Mathematics Level 9

Chapter 1 — Number and Algebra: real numbers, exponents and algebraic expressions · 3 subtopics

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Contents

The real number system: rational and irrational numbers, exact and approximate

1A · Chapter 1 — Number and Algebra: real numbers, exponents and algebraic expressions · Level 9

Content description `VC2M9N01` (word for word): *recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers with and without using digital tools*

Achievement standard anchor (AS 1): *Students recognise and use rational and irrational numbers to solve problems.*

Elaboration ids for linkage: `VC2M9N01.E01`, `VC2M9N01.E02`, `VC2M9N01.E03`, `VC2M9N01.E04`, `VC2M9N01.E05`

Prerequisite pointer: Level 8 number (VC2M8N01, irrational numbers including π and √□; VC2M8N03, fractions and their terminating or recurring decimals) and Level 8 Pythagoras (VC2M8M06) as a tool. These are rehearsed here as tools, not re-taught.

DIGITAL TOOL — this subtopic runs two paths, with and without a digital tool. Your teacher will nominate the tool. Where a decimal is needed, the with-tool path says what to key in and what the display shows; the without-tool path traps the value between whole numbers or keeps it in exact form. Both paths are assessed, and on both paths the answer begins life in exact form.

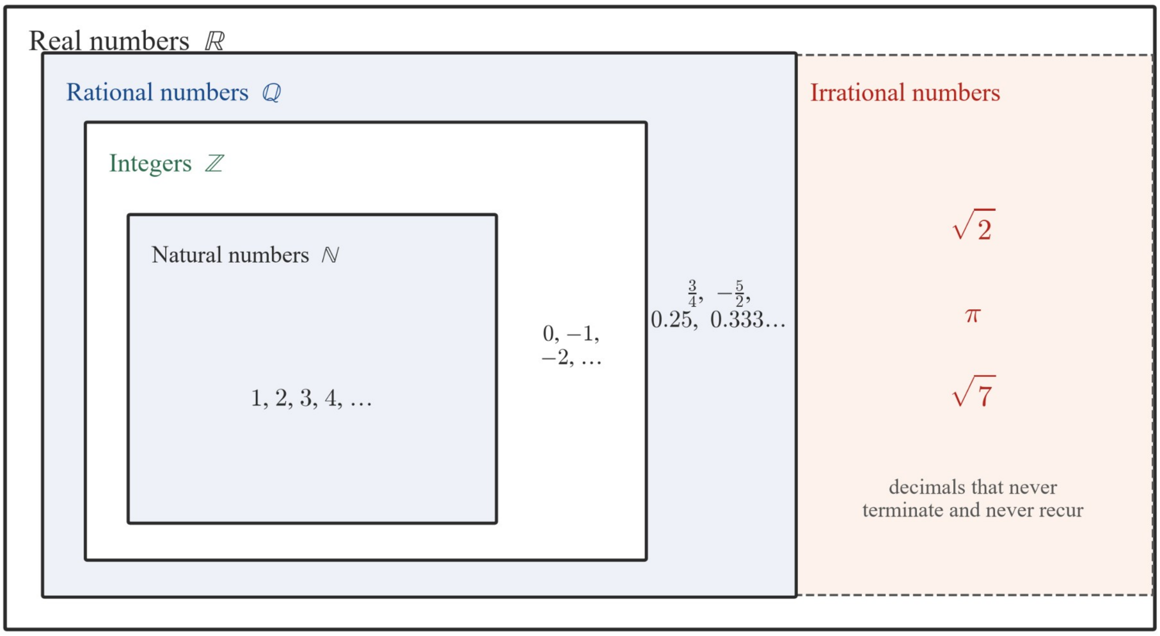
Note on figures: every number line in this section is drawn to scale from computed values ($\sqrt{2}=1.414214\ldots$, $\pi=3.141593\ldots$, $\frac{22}{7}=3.142857\ldots$), every bar carries its verified signed total, and the circle is a true circle of the stated diameter (R7); every context is a generic one, so no real data ships and no source line is owed (R1, R2).

Theory

The families of numbers, nested inside the reals

A **rational number** is any number that can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$. An **irrational number** is a number that cannot be written in that form. Every number on the **real number line** — the line that holds all of these numbers, the **real numbers** — is one or the other, never both.

The families nest. The counting numbers $1, 2, 3, \ldots$ are the **natural numbers**; add $0, -1, -2, \ldots$ and you have the **integers**; add every fraction $\frac{a}{b}$ and you have the rational numbers. Each family sits entirely inside the next one, and the irrational numbers fill whatever is left of the real numbers.

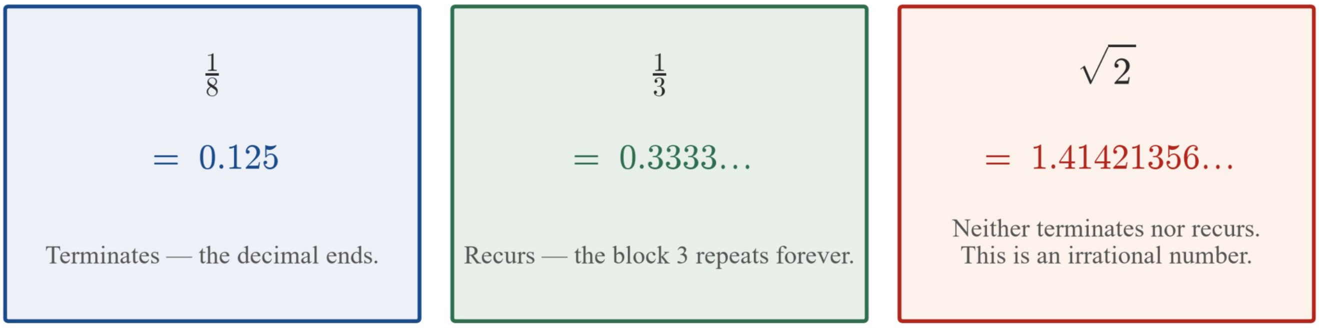


Every real number is either rational or irrational — the two groups never overlap. Each smaller family sits entirely inside the one around it.

Nested boxes: the natural numbers inside the integers, the integers inside the rational numbers, and the rationals inside the real numbers, with the irrational numbers filling the rest of the real box and example members listed in each region

Terminating, recurring, or neither

Level 8 (VC2M8N03) says the test in decimal form: write a rational number as a decimal and the decimal either **terminates** (it ends) or **recurs** (a block of digits repeats forever). An irrational number does neither — its decimal goes on forever with no repeating block. The three cards show the three behaviours.



A rational number always gives a decimal that terminates or recurs. An irrational number gives neither.

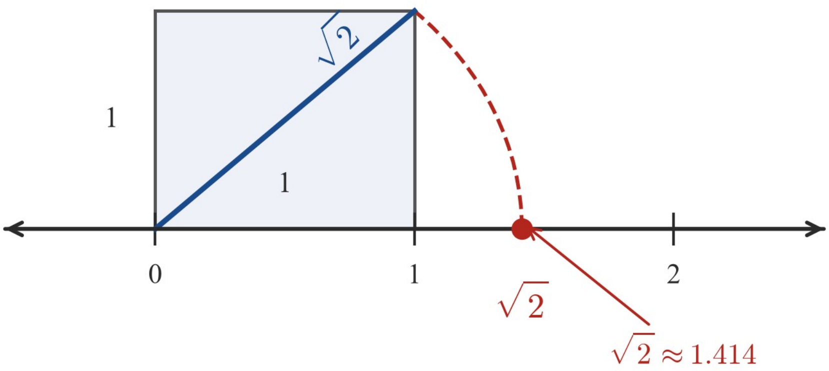
Three cards: one eighth equals 0.125 and terminates; one third equals 0.3333 recurring, with the block 3 repeating forever; the square root of 2 equals 1.41421356 continuing, which neither terminates nor recurs, so it is an irrational number

The test works both ways: a decimal that terminates or recurs can always be rebuilt as a fraction of integers, so “terminates or recurs” and “rational” describe exactly the same set of numbers.

The real number line holds them all

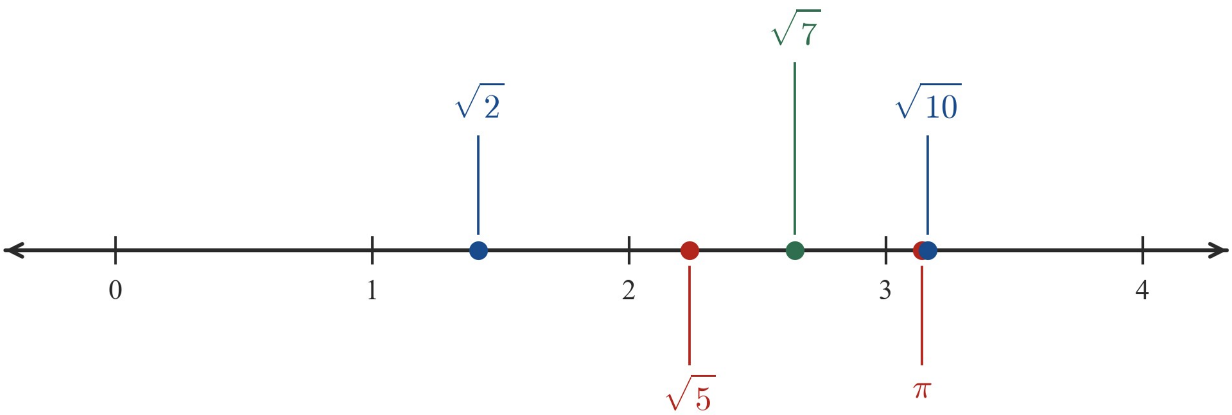
The irrationals are not an abstract leftover — each one is a point on the line with an exact position of its own. Level 8 Pythagoras builds them: a square of side 1 has diagonal d with $d^2 = 1^2 + 1^2 = 2$, so $d = \sqrt{2}$. Swing that length down from 0 with an arc and it lands on the line between 1 and 2 — that point **is** $\sqrt{2}$.

The diagonal of a one-unit square has length $\sqrt{2}$.
Swing an arc of that radius down to the number line.



A unit square standing on the number line, its diagonal labelled the square root of 2, and a dashed arc swinging that length down to mark a point on the line at about 1.414, between 1 and 2

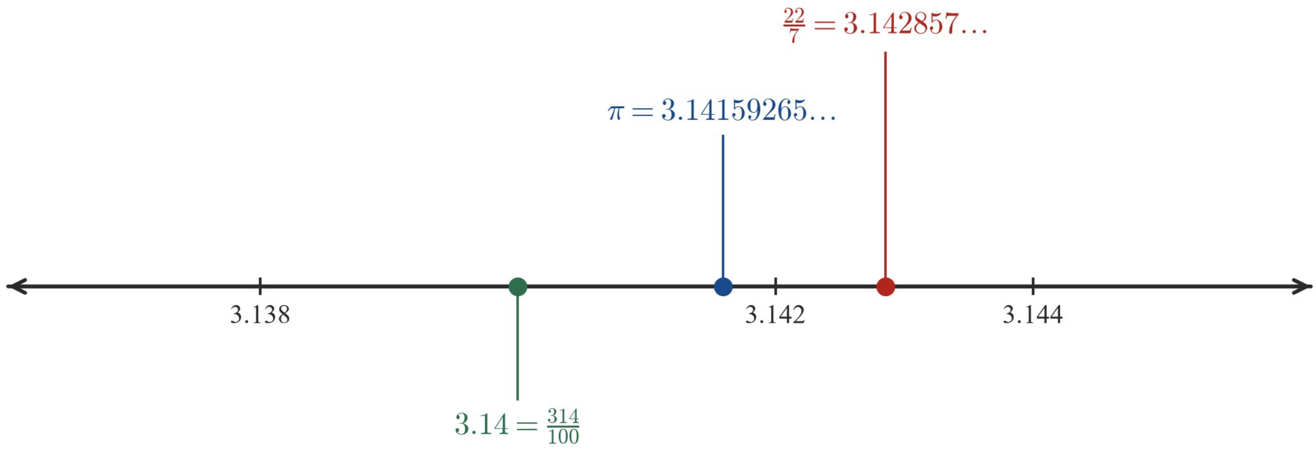
The same move places every square root of a non-square: $\sqrt{5}$ from a 1×2 rectangle ($1^2 + 2^2 = 5$), $\sqrt{10}$ from a 1×3 rectangle, $\sqrt{13}$ from a 2×3 rectangle, and so on. Without a tool, trap a root between the whole numbers either side of it using the perfect squares: $4 < 7 < 9$ means $\sqrt{4} < \sqrt{7} < \sqrt{9}$, that is $2 < \sqrt{7} < 3$. With a tool, the decimal confirms the trap: $\sqrt{7} = 2.645751\ldots \approx 2.646$ (3 d.p.).



Each point sits at its exact value: $\pi \approx 3.14159$ and $\sqrt{10} \approx 3.16228$ are close together, but they are different numbers.

The real number line from 0 to 4 with points marked at their computed positions: the square root of 2 at about 1.414, the square root of 5 at about 2.236, the square root of 7 at about 2.646, pi at about 3.142 and the square root of 10 at about 3.162

Look closely at the two points near 3.14: π and $\sqrt{10}$ are close together but they are different numbers, and both are irrational. So are the rationals that imitate π. Zoom in on the line around 3.14 and three separate points appear: $3.14 = \frac{314}{100}$, then $\pi = 3.14159265\ldots$, then $\frac{22}{7} = 3.142857\ldots$. The fraction $\frac{22}{7}$ differs from π by only about 0.0013 — a handy rational stand-in, but a stand-in, never equal.

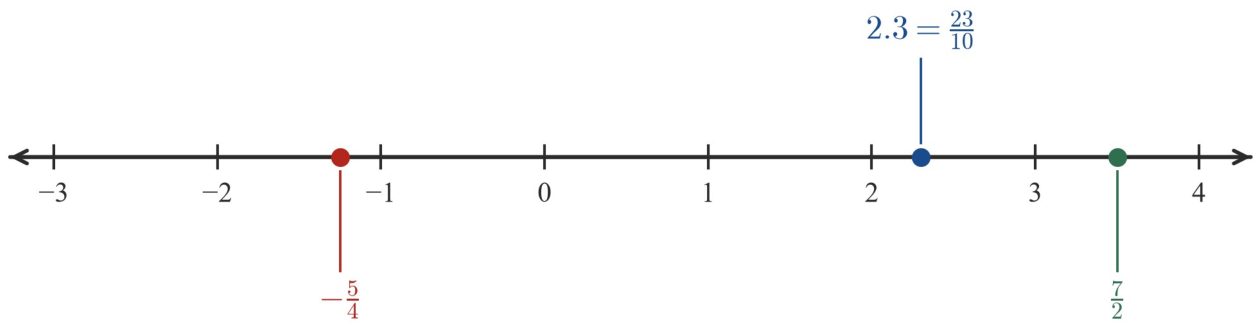


Rational numbers such as 3.14 and $\frac{22}{7}$ sit close to π, but neither of them is equal to π.

A zoomed number line from 3.136 to 3.1462 showing three separate points: 3.14 written as 314 over 100 below the line, pi at 3.14159265 continuing above the line, and 22 sevenths at 3.142857 continuing higher still, with pi between the two rationals

Rationals sit exactly where their fraction says

A fraction names a precise point: $-\frac{5}{4} = -1.25$ is one quarter of the way from -1 to -2; $2.3 = \frac{23}{10}$; $\frac{7}{2} = 3.5$. Between any two rationals, however close, there is always an irrational point as well — the line has no gaps, which is exactly what “the real number system” means.

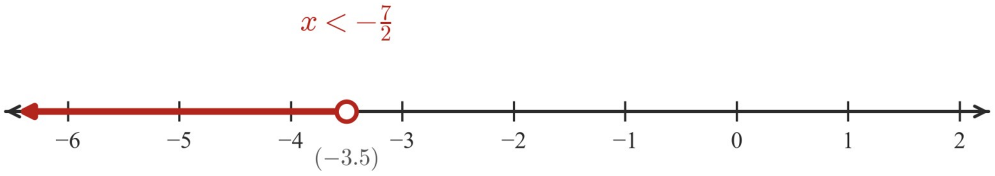


A fraction tells you exactly where the point sits: $-\frac{5}{4} = -1.25$ is one quarter of the way from -1 to -2.

The number line from minus 3 to 4 with three points marked: minus five quarters at minus 1.25 below the line, 2.3 written as 23 over 10 above the line, and seven halves at 3.5 below the line

Intervals on the real number line

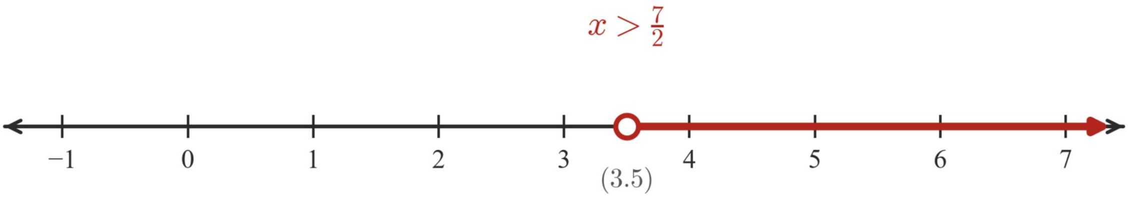
Because the line has no gaps, a whole stretch of it — an **interval** — can be the answer to a question. Level 8 solves the inequality; this subtopic reads the solution as a set of real numbers. The solutions of $2x + 7 < 0$ are every real number to the left of $-\frac{7}{2}$: rational members like -4, irrational members like $-\sqrt{5}$, and the open circle says -3.5 itself is not included.



Every number to the left of -3.5 is a solution. The open circle shows that -3.5 itself is not included.

The number line with an open circle at minus 3.5 and a solid red ray running left from it, labelled x is less than minus seven halves; every number to the left of minus 3.5 is a solution

The solutions of $3x-4.5>6$ are every real number to the right of $\frac{7}{2}$ — again a mix of rational members like 4 and irrational members like $\sqrt{13}\approx 3.606$, while $\pi \approx 3.142$ is **not** a member. An interval of real numbers always contains both kinds.

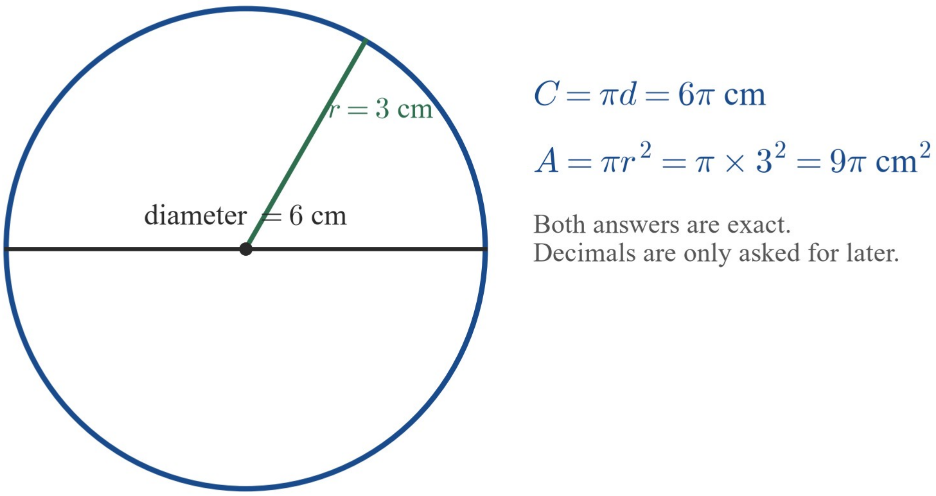


Every number to the right of 3.5 is a solution — rational ones like 4 and irrational ones like $\sqrt{13}$.

The number line with an open circle at 3.5 and a solid red ray running right from it, labelled x is greater than seven halves; every number to the right of 3.5 is a solution, rational ones like 4 and irrational ones like the square root of 13

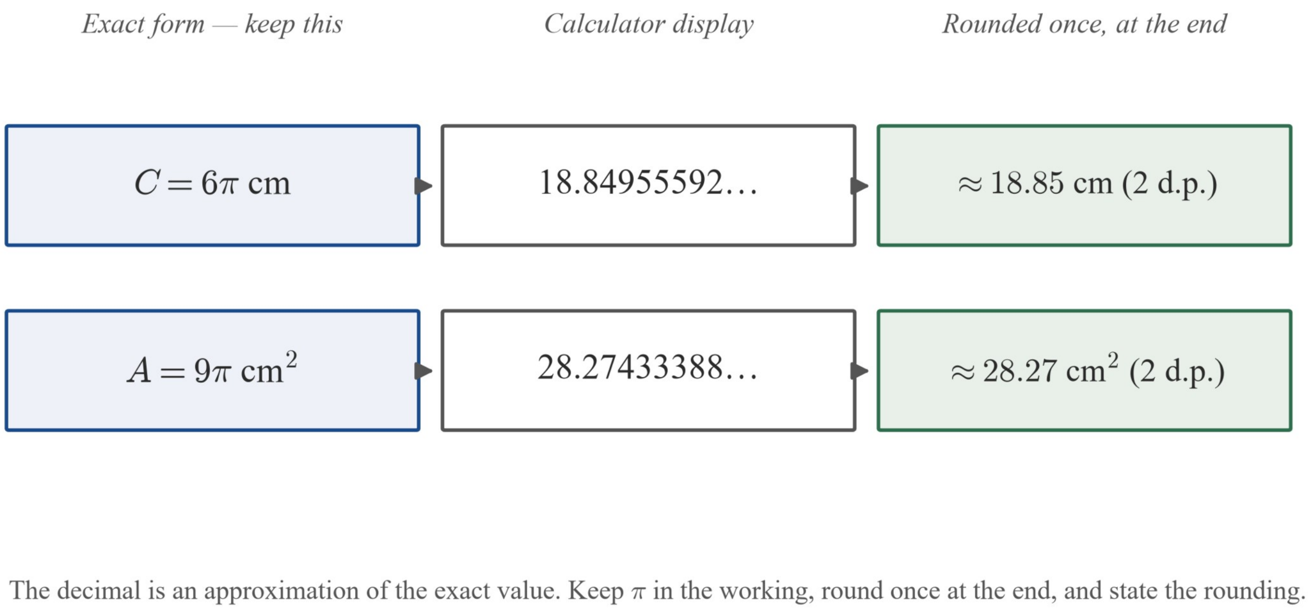
Exact first, approximate last

An **exact** answer keeps its symbols; an **approximation** replaces them with a rounded decimal. For a circle of diameter 6 cm the exact answers are $C=\pi d=6\pi$ cm and $A=\pi r^2=\pi\times 3^2=9\pi$ cm². Nothing is lost in those forms — they are the answers.



A true circle with diameter 6 cm and radius 3 cm marked, beside the exact results C equals pi d equals 6 pi cm and A equals pi r squared equals 9 pi square cm

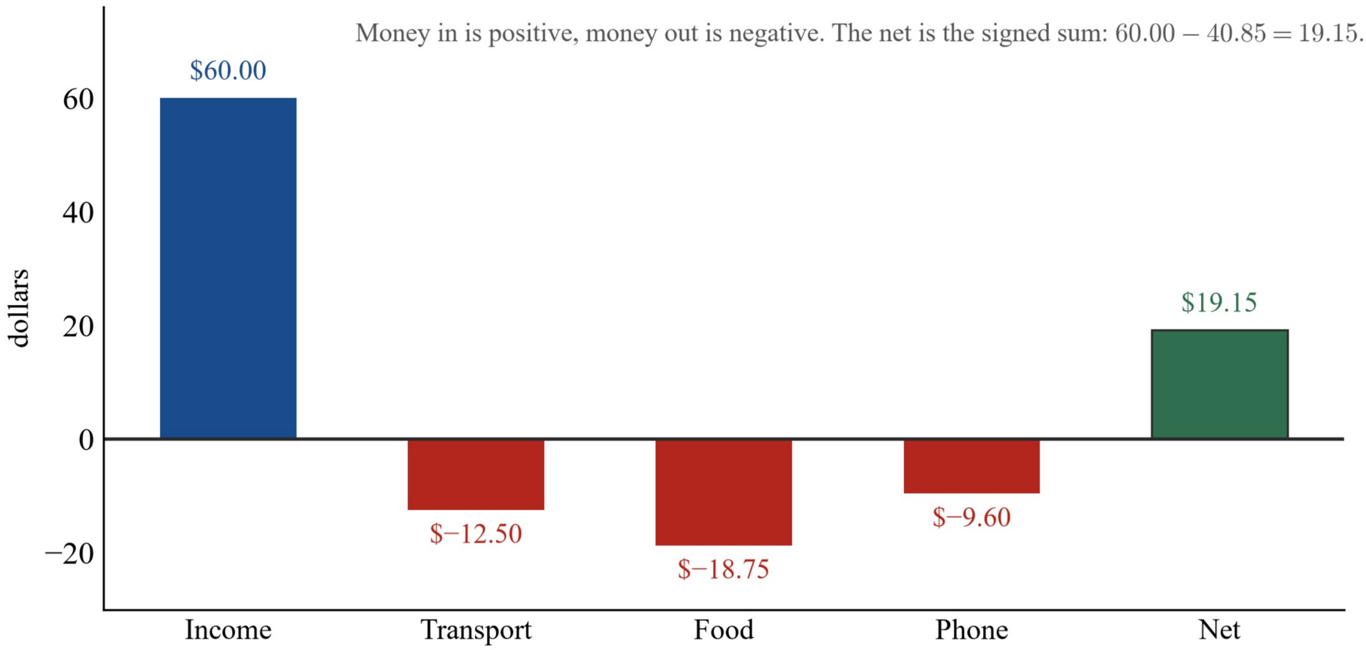
The habit this subtopic builds: keep π symbolic until the last line, then round once and state the rounding. With a tool, key in $6 \times \pi$ and read $18.84955592\ldots$, then ≈ 18.85 cm (2 d.p.). Without a tool, 6π cm stands as the answer, or estimate with $\pi \approx 3.14159$ and still round once, at the end.



Two rows of three boxes: exact form 6 pi cm and 9 pi square cm, the calculator displays 18.84955592 continuing and 28.27433388 continuing, and the values rounded once at the end to about 18.85 cm and about 28.27 square cm, 2 decimal places

Signed rationals in the money context

Rational numbers take signs in money: income is positive, money spent is negative, and the net position is the signed sum. A generic week: $+60.00$ in, then -12.50, -18.75 and -9.60 out. The outgoings total -40.85, so the net is $60.00 - 40.85 = +19.15$ — a positive rational, in the black.



Bar chart of a weekly budget: income plus 60.00 as a tall blue bar, transport minus 12.50, food minus 18.75 and phone minus 9.60 as red bars below the zero line, and the net plus 19.15 as a green bar

One sentence to keep: **the real number line is one unbroken line of rationals and irrationals together — exact symbols name its points precisely, and decimals approximate them when a question asks.**

Worked examples

Example 1 — scaffolded

Classify each number as rational or irrational: $-6, 0.25, \frac{7}{3}, \sqrt{8}, 0.454545\ldots, \pi, \sqrt{25}$**.**

Working	Comment
$-6=\frac{-6}{1}$	A fraction of two integers — rational (and an integer).
$0.25=\frac{25}{100}=\frac{1}{4}$	The decimal terminates, and it rebuilds as a fraction — rational.
$\frac{7}{3}=2.333\ldots$	Already a fraction of integers; the decimal recurs — rational.
$\sqrt{8}$: $4<8<9$, so $2<\sqrt{8}<3$	8 is not a perfect square, so no fraction equals $\sqrt{8}$ — irrational.
$0.454545\ldots=\frac{45}{99}=\frac{5}{11}$	The block 45 repeats forever — recurs, so rational.
$\pi=3.14159265\ldots$	Neither terminates nor recurs — irrational.
$\sqrt{25}=5$	It wears a root sign but equals an integer — rational.

∴ Rational: -6, 0.25, $\frac{7}{3}$, $0.454545\ldots$ and $\sqrt{25}$. Irrational: $\sqrt{8}$ and π. The two traps are $\sqrt{25}$ (a root sign does not make a number irrational) and $0.454545\ldots$ (an endless decimal can still be rational).

Example 2 — scaffolded

A circle has diameter 6 cm. Find its circumference and its area exactly, then as decimals with your tool (2 d.p.).

Working	Comment
$r = \frac{6}{2} = 3$ cm	Radius is half the diameter.
$C = \pi d = 6\pi$ cm	Exact — keep π symbolic.
$A = \pi r^2 = \pi \times 3^2 = 9\pi$ cm²	Exact again; still no decimals.
With tool: $6 \times \pi = 18.84955592\ldots$	Read the full display; do not round yet.
$C \approx 18.85$ cm (2 d.p.)	Round once, at the end, and state the rounding.
With tool: $9 \times \pi = 28.27433388\ldots$; $A \approx 28.27$ cm² (2 d.p.)	The same habit for the area.

∴ Exactly, $C = 6\pi$ cm and $A = 9\pi$ cm²; with the tool and one rounding at the end, $C \approx 18.85$ cm and $A \approx 28.27$ cm² (2 d.p.).

Example 3 — unscaffolded

Without your tool, trap $\sqrt{10}$ between two consecutive whole numbers and construct it as a point on the number line. Then check the position with your tool.

The perfect squares either side of 10 are 9 and 16: $9 < 10 < 16$. Square roots keep the order: $\sqrt{9} < \sqrt{10} < \sqrt{16}$, so $3 < \sqrt{10} < 4$. Construct it: a 1×3 rectangle has diagonal d with $d^2 = 1^2 + 3^2 = 10$, so $d = \sqrt{10}$; swing an arc of that length down from 0 and it lands between 3 and 4. Check with tool: $\sqrt{10} = 3.162277\ldots \approx 3.162$ (3 d.p.) — inside the trap, nearer the 3 end, exactly where the arc landed.

∴ $\sqrt{10}$ is irrational and sits between 3 and 4; the rectangle-and-arc construction places it exactly, and the tool's 3.162 confirms it.

Example 4 — unscaffolded

Mia's week, in dollars: she banks 60.00, then spends 12.50 on transport, 18.75 on food and 9.60 on her phone. Write each amount as a signed rational number, find her net position, then decide what happens when a 25.00 bill arrives.

Income $+60.00$; outgoings -12.50, -18.75, -9.60. Outgoings total: $12.50 + 18.75 + 9.60 = 40.85$, so -40.85. Net: $60.00 - 40.85 = +19.15$ — in the black. The bill: $19.15 - 25.00 = -5.85$.

∴ The net after the three spends is $+19.15$; after the 25.00 bill it is -5.85, so Mia owes 5.85 — the sign of the rational number carries the whole story.

Practice questions

Scaffolded

Q1. Classify each number as rational or irrational, and where it is rational say whether it is also an integer: (i) $\sqrt{16}$, (ii) $\sqrt{11}$, (iii) 0.375, (iv) π.

Q2. Write each fraction as a decimal and say whether it terminates or recurs: (a) $\frac{3}{8}$, (b) $\frac{5}{6}$, (c) $\frac{7}{11}$. (d) Hence explain why none of them could ever equal $\sqrt{2}$.

Q3. Write each number as a decimal and state the two consecutive integers it sits between: (a) $\frac{7}{2}$, (b) $-\frac{5}{4}$, (c) 2.3.

Q4. Without your tool, trap each square root between two consecutive whole numbers: (a) $\sqrt{7}$, (b) $\sqrt{15}$, (c) $\sqrt{40}$.

Q5. The interval figure shows the solutions of $2x + 7 < 0$. (a) Read the boundary value off the figure. (b) Test $x = -4$ by substitution and say whether it is a member. (c) Do the same for $x = 0$.

Q6. A circle has diameter 8 cm. (a) Write its circumference exactly. (b) Write its area exactly. (c) With your tool, give both as decimals rounded once, to 1 d.p.

Unscaffolded

Q7. Classify each number as rational or irrational: $\sqrt{0.09}$; $2.185185\ldots$ (block 185 repeating); $0.101001000\ldots$ (one more zero each time, forever); $\frac{22}{7}$.

Q8. True or false? Give a reason for each; where false, give a **counterexample** — one case that shows the claim fails: (a) every integer is a rational number; (b) every rational number is an integer; (c) the sum of two irrational numbers can never be rational; (d) a decimal that terminates or recurs is always rational.

Q9. List these numbers from smallest to largest: $\sqrt{2}$, $-\sqrt{3}$, 0, $\frac{5}{4}$, -1.5.

Q10. Construct $\sqrt{5}$ on the number line with a rectangle and an arc: state the rectangle's sides, the Pythagoras fact, and the two whole numbers $\sqrt{5}$ falls between. Check the decimal with your tool (3 d.p.).

Q11. A circular table has diameter 1.2 m. Give its circumference and its area exactly, then with your tool to 2 d.p.

Q12. A week in one account: $+55.25$ in; -12.80, -19.95 and -8.50 out. Find the net, then the net after a further -20.50 goes out, and say in words what the final sign means.

Q13. A student writes $\sqrt{9}+\sqrt{16}=\sqrt{25}$. Show by direct calculation that this is false, and give the correct value of the left-hand side.

Q14. A circle has radius 2.35 cm, so $A=\pi\times 2.35^2=5.5225\pi$ cm². With your tool, find A to 2 d.p. keeping π exact until the last line. A student who replaced π with 3.14 at the start got 17.34 cm². Whose answer is right, and what habit went wrong?

Q15. Between which two consecutive integers does $-\sqrt{7}$ lie? Justify with the trap for $\sqrt{7}$, then check with your tool.

Q16. A small business takes $+840.50$ in a week and pays out -315.20, -278.90 and -190.45. Find the net, then the position after a -120.00 part-payment on a larger bill, in dollars and cents.

Q17. The solutions of $3x-4.5>6$ form an interval of real numbers. Find its boundary, then name one rational member and one irrational member, and test with substitution whether π is a member.

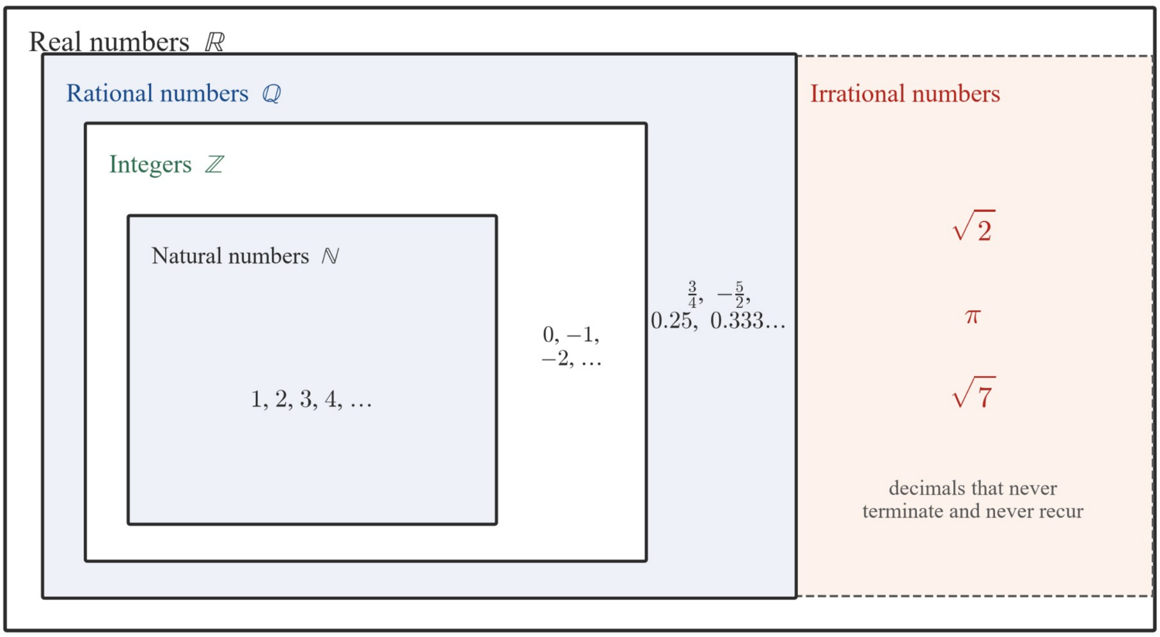
Q18. Write down two rational numbers and two irrational numbers between 3 and 4, and justify each choice in one line.

Answers

Q1 (i) rational, integer ($\sqrt{16}=4$); (ii) irrational; (iii) rational, not an integer ($0.375=\frac{375}{1000}$); (iv) irrational. **Q2** a 0.375, terminates. b $0.833\ldots$, recurs. c $0.6363\ldots$, recurs. d $\sqrt{2}$ neither terminates nor recurs, and every rational decimal does one of those. **Q3** a 3.5, between 3 and 4; b -1.25, between -2 and -1; c 2.3, between 2 and 3. **Q4** a $2<\sqrt{7}<3$; b $3<\sqrt{15}<4$; c $6<\sqrt{40}<7$. **Q5** a -3.5. b Member: $2(-4)+7=-1<0$. c Not a member: $2(0)+7=7>0$. **Q6** a 8π cm. b 16π cm². c ≈ 25.1 cm and ≈ 50.3 cm² (1 d.p.). **Q7** $\sqrt{0.09}=0.3$ rational; $2.185185\ldots$ rational (recurs); $0.101001000\ldots$ irrational (neither terminates nor recurs); $\frac{22}{7}$ rational (a fraction of integers) — close to π, not equal to it. **Q8** a True, $n=\frac{n}{1}$. b False, $\frac{1}{2}$ is rational and not an integer. c False, $\sqrt{2}+(-\sqrt{2})=0$. d True (Level 8: it rebuilds as a fraction). **Q9** $-\sqrt{3}<-1.5<0<\frac{5}{4}<\sqrt{2}$. **Q10** Sides 1 and 2; $1^2+2^2=5$ so the diagonal is $\sqrt{5}$; $2<\sqrt{5}<3$; tool: ≈ 2.236 (3 d.p.). **Q11** $C=1.2\pi\approx 3.77$ m; $A=0.36\pi\approx 1.13$ m² (2 d.p.). **Q12** Net $+14.00$; after the extra out, -6.50 — the account is 6.50 in the red. **Q13** False: $\sqrt{9}+\sqrt{16}=3+4=7$, while $\sqrt{25}=5$; the left-hand side is 7. **Q14** 17.35 cm²; rounding π early gave 17.34 — keep π exact until the last line and round once. **Q15** Between -3 and -2. **Q16** Net $+55.95$; after the part-payment, -64.05. **Q17** Boundary 3.5 ($x>3.5$); rational member 4; irrational member $\sqrt{13}\approx 3.606$; $\pi\approx 3.142$ is not a member. **Q18** For example rationals $\frac{13}{4}$ and $\frac{7}{2}$; irrationals $\sqrt{10}$ and π.

Fully-worked solutions

Q1. (i) $\sqrt{16}=4$: an integer, and every integer n is $\frac{n}{1}$ — rational. (ii) 11 is not a perfect square ($9<11<16$), so $\sqrt{11}$ is irrational. (iii) $0.375=\frac{375}{1000}=\frac{3}{8}$: a fraction of integers — rational, not an integer. (iv) π neither terminates nor recurs — irrational. ∴ (i) and (iii) rational ((i) an integer), (ii) and (iv) irrational — the families figure shows where each sits.



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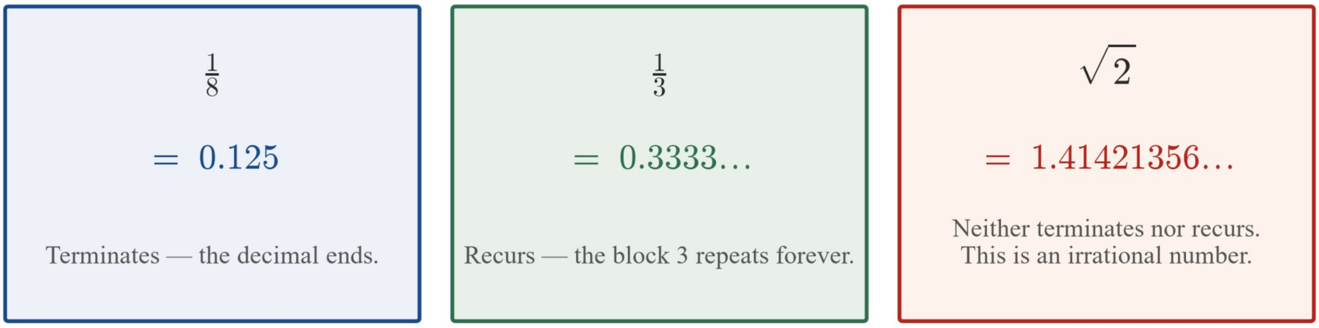
Nested boxes: the natural numbers inside the integers, the integers inside the rational numbers, and the rationals inside the real numbers, with the irrational numbers filling the rest of the real box

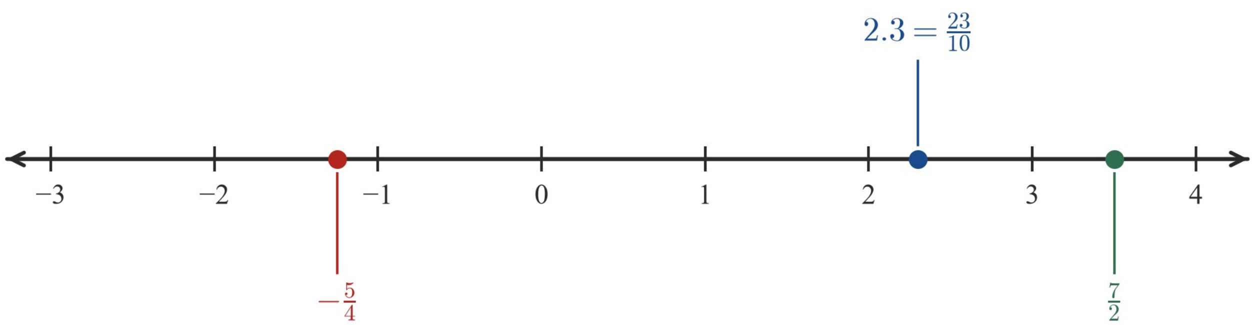
Q2. a $3 \div 8 = 0.375$ — the division ends, so it terminates. b $5 \div 6 = 0.8333\ldots$ — the block 3 repeats, so it recurs. c $7 \div 11 = 0.636363\ldots$ — the block 63 repeats, so it recurs. d Every rational decimal terminates or recurs (Level 8). The decimal of $\sqrt{2}$ does neither, so no terminating or recurring decimal can equal it. $\therefore$ $\frac{3}{8}$ terminates; $\frac{5}{6}$ and $\frac{7}{11}$ recur; $\sqrt{2}$ is in the third card of the decimal figure.



A rational number always gives a decimal that terminates or recurs. An irrational number gives neither.

Three cards: one eighth equals 0.125 and terminates; one third equals 0.3333 recurring; the square root of 2 equals 1.41421356 continuing and neither terminates nor recurs

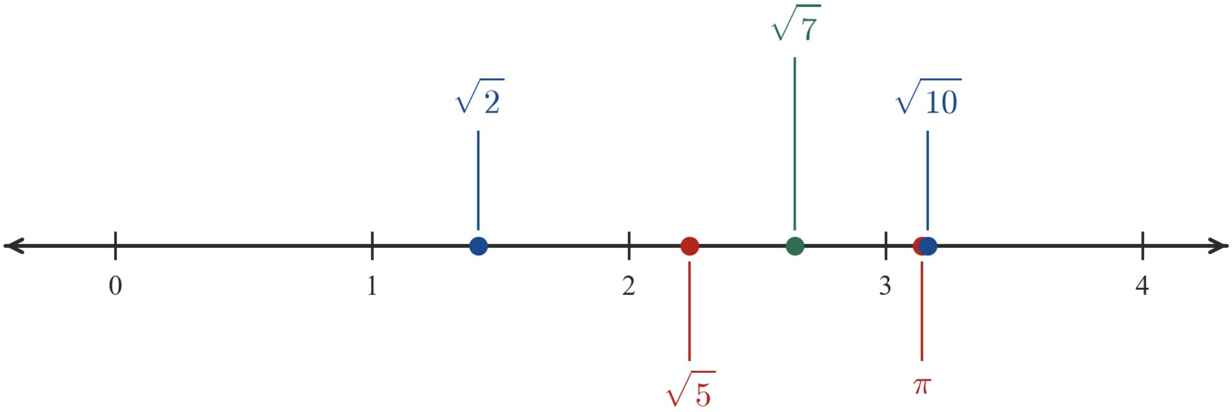
Q3. a $\frac{7}{2} = 3.5$: halfway between 3 and 4. b $-\frac{5}{4} = -1.25$: one quarter of the way from -1 down to -2, so between -2 and -1. c $2.3 = \frac{23}{10}$: between 2 and 3. $\therefore$ 3.5, -1.25, 2.3 — exactly the three points of the rationals-on-the-line figure.



A fraction tells you exactly where the point sits: $-\frac{5}{4} = -1.25$ is one quarter of the way from -1 to -2.

The number line from minus 3 to 4 with points at minus five quarters, 2.3 written as 23 over 10, and seven halves

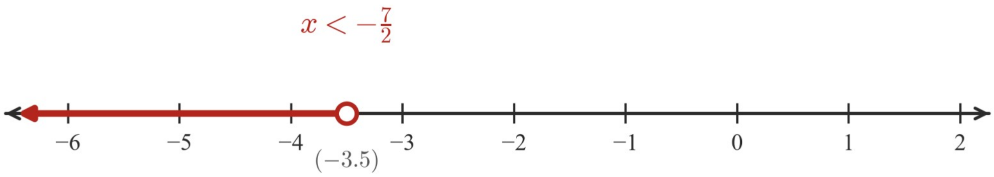
Q4. a $4 < 7 < 9$, so $\sqrt{4} < \sqrt{7} < \sqrt{9}$: $2 < \sqrt{7} < 3$. b $9 < 15 < 16$: $3 < \sqrt{15} < 4$. c $36 < 40 < 49$: $6 < \sqrt{40} < 7$. $\therefore$ The traps are 2–3, 3–4 and 6–7; on the roots figure, $\sqrt{7} \approx 2.646$ and $\sqrt{10} \approx 3.162$ sit inside theirs.



Each point sits at its exact value: $\pi \approx 3.14159$ and $\sqrt{10} \approx 3.16228$ are close together, but they are different numbers.

The real number line from 0 to 4 with points marked at the square roots of 2, 5, 7 and 10 and at pi, each at its computed position

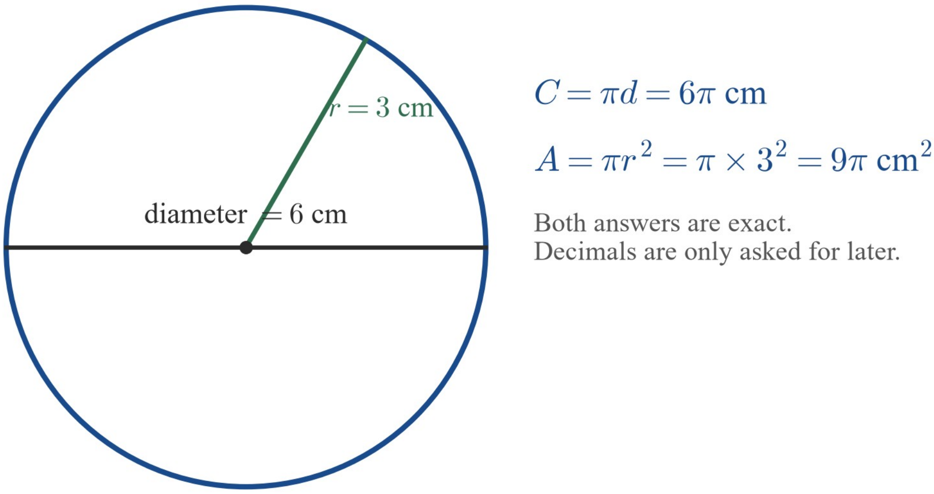
Q5. a The open circle sits at $-\dfrac{7}{2} = -3.5$, the boundary of the interval. b $x = -4$: $2(-4)+7 = -8+7 = -1 < 0$ — true, so -4 is a member (it lies left of -3.5). c $x = 0$: $2(0)+7 = 7 > 0$ — false, so 0 is not a member. ∴ Boundary -3.5; -4 in, 0 out — exactly what the red ray and the open circle show.



Every number to the left of −3.5 is a solution. The open circle shows that −3.5 itself is not included.

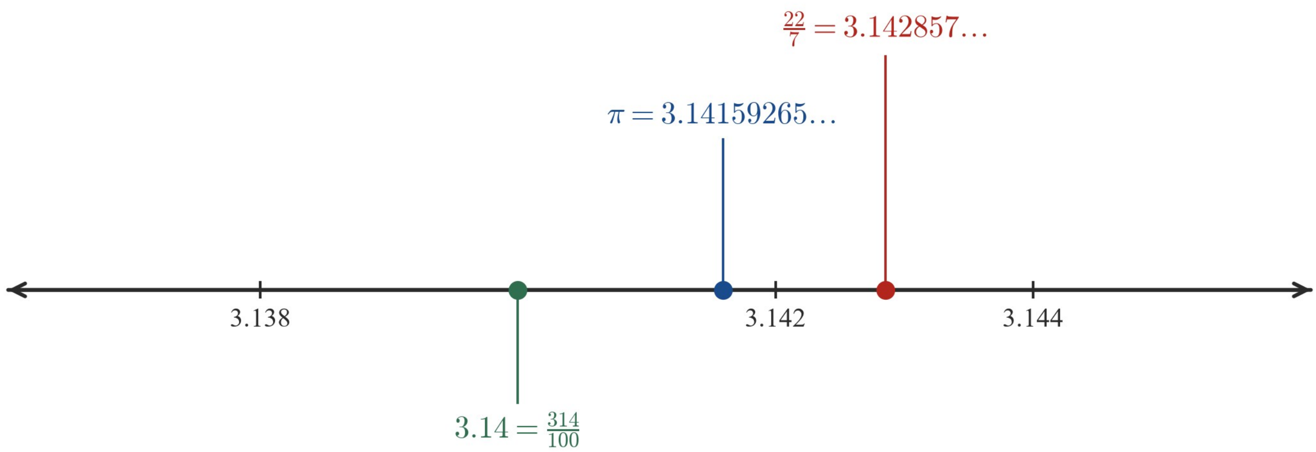
The number line with an open circle at minus 3.5 and a solid red ray running left, showing every solution of 2x plus 7 is less than 0

Q6. $r = \dfrac{8}{2} = 4$ cm. $C = \pi d = 8\pi$ cm; $A = \pi r^2 = \pi \times 4^2 = 16\pi$ cm² — exact. With tool: $8 \times \pi = 25.132741\ldots \approx 25.1$ cm; $16 \times \pi = 50.265482\ldots \approx 50.3$ cm² (1 d.p.). ∴ $C = 8\pi \approx 25.1$ cm and $A = 16\pi \approx 50.3$ cm² (1 d.p.) — the same circle figure's habit with $d = 8$.



A true circle with its diameter and radius marked, beside the exact results C equals pi d and A equals pi r squared

Q7. $\sqrt{0.09}$: $0.3\times0.3=0.09$, so $\sqrt{0.09}=0.3$ — a terminating decimal, rational. $2.185185\ldots$: the block 185 repeats forever — recurs, so rational (it is $2\frac{185}{999}$). $0.101001000\ldots$: the gaps keep growing, so no block ever repeats and it never terminates — irrational. $\frac{22}{7}$: a fraction of two integers — rational by definition, however close its decimal $3.142857\ldots$ sits to π. $\therefore$ Rational: $\sqrt{0.09}$, $2.185185\ldots$, $\frac{22}{7}$; irrational: $0.101001000\ldots$ — the pi-zoom figure shows why $\frac{22}{7}$, however close, stays on the rational side.

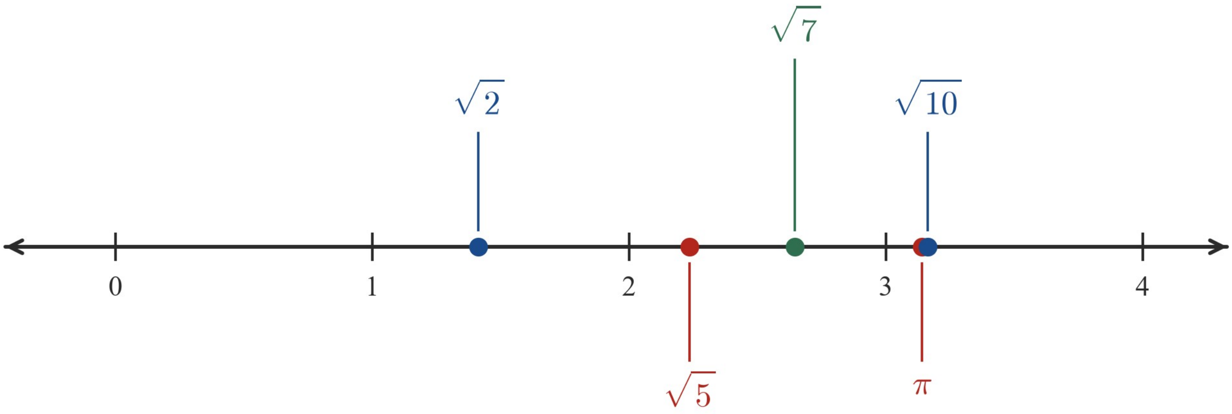


Rational numbers such as 3.14 and $\frac{22}{7}$ sit close to π, but neither of them is equal to π.

A zoomed number line from 3.136 to 3.1462 showing 3.14, pi and 22 sevenths as three separate points, with pi between the two rationals

Q8. a True: any integer n is $\frac{n}{1}$, a fraction of integers. b False: $\frac{1}{2}$ is rational but not an integer — the integer box sits strictly inside the rational box. c False: $\sqrt{2}$ and $-\sqrt{2}$ are both irrational, yet $\sqrt{2}+(-\sqrt{2})=0$, a rational. d True: a terminating or recurring decimal rebuilds as a fraction of integers (Level 8), so it is rational. $\therefore$ a and d true; b and c false, with $\frac{1}{2}$ and 0 as the counterexamples.

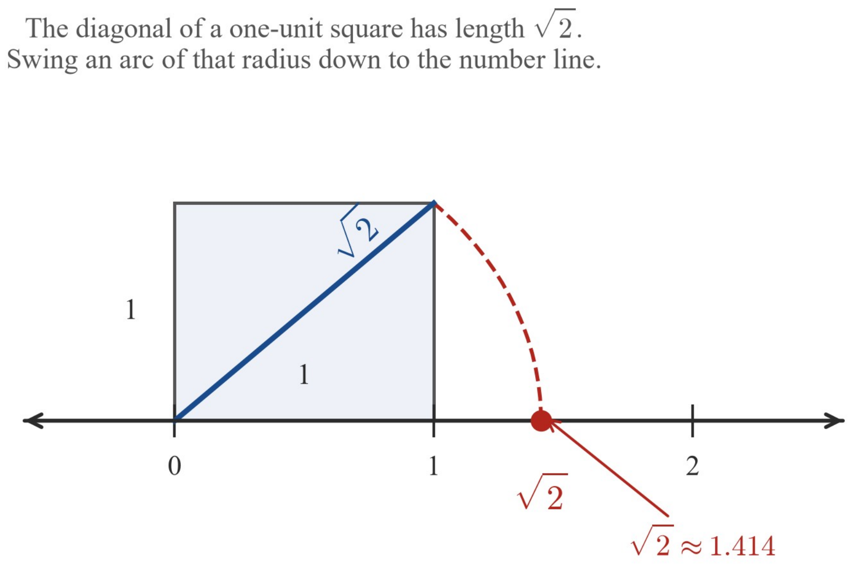
Q9. $-\sqrt{3} \approx -1.732$ (since $1 < 3 < 4$, $1 < \sqrt{3} < 2$, mirrored negative). -1.5; 0; $\frac{5}{4} = 1.25$; $\sqrt{2} \approx 1.414$. Compare: $-1.732 < -1.5 < 0 < 1.25 < 1.414$. $\therefore$ $-\sqrt{3} < -1.5 < 0 < \frac{5}{4} < \sqrt{2}$ — the order the roots figure shows for the two irrational endpoints.



Each point sits at its exact value: $\pi \approx 3.14159$ and $\sqrt{10} \approx 3.16228$ are close together, but they are different numbers.

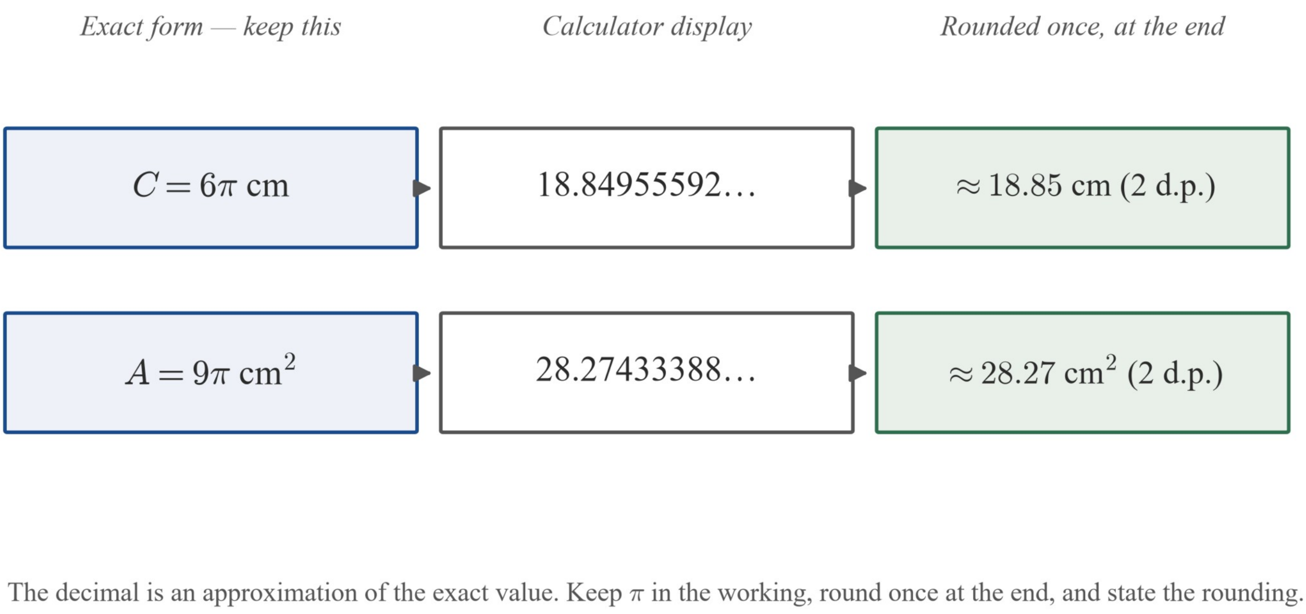
The real number line from 0 to 4 with the square root of 2 marked at about 1.414 and the square root of 5 at about 2.236, among the other marked points

Q10. Take a 1×2 rectangle: diagonal d satisfies $d^2 = 1^2 + 2^2 = 5$, so $d = \sqrt{5}$. Swing an arc of that length from 0 onto the number line — the point it marks is $\sqrt{5}$. Trap: $4 < 5 < 9$, so $2 < \sqrt{5} < 3$; the arc lands between 2 and 3. Check with tool: $\sqrt{5} = 2.236067\ldots \approx 2.236$ (3 d.p.) — inside the trap, where the arc landed. $\therefore$ Sides 1 and 2, diagonal $\sqrt{5}$, between 2 and 3 — the unit-square figure's construction repeated with a longer rectangle.



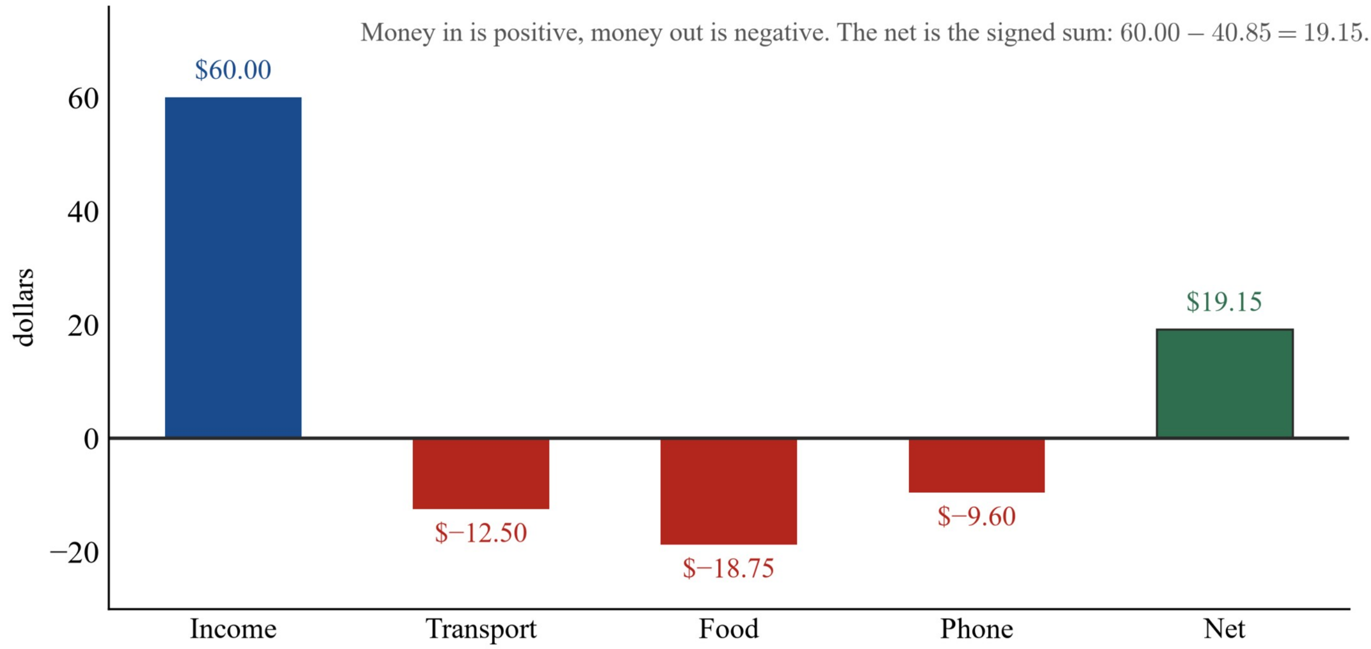
A unit square standing on the number line, its diagonal swung down by a dashed arc to mark the square root of 2 between 1 and 2; a 1 by 2 rectangle used the same way would place the square root of 5

Q11. $r = \frac{1.2}{2} = 0.6$ m. $C = \pi d = 1.2\pi$ m; $A = \pi r^2 = \pi \times 0.6^2 = 0.36\pi$ m² — exact. With tool: $1.2 \times \pi = 3.769911\ldots \approx 3.77$ m; $0.36 \times \pi = 1.130973\ldots \approx 1.13$ m² (2 d.p.). $\therefore$ $C = 1.2\pi \approx 3.77$ m, $A = 0.36\pi \approx 1.13$ m² (2 d.p.) — exact first, one rounding last, as in the exact-versus-approximate figure.



Two rows of three boxes: exact form, calculator display, and the value rounded once at the end, for a circumference and an area in terms of pi

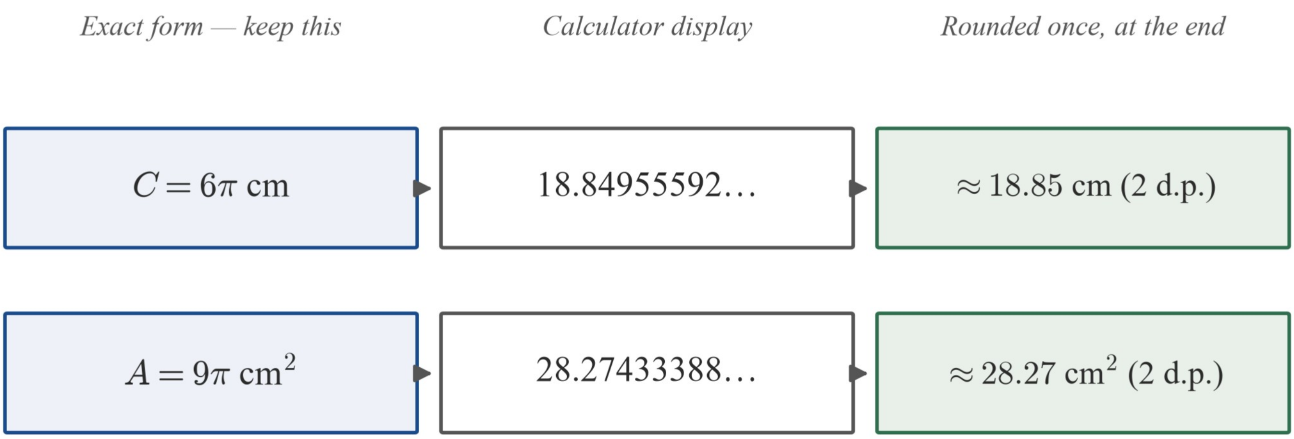
Q12. Outgoings: $12.80 + 19.95 + 8.50 = 41.25$, so -41.25. Net: $55.25 - 41.25 = +14.00$ — in the black. After -20.50: $14.00 - 20.50 = -6.50$. $\therefore$ The week nets $+14.00$; after the extra payment the position is -6.50, a negative rational — the account is 6.50 in the red, the mirror image of the budget figure's positive net.



Bar chart of a weekly budget with a positive income bar, three negative spending bars and a positive net bar; the same signed-sum idea with different values gives a negative final net

Q13. Left-hand side: $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. Right-hand side: $\sqrt{25} = 5$. $7 \neq 5$, so the statement is false; a square root does not split across a sum. $\therefore$ The left-hand side is 7, not 5 — roots of perfect squares are rational numbers here, and they add as ordinary numbers after the roots are evaluated.

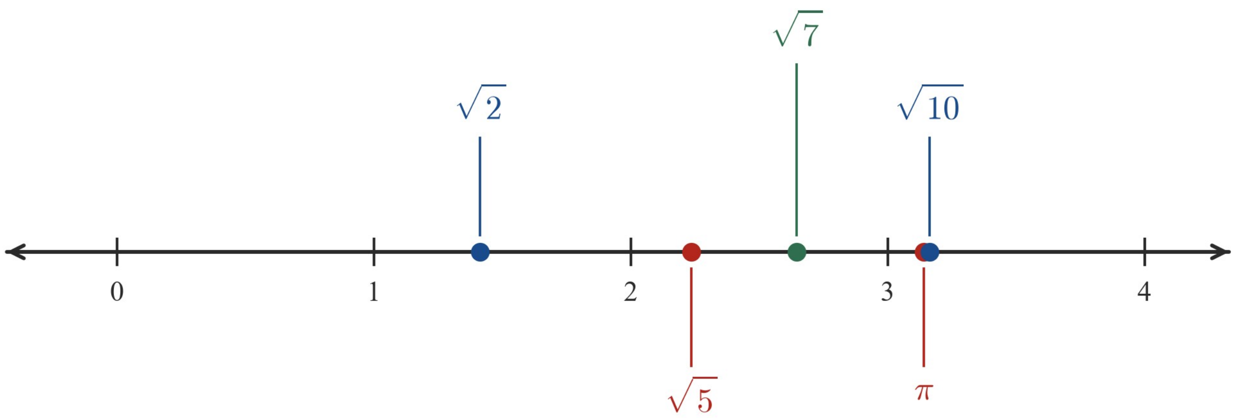
Q14. Keep π exact: $A = 5.5225\pi$ cm². With tool, last line only: $5.5225 \times \pi = 17.34944543\ldots \approx 17.35$ cm² (2 d.p.). The other path: $5.5225 \times 3.14 = 17.34065 \approx 17.34$ cm² — the early replacement of π rounded the answer before the end. $\therefore$ 17.35 cm² is right; the 17.34 came from rounding π too early — the habit is exact until the last line, then one rounding, stated.



The decimal is an approximation of the exact value. Keep π in the working, round once at the end, and state the rounding.

Two rows of three boxes: exact form, calculator display, and the value rounded once at the end, for a circumference and an area in terms of pi

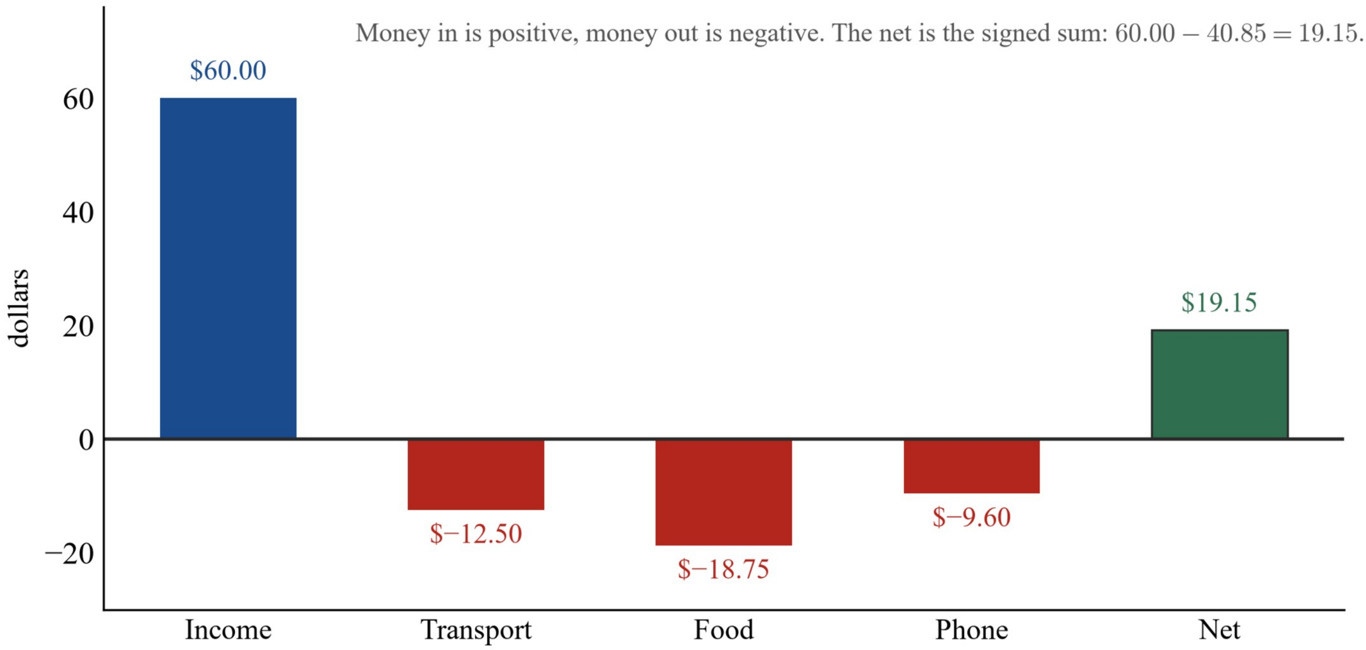
Q15. Trap $\sqrt{7}$: $4 < 7 < 9$, so $2 < \sqrt{7} < 3$; tool check $\sqrt{7} \approx 2.646$. Mirror in 0: $-3 < -\sqrt{7} < -2$. $\therefore$ $-\sqrt{7}$ lies between -3 and -2 — the mirror image of the trap shown for $\sqrt{7}$ on the roots figure.



Each point sits at its exact value: $\pi \approx 3.14159$ and $\sqrt{10} \approx 3.16228$ are close together, but they are different numbers.

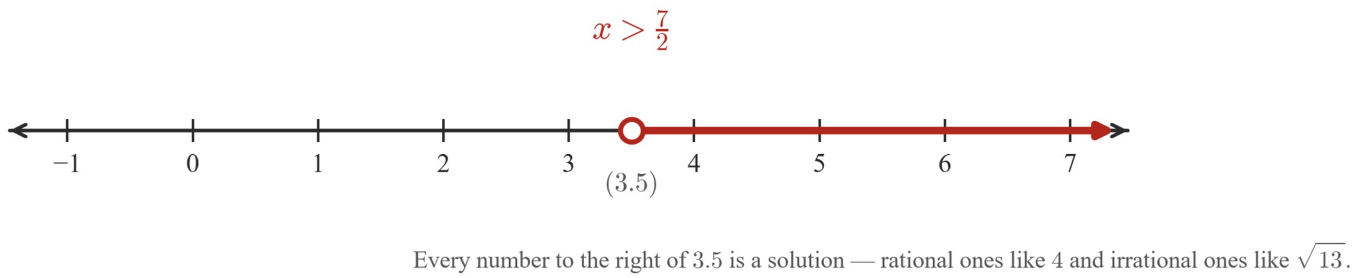
The real number line from 0 to 4 with the square root of 7 marked at about 2.646 between 2 and 3; mirrored in zero, the negative of that point lies between minus 3 and minus 2

Q16. Outgoings: $315.20 + 278.90 + 190.45 = 784.55$, so -784.55. Net: $840.50 - 784.55 = +55.95$. After -120.00: $55.95 - 120.00 = -64.05$. $\therefore$ The week nets $+55.95$; after the part-payment the position is -64.05 — signed rationals, added exactly as the budget figure adds its bars.



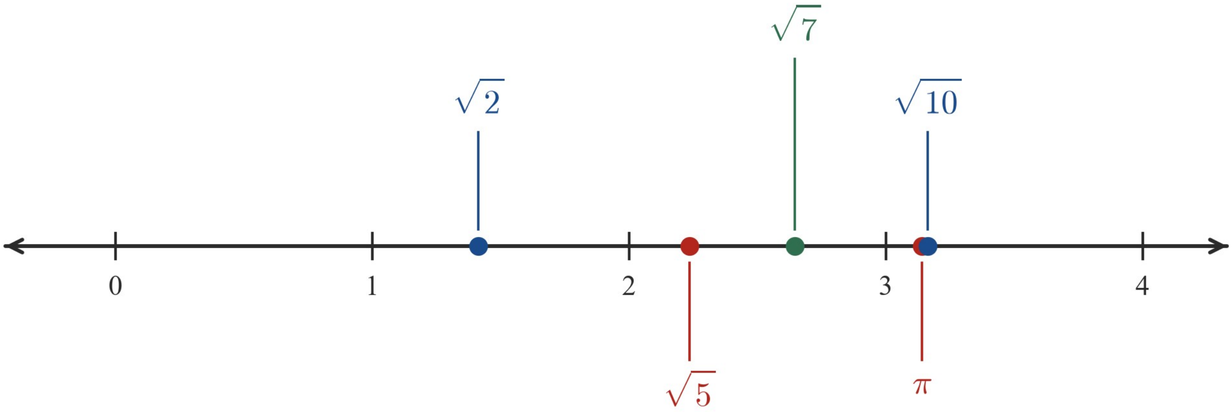
Bar chart of a weekly budget with a positive income bar, three negative spending bars and a net bar; with these values the final net is negative

Q17. Solving (Level 8 skill): $3x>10.5$, so $x>3.5$ — the boundary is 3.5. Rational member: 4, since $3(4)-4.5=7.5>6$. Irrational member: $\sqrt{13}\approx 3.606>3.5$; check $3(3.606)-4.5\approx 6.32>6$. Test π: $\pi\approx 3.14159<3.5$, so $3\pi-4.5\approx 4.92<6$ — not a member. ∴ Boundary 3.5; 4 and $\sqrt{13}$ in, π out — the interval figure shows the ray, and an interval of reals always mixes both kinds of number.



The number line with an open circle at 3.5 and a solid red ray running right, showing every solution of 3x minus 4.5 is greater than 6

Q18. Rationals: $\frac{13}{4}=3.25$ and $\frac{7}{2}=3.5$ — both between 3 and 4, both fractions of integers. Irrationals: $\sqrt{10}$, since $9<10<16$ gives $3<\sqrt{10}<4$; and $\pi=3.14159\ldots$, between 3 and 4 and irrational. ∴ Two rationals $\frac{13}{4},\frac{7}{2}$ and two irrationals $\sqrt{10},\pi$ all live between 3 and 4 — the roots figure shows the irrational two sitting there.



Each point sits at its exact value: $\pi \approx 3.14159$ and $\sqrt{10} \approx 3.16228$ are close together, but they are different numbers.

The real number line from 0 to 4 with pi marked at about 3.142 and the square root of 10 at about 3.162, both between 3 and 4

The exponent laws with integer and zero exponents, extended to variables

1B · Chapter 1 — Number and Algebra: real numbers, exponents and algebraic expressions · Level 9

Curriculum trace — `VC2M9A01` (Algebra).

Content description (word-for-word): *Apply the exponent laws to numerical expressions with integer exponents and the zero exponent, and extend to variables.*

Achievement standard anchor — AS 2 (word-for-word): *Students extend and apply the exponent laws with positive integers and the zero exponent to variables.*

Anchor note (TOC D6). The authority file's AS extract says "positive integers" where the content description says "integer exponents". The content description governs: this section teaches negative integer exponents in full, and the elaborations back that reading — `.E02` works out 5^{-3}, `.E03` writes $2^3 \div 2^5 = 2^{-2}$, and $2^{-2} = \frac{1}{4}$, and `.E06` ends at $8x^2y^{-1}$. Both strings are copied word-for-word above so the difference stays visible.

Elaborations drawn on: `VC2M9A01.E01` (decimals as sums of powers of ten, e.g. $0.475 = 4 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$); `.E02`; `.E03`; `.E04` ($a^0 = 1$, $x^1 = x$, $\frac{1}{w} \times \frac{1}{w} = w^{-2}$); `.E05` ($(a^2)^3 = a^6$, $b^2 b^3 = b^5$, $\frac{y^4}{y^2} = y^2$, $(5a)^2 = 25a^2$); `.E06`; `.E07` (SI prefixes from pico to tera as powers of ten).

Prerequisite pointer (Level 8, word-for-word): *`VC2M8N02` — establish and apply the exponent laws with positive integer exponents and the zero exponent, using exponent notation with numbers.* The laws are revised here as known tools; the new work is extending them to negative integer exponents and to variables. This section feeds 1C, which uses the exponent habits fixed here, and 5B (scientific notation and measurement), where the `.E07` SI-prefix bridge is picked up again.

Source for SI prefixes: BIPM, "SI prefixes", *International System of Units (SI Brochure)*, 9th ed. Retrieved 20 August 2026.

Theory

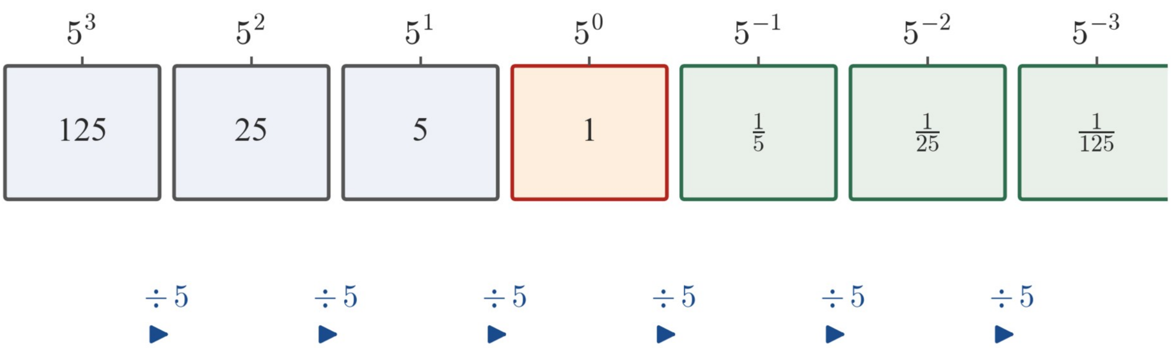
The language, and what Level 8 established. In the power 5^3, the number being multiplied is the **base** (5) and the small raised number is the **exponent** (3). The exponent counts how many times the base is used as a **factor** — a number multiplied in — so $5^3 = 5 \times 5 \times 5$, which is 125. Base and exponent together make a **power**. At Level 8 you established the exponent laws for positive integer exponents and the zero exponent; the table below collects them, one example each. (Some texts call these the *index laws* — this book keeps the curriculum's name, "exponent laws".)

Law	*In general*	*Example*
Product law	$a^m \times a^n = a^{m+n}$	$2^3 \times 2^4 = 2^7$
Quotient law	$a^m \div a^n = a^{m-n}$	$3^7 \div 3^2 = 3^5$
Power of a power	$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
Power of a product	$(ab)^n = a^n b^n$	$(2 \times 3)^4 = 2^4 \times 3^4$
Power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3}$
Zero exponent	$a^0 = 1 \quad (a \neq 0)$	$9^0 = 1$
Negative exponent	$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$	$2^{-3} = \frac{1}{8}$

These hold for every integer exponent (positive, zero or negative) and every non-zero base.

Table of the seven exponent laws, general form and one numerical example per row: product law, quotient law, power of a power, power of a product, power of a quotient, zero exponent, and negative exponent.

Negative exponents: follow the pattern. The next figure writes the powers of 5 in a row, and every step to the right divides by 5.



Each step to the right divides by 5. Past $5^0 = 1$ the values become fractions, and the exponents become negative.

Powers of 5 from 5 cubed down to 5 to the power of negative 3, with values 125 down to 1 over 125 and a divide-by-5 arrow between each step.

Keep dividing past 1 and the values become $\frac{1}{5}$, then $\frac{1}{25}$, then $\frac{1}{125}$, while the exponents keep falling by 1. So $5^{-1} = \frac{1}{5}$, $5^{-2} = \frac{1}{25}$ and $5^{-3} = \frac{1}{125}$. That pattern is the rule to remember: for a nonzero base a and a positive whole number n,

$$a^{-n} = \frac{1}{a^n}$$

Why the rule has to look like that. The laws and plain counting must agree on every answer. Take $2^3 \div 2^5$, done two ways.

$$2^3 \div 2^5$$

Route A: the quotient law

$$2^3 \div 2^5 = 2^{3-5} = 2^{-2}$$

$$= \frac{1}{2^2} = \frac{1}{4}$$

Route B: expand and cancel

$$\frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$$

cancel three 2's top and bottom

$$= \frac{1}{2 \times 2} = \frac{1}{4}$$

$$\frac{1}{4}$$

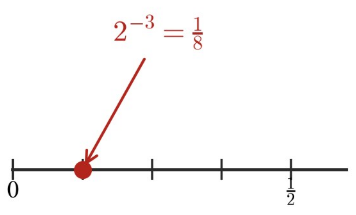
Two routes for 2 cubed divided by 2 to the 5: subtracting exponents gives 2 to the power of negative 2, which is 1 over 4; expanding and cancelling the 2s also gives 1 over 4.

Route 1 uses the quotient law: $2^{3-5} = 2^{-2}$. Route 2 writes the powers out and cancels, leaving $\frac{1}{4}$. Both routes are the same division, so $2^{-2} = \frac{1}{4}$ — exactly the value the pattern gave. The zero exponent is forced the same way: $3^4 \div 3^4$ is plainly 1, and the quotient law writes it as $3^{4-4} = 3^0$, so $3^0 = 1$ for any nonzero base.

The laws now run on every integer exponent. Nothing in the table changes; m and n may be positive, zero or negative. A quick check with a negative exponent: $2^{-2} \times 2^5 = \frac{1}{4} \times 32$, and $\frac{1}{4} \times 32 = 8$, and $8 = 2^3$. The product law says $2^{-2+5} = 2^3$, so it agrees.

Two cautions.

A negative exponent is not a negative number



The exponent is negative but the value sits to the right of 0: it is positive.

Brackets change the base

$$(-2)^4 = (-2)(-2)(-2)(-2) = 16$$

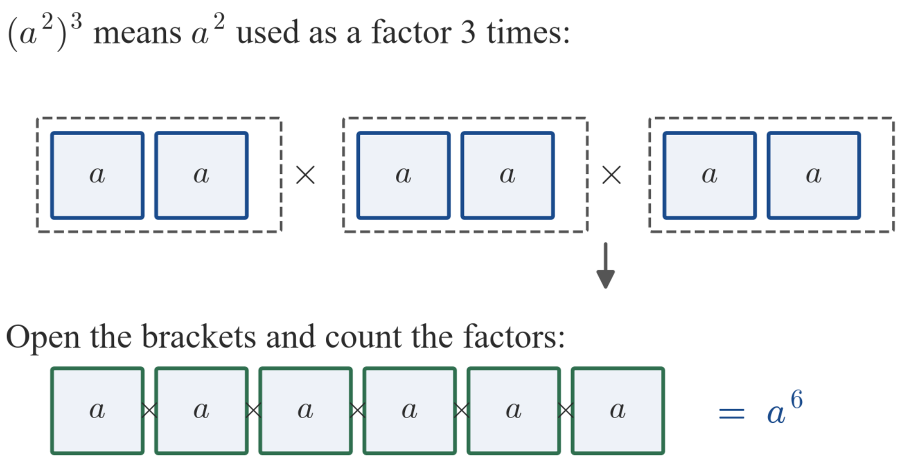
$$-2^4 = -(2 \times 2 \times 2 \times 2) = -16$$

With brackets, the base is -2.
Without them, the power acts on 2 only, and the minus is applied afterwards.

Two caution panels. Left: 2 to the power of negative 3 equals 1 over 8, shown as a positive point on a number line to the right of zero. Right: bracket negative 2, to the 4, equals 16, while the negative of 2 to the 4 equals negative 16.

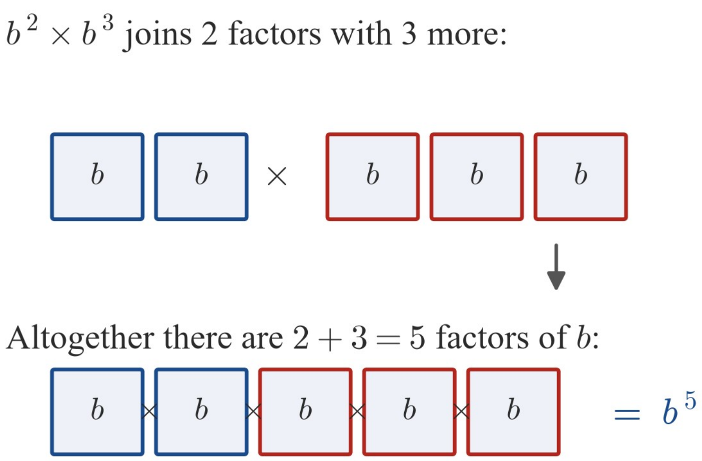
First, a negative exponent does **not** make the value negative: $2^{-3}=\frac{1}{8}$, a small positive number. Second, brackets fix the base: $(-2)^4=16$ because four negative factors multiply to a positive, but $-2^4=-16$ because without brackets only the 2 is raised to the power.

Extending to variables. A **variable** is a letter standing for a number. Every law carries over unchanged, and the next three pictures show why, straight from the meaning of a power.



Bracket a squared, cubed, drawn as three groups of a times a; put together there are six factors of a, so the result is a to the 6.

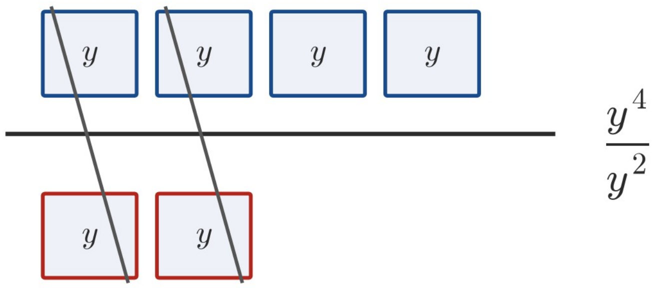
$(a^2)^3$ is three groups of $a \times a$. Put together there are $2 \times 3=6$ factors of a, so $(a^2)^3=a^6$.



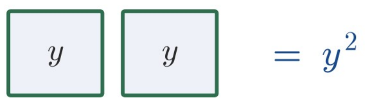
b squared times b cubed drawn as two factors of b joined with three more; altogether there are five factors of b, so the result is b to the 5.

$b^2 \times b^3$ joins 2 factors of b with 3 more. Altogether there are $2+3=5$ factors, so $b^2 \times b^3=b^5$.

Cancel the factors that appear in both rows:



Two y's cancel top and bottom, leaving



y to the 4 over y to the 2 drawn as four factors of y over two factors of y; two pairs cancel, leaving y squared.

$\frac{y^4}{y^2}$ cancels two of the y's on top against the two below, leaving y^2.

Two more rules to keep: $x^1 = x$, and $\frac{1}{w} \times \frac{1}{w} = \frac{1}{w^2}$, and $\frac{1}{w^2} = w^{-2}$. When an answer contains a negative exponent, the same value can be written with positive exponents only — $8ab^{-2}$ and $\frac{8a}{b^2}$ are the same **expression** read two ways. We call the second one the *positive-exponent form*.

Decimals as sums of powers of ten. Each place to the right of the decimal point is the place before it divided by 10, so the places are 10^{-1}, 10^{-2}, 10^{-3} and so on.

Ones	Tenths	Hundredths	Thousandths
0 .	4	7	5
10^0	10^{-1}	10^{-2}	10^{-3}

$$0.475 = 4 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$$

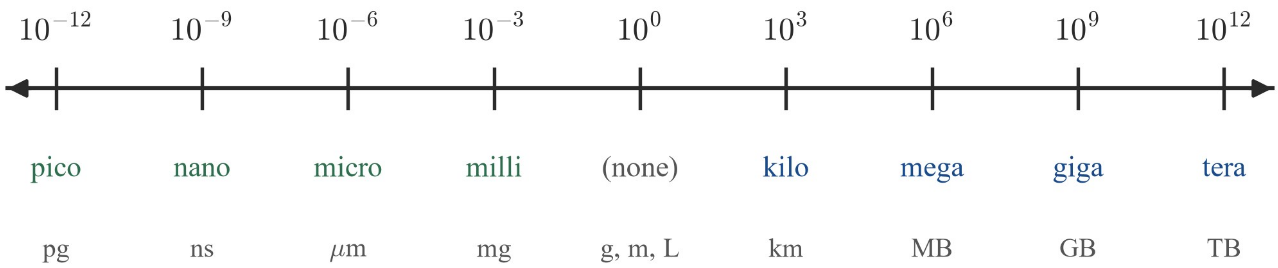
Each digit is multiplied by the power of 10 for its place.

Place-value chart for 0.475: 0 in the ones column, then a decimal point, 4 tenths above 10 to the power of negative 1, 7 hundredths above 10 to the power of negative 2, 5 thousandths above 10 to the power of negative 3, with the sum 0.475 equals 4 times 10 to the negative 1 plus 7 times 10 to the negative 2 plus 5 times 10 to the negative 3.

So $0.475 = 4 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$, and the same reading writes $0.00023 = 23 \times 10^{-5}$. (5B will use this idea again for scientific notation; here it is simply the exponent laws at work on numbers.)

SI prefixes are powers of ten. A **prefix** is a short tag placed before a unit, and each SI prefix means "multiply by this power of ten".

Multiplying by a prefix means multiplying by a power of 10. Steps of $\times 1000$ are shown.



Ladder of powers of 10 from 10 to the negative 12 up to 10 to the 12, labelled with the SI prefixes pico, nano, micro, milli, then the base units, then kilo, mega, giga, tera, with example units such as pg, ns, µm, mg, km, MB, GB and TB.

So pico $=10^{-12}$, nano $=10^{-9}$, micro $=10^{-6}$, milli $=10^{-3}$, kilo $=10^{3}$, mega $=10^{6}$, giga $=10^{9}$ and tera $=10^{12}$: one picogram is 10^{-12} grams and one terabyte is 10^{12} bytes. Converting between prefixed units is a quotient-law job. One teragram is 10^{12} g, and one kilogram is 10^{3} g, so divide: $10^{12} \div 10^{3} = 10^{9}$, which says 1 Tg $=10^{9}$ kg.

Worked examples

Example 1 — scaffolded

Simplify $3^{2} \times 3^{-5} \div 3^{-1}$ and give the value as a fraction.

Working	Comment
$3^{2+(-5)-(-1)}$	One base throughout: product and quotient laws, adding and subtracting the exponents.
$=3^{-2}$	$2+(-5)=-3$, and $-3-(-1)=-2$; subtracting -1 is adding 1.
$=\dfrac{1}{3^{2}}$	Negative exponent: $a^{-n}=\dfrac{1}{a^{n}}$.
$=\dfrac{1}{9}$	$3^{2}=9$.

Check by direct arithmetic: $9 \times \dfrac{1}{243}=\dfrac{1}{27}$. Then $\dfrac{1}{27} \times 3=\dfrac{1}{9}$, the same answer. ✓

Example 2 — scaffolded

Simplify $(4a^{3}b)^{2} \div (2a^{5}b^{4})$, leaving positive exponents.

Working	Comment
$(4a^{3}b)^{2}=16a^{6}b^{2}$	Power over a **product**: $4^{2}=16$, $(a^{3})^{2}=a^{6}$, b^{2}.
$16a^{6}b^{2} \div (2a^{5}b^{4})=8a^{6-5}b^{2-4}$	Numbers first: $16 \div 2=8$; then the quotient law on each letter.
$=8ab^{-2}$	$6-5=1$ and $2-4=-2$.

Working	Comment
$=\dfrac{8a}{b^2}$	Positive-exponent form.

Example 3 — unscaffolded

Write 0.806 as a sum of powers of ten, and write $3\times10^{-2}+5\times10^{-4}$ as a decimal.

In 0.806 the 8 sits in the tenths and the 6 in the thousandths; the hundredths digit is 0 and adds nothing.

$0.806=8\times10^{-1}+6\times10^{-3}$

For the sum, $3\times10^{-2}=0.03$ and $5\times10^{-4}=0.0005$.

$0.03+0.0005=0.0305$

$\therefore$ $0.806=8\times10^{-1}+6\times10^{-3}$, and $3\times10^{-2}+5\times10^{-4}=0.0305$.

Example 4 — unscaffolded

SCENARIO — a laser pulse lasts 20 picoseconds; write this time in seconds. A memory card holds 1 terabyte and each photo stored on it takes 8 megabytes; how many photos fit? (Prefix values as in the figure: pico $=10^{-12}$, mega $=10^{6}$, tera $=10^{12}$.)

The pulse: 20 ps $=20\times10^{-12}$ s.

$20\times10^{-12}=2\times10^{-11}$

so the pulse lasts 2×10^{-11} s.

The photos: the card holds 10^{12} bytes and one photo takes 8×10^{6} bytes.

$10^{12}\div\left(8\times10^{6}\right)=125\,000$

$\therefore$ One pulse lasts 2×10^{-11} s, and the card fits 125 000 photos.

Practice questions

Scaffolded

Q1. Evaluate: (a) 2^6 (b) 9^0 (c) 4^{-2} (d) $(-3)^3$

Q2. Write as a single power: (a) $5^3\times5^6$ (b) $7^8\div7^2$ (c) $\left(2^4\right)^3$ (d) $10^5\times10^{-2}$

Q3. Write with positive exponents, then evaluate where you can: (a) x^{-4} (b) $\left(\dfrac{2}{3}\right)^{-2}$ (c) $m^5\div m^9$

Q4. Expand, then simplify to a single term: (a) $\left(p^2\right)^3$ (b) $n^3\times n^4$ (c) $(3t)^2$

Q5. Write each decimal as a sum of powers of ten, or each sum as a decimal: (a) 0.62 (b) 0.075 (c) $8\times10^{-1}+3\times10^{-3}$

Q6. Write each measure as a number times a power of ten of the base unit: (a) 1 milligram, in grams (b) 1 microsecond, in seconds (c) 5 kilometres, in metres (d) 2 gigabytes, in bytes

Unscaffolded

Q7. Write as a single power: (a) $2^7\times2^{-4}$ (b) $6^2\div6^7$ (c) $\left(3^{-2}\right)^3$ (d) $10^{-3}\times10^3$

Q8. Evaluate: (a) 2^{-5} (b) $\left(\frac{1}{4}\right)^{-2}$ (c) $5^{-1}+5^{0}$ (d) $(-2)^{-3}$

Q9. Simplify to a single power, or to 1: (a) $x^{6}\times x^{-2}$ (b) $a^{3}\div a^{8}$ (c) $(m^{4})^{2}\times m^{-7}$ (d) $y^{-2}\times y^{5}\div y^{3}$

Q10. Write with positive exponents only: (a) $p^{-3}q^{4}$ (b) $(2x)^{-3}$ (c) $\left(\frac{a}{b}\right)^{-2}$ (d) $4m^{-1}n^{0}$

Q11. Simplify, leaving positive exponents: (a) $(3x^{2}y)^{2}$ (b) $\frac{10a^{5}b^{2}}{2a^{2}b^{6}}$ (c) $\frac{(2c^{3})^{3}}{4c^{7}}$ (d) $\frac{6p^{4}q}{2pq^{3}}$

Q12. Simplify fully: (a) $\frac{(2xy)^{3}}{x^{2}y^{5}}$ (b) $\frac{(3m^{2})^{2}n^{3}}{m^{5}n}$ (c) $\frac{(a^{2}b)^{3}}{a^{4}b^{5}}\times b^{2}$

Q13. (a) Write 0.0408 as a sum of powers of ten. (b) Write $9\times10^{-1}+6\times10^{-4}$ as a decimal. (c) Write 0.00056 in the form $n\times10^{-5}$.

Q14. (a) Put in order, smallest first: mega, milli, nano, kilo, micro, pico. (b) Write 400 nanoseconds in seconds as $a\times10^{b}$ with a a one-digit number. (c) How many kilograms are in one teragram? (d) How many metres are in 10^{6} micrometres?

Q15. True or false? Correct each false statement: (a) $x^{3}\times x^{4}=x^{12}$ (b) $(a+b)^{2}=a^{2}+b^{2}$ (c) $7^{-2}=-49$ (d) $(xy)^{3}=x^{3}y^{3}$

Q16. Look at the sequence $100\,000$, $10\,000$, 1000, 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$. (a) Write every term as a power of 10. (b) Describe the step from one term to the next. (c) Write the next two terms as powers of 10.

Q17. Simplify: (a) $(x^{2})^{3}\times x^{-4}\div x^{5}$ (b) $(-3)^{2}\times(-3)^{-5}\div(-3)^{-2}$ (c) $(a^{3}b^{-2})^{2}\times a^{-4}b^{4}$

Q18. SCENARIO — a camera writes photos of 4×10^{6} bytes each onto a 2-terabyte card (tera $=10^{12}$). (a) How many photos fit on the card? (b) A second camera writes photos of 8×10^{6} bytes each; how many of those fit? (c) Write $10^{12}\div10^{6}$ as a single power of 10, and say what that number counts on this card.

Answers

Q1. (a) 64 (b) 1 (c) $\frac{1}{16}$ (d) -27

Q2. (a) 5^9 (b) 7^6 (c) 2^{12} (d) 10^3

Q3. (a) $\frac{1}{x^4}$ (b) $\frac{9}{4}$ (c) $\frac{1}{m^4}$

Q4. (a) p^6 (b) n^7 (c) $9t^2$

Q5. (a) $6\times10^{-1}+2\times10^{-2}$ (b) $7\times10^{-2}+5\times10^{-3}$ (c) 0.803

Q6. (a) 10^{-3} g (b) 10^{-6} s (c) 5×10^3 m (d) 2×10^9 bytes

Q7. (a) 2^3 (b) 6^{-5} (c) 3^{-6} (d) $10^0=1$

Q8. (a) $\frac{1}{32}$ (b) 16 (c) $\frac{6}{5}$ (d) $-\frac{1}{8}$

Q9. (a) x^4 (b) $\frac{1}{a^5}$ (c) m (d) 1

Q10. (a) $\frac{q^4}{p^3}$ (b) $\frac{1}{8x^3}$ (c) $\frac{b^2}{a^2}$ (d) $\frac{4}{m}$

Q11. (a) $9x^4y^2$ (b) $\frac{5a^3}{b^4}$ (c) $2c^2$ (d) $\frac{3p^3}{q^2}$

Q12. (a) $\frac{8x}{y^2}$ (b) $\frac{9n^2}{m}$ (c) a^2

Q13. (a) $4\times10^{-2}+8\times10^{-4}$ (b) 0.9006 (c) 56×10^{-5}

Q14. (a) pico, nano, micro, milli, kilo, mega (b) 4×10^{-7} s (c) 10^9 kg (d) 1 m

Q15. (a) False; $x^3\times x^4=x^7$ (b) False; at $a=1$ and $b=1$ the left side is 4 and the right side is 2 (c) False; $7^{-2}=\frac{1}{49}$ (d) True

Q16. (a) 10^5, 10^4, 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2} (b) divide by 10 (the exponent falls by 1) (c) 10^{-3}, 10^{-4}

Q17. (a) $\frac{1}{x^3}$ (b) $-\frac{1}{3}$ (c) a^2

Q18. (a) $500\,000$ (b) $250\,000$ (c) 10^6 — the number of 10^6-byte blocks the card holds; (a) is that divided by 4 and (b) is that divided by 8

Fully-worked solutions

Q1. (a) $2^6=64$. (b) Any nonzero base to the zero exponent is 1, so $9^0=1$. (c) $4^{-2}=\frac{1}{4^2}$, and $4^2=16$, so the value is $\frac{1}{16}$. (d) Three negative factors give a negative **product**: $(-3)^3=-27$.

Q2. (a) Product law: $3+6=9$, giving 5^9. (b) Quotient law: $8-2=6$, giving 7^6. (c) Power of a power: $4\times 3=12$, giving 2^{12}. (d) Product law: $5+(-2)=3$, giving 10^3.

Q3. (a) $x^{-4}=\frac{1}{x^4}$. (b) $\left(\frac{2}{3}\right)^{-2}=\left(\frac{3}{2}\right)^2$, and $\left(\frac{3}{2}\right)^2=\frac{9}{4}$. (c) Quotient law: $5-9=-4$, so $m^5\div m^9=m^{-4}$, and $m^{-4}=\frac{1}{m^4}$.

Q4. (a) Power of a power: $(p^2)^3=p^6$. (b) Product law: $n^3\times n^4=n^7$. (c) Power over a product: $(3t)^2=3^2t^2$, and $3^2=9$, so $(3t)^2=9t^2$.

Q5. (a) The 6 is in the tenths and the 2 in the hundredths: $0.62=6\times 10^{-1}+2\times 10^{-2}$. (b) The 7 is in the hundredths and the 5 in the thousandths: $0.075=7\times 10^{-2}+5\times 10^{-3}$. (c) $8\times 10^{-1}=0.8$ and $3\times 10^{-3}=0.003$; adding gives 0.803.

Q6. (a) milli $=10^{-3}$, so 1 mg $=10^{-3}$ g. (b) micro $=10^{-6}$, so 1 µs $=10^{-6}$ s. (c) kilo $=10^3$, so 5 km $=5\times 10^3$ m. (d) giga $=10^9$, so 2 GB $=2\times 10^9$ bytes.

Q7. (a) $7+(-4)=3$: 2^3. (b) $2-7=-5$: 6^{-5}. (c) $(-2)\times 3=-6$: 3^{-6}. (d) $-3+3=0$: 10^0, which is 1.

Q8. (a) $2^{-5}=\frac{1}{2^5}$, and $2^5=32$, so the value is $\frac{1}{32}$. (b) $\left(\frac{1}{4}\right)^{-2}=4^2$, which is 16. (c) $5^{-1}=\frac{1}{5}$ and $5^0=1$; $\frac{1}{5}+1=\frac{6}{5}$. (d) $(-2)^{-3}=\frac{1}{(-2)^3}$, and $(-2)^3=-8$, so the value is $-\frac{1}{8}$ — see the second caution panel in the figure above.

Q9. (a) $6+(-2)=4$: x^4. (b) $3-8=-5$: $a^{-5}=\frac{1}{a^5}$. (c) $(m^4)^2=m^8$; then $8+(-7)=1$, giving m. (d) $-2+5-3=0$, giving $y^0=1$.

Q10. (a) $p^{-3}q^4=\frac{q^4}{p^3}$. (b) $(2x)^{-3}=\frac{1}{(2x)^3}$, and $(2x)^3=8x^3$, so the answer is $\frac{1}{8x^3}$. (c) $\left(\frac{a}{b}\right)^{-2}=\left(\frac{b}{a}\right)^2$, which is $\frac{b^2}{a^2}$. (d) $n^0=1$ and $m^{-1}=\frac{1}{m}$, so $4m^{-1}n^0=\frac{4}{m}$.

Q11. (a) Square each factor: $3^2=9$, $(x^2)^2=x^4$, y^2; result $9x^4y^2$. (b) Numbers: $10\div 2=5$. Letters: $a^{5-2}=a^3$ and $b^{2-6}=b^{-4}$; so $\frac{5a^3}{b^4}$. (c) $(2c^3)^3=8c^9$; then $8\div 4=2$ and $c^{9-7}=c^2$, giving $2c^2$. (d) Numbers: $6\div 2=3$. Letters: $p^{4-1}=p^3$ and $q^{1-3}=q^{-2}$; so $\frac{3p^3}{q^2}$.

Q12. (a) $(2xy)^3=8x^3y^3$; then $x^{3-2}=x$ and $y^{3-5}=y^{-2}$, giving $\frac{8x}{y^2}$. (b) $(3m^2)^2=9m^4$; then $m^{4-5}=m^{-1}$ and $n^{3-1}=n^2$, giving $\frac{9n^2}{m}$. (c) $(a^2b)^3=a^6b^3$; then $a^{6-4}=a^2$ and $b^{3-5+2}=b^0$; since $b^0=1$, the result is a^2.

Q13. (a) The 4 is in the hundredths and the 8 in the ten-thousandths: $0.0408=4\times 10^{-2}+8\times 10^{-4}$ — read off the place-value chart in the figure. (b) $9\times 10^{-1}=0.9$ and $6\times 10^{-4}=0.0006$; adding gives 0.9006. (c) $0.00056=56\times 10^{-5}$, since moving the 5 and 6 five places left of the decimal point is dividing by 10^5.

Q14. (a) In powers of ten: mega 10^6, milli 10^{-3}, nano 10^{-9}, kilo 10^3, micro 10^{-6}, pico 10^{-12}. Smallest first: pico, nano, micro, milli, kilo, mega. (b) nano $=10^{-9}$, so 400 ns $=400\times 10^{-9}$ s; and $400\times 10^{-9}=4\times 10^{-7}$, giving 4×10^{-7} s. (c) tera $=10^{12}$, so 1 Tg $=10^{12}$ g; one kilogram is 10^3 g, and $10^{12}\div 10^3=10^9$, so 1 Tg $=10^9$ kg. (d) micro $=10^{-6}$, so 10^6 µm $=10^6\times 10^{-6}$ m, and $10^6\times 10^{-6}=10^0$, which is 1 m.

Q15. (a) False. The product law adds exponents: $x^3 \times x^4 = x^7$, not x^{12}. (b) False. At $a=1$ and $b=1$: left side $(1+1)^2 = 4$; right side $1^2 + 1^2 = 2$; the two differ, so the statement fails. (c) False. $7^{-2} = \frac{1}{7^2}$, which is $\frac{1}{49}$, a positive number — a negative exponent never makes the value negative (first caution panel). (d) True. Power over a product: $(xy)^3 = x^3 y^3$.

Q16. (a) $100\,000 = 10^5$, $10\,000 = 10^4$, $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$, $1 = 10^0$, $\frac{1}{10} = 10^{-1}$, $\frac{1}{100} = 10^{-2}$ — the same shape as the powers-of-5 figure, with base 10. (b) Each step divides by 10, i.e. the exponent falls by 1. (c) Continuing: 10^{-3}, then 10^{-4}.

Q17. (a) $(x^2)^3 = x^6$; then $6 + (-4) - 5 = -3$, giving $x^{-3} = \frac{1}{x^3}$. (b) One base of -3: $2 + (-5) - (-2) = -1$, giving $(-3)^{-1} = \frac{1}{-3}$, which is $-\frac{1}{3}$. (c) $(a^3 b^{-2})^2 = a^6 b^{-4}$; then $a^{6-4} = a^2$ and $b^{-4+4} = b^0$; since $b^0 = 1$, the answer is a^2.

Q18. (a) The card holds 2×10^{12} bytes and one photo takes 4×10^6 bytes. Numbers: $2 \div 4 = \frac{1}{2}$. Powers of ten: $10^{12} \div 10^6 = 10^6$. So the count is $\frac{1}{2} \times 10^6$, which is $500\,000$ photos. (b) Now $2 \div 8 = \frac{1}{4}$, so the count is $\frac{1}{4} \times 10^6 = 250\,000$ photos. (c) $10^{12} \div 10^6 = 10^6$ by the quotient law; it counts the 10^6-byte blocks on the card, and each answer above is that block count divided by the bytes-per-photo number (4 or 8).

Expanding binomial products and factorising monic quadratic expressions

1C · Chapter 1 — Number and Algebra: real numbers, exponents and algebraic expressions · Level 9

Content description `VC2M9A02` (word for word): *simplify algebraic expressions, apply the distributive law to expand algebraic expressions including binomial products, and factorise monic quadratic expressions*

Achievement standard anchor (AS 3): *They expand binomial products and factorise monic quadratic expressions.*

Elaboration ids for linkage: `VC2M9A02.E01`, `VC2M9A02.E02`, `VC2M9A02.E03`

Prerequisite pointer: 1B (the exponent laws with integer and zero exponents, extended to variables). Binomial expansion uses the exponent conventions established in 1B — $x \times x = x^2$ is used here as a tool, not re-taught.

Note on level (D3): every quadratic expression in this section is **monic** — the coefficient of x^2 is 1 — and every number that appears is an integer. Non-monic factorisation, the quadratic formula and completing the square as a solving method are not Level 9 and do not appear; solving quadratic equations by the null factor law is 3A, which reuses the factorisation taught here.

Note on figures: every area, part, sum and product in the figures of this section is computed from its own values (R7); every context is a generic one, so no real data ships and no source line is owed (R1, R2).

Theory

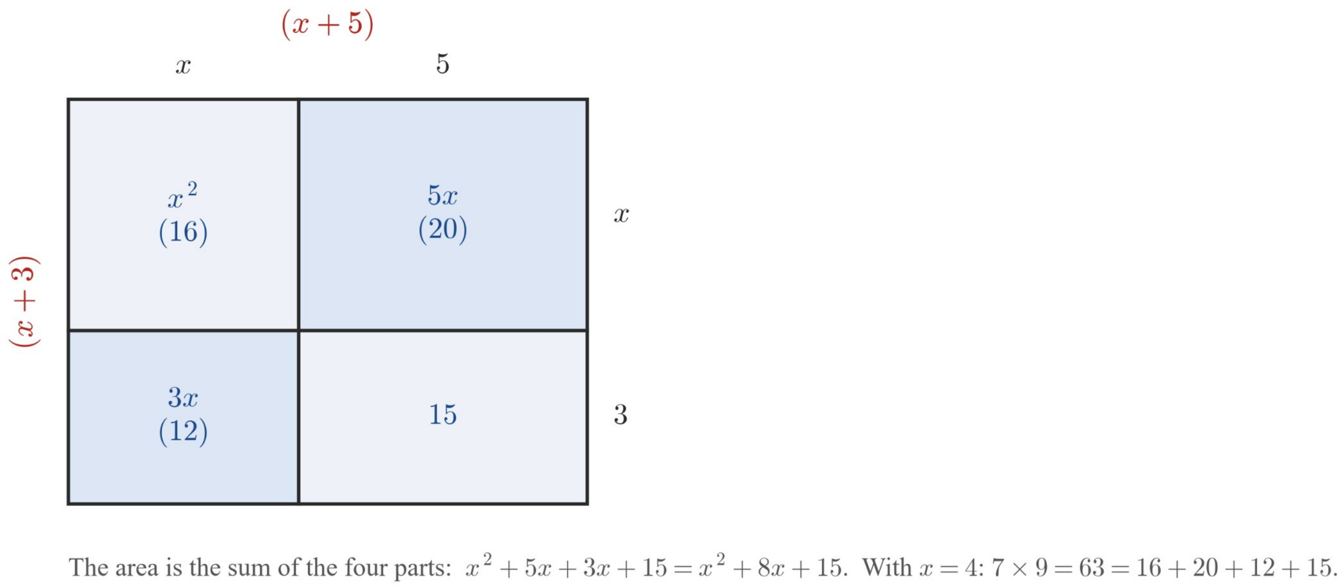
Words this section uses

A **term** is one of the pieces of an expression that are added or subtracted: x^2, $5x$ and 15 are the three terms of $x^2+5x+15$. The **coefficient** of a term is the number multiplying the letter: in $5x$ the coefficient is 5. A term with no letter is a **constant term**. A **binomial** is an expression with two terms, such as $x+3$ or $x-5$. A product of two binomials, $(x+3)(x+5)$, is a **binomial product**. To **expand** means to multiply the brackets out into a sum of terms; to **factorise** means to run that film backwards — to write a sum of terms as a product of brackets. A **quadratic expression** is one whose highest power of x is x^2; when the coefficient of x^2 is 1, as in $x^2+8x+15$, the quadratic expression is **monic**. The three verbs of the content description — *simplify*, *expand*, *factorise* — are the whole of this section.

The distributive law, drawn as an area

The engine of all expansion is the distributive law $a(b+c)=ab+ac$: multiplying a sum means multiplying each term of it. Applied twice, it says that every term in the first bracket multiplies every term in the second bracket.

Draw the product $(x+3)(x+5)$ as a rectangle with sides $x+3$ and $x+5$, cut at the joins. The figure shows it to scale at $x=4$, so the rectangle is 7×9, and the four pieces are $4 \times 4 = 16$, $4 \times 5 = 20$, $3 \times 4 = 12$ and $3 \times 5 = 15$: the pieces add to $16+20+12+15=63$, exactly the whole area $7 \times 9 = 63$.



A rectangle of width x plus 3 and length x plus 5 drawn to scale at x equals 4, cut into four pieces labelled 16, 20, 12 and 15, whose sum 63 equals the whole area 7 times 9

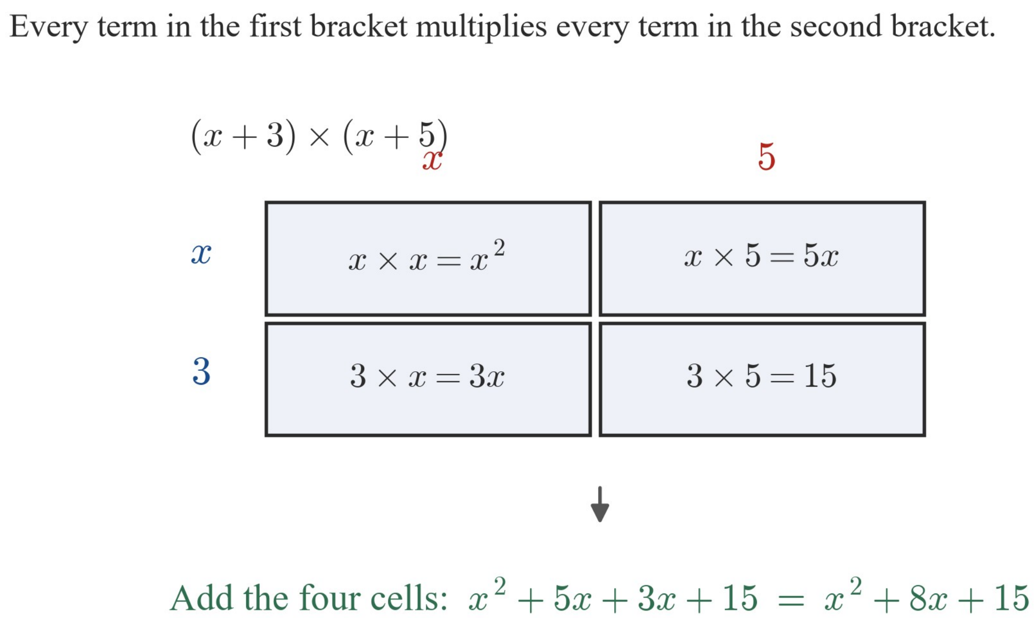
Now drop the numbers and keep the letters. The same four pieces are $x \times x = x^2$ (this is where 1B's $x \times x = x^2$ is used), $x \times 5 = 5x$, $3 \times x = 3x$ and $3 \times 5 = 15$, so

$$(x+3)(x+5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15,$$

after **collecting** the **like terms** $5x + 3x = 8x$ — like terms have the same letter part, so their counts add.

Every term times every term, in a grid

The same four multiplications fit in a grid: head the columns with the terms of one bracket and the rows with the terms of the other, fill each cell with the product of its row and column heads, then add the four cells.



A multiplication grid for the product of x plus 3 and x plus 5: columns headed x and 5, rows headed x and 3, four cells holding x times x equals x squared, x times 5 equals 5x, 3 times x equals 3x and 3 times 5 equals 15, with the sum line x squared plus 5x plus 3x plus 15 equals x squared plus 8x plus 15

Reading the pattern off the grid with general numbers a and b in place of 3 and 5 gives the identity this section returns to again and again:

$$(x+a)(x+b)=x^2+(a+b)x+ab.$$

The coefficient of x is the **sum** $a+b$ of the two numbers; the constant term is their **product** ab. Remember the pattern as *sum and product*.

The four sign combinations (E01)

The numbers a and b may be negative, and the signs travel with them: $x-7$ is $x+(-7)$, so the number is -7. The table expands all four sign combinations of the same two sizes, 7 and 8.

Product	*Expansion*	*Sum (with signs)*	*Product (with signs)*	*Signs*
$(x+7)(x+8)$	$x^2+15x+56$	$7+8=15$	$7\times 8=56$	both +
$(x+7)(x-8)$	x^2-x-56	$7+(-8)=-1$	$7\times(-8)=-56$	signs differ
$(x-7)(x+8)$	x^2+x-56	$-7+8=1$	$(-7)\times 8=-56$	signs differ
$(x-7)(x-8)$	$x^2-15x+56$	$-7+(-8)=-15$	$(-7)\times(-8)=56$	both −

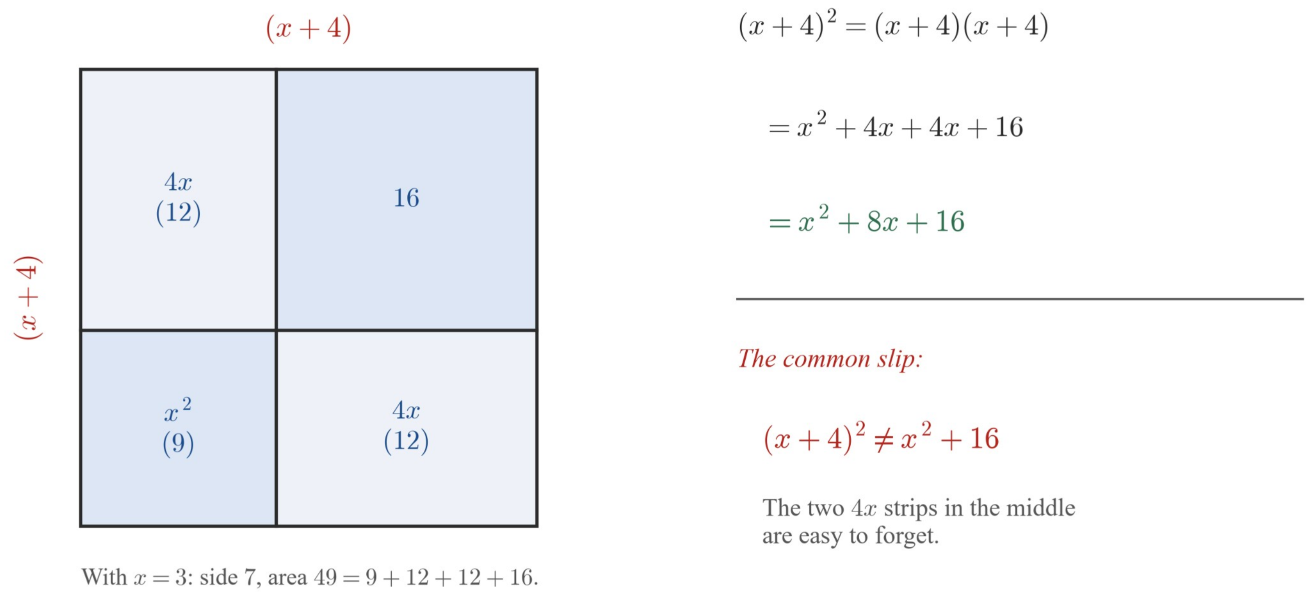
The x-coefficient is always the sum; the constant term is always the product.

A four-row table expanding the products of x plus 7 and x plus 8, x plus 7 and x minus 8, x minus 7 and x plus 8, and x minus 7 and x minus 8, each row showing the expansion, the sum of the two numbers taken with signs, their product taken with signs, and the resulting sign pattern

Read across each row: the x-coefficient is always the sum taken with signs, and the constant term is always the product taken with signs. Both numbers positive: everything positive. Both numbers negative: the x-coefficient negative and the constant term positive, since a negative times a negative is positive. Signs differ: the constant term negative, and the x-coefficient takes the sign of the larger number.

Perfect squares (E02)

When the two brackets are the same, $(x+4)^2=(x+4)(x+4)$, the grid has equal rows and equal columns — the picture is a square. The figure shows it to scale at $x=3$: the pieces 9, 12, 12 and 16 add to $49=7^2$.

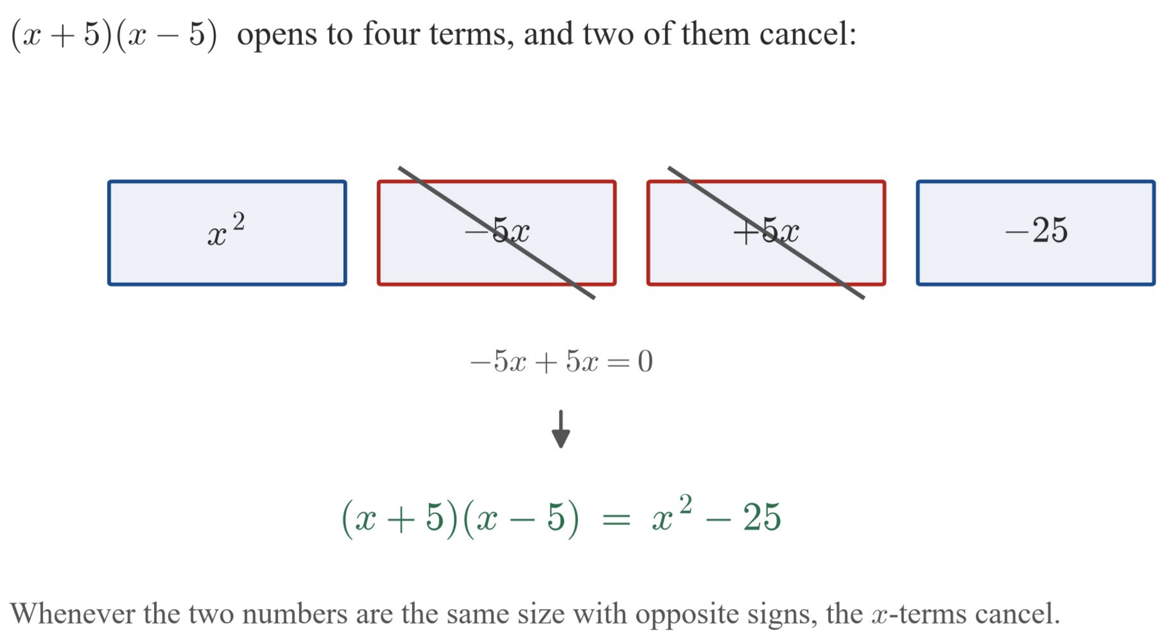


A square of side x plus 4 drawn to scale at x equals 3, cut into pieces 9, 12, 12 and 16 giving x squared plus 8x plus 16, beside a panel marking the common slip that the square of x plus 4 is not x squared plus 16

In letters, $(x+4)^2 = x^2+4x+4x+16 = x^2+8x+16$. The common slip is to write $(x+4)^2 = x^2+16$: the figure shows what that leaves out — the two $4x$ rectangles, the middle term $8x$. A quadratic expression that comes from squaring a binomial, like $x^2+8x+16$, is called a **perfect square**. The same grid with -5 gives $(x-5)^2 = x^2-10x+25$.

Difference of two squares

When the two numbers are the same size with opposite signs, the middle terms cancel:



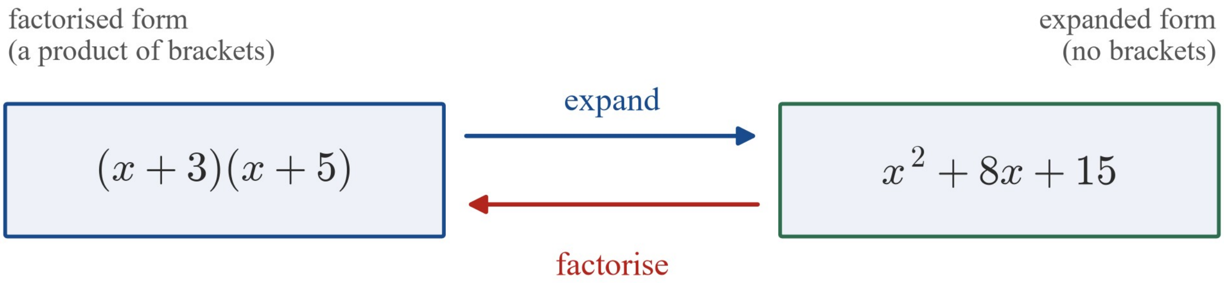
The four term boxes for the product of x plus 5 and x minus 5: x squared, plus 5x, minus 5x and minus 25, with the two middle terms struck out because they cancel, leaving x squared minus 25

$(x+5)(x-5) = x^2+5x-5x-25 = x^2-25$.

The result is one square taken away from another — a **difference of two squares** — and it factorises back the same way: $x^2 - 25 = (x+5)(x-5)$. Any time the x term is missing and the constant term is a negative square, this is the pattern to reach for.

Factorising: expansion in reverse (E03)

Expansion and factorisation are inverse moves — each undoes the other.



The two moves undo each other — each is the check for the other.

Two boxes joined by a pair of arrows: the product of x plus 3 and x plus 5 expands to x squared plus 8x plus 15, and x squared plus 8x plus 15 factorises back to the product of x plus 3 and x plus 5

To factorise a monic quadratic expression $x^2 + mx + n$, run the sum-and-product pattern backwards: find two integers whose sum is m and whose product is n — a **factor pair** — then write the two brackets. The reliable method is a systematic sweep through the factor pairs of n, starting at $1 \times n$ and working inwards, writing each sum until the right one appears. The figure runs the sweep for $x^2 + 9x + 20$.

Factorise $x^2 + 9x + 20$: find two numbers that multiply to 20 and add to 9.

First number	*Second number*	*Sum*	
1	20	21	
2	10	12	
4	5	9	✓ add is 9

$$x^2 + 9x + 20 = (x+4)(x+5)$$

Check by expanding again: $(x+4)(x+5) = x^2 + 9x + 20$. ✓

A factor-pair table for x squared plus 9x plus 20 listing the pairs 1 and 20 with sum 21, 2 and 10 with sum 12, and 4 and 5 with sum 9, the last row highlighted as the pair that works, giving the factorisation as the product of x plus 4 and x plus 5 with an expansion check

The pair $(4, 5)$ multiplies to 20 and adds to 9, so $x^2 + 9x + 20 = (x+4)(x+5)$ — and the check is simply to expand again and get back where you started.

Sign rules for the pair

Before sweeping, read the signs of m and n to know what shape the pair must have; it halves the work.

Form	*Signs of n and m*	*The two numbers*	*Example*
x^2+mx+n	$n>0, m>0$	both numbers positive	$x^2+8x+15=(x+3)(x+5)$
x^2+mx+n	$n>0, m<0$	both numbers negative	$x^2-8x+15=(x-3)(x-5)$
x^2+mx+n	$n<0$	one positive, one negative	$x^2+2x-15=(x+5)(x-3)$

When $n<0$, the sign of m tells you which number has the larger size.

A three-row table for x squared plus mx plus n: when n and m are both positive the two numbers are both positive, example x squared plus 8x plus 15; when n is positive and m negative both numbers are negative, example x squared minus 8x plus 15; when n is negative one number is positive and one negative, example x squared plus 2x minus 15

Constant term positive: the two numbers have the same sign — both positive when $m>0$, both negative when $m<0$. Constant term negative: the numbers have opposite signs, and the sign of m tells you which of the two is the larger in size. When a monic quadratic expression has no integer factor pair at all — the sweep runs out with no sum equal to m — it does not factorise over the integers, and saying so with the completed table as evidence is a full answer.

That completes the theory: expand by multiplying every term by every term and collecting like terms; factorise by hunting the sum-and-product pair; and recognise the two special shapes, perfect squares and differences of two squares, on sight.

Worked examples

Example 1 — scaffolded

Expand $(x+4)(x+7)$.

Working	Comment
$x\times x=x^2$	The first term of each bracket.
$x\times 7=7x$	The x of the first bracket times the 7 of the second.
$4\times x=4x$	The 4 of the first bracket times the x of the second.
$4\times 7=28$	The two constant terms.
$x^2+7x+4x+28=x^2+11x+28$	Collect $7x+4x=11x$.
Check at $x=1$: $(1+4)(1+7)=40$ and $1+11+28=40$	Both sides agree, as they must.

Example 2 — scaffolded

Expand $(x-6)(x+3)$, keeping each sign with its number.

Working	Comment
The numbers are -6 and 3; sum $-6+3=-3$; product $-6\times 3=-18$	Read $x-6$ as $x+(-6)$: the sign travels with the number.
$(x-6)(x+3)=x^2-3x-18$	The pattern $x^2+(\text{sum})x+(\text{product})$.

Working	Comment
Check at $x=1$: $(1-6)(1+3)=-20$ and $1-3-18=-20$	Both sides agree.

Example 3 — scaffolded

Factorise $x^2+9x+20$.

Working	Comment
Need two numbers with sum 9 and product 20	$m=9$, $n=20$; $n>0$ and $m>0$, so both numbers are positive.
$(1,20)\to 21$; $(2,10)\to 12$; $(4,5)\to 9$ ✓	A systematic sweep through the factor pairs of 20.
$x^2+9x+20=(x+4)(x+5)$	The pair $(4,5)$: sum 9, product 20.
Check: $(x+4)(x+5)=x^2+9x+20$	Expanding returns the original — expansion undoes factorisation.

Example 4 — scaffolded

Factorise $x^2-4x-21$.

Working	Comment
Need two numbers with sum -4 and product -21	$n<0$: one number positive, one negative.
$(1,-21)\to -20$; $(-1,21)\to 20$; $(3,-7)\to -4$ ✓	Sweep the pairs of -21; the sum is negative, so the negative number is the larger in size.
$x^2-4x-21=(x+3)(x-7)$	The pair $(3,-7)$.
Check: $(x+3)(x-7)=x^2-4x-21$	Expanding returns the original.

Example 5 — unscaffolded

Expand and simplify $(x+2)(x+6)-x(x+4)$.

Expand the first product: $(x+2)(x+6)=x^2+8x+12$. Expand the second: $x(x+4)=x^2+4x$. Subtract, keeping the second expansion in a bracket: $(x^2+8x+12)-(x^2+4x)=x^2+8x+12-x^2-4x=4x+12$. ∴ $(x+2)(x+6)-x(x+4)=4x+12$ — the x^2 terms cancel and a linear expression is left.

Example 6 — unscaffolded

Factorise x^2-49.

$x^2-49=x^2-7^2$: the x term is missing and the constant term is a negative square — a difference of two squares. The pair is 7 and -7: sum 0, product -49. ∴ $x^2-49=(x+7)(x-7)$.

Example 7 — unscaffolded

Factorise $x^2-12x+36$.

Need two numbers with sum -12 and product 36; $n>0$ and $m<0$, so both are negative. -6 and -6: sum -12, product 36 — the pair repeats, so the expression is a perfect square. ∴ $x^2-12x+36=(x-6)^2$.

Example 8 — unscaffolded

A rectangular garden bed is x m wide and $(x+3)$ m long. Write its area as an expanded expression in x, then evaluate the area when $x=4$.

Area = width × length $=x(x+3)=x^2+3x$ square metres. At $x=4$: $4^2+3\times 4=16+12=28$. Check: width 4 m and length 7 m give $4\times 7=28$ m². ∴ The area is x^2+3x m², and 28 m² when $x=4$.

Practice questions

Scaffolded

Q1. Expand, using the distributive law as in 1B: a. $4(x+5)$ b. $x(x+9)$ c. $-3(x-6)$.

Q2. Expand $(x+2)(x+5)$ with a grid. a. Write the four products $x \times x$, $x \times 5$, $2 \times x$ and 2×5. b. Add them and collect like terms. c. Check at $x=1$ that the bracket form and the expanded form agree.

Q3. Expand $(x+4)(x+6)$ using sum and product. a. State the sum and the product of 4 and 6. b. Hence write the expansion in one line.

Q4. Expand $(x-3)(x+8)$, reading $x-3$ as $x+(-3)$. a. State the sum and the product of -3 and 8. b. Hence write the expansion. c. Check at $x=1$.

Q5. Match each product to its expansion: $(x+7)(x+8)$, $(x+7)(x-8)$, $(x-7)(x+8)$, $(x-7)(x-8)$ against x^2-x-56, $x^2+15x+56$, $x^2-15x+56$, x^2+x-56. a. Which two products have a positive constant term, and why? Match those two first. b. For the remaining two, use the sign of the x-coefficient to finish the matching.

Q6. Expand $(x+6)^2$. a. Write it as a product of two identical brackets. b. Expand. c. Explain in one sentence why $(x+6)^2 \neq x^2+36$.

Q7. Expand $(x+9)(x-9)$. a. Write the four products and the collected result. b. Which two terms cancelled? c. Hence write down $(x+12)(x-12)$ without redoing the grid.

Q8. Expand and simplify $(x+1)(x+4)-(x+2)(x+3)$. a. Expand the first product. b. Expand the second. c. Subtract, keeping the second expansion in a bracket.

Q9. Factorise $x^2+7x+12$. a. List the factor pairs of 12 with their sums. b. Choose the pair whose sum is 7. c. Write the factorisation and check by expanding.

Q10. Factorise $x^2-8x+15$. a. Use the signs of -8 and 15 to state the signs of the two numbers. b. List the possible pairs and their sums. c. Write the factorisation.

Q11. Factorise $x^2+2x-24$. a. Use the sign of -24 to state the signs of the two numbers. b. Sweep the pairs of -24 until a sum of 2 appears. c. Write the factorisation and check by expanding.

Q12. Factorise x^2-81. a. Name the shape of the expression. b. Write the factorisation. c. Check by expanding.

Unscaffolded

Q13. Expand $(x+2)(x+11)$.

Q14. Expand $(x-5)(x+8)$.

Q15. Expand $(x-7)(x-9)$.

Q16. Expand $(x+7)^2$.

Q17. Expand $(x-8)^2$.

Q18. Expand $(x+12)(x-12)$.

Q19. Expand and simplify $(x+3)(x+4)+(x+1)(x+2)$.

Q20. Expand and simplify $(x+5)(x-2)-(x-3)(x+4)$.

Q21. Expand and simplify $x(x-7)+(x+3)(x-2)$.

Q22. Factorise $x^2+8x+12$.

Q23. Factorise $x^2 - 9x + 20$.

Q24. Factorise $x^2 + 3x - 28$.

Q25. Factorise $x^2 - 5x - 36$.

Q26. Factorise $x^2 + 10x + 25$.

Q27. Factorise $x^2 - 64$.

Q28. Factorise $x^2 + 9x$.

Q29. Factorise $x^2 - 13x + 42$.

Q30. Priya writes $(x+3)^2 = x^2 + 9$. Use the square area model to explain her error, and write the correct expansion.

Q31. Find two numbers that add to -1 and multiply to -20, and hence factorise $x^2 - x - 20$.

Q32. A rectangle has area $x^2 + 9x + 18$ cm² and width $(x+3)$ cm. Find its length.

Q33. If $x^2 + mx + 15 = (x+3)(x+5)$, find m.

Q34. A square has side $(x+4)$ cm. Write its area as an expanded expression, then evaluate it at $x = 3$.

Q35. Exactly one of $x^2 + 5x + 6$ and $x^2 + 4x + 5$ factorises over the integers. Show a pair sweep for each to prove which, and factorise the one that does.

Q36. Factorise $x^2 - 11x + 28$ with a full factor-pair table, as in the figure of the theory.

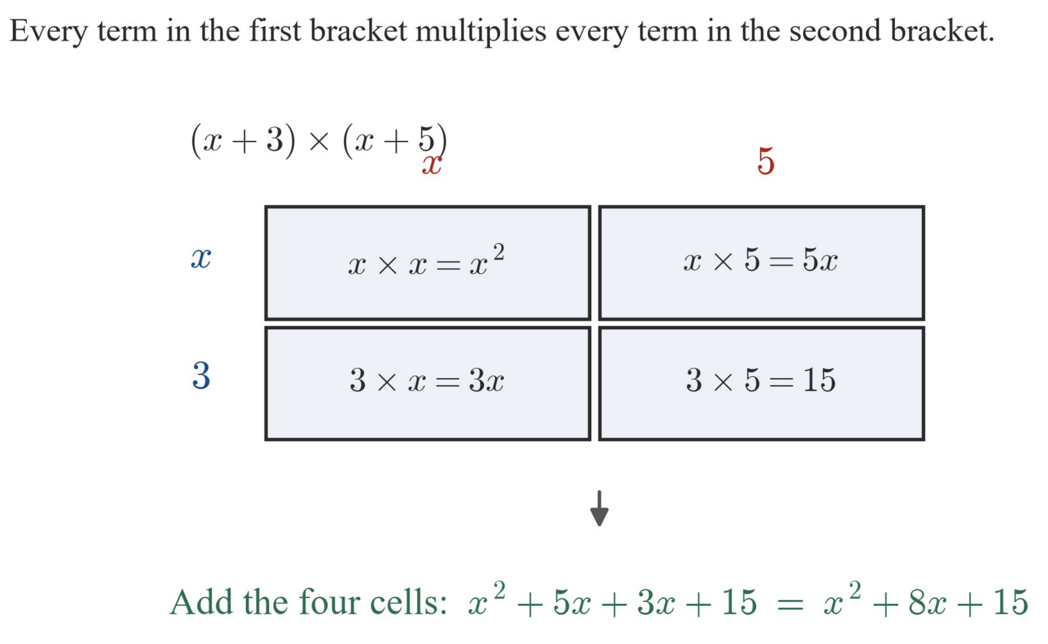
Answers

Q1 a $4x+20$; b x^2+9x; c $-3x+18$. **Q2** a x^2, $5x$, $2x$, 10; b $x^2+7x+10$; c at $x=1$: $(1+2)(1+5)=18$ and $1+7+10=18$. **Q3** a sum 10, product 24; b $x^2+10x+24$. **Q4** a sum 5, product -24; b $x^2+5x-24$; c at $x=1$: $(1-3)(1+8)=-18$ and $1+5-24=-18$. **Q5** $(x+7)(x+8)\to x^2+15x+56$; $(x-7)(x-8)\to x^2-15x+56$; $(x+7)(x-8)\to x^2-x-56$; $(x-7)(x+8)\to x^2+x-56$. **Q6** a $(x+6)(x+6)$; b $x^2+12x+36$; c the two $6x$ cells of the grid — the middle term $12x$ — are left out. **Q7** a $x^2+9x-9x-81=x^2-81$; b $+9x$ and $-9x$; c x^2-144. **Q8** a x^2+5x+4; b x^2+5x+6; c -2. **Q9** a $(1,12)\to 13$, $(2,6)\to 8$, $(3,4)\to 7$; b $(3,4)$; c $(x+3)(x+4)$. **Q10** a both negative; b $(-1,-15)\to -16$, $(-3,-5)\to -8$; c $(x-3)(x-5)$. **Q11** a one positive, one negative; b $(1,-24)\to -23$, $(2,-12)\to -10$, $(3,-8)\to -5$, $(-4,6)\to 2$; c $(x-4)(x+6)$. **Q12** a difference of two squares; b $(x+9)(x-9)$; c $(x+9)(x-9)=x^2+9x-9x-81=x^2-81$. **Q13** $x^2+13x+22$. **Q14** $x^2+3x-40$. **Q15** $x^2-16x+63$. **Q16** $x^2+14x+49$. **Q17** $x^2-16x+64$. **Q18** x^2-144. **Q19** $2x^2+10x+14$. **Q20** $2x+2$. **Q21** $2x^2-6x-6$. **Q22** $(x+2)(x+6)$. **Q23** $(x-4)(x-5)$. **Q24** $(x-4)(x+7)$. **Q25** $(x-9)(x+4)$. **Q26** $(x+5)^2$. **Q27** $(x+8)(x-8)$. **Q28** $x(x+9)$. **Q29** $(x-6)(x-7)$. **Q30** The two $3x$ rectangles are missing: $(x+3)^2=x^2+6x+9$, not x^2+9. **Q31** -5 and 4; $x^2-x-20=(x-5)(x+4)$. **Q32** $(x+6)$ cm. **Q33** $m=8$. **Q34** $x^2+8x+16$ cm²; at $x=3$: 49 cm². **Q35** $x^2+5x+6=(x+2)(x+3)$; x^2+4x+5 does not factorise (pair sums 6 and -6, never 4). **Q36** $(x-4)(x-7)$.

Fully-worked solutions

Q1. a $4(x+5)=4x+20$: the 4 multiplies both terms of the bracket. b $x(x+9)=x^2+9x$: here $x\times x=x^2$, the 1B tool this section leans on. c $-3(x-6)=-3x+18$: note $-3\times -6=+18$. ∴ $4x+20$; x^2+9x; $-3x+18$.

Q2. a The four products: $x\times x=x^2$, $x\times 5=5x$, $2\times x=2x$, $2\times 5=10$. b Add and collect: $x^2+5x+2x+10=x^2+7x+10$. c At $x=1$: bracket form $(3)(6)=18$; expanded form $1+7+10=18$ — equal. ∴ $(x+2)(x+5)=x^2+7x+10$ — the grid of the theory with 2 and 5 in place of 3 and 5.



The multiplication grid of the theory, here with rows and columns headed x, 2 and x, 5

Q3. a Sum $4+6=10$; product $4\times 6=24$. b One line: $(x+4)(x+6)=x^2+10x+24$. ∴ $x^2+10x+24$.

Q4. a The numbers are -3 and 8: sum $-3+8=5$; product $-3\times 8=-24$. b $(x-3)(x+8)=x^2+5x-24$. c At $x=1$: bracket form $(-2)(9)=-18$; expanded form $1+5-24=-18$ — equal. ∴ $x^2+5x-24$.

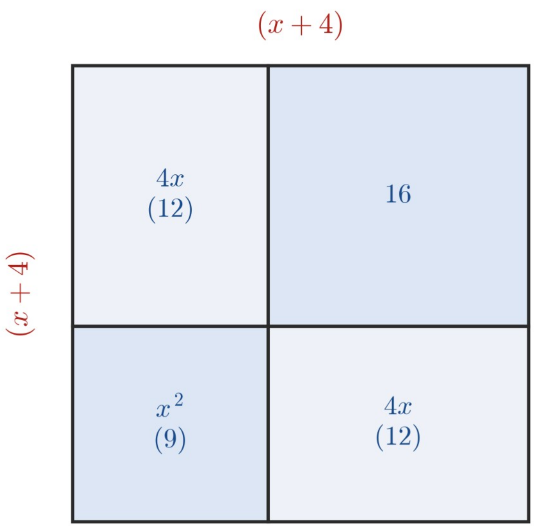
Q5. a A positive constant term needs two numbers of the same sign: $(x+7)(x+8)$ and $(x-7)(x-8)$. Their sums are 15 and -15, so $(x+7)(x+8) \to x^2+15x+56$ and $(x-7)(x-8) \to x^2-15x+56$. b The remaining two have constant term -56; $(x+7)(x-8)$ has sum -1, giving x^2-x-56, and $(x-7)(x+8)$ has sum 1, giving x^2+x-56. ∴ The matching is as in the answer line — exactly the four rows of the sign table in the theory.

Product	*Expansion*	*Sum (with signs)*	*Product (with signs)*	*Signs*
$(x+7)(x+8)$	$x^2+15x+56$	$7+8=15$	$7\times 8=56$	both +
$(x+7)(x-8)$	x^2-x-56	$7+(-8)=-1$	$7\times(-8)=-56$	signs differ
$(x-7)(x+8)$	x^2+x-56	$-7+8=1$	$(-7)\times 8=-56$	signs differ
$(x-7)(x-8)$	$x^2-15x+56$	$-7+(-8)=-15$	$(-7)\times(-8)=56$	both −

The x-coefficient is always the sum; the constant term is always the product.

The four-row sign table of the theory: same four products, same four expansions

Q6. a $(x+6)^2=(x+6)(x+6)$. b Expand: $x^2+6x+6x+36=x^2+12x+36$. c $(x+6)^2 \neq x^2+36$ because the two $6x$ rectangles of the square are part of the area — the middle term $12x$ cannot be dropped. ∴ $x^2+12x+36$.



With $x=3$: side 7, area $49=9+12+12+16$.

$(x+4)^2=(x+4)(x+4)$

$=x^2+4x+4x+16$

$=x^2+8x+16$

The common slip:

$(x+4)^2 \neq x^2+16$

The two $4x$ strips in the middle are easy to forget.

The square area model of the theory: the two middle rectangles are exactly the terms the slip x squared plus 36 forgets

Q7. a $x^2+9x-9x-81=x^2-81$. b $+9x$ and $-9x$ cancelled — same size, opposite signs. c Same shape with 12: $(x+12)(x-12)=x^2-144$. ∴ x^2-81; the cancelled pair $\pm 9x$; x^2-144.

$(x+5)(x-5)$ opens to four terms, and two of them cancel:

$$x^2 \quad \cancel{-5x} \quad \cancel{+5x} \quad -25$$

$$-5x+5x=0$$

↓

$$(x+5)(x-5) \;=\; x^2-25$$

Whenever the two numbers are the same size with opposite signs, the x-terms cancel.

The difference-of-two-squares figure of the theory: the middle terms strike out, leaving x squared minus 25

Q8. a $(x+1)(x+4)=x^2+5x+4$. b $(x+2)(x+3)=x^2+5x+6$. c $(x^2+5x+4)-(x^2+5x+6)=x^2+5x+4-x^2-5x-6=-2$. ∴ The whole expression simplifies to the constant -2; every x term cancels.

Q9. a Pairs of 12: $(1,12)\to 13$; $(2,6)\to 8$; $(3,4)\to 7$. b The pair $(3,4)$ has sum 7. c $x^2+7x+12=(x+3)(x+4)$; check $(x+3)(x+4)=x^2+7x+12$. ∴ $(x+3)(x+4)$.

Factorise $x^2+9x+20$: find two numbers that multiply to 20 and add to 9.

First number	*Second number*	*Sum*	
1	20	21	
2	10	12	
4	5	9	✓ add is 9

↓

$$x^2+9x+20 \;=\; (x+4)(x+5)$$

Check by expanding again: $(x+4)(x+5)=x^2+9x+20$. ✓

The factor-pair table of the theory, run here for 12 instead of 20

Q10. a $n=15>0$ and $m=-8<0$: both numbers negative. b $(-1,-15)\to -16$; $(-3,-5)\to -8$ ✓. c $x^2-8x+15=(x-3)(x-5)$. ∴ $(x-3)(x-5)$.

Q11. a $n=-24<0$: one number positive, one negative. b $(1,-24)\to -23$; $(2,-12)\to -10$; $(3,-8)\to -5$; $(-4,6)\to 2$ ✓. c $x^2+2x-24=(x-4)(x+6)$; check: $x^2+6x-4x-24=x^2+2x-24$. ∴ $(x-4)(x+6)$.

Form	Signs of n and m	The two numbers	Example
x^2+mx+n	$n>0$, $m>0$	both numbers positive	$x^2+8x+15=(x+3)(x+5)$
x^2+mx+n	$n>0$, $m<0$	both numbers negative	$x^2-8x+15=(x-3)(x-5)$
x^2+mx+n	$n<0$	one positive, one negative	$x^2+2x-15=(x+5)(x-3)$

When $n<0$, the sign of m tells you which number has the larger size.

The sign-rules table of the theory, third row: n negative means one positive and one negative

Q12. a $x^2-81=x^2-9^2$: a difference of two squares. b $(x+9)(x-9)$. c Check: $(x+9)(x-9)=x^2+9x-9x-81=x^2-81$. ∴ $(x+9)(x-9)$.

Q13. Sum $2+11=13$; product $2\times11=22$. ∴ $(x+2)(x+11)=x^2+13x+22$.

Q14. Sum $-5+8=3$; product $-5\times8=-40$. ∴ $(x-5)(x+8)=x^2+3x-40$.

Q15. Sum $-7+(-9)=-16$; product $-7\times-9=63$. ∴ $(x-7)(x-9)=x^2-16x+63$.

Q16. $(x+7)^2=x^2+7x+7x+49$. ∴ $x^2+14x+49$ — a perfect square.

Q17. $(x-8)^2=x^2-8x-8x+64$. ∴ $x^2-16x+64$ — a perfect square.

Q18. Difference of two squares: x^2-12^2. ∴ x^2-144.

Q19. $(x+3)(x+4)=x^2+7x+12$ and $(x+1)(x+2)=x^2+3x+2$. Add: $x^2+7x+12+x^2+3x+2=2x^2+10x+14$. ∴ $2x^2+10x+14$.

Q20. $(x+5)(x-2)=x^2+3x-10$ and $(x-3)(x+4)=x^2+x-12$. Subtract: $(x^2+3x-10)-(x^2+x-12)=x^2+3x-10-x^2-x+12=2x+2$. ∴ $2x+2$.

Q21. $x(x-7)=x^2-7x$ and $(x+3)(x-2)=x^2+x-6$. Add: $x^2-7x+x^2+x-6=2x^2-6x-6$. ∴ $2x^2-6x-6$.

Q22. Need sum 8, product 12: $(2,6)$ ✓ since $2+6=8$, $2\times6=12$. ∴ $(x+2)(x+6)$.

Q23. Need sum -9, product 20: both negative; $(-4,-5)$ ✓. ∴ $(x-4)(x-5)$.

Q24. Need sum 3, product -28: opposite signs; $(-4,7)$ ✓ since $-4+7=3$, $-4\times7=-28$. ∴ $(x-4)(x+7)$.

Q25. Need sum -5, product -36: opposite signs; $(-9,4)$ ✓ since $-9+4=-5$, $-9\times4=-36$. ∴ $(x-9)(x+4)$.

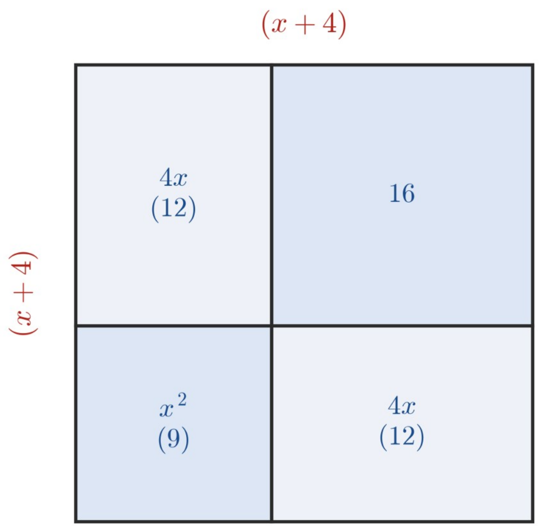
Q26. Need sum 10, product 25: $(5,5)$ ✓ — a repeated pair, so a perfect square. ∴ $(x+5)^2$.

Q27. $x^2-64=x^2-8^2$: difference of two squares. ∴ $(x+8)(x-8)$.

Q28. Both terms share a factor x: $x^2+9x=x(x+9)$ — factorising can begin with a common factor before any pair hunt. ∴ $x(x+9)$.

Q29. Need sum -13, product 42: both negative; $(-6,-7)$ ✓ since $-6+(-7)=-13$, $-6\times-7=42$. ∴ $(x-6)(x-7)$.

Q30. The square of side $x+3$ cuts into x^2, two $3x$ rectangles and 9; Priya's x^2+9 keeps only the corner pieces and drops the two rectangles. Correct expansion: $(x+3)^2=x^2+3x+3x+9=x^2+6x+9$. ∴ $(x+3)^2=x^2+6x+9$; the missing middle term is $6x$.



With $x=3$: side 7, area $49=9+12+12+16$.

$$(x+4)^2=(x+4)(x+4)$$
$$=x^2+4x+4x+16$$
$$=x^2+8x+16$$

The common slip:

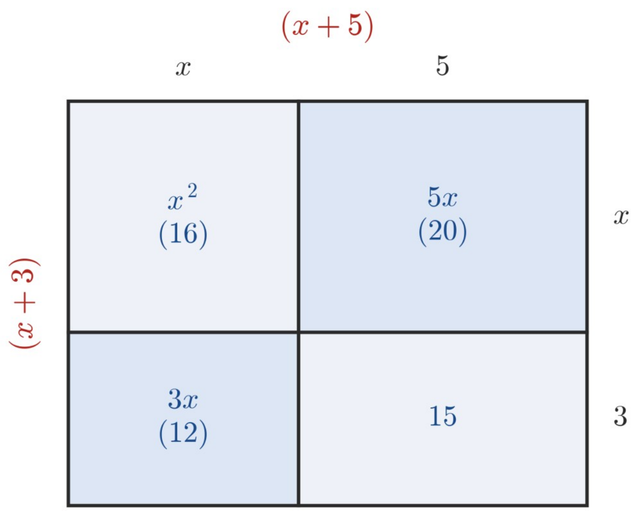
$$(x+4)^2\neq x^2+16$$

The two $4x$ strips in the middle are easy to forget.

The square area model of the theory: Priya's answer keeps the x squared and 16 corner pieces and drops the two middle rectangles

Q31. Product -20 with opposite signs and sum -1: the negative number is the larger in size. -5 and 4: sum -1, product -20 ✓. ∴ $x^2-x-20=(x-5)(x+4)$.

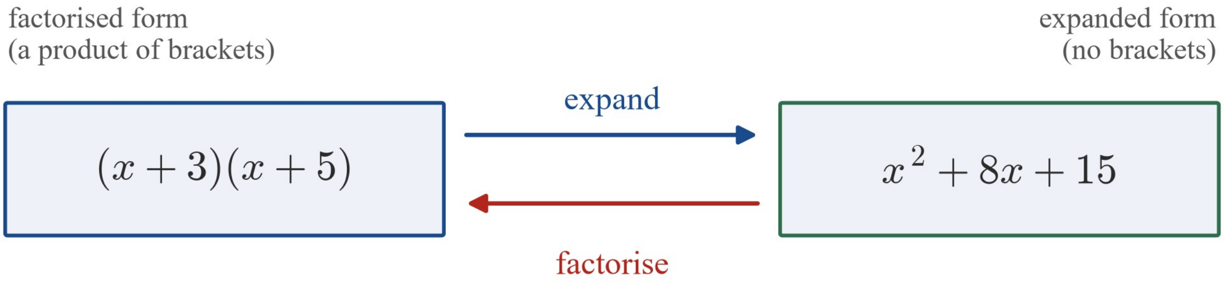
Q32. Length = area ÷ width, so factorise $x^2+9x+18$ with one bracket $(x+3)$: need a pair with product 18 that includes 3: $(3,6)$, sum 9 ✓. $x^2+9x+18=(x+3)(x+6)$. ∴ The length is $(x+6)$ cm.



The area is the sum of the four parts: $x^2+5x+3x+15=x^2+8x+15$. With $x=4$: $7\times9=63=16+20+12+15$.

The area model of the theory: the rectangle x plus 3 by x plus 6 cuts into the four pieces of x squared plus 9x plus 18

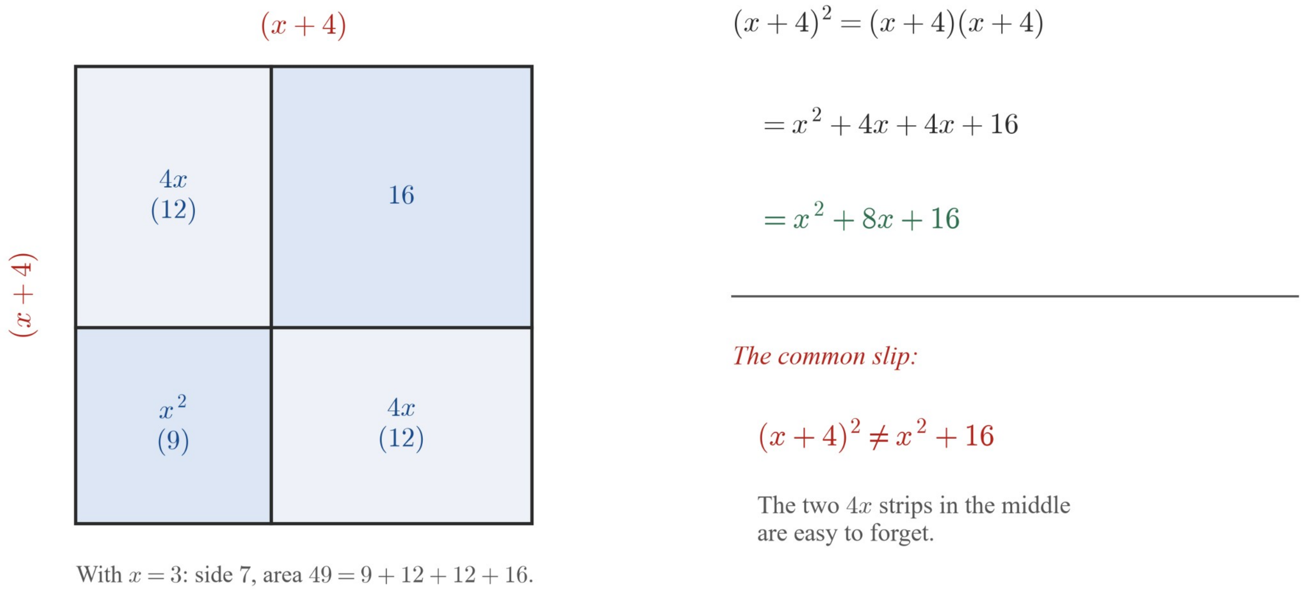
Q33. Expand the right side: $(x+3)(x+5)=x^2+8x+15$. Match the x-coefficient with $x^2+mx+15$: $m=8$. ∴ $m=8$ — expansion and factorisation are inverse moves, so expanding the right side must land exactly on the left.



The two moves undo each other — each is the check for the other.

The inverse figure of the theory: expanding the brackets and factorising the expansion are the same arrow travelled in opposite directions

Q34. Area $=(x+4)^2=x^2+8x+16$ cm². At $x=3$: $9+24+16=49$; check: side 7 cm, area $7\times7=49$ cm². ∴ $x^2+8x+16$ cm², and 49 cm² at $x=3$.



The square area model of the theory at x equals 3: pieces 9, 12, 12 and 16 summing to 49

Q35. For x^2+5x+6: pairs of 6: $(1,6)\to7$; $(2,3)\to5$ ✓ — it factorises as $(x+2)(x+3)$. For x^2+4x+5: pairs of 5: $(1,5)\to6$; $(-1,-5)\to-6$ — no pair sums to 4, and the sweep is complete, so it does not factorise over the integers. ∴ $x^2+5x+6=(x+2)(x+3)$; x^2+4x+5 does not factorise, and the finished table is the proof.

Factorise $x^2 + 9x + 20$: find two numbers that multiply to 20 and add to 9.

First number	*Second number*	*Sum*	
1	20	21	
2	10	12	
4	5	9	✓ add is 9

↓

$$x^2 + 9x + 20 = (x+4)(x+5)$$

Check by expanding again: $(x+4)(x+5) = x^2 + 9x + 20$. ✓

The factor-pair table of the theory: a completed sweep with no matching sum is itself the answer

Q36. Need sum -11, product 28: both negative. Pairs: $(-1,-28) \to -29$; $(-2,-14) \to -16$; $(-4,-7) \to -11$ ✓. $x^2 - 11x + 28 = (x-4)(x-7)$; check: $x^2 - 7x - 4x + 28 = x^2 - 11x + 28$. $\therefore (x-4)(x-7)$.

Factorise $x^2 + 9x + 20$: find two numbers that multiply to 20 and add to 9.

First number	*Second number*	*Sum*	
1	20	21	
2	10	12	
4	5	9	✓ add is 9

↓

$$x^2 + 9x + 20 = (x+4)(x+5)$$

Check by expanding again: $(x+4)(x+5) = x^2 + 9x + 20$. ✓

The factor-pair table of the theory, run here for 28 with both numbers negative

Test

Test T1 · Chapter 1 — Real numbers, exponents, expressions

Mathematics Level 9 (Year 9) · Victorian Curriculum 2.0

Codes assessed: `VC2M9N01` · `VC2M9A01` · `VC2M9A02`

Name: ______________________________ Date: ______________

Total marks	22
Time	27 minutes — 5 minutes reading time, then 1 minute per mark (22 minutes of working time).
Nature of the paper	This is a classroom assessment, not an exam. Attempt every question, and show your working.
Structure	Part A is technology-free. Part B is technology-active.
Equipment	A pen or pencil. Part B also needs a scientific calculator or the digital tool your teacher nominates.
Calculators	No calculator and no digital tool may be used in Part A; Questions 1 and 2 include the without-tools items `VC2M9N01` requires. In Part B a scientific calculator — or the digital tool your teacher nominates — is expected for Questions 5 and 7 and is quickest for Question 6. No CAS anywhere on this paper.

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 9 achievement standard, not against a mark on this paper.

Instructions

- Answer every question, and show your working clearly.
- The marks for each question or part are shown in brackets.
- In Part A, no calculator and no digital tool may be used. Give exact values where a question asks for them.
- In Part B, use your scientific calculator or the digital tool your teacher has nominated. CAS is not permitted.

Subtopic	Content description	Marks
1A The real number system: rational and irrational numbers, exact and approximate	`VC2M9N01`	6
1B The exponent laws with integer and zero exponents, extended to variables	`VC2M9A01`	6
1C Expanding binomial products and factorising monic quadratic expressions	`VC2M9A02`	10
Total		**22**

Part A — technology-free (14 marks)

No calculator and no digital tool may be used in this part.

Question 1 — Rational and irrational numbers (4 marks)

No calculator for this question.

(a) Classify each number as rational or irrational. Give a one-line reason each time. (3 marks)

(b) $\sqrt{36}$

(ii) $\sqrt{19}$

(iii) 0.625

(b) Which one of these numbers is irrational? (1 mark)

A. $\sqrt{49}$

B. $\frac{22}{7}$

C. $\sqrt{20}$

D. $0.3333\ldots$ (the 3 repeats forever)

Question 2 — The exponent laws (5 marks)

No calculator is needed for this question. Give each answer as a single power unless the question asks for a value.

(a) Write each expression as a single power. (2 marks)

(b) $7^3 \times 7^4 \div 7^5$

(ii) $(5^2)^3 \times 5^{-4}$

(b) Evaluate 6^{-2}, giving your answer as a fraction. (1 mark)

(c) Simplify, leaving only positive exponents: $m^5 \times m^{-2} \div m^6$. (1 mark)

(d) One gigabyte is 10^9 bytes and one megabyte is 10^6 bytes. Use a quotient-law step to find how many megabytes are in one gigabyte. (1 mark)

Question 3 — Expanding binomial products (3 marks)

(a) Expand $(x+9)(x-5)$. (1 mark)

(b) Expand $(x-9)^2$. (1 mark)

(c) Expand and simplify $(x+2)(x+8)-x(x+10)$. (1 mark)

Question 4 — Factorising monic quadratic expressions (2 marks)

(a) Factorise x^2+6x+8. (1 mark)

(b) Factorise x^2-121. (1 mark)

Part A total: 14 marks

Part B — technology-active (8 marks)

Use your scientific calculator or the digital tool your teacher has nominated. No CAS.

Question 5 — Exact and approximate (2 marks)

A circle has radius 3.5 cm.

(a) Write the area of the circle exactly, in terms of π. (1 mark)

(b) Use your tool to give the area as a decimal, correct to two decimal places. (1 mark)

Question 6 — A negative exponent as a decimal (1 mark)

Use your tool to write 5^{-6} as an ordinary decimal. Do not leave your answer as a power of ten.

Question 7 — A netball court extended (5 marks)

A full-size netball court is a rectangle 30.5 m long and 15.25 m wide.

Source: Netball Australia, "Official rules of netball", 2024. Retrieved 21 August 2026.

SCENARIO — a Victorian state secondary college is rebuilding its netball court. The new court is x metres longer and x metres wider than a full-size court.

(a) Find the area of a full-size court. (1 mark)

(b) Write an expression for the area of the new court, and expand it. (2 marks)

(c) Use your tool to find the area of the new court when $x = 2.5$. (1 mark)

(d) When $x = 2.5$, how much greater is the area of the new court than the area of a full-size court? (1 mark)

Part B total: 8 marks

Total: 22 marks

Solutions

Teacher-facing. Test T1 is a classroom assessment, not an exam: 27 minutes in total, made up of 5 minutes reading time and 1 minute per mark. Part A is technology-free: Questions 1 and 2 are done by hand, and they carry the without-tools items `VC2M9N01` requires, since its content description asks for problems solved with *and* without digital tools (D9). Part B is technology-active: a scientific calculator — or the tool nominated by the teacher — is expected for Questions 5 and 7 and is quickest for Question 6, and in every case the tool does the decimal arithmetic while the exact form, the expansion and the factorisation stay by-hand work. The calculator status of every question follows the content description behind it, and no CAS is permitted anywhere on the paper (R8).

Coverage and mark map

CD	Subtopic	Part	Questions	Marks
`VC2M9N01`	1A The real number system: rational and irrational numbers, exact and approximate	A and B	Q1 (A), Q5 (B)	6
`VC2M9A01`	1B The exponent laws with integer and zero exponents, extended to variables	A and B	Q2 (A), Q6 (B)	6
`VC2M9A02`	1C Expanding binomial products and factorising monic quadratic expressions	A and B	Q3, Q4 (A), Q7 (B)	10
	Total			**22**

Part A carries 14 marks and Part B carries 8 marks, per the §7 plan. Multiple choice is 1 mark of 22 — Q1(b), four options A–D — used where the judgement genuinely is a selection: picking the irrational number from four candidates. The one multi-part question in a real Victorian context is Q7 (5 marks), carrying its R1 source line; the court dimensions are real and sourced, and the extension by x metres is a labelled SCENARIO built on them (R2).

Marking guide

Criteria are written in the language of the achievement-standard sentence each content description answers to, as corrected by §4 of the plan of record.

CD	Achievement-standard sentence (§4 corrected anchor)	Where it is demonstrated on this paper
`VC2M9N01`	AS 1 — "Students recognise and use rational and irrational numbers to solve problems."	Recognise: Q1 classifies numbers and selects the irrational one. Use: Q5 solves a circle problem exactly and approximately — Q1 the without-tools path, Q5 the with-tools path, as the CD requires (D9).
`VC2M9A01`	AS 2 — "Students extend and apply the exponent laws with positive integers and the zero exponent to variables." † **D6:** the extract reads "positive integers", but the content description reads "integer exponents", and the content description governs — negative integer exponents are assessed here.	Q2 applies the laws to numbers and variables, including negative exponents; Q6 evaluates a negative exponent as a decimal with a tool.
`VC2M9A02`	AS 3 — "They expand binomial products and factorise monic quadratic expressions."	Q3 expands binomial products, including a perfect square and a simplification to a constant; Q4 factorises a monic trinomial and a difference of two squares; Q7 expands a binomial product in a real context and reads the result back into the context.

Scope note (D3): every quadratic expression on this paper is monic and every factorisation has integer constants; there is no quadratic formula, no completing the square and no non-monic factorisation anywhere on the paper.

Answer key

Q	Part	Answer	Marks
1	(a)(i)	rational — $\sqrt{36}=6=\frac{6}{1}$	1
1	(a)(ii)	irrational — 19 is not a perfect square ($16<19<25$)	1
1	(a)(iii)	rational — the decimal terminates; $0.625=\frac{5}{8}$	1
1	(b)	C ($\sqrt{20}$)	1
2	(a)(i)	7^2	1
2	(a)(ii)	5^2	1
2	(b)	$\frac{1}{36}$	1
2	(c)	$\frac{1}{m^3}$	1
2	(d)	$10^3=1000$ megabytes	1

Q	Part	Answer	Marks
3	(a)	$x^2+4x-45$	1
3	(b)	$x^2-18x+81$	1
3	(c)	16	1
4	(a)	$(x+2)(x+4)$	1
4	(b)	$(x+11)(x-11)$	1
5	(a)	12.25π cm² (accept $\frac{49\pi}{4}$ cm²)	1
5	(b)	38.48 cm²	1
6		0.000064	1
7	(a)	465.125 m²	1
7	(b)	$(30.5+x)(15.25+x)=x^2\cdot$	2
7	(c)	585.75 m²	1
7	(d)	120.625 m²	1
	Total		**22**

Fully worked solutions

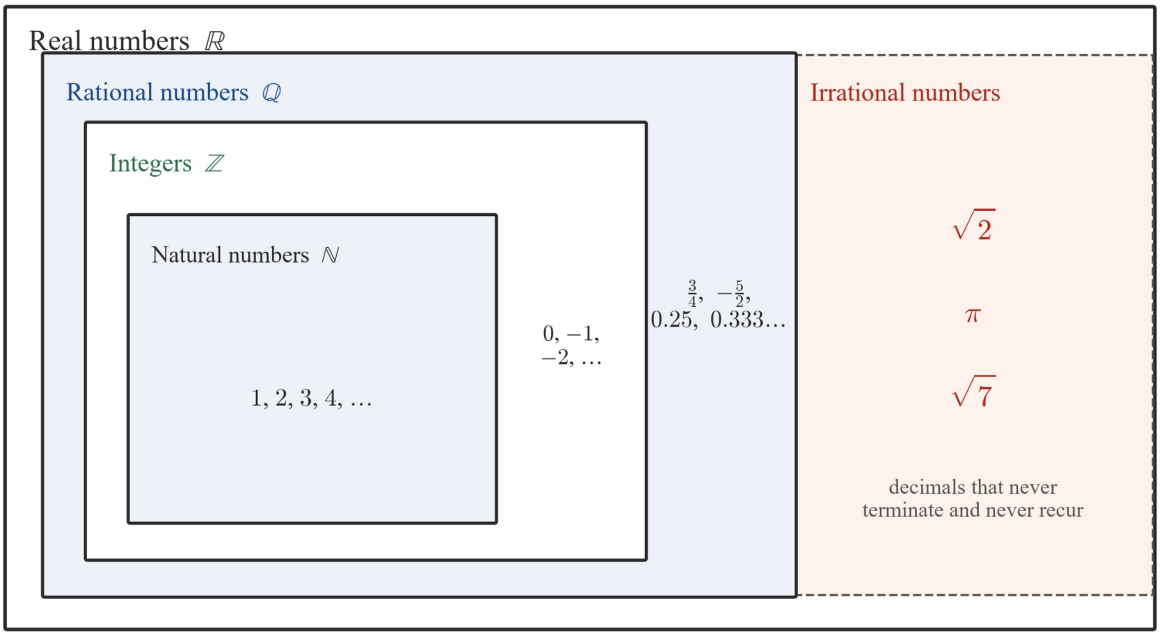
Q1. (a) The families of the real number system decide each label, and the reason is one line.

(i) $\sqrt{36}=6$, and $6=\frac{6}{1}$ is a ratio of integers with a non-zero denominator. ∴ $\sqrt{36}$ is rational.

(ii) $4^2=16$ and $5^2=25$, and $16<19<25$, so $\sqrt{19}$ sits between 4 and 5. The square root of a whole number is rational only when that number is a perfect square, and 19 is not one. ∴ $\sqrt{19}$ is irrational.

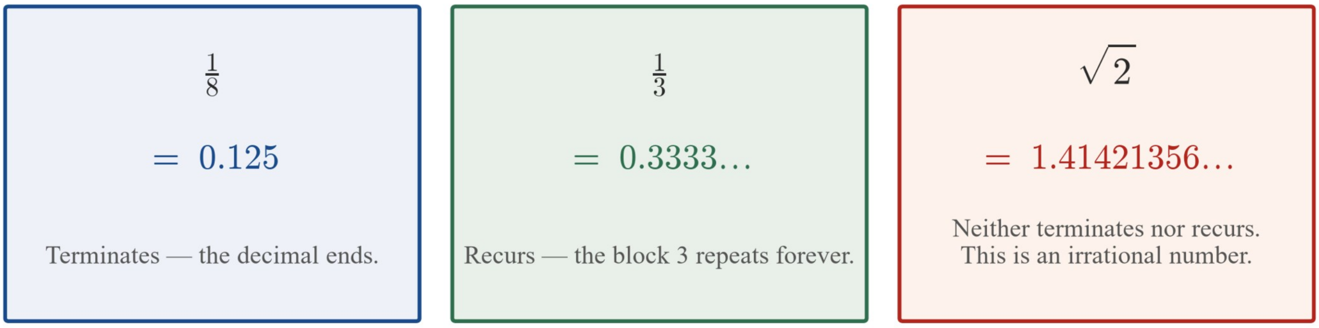
(iii) 0.625 terminates, and every terminating decimal is a fraction: $0.625=\frac{625}{1000}=\frac{5}{8}$. ∴ 0.625 is rational.

(b) Test the four candidates: $\sqrt{49}=7$, rational; $\frac{22}{7}$ is a ratio of integers, rational; $0.3333\ldots$ recurs with block 3, so it is rational ($=\frac{1}{3}$); and $16<20<25$ with 20 not a perfect square, so $\sqrt{20}$ is irrational. ∴ The answer is **C**.



Every real number is either rational or irrational — the two groups never overlap. Each smaller family sits entirely inside the one around it.

Nested boxes: the natural numbers inside the integers, the integers inside the rational numbers, and the rationals inside the real numbers, with the irrational numbers filling the rest of the real box



A rational number always gives a decimal that terminates or recurs. An irrational number gives neither.

Three cards: one eighth equals 0.125 and terminates; one third equals 0.3333 recurring, with the block 3 repeating forever; the square root of 2 equals 1.41421356 continuing, which neither terminates nor recurs, so it is an irrational number

Marks: (a) 1 mark for each number — the label and the reason are both required, and a correct label with no reason scores 0. (b) 1 mark for C.

Q2. All the laws need the same base, and each law changes only the exponent, never the base.

(a)

(i) Product law, then quotient law — add, then subtract, the exponents:

$7^3 \times 7^4 \div 7^5 = 7^{3+4-5} = 7^2$

$\therefore 7^2$.

(ii) Power of a power first, then the product law:

$(5^2)^3 = 5^6$

$5^6 \times 5^{-4} = 5^{6+(-4)} = 5^2$

$\therefore 5^2$.

(b) A negative exponent names the reciprocal of the positive power:

$$6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

$\therefore \frac{1}{36}$.

(c) Collect the exponents on m, then rewrite with a positive exponent:

$$m^5 \times m^{-2} \div m^6 = m^{5+(-2)-6} = m^{-3}$$

$$m^{-3} = \frac{1}{m^3}$$

$\therefore \frac{1}{m^3}$.

(d) Divide the powers of ten — one gigabyte over one megabyte:

$$\frac{10^9}{10^6} = 10^{9-6} = 10^3 = 1000$$

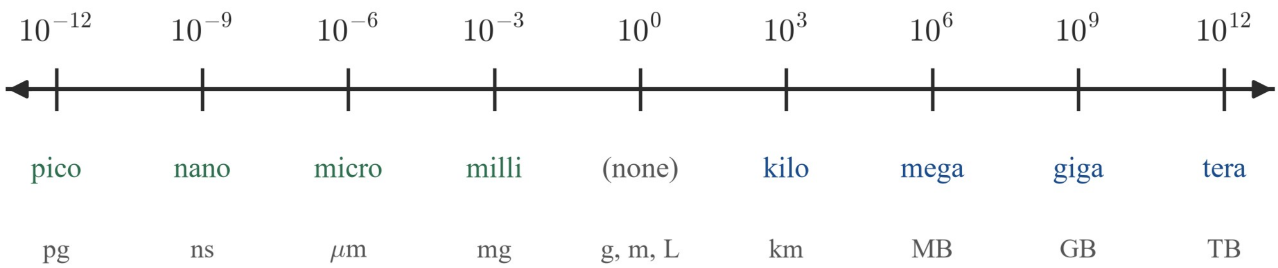
$\therefore$ There are 1000 megabytes in one gigabyte.

Law	*In general*	*Example*
Product law	$a^m \times a^n = a^{m+n}$	$2^3 \times 2^4 = 2^7$
Quotient law	$a^m \div a^n = a^{m-n}$	$3^7 \div 3^2 = 3^5$
Power of a power	$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
Power of a product	$(ab)^n = a^n b^n$	$(2 \times 3)^4 = 2^4 \times 3^4$
Power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{6}{2}\right)^3 = \frac{6^3}{2^3}$
Zero exponent	$a^0 = 1 \quad (a \neq 0)$	$9^0 = 1$
Negative exponent	$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$	$2^{-3} = \frac{1}{8}$

These hold for every integer exponent (positive, zero or negative) and every non-zero base.

Table of the seven exponent laws, general form and one numerical example per row: product law, quotient law, power of a power, power of a product, power of a quotient, zero exponent, and negative exponent.

Multiplying by a prefix means multiplying by a power of 10. Steps of $\times 1000$ are shown.



Ladder of powers of 10 from 10 to the negative 12 up to 10 to the 12, labelled with the SI prefixes pico, nano, micro, milli, then the base units, then kilo, mega, giga, tera, with example units such as pg, ns, µm, mg, km, MB, GB and TB.

Marks: 1 mark for each part; accept any correct ordering of the laws. In (c) the final form must carry a positive exponent to score; in (d) the quotient-law step must be shown, not just 1000.

Q3. Each product is expanded term by term, then like terms are collected.

(a) $(x+9)(x-5)=x^2-5x+9x-45$

$=x^2+4x-45$

$\therefore\ x^2+4x-45.$

(b) Square of a binomial — first squared, twice the product, last squared:

$(x-9)^2=x^2-9x-9x+81$

$=x^2-18x+81$

$\therefore\ x^2-18x+81.$

(c) Expand both products, then subtract the second from the first:

$(x+2)(x+8)=x^2+8x+2x+16=x^2+10x+16$

$x(x+10)=x^2+10x$

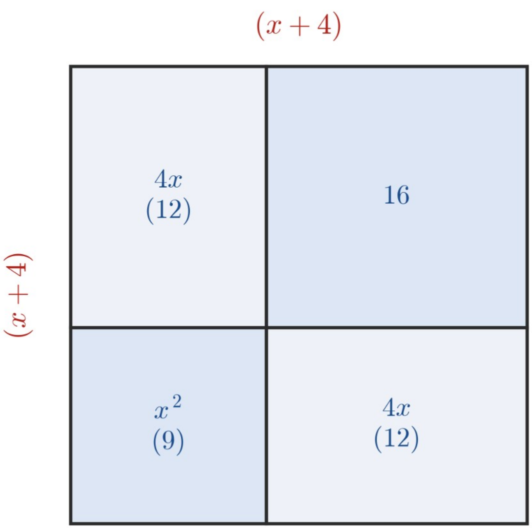
$(x^2+10x+16)-(x^2+10x)=x^2+10x+16-x^2-10x=16$

$\therefore\ 16$ — the x-terms cancel, so the expression is the same number for every x.

Product	Expansion	Sum (with signs)	Product (with signs)	Signs
$(x+7)(x+8)$	$x^2+15x+56$	$7+8=15$	$7\times 8=56$	both +
$(x+7)(x-8)$	x^2-x-56	$7+(-8)=-1$	$7\times(-8)=-56$	signs differ
$(x-7)(x+8)$	x^2+x-56	$-7+8=1$	$(-7)\times 8=-56$	signs differ
$(x-7)(x-8)$	$x^2-15x+56$	$-7+(-8)=-15$	$(-7)\times(-8)=56$	both −

The x-coefficient is always the sum; the constant term is always the product.

A four-row table expanding the products of x plus 7 and x plus 8, x plus 7 and x minus 8, x minus 7 and x plus 8, and x minus 7 and x minus 8, each row showing the expansion, the sum of the two numbers taken with signs, their product taken with signs, and the resulting sign pattern



With $x=3$: side 7, area $49=9+12+12+16$.

$(x+4)^2=(x+4)(x+4)$

$=x^2+4x+4x+16$

$=x^2+8x+16$

The common slip:

$(x+4)^2\neq x^2+16$

The two $4x$ strips in the middle are easy to forget.

A square of side x plus 4 drawn to scale at x equals 3, cut into pieces 9, 12, 12 and 16 giving x squared plus 8x plus 16, beside a panel marking the common slip that the square of x plus 4 is not x squared plus 16

Marks: 1 mark for each part, fully collected and simplified.

Q4. (a) Look for the pair with product 8 and sum 6: 2 and 4.

$x^2+6x+8=(x+2)(x+4)$

Check by expanding: $(x+2)(x+4)=x^2+4x+2x+8=x^2+6x+8$. ✓

$\therefore$ $(x+2)(x+4)$.

(b) A difference of two squares: $121=11^2$.

$x^2 - 121 = x^2 - 11^2 = (x+11)(x-11)$

$\therefore (x+11)(x-11)$.

Factorise $x^2 + 9x + 20$: find two numbers that multiply to 20 and add to 9.

First number	*Second number*	*Sum*	
1	20	21	
2	10	12	
4	5	9	✓ add is 9

$$x^2 + 9x + 20 \;=\; (x+4)(x+5)$$

Check by expanding again: $(x+4)(x+5) = x^2 + 9x + 20$. ✓

A factor-pair table for x squared plus 9x plus 20 listing the pairs 1 and 20 with sum 21, 2 and 10 with sum 12, and 4 and 5 with sum 9, the last row highlighted as the pair that works, giving the factorisation as the product of x plus 4 and x plus 5 with an expansion check

$(x+5)(x-5)$ opens to four terms, and two of them cancel:

$$\boxed{x^2} \quad \boxed{\cancel{-5x}} \quad \boxed{\cancel{+5x}} \quad \boxed{-25}$$

$-5x + 5x = 0$

$$(x+5)(x-5) \;=\; x^2 - 25$$

Whenever the two numbers are the same size with opposite signs, the x-terms cancel.

The four term boxes for the product of x plus 5 and x minus 5: x squared, plus 5x, minus 5x and minus 25, with the two middle terms struck out because they cancel, leaving x squared minus 25

Marks: 1 mark for each factorisation; accept any correct factorised form with integer constants.

Q5. Radius $r = 3.5$ cm, and $A = \pi r^2$.

(a) $A = \pi \times 3.5^2$

$3.5^2 = 12.25$

$\therefore\ A = 12.25\pi$ cm² exactly. (Accept $\dfrac{49\pi}{4}$ cm².)

(b) Keep the exact form until the last step, use the tool, and round once:

$12.25\pi = 38.48451\ldots$

$\therefore\ A \approx 38.48$ cm² (2 d.p.).

Exact form — keep this	*Calculator display*	*Rounded once, at the end*
$C = 6\pi$ cm	$18.84955592\ldots$	≈ 18.85 cm (2 d.p.)
$A = 9\pi\ \text{cm}^2$	$28.27433388\ldots$	$\approx 28.27\ \text{cm}^2$ (2 d.p.)

The decimal is an approximation of the exact value. Keep π in the working, round once at the end, and state the rounding.

Two rows of three boxes: exact form, calculator display, and the value rounded once at the end, for a circumference and an area in terms of pi

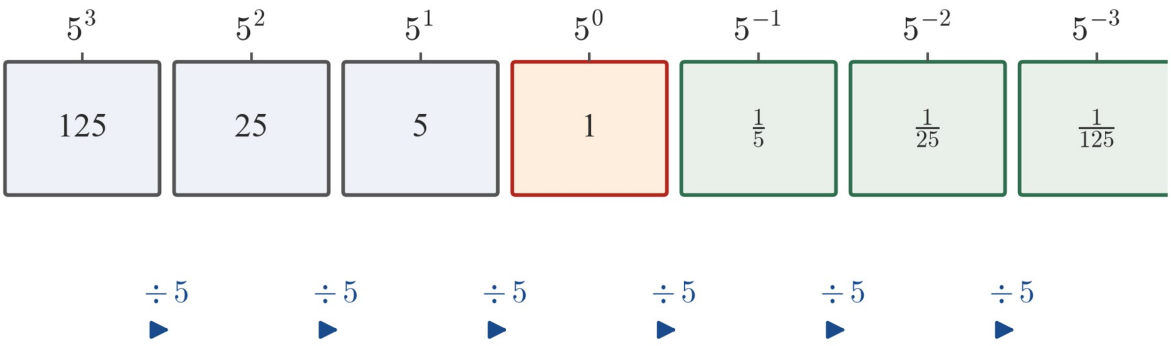
Marks: (a) 1 mark for the exact π form — a decimal scores 0 here. (b) 1 mark for the correctly rounded decimal; accept the working from either 12.25π *or* $\dfrac{49\pi}{4}$.

Q6. The negative exponent names a reciprocal:

$$5^{-6} = \frac{1}{5^6} = \frac{1}{15625}$$

With the tool: $1 \div 15625 = 0.000064$.

$\therefore\ 5^{-6} = 0.000064$.



Each step to the right divides by 5. Past $5^0 = 1$ the values become fractions, and the exponents become negative.

Powers of 5 from 5 cubed down to 5 to the power of negative 3, with values 125 down to 1 over 125 and a divide-by-5 arrow between each step.

Marks: 1 mark for 0.000064; a power of ten scores 0 — the question asked for an ordinary decimal.

Q7. The full-size dimensions are the sourced real data; the extension is the labelled SCENARIO.

(a) Area of the full-size court:

$A = 30.5 \times 15.25$

$= 465.125$

$\therefore$ The full-size court has area 465.125 m².

(b) The new length is $30.5 + x$ and the new width is $15.25 + x$:

$A = (30.5 + x)(15.25 + x)$

$= 30.5 \times 15.25 + 30.5x + 15.25x + x^2$

$= 465.125 + 45.75x + x^2$

$\therefore A = x^2 + 45.75x + 465.125$ m².

(c) Substitute $x = 2.5$ into the expanded form and use the tool:

$A = 2.5^2 + 45.75 \times 2.5 + 465.125$

$= 6.25 + 114.375 + 465.125$

$= 585.75$

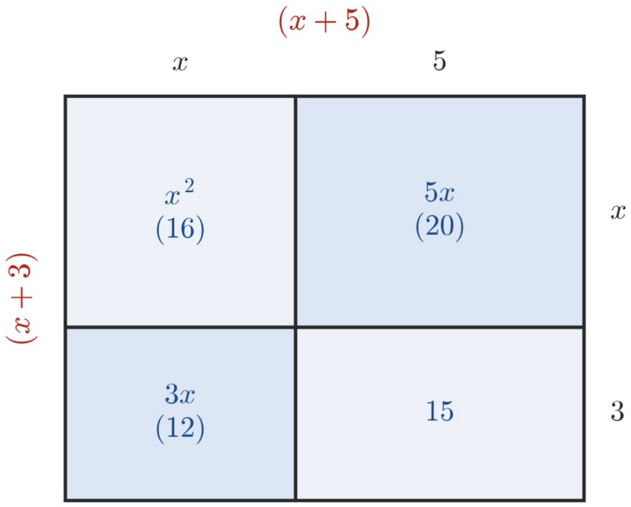
Check from the factorised form: $(30.5 + 2.5)(15.25 + 2.5) = 33 \times 17.75 = 585.75$. ✓

$\therefore$ The new court has area 585.75 m².

(d) Increase = new area − original area:

$585.75 - 465.125 = 120.625$

$\therefore$ The new court is 120.625 m² greater than a full-size court.



The area is the sum of the four parts: $x^2+5x+3x+15=x^2+8x+15$. With $x=4$: $7\times 9=63=16+20+12+15$.

A rectangle of width x plus 3 and length x plus 5 drawn to scale at x equals 4, cut into four pieces labelled 16, 20, 12 and 15, whose sum 63 equals the whole area 7 times 9

Marks: (a) 1 mark for 465.125 m². (b) 2 marks — 1 for the factorised product $(30.5+x)(15.25+x)$ *and 1 for the correct expansion; a slip in one of the four products costs the expansion mark only. (c) 1 mark for 585.75 m². (d) 1 mark for 120.625 m².*