# Mathematics Level 8

## Chapter 1 — Number: integers, exponents, the real number system and percentages

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### Contents

## The four operations with integers and rational numbers

**Content description `VC2M8N04`:** use the 4 operations with integers and with rational numbers, choosing and using efficient mental and written strategies, and digital tools where appropriate, and making estimates for these computations

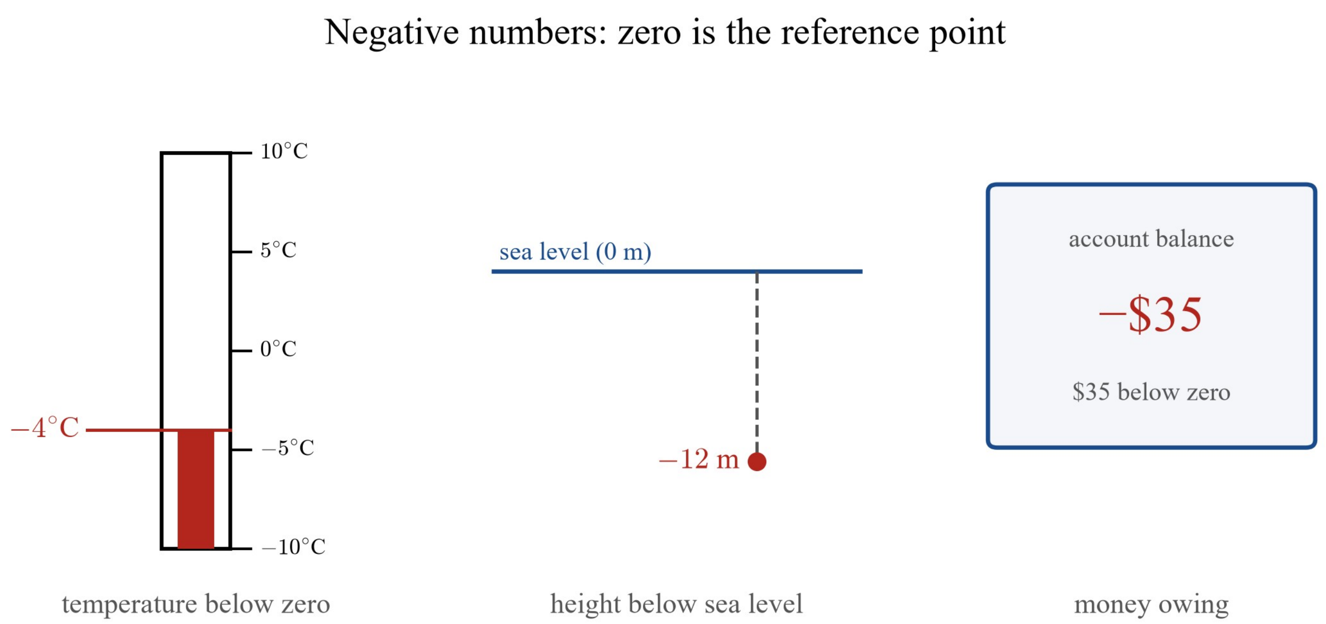
*The morning temperature is −3 °C and by midday it has risen 5 degrees. A bank account can sit below zero, a diver can sit below sea level, and a freezer sits well below the temperature of the room. Numbers like these follow the same rules as the numbers you have always used — once you know what the signs do. In Year 7 you added and subtracted integers, and you worked with positive fractions and decimals. This subtopic joins those two threads: all four operations, with negative numbers, fractions and decimals all in the mix.*

### Theory

#### *What Year 7 gave us, and two names to fix*

In Year 7 you compared, ordered, added and subtracted integers (`VC2M7N08`), and you used the 4 operations with positive fractions and decimals (`VC2M7N06`). This subtopic uses both, so they are recalled here in one line each rather than re-taught: adding and subtracting integers works the way it did last year, and fractions and decimals still multiply, divide, add and subtract exactly as they did when they were positive.

Two names to fix before going on. The **integers** are the whole numbers, positive and negative, together with zero: …, −3, −2, −1, 0, 1, 2, 3, … A **rational number** is any number that can be written as a fraction of two integers, with a non-zero bottom: $\frac{3}{4}$, $-0.6$ (which is $-\frac{6}{10}$), 5 (which is $\frac{5}{1}$) and $-2\frac{1}{2}$ are all rational numbers. Every number in this subtopic is one.




*Three everyday uses of negative numbers with zero as the reference point: a thermometer showing minus 4 degrees, a point 12 metres below sea level, and an account balance of minus 35 dollars*

#### *Multiplying integers — a pattern that fixes the rule*

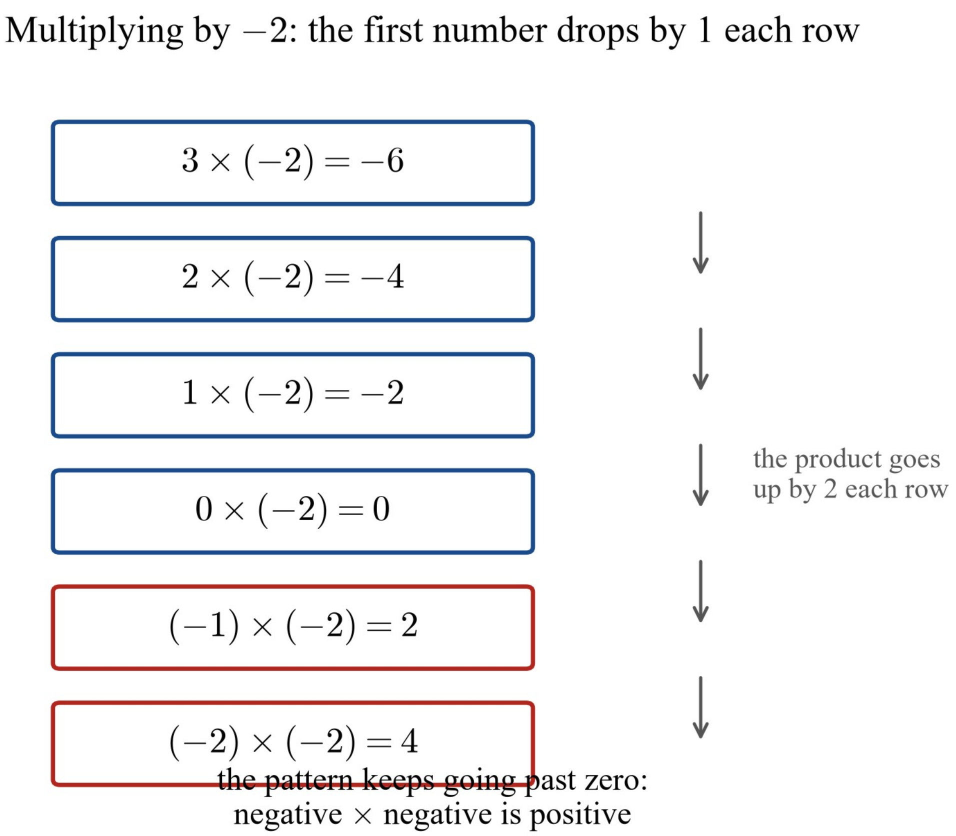
Nobody chooses the sign rules; they are forced on us by keeping the usual patterns going. Start with facts you know, and drop the first number by 1 each row:

$$3\times(-2)=-6, 2\times(-2)=-4, 1\times(-2)=-2, 0\times(-2)=0$$

Each row the product goes **up by 2**. Keep the pattern going past zero:

$$(-1)\times(-2)=2, (-2)\times(-2)=4$$

The pattern only keeps its step if a negative times a negative is a positive. That is the *why* behind the rule (E01's "using patterns to assist in establishing the rules").



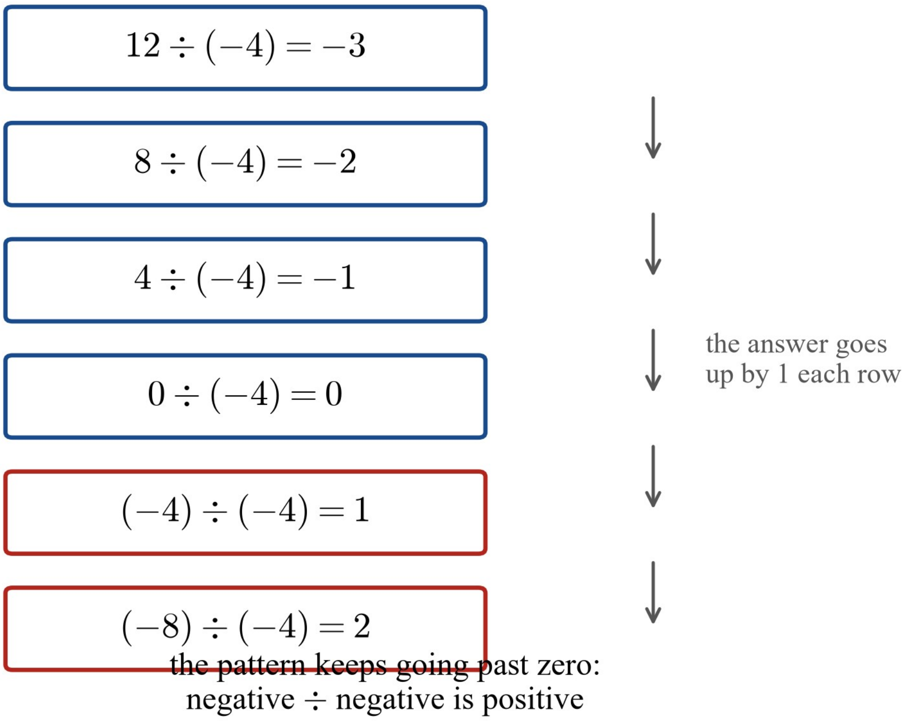

*Six boxes in a column: 3 times minus 2 is minus 6, then 2, 1 and 0 times minus 2, then minus 1 and minus 2 times minus 2 giving 2 and 4; the product goes up by 2 each row, so the pattern keeps going past zero and negative times negative is positive*

#### ***Dividing integers — the same rules, in reverse***

Division undoes multiplication, so its signs must match. The same pattern trick works: divide by −4, dropping the starting number by 4 each row, and the answer goes up by 1 each row — past zero included:

$$12 \div (-4) = -3, \ldots, 0 \div (-4) = 0, (-4) \div (-4) = 1, (-8) \div (-4) = 2$$

Dividing by $-4$: the starting number drops by 4 each row




*Six boxes in a column: 12 divided by minus 4 is minus 3, down to 0 divided by minus 4 is 0, then minus 4 and minus 8 divided by minus 4 giving 1 and 2; the answer goes up by 1 each row, so negative divided by negative is positive*

You can also check division with multiplication. Since $(-6) \times 7 = -42$, dividing the product by either factor returns the other:

$$(-42) \div 7 = -6, (-42) \div (-6) = 7$$

Two edge facts to keep: $0 \div (-4) = 0$ (zero shared any way is still zero), but dividing **by** zero is not allowed — no number times 0 can make a non-zero answer, so a division like $(-4) \div 0$ has no answer at all.

### *The sign rules*

All of the above fits on one card, and the card is the same for × and ÷:

**Sign rules.** When multiplying or dividing two numbers: same signs give a positive answer; different signs give a negative answer.

**both positive**

$$6 \times 7 = 42$$

$$42 \div 6 = 7$$

answer is positive

**different signs**

$$6 \times (-7) = -42$$

$$42 \div (-6) = -7$$

answer is negative

**different signs**

$$(-6) \times 7 = -42$$

$$(-42) \div 6 = -7$$

answer is negative

**both negative**

$$(-6) \times (-7) = 42$$

$$(-42) \div (-6) = 7$$

answer is positive

Same signs give a positive answer. Different signs give a negative answer. The rules are the same for $\times$ and $\div$.

*Four cards: both positive, 6 times 7 is 42 and 42 divided by 6 is 7, answer positive; different signs, 6 times minus 7 is minus 42 and minus 42 divided by 6 is minus 7, answer negative; different signs, minus 6 times 7 is minus 42 and 42 divided by minus 6 is minus 7, answer negative; both negative, minus 6 times minus 7 is 42 and minus 42 divided by minus 6 is 7, answer positive*

The practical habit is: **fix the sign first, then do the number.** Decide + or − from the sign rules, then multiply or divide the sizes as if everything were positive.

### *The effect of signs — counting negative factors*

With more than two numbers multiplied, count the negative factors:

$$(-2)\times 3\times(-5)=30(-2)\times 3\times(-5)\times(-1)=-30$$

The first product has **two** negative factors — an even count — and is positive; adding one more negative factor makes **three** — an odd count — and the product turns negative. The size is just the numbers multiplied; the sign comes from the count.

Count the negative factors

$$\boxed{-2} \times \boxed{3} \times \boxed{-5} \qquad \boxed{=30}$$

2 negative factors — even count

$$\boxed{-2} \times \boxed{3} \times \boxed{-5} \times \boxed{-1} \qquad \boxed{=-30}$$

3 negative factors — odd count

even count: the product is positive
odd count: the product is negative

the size comes from multiplying the numbers; the sign comes from the count

*Two rows of boxed factors: minus 2 times 3 times minus 5 equals 30, with two negative factors, an even count; and minus 2 times 3 times minus 5 times minus 1 equals minus 30, with three negative factors, an odd count; even count gives a positive product, odd count a negative one*

**Effect of signs.** In a product, an even number of negative factors gives a positive answer; an odd number gives a negative answer. (For division, read each division as dividing by that number: it flips the sign the same way a negative factor does.)

The special case of counting $-1$ factors is why $(-1)^4=1$ and $(-1)^5=-1$: four factors of −1 pair up into two pairs of $+1$, while five factors leave an unpaired −1 at the end.

Powers of $-1$: the sign alternates

$$\boxed{(-1)^1 = -1} \rightarrow \boxed{(-1)^2 = 1} \rightarrow \boxed{(-1)^3 = -1} \rightarrow \boxed{(-1)^4 = 1} \rightarrow \boxed{(-1)^5 = -1}$$

each step multiplies by $-1$ again, so the sign flips

$$(-1)^4 = [(-1) \times (-1)] \times [(-1) \times (-1)] = 1 \times 1 = 1$$

$$(-1)^5 = (-1)^4 \times (-1) = 1 \times (-1) = -1$$

even number of negative factors: positive
odd number of negative factors: negative

*Five boxes: minus 1 to the power 1 is minus 1, to the power 2 is 1, to the power 3 is minus 1, to the power 4 is 1, to the power 5 is minus 1, the sign flipping at each step; pairing shows minus 1 to the power 4 equals 1 and minus 1 to the power 5 equals minus 1; an even number of negative factors is positive, an odd number negative*

So: $(-1)$ to an **even** power is 1; to an **odd** power is $-1$. You will use this again in the next subtopic, when powers have negative bases.

### *Negative fractions and decimals — sign first, then the number*

The sign rules were built with integers, but they belong to *numbers*, so fractions and decimals obey them without any new rules. Fix the sign, then work the fraction or decimal exactly as you did in Year 7:

$$\left(-\frac{2}{3}\right) \times \frac{3}{4} : \text{sign: different} \rightarrow -, \text{number:} \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2} \Rightarrow -\frac{1}{2}$$

$$(-2.4) \div (-0.6) : \text{sign: same} \rightarrow +, \text{number:} 2.4 \div 0.6 = 4 \Rightarrow 4$$

Negative rational numbers: sign first, then the number

$\left(-\frac{2}{3}\right) \times \frac{3}{4}$

step 1 — the sign
different signs: negative

step 2 — the number
$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

$\left(-\frac{2}{3}\right) \times \frac{3}{4} = -\frac{1}{2}$

$(-2.4) \div (-0.6)$

step 1 — the sign
same signs: positive

step 2 — the number
$2.4 \div 0.6 = 4$

$(-2.4) \div (-0.6) = 4$

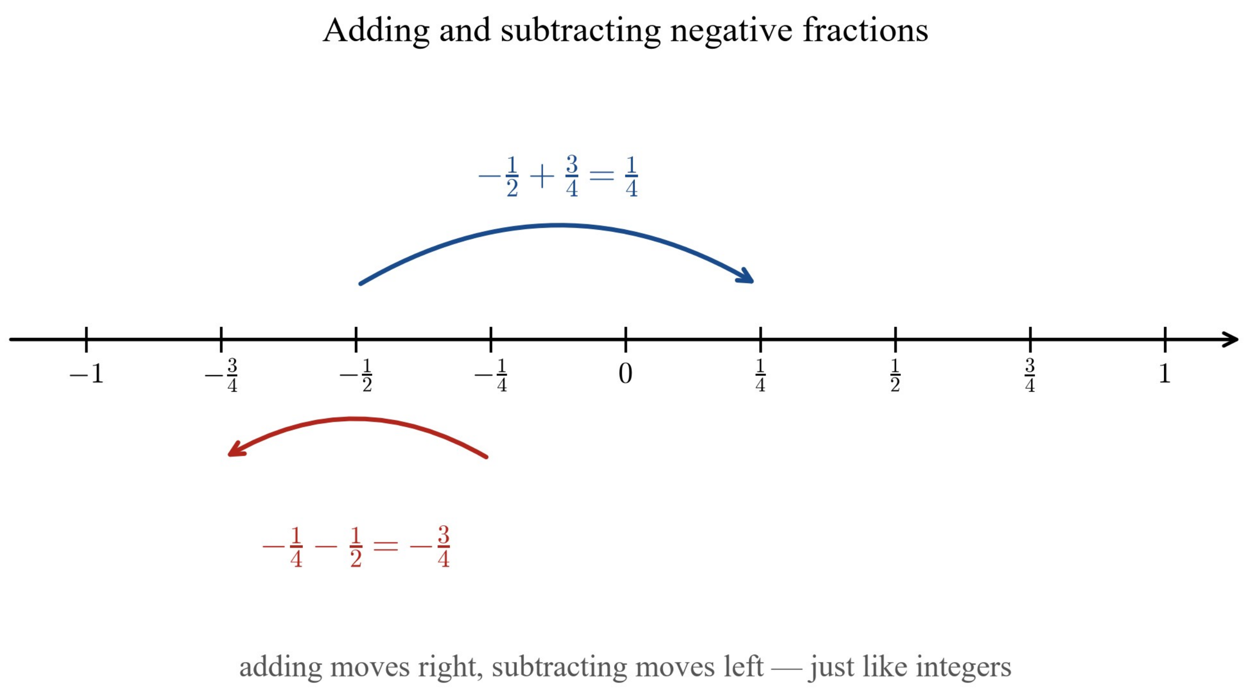
the sign rules for integers work the same way with fractions and decimals

*Two columns. Left: minus two-thirds times three-quarters, step 1 the sign is different so negative, step 2 the number two-thirds times three-quarters is one-half, so the answer is minus one-half. Right: minus 2.4 divided by minus 0.6, step 1 the sign is the same so positive, step 2 the number 2.4 divided by 0.6 is 4, so the answer is 4*

### *Adding and subtracting negative fractions and decimals*

Adding and subtracting integers on a number line was Year 7 work: adding moves you right, subtracting moves you left. The same picture carries fractions and decimals:

$$-\frac{1}{2}+\frac{3}{4}=\frac{1}{4}-\frac{1}{4}-\frac{1}{2}=-\frac{3}{4}$$




*A number line in quarters from minus 1 to 1: a blue arrow moves right from minus one-half to one-quarter, showing minus a half plus three-quarters equals a quarter; a red arrow moves left from minus a quarter to minus three-quarters, showing minus a quarter minus a half equals minus three-quarters; adding moves right and subtracting moves left, just like integers*

Decimals behave the same way: $-1.2+0.5=-0.7$ (start at −1.2, move right by 0.5) and $-0.8-1.4=-2.2$. With same-sign additions you can also fix the sign first and then add the sizes: $-0.8-1.4$ is "0.8 of debt, then 1.4 more", so $-2.2$.

### *Efficient strategies — make the numbers friendly*

The clever strategies you used with positive numbers all survive the move to negative ones (this is E02's list: regrouping, partitioning, place value, known facts and patterning).

**Regroup.** Multiplication lets you reorder and regroup — the **commutative law** (order may be swapped) and the **associative law** (grouping may be changed):

$$(-25)\times 7\times(-4)=(-25)\times(-4)\times 7=100\times 7=700$$

Regroup for friendly numbers

$$(-25) \times 7 \times (-4)$$

$$\downarrow \quad \text{reorder}$$

$$(-25) \times (-4) \times 7$$

$$\downarrow \quad \text{group the pair}$$

$$100 \times 7$$

$$\downarrow$$

$$= 700$$

$(-25) \times (-4) = 100$ is a friendly pair — reorder first, then multiply

*Column of boxes: minus 25 times 7 times minus 4, reordered to minus 25 times minus 4 times 7, then 100 times 7, then 700; the friendly pair minus 25 times minus 4 makes 100, so reorder first and then multiply*

**Partition.** Split a number near a friendly one: $98 \times (-5) = (100 - 2) \times (-5) = -500 + 10 = -490$.

**Place value.** $6 \times 3 = 18$ tells you $0.6 \times (-0.3)$: the digits give 18, the two decimal places give $0.18$, and the signs differ, so $-0.18$.

**Known facts and patterns.** $7 \times 8 = 56$ tells you $(-7) \times (-8) = 56$ at once — the fact supplies the number and the sign rules supply the sign.

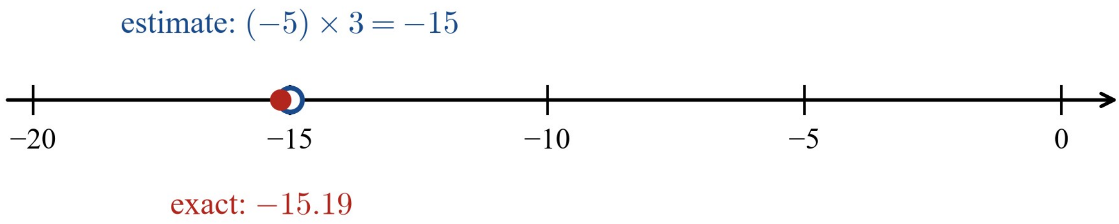
### *Estimate, then check*

The content description asks for estimates *for these computations*, and an estimate is the fastest way to catch a wrong sign or a slipped decimal point. Round each number to one you can work with in your head, and fix the sign as you go:

$$(-4.9) \times 3.1 \approx (-5) \times 3 = -15$$

The exact answer is $-15.19$ — the estimate says the sign should be negative and the size about 15, and $-15.19$ agrees. If a worked answer came out $+15.19$ or $-151.9$, the estimate would catch it at a glance.

Estimate first: $(-4.9) \times 3.1$




the estimate fixes the sign and the rough size — then check the exact answer against it

*A number line from minus 20 to 0 with two points close together: the estimate, minus 5 times 3 equals minus 15, and the exact answer, minus 15.19; the estimate fixes the sign and the rough size, then the exact answer is checked against it*

A calculator or other digital tool is allowed where appropriate — as a **check** after you have estimated and worked by hand. Everything in this subtopic can be done without one, and the estimate is what makes the check meaningful: you only trust a calculator answer that agrees with your estimate.

### *The subtopic on one card*

**Sign rules**

$$(-6)\times(-7)=42$$

same signs: positive
different signs: negative
for $\times$ and $\div$

**Effect of signs**

$$(-1)^4=1,\quad (-1)^5=-1$$

even number of negative factors: positive
odd number: negative

**Efficient strategies**

$$(-25)\times 7\times(-4)=700$$

regroup, partition, place value,
known facts and patterns

**Estimate first**

$$(-4.9)\times 3.1\approx -15$$

check the sign and the size
of the exact answer

All four operations work the same way with negative fractions and decimals.

*Summary card with four panels: the sign rules (same signs positive, different signs negative, for times and divide); the effect of signs (minus 1 to the power 4 is 1, to the power 5 is minus 1, even count of negative factors positive, odd count negative); efficient strategies (minus 25 times 7 times minus 4 is 700 by regrouping, partitioning, place value and known facts); and estimating first (minus 4.9 times 3.1 is about minus 15, checking the sign and size of the exact answer)*

### *Words of this subtopic*

| Word | What it means here |
| --- | --- |
| **integer** | a whole number, positive or negative, or zero: …, −2, −1, 0, 1, 2, … |
| **rational number** | any number that can be written as a fraction of two integers with a non-zero bottom: $\frac{3}{4}$, $-0.6$, 5, $-2\frac{1}{2}$ |
| **sign rules** | same signs give a positive answer, different signs a negative one, for $\times$ and $\div$ |
| **effect of signs** | in a product, an even number of negative factors gives a positive answer, an odd number a negative one |
| **commutative law** | the order may be swapped: $3\times 4=4\times 3$ |
| **associative law** | the grouping may be changed: $(2\times 3)\times 4=2\times(3\times 4)$ |
| **estimate** | a close-enough answer worked out in your head, used to fix the sign and size before checking the exact one |

### Worked examples

#### *Example 1 — with help*

Use the sign rules to find each value.

(a) $(-6)\times 7$
(b) $(-8)\times(-9)$
(c) $(-72)\div 8$
(d) $(-63)\div(-9)$

| Working | Comment |
|---|---|
| (a) signs different → −; $6\times 7=42$; answer $-42$ | Fix the sign first, then do the number. |
| (b) signs the same → +; $8\times 9=72$; answer 72 | Negative times negative is positive. |
| (c) signs different → −; $72\div 8=9$; answer $-9$ | Division uses the same card as multiplication. |
| (d) signs the same → +; $63\div 9=7$; answer 7 | Check: $7\times(-9)=-63$. ✓ |

∴ (a) $-42$; (b) 72; (c) $-9$; (d) 7. Division is checked by multiplying back — the two operations undo each other.

#### *Example 2 — with help*

Sign first, then the number, with fractions and decimals.

(a) $\left(-\frac{2}{3}\right)\times\frac{3}{4}$
(b) $\left(-\frac{3}{4}\right)\div\left(-\frac{1}{2}\right)$
(c) $(-1.2)\times 0.4$
(d) $(-2.4)\div(-0.6)$

| Working | Comment |
|---|---|
| (a) sign −; $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$; answer $-\frac{1}{2}$ | Cancel the 3s top and bottom before multiplying, as in Year 7. |
| (b) sign +; $\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\times 2=\frac{6}{4}=\frac{3}{2}$ | Dividing by $\frac{1}{2}$ is multiplying by 2 — Year 7 fraction division, signs already fixed. |
| (c) sign −; $1.2\times 0.4=0.48$; answer $-0.48$ | One decimal place plus one decimal place gives two. |
| (d) sign +; $2.4\div 0.6=4$; answer 4 | $2.4\div 0.6$ is $24\div 6$ in tenths. |

∴ (a) $-\frac{1}{2}$; (b) $\frac{3}{2}$; (c) $-0.48$; (d) 4. Nothing new is needed: the sign rules do the signs, and Year 7 does the numbers.

#### *Example 3 — with help*

Choose an efficient strategy, then find the value.

(a) $(-25)\times 7\times(-4)$
(b) $98\times(-5)$

| Working | Comment |
|---|---|
| (a) $(-25)\times(-4)\times 7$ | Commutative law: reorder to put the friendly pair together. |
| $=100\times 7=700$ | $(-25)\times(-4)=100$: same signs, and $25\times 4=100$. |
| (b) $(100-2)\times(-5)$ | Partition 98 into $100-2$. |

| Working | Comment |
|---|---|
| $= -500 + 10 = -490$ | Multiply each part: $100 \times (-5) = -500$ and $(-2) \times (-5) = +10$. |

∴ (a) $700$; (b) $-490$. A good strategy turns the working into facts you already know — and the sign rules travel with it.

### *Example 4 — on your own*

Count the negative factors to fix the sign, then find the value.

(a) $(-4) \times 5 \times (-2)$

(b) $(-4) \times 5 \times (-2) \times (-3)$

(c) $(-1)^6$ and $(-1)^9$

(d) Two negative factors — even — so the product is positive: $4 \times 5 \times 2 = 40$, giving $40$.

(e) Three negative factors — odd — so the product is negative: $4 \times 5 \times 2 \times 3 = 120$, giving $-120$. (One extra negative factor flips the answer of (a).)

(f) $(-1)^6$ has six negative factors, an even count, so it is $1$; $(-1)^9$ has an odd count, so it is $-1$.

### *Example 5 — on your own*

Estimate first, then find the exact answer and check it against the estimate.

(a) $(-4.9) \times 3.1$

(b) $-62.4 + 19.8$

(c) Estimate: $(-5) \times 3 = -15$. Exact: $4.9 \times 3.1 = 15.19$ with different signs, so $-15.19$. The exact answer sits right next to the estimate. ✓

(d) Estimate: $-62 + 20 = -42$. Exact: the signs differ, so subtract the sizes and keep the sign of the larger: $62.4 - 19.8 = 42.6$, giving $-42.6$. Again the exact answer matches the estimate's sign and size. ✓

### *Example 6 — on your own*

A diver hovers at −12 m, then descends for 4 minutes at 1.5 m per minute. What depth does the diver reach?

The descent adds $4 \times (-1.5) = -6$ m to the depth. So the new depth is $(-12) + (-6) = -18$ m.

∴ the diver reaches −18 m. Check with an estimate: about $-12 - 6 = -18$. ✓

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## Practice questions

*Every question here can be worked by hand — the numbers are chosen so the sign rules and your number facts do the work. A calculator is allowed as a check, after you have estimated; never as a substitute for the working.*

### *With help*

**Q1.** Find each value using the sign rules.

a. $(-4) \times 5$
b. $(-6) \times (-7)$
c. $(-8) \times 9$
d. $(-6) \times 6$

**Q2.** Find each value using the sign rules.

a. $(-16) \div 2$
b. $(-27) \div (-3)$
c. $45 \div (-5)$
d. $(-48) \div 4$

**Q3.** Find each value. These mix Year 7 addition and subtraction with the new rules.

a. $(-9) + 5$
b. $20 - (-15)$
c. $(-6) \times (-4)$
d. $(-2) + (-6)$

**Q4.** Without finding the value, say whether each product is positive or negative, and how you know.

a. $(-3) \times (-4) \times (-5)$
b. $(-2) \times (-3) \times 4$
c. $(-7) \times 3 \times 2$
d. $(-1) \times (-1) \times (-1) \times (-1)$

**Q5.** Sign first, then the number.

a. $\left(-\frac{1}{2}\right) \times \frac{2}{3}$
b. $\left(-\frac{3}{4}\right) \div \left(-\frac{5}{8}\right)$
c. $\left(-\frac{4}{5}\right) \times \frac{5}{8}$
d. $\left(-\frac{1}{2}\right) \times \frac{1}{4}$

**Q6.** Find each value.

a. $(-1.2) + 2$
b. $(-0.5) + 0.75$
c. $(-0.25) - 0.5$
d. $(-1.3) - 1.5$

**Q7.** Regroup for friendly numbers, then find the value.

a. $(-25) \times 9 \times (-4)$
b. $37 \times 25 \times (-4)$
c. $(-5) \times 17 \times (-20)$

**Q8.** True or false? Correct each false statement.

a. $(-3) \times (-4) = -12$
b. $(-12) \div (-3) = 4$
c. $(-5) \times 0 = -5$
d. $0 \div (-7) = 0$

***On your own***

**Q9.** Use known facts and patterns to find each value.

a. $(-11)\times(-13)$
b. $(-143)\div 11$
c. $(-9)\times 17$
d. $(-195)\div(-13)$

**Q10.** Find each value, choosing an efficient strategy where you see one.

a. $(-6)\times(-4)\times 5$
b. $(-48)\div(-2)$
c. $(-90)\div 3$

**Q11.** Sign first, then the number.

a. $\left(-\frac{1}{4}\right)\times(-2)$
b. $\left(-\frac{5}{4}\right)\div\left(-\frac{1}{2}\right)$
c. $(-0.7)\times 0.06$
d. $(-3.2)\div(-0.8)$

**Q12.** Two students show their work. Find each mistake and give the correct answer.

Sam: $\left(-\frac{2}{3}\right)\times\left(-\frac{3}{5}\right)=-\frac{2}{5}$

Lily: $(-8)\div(-2)=-4$

**Q13.** Estimate first, then find the exact answer and check it against your estimate.

a. $(-4.9)\times 8.2$
b. $-62.5+19.8$
c. $(-54.9)\div 6.1$

**Q14.** Find each value.

a. $13+(-4)$
b. $(-5)-6$
c. $(-24)\div 3$
d. $(-12)\times 9$

**Q15.** Mind the order of operations: multiply and divide before you add and subtract.

a. $(-36)\div 9+(-2)\times(-5)$
b. $(-40)\div(-8)-(-3)\times 4$
c. $(-7)\times(-6)\div(-2)$

**Q16.** Find each value without multiplying everything out.

a. $(-1)^{8}$
b. $(-1)^{15}$
c. $(-1)^{20}$
d. State the rule that lets you answer (a)–(c) at a glance.

### *Maths in real life*

**Q17.** A freezer is at −18 °C. A power fault lets the temperature rise by 2.5 °C each hour for 6 hours. What is the temperature at the end?

**Q18.** A bank account starts at −\$35. The owner banks \$80, then a \$12 fee is charged. What is the new balance?

**Q19.** A diver starts at −120 m and rises for 6 minutes at 15 m per minute. What depth does the diver reach?

**Q20.** Make your own. Write two products that both equal −24, one using exactly one negative factor and one using exactly three. Swap with a partner and check each other's sign counts.

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## Answers

**Q1.** a. −20. b. 42. c. −72. d. −36.

**Q2.** a. −8. b. 9. c. −9. d. −12.

**Q3.** a. −4. b. 35. c. 24. d. −8.

**Q4.** a. Negative (3 negative factors, odd). b. Positive (2, even). c. Negative (1, odd). d. Positive (4, even).

**Q5.** a. $-\frac{1}{3}$. b. $\frac{6}{5}$. c. $-\frac{1}{2}$. d. $-\frac{1}{8}$.

**Q6.** a. 0.8. b. 0.25. c. −0.75. d. −2.8.

**Q7.** a. 900. b. −3700. c. 1700.

**Q8.** a. False; $(-3)\times(-4)=12$. b. True. c. False; $(-5)\times 0=0$. d. True.

**Q9.** a. 143. b. −13. c. −153. d. 15.

**Q10.** a. 120. b. 24. c. −30.

**Q11.** a. $\frac{1}{2}$. b. $\frac{5}{2}$. c. −0.042. d. 4.

**Q12.** Sam: the signs are the same, so the answer is positive: $\frac{2}{5}$. Lily: same signs again, positive: 4.

**Q13.** a. Estimate −40; exact −40.18. b. Estimate −43; exact −42.7. c. Estimate −9; exact −9.

**Q14.** a. 9. b. −11. c. −8. d. −108.

**Q15.** a. 6. b. 17. c. −21.

**Q16.** a. 1. b. −1. c. 1. d. Even power of $-1$ gives 1; odd power gives $-1$.

**Q17.** −3 °C.

**Q18.** $33.

**Q19.** −30 m.

**Q20.** Answers will vary; check the sign counts. Sample: $(-2)\times 3\times 4=-24$ and $(-1)\times(-2)\times 3\times(-4)=-24$.

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## Fully-worked solutions

**Q1.** Fix the sign from the sign rules, then multiply the sizes.

a. Different signs: $-(4\times 5)=-20$.
b. Same signs: $6\times 7=42$.
c. Different signs: $-(8\times 9)=-72$.
d. Different signs: $-(6\times 6)=-36$.

**Q2.** Fix the sign, then divide the sizes.

a. Different signs: $-(16\div 2)=-8$.
b. Same signs: $27\div 3=9$.
c. Different signs: $-(45\div 5)=-9$.
d. Different signs: $-(48\div 4)=-12$.

**Q3.** Addition and subtraction are the Year 7 rules; multiplication uses the new card.

a. $(-9)+5=-4$ (different signs: subtract the sizes, keep the bigger one's sign).
b. $20-(-15)=20+15=35$ (subtracting a negative adds).
c. Same signs: $6\times4=24$.
d. $(-2)+(-6)=-8$ (same signs: add the sizes, keep the sign).

**Q4.** Count the negative factors; even gives positive, odd gives negative.

a. Three negative factors — odd — so negative. (Value: $-60$.)
b. Two — even — so positive. (Value: $24$.)
c. One — odd — so negative. (Value: $-42$.)
d. Four — even — so positive. (Value: $1$.)

**Q5.** Sign first, then the fraction work from Year 7.

a. Sign $-$; $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$; answer $-\frac{1}{3}$.
b. Sign $+$; $\frac{3}{4}\div\frac{5}{8}=\frac{3}{4}\times\frac{8}{5}=\frac{24}{20}=\frac{6}{5}$.
c. Sign $-$; $\frac{4}{5}\times\frac{5}{8}=\frac{20}{40}=\frac{1}{2}$; answer $-\frac{1}{2}$.
d. Sign $-$; $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$; answer $-\frac{1}{8}$.

**Q6.** Move along the number line, or fix the sign and compare the sizes.

a. $-1.2+2=0.8$.
b. $-0.5+0.75=0.25$.
c. $-0.25-0.5=-0.75$.
d. $-1.3-1.5=-2.8$ (two debts add up).

**Q7.** Reorder with the commutative law so the friendly pair multiplies first.

a. $(-25)\times(-4)\times9=100\times9=900$.
b. $25\times(-4)\times37=-100\times37=-3700$.
c. $(-5)\times(-20)\times17=100\times17=1700$.

**Q8.** Test each one against the sign rules.

a. **False.** Same signs give a positive: $(-3)\times(-4)=12$.
b. **True.** Same signs: $(-12)\div(-3)=4$.
c. **False.** Anything times 0 is 0: $(-5)\times0=0$.
d. **True.** Zero divided by anything (non-zero) is 0.

**Q9.** The number comes from a known fact; the sign from the sign rules.

a. Same signs; $11\times13=143$, so $143$.
b. Different signs; $143\div11=13$, so $-13$.
c. Different signs; $9\times17=9\times10+9\times7=153$, so $-153$.
d. Same signs; $195\div13=15$, so $15$.

**Q10.**

a. Two negatives — even — positive: $6\times4\times5=120$.

b. Same signs: $48 \div 2 = 24$.
c. Different signs: $-(90 \div 3) = -30$.

**Q11.**

a. Sign +; $\frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$.
b. Sign +; $\frac{5}{4} \div \frac{1}{2} = \frac{5}{4} \times 2 = \frac{10}{4} = \frac{5}{2}$.
c. Sign −; $7 \times 6 = 42$ with three decimal places: $0.042$; answer $-0.042$.
d. Sign +; $3.2 \div 0.8 = 32 \div 8 = 4$.

**Q12.** Both mistakes are sign mistakes — the number working is fine.

Sam: two negatives multiplied give a positive, so $\left(-\frac{2}{3}\right) \times \left(-\frac{3}{5}\right) = +\frac{2 \times 3}{3 \times 5} = \frac{2}{5}$.

Lily: two negatives divided give a positive, so $(-8) \div (-2) = 4$.

**Q13.** Estimate to fix the sign and the size, then work exactly.

a. Estimate: $(-5) \times 8 = -40$. Exact: $4.9 \times 8.2 = 40.18$, different signs, so $-40.18$. ✓
b. Estimate: $-63 + 20 = -43$. Exact: the negative is the larger, and $62.5 - 19.8 = 42.7$, so $-42.7$. ✓
c. Estimate: $(-54) \div 6 = -9$. Exact: $54.9 \div 6.1 = 9$ (since $6.1 \times 9 = 54.9$), different signs, so $-9$. ✓

**Q14.**

a. $13 + (-4) = 9$.
b. $(-5) - 6 = -11$.
c. Different signs: $-(24 \div 3) = -8$.
d. Different signs: $-(12 \times 9) = -108$.

**Q15.** Multiply and divide first, left to right, then add and subtract.

a. $(-36) \div 9 = -4$ and $(-2) \times (-5) = 10$, so $-4 + 10 = 6$.
b. $(-40) \div (-8) = 5$ and $(-3) \times 4 = -12$, so $5 - (-12) = 5 + 12 = 17$.
c. Left to right: $(-7) \times (-6) = 42$, then $42 \div (-2) = -21$.

**Q16.** Count the factors of −1.

a. Eight factors — even — so 1.
b. Fifteen — odd — so $-1$.
c. Twenty — even — so 1.
d. $(-1)$ to an even power is 1; to an odd power is $-1$, because the negative factors pair up (or leave one unpaired).

**Q17.** The rise adds $6 \times 2.5 = 15$ degrees: $(-18) + 15 = -3$. ∴ the freezer ends at −3 °C.

**Q18.** $(-35) + 80 = 45$, then $45 - 12 = 33$. ∴ the new balance is $33.

**Q19.** The rise adds $6 \times 15 = 90$ m: $(-120) + 90 = -30$. ∴ the diver reaches −30 m.

**Q20.** Answers will vary. A sample pair: $(-2) \times 3 \times 4 = -24$ (one negative factor — odd — negative product) and $(-1) \times (-2) \times 3 \times (-4) = -24$ (three negative factors — odd — negative product). Check: both sign counts are odd, and both products are −24.

## The exponent laws with positive integer exponents and the zero exponent

**Content description `VC2M8N02`:** establish and apply the exponent laws with positive integer exponents and the zero exponent, using exponent notation with numbers

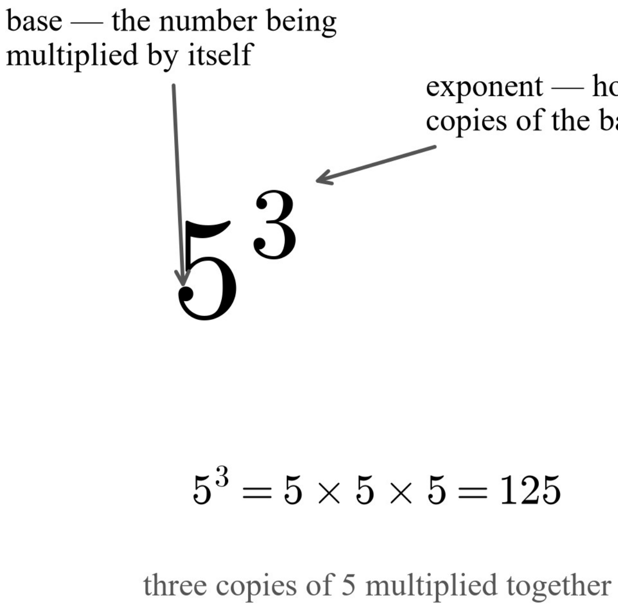
*One fold of a sheet of paper gives 2 layers, two folds give 4, three give 8 — and the count keeps doubling. A cube that is twice as long on every edge holds eight times the volume. Situations like these multiply the same number over and over, and writing every factor out quickly gets messy. Last year you learned the notation that shortens the writing. This subtopic gives you three laws that shorten the working — and one surprise about what a zero exponent must mean.*

### Theory

In Year 7 (`VC2M7N02`) you wrote repeated multiplication in **exponent notation**, and you wrote numbers as products of powers of primes. This subtopic takes that notation and gives you its three **exponent laws**, plus the rule for a zero exponent. They are also called the *index laws* in some other books and in VCE; in this book we use VCAA's name, the **exponent laws**.

#### *Recall: the parts of a power*

In $5^3 = 5 \times 5 \times 5$, the number being multiplied, 5, is the **base**, and the small raised number, 3, is the **exponent**; it counts the factors. Writing $5 \times 5 \times 5$ out in full is called **expanded form**. You met this in Year 7, so this subtopic uses it without re-teaching it.




*The parts of 5 cubed: the base 5, the exponent 3, and the expanded form 5 times 5 times 5*

#### *The product law — multiplying with the same base*

Look at $2^3 \times 2^2$ in expanded form:

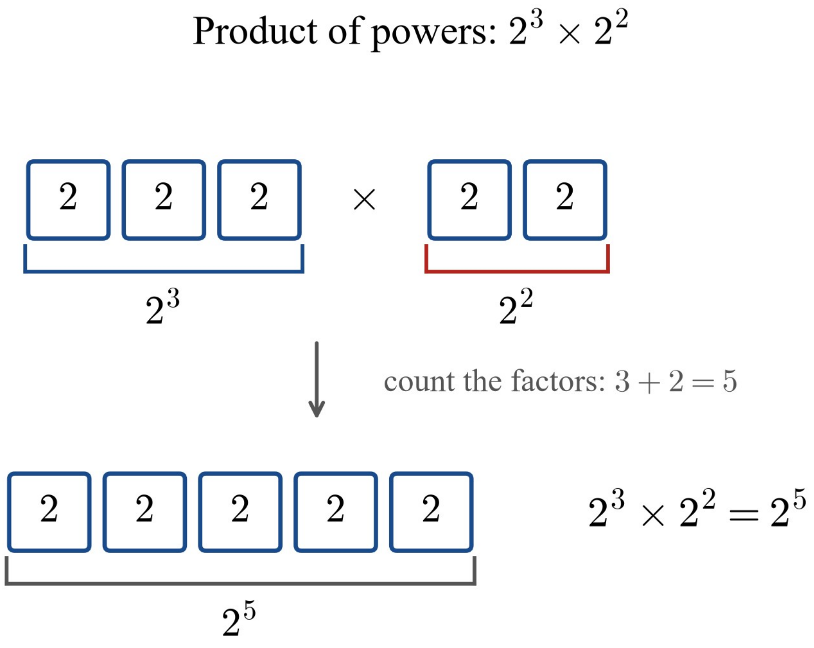
$$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$$

There are 3 factors of 2, then 2 more, so there are $3 + 2 = 5$ factors altogether. That is the first law.

**Product law.** To multiply powers with the same base, add the exponents.

$$5^2 \times 5^4 = 5^{2+4} = 5^6, 3^4 \times 3 = 3^{4+1} = 3^5$$

In the second example, the exponent of $3$ is the invisible 1: $3 = 3^1$.




*Two cubed times two squared in expanded form: three factors of 2 and two more make five factors, so the answer is two to the power five*

### ***The quotient law — dividing with the same base***

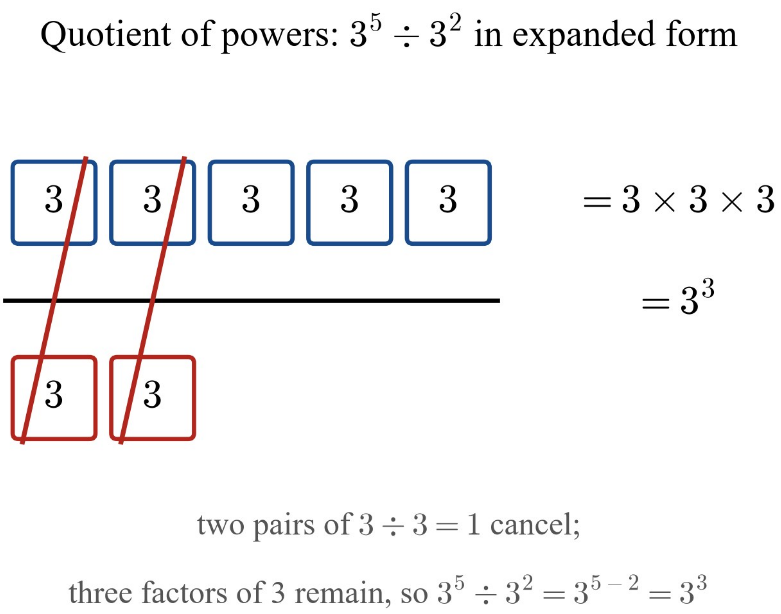
Look at $3^5 \div 3^2$ in expanded form, and cancel the two 3s that appear in both the top and the bottom:

$$3^5 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} = 3 \times 3 \times 3 = 3^3$$

There were 5 factors on top and 2 on the bottom; cancelling leaves $5-2=3$. That is the second law.

**Quotient law.** To divide powers with the same base, subtract the exponent of the bottom from the exponent of the top.

$$7^6 \div 7^4 = 7^{6-4} = 7^2, \frac{10^5}{10^2} = 10^{5-2} = 10^3$$




*Three to the power five divided by three squared in expanded form: two factors of 3 cancel, leaving three factors, so the answer is three cubed*

***The power-of-a-power law — a power raised to an exponent***

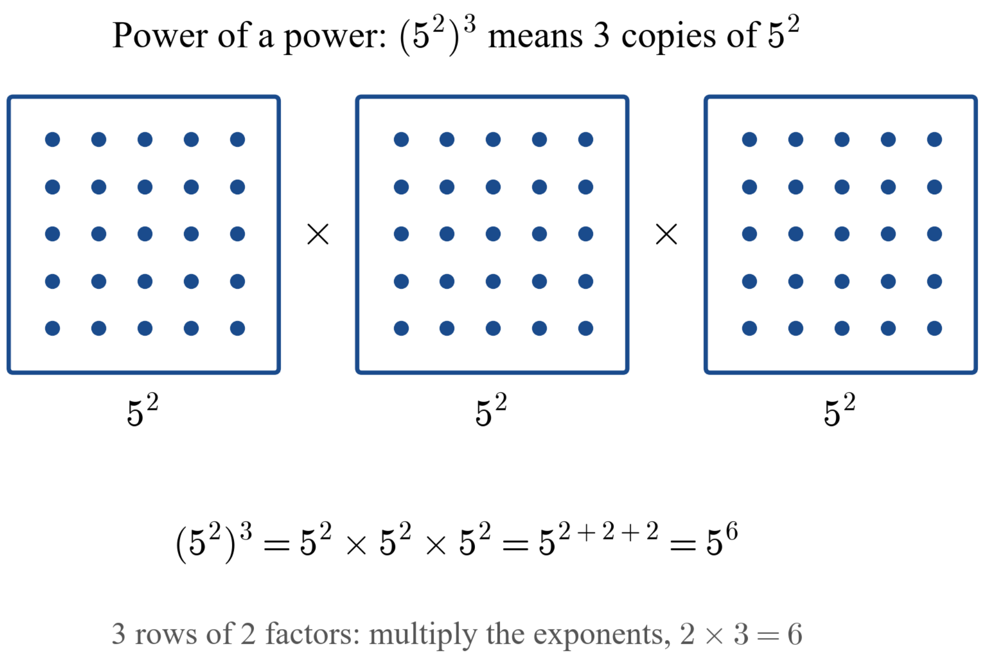
$(5^2)^3$ means $5^2$ multiplied by itself 3 times:

$$(5^2)^3 = 5^2 \times 5^2 \times 5^2 = 5^{2+2+2} = 5^6$$

Adding the same exponent 3 times is multiplying, so:

**Power-of-a-power law.** To raise a power to an exponent, multiply the exponents.

$$(3^3)^2 = 3^{3\times 2} = 3^6, (2^4)^3 = 2^{4\times 3} = 2^{12}$$



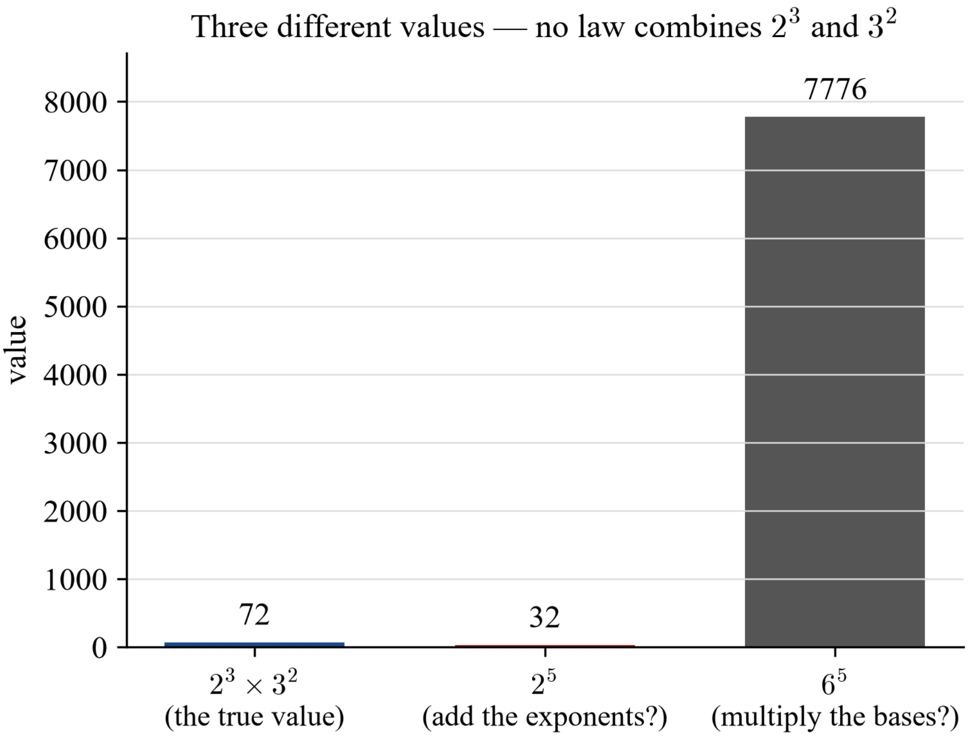

*Five squared, all cubed, shown as three grids of five by five dots: three groups of two factors of 5 make six factors, so the answer is five to the power six*

### ***The bases must be the same***

All three laws work **only when the bases are the same**. Check what goes wrong when they are not:

$$2^3 \times 3^2 = 8 \times 9 = 72\text{, but } 6^5 = 7776$$

So $2^3 \times 3^2$ is **not** $6^5$ — the bases 2 and 3 cannot be joined, and the answer stays as $2^3 \times 3^2$, or is worked out as 72. The same goes for $2^3 \times 2^2$, which is $2^5 = 32$ and **not** $4^5$: adding exponents never changes the base.




*Three bars compared: two cubed times three squared is 72, two cubed times two squared is 32, and six to the power five is 7776 — the bases must match before any law applies*

### *The zero exponent*

Start at $3^4 = 81$ and divide by 3 at each step:

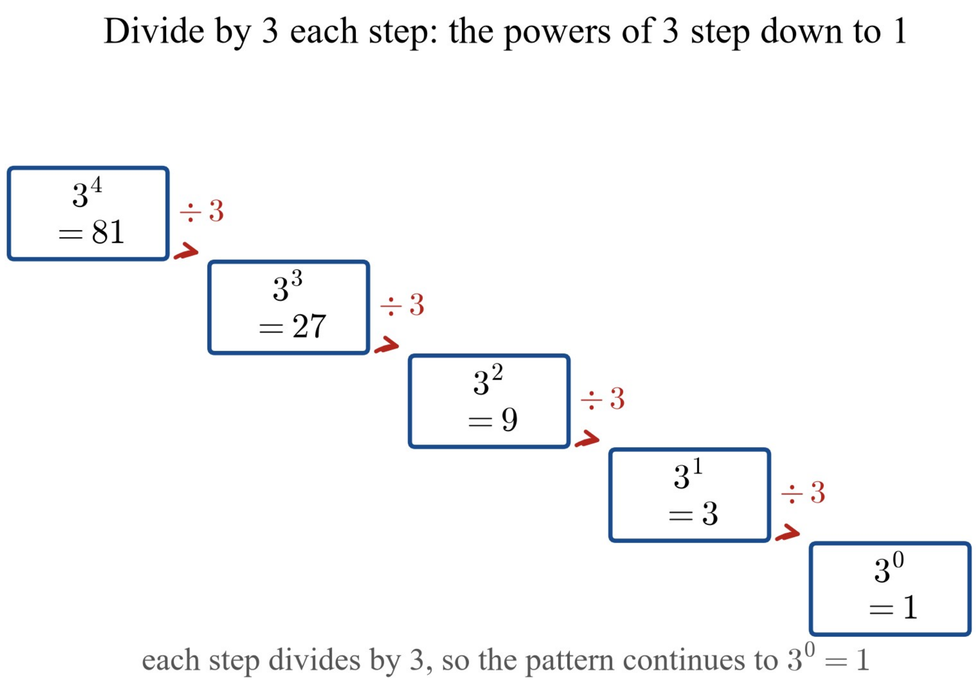
$$3^4 = 81, 3^3 = 27, 3^2 = 9, 3^1 = 3, \ldots$$

Each step down divides by 3, so the next value is $3 \div 3 = 1$. The pattern says $3^0 = 1$.

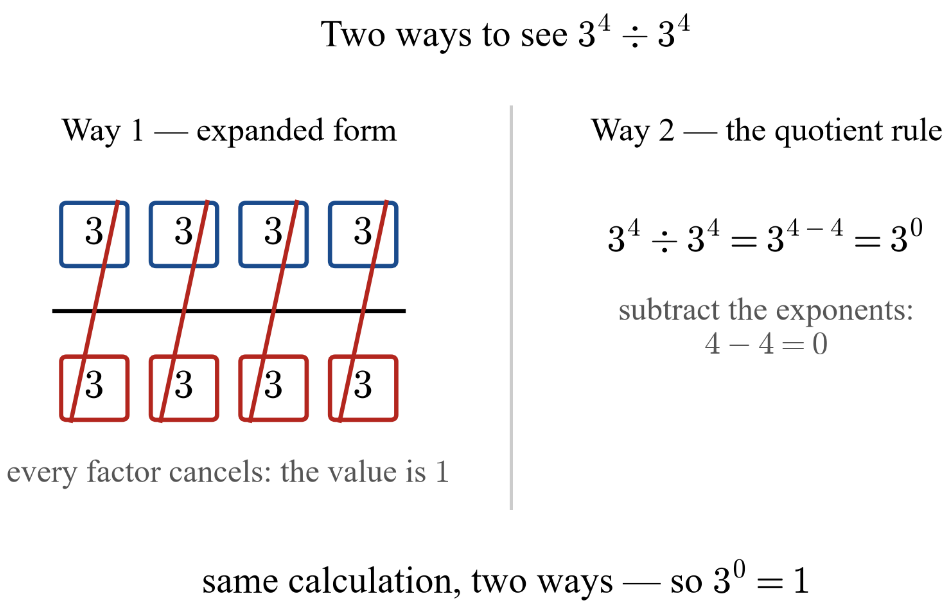
The quotient law says the same thing. Any non-zero number divided by itself is 1, so

$$\frac{3^4}{3^4} = 1, \text{and also} \frac{3^4}{3^4} = 3^{4-4} = 3^0$$

Two ways of reading the same calculation give the same answer, so $3^0 = 1$. VCAA states the rule this way: **for any non-zero natural number** $n$**,** $n^0 = 1$.




*A staircase from three to the power four equals 81 down to three to the power zero equals 1, dividing by 3 at every step*




*Three to the power four divided by itself shown two ways: cancelling every factor gives 1, and the quotient law gives three to the power zero — so three to the power zero is 1*

### *Negative bases — a recall from 1A*

1A owned the sign rules, and 1B uses them. A negative base in brackets raises the whole negative number:

$$(-2)^1 = -2, (-2)^2 = 4, (-2)^3 = -8, (-2)^4 = 16, (-2)^5 = -32$$

The signs alternate, just as 1A showed with $(-1)^4 = 1$ and $(-1)^5 = -1$: an **even** exponent gives a positive answer, and an **odd** exponent gives a negative one. The exponent laws apply to negative bases exactly as before:

$$(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5 = -243$$

Brackets matter. $(-2)^4 = 16$, because four factors of −2 multiply to a positive, but $-2^4 = -16$, because the exponent applies to the 2 only and the minus sign stays out the front.

Powers of a negative base: the sign alternates

$\boxed{(-2)^1 = -2} \rightarrow \boxed{(-2)^2 = 4} \rightarrow \boxed{(-2)^3 = -8} \rightarrow \boxed{(-2)^4 = 16} \rightarrow \boxed{(-2)^5 = -32}$

each step multiplies by $-2$: the size doubles and the sign flips

Brackets change the base:

$$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16, \text{ but } -2^4 = -(2^4) = -16$$

*Powers of negative two from exponent 1 to 5 alternating in sign, with the reminder that negative two, all to the power four, is 16, but negative two to the power four is negative 16*

### ***Combining the laws***

This subtopic is the place where the laws are used together. Work one step at a time, in this order: power of a power first, then the product law, then the quotient law.

$$(2^3)^2 \times 2^5 \div 2^6 = 2^6 \times 2^5 \div 2^6 = 2^{6+5-6} = 2^5 = 32$$

**Product of powers**

$$2^3 \times 2^4 = 2^7$$

same base: add the exponents

**Quotient of powers**

$$5^6 \div 5^2 = 5^4$$

same base: subtract the exponents

**Power of a power**

$$(3^2)^4 = 3^8$$

multiply the exponents

**Zero exponent**

$$7^0 = 1$$

any non-zero number to the exponent 0 equals 1

The product, quotient and power-of-a-power laws work only when the bases are the same.

*Summary card of the four results of this subtopic: multiply with the same base by adding exponents, divide by subtracting them, raise a power to a power by multiplying them, and any non-zero base to the power zero is 1*

In Year 9 (`VC2M9A01`) these same laws are applied to letters as well as numbers, and to negative exponents. This subtopic keeps to numbers, with positive integer exponents and the zero exponent.

### *Words of this subtopic*

| Word | What it means here |
|---|---|
| **base** | the number being multiplied; in $5^3$ the base is 5 |
| **exponent** | the small raised number that counts the factors; in $5^3$ the exponent is 3 |
| **power** | a base with its exponent, written in exponent notation; $5^3$ is a power of 5 |
| **expanded form** | the multiplication written out in full: $5^3 = 5 \times 5 \times 5$ |
| **product law** | multiply powers of the same base by adding the exponents |
| **quotient law** | divide powers of the same base by subtracting the exponents |
| **power-of-a-power law** | raise a power to an exponent by multiplying the exponents |
| **zero exponent** | for any non-zero natural number $n$, $n^0 = 1$ |
| **natural numbers** | the numbers 1, 2, 3, 4, … used for counting |

### Worked examples

#### *Example 1 — with help*

Use the exponent laws to simplify, then find the value.

(a) $5^2 \times 5^4$
(b) $7^6 \div 7^4$

| Working | Comment |
|---|---|
| (a) $5^2 \times 5^4 = 5^{2+4} = 5^6$ | Same base 5, multiplied — add the exponents. |
| $5^6 = 15625$ | Either multiply out $5 \times 5 \times 5 \times 5 \times 5 \times 5$, or use $5^2 \times 5^4 = 25 \times 625$. |
| (b) $7^6 \div 7^4 = 7^{6-4} = 7^2$ | Same base 7, divided — subtract the exponents. |
| $7^2 = 49$ | Two factors of 7. |

∴ (a) $5^6 = 15625$; (b) $7^2 = 49$. Notice how the law turns a long multiplication into a short one.

#### *Example 2 — with help*

Use the exponent laws to simplify, then find the value.

(a) $(3^3)^2$
(b) $9^4 \div 9^4$

| Working | Comment |
|---|---|
| (a) $(3^3)^2 = 3^{3\times 2} = 3^6$ | A power raised to an exponent — multiply the exponents. |
| $3^6 = 729$ | $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$. |
| (b) $9^4 \div 9^4 = 9^{4-4} = 9^0$ | Same base divided — subtract the exponents. |
| $9^0 = 1$ | Any non-zero number to the power zero is 1. (Also: any non-zero number divided by itself is 1.) |

∴ (a) $3^6 = 729$; (b) $9^0 = 1$.

#### *Example 3 — with help*

Simplify, then find the value: $(2^3)^2 \times 2^5 \div 2^6$.

| Working | Comment |
|---|---|
| $(2^3)^2 = 2^6$ | Power of a power first: multiply the exponents. |
| $2^6 \times 2^5 = 2^{11}$ | Now the product law: add the exponents. |
| $2^{11} \div 2^6 = 2^5$ | Now the quotient law: subtract the exponents. |
| $2^5 = 32$ | Five factors of 2. |

∴ $(2^3)^2 \times 2^5 \div 2^6 = 2^5 = 32$. When laws are combined, take one law per step, in the order: power of a power, product, quotient.

#### *Example 4 — on your own*

Simplify, then find the value.

(a) $(-3)^2 \times (-3)^3$
(b) $-3^2 \times 3^3$

Same base −3 in (a), so the product law applies; the answer is a power of −3, and 1A's sign rule decides its sign. In (b) there are no brackets, so $-3^2$ means "the negative of $3^2$".

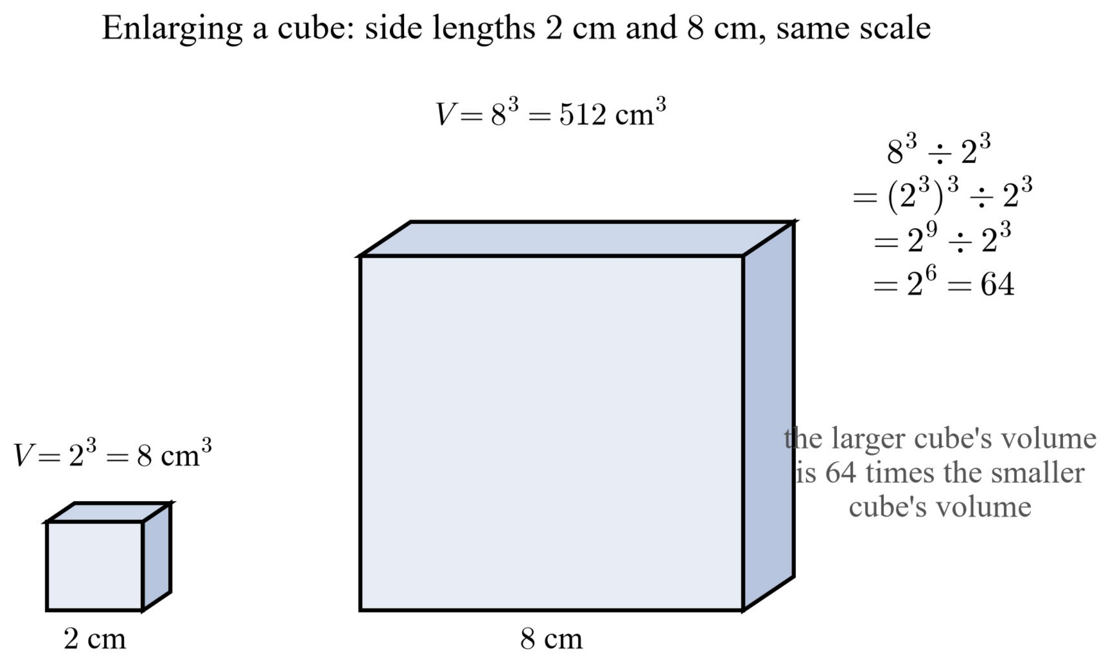
(a) $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5$. The exponent 5 is odd, so the answer is negative: $(-3)^5 = -243$.

(b) $-3^2 \times 3^3 = -9 \times 27 = -243$.

∴ (a) $-243$; (b) $-243$. The two answers are the same number, but for different reasons: in (a) the laws did the work, while in (b) the minus sign stood outside the exponent. Brackets change the meaning, even when the value does not.

### *Example 5 — on your own*

A cube with side length 2 cm is enlarged to a cube with side length 8 cm. How many times larger is the volume?




*A 2 cm cube and an 8 cm cube drawn to scale, with the working 8 cubed divided by 2 cubed equals two to the power nine divided by two cubed equals two to the power six, which is 64*

The volume of a cube is $\text{side}^3$, so the factor is $8^3 \div 2^3$. Write 8 as a power of 2 so the bases match: $8 = 2^3$.

$$8^3 \div 2^3 = (2^3)^3 \div 2^3 = 2^9 \div 2^3 = 2^{9-3} = 2^6 = 64$$

∴ the larger cube's volume is 64 times the smaller cube's volume. Doubling the side length would multiply the volume by $2^3 = 8$; making the side 4 times as long multiplies it by $4^3 = 64$.

### *Example 6 — on your own*

Simplify, then find the value: $(6^4 \times 6^0) \div 6^2$.

The zero exponent goes first: $6^0 = 1$, so multiplying by $6^0$ changes nothing. Then combine the exponents in one line:

$$(6^4 \times 6^0) \div 6^2 = 6^{4+0-2} = 6^2 = 36$$

∴ $(6^4 \times 6^0) \div 6^2 = 6^2 = 36$. The zero exponent fits the laws perfectly: adding 0 to an exponent changes nothing, just as multiplying by 1 changes nothing.

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## Practice questions

*Work these questions by hand. Every number is chosen so the laws do the work — no calculator is needed.*

***With help***

**Q1.** Write each product in exponent form.

a. $7 \times 7 \times 7 \times 7$
b. $(-3) \times (-3) \times (-3)$
c. $10 \times 10 \times 10 \times 10 \times 10$

**Q2.** Find the value.

a. $2^5$
b. $10^4$
c. $(-3)^3$
d. $1^9$

**Q3.** Use the product law to write each expression as a single power.

a. $3^4 \times 3^5$
b. $10^3 \times 10^5$
c. $2^5 \times 2^3$

**Q4.** Use the quotient law to simplify, then find the value.

a. $5^7 \div 5^3$
b. $3^5 \div 3^3$
c. $(-2)^7 \div (-2)^3$

**Q5.** Use the power-of-a-power law to write each expression as a single power.

a. $(3^4)^2$
b. $(7^2)^3$
c. $(10^3)^3$

**Q6.** True or false? Give your reason, and correct each false statement.

a. $2^3 \times 2^4 = 4^7$
b. $5^6 \div 5^2 = 5^4$
c. $(2^3)^2 = 2^6$
d. $7^0 = 0$
e. $2^3 \times 3^2 = 6^5$

**Q7.** Copy and complete each statement with the missing number.

a. $5^3 \times 5^2 = 5^{\square}$
b. $4^7 \div 4^2 = 4^{\square}$
c. $(7^3)^{\square} = 7^9$
d. $2^4 \times 2^{\square} = 2^8$

**Q8.** Find the value.

a. $4^0$
b. $2^6$
c. $(-5)^0$

d. $9^0 + 9^0$

### *On your own*

**Q9.** Simplify, then find the value.

a. $2^5 \div 2$
b. $3^4 \div 3^2$
c. $10^4 \div 10^2$

**Q10.** Find the value. Watch the signs.

a. $(-5)^5$
b. $(-2)^2$
c. $(-4)^3$

**Q11.** Explain why $(-6)^2 = 36$ but $-6^2 = -36$.

**Q12.** Simplify, then find the value. The zero exponent is involved in each one.

a. $2^5 \times 2^0$
b. $3^4 \div 3^2 \times 3^0$
c. $10^3 \times 10^0$
d. $5^0 + 5^1$

**Q13.** You know that $72 = 2^3 \times 3^2$ and $12 = 2^2 \times 3$.

a. Write $72 \times 12$ as a product of powers of primes, then find its value.
b. Use the quotient law on each prime to find $72 \div 12$.

**Q14.** Mia writes $2^3 \times 3^2 = 6^5$. Explain what she has done wrong, and check your explanation by finding the value of each side.

**Q15.** Folding a sheet of paper doubles the number of layers: 1 fold gives 2 layers, 2 folds give 4 layers. (Ignore the fact that real paper gives up long before this.)

a. How many layers are there after 6 folds?
b. How many layers after 7 folds? Write your answer using the product law.
c. How many times more layers are there after 9 folds than after 6 folds?

**Q16.** On day 1, three students hear a rumour. Each day, every student who heard it the day before tells three new students.

a. Write the number of new students on day 4 as a power of 3, and find its value.
b. How many new students hear the rumour altogether on days 1, 2 and 3?
c. How many times more new students hear it on day 4 than on day 2?

### *Maths in real life*

**Q17.** A cube of side length 4 cm is enlarged to a cube of side length 8 cm. How many times larger is the volume? Give your answer by simplifying $8^3 \div 4^3$ with the exponent laws, writing both bases as powers of 2.

**Q18.** In a lab, a culture of bacteria doubles in size every hour, starting from a single unit at time zero.

a. Write the size after 6 hours as a power of 2, and find its value.
b. How many times larger is the culture after 9 hours than after 6 hours?

**Q19.** A memory card holds $2^{25}$ bytes of data. Each photo takes $2^{20}$ bytes. How many photos fit on the card? Simplify $2^{25} \div 2^{20}$ to find out.

**Q20.** Make your own. Write an expression with numbers that needs at least two of the exponent laws to simplify — for example, a power of a power, then a quotient. Simplify it to a single power and find its value. Swap with a partner, simplify each other's expression, and check that you agree.

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## Answers

**Q1.** a. $7^4$. b. $(-3)^3$. c. $10^5$.

**Q2.** a. 32. b. 10000. c. −27. d. 1.

**Q3.** a. $3^9$. b. $10^8$. c. $2^8$.

**Q4.** a. $5^4=625$. b. $3^2=9$. c. $(-2)^4=16$.

**Q5.** a. $3^8$. b. $7^6$. c. $10^9$.

**Q6.** a. False; $2^3\times 2^4=2^7$ — the base does not change. b. True. c. True. d. False; $7^0=1$. e. False; the bases are different, so no law joins them; $2^3\times 3^2=72$ but $6^5=7776$.

**Q7.** a. 5. b. 5. c. 3. d. 4.

**Q8.** a. 1. b. 64. c. 1. d. 2.

**Q9.** a. $2^4=16$. b. $3^2=9$. c. $10^2=100$.

**Q10.** a. −3125. b. 4. c. −64.

**Q11.** $(-6)^2$ squares the −6, giving 36; $-6^2$ squares only the 6 and keeps the minus out the front, giving −36.

**Q12.** a. $2^5=32$. b. $3^2=9$. c. $10^3=1000$. d. $1+5=6$.

**Q13.** a. $2^5\times 3^3=864$. b. $2^1\times 3^1=6$.

**Q14.** The bases 2 and 3 are different, so the product law does not apply. $2^3\times 3^2=72$ but $6^5=7776$.

**Q15.** a. 64 layers. b. 128 layers; $2^6\times 2^1=2^7$. c. 8 times.

**Q16.** a. $3^4=81$. b. $3+9+27=39$. c. 9 times.

**Q17.** 8 times.

**Q18.** a. $2^6=64$ units. b. 8 times.

**Q19.** 32 photos.

**Q20.** Answers will vary. Check: at least two laws are used, the simplifying is correct, and the final value is right.

---

## Fully-worked solutions

**Q1.** Count the equal factors.

a. Four factors of 7: $7\times 7\times 7\times 7=7^4$.
b. Three factors of −3, brackets included: $(-3)\times(-3)\times(-3)=(-3)^3$.
c. Five factors of 10: $10\times 10\times 10\times 10\times 10=10^5$.

∴ a. $7^4$; b. $(-3)^3$; c. $10^5$.

**Q2.** Multiply out the factors; the sign rule from 1A decides the sign in (c).

a. $2^5=2\times 2\times 2\times 2\times 2=32$.
b. $10^4=10\times 10\times 10\times 10=10000$.
c. $(-3)^3=(-3)\times(-3)\times(-3)=9\times(-3)=-27$; the odd exponent gives a negative answer.

d. $1^9 = 1$; multiplying 1 by itself any number of times stays 1.

**Q3.** Multiply powers of the same base by adding the exponents.

a. $3^4 \times 3^5 = 3^{4+5} = 3^9$.
b. $10^3 \times 10^5 = 10^{3+5} = 10^8$.
c. $2^5 \times 2^3 = 2^{5+3} = 2^8$.

**Q4.** Divide powers of the same base by subtracting the exponents.

a. $5^7 \div 5^3 = 5^{7-3} = 5^4 = 625$.
b. $3^5 \div 3^3 = 3^{5-3} = 3^2 = 9$.
c. $(-2)^7 \div (-2)^3 = (-2)^{7-3} = (-2)^4 = 16$; the even exponent gives a positive answer.

**Q5.** Raise a power to an exponent by multiplying the exponents.

a. $(3^4)^2 = 3^{4\times 2} = 3^8$.
b. $(7^2)^3 = 7^{2\times 3} = 7^6$.
c. $(10^3)^3 = 10^{3\times 3} = 10^9$.

**Q6.** Test each statement against the laws.

a. **False.** Multiplying powers of the same base adds the exponents and keeps the base: $2^3 \times 2^4 = 2^7$, not $4^7$.
b. **True.** $5^6 \div 5^2 = 5^{6-2} = 5^4$.
c. **True.** $(2^3)^2 = 2^{3\times 2} = 2^6$.
d. **False.** Any non-zero number to the power zero is 1, so $7^0 = 1$, not 0.
e. **False.** The bases 2 and 3 are different, so no law joins them into a single power. $2^3 \times 3^2 = 8 \times 9 = 72$, but $6^5 = 7776$.

**Q7.** Apply one law in each part.

a. Add: $3 + 2 = 5$, so $5^3 \times 5^2 = 5^5$.
b. Subtract: $7 - 2 = 5$, so $4^7 \div 4^2 = 4^5$.
c. Multiply: $3 \times 3 = 9$, so the missing exponent is 3: $(7^3)^3 = 7^9$.
d. Add to reach 8: $4 + 4 = 8$, so $2^4 \times 2^4 = 2^8$.

**Q8.** Use the zero exponent and multiplication.

a. $4^0 = 1$.
b. $2^6 = 64$.
c. $(-5)^0 = 1$; the base is non-zero, so the zero exponent rule applies, brackets and all.
d. $9^0 + 9^0 = 1 + 1 = 2$; the zero exponent applies to each term separately.

**Q9.** Divide powers of the same base by subtracting the exponents; in (a) the exponent of the bottom 2 is the invisible 1.

a. $2^5 \div 2 = 2^{5-1} = 2^4 = 16$.
b. $3^4 \div 3^2 = 3^{4-2} = 3^2 = 9$.
c. $10^4 \div 10^2 = 10^{4-2} = 10^2 = 100$.

**Q10.** Multiply out the factors; whether the exponent is even or odd decides the sign.

a. $(-5)^5$: five factors of $-5$. The exponent is odd, so the answer is negative: $5^5 = 3125$, so $(-5)^5 = -3125$.

b. $(-2)^2=(-2)\times(-2)=4$; the even exponent gives a positive answer.
c. $(-4)^3=(-4)\times(-4)\times(-4)=16\times(-4)=-64$.

**Q11.** The brackets decide what the exponent applies to.

$(-6)^2=(-6)\times(-6)=36$: the exponent applies to the whole negative number. $-6^2=-(6^2)=-(6\times6)=-36$: with no brackets, the exponent applies to the 6 only, and the minus sign stays out the front.

∴ the brackets make the minus part of the base; without them, the base is 6 and the answer keeps its minus.

**Q12.** Multiply by $n^0$ is multiply by 1.

a. $2^5\times2^0=2^{5+0}=2^5=32$; adding 0 to the exponent changes nothing.
b. $3^4\div3^2\times3^0=3^{4-2+0}=3^2=9$.
c. $10^3\times10^0=10^{3+0}=10^3=1000$.
d. $5^0+5^1=1+5=6$; this is addition, so no law applies — find each value first.

**Q13.** Collect the powers of each prime.

a. $72\times12=2^3\times3^2\times2^2\times3^1=2^{3+2}\times3^{2+1}=2^5\times3^3=32\times27=864$.
b. $72\div12=(2^3\times3^2)\div(2^2\times3^1)=2^{3-2}\times3^{2-1}=2^1\times3^1=6$.

∴ a. $2^5\times3^3=864$, which matches $72\times12=864$; b. 6.

**Q14.** Mia has joined different bases. The product law only works when the bases are the same, and 2 and 3 are not the same:

$$2^3\times3^2=8\times9=72,6^5=7776$$

$72\neq7776$, so $2^3\times3^2\neq6^5$. The expression $2^3\times3^2$ is already as simple as the laws can make it.

**Q15.** Each fold doubles the layers, so the layers are powers of 2.

a. After 6 folds: $2^6=64$ layers.
b. After 7 folds: $2^6\times2^1=2^{6+1}=2^7=128$ layers — one more fold multiplies the layers by $2^1$.
c. $\frac{2^9}{2^6}=2^{9-6}=2^3=8$ times as many.

**Q16.** Each day multiplies the new students by 3.

a. Day 1 is $3^1$, day 2 is $3^2$, day 3 is $3^3$, so day 4 is $3^4=81$ new students.
b. $3+9+27=39$ new students over the first three days.
c. $\frac{3^4}{3^2}=3^{4-2}=3^2=9$ times as many.

**Q17.** The volume factor is $8^3\div4^3$. Write both bases as powers of 2: $8=2^3$ and $4=2^2$.

$$8^3\div4^3=(2^3)^3\div(2^2)^3=2^9\div2^6=2^{9-6}=2^3=8$$

∴ the volume is 8 times larger. (Check: $8^3=512$, $4^3=64$, and $512\div64=8$.)

**Q18.** Each hour multiplies the size by 2.

a. After 6 hours: $2^6=64$ times the starting size — 64 units.
b. $\frac{2^9}{2^6}=2^{9-6}=2^3=8$ times larger.

**Q19.** Divide the total space by the space per photo:

$$2^{25} \div 2^{20} = 2^{25-20} = 2^{5} = 32$$

∴ 32 photos fit on the card.

**Q20.** Answers will vary. A sample: $\left(\left(5^{2}\right)^{3} \times 5^{4}\right) \div 5^{6}$. Power of a power: $\left(5^{2}\right)^{3} = 5^{6}$. Product law: $5^{6} \times 5^{4} = 5^{10}$. Quotient law: $5^{10} \div 5^{6} = 5^{4} = 625$. Check your partner's expression the same way: one law per step, then find the value.

## Fractions as terminating and recurring decimals

**Content description `VC2M8N03`:** convert between fractions and terminating or recurring decimals, using digital tools as appropriate

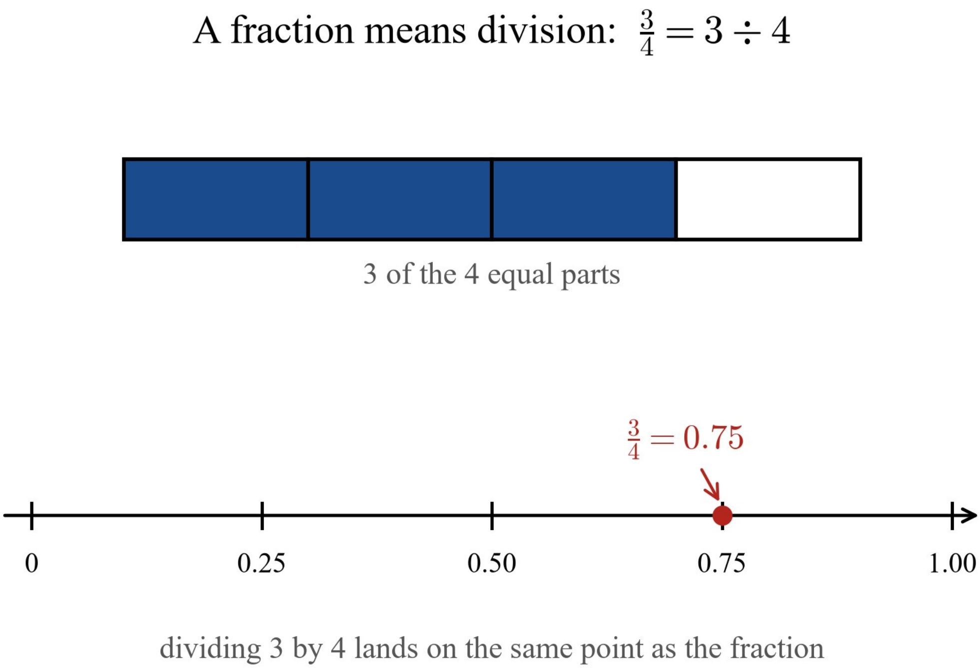
*Share five dollars equally between 4 people and everyone gets exactly one dollar and twenty-five cents. Share it between 6 and the calculator says 0.8333333…, never ending. Same five dollars — two completely different kinds of decimal. Every fraction, divided out, either stops or falls into a loop of digits that repeats forever; and, better still, you can read off the denominator which one it will be before you divide at all. This subtopic runs the conversion both ways: fraction to decimal, and decimal back to fraction.*

### Theory

A fraction is a division waiting to happen: the bar means ÷, so $\frac{3}{4}$ is $3 \div 4$, and dividing it out gives the decimal. In Year 7 (`VC2M7N02`) you also wrote numbers as products of powers of primes — $24 = 2^3 \times 3$ — and 1B used that notation for the exponent laws. Here it becomes a shortcut: the prime factors of the denominator tell you how the decimal behaves before you do any division at all.

#### *Recall: a fraction means division*

Take $\frac{3}{4}$ of a metre of ribbon: the metre is cut into 4 equal parts and 3 of them are kept. The size of that share is exactly $3 \div 4$, and the number line below shows the answer the division gives.

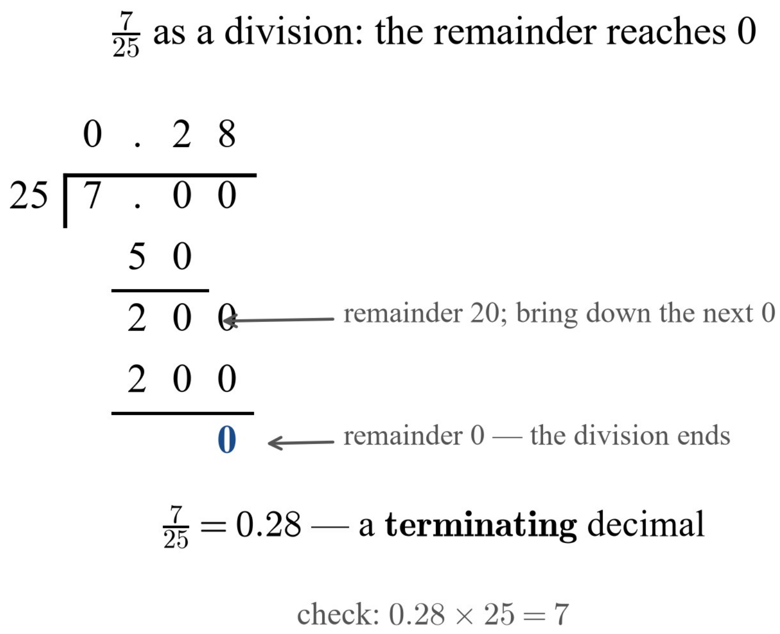



*Three quarters as a division: a bar model of 3 out of 4 equal parts, and a number line from 0 to 1 with a dot at 0.75, showing 3 divided by 4 equals 0.75*

So $\frac{3}{4} = 3 \div 4 = 0.75$. Every fraction works the same way: top divided by bottom. The only question is what the division does in the end.

#### *Terminating decimals*

Divide 7 by 25 and watch the remainders:

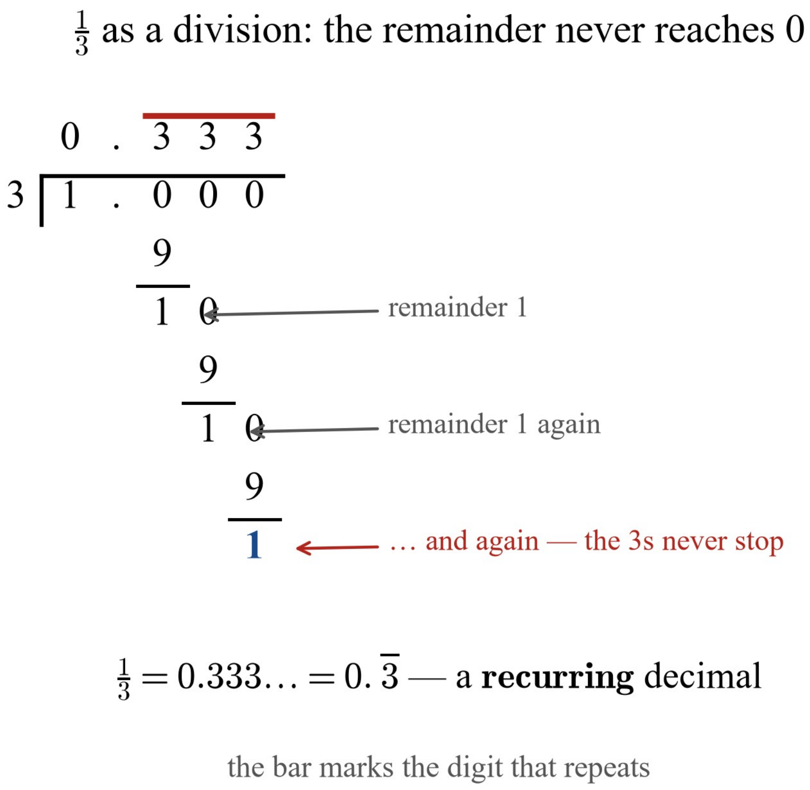



*Long division of 7 by 25: the quotient is 0.28 and the last remainder is 0, so the division ends; 7 over 25 equals 0.28, a terminating decimal; check 0.28 times 25 equals 7*

The remainder reaches $0$, so the division ends. A decimal that ends like this is a **terminating decimal**: $0.28$ is exactly $\frac{7}{25}$, with no rounding at all. You have used terminating decimals all your life — $0.5$, $0.75$, $0.125$ and $1.25$ are all of this kind.

### ***Recurring decimals and the bar***

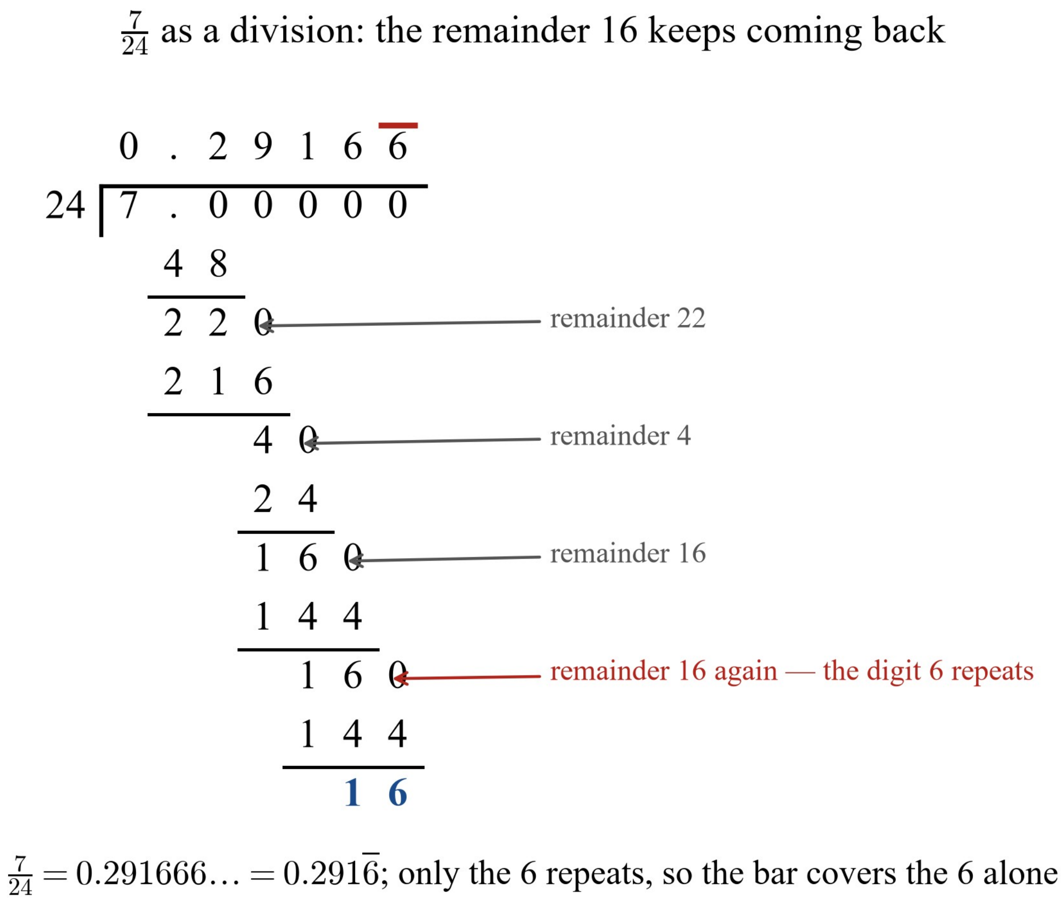
Now divide 1 by 3:




*Long division of 1 by 3: the remainder is 1 at every step, so the 3s never stop; 1 over 3 equals 0.333… written with a bar over the 3, a recurring decimal; the bar marks the digit that repeats*

The remainder never reaches $0$ — it comes back to $1$ at every step, so the same digit keeps coming. A decimal in which a block of digits repeats forever is a **recurring decimal**, and this book writes it with a **bar** over the repeating block: $\frac{1}{3} = 0.\overline{3}$. The bar covers *only* the digits that repeat.

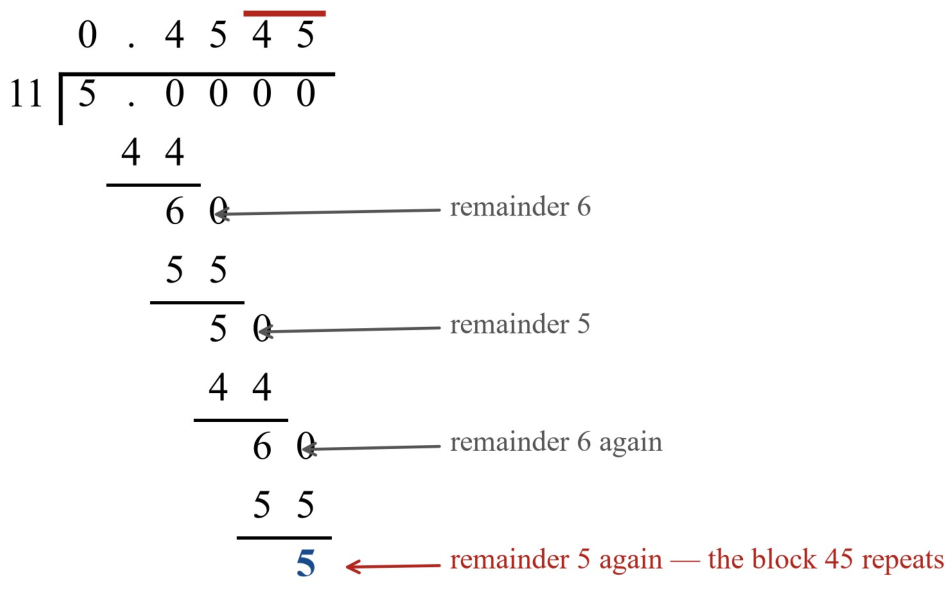
Sometimes a few digits appear once before the block starts. Divide 7 by 24:




*Long division of 7 by 24: the quotient digits are 0.2916 and the remainder 6 returns, so only the 6 repeats; 7 over 24 equals 0.2916 with a bar over the 6*

so $\frac{7}{24} = 0.291\overline{6}$: the digits 2, 9, 1 appear once, then the 6 repeats forever. The repeating block can also hold more than one digit:

$\frac{5}{11}$ as a division: two remainders, two repeating digits




$\frac{5}{11} = 0.4545\ldots = 0.\overline{45}$; the repeating block has two digits

*Long division of 5 by 11: the remainders cycle between 6 and 5, so the two digits 45 repeat; 5 over 11 equals 0.4545... written with a bar over 45*

Negative fractions behave the same way, with the minus carried out the front exactly as 1A's sign rules say: $-\frac{2}{3} = -0.\overline{6}$.

### *The test: read the prime factors of the denominator*

Long division tells you what happens, but there is a shortcut that reads the answer off the denominator before you start. **First write the fraction in simplest form** — top and bottom sharing no factor bigger than 1. Then:

**The test.** A fraction in simplest form gives a terminating decimal if the prime factors of its denominator are only 2s and 5s. If any other prime appears, the decimal recurs.

Read the prime factors of the denominator (simplest form first)

$\frac{7}{25}$

$25 = 5^2$ — only a 5

**terminates**

$\frac{7}{25} = 0.28$

$\frac{7}{24}$

$24 = 2^3 \times 3$ — has a 3

**recurs**

$\frac{7}{24} = 0.291\overline{6}$

only 2s and 5s $\Rightarrow$ terminating; any other prime $\Rightarrow$ recurring

both fractions here are already in simplest form

*Two panels side by side: 7 over 25 with 25 equals 5 squared, only a 5, so it terminates at 0.28; 7 over 24 with 24 equals 2 cubed times 3, has a 3, so it recurs as 0.2916 with a bar over the 6; rule: only 2s and 5s means terminating, any other prime means recurring*

The trap is a fraction that is not in simplest form. $\frac{9}{24}$ looks as if it recurs, because $24 = 2^3 \times 3$ has a $3$ — but $\frac{9}{24} = \frac{3}{8}$, and $8 = 2^3$ is made of 2s only, so the decimal terminates: $\frac{3}{8} = 0.375$. Simplify first, every time.

Why does the test work? A terminating decimal is a count of tenths, hundredths or thousandths — and every power of $10$ is built from $2$s and $5$s only:

## Why the test works

a terminating decimal is a count of tenths, hundredths, thousandths, …

$$\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$$ — the denominator scales up to a power of 10

$10 = 2 \times 5$

$100 = 2^2 \times 5^2$

$1000 = 2^3 \times 5^3$

every power of 10 is built from 2s and 5s only

a denominator with a 3, 7, 11, … in it never scales up to a power of 10 — so its decimal recurs

*Why the test works: 7 over 25 scales to 28 over 100 equals 0.28; boxes show 10 equals 2 times 5, 100 equals 2 squared times 5 squared, 1000 equals 2 cubed times 5 cubed; every power of 10 is built from 2s and 5s only, so a denominator with a 3, 7 or 11 never scales up to a power of 10 and its decimal recurs*

$\frac{7}{25}$ terminates because the denominator scales up to a power of 10 ($25 \times 4 = 100$), turning the fraction into a count of hundredths. A denominator with a 3, a 7 or an 11 in it never scales up to a power of 10 — no whole number fixes it — so the division can never end, and it cannot die out either; the remainders must come back, and the decimal recurs.

### ***Three kinds of decimals***

Putting the test together, dividing an integer by a natural number gives exactly two kinds of decimal — terminating or recurring. There is a third kind on the number line, decimals that never end **and** never repeat; no fraction produces one. That third kind belongs to the irrational numbers, and 1D owns them: here we only note that they exist, and that a fraction never gives one.

| Terminating | Recurring | Neither |
|---|---|---|
| $\frac{3}{4} = 0.75$ | $\frac{1}{3} = 0.\overline{3}$ | $\sqrt{2} = 1.41421356\ldots$ |
| the division ends | one block of digits repeats forever | never ends and never repeats — irrational (1D) |

divide an integer by a natural number and you always get the first or the second kind — never the third

*Three cards: Terminating, 3 over 4 equals 0.75, the division ends; Recurring, 1 over 3 equals 0.3 with a bar, one block of digits repeats forever; Neither, square root of 2 equals 1.41421356…, never ends and never repeats, irrational, studied in 1D; dividing an integer by a natural number always gives the first or the second kind, never the third*

This is also the difference between exact and rounded. $\frac{2}{3} = 0.\overline{6}$ exactly, while $0.67$ is a rounding — close, not equal.

A recurring decimal is not a rounding: $0.\overline{6}$ **is** $\frac{2}{3}$, written in decimal form.

### *From decimal to fraction: terminating decimals*

A terminating decimal is read straight back to a fraction by its last digit's place value:

Reading a terminating decimal by place value

| ones | | tenths | hundredths | thousandths |
|---|---|---|---|---|
| | | | | |
| 0 | . | 3 | 7 | 5 |

0.375 is 375 thousandths: $0.375 = \frac{375}{1000}$

$\downarrow$ divide top and bottom by 125

$$\frac{375}{1000} = \frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

check: $3 \div 8 = 0.375$ — the round trip works

*Place value table for 0.375: 3 tenths, 7 hundredths, 5 thousandths; 0.375 is 375 thousandths, so 375 over 1000, and dividing top and bottom by 125 gives 3 over 8; check 3 divided by 8 equals 0.375*

then simplify. For example $0.65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$.

***From decimal to fraction: recurring decimals***

A recurring decimal never ends, so place value cannot read it. Instead, multiply by a power of 10 that lines up two copies of the repeating part, then subtract — the matching recurring parts cancel:

Multiplying by 10 lines up the recurring parts

$$\begin{aligned} 10 \times 0.\overline{3} &= 3.\overline{3} \\ 1 \times 0.\overline{3} &= 0.\overline{3} \\ \hline 9 \times 0.\overline{3} &= 3 \end{aligned}$$

subtract

the recurring parts match, so subtracting removes them

$$\text{so } 0.\overline{3} = 3 \div 9 = \tfrac{3}{9} = \tfrac{1}{3}$$

a repeating block of two digits needs $\times 100$ instead of $\times 10$

*Multiplying by 10 lines up the recurring parts: 10 times 0.3 with a bar is 3.3 with a bar, and 1 times 0.3 with a bar is 0.3 with a bar; subtracting gives 9 times 0.3 with a bar equals 3, so 0.3 with a bar equals 3 divided by 9 equals 3 over 9 equals 1 over 3; a two-digit block needs times 100 instead of times 10*

A repeating block of two digits needs $\times 100$ instead: $100 \times 0.\overline{45} = 45.\overline{45}$, and subtracting $1 \times 0.\overline{45}$ leaves $99 \times 0.\overline{45} = 45$, so $0.\overline{45} = \frac{45}{99} = \frac{5}{11}$ — the round trip back to the division above. If the block starts after other digits, multiply twice to line the bars up — Example 5 shows how.

### *Summary*

**Fraction → decimal**

$\frac{3}{8} = 0.375$

divide top by bottom;
remainder 0 means terminating

**The test**

$\frac{7}{25} = 0.28,\ \frac{7}{24} = 0.291\overline{6}$

simplest form, then only 2s and 5s
in the denominator means terminating

**Recurring notation**

$\frac{5}{11} = 0.\overline{45}$

the bar covers the repeating block

**Decimal → fraction**

$0.375 = \frac{3}{8},\ 0.\overline{3} = \frac{1}{3}$

place value for terminating;
multiply by a power of 10 and subtract
for recurring

terminating and recurring decimals are exactly the decimals that come from fractions

*Summary card in four panels: fraction to decimal, divide top by bottom and remainder 0 means terminating; the test, simplest form then only 2s and 5s in the denominator means terminating; recurring notation, 5 over 11 equals 0.45 with a bar over 45, the bar covers the repeating block; decimal to fraction, place value for terminating and multiply by a power of 10 then subtract for recurring; terminating and recurring decimals are exactly the decimals that come from fractions*

The third kind of decimal — never ending, never repeating — is where the irrational numbers live. 1D names them and studies them; this subtopic only borrows the fact that no fraction ever produces one.

### *Words of this subtopic*

| Word | What it means here |
|---|---|
| **terminating decimal** | a decimal that ends, because the division reaches remainder 0; 0.28 |
| **recurring decimal** | a decimal in which a block of digits repeats forever; $0.\overline{3}$ |
| **repeating block** | the digits that repeat; in $0.291\overline{6}$ the block is 6; in $0.\overline{45}$ it is 45 |
| **the bar** | the line drawn over the repeating block of a recurring decimal |
| **simplest form** | top and bottom share no factor bigger than 1; $\frac{9}{24} = \frac{3}{8}$ |
| **prime factorisation** | a number written as a product of powers of primes; $24 = 2^3 \times 3$ (Year 7, and 1B) |
| **denominator** | the bottom number of a fraction — the one the test reads |
| **place value** | ones, tenths, hundredths, thousandths; 0.375 is 375 |

| Word | What it means here |
|---|---|
| | thousandths |
| **natural numbers** | the numbers $1, 2, 3, 4, \ldots$ |

## Worked examples

### *Example 1 — with help*

Write each fraction as a decimal, and name the kind of decimal.

(a) $\frac{3}{8}$

(b) $\frac{2}{3}$

| Working | Comment |
|---|---|
| (a) $3 \div 8 = 0.375$ | The remainders run down to $0$, so the division ends. |
| $0.375$ — terminating | A decimal that ends is terminating. |
| (b) $2 \div 3 = 0.666\ldots$ | The remainder stays $2$, so the $6$s never stop. |
| $= 0.\overline{6}$ — recurring | The bar covers the one digit that repeats. |

∴ (a) $\frac{3}{8} = 0.375$, terminating; (b) $\frac{2}{3} = 0.\overline{6}$, recurring. Check with the test: $8 = 2^3$ is only 2s, and $3$ is neither $2$ nor $5$ — the test agrees with the division.

### *Example 2 — with help*

Without dividing, decide whether each fraction terminates or recurs.

(a) $\frac{7}{25}$

(b) $\frac{7}{24}$

(c) $\frac{9}{24}$

| Working | Comment |
|---|---|
| (a) $25 = 5^2$ | Only a $5$ — terminates. |
| (b) $24 = 2^3 \times 3$ | A $3$ appears — recurs. |
| (c) $\frac{9}{24} = \frac{3}{8}$, and $8 = 2^3$ | Simplify **first**: only 2s — terminates. |

∴ (a) terminates; (b) recurs; (c) terminates. Dividing confirms it: $0.28$, $0.291\overline{6}$ and $0.375$. Part (c) is the trap — the test must be applied to the simplest form, not to the fraction as given.

### *Example 3 — with help*

Write each terminating decimal as a fraction in simplest form.

(a) $0.65$

(b) $0.375$

| Working | Comment |
|---|---|
| (a) $0.65 = \frac{65}{100}$ | The last digit sits in the hundredths. |

| Working | Comment |
|---|---|
| $\frac{65}{100}=\frac{65\div5}{100\div5}=\frac{13}{20}$ | Divide top and bottom by 5. |
| (b) $0.375=\frac{375}{1000}$ | The last digit sits in the thousandths. |
| $\frac{375}{1000}=\frac{375\div125}{1000\div125}=\frac{3}{8}$ | Divide top and bottom by 125. |

$\therefore$ (a) $\frac{13}{20}$; (b) $\frac{3}{8}$. Check by dividing: $13\div20=0.65$ and $3\div8=0.375$ — the round trip works.

***Example 4 — on your own***

Write each recurring decimal as a fraction in simplest form.

(a) $0.\overline{3}$
(b) $0.\overline{45}$

One digit repeats in (a), so $\times10$ lines the bars up; two digits repeat in (b), so $\times100$ does.

(a) $10\times0.\overline{3}=3.\overline{3}$. Subtract $1\times0.\overline{3}$: $9\times0.\overline{3}=3$, so $0.\overline{3}=\frac{3}{9}=\frac{1}{3}$.

(b) $100\times0.\overline{45}=45.\overline{45}$. Subtract $1\times0.\overline{45}$: $99\times0.\overline{45}=45$, so $0.\overline{45}=\frac{45}{99}=\frac{5}{11}$.

$\therefore$ (a) $\frac{1}{3}$; (b) $\frac{5}{11}$ — matching the divisions in the theory.

***Example 5 — on your own***

Show that $0.291\overline{6}=\frac{7}{24}$.

Here the block is one digit long but starts three places in, so two multiplications are needed to line the bars up: $\times10000$ puts one copy of the bar just after the point, and $\times1000$ puts the other copy at the same spot.

$$10000\times0.291\overline{6}=2916.\overline{6},1000\times0.291\overline{6}=291.\overline{6}$$

Subtracting cancels the recurring parts:

$$9000\times0.291\overline{6}=2916-291=2625$$

so $0.291\overline{6}=\frac{2625}{9000}=\frac{2625\div375}{9000\div375}=\frac{7}{24}$.

$\therefore$ the round trip returns exactly the fraction the decimal came from, and the test agrees: $24=2^3\times3$ has a 3, so $\frac{7}{24}$ must recur.

***Example 6 — on your own***

Five dollars is shared equally.

(a) Four people share it. What does each person get?

(b) Six people share it. What does each person get, as a decimal?

(c) Which share can be paid exactly in dollars and cents? What is left over in the other case?

(d) $5\div4=1.25$: each person gets $\$1.25$.

(e) $5 \div 6 = 0.8333\ldots = 0.8\overline{3}$: each share is $\$0.8\overline{3}$.

(f) The four-way share terminates, so $\$1.25$ is exact. The six-way share recurs, and coins stop at cents: paying 83 c each uses $6 \times \$0.83 = \$4.98$, leaving 2 c that no exact split can place.

∴ terminating shares pay out exactly; recurring shares do not. This is the same fact as $\frac{5}{4}$ terminating $(4 = 2^2)$ and $\frac{5}{6}$ recurring $(6 = 2 \times 3)$.

---

## Practice questions

*Work these questions by hand — every fraction is chosen so the division or the test does the work. Only Q16 invites a calculator or a spreadsheet, because VC2M8N03 allows digital tools where they are appropriate; there, the pattern is the point.*

### *With help*

**Q1.** Write each fraction as a decimal, and say whether it terminates.

a. $\frac{1}{2}$

b. $\frac{3}{4}$

c. $\frac{1}{4}$

d. $\frac{3}{10}$

**Q2.** Write each fraction as a recurring decimal, using the bar.

a. $\frac{1}{3}$

b. $\frac{2}{9}$

c. $\frac{5}{6}$

**Q3.** Without dividing, use the test to say whether each fraction terminates or recurs.

a. $\frac{3}{8}$

b. $\frac{7}{15}$

c. $\frac{11}{40}$

d. $\frac{5}{12}$

**Q4.** Write each terminating decimal as a fraction in simplest form.

a. 0.7

b. 0.45

c. 0.125

**Q5.** Write each recurring decimal as a fraction in simplest form.

a. $0.\overline{2}$
b. $0.\overline{6}$
c. $0.\overline{18}$

**Q6.** True or false? Give a reason, and correct each false statement.

a. 0.375 is a recurring decimal.
b. $\frac{1}{6}$ gives a terminating decimal.
c. $\frac{3}{24}$ gives a terminating decimal.
d. A fraction in simplest form whose denominator has a 7 among its prime factors recurs.
e. $0.\overline{45} = \frac{45}{99}$.

**Q7.** Copy and complete the table. Remember to simplify first where needed.

| Fraction | Prime factors of the denominator (simplest form) | Terminating or recurring | Decimal |
|---|---|---|---|
| $\frac{3}{16}$ | | | |
| $\frac{7}{15}$ | | | |
| $\frac{9}{24}$ | | | |
| $\frac{11}{40}$ | | | |
| $\frac{5}{12}$ | | | |

**Q8.** Copy and complete each statement with the missing exponents.

a. $10 = 2^{\square} \times 5^{\square}$
b. $100 = 2^{\square} \times 5^{\square}$
c. $1000 = 2^{\square} \times 5^{\square}$
d. What do you notice about the prime factors of every power of 10? How does this explain the test?

### *On your own*

**Q9.** Write each fraction as a decimal, using the bar where the decimal recurs.

a. $\frac{7}{8}$
b. $\frac{5}{12}$
c. $\frac{4}{11}$

**Q10.** Write each fraction as a decimal, using the bar where the decimal recurs. Watch where the block starts.

a. $\frac{5}{9}$

b. $\frac{1}{6}$

c. $\frac{1}{11}$

**Q11.** Without dividing, decide which of $\frac{13}{40}$ and $\frac{13}{45}$ terminates and which recurs. Give the prime factors you read.

**Q12.** Simplify first, then use the test: terminating or recurring?

a. $\frac{6}{15}$

b. $\frac{7}{28}$

c. $\frac{5}{12}$

d. $\frac{9}{40}$

e. $\frac{14}{21}$

**Q13.** Write each recurring decimal as a fraction in simplest form.

a. $0.\overline{7}$
b. $0.\overline{27}$
c. $0.1\overline{6}$

**Q14.** Noah says $\frac{3}{15}$ must recur, because $15 = 3 \times 5$ has a $3$ among its prime factors. Explain his mistake, and find the actual decimal.

**Q15.** The ninths make a pattern: $\frac{1}{9} = 0.\overline{1}$, $\frac{2}{9} = 0.\overline{2}$, $\frac{3}{9} = 0.\overline{3}$.

a. Use the pattern to predict $\frac{5}{9}$ and $\frac{8}{9}$, and check both by division.

b. What must $\frac{9}{9}$ be? What does that tell you about $0.\overline{9}$?

**Q16.** *A calculator or spreadsheet is allowed for this one.* Find $\frac{1}{7}$, $\frac{2}{7}$ and $\frac{3}{7}$ as decimals.

a. What do you notice about the digits of the three answers?
b. What does the test say about sevenths, and does the calculator agree?

### *Maths in real life*

**Q17.** Four dollars and eighty cents is shared equally.

a. Eight people share it. What does each person get?
b. Seven people share it. Write the exact share as a fraction of a dollar, say whether it terminates or recurs, and give the share to the nearest cent.

**Q18.** A cordial label says to mix $1$ part syrup with $8$ parts water. Another brand says $1$ part syrup to $4$ parts water.

a. What fraction of the first drink is syrup? Write it as a decimal.
b. What fraction of the second drink is syrup? Write it as a decimal.
c. Which mix gives a recurring decimal, and which terminates?

**Q19.** Zara makes $7$ of her $8$ free throws; Tom makes $2$ of his $3$.

a. Write each result as a decimal.
b. The scoreboard rounds Tom's result to $0.67$. Is $0.67$ exactly $\frac{2}{3}$? Explain.

**Q20.** Make your own. Choose a fraction of your own. Predict with the test whether it terminates or recurs, then check by division, and write the decimal with the bar if it recurs. Swap with a partner and check each other's work.

---

### Answers

**Q1.** a. $0.5$, terminates. b. $0.75$, terminates. c. $0.25$, terminates. d. $0.3$, terminates.

**Q2.** a. $0.\overline{3}$. b. $0.\overline{2}$. c. $0.8\overline{3}$.

**Q3.** a. Terminates ($8=2^3$). b. Recurs ($15=3\times5$). c. Terminates ($40=2^3\times5$). d. Recurs ($12=2^2\times3$).

**Q4.** a. $\frac{7}{10}$. b. $\frac{9}{20}$. c. $\frac{1}{8}$.

**Q5.** a. $\frac{2}{9}$. b. $\frac{2}{3}$. c. $\frac{2}{11}$.

**Q6.** a. False; it terminates. b. False; it recurs. c. True. d. True. e. True.

**Q7.** $\frac{3}{16}$: $2^4$; terminating; $0.1875$. $\frac{7}{15}$: $3\times5$; recurring; $0.4\overline{6}$. $\frac{9}{24}=\frac{3}{8}$: $2^3$; terminating; $0.375$. $\frac{11}{40}$: $2^3\times5$; terminating; $0.275$. $\frac{5}{12}$: $2^2\times3$; recurring; $0.41\overline{6}$.

**Q8.** a. $1$ and $1$. b. $2$ and $2$. c. $3$ and $3$. d. Only 2s and 5s — see solution.

**Q9.** a. $0.875$. b. $0.41\overline{6}$. c. $0.\overline{36}$.

**Q10.** a. $0.\overline{5}$. b. $0.1\overline{6}$. c. $0.\overline{09}$.

**Q11.** $\frac{13}{40}$ terminates ($40=2^3\times5$); $\frac{13}{45}$ recurs ($45=3^2\times5$).

**Q12.** a. Terminates. b. Terminates. c. Recurs. d. Terminates. e. Recurs.

**Q13.** a. $\frac{7}{9}$. b. $\frac{3}{11}$. c. $\frac{1}{6}$.

**Q14.** The test needs simplest form; $\frac{3}{15}=\frac{1}{5}=0.2$, terminating.

**Q15.** a. $\frac{5}{9}=0.\overline{5}$, $\frac{8}{9}=0.\overline{8}$. b. $\frac{9}{9}=1$, so $0.\overline{9}=1$.

**Q16.** a. The same six digits $142857$, starting in different places. b. Recurs; agrees.

**Q17.** a. $\$0.60$. b. $\frac{24}{35}$ of a dollar, recurs; $\$0.69$ to the nearest cent.

**Q18.** a. $\frac{1}{9}=0.\overline{1}$. b. $\frac{1}{5}=0.2$. c. The $1:8$ mix recurs; the $1:4$ mix terminates.

**Q19.** a. $0.875$ and $0.\overline{6}$. b. No; $0.67$ is a rounding.

**Q20.** Answers will vary — see solution for the checks.

---

### Fully-worked solutions

**Q1.** Divide top by bottom; each division reaches remainder $0$.

a. $1\div2=0.5$ — terminates.
b. $3\div4=0.75$ — terminates.

c. $1 \div 4 = 0.25$ — terminates.
d. $3 \div 10 = 0.3$ — terminates.

**Q2.** Divide top by bottom; the remainder comes back, so a block repeats.

a. $1 \div 3 = 0.333\ldots = 0.\overline{3}$.
b. $2 \div 9 = 0.222\ldots = 0.\overline{2}$.
c. $5 \div 6 = 0.8333\ldots = 0.8\overline{3}$ — the 3 repeats, the 8 does not, so the bar covers the 3 only.

**Q3.** Read the prime factors of the denominator (all four are in simplest form).

a. $8 = 2^3$ — only 2s, terminates.
b. $15 = 3 \times 5$ — a 3 appears, recurs.
c. $40 = 2^3 \times 5$ — only 2s and 5s, terminates.
d. $12 = 2^2 \times 3$ — a 3 appears, recurs.

**Q4.** Read the last digit's place value, then simplify.

a. $0.7 = \frac{7}{10}$ — already in simplest form.
b. $0.45 = \frac{45}{100} = \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$.
c. $0.125 = \frac{125}{1000} = \frac{125 \div 125}{1000 \div 125} = \frac{1}{8}$.

**Q5.** One repeating digit: multiply by 10 and subtract, leaving 9 copies.

a. $9 \times 0.\overline{2} = 2$, so $0.\overline{2} = \frac{2}{9}$.
b. $9 \times 0.\overline{6} = 6$, so $0.\overline{6} = \frac{6}{9} = \frac{2}{3}$.
c. Two repeating digits: $99 \times 0.\overline{18} = 18$, so $0.\overline{18} = \frac{18}{99} = \frac{2}{11}$.

**Q6.** Test each statement.

a. **False.** The division $375 \div 1000$ ends; $0.375$ terminates.
b. **False.** $6 = 2 \times 3$ has a 3, so $\frac{1}{6}$ recurs: $\frac{1}{6} = 0.1\overline{6}$.
c. **True.** $\frac{3}{24} = \frac{1}{8}$, and $8 = 2^3$ is only 2s: $\frac{3}{24} = 0.125$.
d. **True.** A 7 in the simplest form's denominator is "any other prime", so the decimal recurs.
e. **True.** $99 \times 0.\overline{45} = 45$, exactly as Example 4 showed.

**Q7.** Simplify first where needed, read the prime factors, then divide.

| Fraction | Prime factors (simplest form) | Kind | Decimal |
|---|---|---|---|
| $\frac{3}{16}$ | $16 = 2^4$ | terminating | $0.1875$ |
| $\frac{7}{15}$ | $15 = 3 \times 5$ | recurring | $0.4\overline{6}$ |
| $\frac{9}{24} = \frac{3}{8}$ | $8 = 2^3$ | terminating | $0.375$ |

| Fraction | Prime factors (simplest form) | Kind | Decimal |
|---|---|---|---|
| $\frac{11}{40}$ | $40 = 2^3 \times 5$ | terminating | 0.275 |
| $\frac{5}{12}$ | $12 = 2^2 \times 3$ | recurring | $0.41\overline{6}$ |

**Q8.** Write each power of 10 as a product of powers of primes.

a. $10 = 2^1 \times 5^1$.
b. $100 = 2^2 \times 5^2$.
c. $1000 = 2^3 \times 5^3$.
d. Every power of 10 is built from 2s and 5s only. That is why the test works: a terminating decimal is a count of tenths, hundredths or thousandths, so a fraction terminates exactly when its denominator can scale up to a power of 10 — which only 2s and 5s allow.

**Q9.** Divide top by bottom; bar the repeating block.

a. $7 \div 8 = 0.875$ — the division ends.
b. $5 \div 12 = 0.41666\ldots = 0.41\overline{6}$ — the block is the single digit 6, starting two places in.
c. $4 \div 11 = 0.3636\ldots = 0.\overline{36}$ — a two-digit block.

**Q10.** Divide; watch where the block starts.

a. $5 \div 9 = 0.555\ldots = 0.\overline{5}$.
b. $1 \div 6 = 0.1666\ldots = 0.1\overline{6}$ — the 1 appears once, then the 6 repeats.
c. $1 \div 11 = 0.0909\ldots = 0.\overline{09}$ — the block is 09; the zero is part of the block, so the bar must cover it.

**Q11.** Both fractions are in simplest form, so read the denominators. $40 = 2^3 \times 5$ is only 2s and 5s, so $\frac{13}{40}$ terminates (0.325). $45 = 3^2 \times 5$ has a 3, so $\frac{13}{45}$ recurs ($0.2\overline{8}$).

**Q12.** Simplify, then read the new denominator.

a. $\frac{6}{15} = \frac{2}{5}$ — 5 only, terminates (0.4).
b. $\frac{7}{28} = \frac{1}{4}$ — $4 = 2^2$, terminates (0.25).
c. $\frac{5}{12}$ is already in simplest form — $12 = 2^2 \times 3$ has a 3, recurs.
d. $\frac{9}{40}$ is already in simplest form — $40 = 2^3 \times 5$, terminates (0.225).
e. $\frac{14}{21} = \frac{2}{3}$ — a 3, recurs ($0.\overline{6}$).

**Q13.** Multiply by the power of 10 that matches the block, and subtract.

a. $9 \times 0.\overline{7} = 7$, so $0.\overline{7} = \frac{7}{9}$.
b. $99 \times 0.\overline{27} = 27$, so $0.\overline{27} = \frac{27}{99} = \frac{3}{11}$.
c. $100 \times 0.1\overline{6} = 16.\overline{6}$ and $10 \times 0.1\overline{6} = 1.\overline{6}$; subtracting gives $90 \times 0.1\overline{6} = 16 - 1 = 15$, so $0.1\overline{6} = \frac{15}{90} = \frac{1}{6}$.

**Q14.** The test applies to the fraction **in simplest form**, and $\frac{3}{15}$ is not in simplest form: $\frac{3}{15}=\frac{1}{5}$. The denominator 5 is only 5s, so the decimal terminates: $\frac{3}{15}=0.2$. Noah read the prime factors before simplifying — the same trap as $\frac{9}{24}$ in the theory.

**Q15.** The ninths repeat the digit of the numerator.

a. The pattern says $\frac{5}{9}=0.\overline{5}$ and $\frac{8}{9}=0.\overline{8}$; division confirms both ($5\div 9=0.555\ldots$, $8\div 9=0.888\ldots$).

b. $\frac{9}{9}=1$. The pattern would write it as $0.\overline{9}$, so $0.\overline{9}=1$: the recurring decimal $0.999\ldots$ is not close to 1 — it **is** 1. (Check with the method of Example 4: $9\times 0.\overline{9}=9$, so $0.\overline{9}=\frac{9}{9}=1$.)

**Q16.** A calculator or spreadsheet gives, to the display's limit: $\frac{1}{7}=0.142857\ldots$, $\frac{2}{7}=0.285714\ldots$, $\frac{3}{7}=0.428571\ldots$

a. All three answers cycle the same six digits, 142857, just starting at different points of the cycle — the same repeating block, rotated.
b. 7 is neither 2 nor 5, so the test says every seventh recurs — and the calculator's digits never settle, agreeing with the test. (The display cuts the decimal off, but the division itself never ends.)

**Q17.** Divide the dollars by the number of people.

a. $4.80\div 8=0.60$: each person gets exactly $\$0.60$.

b. $4.80\div 7=\frac{480}{7}$ cents $=68.571428\ldots$ cents, i.e. the exact share is $\frac{24}{35}$ of a dollar. The simplest form's denominator $35=5\times 7$ has a 7, so the decimal recurs: $0.6\overline{857142}$. To the nearest cent each person gets $\$0.69$.

**Q18.** The syrup is 1 part of the whole drink: $1+8=9$ parts in the first mix, $1+4=5$ in the second.

a. $\frac{1}{9}=0.\overline{1}$ of the first drink is syrup.

b. $\frac{1}{5}=0.2$ of the second drink is syrup.

c. The $1:8$ mix recurs ($9=3^2$ has a 3); the $1:4$ mix terminates (5 only).

**Q19.** Divide made throws by attempts.

a. Zara: $7\div 8=0.875$. Tom: $2\div 3=0.\overline{6}$.

b. No. $\frac{2}{3}=0.\overline{6}$ exactly, and 0.67 differs from it (it stops, while $0.\overline{6}$ never does). 0.67 is a rounding to two places — fine for a scoreboard, not an equality.

**Q20.** Answers will vary. A sample: $\frac{7}{40}$. The test: $40=2^3\times 5$ is only 2s and 5s, so it should terminate. Division: $7\div 40=0.175$ — it does. Check your partner's work both ways: the test first, then the division, and the bar over exactly the digits that repeat when it recurs.

# Irrational numbers and the real number line

**Content description VC2M8N01:** recognise irrational numbers in applied contexts, including $\pi$ and numbers that develop from the square root of positive real numbers that are not perfect squares, and recognise that irrational numbers cannot develop from the division of integer values by natural numbers

*Cut a square with area $2\,\mathrm{m}^2$. Its side is not 1.4 m, and it is not 1.41 m, and it is not any fraction you care to write down — yet the side is right there, on the edge of the square. Some numbers are not fractions, and their decimals never end and never repeat. This subtopic gives those numbers a name, and finds each one a place on the number line.*

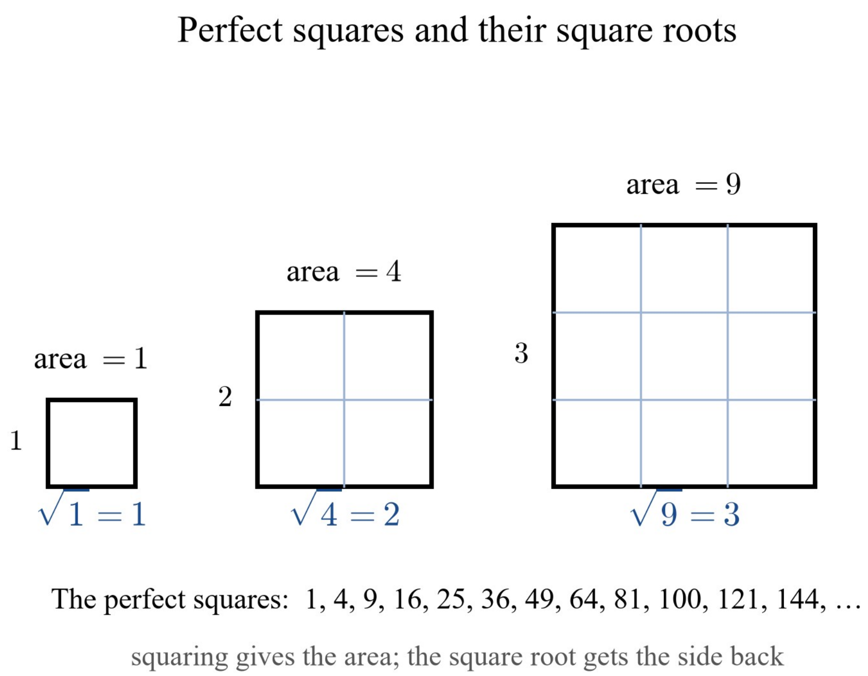
## Theory

### Recall: perfect squares and square roots

In Year 7 (VC2M7N01) you met **perfect squares** and **square roots**. Squaring a number means multiplying it by itself: $3^2=3\times3=9$. The perfect squares are the numbers that come out whole:

$$1,4,9,16,25,36,49,64,81,100,121,144,\ldots$$

The **square root** undoes the squaring: $\sqrt{9}=3$ because $3^2=9$. In Year 7 (VC2M7N03) you also placed fractions and decimals on a number line. This subtopic uses both ideas without re-teaching them.




*Three squares drawn on grid paper with side lengths 1, 2 and 3, labelled with their areas 1, 4 and 9, and the square root of each area giving the side length back again*

### Some square roots are not whole numbers

Where is $\sqrt{2}$? Since $1^2=1$ and $2^2=4$, and 2 sits between 1 and 4, $\sqrt{2}$ sits between 1 and 2. Now squeeze tighter, one decimal place at a time, by squaring:

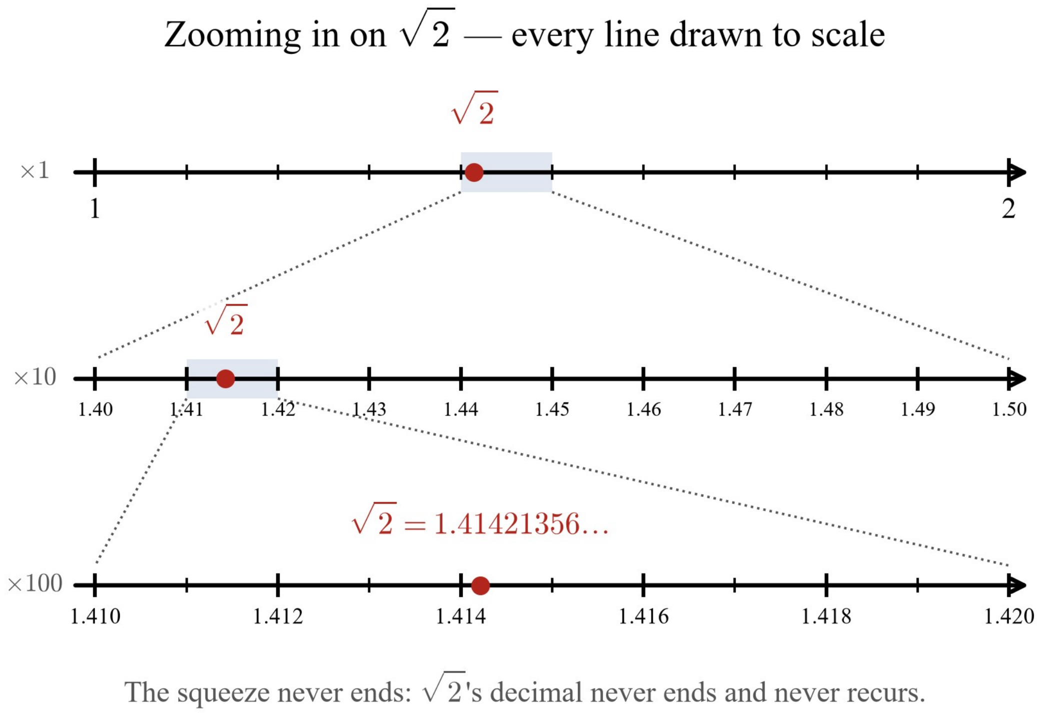
$$1.4^2=1.96 \text{ and } 1.5^2=2.25\text{, so } 1.4<\sqrt{2}<1.5$$

$$1.41^2=1.9881 \text{ and } 1.42^2=2.0164\text{, so } 1.41<\sqrt{2}<1.42$$

The same trick works at the next decimal place, and the next, forever. The squeezing never stops, because

$$\sqrt{2}=1.41421356\ldots$$

is a decimal that **never ends**.




*Three number lines stacked as zooms, from 1 to 2, then 1.4 to 1.5, then 1.41 to 1.42, each drawn to scale, with the point for square root of 2 trapped between narrower pairs of ticks at each zoom*

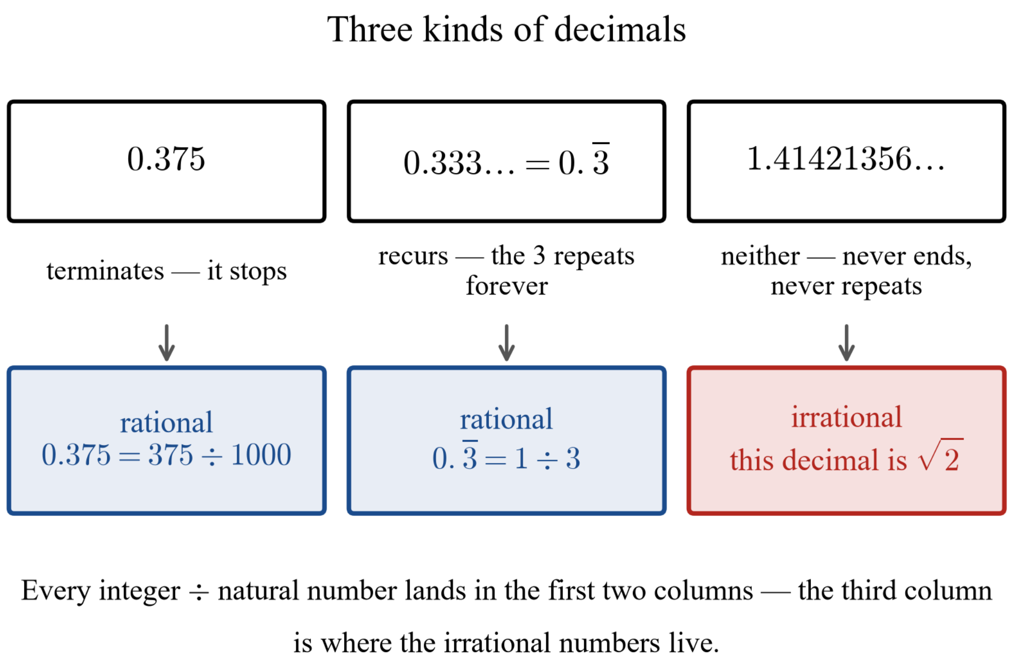
### *Three kinds of decimals*

Subtopic 1C sorted the decimals that come out of division: **terminating** decimals stop, and **recurring** decimals repeat the same block of digits forever. For example

$$\frac{3}{8} = 0.375 \text{ terminates}, \frac{1}{3} = 0.333\ldots = 0.\bar{3} \text{ recurs}.$$

1C also gave a test: look at the prime factorisation of the denominator, and you can tell which way the decimal will go. Every number of the form "integer ÷ natural number" lands in one of these two groups — and nothing else can come out of that division.

There is a third kind of decimal: one that never ends **and** never settles into a repeating block. $\sqrt{2} = 1.41421356\ldots$ is one. Such a decimal cannot come from dividing an integer by a natural number, because every such division terminates or recurs.




*Three decimal cards: 0.375 which terminates, one third which recurs as 0.3 repeating, and square root of 2 which neither terminates nor recurs; the first two are labelled rational and the third irrational*

### *Rational and irrational numbers*

Here are the two words this subtopic is built on.

> **Definitions used in this subtopic.** A **rational number** is any number that can develop from the division of an integer by a natural number. An **irrational number** is a number that cannot develop from such a division. (These are VCAA's own words from the content description, written out here as definitions.)

So 5 is rational, because $5 = 5 \div 1$. The fraction $\frac{-3}{4}$ is rational, because it is $-3 \div 4$. Every terminating decimal and every recurring decimal is rational, since each is the result of such a division.

> **Word watch.** In everyday English, *irrational* means "not sensible". In maths the word is a **subject term** with one fixed meaning: the meaning given above — a number that cannot be written as an integer divided by a natural number. The everyday meaning and the maths meaning are not the same, and this subtopic uses only the maths one.

Which square roots are which? The perfect squares give whole numbers:

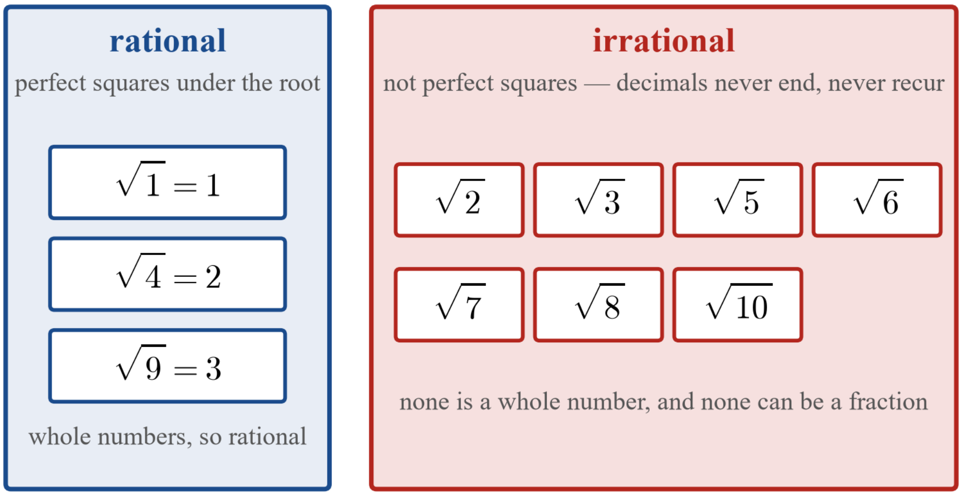
$$\sqrt{1} = 1, \sqrt{4} = 2, \sqrt{9} = 3, \sqrt{16} = 4, \ldots$$

and those are rational. Every other positive whole number under a root sign gives a decimal that never ends and never recurs — an irrational number:

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}, \ldots \text{are all irrational.}$$

Watch two traps. First, the root sign alone does not make a number irrational: $\sqrt{25} = 5$, rational. Second, decimals can be perfect squares too: $0.49 = 0.7 \times 0.7$, so $\sqrt{0.49} = 0.7$, rational.

Which square roots are rational? Only the perfect squares.




The root sign alone tells you nothing — check whether the number inside is a perfect square.

*Ten cards from square root of 1 to square root of 10 sorted into two boxes: the perfect squares, square root of 1, square root of 4 and square root of 9, fall into the rational box as the whole numbers 1, 2 and 3, and the other seven fall into the irrational box*

### $\pi$ *is irrational*

Take any circle. Divide its **circumference** (the distance around the edge) by its **diameter** (the distance across, through the centre). The answer is always the same number, no matter the size of the circle. That number is $\pi$:

$$\pi = 3.14159265\ldots$$

The decimal never ends and never repeats, so $\pi$ **is irrational**.

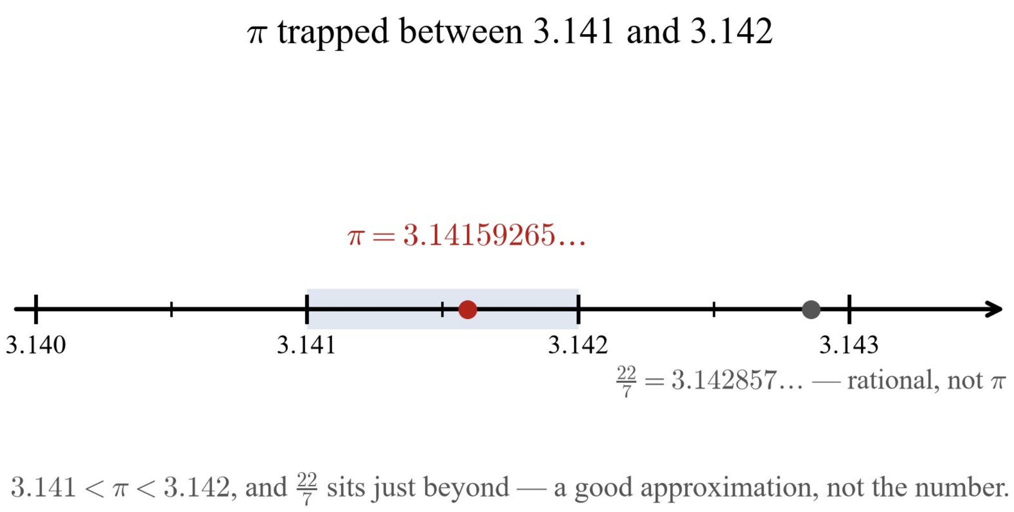
One fact about $\pi$ is worth keeping exactly as the curriculum states it:

$$3.141 < \pi < 3.142$$

Because $\pi$ is irrational, every fraction or stopping decimal used for it is only an **approximation**. The common ones are all rational, and none of them equals $\pi$:

$$3.14 \neq \pi\,,\ \frac{22}{7} = 3.\overline{142857} \neq \pi\,,\ 3.1416 \neq \pi$$

The fraction $\frac{22}{7}$ is a good approximation — it agrees with $\pi$ on 3.14 — but it is a recurring decimal, so it is a rational number, and it sits a little **above** $\pi$ on the number line.




*A number line drawn to scale from 3.140 to 3.1435, with pi marked between 3.141 and 3.142, and twenty-two sevenths, equal to 3.142857 recurring, marked just beyond 3.142*

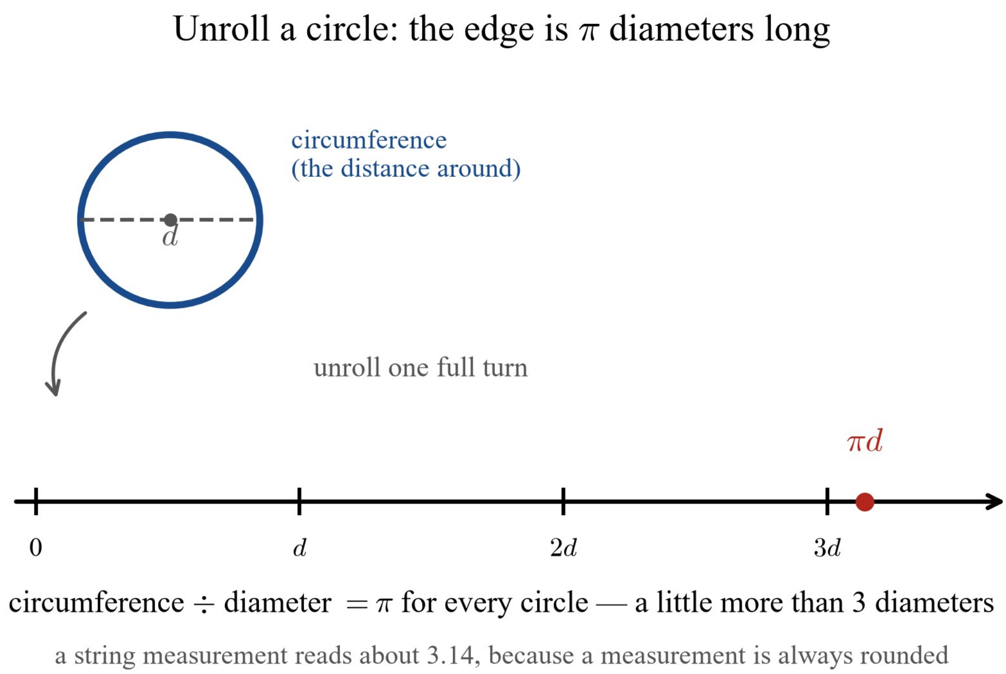
### Measuring $\pi$ with string

Here is the hands-on way to meet $\pi$. Wrap a piece of string once around a circular object, mark the string, straighten it, and measure it: that length is the circumference. Then measure the diameter with a ruler and divide.

For any true circle,

$$\text{circumference} \div \text{diameter} = \pi \text{, exactly.}$$

But a string-and-ruler measurement is always rounded, so your calculation comes out as something like 3.14 or 3.15 — a rational number, close to $\pi$ but not equal to it. The measurement is approximate; $\pi$ itself is exact and irrational. Subtopic 3B will use $\pi$ in the formulas for the circumference and the area of a circle.

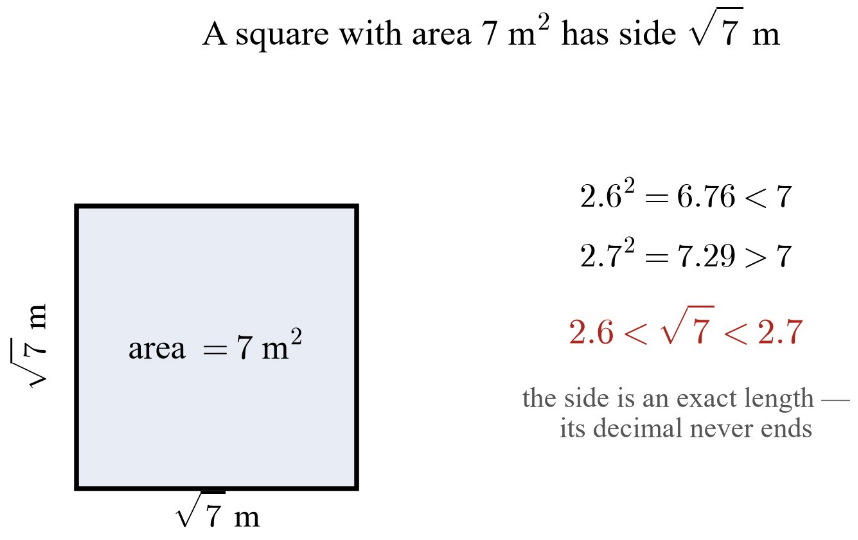



*A circle of diameter d with a string wrapped around its edge, unrolling onto a number line that stretches from 0 to pi, showing the circumference is 3 diameters and a little more, pi times d*

### Where irrational lengths turn up

Irrational numbers are not just a curiosity — they show up as real lengths.

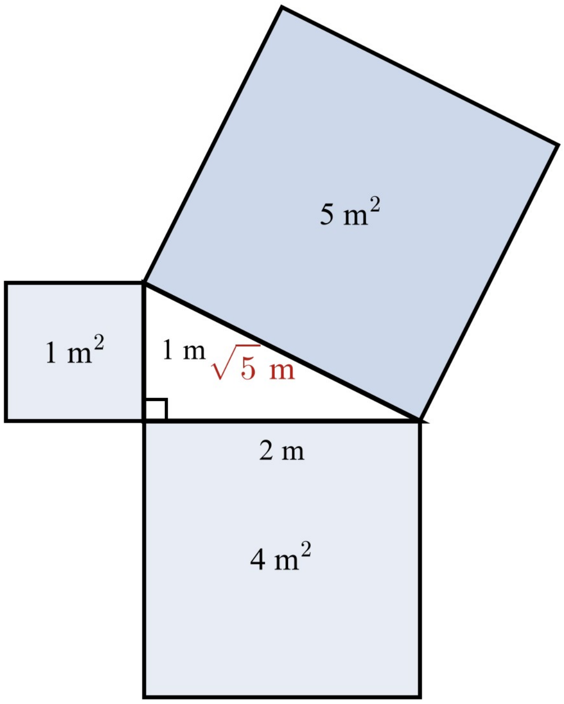
**The side of a square.** A square with area $7\,\mathrm{m}^2$ has side length $\sqrt{7}$ m. Squeeze it: $2.6^2 = 6.76 < 7 < 7.29 = 2.7^2$, so the side is between 2.6 m and 2.7 m. The side is an exact length on the ground; its decimal just never ends.




*A square drawn to scale with area 7 square metres and side length square root of 7 metres, beside the squeeze 2.6 squared is 6.76 and 2.7 squared is 7.29, so square root of 7 lies between 2.6 and 2.7*

**The hypotenuse of a right-angled triangle.** A right-angled triangle with sides 1 m and 2 m has a hypotenuse whose length is $\sqrt{5}$ m. The reason is **Pythagoras' theorem**, which subtopic 3D teaches later in this book: the square on the hypotenuse has area $1^2 + 2^2 = 1 + 4 = 5$, so the hypotenuse is $\sqrt{5}$ m. Squeeze it: $2.2^2 = 4.84 < 5 < 5.29 = 2.3^2$, so $\sqrt{5}$ is between 2.2 and 2.3.

Squares on the sides of a right-angled triangle




$1+4=5$: the hypotenuse is $\sqrt{5}$ m — Pythagoras' theorem comes in 3D.

*A right-angled triangle with sides 1 metre and 2 metres, with a square drawn on each side, the squares having areas 1, 4 and 5 square metres, so the hypotenuse is square root of 5 metres long*

**Paper sizes.** A sheet of A4 paper measures 210 mm by 297 mm. The ratio of the long side to the short side is $297 \div 210 = 1.41428\ldots$, which sits very close to $\sqrt{2} = 1.41421\ldots$. That is by design: the A-series paper standard uses the ratio $\sqrt{2}$ because cutting a sheet in half then gives two sheets with the same shape.

*Source: Wikipedia, "ISO 216"; retrieved 20 August 2026.*

**The golden ratio.** The number

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.61803\ldots$$

is called the **golden ratio** (the symbol $\varphi$ is the Greek letter phi, said "fy"). Because $\sqrt{5}$ is irrational, $\varphi$ is irrational too. In this subtopic, the *golden ratio* is used in the sense given by mathematics: the number $\frac{1+\sqrt{5}}{2}$, about 1.618. Some 20th-century artists and architects, including Le Corbusier and Salvador Dalí, used the golden ratio to size their works.

*Source: Wikipedia, "Golden ratio"; retrieved 20 August 2026.*

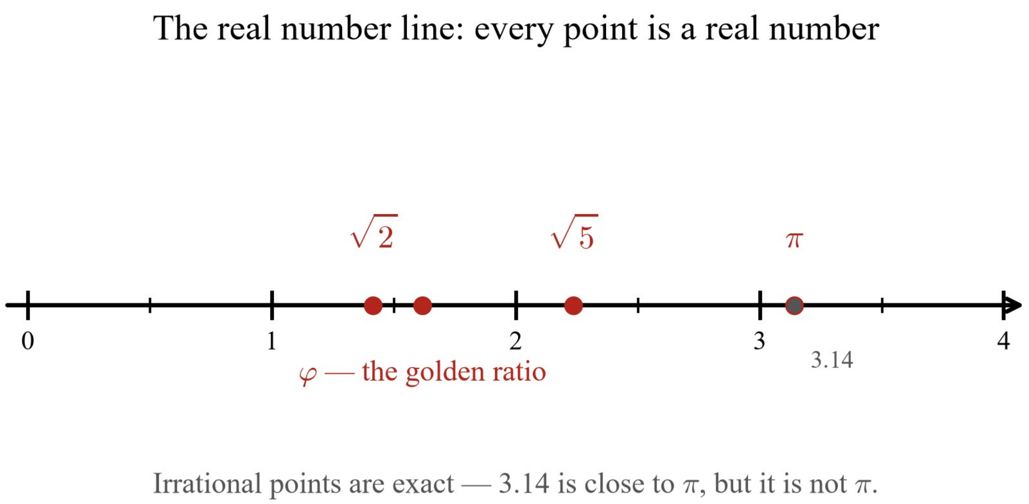
### *The real number line*

Put the rational numbers and the irrational numbers together and you get the **real numbers**. The number line is a picture of the real numbers:

- every point on the line is a real number;
- every real number has its own point, and no two different real numbers share a point.

The rational numbers are everywhere on the line — at every whole number, at every fraction, at every terminating and recurring decimal. The irrational numbers fill in the points that are left. Between any two points on the line, no matter how close, there is always a rational number **and** an irrational number.

The points for irrational numbers are exact, even when we can only write down approximations. The point for $\sqrt{2}$ is exactly the spot between 1.41 and 1.42 that the squeezing on the previous pages points to.



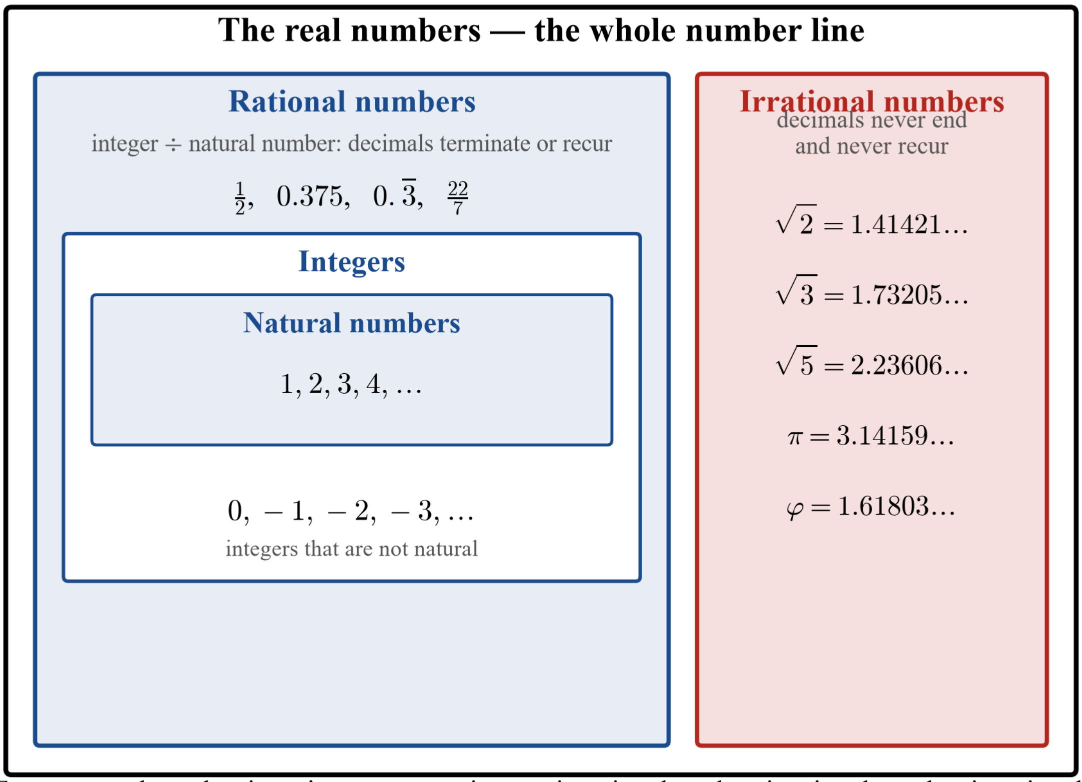

*A number line drawn to scale from 0 to 4 with the irrational numbers square root of 2, the golden ratio, square root of 5 and pi each marked at their approximate positions between the whole numbers*

### *How the number families fit together*

Here is how the sets of numbers sit inside one another.

- The **natural numbers** are the counting numbers $1, 2, 3, 4, \ldots$
- The **integers** add zero and the negatives: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
- The **rational numbers** add every integer ÷ natural number: fractions like $\frac{-3}{4}$, terminating decimals like 0.375, recurring decimals like $0.\overline{3}$.
- The **irrational numbers** are everything else on the line: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\pi$, $\varphi$, and every other decimal that never ends and never recurs.
- The **real numbers** are the rational and irrational numbers together — the whole line.

Every natural number is an integer, and every integer is a rational number ($5 = 5 \div 1$, $-2 = -2 \div 1$). No irrational number is a rational number.




Every natural number is an integer, every integer is rational, and no irrational number is rational.

*A nested diagram of the number families: the real numbers contain the rational numbers and the irrational numbers side by side; inside the rational numbers sit the integers, and inside the integers sit the natural numbers, with example numbers in each family*

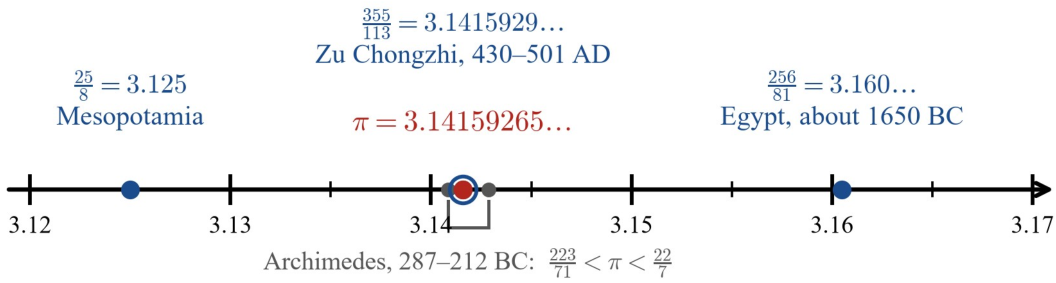
### *A short history of* $\pi$

People have been squeezing $\pi$ for thousands of years, and every culture that worked on it landed on a fraction — a rational approximation, because $\pi$ itself cannot be one.

| Who | Approximation to $\pi$ | As a decimal |
|---|---|---|
| Ancient Mesopotamia | $\frac{25}{8}$ | $3.125$ |
| Ancient Egypt, Rhind Papyrus, about 1650 BC | $4 \times \left(\frac{8}{9}\right)^2 = \frac{256}{81}$ | $3.160\ldots$ |
| Archimedes of Syracuse, 287–212 BC | between $\frac{223}{71}$ and $\frac{22}{7}$ | between $3.1408\ldots$ and $3.1428\ldots$ |
| Zu Chongzhi, 430–501 AD | $\frac{355}{113}$ | $3.1415929\ldots$ |

Notice how the squeezing tightens over time. Archimedes trapped $\pi$ between two fractions. Zu Chongzhi's fraction $\frac{355}{113}$ agrees with $\pi$ all the way to 3.14159 — so close that on any number line you could draw in this book, its point lands exactly on top of the point for $\pi$. Yet $\frac{355}{113}$ is still a rational number, and $\pi$ is still irrational: no fraction ever lands exactly on $\pi$.

Approximations to $\pi$ through history — drawn to scale




Zu Chongzhi's fraction lands on top of $\pi$ at this scale — but every fraction here is rational, and $\pi$ is not.

*A number line drawn to scale from 3.12 to 3.17 with the historical approximations to pi marked: twenty-five eighths at 3.125, two hundred and fifty-six over eighty-one at 3.160, Archimedes' bounds either side of pi, and three hundred and fifty-five over one hundred and thirteen landing on top of pi*

*Source: University of St Andrews, MacTutor History of Mathematics, "Pi through the ages"; retrieved 20 August 2026.*

### *What comes next*

This subtopic recognises irrational numbers, squeezes them between decimals, and places them on the number line. It does not add them, multiply them or simplify them — working with square roots as numbers belongs to Year 9 (`VC2M9N01`), where the real number system is put to work.

Here is the whole subtopic on one card.

**Irrational numbers**

$\sqrt{2},\ \sqrt{5},\ \pi,\ \varphi$

cannot develop from integer ÷ natural number — decimals never end, never recur

**Square roots**

$\sqrt{49} = 7$ rational, $\sqrt{50}$ irrational

$\sqrt{n}$ is rational only when $n$ is a perfect square

$\pi$

$3.141 < \pi < 3.142$

$\frac{22}{7}$ and 3.14 are approximations, not $\pi$

**The real number line**

rationals + irrationals

every point is a real number — irrational points are exact

Squeeze by squaring: trap the number between two perfect squares, then between two squares of tenths.

*A summary card with four boxes: irrational numbers cannot develop from an integer divided by a natural number; a square root is rational only when the number inside is a perfect square; pi lies between 3.141 and 3.142 with 22 sevenths and 3.14 only approximations; and rationals plus irrationals fill the real number line*

### *Words of this subtopic*

| Word | What it means here |
|---|---|
| **rational number** | a number that can develop from the division of an integer by a natural number |
| **irrational number** | a number that cannot develop from the division of an integer by a natural number; its decimal never ends and never recurs |
| **perfect square** | a whole number that is the square of a whole number: 1, 4, 9, 16, … |
| **square root** | $\sqrt{n}$ is the number that squares to give $n$: $\sqrt{9}=3$ |
| **terminating decimal** | a decimal that stops: 0.375 |
| **recurring decimal** | a decimal that repeats the same block of digits forever: $0.\overline{3}$ |
| **real numbers** | the rational and irrational numbers together — every point on the number line |
| **approximation** | a number that is close to, but not exactly equal to, another number |
| $\pi$ **(pi)** | the irrational number equal to circumference ÷ diameter for any circle; $3.141<\pi<3.142$ |
| **golden ratio** | the irrational number $\frac{1+\sqrt{5}}{2} \approx 1.618$ |

## Worked examples

### *Example 1 — with help*

Classify each number as rational or irrational. Give a reason.

(a) $\sqrt{49}$
(b) $\sqrt{50}$
(c) $0.\overline{3}$
(d) 3.14

| Working | Comment |
|---|---|
| (a) $\sqrt{49} = 7$ | 49 is a perfect square: $7^2 = 49$. |
| $7 = 7 \div 1$ | A whole number is an integer ÷ natural number. ∴ **rational**. |
| (b) $7^2 = 49 < 50 < 64 = 8^2$ | 50 sits between two perfect squares, so $\sqrt{50}$ is not a whole number. |
| 50 is not a perfect square | ∴ $\sqrt{50}$ is **irrational**. |
| (c) $0.\overline{3} = 0.333\ldots$ | A recurring decimal comes out of a division: $1 \div 3$. |
| $0.\overline{3} = 1 \div 3$ | ∴ **rational**. |
| (d) $3.14 = 314 \div 100$ | A terminating decimal is an integer ÷ natural number. |
| 3.14 is **not** $\pi$ | ∴ **rational** — it is only an approximation to $\pi$. |

### *Example 2 — with help*

Squeeze $\sqrt{7}$ between two integers, then between two tenths.

| Working | Comment |
|---|---|
| $2^2 = 4 < 7 < 9 = 3^2$ | Find the perfect squares either side of 7. |
| $2 < \sqrt{7} < 3$ | $\sqrt{7}$ lies between those two integers. |
| $2.6^2 = 6.76$ and $2.7^2 = 7.29$ | Try tenths between 2 and 3. |
| $6.76 < 7 < 7.29$ | 7 sits between the two squares. |
| $2.6 < \sqrt{7} < 2.7$ | ∴ $\sqrt{7} \approx 2.6$ to one decimal place. |

Each new decimal place works the same way: square the candidates, and see which two squares trap the number.

### *Example 3 — with help*

A circular plate has diameter 10.0 cm. A string wrapped once around the edge measures 31.4 cm. Find circumference ÷ diameter, and explain why your answer is not exactly $\pi$.

| Working | Comment |
|---|---|
| circumference ÷ diameter $= 31.4 \div 10.0$ | Divide the two measured lengths. |
| $= 3.14$ | Close to $\pi$, as every circle's ratio must be. |
| $3.14 \neq \pi$ | The string and the ruler measure to the nearest mm, so 31.4 cm is already rounded. |

∴ circumference ÷ diameter $= 3.14$. The true circumference of this plate is $10\pi = 31.4159\ldots$ cm; a string can only ever report a rounded, rational value, while $\pi$ itself is irrational.

### *Example 4 — on your own*

A square has area $20\,\mathrm{m}^2$. Its side length is $\sqrt{20}$ m. Squeeze $\sqrt{20}$ between two integers, then between two tenths.

The side of a square whose area is $A$ is $\sqrt{A}$, so the side here is $\sqrt{20}$ m.

Integers first: $4^2=16<20<25=5^2$, so $4<\sqrt{20}<5$.

Now tenths between 4 and 5: $4.4^2=19.36$ and $4.5^2=20.25$. Since $19.36<20<20.25$, the squeeze is $4.4<\sqrt{20}<4.5$ .

∴ the side of the square is between 4.4 m and 4.5 m. One more step gives $4.47^2=19.9809<20<20.0704=4.48^2$, so $\sqrt{20}\approx 4.47$. The exact side is $\sqrt{20}$ m; the decimals only ever approximate it.

### *Example 5 — on your own*

Sort these numbers into rational and irrational:

$$\sqrt{12}, \sqrt{121}, 0.101001000\ldots, 2.75, \pi, \frac{22}{7}$$

Check each one against the definition: can it develop from the division of an integer by a natural number?

- $\sqrt{121}=11=11\div 1$ — **rational** (121 is a perfect square).
- $2.75=275\div 100$ — **rational** (terminating decimal).
- $\frac{22}{7}$ — **rational**: it *is* an integer divided by a natural number. It is a good approximation to $\pi$, but it is not $\pi$.
- $\sqrt{12}$ — **irrational**: 12 is not a perfect square ($3^2=9<12<16=4^2$).
- $0.101001000\ldots$ — **irrational**: the decimal never ends, and the blocks of zeros keep growing, so no block of digits ever recurs.
- $\pi$ — **irrational**: $3.14159265\ldots$ never ends and never recurs.

∴ rational: $\sqrt{121}$, 2.75, $\frac{22}{7}$; irrational: $\sqrt{12}$, $0.101001000\ldots$, $\pi$.

### *Example 6 — on your own*

A right-angled triangle has sides 1 m and 2 m. Pythagoras' theorem (which you will meet in subtopic 3D) says that the square on the hypotenuse has area $1^2+2^2$. Find the length of the hypotenuse, and squeeze it between two tenths.

The square on the hypotenuse has area $1^2+2^2=1+4=5$, so the hypotenuse is $\sqrt{5}$ m.

Squeeze it: $2^2=4<5<9=3^2$, so $2<\sqrt{5}<3$. Now tenths: $2.2^2=4.84$ and $2.3^2=5.29$. Since $4.84<5<5.29$:

$$2.2<\sqrt{5}<2.3$$

∴ the hypotenuse is $\sqrt{5}$ m, between 2.2 m and 2.3 m. One more step gives $2.23^2=4.9729<5<5.0176=2.24^2$, so $\sqrt{5}\approx 2.24$.

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## Practice questions

*Work these questions by hand. Every squeeze is found by squaring — no calculator is needed.*

### *With help*

**Q1.** Classify each number as rational or irrational. Give a reason.

a. $\sqrt{81}$
b. $\sqrt{80}$
c. $0.6$
d. $\pi$

e. $0.\overline{25}$
f. $\sqrt{0.49}$

**Q2.** State the two integers that each number lies between.

a. $\sqrt{3}$
b. $\sqrt{10}$
c. $\sqrt{30}$
d. $\sqrt{101}$

**Q3.** True or false? Give your reason, and correct each false statement.

a. Every square root is irrational.
b. $\pi = \frac{22}{7}$.
c. Every integer is a rational number.
d. The decimal of an irrational number never recurs.
e. $3.14$ is irrational because it is used as an approximation for $\pi$.

**Q4.** Write each number as a fraction, to show that it is rational.

a. $0.7$
b. $2.5$
c. $0.\overline{3}$
d. $1.2$

**Q5.** Squeeze $\sqrt{10}$.

a. Show that $3.1 < \sqrt{10} < 3.2$.
b. Show that $3.16 < \sqrt{10} < 3.17$.

**Q6.** Draw a number line from 0 to 4. Mark the approximate positions of $\sqrt{2}$, $\pi$ and $\sqrt{10}$. (Squeeze each one first.)

**Q7.** Count the rationals.

a. How many of $\sqrt{1}, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{16}$ are rational?
b. How many of $\sqrt{17}, \sqrt{18}, \ldots, \sqrt{50}$ are rational?

**Q8.** Classify each decimal as terminating, recurring, or neither. If it is neither, the number is irrational.

a. $0.125$
b. $0.121212\ldots$
c. $0.120120012000\ldots$ (the blocks of zeros keep growing)
d. $2.0$

### *On your own*

**Q9.** Squeeze $\sqrt{50}$.

a. between two integers
b. between two tenths

**Q10.** Explain why $\sqrt{64}$ is rational but $\sqrt{65}$ is irrational.

**Q11.** A wheel with diameter 1 m rolls one full turn along flat ground. The distance it rolls is the circumference, $\pi$ m. Between which two tenths of a metre does the wheel stop?

**Q12.** Mia says: "$\pi$ is 3.14, and 3.14 is a decimal, so $\pi$ is rational." Explain the error in Mia's reasoning.

**Q13.** The golden ratio is $\varphi = \frac{1+\sqrt{5}}{2}$.

a. Given that $2.23 < \sqrt{5} < 2.24$, show that $1.615 < \varphi < 1.62$.
b. Is $\varphi$ rational or irrational? Give a reason.

**Q14.** Without working out any values, state whether each number is rational or irrational. Give a reason.

a. $\sqrt{0.01}$
b. $\sqrt{200}$
c. $0.314159$
d. $0.\overline{314159}$

**Q15.** Put these numbers in order from smallest to largest: $2.2$, $\sqrt{5}$, $2.24$. (Hint: square each one, and compare with 5.)

**Q16.** A square has area $3\,\mathrm{m}^2$.

a. What is its side length, exactly?
b. Is the side length rational or irrational? Why?
c. Squeeze the side length between two tenths.

### *Maths in real life*

**Q17.** A sheet of A4 paper measures 210 mm by 297 mm. The paper standard chooses the ratio $\sqrt{2}$ for the long side to the short side, because halving a sheet then keeps the same shape.

*Source: Wikipedia, "ISO 216"; retrieved 20 August 2026.*

a. Work out $297 \div 210$ to five decimal places.
b. Given that $\sqrt{2} = 1.41421\ldots$, how many decimal places does your answer agree with $\sqrt{2}$?
c. One of $\frac{297}{210}$ and $\sqrt{2}$ is rational and the other is irrational. Which is which, and why?

**Q18.** A circular cup has diameter 8.0 cm. A string wrapped once around the top measures 25.1 cm.

a. Work out $25.1 \div 8.0$.
b. Explain why your answer is not exactly $\pi$.
c. The true circumference $C$ satisfies $3.141 \times 8 < C < 3.142 \times 8$. Work out these two bounds for $C$.

**Q19.** Four approximations to $\pi$ from history:

| Who | Approximation to $\pi$ |
|---|---|
| Ancient Mesopotamia | $\frac{25}{8}$ |
| Ancient Egypt, Rhind Papyrus, about 1650 BC | $\frac{256}{81}$ |
| Archimedes of Syracuse, 287–212 BC | $\frac{22}{7}$ (the upper bound) |
| Zu Chongzhi, 430–501 AD | $\frac{355}{113}$ |

*Source: University of St Andrews, MacTutor History of Mathematics, "Pi through the ages"; retrieved 20 August 2026.*

a. Write each fraction as a decimal, correct to four decimal places.
b. Given that $\pi = 3.14159\ldots$, which of the four is closest to $\pi$?

c. Explain why all four of these numbers are rational, even though they approximate an irrational number.

**Q20.** Make your own. Choose a whole number between 2 and 30 that is not a perfect square. Squeeze its square root between two integers, then between two tenths, and mark it approximately on a number line. Swap with a partner, check each other's squeezing by squaring, and agree on the position.

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## Answers

**Q1.** a. Rational: $\sqrt{81}=9=9\div 1$. b. Irrational: 80 is not a perfect square. c. Rational: $0.6=6\div 10$. d. Irrational: $\pi$'s decimal never ends and never recurs. e. Rational: $0.\overline{25}=25\div 99$. f. Rational: $\sqrt{0.49}=0.7=7\div 10$.

**Q2.** a. 1 and 2. b. 3 and 4. c. 5 and 6. d. 10 and 11.

**Q3.** a. False: $\sqrt{25}=5$, rational. b. False: $\frac{22}{7}=3.\overline{142857}$ is rational; $\pi$ is irrational; $\frac{22}{7}$ is only an approximation. c. True: any integer $n$ is $n\div 1$. d. True. e. False: $3.14=314\div 100$ is a terminating decimal, so it is rational.

**Q4.** a. $\frac{7}{10}$. b. $\frac{5}{2}$. c. $\frac{1}{3}$. d. $\frac{6}{5}$.

**Q5.** a. $3.1^2=9.61<10<10.24=3.2^2$. b. $3.16^2=9.9856<10<10.0489=3.17^2$.

**Q6.** $\sqrt{2}\approx 1.41$, $\pi\approx 3.14$, $\sqrt{10}\approx 3.16$; $\sqrt{10}$ sits just to the right of $\pi$.

**Q7.** a. 4 ($\sqrt{1}, \sqrt{4}, \sqrt{9}, \sqrt{16}$). b. 3 ($\sqrt{25}, \sqrt{36}, \sqrt{49}$).

**Q8.** a. Terminating (rational). b. Recurring (rational). c. Neither — it never ends and never recurs, so it is irrational. d. Terminating (rational).

**Q9.** a. $7<\sqrt{50}<8$, since $49<50<64$. b. $7.0<\sqrt{50}<7.1$, since $7.0^2=49.00<50<50.41=7.1^2$.

**Q10.** $64=8^2$, so $\sqrt{64}=8=8\div 1$, rational. 65 is not a perfect square (it sits between $8^2=64$ and $9^2=81$), so $\sqrt{65}$ is irrational.

**Q11.** $\pi$ m, which lies between 3.1 m and 3.2 m.

**Q12.** 3.14 is a rounded approximation to $\pi$, and 3.14 itself is rational ($314\div 100$). But $\pi$ is not equal to 3.14: $\pi=3.14159\ldots$ never ends and never recurs, so $\pi$ is irrational.

**Q13.** a. Add 1: $3.23<1+\sqrt{5}<3.24$. Divide by 2: $1.615<\varphi<1.62$. b. Irrational: $\varphi$ is built from $\sqrt{5}$, and its decimal $1.61803\ldots$ never ends and never recurs.

**Q14.** a. Rational: $0.01=0.1^2$, so $\sqrt{0.01}=0.1$. b. Irrational: 200 is not a perfect square ($14^2=196<200<225=15^2$). c. Rational: a terminating decimal, $314159\div 1000000$. d. Rational: a recurring decimal.

**Q15.** $2.2<\sqrt{5}<2.24$. (Squaring: $2.2^2=4.84<5$, and $2.24^2=5.0176>5$.)

**Q16.** a. $\sqrt{3}$ m. b. Irrational: 3 is not a perfect square. c. $1.7<\sqrt{3}<1.8$, since $1.7^2=2.89<3<3.24=1.8^2$.

**Q17.** a. 1.41429 ($297\div 210=1.4142857\ldots$). b. The first four decimal places: both read 1.4142 before they split. c. $\frac{297}{210}$ is rational — it is an integer divided by a natural number. $\sqrt{2}$ is irrational — 2 is not a perfect square.

**Q18.** a. 3.1375. b. The string and the ruler only measure to the nearest mm, so both lengths are rounded; a measurement can only ever give a rational approximation to $\pi$. c. $25.128\,\text{cm}<C<25.136\,\text{cm}$.

**Q19.** a. $\frac{25}{8}=3.1250$; $\frac{256}{81}=3.1605$; $\frac{22}{7}=3.1429$; $\frac{355}{113}=3.1416$. b. $\frac{355}{113}$. c. Each one is an integer divided by a natural number, and every such division gives a terminating or a recurring decimal.

**Q20.** Answers will vary. Check: the number chosen is not a perfect square, each squeeze is justified by squaring both ends, and the marked point sits inside the final squeeze.

### Fully-worked solutions

**Q1.** Test each number against the definition.

a. $\sqrt{81}=9$ because $9^2=81$; and $9=9\div 1$. **Rational.**
b. $8^2=64<80<81=9^2$, so 80 is not a perfect square and $\sqrt{80}$ is not a whole number. **Irrational.**
c. $0.6=6\div 10$. **Rational.**
d. $\pi=3.14159265\ldots$ never ends and never recurs, so it cannot develop from a division of integers. **Irrational.**
e. $0.\overline{25}=0.252525\ldots$ is a recurring decimal, and recurring decimals come from divisions: $0.\overline{25}=25\div 99$. **Rational.**
f. $0.49=0.7\times 0.7$, so $\sqrt{0.49}=0.7=7\div 10$. **Rational.**

**Q2.** Trap each number between perfect squares.

a. $1<3<4$, so $1<\sqrt{3}<2$: **between 1 and 2**.
b. $9<10<16$, so $3<\sqrt{10}<4$: **between 3 and 4**.
c. $25<30<36$, so $5<\sqrt{30}<6$: **between 5 and 6**.
d. $100<101<121$, so $10<\sqrt{101}<11$: **between 10 and 11**.

**Q3.** Test each statement against the definitions.

a. **False.** The square root of a perfect square is a whole number: $\sqrt{25}=5$, which is rational.
b. **False.** $\frac{22}{7}=3.\overline{142857}$ is a recurring decimal, so it is rational; $\pi$ is irrational. $\frac{22}{7}$ is a useful approximation, not the number itself.
c. **True.** Every integer $n$ can be written $n\div 1$.
d. **True.** If the decimal recurs, the number comes from a division of integers and is rational; an irrational decimal never recurs.
e. **False.** $3.14=314\div 100$ terminates, so it is rational. Being close to an irrational number does not make a number irrational.

**Q4.** Write each decimal as a division of an integer by a natural number.

a. $0.7=7\div 10=\frac{7}{10}$.
b. $2.5=25\div 10=\frac{25}{10}=\frac{5}{2}$.
c. $0.\overline{3}=0.333\ldots=1\div 3=\frac{1}{3}$.
d. $1.2=12\div 10=\frac{12}{10}=\frac{6}{5}$.

**Q5.** Square the candidates and compare with 10.

a. $3.1^2=9.61$ and $3.2^2=10.24$. Since $9.61<10<10.24$, $3.1<\sqrt{10}<3.2$.
b. $3.16^2=9.9856$ and $3.17^2=10.0489$. Since $9.9856<10<10.0489$, $3.16<\sqrt{10}<3.17$.

**Q6.** Squeeze first: $1.4<\sqrt{2}<1.5$, $3.141<\pi<3.142$, and $3.1<\sqrt{10}<3.2$. On the line from 0 to 4, mark $\sqrt{2}$ a little right of 1.4, $\pi$ a little right of 3.14, and $\sqrt{10}$ just to the right of $\pi$. Notice how two very different numbers, $\pi$ and $\sqrt{10}$, sit almost on top of one another near 3.14.

**Q7.** A square root $\sqrt{n}$ is rational exactly when $n$ is a perfect square.

a. The perfect squares from 1 to 16 are 1, 4, 9, 16 — **4 of them**.
b. The perfect squares from 17 to 50 are 25, 36, 49 — **3 of them**.

**Q8.** Check whether the decimal stops, repeats, or does neither.

a. $0.125$ stops — **terminating**, rational.
b. $0.121212\ldots$ repeats the block 12 forever — **recurring**, rational.
c. $0.120120012000\ldots$ has a pattern, but the blocks of zeros keep growing, so no block of digits ever repeats exactly — **neither**, irrational.
d. $2.0$ stops — **terminating**, rational.

**Q9.** Trap 50 between squares.

a. $7^2 = 49 < 50 < 64 = 8^2$, so $7 < \sqrt{50} < 8$.
b. $7.0^2 = 49.00$ and $7.1^2 = 50.41$. Since $49.00 < 50 < 50.41$, $7.0 < \sqrt{50} < 7.1$.

**Q10.** Use perfect squares.

$64 = 8^2$, so $\sqrt{64} = 8$, and $8 = 8 \div 1$ develops from the division of an integer by a natural number — rational. For 65: the perfect squares either side are $64 = 8^2$ and $81 = 9^2$, so 65 is not a perfect square, $\sqrt{65}$ is not a whole number, and its decimal never ends and never recurs — irrational.

$\therefore$ $\sqrt{64}$ is rational and $\sqrt{65}$ is irrational because 64 is a perfect square and 65 is not.

**Q11.** One full turn rolls out the circumference.

Distance $= \pi \times 1 = \pi$ m. Since $3.1 < \pi < 3.2$, the wheel stops between the 3.1 m mark and the 3.2 m mark.

**Q12.** Separate the number from its approximation.

3.14 is rational: $3.14 = 314 \div 100$, a terminating decimal. But $\pi \neq 3.14$: $\pi = 3.14159265\ldots$ never ends and never recurs. Mia has taken a rounded, rational approximation and treated it as the number itself. The true value of $\pi$ is irrational.

**Q13.** Squeeze using the given squeeze on $\sqrt{5}$.

a. Start from $2.23 < \sqrt{5} < 2.24$. Add 1 to each part: $3.23 < 1 + \sqrt{5} < 3.24$. Divide each part by 2: $1.615 < \dfrac{1+\sqrt{5}}{2} < 1.62$, that is, $1.615 < \varphi < 1.62$.
b. **Irrational.** $\varphi = 1.61803\ldots$ never ends and never recurs; it is built from $\sqrt{5}$, which is irrational.

**Q14.** Reason from the definitions, without finding values.

a. **Rational.** $0.01 = 0.1 \times 0.1$, so $\sqrt{0.01} = 0.1 = 1 \div 10$.
b. **Irrational.** $14^2 = 196 < 200 < 225 = 15^2$, so 200 is not a perfect square.
c. **Rational.** 0.314159 terminates: it is $314159 \div 1000000$. (Having the same digits as $\pi$ changes nothing — this decimal stops.)
d. **Rational.** $0.\overline{314159}$ recurs, so it comes from a division of integers.

**Q15.** Compare by squaring, since all three numbers are positive.

$2.2^2 = 4.84 < 5$, so $2.2 < \sqrt{5}$. Also $2.24^2 = 5.0176 > 5$, so $\sqrt{5} < 2.24$.

$\therefore$ from smallest to largest: 2.2, $\sqrt{5}$, 2.24.

**Q16.** The side of a square of area $A$ is $\sqrt{A}$.

a. Side $= \sqrt{3}$ m.
b. **Irrational**: 3 is not a perfect square ($1^2 = 1 < 3 < 4 = 2^2$), so $\sqrt{3}$ is not a whole number and its decimal never ends and never recurs.
c. $1.7^2 = 2.89$ and $1.8^2 = 3.24$. Since $2.89 < 3 < 3.24$, $1.7 < \sqrt{3} < 1.8$.

**Q17.** Divide, compare, classify.

a. $297 \div 210 = 1.4142857\ldots = 1.41429$ to five decimal places.

b. $\sqrt{2} = 1.41421\ldots$ Both numbers read $1.4142\ldots$, so they agree for the first **four** decimal places, then split ( $\ldots 28$ against $\ldots 21$).

c. $\frac{297}{210}$ is **rational** — it develops from the division of an integer by a natural number, and its decimal recurs. $\sqrt{2}$ is **irrational** — 2 is not a perfect square, so $\sqrt{2}$ cannot develop from any such division.

**Q18.** Divide the measured lengths, then squeeze the true value.

a. $25.1 \div 8.0 = 3.1375$.

b. The string and the ruler measure to the nearest mm, so 25.1 cm and 8.0 cm are both rounded values. A division of rounded values is itself rounded — a rational approximation, never the exact irrational value $\pi$.

c. $3.141 \times 8 = 25.128$ and $3.142 \times 8 = 25.136$, so $25.128\,\text{cm} < C < 25.136\,\text{cm}$. The measured 25.1 cm sits inside these bounds, as it must.

**Q19.** Long division (or the fraction bar) gives each decimal.

a. $\frac{25}{8} = 3.1250$. $\frac{256}{81} = 3.1604\ldots = 3.1605$ to four decimal places. $\frac{22}{7} = 3.142857\ldots = 3.1429$. $\frac{355}{113} = 3.141592\ldots = 3.1416$.

b. Compare with $\pi = 3.14159\ldots$: 3.1250 is off by about 0.017; 3.1605 is off by about 0.019; 3.1429 is off by about 0.0013; 3.1416 is off by less than 0.0001. $\therefore$ the closest is $\frac{355}{113}$, from Zu Chongzhi.

c. Each number is an integer divided by a natural number — $25 \div 8$, $256 \div 81$, $22 \div 7$, $355 \div 113$ — and every such division terminates or recurs. All four are therefore rational, by definition, even though each sits close to an irrational number.

**Q20.** Answers will vary. A sample: choose 13. Then $3^2 = 9 < 13 < 16 = 4^2$, so $3 < \sqrt{13} < 4$; and $3.6^2 = 12.96 < 13 < 13.69 = 3.7^2$, so $3.6 < \sqrt{13} < 3.7$. Check your partner's work the same way: square both ends of each squeeze, and confirm the chosen number sits between the two squares.

## Percentage increase, percentage decrease and percentage error

**Content description VC2M8N05:** solve problems involving the use of percentages, including percentage increases and decreases and percentage error, with and without digital tools

*A price goes up 10% when GST is added, and down 25% in a sale. A bottle holds a little less than the 250 mL it promises, and a country gains 412,500 people in a year. Every one of these is a percentage change. This subtopic gives you the tools to work them all — with and without a calculator — and to say exactly how big a mistake is, compared with the truth.*

### Theory

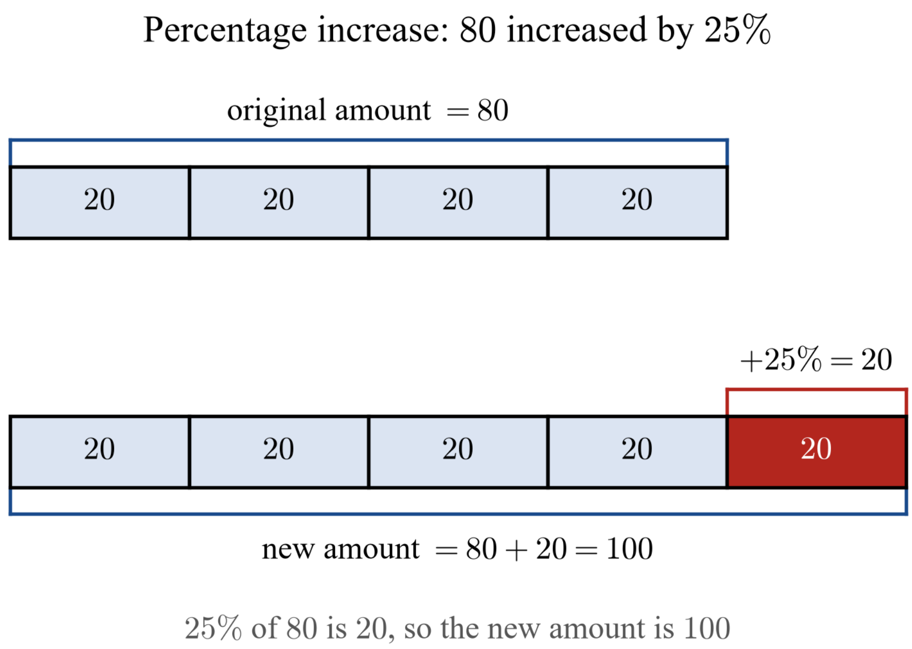
In Year 7 you wrote the same quantity as a fraction, a decimal or a percentage, and you found percentages of quantities (VC2M7N04, VC2M7N07, VC2M7N09, VC2M7N10). This subtopic uses those skills without re-teaching them: every percentage here is still a percentage *of* something, and that "of" still means multiply. This book uses VCAA's names — **percentage increase**, **percentage decrease** and **percentage error** — throughout.

#### *Percentage increase*

To increase 80 by 25%, find 25% of 80 and add it on. 25% is one quarter, and a quarter of 80 is 20, so:

$$80 + 25\,\%\text{ of }80 = 80 + 20 = 100$$

The increase is always a percentage **of the starting amount**.



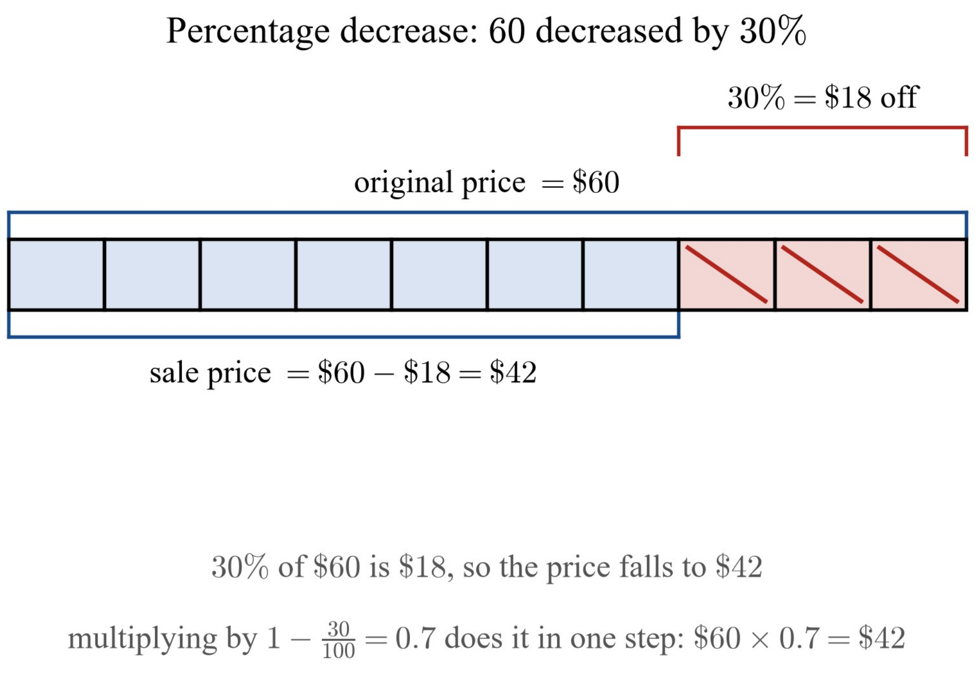

*A strip bar showing 80 as four cells of 20, with one red cell of 20 added for the 25% increase, making 100*

#### *Percentage decrease*

To decrease \$60 by 30%, find 30% of 60 and take it away. 10% of 60 is \$6, so 30% is \$18:

$$60 - 30\,\%\text{ of }60 = 60 - 18 = 42$$

A decrease is also a percentage of the starting amount.



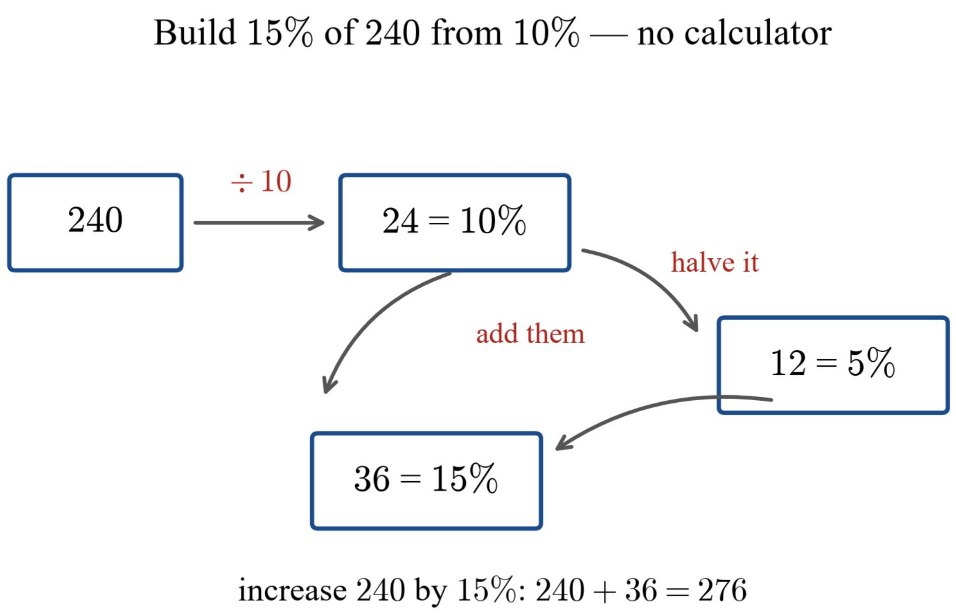

*A strip bar of $60 in ten cells of $6, with three cells crossed out in red for the 30% decrease, leaving $42*

### *The 10% building block — no calculator needed*

Without a calculator, 10% is the key, because 10% is one tenth: just divide by 10. Halve it for 5%, double it for 20%, and build any of these from the pieces. To find 15% of 240:

$$10\,\%\,\text{of}\,240=24\,,5\,\%\,\text{of}\,240=12\,,15\,\%=24+12=36$$

So 240 increased by 15% is $240+36=276$.



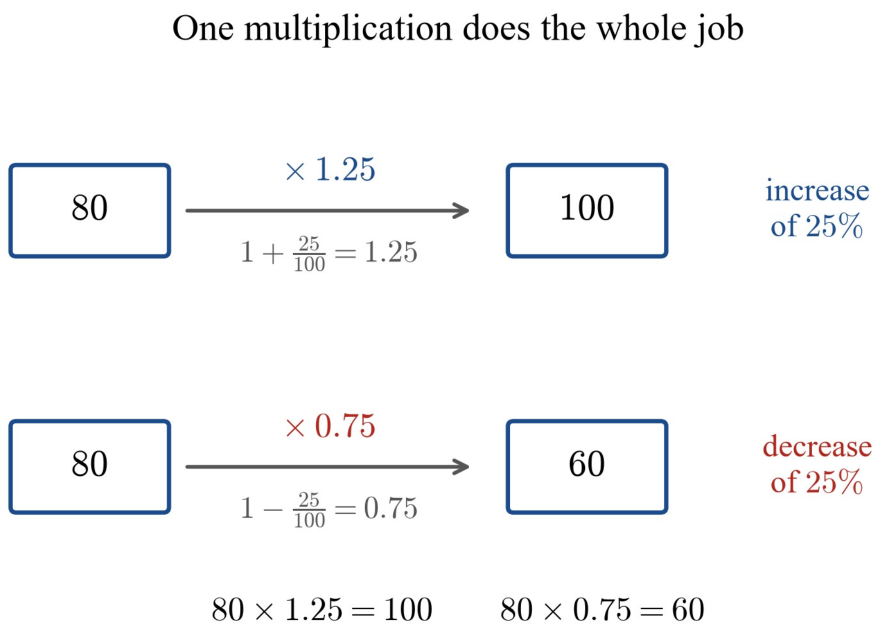

*Building 15% of 240 without a calculator: 10% is 24, 5% is 12, so 15% is 36; and 240 plus 36 is 276*

### *Multipliers — one multiplication does the whole change*

An increase of 25% keeps the whole (100%) and adds 25%, so the answer is 125% of the start: multiply by 1.25. A decrease of 25% keeps 75%: multiply by 0.75.

$$80 \times 1.25 = 100\,,\, 80 \times 0.75 = 60$$

An increase multiplies by a number **greater than 1**; a decrease by a number **less than 1**. The number you multiply by is called the **multiplier**.



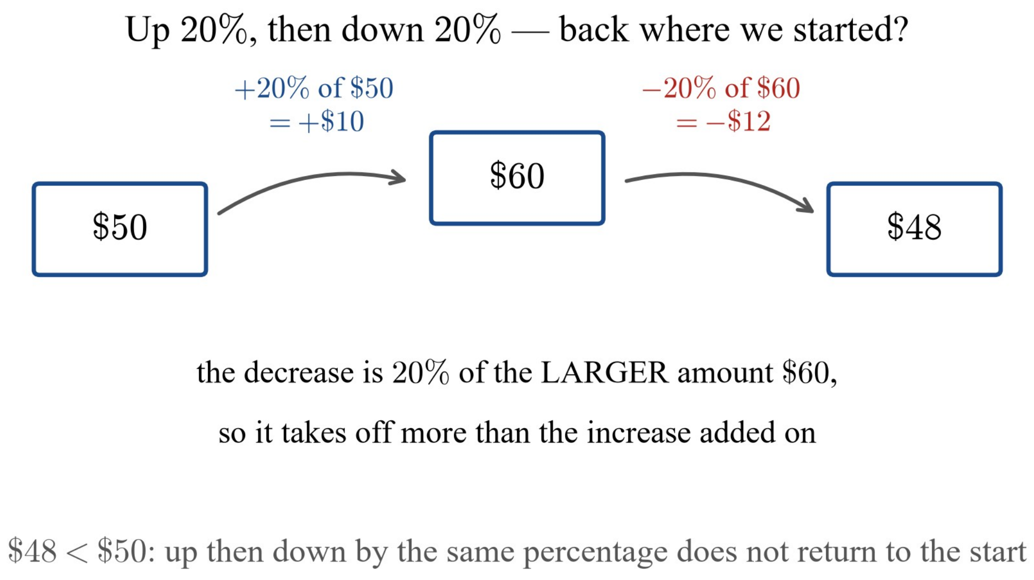

*Two rows: 80 times 1.25 gives 100 for a 25% increase, and 80 times 0.75 gives 60 for a 25% decrease; an increase multiplies by more than 1, a decrease by less than 1*

### ***Up then down does not return to the start***

Increase \$50 by 20%: the increase is \$10, giving \$60. Now decrease \$60 by 20%: the decrease is \$12, giving \$48 — not \$50.

$$50 \times 1.2 = 60\,,\, 60 \times 0.8 = 48$$

The first 20% is taken of 50 and the second of 60. Different starting amounts give different-sized changes, so an increase and the same decrease do not cancel.




*$50 increased by 20% gives $60, then decreased by 20% gives $48, not $50, because the second percentage is taken of the larger amount*

### *With digital tools — and a check that the answer makes sense*

With a calculator, 17% of $84.50 is one multiplication: $0.17 \times 84.50 = 14.365$, which rounds to **$14.37** at the cent. The calculator does the arithmetic; you still choose the multiplication, and you still check the answer makes sense: 17% is a little less than 20%, and 20% of $84.50 is $16.90, so $14.37 is the right size. That check is the point of "with digital tools" — the machine calculates, you judge.

### *GST — a 10% increase you meet in real life*

The **goods and services tax (GST)** is a 10% tax on most goods and services sold in Australia. The rate has been 10% since 1 July 2000, and it is 10% in the 2026–27 financial year. Adding GST to a price is simply a 10% increase:

$$85 \times 1.10 = 93.50$$

so $85 before GST becomes $93.50 with GST. (Most fresh food is GST-free, so no GST is added to it at the checkout.)

Finding the GST already inside a GST-inclusive price runs the increase backwards. The price before GST is 10 equal parts and the GST adds 1 more part, so the inclusive price is 11 parts: divide by 11.

$$66 \div 11 = 6$$

so $66 with GST holds $6 of GST, and the price before GST is $66 − $6 = $60.

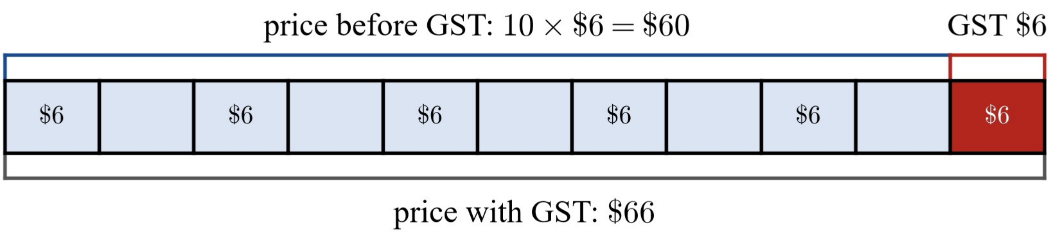
Adding GST: multiply the price by 1.10


$$\$85.00 \times 1.10 = \$93.50$$

the price before GST is 10 parts; GST adds 1 part; the total is 11 parts

*Adding GST: $85.00 before GST times 1.10 gives $93.50 with GST; the price before GST is 10 parts, GST adds 1 part, the total is 11 parts*

The GST inside a $66 price: split into 11 equal parts




$$\$66 \div 11 = \$6 \text{ is one part: the GST}$$

a price with GST is always 11 of these parts

*A bar of $66 with GST split into 11 equal parts of $6: ten parts ($60) are the price before GST and one part ($6) is the GST*

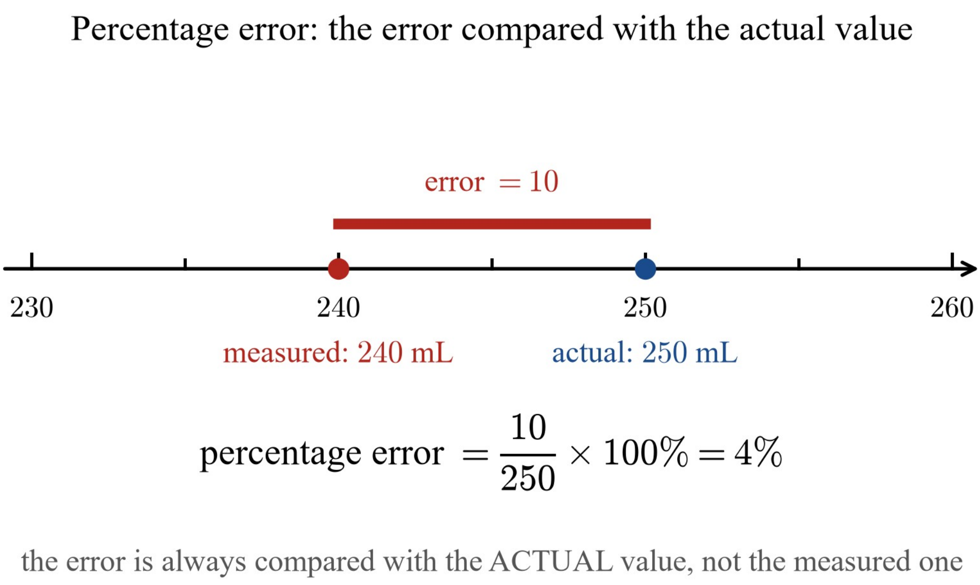
*Source: Australian Taxation Office, "Goods and services tax (GST)"; retrieved 20 August 2026.*

### *Percentage error — how big is the mistake?*

A bottle is labelled 250 mL but holds 240 mL. The error is 10 mL. Is that a bad mistake? Against the actual 250 mL it is small; against a 25 mL bottle it would be huge. The **percentage error** compares the error with the actual value:

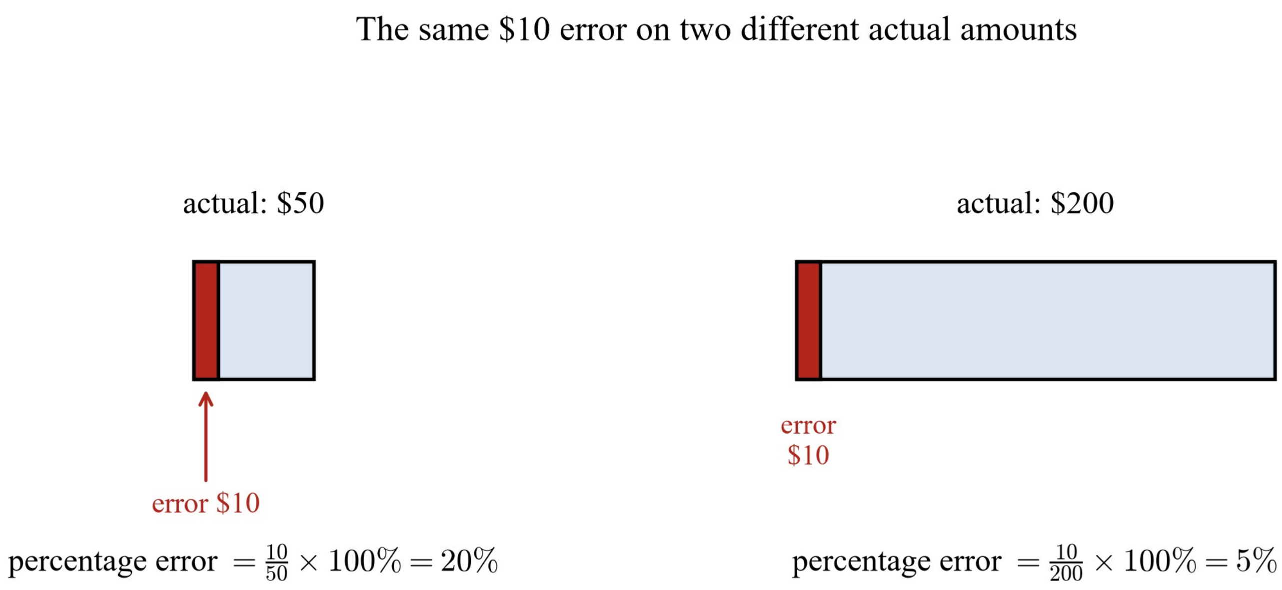
$$\text{percentage error} = \frac{\text{error}}{\text{actual value}} \times 100\,\% = \frac{10}{250} \times 100\,\% = 4\,\%$$

The error is always compared with the **actual** value — the actual value goes under the fraction. Percentage error is this book's only way of measuring the size of a mistake, and it always asks "how big is the error, compared with what is true?"




*A number line from 230 to 260 with the measured 240 mL and the actual 250 mL marked; the error of 10 compared with the actual value gives 10 over 250 times 100% equals 4%*

The same error can be a very different percentage error. A \$10 error on a \$50 item is $\frac{10}{50} \times 100\% = 20\%$; the same \$10 on a \$200 item is only $\frac{10}{200} \times 100\% = 5\%$. The smaller the actual value, the more the same error matters.




*The same \$10 error on a \$50 item is a 20% error, but on a \$200 item only a 5% error — the same error matters more against a smaller actual value*

### *Putting it together*

| **Percentage increase** | **Percentage decrease** | **Percentage error** |
|---|---|---|
| new $=$ original $+$ r% of original | new $=$ original $-$ r% of original | $\dfrac{\text{error}}{\text{actual}} \times 100\%$ |
| $80 \to +25\% \to 100$ (multiply by 1.25) | $\$60 \to -30\% \to \$42$ (multiply by 0.7) | $\dfrac{10}{250} \times 100\% = 4\%$ |

An increase multiplies by a number greater than 1; a decrease by a number less than 1.
Percentage error compares the error with the actual value.

*Summary cards: an increase adds the percentage of the original (80 to 100, multiply by 1.25); a decrease takes it away ($60 to $42, multiply by 0.7); percentage error is the error over the actual value times 100% (10 over 250 gives 4%)*

In Year 9 (`VC2M9M04`) you will use these same ideas again when you work with errors in measurement. This subtopic keeps to percentages of quantities, increases, decreases and percentage error.

### *Words of this subtopic*

| Word | What it means here |
|---|---|
| **percentage increase** | increasing an amount by a given percentage of itself; 80 increased by 25% is 100 |
| **percentage decrease** | decreasing an amount by a given percentage of itself; $60 decreased by 30% is $42 |
| **multiplier** | the one number you multiply by to do the whole change; +25% is ×1.25, −25% is ×0.75 |
| **GST** | the goods and services tax, 10% in Australia; adding GST is a 10% increase |
| **percentage error** | the error compared with the actual value, as a percentage; error ÷ actual value × 100% |
| **actual value** | the true amount, which a measurement or estimate is checked against |

## Worked examples

### *Example 1 — with help*

Increase 80 by 25%, two ways.

| Working | Comment |
|---|---|
| Method 1: $25\% \text{ of } 80 = 20$ | 25% is one quarter; a quarter of 80 is 20. |
| $80 + 20 = 100$ | Add the increase to the starting amount. |
| Method 2: $80 \times 1.25 = 100$ | A 25% increase multiplies by 1.25. |

∴ 80 increased by 25% is 100. Both methods agree — use one, and check with the other.

### *Example 2 — with help*

Decrease \$60 by 30%.

| Working | Comment |
|---|---|
| $10\%\text{ of }60 = 6$ | The 10% building block: one tenth. |
| $30\%\text{ of }60 = 18$ | Three lots of 10%. |
| $60 - 18 = 42$ | Take the decrease away. |
| Check: $60 \times 0.7 = 42$ | A 30% decrease multiplies by 0.7. |

∴ \$60 decreased by 30% is \$42.

### *Example 3 — with help*

Increase 240 by 15%, without a calculator.

| Working | Comment |
|---|---|
| $10\%\text{ of }240 = 24$ | Divide by 10. |
| $5\%\text{ of }240 = 12$ | Half of the 10%. |
| $15\%\text{ of }240 = 24 + 12 = 36$ | Add the two pieces. |
| $240 + 36 = 276$ | Add the increase to the start. |

∴ 240 increased by 15% is 276. Building from 10% and 5% works for any percentage, by hand.

### *Example 4 — on your own*

(a) Add GST to a price of \$85.

(b) A price with GST is \$66. Find the GST inside it, and the price before GST.

(c) Adding GST is a 10% increase: $85 \times 1.10 = 93.50$.

(d) The price with GST is 11 equal parts, so divide by 11: $66 \div 11 = 6$. The GST is \$6, and the price before GST is \$66 − \$6 = 60.

∴ (a) \$93.50; (b) \$6 of GST, and \$60 before GST. Check (b): 10% of 60 is 6, and $60 + 6 = 66$. ✓

### *Example 5 — on your own*

(a) A bottle is labelled 250 mL but holds 240 mL. Find the percentage error.

(b) The same 10 mL error is made on an actual amount of 500 mL. Find the percentage error now, and compare.

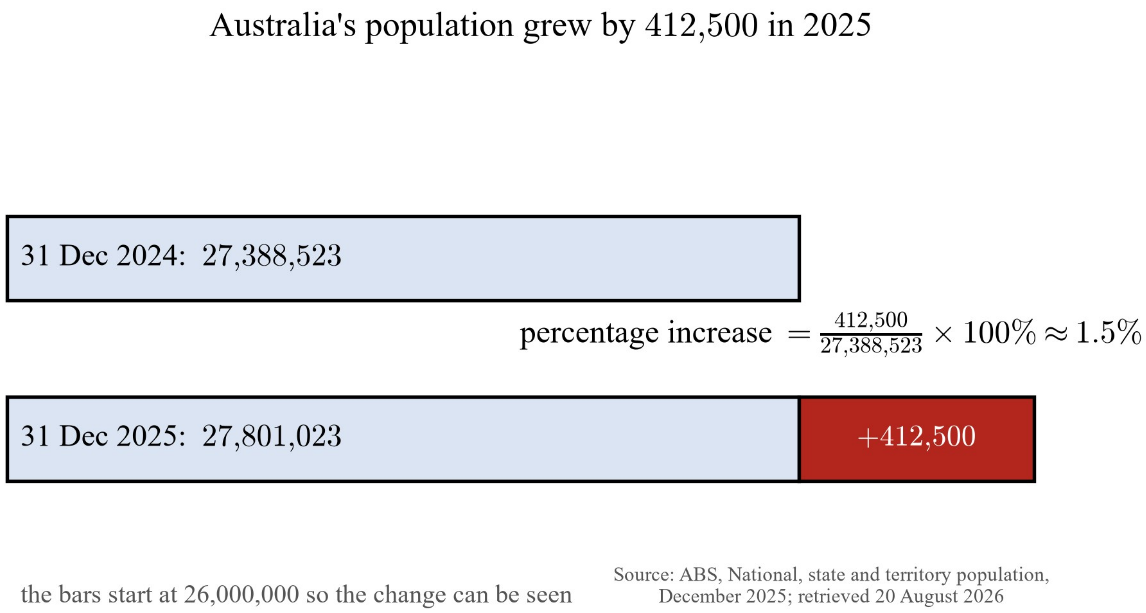
(c) The error is 10 mL: $250 - 240 = 10$. Compare it with the actual value: $\frac{10}{250} \times 100\% = 4\%$.

(d) $\frac{10}{500} \times 100\% = 2\%$.

∴ (a) 4%; (b) 2%. The same 10 mL error is a smaller percentage error against a larger actual amount.

### *Example 6 — on your own, with a calculator*

Australia's population was 27,388,523 on 31 December 2024 and 27,801,023 on 31 December 2025. Find the percentage increase, to one decimal place.




*Two bars: Australia's population of 27,388,523 at 31 December 2024, and the same bar plus 412,500 people to reach 27,801,023 at 31 December 2025, an increase of about 1.5%*

The increase is $27{,}801{,}023 - 27{,}388{,}523 = 412{,}500$ people. Compare it with the starting population:

$$\frac{412{,}500}{27{,}388{,}523} \times 100\% \approx 1.5\%$$

∴ the population increased by about 1.5%. Check: 1% of the starting population is about 274,000, so 1.5% is about 411,000 — the right size for 412,500.

*Source: Australian Bureau of Statistics, "National, state and territory population", December 2025, released 18 June 2026; retrieved 20 August 2026.*

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## Practice questions

*Most of these are by hand — the numbers are chosen so the 10% building block and the multipliers do the work. Where a calculator is allowed, the question says so, and you still check the answer makes sense.*

### *With help*

**Q1.** Find each amount without a calculator, using 10% and 5%.

a. 10% of 70
b. 30% of 70
c. 5% of 70
d. 25% of 70

**Q2.** Increase by the given percentage.

a. 60 by 10%
b. 80 by 25%
c. 175 by 20%

**Q3.** Decrease by the given percentage.

a. 90 by 10%
b. 60 by 30%
c. 400 by 25%

**Q4.** Write the multiplier for each change, then use it.

a. the multiplier for an increase of 25%
b. the multiplier for a decrease of 20%
c. 80 increased by 25%, using your multiplier from (a)
d. 75 decreased by 20%, using your multiplier from (b)

**Q5.** Add 10% GST.

a. the GST on $50
b. the price with GST of an item that is $30 before GST
c. the GST on $20

**Q6.**

a. A price with GST is $33. Find the GST inside it.
b. A 50 mL measure is read as 47 mL. Find the percentage error.

**Q7.** True or false? Give your reason, and correct each false statement.

a. Increasing by 10% is the same as multiplying by 1.1.
b. Decreasing by 20% is the same as multiplying by 0.2.
c. An increase of 50% followed by a decrease of 50% returns to the start.
d. The GST inside $33 is 10% of $33, which is $3.30.

**Q8.** Find each amount without a calculator.

a. 20% of 100
b. 25% of 100
c. 10% of 800
d. 5% of 100

***On your own***

**Q9.**

a. Decrease 100 by 10%.
b. Increase 240 by 15%.
c. Decrease 120 by 10%.

**Q10.** You may use a calculator.

a. 100 increased by 12%
b. 84 decreased by 15%
c. 300 increased by 10%

**Q11.** A jacket was $80. The store takes 10% off the price. What is the new price?

**Q12.** A town has 8,500 people. The population grows by 10%. How many people are there now?

**Q13.** Find the percentage error.

a. measured 95, actual 100
b. measured 204, actual 200

**Q14.** Ash measures a 100 cm length as 110 cm. Binh measures a 200 cm length as 230 cm. Find each percentage error. Whose measurement is closer, in percentage terms?

**Q15.** A game's price with GST is $77. The shop offers 10% off the price with GST. What is the sale price?

**Q16.** A school has 300 students.

a. 18% of the students walk to school. How many students walk?
b. Next year the school grows by 2%. How many students will there be?

### *Maths in real life*

**Q17.** Use a calculator. Australia's population was 27,388,523 on 31 December 2024 and 27,801,023 on 31 December 2025.

a. How many people were added during the year?
b. Show that the increase is about 1.5% of the starting population.

*Source: Australian Bureau of Statistics, "National, state and territory population", December 2025, released 18 June 2026; retrieved 20 August 2026.*

**Q18.** Use a calculator. Victoria's population was 7,004,600 on 31 December 2024 and 7,121,900 on 31 December 2025.

a. How many people were added during the year?
b. Show that the increase is about 1.7% of the starting population.
c. In percentage terms, which grew faster over the year — Australia or Victoria?

*Source: Australian Bureau of Statistics, "National, state and territory population", December 2025, released 18 June 2026; retrieved 20 August 2026.*

**Q19.** A price with GST is $132. Find the GST inside it, and the price before GST.

**Q20.** Make your own. Choose an actual amount and a measured amount, and ask a partner to find the percentage error. Then swap, solve each other's question, and check that you agree.

## Answers

**Q1.** a. 7. b. 21. c. 3.5. d. 17.5.

**Q2.** a. 66. b. 100. c. 210.

**Q3.** a. 81. b. 42. c. 300.

**Q4.** a. 1.25. b. 0.8. c. 100. d. 60.

**Q5.** a. \$5. b. \$33. c. \$2.

**Q6.** a. \$3. b. 6%.

**Q7.** a. True. b. False; it multiplies by 0.8. c. False; 100 → 150 → 75. d. False; the GST inside \$33 is \$3.

**Q8.** a. 20. b. 25. c. 80. d. 5.

**Q9.** a. 90. b. 276. c. 108.

**Q10.** a. 112. b. 71.4. c. 330.

**Q11.** \$72.

**Q12.** 9,350 people.

**Q13.** a. 5%. b. 2%.

**Q14.** Ash: 10%; Binh: 15%. Ash's measurement is closer, in percentage terms.

**Q15.** \$69.30.

**Q16.** a. 54 students. b. 306 students.

**Q17.** a. 412,500 people. b. 412,500 ÷ 27,388,523 × 100% ≈ 1.5%.

**Q18.** a. 117,300 people. b. 117,300 ÷ 7,004,600 × 100% ≈ 1.7%. c. Victoria.

**Q19.** \$12 of GST; \$120 before GST.

**Q20.** Answers will vary. Check: the error is compared with the actual value, and the percentage is right.

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## Fully-worked solutions

**Q1.** Use 10% of 70 = 7, and 5% of 70 = 3.5.

a. 10% of 70 = 7.
b. 30% = 3 × 10%: $3 \times 7 = 21$.
c. 5% of 70 = 3.5.
d. 25% = 10% + 10% + 5%: $7 + 7 + 3.5 = 17.5$.

**Q2.** Find the increase, then add it.

a. 10% of 60 = 6; $60 + 6 = 66$.
b. 25% of 80 = 20; $80 + 20 = 100$.
c. 20% of 175 = 35; $175 + 35 = 210$.

**Q3.** Find the decrease, then take it away.

a. 10% of 90 = 9; $90 - 9 = 81$.
b. 30% of 60 = 18; $60 - 18 = 42$.
c. 25% of 400 = 100; $400 - 100 = 300$.

**Q4.** An increase of r% multiplies by $1+\frac{r}{100}$; a decrease by $1-\frac{r}{100}$.

a. $1+0.25=1.25$.
b. $1-0.2=0.8$.
c. $80\times1.25=100$.
d. $75\times0.8=60$.

**Q5.** GST is 10%, so the GST is one tenth and the price with GST is 11 tenths.

a. 10% of $50 = $5.
b. $30 + 3 = $33.
c. 10% of $20 = $2.

**Q6.**

a. The price with GST is 11 parts: $33\div11=3$. The GST is $3.
b. The error is 3 mL, since $50-47=3$. Percentage error $=\frac{3}{50}\times100\%=6\%$.

**Q7.**

a. **True.** Keeping 100% and adding 10% gives 110%, which is ×1.1.
b. **False.** A 20% decrease keeps 80%, so it multiplies by 0.8, not 0.2.
c. **False.** $100\times1.5=150$, then $150\times0.5=75$; the decrease is taken of the larger amount.
d. **False.** The GST is 10% of the price *before* GST: $33\div11=3$, so the GST is $3, not $3.30.

**Q8.**

a. 20% of 100 = 20.
b. 25% of 100 = 25.
c. 10% of 800 = 80.
d. 5% of 100 = 5.

**Q9.**

a. 10% of 100 = 10; $100-10=90$.
b. 10% of 240 = 24 and 5% = 12, so 15% = 36; $240+36=276$.
c. 10% of 120 = 12; $120-12=108$.

**Q10.** One multiplication each.

a. $100\times1.12=112$.
b. $84\times0.85=71.4$.
c. $300\times1.1=330$.

**Q11.** 10% of 80 = 8; $80-8=72$. The new price is $72. (Check: $80\times0.9=72$.)

**Q12.** 10% of 8,500 = 850; $8{,}500+850=9{,}350$ people.

**Q13.**

a. Error $=100-95=5$; $\frac{5}{100}\times100\%=5\%$.
b. Error $=204-200=4$; $\frac{4}{200}\times100\%=2\%$.

**Q14.** Ash: the error is 10 cm ($110-100=10$); $\frac{10}{100}\times 100\%=10\%$. Binh: the error is 30 cm ($230-200=30$); $\frac{30}{200}\times 100\%=15\%$. Ash's error is the smaller percentage, so Ash's measurement is closer, in percentage terms.

**Q15.** 10% of 77 = 7.70; $77-7.70=69.30$. The sale price is \$69.30. (Check: $77\times 0.9=69.30$.)

**Q16.**

a. 1% of 300 = 3, so 10% = 30, 5% = 15 and 3% = 9; 18% = $30+15+9=54$ students.
b. 2% of 300 = 6; $300+6=306$ students.

**Q17.**

a. $27{,}801{,}023-27{,}388{,}523=412{,}500$ people.
b. $\frac{412{,}500}{27{,}388{,}523}\times 100\%\approx 1.506\%$, which is about 1.5%.

**Q18.**

a. $7{,}121{,}900-7{,}004{,}600=117{,}300$ people.
b. $\frac{117{,}300}{7{,}004{,}600}\times 100\%\approx 1.675\%$, which is about 1.7%.
c. 1.7% > 1.5%, so Victoria grew faster, in percentage terms.

**Q19.** The price with GST is 11 parts: $132\div 11=12$. The GST is \$12, and the price before GST is \$132 − \$12 = \$120. (Check: 10% of 120 is 12, and $120+12=132$.)

**Q20.** Answers will vary. A sample: actual 80, measured 76. Error $=80-76=4$; $\frac{4}{80}\times 100\%=5\%$. Check your partner's question the same way: the error first, then compare it with the actual value.

## Modelling with rational numbers and percentages: profit, loss, GST and income tax

**Content description `VC2M8N06`:** use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts involving profit and loss; formulate problems, choosing efficient mental and written calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the context, reviewing the appropriateness of the model

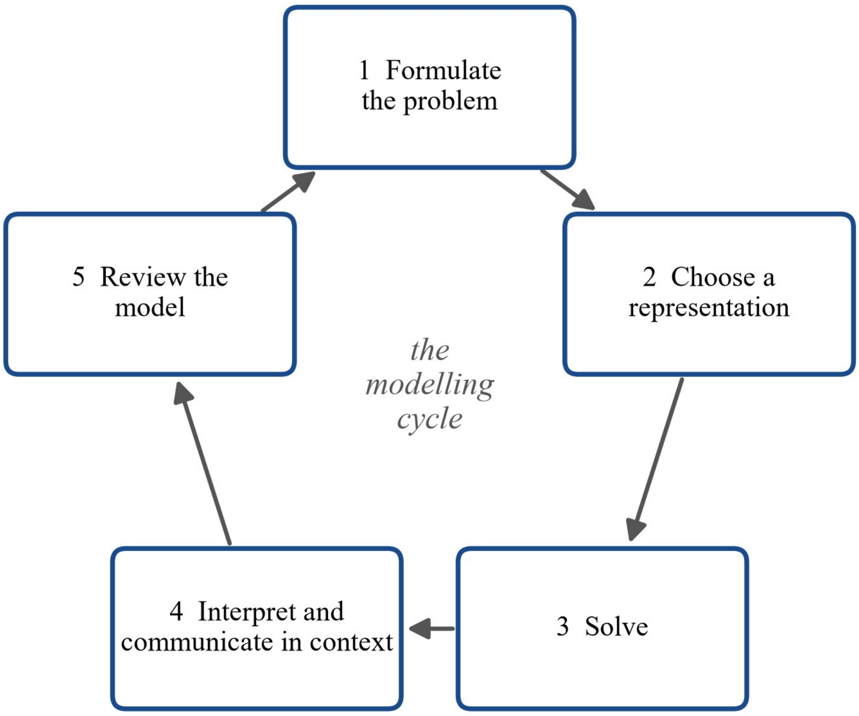
*A market stall buys a crate of oranges for one price and sells the oranges for another — the gap between the two prices is a profit or a loss, and it is best measured as a percentage. A part-time worker wonders what share of a year's pay goes to income tax. A weather report says the temperature rose from below zero to above it. Percentages, and positive and negative rational numbers, are the tools all three situations share. This subtopic puts the modelling cycle to work on profit, loss, GST and income tax — and it ends every model with the question a good modeller always asks: was it appropriate?*

### Theory

In Year 7 you found percentages of quantities (`VC2M7N07`), solved problems with ratios (`VC2M7N09`), used a modelling cycle for practical problems (`VC2M7N10`) and checked your answers for reasonableness (`VC2M7N04`); this subtopic uses all four without re-teaching them, and applies them to money and to positive and negative rational numbers.

#### *The modelling cycle*

Every problem in this subtopic travels the same five-step loop.




*The modelling cycle as a loop of five boxes joined by arrows: 1 formulate the problem, 2 choose a representation, 3 solve, 4 interpret and communicate in context, 5 review the model, with the arrow from review returning to formulate*

**1 Formulate the problem.** Turn the situation into mathematics: decide what is known, what is asked, and which quantities matter — a cost price and a selling price, a price before GST, a taxable income, a starting temperature and a change.

**2 Choose a representation.** A bar model, a number line, a table or a graph may make the structure visible. The content description allows digital tools where appropriate; a spreadsheet, for example, is one natural representation for a table of incomes and tax. Every calculation in this subtopic is also shown by hand.

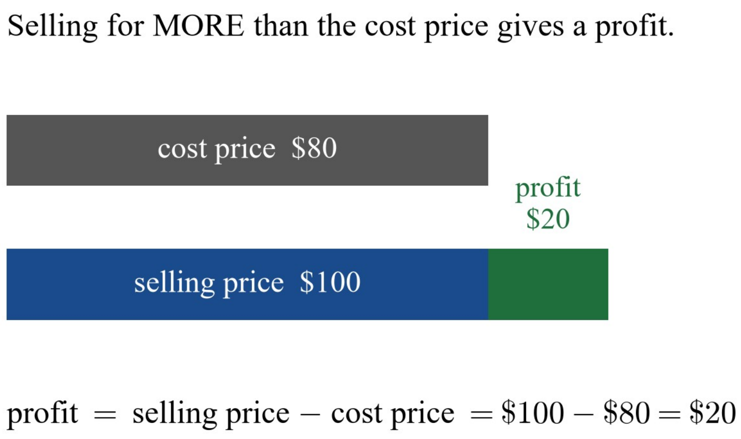
**3 Solve.** Choose efficient mental or written strategies: a percentage of a quantity, a percentage increase or decrease, or addition and subtraction of positive and negative rational numbers.

**4 Interpret and communicate in context.** The answer is not a bare number. “\$20” becomes “the stall makes a profit of \$20 on each crate”, and “14.2%” becomes “about 14.2c of every dollar of income goes to tax”.

**5 Review the model.** Ask whether the model was appropriate: were the rounding and the units right for the context, and what did the model leave out? A profit model that ignores the cost of the stall, or a tax model that ignores the Medicare levy, is still useful — as long as we say so.

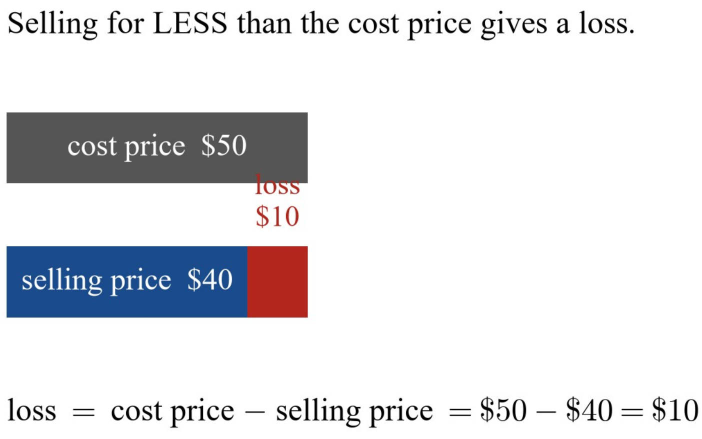
### *Profit and loss*

A seller pays the **cost price** for an item and receives the **selling price** for it. Selling for more than the cost price gives a **profit**; selling for less gives a **loss**.




*Bar model of a profit: a cost price bar of \$80, and below it a selling price bar of \$100 made of the same \$80 plus a green profit part of \$20; below, profit = selling price − cost price = \$100 − \$80 = \$20*

**Profit.** profit = selling price − cost price.




*Bar model of a loss: a cost price bar of \$50, and below it a shorter selling price bar of \$40, with a red loss part of \$10 making up the gap to the cost price; below, loss = cost price − selling price = \$50 − \$40 = \$10*

**Loss.** loss = cost price − selling price.

Profit and loss are usually reported as a **percentage of the cost price** — the seller wants to know what the trade earned relative to what it cost. In the first figure the profit is \$100 − \$80 = \$20, and \$20 ÷ \$80 = 0.25, so the profit is 25% of the cost price. In the second the loss is \$50 − \$40 = \$10, and \$10 ÷ \$50 = 0.2, so the loss is 20% of the cost price.

### *GST as a 10% increase*

Australia’s **goods and services tax (GST)** is 10%. The GST is added to the price before GST, so putting GST on a price is a **percentage increase** of 10%.

GST adds 10% to the price — a percentage increase.


$$\$50 + 10\% \text{ of } \$50 = \$50 + \$5 = \$55$$

*Bar model of GST as a 10% increase: a $50 price bar with a red GST part of $5 added on the end, and below a single bar for the price with GST, $55; the working $50 + 10% of $50 = $50 + $5 = $55*

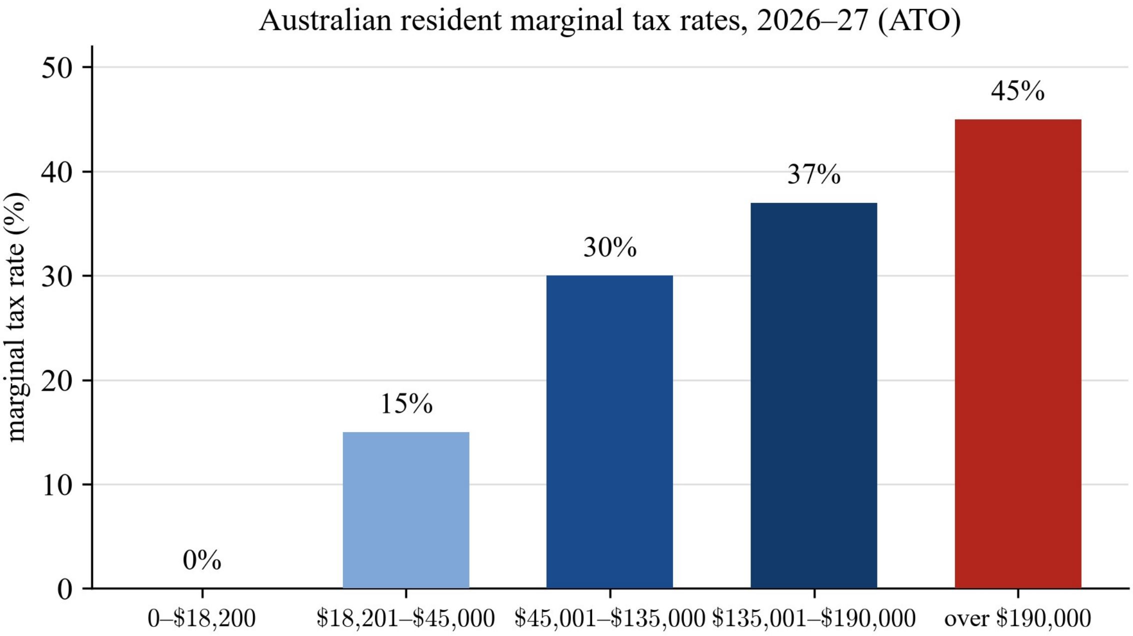
For a price of $50 before GST: the GST is 10% of $50 = $5, and the price with GST is $50 + $5 = $55. Subtopic 1E owns these calculations, including how to extract the GST part from a price that already includes it; here the GST appears inside modelling problems, and 1E’s methods may be used without re-teaching.

### *Income tax as a model*

Australian residents pay income tax on their **taxable income** — the amount of income the tax table applies to. The table works in steps called **brackets**: income in each bracket is taxed at that bracket’s **marginal rate**, the rate paid on each extra dollar earned inside the bracket. Income up to the **tax-free threshold** pays no tax at all. The rates change from one financial year to the next, so a financial year must always be named. The table below is the real table for **2026–27**.

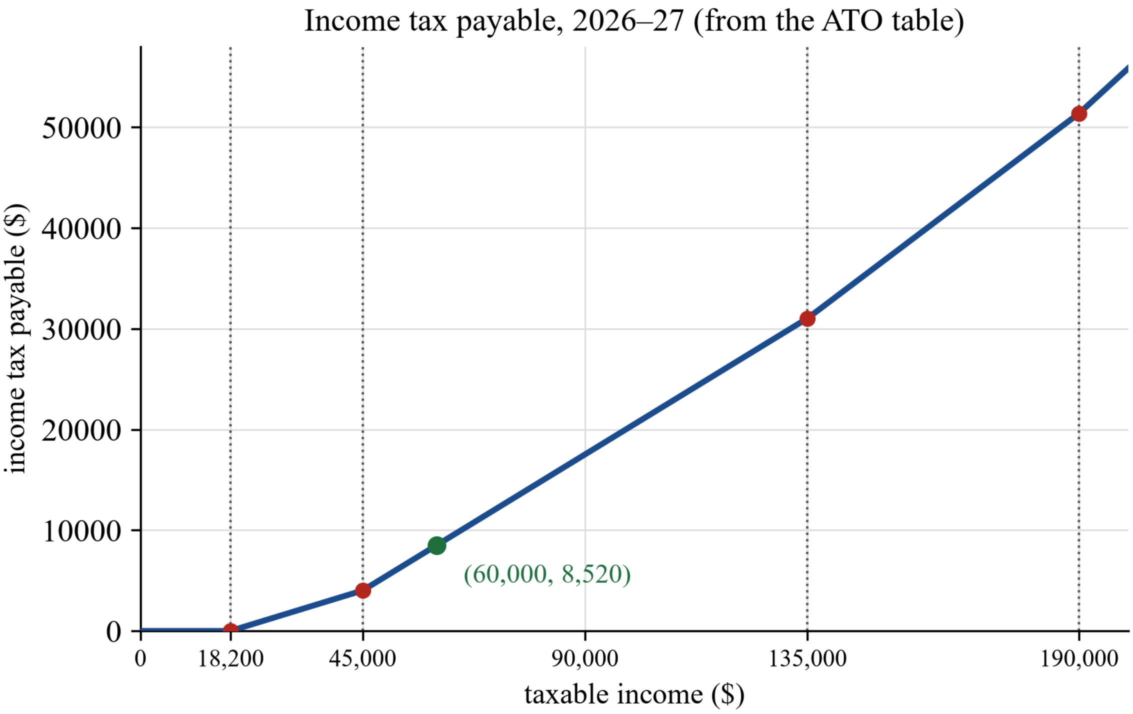
| Taxable income | Income tax payable |
|---|---|
| 0 – $18,200 | Nil |
| $18,201 – $45,000 | 15c for each $1 over $18,200 |
| $45,001 – $135,000 | $4,020 plus 30c for each $1 over $45,000 |
| $135,001 – $190,000 | $31,020 plus 37c for each $1 over $135,000 |
| $190,001 and over | $51,370 plus 45c for each $1 over $190,000 |

*Source: Australian Taxation Office, “Tax rates for Australian residents”, last updated 13 August 2026; retrieved 20 August 2026.*




*Bar chart of the Australian resident marginal tax rates for 2026–27 from the ATO: 0% up to $18,200, then 15%, 30%, 37% and 45% in the brackets $18,201–$45,000, $45,001–$135,000, $135,001–$190,000 and over $190,000*

Read the table on a taxable income of $60,000. The first $18,200 pays Nil. The next $26,800 (up to $45,000) pays 15c per dollar: $26,800 × 0.15 = $4,020. The remaining $15,000 (from $45,000 to $60,000) pays 30c per dollar: $15,000 × 0.30 = $4,500. The tax payable is $4,020 + $4,500 = $8,520.



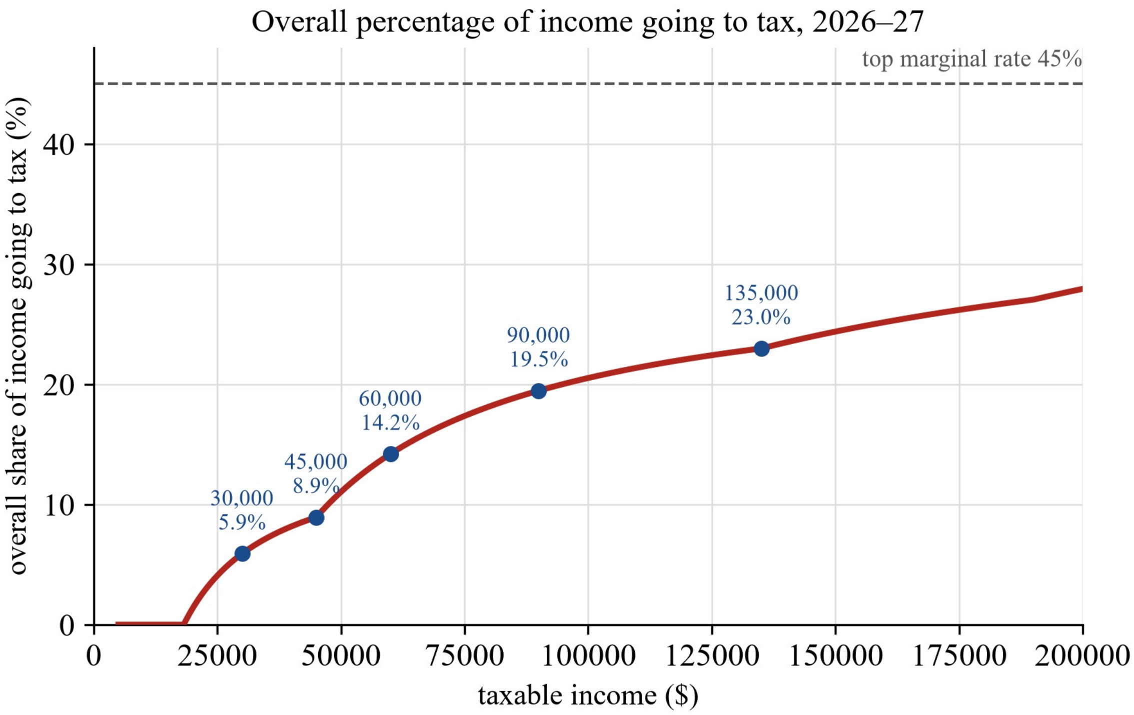

*Graph of income tax payable against taxable income for 2026–27: flat at $0 out to the threshold $18,200, then straight pieces that get steeper at each bracket boundary, with the worked point (60,000, 8,520) marked*

The graph is the model: flat to the threshold, then a straight piece per bracket, steeper each time the marginal rate rises. In Year 9 you will meet graphs built from straight-line pieces like this when you model linear relationships (`VC2M9A06`).

The question E03 asks is then a percentage one: what **overall** share of an income goes to tax? The overall tax rate is tax payable ÷ taxable income, written as a percentage. Working across incomes with the 2026–27 table:

| Taxable income | Income tax payable | Overall share going to tax |
|---|---|---|
| $18,200 | $0 | 0% |
| $30,000 | $1,770 | 5.9% |
| $45,000 | $4,020 | 8.9% |
| $60,000 | $8,520 | 14.2% |
| $90,000 | $17,520 | 19.5% |
| $135,000 | $31,020 | 23.0% |

(For example $1,770 ÷ $30,000 = 0.059 = 5.9%, and $8,520 ÷ $60,000 = 0.142 = 14.2%; the others are rounded to one decimal place.)



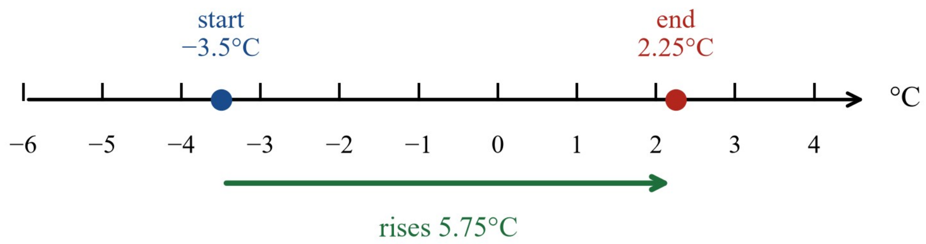

*Graph of the overall percentage of income going to tax against taxable income for 2026–27: a rising curve through the marked points 30,000 at 5.9%, 45,000 at 8.9%, 60,000 at 14.2%, 90,000 at 19.5% and 135,000 at 23.0%, staying below the dashed line at the top marginal rate of 45%*

The overall share rises as income rises — someone on $135,000 sends 23.0% of their income to tax, while someone on $30,000 sends 5.9% — but it always stays below the highest marginal rate reached, because the earlier dollars were taxed more lightly or not taxed at all. That comparison across incomes is the income-tax model of this subtopic.

### *The same cycle in other contexts*

The cycle is not only for money. A temperature of −3.5°C that **rises** 5.75°C is modelled by $-3.5+5.75=2.25$: the new temperature is 2.25°C, above zero. Positive and negative rational numbers carry the context, and the number line makes the move visible.

$$-3.5 + 5.75 = 2.25$$




*Number line in degrees Celsius from −6 to 4: a blue dot at the start −3.5°C, a red dot at the end 2.25°C, and a green arrow underneath labelled rises 5.75°C; above, −3.5 + 5.75 = 2.25*

Percentages can also run over extended time periods, with the change applied again to each new value. A \$64 price that increases by 25% per year rises to \$64 + \$16 = \$80 after one year, and to \$80 + \$20 = \$100 after two years. Two 25% increases give an overall increase of \$36 on \$64, which is 56.25% — not 50%, because the second increase is taken of a larger amount. Reviewing a multi-year percentage model means checking exactly this: what was each percentage taken of?

### *Reviewing the model*

The review step asks three things, in any context.

**Units and rounding.** Money answers are given to the cent, and the overall tax rates in this subtopic are rounded to one decimal place. A temperature answer keeps its °C.

**What the model leaves out.** The 2026–27 tax model leaves out the Medicare levy — a separate 2% charge on most taxable income — and any rebates or offsets; it also applies only to the named financial year, because the table changes. The GST model takes 10% of the price before GST, never of the price with GST. Naming what a model leaves out is part of communicating the solution.

**Reasonableness.** Does the answer fit the context? An overall tax rate must sit between 0% and the top marginal rate; a selling price below the cost price must give a loss; a temperature that rises from below zero may end up on either side of zero. This is the Year 7 reasonableness habit, now applied to models.

### *Words of this subtopic*

| Word | What it means here |
|---|---|
| **cost price** | what a seller pays for an item |
| **selling price** | what a buyer pays the seller for it |
| **profit** | selling price − cost price, when the selling price is higher |
| **loss** | cost price − selling price, when the selling price is lower |
| **GST** | the goods and services tax; 10% in Australia, added to most prices |
| **taxable income** | the amount of income the tax table applies to |
| **tax-free threshold** | income on which no tax is paid; \$18,200 in 2026–27 |
| **bracket** | one step of the tax table, with its own marginal rate |
| **marginal rate** | the rate paid on each extra dollar earned inside a bracket |
| **overall tax rate** | tax payable ÷ taxable income, written as a percentage |
| **review** | the cycle step that asks whether the model was appropriate |

## Worked examples

### *Example 1 — with help*

A sports shop buys a jacket for $80 and sells it for $100. Find the profit, and write it as a percentage of the cost price.

| Working | Comment |
|---|---|
| profit = $100 − $80 = $20 | Selling price − cost price; selling for more gives a profit. |
| $20 ÷ $80 = 0.25 = 25% | The profit as a percentage of the **cost** price. |

∴ the profit is $20, which is 25% of the cost price. Communicated: the shop earns a quarter of what each jacket costs, on every jacket.

### *Example 2 — with help*

A stall buys a toy for $50 and sells it for $40 in a clearance sale. Find the loss, and write it as a percentage of the cost price.

| Working | Comment |
|---|---|
| loss = $50 − $40 = $10 | Cost price − selling price; selling for less gives a loss. |
| $10 ÷ $50 = 0.2 = 20% | The loss as a percentage of the **cost** price. |

∴ the loss is $10, which is 20% of the cost price. The sale clears the stock but gives back only 80% of what the stall paid.

### *Example 3 — with help*

A repair costs $50 before GST. Model the GST as a 10% increase to find the GST and the price with GST.

| Working | Comment |
|---|---|
| GST = 10% of $50 = $5 | A percentage of a quantity (Year 7 skill). |
| price with GST = $50 + $5 = $55 | A 10% increase; 1E's methods apply. |

∴ the GST is $5 and the price with GST is $55. Communicated: the customer pays $55, of which $5 goes to the tax office.

### *Example 4 — on your own*

At 6 am the temperature in a town is −3.5°C. By 2 pm it has risen 5.75°C. Model the change to find the 2 pm temperature.

A rise adds to the starting temperature, and the numbers are signed rationals: $-3.5+5.75=2.25$.

∴ the 2 pm temperature is 2.25°C. Interpreted: the town started below zero and ended above it — the rise crossed zero, which the number line figure in the theory shows clearly.

### *Example 5 — on your own*

A worker's taxable income in 2026–27 is $30,000. Use the ATO table to find the income tax payable, and the overall share of the income going to tax.

The income sits in the second bracket. The first $18,200 pays Nil; the amount over $18,200 is $30,000 − $18,200 = $11,800, taxed at 15c per dollar: $11,800 × 0.15 = $1,770. The overall share is $1,770 ÷ $30,000 = 0.059 = 5.9%.

∴ the tax payable is $1,770, and 5.9% of the income goes to tax. Only the dollars above the threshold are taxed, so the overall share is well below the 15% marginal rate.

### *Example 6 — on your own*

A part-time worker's taxable income in 2026–27 is \$60,000. Run the full modelling cycle: formulate, solve, interpret and communicate, and review.

**Formulate.** The question asks for the tax payable on \$60,000 under the 2026–27 table, and for the overall share of the income going to tax.

**Choose a representation.** The table splits \$60,000 into three pieces: \$18,200 at Nil, \$26,800 at 15c, and \$15,000 at 30c. (The tax-curve figure in the theory marks this point.)

**Solve.** \$26,800 × 0.15 = \$4,020 and \$15,000 × 0.30 = \$4,500, so the tax payable is \$0 + \$4,020 + \$4,500 = \$8,520. The overall share is \$8,520 ÷ \$60,000 = 0.142 = 14.2%.

**Interpret and communicate.** About 14.2c of every dollar of the worker's income goes to income tax in 2026–27.

**Review.** The model uses the named financial year's table and leaves out the Medicare levy and any offsets, so the true amount taken would be a little higher; for comparing incomes, the model is appropriate.

∴ the tax payable is \$8,520, and the overall share of the income going to tax is 14.2%.

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## Practice questions

*Work these questions by hand. The content description allows digital tools where appropriate — a spreadsheet is one good representation for the tax table — but every number here is chosen so the percentages stay friendly on paper.*

### *With help*

**Q1.** A shop buys a jacket for \$60 and sells it for \$75.

a. Find the profit.
b. Write the profit as a percentage of the cost price.

**Q2.** A stall buys a game for \$40 and sells it for \$34.

a. Find the loss.
b. Write the loss as a percentage of the cost price.

**Q3.** Model the GST as a 10% increase. For each price before GST, find the GST and the price with GST.

a. \$80
b. \$45

**Q4.** A trader buys a bicycle for \$250 and sells it for \$300. Find the profit as a percentage of the cost price.

**Q5.** True or false? Correct each false statement.

a. A profit of \$12 on a cost price of \$48 is a 25% profit.
b. Selling for less than the cost price gives a profit.
c. Adding the 10% GST to a \$200 price gives \$220.
d. A loss of \$9 on a cost price of \$90 is a 10% loss.
e. In this book, the profit percentage is taken of the selling price.

**Q6.** Model each temperature change on a number line, and find the new temperature.

a. −1.5°C, falling 2.75°C
b. 3.2°C, falling 4.95°C

**Q7.** Copy and complete the table for 2026–27 using the ATO table in the theory. Give the overall share to one decimal place.

| Taxable income | Income tax payable | Overall share going to tax |
|---|---|---|
| $25,000 | | |
| $50,000 | | |

**Q8.** A $60 price increases by 20% per year for two years.

a. Find the price after one year.
b. Find the price after two years.
c. Find the overall percentage increase from the start to the end. Is it 40%?

### *On your own*

**Q9.** A shop buys a coat for $160 and sells it for $200. Find the profit as a percentage of the cost price.

**Q10.** A stall buys a lamp for $75 and sells it for $60. Find the loss as a percentage of the cost price.

**Q11.** The GST charged on a job is $18. Find the price before GST, and the price with GST.

**Q12.** Find the new temperature.

a. −6.2°C, rising 8.9°C
b. 4.5°C, falling 7.25°C

**Q13.** A worker's taxable income in 2026–27 is $90,000. Find the income tax payable, and the overall share of the income going to tax (one decimal place).

**Q14.** Use the 2026–27 ATO table.

a. Find the tax payable on $18,200, and the overall share going to tax.
b. Find the tax payable on $135,000, and the overall share going to tax (one decimal place).
c. Explain in one sentence why the two overall shares are different.

**Q15.** A bike is worth $800 new. Its value falls by 25% of its value each year.

a. Find its value after one year.
b. Find its value after two years.
c. Find the overall percentage decrease from $800 to the two-year value.

**Q16.** A student checks a $100 price with GST: "add 10% to get $110". Then they say: "to take the GST off, take 10% off $110, which gives $99 — so something has gone wrong." Explain what has gone wrong, and name the correct 1E method for taking the GST off $110.

### *Maths in real life*

**Q17.** A market stall buys a crate of 50 oranges for $20 and sells the oranges at 60c each. All 50 sell.

a. Find the total selling price.
b. Find the profit, and write it as a percentage of the cost price.

**Q18.** A student's part-time job gives a taxable income of $12,000 in 2026–27. Find the income tax payable and the overall share going to tax, and explain the result in one sentence.

**Q19.** In a cold week, the lowest temperature at 6 am is −2.4°C and the highest at 2 pm the same day is 6.8°C. Model the rise from the low to the high.

**Q20.** Run the full modelling cycle for a casual worker with a taxable income of $52,000 in 2026–27: formulate the question, solve for the tax payable and the overall share going to tax (one decimal place), interpret and communicate the answer in one sentence, and review the model by naming one thing it leaves out.

## Answers

**Q1.** a. \$15. b. 25%.

**Q2.** a. \$6. b. 15%.

**Q3.** a. GST \$8; price with GST \$88. b. GST \$4.50; price with GST \$49.50.

**Q4.** 20%.

**Q5.** a. True. b. False; a loss. c. True. d. True. e. False; of the cost price.

**Q6.** a. −4.25°C. b. −1.75°C.

**Q7.** \$25,000: \$1,020; 4.1%. \$50,000: \$5,520; 11.0%.

**Q8.** a. \$72. b. \$86.40. c. 44%; not 40%, because the second increase is taken of a larger amount.

**Q9.** 25%.

**Q10.** 20%.

**Q11.** Before GST \$180; with GST \$198.

**Q12.** a. 2.7°C. b. −2.75°C.

**Q13.** \$17,520; 19.5%.

**Q14.** a. \$0; 0%. b. \$31,020; 23.0%. c. Only the dollars above each threshold are taxed at the higher rates, so the overall share rises with income.

**Q15.** a. \$600. b. \$450. c. 43.75%.

**Q16.** The second 10% is taken of \$110, not of \$100, so it removes \$11, not \$10. Correct method: the GST part of \$110 is \$110 ÷ 11 = \$10, so the price before GST is \$110 − \$10 = \$100.

**Q17.** a. \$30. b. \$10; 50%.

**Q18.** \$0; 0%; the income is below the \$18,200 tax-free threshold.

**Q19.** 9.2°C.

**Q20.** \$6,120; 11.8%. Interpretation and review will vary; a sample is in the solutions.

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## Fully-worked solutions

**Q1.** a. profit = selling price − cost price = \$75 − \$60 = \$15. b. \$15 ÷ \$60 = 0.25 = 25%.

∴ a. \$15; b. 25% of the cost price.

**Q2.** a. loss = cost price − selling price = \$40 − \$34 = \$6. b. \$6 ÷ \$40 = 0.15 = 15%.

∴ a. \$6; b. 15% of the cost price.

**Q3.** a. GST = 10% of \$80 = \$8; price with GST = \$80 + \$8 = \$88. b. GST = 10% of \$45 = \$4.50; price with GST = \$45 + \$4.50 = \$49.50.

**Q4.** profit = \$300 − \$250 = \$50; \$50 ÷ \$250 = 0.2 = 20%.

∴ the profit is 20% of the cost price.

**Q5.** a. **True.** \$12 ÷ \$48 = 0.25 = 25%. b. **False.** Selling for less than the cost price gives a **loss**. c. **True.** 10% of \$200 is \$20, and \$200 + \$20 = \$220. d. **True.** \$9 ÷ \$90 = 0.1 = 10%. e. **False.** In this book the profit percentage is taken of the **cost** price.

**Q6.** a. A fall subtracts: $-1.5-2.75=-4.25$. The new temperature is −4.25°C. b. $3.2-4.95=-1.75$. The fall crosses zero and ends at −1.75°C.

**Q7.** \$25,000: the amount over \$18,200 is \$6,800; \$6,800 × 0.15 = \$1,020. Overall: \$1,020 ÷ \$25,000 = 0.0408 = 4.08% ≈ 4.1%. \$50,000: the first piece gives \$4,020 (as in the theory) and the amount over \$45,000 is \$5,000; \$5,000 × 0.30 = \$1,500. Tax = \$4,020 + \$1,500 = \$5,520. Overall: \$5,520 ÷ \$50,000 = 0.1104 = 11.04% ≈ 11.0%.

**Q8.** a. 20% of \$60 is \$12; \$60 + \$12 = \$72. b. 20% of \$72 is \$14.40; \$72 + \$14.40 = \$86.40. c. The increase is \$86.40 − \$60 = \$26.40; \$26.40 ÷ \$60 = 0.44 = 44%. Not 40%: the second 20% is taken of \$72, not of \$60.

**Q9.** profit = \$200 − \$160 = \$40; \$40 ÷ \$160 = 0.25 = 25%.

∴ the profit is 25% of the cost price.

**Q10.** loss = \$75 − \$60 = \$15; \$15 ÷ \$75 = 0.2 = 20%.

∴ the loss is 20% of the cost price.

**Q11.** The GST is 10% of the price before GST, so the price before GST is \$18 ÷ 0.10 = \$180. The price with GST is \$180 + \$18 = \$198. (1E's check: \$198 ÷ 11 = \$18, the GST part.)

**Q12.** a. $-6.2+8.9=2.7$. The new temperature is 2.7°C. b. $4.5-7.25=-2.75$. The new temperature is −2.75°C.

**Q13.** The income sits in the third bracket. The tax on the first \$45,000 is \$4,020; the amount over \$45,000 is \$90,000 − \$45,000 = \$45,000, taxed at 30c per dollar: \$45,000 × 0.30 = \$13,500. Tax payable = \$4,020 + \$13,500 = \$17,520. Overall: \$17,520 ÷ \$90,000 = 0.19466… ≈ 19.5%.

∴ the tax payable is \$17,520, and about 19.5% of the income goes to tax.

**Q14.** a. \$18,200 is exactly the tax-free threshold: tax \$0, overall share 0%. b. \$135,000 is the top of the third bracket: the table gives \$31,020. Overall: \$31,020 ÷ \$135,000 = 0.22977… ≈ 23.0%. c. The higher rates apply only to the dollars earned above each threshold, so the overall share grows with income but stays below the top marginal rate reached.

**Q15.** a. The value keeps 75%: \$800 × 0.75 = \$600. (Equally: 25% of \$800 is \$200; \$800 − \$200 = \$600.) b. \$600 × 0.75 = \$450. c. The decrease is \$800 − \$450 = \$350; \$350 ÷ \$800 = 0.4375 = 43.75%.

∴ after two years the bike is worth \$450, an overall decrease of 43.75% — not 50%, for the same reason as Q8: each year's 25% is taken of a smaller value.

**Q16.** Adding 10% takes 10% of \$100, which is \$10. Taking 10% off \$110 takes 10% of \$110, which is \$11 — a percentage is always taken *of* something, and the two bases are different, so \$110 − \$11 = \$99 ≠ \$100. The correct 1E method: the GST part of a price with GST is the price ÷ 11, so \$110 ÷ 11 = \$10, and the price before GST is \$110 − \$10 = \$100.

**Q17.** a. 50 × \$0.60 = \$30. b. profit = \$30 − \$20 = \$10; \$10 ÷ \$20 = 0.5 = 50%.

∴ the stall makes a \$10 profit, which is 50% of the cost price.

**Q18.** \$12,000 is below the \$18,200 tax-free threshold, so the tax payable is \$0 and the overall share is 0%. Communicated: in 2026–27 this student's part-time income pays no income tax at all.

**Q19.** The rise is the difference: $6.8-(-2.4)=6.8+2.4=9.2$. Modelled as a change: $-2.4+9.2=6.8$.

∴ the temperature rises 9.2°C from the low to the high.

**Q20. Formulate.** Find the tax payable on \$52,000 under the 2026–27 table, and the overall share going to tax. **Solve.** The first \$45,000 pays \$4,020; the amount over \$45,000 is \$52,000 − \$45,000 = \$7,000, taxed at 30c per dollar: \$7,000 × 0.30 = \$2,100. Tax payable = \$4,020 + \$2,100 = \$6,120. Overall: \$6,120 ÷ \$52,000 = 0.11769… ≈ 11.8%. **Interpret and communicate.** About 11.8c of every dollar of the worker's income goes to income tax in 2026–27. **Review.** The model leaves out the Medicare levy and any offsets, and applies only to the named financial year — for the question asked, it is still appropriate.

∴ the tax payable is \$6,120, and the overall share going to tax is about 11.8%.

# Test

**Test T1 · Chapter 1 — Number: integers, exponents, the real number system and percentages**

**Mathematics Level 8 (Year 8) · Victorian Curriculum 2.0**

**Codes assessed:** `VC2M8N04` · `VC2M8N02` · `VC2M8N03` · `VC2M8N01` · `VC2M8N05` · `VC2M8N06`

Name: ______________________________ Date: ______________

| | |
|---|---|
| **Total marks** | 30 |
| **Time** | 35 minutes — 5 minutes reading time, then 1 minute per mark (30 minutes of working time). |
| **Nature of the paper** | This is a classroom assessment, not an exam. Attempt every question, and show your working where a question asks for it. |
| **Equipment** | A pen or pencil. No other equipment is needed. |
| **Calculators** | Question 5 must be done **without a calculator**. Its content description `VC2M8N05` requires percentage work both *with* and *without* digital tools, and Question 5 assesses the without-tools strand. A scientific calculator is permitted for Questions 1, 3 and 6, whose content descriptions (`VC2M8N04`, `VC2M8N03`, `VC2M8N06`) allow digital tools where appropriate. No calculator is needed or permitted for Questions 2 and 4: their content descriptions (`VC2M8N02`, `VC2M8N01`) name no digital tool, and every number is chosen so the working can be done by hand. |

**Question 1 — The four operations with integers and rational numbers (5 marks)**

*Show your working. A scientific calculator may be used to check your answers, not as a substitute for the working.*

(a) Find each value. (3 marks)

(b) $(-8)\times 7$

(ii) $(-72)\div(-9)$

(iii) $\left(-\frac{2}{5}\right)\times\left(-\frac{5}{6}\right)$ — give your answer in simplest form.

(b) Choose an efficient strategy and find the value of $(-20)\times 13\times(-5)$. (1 mark)

(c) Estimate first, then find the exact value: $(-5.8)\times 4.1$. (1 mark)

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**Question 2 — The exponent laws (5 marks)**

*No calculator is needed for this question. Give each answer as a single number unless the question asks for a single power.*

(a) Write each expression as a single power. (2 marks)

(b) $6^3\times 6^5$

(ii) $(2^3)^4$

(b) Simplify, then find the value: $5^7\div 5^5$. (1 mark)

(c) Find the value of $(-2)^6$. (1 mark)

(d) Find the value of $(7^5 \times 7^0) \div 7^3$. (1 mark)

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**Question 3 — Fractions as terminating and recurring decimals (5 marks)**

(a) Write $\frac{7}{12}$ as a decimal. Use the bar notation where the decimal recurs. (1 mark)

(b) Without dividing, decide whether $\frac{21}{28}$ gives a terminating or a recurring decimal. Show the prime factors you read. (2 marks)

(c) Write $0.\overline{54}$ as a fraction in simplest form. (1 mark)

(d) Which one of these fractions gives a terminating decimal? (1 mark)

A. $\frac{5}{14}$

B. $\frac{5}{15}$

C. $\frac{5}{16}$

D. $\frac{5}{18}$

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**Question 4 — Irrational numbers and the real number line (5 marks)**

*No calculator is needed for this question.*

(a) Classify each number as rational or irrational. Give a reason each time. (2 marks)

(b) $\sqrt{196}$

(ii) $\sqrt{17}$

(b) Squeeze $\sqrt{30}$ between two tenths. Show your working by squaring. (2 marks)

(c) The decimal $3.1416$ is often used as an approximation to $\pi$. Is $3.1416$ rational or irrational, and is it exactly equal to $\pi$? Explain briefly. (1 mark)

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**Question 5 — Percentage increase, percentage decrease and percentage error (5 marks)**

**No calculator for this question.** Show your working.

(a) Increase $180$ by 15%. Build the 15% from 10% and 5%. (2 marks)

(b) Decrease $140 by 25%. (1 mark)

The goods and services tax (GST) is 10% in the 2026–27 financial year.

*Source: Australian Taxation Office, "Goods and services tax (GST)"; retrieved 20 August 2026.*

(c) A game's price with GST is $99. Find the GST inside it, and the price before GST. (1 mark)

(d) A $400$ g weight is measured as $388$ g. Find the percentage error. (1 mark)

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**Question 6 — Modelling with percentages: income tax (5 marks)**

A worker's **taxable income** — the income the tax table applies to — is $75,000 in the 2026–27 financial year. Use the Australian resident tax table below. The table works in steps called **brackets**: the dollars earned inside each bracket are taxed at that bracket's rate, and only the dollars above each threshold are taxed at the higher rates.

| Taxable income | Income tax payable |
|---|---|
| 0 – $18,200 | Nil |
| $18,201 – $45,000 | 15c for each $1 over $18,200 |
| $45,001 – $135,000 | $4,020 plus 30c for each $1 over $45,000 |
| $135,001 – $190,000 | $31,020 plus 37c for each $1 over $135,000 |
| $190,001 and over | $51,370 plus 45c for each $1 over $190,000 |

*Source: Australian Taxation Office, "Tax rates for Australian residents", last updated 13 August 2026; retrieved 20 August 2026.*

(a) Find the income tax payable on $75,000. Show how you split the income across the brackets. (2 marks)

(b) Find the overall percentage of the income that goes to tax, correct to one decimal place. (1 mark)

(c) Communicate your answer in terms of the context. Complete the sentence: "In 2026–27, about ……… of every dollar of this worker's income goes to income tax." (1 mark)

(d) Review the model: name one thing this tax model leaves out. (1 mark)

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**Total: 30 marks**

## Solutions

Teacher-facing. Test T1 is a classroom assessment, not an exam: 35 minutes in total, made up of 5 minutes reading time and 1 minute per mark. Question 5 is calculator-free because `VC2M8N05` requires the percentage work with *and* without digital tools, and T1 carries calculator-free items from 1E for that reason. A scientific calculator is permitted for Questions 1, 3 and 6, and is neither needed nor permitted for Questions 2 and 4 — the calculator status of every question follows the content description behind it.

### Coverage and mark map

| CD | Subtopic | Question | Marks |
|---|---|---|---|
| `VC2M8N04` | 1A The four operations with integers and rational numbers | Q1 | 5 |
| `VC2M8N02` | 1B The exponent laws with positive integer exponents and the zero exponent | Q2 | 5 |
| `VC2M8N03` | 1C Fractions as terminating and recurring decimals | Q3 | 5 |
| `VC2M8N01` | 1D Irrational numbers and the real number line | Q4 | 5 |
| `VC2M8N05` | 1E Percentage increase, percentage decrease and percentage error | Q5 | 5 |
| `VC2M8N06` | 1F Modelling with rational numbers and percentages: profit, loss, GST and income tax | Q6 | 5 |
| | **Total** | | **30** |

Every subtopic is worth exactly 5 marks, per the §7 marking rule. Multiple choice is 1 mark of 30 — Q3(d), four options A–D — used where the judgement genuinely is a selection: picking the terminating decimal from four candidates.

### Answer key

| Q | Part | Answer | Marks |
|---|---|---|---|
| 1 | (a)(i) | $-56$ | 1 |
| 1 | (a)(ii) | $8$ | 1 |
| 1 | (a)(iii) | $\frac{1}{3}$ | 1 |
| 1 | (b) | $1300$, by regrouping $(-20)\times(-5)$ first | 1 |
| 1 | (c) | estimate $-24$; exact $-23.78$ | 1 |
| 2 | (a)(i) | $6^8$ | 1 |
| 2 | (a)(ii) | $2^{12}$ | 1 |
| 2 | (b) | $5^2=25$ | 1 |
| 2 | (c) | $64$ | 1 |
| 2 | (d) | $7^2=49$ | 1 |

| Q | Part | Answer | Marks |
|---|---|---|---|
| 3 | (a) | $0.58\overline{3}$, recurring | 1 |
| 3 | (b) | $\frac{21}{28}=\frac{3}{4}$; $4=2^2$; terminating | 2 |
| 3 | (c) | $\frac{6}{11}$ | 1 |
| 3 | (d) | C ($\frac{5}{16}$) | 1 |
| 4 | (a)(i) | rational — $\sqrt{196}=14$ | 1 |
| 4 | (a)(ii) | irrational — 17 is not a perfect square | 1 |
| 4 | (b) | $5.4<\sqrt{30}<5.5$ | 2 |
| 4 | (c) | rational; no — $\pi$ is irrational | 1 |
| 5 | (a) | 207 | 2 |
| 5 | (b) | \$105 | 1 |
| 5 | (c) | GST \$9; \$90 before GST | 1 |
| 5 | (d) | 3% | 1 |
| 6 | (a) | \$13,020 | 2 |
| 6 | (b) | 17.4% | 1 |
| 6 | (c) | about 17.4c of every dollar | 1 |
| 6 | (d) | for example, the Medicare levy | 1 |

## Worked solutions

**Q1.** (a) Fix the sign first, then work the number.

(i) Different signs give a negative answer: $8\times7=56$, so $(-8)\times7=-56$.

(ii) The same signs give a positive answer: $72\div9=8$, so $(-72)\div(-9)=8$. Check: $8\times(-9)=-72$. ✓

(iii) The signs are the same, so the answer is positive: $\frac{2}{5}\times\frac{5}{6}=\frac{10}{30}=\frac{1}{3}$.

(b) Regroup with the commutative law so the friendly pair multiplies first: $(-20)\times(-5)\times13=100\times13=1300$.

(c) Estimate: $(-6)\times4=-24$. Exact: $5.8\times4.1=23.78$, and the signs differ, so the answer is $-23.78$. The exact value matches the estimate's sign and size. ✓

*Marks: (a) 1 mark for each value. (b) 1 mark for an efficient strategy with the correct value; accept any correct regrouping or partitioning. (c) 1 mark for both the estimate and the exact value.*

**Q2.** All three laws need the same base, and each law only changes the exponent, never the base.

(a)

(i) Product law — add the exponents: $6^3\times6^5=6^{3+5}=6^8$.

(ii) Power-of-a-power law — multiply the exponents: $(2^3)^4=2^{3\times4}=2^{12}$.

(b) Quotient law — subtract the exponents: $5^7\div5^5=5^{7-5}=5^2=25$.

(c) The exponent 6 is even, so the answer is positive: $(-2)^6=2\times2\times2\times2\times2\times2=64$.

(d) The zero exponent gives $7^0 = 1$, so it changes nothing: $(7^5 \times 7^0) \div 7^3 = 7^{5+0-3} = 7^2 = 49$.

*Marks: (a) 1 mark for each single power. (b), (c) and (d) 1 mark each for the correct value.*

**Q3.** (a) Divide top by bottom: $7 \div 12 = 0.58333\ldots$ The digit 3 repeats and the 8 does not, so the bar covers the 3 only: $0.58\overline{3}$. (The test agrees: $12 = 2^2 \times 3$ has a 3, so the decimal recurs.)

(b) Simplify **first**: $\frac{21}{28} = \frac{3}{4}$. The new denominator is $4 = 2^2$ — only 2s — so the decimal **terminates**. (Dividing confirms it: $\frac{3}{4} = 0.75$.) Reading the prime factors of 28 before simplifying is the trap this question tests.

(c) The repeating block has two digits, so multiply by 100 and subtract: $99 \times 0.\overline{54} = 54$, so $0.\overline{54} = \frac{54}{99} = \frac{54 \div 9}{99 \div 9} = \frac{6}{11}$.

(d) Test each denominator in simplest form. A: $14 = 2 \times 7$ — a 7 appears, recurs. B: $\frac{5}{15} = \frac{1}{3}$ — a 3 appears, recurs. C: $16 = 2^4$ — only 2s, **terminates** ($\frac{5}{16} = 0.3125$). D: $18 = 2 \times 3^2$ — a 3 appears, recurs. The answer is **C**.

*Marks: (b) 1 mark for the simplification and the prime factors read, 1 mark for the correct conclusion. (d) 1 mark for C.*

**Q4.** (a) (i) $\sqrt{196} = 14$, because $14^2 = 196$. A whole number is an integer divided by a natural number ($14 = 14 \div 1$), so $\sqrt{196}$ is **rational**.

(ii) $4^2 = 16 < 17 < 25 = 5^2$, so 17 is not a perfect square and $\sqrt{17}$ is not a whole number. Its decimal never ends and never recurs, so $\sqrt{17}$ is **irrational**.

(b) Integers first: $5^2 = 25 < 30 < 36 = 6^2$, so $5 < \sqrt{30} < 6$. Now tenths between 5 and 6: $5.4^2 = 29.16$ and $5.5^2 = 30.25$. Since $29.16 < 30 < 30.25$:

$$5.4 < \sqrt{30} < 5.5$$

(c) $3.1416 = 31416 \div 10000$ is a terminating decimal, so it is **rational**. It is **not** exactly $\pi$: $\pi$ is irrational — its decimal never ends and never recurs — while 3.1416 stops. The number 3.1416 is only an approximation, and a good one: $3.141 < \pi < 3.142$.

*Marks: (a) 1 mark for each classification with a correct reason. (b) 1 mark for the integer squeeze, 1 mark for the tenths squeeze justified by squaring. (c) 1 mark for a response that covers all three parts — the classification, the judgement and the reason.*

**Q5.** Calculators are not used in this question (`VC2M8N05` requires this work without digital tools).

(a) Divide by 10 to get 10%, then halve that to get 5%:

$$10\% \text{ of } 180 = 18, 5\% \text{ of } 180 = 9, 15\% = 18 + 9 = 27$$

Then $180 + 27 = 207$.

(b) 25% is one quarter: $140 \div 4 = 35$, and $140 - 35 = 105$. (Equally: 10% is \$14, so 20% is \$28 and 5% is \$7, giving \$35.) The new amount is \$105.

(c) The price with GST is 11 equal parts — 10 parts price plus 1 part GST — so divide by 11: $99 \div 11 = 9$. The GST is \$9, and the price before GST is \$99 − \$9 = \$90. Check: 10% of \$90 is \$9, and \$90 + \$9 = \$99. ✓

(d) The error is $400 - 388 = 12$ g. Compare it with the actual value: $\frac{12}{400} \times 100\% = 3\%$.

*Marks: (a) 1 mark for building 15% from 10% and 5%, 1 mark for the answer 207. (b) 1 mark for \$105. (c) 1 mark for both the GST (\$9) and the price before GST (\$90). (d) 1 mark for 3%.*

**Q6.** (a) \$75,000 sits in the third bracket. Split it across the brackets: the first \$18,200 pays Nil; the next \$26,800 (from \$18,200 to \$45,000) pays 15c per dollar: \$26,800 × 0.15 = \$4,020; the remaining \$30,000 (from \$45,000 to \$75,000) pays 30c per dollar: \$30,000 × 0.30 = \$9,000. The tax payable is \$4,020 + \$9,000 = \$13,020.

(b) The overall share is tax payable ÷ taxable income: \$13,020 ÷ \$75,000 = 0.1736, which is 17.36%, or **17.4%** to one decimal place.

(c) For example: "In 2026–27, about **17.4c** of every dollar of this worker's income goes to income tax." Accept any sentence that reports the 17.4% share correctly in terms of the context.

(d) Accept any one clearly stated limitation, for example: the model leaves out the Medicare levy — a separate charge on most taxable income; or it leaves out any rebates or offsets; or it applies only to the 2026–27 financial year, because the table changes; or it assumes the whole \$75,000 is taxable income.

*Marks: (a) 1 mark for a correct split across the brackets, 1 mark for \$13,020. (b) 1 mark for 17.4%. (c) 1 mark for a sentence that communicates the result in context. (d) 1 mark for one named limitation.*