Mathematics Level 8

Chapter 3 — Measurement: area, circles, volume and Pythagoras' theorem

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

Significant effort has been made to ensure this content is legal. Please send any areas of concern to admin@vwx.au with appropriate document/legal references so errors can be verified and changes be made.

Contents

Perimeter and area of composite and irregular shapes

> **Content description `VC2M8M01`:** solve problems involving the area and perimeter of irregular and composite shapes using appropriate units

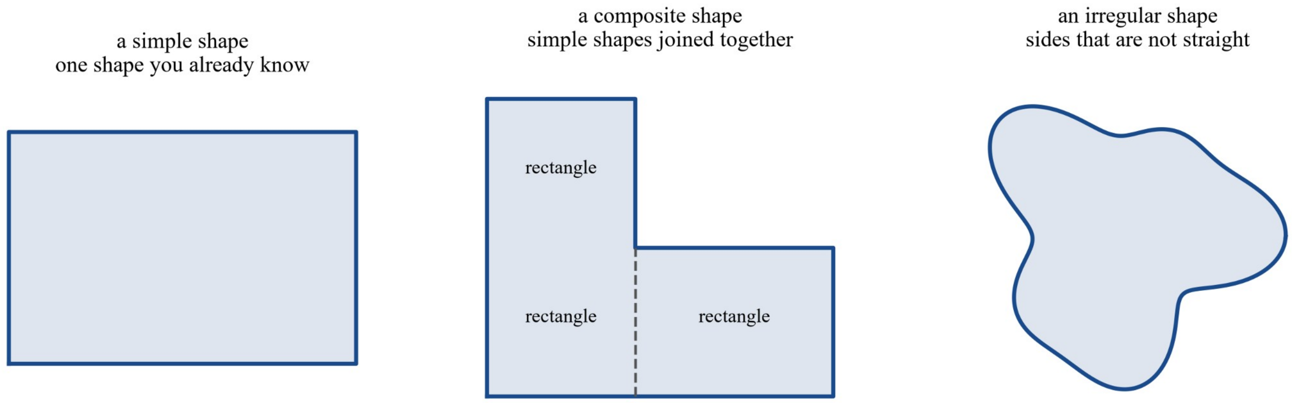
A backyard pool is not a rectangle. A farm paddock is not a rectangle. A leaf is not even close. Yet a builder, a farmer and a scientist all need the area and the perimeter of shapes like these. This subtopic builds two big ideas: composite shapes can be joined or cut into rectangles and triangles, and irregular shapes can be measured as closely as we like with grids and straight pieces — smaller pieces mean closer answers.

Theory

In Year 7 you built the area formulas (`VC2M7M01`): a rectangle's area is length × width, a triangle's area is ½ × base × height, and a parallelogram's area is base × height. The **perimeter** of any shape is the sum of its side lengths. This subtopic uses those facts to measure shapes that are not on that list.

Two kinds of shape

Three kinds of shapes in this subtopic

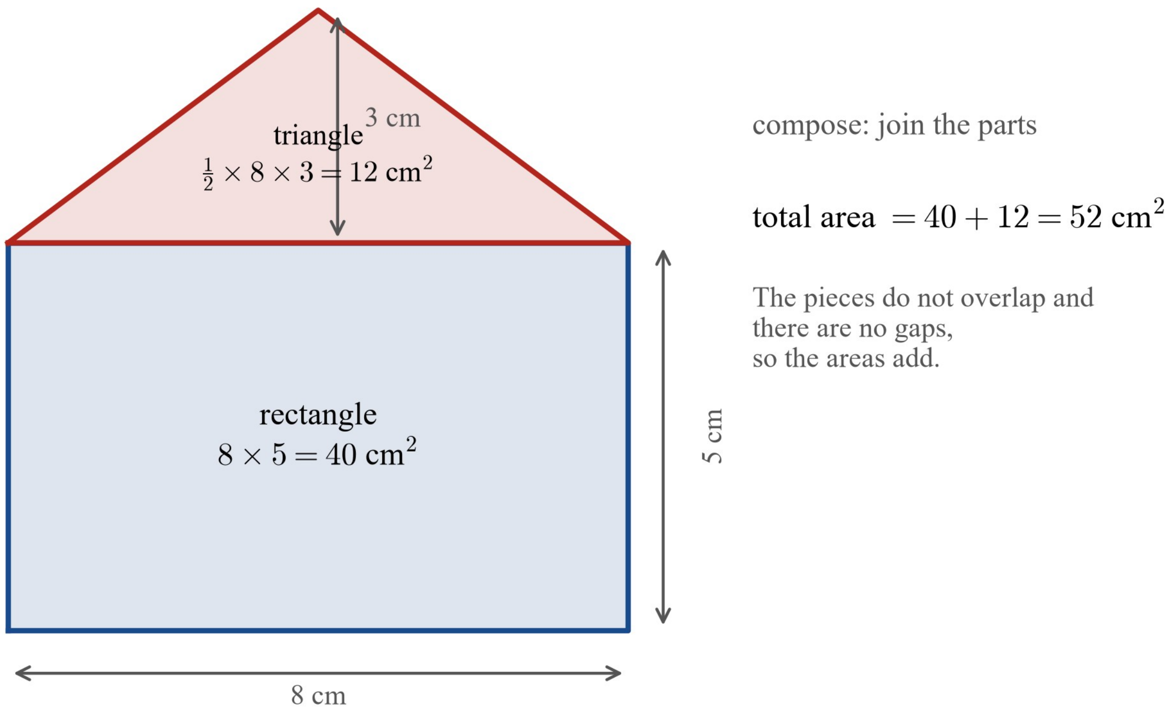


A plain rectangle labelled a simple shape, an L-shape made of two rectangles labelled a composite shape, and a blob outline with curved sides labelled an irregular shape

A **composite shape** is made by joining simple shapes — rectangles, triangles and parallelograms — edge to edge. An **irregular shape** has a curved or uneven **boundary** (its outside edge) that no simple formula fits. Composite shapes are solved exactly, by the three moves below. Irregular shapes are solved by **estimates** that get closer as the measuring pieces get smaller.

Move 1 — compose: join shapes together

Composing — the whole area is the sum of the parts

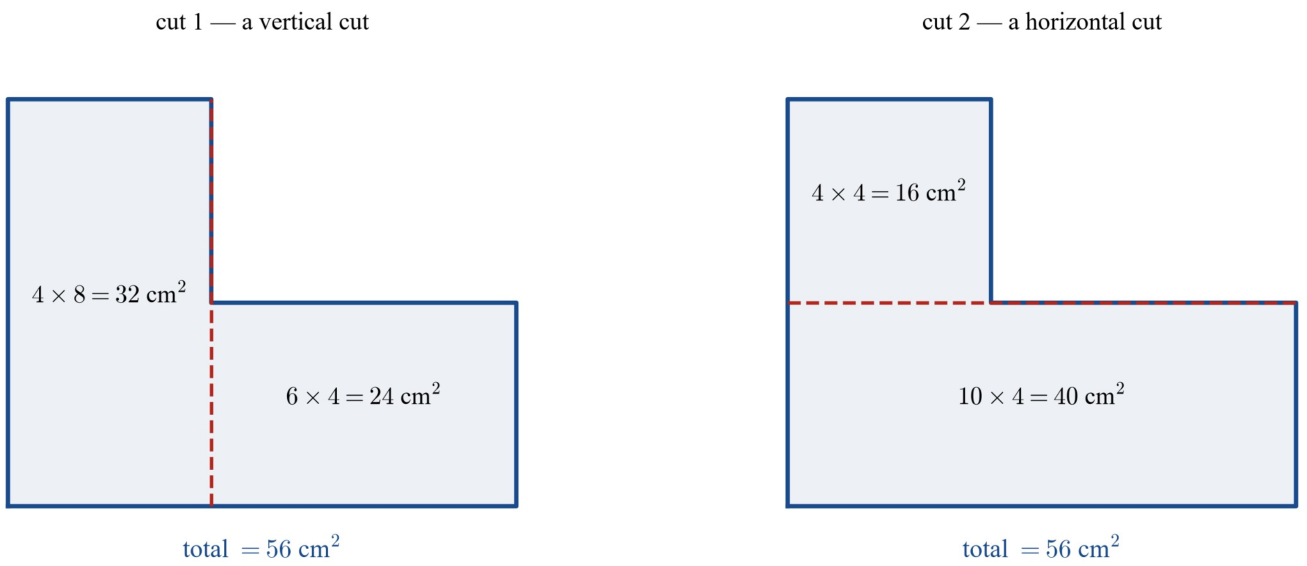


A house shape made of an 8 cm by 5 cm rectangle worth 40 square cm and a triangle on top with base 8 cm and height 3 cm worth 12 square cm, giving a total area of 52 square cm

When simple shapes are joined with no overlaps and no gaps, the whole area is the sum of the parts: $8 \times 5 = 40$ cm², $\frac{1}{2} \times 8 \times 3 = 12$ cm², so the house shape is $40 + 12 = 52$ cm².

Move 2 — decompose: cut the shape up

Decomposing the same L-shape two ways — both cuts give the same total area

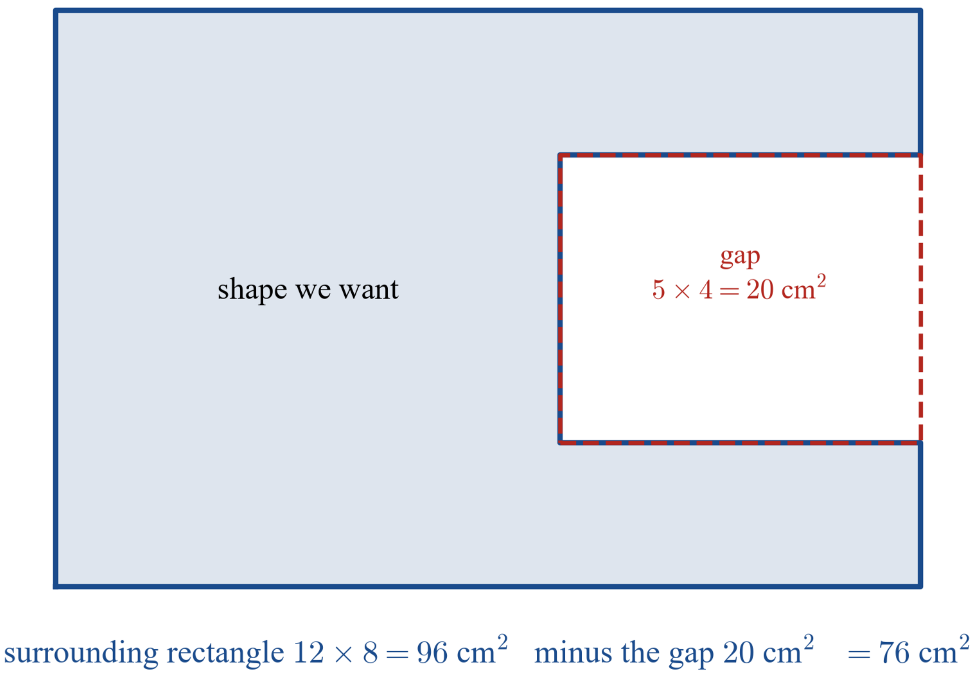


The same L-shape cut two ways: a vertical cut into a 4 by 8 rectangle and a 6 by 4 rectangle giving 32 plus 24, and a horizontal cut into a 4 by 4 square and a 10 by 4 rectangle giving 16 plus 40, both totalling 56 square cm

Cut a composite shape into simple shapes, add the areas. The cut is your choice — every cut that covers the shape with no overlaps gives the same total: $32 + 24 = 56$ cm² and $16 + 40 = 56$ cm².

Move 3 — subtract: big shape minus the gap

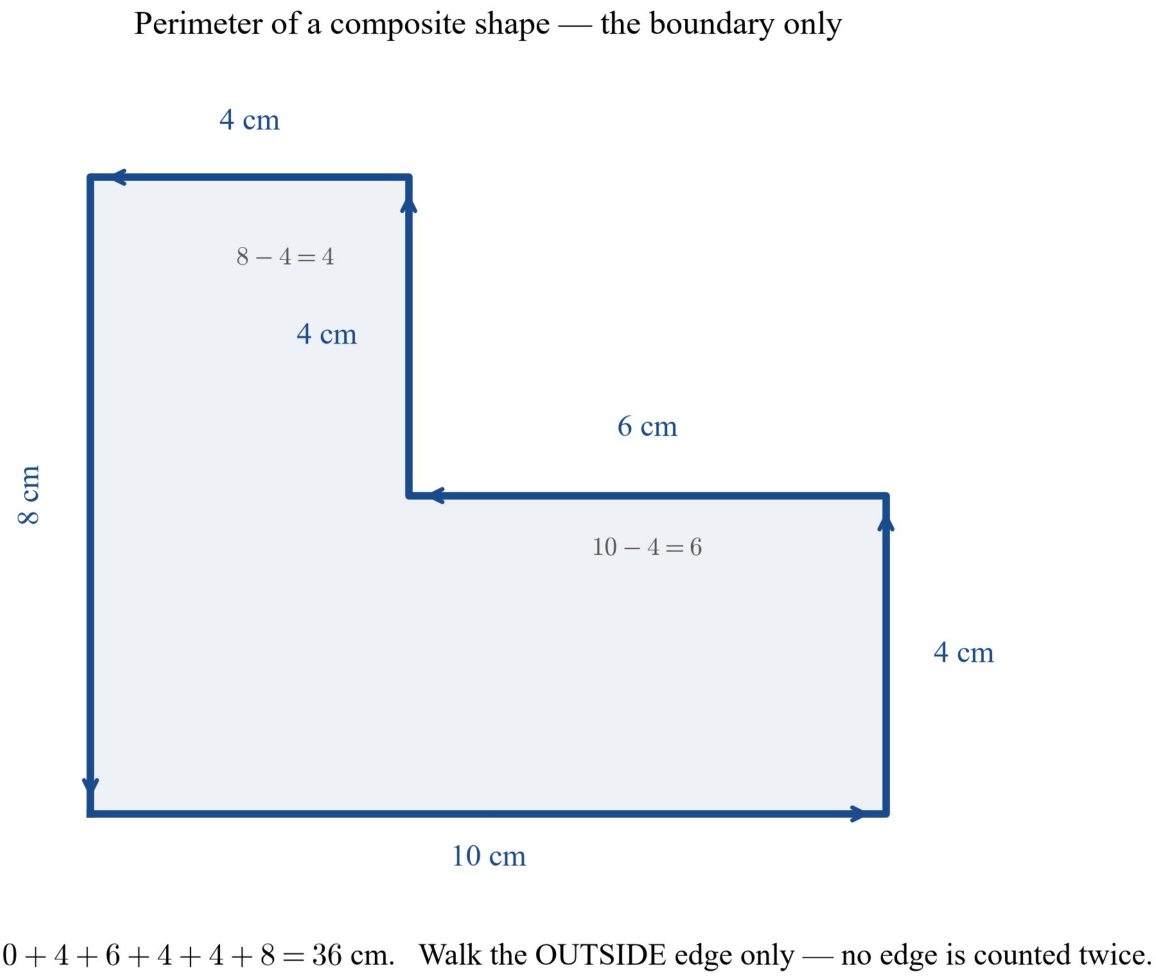
Subtracting — find a big area, take away the gap



A 12 cm by 8 cm rectangle with a 5 cm by 4 cm rectangular gap cut from the right edge: 96 square cm minus 20 square cm equals 76 square cm

Sometimes the quickest cut is no cut: find a big shape that surrounds the whole figure, then take away the piece that is missing. $12 \times 8 = 96$ cm², the gap is $5 \times 4 = 20$ cm², so the shaded shape is $96 - 20 = 76$ cm².

Perimeter of a composite shape — walk the outside edge

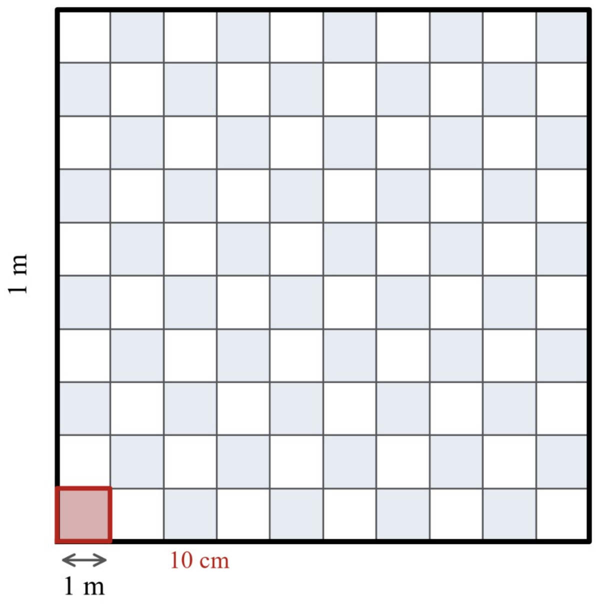


An L-shape with sides 10 cm, 4 cm, 6 cm, 4 cm, 4 cm and 8 cm walked in order with arrows; the two unmarked sides are worked out as 10 minus 4 equals 6 and 8 minus 4 equals 4, and the perimeter is 36 cm

The perimeter is a walk right around the **outside** edge. Sides that are not marked can often be found from the sides that are: here $10-4=6$ cm across and $8-4=4$ cm up, so the perimeter is $10+4+6+4+4+8=36$ cm. Cut lines inside the shape are never part of the perimeter.

Appropriate units

One square metre holds 10 000 square centimetres



The big square measures 1 m by 1 m, so its area is 1 m^2.

Each side is 100 cm long.
Split each side into 10 parts of 10 cm:

$10 \times 10 = 100$ small squares,
each one $10 \times 10 = 100$ cm^2.

$1\ \mathbf{m}^2 = 100 \times 100 = 10\,000\ \mathbf{cm}^2$

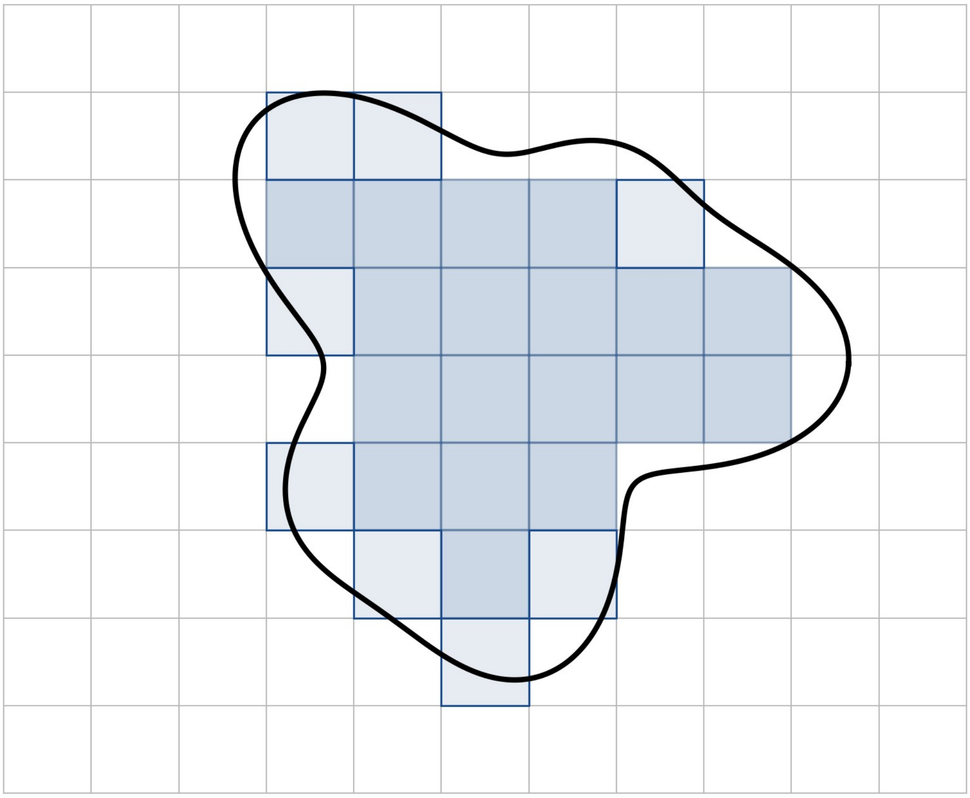
A 1 m by 1 m square split into a 10 by 10 array of smaller squares, each 10 cm by 10 cm, showing 1 square metre equals 100 times 100 equals 10 000 square cm

Lengths come in millimetres, centimetres, metres and kilometres; areas come in their *squares*. Because 1 m $= 100$ cm, one square metre holds $100 \times 100 = 10\,000$ cm^2 — not 100. For large areas such as farms, the **hectare** (ha) is the handy unit: 1 ha $= 10\,000$ m^2, a square 100 m by 100 m. Choosing appropriate units means matching the unit to the job: cm^2 for a sticky note, m^2 for a room, hectares for a paddock.

Irregular shapes — area by counting squares

Lay a grid over the shape and count squares by one rule: **whole squares, plus edge squares that are more than half covered.**

The leaf on a 1 cm grid

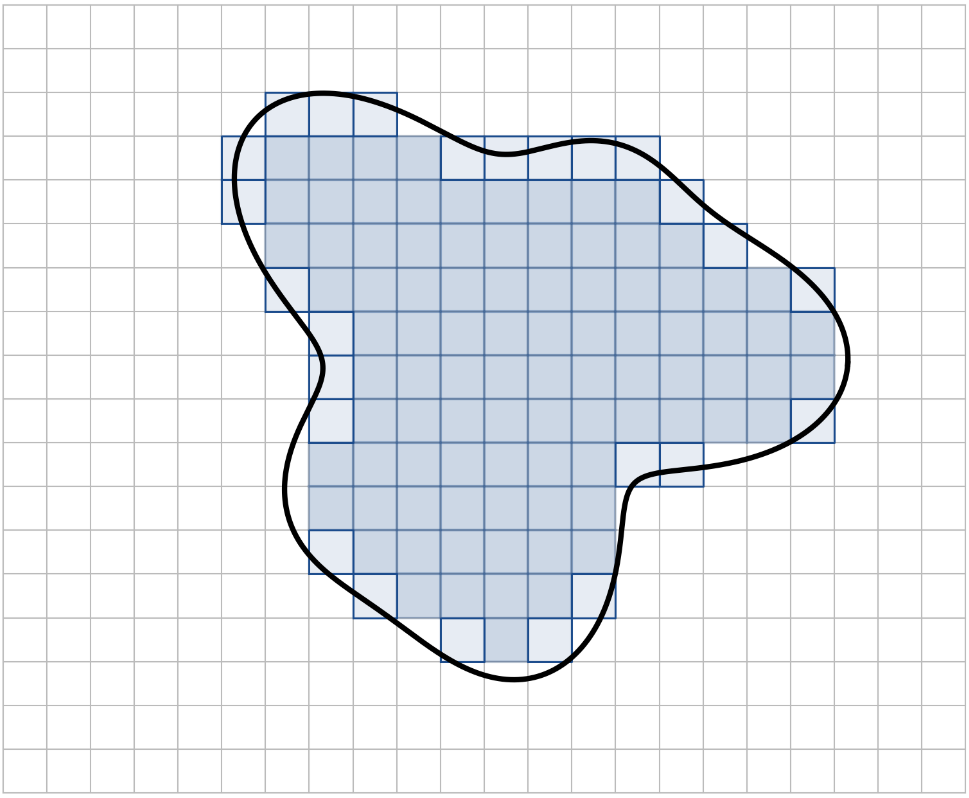


whole squares: 18, edge squares more than half covered: 8 $\Rightarrow$ area $\approx$ 26 cm^2

A leaf outline on a 1 cm grid with 18 whole squares and 8 edge squares more than half covered shaded, giving an area estimate of 26 square cm

On the 1 cm grid: 18 whole squares and 8 edge squares more than half covered, so the leaf is about $18+8=26$ cm². The count also gives a bracket: at least 18 cm² (counting only whole squares) and at most 41 cm² (counting every square the leaf touches).

The same leaf on a 5 mm grid



whole squares: 91, edge squares more than half covered: 25, each 0.25 cm^2 $\Rightarrow$ area $\approx$ 29.00 cm^2

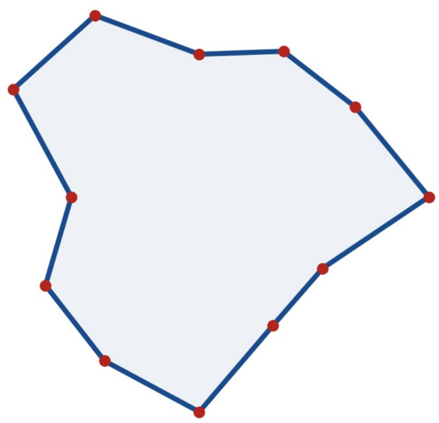
The same leaf on a 5 mm grid with 91 whole squares and 25 edge squares more than half covered shaded, each square 0.25 square cm, giving an area estimate of 29 square cm

On the 5 mm grid each square is $0.5 \times 0.5 = 0.25$ cm². Now 91 whole squares and 25 edge squares are counted: $(91 + 25) \times 0.25 = 29$ cm², inside a tighter bracket of 22.75–35.75 cm². The true area is 29.2 cm². The coarse grid is out by 3.2 cm² — a **percentage error** (subtopic 1E) of about 11% — while the fine grid is out by only 0.2 cm², about 0.7%. *A grid of smaller squares gives a closer estimate*, and the percentage error is how we see it.

Irregular shapes — perimeter by straight pieces

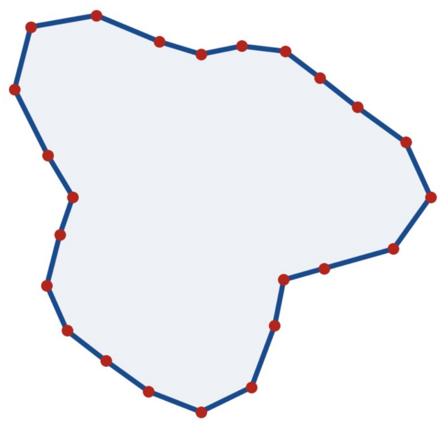
Measuring a curved boundary with straight pieces — more, shorter pieces follow the curve more closely

12 straight pieces along the boundary



perimeter $\approx$ 20.94 cm

24 straight pieces along the boundary

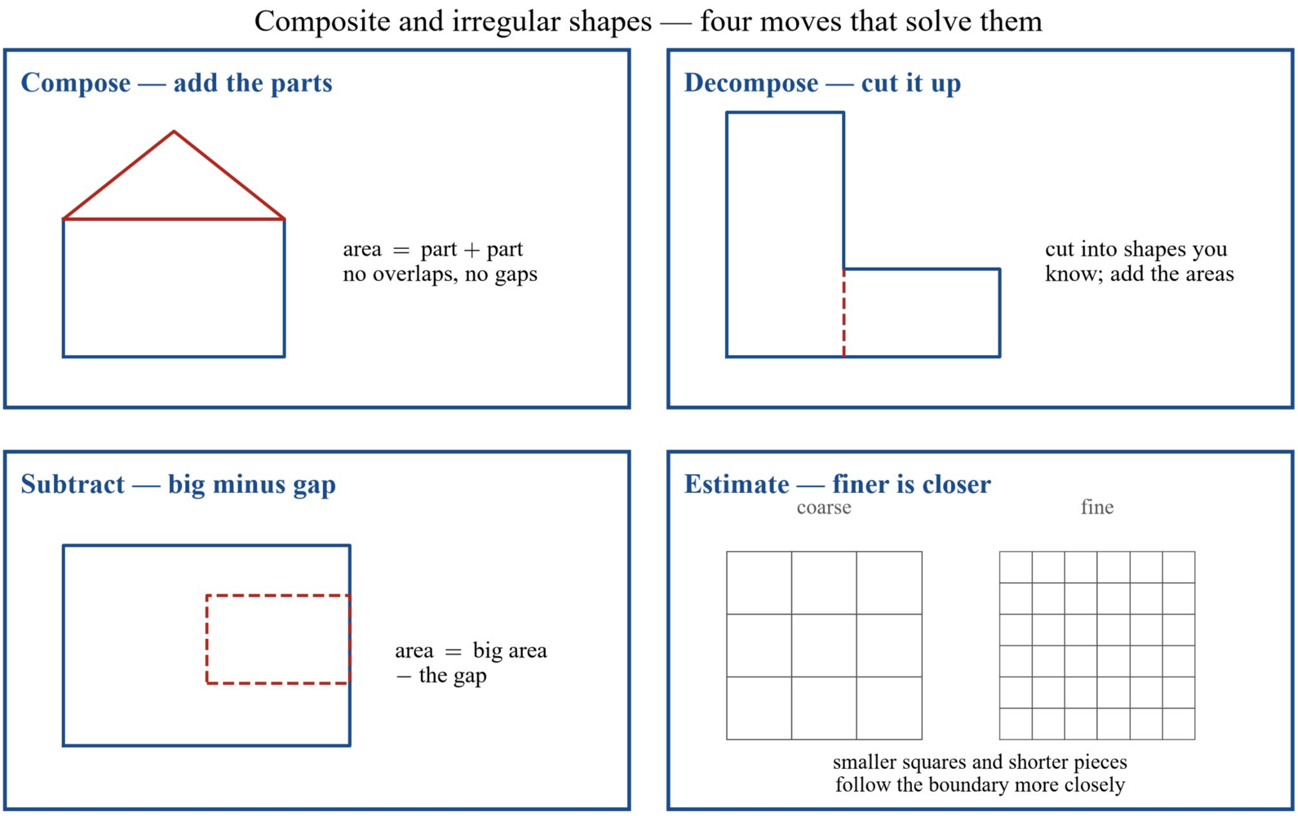


perimeter $\approx$ 21.94 cm

The leaf boundary measured by 12 straight pieces totalling 20.94 cm and by 24 shorter straight pieces totalling 21.94 cm, against a true perimeter of 22.30 cm

A curved boundary is measured the same way: follow it with straight pieces and add their lengths. Twelve pieces give 20.94 cm; twenty-four shorter pieces give 21.94 cm; the true perimeter is 22.30 cm. Each straight piece cuts across the curve a little, so every piece-total sits under the true length — and more, shorter pieces follow the boundary more closely, so the total climbs towards the truth. Smaller pieces give closer answers, for perimeter as for area.

The four moves at a glance



Four summary cards: compose joins parts and adds their areas; decompose cuts the shape into known shapes and adds the areas; subtract takes the gap from the big area; estimate uses smaller squares and shorter pieces to follow the boundary more closely

Words of this subtopic

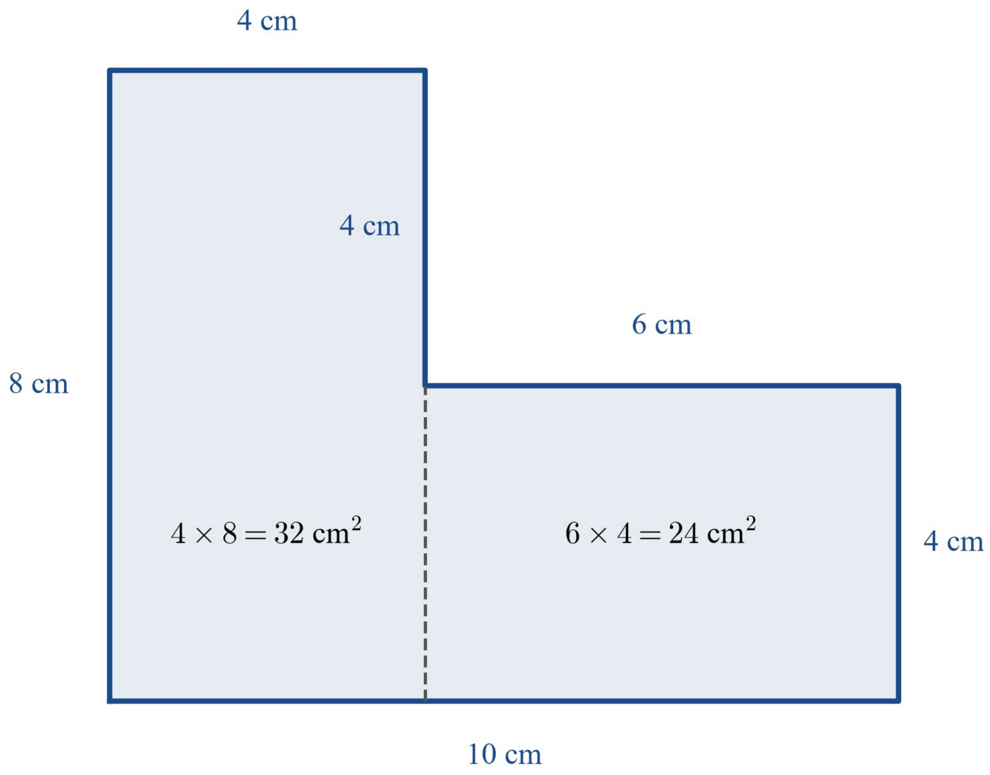
Word	What it means here
composite shape	a shape made by joining simple shapes (rectangles, triangles, parallelograms) edge to edge
irregular shape	a shape with a curved or uneven boundary that no simple formula fits
boundary	the outside edge of a shape — the line you walk to go around it
perimeter	the distance around the outside edge, found by adding the side lengths
area	the amount of flat surface a shape covers, in square units (cm^2, m^2, ha)
compose	join simple shapes to build the shape; the areas add
decompose	cut the shape into simple shapes; the areas add
estimate	a close, sensible answer found by counting or measuring in pieces, not exact
grid	a network of equal squares laid over a shape for counting
hectare (ha)	a unit for large areas: 1 ha = 10 000 m^2, a square 100 m by 100 m
percentage error	the error written as a percentage of the true value (subtopic 1E)

Worked examples

Example 1 — with help

Find the area and the perimeter of this L-shape.

Worked Example 1 — the L-shape



Area $= 32 + 24 = 56$ cm^2 Perimeter $= 10 + 4 + 6 + 4 + 4 + 8 = 36$ cm

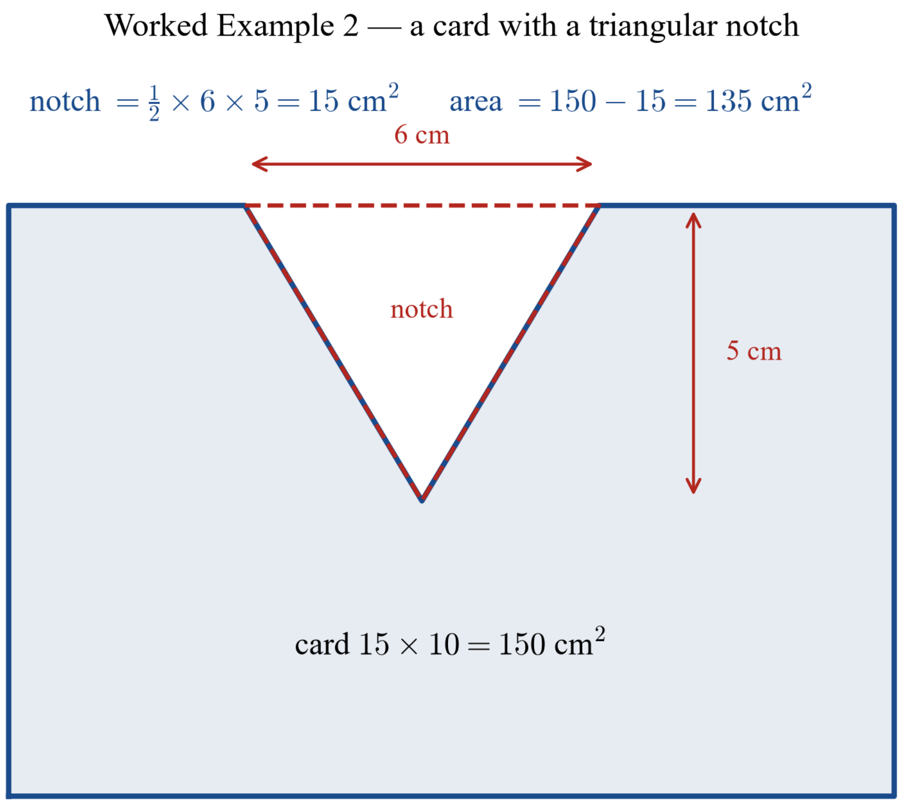
An L-shape with marked sides 10 cm along the bottom, 8 cm up the left, 4 cm across the top, 4 cm down to the step, 6 cm across the step and 4 cm down the right, cut by a dashed vertical line into a 4 by 8 rectangle and a 6 by 4 rectangle

Working	Comment
Cut: left rectangle $4 \times 8 = 32$ cm²	decompose — the cut is your choice
right rectangle $6 \times 4 = 24$ cm²	the step is $10 - 4 = 6$ cm across
Area $= 32 + 24 = 56$ cm²	no overlaps, no gaps, so the areas add
Perimeter $= 10 + 4 + 6 + 4 + 4 + 8 = 36$ cm	the outside edge only — the cut is not counted

∴ the L-shape has area 56 cm² and perimeter 36 cm.

Example 2 — with help

A card 15 cm by 10 cm has a triangular piece cut out of its top edge, 6 cm across the top and 5 cm deep. Find the area of the card that is left.



A 15 cm by 10 cm rectangle with a triangular piece 6 cm wide and 5 cm deep removed from the top edge: the whole card is 150 square cm, the removed triangle is 15 square cm, leaving 135 square cm

Working	Comment
Whole card: $15 \times 10 = 150$ cm²	the big shape that surrounds the figure
Removed triangle: $\frac{1}{2} \times 6 \times 5 = 15$ cm²	base 6 cm across the top, height 5 cm down
Area left $= 150 - 15 = 135$ cm²	subtract — big shape minus the gap

∴ the card has 135 cm² left.

Example 3 — with help

The front wall of a garden shed is a rectangle 6 m wide and 4 m high, with a triangular top piece 2 m high sitting on the full 6 m width. Find the area of the wall, in square metres.

Working	Comment
Rectangle: $6 \times 4 = 24$ m²	length × width
Triangle: $\frac{1}{2} \times 6 \times 2 = 6$ m²	½ × base × height
Wall $= 24 + 6 = 30$ m²	compose — the areas add

∴ the wall has area 30 m². (Metres in, square metres out — the units follow the lengths.)

Example 4 — with help

The leaf from the theory has true area 29.2 cm². Use the two grid counts to show that the fine grid gives the closer estimate, and say how much closer using percentage error.

Working	Comment
Coarse: $18 + 8 = 26$ cm²	whole squares plus edge squares more than half covered
Fine: $(91 + 25) \times 0.25 = 29$ cm²	each 5 mm square is 0.25 cm²
Coarse error: $29.2 - 26 = 3.2$ cm², about 11%	error as a percentage of the true value

Working	Comment
Fine error: $29.2 - 29 = 0.2$ cm², about 0.7%	same rule, smaller squares

∴ the fine grid's error (0.7%) is far smaller than the coarse grid's (11%): a grid of smaller squares gives a closer estimate, and percentage error measures how much closer.

Example 5 — on your own

A picture frame is a rectangle 30 cm by 24 cm on the outside, and the opening in the middle is a rectangle 22 cm by 16 cm. Find the area of the frame itself (the border).

The frame is the big rectangle minus the opening: $30 \times 24 = 720$ cm² and $22 \times 16 = 352$ cm².

∴ the border has area $720 - 352 = 368$ cm² — the subtract move, where the "gap" is the hole in the middle.

Example 6 — on your own

The leaf's true perimeter is 22.30 cm. Twelve straight pieces give 20.94 cm and twenty-four give 21.94 cm. Explain why the 24-piece total is closer to the truth, and why both totals sit under it.

A straight piece joining two points on the curve is shorter than the curve between those points — the straight line is the shortest way between two points. So every piece-total sits under the true perimeter. With 24 pieces, each piece cuts across a smaller bend of the curve, so less is lost on every piece.

∴ more, shorter pieces follow the boundary more closely: 21.94 cm is closer to 22.30 cm than 20.94 cm is, and both are under it.

Practice questions

Work these questions by hand; no calculator is needed. For Q4 and Q15 you will need the printed grid or a sheet of squared paper.

With help

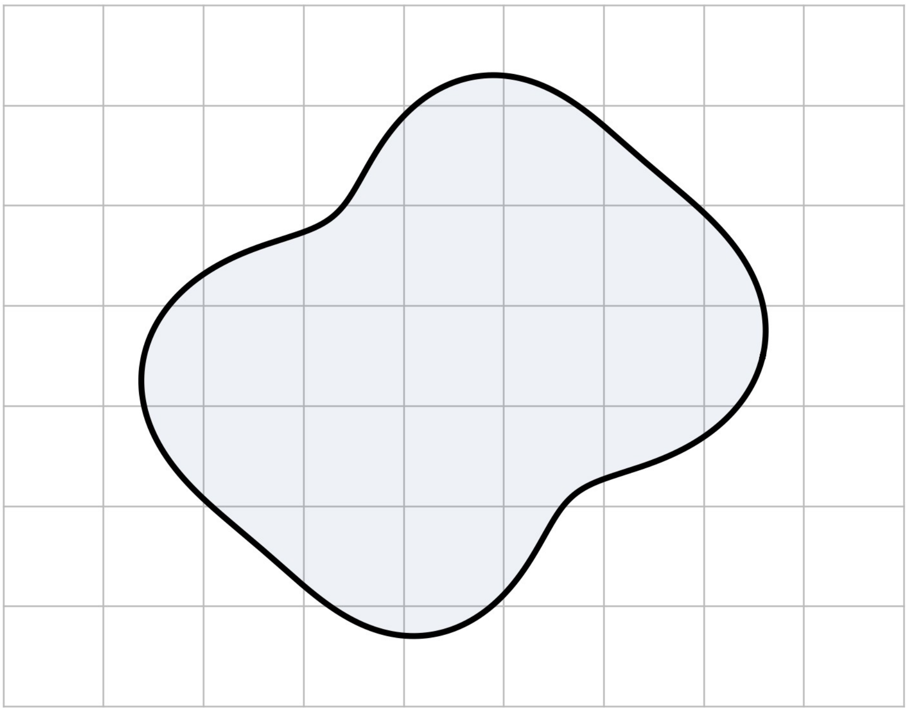
Q1. Say whether each shape is composite or irregular, and why. a. An L-shape made of two rectangles. b. The outline of a leaf. c. A house shape: a rectangle with a triangle on top. d. The outline of a pond on a map.

Q2. Compose: a rectangle 7 cm by 5 cm has a triangle joined across its 7 cm top edge, 3 cm high. Find the total area.

Q3. Subtract: a rectangle 9 cm by 5 cm has a triangular corner piece removed, with sides 4 cm along the top and 3 cm down the edge. Find the area left.

Q4. Estimate the area of this shape by counting squares whose centre lies inside it. Each small square is 1 cm by 1 cm.

Count the squares whose centre lies inside the shape



Each small square is 1 cm by 1 cm.

A curved closed shape drawn on a 1 cm grid of 9 columns by 7 rows, for counting the squares whose centre lies inside the shape

Q5. Choose the more appropriate unit for each measurement. a. The perimeter of your maths book. b. The area of your classroom floor. c. The area of a sticky note. d. The distance between two towns. e. The area of a sheep farm.

Q6. Decompose: a 10 cm by 7 cm rectangle is joined along its 7 cm side to a 3 cm by 7 cm rectangle. Find the total area.

Q7. Convert, using 1 m² $= 10\,000$ cm² and 1 ha $= 10\,000$ m². a. 5 m² to cm². b. 40 000 cm² to m². c. 2 ha to m².

Q8. True or false? Fix the false ones. a. Cutting a composite shape into two pieces halves its area. b. A grid of smaller squares usually gives a closer area estimate. c. The perimeter counts the outside edge only — cut lines are not counted. d. 1 m² $= 100$ cm².

On your own

Q9. A garden is an L-shape: 14 m along the bottom, 10 m up the left side, 2 m across the top and 5 m down the right side. Find its area and its perimeter.

Q10. A wall is a rectangle 8 m wide and 4 m high with a triangular top piece 1 m high sitting on a 4 m base. One litre of paint covers 5 m². How many litres must be bought? (Paint is sold in whole litres.)

Q11. A rectangle 12 cm by 15 cm has a triangle joined along the 12 cm side, 7 cm high. Find the total area.

Q12. A lawn is a rectangle 12 m by 8 m with a square garden bed 4 m by 4 m cut out of it. Find the area of the lawn.

Q13. The leaf's true area is 29.2 cm²; the coarse grid gave 26 cm² and the fine grid gave 29 cm². a. Find the percentage error of the coarse estimate, to the nearest whole percent. b. Find the percentage error of the fine estimate, to one decimal place. c. Which estimate is closer to the truth? How does the percentage error show it?

Q14. A curved garden path is measured with straight pieces: 6 pieces give a total of 11 m, and 12 shorter pieces give 11.5 m. Which total is closer to the true length? Explain why, in one sentence.

Q15. On squared paper, draw a composite shape with area 24 cm² made from two rectangles. Find its perimeter. (One of many answers.)

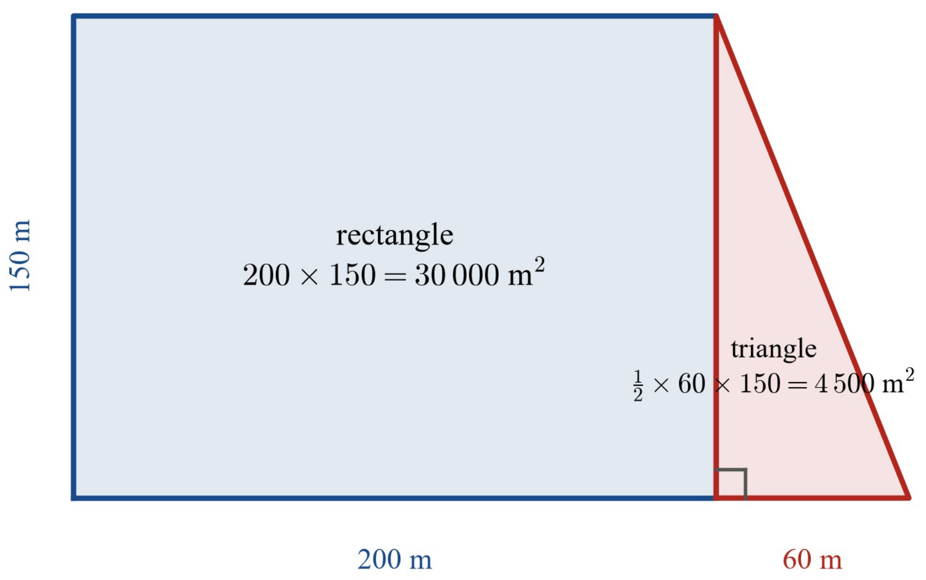
Q16. A rectangle 8 cm by 4 cm has a triangle joined to one long side: the triangle's base along that side is 6 cm and its height is 4 cm. Find the total area.

Maths in real life

Q17. The Melbourne Cricket Ground's playing area is about 174 m by 149 m (Wikipedia, 20 August 2026). a. Estimate the area of the rectangle that surrounds the ground. b. The field inside is closer to a rectangle 120 m by 149 m with a triangle at each end (base 149 m, height 27 m). Estimate the field's area, and write it in hectares to 1 decimal place.

Q18. A paddock is a rectangle 200 m by 150 m with a triangular corner joined on: the triangle has base 60 m along the bottom and the full 150 m height. Fertiliser is spread at 40 kg per hectare. How many kilograms are needed?

The paddock — decompose into a rectangle and a triangle



$$\text{total area } = 30\,000 + 4\,500 = 34\,500\ \mathrm{m}^2 = 3.45\ \mathrm{ha}$$

A paddock made of a 200 m by 150 m rectangle worth 30 000 square m and a triangle with base 60 m and height 150 m worth 4 500 square m, totalling 34 500 square m or 3.45 hectares

Q19. A council checks a swamp for mosquito spray using a grid of 10 m squares, and counts 34 squares covering the swamp. The spray rate is 5 L per 1 000 m². How many litres of spray are needed?

Q20. A builder prices a paved area made of a 4 m by 2 m rectangle joined to a 3 m by 2 m rectangle. Paving costs $35 per m². What is the price?

Answers

Q1. a. Composite — two rectangles joined. b. Irregular — a curved boundary. c. Composite — rectangle plus triangle. d. Irregular — no straight sides to cut along.

Q2. $35 + 10.5 = 45.5$ cm².

Q3. $45 - 6 = 39$ cm².

Q4. 21 cm².

Q5. a. cm. b. m². c. cm². d. km. e. hectares.

Q6. $70 + 21 = 91$ cm².

Q7. a. 50 000 cm². b. 4 m². c. 20 000 m².

Q8. a. False — the pieces' areas still add to the same total. b. True. c. True. d. False — 1 m² $= 10\,000$ cm².

Q9. Area 80 m²; perimeter 48 m.

Q10. 34 m²; $34 \div 5 = 6.8$ L, so buy 7 L.

Q11. $180 + 42 = 222$ cm².

Q12. $96 - 16 = 80$ m².

Q13. a. About 11%. b. About 0.7%. c. The fine one — its percentage error is far smaller.

Q14. 11.5 m — more, shorter pieces follow the curve more closely.

Q15. Answers vary; e.g. a 6 cm by 3 cm rectangle joined to a 3 cm by 2 cm rectangle has area 24 cm² and perimeter 22 cm.

Q16. $32 + 12 = 44$ cm².

Q17. a. 25 926 m². b. 21 903 m² ≈ 2.2 ha.

Q18. 34 500 m² = 3.45 ha; $3.45 \times 40 = 138$ kg.

Q19. 3 400 m²; 17 L.

Q20. 14 m²; $14 \times 35 = \$490$.

Fully-worked solutions

Q1. A composite shape is built from simple shapes; an irregular shape has a boundary no simple formula fits.

a. Two rectangles joined: **composite**.
b. A leaf outline is curved all the way around: **irregular**.
c. Rectangle plus triangle: **composite**.
d. A pond outline is uneven and curved: **irregular**.

Q2. Compose. Rectangle $7 \times 5 = 35$ cm²; triangle ½ $\times 7 \times 3 = 10.5$ cm². Total $35 + 10.5 = 45.5$ cm².

Q3. Subtract. Rectangle $9 \times 5 = 45$ cm²; corner triangle ½ $\times 4 \times 3 = 6$ cm². Left $45 - 6 = 39$ cm².

Q4. Count the squares whose centre lies inside the shape: 21 squares, each 1 cm², so about **21 cm²**. (The centre rule is a quick version of the more-than-half rule — a square whose centre is inside is roughly more than half covered.)

Q5. a. A book's perimeter is a length of a few tens of **cm**. b. A classroom floor is a few tens of **m²**. c. A sticky note is a few tens of **cm²**. d. Town-to-town is a length of many **km**. e. A farm is many **hectares**.

Q6. Decompose. $10 \times 7 = 70$ cm² and $3 \times 7 = 21$ cm²; total $70 + 21 = 91$ cm².

Q7. a. $5 \times 10\,000 = 50\,000$ cm². b. $40\,000 \div 10\,000 = 4$ m². c. $2 \times 10\,000 = 20\,000$ m².

Q8. a. **False** — cutting never loses area: the pieces' areas add to the same total as the whole. b. **True** — smaller squares, closer estimate. c. **True** — the perimeter is the outside walk only. d. **False** — 1 m $= 100$ cm, so 1 m² $= 100 \times 100 = 10\,000$ cm².

Q9. Decompose with a horizontal cut: bottom rectangle $14 \times 5 = 70$ m², top rectangle $2 \times (10 - 5) = 10$ m²; area $70 + 10 = 80$ m². Perimeter: the two unmarked sides are $14 - 2 = 12$ m and $10 - 5 = 5$ m, so $14 + 5 + 12 + 5 + 2 + 10 = 48$ m.

Q10. Rectangle $8 \times 4 = 32$ m²; triangle $½ \times 4 \times 1 = 2$ m²; wall $32 + 2 = 34$ m². Paint: $34 \div 5 = 6.8$ L. Paint sells in whole litres, so buy **7 L** — 6 L would run out.

Q11. Rectangle $12 \times 15 = 180$ cm²; triangle $½ \times 12 \times 7 = 42$ cm²; total $180 + 42 = 222$ cm².

Q12. Subtract. Big rectangle $12 \times 8 = 96$ m²; bed $4 \times 4 = 16$ m²; lawn $96 - 16 = 80$ m².

Q13. a. Error $29.2 - 26 = 3.2$ cm²; $3.2 \div 29.2 \approx 0.11$, about **11%**. b. Error $29.2 - 29 = 0.2$ cm²; $0.2 \div 29.2 \approx 0.007$, about **0.7%**. c. The fine estimate — 0.7% is far smaller than 11%, so the fine grid sits much closer to the true area. That is the smaller-squares observation, made visible.

Q14. The **11.5 m** total. More, shorter pieces cut across smaller bends of the curve, so they follow it more closely (and both totals still sit under the true length).

Q15. One answer: join a 6 cm by 3 cm rectangle (18 cm²) to a 3 cm by 2 cm rectangle (6 cm²): area $18 + 6 = 24$ cm². With the small rectangle on top of the left half, the sides are 6, 3, 3, 2, 3, 5 cm, so perimeter $6 + 3 + 3 + 2 + 3 + 5 = 22$ cm. Check your own shape the same way: the pieces' areas must add to 24 cm², and the perimeter counts the outside edge only.

Q16. Rectangle $8 \times 4 = 32$ cm²; triangle $½ \times 6 \times 4 = 12$ cm²; total $32 + 12 = 44$ cm².

Q17. a. $174 \times 149 = 25\,926$ m². b. Rectangle $120 \times 149 = 17\,880$ m²; two triangles $2 \times (½ \times 27 \times 149) = 27 \times 149 = 4\,023$ m²; field $17\,880 + 4\,023 = 21\,903$ m². In hectares: $21\,903 \div 10\,000 \approx 2.2$ ha.

Q18. Rectangle $200 \times 150 = 30\,000$ m²; triangle $½ \times 60 \times 150 = 4\,500$ m²; paddock $30\,000 + 4\,500 = 34\,500$ m² $= 3.45$ ha. Fertiliser: $3.45 \times 40 = 138$ kg.

Q19. Each grid square is $10 \times 10 = 100$ m², so the swamp is about $34 \times 100 = 3\,400$ m². Spray: $3\,400 \div 1\,000 = 3.4$ lots of 1 000 m², and $3.4 \times 5 = 17$ L.

Q20. Paved area $4 \times 2 + 3 \times 2 = 8 + 6 = 14$ m². Price $14 \times 35 = \$490$.

Circumference and area of a circle

Content description `VC2M8M03`: solve problems involving the circumference and area of a circle using formulas and appropriate units

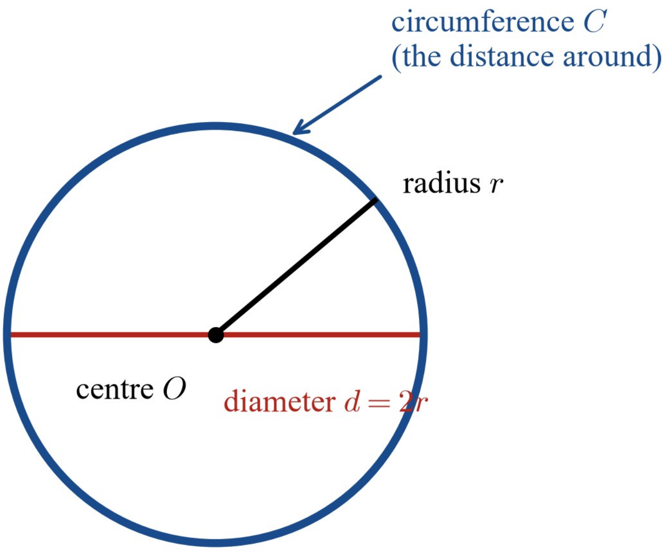
A wheel, a pizza, the circle a sprinkler wets: the world is full of circles, and two numbers describe every one of them — the distance around, and the space inside. Year 7 gave you the multiplier that joins a circle to its distance around. This subtopic uses it to measure the space inside as well — and to work backwards from any one fact about a circle to all the others.

Theory

In Year 7 you met the number π (*pi*): whatever the size of a circle, its distance around is just over 3 times its distance across, and π is the exact times (`VC2M7M03`). In 1D we also saw that π is **irrational** — its decimals never end and never repeat — so any decimal answer that uses π is an approximation. This subtopic uses π ≈ 3.14 throughout, and the sign ≈ is there to remind you of that.

The parts of a circle

Four words name everything we need. The **centre** is the point the circle is drawn about. The **radius** (plural *radii*) is the distance from the centre to the circle. The **diameter** is the distance straight across the circle through the centre. The **circumference** is the distance all the way around the circle.



A circle with its centre O marked, a radius drawn from the centre to the circle, a diameter drawn straight across through the centre, and the circumference labelled as the distance around

Because a diameter is two radii end to end:

$$d = 2r \quad r = d \div 2$$

The circumference

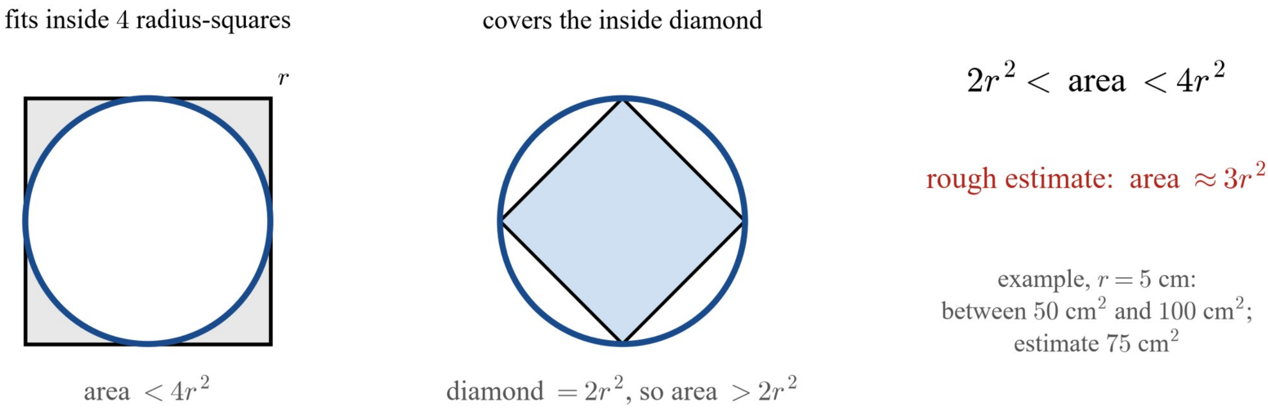
The circumference is π times the diameter, so:

$$C = \pi d \quad C = 2\pi r$$

That is the Year 7 fact, now written as a formula. A circle of diameter 10 cm has $C \approx 3.14 \times 10 = 31.4$ cm — just over three diameters, as it should be.

Trapping the area

Before any formula, here is what the area of a circle must sit between. Call a square with side r a **radius square**; its area is $r \times r = r^2$.



Three panels. Left: a circle of radius r drawn inside a square of side 2r divided into four radius-squares, so the area is less than four radius-squares. Middle: the same circle covering an inside diamond made of two radius-squares, so the area is more than two radius-squares. Right: the summary, area between 2 radius-squares and 4 radius-squares, with the rough estimate area approximately 3 radius-squares and the example r = 5 cm between 50 and 100 square centimetres, estimate 75

- The circle sits inside a big square of side $2r$, made of **four** radius squares, so the area is **less than** $4r^2$.
- The circle covers an inside diamond whose corners touch the circle; the diamond is exactly **two** radius squares (its four triangles pair up into two squares), so the area is **more than** $2r^2$.

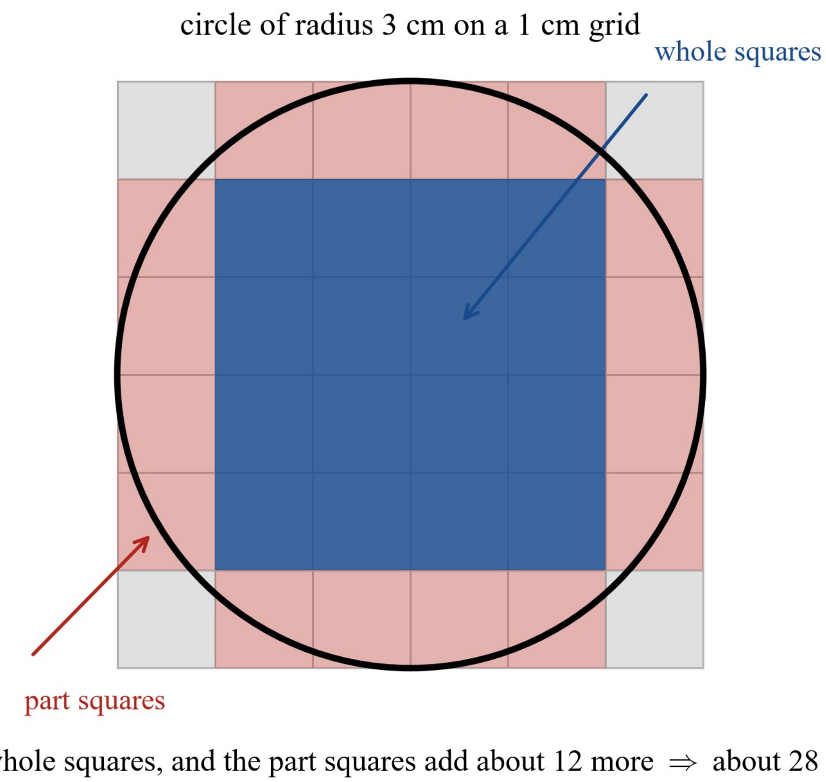
So the area of any circle is trapped:

$$2r^2 < \text{area} < 4r^2\text{, rough estimate: area} \approx 3r^2$$

For a circle of radius 5 cm the area lies between $2 \times 25 = 50$ cm² and $4 \times 25 = 100$ cm², and $3 \times 25 = 75$ cm² is a sensible guess. Keep this trap: any number worked out later for that circle's area must land between 50 and 100, or something has gone wrong.

Counting squares on a grid

A second way to see the size is to draw the circle on 1 cm grid paper and count the squares it covers.

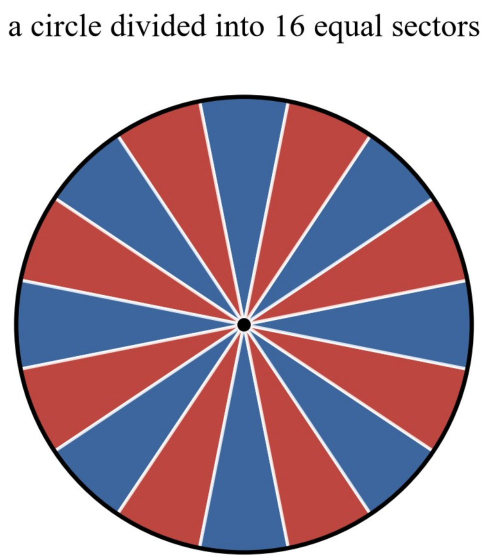


A circle of radius 3 cm drawn on a 1 cm grid: 16 whole squares shaded dark, part squares shaded lighter, and the count 16 whole squares plus part squares adding about 12 more gives about 28 square centimetres

There are 16 whole squares, and the part squares around the rim add up to about 12 more, giving **about 28 cm²**. The rough estimate agrees: $3r^2 = 3 \times 9 = 27$ cm². Keep both numbers — the exact answer we are about to find must land near them.

Rearranging the circle into a near-rectangle

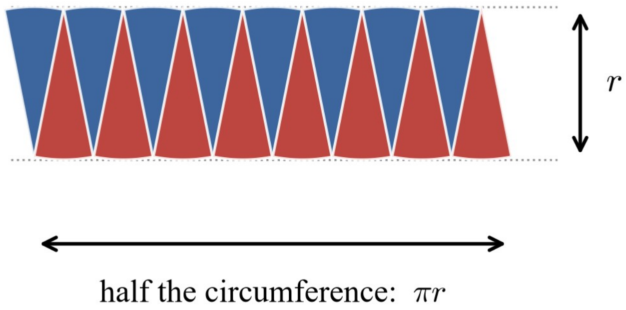
Now the exact answer. Cut the circle into equal **sectors** (pie slices) — sixteen of them here.



A circle divided into 16 equal sectors, coloured alternately blue and red

Then rearrange the sectors, pointing alternately up and down:

16 sectors interleaved — with more and thinner sectors, the shape sits closer and closer to a rectangle



The 16 sectors arranged one up, one down into a near-rectangle: the height is the radius r and the arcs along the bottom (and the top) are half the circumference each, pi r; with more and thinner sectors the shape sits closer and closer to a rectangle

The pieces now make a near-rectangle. Its **height** is a sector's length, the radius r. Its **bottom edge** is made of eight of the sixteen arcs — half of the circle's rim — so the arcs along the bottom add up to half the circumference: $1/2 \times 2\pi r = \pi r$. The top edge is the other half. The area of a rectangle is length × height, so:

$$\text{area} \approx \pi r \times r = \pi r^2$$

With 32 sectors, 64, more and thinner, the wavy edges straighten and the shape becomes closer and closer to a true rectangle of length πr and height r — the answer it closes in on never changes:

$$A = \pi r^2$$

That is why the formula is true, not a rule to memorise. Check it against the circle we counted: $r = 3$ cm gives $A \approx 3.14 \times 9 = 28.26$ cm², in the trap $18 < 28.26 < 36$ and matching the grid count of about 28 cm².

The formulas at a glance

Circumference	Area	Working backwards
$C = \pi d$ $C = 2\pi r$	$A = \pi r^2$ $(\pi \times r \times r)$	$d = 2r, \quad r = d \div 2$ $r = C \div 2\pi$ $r = \sqrt{A \div \pi}$

use $\pi \approx 3.14$ unless told otherwise

Summary cards: circumference C = pi d = 2 pi r; area A = pi r-squared, meaning pi times r times r; working backwards, d = 2r and r = d divided by 2, r = C divided by 2 pi, r = the square root of A divided by pi; with the note use pi approximately 3.14 unless told otherwise

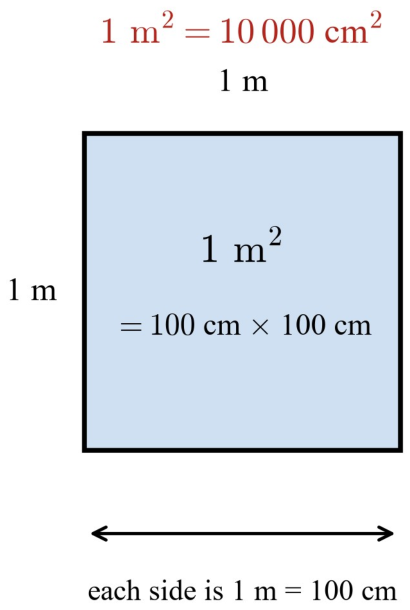
Read $A = \pi r^2$ as $A = \pi \times r \times r$: **square the radius first, then multiply by π.** Two slips to avoid:

- $2\pi r$ is the **circumference**, not the area — one π, one r.
- πr^2 is $\pi \times r \times r$, **not** $\pi \times 2 \times r$.

Appropriate units

A circumference is a length, so it takes length units: mm, cm, m, km. An area takes square units: mm², cm², m², km². Choose units that suit the object — a coin in millimetres, a pool or paddock in metres, a lake in kilometres.

Because square units are lengths squared, they scale by the square of the length change:

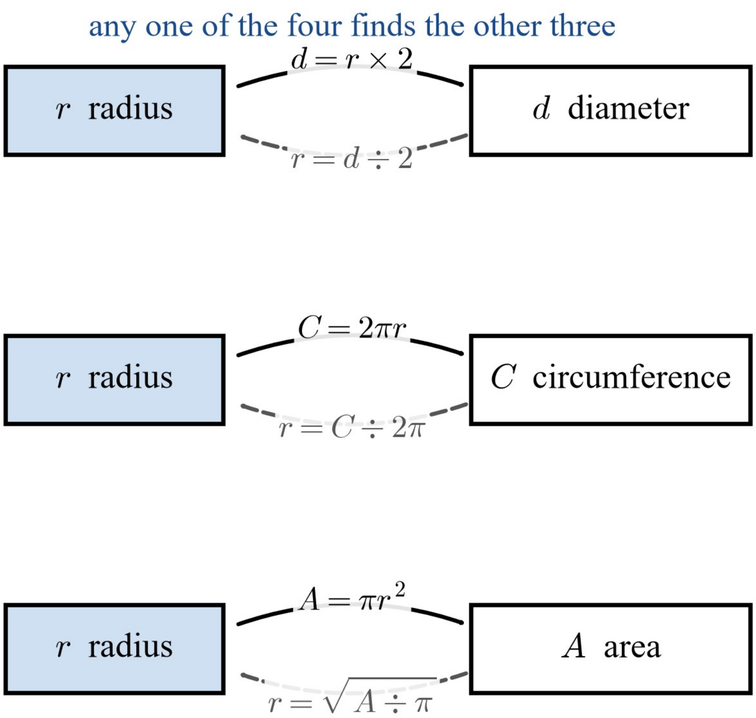


A square with sides labelled 1 m = 100 cm, showing 1 square metre = 100 cm times 100 cm = 10 000 square centimetres

1 m = 100 cm, so 1 m^2 = 100 × 100 = 10 000 cm^2. To change an area from cm^2 to m^2, divide by 10 000.

Working in all four directions

A circle is fixed by any **one** of the four numbers r, d, C, A — every road goes through the radius.



Three lanes joining the radius to each of the other measures: radius to diameter by times 2 and back by divide 2; radius to circumference by C = 2 pi r and back by r = C divided by 2 pi; radius to area by A = pi r-squared and back by r = the square root of A divided by pi; any one of the four finds the other three

The forward steps are the formulas; the backward steps undo them. From a circumference, $r = C \div 2\pi$. From an area, $A \div \pi$ gives r^2, and the square root of that gives r (square roots are Year 7 work, `VC2M7N01`). The classic version is a round tabletop: you are told the **area** of the top and asked for the length of the edging strip around the rim — area, back to radius, forward to circumference.

Before you leave

In Year 9, `VC2M9M01` builds cylinders and solids on top of this subtopic — the circle is the flat piece that work starts from. This subtopic keeps to the flat circle, with numerical answers; leaving an answer "in terms of π" is work for later years.

Words of this subtopic

Word	What it means here
centre	the point the circle is drawn about
radius (radii)	distance from the centre to the circle; r
diameter	distance straight across through the centre; $d = 2r$
circumference	the distance all the way around the circle; $C = \pi d = 2\pi r$
π (pi)	the exact times from diameter to circumference; irrational (1D); use ≈ 3.14 here
radius square	a square of side r, area r^2; the trap $2r^2 < A < 4r^2$ is built from them
sector	a pie slice of the circle; rearranging sectors shows why $A = \pi r^2$
square units	area units such as cm² and m²; 1 m² = 10 000 cm²
≈	"is approximately equal to" — every decimal answer with π is an approximation

Worked examples

Example 1 — with help

A circle has radius 6 cm. Find its circumference and its area (π ≈ 3.14).

Working	Comment
$C = 2\pi r \approx 2 \times 3.14 \times 6$	Circumference from the radius.
$C \approx 37.68$ cm	Length units for a length.
$A = \pi r^2 \approx 3.14 \times 6^2$	Square the radius **first**.
$A \approx 3.14 \times 36 = 113.04$ cm²	Square units for an area.
Trap: $2r^2 = 72$, $4r^2 = 144$, and $72 < 113.04 < 144$ ✓	The area sits in its trap.

Example 2 — with help

A circle has diameter 10 m. Find its circumference and its area.

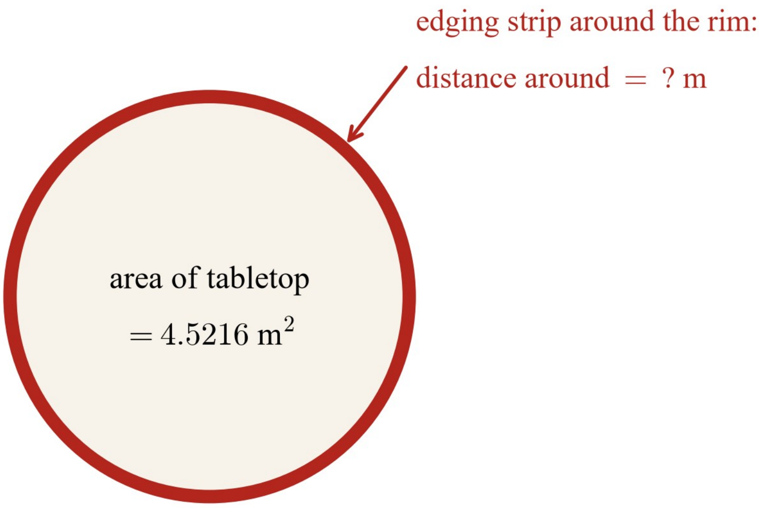
Working	Comment
$r = d \div 2 = 10 \div 2 = 5$ m	The road goes through the radius.
$C = \pi d \approx 3.14 \times 10 = 31.4$ m	With d given, $C = \pi d$ is quickest.
$A = \pi r^2 \approx 3.14 \times 25 = 78.5$ m²	Use the radius for area.

Example 3 — with help

A circle has circumference 62.8 cm. Find its radius, diameter and area.

Working	Comment
$r = C \div 2\pi \approx 62.8 \div 6.28 = 10$ cm	Undo $C = 2\pi r$.
$d = 2r = 20$ cm	Double the radius.
$A = \pi r^2 \approx 3.14 \times 100 = 314$ cm²	Then the area follows.

Example 4 — on your own



A circular tabletop seen from above, its area 4.5216 square metres written inside, and the edging strip around the rim marked as the unknown distance around

The top of a round table is a circle of area 4.5216 m². A metal strip is fitted around the rim. Find the length of the strip.

Work back from the area to the radius: $r^2 = A \div \pi \approx 4.5216 \div 3.14 = 1.44$, so $r = \sqrt{1.44} = 1.2$ m. The strip is the circumference: $C = 2\pi r \approx 2 \times 3.14 \times 1.2 = 7.536$ m.

∴ the strip is about 7.536 m long.

Example 5 — on your own

A bicycle wheel has radius 35 cm. How far does the bicycle travel in one full turn of the wheel?

One turn rolls out exactly one circumference: $C = 2\pi r \approx 2 \times 3.14 \times 35 = 219.8$ cm.

∴ the bicycle travels about 219.8 cm (2.198 m) per turn.

Example 6 — on your own

A circle has radius 50 cm. Find its area in cm², then in m², and check the two answers agree.

$A = \pi r^2 \approx 3.14 \times 50^2 = 3.14 \times 2500 = 7850$ cm². In metres, $r = 0.5$ m, so $A \approx 3.14 \times 0.5^2 = 3.14 \times 0.25 = 0.785$ m². Check: $7850 \div 10\,000 = 0.785$ ✓ — dividing by 10 000 is exactly the m²-to-cm² rule from the theory.

∴ $A \approx 7850$ cm² $= 0.785$ m².

Practice questions

Work by hand with π ≈ 3.14; no calculator is needed. For Q12 you will need 1 cm grid paper and a pair of compasses.

With help

Q1. Copy and complete.

radius	diameter
4 cm	?
7.5 m	?
?	13 mm
?	9 km

Q2. Find the circumference of a circle with:

a. radius 5 cm
b. diameter 12 m
c. radius 2.5 cm

Q3. Find the area of a circle with:

a. radius 4 cm
b. radius 10 m
c. diameter 6 cm

Q4. A circle has radius 7 cm.

a. Between which two numbers (in cm^2) must its area lie?
b. Give the rough estimate $3r^2$ for its area.

Q5. A circle has circumference 31.4 cm. Find its radius and its area.

Q6. A circle has area 200.96 cm^2. Find its radius and its diameter.

Q7. True or false? Correct each false statement.

a. Doubling the radius of a circle doubles its circumference.
b. Doubling the radius of a circle doubles its area.
c. The circumference of any circle is π times its diameter.
d. For every circle, πr^2 and $2\pi r$ are the same number.

Q8. Give the most appropriate unit for each.

a. the circumference of a coin
b. the area of a circular swimming pool
c. the area of a coin
d. the circumference of a circular lake

On your own

Q9. Copy and complete. Each row gives one fact about a circle; find the other three.

radius	diameter	circumference	area
3 cm	?	?	?
?	14 cm	?	?
?	?	94.2 cm	?
?	?	?	314 cm^2

Q10. A garden sprinkler wets a circle of radius 4 m. Find the area of ground it wets.

Q11. A circular garden bed has diameter 6 m. Edging is laid around it. Find the length of edging needed.

Q12. Draw a circle of radius 2 cm on 1 cm grid paper.

a. Estimate its area by counting whole and part squares.

b. Work out $A = \pi r^2$ and compare with your count.

Q13. A wheel has radius 30 cm.

a. Find its circumference.
b. How far does it roll in 10 full turns? Give your answer in metres.

Q14. A round table has radius 60 cm.

a. Find the area of the tabletop in cm^2.
b. Write the same area in m^2.

Q15. A circular rug has area 50.24 m^2. Binding is sewn around its edge. Find the length of binding needed.

Q16. A circle has area 314 cm^2. Find its radius, then decide — with a reason — whether its circumference is more or less than 60 cm.

Maths in real life

Q17. Australian circulating coins are circles. The $1 coin has diameter 25.00 mm and the $2 coin has diameter 20.50 mm. *(Source: Royal Australian Mint, circulating coin pages, retrieved 20 August 2026.)*

a. Find the circumference of the $1 coin.
b. Find the circumference of the $2 coin.
c. How much farther around is the $1 coin than the $2 coin?

Q18. A round canteen table has diameter 1.2 m.

a. Find the distance around the table.
b. Find the area of the tabletop.

Q19. A community garden makes a circular bed of radius 1.5 m.

a. Find the area of the bed.
b. Each seedling needs about 0.5 m^2. About how many seedlings fit?

Q20. One pizza has diameter 20 cm and another has diameter 40 cm. A friend says the bigger one is “twice the pizza”. Is that right? Compare the areas.

Answers

Q1. 8 cm; 15 m; 6.5 mm; 4.5 km.

Q2. a. 31.4 cm. b. 37.68 m. c. 15.7 cm.

Q3. a. 50.24 cm². b. 314 m². c. 28.26 cm².

Q4. a. Between $2 \times 49 = 98$ cm² and $4 \times 49 = 196$ cm². b. $3 \times 49 = 147$ cm².

Q5. $r = 5$ cm; $A = 78.5$ cm².

Q6. $r = 8$ cm; $d = 16$ cm.

Q7. a. True. b. False — the area becomes 4 times as big. c. True. d. False — the two numbers match only when $r = 2$, and they measure different things (area versus length).

Q8. a. mm (or cm). b. m². c. cm² (or mm²). d. km.

Q9. Row 1: 6 cm, 18.84 cm, 28.26 cm². Row 2: 7 cm, 43.96 cm, 153.86 cm². Row 3: 15 cm, 30 cm, 706.5 cm². Row 4: 10 cm, 20 cm, 62.8 cm.

Q10. 50.24 m².

Q11. 18.84 m.

Q12. a. About 12–13 cm². b. 12.56 cm², matching the count.

Q13. a. 188.4 cm. b. 1884 cm = 18.84 m.

Q14. a. 11 304 cm². b. 1.1304 m².

Q15. 25.12 m.

Q16. $r = 10$ cm; $C = 62.8$ cm, which is more than 60 cm.

Q17. a. 78.5 mm. b. 64.37 mm. c. 14.13 mm.

Q18. a. 3.768 m. b. 1.1304 m².

Q19. a. 7.065 m². b. About 14 seedlings.

Q20. No — the bigger pizza has 4 times the area (1256 cm² versus 314 cm²), not twice.

Fully-worked solutions

Q1. $d = 2r$: $4 \times 2 = 8$ cm; $7.5 \times 2 = 15$ m. $r = d \div 2$: $13 \div 2 = 6.5$ mm; $9 \div 2 = 4.5$ km.

Q2. a. $C = 2\pi r \approx 2 \times 3.14 \times 5 = 31.4$ cm. b. $C = \pi d \approx 3.14 \times 12 = 37.68$ m. c. $C \approx 2 \times 3.14 \times 2.5 = 15.7$ cm.

Q3. a. $A \approx 3.14 \times 4^2 = 3.14 \times 16 = 50.24$ cm². b. $A \approx 3.14 \times 100 = 314$ m². c. $r = 6 \div 2 = 3$ cm, $A \approx 3.14 \times 9 = 28.26$ cm².

Q4. a. The trap: $2r^2 = 2 \times 49 = 98$ and $4r^2 = 4 \times 49 = 196$, so the area is between 98 cm² and 196 cm². b. $3r^2 = 3 \times 49 = 147$ cm².

Q5. $r = C \div 2\pi \approx 31.4 \div 6.28 = 5$ cm. Then $A \approx 3.14 \times 25 = 78.5$ cm².

Q6. $r^2 = A \div \pi \approx 200.96 \div 3.14 = 64$, so $r = \sqrt{64} = 8$ cm, and $d = 2r = 16$ cm.

Q7. a. **True**: $C = 2\pi r$, so doubling r doubles C. b. **False**: $A = \pi r^2$, so doubling r gives $\pi \times 2r \times 2r = 4\pi r^2$ — four times the area. c. **True**: $C = \pi d$ is the definition of π. d. **False**: πr^2 is an area and $2\pi r$ a length, so they measure different things; the numbers agree only at $r = 2$ (try $r = 1$: 3.14 versus 6.28).

Q8. a. A coin is small: millimetres (centimetres acceptable). b. A pool is metre-scale: m². c. A coin face: cm² (or mm²). d. A lake is kilometre-scale: km.

Q9. Row 1: $d = 6$ cm; $C \approx 2 \times 3.14 \times 3 = 18.84$ cm; $A \approx 3.14 \times 9 = 28.26$ cm². Row 2: $r = 7$ cm; $C \approx 3.14 \times 14 = 43.96$ cm; $A \approx 3.14 \times 49 = 153.86$ cm². Row 3: $r = 94.2 \div 6.28 = 15$ cm; $d = 30$ cm; $A \approx 3.14 \times 225 = 706.5$ cm². Row 4: $r^2 = 314 \div 3.14 = 100$, $r = 10$ cm; $d = 20$ cm; $C \approx 62.8$ cm.

Q10. $A = \pi r^2 \approx 3.14 \times 16 = 50.24$ m².

Q11. The edging is the circumference: $C = \pi d \approx 3.14 \times 6 = 18.84$ m.

Q12. a. Counting gives 4 whole squares and part squares adding about 8–9 more, so about 12–13 cm². b. $A \approx 3.14 \times 4 = 12.56$ cm² — the count and the formula agree, and the trap $8 < 12.56 < 16$ holds.

Q13. a. $C \approx 2 \times 3.14 \times 30 = 188.4$ cm. b. $10 \times 188.4 = 1884$ cm $= 18.84$ m.

Q14. a. $A \approx 3.14 \times 60^2 = 3.14 \times 3600 = 11\,304$ cm². b. $11\,304 \div 10\,000 = 1.1304$ m².

Q15. $r^2 = 50.24 \div 3.14 = 16$, so $r = 4$ m; the binding is the circumference, $C \approx 2 \times 3.14 \times 4 = 25.12$ m.

Q16. $r^2 = 314 \div 3.14 = 100$, so $r = 10$ cm; $C \approx 2 \times 3.14 \times 10 = 62.8$ cm > 60 cm. (Check with the trap: $2r^2 = 200 < 314 < 400 = 4r^2$ ✓.)

Q17. a. $C = \pi d \approx 3.14 \times 25.00 = 78.5$ mm. b. $C \approx 3.14 \times 20.50 = 64.37$ mm. c. $78.5 - 64.37 = 14.13$ mm.

Q18. a. $C \approx 3.14 \times 1.2 = 3.768$ m. b. $r = 0.6$ m; $A \approx 3.14 \times 0.36 = 1.1304$ m².

Q19. a. $A \approx 3.14 \times 2.25 = 7.065$ m². b. $7.065 \div 0.5 = 14.13$, so about 14 seedlings (the 0.13 of a seedling does not get planted).

Q20. Small: $r = 10$ cm, $A \approx 3.14 \times 100 = 314$ cm². Big: $r = 20$ cm, $A \approx 3.14 \times 400 = 1256$ cm². $1256 \div 314 = 4$: the bigger pizza is **four** times the pizza, not twice. Twice the diameter means four times the area — which is why the bigger one is usually better value.

Volume and capacity of right prisms

> **Content description `VC2M8M02`:** solve problems involving the volume and capacity of right prisms using appropriate units

The juice carton says 250 mL, the fish tank says 100 L, and the concrete truck sells by the cubic metre. Three different units for the same question: how much space does something take up, or how much can it hold? In Year 7 you counted cubes inside boxes and tent-shaped prisms. This subtopic turns that into one formula that works for every right prism, and joins the cubic-centimetre world to the litre world — the conversion you will keep using in maths and in science.

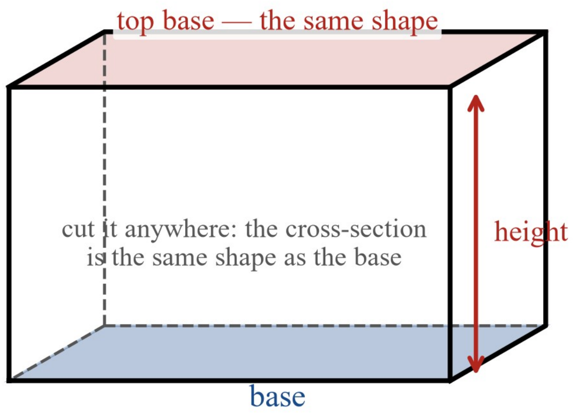
Theory

Recall: volume from Year 7

In Year 7 (`VC2M7M02`) you found the volume of boxes and triangular prisms by counting unit cubes, and you used $V = l \times w \times h$ for a box. This subtopic builds on that; it does not start again.

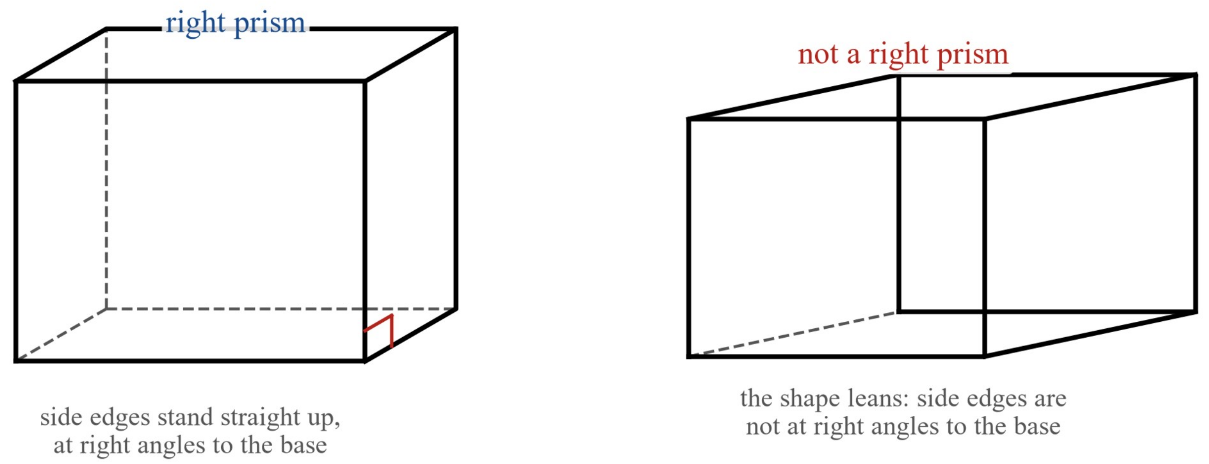
What is a right prism?

A **prism** is a solid with two identical bases facing each other, and the same cross-section all the way through. A **right prism** is a prism whose side edges stand at right angles to the bases, so the top base sits exactly over the bottom one.



A rectangular prism with the bottom base shaded blue and the matching top base shaded red, an arrow marking the height, and the note that a cut at any height gives a cross-section the same shape as the base

The base does not have to be a rectangle. This subtopic uses rectangle, triangle and parallelogram bases. Two kinds of solid are **not** right prisms, and both wait for another subtopic:

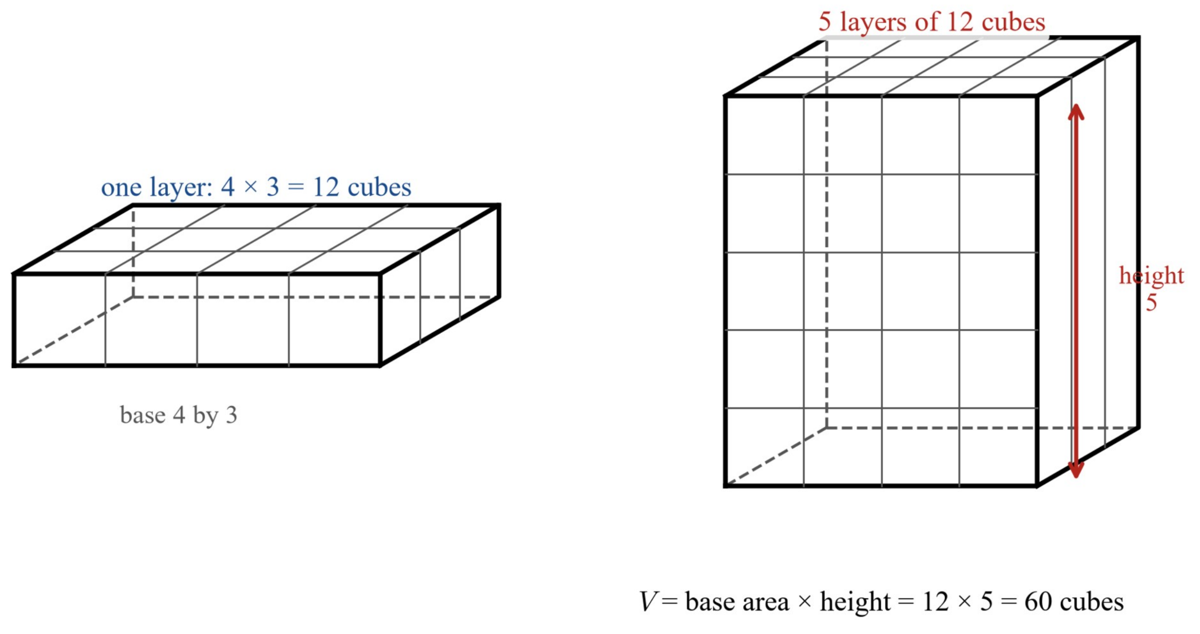


Left, a right prism with a small square corner mark showing a side edge meets the base at right angles; right, a leaning solid labelled not a right prism because its side edges are not at right angles to the base

A solid whose side edges lean is not a right prism. A solid whose base is a circle — a **cylinder**, the tin-can shape — is not a prism at all; it waits for Year 9 (`VC2M9M01`).

One formula for every right prism

Build a prism out of unit cubes. The bottom layer is the base, and every layer above it is exactly the same.



Left, one layer of cubes on a 4 by 3 base, twelve cubes; right, the same base stacked five layers high, with V = base area × height = 12 × 5 = 60 cubes

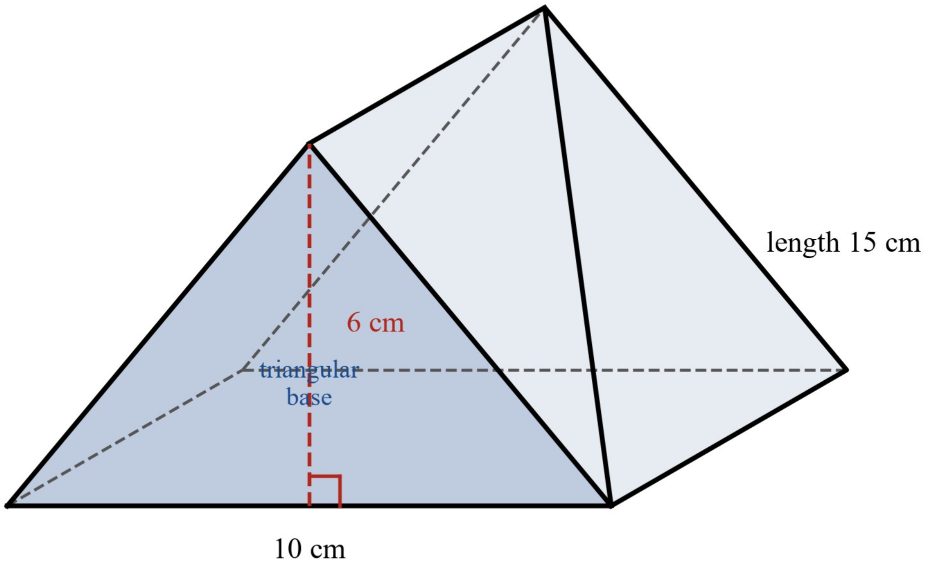
Counting cubes is (cubes in one layer) × (number of layers), and the cubes in one layer is the **area of the base**. So one formula covers every right prism:

Volume of a right prism. $V = A \times h$, where A is the area of the base and h is the height.

For a box, $A = l \times w$, so $V = l \times w \times h$ — the Year 7 rule. The new idea is that the base may be any shape whose area you can find.

Triangular prisms

The triangular base has area $A = \frac{1}{2} \times b \times t$, where b is the base of the triangle and t is the triangle's height, at right angles to b — never a slant side. Then multiply by the length of the prism.

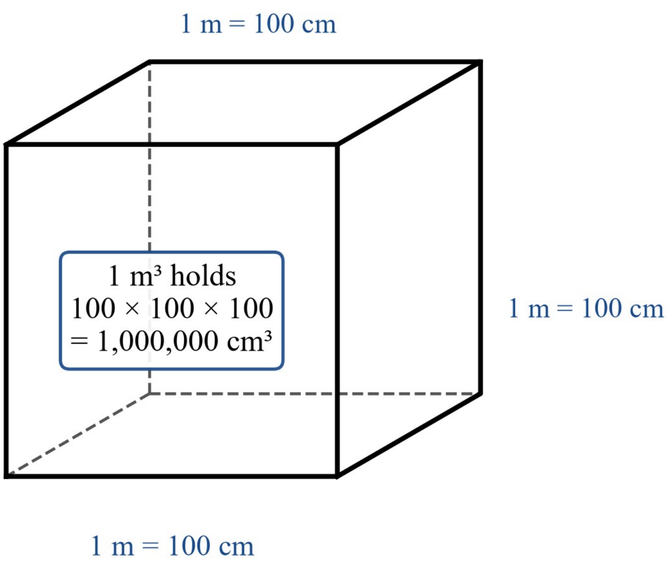


A triangular prism, tent-shaped, with the triangular base shaded, its base 10 cm and height 6 cm marked with a dashed height line and a small square corner mark, and the length 15 cm marked along the prism

$$A = 1/2 \times 10 \times 6 = 30\,\text{cm}^2, V = 30 \times 15 = 450\,\text{cm}^3$$

Volume units: from cm³ to m³

Small volumes are measured in **cubic centimetres** (cm^3) and large ones in **cubic metres** (m^3). How many cm^3 fit in one m^3? Picture a cube one metre tall. Each edge is 100 cm long, so one layer holds $100 \times 100 = 10,000$ cubes, and there are 100 layers:



A cube with every edge labelled 1 m = 100 cm and a panel stating that one cubic metre holds 100 × 100 × 100 = 1,000,000 cubic centimetres

$$1\,\mathrm{m}^3 = 100 \times 100 \times 100 = 1{,}000{,}000\,\mathrm{cm}^3$$

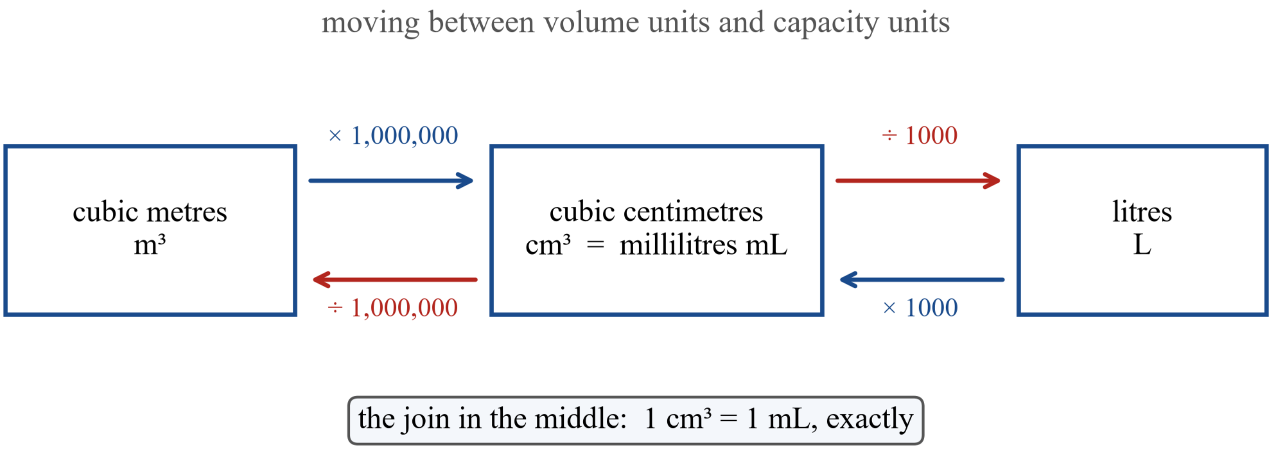
The jump is ×1,000,000 — not ×100 and not ×1000 — because the ×100 happens three times, once for each edge. This is the conversion students most often get wrong.

Capacity: litres and millilitres

Capacity is how much a container can hold. It is measured in **litres** (L) and **millilitres** (mL), with $1\text{ L} = 1000\text{ mL}$. Capacity joins onto volume through one exact fact:

$1\text{ mL} = 1\text{ cm}^3$. A cube 1 cm on each edge holds exactly 1 mL.

From that fact the rest follows: 1000 cm^3 hold $1000\text{ mL} = 1\text{ L}$, so $1\text{ L} = 1000\text{ cm}^3$; and $1\text{ m}^3 = 1{,}000{,}000\text{ cm}^3 = 1{,}000{,}000\text{ mL} = 1000\text{ L}$.



A conversion map with three boxes: cubic metres, cubic centimetres equal to millilitres, and litres; arrows × 1,000,000 and ÷ 1,000,000 between the first two, ÷ 1000 and × 1000 between the last two, and the note that the join in the middle is 1 cm³ = 1 mL exactly

So when a question gives dimensions in cm and asks for a capacity in L: find the volume in cm^3, read it as mL, then divide by 1000.

Choosing the useful measurement

The same object can be measured in different ways, and the job decides which measurement matters.

- Will the fridge fit in the gap under the bench? The **dimensions** — height, width and depth — decide. The capacity is no help.
- How many drinks does the esky hold for the picnic? The **capacity** in litres decides.
- How much water fills the paddling pool? Both views agree: the volume of the water equals the capacity of the pool.

Read what the question asks for, and answer in the unit that fits it.

Where this leads

In Year 9 (`VC2M9M01`) the same $V = A \times h$ idea is carried on to cylinders and to solids built from more than one shape, and the area of the outside faces joins the story. This subtopic stops at right prisms, with volume and capacity only.

3C summary — volume and capacity of right prisms
Volume of any right prism:
$V = A \times h$ (area of base × height)
Capacity link:
1 mL = 1 cm^3 and 1 L = 1000 cm^3
Big unit:
1 m^3 = 1,000,000 cm^3 = 1000 L
Choose units to fit the job:
cm^3 and mL for small amounts, m^3 and L for large

Summary card: volume of any right prism $V = A \times h$; capacity link 1 mL = 1 cm^3 and 1 L = 1000 cm^3; big unit 1 m^3 = 1,000,000 cm^3 = 1000 L; choose units to fit the job

Words of this subtopic

Word	What it means here
prism	a solid with two identical bases facing each other and the same cross-section all the way through
right prism	a prism whose side edges stand at right angles to the bases
base	one of the two identical faces of a prism
cross-section	the shape you see when the prism is cut parallel to the base; the same at every height
volume	the space a solid takes up, in cm^3 or m^3
capacity	how much a container can hold, in mL or L
$V = A \times h$	the volume of any right prism: area of base × height

Word	What it means here
1 mL = 1 cm³	the exact join between capacity and volume
1 L = 1000 cm³	follows from 1 mL = 1 cm³
1 m³ = 1,000,000 cm³ = 1000 L	the big conversion, from the 100 cm cube

Worked examples

Example 1 — with help

A box is 12 cm long, 8 cm wide and 5 cm high. Find its volume.

Working	Comment
$A = 12 \times 8 = 96$ cm²	The base is a 12 cm by 8 cm rectangle.
$V = A \times h = 96 \times 5 = 480$ cm³	Multiply the area of the base by the height.

∴ the volume of the box is 480 cm³. Check with Year 7: $12 \times 8 \times 5 = 480$. ✓

Example 2 — with help

A tent-shaped box is a triangular prism. Its triangular base is 10 cm wide and 6 cm high, and the box is 15 cm long. Find its volume.

Working	Comment
$A = \frac{1}{2} \times 10 \times 6 = 30$ cm²	Area of the triangular base.
$V = A \times h = 30 \times 15 = 450$ cm³	The height of this prism is its length, 15 cm.

∴ the volume of the box is 450 cm³.

Example 3 — with help

A small tank is a rectangular prism 20 cm long, 15 cm wide and 12 cm high. Find the capacity of the full tank in litres.

Working	Comment
$V = 20 \times 15 \times 12 = 3600$ cm³	The volume in cubic centimetres.
3600 cm³ $= 3600$ mL	1 cm³ = 1 mL.
$3600 \div 1000 = 3.6$ L	1000 mL = 1 L.

∴ the full tank holds 3.6 L.

Example 4 — on your own

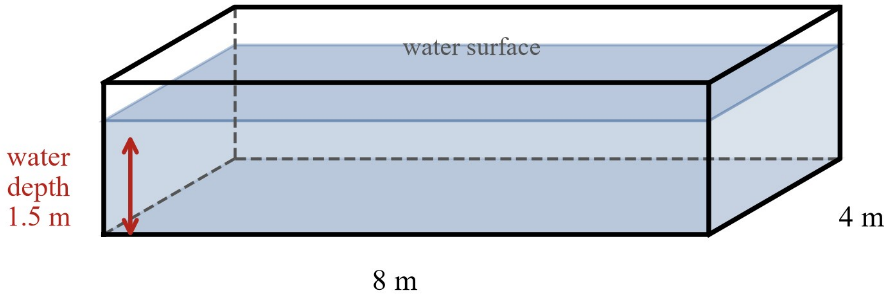
Convert 2.5 m³ into cubic centimetres, and then into litres.

$2.5 \times 1,000,000 = 2,500,000$ cm³. Reading cm³ as mL and dividing by 1000 gives $2,500,000 \div 1000 = 2500$ L.

∴ 2.5 m³ $= 2,500,000$ cm³ $= 2500$ L. Shortcut for next time: 1 m³ $= 1000$ L, so $2.5 \times 1000 = 2500$ L straight away.

Example 5 — on your own

A pool is 8 m long and 4 m wide, filled to a water depth of 1.5 m. A garden tap adds water at 16 L per minute. How long does the tap take to fill the pool to this depth, starting from empty?



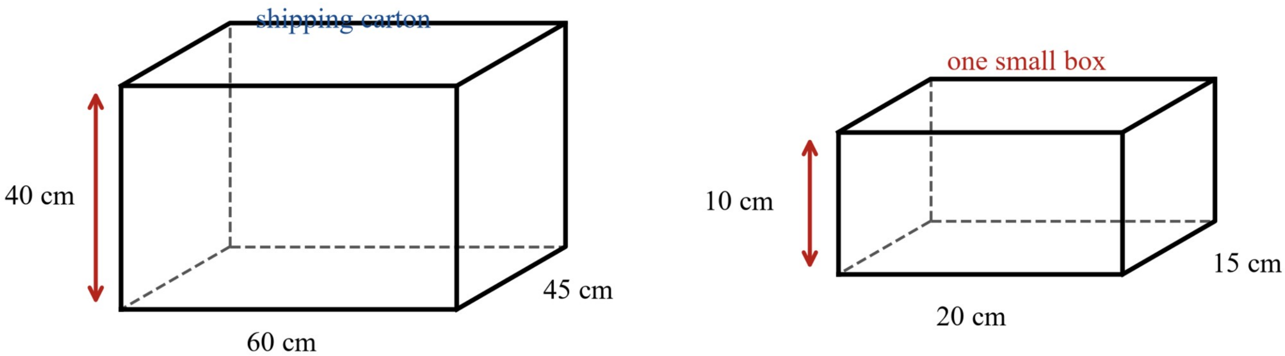
The pool of Worked example 5, an open box 8 m by 4 m, with the water shaded to a depth of 1.5 m

$V = 8 \times 4 \times 1.5 = 48$ m³. Using 1 m³ $= 1000$ L, the pool holds $48 \times 1000 = 48{,}000$ L. Time $= 48{,}000 \div 16 = 3000$ minutes, and $3000 \div 60 = 50$ hours.

∴ the tap takes 3000 minutes, or 50 hours. Filling a pool from a garden tap is a slow job — the volume tells you why.

Example 6 — on your own

A factory packs small boxes measuring 20 cm by 15 cm by 10 cm into a shipping carton measuring 60 cm by 45 cm by 40 cm. The small boxes all face the same way and pack with no gaps. How many small boxes fit in one carton?



The shipping carton 60 cm by 45 cm by 40 cm on the left and one small box 20 cm by 15 cm by 10 cm on the right

Count boxes along each edge: $60 \div 20 = 3$, $45 \div 15 = 3$, $40 \div 10 = 4$. So $3 \times 3 \times 4 = 36$ boxes.

Check with volumes: the carton holds $60 \times 45 \times 40 = 108{,}000$ cm³; each small box is $20 \times 15 \times 10 = 3000$ cm³; and $108{,}000 \div 3000 = 36$. ✓

∴ 36 small boxes fit in one carton.

Practice questions

Every number is chosen so the working runs by hand — no calculator needed.

With help

Q1. Find the volume of each rectangular prism.

a. 9 cm by 4 cm by 5 cm
b. 12 cm by 6 cm by 10 cm

c. a cube of side 7 cm

Q2. Find the volume of each triangular prism.

a. triangular base 8 cm wide and 5 cm high; length 12 cm
b. triangular base 6 cm wide and 9 cm high; length 20 cm

Q3. Convert.

a. 250 cm^3 to mL
b. 3 L to mL
c. 4500 mL to L
d. 8000 cm^3 to L

Q4. Convert.

a. 4 m^3 to cm^3
b. 350,000 cm^3 to m^3
c. 2 m^3 to L
d. 6000 L to m^3

Q5. A tank is a rectangular prism 30 cm long, 20 cm wide and 25 cm high. Find the capacity of the full tank in litres.

Q6. True or false? Give a reason, and correct each false statement.

a. 1 L = 1000 cm^3
b. 1 m^3 = 1000 cm^3
c. 1 mL = 1 cm^3
d. the volume of a right prism is the area of the base plus the height
e. 2500 mL = 2.5 L

Q7. Copy and complete.

a. 1.2 m^3 = $\square$ cm^3
b. 7 L 250 mL = $\square$ mL
c. 4600 mL = $\square$ L
d. 0.5 m^3 = $\square$ L

Q8. For each job, say which measurement matters more — the dimensions, or the capacity — and give your reason.

a. Will the washing machine fit in the gap in the laundry?
b. How much water can a bucket carry for the car wash?
c. Which of two drink bottles holds more for the sports bag?

On your own

Q9. A prism has a parallelogram base 14 cm wide with a height of 8 cm, and the prism is 20 cm tall.

a. Find its volume in cm^3.
b. If it were a water container, what would its capacity be in litres?

Q10. A rectangular prism has a volume of 3000 cm^3. Its base is 25 cm by 12 cm. Find the height of the prism.

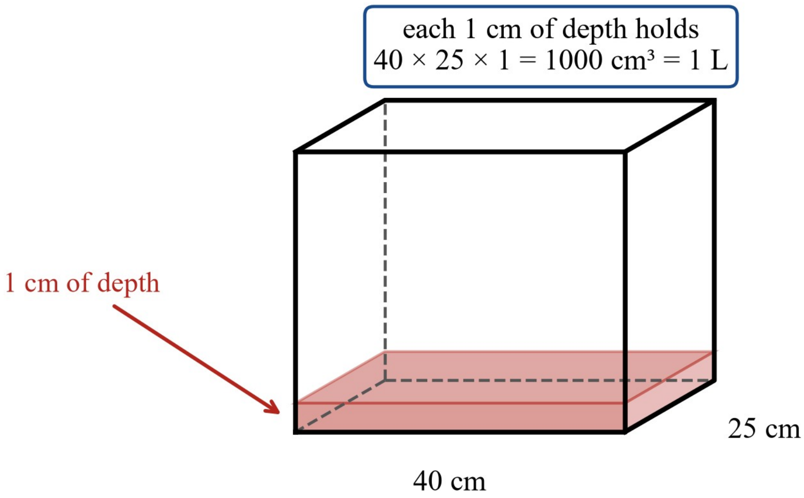
Q11. Container A is a rectangular prism 15 cm by 10 cm by 20 cm. Container B is a triangular prism whose triangular base is 20 cm wide and 12 cm high, with a length of 24 cm. Which container holds more, and by how much? Give the difference in mL.

Q12. Convert.

a. 0.75 m^3 to L
b. 1250 L to m^3

c. 3.05 L to mL
d. 45,000 cm³ to L

Q13. A tank has a rectangular base 40 cm by 25 cm.



The tank of Q13, a box with base 40 cm by 25 cm and a thin shaded slice at the bottom labelled 1 cm of depth, each 1 cm of depth holding 40 × 25 × 1 = 1000 cm³ = 1 L

a. Show that each 1 cm of depth holds 1 L of water.
b. The tank is filled to a depth of 30 cm. How many litres are in it?

Q14. A warehouse packs cartons measuring 10 cm by 6 cm by 4 cm into a crate measuring 60 cm by 30 cm by 20 cm. The cartons all face the same way and pack with no gaps. How many cartons fit in one crate? Check your answer using volumes.

Q15. A tank is 1.2 m long, 0.8 m wide and 0.5 m high. A tap fills it at 12 L per minute. How long does the tap take to fill the tank from empty?

Q16. Tom says: “Because 1 m = 100 cm, one cubic metre must be 100 cm³.” Explain why Tom is wrong, and find the correct number of cm³ in one m³.

Maths in real life

Q17. A home aquarium is 90 cm long and 40 cm wide, filled to a water depth of 45 cm. How many litres of water are in it?

Q18. A rainwater tank has a base area of 1.8 m² and stands 1.6 m high. How many litres does the full tank hold?

Q19. A concrete slab for a garden shed is 6 m long, 4.5 m wide and 0.1 m thick. Find the volume of concrete in the slab, in m³.

Q20. Make your own. A carton must hold exactly 1 L, which is 1000 cm³. Find three different sets of whole-centimetre dimensions (length, width, height) that give a rectangular carton of volume 1000 cm³. Which one would you choose for a juice carton, and why? Swap with a partner and check each other’s volumes.

Answers

Q1. a. 180 cm^3. b. 720 cm^3. c. 343 cm^3.

Q2. a. 240 cm^3. b. 540 cm^3.

Q3. a. 250 mL. b. 3000 mL. c. 4.5 L. d. 8 L.

Q4. a. 4,000,000 cm^3. b. 0.35 m^3. c. 2000 L. d. 6 m^3.

Q5. 15 L.

Q6. a. True. b. False; 1 m^3 = 1,000,000 cm^3. c. True. d. False; it is the area of the base **times** the height. e. True.

Q7. a. 1,200,000. b. 7250. c. 4.6. d. 500.

Q8. a. The dimensions — the gap is about space, not holding. b. The capacity. c. The capacity.

Q9. a. 2240 cm^3. b. 2.24 L.

Q10. 10 cm.

Q11. Container A holds more, by 120 mL.

Q12. a. 750 L. b. 1.25 m^3. c. 3050 mL. d. 45 L.

Q13. a. $40 \times 25 \times 1 = 1000$ cm^3 $= 1000$ mL $= 1$ L. b. 30 L.

Q14. 150 cartons.

Q15. 40 minutes.

Q16. The ×100 must happen once for each of the three edges: 1 m^3 = 1,000,000 cm^3.

Q17. 162 L.

Q18. 2880 L.

Q19. 2.7 m^3.

Q20. Answers will vary; any three sets whose product is 1000, with a sensible choice explained.

Fully-worked solutions

Q1. Use $V = l \times w \times h$.

a. $9 \times 4 \times 5 = 180$ cm^3.
b. $12 \times 6 \times 10 = 720$ cm^3.
c. $7 \times 7 \times 7 = 343$ cm^3.

Q2. Find the area of the triangular base, then multiply by the length.

a. $A = \frac{1}{2} \times 8 \times 5 = 20$ cm^2; $V = 20 \times 12 = 240$ cm^3.

b. $A = \frac{1}{2} \times 6 \times 9 = 27$ cm^2; $V = 27 \times 20 = 540$ cm^3.

Q3. Use 1 cm^3 = 1 mL and 1 L = 1000 mL.

a. 250 cm^3 = 250 mL.
b. 3 L = 3000 mL.
c. 4500 mL = 4.5 L.

d. 8000 cm^3 = 8000 mL = 8 L.

Q4. Use 1 m^3 = 1,000,000 cm^3 and 1 m^3 = 1000 L.

a. 4 m^3 = 4,000,000 cm^3.
b. 350,000 cm^3 = 0.35 m^3 (divide by 1,000,000).
c. 2 m^3 = 2000 L.
d. 6000 L = 6 m^3.

Q5. $V = 30 \times 20 \times 25 = 15,000$ cm^3 $= 15,000$ mL, and $15,000 \div 1000 = 15$ L.

Q6. Test each claim against the unit facts.

a. **True.** 1 L = 1000 mL = 1000 cm^3.
b. **False.** The ×100 happens on each of the three edges: 1 m^3 = 1,000,000 cm^3.
c. **True.** A 1 cm cube holds exactly 1 mL.
d. **False.** Volume is the area of the base **times** the height: $V = A \times h$. Adding would mix up cm^2 with cm.
e. **True.** 2500 mL = 2.5 L.

Q7. One conversion each way.

a. $1.2 \times 1,000,000 = 1,200,000$ cm^3.
b. $7 \times 1000 + 250 = 7250$ mL.
c. $4600 \div 1000 = 4.6$ L.
d. 0.5 m^3 = 500 L, using 1 m^3 = 1000 L.

Q8. Match the measurement to the job.

a. The **dimensions** — the question is whether the machine fits the gap, so height, width and depth decide.
b. The **capacity** — the question is how much water the bucket carries.
c. The **capacity** — the question is which bottle holds more.

Q9. The base is a parallelogram: $A = 14 \times 8 = 112$ cm^2.

a. $V = 112 \times 20 = 2240$ cm^3.
b. 2240 cm^3 = 2240 mL = 2.24 L.

Q10. Work backwards from $V = A \times h$. $A = 25 \times 12 = 300$ cm^2, so $h = 3000 \div 300 = 10$ cm.

Q11. Find each capacity and compare.

A: $15 \times 10 \times 20 = 3000$ cm^3 = 3000 mL. B: $A = \frac{1}{2} \times 20 \times 12 = 120$ cm^2; $V = 120 \times 24 = 2880$ cm^3 = 2880 mL.

$3000 - 2880 = 120$, so container A holds more, by 120 mL.

Q12. a. 0.75 m^3 = 750,000 cm^3 = 750,000 mL = 750 L. (Shortcut: 1 m^3 = 1000 L.) b. 1250 L = 1.25 m^3. c. 3.05 L = 3050 mL. d. 45,000 cm^3 = 45,000 mL = 45 L.

Q13. a. A slice 1 cm deep has volume $40 \times 25 \times 1 = 1000$ cm^3, which is 1000 mL, which is 1 L. So each centimetre of depth holds 1 L. b. 30 cm of depth holds $30 \times 1 = 30$ L.

Q14. Count along each edge: $60 \div 10 = 6$, $30 \div 6 = 5$, $20 \div 4 = 5$, so $6 \times 5 \times 5 = 150$ cartons. Check with volumes: the crate is $60 \times 30 \times 20 = 36,000$ cm^3; each carton is $10 \times 6 \times 4 = 240$ cm^3; $36,000 \div 240 = 150$. ✓

Q15. $V = 1.2 \times 0.8 \times 0.5 = 0.48$ m^3 $= 480$ L. Time $= 480 \div 12 = 40$ minutes.

Q16. A 1 m cube is 100 cm long, 100 cm wide **and** 100 cm high. The ×100 must be applied once for each edge, so 1 m^3 $= 100 \times 100 \times 100 = 1,000,000$ cm^3, not 100 cm^3. Tom applied the length conversion only once.

Q17. $V = 90 \times 40 \times 45 = 162{,}000$ cm³ $= 162{,}000$ mL $= 162$ L.

Q18. $V = A \times h = 1.8 \times 1.6 = 2.88$ m³ $= 2880$ L, using 1 m³ = 1000 L.

Q19. $V = 6 \times 4.5 \times 0.1 = 2.7$ m³.

Q20. Answers will vary. A sample: 10 × 10 × 10, 20 × 10 × 5 and 25 × 8 × 5 all give 1000 cm³. A sensible choice is 20 × 10 × 5 or 25 × 8 × 5, because a juice carton that is 10 cm on every side is short and wide for a shelf, while the taller shapes stand and pour more easily. Check a partner's work by multiplying the three dimensions to get 1000.

Pythagoras’ theorem

> **Content description `VC2M8M06`:** use Pythagoras’ theorem to solve problems involving the side lengths of right-angled triangles

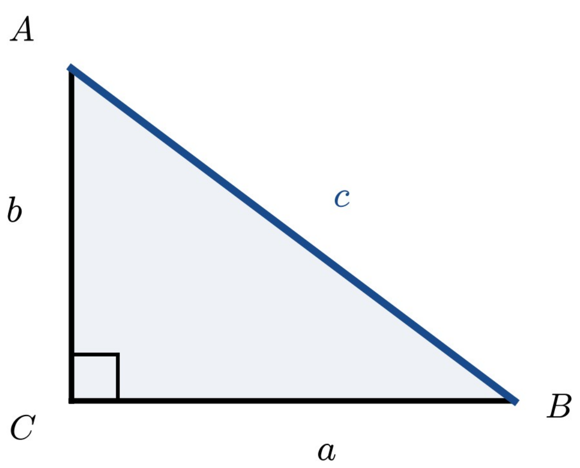
A builder with nothing but a tape measure can still set out a perfectly square corner. A screen’s advertised size is the length of a line you can never lay a ruler along. Both tricks are the same idea, and it is an old one: clay tablets show it was known almost 4000 years ago. This subtopic meets the theorem behind them — what it says, why it is true, and how one equation finds any missing side of a right-angled triangle.

Theory

In Year 7 you classified triangles by their angles (`VC2M7SP02`) — right-angled, acute and obtuse — and you worked with perfect squares and square roots (`VC2M7N01`). This subtopic joins those two ideas with one famous fact about right-angled triangles: **Pythagoras’ theorem**.

The parts of a right-angled triangle

A right-angled triangle has one 90° angle, usually marked with a small square at the corner. The side opposite the right angle is called the **hypotenuse**. It is always the longest side. The other two sides are the two shorter sides.

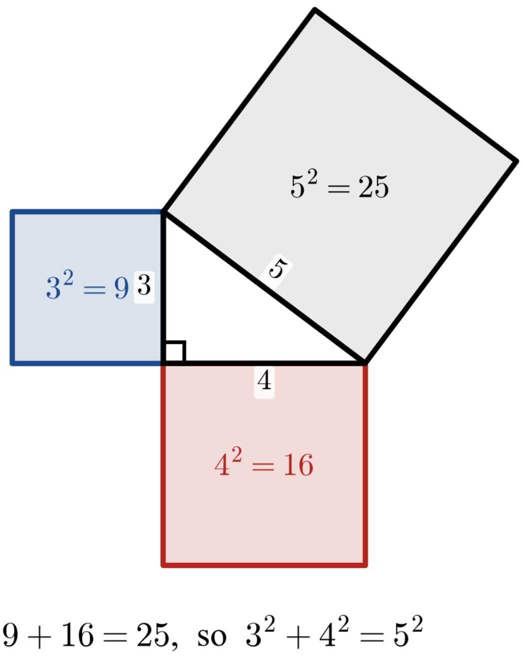


A right-angled triangle ABC with the right angle at C: the hypotenuse c in blue, opposite the right angle, and the two shorter sides a and b

In the picture the right angle is at C, so the hypotenuse is c — the one side that is not part of the right angle. Whatever way the triangle faces, the rule for finding the hypotenuse never changes: look opposite the right angle.

What the theorem says

Draw a square on each side of a right-angled triangle, and compare the areas.



A 3-4-5 right-angled triangle with a square drawn on each side: the squares on the shorter sides have areas 9 and 16, and the square on the hypotenuse has area 25, so 9 + 16 = 25

The picture shows the theorem as areas: the two smaller squares together cover exactly the same area as the big one — $9+16=25$. Written with the side names from the first picture, that is

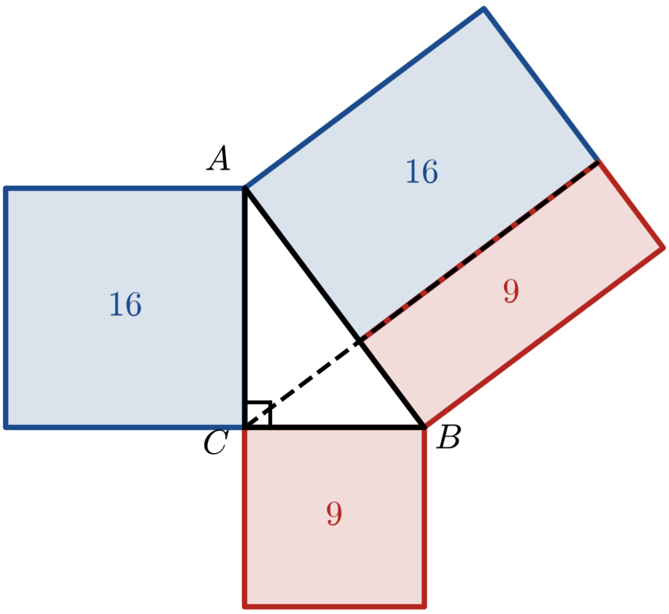
$a^2+b^2=c^2$, where c is the hypotenuse.

Pythagoras' theorem. In a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the two shorter sides.

Read it both ways. Going up, $c^2=a^2+b^2$: square the shorter sides, add, then take the square root to find the hypotenuse. Going down, $a^2=c^2-b^2$: square the hypotenuse, subtract a shorter side's square, then take the square root to find a shorter side. Both directions are used all the time, so this subtopic teaches both.

A proof from ancient Greece

The theorem was proved — not just noticed — many times in history. The most famous proof is in the *Elements*, a mathematics book written around 300 BC by Euclid of Alexandria. It is Book I, Proposition 47. Here is Euclid's idea, using the same 3-4-5 triangle.

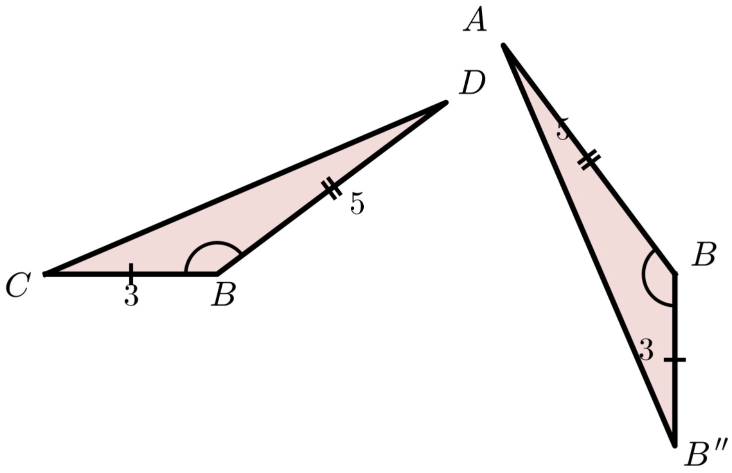


the square on the hypotenuse (area 25) cuts into a 16-rectangle and a 9-rectangle

Euclid's proof picture for a 3-4-5 right-angled triangle: a line through the right angle, at right angles to the hypotenuse, cuts the square on the hypotenuse into two rectangles - a blue rectangle of area 16, the same as the square on one shorter side, and a red rectangle of area 9, the same as the square on the other

Euclid draws the three squares, then a line from the corner of the right angle, at right angles to the hypotenuse, continued across the big square. That line cuts the square on the hypotenuse into two rectangles. His claim: the left rectangle has exactly the area of the square on the left side (16), and the right rectangle exactly the area of the square on the bottom side (9). Add the two rectangles and you have the whole big square — so the two smaller squares' areas add up to the big one.

Why does each rectangle match its square? Euclid's trick is to fit one triangle into both shapes, so that each shape is exactly twice that triangle. The next picture shows the matching pair for the bottom square and its rectangle: two sides and the angle between them are the same in both triangles, so the triangles fit exactly.



two sides and the included angle match — the triangles fit exactly

The matching triangle pair from Euclid's proof, drawn twice: each has a side of 3 and a side of 5 with the same angle between them, so the triangles fit exactly - one is half the small square, the other half the rectangle

A triangle that is half of the square, and a matching triangle that is half of the rectangle, mean the square and the rectangle have equal areas. The same argument on the other side finishes the proof. *(Source: D. Joyce, Euclid's* Elements, *Book I, Proposition 47, online edition, Clark University; and Wikipedia, "Euclid's Elements" (written c. 300 BC by Euclid of Alexandria); both retrieved 20 August 2026.)*

Older than Pythagoras

Long before Euclid, the rule was already written down in Mesopotamia. A clay tablet known as **Plimpton 322**, written around 1800 BC in what is now Iraq, lists rows of numbers we recognise today as sides of right-angled triangles — one row is built on 119 and 169, the triple $(119, 120, 169)$, where $119^2+120^2=169^2$. That is more than a thousand years before Pythagoras, whose name the theorem carries, and other ancient cultures — in India and China too — knew it and found proofs of their own.

(Source: Wikipedia, "Plimpton 322" — tablet held at Columbia University's Rare Book & Manuscript Library, from Senkereh, ancient Larsa, in modern Iraq; retrieved 20 August 2026.)

Finding the hypotenuse

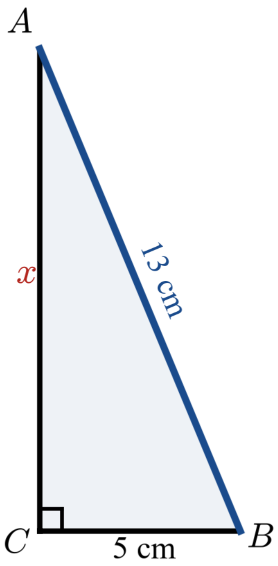
When the two shorter sides a and b are known: square them, add, and take the square root.

$c^2=a^2+b^2$, so $c=\sqrt{a^2+b^2}$.

For a 5-12 triangle: $c^2=5^2+12^2=25+144=169$, and $c=\sqrt{169}=13$. Every step is exact — no measuring, no guessing.

Finding a shorter side

When the hypotenuse c and one shorter side are known, subtract instead of add.



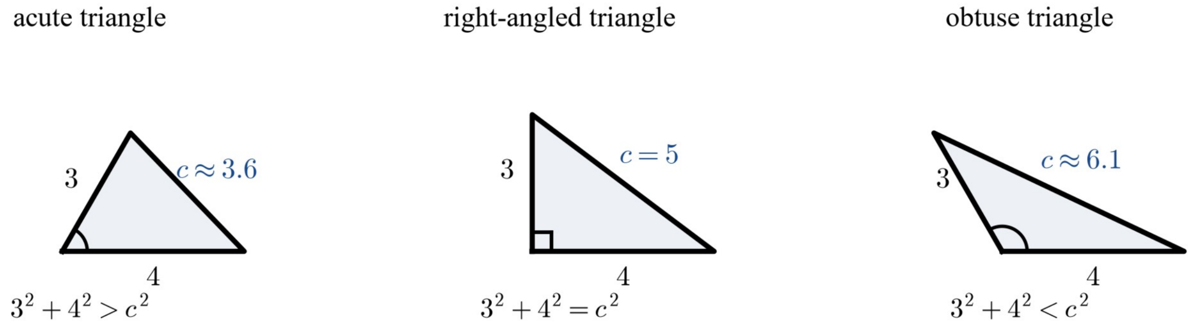
A right-angled triangle with hypotenuse 13 cm and one shorter side 5 cm; the other shorter side is labelled x

Here $x^2 = 13^2 - 5^2 = 169 - 25 = 144$, so $x = \sqrt{144} = 12$ cm. The common mistake is to add — the theorem only ever adds the two *shorter* sides' squares, and here x is not the hypotenuse, so its square is what gets subtracted.

Right-angled, acute or obtuse?

The theorem also works backwards as a test. Take any triangle, call its longest side c, and compare $a^2 + b^2$ with c^2:

- $a^2 + b^2 = c^2$ — the triangle is **right-angled**;
- $a^2 + b^2 > c^2$ — the triangle is **acute** (all angles under 90°);
- $a^2 + b^2 < c^2$ — the triangle is **obtuse** (one angle over 90°).



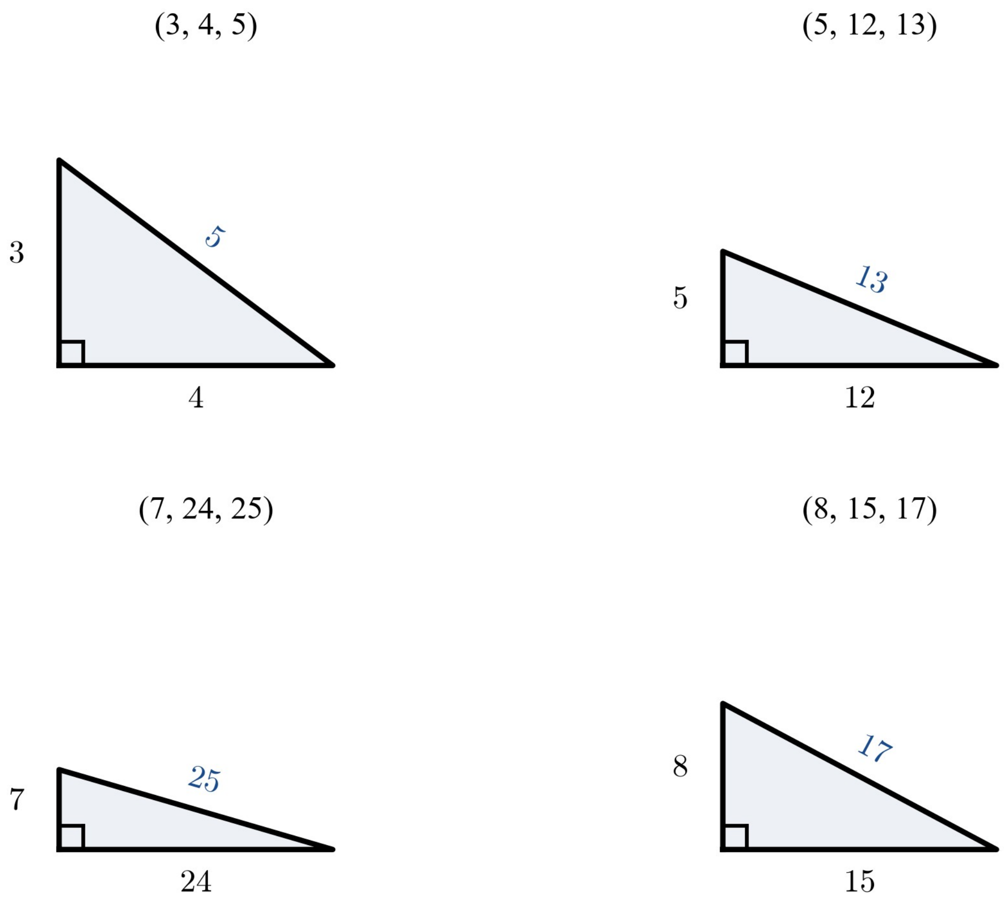
Three triangles each with two sides 3 and 4: an acute triangle whose third side is about 3.6, where 3 squared plus 4 squared is greater than c squared; a right-angled triangle with third side 5, where they are equal; and an obtuse triangle with third side about 6.1, where they are less

The three pictures hold sides 3 and 4 fixed and only open the angle between them. As the angle grows, the third side c grows, and the balance tips from > through = to <. The equals case is exactly Pythagoras' theorem — so the theorem is the boundary between acute and obtuse.

Pythagorean triples

A **Pythagorean triple** is a whole-number triple (a, b, c) that satisfies $a^2 + b^2 = c^2$. The curriculum names four:

$(3, 4, 5)$, $(5, 12, 13)$, $(7, 24, 25)$ and $(8, 15, 17)$.



The four Pythagorean triples named in the curriculum, drawn to scale as right-angled triangles: (3, 4, 5), (5, 12, 13), (7, 24, 25) and (8, 15, 17), each with its hypotenuse opposite the marked right angle

Check one yourself: $7^2+24^2=49+576=625=25^2$. Multiples of a triple are triples too — $(6,8,10)$ is $(3,4,5)$ doubled — and Q14 asks *why* that always works.

The theorem in the world

Builders still set out square corners the oldest way: measure 3 m along one edge and 4 m along the other, and adjust until the diagonal reads exactly 5 m — then the corner is right-angled, guaranteed by the theorem. Screen sizes work the same way: the advertised size is the diagonal, found from the width and height (Q18). Any time a straight distance has to be found from two directions at right angles — a ladder's reach, a shortcut across a park, a rope across a paddock — this is the tool.

In Year 9, `VC2M9M03` and `VC2M9SP01` build on this theorem with right-angled triangles, and `VC2M9A04` uses it to measure distances on the Cartesian plane. This subtopic keeps to the theorem itself.

Words of this subtopic

Word	What it means here
hypotenuse	the longest side of a right-angled triangle, opposite the right angle
right-angled triangle	a triangle with one 90° angle, marked with a small square
acute triangle	a triangle whose angles are all under 90°
obtuse triangle	a triangle with one angle over 90°
Pythagorean triple	three whole numbers (a,b,c) with $a^2+b^2=c^2$, like $(3,4,5)$
perfect square	a whole number that is some whole number squared,

Word	What it means here
	like 144 (12^2)
diagonal	a straight line joining opposite corners of a shape
theorem	a mathematical statement that has been proved true

Worked examples

Example 1 — with help

A right-angled triangle has shorter sides 6 cm and 8 cm. Find the hypotenuse c.

Working	Comment
$c^2 = 6^2 + 8^2$	The square of the hypotenuse equals the sum of the squares of the shorter sides.
$c^2 = 36 + 64 = 100$	Square first, then add.
$c = \sqrt{100} = 10$ cm	The hypotenuse is the square root — and 100 is a perfect square, so the answer is exact.

∴ the hypotenuse is 10 cm. Notice $(6, 8, 10)$ is just $(3, 4, 5)$ doubled.

Example 2 — with help

A right-angled triangle has hypotenuse 15 cm and one shorter side 9 cm. Find the other shorter side x.

Working	Comment
$x^2 = 15^2 - 9^2$	A shorter side: subtract its square's partner from the hypotenuse's square.
$x^2 = 225 - 81 = 144$	Square first, then subtract.
$x = \sqrt{144} = 12$ cm	144 is a perfect square, so the answer is exact.

∴ the other shorter side is 12 cm. Check: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$. ✓

Example 3 — with help

A rectangular lawn is 24 m long and 10 m wide. A path runs straight along a diagonal. How long is the path?

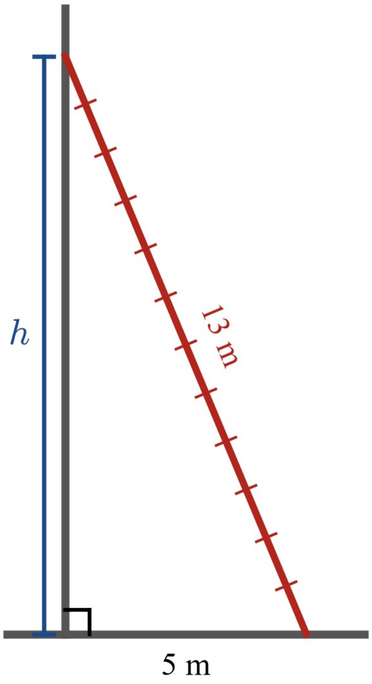
The diagonal cuts the rectangle into two right-angled triangles whose shorter sides are the length and the width.

Working	Comment
$d^2 = 24^2 + 10^2$	The diagonal is the hypotenuse of each triangle.
$d^2 = 576 + 100 = 676$	Square, then add.
$d = \sqrt{676} = 26$ m	$26^2 = 676$, exact.

∴ the path is 26 m long — shorter than the 34 m walk along two edges.

Example 4 — on your own

A 13 m ladder leans against a wall, with its foot 5 m out from the wall's base. How far up the wall does the ladder reach?



A 13 m ladder leaning against a wall with its foot 5 m from the wall, reaching a height h up the wall; the wall and ground meet at a right angle

The wall meets the ground at a right angle, so the ladder is the hypotenuse of a right-angled triangle with one shorter side 5 m and the other h.

$h^2 = 13^2 - 5^2 = 169 - 25 = 144$, so $h = \sqrt{144} = 12$.

$\therefore$ the ladder reaches 12 m up the wall — the triple $(5, 12, 13)$.

Example 5 — on your own

A triangle has sides 8 cm, 15 cm and 16 cm. Is it right-angled, acute or obtuse? Give reasons.

The longest side is 16, so compare $8^2 + 15^2$ with 16^2: $8^2 + 15^2 = 64 + 225 = 289$, and $16^2 = 256$. Since $289 > 256$, the sum of the two smaller squares is *greater* than the biggest square.

$\therefore$ the triangle is acute. (If the longest side were 17 instead, $289 = 289$ and the triangle would be right-angled — $(8, 15, 17)$ is a triple.)

Example 6 — on your own

a. Show that $(7, 24, 25)$ is a Pythagorean triple.

b. Is $(4, 7, 9)$ a Pythagorean triple? If not, what kind of triangle do those sides make?

c. $7^2 + 24^2 = 49 + 576 = 625$, and $25^2 = 625$. The two add up exactly.

d. $4^2 + 7^2 = 16 + 49 = 65$, but $9^2 = 81$. Since $65 < 81$, the sides do not satisfy the theorem, and the balance says obtuse.

$\therefore$ $(7, 24, 25)$ is a triple; $(4, 7, 9)$ is not — it makes an obtuse triangle.

Practice questions

Work these questions by hand; no calculator is needed. Every answer is exact.

With help

Q1. In right-angled triangle ABC, name the hypotenuse.

a. The right angle is at C.
b. The right angle is at B.
c. The right angle is at A.

Q2. Find the hypotenuse c of a right-angled triangle with shorter sides:

a. 5 cm and 12 cm
b. 9 cm and 12 cm
c. 10 cm and 24 cm

Q3. Find the unknown shorter side of a right-angled triangle with:

a. hypotenuse 17 cm and one shorter side 8 cm
b. hypotenuse 20 cm and one shorter side 16 cm
c. hypotenuse 25 cm and one shorter side 7 cm

Q4. Which set of sides makes a right-angled triangle?

A. 4, 6, 8 B. 6, 8, 10 C. 5, 7, 9 D. 8, 10, 12

Q5. True or false? Correct each false statement.

a. The hypotenuse is the longest side of a right-angled triangle.
b. Pythagoras' theorem works for every triangle.
c. $(3,4,5)$ and $(5,12,13)$ are both Pythagorean triples.
d. A triangle with sides 5, 6 and 7 has a right angle.

Q6. Say whether each triangle is right-angled, acute or obtuse.

a. Sides 6, 8, 10
b. Sides 6, 8, 11
c. Sides 6, 8, 9

Q7. Check which of these are Pythagorean triples.

a. $(7,24,25)$
b. $(7,23,25)$
c. $(8,15,17)$

Q8. The squares drawn on the two shorter sides of a right-angled triangle have areas 25 cm² and 144 cm². What is the area of the square on the hypotenuse, and how long is the hypotenuse?

On your own

Q9. Find the hypotenuse of a right-angled triangle with shorter sides:

a. 12 cm and 16 cm
b. 20 cm and 21 cm
c. 9 cm and 40 cm

Q10. A rectangle measures 48 cm by 14 cm. How long is its diagonal?

Q11. A 10 m ladder leans against a wall, with its foot on level ground.

a. The foot is 6 m from the wall. How far up the wall does the ladder reach?
b. The foot is slid out to 8 m from the wall. How far up does it reach now?

Q12. Explain why the hypotenuse of a right-angled triangle is always longer than each of the two shorter sides. (Start from $c^2 = a^2 + b^2$.)

Q13. Say whether each triangle is right-angled, acute or obtuse.

a. Sides 9, 12, 15
b. Sides 9, 12, 16
c. Sides 9, 12, 14

Q14. a. Show that $(15, 36, 39)$ is a Pythagorean triple. b. It is $(5, 12, 13)$ multiplied by 3. Explain why multiplying every number in a triple by the same whole number always gives another triple.

Q15. A square has sides of 1 cm. Use the theorem to work out its diagonal. What kind of number is your answer? (1D has the word.)

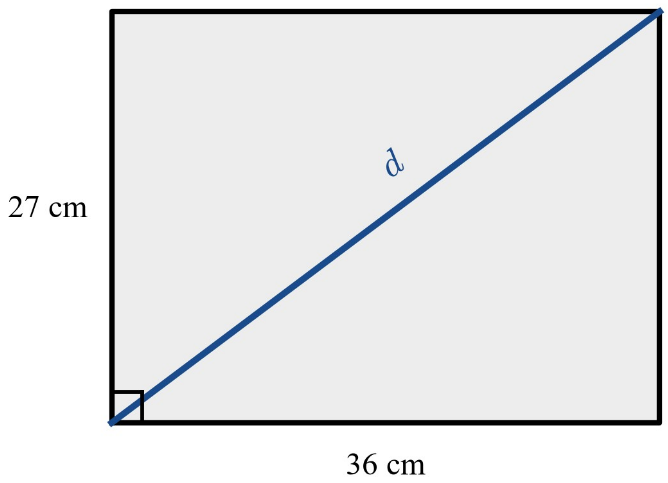
Q16. Mia says a right-angled triangle with shorter sides 9 cm and 12 cm has a hypotenuse of 21 cm, because $9 + 12 = 21$. Explain her mistake and find the correct hypotenuse.

Maths in real life

Q17. The clay tablet Plimpton 322, written around 1800 BC, has a row built on the numbers 119 and 169 — the triple $(119, 120, 169)$. *(Source: Wikipedia, "Plimpton 322" — tablet held at Columbia University's Rare Book & Manuscript Library; retrieved 20 August 2026.)*

a. Check that $119^2 + 120^2 = 169^2$, and say what that means about a triangle with sides 119, 120 and 169.
b. Euclid wrote the *Elements* around 300 BC. About how many years older is the tablet than Euclid's book?

Q18. A screen is 36 cm wide and 27 cm high. Screens are advertised by their diagonal. What size is this screen?



A rectangular screen 36 cm wide and 27 cm high, with its diagonal d drawn in blue from corner to corner; the width and height meet at a right angle

Q19. A builder marks 3 m along one edge of a corner and 4 m along the other. How far apart must the two marks be, for the corner to be exactly right-angled?

Q20. A rectangular park is 60 m long and 25 m wide. Walking the two edges from one corner to the opposite corner takes 85 m; a diagonal path cuts straight across. How much walking does the path save?

Answers

Q1. a. AB. b. AC. c. BC — always the side opposite the right angle.

Q2. a. 13 cm. b. 15 cm. c. 26 cm.

Q3. a. 15 cm. b. 12 cm. c. 24 cm.

Q4. B — $6^2+8^2=100=10^2$.

Q5. a. True. b. False; only for right-angled triangles. c. True. d. False; $5^2+6^2=61\neq 49=7^2$ (the triangle is acute).

Q6. a. Right-angled ($36+64=100$). b. Obtuse ($36+64<121$). c. Acute ($36+64>81$).

Q7. a. Yes ($49+576=625$). b. No ($49+529=578\neq 625$). c. Yes ($64+225=289$).

Q8. Area $25+144=169$ cm²; hypotenuse $\sqrt{169}=13$ cm.

Q9. a. 20 cm. b. 29 cm. c. 41 cm.

Q10. 50 cm.

Q11. a. 8 m. b. 6 m.

Q12. $c^2=a^2+b^2$, and $b^2>0$, so $c^2>a^2$, hence $c>a$; the same argument with a and b swapped gives $c>b$.

Q13. a. Right-angled ($81+144=225$). b. Obtuse ($81+144<256$). c. Acute ($81+144>196$).

Q14. a. $225+1296=1521=39^2$. b. $(3a)^2+(3b)^2=9(a^2+b^2)=9c^2=(3c)^2$.

Q15. $d=\sqrt{2}\approx 1.41$ cm — an irrational number.

Q16. The theorem squares the sides first: $9^2+12^2=225$, hypotenuse 15 cm, not 21 cm.

Q17. a. $14\,161+14\,400=28\,561=169^2$; the triangle is right-angled. b. About 1500 years.

Q18. 45 cm.

Q19. 5 m.

Q20. 20 m (the path is 65 m instead of 85 m).

Fully-worked solutions

Q1. The hypotenuse lies opposite the right angle, so it is the one side that is not part of the right angle.

a. Right angle at C: the hypotenuse is AB.
b. Right angle at B: the hypotenuse is AC.
c. Right angle at A: the hypotenuse is BC.

Q2. Use $c^2=a^2+b^2$, then take the square root.

a. $c^2=25+144=169$, so $c=13$ cm.
b. $c^2=81+144=225$, so $c=15$ cm.
c. $c^2=100+576=676$, so $c=26$ cm.

Q3. Use the subtracting direction: (shorter side)² = (hypotenuse)² − (other side)².

a. $17^2-8^2=289-64=225$, so the side is $\sqrt{225}=15$ cm.
b. $20^2-16^2=400-256=144$, so the side is $\sqrt{144}=12$ cm.

c. $25^2 - 7^2 = 625 - 49 = 576$, so the side is $\sqrt{576} = 24$ cm.

Q4. Test each set with the longest side as c.

A. $4^2 + 6^2 = 52 \neq 64 = 8^2$. No. B. $6^2 + 8^2 = 36 + 64 = 100 = 10^2$. Yes. C. $5^2 + 7^2 = 74 \neq 81 = 9^2$. No. D. $8^2 + 10^2 = 164 \neq 144 = 12^2$. No.

The answer is **B** — $(6, 8, 10)$, which is $(3, 4, 5)$ doubled.

Q5. a. **True** — the hypotenuse is opposite the largest angle (the right angle), and it is the longest side. b. **False** — the theorem is a fact about right-angled triangles; for other triangles the squares do not add up (see Q6). c. **True** — $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. d. **False** — $5^2 + 6^2 = 25 + 36 = 61$, but $7^2 = 49$; $61 > 49$, so the triangle has no right angle (it is acute).

Q6. Compare $a^2 + b^2$ with c^2, where c is the longest side.

a. $6^2 + 8^2 = 100$ and $10^2 = 100$: equal, so right-angled.
b. $6^2 + 8^2 = 100$ and $11^2 = 121$: $100 < 121$, so obtuse.
c. $6^2 + 8^2 = 100$ and $9^2 = 81$: $100 > 81$, so acute.

Q7. Square the two smaller numbers and compare with the biggest square.

a. $7^2 + 24^2 = 49 + 576 = 625 = 25^2$: a triple.
b. $7^2 + 23^2 = 49 + 529 = 578 \neq 625 = 25^2$: not a triple.
c. $8^2 + 15^2 = 64 + 225 = 289 = 17^2$: a triple.

Q8. The square on the hypotenuse equals the two smaller squares' areas added: $25 + 144 = 169$ cm². The hypotenuse is the side of that square: $\sqrt{169} = 13$ cm. This is the theorem read exactly as the areas picture shows it.

Q9. a. $c^2 = 144 + 256 = 400$, so $c = 20$ cm. b. $c^2 = 400 + 441 = 841$, so $c = 29$ cm. c. $c^2 = 81 + 1600 = 1681$, so $c = 41$ cm.

Q10. The diagonal is the hypotenuse of a right-angled triangle with shorter sides 48 cm and 14 cm: $d^2 = 48^2 + 14^2 = 2304 + 196 = 2500$, so $d = \sqrt{2500} = 50$ cm.

Q11. The wall and ground meet at a right angle; the ladder is the hypotenuse, 10 m.

a. $h^2 = 10^2 - 6^2 = 100 - 36 = 64$, so $h = 8$ m.
b. $h^2 = 10^2 - 8^2 = 100 - 64 = 36$, so $h = 6$ m. Sliding the foot out 2 m more makes the top slide down 2 m — the ladder's length never changes, only how it is shared between across and up.

Q12. From the theorem, $c^2 = a^2 + b^2$. A side length is more than zero, so $b^2 > 0$, which makes $c^2 > a^2$, and therefore $c > a$. Swapping the names of the shorter sides gives $c^2 > b^2$, so $c > b$ as well. The hypotenuse beats each shorter side because its square has to pay for *both* squares.

Q13. a. $9^2 + 12^2 = 81 + 144 = 225 = 15^2$: right-angled (a triple). b. $81 + 144 = 225 < 256 = 16^2$: obtuse. c. $81 + 144 = 225 > 196 = 14^2$: acute.

Q14. a. $15^2 + 36^2 = 225 + 1296 = 1521$, and $39^2 = 1521$: a triple. b. If $a^2 + b^2 = c^2$, then multiplying every number by 3 gives sides $3a$, $3b$, $3c$, and $(3a)^2 + (3b)^2 = 9a^2 + 9b^2 = 9(a^2 + b^2) = 9c^2 = (3c)^2$. The squares all pick up the same factor 9, so the equation still balances. (The same argument works for any multiplier — enlarging a right-angled triangle keeps the right angle.)

Q15. The diagonal is the hypotenuse of a right-angled triangle with shorter sides 1 cm and 1 cm: $d^2 = 1^2 + 1^2 = 2$, so $d = \sqrt{2}$ cm. No whole number or fraction squares to 2, so $\sqrt{2}$ is irrational (1D's word) — about 1.41 cm, but never exactly a fraction. This is why the theorem needs irrational numbers: even the unit square's diagonal has them.

Q16. Mia added the sides themselves, but the theorem adds the *squares* of the sides. Correctly: $9^2+12^2=81+144=225$, so the hypotenuse is $\sqrt{225}=15$ cm. (Check: $21^2=441$, but the squares of the shorter sides add to 225 — so 21 cannot be the hypotenuse.)

Q17. a. $119^2=14\,161$ and $120^2=14\,400$; together $28\,561$. And $169^2=28\,561$ — equal, so $(119, 120, 169)$ satisfies the theorem, and a triangle with those sides is right-angled. The tablet's scribe knew this fact about 1500 years before Euclid wrote it down as Proposition 47. b. From c. 1800 BC to c. 300 BC is about $1800-300=1500$ years.

Q18. The width and height meet at a right angle, so the diagonal d satisfies $d^2=36^2+27^2=1296+729=2025$. Then $d=\sqrt{2025}=45$. The screen is advertised as **45 cm** — a $(3, 4, 5)$ triangle multiplied by 9.

Q19. If the corner is right-angled, the two marks and the corner form a right-angled triangle with shorter sides 3 m and 4 m, so the distance between the marks is $\sqrt{3^2+4^2}=\sqrt{25}=5$ m. That is the whole builder's trick: when the diagonal reads 5 m, the corner is square.

Q20. The path is the diagonal: $d^2=60^2+25^2=3600+625=4225$, so $d=\sqrt{4225}=65$ m. The edges take $60+25=85$ m. The path saves $85-65=20$ m of walking — the theorem again, doing its oldest job, shortening the walk between two points.

Test

Mathematics Level 8 · Chapter 3 — Measurement: area, circles, volume and Pythagoras' theorem · Chapter test T3

Marks	20 (5 marks for each subtopic)
Time	25 minutes: 5 minutes reading time, then about 20 minutes working time (1 minute per mark)
Calculators	Not permitted — the content descriptions of Chapter 3 name no digital tools, so every item is to be worked out mentally or in writing. Every item on this paper is calculator-free.
Equipment	Ruler, protractor and pair of compasses

This is a classroom check, not an exam.

Instructions

- Answer every question, and show your working clearly.
- The marks for each question or part are shown in brackets.
- Where a reason is asked for, the reason is part of the marks: a correct answer with no reason does not score full marks.
- Use $\pi \approx 3.14$ for every circle calculation.
- For Question 6 you will need 1 cm grid paper and your pair of compasses.

Subtopic	Content description	Marks
3A Perimeter and area of composite and irregular shapes	`VC2M8M01`	5
3B Circumference and area of a circle	`VC2M8M03`	5
3C Volume and capacity of right prisms	`VC2M8M02`	5
3D Pythagoras' theorem	`VC2M8M06`	5
Total		**20**

3A · Perimeter and area of composite and irregular shapes

Q1. The oval playing field at the Melbourne Cricket Ground fits inside a rectangle about 174 m long and 149 m wide. *(Source: Wikipedia, "Melbourne Cricket Ground"; retrieved 20 August 2026.)* (3 marks)

(a) Find the perimeter of the surrounding rectangle. (1 mark)
(b) Find the area of the surrounding rectangle. (1 mark)
(c) Write this area in hectares, correct to 1 decimal place. (1 mark)

Q2. A rectangular card measures 15 cm by 12 cm. A rectangular piece 6 cm by 5 cm is cut out of one corner. Find the area of the card that is left. (1 mark)

Q3. A leaf is placed on a 1 cm grid. It covers 14 whole squares and 9 edge squares that are more than half covered. Estimate the area of the leaf. (1 mark)

3B · Circumference and area of a circle

Q4. A circular fountain has diameter 12 m. (2 marks)

(a) Find its circumference. (1 mark)
(b) Find its area. (1 mark)

Q5. A circle has area 153.86 cm². Find its radius. (1 mark)

Q6. Use a pair of compasses to draw a circle of radius 2 cm on 1 cm grid paper. (2 marks)

(a) Estimate the area of your circle by counting squares. (1 mark)
(b) Calculate the area using $A = \pi r^2$. (1 mark)

3C · Volume and capacity of right prisms

Q7. A rectangular fish tank is 25 cm long, 20 cm wide and 16 cm high. (2 marks)

(a) Find the volume of the tank in cm^3. (1 mark)
(b) The tank is full. How many litres of water does it hold? (1 mark)

Q8. A triangular prism has a triangular base 8 cm wide and 5 cm high, and a length of 9 cm. Find its volume. (1 mark)

Q9. Convert. (2 marks)

(a) 3 m^3 to cm^3 (1 mark)
(b) 0.75 m^3 to litres (1 mark)

3D · Pythagoras' theorem

Q10. A 17 m ladder leans against a vertical wall. Its foot stands on level ground, 8 m from the base of the wall. How far up the wall does the ladder reach? (2 marks)

Q11. A triangle has sides 7 cm, 24 cm and 26 cm. Is it right-angled, acute or obtuse? Give a reason for your answer. (2 marks)

Q12. A right-angled triangle has shorter sides 9 cm and 40 cm. Find the hypotenuse. (1 mark)

Solutions

Answer key

Q	Answer	Marks
1	(a) 646 m · (b) 25 926 m² · (c) 2.6 ha	3
2	150 cm²	1
3	about 23 cm²	1
4	(a) 37.68 m · (b) 113.04 m²	2
5	7 cm	1
6	(a) about 12 cm² (accept 11–14 cm²) · (b) 12.56 cm²	2
7	(a) 8000 cm³ · (b) 8 L	2
8	180 cm³	1
9	(a) 3 000 000 cm³ · (b) 750 L	2
10	15 m	2
11	Obtuse — $7^2+24^2=625<676=26^2$	2
12	41 cm	1
	Total	**20**

Fully worked solutions

Q1. The surrounding rectangle is 174 m long and 149 m wide.

(a) Perimeter $=2\times(174+149)=2\times323=646$ m.
(b) Area $=174\times149=25\,926$ m² (for example, $174\times150=26\,100$, then $26\,100-174=25\,926$).
(c) 1 ha $=10\,000$ m², so $25\,926\div10\,000=2.5926$ ha, which rounds to 2.6 ha to 1 decimal place.

∴ (a) 646 m; (b) 25 926 m²; (c) 2.6 ha.

Marks: 1 mark for each part.

Q2. The subtract move: big shape minus the gap. Whole card $15\times12=180$ cm²; removed piece $6\times5=30$ cm². Area left $=180-30=150$ cm².

∴ the card has 150 cm² left.

Q3. Count whole squares plus edge squares that are more than half covered: $14+9=23$.

∴ the leaf is about 23 cm².

Q4. The road goes through the radius: $r=12\div2=6$ m.

(a) $C=\pi d\approx3.14\times12=37.68$ m.
(b) $A=\pi r^2\approx3.14\times6^2=3.14\times36=113.04$ m². Check with the trap: $2r^2=72$ and $4r^2=144$, and $72<113.04<144$. ✓

∴ (a) about 37.68 m; (b) about 113.04 m².

Marks: 1 mark for each part.

Q5. Work back from the area: $r^2 = A \div \pi \approx 153.86 \div 3.14 = 49$, so $r = \sqrt{49} = 7$ cm.

∴ the radius is 7 cm.

Q6. Set the compasses to 2 cm and draw the circle on the grid.

(a) A circle of radius 2 cm covers 4 whole squares, with part squares around the rim adding about 8 or 9 more, so the count is about 12 cm². Any sensible count between about 11 and 14 cm² earns the mark.

(b) $A = \pi r^2 \approx 3.14 \times 2^2 = 3.14 \times 4 = 12.56$ cm² — the formula and the count agree.

∴ (a) about 12 cm²; (b) about 12.56 cm².

Marks: 1 mark for a reasonable count in part (a); 1 mark for part (b).

Q7. (a) $V = l \times w \times h = 25 \times 20 \times 16 = 8000$ cm³. (b) 1 cm³ = 1 mL, so 8000 cm³ $= 8000$ mL, and $8000 \div 1000 = 8$ L.

∴ (a) 8000 cm³; (b) 8 L.

Marks: 1 mark for each part.

Q8. Area of the triangular base: $A = 1/2 \times 8 \times 5 = 20$ cm². Then $V = A \times h = 20 \times 9 = 180$ cm³, where the height of this prism is its length.

∴ the volume is 180 cm³.

Q9. (a) The ×100 happens once for each of the three edges, so 1 m³ $= 100 \times 100 \times 100 = 1{,}000{,}000$ cm³. Hence 3 m³ $= 3{,}000{,}000$ cm³. (b) 1 m³ $= 1000$ L, so 0.75 m³ $= 0.75 \times 1000 = 750$ L.

∴ (a) $3{,}000{,}000$ cm³; (b) 750 L.

Marks: 1 mark for each part.

Q10. The wall meets the ground at a right angle, so the ladder is the hypotenuse of a right-angled triangle: $c = 17$ m, one shorter side is 8 m, and the other is the height h up the wall. A shorter side means subtract:

$h^2 = 17^2 - 8^2 = 289 - 64 = 225$, so $h = \sqrt{225} = 15$ m.

∴ the ladder reaches 15 m up the wall — the triple $(8, 15, 17)$, one of the four the curriculum names.

Marks: 1 mark for the correct set-up (subtracting, with 17 as the hypotenuse); 1 mark for 15 m.

Q11. The longest side is 26, so compare $a^2 + b^2$ with c^2:

$7^2 + 24^2 = 49 + 576 = 625$, and $26^2 = 676$. Since $625 < 676$, the sum of the squares on the two shorter sides is *less* than the square on the longest side: $a^2 + b^2 < c^2$.

∴ the triangle is **obtuse**, because $7^2 + 24^2 < 26^2$. (Had the two squares added to exactly 676 the triangle would be right-angled; $(7, 24, 25)$ shows how close this one sits to that boundary.)

Marks: 1 mark for the classification; 1 mark for the stated reason — the comparison $625 < 676$. *A correct classification with no reason scores 1 mark, not 2.*

Q12. The hypotenuse: square the shorter sides, add, then take the square root.

$c^2 = 9^2 + 40^2 = 81 + 1600 = 1681$, so $c = \sqrt{1681} = 41$ cm.

∴ the hypotenuse is 41 cm.