Mathematics Level 9

Chapter 7 — Probability · 3 subtopics

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Contents

Two-step chance experiments, with and without replacement: lists, tree diagrams, tables and arrays

7A · Chapter 7 — Probability · Level 9

Content description `VC2M9P01` (word for word): *list all outcomes for two-step chance experiments both with and without replacement, using lists, tree diagrams, tables or arrays; assign probabilities to outcomes and events*

Achievement standard anchor (AS 18): *Students determine sets of outcomes for two-step chance experiments and represent these in various ways.*

Elaboration ids for linkage: `VC2M9P01.E01`, `VC2M9P01.E02`, `VC2M9P01.E03`, `VC2M9P01.E04`

Prerequisite pointer: Level 8 Probability is assumed — complementary events have a combined probability of one (`VC2M8P01`), all outcome combinations for two events via two-way tables, tree diagrams and Venn diagrams (`VC2M8P02`), and repeated chance experiments and simulations (`VC2M8P03`). The two-way table and the tree are Level 8 tools; 7A adds the list and the array, runs every representation under both replacement rules, and assigns probabilities to outcomes and events.

Note on `VC2M9P01.E03` (extension, recorded): the three-stage tree appears once below, clearly labelled as an extension and not assessed at Level 9 (D3.11); the Level 10 probability ideas that follow it (`VC2M10P01`, `VC2M10P02`) are not taught in this chapter. Two-step experiments only are assessed.

Note on sources: every context that looks like the world — school rolls, team lists, prize bags — is a clearly-labelled SCENARIO with invented parameters (R2); the coins, dice, spinners and counters are mathematical equipment. No real data ships and therefore no source line is owed (R1).

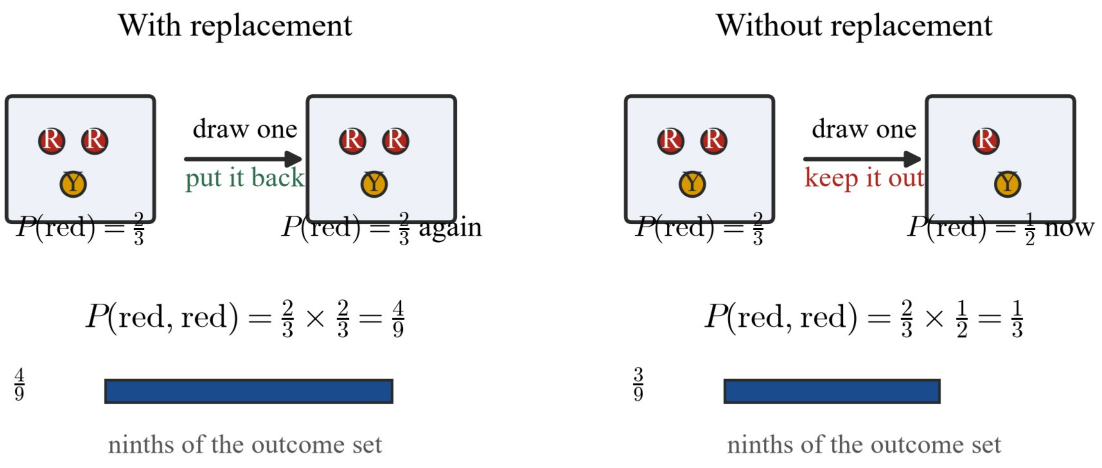
Theory

Two steps, one set of outcomes

A two-step chance experiment does one random thing and then another: flip a coin and roll a die, spin a spinner twice, draw a counter and draw again. One **outcome** records both steps, in order — (heads, 4), (red, blue) — and order matters: (red, blue) and (blue, red) are different outcomes, because "red first, blue second" is a different story from "blue first, red second". The first job in this subtopic is to list **all** the outcomes; the full set is what every representation below shows. Lists are written **systematically**: hold the first step fixed and run through every second-step result, then move on — nothing missed, nothing doubled.

With and without replacement

Draw a counter from a bag and put it back before the second draw, and the experiment runs **with replacement**: the second draw sees exactly the same bag as the first, every first–second pair is possible, and the second-draw chances repeat the first-draw chances. Leave the first counter out, and the experiment runs **without replacement**: the second draw sees a changed bag, some pairs become impossible, and the second-draw chances change. Same bag, same two draws — different sets of outcomes and different probabilities. That contrast is the point of this subtopic, and the figure below is its picture.



Same bag, same event — but replacement changes the second-draw chances: $\frac{4}{9}$ with replacement vs $\frac{3}{9}$ without.

Side-by-side panels showing that with replacement the chance of a second red from a bag of two red and one yellow stays two-thirds, giving two reds probability four-ninths, while without replacement it falls to one-half, giving one-third

SCENARIO. A bag holds 2 red and 1 yellow counter, and two are drawn. With replacement, $P(\text{red, red}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. Without replacement, once a red is out only 1 red and 1 yellow remain, so the second draw's red chance falls to $\frac{1}{2}$ and $P(\text{red, red}) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} = \frac{3}{9}$.

Four ways to show the same experiment

- **List** — every ordered pair written out systematically. Best when the set of outcomes is small: 9 or 12 outcomes read easily.
- **Tree diagram** — branches for the first step, then branches for the second step from each first result. Each branch carries its probability; multiplying along a path gives that outcome's probability; the branches leaving any one node always add to 1, and so do the outcome probabilities at the ends. The tree is the representation that shows *why* the without-replacement chances change.
- **Two-way table** — one step labels the rows and the other the columns; each cell is one outcome. Best when the two steps are different devices, such as a coin and a die.
- **Array** — the square grid of all ordered pairs when the same device runs twice, often written with the sums or products in the cells. Best for two dice or two identical spinners.

All four display the same object — the full set of outcomes — and any one of them can be used to assign probabilities.

Assigning probabilities

An **event** is a collection of outcomes: "the same colour twice", "a sum of 7". When every outcome in the set is equally likely,

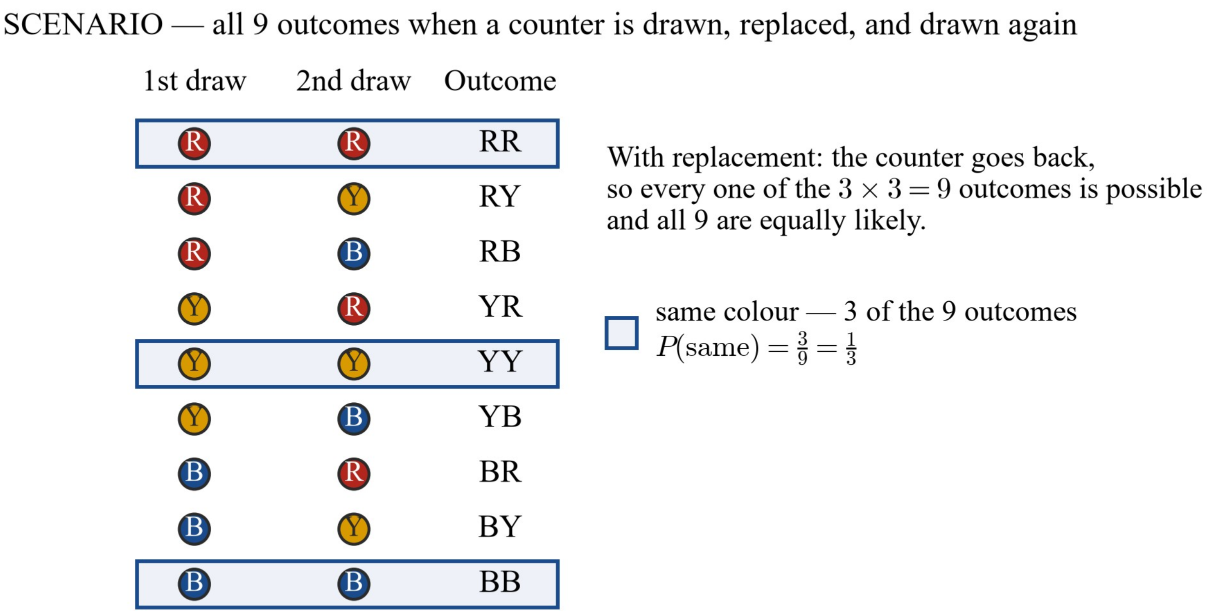
$$P(\text{event}) = \frac{\text{outcomes in the event}}{\text{outcomes in the set}},$$

and the probabilities of all the outcomes add to 1. That makes the Level 8 complement rule (`VC2M8P01`) a shortcut here too: $P(\text{event}) = 1 - P(\text{its complement})$. When the outcomes are not equally likely — a bag with more reds than yellows, say — the tree does the work instead: multiply along each path, then add the paths that belong to the event.

Worked examples

Example 1 — scaffolded

SCENARIO. A bag holds one red, one yellow and one blue counter. A counter is drawn, its colour recorded, put back, and a second is drawn. List all outcomes and find the probability that the two colours are the same.



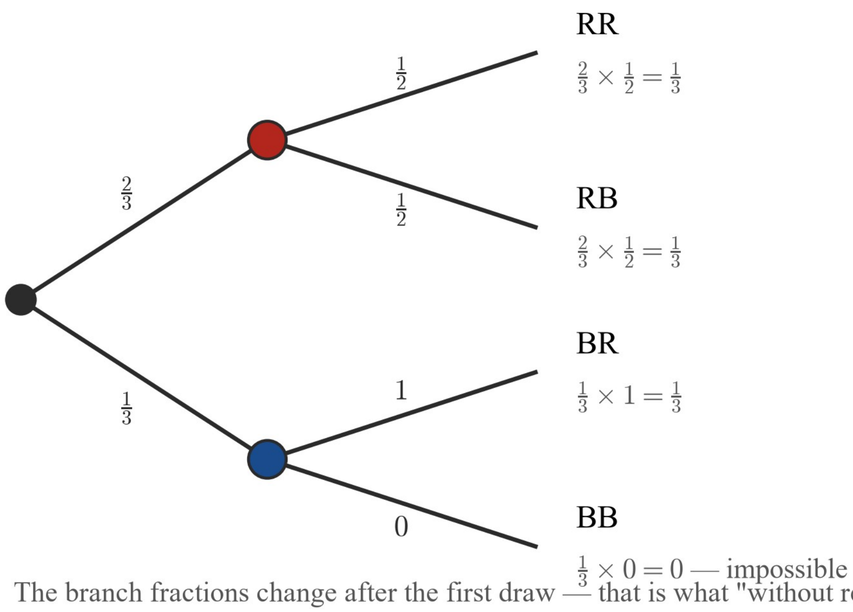
Systematic list of the nine equally likely outcomes when a red, yellow or blue counter is drawn, replaced and drawn again, with the three same-colour outcomes highlighted

Working	Comment
Hold the first draw fixed, vary the second: RR, RY, RB; then YR, YY, YB; then BR, BY, BB	Systematic list: $3 \times 3 = 9$ outcomes, shown in the figure.
Each outcome has probability $\frac{1}{9}$	With replacement nothing changes between the draws, so all 9 are equally likely.
Same colour: RR, YY, BB — 3 outcomes	The highlighted rows in the figure.
$P(\text{same}) = \frac{3}{9} = \frac{1}{3}$	Outcomes in the event ÷ outcomes in the set.
$\therefore P(\text{different}) = 1 - \frac{1}{3} = \frac{2}{3}$	The Level 8 complement rule — and $\frac{6}{9}$ straight from the list agrees.

Example 2 — scaffolded

SCENARIO. A bag holds 2 red and 1 blue counter. Two counters are drawn without replacement. Draw the tree and find the probability of exactly one red.

SCENARIO — tree diagram: 2 red and 1 blue counters, drawn without replacement



The branch fractions change after the first draw — that is what "without replacement" does.

Tree diagram for drawing two counters without replacement from a bag of two red and one blue, with first-draw branch probabilities two-thirds and one-third, second-draw branches one-half and one-half after a red and one and zero after a blue, and outcome probabilities one-third, one-third, one-third and zero

Working	Comment
First draw: $P(R)=\frac{2}{3}$, $P(B)=\frac{1}{3}$	2 of the 3 counters are red.
After a red, 1 red and 1 blue remain: $\frac{1}{2}$ and $\frac{1}{2}$	The bag has changed — that is what "without replacement" means.
After a blue, 2 reds remain: $P(R)=1$, $P(B)=0$	A second blue is impossible.
Multiply along each path: RR $=\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$, RB $=\frac{1}{3}$, BR $=\frac{1}{3}\times 1=\frac{1}{3}$, BB $=\frac{1}{3}\times 0=0$	Path probability = branch × branch, as in the figure.
Exactly one red is the paths RB and BR: $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$	An event's probability is the sum of its outcomes' probabilities.
∴ Check: $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+0=1$	The four outcome probabilities must add to 1 — they do.

Example 3 — scaffolded

SCENARIO. A coin is flipped and a six-sided die is rolled. Record the experiment in a two-way table and find the probability of heads and an even number.

SCENARIO — two-way table: one coin flip and one die roll

Coin \ die	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

heads and an even number: 3 of 12

$2 \times 6 = 12$ equally likely outcomes; $P(\text{heads and even}) = \frac{3}{12} = \frac{1}{4}$

Two-way table of the twelve outcomes when a coin is flipped and a six-sided die is rolled, with the three cells for heads and an even number highlighted

Working	Comment
2 rows (H, T) × 6 columns (1–6) = 12 cells	Each cell is one outcome; all equally likely, each $\frac{1}{12}$.
Heads and even: H2, H4, H6 — 3 cells	The highlighted cells in the figure.
$P(\text{heads and even}) = \frac{3}{12} = \frac{1}{4}$	Count the event, divide by 12.
$\therefore\ P(\text{tails and even}) = \frac{3}{12} = \frac{1}{4}$ as well	The table puts every event one glance away.

Example 4 — scaffolded

SCENARIO. Two six-sided dice are rolled and the scores added. Use an array to find the probability that the sum is 7, and that the sum is greater than 10.

SCENARIO — array of the 36 sums when two dice are rolled

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

sum of 7 — 6 of 36 outcomes

sum greater than 10 — 3 of 36 outcomes

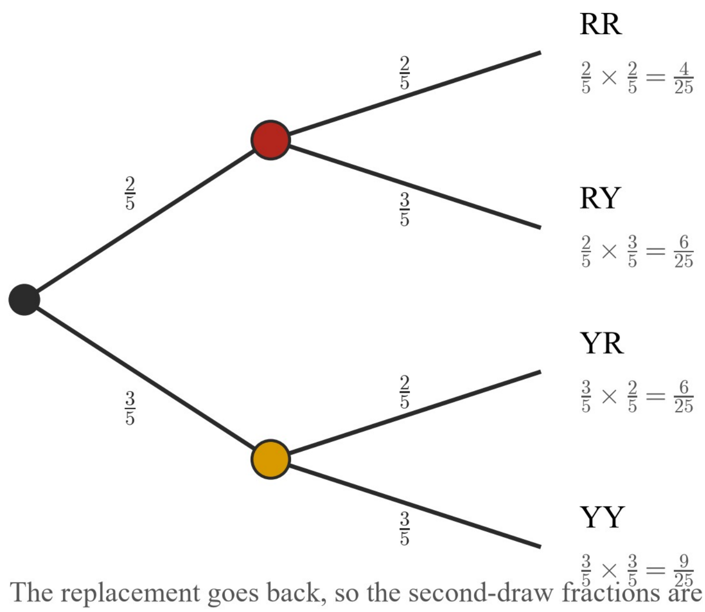
Array of the thirty-six sums of two six-sided dice, with the six cells that sum to seven highlighted in green and the three cells above ten highlighted in red

Working	Comment
$6 \times 6 = 36$ equally likely cells	The array holds every sum at once.
Sum 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) — 6 cells	The green diagonal in the figure.
$P(7) = \frac{6}{36} = \frac{1}{6}$	The most likely single sum — the longest diagonal.
Sum greater than 10: the two 11s and the 12 — 3 cells	The red corner of the figure.
$\therefore P(>10) = \frac{3}{36} = \frac{1}{12}$	Reading an event off an array is counting cells.

Example 5 — unscaffolded

SCENARIO. A bag holds 2 red and 3 yellow counters. A counter is drawn, replaced, and a second drawn. Find the probability that both are red, and the probability of one of each colour.

SCENARIO — tree diagram: 2 red and 3 yellow counters, drawn with replacement



The replacement goes back, so the second-draw fractions are the same as the first-draw ones.

Tree diagram for drawing two counters with replacement from a bag of two red and three yellow, with the same branch probabilities two-fifths and three-fifths on both draws and outcome probabilities four twenty-fifths, six twenty-fifths, six twenty-fifths and nine twenty-fifths

With replacement, both draws see the same bag, so both carry $P(R)=\frac{2}{5}$ and $P(Y)=\frac{3}{5}$ — the tree repeats its branches. Multiplying along the paths: RR $=\frac{2}{5}\times\frac{2}{5}=\frac{4}{25}$; RY = YR $=\frac{2}{5}\times\frac{3}{5}=\frac{6}{25}$; YY $=\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$. One of each colour is the two middle paths: $\frac{6}{25}+\frac{6}{25}=\frac{12}{25}$. ∴ The four outcome probabilities add to $\frac{4+6+6+9}{25}=1$, exactly as they must.

Example 6 — unscaffolded

SCENARIO. The same bag — 2 red and 3 yellow counters — but now the two counters are drawn without replacement. Find the probability that both are red, and the probability of one of each colour, and compare with Example 5.

First draw: $P(R)=\frac{2}{5}$, $P(Y)=\frac{3}{5}$. After a red, 1 red and 3 yellows remain: $P(R)=\frac{1}{4}$, $P(Y)=\frac{3}{4}$. After a yellow, 2 reds and 2 yellows remain: $P(R)=\frac{2}{4}$, $P(Y)=\frac{2}{4}$. Both red: $\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$. One of each: $\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}+\frac{3}{10}=\frac{3}{5}$. Compare Example 5: with replacement, both red was $\frac{4}{25}$; without, it is $\frac{1}{10}$ — taking a red out leaves fewer reds behind, so the second red is harder to draw. ∴ Same bag, same events, different answers: the replacement rule is the choice that changes the probabilities.

Example 7 — unscaffolded

SCENARIO. Four students — Mia, Noah, Ava and Tom — put their names in a bag. Two names are drawn to appoint a captain and a vice-captain, so a name is not replaced. Find the probability that Mia is captain, and that Mia and Noah are the two leaders.

SCENARIO — array for drawing 2 team leaders from 4 names, without replacement

1st draw (captain) ↓
2nd draw (vice-captain) →

	Mia	Noah	Ava	Tom
Mia	×	Mia, Noah	Mia, Ava	Mia, Tom
Noah	Noah, Mia	×	Noah, Ava	Noah, Tom
Ava	Ava, Mia	Ava, Noah	×	Ava, Tom
Tom	Tom, Mia	Tom, Noah	Tom, Ava	×

The diagonal is impossible — a name is not replaced — leaving $4 \times 3 = 12$ equally likely outcomes.
Shaded: Mia and Noah as the two leaders, in either order: $P = \frac{2}{12} = \frac{1}{6}$

Four-by-four array of the ordered pairs of team leaders drawn from the names Mia, Noah, Ava and Tom without replacement, with the impossible diagonal crossed out and the two Mia-and-Noah cells highlighted

The captain's name is not put back, so the diagonal of the array is crossed out: $4 \times 3 = 12$ equally likely outcomes, not 16. Mia as captain is Mia's row, 3 outcomes: $P = \frac{3}{12} = \frac{1}{4}$. Mia and Noah as the two leaders, in either order, are the two highlighted cells: $P = \frac{2}{12} = \frac{1}{6}$. ∴ Without replacement the crossed-out diagonal is the whole story: 12 outcomes, and every count follows from there.

Example 8 — unscaffolded

SCENARIO. A school has 200 students, 50 of them in Year 9. Two names are drawn at random, without replacement, for a student panel. Find the probability that both names are Year 9 students, and compare with drawing with replacement.

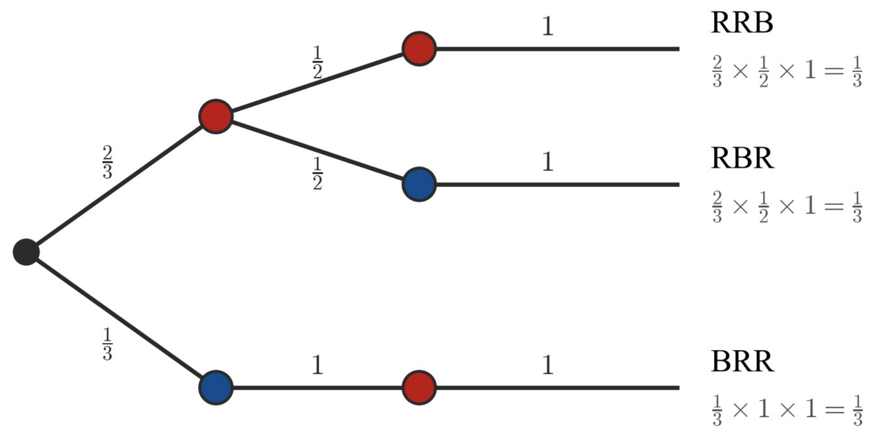
First name Year 9: $\frac{50}{200} = \frac{1}{4}$; then 199 names remain, 49 of them Year 9, so the second is $\frac{49}{199}$. Both Year 9: $\frac{1}{4} \times \frac{49}{199} = \frac{49}{796} \approx 0.062$ — about 6.2%. With replacement instead: $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = 0.0625$ — only slightly larger, because losing one name out of 200 changes very little. ∴ The honest model for two different names is without replacement; with a large population the two answers nearly agree.

Extension — a three-stage tree (labelled extension; not assessed at Level 9)

The content description stops at two steps, and two steps are what this book assesses. `VC2M9P01.E03` goes one stage further, so here is one clearly-labelled look at a three-stage tree — extension material, not assessed.

SCENARIO. A bag holds 2 red and 1 blue counter. All three counters are drawn, one after another, without replacement. What is the probability of each ordering of two reds and one blue?

SCENARIO — Extension: a three-stage tree diagram (labelled extension; not assessed at Level 9)



Three draws empty the bag: the three orderings of two reds and one blue each have probability $\frac{1}{3}$, and $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$.

Three-stage tree diagram for drawing all three counters from a bag of two red and one blue without replacement, showing the three orderings red-red-blue, red-blue-red and blue-red-red each with probability one-third

After two draws only one counter remains, so the last branch on every surviving path is 1: each ordering has probability $\frac{2}{3}\times\frac{1}{2}\times 1=\frac{1}{3}$. The three orderings RRB, RBR and BRR are all there is — the bag is emptied — and $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$. E03's own example works the same way: three cards drawn from a deck, asking for an ace, then a king, then a queen of the same suit — multiply along three branches, one per draw, with the branches changing as cards leave the deck. ∴ A third stage reads exactly like a second — one more layer of branches — but assessment in this book stops at two steps.

Practice questions

Scaffolded

Q1. A coin is flipped twice.

a. List all outcomes.
b. Find the probability of two heads.
c. Find the probability of one head and one tail.

Q2. A spinner with three equal sectors — red, yellow, blue — is spun twice.

a. How many outcomes are in the set?
b. Find the probability of the same colour both times.

Q3. A bag holds 1 red and 1 blue counter. A counter is drawn, replaced, and a second drawn.

a. Find the probability of one counter of each colour.
b. Find the probability of the same colour twice.

Q4. A bag holds 2 red and 2 blue counters. Two counters are drawn without replacement.

a. Find the probability of the same colour twice.
b. Find the probability of two different colours.

Q5. A coin is flipped and a four-sided die, numbered 1–4, is rolled. Record the outcomes in a two-way table.

a. How many outcomes are in the set?
b. Find the probability of heads and an odd number.

Q6. Using the same table as Q5, find the probability of tails and a number greater than 2.

Q7. Two four-sided dice, numbered 1–4, are rolled and the scores added. Use an array.

a. Find the probability that the sum is 5.
b. Find the probability that the sum is 8.

Q8. Using the same array as Q7, find the probability that the sum is even.

Q9. A bag holds 3 green and 1 white counter. A counter is drawn, replaced, and a second drawn.

a. Find the probability that both are green.
b. Find the probability of one of each colour.
c. Find the probability that both are white.

Q10. The same bag as Q9, but the two counters are drawn without replacement.

a. Find the probability that both are green.
b. Find the probability of one of each colour.

Q11. Two coins are flipped; the event is "at least one head".

a. List the outcomes in the event and find its probability.
b. Find the same probability from the complement, using the Level 8 rule.

Q12. A bag holds 2 red and 1 yellow counter, and two counters are drawn.

a. Find the probability of two reds with replacement.
b. Find the probability of two reds without replacement.
c. Which is larger? Give a one-sentence reason.

Unscaffolded

Q13. A coin is flipped and a six-sided die is rolled. How many outcomes are in the set? Find the probability of heads and a 6, and of tails and an even number.

Q14. Two six-sided dice are rolled and the scores added. Find the probability of a sum of 2, a sum of 12, and a sum of 7.

Q15. Two six-sided dice are rolled and the scores added. Find the probability that the sum is at least 10.

Q16. Two six-sided dice are rolled and the difference of the scores is recorded (bigger minus smaller, so never negative). Find the probability that the difference is 0, and that it is 1.

Q17. A bag holds 1 red, 1 yellow and 1 blue counter. Two counters are drawn without replacement. List all outcomes; find the probability of two different colours, and of the same colour twice.

Q18. The same bag as Q17, but the counter is replaced between draws. Find the probability of the same colour twice, and of red then blue.

Q19. A bag holds 4 red and 2 yellow counters. Two counters are drawn without replacement. Find the probability that both are red, and the probability of one of each colour.

Q20. The same bag as Q19, but with replacement. Find the probability that both are red, and the probability of one of each colour. Compare your both-red answer with Q19's.

Q21. SCENARIO. Three students — Zoe, Sam and Ivy — put their names in a bag; a captain and a vice-captain are drawn, so a name is not replaced. Find the probability that Zoe is captain, and that Sam and Ivy are the two leaders.

Q22. SCENARIO. A school has 400 students, 100 of them in Year 9. Two names are drawn at random, without replacement. Find the probability that both are Year 9 students.

Q23. A coin is flipped and a two-colour spinner — yellow, blue — is spun. Find the probability of tails, and of heads and yellow.

Q24. A spinner with three equal sectors — red, yellow, blue — is spun twice. Find the probability of exactly one red.

Q25. A bag holds 3 red and 2 blue counters. Two counters are drawn without replacement. Find the probability of the same colour twice, and of two different colours.

Q26. Two four-sided dice, numbered 1–4, are rolled and the scores multiplied. Find the probability that the product is 6, and that the product is odd.

Q27. A bag holds 3 red and 3 blue counters, and two counters are drawn. Find the probability of two reds with replacement and without, and say which is larger.

Q28. A bag holds 2 red and 3 blue counters. Two counters are drawn without replacement. Use the complement to find the probability of at least one blue.

Q29. A coin is flipped and a four-sided die, numbered 1–4, is rolled. List the outcomes systematically and find the probability of tails and an odd number.

Q30. Two six-sided dice are rolled and the scores added. Find the probability that the sum is less than 5.

Q31. A bag holds 2 red and 1 blue counter. Explain in one sentence why the probability of two reds is different with and without replacement, and calculate both.

Q32. A spinner has 2 red sectors and 1 blue sector, so $P(\text{red})=\frac{2}{3}$ and $P(\text{blue})=\frac{1}{3}$. It is spun twice. Write the probability of each of the four outcomes RR, RB, BR, BB, show that they add to 1, and find the probability of at least one red.

Q33. SCENARIO. Five students each put their name in a bag; a captain and a vice-captain are drawn, so a name is not replaced. How many ordered outcomes are there? Show that the probability a named student ends up with one of the two roles is $\frac{2}{5}$.

Q34. Spinner A has sectors 1 and 2; spinner B has sectors 1, 2 and 3. Both are spun. Find the probability that the sum is 4, and that the product is even.

Q35. A heads-and-tails game: two coins are flipped; Priya scores a point when the faces match, Tom when they differ. Is the game fair? Justify with probabilities. Then consider the variant in which Priya scores only on two heads and Tom scores otherwise — is that fair?

Q36. A bag holds 2 red, 2 yellow and 1 blue counter. Two counters are drawn without replacement.

a. Find the probability that no blue is drawn.
b. Find the probability that the blue comes only on the second draw (first not blue, second blue).
c. Find the probability that the blue appears at least once.

Answers

Q1 a HH, HT, TH, TT; b $\frac{1}{4}$; c $\frac{1}{2}$. **Q2** a 9; b $\frac{1}{3}$. **Q3** a $\frac{1}{2}$; b $\frac{1}{2}$. **Q4** a $\frac{1}{3}$; b $\frac{2}{3}$. **Q5** a 8; b $\frac{1}{4}$. **Q6** $\frac{1}{4}$. **Q7** a $\frac{1}{4}$; b $\frac{1}{16}$. **Q8** $\frac{1}{2}$. **Q9** a $\frac{9}{16}$; b $\frac{3}{8}$; c $\frac{1}{16}$. **Q10** a $\frac{1}{2}$; b $\frac{1}{2}$. **Q11** a HH, HT, TH — $\frac{3}{4}$; b $1-\frac{1}{4}=\frac{3}{4}$. **Q12** a $\frac{4}{9}$; b $\frac{1}{3}$; c with replacement — $\frac{4}{9}>\frac{3}{9}$, because the second draw keeps the full bag. **Q13** 12 outcomes; heads and a 6: $\frac{1}{12}$; tails and even: $\frac{1}{4}$. **Q14** $\frac{1}{36}$; $\frac{1}{36}$; $\frac{1}{6}$. **Q15** $\frac{1}{6}$. **Q16** difference 0: $\frac{1}{6}$; difference 1: $\frac{5}{18}$. **Q17** RY, RB, YR, YB, BR, BY; different colours: 1; same colour: 0. **Q18** $\frac{1}{3}$; $\frac{1}{9}$. **Q19** $\frac{2}{5}$; $\frac{8}{15}$. **Q20** $\frac{4}{9}$; $\frac{4}{9}$; with replacement is the larger both-red chance ($\frac{4}{9}>\frac{2}{5}$). **Q21** $\frac{1}{3}$; $\frac{1}{3}$. **Q22** $\frac{33}{532}\approx 0.062$. **Q23** $\frac{1}{2}$; $\frac{1}{4}$. **Q24** $\frac{4}{9}$. **Q25** $\frac{2}{5}$; $\frac{3}{5}$. **Q26** $\frac{1}{8}$; $\frac{1}{4}$. **Q27** with $\frac{1}{4}$; without $\frac{1}{5}$; with replacement is larger. **Q28** $\frac{9}{10}$. **Q29** $\frac{1}{4}$. **Q30** $\frac{1}{6}$. **Q31** with $\frac{4}{9}$, without $\frac{1}{3}$; without replacement the first red leaves fewer reds behind, so the second is harder to draw. **Q32** RR $\frac{4}{9}$, RB $\frac{2}{9}$, BR $\frac{2}{9}$, BB $\frac{1}{9}$; they add to 1; at least one red $\frac{8}{9}$. **Q33** 20; $\frac{2}{5}$. **Q34** $\frac{1}{3}$; $\frac{2}{3}$. **Q35** first game fair — $\frac{1}{2}$ each; the variant is not — $\frac{1}{4}$ against $\frac{3}{4}$. **Q36** a $\frac{3}{5}$; b $\frac{1}{5}$; c $\frac{2}{5}$.

Fully-worked solutions

Q1. a Each flip has 2 faces, so two flips give HH, HT, TH, TT — 4 outcomes. b Two heads is 1 of the 4: $\frac{1}{4}$. c One of each is HT and TH: $\frac{2}{4}=\frac{1}{2}$. ∴ 4 outcomes; two heads $\frac{1}{4}$; one of each $\frac{1}{2}$.

Q2. a $3\times 3=9$ outcomes, listed systematically as RR, RY, RB, YR, YY, YB, BR, BY, BB. b Same colour: RR, YY, BB — $\frac{3}{9}=\frac{1}{3}$. ∴ 9 outcomes, and the same-colour probability is $\frac{1}{3}$.

Q3. a With replacement each draw is $\frac{1}{2}$–$\frac{1}{2}$; one of each is RB + BR $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$. b Same colour is RR + BB $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ — here the two events split the set evenly. ∴ One of each $\frac{1}{2}$; same colour $\frac{1}{2}$.

Q4. a First draw is $\frac{2}{4}$ either colour; after one counter, 3 remain with 1 of its colour and 2 of the other. Same colour: $\frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$. b Different colours: $1-\frac{1}{3}=\frac{2}{3}$. ∴ Same colour $\frac{1}{3}$; different $\frac{2}{3}$.

Q5. a 2 rows × 4 columns = 8 outcomes. b Heads and odd: H1, H3 — $\frac{2}{8}=\frac{1}{4}$. ∴ 8 outcomes, and heads-with-odd is $\frac{1}{4}$.

Q6. Tails and greater than 2: T3, T4 — 2 of the 8 cells, $\frac{2}{8}=\frac{1}{4}$. ∴ $\frac{1}{4}$.

Q7. a The array has 16 cells; sum 5 comes from (1,4), (2,3), (3,2), (4,1) — $\frac{4}{16}=\frac{1}{4}$. b Sum 8 needs (4,4) — $\frac{1}{16}$. ∴ Sum 5: $\frac{1}{4}$; sum 8: $\frac{1}{16}$.

Q8. The sum is even when both scores are odd or both even: odd–odd gives $2\times2=4$ cells, even–even another 4, so 8 of 16: $\frac{8}{16}=\frac{1}{2}$. ∴ $\frac{1}{2}$.

Q9. a $P(G)=\frac{3}{4}$ both draws: $\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$. b One of each: GW + WG = $\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}=\frac{6}{16}=\frac{3}{8}$. c WW: $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$. ∴ $\frac{9}{16}$, $\frac{3}{8}$, $\frac{1}{16}$ — and they add to 1.

Q10. a Both green: $\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$ — after a green, 2 greens and 1 white remain. b One of each: GW = $\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$ and WG = $\frac{1}{4}\times1=\frac{1}{4}$, together $\frac{1}{2}$. ∴ Both green $\frac{1}{2}$; one of each $\frac{1}{2}$ — and WW is impossible, $\frac{1}{4}\times0=0$.

Q11. a The set is HH, HT, TH, TT; the event holds HH, HT, TH — 3 of 4, so $\frac{3}{4}$. b The complement is TT alone, $\frac{1}{4}$; $1-\frac{1}{4}=\frac{3}{4}$ — the Level 8 rule agrees. ∴ $P(\text{at least one head})=\frac{3}{4}$, listed or complemented.

Q12. a With replacement: $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$. b Without: $\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$. c $\frac{4}{9}>\frac{3}{9}$: with replacement the second draw keeps the full bag, so the second red stays as easy as the first. ∴ $\frac{4}{9}$ with, $\frac{1}{3}$ without — replacement raises the chance here.

Q13. 2 coin faces × 6 die scores = 12 equally likely outcomes. Heads and a 6 is one cell: $\frac{1}{12}$. Tails and even: T2, T4, T6 — $\frac{3}{12}=\frac{1}{4}$. ∴ 12 outcomes; $\frac{1}{12}$ and $\frac{1}{4}$.

Q14. Sum 2 is (1,1) and sum 12 is (6,6): one cell each of 36, so $\frac{1}{36}$ each. Sum 7 has 6 cells: $\frac{6}{36}=\frac{1}{6}$. ∴ $\frac{1}{36}$, $\frac{1}{36}$, $\frac{1}{6}$ — the middle sum dwarfs the ends.

Q15. Sum 10: (4,6), (5,5), (6,4); sum 11: (5,6), (6,5); sum 12: (6,6) — 6 cells of 36. ∴ $P(\text{sum}\geq10)=\frac{6}{36}=\frac{1}{6}$.

Q16. Difference 0 means doubles: 6 of 36, $\frac{1}{6}$. Difference 1: (1,2), (2,3), (3,4), (4,5), (5,6) and their reversals — 10 of 36, $\frac{10}{36}=\frac{5}{18}$. ∴ Doubles $\frac{1}{6}$; difference 1 $\frac{5}{18}$.

Q17. Without replacement the list is RY, RB, YR, YB, BR, BY — 6 outcomes. Every one of the 6 has two different colours, so $P(\text{different})=\frac{6}{6}=1$, and $P(\text{same})=0$ — with one counter of each colour, a repeat is impossible. ∴ 6 outcomes; different colours is certain, same colour impossible.

Q18. With replacement there are $3\times3=9$ equally likely outcomes. Same colour: RR, YY, BB — $\frac{3}{9}=\frac{1}{3}$. Red then blue is one outcome: $\frac{1}{9}$. ∴ $\frac{1}{3}$ and $\frac{1}{9}$ — compare Q17, where same colour was impossible.

Q19. Both red: $\frac{4}{6}\times\frac{3}{5}=\frac{12}{30}=\frac{2}{5}$. One of each: RY = $\frac{4}{6}\times\frac{2}{5}=\frac{4}{15}$ and YR = $\frac{2}{6}\times\frac{4}{5}=\frac{4}{15}$, together $\frac{8}{15}$. ∴ Both red $\frac{2}{5}$; one of each $\frac{8}{15}$.

Q20. With replacement: both red $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$; one of each $2\times\frac{2}{3}\times\frac{1}{3}=\frac{4}{9}$. Against Q19's $\frac{2}{5}=\frac{18}{45}$, the with-replacement $\frac{4}{9}=\frac{20}{45}$ is larger — keeping the reds in the bag keeps the second red easy. ∴ Both red $\frac{4}{9}$, one of each $\frac{4}{9}$, and here with replacement beats without.

Q21. Without replacement there are $3\times 2=6$ ordered outcomes: ZS, ZI, SZ, SI, IZ, IS. Zoe captain is ZS, ZI: $\frac{2}{6}=\frac{1}{3}$. Sam and Ivy as the two leaders is SI, IS: $\frac{2}{6}=\frac{1}{3}$. ∴ Both probabilities are $\frac{1}{3}$.

Q22. First name Year 9: $\frac{100}{400}=\frac{1}{4}$; then 399 names remain, 99 of them Year 9. Both Year 9: $\frac{1}{4}\times\frac{99}{399}=\frac{99}{1596}=\frac{33}{532}\approx 0.062$ — about 6.2%. ∴ $\frac{33}{532}$; the large school makes the with-replacement answer $\frac{1}{16}=0.0625$ almost identical.

Q23. The set is HY, HB, TY, TB — 4 equally likely outcomes. Tails is TY, TB: $\frac{2}{4}=\frac{1}{2}$. Heads and yellow is HY: $\frac{1}{4}$. ∴ $\frac{1}{2}$ and $\frac{1}{4}$.

Q24. Exactly one red is R-then-not-red or not-red-then-R: $\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}=\frac{2}{9}+\frac{2}{9}=\frac{4}{9}$. ∴ $\frac{4}{9}$ — the two one-red paths carry equal weight.

Q25. Same colour: RR $=\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$ plus BB $=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$, together $\frac{4}{10}=\frac{2}{5}$. Different colours: $1-\frac{2}{5}=\frac{3}{5}$. ∴ Same $\frac{2}{5}$; different $\frac{3}{5}$.

Q26. Product 6: (2,3) and (3,2) — 2 of 16, $\frac{2}{16}=\frac{1}{8}$. Product odd needs both scores odd: $2\times 2=4$ of 16, $\frac{1}{4}$. ∴ $\frac{1}{8}$ and $\frac{1}{4}$.

Q27. With replacement: $\frac{3}{6}\times\frac{3}{6}=\frac{1}{4}$. Without: $\frac{3}{6}\times\frac{2}{5}=\frac{6}{30}=\frac{1}{5}$. $\frac{1}{4}>\frac{1}{5}$ — with replacement the second draw keeps all three reds. ∴ $\frac{1}{4}$ with, $\frac{1}{5}$ without; with replacement is the larger.

Q28. The complement of "at least one blue" is "no blue", i.e. two reds: $\frac{2}{5}\times\frac{1}{4}=\frac{2}{20}=\frac{1}{10}$. So $P(\text{at least one blue})=1-\frac{1}{10}=\frac{9}{10}$. ∴ $\frac{9}{10}$ — one complement saves a four-path tree.

Q29. Systematically: H1, H2, H3, H4; T1, T2, T3, T4 — 8 outcomes. Tails and odd: T1, T3 — $\frac{2}{8}=\frac{1}{4}$. ∴ $\frac{1}{4}$.

Q30. Sum less than 5: sum 2 gives (1,1); sum 3 gives (1,2), (2,1); sum 4 gives (1,3), (2,2), (3,1) — 6 cells of 36. ∴ $P(<5)=\frac{6}{36}=\frac{1}{6}$.

Q31. Without replacement, the first red leaves fewer reds in the bag, so the second draw's red chance falls; with replacement it does not. With: $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$. Without: $\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$. $\therefore$ $\frac{4}{9}$ against $\frac{1}{3}$ — the changed second draw is the whole difference.

Q32. RR $=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$; RB = BR $=\frac{2}{3}\times\frac{1}{3}=\frac{2}{9}$; BB $=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$. Check: $\frac{4}{9}+\frac{2}{9}+\frac{2}{9}+\frac{1}{9}=\frac{9}{9}=1$. At least one red is everything but BB: $1-\frac{1}{9}=\frac{8}{9}$. $\therefore$ $\frac{4}{9},\frac{2}{9},\frac{2}{9},\frac{1}{9}$, summing to 1; at least one red $\frac{8}{9}$.

Q33. Captain then vice-captain: $5\times4=20$ ordered outcomes. A named student — say Mia — is captain in 4 outcomes and vice-captain in 4, so 8 of 20: $\frac{8}{20}=\frac{2}{5}$. Check by complement: $P(\text{no role})=\frac{4}{5}\times\frac{3}{4}=\frac{3}{5}$, and $1-\frac{3}{5}=\frac{2}{5}$. $\therefore$ 20 outcomes, and the named student's chance is $\frac{2}{5}$.

Q34. The set is (1,1), (1,2), (1,3), (2,1), (2,2), (2,3) — 6 equally likely outcomes. Sum 4: (1,3) and (2,2) — $\frac{2}{6}=\frac{1}{3}$. Product even: (1,2), (2,1), (2,2), (2,3) — $\frac{4}{6}=\frac{2}{3}$. $\therefore$ Sum 4: $\frac{1}{3}$; product even: $\frac{2}{3}$.

Q35. The set is HH, HT, TH, TT. Matching faces: HH, TT — $\frac{2}{4}=\frac{1}{2}$; differing: $\frac{1}{2}$ — so Priya and Tom score equally, and the game is fair. The variant: two heads is $\frac{1}{4}$ for Priya; Tom's "otherwise" is $1-\frac{1}{4}=\frac{3}{4}$ — three times Priya's share. $\therefore$ The matching game is fair at $\frac{1}{2}$ each; the two-heads variant is not.

Q36. a No blue means two draws from the 4 non-blue counters: $\frac{4}{5}\times\frac{3}{4}=\frac{12}{20}=\frac{3}{5}$. b Blue only on the second draw: not blue first ($\frac{4}{5}$), then blue from the 4 left ($\frac{1}{4}$): $\frac{4}{5}\times\frac{1}{4}=\frac{1}{5}$. c At least once is the complement of "no blue": $1-\frac{3}{5}=\frac{2}{5}$ — matching the count: there are 10 unordered pairs of counters, and the blue sits in 4 of them. $\therefore$ $\frac{3}{5},\frac{1}{5},\frac{2}{5}$.

Relative frequency, and events joined by ‘and’, inclusive ‘or’ and exclusive ‘or’

7B · Chapter 7 — Probability · Level 9

Content description `VC2M9P02` (word for word): *calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’, inclusive ‘or’ and exclusive ‘or’*

Achievement standard anchor (AS 19): *They assign probabilities to the outcomes of two-step chance experiments.*

Elaboration ids for linkage: `VC2M9P02.E01`, `VC2M9P02.E02`

Prerequisite pointer: 7A — ‘and’/‘or’ events are assigned over the outcome sets built in 7A. Level 8, `VC2M8P01` (complementary events), `VC2M8P02` (two-way tables, tree diagrams and Venn diagrams) and `VC2M8P03` (repeated chance experiments and simulations) are assumed knowledge: used here, not re-taught.

Note on the word ‘or’: everyday English uses ‘or’ in two different senses — *soup or salad* usually means pick one, not both, while *owners or renters* counts everyone. Probability keeps the two senses apart by naming them: **inclusive ‘or’** means *at least one* of the events happens (both together still counts), and **exclusive ‘or’** means *exactly one* happens, not both. They are different events with different probabilities. Where a question in this book writes plain ‘or’ about events, read it as the inclusive ‘or’ and say so in your answer.

Note on sources and scenarios: the July 2026 Melbourne weather table is real data — Source: Bureau of Meteorology, Daily Weather Observations for Melbourne (Olympic Park), Victoria, July 2026, 2026. Retrieved 20 August 2026. Every other context is a labelled SCENARIO with invented numbers and generic people, so no further source line is owed.

Note on figures: every running-frequency curve is drawn from its own recorded trials, every table and Venn region from its own counts, and every bar from its own data (R7).

Theory

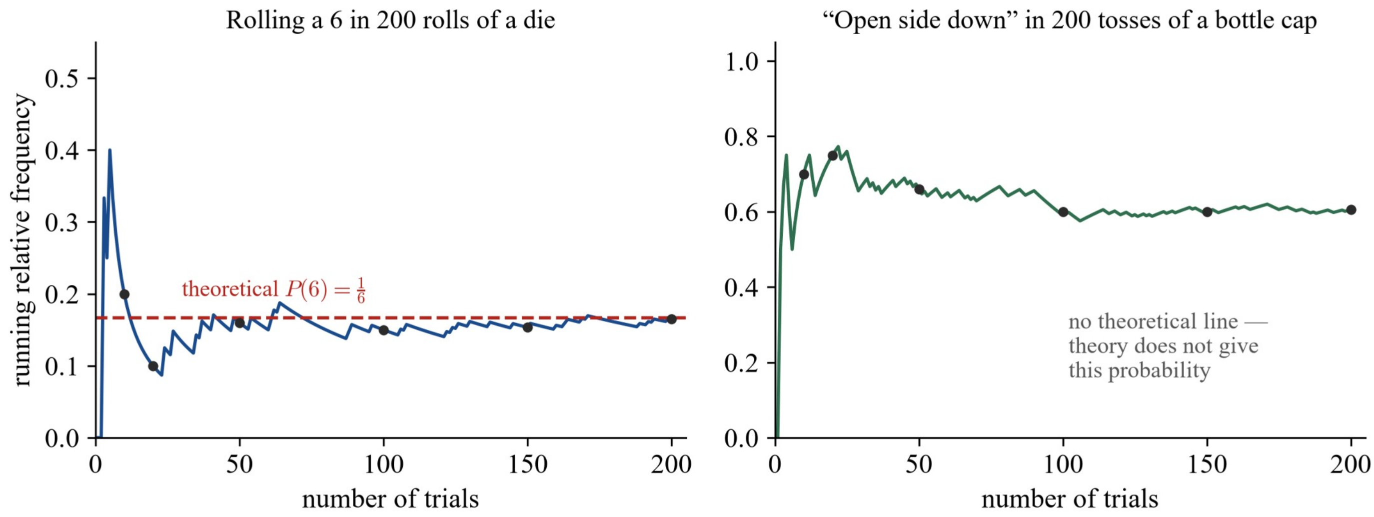
Relative frequency

Roll a die 200 times and count the sixes, or toss a bottle cap 200 times and count how often it lands open side down. After any number of trials, the **relative frequency** of the outcome is

relative frequency = (number of times the outcome happens) ÷ (number of trials).

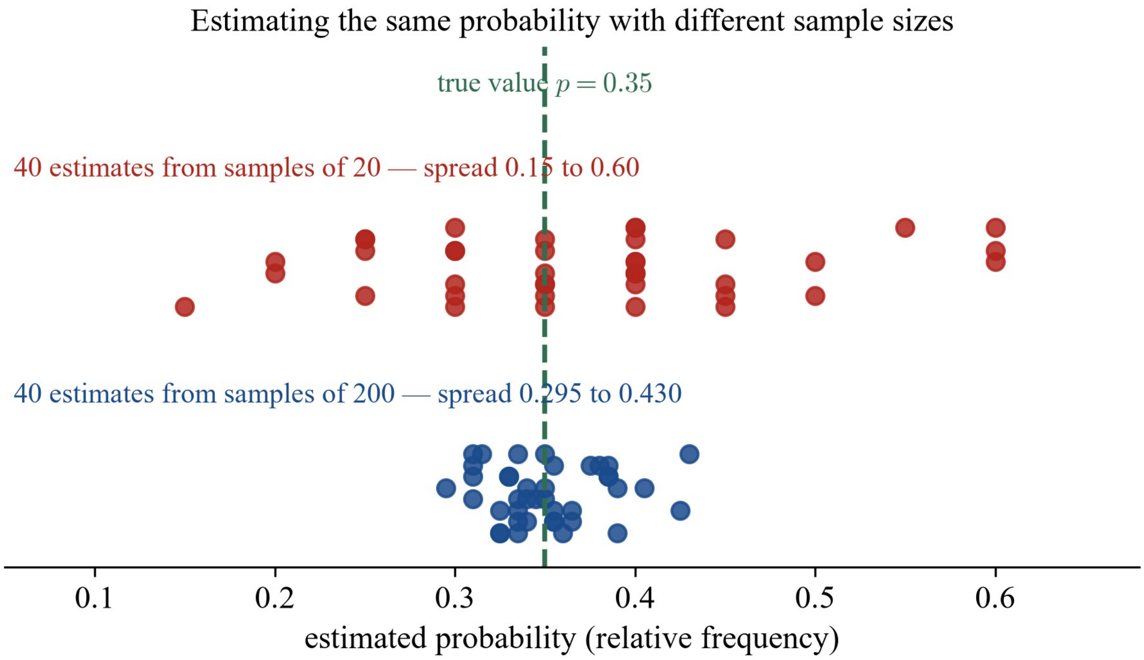
After 50 cap tosses with 33 open-side-down landings, the relative frequency is $33 \div 50 = 0.66$. The same idea works as a fraction, a decimal or a percentage: $0.66 = 66\,\%$.

For a fair die, the six equally likely outcomes of 7A give the probability without any rolling: $P(6) = \frac{1}{6}$. A bottle cap has no equally likely outcomes — its two sides are different shapes — so theory cannot give the answer and data must. The key fact (the first elaboration of this content description) is that as the number of trials grows, the **running relative frequency** — the relative frequency recalculated after every trial — jumps about a lot at first and then settles close to one value. That settled value is the **estimate** of the probability, and a **prediction** is the estimate multiplied by the number of future trials.



Two panels of running relative frequency against number of trials: rolling a 6 in 200 die rolls settling close to the dashed theoretical line at 1/6, and "open side down" in 200 bottle-cap tosses settling near 0.6 with no theoretical line

The size of the data matters. The next figure shows 40 estimates of the *same* probability, $p = 0.35$: forty samples of only 20 trials spread from 0.15 to 0.60, while forty samples of 200 trials stay close together, from 0.295 to 0.430. Bigger samples give steadier estimates, which is why the cap curve above is trusted at 200 tosses in a way its first 20 tosses are not.



Dot plot of 40 estimates of the same probability p = 0.35: samples of 20 spread from 0.15 to 0.60 around the dashed true line, samples of 200 cluster from 0.295 to 0.430

Events joined by 'and'

An **event** is a set of outcomes, as in 7A. Joining two events with 'and' makes a new event: *A and B* happens when **both** happen in the one trial. In a Venn diagram it is the overlap of the two circles; in a two-way table it is the single cell where the row and column meet. If one in four days in a table is both rainy and windy, then $P(\text{rainy and windy}) = \frac{1}{4}$, read straight off the shared cell.

Events joined by inclusive 'or'

A or B in the **inclusive** sense happens when at least one of the two happens — A alone, B alone, or both together. The Venn shading is all of both circles; the two-way table shading is three cells. The trap is adding $P(A) + P(B)$: the overlap is counted twice, once inside each event. In a group of 120 where 48 walk and 75 arrive early, of whom 34 do both, $\frac{48}{120}+\frac{75}{120}=\frac{123}{120}$, more than 1 — impossible for a probability, because the 34 double-counted students push the total past the group. The fix subtracts the overlap once:

$$P(A \text{ or } B \text{ inclusive}) = P(A) + P(B) - P(A \text{ and } B).$$

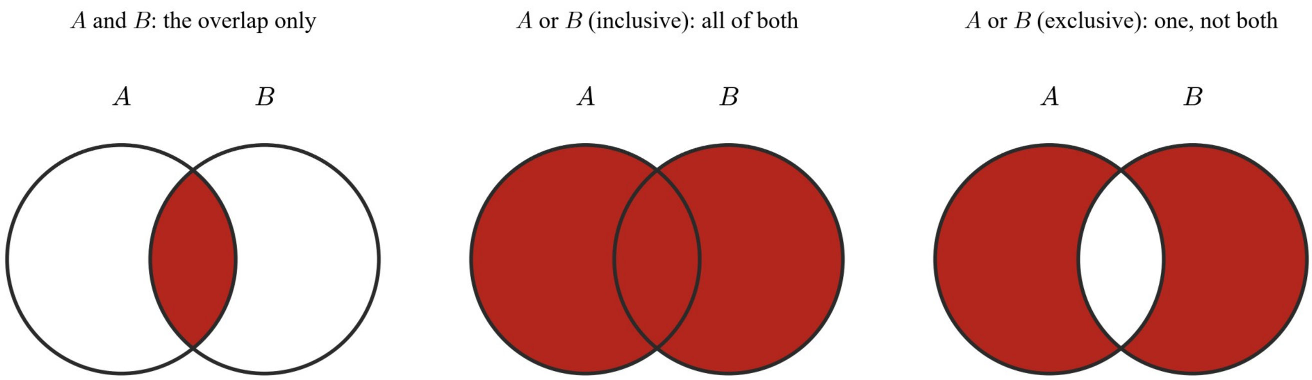
Events joined by exclusive 'or'

The **exclusive** 'or' is a different event: *exactly one* of A, B happens, not both. The Venn shading is both circles with the overlap left blank; the table shading is the two 'only' cells. Starting from the inclusive 'or', remove the overlap a second time:

$$P(A \text{ or } B \text{ exclusive}) = P(A) + P(B) - 2 \times P(A \text{ and } B).$$

The gap between the two 'or' probabilities is exactly the overlap. When the overlap is empty — the events cannot happen together, like *roll a 1* and *roll a 6* on one die — inclusive and exclusive 'or' agree, and $P(A \text{ or } B) = P(A) + P(B)$ with nothing to subtract.

The three connectives side by side, in Venn and table form:



Three Venn panels: A and B shading only the overlap; A or B inclusive shading all of both circles; A or B exclusive shading both circles except the overlap

'and': walks AND arrives early

	Early	Not	Total
Walks	34	14	48
Does not	41	31	72
Total	75	45	120

inclusive 'or': walks OR arrives early (or both)

	Early	Not	Total
Walks	34	14	48
Does not	41	31	72
Total	75	45	120

exclusive 'or': walks OR arrives early, but not both

	Early	Not	Total
Walks	34	14	48
Does not	41	31	72
Total	75	45	120

Three panels of the same 120-student two-way table: 'and' shading the one shared cell of 34; inclusive 'or' shading 34, 14 and 41; exclusive 'or' shading only 14 and 41

Word	Means	In a two-way table	In a Venn diagram
A and B	both happen	the one shared cell	the overlap
A or B (inclusive)	at least one happens	three cells, overlap counted once	all of both circles
A or B (exclusive)	exactly one happens	the two 'only' cells	both circles minus the

Word	Means	In a two-way table	In a Venn diagram
			overlap

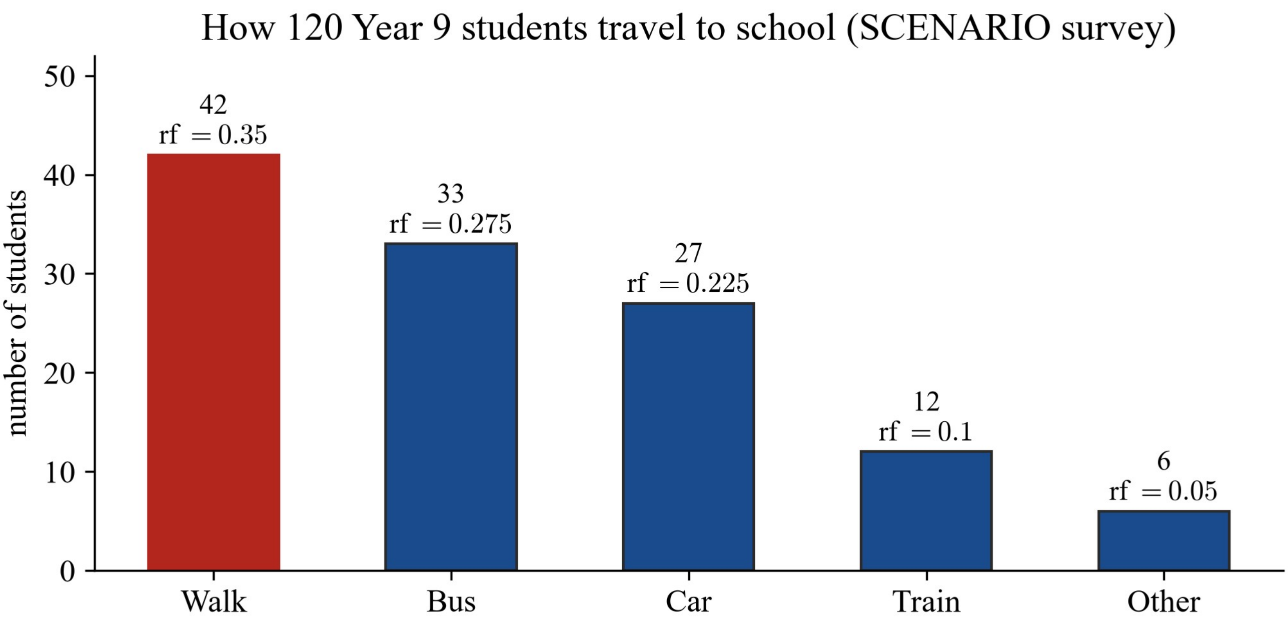
Worked examples

Example 1 — scaffolded

A Year 9 group surveyed how its 120 students travel to school: 42 walk, 33 catch a bus, 27 come by car, 12 by train and 6 other ways. (SCENARIO) Estimate the probability that a student from this group walks, and predict the walkers in a whole school of 900.

Working	Comment
rf(walk) $=42\div 120=0.35$	Relative frequency = count ÷ number of trials (here, students).
rf(bus) $=33\div 120=0.275$, rf(car) $=0.225$, rf(train) $=0.1$, rf(other) $=0.05$	Every category gets its own relative frequency.
$0.35+0.275+0.225+0.1+0.05=1$	The relative frequencies of all outcomes must add to 1 — a check.
$P(\text{walk})\approx 0.35$; prediction $=900\times 0.35=315$	The relative frequency estimates the probability; the estimate times 900 predicts.

$\therefore$ $P(\text{walk})\approx 0.35$, and about 315 of the 900 students would be expected to walk.



Bar graph of the 120-student transport survey: walk 42 (rf 0.35), bus 33 (0.275), car 27 (0.225), train 12 (0.1), other 6 (0.05)

Example 2 — scaffolded

The table records two long runs of trials: a die rolled 200 times counting sixes, and a bottle cap tossed 200 times counting open-side-down landings. (SCENARIO) Estimate $P(6)$ and the cap probability, and say which estimate the theory of equally likely outcomes supports.

Working	Comment
Die: rf $=\frac{2}{10}=0.2$ at 10 rolls; 0.1 at 20; 0.16 at 50; 0.15 at 100; ≈ 0.153 at 150; 0.165 at 200.	The running relative frequency jumps about early and settles later.
Cap: rf $=0.7$ at 10 tosses; 0.75 at 20; 0.66 at 50; 0.6 at	The cap settles near 0.6.

Working	Comment
100 and 150; 0.605 at 200.	
$P(6)\approx 0.165$; theory gives $\frac{1}{6}\approx 0.167$.	The 200-roll estimate sits almost on the theoretical value.
$P(\text{open side down})\approx 0.605$; no theoretical value exists.	The cap's sides are not equally likely, so only data estimates it.

$\therefore$ $P(6)\approx 0.165$, close to the theoretical $\frac{1}{6}$; $P(\text{open side down})\approx 0.605$, known from data alone.

Example 3 — scaffolded

The two-way table shows the 31 days of July 2026 at Melbourne (Olympic Park): a day is *rainy* with at least 1 mm of rain and *windy* with a strongest gust of at least 40 km/h. Estimate the probability that a July day there was rainy and windy.

Working	Comment
Rainy and windy days: the shared cell $=4$.	'And' is the one cell where the row and column meet.
$P(\text{rainy and windy})=\frac{4}{31}$	The count over the 31 days.
$\frac{4}{31}\approx 0.13$	About 13% of July days were both.

$\therefore$ $P(\text{rainy and windy})=\frac{4}{31}\approx 0.13$.

Melbourne (Olympic Park), July 2026 — 31 days (Bureau of Meteorology)

	Windy day (gust $\geq$ 40 km/h)	Not windy	Total
Rainy day (rain $\geq$ 1 mm)	4	5	9
Not rainy	5	17	22
Total	9	22	31

Two-way table of the 31 July 2026 Melbourne days: rainy and windy 4, rainy only 5, windy only 5, neither 17, with row and column totals 9, 9 and 31

Example 4 — scaffolded

In a group of 120 students, 48 walk to school and 75 arrive early, and 34 do both. (SCENARIO) Use the correction formula to estimate the probability that a student walks or arrives early, in the inclusive sense.

Working	Comment
$P(\text{walk})=\frac{48}{120}$, $P(\text{early})=\frac{75}{120}$, $P(\text{walk and early})=\frac{34}{120}$	Three counts from the table.

Working	Comment
$\frac{48}{120}+\frac{75}{120}=\frac{123}{120}>1$	Plain adding double-counts the 34 who do both.
$\frac{48}{120}+\frac{75}{120}-\frac{34}{120}=\frac{89}{120}$	Subtract the overlap once.
Check on the table: $34+14+41=89$ ✓	The three shaded cells give the same count.

$\therefore P(\text{walk or early, inclusive})=\frac{89}{120}\approx 0.74$.

Example 5 — unscaffolded

Same group as Example 4. Estimate the probability that a student walks or arrives early in the exclusive sense — one of the two, not both.

Exclusive ‘or’ removes the overlap a second time: $P=\frac{48}{120}+\frac{75}{120}-2\times\frac{34}{120}=\frac{48+75-68}{120}=\frac{55}{120}=\frac{11}{24}$. Check on the table: the two ‘only’ cells, $14+41=55$ ✓. The inclusive answer $\frac{89}{120}$ minus the exclusive answer $\frac{55}{120}$ is $\frac{34}{120}$, exactly the overlap, as it must be.

$\therefore P(\text{walk or early, exclusive})=\frac{11}{24}\approx 0.46$.

Example 6 — unscaffolded

Two fair dice are rolled. Let A be ‘the sum is 8’ and B be ‘the roll is a double’. From the 36-outcome array, find $P(A \text{ and } B)$, $P(A \text{ or } B \text{ inclusive})$ and $P(A \text{ or } B \text{ exclusive})$.

The array shows five shaded sum-8 outcomes — $(2,6),(3,5),(4,4),(5,3),(6,2)$ — and six bordered doubles, with $(4,4)$ the only outcome in both. $P(A \text{ and } B)=\frac{1}{36}$. Inclusive: $\frac{5}{36}+\frac{6}{36}-\frac{1}{36}=\frac{10}{36}=\frac{5}{18}$. Exclusive: $\frac{5}{36}+\frac{6}{36}-2\times\frac{1}{36}=\frac{9}{36}=\frac{1}{4}$.

$\therefore P(A \text{ and } B)=\frac{1}{36}$; inclusive ‘or’ $=\frac{5}{18}$; exclusive ‘or’ $=\frac{1}{4}$.

First die a across the top, second die b down the side

	1	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

shaded: sum of 8 (event A) — 5 outcomes

red border: doubles (event B) — 6 outcomes

green border too: A and B — the outcome $(4, 4)$

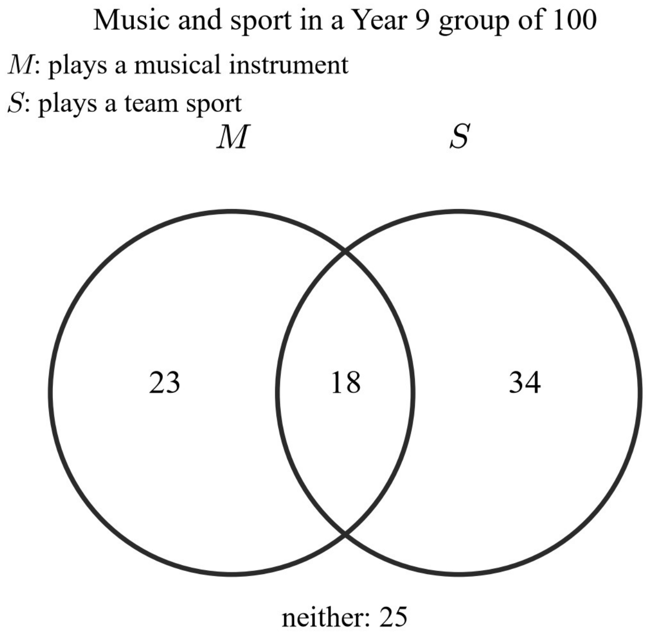
Six-by-six array of the 36 two-dice outcomes with the five sum-8 cells shaded, the six doubles bordered in red, and (4,4) carrying both marks as the only shared outcome

Example 7 — unscaffolded

In a Year 9 group of 100, the Venn diagram shows 23 who play a musical instrument only, 18 who do both, 34 who play a team sport only and 25 who do neither. (SCENARIO) Read off $P(M \text{ and } S)$, the inclusive ‘or’ and the exclusive ‘or’.

$P(M \text{ and } S) = \frac{18}{100} = \frac{9}{50}$, the overlap. Inclusive ‘or’: $\frac{23+18+34}{100} = \frac{75}{100} = \frac{3}{4}$, all of both circles. Exclusive ‘or’: $\frac{23+34}{100} = \frac{57}{100}$, the two ‘only’ regions. Check by formula: $P(M) = \frac{41}{100}$, $P(S) = \frac{52}{100}$, so $\frac{41}{100} + \frac{52}{100} - \frac{18}{100} = \frac{75}{100}$ ✓.

$\therefore P(M \text{ and } S) = \frac{9}{50}$; inclusive ‘or’ $= \frac{3}{4}$; exclusive ‘or’ $= \frac{57}{100}$.



Venn diagram for 100 students with regions 23 (instrument only), 18 (both), 34 (team sport only) and neither 25

Example 8 — unscaffolded

After 10 tosses of a bottle cap, the relative frequency of open side down was $\frac{7}{10}$. After 200 tosses it was $\frac{121}{200} = 0.605$. (SCENARIO) Which value should estimate the probability, and what does it predict for 400 more tosses?

The 10-toss value $\frac{7}{10} = 0.7$ comes from a small sample, and small samples spread widely — the figure above shows samples of 20 landing anywhere from 0.15 to 0.60 for the same true probability. The 200-toss value 0.605 is the steadier estimate, so use it. Prediction: $400 \times 0.605 = 242$.

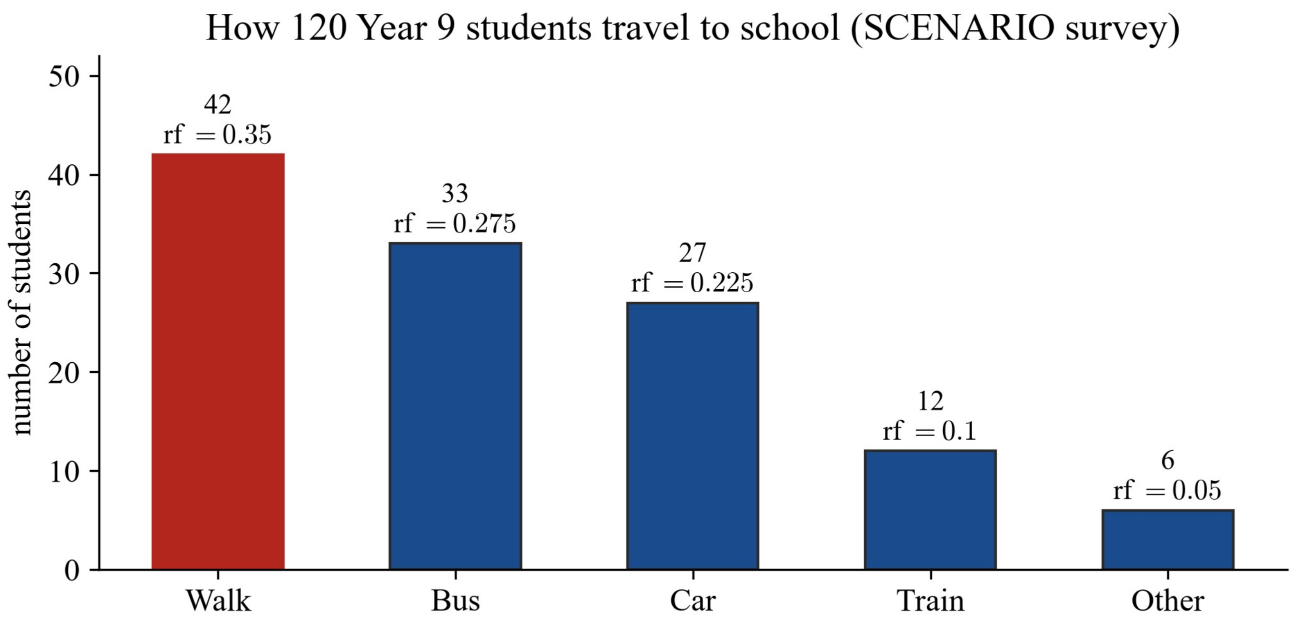
∴ Trust the 200-toss estimate 0.605; expect about 242 open-side-down landings in 400 tosses.

Practice questions

Scaffolded

Q1. The bar graph shows the transport survey of Example 1: 42 walk, 33 bus, 27 car, 12 train, 6 other, out of 120 students.

a. Calculate the relative frequency of walking.
b. Calculate the relative frequency of travelling by train.
c. Show that the five relative frequencies add to 1, and say why they must.



Bar graph of the 120-student transport survey: walk 42, bus 33, car 27, train 12, other 6, each bar labelled with its relative frequency

Q2. The table below records a die rolled 200 times, counting sixes.

Trials	10	20	50	100	150	200
Sixes	2	2	8	15	23	33

a. Calculate the relative frequency of a six after 50 rolls.
b. Calculate it after 200 rolls.
c. The theoretical probability is $\frac{1}{6} \approx 0.167$. Which of your two estimates is closer to it, and why does that make sense?

Q3. A factory check finds 41 of a batch of 50 chargers pass a safety test. (SCENARIO)

a. Calculate the relative frequency of passing.
b. Use it to predict how many of the next 200 chargers will pass.

Q4. A coin is tossed and a fair die rolled — 12 equally likely outcomes. Let H be ‘head’ and E be ‘an even number’.

a. List the outcomes of H and E, and find $P(H \text{ and } E)$.
b. Find $P(H \text{ or } E)$ in the inclusive sense.
c. Find $P(H \text{ or } E)$ in the exclusive sense.

Q5. Two fair dice are rolled. Let A be ‘the sum is 5’ and B be ‘the roll is a double’.

a. List the outcomes of A. Can a double sum to 5? What is $P(A \text{ and } B)$?
b. Find $P(A \text{ or } B)$ inclusive.
c. Find $P(A \text{ or } B)$ exclusive, and explain why it matches part b.

Q6. The two-way table shows how 120 students travel and arrive. (SCENARIO)

	Arrives early	Does not	Total
Walks	34	14	48
Does not walk	41	31	72
Total	75	45	120

a. Find $P(\text{walks and early})$.

b. Find $P(\text{does not walk and does not arrive early})$.

Q7. Using the table of Q6:

a. Explain in one sentence why $\frac{48}{120}+\frac{75}{120}$ cannot be the probability of ‘walks or arrives early’.

b. Apply the correction and find $P(\text{walks or early, inclusive})$.

Q8. Using the table of Q6, find $P(\text{walks or early, exclusive})$ two ways: from the formula, and by adding the two ‘only’ cells.

Q9. A Venn diagram of 28 households shows 5 with a cat only, 8 with a dog only, 7 with both and 8 with neither. (SCENARIO)

a. Check the four regions add to 28.
b. Find $P(\text{cat and dog})$.
c. Find $P(\text{neither})$.

Q10. A fair die is rolled. Let A be ‘a 1 or a 2’ and B be ‘a 5 or a 6’.

a. Can A and B happen together? What is $P(A \text{ and } B)$?
b. Find $P(A \text{ or } B)$ inclusive.
c. Find $P(A \text{ or } B)$ exclusive, and comment.

Q11. From the July 2026 Melbourne weather table of Example 3 (9 rainy days, 9 windy days, 4 both, out of 31 days):

a. Find $P(\text{rainy and windy})$.
b. Find $P(\text{rainy or windy, inclusive})$.

Q12. Using the same weather table:

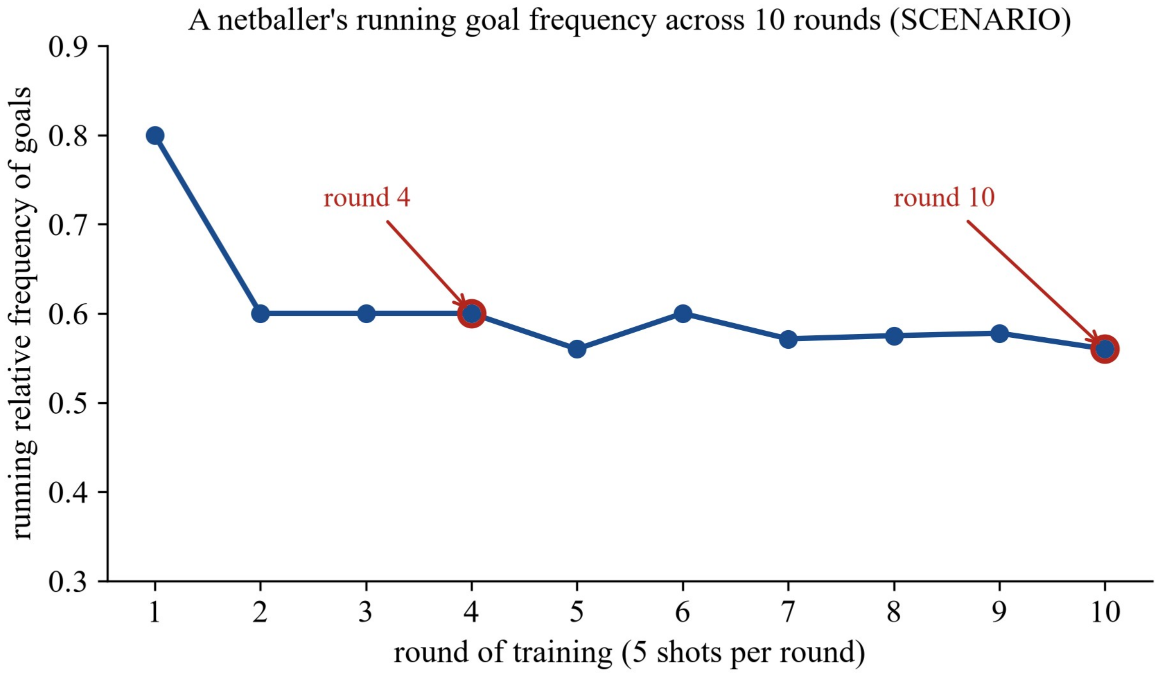
a. Find $P(\text{rainy or windy, exclusive})$ and $P(\text{neither})$.
b. A school term runs 93 days. Use the data to predict how many rainy days it will contain.

Unscaffolded

Q13. A tennis player lands 156 of 240 first serves in a practice match. (SCENARIO) Estimate the probability of a landed first serve, and predict the landings in 500 serves.

Q14. The graph shows a netball player’s running relative frequency of goals, five shots per round across 10 rounds. (SCENARIO)

a. Read the relative frequency after round 4 and after round 10.
b. Using the better estimate, predict her goals from 40 shots at goal.



Line graph of a netball player's running relative frequency of goals across 10 rounds, settling from 0.8 after round 1 towards 0.56 after round 10, with rounds 4 and 10 circled

Q15. A three-colour spinner is spun 150 times: red 66, blue 45, green 39. (SCENARIO)

a. Calculate the three relative frequencies and check they add to 1.
b. Which colour is the spinner least likely to land on?

Q16. In a street of 28 households, 12 have a cat, 15 have a dog and 7 have both. (SCENARIO) Find $P(\text{cat or dog})$ in the inclusive sense, in the exclusive sense, and $P(\text{cat and dog})$.

Q17. In a batch of 200 tiles, 9 are cracked, 6 are chipped and 3 are both. (SCENARIO) Find the probability that a tile is cracked or chipped (inclusive), cracked or chipped (exclusive), and neither fault.

Q18. In a group of 90 students, 38 study a language other than English, 47 play a team sport and 15 do both. (SCENARIO) Find the number who do neither, then $P(\text{language or sport})$ inclusive and exclusive, and $P(\text{language and sport})$.

Q19. Two fair dice are rolled. Let A be 'the sum is 9' and B be 'the roll is a double'. Find $P(A \text{ and } B)$, then the inclusive and exclusive 'or', and explain the relationship between your two 'or' answers.

Q20. A coin is tossed and a fair die rolled. Let T be 'tail' and O be 'an odd number'. Find $P(T \text{ and } O)$, and the inclusive and exclusive 'or'.

Q21. A card is drawn at random from a 52-card pack. Let H be 'a heart' and C be 'a court card (J, Q, K)'. Find $P(H \text{ and } C)$, and the inclusive and exclusive 'or'.

Q22. A card is drawn at random from a 52-card pack. Let R be 'a red card' and A be 'an ace'. Find $P(R \text{ and } A)$, and the inclusive and exclusive 'or'.

Q23. Of 80 gym members, 35 use the gym in the morning, 41 in the evening, and 12 at neither time; some use it at both. (SCENARIO) Find how many use it at both times, then $P(\text{morning or evening})$ inclusive and exclusive, and $P(\text{both})$.

Q24. Using the July 2026 Melbourne weather table: predict the number of rainy days and the number of windy days in a 93-day term, and explain why the two predictions are the same.

Q25. Two estimates exist for the probability that a seed of a certain plant sprouts: $\frac{3}{5}$ from 5 trials, and $\frac{58}{100}=0.58$ from 100 trials. (SCENARIO) Say which estimate to trust and why, then use it to predict sprouting seeds out of 250 planted.

Q26. At a cafe, 26 of 60 morning customers order tea, 39 order coffee and 17 order both. (SCENARIO) Find $P(\text{tea or coffee})$ inclusive and exclusive, and $P(\text{neither})$.

Q27. The cap-tossing run of Example 2 gave relative frequency 0.6 after 100 tosses and 0.605 after 200. (SCENARIO) Predict the open-side-down landings in 1000 tosses, saying which value you use.

Q28. Priya read a two-way table showing 22 of 60 students with a sister, 19 with a brother and 7 with both, and found $P(\text{sister or brother})=\frac{22}{60}+\frac{19}{60}=\frac{41}{60}$. (SCENARIO) Find her mistake and give the correct inclusive 'or'.

Q29. A coin is tossed and a fair three-section spinner (red, blue, green) is spun — 6 equally likely outcomes. Let H be 'head' and R be 'red'. Find $P(H \text{ and } R)$, and the inclusive and exclusive 'or'.

Q30. After 500 drops, a drawing pin has landed point up with relative frequency 0.28. (SCENARIO) Estimate the probability that it lands point down, naming the Level 8 idea you are using.

Q31. A shooter records her running relative frequency of scoring: $\frac{5}{8}$ after the first quarter of her shots, $\frac{9}{15}$ at half time, $\frac{16}{24}$ after three quarters and $\frac{19}{30}$ at the end. (SCENARIO) After which quarter was her running relative frequency highest, and what is the best single estimate of her scoring probability?

Q32. In a year level of 100 students, 55 play sport, 32 have a part-time job and 18 do both. (SCENARIO) Find the number who play sport only, the number with a job only, the number who do neither, then $P(\text{sport or job})$ inclusive, exclusive, and $P(\text{sport and job})$.

Q33. Could each set of values describe two events A and B? Give a reason each time.

a. $P(A)=0.4$, $P(B)=0.5$, $P(A \text{ and } B)=0.2$
b. $P(A)=0.5$, $P(B)=0.6$, $P(A \text{ and } B)=0.05$

Q34. A Venn diagram's four regions hold x, $x+3$, $2x-5$ and 34 students, and the group has 120. (SCENARIO) Find x and the four region counts, then $P(A \text{ or } B)$ inclusive and exclusive, and $P(A \text{ and } B)$, where the overlap holds $x+3$.

Q35. In a group of 150 students, 88 follow football, 71 follow cricket and 44 follow both. (SCENARIO) Find $P(\text{football or cricket})$ inclusive, exclusive, and $P(\text{neither})$.

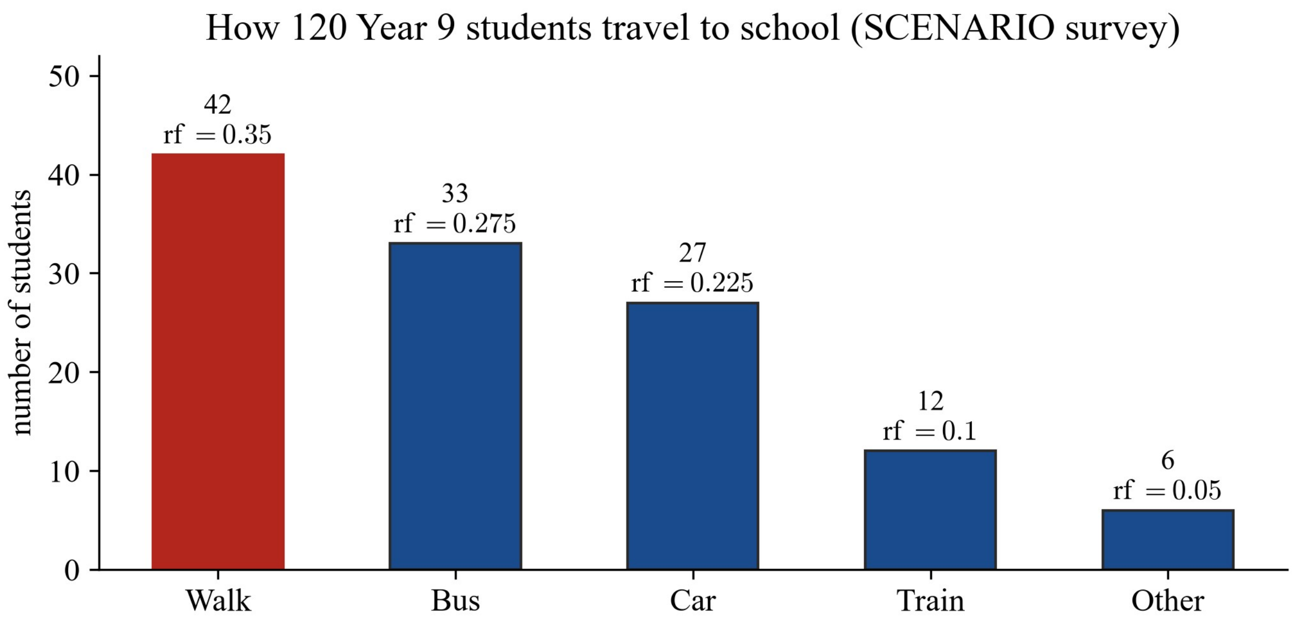
Q36. The group of Q35 is part of a school of 600. Using the relative frequencies from Q35, predict for the whole school the number who follow football or cricket (inclusive), the number who follow exactly one, and the number who follow neither.

Answers

Q1 a 0.35; b 0.1; c $0.35+0.275+0.225+0.1+0.05=1$ — every student falls in exactly one category. **Q2** a 0.16; b 0.165; c the 200-roll value, $|0.165-0.167|<|0.16-0.167|$; longer runs settle closer to the probability. **Q3** a 0.82; b 164. **Q4** a $(H,2),(H,4),(H,6)$; $\frac{1}{4}$; b $\frac{3}{4}$; c $\frac{1}{2}$. **Q5** a $(1,4),(2,3),(3,2),(4,1)$; no; 0; b $\frac{5}{18}$; c $\frac{5}{18}$ — the overlap is empty, so the two ‘or’s agree. **Q6** a $\frac{17}{60}$; b $\frac{31}{120}$. **Q7** a it is $\frac{123}{120}>1$, because the 34 who do both are counted twice; b $\frac{89}{120}$. **Q8** $\frac{11}{24}$ both ways. **Q9** a $5+8+7+8=28$ ✓; b $\frac{1}{4}$; c $\frac{2}{7}$. **Q10** a no; 0; b $\frac{2}{3}$; c $\frac{2}{3}$ — with an empty overlap the two ’or’s agree. **Q11** a $\frac{4}{31}$; b $\frac{14}{31}$. **Q12** a $\frac{10}{31}$ and $\frac{17}{31}$; b 27. **Q13** 0.65; 325. **Q14** a 0.6 and 0.56; b about 22. **Q15** a $0.44, 0.3, 0.26$, sum 1; b green. **Q16** inclusive $\frac{5}{7}$; exclusive $\frac{13}{28}$; and $\frac{1}{4}$. **Q17** inclusive $\frac{3}{50}$; exclusive $\frac{3}{100}$; neither $\frac{47}{50}$. **Q18** neither 20; inclusive $\frac{7}{9}$; exclusive $\frac{11}{18}$; and $\frac{1}{6}$. **Q19** and 0; inclusive $\frac{5}{18}$; exclusive $\frac{5}{18}$ — no double sums to 9, so the overlap is empty and the two ’or’s agree. **Q20** and $\frac{1}{4}$; inclusive $\frac{3}{4}$; exclusive $\frac{1}{2}$. **Q21** and $\frac{3}{52}$; inclusive $\frac{11}{26}$; exclusive $\frac{19}{52}$. **Q22** and $\frac{1}{26}$; inclusive $\frac{7}{13}$; exclusive $\frac{1}{2}$. **Q23** both 8; inclusive $\frac{17}{20}$; exclusive $\frac{3}{4}$; and $\frac{1}{10}$. **Q24** 27 rainy and 27 windy — the table has 9 of each out of 31, and $93\times\frac{9}{31}=27$. **Q25** 0.58 from 100 trials; 145. **Q26** inclusive $\frac{4}{5}$; exclusive $\frac{11}{30}$; neither $\frac{1}{5}$. **Q27** 0.605 (the 200-toss value); 605. **Q28** she added without subtracting the 7 who have both; $\frac{34}{60}=\frac{17}{30}$. **Q29** and $\frac{1}{6}$; inclusive $\frac{2}{3}$; exclusive $\frac{1}{2}$. **Q30** $1-0.28=0.72$, complementary events. **Q31** after three quarters, $\frac{2}{3}$; best estimate $\frac{19}{30}\approx 0.63$. **Q32** 37, 14, 31; inclusive $\frac{69}{100}$; exclusive $\frac{51}{100}$; and $\frac{9}{50}$. **Q33** a yes: inclusive ’or’ $=0.4+0.5-0.2=0.7\leq 1$; b no: $0.5+0.6-0.05=1.05>1$. **Q34** $x=22$; regions 22, 25, 39, 34; inclusive $\frac{43}{60}$; exclusive $\frac{61}{120}$; and $\frac{5}{24}$. **Q35** inclusive $\frac{23}{30}$; exclusive $\frac{71}{150}$; neither $\frac{7}{30}$. **Q36** 460, 284, 140.

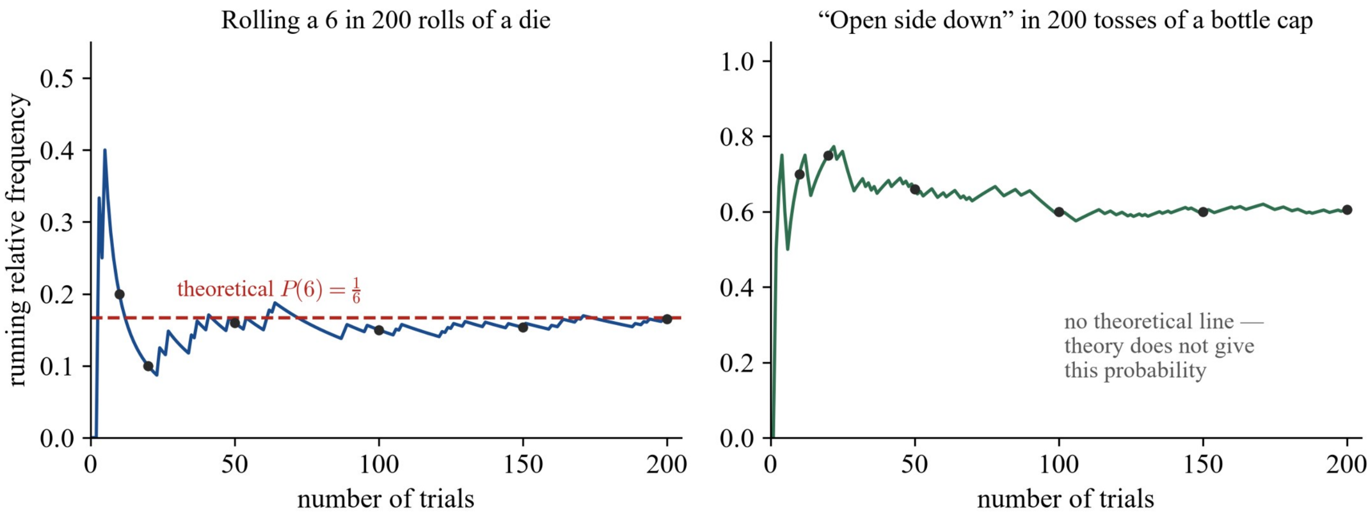
Fully-worked solutions

Q1. a rf(walk) $=42\div 120=0.35$. b rf(train) $=12\div 120=0.1$. c $0.35+0.275+0.225+0.1+0.05=1$. They must add to 1 because each of the 120 students falls in exactly one category, so the five counts add to 120 and the five relative frequencies add to $120\div 120=1$. ∴ 0.35; 0.1; sum 1, one category per student.



The transport survey bars with counts and relative frequencies — the same data Q1 reads

Q2. a After 50 rolls: $8 \div 50 = 0.16$. b After 200 rolls: $33 \div 200 = 0.165$. c $\frac{1}{6} \approx 0.167$; 0.165 is the closer estimate. Running relative frequencies settle as trials grow, so the longer run sits closer to the theoretical probability. $\therefore$ 0.16; 0.165; the 200-roll estimate.



The running relative frequency of sixes over 200 rolls settling towards the dashed 1/6 line — the same long run Q2's table records

Q3. a rf $= 41 \div 50 = 0.82$. b $200 \times 0.82 = 164$. $\therefore$ 0.82; about 164 chargers.

Q4. a H and E: $(H, 2), (H, 4), (H, 6)$ — 3 of the 12 outcomes, $P = \frac{3}{12} = \frac{1}{4}$. b $P(H) = \frac{6}{12}$, $P(E) = \frac{6}{12}$; inclusive 'or' $= \frac{6}{12} + \frac{6}{12} - \frac{3}{12} = \frac{9}{12} = \frac{3}{4}$. c Exclusive 'or' $= \frac{6}{12} + \frac{6}{12} - 2 \times \frac{3}{12} = \frac{6}{12} = \frac{1}{2}$. $\therefore$ $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{2}$.

Q5. a $A = \{(1,4), (2,3), (3,2), (4,1)\}$, four outcomes. A double gives sums $2, 4, 6, 8, 10, 12$ — never 5 — so A and B share no outcome and $P(A \text{ and } B) = 0$. b Inclusive: $\frac{4}{36} + \frac{6}{36} - 0 = \frac{10}{36} = \frac{5}{18}$. c Exclusive: $\frac{4}{36} + \frac{6}{36} - 2 \times 0 = \frac{5}{18}$ — the same, because there is no overlap to remove. $\therefore$ $P(A \text{ and } B) = 0$; both 'or's $= \frac{5}{18}$.

Q6. a The shared cell: $\frac{34}{120}=\frac{17}{60}$. b The ‘neither’ cell: $\frac{31}{120}$. ∴ $\frac{17}{60}$; $\frac{31}{120}$.

'and': walks AND arrives early

	Early	Not	Total
Walks	34	14	48
Does not	41	31	72
Total	75	45	120

inclusive 'or': walks OR arrives early (or both)

	Early	Not	Total
Walks	34	14	48
Does not	41	31	72
Total	75	45	120

exclusive 'or': walks OR arrives early, but not both

	Early	Not	Total
Walks	34	14	48
Does not	41	31	72
Total	75	45	120

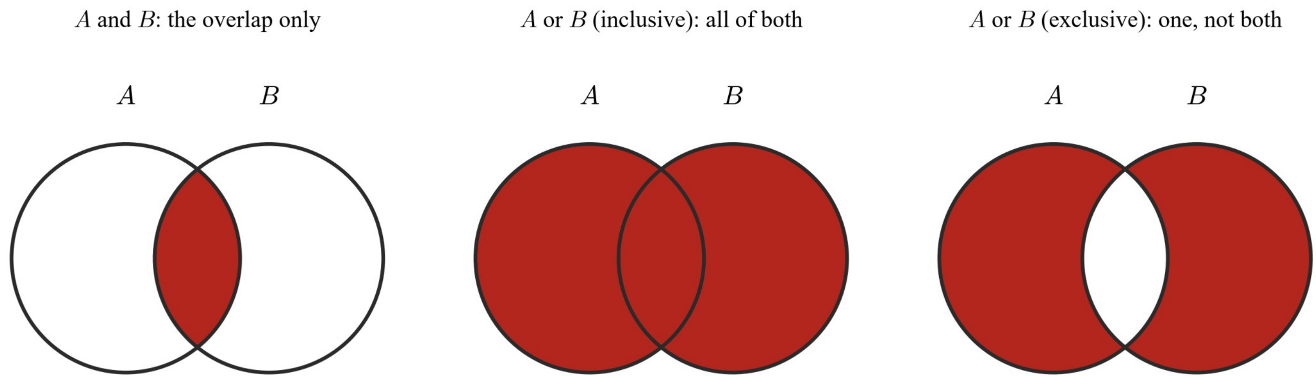
The 120-student travel/arrival two-way table with its four cells — the table Q6 reads

Q7. a $\frac{48}{120}+\frac{75}{120}=\frac{123}{120}$, greater than 1: no probability exceeds 1, and the sum counts the 34 students who both walk and arrive early twice. b $\frac{48}{120}+\frac{75}{120}-\frac{34}{120}=\frac{89}{120}$. ∴ The plain sum double-counts; the inclusive ‘or’ is $\frac{89}{120}$.

Q8. Formula: $\frac{48}{120}+\frac{75}{120}-2\times\frac{34}{120}=\frac{55}{120}=\frac{11}{24}$. Table: walks-only 14 + early-only $41=55$, so $\frac{55}{120}=\frac{11}{24}$ ✓. ∴ $\frac{11}{24}$.

Q9. a $5+8+7+8=28$ ✓. b $P(\text{cat and dog})=\frac{7}{28}=\frac{1}{4}$. c $P(\text{neither})=\frac{8}{28}=\frac{2}{7}$. ∴ Regions check; $\frac{1}{4}$; $\frac{2}{7}$.

Q10. a One roll cannot show both a 1-or-2 and a 5-or-6: $P(A \text{ and } B)=0$. b Inclusive: $\frac{2}{6}+\frac{2}{6}-0=\frac{4}{6}=\frac{2}{3}$. c Exclusive: $\frac{2}{6}+\frac{2}{6}-0=\frac{2}{3}$ — the same, since the overlap is empty. ∴ 0; $\frac{2}{3}$; $\frac{2}{3}$, and the match is expected when events cannot happen together.



Three Venn panels for ‘and’, inclusive ‘or’ and exclusive ‘or’ — when the circles do not overlap at all, the last two panels would shade the same way, as in Q10

Q11. a $P(\text{rainy and windy})=\frac{4}{31}$, the shared cell. b Inclusive: $\frac{9}{31}+\frac{9}{31}-\frac{4}{31}=\frac{14}{31}$. ∴ $\frac{4}{31}$; $\frac{14}{31}$.

Melbourne (Olympic Park), July 2026 — 31 days (Bureau of Meteorology)

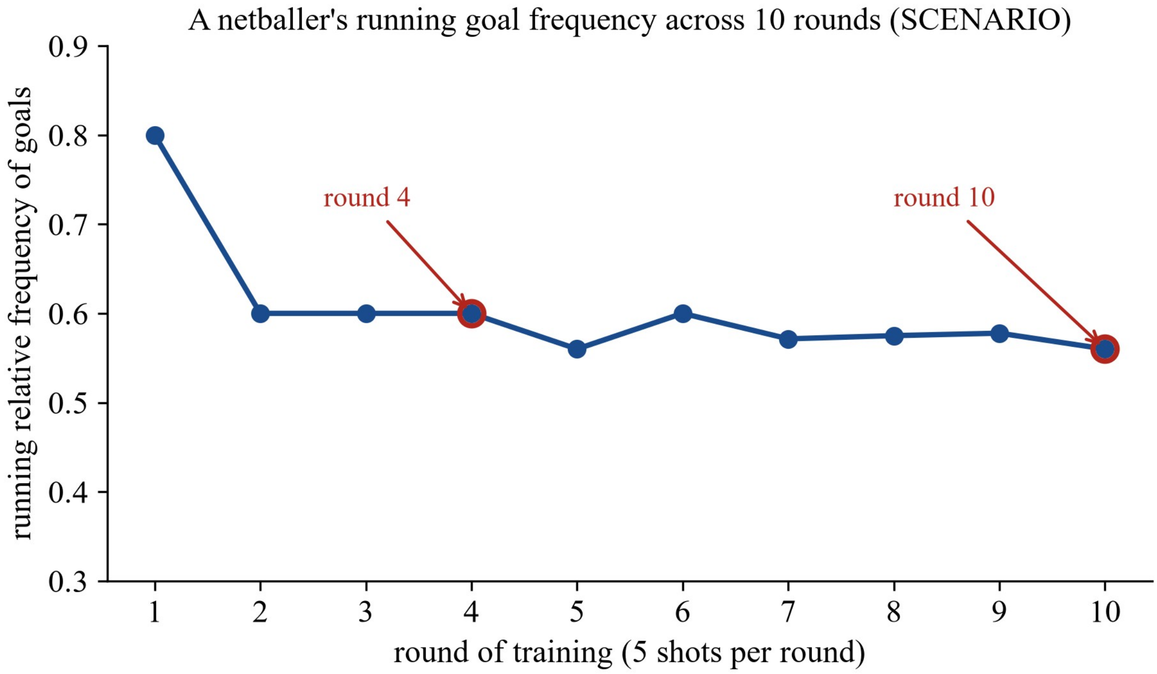
	Windy day (gust $\geq$ 40 km/h)	Not windy	Total
Rainy day (rain $\geq$ 1 mm)	4	5	9
Not rainy	5	17	22
Total	9	22	31

The BOM two-way table of July 2026 Melbourne days — the table Q11 reads

Q12. a Exclusive: $\frac{9}{31}+\frac{9}{31}-2\times\frac{4}{31}=\frac{10}{31}$; neither: $31-4-5-5=17$ days, $\frac{17}{31}$. b $P(\text{rainy})=\frac{9}{31}$; $93\times\frac{9}{31}=27$. ∴ $\frac{10}{31}$; $\frac{17}{31}$; about 27 rainy days.

Q13. rf $=156\div240=0.65$; prediction $=500\times0.65=325$. ∴ 0.65; about 325 landed serves.

Q14. a After round 4: $12\div20=0.6$; after round 10: $28\div50=0.56$. b The round-10 value uses all 50 shots, the steadier estimate: $40\times0.56=22.4$, about 22 goals. ∴ 0.6, 0.56; about 22 goals.



The netball player's running relative frequency graph — Q14 reads rounds 4 and 10 from it

Q15. a $66\div150=0.44$; $45\div150=0.3$; $39\div150=0.26$; and $0.44+0.3+0.26=1$ ✓. b Green, with the smallest relative frequency, 0.26. ∴ 0.44, 0.3, 0.26; green.

Q16. $P(\text{cat})=\frac{12}{28}$, $P(\text{dog})=\frac{15}{28}$, $P(\text{both})=\frac{7}{28}$. Inclusive: $\frac{12}{28}+\frac{15}{28}-\frac{7}{28}=\frac{20}{28}=\frac{5}{7}$. Exclusive: $\frac{12}{28}+\frac{15}{28}-2\times\frac{7}{28}=\frac{13}{28}$. And: $\frac{7}{28}=\frac{1}{4}$. ∴ $\frac{5}{7}$; $\frac{13}{28}$; $\frac{1}{4}$.

Q17. Inclusive: $\frac{9}{200}+\frac{6}{200}-\frac{3}{200}=\frac{12}{200}=\frac{3}{50}$. Exclusive: $\frac{9}{200}+\frac{6}{200}-2\times\frac{3}{200}=\frac{6}{200}=\frac{3}{100}$. Neither: $200-12=188$ tiles, $\frac{188}{200}=\frac{47}{50}$. $\therefore$ $\frac{3}{50}$; $\frac{3}{100}$; $\frac{47}{50}$.

Q18. Neither: $90-(38+47-15)=90-70=20$. Inclusive: $\frac{38}{90}+\frac{47}{90}-\frac{15}{90}=\frac{70}{90}=\frac{7}{9}$. Exclusive: $\frac{38}{90}+\frac{47}{90}-2\times\frac{15}{90}=\frac{55}{90}=\frac{11}{18}$. And: $\frac{15}{90}=\frac{1}{6}$. $\therefore$ 20; $\frac{7}{9}$; $\frac{11}{18}$; $\frac{1}{6}$.

Q19. Sum-9 outcomes: $(3,6),(4,5),(5,4),(6,3)$ — four. Doubles give only even sums, so no double sums to 9: $P(A \text{ and } B)=0$. Inclusive: $\frac{4}{36}+\frac{6}{36}=\frac{10}{36}=\frac{5}{18}$; exclusive: the same, $\frac{5}{18}$. The two ‘or’ answers agree because the overlap is empty — inclusive and exclusive ‘or’ differ only by the overlap. $\therefore$ 0; $\frac{5}{18}$; $\frac{5}{18}$.

First die a across the top, second die b down the side

	1	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

shaded: sum of 8 (event A) — 5 outcomes

red border: doubles (event B) — 6 outcomes

green border too: A and B — the outcome $(4, 4)$

The two-dice array with sum-8 shading and double borders — Q19 repeats the method with sum 9, whose four cells share none of the bordered doubles

Q20. T and O: $(T,1),(T,3),(T,5)$ — $\frac{3}{12}=\frac{1}{4}$. Inclusive: $\frac{6}{12}+\frac{6}{12}-\frac{3}{12}=\frac{9}{12}=\frac{3}{4}$. Exclusive: $\frac{6}{12}+\frac{6}{12}-2\times\frac{3}{12}=\frac{6}{12}=\frac{1}{2}$. $\therefore$ $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{2}$.

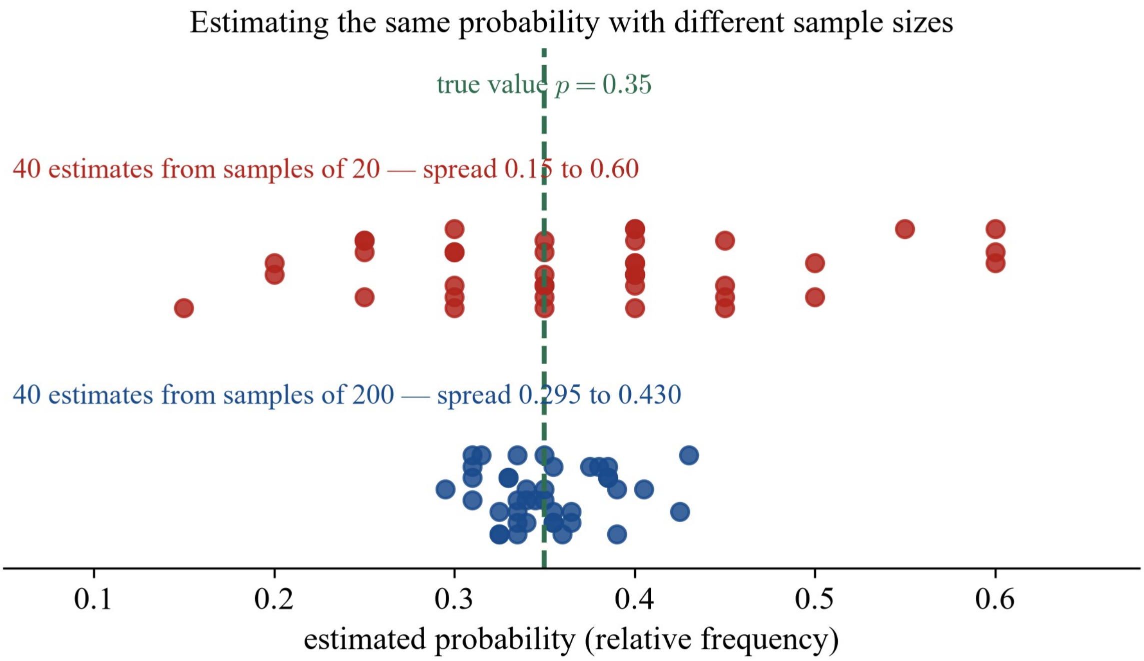
Q21. Hearts: 13; court cards: 12; heart court cards (J, Q, K of hearts): 3. And: $\frac{3}{52}$. Inclusive: $\frac{13}{52}+\frac{12}{52}-\frac{3}{52}=\frac{22}{52}=\frac{11}{26}$. Exclusive: $\frac{13}{52}+\frac{12}{52}-2\times\frac{3}{52}=\frac{19}{52}$. $\therefore$ $\frac{3}{52}$; $\frac{11}{26}$; $\frac{19}{52}$.

Q22. Red: 26; aces: 4; red aces: 2. And: $\frac{2}{52}=\frac{1}{26}$. Inclusive: $\frac{26}{52}+\frac{4}{52}-\frac{2}{52}=\frac{28}{52}=\frac{7}{13}$. Exclusive: $\frac{26}{52}+\frac{4}{52}-2\times\frac{2}{52}=\frac{26}{52}=\frac{1}{2}$. ∴ $\frac{1}{26}$; $\frac{7}{13}$; $\frac{1}{2}$.

Q23. Members using morning or evening: $80-12=68$; both $=35+41-68=8$. Inclusive: $\frac{68}{80}=\frac{17}{20}$. Exclusive: $\frac{35+41-2\times 8}{80}=\frac{60}{80}=\frac{3}{4}$. And: $\frac{8}{80}=\frac{1}{10}$. ∴ 8; $\frac{17}{20}$; $\frac{3}{4}$; $\frac{1}{10}$.

Q24. Rainy days: $P=\frac{9}{31}$, so $93\times\frac{9}{31}=27$. Windy days: also $\frac{9}{31}$, so also 27. The predictions match because July had the same count of rainy and windy days, 9 each. ∴ 27 and 27.

Q25. The 100-trial value 0.58 is the better estimate: five trials are far too few, and small samples spread widely. Prediction: $250\times 0.58=145$. ∴ 0.58; about 145 seeds.



Dot plot of 40 small-sample and 40 large-sample estimates of the same probability — the spread shown is why Q25 trusts the 100-trial estimate

Q26. Inclusive: $\frac{26}{60}+\frac{39}{60}-\frac{17}{60}=\frac{48}{60}=\frac{4}{5}$. Exclusive: $\frac{26}{60}+\frac{39}{60}-2\times\frac{17}{60}=\frac{22}{60}=\frac{11}{30}$. Neither: $60-48=12$ customers, $\frac{12}{60}=\frac{1}{5}$. ∴ $\frac{4}{5}$; $\frac{11}{30}$; $\frac{1}{5}$.

Q27. The 200-toss value 0.605 is the steadier estimate. Prediction: $1000\times 0.605=605$. ∴ 605 open-side-down landings.

Q28. Priya added the two probabilities without subtracting the students counted twice — the 7 with both a sister and a brother. Correctly: $\frac{22}{60}+\frac{19}{60}-\frac{7}{60}=\frac{34}{60}=\frac{17}{30}$. ∴ The overlap was not subtracted; $P=\frac{17}{30}$.

Q29. H and R: one outcome of the six, $P=\frac{1}{6}$. Inclusive: $\frac{3}{6}+\frac{2}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$. Exclusive: $\frac{3}{6}+\frac{2}{6}-2\times\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$. ∴ $\frac{1}{6}$; $\frac{2}{3}$; $\frac{1}{2}$.

Q30. Point down is the complement of point up: $1-0.28=0.72$ (complementary events, Level 8). ∴ 0.72.

Q31. $\frac{5}{8}=0.625$; $\frac{9}{15}=0.6$; $\frac{16}{24}=\frac{2}{3}\approx 0.67$; $\frac{19}{30}\approx 0.63$. Highest: $\frac{2}{3}$, after three quarters. The best single estimate is the value from all the shots, $\frac{19}{30}\approx 0.63$ — the whole data set is the largest sample. ∴ After three quarters; $\frac{19}{30}\approx 0.63$.

Q32. Sport only: $55-18=37$; job only: $32-18=14$; neither: $100-(55+32-18)=31$. Inclusive: $\frac{69}{100}$; exclusive: $\frac{37+14}{100}=\frac{51}{100}$; and: $\frac{18}{100}=\frac{9}{50}$. ∴ 37, 14, 31; $\frac{69}{100}$; $\frac{51}{100}$; $\frac{9}{50}$.

Q33. a Inclusive 'or' would be $0.4+0.5-0.2=0.7$, at most 1, and 0.2 is no bigger than either 0.4 or 0.5 — consistent. b Inclusive 'or' would be $0.5+0.6-0.05=1.05$, more than 1 — impossible, so no such events exist. ∴ a possible; b impossible.

Q34. $x+(x+3)+(2x-5)+34=120$, so $4x+32=120$, $4x=88$, $x=22$. Regions: 22, 25, 39, 34; the overlap is $x+3=25$. $n(A)=22+25=47$; $n(B)=25+39=64$. Inclusive: $\frac{47}{120}+\frac{64}{120}-\frac{25}{120}=\frac{86}{120}=\frac{43}{60}$; the same as the three regions $22+25+39=86$ ✓. Exclusive: $\frac{22+39}{120}=\frac{61}{120}$. And: $\frac{25}{120}=\frac{5}{24}$. ∴ $x=22$; regions 22, 25, 39, 34; inclusive $\frac{43}{60}$; exclusive $\frac{61}{120}$; and $\frac{5}{24}$.

Q35. Inclusive: $\frac{88}{150}+\frac{71}{150}-\frac{44}{150}=\frac{115}{150}=\frac{23}{30}$. Exclusive: $\frac{88}{150}+\frac{71}{150}-2\times\frac{44}{150}=\frac{71}{150}$. Neither: $150-115=35$ students, $\frac{35}{150}=\frac{7}{30}$. ∴ $\frac{23}{30}$; $\frac{71}{150}$; $\frac{7}{30}$.

Q36. Both: $600\times\frac{44}{150}=176$. Inclusive: $600\times\frac{23}{30}=460$. Exactly one: $600\times\frac{71}{150}=284$. Neither: $600\times\frac{7}{30}=140$. Check: $284+176+140=600$ ✓. ∴ 460; 284; 140.

Repeated chance experiments and simulations with digital tools

7C · Chapter 7 — Probability: theoretical probability, relative frequency and simulation · Level 9

Content description `VC2M9P03` (word for word): *design and conduct repeated chance experiments and simulations using digital tools to estimate probabilities that cannot be determined exactly*

Achievement standard anchor (AS 20): *They design and conduct experiments or simulations for combined events using digital tools.*

Elaboration ids for linkage: `VC2M9P03.E01`, `VC2M9P03.E02`, `VC2M9P03.E03`

Elaboration not drawn on: `VC2M9P03.E04` names repeated trials of Aboriginal and/or Torres Strait Islander children's games. First Nations content is excluded from this book, so this subtopic teaches the same idea — estimating a probability of winning and not winning by repeated trials — with neutral devices instead. The omission is deliberate, not an oversight.

Prerequisite pointer: 7A (listing outcomes and theoretical probability) and 7B (relative frequency from experiments). A simulation is compared here against the exact probabilities of 7A and the long-run idea of 7B; both are rehearsed as tools, not re-taught.

DIGITAL TOOL — this task assumes a spreadsheet or a general-purpose programming language. Your teacher will nominate the tool. Every experiment below says exactly what random numbers to produce and what you should see, and every figure shows a real, reproducible run, so the same experiment can also be followed with paper and a list of numbers if the tool is not in front of you. In a spreadsheet, a function such as `=RANDBETWEEN(1,5)` returns a random whole number from 1 to 5; in a programming language, use the matching random-integer command.

Note on figures: every count, proportion and running frequency plotted here comes from a seeded run — a run whose every number is fixed in advance by its seed and is therefore reproducible (R7) — and is cross-checked in `src/verify_VC2M9P03.py`. Every context is a generic, clearly-labelled SCENARIO with invented parameters, so no real data ships and no source line is owed (R1, R2).

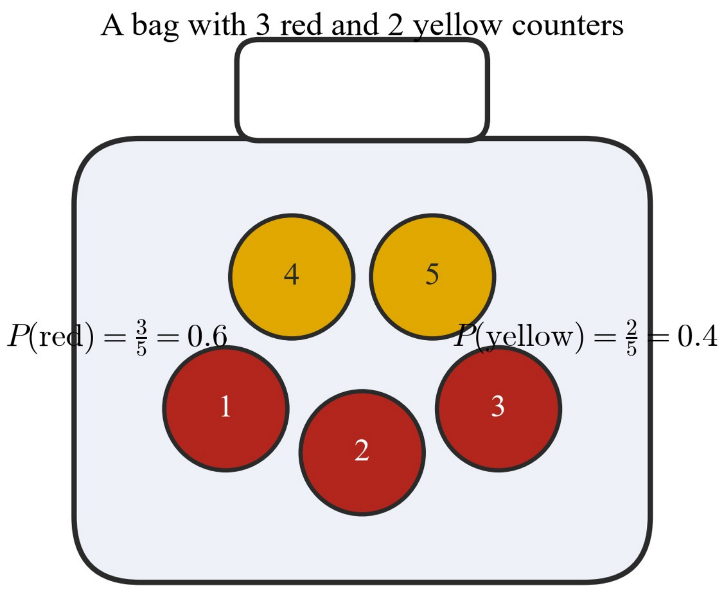
Theory

When an exact answer is possible, and when it is not

In 7A you met chance experiments whose probability can be found exactly by counting. Roll a fair die: six **outcomes** (the single results of one run of the experiment), all equally likely, one of them **favourable** to "a 6", so the probability is $\frac{1}{6}$. Spin a fair spinner with eight equal sectors and ask for a 2: $\frac{1}{8}$. The method works because the device is fair and the outcomes can be listed.

Now ask: what is the probability that a seed from a particular batch will sprout? There is no fair device to count. The seeds are not equally likely to do anything — the batch simply behaves a certain way, and the only way to know is to plant a lot of seeds and see. The same is true of "it rains on Saturday" or "a player makes a free throw". For these, the probability cannot be determined exactly from a list of outcomes; it has to be **estimated** — approximated — from data.

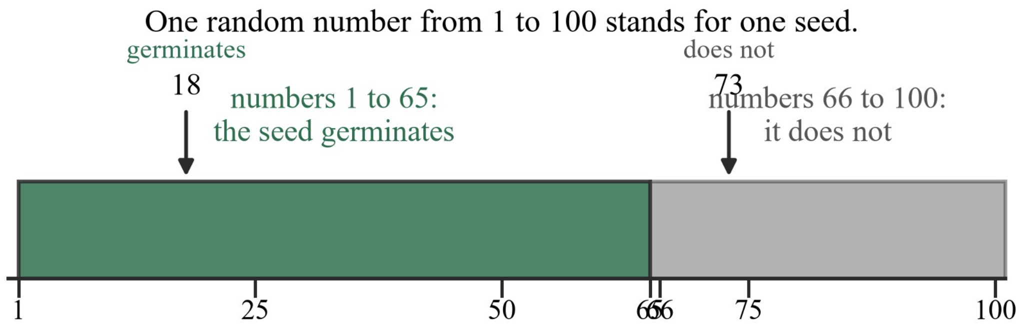
That is the job of this subtopic. When an event has no fair device behind it, we build one that behaves the same way, run it many times with a digital tool, and let the long-run **relative frequency** (the count of favourable outcomes divided by the number of **trials**, the individual runs) stand as our estimate. Running such a stand-in experiment on a tool is a **simulation** — a model of the real event, made of random numbers.



A bag holding five counters numbered 1 to 5, three red and two yellow, with P(red) = 3/5 = 0.6 and P(yellow) = 2/5 = 0.4 labelled

Mapping the real event onto random numbers

A simulation begins with a **model**: a rule that turns each random number into one outcome of the real event, at the right proportions. Suppose a batch of seeds sprouts 65 times in every 100, on average. Ask the tool for a random whole number from 1 to 100. Let the numbers 1 to 65 stand for "the seed sprouts" and 66 to 100 for "it does not". Because 65 of the 100 possible numbers mean "sprouts", the model behaves like the batch. Each number produced is one **trial**.



A number line from 1 to 100 with 1 to 65 shaded green as "the seed sprouts" and 66 to 100 shaded grey as "it does not", with example numbers 18 (sprouts) and 73 (does not) marked

The same idea scales to any proportion. A player who makes 70% of her shots is modelled by a random number from 1 to 10 with 1 to 7 meaning "makes it". The map is the model; choosing it is the "design" step the content description speaks of.

Running the simulation and reading the estimate

Run the model many times and keep a running count. Here is a real 50-trial run of the bag from the first figure (random number 1 to 5, with 1 to 3 meaning red), set out as a spreadsheet would show it.

A simulation of 50 draws, run in a spreadsheet (seed 18, so these numbers are reproducible)

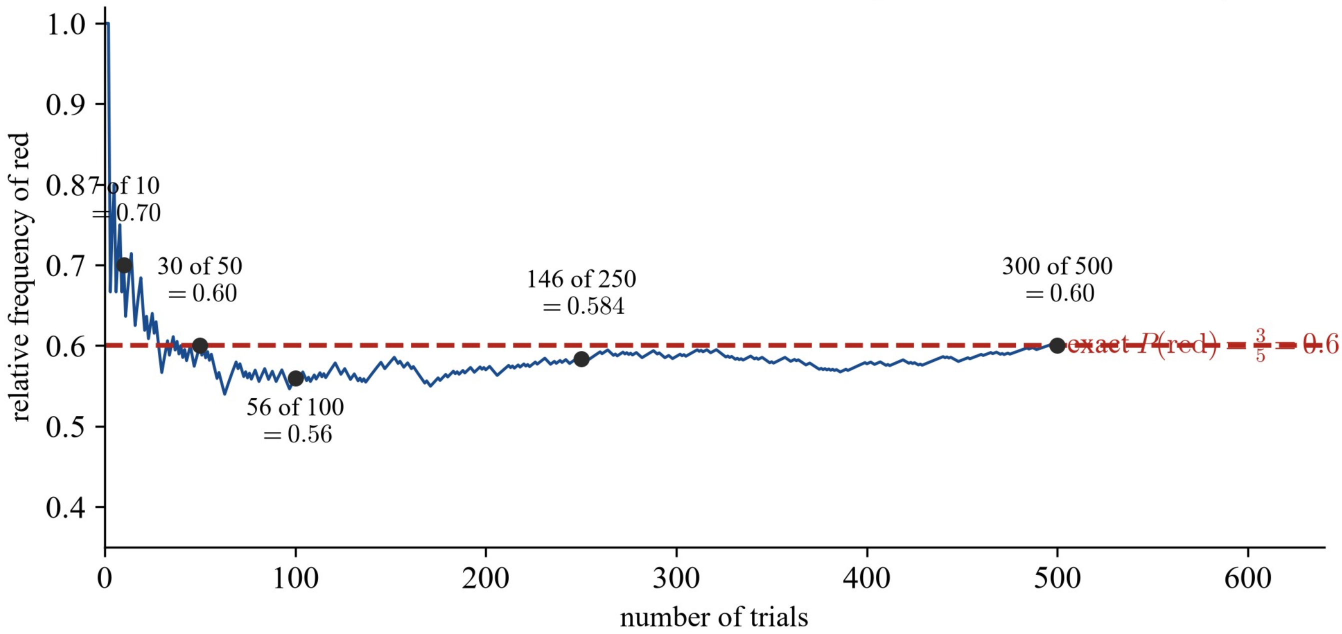
Trial	Random number	Outcome	Reds so far	Relative frequency
1	2	red	1	1.00
2	1	red	2	1.00
3	4	yellow	2	0.67
4	3	red	3	0.75
5	2	red	4	0.80
6	2	red	5	0.83
7	4	yellow	5	0.71
8	4	yellow	5	0.62
20	2	red	14	0.70
50	5	yellow	31	0.62

The last row: 31 reds in 50 trials, so the estimate is $\frac{31}{50} = 0.62$.

A spreadsheet-style table of a 50-trial run, columns Trial, Random number, Outcome, Reds so far, Relative frequency, with the first eight rows shown, then row 20 and the final row 50 reading 31 reds and relative frequency 0.62

The last row divides 31 by 50 to give the estimate 0.62. With only 50 trials the estimate wanders; with more, it settles. The next figure runs the same model for 500 trials and plots the running relative frequency after each trial.

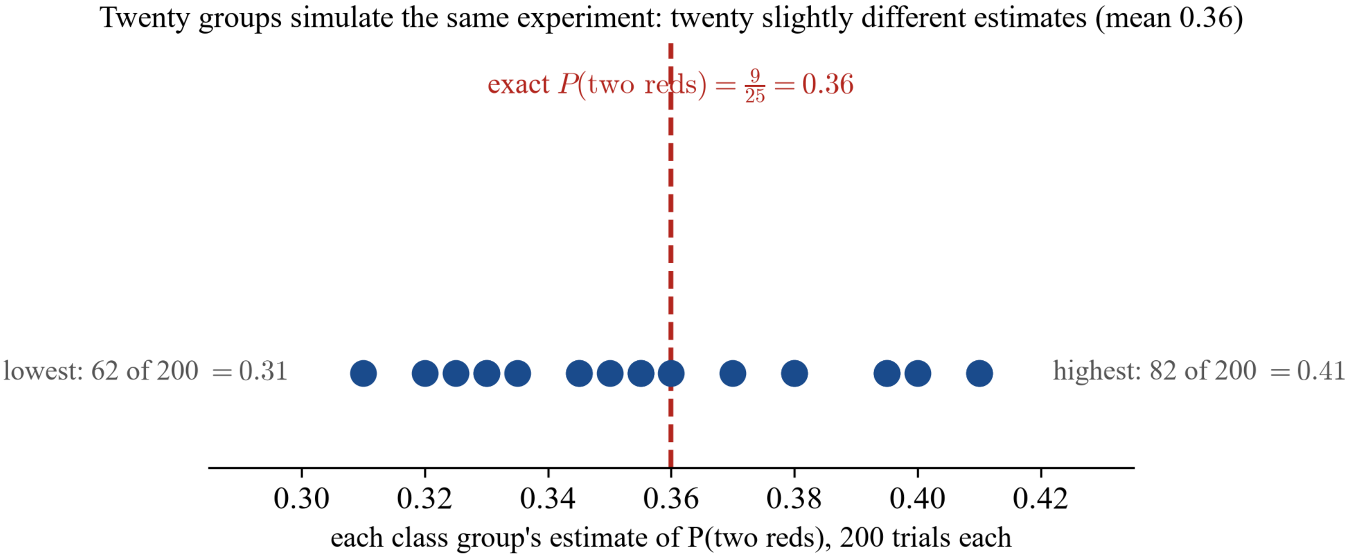
The estimate moves about at first, then settles near 0.6 (one run of 500 trials, seed 7)



A line graph of the running relative frequency of red across 500 trials, jumping about early (0.7 at 10 trials, 0.6 at 50, 0.56 at 100) then settling, with a dashed red line at the exact value 0.6

Read the milestones: after 10 trials the frequency is $\frac{7}{10} = 0.7$; after 100 it is $\frac{56}{100} = 0.56$; after 500 it is $\frac{300}{500} = 0.6$, exactly the theoretical value. The estimate moves about at first and settles near the true probability as trials accumulate — the long-run idea of 7B, now produced by a tool.

And yet one run is not the whole story. If a whole class runs the same 200-trial experiment, each group gets a slightly different estimate. The next figure shows twenty such runs of the two-red experiment below: the estimates spread from 0.31 to 0.41 around the exact $\frac{9}{25}=0.36$, and their mean is exactly 0.36. So report an estimate as "around" a value, and trust it more when it comes from many trials.



A dot plot of twenty class estimates of P(two reds) from 200 trials each, dots spread from 0.31 to 0.41 around a dashed red line at the exact value 0.36

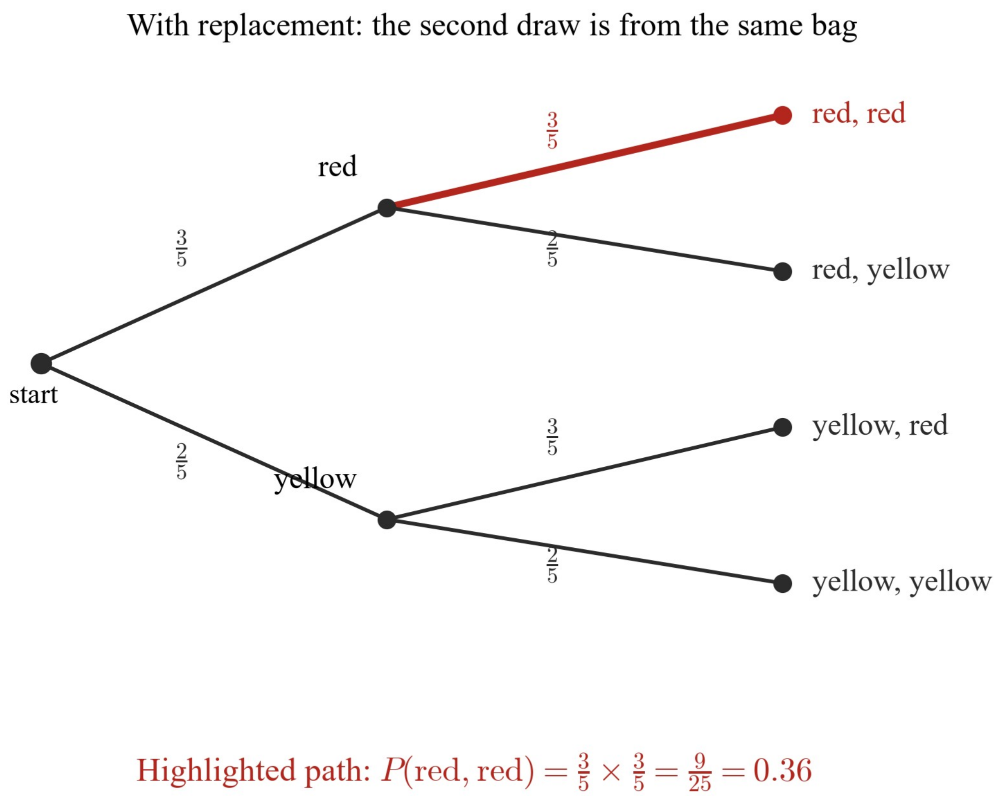
Two-step experiments: with and without replacement

Now a **combined** (two-step) event, from a bag with 3 red and 2 yellow counters. Draw two counters and ask for the probability both are red. There are two different experiments, differing in nothing except one rule.

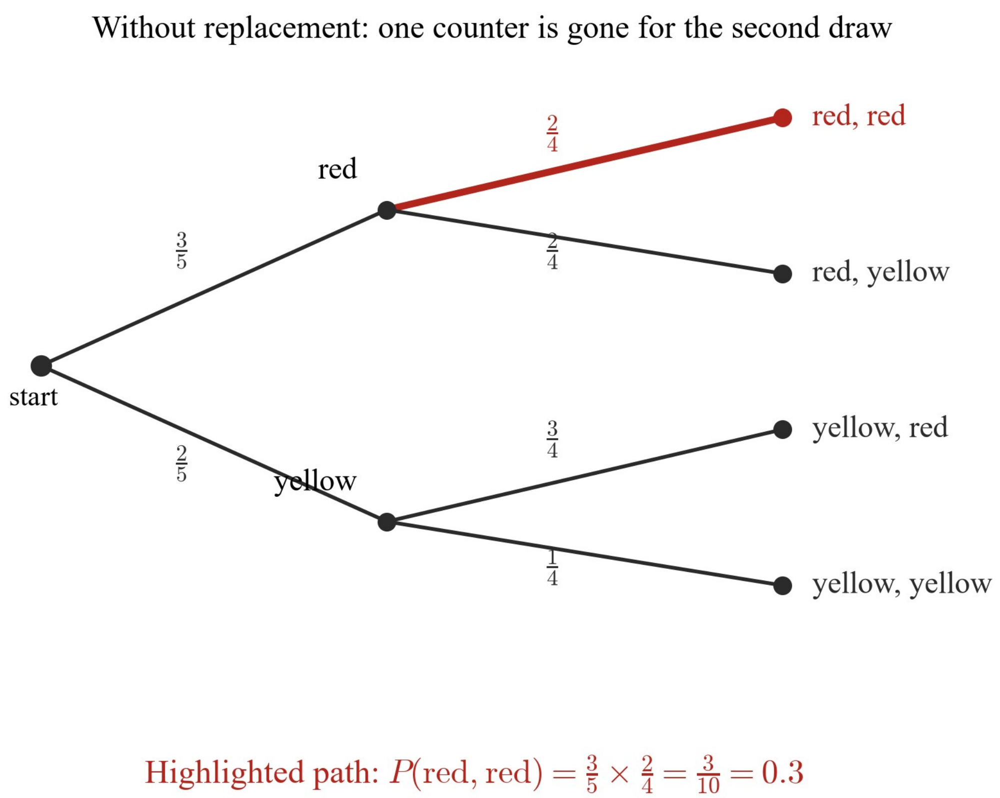
- **With replacement**: after the first draw, put the counter back before the second draw. The second draw is from the same bag, so $P(red)=\frac{3}{5}$ again.
- **Without replacement**: keep the first counter out. One counter is gone, so the second draw is from 4 counters, and the probabilities change with what was removed.

The tree diagrams (the systematic listing method of 7A) give the exact answers. Following the highlighted red-red path and multiplying along the branches:

- with replacement: $\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}=0.36$;
- without replacement: $\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}=0.3$.

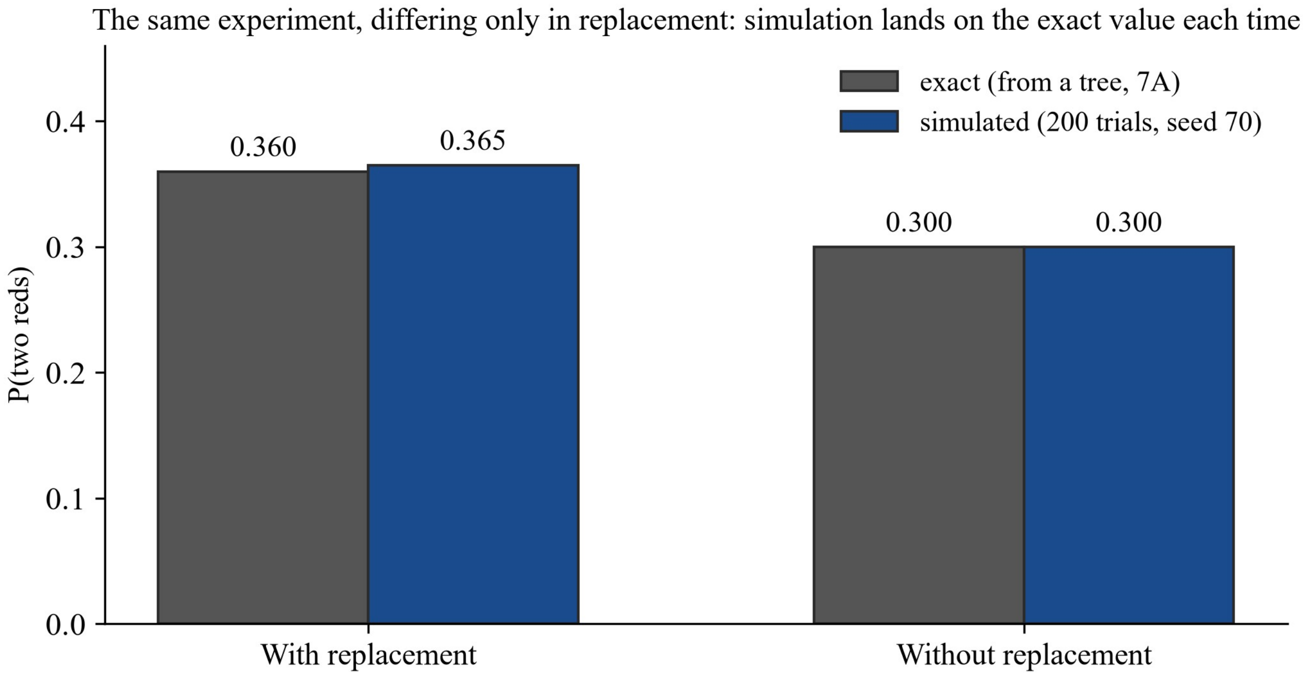


A two-step tree for drawing with replacement: first draw red 3/5 or yellow 2/5, second draw the same 3/5 and 2/5 from each branch, with the red-red path highlighted



A two-step tree for drawing without replacement: first draw red 3/5 or yellow 2/5, then from a red first draw 2/4 red and 2/4 yellow and from a yellow first draw 3/4 red and 1/4 yellow, with the red-red path highlighted

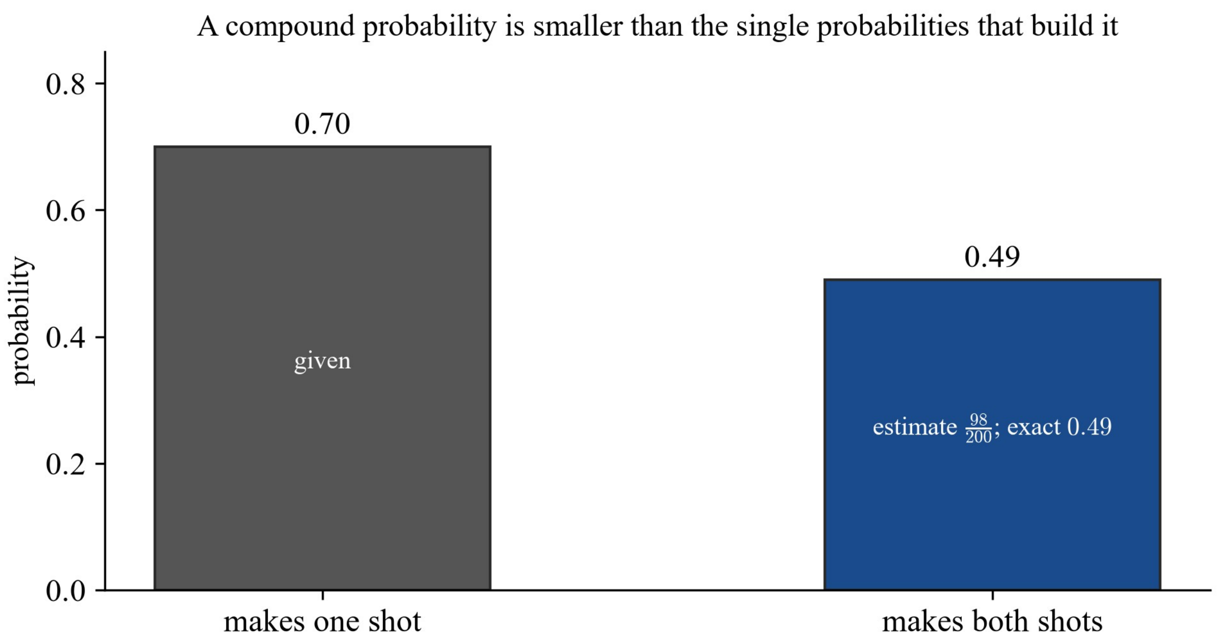
Running 200 trials of each experiment from the same stream of random numbers lands on the exact value both times — 0.365 against 0.36, and 0.30 against 0.30 — which is how we know the model is faithful before we trust it on questions with no exact answer.



A paired bar chart comparing exact and simulated probabilities of two reds: with replacement 0.36 exact and 0.365 simulated, without replacement 0.30 exact and 0.30 simulated

A compound probability is smaller than the probabilities that build it

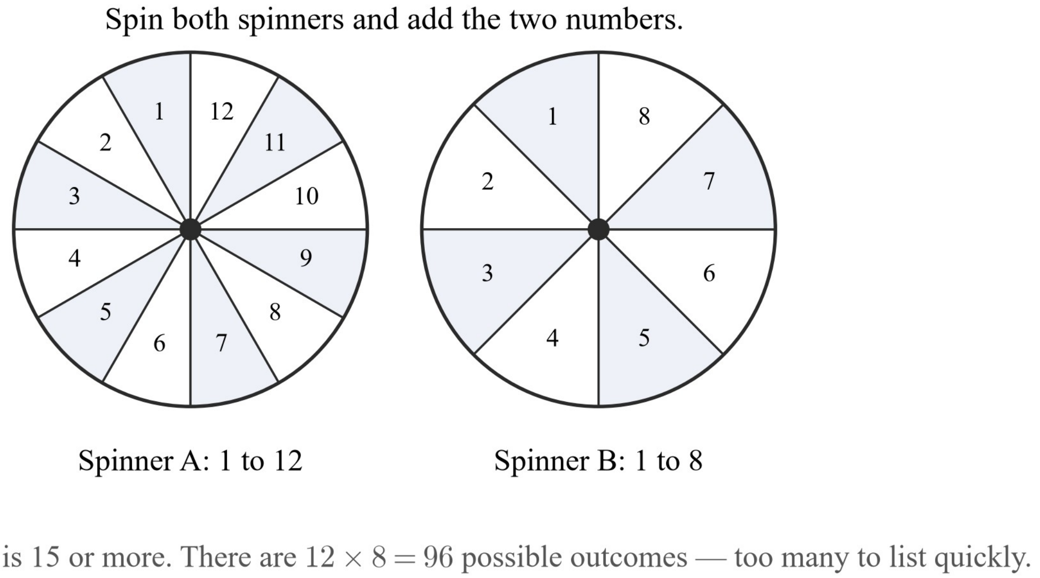
For a combined event joined by "and", the probability is found by multiplying along the branches, and multiplying by numbers less than 1 makes things smaller. A player who makes 0.7 of her shots makes both of two shots with probability $0.7\times0.7=0.49$ — smaller than the single-shot probability. A simulation (98 both-made in 200 pairs of trials) confirms $\frac{98}{200}=0.49$.



A bar chart with one bar at 0.70 labelled "makes one shot" and a shorter bar at 0.49 labelled "makes both shots", showing the compound probability is smaller

When there are too many outcomes to list

Sometimes an exact answer exists in principle but the list is too big to make in practice. Spin a 12-sector spinner and an 8-sector spinner and add the numbers: you win with a total of 15 or more. There are $12\times8=96$ outcomes. An array would list them all, but it is slow and error-prone; a simulation gives a working estimate in seconds, and for larger games no array fits at all. That is precisely the situation the content description targets — a probability that, for us, cannot be determined exactly.



Two spinners side by side, one with twelve equal sectors numbered 1 to 12 and one with eight equal sectors numbered 1 to 8, under the rule "spin both and add; you win if the total is 15 or more", noting 12 x 8 = 96 outcomes

Worked examples

Example 1 — scaffolded

A bag holds 3 red and 2 yellow counters. Simulate 50 draws with replacement (random number 1 to 5, with 1 to 3 red), and compare the estimate for red with the exact probability.

Working	Comment
Model: one random number 1 to 5 per trial; 1 to 3 red, 4 to 5 yellow	3 of the 5 numbers mean red, matching $\frac{3}{5}$.
First ten trials: 2,1,4,3,2,2,4,4,2,4 → R,R,Y,R,R,R,Y,Y,R,Y	Read each number off the model; six reds in ten.
After 50 trials: 31 reds	The full run is in the spreadsheet figure.
Estimate $=\frac{31}{50}=0.62$	Relative frequency = count ÷ trials.
Exact (7A): $P(red)=\frac{3}{5}=0.6$	Counted, not simulated.
Check: 0.62 against 0.6	The estimate sits 0.02 from the exact value.

∴ The simulation gives 0.62, close to the exact 0.6; with more trials the estimate settles closer still.

Example 2 — scaffolded

From the same bag, draw two counters. Find the exact probability of two reds (i) with and (ii) without replacement using trees, then check both against a 200-trial simulation of each.

Working	Comment
(i) With: $\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}=0.36$	Second draw is from the same bag.

Working	Comment
(ii) Without: $\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}=0.3$	One counter is gone; 2 reds of 4 remain after a red.
First ten (i), R = red, Y = yellow: RR,YY,RR,RR,YR,YY,RY,RY,RR,YY → 4 two-red	Systematic read of each pair.
First ten (ii): YR,RR,YR,RY,RR,YR,YR,RY,RY,YR → 2 two-red	The pairs differ; so does the count.
Simulated, 200 trials each: (i) $\frac{73}{200}=0.365$; (ii) $\frac{60}{200}=0.3$	Same stream of numbers, two different rules.
Check: $0.365\approx 0.36$ and $0.3=0.3$	The model reproduces both exact values.

∴ The exact values are $\frac{9}{25}$ with and $\frac{3}{10}$ without replacement, and the simulations confirm them; dropping the replacement rule lowers the two-red probability.

Example 3 — unscaffolded

SCENARIO: a batch of seeds sprouts 65 times in 100. Two seeds are planted. Estimate the probability that at least one sprouts by simulating 500 pairs of trials (two random numbers 1 to 100 per trial; 1 to 65 sprouts), and compare with the model value.

Model: each random number 1 to 100 is one seed; 1 to 65 sprouts. A trial is a pair of numbers; it is favourable if at least one number is 65 or less.

The 500-trial run gives 436 favourable trials, so the estimate is $\frac{436}{500}=0.872$. At the 100-trial mark the running estimate was $\frac{86}{100}=0.86$, already close.

Model value (for comparison): “at least one sprouts” fails only when both fail, and each fails with probability $\frac{35}{100}$, so the model value is $1-\frac{35}{100}\times\frac{35}{100}=1-0.1225=0.8775$. The real batch’s true probability is not exactly 0.8775 — the 0.65 itself came from data — which is why the answer must be an estimate.

∴ The simulation estimates $P(\text{at least one sprouts})\approx 0.872$, near the model value 0.8775.

Example 4 — unscaffolded

SCENARIO: at a fete, you spin the 12-sector and the 8-sector spinners and win if the total is 15 or more. Simulate 400 trials to estimate the chance of winning, and explain why listing outcomes is not the method of choice.

Model: one random number 1 to 12 and one 1 to 8 per trial; favourable if the sum is 15 or more.

First eight trials: (4,1)=5, (9,1)=10, (3,4)=7, (1,1)=2, (11,3)=14, (12,6)=18, (4,2)=6, (6,8)=14 — one win, the (12,6).

The 400-trial run gives 88 wins, so the estimate is $\frac{88}{400}=0.22$.

Why not list? There are $12\times 8=96$ outcomes; an array can count 21 favourable ones, giving $\frac{21}{96}=\frac{7}{32}=0.21875$, and the estimate 0.22 sits close to it. But building that array is slow and easy to miscount, and the moment the game grows (a third spinner, a changing rule) no array fits — while the simulation stays almost the same. That is the point of simulating.

$\therefore$ The estimate is $\frac{88}{400}=0.22$, in line with the exact $\frac{7}{32}\approx 0.219$; simulation is the practical tool when the outcome list is too large.

Practice questions

Scaffolded

Q1. Decide for each whether the probability can be found exactly by listing equally-likely outcomes (7A), or must be estimated from data or a simulation.

a. Rolling a fair die and getting a 6.
b. A seed from a particular batch sprouting.
c. Spinning a fair 8-sector spinner and getting a 2.
d. It raining on Saturday.

Q2. A fair 20-sector spinner has red on 1 to 10, blue on 11 to 16 and yellow on 17 to 20. A simulation produced these 20 numbers: 20, 9, 12, 17, 1, 15, 8, 2, 6, 4, 12, 16, 8, 13, 18, 4, 19, 8, 1, 7.

a. Tally the outcomes into red, blue and yellow.
b. Write the relative frequency of each colour.
c. Write the exact probability of each colour and compare.

Q3. A spinner wins when it lands on 1 or 2 of 8 equal sectors, so $P(win)=\frac{2}{8}=0.25$. A simulation recorded 11 wins after 50 spins, 26 after 100 and 50 after 200.

a. Write the relative frequency of a win at each stage.
b. Which of the three is the most reliable estimate, and why?

Q4. A bag holds 4 blue and 1 green counter. Two counters are drawn **with** replacement; the event is "blue then green".

a. Use a tree to show the exact probability is $\frac{4}{5}\times\frac{1}{5}=\frac{4}{25}=0.16$.
b. A 100-trial simulation gave 17 blue-then-green trials. Write the estimate and compare with 0.16.

Q5. The same bag, now drawing two counters **without** replacement; again the event is "blue then green".

a. Use a tree to show the exact probability is $\frac{4}{5}\times\frac{1}{4}=\frac{1}{5}=0.2$.
b. A 100-trial simulation gave 22. Write the estimate and compare with 0.2.
c. In one sentence: why is the without-replacement probability larger here?

Q6. SCENARIO: the chance of rain on any one day is 0.3. Model a weekend (Saturday and Sunday) with two random numbers 1 to 10 per weekend, where 1 to 3 means rain. Here are ten simulated weekends, as (Saturday, Sunday) pairs: (4,5), (2,7), (8,3), (2,2), (1,7), (9,5), (1,4), (9,9), (6,5), (3,2).

a. Count the weekends with rain on both days.
b. Count the weekends with rain on at least one day.
c. Write both estimates, and compare the both-days estimate with the model value $0.3\times 0.3=0.09$.

Unscaffolded

Q7. A student says every probability can be found exactly by listing outcomes. Give two examples of events for which this fails, and say in one sentence each why.

Q8. SCENARIO: a seed sprouts 80 times in 100. Two seeds are planted; model each with a random number 1 to 5 where 1 to 4 sprouts. These 20 pairs were simulated: (2,3), (1,4), (4,2), (1,1), (1,4), (5,3), (1,2), (5,5), (3,3), (2,1), (3,2), (1,3), (3,2), (2,3), (3,3), (1,5), (3,4), (5,2), (2,2), (4,3).

Estimate the probability that at least one sprouts, and compare with the model value $1-0.2\times0.2=0.96$.

Q9. A simulation of an event with true probability 0.6 was run for 5000 trials. The running relative frequency was 0.57 at 100 trials, 0.588 at 500, 0.594 at 1000 and 0.600 at 5000. Describe in one sentence what happens to the estimate as the trials grow, and name the idea.

Q10. To simulate drawing two counters without replacement from a 5-counter bag, a student takes a first random number 1 to 5, then — for the second counter — asks the tool for a fresh random number 1 to 5. Explain what experiment this actually models, and what the second draw should use instead.

Q11. SCENARIO: a player makes 0.7 of her free throws. Model one shot with a random number 1 to 10 where 1 to 7 makes it. In 200 pairs of trials, both shots were made 98 times. Write the estimate for "makes both" and compare it with 0.7. Which is larger, and why must that be so?

Q12. Using the same 200 pairs of trials as Q11, at least one shot was made in 183 pairs. Write the estimate for "makes at least one" and compare with the model value $1-0.3\times0.3=0.91$.

Q13. For the fete game of Example 4 (12-sector and 8-sector spinners, win on a total of 15 or more), a 400-trial simulation gave 88 wins.

a. Write the estimate.
b. Explain in one sentence why the exact value is impractical to list by hand, even though it exists.

Q14. SCENARIO: a pack of 6 cards holds 4 winning cards. Two cards are drawn without replacement; the event is "both win". The exact probability is $\frac{4}{6}\times\frac{3}{5}=\frac{2}{5}=0.4$. A 150-trial simulation gave 61. Write the estimate and compare with 0.4.

Q15. Two students each simulate drawing two counters with replacement from the 3-red-2-yellow bag, 200 trials each, counting two reds (exact value $\frac{9}{25}=0.36$). Student A's seed gives 69; student B's gives 75. Write both estimates and say, in one sentence, whether the difference means one of them is wrong.

Q16. SCENARIO: a goal-kicker converts 4 attempts in 5. Design (do not run) a simulation to estimate the probability of converting both of two attempts: state the tool input for one trial, which numbers mean a conversion, and how you would turn the finished run into an estimate.

Q17. A bag holds 2 red and 3 yellow counters. Two are drawn; the event is "two yellows".

a. Show the exact probability is $\frac{9}{25}=0.36$ with replacement and $\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}=0.3$ without.
b. Simulations of 200 trials each gave 72 (with) and 60 (without). Write both estimates and check them against the exact values.

Q18. A report claims that, because one 200-trial simulation of two reds gave 82 favourable outcomes, the probability is "exactly 0.41". Use the class dot-plot figure to explain why "exactly" is the wrong word, and rewrite the claim correctly.

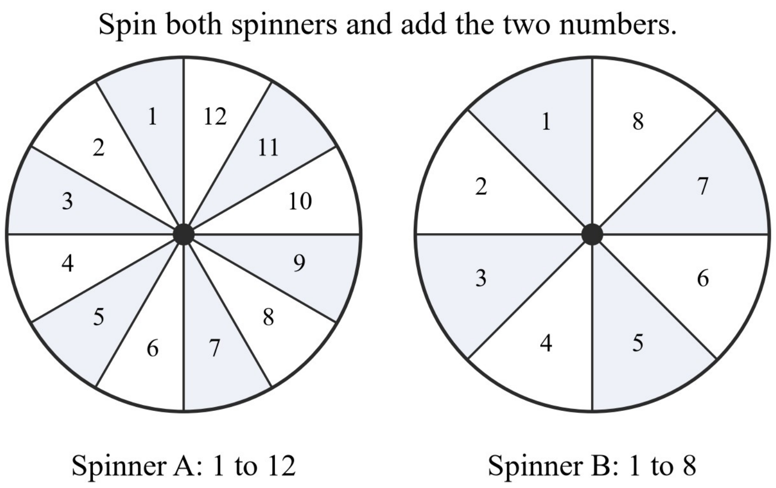
Answers

Q1 a exact, $\frac{1}{6}$; b estimate; c exact, $\frac{1}{8}$; d estimate. **Q2** a red 11, blue 5, yellow 4; b 0.55, 0.25, 0.20; c red $\frac{1}{2}$, blue $\frac{3}{10}$, yellow $\frac{1}{5}$ — each estimate sits within 0.05 of its exact value. **Q3** a 0.22, 0.26, 0.25; b the 200-spin one — more trials, so the estimate has settled closer to 0.25. **Q4** b $\frac{17}{100}=0.17$, within 0.01 of 0.16. **Q5** b $\frac{22}{100}=0.22$, within 0.02 of 0.2; c removing a blue leaves a higher share of green, so green is more likely on the second draw. **Q6** a 2; b 6; c both days $\frac{2}{10}=0.2$ (model 0.09), at least one $\frac{6}{10}=0.6$ (model 0.51) — ten weekends is too few for a tight estimate. **Q7** Any two of: a seed sprouting (no fair device; only data), rain (weather is not equally-likely outcomes), a player making a shot (skill, not a fair device). Each fails listing because the outcomes are not equally likely. **Q8** 19 of 20, $\frac{19}{20}=0.95$, near the model 0.96. **Q9** The estimate moves about early, then settles towards 0.6 as trials grow — the long-run relative-frequency idea. **Q10** It models drawing **with** replacement; the second draw should use a number 1 to 4 over the relabelled remaining counters. **Q11** $\frac{98}{200}=0.49$; the single-shot 0.7 is larger, because multiplying by 0.7 again can only shrink it. **Q12** $\frac{183}{200}=0.915$, near 0.91. **Q13** a $\frac{88}{400}=0.22$; b 96 outcomes is a long, error-prone array, and bigger games have none. **Q14** $\frac{61}{150}\approx0.407$, near 0.4. **Q15** A $\frac{69}{200}=0.345$; B $\frac{75}{200}=0.375$; no — both sit either side of 0.36; run-to-run spread is expected. **Q16** One trial = two random numbers 1 to 5; 1 to 4 = conversion; estimate = favourable pairs ÷ trials. **Q17** b $\frac{72}{200}=0.36$ and $\frac{60}{200}=0.3$ — exactly the tree values. **Q18** The dot plot shows twenty honest runs spread from 0.31 to 0.41 around the exact 0.36; $\frac{82}{200}=0.41$ is just the top dot — one estimate, not the value. Rewrite: "the probability is about 0.36; this run estimated it at 0.41".

Fully-worked solutions

Q1. a A fair die has six equally-likely outcomes, one favourable: exact, $\frac{1}{6}$. b A seed either sprouts or not, but the two outcomes are not equally likely — only planting and counting tells you. Estimate. c Eight equal sectors, one favourable: exact, $\frac{1}{8}$. d Rain is weather, not a fair device; its probability comes from records. Estimate. ∴ a and c exact; b and d estimated — a fair, listable device is what separates the two.

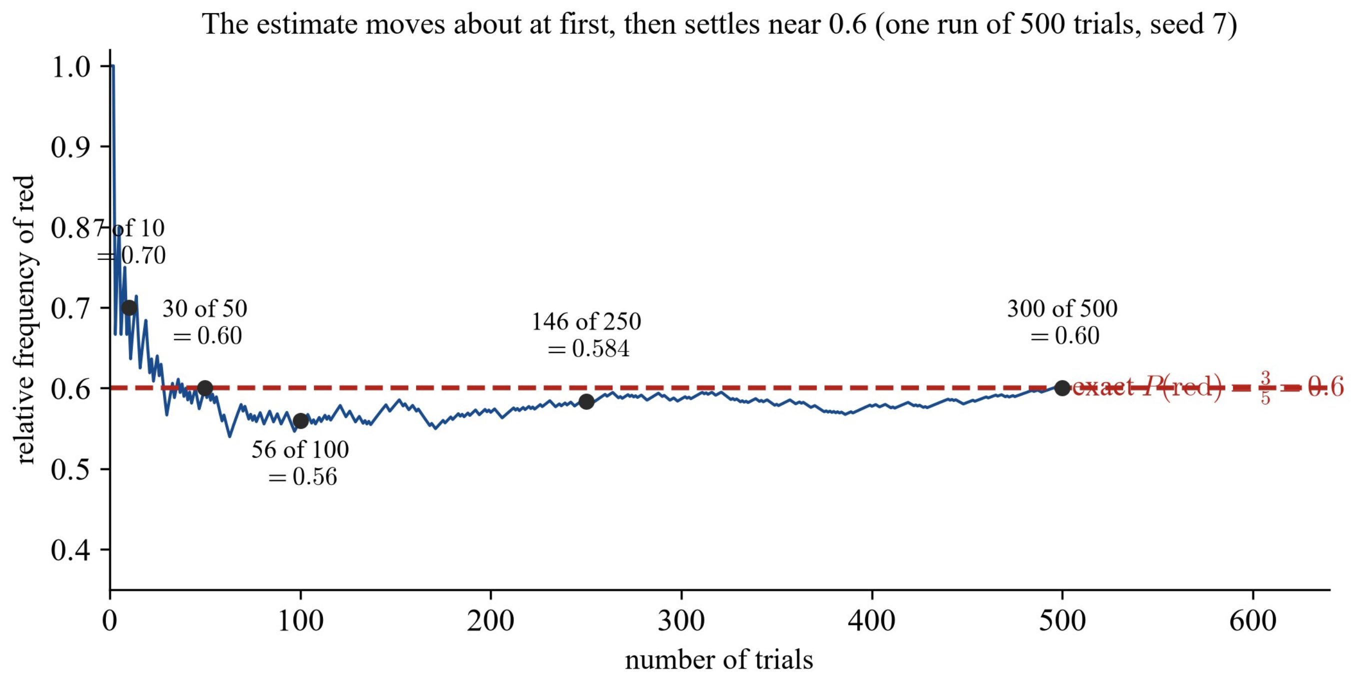
Q2. a Red (1 to 10): 9,1,8,2,6,4,8,4,8,1,7 → 11. Blue (11 to 16): 12,15,12,16,13 → 5. Yellow (17 to 20): 20,17,18,19 → 4. b $\frac{11}{20}=0.55$; $\frac{5}{20}=0.25$; $\frac{4}{20}=0.20$. c Exact: red $\frac{10}{20}=\frac{1}{2}$, blue $\frac{6}{20}=\frac{3}{10}$, yellow $\frac{4}{20}=\frac{1}{5}$. ∴ Each 20-number relative frequency sits within 0.05 of its exact value; twenty trials is few, hence the small gaps.



You win if the total is 15 or more. There are $12 \times 8 = 96$ possible outcomes — too many to list quickly.

Two spinners side by side with the win rule for the fete game, showing how a many-outcome device is simulated rather than listed

Q3. a $\frac{11}{50}=0.22$; $\frac{26}{100}=0.26$; $\frac{50}{200}=0.25$. b The 200-spin value: more trials means the running frequency has settled closer to the exact 0.25 (here it lands exactly on it). $\therefore$ $0.22 \rightarrow 0.26 \rightarrow 0.25$: the estimate tightens onto 0.25 as trials grow, as in the convergence figure.



A line graph of a running relative frequency settling onto a dashed exact-value line as trials accumulate

Q4. a With replacement the second draw is from the same bag: $P(blue)=\frac{4}{5}$, $P(green)=\frac{1}{5}$, so $\frac{4}{5}\times\frac{1}{5}=\frac{4}{25}=0.16$. b Estimate $\frac{17}{100}=0.17$, within 0.01 of 0.16. $\therefore$ The simulation confirms the exact $\frac{4}{25}$.

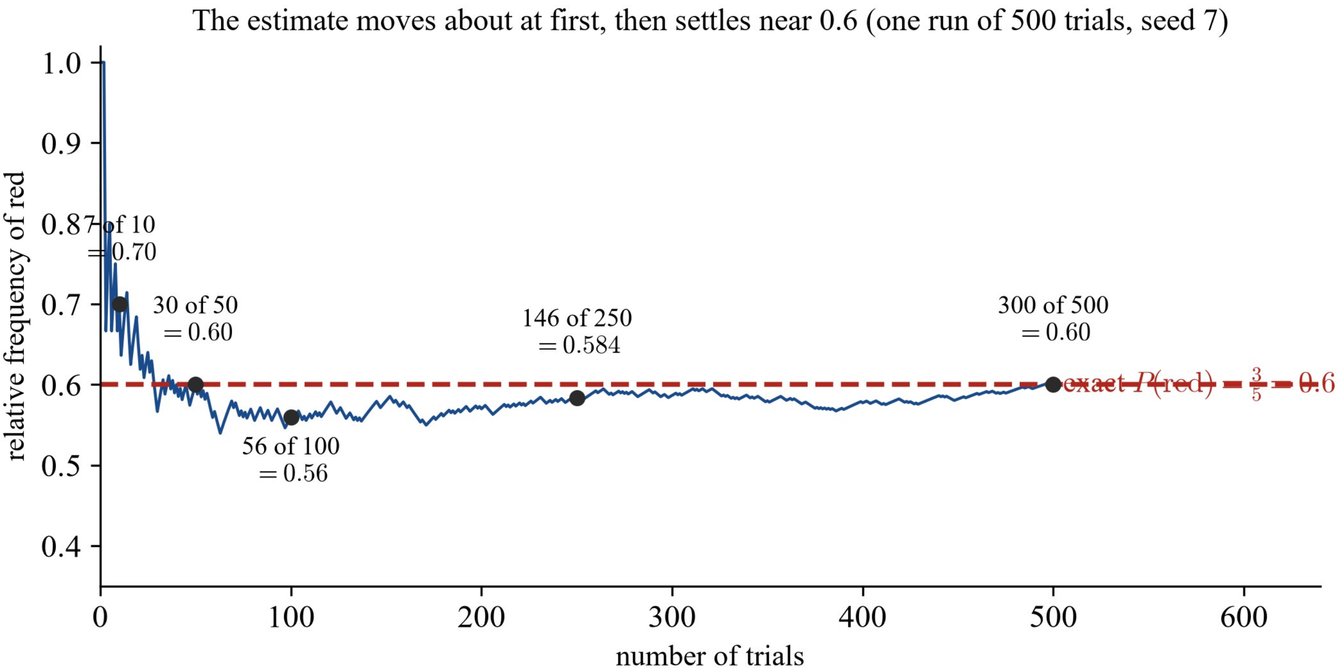
Q5. a Without replacement, after a blue the bag is 3 blue 1 green: $\frac{4}{5}\times\frac{1}{4}=\frac{1}{5}=0.2$. b Estimate $\frac{22}{100}=0.22$, within 0.02 of 0.2. c Removing a blue leaves 1 green in 4, a larger share than 1 in 5, so the second-draw green probability rises. ∴ Without replacement the event is $\frac{1}{5}$, larger than $\frac{4}{25}$ — the replacement rule is the only difference between the two experiments.

Q6. a Both days (both numbers 1 to 3): (2,2) and (3,2) → 2. b At least one day: (2,7), (8,3), (2,2), (1,7), (1,4), (3,2) → 6. c Both $\frac{2}{10}=0.2$ against model 0.09; at least one $\frac{6}{10}=0.6$ against model 0.51. ∴ With only ten weekends the estimates wander well off the model values — the runs-vary figure shows why small runs mislead.

Q7. A seed sprouting: the two outcomes are not equally likely, so counting cannot give the probability; only data can. Rain on a day: weather has no fair device; its probability is read from records, not listed. ∴ Listing needs equally-likely outcomes; events driven by the real world do not have them.

Q8. “At least one sprouts” fails only when both numbers are 5: the single pair (5,5). So 19 of 20 are favourable: $\frac{19}{20}=0.95$. Model value: $1-0.2\times0.2=1-0.04=0.96$. ∴ Estimate 0.95, within 0.01 of the model 0.96.

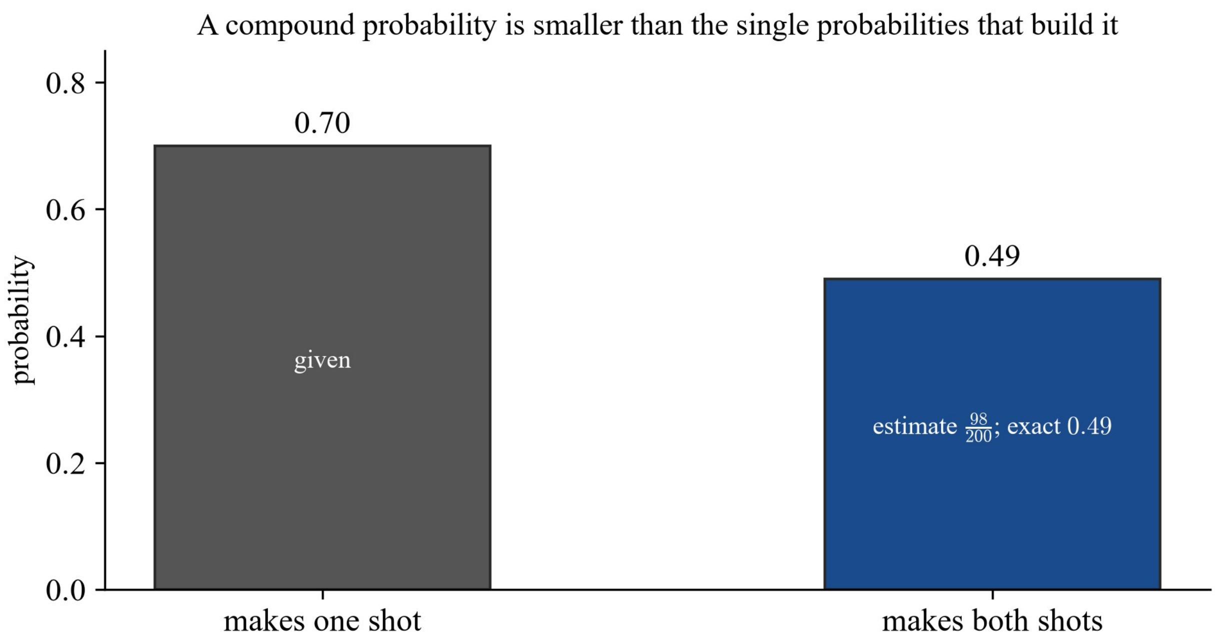
Q9. The sequence $0.57, 0.588, 0.594, 0.600$ creeps towards 0.6: the estimate wobbles with few trials and settles near the true probability with many. ∴ That settling is the long-run relative-frequency idea of 7B, produced here by a tool.



A line graph of a running relative frequency settling onto the exact value as trials accumulate

Q10. A fresh number 1 to 5 for the second counter puts the removed counter back in the bag — the model draws **with** replacement, not without. The second draw should use a random number 1 to 4 read against the four remaining counters (relabelled after the first draw). ∴ The student’s code simulates the wrong experiment; the fix is a 1-to-4 second draw over what is left.

Q11. Estimate $\frac{98}{200}=0.49$. The single-shot probability 0.7 is larger: “makes both” is $0.7\times0.7=0.49$, and multiplying by a number below 1 shrinks. ∴ A compound “and” probability is smaller than the single probabilities that build it, as the bar figure shows.



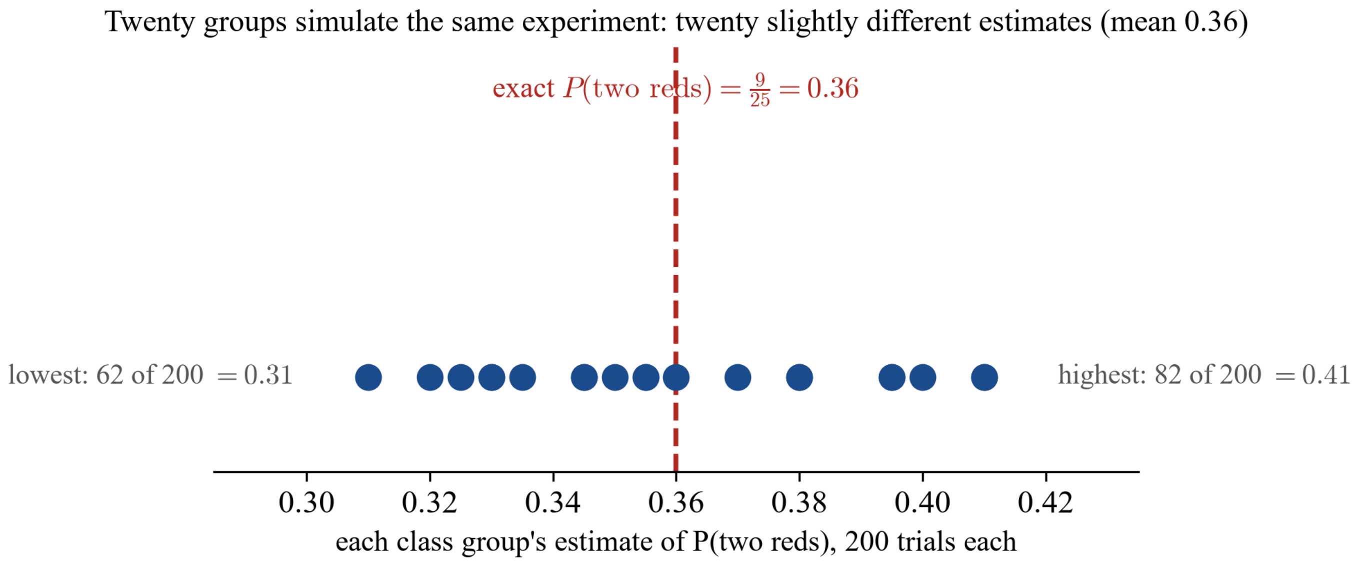
A bar chart with a 0.70 bar for one made shot beside a shorter 0.49 bar for both made shots

Q12. Estimate $\frac{183}{200} = 0.915$. Model value $1 - 0.3 \times 0.3 = 1 - 0.09 = 0.91$. $\therefore$ 0.915 sits within 0.005 of 0.91; "at least one" is the large probability, "both fails" the small one.

Q13. a $\frac{88}{400} = 0.22$. b The exact value lives in a $12 \times 8 = 96$-cell array — slow and easy to miscount — and a slightly bigger game has no array at all, so simulation is the working method. $\therefore$ Estimate 0.22, against the exact $\frac{7}{32} \approx 0.219$ the array would eventually give.

Q14. Estimate $\frac{61}{150} \approx 0.407$; exact $\frac{2}{5} = 0.4$. $\therefore$ Within 0.01: the model is faithful.

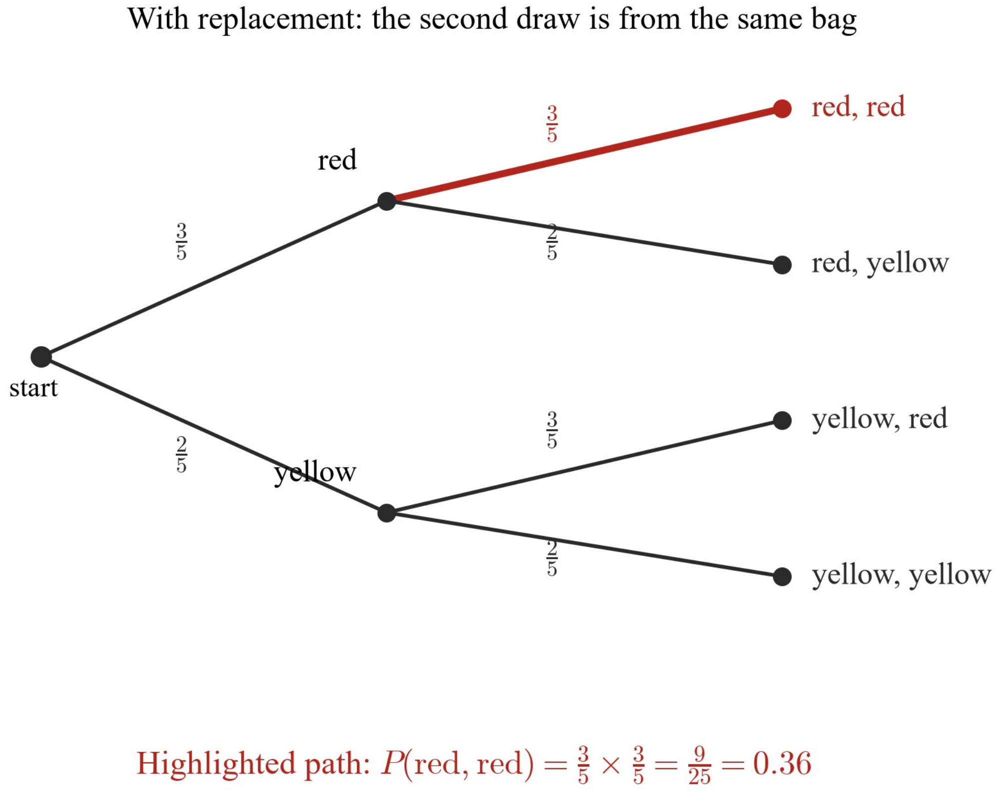
Q15. A: $\frac{69}{200} = 0.345$. B: $\frac{75}{200} = 0.375$. Neither is wrong: the two estimates sit 0.015 either side of the exact 0.36, exactly the run-to-run spread the dot-plot figure displays. $\therefore$ Different seeds give different honest estimates; agreement to within a few hundredths is what "correct" looks like.



A dot plot of twenty class estimates spread around the exact value, showing normal run-to-run spread

Q16. One trial: two random numbers 1 to 5 (the tool produces both, one per attempt). Conversion: numbers 1 to 4 (4 of the 5 mean a conversion, matching $\frac{4}{5}$). Estimate: count trials where both numbers are 1 to 4, and divide by the number of trials. ∴ That design mirrors Example 3's structure with the proportions changed; running it would settle near $\frac{4}{5}\times\frac{4}{5}=0.64$.

Q17. a With: $\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}=0.36$. Without: after a yellow, 2 of the 4 left are yellow, $\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}=0.3$. b $\frac{72}{200}=0.36$ and $\frac{60}{200}=0.3$. ∴ Both simulations land exactly on the tree values; as ever, removing the replacement rule lowers the same-colour probability.



The with-replacement tree for the two-red path, the same structure used for two yellows

Q18. The dot-plot figure shows twenty honest 200-trial runs of this same experiment spreading from 0.31 to 0.41 about the exact 0.36; the run giving $\frac{82}{200}=0.41$ is just the top dot in that spread — one estimate, not the probability. Correct wording: "this run estimates the probability at 0.41; the probability itself is about 0.36". ∴ A simulation yields an estimate that tightens with more trials; it never yields an exact value.

Test

Chapter test T7 — Chapter 7: Probability

Mathematics Level 9

This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 9 achievement standard, not against a mark on this paper.

Time	31 minutes: 5 minutes reading time, then 26 minutes working time (1 minute per mark)
Total marks	26
Part A	Technology-free — 10 marks. No calculators or digital tools: exact values and by-hand computation.
Part B	Technology-active — 16 marks. A scientific calculator is permitted throughout Part B, and Question 5 **requires access to a simulation** — a spreadsheet with a random whole number function, or a general-purpose programming language, as nominated by your teacher.
Equipment	Access to a simulation for Part B (Question 5). No CAS.

Instructions

- Answer all questions. The mark for each part is shown in brackets.
- Show all your working — working earns credit even when a final number slips.
- Probabilities may be given as fractions, decimals or percentages unless a part says otherwise. Give fractions in simplest form, and give exact fractions where a part asks for them.
- In Part A, do all working by hand. In Part B, you may use the tools described above. A computer algebra system (CAS) is not permitted anywhere.

Part A — Technology-free (10 marks)

Question 1 (7A · `VC2M9P01`) — 6 marks

SCENARIO. A bag holds 4 red counters and 3 yellow counters. Two counters are drawn at random, one after another, **without replacement** — the first counter is kept out when the second is drawn.

(a) Draw a tree diagram for the experiment. Show the probability on every branch. **[2]**

(b) Find the probability that both counters are red. Give your answer as an exact fraction in simplest form. **[1]**

(c) Find the probability that the two counters are different colours. Give your answer as an exact fraction in simplest form. **[1]**

(d) Use the complement to find the probability that at least one of the counters is yellow. **[1]**

(e) Suppose instead that the first counter is **replaced** before the second draw. State whether the probability of two reds increases, decreases or stays the same, and give the new value as an exact fraction. **[1]**

Question 2 (7B · `VC2M9P02`) — 4 marks

SCENARIO. Of the 40 students in a Year 9 group, 22 play a team sport, 25 play chess, and 12 do both. One student is chosen at random.

(a) Find the number of students who do neither activity. **[1]**

(b) Find $P(\text{the student plays a team sport and plays chess})$. Give your answer as a fraction in simplest form. **[1]**

(c) Find $P(\text{the student plays a team sport or plays chess})$ in the **inclusive** sense — at least one of the two happens, and doing both still counts. Give your answer as a fraction in simplest form. **[1]**

(d) Find $P(\text{the student does exactly one of the two activities})$ — the **exclusive** 'or'. Give your answer as a fraction in simplest form. **[1]**

Part B — Technology-active (16 marks)

A scientific calculator is permitted throughout Part B. Question 5 also requires access to a simulation — a spreadsheet with a random whole number function, or a general-purpose programming language, as nominated by your teacher. No CAS.

Question 3 (7A · `VC2M9P01`) — 4 marks

Two fair six-sided dice are rolled and the two scores recorded as an ordered pair, so there are $6\times6=36$ equally likely outcomes.

(a) The 36 outcomes of an experiment in which the same device runs twice can be displayed in a square grid, with one score labelling the rows and the other labelling the columns. Which name does 7A give to this representation? **[1]**

A. A systematic list B. A tree diagram C. A two-way table D. An array

(b) Find the probability that the product of the two scores is even. **[1]**

(c) Find the probability that the difference between the two scores is exactly 3 — so $(1,4)$ counts, and $(4,1)$ counts. **[1]**

(d) Find the probability that the sum of the two scores is at least 11. **[1]**

Question 4 (7B · `VC2M9P02`) — 6 marks

The table shows the 31 days of July 2026 at Ballarat Aerodrome, Victoria. A day is called **rainy** when the rainfall is at least 1 mm, and **windy** when the strongest gust (the fastest burst of wind recorded that day) is at least 50 km/h. One of the 31 days is chosen at random.

Source: Bureau of Meteorology, Daily Weather Observations for Ballarat, Victoria, July 2026, 2026. Retrieved 20 August 2026.

	Windy day	Not a windy day	Total
Rainy day	8	5	13
Not a rainy day	3	?	18
Total	11	20	31

(a) Find the missing number in the table. **[1]**

(b) Find $P(\text{the day is rainy and windy})$. **[1]**

(c) Find $P(\text{the day is rainy or windy})$ in the inclusive sense. Show why adding $P(\text{rainy})+P(\text{windy})$ counts some days twice, and correct it. **[2]**

(d) A school term runs for 93 days. Use the relative frequency of rainy days in the table to predict the number of rainy days in the term. **[2]**

Question 5 (7C · `VC2M9P03`) — 6 marks

DIGITAL TOOL — this question assumes a spreadsheet or a general-purpose programming language. Your teacher will nominate the tool.

SCENARIO. A soccer player's record shows that she scores 6 of every 10 penalty kicks. She takes two penalty kicks. There is no fair device behind "she scores", so the probability cannot be determined exactly — it must be estimated by simulation.

(a) To simulate one kick, the tool produces a random whole number from 1 to 10. State which numbers should stand for a goal and which should stand for a miss, and say why this mapping matches the player's record. **[1]**

(b) One trial is one pair of kicks, so each trial uses two random numbers. Run your simulation for 50 trials and record the number of trials in which both kicks were scored. **[1]**

(c) From your run, find the relative frequency of "both kicks scored". **[1]**

(d) If each kick keeps the same 6-in-10 chance, the model value of "both kicks scored" is found by multiplying along a two-step tree. Find the model value. How does your estimate from part (c) compare with it? **[1]**

(e) Another group runs the same simulation for 500 trials and records 171 trials in which both kicks were scored — a relative frequency of $171 \div 500 = 0.342$.

i. Which estimate is better — the other group's 500-trial estimate or your 50-trial estimate? Give a reason. **[1]**

ii. Even the 500-trial result is an estimate, not an exact probability. Give one reason why. **[1]**

End of test — 26 marks

Solutions

Answer key

Final values only. Fully worked solutions follow.

Question	Answer	Marks
1a	Tree: first draw $P(R)=\frac{4}{7}$, $P(Y)=\frac{3}{7}$; after a red $P(R)=\frac{3}{6}$, $P(Y)=\frac{3}{6}$; after a yellow $P(R)=\frac{4}{6}$, $P(Y)=\frac{2}{6}$	2
1b	$\frac{2}{7}$	1
1c	$\frac{4}{7}$	1
1d	$\frac{5}{7}$	1
1e	Increases; $\frac{16}{49}$	1
2a	5 students	1
2b	$\frac{3}{10}$	1
2c	$\frac{7}{8}$	1
2d	$\frac{23}{40}$	1
3a	D — an array	1
3b	$\frac{3}{4}$	1
3c	$\frac{1}{6}$	1
3d	$\frac{1}{12}$	1
4a	15	1
4b	$\frac{8}{31}$	1
4c	$\frac{16}{31}$	2
4d	39 rainy days	2
5a	1–6 stand for a goal, 7–10 for a miss; 6 of the 10 numbers are goals, matching 6 in 10	1
5b	Answers will vary (student's own run)	1
5c	Answers will vary (follow-	1

Question	Answer	Marks
	through from 5b)	
5d	0.36; the estimate sits near it	1
5e i	The 500-trial estimate — more trials, so the relative frequency has settled closer	1
5e ii	A simulation yields an estimate that varies from run to run; also 0.6 itself came from data	1
Total		**26**

Fully worked solutions

Question 1 (6 marks)

(a) First draw: 4 of the 7 counters are red and 3 are yellow, so $P(R)=\frac{4}{7}$ and $P(Y)=\frac{3}{7}$. Without replacement, the bag has changed by the second draw:

- after a red, 3 red and 3 yellow remain: $P(R)=\frac{3}{6}$, $P(Y)=\frac{3}{6}$;
- after a yellow, 4 red and 2 yellow remain: $P(R)=\frac{4}{6}$, $P(Y)=\frac{2}{6}$.

The tree (R = red, Y = yellow):

```
                  ┌─ R  3/6   →  RR
       ┌─ R 4/7 ─┤
       │          └─ Y  3/6   →  RY
start ─┤
       │          ┌─ R  4/6   →  YR
       └─ Y 3/7 ─┤
                  └─ Y  2/6   →  YY
```

One mark for the correct first-draw branches, one mark for the correct second-draw branches (the changed bag). **(2 marks)**

(b) Both red is the RR path — multiply along the branches:

$$P(\text{two reds})=\frac{4}{7}\times\frac{3}{6}=\frac{12}{42}=\frac{2}{7}$$

$\therefore \frac{2}{7}$. **(1 mark)**

(c) Different colours is the two middle paths, RY and YR:

$P(RY)=\frac{4}{7}\times\frac{3}{6}=\frac{2}{7}$ and $P(YR)=\frac{3}{7}\times\frac{4}{6}=\frac{2}{7}$, so $P(\text{different})=\frac{2}{7}+\frac{2}{7}=\frac{4}{7}$.

$\therefore \frac{4}{7}$. Check: $\frac{2}{7}+\frac{4}{7}+P(YY)=\frac{2}{7}+\frac{4}{7}+\frac{1}{7}=1$ ✓ **(1 mark)**

(d) “At least one yellow” fails only when both counters are red, so by the Level 8 complement rule:

$$P(\text{at least one yellow})=1-P(\text{two reds})=1-\frac{2}{7}=\frac{5}{7}$$

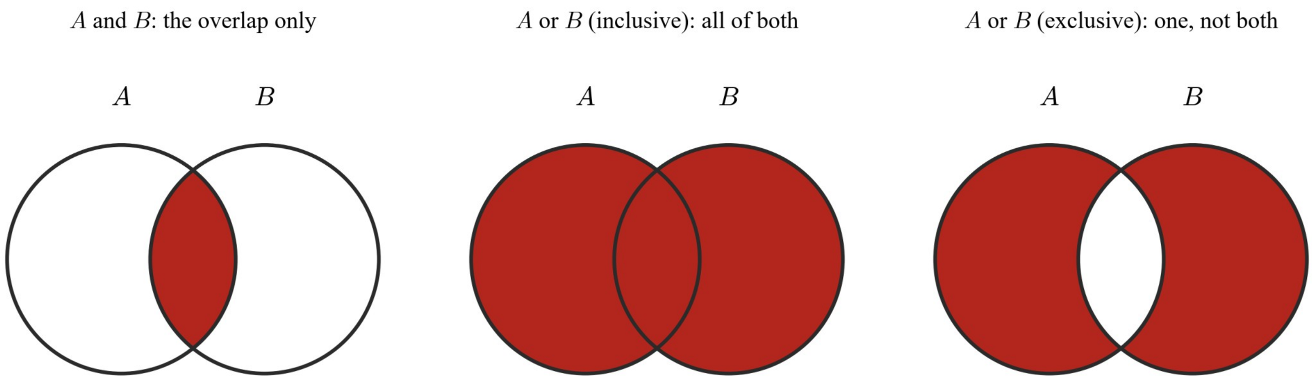
$\therefore \frac{5}{7}$. **(1 mark)**

(e) With replacement, the second draw sees the full bag again, so both draws carry $P(R) = \frac{4}{7}$:

$P(\text{two reds}) = \frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$

Since $\frac{16}{49} > \frac{14}{49} = \frac{2}{7}$, the probability **increases**. Putting the red counter back keeps all four reds available for the second draw. **(1 mark)**

Question 2 (4 marks)



Three Venn panels: A and B shading only the overlap; A or B inclusive shading all of both circles; A or B exclusive shading both circles except the overlap

The three parts (b), (c) and (d) shade the Venn diagram exactly as the three panels above do: the overlap, all of both circles, and the two ‘only’ regions.

(a) Students doing at least one activity: $22 + 25 - 12 = 35$ — the 12 who do both are in both counts, so they are subtracted once. Neither:

$40 - 35 = 5$ students. **(1 mark)**

(b) 12 of the 40 students are in the overlap:

$P(\text{sport and chess}) = \frac{12}{40} = \frac{3}{10}$. **(1 mark)**

(c) Inclusive ‘or’ counts everyone in at least one circle. Plain adding counts the 12 who do both twice, so subtract the overlap once:

$P(\text{sport or chess}) = \frac{22}{40} + \frac{25}{40} - \frac{12}{40} = \frac{35}{40} = \frac{7}{8}$.

Check from the regions: sport only $22 - 12 = 10$, both 12, chess only $25 - 12 = 13$, and $10 + 12 + 13 = 35$ ✓ **(1 mark)**

(d) Exclusive ‘or’ counts exactly one activity — the two ‘only’ regions, with the overlap removed a second time:

$P(\text{exactly one}) = \frac{10 + 13}{40} = \frac{23}{40}$,

equivalently $\frac{22}{40} + \frac{25}{40} - 2 \times \frac{12}{40} = \frac{23}{40}$. **(1 mark)**

Question 3 (4 marks)

(a) **D — an array.** In 7A, the array is the square grid of all ordered pairs when the same device runs twice — two dice here. A two-way table is used when the two steps are different devices, a tree diagram shows steps one after another, and a systematic list is a written-out row of outcomes, not a grid. **(1 mark)**

(b) The product is odd only when **both** scores are odd — 3 odd scores out of 6 on each die, giving $3 \times 3 = 9$ odd-product outcomes. So the even products number $36 - 9 = 27$:

$P(\text{product even}) = \frac{27}{36} = \frac{3}{4}$. **(1 mark)**

(c) Difference exactly 3: $(1,4),(2,5),(3,6)$ and the three reversals $(4,1),(5,2),(6,3)$ — 6 outcomes of 36:

$P(\text{difference}\,3) = \frac{6}{36} = \frac{1}{6}$. **(1 mark)**

(d) Sum at least 11: sum 11 comes from $(5,6),(6,5)$ and sum 12 from $(6,6)$ — 3 outcomes of 36:

$P(\text{sum} \geq 11) = \frac{3}{36} = \frac{1}{12}$. **(1 mark)**

Question 4 (6 marks)

(a) The ‘not a rainy day’ row totals 18 and already shows 3 windy days, so the missing cell is

$18 - 3 = 15$

(check: $8 + 5 + 3 + 15 = 31$ ✓). **(1 mark)**

(b) ‘And’ is the one shared cell — rainy **and** windy days number 8 of the 31:

$P(\text{rainy and windy}) = \frac{8}{31}$. **(1 mark)**

(c) $P(\text{rainy}) = \frac{13}{31}$ and $P(\text{windy}) = \frac{11}{31}$. Adding them gives $\frac{24}{31}$, but the 8 days that are both rainy and windy sit inside **both** counts, so they have been counted twice. Subtract the overlap once:

$P(\text{rainy or windy}) = \frac{13}{31} + \frac{11}{31} - \frac{8}{31} = \frac{16}{31}$.

Check from the three shaded regions: rainy only 5, both 8, windy only 3 — $5 + 8 + 3 = 16$ ✓

One mark for identifying the double count, one mark for $\frac{16}{31}$. **(2 marks)**

(d) The relative frequency of a rainy day in the data is

$\text{rf}(\text{rainy}) = \frac{13}{31}$,

and a prediction is the estimate multiplied by the number of future days:

$93 \times \frac{13}{31} = 3 \times 13 = 39$.

∴ About **39 rainy days** in a 93-day term. One mark for the relative frequency, one mark for the prediction. **(2 marks)**

Question 5 (6 marks)

(a) Let the numbers **1 to 6 stand for a goal** and **7 to 10 stand for a miss**. Six of the ten possible numbers mean a goal, matching the player's 6-in-10 record, so the model behaves like the player. **(1 mark)**

(b) Answers will vary from run to run — that is expected, because a simulation uses random numbers. Award the mark for any count recorded from the student's own 50-trial run. **(1 mark)**

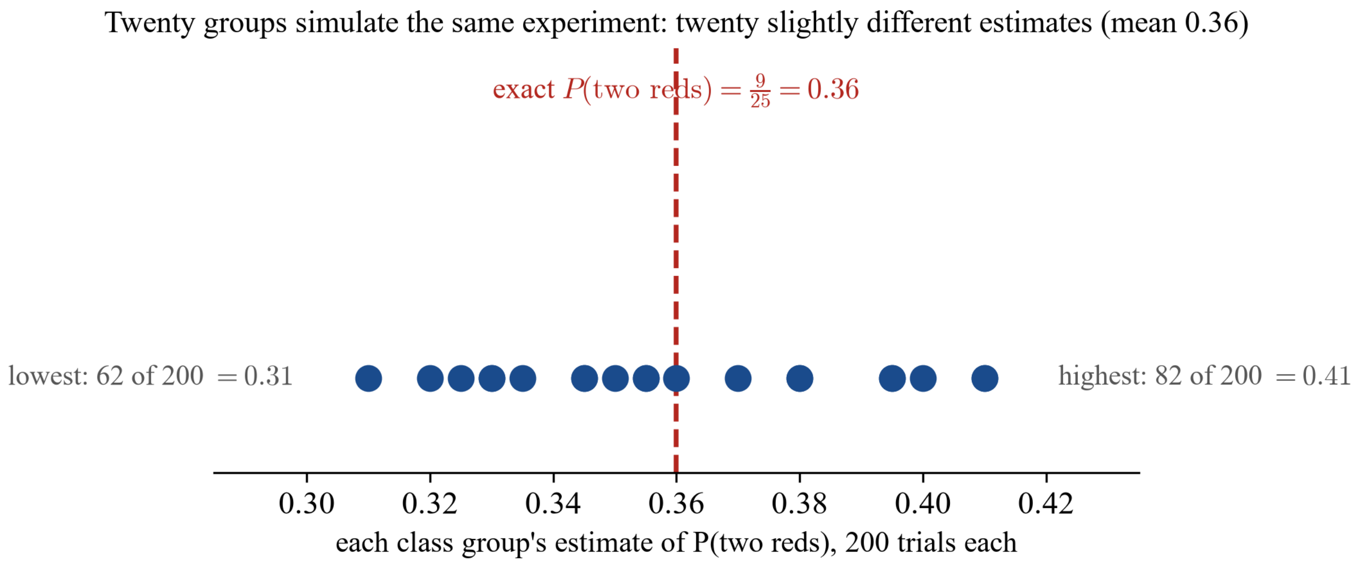
(c) Relative frequency = favourable trials ÷ number of trials, using the student's own count from part (b). Award the mark for a correct division by 50. **(1 mark)**

(d) Multiply along the two-step tree: $0.6 \times 0.6 = 0.36$. The student's estimate from part (c) should sit somewhere near 0.36 — with only 50 trials it need not be close, as the runs-vary figure below shows. One mark for 0.36 with a comparison. **(1 mark)**

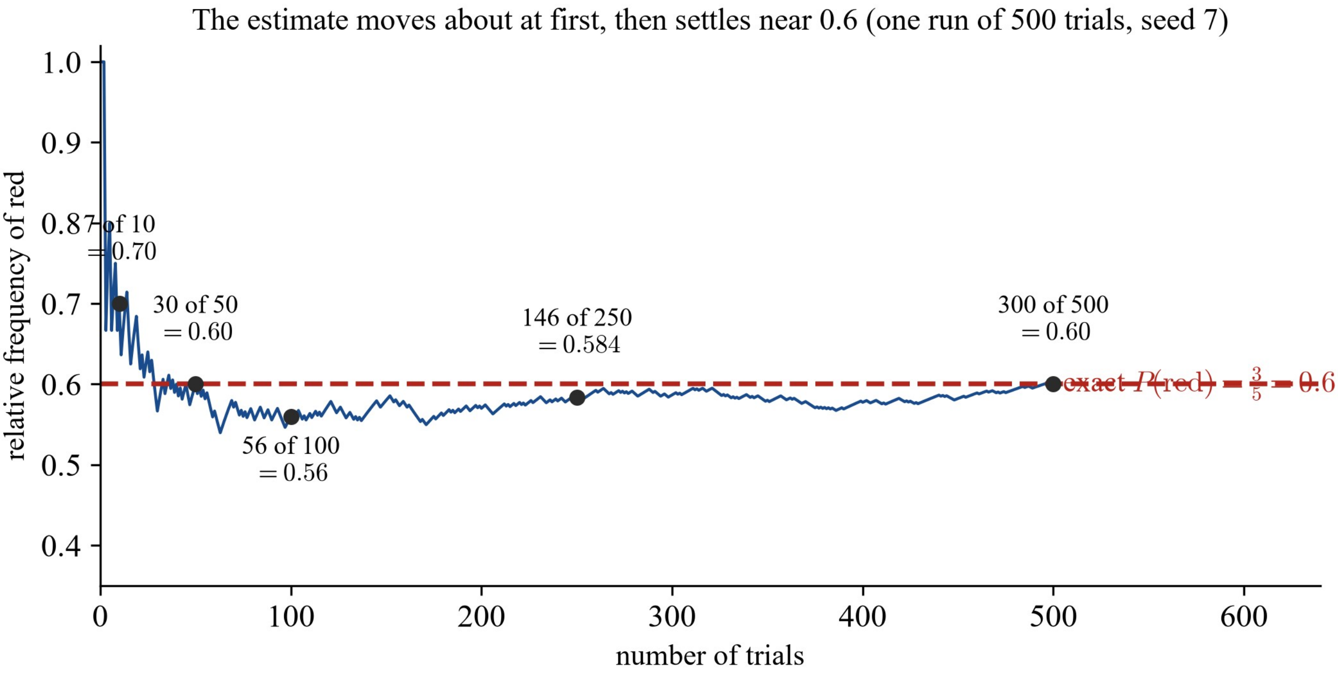
(e)

i. The **500-trial estimate** is better: running relative frequencies jump about with few trials and settle closer to the probability as the number of trials grows, so 500 trials gives a steadier estimate than 50. **(1 mark)**

ii. Any one correct reason earns the mark: a simulation produces an **estimate** that varies from run to run, never an exact value — the dot plot below shows twenty honest runs of one 200-trial simulation spread from 0.31 to 0.41; or the 0.6 itself came from the player's recorded data, so the true probability of "both scored" cannot be determined exactly in the first place. **(1 mark)**



A dot plot of twenty class estimates of P(two reds) from 200 trials each, dots spread from 0.31 to 0.41 around a dashed red line at the exact value 0.36 — the same model value as "both kicks scored" in Question 5, and the spread is the point: honest runs of the same simulation land at different values



A line graph of the running relative frequency of red across 500 trials, jumping about early then settling, with a dashed red line at the exact value 0.6 — the settling shown here is why Question 5e trusts the 500-trial estimate