Mathematics Level 9

Chapter 5 — Measurement · 5 subtopics

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Contents

Section	Page
Volume and surface area of right prisms, cylinders and composite objects	2
Scientific notation: very large and very small measurements, timescales and intervals	19
Spatial problems: angle properties, scale, similarity, ratio, Pythagoras' theorem and trigonometry	36
Absolute, relative and percentage error in measurements	52
Modelling with direct proportion, rates, ratio and scale, including financial contexts	66
Test	79

Volume and surface area of right prisms, cylinders and composite objects

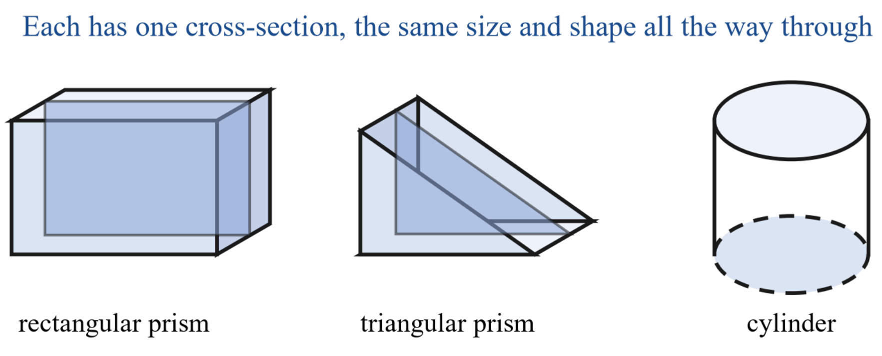
5A · Chapter 5 — Measurement · Level 9

> **Content description `VC2M9M01` (word for word):** *solve problems involving the volume and surface area of right prisms, cylinders and composite objects using appropriate units*
>
> **Achievement standard anchor (AS 8):** *Students apply formulas to solve problems involving the surface area and volume of right prisms, cylinders and composite shapes.*
>
> **Elaboration ids for linkage:** `VC2M9M01.E01`, `VC2M9M01.E02`, `VC2M9M01.E03`, `VC2M9M01.E04`
>
> **Prerequisite pointer:** Level 8 measurement — area and perimeter of irregular and composite shapes (`VC2M8M01`), volume and capacity of right prisms (`VC2M8M02`), and circumference and area of a circle (`VC2M8M03`). All three are reviewed in the opening theory and then used as tools; they are not re-taught.
>
> **Note on `VC2M9M01.E04` (exclusion recorded):** the elaboration "investigating objects and technologies of Aboriginal and Torres Strait Islander Peoples" is excluded from this book under CONTENT_POLICY Rule 1 (D10). The content description is taught in full through `E01`–`E03`, which carry the net-to-formula work, the material-planning task and the same-volume–different-surface-area conjecturing.
>
> **Note on the conjecturing limb (`VC2M9M01.E03`):** finding prisms with the same volume but different surface areas, and conjecturing which shape has the smallest surface area, is taught and assessed here (Chapter 5 note in the TOC) — it is the mathematics behind the surface-area-to-volume argument used later in science, so it is done in full, not mentioned in passing.
>
> **Note on sources:** every quantitative context in this section is a clearly-labelled SCENARIO with invented numbers (R2). No real data ships and therefore no source line is owed (R1).

Theory

Right prisms, cylinders and their cross-sections

A **right prism** is a solid with one cross-section, the same size and shape, all the way through, and with sides that stand at right angles to that cross-section. A **cylinder** behaves like a prism whose cross-section is a circle. Slice any of them parallel to the ends and every slice is the same shape — that one repeated shape is the **cross-section**.

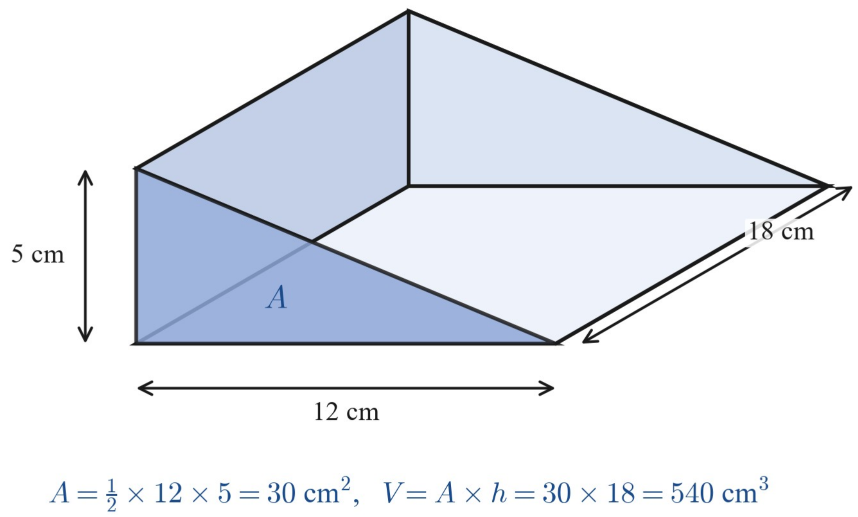


A rectangular prism, a triangular prism and a cylinder in a row, each with a shaded slice parallel to its ends showing one cross-section the same size and shape all the way through

Because the solid is a stack of identical slices, its **volume** is the area of the cross-section multiplied by how long the solid is:

$V = A \times h$, where A is the area of the cross-section and h is the length (the height the solid is stacked to).

For a cuboid this is the familiar $V = l \times w \times h$; for a cylinder the cross-section is a circle of area πr^2, so $V = \pi r^2 h$.

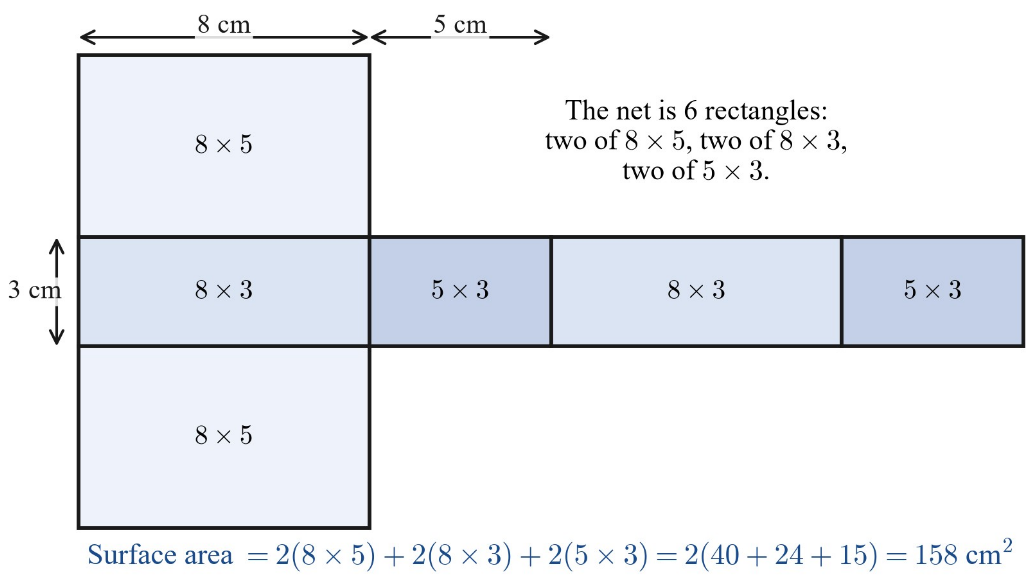


A triangular prism lying on a rectangular face, with the right-triangle end shaded and labelled A, base 12 cm, height 5 cm, and length 18 cm; below, A = 1/2 × 12 × 5 = 30 cm² and V = A × h = 30 × 18 = 540 cm³

The figure shows the idea on a triangular prism: the cross-section has area $A=\frac{1}{2}\times 12\times 5=30$ cm², and the length is 18 cm, so $V=30\times 18=540$ cm³. The same two steps — area of cross-section, then multiply by length — work for every prism in this subtopic.

Surface area from nets (`VC2M9M01.E01`)

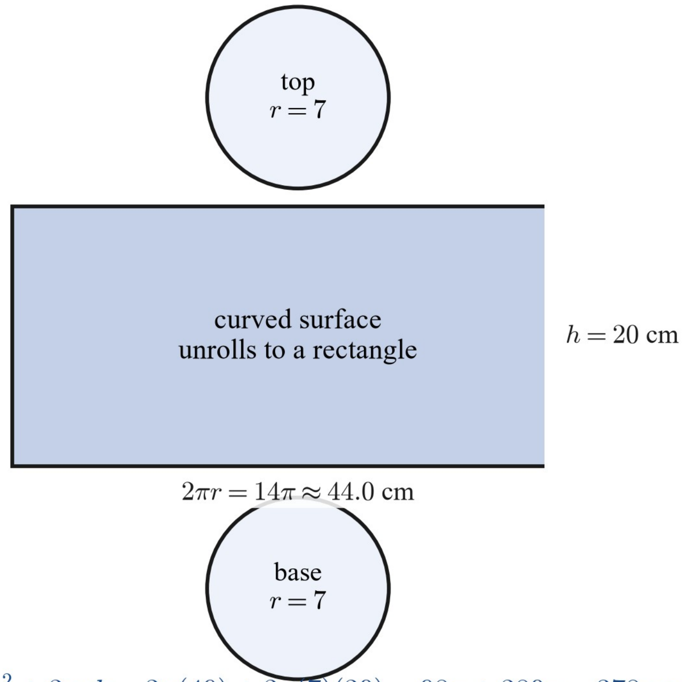
The **surface area** of a solid is the total area of its outside faces. The reliable way to see every face exactly once is to unfold the solid into its **net**. Unfolding a cuboid gives six rectangles in three matching pairs, so the surface area is a sum of three doubled rectangles.



The net of an 8 cm by 5 cm by 3 cm cuboid: a row of four rectangles labelled 8×3, 5×3, 8×3, 5×3 with an 8×5 rectangle above and another below the first; outside dimensions 8 cm, 5 cm and 3 cm marked; below, surface area = 2(8×5) + 2(8×3) + 2(5×3) = 2(40+24+15) = 158 cm²

Reading the net: two faces of 8×5, two of 8×3 and two of 5×3, so the surface area is $2(40+24+15)=158$ cm². Doing the same unfolding with labels l, w, h instead of numbers gives the cuboid formula $SA=2(lw+lh+wh)$ — the net *establishes* the formula rather than the formula being handed down (`E01`).

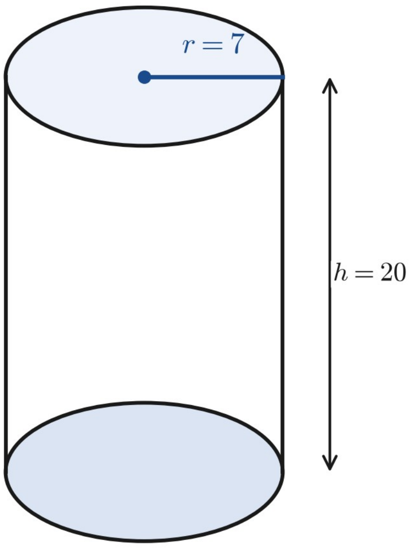
A cylinder unfolds to two circles and one rectangle: the rectangle is the curved surface unrolled, its width equal to the circumference $2\pi r$ and its height equal to h.



Surface area $= 2\pi r^2 + 2\pi rh = 2\pi(49) + 2\pi(7)(20) = 98\pi + 280\pi = 378\pi \approx 1187.5\ \text{cm}^2$

The net of a cylinder of radius 7 cm and height 20 cm: a circle labelled top above a rectangle labelled curved surface unrolls to a rectangle, and a circle labelled base below it; the rectangle is marked 2πr = 14π ≈ 44.0 cm wide and h = 20 cm high; below, surface area = 2πr² + 2πrh = 98π + 280π = 378π ≈ 1187.5 cm²

So the cylinder's surface area is the two circles plus the rectangle: $SA = 2\pi r^2 + 2\pi r h$. For $r = 7$, $h = 20$: $2\pi(49) + 2\pi(7)(20) = 98\pi + 280\pi = 378\pi \approx 1187.5$ cm².



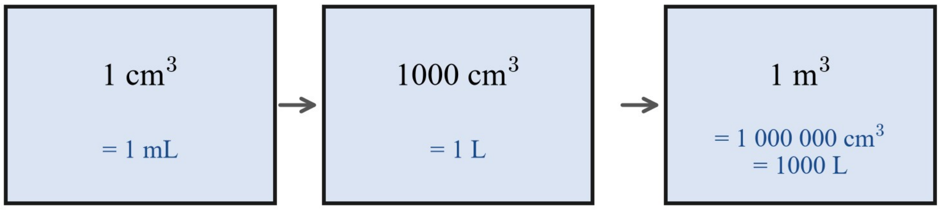
$V = \pi r^2 h = \pi \times 49 \times 20 = 980\pi \approx 3078.8\ \text{cm}^3$

A standing cylinder with a dot at the centre of the top circle, a radius marked r = 7 and a height marked h = 20; below, V = πr²h = π × 49 × 20 = 980π ≈ 3078.8 cm³

and its volume, from the circular cross-section, is $V = \pi r^2 h = \pi \times 49 \times 20 = 980\pi \approx 3078.8$ cm³. Keep answers in exact π form and round once at the end.

Units of volume and capacity

Volume is measured in cubic units and capacity in litres and millilitres, and the two scales lock together: a 1 cm cube holds exactly 1 mL.



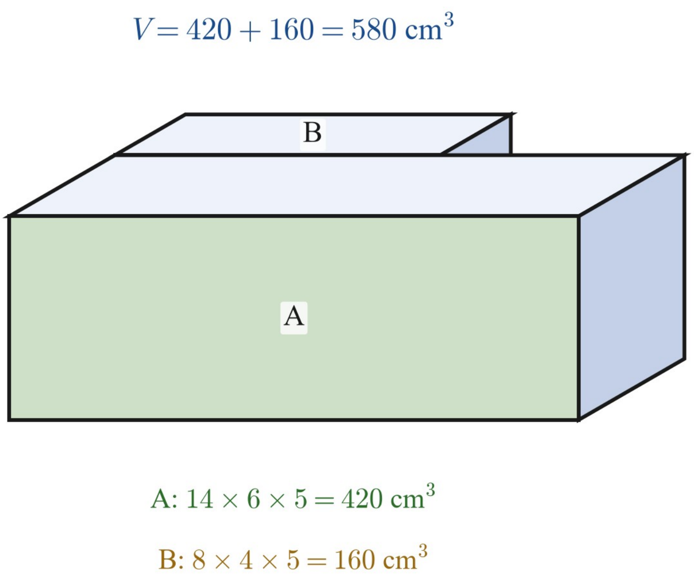
Volume is measured in cubic units; capacity in litres and millilitres.

Three boxes joined by arrows: 1 cm³ = 1 mL, then 1000 cm³ = 1 L, then 1 m³ = 1 000 000 cm³ = 1000 L; caption: volume is measured in cubic units, capacity in litres and millilitres

The three conversions worth knowing by heart are 1 cm^3 = 1 mL, 1000 cm^3 = 1 L, and 1 m^3 = 1000 L. The last one surprises people: a cubic metre is a thousand litres, not a hundred.

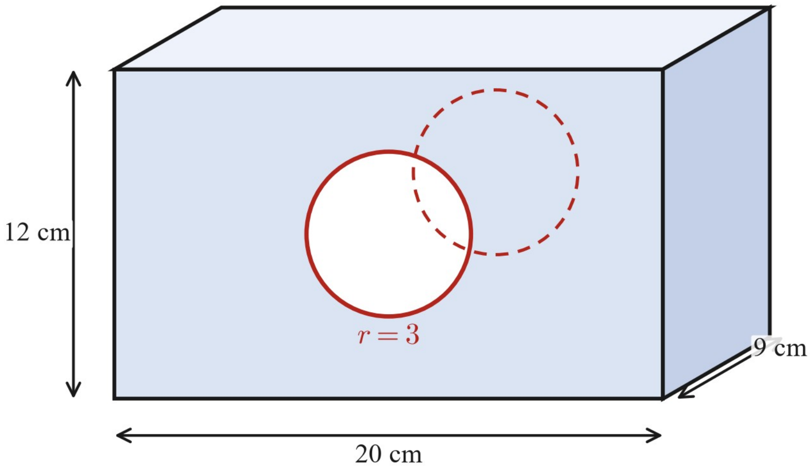
Composite objects

A **composite object** is a solid built from simpler solids. Its volume is found by **adding** the pieces, or by **subtracting** a hole or cut-out from a bigger solid — either route gives the answer, and a good habit is to check one with the other.



An L-shaped block drawn as two cuboids, the front one shaded green and labelled A with 14 × 6 × 5 = 420 cm³ and the back one shaded orange and labelled B with 8 × 4 × 5 = 160 cm³; above, V = 420 + 160 = 580 cm³

Adding: the L-block splits into cuboid A, $14 \times 6 \times 5 = 420$ cm^3, and cuboid B, $8 \times 4 \times 5 = 160$ cm^3, giving $V = 580$ cm^3. (Check by subtracting: the enclosing $14 \times 10 \times 5$ block is 700 cm^3, and the missing corner is $6 \times 4 \times 5 = 120$ cm^3; $700 - 120 = 580$ cm^3. ✓)

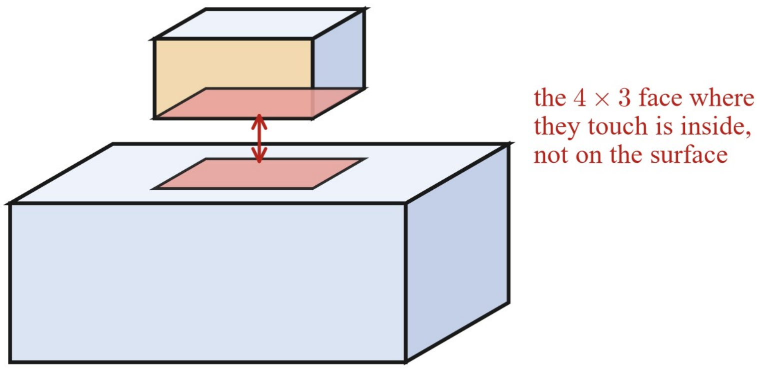


$V = 20 \times 12 \times 9 - \pi \times 3^2 \times 9 = 2160 - 81\pi \approx 1905.5 \text{ cm}^3$

A 20 cm by 12 cm by 9 cm cuboid with a circular hole of radius 3 cm cut all the way through the 9 cm depth, the front rim solid red and the back rim dashed red; below, V = 20 × 12 × 9 − π × 3² × 9 = 2160 − 81π ≈ 1905.5 cm³

Subtracting: a block with a cylindrical hole keeps the block's volume minus the cylinder's: $2160 - 81\pi \approx 1905.5$ cm³.

Surface area needs one extra idea: faces that are glued together or pressed against each other are **inside** the solid, not on its surface, and must not be counted.



$SA = SA_A + SA_B - 2 \times (4 \times 3) = 248 + 52 - 24 = 276 \text{ cm}^2$

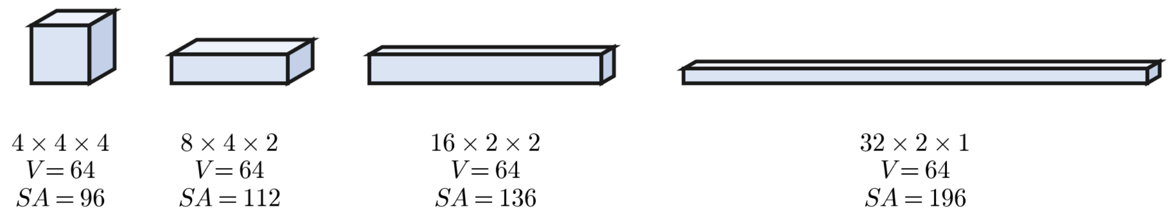
A small 4 by 3 by 2 cuboid lifted just above the centre of a 10 by 6 by 4 cuboid; the 4 by 3 face on the underside of the small cuboid and the matching patch on the top of the large one are both shaded red and joined by a double arrow; label: the 4 × 3 face where they touch is inside, not on the surface; below, SA = 248 + 52 − 2 × (4 × 3) = 276 cm²

Stacking the $4 \times 3 \times 2$ block on the $10 \times 6 \times 4$ block hides the 4×3 contact face **twice** — once from each block — so the composite surface area is $SA_A + SA_B - 2 \times (4 \times 3) = 248 + 52 - 24 = 276$ cm².

Same volume, different surface area (`VC2M9M01.E03`)

Volume and surface area do not travel together. Four cuboids can hold exactly the same amount and still show very different areas of outside.

Same volume (64 cm^3), but the long thin prism has more than twice the surface area of the cube



Four cuboids drawn to scale in a row, all with V = 64 cm³: the cube 4 × 4 × 4 with SA = 96, then 8 × 4 × 2 with SA = 112, then 16 × 2 × 2 with SA = 136, then the long flat 32 × 2 × 1 with SA = 196; caption: same volume, but the long thin prism has more than twice the surface area of the cube

All four solids have $V = 64$ cm³, yet the surface areas run $96, 112, 136, 196$ cm². Stretching the same volume into a long thin shape nearly doubles the surface it shows. This is the start of a proper conjecture: **for a fixed volume, the cube has the smallest surface area** among cuboids — and the matching habit is to compare solids by the **ratio** of surface area to volume, $SA : V$, which here falls from $196 \div 64 \approx 3.06$ for the flat slab to $96 \div 64 = 1.5$ for the cube. Ratios like these are exactly why the size and shape of an object decides how fast it heats, cools, dries or absorbs — the same-volume–different-surface-area fact is doing real work, not just arithmetic.

Worked examples

Example 1 — scaffolded

Find the volume of a triangular prism whose cross-section is a right triangle with base 10 cm and height 6 cm, and whose length is 14 cm.

Working	Comment
$A = \frac{1}{2} \times 10 \times 6 = 30$ cm²	Area of the cross-section first.
$V = A \times h = 30 \times 14$	Then multiply by the length of the prism.
$V = 420$ cm³	One rounding, at the end — none needed here.

The two-step pattern (area of cross-section, then × length) is the whole method; every prism volume in this subtopic is these two steps.

Example 2 — scaffolded

A cylinder has radius 7 cm and height 20 cm. Find its volume and its surface area, exact in π and to one decimal place.

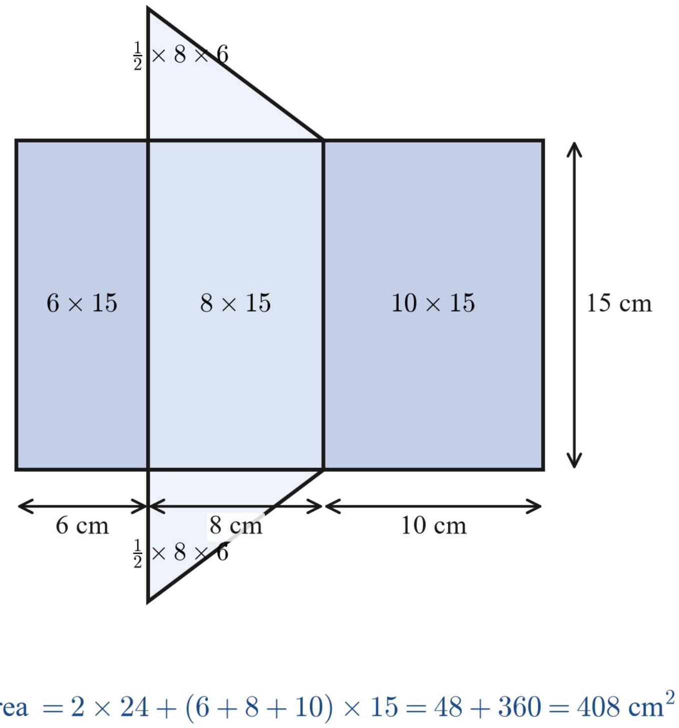
Working	Comment
$V = \pi r^2 h = \pi \times 49 \times 20 = 980\pi$	Circular cross-section, area πr^2, times the height.
$V = 980\pi \approx 3078.8$ cm³	Round once, at the end.
$SA = 2\pi r^2 + 2\pi r h$	Two circles plus the unrolled curved surface (the net).
$= 2\pi(49) + 2\pi(7)(20) = 98\pi + 280\pi$	Ends first, then the curved rectangle.
$= 378\pi \approx 1187.5$ cm²	Exact form kept until the last line.

The net is the memory aid: whatever you forget about formulas, the picture of two circles and a $2\pi r \times h$ rectangle rebuilds them.

Example 3 — scaffolded

A triangular prism has right-triangle ends with sides 6 cm, 8 cm and 10 cm, and length 15 cm. Use its net to find the surface area, and find the volume.

Working	Comment
Ends: $2\times\left(\frac{1}{2}\times 8\times 6\right)=2\times 24=48$ cm²	The two triangular cross-sections.
Sides: $\left(6+8+10\right)\times 15=24\times 15=360$ cm²	The three rectangles: perimeter of the end × length.
$S\,A=48+360=408$ cm²	The net counts every face exactly once (`E01`).
$V=24\times 15=360$ cm³	Cross-section area × length, as always.



The net of the triangular prism: three rectangles in a row labelled 6×15, 8×15 and 10×15 with a right triangle above and below the middle rectangle, each 1/2 × 8 × 6; outside dimensions 6 cm, 8 cm, 10 cm and 15 cm marked; below, surface area = 2 × 24 + (6+8+10) × 15 = 48 + 360 = 408 cm²

Notice the side-area short cut: the three rectangles together are one big rectangle, perimeter of the end × length. That short cut, read off the net, is what `E01` means by establishing the formula.

Example 4 — scaffolded

SCENARIO — a class fundraising project (`VC2M9M01.E02`). A class is making 30 can-coolers, each an open-topped cylinder of radius 4 cm and height 12 cm, from foam sheet sold by the square metre. Find the foam needed for one cooler, then for all 30, in m² to two decimal places.

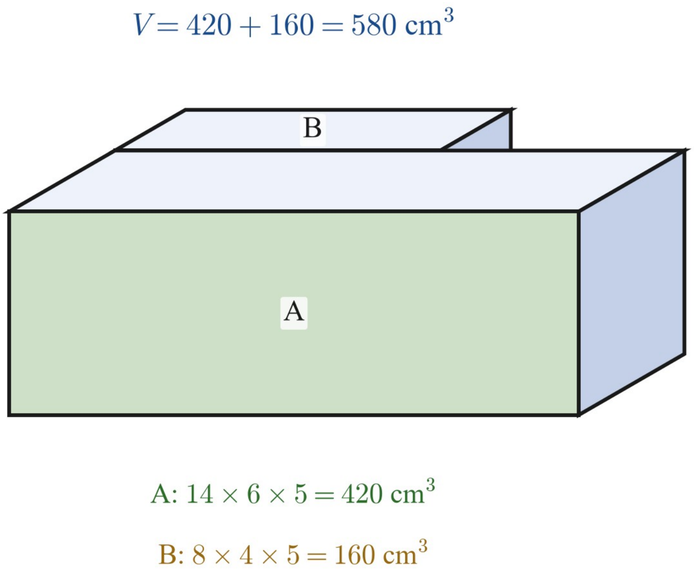
Working	Comment
One cooler: curved surface $2\pi r\,h=2\pi\left(4\right)\left(12\right)=96\pi$	No top — a can-cooler is open at the top.
plus base $\pi r^{2}=\pi\left(16\right)=16\pi$	One circle only.
$=96\pi+16\pi=112\pi\approx 351.9$ cm²	Foam per cooler.
Thirty: $30\times 112\pi=3360\pi\approx 10\,555.8$ cm²	Multiply before converting or rounding.
$=3360\pi\div 10\,000\approx 1.06$ m²	1 m² = 10 000 cm².

The class should order about 1.06 m² of foam before worrying about waste; laying the circles and rectangles onto the roll with as little waste as possible is the cost-efficient cutting step of `E02`, and the number above is what it must beat.

Example 5 — unscaffolded

Find the volume of the L-block: an enclosing cuboid 14 cm by 10 cm by 5 cm with a 6 cm by 4 cm corner notch cut out of the top view, full height.

Split the block into cuboid A ($14 \times 6 \times 5$) at the front and cuboid B ($8 \times 4 \times 5$) behind it. $V_A = 14 \times 6 \times 5 = 420$ cm³. $V_B = 8 \times 4 \times 5 = 160$ cm³. $V = 420 + 160 = 580$ cm³. Check by subtraction: $14 \times 10 \times 5 = 700$ cm³ enclosing, notch $6 \times 4 \times 5 = 120$ cm³, and $700 - 120 = 580$ cm³. ✓ ∴ $V = 580$ cm³, and the two routes agree.

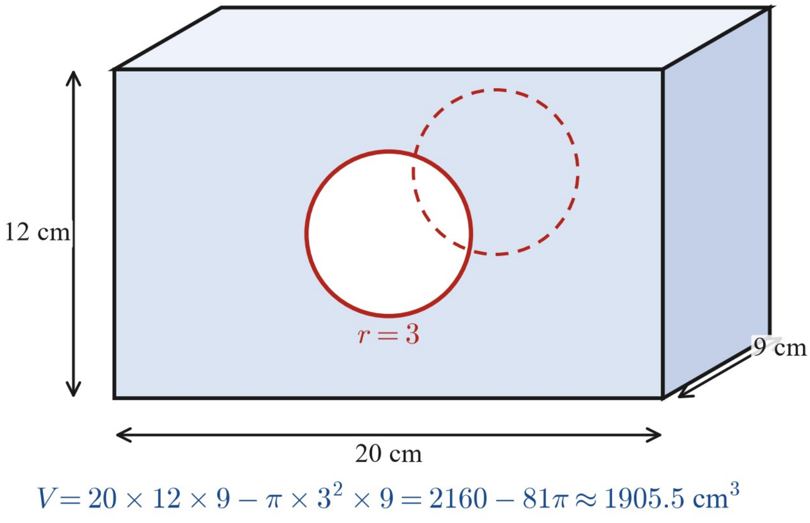


The L-block as two cuboids, A shaded green (14 × 6 × 5 = 420 cm³) and B shaded orange behind it (8 × 4 × 5 = 160 cm³), with V = 420 + 160 = 580 cm³ above

Example 6 — unscaffolded

A 20 cm by 12 cm by 9 cm block has a cylindrical hole of radius 3 cm drilled straight through the 9 cm depth. Find the volume of the block that remains, exact in π and to one decimal place.

Block: $20 \times 12 \times 9 = 2160$ cm³. Hole: a cylinder of radius 3 and height 9: $\pi \times 3^2 \times 9 = 81\pi$ cm³. Remaining: $2160 - 81\pi$ cm³. $2160 - 81\pi \approx 1905.5$ cm³. ∴ The drilled block has volume $2160 - 81\pi \approx 1905.5$ cm³.

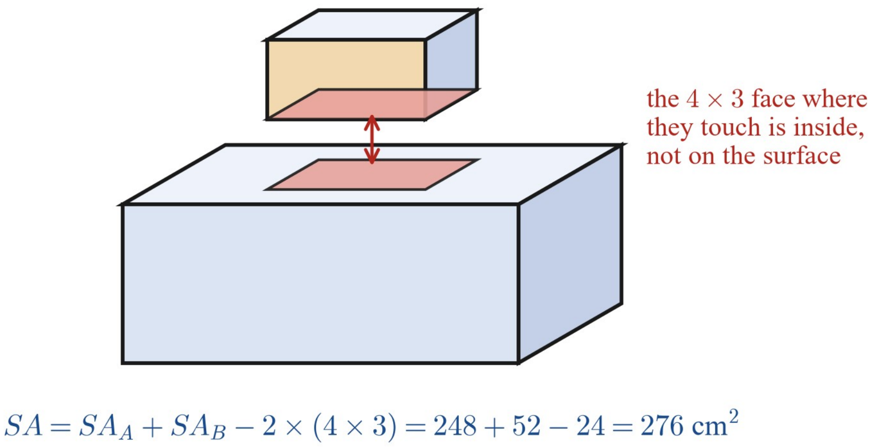


The 20 × 12 × 9 block with the radius 3 cm hole through the 9 cm depth, front rim solid red and back rim dashed; V = 2160 − 81π ≈ 1905.5 cm³

Example 7 — unscaffolded

A 4 cm by 3 cm by 2 cm block sits centred on top of a 10 cm by 6 cm by 4 cm block. Find the surface area of the composite solid.

Large block: $SA_A = 2(10\times6+10\times4+6\times4) = 2(60+40+24) = 248$ cm². Small block: $SA_B = 2(4\times3+4\times2+3\times2) = 2(12+8+6) = 52$ cm². Contact face: $4\times3 = 12$ cm², hidden twice — once from each block. Composite: $248+52-2\times12 = 276$ cm². ∴ The composite surface area is 276 cm² — not $248+52 = 300$, because the glued faces are inside the solid.



The small block lifted above the large one with both 4 × 3 contact faces shaded red and a double arrow between them; SA = 248 + 52 − 2 × (4 × 3) = 276 cm²

Example 8 — unscaffolded

A rectangular tank is 60 cm long, 35 cm wide and 40 cm high. (a) What is its capacity in litres? (b) When the tank holds 63 L of water, how deep is the water?

(a) $V = 60\times35\times40 = 84\,000$ cm³ $= 84$ L, using 1000 cm³ $= 1$ L.

(b) 63 L = 63 000 cm³. The water sits as a cuboid on the base $60\times35=2100$ cm². Depth = $63\,000\div2100=30$ cm; check: $30<40$, so the water fits inside the tank. ✓ ∴ (a) 84 L; (b) the water is 30 cm deep.

Practice questions

Where a question is labelled **SCENARIO**, the context and its numbers are invented for this book (R2).

Scaffolded

Q1. Find the volume of each prism.

a. A triangular prism: cross-section a right triangle with base 10 cm and height 6 cm, length 12 cm — start with $A=\frac{1}{2}\times10\times6$.

b. A cuboid 5 cm by 5 cm by 11 cm.

Q2. Find the volume of each cylinder, exact in π and to one decimal place.

a. $r=3$ cm, $h=10$ cm.
b. $r=5$ cm, $h=6$ cm.
c. $r=7$ cm, $h=10$ cm.

Q3. The theory figure unfolds an 8 cm by 5 cm by 3 cm cuboid into its net.

a. Write the areas of the three pairs of faces.
b. Add the six faces to find the surface area.
c. Check your total against $SA=2(lw+lh+wh)$.

Q4. Find the surface area of each cylinder, exact in π and to one decimal place.

a. $r=4$ cm, $h=10$ cm.
b. $r=6$ cm, $h=15$ cm.

Q5. Convert, using 1 cm³ = 1 mL, 1000 cm³ = 1 L and 1 m³ = 1000 L.

a. 4500 cm³ to litres.
b. 2.3 L to cm³.
c. 750 mL to cm³.
d. 0.6 m³ to litres.

Q6. A prism has a trapezium cross-section with parallel sides 8 cm and 12 cm and height 5 cm, and length 20 cm.

a. Show the cross-section has area 50 cm².
b. Hence find the volume.

Q7. The L-block of Example 5 has volume 580 cm³ by adding.

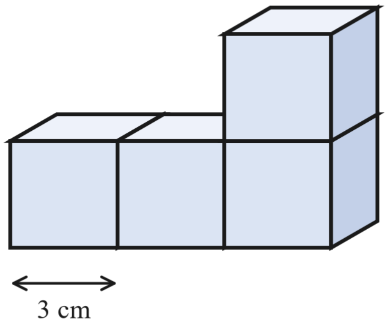
a. Find the volume of the enclosing cuboid, 14 cm by 10 cm by 5 cm.
b. Find the volume of the missing corner, 6 cm by 4 cm by 5 cm.
c. Subtract, and compare with Example 5.

Q8. A 12 cm by 8 cm by 10 cm block has a square tunnel 4 cm by 4 cm cut straight through the 10 cm length.

a. Volume of the whole block.
b. Volume of the tunnel.
c. Volume of the block that remains.

Q9. Four cubes, each 3 cm, are joined in an L, as in the figure.

a. Find the volume of one cube, and of the solid.
b. Each join hides 2 faces. How many faces of the 24 are hidden, and how many show?
c. Hence find the surface area of the solid.



Four cubes, each 3 cm. Count the faces you can see and the faces you cannot: each join hides 2 faces.

Four 3 cm cubes joined in an L shape — three in a row with one on top of the end cube — with one edge marked 3 cm

Q10. Four cuboids each have volume 64 cm³. Complete the surface-area column.

Dimensions (cm)	Volume (cm³)	Surface area (cm²)
$16 \times 2 \times 2$	64	
$8 \times 4 \times 2$	64	
$8 \times 8 \times 1$	64	
$4 \times 4 \times 4$	64	

a. Fill in the four surface areas.
b. Which cuboid has the smallest surface area?
c. What happens to the surface area as the shape gets longer and thinner (`E03`)?

Q11. A label is to wrap right around the curved surface of a cylinder of radius 2 cm and height 9 cm (no ends).

a. Find the area of the label, exact in π and to one decimal place.
b. Explain in one sentence why the two circles are not part of the label.

Q12. Work backwards.

a. A prism has volume 240 cm³ and cross-section area 24 cm². Find its length.
b. A cylinder has volume 500π cm³ and radius 10 cm. Find its height.

Unscaffolded

Q13. Find the volume of each solid, exact in π and to one decimal place where π appears: (a) a prism with trapezium cross-section, parallel sides 9 cm and 8 cm, cross-section height 4 cm, length 12 cm; (b) a cylinder, $r = 4.5$ cm, $h = 14$ cm; (c) a triangular prism with right-triangle cross-section of legs 9 cm and 12 cm, length 20 cm.

Q14. Find the surface area of each solid, exact in π and to one decimal place where π appears: (a) a cuboid 12 cm by 7 cm by 5 cm; (b) a prism with right-triangle ends 5 cm, 12 cm, 13 cm and length 20 cm; (c) a cylinder, $r = 10$ cm, $h = 8$ cm.

Q15. SCENARIO — a fish tank. A rectangular tank measures 50 cm by 40 cm by 30 cm inside. (a) Find its capacity in litres. (b) How many 2.5 L bottles fill it from empty?

Q16. A cylinder has volume 2000 cm³ and radius 10 cm. Find its height, exact in π and to two decimal places.

Q17. A prism has a cross-shaped cross-section made from a 20 cm by 4 cm rectangle and a 12 cm by 4 cm rectangle centred on each other, and length 25 cm. (a) Show the cross-section has area 112 cm². (b) Hence find the volume.

Q18. SCENARIO — a can in a box. A cylindrical can of radius 5 cm and height 18 cm stands inside a 10 cm by 10 cm by 18 cm box. (a) Volume of the can, exact and to one decimal place. (b) Volume of the box. (c) Volume of the air gap between can and box, to one decimal place. (d) Surface area of the can, exact and to one decimal place.

Q19. SCENARIO — an open tank. A rectangular tank 80 cm by 50 cm by 40 cm high has no lid. Find the area of metal in the five faces, in cm² and in m².

Q20. A student finds the surface area of a $3\times2\times2$ block sitting on an $8\times5\times3$ block by writing $158+32=190$ cm². Find the error and give the correct surface area.

Q21. Four cuboids each have volume 216 cm³: $6\times6\times6$, $12\times6\times3$, $18\times4\times3$ and $36\times3\times2$. For each, find the surface area and the ratio $SA:V$ as a decimal to two places, then make a conjecture about which shape gives the smallest surface area for a fixed volume (`E03`).

Q22. A half-cylinder — a cylinder of radius 6 cm cut in half lengthways — has length 30 cm. Find its volume, exact in π and to one decimal place.

Q23. Convert: (a) 3.2 m³ to litres; (b) 45 000 cm³ to m³; (c) 2.4 m³ to cm³ and to litres.

Q24. SCENARIO — a water tank. A cylindrical tank has radius 0.5 m and height 1.6 m. Find its capacity, exact in π in cubic metres, and in litres to one decimal place.

Q25. Cylinder A has $r=4$ cm, $h=9$ cm; cylinder B has $r=6$ cm, $h=4$ cm. Show they have the same volume, then find both surface areas exact in π. Which holds the same amount with less material?

Q26. A 16 cm by 10 cm by 6 cm block has a cylindrical hole of radius 2 cm drilled through the 6 cm depth. Find the remaining volume, exact in π and to one decimal place.

Q27. SCENARIO — wrapping a gift. A box 30 cm by 20 cm by 12 cm is wrapped with 10% extra paper for overlaps. What area of paper is used, in cm²?

Q28. SCENARIO — a water tower. A cuboid tower 12 m by 8 m by 5 m carries a cylindrical tank of radius 3 m and height 6 m on its top face. Find the surface area of the outside of the structure (do not count the circle where tank and tower meet, on either solid), exact in π and to one decimal place.

Q29. SCENARIO — a stone in a tank. A rectangular tank has a base 40 cm by 25 cm. (a) 12 L of water are poured in; find the depth of the water. (b) A stone is lowered in, fully covered, and the water rises 3 cm; find the volume of the stone.

Q30. SCENARIO — a candle order. A candle is a cylinder of radius 3 cm and height 8 cm. An order of 40 candles is poured. (a) Find the wax needed, exact in π. (b) Give it in cm³ and in litres, to one and two decimal places respectively.

Q31. A cube of side 3 cm is doubled to a cube of side 6 cm. (a) By what factor does the surface area grow? (b) By what factor does the volume grow? (c) The surface-area-to-volume ratio falls from 2 to 1. Explain, in words, why doubling a shape always halves this ratio.

Q32. True or false? Correct each false statement. (a) Doubling the radius of a cylinder doubles its volume. (b) Two solids with the same volume have the same surface area. (c) A cube of side 1 m holds 1000 L.

Q33. A 60 cm wire is bent into a triangle with sides 15 cm, 20 cm and 25 cm, which becomes the cross-section of a prism of length 30 cm. (a) Show the cross-section is a right triangle and find its area. (b) Find the volume. (c) Find the surface area.

Q34. SCENARIO — two pools. Pool A is a cylinder of radius 2 m and depth 1.2 m; pool B is a rectangular pool 6 m by 3 m by 1.2 m deep. (a) Find the capacity of each pool in litres, to one decimal place where needed. (b) How many more litres does the bigger pool hold?

Q35. For volume 64 cm^3, the theory figure compares four cuboids. Using at least three cuboids of your own with whole-number edges and volume 64 cm^3, together with the four in the figure, state and defend a conjecture about which cuboid has the smallest surface area (`E03`).

Q36. A cube of side 12 cm has a cylindrical hole of radius 3 cm drilled straight through, from the centre of one face to the centre of the opposite face. (a) Find the volume that remains, exact in π and to one decimal place. (b) Find the surface area of the drilled cube, exact in π and to one decimal place — remember the two circles lost from the faces and the tube gained inside.

Answers

Q1 a 360 cm³; b 275 cm³. **Q2** a $90\pi \approx 282.7$ cm³; b $150\pi \approx 471.2$ cm³; c $490\pi \approx 1539.4$ cm³. **Q3** a $40, 24, 15$ cm² per pair; b 158 cm²; c $2(40+24+15)=158$ cm² ✓. **Q4** a $112\pi \approx 351.9$ cm²; b $252\pi \approx 791.7$ cm². **Q5** a 4.5 L; b 2300 cm³; c 750 cm³; d 600 L. **Q6** a $\frac{1}{2}(8+12) \times 5 = 50$ cm²; b 1000 cm³. **Q7** a 700 cm³; b 120 cm³; c $700-120=580$ cm³ — same as adding. **Q8** a 960 cm³; b 160 cm³; c 800 cm³. **Q9** a 27 cm³; 108 cm³; b 6 hidden, 18 showing; c 162 cm². **Q10** a $136, 112, 160, 96$ cm²; b the cube; c the longer and thinner the shape, the bigger the surface area. **Q11** a $36\pi \approx 113.1$ cm²; b the label wraps the curved surface only — the ends stay open. **Q12** a 10 cm; b 5 cm. **Q13** a 408 cm³; b $283.5\pi \approx 890.6$ cm³; c 1080 cm³. **Q14** a 358 cm²; b 660 cm²; c $360\pi \approx 1131.0$ cm². **Q15** a 60 L; b 24 bottles. **Q16** $20/\pi \approx 6.37$ cm. **Q17** a $80+48-16=112$ cm²; b 2800 cm³. **Q18** a $450\pi \approx 1413.7$ cm³; b 1800 cm³; c ≈ 386.3 cm³; d $230\pi \approx 722.6$ cm². **Q19** 14 400 cm² = 1.44 m². **Q20** The glued 3×2 face is counted twice: $158+32-2 \times 6=178$ cm². **Q21** $216, 252, 276, 372$ cm²; ratios $1, 1.17, 1.28, 1.72$; conjecture: the cube has the smallest surface area for a fixed volume. **Q22** $540\pi \approx 1696.5$ cm³. **Q23** a 3200 L; b 0.045 m³; c 2 400 000 cm³ = 2400 L. **Q24** 0.4π m³ ≈ 1256.6 L. **Q25** Both 144π cm³; $SA_A = 104\pi \approx 326.7$ cm², $SA_B = 120\pi \approx 377.0$ cm²; A. **Q26** $960-24\pi \approx 884.6$ cm³. **Q27** 2640 cm². **Q28** $392+36\pi \approx 505.1$ m². **Q29** a 12 cm; b 3000 cm³. **Q30** a 2880π cm³; b ≈ 9047.8 cm³ ≈ 9.05 L. **Q31** a ×4; b ×8; c both area and volume multiply, but volume grows one power faster, so the ratio halves. **Q32** a false — it quadruples; b false — see Q10; c true. **Q33** a 150 cm²; b 4500 cm³; c 2100 cm². **Q34** a A: $4800\pi \approx 15\,079.6$ L; B: 21 600 L; b ≈ 6520.4 L. **Q35** The cube, 96 cm²; every stretched alternative measured comes out larger. **Q36** a $1728-108\pi \approx 1388.7$ cm³; b $864+54\pi \approx 1033.6$ cm².

Fully-worked solutions

Q1. a $A=\frac{1}{2} \times 10 \times 6 = 30$ cm²; $V = 30 \times 12 = 360$ cm³. b $V = 5 \times 5 \times 11 = 275$ cm³. ∴ 360 cm³ and 275 cm³ — same two steps: cross-section, then × length.

Q2. a $V = \pi \times 3^2 \times 10 = 90\pi \approx 282.7$ cm³. b $V = \pi \times 5^2 \times 6 = 150\pi \approx 471.2$ cm³. c $V = \pi \times 7^2 \times 10 = 490\pi \approx 1539.4$ cm³. ∴ Exact in π first; one rounding at the end.

Q3. a Pairs: $8 \times 5 = 40$, $8 \times 3 = 24$, $5 \times 3 = 15$ cm². b $2(40+24+15) = 2 \times 79 = 158$ cm². c $SA = 2(lw+lh+wh) = 2(8 \times 5 + 8 \times 3 + 5 \times 3) = 158$ cm² — the formula is the net, shortened (`E01`). ∴ 158 cm², both routes.

Q4. a $SA = 2\pi(4^2) + 2\pi(4)(10) = 32\pi + 80\pi = 112\pi \approx 351.9$ cm². b $SA = 2\pi(6^2) + 2\pi(6)(15) = 72\pi + 180\pi = 252\pi \approx 791.7$ cm². ∴ 112π and 252π cm² — ends plus curved rectangle, every time.

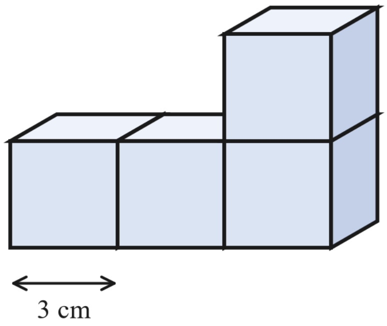
Q5. a $4500 \div 1000 = 4.5$ L. b $2.3 \times 1000 = 2300$ cm³. c 750 mL $= 750$ cm³, since 1 cm³ = 1 mL. d $0.6 \times 1000 = 600$ L. ∴ Multiply or divide by 1000 at each step of the ladder.

Q6. a $A = \frac{1}{2}(8+12) \times 5 = \frac{1}{2} \times 20 \times 5 = 50$ cm². b $V = 50 \times 20 = 1000$ cm³. ∴ A trapezium cross-section changes nothing: area, then × length.

Q7. a $14 \times 10 \times 5 = 700$ cm³. b $6 \times 4 \times 5 = 120$ cm³. c $700 - 120 = 580$ cm³ — the same 580 cm³ Example 5 found by adding. ∴ Adding pieces and subtracting the notch agree; either checks the other.

Q8. a $12 \times 8 \times 10 = 960$ cm³. b The tunnel is a $4 \times 4 \times 10$ square prism: 160 cm³. c $960 - 160 = 800$ cm³. ∴ 800 cm³ remains.

Q9. a One cube: $3^3 = 27$ cm³; solid: $4 \times 27 = 108$ cm³. b Three joins hide $3 \times 2 = 6$ of the 24 faces; $24 - 6 = 18$ faces show. c Each face is $3 \times 3 = 9$ cm², so $SA = 18 \times 9 = 162$ cm². ∴ $V = 108$ cm³, $SA = 162$ cm².

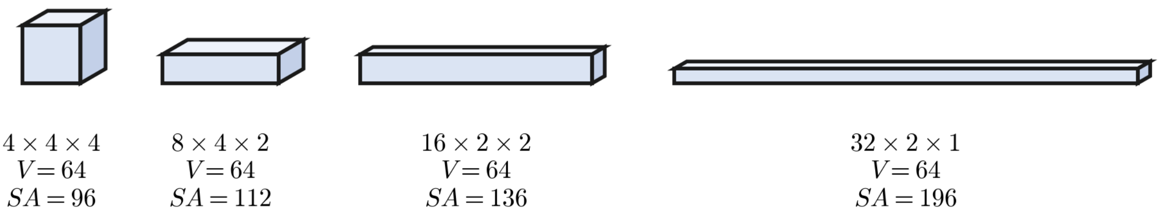


Four cubes, each 3 cm. Count the faces you can see and the faces you cannot: each join hides 2 faces.

Four 3 cm cubes joined in an L, three in a row and one on top of the end cube; each join hides two faces

Q10. a $16 \times 2 \times 2$: $2(32+32+4)=136$; $8 \times 4 \times 2$: $2(32+16+8)=112$; $8 \times 8 \times 1$: $2(64+8+8)=160$; $4 \times 4 \times 4$: $6 \times 16 = 96$ cm². b The cube, at 96 cm². c Stretching the same volume longer and thinner makes the surface area grow: 96 → 112 → 136 → 160. ∴ Same volume, different surface area — the cube wins (`E03`).

Same volume (64 cm^3), but the long thin prism has more than twice the surface area of the cube



Four cuboids to scale, all V = 64 cm³: 4×4×4 with SA 96, 8×4×2 with 112, 16×2×2 with 136, 32×2×1 with 196

Q11. a Curved surface only: $2\pi r h = 2\pi(2)(9) = 36\pi \approx 113.1$ cm². b The label goes around the side; the two circular ends are not covered, so they are not paid for. ∴ $36\pi \approx 113.1$ cm² of label.

Q12. a $h = V \div A = 240 \div 24 = 10$ cm. b $500\pi = \pi \times 10^2 \times h = 100\pi h$, so $h = 500\pi \div 100\pi = 5$ cm. ∴ 10 cm and 5 cm — the volume formula rearranged.

Q13. a $A = \frac{1}{2}(9+8) \times 4 = 34$ cm²; $V = 34 \times 12 = 408$ cm³. b $V = \pi \times 4.5^2 \times 14 = 283.5\pi \approx 890.6$ cm³. c $A = \frac{1}{2} \times 9 \times 12 = 54$ cm²; $V = 54 \times 20 = 1080$ cm³. ∴ 408 cm³, $283.5\pi \approx 890.6$ cm³, 1080 cm³.

Q14. a $2(12 \times 7 + 12 \times 5 + 7 \times 5) = 2(84+60+35) = 358$ cm². b Ends: $2 \times \frac{1}{2} \times 5 \times 12 = 60$; sides: $(5+12+13) \times 20 = 600$; $SA = 660$ cm². c $2\pi(10^2) + 2\pi(10)(8) = 200\pi + 160\pi = 360\pi \approx 1131.0$ cm². ∴ 358 cm², 660 cm², $360\pi \approx 1131.0$ cm².

Q15. a $V = 50 \times 40 \times 30 = 60\,000$ cm³ $= 60$ L. b $60 \div 2.5 = 24$ bottles. ∴ 60 L; 24 bottles.

Q16. $2000 = \pi \times 10^2 \times h = 100\pi h$, so $h = 2000 \div 100\pi = 20/\pi$. $20/\pi \approx 6.37$ cm. ∴ $h = 20/\pi \approx 6.37$ cm.

Q17. a Vertical rectangle $20 \times 4 = 80$; horizontal $12 \times 4 = 48$; the $4 \times 4 = 16$ cm² centre is counted twice, so $A = 80 + 48 - 16 = 112$ cm². b $V = 112 \times 25 = 2800$ cm³. ∴ 112 cm² and 2800 cm³ — overlap counted once.

Q18. a $V=\pi\times5^2\times18=450\pi\approx1413.7$ cm³. b $V=10\times10\times18=1800$ cm³. c $1800-450\pi\approx386.3$ cm³ of air. d $SA=2\pi(5^2)+2\pi(5)(18)=50\pi+180\pi=230\pi\approx722.6$ cm². ∴ The gap is $1800-450\pi\approx386.3$ cm³ — the box is a fifth air.

Q19. Base: $80\times50=4000$; two 80×40 faces: 6400; two 50×40 faces: 4000. Total: $4000+6400+4000=14\,400$ cm². $14\,400\div10\,000=1.44$ m². ∴ Five faces, no lid: 14 400 cm² = 1.44 m².

Q20. $SA_A=2(8\times5+8\times3+5\times3)=158$; $SA_B=2(3\times2+3\times2+2\times2)=32$ cm². The $3\times2=6$ cm² contact face is inside the solid and must be removed twice: $158+32-12=178$ cm². ∴ The student counted the glued faces; the correct area is 178 cm².

Q21. $6\times6\times6$: $SA=216$, ratio $216\div216=1$. $12\times6\times3$: $2(72+36+18)=252$, ratio $252\div216\approx1.17$. $18\times4\times3$: $2(72+54+12)=276$, ratio $276\div216\approx1.28$. $36\times3\times2$: $2(108+72+6)=372$, ratio $372\div216\approx1.72$. ∴ Conjecture: for a fixed volume, the cube has the smallest surface area — and the more a cuboid is stretched from the cube, the bigger its surface area and its $SA:V$ ratio (`E03`).

Q22. Full cylinder: $\pi\times6^2\times30=1080\pi$; half of it: 540π. $540\pi\approx1696.5$ cm³. ∴ $540\pi\approx1696.5$ cm³ — halving the cylinder halves the volume.

Q23. a $3.2\times1000=3200$ L. b $45\,000\div1\,000\,000=0.045$ m³. c $2.4\times1\,000\,000=2\,400\,000$ cm³; $2.4\times1000=2400$ L. ∴ 3200 L; 0.045 m³; 2 400 000 cm³ = 2400 L.

Q24. $V=\pi\times0.5^2\times1.6=0.4\pi$ m³. $0.4\pi\times1000=400\pi\approx1256.6$ L. ∴ 0.4π m³, about 1256.6 L.

Q25. $V_A=\pi\times4^2\times9=144\pi$; $V_B=\pi\times6^2\times4=144\pi$ — equal.
$SA_A=2\pi(16)+2\pi(4)(9)=32\pi+72\pi=104\pi\approx326.7$ cm².
$SA_B=2\pi(36)+2\pi(6)(4)=72\pi+48\pi=120\pi\approx377.0$ cm². ∴ Same volume, different surface area: A shows 104π cm² against B's 120π cm², so A holds the same amount with less material (`E03`).

Q26. Block: $16\times10\times6=960$ cm³; hole: $\pi\times2^2\times6=24\pi$ cm³. $960-24\pi\approx884.6$ cm³. ∴ $960-24\pi\approx884.6$ cm³.

Q27. $SA=2(30\times20+30\times12+20\times12)=2(600+360+240)=2400$ cm². With 10% extra: $2400\times1.1=2640$ cm². ∴ 2640 cm² of paper.

Q28. Tower: full cuboid $2(12\times8+12\times5+8\times5)=392$ m²; the tank covers a circle of area 9π on its top, so the tower shows $392-9\pi$. Tank: curved surface $2\pi(3)(6)=36\pi$ plus its top circle 9π: 45π. Total: $392-9\pi+45\pi=392+36\pi$. $392+36\pi\approx505.1$ m². ∴ $392+36\pi\approx505.1$ m² — the covered circle cancels the tank's bottom circle.

Q29. a 12 L = $12\,000$ cm³; base $40\times25=1000$ cm²; depth $12\,000\div1000=12$ cm. b The water rises by exactly the stone's own volume: $1000\times3=3000$ cm³. ∴ Depth 12 cm; the stone is 3000 cm³.

Q30. a One candle: $\pi\times3^2\times8=72\pi$ cm³; forty: $40\times72\pi=2880\pi$ cm³. b $2880\pi\approx9047.8$ cm³; $\div1000\approx9.05$ L. ∴ $2880\pi\approx9047.8$ cm³ ≈ 9.05 L of wax.

Q31. a Surface areas: $6\times3^2=54$ cm² and $6\times6^2=216$ cm²; $216\div54=4$: factor 4. b Volumes: 27 and 216: factor 8. c Surface area grows by $2^2=4$ and volume by $2^3=8$, so the ratio is multiplied by $4\div8=\frac{1}{2}$: doubling any cube halves the ratio. ∴ ×4, ×8, and the ratio halves — area keeps up with size more slowly than volume does.

Q32. a False: $V=\pi r^2h$, so doubling r multiplies V by 4 — it quadruples. b False: Q10 and Q25 show equal volumes with different surface areas. c True: 1 m³ = 1000 L, so the cube holds 1000 L. ∴ False, false, true.

Q33. a $15^2+20^2=225+400=625=25^2$, so the triangle is right-angled with legs 15 cm and 20 cm; $A=\frac{1}{2}\times 15\times 20=150$ cm². b $V=150\times 30=4500$ cm³. c Ends: $2\times 150=300$; sides: $(15+20+25)\times 30=60\times 30=1800$; $SA=2100$ cm². ∴ 150 cm², 4500 cm³, 2100 cm².

Q34. a Pool A: $V=\pi\times 2^2\times 1.2=4.8\pi$ m³ $=4800\pi\approx 15\,079.6$ L. Pool B: $6\times 3\times 1.2=21.6$ m³ $=21\,600$ L. b $21\,600-4800\pi\approx 6520.4$ L. ∴ Pool B holds about 6520.4 L more.

Q35. Examples beyond the figure: $64\times 1\times 1$ gives $SA=2(64+64+1)=258$; $32\times 2\times 1$ gives 196; $16\times 4\times 1$ gives $2(64+16+4)=168$; $8\times 4\times 2$ gives 112; $4\times 4\times 4$ gives 96 cm². Every move away from the cube raises the surface area, and every move toward the cube lowers it. ∴ Conjecture: among cuboids of a fixed volume, the cube has the smallest surface area (`E03`).

Q36. a Hole: $\pi\times 3^2\times 12=108\pi$; remaining: $1728-108\pi\approx 1388.7$ cm³. b Cube: $6\times 12^2=864$; lose two circles: $-2\times 9\pi=-18\pi$; gain the tube: $2\pi(3)(12)=72\pi$. $SA=864-18\pi+72\pi=864+54\pi\approx 1033.6$ cm². ∴ $1728-108\pi\approx 1388.7$ cm³ and $864+54\pi\approx 1033.6$ cm² — drilling *adds* surface area even though it removes volume.

Scientific notation: very large and very small measurements, timescales and intervals

5B · Chapter 5 — Measurement · Level 9

Content description `VC2M9M02` (word for word): *solve problems involving very small and very large measurements, timescales and intervals expressed in scientific notation*

Achievement standard anchor (AS 13): *Students express small and large numbers in scientific notation.*

Elaboration ids for linkage: `VC2M9M02.E01`, `VC2M9M02.E02`, `VC2M9M02.E03`

Prerequisite pointer: 1B (exponent laws and SI prefixes). Scientific notation is the exponent laws of 1B applied to measurement; the laws are used here as tools, not re-taught. Teach 5B and 5D consecutively: the instrument accuracy examined here sets up absolute and relative error there.

Note on sources: the age of the Earth (about 4.6×10^{9} years) and the mass of a glucose molecule (2.99×10^{-22} g) are the VCAA's own examples in `VC2M9M02.E01`; the speed of light $c = 299\,792\,458$ m/s and the Avogadro constant $6.022\,140\,76 \times 10^{23}$ are exact SI defining constants. Source: VCAA, Victorian Curriculum 2.0 Mathematics elaboration export (VC2M9M02), 2026. Retrieved 29 July 2026. Source: BIPM, SI Brochure: The International System of Units, 9th edition, 2019. Retrieved 20 August 2026. Every other quantity is arithmetic from definitions (86 400 s in a day, the 365-day year, SI prefixes) or a clearly-labelled SCENARIO with generic actors, so no further source line is owed (R1, R2).

Note on figures: every number drawn in a figure is a value checked in `src/verify_VC2M9M02.py` (R7).

Theory

Why the zeros fail us

Write the age of the Earth in seconds, or the mass of a molecule in grams, in ordinary form and the number drowns in zeros. Counting places by eye is exactly where mistakes happen, and two measurements of very different sizes end up looking alike on the page. Scientific notation removes the zeros from the job: a short number carries the digits, and a power of 10 carries the size.

The age of the Earth in seconds,

written out:

145 000 000 000 000 000 s

18 digits — easy to miscount,

so we write

1.45×10^{17} s

The mass of a glucose molecule,

written out:

0.000 000 000 000 000 000 000 299 g

21 zeros between the point and the 299 —

so we write

2.99×10^{-22} g

Two panels: on the left the age of the Earth in seconds written out as 145 000 000 000 000 000 s and then rewritten as 1.45 times 10 to the 17 s; on the right the mass of a glucose molecule written out as 0.000 000 000 000 000 000 000 299 g and then rewritten as 2.99 times 10 to the minus 22 g

The shape of scientific notation

A number is written in **scientific notation** when it has the shape

$a \times 10^{n}$, where $1 \le a < 10$ and n is an integer.

The number a carries the digits; the **exponent** n says how many places the decimal point has moved. Both parts are checked when you read or write the form: a must sit between 1 and 10 (1 allowed, 10 not), and n must be a whole number, positive, negative or zero.

Every number in scientific notation has the same shape:

$$3.46 \times 10^5$$

$a = 3.46$ — at least 1, under 10

10^n with $n = 5$ — a power of 10; n an integer

$$3.46 \times 10^5 = 3.46 \times 100\,000 = 346\,000$$

The power of 10 only moves the decimal point; the digits stay 3, 4, 6.

The number 3.46 times 10 to the 5 with the two parts labelled: a = 3.46, at least 1 and under 10, in blue; 10 to the n with n = 5, a power of 10 with n an integer, in green; below, 3.46 times 10 to the 5 = 3.46 times 100 000 = 346 000

The power of 10 only moves the decimal point; it never changes the digits. That is the whole idea, and it is why the exponent laws of 1B do the operating in this section.

From scientific notation to decimal form, and back

To write $a \times 10^n$ in decimal form, move the decimal point n places: right when n is positive (the number gets bigger), left when n is negative (the number gets smaller), filling any empty places with zeros.

10^4	10^3	10^2	10^1	10^0
2	7.	0	0	0

move the point 4 places right

$2.7 \times 10^4 = 27\,000$ — the 4 sends the digits 4 places to the left, filling empty places with zeros.

Place-value chart with columns 10 to the 4 down to 10 to the 0 holding the digits 2, 7, 0, 0, 0 and a red arrow showing the decimal point moving 4 places right: 2.7 times 10 to the 4 = 27 000

10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
0 .	0	0	5	0	8

move the point 3 places left

$5.08 \times 10^{-3} = 0.005\,08$ — the −3 sends the digits 3 places to the right, filling empty places with zeros.

Place-value chart with columns 10 to the 0 down to 10 to the minus 5 holding the digits 0.005 08 and a red arrow showing the decimal point moving 3 places left: 5.08 times 10 to the minus 3 = 0.005 08

Going back the other way, count the moves. For $42\,000$ the point moves 4 places left to sit after the 4, so $42\,000 = 4.2 \times 10^4$. For 0.0071 the point moves 3 places right to sit after the 7, so $0.0071 = 7.1 \times 10^{-3}$. And a number between 1 and 10 needs no move at all: $9 = 9 \times 10^0$, since $10^0 = 1$.

Is it scientific notation? Two tests

Test 1: is $1 \le a < 10$? Test 2: is n an integer? If either test fails, the writing is not scientific notation — though a near-miss on Test 1 is always one shift of the point away from being correct.

Scientific notation needs $1 \le a < 10$ and an integer exponent n.

×	34×10^3	$a = 34$ is 10 or more — shift the point: 3.4×10^4
×	0.5×10^6	$a = 0.5$ is under 1 — shift the point: 5×10^5
×	$4.2 \times 10^{2.5}$	the exponent 2.5 is not an integer — not scientific notation
✓	7.1×10^{-4}	$1 \le 7.1 < 10$ and -4 is an integer — correct

Four rows testing the two rules: 34 times 10 to the 3 fails because a = 34 is 10 or more and becomes 3.4 times 10 to the 4; 0.5 times 10 to the 6 fails because a = 0.5 is under 1 and becomes 5 times 10 to the 5; 4.2 times 10 to the 2.5 fails because the exponent is not an integer; 7.1 times 10 to the minus 4 passes both tests

Shifting the point one place right divides a by 10 and so adds 1 to n; one place left multiplies a by 10 and subtracts 1 from n. The number itself never changes — only its writing.

Multiplying and dividing in scientific notation

Multiply or divide the number parts, and let the exponent laws of 1B handle the powers of 10: the product law $10^m \times 10^n = 10^{m+n}$ and the quotient law $10^m \div 10^n = 10^{m-n}$.

$$(3 \times 10^4) \times (2 \times 10^6)$$

multiply the numbers:
$3 \times 2 = 6$

apply the product law:
$10^4 \times 10^6 = 10^{4+6} = 10^{10}$

$$6 \times 10^{10}$$

6 already sits between 1 and 10, so the answer is in scientific notation.

Check: $3 \times 10^4 = 30\,000$ and $2 \times 10^6 = 2\,000\,000$; their product is $60\,000\,000\,000 = 6 \times 10^{10}$.

Flow diagram: (3 times 10 to the 4) times (2 times 10 to the 6) splits into two boxes, multiply the numbers 3 times 2 = 6 and apply the product law 10 to the 4 times 10 to the 6 = 10 to the 10, which combine to 6 times 10 to the 10, with a decimal check line below

$$(8.4 \times 10^7) \div (2.1 \times 10^3)$$

divide the numbers: $8.4 \div 2.1 = 4$

apply the quotient law: $10^7 \div 10^3 = 10^{7-3} = 10^4$

$$4 \times 10^4$$

When the number part falls outside 1 to 10, fix it at the end:

$$(6 \times 10^5) \div (8 \times 10^2) = 0.75 \times 10^3$$

$$\downarrow$$

0.75 is under 1, so shift the point one place: $0.75 \times 10^3 = 7.5 \times 10^2$

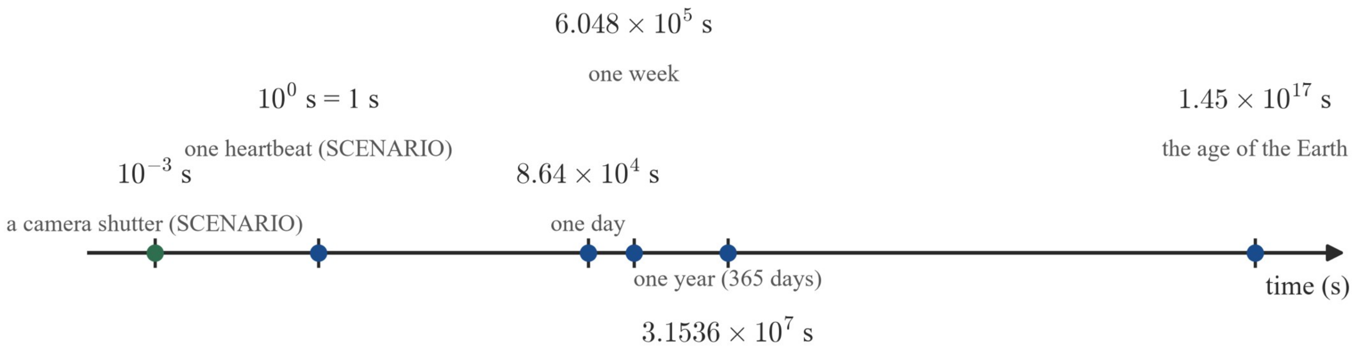
Both equal 750 — the shift changes the writing, not the number.

Flow diagram: (8.4 times 10 to the 7) divided by (2.1 times 10 to the 3) splits into divide the numbers 8.4 divided by 2.1 = 4 and apply the quotient law 10 to the 7 divided by 10 to the 3 = 10 to the 4, combining to 4 times 10 to the 4; below, the normalising case (6 times 10 to the 5) divided by (8 times 10 to the 2) = 0.75 times 10 to the 3 rewritten as 7.5 times 10 to the 2

If the number part of the answer lands outside 1 to 10, normalise it at the end with one shift, exactly as the near-miss test above does.

Timescales and intervals

The second is the base unit of time, and scientific notation makes a ladder of intervals easy to read. One day is $24 \times 60 \times 60 = 86\,400$ s $= 8.64 \times 10^4$ s; one week is $7 \times 86\,400 = 604\,800$ s $= 6.048 \times 10^5$ s; one year of 365 days is $365 \times 86\,400 = 31\,536\,000$ s $= 3.1536 \times 10^7$ s.



Each step is a factor of 10. The spacing on this line is by powers of 10, so the age of the Earth sits only about 10 marks from a single heartbeat.

Horizontal line of time in seconds spaced by powers of 10, marked at 10 to the minus 3 s (a camera shutter, SCENARIO), 1 s (one heartbeat, SCENARIO), 8.64 times 10 to the 4 s (one day), 6.048 times 10 to the 5 s (one week), 3.1536 times 10 to the 7 s (one year of 365 days) and 1.45 times 10 to the 17 s (the age of the Earth)

Light is the great measuring stick for large distances and tiny intervals. Light travels at $c = 299\,792\,458$ m/s, so in one microsecond (10^{-6} s) it travels about 300 m, and the time light takes to cross 1 km is about 3.3×10^{-6} s. Worked example 2 below turns the same fact around to measure a light-year in metres.

Prefixes are powers of 10

1B named the SI prefixes; here they are read straight into scientific notation. A prefix is only a shortcut for a power of 10, so removing it leaves a number in scientific notation.

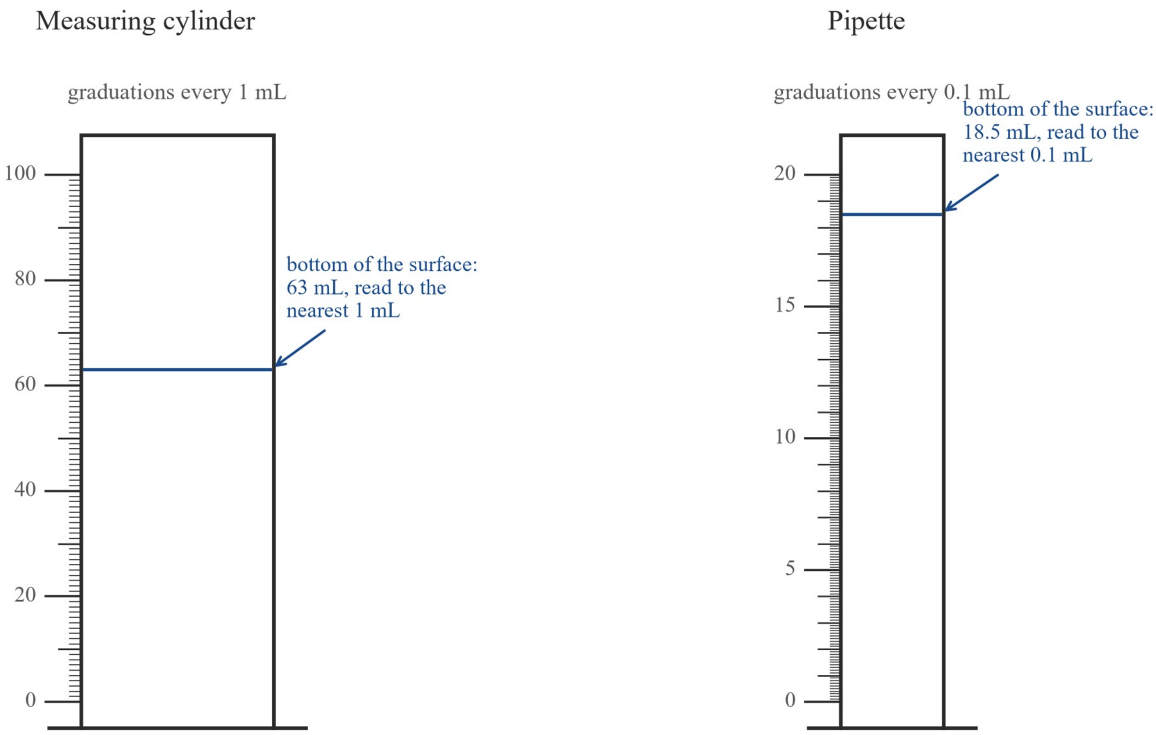
Power of 10	*Prefix*	*Rewritten in scientific notation*
10^{-12}	pico (p)	$3\,\text{pm} = 3 \times 10^{-12}$ m
10^{-9}	nano (n)	$450\,\text{nm} = 4.5 \times 10^{-7}$ m
10^{-6}	micro (μ)	$3\,\mu\text{m} = 3 \times 10^{-6}$ m
10^{-3}	milli (m)	$25\,\text{mm} = 2.5 \times 10^{-2}$ m
10^{0}	(none)	—
10^{3}	kilo (k)	$7\,\text{km} = 7 \times 10^{3}$ m

A prefix is a shortcut for a power of 10 — removing it leaves a number in scientific notation.

Table from 10 to the minus 12 (pico) to 10 to the 3 (kilo) with one worked conversion per row: 3 pm = 3 times 10 to the minus 12 m, 450 nm = 4.5 times 10 to the minus 7 m, 3 micrometres = 3 times 10 to the minus 6 m, 25 mm = 2.5 times 10 to the minus 2 m, 7 km = 7 times 10 to the 3 m

The laboratory: how fine is the instrument?

SCENARIO — a school science laboratory. Two instruments hold water: a measuring cylinder with a graduation every 1 mL, and a pipette readable to 0.1 mL. Read each to the bottom of the water's surface, and record the value to the accuracy the instrument allows.



A measuring cylinder with a graduation every 1 mL reading 63 mL beside a narrow pipette with a graduation every 0.1 mL reading 18.5 mL, each reading taken at the bottom of the water's surface

In litres, the graduations are 1 mL $= 10^{-3}$ L on the cylinder and 0.1 mL $= 10^{-4}$ L on the pipette: the pipette sees intervals ten times smaller. The readings, recorded in scientific notation, are 63 mL $= 6.3 \times 10^{-2}$ L and 18.5 mL $= 1.85 \times 10^{-2}$ L. A single drop of about 0.5 mL (5×10^{-4} L) changes the pipette's reading but is invisible to the cylinder — which is exactly why the instrument sets the degree of accuracy of the recorded value.

The section in one figure

Scientific notation puts every measurement on the same shelf: the exponent says how far the number sits from 1, whether the measurement is a molecule's mass or the age of the Earth.

Length (m)

10^{-12} pm $\to$ 10^{-9} nm $\to$ 10^{-6} μm $\to$ 10^{-3} mm $\to$ 1 (about a stride)
$\to$ 10^{3} km $\to$ 9.5×10^{15} (one light-year)

Time (s)

10^{-3} (a camera shutter) $\to$ 1 (one heartbeat) $\to$ 8.64×10^{4} (one day)
$\to$ 3.1536×10^{7} (one year) $\to$ 1.45×10^{17} (the age of the Earth)

Mass (g)

2.99×10^{-22} (one glucose molecule) $\to$ 10^{-3} (one milligram)
$\to$ 1 (a paperclip, SCENARIO) $\to$ 10^{3} (one kilogram)

Each arrow moves to a larger power of 10; the exponent says how far the measurement sits from 1.

Three rows, for length in metres, time in seconds and mass in grams, each listing anchors from smallest to largest power of 10: from 10 to the minus 12 (the picometre) to 9.5 times 10 to the 15 (one light-year); from 10 to the minus 3 (a camera shutter) to 1.45 times 10 to the 17 (the age of the Earth); from 2.99 times 10 to the minus 22 (one glucose molecule) to 10 to the 3 (one kilogram)

That is the achievement standard's sentence in practice: students express small and large numbers in scientific notation.

Worked examples

Example 1 — scaffolded

Write (i) $0.000\,004\,6$ and (ii) $3\,800\,000\,000$ in scientific notation; write (iii) 2.99×10^{-22} and (iv) 4.6×10^{9} in decimal form.

Working	Comment
(i) move the point 6 places right to sit after the 4: 4.6×10^{-6}	A number under 1 gets a negative exponent; count the moves.
(ii) move the point 9 places left to sit after the 3: 3.8×10^{9}	A number over 10 gets a positive exponent.
(iii) $2.99\times10^{-22}=0.000\,000\,000\,000\,000\,000\,000\,299$	The -22 puts the first digit 22 places in: 21 zeros, then 299.
(iv) $4.6\times10^{9}=4\,600\,000\,000$	The 9 moves the point 9 places right; fill empty places with zeros.
Check (i): $4.6\div1\,000\,000=0.000\,004\,6$	Going back the other way returns the starting number.

Example 2 — scaffolded

Light travels at about 3.0×10^{8} m/s. Find the distance light travels in one year of 365 days, in scientific notation rounded to two significant figures.

Working	Comment
$365\times86\,400=31\,536\,000$ s $=3.1536\times10^{7}$ s	Write the year in seconds, in scientific notation.

Working	Comment
distance = speed × time $=(3.0\times10^{8})\times(3.1536\times10^{7})$	The measurement formula from science, set in scientific notation.
$=(3.0\times3.1536)\times(10^{8}\times10^{7})$	Numbers with numbers; powers of 10 with powers of 10.
$=9.4608\times10^{15}$ m	Product law: $10^{8}\times10^{7}=10^{15}$.
$\approx9.5\times10^{15}$ m	Round once, at the end, to two significant figures.

Using the exact value of c gives $9\,454\,254\,955\,488\,000$ m, which rounds to the same 9.5×10^{15} m — the two-significant-figure answer does not depend on the small rounding of the speed.

Example 3 — unscaffolded

A glucose molecule has a mass of about 2.99×10^{-22} g. Find the mass of 5×10^{6} glucose molecules, in scientific notation.

The mass is $(2.99\times10^{-22})\times(5\times10^{6})$ g. Multiply the number parts: $2.99\times5=14.95$. Apply the product law: $10^{-22}\times10^{6}=10^{-16}$. So the mass is 14.95×10^{-16} g, and 14.95 is 10 or more: one shift of the point gives 1.495×10^{-15} g.

∴ The mass of 5×10^{6} glucose molecules is about 1.495×10^{-15} g.

Example 4 — unscaffolded

The age of the Earth is about 4.6×10^{9} years. Express this timescale in seconds in scientific notation (two significant figures), and find how many 80-year lifetimes fit into the age of the Earth.

One year of 365 days is 3.1536×10^{7} s, so the age in seconds is $(4.6\times10^{9})\times(3.1536\times10^{7})=(4.6\times3.1536)\times10^{16}=14.50656\times10^{16}=1.450656\times10^{17}$ s, which rounds to 1.5×10^{17} s. For the lifetimes, write 80 as 8×10^{1} and divide: $(4.6\times10^{9})\div(8\times10^{1})=(4.6\div8)\times10^{8}=0.575\times10^{8}=5.75\times10^{7}$.

∴ The age of the Earth is about 1.5×10^{17} s, and about 5.75×10^{7} (57.5 million) 80-year lifetimes fit into it.

Practice questions

Scaffolded

Q1. Read each number in scientific notation: (i) 5.2×10^{3}, (ii) 7×10^{-2}, (iii) 1.06×10^{5}.

a. For each, state a and n, and say which of the two tests ($1\le a<10$; n an integer) each part passes.
b. Write each number in decimal form by moving the point.

Q2. Write each number in scientific notation, counting the moves of the point.

a. $42\,000$
b. 0.0071
c. 9
d. 0.6×10^{4} (the digits must move once more: what happens to n?)

Q3. Each number below fails one test, both tests, or neither. Fix the ones that need fixing, and for any that are already correct, say so and why.

a. 52×10^{3}
b. 0.99×10^{-6}

c. 3.0×10^{0}
d. 0.5×10^{6}

Q4. Multiply, leaving the answer in scientific notation: number parts together, then the product law.

a. $\left(2 \times 10^{3}\right) \times \left(4 \times 10^{5}\right)$
b. $\left(3 \times 10^{-2}\right) \times \left(5 \times 10^{6}\right)$
c. $\left(6 \times 10^{4}\right) \times \left(7 \times 10^{2}\right)$

Q5. Divide, leaving the answer in scientific notation: number parts together, then the quotient law.

a. $\left(8 \times 10^{9}\right) \div \left(2 \times 10^{3}\right)$
b. $\left(9 \times 10^{4}\right) \div \left(3 \times 10^{-2}\right)$
c. $\left(5 \times 10^{3}\right) \div \left(4 \times 10^{5}\right)$

Q6. Arrange 3.1×10^{-4}, 1.2×10^{5}, 9×10^{3}, 3.1×10^{4} from smallest to largest.

a. Which is the only number with a negative exponent, and what does that tell you about its size compared with the others?
b. Compare the remaining three by their exponents first.
c. Write the four numbers in order, smallest first.

Unscaffolded

Q7. Write in scientific notation: (a) $150\,000\,000$; (b) $0.000\,0508$; (c) $1\,000\,000\,000$.

Q8. Write in decimal form: (a) 2.2×10^{-5}; (b) $6.02214076 \times 10^{23}$ (the Avogadro constant, written out in full); (c) 4.6×10^{9} (the age of the Earth in years).

Q9. Rewrite in correct scientific notation: (a) 0.0045×10^{8}; (b) 27.3×10^{-5}; (c) 92×10^{-3}.

Q10. Evaluate, leaving each answer in scientific notation: (a) $\left(3 \times 10^{6}\right) \times \left(4 \times 10^{-2}\right)$; (b) $\left(7.2 \times 10^{5}\right) \div \left(9 \times 10^{2}\right)$; (c) $\left(2 \times 10^{-3}\right) \times \left(8 \times 10^{-4}\right)$.

Q11. Timescales of a week. (a) One week is $604\,800$ s; write this in scientific notation. (b) SCENARIO: a camera shutter is open for 10^{-3} s. How many shutter-times fit in one second? (c) Is one week more or less than 10^{6} (one million) seconds? How does the exponent settle it without any long division?

Q12. Write each length in metres, in scientific notation: (a) 7 km; (b) 25 mm; (c) 3 μm; (d) 450 nm.

Q13. The light-minute. (a) Light travels at about 3.0×10^{8} m/s; find the distance it travels in 60 s, in scientific notation. (b) SCENARIO: a space probe is 4.5×10^{10} m from Earth. How many light-minutes away is that?

Q14. SCENARIO — the school laboratory of the theory figure. (a) The measuring cylinder's smallest graduation is 1 mL; write this in litres in scientific notation. (b) The pipette reads to 0.1 mL; write this in litres in scientific notation. (c) Record the pipette reading of 18.5 mL in litres, in scientific notation. (d) Which instrument gives the more accurate reading, and how many times smaller is its smallest graduation?

Q15. Arrange from smallest to largest: 2×10^{-4}, 1×10^{-3}, 3×10^{-5}, 5.5×10^{-5}.

Q16. SCENARIO: a school canteen uses 2.4×10^{4} bottles in a day, each holding 5×10^{-2} L. (a) Find the volume of drink used per day, in litres, in scientific notation. (b) Hence find the volume used over 3×10^{2} days, in scientific notation.

Q17. SCENARIO assumption: a resting heart beats about once per second. Estimate the number of heartbeats in one year of 365 days, in scientific notation, and state the assumption in your answer.

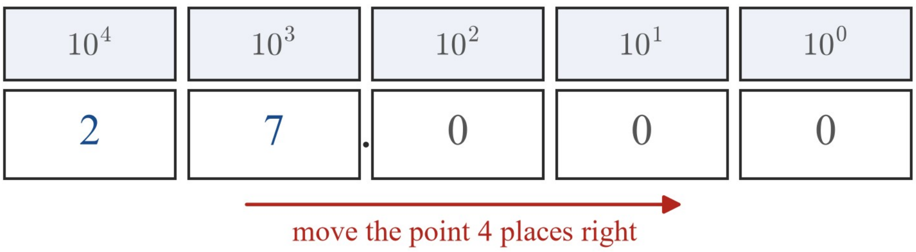
Q18. Find the integer n such that $\left(2\times 10^{n}\right)\times\left(5\times 10^{4}\right)=10^{9}$.

Answers

Q1 a (i) $a=5.2$, $n=3$; (ii) $a=7$, $n=-2$; (iii) $a=1.06$, $n=5$; each a sits between 1 and 10 and each n is an integer. b (i) 5200; (ii) 0.07; (iii) 106 000. **Q2** a 4.2×10^{4}; b 7.1×10^{-3}; c 9×10^{0}; d 6×10^{3}. **Q3** a 5.2×10^{4}; b 9.9×10^{-7}; c already correct: $1\le3.0<10$ and 0 is an integer; d 5×10^{5}. **Q4** a 8×10^{8}; b 1.5×10^{5}; c 4.2×10^{7}. **Q5** a 4×10^{6}; b 3×10^{6}; c 1.25×10^{-2}. **Q6** $3.1\times10^{-4}<9\times10^{3}<3.1\times10^{4}<1.2\times10^{5}$. **Q7** a 1.5×10^{8}; b 5.08×10^{-5}; c 1×10^{9}. **Q8** a 0.000 022; b 602 214 076 000 000 000 000 000; c 4 600 000 000. **Q9** a 4.5×10^{5}; b 2.73×10^{-4}; c 9.2×10^{-2}. **Q10** a 1.2×10^{5}; b 8×10^{2}; c 1.6×10^{-6}. **Q11** a 6.048×10^{5} s; b 10^{3}; c less: $6.048\times10^{5}<10^{6}$, since the exponent 5 is below 6. **Q12** a 7×10^{3} m; b 2.5×10^{-2} m; c 3×10^{-6} m; d 4.5×10^{-7} m. **Q13** a 1.8×10^{10} m; b 2.5 light-minutes. **Q14** a 10^{-3} L; b 10^{-4} L; c 1.85×10^{-2} L; d the pipette; its smallest graduation is 10 times smaller. **Q15** $3\times10^{-5}<5.5\times10^{-5}<2\times10^{-4}<1\times10^{-3}$. **Q16** a 1.2×10^{3} L; b 3.6×10^{5} L. **Q17** 3.1536×10^{7} beats (assumption: one beat per second). **Q18** $n=4$.

Fully-worked solutions

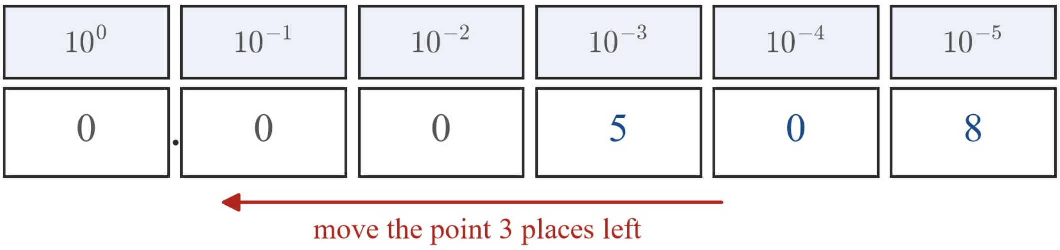
Q1 a (i) $a=5.2$ (between 1 and 10) and $n=3$ (an integer); (ii) $a=7$, $n=-2$; (iii) $a=1.06$, $n=5$. Both tests pass in every case, so all three are in scientific notation. b Move the point n places: (i) 3 right: $5.2\times10^{3}=5200$; (ii) 2 left: $7\times10^{-2}=0.07$; (iii) 5 right: $1.06\times10^{5}=106\,000$.



$2.7\times10^{4}=27\,000$ — the 4 sends the digits 4 places to the left, filling empty places with zeros.

Place-value chart with columns 10 to the 4 down to 10 to the 0 holding the digits 2, 7, 0, 0, 0 and a red arrow showing the decimal point moving 4 places right: 2.7 times 10 to the 4 = 27 000

Q2 a The point moves 4 left: $42\,000=4.2\times10^{4}$. b The point moves 3 right: $0.0071=7.1\times10^{-3}$. c No move is needed: $9=9\times10^{0}$. d 0.6×10^{4}: shift the point one right to make $a=6$, which subtracts 1 from the exponent: 6×10^{3}.



$5.08 \times 10^{-3} = 0.005\,08$ — the -3 sends the digits 3 places to the right, filling empty places with zeros.

Place-value chart with columns 10 to the 0 down to 10 to the minus 5 holding the digits 0.005 08 and a red arrow showing the decimal point moving 3 places left: 5.08 times 10 to the minus 3 = 0.005 08

Q3 a $a = 52 \geq 10$: shift one place, $52 \times 10^3 = 5.2 \times 10^4$. b $a = 0.99 < 1$: shift one place, $0.99 \times 10^{-6} = 9.9 \times 10^{-7}$. c 3.0×10^0 passes both tests ($1 \leq 3.0 < 10$, $n = 0$ an integer) — it is already scientific notation, and equals 3. d $a = 0.5 < 1$: shift one place, $0.5 \times 10^6 = 5 \times 10^5$.

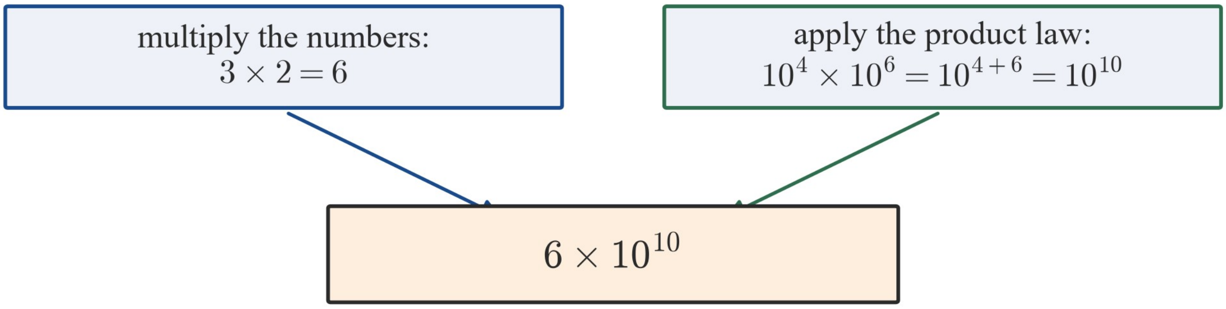
Scientific notation needs $1 \leq a < 10$ and an integer exponent n.

×	34×10^3	$a = 34$ is 10 or more — shift the point: 3.4×10^4
×	0.5×10^6	$a = 0.5$ is under 1 — shift the point: 5×10^5
×	$4.2 \times 10^{2.5}$	the exponent 2.5 is not an integer — not scientific notation
✓	7.1×10^{-4}	$1 \leq 7.1 < 10$ and -4 is an integer — correct

Four rows testing the two rules: 34 times 10 to the 3 fails because a = 34 is 10 or more and becomes 3.4 times 10 to the 4; 0.5 times 10 to the 6 fails because a = 0.5 is under 1 and becomes 5 times 10 to the 5; 4.2 times 10 to the 2.5 fails because the exponent is not an integer; 7.1 times 10 to the minus 4 passes both tests

Q4 a $(2 \times 4) \times 10^{3+5} = 8 \times 10^8$. b $(3 \times 5) \times 10^{-2+6} = 15 \times 10^4 = 1.5 \times 10^5$, normalised with one shift. c $(6 \times 7) \times 10^{4+2} = 42 \times 10^6 = 4.2 \times 10^7$.

$$(3 \times 10^4) \times (2 \times 10^6)$$



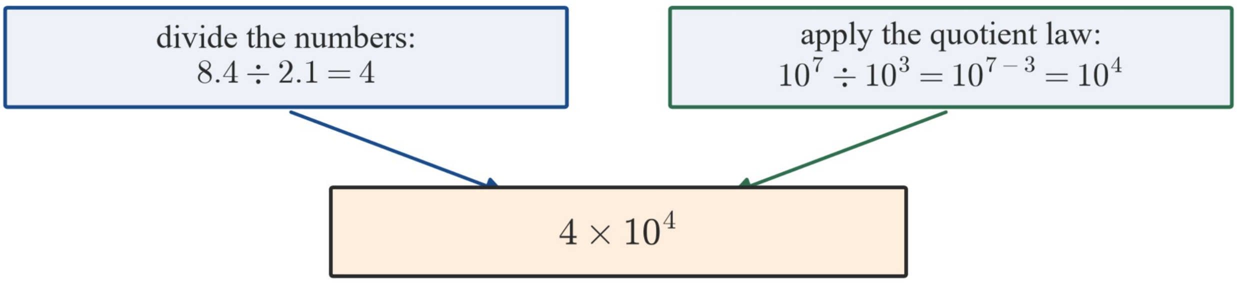
6 already sits between 1 and 10, so the answer is in scientific notation.

Check: $3 \times 10^4 = 30\,000$ and $2 \times 10^6 = 2\,000\,000$; their product is $60\,000\,000\,000 = 6 \times 10^{10}$.

Flow diagram: (3 times 10 to the 4) times (2 times 10 to the 6) splits into two boxes, multiply the numbers 3 times 2 = 6 and apply the product law 10 to the 4 times 10 to the 6 = 10 to the 10, which combine to 6 times 10 to the 10, with a decimal check line below

Q5 a $(8 \div 2) \times 10^{9-3} = 4 \times 10^6$. b $(9 \div 3) \times 10^{4-(-2)} = 3 \times 10^6$. c $(5 \div 4) \times 10^{3-5} = 1.25 \times 10^{-2}$, already between 1 and 10.

$$(8.4 \times 10^7) \div (2.1 \times 10^3)$$



When the number part falls outside 1 to 10, fix it at the end:

$$(6 \times 10^5) \div (8 \times 10^2) = 0.75 \times 10^3$$

$$\downarrow$$

0.75 is under 1, so shift the point one place: $0.75 \times 10^3 = 7.5 \times 10^2$

Both equal 750 — the shift changes the writing, not the number.

Flow diagram: (8.4 times 10 to the 7) divided by (2.1 times 10 to the 3) splits into divide the numbers 8.4 divided by 2.1 = 4 and apply the quotient law 10 to the 7 divided by 10 to the 3 = 10 to the 4, combining to 4 times 10 to the 4;

below, the normalising case (6 times 10 to the 5) divided by (8 times 10 to the 2) = 0.75 times 10 to the 3 rewritten as 7.5 times 10 to the 2

Q6 a 3.1×10^{-4} is the only one with a negative exponent, so it is the only one under 1 — the smallest. b The rest have exponents 3, 4, 5: 9×10^3, then 3.1×10^4, then 1.2×10^5. c $3.1 \times 10^{-4} < 9 \times 10^3 < 3.1 \times 10^4 < 1.2 \times 10^5$.

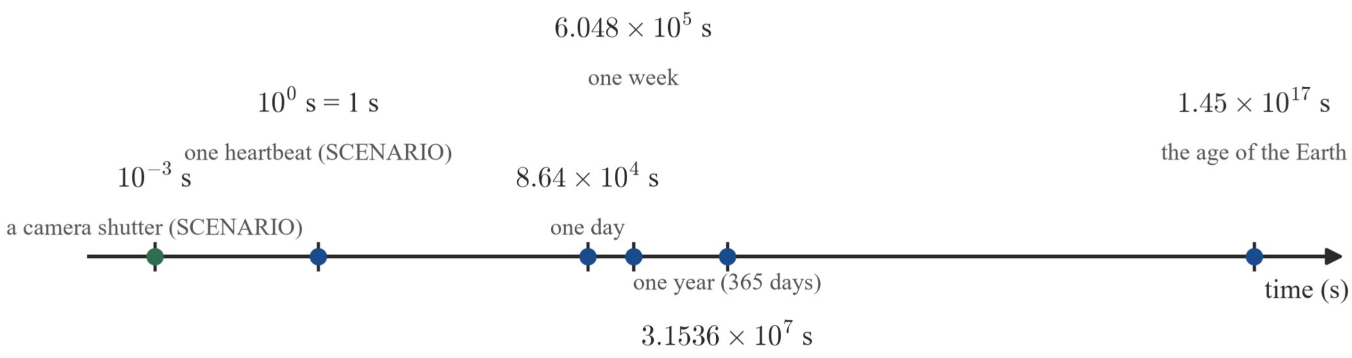
Q7 a Point 8 left: 1.5×10^8. b Point 5 right: 5.08×10^{-5}. c Point 9 left: 1×10^9.

Q8 a Point 5 left: $0.000\,022$. b Point 23 right: $602\,214\,076\,000\,000\,000\,000\,000$ — the digits 602214076 followed by 15 zeros. c Point 9 right: $4\,600\,000\,000$.

Q9 a $0.0045 \times 10^8 = 4.5 \times 10^{-3} \times 10^8 = 4.5 \times 10^5$. b $27.3 \times 10^{-5} = 2.73 \times 10^1 \times 10^{-5} = 2.73 \times 10^{-4}$. c $92 \times 10^{-3} = 9.2 \times 10^1 \times 10^{-3} = 9.2 \times 10^{-2}$.

Q10 a $(3 \times 4) \times 10^{6+(-2)} = 12 \times 10^4 = 1.2 \times 10^5$. b $(7.2 \div 9) \times 10^{5-2} = 0.8 \times 10^3 = 8 \times 10^2$. c $(2 \times 8) \times 10^{-3+(-4)} = 16 \times 10^{-7} = 1.6 \times 10^{-6}$.

Q11 a $604\,800 = 6.048 \times 10^5$ s. b $1 \div 10^{-3} = 10^3$ shutter-times in one second. c Less. 10^6 has exponent 6 and the week has exponent 5, so $6.048 \times 10^5 < 10^6$ without any further working — a week is a little over half a million seconds, not a million.



Each step is a factor of 10. The spacing on this line is by powers of 10, so the age of the Earth sits only about 10 marks from a single heartbeat.

Horizontal line of time in seconds spaced by powers of 10, marked at 10 to the minus 3 s (a camera shutter, SCENARIO), 1 s (one heartbeat, SCENARIO), 8.64 times 10 to the 4 s (one day), 6.048 times 10 to the 5 s (one week), 3.1536 times 10 to the 7 s (one year of 365 days) and 1.45 times 10 to the 17 s (the age of the Earth)

Q12 Replace each prefix with its power of 10: a kilo $= 10^3$, so 7 km $= 7 \times 10^3$ m. b milli $= 10^{-3}$, so 25 mm $= 25 \times 10^{-3}$ m $= 2.5 \times 10^{-2}$ m. c micro $= 10^{-6}$, so 3 μm $= 3 \times 10^{-6}$ m. d nano $= 10^{-9}$, so 450 nm $= 450 \times 10^{-9}$ m $= 4.5 \times 10^{-7}$ m.

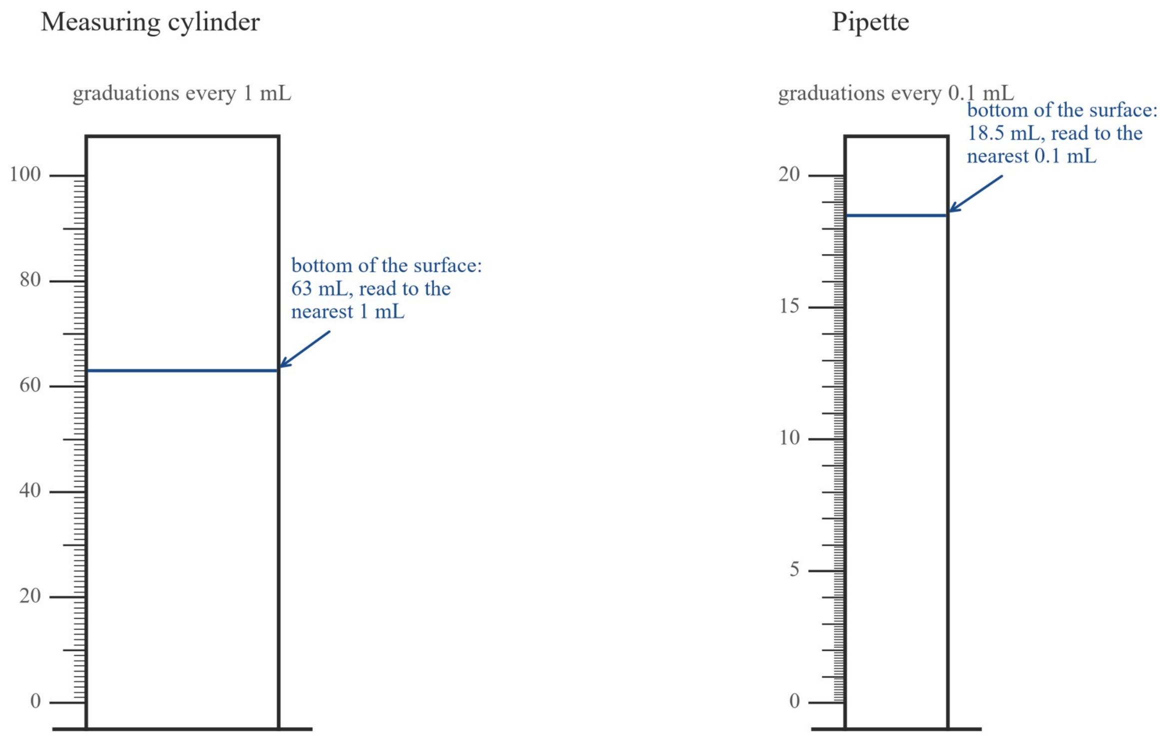
Power of 10	*Prefix*	*Rewritten in scientific notation*
10^{-12}	pico (p)	$3\,\text{pm} = 3 \times 10^{-12}$ m
10^{-9}	nano (n)	$450\,\text{nm} = 4.5 \times 10^{-7}$ m
10^{-6}	micro (μ)	$3\,\mu\text{m} = 3 \times 10^{-6}$ m
10^{-3}	milli (m)	$25\,\text{mm} = 2.5 \times 10^{-2}$ m
10^{0}	(none)	—
10^{3}	kilo (k)	$7\,\text{km} = 7 \times 10^{3}$ m

A prefix is a shortcut for a power of 10 — removing it leaves a number in scientific notation.

Table from 10 to the minus 12 (pico) to 10 to the 3 (kilo) with one worked conversion per row: 3 pm = 3 times 10 to the minus 12 m, 450 nm = 4.5 times 10 to the minus 7 m, 3 micrometres = 3 times 10 to the minus 6 m, 25 mm = 2.5 times 10 to the minus 2 m, 7 km = 7 times 10 to the 3 m

Q13 a $(3.0 \times 10^{8}) \times 60 = (3.0 \times 6) \times 10^{9} = 18 \times 10^{9} = 1.8 \times 10^{10}$ m. b $(4.5 \times 10^{10}) \div (1.8 \times 10^{10}) = (4.5 \div 1.8) \times 10^{0} = 2.5$. The probe is 2.5 light-minutes from Earth.

Q14 a 1 mL $= 10^{-3}$ L. b 0.1 mL $= 10^{-4}$ L. c 18.5 mL $= 18.5 \times 10^{-3}$ L $= 1.85 \times 10^{-2}$ L. d The pipette: 10^{-4} L is one tenth of 10^{-3} L, so its graduations are 10 times smaller — the cylinder cannot even show the 0.5 in 18.5.



A measuring cylinder with a graduation every 1 mL reading 63 mL beside a narrow pipette with a graduation every 0.1 mL reading 18.5 mL, each reading taken at the bottom of the water's surface

Q15 Compare exponents first: $-5, -5, -4, -3$. The two with exponent -5 compare by their number parts: $3 < 5.5$. So $3 \times 10^{-5} < 5.5 \times 10^{-5} < 2 \times 10^{-4} < 1 \times 10^{-3}$.

Q16 a $(2.4\times10^4)\times(5\times10^{-2})=(2.4\times5)\times10^2=12\times10^2=1.2\times10^3$ L per day. b $(1.2\times10^3)\times(3\times10^2)=3.6\times10^5$ L over the 3×10^2 days.

Q17 At one beat per second, the number of beats in a year equals the number of seconds in a year of 365 days: $31\,536\,000=3.1536\times10^7$. ∴ About 3.1536×10^7 heartbeats, on the stated assumption.

Length (m)

10^{-12} pm $\rightarrow$ 10^{-9} nm $\rightarrow$ 10^{-6} μm $\rightarrow$ 10^{-3} mm $\rightarrow$ 1 (about a stride)
$\rightarrow$ 10^3 km $\rightarrow$ 9.5×10^{15} (one light-year)

Time (s)

10^{-3} (a camera shutter) $\rightarrow$ 1 (one heartbeat) $\rightarrow$ 8.64×10^4 (one day)
$\rightarrow$ 3.1536×10^7 (one year) $\rightarrow$ 1.45×10^{17} (the age of the Earth)

Mass (g)

2.99×10^{-22} (one glucose molecule) $\rightarrow$ 10^{-3} (one milligram)
$\rightarrow$ 1 (a paperclip, SCENARIO) $\rightarrow$ 10^3 (one kilogram)

Each arrow moves to a larger power of 10; the exponent says how far the measurement sits from 1.

Three rows, for length in metres, time in seconds and mass in grams, each listing anchors from smallest to largest power of 10: from 10 to the minus 12 (the picometre) to 9.5 times 10 to the 15 (one light-year); from 10 to the minus 3 (a camera shutter) to 1.45 times 10 to the 17 (the age of the Earth); from 2.99 times 10 to the minus 22 (one glucose molecule) to 10 to the 3 (one kilogram)

Q18 $(2\times10^n)\times(5\times10^4)=(2\times5)\times10^{n+4}=10\times10^{n+4}=10^{n+5}$. Setting $10^{n+5}=10^9$ gives $n+5=9$, so $n=4$. Check: $(2\times10^4)\times(5\times10^4)=10\times10^8=10^9$. ∴ $n=4$.

Spatial problems: angle properties, scale, similarity, ratio, Pythagoras' theorem and trigonometry

5C · Chapter 5 — Measurement · Level 9

Content description `VC2M9M03` (word for word): *solve spatial problems, applying angle properties, scale, similarity, ratio, Pythagoras' theorem and trigonometry in right-angled triangles*

Achievement standard anchors (AS 9 and AS 11 — AS 11 orphan restored, D7): *They solve problems involving ratio, similarity and scale in two-dimensional situations.* (AS 9) *Students apply Pythagoras' theorem and use trigonometric ratios to solve problems involving right-angled triangles.* (AS 11)

Elaboration ids for linkage: `VC2M9M03.E01`, `VC2M9M03.E02`, `VC2M9M03.E03`, `VC2M9M03.E04`, `VC2M9M03.E05`, `VC2M9M03.E06`

Prerequisite pointer: 4A derives the trigonometric ratios this subtopic applies; 4B supplies the scale factor and the $k \to k^2, k^3$ rule for areas and volumes; 2B supplies the distance result on the plane. All three are assumed here — they are rehearsed as tools, not re-taught. Pythagoras' theorem itself is assumed from Level 8 (`VC2M8M06`) and is likewise restated, not re-taught.

Scope guard: every triangle in this subtopic is right-angled. Angles of elevation and depression, and bearings, belong to Level 10 (`VC2M10M03`) and never appear here; the sine and cosine rules belong to VCE and never appear here. Where `VC2M9M03.E05` invites investigation of conjectures, we conjecture and test; formal proof belongs to Level 10.

Note on sources: every context in this subtopic is a SCENARIO — an invented situation whose numbers are given in the question. No real-world data is quoted, so no source lines are owed.

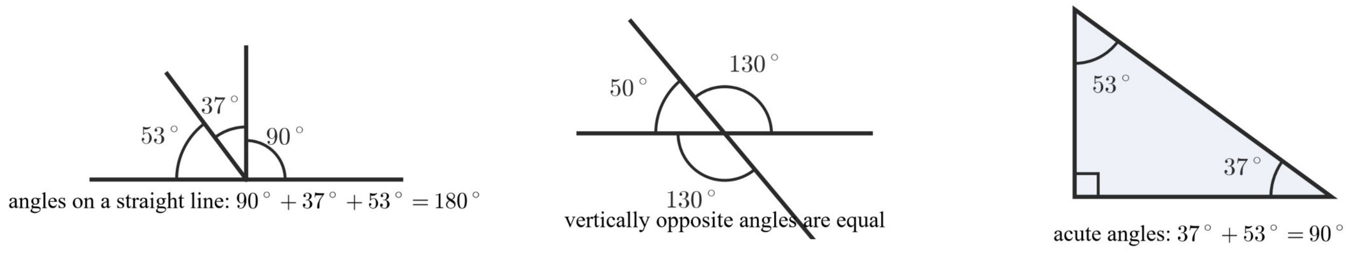
Note on digital tools: a scientific calculator is used for the trigonometric ratios and their inverses; everything else is by hand.

Theory

The angle facts we lean on

Three facts about angles do quiet work in almost every spatial problem. They are Level 8 tools, restated here so that every working below can lean on them.

- **Angles on a straight line add to 180°.**
- **Vertically opposite angles are equal.**
- **The two acute angles of a right-angled triangle add to 90°** — the three angles of any triangle add to 180°, and the right angle has already taken 90°.



Three panels: angles 90 degrees, 37 degrees and 53 degrees on a straight line; vertically opposite angles 130 degrees and 50 degrees at crossing lines; a right-angled triangle whose acute angles are 37 degrees and 53 degrees

In the first panel, $90° + 37° + 53° = 180°$; in the second, each 130° angle sits opposite the other, as do the two 50° angles; in the third, $37° + 53° = 90°$.

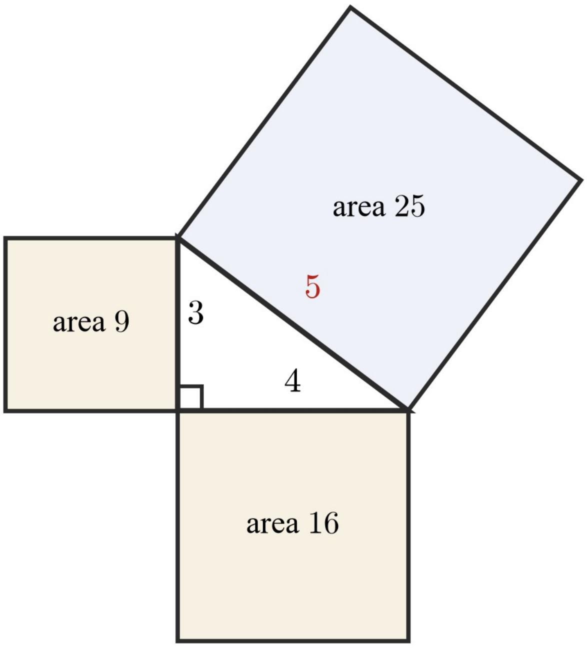
Pythagoras' theorem, restated

In a right-angled triangle, the side opposite the right angle is the **hypotenuse** — always the longest side. Call it c, and call the two shorter sides a and b. Pythagoras' theorem says

$$a^2 + b^2 = c^2$$

— the square on the hypotenuse equals the sum of the squares on the other two sides. To find a shorter side, rearrange first and then take one square root: $a^2 = c^2 - b^2$.

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$



Right-angled triangle with sides 3 and 4 and hypotenuse 5, with a square of area 9 on one side, a square of area 16 on the other, and a square of area 25 on the hypotenuse

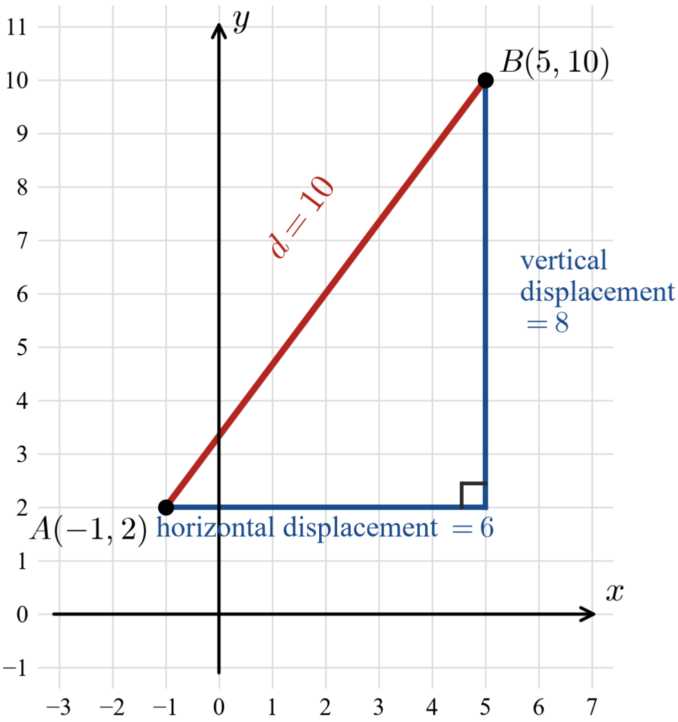
The picture is the theorem itself for the $(3, 4, 5)$ triangle: $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Distance on the Cartesian plane

Given two points on the plane, the **horizontal displacement** (how far across) and the **vertical displacement** (how far up) are the two shorter sides of a right-angled triangle, and the straight-line distance between the points is its hypotenuse. So

$$d^2 = (\text{horizontal displacement})^2 + (\text{vertical displacement})^2$$

which is the distance result from 2B, restated. 2B owns it; 5C applies it.

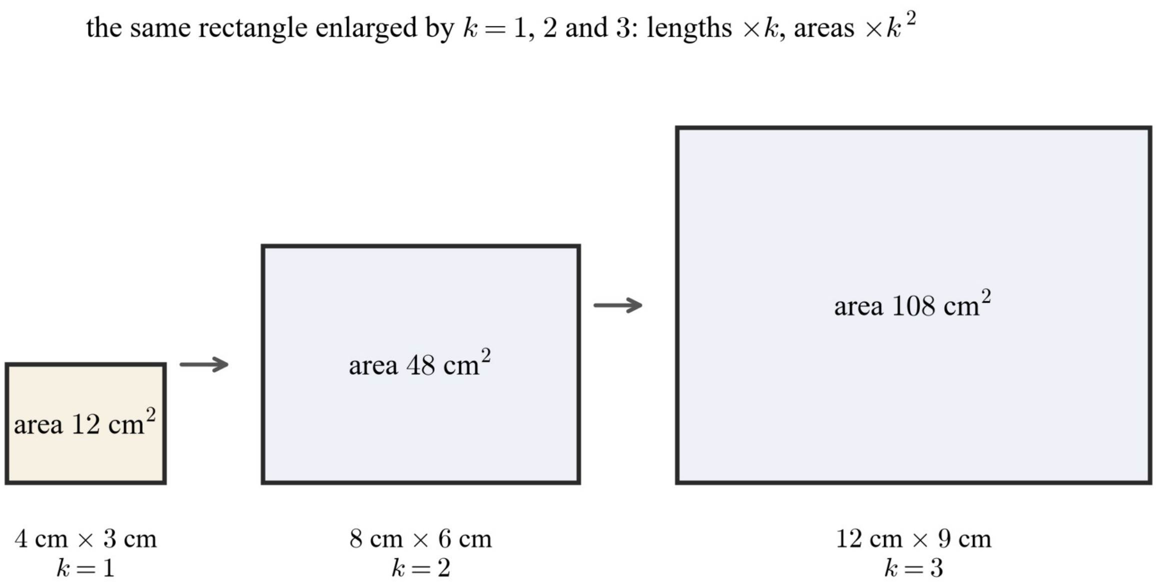


Cartesian plane with A at (-1, 2) and B at (5, 10); the horizontal displacement is 6, the vertical displacement is 8, and the distance from A to B is 10

From $A(-1,2)$ to $B(5,10)$: across, $5-(-1)=6$; up, $10-2=8$; and $6^2+8^2=36+64=100$, so $d=10$.

Scale factor: lengths, areas and volumes

An enlargement by **scale factor** k multiplies every length by k. Because area is length × length, it multiplies every area by k^2; because volume is length × length × length, it multiplies every volume by k^3. Both facts come from 4B and are used here without re-derivation.

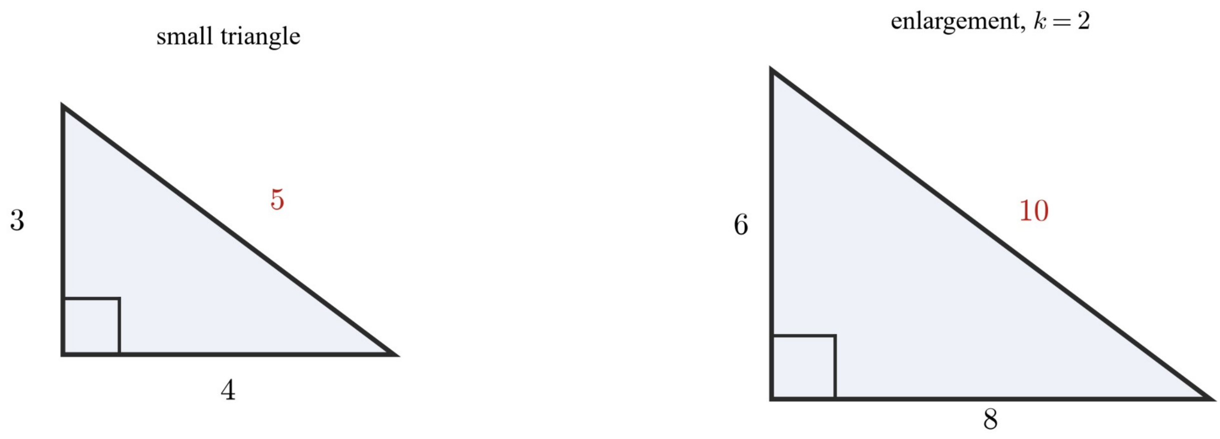


A 4 cm by 3 cm rectangle enlarged by scale factors 1, 2 and 3, giving areas 12, 48 and 108 square cm

The same 4 cm × 3 cm rectangle at $k=1,2,3$ has areas 12, 48 and 108 cm²: lengths ×2 gives area ×4, and lengths × 3 gives area ×9.

Similar figures and similar right-angled triangles

Two figures are **similar** when one is an enlargement of the other: corresponding angles are equal and corresponding sides are in the same ratio, the scale factor k. For similar triangles, finding k is one division of corresponding sides, and every other pair of sides then gives the same ratio.



$$\text{corresponding sides: } 6 \div 3 = 8 \div 4 = 10 \div 5 = 2$$

Two similar right-angled triangles, one with sides 3, 4 and 5 and its enlargement with sides 6, 8 and 10; each side of the larger is twice the matching side of the smaller

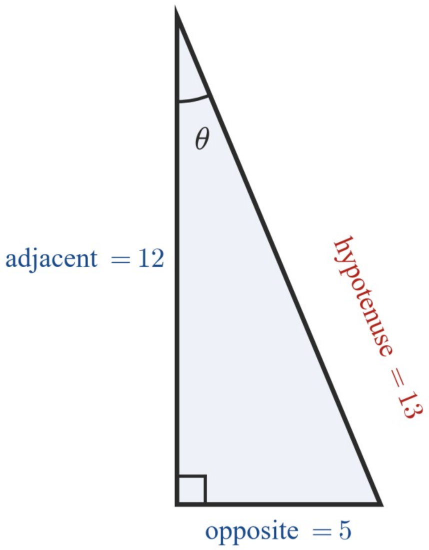
Here $6 \div 3 = 8 \div 4 = 10 \div 5 = 2$, so the larger triangle is the smaller enlarged by $k = 2$ — and its area is $2^2 = 4$ times as big.

The trigonometric ratios, restated

In a right-angled triangle, pick one acute angle and call it θ. The **opposite** side is across from θ; the **adjacent** side is next to θ; the hypotenuse is opposite the right angle. 4A showed that the three ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

depend only on the angle, not on the size of the triangle — similar triangles have the same ratios, and that constancy is what makes the ratios useful.



sine $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{5}{13}$

cosine $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{12}{13}$

tangent $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{5}{12}$

Right-angled triangle with sides 5, 12 and 13 and angle theta at the top, beside the three ratios: sine theta equals 5 over 13, cosine theta equals 12 over 13, tangent theta equals 5 over 12

To find a **side**, pick the ratio that uses what you know and what you want, then solve the equation. To find an **angle** from a ratio, use the inverse button on the calculator: if $\sin\theta = 0.5$, then $\theta = \sin^{-1}(0.5) = 30°$.

Choosing the right tool

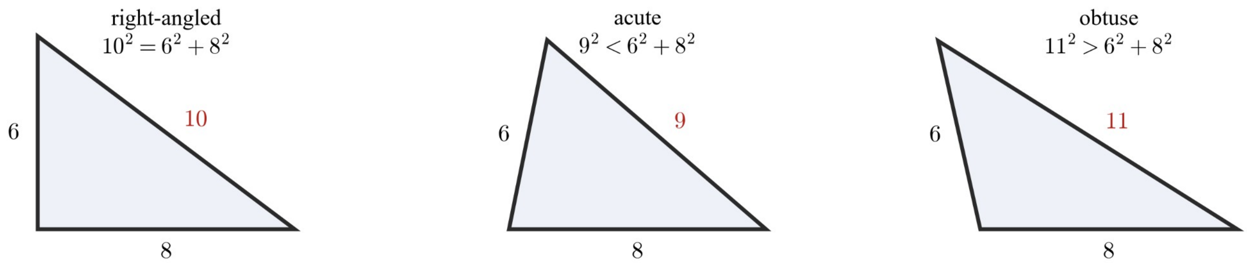
Most spatial problems announce their own method. A working checklist:

1. Only angles, no sides wanted — the angle facts.
2. Two sides known, third side wanted — Pythagoras' theorem.
3. One side and one angle known, another side or angle wanted — a trigonometric ratio.
4. A map, a model, a photo, an enlargement — scale and similarity.
5. Two points on a grid — displacements, then Pythagoras' theorem.
6. A height or width you cannot measure directly — similar triangles in the field (below).

Testing a triangle: right-angled, acute or obtuse

Run Pythagoras' theorem backwards as a test. Call the longest side L and the other two a and b, and compare L^2 with $a^2 + b^2$:

- $L^2 = a^2 + b^2$ — **right-angled**, opposite the longest side;
- $L^2 < a^2 + b^2$ — **acute** (all three angles less than 90°);
- $L^2 > a^2 + b^2$ — **obtuse** (the angle opposite the longest side is more than 90°).



Three triangles sharing sides 6 and 8: with third side 10 the triangle is right-angled because 10 squared equals 6 squared plus 8 squared; with 9 it is acute because 9 squared is less; with 11 it is obtuse because 11 squared is more

The first line of the test is the **converse** of Pythagoras' theorem — the theorem run from sides back to angles.

How much information is enough?

SCENARIO: a triangle is drawn but smudged, and only some of its measures survive. Which measures pin the triangle down completely? Conjecture, then test by constructing:

- **Two sides of a right-angled triangle** fix the third (Pythagoras) and then fix every angle (a ratio).
- **One side and one acute angle of a right-angled triangle** fix everything else: the right angle and the angle facts give the third angle, and the ratios give the sides.
- **Two angles only** fix the *shape* but not the size — every triangle with those angles is similar to every other, and the size is still free.

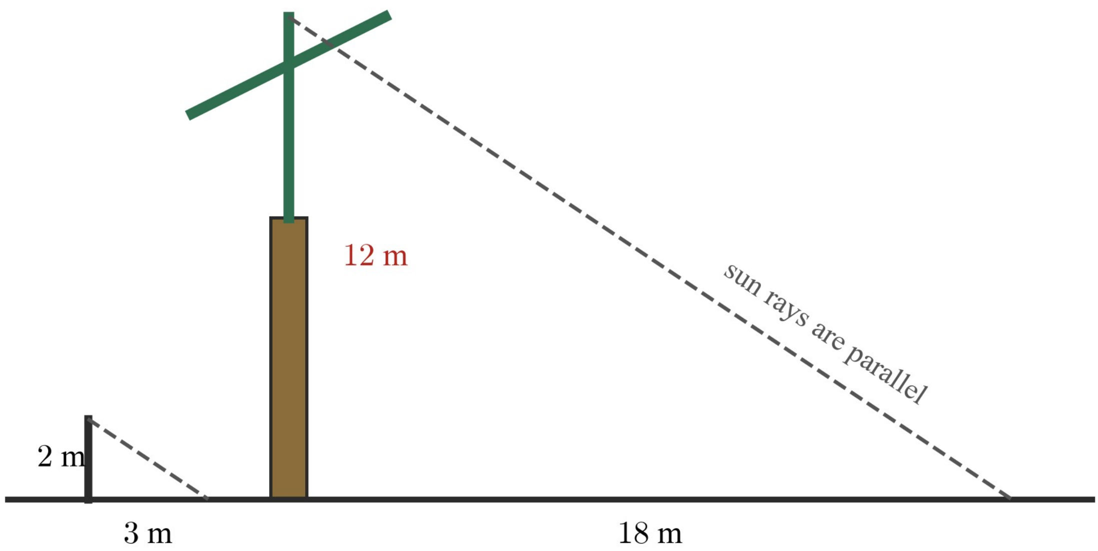
Testing the conjecture on drawn cases confirms all three claims. That is conjecture-and-test; a formal proof that these are exactly the minimal sets belongs to Level 10.

Measuring without a tape: similarity in the field

Some lengths cannot be measured directly — the height of a tree, the width of a creek. Similar triangles carry the measure across.

Shadows. The sun is so far away that its rays reach us parallel. A pole and a tree standing on level ground therefore make two similar right-angled triangles with their shadows, and one ratio reads the tree's height.

$$\text{similar triangles: } h \div 18 = 2 \div 3, \text{ so } h = 12 \text{ m}$$

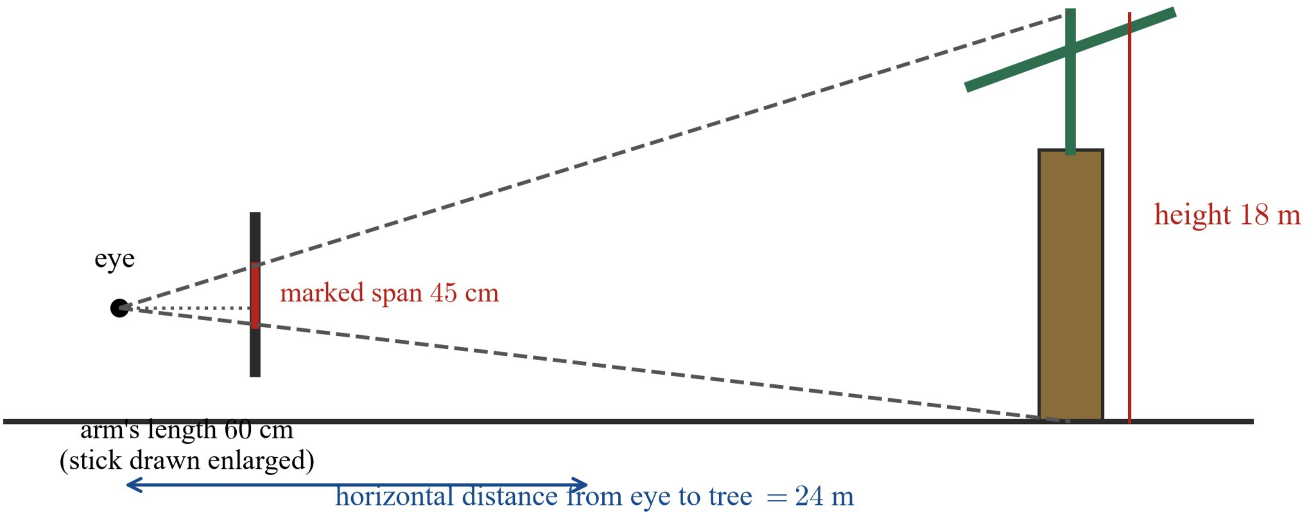


A 2 m pole casting a 3 m shadow beside a tree casting an 18 m shadow under parallel sun rays; similar triangles give a tree height of 12 m

Photographs. A photograph records everything in it at one scale. If a person of known height stands in front of a tree, the ratio of the two heights in the photo is the ratio in reality — an estimate of the tree's height from a single image.

A Biltmore stick. Hold a marked stick at arm's length and sight a tree past it. The sight lines from your eye make two similar triangles: one on the stick, one on the tree. The marked span on the stick, the arm's length, and the paced distance to the tree turn one division and one multiplication into the tree's diameter or height.

$$\text{similar triangles: } 24 \times 45 \div 60 = 18 \text{ m}$$



An eye sighting the top and the base of a tree 24 m away past a stick held at arm's length 60 cm; the marked span of 45 cm on the stick corresponds to the tree's height of 18 m

In the picture, span ÷ arm's length $= 45 \div 60$, and the tree's height is $24 \times 45 \div 60 = 18$ m. Notice what is *not* in any of the three methods: no angle of the sun or of the sight line is ever named. The working is a ratio of corresponding sides, which is exactly where Level 9 leaves it.

Worked examples

Example 1 — scaffolded

A rectangular lawn is 24 m long and 10 m wide. A path runs straight along a diagonal. How long is the path?

Working	Comment
The diagonal is the hypotenuse of a right-angled triangle with shorter sides 24 and 10.	The length and the width meet at a right angle.
$d^2 = 24^2 + 10^2$	Pythagoras' theorem.
$d^2 = 576 + 100 = 676$	Square, then add.
$d = 26$	One square root; $26^2 = 676$.

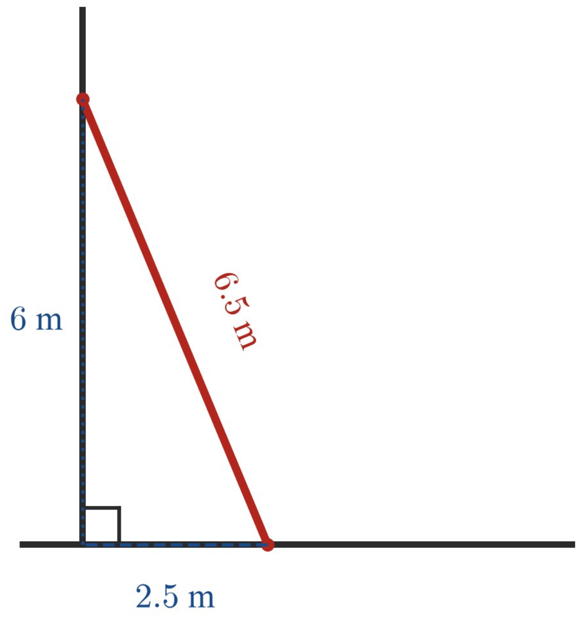
∴ The path is 26 m long — shorter than the $24 + 10 = 34$ m walk around the two sides.

Example 2 — scaffolded

A 6.5 m ladder leans against a wall. Its foot is 2.5 m out from the wall. How far up the wall does the ladder reach?

Working	Comment
The ladder is the hypotenuse; the wall height h is a shorter side.	Wall and ground meet at a right angle.
$h^2 = 6.5^2 - 2.5^2$	Rearrange before taking the root.
$h^2 = 42.25 - 6.25 = 36$	Square, then subtract.
$h = 6$	$6^2 = 36$.

∴ The ladder reaches 6 m up the wall.



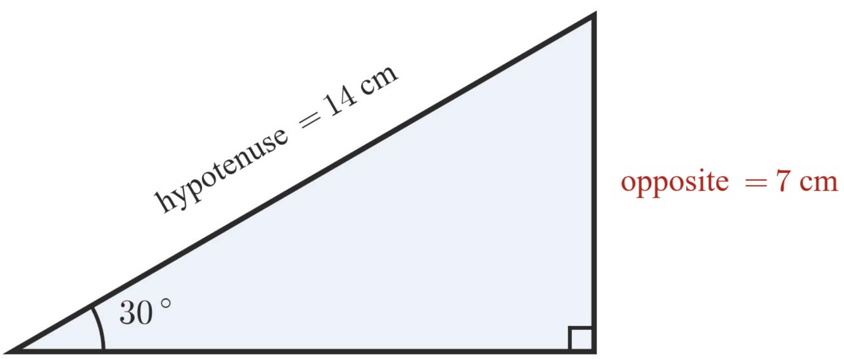
A ladder 6.5 m long leaning against a wall, its foot 2.5 m from the wall, reaching 6 m up the wall

Example 3 — scaffolded

In a right-angled triangle, one angle is 30° and the hypotenuse is 14 cm. Find the side opposite the 30° angle.

Working	Comment
$\sin 30° = \frac{\text{opposite}}{14}$	Sine uses opposite and hypotenuse — the pair involved.
$\text{opposite} = 14 \times \sin 30°$	Multiply both sides by 14.
$\text{opposite} = 14 \times 0.5 = 7$	$\sin 30° = 0.5$ exactly.

∴ The opposite side is 7 cm — exactly half the hypotenuse, as $\sin 30° = 0.5$ says it must be.



Right-angled triangle with a 30 degree angle, hypotenuse 14 cm, and the side opposite the 30 degree angle equal to 7 cm

Example 4 — scaffolded

SCENARIO: a map has a scale of 1:25 000. Two towns are 6.4 cm apart on the map. How far apart are they in reality? Give your answer in kilometres.

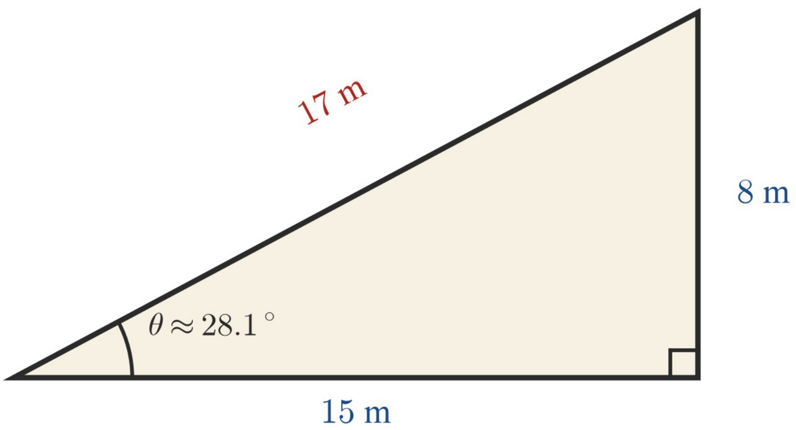
Working	Comment
Reality $= 6.4 \times 25\,000$ cm	The scale factor multiplies map lengths.
$= 160\,000$ cm	$6.4 \times 25\,000 = 160\,000$.
$= 1600$ m $= 1.6$ km	Divide by 100, then by 1000.

∴ The towns are 1.6 km apart in reality.

Example 5 — unscaffolded

SCENARIO: a ramp is built across a horizontal run of 15 m, and its sloping surface is 17 m long. Find the height the ramp rises, and the angle the ramp makes with the ground (to 1 decimal place).

The run and the rise meet at a right angle, and the sloping surface is the hypotenuse. $\text{rise}^2 = 17^2 - 15^2$. $\text{rise}^2 = 289 - 225 = 64$. $\text{rise} = 8$ m. For the angle θ between the ramp and the ground, the run is adjacent and the slope is the hypotenuse: $\cos\theta = 15 \div 17$. $\theta = \cos^{-1}(15 \div 17) \approx 28.07°$. ∴ The ramp rises 8 m and makes an angle of about 28.1° with the ground.



Right-angled triangle modelling the ramp: horizontal run 15 m, rise 8 m, sloping surface 17 m, and the angle between the ramp and the ground about 28.1 degrees

Example 6 — unscaffolded

A photograph 9 cm by 6 cm is enlarged so that its width becomes 36 cm. Find the height of the enlargement, and compare the two areas.

The enlargement is similar to the original, so $k = 36 \div 9 = 4$. Height $= 6 \times 4 = 24$ cm. Original area $= 9 \times 6 = 54$ cm². Enlarged area $= 36 \times 24 = 864$ cm². $864 \div 54 = 16$, and $k^2 = 4^2 = 16$ — the area grew by k^2, exactly as 4B says it must. ∴ The enlargement is 36 cm by 24 cm, and its area is 16 times the original area.

Example 7 — unscaffolded

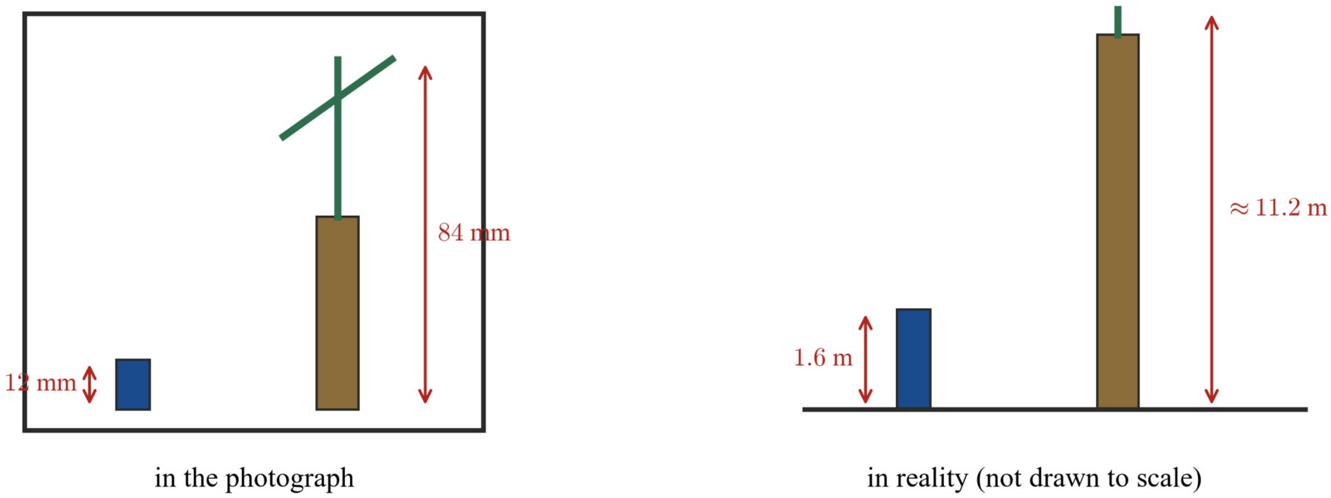
Find the distance between $A(-3, -2)$ and $B(5, 4)$.

Horizontal displacement: $5 - (-3) = 8$. Vertical displacement: $4 - (-2) = 6$. $d^2 = 8^2 + 6^2 = 64 + 36 = 100$. $d = 10$. ∴ The distance from A to B is 10 units.

Example 8 — unscaffolded

SCENARIO: in a photograph, a person of height 1.6 m measures 12 mm, and a tree behind them measures 84 mm. Estimate the height of the tree, and give one way to improve the estimate.

The photo records both at the same scale, so the ratio of the real heights equals the ratio of the photo heights. $84 \div 12 = 7$ — the tree measures 7 times the person in the photo. Tree height $\approx 7 \times 1.6 = 11.2$ m. One improvement: take the photo square-on, with the person standing at the base of the tree, so that both are the same distance from the camera — otherwise the farther object looks smaller and the estimate runs low. $\therefore$ The tree is about 11.2 m tall.



$$84 \div 12 = 7\text{, so the tree is about } 7 \times 1.6 = 11.2 \text{ m tall}$$

Two panels. In the photograph the person measures 12 mm and the tree 84 mm; in reality the person is 1.6 m and the tree about 11.2 m

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Practice questions

Scaffolded

Q1. Three angles on a straight line are 90°, 37° and x. (a) What do the three angles add to? (b) Hence find x.

Q2. One acute angle of a right-angled triangle is 38°. (a) What do the two acute angles add to? (b) Hence find the other acute angle.

Q3. The two shorter sides of a right-angled triangle are 9 cm and 12 cm. (a) Write c^2 as a sum of two squares. (b) Hence find the hypotenuse c.

Q4. The hypotenuse of a right-angled triangle is 13 cm and one other side is 5 cm. (a) Rearrange Pythagoras' theorem to write the unknown side squared as a difference. (b) Hence find the unknown side.

Q5. Is a triangle with sides 7 cm, 24 cm and 25 cm right-angled? (a) Which side must be L? (b) Compute a^2+b^2 and L^2, and compare.

Q6. Find the distance from the origin $O(0,0)$ to $P(4,3)$. (a) Write down the horizontal and the vertical displacement. (b) Apply Pythagoras' theorem.

Q7. Two similar triangles have corresponding sides 3 cm and 12 cm. (a) Find the scale factor k. (b) Hence find the side EF that corresponds to a 5 cm side of the smaller triangle.

Q8. SCENARIO: a map has a scale of 1:50 000. A road measures 7 cm on the map. (a) How long is the road in centimetres in reality? (b) Write the length in kilometres.

Q9. Two similar shapes have scale factor $k=2$. The smaller has area 15 cm². (a) By what factor does the area grow? (b) Hence find the larger area.

Q10. A triangle has sides 5 cm, 12 cm and 13 cm. (a) Which side must be L? (b) Run the right-angle test and state the result.

Q11. In a right-angled triangle, one angle is 40° and the hypotenuse is 10 cm. (a) Which ratio uses the opposite side and the hypotenuse? (b) Hence find the opposite side, to 1 decimal place.

Q12. In a right-angled triangle, the side opposite an angle θ is 5 cm and the hypotenuse is 13 cm. (a) What is $\sin\theta$? (b) Hence find θ, to 1 decimal place.

Unscaffolded

Q13. Find the hypotenuse of a right-angled triangle with shorter sides: (a) 8 cm and 15 cm; (b) 12 cm and 16 cm; (c) 20 cm and 21 cm.

Q14. Find the unknown shorter side of a right-angled triangle with: (a) hypotenuse 20 cm, one side 12 cm; (b) hypotenuse 10 cm, one side 6 cm; (c) hypotenuse 25 cm, one side 7 cm.

Q15. Decide whether each triangle is right-angled: (a) 9 cm, 40 cm, 41 cm; (b) 6 cm, 7 cm, 8 cm; (c) 10 cm, 24 cm, 26 cm.

Q16. SCENARIO: a builder lays a concrete slab 12 m by 5 m. (a) How long is the diagonal of the slab? (b) To check that a corner is square, the builder marks 3 m along one edge and 4 m along the other. What should the distance between the marks be?

Q17. A 4.1 m ladder leans against a wall with its foot 0.9 m out from the wall. How far up the wall does it reach?

Q18. Find the distance between each pair of points: (a) $(2,3)$ and $(6,6)$; (b) $(-1,1)$ and $(4,13)$; (c) $(-2,-1)$ and $(4,7)$.

Q19. A triangle has corners $A(1,2)$, $B(7,2)$ and $C(7,10)$. Find the three side lengths and the perimeter, and name the right angle.

Q20. A triangle has sides 5 cm, 7 cm and 9 cm. A similar triangle's shortest side is 15 cm. Find the other two sides of the similar triangle.

Q21. (a) Two similar shapes have scale factor 4. The smaller has area 7 cm². Find the larger area. (b) Two similar solids have scale factor 2. The smaller has volume 500 cm³. Find the larger volume.

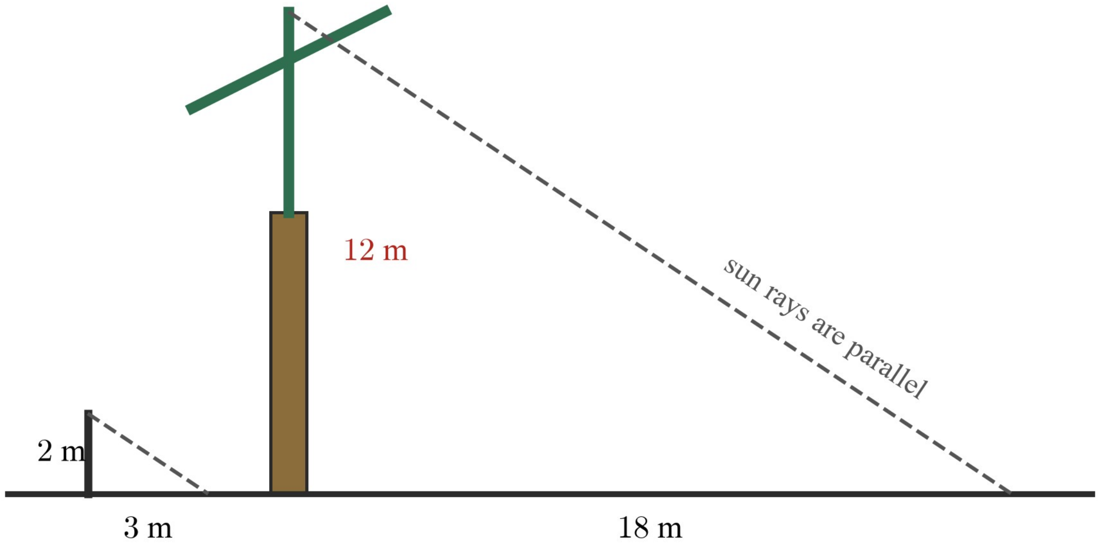
Q22. SCENARIO: a plan is drawn at a scale of 1:150. (a) A wall measures 22 cm on the plan. How long is it in reality, in metres? (b) A path is 45 m long in reality. How long is it on the plan, in centimetres?

Q23. Find the marked side, to 1 decimal place: (a) hypotenuse 20 cm, angle 36°, the side opposite the angle; (b) adjacent side 9 cm, angle 52°, the side opposite the angle; (c) adjacent side 12 cm, angle 65°, the hypotenuse.

Q24. Find the angle θ, to 1 decimal place: (a) opposite 7, adjacent 24; (b) adjacent 11, hypotenuse 14; (c) opposite 9, hypotenuse 10.

Q25. SCENARIO: on level ground, a 2 m pole casts a 3 m shadow. At the same time, a tree casts an 18 m shadow. Find the height of the tree.

similar triangles: $h \div 18 = 2 \div 3$, so $h = 12$ m



A 2 m pole with a 3 m shadow and a tree with an 18 m shadow; the parallel sun rays make similar triangles, and the tree's height is 12 m

Q26. SCENARIO: in a photograph taken square-on, a person of height 1.5 m measures 18 mm and a tree beside them measures 132 mm. Estimate the height of the tree.

Q27. Classify each triangle as right-angled, acute or obtuse: $(6,8,11)$; $(6,8,9)$; $(6,8,10)$; $(5,5,8)$.

Q28. Two angles of a triangle are 40° and 60°. Find the third angle, and classify the triangle as right-angled, acute or obtuse.

Q29. SCENARIO (the Biltmore stick of the theory): a forester holds a stick at arm's length 60 cm and stands 24 m from a tree. (a) The sight lines across the trunk's width cut a marked span of 5 cm on the stick. Estimate the diameter of the trunk. (b) The sight lines to the top and the base of the tree cut a marked span of 45 cm. Estimate the height of the tree. (c) Model the trunk as a cylinder with the diameter from (a) and the height from (b). Show that its volume is 18π m³, and give this to 1 decimal place. (d) The dry density of the wood is 600 kg per m³. Estimate the dry mass of the trunk to the nearest kilogram. (e) 40% of the dry mass becomes paper. Estimate the mass of paper to the nearest kilogram.

Q30. SCENARIO: a rectangular courtyard is 28 m by 21 m. (a) How long is its diagonal? (b) A row of bricks is laid along the diagonal, with 120 bricks per metre. How many bricks are in the row?

Q31. A right-angled triangle has opposite 3 and hypotenuse 5 for an angle θ. Write $\sin\theta$ as a decimal. A similar triangle has hypotenuse 12.5; find its side opposite the matching angle.

Q32. SCENARIO: a scale drawing uses 1:200. (a) A room is 5.4 m by 4.2 m in reality. What are its dimensions on the drawing, in centimetres? (b) A line on the drawing measures 4 mm. What length is that in reality, in metres?

Q33. Two similar rectangles measure 5 cm by 2 cm and 15 cm by 6 cm. Compute both areas, and check that the area factor is the square of the scale factor.

Q34. SCENARIO: an access ramp has a sloping surface of 17 m and rises 8 m. Find the angle the ramp makes with the ground, to 1 decimal place.

Q35. SCENARIO: a surveyor stands at a point directly opposite a tree on the far bank of a straight creek, so the line to the tree is at right angles to the bank. She walks 30 m along the bank, and the straight-line distance from her new spot back to the tree is 34 m. How wide is the creek?

Q36. A triangle has sides 6 cm and 8 cm, and the third side x is a whole number of centimetres. (a) Test $x = 2$ and $x = 14$ by drawing or by the straight-line case, and state what happens. (b) Conjecture the range of values of x that give a triangle. (c) How many whole-number values are there?

Answers

Q1 a 180°; b 53°. **Q2** a 90°; b 52°. **Q3** a $c^2=9^2+12^2$; b 15 cm. **Q4** a $a^2=13^2-5^2$; b 12 cm. **Q5** a 25; b $7^2+24^2=625=25^2$ — yes, right-angled. **Q6** a 4 and 3; b 5. **Q7** a $k=4$; b $EF=20$ cm. **Q8** a 350 000 cm; b 3.5 km. **Q9** a $k^2=4$; b 60 cm². **Q10** a 13; b $5^2+12^2=169=13^2$ — right-angled. **Q11** a sine; b 6.4 cm. **Q12** a 5 / 13; b 22.6°. **Q13** a 17 cm; b 20 cm; c 29 cm. **Q14** a 16 cm; b 8 cm; c 24 cm. **Q15** a right-angled; b not right-angled; c right-angled. **Q16** a 13 m; b 5 m. **Q17** 4 m. **Q18** a 5; b 13; c 10. **Q19** $AB=6$, $BC=8$, $AC=10$; perimeter 24; right angle at B. **Q20** 21 cm and 27 cm. **Q21** a 112 cm²; b 4000 cm³. **Q22** a 33 m; b 30 cm. **Q23** a 11.8 cm; b 11.5 cm; c 28.4 cm. **Q24** a 16.3°; b 38.2°; c 64.2°. **Q25** 12 m. **Q26** 11 m. **Q27** Obtuse; acute; right-angled; obtuse. **Q28** 80°; acute. **Q29** a 2 m; b 18 m; c $18\pi\approx56.5$ m³; d 33 929 kg; e 13 572 kg. **Q30** a 35 m; b 4200 bricks. **Q31** 0.6; 7.5. **Q32** a 2.7 cm by 2.1 cm; b 0.8 m. **Q33** 10 cm² and 90 cm²; factor $9=3^2$. **Q34** 28.1°. **Q35** 16 m. **Q36** a both lie flat — no triangle; b $2<x<14$; c 11 values.

Fully-worked solutions

Q1. The three angles on the straight line add to 180°. $90°+37°+x=180°$. $x=180°-127°=53°$. $\therefore$ $x=53°$.

Q2. The acute angles of a right-angled triangle add to 90°. $90°-38°=52°$. $\therefore$ The other acute angle is 52°.

Q3. $c^2=9^2+12^2=81+144=225$. $c=15$. $\therefore$ The hypotenuse is 15 cm.

Q4. $a^2=13^2-5^2=169-25=144$. $a=12$. $\therefore$ The unknown side is 12 cm.

Q5. $L=25$; the other sides are 7 and 24. $7^2+24^2=49+576=625$. $L^2=25^2=625$. The two are equal. $\therefore$ The triangle is right-angled.

Q6. Horizontal displacement 4; vertical displacement 3. $d^2=4^2+3^2=16+9=25$. $d=5$. $\therefore$ The distance is 5 units.

Q7. $k=12\div3=4$. $EF=5\times4=20$. $\therefore$ $EF=20$ cm.

Q8. Reality $=7\times50\,000=350\,000$ cm. $350\,000$ cm $=3500$ m $=3.5$ km. $\therefore$ The road is 3.5 km long.

Q9. Areas grow by $k^2=2^2=4$. $15\times4=60$. $\therefore$ The larger area is 60 cm².

Q10. $L=13$; the other sides are 5 and 12. $5^2+12^2=25+144=169=13^2$. $\therefore$ The triangle is right-angled, opposite the 13 cm side.

Q11. $\sin 40°=\text{opposite}\div10$. $\text{opposite}=10\times\sin40°\approx6.43$. $\therefore$ The opposite side is about 6.4 cm.

Q12. $\sin\theta=5\div13$. $\theta=\sin^{-1}(5\div13)\approx22.62°$. $\therefore$ $\theta\approx22.6°$.

Q13. (a) $8^2+15^2=64+225=289$; $\sqrt{289}=17$. (b) $12^2+16^2=144+256=400$; $\sqrt{400}=20$. (c) $20^2+21^2=400+441=841$; $\sqrt{841}=29$. $\therefore$ The hypotenuses are 17 cm, 20 cm and 29 cm.

Q14. (a) $20^2-12^2=400-144=256$; $\sqrt{256}=16$. (b) $10^2-6^2=100-36=64$; $\sqrt{64}=8$. (c) $25^2-7^2=625-49=576$; $\sqrt{576}=24$. $\therefore$ The unknown sides are 16 cm, 8 cm and 24 cm.

Q15. (a) $9^2+40^2=81+1600=1681=41^2$ — right-angled. (b) $6^2+7^2=36+49=85\neq64=8^2$ — not right-angled. (c) $10^2+24^2=100+576=676=26^2$ — right-angled. $\therefore$ (a) and (c) are right-angled; (b) is not.

Q16. (a) $12^2+5^2=144+25=169$; the diagonal is 13 m. (b) The corner triangle has shorter sides 3 and 4: $3^2+4^2=9+16=25$; the distance is 5 m. $\therefore$ The diagonal is 13 m, and the marks must be 5 m apart for a square corner.

Q17. $h^2=4.1^2-0.9^2=16.81-0.81=16$. $h=4$. $\therefore$ The ladder reaches 4 m up the wall.

Q18. (a) Across $6-2=4$; up $6-3=3$; $d^2=16+9=25$; $d=5$. (b) Across $4-(-1)=5$; up $13-1=12$; $d^2=25+144=169$; $d=13$. (c) Across $4-(-2)=6$; up $7-(-1)=8$; $d^2=36+64=100$; $d=10$. ∴ The distances are 5, 13 and 10 units.

Q19. $AB=7-1=6$; $BC=10-2=8$. $AC^2=6^2+8^2=100$; $AC=10$. Perimeter $=6+8+10=24$. $6^2+8^2=10^2$, so the right angle is at B, between the horizontal and the vertical side. ∴ The sides are 6, 8 and 10, the perimeter is 24, and the right angle is at B.

Q20. $k=15\div5=3$. $7\times3=21$; $9\times3=27$. ∴ The other two sides are 21 cm and 27 cm.

Q21. (a) Areas grow by $4^2=16$: $7\times16=112$ cm². (b) Volumes grow by $2^3=8$: $500\times8=4000$ cm³. ∴ 112 cm² and 4000 cm³.

Q22. (a) Reality $=22\times150=3300$ cm $=33$ m. (b) 45 m $=4500$ cm; plan $=4500\div150=30$ cm. ∴ The wall is 33 m long, and the path measures 30 cm on the plan.

Q23. (a) $\text{opposite}=20\times\sin36°\approx11.76$ — 11.8 cm. (b) $\text{opposite}=9\times\tan52°\approx11.52$ — 11.5 cm. (c) $\text{hypotenuse}=12\div\cos65°\approx28.39$ — 28.4 cm. ∴ 11.8 cm, 11.5 cm and 28.4 cm.

Q24. (a) $\tan\theta=7\div24$; $\theta=\tan^{-1}(7\div24)\approx16.26°$ — 16.3°. (b) $\cos\theta=11\div14$; $\theta=\cos^{-1}(11\div14)\approx38.21°$ — 38.2°. (c) $\sin\theta=9\div10$; $\theta=\sin^{-1}(9\div10)\approx64.16°$ — 64.2°. ∴ 16.3°, 38.2° and 64.2°.

Q25. Parallel sun rays make the pole triangle and the tree triangle similar. Height ÷ shadow is the same ratio for both: $h\div18=2\div3$. $h=18\times2\div3=12$. ∴ The tree is 12 m tall.

Q26. The photo scale is the same for both, so tree height $=132\div18\times1.5$. $132\div18\times1.5=11$. ∴ The tree is about 11 m tall.

Q27. $(6,8,11)$: $6^2+8^2=100<121=11^2$ — obtuse. $(6,8,9)$: $100>81=9^2$ — acute. $(6,8,10)$: $100=100$ — right-angled. $(5,5,8)$: $5^2+5^2=50<64=8^2$ — obtuse. ∴ Obtuse, acute, right-angled, obtuse.

Q28. $180°-40°-60°=80°$. All three angles — 40°, 60°, 80° — are less than 90°. ∴ The third angle is 80°, and the triangle is acute.

Q29. (a) Diameter $=24\times5\div60=2$ m. (b) Height $=24\times45\div60=18$ m. (c) Radius $=1$ m; volume $=\pi\times1^2\times18=18\pi\approx56.5$ m³. (d) Mass $=18\pi\times600=10\,800\pi\approx33\,929.2$ — 33 929 kg to the nearest kilogram. (e) Paper $=10\,800\pi\times0.40=4320\pi\approx13\,571.7$ — 13 572 kg to the nearest kilogram. ∴ Diameter 2 m, height 18 m, volume ≈ 56.5 m³, dry mass ≈ 33 929 kg, paper ≈ 13 572 kg.

Q30. (a) $28^2+21^2=784+441=1225$; the diagonal is 35 m. (b) $35\times120=4200$. ∴ The diagonal is 35 m and the row uses 4200 bricks.

Q31. $\sin\theta=3\div5=0.6$. Similar triangles keep the ratio: opposite $=12.5\times0.6=7.5$. ∴ $\sin\theta=0.6$, and the matching opposite side is 7.5.

Q32. (a) 5.4 m $=540$ cm; $540\div200=2.7$ cm. 4.2 m $=420$ cm; $420\div200=2.1$ cm. (b) 4 mm $\times200=800$ mm $=0.8$ m. ∴ The room is 2.7 cm by 2.1 cm on the drawing, and the 4 mm line is 0.8 m in reality.

Q33. Small area $=5\times2=10$ cm²; large area $=15\times6=90$ cm². $90\div10=9$; scale factor $k=15\div5=3$, and $3^2=9$. ∴ The areas are 10 cm² and 90 cm², and the area factor 9 is k^2.

Q34. The rise is opposite the angle and the sloping surface is the hypotenuse. $\sin\theta=8\div17$. $\theta=\sin^{-1}(8\div17)\approx28.07°$. ∴ The ramp makes an angle of about 28.1° with the ground.

Q35. The width w and the 30 m walk are the shorter sides; the 34 m sight line is the hypotenuse. $w^2=34^2-30^2=1156-900=256$. $w=16$. ∴ The creek is 16 m wide.

Q36. (a) $x = 2$: $2 + 6 = 8$ — the two shorter sides only reach the end of the 8 cm side, and the three lie flat. $x = 14$: $6 + 8 = 14$ — flat again. Neither gives a triangle. (b) Conjecture: the third side must be more than the difference, $8 - 6 = 2$, and less than the sum, $8 + 6 = 14$ — that is, $2 < x < 14$. Test $x = 3$: $3 + 6 = 9 > 8$ — a triangle; test $x = 13$: $6 + 8 = 14 > 13$ — a triangle. The conjecture holds on the tests. (c) Whole numbers from 3 to 13: $13 - 3 + 1 = 11$. ∴ There are 11 whole-number values of x.

Absolute, relative and percentage error in measurements

5D · Chapter 5 — Measurement · Level 9

Content description `VC2M9M04` (word for word): *calculate and interpret absolute, relative and percentage errors in measurements*

Achievement standard anchor (AS 10): *They determine percentage errors in measurements.*

Elaboration ids for linkage: `VC2M9M04.E01`, `VC2M9M04.E02`, `VC2M9M04.E03`, `VC2M9M04.E04`

Prerequisite pointer: 5B (instrument accuracy and recorded precision). That every recorded measurement carries an accuracy limit is assumed from 5B and is turned here into bounds and into a maximum error we can put a number on.

Note on figures: every number line, bar and scale in this section is computed from its own values (R7); every context is a generic one, so no real data ships and no source line is owed (R1, R2).

Theory

1 · Error — the gap between what we have and what is true

Every estimate and every measurement carries a gap between the value we have and the true or actual value. That gap is the **error**.

A club guesses that 240 people will come to a night market; in the end 250 come.

$$\text{signed error} = \text{value we have} - \text{actual value} = 240 - 250 = -10$$

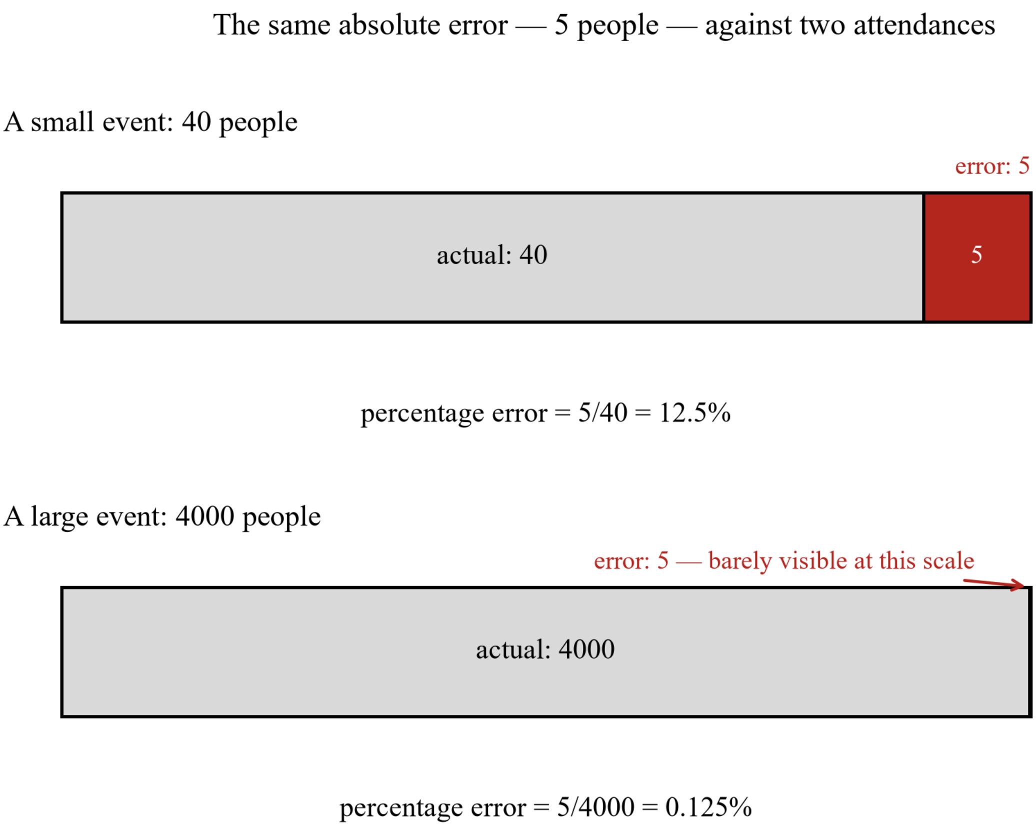
The **signed error** is -10: the minus records that the guess fell under the actual number. The **absolute error** drops the sign and keeps only the size:

$$\text{absolute error} = |240 - 250| = |-10| = 10\ \text{people}$$

Two names, two jobs: the signed error says *which way* we were wrong, and the absolute error says *how far* we were wrong.

2 · Why the size alone can mislead — relative and percentage error

Is an absolute error of 5 people large? It depends on the size of the thing measured.



Two bar diagrams each showing an actual attendance in grey with the same red error block of 5 people on the end: against 40 the block is large and the percentage error is 12.5 percent, while against 4000 the block is barely visible and the percentage error is 0.125 percent

The same absolute error, 5 people, against two actual attendances:

against $40: 5 \div 40 = 0.125$, so the percentage error is 12.5%
against $4000: 5 \div 4000 = 0.00125$, so the percentage error is 0.125%

The **relative error** is the absolute error divided by the actual value — the error as a fraction of what is real. Multiplied by 100%, it becomes the **percentage error**:

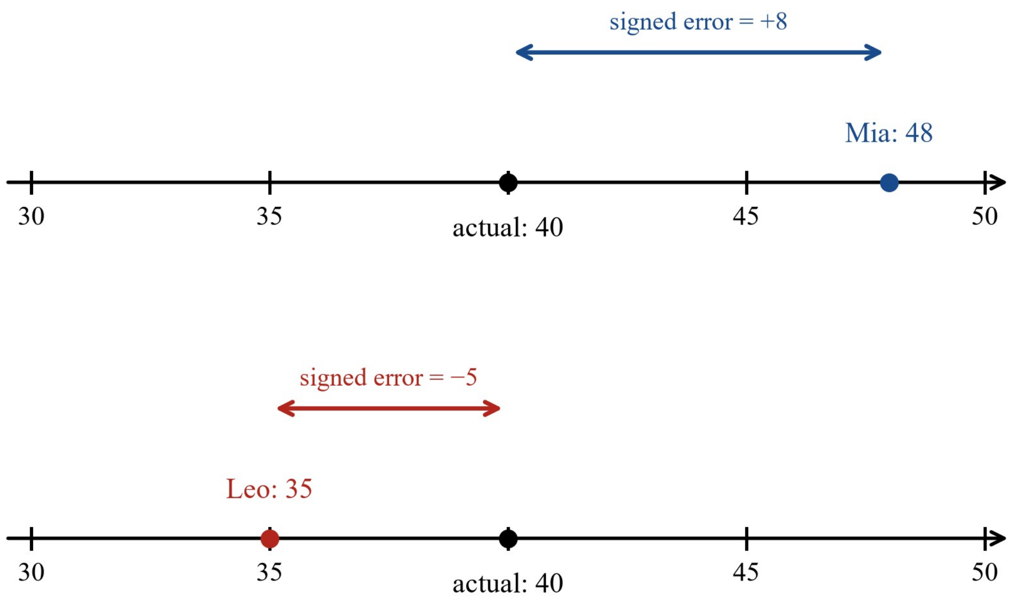
$$\text{percentage error} = \frac{\text{absolute error}}{\text{actual value}} \times 100\%$$

Missing 5 people at a 40-person event (12.5%) is a poor guess; missing 5 at a 4000-person event (0.125%) is an excellent one. The absolute error cannot tell those apart, and the percentage error can. To interpret a percentage error is exactly this: to say how large the error is *relative to* the thing being measured.

3 · Keeping the sign or dropping it (E02)

Two students estimate the number of chairs set up in a hall; the actual number is 40.

Signed errors record the direction as well as the size



Two number lines from 30 to 50 with the actual value 40 marked in black; on the first, Mia's estimate 48 sits to the right with a blue two-headed arrow labelled signed error equals plus 8, and on the second, Leo's estimate 35 sits to the left with a red two-headed arrow labelled signed error equals minus 5

Mia: $48 - 40 = +8$, and $8 \div 40 \times 100\,\% = 20\,\%$ — an overestimate. Leo: $35 - 40 = -5$, and $5 \div 40 \times 100\,\% = 12.5\,\%$ — an underestimate.

Whether the sign stays depends on the question being asked.

- If the question is *which way, and by how much* — did the committee budget too much or too little? did the guess run over or under? — keep the **signed error**.
- If the question is *how accurate* — how close were we, never mind the direction? — the sign is not wanted, so take the absolute value.

That is exactly the job of the absolute-value bars in the standing formula:

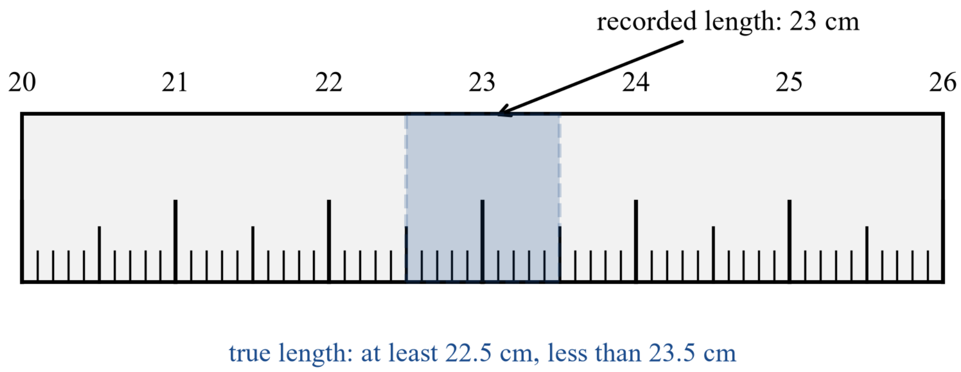
$$\text{percentage error} = \frac{|\text{value} - \text{actual}|}{\text{actual}} \times 100\,\%$$

Without the bars the formula returns a signed percentage error ($+20\,\%$ for Mia, $-12.5\,\%$ for Leo), which some contexts want; with the bars it returns the accuracy number. The context chooses, not habit.

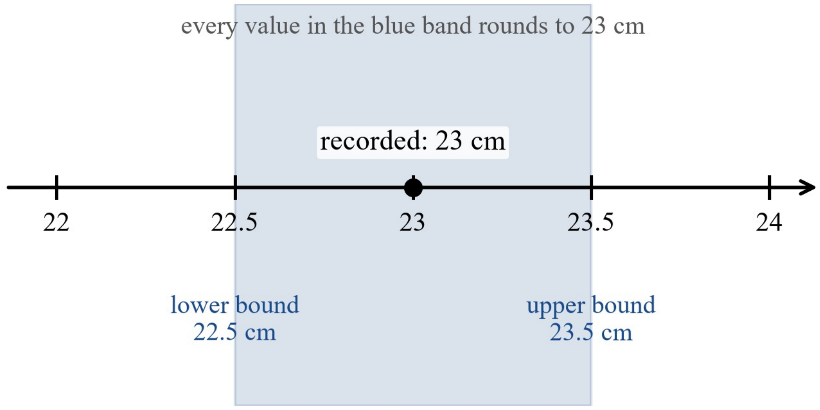
4 · When there is no exact value — bounds and the half-unit rule (E04)

With a guess about a market, the actual number can be counted later. With a measurement there is often no exact value to compare against: the instrument itself runs out of accuracy. Section 5B said that every recording has an accuracy limit; here we put numbers on that limit.

A stick is recorded as 23 cm long, measured to the nearest 1 cm.



A ruler from 20 to 26 cm with millimetre marks, the band from 22.5 to 23.5 cm shaded blue as the lengths that record as 23 cm, and the true length stated as at least 22.5 cm and less than 23.5 cm



Number line from 22 to 24 with the band from 22.5 to 23.5 shaded blue around the recorded value 23 cm, labelled lower bound 22.5 cm and upper bound 23.5 cm

Every true length from 22.5 cm up to (but not including) 23.5 cm rounds to 23 cm, so the true length lies in that band:

$$\text{lower bound} = 22.5\,\text{cm}\,, \text{upper bound} = 23.5\,\text{cm}$$

The error is therefore at most half a unit either way:

$$\text{maximum absolute error} = 0.5\,\text{cm}$$

and dividing by the recorded 23 cm (the true value is unknown, so the recorded value stands in for it):

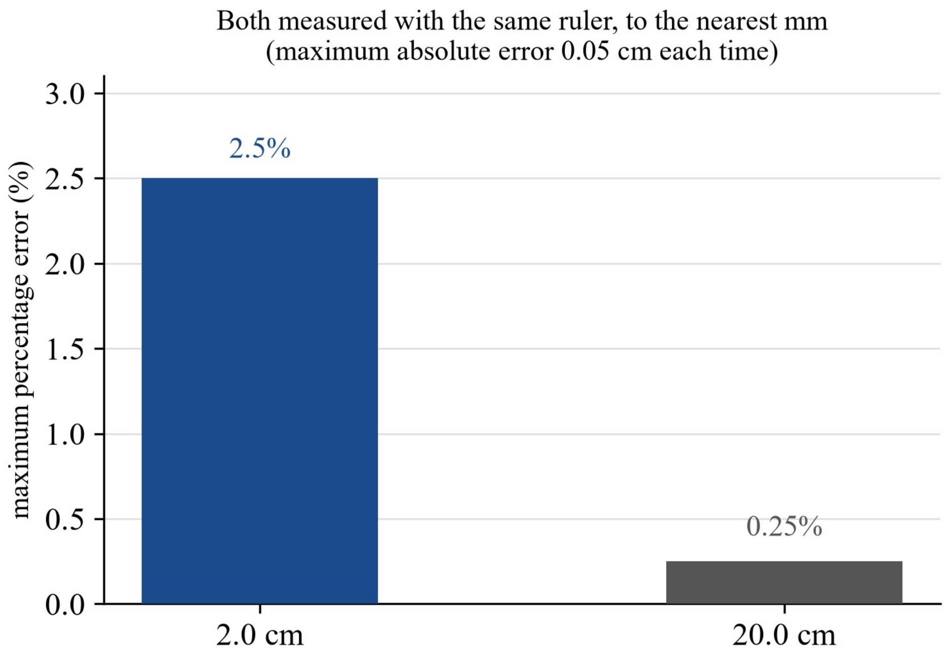
$$\text{maximum percentage error} = 0.5 \div 23 \times 100\,\% \approx 2.2\,\%$$

The rule: **measured to the nearest u, the maximum absolute error is** $\frac{1}{2}u$ — half the unit of accuracy — and the bounds are the recorded value minus and plus that half-unit.

5 · The same instrument does better work on bigger amounts

A mass recorded as 70 kg, to the nearest 5 kg, has bounds 67.5–72.5 kg and a maximum absolute error of 2.5 kg — five times the stick's — yet as a percentage it is $2.5 \div 70 \times 100\,\% \approx 3.6\,\%$, only a little worse than the stick's $2.2\,\%$ — a much bigger absolute error with a much smaller change in the percentage. Now hold the instrument fixed and change the size.

One ruler, reading to the nearest millimetre (0.1 cm), measures two sticks: one records as 2.0 cm, one as 20.0 cm. Each recording has the same maximum absolute error, 0.05 cm.



Bar chart of maximum percentage error for two lengths measured with the same ruler to the nearest millimetre: 2.5 percent for the 2.0 cm length and 0.25 percent for the 20.0 cm length

$2.0\,\text{cm}: 0.05 \div 2.0 \times 100\,\% = 2.5\,\%$ $20.0\,\text{cm}: 0.05 \div 20.0 \times 100\,\% = 0.25\,\%$

Ten times the length gives one tenth the percentage error, from the same ruler. So the honest report of a measurement is not “the error was 0.05 cm” but “the error was at most 0.05 cm, which is 2.5% of what I measured” — the percentage error is the number that carries the accuracy.

Reading any measurement — three questions, in order

1. What was it measured (or counted, or guessed) to the nearest of? That sets the unit u.
2. What are the bounds, and the maximum absolute error $\frac{1}{2}u$?
3. What is that error as a percentage of the recorded value — and is that good enough for the job?

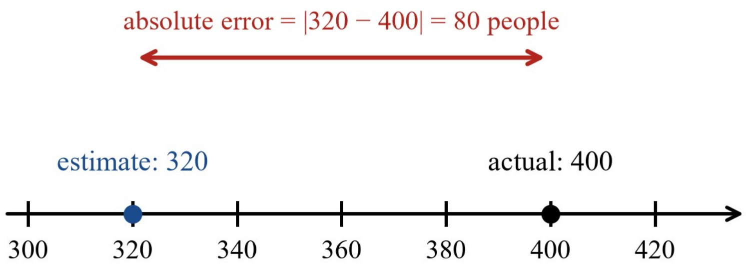
Worked examples

Example 1 — scaffolded

The fete committee estimates that 320 people will attend; on the day, 400 attend. Find the signed error, the absolute error, the relative error and the percentage error of the estimate.

Working	Comment
$320 - 400 = -80$	Signed error = estimate − actual; the minus says the estimate ran under.
$\lvert -80 \rvert = 80$ people	Absolute error: the size only.
$80 \div 400 = \frac{1}{5} = 0.2$	Relative error: absolute error as a fraction of the actual value.
$0.2 \times 100\,\% = 20\,\%$	Percentage error.
Check: $20\,\%$ of $400 = 80$	Back to the absolute error — the chain agrees.

The figure shows the estimate and the actual value on one number line, with the absolute error as the gap between them.



Number line from 300 to 420 in steps of 20 with a blue dot at the estimate 320 and a black dot at the actual 400, joined above by a red two-headed arrow labelled absolute error equals 80 people

Example 2 — scaffolded

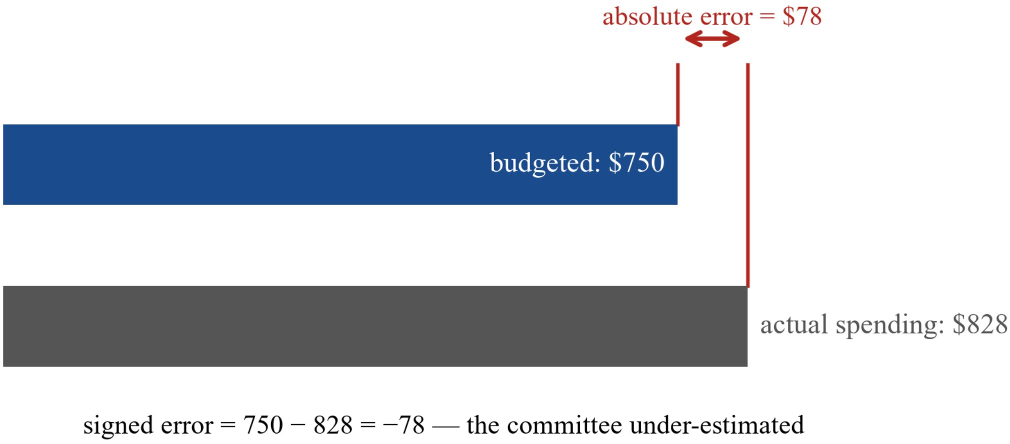
A bag of flour is recorded as 2.5 kg, measured to the nearest 0.1 kg. Give the lower and upper bounds of the true mass, the maximum absolute error, and the maximum percentage error.

Working	Comment
Half-unit: $0.1 \div 2 = 0.05$ kg	Nearest 0.1 kg → maximum absolute error 0.05 kg.
$2.5 - 0.05 = 2.45$ and $2.5 + 0.05 = 2.55$	The true mass lies between 2.45 kg and 2.55 kg.
$0.05 \div 2.5 \times 100\,\% = 2\,\%$	Maximum percentage error, dividing by the recorded value.
Check: $2\,\%$ of $2.5 = 0.05$	The chain agrees.

Example 3 — unscaffolded

A committee budgets 750 dollars for a concert; the actual spending is 828 dollars. Find the signed error, the absolute error and the percentage error of the budget, and say what the sign means here.

Signed error: $750 - 828 = -78$. Absolute error: $|-78| = 78$ dollars. Percentage error: $78 \div 828 \times 100\,\% \approx 9.4\,\%$ (1 d.p.). The sign is negative: the budget ran under the actual spending, so the committee under-estimated.



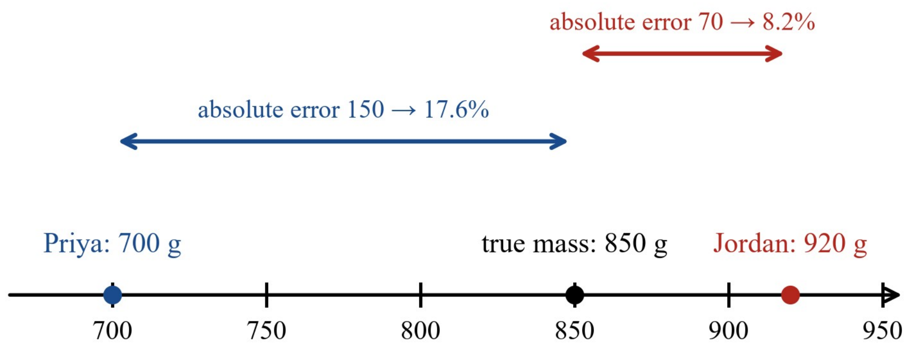
Two horizontal bars, a blue bar for the budgeted 750 dollars and a longer grey bar for the actual spending of 828 dollars, with a red two-headed arrow over the gap labelled absolute error equals 78 dollars and the signed error stated as minus 78

∴ Signed error -78 (an underestimate), absolute error 78, percentage error $\approx 9.4\,\%$.

Example 4 — unscaffolded

The true mass of a rock is 850 g. Priya estimates 700 g and Jordan estimates 920 g. Which estimate is the better one? Justify your answer with percentage errors.

Priya: $|700-850|=150$ g, so $150 \div 850 \times 100\% \approx 17.6\%$. Jordan: $|920-850|=70$ g, so $70 \div 850 \times 100\% \approx 8.2\%$. Jordan's absolute error (70 g) is smaller than Priya's (150 g), and the percentage errors agree: $8.2\% < 17.6\%$.



Number line from 700 to 950 with the true mass 850 g in black, Priya's estimate 700 g in blue with a two-headed arrow showing absolute error 150 giving 17.6 percent, and Jordan's estimate 920 g in red with a two-headed arrow showing absolute error 70 giving 8.2 percent

∴ Jordan's estimate is the better one: percentage error $\approx 8.2\%$ against Priya's $\approx 17.6\%$.

Practice questions

Percentages: round to 1 decimal place unless the question says otherwise.

Scaffolded

Q1. A stallholder estimates selling 260 cups; the actual number is 250. Find (a) the signed error, (b) the absolute error, (c) the percentage error.

Q2. The actual number of goals kicked in a season is 60. Three predictions are 72, 51 and 60. Copy the table and complete it.

Prediction	Signed error	Absolute error	Percentage error
72			
51			
60			

Q3. Give the lower bound, the upper bound and the maximum absolute error for each recording. (a) 17 cm, to the nearest 1 cm. (b) 3.8 kg, to the nearest 0.1 kg. (c) 25 min, to the nearest 5 min.

Q4. A fundraiser estimates 480 dollars in sales; the actual takings are 528 dollars. Find (a) the signed error, (b) the absolute error, (c) the percentage error.

Q5. The actual attendance at a club night is 120. One estimate is 102 and another is 141. Find the absolute error and the percentage error of each, and say which estimate was the better one.

Q6. A measuring cylinder reads to the nearest 1 mL and the water level is recorded as 36 mL. Give (a) the bounds of the true volume, (b) the maximum absolute error, (c) the maximum percentage error.

Unscaffolded

Q7. A bolt is measured as 24 mm long; its true length is 18 mm. Find the absolute error and the percentage error of the measurement.

Q8. The same absolute error, 12, is made on two true values: 80 and 600. Find the percentage error in each case, and write one sentence saying what the two answers show.

Q9. Give the lower and upper bounds of the true value in each case. (a) 54 g, to the nearest 1 g. (b) 6.25 s, to the nearest 0.01 s. (c) 150 km, to the nearest 10 km. (d) 2.4 L, to the nearest 0.5 L.

Q10. A ruler reads to the nearest 0.1 cm. Find the maximum percentage error when it measures (a) a 1.8 cm stick, to 1 d.p., and (b) an 18.0 cm stick, to 2 d.p. What does the pair of answers say about measuring small and large things with one ruler?

Q11. A caterer prepares 2000 meals; the actual number needed is 1875. Find the signed error and the percentage error of the caterer's number.

Q12. Say whether each statement is true or false, with a reason. (a) An absolute error of 10 is equally serious against a true value of 50 and against a true value of 200. (b) Recording a true value of 20 as 45 gives a percentage error of 125%. (c) A length recorded as 8 cm, to the nearest 1 cm, could have a true value of 8.5 cm.

Q13. The actual cost of an excursion is 240 dollars. Write down the estimate that is (a) 15% too high, (b) 15% too low.

Q14. A beaker reads to the nearest 10 mL and records 50 mL; a measuring cylinder reads to the nearest 1 mL and records 49 mL. Find the maximum percentage error of each reading, and say which instrument gives the more accurate reading.

Q15. A guess puts the number of tiles in a stack at 180; the actual number is 225. Find the absolute error and the percentage error of the guess.

Q16. A travel app promises that its time estimates are within 5% of the actual time. For one trip it estimates 43 min and the trip takes 40 min. Was the promise kept on this trip?

Q17. A ruler reads to the nearest 0.1 cm, so every recording it makes carries a maximum absolute error of 0.05 cm. Find the maximum percentage error when the recorded length is (a) 5 cm, (b) 10 cm, (c) 25 cm, and write one sentence generalising.

Q18. A fair organisers' report makes two predictions. They estimate the attendance at 640 people, and 725 people turn up. They budget 1200 dollars, and the actual spending is 1134 dollars. (a) Find the signed error and the percentage error of the attendance estimate. (b) Find the signed error and the percentage error of the budget. (c) Which of the two predictions was closer, relatively speaking? (d) Next year the actual attendance is 750. Which whole-number estimates would have been within 5% of it?

Answers

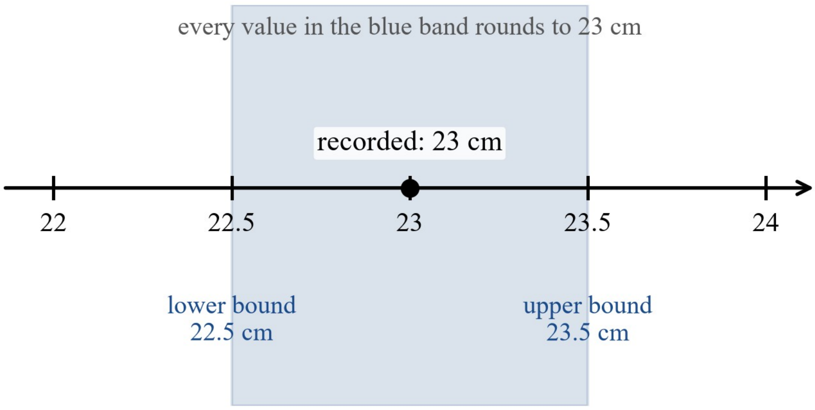
Q1 (a) $+10$; (b) 10; (c) $4\,\%$. **Q2** 72: $+12$, 12, $20\,\%$; 51: -9, 9, $15\,\%$; 60: 0, 0, $0\,\%$. **Q3** (a) 16.5–17.5 cm, max 0.5 cm; (b) 3.75–3.85 kg, max 0.05 kg; (c) 22.5–27.5 min, max 2.5 min. **Q4** (a) -48; (b) 48; (c) $\approx 9.1\,\%$. **Q5** 102: 18, $15\,\%$; 141: 21, $17.5\,\%$; the 102 estimate is better. **Q6** (a) 35.5–36.5 mL; (b) 0.5 mL; (c) $\approx 1.4\,\%$. **Q7** 6 mm; $\approx 33.3\,\%$. **Q8** $15\,\%$; $2\,\%$; the same absolute error is far less serious against the larger true value. **Q9** (a) 53.5–54.5 g; (b) 6.245–6.255 s; (c) 145–155 km; (d) 2.15–2.65 L. **Q10** (a) $\approx 2.8\,\%$; (b) $\approx 0.28\,\%$; ten times the length gives one tenth the percentage error. **Q11** $+125$; $\approx 6.7\,\%$. **Q12** (a) False — $20\,\%$ against $5\,\%$; (b) True — $25 \div 20 = 125\,\%$; (c) False — 8.5 rounds to 9, so the bounds of 8 are 7.5–8.5. **Q13** (a) 276 dollars; (b) 204 dollars. **Q14** beaker $10\,\%$; cylinder $\approx 1.0\,\%$; the cylinder reading is the more accurate. **Q15** 45; $20\,\%$. **Q16** No — the error is 3 min, which is $7.5\,\% > 5\,\%$. **Q17** (a) $1\,\%$; (b) $0.5\,\%$; (c) $0.2\,\%$; the larger the recorded length, the smaller the maximum percentage error. **Q18** (a) -85, $\approx 11.7\,\%$; (b) $+66$, $\approx 5.8\,\%$; (c) the budget; (d) 713 to 787 people.

Fully-worked solutions

Q1. (a) Signed error $= 260 - 250 = +10$. (b) Absolute error $= |+10| = 10$ cups. (c) Percentage error $= 10 \div 250 \times 100\,\% = 4\,\%$. $\therefore$ $+10$; 10; $4\,\%$ — a small overestimate.

Q2. 72: signed $72 - 60 = +12$; absolute 12; percentage $12 \div 60 \times 100\,\% = 20\,\%$. 51: signed $51 - 60 = -9$; absolute 9; percentage $9 \div 60 \times 100\,\% = 15\,\%$. 60: signed 0; absolute 0; percentage $0\,\%$ — a perfect prediction. $\therefore$ The completed table reads $+12, 12, 20\,\%$; $-9, 9, 15\,\%$; $0, 0, 0\,\%$.

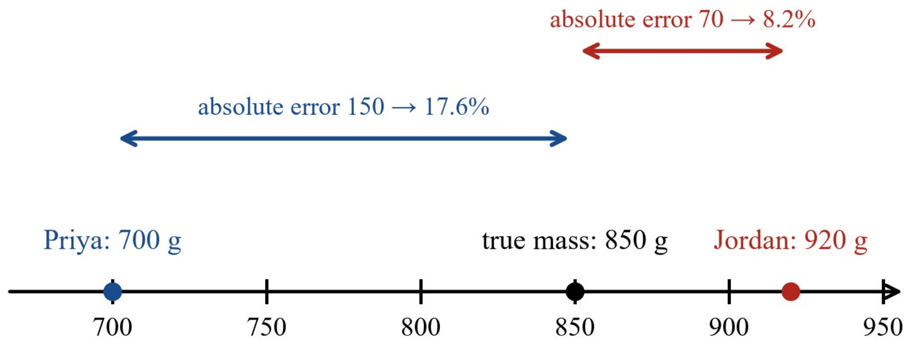
Q3. (a) Nearest 1 cm → half-unit 0.5: bounds $17 - 0.5 = 16.5$ and $17 + 0.5 = 17.5$ cm; maximum absolute error 0.5 cm. (b) Nearest 0.1 kg → half-unit 0.05: bounds 3.75 and 3.85 kg; maximum absolute error 0.05 kg. (c) Nearest 5 min → half-unit 2.5: bounds 22.5 and 27.5 min; maximum absolute error 2.5 min. $\therefore$ Each pair of bounds is the recorded value ± half the unit of accuracy, as on the bounds number line.



Number line from 22 to 24 with the band from 22.5 to 23.5 shaded blue around the recorded value 23 cm, labelled lower bound 22.5 cm and upper bound 23.5 cm; Q3 uses the same picture with different numbers

Q4. (a) Signed error $= 480 - 528 = -48$. (b) Absolute error $= |-48| = 48$ dollars. (c) Percentage error $= 48 \div 528 \times 100\,\% = 9.0909\ldots\,\% \approx 9.1\,\%$. $\therefore$ -48; 48; $\approx 9.1\,\%$ — the estimate ran under, as in the concert budget of Example 3.

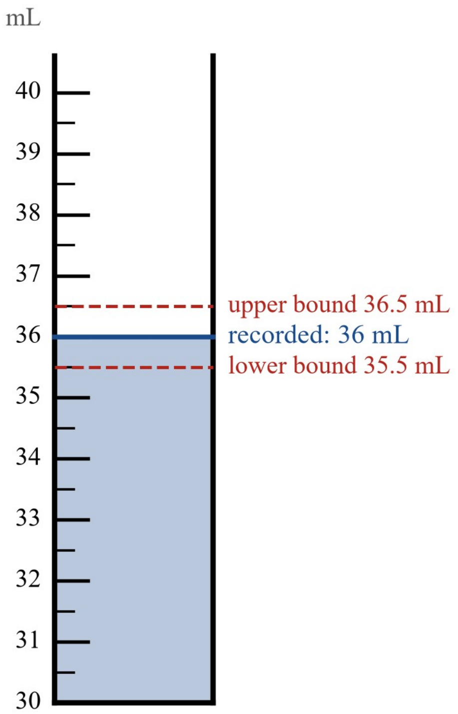
Q5. 102: absolute error $|102 - 120| = 18$; percentage $18 \div 120 \times 100\,\% = 15\,\%$. 141: absolute error $|141 - 120| = 21$; percentage $21 \div 120 \times 100\,\% = 17.5\,\%$. Both errors are compared against the same actual value, so the smaller absolute error already wins; the percentages confirm it. $\therefore$ The 102 estimate is better: $15\,\%$ against $17.5\,\%$ — the same comparison as the two-estimates figure.



Number line from 700 to 950 with the true mass 850 g in black and two estimates either side, each with its own absolute-error arrow and percentage error; Q5 makes the same comparison with different numbers

Q6. (a) Nearest 1 mL → bounds $36-0.5=35.5$ and $36+0.5=36.5$ mL. (b) Maximum absolute error $=0.5$ mL. (c) Maximum percentage error $=0.5\div 36\times 100\,\%=1.388\ldots\%\approx 1.4\,\%$. ∴ Bounds 35.5–36.5 mL; maximum absolute error 0.5 mL; maximum percentage error $\approx 1.4\,\%$.

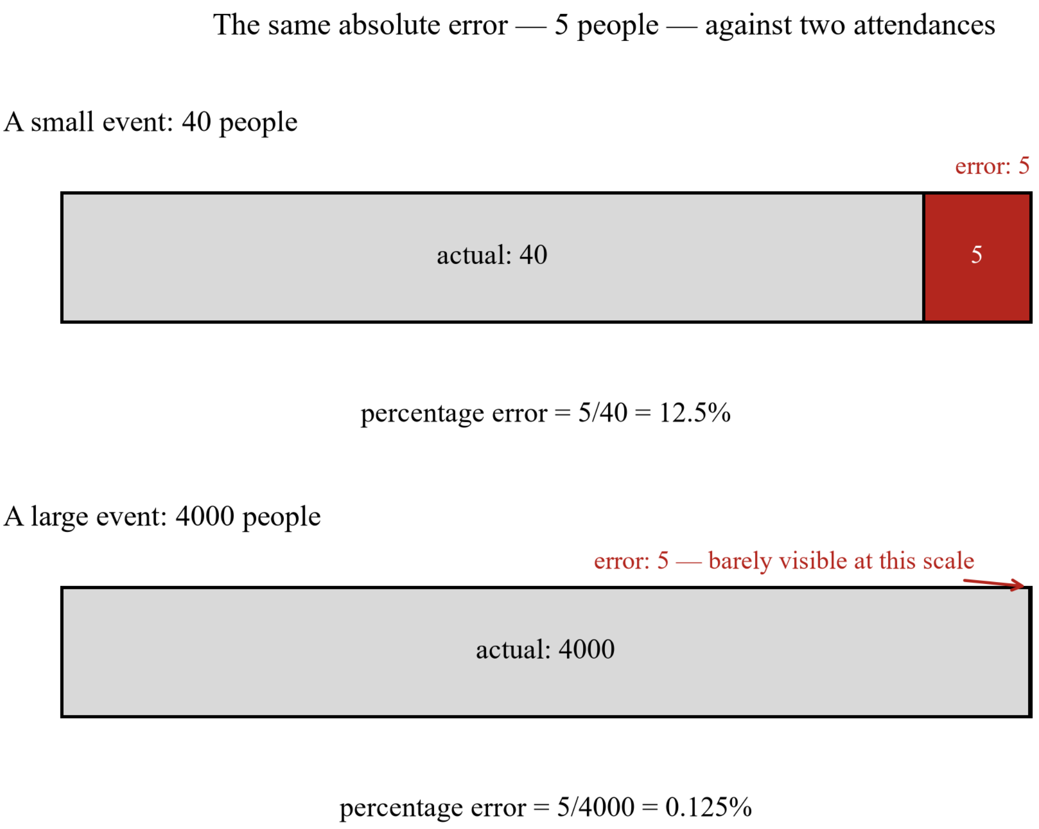
Water measured to the nearest 1 mL



A measuring cylinder graduated from 30 to 40 mL with the water level recorded at 36 mL and dashed red lines at the lower bound 35.5 mL and the upper bound 36.5 mL

Q7. Absolute error $=|24-18|=6$ mm. Percentage error $=6\div 18\times 100\,\%=33.333\ldots\%\approx 33.3\,\%$. ∴ 6 mm, $\approx 33.3\,\%$ — a third of the true length: a very poor measurement.

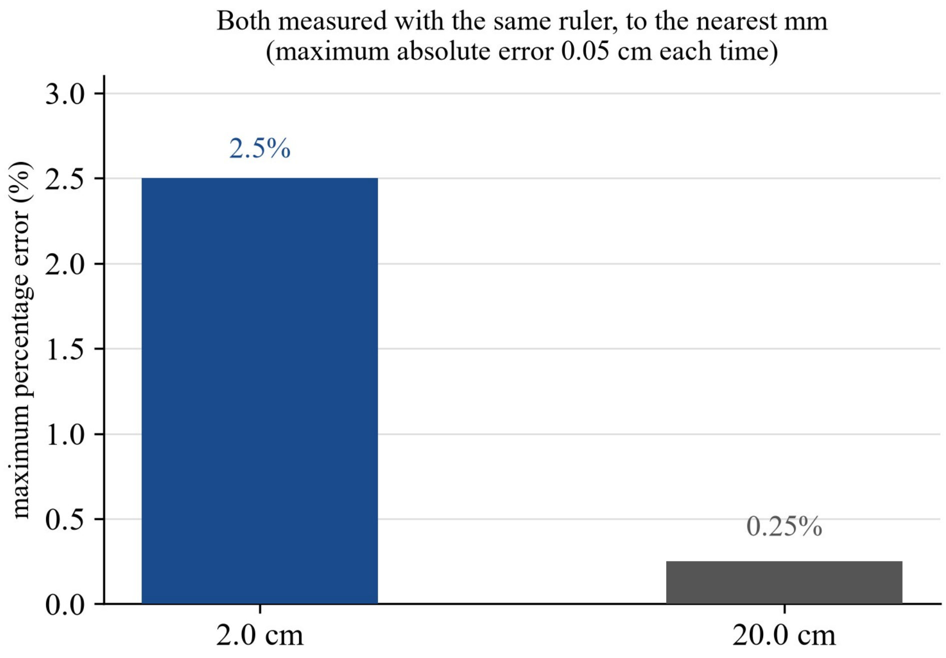
Q8. Against 80: $12\div 80\times 100\,\%=15\,\%$. Against 600: $12\div 600\times 100\,\%=2\,\%$. ∴ $15\,\%$ and $2\,\%$: one absolute error, two very different stories — the percentage error is what tells us how serious the error is, exactly the point of the two-attendances figure.



Two bar diagrams each showing an actual attendance in grey with the same red error block of 5 people on the end: against 40 the percentage error is 12.5 percent and against 4000 it is 0.125 percent; Q8 repeats the idea with 12 against 80 and 600

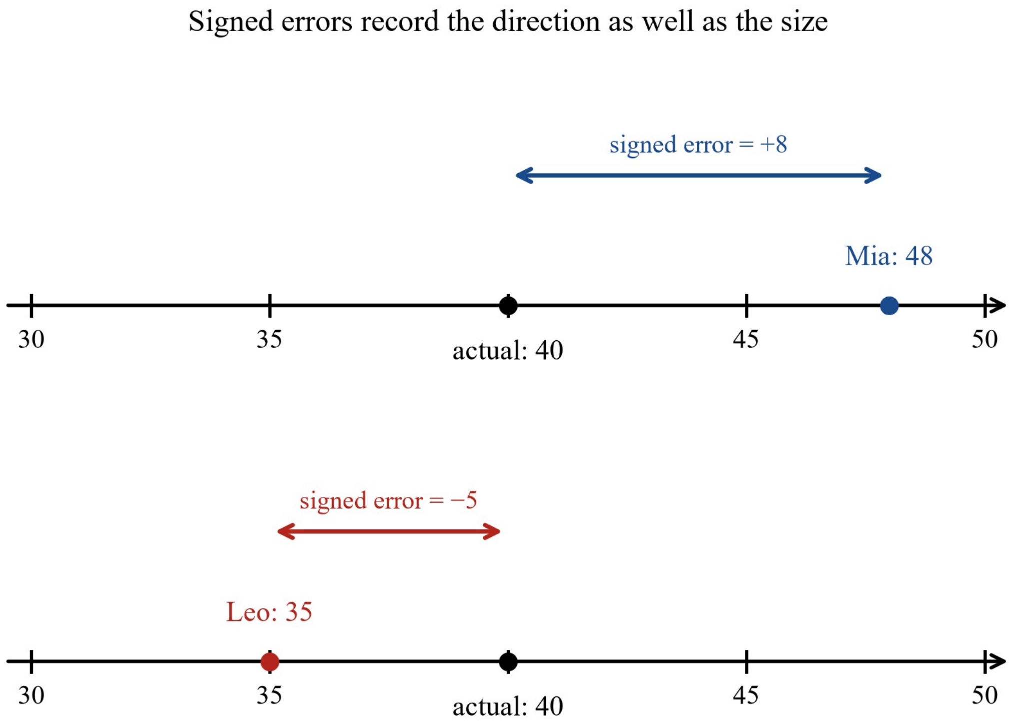
Q9. (a) Nearest 1 g → half-unit 0.5: bounds 53.5 and 54.5 g. (b) Nearest 0.01 s → half-unit 0.005: bounds 6.245 and 6.255 s. (c) Nearest 10 km → half-unit 5: bounds 145 and 155 km. (d) Nearest 0.5 L → half-unit 0.25: bounds 2.15 and 2.65 L. ∴ In every case the bounds are the recorded value ± half the unit of accuracy.

Q10. Maximum absolute error $= 0.05$ cm in both recordings. (a) $0.05 \div 1.8 \times 100\,\% = 2.777\ldots\% \approx 2.8\,\%$. (b) $0.05 \div 18.0 \times 100\,\% = 0.2777\ldots\% \approx 0.28\,\%$. $\therefore$ $\approx 2.8\,\%$ and $\approx 0.28\,\%$: the same ruler is ten times more accurate, in percentage terms, on the longer stick — the size effect shown in the bar chart.



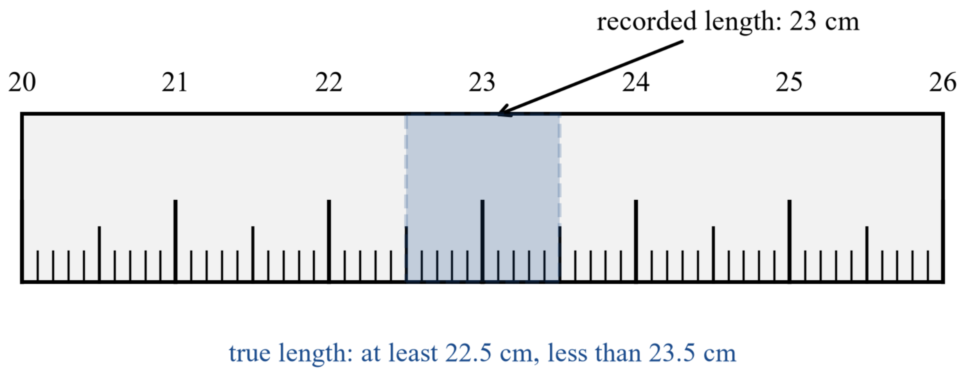
Bar chart of maximum percentage error for two lengths measured with the same ruler to the nearest millimetre: 2.5 percent for the 2.0 cm length and 0.25 percent for the 20.0 cm length; Q10 shows the same effect with 1.8 cm and 18.0 cm

Q11. Signed error $= 2000 - 1875 = +125$ meals. Absolute error $= 125$. Percentage error $= 125 \div 1875 \times 100\,\% = 6.666\ldots\% \approx 6.7\,\%$. $\therefore$ $+125$ and $\approx 6.7\,\%$ — an overestimate, so the signed error is positive, as with Mia's chairs in the signed-error figure.



Two number lines from 30 to 50 showing an overestimate with a positive signed error and an underestimate with a negative one; Q11's caterer overestimates like the first panel

Q12. (a) Against 50: $10 \div 50 \times 100\,\% = 20\,\%$; against 200: $10 \div 200 \times 100\,\% = 5\,\%$. Not equally serious: False. (b) $|45 - 20| = 25$; $25 \div 20 \times 100\,\% = 125\,\%$. True. (c) Recorded 8 cm to the nearest 1 cm has bounds 7.5–8.5 cm, and 8.5 itself rounds *up* to 9, so a true length of 8.5 cm records as 9 cm, not 8 cm. False. $\therefore$ (a) False; (b) True; (c) False — the upper bound is never included, as the ruler band shows.

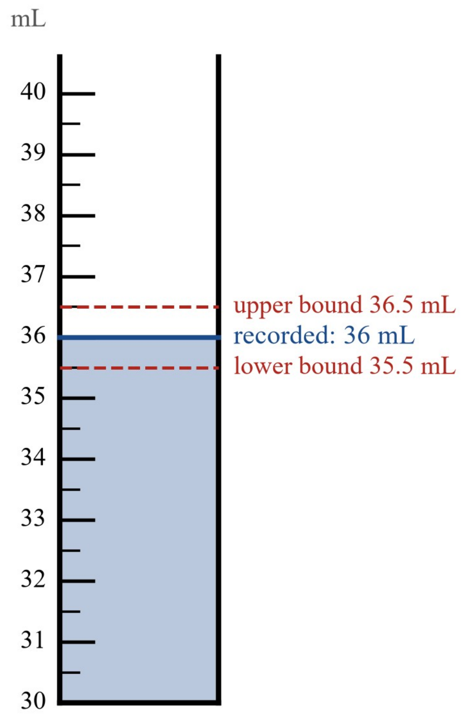


A ruler from 20 to 26 cm with millimetre marks and the band of lengths that record as 23 cm shaded blue; Q12(c) uses the same half-up rounding at the edge of the band

Q13. (a) 15% too high: $240 \times 1.15 = 276$ dollars. (b) 15% too low: $240 \times 0.85 = 204$ dollars. Check: $(276 - 240) \div 240 = 15\,\%$, and $(240 - 204) \div 240 = 15\,\%$. ∴ 276 dollars and 204 dollars.

Q14. Beaker: nearest 10 mL → maximum absolute error 5 mL; $5 \div 50 \times 100\,\% = 10\,\%$. Cylinder: nearest 1 mL → maximum absolute error 0.5 mL; $0.5 \div 49 \times 100\,\% = 1.0204\ldots\,\% \approx 1.0\,\%$. ∴ $10\,\%$ against $\approx 1.0\,\%$: the cylinder reading is the more accurate, even though the two volumes are almost the same — the instrument's unit of accuracy decides it.

Water measured to the nearest 1 mL

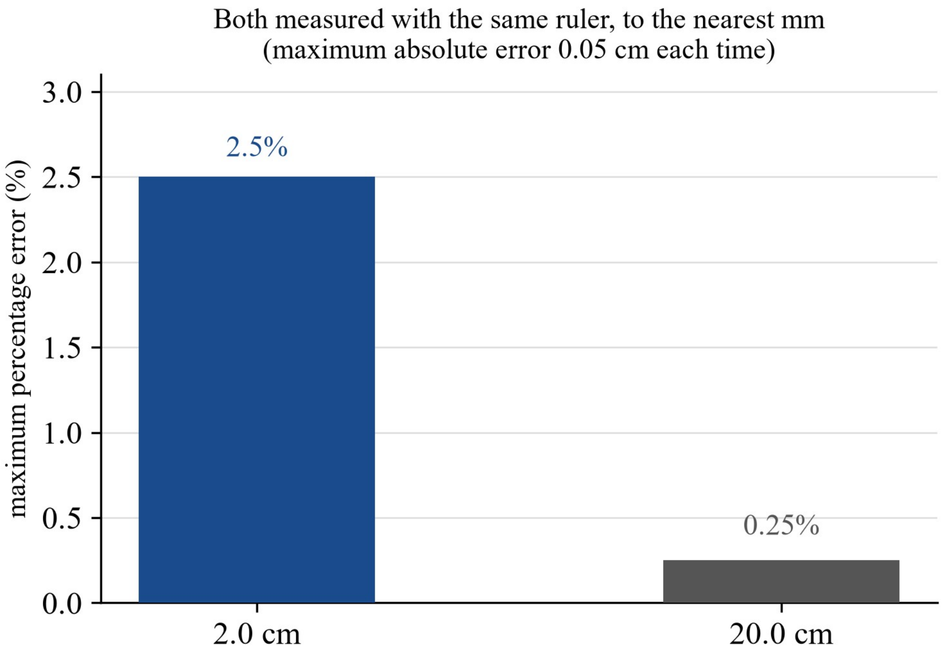


A measuring cylinder graduated from 30 to 40 mL with the water level recorded at 36 mL and dashed red lines at the bounds 35.5 mL and 36.5 mL; the cylinder in Q14 carries the same half-unit idea

Q15. Absolute error $= |180 - 225| = 45$ tiles. Percentage error $= 45 \div 225 \times 100\,\% = 20\,\%$. ∴ 45 and $20\,\%$ — the same shape of calculation as the fete estimate of Example 1.

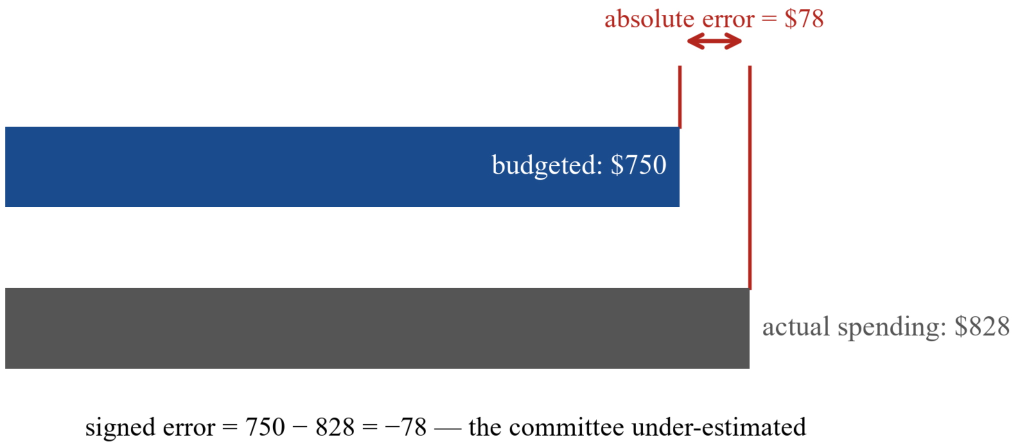
Q16. Absolute error $= |43 - 40| = 3$ min. Percentage error $= 3 \div 40 \times 100\,\% = 7.5\,\%$. The promise allows $5\,\%$ of $40 = 2$ min; the error was 3 min. ∴ The promise was not kept: $7.5\,\% > 5\,\%$ (equally, 3 min > 2 min).

Q17. (a) $0.05 \div 5 \times 100\,\% = 1\,\%$. (b) $0.05 \div 10 \times 100\,\% = 0.5\,\%$. (c) $0.05 \div 25 \times 100\,\% = 0.2\,\%$. $\therefore$ $1\,\%$, $0.5\,\%$, $0.2\,\%$: for a fixed unit of accuracy, the maximum percentage error shrinks as the recorded length grows — the same pattern as the 2.0 cm and 20.0 cm sticks.



Bar chart of maximum percentage error for two lengths measured with the same ruler to the nearest millimetre: 2.5 percent for the 2.0 cm length and 0.25 percent for the 20.0 cm length; Q17's three lengths follow the same falling pattern

Q18. (a) Signed error $= 640 - 725 = -85$; percentage $85 \div 725 \times 100\,\% = 11.724\ldots\,\% \approx 11.7\,\%$. (b) Signed error $= 1200 - 1134 = +66$; percentage $66 \div 1134 \times 100\,\% = 5.819\ldots\,\% \approx 5.8\,\%$. (c) $5.8\,\% < 11.7\,\%$: relatively speaking, the budget was the closer prediction. (d) $5\,\%$ of $750 = 37.5$, so acceptable estimates run from $750 - 37.5 = 712.5$ to $750 + 37.5 = 787.5$; in whole people, 713 to 787. $\therefore$ (a) -85, $\approx 11.7\,\%$; (b) $+66$, $\approx 5.8\,\%$; (c) the budget; (d) 713–787 people.



Two horizontal bars, a blue bar for a budgeted amount and a longer grey bar for the actual spending with the gap marked as the absolute error; Q18's two predictions are compared in the same way

Modelling with direct proportion, rates, ratio and scale, including financial contexts

5E · Chapter 5 — Measurement · Level 9

Content description `VC2M9M05` (word for word): *use mathematical modelling to solve practical problems involving direct proportion, rates, ratio and scale, including financial contexts; formulate the problems and interpret solutions in terms of the situation; evaluate the model and report methods and findings*

Achievement standard anchor (AS 12, transposed in the authority file — order corrected per decision D7): *They use mathematical modelling to solve practical problems involving direct and indirect proportion, ratio and scale, evaluating the model and communicating their methods and findings.* At Level 9 we model **direct** proportion only: inverse proportion and compound interest belong to Level 10 (`VC2M10M04`), and simple interest is taught in 3C (`VC2M9A06`). Where the AS wording and the content description differ, the content description governs (D3.12).

Elaboration ids for linkage: `VC2M9M05.E01`, `VC2M9M05.E02`, `VC2M9M05.E03`, `VC2M9M05.E04`

Exclusion note (CONTENT_POLICY Rule 1): `VC2M9M05.E05` (fire in land management as a rate context) is not used in this book; Aboriginal and Torres Strait Islander content is excluded. The remaining elaborations are covered: pay and exchange rates, quotes and Hooke's law (E01), image resizing and aspect distortion (E02), staircases and escalators (E03), and density, birth, flow and heart rates (E04).

Prerequisite pointers: 4B (scale factor and similar figures — reused here for map scale, plans and aspect ratio) and 2A (linear graphs — a straight line through the origin is the graph of direct proportion).

Sources: exchange rates are real data — Reserve Bank of Australia, "Exchange Rates" statistical table, as at 19 August 2026 (snapshot `_audit/fx_snapshot.html`). The GST rate of 10% follows house Rule 4. The staircase rule and the fuel figures are invented and labelled SCENARIO. Every other number is worked from these or from stated data.

Assessment pointer: the full modelling cycle practised here is assessed by extended task X2 (§7).

Digital tools: none required for 5E (D9).

Theory

Direct proportion

Two quantities are in **direct proportion** when one is a constant multiple of the other. Double one and the other doubles; treble one and the other trebles. Writing the quantities as x and y,

$$y = k\,x$$

where the fixed number k is the **constant of proportionality**. Every direct proportion in this section is a rule of this shape, and k always carries a meaning — a pay rate, a speed, a price per unit, a scale.

The **table test** for direct proportion: compute $y \div x$ for every row. If the quotient is the same number k every time, then $y = k\,x$ and the quantities are in direct proportion; if the quotient changes, they are not.

Direct proportion: y/x is constant

x	y	$y \div x$
2	5	$5/2 = 2.5$
4	10	$10/4 = 2.5$
6	15	$15/6 = 2.5$
8	20	$20/8 = 2.5$

$k = 2.5$ every time — so $y = 2.5x$

Not direct proportion: y/x changes

x	y	$y \div x$
1	4	$4/1 = 4$
2	5	$5/2 = 2.5$
3	6	$6/3 = 2$
4	7	$7/4 = 1.75$

no single k — not proportional

Two tables side by side: on the left, y divided by x equals 2.5 in every row, so y is directly proportional to x; on the right, the quotient changes from row to row, so it is not

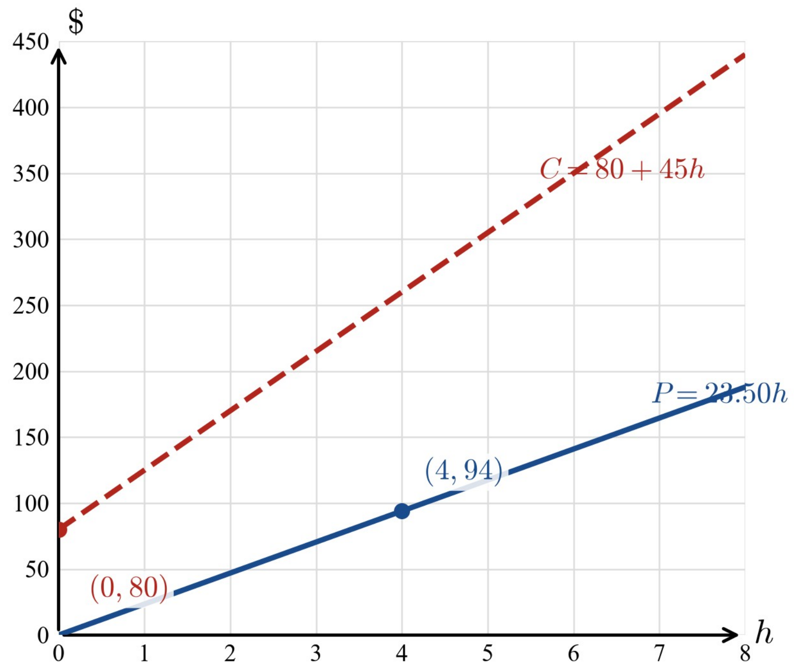
A pay table makes the test concrete. A casual job pays $23.50 per hour, pro rata.

Hours, h	2	4	6	8
Pay, P ($)	47.00	94.00	141.00	188.00
$P \div h$	23.50	23.50	23.50	23.50

The quotient is 23.50 in every row — the hourly rate — so $P = 23.50\,h$: pay is directly proportional to hours.

Now compare a plumber who charges a $80 call-out fee plus $45 per hour: $C = 80 + 45\,h$. At $h = 2$, $C = 170$; at $h = 4$, $C = 260$. The quotients $170 \div 2 = 85$ and $260 \div 4 = 65$ are different, so C is **not** directly proportional to h, even though the rule looks almost the same. The tell-tale sign: a direct proportion always gives $y = 0$ when $x = 0$, and the plumber's rule gives $C = 80$ at $h = 0$.

The same test appears on a graph. The graph of $y = k\,x$ is a **straight line through the origin**; k is its gradient. Any straight line that misses the origin — like $C = 80 + 45\,h$, which meets the vertical axis at $(0, 80)$ — is not a direct proportion.



Graph of pay against hours: the line P equals 23.50 h passes through the origin and the point (4, 94), while the line C equals 80 plus 45 h is straight but meets the vertical axis at (0, 80), so it is not a direct proportion

Ratio

A **ratio** compares the sizes of two or more parts, written with a colon and no spaces, as $2{:}3$. A ratio is simplified by dividing every part by the same number, just as a fraction is: $1200{:}900 = 4{:}3$ (dividing by 300).

Ratios split a total into shares. To split \$480 in the ratio $3{:}5$: the shares make $3+5=8$ equal parts, one part is $480 \div 8 = 60$, and the shares are $3 \times 60 = 180$ and $5 \times 60 = 300$. Check: $180+300=480$.

A ratio of two quantities in the same units is also the scale factor of 4B: $4{:}3$ says the first quantity is $4/3$ of the second.

Rates

A **rate** is a ratio in which the two sides have different units: \$23.50 per hour, 8 litres per minute, 72 beats per minute, 112.63 yen per Australian dollar. When a rate stays constant, the quantity and the driver are in direct proportion — 8 L/min means $V=8t$, and the constant of proportionality *is* the rate. Rates let us compare like with like: "102 births per 8500 people" becomes easier to read as a rate per 1000 people, $102 \div 8.5 = 12$ per 1000.

Gradient is a rate too — rise per unit of run. An escalator that climbs 4 m over a horizontal run of 9 m has gradient $4 \div 9 = 4/9$, and over an 18 m run it climbs $18 \times 4/9 = 8$ m. Comparing steepness by gradient is comparing rates; no angles are needed.

Scale

A **scale** states how a drawing, map or model relates to the real thing, in the same units on both sides. A map at scale $1{:}25\,000$ says 1 cm on the map stands for 25 000 cm — that is, 250 m — in real life. Lengths scale by the factor 25 000; **areas scale by the factor squared**, $25\,000^2$, so 1 cm² of that map stands for $62\,500$ m² of ground, which is 6.25 hectares. Forgetting to square the scale factor is the classic scale error, and the questions below check it twice.

The same idea resizes images. A photo that is 1200×900 pixels has **aspect ratio** $1200{:}900 = 4{:}3$. Resizing it to width 1600 while keeping the shape means the height becomes $1600 \times 3/4 = 1200$ — both sides multiplied by the same scale factor. Setting the height to anything else stretches one direction only, and circles in the photo turn into ellipses.

Financial contexts

An **exchange rate** is a rate in the sense above: the Reserve Bank of Australia publishes how many units of each foreign currency one Australian dollar buys. On 19 August 2026 these rates were:

1 AUD buys	USD	NZD	JPY	EUR	GBP
units	0.7070	1.2042	112.63	0.6101	0.5220

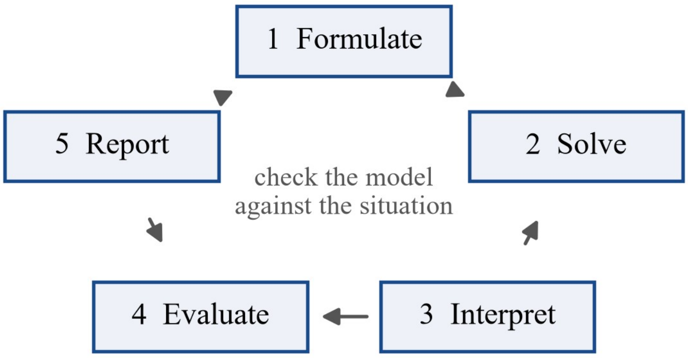
Source: Reserve Bank of Australia, "Exchange Rates" statistical table, 19 August 2026. Retrieved 20 August 2026.

Converting AUD to foreign currency multiplies by the rate ($U = 0.7070\,A$ for US dollars); converting back divides by it. Because both directions are $y = k\,x$, exchange is direct proportion in each direction — with a *different* k each way.

The goods and services tax (GST) is 10%. Adding GST multiplies a price by 1.1; removing GST divides by 1.1 (in other words, the GST contained in a GST-inclusive price is that price $\div 11$). Unit pricing — dollars per kilogram, cents per 100 g — is a rate used to compare value, and pro rata pay is the direct proportion $P = k\,h$ with the hourly rate as k.

The modelling cycle

Every problem in this section runs the same five-move cycle.



Flow chart of the modelling cycle: 1 Formulate leads to 2 Solve, then 3 Interpret, then 4 Evaluate, then 5 Report, and back to Formulate, with the note "check the model against the situation" in the centre

1. **Formulate.** Decide what to ignore and name the variables; write the proportional model $y = k\,x$, stating k and its units, and state the **assumptions** — for example, "the rate stays constant".
2. **Solve.** Carry out the calculation, one step per line.
3. **Interpret.** Translate the number back into the situation, with units: not "1.6" but "1.6 km of walking track". Round to what the situation allows — you cannot send 7.5 eggs to market.
4. **Evaluate.** Check the model against the situation. Does the answer make sense? Where would the model break — does the rate really stay constant that far?
5. **Report.** State the method and the finding in a sentence someone outside the maths room could read.

The cycle is the spine of the worked examples that follow: WE8 runs all five moves in full.

Worked examples

Example 1 — scaffolded

A casual job pays \$23.50 per hour, pro rata. Model the pay, then find the pay for 7.5 hours.

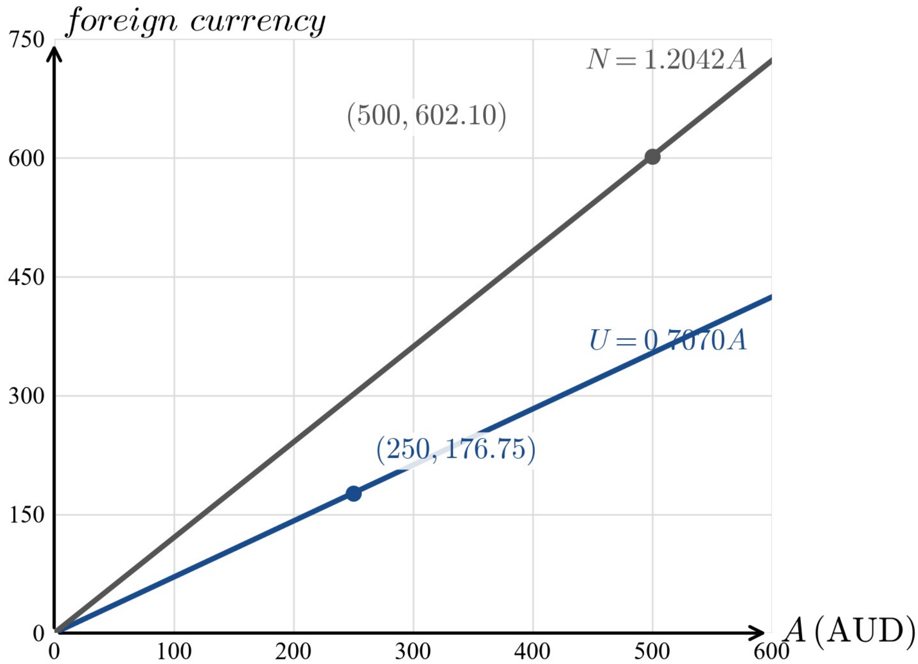
Working	Comment
Assume the rate is constant and there is no loading	Formulate: state the assumption.
$P = 23.50\,h$, with P in dollars and h in hours	The model: direct proportion with $k = 23.50$ dollars per hour.
$P = 23.50 \times 7.5$	Solve: substitute $h = 7.5$.
$P = 176.25$	One step at a time.
Check: $P \div h = 176.25 \div 7.5 = 23.50$	The quotient is the rate, so the point sits on $P = 23.50\,h$.
∴ The pay for 7.5 hours is \$176.25.	Interpret: dollars and hours, as the job quotes them.

Example 2 — scaffolded

Using the RBA rates of 19 August 2026, convert AUD 250 to US dollars, and convert USD 141.40 back to Australian dollars.

Working	Comment
$U = 0.7070\,A$	Formulate: 1 AUD buys 0.7070 USD, so the model is direct proportion with $k = 0.7070$.
$U = 0.7070 \times 250$	Solve: substitute $A = 250$.
$U = 176.75$	AUD 250 buys USD 176.75.

Working	Comment
$A = 141.40 \div 0.7070$	Converting back divides by the same k.
$A = 200$	
Check: $1 \div 0.7070 \approx 1.4144$	The reverse rate: 1 USD is worth about 1.4144 AUD — a different k, as the theory warned.
∴ AUD 250 = USD 176.75, and USD 141.40 = AUD 200, at these rates.	Interpret in the currency each side names.



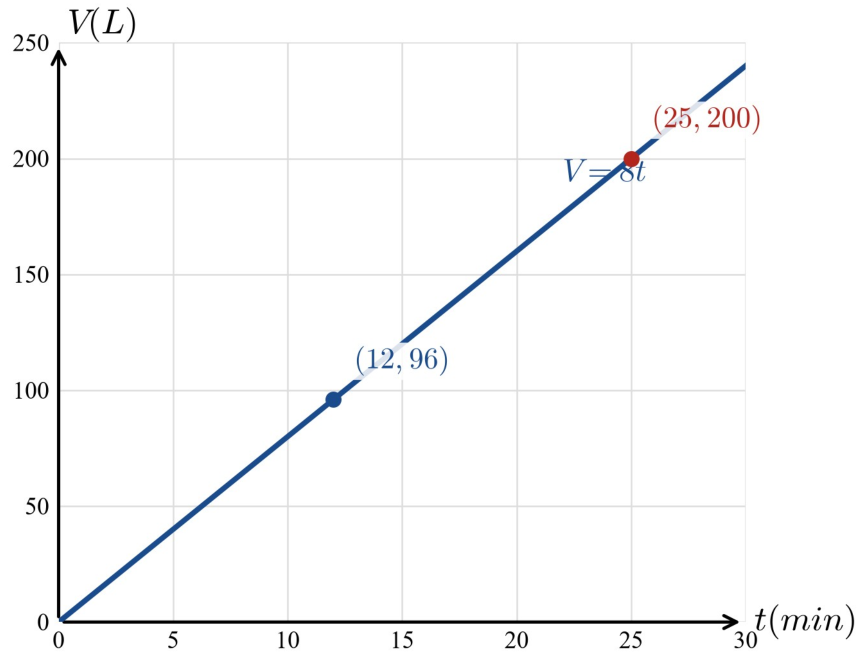
Graph of foreign currency against Australian dollars: the line U equals 0.7070 A passes through (250, 176.75) and the line N equals 1.2042 A passes through (500, 602.10), both straight lines through the origin

Example 3 — scaffolded

A tap fills a tank at 8 L per minute. How much water flows in 12 minutes, and how long does 200 L take?

Working	Comment
Assume the flow rate stays constant.	Formulate.
$V = 8t$, with V in litres and t in minutes	The rate is the constant of proportionality.
$V = 8 \times 12 = 96$	Solve the first question.
$t = 200 \div 8 = 25$	The second question solves the same model for t.
∴ 96 L flows in 12 minutes, and 200 L takes 25 minutes.	Interpret with units.

The same model shape reads a heart rate. At rest, a heart beats about 72 times per minute: $B = 72t$. In 15 minutes it beats $72 \times 15 = 1080$ times, and in one hour $72 \times 60 = 4320$ times. The assumption — a steady resting rate — is what the evaluate move would question after exercise.

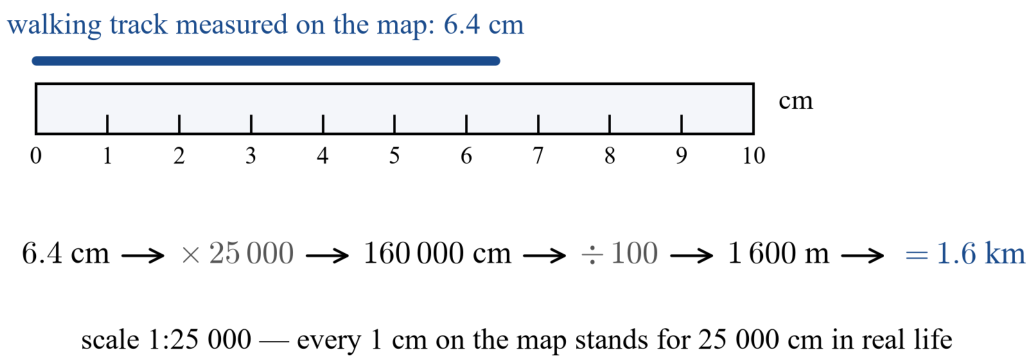


Graph of volume against time for the tap: the line V equals 8 t passes through (12, 96) and (25, 200)

Example 4 — scaffolded

A walking track measures 6.4 cm on a map at scale 1:25 000. How long is the track in real life? And how many centimetres on the map show a 4.5 km section?

Working	Comment
1 cm on the map = 25 000 cm in real life	Formulate: the scale is the constant of proportionality.
$6.4 \times 25\,000 = 160\,000$ cm	Solve: map length to real length.
$160\,000 \div 100 = 1600$ m $= 1.6$ km	Interpret: change to sensible units.
4.5 km $= 450\,000$ cm	The other direction: real length to cm first.
$450\,000 \div 25\,000 = 18$ cm	Divide by the scale factor.
∴ The track is 1.6 km long, and the 4.5 km section measures 18 cm on the map.	Both directions use the same k.



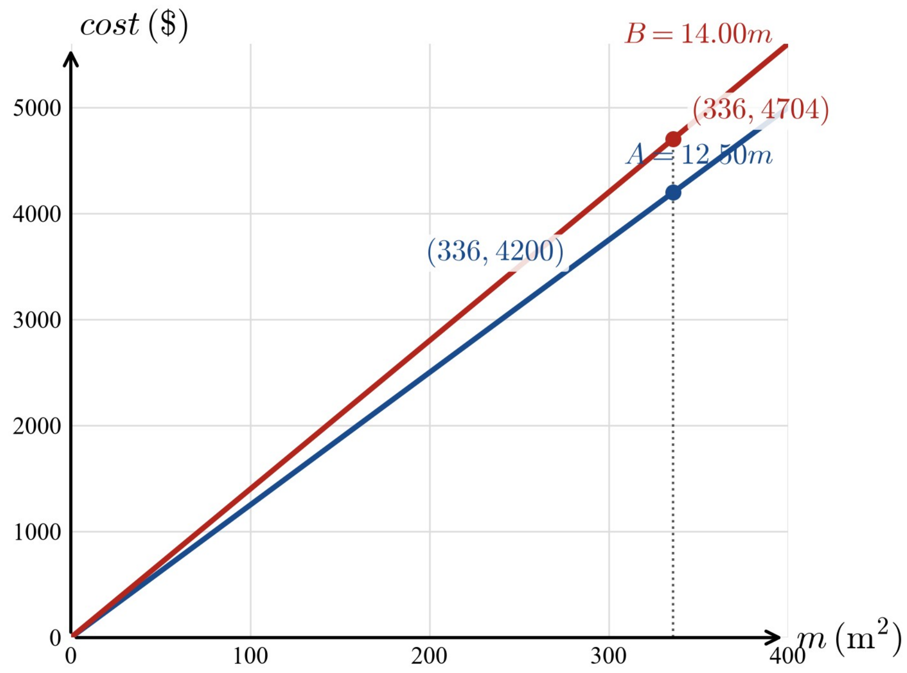
A ruler marked 0 to 10 cm with a 6.4 cm segment above it, and the conversion chain: 6.4 cm, times 25 000, gives 160 000 cm, divided by 100 gives 1600 m, which equals 1.6 km

Example 5 — unscaffolded

A school gym floor is 24 m by 14 m. Two contractors quote by the square metre: A charges \$12.50 per m² and B charges \$14.00 per m². Which quote is cheaper, and by how much?

Formulate: assume each quote is purely proportional to the area, with no fixed fee. Area: $24 \times 14 = 336$ m². Quote A: $12.50 \times 336 = 4200$, so \$4200. Quote B: $14.00 \times 336 = 4704$, so \$4704. Difference: $4704 - 4200 = 504$. Evaluate:

the difference is \$1.50 per m² — $336 \times 1.50 = 504$ agrees, so B is dearer whatever the area, as long as both quotes stay proportional.



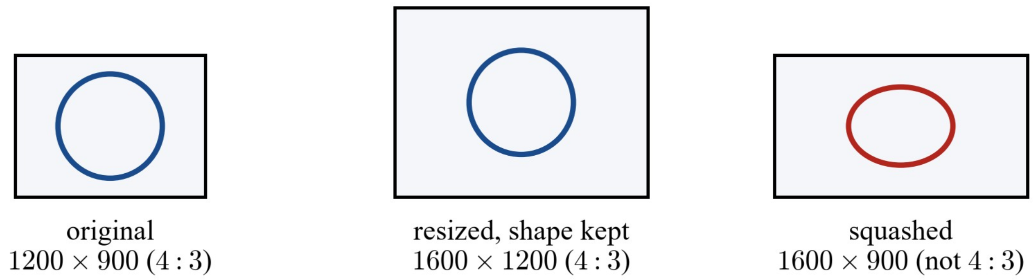
Graph of cost against area for the two quotes: the line B equals 14.00 m lies above the line A equals 12.50 m everywhere, with the points (336, 4200) and (336, 4704) marked

∴ Quote A is cheaper, by \$504 for this floor.

Example 6 — unscaffolded

A photo is 1200 × 900 pixels. It is resized to width 1600 pixels. What height keeps the shape? Describe what goes wrong if the height is left at 900.

Formulate: keeping the shape means multiplying both sides by the same scale factor — the aspect ratio $1200 : 900 = 4 : 3$ must not change. Scale factor: $1600 \div 1200 = 4/3 \approx 1.333$. Height that keeps the shape: $900 \times 4/3 = 1200$ pixels. If the height stays 900: the width is stretched by $4/3$ while the height is stretched by $900 \div 900 = 1$. The two directions use different factors, so the aspect ratio becomes $1600 : 900$, not $4 : 3$ — every circle in the photo is pulled into a horizontal ellipse.



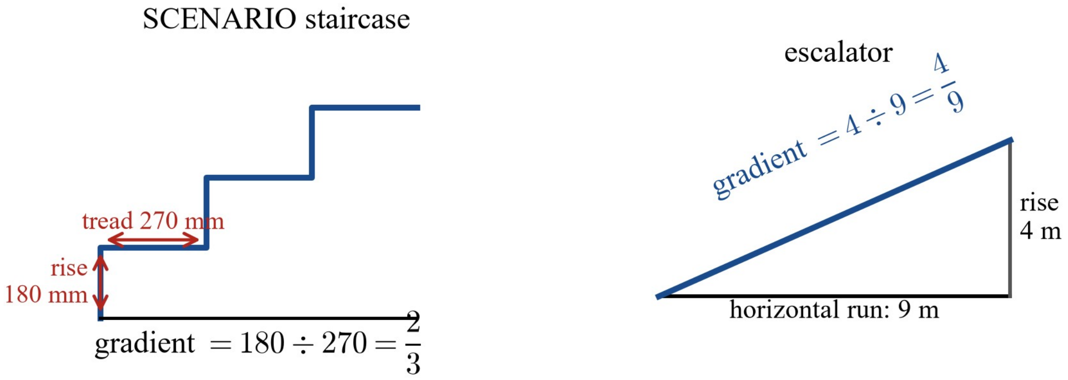
Three panels: a 1200 by 900 image with a circle inside; the same image resized to 1600 by 1200 with the circle still round; and the image squashed to 1600 by 900 with the circle stretched into an ellipse

∴ The height must be 1200 pixels; leaving it at 900 stretches the picture horizontally by the factor $4/3$ and distorts every shape.

Example 7 — unscaffolded

SCENARIO building guidance: a staircase is comfortable when the rise R and tread G (in mm) satisfy $600 \le 2R + G \le 650$. A stair has rise 180 mm and tread 270 mm. Find its gradient as a ratio, test the rule, and compare with an escalator that rises 4 m over a 9 m run.

Formulate: model each slope by its gradient, rise ÷ tread — a rate, with both lengths in the same units. Stair gradient: $180 \div 270 = 2/3 \approx 0.667$, that is, the ratio $2:3$. Rule test: $2 \times 180 + 270 = 630$, and $600 \le 630 \le 650$ — the stair meets the SCENARIO guidance. Escalator gradient: $4 \div 9 = 4/9 \approx 0.444$. Compare: $2/3 = 6/9 > 4/9$ — the stair is steeper, which is why escalators feel easier. Interpret: over an 18 m horizontal run the escalator rises $18 \times 4/9 = 8$ m.



Left: a staircase profile with rise 180 mm and tread 270 mm marked, gradient 180 divided by 270 equals two thirds. Right: an escalator as a triangle with rise 4 m and horizontal run 9 m, gradient 4 divided by 9

∴ The stair's gradient is $2:3$ and it satisfies the SCENARIO rule; the escalator's gentler gradient $4/9$ climbs 8 m over an 18 m run.

Example 8 — unscaffolded

A spring is loaded with masses and the extension recorded:

Load L (N)	0	1	2	3	4	5
Extension E (cm)	0	2.4	4.8	7.2	9.6	13.1

Run the full modelling cycle on this data.

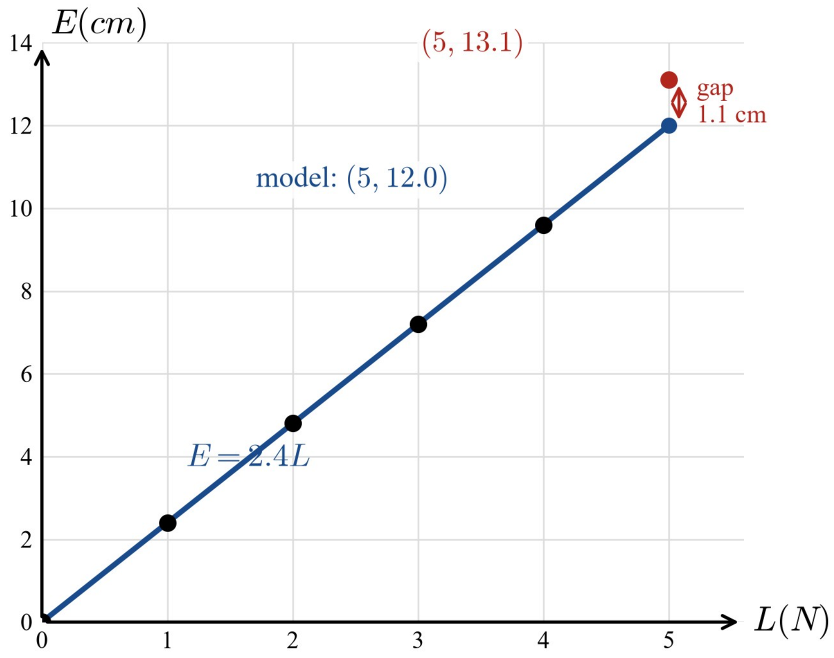
Formulate. The first five rows give $E \div L = 2.4$ every time, so propose the model $E = 2.4L$ — Hooke's law, extension directly proportional to load, with $k = 2.4$ cm/N. Assumption: the proportion holds for every load in the table.

Solve. The model predicts $E = 2.4 \times 5 = 12.0$ cm at 5 N; the table records 13.1 cm. It predicts $E = 2.4 \times 3.5 = 8.4$ cm at 3.5 N, and $E = 10.8$ cm comes from $L = 10.8 \div 2.4 = 4.5$ N.

Interpret. Up to 4 N the spring extends 2.4 cm per newton; a load of 4.5 N gives 10.8 cm of extension.

Evaluate. At 5 N the data leaves the model by $13.1 - 12.0 = 1.1$ cm — beyond about 4 N the spring stretches more than the proportion allows (it has been pulled past its elastic limit), so the assumption fails there and the model is trustworthy only up to about 4 N.

Report. Extension is proportional to load, $E = 2.4L$, for loads up to 4 N; at 5 N the spring extends 1.1 cm more than the model predicts, so the proportional model must not be used past 4 N.



Graph of extension against load: the points up to 4 N lie on the line E equals 2.4 L, while the 5 N point at (5, 13.1) sits 1.1 cm above the line's value of 12.0

∴ The model $E = 2.4\,L$ fits loads up to 4 N and breaks down at 5 N, where the spring extends 1.1 cm beyond prediction.

Practice questions

Use the RBA rates of 19 August 2026 (theory table) wherever a question needs an exchange rate.

Scaffolded

Q1. Apply the table test to each table. If y is directly proportional to x, give k.

a. x: 2, 4, 6, 8 with y: 5, 10, 15, 20
b. x: 1, 2, 3, 4 with y: 4, 5, 6, 7
c. x: 5, 10, 15 with y: 1, 2, 3

Q2. y is directly proportional to x, and $y = 30$ when $x = 4$.

a. Find k.
b. Find y when $x = 9$.
c. Find x when $y = 90$.

Q3. y is directly proportional to x, and $y = 12$ when $x = 5$.

a. Find k and write the model.
b. Find y when $x = 8$.
c. Explain in one sentence why $y = 2x + 3$ is not a direct proportion.

Q4. Convert using the RBA rates.

a. AUD 500 to New Zealand dollars.
b. AUD 320 to Japanese yen, to the nearest yen.

Q5. Convert using the RBA rates.

a. GBP 26.10 to Australian dollars.

b. EUR 122.02 to Australian dollars.

Q6. A resting heart rate is about 72 beats per minute.

a. How many beats in one hour?
b. How many minutes for 5400 beats?

Q7. A tap fills a tank at 8 L per minute.

a. What volume flows in 12 minutes?
b. How long does 200 L take?

Q8. A map is at scale 1:25 000.

a. A path measures 9.2 cm on the map. How long is it, in km?
b. A straight section is 750 m in real life. How many cm on the map?

Q9. Split $480 between two people in the ratio 3:5.

a. How many equal parts do the shares make?
b. What is one part worth?
c. Write the two shares and check they add to $480.

Q10. A video frame is 1920 × 1080 pixels.

a. Simplify the ratio 1920:1080.
b. A screen shows the video at width 80 cm with the shape kept. How high is the picture?

Q11. GST is 10%.

a. Find the GST on a price of $80 (before GST).
b. What is the price with GST included?
c. A price of $132 includes GST. Find the price before GST, and the GST contained in it.

Q12. One stair has rise 175 mm and tread 250 mm; the stair of Example 7 has gradient $2/3$.

a. Find the gradient of the first stair.
b. Which stair is steeper?

Unscaffolded

Q13. A spring extends 7.5 cm under a 3 N load, and extension is directly proportional to load. Find the extension under 6.4 N, and the load that gives 22 cm.

Q14. Sound travels through air at about 340 m per second, so wavelength equals speed ÷ frequency. Find the wavelength of a 170 Hz tone and of a 340 Hz tone, and state what happens to the product frequency × wavelength. (Finding one quantity by dividing by the other is a rate calculation; the inverse relationship itself belongs to Level 10.)

Q15. A train covers 240 km in 3 hours at a steady speed. Find the speed, the time for 100 km, and the distance covered in 45 minutes.

Q16. A steel block of volume 30 cm^3 has mass 237 g. Find the density of steel in g/cm^3, then the mass of 45 cm^3 of aluminium, whose density is 2.7 g/cm^3.

Q17. A town records 102 births per 8500 people in a year. Write this as a rate per 1000 people, then find the expected births in a city of 20 000.

Q18. Brand A coffee costs $6.25 for 250 g; brand B costs $9.60 for 400 g. Find each price per kilogram and say which is the better value.

Q19. Split $1260 in the ratio 2:3:4.

Q20. Find the missing value: a. $5:8=15:x$; b. $3:7=x:42$; c. simplify $48:120$.

Q21. A rectangular paddock measures 4 cm by 2 cm on a map at scale 1:25 000. Find its real area in km², and in hectares (1 km² = 100 ha).

Q22. A model railway is built at scale 1:87. A real carriage is 19.2 m long; find the model length in mm, to one decimal place. A model tank holds 25 mL; find the real volume in litres. (Volumes scale by the factor cubed: $87^3 = 658\,503$.)

Q23. A printer places 1000 pixels across 25 cm horizontally, but 800 pixels across 16 cm vertically. Find the horizontal and vertical cm-per-pixel rates, the horizontal stretch factor, and say what a printed circle looks like.

Q24. Test each stair against the SCENARIO rule of Example 7 (rise at most 190 mm, and $600 \le 2R + G \le 650$): a. rise 190 mm, tread 240 mm; b. rise 200 mm, tread 250 mm. Report which stair passes and which fails, and on what test.

Q25. A customer changes AUD 500 to US dollars at the RBA rate, then changes the whole amount back to Australian dollars at the same rate. What do they get, and why is it exactly the starting amount in this model? What does a real exchange desk add that the model leaves out?

Q26. Job A pays $23.50 per hour and job B pays $28.20 per hour, both pro rata. How many hours at B earn what 6 hours at A earn?

Q27. A paint mixer uses the ratio 4:1 of paint to water. How much paint and how much water go into 7.5 L of mix?

Q28. The contractors of Example 5 quote for a smaller job of 200 m². Compute both quotes, and explain why B is exactly $1.50 per m² dearer at every possible area.

Q29. At 72 beats per minute, how many times does a resting heart beat in 15 minutes, and in one hour?

Q30. Estimate the beats in a year at a steady 72 per minute. State one reason the real count would differ.

Q31. A floor plan is drawn at scale 1:50. A room is 420 cm by 360 cm. Find the drawing dimensions in cm.

Q32. SCENARIO: a car uses 7.5 L of fuel per 100 km, and fuel costs $1.86 per litre. Find the fuel used and the cost for a 340 km trip.

Q33. A recipe for 8 people uses 500 g of flour and 3 eggs. Scale it to 20 people, and interpret the egg count the way a cook must.

Q34. Convert 90 km/h to m/s. A sprinter runs 100 m in 12.5 s; find the speed in m/s, then in km/h, and compare with the 90 km/h car.

Q35. A phone gains 20% charge in 12 minutes. A proportional model predicts the time to full charge. Find it, then evaluate the assumption for a real phone.

Q36. A market stall sells portions for $6.00 each; each portion costs $3.60 to make. Write the profit for n portions as a direct proportion, and find the profit, the income and the cost for 40 portions.

Answers

Q1 a yes, $k=2.5$; b no; c yes, $k=0.2$. **Q2** a 7.5; b 67.5; c 12. **Q3** a $k=2.4$, $y=2.4x$; b 19.2; c at $x=0$ it gives $y=3$, not 0. **Q4** a NZD 602.10; b ¥36 042. **Q5** a AUD 50.00; b AUD 200.00. **Q6** a 4320; b 75 minutes. **Q7** a 96 L; b 25 minutes. **Q8** a 2.3 km; b 3 cm. **Q9** a 8; b $60; c $180 and $300. **Q10** a 16:9; b 45 cm. **Q11** a $8; b $88; c $120 before GST, $12 of GST. **Q12** a 0.7; b the 175/250 stair. **Q13** 16 cm; 8.8 N. **Q14** 2 m; 1 m; the product is 340 both times. **Q15** 80 km/h; 1.25 h; 60 km. **Q16** 7.9 g/cm³; 121.5 g. **Q17** 12 per 1000; 240 births. **Q18** A $25/kg; B $24/kg; B. **Q19** $280, $420, $560. **Q20** a 24; b 18; c 2:5. **Q21** 0.5 km²; 50 ha. **Q22** 220.7 mm; 16 462.575 L. **Q23** 0.025 cm/px; 0.02 cm/px; 1.25; circles look stretched sideways. **Q24** a passes both tests; b fails the rise test (200 > 190) although $2R+G=650$ passes. **Q25** AUD 500; converting multiplies then divides by the same k; a real desk adds a fee or a spread. **Q26** 5 hours. **Q27** 6 L of paint, 1.5 L of water. **Q28** $2500 and $2800; $14.00-12.50=1.50$ per m². **Q29** 1080; 4320. **Q30** 37 843 200; a real heart rate is not steady all year. **Q31** 8.4 cm by 7.2 cm. **Q32** 25.5 L; $47.43. **Q33** 1250 g of flour; 7.5 eggs by the model, so 8 in the kitchen. **Q34** 25 m/s; 8 m/s; 28.8 km/h; the car is much faster. **Q35** 60 minutes; real charging slows near full, so the model understates the time. **Q36** $2.40n; profit $96; income $240; cost $144.

Fully-worked solutions

Q1. a $5\div 2=2.5$, $10\div 4=2.5$, $15\div 6=2.5$, $20\div 8=2.5$ — constant, so yes, $k=2.5$. b $4\div 1=4$, $5\div 2=2.5$, $6\div 3=2$, $7\div 4=1.75$ — the quotient changes, so no. c $1\div 5=0.2$, $2\div 10=0.2$, $3\div 15=0.2$ — constant, so yes, $k=0.2$.

Q2. a $k=30\div 4=7.5$. b $y=7.5\times 9=67.5$. c $x=90\div 7.5=12$.

Q3. a $k=12\div 5=2.4$; the model is $y=2.4x$. b $y=2.4\times 8=19.2$. c At $x=0$, $y=2\times 0+3=3$, but a direct proportion must give 0 at $x=0$.

Q4. a $500\times 1.2042=602.10$ — NZD 602.10. b $320\times 112.63=36\,041.6$ — to the nearest yen, ¥36 042.

Q5. a $26.10\div 0.5220=50$ — AUD 50.00. b $122.02\div 0.6101=200$ — AUD 200.00.

Q6. a $72\times 60=4320$ beats. b $5400\div 72=75$ minutes.

Q7. a $V=8\times 12=96$ L. b $t=200\div 8=25$ minutes.

Q8. a $9.2\times 25\,000=230\,000$ cm $=2300$ m $=2.3$ km. b 750 m $=75\,000$ cm; $75\,000\div 25\,000=3$ cm.

Q9. a $3+5=8$ parts. b $480\div 8=60$ per part. c $3\times 60=180$ and $5\times 60=300$; check $180+300=480$.

Q10. a Divide both by 120: $1920:1080=16:9$. b Height $=80\times 9/16=45$ cm.

Q11. a $80\div 10=8$ — the GST is $8. b $80+8=88$. c $132\div 1.1=120$ before GST; the GST is $132-120=12$ (check: $132\div 11=12$).

Q12. a $175\div 250=0.7$. b $0.7>2/3\approx 0.667$ — the 175/250 stair is steeper.

Q13. $k=7.5\div 3=2.5$ cm/N, so $E=2.5L$. $E=2.5\times 6.4=16$ cm. $L=22\div 2.5=8.8$ N. ∴ The extension is 16 cm and the load is 8.8 N.

Q14. 170 Hz: $340\div 170=2$ m. 340 Hz: $340\div 340=1$ m. Product: $170\times 2=340$ and $340\times 1=340$ — the speed, both times. ∴ The wavelengths are 2 m and 1 m; the product frequency × wavelength stays 340 m/s.

Q15. Speed: $240\div 3=80$ km/h. Time for 100 km: $100\div 80=1.25$ h. 45 min $=0.75$ h: $80\times 0.75=60$ km. ∴ 80 km/h; 1.25 h; 60 km.

Q16. Density: $237\div 30=7.9$ g/cm³. Aluminium: $2.7\times 45=121.5$ g. ∴ Steel is 7.9 g/cm³; the aluminium piece has mass 121.5 g.

Q17. $102\div 8.5=12$ births per 1000 people. $12\times 20=240$ births in 20 000. ∴ 12 per 1000; about 240 births.

Q18. A: $6.25\div 0.25=25$ per kg. B: $9.60\div 0.4=24$ per kg. ∴ Brand B is the better value at $24/kg.

Q19. $2+3+4=9$ parts; one part $1260 \div 9 = 140$. Shares: 280, 420, 560; check $280+420+560=1260$. ∴ \$280, \$420 and \$560.

Q20. a $x = 15 \times 8/5 = 24$. b $x = 42 \times 3/7 = 18$. c $48:120 = 2:5$ (divide by 24).

Q21. Length: $4 \times 25\,000 = 100\,000$ cm = 1 km. Width: $2 \times 25\,000 = 50\,000$ cm = 0.5 km. Area: $1 \times 0.5 = 0.5$ km² = 50 ha. ∴ 0.5 km², that is, 50 hectares.

Q22. Model length: 19.2 m = 19 200 mm; $19\,200 \div 87 \approx 220.7$ mm. Real volume: $25 \times 658\,503 = 16\,462\,575$ mL = 16 462.575 L. ∴ The model carriage is about 220.7 mm long, and the real tank holds 16 462.575 L.

Q23. Horizontal: $25 \div 1000 = 0.025$ cm per pixel. Vertical: $16 \div 800 = 0.02$ cm per pixel. Stretch factor: $0.025 \div 0.02 = 1.25$. ∴ Circles print 1.25 times as wide as they are tall — stretched sideways.

Q24. a $2 \times 190 + 240 = 620$, inside 600–650, and the rise 190 is not more than 190 — passes both tests. b $2 \times 200 + 250 = 650$, inside the band, but the rise 200 mm is more than the 190 mm maximum — fails the rise test. ∴ Stair (a) passes; stair (b) fails on rise even though $2R+G$ sits on the band's edge.

Q25. Out: $500 \times 0.7070 = 353.50$ USD. Back: $353.50 \div 0.7070 = 500$ AUD. The model multiplies by k and then divides by the same k, which undoes itself exactly. A real desk charges a fee or quotes a worse rate in one direction — a spread the model leaves out. ∴ The customer gets back exactly AUD 500 in this model.

Q26. 6 h at A: $23.50 \times 6 = 141$. Hours at B: $141 \div 28.20 = 5$. ∴ 5 hours at B earn what 6 hours at A earn.

Q27. $4+1=5$ parts; one part $7.5 \div 5 = 1.5$ L. Paint: $4 \times 1.5 = 6$ L; water: $1 \times 1.5 = 1.5$ L. ∴ 6 L of paint and 1.5 L of water.

Q28. A: $12.50 \times 200 = 2500$. B: $14.00 \times 200 = 2800$. Both quotes are $y = kx$ with k differing by $14.00 - 12.50 = 1.50$, so at any area m the difference is $1.50m$ — at 336 m² that is the \$504 of Example 5. ∴ \$2500 and \$2800; B is \$1.50 per m² dearer at every area.

Q29. $72 \times 15 = 1080$ beats in 15 minutes. $72 \times 60 = 4320$ beats in one hour. ∴ 1080 and 4320.

Q30. $72 \times 60 \times 24 \times 365 = 37\,843\,200$ beats in a year. Evaluate: a real heart rate rises with activity and falls in sleep, so the steady-rate model only estimates. ∴ About 37 843 200 beats, as an estimate from a constant-rate model.

Q31. $420 \div 50 = 8.4$ cm; $360 \div 50 = 7.2$ cm. ∴ The room draws as 8.4 cm by 7.2 cm.

Q32. Fuel: $7.5 \times 340 \div 100 = 25.5$ L. Cost: $25.5 \times 1.86 = 47.43$. ∴ The trip uses 25.5 L and costs \$47.43 (SCENARIO figures).

Q33. Scale factor: $20 \div 8 = 2.5$. Flour: $500 \times 2.5 = 1250$ g. Eggs: $3 \times 2.5 = 7.5$ — the model says 7.5; a cook interprets that as 8 eggs. ∴ 1250 g of flour and (interpreting 7.5) 8 eggs.

Q34. 90 km/h = $90 \times 1000 \div 3600 = 25$ m/s. Sprinter: $100 \div 12.5 = 8$ m/s. $8 \times 3600 \div 1000 = 28.8$ km/h. ∴ The car's 25 m/s is far beyond the sprinter's 8 m/s (28.8 km/h).

Q35. Rate: $20 \div 12 = 5/3$ percent per minute. Full charge: $100 \div 5/3 = 60$ minutes. Evaluate: real phones charge quickly at low charge and slowly near full, so the rate is not constant and 60 minutes understates the true time. ∴ The proportional model predicts 60 minutes; a real phone takes longer.

Q36. Profit per portion: $6.00 - 3.60 = 2.40$, so profit $= 2.40n$ — direct proportion with $k = 2.40$. At $n = 40$: profit $2.40 \times 40 = 96$; income $6.00 \times 40 = 240$; cost $3.60 \times 40 = 144$. Check: $240 - 144 = 96$. ∴ Profit is \$2.40n; for 40 portions the profit is \$96 on income \$240 and cost \$144.

Test

Test T5 · Chapter 5 — Measurement · Level 9

This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 9 achievement standard, not against a mark on this paper.

Time: 41 minutes — 5 minutes reading time plus 36 minutes writing time (1 minute per mark).

Structure: two parts, as the curriculum itself is split (D9). **Part A is technology-free** (14 marks): no calculators and no digital tools. **Part B is technology-active** (22 marks): a scientific calculator, spreadsheet, graphing tool, dynamic geometry software or a programming language, as nominated by the teacher. **No CAS.** Answer all questions.

Equipment: ruler, protractor and pair of compasses.

Reasoning: in this paper, reasoning is marked separately from the answer where the content description requires an explanation (`VC2M9M03`). On those items (Q3b and Q8a) a correct number with no stated reason does not score full marks.

Cumulative reach: this test is cumulative over the chapters before Chapter 5. Items draw on the exponent laws of 1B (Q2, Q9), the trigonometric ratios of 4A and the scale-factor rules of 4B (Q4b, Q6, Q8), the distance and Pythagoras'-theorem work behind 5C (Q3), and the straight-line-through-the-origin graph of direct proportion from 2A (Q12). These skills are used as tools, exactly as the chapter used them.

Dated items (R3): the exchange rate in Q12 is as at August 2026; the rainfall record in Q11 is for July 2026. Both are listed here so a reissue can refresh them without re-reading the paper.

Mark schedule — standard subtopic 6 marks, extended subtopic 10 marks, full-cycle CDs 4 test marks (§7):

Subtopic	CD	Part A	Part B	Total	Items
5A	`VC2M9M01`	3	7	**10**	Q1, Q7, Q11b, Q11c
5B	`VC2M9M02`	2	4	**6**	Q2, Q9, Q11a
5C	`VC2M9M03`	7	3	**10**	Q3, Q4, Q6, Q8
5D	`VC2M9M04`	2	4	**6**	Q5, Q10, Q11d
5E	`VC2M9M05`	0	4	**4**	Q12
Total		**14**	**22**	**36**	

The real data in Q11 and Q12 carries its source line at the point of use (R1). Every other context is a clearly-labelled SCENARIO whose numbers are given in the question (R2).

Part A — technology-free (14 marks)

Q1 (3 marks). a. A triangular prism has a right-triangle cross-section with base 8 cm and height 5 cm, and length 12 cm. Find its volume. **(1 mark)** b. Convert: (i) 2.6 L to cm³; (ii) 45 000 cm³ to litres. **(1 mark)** c. Find the surface area of a cuboid 7 cm by 5 cm by 3 cm. **(1 mark)**

Q2 (2 marks). a. Write $0.000\,072$ in scientific notation. **(1 mark)** b. Which one of the following is in scientific notation? **(1 mark)**

A 45×10^3 B 0.45×10^6 C $4.5 \times 10^{2.5}$ D 4.5×10^{-4}

Q3 (3 marks). a. A right-angled triangle has hypotenuse 25 cm and one shorter side 7 cm. Find the other shorter side. **(1 mark)** b. A triangle has sides 8 cm, 15 cm and 17 cm. Is it right-angled? Give the reason for your answer. **(2 marks)**

Q4 (2 marks). a. One acute angle of a right-angled triangle is 34°. Find the other acute angle. **(1 mark)** b. A right-angled triangle has hypotenuse 16 cm and one angle of 30°. Without using a calculator, find the length of the side opposite the 30° angle. **(1 mark)**

Q5 (2 marks). SCENARIO — a market stall. A stallholder estimates that 360 cups will be sold; the actual number sold is 300. a. Find the signed error of the estimate. **(1 mark)** b. Find the percentage error of the estimate. **(1 mark)**

Q6 (2 marks). Two similar solids have scale factor $k=3$ from the smaller to the larger. a. The smaller solid has surface area 6 cm². Find the surface area of the larger solid. **(1 mark)** b. The smaller solid has volume 4 cm³. Find the volume of the larger solid. **(1 mark)**

Part B — technology-active (22 marks)

Q7 (3 marks). A cylinder has radius 6 cm and height 15 cm. a. Find its volume, exact in π and to one decimal place. **(2 marks)** b. Find its surface area, exact in π and to one decimal place. **(1 mark)**

Q8 (3 marks). a. A straight ramp is 9.5 m long and makes an angle of 32° with the ground. Find the height the ramp rises, to one decimal place. **(2 marks)** b. In a right-angled triangle, the side opposite an angle θ is 7 cm and the hypotenuse is 15 cm. Find θ, to one decimal place. **(1 mark)**

Q9 (3 marks). a. Rewrite 48×10^{-5} in scientific notation. **(1 mark)** b. Evaluate $(2.4\times10^{6})\div(8\times10^{-2})$, leaving your answer in scientific notation. **(1 mark)** c. A gap is 0.045 mm wide. Write this width in metres, in scientific notation. **(1 mark)**

Q10 (2 marks). The mass of a parcel is recorded as 12.4 kg, measured to the nearest 0.1 kg. a. Give the lower and upper bounds of the true mass. **(1 mark)** b. Find the maximum percentage error of the recording, to one decimal place. **(1 mark)**

Q11 (7 marks). Real Victorian context — July rainfall in Melbourne.

The Bureau of Meteorology recorded the following rainfall at Melbourne (Olympic Park) in July 2026. These were the only days of the month with measurable rain; every other day recorded 0.0 mm. The month totalled **44.4 mm**.

Day of July	1	2	3	4	5	9	12	13	14	15	16	23	26	29	30
Rainfall (mm)	6.6	4.0	7.0	0.2	1.6	0.2	14.6	1.4	0.4	0.2	0.2	3.0	0.6	2.2	2.2

Source: Bureau of Meteorology, "Daily Weather Observations for Melbourne (Olympic Park), Victoria, July 2026" (station 086338), prepared 14 August 2026. Retrieved 20 August 2026.

SCENARIO — a house in Melbourne has a flat rectangular roof, 15 m by 8 m. All the rain that falls on the roof drains into one cylindrical tank of radius 1.2 m and height 1.5 m. Take 1 mm of rain on 1 m² of roof to collect 1 L of water.

a. Write the July rainfall total, 44.4 mm, in metres, in scientific notation. **(1 mark)**
b. Find the area of the roof, and hence the volume of rainwater, in litres, collected from the roof over July. **(2 marks)**
c. Find the capacity of the tank in litres, to the nearest litre. Did the tank hold all of the water collected in July? Give a reason. **(2 marks)**
d. Before July began, a weather app predicted a total of 35 mm of rain for the month. Find (i) the absolute error and (ii) the percentage error of the prediction, to one decimal place. **(2 marks)**

Q12 (4 marks). Real Victorian context — exchange.

As at 19 August 2026, 1 Australian dollar (AUD) bought 112.63 Japanese yen (JPY).

Source: Reserve Bank of Australia, "Exchange Rates" statistical table, as at August 2026. Retrieved 20 August 2026.

SCENARIO — a student saves AUD 400 for a trip to Japan, and returns home with 22 526 JPY unspent. Model the exchange as a direct proportion.

a. Write the direct proportion model connecting J yen with A Australian dollars, and state the constant of proportionality and what it means. **(1 mark)**
b. Convert the AUD 400 saving to yen. **(1 mark)**
c. Convert the 22 526 JPY left over back to Australian dollars. **(1 mark)**
d. Give one limitation of this model for a real trip. **(1 mark)**

Solutions

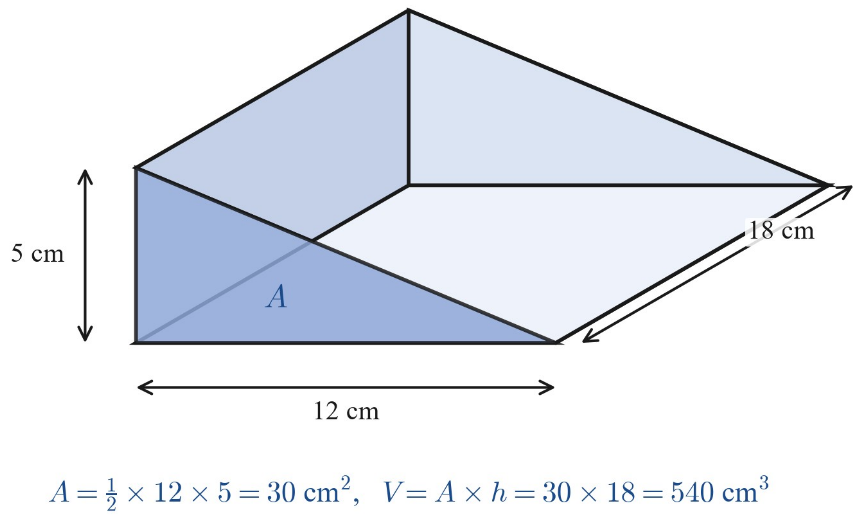
Answer key

Item	Part	Subtopic	Answer	Marks
Q1a	A	5A	240 cm^3	1
Q1b	A	5A	(i) 2600 cm^3; (ii) 45 L	1
Q1c	A	5A	142 cm^2	1
Q2a	A	5B	7.2×10^{-5}	1
Q2b	A	5B	D	1
Q3a	A	5C	24 cm	1
Q3b	A	5C	right-angled, since $8^2 + 15^2 = 17^2$	2
Q4a	A	5C	56°	1
Q4b	A	5C	8 cm	1
Q5a	A	5D	$+60$ cups	1
Q5b	A	5D	20%	1
Q6a	A	5C	54 cm^2	1
Q6b	A	5C	108 cm^3	1
Q7a	B	5A	$540\pi \approx 1696.5$ cm^3	2
Q7b	B	5A	$252\pi \approx 791.7$ cm^2	1
Q8a	B	5C	5.0 m	2
Q8b	B	5C	27.8°	1
Q9a	B	5B	4.8×10^{-4}	1
Q9b	B	5B	3×10^7	1
Q9c	B	5B	4.5×10^{-5} m	1
Q10a	B	5D	12.35 kg and 12.45 kg	1
Q10b	B	5D	≈ 0.4%	1
Q11a	B	5B	4.44×10^{-2} m	1
Q11b	B	5A	120 m^2; 5328 L	2
Q11c	B	5A	6786 L; yes, 5328 L < 6786 L	2
Q11d	B	5D	(i) 9.4 mm; (ii) ≈ 21.2%	2
Q12a	B	5E	$J = 112.63A$; 112.63 yen per AUD	1
Q12b	B	5E	45 052 JPY	1
Q12c	B	5E	AUD 200	1
Q12d	B	5E	any one valid limitation	1

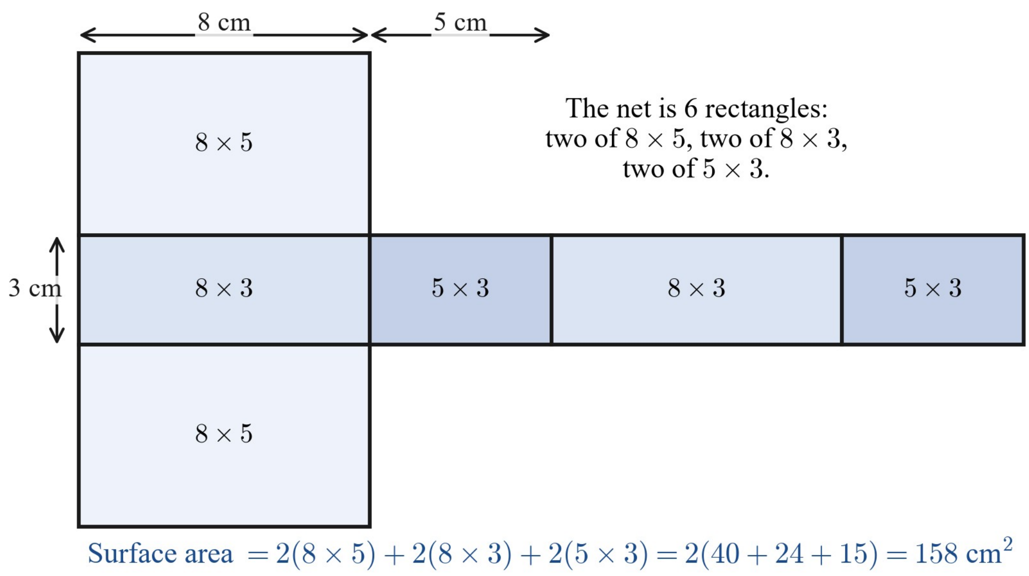
Item	Part	Subtopic	Answer	Marks
			Total	**36**

Fully-worked solutions

Q1. a Area of the cross-section first: $A=\frac{1}{2}\times 8\times 5=20$ cm². Volume: $V=A\times h=20\times 12=240$ cm³. b (i) $2.6\times 1000=2600$ cm³, using 1000 cm³ $=1$ L. (ii) $45\,000\div 1000=45$ L. c Three pairs of faces: $2(7\times 5+7\times 3+5\times 3)=2(35+21+15)=2\times 71=142$ cm². ∴ 240 cm³; 2600 cm³ and 45 L; 142 cm² — cross-section then × length, and the net in three doubled pairs.



A triangular prism lying on a rectangular face, with the right-triangle end shaded and labelled A; Q1a uses the same figure with different numbers



The net of a cuboid as a row of four rectangles with one rectangle above and one below; Q1c uses the same net with different numbers

Q2. a The point moves 5 places right to sit after the 7: $0.000\,072=7.2\times 10^{-5}$. b Test 1 asks whether $1\le a<10$; Test 2 asks whether n is an integer. A: $a=45$ is 10 or more — fails Test 1. B: $a=0.45$ is under 1 — fails Test 1. C: $n=2.5$ is not an integer — fails Test 2. D: $1\le 4.5<10$ and -4 is an integer — passes both tests. ∴ 7.2×10^{-5}; and the answer is **D**.

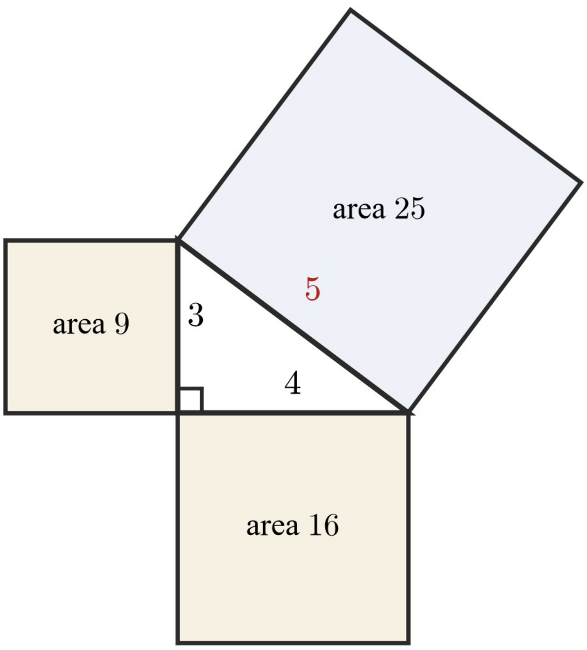
Scientific notation needs $1 \leq a < 10$ and an integer exponent n.

×	34×10^3	$a = 34$ is 10 or more — shift the point: 3.4×10^4
×	0.5×10^6	$a = 0.5$ is under 1 — shift the point: 5×10^5
×	$4.2 \times 10^{2.5}$	the exponent 2.5 is not an integer — not scientific notation
✓	7.1×10^{-4}	$1 \leq 7.1 < 10$ and -4 is an integer — correct

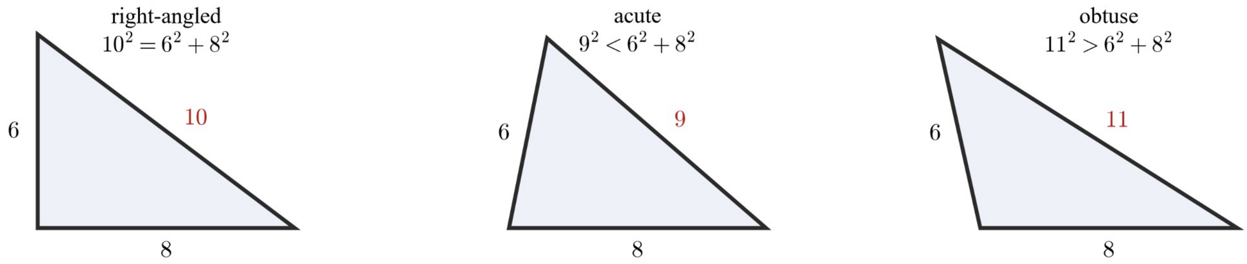
Four rows testing the two rules of scientific notation: 34 times 10 to the 3 fails because a is 10 or more; 0.5 times 10 to the 6 fails because a is under 1; 4.2 times 10 to the 2.5 fails because the exponent is not an integer; 7.1 times 10 to the minus 4 passes both tests; Q2b uses the same two tests with different numbers

Q3. a The unknown side x is a shorter side, so rearrange before taking the root: $x^2 = 25^2 - 7^2$. $x^2 = 625 - 49 = 576$. $x = 24$ cm, since $24^2 = 576$. b Take the longest side as $L = 17$ and compare L^2 with the sum of the other two squares: $a^2 + b^2 = 8^2 + 15^2 = 64 + 225 = 289$. $L^2 = 17^2 = 289$. Since $L^2 = a^2 + b^2$, the triangle is right-angled, opposite the 17 cm side. ∴ 24 cm; and yes, right-angled, because $8^2 + 15^2 = 289 = 17^2$ — the reason carries one of the two marks.

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

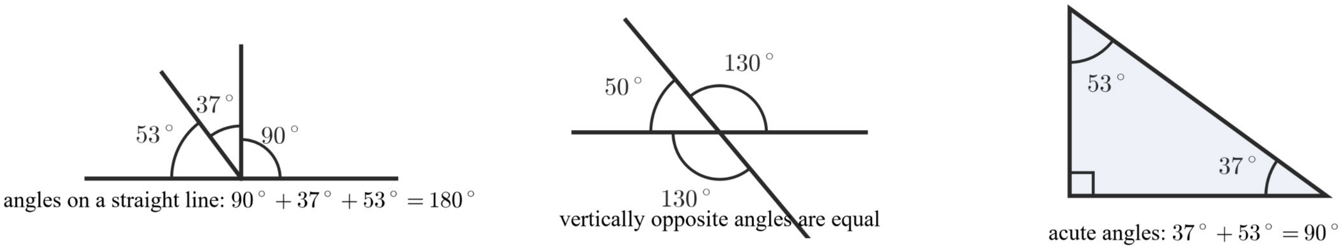


Right-angled triangle with sides 3 and 4 and hypotenuse 5, with a square on each side; Q3a uses the same picture with different numbers

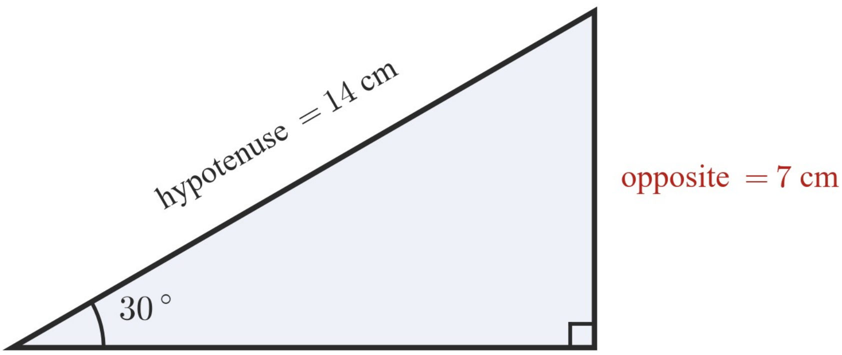


Three triangles sharing two sides: with the third side matching the squares the triangle is right-angled, with less it is acute, with more it is obtuse; Q3b runs the same test with sides 8, 15 and 17

Q4. a The two acute angles of a right-angled triangle add to 90°: $90° - 34° = 56°$. b The side opposite 30° and the hypotenuse call for the sine ratio: $\sin 30° = \frac{\text{opposite}}{16}$. $\text{opposite} = 16 \times \sin 30° = 16 \times 0.5 = 8$ cm, since $\sin 30° = 0.5$ exactly — no calculator needed. ∴ 56°; 8 cm.

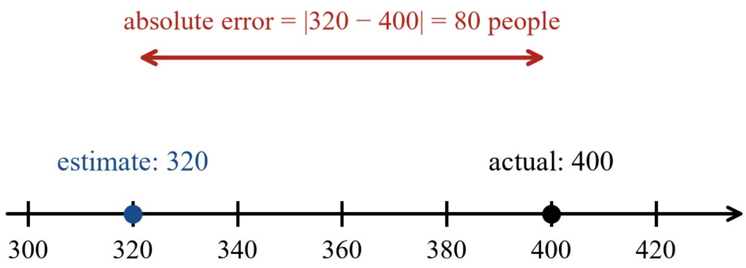


Three panels: angles on a straight line adding to 180 degrees, vertically opposite angles equal, and a right-angled triangle whose acute angles add to 90 degrees; Q4a uses the third panel with different numbers



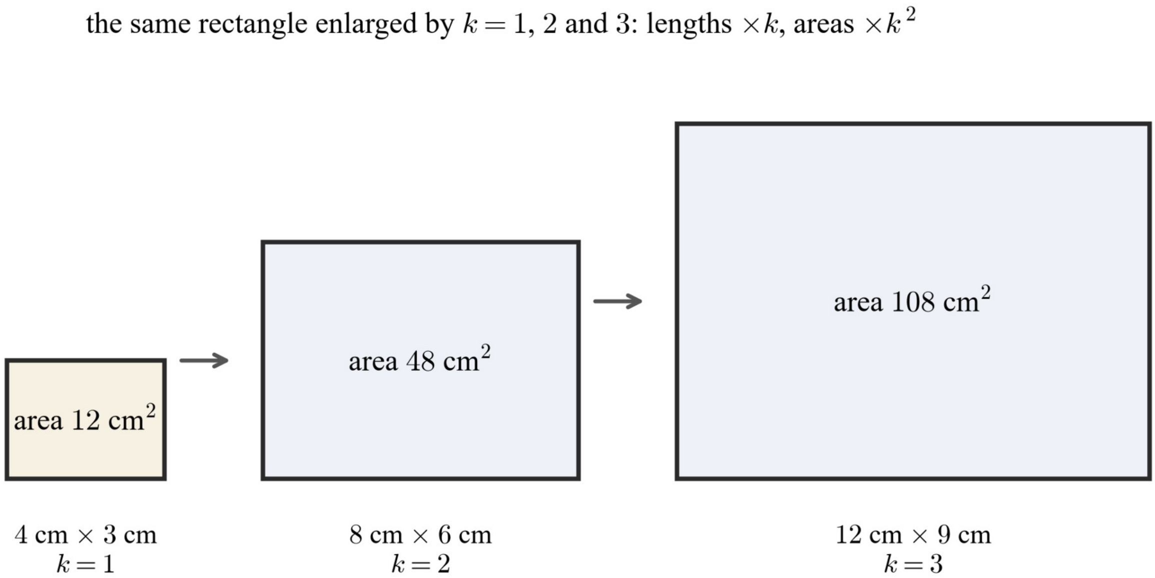
Right-angled triangle with a 30 degree angle and hypotenuse 14 cm beside the sine ratio; Q4b uses the same figure with different numbers

Q5. a Signed error $= \text{estimate} - \text{actual} = 360 - 300 = +60$ cups; the plus says the estimate ran over. b Percentage error $= 60 \div 300 \times 100\% = 20\%$. ∴ $+60$ cups, a 20% overestimate.



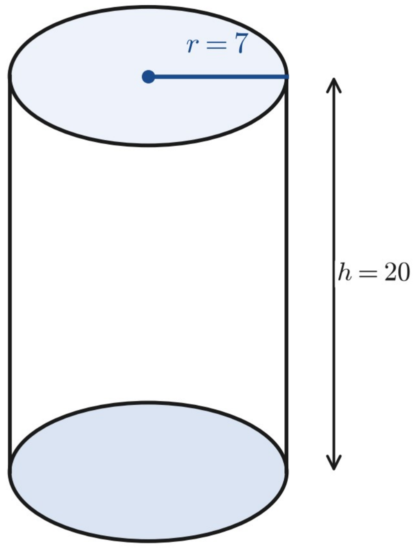
Number line with a dot at the estimate and a dot at the actual value, joined by a two-headed arrow labelled with the absolute error; Q5 uses the same picture with different numbers

Q6. a Areas multiply by k^2: $6 \times 3^2 = 6 \times 9 = 54$ cm². b Volumes multiply by k^3: $4 \times 3^3 = 4 \times 27 = 108$ cm³. ∴ 54 cm² and 108 cm³ — lengths ×3, areas ×9, volumes ×27, as 4B established.



A rectangle enlarged by scale factors 1, 2 and 3, with the areas growing by the square of the scale factor; Q6 uses the same rule with different numbers

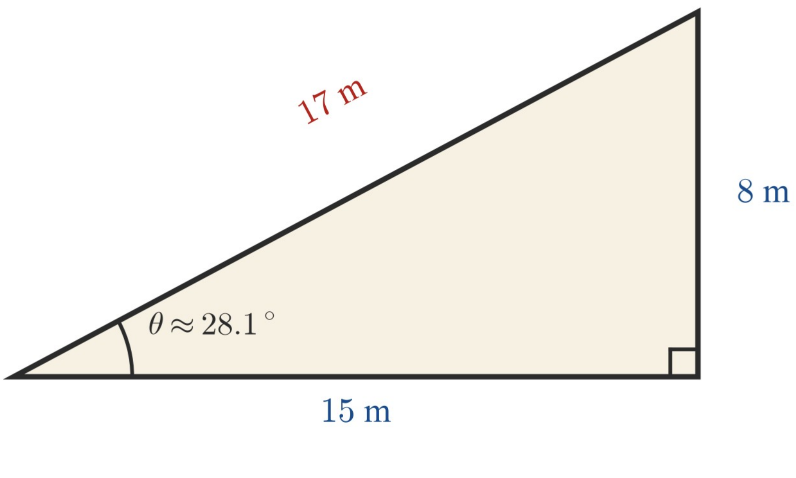
Q7. a $V = \pi r^2 h = \pi \times 6^2 \times 15 = 540\pi$ cm³. $540\pi \approx 1696.5$ cm³ (1 d.p.) — round once, at the end. b Two circles plus the unrolled curved surface: $SA = 2\pi r^2 + 2\pi r h = 2\pi(36) + 2\pi(6)(15) = 72\pi + 180\pi = 252\pi$ cm². $252\pi \approx 791.7$ cm² (1 d.p.). ∴ $V = 540\pi \approx 1696.5$ cm³; $SA = 252\pi \approx 791.7$ cm².



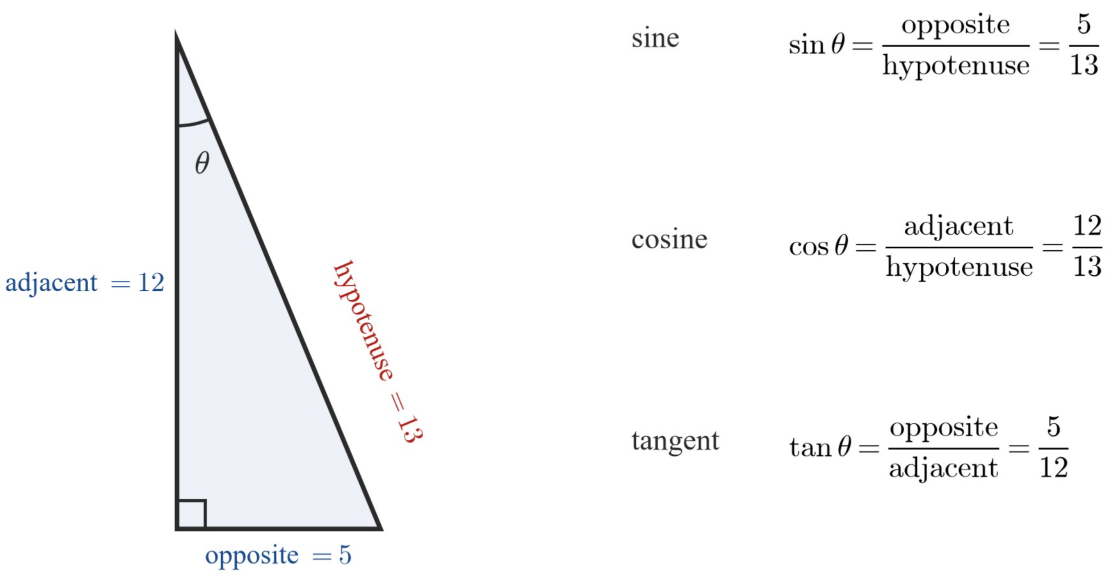
$V = \pi r^2 h = \pi \times 49 \times 20 = 980\pi \approx 3078.8\ \text{cm}^3$

A standing cylinder with a dot at the centre of the top circle, a radius marked r and a height marked h; Q7 uses the same figure with r = 6 and h = 15

Q8. a The run, the rise and the ramp form a right-angled triangle, with the ramp as the hypotenuse. The height is opposite the 32° angle, so the sine ratio is the one that connects the known hypotenuse to the wanted side: $\sin 32° = \dfrac{h}{9.5}$. $h = 9.5 \times \sin 32° \approx 9.5 \times 0.5299 \approx 5.03$ m. $h \approx 5.0$ m (1 d.p.). b $\sin\theta = 7 \div 15 = 0.4667$ (4 s.f.). $\theta = \sin^{-1}(0.4667) \approx 27.8°$ (1 d.p.). ∴ The ramp rises about 5.0 m; $\theta \approx 27.8°$ — in (a) the choice of ratio and the stated reason carry one mark, the value carries the other.

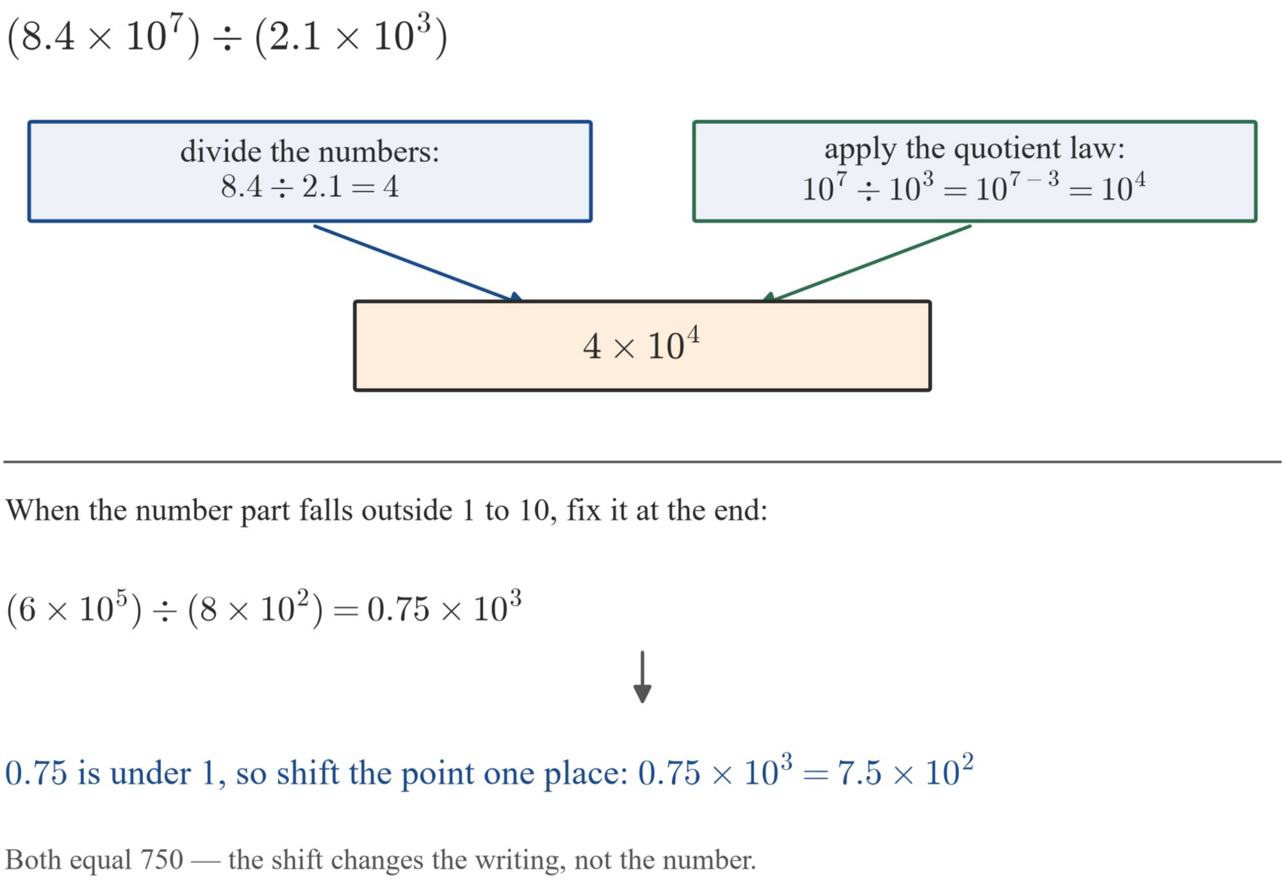


Right-angled triangle modelling a ramp: horizontal run, rise, and sloping surface with the angle between ramp and ground marked; Q8a uses the same figure with different numbers



Right-angled triangle with sides labelled beside the three ratios sine, cosine and tangent; Q8b uses the sine ratio from the same figure

Q9. a Shift the point one place right to make $a = 4.8$, which subtracts 1 from the exponent: $48 \times 10^{-5} = 4.8 \times 10^{-4}$. b Number parts with number parts, powers of 10 with powers of 10: $(2.4 \div 8) \times 10^{6-(-2)} = 0.3 \times 10^{8}$. Normalise with one shift: $0.3 \times 10^{8} = 3 \times 10^{7}$. c milli $= 10^{-3}$, so 0.045 mm $= 0.045 \times 10^{-3}$ m $= 4.5 \times 10^{-5}$ m. ∴ 4.8×10^{-4}; 3×10^{7}; 4.5×10^{-5} m — the exponent laws of 1B doing the operating.



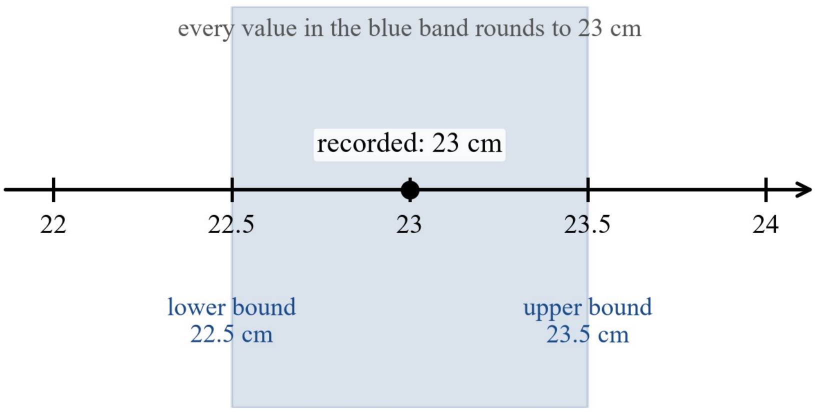
Flow diagram of a division in scientific notation: divide the number parts, apply the quotient law, then normalise with one shift; Q9b uses the same method with different numbers

Power of 10	Prefix	Rewritten in scientific notation
10^{-12}	pico (p)	$3\,\text{pm} = 3 \times 10^{-12}$ m
10^{-9}	nano (n)	$450\,\text{nm} = 4.5 \times 10^{-7}$ m
10^{-6}	micro (μ)	$3\,\mu\text{m} = 3 \times 10^{-6}$ m
10^{-3}	milli (m)	$25\,\text{mm} = 2.5 \times 10^{-2}$ m
10^{0}	(none)	—
10^{3}	kilo (k)	$7\,\text{km} = 7 \times 10^{3}$ m

A prefix is a shortcut for a power of 10 — removing it leaves a number in scientific notation.

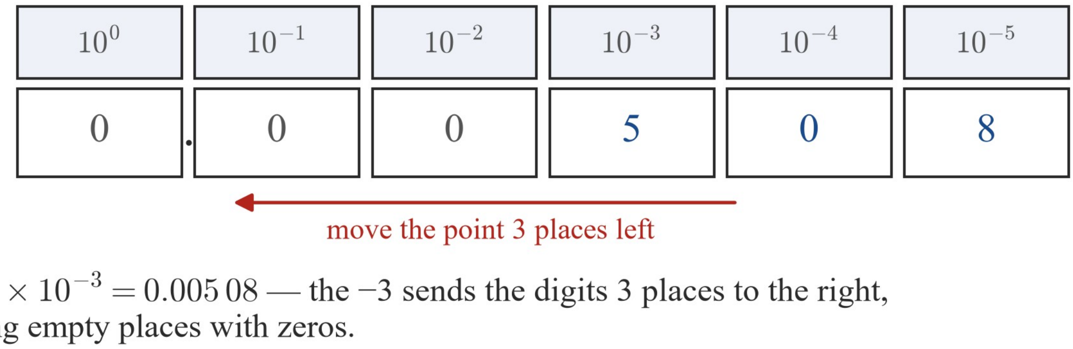
Table of SI prefixes as powers of 10 with one worked conversion per row; Q9c uses the same prefix bridge with different numbers

Q10. a Nearest 0.1 kg → half-unit $0.1 \div 2 = 0.05$ kg: lower bound $= 12.4 - 0.05 = 12.35$ kg, upper bound $= 12.4 + 0.05 = 12.45$ kg. b Maximum absolute error $= 0.05$ kg, and the recorded value stands in for the unknown true value: maximum percentage error $= 0.05 \div 12.4 \times 100\,\% = 0.4032\ldots\% \approx 0.4\,\%$ (1 d.p.). ∴ Bounds 12.35–12.45 kg; maximum percentage error ≈ 0.4%.

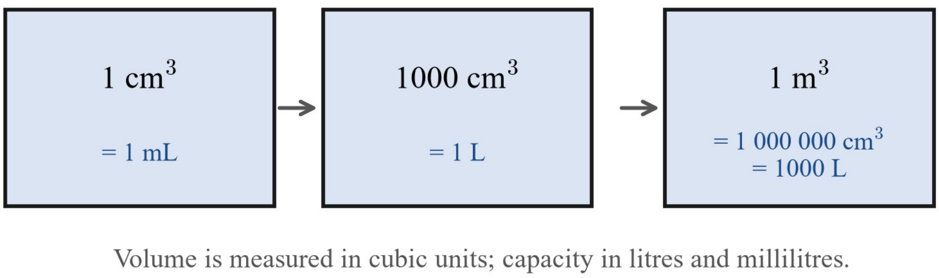


Number line with the band from the lower bound to the upper bound shaded around the recorded value; Q10 uses the same picture with different numbers

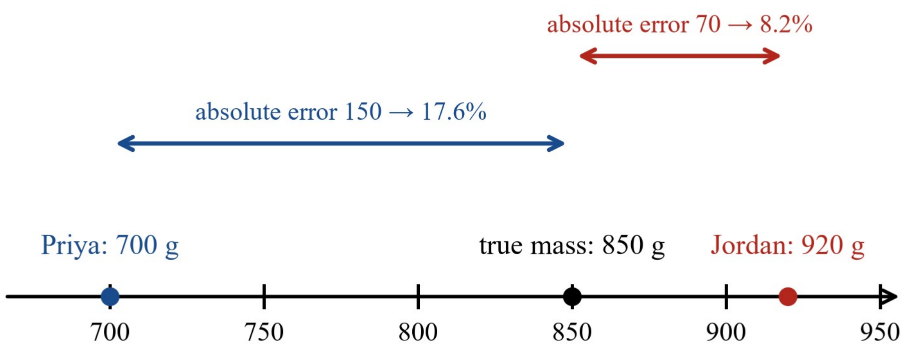
Q11. a 44.4 mm $= 44.4 \times 10^{-3}$ m. One shift of the point gives 4.44×10^{-2} m. b Area of the roof: $15 \times 8 = 120$ m². At 1 mm per 1 m² collecting 1 L, the July collection is $120 \times 44.4 = 5328$ L. c Tank: $V = \pi r^2 h = \pi \times 1.2^2 \times 1.5 = 2.16\pi$ m³. $2.16\pi \approx 6.7858$ m³ $= 6785.8\ldots$ L, which rounds to 6786 L. Comparison: $5328 < 6786$, so yes — the tank held all of the water collected in July. d Signed error $= 35 - 44.4 = -9.4$, an under-prediction; absolute error $= 9.4$ mm. Percentage error $= 9.4 \div 44.4 \times 100\,\% = 21.17\ldots\% \approx 21.2\,\%$ (1 d.p.). ∴ 4.44×10^{-2} m; 120 m² and 5328 L; 6786 L, yes, because 5328 L < 6786 L; absolute error 9.4 mm, percentage error ≈ 21.2%.



Place-value chart with the decimal point moving left to rewrite a number in scientific notation; Q11a uses the same shift with different numbers

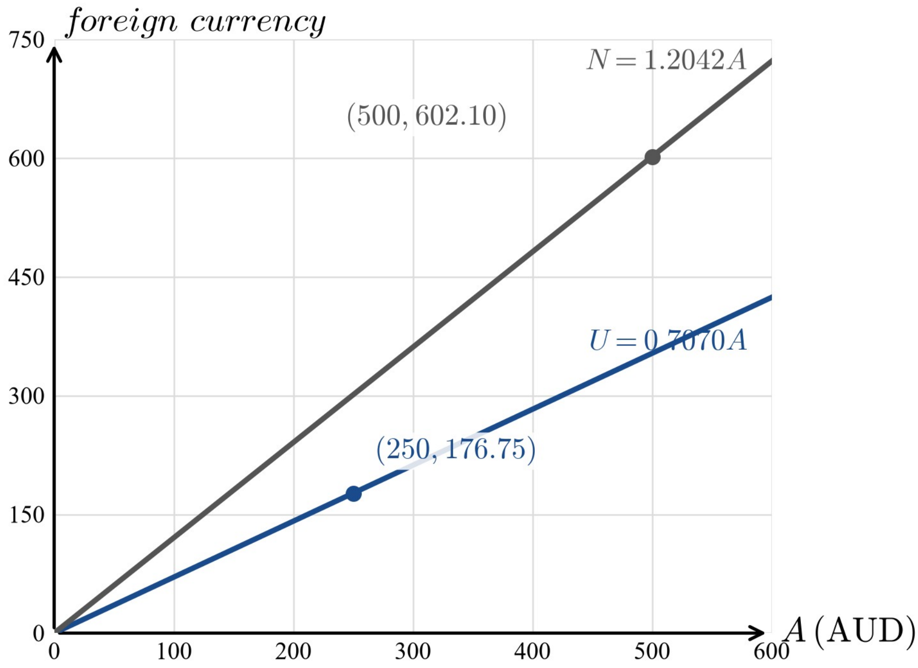


Three boxes joining cubic units to capacity: 1 cm³ = 1 mL, 1000 cm³ = 1 L, 1 m³ = 1000 L; Q11b and Q11c use the same unit ladder



Number line with the true value marked and two estimates either side, each with its own absolute-error arrow and percentage error; Q11d makes the same comparison between the prediction and the actual total

Q12. a 1 AUD buys 112.63 JPY, so the model is $J = 112.63\,A$. The constant of proportionality is 112.63, and it means each Australian dollar buys 112.63 yen — the rate. b $J = 112.63 \times 400 = 45\,052$ JPY. c Converting back divides by the same constant: $A = 22\,526 \div 112.63 = 200$. d One limitation: the rate is for one date — real exchange rates move day by day, and banks or exchange bureaus charge fees or offer their own rate, so the amount actually received is less than the model says. (Any one of these, stated as a limitation of the model, scores the mark.) $\therefore$ $J = 112.63\,A$; 45 052 JPY; AUD 200; one valid limitation. The graph of $J = 112.63\,A$ is a straight line through the origin, as direct proportion requires (2A), and the two directions use different constants: multiplying by 112.63 going there, dividing by it coming back.



Graph of foreign currency against Australian dollars: straight lines through the origin for two exchange rates; Q12 uses the same picture with the JPY rate