Mathematics Level 8

Chapter 5 — Space: congruence, similarity and position in three dimensions

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Contents

Conditions for congruence and similarity

> **Content description `VC2M8SP01`:** identify the conditions for congruence and similarity of triangles and explain the conditions for other sets of common shapes to be congruent or similar, including those formed by transformations

Hold two triangles up to the light. If one can slide, turn or flip until it lies exactly on top of the other, the triangles are congruent — every side matches and every angle matches. If one is simply an enlarged copy of the other, the triangles are similar — the same shape, but a different size. This subtopic works out exactly how much you need to measure to know which is which — and why three well-chosen measurements can do the work of six.

Theory

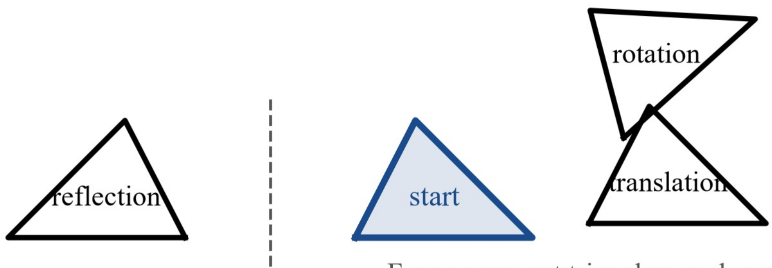
Recall: shapes and transformations from Year 7

In Year 7 you classified triangles and four-sided shapes by their sides and angles (`VC2M7SP02`): equilateral, isosceles and right-angled triangles; squares, rectangles, parallelograms and rhombuses. You moved shapes by **translations** (slides), **reflections** (flips) and **rotations** (turns) (`VC2M7SP03`). And you know the angle facts from `VC2M7M04` and `VC2M7M05`: the three angles of any triangle add to 180°, and parallel lines make equal corresponding and alternate angles. This subtopic uses all of these ideas without re-teaching them.

What congruence means

> **Definitions used in this subtopic.** Two shapes are **congruent** if one can lie exactly on top of the other after one or more transformations — translations, reflections and rotations. When one shape lies on top of the other, the matching sides are equal and the matching angles are equal.

This is the meaning that transformations give us: a slide, a flip or a turn never stretches or shrinks anything, so the shape that lands is exactly the same size and the same shape as the shape that started.



Four congruent triangles: each copy is the same triangle, moved.

Four congruent triangles: the same triangle shown once as a starting position, then as a translated copy, a reflected copy drawn across a dashed mirror line, and a rotated copy

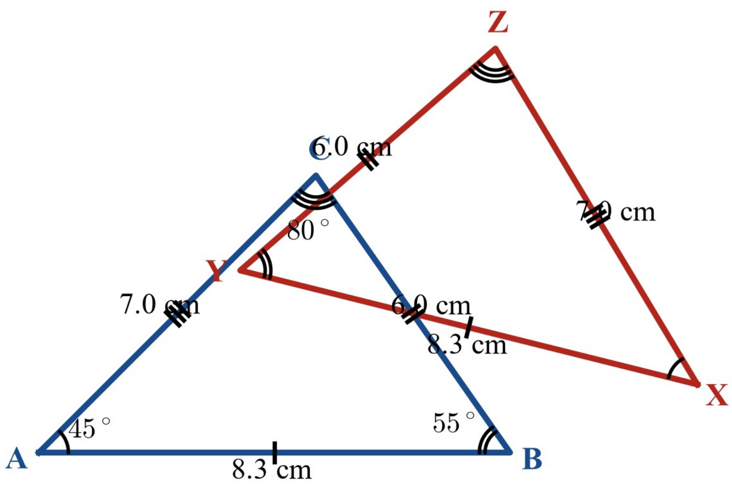
The four triangles in the diagram are all congruent. Each copy is the very same triangle — only its position has changed. If you cut any one of them out of paper, it would fit exactly on top of any other.

The parts that land on each other when one shape lies on top of the other are called **matching parts** or **corresponding parts**. When matching parts line up, each side equals its matching side and each angle equals its matching angle.

> **Word watch.** In everyday English, *similar* means "roughly alike". In maths the word has a strict meaning that is defined below — same shape, with sides enlarged by the same number. *Congruent* is used with one fixed meaning in this book — the meaning given above: same shape **and** same size, so one shape lies exactly on the other. Saying two shapes are "the same" is not careful enough, because two shapes can have the same shape but different sizes.

The six matching parts of a triangle

A triangle has six parts you can measure: **three sides and three angles**. If two triangles are congruent, then all six matching parts are equal — three pairs of equal sides and three pairs of equal angles.



Two congruent triangles drawn in different positions and turned different ways, with all six matching parts labelled: sides 8.3 cm, 7.0 cm and 6.0 cm, and angles 45, 55 and 80 degrees marked with matching tick marks and matching arc marks

In the diagram, $\triangle ABC$ and $\triangle XYZ$ are congruent. The matching parts are:

Part of $\triangle ABC$	Matching part of $\triangle XYZ$
$AB = 8.3\,\text{cm}$	$XY = 8.3\,\text{cm}$
$BC = 6.0\,\text{cm}$	$YZ = 6.0\,\text{cm}$
$CA = 7.0\,\text{cm}$	$ZX = 7.0\,\text{cm}$
$\angle A = 45^\circ$	$\angle X = 45^\circ$
$\angle B = 55^\circ$	$\angle Y = 55^\circ$
$\angle C = 80^\circ$	$\angle Z = 80^\circ$

(As a check: $45^\circ + 55^\circ + 80^\circ = 180^\circ$, as the angles of a triangle must.)

There is a shorthand for this. We write

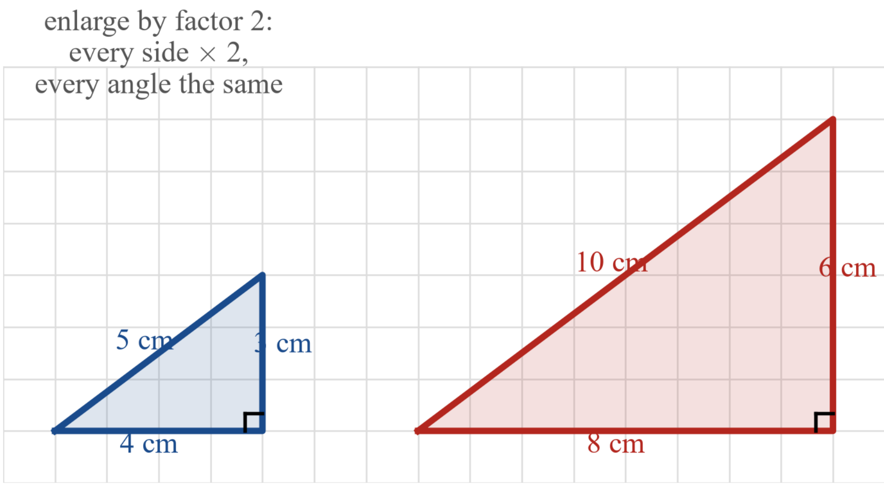
$$\triangle ABC \cong \triangle XYZ$$

and read it as "triangle ABC is congruent to triangle XYZ". The **order of the letters tells you the matching**: the first letter matches the first (A matches X), the second matches the second (B matches Y), and the third matches the third (C matches Z). Writing the letters in the matching order is part of the mathematics — it says which side equals which side, and which angle equals which angle.

What similarity means

> **Definitions used in this subtopic.** Two shapes are **similar** if one is an enlarged or shrunk copy of the other: the matching angles are equal, and the matching sides are all multiplied by the same number. That number is called the **scale factor**.

Enlarging a shape means multiplying **every** length by the same number. When every side is multiplied by the same number, the shape keeps its exact shape — it only changes its size — so every angle stays the same.



A right-angled triangle with sides 3 cm, 4 cm and 5 cm drawn on a light grid, beside an enlarged copy with sides 6 cm, 8 cm and 10 cm, every side multiplied by 2 while every angle stays the same

In the diagram, the large triangle is an enlargement of the small one with scale factor 2: every side is multiplied by 2 ($3 \times 2 = 6$, $4 \times 2 = 8$, $5 \times 2 = 10$). You can check on the grid that the angles have not changed — both triangles have the same right angle, and the other two angles match too. The two triangles are **similar**. They are **not congruent**, because the sides are different lengths.

You can make a whole family of similar shapes this way: start with one shape and enlarge it by scale factor 2, then 3, then 4, and so on. Every member of the family is similar to every other. (At Level 9, enlargement becomes a formal transformation with a centre and coordinates, `VC2M9SP02`. Here we only use the idea of multiplying every length by the same number.)

Two things follow straight from the definitions:

- **Every congruent pair is also similar** — a congruent copy is an enlargement with scale factor 1, because multiplying every side by 1 changes nothing.
- **A similar pair is congruent only when the scale factor is 1.**

What changes? What stays the same?

Here is the whole story of this subtopic in one table. For each transformation, ask: what happens to the lengths of the sides, and what happens to the angles?

Transformation	Lengths of sides	Angles	Result
Translation (slide)	stay the same	stay the same	congruent
Reflection (flip)	stay the same	stay the same	congruent
Rotation (turn)	stay the same	stay the same	congruent
Enlargement (scale factor k)	all multiplied by k	stay the same	similar; congruent only when $k = 1$

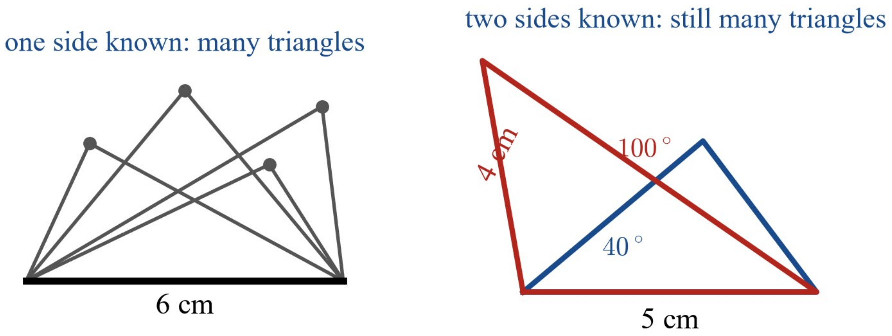
Translations, reflections and rotations keep **everything** the same, so they always produce congruent shapes. Enlargement keeps the angles but multiplies the lengths, so it produces shapes that are similar — but congruent only when the scale factor is 1.

This table answers the two questions that organise the subtopic: *What changes?* and *What stays the same?*

How much do you need to know?

Suppose you are given two triangles and want to find out whether they are congruent. The brute-force way is to measure all six matching parts — three sides and three angles. If all six match, the triangles are certainly congruent.

But six measurements are more than you need. Some smaller sets of parts already fix a triangle completely — once you know them, only one triangle is possible. And some sets are not enough. Look at what goes wrong with one part or two parts.



Left panel: many different triangles that all share the same base of length 6 cm. Right panel: two different triangles that both have sides 5 cm and 4 cm, with angles of 40 degrees and 100 degrees between those sides

- **One part is not enough.** Knowing one side (say 6 cm) leaves the other two sides and all three angles free. Many different triangles share that one side.
- **Two parts are not enough.** Knowing two sides (say 5 cm and 4 cm) still leaves the angle between them free. Change that angle and you get a different triangle — the right-hand panel shows two of them.

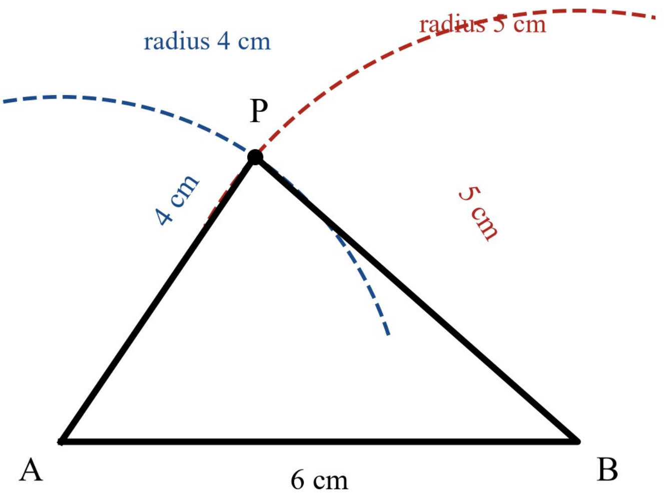
So at least **three** parts are needed. But not just any three parts work — two sets of three parts fail, and you will meet them after the four tests that succeed.

Four tests for congruent triangles

Each of the following tests gives three pieces of information that **fix a triangle completely**: once you know them, only one triangle is possible, so any triangle with those parts must be congruent to any other triangle with the same parts. Each test comes with the reason it works.

SSS — side, side, side. If the three sides of one triangle are equal to the three sides of another triangle, the triangles are congruent.

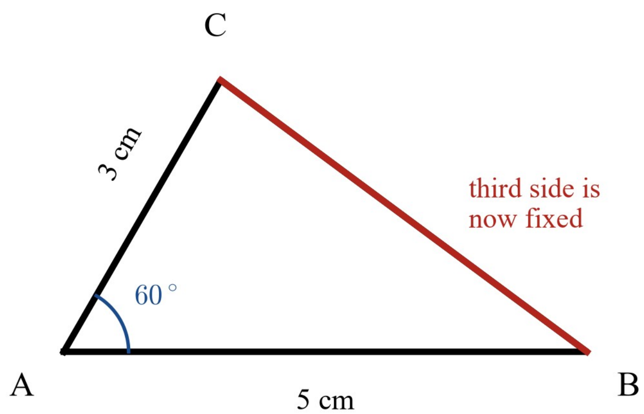
Why it works: draw one side as a base. The third vertex (corner point) must sit at the right distance from **both** ends of the base. The points at that distance from one end form an arc; the points at that distance from the other end form another arc; and the two arcs meet at exactly one point on each side of the base. Those two meeting points are mirror images of each other, so they give congruent triangles. Three sides leave no freedom at all.



A base of length 6 cm with a dashed arc of radius 4 cm drawn from one end and a dashed arc of radius 5 cm drawn from the other end, the two arcs meeting at exactly one point above the base to fix a triangle with sides 6 cm, 4 cm and 5 cm

SAS — side, angle, side (with the angle between the two sides). If two sides of one triangle, and the angle between them, are equal to two sides of another triangle and the angle between them, the triangles are congruent. The angle must be the **included angle** — the angle sitting between the two sides.

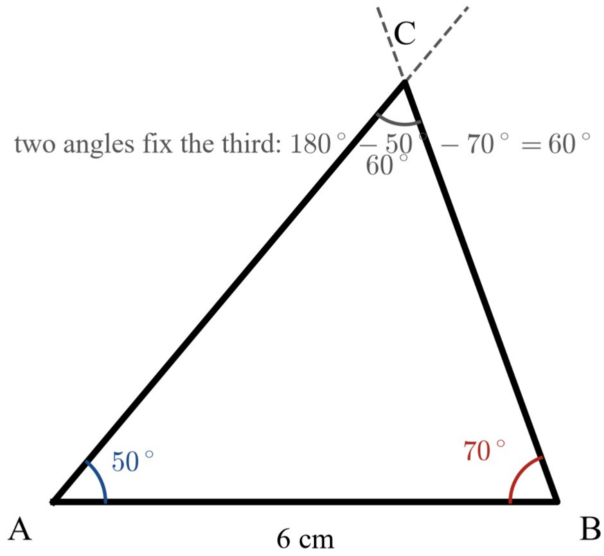
Why it works: two sides of fixed length standing at a fixed angle to each other can only reach one point for the third vertex. The third side — the join back to the start — is then fixed too, because its two endpoints are fixed.



Two sides of length 5 cm and 3 cm meeting at an angle of 60 degrees, with the third side joining their free ends; because the two sides and the included angle are fixed, the third side is fixed too

ASA — angle, side, angle. If two angles of one triangle, and any side, are equal to two angles of another triangle and the matching side, the triangles are congruent.

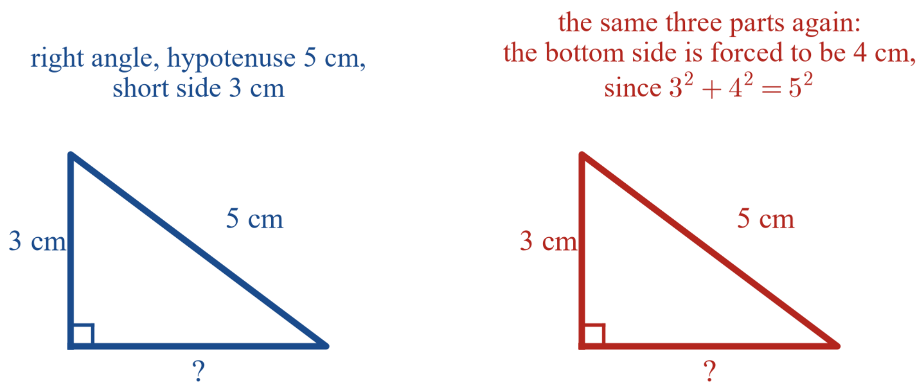
Why it works: two angles fix the third, because the angles of a triangle always add to 180°. So two angles really give all three angles, which fix the **shape**. The one side then fixes the **size**: a shape with one side of a known length can only be one triangle.



A base of length 6 cm with a ray rising from each end, one making an angle of 50 degrees and the other 70 degrees, the two rays meeting to form a triangle whose third angle must be 60 degrees

RHS — right angle, hypotenuse, side. If two right-angled triangles have equal hypotenuses and one equal pair of shorter sides, the triangles are congruent.

Why it works: in a right-angled triangle, the two shorter sides and the hypotenuse are tied together by Pythagoras' theorem (3D). So if the hypotenuse and one short side are fixed, the other short side is fixed too — and the triangle has all three sides fixed, which is SSS. For example, if the hypotenuse is 5 cm and one short side is 3 cm, the other short side must be 4 cm, because $3^2+4^2=5^2$.



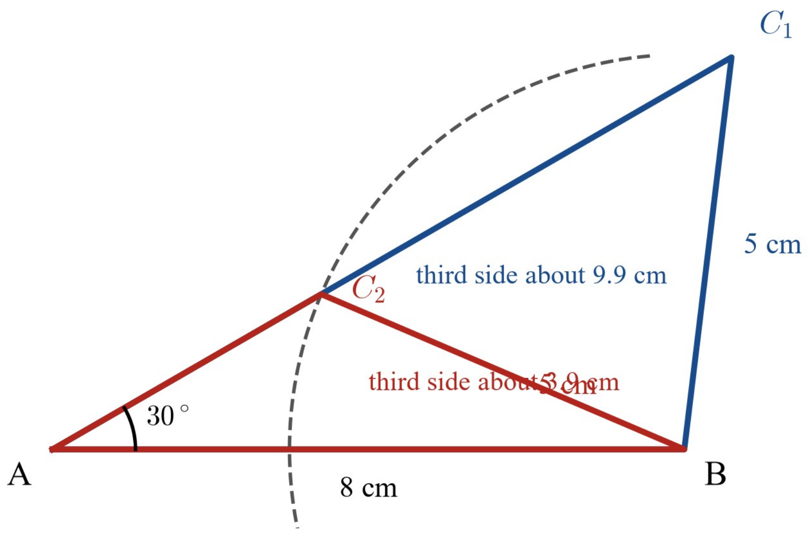
Two right-angled triangles, each with hypotenuse 5 cm and one short side 3 cm, marked with right-angle symbols; in each triangle the remaining short side is forced to be 4 cm, since 3 squared plus 4 squared equals 5 squared

Two things that do not work

Not every set of three parts fixes a triangle. Two famous sets fail.

AAA does not give congruence. Three angles fix the shape but not the size: triangles with angles 40°, 60° and 80° come in every size. AAA is a test for **similarity**, not for congruence.

Two sides and a non-included angle do not work. If you know two sides and an angle that is **not** between them, two different triangles can fit the information. In the diagram below, both triangles have a base of 8 cm, a side of 5 cm, and a 30° angle that is not between those sides — yet they are different triangles. So this combination of parts can never prove congruence.



Two different triangles sharing the same base of 8 cm, each with a side of 5 cm from the right-hand end of the base to a point on the same ray from the left-hand end, which makes an angle of 30 degrees with the base; a dashed arc of radius 5 cm cuts the ray in two places, giving a larger triangle with third side about 9.9 cm and a smaller triangle with third side about 3.9 cm

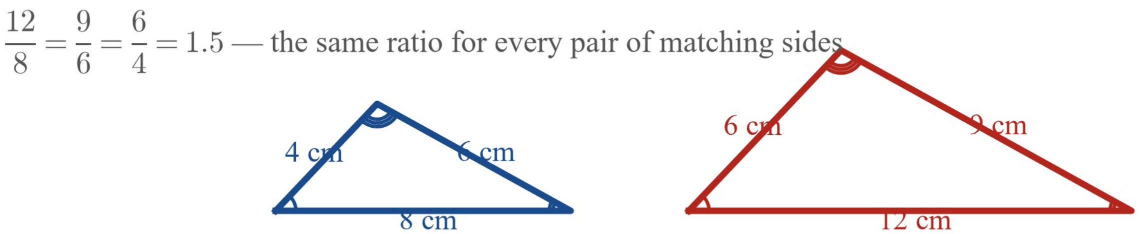
The dashed arc is the set of all points 5 cm from the right-hand end of the base. It cuts the ray in **two** places, and each one gives a different triangle. (In SAS the angle sits *between* the two sides, and the third vertex is fixed; here it does not, and it is not.)

Similar triangles

For triangles, similarity has a simple test.

> **Test for similar triangles.** Two triangles are similar if the three angles of one are equal to the three angles of the other. When the angles match, the matching sides are automatically in the same ratio — each side of one triangle is the same multiple of its matching side in the other.

Notice how different this is from congruence: for similarity, **three angles are enough** — no side measurements at all. That is because angles fix the shape, and similarity is exactly about shape.



Two similar triangles, one with sides 4 cm, 6 cm and 8 cm and the other with sides 6 cm, 9 cm and 12 cm, with matching angles marked by the same arc marks and the matching sides in the constant ratio 12 over 8 equals 9 over 6 equals 6 over 4 equals 1.5

In the diagram, each side of the larger triangle is 1.5 times its matching side:

$$\frac{12}{8} = \frac{9}{6} = \frac{6}{4} = 1.5$$

The matching sides are the ones **opposite equal angles** — the angle marks show you the matching. We write $\triangle ABC \sim \triangle XYZ$ for "triangle ABC is similar to triangle XYZ", again with the letters in matching order.

Conditions for other common shapes

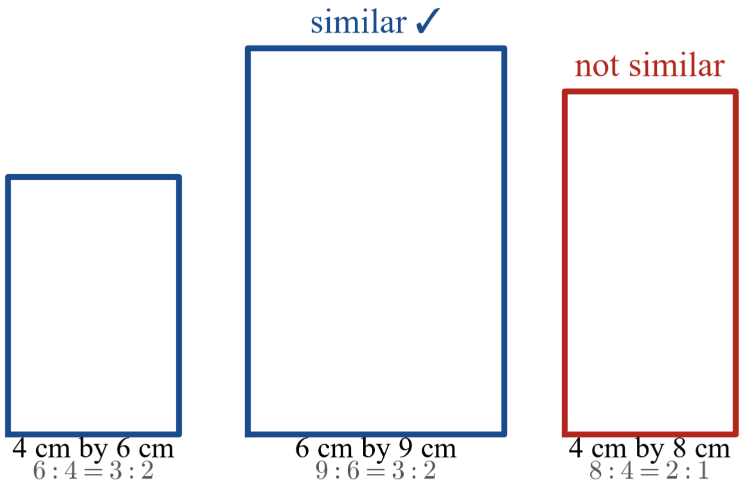
Triangles are not the only shapes that can be congruent or similar. Here are the conditions for other common shapes.

Squares. All squares are similar: every square has four right angles and four equal sides, so any square is an enlarged copy of any other. Two squares are congruent only if their sides are equal.

Circles. All circles are similar: a circle has no angles and no straight sides to get in the way — every circle is an enlarged copy of every other. Two circles are congruent only if their radii are equal.

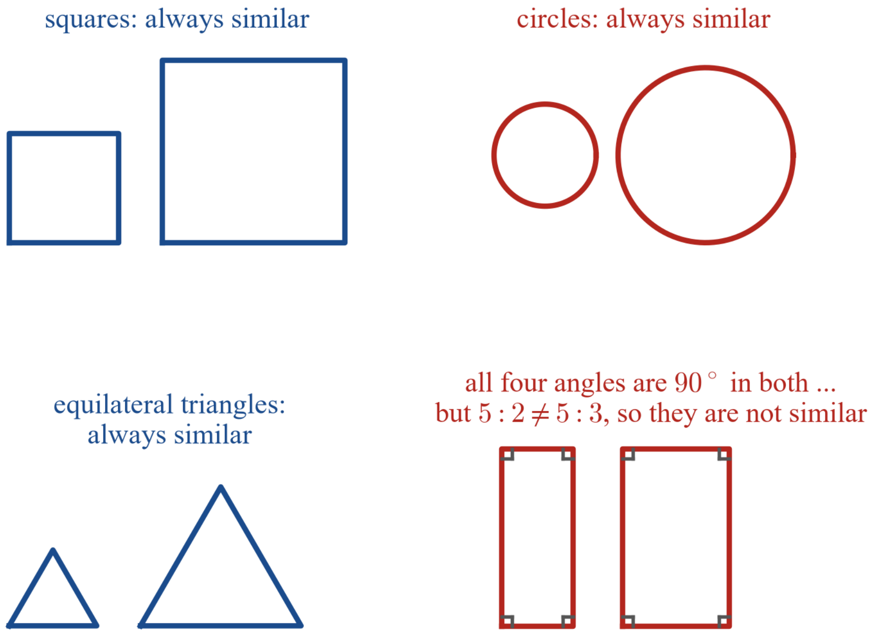
Equilateral triangles. All equilateral triangles (triangles with all three sides equal, and so all three angles 60°) are similar. Two equilateral triangles are congruent only if their sides are equal.

Rectangles. Rectangles are fussier. All rectangles have four right angles, so the angles can never tell two rectangles apart. Two rectangles are similar **only if their long side and short side are in the same ratio**.



Three rectangles drawn to scale: a 4 cm by 6 cm rectangle and a 6 cm by 9 cm rectangle, both with long side to short side in the ratio 3:2 and marked similar, beside a 4 cm by 8 cm rectangle with ratio 2:1 marked not similar to them

In the diagram, $6:4=3:2$ and $9:6=3:2$, so the first two rectangles are similar. But $8:4=2:1$, which is a different ratio, so the third rectangle is not similar to them — even though all three have four right angles.

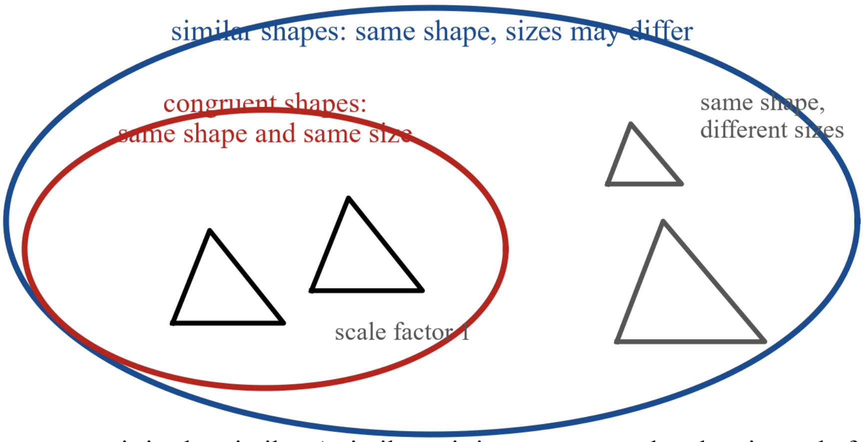


Four panels: two squares of different sizes (always similar), two circles of different sizes (always similar), two equilateral triangles of different sizes (always similar), and two rectangles of height 5 with widths 2 and 3, where all four angles are right angles in both but the ratios 5:2 and 5:3 differ, so they are not similar

The last panel is the warning to keep: **for four-sided shapes, equal angles alone do not make shapes similar**. Squares, circles and equilateral triangles are special — for them, the shape is fixed, so every copy is similar to every other.

Congruent and similar together

One picture holds the whole subtopic together.



Every congruent pair is also similar. A similar pair is congruent only when its scale factor is 1.

A diagram of two nested ovals: the outer oval contains all similar shapes, labelled same shape with sizes that may differ, and the inner oval contains the congruent shapes, labelled same shape and same size; a pair of congruent triangles sits inside the inner oval marked scale factor 1, and a pair of triangles in the same shape but different sizes sits between the ovals

- **Congruent** shapes sit in the smaller set: same shape **and** same size — scale factor 1.
- **Similar** shapes fill the larger set: same shape, but the size may differ.

Every congruent pair is automatically similar (scale factor 1). A similar pair is congruent only when its scale factor is 1. "Congruent" is the stronger statement.

What comes next

In 5B you will use these tests in logical reasoning and proofs about plane shapes; in 5C you will design and test algorithms that check for congruence and similarity; and at Level 9, trigonometry — the topic that uses ratios of sides to find lengths and angles in right-angled triangles (`VC2M9SP01`, `VC2M9M03`) — and enlargement as a formal transformation (`VC2M9SP02`) build directly on this subtopic.

Words of this subtopic

Word	What it means here
congruent	one shape can lie exactly on top of the other after one or more transformations; the matching sides and matching angles are equal
similar	one shape is an enlarged or shrunk copy of the other: matching angles are equal and matching sides are all multiplied by the same number
corresponding parts (matching parts)	the parts that land on each other when one shape lies on top of the other
scale factor	the number that every side is multiplied by in an enlargement
included angle	the angle sitting between two given sides
hypotenuse	the longest side of a right-angled triangle, opposite the right angle

Word	What it means here
vertex (vertices)	a corner point of a shape, where two sides meet
equilateral triangle	a triangle with all three sides equal (and so all three angles 60°)
protractor	the measuring tool used for measuring and drawing angles
compasses	the drawing tool used for drawing circles and arcs (a pair of compasses)
SSS, SAS, ASA, RHS	the four tests for congruent triangles: three sides; two sides and the included angle; two angles and a side; right angle, hypotenuse and one short side

Worked examples

Example 1 — with help

$\triangle ABC$ and $\triangle XYZ$ are the congruent triangles from the theory, with $\triangle ABC \cong \triangle XYZ$. You are given $\angle A=45^\circ$, $\angle B=55^\circ$, $AB=8.3\,\text{cm}$ and $BC=6.0\,\text{cm}$.

(a) Find $\angle Z$.
(b) Find YZ.

Working	Comment
(a) $\angle C=180^\circ-45^\circ-55^\circ=80^\circ$	Angles of a triangle add to 180°.
C matches Z	Third letters match, because $\triangle ABC \cong \triangle XYZ$.
$\angle Z=80^\circ$	Matching angles of congruent triangles are equal.
(b) BC matches YZ	Second-and-third letters: B matches Y, C matches Z.
$YZ=6.0\,\text{cm}$	Matching sides of congruent triangles are equal.

Example 2 — with help

A photo is 6 cm wide and 9 cm tall. It is enlarged by scale factor 3.

(a) Find the new width and height.
(b) Show that the enlarged photo is similar to the original, and explain why it is not congruent to it.

Working	Comment
(a) New width $=6\times3=18\,\text{cm}$	Enlarging by scale factor 3 multiplies every length by 3.
New height $=9\times3=27\,\text{cm}$	Multiply the height by 3 as well.
(b) Original ratio, height : width $=9:6=3:2$	Compare the two ratios.
New ratio, height : width $=27:18=3:2$	The ratios are equal, so the shape has not changed.
Similar, with scale factor 3	Matching angles are equal (all 90°) and matching sides are all multiplied by 3.
Not congruent	The sides are different lengths ($18\neq6$, $27\neq9$), so one cannot lie exactly on the other.

Example 3 — with help

Two triangles are similar. The sides of the smaller triangle are 4 cm, 6 cm and 8 cm. The shortest and middle sides of the larger triangle are 6 cm and 9 cm. Find the longest side x of the larger triangle.

Working	Comment
Match shortest with shortest: $4\to6$	In similar figures, matching sides are opposite equal angles — the shortest side always matches the shortest

Working	Comment
	side.
Scale factor $= 6 \div 4 = 1.5$	Find the number each side is multiplied by.
Check: $6 \times 1.5 = 9$ ✓	The middle side confirms the scale factor.
$x = 8 \times 1.5 = 12\,\text{cm}$	Multiply the longest side by the same scale factor.

Example 4 — on your own

Decide whether each set of information is enough to prove that two triangles are congruent. If it is, name the test. If it is not, say why not.

(a) All three sides of one triangle are equal to all three sides of the other.
(b) Two sides and the angle between them are equal in both triangles.
(c) Two angles and one side are equal in both triangles.
(d) Both triangles are right-angled, with equal hypotenuses and one equal pair of shorter sides.
(e) Two sides and an angle that is **not** between them are equal in both triangles.

Solution.

(a) Yes — **SSS**. Three sides fix a triangle completely: with the base drawn, the two arcs for the third vertex meet at exactly one point on each side of the base, and those two points are mirror images.

(b) Yes — **SAS**. Two sides with a fixed angle between them can only reach one point for the third vertex, so the third side is fixed too.

(c) Yes — **ASA**. Two angles fix the third (angles add to 180°), so the shape is fixed, and the one side fixes the size.

(d) Yes — **RHS**. Pythagoras' theorem fixes the remaining short side, so all three sides are fixed — which is SSS again.

(e) **No.** Two sides and a non-included angle can fit two different triangles (see the diagram in "Two things that do not work"), so this information can never prove congruence. It is not a test.

Example 5 — on your own

Are the following pairs of shapes similar? Give reasons.

(a) A rectangle 10 cm by 15 cm, and a rectangle 16 cm by 24 cm.
(b) A triangle with angles 35°, 65°, 80°, and a triangle with angles 65°, 80°, 35°.

Solution.

(a) Yes. Both rectangles have long side : short side in the same ratio: $15:10=3:2$ and $24:16=3:2$. Equal ratios mean the rectangles are similar. (The order of the angles and sides never matters for rectangles: every rectangle has four right angles.)

(b) Yes. Both triangles have the same three angles — the sets $\{35^\circ, 65^\circ, 80^\circ\}$ and $\{65^\circ, 80^\circ, 35^\circ\}$ are the same three numbers. For triangles, equal angles are enough: the matching sides must come out in a constant ratio. The triangles are similar.

Example 6 — on your own

Three garden beds are each marked out as a triangle by three sides: bed A has sides 3 m, 4 m and 5 m, bed B has sides 6 m, 8 m and 10 m, and bed C has sides 6 m, 8 m and 9 m.

(a) Are beds A and B similar? Are they congruent?
(b) Is bed C similar to bed A?

Solution.

(a) Similar, yes; congruent, no. Compare the matching (shortest, middle, longest) sides: $6 \div 3 = 2$, $8 \div 4 = 2$ and $10 \div 5 = 2$. All three ratios are equal, so bed B is an enlargement of bed A with scale factor 2 — the beds are similar. They are not congruent, because the scale factor is not 1: the sides are different lengths. (The right angle in bed A matches the right angle in bed B, because $3^2 + 4^2 = 5^2$ and $6^2 + 8^2 = 10^2$ — both beds are right-angled.)

(b) No. Compare the ratios shortest to shortest, middle to middle, longest to longest: $6 \div 3 = 2$, $8 \div 4 = 2$, but $9 \div 5 = 1.8$. The ratios are not all the same, so bed C is not an enlargement of bed A. The beds are not similar.

Practice questions

Work these questions by hand. A ruler, a protractor (the tool for measuring angles) and a pair of compasses (the tool for drawing arcs and circles) are needed for some questions — Q16 says where. No calculator is needed for any question.

With help

Q1. $\triangle ABC \cong \triangle DEF$, with $AB = 7\,\text{cm}$, $\angle A = 50^\circ$ and $\angle B = 60^\circ$.

a. Find DE.
b. Find $\angle D$.
c. Find $\angle F$.

Q2. True or false? Give a reason for each answer.

a. If two shapes are congruent, then they are similar.
b. If two shapes are similar, then they are congruent.
c. Two squares with different side lengths are similar.
d. Any two rectangles are similar.

Q3. Copy and complete this table, which compares the four transformations.

Transformation	Lengths of sides	Angles	The result is …
Translation			congruent
Reflection			
Rotation		stay the same	
Enlargement (scale factor k)			similar

Q4. $\triangle KLM \cong \triangle PQR$, with $\angle K = 72^\circ$ and $\angle L = 38^\circ$. Find $\angle R$. Give the steps of your reasoning.

Q5. A triangle has sides 3 cm, 5 cm and 7 cm. It is enlarged by scale factor 2.

a. Find the three sides of the enlarged triangle.
b. Is the enlarged triangle similar to the original? Is it congruent to the original? Answer both, with reasons.

Q6. In two similar triangles, a side of 5 cm in the smaller triangle matches a side of 20 cm in the larger triangle. Find the scale factor, and use it to find the side of the larger triangle that matches a side of 8 cm in the smaller one.

Q7. Two similar rectangles have matching sides in the ratio $4:10$ (small to large). The smaller rectangle has a longer side of 6 cm. Find the longer side x of the larger rectangle.

Q8. Match each set of information to the congruence test it belongs to: SSS, SAS, ASA or RHS.

a. Three pairs of equal sides.
b. Two pairs of equal sides and the equal angle between them.
c. Two pairs of equal angles and one pair of equal sides.

d. Right angles in both triangles, equal hypotenuses, and one equal pair of shorter sides.

On your own

Q9. Explain why “two sides and a non-included angle” is not a test for congruence. Your explanation should say what can go wrong.

Q10. Decide whether each pair of rectangles is similar. Give the ratios you compare.

a. 8 cm by 12 cm, and 10 cm by 15 cm.
b. 5 cm by 8 cm, and 10 cm by 16 cm.
c. 6 cm by 10 cm, and 9 cm by 12 cm.

Q11. A triangle has sides 5 cm, 7 cm and 9 cm. A similar triangle has its shortest side equal to 15 cm. Find the other two sides of the similar triangle.

Q12. Answer *always*, *sometimes* or *never*, and give a reason each time.

a. Two squares are similar.
b. Two squares are congruent.
c. Two circles are similar.
d. Two equilateral triangles are similar.
e. Two rectangles are similar.

Q13. Jia has two triangles drawn on separate cards. She has checked that one pair of matching sides is equal. She is allowed exactly two more measurements. Give **two different choices** of what she could measure that would let her prove the triangles are congruent, and name the test each choice uses.

Q14. A photo is 4 cm wide and 6 cm tall.

a. It is printed at 10 cm wide and 15 cm tall. Is the print similar to the original? Show the ratios you compare.
b. Another print is 10 cm wide and 14 cm tall. Is this print similar to the original? What would the photo look like?

Q15. Two triangles both have angles 40°, 60° and 80°. In one triangle the shortest side is 5 cm; in the other it is 10 cm.

a. Must the triangles be similar? Give the reason.
b. Must the triangles be congruent? Give the reason.

Q16. You will need a ruler and a pair of compasses.

a. Draw a line segment 6 cm long and label it AB. With your compasses, draw an arc of radius 5 cm centred on A and an arc of radius 7 cm centred on B, both above the segment. Label the point where the arcs meet C, and join AC and BC.
b. Repeat the whole construction again, somewhere else on the page.
c. Compare your two triangles. Are they congruent? Which test says so, and why does the test guarantee your answer?

Maths in real life

Q17. A photo is 12 cm wide and 8 cm tall. It is enlarged so that the new width is 30 cm. Find the new height, and state the scale factor of the enlargement.

Q18. A 1 m stick standing upright casts a shadow 1.5 m long. At the same time, a flagpole casts a shadow 9 m long. The sun’s rays reach the stick and the flagpole at the same angle, because the sun is so far away.

a. Explain why the triangle formed by the stick and its shadow is similar to the triangle formed by the flagpole and its shadow.
b. Find the height of the flagpole.

Q19. A4 paper is 210 mm wide and 297 mm long. When a sheet of A4 is cut in half parallel to its shorter sides, it makes two sheets of A5.

a. What are the width and length of a sheet of A5?
b. Find the ratio long side ÷ short side for A4 and for A5 (give each to 2 decimal places). What do you notice?
c. The A-series is designed so that long side ÷ short side $= \sqrt{2} \approx 1.4142$ — a number you met in 1D. Use this to explain why every A-size is similar to every other A-size.

Source: Wikipedia, "ISO 216"; retrieved 20 August 2026.

Q20. Draw any triangle and label its angles. Now draw a similar triangle with scale factor 2: multiply each side by 2. Measure all three angles of your second triangle.

a. Compare the angles of the two triangles. What do you find?
b. Swap your two triangles with a partner and check theirs. Does everyone's pair of triangles behave the same way? Write one sentence about what this activity shows about enlargement.

Answers

Q1. a. $DE = 7\,\text{cm}$. b. $\angle D = 50^\circ$. c. $\angle F = 70^\circ$.

Q2. a. True: congruent shapes have the same shape and the same size, so they are similar with scale factor 1. b. False: similar shapes can have different sizes, such as a square of side 1 cm and a square of side 2 cm. c. True: all squares are similar. d. False: rectangles 4 cm by 6 cm and 4 cm by 8 cm both have four right angles, but $6:4 = 3:2$ while $8:4 = 2:1$, so they are not similar.

Q3. Translation: stay the same; stay the same; congruent. Reflection: stay the same; stay the same; congruent. Rotation: stay the same; congruent. Enlargement: all multiplied by k; stay the same; similar.

Q4. $\angle R = 70^\circ$.

Q5. a. 6 cm, 10 cm, 14 cm. b. Similar, yes (scale factor 2); congruent, no (the sides are different lengths).

Q6. Scale factor 4; matching side $= 8 \times 4 = 32\,\text{cm}$.

Q7. $x = 15\,\text{cm}$.

Q8. a. SSS. b. SAS. c. ASA. d. RHS.

Q9. Two different triangles can fit the same information (see the diagram in "Two things that do not work"), so the information does not fix the triangle.

Q10. a. Similar: $12:8 = 3:2$ and $15:10 = 3:2$. b. Similar: $8:5$ and $16:10 = 8:5$. c. Not similar: $10:6 = 5:3$ but $12:9 = 4:3$.

Q11. 21 cm and 27 cm.

Q12. a. Always. b. Sometimes. c. Always. d. Always. e. Sometimes.

Q13. Any two of: the included angle and one more side (SAS); the two angles at the ends of the known side (ASA); for right-angled triangles where the known equal side is a short side, the hypotenuse plus the right angle (RHS). See the fully-worked solutions.

Q14. a. Similar: $4:6 = 2:3$ and $10:15 = 2:3$. b. Not similar: $10:14 = 5:7 \neq 2:3$; the photo would look stretched.

Q15. a. Yes — the three angles are equal, which is the test for similar triangles. b. No — the shortest sides are 5 cm and 10 cm, so the sizes differ.

Q16. c. Congruent, by SSS: both triangles have sides 5 cm, 6 cm and 7 cm.

Q17. New height $20\,\text{cm}$; scale factor 2.5.

Q18. b. $6\,\text{m}$.

Q19. a. 148.5 mm by 210 mm. b. A4: $297 \div 210 \approx 1.41$; A5: $210 \div 148.5 \approx 1.41$. The ratios are equal (to 2 decimal places) — both are close to $\sqrt{2} \approx 1.4142$.

Q20. a. The angles are equal.

Fully-worked solutions

Q1. Use the matching order in $\triangle ABC \cong \triangle DEF$: A matches D, B matches E, C matches F.

a. AB matches DE, so $DE = AB = 7\,\text{cm}$.
b. A matches D, so $\angle D = \angle A = 50^\circ$.
c. First find $\angle C = 180^\circ - 50^\circ - 60^\circ = 70^\circ$. Then C matches F, so $\angle F = 70^\circ$.

Q2. Apply the definitions.

a. **True.** Congruent shapes have the same shape and the same size — that is exactly similarity with scale factor 1.
b. **False.** Similar shapes have the same shape, but their sizes may differ. A square of side 1 cm and a square of side 2 cm are similar but not congruent.
c. **True.** All squares have four right angles and four equal sides, so any square is an enlargement of any other.
d. **False.** All rectangles have four right angles, but the ratios can differ: $6:4=3:2$ for a 4 cm by 6 cm rectangle, but $8:4=2:1$ for a 4 cm by 8 cm rectangle. Different ratios mean not similar.

Q3. Translations, reflections and rotations keep all lengths and all angles the same, so each one produces congruent shapes. Enlargement multiplies all lengths by the scale factor k but keeps all angles the same, so it produces similar shapes.

Transformation	Lengths of sides	Angles	The result is …
Translation	stay the same	stay the same	congruent
Reflection	stay the same	stay the same	congruent
Rotation	stay the same	stay the same	congruent
Enlargement (scale factor k)	all multiplied by k	stay the same	similar

Q4. Find the third angle, then match.

$\angle M=180^\circ-72^\circ-38^\circ=70^\circ$. In $\triangle KLM\cong\triangle PQR$, the third letters match: M matches R. So $\angle R=\angle M=70^\circ$.

Q5. Apply the enlargement, then compare.

a. Multiply each side by 2: $3\times2=6$, $5\times2=10$, $7\times2=14$. The sides are 6 cm, 10 cm and 14 cm.
b. The enlarged triangle is **similar** to the original: the matching angles are equal and every side is multiplied by the same number, 2. It is **not congruent**: congruent shapes must have equal sides, and $6\neq3$, $10\neq5$, $14\neq7$.

Q6. Divide a matching pair, then multiply.

Scale factor $=20\div5=4$. The matching side $=8\times4=32\,\text{cm}$.

Q7. Use the ratio of matching sides.

The ratio of matching sides (small to large) is $4:10$, so each side of the larger rectangle is $10\div4=2.5$ times the matching side of the smaller one. $x=6\times2.5=15\,\text{cm}$.

Q8. Read off each test from its definition.

a. Three pairs of equal sides — **SSS**.
b. Two pairs of equal sides with the angle between them — **SAS**.
c. Two pairs of equal angles with one pair of equal sides — **ASA**.
d. Right angles, equal hypotenuses and one equal pair of shorter sides — **RHS**.

Q9. Build the explanation around what fails.

A test for congruence must fix the triangle completely — only one triangle may be possible. With two sides and an angle that is not between them, two different triangles can fit the same information: the arc of points at the given distance from one endpoint can cut the ray from the other endpoint in two places, giving one larger triangle and one smaller. Since two different triangles are possible, the information does not prove congruence, so it is not a test.

Q10. Compare long side : short side for each rectangle in the pair.

a. $12:8=3:2$ and $15:10=3:2$. Same ratio — **similar**.
b. $8:5$, and $16:10=8:5$. Same ratio — **similar**.
c. $10:6=5:3$, but $12:9=4:3$. Different ratios — **not similar**.

Q11. Find the scale factor from the matching shortest sides.

Scale factor $= 15 \div 5 = 3$. The other sides are $7 \times 3 = 21\,\text{cm}$ and $9 \times 3 = 27\,\text{cm}$.

Q12. Apply the conditions for each shape.

a. **Always.** Every square has four right angles and four equal sides, so any square is an enlargement of any other.
b. **Sometimes.** Two squares are congruent only when their sides are equal. Squares of side 3 cm and 5 cm are not congruent; two squares both of side 3 cm are.
c. **Always.** Every circle is an enlargement of every other — a circle has no angles or sides that could differ.
d. **Always.** Every equilateral triangle has angles 60°, 60°, 60°, so any two are similar.
e. **Sometimes.** Two rectangles are similar only when long side : short side is the same ratio for both. $6:4=3:2$ and $9:6=3:2$ match; $6:4$ and $8:4$ do not.

Q13. Choose measurements that complete one of the tests, starting from one known pair of equal sides. Two clean choices:

- Measure the **angle at each end of the known side** in both triangles. If both pairs match, the triangles are congruent by **ASA** (two angles and the side between them).
- Measure the **included angle** (the angle between the known side and a second side) and the **second side** in both triangles. If both match, the triangles are congruent by **SAS**.

(A third route exists for right-angled triangles: if the known equal side is a short side, measure the hypotenuse and check the right angle — giving **RHS**.)

Q14. Compare width : height ratios.

a. Original: $4:6=2:3$. Print: $10:15=2:3$. The ratios are equal, so the print is **similar** to the original (scale factor 2.5).
b. Second print: $10:14=5:7$, which is not $2:3$. The print is **not similar**: because the width and height were multiplied by different numbers, the photo would look stretched.

Q15. Apply the tests.

a. **Yes.** The three angles of one triangle equal the three angles of the other (both have 40°, 60°, 80°), and equal angles are the test for similar triangles.
b. **No.** Congruent triangles have equal matching sides; here the shortest sides are 5 cm and 10 cm. The triangles are similar but not congruent.

Q16. The construction draws two triangles with the same three sides.

c. Both triangles have sides 5 cm, 6 cm and 7 cm — the construction used arcs of radius 5 cm and 7 cm from the ends of a 6 cm base, so every side is forced. By **SSS**, any two triangles with the same three sides are congruent. The test guarantees the answer because the two arcs meet at exactly one point on each side of the base, and those two points are mirror images — so only one triangle (up to a flip) can have those three sides.

Q17. Find the scale factor from the widths, then apply it to the height.

Scale factor $= 30 \div 12 = 2.5$. New height $= 8 \times 2.5 = 20\,\text{cm}$.

Q18. Use similar triangles from the shared sun angle.

a. Both triangles have a right angle (the stick and the flagpole stand upright) and the sun's rays reach both at the same angle, so two angles of one triangle equal two angles of the other. The third angles must be equal too (angles add to 180°). Equal angles mean the triangles are similar.
b. With similar triangles, matching sides are in the same ratio:

$$\frac{\text{flagpole height}}{9} = \frac{1}{1.5}$$

so flagpole height = $9 \div 1.5 = 6\,\text{m}$.

Q19. Use the halving rule of A-series paper.

a. Cutting parallel to the shorter sides halves the longer side: A5 is $297 \div 2 = 148.5\,\text{mm}$ by $210\,\text{mm}$.
b. A4: $297 \div 210 \approx 1.41$. A5: $210 \div 148.5 \approx 1.41$. Both ratios are the same to 2 decimal places. Each is close to the exact design value $\sqrt{2} \approx 1.4142$; the small difference from $\sqrt{2}$ is there because paper sizes are rounded to the nearest millimetre.
c. The design rule is long side ÷ short side = $\sqrt{2}$. When the sheet is cut in half parallel to its shorter sides, the new sheet's long side is the old short side, and its short side is half the old long side. Because the old long side was $\sqrt{2}$ times the old short side, the new ratio works out to $\sqrt{2}$ again. Every A-size has the same long : short ratio, so every A-size is similar to every other.

Q20. The activity shows what enlargement does to angles.

a. Each angle of the larger triangle is equal to the matching angle of the smaller one. Multiplying every side by 2 changed the lengths but not the angles.
b. Everyone's pair behaves the same way. This shows that enlargement multiplies every length by the same number while keeping every angle the same — which is exactly why an enlarged copy is always similar to the original.

Properties of quadrilaterals proved from congruent triangles

> **Content description `VC2M8SP02`:** establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems explaining reasoning

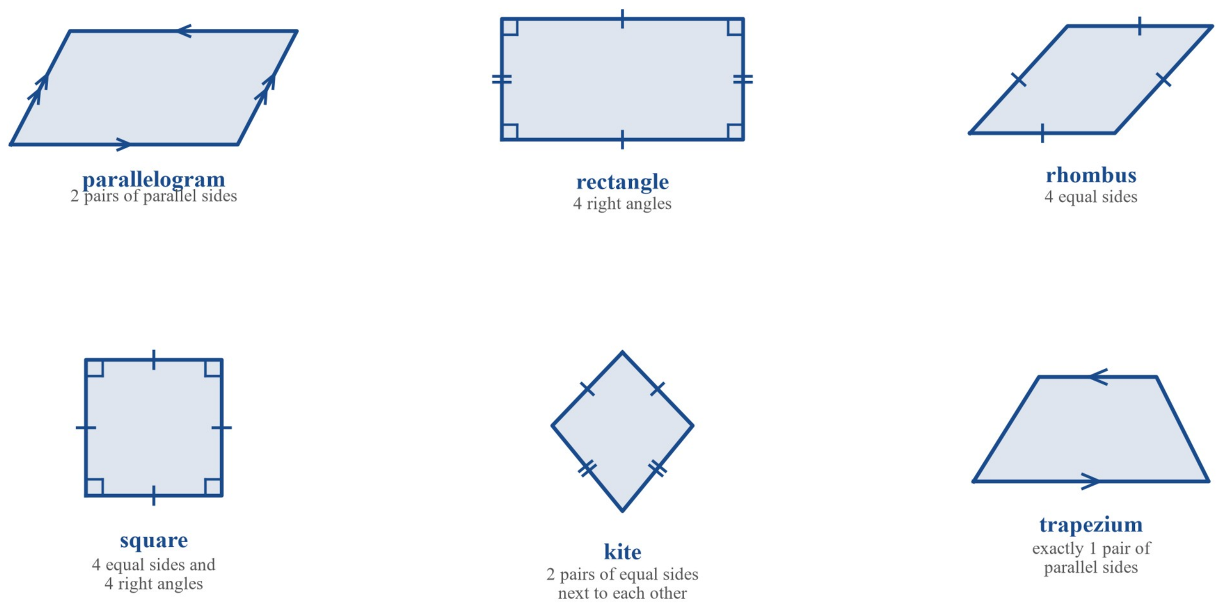
In Year 7 you sorted the quadrilateral families and spotted what seems to be true about them — the opposite sides of a parallelogram look equal, the diagonals of a rectangle look the same length, a kite seems to fold in half along one diagonal. "Looks equal" is not yet "is equal". This subtopic turns those spotted facts into proved facts, using the one tool that can do it: congruent triangles. Every result below is earned the same way — cut the shape into triangles, show the triangles are congruent, and read the matching parts off.

Theory

The six families, and the marks we use

You met these families in Year 7 and met them again in 5A. This subtopic's job is to **prove** the properties you were asked to spot then.

The six quadrilateral families — ticks mark equal sides, arrows mark parallel sides, squares mark right angles



The six quadrilateral families drawn with house marking conventions: a leaning parallelogram with one arrow on each of one pair of parallel sides and two on the other; a rectangle with right angle squares at all four corners and matching ticks on opposite sides; a rhombus with one tick on each of its four equal sides; a square with ticks on all four sides and right angle squares at the corners; a kite with one tick on each upper pair of sides and two ticks on each lower pair; and a trapezium with arrows on its one pair of parallel sides only

The marks carry the whole story in this subtopic, so read them the same way every time:

Mark	Meaning
tick(s) on a side	sides with the same number of ticks are equal in length
arrow(s) on a side	sides with the same number of arrows are parallel
small square at a corner	a right angle (90°)
same number of arcs at two corners	those two angles are equal

The tools we are allowed to use

Two toolkits unlock every proof in this subtopic.

Toolkit 1 — the four congruence tests from 5A. If two triangles match under any one of these, they are congruent ($\cong$), and then *all* matching sides and angles are equal ("matching parts of congruent triangles"):

Test	What must match
SSS	three sides
SAS	two sides and the angle *between* them
ASA	two angles and the side *between* them
RHS	right angle, hypotenuse and one other side

Toolkit 2 — the Year 7 angle facts. You learnt these in Year 7. They are not the topic here — they are tools you may give as a reason, exactly like a congruence test:

- angles on a straight line add to 180°; angles at a point add to 360°
- vertically opposite angles are equal (like the angles where two straight lines cross)
- corresponding angles are equal (parallel lines); alternate angles are equal (parallel lines); co-interior angles add to 180° (parallel lines)
- the angle sum of a triangle is 180°; the angle sum of a quadrilateral is 360°
- equal alternate angles (or equal corresponding angles) also work *backwards*: they tell you two lines are parallel

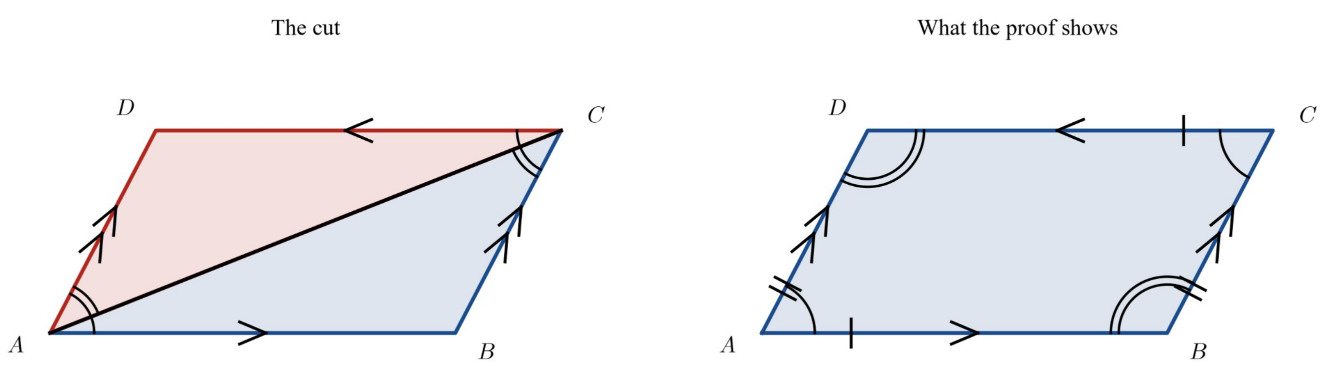
The shape of a proof here

Every proof in this subtopic has the same three beats: **statement** · **reason** · **conclusion**. Each statement sits in a table next to its reason — a statement with no reason is just a claim. The conclusion says what has now been shown, marked ∴ ("therefore"). Watch this pattern in Result 1; it is the pattern you will copy in the questions.

Result 1 — a diagonal cuts a parallelogram into two congruent triangles

This is the sighting 5A left you with. Now the proof.

A diagonal of a parallelogram makes two congruent triangles



the diagonal AC cuts the parallelogram into $\triangle ABC$ and $\triangle CDA$ opposite sides are equal; opposite angles are equal

Two panels. Left, "The cut": parallelogram ABCD with the diagonal AC drawn, shading triangle ABC blue and triangle CDA red; one arrow on AB and on CD, two on AD and on BC, a single arc at angle BAC and at angle DCA, and a double arc at angle BCA and at angle DAC. Right, "What the proof shows": the same parallelogram with one tick on AB and CD, two ticks on AD and BC, a single arc at the whole angles A and C and a double arc at the whole angles B and D

Result 1. *A diagonal of a parallelogram splits it into two congruent triangles. Hence the opposite sides of a parallelogram are equal, and its opposite angles are equal.*

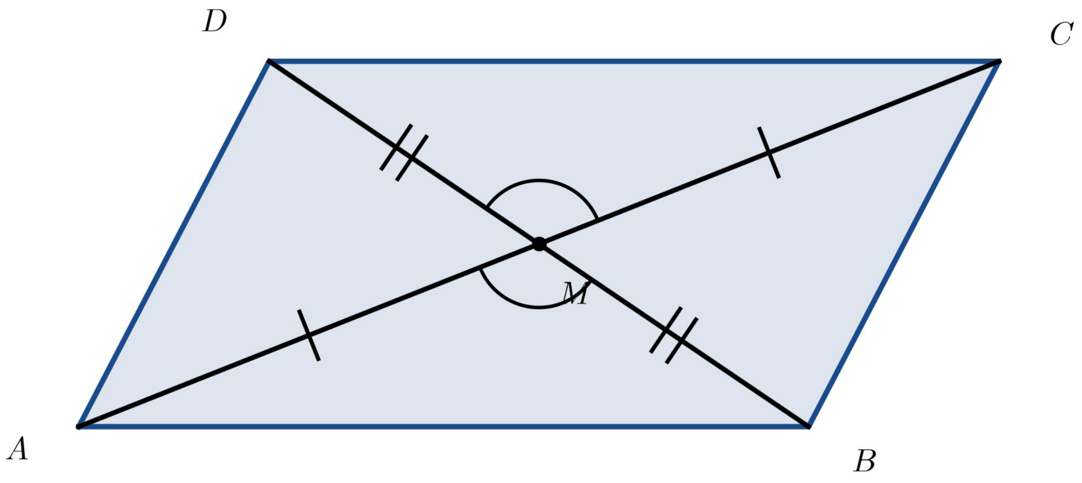
In parallelogram $ABCD$, draw the diagonal AC.

Statement	Reason
$\angle BAC = \angle DCA$	alternate angles, $AB \parallel DC$
$\angle BCA = \angle DAC$	alternate angles, $BC \parallel AD$
$AC = CA$	common side
$\triangle ABC \cong \triangle CDA$	ASA
$AB = CD$ and $BC = DA$	matching sides of congruent triangles
$\angle ABC = \angle CDA$	matching angles of congruent triangles
$\angle BAD = \angle BAC + \angle DAC = \angle DCA + \angle BCA =$	equal angles added to equal angles
$\therefore$ the opposite sides are equal and the opposite angles are equal	conclusion

Notice the logic's direction: the *parallel* sides (given, by definition) give equal angles, the equal angles plus a common side give congruent triangles, and the congruent triangles give the equal *lengths* we could not see any other way. That is the whole game of this subtopic.

Result 2 — the diagonals of a parallelogram bisect each other

The diagonals of a parallelogram



$AM = CM$ and $BM = DM$: the diagonals bisect each other

Parallelogram ABCD with both diagonals drawn, meeting at M; one tick on AM and on MC, two ticks on BM and on MD, and matching single arcs at the vertically opposite angles AMB and CMD

Result 2. *The diagonals of a parallelogram bisect each other.*

In parallelogram $ABCD$, let the diagonals AC and BD meet at M.

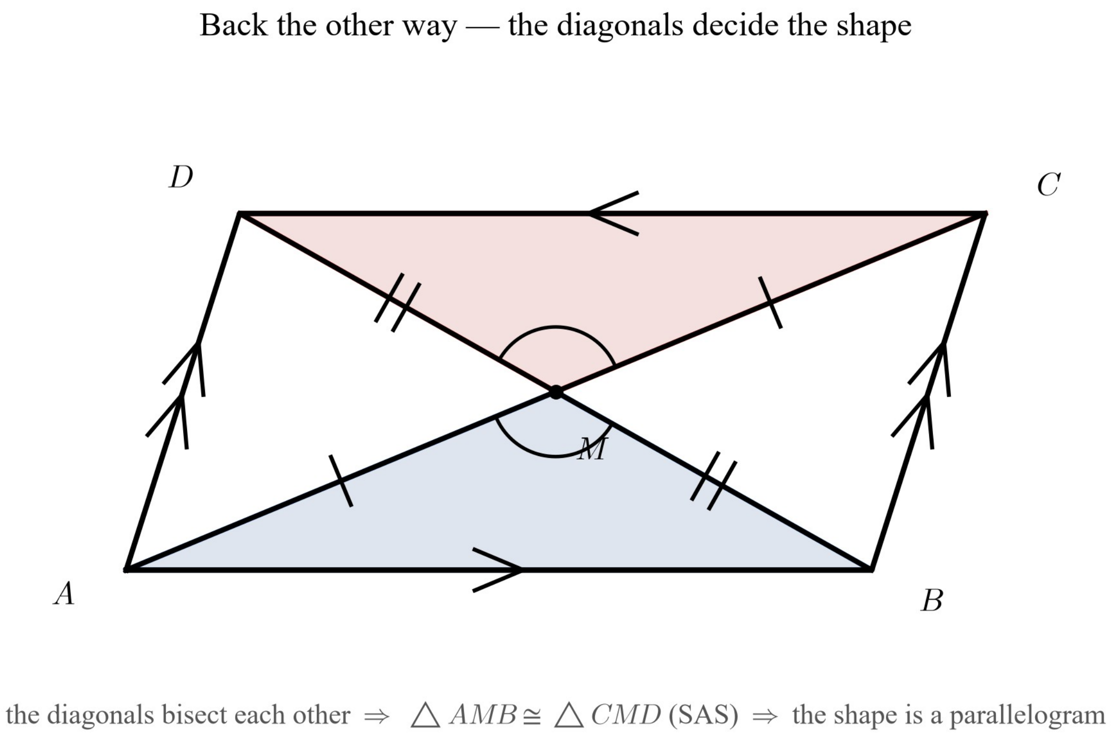
Statement	Reason
$\angle MAB = \angle MCD$	alternate angles, $AB \parallel DC$
$\angle MBA = \angle MDC$	alternate angles, $AB \parallel DC$

Statement	Reason
$AB = CD$	opposite sides of a parallelogram (Result 1)
$\triangle AMB \cong \triangle CMD$	ASA
$AM = CM$ and $BM = DM$	matching sides of congruent triangles
$\therefore$ the diagonals bisect each other	conclusion

(“Bisect” means “cut into two equal halves” — each diagonal cuts the other in half.)

Result 3 — back the other way: the diagonals can prove the shape

Results 1 and 2 say *if* the shape is a parallelogram, *then* the diagonals bisect each other. The **converse** swaps the “if” and the “then”: *if* the diagonals bisect each other, *then* the shape is a parallelogram. A converse is a new claim — it is not true just because the first claim is, so it needs its own proof. This one has a proof.



Quadrilateral ABCD with diagonals meeting at M, one tick on AM and MC, two ticks on BM and MD, matching arcs at the vertically opposite angles AMB and CMD, triangle AMB shaded blue and triangle CMD shaded red; arrows mark AB parallel to DC and AD parallel to BC as the conclusion

Result 3. *If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.*

In quadrilateral $ABCD$, suppose the diagonals meet at M with $AM = CM$ and $BM = DM$.

Statement	Reason
$AM = CM$	given
$\angle AMB = \angle CMD$	vertically opposite angles
$BM = DM$	given
$\triangle AMB \cong \triangle CMD$	SAS
$\angle MAB = \angle MCD$	matching angles of congruent triangles
$AB \parallel DC$	alternate angles equal, working backwards
$\triangle AMD \cong \triangle CMB$	the same three steps with D and B swapped (SAS)

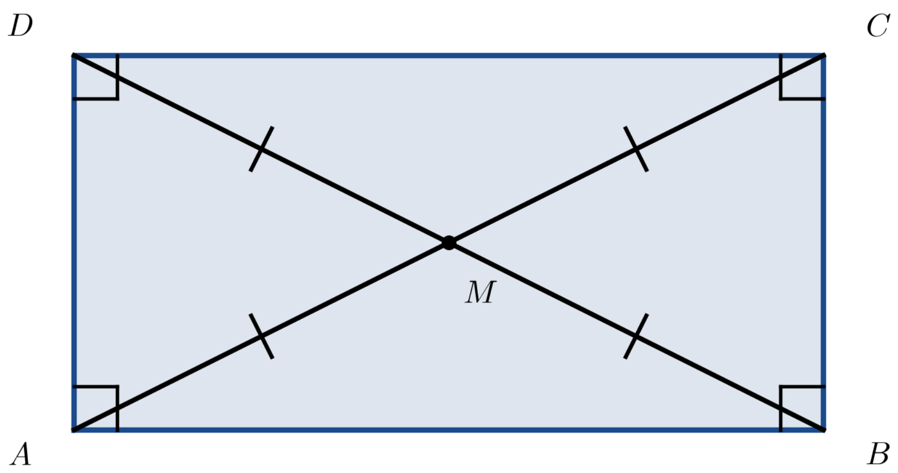
Statement	Reason
$AD \parallel BC$	alternate angles equal, working backwards
$\therefore\ ABCD$ has both pairs of opposite sides parallel: it is a parallelogram	conclusion

Result 3 is the most useful *test* in this subtopic: measure the diagonals, and if they cut each other in half, you are looking at a parallelogram — no angle measuring needed. (By the same SSS steps, a quadrilateral whose opposite sides are equal is also a parallelogram.)

Result 4 — the diagonals of a rectangle are equal

A rectangle is a parallelogram with four right angles, so Results 1–3 all apply to it — and the right angles buy one extra fact.

The diagonals of a rectangle



$AC = BD$, and the diagonals bisect each other: $AM = BM = CM = DM$

Rectangle ABCD with both diagonals drawn meeting at M, right angle squares at all four corners, and one tick on each of the four half-diagonals AM, BM, CM and DM

Result 4. *The diagonals of a rectangle are equal, and they bisect each other into four equal halves.*

In rectangle $ABCD$, with diagonals meeting at M:

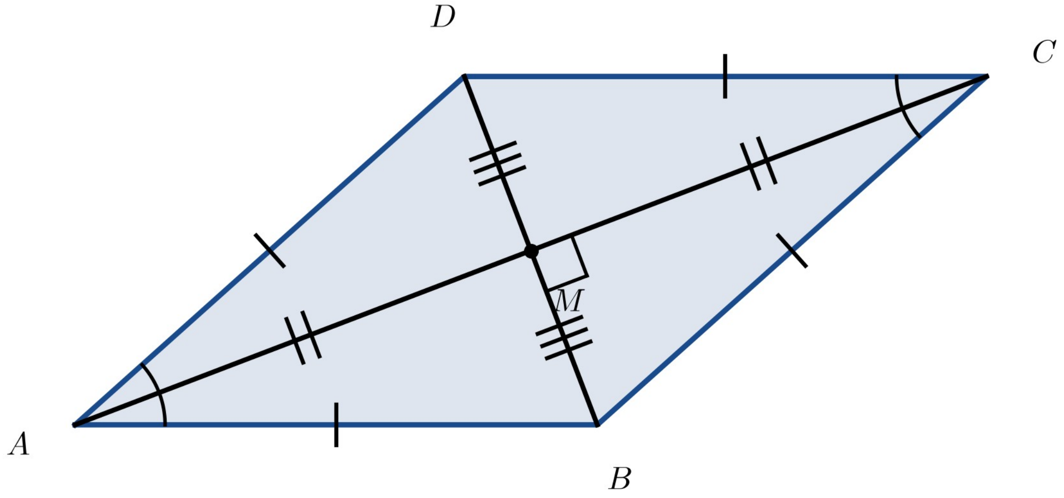
Statement	Reason
$AB = DC$	opposite sides of a parallelogram (Result 1)
$\angle ABC = \angle DCB = 90^\circ$	right angles of the rectangle
$BC = CB$	common side
$\triangle ABC \cong \triangle DCB$	SAS
$AC = DB$	matching sides (the hypotenuses) of congruent triangles
$AM = CM$ and $BM = DM$	diagonals of a parallelogram bisect each other (Result 2)
$\therefore\ AC = BD$, and $AM = BM = CM = DM$	equal wholes cut into equal halves

That last line is why rectangles are everywhere in building: the point M is the *same distance* from all four corners.

Result 5 — the diagonals of a rhombus meet at right angles

A rhombus is a parallelogram with four equal sides, so its diagonals already bisect each other (Result 2). The equal sides then force the diagonals to cross at right angles, and to bisect the corner angles.

The diagonals of a rhombus



the diagonals meet at right angles and bisect each other; each diagonal bisects the angles at its ends

Rhombus ABCD with both diagonals meeting at M, one tick on each of the four sides, a right angle square at M, two ticks on AM and MC, three ticks on BM and MD, and matching single arcs showing that diagonal AC splits the angles at A and at C into two equal halves

Result 5. *The diagonals of a rhombus meet at right angles, and each diagonal bisects the corner angles at its ends.*

In rhombus $ABCD$, let the diagonals meet at M.

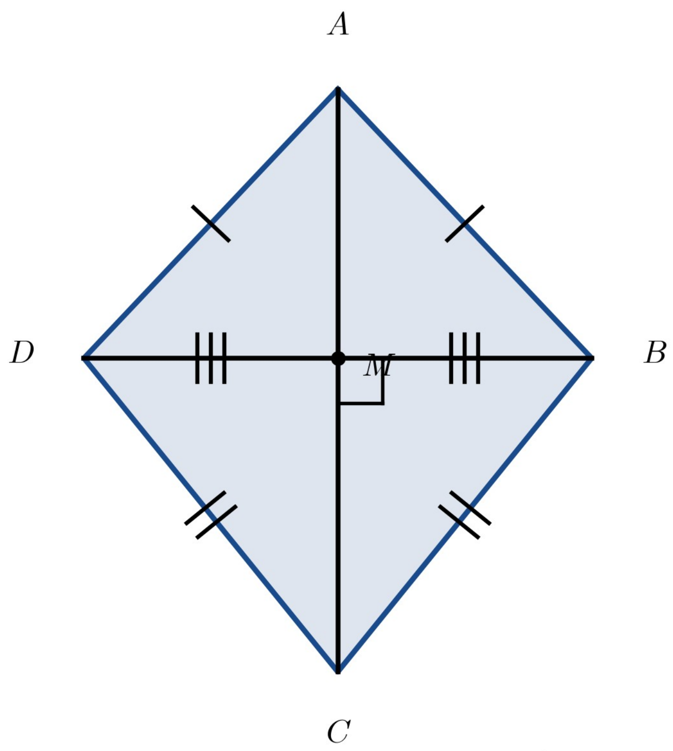
Statement	Reason
$AB = AD$	sides of the rhombus
$BM = DM$	diagonals bisect each other (Result 2)
$AM = AM$	common side
$\triangle ABM \cong \triangle ADM$	SSS
$\angle AMB = \angle AMD$	matching angles of congruent triangles
$\angle AMB + \angle AMD = 180^\circ$, so each is 90°	angles on a straight line (B, M, D lie on one line)
$\angle BAM = \angle DAM$	matching angles of congruent triangles
$\therefore$ the diagonals meet at right angles, and AC bisects the angles at A and at C (repeat the steps at C)	conclusion

Repeating the same steps with the other diagonal shows BD bisects the angles at B and D.

Result 6 — the diagonals of a kite

A kite is *not* a parallelogram, so Result 2 cannot be used — we start from the two pairs of equal sides instead. The result ends up looking like half of Result 5: one diagonal cuts the other in half at right angles, but the diagonals do **not** both get cut in half.

The diagonals of a kite



the diagonals meet at right angles; the axis diagonal AC bisects BD — but AC is not bisected ($AM \neq MC$)

Kite ABCD with AB = AD and CB = CD, diagonals meeting at M; one tick on AB and AD, two ticks on CB and CD, three ticks on BM and MD, a right angle square at M; the longer diagonal AC is visibly not cut in half

Result 6. *In a kite, the diagonals meet at right angles, and the diagonal joining the vertices where the equal sides meet bisects the other diagonal and the two vertex angles.*

In kite $ABCD$ with $AB = AD$ and $CB = CD$, let the diagonals meet at M.

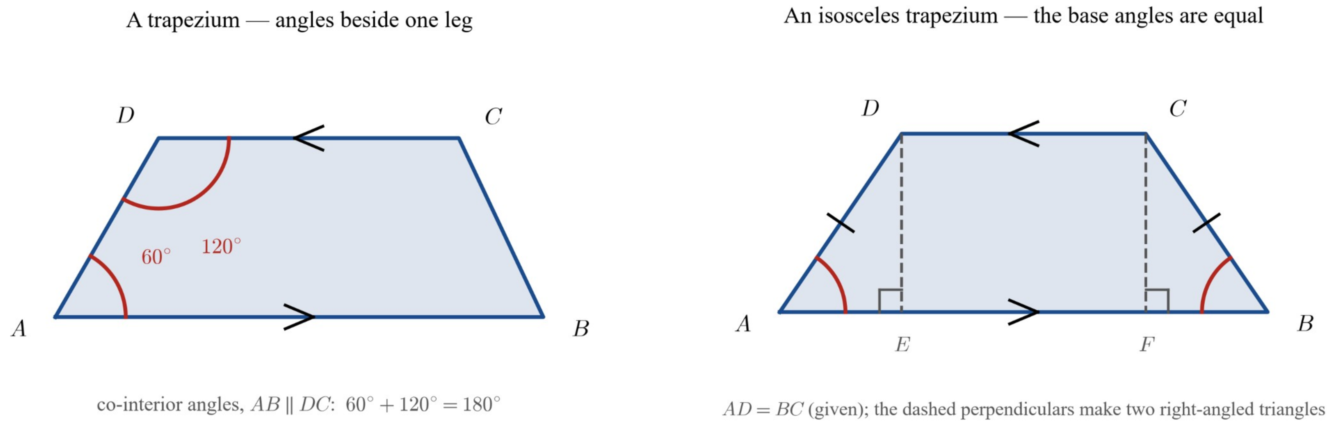
Statement	Reason
$AB = AD$	given
$CB = CD$	given
$AC = AC$	common side
$\triangle ABC \cong \triangle ADC$	SSS
$\angle BAM = \angle DAM$	matching angles of congruent triangles
$AB = AD$, $\angle BAM = \angle DAM$, $AM = AM$	from above, and common side
$\triangle ABM \cong \triangle ADM$	SAS
$BM = DM$, and $\angle AMB = \angle AMD = 90^\circ$	matching sides; matching angles on a straight line
$\therefore$ AC cuts BD in half at right angles, and bisects the angles at A and C	conclusion

But $AM \neq MC$ in general — the kite's diagonals do not bisect *each other*, which is exactly why a kite is not a parallelogram (Result 3 would otherwise make it one).

Result 7 — trapeziums: co-interior angles, and the isosceles case

A trapezium has **exactly one** pair of parallel sides (this book's definition, as in Year 7). The parallel pair makes the angle facts work along each leg.

Trapeziums: one pair of parallel sides



Two panels. Left: trapezium ABCD with AB parallel to DC, a single arc at angle A labelled 60 degrees and a single arc at angle D labelled 120 degrees, the pair adding to 180 degrees. Right: an isosceles trapezium ABCD with AD = BC ticked, dashed perpendiculars dropped from D and C to points E and F on AB with right angle squares at E and F, and matching arcs at the base angles A and B

Result 7a. *In a trapezium, the two angles beside each leg add to 180°.*

With $AB \parallel DC$, the angles at A and D are co-interior angles between the parallel sides, so $\angle A + \angle D = 180^\circ$; the same holds at B and C. (That is a Year 7 angle fact applied to the parallel pair — no congruence needed.)

An **isosceles trapezium** is a trapezium whose legs (the non-parallel sides) are equal. It is the trapezium with a mirror line, and congruent triangles pin down its base angles.

Result 7b. *The base angles of an isosceles trapezium are equal.*

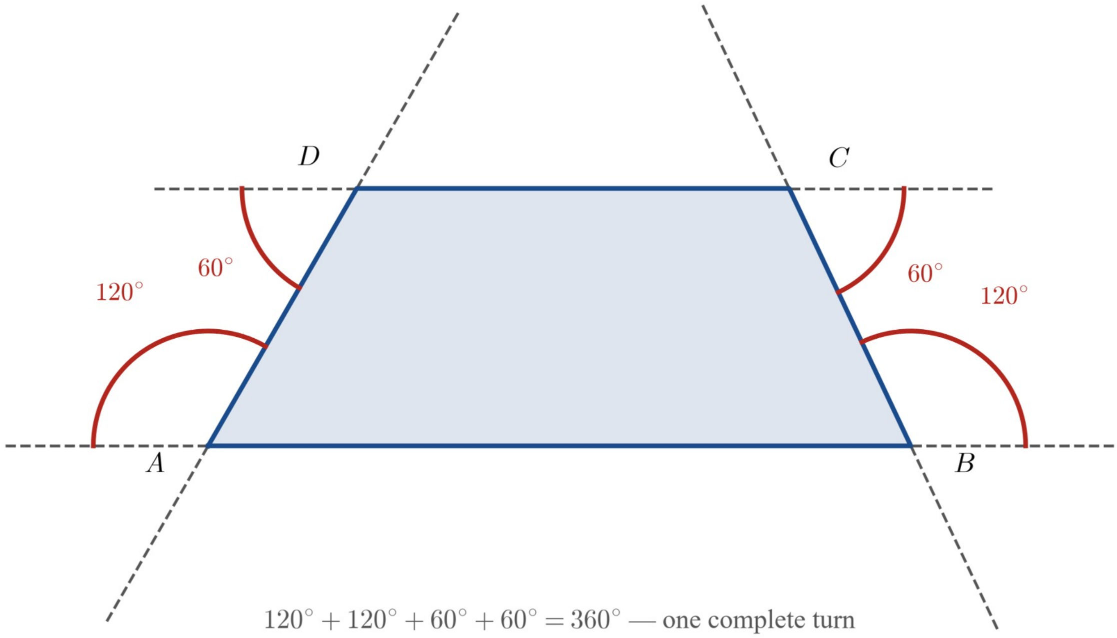
In isosceles trapezium $ABCD$ ($AB \parallel DC$, $AD = BC$), drop perpendiculars from D and C to E and F on AB.

Statement	Reason
$DE \parallel CF$	both perpendicular to AB (equal corresponding angles)
$DC \parallel EF$	part of $DC \parallel AB$
$DEFC$ is a parallelogram, so $DE = CF$	definition of a parallelogram; Result 1
$\angle AED = \angle BFC = 90^\circ$	construction
$AD = BC$	given (isosceles)
$DE = CF$	shown above
$\triangle ADE \cong \triangle BCF$	RHS
$\therefore \angle A = \angle B$ (and hence $\angle D = \angle C$, by Result 7a)	matching angles; co-interior angles

Result 8 — the exterior angles of any quadrilateral make one complete turn

Walk right around any quadrilateral, turning at each corner so you keep going along the next side. When you face the way you started, you have turned one complete turn — 360°. Each turn is an **exterior angle** (the angle between a side, extended, and the next side). The picture shows it for one trapezium; the proof below works for *every* quadrilateral.

The four exterior angles of a quadrilateral



Trapezium ABCD with every side extended as a dashed line beyond each vertex; the four exterior angles are arced and labelled 120 degrees at A, 120 degrees at B, 60 degrees at C and 60 degrees at D, and the caption records 120 + 120 + 60 + 60 = 360 degrees, one complete turn

Result 8. *The exterior angles of a quadrilateral add to 360°.*

Statement	Reason
at each vertex, interior angle + exterior angle $= 180^\circ$	angles on a straight line
total of the four interior + four exterior angles $= 4 \times 180^\circ = 720^\circ$	adding the four straight lines
the four interior angles add to 360°	angle sum of a quadrilateral (Year 7)
exterior angles $= 720^\circ - 360^\circ = 360^\circ$	subtraction
$\therefore$ the exterior angles make one complete turn	conclusion

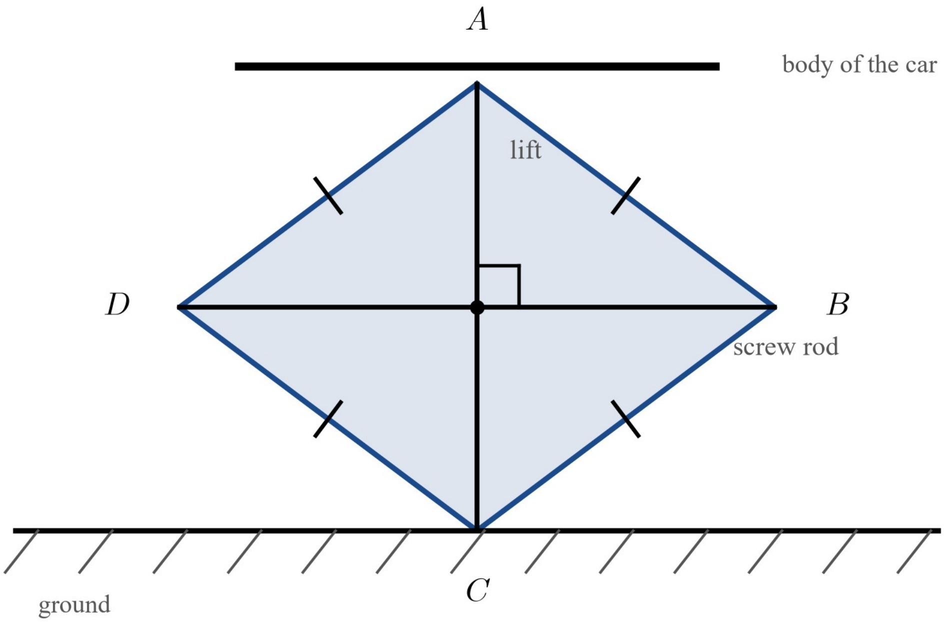
(The same walking idea works for any polygon — the exterior angles of a pentagon, a hexagon, or a twenty-sided shape all add to 360°. Try it on the trapezium in the figure: $120^\circ + 120^\circ + 60^\circ + 60^\circ = 360^\circ$.)

Why builders love these results

The results are not just paper facts. Three working examples from construction:

The car jack. A scissor-style car jack is a rhombus of four equal arms. Turning the screw shortens the horizontal diagonal — and Result 5 says the diagonals of a rhombus meet at right angles, so the top corner stays *directly above* the bottom corner at every height. The car rises straight up, not sideways.

A car jack

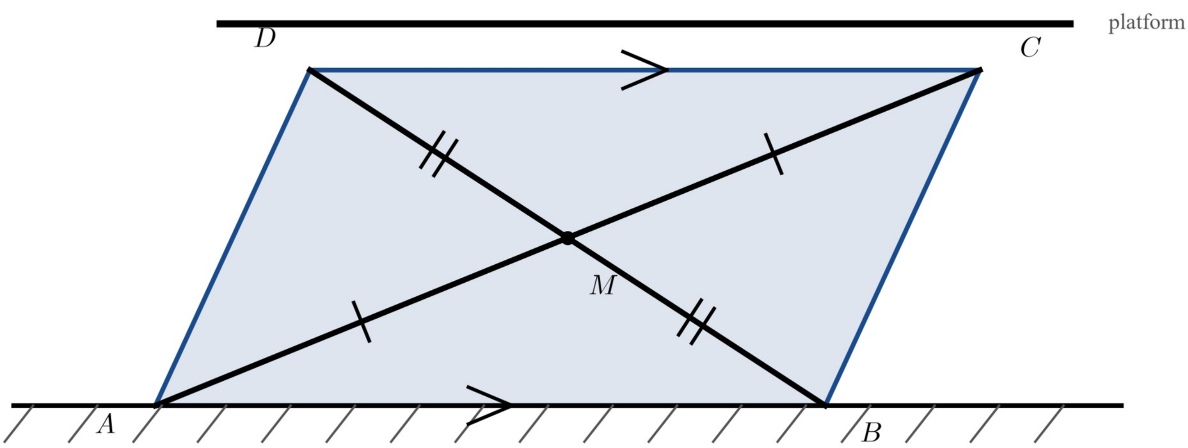


4 equal arms $\Rightarrow$ a rhombus: the diagonals meet at right angles,
so the top rises straight above the bottom

A car jack drawn as a rhombus ABCD standing on a hatched ground line with the body of the car as a heavy line above the top corner; the four arms carry one tick each, the diagonals are drawn with a right angle square at their crossing M, the horizontal diagonal labelled "screw rod" and the vertical diagonal labelled "lift"

The scissor lift. A scissor lift is a stack of X-frames whose two arms are pinned at their middle points. Result 3 — the converse — now does the work: the arms are the diagonals of quadrilateral $ABCD$, they bisect each other, so $ABCD$ is *always* a parallelogram however the arms swing, and the platform (side DC) stays parallel to the base (side AB) at every height.

A scissor lift



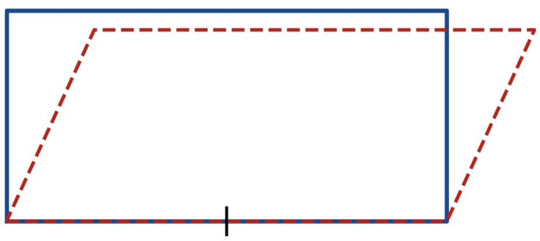
the equal arms cross at their middle points: $AM = CM$ and $BM = DM$,
so $ABCD$ is a parallelogram and the platform stays parallel to the base

A scissor lift: parallelogram ABCD with the base AB on a hatched ground line and a heavy platform line above DC; the arms AC and BD cross at M with one tick on AM and MC and two ticks on BM and MD, and matching arrows show AB parallel to DC

The gate brace. Year 7 taught you that triangles are rigid but quadrilaterals flex: four fixed side lengths still let a rectangle sag into a leaning parallelogram. That is why a gate carries a brace from corner to corner — the brace is a diagonal, and it cuts the gate into two triangles whose three sides can no longer change (SSS). Congruence explains the brace: with three sides fixed, every triangle in the gate is congruent to the one the builder made, forever.

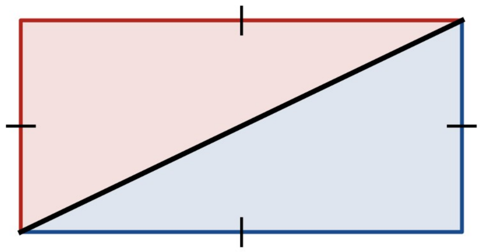
Why gates carry a brace from corner to corner

Without a brace, a rectangle can sag



the same 4 lengths — but the corner angles have changed

With a brace, the gate is fixed

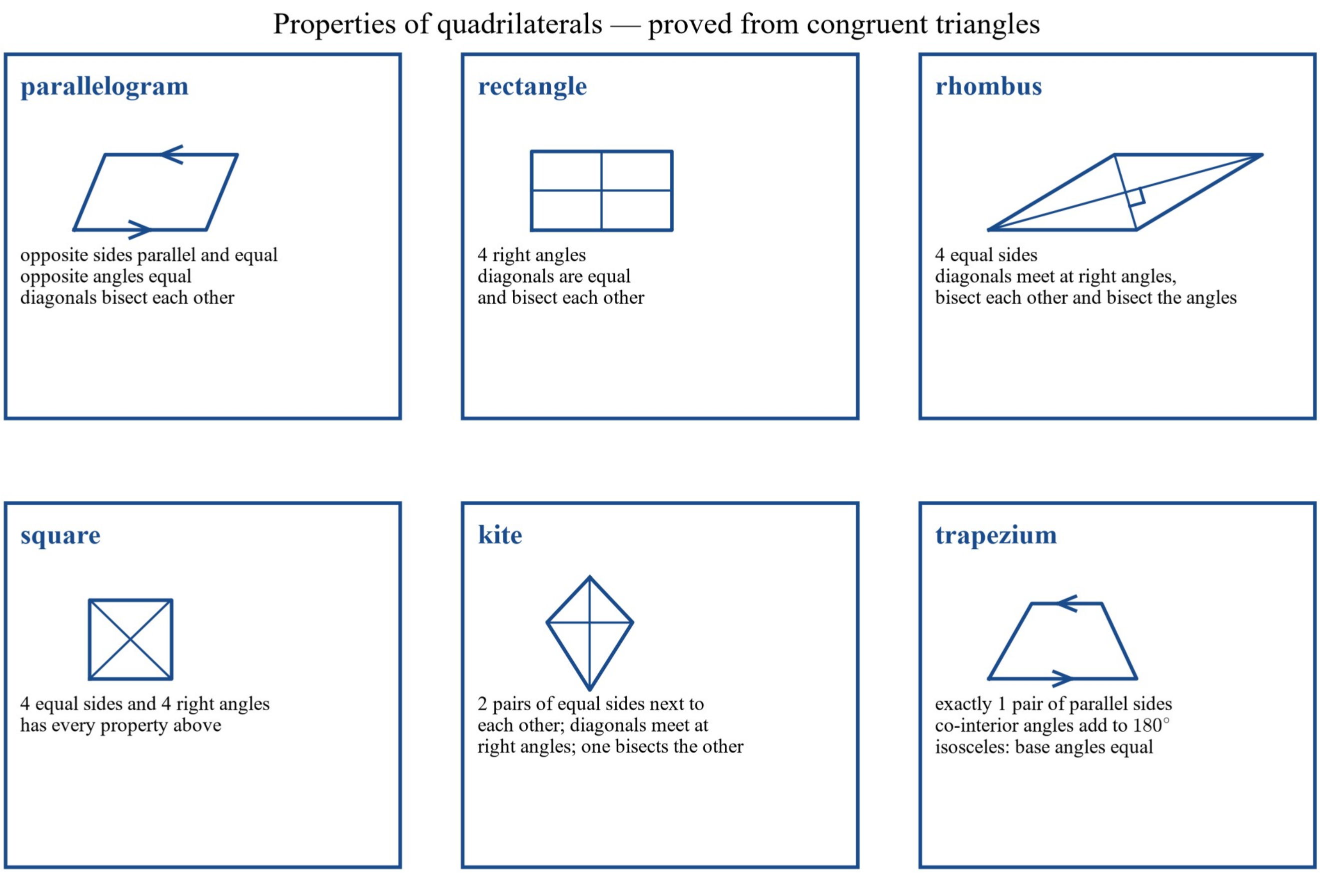


the brace makes 2 triangles: 3 sides fix each triangle (SSS),
so the shape cannot change

Two panels. Left, "Without a brace, a rectangle can sag": a blue rectangle overlaid by a red dashed parallelogram built from the same four side lengths, with matching ticks on the two bottom edges. Right, "With a brace, the gate is fixed": the rectangle with a heavy diagonal brace from corner to corner, the two resulting triangles shaded, and one tick on each side

Folding umbrellas, cherry pickers and toolbox lids run on the same two facts — rhombus diagonals meet at right angles, and quadrilaterals whose diagonals bisect each other are parallelograms.

Everything proved, on one card



Summary card of six titled boxes. Parallelogram: opposite sides parallel and equal, opposite angles equal, diagonals bisect each other. Rectangle: 4 right angles, diagonals are equal and bisect each other. Rhombus: 4 equal sides, diagonals meet at right angles, bisect each other and bisect the angles. Square: 4 equal sides and 4 right angles, has every property above. Kite: 2 pairs of equal sides next to each other, diagonals meet at right angles, one bisects the other. Trapezium: exactly 1 pair of parallel sides, co-interior angles add to 180 degrees, isosceles means base angles equal

Notice the square's box: a square is a rectangle *and* a rhombus, so it has every property on the card. That is the hierarchy Year 7 drew, now with proofs underneath it.

Words of this subtopic

Word	What it means here
diagonal	a line joining two opposite corners of a shape
bisect	cut into two equal halves
congruent (≅)	same shape and same size — an exact copy, allowing slides, flips and turns
matching parts	the sides and angles of congruent triangles that line up when one triangle is placed on the other (also called *corresponding* parts)
converse	a claim made by swapping the "if" and "then" of another claim; it needs its own proof
leg (of a trapezium)	one of the two non-parallel sides
isosceles trapezium	a trapezium whose two legs are equal
base angles	the two angles on the longer parallel side of an isosceles trapezium
exterior angle	the angle between one side of a shape (extended) and the

Word	What it means here
	next side
vertex angle (of a kite)	an angle where the two pairs of equal sides meet
rigid / flex	a triangle’s shape is fixed by its three side lengths (rigid); a quadrilateral’s is not (it flexes)

Worked examples

Example 1 (with help)

$ABCD$ is a parallelogram with $\angle A = 68^\circ$. Find the other three angles.

Working	Comment
$\angle C = 68^\circ$	opposite angles of a parallelogram are equal (Result 1)
$\angle B = 180^\circ - 68^\circ = 112^\circ$	co-interior angles add to 180° ($AD \parallel BC$)
$\angle D = \angle B = 112^\circ$	opposite angles equal (Result 1)

Check: $68^\circ + 112^\circ + 68^\circ + 112^\circ = 360^\circ$ — the quadrilateral angle sum. ✓

Example 2 (with help)

The diagonals of rectangle $ABCD$ meet at M, and $\angle MAB = 35^\circ$. Find $\angle MBA$, $\angle AMB$ and $\angle MAD$.

Working	Comment
$AM = BM$	the diagonals of a rectangle cut into four equal halves (Result 4)
$\angle MBA = \angle MAB = 35^\circ$	base angles of the isosceles triangle AMB
$\angle AMB = 180^\circ - 35^\circ - 35^\circ = 110^\circ$	angle sum of $\triangle AMB$
$\angle MAD = 90^\circ - 35^\circ = 55^\circ$	the corner angle $\angle BAD$ is 90°

Example 3 (with help)

A kite $ABCD$ has $AB = AD$ and $CB = CD$, with $\angle A = 84^\circ$ and $\angle C = 52^\circ$. Find $\angle ABC$.

Working	Comment
$\angle ABC = \angle ADC$	$\triangle ABC \cong \triangle ADC$ (SSS), so matching angles are equal (Result 6)
$\angle ABC + \angle ADC = 360^\circ - 84^\circ - 52^\circ = 224^\circ$	angle sum of a quadrilateral
$\angle ABC = 224^\circ \div 2 = 112^\circ$	the two unknown angles are equal

Example 4 (on your own)

The diagonal AC of rhombus $ABCD$ makes an angle of 28° with the side AB. Find (a) $\angle BAD$, (b) $\angle ABC$, (c) $\angle AMB$, where M is the point where the diagonals meet.

Example 5 (on your own)

An isosceles trapezium $ABCD$ has $AB \parallel DC$, $AD = BC$, and $\angle A = 70^\circ$. Find the other three angles, giving a reason for each.

Example 6 (on your own)

The diagonals of quadrilateral $ABCD$ meet at M, with $AM = CM = 5$ cm and $BM = DM = 3$ cm. (a) Explain why $ABCD$ must be a parallelogram. (b) If $\angle AMB = 70^\circ$, find $\angle CMD$ and $\angle AMD$.

Practice questions

With help

Q1. Name the family described. (a) exactly one pair of parallel sides (b) four equal sides, no right angles marked (c) two pairs of equal sides next to each other (d) four right angles

Q2. The diagonals of a quadrilateral bisect each other at right angles, and the two diagonals have different lengths. The quadrilateral is a: **A** rectangle **B** rhombus **C** kite **D** trapezium

Q3. $ABCD$ is a parallelogram with $\angle A = 55^\circ$. Find $\angle B$, $\angle C$ and $\angle D$, with a reason for each.

Q4. In parallelogram $ABCD$ the diagonal AC is drawn. $\angle BAC = \angle DCA$ (alternate angles), $\angle BCA = \angle DAC$ (alternate angles), and AC is a common side. Name the congruence test that shows $\triangle ABC \cong \triangle CDA$.

Q5. In trapezium $ABCD$, $AB \parallel DC$, $\angle A = x^\circ$ and $\angle D = (x+14)^\circ$. Find x.

Q6. In trapezium $ABCD$, $AB \parallel DC$, $\angle A = 73^\circ$ and $\angle B = 81^\circ$. Find $\angle D$ and $\angle C$.

Q7. A kite $ABCD$ has $AB = AD$ and $CB = CD$, with $\angle A = 62^\circ$ and $\angle C = 56^\circ$. Find $\angle B$.

Q8. These five statements prove Result 2, but they are out of order. Write them in a logical order. 1. So $AM = CM$ and $BM = DM$ (matching sides of congruent triangles). 2. In $\triangle AMB$ and $\triangle CMD$: $\angle MAB = \angle MCD$ and $\angle MBA = \angle MDC$ (alternate angles, $AB \parallel DC$). 3. Draw the diagonals AC and BD, meeting at M. 4. $AB = CD$ (opposite sides of a parallelogram, Result 1). 5. So $\triangle AMB \cong \triangle CMD$ (ASA).

On your own

Q9. In parallelogram $ABCD$, $AB = (3x-2)$ cm and $CD = (x+6)$ cm. Find x and the length of AB.

Q10. The diagonals of rectangle $ABCD$ meet at M, and $\angle MAB = 32^\circ$. Find $\angle AMB$ and $\angle AMD$.

Q11. In rhombus $ABCD$ the diagonal AC makes $\angle BAC = 38^\circ$. Find $\angle DAC$, $\angle BCA$ and $\angle ABC$.

Q12. Prove Result 1's partner fact in full: *the opposite angles of a parallelogram are equal.* Set out a two-column proof (statement · reason), starting "In parallelogram $ABCD$, draw the diagonal AC."

Q13. An isosceles trapezium has $\angle A = 71^\circ$ on its longer parallel side. Find all four angles.

Q14. Write down the number of lines of symmetry of each: square, rectangle, rhombus, kite, (general) parallelogram.

Q15. Prove the first half of Result 6: *in kite $ABCD$ with $AB = AD$ and $CB = CD$, the diagonal AC bisects the angles at A and at C.* (One congruent pair is enough.)

Q16. Three exterior angles of a quadrilateral are 85°, 95° and 100°. Find the fourth exterior angle.

Maths in real life

Q17. The car jack. In the jack shown in the theory, each arm makes an angle of 52° with the screw rod. (a) Find the angle between the two arms at the side hinge (the vertex on the screw rod). (b) Find the angle between the two arms at the top hinge. (c) Explain, naming the result, why the top of the jack rises straight above the bottom.

Q18. The scissor lift. In the scissor lift shown in the theory, the two arms AC and BD are the same length and are pinned together at their middle point M. Explain, naming the result, why the platform stays parallel to the base as the lift rises.

Q19. The gate brace. A garden gate is a rectangle 2 m by 1 m. Left unbraced, it sags after a few years. (a) Explain why four fixed side lengths cannot hold the rectangle's shape. (b) A brace is screwed from corner to corner. Explain, naming a congruence test, why the gate can no longer sag.

Q20. The cherry picker. The arms of a cherry picker's frame are PR and QS, pinned at their common middle point M, so that $PM = RM$ and $QM = SM$. The boom is PQ and the base is RS. Prove that the boom stays parallel to the base: show $\triangle PMQ \cong \triangle RMS$, and hence $PQ \parallel RS$.

Answers

Q1. (a) trapezium (b) rhombus (c) kite (d) rectangle **Q2.** B (rhombus) — diagonals that bisect each other give a parallelogram (Result 3), and a parallelogram whose diagonals cross at right angles is a rhombus (the steps of Result 5 run in reverse); the unequal diagonals rule out a square. **Q3.** $\angle B = 125^\circ$ (co-interior with $\angle A$), $\angle C = 55^\circ$, $\angle D = 125^\circ$ (opposite angles equal, Result 1) **Q4.** ASA (two angles and the side between them) **Q5.** $x = 83$ (co-interior angles: $x + x + 14 = 180$) **Q6.** $\angle D = 107^\circ$, $\angle C = 99^\circ$ (co-interior angles, Result 7a) **Q7.** $\angle B = 121^\circ$ (angle sum 360°, and $\angle B = \angle D$ by Result 6) **Q8.** 3, 2, 4, 5, 1 **Q9.** $x = 4$; $AB = 10$ cm **Q10.** $\angle AMB = 116^\circ$; $\angle AMD = 64^\circ$ **Q11.** $\angle DAC = 38^\circ$, $\angle BCA = 38^\circ$, $\angle ABC = 104^\circ$ **Q12.** Proof — see solutions. **Q13.** $71^\circ, 71^\circ, 109^\circ, 109^\circ$ **Q14.** square 4; rectangle 2; rhombus 2; kite 1; parallelogram 0 **Q15.** Proof — see solutions. **Q16.** 80° **Q17.** (a) 104° (b) 76° (c) the arms form a rhombus; its diagonals meet at right angles (Result 5), so the lift diagonal is perpendicular to the (horizontal) screw rod **Q18.** the arms are diagonals that bisect each other, so the frame is a parallelogram (Result 3), and $DC \parallel AB$ **Q19.** (a) quadrilaterals flex: four side lengths do not fix the angles (Year 7) (b) the brace is a diagonal; each triangle then has three fixed sides, so each is fixed by SSS **Q20.** Proof — see solutions.

Worked examples (on your own): Ex 4. (a) 56° (b) 124° (c) 90° **Ex 5.** $\angle B = 70^\circ$ (base angles, Result 7b); $\angle C = \angle D = 110^\circ$ (co-interior, Result 7a) **Ex 6.** (a) the diagonals bisect each other, so Result 3 gives a parallelogram (b) $\angle CMD = 70^\circ$ (vertically opposite), $\angle AMD = 110^\circ$ (angles on a straight line)

Fully-worked solutions

Q12. In parallelogram $ABCD$, draw the diagonal AC.

Statement	Reason
$\angle BAC = \angle DCA$	alternate angles, $AB \parallel DC$
$\angle BCA = \angle DAC$	alternate angles, $BC \parallel AD$
$AC = CA$	common side
$\triangle ABC \cong \triangle CDA$	ASA
$\angle B = \angle D$	matching angles of congruent triangles
$\angle BAD = \angle BAC + \angle DAC = \angle DCA + \angle BCA =$	adding equal angles
∴ the opposite angles of a parallelogram are equal	conclusion

Q15. In kite $ABCD$, $AB = AD$ (given), $CB = CD$ (given), $AC = AC$ (common side), so $\triangle ABC \cong \triangle ADC$ (SSS). Hence $\angle BAC = \angle DAC$ and $\angle BCA = \angle DCA$ (matching angles) — that is, AC bisects the angles at A and at C. ∴ the vertex-to-vertex diagonal bisects the vertex angles.

Q20. In $\triangle PMQ$ and $\triangle RMS$: $PM = RM$ (given, M is the middle of PR); $\angle PMQ = \angle RMS$ (vertically opposite angles); $QM = SM$ (given, M is the middle of QS). So $\triangle PMQ \cong \triangle RMS$ (SAS). Hence $\angle MPQ = \angle MRS$ (matching angles). These are equal alternate angles for the lines PQ and RS cut by the line PR, so $PQ \parallel RS$. ∴ the boom stays parallel to the base at every height.

Q3 (model write-up). $\angle B = 180^\circ - 55^\circ = 125^\circ$ (co-interior angles, $AD \parallel BC$). $\angle C = \angle A = 55^\circ$ and $\angle D = \angle B = 125^\circ$ (opposite angles of a parallelogram are equal, Result 1).

Q8 (why that order). Statement 3 sets up the figure; 2 collects the two equal angle pairs; 4 supplies the equal side *between* them; 5 applies ASA; 1 reads off the matching sides, which is exactly what "bisect each other" means.

Ex 4. (a) AC bisects $\angle BAD$ (Result 5), so $\angle BAD = 2 \times 28^\circ = 56^\circ$. (b) $\angle ABC = 180^\circ - 56^\circ = 124^\circ$ (co-interior angles, $AD \parallel BC$). (c) $\angle AMB = 90^\circ$ (the diagonals of a rhombus meet at right angles, Result 5).

Ex 5. $\angle B = \angle A = 70^\circ$ (base angles of an isosceles trapezium, Result 7b). $\angle D = 180^\circ - 70^\circ = 110^\circ$ and $\angle C = 180^\circ - 70^\circ = 110^\circ$ (co-interior angles beside each leg, Result 7a). Check: $70^\circ + 70^\circ + 110^\circ + 110^\circ = 360^\circ$. ✓

Ex 6. (a) $AM = CM$ and $BM = DM$ mean the diagonals bisect each other, so by Result 3 (the converse), $ABCD$ is a parallelogram. (b) $\angle CMD = \angle AMB = 70^\circ$ (vertically opposite angles). $\angle AMD = 180^\circ - 70^\circ = 110^\circ$ (angles on a straight line — AC is one straight diagonal).

Algorithms that test for congruence and similarity

Content description `VC2M8SP04`: design and test algorithms involving a sequence of steps and decisions that identify congruency or similarity of shapes, and describe how the algorithm works

An algorithm is a rule you can follow exactly: a sequence of steps and decisions with no guesswork. This subtopic designs one such rule — a sorting machine for pairs of shapes — whose output is one of three labels: congruent, similar but not congruent, or neither. You will draw the rule as a flow chart, trace it on real triangles, test it on cases whose answers are already known, and catch it when a careless version of it gets a case wrong.

Theory

Recall: algorithms from Year 7

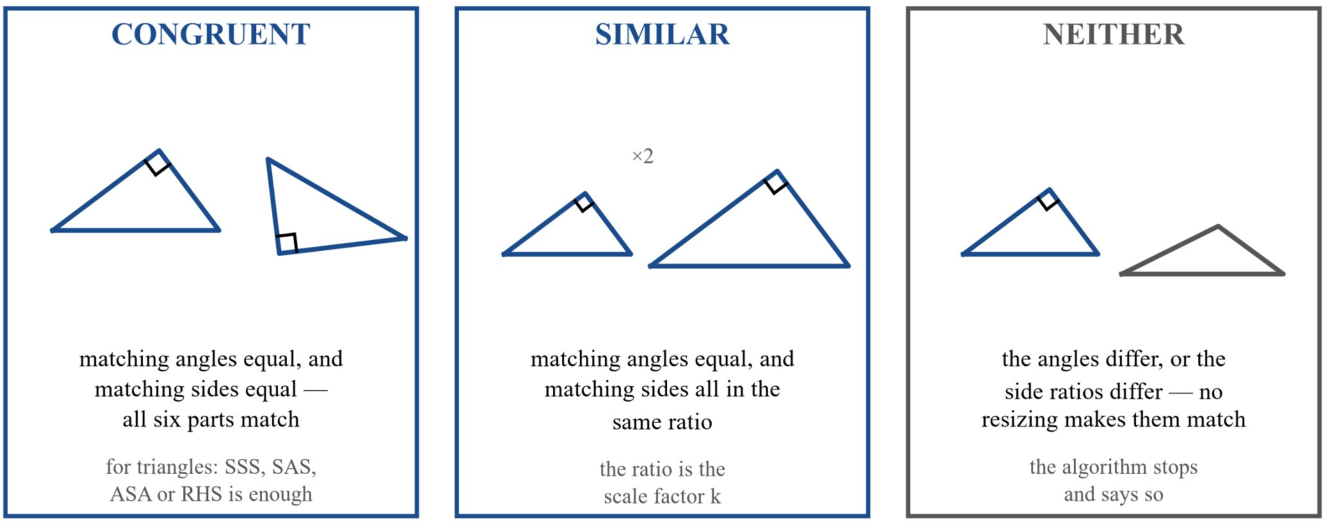
In Year 7 you designed and tested **algorithms** — exact sequences of steps and decisions — for sorting and classifying shapes (`VC2M7SP04`). You know the three ingredients this subtopic uses again:

- a **step** is one instruction the rule carries out, in order;
- a **decision** is a yes/no question where the rule branches;
- a **flow chart** is a diagram of the rule: boxes hold steps, diamonds hold decisions, and arrows show the order.

Here the job is bigger: the algorithm must sort *pairs* of shapes into congruent, similar, or neither, using the conditions that 5A proved.

The conditions the algorithm will use

An algorithm is only as good as the mathematics inside its decisions. This algorithm uses exactly the conditions from 5A.



Three cards. The CONGRUENT card shows two right-angled triangles with sides 3 cm, 4 cm and 5 cm, the same size with one turned, and notes that all six matching parts are equal, and that for triangles SSS, SAS, ASA or RHS is enough. The SIMILAR card shows a 3-4-5 triangle beside its copy at twice the size, and notes that the matching angles are equal and the matching sides are all in the same ratio, the scale factor k. The NEITHER card shows a 3-4-5 triangle beside a flatter 3-4-6 triangle, and notes that the angles differ or the side ratios differ, so no resizing makes them match.

- **Congruent** means same shape and same size: the matching angles are equal *and* the matching sides are equal. For triangles, 5A's four tests - SSS, SAS, ASA, RHS - are shortcuts that guarantee this.
- **Similar** means same shape, size allowed to differ: the matching angles are equal *and* the matching sides are all in the same ratio. That ratio is the **scale factor** k.
- **Neither** means no resizing of one shape will ever make it match the other: the angles differ, or the side ratios differ.

And two facts from 5A sit behind everything the algorithm does:

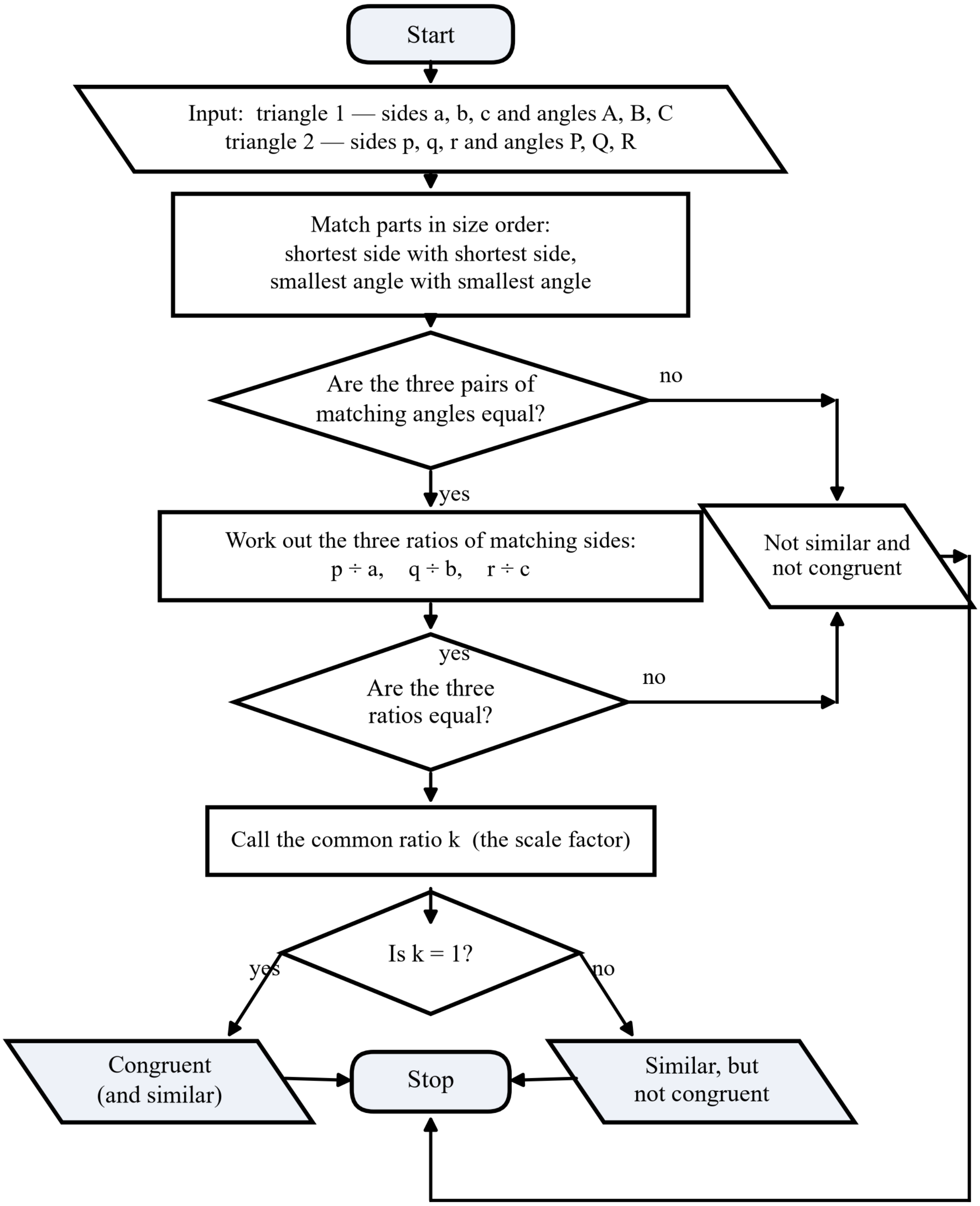
- every congruent pair is also similar, with scale factor 1;
- for triangles, equal angles alone force similarity - but they never force congruence.

Designing the algorithm

The algorithm takes two triangles as **input** - three sides and three angles each - and **outputs** one label: congruent, similar but not congruent, or neither. As numbered steps:

1. **Input.** Triangle 1: sides a, b, c and angles A, B, C. Triangle 2: sides p, q, r and angles P, Q, R.
2. **Match the parts in size order.** Shortest side with shortest side, middle with middle, longest with longest; smallest angle with smallest, middle with middle, largest with largest.
3. **Decision: are the three pairs of matching angles equal?** No - output *not similar and not congruent*, and stop. Yes - go to step 4.
4. **Work out the three ratios of matching sides:** $p \div a$, $q \div b$, $r \div c$.
5. **Decision: are the three ratios equal?** No - output *not similar and not congruent*, and stop. Yes - the triangles are similar; call the common ratio k.
6. **Decision: is $k = 1$?** Yes - output *congruent (and similar)*. No - output *similar, but not congruent*.

The same rule as a flow chart:



Flow chart for sorting two triangles. From Start, an input box lists triangle 1 with sides a, b, c and angles A, B, C and triangle 2 with sides p, q, r and angles P, Q, R. A step box matches parts in size order. A decision diamond asks whether the three pairs of matching angles are equal: no leads right to an output box "Not similar and not congruent", yes leads down to a step box working out the three ratios of matching sides, p divided by a, q divided by b, r divided by c. A second decision diamond asks whether the three ratios are equal: no leads right and up to the same "Not similar and not congruent" box, yes leads down to a step box calling the common ratio k, the scale factor. A third decision diamond asks whether k equals 1: yes leads to the output "Congruent (and similar)", no to "Similar, but not congruent"; every output box then leads to Stop, including the neither box by a line down the right-hand margin.

Read the chart the way you would read any flow chart: enter at Start, follow the arrows, answer each diamond honestly, and stop where an output box sends you. Every run ends at exactly one of the three outputs - that is what makes it an algorithm and not a suggestion.

How the algorithm works

Each design choice has a reason.

Match in size order, because matching parts sit opposite equal angles. In a triangle the shortest side lies opposite the smallest angle, and the longest opposite the largest. Sorting by size is therefore a quick, exact way to pair each side and angle with its matching part - the pairing that 5A's $\cong$ and $\sim$ notation records with letter order.

Check the angles first, because angles are the shape. If the angles differ, the shapes are different shapes, and no ratio of sides can rescue them: the first decision sends such a pair straight to *neither*.

Then check the side ratios, because equal ratios are exactly "one enlargement". If the three ratios of matching sides are all the same number k, then every side of triangle 2 is k times its matching side - triangle 2 is an enlargement of triangle 1, which is what similar means. If the ratios differ, no single enlargement works, so the pair is *neither*.

Finally ask $k = 1$?, because congruent is similar at scale factor 1. Equal ratios of 1 mean every side equals its matching side: same shape and same size.

A sharp reader will notice something odd: for *triangles*, step 5 can never say no once step 3 has said yes, because 5A proved that equal angles force the sides into one ratio. The ratio step is kept anyway, for two reasons. First, measurements taken by hand carry error, and checking both angles and ratios catches mistakes - a pair that passes one check but fails the other was probably measured wrong. Second, the same two-check design works for shapes where equal angles do **not** force equal ratios, such as rectangles — which is exactly the sorter in "Sorting other shapes" below.

The whole rule at a glance:

The triangle-sorting algorithm

1 Match the parts in size order

2 Are the three pairs of matching angles equal?

No → not similar, and not congruent

3 Work out the three ratios of matching sides

Unequal → not similar, and not congruent

4 Equal ratios → similar; call the ratio k

5 k = 1 → congruent k ≠ 1 → similar only

The three possible outputs

CONGRUENT

same shape, same size
(k = 1)

SIMILAR ONLY

same shape, different size
(k ≠ 1)

NEITHER

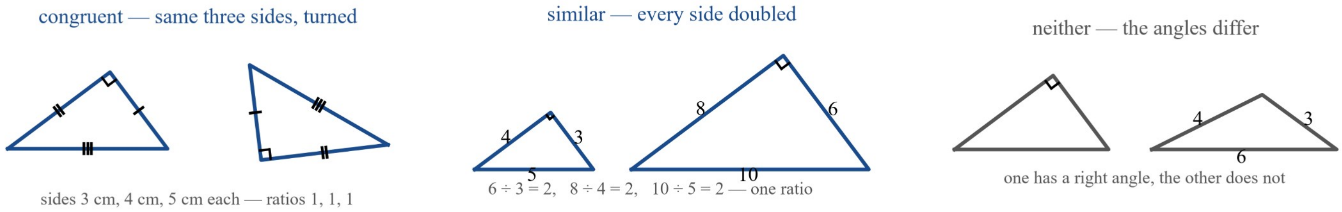
angles differ, or the
ratios differ

For any shape: check every matching angle, then every matching side ratio.

The triangle-sorting algorithm at a glance, as a numbered list beside its three possible outputs. Steps: 1 match the parts in size order; 2 are the three pairs of matching angles equal, no means not similar and not congruent; 3 work out the three ratios of matching sides, unequal means not similar and not congruent; 4 equal ratios mean similar, call the ratio k; 5 k = 1 means congruent, k not 1 means similar only. Output boxes: CONGRUENT, same shape same size with k = 1; SIMILAR ONLY, same shape different size with k not 1; NEITHER, angles differ or the ratios differ. A line below adds: for any shape, check every matching angle, then every matching side ratio.

Tracing the algorithm on three pairs

To **trace** an algorithm means to follow it step by step on one particular input, writing down what happens at every step. Here are the three traces that matter, one for each output.



Three panels drawn to true scale. Left: a triangle with sides 3 cm, 4 cm, 5 cm beside a turned copy with the same three sides, matching sides carrying matching tick marks - congruent, ratios 1, 1, 1. Middle: a 3-4-5 triangle beside a triangle with sides 6 cm, 8 cm, 10 cm - similar, every side doubled, 6 divided by 3 equals 2, 8 divided by 4 equals 2, 10 divided by 5 equals 2, one ratio. Right: a 3-4-5 triangle with its right angle marked beside a flatter triangle with sides 3 cm, 4 cm, 6 cm - neither, one has a right angle and the other does not.

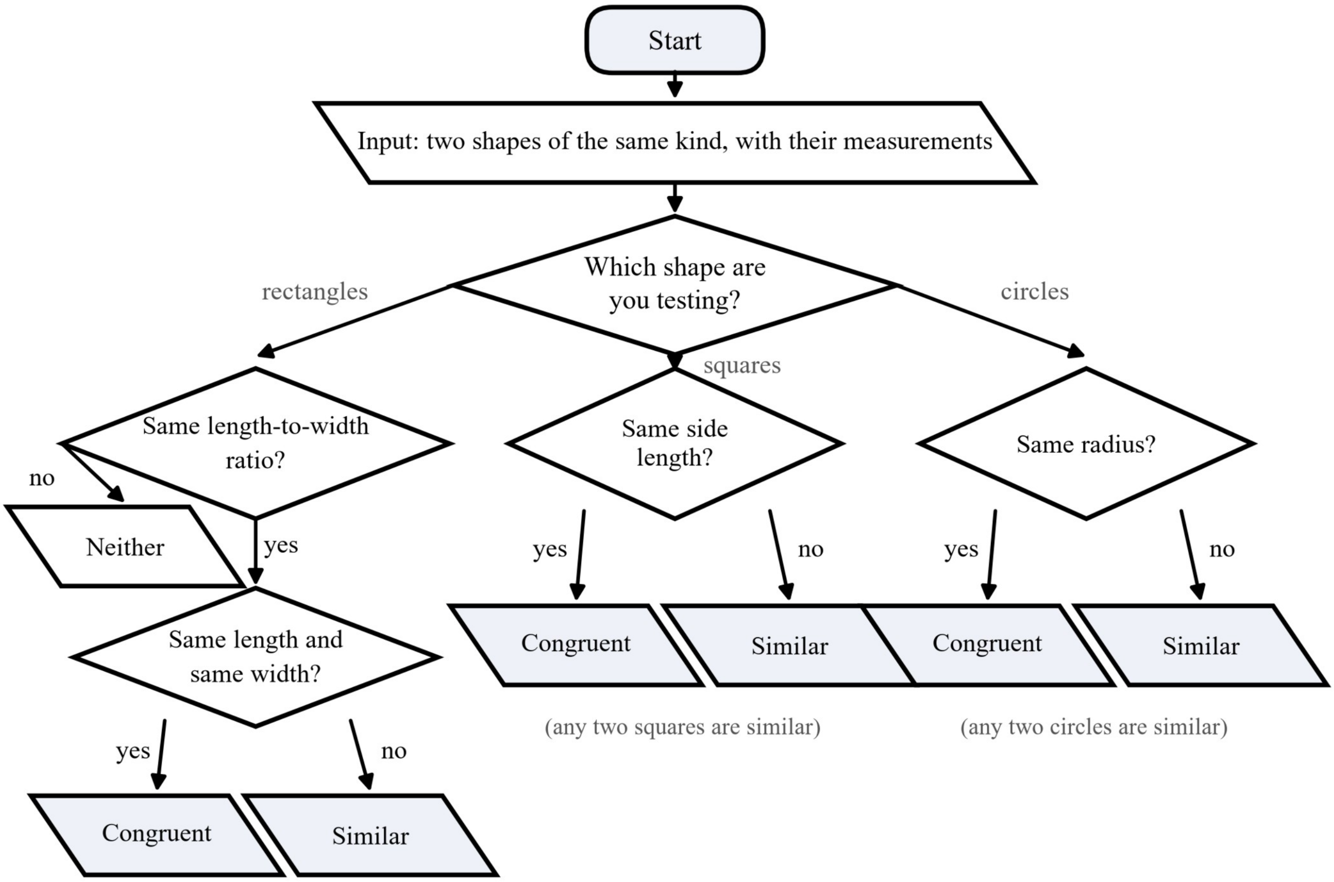
Trace 1 - congruent. Sides 3 cm, 4 cm, 5 cm and 3 cm, 4 cm, 5 cm; angles $37^\circ, 53^\circ, 90^\circ$ in both. Step 2 pairs 3 with 3, 4 with 4, 5 with 5. Step 3: the angle pairs $37^\circ = 37^\circ$, $53^\circ = 53^\circ$, $90^\circ = 90^\circ$ are equal - yes. Step 4: the ratios are $3 \div 3 = 1$, $4 \div 4 = 1$, $5 \div 5 = 1$. Step 5: equal - yes, so $k = 1$. Step 6: $k = 1$ - yes. Output: **congruent (and similar)**.

Trace 2 - similar, not congruent. Sides 3 cm, 4 cm, 5 cm and 6 cm, 8 cm, 10 cm; angles $37^\circ, 53^\circ, 90^\circ$ in both. Step 3: yes. Step 4: $6 \div 3 = 2$, $8 \div 4 = 2$, $10 \div 5 = 2$. Step 5: equal - yes, $k = 2$. Step 6: $k \neq 1$ - no. Output: **similar, but not congruent**.

Trace 3 - neither. Sides 3 cm, 4 cm, 5 cm (angles $37^\circ, 53^\circ, 90^\circ$) and 3 cm, 4 cm, 6 cm (angles $26^\circ, 36^\circ, 117^\circ$). Step 3: the smallest angles give $37^\circ \neq 26^\circ$ - no. Output: **not similar and not congruent**, without ever computing a ratio.

Sorting other shapes

The triangle sorter is one instance of a general method: for any family of shapes, write the 5A conditions for that family as decisions. Here is the sorter for rectangles, squares and circles.



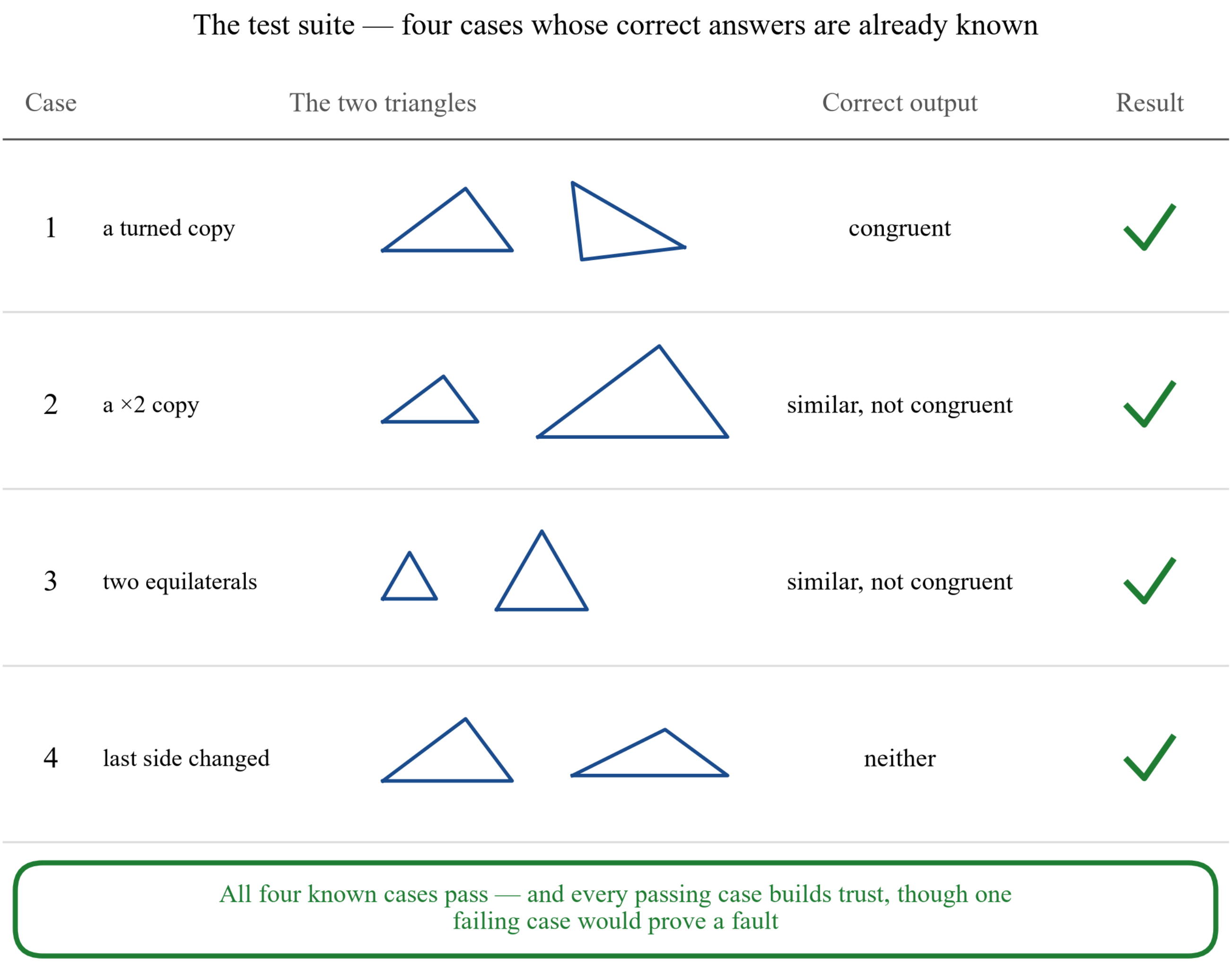
Every path ends at exactly one of the three outputs.

Decision tree for two shapes of the same kind. From Start and an input box, a decision diamond asks which shape is being tested, branching to rectangles, squares or circles. Rectangles: a diamond asks whether the length-to-width ratio is the same - no gives Neither; yes leads to a diamond asking whether the length and width are the same - yes gives Congruent, no gives Similar. Squares: a diamond asks whether the side length is the same - yes Congruent, no Similar, since any two squares are similar. Circles: a diamond asks whether the radius is the same - yes Congruent, no Similar, since any two circles are similar. Every path ends at exactly one of the three outputs.

The rectangles branch is the interesting one, because it is the two-check design again: all rectangles have four right angles, so equal angles can never tell two rectangles apart, and the length-to-width ratio has to do the sorting. The squares and circles branches are short because 5A showed those families are always similar - only the size decision remains.

Testing the algorithm

Designing an algorithm is only half of `VC2M8SP04`; the CD says *test* it. Testing means running the algorithm on **test cases** - inputs whose correct output is already known from the mathematics, not from the algorithm — and checking that the two agree.



A test suite in four rows, each row showing the two triangles, the correct output and a green tick because the algorithm's output matched it. Case 1: a triangle and a turned copy - congruent. Case 2: a triangle and its copy at twice the size - similar, not congruent. Case 3: two equilateral triangles of different sides - similar, not congruent. Case 4: a 3-4-5 triangle and a 3-4-6 triangle - neither. A box below reads: all four known cases pass, and every passing case builds trust, though one failing case would prove a fault.

Cases 1 and 2 are the traces above; case 3 is two equilateral triangles (all angles 60°, so similar; different sides, so not congruent); case 4 is trace 3. The algorithm as designed agrees with the known answer on all four.

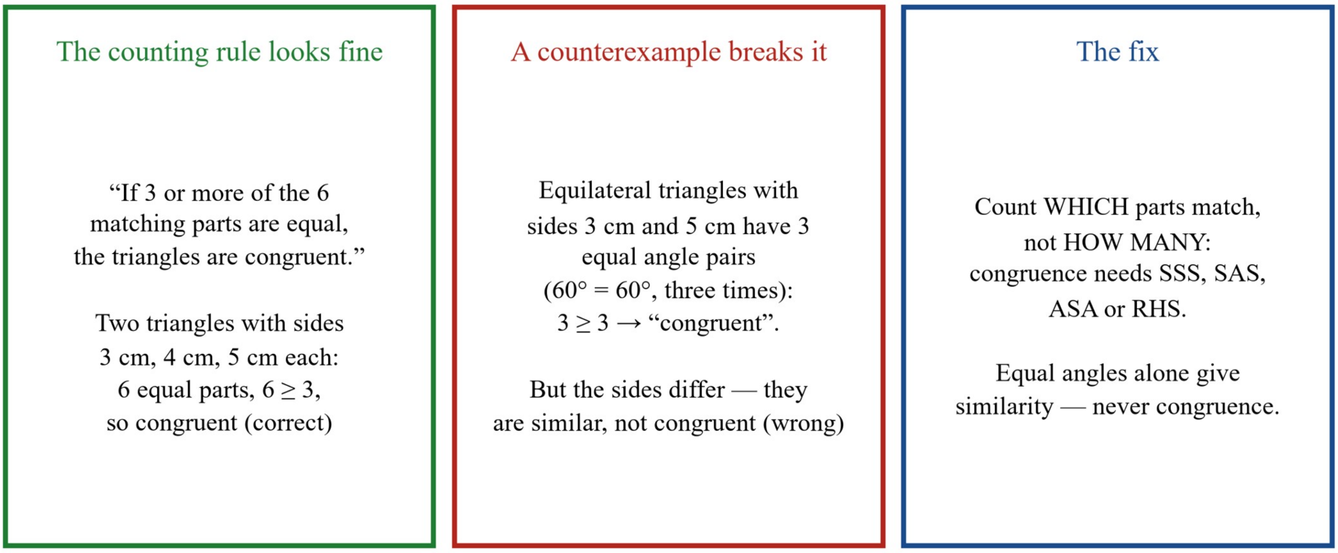
Two facts about testing are worth keeping:

- **One failing case proves a fault.** A hundred passing cases build trust, but a single input where the algorithm's output differs from the known answer shows the rule is wrong.
- **Passing cases never prove perfection.** The suite has four cases; there are infinitely many pairs of triangles. Testing builds confidence, not certainty - which is why the evaluation below matters.

Evaluating the algorithm: a faulty rule and its fix

Evaluation asks whether the algorithm classifies *accurately* - whether its outputs are the mathematically correct ones, on every input, not just on the cases tested. The quickest way to evaluate is to break a rule and watch it fail. Consider this careless sorting rule:

"If 3 or more of the 6 matching parts are equal, the triangles are congruent."



Three cards. The first, in green, states the counting rule and applies it to two triangles with sides 3 cm, 4 cm, 5 cm each: 6 equal parts, 6 is at least 3, so congruent, marked correct. The second, in red, breaks the rule: equilateral triangles with sides 3 cm and 5 cm have 3 equal angle pairs, 3 is at least 3, so the rule says congruent - but the sides differ, they are similar, not congruent, marked wrong. The third, in blue, gives the fix: count WHICH parts match, not HOW MANY - congruence needs SSS, SAS, ASA or RHS, and equal angles alone give similarity, never congruence.

On the first card the rule looks fine. On the second card it fails: two equilateral triangles with sides 3 cm and 5 cm have three equal angle pairs ($60^\circ = 60^\circ$, three times), so the rule counts 3 and declares *congruent* — while the sides plainly differ. The correct label is *similar, not congruent*. One failing case, and the rule is proved faulty.

The fault is precise: the rule counts **how many** parts match instead of checking **which** parts match. Congruence needs the right *kinds* of parts - SSS, SAS, ASA or RHS - and equal angles alone give similarity, never congruence. The designed algorithm cannot make this mistake, because its decisions check the angles *and* the side ratios: on the two equilateral triangles its step 5 finds $k = 5 \div 3 \neq 1$ and correctly outputs *similar, but not congruent*.

What comes next

At Level 9 you will test and refine algorithms like this one - comparing different rules for the same job and improving them (`VC2M9SP03`); and 5B’s proofs are where the conditions inside our decisions get used as reasons in logical arguments.

Words of this subtopic

Word	What it means here
algorithm	an exact sequence of steps and decisions that can be followed to complete a task, with no guesswork
flow chart	a diagram of an algorithm: boxes hold steps, diamonds hold decisions, arrows show the order
input / output	what the algorithm is given (input), and the answer it produces (output)
decision	a yes/no question where the algorithm branches
trace	following an algorithm step by step on one particular input, writing down what happens at each step
test case	an input whose correct output is already known, used to check an algorithm
matching parts	(5A) the sides and angles that land on each other when one shape lies on top of the other

Word	What it means here
scale factor	(5A) the number every side is multiplied by in an enlargement; the common ratio k in the algorithm
congruent / similar	(5A) same shape and same size / same shape with matching sides all in the same ratio
SSS, SAS, ASA, RHS	(5A) the four tests for congruent triangles

Worked examples

Example 1 — with help

Triangle 1 has sides 3 cm, 4 cm, 5 cm and angles $37^\circ, 53^\circ, 90^\circ$. Triangle 2 is a turned copy: sides 3 cm, 4 cm, 5 cm and angles $37^\circ, 53^\circ, 90^\circ$. Trace the sorting algorithm and state its output.

Working	Comment
Step 2: pairs are 3 with 3, 4 with 4, 5 with 5	Match shortest with shortest, middle with middle, longest with longest.
Step 3: $37^\circ = 37^\circ$, $53^\circ = 53^\circ$, $90^\circ = 90^\circ$ - yes	All three pairs of matching angles are equal.
Step 4: $3 \div 3 = 1$, $4 \div 4 = 1$, $5 \div 5 = 1$	The three ratios of matching sides.
Step 5: the ratios are equal - yes, $k = 1$	One common ratio, so the triangles are similar.
Step 6: $k = 1$ - yes	Scale factor 1 means same size as well as same shape.
Output: **congruent (and similar)**	The turned copy lies exactly on the original.

Example 2 — with help

Triangle 1 has sides 5 cm, 12 cm, 13 cm and angles $23^\circ, 67^\circ, 90^\circ$. Triangle 2 has sides 10 cm, 24 cm, 26 cm and angles $23^\circ, 67^\circ, 90^\circ$. Trace the sorting algorithm and state its output.

Working	Comment
Step 2: pairs are 5 with 10, 12 with 24, 13 with 26	Size order does the matching.
Step 3: the three angle pairs are equal - yes	$23^\circ, 67^\circ, 90^\circ$ in both.
Step 4: $10 \div 5 = 2$, $24 \div 12 = 2$, $26 \div 13 = 2$	Each side of triangle 2 is twice its matching side.
Step 5: equal - yes, $k = 2$	One common ratio: similar.
Step 6: $k = 1$? no	The sizes differ.
Output: **similar, but not congruent**	An enlargement with scale factor 2.

Example 3 — with help

Triangle 1 has sides 5 cm, 12 cm, 13 cm and angles $23^\circ, 67^\circ, 90^\circ$. Triangle 2 has sides 5 cm, 12 cm, 14 cm and angles $20^\circ, 57^\circ, 103^\circ$. Trace the sorting algorithm and state its output.

Working	Comment
Step 2: pairs are 5 with 5, 12 with 12, 13 with 14	Size order again.
Step 3: smallest angles give $23^\circ \neq 20^\circ$ - no	One unequal pair of matching angles is enough.
Output: **not similar and not congruent**	The algorithm stops at the first decision; no ratios are needed.

Example 4 — on your own

A careless sorting rule says: *"If 3 or more of the 6 matching parts are equal, the triangles are congruent."*

(a) Show that the rule gives the answer *congruent* for an equilateral triangle of side 3 cm and an equilateral triangle of side 5 cm.

(b) Explain why that answer is wrong, giving the correct output.
(c) State the fix in one sentence.

Solution.

(a) Every angle of each equilateral triangle is 60°, so the three angle pairs are equal: $60^\circ = 60^\circ$ three times. The rule counts 3 equal parts, 3 is at least 3, so it outputs *congruent*.

(b) The sides differ (3 cm against 5 cm), so one triangle cannot lie exactly on the other - they are not congruent. The correct output is *similar, not congruent*: the matching angles are equal and every side of the larger is $\frac{5}{3}$ times its matching side. The failing case proves the rule faulty.

(c) Count **which** parts match, not **how many**: congruence needs SSS, SAS, ASA or RHS, because equal angles alone give similarity, never congruence.

Example 5 — on your own

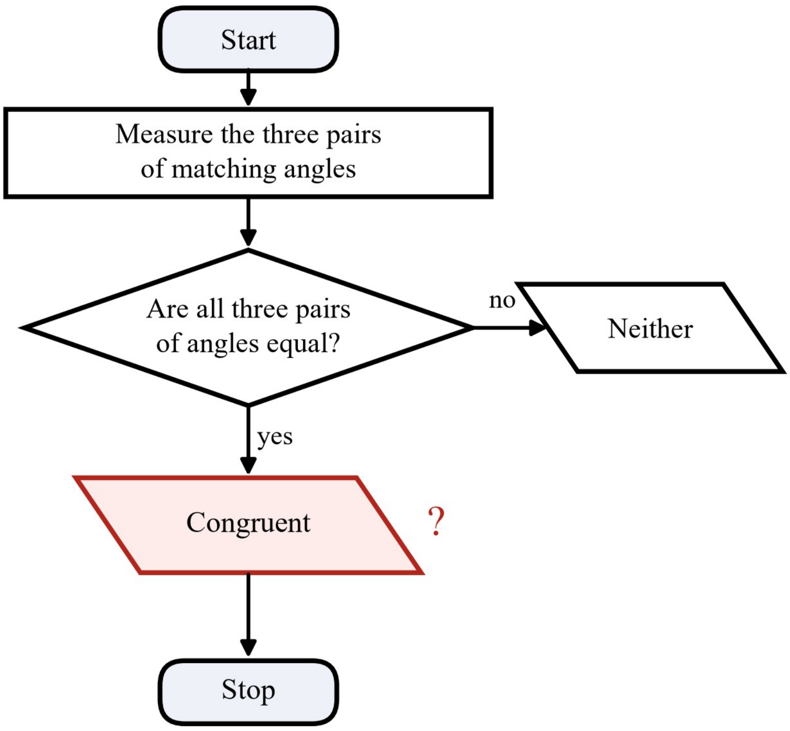
Use the rectangles branch of the shape sorter on a rectangle 10 cm by 6 cm and a rectangle 15 cm by 9 cm. State the output and the scale factor.

Solution.

Length-to-width ratios: $10:6 = 5:3$ and $15:9 = 5:3$. The ratios are the same, so the first decision is yes. Same length and same width? $15 \neq 10$ (and $9 \neq 6$) - no. Output: **similar, not congruent**. Scale factor $= 15 \div 10 = 1.5$ (check: $6 \times 1.5 = 9$).

Example 6 — on your own

Here is a faulty flow chart for sorting two triangles.



Faulty flow chart: from Start, a step box measures the three pairs of matching angles; a decision diamond asks whether all three pairs of angles are equal; no leads to an output box Neither; yes leads straight to an output box Congruent, marked with a red question mark; then Stop.

(a) Run the faulty chart on a 3-4-5 triangle and its copy at twice the size (6 cm, 8 cm, 10 cm). What does it output, and why is that wrong?

(b) Describe the steps that must be inserted before the chart can be trusted, so that it ends with the three correct outputs.

Solution.

(a) Both triangles have angles $37^\circ, 53^\circ, 90^\circ$, so the diamond says yes and the chart outputs *congruent* - wrong: the sides differ, so the correct output is *similar, but not congruent*. The chart makes exactly the counting-rule mistake: equal angles alone give similarity, never congruence.

(b) After the yes branch, insert: work out the three ratios of matching sides; if they are not equal, output *not similar and not congruent*; if they are equal, call the ratio k, and decide *is* $k = 1$*?* - yes gives *congruent (and similar)*, no gives *similar, but not congruent*. That is the lower half of the designed flow chart.

Practice questions

Work these questions by hand. No calculator, ruler or protractor is needed for any question - the algorithm does the sorting, and you do the reasoning. Where a question asks for a reason, the reason carries marks.

With help

Q1. The algorithm matches parts in size order. One triangle has sides 7 cm, 3 cm and 5 cm; the other has sides 5 cm, 7 cm and 3 cm. Write the three pairs of matching sides.

Q2. True or false? Give a reason for each answer.

a. If the three pairs of matching angles are not equal, the triangles cannot be congruent.
b. If the three ratios of matching sides are all 1, the triangles are similar as well as congruent.
c. Two triangles with three equal pairs of angles must be congruent.

Q3. Trace the algorithm on triangles with sides 6 cm, 8 cm, 10 cm and 8 cm, 6 cm, 10 cm, both with angles $37^\circ, 53^\circ, 90^\circ$. State the output.

Q4. A pair of triangles has sides 4 cm, 5 cm, 6 cm and 8 cm, 10 cm, 12 cm, with equal matching angles. Work out the three ratios and state the algorithm's output, including the value of k.

Q5. In the flow chart, which decision sends the pair to *neither*: a triangle with sides 3 cm, 4 cm, 5 cm (angles $37^\circ, 53^\circ, 90^\circ$) against a triangle with sides 3 cm, 4 cm, 6 cm (angles $26^\circ, 36^\circ, 117^\circ$)? Give the reason the decision gives.

Q6. Use the squares branch of the shape sorter on squares of side 5 cm and 9 cm. State the output with a reason.

Q7. Use the circles branch on two circles, both of radius 4 cm. State the output with a reason.

Q8. Use the rectangles branch on a rectangle 6 cm by 4 cm and a rectangle 9 cm by 6 cm. State the output, with the two ratios you compare.

On your own

Q9. Trace the algorithm on triangles with sides 5 cm, 12 cm, 13 cm and 10 cm, 24 cm, 26 cm, both with angles $23^\circ, 67^\circ, 90^\circ$. Show every step and state the output.

Q10. One triangle is written with sides 7 cm, 24 cm, 25 cm and the other with sides 25 cm, 7 cm, 24 cm; both have angles $16^\circ, 74^\circ, 90^\circ$. Trace the algorithm and state the output.

Q11. One triangle has sides 4 cm, 5 cm, 6 cm with angles $41^\circ, 56^\circ, 83^\circ$; the other has sides 4 cm, 5 cm, 7 cm with angles $34^\circ, 44^\circ, 102^\circ$. Trace the algorithm and state the output.

Q12. Explain why step 2 of the algorithm matches sides in size order. Your explanation should say what can go wrong without this step.

Q13. A table of test cases labels the pair (3 cm, 4 cm, 5 cm) and (6 cm, 8 cm, 9 cm) as *similar*. Run the algorithm on this pair and decide whether the label is correct. Give the correct output.

Q14. Use the rectangles branch on a rectangle 12 cm by 8 cm and a rectangle 18 cm by 12 cm. State the output and the scale factor.

Q15. A classmate proposes the counting rule of Example 4 as a shortcut inside the algorithm. Use the two equilateral triangles of sides 3 cm and 5 cm to show that the shortcut mislabels a pair, and name the kind of mistake it makes.

Q16. Describe, in three or four sentences of your own, how the triangle-sorting algorithm works. Your description should say what the algorithm checks, in what order, and how it chooses between its three outputs.

Maths in real life

Q17. The Australian national flag is, by law, twice as long as it is wide. One flag is 1 m wide and another is 1.5 m wide, both made to the legal ratio.

a. Write the length of each flag.
b. Are the two flags similar? Are they congruent? Answer both, with reasons.

Source: Flags Act 1953 (Cth) - the flag's length is twice its width (ratio 2:1); retrieved 20 August 2026.

Q18. A photo is 10 cm wide and 15 cm tall.

a. It is enlarged to 30 cm wide and 45 cm tall. Is the enlargement similar to the original? Show the ratios you compare, and state the scale factor.
b. A different print is 30 cm wide and 40 cm tall. Is this print similar to the original? What would the photo look like?

Q19. A poster is 20 cm wide and 30 cm tall. It is enlarged so that the new width is 50 cm. Find the new height that keeps the poster similar to the original, and state the scale factor.

Q20. Two screens are compared. Screen A is 80 cm wide and 45 cm tall; screen B is 120 cm wide and 67.5 cm tall; screen C is 120 cm wide and 70 cm tall.

a. Show that screens A and B are similar, and state the scale factor.
b. Show that screen C is not similar to screen A, and say what a picture designed for screen A would look like on screen C.

Answers

Q1. 3 cm with 3 cm, 5 cm with 5 cm, 7 cm with 7 cm.

Q2. a. True. b. True. c. False.

Q3. Congruent (and similar).

Q4. Ratios 2, 2, 2; output similar, but not congruent, with $k = 2$.

Q5. The first decision (the angles): $37^\circ \neq 26^\circ$, so no - neither.

Q6. Similar, not congruent: the sides differ.

Q7. Congruent: equal radii.

Q8. Similar, not congruent: both ratios are 3:2.

Q9. Similar, but not congruent ($k = 2$).

Q10. Congruent (and similar).

Q11. Not similar and not congruent.

Q12. Matching by size pairs each side with its true matching part; without it, equal ratios can be missed and a similar pair can be wrongly labelled neither.

Q13. The label is wrong: the correct output is neither.

Q14. Similar, not congruent; scale factor 1.5.

Q15. The rule says congruent (3 equal angle pairs), but the pair is similar, not congruent; it counts how many parts match instead of which parts match.

Q16. See fully-worked solutions.

Q17. a. 2 m and 3 m. b. Similar, yes; congruent, no.

Q18. a. Similar, scale factor 3. b. Not similar; stretched.

Q19. 75 cm; scale factor 2.5.

Q20. a. Similar, scale factor 1.5. b. Not similar; stretched.

Fully-worked solutions

Q1. Sort each triangle's sides: 3 cm, 5 cm, 7 cm and 3 cm, 5 cm, 7 cm. The matching pairs are 3 cm with 3 cm, 5 cm with 5 cm, and 7 cm with 7 cm. Size order ignores the order the sides were written in - which is exactly its job.

Q2. Apply the algorithm's decisions.

a. **True.** The first decision sends any pair with unequal matching angles to *neither*, and congruent triangles have equal matching angles.
b. **True.** Ratios all 1 mean every side equals its matching side - congruent - and every congruent pair is similar with scale factor 1.
c. **False.** Equal angles give similarity, never congruence: equilateral triangles of sides 3 cm and 5 cm have three equal angle pairs but different sides.

Q3. Step 2 pairs 6 with 6, 8 with 8, 10 with 10 (size order undoes the scrambled writing). Step 3: the angle pairs are equal - yes. Step 4: $6 \div 6 = 1$, $8 \div 8 = 1$, $10 \div 10 = 1$. Step 5: yes, $k = 1$. Step 6: yes. Output: **congruent (and similar)**.

Q4. $8 \div 4 = 2$, $10 \div 5 = 2$, $12 \div 6 = 2$. The ratios are equal, so the triangles are similar with $k = 2$; and $k \neq 1$, so the output is **similar, but not congruent**.

Q5. The **first** decision, the angles. The smallest angles give $37^\circ \neq 26^\circ$ (and the largest give $90^\circ \neq 117^\circ$), so the answer is no and the output is *not similar and not congruent*. The pair never reaches the ratio step.

Q6. Same side length? $5 \neq 9$ - no. Output: **similar, not congruent**, because any two squares are similar (four right angles and four equal sides in both).

Q7. Same radius? $4 = 4$ - yes. Output: **congruent**, because equal radii give equal circles - one lies exactly on the other.

Q8. Length-to-width ratios: $6:4 = 3:2$ and $9:6 = 3:2$ - the same, so yes. Same length and same width? No. Output: **similar, not congruent**.

Q9. Step 2: 5 with 10, 12 with 24, 13 with 26. Step 3: $23^\circ = 23^\circ$, $67^\circ = 67^\circ$, $90^\circ = 90^\circ$ - yes. Step 4: $10 \div 5 = 2$, $24 \div 12 = 2$, $26 \div 13 = 2$. Step 5: equal - yes, $k = 2$. Step 6: $k = 1$? no. Output: **similar, but not congruent**.

Q10. Step 2 sorts both triangles to 7 cm, 24 cm, 25 cm, pairing like with like. Step 3: the angle pairs are equal - yes. Step 4: $7 \div 7 = 1$, $24 \div 24 = 1$, $25 \div 25 = 1$. Step 5: yes, $k = 1$. Step 6: yes. Output: **congruent (and similar)** - the scrambled writing changed nothing, because size order does the matching.

Q11. Step 3 fails first: the smallest angles give $41^\circ \neq 34^\circ$. Output: **not similar and not congruent**. (The ratios would fail too: $4 \div 4 = 1$, $5 \div 5 = 1$, but $7 \div 6 \neq 1$.)

Q12. Matching parts are the parts opposite equal angles, and in any triangle the shortest side lies opposite the smallest angle. Sorting by size therefore pairs each part with its true partner. Without this step, the ratios can be computed from the wrong pairing: for a 3-4-5 triangle against a 6-8-10 triangle written as (5, 3, 4) and (6, 8, 10), dividing in written order gives $6 \div 5 = 1.2$, $8 \div 3 \neq 1.2$, $10 \div 4 \neq 1.2$ - unequal ratios, and a truly similar pair would be mislabelled *neither*.

Q13. Step 2: 3 with 6, 4 with 8, 5 with 9. Step 3: the angles of (3, 4, 5) are $37^\circ, 53^\circ, 90^\circ$; the angles of (6, 8, 9) are about $41^\circ, 61^\circ, 79^\circ$ - not equal, so no. Output: **not similar and not congruent**. The label *similar* is wrong. (The ratios agree with this: $6 \div 3 = 2$, $8 \div 4 = 2$, but $9 \div 5 = 1.8$ - unequal.)

Q14. Ratios: $12:8 = 3:2$ and $18:12 = 3:2$ - the same, yes. Same length and width? No. Output: **similar, not congruent**, with scale factor $18 \div 12 = 1.5$ (check $8 \times 1.5 = 12$).

Q15. Each equilateral triangle has three 60° angles, so the counting rule finds 3 equal parts and outputs *congruent*. But the sides differ (3 cm against 5 cm), so the correct output is *similar, not congruent*. The mistake is counting **how many** parts match instead of checking **which** parts match - equal angles alone give similarity, never congruence.

Q16. The algorithm first matches the parts of the two triangles in size order, so that each side and angle is compared with its true matching part. It then checks the three pairs of matching angles, because the angles are the shape: if they differ, the pair is neither similar nor congruent and the algorithm stops. If the angles agree, it computes the three ratios of matching sides; a single common ratio k means one triangle is an enlargement of the other, so the pair is similar, while unequal ratios mean neither. Finally it asks whether $k = 1$: scale factor 1 means same size as well as same shape, so the pair is congruent; otherwise it is similar but not congruent.

Q17. a. Length is twice width: $1 \times 2 = 2\,\text{m}$ and $1.5 \times 2 = 3\,\text{m}$.

b. Similar, yes: length-to-width ratios are $2:1$ and $3:1.5 = 2:1$, the same ratio (that is, every length is multiplied by 1.5). Congruent, no: the sizes differ ($3 \neq 2$), so the scale factor is 1.5, not 1.

Q18. a. Widths: $30 \div 10 = 3$; heights: $45 \div 15 = 3$. One common ratio, so the enlargement is **similar**, scale factor 3. (Ratios of sides within each rectangle: $15:10 = 3:2$ and $45:30 = 3:2$.)

b. Widths: $30 \div 10 = 3$; heights: $40 \div 15 \approx 2.7$. The ratios differ, so the print is **not similar** to the original - the photo would look stretched, because the width and height were multiplied by different numbers.

Q19. Scale factor $= 50 \div 20 = 2.5$. New height $= 30 \times 2.5 = 75\,\text{cm}$. (Check the within-poster ratios: $30:20 = 3:2$ and $75:50 = 3:2$.)

Q20. a. Widths: $120 \div 80 = 1.5$; heights: $67.5 \div 45 = 1.5$. One common ratio, so A and B are **similar**, scale factor 1.5.

b. Widths: $120 \div 80 = 1.5$; heights: $70 \div 45 \approx 1.6$. The ratios differ, so C is **not similar** to A. A picture made for screen A would look slightly stretched vertically on screen C, because the height was multiplied by a different number from the width.

Position and location in three dimensions

Content description `VC2M8SP03`: describe in different ways the position and location of three-dimensional objects in 3 dimensions, including using a three-dimensional Cartesian coordinate system with the use of dynamic geometry software or other digital tools

A map tells you where a building is, but not how tall it is. A photo tells you what a drone looks like, but not how high it is flying. Two numbers were never going to be enough for a three-dimensional world. This subtopic adds the third number — the one that says how high — and with it a whole coordinate system that can pin down anything from a car park bay to an aircraft.

Theory

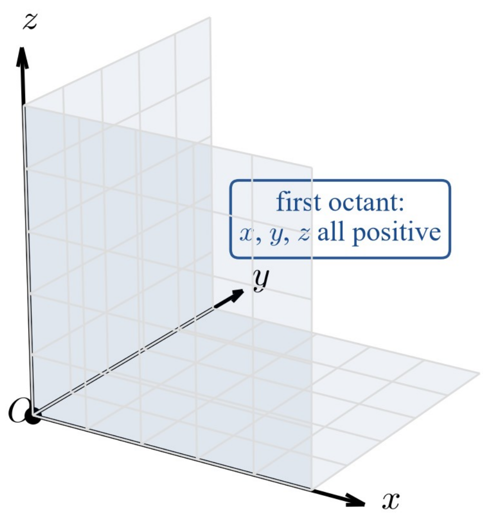
Recall: two dimensions first

In Year 7 and in subtopic 2B of this book you met the **Cartesian coordinate system** in two dimensions (`VC2M7A05`): two number lines, the x-axis and the y-axis, crossing at right angles at the **origin** O, with every point located by an ordered pair such as $(3,4)$ — 3 across along x, then 4 up parallel to y. This subtopic keeps all of that and adds one more axis.

The three-dimensional Cartesian coordinate system

Add a third axis, the z-axis, through the origin at right angles to both the x-axis and the y-axis. Now every point in space is located by an **ordered triple** — three numbers in a fixed order, written (x,y,z):

- the first number: how far along the x-axis,
- the second number: how far across, parallel to the y-axis,
- the third number: how far up, parallel to the z-axis.

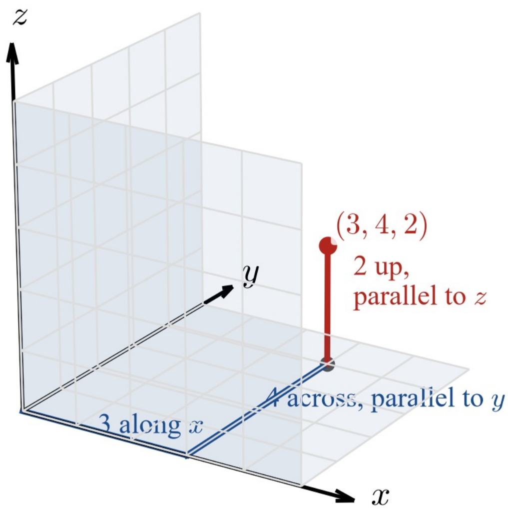


The three-dimensional Cartesian coordinate system: three axes x, y and z meeting at right angles at the origin O, with a floor grid and two back walls shading the corner of space where x, y and z are all positive, labelled the first octant

This book works only in the corner of space shown in the diagram, where x, y and z are all positive (or zero). Space is cut into eight such corners by the three axes, and each corner is called an **octant** (think "oct", as in octopus: eight). The corner where all three coordinates are positive is the **first octant**. (Some materials, including one VCAA elaboration, borrow the 2D word and call it the "first quadrant" of 3D space; this book uses the standard 3D word, octant.)

Plotting a point

To plot $(3,4,2)$: start at the origin, go 3 along x, then 4 across parallel to y, then 2 up parallel to z.

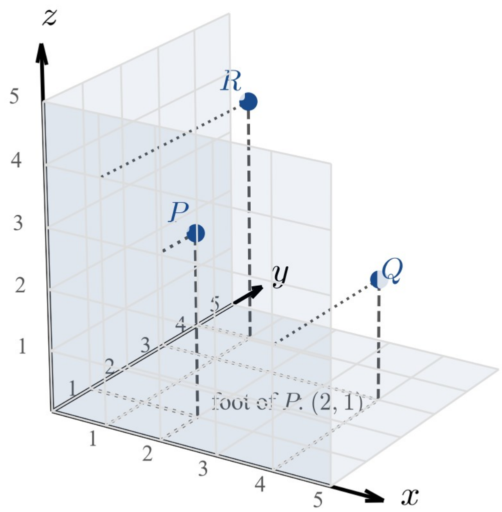


Plotting the point (3, 4, 2): a blue path runs 3 along the x-axis, then 4 across parallel to the y-axis to the point's foot on the floor, then a red line rises 2 up parallel to the z-axis to the plotted point

The dashed drop-line in the diagram shows the point's **foot** — the point $(3, 4, 0)$ on the floor directly below it. The foot needs only two numbers, exactly like a 2D point; the third number is the height of the point above its foot.

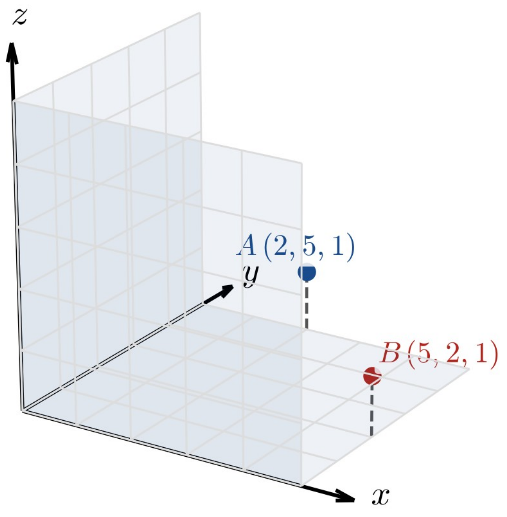
Reading coordinates, and why order matters

Reading is plotting in reverse: drop straight down to the floor to find the first two numbers, then read the height. In the next diagram, P is read for you as a model — its foot sits at $(2, 1)$ on the floor and its height is 3, so P is $(2, 1, 3)$.



Three points P, Q and R plotted in the first octant, each with a dashed drop-line to the floor, dotted lines from the foot to the x- and y-axes, a dotted level line to the back wall, and numbered ticks on all three axes; P is labelled with its foot (2, 1) as the worked model

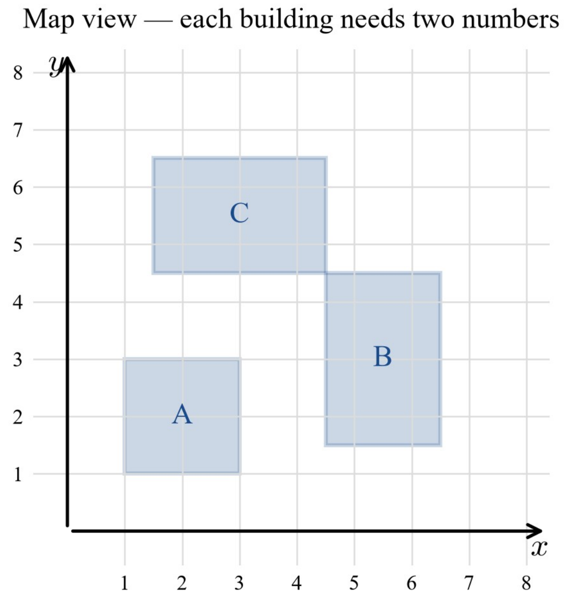
The order of the three numbers is part of the mathematics. Swapping numbers moves the point: $(2, 5, 1)$ and $(5, 2, 1)$ are two different points, as the diagram shows.

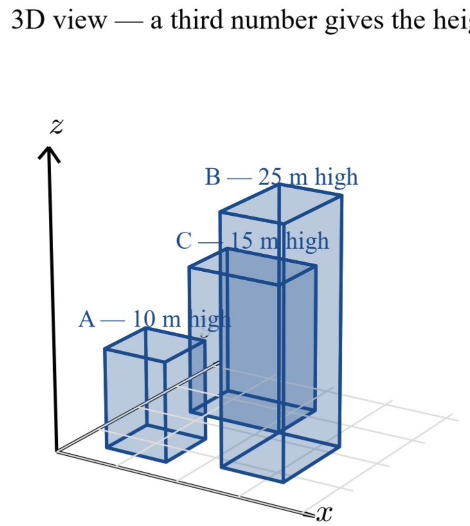


Two different points in the first octant, A (2, 5, 1) and B (5, 2, 1), each with a dashed drop-line to the floor, showing that swapping the first two coordinates gives a different point

Same and different: 2D and 3D

Look at the same small city two ways. On a flat map, each building is located by two numbers. In a 3D view, a third number records the height.



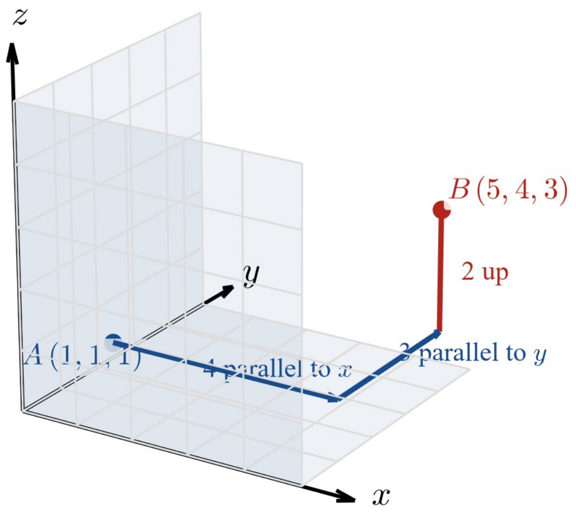


The same three buildings shown twice: left, a flat map view with x and y axes where each building A, B, C is a rectangle located by two numbers; right, a 3D view with x, y and z axes where a third number gives each height, A 10 m high, B 25 m high, C 15 m high

Same as 2D: perpendicular axes meeting at an origin; numbers written in a fixed order; the same plotting idea (along, across, then up). **Different from 2D:** three axes and three numbers instead of two; the third number gives height; and space has eight octants where the plane had four quadrants. The same comparison is why a flat virtual map and a street-level view of a city feel different: the flat view drops the third number, and the street view puts you inside it.

Describing moves parallel to the axes

A position can also be described by the moves that get you there, each move parallel to one axis. From $A(1,1,1)$ to $B(5,4,3)$: move 4 parallel to x, then 3 parallel to y, then 2 up.

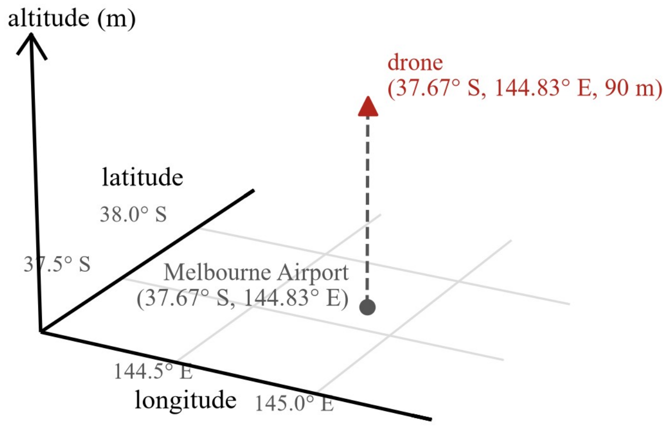


The move from A (1, 1, 1) to B (5, 4, 3) drawn as three arrows: a blue arrow 4 units parallel to the x-axis, a blue arrow 3 units parallel to the y-axis, and a red arrow 2 units up parallel to the z-axis

Check: $1+4=5$, $1+3=4$ and $1+2=3$, giving $B(5,4,3)$. Each move changes exactly one of the three numbers.

Position in the real world: latitude, longitude and altitude

Aircraft and drones are located the same way, with three numbers that are not all in the same units: **latitude** (how far south or north, in degrees), **longitude** (how far east or west, in degrees) and **altitude** (height, in metres).



A ground grid labelled in degrees of longitude east and degrees of latitude south with a vertical altitude axis in metres; Melbourne Airport is marked on the ground at 37.67 degrees south, 144.83 degrees east, and a red drone marker sits 90 m straight above it, labelled drone (37.67 degrees S, 144.83 degrees E, 90 m)

The drone in the diagram is directly above Melbourne Airport, so it keeps the airport's latitude and longitude and adds a height: (37.67° S, 144.83° E, 90 m). Here are the real coordinates of four Victorian stations:

Station	Latitude	Longitude	Elevation above sea level
Melbourne Airport	37.67° S	144.83° E	113 m
Moorabbin Airport	37.98° S	145.10° E	12 m
Mildura Airport	34.24° S	142.09° E	50 m
Mount Buller	37.15° S	146.44° E	1707 m

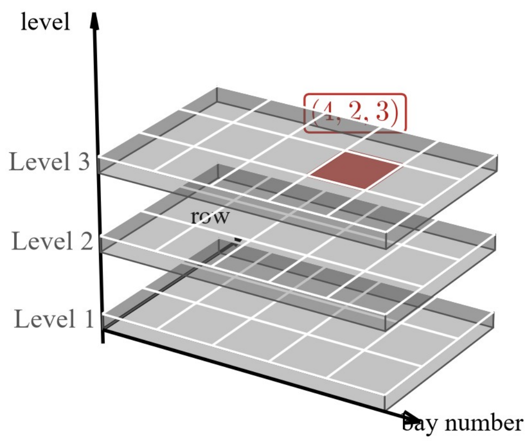
Source: Bureau of Meteorology, climate statistics for Australian locations, station pages 086282, 086077, 076031 and 083024; retrieved 20 August 2026.

Notice two things. Smaller south-latitudes are further north: Mildura, at 34.24° S, is the furthest north of the four. And compare the units: latitude and longitude are measured in degrees while altitude is measured in metres, whereas in the Cartesian system all three numbers are counted in the same units. Both are three-number descriptions of position — the same big idea.

Everyday codes for 3D position

Plenty of everyday systems locate things with three numbers, each system choosing its own order and its own words.

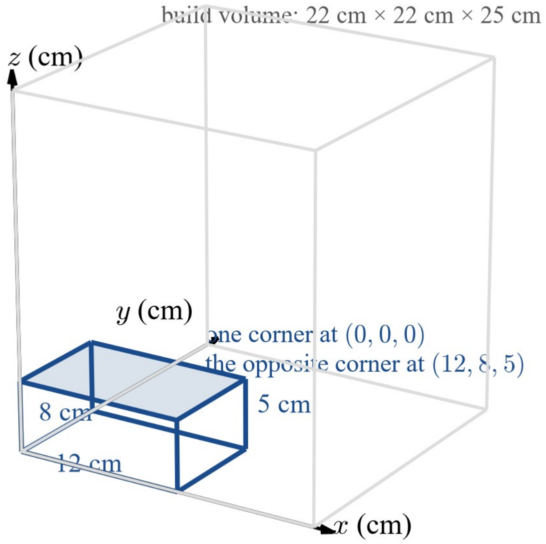
A multistorey car park. Your bay is fixed by its **bay number** along one direction, its **row** across, and its **level** up. This book writes the triple as (bay, row, level).



A car park with three levels drawn as three stacked grids of 5 bays by 3 rows; the levels are numbered 1, 2, 3 on a vertical level axis, the other two axes are labelled bay number and row, and one bay on level 3 is coloured red and labelled (4, 2, 3)

The red bay is bay 4 in row 2 on level 3, written $(4, 2, 3)$.

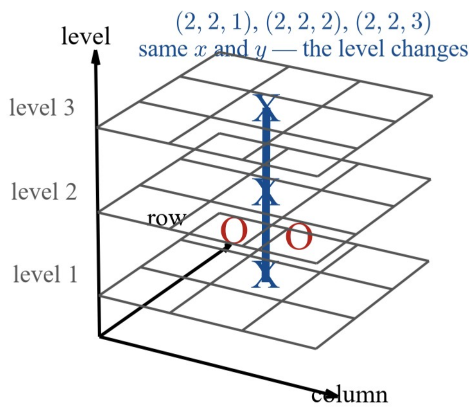
A 3D printer. The printer's build volume is a box measured in centimetres along x, along y, and up z. A printed object is pinned down by the coordinates of its corners.



A 3D printer build volume drawn as a faint wireframe box 22 cm by 22 cm by 25 cm with x, y and z axes in centimetres; inside it at the corner, a blue printed box 12 cm by 8 cm by 5 cm with one corner at (0, 0, 0) and the opposite corner at (12, 8, 5)

The printed box has one corner at $(0,0,0)$ and the opposite corner at $(12,8,5)$: it reaches 12 cm along x, 8 cm along y and 5 cm up — comfortably inside the 22 cm × 22 cm × 25 cm build volume.

Three-dimensional noughts and crosses. The grid is 3 × 3 × 3: a column, a row and a level. A winning line is three in a row in any direction — across a layer, or straight up through the layers.



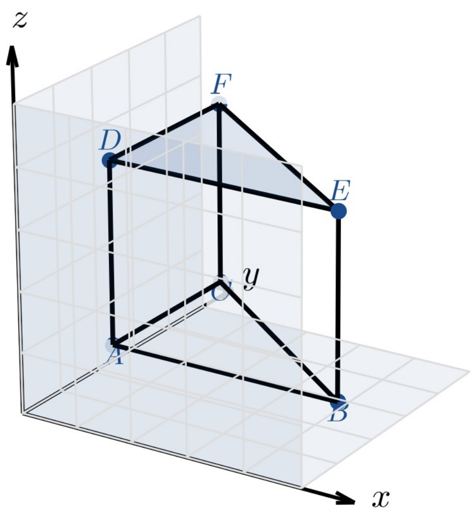
Three-dimensional noughts and crosses: three stacked 3 by 3 grids labelled level 1, level 2, level 3 on a vertical axis with column and row axes below; X marks the centre cell of every level with a thick blue winning line through the three X's, labelled (2, 2, 1), (2, 2, 2), (2, 2, 3), same x and y, the level changes, and two O's sit in other cells

The three X's are at $(2,2,1)$, $(2,2,2)$ and $(2,2,3)$ — same column, same row, level changing: a vertical winning line.

Digital tools are part of the job

The content description says it plainly: 3D coordinates are meant to be used with **dynamic geometry software or other digital tools**. On a screen, the first octant is drawn just like the diagrams in this subtopic, and you can rotate the whole scene — a point that looks hidden behind another from one direction is clearly separate from another. This is also how design software works: a 3D printer's model, a game world, a drone's flight path are all lists of coordinates.

A solid is fixed by listing its **vertices** (corner points). The triangular **prism** below — a solid with two identical triangle faces joined by rectangles — is completely described by six triples: $A(1,1,1)$, $B(5,1,1)$, $C(1,4,1)$, $D(1,1,4)$, $E(5,1,4)$, $F(1,4,4)$.



A triangular prism in the first octant with its six vertices labelled: A (1, 1, 1), B (5, 1, 1) and C (1, 4, 1) on the bottom triangle, and D (1, 1, 4), E (5, 1, 4), F (1, 4, 4) directly above them

You will build points and solids like this in software in Q11. Rotating the view is also the best check that you have read or plotted a point correctly.

Putting it together

One point, three numbers

A position in three dimensions is written as an ordered triple (x, y, z).
x along the x-axis, then y parallel to y, then z up, parallel to the z-axis.
Order matters: $(2, 5, 1) \neq (5, 2, 1)$.

Same as 2D — and different

Same: perpendicular axes from an origin; numbers in a fixed order.
Different: three axes and three numbers; height z; eight octants — this book works in the first octant, where x, y, z are positive.

Position in the real world

Aircraft and drones: latitude, longitude, altitude (degrees, degrees, metres).
Multistorey car park: bay, row, level.
3D printer: centimetres along the plate and centimetres of height.

Digital tools are part of the job

Build points and solids in 3D graphing or dynamic geometry software; rotate the view and read coordinates from any direction.
A solid is fixed by listing its vertices — six triples fix a triangular prism.

Four summary cards. Card 1, one point three numbers: a position in three dimensions is an ordered triple (x, y, z), along x then across parallel to y then up parallel to z, and order matters because (2, 5, 1) is not (5, 2, 1). Card 2, same as 2D and different: perpendicular axes from an origin and numbers in a fixed order, but three axes, a height, and eight octants, with this book working in the first octant. Card 3, position in the real world: aircraft and drones use latitude, longitude, altitude; a car park uses bay, row, level; a 3D printer uses centimetres along the plate and of height. Card 4, digital tools are part of the job: build points and solids in 3D graphing or dynamic geometry software, rotate the view, and a solid is fixed by listing its vertices

Words of this subtopic

Word	What it means here
Cartesian coordinate system	a coordinate system of perpendicular number lines (axes) meeting at an origin; in 3D there are three axes, x , y and z
origin	the point O where the axes meet, $(0,0,0)$ in 3D
axis (axes)	a number line of the coordinate system; the three axes of 3D are the x-axis, y-axis and z-axis
coordinates / ordered triple	the three numbers (x, y, z) that locate a point, written in a fixed order
octant / first octant	one of the eight corners of space cut out by the three axes; the first octant is where x, y and z are all positive
vertex (vertices)	a corner point of a solid; a solid is fixed by listing the coordinates of its vertices
prism	a solid with two identical parallel faces (its ends) joined by rectangles
cuboid	a box-shaped solid: six rectangle faces meeting at right angles
latitude	how far south or north a place is, measured in degrees (°

Word	What it means here
	S or ° N)
longitude	how far east or west a place is, measured in degrees (° E or ° W)
altitude / elevation	height above the ground, or above sea level, measured in metres
dynamic geometry software	screen software that draws and lets you drag or rotate shapes and coordinate systems; a 3D graphing tool does the same for coordinates

Worked examples

Example 1 — with help

Use the reading diagram from the theory. P is $(2,1,3)$. Read the coordinates of Q and R.

Working	Comment
Q: drop to the floor; the foot is 4 along x and 3 across y	The first two numbers come from the foot.
height of Q is 2	The third number is the height above the foot.
$Q=(4,3,2)$	Three numbers, in the fixed order (x,y,z).
R: foot is 1 along x and 4 across y; height is 4	Read R the same way.
$R=(1,4,4)$	Here the second and third numbers happen to be equal — that is allowed.

Example 2 — with help

Plot the point $(5,2,4)$ in the first octant. Describe each step.

Working	Comment
Start at the origin O.	Every point is located from O.
Move 5 along the x-axis.	The first number.
Move 2 across, parallel to the y-axis.	The second number; you are now at the foot $(5,2,0)$.
Move 4 up, parallel to the z-axis, and plot the point.	The third number is the height.

Example 3 — with help

Use the car-park diagram, with triples written as (bay, row, level).

(a) Write the triple for the bay in **bay 6, row 3, level 2**.
(b) Describe the red bay $(4,2,3)$ in words.

Working	Comment
(a) $(6,3,2)$	The words go into the triple in the fixed order (bay, row, level).
(b) Bay 4, row 2, level 3	Reading a triple back is the same order in reverse.

Example 4 — on your own

A drone hovers directly above Moorabbin Airport at a height of 120 m. Using the table in the theory, write the drone's position as (latitude, longitude, height).

Solution.

The drone is directly above the airport, so it has the airport's latitude and longitude: 37.98° S and 145.10° E. Its height is 120 m.

∴ the drone's position is (37.98° S, 145.10° E, 120 m).

Example 5 — on your own

Start at the point $(2,1,3)$. Move 4 parallel to the x-axis, then 3 parallel to the y-axis, then 2 **down**. Find the coordinates of the point you reach.

Solution.

Each move changes exactly one number: $2+4=6$, then $1+3=4$, then $3-2=1$.

∴ the point you reach is $(6,4,1)$.

Example 6 — on your own

A 3D printer has a build volume 22 cm × 22 cm × 25 cm. A model box is printed with one corner at $(0,0,0)$ and the opposite corner at $(12,8,5)$. Explain, using coordinates, why the box fits inside the build volume.

Solution.

The box reaches from 0 to 12 along x, from 0 to 8 along y, and from 0 to 5 up. Compare each reach with the build volume: $12\le 22$, $8\le 22$ and $5\le 25$. Every coordinate of every corner of the box lies inside the build volume.

∴ the box fits, with room to spare in all three directions.

Practice questions

Work these questions by hand unless a question says otherwise. Q11 needs access to dynamic geometry software or a 3D graphing tool — a digital tool is part of this subtopic. No calculator is needed for any question.

With help

Q1. Write each description as an ordered triple (x,y,z).

a. 3 along x, 2 across, 5 up.
b. 1 along x, 0 across, 4 up.
c. the origin.

Q2. The diagram in "Reading coordinates" shows $A(2,5,1)$ and $B(5,2,1)$. Explain in a sentence why they are different points, even though they use the same three numbers.

Q3. True or false? Give a reason.

a. The point $(0,0,4)$ lies on the z-axis.
b. The point $(3,0,0)$ lies on the y-axis.
c. Every point in the first octant has three positive coordinates.

Q4. Copy and complete this table comparing the 2D and 3D Cartesian systems.

	2D (the plane)	3D (space)
Number of axes	2	
Numbers needed for one point	2	
Name of a region where all coordinates are positive	first quadrant	
What the last number gives	height above the x-axis	

Q5. Use the car-park triple order (bay, row, level).

a. Write the triple for the bay in bay 5, row 2, level 1.
b. Describe the bay $(3, 1, 2)$ in words.

Q6. Start at $(1, 2, 1)$. Move 3 parallel to the x-axis and then 2 up. Write the coordinates of the point you reach.

Q7. Use the table of stations in the theory.

a. Which station is the furthest north?
b. Which station has the greatest elevation above sea level?

Q8. A drone hovers directly above Melbourne Airport at a height of 90 m. Write its position as (latitude, longitude, height).

On your own

Q9. Write the coordinates of the point directly above $(2, 1, 3)$ at height 5.

Q10. Sam says $(4, 3, 2)$ and $(3, 4, 2)$ must be the same point, "because they use the same numbers". Explain why Sam is wrong.

Q11. *You will need dynamic geometry software or a 3D graphing tool.* Plot the points $(1, 1, 1)$, $(4, 1, 1)$, $(4, 3, 1)$, $(1, 3, 1)$, $(1, 1, 3)$, $(4, 1, 3)$, $(4, 3, 3)$ and $(1, 3, 3)$, and join them to form a cuboid. Rotate the view.

a. What are the cuboid's lengths in the x, y and z directions?
b. Write the coordinates of the vertex opposite $(1, 1, 1)$.

Q12. Describe the move from $A(1, 1, 1)$ to $B(5, 4, 3)$ in words, as in the moves diagram.

Q13. Write Mildura Airport's position from the theory table as a triple (latitude, longitude, elevation).

Q14. The point $(6, 0, 2)$ has $y = 0$. Describe where it sits in the first octant (on the floor, on a wall, or on an axis).

Q15. In 3D noughts and crosses, explain why $(2, 2, 1)$, $(2, 2, 2)$ and $(2, 2, 3)$ form a winning line.

Q16. A printed box has one corner at $(0, 0, 0)$ and the opposite corner at $(9, 6, 4)$. Write its dimensions in centimetres.

Maths in real life

Q17. A helicopter hovers 250 m above the ground at Mount Buller. Using the theory table, write its position as (latitude, longitude, height).

Q18. A city car park has 4 levels; each level has 6 rows of bays; each row has 10 bays. Use the triple order (bay, row, level).

a. How many bays does the car park have?
b. Write the triple for the last bay on the top row of the top level.

Q19. A smaller 3D printer has a build volume 18 cm × 18 cm × 16 cm. A model box measures 20 cm × 10 cm × 9 cm. Can the box be printed on this printer, in any orientation? Give reasons.

Q20. In 3D noughts and crosses your opponent has O at $(1, 3, 1)$ and $(1, 3, 2)$, and is one move from a vertical winning line. Where must you play to block it?

Answers

Q1. a. $(3,2,5)$. b. $(1,0,4)$. c. $(0,0,0)$.

Q2. The order is part of the mathematics: in A the 5 is the y-coordinate, in B it is the x-coordinate, so the points sit in different places.

Q3. a. True. b. False. c. True.

Q4. 3; 3; first octant; height above the xy-floor.

Q5. a. $(5,2,1)$. b. Bay 3, row 1, level 2.

Q6. $(4,2,3)$.

Q7. a. Mildura Airport. b. Mount Buller.

Q8. (37.67° S, 144.83° E, 90 m).

Q9. $(2,1,5)$.

Q10. In $(4,3,2)$ the foot is at $(4,3)$; in $(3,4,2)$ the foot is at $(3,4)$ — different feet, different points.

Q11. a. 3 in x, 2 in y, 2 in z. b. $(4,3,3)$.

Q12. 4 parallel to x, then 3 parallel to y, then 2 up.

Q13. (34.24° S, 142.09° E, 50 m).

Q14. On the back wall — the wall that contains the x-axis and the z-axis.

Q15. Same column and same row, with the level changing 1, 2, 3: three in a row straight up.

Q16. 9 cm × 6 cm × 4 cm.

Q17. (37.15° S, 146.44° E, 250 m).

Q18. a. 240. b. $(10,6,4)$.

Q19. No.

Q20. $(1,3,3)$.

Fully-worked solutions

Q1. Read each description straight into the fixed order (x,y,z).

a. $(3,2,5)$.
b. $(1,0,4)$ — a zero is a perfectly good coordinate: this point sits on the wall containing the x- and z-axes.
c. The origin is $(0,0,0)$: no move in any direction.

Q2. Coordinates are an *ordered* triple. $A(2,5,1)$ has foot $(2,5)$ while $B(5,2,1)$ has foot $(5,2)$: the 2 and the 5 have swapped which axis they belong to, so the points sit in different places in the diagram.

Q3. Apply the meaning of each coordinate.

a. **True.** $(0,0,4)$ makes no move along x or y and rises 4: that is on the z-axis.
b. **False.** $(3,0,0)$ moves 3 along x and not at all otherwise: it lies on the x-axis, not the y-axis.
c. **True.** The first octant is exactly the corner where x, y and z are all positive.

Q4. Adding the third axis changes every row except the idea of an origin.

	2D (the plane)	3D (space)
Number of axes	2	3
Numbers needed for one point	2	3
Name of a region where all coordinates are positive	first quadrant	first octant
What the last number gives	height above the x-axis	height above the $x\,y$-floor

Q5. Use the fixed order (bay, row, level).

a. Bay 5, row 2, level 1 gives $(5,2,1)$.
b. $(3,1,2)$ reads back as bay 3, row 1, level 2.

Q6. One move changes one number: $1+3=4$ along x, y stays 2, and $1+2=3$ up. The point is $(4,2,3)$.

Q7. Read the table with the two rules from the theory.

a. Further north means a smaller south-latitude. Mildura's 34.24° S is the smallest, so Mildura Airport is furthest north.
b. Mount Buller, at 1707 m above sea level.

Q8. Directly above the airport, latitude and longitude are unchanged; the height is 90 m: (37.67° S, 144.83° E, 90 m) — the drone marked in the theory diagram.

Q9. "Directly above" keeps the first two numbers and changes only the height: $(2,1,5)$.

Q10. The first two numbers give the foot on the floor. $(4,3,2)$ has its foot at $(4,3)$ and $(3,4,2)$ has its foot at $(3,4)$ — two different floor positions, so two different points (just like A and B in the order diagram).

Q11. Build and rotate in the software; then read lengths off the coordinates.

a. x: from 1 to 4, a length of $4-1=3$; y: from 1 to 3, a length of 2; z: from 1 to 3, a length of 2. The cuboid is 3 × 2 × 2.
b. The opposite vertex takes the far value in all three directions: $(4,3,3)$. Rotating the view confirms it is the corner diagonally through the cuboid from $(1,1,1)$.

Q12. x goes from 1 to 5 (4 parallel to x), y from 1 to 4 (3 parallel to y), z from 1 to 3 (2 up) — exactly the three arrows in the moves diagram.

Q13. Copy the row in the fixed order (latitude, longitude, elevation): (34.24° S, 142.09° E, 50 m).

Q14. With $y=0$ there is no move away from the back wall, so the point sits on the wall that contains the x-axis and the z-axis: 6 along and 2 up on that wall. (It is not on an axis, because both x and z are non-zero.)

Q15. All three marks have column 2 and row 2; only the level changes, through 1, 2, 3. That is three cells in a straight line running vertically through the layers — a winning line.

Q16. The opposite corner gives the reaches: 9 along x, 6 along y, 4 up. The dimensions are 9 cm × 6 cm × 4 cm.

Q17. Above the ground at Mount Buller, latitude and longitude are the station's: 37.15° S and 146.44° E; the height is 250 m. The position is (37.15° S, 146.44° E, 250 m).

Q18. Count and then code.

a. $4\times6\times10=240$ bays — one for every choice of level, row and bay.
b. Top level is 4, top row is 6, last bay is 10: in the order (bay, row, level), $(10,6,4)$.

Q19. Try every orientation. The longest side of the box, 20 cm, must lie along one of the printer's three directions, whose limits are 18 cm, 18 cm and 16 cm. But $20>18$ and $20>16$, so the 20 cm side fits in no direction.

$\therefore$ the box cannot be printed on this printer in any orientation.

Q20. The two O's share column 1 and row 3, with levels 1 and 2: the threatened winning line is the vertical line through $(1,3,1)$, $(1,3,2)$ and $(1,3,3)$. Playing $(1,3,3)$ blocks it.

Test

Mathematics Level 8 · Chapter 5 — Space: congruence, similarity and position in three dimensions · Chapter test T5

Marks	20 (5 marks for each subtopic)
Time	25 minutes: 5 minutes reading time, then about 20 minutes working time (1 minute per mark)
Calculators	Not permitted — none of the content descriptions of Chapter 5 names a calculator, so every item is to be worked out mentally or in writing. `VC2M8SP03` names dynamic geometry software — see Equipment.
Equipment	Ruler, protractor and pair of compasses (Question 3); dynamic geometry software or another 3D graphing tool (Question 12 — required by `VC2M8SP03`)

This is a classroom check, not an exam.

Instructions

- Answer every question, and show your working clearly.
- The marks for each question or part are shown in brackets.
- Where a question asks for a reason, the reason carries marks: a correct answer with no reason does not score full marks.
- Question 3 needs your ruler, pair of compasses and protractor. Question 12 is to be done in dynamic geometry software or a 3D graphing tool.

Subtopic	Content description	Marks
5A Conditions for congruence and similarity	`VC2M8SP01`	5
5B Properties of quadrilaterals proved from congruent triangles	`VC2M8SP02`	5
5C Algorithms that test for congruence and similarity	`VC2M8SP04`	5
5D Position and location in three dimensions	`VC2M8SP03`	5
Total		**20**

5A · Conditions for congruence and similarity

Q1. In $\triangle ABC$ and $\triangle DEF$, $AB = DE = 6\,\text{cm}$, $\angle B = \angle E = 90^\circ$, and $AC = DF = 10\,\text{cm}$. The **hypotenuse** of a right-angled triangle is the side opposite the right angle. Which test proves that the two triangles are congruent? (1 mark)

A SSS **B** SAS **C** ASA **D** RHS

Q2. Two triangles are similar: their matching angles are equal, and their matching sides are all multiplied by the same number, the **scale factor**. The sides of the smaller triangle are 4 cm, 6 cm and 9 cm. The shortest side of the larger triangle is 12 cm. (2 marks)

(a) Find the scale factor from the smaller triangle to the larger triangle. (1 mark)

(b) Find the longest side of the larger triangle. (1 mark)

Q3. You will need your ruler, your pair of compasses and your protractor. (2 marks)

(a) Draw a line segment AB of length 5 cm. With your compasses, draw an arc of radius 3 cm centred on A and an arc of radius 4 cm centred on B, both above the segment. Label the point where the arcs meet C, and join AC and BC. Measure $\angle ACB$ with your protractor and record it. (1 mark)

(b) A classmate constructs a triangle with the same three side lengths — 3 cm, 4 cm and 5 cm — starting in a different direction. Must their triangle be congruent to yours? Name the congruence test, and give the reason the test guarantees your answer. (1 mark)

5B · Properties of quadrilaterals proved from congruent triangles

Q4. $ABCD$ is a parallelogram with $\angle A = 62^\circ$. Find $\angle B$ and $\angle D$, giving a reason for each answer. (2 marks)

Q5. The diagonals of rectangle $ABCD$ meet at M, and $\angle MAB = 34^\circ$. Find $\angle AMB$, giving a reason for each step of your working. (2 marks)

Q6. The diagonals of quadrilateral $PQRS$ bisect each other — each cuts the other into two equal halves. What must $PQRS$ be? Give the reason. (1 mark)

5C · Algorithms that test for congruence and similarity

Q7. Triangle 1 has sides 6 cm, 8 cm, 10 cm and angles $37^\circ, 53^\circ, 90^\circ$. Triangle 2 has sides 9 cm, 12 cm, 15 cm and angles $37^\circ, 53^\circ, 90^\circ$. Trace the sorting algorithm from 5C — follow it step by step: match the parts in size order, check the three pairs of matching angles, work out the three ratios of matching sides — and state the algorithm's output. (2 marks)

Q8. A careless sorting rule says: *"If 3 or more of the 6 matching parts are equal, the triangles are congruent."* Consider an equilateral triangle of side 2 cm and an equilateral triangle of side 6 cm. (2 marks)

(a) Show that the rule gives the answer *congruent* for this pair. (1 mark)

(b) Explain why that answer is wrong, and give the correct output. (1 mark)

Q9. The sorting algorithm matches the parts in size order before it compares them. Explain why this step is needed. (1 mark)

5D · Position and location in three dimensions

Q10. Write this description as an **ordered triple** — three numbers in a fixed order, (x, y, z): 5 along x, 3 across, 2 up. (1 mark)

Q11. Use the table below, which gives the real coordinates of a Victorian station. (2 marks)

Station	Latitude	Longitude	Elevation above sea level
Mildura Airport	34.24° S	142.09° E	50 m

Source: Bureau of Meteorology, climate statistics for Australian locations, station page 076031 (Mildura Airport); retrieved 20 August 2026.

(a) Start at the point $(2, 1, 5)$. Move 3 parallel to the x-axis, then 2 parallel to the y-axis, then 4 down. Write the coordinates of the point you reach. (1 mark)

(b) A drone hovers directly above Mildura Airport at a height of 200 m. Write the drone's position as (latitude, longitude, height). (1 mark)

Q12. *You will need dynamic geometry software or a 3D graphing tool.* Plot the points $P(1, 2, 1)$, $Q(5, 2, 1)$, $R(5, 4, 1)$ and $S(1, 4, 1)$, and join them to form the base of a cuboid (a box-shaped solid). The cuboid has height 3. (2 marks)

(a) Construct the top face of the cuboid in the software, and rotate the view. Write the coordinates of the four vertices of the top face. (1 mark)

(b) State the cuboid's lengths in the x, y and z directions. (1 mark)

Total: 20 marks

Solutions

Teacher-facing. T5 is a classroom check, not an exam: 25 minutes of guidance (5 minutes reading plus about 1 minute per mark), a ruler, a protractor, a pair of compasses and — for Question 12 — dynamic geometry software or a 3D graphing tool, as stated on the paper. No calculator is permitted: none of the four Chapter 5 content descriptions names a calculator (D10), and `VC2M8SP03` makes a digital tool compulsory, which Q12 delivers. Every item is newly authored for this paper; the only real data is the Mildura Airport station row, re-cited from 5D with its source line. Reasoning is marked separately from the answer throughout (TOC section 7.1): wherever a reason is asked for, a correct number with no reason does not score full marks.

Marking guide

Marks by content description. Each of the four subtopics carries exactly 5 marks (TOC section 7 marking rule: 5 marks per subtopic).

Item	CD code	Subtopic	Type	Marks
Q1–Q3	`VC2M8SP01`	5A	MC (A–D, 1 mark — used sparingly; choosing the congruence test is the selection task TOC section 7.1 names), short answer, worked response with a construction	5
Q4–Q6	`VC2M8SP02`	5B	Short answer and worked response; every answer carries its reason	5
Q7–Q9	`VC2M8SP04`	5C	Worked response: trace the algorithm, evaluate a faulty rule, describe a design choice	5
Q10–Q12	`VC2M8SP03`	5D	Short answer and worked response; Q12 in dynamic geometry software	5
			Total	**20**

MC total: 1 mark of 20, sparingly per TOC section 7.1.

Reason marks. Per TOC section 7.1, reasoning is marked separately in T5, and an item worth 5 marks in these subtopics carries at least 1 mark for the stated reason. Here: Q3(b) requires the test named *and* the reason it works; every 5B answer requires its property or angle fact stated; Q7's output requires the reason (equal ratios, $k \neq 1$); Q8(b) and Q9 are explanation marks. A correct number with no reason does not score full marks.

Achievement-standard extracts (quoted verbatim from `maths_8.json`; the standard spells "congruency" — this book uses "congruence" in prose, D13, and quotes the standard as VCAA wrote it):

- `VC2M8SP01` (5A): *"They identify conditions for congruency and similarity in triangles and other common shapes, and design and test algorithms to test for congruency and similarity."* — the identify-conditions half is scored here; the algorithm half belongs to 5C.
- `VC2M8SP02` (5B): *"Students apply the properties of quadrilaterals to solve problems."*
- `VC2M8SP03` (5D): *"Students use 3 dimensions to locate and describe position."*
- `VC2M8SP04` (5C): *"They identify conditions for congruency and similarity in triangles and other common shapes, and design and test algorithms to test for congruency and similarity."* — the algorithm half is scored here.

General marking notes.

- Q1: one mark for the correct letter; no part marks. Accept RHS only.
- Q2(a): the scale factor must come from the matching shortest sides ($12 \div 4$); a ratio taken from non-matching sides earns 0.

- Q3(a): allow 1° of protractor wobble on the measured angle, as in 5A. The mark needs the constructed triangle and the recorded angle.
- Q3(b): the mark needs the test (SSS) *and* the reason; a bare "yes" earns 0.
- Q4: each mark is answer + reason together; either part with no reason earns 0 for that part.
- Q5: a bare 112° with no reasons earns at most 1 of the 2 marks.
- Q6: the mark needs the shape *and* the reason; either alone earns 0.
- Q7: accept the trace set out in any order that shows the three ratios and the output; the ratios must be computed from size-ordered matching sides.
- Q8(a): the student should state that the three angle pairs are equal (60° each) and that 3 is at least 3.
- Q9: accept any explanation that says both why size order pairs each side with its true matching part and what can go wrong without it.
- Q10 and Q11(a): the order of the three numbers is part of the mathematics; a swapped order earns 0.
- Q11(b): the height recorded is the 200 m given, as in 5D's worked examples; do not add the station's 50 m elevation.
- Q12(a): all four top vertices must be correct for the mark; each keeps its x and y from the base and gains $z=4$.
- Q12(b): lengths are read off the coordinates along the axes ($5-1=4$, $4-2=2$, and the height 3); no distance formula is taught at this level (D5).

Answer key

Q	Answer	Marks
1	D — RHS	1
2	(a) 3 · (b) 27 cm	2
3	(a) triangle with sides 3 cm, 4 cm, 5 cm; $\angle ACB=90^{\circ}$ (accept 89–91°) · (b) yes — SSS	2
4	$\angle B=118^{\circ}$ (co-interior angles) · $\angle D=62^{\circ}$ (opposite angles of a parallelogram are equal)	2
5	$\angle AMB=112^{\circ}$ — $AM=BM$ (diagonals of a rectangle are equal and bisect each other), isosceles triangle, angle sum	2
6	parallelogram — a quadrilateral whose diagonals bisect each other is a parallelogram	1
7	ratios $1.5, 1.5, 1.5$; $k=1.5$ — similar, but not congruent	2
8	(a) 3 equal angle pairs, and 3 is at least 3 · (b) the sides differ — similar, not congruent	2
9	size order pairs each side with the side opposite the equal angle; without it, a similar pair can be labelled neither	1
10	$(5,3,2)$	1
11	(a) $(5,3,1)$ · (b) (34.24° S, 142.09° E, 200 m)	2
12	(a) $(1,2,4)$, $(5,2,4)$,	2

Q	Answer	Marks
	$(5,4,4),(1,4,4)$ · (b) 4 in x, 2 in y, 3 in z	
	Total	**20**

Fully-worked solutions

Q1. Sort the given parts. $\angle B$ and $\angle E$ are the right angles. The sides opposite them, AC and DF, are the hypotenuses, and they are equal (10 cm each). $AB = DE = 6$ cm is one equal pair of shorter sides. That is right angle, hypotenuse, side — **RHS**. The others do not fit: SSS needs three sides and only two are known; SAS needs the angle *between* two known sides, but $\angle B$ sits between AB and BC, not between AB and AC; ASA needs two angles and only one is known. ∴ **D, RHS**.

Marks: 1 mark for D. No part marks.

Q2. In similar triangles, matching sides sit opposite equal angles, so the shortest side matches the shortest side and the longest matches the longest.

(a) Scale factor $= 12 \div 4 = 3$.

(b) Longest side of the larger triangle $= 9 \times 3 = 27$ cm.

∴ (a) 3; (b) 27 cm. (Check the middle sides as well: $6 \times 3 = 18$ cm — every side is multiplied by the same number, which is what similar means.)

Marks: 1 mark for each part.

Q3. (a) The two arcs meet at exactly one point above the base. The triangle has sides 3 cm, 4 cm and 5 cm, and the measured angle is $\angle ACB = 90^\circ$ (accept 89–91° — protractor wobble). It is a right angle because $3^2 + 4^2 = 5^2$: the side opposite $\angle C$ is the longest side, and the triangle satisfies Pythagoras' theorem (3D).

(b) **Yes — SSS.** The reason the test works: three sides fix a triangle completely. With the base drawn, the arc of radius 3 cm from A and the arc of radius 4 cm from B meet at exactly one point on each side of the base, and those two meeting points are mirror images of each other. So any triangle with sides 3 cm, 4 cm and 5 cm is the very same triangle, up to a flip — your classmate's must be congruent to yours.

Marks: (a) 1 mark for the construction and the recorded angle; (b) 1 mark for SSS with the reason — a bare "yes" with no test and no reason earns 0.

Q4. $\angle A$ and $\angle B$ are co-interior angles between the parallel sides AD and BC (with AB crossing them), so they add to 180°: $\angle B = 180^\circ - 62^\circ = 118^\circ$ (*co-interior angles add to* 180°, $AD \parallel BC$). $\angle D$ sits opposite $\angle A$, and the opposite angles of a parallelogram are equal — proved in 5B from congruent triangles — so $\angle D = 62^\circ$ (*opposite angles of a parallelogram are equal*). Check: $62^\circ + 118^\circ + 62^\circ + 118^\circ = 360^\circ$, the angle sum of a quadrilateral ✓.

Marks: 1 mark for $\angle B$ with its reason; 1 mark for $\angle D$ with its reason. A bare number with no reason earns 0 for that part.

Q5. The diagonals of a rectangle are equal and bisect each other, so they cut each other into four equal halves (*Result 4 of 5B*): $AM = BM$. Triangle AMB is therefore isosceles, and the angles opposite the equal sides are equal: $\angle MBA = \angle MAB = 34^\circ$. The angles of a triangle add to 180°:

$$\angle AMB = 180^\circ - 34^\circ - 34^\circ = 112^\circ$$

∴ $\angle AMB = 112^\circ$.

Marks: 1 mark for $AM = BM$ with its reason (diagonals of a rectangle are equal and bisect each other); 1 mark for 112° with the isosceles and angle-sum reasons. A bare 112° with no reasons earns at most 1 of the 2 marks.

Q6. $PQRS$ is a **parallelogram**. Reason: this is the converse proved in 5B (*Result 3*) — if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.

Marks: 1 mark for the shape with the reason; either alone earns 0.

Q7. Follow the algorithm from 5C.

- Match the parts in size order: shortest with shortest, middle with middle, longest with longest — 6 with 9, 8 with 12, 10 with 15 — and the angles smallest with smallest, middle with middle, largest with largest.
- Are the three pairs of matching angles equal? Yes — both triangles have $37^\circ, 53^\circ, 90^\circ$.
- Work out the three ratios of matching sides: $9 \div 6 = 1.5$, $12 \div 8 = 1.5$, $15 \div 10 = 1.5$. The ratios are equal, so the triangles are similar; call the common ratio $k = 1.5$.
- Is $k = 1$? No — the sizes differ.

Output: **similar, but not congruent**.

Marks: 1 mark for the three ratios and $k = 1.5$; 1 mark for the output with its reason — equal ratios mean similar, and $k \neq 1$ means not congruent. The ratios must come from size-ordered matching sides.

Q8. (a) Every angle of an equilateral triangle is 60°, so the three pairs of matching angles are all equal: $60^\circ = 60^\circ$, three times. The rule counts 3 equal parts, and 3 is at least 3, so the rule outputs *congruent*.

(b) The sides differ (2 cm against 6 cm), so one triangle cannot lie exactly on top of the other — the pair is **not congruent**. The correct output is *similar, but not congruent*: the matching angles are equal, and every side of the larger triangle is 3 times its matching side. The mistake is counting **how many** parts match instead of checking **which** parts match — equal angles alone give similarity, never congruence.

Marks: 1 mark for each part; part (b) needs the correct output and the reason.

Q9. Matching sides sit opposite equal angles, and in any triangle the shortest side lies opposite the smallest angle. Sorting by size therefore pairs each side with its true matching part. Without this step, the ratios could be computed from the wrong pairing — dividing the wrong sides gives unequal ratios, and a truly similar pair would be labelled *neither*.

Marks: 1 mark for an explanation that says both why size order gives the correct pairing (sides opposite equal angles) and what can go wrong without it.

Q10. Read the description into the fixed order (x, y, z): 5 along x, 3 across, 2 up gives $(5, 3, 2)$.

Marks: 1 mark; the order of the three numbers must be exact.

Q11. (a) Each move changes exactly one number: x: $2+3=5$; y: $1+2=3$; z: $5-4=1$. The point reached is $(5, 3, 1)$.

(b) Directly above the airport, the drone keeps the airport's latitude and longitude and adds its height: **(34.24° S, 142.09° E, 200 m)**. (The station's 50 m is its elevation above sea level; the 200 m is the height given, measured from the ground — recorded as given, exactly as in 5D's worked examples.)

Marks: 1 mark for each part; in part (b) the three entries must be in the order (latitude, longitude, height).

Q12. (a) The top face sits 3 units above the base, so every top vertex keeps its x and its y from the base and gains $z = 1+3 = 4$. The four top vertices are $(1, 2, 4)$, $(5, 2, 4)$, $(5, 4, 4)$ and $(1, 4, 4)$. Rotating the view in the software confirms the cuboid.

(b) Read each length off the coordinates along its axis, as in 5D: x: from 1 to 5, a length of $5-1=4$; y: from 2 to 4, a length of $4-2=2$; z: the height is 3. The cuboid's lengths are **4 in x, 2 in y and 3 in z**.

Marks: (a) 1 mark for all four top vertices; (b) 1 mark for all three lengths.