Mathematics Level 8

Chapter 7 — Probability: complementary events and combinations of two events

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Contents

Complementary events

> **Content description `VC2M8P01`:** recognise that complementary events have a combined probability of one; use this relationship to calculate probabilities in applied contexts

Every weather forecast makes two statements at once. "There is an 80% chance of rain tomorrow" is also telling you the chance that it does not rain — you need only one fact about probability to find it. This subtopic names that pair of statements, an event and its ***complement****, and turns the fact into a rule you can use with fractions, decimals and percentages.*

Theory

Recall: probability from Year 7

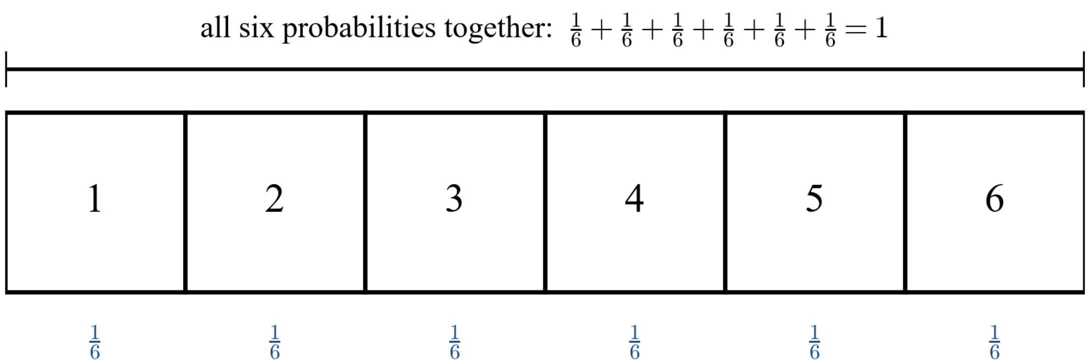
In Year 7 (`VC2M7P01`) you ran single-stage experiments — rolling a dice, spinning a spinner, drawing a card — and listed every possible result. That complete list is called the **sample space**. An **event** is a result, or a collection of results, that we are interested in, and a **probability** is the number from 0 to 1 that measures how likely an event is to happen.

Two facts from Year 7 sit underneath everything in this subtopic.

Fact 1. When the outcomes are **equally likely**, they share the probability equally. A fair six-sided dice has six equally likely outcomes, so each outcome has probability $\frac{1}{6}$.

Fact 2. All the probabilities in a sample space add to 1. For the fair dice:

$$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=1$$



Six faces of a fair dice, each labelled with its probability one-sixth, with a bracket underneath all six showing that the probabilities add to one

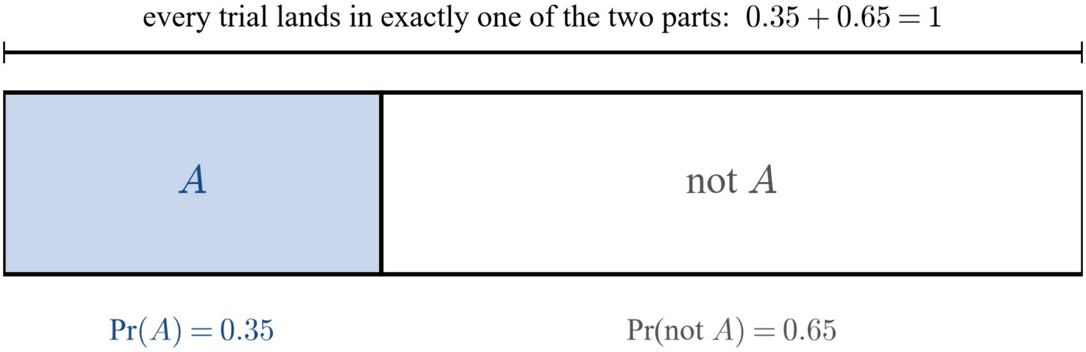
An event that cannot happen has probability 0 — it is **impossible**. An event that must happen has probability 1 — it is **certain**. Every other probability sits between 0 and 1.

What complementary events are

Think again about the fair dice. Call the event "roll a 6" event A. Its opposite, "do not roll a 6", is called the **complement** of A, written **not** A. These two events have a special relationship:

- they can never happen together — a single roll cannot be a 6 and not a 6;
- between them they cover every outcome — a roll is either a 6 or not a 6, with no third possibility;
- so exactly one of them must happen on every roll.

> **Definition used in this subtopic.** Two events are **complementary** if exactly one of them must happen every time. The complement of an event A is the event that A does not happen, written **not** A.



A bar representing all possible outcomes, split into two parts: an event labelled A on the left and its complement labelled not A on the right, together making the whole bar

The bar in the diagram stands for all the outcomes — the whole, with probability 1. The event A and its complement not A share out that whole between them, with nothing left over and no overlap. If one part has probability p, the other part has the rest.

The impossible event and the certain event are complements of each other: if $Pr(A)=0$, then not A must happen, so $Pr(\text{not } A)=1$ — and the other way around.

The rule

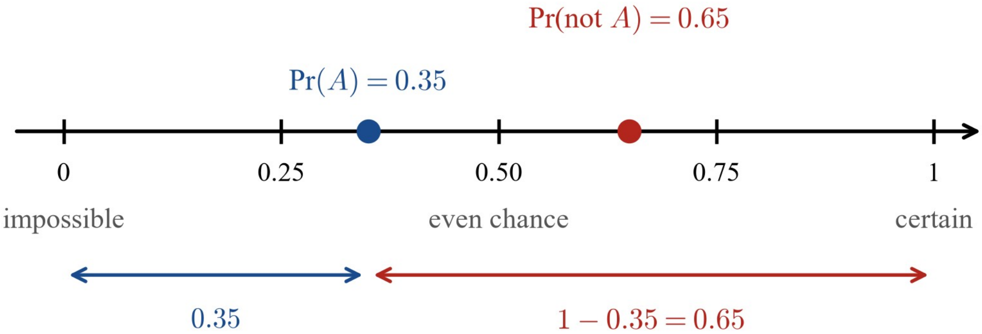
Because A and not A share out the whole, their probabilities add to 1:

$$Pr(A)+Pr(\text{not } A)=1$$

Rearranging gives the two forms you will use most often:

$Pr(\text{not } A)=1-Pr(A)$ and $Pr(A)=1-Pr(\text{not } A)$

The rule works in fractions, decimals and percentages — just keep the whole as 1, as 1.00 or as $100\,\%$, to match the form the question gives you.

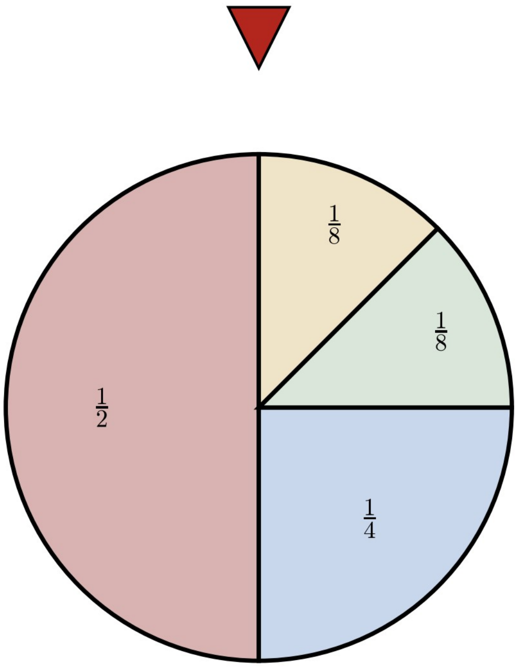


A number line from zero to one with a dot at 0.35 labelled Pr(A) in blue and a dot at 0.65 labelled Pr(not A) in red, with arrows underneath showing that 0.35 plus one minus 0.35 equals 0.65 makes the whole distance from zero to one

In the diagram, $Pr(A)=0.35$. The complement is the rest of the way from that dot to 1: $Pr(\text{not } A)=1-0.35=0.65$.

The rule works even when outcomes are not equally likely

Look back at the rule. It does not ask the outcomes to be equally likely. It only asks the two events to share out the whole. So the rule works just as well for spinners, promotions and weather forecasts, where some outcomes are more likely than others.



the red sector is half the spinner, so red wins half the trials

A spinner divided into four unequal wedges: red one-half, blue one-quarter, green one-eighth and yellow one-eighth, with a pointer above it

This spinner has four wedges: red $\frac{1}{2}$, blue $\frac{1}{4}$, green $\frac{1}{8}$ and yellow $\frac{1}{8}$. The outcomes are not equally likely — the red wedge is twice as likely as the blue wedge. Still, “red” and “not red” are complementary events, so

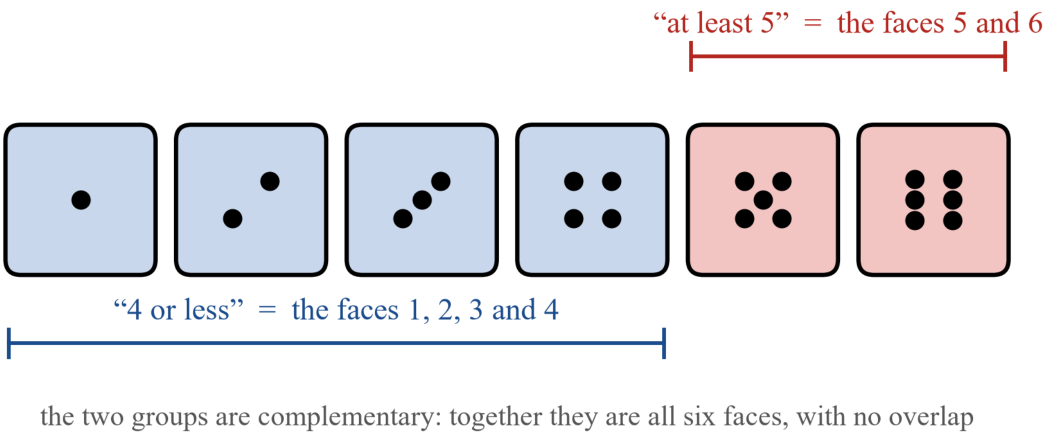
$$Pr(\text{not red}) = 1 - Pr(\text{red}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Notice what you did not need: you did not need to add up the blue, green and yellow probabilities. The rule goes straight from $Pr(A)$ to $Pr(\text{not } A)$.

Saying the complement

In probability questions, the complement is often described in words rather than with “not”. Learning the matching pairs saves time and stops mistakes.

An event …	has the complement …
at least 5	4 or less
more than 2	2 or less
less than 6	at least 6 (6 or more)
win	not win (draw or lose)

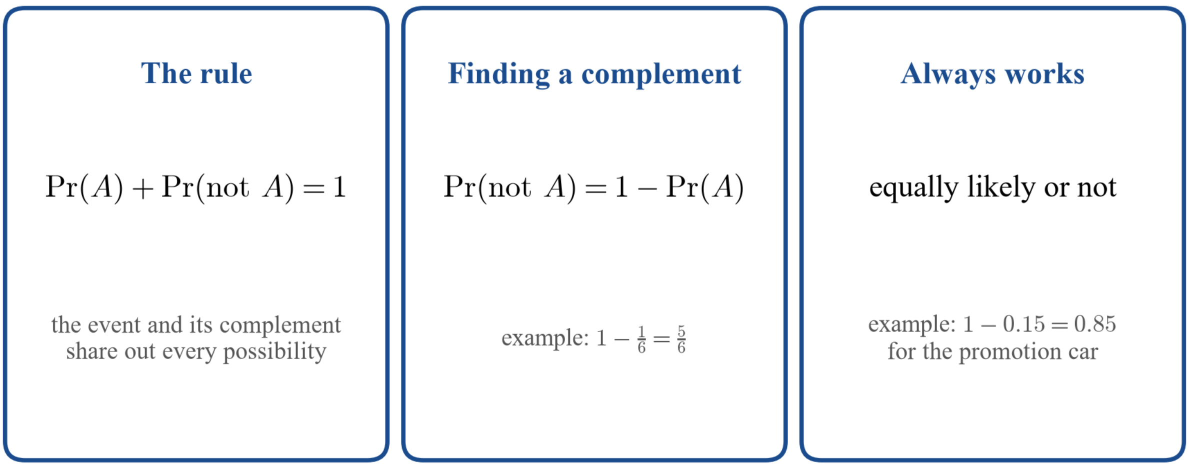


Two rows of dice faces: the top row brackets the faces 5 and 6 under the label at least 5, and the bottom row brackets the faces 1, 2, 3 and 4 under the label 4 or less, showing that the two events are complementary

In the diagram, "at least 5" means 5 or more, so its outcomes are 5 and 6. Its complement is "4 or less", with outcomes 1, 2, 3 and 4. Between them the two events use all six faces exactly once — so they are complementary, and their probabilities add to 1.

Watch the boundary. "At least 5" includes 5, so its complement must stop at 4. "More than 2" does not include 2, so its complement must include 2. When in doubt, list the outcomes and check that the two lists share out the sample space with no overlap.

At a glance



Three summary cards: the first states the rule Pr(A) plus Pr(not A) equals one, the second states that to find a complement you subtract the probability from one, and the third states that the rule always works because an event and its complement share out all the outcomes

- **The rule:** $Pr(A) + Pr(\text{not } A) = 1$.
- **Finding a complement:** subtract the probability from 1, in whatever form it is given — fraction, decimal or percentage.
- **Why it always works:** an event and its complement never happen together, and between them they cover every outcome. Equally likely or not, they share out the whole.

What comes next

In 7B you will combine two events — "or", "and" and "at least one" — using two-way tables, tree diagrams and Venn diagrams (`VC2M8P02`), and the rule from this subtopic still powers those calculations; at Level 9 the same rule is used again inside two-stage experiments (`VC2M9P01`).

Words of this subtopic

Word	What it means here
outcome	one possible result of an experiment
sample space	the complete list of all possible outcomes
event	an outcome, or a collection of outcomes, that we are interested in
probability	the number from 0 to 1 that measures how likely an event is
equally likely	outcomes that all have the same chance of happening
complementary events	two events where exactly one of them must happen every time
complement of A (not A)	the event that A does not happen
impossible	probability 0 — the event cannot happen
certain	probability 1 — the event must happen

Worked examples

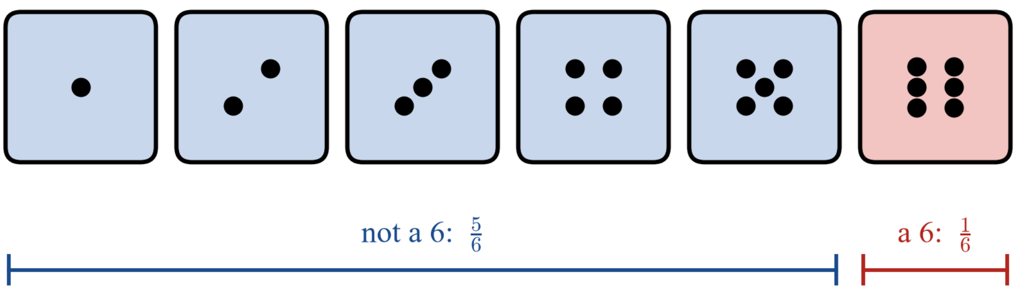
Example 1 — with help

A fair six-sided dice is rolled once. Event A is "roll a 6".

(a) Find $Pr(A)$.

(b) Find $Pr(\text{not } A)$.

one face out of six is a 6; the other five are not



Six faces of a fair dice, with the face showing 6 marked red and labelled a 6 probability one-sixth, and the five other faces marked blue and labelled not a 6 probability five-sixths

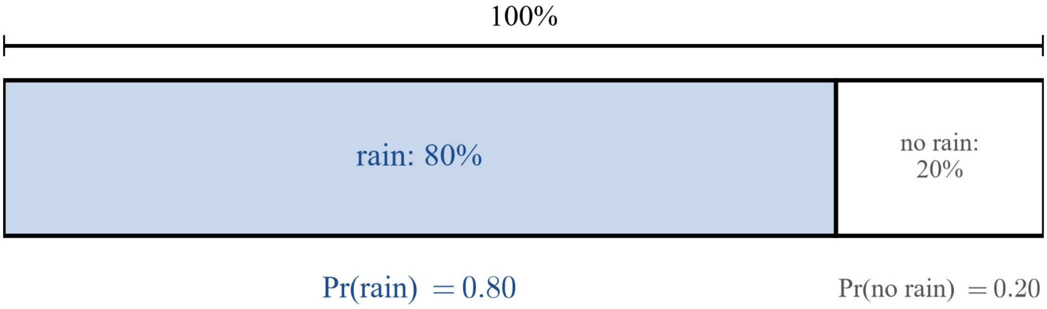
Working	Comment
(a) $Pr(A)=\dfrac{1}{6}$	The dice is fair, so the six outcomes are equally likely — each has probability $\dfrac{1}{6}$.
(b) A and not A are complementary events	A roll is either a 6 or not a 6 — exactly one must happen.
$Pr(\text{not } A)=1-Pr(A)=1-\dfrac{1}{6}=\dfrac{5}{6}$	Complementary events add to 1.

Example 2 — with help

A weather forecast says there is an 80% chance of rain tomorrow.

(a) Find the probability that it does not rain tomorrow.

(b) Write this probability as a decimal.



A bar representing tomorrow's weather, split into two parts: an 80% section labelled rain and a 20% section labelled no rain

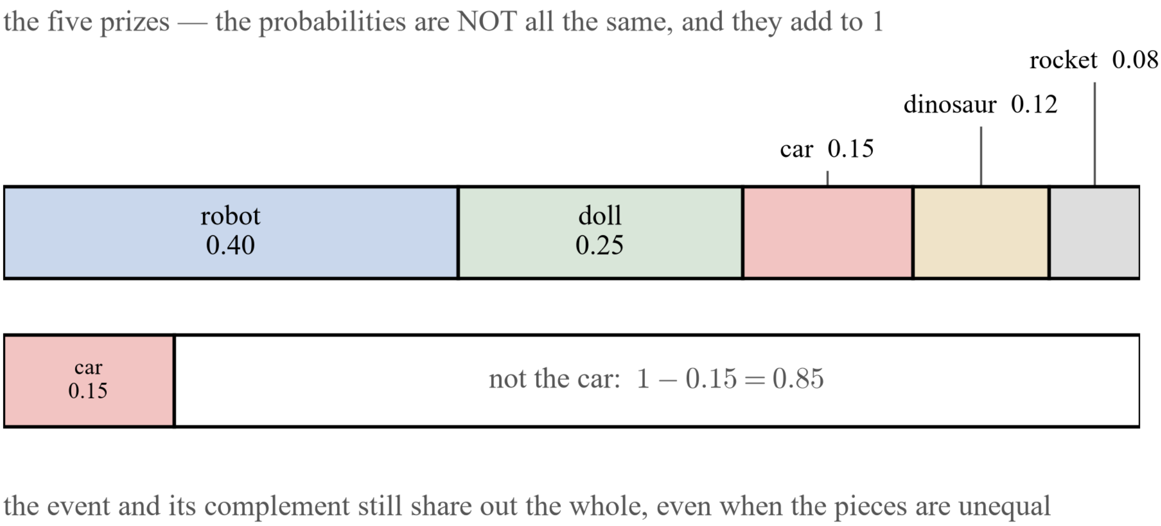
Working	Comment
(a) "Rain" and "no rain" are complementary events	Tomorrow either rains or it does not — exactly one must happen.
$Pr(\text{no rain}) = 100\% - 80\% = 20\%$	Complementary events add to the whole. In percentages, the whole is 100%.
(b) $Pr(\text{no rain}) = 1 - 0.80 = 0.20$	In decimals, the whole is 1. The two answers, 20% and 0.20, are the same probability.

Example 3 — with help

A supermarket promotion puts one toy in every box of cereal. The probabilities of getting each toy are:

Toy	Robot	Doll	Car	Dinosaur	Rocket
Probability	0.40	0.25	0.15	0.12	?

(a) Find the probability of getting the rocket.
(b) Find the probability of not getting the car.



Two horizontal bars for the promotion: the top bar split into five sections for robot 0.40, doll 0.25, car 0.15, dinosaur 0.12 and rocket 0.08, and the bottom bar split into the car 0.15 and its complement not the car 0.85

Working	Comment
(a) $0.40 + 0.25 + 0.15 + 0.12 = 0.92$	The other four probabilities add to 0.92.
$Pr(\text{rocket}) = 1 - 0.92 = 0.08$	All the probabilities in the sample space add to 1.

Working	Comment
(b) “Car” and “not car” are complementary events	You either get the car or you do not.
$Pr(\text{not car}) = 1 - 0.15 = 0.85$	Complementary events add to 1. The outcomes are not equally likely, but the rule still works.

Example 4 — on your own

The spinner in the theory has wedges red $\frac{1}{2}$, blue $\frac{1}{4}$, green $\frac{1}{8}$ and yellow $\frac{1}{8}$. It is spun once.

(a) Find $Pr(\text{not red})$.
(b) Find $Pr(\text{not green})$.

Solution.

(a) “Red” and “not red” are complementary events, so

$$Pr(\text{not red}) = 1 - Pr(\text{red}) = 1 - \frac{1}{2} = \frac{1}{2}$$

(b) “Green” and “not green” are complementary events, so

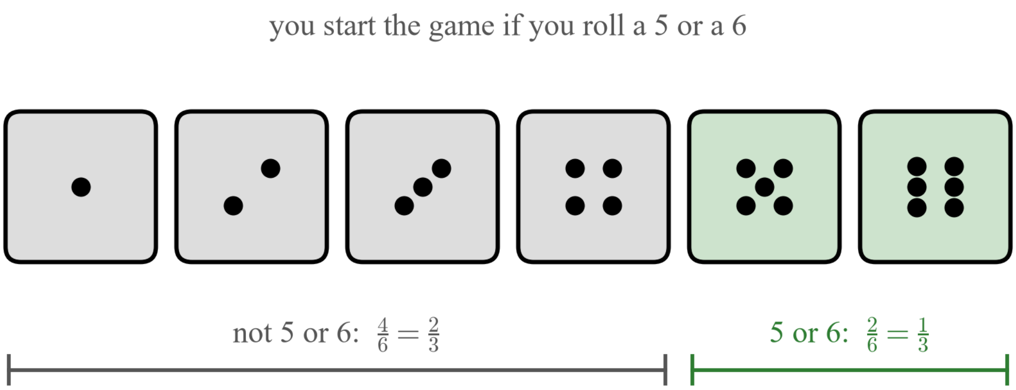
$$Pr(\text{not green}) = 1 - Pr(\text{green}) = 1 - \frac{1}{8} = \frac{7}{8}$$

Notice that not green covers three wedges — red, blue and yellow — but you did not need to add them one by one. The rule does the work in one step.

Example 5 — on your own

In a board game, you start the game when you throw a 5 or a 6 on a fair dice.

(a) Find the probability that you start the game.
(b) Find the probability that you do not start the game.



Six faces of a fair dice: the faces 5 and 6 marked green and labelled 5 or 6 probability two-sixths equals one-third, and the faces 1, 2, 3 and 4 marked grey and labelled not 5 or 6 probability four-sixths equals two-thirds

Solution.

(a) The outcomes 5 and 6 are equally likely, each with probability $\frac{1}{6}$:

$$Pr(5 \text{ or } 6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

(b) “Start” and “not start” are complementary events, so

$Pr(\text{not start}) = 1 - \frac{1}{3} = \frac{2}{3}$

Check: not starting means throwing 1, 2, 3 or 4, and $\frac{4}{6} = \frac{2}{3}$. The two answers add to 1, as they must.

Example 6 — on your own

A fair dice is rolled once. For each event, state the complement in words and find both probabilities.

(a) E: roll at least 5.
(b) F: roll more than 2.

Solution.

(a) "At least 5" means 5 or more, so its outcomes are 5 and 6. The complement is **4 or less** (the outcomes 1, 2, 3, 4).

$Pr(E) = \frac{2}{6} = \frac{1}{3}$, so $Pr(\text{not } E) = 1 - \frac{1}{3} = \frac{2}{3}$.

(b) "More than 2" means 3, 4, 5 or 6 — it does not include 2. The complement is **2 or less** (the outcomes 1 and 2).

$Pr(F) = \frac{4}{6} = \frac{2}{3}$, so $Pr(\text{not } F) = 1 - \frac{2}{3} = \frac{1}{3}$.

Practice questions

Work these questions by hand. You do not need a calculator for any question. Remember the rule from this subtopic: complementary events add to 1.

With help

Q1. Write the complement of each event.

a. It rains on Saturday.
b. The 8:15 bus arrives on time.
c. Your team wins the match.

Q2. Find $Pr(\text{not } A)$, given $Pr(A)$. Give each answer as a fraction.

a. $Pr(A) = \frac{1}{4}$

b. $Pr(A) = \frac{3}{7}$

c. $Pr(A) = \frac{5}{8}$

Q3. Find $Pr(\text{not } A)$, given $Pr(A)$. Give each answer as a decimal.

a. $Pr(A) = 0.3$
b. $Pr(A) = 0.72$
c. $Pr(A) = 0.05$

Q4. Find $Pr(\text{not } A)$, given $Pr(A)$. Give each answer as a percentage.

a. $Pr(A) = 15\%$

b. $Pr(A) = 62\%$
c. $Pr(A) = 90\%$

Q5. A fair six-sided dice is rolled once.

a. Find $Pr(\text{roll a } 3)$.
b. Find $Pr(\text{not roll a } 3)$.
c. Find $Pr(\text{roll an even number})$.
d. Find $Pr(\text{not roll an even number})$.

Q6. A bag contains 4 red marbles and 6 blue marbles. One marble is drawn at random.

a. Find $Pr(\text{red})$.
b. Find $Pr(\text{not red})$.

Q7. True or false? Give a reason for each answer.

a. If $Pr(A) = 0$, then A cannot happen.
b. $Pr(A) + Pr(\text{not } A) = 1$ only works when the outcomes are equally likely.
c. An event can have probability 1.4.

Q8. A spinner has four colours with these probabilities:

Colour	Red	Blue	Green	Yellow
Probability	0.50	0.20	0.15	?

a. Find $Pr(\text{yellow})$.
b. Find $Pr(\text{not green})$.

On your own

Q9. Find $Pr(\text{not } A)$.

a. $Pr(A) = \frac{7}{12}$
b. $Pr(A) = 0.46$
c. $Pr(A) = 33\%$
d. $Pr(A) = 1$

Q10. A weather forecast gives a 65% chance of rain tomorrow. Write the probability that it does not rain tomorrow as a decimal.

Q11. A fair six-sided dice is rolled once.

a. Find $Pr(\text{not roll a } 2)$.
b. Find $Pr(\text{not roll an even number})$.
c. Find $Pr(\text{not roll more than } 4)$.

Q12. A wheel is divided into 8 equal sectors (pie-shaped slices), numbered 1 to 8, and spun once. The probability of the event "more than 6" is $\frac{2}{8}$. Find the probability that the result is not more than 6. Give your answer as a fraction in simplest form.

Q13. The Year 8 fundraiser sells 200 raffle tickets, and you buy 5 of them. One ticket is drawn at random to win the prize.

a. Find $Pr(\text{you win})$. Give your answer as a fraction in simplest form.

b. Find $Pr(\text{you do not win})$.

Q14. A fair dice is rolled once. Event E is “roll at least 3”.

a. State the complement of E in words.
b. Find $Pr(E)$.
c. Find $Pr(\text{not } E)$.

Q15. At a set of traffic lights, the probabilities of the lights being red and yellow at a random moment are 0.45 and 0.10.

a. Find the probability that the lights are green.
b. Find the probability that the lights are not yellow.

Q16. Mia says: “$Pr(\text{rain}) + Pr(\text{no rain})$ must be 1, because it either rains or it doesn’t.” Explain, in your own words, the two facts about “rain” and “no rain” that make Mia’s argument work, and why the same argument works for any event and its complement.

Maths in real life

Q17. At the school fete, the lucky dip always returns one of four prizes: a mug (probability 0.35), a badge (probability 0.30), a keyring (probability 0.20) or a sticker.

a. Find the probability of winning the sticker.
b. Find the probability of not winning the mug.

Q18. A goalkeeper saves 25% of the penalty kicks taken against them. Find the probability that the ball goes in from the next penalty. Give your answer as a percentage and as a decimal.

Q19. A factory checks its light globes and finds that 3% of them are faulty. One globe is chosen at random.

a. Write $Pr(\text{faulty})$ as a decimal.
b. Find $Pr(\text{not faulty})$ as a decimal, and as a percentage.

Q20. A standard pack of 52 playing cards contains 13 hearts. One card is drawn at random from a shuffled pack.

a. Find $Pr(\text{heart})$. Give your answer as a fraction in simplest form.
b. Find $Pr(\text{not heart})$.

Answers

Q1. a. It does not rain on Saturday. b. The 8:15 bus is late. c. Your team does not win the match (it draws or loses).

Q2. a. $\frac{3}{4}$. b. $\frac{4}{7}$. c. $\frac{3}{8}$.

Q3. a. 0.7. b. 0.28. c. 0.95.

Q4. a. $85\,\%$. b. $38\,\%$. c. $10\,\%$.

Q5. a. $\frac{1}{6}$. b. $\frac{5}{6}$. c. $\frac{1}{2}$. d. $\frac{1}{2}$.

Q6. a. $\frac{2}{5}$. b. $\frac{3}{5}$.

Q7. a. True. b. False. c. False. See the fully-worked solutions for reasons.

Q8. a. 0.15. b. 0.85.

Q9. a. $\frac{5}{12}$. b. 0.54. c. $67\,\%$. d. 0.

Q10. 0.35.

Q11. a. $\frac{5}{6}$. b. $\frac{1}{2}$. c. $\frac{2}{3}$.

Q12. $\frac{3}{4}$.

Q13. a. $\frac{1}{40}$. b. $\frac{39}{40}$.

Q14. a. Roll 2 or less. b. $\frac{2}{3}$. c. $\frac{1}{3}$.

Q15. a. 0.45. b. 0.90.

Q16. The two events cannot happen together, and between them they cover every possibility — so their probabilities must add to 1.

Q17. a. 0.15. b. 0.65.

Q18. $75\,\%$, or 0.75.

Q19. a. 0.03. b. 0.97, or $97\,\%$.

Q20. a. $\frac{1}{4}$. b. $\frac{3}{4}$.

Fully-worked solutions

Q1. The complement of an event is the event not happening — and for each pair, exactly one of the two must happen.

a. It does not rain on Saturday.
b. The 8:15 bus is late.
c. Your team does not win the match — it draws or loses.

Q2. Subtract the probability from 1.

a. $Pr(\text{not } A) = 1 - \frac{1}{4} = \frac{3}{4}$.

b. $Pr(\text{not } A) = 1 - \frac{3}{7} = \frac{4}{7}$.

c. $Pr(\text{not } A) = 1 - \frac{5}{8} = \frac{3}{8}$.

Q3. Subtract from 1, keeping decimal form.

a. $Pr(\text{not } A) = 1 - 0.3 = 0.7$.
b. $Pr(\text{not } A) = 1 - 0.72 = 0.28$.
c. $Pr(\text{not } A) = 1 - 0.05 = 0.95$.

Q4. Subtract from $100\,\%$.

a. $Pr(\text{not } A) = 100\,\% - 15\,\% = 85\,\%$.
b. $Pr(\text{not } A) = 100\,\% - 62\,\% = 38\,\%$.
c. $Pr(\text{not } A) = 100\,\% - 90\,\% = 10\,\%$.

Q5. The dice is fair, so all six outcomes are equally likely with probability $\frac{1}{6}$ each.

a. Only one outcome is a 3, so $Pr(\text{roll a } 3) = \frac{1}{6}$.

b. Complementary events: $Pr(\text{not roll a } 3) = 1 - \frac{1}{6} = \frac{5}{6}$.

c. The even numbers are 2, 4 and 6 — three outcomes — so $Pr(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

d. Complementary events: $Pr(\text{not even}) = 1 - \frac{1}{2} = \frac{1}{2}$.

Q6. There are $4 + 6 = 10$ marbles, all equally likely to be drawn.

a. $Pr(\text{red}) = \frac{4}{10} = \frac{2}{5}$.

b. “Red” and “not red” are complementary: $Pr(\text{not red}) = 1 - \frac{2}{5} = \frac{3}{5}$. (You could also count the 6 blue marbles directly: $\frac{6}{10} = \frac{3}{5}$.)

Q7. Apply the definitions.

a. **True.** Probability 0 means impossible — the event cannot happen.
b. **False.** The rule works for any event, equally likely outcomes or not. All it needs is for the event and its complement to share out every outcome, which is always true by the meaning of complement. (Example 3 used a promotion where the outcomes were not equally likely.)
c. **False.** A probability measures part of the whole, and the whole is 1, so every probability lies between 0 and 1. There is nothing left to give to an event beyond “must happen”.

Q8. The four colours share out every spin.

a. The known probabilities add to $0.50 + 0.20 + 0.15 = 0.85$, so $Pr(\text{yellow}) = 1 - 0.85 = 0.15$.
b. “Green” and “not green” are complementary: $Pr(\text{not green}) = 1 - 0.15 = 0.85$.

Q9. Subtract from 1, matching the form given.

a. $Pr(\text{not } A) = 1 - \frac{7}{12} = \frac{5}{12}$.

b. $Pr(\text{not } A) = 1 - 0.46 = 0.54$.

c. $Pr(\text{not } A) = 100\% - 33\% = 67\%$.

d. $Pr(\text{not } A) = 1 - 1 = 0$. Here A is certain — it must happen — so not A is impossible.

Q10. "Rain" and "no rain" are complementary events, so the probabilities add to 1.

$Pr(\text{no rain}) = 100\% - 65\% = 35\% = 0.35$.

Q11. Six equally likely outcomes, probability $\frac{1}{6}$ each.

a. Complementary events: $Pr(\text{not roll a } 2) = 1 - \frac{1}{6} = \frac{5}{6}$.

b. Even means 2, 4 or 6, so $Pr(\text{even}) = \frac{3}{6} = \frac{1}{2}$, and $Pr(\text{not even}) = 1 - \frac{1}{2} = \frac{1}{2}$.

c. "Not more than 4" means "4 or less": the outcomes 1, 2, 3, 4. So $Pr(\text{not roll more than } 4) = \frac{4}{6} = \frac{2}{3}$.

Q12. "More than 6" and "not more than 6" are complementary events.

$Pr(\text{not more than } 6) = 1 - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$.

Q13. The draw picks 1 ticket from 200, and all tickets are equally likely.

a. You hold 5 of the 200 tickets: $Pr(\text{you win}) = \frac{5}{200} = \frac{1}{40}$.

b. "Win" and "do not win" are complementary: $Pr(\text{you do not win}) = 1 - \frac{1}{40} = \frac{39}{40}$.

Q14. Find the outcomes first.

a. "At least 3" means 3 or more — the outcomes 3, 4, 5, 6. The complement is **2 or less** — the outcomes 1 and 2.

b. $Pr(E) = \frac{4}{6} = \frac{2}{3}$.

c. Complementary events: $Pr(\text{not } E) = 1 - \frac{2}{3} = \frac{1}{3}$.

Q15. The lights are red, yellow or green — the three outcomes share out every possibility.

a. Red and yellow together: $0.45 + 0.10 = 0.55$. So $Pr(\text{green}) = 1 - 0.55 = 0.45$.

b. "Yellow" and "not yellow" are complementary: $Pr(\text{not yellow}) = 1 - 0.10 = 0.90$.

Q16. Build the argument from the meaning of complement.

Two facts make Mia's argument work. First, "rain" and "no rain" can never happen together — tomorrow cannot both rain and not rain. Second, between them they cover every possibility — there is no third sort of tomorrow. So the two events share out the whole, and their probabilities must add to 1. The same two facts hold for any event and its complement by definition, so $Pr(A) + Pr(\text{not } A) = 1$ for every event A.

Q17. The four prizes share out every lucky dip result.

a. The known prizes add to $0.35 + 0.30 + 0.20 = 0.85$, so $Pr(\text{sticker}) = 1 - 0.85 = 0.15$.

b. "Mug" and "not mug" are complementary: $Pr(\text{not mug}) = 1 - 0.35 = 0.65$.

Q18. For each penalty, “save” and “ball goes in” are complementary events.

$Pr(\text{ball goes in}) = 100\% - 25\% = 75\% = 0.75.$

Q19. Each globe is either faulty or not faulty.

a. $Pr(\text{faulty}) = 3\% = 0.03.$

b. Complementary events: $Pr(\text{not faulty}) = 1 - 0.03 = 0.97$, which is 97%.

Q20. All 52 cards are equally likely to be drawn.

a. There are 13 hearts: $Pr(\text{heart}) = \frac{13}{52} = \frac{1}{4}.$

b. “Heart” and “not heart” are complementary: $Pr(\text{not heart}) = 1 - \frac{1}{4} = \frac{3}{4}.$

Two events: two-way tables, tree diagrams and Venn diagrams

Content description `VC2M8P02`: determine all possible outcome combinations for 2 events, using two-way tables, tree diagrams and Venn diagrams, and use these to determine probabilities of specific events in practical situations

Flip a coin and roll a die, and two things happen at once. What is the chance the coin shows a head ***and*** *the die shows a 4? Questions about two events need one careful first step: list every combination the two events can produce, so nothing is missed and nothing is counted twice. This subtopic builds the three tools the curriculum names for that job — two-way tables, tree diagrams and Venn diagrams — and uses each one to turn a careful list into a probability.*

Theory

Recall: probability from Year 7

In Year 7 you measured the chance of an event with a probability between 0 and 1 (`VC2M7P01`). When every outcome is **equally likely**, the probability of an event is

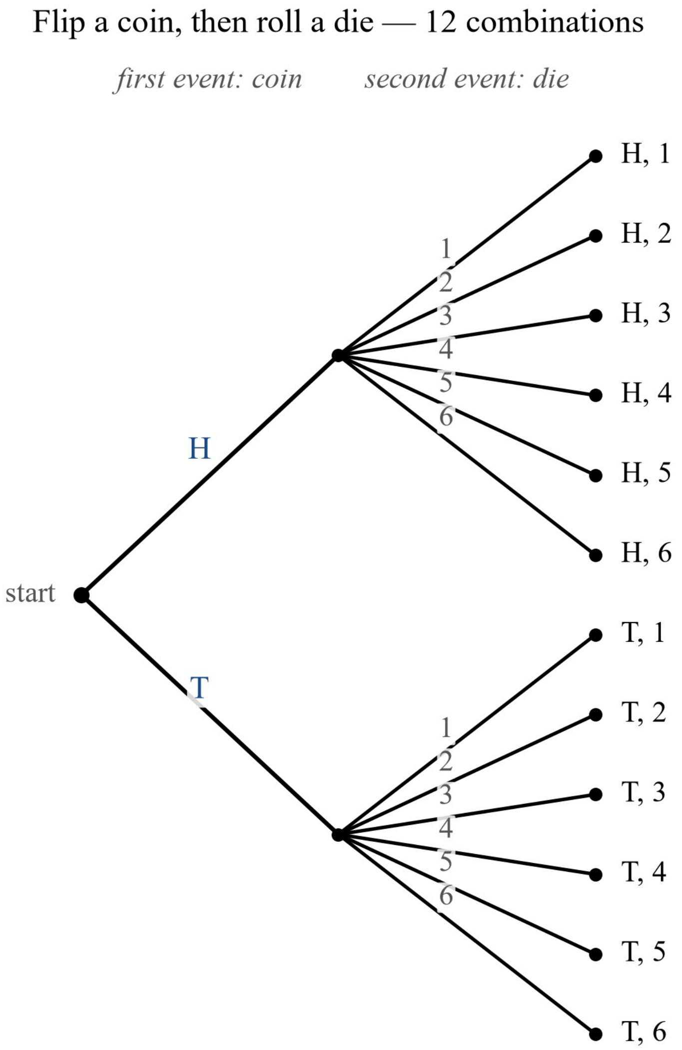
$$\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes overall}}$$

so one fair die gives $\Pr(\text{even}) = \frac{3}{6} = \frac{1}{2}$, and an event that cannot happen has probability 0 while an event that must happen has probability 1. This subtopic keeps that same fraction — the only thing that changes is that outcomes are now **combinations** of the results of two events, and the three diagrams here are three ways of listing those combinations without missing any.

Two events and their combinations

An **outcome** is one result of a chance situation; an **event** is a set of outcomes we care about, described by a statement such as "the die shows an even number". With **two events** — a coin flip and a die roll, two spins of a spinner, or two questions in a survey — a single outcome is a **combination**: the result of the first event paired with the result of the second.

A **tree diagram** lists combinations by following every path from the start to an end.



A horizontal tree diagram for flipping a coin then rolling a die: from the start point two branches labelled H and T lead to two nodes, and from each node six branches labelled 1 to 6 lead to twelve end points, listed as H, 1 down to T, 6

Each path through the tree is one combination, so the tree shows all $2 \times 6 = 12$ combinations: H with each of 1-6, and T with each of 1-6. Because the coin and the die are fair, all twelve combinations are equally likely, and each one has probability $\frac{1}{12}$.

> **Key idea.** To find the probability of an event that involves two events, first list all the combinations — with a tree, a table or a Venn diagram — then count how many of them belong to the event. The probability is that count over the total count, provided the combinations are equally likely.

The same combinations in a two-way table

A **two-way table** lays the same list out in rows and columns: one event supplies the rows, the other supplies the columns, and each cell is one combination.

The same 12 combinations as the tree, in a two-way table

	1	2	3	4	5	6
Head (H)	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
Tail (T)	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

result on the die

A two-way table with rows Head (H) and Tail (T) and columns 1 to 6, whose twelve cells read H, 1 to H, 6 and T, 1 to T, 6

The table holds exactly the twelve combinations the tree showed — the tree reads them along its paths, the table reads them across its cells. Use whichever shows the list more clearly for the question you are asking: trees suit events that happen one after another, tables suit two results recorded side by side.

When both events are dice, the table becomes a grid of all 36 combinations, and writing the **sum** in each cell makes patterns easy to see.

The 36 combinations when two dice are rolled (each cell shows the total; shaded = total of 7)

result on the first die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

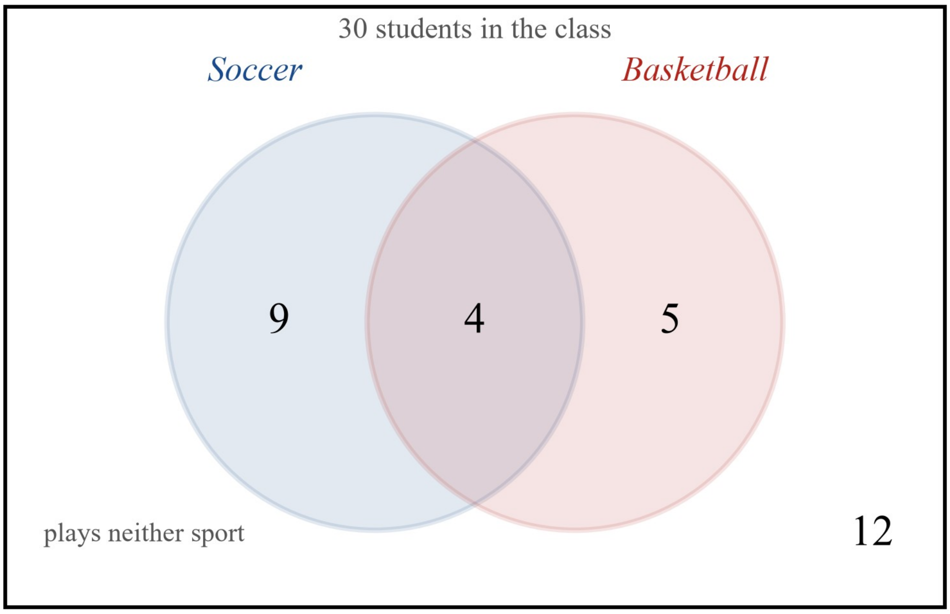
result on the second die

A 6 by 6 grid for two dice with rows and columns labelled 1 to 6; each cell shows the sum of the two dice, from 2 to 12, and the six cells whose sum is 7 are shaded dark, running in a diagonal from top right to bottom left

The shaded cells are the six combinations with a sum of 7, so $\Pr(\text{sum is } 7) = \frac{6}{36} = \frac{1}{6}$. The sums run along diagonals — one reason a table is the clearest representation for questions about totals.

Venn diagrams: and, or, neither

A tree or a table lists combinations of two *chance* events. A **Venn diagram** is the right tool when the two events are two yes-or-no questions about the same people or things — “plays soccer” and “plays basketball” — because every member of the group sits in exactly one of four regions.



A Venn diagram inside a rectangle labelled 30 students in the class: a blue circle labelled Soccer and a red circle labelled Basketball overlap; the soccer-only region holds 9, the overlap holds 4, the basketball-only region holds 5, and the outside region holds 12, labelled plays neither sport

Read the four regions with the words **and**, **or** and **neither**:

- the overlap is soccer **and** basketball: 4 students;
- soccer **only** (soccer and not basketball) holds 9; basketball only holds 5;
- **or** means at least one: soccer or basketball is $9+4+5=18$ students;
- outside both circles is **neither**: 12 students.

Every student sits in exactly one region, so the four counts must add to the total: $9+4+5+12=30$ ✓. That check works in every Venn diagram.

The same four numbers fill a two-way table, because a Venn diagram and a two-way table are the same list drawn two ways:

The class survey as a two-way table

	Plays basketball	Does not play basketball	Total
Plays soccer	4	9	13
Does not play soccer	5	12	17
Total	9	21	30

A two-way table of the class survey: rows Plays soccer and Does not play soccer, columns Plays basketball and Does not play basketball; the cells are 4, 9, 5 and 12, the row totals are 13 and 17, the column totals are 9 and 21, and the grand total is 30

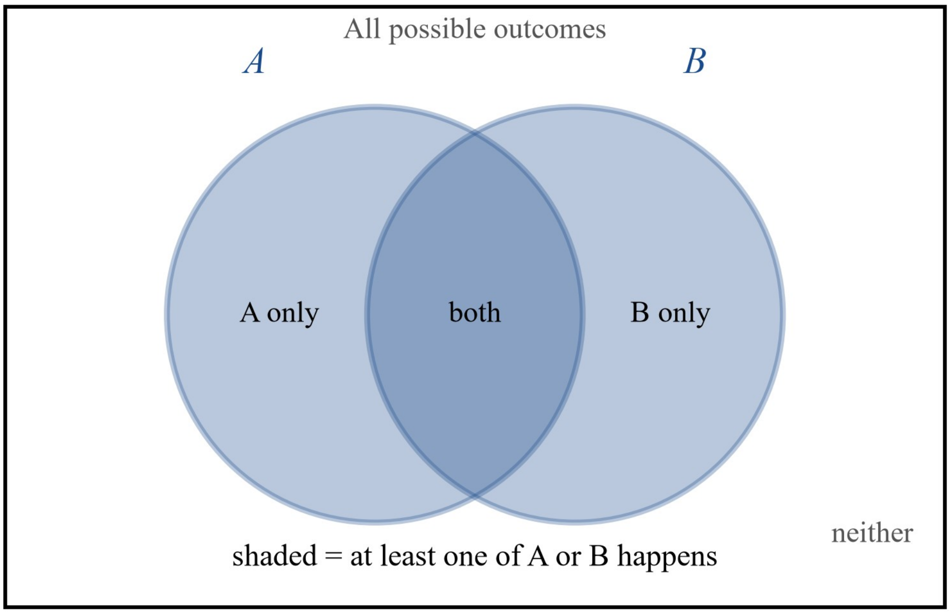
The overlap of the Venn (4) is the top-left cell; the soccer-only and basketball-only regions are the other off-diagonal cells; the neither region is the bottom-right cell. Moving between the two views is just moving the same four counts. From either view, choosing one student at random from the 30:

$$\Pr(\text{plays both}) = \frac{4}{30} = \frac{2}{15}, \Pr(\text{plays neither}) = \frac{12}{30} = \frac{2}{5}.$$

At least one, either but not both, and mutually exclusive

Three phrases do most of the work in two-event questions, and each names a precise set of regions:

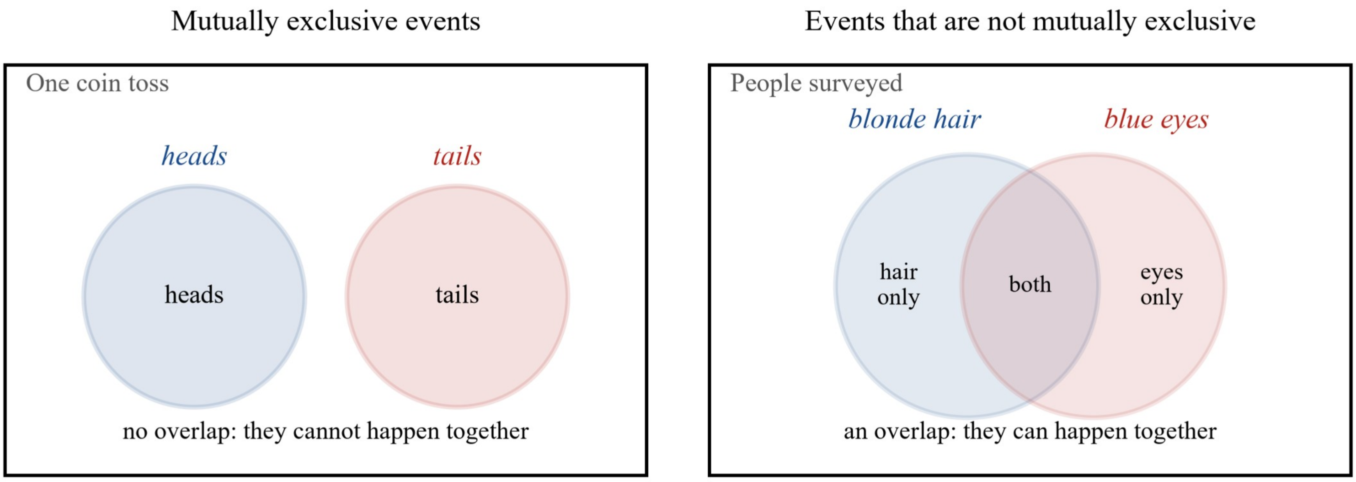
- **at least one** of A or B (inclusive or — *or* that includes the “both” case) means A only, B only, or both: the whole overlap picture shaded;
- **either but not both** (exclusive or) means A only or B only, leaving the overlap out;
- **and** means the overlap alone.



A Venn diagram with circles A and B inside a rectangle of all possible outcomes, in which the whole of both circles is shaded; the regions are labelled A only, both and B only, with neither outside, and the caption shaded = at least one of A or B happens

In the class survey, "at least one sport" is $\frac{18}{30}=\frac{3}{5}$ while "exactly one sport" is $\frac{9+5}{30}=\frac{14}{30}=\frac{7}{15}$. The two questions differ by exactly the overlap.

Two events are **mutually exclusive** when they cannot happen together — when the overlap is empty. Compare tossing one coin with surveying people:



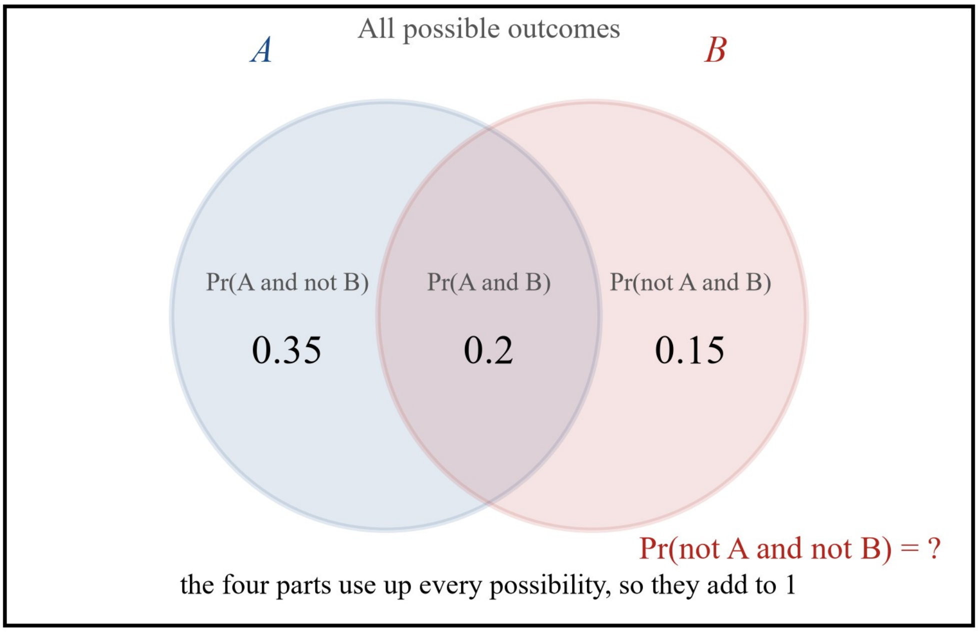
Two panels. Left, one coin toss: two separate circles labelled heads and tails that do not overlap, captioned no overlap: they cannot happen together, titled Mutually exclusive events. Right, people surveyed: two overlapping circles labelled blonde hair and blue eyes with regions hair only, both and eyes only, captioned an overlap: they can happen together, titled Events that are not mutually exclusive

Heads and tails are mutually exclusive: one toss cannot land both, so $\Pr(\text{heads and tails})=0$ and the circles do not overlap. Blonde hair and blue eyes are not mutually exclusive: a person can have both, so the circles overlap and the overlap holds people. For mutually exclusive events, "at least one" is just the two regions added: $\Pr(\text{A or B})=\Pr(\text{A})+\Pr(\text{B})$.

The four parts add to 1

However the two events overlap, the four regions — A and B, A and not B, not A and B, not A and not B — use up every possibility, so their probabilities add to 1:

$$\Pr\left(A \text{ and } B\right)+\Pr\left(A \text{ and not } B\right)+\Pr(\text{not } A \text{ and } B)+\Pr(\text{not } A \text{ and not } B)=1$$

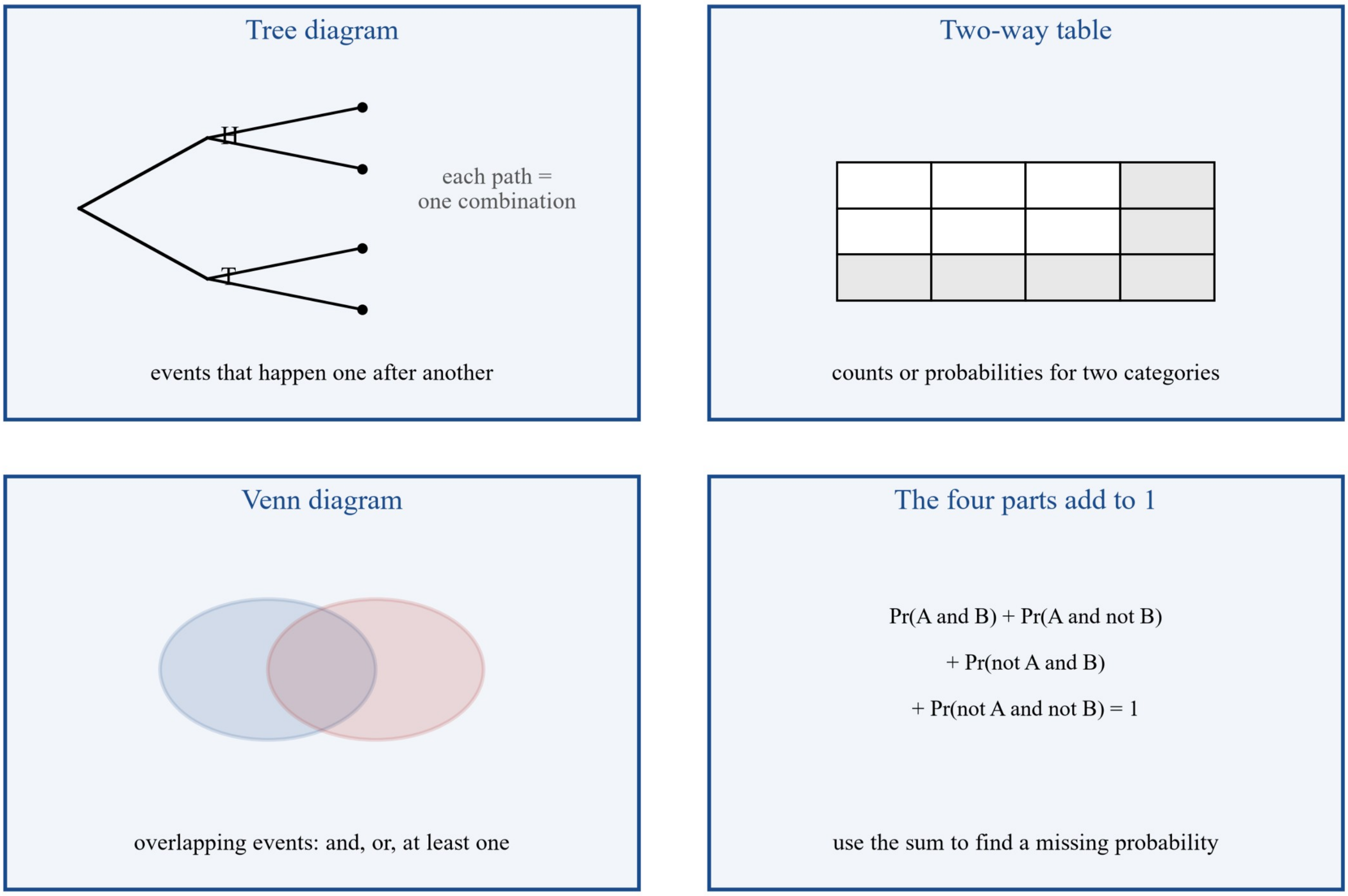


A Venn diagram with circles A and B inside a rectangle of all possible outcomes: the A-only region is labelled Pr(A and not B) with 0.35, the overlap Pr(A and B) with 0.2, the B-only region Pr(not A and B) with 0.15, and the outside region is labelled Pr(not A and not B) = ? in red; caption the four parts use up every possibility, so they add to 1

Here $0.35+0.2+0.15=0.7$, so the missing part is $1-0.7=0.3$: the probability that neither A nor B happens is 0.3. The same sum gives “at least one” directly: $1-0.3=0.7$. This four-part sum is the quickest road into most two-event questions where the probabilities (rather than counts) are given.

Choosing a representation

The three tools are three views of the same idea — a careful list of combinations.



Four summary cards. Tree diagram: a small sketch of a branching tree, captioned events that happen one after another, each path one combination. Two-way table: a small empty table with shaded total row and column, captioned counts or probabilities for two categories. Venn diagram: two overlapping circles, captioned overlapping events: and, or, at least one. The four parts add to 1: the sum Pr(A and B) + Pr(A and not B) + Pr(not A and B) + Pr(not A and not B) = 1, captioned use the sum to find a missing probability

- A **tree diagram** suits events that happen one after another (a flip, then a roll; a spin, then a spin).
- A **two-way table** suits two results recorded side by side, and makes totals and sums easy to see.
- A **Venn diagram** suits two yes-or-no questions about one group, and makes *and*, *or* and *neither* visible.

If one representation feels awkward, redraw the same combinations in another — the survey Venn and table above hold the same four counts.

Before you leave

In Year 9, `VC2M9P01` puts probabilities **on** the branches of a tree and multiplies along them — needed when the question asks about chances of chances — and `VC2M9P02` takes up Venn diagrams and two-way tables again with set notation. This subtopic keeps to counting equally-likely combinations and to probabilities given for the four regions; both Year 9 steps stand on the lists of combinations built here.

Words of this subtopic

Word	What it means here
outcome	one result of a chance situation
event	a set of outcomes described by a statement, such as “the die shows an even number”
combination	the result of the first event paired with the result of the second
tree diagram	a branching picture whose paths list all the combinations
two-way table	a rows-and-columns table whose cells list all the

Word	What it means here
	combinations
Venn diagram	overlapping circles in a rectangle; the four regions are *and*, the two *only* parts, and *neither*
mutually exclusive	cannot happen together; the overlap is empty and $\Pr(A \text{ and } B) = 0$
at least one	A or B or both — everything except the neither region
either but not both	A only or B only — the overlap left out
neither	not A and not B — the region outside both circles

Worked examples

Example 1 — with help

A fair coin is flipped and a fair die is rolled. Use the tree diagram in the theory to find:

a) the number of combinations; b) Pr(head and a 4); c) Pr(tail and a number greater than 4).

Working	Comment
a) $2 \times 6 = 12$ combinations, all equally likely.	Each of the 2 coin results pairs with each of the 6 die results; the tree shows one path per combination.
b) $\Pr(\text{H and 4}) = \frac{1}{12}$	One path — H, 4 — out of the twelve.
c) The paths T, 5 and T, 6: 2 combinations. $\Pr = \frac{2}{12} = \frac{1}{6}$	"Greater than 4" means 5 or 6, paired with T: two combinations. Reduce the fraction at the end.

Example 2 — with help

In a class of 30 students, 13 play soccer, 9 play basketball and 4 play both. Complete the Venn diagram and find Pr(neither sport) and Pr(exactly one sport).

Working	Comment
Overlap: 4. Soccer only: $13 - 4 = 9$.	The 13 who play soccer include the 4 who play both.
Basketball only: $9 - 4 = 5$.	Same move for basketball.
Neither: $30 - (9 + 4 + 5) = 12$.	The four regions add to 30 — the same counts as the survey figure.
$\Pr(\text{neither}) = \frac{12}{30} = \frac{2}{5}$	Count the neither region over the whole class.
$\Pr(\text{exactly one}) = \frac{9+5}{30} = \frac{14}{30} = \frac{7}{15}$	Exactly one leaves the overlap out.

Example 3 — with help

Two fair dice are rolled. Use the 36-cell grid in the theory to find:

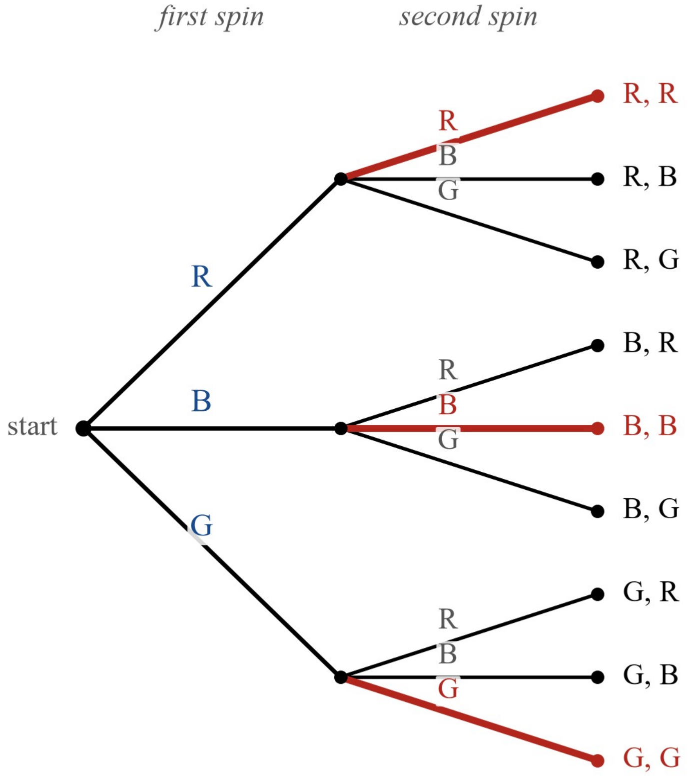
a) Pr(the sum is 7); b) Pr(doubles); c) Pr(the sum is at least 10).

Working	Comment
a) Six shaded cells: $\frac{6}{36} = \frac{1}{6}$	The sum-7 diagonal.
b) Doubles are (1,1) to (6,6): 6 cells, $\frac{6}{36} = \frac{1}{6}$	Doubles run down the other diagonal of the grid.
c) Sums 10, 11, 12: $3 + 2 + 1 = 6$ cells, $\frac{6}{36} = \frac{1}{6}$	Count each sum separately, then add the counts.

Example 4 — on your own

A spinner with three equal sectors — red (R), blue (B) and green (G) — is spun twice. The tree diagram below lists the 9 combinations; the red paths are the ones that land on the same colour twice.

Spin a 3-colour spinner twice — 9 combinations (red paths land on the same colour twice)



A horizontal tree diagram for two spins of a three-colour spinner: from the start, three branches R, B and G lead to three nodes, and from each node three branches R, B and G lead to nine end points R, R down to G, G; the three paths R-R, B-B and G-G are drawn in red

Find: a) Pr(the same colour twice); b) Pr(at least one red); c) Pr(no red).

a) The three red paths out of 9: $\frac{3}{9}=\frac{1}{3}$.

b) The combinations with at least one R are R,R; R,B; R,G; B,R; G,R — five of them: $\frac{5}{9}$.

c) No red means both spins are B or G: B,B; B,G; G,B; G,G — four of them: $\frac{4}{9}$. Check: $\frac{5}{9}+\frac{4}{9}=1$ ✓ — "at least one red" and "no red" use up every possibility.

Example 5 — on your own

For two events A and B, $\Pr(A \text{ and } B)=0.2$, $\Pr(A \text{ and not } B)=0.35$ and $\Pr(\text{not } A \text{ and } B)=0.15$ — the four-region figure in the theory. Find:

a) Pr(neither A nor B); b) Pr(A); c) Pr(at least one of A or B).

b) The four parts add to 1, so $\Pr(\text{neither})=1-(0.2+0.35+0.15)=1-0.7=0.3$.

c) A happens in two regions — A and B, and A and not B: $\Pr(A)=0.2+0.35=0.55$.

d) At least one is everything except neither: $1-0.3=0.7$. (The same as $0.35+0.2+0.15$.)

Example 6 — on your own

A box holds only red, blue and green marbles. One marble is drawn at random. Pr(red) $= 0.4$ and Pr(blue) $= 0.35$.

a) Explain why "red" and "blue" are mutually exclusive. b) Find Pr(green).

b) Find Pr(red or blue).

c) One drawn marble has only one colour, so red and blue cannot happen together — the overlap is empty and $\Pr(\text{red and blue}) = 0$.

d) The three colours use up every possibility, so $\Pr(\text{green}) = 1 - (0.4 + 0.35) = 0.25$.

e) Mutually exclusive events add: $\Pr(\text{red or blue}) = 0.4 + 0.35 = 0.75$.

Practice questions

No calculator is needed: every probability here reduces by hand to a simple fraction or a one- or two-decimal number. Keep the key idea in front of you — list the combinations first, then count.

With help

Q1. Complete each sentence in your own words.

- A combination of two events is ______.
- Two events are mutually exclusive when ______.
- "At least one of A or B" includes the regions ______.

Q2. A fair coin is flipped and a fair die is rolled (see the tree and the table in the theory).

a) How many combinations are there? b) Find Pr(head and a 6). c) Find Pr(head and an even number).

Q3. Use the survey Venn diagram in the theory (30 students).

a) How many students play both sports? b) Find Pr(a randomly chosen student plays soccer). c) Find Pr(plays neither sport).

Q4. Copy and complete this two-way table of the same survey, then check the four region counts 4, 9, 5, 12 add to 30.

	Plays basketball	Does not play basketball	Total
Plays soccer	4	?	13
Does not play soccer	?	12	?
Total	?	21	30

Q5. Two fair dice are rolled (see the 36-cell grid).

a) Find Pr(the sum is 2). b) Find Pr(the sum is 5). c) Which sum is most likely, and what is its probability?

Q6. State whether each pair of events is **mutually exclusive**, with a reason.

a) One coin toss lands heads; the same toss lands tails.
b) One die roll shows an even number; the same roll shows a number greater than 4.
c) A randomly chosen student has blonde hair; the same student has blue eyes.

Q7. The spinner tree in Example 4 lists 9 combinations.

a) Find Pr(the first spin is red). b) Find Pr(two different colours). c) Which representation — the tree or a 3-by-3 two-way table — shows the "same colour twice" combinations more clearly? Give a reason.

Q8. For two events A and B: $\Pr(A \text{ and } B) = 0.1$, $\Pr(A \text{ and not } B) = 0.4$, $\Pr(\text{not } A \text{ and } B) = 0.2$.

a) Find Pr(neither A nor B). b) Find Pr(at least one of A or B).

On your own

Q9. A fair coin is flipped twice. The combinations are HH, HT, TH, TT.

a) Find Pr(two heads). b) Find Pr(exactly one head). c) Find Pr(at least one head).

Q10. In a group of 25 students, 14 wore a hat on sports day, 16 wore sunscreen and 9 did both.

a) Complete a Venn diagram for the group. b) Find Pr(a randomly chosen student did neither). c) Find Pr(did at least one of the two).

Q11. Redraw the Q10 data as a two-way table (rows: wore a hat / did not; columns: wore sunscreen / did not), with row and column totals. Which of the two drawings — Venn or table — would you choose to show "did both", and why?

Q12. Of 32 students, 18 ride a bike to school, 5 both ride a bike and skateboard, and 9 do neither.

a) Find how many skateboard only. b) How many skateboard in total? c) Find Pr(a randomly chosen student skateboards).

Q13. A team's next game: Pr(win) $= 0.35$, Pr(draw) $= 0.4$. A game is won, drawn or lost.

a) Why are "win" and "draw" mutually exclusive? b) Find Pr(lose). c) Find Pr(win or draw).

Q14. For two events A and B: $\Pr(A \text{ and } B) = 0.15$, $\Pr(A \text{ and not } B) = 0.45$, $\Pr(\text{not } A \text{ and } B) = 0.1$. Find: a) Pr(neither); b) Pr(A); c) Pr(B).

Q15. The four regions of a two-event Venn diagram have probabilities $\frac{1}{5}, \frac{1}{2}$ and $\frac{1}{10}$ for three of them. Find the fourth.

Q16. Back to the class survey (30 students: 9 soccer only, 4 both, 5 basketball only, 12 neither).

a) Find Pr(exactly one sport). b) Find Pr(at least one sport). c) By how much do the two probabilities differ, and which region of the Venn diagram is that difference?

Maths in real life

Q17. A fete game: roll two dice; you win a prize if the sum is 7 **or** you roll doubles.

a) Explain why "sum is 7" and "doubles" are mutually exclusive. b) Find Pr(winning a prize on one roll).

Q18. The canteen surveys 40 students: 22 bought a lunch, 25 bought a drink, and 15 bought both.

a) Find Pr(a randomly chosen survey student bought at least one). b) Find Pr(bought neither).

Q19. A forecast for Saturday: Pr(rain) $= 0.6$, Pr(wind) $= 0.4$, Pr(rain and wind) $= 0.2$.

a) Find Pr(neither rain nor wind). b) Find Pr(rain only). c) Find Pr(at least one of rain or wind).

Q20. A game at the school fair: spin the three-colour spinner of Example 4 once and flip a coin once. A player wins when the spinner shows red **and** the coin shows a head.

a) List the combinations and say how many there are. b) Find Pr(winning). c) The fete runners expect 120 players. About how many winners should they expect? Explain why the actual number may differ.

Answers

Q1. A combination of two events is the result of the first event paired with the result of the second. Two events are mutually exclusive when they cannot happen together. “At least one of A or B” includes the A-only region, the B-only region and the overlap (both).

Q2. a) 12. b) $\frac{1}{12}$. c) $\frac{3}{12}=\frac{1}{4}$.

Q3. a) 4. b) $\frac{13}{30}$. c) $\frac{12}{30}=\frac{2}{5}$.

Q4. Soccer-and-not-basketball: 9; not-soccer-and-basketball: 5; not-soccer total: 17; basketball column total: 9. Check: $4+9+5+12=30$ ✓.

Q5. a) $\frac{1}{36}$. b) $\frac{4}{36}=\frac{1}{9}$. c) 7, with probability $\frac{6}{36}=\frac{1}{6}$.

Q6. a) Mutually exclusive — one toss cannot land heads and tails. b) Not mutually exclusive — a 6 is even and greater than 4. c) Not mutually exclusive — one student can have both.

Q7. a) $\frac{3}{9}=\frac{1}{3}$. b) $\frac{6}{9}=\frac{2}{3}$. c) Any reasoned answer; for example the tree, because the same-colour paths R-R, B-B and G-G can be marked in one colour and seen at a glance (or the table, because they lie along one diagonal).

Q8. a) $1-(0.1+0.4+0.2)=0.3$. b) $0.1+0.4+0.2=0.7$.

Q9. a) $\frac{1}{4}$. b) $\frac{2}{4}=\frac{1}{2}$. c) $\frac{3}{4}$.

Q10. a) Hat only 5, sunscreen only 7, both 9, neither 4. b) $\frac{4}{25}$. c) $\frac{21}{25}$.

Q11. Table cells: 9, 5, 7, 4; row totals 14 and 11; column totals 16 and 9; grand total 25. Any reasoned choice; the Venn shows “did both” as the visible overlap.

Q12. a) 5. b) 10. c) $\frac{10}{32}=\frac{5}{16}$.

Q13. a) One game has only one result, so win and draw cannot happen together. b) $1-(0.35+0.4)=0.25$. c) $0.35+0.4=0.75$.

Q14. a) 0.3. b) 0.6. c) 0.25.

Q15. $\frac{1}{5}$.

Q16. a) $\frac{14}{30}=\frac{7}{15}$. b) $\frac{18}{30}=\frac{3}{5}$. c) $\frac{3}{5}-\frac{7}{15}=\frac{4}{30}=\frac{2}{15}$ — the overlap region (both sports).

Q17. a) A double has two equal numbers, so its sum is even — it cannot be 7. b) $\frac{6+6}{36}=\frac{12}{36}=\frac{1}{3}$.

Q18. a) $\frac{22+25-15}{40}=\frac{32}{40}=\frac{4}{5}$. b) $\frac{8}{40}=\frac{1}{5}$.

Q19. a) $1-(0.6+0.4-0.2)=0.2$. b) $0.6-0.2=0.4$. c) 0.8.

Q20. a) 6: R-H, R-T, B-H, B-T, G-H, G-T. b) $\frac{1}{6}$. c) About $120 \times \frac{1}{6} = 20$ winners; chance varies from group to group, so the actual count will usually sit near 20 but rarely land on it exactly.

Fully-worked solutions

Q1. A combination of two events is the result of the first event paired with the result of the second — one path on a tree, one cell of a table, one position in a Venn diagram. Two events are mutually exclusive when they cannot happen together, so the circles of a Venn diagram would not overlap and Pr(A and B) $= 0$. “At least one of A or B” includes three of the four regions: A only, B only, and the overlap where both happen; the only region left out is neither.

Q2. a) Each of the 2 coin results pairs with each of the 6 die results, so there are $2 \times 6 = 12$ combinations — the twelve paths of the tree and the twelve cells of the table. b) Exactly one combination is head and a 6, so the probability is $\frac{1}{12}$. c) Head with an even number means H-2, H-4 or H-6: three combinations, so $\frac{3}{12} = \frac{1}{4}$.

Q3. a) The overlap of the two circles holds 4 students. b) The soccer circle holds $9 + 4 = 13$ students, so the probability is $\frac{13}{30}$. c) The neither region holds 12 of the 30, so the probability is $\frac{12}{30} = \frac{2}{5}$.

Q4. Work from the totals given. Top-right cell: the soccer row totals 13, so soccer-and-not-basketball is $13 - 4 = 9$. Bottom row total: $30 - 13 = 17$, so the bottom-left cell is $17 - 12 = 5$. Basketball column total: $30 - 21 = 9$. Check: the column also gives $4 + 5 = 9$ ✓, the four region counts $4 + 9 + 5 + 12 = 30$ ✓, the row totals $13 + 17 = 30$ ✓ and the column totals $9 + 21 = 30$ ✓ — every way of adding up gives the class size.

Q5. a) Only (1,1) gives a sum of 2: one cell of the 36, so $\frac{1}{36}$. b) A sum of 5 comes from (1,4), (2,3), (3,2), (4,1): four cells, so $\frac{4}{36} = \frac{1}{9}$. c) Reading the grid, the sum 7 has the most cells — six — so 7 is the most likely sum, with probability $\frac{6}{36} = \frac{1}{6}$.

Q6. a) Mutually exclusive: the one tossed coin lands on one side only, so heads and tails cannot happen together. b) Not mutually exclusive: rolling a 6 makes both statements true at once (6 is even and greater than 4), so the events overlap. c) Not mutually exclusive: one student can have blonde hair and blue eyes at the same time — that overlap is exactly the right-hand panel of the exclusive-or figure.

Q7. a) The first spin is red on the three paths R-R, R-B, R-G: $\frac{3}{9} = \frac{1}{3}$. b) Two different colours means not the same colour: $1 - \frac{3}{9} = \frac{6}{9} = \frac{2}{3}$ (or count the six off-diagonal paths). c) Any reasoned answer earns the mark. For example: the tree, because the three same-colour paths can be drawn in red and seen as three whole paths; or the table, because the same-colour cells lie along one diagonal, as the sums of 7 did in the dice grid. The judgement the test looks for is that the representation should make the combinations of interest easy to see.

Q8. a) The four parts add to 1, so Pr(neither) $= 1 - (0.1 + 0.4 + 0.2) = 1 - 0.7 = 0.3$. b) “At least one” is the three given regions together: $0.1 + 0.4 + 0.2 = 0.7$ — the same as $1 - 0.3$.

Q9. The four combinations HH, HT, TH, TT are equally likely. a) Two heads is HH alone: $\frac{1}{4}$. b) Exactly one head is HT and TH: $\frac{2}{4} = \frac{1}{2}$. c) At least one head is HT, TH, HH: $\frac{3}{4}$ — or $1 - \mathrm{Pr(TT)} = 1 - \frac{1}{4}$.

Q10. a) Both: 9. Hat only: $14-9=5$. Sunscreen only: $16-9=7$. Neither: $25-(5+9+7)=4$. b) $\frac{4}{25}$. c) At least one is $5+9+7=21$ students: $\frac{21}{25}$ — check: $1-\frac{4}{25}=\frac{21}{25}$ ✓.

Q11. The table: wore a hat and sunscreen 9; hat and not sunscreen 5; not hat and sunscreen 7; neither 4. Row totals: $9+5=14$ and $7+4=11$; column totals: $9+7=16$ and $5+4=9$; grand total 25 ✓. The choice is a judgement: the Venn diagram shows "did both" as the visible overlap of the circles, which is why many people choose it; the table shows the same number in the top-left cell and makes the totals easier to check. Either reason, clearly given, earns the mark.

Q12. a) Bike only is $18-5=13$. The four regions add to 32, so skateboard only is $32-13-5-9=5$. b) Skateboard in total is $5+5=10$. c) $\frac{10}{32}=\frac{5}{16}$.

Q13. a) The game has exactly one result — win, draw or lose — so win and draw cannot happen together; the circles would not overlap. b) The three results use up every possibility: Pr(lose) $=1-(0.35+0.4)=0.25$. c) Mutually exclusive events add: Pr(win or draw) $=0.35+0.4=0.75$.

Q14. a) The four parts add to 1: Pr(neither) $=1-(0.15+0.45+0.1)=1-0.7=0.3$. b) A happens in the regions "A and B" and "A and not B": Pr(A) $=0.15+0.45=0.6$. c) B happens in "A and B" and "not A and B": Pr(B) $=0.15+0.1=0.25$.

Q15. Three regions sum to $\frac{1}{5}+\frac{1}{2}+\frac{1}{10}=\frac{2}{10}+\frac{5}{10}+\frac{1}{10}=\frac{8}{10}=\frac{4}{5}$. The four parts add to 1, so the fourth region is $1-\frac{4}{5}=\frac{1}{5}$.

Q16. a) Exactly one sport is soccer only plus basketball only: $9+5=14$ students, $\frac{14}{30}=\frac{7}{15}$. b) At least one adds the overlap: $14+4=18$, $\frac{18}{30}=\frac{3}{5}$. c) $\frac{3}{5}-\frac{7}{15}=\frac{18}{30}-\frac{14}{30}=\frac{4}{30}=\frac{2}{15}$ — exactly the overlap region, the 4 students who play both. "At least one" and "exactly one" always differ by the overlap.

Q17. a) A double shows the same number twice, so its sum is $2\times$ that number — an even number — and 7 is odd; no roll is both, so the two events are mutually exclusive. b) Mutually exclusive events add their counts: 6 sum-7 cells plus 6 double cells is 12 of the 36, so Pr(win) $=\frac{12}{36}=\frac{1}{3}$.

Q18. a) At least one is lunch plus drink minus both (the 15 were counted twice): $22+25-15=32$, so the probability is $\frac{32}{40}=\frac{4}{5}$. b) Neither is $40-32=8$ students: $\frac{8}{40}=\frac{1}{5}$ — check: $\frac{4}{5}+\frac{1}{5}=1$ ✓.

Q19. a) At least one is $0.6+0.4-0.2=0.8$, so neither is $1-0.8=0.2$. b) Rain only leaves the overlap out of the rain probability: $0.6-0.2=0.4$. c) 0.8, as found in part a.

Q20. a) Each of the 3 spinner results pairs with each of the 2 coin results: 6 combinations — R-H, R-T, B-H, B-T, G-H, G-T. b) One of the six is red and head, so Pr(win) $=\frac{1}{6}$. c) With 120 players the expected count of winners is $120\times\frac{1}{6}=20$. The actual number will vary — 120 players is a group of chance outcomes, and probability says what happens on average over many groups, not what any one group of 120 must show.

Simulating compound events

> **Content description `VC2M8P03`:** conduct repeated chance experiments and simulations, using digital tools to determine probabilities for compound events, and describe results

One roll of two dice can give any difference from 0 to 5 — but the differences are not equal rivals. In this subtopic you will settle the argument the only honest way: make a computer roll the dice thousands of times, watch the relative frequencies settle down, and describe what the long run says. The same trick answers any "how often does this really happen?" question about two chance events together.

Theory

Recall: chance from Year 7

In Year 7 you listed the possible outcomes of a chance experiment — its **sample space** — and assigned probabilities between 0 and 1, with equally likely outcomes sharing the probability evenly (`VC2M7P01`). You also ran **repeated chance experiments**: you repeated an experiment many times, compared the observed results with what was predicted, and saw that larger numbers of trials give results closer to the prediction (`VC2M7P02`). This subtopic uses both ideas without re-teaching them, and adds one new tool to do the repeating for you: a **digital simulation**.

You also have two results from earlier in this chapter. From 7A, if A is any event, then

$$Pr(A) + Pr(\text{not } A) = 1$$

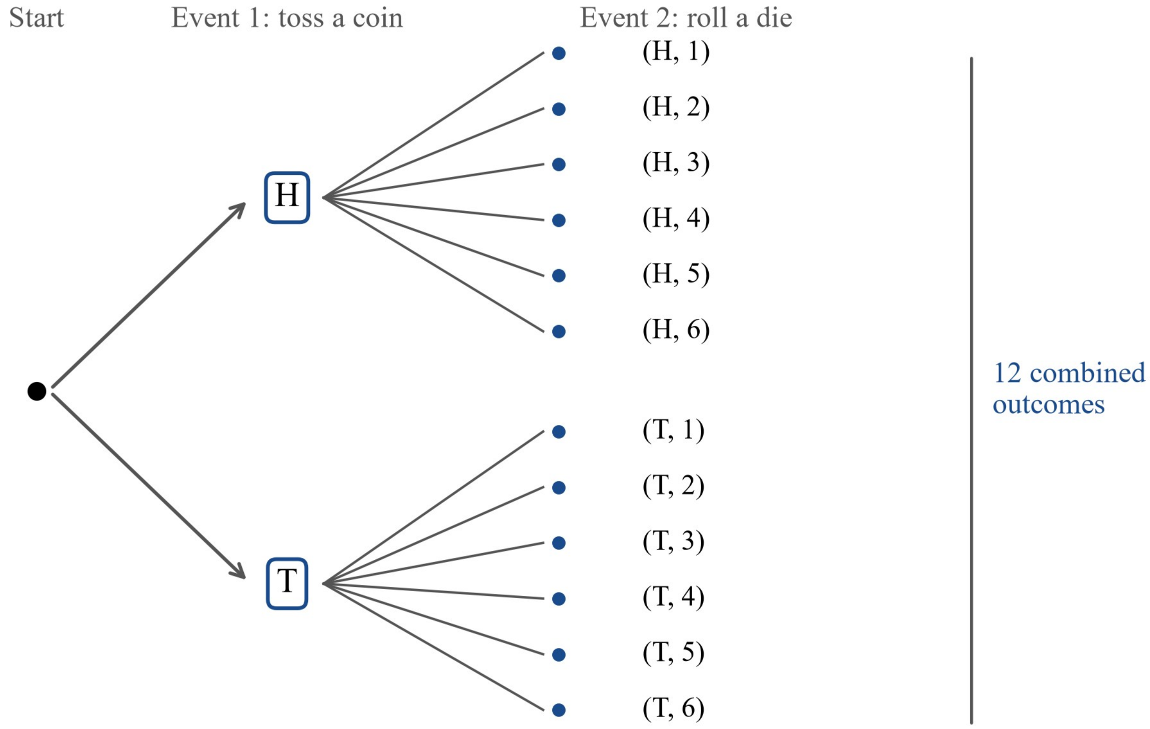
and from 7B, you can list **all** the outcome combinations of two events using a two-way table, a tree diagram or an organised list.

Compound events

A **compound event** is an event built from two or more chance activities. Tossing a coin **and** rolling a die is a compound event; so is rolling two dice and looking at their sum, or tossing two coins and asking whether at least one is a head.

At Level 8, you work out **all the possible outcome combinations** for the two events, and then count the combinations that give the event you care about.

A compound event: two single events combined in one trial



A tree diagram of a compound event: a coin toss branches to heads and tails, and each of those branches then splits into the six results of a die roll, giving twelve combined outcomes in all

In the diagram, the coin has 2 possible results and the die has 6, so the compound experiment has $2 \times 6 = 12$ outcome combinations:

$$(\mathrm{H},1),(\mathrm{H},2),(\mathrm{H},3),(\mathrm{H},4),(\mathrm{H},5),(\mathrm{H},6),(\mathrm{T},1),(\mathrm{T},2),(\mathrm{T},3),(\mathrm{T},4),(\mathrm{T},5),(\mathrm{T},6)$$

If the coin is fair and the die is fair, all 12 combinations are **equally likely**, so each one has probability $\frac{1}{12}$.

The sample space of two dice

Rolling two dice is the compound experiment this subtopic keeps coming back to. Each die can land 1, 2, 3, 4, 5 or 6, so there are $6 \times 6 = 36$ outcome combinations, all equally likely. The grid below is the sample space, with the first die reading across and the second die reading down.

The sample space of two dice: 6 × 6 = 36 equally likely combinations
Shaded cells are the 6 combinations whose sum is 7 → Pr(sum is 7) = 6/36 = 1/6

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	**(1, 6)**
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	**(2, 5)**	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	**(3, 4)**	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	**(4, 3)**	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	**(5, 2)**	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	**(6, 1)**	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The sample space of rolling two dice: a six-by-six grid of the 36 equally likely outcome combinations, with the six combinations whose sum is 7 shaded, so the probability of a sum of 7 is 6 out of 36, which equals one-sixth

The shaded cells are the combinations with **sum 7**. There are 6 of them out of 36, so

$$Pr(\text{sum is } 7) = \frac{6}{36} = \frac{1}{6}$$

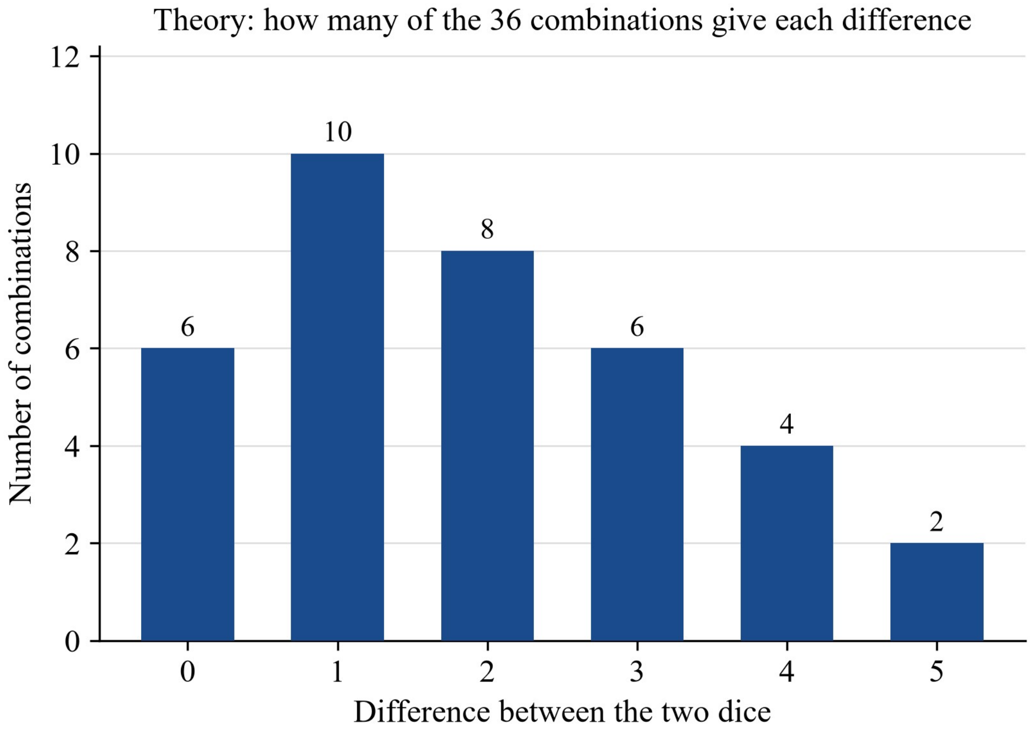
The grid is worth memorising as a habit, not as a picture: whenever an event about two dice seems hard, count its cells in the grid. Counting combinations is the whole skill — no new probability rules are needed.

The difference game

Here is the question that organises this subtopic. Roll two dice, subtract the smaller number from the larger, and call the answer the **difference** (if the dice match, the difference is 0). The difference is always one of 0, 1, 2, 3, 4, 5.

Which difference is likely to occur more often?

Counting the cells of the grid gives the answer. The diagram below shows how many of the 36 combinations give each difference.



A bar chart of the theoretical counts for the difference between two dice: difference 0 has 6 combinations, difference 1 has 10, difference 2 has 8, difference 3 has 6, difference 4 has 4, and difference 5 has 2, out of 36 in all

So the theoretical probabilities are

$$Pr(\text{difference } 0)=\frac{6}{36}, Pr(\text{difference } 1)=\frac{10}{36}, Pr(\text{difference } 2)=\frac{8}{36}, Pr(\text{difference } 3)=\frac{6}{36}, Pr(\text{difference } 4)=\frac{4}{36}, P$$

and **difference 1 is the most likely**, with $\frac{10}{36}\approx 0.278$, while difference 5 is the least likely, with $\frac{2}{36}\approx 0.056$.

That settles it exactly — but this subtopic is about what happens when you **run the experiment**, and whether the real results agree. Time to simulate.

What a simulation is

A **simulation** is an experiment in which **random numbers stand in for the real thing**. Instead of really rolling two dice a thousand times by hand, you let a **random number generator** — the one built into a spreadsheet or a simulation app — produce the numbers, and you agree on what each number means.

The one rule that makes a simulation honest: **the random numbers must match the chances of the real experiment**. A fair die has six equally likely results, so you use random whole numbers from 1 to 6 — never 1 to 5, and never a list where some numbers appear more often than others.

Every simulation in this book follows the same six steps.

Planning a simulation

1. Ask the question. Define one trial and the event to count.
2. Map random numbers to outcomes so the chances match.
3. Run many trials with a digital random number generator.
4. Tally how many times the event happens.
5. Relative frequency = event count ÷ number of trials.
6. Describe the result and compare it with the theory.

A flowchart of the six steps for planning a simulation: ask the question and define one trial, map random numbers to outcomes so the chances match, run many trials with a digital random number generator, tally how many times the event happens, compute relative frequency as event count divided by number of trials, and describe the result and compare it with the theory

Step 2 is the one that goes wrong most often in practice — check the mapping before you trust any result. And step 3 is where the digital tool earns its place: a computer can run 1000 trials in a second, which is exactly why simulation works (D10 note: a simulation for this content description **must** use a digital tool).

> **Run it yourself.** Using the random number generator in a spreadsheet or a simulation app, run the difference experiment for 100 trials: generate two random whole numbers from 1 to 6 for each trial, find the difference, and tally the results. Then read on and compare your 100-trial run with the runs below. (Every run shown in this subtopic is one real simulation, produced by a seeded random number generator, so the numbers always match the diagrams.)

Running the experiment: 10 trials, then 1000

Here are the first 10 trials of the simulated difference experiment.

The first 10 trials of the two-dice-difference run: difference 1 appeared 2 times in 10 — is that enough to decide anything?

Trial	1	2	3	4	5	6	7	8	9	10
Die 1	6	2	1	4	3	3	1	3	4	3
Die 2	3	6	5	2	4	6	6	2	2	1
Difference	3	4	4	2	**1**	3	5	**1**	2	2

Tally of the 10 differences: 0 → none, 1 → ||, 2 → |||, 3 → ||, 4 → ||, 5 → |

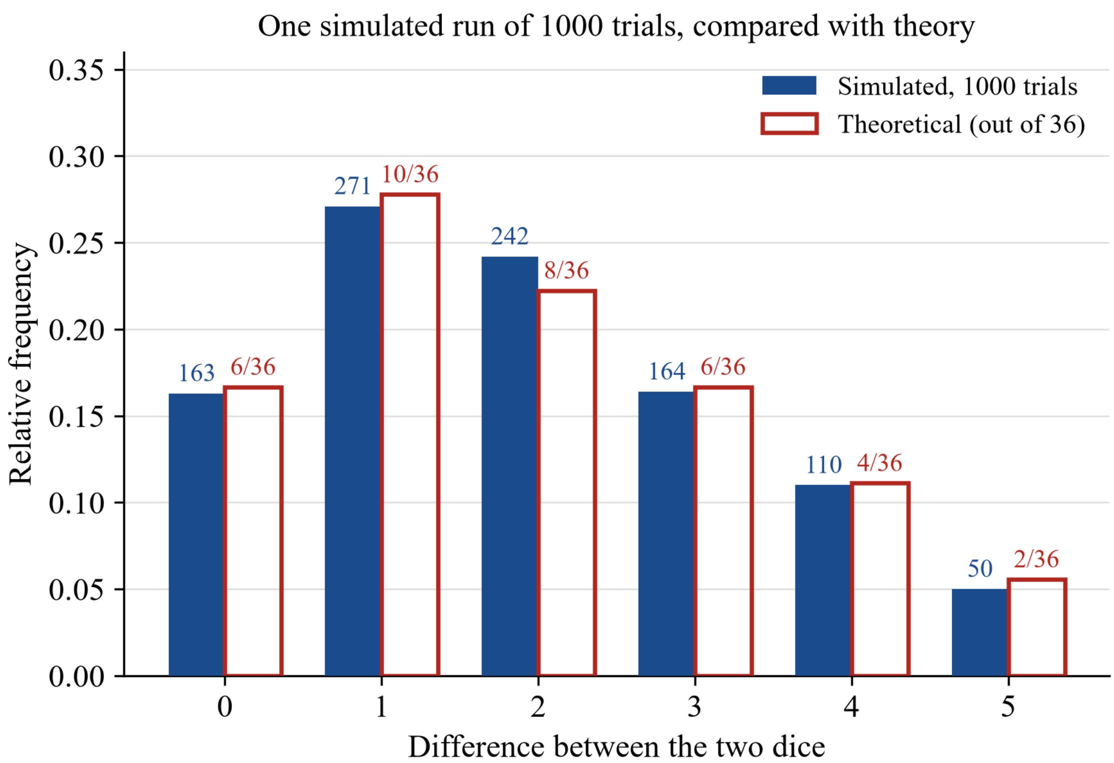
A table of the first ten trials of the dice-difference simulation, showing the two dice and the difference for each trial, with the tally of the ten differences: difference 1 twice, difference 2 three times, difference 3 twice, difference 4 twice, difference 5 once, and difference 0 not at all

The tally for these 10 trials is:

Difference	0	1	2	3	4	5
Count	0	2	3	2	2	1

After only 10 trials, difference 2 is "winning" with 3 hits. Does that mean the theory was wrong and difference 2 is the most likely? No — 10 trials is far too few, and small runs jump around. This is the same message as `VC2M7P02`: **small numbers of trials give unreliable pictures**.

So the simulation kept going — to 1000 trials.



A bar chart comparing the observed counts from 1000 simulated trials, 163, 271, 242, 164, 110 and 50 for differences 0 to 5, with the theoretical probabilities 6, 10, 8, 6, 4 and 2 out of 36 drawn alongside as fractions

The 1000-trial counts, and the relative frequencies they give:

Difference	0	1	2	3	4	5
Count (of 1000)	163	271	242	164	110	50
Relative frequency	0.163	0.271	0.242	0.164	0.110	0.050
Theoretical probability	$\frac{6}{36} \approx 0.167$	$\frac{10}{36} \approx 0.278$	$\frac{8}{36} \approx 0.222$	$\frac{6}{36} \approx 0.167$	$\frac{4}{36} \approx 0.111$	$\frac{2}{36} \approx 0.056$

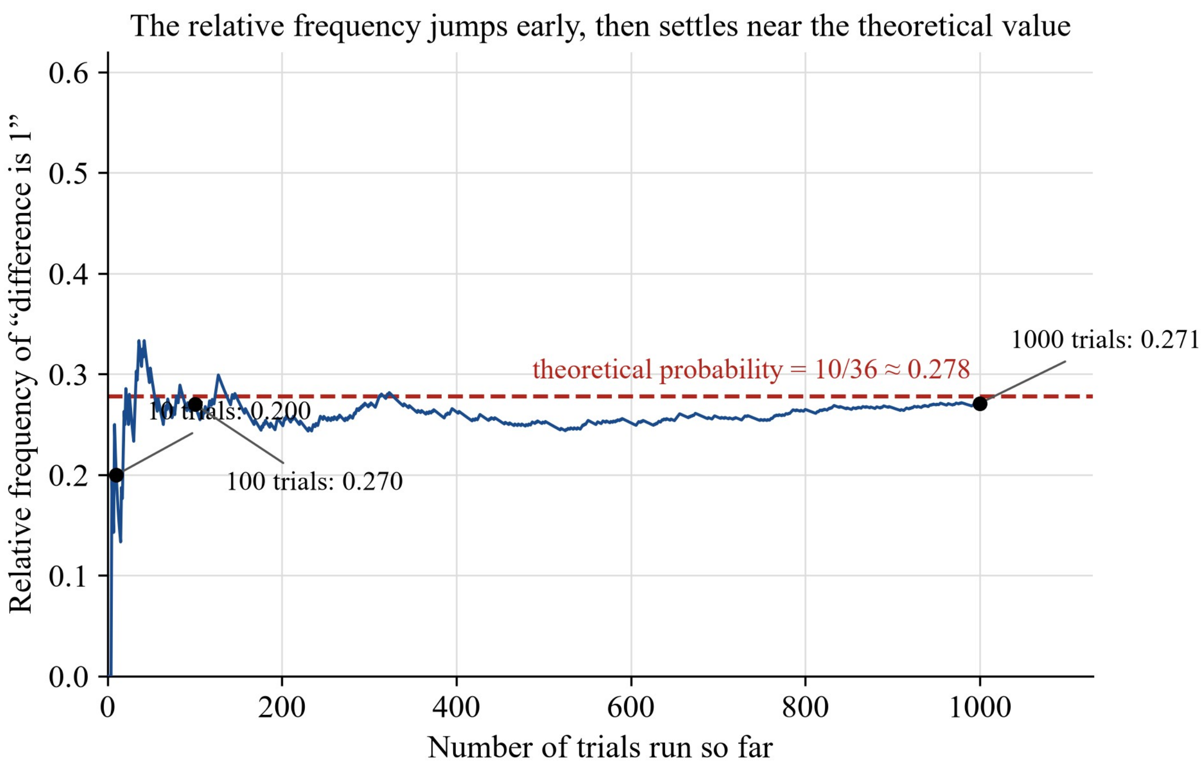
Now the picture matches the theory: difference 1 occurred most often, exactly as the counting predicted. The **relative frequency** of an event is

$$\text{relative frequency} = \frac{\text{number of times the event happens}}{\text{number of trials}}$$

and it is the simulation's answer to "how often does this happen?".

The long run

The best view of the whole subtopic is a single curve. The diagram below tracks the relative frequency of **difference 1** as the simulation runs — recalculating it after every trial.



The running relative frequency of difference 1 across 1000 trials, jumping widely at first and settling closer and closer to the theoretical probability 10 over 36, about 0.278, which is drawn as a dashed line; after 10 trials it sits at 0.200, after 100 trials at 0.270, and after 1000 trials at 0.271

Read the checkpoints from left to right:

- after 10 trials: $\frac{2}{10} = 0.200$ — well away from $\frac{10}{36} \approx 0.278$;
- after 100 trials: $\frac{27}{100} = 0.270$ — already close;
- after 1000 trials: $\frac{271}{1000} = 0.271$ — very close.

The curve jumps around a lot at first, then **settles down** and stays close to the theoretical probability. This settled behaviour after many trials is called the **long run**, and it is the whole reason simulation works: in the long run, relative frequencies settle near the theoretical probabilities.

Notice what this section is doing, because it is the deliverable of this subtopic — **describing the result**:

> **Describing the result of the 1000-trial run.** In 1000 simulated rolls of two dice, difference 1 occurred 271 times, a relative frequency of 0.271. The theoretical probability is $\frac{10}{36} \approx 0.278$. The relative frequency is close to the theoretical probability, and the running curve settles near it, so the simulation agrees with the theory: difference 1 occurs more often than any other difference.

A good description always names the event, the number of trials, the relative frequency, what it settles near, and how that compares with the theoretical probability.

Complementary events in the long run

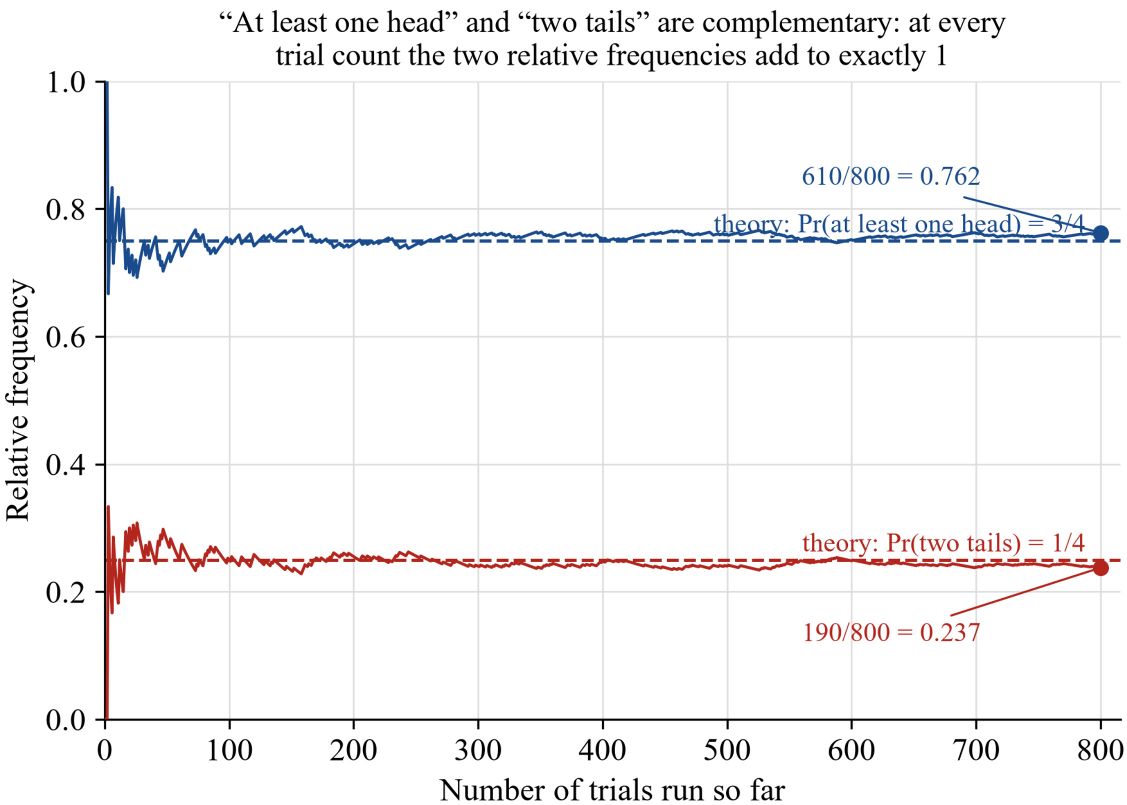
The same ideas answer a second question: do **complementary events** behave in the long run the way the theory says they should?

Toss two coins. The event "**at least one head**" and the event "**two tails**" are complementary: they can never both happen, and one of them must happen every time. The four equally likely combinations are HH, HT, TH, TT, so

$$Pr(\text{at least one head}) = \frac{3}{4} = 0.75, \; Pr(\text{two tails}) = \frac{1}{4} = 0.25$$

and, as 7A says, they add to 1.

A simulation of 800 two-coin trials tracked both events at once.



The running relative frequencies of two complementary events across 800 simulated two-coin trials: at least one head stays close to three-quarters and two tails stays close to one-quarter, finishing at 610 out of 800 and 190 out of 800

In the full run, at least one head happened 610 times and two tails happened 190 times:

$$\frac{610}{800}=0.7625\,\text{and}\,\frac{190}{800}=0.2375$$

Both are close to the theory (0.75 and 0.25), and they add to exactly 1 — as they must, trial by trial, because on every single trial exactly one of the two events happened. That is the long-run picture of complementary events from `VC2M8P01`: **the relative frequencies of complementary events settle at probabilities that add to 1**.

Be careful: not every pair of events is complementary. "Difference 0" and "difference 1" can never both happen, but they do not cover every trial — most trials give neither — so they are **not** complementary, and their probabilities do not add to 1.

The subtopic in one picture

The six ideas of this subtopic fit on six cards.

Simulating compound events — key ideas

Compound event

two or more single events combined in one trial; its outcomes are the combinations of the single outcomes

Sample space of two dice

$6 \times 6 = 36$ equally likely combinations; count the favourable ones to find the theoretical probability

Relative frequency

event count ÷ number of trials; it changes from run to run, especially for small trial counts

Long run

the more trials, the closer the relative frequency sits to the theoretical probability

Simulation

a digital random number generator stands in for the real experiment so thousands of trials take seconds

Complementary events

exactly one of the pair happens on every trial; the two probabilities (and the two relative frequencies) add to 1

Six summary cards for this subtopic: compound event, the sample space of two dice, relative frequency, the long run, simulation, and complementary events, each with a one-line description

If you can say what each card means, you can run — and describe — any simulation at this level.

How to describe the result of a simulation

Keep this checklist. Any question that says *describe the result* wants every line of it:

1. Name the event and the number of trials.
2. Give the count and the relative frequency (count ÷ trials).
3. Say what the relative frequency settles near as trials increase.
4. If a theoretical probability exists, compare: close, or not close?

5. Comment on the number of trials — was it large enough to trust?

What comes next

At Level 9 (`VC2M9P03`), you will use exactly these simulation skills to **estimate probabilities that cannot be worked out exactly** — when there is no neat sample space to count, the long run becomes the answer.

Words of this subtopic

Word	What it means here
compound event	an event built from two or more chance activities
trial	one run of the experiment, from start to result
sample space	the list of all possible outcomes (for two dice: 36 combinations)
equally likely	every outcome has the same chance
simulation	an experiment in which random numbers stand in for the real thing
random number generator	a digital tool that produces unpredictable numbers, built into spreadsheets and simulation apps
relative frequency	number of times the event happens ÷ number of trials
long run	the settled behaviour of relative frequencies after many trials
theoretical probability	the exact probability found by counting the sample space
complementary events	two events that can never both happen, but one of which must happen every trial; their probabilities add to 1
tally	a running count of how many times each result occurs

Worked examples

Example 1 — with help

Two fair dice are rolled.

(a) Find the probability that the sum is 7.
(b) Find the probability that the sum is greater than 9.

Working	Comment
(a) There are $6\times6=36$ equally likely combinations.	List all outcome combinations for the two events.
Sum 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) — 6 combinations	Count the cells of the grid that give the event.
$Pr(\text{sum is }7)=\frac{6}{36}=\frac{1}{6}$	Count ÷ total.
(b) Sum 10: (4,6), (5,5), (6,4) — 3 combinations	Sum greater than 9 means 10, 11 or 12. Count each.
Sum 11: (5,6), (6,5) — 2 combinations	
Sum 12: (6,6) — 1 combination	
Total: $3+2+1=6$ combinations	
$Pr(\text{sum}>9)=\frac{6}{36}=\frac{1}{6}$	A different event can have the same probability.

Example 2 — with help

At a school fete, a game tosses two fair coins. You win if **at least one coin shows a head**. Design a simulation, using a digital random number generator, to estimate the chance of winning.

Working	Comment
1. One trial = tossing two coins. The event to count is "at least one head".	Ask the question: define one trial and the event.
2. Let 1 stand for a head and 0 for a tail; generate random whole numbers 0 and 1 with equal chances.	Map random numbers to outcomes so the chances match — a fair coin has two equally likely results.
3. Generate two numbers per trial (one per coin); run a large number of trials, say 500.	A digital random number generator does the repeating; many trials give the long-run picture.
4. Count the trials where at least one number is 1.	Tally the event.
5. Relative frequency = wins ÷ 500.	This estimates the chance of winning.
6. Compare with the theory: $Pr(\text{at least one head})=\frac{3}{4}=0.75$.	HH, HT, TH win; only TT loses. Describe the result and compare.

Example 3 — with help

The 1000-trial simulation in the theory gave difference 2 exactly 242 times.

(a) Find the relative frequency of difference 2 in this run.
(b) Describe how the result compares with the theoretical probability.

Working	Comment
(a) Relative frequency $=242\div 1000=0.242$	Count ÷ number of trials.
(b) Theoretical probability: $\frac{8}{36}\approx 0.222$	Count the grid: 8 of the 36 combinations give difference 2.
0.242 is close to 0.222	Difference of about 0.02.
In 1000 trials, difference 2 occurred with relative frequency 0.242, which is close to the theoretical probability $\frac{8}{36}\approx 0.222$, so the run agrees with the theory.	A full description names the event, the trials, the relative frequency and the comparison.

Example 4 — on your own

In the dice-difference simulation, difference 1 occurred 2 times in the first 10 trials and 271 times in the first 1000 trials.

(a) Find the relative frequency of difference 1 after 10 trials and after 1000 trials.
(b) Which result should you trust more? Explain why.

Solution.

(a) After 10 trials: $\frac{2}{10}=0.2$. After 1000 trials: $\frac{271}{1000}=0.271$.
(b) Trust the 1000-trial result. Small numbers of trials jump around — after 10 trials, the relative frequency 0.2 is well away from the theoretical probability $\frac{10}{36}\approx 0.278$. After 1000 trials, 0.271 is very close to it. The more trials, the closer relative frequencies settle to the theoretical probabilities — that is the long run.

Example 5 — on your own

In the two-coin simulation, "at least one head" happened 610 times in 800 trials and "two tails" happened 190 times.

(a) Show that the two relative frequencies add to 1.
(b) Explain why they **must** add to 1.
(c) Compare each with its theoretical probability.

Solution.

(a) $\frac{610}{800}+\frac{190}{800}=\frac{800}{800}=1$.

(b) The two events are **complementary**: on every trial, either at least one head shows or both coins are tails — never both at once, and never neither. So every one of the 800 trials is counted exactly once between them, and the two counts must add to 800. This is the rule from 7A: $Pr(A)+Pr(\text{not } A)=1$.

(c) 0.7625 is close to $\frac{3}{4}=0.75$, and 0.2375 is close to $\frac{1}{4}=0.25$. Both relative frequencies settle near their theoretical probabilities — the long-run behaviour of complementary events.

Example 6 — on your own

A student simulates rolling two dice 900 times and counts how often the sum is even. The event happens 441 times.

(a) Find the relative frequency of an even sum in this run.
(b) Find the theoretical probability that the sum of two dice is even.
(c) Describe the result of the simulation.

Solution.

(a) Relative frequency $=441 \div 900=0.49$.

(b) The sum is even when both dice are odd or both dice are even. Counting the grid: $3 \times 3=9$ odd–odd combinations and $3 \times 3=9$ even–even combinations, so 18 of the 36 combinations give an even sum. $Pr(\text{sum is even})=\frac{18}{36}=0.5$.

(c) In 900 trials, an even sum occurred 441 times, a relative frequency of 0.49. This is close to the theoretical probability 0.5, so the simulation agrees with the theory: about half of all trials give an even sum.

Practice questions

A digital tool with a random number generator — a spreadsheet or a simulation app — is needed for Q9, Q17c, Q18d and Q19d, where you design or check a simulation. The other questions can be done by hand with the grid and the theory. The simulation data quoted in the questions come from the same seeded run as the diagrams in this subtopic.

With help

Q1. Copy and complete each sentence.

a. An event built from two or more chance activities is called a … event.
b. The relative frequency of an event is the number of times it happens divided by the number of … .
c. Two events are … if they can never both happen, but one of them must happen every trial.
d. The settled behaviour of relative frequencies after many trials is called the … .

Q2. A coin is tossed, and then a fair die is rolled.

a. How many outcome combinations are there in all?
b. List the combinations where the coin shows a head **and** the die shows an even number.
c. Find the probability of that event.

Q3. Two fair dice are rolled. Use the 36-combination sample space.

a. Find the probability that the sum is 7.
b. Find the probability that the dice show the same number (a **double**).
c. Find the probability that the sum is 2.

Q4. Which one of these is a compound event?

A. Rolling one die and getting a 6. B. Tossing two coins and getting at least one head. C. Drawing one card from a deck and getting a red card. D. Spinning one spinner and getting a 3.

Q5. The first 10 trials of the dice-difference simulation gave this tally:

Difference	0	1	2	3	4	5
Count	0	2	3	2	2	1

a. Find the relative frequency of difference 1 in these 10 trials.
b. Find the relative frequency of difference 2 in these 10 trials.
c. In this tally, difference 2 has the highest count. Explain why these 10 trials are still too few to decide which difference is most likely.

Q6. The six steps of planning a simulation are listed below out of order. Write them in the correct order (the same order as the flowchart in the theory).

A. Tally how many times the event happens. B. Ask the question: define one trial and the event to count. C. Describe the result and compare it with the theory. D. Run many trials with a digital random number generator. E. Map random numbers to outcomes so the chances match. F. Relative frequency = event count ÷ number of trials.

Q7. Say whether each pair of events is complementary. Give a reason each time.

a. Rolling two dice: “the sum is even” and “the sum is odd”.
b. Rolling two dice: “the sum is 7” and “the sum is 8”.
c. Tossing two coins: “at least one head” and “two tails”.
d. Tossing two coins: “at least one head” and “two heads”.

Q8. Two fair dice are rolled. Give each probability as a fraction out of 36.

a. $Pr(\text{difference } 0)$
b. $Pr(\text{difference } 4)$
c. $Pr(\text{difference } 5)$

On your own

Q9. You will need a digital tool with a random number generator. Use the six steps from the theory to design a simulation for the fete game of Example 2 (toss two coins; win if at least one is a head). Your plan must say:

a. what one trial is, and which event you will count;
b. how you will map random numbers to heads and tails, and why the chances match;
c. how many trials you will run, and why you chose that many;
d. what you will calculate at the end;
e. the theoretical probability you will compare your result with.

Then run your simulation and report the result.

Q10. In the 1000-trial dice-difference simulation, the counts were:

Difference	0	1	2	3	4	5
Count	163	271	242	164	110	50

a. Which difference occurred most often, and how many times?
b. Find its relative frequency.
c. Find the theoretical probability of that difference.
d. Describe the result, comparing the relative frequency with the theoretical probability.

Q11. In the dice-difference simulation, the relative frequency of difference 1 was 0.200 after 10 trials, 0.270 after 100 trials and 0.271 after 1000 trials. The theoretical probability is $\frac{10}{36} \approx 0.278$. In two or three sentences, describe what happens to the relative frequency as the number of trials increases.

Q12. A friend rolls two dice 10 times, gets difference 1 four times, and says: "The probability of difference 1 is 0.4, not $\frac{10}{36}$." Write a short reply using the ideas of this subtopic (number of trials, long run, theoretical probability).

Q13. In 800 simulated trials of tossing two coins, "at least one head" happened 610 times and "two tails" happened 190 times.

a. Find the relative frequency of each event.
b. Show that the two relative frequencies add to 1, and explain why they must.
c. Are the two events complementary? Give both reasons from the definition.
d. Compare each relative frequency with its theoretical probability.

Q14. At the fete, the dice-difference game pays a prize when the difference is 0, 1 or 2.

a. Find the probability of winning. (Hint: add the counts from the theoretical bar chart.)
b. Find the probability of losing, two ways: by counting, and by using the rule from 7A.
c. Is it more likely to win or to lose? Explain.

Q15. A simulation rolls two dice 900 times and counts the trials where the sum is even. The event happens 441 times.

a. Find the relative frequency of an even sum.
b. Find the theoretical probability that the sum is even.
c. Describe the result of the simulation.

Q16. Two students each run their own 1000-trial simulation of the dice-difference experiment. One gets difference 1 exactly 271 times; the other gets it 289 times.

a. Is either student wrong? Explain.
b. Both students compare their relative frequencies with $\frac{10}{36} \approx 0.278$. What does the fact that their runs differ show about simulation?

Maths in real life

Q17. In a board game, you roll two dice on each turn, and if the dice show a double you take an extra turn.

a. Find the probability of rolling a double.
b. In 60 turns, about how many extra turns would you expect?
c. A player claims the dice are not fair, because they rolled 8 doubles in their first 30 turns. Comment on the claim, and describe how you could use a simulation to check it.

Q18. At the school fete, a game uses two spinners: spinner A has 3 equal sectors numbered 1, 2, 3, and spinner B has 4 equal sectors numbered 1, 2, 3, 4. You spin both, and win a prize if the two numbers add to 5.

a. How many outcome combinations are there in all?
b. List the winning combinations and find the probability of winning.
c. If 240 people play the game, about how many would you expect to win?
d. Describe how you would use a digital simulation to check your answer to b.

Q19. Two netball teams decide who takes the first centre pass by tossing two coins. If the coins match (both heads or both tails), team A goes first; if they differ, team B goes first.

a. List the four equally likely outcome combinations.
b. Find the probability that team A goes first, and the probability that team B goes first.

c. Is the method fair? Explain.
d. Describe a simulation you could run to check your answer to c.

Q20. In a race game, each player chooses one "lucky difference" from 0 to 5. On each turn, two dice are rolled, and any player whose lucky difference matches the difference of the dice moves forward.

a. Which lucky difference would you choose? Use the theoretical counts to explain why.
b. Your friend chose 5. After 20 turns with no 5, they say "a difference of 5 is due — it has to come up soon." Explain what is wrong with this thinking.

Answers

Q1. a. compound. b. trials. c. complementary. d. long run.

Q2. a. 12. b. (H, 2), (H, 4), (H, 6). c. $\frac{3}{12}=\frac{1}{4}$.

Q3. a. $\frac{6}{36}=\frac{1}{6}$. b. $\frac{6}{36}=\frac{1}{6}$. c. $\frac{1}{36}$.

Q4. B.

Q5. a. 0.2. b. 0.3. c. See the fully-worked solution.

Q6. B, E, D, A, F, C.

Q7. a. Yes. b. No. c. Yes. d. No.

Q8. a. $\frac{6}{36}$. b. $\frac{4}{36}$. c. $\frac{2}{36}$.

Q9. Answers may vary; see the fully-worked solution for a complete plan.

Q10. a. Difference 1, 271 times. b. 0.271. c. $\frac{10}{36}\approx 0.278$. d. See the fully-worked solution.

Q11. See the fully-worked solution.

Q12. See the fully-worked solution.

Q13. a. 0.7625 and 0.2375. b. See the fully-worked solution. c. Yes. d. Close to 0.75 and 0.25.

Q14. a. $\frac{24}{36}=\frac{2}{3}$. b. $\frac{12}{36}=\frac{1}{3}$, and $1-\frac{2}{3}=\frac{1}{3}$. c. Win.

Q15. a. 0.49. b. 0.5. c. See the fully-worked solution.

Q16. a. No. b. See the fully-worked solution.

Q17. a. $\frac{1}{6}$. b. About 10. c. See the fully-worked solution.

Q18. a. 12. b. (1,4), (2,3), (3,2); $\frac{3}{12}=\frac{1}{4}$. c. About 60. d. See the fully-worked solution.

Q19. a. HH, HT, TH, TT. b. $\frac{1}{2}$ each. c. Yes. d. See the fully-worked solution.

Q20. a. 1. b. See the fully-worked solution.

Fully-worked solutions

Q1. Apply the definitions from "Words of this subtopic".

a. compound — an event built from two or more chance activities.
b. trials — relative frequency = event count ÷ number of trials.
c. complementary — never both, always one; their probabilities add to 1.
d. long run — relative frequencies settle near the theoretical probabilities.

Q2. Count the combinations, then the winning ones.

a. The coin has 2 results and the die has 6, so there are $2\times 6=12$ equally likely combinations.

b. Head on the coin and an even number on the die: (H, 2), (H, 4), (H, 6) — 3 combinations.

c. $Pr = \frac{3}{12} = \frac{1}{4}$.

Q3. Count cells of the 36-combination grid.

a. Sum 7 has 6 combinations: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). $Pr = \frac{6}{36} = \frac{1}{6}$.

b. The doubles are (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) — 6 combinations. $Pr = \frac{6}{36} = \frac{1}{6}$.

c. Only (1,1) gives sum 2. $Pr = \frac{1}{36}$.

Q4. A compound event is built from two or more chance activities. A, C and D each use one activity (one die, one card, one spinner). B uses two coins — it is the compound event.

Q5. Use the definition of relative frequency, then the lesson about small runs.

a. Difference 1 occurred 2 times in 10 trials: $\frac{2}{10} = 0.2$.

b. Difference 2 occurred 3 times in 10 trials: $\frac{3}{10} = 0.3$.

c. Ten trials is far too few. With only 10 trials, the counts jump around from one possible set of results to the next — here difference 2 leads by a single hit. The theoretical count says difference 1 has 10 of the 36 combinations while difference 2 has only 8, so in the long run difference 1 must come out ahead. You need many trials — like the 1000-trial run — before the picture can be trusted.

Q6. Follow the flowchart in the theory: first ask the question (B), then map the random numbers (E), then run the trials (D), then tally the event (A), then compute the relative frequency (F), then describe and compare (C). Order: **B, E, D, A, F, C**.

Q7. Complementary means: never both, and always one.

a. **Yes.** A sum cannot be both even and odd, and every sum is either even or odd — one of the two events happens every trial.
b. **No.** They can never both happen, but most trials give neither (a sum of 2, for instance), so one of them does not have to happen. They are not complementary.
c. **Yes.** “At least one head” covers HH, HT, TH; “two tails” covers TT. Every trial gives exactly one of the two.
d. **No.** HH makes both events happen at once (HH has at least one head, and is also two heads), and complementary events can never both happen.

Q8. Read the counts from the theoretical bar chart.

a. Difference 0: 6 combinations — $\frac{6}{36}$.

b. Difference 4: 4 combinations — $\frac{4}{36}$.

c. Difference 5: 2 combinations — $\frac{2}{36}$.

Q9. One complete plan (other correct plans are possible):

a. One trial is tossing two coins. The event to count is “at least one head” (a win).
b. Map random whole numbers 0 and 1 to the coins — say 1 = head, 0 = tail, with the generator giving each with equal chance. This matches a fair coin, which has two equally likely results. Generate two numbers per trial, one for each coin.

c. Run a large number of trials — say 500 or 1000. Many trials are needed because small runs jump around; the long run is what settles near the true probability.

d. Calculate the relative frequency of wins: wins ÷ number of trials.

e. Compare with the theoretical probability $\frac{3}{4}=0.75$ (three of the four combinations HH, HT, TH, TT win).

A correct run should finish with a relative frequency near 0.75, and the description should say so and name the number of trials.

Q10. Read the table, then describe.

a. Difference 1 occurred most often: 271 times.

b. Relative frequency $=\frac{271}{1000}=0.271$.

c. Difference 1 has 10 of the 36 combinations, so the theoretical probability is $\frac{10}{36}\approx 0.278$.

d. In 1000 trials, difference 1 occurred 271 times, a relative frequency of 0.271. This is close to the theoretical probability $\frac{10}{36}\approx 0.278$, so the simulation agrees with the theory: difference 1 occurs more often than any other difference.

Q11. The relative frequency jumps well away from the theoretical probability when there are only 10 trials (0.200), then moves close to it by 100 trials (0.270) and stays close at 1000 trials (0.271). As the number of trials increases, the relative frequency settles near the theoretical probability 0.278 — that is the long run.

Q12. A complete reply:

Ten trials is far too few to measure a probability. In small runs the relative frequencies jump around — your 0.4 is one of many values it could take. The theoretical probability, counted from the 36 equally likely combinations, is $\frac{10}{36}\approx 0.278$, and when the experiment is simulated for 1000 trials the relative frequency settles close to that (0.271). Run many more trials and yours will move towards 0.278 as well.

Q13. Use the definitions, then compare.

a. At least one head: $\frac{610}{800}=0.7625$. Two tails: $\frac{190}{800}=0.2375$.

b. $0.7625+0.2375=1$. They must add to 1 because every trial gives exactly one of the two events: the counts 610 and 190 add to 800, the total number of trials.

c. Yes. The two reasons: they can never both happen (a trial with at least one head is not two tails), and one of them must happen every trial (any pair of coins either has a head or is two tails).

d. 0.7625 is close to $\frac{3}{4}=0.75$, and 0.2375 is close to $\frac{1}{4}=0.25$ — the long-run behaviour of complementary events.

Q14. Count the winning combinations.

a. Winning differences are 0, 1 and 2, with theoretical counts 6, 10 and 8. So $6+10+8=24$ of the 36 combinations win: $Pr(\text{win})=\frac{24}{36}=\frac{2}{3}$.

b. By counting: $36-24=12$ combinations lose, so $Pr(\text{lose})=\frac{12}{36}=\frac{1}{3}$. By the rule from 7A: winning and losing are complementary, so $Pr(\text{lose})=1-\frac{2}{3}=\frac{1}{3}$.

c. It is more likely to win, because $\frac{2}{3}$ is greater than $\frac{1}{3}$ — winning covers twice as many combinations as losing.

Q15. Compute, then describe.

a. Relative frequency $= \dfrac{441}{900} = 0.49$.

b. The sum is even when both dice are odd (9 combinations) or both are even (9 combinations): 18 of the 36 combinations. $Pr(\text{even sum}) = \dfrac{18}{36} = 0.5$.

c. In 900 trials, an even sum occurred 441 times, a relative frequency of 0.49. This is close to the theoretical probability 0.5, so the simulation agrees with the theory: about half of all trials give an even sum.

Q16. Think about what simulation does.

a. Neither student is wrong. A simulation uses random numbers, so two runs of the same experiment give slightly different counts — that is normal. Both 271 and 289 are close to the expected count near $1000 \times \dfrac{10}{36} \approx 278$.

b. It shows that single runs vary: no two runs of the same experiment land on exactly the same counts. What stays the same is the long-run behaviour — both relative frequencies (0.271 and 0.289) sit close to the theoretical probability 0.278, and more trials would hold them close to it.

Q17. Count the doubles, then reason about the claim.

a. There are 6 doubles out of the 36 combinations: $Pr(\text{double}) = \dfrac{6}{36} = \dfrac{1}{6}$.

b. About $\dfrac{1}{6}$ of the turns: $60 \times \dfrac{1}{6} = 10$ extra turns.

c. 8 doubles in 30 turns is a relative frequency of $\dfrac{8}{30} \approx 0.267$, well above $\dfrac{1}{6} \approx 0.167$ — but 30 turns is a small run, and small runs jump around, so this alone does not prove the dice are unfair. To check with a simulation: use a random number generator to roll two fair dice 30 times, count the doubles, and repeat this whole 30-turn run many times. If 8 or more doubles comes up often in the fair runs, the player's result can easily happen by chance; if it almost never happens, the claim deserves a closer look.

Q18. Count combinations, then simulate to check.

a. Spinner A has 3 results and spinner B has 4, so there are $3 \times 4 = 12$ equally likely combinations.

b. Sum 5: (1,4), (2,3), (3,2) — 3 combinations. $Pr(\text{win}) = \dfrac{3}{12} = \dfrac{1}{4}$.

c. About $\dfrac{1}{4}$ of 240: $240 \times \dfrac{1}{4} = 60$ winners.

d. Map random whole numbers 1–3 to spinner A and 1–4 to spinner B, with each number equally likely — this matches the equal sectors of the spinners. Each trial: generate one number for each spinner and add them. Run many trials (say 600), tally the trials where the sum is 5, and compute the relative frequency. It should settle near $\dfrac{1}{4}$, confirming the count.

Q19. List the sample space, then judge fairness.

a. HH, HT, TH, TT — four equally likely combinations.

b. Team A goes first when the coins match: HH, TT — 2 of the 4 combinations, so $Pr(A) = \dfrac{2}{4} = \dfrac{1}{2}$. Team B goes first when they differ: HT, TH — also $\dfrac{2}{4} = \dfrac{1}{2}$.

c. Yes — both teams have the same probability, $\dfrac{1}{2}$, of going first. (Matching and differing are also complementary: one of them happens every toss.)

d. Simulate many trials of two coins with a random number generator (for instance, 0 = tail and 1 = head, two numbers per trial), tally the matching trials and the differing trials, and compute both relative frequencies. With many trials they should both settle near $\frac{1}{2}$.

Q20. Use the theoretical counts, then fix the "due" mistake.

a. Choose difference 1. It has 10 of the 36 combinations — more than any other difference (difference 5 has only 2), so in the long run it occurs most often: $Pr(\text{difference } 1) = \frac{10}{36} \approx 0.278$.

b. The dice have no memory. Each roll starts fresh, so the probability of a difference of 5 is still $\frac{2}{36}$ on the next roll, no matter what the last 20 rolls did. "It is due" confuses the long run — difference 5 will occur about $\frac{2}{36}$ of all rolls in a very long game — with what must happen next, which is never forced by past rolls.

Test

Chapter test T7 — Chapter 7: Probability: complementary events and combinations of two events

Mathematics Level 8

Time	20 minutes: 5 minutes reading time, then 15 minutes working time
Total marks	15
Equipment	No special equipment is needed for Questions 1 and 2. Question 3 **requires access to a simulation tool** — a spreadsheet or a simulation app with a random number generator.
Calculators	Questions 1 and 2 are **calculator-free**. For Question 3, the simulation tool does the repeating.

Instructions

- Answer all three questions. The mark for each part is shown in brackets.
- Show all your working.
- Probabilities may be given as fractions, decimals or percentages unless a part says otherwise. Give fractions in simplest form.

Question 1 (5 marks) — Complementary events (7A)

(a) $Pr(A)=\frac{2}{9}$. Find $Pr(\text{not } A)$. Give your answer as a fraction. (1 mark)

(b) $Pr(A)=0.47$. Find $Pr(\text{not } A)$. Give your answer as a decimal. (1 mark)

(c) A weather forecast gives a 30% chance of rain tomorrow. Find the probability that it does **not** rain tomorrow. Give your answer as a percentage. (1 mark)

(d) A fair six-sided dice is rolled once. Event E is “roll at least 4”. State the complement of E in words. (1 mark)

(e) A spinner lands on one of four colours: red, blue, green or yellow. The probabilities of red, blue and green are 0.45, 0.30 and 0.10. Find the probability of yellow. (1 mark)

Question 2 (5 marks) — Two events (7B)

In a class of 24 students, 14 own a dog, 10 own a cat, and 6 own both a dog and a cat. One student is chosen at random.

(a) Which representation best shows how the two events “owns a dog” and “owns a cat” overlap in this class? (1 mark)

A. A tree diagram B. A Venn diagram C. A line graph D. A pie chart

(b) How many students own a dog but **not** a cat? (1 mark)

(c) How many students own **neither** pet? (1 mark)

(d) Find $Pr(\text{the student owns both a dog and a cat})$. Give your answer as a fraction in simplest form. (1 mark)

(e) Find $Pr(\text{the student owns at least one of the two pets})$. Give your answer as a fraction in simplest form. (1 mark)

Question 3 (5 marks) — Simulating a compound event (7C)

In a game, two fair coins are flipped. A player scores a point when the two coins **match** — both heads or both tails.

(a) List the four equally likely outcome combinations, and find the theoretical probability that the coins match. (1 mark)

(b) You simulate the game using the random number generator in a spreadsheet or a simulation app. Describe how random numbers can stand in for one trial of the game, so that the chances match the real coins. Say what each random number stands for. (1 mark)

(c) Run your simulation for 50 trials. Record how many of the 50 trials give a match. (1 mark)

(d) Find the relative frequency of a match in your 50 trials. (1 mark)

(e) Describe the result of your simulation: compare your relative frequency with the theoretical probability from part (a), and say what you expect to happen to the relative frequency as the number of trials gets very large. (1 mark)

End of test — 15 marks

Solutions

Worked solutions

Question 1 (5 marks)

(a) A and not A are complementary events — between them they cover every outcome, and they can never happen together. Complementary events add to 1:

$Pr(\text{not } A) = 1 - Pr(A) = 1 - \frac{2}{9} = \frac{7}{9}$ (1 mark)

(b) Keep the whole as 1 in decimal form:

$Pr(\text{not } A) = 1 - 0.47 = 0.53$ (1 mark)

(c) "Rain" and "no rain" are complementary events. In percentages, the whole is $100\,\%$:

$Pr(\text{no rain}) = 100\,\% - 30\,\% = 70\,\%$ (1 mark)

(d) "At least 4" means 4 or more — the outcomes 4, 5 and 6. The complement covers the remaining outcomes, 1, 2 and 3, so the complement of E is **"3 or less"** (the same as "less than 4"). (1 mark)

(e) The four colours share out every spin, so all four probabilities add to 1. The three known probabilities add to

$0.45 + 0.30 + 0.10 = 0.85$

so

$Pr(\text{yellow}) = 1 - 0.85 = 0.15$ (1 mark)

Question 2 (5 marks)

(a) **B — a Venn diagram.** The two events are two yes-or-no questions about the same group of students. A Venn diagram makes *and*, *or* and *neither* visible: "owns both pets" is the visible overlap of the two circles. A tree diagram suits events that happen one after another, and a line graph and a pie chart do not show the overlap of two events. (1 mark)

(b) The 14 dog owners include the 6 students who own both pets, so the dog-only count is

$14 - 6 = 8$ students. (1 mark)

(c) Cat only: $10 - 6 = 4$. The four regions — dog only, both, cat only, neither — add to the class total of 24, so

$\text{neither} = 24 - (8 + 6 + 4) = 24 - 18 = 6$ students. (1 mark)

(d) 6 of the 24 students are in the overlap, and every student is equally likely to be chosen:

$Pr(\text{both}) = \frac{6}{24} = \frac{1}{4}$ (1 mark)

(e) "At least one pet" is every student except the 6 in the neither region: $24 - 6 = 18$ students, so

$Pr(\text{at least one}) = \frac{18}{24} = \frac{3}{4}$

Check using the complement: $1 - \frac{1}{4} = \frac{3}{4}$ ✓ (1 mark)

Question 3 (5 marks)

(a) The four equally likely combinations are HH, HT, TH and TT. The coins match on HH and TT — 2 of the 4 combinations — so

$Pr(\text{match}) = \frac{2}{4} = \frac{1}{2}$ (1 mark)

(b) Example answer: let the random number generator produce 0s and 1s with equal chances, with 0 standing for a tail and 1 standing for a head. Generate two numbers for each trial, one for each coin. The chances match the real game because a fair coin has two equally likely results. (1 mark)

(c) Answers will vary from run to run — that is expected, because a simulation uses random numbers. Example: a 50-trial run gives 27 matches. Award the mark for any count recorded from the student's own run. (1 mark)

(d) Relative frequency = number of matches ÷ number of trials, using the student's own count from part (c). Example:

$\frac{27}{50} = 0.54$ (1 mark)

(e) Example description: in 50 trials, the coins matched 27 times, a relative frequency of 0.54. This is close to the theoretical probability $\frac{1}{2} = 0.5$. As the number of trials gets very large, the relative frequency is expected to settle near the theoretical probability — that is the long run. Award the mark for a description that both compares with $\frac{1}{2}$ and names the long-run settling. (1 mark)

Answer key

Question	Answer	Marks
1a	$\frac{7}{9}$	1
1b	0.53	1
1c	70%	1
1d	"3 or less" (the same as "less than 4")	1
1e	0.15	1
2a	B (a Venn diagram)	1
2b	8 students	1
2c	6 students	1
2d	$\frac{1}{4}$	1
2e	$\frac{3}{4}$	1
3a	$\frac{1}{2}$	1
3b	0 = tail, 1 = head, equal chances; two numbers per trial	1
3c	Answers will vary (student's own run)	1
3d	Answers will vary (follow-through from 3c)	1
3e	Close to $\frac{1}{2}$; settles near it in the long run	1
Total		**15**