

VCE VM Numeracy Unit 1

Chapter 5 — Relationships

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Chapter 5 Relationships — 5A Rates in familiar situations

About the data in this section. Every price, trip and measurement here is **made up** (not real): pretend shops, pretend trips and pretend numbers, chosen so the arithmetic stays friendly. The methods are the real part — they work the same way on real numbers.

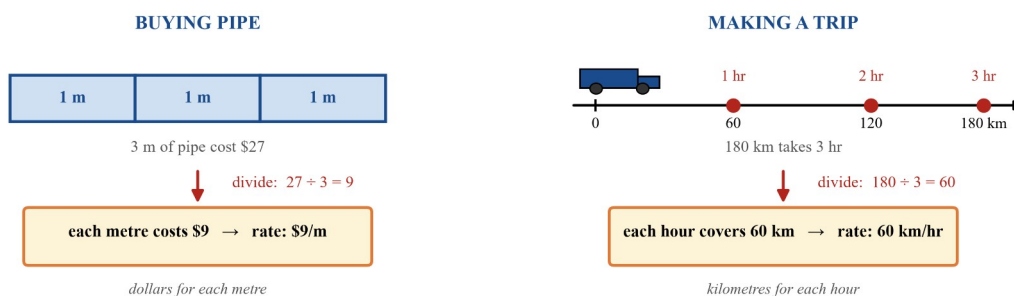
Theory

In the study design (Unit 1, Area of Study 4), the key skill for this section is using rates in everyday situations you know, for example km/hr and \$/m.

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-08.

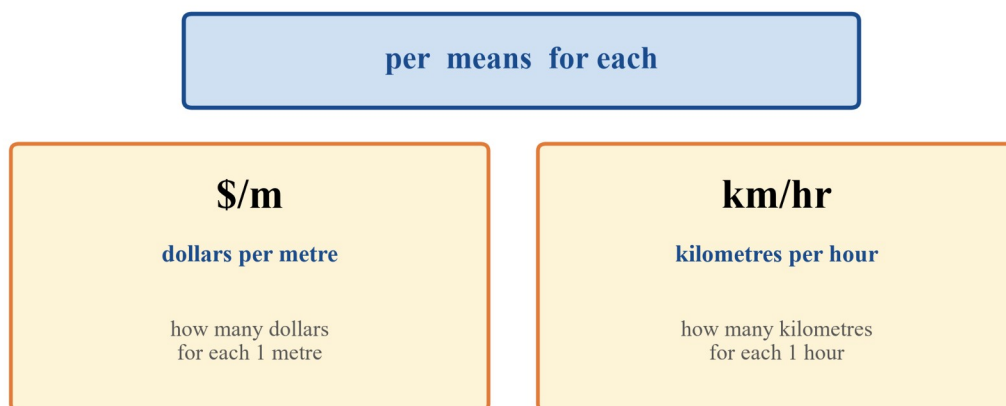
What is a rate? A rate compares two different kinds of quantity and tells you the amount of one of them **for each single one** of the other. You have met this idea before as the value of one: if 3 tubes of glue cost \$ 12, then one tube costs \$ 4. A rate is the value of one with **two units attached**, because two kinds of quantity are being compared.

A rate compares two different kinds of quantity



On the left, \$ 27 buys 3 m of pipe, so each single metre costs $27 \div 3 = \$9$: the rate is \$ 9/m. On the right, a van covers 180 km in 3 hr, so each single hour covers $180 \div 3 = 60$ km: the rate is 60 km/hr. Same idea, two different dressings.

The word “per”. In a rate, **per** means **for each**. The slash in \$ 9/m is read “per”: dollars **per** metre, meaning dollars **for each** metre. This section uses two rates only.



The slash / means per. Road signs sometimes write km/h — same rate.

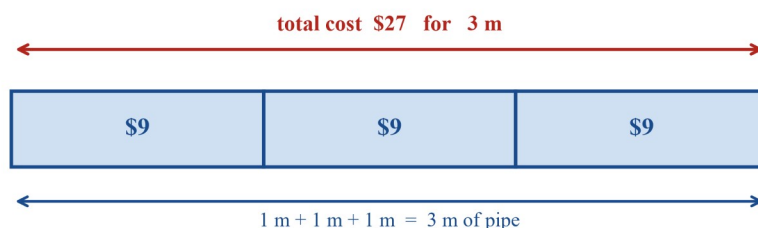
Rate	Read as	Tells you
\$/m	dollars per metre	how many dollars for each 1 metre

Rate	Read as	Tells you
km/hr	kilometres per hour	how many kilometres for each 1 hour

Finding a rate: divide. To find a rate, divide the total by the number of ones you are splitting it across.

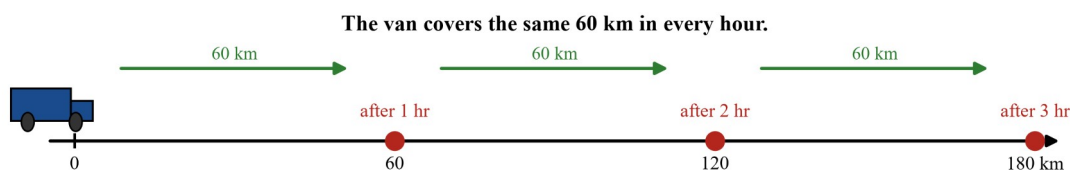
price per metre = total cost ÷ number of metres

speed = distance ÷ time



price per metre = total cost ÷ number of metres = $27 \div 3 = 9$

So the pipe costs \$9 for each metre — written \$9/m.



speed = distance ÷ time = $180 \div 3 = 60$

The speed is 60 km/hr — sixty kilometres for each hour.

Let the units tell you the order. The rate you want names the division. \$9/m is dollars per metre, so it is dollars ÷ metres: $27 \div 3 = 9$, not $3 \div 27$. Likewise 60 km/hr is kilometres ÷ hours: $180 \div 3 = 60$. If your answer comes out smaller than you expect, check you divided the right way round.

Refresher: dividing by a decimal. Rates often need division by a decimal such as 1.5 or 2.5. Shift **both** numbers by the same power of ten until the divisor is a whole number — the answer does not change.

Division	Shifted	Answer
$9 \div 0.5$	$90 \div 5$	18
$24 \div 1.5$	$240 \div 15$	16
$150 \div 2.5$	$1500 \div 25$	60
$27 \div 0.75$	$2700 \div 75$	36

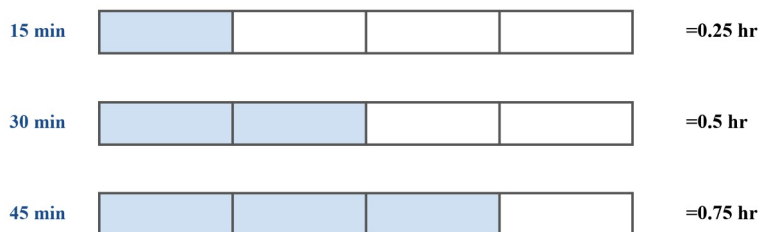
Refresher: time as a decimal number of hours. A speed in km/hr needs the time in **hours**. There are 60 minutes in an hour, so minutes ÷ 60 gives the decimal part of an hour.

Time	As a decimal number of hours
6 min	0.1 hr

12 min	0.2 hr
15 min	0.25 hr
30 min	0.5 hr
45 min	0.75 hr
1 hr 30 min	1.5 hr
2 hr 45 min	2.75 hr

Reading the table backwards: 0.25 hr is 15 min, 0.5 hr is 30 min, 0.75 hr is 45 min, and 0.15 hr is 9 min.

One hour, cut into four quarters



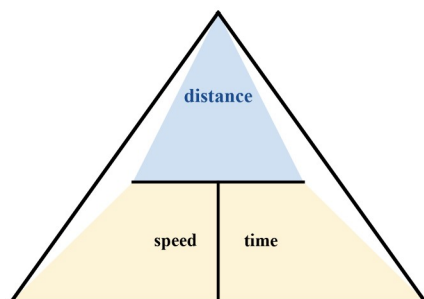
Each quarter is 15 minutes. Minutes $\div$ 60 gives the decimal.

Using a rate: multiply or divide. Once you have a rate, it becomes a tool. Multiply by the number of ones to get a total; divide a total by the rate to get the number of ones.

You know	You want	Working	Answer
hose at \$ 17.50/m, 6 m	total cost	17.50×6	\$ 105.00
pipe at \$ 9/m, \$ 45 to spend	metres	$45 \div 9$	5 m
car at 70 km/hr, 2 hr	distance	70×2	140 km
trip of 210 km at 70 km/hr	time	$210 \div 70$	3 hr

The word triangles hold all six facts each. Cover the word you want and the triangle shows the working.

TRIPS

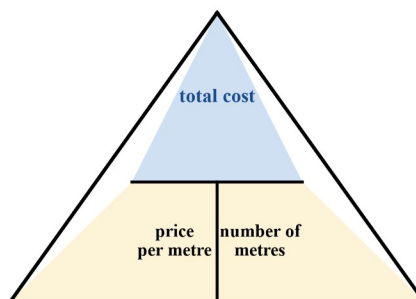


cover distance $\rightarrow$ speed $\times$ time

cover speed $\rightarrow$ distance $\div$ time

cover time $\rightarrow$ distance $\div$ speed

BUYING BY THE METRE



cover total cost $\rightarrow$ price per metre $\times$ metres

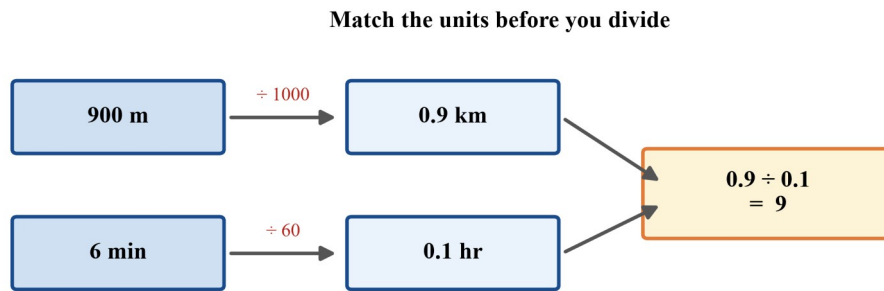
cover price per metre $\rightarrow$ total cost $\div$ metres

cover metres $\rightarrow$ total cost $\div$ price per metre

Refresher: metres and kilometres — match the units first. A rate in km/hr needs **kilometres** and **hours**. If your numbers are in metres and minutes, change them first — one step each. Metres to kilometres: $\div 1000$. Kilometres to metres: $\times 1000$.

This	Is the same as
900 m	0.9 km
2 400 m	2.4 km
1.8 km	1 800 m
0.5 km	500 m

Rex jogs 900 m in 6 min. In the units km/hr asks for, that is 0.9 km in 0.1 hr, so his rate is $0.9 \div 0.1 = 9$ km/hr.



Rex jogs 900 m in 6 min. His rate is 9 km/hr.

km/hr needs kilometres AND hours — convert first (one step each).

Comparing rates. To compare two offers, or two trips, write both rates in the **same units**, then compare the numbers. For prices per metre, the smaller number is the cheaper buy; for speeds, the bigger number is the faster trip. Roll A of garden mesh costs \$84 for 10 m and roll B costs \$96 for 12 m:

$$84 \div 10 = 8.40 \text{ and } 96 \div 12 = 8.00$$

Roll B is the better buy: \$8.00/m against \$8.40/m, saving $8.40 - 8.00 = \$0.40$ on every metre.

When the division does not come out exact. Some rates do not divide to a tidy number. \$20 for 3 m of chain gives $20 \div 3 = 6.666\dots$, which as money is \$6.67/m to the nearest cent. Always round to the accuracy the question asks for, say so in your answer, and keep the full number in your working until the last step.

Refresher: rounding to two decimal places. Look at the third decimal place. If it is 5 or more, round the second place up; if it is 4 or less, leave the second place alone.

- $25 \div 6 = 4.1666\dots$ rounds to 4.17 (to 2 d.p.), because the third decimal is 6.
- $20 \div 3 = 6.666\dots$ rounds to 6.67 (to 2 d.p.), for the same reason.

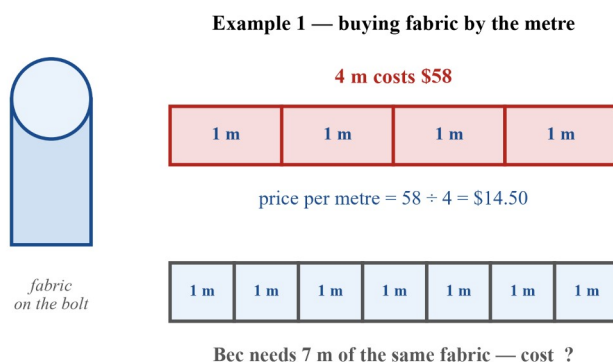
Every trip in this section is steady. Each vehicle, walker and boat here moves at a **steady** speed: the same distance in every hour, with no stopping and no speeding up. That is what lets one division describe the whole trip.

Key ideas.

- A rate compares two kinds of quantity and gives the amount of one **for each single one** of the other.
- **per** means **for each**; the slash in \$/m and km/hr is read “per”.
- Finding a rate is a division: $\text{total} \div \text{number of ones} = \text{rate}$. The units of the rate tell you the order.
- Using a rate is a multiplication ($\text{rate} \times \text{ones} = \text{total}$) or a division ($\text{total} \div \text{rate} = \text{ones}$).
- Match the units before you divide: km/hr needs kilometres and hours.
- To compare, put both rates in the same units first. Round only at the end, and say to what accuracy.

Worked examples

Example 1. Bec buys curtain fabric from a bolt (a big roll). A 4 m cut costs \$58.



(a) Find the price per metre. (b) Hence find the cost of 7 m of the same fabric. (c) Explain in words what “\$ 14.50 per metre” means.

Working

Comment

$$58 \div 4 = 14.50$$

(a) price per metre = total cost $\div$ metres

$$\text{check: } 14.50 \times 4 = 58$$

multiplying back returns the \$58 tag

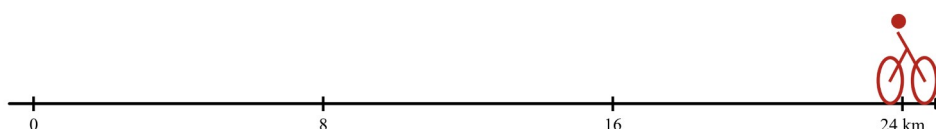
$$14.50 \times 7 = 101.50$$

(b) 7 m at \$14.50 for each metre

(c) Each single metre of this fabric costs \$14.50, so 7 metres cost 7 lots of \$14.50.

Example 2. Liam rides his bike 24 km in 1 hr 30 min, at a steady pace.

Liam rides 24 km in 1 hr 30 min — at a steady pace.



$$1 \text{ hr } 30 \text{ min} = 1.5 \text{ hr}$$

$$\text{speed} = \text{distance} \div \text{time} = 24 \div 1.5 = ?$$

(a) Write the time as a decimal number of hours. (b) Find his speed in km/hr. (c) How far does he ride in 2 hr at this speed?

Working

Comment

$$1 \text{ hr } 30 \text{ min} = 1.5 \text{ hr}$$

(a) 30 min is half an hour

$$24 \div 1.5 = 16$$

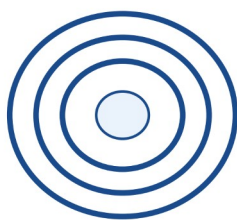
(b) speed = distance $\div$ time

$$16 \times 2 = 32$$

(c) 2 hr at 16 km for each hour

Example 3. Irrigation pipe (pipe that waters a garden) is priced at \$6.80/m at the rural store.

Example 3 — pipe sold by the metre



a coil of irrigation pipe

\$6.80 per metre

written \$6.80/m

Jess needs 12.5 m for her veggie garden.

Her neighbour paid \$51.00 at the same rate.

(a) Find the cost of 12.5 m of this pipe. **(b)** Jess's neighbour paid \$51.00 for the same pipe. How many metres did the neighbour buy? **(c)** Jess has \$150. Is that enough for 20 m of this pipe? If yes, what change does she get?

(a) $6.80 \times 12.5 = 85$

The 12.5 m of pipe costs \$85.00.

(b) $51 \div 6.80 = 7.5$

The neighbour bought 7.5 m of pipe.

(c) $6.80 \times 20 = 136$, and 136 is less than 150, so yes.

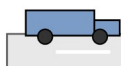
$$150 - 136 = 14$$

$\therefore$ yes, \$150 is enough for 20 m, with \$14.00 change.

Example 4. Two delivery vans make highway runs, each at a steady speed. Mara's van travels 195 km in 3 hr.

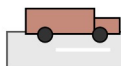
Two delivery vans on highway runs

Mara



195 km in 3 hr

Sonny



165 km in 2.5 hr

Each van travels at a steady speed. Whose van is faster?

(a) Show that Mara's van travels at 65 km/hr. (b) Sonny's van travels 165 km in 2.5 hr. Find Sonny's speed in km/hr. (c) Whose van is faster, and by how much?

(a) $195 \div 3 = 65$

Mara's van travels at 65 km/hr.

(b) $165 \div 2.5 = 66$

Sonny's van travels at 66 km/hr.

(c) $66 - 65 = 1$

$\therefore$ Sonny's van is faster, by 1 km/hr.

Practice questions

Scaffolded

Q1. Find each rate. (a) A coach travels 150 km in 3 hr — speed in km/hr. (b) 6 m of rope costs \$ 45 — price per metre. (c) A car travels 350 km in 5 hr — speed in km/hr.

Q2. (a) 5 m of garden edging costs \$33.50. Find the price per metre. (b) 7 m of chain costs \$ 50. Find the price per metre to the nearest cent. (c) Write 2 hr 45 min as a decimal number of hours.

Q3. A road sign reads 40 km/hr, and a driver holds exactly that steady speed. (a) Explain in your own words what “40 km/hr” tells you about the trip. (b) How far does the car travel in 2 hr? (c) How long does 180 km take at this speed?

Q4. A fabric shop prices three fabrics by the metre.

Fabric	Length of cut	Price of the cut
Cotton	4 m	\$ 54.00
Denim	6 m	\$ 102.00
Canvas	5 m	\$ 47.50

Made-up data: pretend shop prices for this question.

(a) Find the price per metre of the cotton. (b) Find the price per metre of the denim. (c) Which fabric is the cheapest per metre, and what is its price per metre?

Q5. Electrical cable is priced at \$ 8.20/m. (a) Find the cost of 10 m. (b) Find the cost of 2.5 m. (c) A customer pays \$ 57.40. How many metres of cable did they buy?

Q6. A coach (a large bus) travels at a steady 75 km/hr. (a) How far does it travel in 3 hr? (b) How far does it travel in 30 min? (c) How long does 150 km take?

Unscaffolded

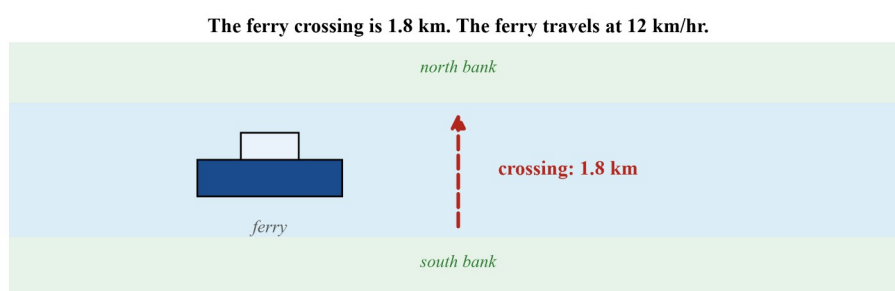
Q7. A hardware yard charges \$ 216 for a 12 m length of timber. (a) Find the price per metre. (b) Find the cost of 22 m of the same timber. (c) How many metres can be bought for \$ 162?

Q8. Callum walks 3 km in 30 min at a steady pace. (a) Write 30 min as a decimal number of hours. (b) Find his walking speed in km/hr. (c) How far does he walk in 1.5 hr at this pace?

Q9. Two rolls of garden mesh (a net-like fabric) are on sale: roll A is 20 m for \$ 63, roll B is 15 m for \$ 48. (a) Find the price per metre of roll A. (b) Find the price per metre of roll B. (c) Which roll is the better buy per metre, and by how much per metre?

Q10. Nikki cycles 2 400 m in 12 min at a steady pace. (a) Write 2 400 m in km. (b) Write 12 min as a decimal number of hours. (c) Find her speed in km/hr.

- Q11.** A delivery van travels at a steady 85 km/hr. **(a)** How long does a 255 km run take? **(b)** How far does the van travel in 1.5 hr? **(c)** A second van covers the same 255 km in 2.5 hr at a steady speed. Which van is faster?
- Q12.** Marty's scooter travels 27 km in 45 min at a steady speed. **(a)** Write 45 min as a decimal number of hours. **(b)** Show that the scooter's speed is 36 km/hr. **(c)** How far does the scooter travel in 1.25 hr at this speed?
- Q13.** A rope is priced at \$36 for 8 m. Wes works out $8 \div 36 = 0.22$ (to 2 d.p.) and calls that the price per metre. **(a)** Explain what Wes's 0.22 actually measures, and why it is not the price per metre. **(b)** Find the correct price per metre. **(c)** Find the cost of 15 m of this rope.
- Q14.** Josie walks 5 km in 1 hr 15 min at a steady pace. **(a)** Write the time as a decimal number of hours. **(b)** Find her walking speed in km/hr. **(c)** How long does 7 km take at this pace? Give your answer in hours and minutes.
- Q15.** Shade cloth (fabric that blocks sun) is priced at \$12.40/m. Otto has \$100. **(a)** Find the cost of 6.5 m of shade cloth. **(b)** How many **whole** metres of shade cloth can Otto buy? **(c)** If he buys that many whole metres, what change does he get from \$100?
- Q16.** A ferry (a boat that carries people and cars across water) makes a 1.8 km crossing at a steady 12 km/hr.



A ferry is a boat that carries people and cars across water.

- (a)** How long does the crossing take, in hours? **(b)** Write that time in minutes. **(c)** A second ferry makes the same 1.8 km crossing in 15 min at a steady speed. Find its speed in km/hr.

- Q17.** A job sheet lists three brands of weed mat, each sold in rolls.

Brand	Roll length	Roll price
Supco	25 m	\$190.00
Fixit	30 m	\$210.00
DuraRoll	20 m	\$166.00

Made-up data: pretend supplier prices for this question.

- (a)** Find the price per metre of Fixit. **(b)** Find the price per metre of DuraRoll. **(c)** Which brand is the cheapest per metre, and what would 45 m of that brand cost?

- Q18.** Ella rides her motorbike in two legs, each at a steady speed: 50 km at 20 km/hr, then 27 km at 12 km/hr. **(a)** How long does the first leg take? **(b)** How long does the second leg take? **(c)** How long does the whole ride take?

Answers

Q1. (a) 50 km/hr (b) \$ 7.50/m (c) 70 km/hr **Q2.** (a) \$ 6.70/m (b) \$ 7.14/m (to the nearest cent) (c) 2.75 hr **Q3.** (a) Each hour of the trip covers 40 km at this steady speed (b) 80 km (c) 4.5 hr **Q4.** (a) \$ 13.50/m (b) \$ 17.00/m (c) canvas, at \$ 9.50/m **Q5.** (a) \$ 82.00 (b) \$ 20.50 (c) 7 m **Q6.** (a) 225 km (b) 37.5 km (c) 2 hr **Q7.** (a) \$ 18/m (b) \$ 396.00 (c) 9 m **Q8.** (a) 0.5 hr (b) 6 km/hr (c) 9 km **Q9.** (a) \$ 3.15/m (b) \$ 3.20/m (c) roll A, by \$ 0.05 per metre **Q10.** (a) 2.4 km (b) 0.2 hr (c) 12 km/hr **Q11.** (a) 3 hr (b) 127.5 km (c) the second van, at 102 km/hr **Q12.** (a) 0.75 hr (b) 36 km/hr (c) 45 km **Q13.** (a) Metres of rope per dollar — how much rope one dollar buys — not dollars per metre (b) \$ 4.50/m (c) \$ 67.50 **Q14.** (a) 1.25 hr (b) 4 km/hr (c) 1.75 hr, that is 1 hr 45 min **Q15.** (a) \$ 80.60 (b) 8 whole metres (c) \$ 0.80 **Q16.** (a) 0.15 hr (b) 9 min (c) 7.2 km/hr **Q17.** (a) \$ 7.00/m (b) \$ 8.30/m (c) Fixit; 45 m costs \$ 315.00 **Q18.** (a) 2.5 hr (b) 2.25 hr (c) 4.75 hr

Fully-worked solutions

Q1. (a) speed = distance $\div$ time: $150 \div 3 = 50$, so 50 km/hr. (b) price per metre = total cost $\div$ metres: $45 \div 6 = 7.50$, so \$ 7.50/m. (c) $350 \div 5 = 70$, so 70 km/hr.

Q2. (a) $33.50 \div 5 = 6.70$, so \$ 6.70/m. (b) $50 \div 7 = 7.142857 \dots$, which rounds to \$ 7.14/m to the nearest cent. (c) 45 min is 0.75 hr, so 2 hr 45 min is 2.75 hr.

Q3. (a) At this steady speed the car covers 40 km in every hour of the trip. (b) $40 \times 2 = 80$ km. (c) $180 \div 40 = 4.5$ hr.

Q4. (a) $54 \div 4 = 13.50$, so \$ 13.50/m. (b) $102 \div 6 = 17.00$, so \$ 17.00/m. (c) Canvas: $47.50 \div 5 = 9.50$, so \$ 9.50/m — cheaper than \$ 13.50/m and \$ 17.00/m.

Q5. (a) $8.20 \times 10 = 82$, so \$ 82.00. (b) $8.20 \times 2.5 = 20.50$, so \$ 20.50. (c) $57.40 \div 8.20 = 7$, so 7 m.

Q6. (a) $75 \times 3 = 225$ km. (b) 30 min is 0.5 hr, and $75 \times 0.5 = 37.5$ km. (c) $150 \div 75 = 2$ hr.

Q7. (a) $216 \div 12 = 18$, so \$ 18/m. (b) $18 \times 22 = 396$, so \$ 396.00. (c) $162 \div 18 = 9$ m.

Q8. (a) $30 \div 60 = 0.5$ hr. (b) $3 \div 0.5 = 6$ km/hr. (c) $6 \times 1.5 = 9$ km.

Q9. (a) $63 \div 20 = 3.15$, so \$ 3.15/m. (b) $48 \div 15 = 3.20$, so \$ 3.20/m. (c) Roll A: $3.20 - 3.15 = 0.05$, so roll A is the better buy by \$ 0.05 per metre.

Q10. (a) $2400 \div 1000 = 2.4$ km. (b) $12 \div 60 = 0.2$ hr. (c) $2.4 \div 0.2 = 12$ km/hr.

Q11. (a) $255 \div 85 = 3$ hr. (b) $85 \times 1.5 = 127.5$ km. (c) Second van: $255 \div 2.5 = 102$ km/hr, and $102 > 85$, so the second van is faster.

Q12. (a) $45 \div 60 = 0.75$ hr. (b) $27 \div 0.75 = 2700 \div 75 = 36$, so the speed is 36 km/hr. (c) $36 \times 1.25 = 45$ km.

Q13. (a) Wes divided metres by dollars, so his 0.22 is **metres of rope per dollar** — how much rope one dollar buys. The price per metre must be dollars $\div$ metres. (b) $36 \div 8 = 4.50$, so \$ 4.50/m. (c) $4.50 \times 15 = 67.50$, so \$ 67.50.

Q14. (a) 15 min is 0.25 hr, so 1 hr 15 min is 1.25 hr. (b) $5 \div 1.25 = 4$ km/hr. (c) $7 \div 4 = 1.75$ hr; 0.75 hr is 45 min, so 1 hr 45 min.

Q15. (a) $12.40 \times 6.5 = 80.60$, so \$ 80.60. (b) $100 \div 12.40 = 8.06 \dots$ m. Check: 8 m costs $12.40 \times 8 = 99.20$ (fits \$ 100) but 9 m costs $12.40 \times 9 = 111.60$ (too much), so 8 whole metres. (c) $100 - 99.20 = 0.80$, so \$ 0.80 change.

Q16. (a) $1.8 \div 12 = 0.15$ hr. (b) $0.15 \times 60 = 9$ min. (c) 15 min is 0.25 hr, and $1.8 \div 0.25 = 7.2$ km/hr.

Q17. (a) $210 \div 30 = 7.00$, so \$ 7.00/m. (b) $166 \div 20 = 8.30$, so \$ 8.30/m. (c) Supco: $190 \div 25 = 7.60$, so \$ 7.60/m. The three rates are \$ 7.60/m, \$ 7.00/m and \$ 8.30/m, so Fixit is cheapest. 45 m of Fixit: $7.00 \times 45 = 315$, so \$ 315.00.

Q18. (a) $50 \div 20 = 2.5$ hr. (b) $27 \div 12 = 2.25$ hr. (c) $2.5 + 2.25 = 4.75$ hr.

Chapter 5 Relationships — 5B Simple formulae: area, perimeter and cost per hour

About the data in this section. The price cards, receipts, quotes and measurements here are **made up** (not real): pretend hire shops and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

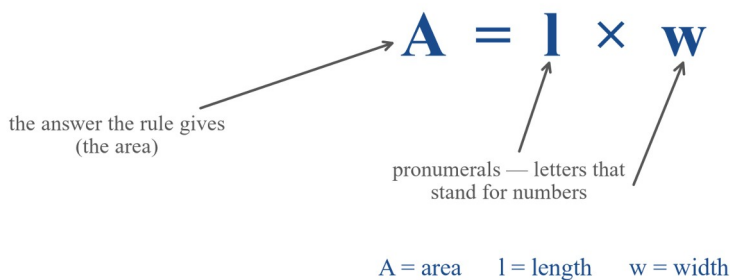
Theory

A formula is a rule written with letters.

Section 4B found perimeters and areas by adding and multiplying the numbers printed on a plan. This section writes those rules down once, with letters, so that one rule works for every rectangle — and then uses the same idea for money: cost per hour.

A **formula** is a rule for working something out, written with letters as well as numbers. Each letter in a formula stands for a number, and a letter that stands for a number is called a **pronumeral** (say “pro-numeral”). In the formula $A = l \times w$ the pronumerals are A , l and w : A stands for the area, l for the length and w for the width. Putting a number in place of a pronumeral is called **substitution** — it is what turns a formula into a calculation you can finish.

A formula is a rule written with letters



Every letter is defined — and the one rule works for EVERY rectangle.

The same rules, written two ways.

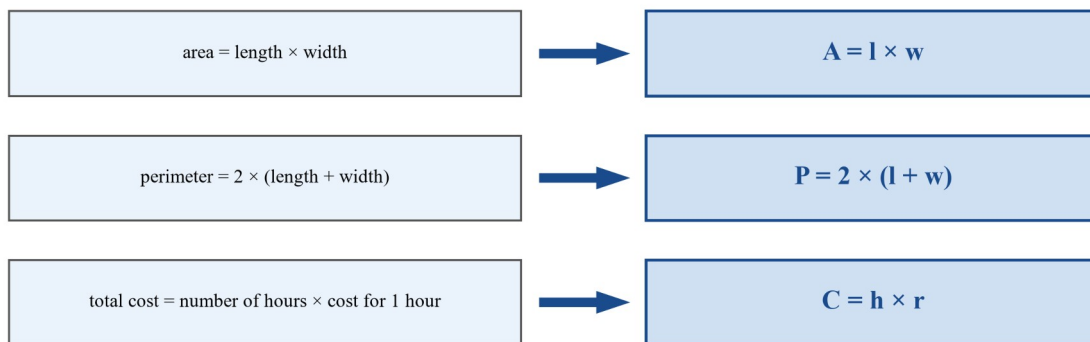
Every formula in this section says in letters something you already say in words.

In words	With letters
area = length $\times$ width	$A = l \times w$
perimeter = $2 \times (\text{length} + \text{width})$	$P = 2 \times (l + w)$
total cost = number of hours $\times$ cost for 1 hour	$C = h \times r$

The same rules, written two ways

in words

with letters



The letter version is shorter — and it says exactly the same thing.

The letters used in this section are A = area, P = perimeter, l = length, w = width, C = total cost, h = number of hours, r = cost for 1 hour and f = call-out fee. A formula always tells you what each of its letters means — never substitute into a letter that has not been defined.

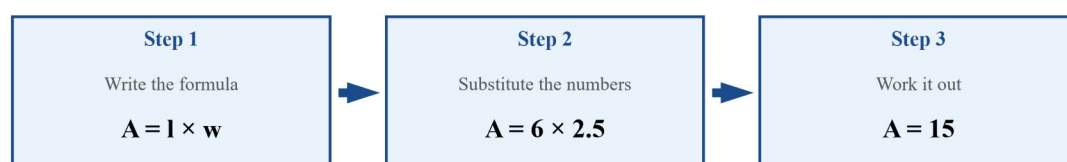
Substitution — three steps.

Worked model. A rectangle is 6 m long and 2.5 m wide. Find its area.

- Step 1 — write the formula: $A = l \times w$.
- Step 2 — substitute the numbers: $A = 6 \times 2.5$.
- Step 3 — work it out: $A = 15$.

The area is 15 m². The answer takes its units from the letters you substituted: metres in, square metres out.

Substituting: replace each letter with its number



$w = 2.5 \text{ m}$



$l = 6 \text{ m}$

*The area is 15 m².
The answer takes the
units of the letters.*

Units must match before you substitute. A formula multiplies and adds the numbers you put into it, so those numbers must be in the same unit first. A plan that says 5.4 m on one side and 320 cm on the other cannot be substituted straight in: 320 cm = 3.2 m ($\div 100$, Section 4A), and only then do 5.4 and 3.2 go into the formula.

Refresher: multiplying money. Multiply a dollars-and-cents amount just like a decimal: $\$38.50 \times 3 = \$115.50$ — two decimal places in the amount, two in the answer. Costings need this every time a price with cents gets multiplied.

Refresher: hours as decimals. Hire and pay are priced per hour, and parts of an hour are written as decimals: half an hour is 0.5 h and a quarter of an hour is 0.25 h (Section 2E: $\frac{1}{4} = 0.25$). To see what a decimal of an hour means in minutes, multiply it by 60 (Section 4D):

Hours	Multiply by 60	Minutes
0.5 h	0.5×60	30 min
0.25 h	0.25×60	15 min
0.75 h	0.75×60	45 min
1.5 h	1.5×60	90 min
1.25 h	1.25×60	75 min

Cost per hour — the money formula. When a hire costs the same for every hour, the total cost is the number of hours multiplied by the cost for 1 hour:

$$C = h \times r$$

Here h is the number of hours, r is the cost for 1 hour and C is the total cost. A trailer hired for 4 hours at \$32 per hour costs $C = 4 \times 32 = 128$, so \$128 altogether.

A trailer hired for 4 hours at \$32 per hour



$$C = h \times r = 4 \times 32 = \$128$$

$h = \text{number of hours}$ $r = \text{cost for 1 hour}$ $C = \text{total cost}$

Working backwards — division undoes multiplication. The same formula runs in reverse. If you know the total cost and the number of hours, then the cost for 1 hour is the total divided by the hours: \$167.50 paid for 5 hours of hire means $167.50 \div 5 = 33.50$, so \$33.50 per hour. Division undoes the multiplication in $C = h \times r$ — one step backwards, the same idea as finding a missing side from a perimeter in Section 4B.

\$167.50 paid for 5 hours of hire

total cost $C = \$167.50$



$$\text{cost for 1 hour} = \text{total cost} \div \text{number of hours} = 167.50 \div 5 = \$33.50$$

Division undoes multiplication — that is why it finds the cost for 1 hour.

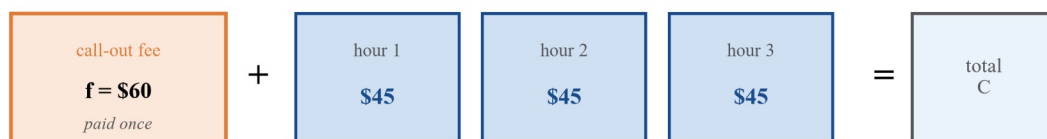
Refresher: dividing money and rounding to the nearest cent. Not every total divides evenly. \$100 spread over 3 hours: $100 \div 3$ does not come out exactly, and to the nearest cent it is \$33.33 (Section 2C — the next digit is 3, less than 5, so round down). Money answers are rounded to the nearest cent, which is two decimal places, and the precision is always stated.

A call-out fee plus an hourly rate. Many trades charge a fixed **call-out fee** once, just for coming to the job, plus a cost for every hour worked. The formula has both parts in it:

$$C = f + h \times r$$

Here f is the call-out fee, paid once whatever the hours. A job with a \$ 60 call-out fee and 3 hours at \$ 45 per hour costs $C = 60 + 3 \times 45 = 60 + 135 = 195$, so \$ 195.

A job of 3 hours, with a \$60 call-out fee and \$45 per hour



$$C = f + h \times r = 60 + 3 \times 45 = 60 + 135 = \$195$$

Multiply first (3×45), then add the fee once — order of operations.

Refresher: order of operations in a formula. In $C = f + h \times r$ the multiplication is done first (Section 2D): $3 \times 45 = 135$, then the \$ 60 fee is added once. Adding first changes the answer completely: $(60 + 3) \times 45 = 2835$ — that mistake counts the call-out fee once per hour instead of once for the job.

Key ideas.

- A formula is a rule written with letters; each letter is a **pronumeral** standing for a number, and every letter is defined.
- Substitution: write the formula, replace each letter with its number (same units first), then work it out.
- Rectangle: $A = l \times w$ and $P = 2 \times (l + w)$ — the Section 4B rules, now written once with letters so they work for every rectangle.
- Cost per hour: $C = h \times r$. Backwards: cost for 1 hour = total cost $\div$ number of hours, because division undoes multiplication.
- Fee plus hourly rate: $C = f + h \times r$ — multiply $h \times r$ first, then add the fee once.
- Parts of an hour are decimals (0.5 h, 0.25 h, 1.25 h); multiplying by 60 turns them into minutes.
- Round money only at the end, to the stated precision — the nearest cent is two decimal places.

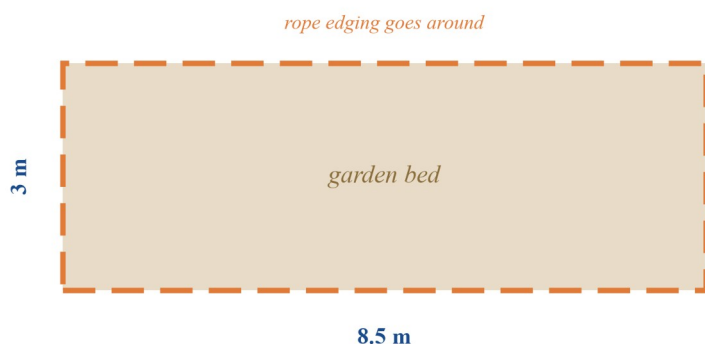
Worked examples

Example 1 — scaffolded

Marlee is building a rectangular garden bed 8.5 m long and 3 m wide. Mulch covers the surface inside the bed, and rope edging goes around the edge.

- Find the area the mulch must cover.
- Find the length of rope edging Marlee needs.

(c) One of your answers is an area and one is a length. Which is which, and how do the units tell you?



Mulch covers the surface; the rope goes around the edge.

Working	Comment
$A = l \times w = 8.5 \times 3 = 25.5 \text{ m}^2$	mulch covers the surface: length $\times$ width
$l + w = 8.5 + 3 = 11.5$	add the two different sides first
$P = 2 \times 11.5 = 23 \text{ m}$	edging goes around all four sides: twice (length + width)
Mulch: 25.5 m^2 . Edging: 23 m	the ^{2} marks the area; the edging stays in plain metres

Example 2 — scaffolded

A hire shop's price card prices its carpet cleaner at \$28 per hour. Caleb hires the cleaner for 3.5 hours.

- Write down the values of h and r from the card.
- Substitute into $C = h \times r$ to find the cost of the hire.
- The shop's receipt prints \$98.00. Check the cost for 1 hour by dividing the total by the hours.



Working	Comment
$h = 3.5, r = 28$	3.5 h is three and a half hours; \$28 is the cost for 1 hour
$C = h \times r = 3.5 \times 28 = 98$	substitute the numbers, then multiply
the hire costs \$98.00	money is written to two decimal places
check: $98 \div 3.5 = 28$	division undoes the multiplication — back to the card's rate ✓

Example 3 — unscaffolded

Kira hires a ride-on mower. The receipt shows 6 hours of hire and a total of \$175.00, with the same price for every hour.

- (a) Find the cost for 1 hour, correct to the nearest cent.
- (b) Kira's neighbour hires the same mower for 3 hours at the same rate. Find the neighbour's cost.

EQUIPMENT HIRE	
- RECEIPT -	
Ride-on mower	6 hours
TOTAL	\$175.00
Same price for every hour.	
The receipt shows the TOTAL — not the cost for 1 hour.	

- (a) The cost for 1 hour is the total divided by the hours. $175 \div 6$ does not come out exactly; to the nearest cent it is \$29.17 (the next digit is 6, so round up).

$\therefore$ the cost for 1 hour is \$29.17, correct to the nearest cent.

- (b) 3 hours is half of 6 hours, so the neighbour's cost is half of the total: $175 \div 2 = 87.5$.

$\therefore$ the neighbour pays \$87.50.

Careful with rounding here: using the rounded rate from (a), $29.17 \times 3 = 87.51$, which is one cent out. Carry the exact rate through the working and round only the answer you report.

Example 4 — unscaffolded

Frankie gets two quotes for the same electrical job. Electrician 1 charges a call-out fee of \$70 plus \$55 per hour. Electrician 2 charges no call-out fee and \$80 per hour.

- (a) Write a formula for the total cost C of each quote, using h for the number of hours.
- (b) Find the cost of each quote for a 2-hour job.
- (c) The job takes 3.5 hours. Which electrician costs less, and by how much?

Two quotes for the same job

Electrician 1
call-out fee \$70
+ \$55 per hour
Formula: $C = f + h \times r$
$f = \text{call-out fee}$ $h = \text{hours}$ $r = \text{cost for 1 hour}$

Electrician 2
no call-out fee
\$80 per hour
Formula: $C = h \times r$
$f = \text{call-out fee}$ $h = \text{hours}$ $r = \text{cost for 1 hour}$

- (a) Electrician 1 has a fee plus an hourly rate: $C = f + h \times r$, so $C = 70 + h \times 55$. Electrician 2 has no fee: $C = h \times r$, so $C = h \times 80$.

(b) Electrician 1: $C = 70 + 2 \times 55 = 70 + 110 = 180$, so \$ 180. Electrician 2: $C = 2 \times 80 = 160$, so \$ 160.

(c) Electrician 1: $C = 70 + 3.5 \times 55 = 70 + 192.5 = 262.5$, so \$ 262.50. Electrician 2: $C = 3.5 \times 80 = 280$, so \$ 280.00. The difference is $280 - 262.5 = 17.5$.

$\therefore$ for the 3.5-hour job Electrician 1 costs less, by \$ 17.50 — even though Electrician 2 was cheaper for the 2-hour job, the call-out fee is paid only once however long the job runs.

Practice questions

Scaffolded

Q1. Substitute into $A = l \times w$ to find the area of each rectangle.

(a) $l = 7$ m, $w = 4$ m

(b) $l = 9$ m, $w = 2.5$ m

(c) $l = 12$ m, $w = 3.6$ m

Q2. Substitute into $P = 2 \times (l + w)$ to find the perimeter of each rectangle.

(a) $l = 10$ m, $w = 6$ m

(b) $l = 8.5$ m, $w = 4$ m

(c) $l = 15$ m, $w = 7.5$ m

Q3. A hire item costs the same for every hour. Substitute into $C = h \times r$ to find the total cost.

(a) $h = 5$, $r = \$ 40$

(b) $h = 3$, $r = \$ 48.50$

(c) $h = 2.5$, $r = \$ 36$

Q4. Holly's market stall hires a popcorn machine at \$ 24 per hour.

(a) Write down the value of r .

(b) Write the formula $C = h \times r$ with r replaced by its number.

(c) Find the cost of 5 hours of hire.

Q5. Nik the handyman charges the same rate for every hour of work, and invoices \$ 190 for 4 hours.

(a) Find Nik's cost for 1 hour.

(b) Find the cost of 7 hours of work at that rate.

(c) Check your answer to (b) another way: 7 hours is the first 4 hours plus 3 more hours at the rate from (a).

Q6. A rectangular concrete slab is 4.8 m long and 3.5 m wide.

(a) Find the area of the top of the slab.

(b) Find the perimeter of the slab.

(c) One of your two answers is in square units. Which one, and what is that measurement called?

Un scaffolded

Q7. A rectangular piece of artificial turf is 7.25 m long and 4.75 m wide. Binding tape is sewn around the edge of the turf, and the tape costs \$6 per metre.

- (a) Find the area of the turf, to 2 decimal places.
- (b) Find the length of tape needed.
- (c) Find the cost of the tape.

Q8. Tama hires a wood chipper at \$46 per hour.

- (a) Find the cost of 2.5 hours of hire.
- (b) How many minutes is 2.5 hours?
- (c) Find the cost of 1.25 hours of hire.

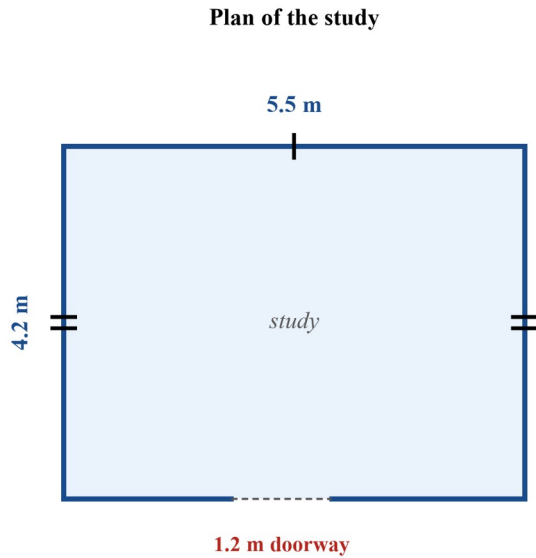
Q9. An excavator is hired for 7 hours at a total cost of \$250, with the same price for every hour.

- (a) Find the cost for 1 hour, correct to the nearest cent.
- (b) Find the cost of 2 hours at the exact rate, correct to the nearest cent.
- (c) A student doubles the rounded rate from (a) and gets \$71.42. Explain the one-cent difference between the two answers.

Q10. A rectangular vegetable garden is 6.4 m long and 2.9 m wide. Edging runs right around the garden, and the edging costs \$11.50 per metre.

- (a) Show that the perimeter of the garden is 18.6 m.
- (b) Find the area of the garden.
- (c) Find the cost of the edging.

Q11. The plan below shows a rectangular study. Only two measurements are printed on the plan, and a 1.2 m doorway has no skirting board across it.



- (a) Find the perimeter of the study.
- (b) Find the area of the study floor.
- (c) Skirting board runs around the base of the walls except across the doorway. Find the length of skirting board needed.

Q12. A tow truck charges a call-out fee of \$ 85 plus \$ 60 per hour.

- (a) Write the formula $C = f + h \times r$ with f and r replaced by their numbers.
- (b) Find the cost of a 1.5-hour job.
- (c) Find the cost of a 2.75-hour job.

Q13. A 4-hour job can be priced two ways. Option A: a call-out fee of \$ 40 plus \$ 35 per hour. Option B: no call-out fee and \$ 50 per hour.

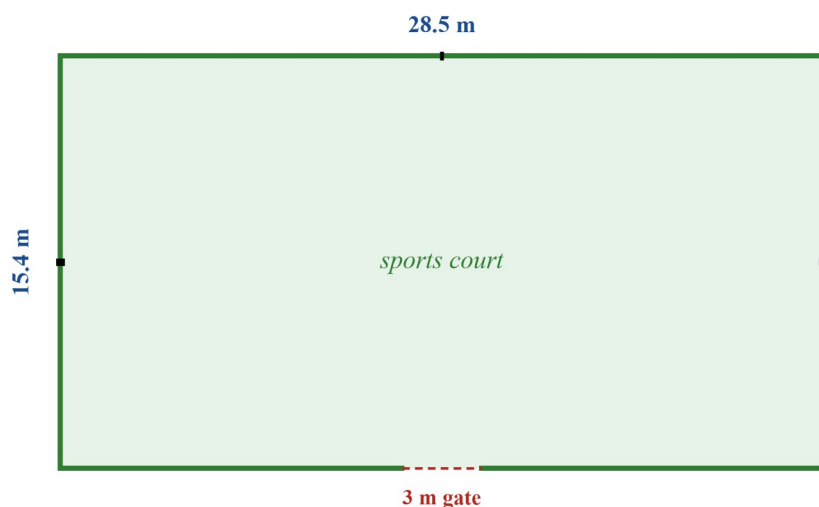
- (a) Find the cost of Option A for this job.
- (b) Find the cost of Option B for this job.
- (c) Which option is cheaper for this job, and by how much?

Q14. Ollie's casual job pays \$ 29.75 per hour.

- (a) Find Ollie's pay for 5 hours of work.
- (b) Find Ollie's pay for 6.5 hours of work, correct to the nearest cent.
- (c) One week Ollie's pay is \$ 238. How many hours did Ollie work?

Q15. The plan below shows a rectangular sports court. Mesh fencing runs around the edge of the court except for a 3 m gate, and the mesh costs \$8.75 per metre.

Plan of the sports court



Mesh fencing runs around the whole court except the gate.

- Find the perimeter of the court.
- Find the length of mesh fencing needed.
- Find the cost of the mesh fencing.

Q16. A hire shop's price table for a trailer shows the same price for every hour of hire.

Hours of hire	Total cost
1	\$ 36
2	\$ 72
3	\$ 108

Made-up data: the hours and every price in the table are made up for this section.

- Find the cost for 1 hour, r . Check it against the other rows of the table.
- Find the cost of 5 hours of hire.
- A customer pays \$ 252 for a hire. How many hours was the trailer hired?

Q17. A student is asked to find the perimeter of a rectangle 9 m long and 4 m wide. They write $2 \times 9 + 4 = 22$ and give the answer as 22 m.

- Work out the student's expression. Which sides of the rectangle does it add, and which side does it miss?
- Explain what is missing from the student's formula.
- Work out the perimeter of the rectangle correctly.

Q18. A rectangular hall floor is 12.5 m long and 8.4 m wide. For a 4.5-hour job, a floor polisher can be hired at \$ 34 per hour, or a cleaning service can be booked for a \$ 50 call-out fee plus \$ 45 per hour.

- Find the area of the hall floor.
- Find the cost of hiring the polisher for 4.5 hours.
- Find the cost of the cleaning service for 4.5 hours, and say which of the two costs less.

Answers

Q1. (a) 28 m^2 . (b) 22.5 m^2 . (c) 43.2 m^2 .

Q2. (a) 32 m. (b) 25 m. (c) 45 m.

Q3. (a) \$200. (b) \$145.50. (c) \$90.

Q4. (a) $r = 24$. (b) $C = h \times 24$. (c) \$120.

Q5. (a) \$47.50. (b) \$332.50. (c) $190 + 3 \times 47.50 = 190 + 142.50 = 332.50$ ✓ same as (b).

Q6. (a) 16.8 m^2 . (b) 16.6 m. (c) the area, 16.8 m^2 — square units mark an area.

Q7. (a) 34.44 m^2 (to 2 d.p.). (b) 24 m. (c) \$144.

Q8. (a) \$115. (b) 150 min. (c) \$57.50.

Q9. (a) \$35.71 (to the nearest cent). (b) \$71.43 (to the nearest cent). (c) $2 \times 35.71 = 71.42$ uses the rate after it was rounded; the exact rate gives 71.43 — round only at the end.

Q10. (a) $6.4 + 2.9 = 9.3$, then $2 \times 9.3 = 18.6 \text{ m}$, shown. (b) 18.56 m^2 . (c) \$213.90.

Q11. (a) 19.4 m. (b) 23.1 m^2 . (c) 18.2 m.

Q12. (a) $C = 85 + h \times 60$. (b) \$175. (c) \$250.

Q13. (a) \$180. (b) \$200. (c) Option A, by \$20.

Q14. (a) \$148.75. (b) \$193.38 (to the nearest cent). (c) 8 hours.

Q15. (a) 87.8 m. (b) 84.8 m. (c) \$742.

Q16. (a) \$36; $72 \div 2 = 36$ and $108 \div 3 = 36$ ✓. (b) \$180. (c) 7 hours.

Q17. (a) 22; it adds both lengths and one width ($9 + 9 + 4$), missing the second width. (b) brackets — $2 \times (9 + 4)$ doubles the length and the width. (c) 26 m.

Q18. (a) 105 m^2 . (b) \$153. (c) \$252.50; the polisher costs less.

Fully-worked solutions

Q1. Area covers the surface: $A = l \times w$.

(a) $A = 7 \times 4 = 28 \text{ m}^2$.

(b) $A = 9 \times 2.5 = 22.5 \text{ m}^2$.

(c) $A = 12 \times 3.6 = 43.2 \text{ m}^2$.

Q2. Perimeter goes around the outside: $P = 2 \times (l + w)$.

(a) $P = 2 \times (10 + 6) = 2 \times 16 = 32 \text{ m}$.

(b) $P = 2 \times (8.5 + 4) = 2 \times 12.5 = 25 \text{ m}$.

(c) $P = 2 \times (15 + 7.5) = 2 \times 22.5 = 45 \text{ m}$.

Q3. Total cost with cost per hour: $C = h \times r$.

(a) $C = 5 \times 40 = \$200$.

(b) $C = 3 \times 48.50 = \$145.50$.

(c) $C = 2.5 \times 36 = \$90$.

Q4. (a) The cost for 1 hour is $r = 24$.

(b) $C = h \times 24$.

(c) $C = 5 \times 24 = \$120$.

Q5. (a) Cost for 1 hour = total $\div$ hours: $190 \div 4 = 47.5$, so \$47.50.

(b) $C = h \times r = 7 \times 47.50 = \$332.50$.

(c) 4 hours cost \$190 and 3 more hours cost $3 \times 47.50 = 142.50$; $190 + 142.50 = 332.50$ ✓ — the same as (b).

Q6. (a) $A = l \times w = 4.8 \times 3.5 = 16.8 \text{ m}^2$.

(b) $P = 2 \times (l + w) = 2 \times (4.8 + 3.5) = 2 \times 8.3 = 16.6 \text{ m}$.

(c) The area, 16.8 m^2 . The ^{2} (square units) is the sign of an area; the perimeter stays in plain length units.

Q7. (a) $A = l \times w = 7.25 \times 4.75 = 34.4375 \text{ m}^2$; to 2 decimal places, 34.44 m^2 (the third decimal is 7, so round up).

(b) Tape goes around the edge: $P = 2 \times (7.25 + 4.75) = 2 \times 12 = 24 \text{ m}$.

(c) $24 \times \$6 = \$144$.

Q8. (a) $C = h \times r = 2.5 \times 46 = \$115$.

(b) $2.5 \times 60 = 150$, so 150 min.

(c) $C = 1.25 \times 46 = \$57.50$.

Q9. (a) Cost for 1 hour = total $\div$ hours: $250 \div 7$ does not come out exactly; to the nearest cent it is \$35.71 (the third decimal is 4, less than 5, so round down).

(b) 2 hours at the exact rate is $2 \times 250 \div 7$; keeping full precision and rounding only at the end gives \$71.43 to the nearest cent.

(c) $2 \times 35.71 = 71.42$ multiplies a rate that was already rounded down, so the answer is one cent light. The exact rate gives 71.43. Round once, at the end, at the stated precision.

Q10. (a) One step per line:

$$6.4 + 2.9 = 9.3$$

$$2 \times 9.3 = 18.6$$

$\therefore$ the perimeter is 18.6 m.

(b) $A = l \times w = 6.4 \times 2.9 = 18.56 \text{ m}^2$.

(c) $18.6 \times \$11.50 = \$213.90$.

Q11. (a) $P = 2 \times (l + w) = 2 \times (5.5 + 4.2) = 2 \times 9.7 = 19.4 \text{ m}$.

(b) $A = l \times w = 5.5 \times 4.2 = 23.1 \text{ m}^2$.

(c) The doorway replaces 1.2 m of skirting: $19.4 - 1.2 = 18.2 \text{ m}$.

Q12. (a) $C = 85 + h \times 60$.

(b) $C = 85 + 1.5 \times 60 = 85 + 90 = 175$, so \$175.

(c) $C = 85 + 2.75 \times 60 = 85 + 165 = 250$, so \$250.

Q13. (a) Option A: $C = 40 + 4 \times 35 = 40 + 140 = 180$, so \$ 180.

(b) Option B: $C = 4 \times 50 = 200$, so \$ 200.

(c) $180 < 200$: Option A is cheaper, by $200 - 180 = \$20$.

Q14. (a) $C = h \times r = 5 \times 29.75 = \$148.75$.

(b) $6.5 \times 29.75 = 193.375$; to the nearest cent, \$ 193.38 (the third decimal is 5, so round up).

(c) Hours = total $\div$ rate: $238 \div 29.75 = 8$, so Ollie worked 8 hours.

Q15. (a) $P = 2 \times (l + w) = 2 \times (28.5 + 15.4) = 2 \times 43.9 = 87.8$ m.

(b) The gate replaces 3 m of mesh: $87.8 - 3 = 84.8$ m.

(c) $84.8 \times \$8.75 = \$742$.

Q16. (a) Row 1: $36 \div 1 = 36$. Check row 2: $72 \div 2 = 36$ ✓ and row 3: $108 \div 3 = 36$ ✓. So $r = \$36$ per hour.

(b) $C = 5 \times 36 = \$180$.

(c) $252 \div 36 = 7$, so the trailer was hired for 7 hours.

Q17. (a) $2 \times 9 + 4 = 22$. It adds the two lengths and one width ($9 + 9 + 4$) — the second width is missing.

(b) The brackets. $P = 2 \times (l + w)$ doubles the length and the width; without brackets only the length is doubled.

(c) $P = 2 \times (9 + 4) = 2 \times 13 = 26$ m.

Q18. (a) $A = l \times w = 12.5 \times 8.4 = 105$ m².

(b) Polisher: $C = h \times r = 4.5 \times 34 = \$153$.

(c) Cleaning service: $C = f + h \times r = 50 + 4.5 \times 45 = 50 + 202.5 = 252.5$, so \$ 252.50. $153 < 252.50$, so the polisher costs less.

Chapter 5 Relationships — 5C Conventions for writing simple algebra

About the data in this section. The job cards, price lists and order sheets here are **made up** (not real): pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

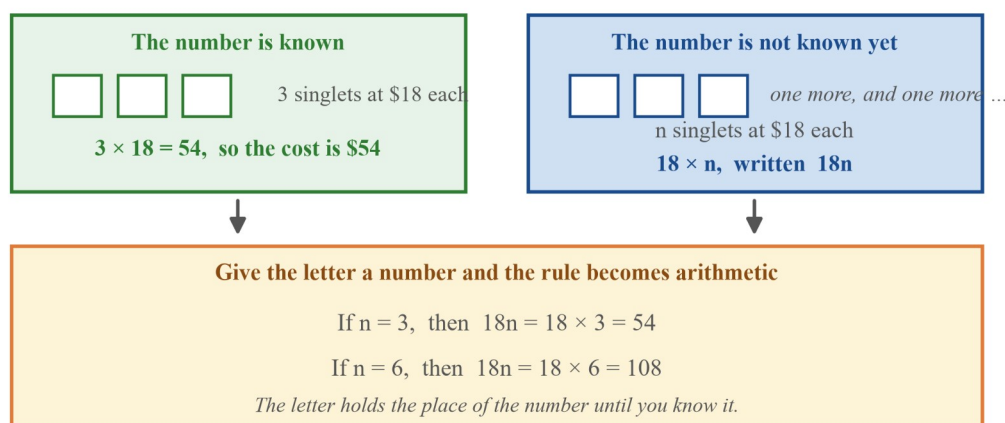
Theory

A letter stands in for a number.

At work, rules about money and materials get written down so the next person can use them. Bright Leaf Printing charges \$18 for each singlet it screen-prints. Three singlets cost $3 \times 18 = \$54$. Six singlets cost $6 \times 18 = \$108$. The rule is always “the number of singlets, times 18” — but an order can be any number of singlets, and before the order is placed that number is not known yet.

So give the unknown number a letter. Let n = the number of singlets ordered. The printing cost is then $18 \times n$ dollars, and the letter n holds the place of the number until the order is known.

A letter stands for a number

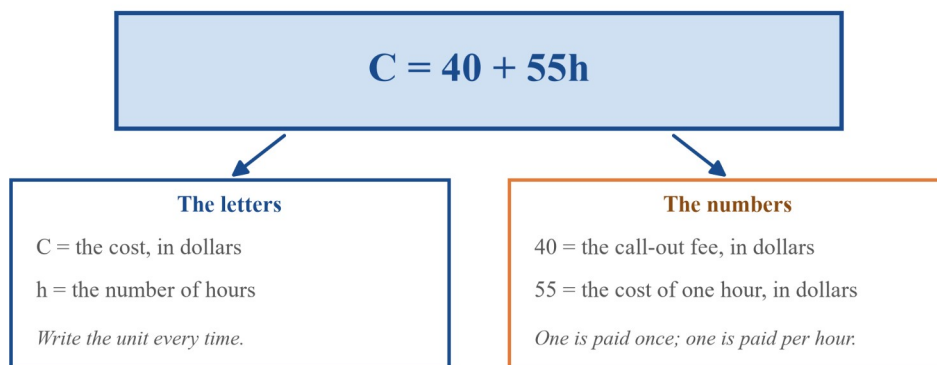


Left card: three singlets at 18 dollars each cost $3 \times 18 = 54$ dollars. Right card: n singlets at 18 dollars each cost $18 \times n$, written $18n$. Bottom card: at $n = 3$ the rule gives 54, at $n = 6$ it gives 108.

A letter that stands in for a number is called a **variable**, because the number it stands for can vary from job to job. Writing rules with letters is called **algebra**, and the agreed ways of writing those rules are called **conventions**. Conventions matter for the same reason road rules matter: everyone reading the rule must get the same meaning from it.

Say what every letter stands for.

A rule on its own is not finished until the reader knows what each letter means and what unit it is measured in. Write this as a **key** under the rule. For example, a plumber's rule $C = 40 + 55h$ needs the key: C = the cost, in dollars; h = the number of hours of work. The numbers need meanings too: 40 is the call-out fee in dollars (paid once) and 55 is the cost of one hour in dollars (paid per hour).



A rule like this is called a formula. The key tells the reader what every letter means.

A formula card reading $C = 40 + 55h$, with two key boxes: the letters (C = the cost in dollars, h = the number of hours) and the numbers (40 = the call-out fee in dollars, 55 = the cost of one hour in dollars).

A rule written with letters, numbers and an = sign is called a **formula**.

Refresher: clue words for the four operations. Words in a job description tell you which operation the rule needs (Section 2A).

Clue words	Operation	Example
each, per, for every	$\times$	4 punnets at \$ 12 each: $4 \times 12 = \$ 48$
plus, altogether, in total	+	\$ 9 and \$ 6 altogether: $9 + 6 = \$ 15$
left, less, cheaper	-	\$ 15 with \$ 4 spent: $15 - 4 = \$ 11$
shared equally, split between	$\div$	\$ 36 between 3 people: $36 \div 3 = \$ 12$

Leaving out the multiplication sign.

Between a number and a letter, the $\times$ sign can be left out: $4 \times n$ is written $4n$, and both mean “4 times whatever n is”. This is the shorthand you will see on job sheets and in pay guides. Between two *numbers* the sign must stay, because $3 \times 4 = 12$ and writing it as 34 would mean thirty-four — a completely different number.

Leaving out the multiplication sign

$4 \times n$	→	$4n$	✓	a number and a letter
$n \times 12$	→	$12n$	✓	the number moves to the front
3×4	→	12	✗	two numbers — the sign stays

$3 \times 4 = 12$. Writing it as 34 would mean thirty-four.

Between a number and a letter the sign can go. Between two numbers it cannot.

Three rows. $4 \times n$ becomes $4n$ (tick): a number and a letter. $n \times 12$ becomes $12n$ (tick): the number moves to the front. 3×4 becomes 12 (cross on dropping the sign): two numbers, the sign stays. Warning card: $3 \times 4 = 12$, writing it as 34 would mean thirty-four.

Number first, and the 1 is left out.

When the sign is left out, the number is written *before* the letter: $5 \times m$ becomes $5m$, never $m5$. Writing $m5$ looks like a label — a machine number or a seat number — not a rule. And because $1 \times k$ is just k , the 1 is left out altogether: write k , not $1k$.

Number first, and the 1 is left out

Write it this way			Not this way	
✓	$5m$	$5 \times m$	✗	$m5$
✓	k	$1 \times k$	✗	$1k$
✓	$12n$	$n \times 12$	✗	$n12$

$m5$ reads like a label — a machine number or a seat number — not a rule.

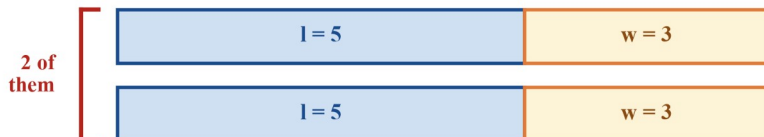
$1 \times k$ and k are the same number, so the 1 is dropped: write k .

Two cards. Write it this way (ticks): $5m$, k , $12n$. Not this way (crosses): $m5$, $1k$, $n12$. Footnote: $m5$ reads like a label, not a rule; $1 \times k$ and k are the same number, so the 1 is dropped.

Brackets group things together.

Brackets say “work this out first, as one lot”. In Section 4B the perimeter of a rectangle was written $P = 2 \times (l + w)$: add the length and the width, then take two of that lot. With the sign left out, the same rule is written $P = 2(l + w)$. The number in front of the brackets multiplies *everything* inside them, so $2(l + w)$ is the same rule as $2l + 2w$.

Brackets group: the number in front multiplies everything inside



one $(l + w)$ is $5 + 3 = 8$

$$2(l + w) = 2 \times (5 + 3) = 2 \times 8 = 16$$

The same rule with the brackets multiplied out:

$$2l + 2w = 2 \times 5 + 2 \times 3 = 16$$

Both give 16 — they are the same rule written two ways.

Two bars, each made of an $l = 5$ piece and a $w = 3$ piece, bracketed as 2 of them. $2(l + w) = 2 \times (5 + 3) = 2 \times 8 = 16$, and multiplied out $2l + 2w = 2 \times 5 + 2 \times 3 = 16$.

Refresher: order of operations. Multiplication is done before addition (Section 2D). In $C = 20 + 5n$ with $n = 7$: first $5 \times 7 = 35$, then $20 + 35 = 55$. The brackets in $2(l + w)$ are what tell you to add first instead: $(20 + 5) \times 7 = 175$ is a different rule, not the same one.

Like terms.

Parts of a rule that count the *same* thing can be joined into one part. $3a + 5a$ is three lots of a plus five lots of a , which is eight lots of a : $3a + 5a = 8a$. Check with $a = 2$: the long way, $3 \times 2 + 5 \times 2 = 16$; the short way, $8 \times 2 = 16$. The same idea gives $n + n = 2n$ (one lot plus one lot is two lots) and $7m - 2m = 5m$.

Parts that count *different* things cannot be joined. $3a + 5b$ stays as $3a + 5b$, because a and b are different quantities. Check with $a = 2$ and $b = 1$: $3 \times 2 + 5 \times 1 = 11$, while joining them wrongly into $8ab$ would give $8 \times 2 \times 1 = 16$ — not the same number, so the join is not allowed. Parts with the same letter are called **like terms**; only like terms can be joined.

Only like terms can be joined

$$\boxed{a} \boxed{a} \boxed{a} + \boxed{a} \boxed{a} \boxed{a} \boxed{a} \boxed{a} = \boxed{a} \boxed{a} \boxed{a} \boxed{a} \boxed{a} \boxed{a} \boxed{a} \boxed{a}$$

$$3a + 5a = 8a \quad (\text{eight of the same thing})$$

$$\text{Check at } a = 2: 3 \times 2 + 5 \times 2 = 16 \quad \text{and} \quad 8 \times 2 = 16$$

$$\boxed{a} \boxed{a} \boxed{a} + \boxed{b} \boxed{b} \boxed{b} \boxed{b} \boxed{b} \neq \boxed{8ab}$$

$$3a + 5b \text{ stays as it is — } a \text{ and } b \text{ count different things}$$

$$\text{Check at } a = 2 \text{ and } b = 1: 3 \times 2 + 5 \times 1 = 11, \quad \text{but} \quad 8ab = 8 \times 2 \times 1 = 16$$

Like terms have the same letter. Join those, and leave the rest alone.

Top row: three tiles labelled a plus five tiles labelled a equal eight tiles labelled a , so $3a + 5a = 8a$, checked at $a = 2$.
Bottom row: three tiles a plus five tiles b cannot be joined, so $3a + 5b$ stays as it is, checked at $a = 2$ and $b = 1$.

Refresher: multiplying a decimal by a whole number. Multiply as if the decimal point were not there, then put it back so the answer has the same number of decimal places as the decimal you started with (Section 2B). So $0.40 \times 30 = 12$, $4.50 \times 12 = 54$ and $2.5 \times 6 = 15$.

Refresher: rounding money to the nearest cent. Money answers are rounded to the nearest cent, which is 2 decimal places (Section 2C). Look at the third decimal: 5 or more rounds up, less than 5 rounds down. A share of \$11.625 has a 5 in the third place, so it rounds up to \$11.63. Always say how you rounded: write “to the nearest cent” next to the answer.

Division has two forms.

Refresher: a fraction bar is a division. The bar in a fraction says divide (Section 2D): $\frac{640}{2}$ means $640 \div 2$. So

division can be written with the $\div$ sign or with a fraction bar, and the two mean the same thing: $n \div 4$ and $\frac{n}{4}$ both say “

n shared into 4 equal parts”. The fraction bar is handy on a job sheet: half the flour in a recipe is $\frac{f}{2}$, where f is the

grams of flour, so with $f = 640$ it is $\frac{640}{2} = 320$ g. A third of the flour is $\frac{f}{3}$, so with $f = 900$ it is $\frac{900}{3} = 300$ g.

Units are not letters.

A measurement keeps a space between the number and its unit: 5 m means five metres. With no space, $5m$ means $5 \times m$ — a rule, not a measurement. The space is the whole difference, so in this book every measurement is written with a space (35 km, 2.5 m, 5 m) and every rule is written with no space ($5m$, $18n$).

A unit and a letter are not the same thing

5 m

five metres — a measurement

1

1

1

1

1

note the space between the 5 and the m

m here means metres, not a number

5m

5 × m — a rule

no space, so the m is a letter

If m = 3, then

5m = 5 × 3 = 15

Same letters, different meanings — the space is what tells them apart.

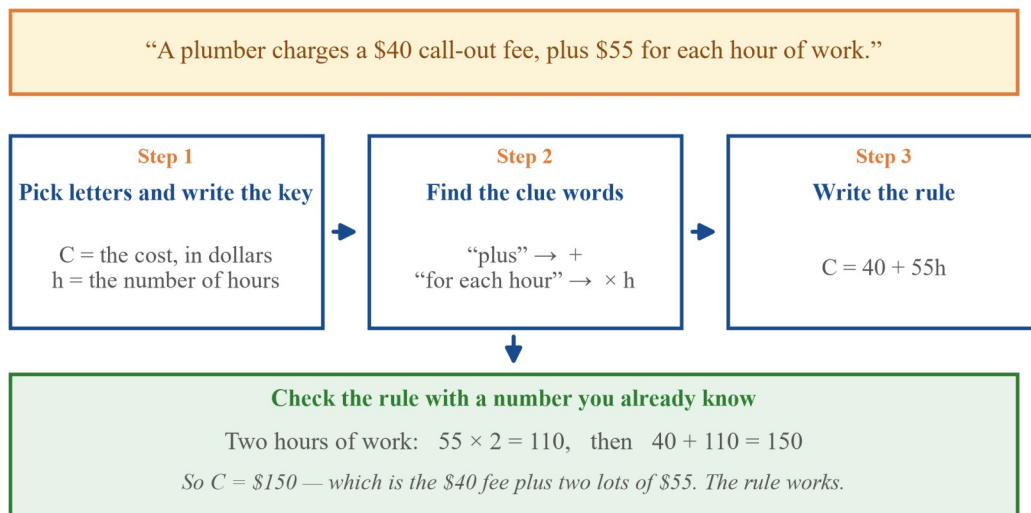
In this book a measurement is always written with a space: 35 km, 2.5 m, 5 m.

Left card: 5 m is five metres, a measurement, drawn as five 1-metre lengths with a space between the 5 and the m.
Right card: 5m is 5 × m, a rule; at m = 3 it gives 5 × 3 = 15.

From words to a formula.

Turning a sentence into a formula is three steps: pick the letters and write the key, find the clue words that give the operations, then write the rule. The plumber's sentence "a \$40 call-out fee, plus \$55 for each hour of work" becomes $C = 40 + 55h$.

From words to a formula



Flow chart. The sentence about the plumber's fee, then Step 1 pick letters and write the key, Step 2 find the clue words (plus gives +, for each hour gives × h), Step 3 write the rule $C = 40 + 55h$, then a check: two hours gives $55 \times 2 = 110$ and $40 + 110 = 150$, so $C = 150$ dollars.

Refresher: what the = sign says. The = sign says the two sides are the same number (Section 1D). That gives you a free check on any formula: put in a number you already know and see if the rule agrees. For $C = 40 + 55h$ with $h = 2$: the right side is $55 \times 2 = 110$, then $40 + 110 = 150$, and indeed a \$40 fee plus two hours at \$55 is \$150. The rule passes the check.

On paper and on a screen.

The same rule can be written two ways depending on where it lives. On paper, $T = 4.50n$. In a spreadsheet the keyboard has no × or ÷ key, so the screen uses * for × and / for ÷: the same rule is typed =4.50*A2, and with 12 items at \$4.50 each the cell =A2*B2 shows \$54.00. Reading a screen rule is just translating: * back to ×, / back to ÷.

On paper and on a screen

On paper

$T = 4.50n$

$T = 4.50 \times n$

n = the number of items
 T = the total, in dollars

same rule

In a spreadsheet

	A	B	C
1	n	price	total
2	12	4.50	=A2*B2
			54.00

the * is the multiplication sign

A screen cannot read $\times$ or $\div$, so it uses * for $\times$ and / for $\div$

With 12 items at \$4.50 each, =A2*B2 gives \$54.00. And =4*A2 gives 36 when A2 holds 9.

Left card, on paper: $T = 4.50n$, that is $T = 4.50 \times n$. Right card, in a spreadsheet: a small grid with A2 = 12, B2 = 4.50 and C2 = =A2B2 giving 54.00. Footnote: a screen cannot read $\times$ or $\div$, so it uses * for $\times$ and / for $\div$; =4*A2 gives 36 when A2 holds 9.

Key ideas.

- A letter standing in for a number is a variable; a rule with letters, numbers and an = sign is a formula.
- Write a key: say what each letter stands for, and give its unit.
- Between a number and a letter the $\times$ can be left out ($4n$); between two numbers it stays ($3 \times 4 = 12$, not 34).
- The number goes first ($5m$, not $m5$) and the 1 is left out (k , not $1k$).
- Brackets group: the number in front multiplies everything inside, so $2(l + w) = 2l + 2w$.
- Only like terms join: $3a + 5a = 8a$, while $3a + 5b$ stays as it is.
- Division is $n \div 4$ or $\frac{n}{4}$; a measurement keeps its space (5 m), a rule does not ($5m$).
- Check any formula by putting in a number you already know.

CONVENTIONS CARD

1 A letter stands for a number. 2 Say what each letter stands for, and give its unit. 3 Leave the $\times$ out between a number and a letter: $4n$. 4 Keep the $\times$ between two numbers: $3 \times 4 = 12$, not 34. 5 Write the number first: $5m$, not $m5$. 6 Leave out the 1: k, not $1k$.	7 Brackets group: $2(l + w) = 2l + 2w$. 8 Join like terms only: $3a + 5a = 8a$; $3a + 5b$ stays. 9 Division two ways: $n \div 4$ or $n/4$. 10 A formula has an = sign: $C = 40 + 55h$. 11 Keep a space before a unit: 5 m is five metres. 12 Check a rule by trying a number you already know.
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These are the conventions — the agreed ways of writing algebra so that everyone reads a rule the same way.

A conventions card listing all twelve writing conventions of this section in two columns.

Worked examples

Example 1 — scaffolded

Jodi takes orders at Bright Leaf Printing. The job card says: screen set-up \$ 45 (once only), and printing at \$ 18 per singlet.

- Write the cost rule with the multiplication sign shown.
- Write the same rule in shorthand.
- Find the cost of an order for 6 singlets, using each writing of the rule.

BRIGHT LEAF PRINTING — JOB CARD	
Screen set-up (once only)	\$45
Printing, per singlet	\$18
Number of singlets ordered	n
$C = 45 + 18 \times n$ which is written $C = 45 + 18n$	
For $n = 6$: $18 \times 6 = 108$, then $45 + 108 = 153$ $C = \$153$	

Bright Leaf Printing job card: screen set-up 45 dollars once only, printing 18 dollars per singlet, number of singlets n ; the rule $C = 45 + 18 \times n$, written $C = 45 + 18n$, and at $n = 6$ the cost is 153 dollars.

Working	Comment
$C = 45 + 18 \times n$	the set-up once, plus \$ 18 for each of the n singlets
$C = 45 + 18n$	shorthand: number first, $\times$ left out
$18 \times 6 = 108$	multiplication before addition (Section 2D)
$45 + 108 = 153$	then add the one-off set-up
$C = \$153$ from both writings	the two writings are one rule, so they agree

Example 2 — scaffolded

Sione reads an order sheet at Pinewood Carpentry. The sheet lists the timber and fixings as $7m + 3m + 4s$, where m = metres of pine and s = boxes of screws.

- Explain why $7m$ and $3m$ can be joined into one part but $4s$ cannot.
- Write the order sheet as simply as it can be written.
- A trainee copies the sheet as $m7 + m3 + s4$. Which conventions has the trainee broken?

Working	Comment
$7m$ and $3m$ both count metres of pine	like terms — the same letter, so the same thing
$4s$ counts boxes of screws, a different thing	unlike terms stay as they are
$7m + 3m = 10m$, so the sheet is $10m + 4s$	$7 + 3 = 10$ lots of m
$m7, m3, s4$ put the number last	the number goes first: $7m, 3m, 4s$

Example 3 — unscaffolded

Tessa hires out a mower trailer at Bayview Mowers. The hire card says: trailer fee \$ 60, plus \$ 35 per hour of use.

- Write a formula for the cost, with a key for each letter.
- Find the cost of 3 hours of use.

(c) Say what the 60 and the 35 in your formula stand for.

The cost is the one-off trailer fee plus \$35 for each hour, so with C = the cost in dollars and h = the number of hours:

$$C = 60 + 35 \times h = 60 + 35h$$

For 3 hours, multiply before adding:

$$35 \times 3 = 105$$

$$60 + 105 = 165$$

$\therefore$ the cost of 3 hours of use is $C = \$165$.

The 60 is the trailer fee in dollars, paid once no matter how long the trailer is out. The 35 is the cost of one hour in dollars, paid once per hour — it is the number that gets multiplied by h .

Example 4 — unscaffolded

Ollie is training at Ridgeline Roofing and writes up a job card. His card reads:

- cost of the job: $C = 60 + n \times 25$
- sheets of metal: $S = 1s$
- tins of sealant: $T = m8$
- note: “5m of gutter still to fit”

where n = hours of work, s = sheets of metal and m = tins of sealant.

- Rewrite the three formula lines so they follow the conventions.
- Show that the corrected cost line gives \$160 for 4 hours of work, using both writings of the rule.
- What does Ollie’s note “5m of gutter” actually say, and how should five metres of gutter be written?

Fixing up a job card

The trainee wrote		The card should read
$C = 60 + n \times 25$	✗	$C = 60 + 25n$ ✓
$S = 1s$	✗	$S = s$ ✓
$T = m8$	✗	$T = 8m$ ✓
<i>each line breaks a convention</i>		
<i>number first, and no 1</i>		

Check the corrected rule at $n = 4$: $60 + 4 \times 25 = 160$ and $60 + 25 \times 4 = 160$

The note “5m of gutter” means $5 \times m$ — for five metres, write 5 m with a space.

Two job cards side by side. The trainee wrote $C = 60 + n \times 25$, $S = 1s$ and $T = m8$ (each crossed); the corrected card reads $C = 60 + 25n$, $S = s$ and $T = 8m$ (each ticked). At $n = 4$ both writings give 160 dollars.

- Number first, and the 1 left out: $C = 60 + 25n$, $S = s$ and $T = 8m$.
- One step per line, each way round.

With the sign shown:

$$4 \times 25 = 100$$

$$60 + 100 = 160$$

In shorthand:

$$25 \times 4 = 100$$

$$60 + 100 = 160$$

$\therefore$ both writings give $C = \$160$ for 4 hours of work.

- (c) With no space, $5m$ reads as $5 \times m$ — five times the tins of sealant, which is not what Ollie meant. Five metres of gutter is a measurement, so it is written with the space: 5 m.

Practice questions

Scaffolded

Q1. Write each of these in shorthand, with the multiplication sign left out.

- (a) $7 \times p$
- (b) $n \times 12$
- (c) $1 \times k$

Q2. Put the multiplication sign back into each of these.

- (a) $9c$
- (b) $4m + 15$
- (c) $2(a + b)$

Q3. Simplify the ones that can be simplified, and say why any that cannot, cannot.

- (a) $6x + 2x$
- (b) $6x + 2y$
- (c) $9m - 4m$

Q4. The Copper Kettle Café writes its muffin rule as $C = 4n$, where C = the cost in dollars and n = the number of muffins.

- (a) Say what the letter n stands for.
- (b) Say what the number 4 stands for.
- (c) Find the cost of 7 muffins.

Q5. Section 4B wrote the perimeter of a rectangle as $P = 2 \times (l + w)$. A garden bed is $l = 6$ m long and $w = 2.5$ m wide.

- (a) Write the same rule in shorthand.
- (b) Find the perimeter using $2(l + w)$.
- (c) Find the perimeter again using $2l + 2w$, and check the two agree.

Q6. A job sheet shows 5 m in one place and $5m$ in another.

- (a) Which one is a measurement and which one is a rule?
- (b) If $m = 3$, what number does $5m$ stand for?
- (c) Explain why the space matters.

Unscaffolded

Q7. Fernhill Nursery delivers punnets of seedlings: delivery \$25, plus \$12 per punnet. The nursery's rule is $C = 25 + 12p$, where p = the number of punnets.

- (a) Write the rule with the multiplication sign shown.
- (b) Find the cost of 8 punnets.
- (c) Say what the 12 stands for.

Q8. A phone plan card reads: \$25 each month, plus \$0.40 for each minute of calls. The plan's rule is $C = 25 + 0.40t$

TALK-EASY MOBILE PLAN
\$25 each month, plus \$0.40 for each minute of calls
t = the number of minutes of calls C = the cost for the month, in dollars
$C = 25 + 0.40t$
written the other way round: $C = 25 + t \times 0.40$
For $t = 30$: $0.40 \times 30 = 12$, then $25 + 12 = 37$ $C = \\$37.00$ (to the nearest cent)

Talk-easy mobile plan card: 25 dollars each month plus 0.40 dollars for each minute of calls; t = minutes, C = cost for the month in dollars, rule $C = 25 + 0.40t$, and at $t = 30$ the cost is 37.00 dollars to the nearest cent.

- (a) Say what t and C stand for, with units.
- (b) Find the cost of a month with 30 minutes of calls, to the nearest cent.
- (c) A friend writes the rule as $C = 25 + t \times 0.40$. Is that the same rule? Explain.

Q9. Westgate Tool Hire's order sheet lists a digger for $4d + 7d$ days and a walker for $3w$ days, where d and w count days of each machine.

- (a) Write the sheet as simply as it can be written.
- (b) Explain why the d parts and the w part cannot be joined into one part.
- (c) For $d = 2$ and $w = 3$, find the total number of hire days on the sheet.

Q10. Marco builds a rectangular garden bed 4.5 m long and 1.8 m wide.

- (a) Write the area rule $A = l \times w$ in shorthand.
- (b) Find the area of the garden bed.
- (c) Write the perimeter rule in shorthand and find the perimeter.

Q11. Four flatmates share a power bill of P dollars equally. The share per person is $S = P \div n$, where n = the number of people.

- (a) Write the same rule with a fraction bar.
- (b) For a bill of \$46.50 shared by 4 people, find each share to the nearest cent.
- (c) Explain why $n \div P$ would not give a sensible share.

Q12. A cake box order form uses the rule $C = 3o + 50$, where o = the number of orders and C = the cost in dollars.

- (a) Why is o a risky letter to choose? (Think about how it looks in handwriting.)
- (b) Rewrite the rule using the letter n instead.
- (c) Find the cost of 12 orders.

Q13. A builder's card says: "4 bags of cement at \$9 each: 49 dollars".

- (a) Explain the mistake on the card.
- (b) Write the correct working for the cost of the cement.
- (c) Write a rule for the cost of b bags at \$9 each, in shorthand.

Q14. Rosa prices tool hire in a spreadsheet. Column A holds the number of items, column B holds the price per item in dollars, and column C must show the total. On paper her rule is $T = 4.50n$.

- (a) Write what T and n stand for, with units.
- (b) What should Rosa type into cell C2?
- (c) For 12 items at \$4.50 each, what total does the spreadsheet show?

Q15. A school camp costs \$180 for the bus plus \$22 per person. The camp's rule is $C = 180 + 22n$, where n = the number of people.

- (a) Say what the 180 and the 22 stand for.
- (b) Find the cost for 15 people.
- (c) The year group has \$700 to spend. Can 22 people go, and how much of the \$700 would be left over?

Q16. Sunrise Fitness prints posters: set-up \$95, and \$6 to print each poster.

- (a) Write the cost rule with the multiplication sign shown.
- (b) Write the same rule in shorthand.
- (c) Say in words what your rule means.

Q17. A bakery recipe uses f grams of flour. The baker keeps half the flour for the dough and divides the rest.

- (a) Write "half the flour" two ways: once with $\div$ and once with a fraction bar.
- (b) For $f = 750$, find half the flour in grams.
- (c) The sugar is $S = \frac{f}{3}$ grams. For $f = 900$, find the sugar in grams.

Q18. A market stall writes its takings rule as $C = 8n + 5$, where n = the number of baskets sold, and a note on the same card says "5m of cloth for the stall table".

- (a) Put the multiplication sign back into the rule.
- (b) Find C when 6 baskets are sold.
- (c) What does 5 m on the note mean as written, and how should five metres of cloth be written?

Answers

Q1. (a) $7p$. (b) $12n$. (c) k .

Q2. (a) $9 \times c$. (b) $4 \times m + 15$. (c) $2 \times (a + b)$.

Q3. (a) $8x$. (b) cannot — x and y count different things. (c) $5m$.

Q4. (a) the number of muffins. (b) the price of one muffin, in dollars. (c) \$28.

Q5. (a) $P = 2(l + w)$. (b) 17 m. (c) $12 + 5 = 17$ m ✓ same both ways.

Q6. (a) 5 m is the measurement; $5m$ is the rule. (b) 15. (c) the space says “metres”; no space says “times m ”.

Q7. (a) $C = 25 + 12 \times p$. (b) \$121. (c) the cost of one punnet, in dollars.

Q8. (a) t = minutes of calls; C = cost for the month, in dollars. (b) \$37.00 to the nearest cent. (c) yes — same rule, the $\times$ is just written after the letter.

Q9. (a) $11d + 3w$. (b) d and w count different machines, so they are unlike terms. (c) 31 days.

Q10. (a) $A = lw$. (b) 8.1 m^2 . (c) $P = 2(l + w) = 12.6$ m.

Q11. (a) $S = \frac{P}{n}$. (b) \$11.63 to the nearest cent. (c) it would share the number of people between the dollars — a share must be dollars per person.

Q12. (a) o looks like the digit zero in handwriting and on screens. (b) $C = 3n + 50$. (c) \$86.

Q13. (a) 49 reads as forty-nine; the $\times$ between two numbers was dropped. (b) $4 \times 9 = \$36$. (c) $C = 9b$.

Q14. (a) T = the total, in dollars; n = the number of items. (b) $= A2 \times B2$. (c) \$54.00.

Q15. (a) 180 = the bus cost in dollars (once); 22 = the cost per person in dollars. (b) \$510. (c) yes — \$664, leaving \$36 of the \$700.

Q16. (a) $C = 95 + 6 \times p$. (b) $C = 95 + 6p$. (c) the set-up is paid once and \$6 is paid for every poster.

Q17. (a) $f \div 2$ or $\frac{f}{2}$. (b) 375 g. (c) 300 g.

Q18. (a) $C = 8 \times n + 5$. (b) \$53. (c) as written it means $5 \times m$; five metres is written 5 m.

Fully-worked solutions

Q1. Leave the $\times$ out between a number and a letter; the number goes first; a 1 is dropped.

(a) $7 \times p = 7p$.

(b) $n \times 12 = 12n$ (number first).

(c) $1 \times k = k$ (the 1 is left out).

Q2. Put the $\times$ back where it was left out.

(a) $9c = 9 \times c$.

(b) $4m + 15 = 4 \times m + 15$.

(c) $2(a + b) = 2 \times (a + b)$ — the 2 multiplies everything in the brackets.

Q3. Only like terms join.

- (a) $6x + 2x = 8x$ (six lots of x plus two lots of x).
- (b) $6x + 2y$ cannot be simplified: x and y count different things, so these are unlike terms.
- (c) $9m - 4m = 5m$ (nine lots take away four lots leaves five lots).

Q4. Read the rule against its key.

- (a) n = the number of muffins bought.
- (b) 4 = the price of one muffin, in dollars — the number that gets multiplied by n .
- (c) $C = 4 \times 7 = \$28$.

Q5. Both writings are the same rule.

- (a) $P = 2(l + w)$.
- (b) $l + w = 6 + 2.5 = 8.5$, then $P = 2 \times 8.5 = 17$ m.
- (c) $2l + 2w = 2 \times 6 + 2 \times 2.5 = 12 + 5 = 17$ m ✓ the two writings agree.

Q6. The space is the whole difference.

- (a) 5 m (with the space) is a measurement of five metres; $5m$ (no space) is the rule $5 \times m$.
- (b) $5m = 5 \times 3 = 15$.
- (c) With the space, the m is the unit “metres”. Without the space, the m is a letter standing for a number, so $5m$ means five times that number.

Q7. The rule is a one-off fee plus a per-punnet cost.

- (a) $C = 25 + 12 \times p$.
- (b) $12 \times 8 = 96$, then $25 + 96 = 121$, so $C = \$121$.
- (c) 12 = the cost of one punnet, in dollars.

Q8. Read the card, then substitute.

- (a) t = the number of minutes of calls; C = the cost for the month, in dollars.
- (b) $0.40 \times 30 = 12$, then $25 + 12 = 37$, so $C = \$37.00$ to the nearest cent (the working came out exactly at 37, so the cents are 00).
- (c) Yes. $25 + t \times 0.40$ multiplies the same two numbers, so it is the same rule; the convention simply prefers the number first, $0.40t$.

Q9. Join like terms only.

- (a) $4d + 7d = 11d$, so the sheet is $11d + 3w$.
- (b) d counts digger days and w counts walker days — different things, so they are unlike terms and stay separate.
- (c) $11d = 11 \times 2 = 22$ and $3w = 3 \times 3 = 9$, so $22 + 9 = 31$ days.

Q10. Area covers, perimeter goes around.

- (a) $A = lw$.
- (b) $A = 4.5 \times 1.8 = 8.1 \text{ m}^2$.
- (c) $P = 2(l + w)$; $l + w = 4.5 + 1.8 = 6.3$, then $P = 2 \times 6.3 = 12.6$ m.

Q11. A share is the bill split between the people.

(a) $S = \frac{P}{n}$.

(b) $\frac{46.50}{4} = 11.625$, which rounds to \$ 11.63 to the nearest cent (the third decimal is 5, so round up).

(c) $n \div P$ would share 4 people between \$ 46.50, giving a tiny number of “people per dollar” — not a money share at all. The bill must be the number that gets divided.

Q12. Choose letters that cannot be misread.

(a) A written *o* is easily mistaken for the digit zero, so a reader can take the letter for a number and misread the whole rule.

(b) $C = 3n + 50$.

(c) $3 \times 12 = 36$, then $36 + 50 = 86$, so $C = \$86$.

Q13. The $\times$ between two numbers must stay.

(a) Dropping the sign turned 4×9 into 49, which reads as forty-nine dollars — \$ 13 too much.

(b) $4 \times 9 = \$36$.

(c) $C = 9b$.

Q14. Translate between paper and screen.

(a) T = the total cost, in dollars; n = the number of items.

(b) =A2*B2 — the screen’s * is the multiplication sign.

(c) $12 \times 4.50 = 54$, so the cell shows \$ 54.00.

Q15. One-off cost plus a per-person cost.

(a) 180 = the bus cost in dollars, paid once; 22 = the cost per person in dollars, paid once per person.

(b) $22 \times 15 = 330$, then $180 + 330 = 510$, so $C = \$510$.

(c) $22 \times 22 = 484$, then $180 + 484 = 664$, so $C = \$664$. That is under \$ 700, so 22 people can go, with $700 - 664 = \$36$ left over.

Q16. Write the rule two ways and say it in words.

(a) $C = 95 + 6 \times p$, where p = the number of posters.

(b) $C = 95 + 6p$.

(c) The set-up of \$ 95 is paid once, and \$ 6 is paid for every poster printed.

Q17. Division, two forms.

(a) $f \div 2$ or $\frac{f}{2}$.

(b) $\frac{750}{2} = 375$ g.

(c) $S = \frac{900}{3} = 300$ g.

Q18. Read the rule and the note separately.

(a) $C = 8 \times n + 5$.

(b) $8 \times 6 = 48$, then $48 + 5 = 53$, so $C = \$53$.

(c) With no space, $5m$ means $5 \times m$ — five times whatever m is. Five metres of cloth is a measurement, so it is written 5 m.

Chapter 5 Relationships — 5D Tables, charts and graphs of change

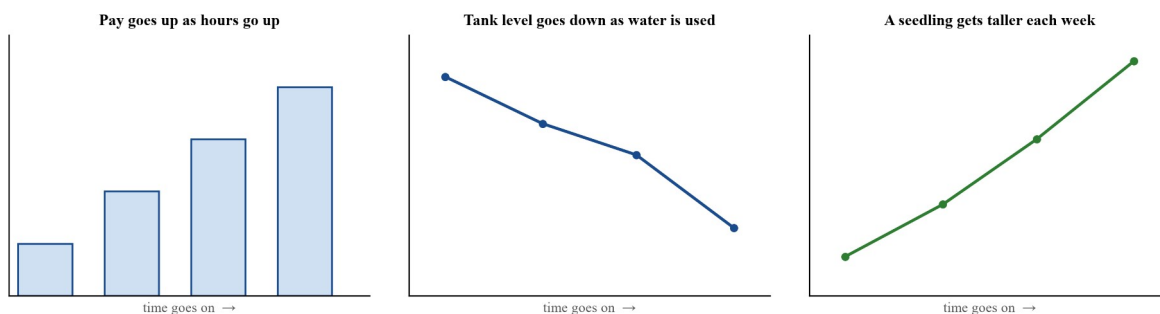
About the data in this section. The tables, charts and graphs here are **made up** (not real): pretend businesses and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Theory

Where this section sits.

Section 5A looked at rates such as \$6.40 per metre and 80 km/hr — how fast one quantity changes against another. This section makes that kind of change something you can **see**: the two quantities are recorded together in a **table**, a **bar chart** or a **line graph**, so a reader can watch the change happen without doing any sums first.

Change happens in pairs.



Almost every change pulls a second change along with it. Work more hours and your pay goes up. Use water from a tank and the tank level goes down. Weeks pass and a seedling gets taller. To show a change like these, you record the two quantities **together** — hours *and* pay, days *and* water level, weeks *and* height — and set the records out in one of three ways: a table, a bar chart or a line graph. Each of the three is a **representation** of the change (Section 1D used this word for any way of setting out results).

Refresher. Multiplying money by a whole number. Tables of change are filled in by multiplying the same amount again and again, so this multiplication needs to be quick and tidy. Multiply the dollars-and-cents number as it stands, and keep **two decimal places** in the answer:

- $3 \times \$8.50 = \$25.50$;
- $7 \times \$4.35 = \$30.45$;
- $4 \times \$6.40 = \$25.60$.

If a product comes out with only one decimal place, add the zero: $4 \times \$6.40$ gives \$25.6, but money is written \$25.60 so the cents place is always visible.

Tables of change.

A **table of change** lines the two quantities up so that each pair sits together. The next figure takes apart one example: a wages table for a job paying \$25 per hour.

title — says what the table is about



Pay at \$25 per hour

Hours worked (h)	Pay (dollars)
0	\$0
1	\$25
2	\$50
3	\$75
4	\$100

column headings — name each quantity and give its units

one row is one pair:
2 h of work goes with \$50 of pay

As the hours go up, the pay goes up — the table shows the change.

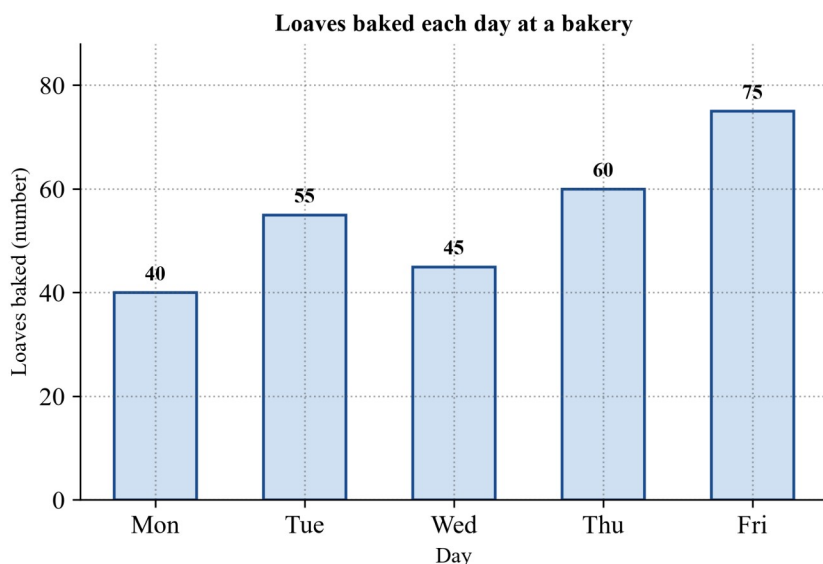
A good table carries a **title**, **column headings with units**, and one **pair** per row: 2 hours of work goes with \$50 of pay. As the hours go up, the pay goes up with them — the table shows the change.

Read the change between two rows by subtracting. From 1 hour to 2 hours: $\$50 - \$25 = \$25$. The same \$25 rise happens at every step of this table, because each extra hour adds one more lot of \$25. A change that rises (or falls) by the **same amount every step** is called a **steady change**.

A table can also be described by a **rule in words**: for the wages table, “start at \$0, then add \$25 for each hour worked”. The rule tells you how to build any row of the table.

Bar charts — comparing amounts.

A **bar chart** draws one bar for each category — here, one bar for each day of the week. A taller bar means a bigger amount.



Refresher. The four checks before you read a graph. Section 1D set these out; they protect every reading in this section. Before you trust a graph, check:

1. the **title** — what is being counted, and over what period;
2. the **axis labels**, including the **units** (number, dollars, litres, days);
3. the **scale** — what each gridline step is worth;
4. the **values** — read from the labelled bars or points, or from the gridlines.

Applying the four checks to the bakery chart: the title says loaves baked each day, the vertical axis counts loaves, and every bar is labelled. Reading the values: the tallest bar is Friday, so Friday baked the most — 75 loaves. Thursday baked $60 - 45 = 15$ more loaves than Wednesday. The week total is $40 + 55 + 45 + 60 + 75 = 275$ loaves.

Line graphs — following change over time.

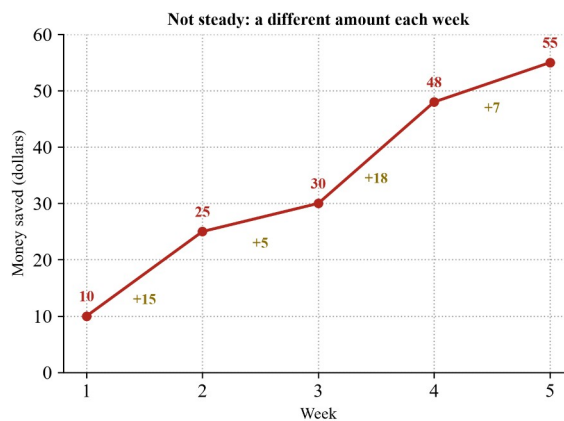
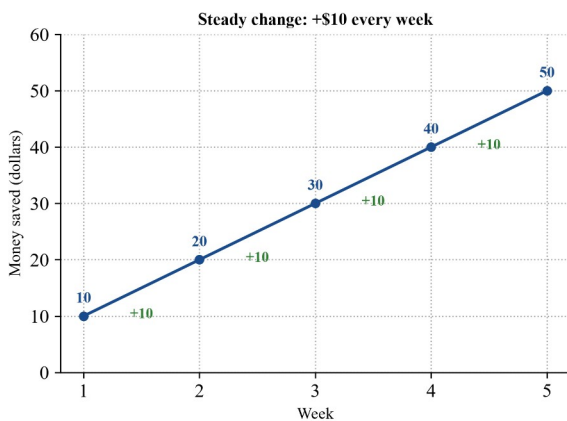
A **line graph** marks one point for each recorded pair and joins the points in order. The shape of the line tells the story of the change:

- the line goes **up** — the quantity is increasing;
- the line is **flat** — no change over that stretch;
- the line goes **down** — the quantity is decreasing.

Worked model — finding a change. A savings account held \$80 in week 2 and \$200 in week 5. To find the change, subtract the two readings: $\$200 - \$80 = \$120$. The later reading is the bigger one, so the savings **rose by** \$120 between week 2 and week 5. When the later reading is the smaller one, the same subtraction tells you how much the quantity **fell by**.

Refresher. Reading the scale on a graph. Points do not always land on a labelled gridline. First find what one step is worth: if gridlines are labelled 0 and 50 with 5 steps between them, one step is $(50 - 0) \div 5 = 10$. Then count steps from the nearest labelled gridline: a point one step above the 40 gridline is at $40 + 10 = 50$.

Steady change and changing change.



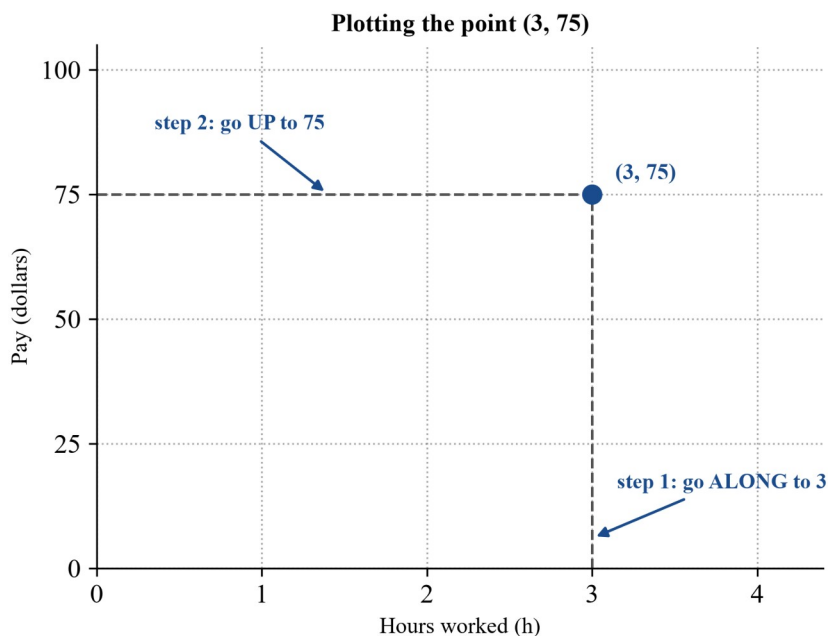
The tin on the left gains \$10 every week: the rises are 10, 10, 10, 10 — all the same. That is a **steady change**, and on a line graph a steady change makes a **straight line**. The tin on the right gains \$15, then \$5, then \$18, then \$7 — a different amount each week. That change is **not steady**, and its line graph is not straight: it climbs hard where the rise is big and eases off where the rise is small.

Drawing a line graph from a table.

Four steps turn a table into a line graph:

1. **label the axes** — one axis for each quantity, with units (Week; Height (cm));
2. **choose a scale** — mark the gridlines in equal, easy steps;
3. **plot the points** — one point for each pair in the table;
4. **join the points** in order.

Refresher. Plotting a point on a grid. A point such as (3, 75) is one pair from a table — here, 3 hours of work at \$25 per hour, which pays $3 \times \$25 = \$75$. Plot it in two moves, **along first, then up**: go along the bottom axis to 3, then go straight up to 75 and mark the point.



Choosing the representation.

You want to show	Best choice
exact values someone may reuse	a table
a comparison between categories, day against day	a bar chart
how a quantity changes over time	a line graph

The same data can be set out in all three ways. Pick the one that answers the question being asked.

Worked examples

Example 1 — scaffolded

A dressmaker buys fabric at \$6.40 per metre. She writes a table of costs for lengths from 1 m to 5 m.

(a) Copy and complete the table.

Length (m)	1	2	3	4	5
Cost (dollars)	?	?	?	?	?

Made-up data: the price and every number in this table are made up for this section.

(b) By how much does the cost rise for each extra metre?

(c) Use the pattern to find the cost of 8 m of fabric.

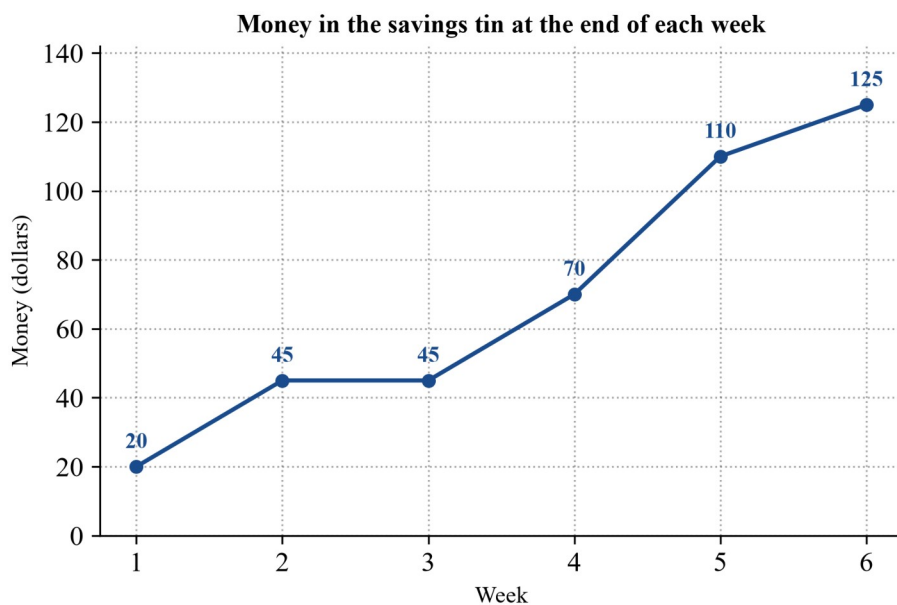
Working	Comment
$1 \times \$6.40 = \$6.40$	every row is: length $\times$ price per metre
$2 \times \$6.40 = \$12.80$	two lots of \$6.40
$3 \times \$6.40 = \$19.20$	three lots of \$6.40
$4 \times \$6.40 = \$25.60$	money keeps two decimal places
$5 \times \$6.40 = \$32.00$	\$32 is written \$32.00 — the cents place stays visible
rises: $\$12.80 - \$6.40 = \$6.40$, and the same at every step	each extra metre adds one more lot of \$6.40 — a steady change
$8 \times \$6.40 = \$51.20$	the pattern keeps going: 8 lots of \$6.40

The completed table:

Length (m)	1	2	3	4	5
Cost (dollars)	\$ 6.40	\$ 12.80	\$ 19.20	\$ 25.60	\$ 32.00

Example 2 — scaffolded

The line graph shows the money in a savings tin at the end of each week.



- (a) How much money was in the tin at the end of week 3?
- (b) Between which two weeks did the amount in the tin not change?
- (c) Between which two weeks did the money rise the most?

(a) the week-3 point is labelled \$ 45

read the value on the labelled point

(b) weeks 2 and 3 — both labelled \$ 45

the line is flat there: no rise and no fall

(c) the rises, week by week: \$ 25, \$ 0, \$ 25, \$ 40, \$ 15

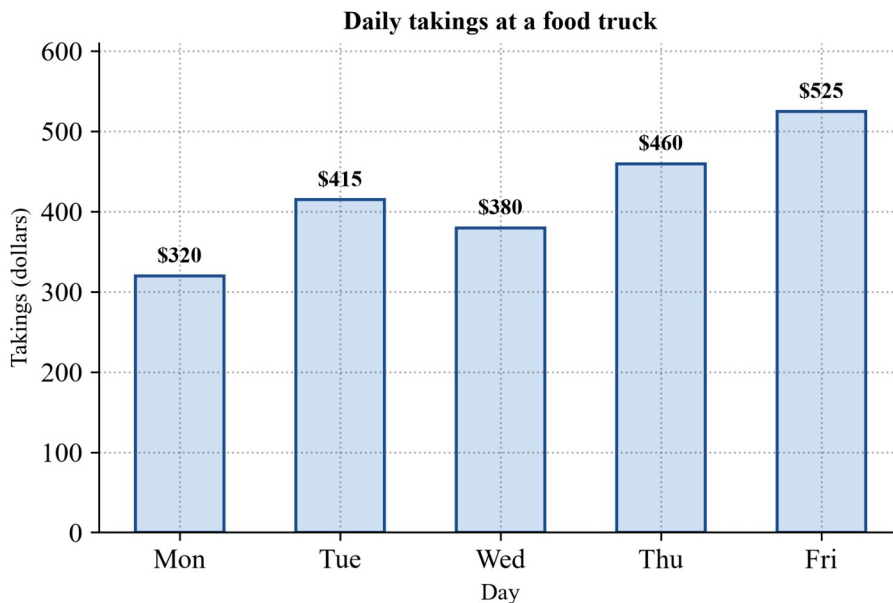
list the rise between each pair of weeks

the biggest rise is \$ 40: weeks 4 to 5

pick the largest number in the list of rises

Example 3 — unscaffolded

The bar chart shows the daily takings at a food truck for one week.



(a) Which day had the biggest takings, and how much were they?

(b) Find the total takings for the five days.

(c) How much more did the truck take on Friday than on Monday?

(d) The tallest bar is Friday's, labelled \$ 525.

∴ Friday had the biggest takings: \$ 525.

(b) Add the five labelled values:

$$\$320 + \$415 + \$380 + \$460 + \$525 = \$2100$$

∴ the total takings for the week were \$ 2 100 (exact — no rounding needed).

(c) Subtract Monday's takings from Friday's:

$$\$525 - \$320 = \$205$$

∴ the truck took \$ 205 more on Friday than on Monday.

Example 4 — unscaffolded

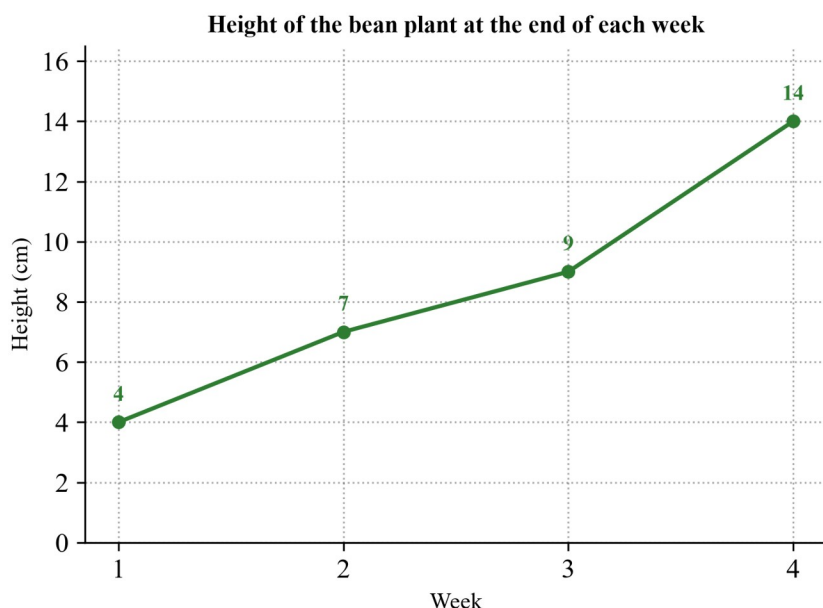
A gardener measures a bean plant at the end of each week for four weeks.

Week	1	2	3	4
Height (cm)	4	7	9	14

Made-up data: the labels and every number in this table are made up for this section.

(a) Draw a line graph of height against week.

- (b) How much did the plant grow in total, from week 1 to week 4?
- (c) Between which two weeks did the plant grow the most?
- (d) Label the axes Week and Height (cm), choose a scale, plot the four points $(1, 4)$, $(2, 7)$, $(3, 9)$ and $(4, 14)$, and join them in order.



- (b) The total growth is the week-4 height minus the week-1 height:

$$14 - 4 = 10$$

$\therefore$ the plant grew 10 cm in total from week 1 to week 4.

- (c) The growth in each week is the rise between neighbouring points: $7 - 4 = 3$ cm, $9 - 7 = 2$ cm, $14 - 9 = 5$ cm.
The largest of these is 5 cm.

$\therefore$ the plant grew the most between weeks 3 and 4.

Practice questions

Scaffolded

Q1. Cinema tickets for a group booking cost \$14 each.

- (a) Copy and complete the table for 2, 3 and 4 tickets.

Tickets (number)	1	2	3	4
Cost (dollars)	14	?	?	?

Made-up data: the labels and every number in this table are made up for this section.

- (b) How much do 5 tickets cost?
- (c) By how much does the cost rise for each extra ticket?

Q2. A cafe job pays \$28 per hour.

(a) Copy and complete the table for 2, 3 and 4 hours.

Hours worked

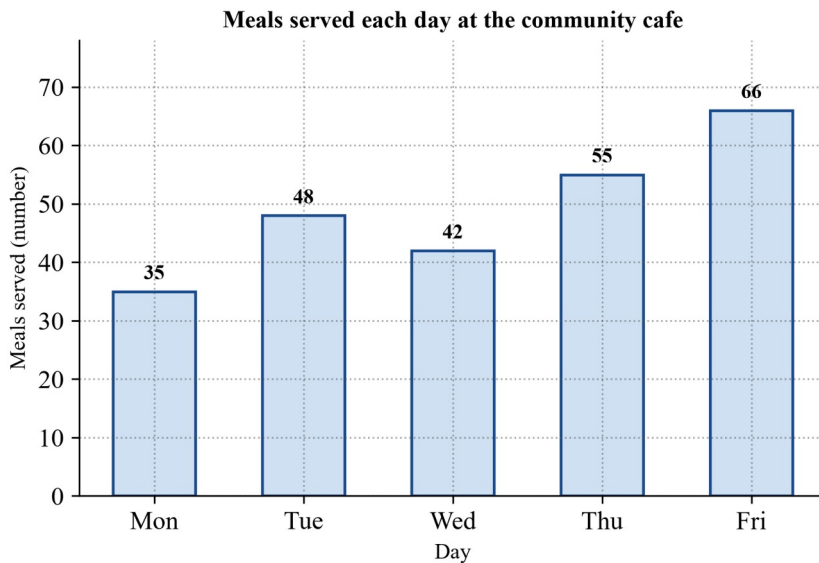
(h)	1	2	3	4
Pay (dollars)	28	?	?	?

Made-up data: the labels and every number in this table are made up for this section.

(b) How much more is 4 hours of work worth than 3 hours?

(c) Is the change in the table steady? Explain your answer.

Q3. The bar chart shows the meals served each day at a community cafe.

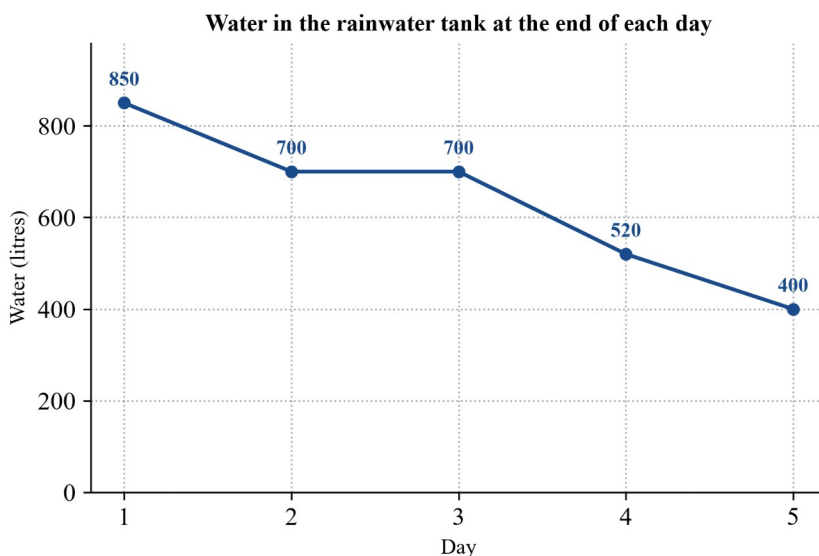


(a) How many meals were served on Wednesday?

(b) Which day was the busiest?

(c) How many more meals were served on Friday than on Monday?

Q4. The line graph shows the water in a rainwater tank at the end of each day of one dry week.



(a) How much water was in the tank at the end of day 2?

(b) Between which two days was the line flat?

(c) By how much did the water fall from day 1 to day 5?

Q5. A hardware store sells shade cloth (cloth used to shade garden beds) at \$4.60 per metre.

- (a) Find the cost of 2 m.
- (b) Find the cost of 5 m.
- (c) Find the cost of 8 m.

Q6. A plant nursery (a place that grows plants for sale) counts the seedlings sold each week.

Week	1	2	3	4	5
Seedlings sold (number)	20	25	30	35	40

Made-up data: the labels and every number in this table are made up for this section.

- (a) How many seedlings were sold in week 3?
- (b) How many more seedlings were sold in week 5 than in week 2?
- (c) Is the change steady? Explain your answer.

Unscaffolded

Q7. A car travels at a steady 80 km/hr.

- (a) How far does the car travel in 3 hours?
- (b) How far does the car travel in 4 hours?
- (c) By how much does the distance rise for each extra hour of travel?

Q8. A tomato plant is measured at the end of each week for four weeks.

Week	1	2	3	4
Height (cm)	10	15	25	30

Made-up data: the labels and every number in this table are made up for this section.

- (a) Draw a line graph of height against week.
- (b) How much did the plant grow in total, from week 1 to week 4?
- (c) Between which two weeks did the plant grow the most?

Q9. The table shows the charge left in a phone battery as it is used.

Time of use (h)	0	1	2	3	4
Charge (%)	100	85	70	40	25

Made-up data: the labels and every number in this table are made up for this section.

- (a) What was the charge after 2 hours of use?
- (b) By how much did the charge fall between hour 2 and hour 3?
- (c) Did the charge fall by the same amount every hour? Explain your answer.

Q10. A sports club ran a sausage sizzle (a fund-raiser where sausages are cooked and sold) on each day of one week. The table shows the daily takings.

Day	Mon	Tue	Wed	Thu	Fri
Takings (dollars)	214.50	196.25	231.75	188.50	265.45

Made-up data: the labels and every number in this table are made up for this section.

- Which day had the biggest takings?
- Find the total takings for the five days.
- Round your answer to part (b) **to the nearest dollar**.

Q11. A printing machine runs at a steady rate. Two numbers are missing from the table.

Time (h)	1	2	3	4	5
Flyers printed (number)	120	240	?	480	?

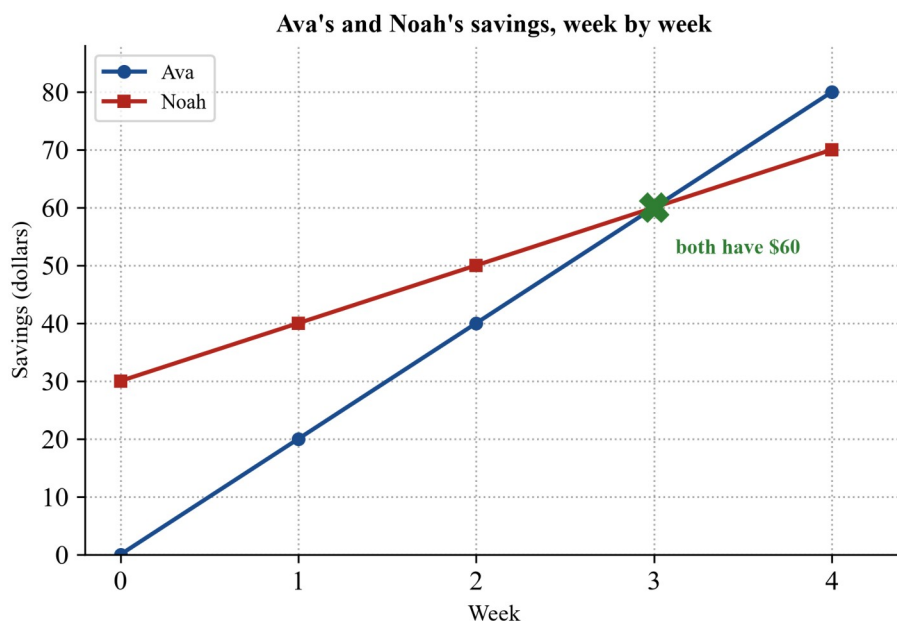
Made-up data: the labels and every number in this table are made up for this section.

- Copy and complete the table — fill the two gaps.
- How many more flyers does the machine finish for each extra hour?
- Does the table show a steady change? Explain your answer.

Q12. A tutor charges \$40 per hour.

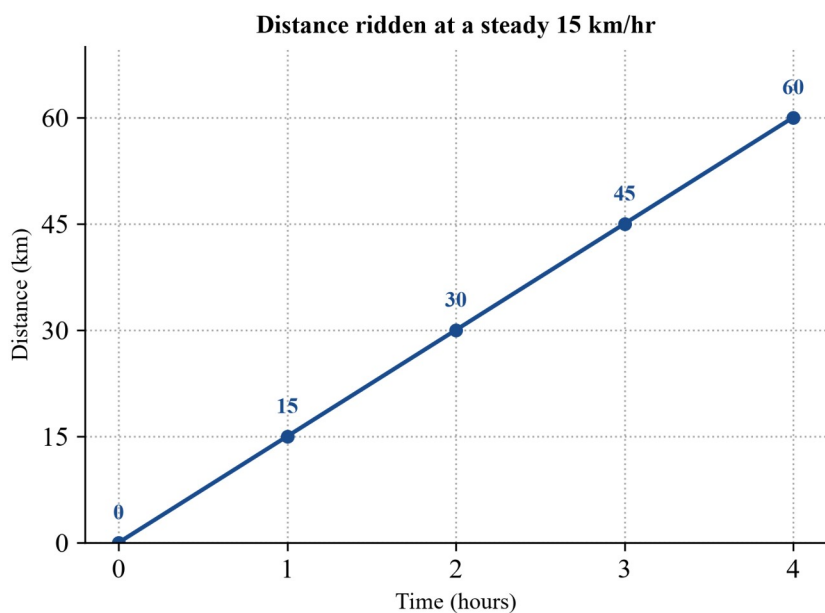
- Copy and complete a table of costs for 1, 2, 3 and 4 hours of tutoring.
- Josh has \$100. How many **whole** hours of tutoring can he afford? Explain your answer.
- How much change would Josh receive from \$100 after 2 hours of tutoring?

Q13. Ava starts with \$0 and saves \$20 each week. Noah starts with \$30 and saves \$10 each week. The line graph shows their savings.



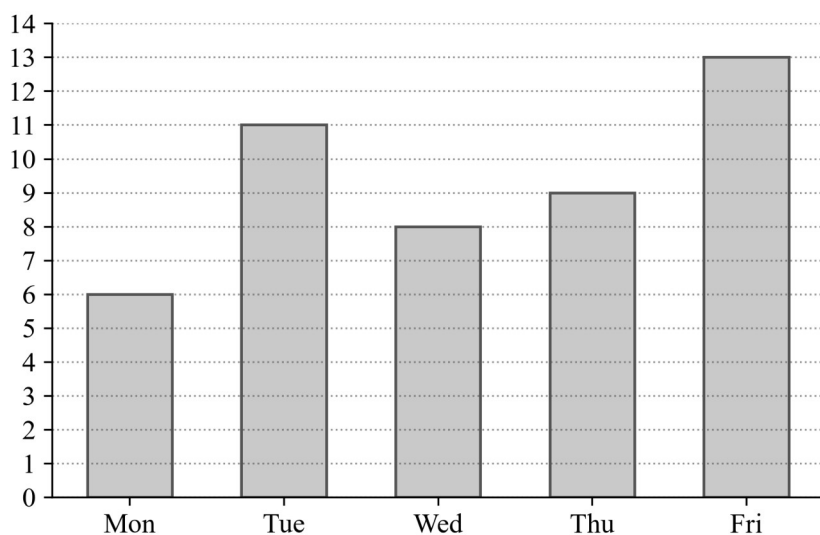
- How much did Ava have at the end of week 1?
- In which week did Ava and Noah have the same amount of savings? State the week and the amount.
- At the end of week 4, how much more did Ava have than Noah?

Q14. The line graph shows how far a rider travelled from home on a bike ride at a steady 15 km/hr.



- How far had the rider travelled after 2 hours?
- At what time had the rider travelled 45 km?
- Describe the change shown by the graph.

Q15. A work crew's poster shows only this graph of the jobs finished each day.



- How many jobs were finished on Thursday?
- Name two things missing from this graph that a reader needs.
- Which day was the busiest, and how many jobs were finished that day?

Q16. A swim club charges a \$30 joining fee, then \$8 for each swim.

- Show that the cost of 2 swims is \$46, one step per line.
- Find the cost of 4 swims.
- How much more do 4 swims cost than 2 swims?

Q17. An oven heats at a steady rate. The table shows its temperature.

Time (min)	0	10	20	30
Temperature (°C)	20	60	100	140

Made-up data: the labels and every number in this table are made up for this section.

- By how many degrees does the temperature rise every 10 minutes?
- The pattern continues. Predict the temperature after 40 minutes.
- Would a line graph of this table be a straight line? Explain your answer.

Q18. A puppy is weighed at the end of each month for six months.

Month	1	2	3	4	5	6
Weight (kg)	2.4	3.1	3.8	4.7	5.2	6.0

Made-up data: the labels and every number in this table are made up for this section.

- How much weight did the puppy gain in total, from month 1 to month 6?
- How much weight did the puppy gain from month 2 to month 5?
- Between which two months was the weight gain the smallest?

Answers

Q1. (a) \$28, \$42, \$56. (b) \$70. (c) \$14 — the same rise for every extra ticket. **Q2.** (a) \$56, \$84, \$112. (b) \$28. (c) yes — the pay rises by \$28 at every step. **Q3.** (a) 42 meals. (b) Friday. (c) 31 meals. **Q4.** (a) 700 L. (b) days 2 and 3. (c) 450 L. **Q5.** (a) \$9.20. (b) \$23.00. (c) \$36.80. **Q6.** (a) 30 seedlings. (b) 15 seedlings. (c) yes — sales rise by 5 seedlings each week. **Q7.** (a) 240 km. (b) 320 km. (c) 80 km for each extra hour. **Q8.** (a) drawn in solutions. (b) 20 cm. (c) weeks 2 and 3 (10 cm). **Q9.** (a) 70 %. (b) 30 %. (c) no — the falls are 15 %, 15 %, 30 %, 15 %. **Q10.** (a) Friday (\$265.45). (b) \$1096.45 (exact). (c) \$1096, to the nearest dollar. **Q11.** (a) 360 and 600. (b) 120 flyers. (c) yes — the count rises by 120 each hour. **Q12.** (a) \$40, \$80, \$120, \$160. (b) 2 hours — \$80 fits in \$100, \$120 does not. (c) \$20. **Q13.** (a) \$20. (b) week 3 — both have \$60. (c) \$10. **Q14.** (a) 30 km. (b) after 3 hours. (c) steady — the distance rises by 15 km every hour, so the graph is a straight line. **Q15.** (a) 9 jobs. (b) a title; axis labels with units. (c) Friday, 13 jobs. **Q16.** (a) shown in solutions. (b) \$62. (c) \$16. **Q17.** (a) 40°C. (b) 180°C. (c) yes — the rise is the same 40°C every 10 minutes. **Q18.** (a) 3.6 kg. (b) 2.1 kg. (c) months 4 and 5 (a gain of 0.5 kg).

Fully-worked solutions

Q1. (a) Every row is (tickets) $\times$ (\$14): $2 \times \$14 = \$28$, $3 \times \$14 = \$42$, $4 \times \$14 = \$56$. (b) The pattern continues past the table: $5 \times \$14 = \$70$. (c) The cost rises by \$14 for each extra ticket — one more ticket always adds one more lot of \$14, so the rise is the same at every step.

Q2. (a) Every row is (hours) $\times$ (\$28): $2 \times \$28 = \$56$, $3 \times \$28 = \$84$, $4 \times \$28 = \$112$. (b) $\$112 - \$84 = \$28$. (c) Yes. The pay rises by \$28 at every step ($\$56 - \$28 = \$28$, $\$84 - \$56 = \$28$, $\$112 - \$84 = \$28$), and the same rise at every step is a steady change.

Q3. (a) The Wednesday bar is labelled 42: 42 meals. (b) The tallest bar is Friday's (66 meals), so Friday was the busiest. (c) Friday 66, Monday 35: $66 - 35 = 31$ more meals on Friday.

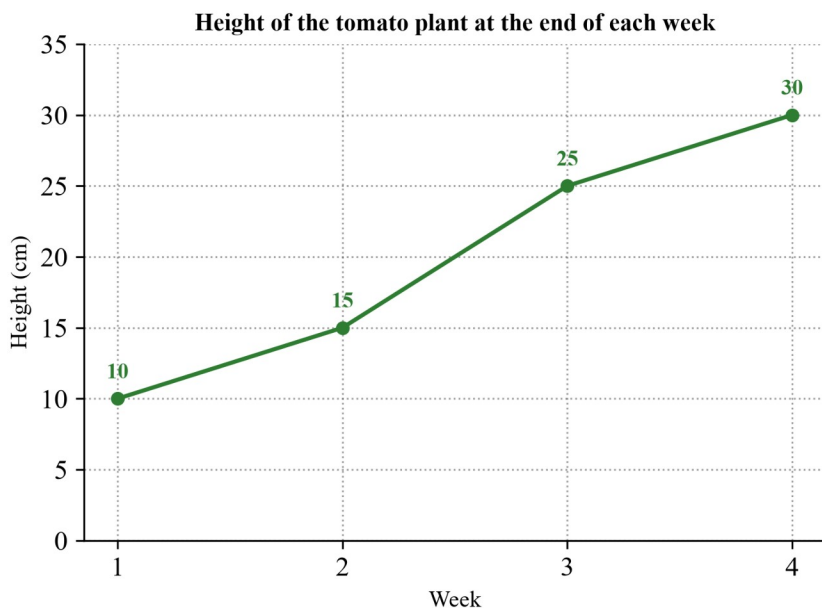
Q4. (a) The point above day 2 is labelled 700: 700 L. (b) Between days 2 and 3 the line is flat — both points are 700 L, so the amount of water did not change. (c) Day 1: 850 L; day 5: 400 L. $850 - 400 = 450$: the water fell by 450 L.

Q5. Each cost is (metres) $\times$ (\$4.60), exact to the cent. (a) $2 \times \$4.60 = \$9.20$. (b) $5 \times \$4.60 = \$23.00$ — written with both decimal places. (c) $8 \times \$4.60 = \$36.80$.

Q6. (a) The week-3 column reads 30: 30 seedlings. (b) Week 5: 40; week 2: 25. $40 - 25 = 15$: 15 more seedlings. (c) Yes. The rises are $25 - 20 = 5$, $30 - 25 = 5$, $35 - 30 = 5$, $40 - 35 = 5$ — the same rise of 5 seedlings every week, so the change is steady.

Q7. Distance = (hours) $\times$ (80 km each hour). (a) $3 \times 80 = 240$: 240 km. (b) $4 \times 80 = 320$: 320 km. (c) The distance rises by 80 km for each extra hour — at a steady 80 km/hr, one more hour always adds 80 km.

Q8. (a) Label the axes Week and Height (cm), choose a scale, plot $(1, 10)$, $(2, 15)$, $(3, 25)$ and $(4, 30)$, and join the points in order:



(b) $30 - 10 = 20$: the plant grew 20 cm in total.

(c) The growth each week: $15 - 10 = 5$ cm, $25 - 15 = 10$ cm, $30 - 25 = 5$ cm. The biggest is 10 cm, so the plant grew the most between weeks 2 and 3.

Q9. (a) The 2-hour column reads 70: the charge was 70 %. (b) Hour 2: 70 %; hour 3: 40 %. $70 - 40 = 30$: the charge fell by 30 %. (c) No. The falls, hour by hour, are $100 - 85 = 15$, $85 - 70 = 15$, $70 - 40 = 30$, $40 - 25 = 15$ (all %). The fall from hour 2 to hour 3 is bigger than the others, so the charge did not fall by the same amount every hour.

Q10. (a) The largest takings in the table is \$265.45, on Friday. (b) $\$214.50 + \$196.25 + \$231.75 + \$188.50 + \$265.45 = \$1096.45$ — exact to the cent. (c) To the nearest dollar, look at the first decimal place: it is 4, which is less than 5, so round down. \$1096.45 rounds to \$1096, to the nearest dollar.

Q11. (a) The count rises by 120 each hour ($240 - 120 = 120$). Hour 3: $240 + 120 = 360$. Check against the table: $360 + 120 = 480$, which matches the hour-4 value ✓. Hour 5: $480 + 120 = 600$. The gaps are 360 and 600. (b) 120 more flyers for each extra hour. (c) Yes — the count rises by the same 120 at every step, so the change is steady.

Q12. (a) Every row is (hours) $\times$ (\$40):

Hours of tutoring (h)	1	2	3	4
Cost (dollars)	\$40	\$80	\$120	\$160

(b) 2 hours cost \$80, which fits in \$100; 3 hours cost \$120, which is more than \$100. So Josh can afford 2 whole hours.

(c) $\$100 - \$80 = \$20$: Josh would receive \$20 change.

Q13. (a) Ava's line at week 1 is labelled \$20. (b) The two lines meet at week 3, where both are \$60. Check with the rules: Ava saves $3 \times \$20 = \$60$; Noah has $\$30 + 3 \times \$10 = \$60$. (c) At week 4: Ava has \$80, Noah has \$70. $\$80 - \$70 = \$10$: Ava had \$10 more.

Q14. (a) The point above 2 hours is labelled 30: 30 km. (b) Find 45 on the distance axis, go across to the line and down to the time axis: 45 km happens after 3 hours. (c) The change is steady: the distance rises by 15 km for every extra hour (15 km/hr), and the points form a straight line.

Q15. (a) The Thursday bar reaches 9 on the scale (gridlines every 1): 9 jobs. (b) The graph has **no title**, so a reader cannot say what is being counted; and the **axes have no labels or units**, so nothing says the bars are jobs or the categories are days of the week. (c) The tallest bar is Friday's, at 13: Friday was the busiest, with 13 jobs.

Q16. (a) One step per line:

$$2 \times \$8 = \$16$$

$$\$30 + \$16 = \$46$$

$\therefore$ the cost of 2 swims is \$46.

(b) $4 \times \$8 = \$32$, and $\$30 + \$32 = \$62$: the cost of 4 swims is \$62.

(c) $\$62 - \$46 = \$16$: 4 swims cost \$16 more than 2 swims.

Q17. (a) The rises: $60 - 20 = 40$, $100 - 60 = 40$, $140 - 100 = 40$ (all $^{\circ}\text{C}$). The temperature rises by 40°C every 10 minutes. (b) The pattern continues one more step: $140 + 40 = 180$, so after 40 minutes the temperature is 180°C . (c) Yes — the rise is the same 40°C at every step, and a steady change graphs as a straight line.

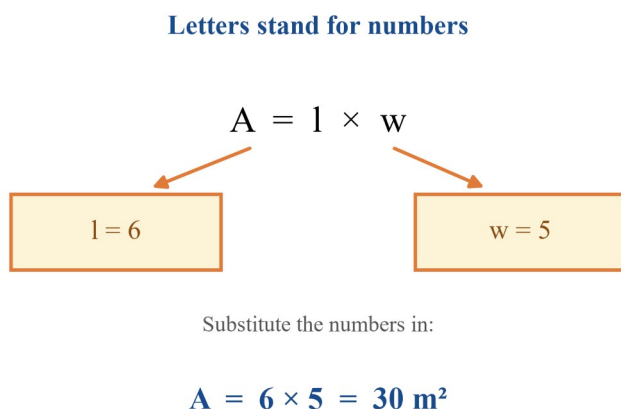
Q18. (a) $6.0 - 2.4 = 3.6$: the puppy gained 3.6 kg in total. (b) $5.2 - 3.1 = 2.1$: the puppy gained 2.1 kg from month 2 to month 5. (c) The gains, month by month: $3.1 - 2.4 = 0.7$, $3.8 - 3.1 = 0.7$, $4.7 - 3.8 = 0.9$, $5.2 - 4.7 = 0.5$, $6.0 - 5.2 = 0.8$ (all kg). The smallest gain is 0.5 kg, between months 4 and 5.

Chapter 5 Relationships — 5E Simple substitution

About the data in this section. The prices, pay rates, job cards, spreadsheets and tables here are **made up** (not real): pretend businesses and pretend numbers, built for this book so you can practise substitution. Nothing here is real data about a real business, person or place.

Theory

Substitution means putting the numbers in. A formula is a rule written in letters (Section 5B). The letters are placeholders: once you are told what number each letter stands for, you **substitute** — you replace each letter with its number — and then work out the answer. That whole process is called **substitution**, and it is the skill that makes every formula in this book useful. The figure below shows the idea in one picture: the formula $A = l \times w$, the key $l = 6$ and $w = 5$, and the swap that gives the answer.



With $l = 6$ and $w = 5$:

$$A = l \times w = 6 \times 5 = 30$$

The area is $A = 30 \text{ m}^2$.

The three-step method. Every substitution in this section works the same way, and the same three steps never let you down:

- **Step 1** — write the formula down, exactly as given.
- **Step 2** — replace each letter with its number, and **write the $\times$ back in** wherever a number sits next to a letter.
- **Step 3** — work out the calculation in the correct order, and put the unit on the answer.

Worked model. Find the perimeter of a rectangle with $l = 5.5 \text{ m}$ and $w = 3 \text{ m}$, using $P = 2 \times (l + w)$.

$$P = 2 \times (l + w)$$

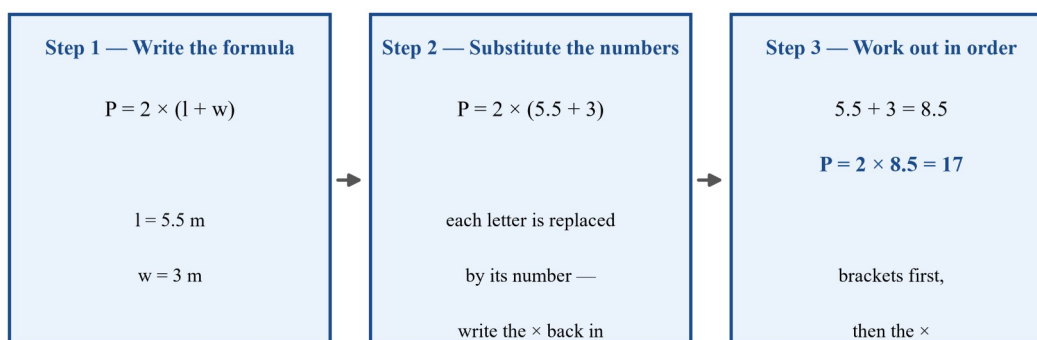
$$P = 2 \times (5.5 + 3)$$

$$5.5 + 3 = 8.5$$

$$P = 2 \times 8.5 = 17$$

The perimeter is $P = 17 \text{ m}$. Step 2 kept the brackets and wrote the $\times$ back in; Step 3 did the bracket first, then the multiplying, and the unit (m) went on at the end.

Substitution in three steps



Answer: $P = 17 \text{ m}$ — put the unit back on at the end.

Refresher: order of operations (Section 2D). When a calculation mixes operations, do **brackets first**, then $\times$ and $\div$, then $+$ and $-$, working left to right. Brackets can change the answer: $5 + 2 \times 3 = 11$, but $(5 + 2) \times 3 = 21$. That is why Step 2 keeps brackets in place, and why in a rule like $C = 95 + 26h$ you multiply before you add.

Refresher: multiplying decimals (Section 2A). Multiply as if the decimal points were not there, then count the decimal places in the question and give the answer that many decimal places. For example, 2.5×1.2 : ignore the points, $25 \times 12 = 300$; the question has $1 + 1 = 2$ decimal places, so the answer is 3.00, which is just 3. Quick ones: $3.5 \times 4 = 14$ and $0.6 \times 5 = 3$.

Decimal hours. Times in this section can be decimals: 6.5 h means 6 hours and 30 minutes, because the .5 is half an hour ($0.5 \times 60 = 30$ minutes) — it does *not* mean 5 minutes.

Short form — and the classic trap. Algebra leaves the $\times$ sign out to keep writing short: a number and a letter written together mean multiply. **Substitution puts the $\times$ back in.** When you substitute, never leave two values sitting side by side.

Refresher: letters stand for numbers, and the $\times$ sign is left out (Section 5C). Before you work anything out, read the short form and write what it means:

Short form	What it means
$13n$	$13 \times n$
lw	$l \times w$
$2(l + w)$	$2 \times (l + w)$
$95 + 26h$	$95 + 26 \times h$

Worked model. For $C = 13n$ with $n = 4$:

$$C = 13 \times 4 = 52$$

The trap. $C = 13n$ with $n = 4$ is **not** $C = 134$. Writing the 13 and the 4 side by side is not multiplying — $13n$ means $13 \times n$, so the answer is 52. If you always write the $\times$ back in (Step 2), this trap cannot catch you.

When the $\times$ is left out

$C = 13n$

means $C = 13 \times n$

If $n = 4$:

$C = 13 \times 4 = 52$

~~**$C = 134 ?$**~~

WRONG

A number and a letter sitting together still means multiply — they never join into one number.

Two-part rules. Many everyday rules have two parts: a **fixed** amount that never changes, and a **per-hour** (or per-item) amount that grows with the work. In $C = 95 + 26h$, the \$95 is a fixed fee — charged once whatever the job — and the $26h$ part means \$26 for each hour h . Substitute h , multiply first (order of operations), then add.

Worked model. For $C = 95 + 26h$ with $h = 2$:

$$C = 95 + 26 \times 2$$

$$26 \times 2 = 52$$

$$C = 95 + 52 = 147$$

The cost is \$147: the fixed \$95 plus 2 hours at \$26 each.

A rule with two parts: $C = 95 + 26h$

fixed part \$95 call-out fee	part that grows with h \$26 for each hour $\rightarrow 26 \times h$
--	--

$$C = \text{fixed part} + \text{hourly part}$$

Substitute $h = 2$:

$$C = 95 + 26 \times 2 = 95 + 52 = 147$$

Multiply the hourly part first, then add the fixed \$95.

The letters you will meet. Each rule comes with a key that says what its letters stand for. These are the usual ones in this section:

Letter	Usually stands for	Usual units
A	area	m^2, cm^2
P	perimeter	m, cm
V	volume	m^3
C	cost or pay	dollars
D	distance	km
l, w, h	length, width, height	m, cm
s	speed	km/hr
t	time	hours (h)
n	number of items	a count
r	rate per hour	\$ per hour

The same letter can stand for different things in different rules: h is the **height** in $V = l \times w \times h$ (Section 4C), but **hours** in $C = 23h$. Always read the key that comes with the rule.

The distance rule. Distance travelled equals speed multiplied by time:

$$D = s \times t$$

where D is the distance in km, s is the speed in km/hr and t is the time in hours (h). **Worked model.** A car travels at $s = 90$ km/hr for $t = 1.5$ h:

$$D = 90 \times 1.5 = 135$$

The car travels 135 km.

Checking a claim by substitution. Substitution does not only *find* values — it also *tests* them. If someone claims that $C = 13n$ gives $C = 52$ when $n = 4$, substitute and see: $13 \times 4 = 52$ ✓ — the claim checks out. If the substituted value does not match the claim, the claim is wrong. Several questions below ask you to check claims this way.

Building a table from a rule. In Section 5D you read tables of changing values. Here you build the table yourself: take the rule, substitute each input in turn, and write the outputs in a row. For $C = 95 + 26h$: when $h = 1$, $C = 95 + 26 = 121$; when $h = 2$, $C = 95 + 52 = 147$; when $h = 3$, $C = 95 + 78 = 173$.

A substitution table for $C = 95 + 26h$

h	1	2	3
$C = 95 + 26h$	121	147	173

Same rule, new number in each row: put h in, work out C .

Spreadsheets do the substitution for you. A spreadsheet cell can hold a formula, and the spreadsheet substitutes for you every time the input changes (cells and formulas: Section 1F). In the figure below, cell A2 holds the hours h , and cell B2 holds the formula $=95+26*A2$ — the $*$ is the spreadsheet's $\times$ sign. With 2 typed into A2, the spreadsheet substitutes $h = 2$ and B2 shows 147. Change A2 and B2 updates straight away: you do the thinking once, and the machine repeats it as often as you like. Every substitution in this section can be done with pen and paper — the spreadsheet just does the repetitive ones faster.

Substitution in a spreadsheet

B2	fx	$=95+26*A2$
----	------	-------------

A	B
h	C
2	147

Type the formula once into B2. Change h in A2 and the answer updates — the spreadsheet does the substituting for you.

Units and rounding. A number on its own is not an answer. Put the unit back on at the end: 17 is not a perimeter, 17 m is. For money, write answers to the cent. Sometimes the calculation leaves extra decimals — when that happens, round to the nearest cent and say that you did (as in Question 17).

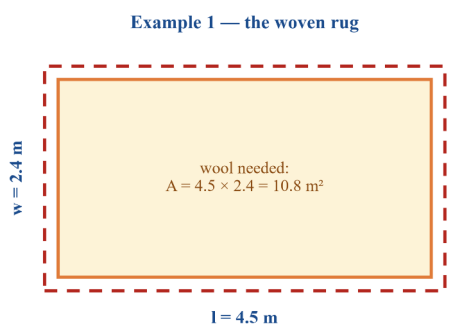
Refresher: rounding money to the nearest cent (Section 2C). Look at the **third** decimal place. If it is 5 or more, round the second decimal place **up**; if it is less than 5, leave it alone. So \$12.346 rounds to \$12.35 (decider 6, round up), and \$7.854 rounds to \$7.85 (decider 4, leave it).

Worked examples

Example 1 — scaffolded

The class rug. A class is weaving a rectangular rug for the school office. The rug is 4.5 m long and 2.4 m wide. The wool covers the surface; binding (a strip of cloth) runs around the edge.

- Write $A = l \times w$ with the numbers substituted in (the area formula is from Section 4B).
- Work out the area of wool needed.
- The binding runs all the way around the edge. Substitute into $P = 2 \times (l + w)$ and work out the length of binding needed.



binding around the edge (dashed): $P = 2 \times (4.5 + 2.4) = 13.8 \text{ m}$

Working

$$A = l \times w = 4.5 \times 2.4$$

$$45 \times 24 = 1080, \text{ so } 4.5 \times 2.4 = 10.80 = 10.8$$

$$A = 10.8 \text{ m}^2$$

$$P = 2 \times (4.5 + 2.4) = 2 \times 6.9 = 13.8$$

Binding: 13.8 m

Comment

Step 2 — replace l with 4.5 and w with 2.4; write the $\times$ back in

multiply without the points, then $1 + 1 = 2$ decimal places

area is in square units — 10.8 m^2 of wool

brackets first (6.9), then multiply

perimeter is a length, so the unit is m

Example 2 — scaffolded

Saturday shift pay. Tessa works Saturday shifts at a plant nursery. Her pay follows the rule $C = 23h$, where C is her pay in dollars and h is the number of hours she works.

- Write the substitution for a 6-hour shift.
- How much does Tessa earn for 6 hours?

- (c) One Saturday she works 6.5 hours. How much more does she earn than for the 6-hour shift?

Example 2 — Tessa's Saturday job

Plant nursery — pay: \$23 per hour	
Formula: $C = 23h$ (C = pay in dollars, h = hours)	
$h = 6$:	$C = 23 \times 6 = \$138$
$h = 6.5$:	$C = 23 \times 6.5 = \$149.50$
extra half-hour:	$\$149.50 - \$138 = \$11.50$

Working

$$C = 23h \text{ means } C = 23 \times h$$

$$C = 23 \times 6 = \$138$$

$$C = 23 \times 6.5 = \$149.50$$

$$\$149.50 - \$138 = \$11.50$$

$$\text{Check: } 23 \times 0.5 = \$11.50 \checkmark$$

Comment

a number next to a letter means multiply

the 6-hour shift pays \$138

6.5 h is 6 hours 30 minutes; money is written to the cent

the longer shift pays \$11.50 more

the extra half hour is half of \$23 — it matches

Example 3 — unscaffolded

The mechanic's job card. A mobile mechanic charges a \$65 call-out fee (a fixed amount just for coming to you) plus \$40 for each hour of work. The rule is $C = 65 + 40h$, where C is the total cost in dollars and h is the number of hours worked. Della books the mechanic for a job.

- The job takes 3 hours. Find the cost.
- Find the cost of a 5-hour job.
- Della has \$300 to spend. Can she afford a 6-hour job? Show your substitution and give a reason.

Example 3 — the mechanic's rates

Call-out fee: \$65 Hourly rate: \$40 per hour	
Formula: $C = 65 + 40h$	
$h = 3$:	$C = 65 + 40 \times 3 = 65 + 120 = \$185$
$h = 5$:	$C = 65 + 40 \times 5 = 65 + 200 = \$265$
$h = 6$:	$C = 65 + 40 \times 6 = 65 + 240 = \$305$
Budget \$300 for 6 hours? $\$305 > \$300$ — no, it is \$5 over.	

The rule is $C = 65 + 40h$ — multiply first, then add.

- Substitute $h = 3$:

$$C = 65 + 40 \times 3$$

$$40 \times 3 = 120$$

$$C = 65 + 120 = 185$$

$\therefore$ the 3-hour job costs \$185.

(b) Substitute $h=5$:

$$C = 65 + 40 \times 5$$

$$40 \times 5 = 200$$

$$C = 65 + 200 = 265$$

$\therefore$ a 5-hour job costs \$265.

(c) Substitute $h=6$:

$$C = 65 + 40 \times 6$$

$$40 \times 6 = 240$$

$$C = 65 + 240 = 305$$

\$305 is more than Della's \$300 budget — over by $\$305 - \$300 = \$5$.

$\therefore$ no, Della cannot afford the 6-hour job: it costs \$305, which is \$5 over her \$300 budget.

Example 4 — unscaffolded

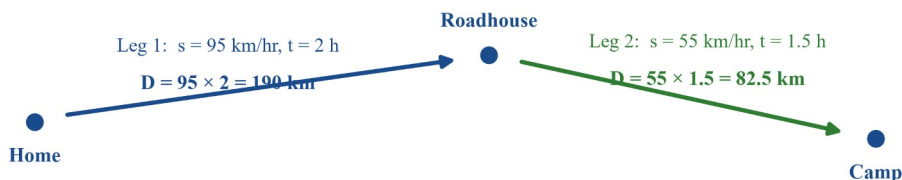
Two legs of a trip. A family road trip uses the rule $D = s \times t$, where D is the distance in km, s is the speed in km/hr and t is the time in hours (h). Each leg is one part of the trip. Leg 1: 95 km/hr on the highway for 2 h. Leg 2: 55 km/hr on a country road for 1.5 h.

(a) Show that leg 1 covers 190 km.

(b) Find the distance covered on leg 2.

(c) Find the total distance for the whole trip.

Example 4 — the two-leg trip



Whole trip: $D = 190 + 82.5 = 272.5$ km

The rule is $D = s \times t$.

(a) Substitute $s=95$ and $t=2$:

$$D = 95 \times 2$$

$$95 \times 2 = 190$$

$\therefore$ leg 1 covers 190 km, which is what we were asked to show.

(b) Substitute $s=55$ and $t=1.5$:

$$D = 55 \times 1.5 = 82.5$$

$\therefore$ leg 2 covers 82.5 km.

(c) Add the two legs:

$$190 + 82.5 = 272.5$$

∴ the whole trip covers 272.5 km.

Practice questions

Scaffolded

Q1. Find the value of each expression.

- (a) $6n$ when $n=3$
- (b) $4x+3$ when $x=6$
- (c) $h \div 4$ when $h=36$

Q2. A framed print is 30 cm long and 20 cm wide.

- (a) Write $A = l \times w$ with the numbers substituted in.
- (b) Find the area of the print.
- (c) The shop also sells a square print 30 cm by 30 cm. Find its area.

Q3. A rectangular veggie patch is 6 m long and 4 m wide. Its perimeter rule is written in short form: $P = 2l + 2w$.

- (a) Write the rule with $l=6$ and $w=4$ substituted in.
- (b) Find the perimeter.
- (c) Check your answer by substituting into the bracket form $P = 2 \times (l + w)$ instead.

Q4. A lawn-mowing job pays by the rule $C = 31h$, where C is the pay in dollars and h is the number of hours worked.

- (a) Find the pay for 2 h.
- (b) Find the pay for 3.5 h.
- (c) Find the pay for a 0.5 h job.

Q5. Drinks at the school fete stall cost \$6 each, so the takings follow $C = 6n$, where C is the takings in dollars and n is the number of drinks sold. Copy and complete the substitution table.

n	4	7	10
$C = 6n$			

Made-up data: the stall prices and every number in the table are made up for this section.

Q6. A compost bin is shaped like a rectangular box. Its volume follows $V = l \times w \times h$, where V is the volume in m^3 (the volume formula is from Section 4C). This bin is 5 m long, 2 m wide and 1.5 m high.

- (a) Write the rule with the numbers substituted in.
- (b) Find the volume of the bin.
- (c) A deeper bin has the same length and width but $h = 2.5$ m. Find its volume.

Unscaffolded

Q7. A hardware store hires out pressure cleaners. The rule is $C = 41 + 17h$: a \$41 hire fee plus \$17 for each hour h .

- (a) Find the cost of hiring for 3 h.
- (b) Find the cost of hiring for 5 h.
- (c) Find the cost of hiring for 2.5 h.

Q8. A car travels at a steady $s = 50$ km/hr. Distances follow the rule $D = s \times t$, where t is the time in hours (h).

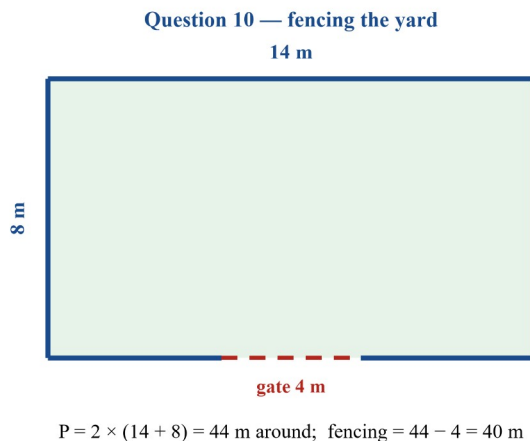
- (a) Find the distance travelled in $t = 2$ h.
- (b) Find the distance travelled in $t = 3.5$ h.
- (c) Find the total distance travelled in the two trips.

Q9. A tutoring poster says: “4 hours of tutoring — \$192!” The prices follow the rule $C = 48h$, where C is the cost in dollars and h is the number of hours.

- (a) Substitute $h = 4$ and check whether the \$192 price is correct.
- (b) The poster also claims that 5 hours costs \$230. Substitute $h = 5$ and check this claim.
- (c) What is the correct cost of 5 hours of tutoring?

Q10. A rectangular yard is 14 m long and 8 m wide. It is fenced on all four sides except for a 4 m gate. The fencing costs \$9 per metre.

- (a) Substitute into $P = 2 \times (l + w)$ and find the distance around the yard.
- (b) Find how many metres of fencing are needed.
- (c) Find the cost of the fencing.



Q11. Joel is finding the perimeter of a rectangular strip of lawn 2 m wide and 9 m long. The rule is $P = 2 \times (l + w)$. Joel writes: $P = 2 \times 2 + 9 = 13$.

- (a) What has Joel done wrong?
- (b) Substitute correctly and find the perimeter.
- (c) Check your answer by adding the four sides.

Q12. Ravi’s workshop tracks machine hire in a spreadsheet. Cell B2 holds the number of hours. Cell C2 holds the formula $=44*B2$ and shows the cost C in dollars.

- When B2 holds 5, what value shows in C2?
- When B2 holds 6.5, what value shows in C2?
- Without the spreadsheet, write the substitution for 6.5 hours and work it out on paper.

Question 12 — hire tracked in a spreadsheet

C2	f_x	$=44*B2$
----	-------	----------

A	B	C
	hours	cost
	5	220
	6.5	286

B2 holds the hours; C2 substitutes them into $C = 44 \times h$.

Q13. A rectangular garden pond is 4 m long, 2.5 m wide and 0.8 m deep. Its volume follows $V = l \times w \times h$, where V is the volume in m^3 .

- Show that the pond holds $8 m^3$ of water.
- A rectangular swimming pool is 8 m long, 4 m wide and has an average depth of 1.5 m. Find the volume of water it holds.
- After a dry week the pond is only one-quarter full. How many m^3 of water are in it?

Q14. A print shop charges a \$70 set-up fee (a fixed amount charged once, before any printing) plus \$11 for each newsletter printed: $C = 70 + 11n$, where C is the cost in dollars and n is the number of copies.

- Copy and complete the substitution table.

n	5	15	25
$C = 70 + 11n$			

Made-up data: the print-shop prices and every number in the table are made up for this section.

- A club member says 15 copies will cost \$205. Use your table to explain why they are wrong.
- Find the cost of 20 copies.

Q15. A country train runs at a steady $s = 72$ km/hr. Distances follow the rule $D = s \times t$, where t is the time in hours (h).

- Show that the train travels 90 km in $t = 1.25$ h.
- Find the distance it travels in $t = 0.5$ h.
- Check that your answer to part (a) is reasonable: round the speed to 70 km/hr and estimate the distance.

Q16. Three workers at a repair shop are paid by the rule $C = r \times h$, where C is the pay in dollars, r is the worker's rate in dollars per hour and h is the number of hours worked this week.

Worker	r (\$/hr)	h
Cara	35	4
Omar	49	2.5
Miri	38	5.5

Made-up data: the worker names and every number in the table are made up for this section.

- (a) Find Cara's pay for the week.
- (b) Find Omar's pay for the week.
- (c) Find Miri's pay for the week.

Q17. A casual worker — someone paid only for the hours worked — is paid by the rule $C = 34.75 h$, where C is the pay in dollars and h is the number of hours worked.

- (a) Find the pay for $h = 7.5$, correct to the nearest cent.
- (b) Find the pay for $h = 12$.
- (c) Check that your answer to part (a) is reasonable: round the rate to \$ 35 per hour and estimate the pay for 7.5 h.

Q18. Booking the rooms for the school dinner costs \$ 120, plus \$ 8 per person for food. The rule is $C = 120 + 8n$, where C is the total cost in dollars and n is the number of people.

- (a) Find the cost for $n = 25$.
- (b) Find the cost for $n = 45$.
- (c) The organisers have a \$ 500 budget. Can they afford the dinner for $n = 48$ people? Show your substitution and give a reason.

Answers

Q1. (a) 18. (b) 27. (c) 9.

Q2. (a) $A = 30 \times 20$. (b) 600 cm^2 . (c) 900 cm^2 .

Q3. (a) $P = 2 \times 6 + 2 \times 4$. (b) 20 m. (c) $2 \times (6 + 4) = 2 \times 10 = 20 \text{ m}$ — same ✓.

Q4. (a) \$ 62. (b) \$ 108.50. (c) \$ 15.50.

Q5. C: \$ 24, \$ 42, \$ 60.

Q6. (a) $V = 5 \times 2 \times 1.5$. (b) 15 m^3 . (c) 25 m^3 .

Q7. (a) \$ 92. (b) \$ 126. (c) \$ 83.50.

Q8. (a) 100 km. (b) 175 km. (c) 275 km.

Q9. (a) $48 \times 4 = 192$ — the \$ 192 price is correct ✓. (b) $48 \times 5 = 240$, not \$ 230 — the claim is wrong. (c) \$ 240.

Q10. (a) 44 m. (b) 40 m. (c) \$ 360.

Q11. (a) He dropped the brackets — $2 \times 2 + 9$ doubles only one side. (b) 22 m. (c) $2 + 9 + 2 + 9 = 22 \text{ m}$ ✓.

Q12. (a) 220. (b) 286. (c) $C = 44 \times 6.5 = 286$ ✓ matches.

Q13. (a) Shown: $4 \times 2.5 = 10$; $10 \times 0.8 = 8$. (b) 48 m^3 . (c) 2 m^3 .

Q14. (a) \$ 125, \$ 235, \$ 345. (b) The table gives \$ 235 for 15 copies, not \$ 205. (c) \$ 290.

Q15. (a) Shown: $72 \times 1.25 = 90$. (b) 36 km. (c) $70 \times 1.25 = 87.5 \text{ km}$ — close to 90 km ✓.

Q16. (a) \$ 140. (b) \$ 122.50. (c) \$ 209.

Q17. (a) \$ 260.63 (to the nearest cent). (b) \$ 417. (c) $35 \times 7.5 = \$ 262.50$ — close to \$ 260.63 ✓.

Q18. (a) \$ 320. (b) \$ 480. (c) $C = 120 + 8 \times 48 = \$ 504$; \$ 504 is more than \$ 500 — no, \$ 4 over budget.

Fully-worked solutions

Q1. Substitute and work out — write the $\times$ back in first.

(a) $6n$ means $6 \times n$. With $n = 3$: $6 \times 3 = 18$.

(b) $4x + 3$ means $4 \times x + 3$. With $x = 6$: $4 \times 6 = 24$, then $24 + 3 = 27$.

(c) $h \div 4$ with $h = 36$: $36 \div 4 = 9$.

Q2. The print is 30 cm by 20 cm.

(a) $A = l \times w = 30 \times 20$.

(b) $30 \times 20 = 600$, so the area is 600 cm^2 .

(c) The square print: $A = 30 \times 30 = 900 \text{ cm}^2$.

Q3. $P = 2l + 2w$ with $l = 6$ and $w = 4$.

(a) $P = 2 \times 6 + 2 \times 4$ — the short form means two lots of l plus two lots of w .

(b) $2 \times 6 = 12$ and $2 \times 4 = 8$, so $P = 12 + 8 = 20 \text{ m}$.

(c) Bracket form: $P = 2 \times (6 + 4) = 2 \times 10 = 20 \text{ m}$ — the same answer ✓.

Q4. $C = 31h$ — one-step substitution, money to the cent.

- (a) $h = 2$: $C = 31 \times 2 = \$62$.
- (b) $h = 3.5$: $C = 31 \times 3.5 = \$108.50$.
- (c) $h = 0.5$: $C = 31 \times 0.5 = \$15.50$.

Q5. Substitute each n into $C = 6n$:

- $n = 4$: $C = 6 \times 4 = \$24$
- $n = 7$: $C = 6 \times 7 = \$42$
- $n = 10$: $C = 6 \times 10 = \$60$

n	4	7	10
$C = 6n$	24	42	60

Q6. $V = l \times w \times h$ with $l = 5$, $w = 2$ and $h = 1.5$.

- (a) $V = 5 \times 2 \times 1.5$.
- (b) $5 \times 2 = 10$, then $10 \times 1.5 = 15$, so $V = 15 \text{ m}^3$.
- (c) With $h = 2.5$: $V = 5 \times 2 \times 2.5 = 10 \times 2.5 = 25 \text{ m}^3$.

Q7. $C = 41 + 17h$ — multiply first, then add.

- (a) $h = 3$: $C = 41 + 17 \times 3$. $17 \times 3 = 51$, so $C = 41 + 51 = \$92$.
- (b) $h = 5$: $17 \times 5 = 85$, so $C = 41 + 85 = \$126$.
- (c) $h = 2.5$: $17 \times 2.5 = 42.5$, so $C = 41 + 42.5 = \$83.50$.

Q8. $D = s \times t$ with $s = 50$.

- (a) $t = 2$: $D = 50 \times 2 = 100 \text{ km}$.
- (b) $t = 3.5$: $D = 50 \times 3.5 = 175 \text{ km}$.
- (c) Total: $100 + 175 = 275 \text{ km}$.

Q9. $C = 48h$.

- (a) Substitute $h = 4$: $C = 48 \times 4 = 192$. That matches the poster, so the \$192 price is correct ✓.
- (b) Substitute $h = 5$: $C = 48 \times 5 = 240$. \$240 is not \$230, so the claim is wrong.
- (c) The correct cost of 5 hours is \$240.

Q10. $l = 14 \text{ m}$, $w = 8 \text{ m}$, gate 4 m, fencing at \$9 per metre.

- (a) $P = 2 \times (14 + 8)$. $14 + 8 = 22$, so $P = 2 \times 22 = 44 \text{ m}$.
- (b) No fencing across the gate: $44 - 4 = 40 \text{ m}$ of fencing.
- (c) Cost = $9 \times 40 = \$360$.

Q11. $P = 2 \times (l + w)$ for a strip 2 m by 9 m.

- (a) Joel dropped the brackets. The rule multiplies 2 by the **whole** sum $(l + w)$; Joel's line $2 \times 2 + 9 = 13$ doubles only one side and adds the other once, so it is too small.
- (b) $P = 2 \times (2 + 9)$. $2 + 9 = 11$, so $P = 2 \times 11 = 22 \text{ m}$.
- (c) Four sides: $2 + 9 + 2 + 9 = 22 \text{ m}$ ✓ — the same answer.

Q12. Cell C2 holds $=44*B2$, so the spreadsheet substitutes $C = 44 \times h$ with h from B2.

- (a) $B2 = 5$: $44 \times 5 = 220$, so C2 shows 220.
- (b) $B2 = 6.5$: $44 \times 6.5 = 286$, so C2 shows 286.
- (c) On paper: $C = 44 \times 6.5 = 286$ ✓ — the same as the spreadsheet.

Q13. $V = l \times w \times h$.

- (a) Substitute $l = 4$, $w = 2.5$ and $h = 0.8$:

$$V = 4 \times 2.5 \times 0.8$$

$$4 \times 2.5 = 10$$

$$10 \times 0.8 = 8$$

∴ the pond holds 8 m^3 , which is what we were asked to show.

- (b) $V = 8 \times 4 \times 1.5$. $8 \times 4 = 32$, then $32 \times 1.5 = 48$, so the pool holds 48 m^3 .
- (c) One-quarter of 8 m^3 : $8 \div 4 = 2 \text{ m}^3$.

Q14. $C = 70 + 11n$ — multiply first, then add.

- (a) Substitute each n :
 - $n = 5$: $11 \times 5 = 55$, so $C = 70 + 55 = \$125$
 - $n = 15$: $11 \times 15 = 165$, so $C = 70 + 165 = \$235$
 - $n = 25$: $11 \times 25 = 275$, so $C = 70 + 275 = \$345$

n	5	15	25
$C = 70 + 11n$	125	235	345

- (b) The table gives \$235 for 15 copies. \$235 is not \$205, so the member is wrong.
- (c) $n = 20$: $11 \times 20 = 220$, so $C = 70 + 220 = \$290$.

Q15. $D = s \times t$ with $s = 72$.

- (a) Substitute $s = 72$ and $t = 1.25$:

$$D = 72 \times 1.25$$

$$72 \times 1.25 = 90$$

∴ the train travels 90 km in 1.25 h, which is what we were asked to show.

- (b) $t = 0.5$: $D = 72 \times 0.5 = 36 \text{ km}$.
- (c) Estimate with $s = 70$: $70 \times 1.25 = 87.5 \text{ km}$. 87.5 is close to 90, so the answer in part (a) is reasonable ✓.

Q16. $C = r \times h$ — read each worker's r and h from the table.

- (a) Cara: $r = 35$, $h = 4$. $C = 35 \times 4 = \$140$.
- (b) Omar: $r = 49$, $h = 2.5$. $C = 49 \times 2.5 = \$122.50$.
- (c) Miri: $r = 38$, $h = 5.5$. $C = 38 \times 5.5 = \$209$.

Q17. $C = 34.75h$.

- (a) Substitute $h = 7.5$: $C = 34.75 \times 7.5 = 260.625$. The third decimal place is 5, so round up: the pay is \$260.63 to the nearest cent.
- (b) $h = 12$: $C = 34.75 \times 12 = \$417$ — exactly \$417.00, no rounding needed.
- (c) Estimate with the rate rounded to \$35: $35 \times 7.5 = \$262.50$. That is close to \$260.63, so the answer in part (a) is reasonable ✓.

Q18. $C = 120 + 8n$ — multiply first, then add.

- (a) $n = 25$: $8 \times 25 = 200$, so $C = 120 + 200 = \$320$.
- (b) $n = 45$: $8 \times 45 = 360$, so $C = 120 + 360 = \$480$.
- (c) $n = 48$: $8 \times 48 = 384$, so $C = 120 + 384 = \$504$. \$504 is more than the \$500 budget — over by $\$504 - \$500 = \$4$.

∴ no, they cannot afford the dinner for 48 people: it costs \$504, which is \$4 over the \$500 budget.

Chapter 05 — Test A (multiple choice)

Chapter 5 (Relationships). Materials: paper and a basic calculator. There are 12 multiple-choice questions, each worth 1 mark (12 marks). Approximate time: 25 minutes. Choose the one correct response for each question. Every business, price and measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1

A hardware store charges \$46.80 for a 4 m length of clothesline. What is the price per metre, to the nearest cent?

- A. \$0.09
- B. \$11.70
- C. \$50.80
- D. \$187.20

Question 2

Anh rides her bike 9 km to her grandmother's house at a steady speed. The ride takes 45 minutes. What is her speed, in km/hr?

- A. 0.2 km/hr
- B. 20 km/hr
- C. 12 km/hr
- D. 405 km/hr

Question 3

Dripper pipe for a garden costs \$5.80 per metre. What is the cost of 4.5 m of this pipe?

- A. \$23.20
- B. \$10.30
- C. \$261.00
- D. \$26.10

Question 4

A rectangular concrete path is 7.5 m long and 3.2 m wide. What is the area of the path?

- A. 21.4 m
- B. 24 m²
- C. 10.7 m²
- D. 240 m²

Question 5

A mobile dog groomer charges a \$75 call-out fee plus \$48 for each hour of work. What is the total cost of a job that takes 3.5 hours?

- A. \$430.50
- B. \$310.50
- C. \$243.00
- D. \$123.00

Question 6

A floor sander is hired for 4.8 hours at a total cost of \$168. Every hour is charged at the same price. What is the cost of 1 hour?

- A. \$21.00
- B. \$806.40
- C. \$163.20
- D. \$35.00

Question 7

Simplify $6m + 2m$.

- A. $8m$
- B. $12m$
- C. 8
- D. $4m$

Question 8

Mia keeps track of her market stall takings in a spreadsheet. Cell A2 holds the number of items sold and cell B2 holds the price of one item, in dollars. Cell C2 must show the total takings. Which rule should Mia type into cell C2?

- A. $=A2+B2$
- B. $=A2 \times B2$
- C. $=B2 * B2$
- D. $=A2 * B2$

Question 9

A printing business charges a \$150 set-up fee plus \$25 to print each T-shirt. Which rule gives the cost C , in dollars, of printing n T-shirts?

- A. $C = 150 + 25n$
- B. $C = 175n$
- C. $C = 25 + 150n$
- D. $C = 25n$

Question 10

A car wash pays \$27 for each hour worked. The table shows the pay for different numbers of hours.

Hours worked	1	2	3	4
Pay (\$)	27	54	?	108

Made-up data: the labels and every number in this table are made up for this test.

What pay belongs in the gap?

- A. \$27
- B. \$81
- C. \$108
- D. \$162

Question 11

A sports club is saving for new uniforms. The club starts with \$35 in a tin and adds \$18 to the tin at the end of every week. How much is in the tin at the end of week 4?

- A. \$107
- B. \$89
- C. \$72
- D. \$630

Question 12

A market stall sells bags of nuts for \$14 each. The takings rule is $C = 14n$, where n is the number of bags sold and C is the takings, in dollars. What is C when $n = 6$?

- A. \$146.00
- B. \$20.00
- C. \$84.00
- D. \$8.40

End of Test A

Test A — Answers

Que stio n	1	2	3	4	5	6	7	8	9	10	11	12
Ans wer	B	C	D	B	C	D	A	D	A	B	A	C

Test A — Fully-worked solutions

Question 1. Price per metre = total cost $\div$ metres = $46.80 \div 4 = 11.70$, so the price is \$11.70 per metre. (\$187.20 multiplies instead of dividing, \$50.80 adds the two numbers, and \$0.09 divides the wrong way round — $4 \div 46.80$ is about 0.09, which would be metres per dollar, not dollars per metre.) **Answer B.**

Question 2. First change the time to hours: 45 minutes is 0.75 hr. Speed = distance $\div$ time = $9 \div 0.75 = 12$, so the speed is 12 km/hr. (20 km/hr comes from treating 45 minutes as 0.45 hr, 0.2 km/hr divides kilometres by minutes without converting, and 405 multiplies instead of dividing.) **Answer C.**

Question 3. Cost = rate $\times$ length = $5.80 \times 4.5 = 26.10$, so the cost is \$26.10. (\$23.20 multiplies by 4 and forgets the extra half metre, \$10.30 adds the two numbers, and \$261.00 slips the decimal point — it is 58×4.5 , not 5.80×4.5 .) **Answer D.**

Question 4. Area = length $\times$ width = $7.5 \times 3.2 = 24$, so the area is 24 m². (21.4 m is the perimeter, not the area, 10.7 m² adds the length and the width, and 240 m² slips the decimal point — it is 7.5×32 .) **Answer B.**

Question 5. Multiply the hours by the hourly rate first, then add the fee once: $48 \times 3.5 = 168$, then $75 + 168 = 243$, so the cost is \$243.00. (\$430.50 wrongly adds $75 + 48$ first and multiplies the lot by 3.5, \$310.50 multiplies the fee by the hours and adds the hourly rate once, and \$123.00 adds the two numbers and forgets the 3.5 hours.) **Answer C.**

Question 6. Cost of 1 hour = total cost $\div$ hours = $168 \div 4.8 = 35$, so the cost is \$35.00 per hour. (\$21.00 divides by 8 instead of 4.8, \$806.40 multiplies instead of dividing, and \$163.20 subtracts.) **Answer D.**

Question 7. 6 m and 2 m are like terms — both count m — so the numbers add and the letter stays: $6m + 2m = 8m$. (12 m multiplies the numbers, 4 m subtracts them, and 8 drops the letter, which loses the units.) **Answer A.**

Question 8. Total takings = number of items $\times$ price per item, and a spreadsheet writes $\times$ as $*$, so the rule is $=A2*B2$. ($=A2+B2$ adds instead of multiplying, $=A2*B2$ uses a times sign a spreadsheet does not understand, and $=B2*B2$ multiplies the price by itself.) **Answer D.**

Question 9. The \$150 set-up fee is paid once, whatever the number of shirts, and \$25 is charged for each shirt, so $C = 150 + 25n$. ($C = 25 + 150n$ swaps the fee and the rate, $C = 175n$ adds the two numbers into one rate and charges the fee per shirt, and $C = 25n$ leaves out the set-up fee.) **Answer A.**

Question 10. The pay rises by \$27 at every step: $54 + 27 = 81$. Check with the next step: $81 + 27 = 108$, which matches the table, so the gap is \$81. (Equally, 3 hours at \$27 per hour is $27 \times 3 = 81$.) (\$27 subtracts $54 - 27$, \$108 copies the hour-4 value into the gap, and \$162 multiplies the hour-2 pay by 3.) **Answer B.**

Question 11. By the end of week 4 the club has added \$18 four times: $4 \times 18 = 72$, then add the starting \$35 once: $35 + 72 = 107$, so the tin holds \$107. (\$89 adds only three lots of \$18, \$72 forgets the starting \$35, and \$630 multiplies 35×18 .) **Answer A.**

Question 12. $C = 14n$ means $14 \times n$. Substitute $n = 6$: $C = 14 \times 6 = 84$, so the takings are \$84.00. (\$146.00 writes the 14 and the 6 side by side — but $14n$ means $14 \times n$, not the number one hundred and forty-six, \$20.00 adds $14 + 6$, and \$8.40 slips the decimal point.) **Answer C.**

Chapter 05 — Test B (written response)

Chapter 5 (Relationships). Materials: paper and a basic calculator. There are 8 written questions, worth 42 marks in total. Approximate time: 40 minutes. Answer every question in the spaces provided and show your working. Where an answer needs rounding, the question states the precision — round only as instructed. Every business, price and measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1 (5 marks)

A market stall sells ribbon by the metre. Every metre costs the same. A customer buys 6 m of ribbon for \$31.50.

- a.** Find the price of 1 metre of ribbon. 1 mark

- b.** Find the cost of 4.5 m of ribbon at this price, to the nearest cent. 2 marks

- c.** Another customer spends \$47.25 on ribbon at this price. How many metres of ribbon does she get? 2 marks

Question 2 (5 marks)

A delivery van travels 126 km at a steady speed. The trip takes 2 hours 15 minutes.

- a.** Write 2 hours 15 minutes as a decimal number of hours. 1 mark

- b.** Show that the van's speed is 56 km/hr. 2 marks

- c.** At this speed, how far does the van travel in 45 minutes? 2 marks

Question 3 (5 marks)

A rectangular concrete slab is poured for a garden shed. The slab is 5.4 m long and 3.6 m wide. Use the formulae $A = l \times w$ and $P = 2 \times (l + w)$.

a. Find the area of the slab, in m^2 .

2 marks

b. Find the perimeter of the slab, in metres.

1 mark

c. Timber formwork is placed all the way around the slab before the concrete is poured. The formwork costs \$9.80 per metre. Find the cost of the formwork.

2 marks

Question 4 (6 marks)

Two hire shops rent out wood chippers.

- **Option A:** a \$88 delivery fee plus \$24 for each hour.
- **Option B:** no fee, \$57 for each hour.

a. Write a formula for the cost C , in dollars, of hiring under Option A for h hours.

1 mark

b. Find the cost of each option for a 2-hour job.

2 marks

c. The job actually takes 6 hours. Which option is cheaper for a 6-hour job, and by how much?

3 marks

Question 5 (5 marks)

Sam is training at a sign-making shop. His job card reads:

Cost of the job: $C = 45 + n \times 30$, where n is the number of hours of work.

Timber: $4b + 3b + 2c$, where b is a bag of cement mix and c is a carton of sealant.

a. Rewrite the cost rule using the shorthand conventions.

1 mark

b. Write $4b + 3b + 2c$ as simply as possible.

1 mark

c. Explain why the $2c$ cannot be joined onto the b terms.

1 mark

d. On a delivery note Sam writes the cost of 3 tins of paint at \$8 each as \$38. Explain his mistake and find the correct cost.

2 marks

Question 6 (5 marks)

A bottling machine fills bottles at a steady pace of 15 bottles per minute. The table shows the number of bottles filled after different numbers of minutes.

Time (min)	0	5	10	15	20
Bottles filled	0	75	150	?	?

Made-up data: the labels and every number in this table are made up for this test.

a. Copy the table into the space below and fill in both gaps.

2 marks

b. How many more bottles are filled for each extra 5 minutes?

1 mark

c. Is the change in this table steady? Explain your answer.

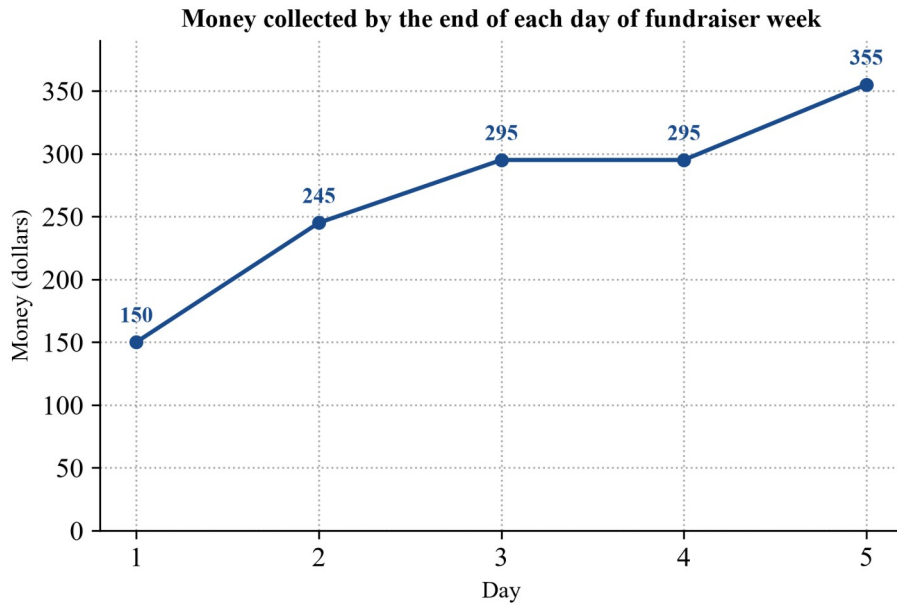
1 mark

d. Write a rule in words for the number of bottles filled.

1 mark

Question 7 (5 marks)

The line graph shows the money collected by a school fundraiser at the end of each day of its week.



a. How much money had been collected by the end of day 2?

1 mark

b. Between which two days was the line flat?

1 mark

c. Between which two days did the money rise the most? Show how you checked.

2 marks

d. Did the money rise by the same amount each day? Explain your answer.

1 mark

Question 8 (6 marks)

A coach travels at a steady speed. The distance rule is $D = s \times t$, where D is the distance travelled in km, s is the speed in km/hr and t is the time in hours.

a. The coach travels at $s = 85$ km/hr for $t = 2$ hours. Show that $D = 170$ km.

2 marks

b. On the next leg, $s = 74$ km/hr and $t = 1.25$ hours. Find D .

2 marks

c. A student says the coach travels 200 km in 2.5 hours at a steady speed of 85 km/hr. Use substitution to check this claim.

2 marks

End of Test B

Test B — Answers

Q1. (a) \$5.25 per metre (b) \$23.63 (to the nearest cent) (c) 9 m. **Q2.** (a) 2.25 hr (b) $126 \div 2.25 = 56$ km/hr, shown (c) 42 km. **Q3.** (a) 19.44 m^2 (b) 18 m (c) \$176.40. **Q4.** (a) $C = 88 + 24h$ (b) Option A \$136, Option B \$114 (c) Option A, by \$110. **Q5.** (a) $C = 45 + 30n$ (b) $7b + 2c$ (c) $2c$ counts cartons, not bags — only like terms can be joined (d) the $\times$ between two numbers was dropped; \$38 reads as thirty-eight, and the correct cost is $3 \times 8 = \$24$. **Q6.** (a) 225 and 300 (b) 75 more bottles (c) yes — the rise is 75 at every step (d) start at 0 and add 75 for every extra 5 minutes (or: bottles = $15 \times$ minutes). **Q7.** (a) \$245 (b) days 3 and 4 (c) days 1 to 2 — a rise of \$95 (d) no — the rises are \$95, \$50, \$0 and \$60, which are not all the same. **Q8.** (a) $D = 85 \times 2 = 170$ km, shown (b) 92.5 km (c) $85 \times 2.5 = 212.5$ km, not 200 km — the claim is wrong.

Test B — Fully-worked solutions

Question 1.

a. Price per metre = total cost $\div$ metres:

$$31.50 \div 6 = 5.25$$

The price is \$5.25 per metre.

b. Cost = rate $\times$ length:

$$5.25 \times 4.5 = 23.625$$

Carry the full precision and round only at the end: to the nearest cent, the third decimal place is 5, so round up. The cost is \$23.63.

c. Length = total cost $\div$ price per metre:

$$47.25 \div 5.25 = 9$$

Check: $5.25 \times 9 = 47.25$ ✓. She gets 9 m of ribbon.

Question 2.

a. 15 minutes is $15 \div 60 = 0.25$ hr, so 2 hours 15 minutes is 2.25 hr.

b. Speed = distance $\div$ time. The distance is 126 km and the time is 2.25 hr:

$$126 \div 2.25 = 56$$

So the speed is 56 km/hr, which is what we were asked to show.

c. 45 minutes is 0.75 hr. Distance = speed $\times$ time:

$$56 \times 0.75 = 42$$

The van travels 42 km.

Question 3.

a. $A = l \times w = 5.4 \times 3.6 = 19.44$

The area is 19.44 m².

b. $P = 2 \times (l + w) = 2 \times (5.4 + 3.6) = 2 \times 9 = 18$

The perimeter is 18 m.

c. The formwork runs all the way around the slab, so its length is the perimeter, 18 m. Cost = rate $\times$ length:

$$18 \times 9.80 = 176.40$$

The formwork costs \$176.40.

Question 4.

a. The \$88 fee is paid once and \$24 is charged for each hour, so $C = 88 + 24h$.

b. Option A: $C = 88 + 24 \times 2 = 88 + 48 = 136$, so \$136. Option B: $57 \times 2 = 114$, so \$114.

c. Option A: $C = 88 + 24 \times 6 = 88 + 144 = 232$, so \$232. Option B: $57 \times 6 = 342$, so \$342. Compare: $232 < 342$, so Option A is cheaper, by $342 - 232 = \$110$.

Question 5.

- a. Write the number first and drop the $\times$ between the number and the letter: $n \times 30$ becomes $30n$. The rule is $C = 45 + 30n$.
- b. $4b$ and $3b$ are like terms — both count bags — so $4b + 3b = 7b$. The $2c$ counts something else, so it stays separate: $7b + 2c$.
- c. Only like terms can be joined. The b terms count bags of cement mix and the $2c$ counts cartons of sealant — different things — so $7b$ and $2c$ cannot be added into one term. $7b + 2c$ is already as simple as it gets.
- d. The $\times$ between two numbers was dropped. As written, \$38 reads as thirty-eight dollars. Sam meant 3 times \$8: $3 \times 8 = 24$, so the correct cost is \$24. The $\times$ can only be dropped between a number and a letter — never between two numbers.

Question 6.

- a. The machine fills 15 bottles per minute, so every 5 minutes adds $15 \times 5 = 75$ bottles:

Time (min)	0	5	10	15	20
Bottles filled	0	75	150	225	300

The gaps are 225 and 300. Check: $150 + 75 = 225$ and $225 + 75 = 300$ ✓.

- b. 75 more bottles for each extra 5 minutes.
- c. Yes — the change is steady, because the count rises by the same amount, 75, at every step of the table.
- d. Start at 0 and add 75 for every extra 5 minutes. (Equally: bottles filled = $15 \times$ number of minutes.)

Question 7.

- a. The point above day 2 is labelled 245, so \$245 had been collected by the end of day 2.
- b. The line is flat between days 3 and 4 — the value stays at \$295.
- c. Work out the rise between each pair of days: day 1 to day 2: $245 - 150 = 95$; day 2 to day 3: $295 - 245 = 50$; day 3 to day 4: $295 - 295 = 0$; day 4 to day 5: $355 - 295 = 60$. The biggest rise is \$95, so the money rose most between days 1 and 2.
- d. No — the rises are \$95, \$50, \$0 and \$60, which are not all the same, so the money did not rise by the same amount each day. The change is not steady.

Question 8.

- a. Substitute $s = 85$ and $t = 2$ into $D = s \times t$:

$$D = 85 \times 2 = 170$$

So $D = 170$ km, which is what we were asked to show.

- b. Substitute $s = 74$ and $t = 1.25$:

$$D = 74 \times 1.25 = 92.5$$

The coach travels 92.5 km.

- c. Substitute $s = 85$ and $t = 2.5$ into $D = s \times t$:

$$D = 85 \times 2.5 = 212.5$$

The rule gives 212.5 km, not 200 km, so the student's claim is wrong — the coach would actually travel 212.5 km.