Q4. Square floor tiles have sides of 20 cm. (a) Find the area of one tile, in cm^2 . (b) A square floor has sides of 240 cm. Find its area, in cm^2 . (c) How many of these tiles cover the floor exactly?

Q5. For each number, first trap its square root between two whole numbers, then use a calculator and give the root to 2 decimal places. (a) $\sqrt{30}$ (b) $\sqrt{60}$ (c) $\sqrt{85}$

Q6. Cube-shaped gift boxes have edges of 12 cm. (a) Find the volume of one box, in cm^3 . (b) Find the total volume of 27 boxes, in cm^3 . (c) The 27 boxes pack exactly into a cube-shaped carton with 3 boxes along each edge. Find the carton's edge length, then use it to check your answer to (b).

Un scaffolded

Q7. Tam bakes a square slice tray with sides of 24 cm and cuts it into 16 equal square slices. (a) Find the area of the tray, in cm^2 . (b) Find the area of one slice, in cm^2 . (c) Find the side length of one slice, and check it against the tray's side.

Q8. Two cube-shaped water tanks stand side by side. The small tank has edges of 1.5 m; the large tank has edges of 3 m. (a) Find the volume of the small tank, in m^3 . (b) Find the volume of the large tank, in m^3 . (c) How many times more water does the large tank hold than the small one? (It is not 2.)

Q9. Nadia prints a square photo with an area of 324 cm^2 and glues trim around its edge. Trim costs \$2.40 per 10 cm. (a) Find the side length of the photo. (b) Find the length of trim needed. (c) Find the cost of the trim, correct to the nearest cent.

Q10. A square yoga mat has an area of 2 m^2 . (a) Between which two whole metres is the mat's side length? (b) Find the side length to 2 decimal places. (c) The mat must slide into a gap 1.5 m wide. Will it fit? Use your answer to (b) to explain.

Q11. A game called Power Climb pays points in powers of 10: level 1 pays 10 points, level 2 pays 100 points and level 3 pays 1000 points. (a) Write the level 2 and level 3 payouts as powers of 10. (b) A player clears all three levels once. How many points do they score in total? (c) How many times bigger is the level 3 payout than the level 1 payout?

Q12. (a) Show that $14^2 = 196$ without a calculator, one step per line. (b) Hence write down $\sqrt{196}$. (c) Without a calculator, find $\sqrt{19600}$, and check your answer by squaring it.

Q13. Ron is paving a square patio with sides of 3.6 m. One bag of paving mix covers 3 m^2 and costs \$8.90. Bags are sold whole. (a) Find the area of the patio, in m^2 , to 2 decimal places. (b) How many whole bags must Ron buy? (c) Find the total cost of the bags, then the cost per m^2 of patio, correct to the nearest cent.

Q14. A school buys square shade sails. A small sail has sides of 3 m; a large sail has sides of 4.5 m. (a) Find the area of each sail, in m^2 . (b) How many times the small sail's area is the large sail's area? (c) Fabric for the small sail costs \$54. At the same rate per m^2 , find the fabric cost of the large sail, correct to the nearest cent.

Q15. A community hall has a square floor with an area of 150 m^2 . Chairs will line one wall, about 1 m of wall per chair. (a) Between which two whole metres is the side of the floor? (b) Find the side of the floor to the nearest whole metre. (c) About how many chairs fit along one wall? State the assumption you are using.

Q16. A builder's job sheet shows two calculations that look alike: 3×4^2 and $(3 \times 4)^2$. (a) Find 3×4^2 . (b) Find $(3 \times 4)^2$. (c) Explain in one sentence why the two answers differ.

Q17. Dice are cubes with edges of 16 mm. They are packed into a cube-shaped box with edges of 80 mm. (a) Find the volume of one die, in mm^3 . (b) Find the volume of the box, in mm^3 . (c) How many dice fit in the box? Check by counting dice along each edge.

Q18. Bess sews a square quilt from 25 square patches arranged 5 by 5. Each patch has sides of 30 cm. Binding tape goes all the way around the edge of the quilt. (a) Find the side length of the quilt, in cm. (b) Find the area of the quilt, in cm^2 , and check it by adding up the patches' areas. (c) Find the length of binding tape needed, in cm and in m.

Answers

Q1. (a) $6 \times 6 = 36$. (b) $5 \times 5 \times 5 = 125$. (c) $15 \times 15 = 225$. **Q2.** (a) 7. (b) 12. (c) 0.5. **Q3.** (a) 37. (b) $3^2 = 9$ beats $2^3 = 8$, so 3^2 is bigger. (c) 16. **Q4.** (a) 400 cm^2 . (b) $57\,600 \text{ cm}^2$. (c) 144 tiles. **Q5.** (a) between 5 and 6; 5.48. (b) between 7 and 8; 7.75. (c) between 9 and 10; 9.22. (roots to 2 d.p.) **Q6.** (a) 1728 cm^3 . (b) $46\,656 \text{ cm}^3$. (c) edge $3 \times 12 = 36 \text{ cm}$; $36^3 = 46\,656 \text{ cm}^3$, which matches (b). **Q7.** (a) 576 cm^2 . (b) 36 cm^2 . (c) 6 cm; check $24 \div 6 = 4$ slices per row and $4 \times 4 = 16$ slices. **Q8.** (a) 3.375 m^3 . (b) 27 m^3 . (c) 8 times. **Q9.** (a) 18 cm. (b) 72 cm. (c) \$17.28. **Q10.** (a) between 1 m and 2 m. (b) 1.41 m. (c) yes — $1.41 \text{ m} < 1.5 \text{ m}$, so the mat fits the gap. **Q11.** (a) 10^2 and 10^3 . (b) 1110 points. (c) 100 times. **Q12.** (a) see the solution. (b) 14. (c) 140; check $140 \times 140 = 19\,600$. **Q13.** (a) 12.96 m^2 . (b) 5 bags. (c) \$44.50 in total; \$3.43 per m^2 . **Q14.** (a) 9 m^2 and 20.25 m^2 . (b) 2.25 times. (c) \$121.50. **Q15.** (a) between 12 m and 13 m. (b) 12 m. (c) about 12 chairs, assuming each chair needs about 1 m of wall. **Q16.** (a) 48. (b) 144. (c) in (a) only the 4 is squared; in (b) the brackets put the 12 inside the power. **Q17.** (a) 4096 mm^3 . (b) $512\,000 \text{ mm}^3$. (c) 125 dice; check $5 \times 5 \times 5 = 125$. **Q18.** (a) 150 cm. (b) $22\,500 \text{ cm}^2$; check $25 \times 900 = 22\,500 \text{ cm}^2$. (c) $600 \text{ cm} = 6 \text{ m}$.

Fully-worked solutions

Q1. (a) $6^2 = 6 \times 6 = 36$. (b) $5^3 = 5 \times 5 \times 5 = 125$. (c) $15^2 = 15 \times 15 = 225$.

Q2. (a) $\sqrt{49} = 7$, because $7 \times 7 = 49$. (b) $\sqrt{144} = 12$, because $12 \times 12 = 144$. (c) $\sqrt{0.25} = 0.5$, because $0.5 \times 0.5 = 0.25$.

Q3. (a) $4^3 = 64$ and $3^3 = 27$, so $4^3 - 3^3 = 64 - 27 = 37$. (b) $2^3 = 8$ and $3^2 = 9$; $9 > 8$, so 3^2 is bigger. (c) The minus is inside the brackets, so it gets squared too: $(-4) \times (-4) = 16$.

Q4. (a) $20^2 = 400 \text{ cm}^2$. (b) $240^2 = 240 \times 240 = 57\,600 \text{ cm}^2$. (c) $57\,600 \div 400 = 144$ tiles — exact, with no cutting, because $144 \times 400 = 57\,600$.

Q5. (a) $5^2 = 25 < 30 < 36 = 6^2$, so $\sqrt{30}$ is between 5 and 6; calculator: 5.48 to 2 decimal places. (b) $7^2 = 49 < 60 < 64 = 8^2$, so between 7 and 8; 7.75 to 2 decimal places. (c) $9^2 = 81 < 85 < 100 = 10^2$, so between 9 and 10; 9.22 to 2 decimal places.

Q6. (a) $12^3 = 12 \times 12 \times 12 = 1728 \text{ cm}^3$. (b) $27 \times 1728 = 46\,656 \text{ cm}^3$. (c) Three boxes of 12 cm along each edge: $3 \times 12 = 36 \text{ cm}$. Check: $36^3 = 46\,656 \text{ cm}^3$, the same total as (b).

Q7. (a) $24^2 = 576 \text{ cm}^2$. (b) $576 \div 16 = 36 \text{ cm}^2$ per slice. (c) $\sqrt{36} = 6 \text{ cm}$. Check: $24 \div 6 = 4$ slices per row, and $4 \times 4 = 16$ slices — the number cut.

Q8. (a) $1.5^3 = 1.5 \times 1.5 \times 1.5 = 3.375 \text{ m}^3$. (b) $3^3 = 27 \text{ m}^3$. (c) $27 \div 3.375 = 8$. Doubling the edge multiplies the volume by $2 \times 2 \times 2 = 8$, not by 2.

Q9. (a) $\sqrt{324} = 18 \text{ cm}$, because $18 \times 18 = 324$. (b) Perimeter $= 4 \times 18 = 72 \text{ cm}$. (c) 72 cm is $72 \div 10 = 7.2$ lots of 10 cm, so the trim costs $7.2 \times \$2.40 = \$17.28$ — already exact to the cent.

Q10. (a) $1^2 = 1 < 2 < 4 = 2^2$, so the side is between 1 m and 2 m. (b) $\sqrt{2} = 1.41 \text{ m}$ to 2 decimal places. (c) Yes: $1.41 \text{ m} < 1.5 \text{ m}$, so the mat's side fits inside the gap.

Q11. (a) Level 2: $100 = 10^2$. Level 3: $1000 = 10^3$. (b) $10 + 100 + 1000 = 1110$ points. (c) $1000 \div 10 = 100$ times bigger.

Q12. (a) One step per line: 14×14
 $= 14 \times (10 + 4)$
 $= (14 \times 10) + (14 \times 4)$
 $= 140 + 56$
 $= 196$ so $14^2 = 196$, as required. (b) $\sqrt{196} = 14$, because part (a) showed $14 \times 14 = 196$. (c) $19\,600 = 196 \times 100$. From

(a), $14 \times 14 = 196$, and $10 \times 10 = 100$, so $(14 \times 10) \times (14 \times 10) = 19\,600$, that is $140 \times 140 = 19\,600$. Hence $\sqrt{19\,600} = 140$. Check: $140 \times 140 = 19\,600$.

Q13. (a) $3.6^2 = 3.6 \times 3.6 = 12.96 \text{ m}^2$, exact, so 12.96 m^2 to 2 decimal places. (b) Bags: $12.96 \div 3 = 4.32$, and bags are sold whole, so round up: 5 bags. (c) Total: $5 \times \$8.90 = \$44.50$. Cost per m^2 : $\$44.50 \div 12.96 = \$3.43$ correct to the nearest cent.

Q14. (a) Small: $3^2 = 9 \text{ m}^2$. Large: $4.5^2 = 4.5 \times 4.5 = 20.25 \text{ m}^2$. (b) $20.25 \div 9 = 2.25$ times the small sail's area. (c) Rate: $\$54 \div 9 = \$6$ per m^2 . Large sail: $20.25 \times \$6 = \$121.50$, correct to the nearest cent. Check by ratio: $2.25 \times \$54 = \$121.50$.

Q15. (a) $12^2 = 144 < 150 < 169 = 13^2$, so the side is between 12 m and 13 m. (b) $\sqrt{150} = 12.25$ to 2 decimal places, so 12 m to the nearest whole metre. (c) About 12 chairs, using the assumption that each chair needs about 1 m of wall and the side rounds to 12 m.

Q16. (a) Power first: $4^2 = 16$, then $3 \times 16 = 48$. (b) Brackets first: $3 \times 4 = 12$, then $12^2 = 144$. (c) In (a) the index sits on the 4 only, while in (b) the brackets put the whole product 12 inside the power — so different numbers get squared.

Q17. (a) $16^3 = 16 \times 16 \times 16 = 4096 \text{ mm}^3$. (b) $80^3 = 80 \times 80 \times 80 = 512\,000 \text{ mm}^3$. (c) $512\,000 \div 4096 = 125$ dice. Check along the edges: $80 \div 16 = 5$ dice per edge, and $5 \times 5 \times 5 = 125$ — the same count.

Q18. (a) 5 patches of 30 cm across: $5 \times 30 = 150 \text{ cm}$. (b) $150^2 = 22\,500 \text{ cm}^2$. Check from the patches: one patch is $30^2 = 900 \text{ cm}^2$, and $25 \times 900 = 22\,500 \text{ cm}^2$ — the same. (c) Perimeter $= 4 \times 150 = 600 \text{ cm}$, which is 6 m of binding tape.

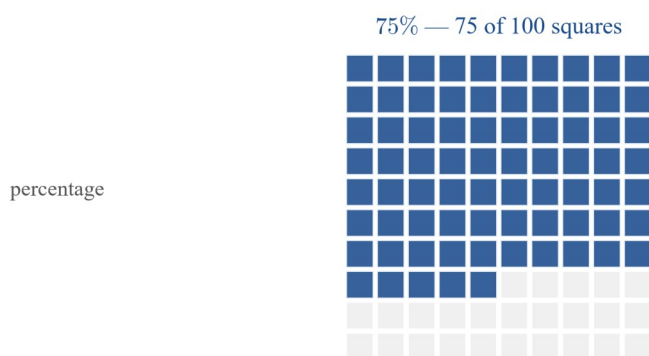
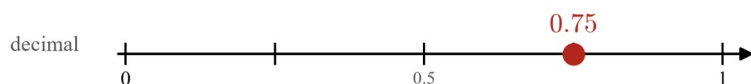
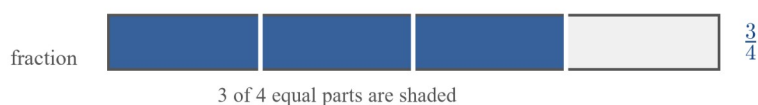
Chapter 2 Number — 2F Fractions, decimals and percentages: equivalence and operations

Theory

One number can be written in three ways. The fraction $\frac{3}{4}$, the decimal 0.75 and the percentage 75 % all name the same amount. These three forms are called **equivalent** — equal in value, even though they look different. Moving between the forms lets you compare numbers, use a calculator, and make sense of sales, payslips, recipes, labels and news reports.

The three forms.

One number, three faces



$$\frac{3}{4} = 0.75 = 75\%$$

- A **fraction** splits a whole into equal parts. In $\frac{3}{4}$, the **numerator** (the top number, 3) counts the parts you have. The **denominator** (the bottom number, 4) says how many equal parts make one whole.
- A **decimal** uses place value, like the numbers in §2A. After the decimal point come tenths, hundredths and thousandths. 0.75 means 7 tenths and 5 hundredths.
- A **percentage** counts parts out of 100. *Per cent* means “for each hundred”, so $75\% = \frac{75}{100}$.

Converting between the forms.

From	To	Do this	Example
fraction	decimal	divide the numerator by the denominator	$\frac{3}{4} = 3 \div 4 = 0.75$
decimal	fraction	use place value, then simplify	$0.6 = \frac{6}{10} = \frac{3}{5}$
decimal	percentage	multiply by 100	$0.75 \times 100 = 75\%$

From	To	Do this	Example
percentage	decimal	divide by 100	$62\% \div 100 = 0.62$
fraction	percentage	divide, then multiply by 100	$\frac{3}{5} = 0.6 = 60\%$
percentage	fraction	write over 100, then simplify	$20\% = \frac{20}{100} = \frac{1}{5}$

If the denominator divides into 100 (such as 4, 5, 10, 20, 25 or 50), you can scale the fraction up to a denominator of 100 and read the percentage straight off: $\frac{21}{25} = \frac{84}{100} = 84\%$ (multiply top and bottom by 4).

Benchmarks worth knowing by heart.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{3}$	$\frac{2}{3}$
Decimal	0.5	0.25	0.75	0.2	0.1	0.125	0.333...	0.666...
Percentage	50%	25%	75%	20%	10%	12.5%	about 33%	about 67%

When the decimal never ends. $1 \div 3 = 0.333\ldots$ — the 3s go on forever, so $\frac{1}{3}$ cannot be written exactly as a decimal.

When you need a decimal like this, round it and say the precision (§2C): $\frac{1}{3} = 0.33$ to 2 decimal places, or 0.333 to 3 decimal places. As percentages, $\frac{1}{3} \approx 33\%$ and $\frac{2}{3} \approx 67\%$ to the nearest whole per cent; more precisely, $\frac{2}{3} = 66.7\%$ to 1 decimal place. The fraction itself is always exact — that is why we keep fractions in the working and only change form when it helps.

Equivalent fractions and simplifying.

Multiply the numerator and the denominator by the same number and the value does not change: $\frac{2}{3} = \frac{8}{12}$ (both parts $\times 4$). Divide both parts by the same number and the value still does not change — this is **simplifying**. A fraction is in its **simplest form** when no number bigger than 1 divides into both parts. Use the **highest common factor (HCF)** — the biggest number that divides into both (from §2B). For $\frac{12}{18}$, the HCF of 12 and 18 is 6, so $\frac{12}{18} = \frac{2}{3}$.

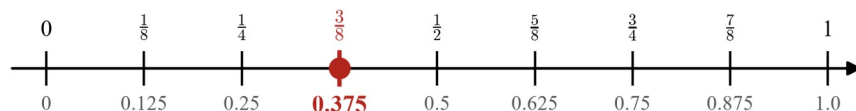
Mixed numbers and improper fractions.

A **mixed number** has a whole number and a fraction, like $2\frac{3}{5}$. An **improper fraction** has a numerator bigger than its denominator, like $\frac{13}{5}$ — it is the same number. To go from mixed to improper, multiply and add: $2\frac{3}{5} = \frac{2 \times 5 + 3}{5} = \frac{13}{5}$. To go back, divide: $13 \div 5 = 2$ remainder 3, so $\frac{13}{5} = 2\frac{3}{5}$. Improper fractions are easier to calculate with; mixed numbers are easier to picture.

Comparing and ordering.

Mixed forms are hard to compare as they stand. Convert everything to the same form first — decimals are usually quickest, and a calculator helps.

$$\frac{3}{8} = 0.375$$

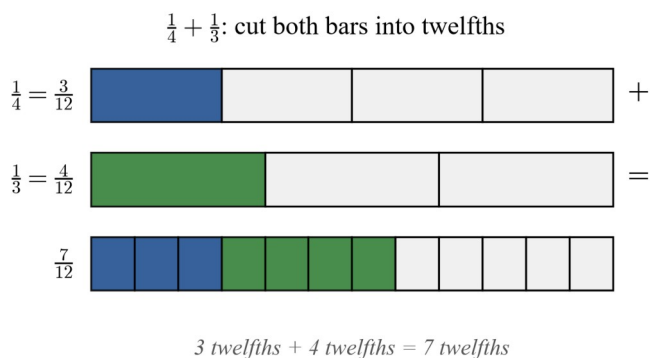


Which is larger, $\frac{3}{8}$ or 0.4? Convert: $\frac{3}{8} = 3 \div 8 = 0.375$, and $0.375 < 0.4$, so 0.4 is larger. The number line above shows where the eighths sit between 0 and 1.

Adding and subtracting fractions.

Fractions can only be added or subtracted directly when the denominators are the same: $\frac{5}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$ (add the numerators, keep the denominator, then simplify).

When the denominators differ, first rewrite the fractions with a **common denominator** — the **lowest common multiple (LCM)** of the denominators (from §2B) is the smallest one that works.



Worked model: $\frac{1}{4} + \frac{1}{3}$. The LCM of 4 and 3 is 12, so $\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{3} = \frac{4}{12}$. Then $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$. The bars in the figure show why: both strips are cut into twelfths before the shaded parts are joined.

Subtraction works the same way: $\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$. With mixed numbers, convert to improper fractions first:

$$3\frac{1}{2} - 1\frac{1}{4} = \frac{7}{2} - \frac{5}{4} = \frac{14}{4} - \frac{5}{4} = \frac{9}{4} = 2\frac{1}{4}.$$

Multiplying and dividing fractions.

- In maths, **of** means multiply. $\frac{3}{4}$ of 20 = $\frac{3}{4} \times 20 = 15$.
- To multiply fractions, multiply the numerators and multiply the denominators, then simplify: $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.
- To divide by a fraction, multiply by its **reciprocal** — the same fraction turned upside down (the numerator and denominator swap). The reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$.

Worked models: $3 \div \frac{1}{2} = 3 \times 2 = 6$ (there are six halves in three wholes), and $\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times 4 = \frac{20}{8} = \frac{5}{2} = 2\frac{1}{2}$.

Everyday decimal arithmetic.

- **Add and subtract:** line up the decimal points. $2.75 + 1.4 = 4.15$ (think of 1.4 as 1.40).
- **Multiply:** multiply as if whole, then count decimal places. 0.6×0.3 : $6 \times 3 = 18$, and one place plus one place gives two places, so $0.6 \times 0.3 = 0.18$.
- **Divide money by a whole number:** $\$12.60 \div 4 = \$3.15$.

A calculator handles all of this quickly — but estimate first and check the answer is reasonable (§1C). A decimal point in the wrong place is the classic slip.

Finding a percentage of an amount.

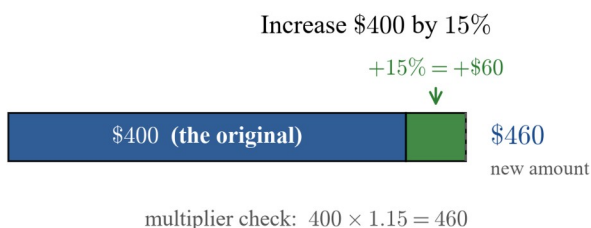
Convert the percentage to a decimal or a fraction, then multiply: 15 % of 80 = $0.15 \times 80 = 12$.

Without a calculator, build from 10 %: to find 10 %, move the decimal point one place left. 10 % of \$ 45 = \$ 4.50. Halve that for 5 % (\$ 2.25), or double it for 20 % (\$ 9.00), then add pieces together as needed.

Percentage increase and decrease.

Two steps work every time:

1. Find the percentage of the original amount.
2. **Add** it for an increase, or **subtract** it for a decrease.



Worked model: increase \$ 400 by 15 %. Step 1: 15 % of \$ 400 = $0.15 \times 400 = \$60$. Step 2 (increase, so add): $\$400 + \$60 = \$460$.

The multiplier shortcut. An increase of 15 % keeps 100 % and adds 15 %, so the new amount is 115 % of the original: multiply by 1.15. The number you multiply by (1.15 here) is called the **multiplier**. Check: $400 \times 1.15 = 460$. ✓ A decrease of 20 % keeps $100\% - 20\% = 80\%$: multiply by 0.80. Check: \$ 85 shoes at 20 % off cost $85 \times 0.80 = \$68$.

Finding the percentage change. When you know the old and new amounts:

$$\text{percentage change} = \frac{\text{change}}{\text{original amount}} \times 100$$

Worked model: a bill rises from \$ 240 to \$ 276. The change is \$ 36, so the percentage increase is $\frac{36}{240} \times 100 = 15\%$.

(Notice $\frac{36}{240} = \frac{3}{20} = \frac{15}{100}$ — simplifying the fraction first often makes the percentage obvious.) Always divide by the **original** amount, never the new one.

Going backwards: finding the original amount. After a 10 % rise, a worker earns \$ 27.50 an hour. The new pay is 110 % of the old pay, so old pay = $\$27.50 \div 1.10 = \$25.00$. Backwards problems **divide** by the multiplier — do not subtract 10 % from \$ 27.50 (10 % of a bigger number is a bigger amount, so that gives the wrong answer).

Tools.

Calculators are allowed in this subject, and the percentage key (%) differs between models, so the safest route is always the same: convert the percentage to a decimal, then multiply. Spreadsheets and phone apps do these

calculations too, but every question in this book can be done with pen, paper and a basic calculator. Estimate first, then check your answer makes sense (§1C).

Precision.

Carry full precision through the working and round only where the question tells you to — and the question always says how far (“correct to the nearest cent”, “to 2 decimal places”). Money is rounded to the nearest cent (§2C).

About the data in this section. The prices, wages, bills and statistics here are **made up** (not real): pretend people, pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

Minh scores 21 out of 25 on a workplace safety quiz. Write his score as a decimal and as a percentage.

Working	Comment
Score as a fraction: $\frac{21}{25}$.	Numerator = marks scored; denominator = marks possible.
HCF of 21 and 25 is 1, so the fraction is already in simplest form.	Check for simplifying before converting.
Decimal: $21 \div 25 = 0.84$.	A fraction bar means “divide”.
Shortcut: $\frac{21}{25} = \frac{21 \times 4}{25 \times 4} = \frac{84}{100}$.	25 divides into 100, so scale up to hundredths.
Percentage: $0.84 \times 100 = 84\%$.	Decimal to percentage: multiply by 100.
Minh’s score is 0.84 as a decimal and 84 % as a percentage.	The fraction $\frac{84}{100}$ shows the 84 % directly.

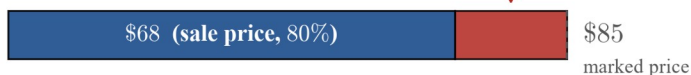
Example 2 — scaffolded

A pair of shoes is marked \$85. A sale sign says 20 % off. Find the discount and the sale price.

Working	Comment
$20\% = 0.20 = \frac{20}{100} = \frac{1}{5}$.	Convert the percentage first — decimal or fraction, your choice.
Discount: $0.20 \times \$85 = \$17$.	“Of” means multiply: 20 % of the marked price.
Sale price: $\$85 - \$17 = \$68$.	A discount comes <i>off</i> , so subtract.
Multiplier check: $100\% - 20\% = 80\%$, and $0.80 \times \$85 = \$68$. ✓	The one-step method must agree with the two-step method.
Reasonableness: 25 % off \$85 would be $\$85 - \$21.25 = \$63.75$, and a 20 % discount is slightly smaller.	Estimate and check (§1C): the sale price should be a little above \$63.75 — \$68 fits, and it sits below \$85 as a discount must.

Decrease \$85 by 20%

$$-20\% = -\$17$$



multiplier check: $85 \times 0.80 = 68$

Example 3 — unscaffolded

A carpenter has a 5 m length of timber. He cuts it into pieces that are each $\frac{3}{5}$ m long. How many whole pieces does he get, and how long is the offcut (the leftover timber)?

The number of pieces is $5 \div \frac{3}{5}$. Dividing by a fraction means multiplying by its reciprocal:

$$5 \div \frac{3}{5} = 5 \times \frac{5}{3} = \frac{25}{3} = 8\frac{1}{3}$$

Only whole pieces count, so the carpenter gets 8 pieces (9 pieces would need $9 \times \frac{3}{5} = \frac{27}{5} = 5.4$ m, which is more than the 5 m he has). The timber used by 8 pieces is $8 \times \frac{3}{5} = \frac{24}{5} = 4\frac{4}{5}$ m. The offcut is $5 - 4\frac{4}{5} = \frac{1}{5}$ m.

$\therefore$ he gets 8 whole pieces, with an offcut of $\frac{1}{5}$ m.

Example 4 — unscaffolded

Between two quarters, a household's electricity bill rises from \$240 to \$276. Find the percentage increase.

The change in dollars is $\$276 - \$240 = \$36$. The percentage change divides the change by the **original** amount and multiplies by 100:

$$\frac{36}{240} \times 100$$

Simplify the fraction first: $\frac{36}{240} = \frac{3}{20}$ (both parts $\div 12$). Then $\frac{3}{20} = \frac{15}{100} = 15\%$, because $\frac{36}{240} \times 100 = \frac{3600}{240} = 15$.

Check with the multiplier: $240 \times 1.15 = 276$. ✓

$\therefore$ the bill increased by 15%.

Practice questions

Scaffolded

Q1. Write each fraction as (i) a decimal and (ii) a percentage. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$

Q2. (a) Simplify $\frac{12}{18}$. (b) Write $\frac{3}{5}$ as an equivalent fraction with a denominator of 10, and then as a decimal. (c) Write your decimal from (b) as a percentage.

Q3. A sign-writer mixes $\frac{2}{3}$ L of white paint with $\frac{1}{4}$ L of blue paint. (a) Find the LCM of 3 and 4. (b) How many litres of paint does she mix altogether? Give your answer as a fraction. (c) She then pours $\frac{2}{3}$ L from a $1\frac{1}{2}$ L tin of white paint. How much is left in the tin? Give your answer as a fraction.

Q4. (a) A builder stacks $\frac{3}{4}$ of a load of 24 plywood sheets on the roof. How many sheets is that? (b) A 10 L bucket of sealant is $\frac{2}{5}$ used. How many litres have been used? (c) A scone recipe uses $\frac{3}{4}$ cup of oats per batch. How many cups of oats are needed for 3 batches?

Q5. A game marked \$45 is reduced by 15 % in a sale. Work **without** a calculator, using 10 % and 5 %. (a) Find 10 % of \$45. (b) Hence find 5 % of \$45. (c) Hence find 15 % of \$45, and then the sale price.

Q6. Sarah earns \$24.50 an hour. Her pay rises by 4 %. (a) Find the increase per hour. (b) Find her new hourly rate. (c) Check your answer to (b) by multiplying \$24.50 by 1.04.

Unscaffolded

Q7. Three cards show the numbers $\frac{5}{8}$, 0.6 and 62 %. (a) Write each number as a percentage. (b) List the three numbers from smallest to largest. (c) $\frac{2}{3} = 66.7\%$ to 1 decimal place. Which of the three numbers is closest to $\frac{2}{3}$?

Q8. A community kitchen has $4\frac{1}{2}$ L of soup. It fills containers that each hold $\frac{3}{4}$ L. How many containers can be filled? Show that there is no soup left over.

Q9. Hana buys milk for \$2.95, bread for \$4.30, eggs for \$6.15 and cheese for \$8.50. (a) Find the exact total. (b) Find her change from \$50. (c) The bananas weigh 1.85 kg and cost \$3.60 per kg. Find the exact cost of the bananas.

Q10. Two stores advertise the same \$120 pair of work boots. Store A takes 25 % off. Store B takes $\frac{1}{3}$ off.

STEPZ	BOOTCO
work boots	work boots
\$120	\$120
-----	-----
25% OFF!	$\frac{1}{3}$ OFF!

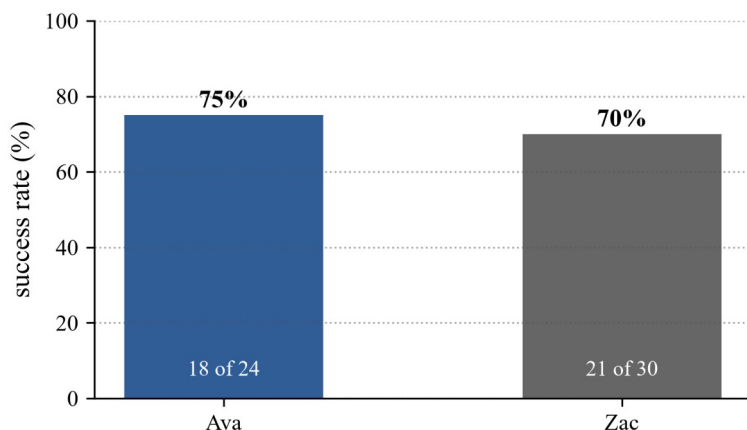
- (a) Find the discount in dollars at each store.
- (b) Find the sale price at each store.
- (c) Which store is cheaper, and by how much?

Q11. Aisha's rent rises from \$380 to \$418 per week. (a) Find the increase in dollars per week. (b) Find the percentage increase. (c) Find the extra rent she pays over 52 weeks.

Q12. Two players' netball shooting records for the season are shown.

Player	Goals	Attempts
Ava	18	24
Zac	21	30

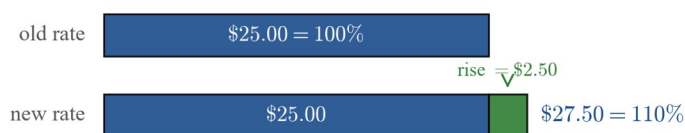
Made-up data: the names and every number in this table are made up for this section.



- Write each player's success rate as a percentage.
- Which player has the higher success rate?
- Ava then scores with all of her next 6 attempts. Find her new percentage from 30 attempts.

Q13. After a 10% pay rise, Elena earns \$27.50 an hour. (a) Explain why \$27.50 is 110% of her old hourly rate. (b) Find her old hourly rate. (c) Find the increase in dollars per hour.

After a 10% rise, Elena earns \$27.50 an hour



$$\$27.50 \div 1.10 = \$25.00$$

Q14. The smoothie card below serves 2: $\frac{2}{3}$ cup of milk, $\frac{1}{4}$ cup of rolled oats, $1\frac{1}{2}$ bananas and $\frac{1}{2}$ cup of ice. Kai wants to serve 3, so he multiplies every amount by $\frac{3}{2}$. Find the amount of each ingredient he needs for 3 servings.

Smoothie
serves 2

- $\frac{2}{3}$ cup milk
- $\frac{1}{4}$ cup rolled oats
- $1\frac{1}{2}$ bananas
- $\frac{1}{2}$ cup ice

Q15. Rosa's car uses 7.4 L of fuel per 100 km. She drives 340 km, and fuel costs \$1.95 per litre. (a) How many litres of fuel does the trip use? (b) Find the fuel cost for the trip, correct to the nearest cent. (c) Rosa drives home again, so the total trip is 680 km. Find the fuel cost for 680 km, correct to the nearest cent.

Q16. The label on a 100 mL bottle of cough mixture says: *adult dose 10 mL; child dose half the adult dose*. A child takes one dose 3 times a day for 5 days. (a) How many mL is one child dose? (b) How many mL does the child take in total over the 5 days? (c) What fraction of the bottle does the 5-day course use? Simplify your answer.

Q17. In a made-up council election, 875 of the 1 250 enrolled voters vote. (a) Find the turnout (the percentage of enrolled voters who voted). (b) Show that more than $\frac{2}{3}$ of the enrolled voters voted. (c) Find the percentage of enrolled voters who did **not** vote.

Q18. Jai earns \$26.40 an hour. He receives a 3.5 % pay rise, and he works 12 hours a week. (a) Find the increase per hour, correct to the nearest cent. (b) Find his new hourly rate, correct to the nearest cent. (c) Using your answer to (b), find how much more Jai earns for a 12-hour week at the new rate than at the old rate, correct to the nearest cent.

Answers

Q1. (a) 0.5; 50 %. (b) 0.25; 25 %. (c) 0.2; 20 %. **Q2.** (a) $\frac{2}{3}$. (b) $\frac{6}{10}$; 0.6. (c) 60 %. **Q3.** (a) 12. (b) $\frac{11}{12}$ L. (c) $\frac{5}{6}$ L.
Q4. (a) 18 sheets. (b) 4 L. (c) $2\frac{1}{4}$ cups. **Q5.** (a) \$4.50. (b) \$2.25. (c) \$6.75 off; sale price \$38.25. **Q6.** (a) \$0.98. (b) \$25.48. (c) $24.50 \times 1.04 = 25.48$ ✓. **Q7.** (a) $\frac{5}{8} = 62.5\%$; $0.6 = 60\%$; third card 62 %. (b) 0.6, 62 %, $\frac{5}{8}$. (c) $\frac{5}{8}$. **Q8.** 6 containers; $6 \times \frac{3}{4} = 4\frac{1}{2}$ L exactly. **Q9.** (a) \$21.90. (b) \$28.10. (c) \$6.66. **Q10.** (a) A: \$30; B: \$40. (b) A: \$90; B: \$80. (c) Store B, by \$10. **Q11.** (a) \$38. (b) 10 %. (c) \$1 976. **Q12.** (a) Ava 75 %; Zac 70 %. (b) Ava. (c) 80 %. **Q13.** (a) old rate + 10 % of old rate = 110 % of old rate. (b) \$25.00. (c) \$2.50. **Q14.** milk 1 cup; oats $\frac{3}{8}$ cup; banana $2\frac{1}{4}$; ice $\frac{3}{4}$ cup. **Q15.** (a) 25.16 L. (b) \$49.06 (to the nearest cent). (c) \$98.12 (to the nearest cent). **Q16.** (a) 5 mL. (b) 75 mL. (c) $\frac{3}{4}$. **Q17.** (a) 70 %. (b) $\frac{2}{3}$ of 1 250 is $833\frac{1}{3}$; $875 > 833\frac{1}{3}$. (c) 30 %. **Q18.** (a) \$0.92 (to the nearest cent). (b) \$27.32 (to the nearest cent). (c) \$11.04 (to the nearest cent).

Fully-worked solutions

Q1. (a) $1 \div 2 = 0.5$; $0.5 \times 100 = 50\%$. (b) $1 \div 4 = 0.25$; $0.25 \times 100 = 25\%$. (c) $1 \div 5 = 0.2$; $0.2 \times 100 = 20\%$.

Q2. (a) The HCF of 12 and 18 is 6. Divide top and bottom by 6: $\frac{12}{18} = \frac{2}{3}$. (b) Multiply top and bottom by 2: $\frac{3}{5} = \frac{6}{10}$. Six tenths is 0.6. (c) $0.6 \times 100 = 60\%$.

Q3. (a) Multiples of 3: 3, 6, 9, 12, ... Multiples of 4: 4, 8, 12, ... The lowest common multiple is 12. (b) Common denominator 12: $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$. Total: $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$ L. (c) $1\frac{1}{2} - \frac{2}{3} = \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$ L left in the tin.

Q4. (a) “Of” means multiply: $\frac{3}{4} \times 24 = \frac{72}{4} = 18$ sheets. (b) $\frac{2}{5} \times 10 = \frac{20}{5} = 4$ L used. (c) $\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$ cups of oats.

Q5. (a) Move the decimal point one place left: 10 % of \$45 = \$4.50. (b) 5 % is half of 10 %: $\$4.50 \div 2 = \$2.25$. (c) $15\% = 10\% + 5\%$: $\$4.50 + \$2.25 = \$6.75$ off. Sale price: $\$45 - \$6.75 = \$38.25$.

Q6. (a) $4\% = 0.04$: $0.04 \times \$24.50 = \$0.98$. (b) $\$24.50 + \$0.98 = \$25.48$ per hour. (c) Multiplier for + 4 % is 1.04 : $\$24.50 \times 1.04 = \$25.48$ ✓ — the same answer as (b).

Q7. (a) $\frac{5}{8} = 5 \div 8 = 0.625$, and $0.625 \times 100 = 62.5\%$; $0.6 \times 100 = 60\%$; the third card is already a percentage, 62 %.

(b) Compare the percentages: $60 < 62 < 62.5$, so from smallest to largest: 0.6, 62 %, $\frac{5}{8}$. (c) Gaps from 66.7 %:

$66.7 - 62.5 = 4.2$ points; $66.7 - 62 = 4.7$ points; $66.7 - 60 = 6.7$ points. The smallest gap is 4.2, so $\frac{5}{8}$ (62.5 %) is closest to $\frac{2}{3}$.

Q8. $4\frac{1}{2} \div \frac{3}{4} = \frac{9}{2} \times \frac{4}{3} = \frac{36}{6} = 6$. Check the leftover: $6 \times \frac{3}{4} = \frac{18}{4} = 4\frac{1}{2}$ L, which is exactly the soup available. So 6 containers are filled and nothing is left over.

Q9. (a) Line up the decimal points: $\$2.95 + \$4.30 + \$6.15 + \$8.50 = \$21.90$. (b) $\$50.00 - \$21.90 = \$28.10$ change. (c) $1.85 \times \$3.60 = \$6.66$ (multiply $185 \times 360 = 66\,600$, then put back $2 + 2 = 4$ decimal places). The cost is exactly \$6.66 — no rounding needed.

Q10. (a) Store A: 25% of $\$120 = 0.25 \times 120 = \$30$. Store B: $\frac{1}{3}$ of $\$120 = \$120 \div 3 = \$40$. (b) Store A: $\$120 - \$30 = \$90$. Store B: $\$120 - \$40 = \$80$. (c) Store B is cheaper, by $\$90 - \$80 = \$10$. Notice $\frac{1}{3} \approx 33.3\%$ to 1 decimal place, which is more than 25% — so Store B was always going to give the bigger discount.

Q11. (a) $\$418 - \$380 = \$38$ per week. (b) $\frac{38}{380} \times 100 = \frac{1}{10} \times 100 = 10\%$. (c) $52 \times \$38 = \$1976$ extra over the year.

Q12. (a) Ava: $\frac{18}{24} = \frac{3}{4} = 75\%$. Zac: $\frac{21}{30} = \frac{7}{10} = 70\%$. (b) $75\% > 70\%$, so Ava has the higher success rate — even though Zac scored more goals (21 vs 18), he needed more attempts to get them. (c) New record: $18 + 6 = 24$ goals from $24 + 6 = 30$ attempts. $\frac{24}{30} = \frac{4}{5} = 80\%$.

Q13. (a) Her new rate is the old rate (100% of it) plus the rise (10% of it), so new rate = $100\% + 10\% = 110\%$ of the old rate. (b) 110% is the multiplier 1.10: old rate = $\$27.50 \div 1.10 = \$25.00$. (c) $\$27.50 - \$25.00 = \$2.50$ per hour. Check: 10% of $\$25.00 = \$2.50$ ✓.

Q14. Multiply each amount by $\frac{3}{2}$:

- milk: $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$ cup
- rolled oats: $\frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$ cup
- banana: $1\frac{1}{2} \times \frac{3}{2} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$ bananas
- ice: $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ cup

Q15. (a) $340 \div 100 = 3.4$ hundreds of km, so fuel used = $3.4 \times 7.4 = 25.16$ L (exact). (b) Cost = $25.16 \times \$1.95 = \$49.062$. Correct to the nearest cent: **\$49.06**. (c) For 680 km: $6.8 \times 7.4 = 50.32$ L, and cost = $50.32 \times \$1.95 = \$98.124$. Correct to the nearest cent: **\$98.12**. Carrying full precision matters: $2 \times \$49.062 = \$98.124$ gives the same answer. Doubling the rounded \$49.06 also gives \$98.12 here, but on other numbers that shortcut can be a cent out — always keep the unrounded value until the final step.

Q16. (a) Half of the adult dose: $\frac{1}{2} \times 10 = 5$ mL. (b) $5 \text{ mL} \times 3 \text{ doses a day} \times 5 \text{ days} = 75 \text{ mL}$. (c) $\frac{75}{100} = \frac{3}{4}$ of the bottle (divide top and bottom by the HCF, 25).

Q17. (a) Turnout = $\frac{875}{1250} \times 100 = 0.7 \times 100 = 70\%$. (b) Show that more than $\frac{2}{3}$ voted:

$$\frac{2}{3} \text{ of } 1250 = \frac{2}{3} \times 1250$$

$$= \frac{2500}{3}$$

$$= 833\frac{1}{3}$$

$$\text{and } 875 > 833\frac{1}{3}.$$

$\therefore$ more than $\frac{2}{3}$ of the enrolled voters voted. (c) Did not vote: $1\,250 - 875 = 375$ people. Percentage:

$$\frac{375}{1250} \times 100 = 30\% \text{ (or simply } 100\% - 70\% = 30\%).$$

Q18. (a) $3.5\% = 0.035$: $0.035 \times \$26.40 = \$0.924$. Correct to the nearest cent: **\$0.92**. (b) New rate $= \$26.40 + \$0.924 = \$27.324$. Correct to the nearest cent: **\$27.32**. (Pay is quoted in whole cents, so we use \$27.32 from here on.) (c) Using the rate from (b): extra per hour $= \$27.32 - \$26.40 = \$0.92$. Over a 12-hour week: $12 \times \$0.92 = \$11.04$ more per week, correct to the nearest cent.

Chapter 2 Number — 2G Simple ratio and proportion

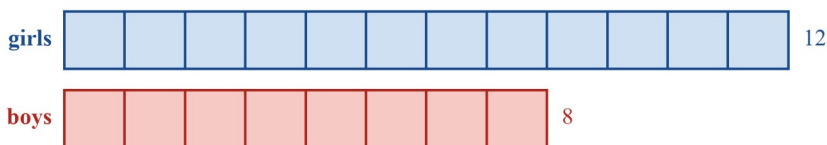
Theory

What a ratio is.

At soccer training one evening there are 12 girls and 8 boys. That comparison — girls to boys — is a **ratio**. A ratio compares two or more quantities of the same kind. We write it with a colon:

12:8, read aloud as “12 to 8”.

The numbers in a ratio are called its **terms**. The order of the terms matters. Girls to boys is 12:8, while boys to girls is 8:12 — a different ratio, because it compares the other way round.



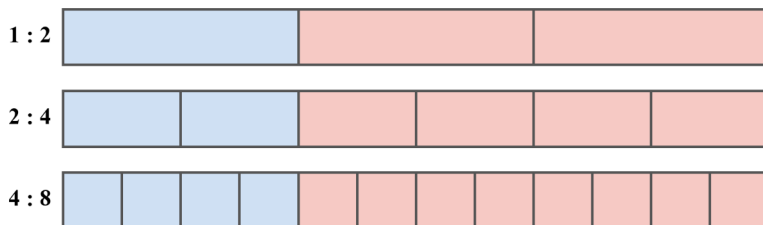
$$\text{girls : boys} = 12 : 8 = 3 : 2$$

each square is one person; the two rows compare like with like

Equivalent ratios and simplest form.

Multiply or divide both terms of a ratio by the same number and you get an **equivalent ratio** — the same comparison, written with different numbers. For example:

$$1:2 = 2:4 = 4:8, \text{ and } 12:8 = 6:4 = 3:2$$



all three bars are the same length with the same part shaded —
so $1:2 = 2:4 = 4:8$ are equivalent ratios

Each bar above is the same length with the same part shaded, so 1:2, 2:4 and 4:8 are all the same comparison.

A ratio is in **simplest form** when its terms are whole numbers with no common factor bigger than 1. To get there, divide both terms by their **highest common factor** — the biggest number that divides into both (see §2B). For 12 and 8 that number is 4, so $12:8 = 3:2$ in one step.



or ÷ 4 in one step (4 is the highest common factor)

Same units first.

A ratio only makes sense when both terms are measured in the same units. To write 500 g to 2 kg as a ratio, first change 2 kg into 2000 g. The ratio is then 500 : 2000, which simplifies to 1 : 4. Once the units match, they disappear: 1 : 4 carries no units at all.

Sharing a quantity in a given ratio.

Suppose \$600 must be shared between two people in the ratio 2 : 3. Think in **parts**: the first person gets 2 equal parts, the second gets 3.

1. **Add the terms** to find the total number of parts: $2 + 3 = 5$ parts.
2. **Divide the quantity by the total parts** to find one part: $\$600 \div 5 = \$120$.
3. **Multiply each term by one part**: $2 \times \$120 = \$240$ and $3 \times \$120 = \$360$.
4. **Check both ways**: the shares add back to the amount, $\$240 + \$360 = \$600$, and the shares are in the ratio asked for, $240 : 360 = 2 : 3$.

share \$600 in the ratio 2 : 3

\$120	\$120	\$120	\$120	\$120
-------	-------	-------	-------	-------

2 parts: $2 \times \$120 = \$240$

3 parts: $3 \times \$120 = \$360$

5 parts in total; one part = $\$600 \div 5 = \$120$

check: $\$240 + \$360 = \$600$ and $240 : 360 = 2 : 3$

Three-way splits work exactly the same way — the ratio 2 : 3 : 4 simply has $2 + 3 + 4 = 9$ parts.

Ratios and fractions.

A ratio can also be read as fractions of the whole. In a 2 : 3 split the whole is 5 parts, so the first share is $\frac{2}{5}$ of the whole and the second share is $\frac{3}{5}$ of it (see §2F for fractions).

Proportion: two ratios that are equal.

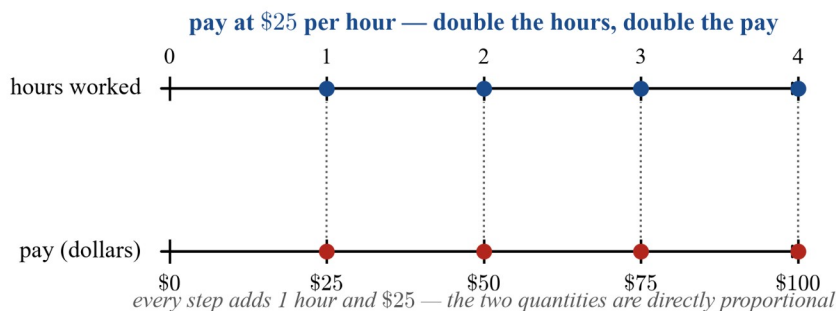
A **proportion** is a statement that two ratios are equal. For example:

$$3 : 5 = 12 : 20, \text{ that is } \frac{3}{5} = \frac{12}{20}$$

To find a missing term in a proportion, look at what happened to the term you know, and do the same to the other one. Here 3 became 12, which is $\times 4$, so 5 becomes $5 \times 4 = 20$.

Direct proportion and the unitary method.

Two quantities are in **direct proportion** when one is always the same multiple of the other. Double one and the other doubles; halve one and the other halves. Pay at \$25 an hour is in direct proportion with the hours worked:



Most proportion problems are solved by the **unitary method**: find the value of **one** unit first, then scale up or down. If a machine makes 350 bolts in 5 hours, then one hour makes $350 \div 5 = 70$ bolts — and from that single fact you can answer any question about the machine.

Unit price.

The **unit price** is what one unit costs: one item, 100 g, one litre. Unit prices make packs of different sizes easy to compare. A 500 g pack at \$3.45 costs $\$3.45 \div 5 = \$0.69$ per 100 g. A 2 kg bag at \$13.20 costs $\$13.20 \div 20 = \$0.66$ per 100 g. The bigger bag is cheaper per 100 g, so it is the **better buy**.

When an answer needs rounding.

Some divisions do not land on whole cents. For example $\$2.00 \div 3 = \$0.6666\dots$, which is \$0.67 correct to the nearest cent. Whenever an answer in this book needs rounding, the question says so — “correct to the nearest cent”, “to the nearest whole number”. Sometimes the situation decides the rounding instead: whole boxes, whole buses and whole tables must be rounded *up*, because part of one does not do the job (see §1C).

Where ratio and proportion lead.

The same skills return in §5B (rates such as pay per hour), §5C (scales on maps and plans, mixing, magnification factors) and §6A (comparing prices while shopping). This section builds the number skill those sections rely on.

About the data in this section. The prices, wages, prizes and quantities here are **made up** (not real): pretend people, pretend shops and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

Write each ratio in simplest form: (a) 12 : 18 (b) 500 g : 2 kg (c) 45 minutes : 1 hour.

Working	Comment
(a) The highest common factor of 12 and 18 is 6. $12 \div 6 = 2$ and $18 \div 6 = 3$, so $12 : 18 = 2 : 3$.	Divide both terms by the biggest number that goes into both.
(b) Change to the same units: 2 kg is 2000 g, so the ratio is 500 : 2000. The highest common factor is 500, so $500 : 2000 = 1 : 4$.	Same units first — grams in this case.
(c) Change to the same units: 1 hour is 60 minutes, so the ratio is 45 : 60. The highest common factor is 15, so $45 : 60 = 3 : 4$.	Minutes in this case. Check: $3 \times 15 = 45$ and $4 \times 15 = 60$.

Example 2 — scaffolded

A community group shares \$910 of raffle takings between an animal shelter and a wildlife rescue in the ratio 3 : 4. How much does each charity receive?

Understand: \$910 shared in the ratio 3 : 4.

Write down the amount and the ratio.

Total parts: $3 + 4 = 7$ parts.

Add the terms of the ratio.

One part: $\$910 \div 7 = \$130$.

Divide the amount by the total number of parts.

Shares: shelter $3 \times \$130 = \$390$; wildlife rescue
 $4 \times \$130 = \$520$.

Multiply each term by the value of one part.

Check: $\$390 + \$520 = \$910$, and $390 : 520 = 3 : 4$.
 Both checks pass.

The shares add back to \$910 and sit in the ratio asked for.

\$910 shared in the ratio 3 : 4



animal shelter: 3 parts = \$390

wildlife rescue: 4 parts = \$520

7 parts in total; one part = $\$910 \div 7 = \$130$

check: $\$390 + \$520 = \$910$ and $390 : 520 = 3 : 4$

Example 3 — unscaffolded

A machine in a small factory makes 350 bolts in 5 hours, working at a steady speed. (a) How many bolts does it make in 8 hours? (b) How long does it take to make 700 bolts?

Use the unitary method: find what one hour makes first.

One hour makes $350 \div 5 = 70$ bolts.

(a) Eight hours make $8 \times 70 = 560$ bolts. A quick direction check from §1C: 8 hours is longer than 5 hours, so the answer must be more than 350 bolts — and 560 is.

(b) For 700 bolts, divide by the hourly amount: $700 \div 70 = 10$ hours.



the unitary method: find ONE hour first, then scale up

going the other way:

$700 \div 70 = 10$ hours for 700 bolts

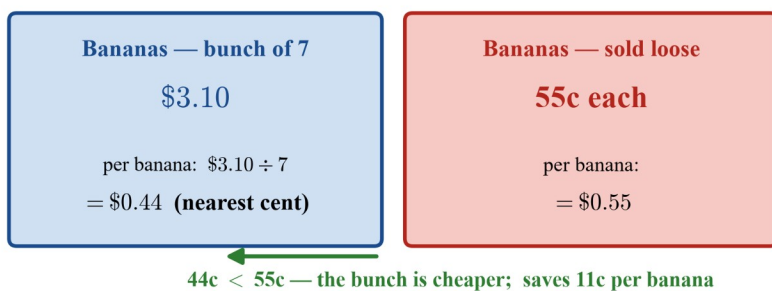
$\therefore$ the machine makes 560 bolts in 8 hours, and 700 bolts take 10 hours.

Example 4 — unscaffolded

A fruit shop sells bananas two ways: in bunches of 7 for \$3.10, or loose at 55c each. Which way is cheaper per banana? Give the bunch price per banana correct to the nearest cent.

The bunch: $\$3.10 \div 7 = \$0.4428\dots$ per banana, which is \$0.44 correct to the nearest cent. Loose bananas cost 55c, that is \$0.55, each.

Comparing the two unit prices, $44c < 55c$, so the bunch is cheaper — by 11c a banana. A backwards check agrees: seven loose bananas would cost $7 \times \$0.55 = \$3.85$, which is \$0.75 more than the \$3.10 bunch.



∴ the bunch is the better buy: about \$0.44 a banana correct to the nearest cent, 11c cheaper per banana than buying loose.

Practice questions

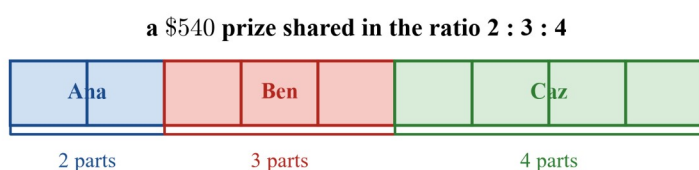
Scaffolded

Q1. Write each ratio in simplest form. (a) 8 : 12 (b) 15 : 25 (c) 18 : 24

Q2. Put both quantities in the same units first, then write each ratio in simplest form. (a) 40 cm : 2 m (b) 250 g : 1 kg (c) 20 minutes : 2 hours

Q3. Jo and Sam share \$48 of garage-sale takings in the ratio 5 : 3, because Jo did more of the work. (a) How many parts are there in total? (b) What is one part worth? (c) How much does each person get? Check your answer two ways.

Q4. Three friends — Ana, Ben and Caz — share a \$540 prize in the ratio 2 : 3 : 4.



How many equal parts in total? What is one part worth?

- (a) How many parts are there in total?
- (b) What is one part worth?
- (c) How much does each friend receive?

Q5. Find the missing number x so that each pair of ratios is a proportion. (a) $2:5 = 6:x$ (b) $x:7 = 24:56$ (c) $4:9 = x:45$

Q6. Five identical T-shirts cost \$87.50 altogether. (a) What does one T-shirt cost? (b) What do eight of these T-shirts cost? (c) How many of these T-shirts can be bought for \$210?

Un scaffolded

Q7. A youth group has 18 girls and 12 boys. (a) Write the ratio of girls to boys in simplest form. (b) What fraction of the whole group is girls? (c) What fraction of the whole group is boys? Check that your two fractions add to 1.

Q8. Two flatmates share rent of \$1 560 a month in the ratio 7 : 5, because one bedroom is much larger than the other. (a) How many parts are there in total, and what is one part worth? (b) How much rent does each flatmate pay? (c) How much more per month does the flatmate with the larger room pay?

Q9. A pancake recipe for 4 people uses 2 cups of flour, 2 eggs, 300 mL of milk and 1 tablespoon of sugar. (a) What number do you multiply by to feed 10 people instead of 4? (b) How much of each ingredient is needed for 10 people? (c) How much milk is needed for 6 people?

Q10. A car uses 6 litres of fuel for every 100 km. Complete the table, then answer parts (b) and (c).

Distance (km)	100	200	350	500
Fuel (L)	6	12		

Made-up data: the numbers in this table are made up for this section.

- (a) Fill in the two empty cells.
- (b) How much fuel does a 250 km trip use?
- (c) Is the fuel used in direct proportion to the distance travelled? Explain how you know.

Q11. Two bags of the same rice sit on one shelf.

Rice — small bag 2 kg \$5.60	Rice — big bag 5 kg \$13.25
---	--

- (a) Find the price per kilogram of each bag.
- (b) Which bag is the better buy? State the saving per kilogram.
- (c) A third option is a 375 g box for \$2.50. Find its price per 100 g, correct to the nearest cent.

Q12. A trade supplier sells decking timber at a fixed price per metre. Six metres cost \$54.30. (a) What is the price per metre? (b) What does 15 m cost? (c) How many metres of timber can a buyer get for \$271.50?

Q13. The three owners of a small business share the year's profit in the ratio 2 : 3 : 4. The smallest share is \$3 600. (a) What is one part worth? (b) What are the other two shares? (c) What was the profit for the year? Check by adding the three shares.

Q14. Write each ratio in simplest form. (a) 1.5 : 6 (b) 0.25 : 0.75 (c) 2.4 km : 800 m

Q15. Show that sharing \$405 in the ratio 4 : 5 gives shares of \$180 and \$225. Write one step per line.

Q16. Priya is paid \$354 for a 12-hour week. (a) What is her hourly rate of pay? (b) What would she earn for a 20-hour week at the same rate? (c) How many hours must she work to earn \$885 at the same rate?

Q17. Floor tiles are sold in boxes. Eight boxes cover 24 m² of floor. (a) What area does one box cover? (b) How many boxes are needed for 45 m² of floor? (c) Tiles are sold only in whole boxes. How many boxes must be bought for a 50 m² floor? Explain why you rounded the way you did.

Q18. Hana and Marco run a market stall together and share the takings in the ratio 5:7, because Marco works the longer hours. In one month the difference between their two shares is \$86.

market-stall takings shared in the ratio 5 : 7



the difference = \$86

12 equal parts in total — what is one part worth?

- The difference between the shares is how many parts?
- What is one part worth?
- How much does each person receive that month?
- What were the total takings for the month? Check your answer two ways.

Answers

Q1. (a) 2:3. (b) 3:5. (c) 3:4. **Q2.** (a) 1:5. (b) 1:4. (c) 1:6. **Q3.** (a) 8 parts. (b) \$6. (c) Jo \$30, Sam \$18; checks: $\$30 + \$18 = \$48$ and $30:18 = 5:3$. **Q4.** (a) 9 parts. (b) \$60. (c) Ana \$120, Ben \$180, Caz \$240. **Q5.** (a) $x = 15$. (b) $x = 3$. (c) $x = 20$. **Q6.** (a) \$17.50. (b) \$140.00. (c) 12 T-shirts. **Q7.** (a) 3:2. (b) $\frac{3}{5}$. (c) $\frac{2}{5}$; and $\frac{3}{5} + \frac{2}{5} = 1$. **Q8.** (a) 12 parts; \$130. (b) \$910 and \$650. (c) \$260 more. **Q9.** (a) 2.5. (b) 5 cups of flour, 5 eggs, 750 mL of milk, 2.5 tablespoons of sugar. (c) 450 mL. **Q10.** (a) 21 L and 30 L. (b) 15 L. (c) yes — double the distance and the fuel doubles; every row keeps the same ratio, 6 L : 100 km. **Q11.** (a) \$2.80 and \$2.65 per kg. (b) the 5 kg bag, saving \$0.15 per kg. (c) \$0.67 per 100 g, correct to the nearest cent. **Q12.** (a) \$9.05. (b) \$135.75. (c) 30 m. **Q13.** (a) \$1 800. (b) \$5 400 and \$7 200. (c) \$16 200. **Q14.** (a) 1:4. (b) 1:3. (c) 3:1. **Q15.** 9 parts; one part \$45; $4 \times \$45 = \$180$; $5 \times \$45 = \$225$; $\$180 + \$225 = \$405$. **Q16.** (a) \$29.50. (b) \$590.00. (c) 30 hours. **Q17.** (a) 3 m². (b) 15 boxes. (c) 17 boxes — 16 boxes cover only 48 m², so round up. **Q18.** (a) 2 parts. (b) \$43. (c) Hana \$215, Marco \$301. (d) \$516.

Fully-worked solutions

Q1. (a) The highest common factor of 8 and 12 is 4: $8:12 = 2:3$. (b) The highest common factor of 15 and 25 is 5: $15:25 = 3:5$. (c) The highest common factor of 18 and 24 is 6: $18:24 = 3:4$.

Q2. (a) 2 m is 200 cm, so the ratio is 40:200; dividing by 40 gives 1:5. (b) 1 kg is 1000 g, so the ratio is 250:1000; dividing by 250 gives 1:4. (c) 2 hours is 120 minutes, so the ratio is 20:120; dividing by 20 gives 1:6.

Q3. (a) $5 + 3 = 8$ parts. (b) $\$48 \div 8 = \$6$ per part. (c) Jo: $5 \times \$6 = \$30$. Sam: $3 \times \$6 = \$18$. Check one: the shares add back, $\$30 + \$18 = \$48$. Check two: the shares are in the ratio asked for, $30:18 = 5:3$ (dividing both by 6).

Q4. (a) $2 + 3 + 4 = 9$ parts. (b) $\$540 \div 9 = \$60$ per part. (c) Ana: $2 \times \$60 = \$120$. Ben: $3 \times \$60 = \$180$. Caz: $4 \times \$60 = \$240$. Check: $\$120 + \$180 + \$240 = \$540$.

Q5. (a) 2 became 6, which is $\times 3$, so $x = 5 \times 3 = 15$. Check: $2:5 = 6:15$. (b) 56 became 7, which is $\div 8$, so $x = 24 \div 8 = 3$. Check: $3:7 = 24:56$. (c) 9 became 45, which is $\times 5$, so $x = 4 \times 5 = 20$. Check: $4:9 = 20:45$.

Q6. (a) $\$87.50 \div 5 = \$17.50$ per T-shirt. (b) $8 \times \$17.50 = \$140.00$. (c) $\$210 \div \$17.50 = 12$, so 12 T-shirts.

Q7. (a) 18:12; dividing by 6 gives 3:2. (b) The whole group is $18 + 12 = 30$ people, so girls are $\frac{18}{30} = \frac{3}{5}$ of the group.
(c) Boys are $\frac{12}{30} = \frac{2}{5}$ of the group. Check: $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$ — the two fractions cover the whole group.

Q8. (a) $7 + 5 = 12$ parts; $\$1560 \div 12 = \$130$ per part. (b) Larger room: $7 \times \$130 = \$910$. Smaller room: $5 \times \$130 = \$650$. Check: $\$910 + \$650 = \$1560$. (c) $\$910 - \$650 = \$260$ more per month. (That is the 2 extra parts: $2 \times \$130 = \$260$.)

Q9. (a) $10 \div 4 = 2.5$, so multiply every amount by 2.5. (b) Flour: $2 \times 2.5 = 5$ cups. Eggs: $2 \times 2.5 = 5$ eggs. Milk: $300 \times 2.5 = 750$ mL. Sugar: $1 \times 2.5 = 2.5$ tablespoons. (c) For 6 people multiply by $6 \div 4 = 1.5$: milk is $300 \times 1.5 = 450$ mL.

Q10. (a) 350 km is 3.5 lots of 100 km, so $3.5 \times 6 = 21$ L. 500 km is 5 lots of 100 km, so $5 \times 6 = 30$ L. (b) 250 km is 2.5 lots of 100 km: $2.5 \times 6 = 15$ L. (c) Yes. Every time the distance doubles, the fuel doubles (100 km uses 6 L, 200 km uses 12 L, 500 km uses 30 L), and every row of the table keeps the same ratio of fuel to distance, 6 L : 100 km. That is what direct proportion means.

Q11. (a) Small bag: $\$5.60 \div 2 = \$2.80$ per kg. Big bag: $\$13.25 \div 5 = \$2.65$ per kg. (b) The 5 kg bag is the better buy, because \$2.65 per kg is less than \$2.80 per kg. The saving is $\$2.80 - \$2.65 = \$0.15$ per kg. (c) 375 g is 3.75 lots of 100 g, so the price per 100 g is $\$2.50 \div 3.75 = \$0.6666\dots$, which is \$0.67 correct to the nearest cent.

Q12. (a) $\$54.30 \div 6 = \$9.05$ per metre. (b) $15 \times \$9.05 = \$135.75$. (c) $\$271.50 \div \$9.05 = 30$, so 30 m of timber.

Q13. (a) The smallest share is the 2-part share, so one part is $\$3600 \div 2 = \$1800$. (b) The other shares are $3 \times \$1800 = \$5400$ and $4 \times \$1800 = \$7200$. (c) The profit is 9 parts: $9 \times \$1800 = \$16200$. Check: $\$3600 + \$5400 + \$7200 = \$16200$.

Q14. (a) Multiply both terms by 2 to clear the decimal: $1.5:6 = 3:12$, then divide by 3: $1:4$. (b) Multiply both terms by 4: $0.25:0.75 = 1:3$, already in simplest form. (c) Same units: 2.4 km is 2400 m, so the ratio is $2400:800$; dividing by 800 gives $3:1$.

Q15. Add the terms: $4 + 5 = 9$ parts. One part: $\$405 \div 9 = \$45$. First share: $4 \times \$45 = \$180$. Second share: $5 \times \$45 = \$225$. Check: $\$180 + \$225 = \$405$, and $180:225 = 4:5$ (dividing both by 45). So sharing \$405 in the ratio 4:5 does give \$180 and \$225.

Q16. (a) $\$354 \div 12 = \$29.50$ per hour. (b) $20 \times \$29.50 = \$590.00$. (c) $\$885 \div \$29.50 = 30$, so 30 hours.

Q17. (a) $24 \div 8 = 3$, so one box covers 3 m^2 . (b) $45 \div 3 = 15$ boxes. (c) $50 \div 3 = 16.66\dots$ boxes. Boxes are sold whole, and 16 boxes cover only $16 \times 3 = 48 \text{ m}^2$, leaving 2 m^2 of bare floor — so round **up** to 17 boxes, which cover $17 \times 3 = 51 \text{ m}^2$ with a little left over. The situation, not the nearest-number rule, decides the rounding here (see §1C).

Q18. (a) The shares are 5 parts and 7 parts, so the difference is $7 - 5 = 2$ parts. (b) Those 2 parts are worth \$86, so one part is $\$86 \div 2 = \$43$. (c) Hana: $5 \times \$43 = \$215$. Marco: $7 \times \$43 = \$301$. (d) Total: 12 parts, $12 \times \$43 = \$516$. Check one: $\$215 + \$301 = \$516$. Check two: the difference is $\$301 - \$215 = \$86$, as stated, and $215:301 = 5:7$ (dividing both by 43).

Chapter 02 — Test A (multiple choice)

Chapter 2 (Number), sections 2A to 2G. Materials: pen, paper; a scientific calculator may be used. Test A: 12 multiple-choice questions (12 marks). Approximate time: 20 minutes.

About the data in this test. The prices, times and other numbers here are **made up** (not real): pretend people, pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Choose the response that is correct for the question.

Question 1

In the number 8 305 217.46, the value of the digit 3 is

- A. 300 000
- B. 30 000
- C. 3 000 000
- D. 300

Question 2

Four parcels are weighed at the post office. Their masses, in kilograms, are 0.54, 0.45, 0.504 and 0.405. Which list shows the masses from lightest to heaviest?

- A. 0.54, 0.504, 0.45, 0.405
- B. 0.405, 0.45, 0.504, 0.54
- C. 0.45, 0.405, 0.54, 0.504
- D. 0.405, 0.504, 0.45, 0.54

Question 3

A rectangular floor measuring 42 cm by 56 cm is to be covered with identical square tiles. Tiles cannot be cut. The largest possible side length of a tile is

- A. 7 cm
- B. 21 cm
- C. 14 cm
- D. 168 cm

Question 4

Two warning lights at a railway crossing start flashing together. One light flashes every 12 seconds and the other flashes every 18 seconds. The number of seconds until they next flash together is

- A. 6
- B. 30
- C. 216
- D. 36

Question 5

At the market, Priya buys 1.75 kg of potatoes at \$2.38 per kilogram. The cost, correct to the nearest cent, is

- A. \$4.17
- B. \$4.16
- C. \$4.165
- D. \$4.20

Question 6

A football club records an attendance of 56 482 at a match. Rounded to the nearest hundred, the attendance is

- A. 56 400
- B. 56 480
- C. 56 500
- D. 60 000

Question 7

In a computer game, a player scores 4 points for each gem collected and -3 points for each trap hit. Kai collects 9 gems and hits 6 traps. Kai's total score is

- A. 54
- B. 18
- C. -18
- D. 27

Question 8

Mia's bank account is overdrawn — the bank is owed money. Her balance is $-\$45$. She deposits \$80, and the bank then deducts a \$12 account-keeping fee. Mia's balance is now

- A. \$23
- B. \$47
- C. $-\$23$
- D. \$137

Question 9

A cube-shaped storage box has edges of 15 cm. Its volume, in cm^3 , is

- A. 45
- B. 225
- C. 675
- D. 3375

Question 10

A square vegetable garden has an area of 90 m^2 . The side length of the garden lies between

- A. 8 m and 9 m
- B. 9 m and 10 m
- C. 10 m and 11 m
- D. 7 m and 8 m

Question 11

A town's water bill increases from \$360 to \$396 per quarter. The percentage increase is

- A. 36 %
- B. 9 %
- C. 10 %
- D. 11 %

Question 12

Rice is sold in a 3 kg bag for \$7.95 and in a 5 kg bag for \$12.75. The better buy, and its price per kilogram, is

- A. the 3 kg bag, because it costs less in total
- B. the 3 kg bag, at \$2.65 per kg
- C. the 5 kg bag, at \$2.65 per kg
- D. the 5 kg bag, at \$2.55 per kg

End of Test A

Test A — Answers

Question	1	2	3	4	5	6	7	8	9	10	11	12
Answer	A	B	C	D	A	C	B	A	D	B	C	D

Test A — Fully-worked solutions

Question	Answer	Working
1	A	Counting places from the decimal point: 8 millions, 3 hundred-thousands, 0 ten-thousands, 5 thousands, 2 hundreds, 1 tens, 7 ones. The 3 sits in the hundred-thousands place, so its value is $3 \times 100\,000 = 300\,000$.
2	B	More digits does not mean bigger. Compare place by place: $0.405 < 0.45 (= 0.450) < 0.504 < 0.54$. Lightest to heaviest: 0.405, 0.45, 0.504, 0.54. Option A is heaviest to lightest; option C compares as if the decimals were whole numbers ($45 < 405$), which is wrong.
3	C	The tile side must divide both 42 and 56 exactly, so it must be a common factor; the largest possible tile uses the HCF. $42 = 2 \times 3 \times 7$ and $56 = 2^3 \times 7$, so $\text{HCF} = 2 \times 7 = 14$. Check: $42 \div 14 = 3$ and $56 \div 14 = 4$, so 12 tiles fit with none cut. Option B (21) divides 42 but not 56; option D (168) is the LCM, far bigger than the floor width.
4	D	They next flash together after a whole number of both 12-second and 18-second cycles, so find the LCM. Multiples of 12: 12, 24, 36, ...; multiples of 18: 18, 36, ...; $\text{LCM}(12, 18) = 36$ seconds. Option A (6) is the HCF, option B (30) is the sum and option C (216) is the product 12×18 — none is the first shared flash time.
5	A	$\text{Cost} = 1.75 \times 2.38 = \$4.165$ exactly. Round to the nearest cent (2 decimal places): the deciding digit is the 5 in the thousandths place, and exactly halfway rounds up, so the 6 becomes 7: \$4.17. Option B truncates instead of rounding; option C is the unrounded value, which is not a cent amount; option D rounds the wrong place.
6	C	Rounding to the nearest hundred: the deciding digit is the tens digit, $8 \geq 5$, so round up: $56\,482 \rightarrow 56\,500$. Option A rounds down, option B rounds to the nearest ten, option D

Question	Answer	Working
7	B	rounds to the wrong place. Gems: $9 \times 4 = 36$ points. Traps: $6 \times (-3) = -18$ points. Total: $36 - 18 = 18$. Option A (54) adds $36 + 18$, forgetting that the traps score negative points.
8	A	Start at -45, deposit 80: $-45 + 80 = 35$. Fee of 12 comes out: $35 - 12 = 23$. Balance = \$ 23. Option B (47) adds the fee instead of deducting it; option D (137) ignores the overdraft and adds $45 + 80 + 12$.
9	D	Volume of a cube = $\text{edge}^3 = 15^3 = 15 \times 15 \times 15 = 3375 \text{ cm}^3$. Option A is 3×15, option B is 15^2 (the area of one face) and option C is 3×15^2 — cubing means three factors of 15, not multiplying by 3.
10	B	Estimate first: $9^2 = 81$ and $10^2 = 100$. Since $81 < 90 < 100$, $\sqrt{90}$ lies between 9 and 10 (about 9.5 m).
11	C	Change = $396 - 360 = 36$. Percentage change divides by the original amount: $\frac{36}{360} \times 100 = 10\%$. Option B (9%) divides by the new amount ($36 \div 396 \approx 9\%$) — always divide by the original.
12	D	Unit prices: 3 kg bag: $7.95 \div 3 = \\$2.65$ per kg; 5 kg bag: $12.75 \div 5 = \\$2.55$ per kg. $\\$2.55 < \\$2.65$, so the 5 kg bag is the better buy at \$ 2.55 per kg, even though it costs more in total (which is why option A is wrong).

Chapter 02 — Test B (written response)

Chapter 2 (Number), sections 2A to 2G. Materials: pen, paper; a scientific calculator may be used. Test B: 7 questions (33 marks). Approximate time: 45 minutes. Answer all questions in the spaces provided. In all questions where a numerical answer is required, you should only round when instructed to do so, and then to the precision stated. Where more than one mark is available, appropriate working must be shown.

About the data in this test. The prices, times and other numbers here are **made up** (not real): pretend people, pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Question 1 (4 marks)

The Fernvale community newsletter prints numbers both in words and in numerals.

a. Write 47 062 in words. 1 mark

b. Write “six hundred and five thousand and eighty” in numerals. 1 mark

c. A fish caught in Fernvale Lake is weighed at 4.062 kg. Write this mass in words, using place form. 1 mark

d. List 6.5, 6.05 and 6.055 in ascending order (smallest first). 1 mark

Question 2 (5 marks)

The Lakeside community fair committee is packing treats and running a shuttle.

a. List all the factors of 40. 1 mark

b. Find the highest common factor (HCF) of 40 and 56. 1 mark

c. The committee has 40 cupcakes and 56 biscuits. All the treats are packed into identical boxes so that every box holds the same number of treats, cupcakes and biscuits are never mixed, and the boxes are as large as possible. How many treats go in each box, and how many boxes are used in total? 2 marks

d. A shuttle bus leaves the stop every 15 minutes and a ferry leaves every 20 minutes. A bus and a ferry leave together at 10:00 am. At what time will a bus and a ferry next leave together? 1 mark

Question 3 (4 marks)

a. Round 28 745 to the nearest thousand. 1 mark

b. Write \$ 7.385 correct to the nearest cent. 1 mark

c. Round 2.964 to one decimal place. 1 mark

d. Three workers share a \$ 157 dinner bill equally. Find each person's share, correct to the nearest cent. 1 mark

Question 4 (5 marks)

Nadia's bank account is overdrawn — the bank is owed money. Her balance is $-\$75$.

a. On Monday, Nadia deposits \$ 140. Find her balance. 1 mark

b. On Tuesday, a rent payment of \$ 200 goes out of the account. Find her balance. 1 mark

c. Use $<$ or $>$ to compare -135 and -75 . Which of Nadia's two balances is the deeper overdraft? 1 mark

d. On Wednesday, Nadia deposits \$ 200 and the bank reverses (gives back) an old \$ 15 fee. Find her balance. 1 mark

e. Over the next four days, Nadia's small purchases reduce her balance by \$ 18 each day. Write the total change over the four days as a product involving a negative number, and find her final balance. 1 mark

Question 5 (5 marks)

A community garden is square, with an area of 169 m^2 . The garden group also builds cube-shaped planter boxes.

a. Find the side length of the garden. 1 mark

b. Fencing runs all the way around the garden. Find the length of fencing needed. 1 mark

c. One planter box has edges of 25 cm. Find its volume, in cm^3 . 1 mark

d. A job sheet lists two calculations: 5×3^2 and $(5 \times 3)^2$. Evaluate each of them. 2 marks

Question 6 (5 marks)

a. Write 0.65 as a fraction in simplest form, and as a percentage.

1 mark

b. Find $\frac{3}{5}$ of \$ 85.

1 mark

c. A painter uses $\frac{5}{8}$ L of paint on the first coat of a wall and $\frac{1}{4}$ L on the second coat. How many litres of paint are used in total? Give your answer as a fraction.

1 mark

d. A gym membership increases from \$ 60 per month to \$ 69 per month. Find the percentage increase.

1 mark

e. After a pay rise of 20 %, Tomas earns \$ 31.50 per hour. Find his hourly rate before the rise.

1 mark

Question 7 (5 marks)

a. Write the ratio 45 minutes : 1.5 hours in simplest form.

1 mark

b. Two concrete mixes are in proportion: 6 buckets of cement to x buckets of sand matches 15 buckets of cement to 20 buckets of sand. Find x .

1 mark

c. A machine packs 450 cans in 6 hours, working at a steady rate. How many cans does it pack in 10 hours?

1 mark

d. Juice is sold in a 4-pack for \$ 5.20 and in a 6-pack for \$ 7.50. Find the price per carton for each pack, and state which pack is the better buy.

1 mark

e. Three siblings share \$ 720 in the ratio 3 : 4 : 5. Find each share.

1 mark

End of Test B

Test B — Answers

Q1. a. forty-seven thousand and sixty-two b. 605 080 c. four and sixty-two thousandths kilograms d. 6.05, 6.055, 6.5

Q2. a. 1, 2, 4, 5, 8, 10, 20, 40 b. 8 c. 8 treats per box; 12 boxes in total d. 11:00 am

Q3. a. 29 000 (nearest thousand) b. \$ 7.39 (nearest cent) c. 3.0 (1 decimal place) d. \$ 52.33 (nearest cent)

Q4. a. \$ 65 b. – \$ 135 c. $-135 < -75$; Tuesday's balance (– \$ 135) is the deeper overdraft d. \$ 80 e. $-18 \times 4 = -72$; final balance \$ 8

Q5. a. 13 m b. 52 m c. $15\,625\text{ cm}^3$ d. $5 \times 3^2 = 45$; $(5 \times 3)^2 = 225$

Q6. a. $\frac{13}{20}$; 65 % b. \$ 51 c. $\frac{7}{8}$ L d. 15 % e. \$ 26.25

Q7. a. 1 : 2 b. $x = 8$ c. 750 cans d. \$ 1.30 and \$ 1.25 per carton; the 6-pack is the better buy e. \$ 180, \$ 240 and \$ 300

Test B — Fully-worked solutions

Question 1.

- a. Split into groups: 47 thousand, then the last group (below one hundred) joins with “and”: **forty-seven thousand and sixty-two**.
- b. Six hundred and five thousand = 605 000; “and eighty” adds 80, with a zero in the hundreds place as a placeholder: $605\,000 + 80 = 605\,080$. Answer: **605 080**.
- c. 4.062 kg: the decimal part is 0.062, which is 62 thousandths, so in place form: **four and sixty-two thousandths kilograms**.
- d. Write each number with the same number of decimal places: 6.500, 6.050, 6.055. Compare from the left: $6.050 < 6.055 < 6.500$, so ascending order is **6.05, 6.055, 6.5**. More decimal digits does not mean bigger.

Question 2.

- a. Factor pairs in order: 1×40 , 2×20 , 4×10 , 5×8 . Factors of 40: **1, 2, 4, 5, 8, 10, 20, 40**.
- b. Prime factorise: $40 = 2^3 \times 5$ and $56 = 2^3 \times 7$. The shared part is $2^3 = 8$, so $\text{HCF}(40, 56) = 8$.
- c. “Identical boxes, as large as possible” means the box size is the HCF: **8** treats per box. Cupcakes: $40 \div 8 = 5$ boxes. Biscuits: $56 \div 8 = 7$ boxes. Total: $5 + 7 = 12$, so **12 boxes**. Check: $5 \times 8 = 40$ cupcakes and $7 \times 8 = 56$ biscuits — every treat is used.
- d. “Next time together” means the LCM. Multiples of 15: 15, 30, 45, 60, ...; multiples of 20: 20, 40, 60, ...; so $\text{LCM}(15, 20) = 60$ minutes. 10:00 am plus 60 minutes is **11:00 am**.

Question 3.

- a. Nearest thousand: the deciding digit is the hundreds digit, $7 \geq 5$, so round up. 28 745 rounds to **29 000**.
- b. Nearest cent means 2 decimal places. The deciding digit is the thousandths digit, 5; exactly halfway rounds up, so \$7.385 rounds to **\$7.39**.
- c. One decimal place: the deciding digit is $6 \geq 5$, so the 9 rounds up to 10 and carries into the ones: 2.9 becomes 3.0. Answer: **3.0** — keep the 0 to show the rounding is to one decimal place.
- d. $157 \div 3 = 52.3333\dots$ Correct to the nearest cent, the deciding digit is $3 < 5$, so round down: **\$52.33** each. Check: $3 \times 52.33 = 156.99$, just under the bill, as expected when rounding down.

Question 4.

- a. $-75 + 140 = 65$, so Nadia’s balance is **\$65**. (From -75 , moving up 75 reaches 0; the remaining 65 takes the balance to $+65$.)
- b. $65 - 200 = -135$, so her balance is **-\$135** — the account is overdrawn again.
- c. $-135 < -75$. On the number line -135 is further left, so **Tuesday’s balance** ($-\$135$) is the deeper overdraft — a bigger number after the minus sign means more is owed.
- d. Deposit first: $-135 + 200 = 65$; then the reversed fee adds back \$15: $65 + 15 = 80$. Her balance is **\$80**.
- e. A drop of \$18 each day for 4 days: change = $-18 \times 4 = -72$, so the balance drops by \$72. Final balance = $80 - 72 = 8$: **\$8**.

Question 5.

- a. Side length = $\sqrt{169}$. Since $13 \times 13 = 169$, the side is **13 m**.
- b. Perimeter of a square = $4 \times \text{side} = 4 \times 13 = 52$: **52 m** of fencing.
- c. Volume of a cube = $\text{edge}^3 = 25^3 = 25 \times 25 \times 25 = 625 \times 25 = 15\,625$: **15 625 cm³**.
- d. 5×3^2 : the index applies only to the 3, so do the power first: $3^2 = 9$, then $5 \times 9 = 45$. $(5 \times 3)^2$: the brackets come first, so the index applies to the whole product: $5 \times 3 = 15$, then $15^2 = 225$. The brackets change what the index applies to, which is why the two answers are different.

Question 6.

- a. $0.65 = \frac{65}{100}$. Divide top and bottom by the HCF, 5: $\frac{65 \div 5}{100 \div 5} = \frac{13}{20}$. As a percentage: $0.65 \times 100 = 65\%$. So $0.65 = \frac{13}{20} = 65\%$.
- b. "Of" means multiply: $\frac{3}{5} \times 85 = \frac{3 \times 85}{5} = \frac{255}{5} = 51$. Answer: **\$51**. (Or: $85 \div 5 = 17$, then $3 \times 17 = 51$.)
- c. Common denominator (the LCM of 8 and 4 is 8): $\frac{1}{4} = \frac{2}{8}$. Total = $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$, so **7/8 L** in total.
- d. Change = $69 - 60 = 9$. Percentage change divides by the original: $\frac{9}{60} \times 100 = 15$, a **15%** increase. Check with the multiplier: $60 \times 1.15 = 69$. ✓
- e. Going backwards means dividing by the multiplier. A 20% rise multiplies the old rate by 1.20, so the old rate = $31.50 \div 1.20 = 26.25$: **\$26.25** per hour. Check: $26.25 \times 1.20 = 31.50$. ✓

Question 7.

- a. Same units first: 1.5 hours = 90 minutes, so the ratio is 45:90. Divide both terms by the HCF, 45: **1 : 2**.
- b. Proportion means equal ratios: $\frac{6}{15} = \frac{x}{20}$. So $x = \frac{6 \times 20}{15} = \frac{120}{15} = 8$. Check: $6:8 = 3:4$ and $15:20 = 3:4$. ✓ Answer: $x = 8$.
- c. Unitary method — find the rate for one hour first: $450 \div 6 = 75$ cans per hour. Then $75 \times 10 = 750$: **750 cans** in 10 hours.
- d. 4-pack: $5.20 \div 4 = \$1.30$ per carton. 6-pack: $7.50 \div 6 = \$1.25$ per carton. Since $\$1.25 < \$1.30$, the **6-pack is the better buy**.
- e. Total parts = $3 + 4 + 5 = 12$. One part = $720 \div 12 = 60$. Shares: $3 \times 60 = \mathbf{\$180}$, $4 \times 60 = \mathbf{\$240}$ and $5 \times 60 = \mathbf{\$300}$. Check both ways: $180 + 240 + 300 = 720$ ✓ and $180:240:300 = 3:4:5$ ✓.