Mathematics Level 9

Chapter 2 — Algebra: linear relationships and the Cartesian plane · 2 subtopics

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

Significant effort has been made to ensure this content is legal. Please send any areas of concern to admin@vwx.au with appropriate document/legal references so errors can be verified and changes be made.

Contents

Section	Page
Sketching linear graphs from 2 points, and solving linear equations	2
Gradient, midpoint and distance on the Cartesian plane	23
Test	40

Sketching linear graphs from 2 points, and solving linear equations

2A · Chapter 2 — Algebra: linear relationships and the Cartesian plane · Level 9

> **Content description `VC2M9A03` (word for word):** *sketch linear graphs of equations in various algebraic forms, using the coordinates of 2 points, and solve linear equations*
>
> **Achievement standard anchor (AS 4):** *They find the distance between 2 points on the Cartesian plane, sketch linear graphs and find the gradient and midpoint of a line segment.*
>
> **Elaboration id for linkage:** `VC2M9A03.E01`
>
> **Prerequisite pointer:** Level 8, `VC2M8A02` (assumed prior knowledge): graphing linear relations and solving linear equations. Both are extended here to new algebraic forms, not re-taught. The distance, gradient and midpoint named in the AS sentence are taught in 2B.
>
> **Note on figures:** every line in this section is drawn from its own rule and every marked point from its own coordinates (R7); every context is a generic one, so no real data ships and no source line is owed (R1, R2).

Theory

A line is fixed by two points

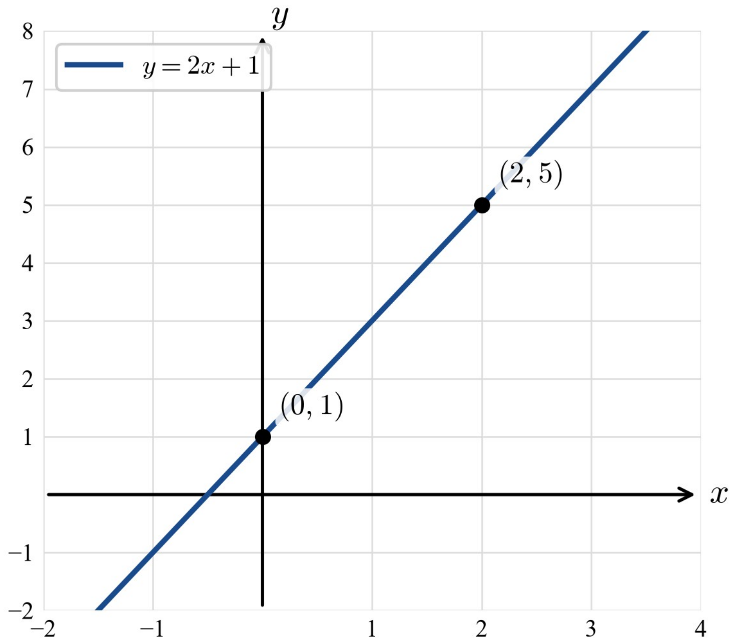
The graph of every linear equation is a straight line, and a straight line is fixed by any two of its points. That is the whole method: choose two values of x, work out the matching y-values from the equation, plot the two points, and join them with a straight edge. A third point, not used to draw the line, is the check — if it does not land on the line, an arithmetic slip happened.

Model it on $y=2x+1$. Choose $x=0$ and $x=2$:

$x=0$ gives $y=2\times0+1=1$, the point $(0,1)$.

$x=2$ gives $y=2\times2+1=5$, the point $(2,5)$.

Plot both, join, done. The check point $x=1$ gives $y=3$, and $(1,3)$ does sit on the line.



The straight line y = 2x + 1 on a set of axes, with the two points used to draw it, (0, 1) and (2, 5), marked

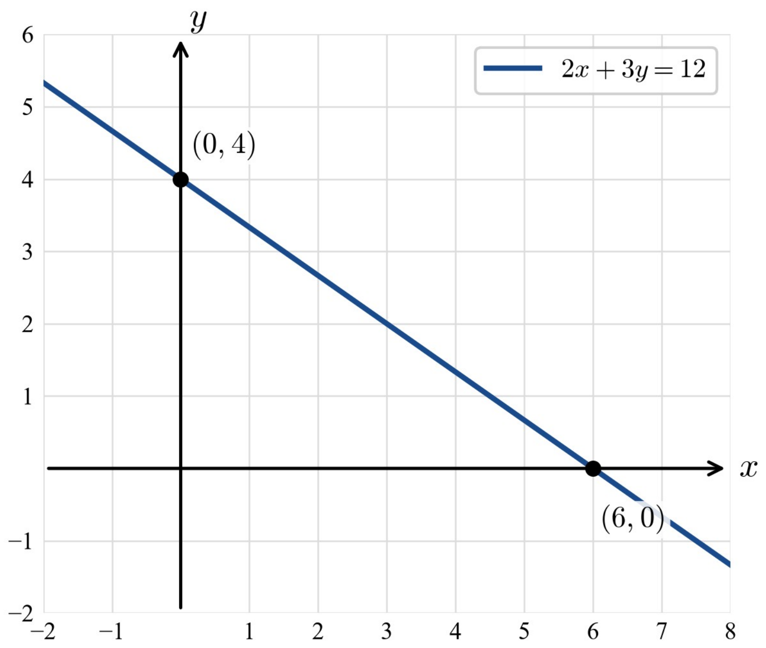
The four algebraic forms

The content description says the equations come *in various algebraic forms*. Four forms appear at this level, and each has its own quick way of choosing two points.

Form	What it looks like	Two good points
$y=mx+c$	$y=2x+1$	$x=0$ gives $(0,c)$; pick any other x
$ax+by=c$	$2x+3y=12$	the two intercepts
$x=a$	$x=3$	vertical line: two points with $x=a$
$y=a$	$y=-2$	horizontal line: two points with $y=a$

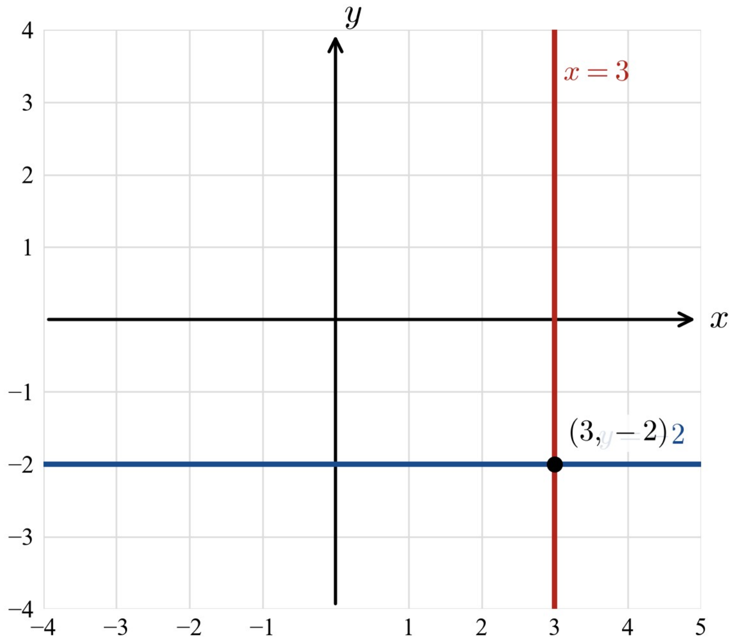
The form $y=mx+c$ shows two things at a glance: c is the y-**intercept**, the y-value where the line crosses the y-axis, and m is the **gradient**, the number that measures how steep the line is — positive when the line rises from left to right, negative when it falls. The full story of gradient belongs to 2B; here m and c simply help you choose your two points.

An **intercept** is a crossing with an axis. The y-intercept is where the line crosses the y-axis, so $x=0$ there; the x-intercept is where it crosses the x-axis, so $y=0$ there. That makes the intercepts the natural two points for the form $ax+by=c$, where y has not been sorted out on its own. In $2x+3y=12$: put $x=0$ and $3y=12$, so $y=4$ — the point $(0,4)$; put $y=0$ and $2x=12$, so $x=6$ — the point $(6,0)$.



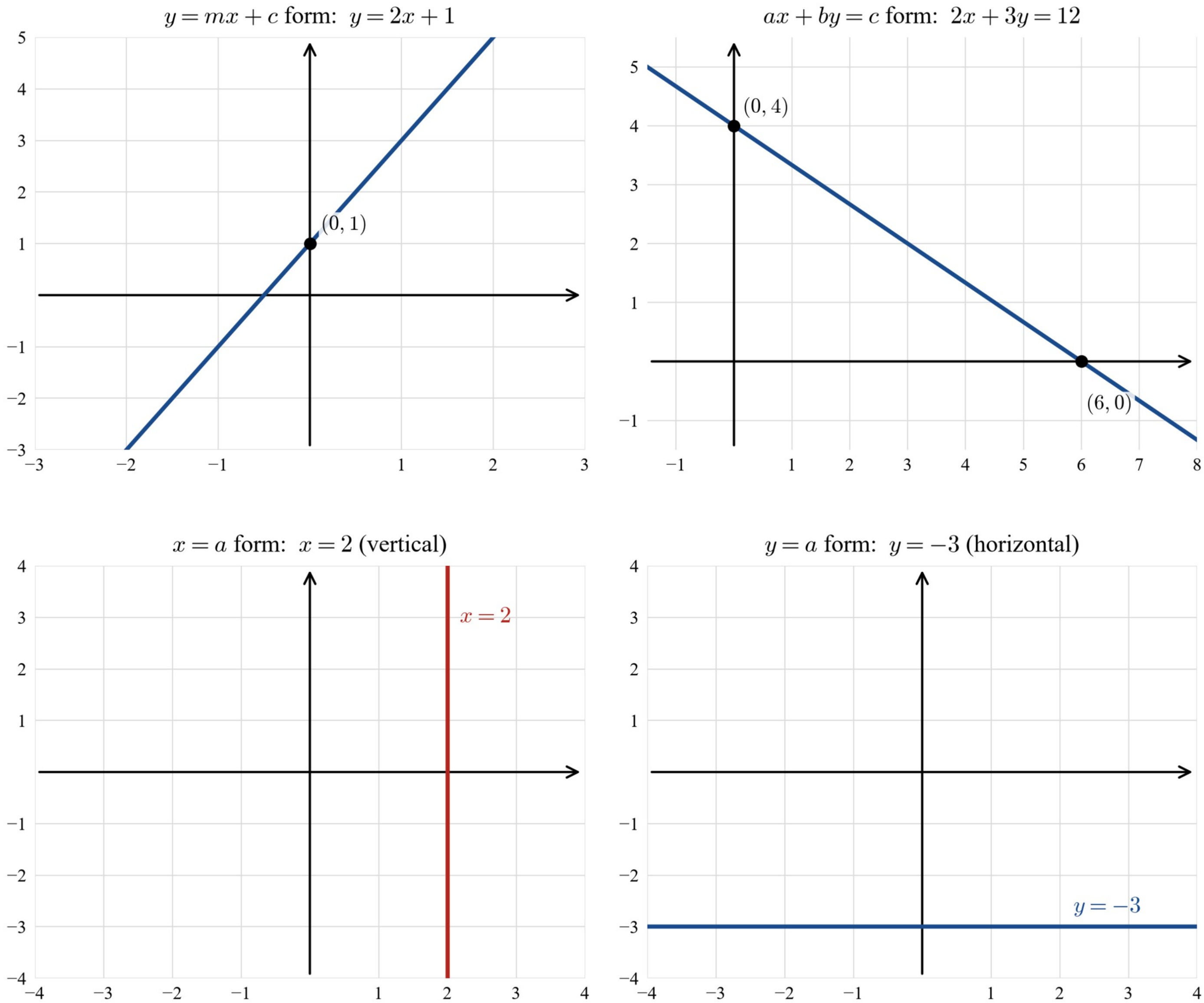
The straight line 2x + 3y = 12 drawn through its two intercepts, (0, 4) on the y-axis and (6, 0) on the x-axis, both marked

The last two forms are the ones most often read the wrong way around. The equation $x=3$ is not a single point: it says *every* point whose x-value is 3, whatever the y-value — a **vertical** line. The equation $y=-2$ says every point whose y-value is -2 — a **horizontal** line. Where the two cross, both are true at once: the point $(3,-2)$.



A vertical line x = 3 in red and a horizontal line y = minus 2 in blue on one set of axes, crossing at the marked point (3, minus 2)

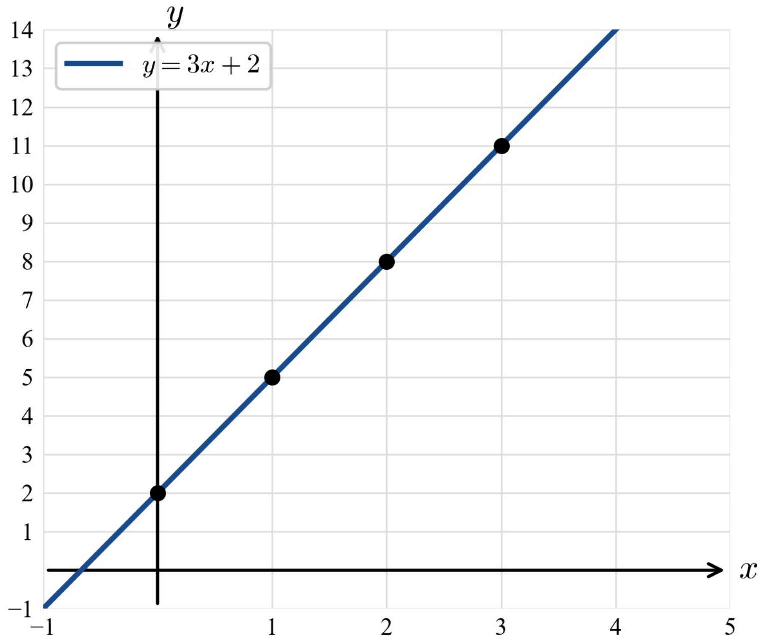
The four forms side by side, so the eye learns them:



Four panels: the y = mx + c form as the rising line y = 2x + 1 with (0, 1) marked; the ax + by = c form as the falling line 2x + 3y = 12 with its intercepts (0, 4) and (6, 0) marked; the x = a form as the vertical red line x = 2; the y = a form as the horizontal blue line y = minus 3

From a table or a graph back to the rule

The connection between rule and graph runs in both directions. Given a table of values that came from a line, the y-values step by the same amount at every step of 1 in x — that steady step is exactly what makes the graph straight. Here the y-values run 2, 5, 8, 11: the line starts at 2 when $x=0$, and 3 is added at every step, so in words the rule is *start at 2 and add 3 for every step of 1 in x*, and in algebra it is $y=3x+2$. The two descriptions say the same thing.



The line y = 3x + 2 with the four table points (0, 2), (1, 5), (2, 8) and (3, 11) marked on it

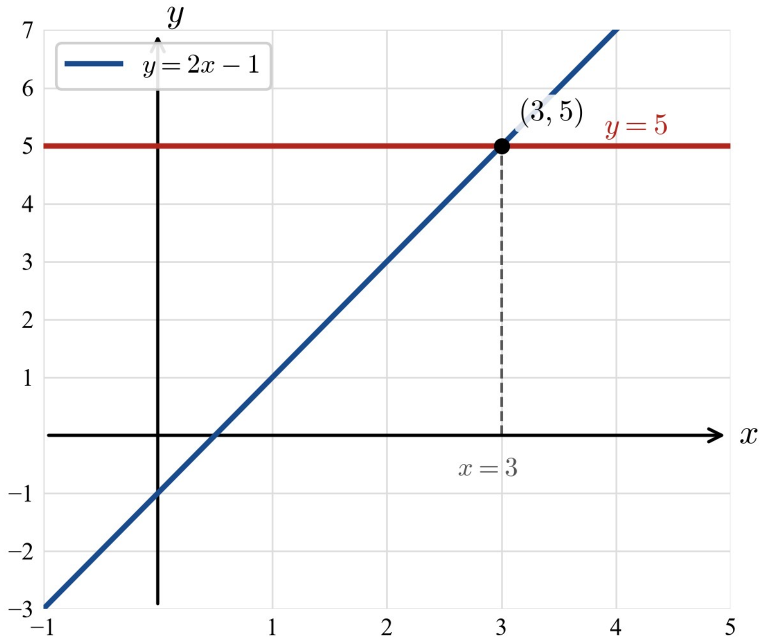
The same move reads a rule off a drawn line: pick two points with whole-number coordinates, starting at the y-intercept when it is a clean point, note the y-value at $x=0$ and the steady step in y per step of 1 in x, and write the rule in words first and then in algebra.

Solving linear equations

The second half of the content description. A linear equation is solved by keeping both sides equal while you untangle the unknown: whatever is done to one side is done to the other, one step written per line. The working order that never fails:

- expand any brackets first;
- gather the unknown terms on one side and the numbers on the other;
- if a fraction appears, multiply both sides by the denominator first;
- finish by dividing by whatever multiplies the unknown;
- check by putting the answer back into the original equation.

The two halves of this subtopic are one idea. Solving $2x-1=5$ asks: at what x-value does the line $y=2x-1$ reach height 5? The figure answers it by eye — the line meets the horizontal line $y=5$ at $(3,5)$, so the solution is $x=3$ — and algebra agrees: $2x=6$, so $x=3$. A graph will not always read as cleanly as this, so the algebra is the tool; but the picture is what the algebra means.



The line y = 2x minus 1 crossing the horizontal red line y = 5 at the marked point (3, 5), with a dashed grey drop line showing x = 3

Worked examples

Example 1 — scaffolded

Sketch $y=3x-4$ using the coordinates of two points, and check with a third.

Working	Comment
$x=0$: $y=3\times 0-4=-4$	The y-intercept, $(0,-4)$.
$x=2$: $y=3\times 2-4=2$	A second point, $(2,2)$.
Plot $(0,-4)$ and $(2,2)$; join with a straight edge.	Two points fix the line.
Check: $x=3$ gives $y=3\times 3-4=5$; $(3,5)$ lands on the line.	A third point tests the arithmetic.

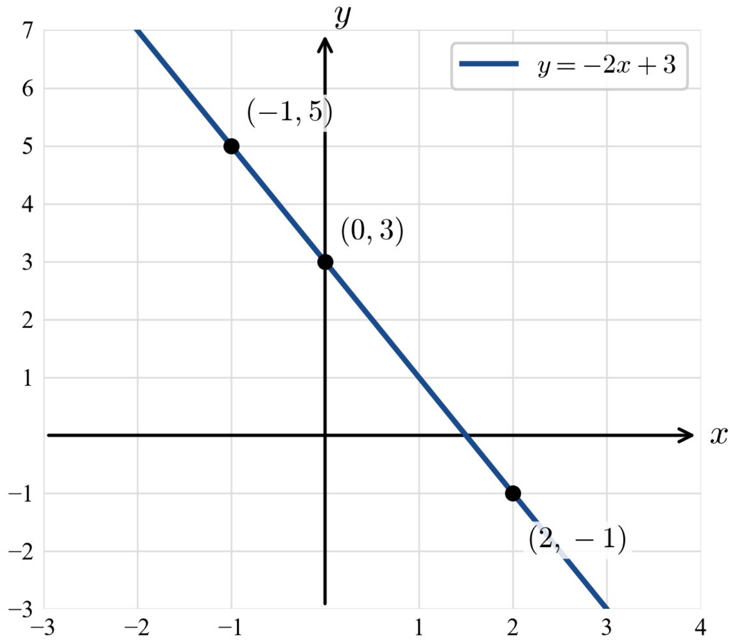
∴ The line through $(0,-4)$ and $(2,2)$ is the graph of $y=3x-4$.

Example 2 — scaffolded

Sketch $y=-2x+3$, choosing one negative x-value and one positive one.

Working	Comment
$x=-1$: $y=-2\times(-1)+3=2+3=5$	A negative x tests the sign work: $-2\times -1=2$.
$x=2$: $y=-2\times 2+3=-4+3=-1$	A positive x.
Plot $(-1,5)$ and $(2,-1)$; join.	The line falls from left to right, because the gradient -2 is negative.
Check: $x=0$ gives $y=3$; $(0,3)$ sits on the line.	The y-intercept matches $c=3$.

∴ The line through $(-1,5)$ and $(2,-1)$ is the graph of $y=-2x+3$.



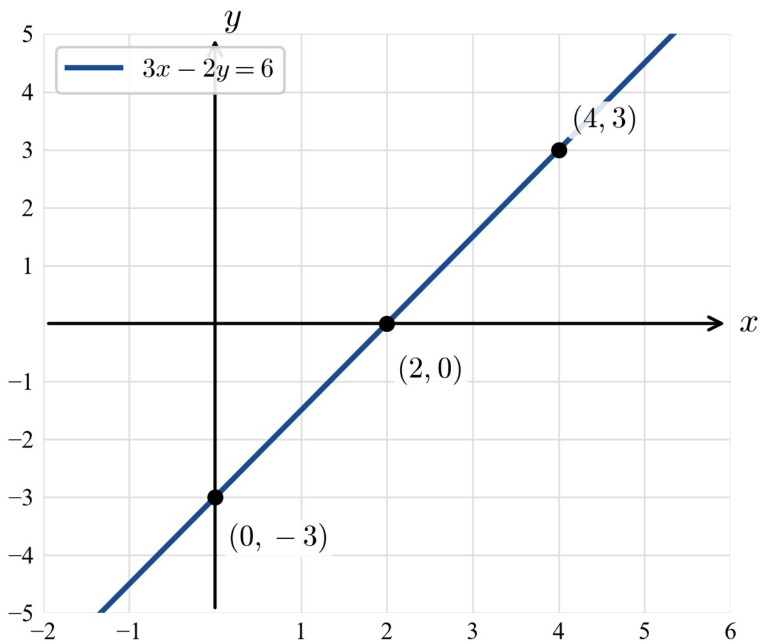
The falling line y = minus 2x + 3 with the points (minus 1, 5) and (2, minus 1) used to draw it and the check point (0, 3) marked

Example 3 — scaffolded

Sketch $3x-2y=6$ **using its intercepts.**

Working	Comment
$x=0$: $-2y=6$, so $y=-3$	The y-intercept, $(0,-3)$.
$y=0$: $3x=6$, so $x=2$	The x-intercept, $(2,0)$.
Plot $(0,-3)$ and $(2,0)$; join.	The form $ax+by=c$ is sketched from its intercepts.
Check: $(4,3)$: $3\times4-2\times3=12-6=6$ ✓	Both sides equal 6, so the check point lies on the line.

∴ The line through $(2,0)$ and $(0,-3)$ is the graph of $3x-2y=6$.



The line 3x minus 2y = 6 drawn through its intercepts (2, 0) and (0, minus 3), with the check point (4, 3) marked

Example 4 — scaffolded

Solve $4(x+2)=3x+11$.

Working	Comment
$4x+8=3x+11$	Expand the bracket: $4\times x+4\times 2$.
$x+8=11$	Subtract $3x$ from both sides.
$x=3$	Subtract 8 from both sides.
Check: $4(3+2)=20$ and $3\times 3+11=20$ ✓	Both sides equal, so the solution stands.

∴ The solution is $x=3$.

Example 5 — unscaffolded

A straight line passes through $(0,-2)$ and $(3,4)$. Describe its rule in words and in algebra.

At $x=0$ the y-value is -2, so the line starts at -2. From $(0,-2)$ to $(3,4)$, y rises by $4-(-2)=6$ while x runs on by 3 — a rise of 2 for every step of 1 in x. In words: start at -2 and add 2 for every step of 1 in x. In algebra: $y=2x-2$. Check: $x=3$ gives $2\times 3-2=4$ ✓.

∴ The rule is $y=2x-2$: start at -2, add 2 per step.

Example 6 — unscaffolded

Solve $\frac{2x+6}{4}=5$.

Multiply both sides by 4: $2x+6=20$. Subtract 6: $2x=14$. Divide by 2: $x=7$. Check: $\frac{2\times 7+6}{4}=\frac{20}{4}=5$ ✓.

∴ $x=7$.

Example 7 — unscaffolded

Solve $3-2x=11$.

Subtract 3 from both sides: $-2x=8$. Divide by -2: $x=-4$. Check: $3-2\times(-4)=3+8=11$ ✓.

∴ $x=-4$.

Example 8 — unscaffolded

Solve $3(x-2)=x+4$.

Expand: $3x-6=x+4$. Subtract x: $2x-6=4$. Add 6: $2x=10$. Divide by 2: $x=5$. Check: $3(5-2)=9$ and $5+4=9$ ✓.

∴ $x=5$.

Practice questions

Scaffolded

Q1. Copy and complete a table of values for $y=3x-2$ at $x=0,1,2$.

a. What are the three y-values?
b. Which two points will you plot?
c. Sketch the line and confirm with your third point that it sits on the line.

Q2. Consider the line $y = -2x + 6$.

a. Put $x = 0$ to find the y-intercept.
b. Put $x = 3$ and find the matching y-value.
c. Sketch the line through your two points.

Q3. Four equations: (i) $y = 4x - 1$, (ii) $2x + y = 8$, (iii) $x = 5$, (iv) $y = -3$.

a. Which one is already in the form $y = mx + c$?
b. Which one is in the form $ax + by = c$?
c. Which two are $x = a$ and $y = a$, and will their graphs be vertical or horizontal?

Q4. Sketch $4x + 2y = 8$ using its intercepts.

a. Put $y = 0$ and find the x-intercept.
b. Put $x = 0$ and find the y-intercept.
c. Plot both and join.

Q5. Sketch $x - 3y = 6$ using its intercepts.

a. Find the x-intercept.
b. Find the y-intercept.
c. Plot both and join, then test the point $(3, -1)$ on your line.

Q6. On one set of axes:

a. draw the line $x = -1$;
b. draw the line $y = 2$;
c. write down the coordinates of the point where the two lines cross.

Q7. A table of values came from a line: $y = 4, 7, 10, 13$ at $x = 0, 1, 2, 3$.

a. By how much does y change at each step of 1 in x?
b. Write the rule in words and then in algebra.
c. Use the rule to find y when $x = 10$.

Q8. Solve each equation, one step per line.

a. $x + 7 = 19$
b. $3x = 21$
c. $5x - 4 = 16$

Q9. Solve, gathering the unknown on one side first.

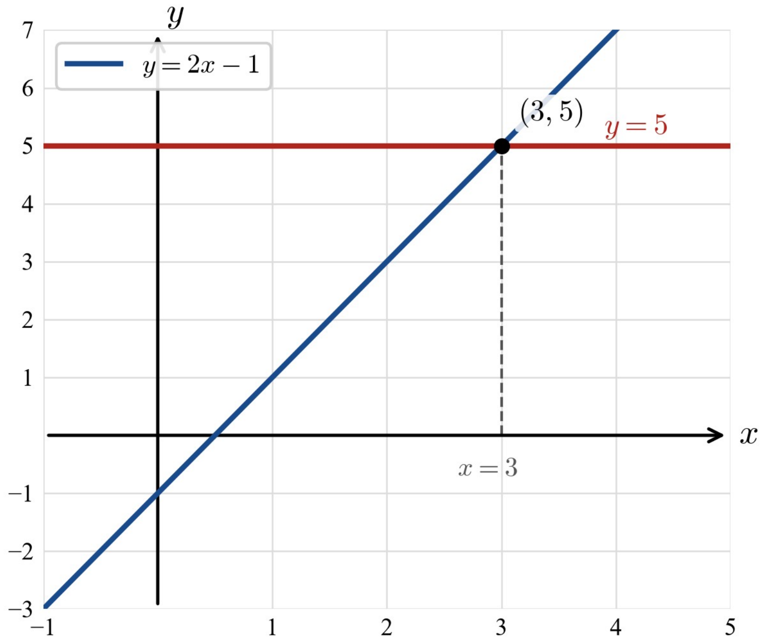
a. $2x + 3 = x + 9$
b. $7x - 5 = 4x + 10$

Q10. Solve, expanding brackets first.

a. $3(x + 4) = 27$
b. $2(x - 3) = 5x - 15$

Q11. The figure shows the line $y = 2x - 1$ and the horizontal line $y = 5$.

a. Write down the equation that the crossing point solves.
b. Read the solution off the figure.
c. Solve $2x - 1 = 5$ algebraically and compare.



The line y = 2x minus 1 crossing the horizontal red line y = 5 at the marked point (3, 5), with a dashed grey drop line at x = 3

Q12. A line passes through $(0,3)$ and $(2,7)$.

a. State the y-value at $x=0$.
b. Work out the rise in y from $x=0$ to $x=2$, and the rise per step of 1.
c. Write the rule in algebra.

Unscaffolded

Q13. Sketch $y=4x-3$ from two points of your own choosing.

Q14. Sketch $y=-x-2$ from two points of your own choosing.

Q15. Sketch $y=\frac{1}{2}x+1$, choosing x-values that keep the y-values whole.

Q16. Sketch $x+2y=8$ using its intercepts.

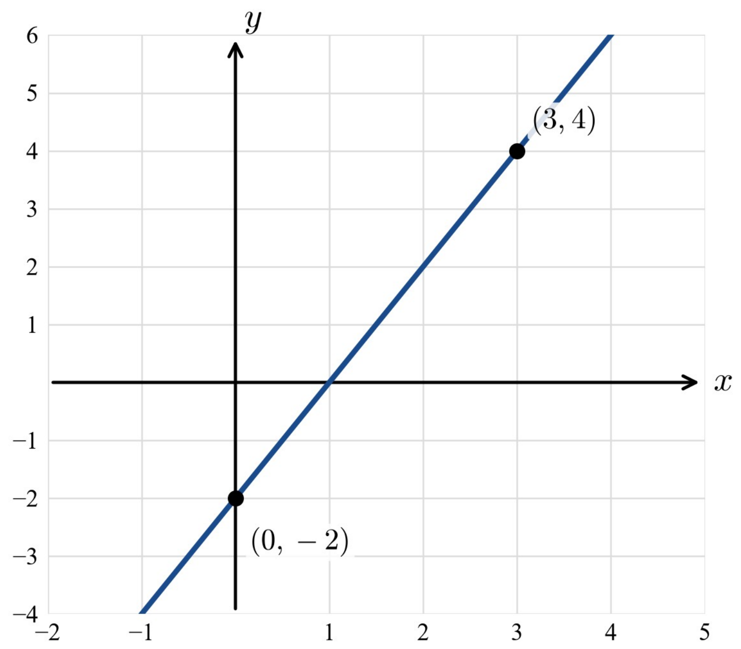
Q17. Sketch $5x-2y=10$ using its intercepts.

Q18. On one set of axes draw $x=4$ and $y=-1$, and write down the coordinates of their crossing point.

Q19. Exactly one of the lines $y=2x$, $x=2$ and $y=2$ passes through the point $(2,5)$. Which one, and why do the other two fail?

Q20. A line passes through $(0,5)$ and $(2,1)$. Write its rule in algebra.

Q21. The figure shows a straight line with two of its points marked. Describe its rule in words and then write it in algebra.



A straight line rising from left to right with two marked points, (0, minus 2) on the y-axis and (3, 4)

Q22. Copy a table of values for $y=-3x+6$ at $x=0,1,2$, and sketch the line. What do you notice about the point at $x=2$?

Q23. Solve $4x+9=2x+25$.

Q24. Solve $6(x-2)=4x+8$.

Q25. Solve $5-3x=17$.

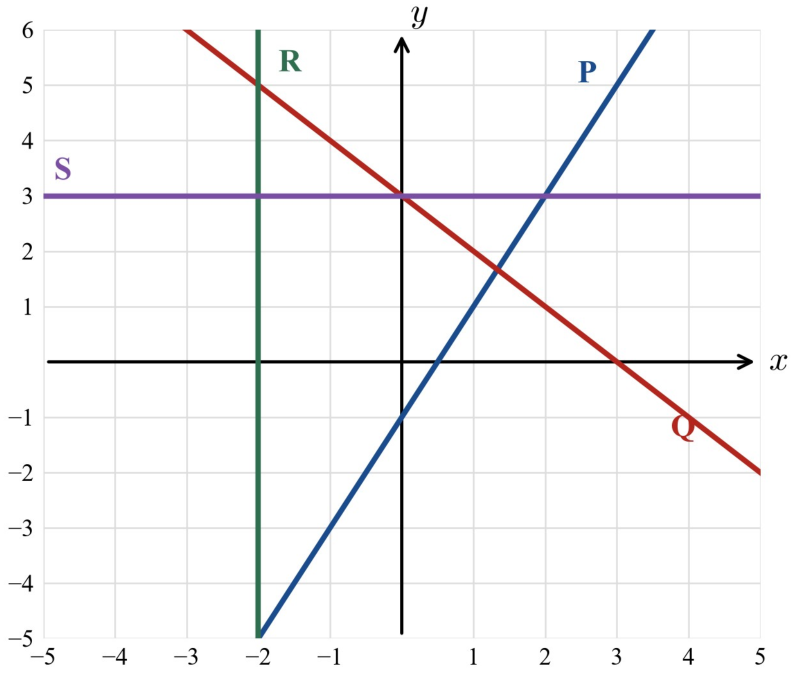
Q26. Solve $\frac{2x+8}{3}=6$.

Q27. Solve $\frac{x}{3}-2=4$.

Q28. Solve $3(2x-1)=5x+9$.

Q29. When a number is multiplied by 3 and then 7 is added, the answer is the same as when the number is multiplied by 5 and then 1 is taken away. Find the number.

Q30. The figure shows four lines P, Q, R and S. Match each line to one of the rules $y=2x-1$, $y=-x+3$, $x=-2$, $y=3$, giving one reason for each match.



Four lines labelled P, Q, R and S on one set of axes: P rising steeply, Q falling, R vertical and S horizontal, for matching to rules

Q31. A tank holds 50 L of water and is filled at a steady 20 L per minute, so after t minutes the volume is $V = 20t + 50$ litres. Find the volume after 10 minutes, and find how long the tank has been filling when it holds 170 L.

Q32. Sketch $3x + 4y = 24$ using its intercepts, then decide whether the point $(4, 3)$ lies on your line.

Q33. Sketch $y = 4 - x$, and use your sketch to solve $4 - x = -2$ by reading where the line reaches height -2.

Q34. Zoe solved $2(x + 3) = 14$ by writing $2x + 3 = 14$, then $2x = 11$, then $x = 5.5$. Find her mistake and solve the equation correctly.

Q35. A rectangle is 3 cm longer than it is wide, and its perimeter is 26 cm. Writing the width as x cm, find the width and the length.

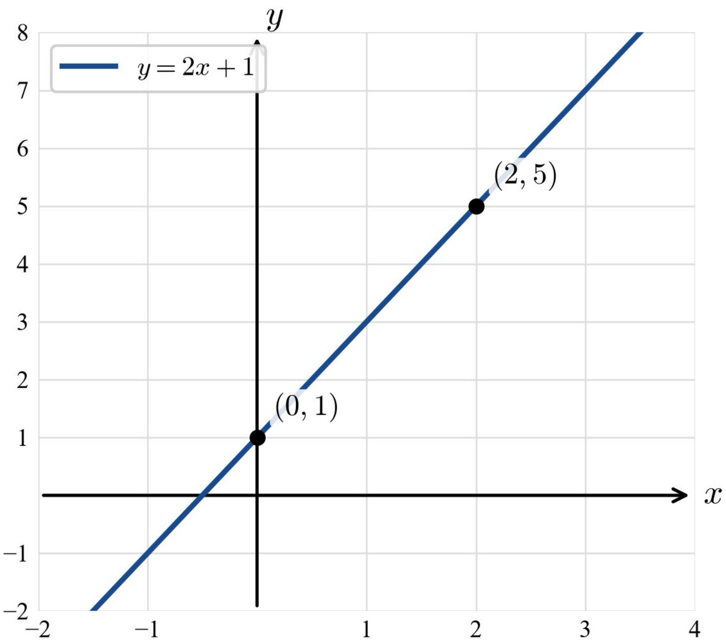
Q36. A line passes through $(-2, 0)$ and $(0, 4)$. Write its rule in algebra, use the rule to find the y-value when $x = 7$, and find the x-value when $y = 0$. What does each answer show on your graph?

Answers

Q1 a $-2,1,4$; b $(0,-2)$ and $(1,1)$; c the line through them, with $(2,4)$ on it. **Q2** a $(0,6)$; b $(3,0)$; c the line through both. **Q3** a (i); b (ii); c (iii) is $x=a$, vertical; (iv) is $y=a$, horizontal. **Q4** a $(2,0)$; b $(0,4)$; c the line through both. **Q5** a $(6,0)$; b $(0,-2)$; c the line; $(3,-1)$ sits on it. **Q6** c $(-1,2)$. **Q7** a up by 3 each step; b start at 4 and add 3 per step: $y=3x+4$; c 34. **Q8** a $x=12$; b $x=7$; c $x=4$. **Q9** a $x=6$; b $x=5$. **Q10** a $x=5$; b $x=3$. **Q11** a $2x-1=5$; b $x=3$; c $2x=6$, $x=3$ — the same. **Q12** a 3; b 4 over a run of 2, so 2 per step; c $y=2x+3$. **Q13** e.g. $(0,-3)$ and $(1,1)$, joined. **Q14** e.g. $(0,-2)$ and $(-2,0)$, joined. **Q15** e.g. $(0,1)$ and $(4,3)$, joined. **Q16** $(8,0)$ and $(0,4)$, joined. **Q17** $(2,0)$ and $(0,-5)$, joined. **Q18** $(4,-1)$. **Q19** $x=2$; $y=2x$ gives $y=4\neq 5$ there, and $y=2$ gives $y=2\neq 5$. **Q20** $y=-2x+5$. **Q21** start at -2 and add 2 per step: $y=2x-2$. **Q22** $6,3,0$; the point at $x=2$ is $(2,0)$ — the x-intercept. **Q23** $x=8$. **Q24** $x=10$. **Q25** $x=-4$. **Q26** $x=5$. **Q27** $x=18$. **Q28** $x=12$. **Q29** 4. **Q30** P: $y=2x-1$; Q: $y=-x+3$; R: $x=-2$; S: $y=3$. **Q31** 250 L; 6 minutes. **Q32** intercepts $(8,0)$ and $(0,6)$; yes, $(4,3)$ lies on it. **Q33** $x=6$. **Q34** she expanded $2(x+3)$ as $2x+3$; correctly, $x=4$. **Q35** width 5 cm, length 8 cm. **Q36** $y=2x+4$; 18; $x=-2$ — the second point the line was drawn from.

Fully-worked solutions

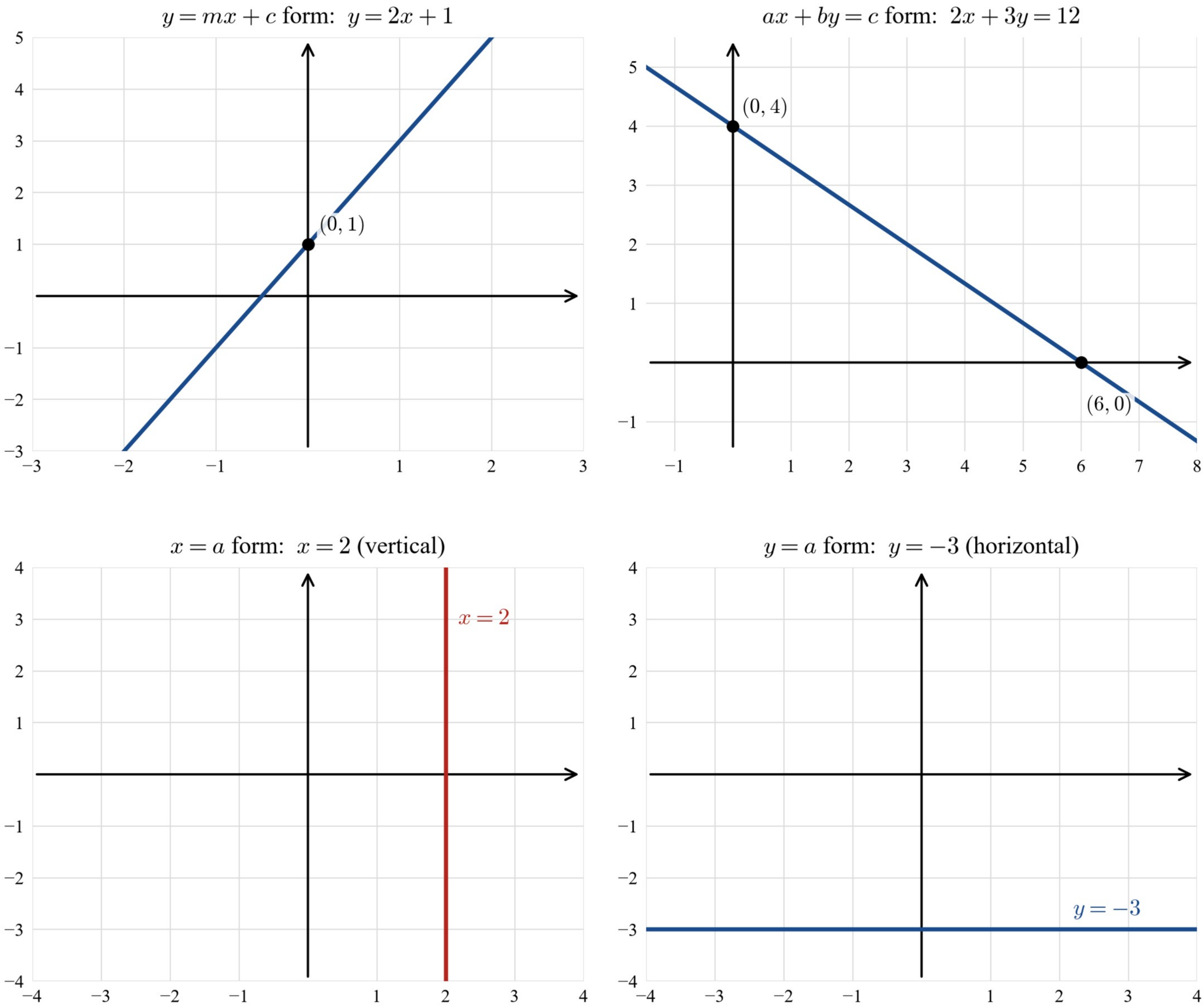
Q1. a $x=0$: $3\times 0-2=-2$; $x=1$: $3-2=1$; $x=2$: $6-2=4$. b Plot $(0,-2)$ and $(1,1)$. c The straight edge joins them; the unused point $(2,4)$ lands on the line, so the sketch is right. ∴ Table $-2,1,4$; line through $(0,-2)$ and $(1,1)$.



The two-point method: a line drawn through two marked points, here y = 2x + 1 through (0, 1) and (2, 5) — the same method Q1 uses with different numbers

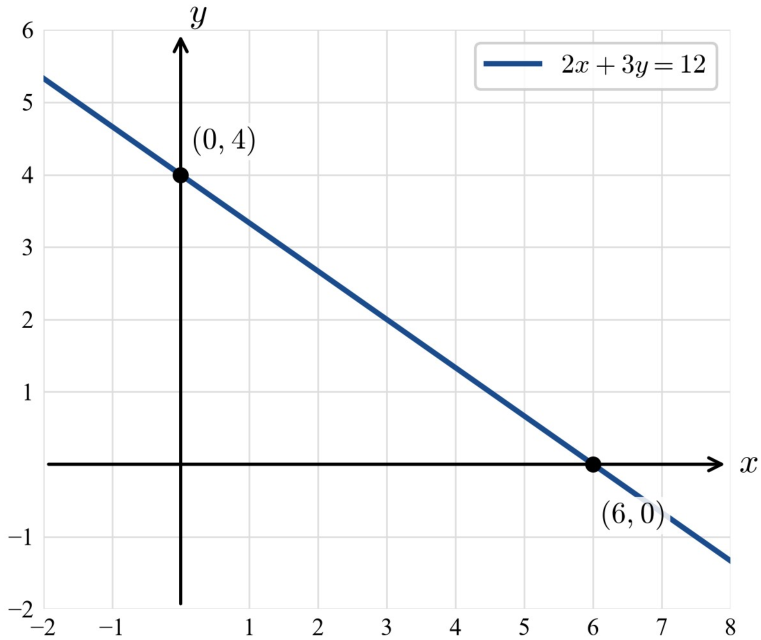
Q2. a $x=0$: $y=-2\times 0+6=6$, the point $(0,6)$. b $x=3$: $y=-2\times 3+6=0$, the point $(3,0)$ — this line's x-intercept. c Joining the two gives a falling line, since the gradient -2 is negative. ∴ The line through $(0,6)$ and $(3,0)$.

Q3. a (i), already written as $y=mx+c$ with $m=4$, $c=-1$. b (ii): $2x+y=8$ is $ax+by=c$ with $a=2$, $b=1$, $c=8$. c (iii) is $x=a$ with $a=5$ — a vertical line; (iv) is $y=a$ with $a=-3$ — a horizontal line. ∴ (i) gradient–intercept form; (ii) general form; (iii) vertical; (iv) horizontal.



The four algebraic forms in four panels: a rising line, a falling line through its intercepts, a vertical line and a horizontal line

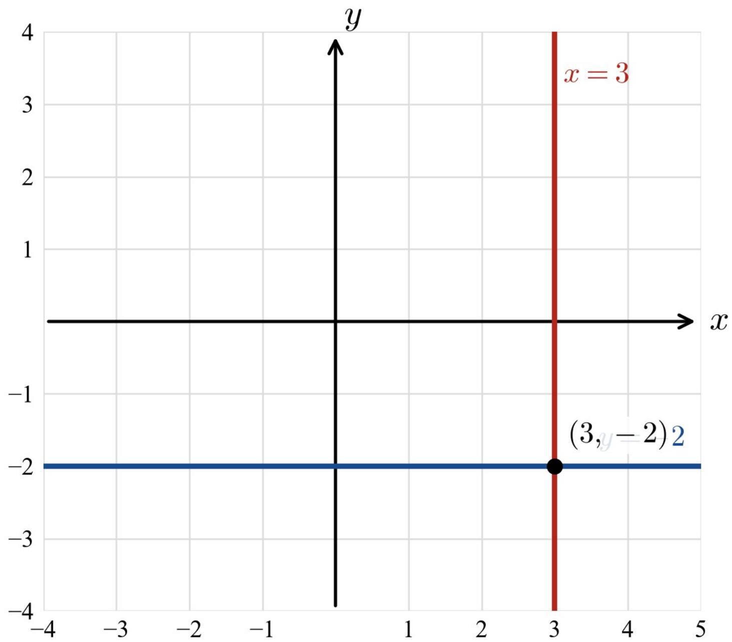
Q4. a $y=0$: $4x=8$, so $x=2$ — the point $(2, 0)$. b $x=0$: $2y=8$, so $y=4$ — the point $(0, 4)$. c Join the two intercepts. ∴ The line through $(2, 0)$ and $(0, 4)$.



A line drawn through its two intercepts, (0, 4) and (6, 0) — the same intercept method Q4 uses with different numbers

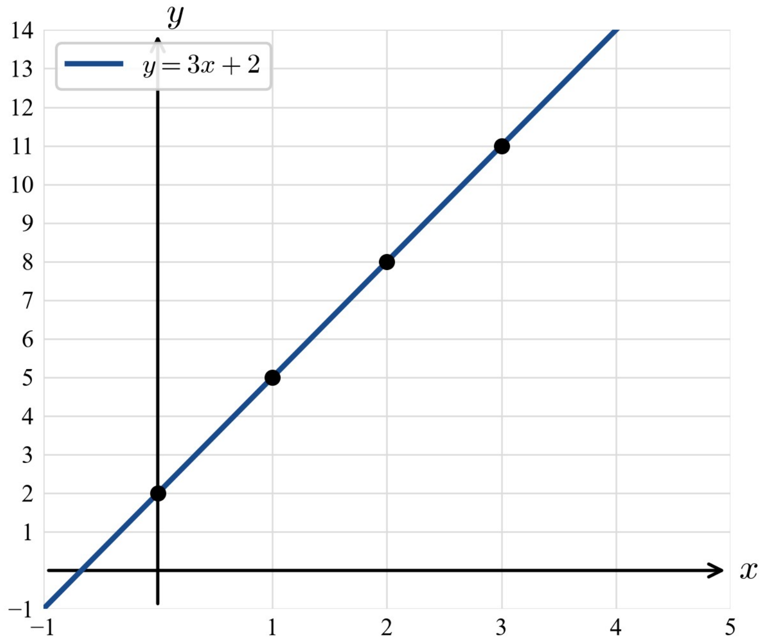
Q5. a $y=0$: $x=6$ — the point $(6,0)$. b $x=0$: $-3y=6$, so $y=-2$ — the point $(0,-2)$. c Join them; the test point $(3,-1)$: $3-3\times(-1)=3+3=6$ ✓, so it lies on the line. ∴ The line through $(6,0)$ and $(0,-2)$, with $(3,-1)$ on it.

Q6. a $x=-1$ is vertical: every point with $x=-1$. b $y=2$ is horizontal: every point with $y=2$. c At the crossing, both hold at once: $x=-1$ and $y=2$. ∴ The lines cross at $(-1,2)$.



A vertical line and a horizontal line on one set of axes crossing at a single marked point — the same picture Q6 draws with x = minus 1 and y = 2

Q7. a $7-4=3$, $10-7=3$, $13-10=3$: up by 3 at every step. b At $x=0$, $y=4$; so in words, start at 4 and add 3 per step — in algebra, $y=3x+4$. c $x=10$: $3\times10+4=34$. ∴ Step 3; rule $y=3x+4$; $y=34$ at $x=10$.



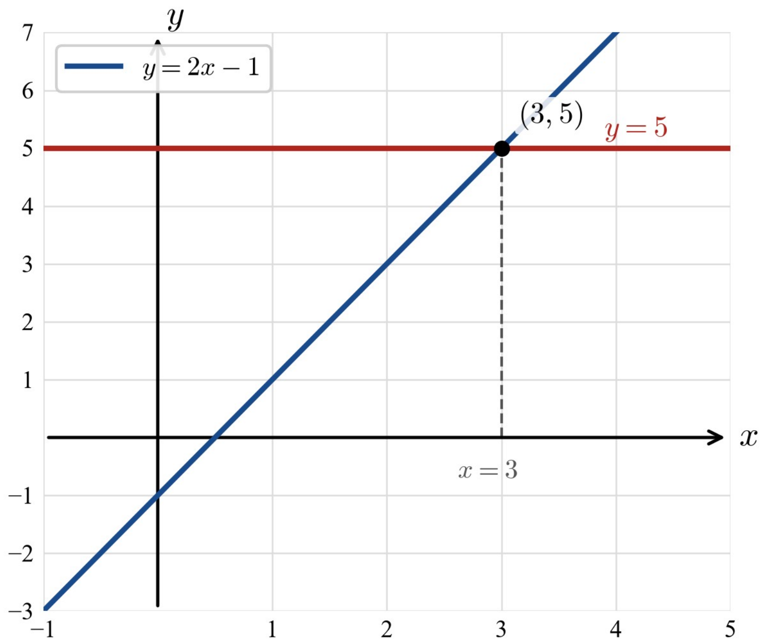
A line with four equally spaced table points marked on it, y = 3x + 2 through (0, 2), (1, 5), (2, 8) and (3, 11) — the same steady-step reading Q7 uses

Q8. a Subtract 7: $x=12$. b Divide by 3: $x=7$. c Add 4: $5x=20$; divide by 5: $x=4$. ∴ 12, 7, 4.

Q9. a Subtract x: $x+3=9$; subtract 3: $x=6$. b Subtract $4x$: $3x-5=10$; add 5: $3x=15$; divide by 3: $x=5$. ∴ 6 and 5.

Q10. a $3x+12=27$; subtract 12: $3x=15$; divide by 3: $x=5$. b $2x-6=5x-15$; add 15: $2x+9=5x$; subtract $2x$: $9=3x$; divide by 3: $x=3$. ∴ 5 and 3.

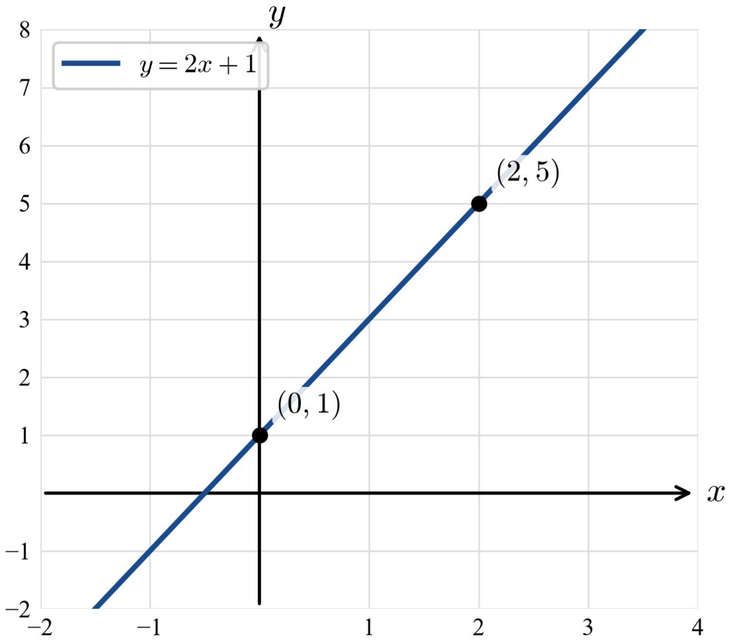
Q11. a The crossing is where the line's height equals 5: $2x-1=5$. b The figure marks the crossing at $(3,5)$, so $x=3$. c Add 1: $2x=6$; divide by 2: $x=3$ — the graph and the algebra agree. ∴ $x=3$.



The line y = 2x minus 1 meeting y = 5 at (3, 5): the crossing point is the solution of 2x minus 1 = 5

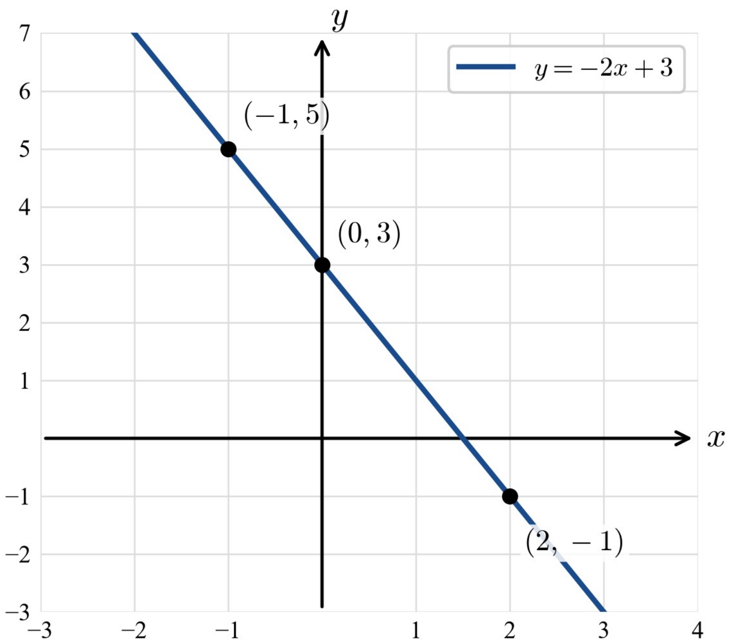
Q12. a At $x=0$, $y=3$. b $7-3=4$ over a run of $2-0=2$; so y rises 2 per step of 1. c Start at 3, add 2 per step: $y=2x+3$. ∴ $y=2x+3$.

Q13. $x=0$: $y=-3$; $x=1$: $y=1$. Join $(0,-3)$ and $(1,1)$; the rise of 4 per step shows the gradient 4. $\therefore$ The line through $(0,-3)$ and $(1,1)$.



The two-point method again: two marked points fix the straight line

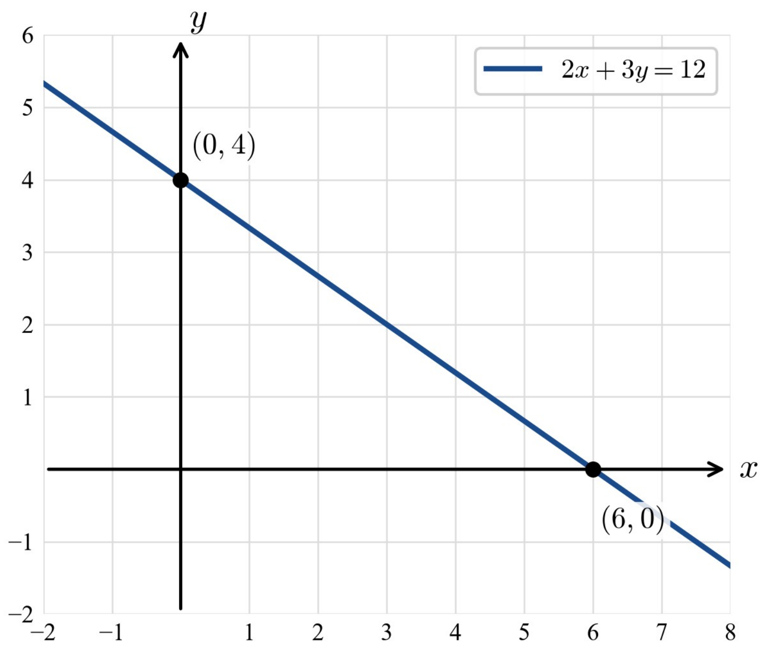
Q14. $x=0$: $y=-2$; $x=-2$: $y=0$. Join $(0,-2)$ and $(-2,0)$; a falling line, since the gradient -1 is negative. $\therefore$ The line through $(0,-2)$ and $(-2,0)$.



A falling line drawn from two points, y = minus 2x + 3 through (minus 1, 5) and (2, minus 1) — Q14's line falls the same way with gradient minus 1

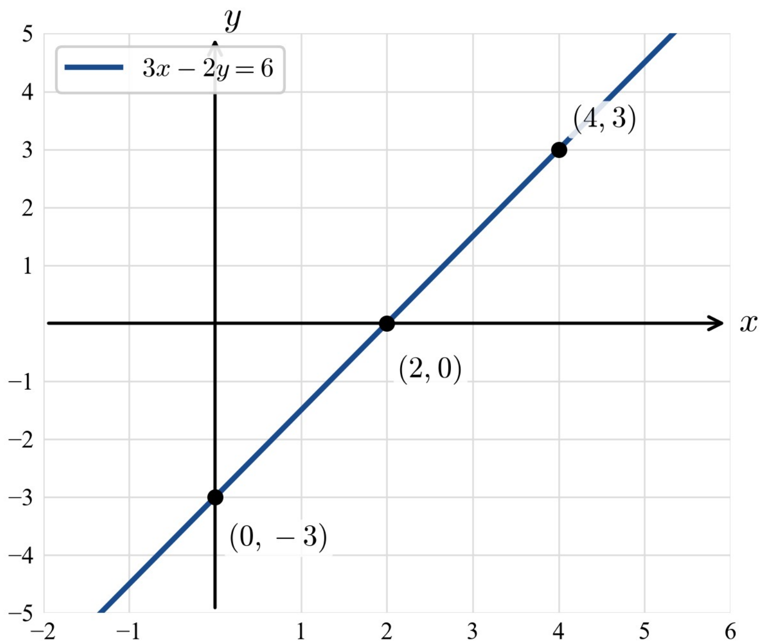
Q15. Choose even x-values so halves are whole: $x=0$ gives $y=1$; $x=4$ gives $y=3$. Join $(0,1)$ and $(4,3)$. $\therefore$ The line through $(0,1)$ and $(4,3)$.

Q16. $y=0$: $x=8$; $x=0$: $2y=8$, so $y=4$. Join $(8,0)$ and $(0,4)$. $\therefore$ The line through $(8,0)$ and $(0,4)$.



The intercept method: a line drawn through its intercepts on the two axes

Q17. $y=0$: $5x=10$, so $x=2$; $x=0$: $-2y=10$, so $y=-5$. Join $(2,0)$ and $(0,-5)$. $\therefore$ The line through $(2,0)$ and $(0,-5)$.



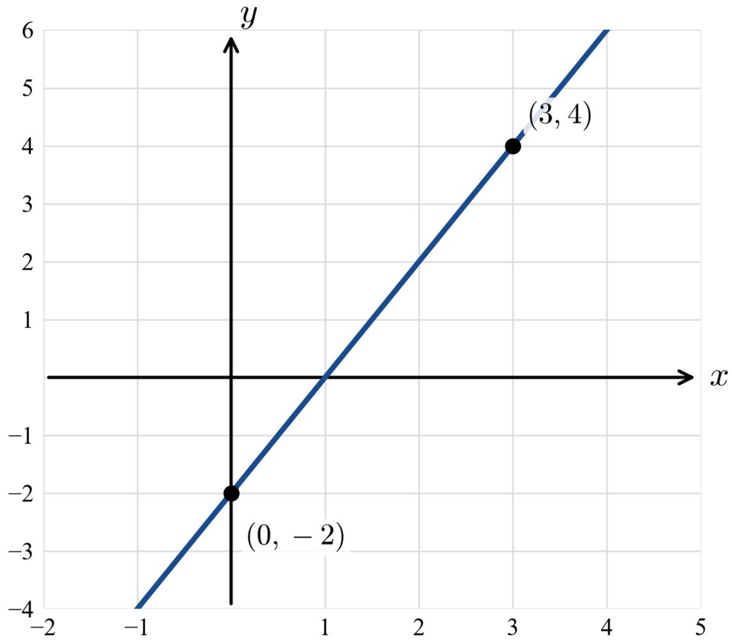
The intercept method with a negative y-intercept: 3x minus 2y = 6 through (2, 0) and (0, minus 3) — Q17 works the same way

Q18. $x=4$ is vertical; $y=-1$ is horizontal; at the crossing both hold. $\therefore$ The lines cross at $(4,-1)$.

Q19. At $(2,5)$ the x-value is 2 and the y-value is 5. $y=2x$ fails: $2\times 2=4\neq 5$. $y=2$ fails: the y-value would have to be 2. $x=2$ holds: the x-value is 2, and every y-value is allowed. $\therefore$ Only $x=2$ passes through $(2,5)$.

Q20. At $x=0$, $y=5$. From $(0,5)$ to $(2,1)$, y falls 4 over a run of 2 — down 2 per step. $\therefore$ $y=-2x+5$.

Q21. The line starts at -2 on the y-axis; from $(0,-2)$ to $(3,4)$, y rises 6 over a run of 3 — up 2 per step. In words: start at -2 and add 2 per step; in algebra: $y=2x-2$. $\therefore$ $y=2x-2$.



The figure's line read back to its rule: through the marked points (0, minus 2) and (3, 4), the rule is y = 2x minus 2

Q22. $x=0$: 6; $x=1$: 3; $x=2$: 0. The sketch falls through $(0,6)$, $(1,3)$ and $(2,0)$. The point at $x=2$ is $(2,0)$ — on the x-axis, so it is the x-intercept. ∴ Table 6, 3, 0; the third point is the x-intercept.

Q23. Subtract $2x$: $2x+9=25$; subtract 9: $2x=16$; divide by 2: $x=8$. Check: $4\times8+9=41$ and $2\times8+25=41$ ✓. ∴ $x=8$.

Q24. $6x-12=4x+8$; subtract $4x$: $2x-12=8$; add 12: $2x=20$; divide by 2: $x=10$. Check: $6(10-2)=48$ and $4\times10+8=48$ ✓. ∴ $x=10$.

Q25. Subtract 5: $-3x=12$; divide by -3: $x=-4$. Check: $5-3\times(-4)=5+12=17$ ✓. ∴ $x=-4$.

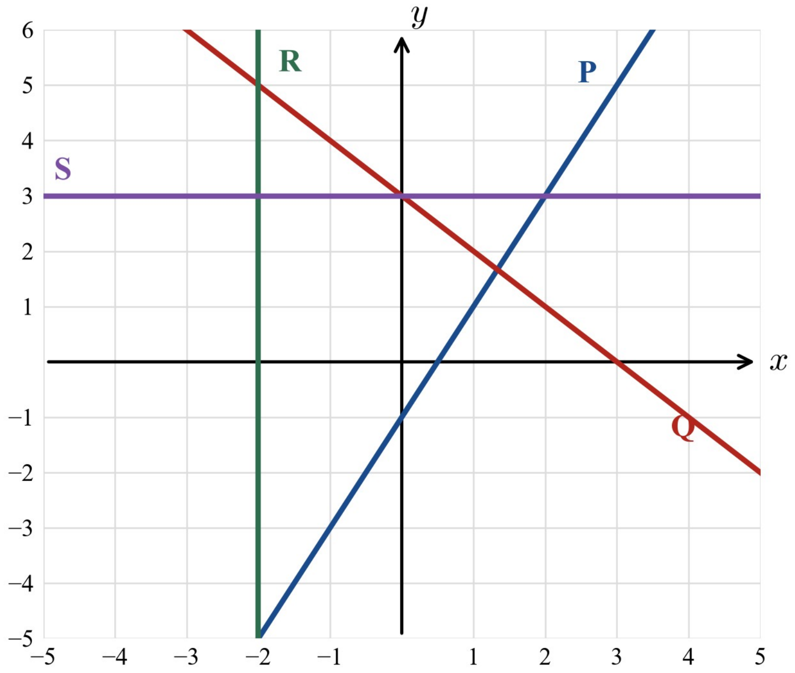
Q26. Multiply by 3: $2x+8=18$; subtract 8: $2x=10$; divide by 2: $x=5$. Check: $\frac{2\times5+8}{3}=\frac{18}{3}=6$ ✓. ∴ $x=5$.

Q27. Add 2: $\frac{x}{3}=6$; multiply by 3: $x=18$. Check: $\frac{18}{3}-2=6-2=4$ ✓. ∴ $x=18$.

Q28. $6x-3=5x+9$; subtract $5x$: $x-3=9$; add 3: $x=12$. Check: $3(2\times12-1)=3\times23=69$ and $5\times12+9=69$ ✓. ∴ $x=12$.

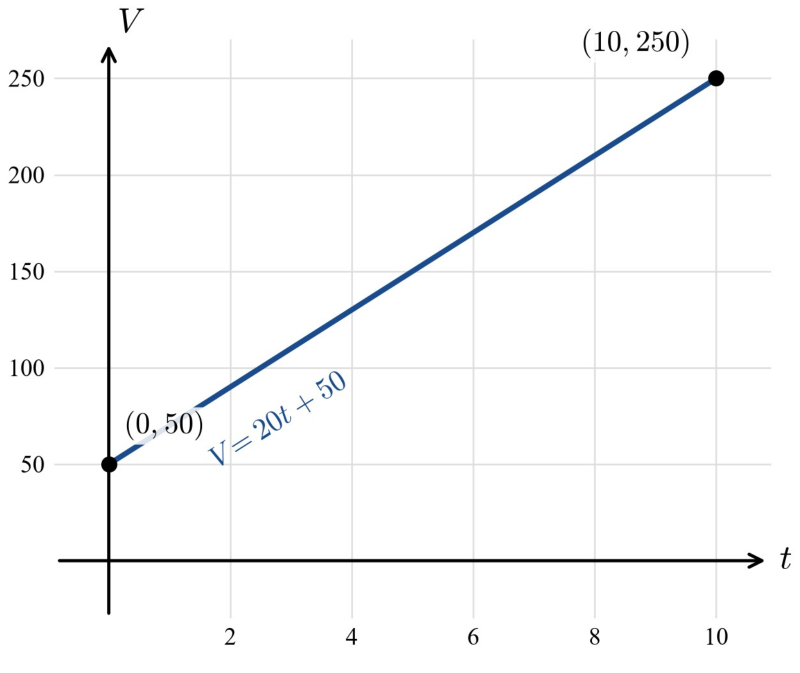
Q29. Let the number be n: $3n+7=5n-1$. Add 1: $3n+8=5n$; subtract $3n$: $8=2n$; divide by 2: $n=4$. Check: $3\times4+7=19$ and $5\times4-1=19$ ✓. ∴ The number is 4.

Q30. P rises and crosses the y-axis at -1: $y=2x-1$. Q falls and crosses the y-axis at 3: $y=-x+3$. R is vertical at $x=-2$: $x=-2$. S is horizontal at $y=3$: $y=3$. ∴ P ↔ $y=2x-1$, Q ↔ $y=-x+3$, R ↔ $x=-2$, S ↔ $y=3$.



Four lines P, Q, R, S matched to their rules by direction, intercepts and orientation

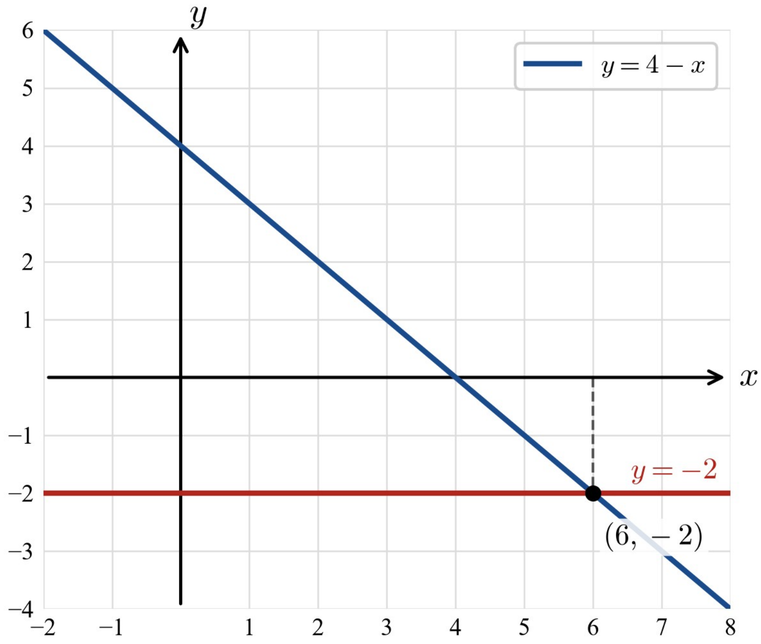
Q31. $t = 10$: $V = 20 \times 10 + 50 = 250$. $V = 170$: $20t + 50 = 170$; subtract 50: $20t = 120$; divide by 20: $t = 6$. ∴ 250 L after 10 minutes; 6 minutes to reach 170 L.



The tank line V = 20t + 50 from (0, 50) to (10, 250) on time–volume axes

Q32. $y = 0$: $3x = 24$, so $x = 8$; $x = 0$: $4y = 24$, so $y = 6$. Join $(8, 0)$ and $(0, 6)$. Test $(4, 3)$: $3 \times 4 + 4 \times 3 = 12 + 12 = 24$ ✓ — both sides equal, so the point is on the line. ∴ Intercepts $(8, 0)$ and $(0, 6)$; yes, $(4, 3)$ lies on the line.

Q33. $x = 0$: $y = 4$; $x = 4$: $y = 0$; join $(0, 4)$ and $(4, 0)$. The line reaches height -2 at the point $(6, -2)$, so $4 - x = -2$ has solution $x = 6$. Algebra: subtract 4: $-x = -6$; divide by -1: $x = 6$ ✓. ∴ $x = 6$.



The falling line y = 4 minus x crossing the horizontal red line y = minus 2 at the marked point (6, minus 2), with a dashed drop line at x = 6

Q34. Her first line expanded $2(x+3)$ as $2x+3$ — the 2 was multiplied by the x only, not by the 3. Correctly: $2x+6=14$; subtract 6: $2x=8$; divide by 2: $x=4$. Check: $2(4+3)=2\times7=14$ ✓. ∴ The mistake is in the expansion; $x=4$.

Q35. Width x cm, length $(x+3)$ cm; perimeter $2(x+3)+2x=26$. $2x+6+2x=26$, that is $4x+6=26$; subtract 6: $4x=20$; divide by 4: $x=5$. Length $5+3=8$. Check: $2\times8+2\times5=16+10=26$ ✓. ∴ Width 5 cm, length 8 cm.

Q36. Start at 4 (the y-value at $x=0$); from $(-2,0)$ to $(0,4)$, y rises 4 over a run of 2 — up 2 per step. Rule: $y=2x+4$. $x=7$: $y=2\times7+4=18$. $y=0$: $2x+4=0$; subtract 4: $2x=-4$; divide by 2: $x=-2$. The first answer is a point far along the line; the second is the line's x-intercept, $(-2,0)$ — one of the two points the line was drawn from, as it must be. ∴ $y=2x+4$; $y=18$ at $x=7$; $x=-2$, the x-intercept.

Gradient, midpoint and distance on the Cartesian plane

2B · Chapter 2 — Algebra: linear relationships and the Cartesian plane · Level 9

Content description `VC2M9A04` (word for word): *find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane*

Achievement standard anchor (AS 4): *They find the distance between 2 points on the Cartesian plane, sketch linear graphs and find the gradient and midpoint of a line segment.*

Elaboration ids for linkage: `VC2M9A04.E01`, `VC2M9A04.E02`, `VC2M9A04.E03`, `VC2M9A04.E04`, `VC2M9A04.E05`

Prerequisite pointer: 2A (sketching linear graphs from two points) and Pythagoras' theorem from Level 8. Both are rehearsed here as tools, not re-taught.

Note on figures: every line in this section is drawn from its own rule and every marked point from its own coordinates (R7); every context is a generic one, so no real data ships and no source line is owed (R1, R2).

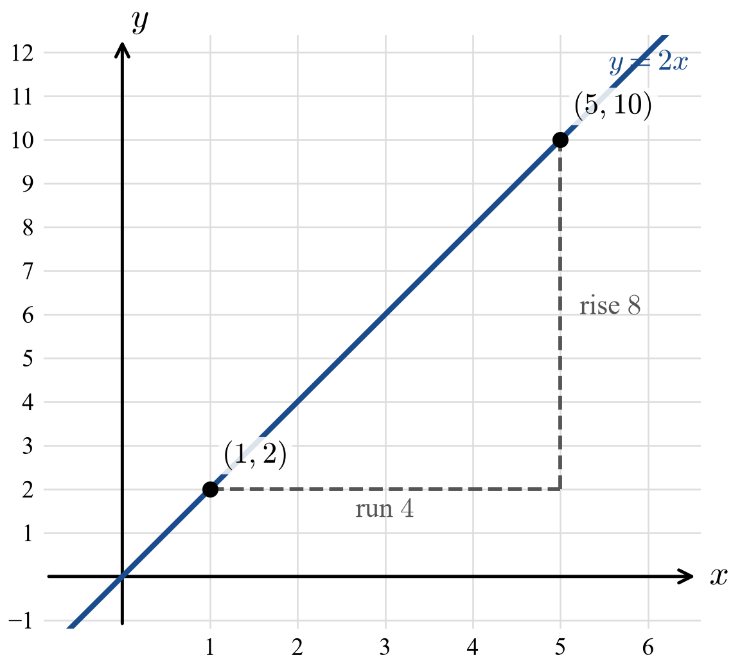
Theory

Gradient — rise over run

2A sketched a line from two of its points. This subtopic puts numbers on how steep the line is. A **line segment** is the part of a line between two end points, and every segment that is not vertical has a **gradient**: a measure of its steepness and its direction.

Move along the segment from the first point to the second. The change across is the **run**, $x_2 - x_1$; the change up is the **rise**, $y_2 - y_1$. The gradient is the rise divided by the run:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

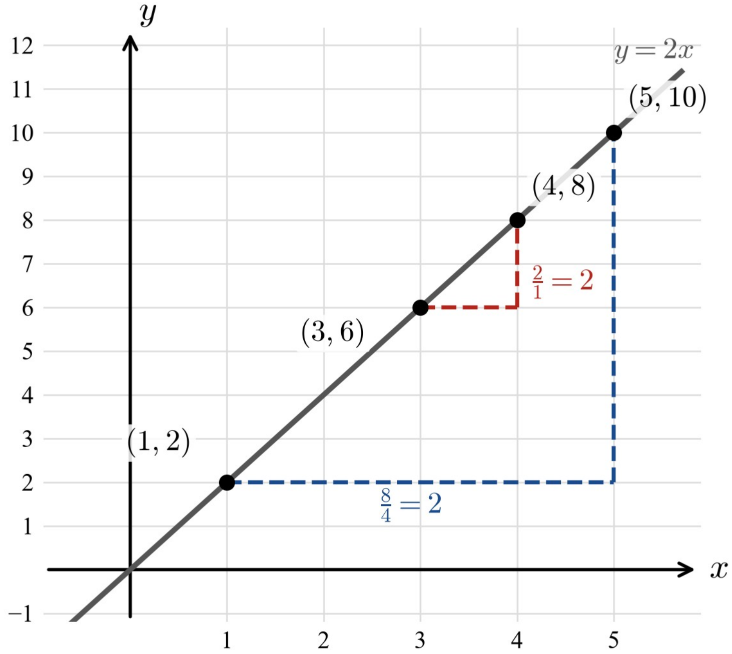


The line y = 2x with the points (1, 2) and (5, 10) marked and a dashed right-angled triangle showing a run of 4 and a rise of 8

On the figure, from $(1,2)$ to $(5,10)$: the run is $5-1=4$ and the rise is $10-2=8$, so $m=\frac{8}{4}=2$. It does not matter which point you start from, as long as you keep the same order on top and on the bottom: $\frac{2-10}{1-5}=\frac{-8}{-4}=2$ as well.

Any two points give the same gradient (E01)

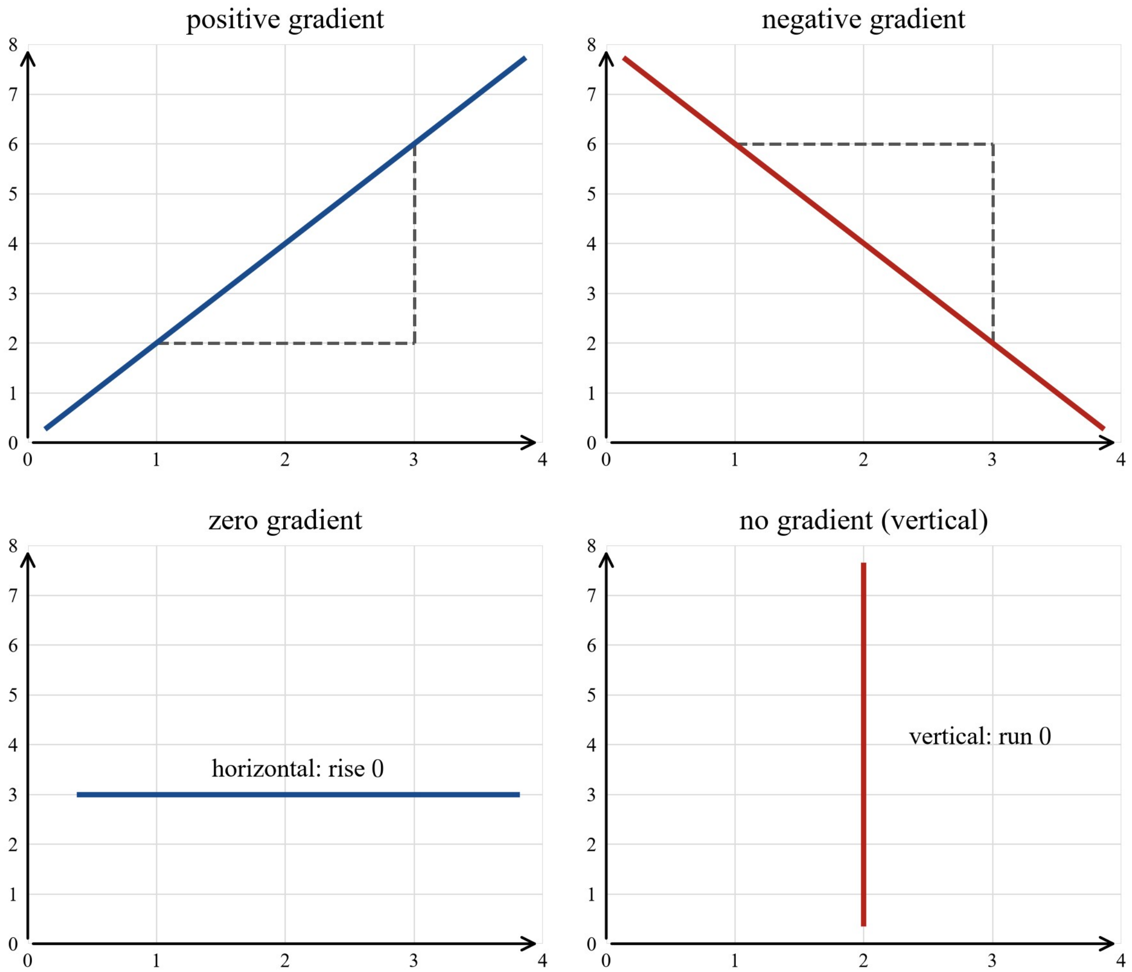
The gradient is a fact about the *line*, not about the pair of points you happen to choose. The figure checks this on the line $y=2x$ with a second pair of points, $(3,6)$ and $(4,8)$: there the run is 1 and the rise is 2, and $m=\frac{2}{1}=2$ — the same answer.



The line y = 2x with two pairs of points on it, (1, 2) to (5, 10) in blue and (3, 6) to (4, 8) in red, each with its own dashed triangle; both triangles give gradient 2

Why must this always happen? A straight line rises evenly, so on it the rise is proportional to the run: double the run and you double the rise. The two dashed triangles in the figure are the same shape, one a bigger copy of the other, and the ratio $\frac{\text{rise}}{\text{run}}$ is the same in both. That is the insight to keep: **the gradient of a line is the gradient of any line segment on that line, so any two distinct points on the line may be used to calculate it.**

The four cases — sign of the gradient

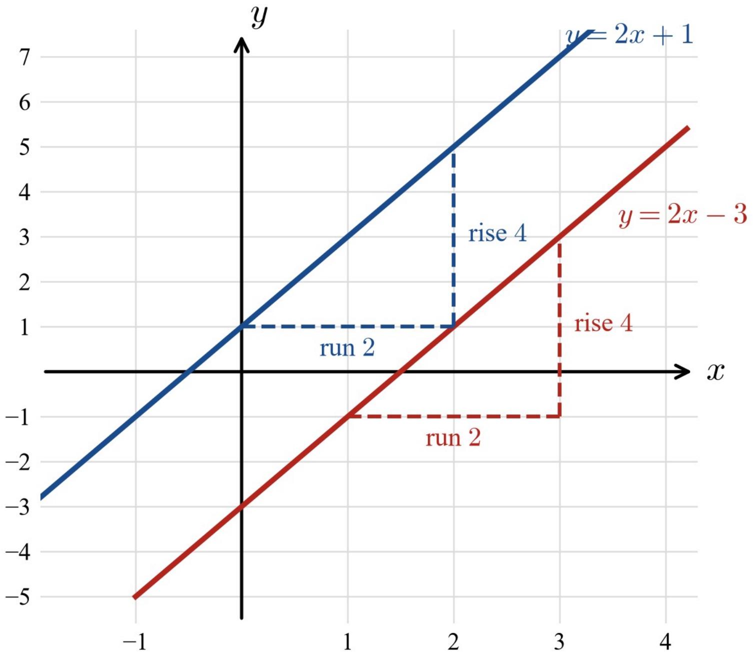


Four panels: a rising line labelled positive gradient, a falling line labelled negative gradient, a horizontal line labelled zero gradient, and a vertical line labelled no gradient because its run is 0

- **Positive gradient:** the line rises from left to right; rise and run have the same sign.
- **Negative gradient:** the line falls from left to right; the rise is negative while the run is positive.
- **Zero gradient:** a horizontal line has rise 0, so $m=0$.
- **No gradient:** a vertical line has run 0, and division by 0 is not allowed — a vertical line has no gradient.

Parallel and perpendicular gradients (E02)

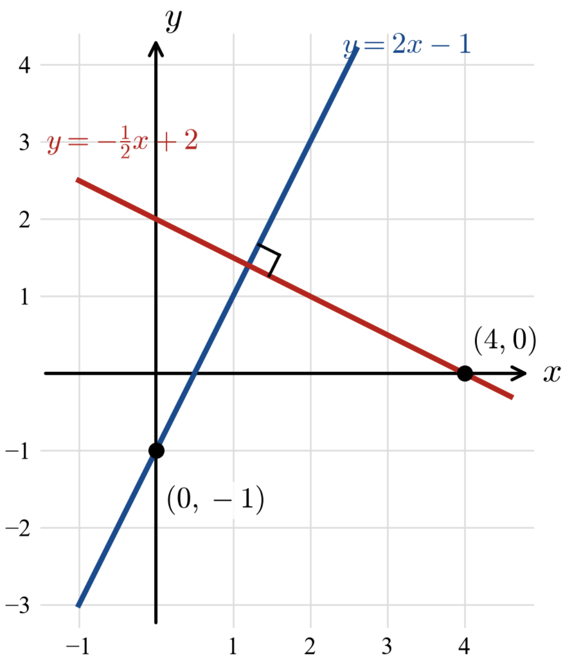
Parallel lines are the same steepness in the same direction, so they should have the same gradient — and they do. The figure shows $y=2x+1$ and $y=2x-3$; each dashed triangle is a rise of 4 over a run of 2, so both lines have gradient 2.



The parallel lines y = 2x + 1 in blue and y = 2x − 3 in red, each with a dashed triangle of run 2 and rise 4, showing that both have gradient 2

Parallel lines have equal gradients; and two lines with equal gradients are parallel.

Perpendicular lines meet at a right angle, and the relationship between their gradients is a product: the figure shows $y = 2x - 1$ and $y = -\frac{1}{2}x + 2$ crossing at a right angle, and $2 \times \left(-\frac{1}{2}\right) = -1$.

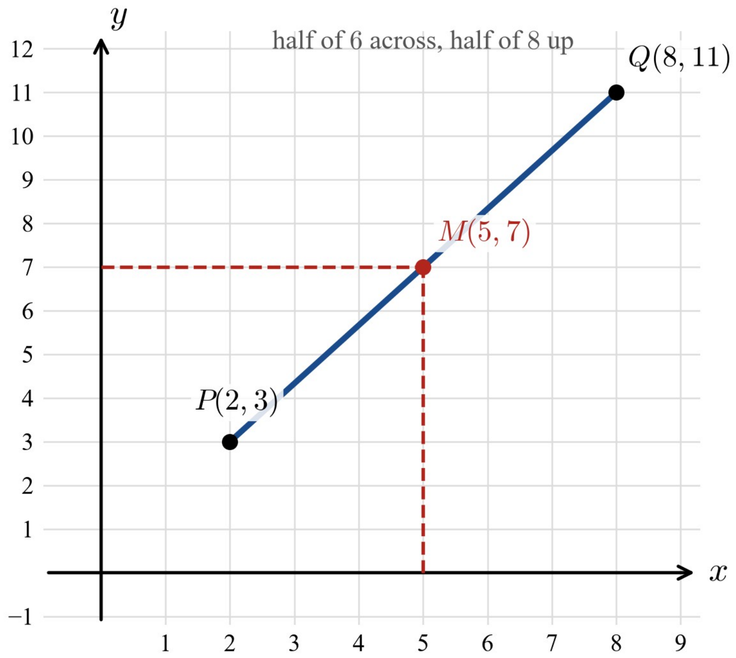


The line y = 2x − 1 in blue and the line y = −½x + 2 in red crossing at a right angle, marked with a small square at the crossing point; the blue line crosses the y-axis at (0, −1) and the red line crosses the x-axis at (4, 0)

Two lines are perpendicular exactly when the product of their gradients is -1. Both facts are demonstrated here by computation; if a graphing tool is in front of you, typing each pair of rules and looking at the crossing is a quick way to see them. Solving problems with these facts (for example, finding the rule of the perpendicular through a given point) is Level 10 work — here they are facts to know and to check.

The midpoint of a segment (E04)

The **midpoint** of a segment is the point halfway between its ends — half of the horizontal distance across, and half of the vertical distance up. On the figure, from $P(2,3)$ to $Q(8,11)$ the way across is 6 and the way up is 8; half of 6 is 3 and half of 8 is 4, so the midpoint is $(2+3,3+4)=(5,7)$.



The segment from P(2, 3) to Q(8, 11) with the midpoint M(5, 7) marked in red and dashed guides from M to both axes; half of 6 across and half of 8 up

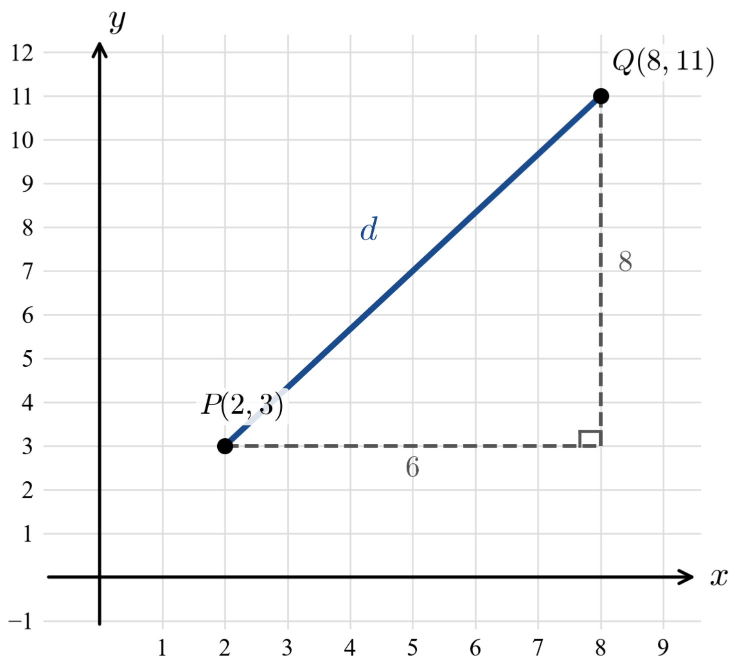
Doing the same move in symbols gives the midpoint rule: halfway across is the average of the x-coordinates, and halfway up is the average of the y-coordinates,

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right).$$

The distance between two points (E03)

How far is it from $P(2,3)$ to $Q(8,11)$? The segment PQ is the **hypotenuse** of a right-angled triangle whose other two sides are the horizontal distance (6) and the vertical distance (8). Pythagoras' theorem does the rest:

$d^2=6^2+8^2=36+64=100$, so $d=\sqrt{100}=10$.



The segment from P(2, 3) to Q(8, 11) drawn as the hypotenuse d of a right-angled triangle with horizontal leg 6 and vertical leg 8, the right angle marked with a small square at the corner (8, 3)

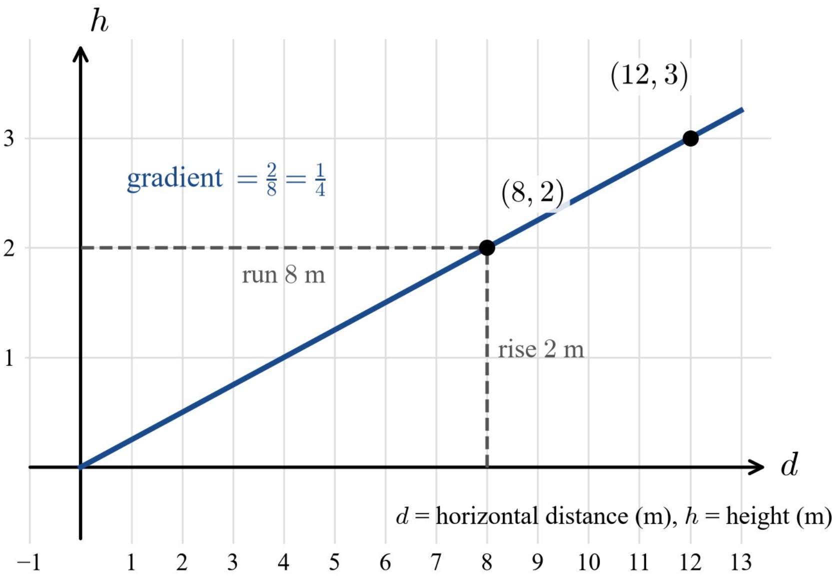
In symbols, the legs are $x_2 - x_1$ and $y_2 - y_1$, so the distance rule is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

— not a formula to memorise, but Pythagoras' theorem standing up on the Cartesian plane. This result travels: 5C will take it off the plane and into space.

Gradient as a constant rate of change (E05)

A gradient of 2 means a rise of 2 for every 1 across — the same deal at every point of the line. That is what *constant* adds to *rate of change*: on a line, the rate never varies. The figure reads a ramp with gradient $\frac{2}{8} = \frac{1}{4}$ as a rate: for every 4 m forward, the ramp rises 1 m, however far along it you measure. At 12 m forward the height is $\frac{12}{4} = 3$ m, exactly where the figure shows it, and the rule of the line is $h = \frac{1}{4}d$.



A ramp drawn as the line h = d/4 through the origin, with the points (8, 2) and (12, 3) marked and dashed guides showing a run of 8 m and a rise of 2 m; the horizontal axis is d, horizontal distance in metres, and the vertical axis is h, height in metres

The gradient of a line is its constant rate of change — the rise per unit of run, the same between any two points of the line. Read that way, a gradient is never just a number: 5 L per minute filling a tank, -2 cm per hour as a candle burns down, $\frac{1}{4}$ m of height per 4 m of ramp. Every rate you have ever read off a straight graph was a gradient.

Worked examples

Example 1 — scaffolded

Find the gradient of the segment joining $A(1,2)$ and $B(5,10)$, then check that a different pair of points on the same line gives the same gradient.

Working	Comment
Run $=5-1=4$; rise $=10-2=8$	The change across, then the change up.
$m=\frac{8}{4}=2$	Gradient = rise over run.
The line has rule $y=2x$: at $x=1$, $y=2$; at $x=5$, $y=10$	Both given points sit on the line $y=2x$.
A second pair on $y=2x$: $(3,6)$ and $(4,8)$, with $m=\frac{8-6}{4-3}=\frac{2}{1}=2$	Same line, different pair — the same gradient (E01).
Starting from B instead: $m=\frac{2-10}{1-5}=\frac{-8}{-4}=2$	The order of the points does not matter, as long as it matches on top and on the bottom.

Example 2 — scaffolded

Find the midpoint and the distance between $P(2,3)$ and $Q(8,11)$.

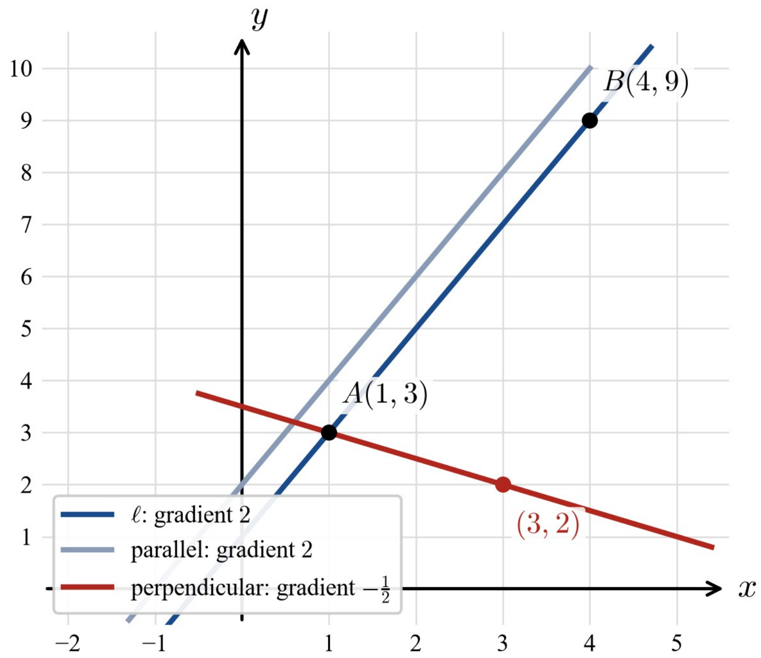
Working	Comment
Midpoint $=\left(\frac{2+8}{2},\frac{3+11}{2}\right)=(5,7)$	The average of the x-coordinates; the average of the y-coordinates.
Horizontal distance $=8-2=6$; vertical distance $=11-3=8$	The two legs of the right-angled triangle.
$d^2=6^2+8^2=36+64=100$	Pythagoras' theorem.

Working	Comment
$d=\sqrt{100}=10$	A distance is positive.

Example 3 — unscaffolded

The line ℓ passes through $A(1,3)$ and $B(4,9)$. Find the gradient of ℓ, the gradient of any line parallel to ℓ, and the gradient of any line perpendicular to ℓ, and check the perpendicular relationship as a product.

The gradient of ℓ is $m=\dfrac{9-3}{4-1}=\dfrac{6}{3}=2$. A line parallel to ℓ has the same gradient, 2. A line perpendicular to ℓ has a gradient whose product with 2 is -1, namely $-\dfrac{1}{2}$. Check: $2\times\left(-\dfrac{1}{2}\right)=-1$, the required product. The figure shows all three lines; from $A(1,3)$, one run of 2 and one rise of -1 lands on $(3,2)$, a second point of the perpendicular.



The line through A(1, 3) and B(4, 9) with gradient 2 in blue, a parallel line of gradient 2 in light blue, and a perpendicular line of gradient −½ in red passing through A(1, 3) and (3, 2)

$\therefore$ ℓ has gradient 2; parallel lines have gradient 2; perpendicular lines have gradient $-\dfrac{1}{2}$, since $2\times\left(-\dfrac{1}{2}\right)=-1$.

Example 4 — unscaffolded

A ramp rises 2 m over a horizontal distance of 8 m, at a steady rate. Find the gradient of the ramp, write it as a rate of change, and hence find the height of the ramp 12 m along from the start.

The gradient is $\dfrac{2}{8}=\dfrac{1}{4}$. As a rate: $\dfrac{1}{4}$ is 1 m of height for every 4 m forward — the constant rate of change of height with horizontal distance. At 12 m forward the height is $\dfrac{12}{4}=3$ m. The rule connecting height h and horizontal distance d is $h=\dfrac{1}{4}d$, the rule drawn in the ramp figure of the theory.

$\therefore$ The gradient is $\dfrac{1}{4}$ — a constant rate of change of 1 m up per 4 m forward — and the height at 12 m is 3 m.

Practice questions

Scaffolded

Q1. Find the gradient of each segment. First write the run and the rise, then divide the rise by the run.

a. $(1,2)$ to $(5,10)$
b. $(2,1)$ to $(4,7)$
c. $(0,5)$ to $(5,5)$ — once you have the gradient, say what the segment looks like.

Q2. Find each gradient, keeping the signs careful.

a. $(0,6)$ to $(3,0)$
b. $(1,1)$ to $(4,4)$
c. $(2,3)$ to $(2,8)$ — work out the run first, and say what it means for the gradient.

Q3. Find the midpoint of each segment: average the x-coordinates, then average the y-coordinates.

a. $(2,4)$ and $(6,10)$
b. $(1,1)$ and $(7,5)$
c. $(0,6)$ and $(8,2)$

Q4. Find the distance between each pair of points. First write the horizontal and the vertical distance, then use Pythagoras' theorem.

a. $(0,0)$ and $(3,4)$
b. $(1,1)$ and $(4,5)$
c. $(2,2)$ and $(8,10)$

Q5. A line has the gradient given. Write the gradient of a line parallel to it, and the gradient of a line perpendicular to it.

a. 3
b. -2
c. $\frac{1}{2}$

Q6. A tank fills at a steady rate. Time t (minutes): $0,2,4,6$. Volume V (litres): $0,10,20,30$.

a. Show that the rise in V is the same from each row of the table to the next.
b. Hence write the rate of change in litres per minute.
c. Write V in terms of t.

Unscaffolded

Q7. Find the gradient of the segment joining $(-3,1)$ and $(3,7)$.

Q8. Show that the points $(0,1)$, $(2,5)$ and $(4,9)$ all lie on one straight line.

Q9. The segment from $(1,2)$ to $(5,k)$ has gradient 3. Find k.

Q10. Find the midpoint of the segment joining $(-3,-2)$ and $(5,6)$.

Q11. $M(3,4)$ is the midpoint of a segment from $A(1,7)$ to B. Find the coordinates of B.

Q12. Find the exact distance between $(2,1)$ and $(5,4)$, leaving your answer as a surd, and give it to 2 d.p. as well.

Q13. A triangle has vertices $(0,0)$, $(6,0)$ and $(0,8)$. Find the lengths of its three sides and its perimeter, and show that the triangle is right-angled.

Q14. Show that $A(1,1)$, $B(5,2)$, $C(6,6)$ and $D(2,5)$, taken in that order, form a parallelogram, using the midpoints of the diagonals AC and BD.

Q15. Decide whether the segment from $(0,0)$ to $(4,2)$ and the segment from $(1,5)$ to $(3,1)$ are perpendicular.

Q16. A candle burns at a steady rate. Time t (hours): $0,2,4$. Length L (cm): $20,16,12$. Find the rate of change of the length, and say in a sentence what its sign is telling you.

Q17. For $A(-2,1)$ and $B(6,7)$, find the gradient of AB, the midpoint of AB, and the length of AB.

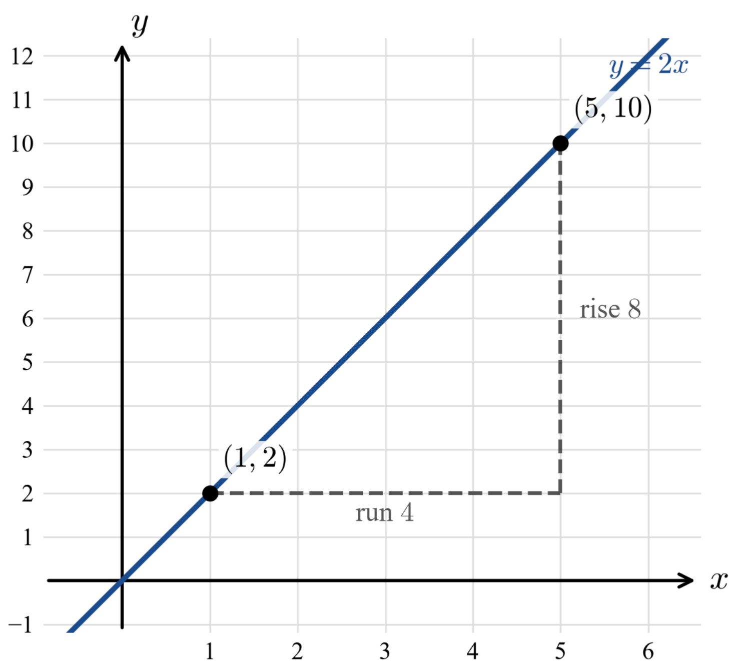
Q18. A triangle has vertices $A(-1,2)$, $B(5,2)$ and $C(2,6)$. Show that it is isosceles by finding the lengths of its three sides.

Answers

Q1 a 2; b 3; c 0 — the segment is horizontal. **Q2** a -2; b 1; c the run is 0, so the segment is vertical and has no gradient. **Q3** a $(4,7)$; b $(4,3)$; c $(4,4)$. **Q4** a 5; b 5; c 10. **Q5** a parallel 3, perpendicular $-\frac{1}{3}$; b parallel -2, perpendicular $\frac{1}{2}$; c parallel $\frac{1}{2}$, perpendicular -2. **Q6** a 10 each time; b 5 L per minute; c $V=5t$. **Q7** 1. **Q8** Both segments have gradient 2 and share the point $(2,5)$, so all three points lie on the one line, $y=2x+1$. **Q9** 14. **Q10** $(1,2)$. **Q11** $(5,1)$. **Q12** $\sqrt{18}=3\sqrt{2}\approx 4.24$. **Q13** Sides 6, 8, 10; perimeter 24; $6^2+8^2=10^2$, so the angle at $(0,0)$ is a right angle. **Q14** Both diagonals have midpoint $\left(\frac{7}{2},\frac{7}{2}\right)$, so they cut each other in half — a parallelogram. **Q15** Gradients $\frac{1}{2}$ and -2; the product is -1, so the segments are perpendicular. **Q16** -2 cm per hour; the negative sign says the length is falling by 2 cm every hour. **Q17** Gradient $\frac{3}{4}$; midpoint $(2,4)$; length 10. **Q18** $AC=BC=5$ and $AB=6$: two equal sides, so the triangle is isosceles.

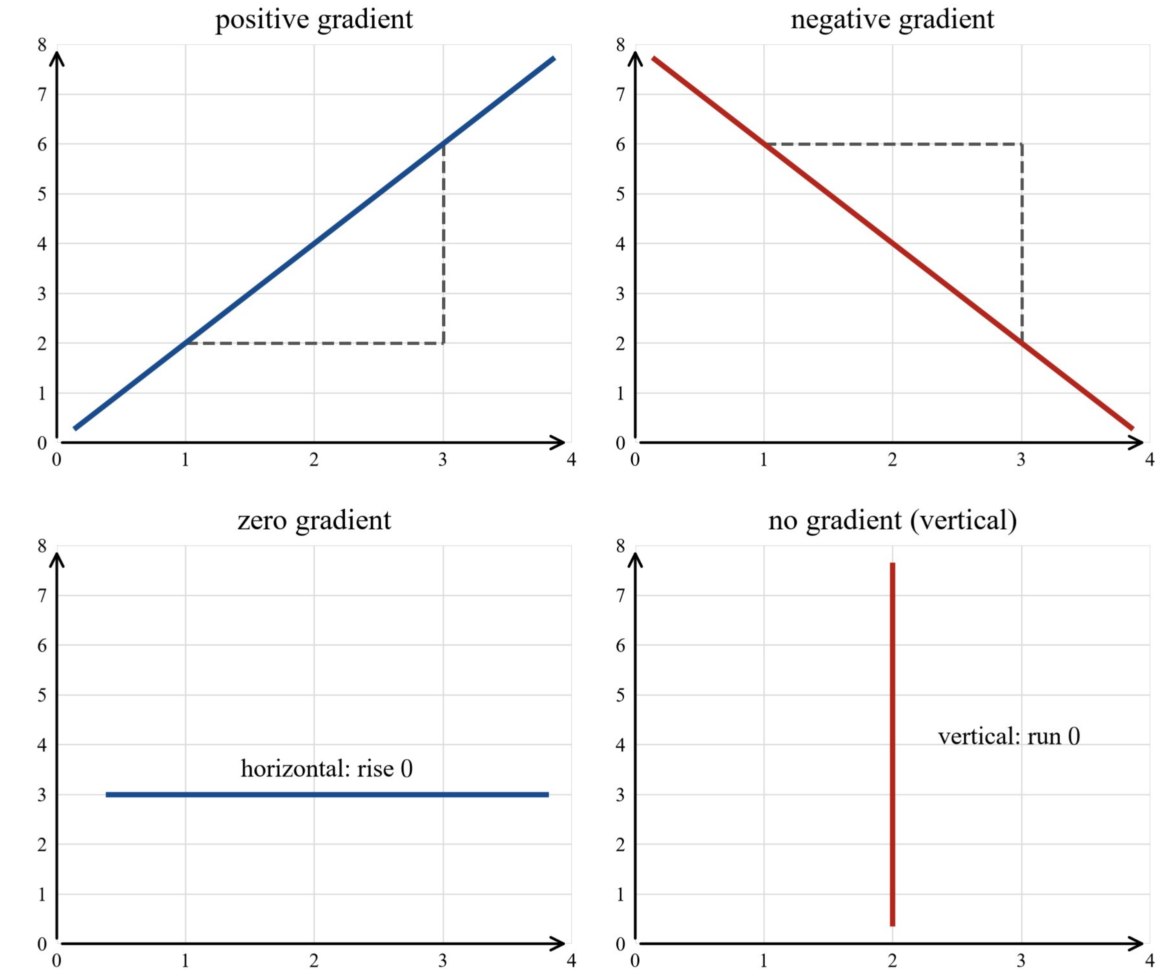
Fully-worked solutions

Q1. a Run $5-1=4$, rise $10-2=8$: $m=\frac{8}{4}=2$. b Run $4-2=2$, rise $7-1=6$: $m=\frac{6}{2}=3$. c Run $5-0=5$, rise $5-5=0$: $m=\frac{0}{5}=0$. A gradient of 0 means the segment is horizontal. ∴ The gradients are 2, 3 and 0; part a is the very pair drawn in the first figure of the theory.



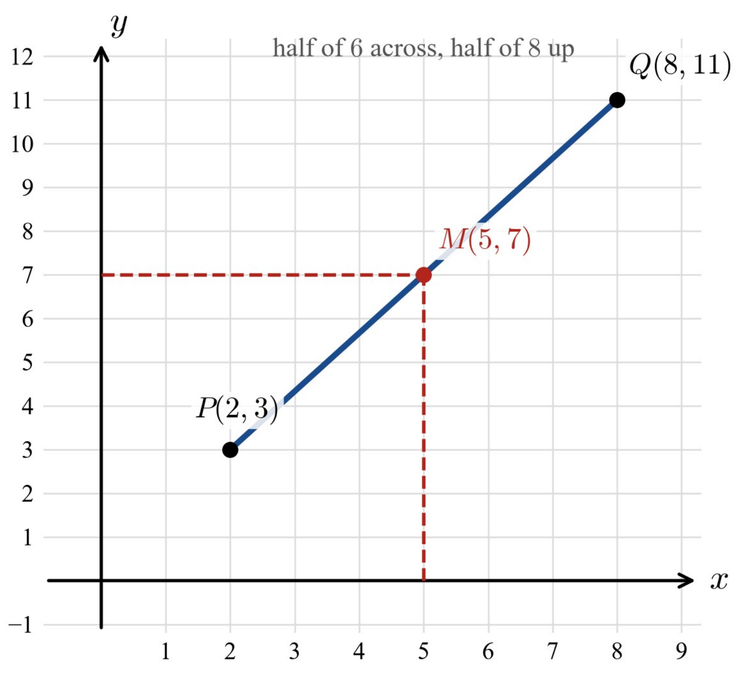
The line y = 2x with the points (1, 2) and (5, 10) and a dashed triangle of run 4 and rise 8 — the segment of Q1a

Q2. a Run $3-0=3$, rise $0-6=-6$: $m=\frac{-6}{3}=-2$. b Run $4-1=3$, rise $4-1=3$: $m=\frac{3}{3}=1$. c Run $2-2=0$: the segment is vertical, and division by 0 is not allowed, so it has no gradient. ∴ -2, 1, and no gradient — the falling, rising and vertical cases of the four-cases figure.



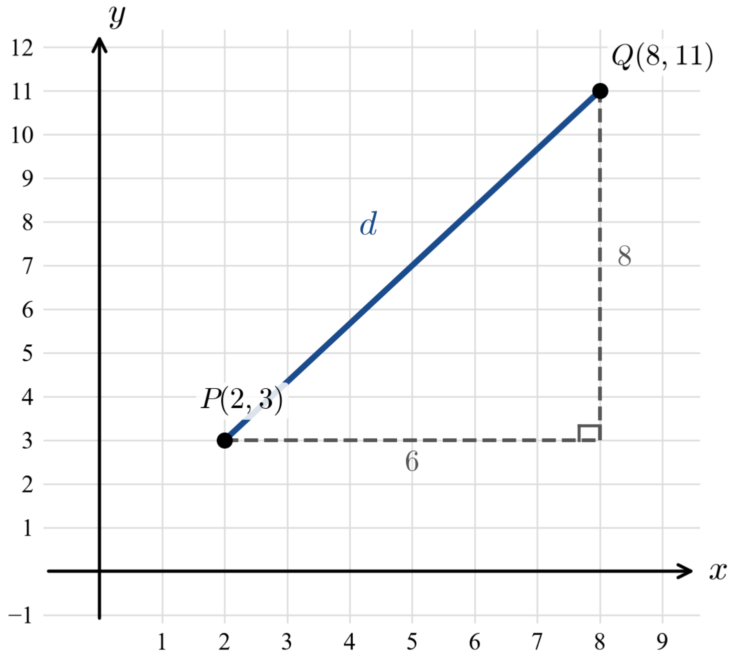
Four panels showing the positive, negative, zero and no-gradient cases; Q2a falls, Q2b rises, Q2c is vertical

Q3. a $\left(\frac{2+6}{2},\frac{4+10}{2}\right)=(4,7)$. b $\left(\frac{1+7}{2},\frac{1+5}{2}\right)=(4,3)$. c $\left(\frac{0+8}{2},\frac{6+2}{2}\right)=(4,4)$. $\therefore$ $(4,7)$, $(4,3)$, $(4,4)$ — each is half of the horizontal distance across and half of the vertical distance up, as in the midpoint figure.



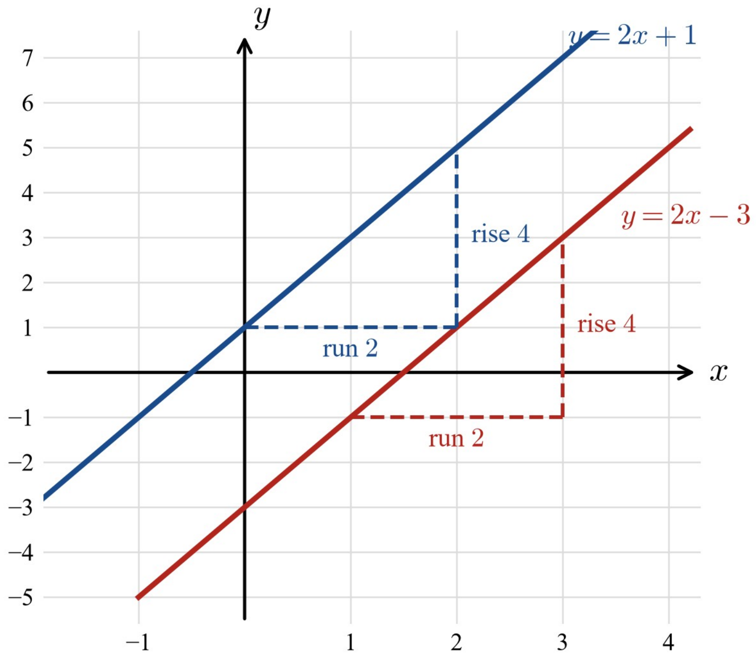
The segment from P(2, 3) to Q(8, 11) with midpoint M(5, 7): the same halfway-across, halfway-up move as Q3

Q4. a Horizontal 3, vertical 4: $d^2=3^2+4^2=25$, $d=5$. b Horizontal 3, vertical 4: $d=5$ again — the same triangle, slid. c Horizontal 6, vertical 8: $d^2=36+64=100$, $d=10$. $\therefore$ 5, 5, 10; part c is the very triangle of the distance figure.

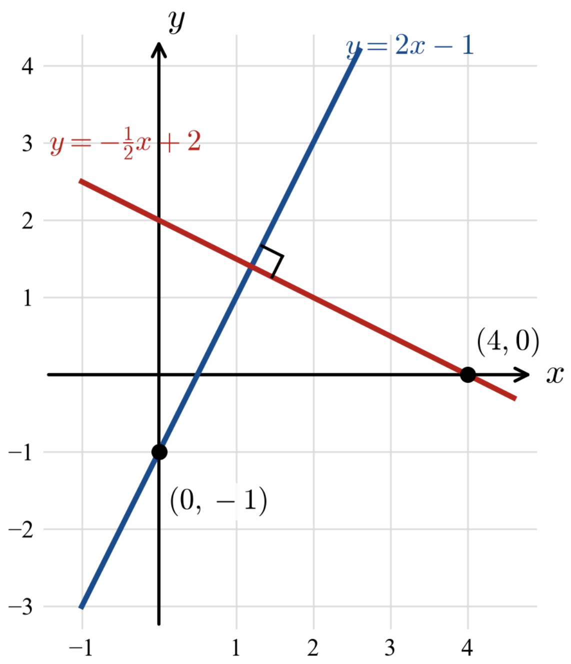


The right-angled triangle with legs 6 and 8 and hypotenuse 10 — the triangle of Q4c

Q5. Parallel lines have the same gradient; perpendicular gradients multiply to -1. a Parallel: 3; perpendicular: $-\frac{1}{3}$, since $3\times\left(-\frac{1}{3}\right)=-1$. b Parallel: -2; perpendicular: $\frac{1}{2}$, since $-2\times\frac{1}{2}=-1$. c Parallel: $\frac{1}{2}$; perpendicular: -2, since $\frac{1}{2}\times(-2)=-1$. ∴ Same gradient for parallel; product -1 for perpendicular — the two facts demonstrated in the theory figures.

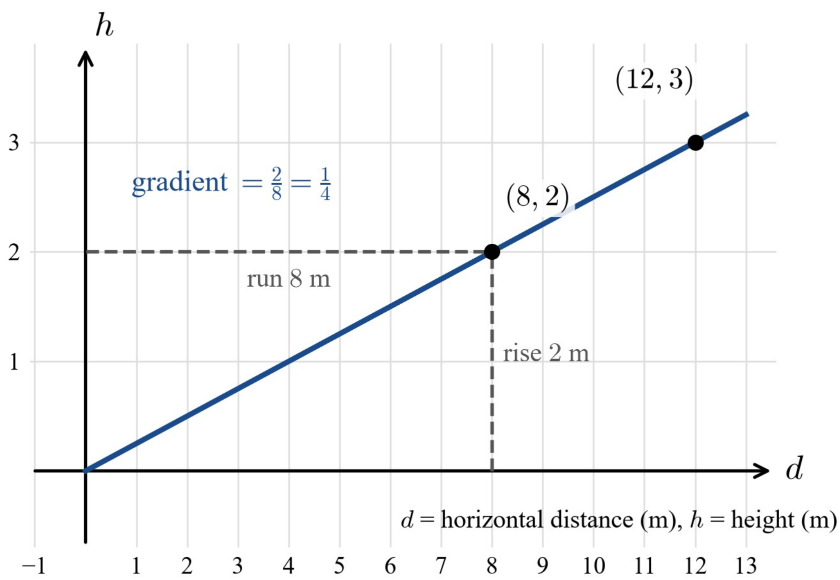


The parallel lines y = 2x + 1 and y = 2x − 3 with equal gradients, as Q5 uses for any gradient



The perpendicular lines y = 2x − 1 and y = −½x + 2 whose gradients multiply to −1

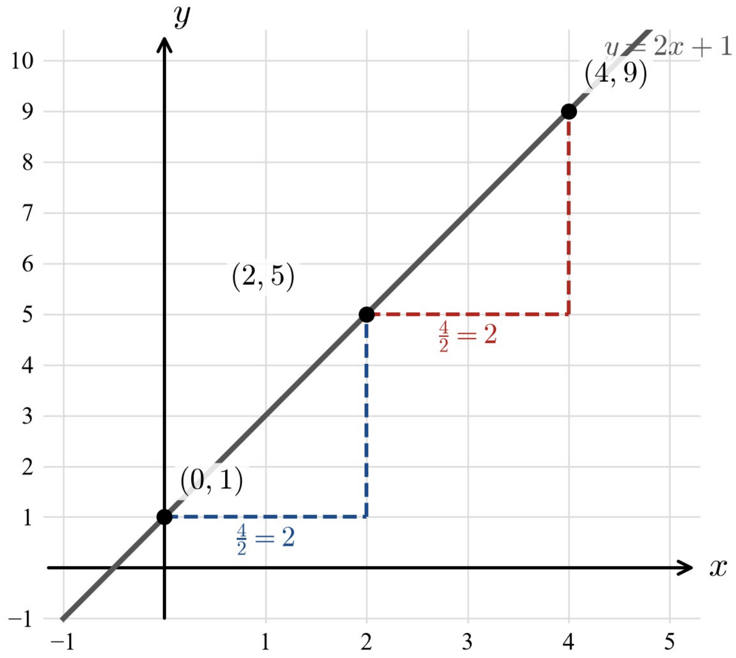
Q6. a From row to row, V rises $10-0=10$, $20-10=10$, $30-20=10$ — the same rise, 10, each time. b 10 L in 2 minutes is $\frac{10}{2}=5$ L per minute. c $V=5t$. ∴ A constant rise in the table means a constant rate of change — a gradient of 5, read exactly like the ramp's $\frac{1}{4}$.



The ramp line h = d/4: a constant rate of change read off a graph, the same idea as the constant rate read off the table in Q6

Q7. Run $3-(-3)=6$; rise $7-1=6$; $m=\frac{6}{6}=1$. ∴ $m=1$; the negatives cancel in the run exactly as they would on the number line.

Q8. From $(0,1)$ to $(2,5)$: $m=\frac{5-1}{2-0}=\frac{4}{2}=2$. From $(2,5)$ to $(4,9)$: $m=\frac{9-5}{4-2}=\frac{4}{2}=2$. The two segments have the same gradient and share the point $(2,5)$, so they are two parts of one straight line, $y=2x+1$. ∴ All three points lie on the one line — equal gradients from a shared point is exactly the E01 idea in reverse.



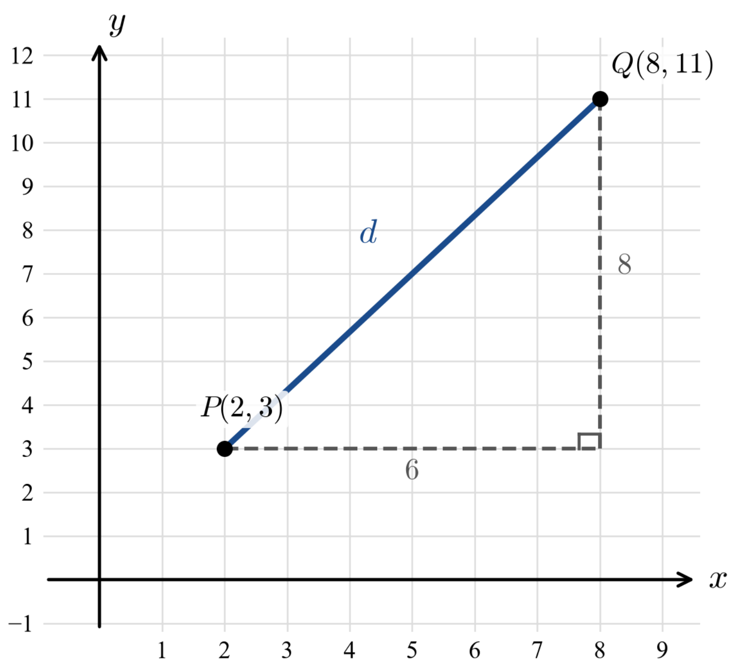
The line y = 2x + 1 through (0, 1), (2, 5) and (4, 9), with two dashed triangles each giving gradient 2

Q9. The gradient is $\frac{k-2}{5-1}=\frac{k-2}{4}$, and this equals 3. So $k-2=12$, giving $k=14$. Check: from $(1,2)$ to $(5,14)$, rise 12 over run 4 is 3. $\therefore$ $k=14$.

Q10. $\left(\frac{-3+5}{2},\frac{-2+6}{2}\right)=\left(\frac{2}{2},\frac{4}{2}\right)=(1,2)$. $\therefore$ The midpoint is $(1,2)$.

Q11. The midpoint averages the coordinates, so B is as far beyond M as A is behind it. Across: from 1 to 3 is 2, so B is at $3+2=5$. Up–down: from 7 to 4 is 3 down, so B is at $4-3=1$. Check: $\left(\frac{1+5}{2},\frac{7+1}{2}\right)=(3,4)=M$. $\therefore$ $B(5,1)$.

Q12. Horizontal $5-2=3$; vertical $4-1=3$. $d^2=3^2+3^2=18$, so $d=\sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}$. $3\sqrt{2}\approx 4.24$ to 2 d.p. $\therefore$ $d=3\sqrt{2}\approx 4.24$ — the same right-angled triangle as the distance figure, with legs of 3.

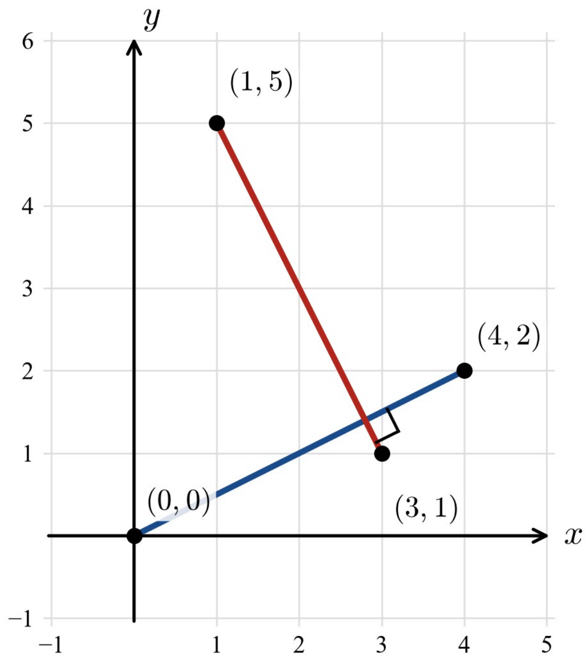


The distance figure: legs 6 and 8 with hypotenuse 10; Q12 is the same construction with legs 3 and 3

Q13. From $(0,0)$ to $(6,0)$: 6. From $(0,0)$ to $(0,8)$: 8. From $(6,0)$ to $(0,8)$: $d^2 = 6^2 + 8^2 = 100$, $d = 10$. Perimeter $= 6 + 8 + 10 = 24$. $6^2 + 8^2 = 100 = 10^2$, so the two shorter sides meet at a right angle, at $(0,0)$. $\therefore$ Sides $6, 8, 10$; perimeter 24; right-angled at the origin.

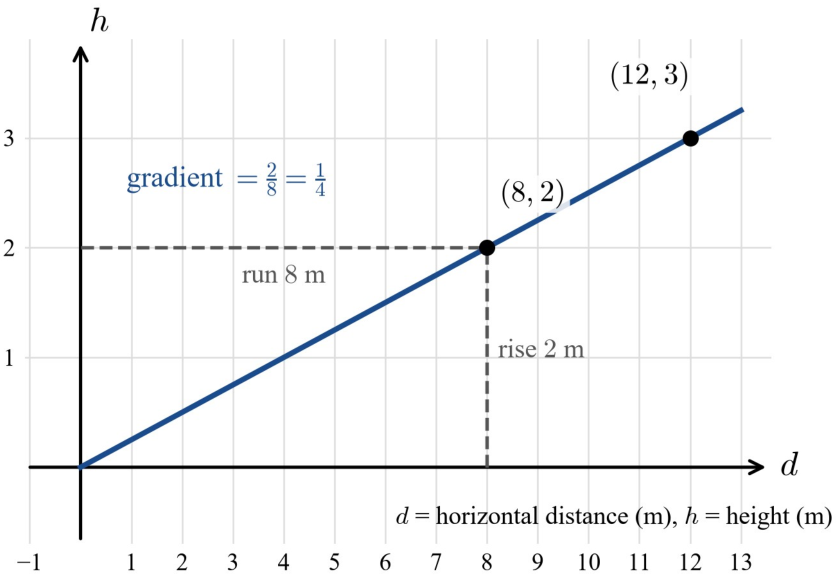
Q14. Midpoint of AC: $\left(\frac{1+6}{2}, \frac{1+6}{2}\right) = \left(\frac{7}{2}, \frac{7}{2}\right)$. Midpoint of BD: $\left(\frac{5+2}{2}, \frac{2+5}{2}\right) = \left(\frac{7}{2}, \frac{7}{2}\right)$. The diagonals have the same midpoint, so each cuts the other in half — and a quadrilateral whose diagonals cut each other in half is a parallelogram. $\therefore$ $ABCD$ is a parallelogram.

Q15. First segment: $m = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$. Second segment: $m = \frac{1-5}{3-1} = \frac{-4}{2} = -2$. Product: $\frac{1}{2} \times (-2) = -1$, so the segments are perpendicular — the figure shows the right angle where they cross. $\therefore$ Perpendicular, since the product of the gradients is -1.



The segment from (0, 0) to (4, 2) and the segment from (1, 5) to (3, 1) crossing at a right angle, marked with a small square

Q16. From $t=0$ to $t=2$: $\frac{16-20}{2-0} = \frac{-4}{2} = -2$; from $t=2$ to $t=4$: $\frac{12-16}{4-2} = -2$ — a constant rate. The rate of change is -2 cm per hour. The negative sign says the length is falling: the candle loses 2 cm of length every hour. $\therefore$ -2 cm per hour — a negative gradient read as a rate, the falling-side twin of the ramp's positive one.



The ramp line with positive gradient one-quarter read as a rate; the candle's rate is the same idea with a negative gradient

Q17. Gradient: $\frac{7-1}{6-(-2)}=\frac{6}{8}=\frac{3}{4}$. Midpoint: $\left(\frac{-2+6}{2},\frac{1+7}{2}\right)=(2,4)$. Distance: horizontal 8, vertical 6; $d^2=8^2+6^2=100$, $d=10$. ∴ Gradient $\frac{3}{4}$, midpoint $(2,4)$, length 10 — all three tools of this subtopic on one segment.

Q18. AB: horizontal $5-(-1)=6$, vertical 0: $AB=6$. AC: horizontal $2-(-1)=3$, vertical $6-2=4$: $AC^2=9+16=25$, $AC=5$. BC: horizontal $5-2=3$, vertical $6-2=4$: $BC^2=9+16=25$, $BC=5$. Two sides are equal, $AC=BC=5$, while $AB=6$ is different. ∴ The triangle is isosceles, with two sides of 5 and one of 6.

Test

Mathematics Level 9 — Chapter test T2

Chapter 2 · Algebra: linear relationships and the Cartesian plane

Codes assessed: `VC2M9A03` · `VC2M9A04`

Name: ______________________________ Date: ______________

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 9 achievement standard, not against a mark on this paper.

Total marks	16
Time	21 minutes — 5 minutes reading time, then 16 minutes working
Structure	Two parts. **Part A is technology-free. Part B is technology-active.**
Equipment	A ruler. For Part B, the digital tool your teacher nominates (a graphing tool or a spreadsheet) and a scientific calculator.
Technology rules	Part A: no calculator and no digital tool — exact values and by-hand working only. Part B: a scientific calculator and a graphing tool or spreadsheet, as nominated by your teacher. **No CAS.**

Attempt every question. Show your working — working earns marks even when a final number slips. Where a question says *check*, substitute your answer back into the original statement. Write one step per line.

Part A — Technology-free (10 marks)

Question 1 (2A) — 5 marks · technology-free

(a) Solve $4x-9=23$. **[1]**

(b) Solve $5(x-3)=3x+7$. Check your answer by substitution. **[2]**

(c) The line $2x+y=8$ crosses the y-axis and the x-axis.

i. Find the coordinates of both intercepts.
ii. Use the two intercepts to sketch the line.

[2]

Question 2 (2B) — 5 marks · technology-free

(a) Find the gradient of the segment joining $A(1,3)$ and $B(5,11)$. **[1]**

(b) Find the midpoint of the segment joining $(2,1)$ and $(8,7)$. **[1]**

(c) Find the exact distance between $(0,0)$ and $(2,3)$. Leave your answer as a surd (an exact square root) — do not round. **[1]**

(d) A line has gradient 2. Write down the gradient of a line parallel to it, and the gradient of a line perpendicular to it. **[2]**

Part B — Technology-active (6 marks)

Question 3 (2A) — 5 marks · technology-active

The Great Ocean Road runs about 243 km along the Victorian coast, from Torquay to Allansford.

Source: Visit Victoria, *Great Ocean Road*, 2026. Retrieved 23 August 2026.

SCENARIO: a car leaves Torquay and travels along the road at a steady 60 km/h. Let d be the distance travelled, in km, after t hours.

(a) Write a rule connecting d and t. **[1]**

(b) Copy and complete a table of values for $t = 0, 1, 2, 3$. **[1]**

(c) Use a graphing tool to draw the graph of your rule, for t from 0 to 4. **[1]**

(d) Use your graph to find the distance travelled after 2.5 hours, then confirm your reading using the rule. **[1]**

(e) The road is 243 km long. Find how long it takes the car to travel the whole road at 60 km/h. Give your answer in hours and minutes. **[1]**

Question 4 (2B) — 1 mark · technology-active

(a) Find the distance between $P(-3, 5)$ and $Q(4, -2)$. Give your answer correct to 2 decimal places. **[1]**

End of test — total 16 marks

Solutions

Teacher-facing. Test T2 is a classroom paper, not an exam: 21 minutes (1 minute per mark plus 5 minutes reading). Part A is technology-free; Part B is technology-active with a scientific calculator and a graphing tool or spreadsheet as nominated by the teacher. No CAS. Every value on this paper was verified with sympy before writing (R7).

Coverage and mark map

CD	Subtopic	Questions	Part A	Part B	Marks
`VC2M9A03`	2A Sketching linear graphs from 2 points, and solving linear equations	Q1, Q3	5	5	10
`VC2M9A04`	2B Gradient, midpoint and distance on the Cartesian plane	Q2, Q4	5	1	6
	Total		**10**	**6**	**16**

Per the §7 marking rule, the extended subtopic 2A scores 10 marks and the standard subtopic 2B scores 6 marks. The §7 multi-part question in a real Victorian context is Q3 (5 marks, Great Ocean Road). Multiple choice is 0 of 16: §7 reserves MC for genuine selection tasks (6B, 7A) and none arises in Chapter 2.

Achievement-standard anchor carried by both CDs (AS 4, teacher/audit metadata): *They find the distance between 2 points on the Cartesian plane, sketch linear graphs and find the gradient and midpoint of a line segment.*

Question 1

(a) Undo the operations in reverse order — add 9, then divide by 4:

$4x-9=23$, so $4x=32$, so $x=8$.

Check: $4\times8-9=32-9=23$ ✓ $\therefore$ $x=8$.

(b) Expand the bracket first, then gather the unknown terms on one side:

$5(x-3)=3x+7$ $5x-15=3x+7$ (the 5 multiplies both x and -3) $2x-15=7$ (subtract $3x$ from both sides) $2x=22$ (add 15 to both sides) $x=11$ (divide by 2)

Check: left side $5(11-3)=5\times8=40$; right side $3\times11+7=33+7=40$. Both sides return 40 ✓ $\therefore$ $x=11$.

(c) An intercept is a crossing with an axis. The y-intercept has $x=0$; the x-intercept has $y=0$.

i. Put $x=0$: $2\times0+y=8$, so $y=8$ — the point $(0,8)$. Put $y=0$: $2x+0=8$, so $2x=8$ and $x=4$ — the point $(4,0)$. $\therefore$ The intercepts are $(0,8)$ and $(4,0)$.

ii. Plot $(0,8)$ and $(4,0)$ and join them with a straight edge. The line falls from left to right. A check point in between: $x=2$ gives $2\times2+y=8$, so $y=4$; $(2,4)$ sits on the line ✓

Question 2

(a) Gradient is the rise over the run, $\dfrac{y_2-y_1}{x_2-x_1}$:

$m=\dfrac{11-3}{5-1}=\dfrac{8}{4}=2$ $\therefore$ **gradient** $=2$.

(b) The midpoint averages the x-coordinates and the y-coordinates:

$\left(\frac{2+8}{2}, \frac{1+7}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$ ∴ **midpoint** $= (5, 4)$.

(c) The horizontal distance is $2 - 0 = 2$ and the vertical distance is $3 - 0 = 3$. Pythagoras' theorem gives the distance:

$d^2 = 2^2 + 3^2 = 4 + 9 = 13$, so $d = \sqrt{13}$.

$\sqrt{13}$ is not a perfect square, so it stays as a surd. ∴ $d = \sqrt{13}$.

(d) Parallel lines have equal gradients, and perpendicular gradients multiply to -1.

A line parallel to gradient 2 has gradient 2. A line perpendicular to gradient 2 has gradient $-\frac{1}{2}$, since $2 \times \left(-\frac{1}{2}\right) = -1$ ✓ ∴ **parallel** 2**; perpendicular** $-\frac{1}{2}$.

Question 3

(a) Distance is speed multiplied by time, and the speed is the steady 60 km/h:

∴ $d = 60t$.

(b) Substitute each t into $d = 60t$:

t (hours)	0	1	2	3
d (km)	0	60	120	180

(c) In the graphing tool, enter the rule $d = 60t$ and draw it for t from 0 to 4. The graph is a straight line through the origin, passing through $(1, 60)$, $(2, 120)$, $(3, 180)$ and $(4, 240)$. The steady rise of 60 km for every 1 hour is the gradient 60 — the car's constant speed.

(d) From the graph, the point above $t = 2.5$ sits at $d = 150$. Confirm with the rule: $d = 60 \times 2.5 = 150$ ✓ ∴ 150 **km**.

(e) Set $d = 243$ in the rule and solve for t:

$60t = 243$ $t = \frac{243}{60} = 4.05$ hours.

0.05 of an hour is $0.05 \times 60 = 3$ minutes. ∴ **4 hours 3 minutes**.

Question 4

(a) The horizontal distance is $4 - (-3) = 7$ and the vertical distance is $-2 - 5 = -7$. Pythagoras' theorem gives the distance:

$d^2 = 7^2 + (-7)^2 = 49 + 49 = 98$ $d = \sqrt{98} \approx 9.899\ldots$

Rounded to 2 decimal places, ∴ $d \approx 9.90$.

Answer key

Q	Part	Answer	Marks
1	(a)	$x = 8$	1
1	(b)	$x = 11$	2
1	(c)	i $(0, 8)$ and $(4, 0)$; ii line through both intercepts	2
2	(a)	2	1
2	(b)	$(5, 4)$	1

Q	Part	Answer	Marks
2	(c)	$\sqrt{13}$	1
2	(d)	parallel 2; perpendicular $-\frac{1}{2}$	2
3	(a)	$d = 60t$	1
3	(b)	$0, 60, 120, 180$	1
3	(c)	straight line through the origin, gradient 60	1
3	(d)	150 km	1
3	(e)	4 hours 3 minutes	1
4	(a)	9.90	1
		Total	**16**