

VCE VM Numeracy Unit 1

Chapter 3 — Shape

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Chapter 3 Shape — 3A Two-dimensional shapes: properties and names

About the data in this section. The job sheets, floor plans and measurements here are **made up** (not real): pretend jobs and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Theory

Why shapes.

Every trade and every workplace runs on shapes: a floor tile, a door, a road sign, a panel of glass, a garden bed, a nut that fits a bolt. To order the right piece, check a job, or follow a plan, you need to say exactly which shape you mean. Naming a shape is not just a label — the name carries the shape's **properties**, the list of things that are always true about it. Once you know the properties, you can check a piece of material before you cut it, and you can spot a mistake when a piece comes back wrong.

Two-dimensional shapes.

A **two-dimensional shape** (a **2D shape**) is a flat shape. It has length and width but no thickness at all, so it lies completely flat on a sheet of paper. The flat face of an object is a 2D shape even when the object itself is solid: a floor tile is a square face, a door has a rectangle face, a clock has a circle face. This section is about those flat faces — the shapes you can draw on paper without lifting your pencil off the page.

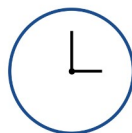
Flat faces of everyday objects



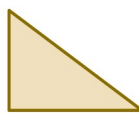
square
floor tile



rectangle
door face



circle
clock face



triangle
sandwich cut
corner to corner



hexagon
nut for a bolt

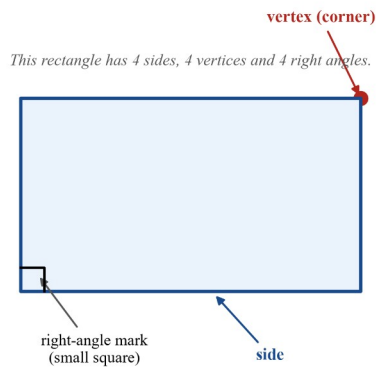
The parts of a shape: sides, vertices and right angles.

Three words do most of the work in this section.

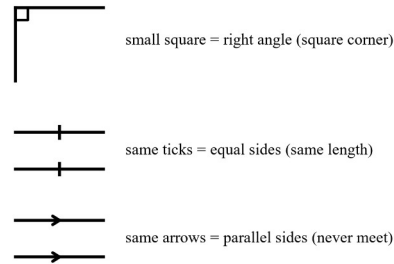
- A **side** is one straight edge of a shape.
- A **vertex** (say “VER-tex”) is a corner where two sides meet. More than one corner is **vertices** (“VER-tih-seez”).
- A **right angle** is a square corner — the corner shape of a door or a sheet of paper. In drawings a right angle is marked with a **small square** in the corner.

Drawings also use two shortcuts. **Tick marks** (short lines across a side) show equal sides: sides with the same ticks are the same length. **Arrow marks** on two sides show **parallel** sides (explained below). A **property** of a shape is anything you can say that is always true about it, such as “a square has four equal sides”.

Two sides of a shape are **opposite sides** when they face each other across the shape, and **next to each other** when they meet at a corner.



Marks used in drawings

**Refresher. Counting around a shape.**

To count sides or vertices without missing one or counting one twice: put your pencil on one corner and mark it lightly, travel once around the shape in one direction, count each side as you cross it and each corner as you reach it, and stop the moment you return to your mark. Try it on a hexagon: starting at any corner and travelling once around gives 6 sides and 6 vertices — and this always happens with polygons, because every side ends at the next vertex, so the two counts match.

Curved shapes: the circle.

A **circle** is the one shape in this section with no straight sides at all: its edge is one smooth curve that closes up on itself. A circle has 0 straight sides and 0 vertices. Because it has no straight sides, a circle is not a **polygon** — the family name for closed shapes made only of straight sides.

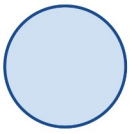
Polygons: shapes with straight sides.

A **polygon** is a closed shape whose sides are all straight. Polygons are named by how many sides they have, and the vertex count is always the same number:

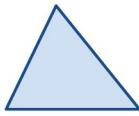
Straight sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
8	octagon

Two memory hooks: **hexagon** and **six** both start with a hissing sound and both mean 6; an **octopus** has 8 arms and an **octagon** has 8 sides. The gallery below shows every shape named in this section, in the order of the table plus the circle and the two four-sided shapes that come next.

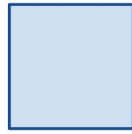
The 2D shapes named in this section



circle



triangle



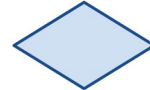
square



rectangle



parallelogram



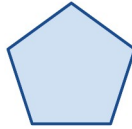
rhombus



trapezium



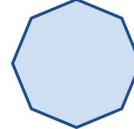
kite



pentagon



hexagon



octagon

Triangles.

A **triangle** is a polygon with 3 sides and 3 vertices — the smallest number of sides a closed shape can have. Triangles come in many looks: wide and flat, tall and thin, or tilted over. The look does not change the name: 3 straight sides and 3 vertices means triangle, whatever the angle of the tilt.

Parallel sides.

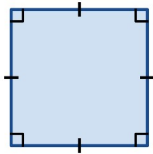
Two sides are **parallel** when they stay the same distance apart along their whole length, so they never meet however far you keep drawing them longer. Think of the two rails of a railway track: side by side, the same gap all the way, never touching. In drawings, parallel sides carry the same **arrow mark**; a second pair of parallel sides on the same shape gets double arrows. When a shape's sides are not parallel, they will meet somewhere if you keep drawing them longer.

The quadrilateral family.

A **quadrilateral** is any polygon with 4 sides and 4 vertices. The six quadrilaterals in this section are told apart by three properties only: right angles, equal sides, and parallel sides.

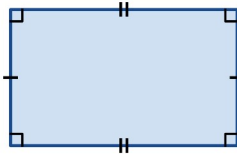
Shape	Properties (always true)
square	4 equal sides, 4 right angles
rectangle	4 right angles, opposite sides equal
parallelogram	opposite sides equal and parallel, no right angles
rhombus	4 equal sides, opposite sides parallel, no right angles
trapezium	exactly 1 pair of parallel sides
kite	2 pairs of equal sides next to each other, no parallel sides

The quadrilateral family: names come from properties



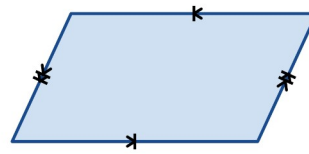
square

4 equal sides, 4 right angles



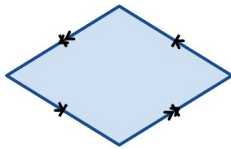
rectangle

4 right angles, opposite sides equal



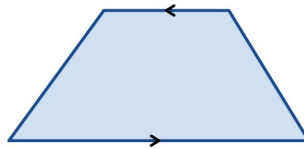
parallelogram

opposite sides equal and parallel, no right angles



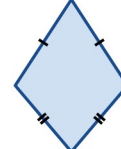
rhombus

4 equal sides, opposite sides parallel, no right angles



trapezium

exactly 1 pair of parallel sides



kite

2 pairs of equal sides next to each other, no parallel sides

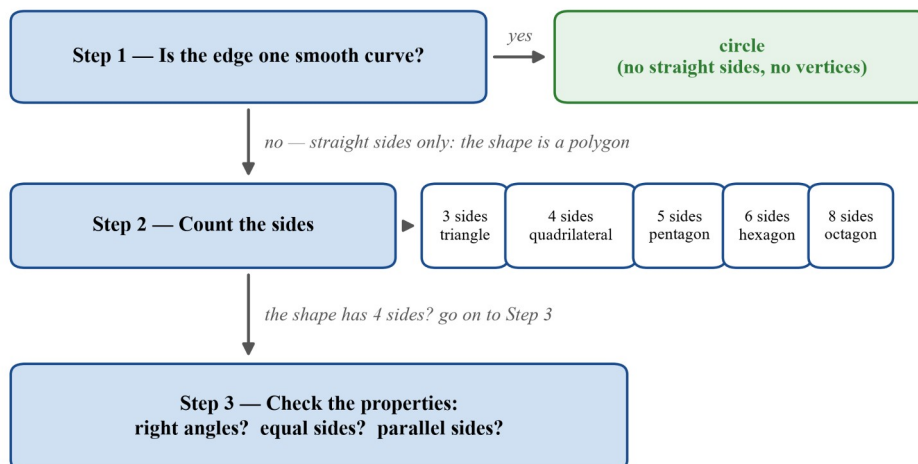
Three naming rules keep the family tidy in this section. A **trapezium** means a quadrilateral with **exactly one** pair of parallel sides — a shape with two parallel pairs is a parallelogram or a rhombus, not a trapezium. The **parallelogram** and the **rhombus** drawn and named here have corners that are not right angles; if a shape like that grew square corners it would be a rectangle or a square instead. And a **square is a special rectangle**: it has everything a rectangle has, plus all four sides equal, so “rectangle” is never wrong for a square — “square” is simply the more exact name.

How to classify any 2D shape.

To **classify** a shape means to put its exact name to it. Three steps are enough for every shape in this section.

1. **Look at the edge.** One smooth curve and no corners? The shape is a circle.
2. **Count the sides** (counting around once, as in the refresher). The count gives the family: 3 triangle, 4 quadrilateral, 5 pentagon, 6 hexagon, 8 octagon.
3. **If the shape has 4 sides, check the properties** — right angles? equal sides? parallel sides? — and read the name off the quadrilateral table.

Classify any 2D shape in three steps

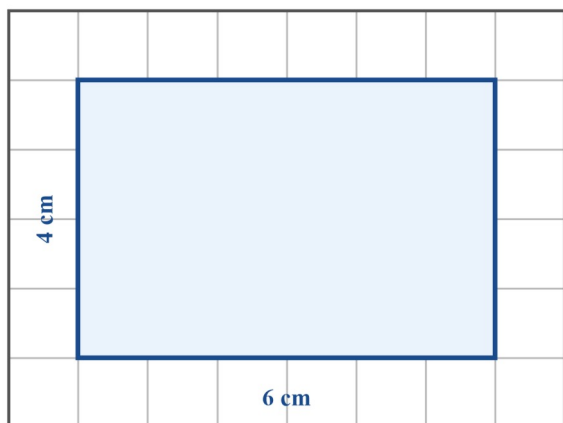


- **square** 4 equal sides + 4 right angles
 - **rectangle** 4 right angles, opposite sides equal
 - **parallelogram** opposite sides equal and parallel
 - **rhombus** 4 equal sides, no right angles
 - **trapezium** exactly 1 pair of parallel sides
 - **kite** 2 pairs of equal sides next to each other
- A square is a special rectangle: a rectangle with all 4 sides equal.*

Refresher. Reading lengths on grid paper.

On grid paper where each small square is 1 cm wide and 1 cm tall, a length is read by counting squares, not corners: put your pencil on one end of the side, count one for every square you cross, and stop at the other end. On the rectangle below the bottom side crosses 6 squares, so it is 6 cm long; the left side crosses 4 squares, so it is 4 cm tall. Counting corners instead of squares is the classic mistake — it always gives one too many.

A rectangle drawn on grid paper

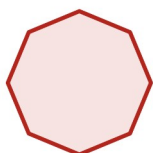


Each small square is 1 cm wide and 1 cm tall.

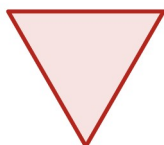
Shapes in signs.

Street signs are sorted by shape before they are sorted by colour, because the shape carries the meaning. A **STOP sign** is an **octagon** — 8 sides. A **GIVE WAY sign** is a **triangle pointing down** — 3 sides. **Pedestrian signs** are **diamonds**, which is a square standing on one corner: 4 equal sides and 4 right angles, just tilted. **Information signs** are **rectangles**. Knowing the shape names lets you describe a sign to someone without pointing at it.

Sign shapes in the street



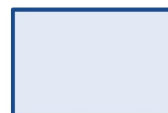
STOP sign
octagon — 8 sides



GIVE WAY sign
triangle — 3 sides



pedestrian signs
diamond — a square
standing on one corner



information signs
rectangle

Technology for drawing and checking shapes.

A drawing app or a geometry program on a phone, tablet or computer can draw these shapes for you and let you check them: drag a corner and watch the side counts and parallel marks update, or ask the program to measure a side. These tools sit in the digital toolkit from Chapter 1 (§1F), alongside the calculators and apps you already use. Technology is a shortcut, never a requirement: every drawing and every check in this section can be done with pencil, ruler and grid paper, and a sketch with the right marks (ticks, arrows, small squares) says everything an app would say.

Key ideas.

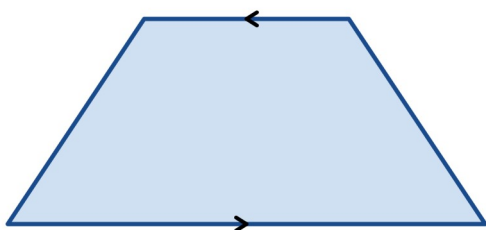
- A 2D shape is flat: length and width, no thickness. The flat face of an object is a 2D shape.
- Sides are the straight edges; vertices are the corners where two sides meet; a right angle is a square corner, marked with a small square in drawings.
- Tick marks show equal sides; arrow marks show parallel sides, which stay the same distance apart and never meet.

- A circle has 0 straight sides and 0 vertices. Every other shape named here is a polygon: closed, straight sides only.
- Family names come from the side count: 3 triangle, 4 quadrilateral, 5 pentagon, 6 hexagon, 8 octagon — and the vertex count matches the side count.
- Quadrilateral names come from properties: right angles, equal sides, parallel sides. A trapezium has exactly 1 pair of parallel sides; a square is a special rectangle.
- On 1 cm grid paper, count squares (not corners) to read a length.
- A drawing app or geometry program can draw and check shapes, but pencil, ruler and grid paper are always enough.

Worked examples

Example 1 — scaffolded

A worker cuts a piece from a board and the leftover piece has the shape below. The matching arrow marks show its only pair of parallel sides. (a) How many sides and vertices does the piece have? (b) Which family does it belong to? (c) Give the most exact name for the shape. (d) Does the piece have any right angles? How do you know from the drawing?



Working

Comment

(a) counting once around: 4 sides and 4 vertices

mark one corner, count each side and corner once, stop back at the mark

(b) 4 sides, so the quadrilateral family

the family name comes from the side count

(c) exactly 1 pair of parallel sides (the arrowed pair), so a trapezium

the other two sides would meet if drawn longer, so there is only the one parallel pair

(d) no right angles: no corner carries the small-square mark

a right angle is always marked with a small square; this drawing has none

Example 2 — scaffolded

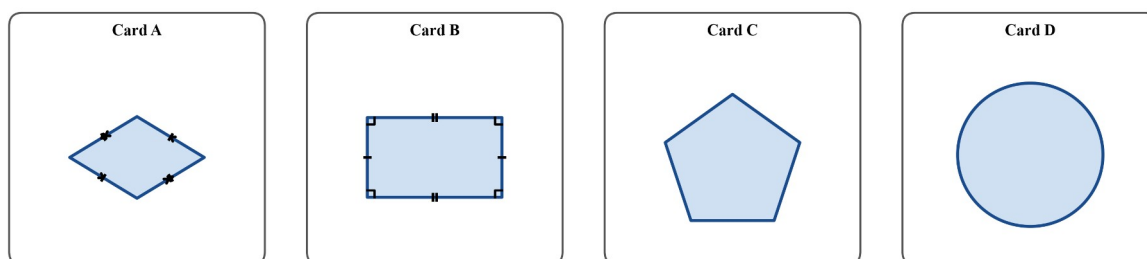
Each clue below describes exactly one shape from the gallery in the theory. (a) I have 4 sides, all four the same length, and 4 right angles. (b) I have 6 sides and 6 vertices. (c) I am a quadrilateral with exactly 1 pair of parallel sides. (d) True or false: a square has every property a rectangle has. Explain.

Working	Comment
(a) 4 equal sides + 4 right angles = square	both properties are needed: a rhombus has 4 equal sides but no right angles
(b) 6 sides and 6 vertices = hexagon	in a polygon the vertex count always matches the side count
(c) exactly 1 parallel pair = trapezium	“exactly 1” rules out the square, rectangle, parallelogram and rhombus, which all have 2 parallel pairs
(d) true: a square has 4 right angles and equal opposite sides, so it meets every rectangle property	a square is a special rectangle — one with all 4 sides equal

Example 3 — unscaffolded

A new worker sorts four shape cards into a teaching kit. The cards use the marks from the theory: ticks for equal sides, arrows for parallel sides, small squares for right angles. (a) Name the shape on each card. (b) Which cards show quadrilaterals? (c) Which card shows a shape that is not a polygon? Explain why.

Four shape cards



Card A has 4 sides, all carrying the same single tick, so all 4 sides are equal; both pairs carry arrow marks, so both pairs are parallel; and no corner has a small square, so there are no right angles. Four equal sides, parallel opposite sides, no right angles: Card A is a **rhombus**. Card B has 4 sides with a small square in every corner, so 4 right angles, and two styles of tick (double on top and bottom, single on the ends), so the opposite sides are equal but not all four: Card B is a **rectangle**. Card C has 5 straight sides: a **pentagon**. Card D’s edge is one smooth curve with no straight sides and no vertices: a **circle**.

(b) Quadrilaterals are the 4-sided shapes, so Card A and Card B. Card C has 5 sides and Card D has none.

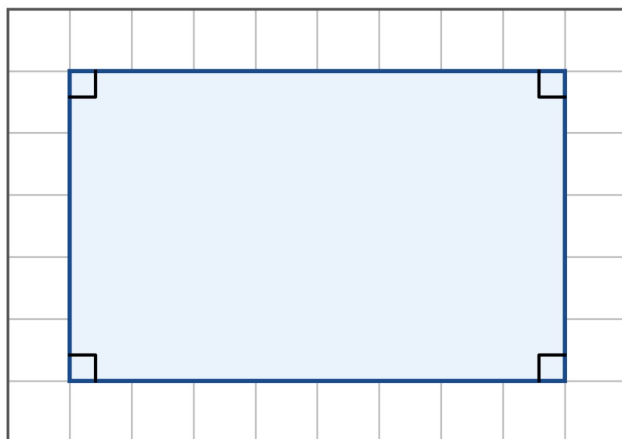
(c) A polygon is closed and made of straight sides only. Card D’s edge is a curve, so the circle on Card D is not a polygon.

∴ Card A is a rhombus, B a rectangle, C a pentagon and D a circle; the quadrilaterals are A and B; and D is not a polygon because its edge is one smooth curve, not straight sides.

Example 4 — unscaffolded

A worker marks out a piece of glass on grid paper before cutting it. Each small square on the grid is 1 cm. (a) Read the width and the height of the piece from the grid. (b) Name the shape as exactly as you can. (c) The job changes: the piece must now be as tall as it is wide, 8 cm by 8 cm. What is the most exact name for the new shape?

The glass panel on grid paper



Each small square = 1 cm.

- (a) Counting squares along the bottom side gives 8, so the piece is 8 cm wide. Counting up the left side gives 5, so it is 5 cm tall.
- (b) The piece has 4 sides, a small-square mark in all 4 corners (so 4 right angles), and opposite sides equal — 8 cm top and bottom, 5 cm at the ends — but not all four sides equal. That is exactly the rectangle row of the table.
- (c) At 8 cm by 8 cm the piece still has 4 right angles, and now all 4 sides are equal as well. A rectangle with all four sides equal is a square, and “square” is the more exact name.

∴ the marked piece is 8 cm wide and 5 cm tall and is a rectangle; redrawn 8 cm by 8 cm it has 4 right angles and 4 equal sides, so its most exact name is square.

Practice questions

Scaffolded

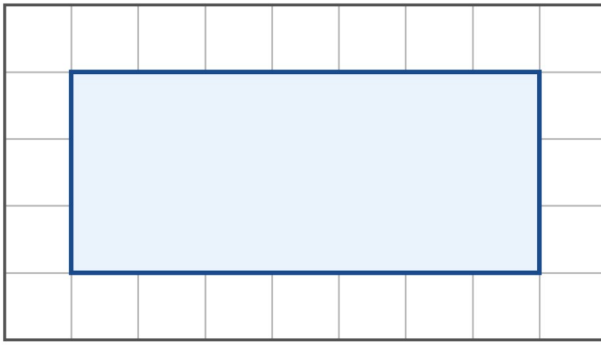
Q1. Name the shape from its properties. (a) 4 right angles and all 4 sides equal. (b) 5 straight sides. (c) No straight sides and no vertices.

Q2. (a) How many sides does a hexagon have? (b) How many vertices does an octagon have? (c) How many sides does a triangle have?

Q3. Choose from this word bank: square, rectangle, parallelogram, trapezium. (a) A quadrilateral with exactly 1 pair of parallel sides. (b) A quadrilateral with opposite sides equal and parallel, and no right angles. (c) A quadrilateral with 4 right angles, 2 long sides and 2 shorter sides.

Q4. The shape below is drawn on grid paper. Each small square is 1 cm.

The shape drawn on grid paper



Each small square = 1 cm.

- (a) Read the length of the bottom side in cm.
- (b) Read the length of a short side in cm.
- (c) Name the shape as exactly as you can.

Q5. Say whether each statement is true or false, and give a reason. (a) Every square is a rectangle. (b) Every rectangle is a square. (c) A circle is a polygon.

Q6. (a) A STOP sign is an octagon. How many sides does it have? (b) A GIVE WAY sign is a triangle. How many vertices does it have? (c) What is the family name for shapes with 3 sides?

Un scaffolded

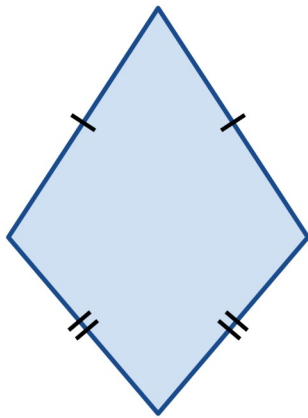
Q7. (a) Name the shape with 8 sides and 8 vertices. (b) How many more sides does an octagon have than a pentagon? (c) Name the quadrilateral with 4 equal sides and no right angles.

Q8. Here is a list of shapes: triangle, square, circle, trapezium, hexagon, rectangle. (a) Write down the quadrilaterals in the list. How many are there? (b) Which shape in the list is not a polygon? (c) How many shapes in the list have 6 sides?

Q9. Draw, or describe carefully, a shape with all of these properties: opposite sides equal and parallel, no right angles, and not all four sides equal. (a) Sketch the shape (or describe it in words). (b) Name the shape you drew. (c) How many vertices does it have?

Q10. A floor tile is a square with sides of 10 cm. (a) How many right angles does the tile have? (b) How many of its sides are 10 cm long? (c) A rectangle tile also has 4 right angles. How are its sides different from the square tile's sides?

Q11. The shape below has tick marks showing which sides are equal.



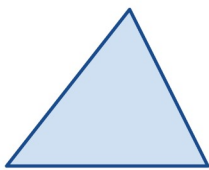
- (a) Name the shape.
- (b) How many pairs of equal sides do the tick marks show?
- (c) Does the drawing show any right-angle marks?

Q12. Compare a square and a rhombus. (a) What do they both have in common about their sides? (b) What does the square have that the rhombus does not? (c) What family name do they both belong to?

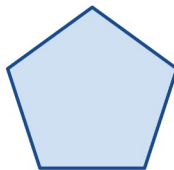
Q13. A workshop job sheet describes three pieces. Name the shape of each. (a) “Round opening, no corners at all.” (b) “Plate with 6 straight sides.” (c) “Panel with 4 square corners, 2 long sides and 2 shorter sides.”

Q14. A drawing app on a tablet shows a shape with opposite sides equal and parallel, corners that are not right angles, and not all four sides equal. (a) Name the shape. (b) Name one everyday tool, with no screen, that you could use to draw this shape on paper. (c) Name one kind of technology that could draw or check this shape.

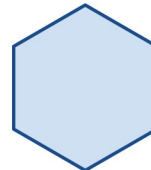
Q15. Three shapes are drawn below.



Shape X



Shape Y



Shape Z

- (a) Count the vertices of Shape X, Shape Y and Shape Z.
- (b) Put the shapes in order from fewest sides to most sides.
- (c) Which of the three shapes are quadrilaterals?

Q16. The face of a door is 2 m tall and 1 m wide, with square corners. (a) Name the shape of the door face. (b) How many right angles does it have? (c) Are all four sides equal? Explain.

Q17. Copy and complete the table.

Shape	Straight sides	Vertices
triangle	3	(a)
hexagon	(b)	6
circle	(c)	0

Q18. A garden plan shows three garden beds: Bed A is a square, Bed B is a trapezium and Bed C is a circle. (a) How many vertices does each bed's shape have? (b) Which bed's shape has no vertices at all? (c) How many vertices do Bed A and Bed B have altogether?

Answers

Q1. (a) square. (b) pentagon. (c) circle. **Q2.** (a) 6. (b) 8. (c) 3. **Q3.** (a) trapezium. (b) parallelogram. (c) rectangle. **Q4.** (a) 7 cm. (b) 3 cm. (c) rectangle. **Q5.** (a) true — a square has all 4 right angles and equal opposite sides, so it meets every rectangle property. (b) false — a rectangle with 2 long and 2 shorter sides does not have 4 equal sides, so it is not a square. (c) false — a polygon has straight sides only, and a circle's edge is one smooth curve. **Q6.** (a) 8. (b) 3. (c) triangle. **Q7.** (a) octagon. (b) $8 - 5 = 3$. (c) rhombus. **Q8.** (a) square, trapezium, rectangle — 3 of them. (b) the circle. (c) 1 (the hexagon). **Q9.** (a) see the sketch in the solutions. (b) parallelogram. (c) 4. **Q10.** (a) 4. (b) 4. (c) the rectangle tile has 2 longer and 2 shorter sides; the square tile's 4 sides are all the same length. **Q11.** (a) kite. (b) 2 pairs. (c) no — no corner carries the small-square mark. **Q12.** (a) both have 4 equal sides. (b) the square has 4 right angles; the rhombus has none. (c) quadrilateral. **Q13.** (a) circle. (b) hexagon. (c) rectangle. **Q14.** (a) parallelogram. (b) pencil and ruler on paper (grid paper works too). (c) a drawing app or a geometry program. **Q15.** (a) X 3, Y 5, Z 6. (b) X, Y, Z. (c) none — no shape in the figure has 4 sides. **Q16.** (a) rectangle. (b) 4. (c) no — the opposite sides are equal (2 sides of 2 m and 2 sides of 1 m), so not all four are equal. **Q17.** (a) 3. (b) 6. (c) 0. **Q18.** (a) A 4, B 4, C 0. (b) Bed C. (c) $4 + 4 = 8$.

Fully-worked solutions

Q1. (a) 4 right angles and 4 equal sides is the square row of the table. (b) 5 straight sides means pentagon. (c) No straight sides and no vertices means the circle.

Q2. (a) A hexagon has 6 sides (hex- means 6). (b) An octagon has 8 vertices, matching its 8 sides. (c) A triangle has 3 sides.

Q3. (a) Exactly 1 pair of parallel sides is the trapezium property. (b) Opposite sides equal and parallel with no right angles is the parallelogram property. (c) 4 right angles with 2 long and 2 shorter sides is the rectangle property (a square would need all four sides equal).

Q4. (a) Counting squares along the bottom side: 7 squares, so 7 cm. (b) Counting squares up a short side: 3 squares, so 3 cm. (c) The shape has 4 sides, 4 right angles and opposite sides equal (7 cm and 3 cm), but not all four sides equal: a rectangle.

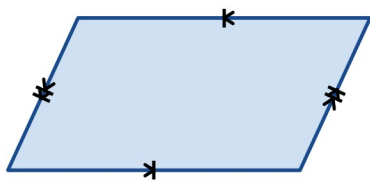
Q5. (a) True. A rectangle needs 4 right angles and equal opposite sides; a square has both, plus more, so every square counts as a rectangle. (b) False. A rectangle only needs its opposite sides equal, so a rectangle with 2 long and 2 shorter sides is not a square. Squares are the special case, not the whole family. (c) False. A polygon is a closed shape with straight sides only. A circle's edge is one smooth curve with 0 straight sides, so it is not a polygon.

Q6. (a) An octagon has 8 sides, so the STOP sign has 8. (b) A triangle has 3 vertices, so the GIVE WAY sign has 3. (c) The 3-sided family is the triangle family.

Q7. (a) 8 sides and 8 vertices: octagon. (b) Octagon 8 sides, pentagon 5 sides: $8 - 5 = 3$ more sides. (c) 4 equal sides with no right angles: rhombus (a square would need the right angles).

Q8. (a) Quadrilaterals have 4 sides: square, trapezium and rectangle — 3 shapes. (b) The circle has no straight sides, so it is the one shape that is not a polygon. (c) Only the hexagon has 6 sides: 1 shape.

Q9. (a) One possible sketch is below: a tilted four-sided shape with the top and bottom parallel and equal, the two ends parallel and equal, no small-square marks, and the ends shorter than the top and bottom.



One possible answer

- (b) Opposite sides equal and parallel, no right angles, and not all four sides equal is exactly the parallelogram row of the table. (A rhombus is ruled out because it has all four sides equal.)
 (c) Every quadrilateral has 4 vertices, so the shape has 4.

Q10. (a) A square has 4 right angles, so the tile has 4. (b) All 4 sides of a square are equal, so all 4 sides are 10 cm long. (c) A rectangle has only its opposite sides equal: 2 longer sides and 2 shorter sides. The square tile is the special rectangle whose four sides are all the same length.

Q11. (a) The shape has 4 sides, 2 pairs of equal sides next to each other (single ticks on the top pair, double ticks on the bottom pair) and no parallel sides: a kite. (b) Two pairs: the single-tick pair and the double-tick pair. (c) No. A right angle would carry a small-square mark in the corner, and the drawing shows none.

Q12. (a) Both shapes have 4 sides all the same length. (b) The square has 4 right angles; the rhombus has none (its corners are not square corners). (c) Both have 4 sides, so both are quadrilaterals.

Q13. (a) Round with no corners: 0 straight sides and 0 vertices — a circle. (b) 6 straight sides — a hexagon. (c) 4 sides with 4 square corners and 2 long plus 2 shorter sides — a rectangle.

Q14. (a) Opposite sides equal and parallel, corners not right angles, not all four sides equal: parallelogram. (b) Pencil and ruler on plain paper, or grid paper to keep the opposite sides equal — no screen needed. (c) A drawing app or a geometry program on a phone, tablet or computer could draw the shape and check its sides and parallel marks.

Q15. (a) Counting around each shape: Shape X (triangle) 3 vertices, Shape Y (pentagon) 5 vertices, Shape Z (hexagon) 6 vertices. (b) Fewest sides to most: X (3), then Y (5), then Z (6). (c) None. A quadrilateral has 4 sides, and the side counts here are 3, 5 and 6.

Q16. (a) 4 sides, square corners, and two different side lengths (2 m and 1 m): a rectangle. (b) A rectangle has 4 right angles, one in each corner. (c) No. Opposite sides are equal — 2 sides of 2 m and 2 sides of 1 m — so the four sides are not all the same length. (If they were, the door face would be a square.)

Q17. (a) A triangle has 3 vertices, matching its 3 sides. (b) A hexagon has 6 sides, matching its 6 vertices. (c) A circle has 0 straight sides (its edge is one smooth curve).

Q18. (a) Bed A (square) has 4 vertices; Bed B (trapezium) has 4 vertices; Bed C (circle) has 0 vertices. (b) Bed C: a circle has no corners at all. (c) Bed A and Bed B together: $4 + 4 = 8$ vertices.

Chapter 3 Shape — 3B Everyday three-dimensional objects

Theory

Solids all around you.

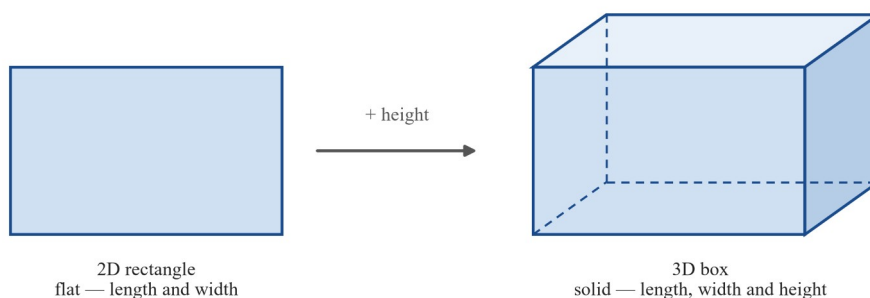
Look around any room, workshop or shop. Most of the objects you can pick up are **three-dimensional** solids: boxes on a truck, drink cans, bricks, tents, pencils, drums. This section names those everyday solids, counts their parts, and shows how every one of them is connected to flat two-dimensional shapes.

Two-dimensional and three-dimensional.

A **two-dimensional** (2D) shape is flat, like a drawing on a page. It has two measurements: length and width.

A **three-dimensional** (3D) solid is thick as well as long and wide. It has three measurements: length, width and height. You can hold a 3D solid in your hand. A 2D shape can only be looked at.

A 2D shape is flat; a 3D solid has thickness



On the left is a 2D rectangle — flat, like a drawing of a box on paper. On the right is a 3D box — a solid you can pack things into.

Refresher. The 2D shapes you need here.

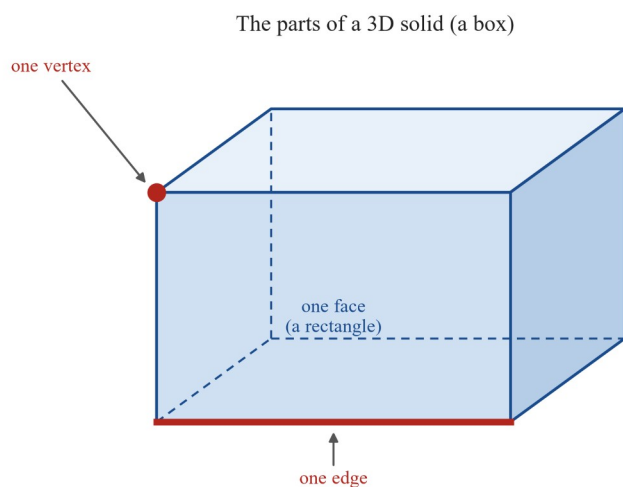
Section 3A names the 2D shapes. This quick table refreshes the five shapes this section uses:

2D shape	Sides	Everyday example
square	4 straight sides, all the same length	the face of a die
rectangle	4 straight sides, opposite sides the same length	the face of a box
triangle	3 straight sides	the end of a camping tent
circle	no straight sides — one round edge	the lid of a can
hexagon	6 straight sides	the end of a pencil

Faces, edges and vertices.

Every 3D solid is built from parts you can count.

- A **face** is one surface of a solid. Each face of a box is flat and is a rectangle.
- An **edge** is a line where two faces meet.
- A **vertex** is a corner point where edges meet. The plural of vertex is **vertices**.



The picture shows one face, one edge and one vertex of a box.

Refresher. Count without missing any.

Counting the parts of a solid goes wrong in two ways: a part gets missed, or a part gets counted twice. The fix is to count **in order**, and to tick each part off as you go. Two orders that work:

- count in matching pairs: top and bottom, front and back, left and right;
- count in groups: the edges around the top, then around the bottom, then the standing edges that join them.

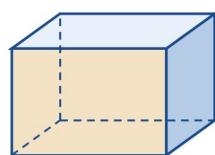
Prisms.

A **prism** is a 3D solid that:

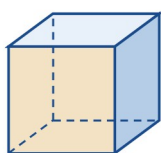
- has two ends that are the same 2D shape and the same size;
- is the same thickness all the way through — a slice across the solid, anywhere along it, is the same shape and size as the end;
- has sides that are all rectangles.

A prism is named after the shape of its ends:

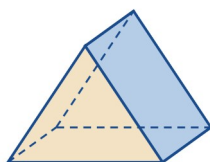
Prisms are named after the shape of their ends (shaded)



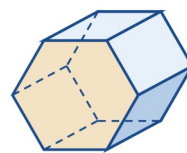
Rectangular prism (box)
ends: rectangles



Cube
ends: squares



Triangular prism
ends: triangles



Hexagonal prism
ends: hexagons

In each picture the shaded face is one of the two matching ends.

Solid	Shape of the ends	Everyday examples
rectangular prism	rectangle	cereal box, brick, shipping container

Solid	Shape of the ends	Everyday examples
cube	square	die, some gift boxes
triangular prism	triangle	camping tent, some chocolate boxes
hexagonal prism	hexagon	pencil, some candles

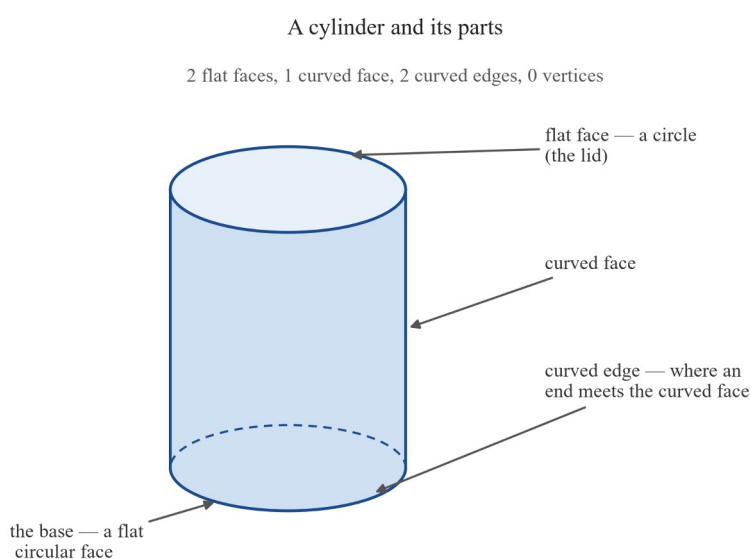
A **box** is the everyday name for a rectangular prism. A **cube** is a special box: all 6 of its faces are the same square.

When the end of a prism is a shape with all sides the same length — a square, or a hexagon with equal sides — the solid is called a **regular prism**. A cube is a regular prism, and so is a pencil whose hexagon ends have equal sides.

Cylinders.

A **cylinder** is a 3D solid with:

- two ends that are the same circle, the same size;
- the same thickness all the way through — a slice across it is always a circle;
- one curved face joining the two ends.



Drink cans, paint tins, drums and round water tanks are cylinders.

A cylinder is like a prism because it is the same thickness all the way through, but its long face is curved instead of being made of rectangles, so it gets its own name. The two flat ends are faces, and each of these faces is a circle.

Where each flat end meets the curved face there is a **curved edge**: a cylinder has 2 curved edges. A cylinder has **no vertices** — there is no corner point anywhere on it.

Faces, edges and vertices: the summary table.

Solid	Faces	Edges	Vertices
box (rectangular prism)	6 rectangles	12	8
cube	6 squares	12	8
triangular prism	2 triangles + 3 rectangles = 5	9	6
hexagonal prism	2 hexagons + 6 rectangles = 8	18	12
cylinder	2 flat circles + 1 curved = 3	2 curved	0

Here is the box counted in order, the way the refresher above suggests. The faces come in 3 matching pairs — top and bottom, front and back, left and right — so the box has $3 \times 2 = 6$ faces. The edges come in 3 groups of 4 — around the top, around the bottom, and the standing edges that join them — so $4 + 4 + 4 = 12$ edges. The vertices come in 2 layers of 4 — 4 on top and 4 on the bottom — so $4 + 4 = 8$ vertices.

Any prism can be counted the same way. A hexagonal prism has a hexagon at each end: 6 edges around the top end, 6 around the bottom end and 6 standing edges give $6 + 6 + 6 = 18$ edges, and 6 vertices on top with 6 on the bottom give $6 + 6 = 12$ vertices.

Refresher. Repeated addition is multiplication.

When several identical solids are counted together, the total can be found by adding the same number again and again — or by multiplying. The two ways always agree:

$$6 + 6 + 6 = 3 \times 6 = 18$$

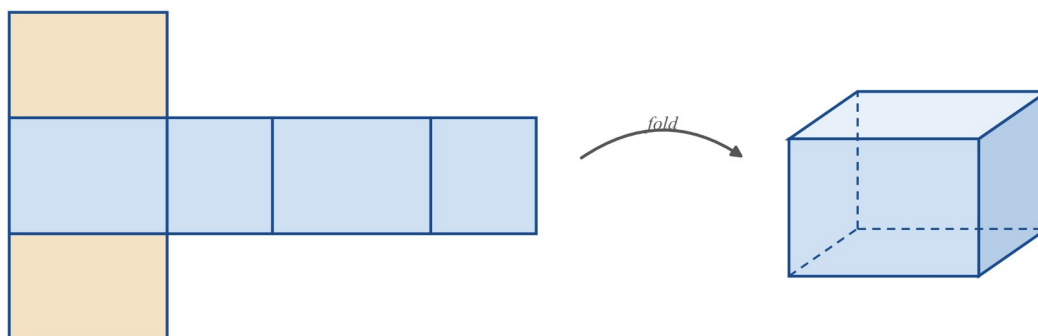
So 3 boxes have $3 \times 6 = 18$ faces altogether. Multiplication is the quick way to write repeated addition (the multiplication facts from 2B do the work).

Nets: a 3D solid folded flat.

A **net** is a 2D pattern that folds up to make a 3D solid. Every face of the solid appears as one 2D shape in the net. The solid is made by folding the net along the lines where the shapes join.

The net of a box is 6 rectangles:

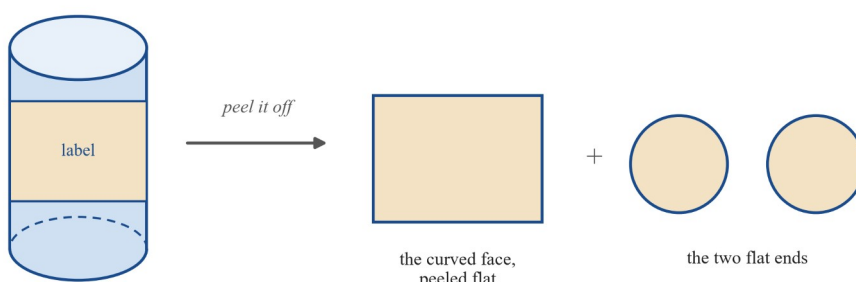
Folding the net along the joins makes the solid



The net of a box: 6 rectangles

The net of a cylinder is 1 rectangle and 2 circles. You can see this on any can: the paper label wrapped around the can is a rectangle once it is peeled off and laid flat, and the lid and the base are the 2 circles.

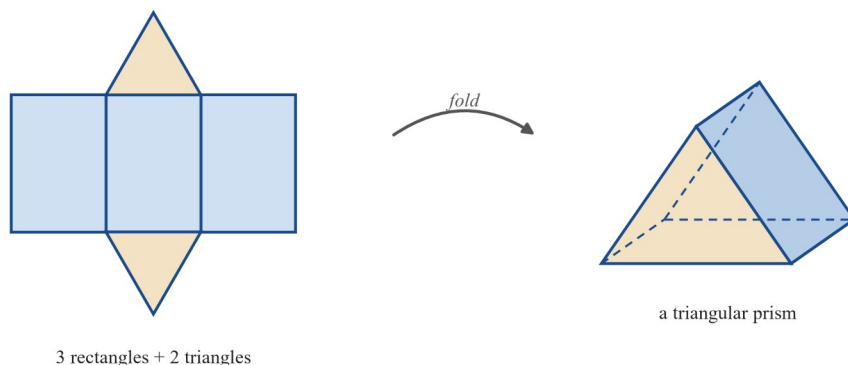
A cylinder and its net



The net of a cylinder: 1 rectangle + 2 circles

The net of a triangular prism is 3 rectangles and 2 triangles:

The net of a triangular prism

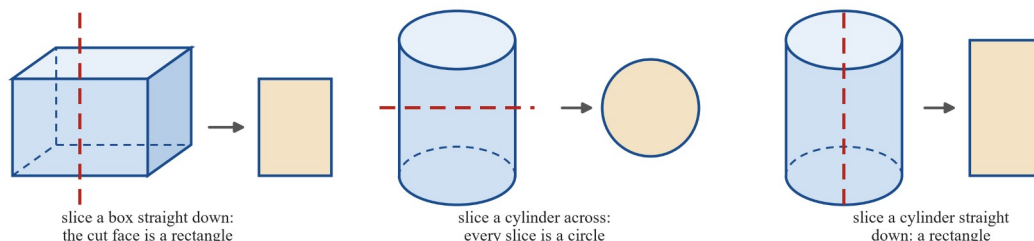


Nets show why 2D and 3D shapes belong together: **every face of a 3D solid is a 2D shape**. Count the 2D shapes in a net and you have counted the faces of the solid — 6 shapes for the box, 3 for the cylinder and 5 for the triangular prism.

Slices: 2D shapes inside 3D solids.

Cutting straight through a solid shows a flat 2D shape on the cut face. This slice is also called a **cross-section**: it is a 2D shape sitting inside a 3D solid.

Straight cuts show 2D shapes inside 3D solids



- Slice straight down through a box, the way a stick of butter is cut: the cut face is a **rectangle**.
- Slice across a cylinder, the way a cucumber is cut into rounds: every slice is a **circle**.
- Slice straight down through the middle of a cylinder: the cut face is a **rectangle**.

Slicing is the second connection between 2D and 3D: a net shows the outside faces as 2D shapes, and a slice shows a 2D shape on the inside too.

Creating 3D shapes.

There are three simple ways to create the solids of this section.

- **Fold a net.** Draw the net on paper or cardboard, cut it out, fold along the joins, and tape the edges together.
- **Draw it.** Draw the front face first, then draw the depth lines going back and join them up — that is how the pictures in this section are drawn.
- **Use technology.** A drawing app on a computer or tablet can create these solids and turn them around, so you can look at every face.

Every task in this section can also be done with pen, paper, scissors and tape — no device is ever needed.

Key ideas.

- A 2D shape is flat (length and width); a 3D solid has thickness as well (length, width and height).
- The parts of a solid: a **face** is a surface, an **edge** is a line where two faces meet, and a **vertex** is a corner point where edges meet (plural: **vertices**).
- A **prism** has two ends that are the same 2D shape and size, is the same thickness all the way through, and has rectangle sides. It is named by its end: rectangular, triangular or hexagonal prism. A box is a rectangular prism; a cube has 6 square faces.
- A **cylinder** has 2 matching circular ends, 1 curved face, 2 curved edges and 0 vertices.
- Box: 6 faces, 12 edges, 8 vertices. Triangular prism: 5 faces, 9 edges, 6 vertices. Hexagonal prism: 8 faces, 18 edges, 12 vertices.
- A **net** is a 2D pattern that folds into a 3D solid. Box net = 6 rectangles. Cylinder net = 1 rectangle + 2 circles. Triangular-prism net = 3 rectangles + 2 triangles. Every face of the solid is one 2D shape in the net.
- A straight cut shows a **slice** (cross-section): a box sliced straight down gives a rectangle; a cylinder sliced across gives a circle; a cylinder sliced straight down through the middle gives a rectangle.

Worked examples

Example 1 — scaffolded

A family is moving house. One moving box is a rectangular prism. (a) How many faces does the box have? (b) How many edges does it have? (c) How many vertices does it have? (d) What 2D shape is each face?

Working	Comment
(a) faces come in matching pairs: top and bottom, front and back, left and right — 3 pairs, so $3 \times 2 = 6$ faces	Counting faces in pairs stops you missing one and stops you counting one twice.
(b) 4 edges around the top, 4 around the bottom, 4 standing edges joining them: $4 + 4 + 4 = 12$ edges	Sort the edges into 3 groups of 4.
(c) 4 vertices on the top face and 4 on the bottom face: $4 + 4 = 8$ vertices	The corners come in a top layer and a bottom layer.
(d) every face is a rectangle	The solid is named after this shape: a rectangular prism.

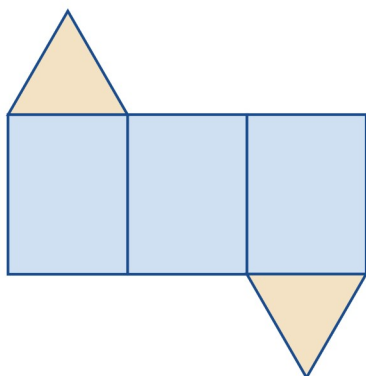
Example 2 — scaffolded

A tin of paint sits on a shelf. The tin is a cylinder. (a) How many flat faces does it have, and what 2D shape is each one? (b) How many curved faces does it have? (c) How many edges does it have, and what kind are they? (d) How many vertices does it have?

Working	Comment
(a) 2 flat faces — the lid and the base — and each one is a circle	The two ends of a cylinder are matching circles.
(b) 1 curved face, wrapped around between the two ends	The curved face joins the lid to the base.
(c) 2 edges, both curved: one where the lid meets the curved face, and one where the base meets it	An edge is a line where two faces meet.
(d) 0 vertices	A cylinder has no corner points anywhere.

Example 3 — unscaffolded

A net is cut from cardboard and folded along its joins to make a closed 3D solid.



Name the solid this net makes, and say how the number of 2D shapes in the net is connected to the solid.

The net holds 3 rectangles and 2 triangles, so it has $3 + 2 = 5$ flat shapes altogether.

The 2 triangles are the same size and shape, and they sit at the top and bottom of the net, so they fold up to cap the two ends of the solid. A solid with two matching triangular ends and rectangle sides is a triangular prism. The 3 rectangles wrap around to become its 3 sides.

A triangular prism has 5 faces, and the net has 5 shapes: each 2D shape in the net folds up to become one face of the solid.

$\therefore$ the net folds into a triangular prism, and its 5 2D shapes are exactly the 5 faces of the solid — every face of the solid is one shape in the net.

Example 4 — unscaffolded

At a market stall, 3 identical cube-shaped crates are stacked together. (a) What 2D shape is each face of a crate? (b) How many faces do the 3 crates have altogether? (c) How many edges do the 3 crates have altogether?

Every face of a cube is a square, and a cube has 6 faces, so each crate has 6 square faces.

For the totals, use repeated addition or multiplication — the two agree. One cube has 6 faces, so

$$6 + 6 + 6 = 3 \times 6 = 18 \text{ faces}$$

One cube has 12 edges, so

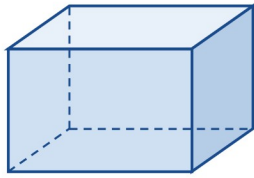
$$12 + 12 + 12 = 3 \times 12 = 36 \text{ edges}$$

$\therefore$ each face of a crate is a square, and the 3 crates have 18 faces and 36 edges altogether.

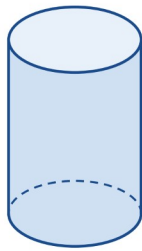
Practice questions

Scaffolded

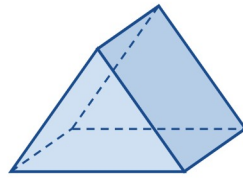
Q1. Name each solid in the picture.



A



B



C

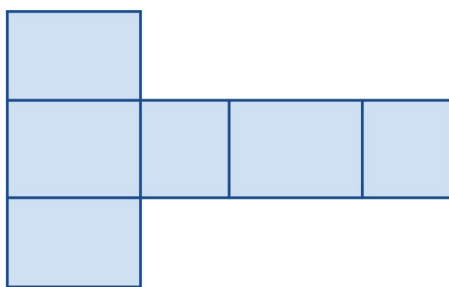
- (a) Solid A
- (b) Solid B
- (c) Solid C

Q2. A die is a cube. (a) What 2D shape is each face of the die? (b) How many faces does the die have? (c) How many vertices does the die have?

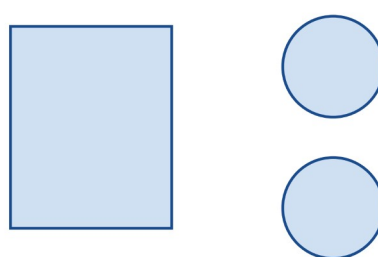
Q3. A cereal box is a rectangular prism. (a) How many edges does one cereal box have? (b) How many faces does one cereal box have? (c) How many faces do 2 of these cereal boxes have altogether?

Q4. A tin of soup is a cylinder. (a) How many flat faces does the tin have, and what 2D shape is each one? (b) How many curved faces does the tin have? (c) How many vertices does the tin have?

Q5. The picture shows two nets.



Net A



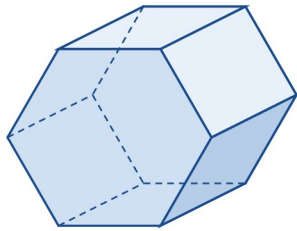
Net B

- (a) Which 3D solid does Net A fold up to make?
- (b) How many rectangles are in Net A?
- (c) Name the 2D shapes in Net B.

Q6. Think about slicing solids with one straight cut. (a) A box is sliced straight down, the way a stick of butter is cut. What 2D shape is the cut face? (b) A cylinder is sliced across, the way a cucumber is cut into rounds. What 2D shape is every slice? (c) A cylinder is sliced straight down through the middle. What 2D shape is the cut face?

Unscaffolded

Q7. A gift box is a hexagonal prism.



- (a) How many faces does the gift box have altogether?
- (b) How many of its faces are rectangles, and how many are hexagons?
- (c) How many edges does the gift box have?

Q8. A camping tent is a triangular prism. (a) How many triangular faces does the tent have? (b) How many rectangular faces does the tent have? (c) How many vertices does the tent have altogether?

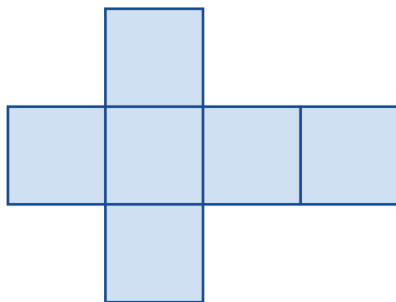
Q9. A farm water tank is a cylinder. (a) How many curved edges does the tank have? (b) The tank is cut straight across. What 2D shape is the cut face? (c) How many flat faces does the tank have?

Q10. The paper label is peeled off a cylindrical tin and laid flat. (a) What 2D shape is the label? (b) What 2D shape is each of the tin's two lids? (c) The label and the two lids together are the net of which 3D solid?

Q11. A moving truck carries 5 identical box-shaped crates and 1 cylindrical drum. (a) Show that the 5 crates have 60 edges altogether, one step per line. (b) How many vertices do the 5 crates have altogether? (c) How many edges does the drum have?

Q12. Say whether each sentence is true or false. For a false sentence, give the correct fact. (a) A cylinder has 2 vertices. (b) The two ends of a prism are the same size and the same shape. (c) Every face of a rectangular prism is a rectangle.

Q13. The net below is made of identical squares.



- (a) Name the 3D solid this net folds up to make.
- (b) How many squares are in the net?
- (c) Say how the squares in the net are connected to the faces of the solid.

Q14. In a kitchen, a cucumber is shaped like a cylinder and a block of butter is shaped like a rectangular prism. (a) The cucumber is cut into rounds. What 2D shape is every round? (b) The butter is cut straight down. What 2D shape is the cut face? (c) Name the 3D solid each food is shaped like.

Q15. Six everyday objects are: a tin of tomatoes, a brick, a camping tent, a pencil, a drum, and a cereal box. (a) Which two objects are cylinders? (b) Which two objects are rectangular prisms? (c) Name the 3D solid of the camping tent, and of the pencil.

Q16. Mai draws a net for a triangular prism. (a) How many rectangles will her net have? (b) How many triangles will her net have? (c) After the net is drawn and cut out, what does she do to make the 3D solid?

Q17. (a) What 2D shape is every face of a cube? (b) A triangular prism has 5 faces. How many 2D shapes are in its net? (c) In one sentence, say how a net shows the relationship between 2D shapes and a 3D solid.

Q18. A shop window display stacks 4 identical cube-shaped boxes, with 2 cylindrical paint tins standing beside them. (a) Name the two different 3D solids in the display. (b) How many edges do the 4 cube-shaped boxes have altogether? (c) How many flat faces do the 2 paint tins have altogether?

Answers

Q1. (a) A: rectangular prism (a box). (b) B: cylinder. (c) C: triangular prism. **Q2.** (a) square. (b) 6. (c) 8. **Q3.** (a) 12. (b) 6. (c) 12 ($2 \times 6 = 12$). **Q4.** (a) 2 flat faces, each a circle. (b) 1. (c) 0. **Q5.** (a) rectangular prism (a box). (b) 6. (c) 1 rectangle and 2 circles. **Q6.** (a) rectangle. (b) circle. (c) rectangle. **Q7.** (a) 8. (b) 6 rectangles and 2 hexagons. (c) 18. **Q8.** (a) 2. (b) 3. (c) 6. **Q9.** (a) 2. (b) circle. (c) 2. **Q10.** (a) rectangle. (b) circle. (c) cylinder. **Q11.** (a) shown in solutions ($5 \times 12 = 60$). (b) 40. (c) 2. **Q12.** (a) false — a cylinder has 0 vertices. (b) true. (c) true. **Q13.** (a) cube. (b) 6. (c) each square folds up to become one face; 6 squares = 6 faces. **Q14.** (a) circle. (b) rectangle. (c) cucumber: cylinder; butter: rectangular prism (a box). **Q15.** (a) the tin of tomatoes and the drum. (b) the brick and the cereal box. (c) triangular prism; hexagonal prism. **Q16.** (a) 3. (b) 2. (c) fold it along the joins so the rectangles wrap around and the triangles cap the ends, then tape the edges together. **Q17.** (a) square. (b) 5 — one 2D shape for each face. (c) every face of the solid is one 2D shape in the net, so folding the net makes the solid. **Q18.** (a) cube and cylinder. (b) 48 ($4 \times 12 = 48$). (c) 4 ($2 \times 2 = 4$).

Fully-worked solutions

Q1. (a) Solid A has rectangle faces and two matching rectangle ends: it is a rectangular prism — the everyday name is a box. (b) Solid B has two circular ends and one curved face: it is a cylinder. (c) Solid C has two matching triangular ends and rectangle sides: it is a triangular prism.

Q2. (a) Every face of a cube is the same square. (b) A cube has 6 faces — 3 matching pairs. (c) A cube has 8 vertices: 4 on the top layer and 4 on the bottom layer, $4 + 4 = 8$.

Q3. (a) A rectangular prism has 12 edges: 4 around the top, 4 around the bottom and 4 standing edges joining them, $4 + 4 + 4 = 12$. (b) It has 6 faces: 3 matching pairs (top and bottom, front and back, left and right). (c) $2 \times 6 = 12$, so 2 cereal boxes have 12 faces altogether.

Q4. (a) The tin has 2 flat faces — the lid and the base — and each one is a circle. (b) It has 1 curved face, wrapped around between the two ends. (c) It has 0 vertices: a cylinder has no corner points anywhere.

Q5. (a) Net A is made of 6 rectangles, and it folds into a rectangular prism — a box. (b) Net A has 6 rectangles: 4 wrap around the sides and 2 cap the ends. (c) Net B has 1 rectangle and 2 circles — it is the net of a cylinder.

Q6. (a) The cut face of the box is a rectangle. (b) Every round of cucumber is a circle. (c) Cutting a cylinder straight down through the middle shows a rectangle.

Q7. (a) The ends are hexagons and the sides are rectangles: 6 rectangles + 2 hexagons = $6 + 2 = 8$ faces. (b) 6 faces are rectangles and 2 faces are hexagons. (c) Count edges in groups: 6 around the top end, 6 around the bottom end and 6 standing edges, so $6 + 6 + 6 = 18$ edges.

Q8. (a) The two ends of the tent are triangles: 2 triangular faces. (b) The three sides are rectangles: 3 rectangular faces. (c) The vertices come in two layers: 3 on the front triangle and 3 on the back triangle, so $3 + 3 = 6$ vertices.

Q9. (a) A cylinder has 2 curved edges: one where the top end meets the curved face, and one where the bottom end meets it. (b) A slice straight across a cylinder is a circle. (c) A cylinder has 2 flat faces — the two circular ends.

Q10. (a) Peeled off and laid flat, the label is a rectangle. (b) Each lid is a circle. (c) The label and the 2 lids make 1 rectangle and 2 circles — that is the net of a cylinder.

Q11. (a) One step per line:

A crate is a box, and a box has 12 edges.

5 crates: $5 \times 12 = 60$

$\therefore$ the 5 crates have 60 edges altogether.

(b) A box has 8 vertices, so $5 \times 8 = 40$: the 5 crates have 40 vertices altogether.

(c) The drum is a cylinder, and a cylinder has 2 curved edges, so the drum has 2 edges.

Q12. (a) False. A cylinder has 0 vertices — it has no corner points. (b) True. The two ends of a prism are the same 2D shape and the same size. (c) True. Every face of a rectangular prism is a rectangle — that is where its name comes from.

Q13. (a) The net is made of squares, and it folds into a cube. (b) Count them in order: 4 in the row, 1 above and 1 below — 6 squares. (c) Each square folds up to become one face of the cube: 6 squares = 6 faces.

Q14. (a) The cucumber is a cylinder, so every round slice is a circle. (b) The butter is a box, so the straight cut face is a rectangle. (c) The cucumber is shaped like a cylinder; the butter is shaped like a rectangular prism (a box).

Q15. (a) The cylinders are the tin of tomatoes and the drum — each has two circular ends and one curved face. (b) The rectangular prisms are the brick and the cereal box — each is a box with 6 rectangle faces. (c) The camping tent is a triangular prism; the pencil is a hexagonal prism.

Q16. (a) 3 rectangles — one for each side of the prism. (b) 2 triangles — the two matching ends. (c) She folds the net along the joins, so the 3 rectangles wrap around the sides and the 2 triangles cap the two ends, then tapes the edges together.

Q17. (a) A cube has 6 faces, and every face is a square. (b) A triangular prism has 5 faces (2 triangles + 3 rectangles), so its net has 5 2D shapes — one for each face. (c) Every face of the 3D solid appears as one 2D shape in the net, so folding the 2D net along its joins makes the 3D solid.

Q18. (a) The cube-shaped boxes are cubes, and the paint tins are cylinders. (b) One cube has 12 edges, so $4 \times 12 = 48$: the 4 boxes have 48 edges altogether. (c) One cylinder has 2 flat faces, so $2 \times 2 = 4$: the 2 paint tins have 4 flat faces altogether.

Theory

The study design lists this topic as key knowledge for the unit: simple reflection, rotation and symmetry, worked with shapes you know from everyday life (Unit 1, Area of Study 2).

Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-08.

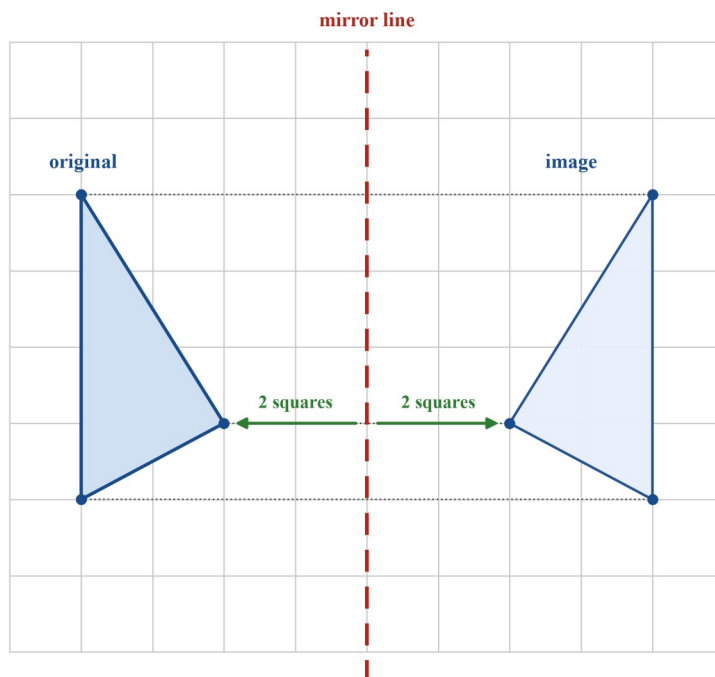
Reflection, rotation and symmetry are the ways a shape can be flipped, turned, or seen to match itself. *Reflection* is a flip in a mirror line; *rotation* is a turn about a point; and when a shape folds onto itself, or turns onto itself part-way through a full turn, the shape is said to have **symmetry** — mirror-line symmetry when the fold works, turn symmetry when the turn works. They are everywhere in ordinary life: a logo that folds neatly in half, a tile that looks the same after a turn, a word on a T-shirt that reads the same in a mirror. This section teaches all three ideas with everyday shapes, on square grid paper, one step at a time.

About the data in this section. The logos, cards, tile walls and fabrics in this section are **made up** (not real): pretend shops, pretend games and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Refresher: the shapes you need. This section uses four familiar shapes. A *corner* is where two sides of a shape meet.

- **Square** — 4 sides, 4 corners, all sides the same length.
- **Rectangle** — 4 sides, 4 corners, opposite sides the same length, every corner a square corner.
- **Triangle** — 3 sides, 3 corners. A triangle with **all sides equal** has 3 equal sides; a triangle with **two sides equal** has a pair of equal sides and one different side.
- **Hexagon** — 6 sides, 6 corners. The hexagons in this section have all sides the same length.

Reflection: flipping a shape in a mirror line. To *reflect* a shape is to flip it across a line, called the **mirror line**. Every corner of the shape jumps straight across the mirror line to the other side, and lands the **same distance** from the line as it started. The flipped shape is called the **image**.



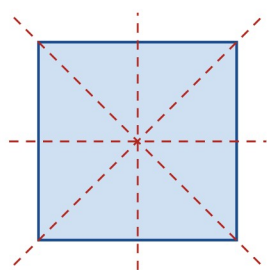
The tip is 2 squares left of the mirror line, so its image is 2 squares right.

Read the picture above on the square grid. The tip of the flag sits 2 squares left of the mirror line, so the tip of the image sits 2 squares right of the mirror line, on the same row. The other two corners do the same jump: straight across, same distance, same row. Three things are always true of a flip:

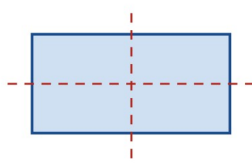
- the image is the **same size and same shape** as the original;
- the image faces the **opposite way**, like a mirror picture;
- every corner keeps its **distance** from the mirror line — only the side changes.

Refresher: reading a square grid. Grid paper is made of little squares. To say how far a point is from a line, count whole squares in a straight line — across for a left-right distance, up or down for a top-bottom distance. Points in this section sit where grid lines cross, so every distance is a whole number of squares. “3 squares left of the mirror line” always means: start at the point, count 3 squares straight across, and you are on the mirror line.

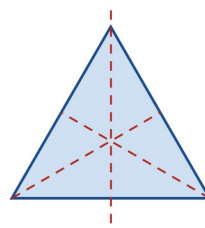
Mirror lines: folding a shape onto itself. A **mirror line** of a shape is a line you could fold the shape along so the two halves land exactly on each other. A shape can have several mirror lines, one, or none at all. The fold test is the quickest way to find them: picture the fold, and check that every part of the shape has a matching part on the other side.



square
4 mirror lines



rectangle
2 mirror lines



triangle with all sides equal
3 mirror lines

The counts for the familiar shapes are worth knowing:

Shape	Mirror lines	Times it matches itself in one full turn
square	4	4
rectangle	2	2
triangle with all sides equal	3	3
triangle with two sides equal	1	1 (only when the full turn finishes)
hexagon with all sides equal	6	6

Rotation: turning a shape. To *rotate* a shape is to turn it about a point. In this book turns are named by the fraction of a full turn they cover, and each name has a degree name as well:

Turn	Degree name
quarter turn	90°
half turn	180°
three-quarter turn	270°
full turn	360°

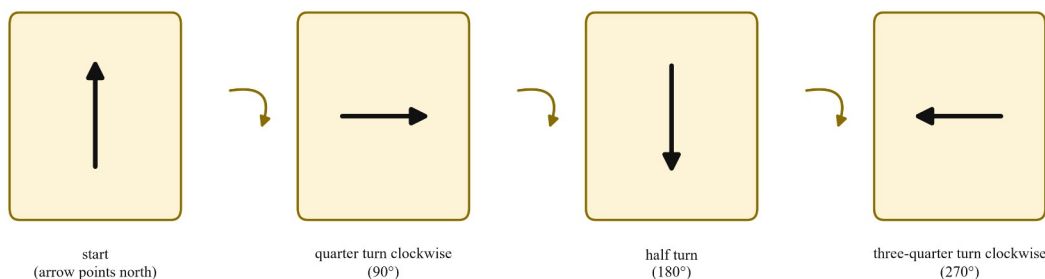
In this section the degree names are only *names* for turn sizes — a quarter turn is a 90° turn because a full turn is called 360° . There is no angle measuring here: no tool, no protractor, no reading scales. Every turn in this section is a quarter, half, three-quarter or full turn, and you can see it on a clock face.

Refresher: clockwise and anticlockwise. The hands of a clock move one way — from 12 to 3 to 6 to 9. That way is called **clockwise**. The opposite way is called **anticlockwise**. A quarter turn clockwise takes the clock hand from 12 to 3; a half turn takes it from 12 to 6; a full turn brings it home to 12. When a question says a turn is clockwise, picture the clock hands; when it says anticlockwise, picture them running backwards.

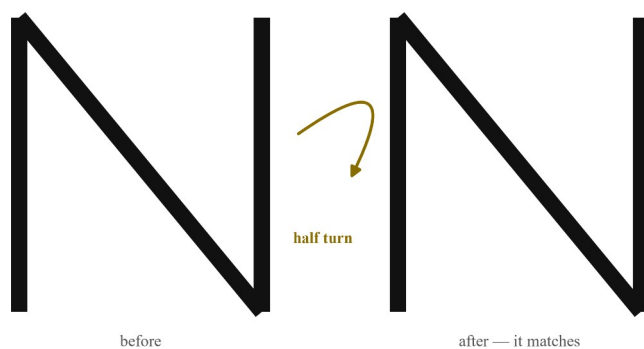
Refresher: halves and quarters. A **half** is one of two equal parts; a **quarter** is one of four equal parts. Two quarters make a half, and four quarters make a whole. That is all a turn name means: a quarter turn is a quarter of a full turn, a half turn is a half of a full turn, and a three-quarter turn is three quarters of a full turn.

The tile below carries an arrow so you can watch a turn happen. Start with the arrow pointing north, then turn the tile clockwise.

Each turn is clockwise — the way clock hands move.



Matching itself under a turn. Some shapes look exactly the same part-way through a full turn. Turn the shape about its centre and watch for the moments it matches its own starting outline. The block letter N below has no mirror line at all, yet a half turn maps it exactly onto itself.



The block letter N has no mirror line, but a half turn maps it onto itself.

The table earlier in this section lists, for each familiar shape, how many times it matches itself in one full turn (counting the finish of the turn). A square matches at every quarter turn, so 4 times. A rectangle matches only at the half turn and at the finish, so 2 times — a quarter turn leaves a rectangle standing on its short ends, which does not match. A hexagon with all sides equal matches at every sixth of a turn, so 6 times — and, like the rectangle, it does **not** match at a quarter turn.

Flip or turn? Two tests to keep apart. Reflection and rotation answer different questions, and a shape can pass one test, both, or neither.

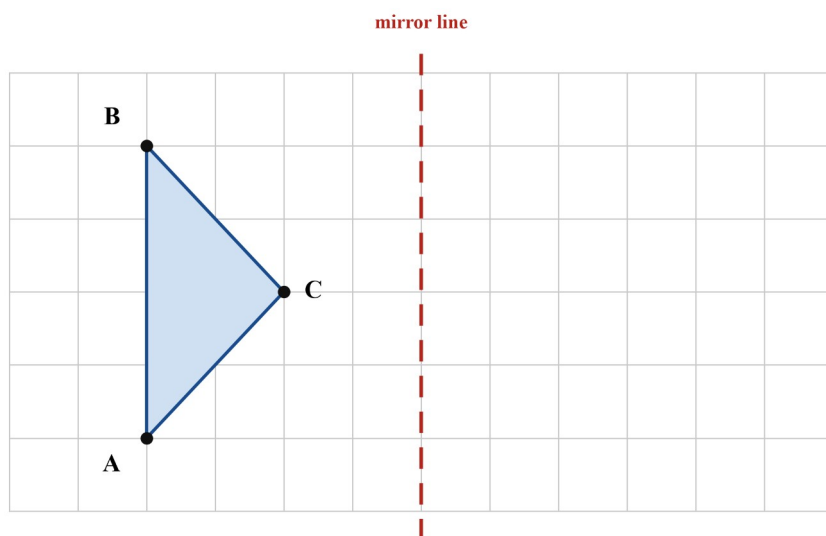
Ask this	If yes, then ...
Can I fold the shape along a line so the two halves match exactly?	the shape has a mirror line (that fold line)
Does the shape match its own outline part-way through a full turn?	the shape matches itself under a turn (that size of turn)

Some shapes pass both tests: the square, the hexagon, and the block letters H and 8. Some pass only the fold test: the block letter A folds neatly down its middle but turns upside down under a half turn. Some pass only the turn test: the block letters N, S and Z have no mirror line but all match under a half turn. Keeping the two tests apart is the whole skill of this section.

Worked examples

Example 1 — scaffolded

A triangle is drawn on square grid paper with a mirror line beside it.



- Count how many squares each corner A, B and C sits from the mirror line.
- The triangle is reflected in the mirror line. How many squares from the mirror line does each image corner sit, and on which side?
- How many corners does the image triangle have?

Working

Comment

(a) A: 4 squares. B: 4 squares. C: 2 squares.

Count straight across from each corner to the mirror line.

(b) Image of A: 4 squares right, on the same row as A.
Image of B: 4 squares right, same row as B. Image of C:
2 squares right, same row as C.

A flip keeps the distance and the row; only the side of the line changes.

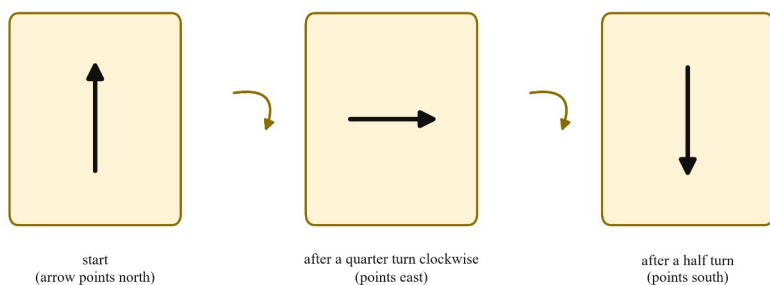
(c) 3 corners — one for each corner of the original triangle.

A flip never adds or takes away corners.

The image is the same triangle, the same size, facing the opposite way — a true mirror picture of the original.

Example 2 — scaffolded

A floor tile has an arrow painted on it. The arrow starts pointing north.



- The tile is given a quarter turn clockwise. Which way does the arrow point now?
- Starting again from north, the tile is given a half turn. Which way does the arrow point now?

(c) How many quarter turns clockwise does the tile need to face north again?

Working	Comment
(a) North, then a quarter turn clockwise, gives east.	A quarter turn clockwise is the clock hand going from 12 to 3.
(b) North, then a half turn, gives south.	A half turn is two quarter turns: north to east, then east to south.
(c) 4 quarter turns.	Four quarters make one full turn, and a full turn brings the arrow home to north.

So a quarter turn clockwise sends the arrow east, a half turn sends it south, and four quarter turns bring it back to north.

Example 3 — unscaffolded

Five block letters are painted on a set of cards: H, A, N, S and Z.



For each letter, decide whether it has a mirror line (and how many), and decide whether it matches itself under a half turn.

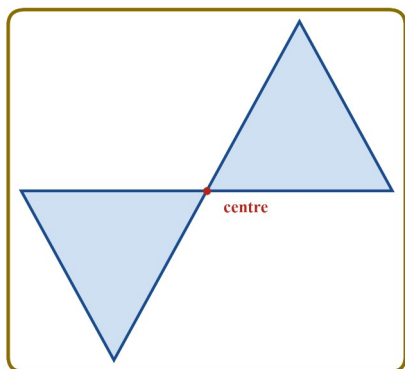
Solution. Test the folds first. H folds two ways — down the middle from top to bottom, and across the middle from left to right — so H has 2 mirror lines. A folds one way only, down the middle from top to bottom, so A has 1 mirror line; a fold across the middle of A puts the point at the bottom, which does not match. N, S and Z have no fold that works: every fold leaves strokes that do not line up, so each of N, S and Z has 0 mirror lines.

Now test the half turn. H turned a half way about its middle lands stroke for stroke on itself, so H matches. A turned a half way stands upside down — point at the bottom — so A does not match. N turned a half way lands exactly on itself: the two uprights swap places and the slant still runs the same way, so N matches. The same half turn maps S onto S and Z onto Z, so both match.

∴ Mirror lines: H has 2, A has 1, and N, S and Z have none. Half-turn matches: H, N, S and Z match themselves under a half turn, and A does not.

Example 4 — unscaffolded

A pretend coffee shop prints its coasters on square card. The design is two triangles: one pointing up in the top-right quarter of the coaster, and one pointing down in the bottom-left quarter.



*One triangle points up in the top-right quarter;
the other points down in the bottom-left quarter.*

The owner makes two claims: that the design has a mirror line, and that the design matches itself under a half turn about the centre of the coaster. Check each claim, and decide whether a quarter turn about the centre matches the design.

Solution. Fold test first. A fold down the middle from top to bottom would carry the top-right triangle onto the top-left quarter, which is empty — no match. A fold across the middle from left to right would carry the top-right triangle onto the bottom-right quarter, which is also empty — no match. A corner-to-corner fold is no better: folding along either corner-to-corner line lands an up-pointing triangle sideways, and the design has no sideways triangles. No fold line works, so the design has no mirror line and the first claim fails.

Turn test next. A half turn about the centre carries the top-right quarter to the bottom-left quarter, and an up-pointing triangle to a down-pointing one — which is exactly the second triangle. At the same time the bottom-left triangle spins onto the top-right one. Every part lands on a matching part, so the half turn matches and the second claim holds. A quarter turn about the centre sends the two triangles into the other two corners (top-left and bottom-right) and turns them sideways, so a quarter turn does not match.

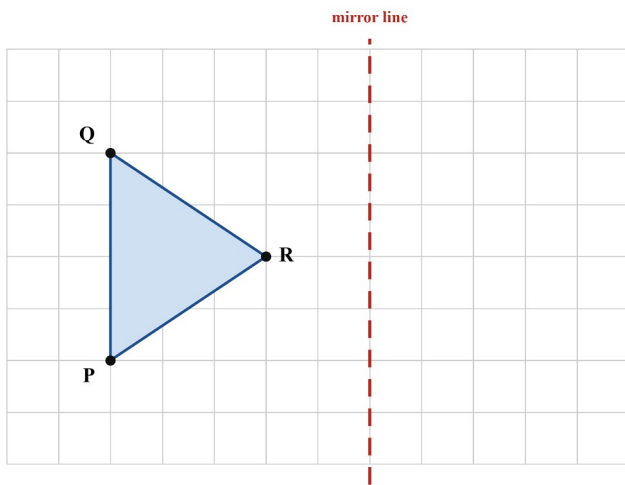
∴ The owner is half right: the coaster design has no mirror line, but it does match itself under a half turn about the centre. A quarter turn does not match it.

Practice questions

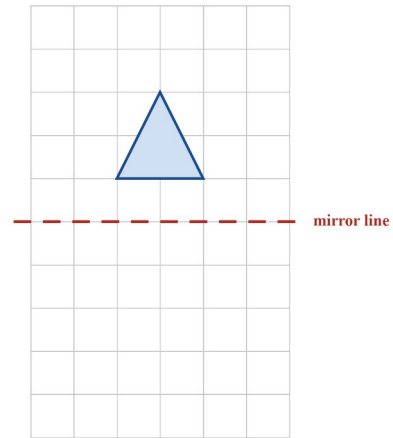
Scaffolded

For Q1–Q6, use the two tests from the theory: count squares straight across for a flip, and step quarter turns on a clock face for a turn.

Q1. A triangle PQR is drawn on square grid paper with a mirror line. Look at grid (i) in the figure below.



(i)



(ii)

- Count how many squares each corner P, Q and R sits from the mirror line.
- The triangle is reflected in the mirror line. How many squares from the mirror line does each image corner sit, and on which side?
- How many corners does the image triangle have?

Q2. Look at grid (ii) in the figure above. A triangle sits above a horizontal mirror line.

- How many squares above the mirror line is the top corner of the triangle?
- The triangle is reflected in the mirror line. How many squares below the mirror line is the image of that top corner?
- Counting the original triangle and its image together, how many corners does the whole picture have?

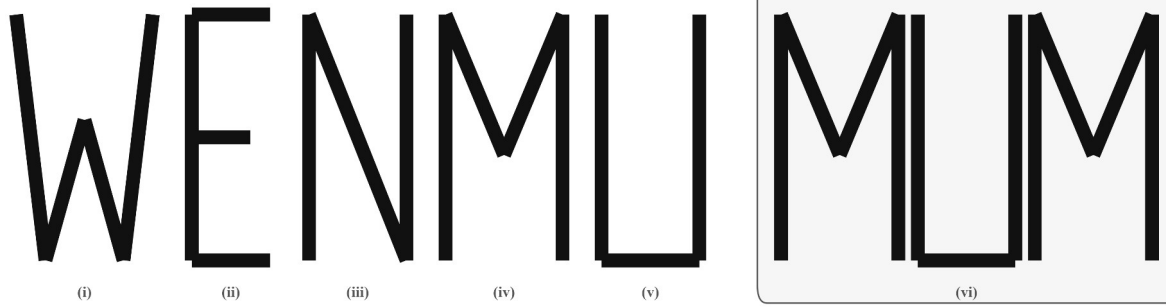
Q3. A tile arrow starts pointing north.

- Which way does it point after a quarter turn clockwise?
- Starting again from north, which way does it point after a quarter turn anticlockwise?
- Starting again from north, which way does it point after a half turn?

Q4. How many mirror lines does each shape have?

- a square
- a rectangle
- a triangle with all sides equal

Q5. Look at letters (i), (ii) and (iii) in the figure below.



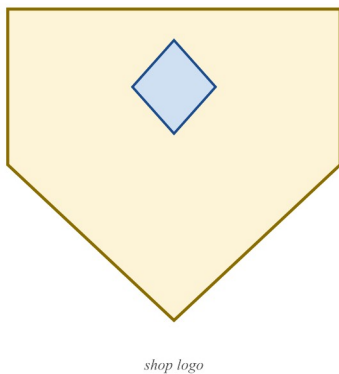
- (a) Does the letter W in (i) have a mirror line? If yes, how many, and which way does it run?
- (b) Does the letter E in (ii) have a mirror line? If yes, how many, and which way does it run?
- (c) Does the letter N in (iii) match itself under a half turn?

Q6. Name each turn in degrees.

- (a) a quarter turn
- (b) a half turn
- (c) How many quarter turns make one full turn?

Unscaffolded

Q7. A pretend bakery uses a logo shaped like a shield, with a diamond centred near the top.

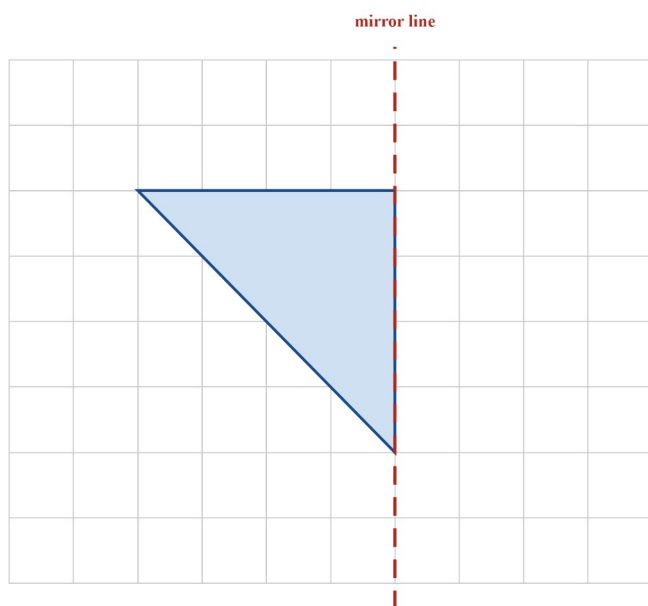


- (a) How many mirror lines does the logo have?
- (b) Does the logo match itself under a half turn?

Q8. A wall is tiled with square tiles. Every tile carries the same design: one triangle pointing up, with the triangle's base along the bottom edge of the tile.

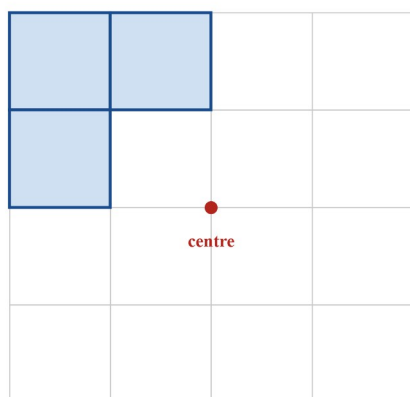
- (a) How many mirror lines does one tile design have?
- (b) The wall is 5 tiles wide and 4 tiles high. How many tiles are on the wall?
- (c) If every tile on the wall were given a half turn, would the wall look the same as before? Explain in one sentence.

Q9. Half of a triangle is drawn against a mirror line on square grid paper.



- The triangle is reflected in the mirror line to complete the picture. The left corner of the half-triangle sits 4 squares left of the mirror line. How many squares right of the mirror line does its image sit?
- Once the picture is complete, the two halves join into one big triangle. How many sides does the whole outline have?

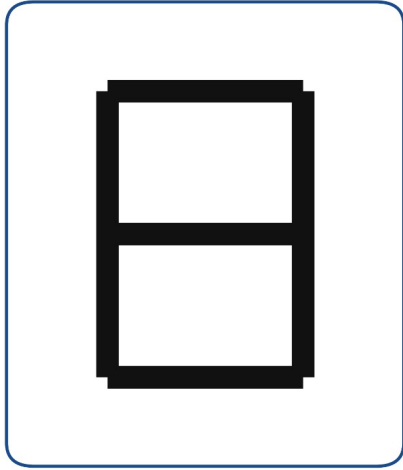
Q10. An L-shape of three shaded squares sits in the top-left corner of a 4-by-4 grid. The centre of the grid is marked.



The L-shape turns about the centre of the grid.

- The L-shape is turned a quarter turn clockwise about the centre of the grid. Which corner of the grid does it sit in now?
- Starting again from the top-left, the L-shape is turned a half turn about the centre. Which corner of the grid does it sit in now?
- How many quarter turns clockwise about the centre bring the L-shape back to its start? Do 1, 2 or 3 quarter turns put it back?

Q11. A pretend card game uses cards that show one big block digit. One card shows the digit 8.



one card from a pretend card game

- (a) How many mirror lines does the block digit 8 have?
- (b) Does the block digit 8 match itself under a half turn?
- (c) The card is given a half turn. What digit do you read now?

Q12. Sam says: “A rectangle and a square both have 4 corners, so they must both have 4 mirror lines.”

- (a) Is Sam right? Use the mirror-line counts for a square and a rectangle to support your answer.
- (b) How many times does a rectangle match itself in one full turn?

Q13. A ceiling fan has 4 blades of the same shape, spaced evenly around the centre, all leaning the same way like a paper windmill toy.

- (a) How many times does the fan match itself in one full turn?
- (b) What is the smallest turn that matches the fan?
- (c) Does a half turn match the fan?

Q14. A pretend fabric shop sells a strip fabric. The pattern is pairs of birds facing each other, with a mirror line between the two birds of each pair. Each pair takes 25 cm of strip.

- (a) Where is the mirror line in one pair of birds?
- (b) A strip is 100 cm long. How many pairs of birds are on the strip?
- (c) A cushion needs 3 pairs of birds. How many centimetres of strip is that?

Q15. Look at letters (iv), (v) and (vi) in the Q5 figure: the letters M and U, and the word MUM printed on a panel for a charity fun-run T-shirt.

- (a) Does the letter M have a mirror line? If yes, how many, and which way does it run?
- (b) Does the letter U have a mirror line? If yes, how many, and which way does it run?
- (c) Does the word MUM have a mirror line? If yes, which way does it run — and does it also have a mirror line the other way?

Q16. On square grid paper, half of a butterfly design is drawn next to a mirror line. The half design has one wing tip sitting 3 squares from the mirror line.

- (a) State the rule that places each corner of the image when the half design is reflected in the mirror line.
- (b) How many squares from the mirror line does the image of the wing tip sit?
- (c) Once the design is complete, it always has at least one mirror line. Which line is it?

Q17. The head of a bolt is a hexagon with all sides equal.

- (a) How many mirror lines does the hexagon bolt head have?
- (b) How many times does it match itself in one full turn?
- (c) Does a quarter turn match the hexagon bolt head?

Q18. A square path tile has one corner painted dark so the layer can see it turn. The painted corner starts at the top-left.

- (a) The tile is turned a quarter turn clockwise. Which corner is the painted corner in now?
- (b) Starting again from the top-left, the tile is turned a half turn. Which corner is the painted corner in now?
- (c) Counting the start, how many different corner positions can the painted corner sit in as the tile turns?

Answers

Q1. (a) P 5 squares, Q 5 squares, R 2 squares (b) each image corner the same count on the right: 5, 5 and 2 squares, same rows (c) 3 corners **Q2.** (a) 3 squares above (b) 3 squares below (c) 6 corners **Q3.** (a) east (b) west (c) south **Q4.** (a) 4 (b) 2 (c) 3 **Q5.** (a) yes — 1 mirror line, running top to bottom (b) yes — 1 mirror line, running left to right (c) yes **Q6.** (a) 90° (b) 180° (c) 4 **Q7.** (a) 1 mirror line (top to bottom, through the diamond and the shield point) (b) no **Q8.** (a) 1 mirror line (top to bottom through the middle of the triangle) (b) 20 tiles (c) no — every triangle would point down **Q9.** (a) 4 squares right (b) 3 sides **Q10.** (a) the top-right corner (b) the bottom-right corner (c) 4 quarter turns; no, 1, 2 and 3 quarter turns do not put it back **Q11.** (a) 2 (b) yes (c) 8 **Q12.** (a) no — a square has 4 mirror lines but a rectangle has only 2 (b) 2 times **Q13.** (a) 4 times (b) a quarter turn (c) yes **Q14.** (a) straight down the middle between the two facing birds (b) 4 pairs (c) 75 cm **Q15.** (a) yes — 1 mirror line, top to bottom (b) yes — 1 mirror line, top to bottom (c) yes — 1 mirror line top to bottom, through the middle of the U; no mirror line left to right **Q16.** (a) each corner jumps straight across the mirror line to the same distance on the other side, on the same row (b) 3 squares (c) the mirror line the design was flipped across **Q17.** (a) 6 (b) 6 times (c) no **Q18.** (a) top-right (b) bottom-right (c) 4 positions

Fully-worked solutions

Q1. (a) Counting straight across to the mirror line: P sits 5 squares from it, Q sits 5 squares from it, and R sits 2 squares from it. (b) A flip keeps each distance and each row, and changes only the side: the image of P sits 5 squares right of the mirror line on P's row, the image of Q sits 5 squares right on Q's row, and the image of R sits 2 squares right on R's row. (c) The image has 3 corners, one for each corner of the original — a flip never changes the number of corners.

Q2. (a) The top corner of the triangle sits 3 squares above the mirror line. (b) Its image sits the same distance on the other side: 3 squares below the mirror line, in the same column. (c) The original triangle has 3 corners and the image has 3 corners, and none of them coincide, so the whole picture shows 6 corners.

Q3. (a) A quarter turn clockwise from north is the clock hand going from 12 to 3, so the arrow points east. (b) A quarter turn anticlockwise runs the clock hand backwards from 12 to 9, so the arrow points west. (c) A half turn is two quarter turns: north to east, then east to south, so the arrow points south.

Q4. (a) A square folds four ways — top to bottom, left to right, and along both corner-to-corner lines — so it has 4 mirror lines. (b) A rectangle folds only top to bottom and left to right; the corner-to-corner folds leave the short sides over the long sides, so it has 2 mirror lines. (c) A triangle with all sides equal folds from each corner to the middle of the opposite side, so it has 3 mirror lines.

Q5. (a) W folds one way, down the middle from top to bottom, so it has 1 mirror line running top to bottom; a fold left to right puts the point of the W at the top, which does not match. (b) E folds one way, across the middle from left to right, so it has 1 mirror line running left to right; a fold top to bottom puts the three arms backwards, which does not match. (c) N has no mirror line, but a half turn swaps its two uprights and keeps the slant running the same way, so yes — N matches itself under a half turn.

Q6. (a) A quarter turn is a quarter of a full turn, named 90° . (b) A half turn is named 180° . (c) Four quarters make a whole, so 4 quarter turns make one full turn (named 360°).

Q7. (a) The logo folds one way only: straight down the middle, through the centre of the diamond and the point of the shield. The two halves match, and no other fold works, so the logo has 1 mirror line. (b) A half turn turns the shield upside down — point at the top instead of the bottom — so the logo does not match itself under a half turn.

Q8. (a) One tile design folds one way, straight down the middle of the up-pointing triangle, so it has 1 mirror line. (b) The wall is 5 tiles wide and 4 tiles high, so it holds $5 \times 4 = 20$ tiles. (c) No — a half turn flips every up-pointing triangle to point down, so the wall would read as a wall of down-pointing triangles, not the same wall.

Q9. (a) A flip keeps the distance from the mirror line and changes only the side, so the image of the left corner sits 4 squares right of the mirror line, on the same row. (b) The completed picture is one big triangle: the two top edges join into a single straight top edge, the two slant edges become the two sides, and the shared edge along the mirror line disappears inside the shape. The whole outline has 3 sides.

Q10. (a) A quarter turn clockwise about the centre carries the top-left corner of the grid to the top-right corner, so the L-shape sits in the top-right corner. (b) A half turn carries the top-left corner to the bottom-right corner, so the L-shape sits in the bottom-right corner. (c) The L-shape returns to its start after 4 quarter turns, because 4 quarter turns make one full turn. No — 1, 2 and 3 quarter turns leave it in the top-right, bottom-right and bottom-left corners in turn, so none of them puts it back.

Q11. (a) The block digit 8 folds two ways — down the middle from top to bottom, and across the middle from left to right — so it has 2 mirror lines. (b) Yes: a half turn swaps the top and bottom halves and swaps the two uprights, and the 8 lands exactly on itself. (c) The digit still reads 8 — that is what matching itself under a half turn means.

Q12. (a) Sam is wrong. A square does have 4 mirror lines, but a rectangle has only 2: it folds top to bottom and left to right, and its corner-to-corner folds do not match because the short sides land over the long sides. The number of corners does not decide the number of mirror lines. (b) A rectangle matches itself at the half turn and at the finish of the turn, so it matches 2 times in one full turn; a quarter turn stands it on its short ends, which does not match.

Q13. (a) The fan has 4 blades spaced evenly around the centre, so it matches itself once per blade in one full turn: 4 times. (b) The smallest matching turn is the turn from one blade to the next, which is a quarter turn. (c) Yes — a half turn is two quarter turns, and each quarter turn matches, so a half turn matches too.

Q14. (a) The mirror line runs straight down the middle of each pair, between the two facing birds: each bird is the flip of the other across that line. (b) Each pair takes 25 cm and the strip is 100 cm, so the strip holds $100 \div 25 = 4$ pairs. (c) Three pairs take $3 \times 25 = 75$ cm of strip.

Q15. (a) M folds one way, down the middle from top to bottom, so it has 1 mirror line running top to bottom. (b) U folds the same one way, so it has 1 mirror line running top to bottom. (c) The word MUM has 1 mirror line running top to bottom, through the middle of the U: the two Ms are flips of each other and the M in the middle folds onto itself. It has no mirror line running left to right — that fold would turn every letter upside down, and none of M, U or M matches upside down.

Q16. (a) The rule is the flip rule: each corner of the image sits straight across the mirror line from its original corner, the same number of squares from the line, on the other side and on the same row. (b) The wing tip sits 3 squares from the mirror line, so its image sits 3 squares from the mirror line on the other side. (c) The mirror line the half design was flipped across — once both halves are drawn, folding along that line lands one half exactly on the other, so it is a mirror line of the finished design.

Q17. (a) A hexagon with all sides equal folds from each corner to the corner opposite, and from the middle of each side to the middle of the side opposite: 6 mirror lines. (b) It matches itself once per side in one full turn, so 6 times. (c) No — a quarter turn puts the flat sides where the corners were, so the outline does not match; the smallest matching turn is a sixth of a full turn.

Q18. (a) A quarter turn clockwise carries the top-left corner to the top-right corner, so the painted corner is at the top-right. (b) A half turn carries the top-left corner to the bottom-right corner, so the painted corner is at the bottom-right. (c) The painted corner visits top-left, top-right, bottom-right and bottom-left as the tile turns, so counting the start it sits in 4 different corner positions.

Chapter 3 Shape — 3D Patterns in shapes and shape technologies

Theory

What is a pattern?

A **pattern** is something that repeats in a way you can predict. The part that gets copied over and over is called the **repeat**. A **rule** is a description of how the pattern goes on — in words, like “square, triangle, circle, over and over”, or “start at 4 and add 3 each time”.

Once you know the repeat, you can count a long pattern without counting every shape one by one.

The border repeats: square, triangle, circle



The border above repeats **square, triangle, circle**. One repeat holds 3 shapes, so 8 repeats hold $8 \times 3 = 24$ shapes. One of the three shapes in each repeat is a triangle, so 8 repeats hold 8 triangles (and 8 squares, and 8 circles).

Which shape is in a given spot? Divide the position number by the size of the repeat, and let the remainder point into the repeat.

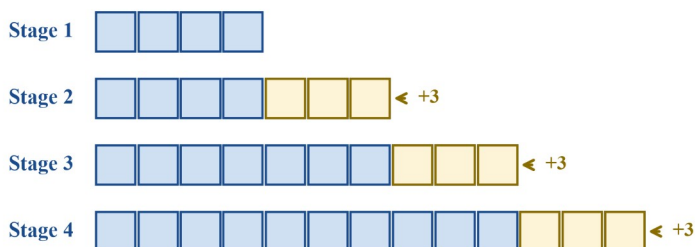
Refresher: division with a remainder (Section 2B). When a whole number does not divide exactly, the left-over part is the **remainder**, written like this: $47 \div 6 = 7$ remainder 5, because $6 \times 7 = 42$ and $47 - 42 = 5$.

- The 20th shape: $20 \div 3 = 6$ remainder 2. Remainder 2 means the 2nd shape of the repeat — a **triangle**.
- The 18th shape: $18 \div 3 = 6$ remainder 0. Remainder 0 means the repeat finishes exactly on that shape, so it is the **last** shape of the repeat — a **circle**.

Growing patterns.

Some patterns do not repeat — they **grow** by the same amount each time. Each step of a growing pattern is called a **stage**.

A growing pattern: start with 4 tiles, add 3 tiles each stage



Blue = stage 1 tiles Amber = tiles added at the next stage

The rule in words: **start with 4 tiles, add 3 tiles each stage**. So the stages go 4, 7, 10, 13, ...

Refresher: skip counting and multiples (Section 2B). Skip counting is counting on by the same number every time — counting on by 3s from 4 gives 4, 7, 10, 13, 16, 19, ... The multiples of a number are what you say when you skip count from that number: the multiples of 3 are 3, 6, 9, 12, 15, ...

Jumping straight to a far stage. To find stage 6 without drawing stages 2,3,4,5: from stage 1 to stage 6 is 5 jumps of 3 tiles.

- Jumps first: $5 \times 3 = 15$ tiles added.
- Then add the start: $15 + 4 = 19$ tiles.

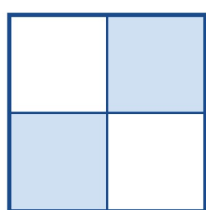
One number sentence: $4 + 3 \times 5 = 19$.

Refresher: order of operations (Section 2D). In a number sentence with $+$ and $\times$, do the multiplication first. In $4 + 3 \times 5$ the $\times$ happens before the $+$: $3 \times 5 = 15$, then $4 + 15 = 19$. Adding first ($4 + 3 = 7$, then $7 \times 5 = 35$) is the classic trap — it gives the wrong answer.

Patterns between shapes — shapes made of shapes.

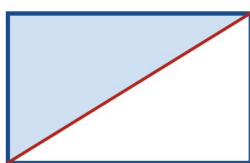
Familiar shapes hide inside each other, and naming the hiding places is a pattern too.

Square cut both ways



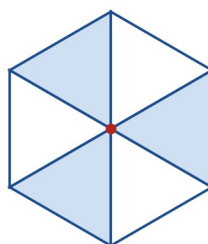
4 small squares

Rectangle cut corner to corner



2 identical triangles

Hexagon cut from the centre



6 same-size triangles

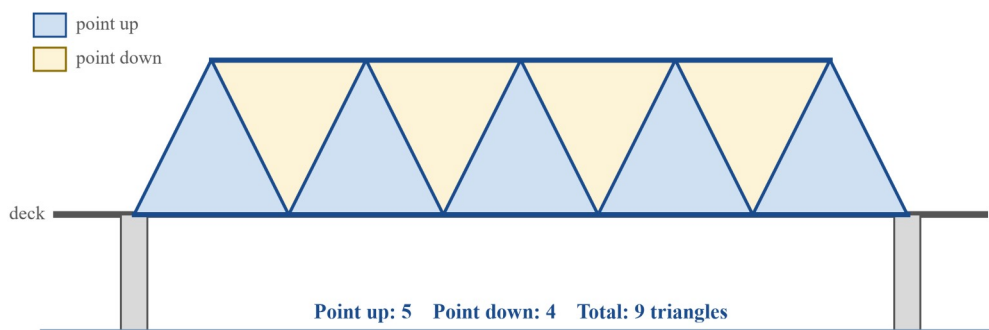
- Cut a square both ways through the middle: 4 small squares.
- Cut a rectangle corner to corner: 2 identical triangles.
- Cut a hexagon from its centre to every corner: 6 same-size triangles (each side of every triangle is the same length as a side of the hexagon).

So a mural of 8 hexagons cut this way is really $8 \times 6 = 48$ triangles.

Patterns in engineering, architecture and design.

Bridges, buildings and sculptures are full of repeating shape patterns.

A truss bridge: the triangles repeat along the span



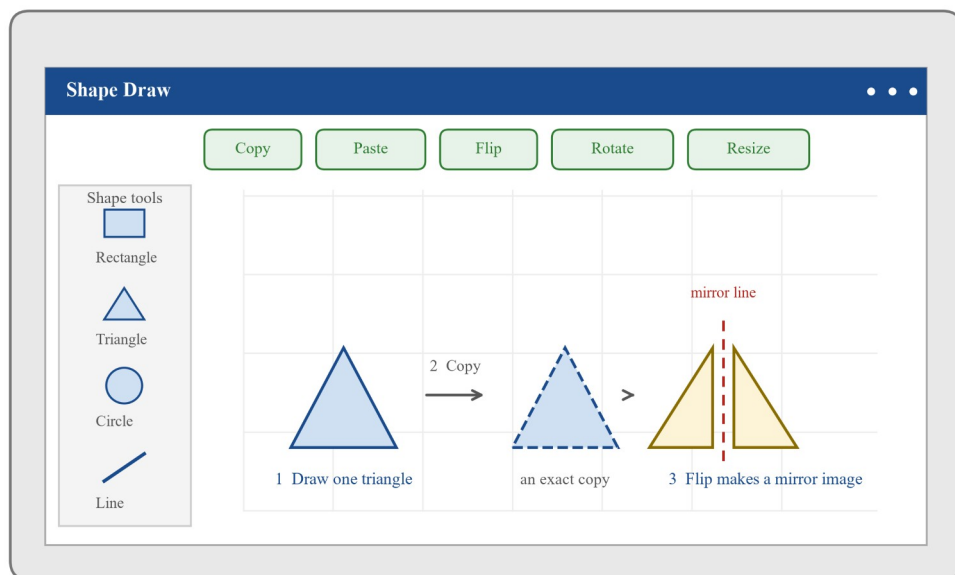
The frame above is a **truss** — a frame of bars built only from triangles. In this one frame the triangles repeat along the span: 5 point up, 4 point down, 9 triangles altogether. Builders use triangles because a triangle keeps its shape when you push on it. A square frame with the same four bars can lean into a diamond without any bar changing length; add one diagonal bar and the square becomes two triangles, which hold firm. That is why bridge spans, roof frames and tower cranes are stitched with triangles.

Buildings repeat shapes too: bricks lie in equal rows (called **courses**), windows sit in equal rows and columns, and a **mosaic** — a picture made of small tiles — is built from a repeating block of tiles. Sculptures often repeat one group of solids over and over along a base.

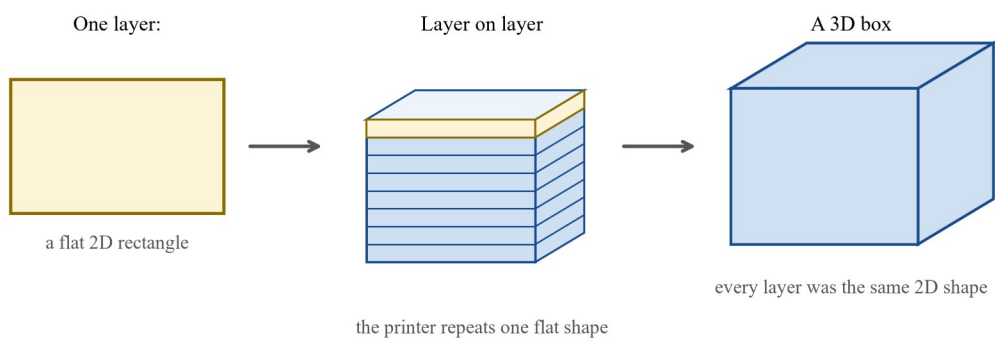
Refresher: arrays — rows and columns (Section 2B). An **array** is a set of things lined up in equal rows. Count an array by multiplying: rows $\times$ the number in each row. 3 rows of 5 windows is $3 \times 5 = 15$ windows.

Technologies that create and manipulate shapes.

To **manipulate** a shape means to change it or move it — flip it, turn it, make it bigger or smaller, copy it.



A simple drawing app creates shapes with the shape tools (rectangle, triangle, circle, line), then manipulates them: **copy** and **paste** make exact copies, **flip** makes a mirror image, **rotate** turns a shape, and **resize** grows or shrinks it. Off screen, a **stencil** or **template** — a cut-out shape you draw around — does the same job: trace it again and again for exact copies. A **laser cutter** follows a 2D outline drawn on a computer and cuts that same outline out of sheet material as many times as you ask.



A 3D printer builds a solid object by laying the same flat 2D shape layer on layer, so a 3D box grows out of a repeated 2D rectangle.

	A	B
1	Stage	Tiles
2	1	3
3	2	6
4	3	9
5	4	12
6	5	15
7	6	18
8	7	

drag the handle down to fill

The sheet keeps the rule: tiles = 3 x stage number

A spreadsheet extends a number pattern for you. In the sheet above, column B keeps the rule *tiles = 3 × stage number*: 3, 6, 9, 12, 15, 18. Dragging the small handle at the corner of the selected cell **down** (fill down) keeps the rule going into the empty rows.

Pencil and paper is enough. Every task in this section can be done by hand — drawing the shapes, copying them, flipping them and counting them on paper. The apps, cutters, printers and spreadsheets here are real tools that do those same jobs faster; nothing in this section needs a device.

Words to know. A **truss** is a frame of bars built from triangles. A **course** is one horizontal row of bricks in a wall. A **mosaic** is a picture made of small tiles. A **stencil** or **template** is a cut-out shape you draw around to copy it. A **laser cutter** is a machine that cuts a drawn 2D outline out of sheet material.

About the data in this section. The walls, bridges, mosaics, sculptures and prices here are **made up** (not real): pretend buildings, pretend bridges and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

A tile border repeats **square, circle, triangle, circle**. The border has 7 repeats. (a) How many shapes are in the whole border? (b) How many of them are circles? (c) What is the 26th shape?

Working	Comment
Shapes in one repeat: 4. Whole border: $7 \times 4 = 28$ shapes.	Repeats × shapes per repeat counts the lot at once.
Circles in one repeat: 2. Circles: $7 \times 2 = 14$.	Count only the circles inside one repeat, then multiply by the repeats.
26th shape: $26 \div 4 = 6$ remainder 2 → the 2nd shape of the repeat, a circle .	Remainder 2 points at the 2nd shape of the repeat; remainder 0 would point at the last one.
Check: 6 full repeats use $6 \times 4 = 24$ shapes; 2 more shapes (square, then circle) reach 26 ✓.	Counting on from a whole number of repeats agrees with the remainder rule.

Example 2 — scaffolded

A growing pattern of tiles starts with 5 tiles and adds 4 tiles each stage. (a) How many tiles are in stage 2 and stage 3? (b) How many tiles are in stage 5, without drawing it? (c) Make a table of the tiles for stages 1 to 5.

Working	Comment
Stage 2: $5 + 4 = 9$ tiles. Stage 3: $9 + 4 = 13$ tiles.	Each stage adds 4 tiles to the stage before it.
Stage 5: 4 jumps from stage 1; $4 \times 4 = 16$; $16 + 5 = 21$ tiles.	Jumps $\times$ the amount added, then add the start.
One number sentence: $5 + 4 \times 4 = 21$.	Order of operations (Section 2D): the multiplication happens first.
Table: stages 1–5 are 5, 9, 13, 17, 21.	Skip counting on by 4s from 5 (Section 2B).

Stage 5 has 21 tiles.

Example 3 — unscaffolded

A jetty railing is built from 7 sections. Each section is a truss frame like the one in the theory figure: 9 triangles, of which 5 point up and 4 point down. (a) How many triangles are in the whole railing? (b) How many of them point up?

Triangles in one frame: $5 + 4 = 9$.

Whole railing: $7 \times 9 = 63$ triangles.

Pointing up: $7 \times 5 = 35$.

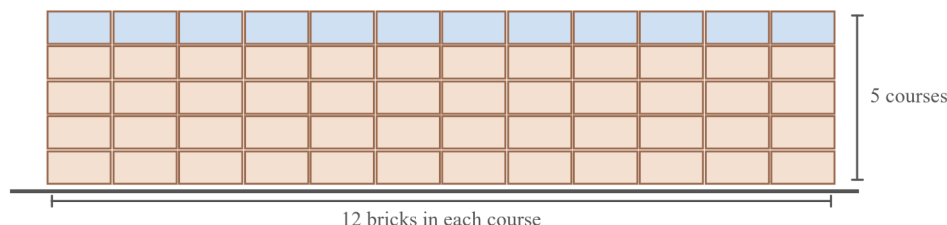
Check with the point-down triangles: $7 \times 4 = 28$, and $35 + 28 = 63$ ✓.

$\therefore$ the railing uses 63 triangles, and 35 of them point up.

Example 4 — unscaffolded

A garden wall is built from bricks in equal courses.

A garden wall: 5 courses of 12 bricks



The wall has 5 courses with 12 bricks in each course. Each brick costs \$1.80. What do the bricks for this wall cost?

Bricks: $5 \times 12 = 60$ bricks.

Cost: $60 \times \$1.80 = \$108.00$.

Check by splitting \$1.80 into \$1 + \$0.80: $60 \times \$1 = \$60$ and $60 \times \$0.80 = \$48.00$; then $60 + 48 = 108$ ✓.

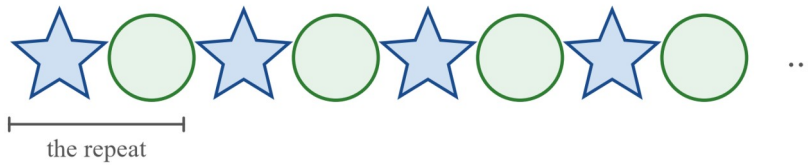
$\therefore$ the bricks for the wall cost \$108.00.

Practice questions

Scaffolded

Q1. Look at the painted strip on the garden shed.

The painted strip on the garden shed



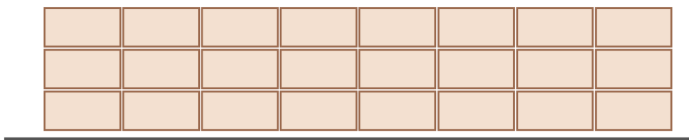
- (a) Write down the repeat.
- (b) Write down the next four shapes.
- (c) How many stars are in 10 repeats?

Q2. A growing pattern of buttons starts with 2 buttons and adds 5 buttons each stage. (So stage 2 has 7 buttons.) (a) How many buttons are in stage 3? (b) How many buttons are in stage 4? (c) How many buttons are in stage 5 and stage 6?

Q3. Use the shapes-made-of-shapes figure in the theory. (a) How many same-size triangles make up the hexagon? (b) How many small squares make up the big square? (c) A mural uses 3 hexagons cut like the one in the figure. How many triangles are in the mural?

Q4. Rosa builds a low garden wall.

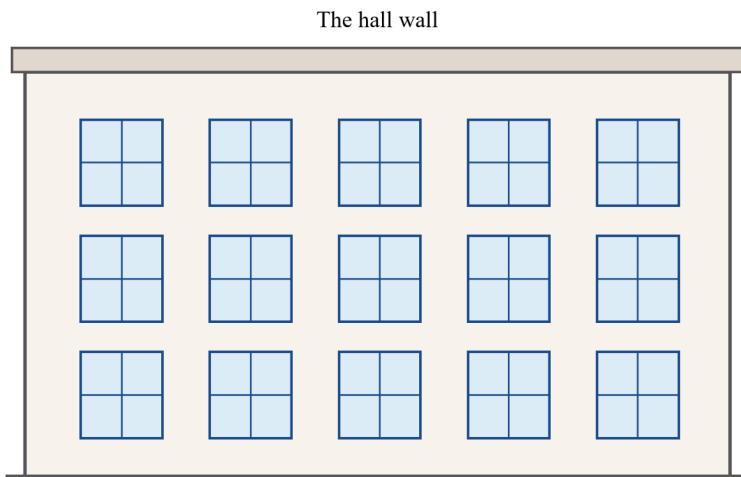
Rosa's low garden wall



- (a) How many courses are in the wall?
- (b) How many bricks are in each course?
- (c) How many bricks are in the wall altogether?

Q5. In a drawing app, Dan draws one rectangle and then pastes 5 more copies of it. He cuts every rectangle corner to corner into 2 identical triangles. (a) How many rectangles does Dan have? (b) How many triangles does each rectangle make? (c) How many triangles does he have altogether?

Q6. Look at the hall wall.



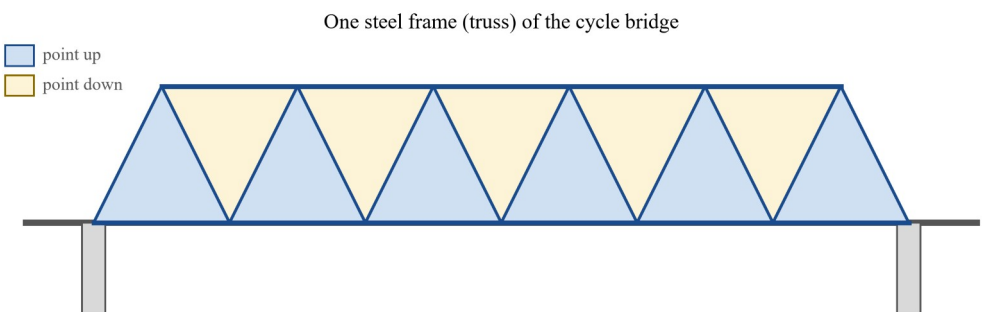
- (a) How many windows are in each row?
- (b) How many rows of windows are there?
- (c) How many windows are on the wall altogether?

Unscaffolded

Q7. A welcome banner repeats **star, triangle, circle**. The banner has 9 repeats. (a) How many shapes are on the banner? (b) How many of them are triangles? (c) What is the 25th shape on the banner?

Q8. A shell pattern grows: stage 1 has 6 shells, and each stage adds 4 shells. (a) How many shells are in stage 3? (b) How many shells are in stage 6?

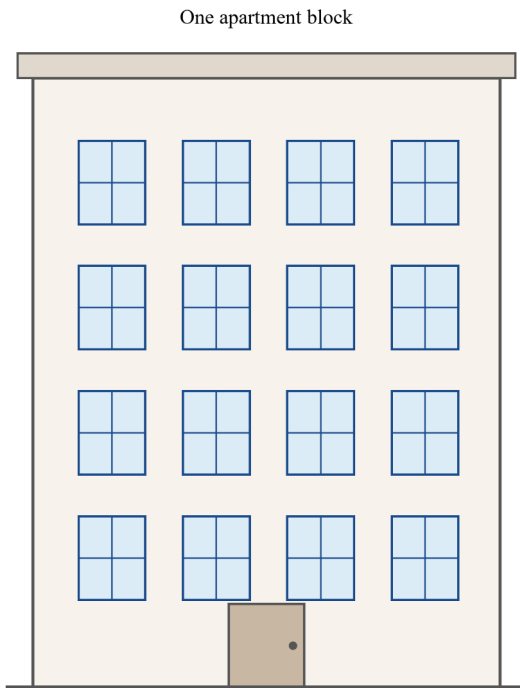
Q9. One steel frame (truss) of the cycle bridge is shown below. The bridge uses 8 frames like this one.



- (a) Count the triangles in one frame.
- (b) How many triangles are in the whole bridge?

Q10. A mosaic panel is an array of 8 rows of 7 tiles. An artist makes 3 panels like this. Tiles cost \$ 1.50 each. (a) How many tiles are in one panel? (b) How many tiles does the artist use altogether? (c) What do the tiles cost altogether?

Q11. Look at one apartment block.



- (a) How many windows are on one floor?
- (b) How many windows are on the whole front of the block?
- (c) The street has 3 identical blocks. How many windows are on the fronts of the 3 blocks altogether?

Q12. A 3D printer lays 24 layers to print a box. Each layer takes 3 minutes. How many minutes does the print take?

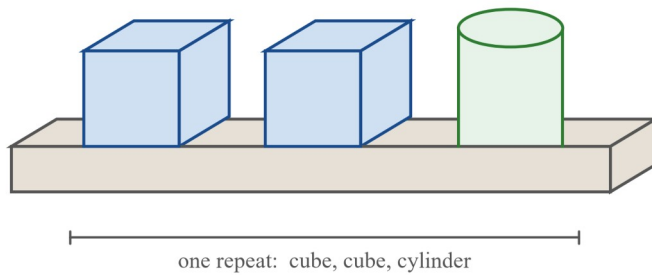
Q13. The spreadsheet in the theory keeps the rule $\text{tiles} = 3 \times \text{stage number}$. (a) Write the rule in words, as a skip-counting rule. (b) What number appears in cell B8 (stage 7)? (c) How many tiles are in stage 10?

Q14. A paver path has 8 rows of 10 pavers. In every row the pavers alternate grey, red, grey, red, ... , starting with grey and ending with red. (a) How many red pavers are in one row? (b) How many pavers are in the whole path? (c) How many red pavers are in the whole path?

Q15. A brick wall has 9 courses with 14 bricks in each course. The builder has 100 bricks in the yard. (a) Show that the wall needs 126 bricks. (b) How many more bricks does the builder need?

Q16. The garden sculpture repeats the group **cube, cube, cylinder** along its base.

The garden sculpture: the same group, over and over



The finished sculpture has 12 repeats of the group. (a) How many pieces are in the sculpture? (b) How many of them are cylinders? (c) How many of them are cubes?

Q17. Toni says: “This border repeats **square, triangle**. So the 23rd shape is a triangle.” (a) Divide 23 by the size of the repeat. What is the remainder? (b) Which shape is the 23rd? Is Toni right?

Q18. A mosaic gift pattern starts with 8 tiles and adds 5 tiles each stage. Tiles cost \$0.60 each. (a) How many tiles are in stage 3? (b) How many tiles are in stage 4? (c) What do the tiles for stage 4 cost?

Answers

Q1. (a) star, circle. (b) star, circle, star, circle. (c) 10 stars. **Q2.** (a) 12. (b) 17. (c) 22 and 27. **Q3.** (a) 6. (b) 4. (c) 18. **Q4.** (a) 3 courses. (b) 8 bricks. (c) 24 bricks. **Q5.** (a) 6 rectangles. (b) 2. (c) 12 triangles. **Q6.** (a) 5. (b) 3. (c) 15. **Q7.** (a) 27. (b) 9. (c) star. **Q8.** (a) 14. (b) 26. **Q9.** (a) 11. (b) 88. **Q10.** (a) 56. (b) 168. (c) \$ 252.00. **Q11.** (a) 4. (b) 16. (c) 48. **Q12.** 72 minutes. **Q13.** (a) count on by 3s each stage: tiles = $3 \times$ stage number. (b) 21. (c) 30. **Q14.** (a) 5. (b) 80. (c) 40. **Q15.** (a) shown: $9 \times 14 = 126$. (b) 26 bricks. **Q16.** (a) 36. (b) 12. (c) 24. **Q17.** (a) $23 \div 2 = 11$ remainder 1. (b) square — Toni is not right. **Q18.** (a) 18. (b) 23. (c) \$ 13.80.

Fully-worked solutions

Q1. (a) The group that copies is star, circle. (b) Carry the repeat on: star, circle, star, circle. (c) One repeat holds 1 star, so 10 repeats hold $10 \times 1 = 10$ stars.

Q2. (a) $7 + 5 = 12$ buttons. (b) $12 + 5 = 17$ buttons. (c) Stage 5: $17 + 5 = 22$; stage 6: $22 + 5 = 27$ buttons.

Q3. (a) The hexagon cuts from its centre into 6 same-size triangles. (b) The square cuts both ways into 4 small squares. (c) $3 \times 6 = 18$ triangles.

Q4. (a) 3 courses. (b) 8 bricks in each course. (c) $3 \times 8 = 24$ bricks.

Q5. (a) $1 + 5 = 6$ rectangles. (b) One cut corner to corner makes 2 identical triangles. (c) $6 \times 2 = 12$ triangles.

Q6. (a) 5 windows in each row. (b) 3 rows. (c) $3 \times 5 = 15$ windows.

Q7. (a) $9 \times 3 = 27$ shapes. (b) One repeat holds 1 triangle: $9 \times 1 = 9$ triangles. (c) $25 \div 3 = 8$ remainder 1; remainder 1 is the 1st shape of the repeat — a star.

Q8. (a) Stage 3 is 2 jumps on: $6 + 5 \times 2$ — multiply first: $5 \times 2 = 10$, then $6 + 10 = 14$ shells. (b) Stage 6 is 5 jumps on: $5 \times 4 = 20$ added, then $20 + 6 = 26$ shells.

Q9. (a) 6 point up and 5 point down: $6 + 5 = 11$ triangles. (b) $8 \times 11 = 88$ triangles.

Q10. (a) $8 \times 7 = 56$ tiles. (b) $3 \times 56 = 168$ tiles. (c) $168 \times \$ 1.50 = \$ 252.00$.

Q11. (a) 4 windows on one floor. (b) 4 floors of 4: $4 \times 4 = 16$ windows. (c) $3 \times 16 = 48$ windows.

Q12. $24 \times 3 = 72$ minutes.

Q13. (a) The tiles count on by 3s each stage: tiles = $3 \times$ the stage number. (b) Stage 7: $3 \times 7 = 21$. (c) Stage 10: $3 \times 10 = 30$.

Q14. (a) The row alternates grey, red 5 times, so 5 red pavers. (b) $8 \times 10 = 80$ pavers. (c) $8 \times 5 = 40$ red pavers.

Q15. (a)

$$9 \times 14 = 126$$

$\therefore$ the wall needs 126 bricks. (b) $126 - 100 = 26$ more bricks.

Q16. (a) One repeat holds 3 pieces: $12 \times 3 = 36$ pieces. (b) One repeat holds 1 cylinder: $12 \times 1 = 12$ cylinders. (c) One repeat holds 2 cubes: $12 \times 2 = 24$ cubes.

Q17. (a) The repeat has 2 shapes: $23 \div 2 = 11$ remainder 1. (b) Remainder 1 is the 1st shape of the repeat — a square. Toni is not right: the 23rd shape is a square, not a triangle.

Q18. (a) Stage 3 is 2 jumps on: $5 \times 2 = 10$ added, then $8 + 10 = 18$ tiles. (b) Stage 4 is 3 jumps on: $5 \times 3 = 15$ added, then $8 + 15 = 23$ tiles. (c) $23 \times \$ 0.60 = \$ 13.80$.

Theory

Same shape, different size.

Sometimes you need a bigger or a smaller copy of a picture. A builder draws a whole house on one sheet of paper. A photo shop prints a phone photo as a poster. A club shrinks its logo so it fits on a badge.

Each copy should look just like the original — only bigger or smaller. This section is about getting the numbers right so that happens. The study design calls this simple scaling: making a picture bigger or smaller, for example a photo, a plan or a diagram (Unit 1, Area of Study 2). Source: VCAA, VCE Vocational Major Numeracy Study Design, retrieved 2026-10-08.

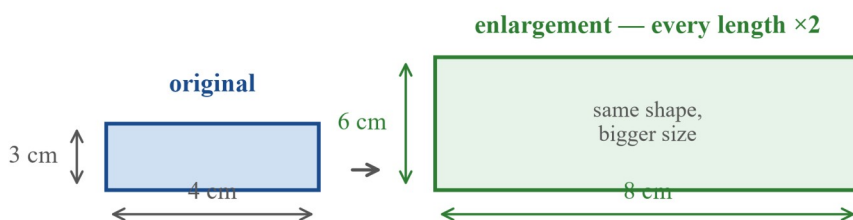
Enlargement and reduction.

An **enlargement** makes a bigger copy. A **reduction** makes a smaller copy. Either one is called **scaling** a shape.

There is one golden rule:

Multiply **every** length by the **same** number.

When every length is multiplied by the same number, the copy has the same shape as the original. When different lengths are multiplied by different numbers, the copy is stretched and looks wrong.



The rectangle on the right is an enlargement of the rectangle on the left. The width goes from 4 cm to 8 cm and the height goes from 3 cm to 6 cm — both lengths are multiplied by 2, so the shape does not change. Same shape, bigger size.

Scale factor.

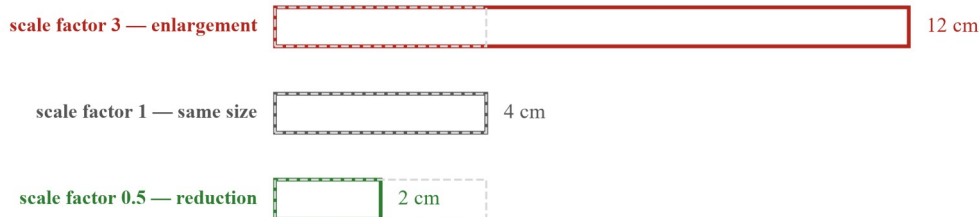
The number you multiply every length by is called the **scale factor**.

$$\text{new length} = \text{original length} \times \text{scale factor}$$

- A scale factor **bigger than 1** gives an **enlargement**. $\times 3$ makes every length three times as long.

- A scale factor **smaller than 1** gives a **reduction**. $\times 0.5$ halves every length.
- A scale factor **equal to 1** gives a copy the **same size** as the original.

The same 4 cm bar, scaled three ways



To go backwards — from the new length to the original length — divide:

$$\text{original length} = \text{new length} \div \text{scale factor}$$

You can also **find** a scale factor with division:

$$\text{scale factor} = \text{new length} \div \text{original length}$$

Refresher: division tells you how many times as big. $12 \div 4 = 3$ means “12 is 3 times as big as 4”. So a line that grows from 4 cm to 12 cm has scale factor 3 — an enlargement. A line that shrinks from 10 cm to 5 cm has scale factor $5 \div 10 = 0.5$ — a reduction.

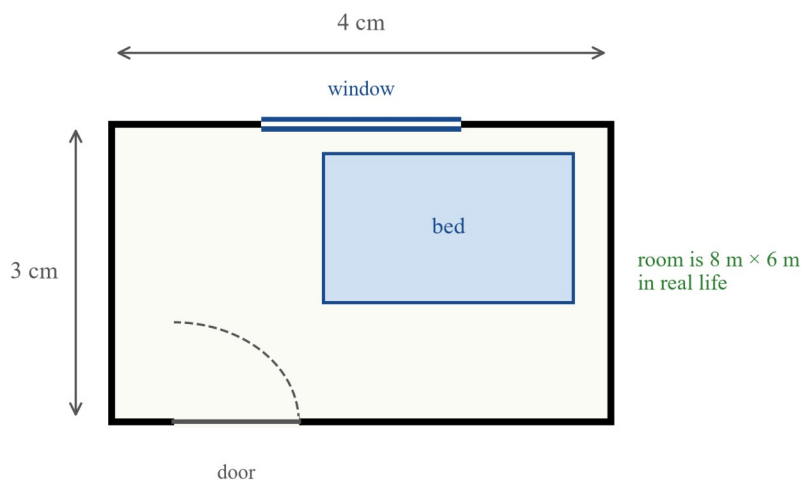
Scaling plans.

A **plan** is a drawing that shows something at a smaller size — a room, a house or a garden drawn on paper. A plan of a room cannot be room-sized, so every plan comes with a rule. The rule might say:

1 cm represents 2 m

This means: every 1 cm you measure **on the plan** stands for 2 m **in real life**. The word **represents** means “stands for”.

Bedroom plan — Scale: 1 cm represents 2 m



- **Plan to real life: multiply.** A wall that is 3 cm on the plan is $3 \times 2 = 6$ m in real life.
- **Real life to plan: divide.** A real wall of 10 m is drawn $10 \div 2 = 5$ cm long on the plan.

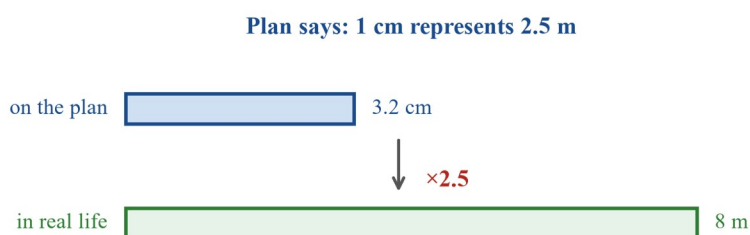
Refresher: metric lengths. 1 m = 100 cm. To change metres into centimetres, multiply by 100 (3 m = 300 cm). To change centimetres into metres, divide by 100 (250 cm = 2.5 m).

Refresher: moving the decimal point with 10 and 100.

- Multiply by 100: the point jumps two places right — $6.5 \times 100 = 650$.
- Divide by 100: the point jumps two places left — $240 \div 100 = 2.4$.

Worked model. A kitchen plan says 1 cm represents 2.5 m. A wall is 3.2 cm on the plan. How long is the real wall?

- Plan to real life, so multiply: $3.2 \times 2.5 = 8$.
- The real wall is 8 m long.

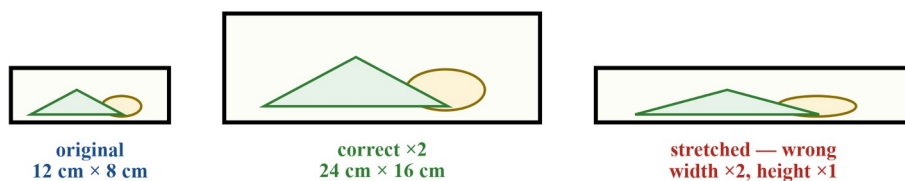


Refresher: multiplying decimals. Do the multiplication with no decimal points first, then put the point back. For 3.2×2.5 : $32 \times 25 = 800$. The two numbers carry two decimal places between them, so the answer keeps two decimal places — 8.00, which is just 8.

Scaling photographs and diagrams.

When you zoom a photo on a phone, the phone is making an enlargement. When a shop prints a poster from a small photo, it is making an enlargement too. The rule is the same: multiply the width **and** the height by the same scale factor.

Multiply the width AND the height by the same scale factor



The copy on the left is right: the width and the height are both $\times 2$. The copy on the right is stretched: the width is $\times 2$ but the height stayed the same ($\times 1$). Same photo — wrong shape.

On grid paper you can scale a diagram by counting squares. To enlarge a drawing with scale factor 3, redraw every part using 3 times as many squares across and 3 times as many squares down. The copy has the same shape and is three times as big in every direction.

Words to know. An **enlargement** is a bigger copy; a **reduction** is a smaller copy. The **scale factor** is the number every length is multiplied by. A **plan** is a smaller-size drawing of a room, a building or an object. **Represents** means “stands for”. The **original** is the shape you started with.

About the data in this section. The plans, photos and measurements here are **made up** (not real): pretend rooms, pretend photo shops and pretend numbers, built for this book so you can practise the maths. Nothing here is real data about a real business, person or place.

Worked examples

Example 1 — scaffolded

A kitchen plan says 1 cm represents 2 m. (a) The bench is 3.5 cm on the plan. How long is the real bench? (b) The window is 1.5 cm on the plan. How wide is the real window? (c) A wall is 9 m in real life. How long is it on the plan?

Working	Comment
(a) $3.5 \times 2 = 7$. The real bench is 7 m long.	Plan to real life: multiply by what 1 cm represents.
(b) $1.5 \times 2 = 3$. The real window is 3 m wide.	Same rule again — every 1 cm on the plan stands for 2 m.
(c) $9 \div 2 = 4.5$. The wall is 4.5 cm on the plan.	Real life to plan: divide. Division undoes multiplication.
Check (c): $4.5 \times 2 = 9$ ✓	Multiply the plan length back — you should get the real length.

Example 2 — scaffolded

A photo is 12 cm wide and 8 cm high. A shop prints it as a poster with scale factor 2.5. Find the width and the height of the poster.

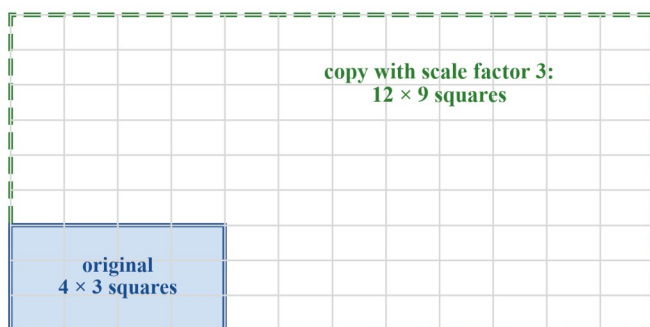
Working	Comment
Width: $12 \times 2.5 = 30$ cm.	Enlargement: multiply every length by the scale factor.
Height: $8 \times 2.5 = 20$ cm.	The height must use the same factor as the width, or the photo stretches.
Check: $30 \div 12 = 2.5$ and $20 \div 8 = 2.5$ ✓	Dividing each new length by its original length gives the same scale factor both times.

The poster is 30 cm wide and 20 cm high.

Example 3 — unscaffolded

A drawing on grid paper is 4 squares wide and 3 squares high. Each square is 1 cm across. The drawing is enlarged with scale factor 3.

Each grid square is 1 cm across — count squares to scale



How many squares wide and high is the copy, and what are its width and height in cm?

Width in squares: $4 \times 3 = 12$ squares.

Height in squares: $3 \times 3 = 9$ squares.

Each square is 1 cm across, so the copy is 12 cm wide and 9 cm high.

Check: $12 \div 4 = 3$ and $9 \div 3 = 3$ ✓ — the same scale factor both ways, so the shape has not stretched.

∴ the copy is 12 squares (12 cm) wide and 9 squares (9 cm) high.

Example 4 — unscaffolded

A tool shed is 3 m long and 2 m wide in real life. On a plan, the shed is drawn 6 cm long and 4 cm wide. What does 1 cm represent on this plan?

Change the real length to cm: $3 \text{ m} = 3 \times 100 = 300 \text{ cm}$.

Divide the real length by the plan length: $300 \div 6 = 50$.

Check with the width: $2 \text{ m} = 200 \text{ cm}$, and $200 \div 4 = 50$ ✓ — the same answer from both sides of the shed.

∴ 1 cm on the plan represents 50 cm (which is 0.5 m).

Practice questions

Scaffolded

Q1. Multiply each length by the scale factor given. (a) 5 cm with scale factor 3. (b) 8 cm with scale factor 0.5. (c) 12 cm with scale factor 0.25.

Q2. A line is 20 cm long. (a) Find the new length when the line is enlarged with scale factor 2. (b) Find the new length when the line is reduced with scale factor 0.25. (c) How much longer is the line from (a) than the line from (b)?

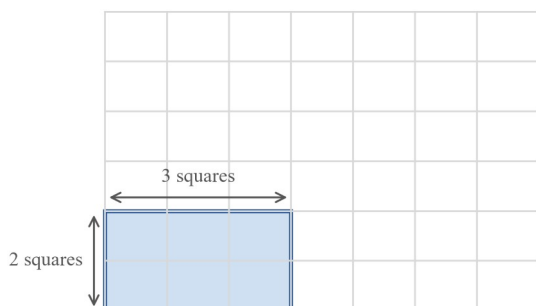
Q3. Find the scale factor in each case, and say whether it is an enlargement or a reduction. (a) 4 cm becomes 12 cm. (b) 10 cm becomes 5 cm. (c) 6 cm becomes 15 cm.

Q4. A garden plan says 1 cm represents 3 m. (a) A path is 2 cm on the plan. How long is the real path? (b) A driveway is 4.5 cm on the plan. How long is the real driveway? (c) A real fence is 24 m long. How long is it on the plan?

Q5. A photo is 9 cm wide and 12 cm high. It is printed with scale factor 2. (a) Find the width of the print. (b) Find the height of the print. (c) Explain why the shop multiplies the width and the height by the same number.

Q6. Look at the shape on the grid. Each square is 1 cm across.

Each square is 1 cm across



The shape is enlarged with scale factor 2. (a) How many squares wide is the copy, and how many cm is that? (b) How many squares high is the copy, and how many cm is that? (c) Use division to check that your copy has scale factor 2.

Un scaffolded

Q7. A bedroom plan says 1 cm represents 0.5 m. (a) The bed is 4 cm long on the plan. What is its real length? (b) The room is 3 m wide in real life. How wide is it on the plan? (c) A wardrobe is 1.25 m wide in real life. How wide is it on the plan?

Q8. A phone photo is 8 cm wide and 6 cm high. It is enlarged with scale factor 2.5 for a print. (a) Find the width of the print. (b) Find the height of the print. (c) A frame is 21 cm wide and 16 cm high. Does the print fit in the frame? Give a reason.

Q9. A carpenter draws a 240 cm shelf for an ad. The drawing uses scale factor 0.25. (a) How long is the shelf in the drawing? (b) Is the drawing an enlargement or a reduction? How do you know? (c) A second shelf is 180 cm long. How long is it in the drawing?

Q10. On a floor plan, a real wall of 5 m is drawn 10 cm long. (a) Change 5 m into cm. (b) What does 1 cm represent on this plan? (c) A window in the same room is 1.6 m wide in real life. How wide is the window on the plan?

Q11. A vegetable garden is drawn on grid paper. Each square on the grid stands for 0.5 m. The garden bed is 4 squares long and 3 squares wide. (a) What is the real length of the garden bed? (b) The bed is enlarged with scale factor 3. How many squares long is the copy? (c) What is the real length of the enlarged bed?

Q12. Tia enlarges a photo that is 6 cm wide and 4 cm high. She multiplies the width by 3 and the height by 2. (a) What are Tia's new width and new height? (b) Explain what is wrong with Tia's copy. (c) What should the height be if the width is multiplied by 3?

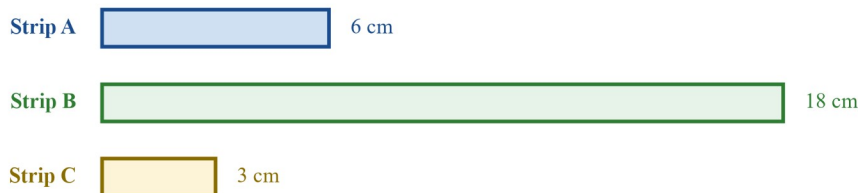
Q13. A poster is 80 cm wide and 60 cm high. It is reduced with scale factor 0.75 for a flyer. (a) Find the width of the flyer. (b) Find the height of the flyer. (c) Is this an enlargement or a reduction? How do you know?

Q14. A community garden plan says 1 cm represents 2 m. One plot is 7.5 cm long and 4 cm wide on the plan. (a) Find the real length of the plot. (b) Find the real width of the plot. (c) How much longer than wide is the plot, in metres?

Q15. (a) Show that a 15 cm line reduced with scale factor 0.4 gives a 6 cm line. One step per line, with the target value at the end. (b) Show that enlarging the 6 cm line from (a) with scale factor 2.5 gives 15 cm back.

Q16. Look at the three strips. Strip A is 6 cm long, strip B is 18 cm long and strip C is 3 cm long.

The three strips (drawn to scale)



- (a) Which strip is an enlargement of strip A with scale factor 3?
- (b) Which strip is a reduction of strip A with scale factor 0.5?
- (c) Find the scale factor that enlarges strip C to strip A.

Q17. A class paints a wall picture. Their design is 30 cm wide on paper. The wall is 4.5 m wide. (a) Change 4.5 m into cm. (b) Find the scale factor from the design to the wall. (c) The design is 20 cm high. How high is the picture on the wall, in metres?

Q18. A plan says 1 cm represents 1.5 m. (a) A wall is 4 cm on the plan. How long is the real wall? (b) Another wall is 5 m in real life. How long is it on the plan? Give your answer to 2 decimal places. (c) Write a “ $\therefore$ ” sentence giving the plan length from (b).

Answers

Q1. (a) 15 cm. (b) 4 cm. (c) 3 cm. **Q2.** (a) 40 cm. (b) 5 cm. (c) 35 cm. **Q3.** (a) 3 — enlargement. (b) 0.5 — reduction. (c) 2.5 — enlargement. **Q4.** (a) 6 m. (b) 13.5 m. (c) 8 cm. **Q5.** (a) 18 cm. (b) 24 cm. (c) the same scale factor both ways keeps the shape — different factors stretch the photo. **Q6.** (a) 6 squares, 6 cm. (b) 4 squares, 4 cm. (c) $6 \div 3 = 2$ ✓. **Q7.** (a) 2 m. (b) 6 cm. (c) 2.5 cm. **Q8.** (a) 20 cm. (b) 15 cm. (c) yes — $20 < 21$ and $15 < 16$, so the print fits. **Q9.** (a) 60 cm. (b) reduction — the scale factor 0.25 is smaller than 1. (c) 45 cm. **Q10.** (a) 500 cm. (b) 1 cm represents 50 cm. (c) 3.2 cm. **Q11.** (a) 2 m. (b) 12 squares. (c) 6 m. **Q12.** (a) 18 cm and 8 cm. (b) the width and height used different scale factors, so the copy is stretched. (c) 12 cm. **Q13.** (a) 60 cm. (b) 45 cm. (c) reduction — 0.75 is smaller than 1. **Q14.** (a) 15 m. (b) 8 m. (c) 7 m. **Q15.** (a) shown: $15 \times 0.4 = 6$. (b) shown: $6 \times 2.5 = 15$. **Q16.** (a) B. (b) C. (c) 2. **Q17.** (a) 450 cm. (b) 15. (c) 3 m. **Q18.** (a) 6 m. (b) 3.33 cm (to 2 decimal places). (c) $\therefore$ the wall is 3.33 cm long on the plan.

Fully-worked solutions

Q1. Multiply the length by the scale factor. (a) $5 \times 3 = 15$ cm. (b) $8 \times 0.5 = 4$ cm. (c) $12 \times 0.25 = 3$ cm.

Q2. (a) $20 \times 2 = 40$ cm. (b) $20 \times 0.25 = 5$ cm. (c) $40 - 5 = 35$ cm longer.

Q3. Scale factor = new length $\div$ original length. (a) $12 \div 4 = 3$ — bigger than 1, so an enlargement. (b) $5 \div 10 = 0.5$ — smaller than 1, so a reduction. (c) $15 \div 6 = 2.5$ — bigger than 1, so an enlargement.

Q4. 1 cm represents 3 m. (a) Plan to real life: $2 \times 3 = 6$ m. (b) $4.5 \times 3 = 13.5$ m. (c) Real life to plan: $24 \div 3 = 8$ cm.

Q5. (a) $9 \times 2 = 18$ cm. (b) $12 \times 2 = 24$ cm. (c) Multiplying both lengths by the same number keeps the same shape. If the width and the height used different numbers, the print would be stretched.

Q6. The shape is 3 squares wide and 2 squares high. (a) $3 \times 2 = 6$ squares wide; each square is 1 cm, so 6 cm. (b) $2 \times 2 = 4$ squares high, so 4 cm. (c) $6 \div 3 = 2$ ✓ — the copy is 2 times as wide as the original.

Q7. 1 cm represents 0.5 m. (a) $4 \times 0.5 = 2$ m. (b) $3 \div 0.5 = 6$ cm. (c) $1.25 \div 0.5 = 2.5$ cm.

Q8. (a) $8 \times 2.5 = 20$ cm. (b) $6 \times 2.5 = 15$ cm. (c) Yes — the print is 20 cm wide and $21 > 20$, and the print is 15 cm high and $16 > 15$, so the print fits in the frame.

Q9. (a) $240 \times 0.25 = 60$ cm. (b) A reduction — the scale factor 0.25 is smaller than 1, so the drawing is smaller than the real shelf. (c) $180 \times 0.25 = 45$ cm.

Q10. (a) $5 \text{ m} = 5 \times 100 = 500$ cm. (b) $500 \div 10 = 50$, so 1 cm represents 50 cm. (c) $1.6 \text{ m} = 160$ cm; $160 \div 50 = 3.2$ cm on the plan.

Q11. (a) $4 \times 0.5 = 2$ m. (b) $4 \times 3 = 12$ squares. (c) $12 \times 0.5 = 6$ m.

Q12. (a) Width: $6 \times 3 = 18$ cm. Height: $4 \times 2 = 8$ cm. (b) A proper enlargement multiplies every length by the same scale factor. Tia used 3 for the width but 2 for the height, so her copy is stretched, not an enlargement. (c) $4 \times 3 = 12$ cm.

Q13. (a) $80 \times 0.75 = 60$ cm. (b) $60 \times 0.75 = 45$ cm. (c) A reduction — the scale factor 0.75 is smaller than 1, so the flyer is smaller than the poster.

Q14. 1 cm represents 2 m. (a) $7.5 \times 2 = 15$ m. (b) $4 \times 2 = 8$ m. (c) $15 - 8 = 7$ m longer than wide.

Q15. (a)

$$15 \times 0.4 = 6$$

$\therefore$ a 15 cm line reduced with scale factor 0.4 gives a 6 cm line. (b)

$$6 \times 2.5 = 15$$

$\therefore$ enlarging the 6 cm line with scale factor 2.5 gives 15 cm back.

Q16. (a) $6 \times 3 = 18$ cm — that is strip B. (b) $6 \times 0.5 = 3$ cm — that is strip C. (c) $6 \div 3 = 2$ — the scale factor from C to A is 2.

Q17. (a) $4.5 \text{ m} = 4.5 \times 100 = 450$ cm. (b) $450 \div 30 = 15$ — the scale factor is 15. (c) $20 \times 15 = 300$ cm; $300 \div 100 = 3$ m high.

Q18. (a) $4 \times 1.5 = 6$ m. (b) $5 \div 1.5 = 3.333\dots$; to 2 decimal places, look at the third decimal digit (3, less than 5), so the plan length is 3.33 cm. (c) $\therefore$ the wall is 3.33 cm long on the plan (to 2 decimal places).

Chapter 03 — Test A (multiple choice)

Chapter 3 (Shape). Materials: one scientific calculator and paper. There are 12 multiple-choice questions, each worth 1 mark (12 marks). Approximate time: 25 minutes. Choose the one correct response for each question. Shapes described in words are not drawn to scale. Every measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1

A polygon has 8 sides. How many vertices does it have?

- A. 8
- B. 4
- C. 6
- D. 16

Question 2

Which quadrilateral has 4 equal sides but no right angles?

- A. Square
- B. Rectangle
- C. Trapezium
- D. Rhombus

Question 3

How many faces does a cube have?

- A. 4
- B. 6
- C. 8
- D. 12

Question 4

How many edges does a triangular prism have?

- A. 5
- B. 6
- C. 9
- D. 12

Question 5

A net is folded to make a cylinder. Which 2D shapes make up this net?

- A. 1 rectangle and 2 circles
- B. 3 rectangles
- C. 2 rectangles and 1 circle
- D. 1 rectangle and 1 circle

Question 6

How many mirror lines does a square have?

- A. 1
- B. 2
- C. 8
- D. 4

Question 7

An arrow points north. It makes a quarter turn clockwise. Which direction does the arrow point now?

- A. North
- B. West
- C. East
- D. South

Question 8

A pattern repeats these four shapes in order: star, circle, square, triangle. It keeps repeating in the same order. What shape is in position 14?

- A. Star
- B. Circle
- C. Square
- D. Triangle

Question 9

A pattern has 5 tiles at stage 1. Each later stage has 3 more tiles than the stage before it. How many tiles are there at stage 7?

- A. 20
- B. 26
- C. 35
- D. 23

Question 10

A square piece of paper is cut straight down the middle and then straight across the middle. How many smaller squares does this make?

- A. 4
- B. 2
- C. 3
- D. 8

Question 11

A photo is enlarged with scale factor 2. One side is 5 cm long. How long is this side after the enlargement?

- A. 7 cm
- B. 10 cm
- C. 2.5 cm
- D. 25 cm

Question 12

A plan uses the scale 1 cm represents 2 m. On the plan, a garden is 5 cm long. How long is the real garden?

- A. 2.5 m
- B. 5 m
- C. 10 m
- D. 7 m

End of Test A

Test A — Answers

Que
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Ans
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1	2	3	4	5	6	7	8	9	10	11	12
A	D	B	C	A	D	C	B	D	A	B	C

Test A — Fully-worked solutions

Question 1. A polygon always has the same number of vertices as sides. A polygon with 8 sides has 8 vertices. **Answer A.**

Question 2. A rhombus has 4 equal sides and opposite sides parallel, but no right angles. (A square has 4 equal sides and 4 right angles, so it is not the answer.) The shape is a **rhombus**. **Answer D.**

Question 3. A cube has 6 square faces. **Answer B.**

Question 4. A triangular prism has 2 triangular ends and 3 rectangular sides. Count the edges: $2 \times 3 = 6$ edges around the two ends, plus 3 edges joining them, so $6 + 3 = 9$ edges. **Answer C.**

Question 5. The net of a cylinder is one rectangle (the curved surface unrolled) and two circles (the top and bottom ends). **Answer A.**

Question 6. A square folds onto itself along 4 lines: top-to-bottom, left-to-right and the two diagonals. So a square has 4 mirror lines. **Answer D.**

Question 7. Clockwise is the way the hands of a clock move: from north, to east, to south, to west. A quarter turn clockwise from north points the arrow **east**. **Answer C.**

Question 8. The repeat has 4 shapes. Divide the position by 4: $14 \div 4 = 3$ remainder 2. The remainder 2 points to the 2nd shape in the repeat, which is **circle**. **Answer B.**

Question 9. Stage 1 has 5 tiles. From stage 1 to stage 7 there are 6 jumps, and each jump adds 3 tiles: $5 + 3 \times 6 = 5 + 18 = 23$ tiles. **Answer D.**

Question 10. Cutting down the middle makes 2 pieces. Cutting across the middle splits each of those into 2 more, so $2 \times 2 = 4$ smaller squares. **Answer A.**

Question 11. New length = original $\times$ scale factor = $5 \times 2 = 10$ cm. **Answer B.**

Question 12. On the plan, 1 cm represents 2 m, so going from the plan to real life means multiplying by 2: real length = $5 \times 2 = 10$ m. **Answer C.**

Chapter 03 — Test B (written response)

Chapter 3 (Shape). Materials: one scientific calculator and paper. There are 7 written questions, worth 28 marks in total. Approximate time: 40 minutes. Answer every question in the spaces provided and show your working. Where an answer needs rounding, the question states the precision — round only as instructed. Every measurement in this test is made up (not real): it is used only to give clean numbers for practising the mathematics.

Question 1 (4 marks)

This question is about 2D shapes.

a. How many sides does a pentagon have? 1 mark

b. How many vertices does an octagon have? 1 mark

c. How many straight sides does a circle have? 1 mark

d. Give the most exact name for the quadrilateral that has exactly one pair of parallel sides. 1 mark

Question 2 (4 marks)

A hexagonal prism has two hexagonal ends and 6 rectangular sides.

a. State the number of faces of the hexagonal prism. 1 mark

b. State the number of edges of the hexagonal prism. 1 mark

c. State the number of vertices of the hexagonal prism. 1 mark

d. A box (rectangular prism) has 6 faces. State the total number of faces on 4 boxes. 1 mark

Question 3 (4 marks)

This question is about mirror lines and turns.

a. State the number of mirror lines a rectangle has. 1 mark

b. A triangle has three equal sides. State the number of mirror lines it has. 1 mark

c. Give the name of a turn of 90° . 1 mark

d. An arrow points north. It makes a half turn. State the direction the arrow now points. 1 mark

Question 4 (4 marks)

A pattern repeats these three shapes in order: circle, star, heart. It keeps repeating in the same order.

a. State the shape in position 9.

1 mark

b. State the shape in position 14.

1 mark

c. A different pattern has 6 tiles at stage 1. Each later stage has 4 more tiles than the stage before it. Find the number of tiles at stage 5. Show your working.

2 marks

Question 5 (4 marks)

This question is about patterns and arrays.

a. A brick wall has 8 courses (rows). Each course has 12 bricks. Find the total number of bricks in the wall. Show your working.

2 marks

b. Calculate $5 + 3 \times 4$.

1 mark

c. A hexagon is cut from its centre to each of its 6 corners. State the number of triangles this makes.

1 mark

Question 6 (4 marks)

This question is about scale factors and plans.

a. A shape is enlarged with scale factor 3. One side is 6 cm long. Find the new length of this side.

1 mark

b. A side is reduced from 20 cm to 5 cm. Find the scale factor. Show your working.

2 marks

c. A plan uses the scale 1 cm represents 3 m. A room is 4 cm wide on the plan. Find the real width of the room.

1 mark

Question 7 (4 marks)

A rectangular photo is 8 cm wide and 5 cm tall. It is reduced with scale factor 0.5.

a. Find the new width of the photo.

1 mark

b. Find the new height of the photo.

1 mark

c. A different shape is enlarged with scale factor 3. The new length of one side is 15 cm. Find the original length of this side. Show your working.

2 marks

End of Test B

Test B — Answers

Q1. (a) 5 (b) 8 (c) 0 (d) trapezium. **Q2.** (a) 8 faces (b) 18 edges (c) 12 vertices (d) 24 faces. **Q3.** (a) 2 (b) 3 (c) quarter turn (d) south. **Q4.** (a) heart (b) star (c) 22 tiles. **Q5.** (a) 96 bricks (b) 17 (c) 6 triangles. **Q6.** (a) 18 cm (b) 0.25 (c) 12 m. **Q7.** (a) 4 cm (b) 2.5 cm (c) 5 cm.

Test B — Fully-worked solutions

Question 1.

- a. A pentagon is a polygon with 5 sides.
- b. A polygon has the same number of vertices as sides, so an octagon (8 sides) has 8 vertices.
- c. A circle has a curved edge and no straight sides, so it has 0 straight sides. (A circle is not a polygon.)
- d. A quadrilateral with exactly one pair of parallel sides is a **trapezium**.

Question 2.

- a. Faces: 2 hexagonal ends + 6 rectangular sides = 8 faces.
- b. Edges: each hexagonal end has 6 edges, so the two ends give $2 \times 6 = 12$, and 6 more edges join the two ends, giving $12 + 6 = 18$ edges.
- c. Vertices: each hexagonal end has 6 vertices, so $6 + 6 = 12$ vertices.
- d. Each box has 6 faces, so 4 boxes have $4 \times 6 = 24$ faces.

Question 3.

- a. A rectangle folds onto itself along 2 lines (joining the midpoints of opposite sides), so it has 2 mirror lines.
- b. A triangle with three equal sides folds onto itself along 3 lines (each from a corner to the middle of the opposite side), so it has 3 mirror lines.
- c. A full turn is 360° , and 90° is one quarter of it, so a turn of 90° is a **quarter turn**.
- d. A half turn is 180° . Turning north through a half turn points the arrow **south**.

Question 4.

- a. The repeat has 3 shapes. Divide the position by 3: $9 \div 3 = 3$ remainder 0. A remainder of 0 means the last shape in the repeat, which is **heart**.
- b. $14 \div 3 = 4$ remainder 2. The remainder 2 points to the 2nd shape in the repeat, which is **star**.
- c. From stage 1 to stage 5 there are 4 jumps, and each jump adds 4 tiles: $6 + 4 \times 4 = 6 + 16 = 22$ tiles.

Question 5.

- a. Total bricks = number of courses $\times$ bricks per course = $8 \times 12 = 96$ bricks.
- b. Multiply before adding: $3 \times 4 = 12$, then $5 + 12 = 17$.
- c. Cutting from the centre to each of the 6 corners splits the hexagon into 6 triangles.

Question 6.

- a. New length = original $\times$ scale factor = $6 \times 3 = 18$ cm.
- b. Scale factor = new length $\div$ original length = $5 \div 20 = 0.25$. It is less than 1, so this is a reduction.
- c. Plan to real means multiplying by 3 (since 1 cm represents 3 m): real width = $4 \times 3 = 12$ m.

Question 7.

a. New width = $8 \times 0.5 = 4$ cm.

b. New height = $5 \times 0.5 = 2.5$ cm.

c. Original length = new length $\div$ scale factor = $15 \div 3 = 5$ cm.