

# Mathematics Level 7

## Chapter 5 — Space

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## Representing three-dimensional objects in two dimensions

**Content description VC2M7SP01:** represent three-dimensional objects in 2 dimensions; discuss and reason about the advantages and disadvantages of different representations

**Achievement standard (extract):** They represent objects two-dimensionally in different ways, describing the usefulness of these representations.

### Theory

**Three dimensions on a flat page.** Everything you can hold or walk into is a three-dimensional (3-D) object: it has length, width and height. This page is flat — two-dimensional (2-D). To **represent** a 3-D object in 2-D means to draw it flat on the page. There is more than one way to do it, and each way shows different things. This subtopic uses four ways: **nets**, **plan and side views**, **isometric drawings** and **perspective drawings**. For each one you will learn what it shows well and what it shows badly — its advantages and disadvantages.

**Faces, edges and vertices.** The flat sides of a solid are its **faces**. Where two faces meet is an **edge**, and where edges meet is a **vertex** (more than one: **vertices**). Figure 1 labels all three on a rectangular prism (also called a **cuboid**): it has 6 faces, 12 edges and 8 vertices. Recall from Level 6 (VC2M6SP01) that a **prism** is a solid with the same shape all the way through, like this box — this subtopic names prisms but does not re-teach them.

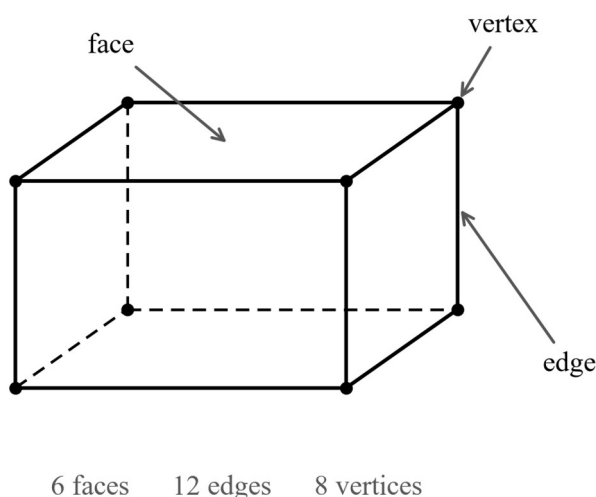


Figure 1. A rectangular prism with a face, an edge and a vertex labelled, and the counts 6 faces, 12 edges and 8 vertices written below.

**Scale.** Because the page is smaller than the object, drawings are often made to a **scale**, such as 1 square : 1 cm on squared paper. A scale can also be written as a ratio like 1:50 (ratio notation was covered in 2F, and the scaling methods themselves belong to 2F and 4F). Every drawing in this subtopic tells you its scale, and yours should too.

### Key words.

| Term              | Meaning here                                  | Related words                    |
|-------------------|-----------------------------------------------|----------------------------------|
| <b>net</b>        | a flat pattern that folds to make a solid     | packaging, cut-out               |
| <b>plan view</b>  | what you see looking straight down from above | aerial view, footprint, top view |
| <b>front view</b> | what you see looking straight at the front    | view from the front              |
| <b>side view</b>  | what you see looking straight at one side     | view from the side               |

| Term                       | Meaning here                                                                | Related words       |
|----------------------------|-----------------------------------------------------------------------------|---------------------|
| <b>isometric drawing</b>   | a 3-D picture on dot paper, with true lengths in three directions           | dot-paper drawing   |
| <b>perspective drawing</b> | a picture as the eye sees it, with a vanishing point                        | picture as it looks |
| <b>vanishing point</b>     | the point on the horizon where edges running into the distance seem to meet | horizon             |
| <b>apex</b>                | the top vertex of a pyramid                                                 | point of a pyramid  |

**Nets.** A **net** is a flat pattern that folds up to make a solid. Every piece of packaging — every box on a supermarket shelf — starts life as a net cut from flat card, so a net is the right representation when something must be *made*. Figure 2 shows one net of a cube with its faces numbered: when it folds up, faces 1 and 4, 2 and 6, and 3 and 5 end up **opposite** each other. A cube has 11 different nets altogether; Figure 2 is just one of them. The advantage of a net is that every face is shown flat at its true size and shape; the disadvantage is that you cannot see what the folded solid will look like.

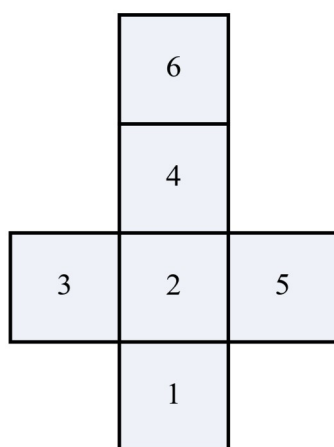


Figure 2. One net of a cube: six numbered squares in a cross, with 1, 2, 4 and 6 in the column and 3 and 5 on the arms.

**Plan and side views.** Imagine flying above a house and looking straight down: that **aerial view** shows the **footprint** of the house on the ground. Flat views like this are the second representation. The **plan view** looks straight down from above, the **front view** looks straight at the front, and the **side view** looks straight at one side. Figure 3 shows all three for a box 4 cm long, 3 cm wide and 2 cm high: the plan view is a 4 cm × 3 cm rectangle, the front view is 4 cm × 2 cm, and the side view is 3 cm × 2 cm. No single view shows the whole solid, but together the three views give every length — which is why builders and designers work from them. The advantage is that each view shows true lengths; the disadvantage is that you must look at three drawings and picture the solid yourself.

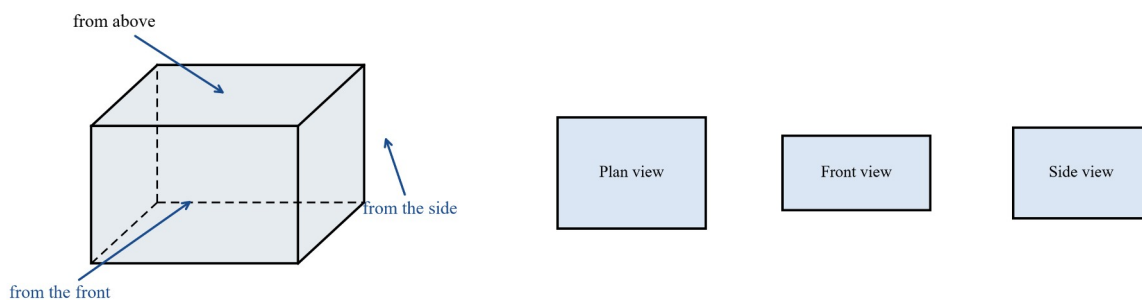


Figure 3. A box with arrows marked from above, from the front and from the side, next to its three flat views: a plan view 4 by 3, a front view 4 by 2 and a side view 3 by 2.

**Isometric drawings.** Isometric dot paper has dots in a pattern that gives three directions from any dot: two at 30 degrees to the horizontal and one straight up (Figure 4). Drawing a solid along those directions gives an **isometric drawing**: one picture of the whole solid with true lengths in all three directions, so you can measure from it. That is its advantage. Its disadvantage is that right angles do not *look* like right angles — the cube in Figure 4 has square faces, yet none of them is drawn as a square.

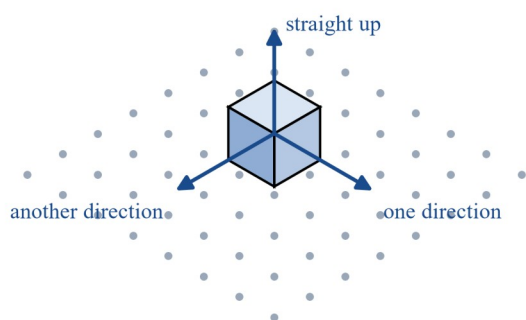


Figure 4. Isometric dot paper with three arrows from one dot — two at 30 degrees and one straight up — and one cube drawn along the dots.

**Perspective drawings.** Stand at the end of a long straight road and the two sides of the road seem to come together at one point on the **horizon**. A **perspective drawing** copies what the eye sees: edges that run into the distance are drawn heading towards one point, the **vanishing point** (Figure 5). Edges that are the same length in real life are drawn shorter as they run into the distance, so the back face of the box in Figure 5 is drawn smaller than the front face even though it is the same size. The advantage is that a perspective drawing looks the way the object really looks to the eye; the disadvantage is that lengths and angles are not true, so you cannot measure from it.

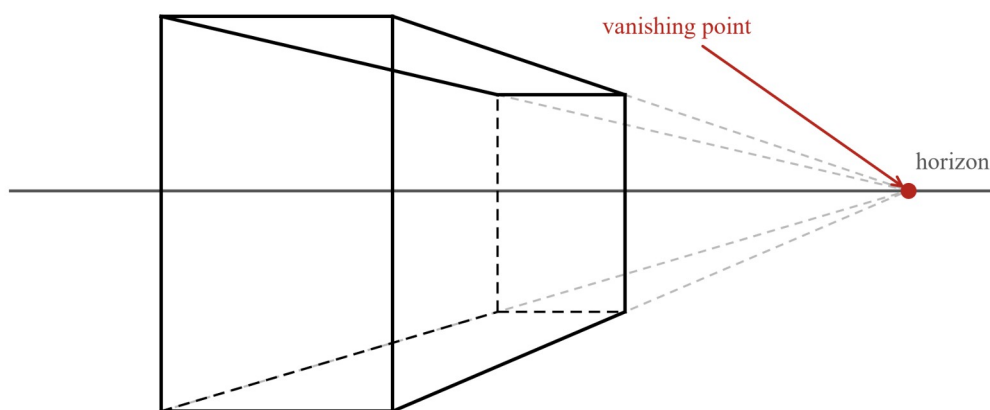


Figure 5. A box drawn in one-point perspective: the edges that run into the distance head towards a vanishing point on the horizon, and the hidden edges are dashed.

**Choosing a representation.** The four representations, side by side:

| Representation            | Advantage — good for                                           | Disadvantage — poor at                                       |
|---------------------------|----------------------------------------------------------------|--------------------------------------------------------------|
| Net                       | shows every face flat at true size;<br>folds to make the solid | does not show what the solid looks like                      |
| Plan + front + side views | true lengths in every direction;<br>enough to build from       | three separate drawings; you must picture the solid yourself |
| Isometric drawing         | the whole solid in one picture, with true lengths              | right angles do not look like right angles                   |
| Perspective drawing       | looks the way the eye sees it                                  | lengths are not true; you cannot measure from it             |

## Worked examples

### Example 1 — scaffolded

A parcel box is a cuboid 12 cm long, 8 cm wide and 5 cm high. Figure 6 shows its net at true size, with one face shaded. (a) How many faces has the box? (b) Give the sizes of the three pairs of matching faces. (c) The shaded face is  $8\text{ cm} \times 5\text{ cm}$ . Which face of the net is opposite it when the box is folded?

| Working                                                                                                                                      | Comment                                                                                                              |
|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| (a) 6 faces.                                                                                                                                 | A cuboid always has 6 faces.                                                                                         |
| (b) Two faces $12\text{ cm} \times 8\text{ cm}$ , two faces $12\text{ cm} \times 5\text{ cm}$ , two faces $8\text{ cm} \times 5\text{ cm}$ . | Matching faces of a cuboid have the same size and shape.                                                             |
| (c) The other $8\text{ cm} \times 5\text{ cm}$ face — the second face of the band of four.                                                   | In the band, the 1st and 3rd faces end up opposite, and the 2nd and 4th end up opposite; the shaded face is the 4th. |

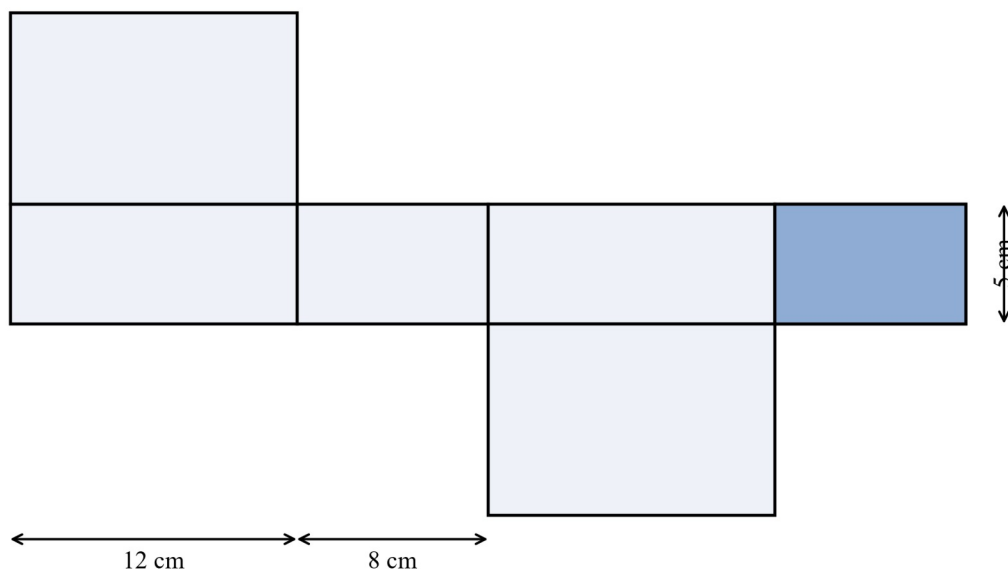


Figure 6. The net of the parcel box: a band of four rectangles 12 cm, 8 cm, 12 cm and 8 cm wide, all 5 cm tall, with a 12 cm by 8 cm lid above the first face and below the third face, and the last 8 cm by 5 cm face shaded.

### Example 2 — scaffolded

Figure 7 shows a net: a square 6 cm by 6 cm with four triangles of the same size and shape, one on each side of the square. (a) Name the solid the net folds into. (b) Count the faces, edges and vertices of the solid.

#### Working

(a) A square-based pyramid.

(b) Faces:  $1 + 4 = 5$ . Edges:  $4 + 4 = 8$ . Vertices:  $4 + 1 = 5$ .

#### Comment

The square is the base; the four triangles fold up to meet at one vertex, the apex.

Four base edges and four slant edges; four base corners and the apex.

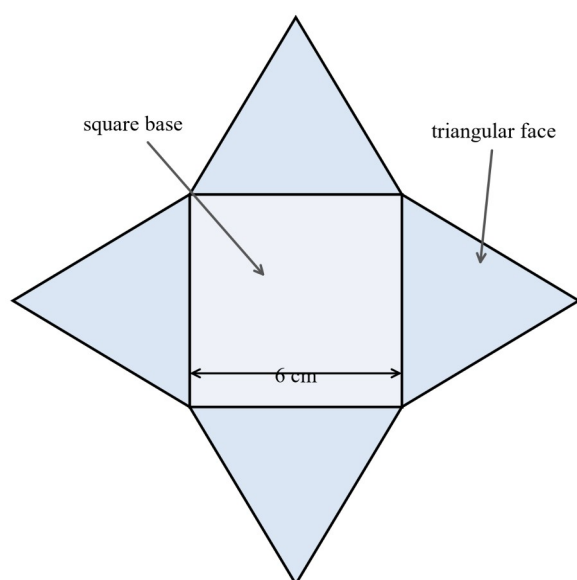


Figure 7. A net made of a square base 6 cm wide and four triangular faces, one on each side of the square.

### Example 3 — unscaffolded

Figure 8 shows a solid built from five 1 cm cubes; the arrow marks the front. On squared paper, at the scale 1 square : 1 cm, draw the plan view, the front view and the side view of the solid.

Looking straight down, the footprint is a row of three squares with one square behind the middle square. Looking straight at the front, there are two columns: one 2 squares high and one 1 square high. Looking straight at the side, there are three columns: the first 2 squares high, the other two 1 square high. Figure 9 shows the three views; a mirror image of the side view is also correct, depending which side you look from.

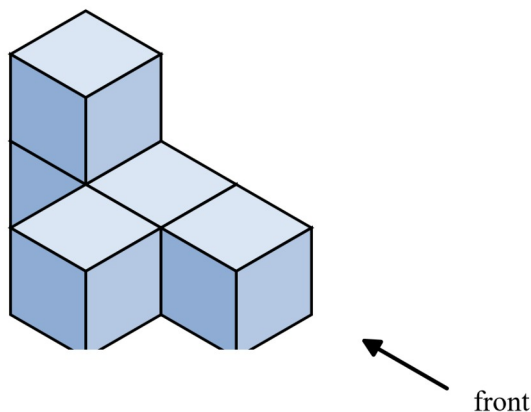


Figure 8. A solid made of five 1 cm cubes: a row of three cubes with one cube behind the middle cube and one cube on top of the left cube, with an arrow marking the front.

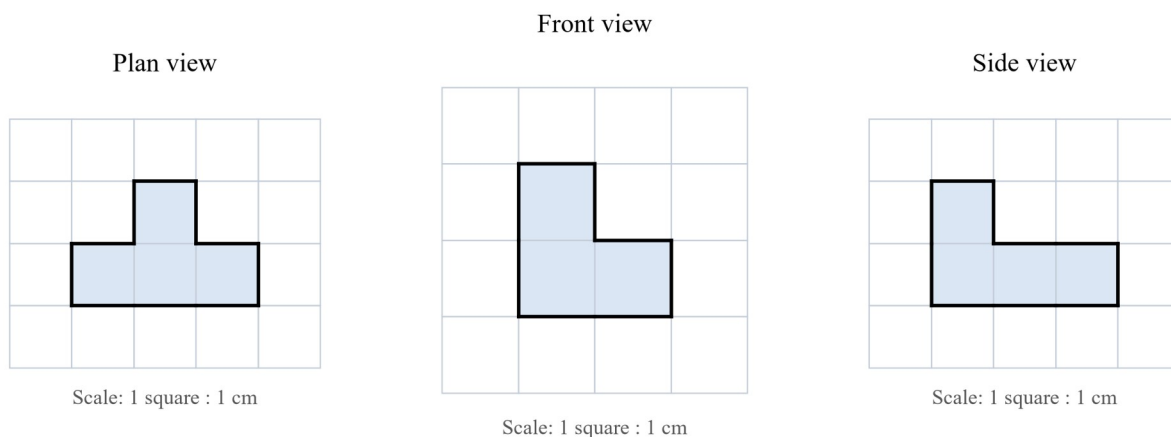


Figure 9. The three flat views of the solid in Figure 8 on squared paper at the scale 1 square to 1 cm: a plan view of four squares, a front view of three squares and a side view of four squares.

#### Example 4 — unscaffolded

Figure 10 shows a prism drawn on isometric dot paper. (a) Read off its length, width and height. (b) On squared paper at the scale 1 square : 1 cm, draw its plan view.

- Reading along the three directions: length 4 cm, width 2 cm, height 3 cm.
- From above you see the top face, a 4 cm  $\times$  2 cm rectangle, so the plan view is a 4 by 2 rectangle of 8 squares at the scale 1 square : 1 cm.

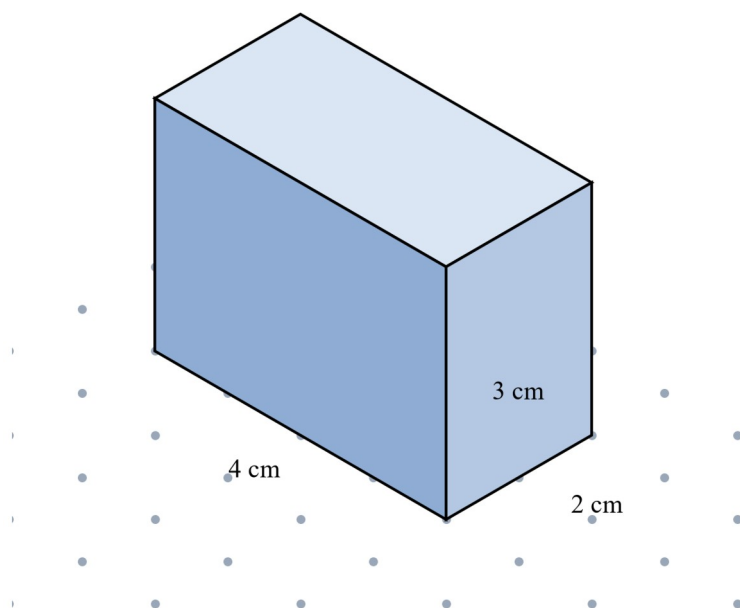


Figure 10. A prism 4 cm by 2 cm by 3 cm drawn on isometric dot paper, with the three lengths labelled.

## Practice questions

### Scaffolded

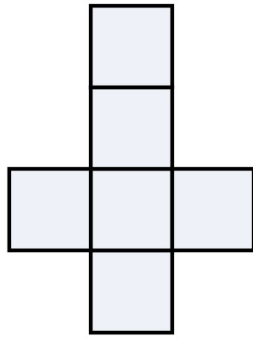
**Q1.** Copy the table and fill in the number of faces, edges and vertices of each solid.

| Solid                | Faces | Edges | Vertices |
|----------------------|-------|-------|----------|
| Rectangular prism    |       |       |          |
| Triangular prism     |       |       |          |
| Square-based pyramid |       |       |          |

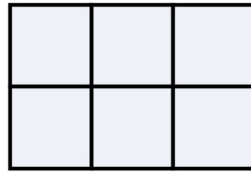
**Q2.** Use the net in Figure 6. (a) How many faces of the box are  $12\text{ cm} \times 5\text{ cm}$ ? (b) Which two lengths make the lid faces? (c) When the net is folded into the box, which face is opposite the top lid?

**Q3.** A triangular prism is a prism whose ends are triangles. Describe its net: how many triangles and how many rectangles does it have, and which faces are they?

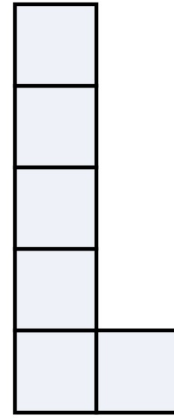
**Q4.** Figure 11 shows three flat shapes made of six squares each. Only one of them folds into a cube. Say which one, and give a reason why each of the other two fails.



A



B



C

Figure 11. Three flat shapes made of six squares each, labelled A, B and C: A is a strip of four squares with one square on each side, B is a 2 by 3 block, and C is a strip of five squares with one square on the end.

**Q5.** The building in Figure 12 is made of two blocks, each 3 m tall: a main block 8 m by 6 m and a wing 3 m by 4 m joined to one side. On squared paper at the scale 1 square : 1 m, sketch the plan view (the footprint) of the building, and give the overall length and width of the footprint.

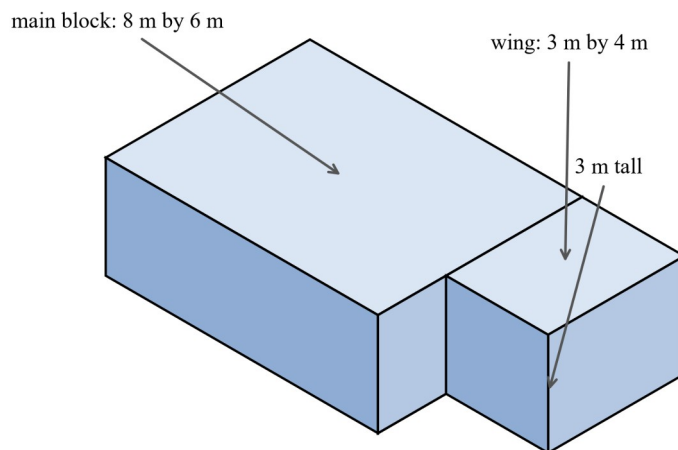


Figure 12. A building made of two blocks 3 m tall: a main block 8 m by 6 m and a wing 3 m by 4 m joined to one side.

**Q6.** Figure 13 shows a solid built from six 1 cm cubes; the arrow marks the front. On squared paper at the scale 1 square : 1 cm, draw the plan view, the front view and the side view of the solid.

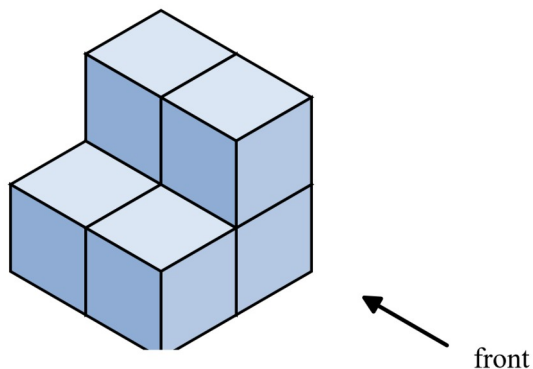


Figure 13. A solid made of six 1 cm cubes: a 2 by 2 layer of four cubes with two more cubes on top of one row, with an arrow marking the front.

### Un scaffolded

**Q7.** Say whether each statement is true or false. Correct every false statement. (a) A net of a cube is made of exactly six squares. (b) Any flat shape made of six squares folds into a cube. (c) The plan view of a solid is what you see looking straight down from above. (d) A perspective drawing shows true lengths, so a builder could measure from it.

**Q8.** For each job, choose the most useful representation. **A** a net **B** plan + front + side views **C** an isometric drawing **D** a perspective drawing (a) A company makes cardboard parcels and needs the flat pattern to cut from card. (b) A builder needs to mark on the ground exactly where the walls of a building go. (c) An artist wants to show a street the way it looks to a person standing in it.

**Q9.** A plan is drawn at the scale 1:50. (a) A wall is 4 m long. How long is the wall on the drawing? (b) A door is 6 cm wide on the drawing. How wide is the real door?

**Q10.** A student says: “A net of a cube must have six squares, so any shape made of six squares is a net of a cube.” Use net B in Figure 11 to explain why this reasoning fails.

**Q11.** On isometric dot paper, draw a prism 3 cm long, 2 cm wide and 2 cm high.

**Q12.** Look at Figure 5. (a) The back face of the box is the same size as the front face, yet it is drawn smaller. Why? (b) Which edges of the box, if kept going, would pass through the vanishing point?

**Q13.** Give one advantage and one disadvantage of using a net to represent a solid.

**Q14.** The building in Figure 12 is made of two prisms: one  $8\text{ m} \times 6\text{ m} \times 3\text{ m}$  and one  $3\text{ m} \times 4\text{ m} \times 3\text{ m}$ . A builder marking out the walls on the ground works from the plan view, not from a perspective drawing of the building. Give a reason why.

**Q15.** Build your own solid from four 1 cm cubes, joined face to face. On squared paper at the scale 1 square : 1 cm, draw its plan view and its front view.

**Q16.** For each purpose, choose the best representation — net, plan + front + side views, isometric drawing or perspective drawing — and give a reason for your choice. (a) Making the packaging for a new biscuit box. (b) Drawing a cube stack so that a friend can rebuild it exactly. (c) A picture of a building for a postcard.

## Answers

**Q1.** Rectangular prism: 6, 12, 8. Triangular prism: 5, 9, 6. Square-based pyramid: 5, 8, 5.

**Q2.** (a) 2; (b) 12 cm and 8 cm; (c) the other  $12\text{ cm} \times 8\text{ cm}$  lid (the one below the third face of the band).

**Q3.** Two triangles (the matching ends) and three rectangles (the sides joining the ends).

**Q4.** Net A folds into a cube. Net B fails: when folded, two squares land on top of each other and one face of the cube is left open. Net C fails: the strip of five squares wraps all the way around, so the fifth square lands back on the first.

**Q5.** A footprint shaped like an 11 m by 6 m rectangle with a 3 m by 2 m corner cut out; overall 11 m long and 6 m wide.

**Q6.** Plan view: a 2 by 2 square of four squares. Front view: two columns, one 2 squares high and one 1 square high. Side view: a 2 by 2 square of four squares.

**Q7.** (a) true; (b) false — nets B and C in Figure 11 have six squares but do not fold into a cube; (c) true; (d) false — a perspective drawing does not show true lengths, so a builder uses views or an isometric drawing instead.

**Q8.** (a) A; (b) B; (c) D.

**Q9.** (a) 8 cm; (b) 3 m.

**Q10.** Six squares is needed, but it is not enough: folding net B puts two squares on the same face and leaves one face open.

**Q11.** A correct drawing has a 3 by 2 top face along the two slanted directions and a height of 2 dots straight up.

**Q12.** (a) It is further from the eye, and in a perspective drawing things further away are drawn smaller. (b) The four edges that run into the distance (the depth edges), not the edges of the front face.

**Q13.** Advantage: every face is shown flat at its true size and shape, so the net can be cut out and folded to make the solid. Disadvantage: it does not show what the folded solid looks like.

**Q14.** The plan view shows true lengths and right angles on the ground, so the walls can be marked out exactly; a perspective drawing does not show true lengths.

**Q15.** Any solid of four cubes with matching views. Example: a row of three cubes with one on top of the middle cube gives a plan view of three squares in a row and a front view of three squares in a row with one square on top of the middle.

**Q16.** (a) A net — it folds into the box and shows each face flat at true size for printing. (b) Plan + front + side views (or an isometric drawing) — they show true lengths in every direction, so the stack can be rebuilt exactly. (c) A perspective drawing — it looks the way the building looks to the eye.

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## Fully-worked solutions

**Q1.** Count on each solid. A rectangular prism is a cuboid: 6 faces, 12 edges, 8 vertices. A triangular prism has two triangle ends and three rectangle sides: 5 faces;  $3 + 3 + 3 = 9$  edges;  $3 + 3 = 6$  vertices. A square-based pyramid has a square base and four triangles: 5 faces;  $4 + 4 = 8$  edges;  $4 + 1 = 5$  vertices.

**Q2.** (a) The band of four side faces alternates 12 cm, 8 cm, 12 cm, 8 cm, so two faces are  $12\text{ cm} \times 5\text{ cm}$ . (b) The lids are the top and bottom faces: 12 cm by 8 cm. (c) The two lids end up opposite each other, so the face opposite the top lid is the other  $12\text{ cm} \times 8\text{ cm}$  lid.

**Q3.** The two matching triangle ends are the two triangles, and the three rectangles join the matching sides of the triangles, so the net has 2 triangles and 3 rectangles.

**Q4.** Net A is a proper cube net: the strip of four squares wraps around the sides and the two arm squares close the top and bottom. Net B is a 2 by 3 block: folding it around puts two squares on top of each other and leaves one face open,

so it cannot close into a cube. Net C is a strip of five squares with one extra: wrapping the strip, the fourth square returns to the start, so the fifth square lands on the first square instead of a new face. In each failed net, some face of the cube is covered twice and another face not at all.

**Q5.** Looking straight down, the main block gives an 8 m by 6 m rectangle and the wing adds a 3 m by 4 m rectangle on one side, so the footprint is an 11 m by 6 m rectangle with a 3 m by 2 m corner cut out. The overall length is  $8\text{ m} + 3\text{ m} = 11\text{ m}$  and the overall width is 6 m. At the scale 1 square : 1 m, the footprint covers  $11 \times 6 - 3 \times 2 = 60$  squares.

**Q6.** From above, all four columns of cubes sit over a 2 by 2 footprint, so the plan view is a 2 by 2 square. From the front, the row with the extra cubes is 2 high and the other row is 1 high: two columns, 2 and 1. From the side, both columns carry a top cube, so the side view is a 2 by 2 square.

**Q7.** (a) **True:** folding a cube net always gives the 6 faces of the cube. (b) **False:** Figure 11 shows six-square shapes (B and C) that overlap or leave a face open when folded; a cube net needs six squares *in the right layout*. (c) **True:** that is what a plan view is. (d) **False:** in a perspective drawing, lengths running into the distance are drawn shorter, so you cannot measure true lengths from it; a builder measures from views or an isometric drawing.

**Q8.** (a) **A:** the parcel is made of flat card, and a net is the flat pattern to cut and fold. (b) **B:** the plan view shows the footprint at true lengths, so the walls can be marked on the ground exactly. (c) **D:** a perspective drawing shows the street the way the eye sees it.

**Q9.** (a)  $4\text{ m} = 400\text{ cm}$ , and  $400 \div 50 = 8$ , so the wall is 8 cm on the drawing. (b) 6 cm on the drawing stands for  $6 \times 50 = 300\text{ cm} = 3\text{ m}$ , so the real door is 3 m wide.

**Q10.** Having six squares is a fact every cube net must have, but net B shows it is not enough. B has six squares, yet folding it around puts two squares on the same face of the cube and leaves the opposite face open, so B is not a cube net. The squares must also sit in a layout that covers all six faces once each.

**Q11.** On isometric dot paper, count 3 dots along one slanted direction for the length, 2 dots along the other slanted direction for the width, and 2 dots straight up for the height, then join the dots to complete the three visible faces of the prism.

**Q12.** (a) In a perspective drawing, edges running into the distance are drawn shorter, so a face that is further from the eye is drawn smaller even when it is the same size. (b) The four depth edges — the ones running from the front face towards the back face — head towards the vanishing point; if kept going, they would all pass through it. The four edges of the front face would not.

**Q13.** Advantage: a net shows every face flat at its true size and shape, which is exactly what is needed to cut out and make the solid (as packaging is). Disadvantage: a net does not show what the folded solid looks like, so it is a poor choice for picturing the solid.

**Q14.** The plan view is drawn to scale with true lengths and right angles, so the builder can mark the exact footprint — 11 m by 6 m overall with the 3 m by 2 m corner missing — on the ground. A perspective drawing shows the building the way the eye sees it, with lengths drawn shorter as they run into the distance, so no true length can be taken from it.

**Q15.** Example solid: a row of three cubes with a fourth on top of the middle cube. From above, only the row shows: the plan view is three squares in a row. From the front, the row shows with the extra cube on top of the middle: the front view is three squares in a row with one square above the middle. Any four-cube solid is acceptable if the two views match the solid at the scale 1 square : 1 cm.

**Q16.** (a) **Net:** the box is made from flat card, the net folds into the box, and each face is shown flat at true size for printing the design. (b) **Plan + front + side views** (an isometric drawing also earns the mark): they give the true lengths in every direction, so a friend can rebuild the stack exactly. A perspective drawing would not, because its lengths are not true. (c) **Perspective drawing:** the postcard should show the building the way it looks to the eye, which is exactly what perspective does. In each case the mark for the reason is earned by naming what the representation shows well or badly.

## Teacher note

This subtopic compares the four representations and states what each is good and poor at, per the chapter plan — the advantage/disadvantage reasoning is the core of VC2M7SP01, not an add-on. The formal term *elevation* sits at Yr9 on the vocabulary ladder, so the student text and the figure labels say *front view* and *side view*; the term may be introduced orally for students who need it. Elaboration VC2M7SP01 . E06 (First Nations artworks and maps) is excluded per CONTENT\_POLICY Rule 1, with no replacement activity. The Level 6 prism idea (VC2M6SP01) is recalled in one sentence and not re-taught, and no question demands surface area or volume — those belong to 4A and 4B. Per D4.17 there are no 3-D coordinates anywhere in the section. Per P6 every drawing states a scale as a convention (ratio notation cross-references 2F; the scaling technique itself remains with 2F and 4F). Reasoning marks follow the assessment plan: Q4, Q7, Q10, Q12, Q13, Q14 and Q16 each carry at least one mark for a stated reason, and Q8 is the multiple-choice item the plan allows for “which representation of a solid is most useful”. All thirteen figures are computed from the real geometry they show (see `src/gen_5a.py`): views are projected cell by cell from the cube lists, so the answer keys match the pictures exactly, and mirror images of side views are accepted.

## Classifying triangles, quadrilaterals and other polygons

**Content description VC2M7SP02:** classify triangles, quadrilaterals and other polygons according to their side and angle properties; identify and reason about relationships

**Achievement standard (extract):** Students classify polygons according to their features and design an algorithm to sort and classify shapes.

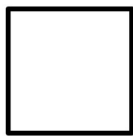
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### Theory

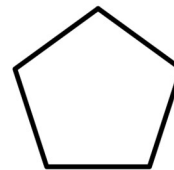
**Naming polygons.** A **polygon** is a closed shape in a plane whose sides are all straight, and a polygon is named by its number of sides. Figure 1 shows the names this book uses most.



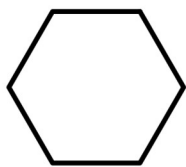
**triangle**  
3 sides



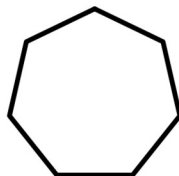
**quadrilateral**  
4 sides



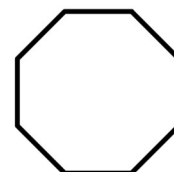
**pentagon**  
5 sides



**hexagon**  
6 sides



**heptagon**  
7 sides



**octagon**  
8 sides

*Figure 1. Six polygons named by their number of sides, in two rows: triangle (3 sides), quadrilateral (4 sides) and pentagon (5 sides) above hexagon (6 sides), heptagon (7 sides) and octagon (8 sides); each name sits under its shape with the side count beneath.*

Quadrilaterals — the 4-sided polygons — are the family this subtopic looks at most closely, but the same two ideas, side properties and angle properties, sort every polygon.

**Triangles by their sides.** A triangle is sorted by its sides into one of three families (Figure 2). A **scalene** triangle has no equal sides, an **isosceles** triangle has 2 equal sides, and an **equilateral** triangle has 3 equal sides. Equal sides carry the same ticks — *same ticks, same length* — and a side with no tick is a different length. On that wording an equilateral triangle is a special isosceles triangle: it has 2 equal sides and more. When a shape fits more than one family, give it its most special name.

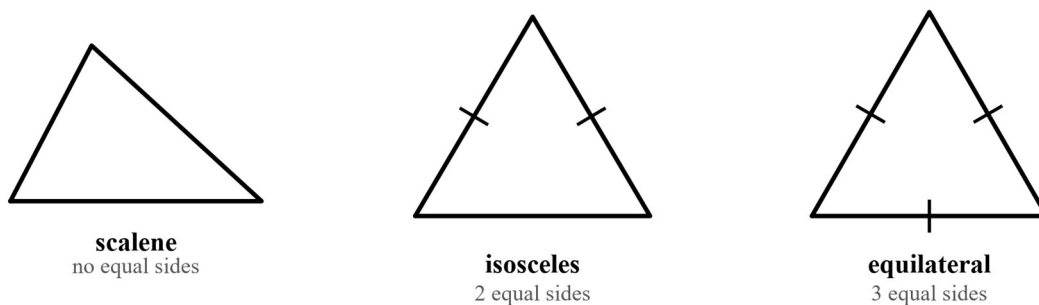


Figure 2. Three triangles sorted by their sides: a scalene triangle with no equal sides and no ticks; an isosceles triangle with one tick on each of its 2 equal sides; an equilateral triangle with one tick on each of its 3 equal sides; each name and its rule sits under its shape.

**Triangles by their angles.** The same triangle is sorted a second way, by its angles (Figure 3). An **acute** triangle has all 3 angles less than  $90^\circ$ , a **right-angled** triangle has one angle of exactly  $90^\circ$ , marked with the small square, and an **obtuse** triangle has one angle greater than  $90^\circ$ . A triangle holds at most one right angle and at most one obtuse angle: the 3 angles of a triangle add to  $180^\circ$  (subtopic 4E), and two right angles alone would already use all of it. The two sorts are independent, so one triangle carries both names — a right-angled isosceles triangle, for example.

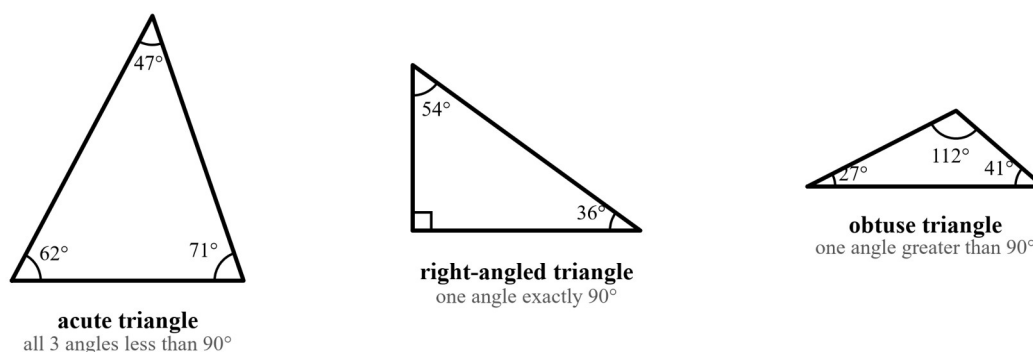


Figure 3. Three triangles sorted by their angles, every angle labelled with its size: an acute triangle with angles 62, 71 and 47 degrees; a right-angled triangle with the right angle marked by a small square and the other angles 36 and 54 degrees; an obtuse triangle with angles 27, 41 and 112 degrees; each name and its rule sits under its shape.

**Can any 3 lengths make a triangle?** Take 3 strips and try to join them end to end. Strips of 3 cm, 4 cm and 5 cm close into a triangle (Figure 4, left). Strips of 3 cm, 4 cm and 8 cm can never meet (Figure 4, right): even resting flat along the 8 cm strip, the two short strips only reach  $3 + 4 = 7$  cm. The rule: *the two shorter lengths must add to more than the longest length*. When they do, the 3 lengths close into a triangle; when they do not, no triangle is possible.

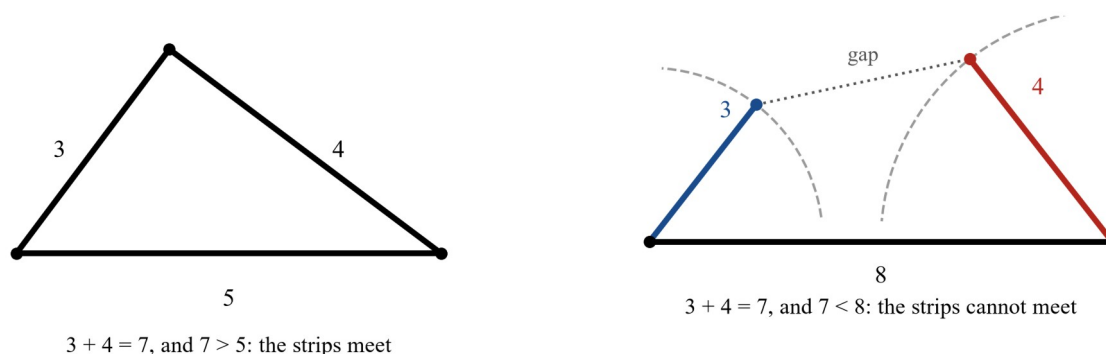
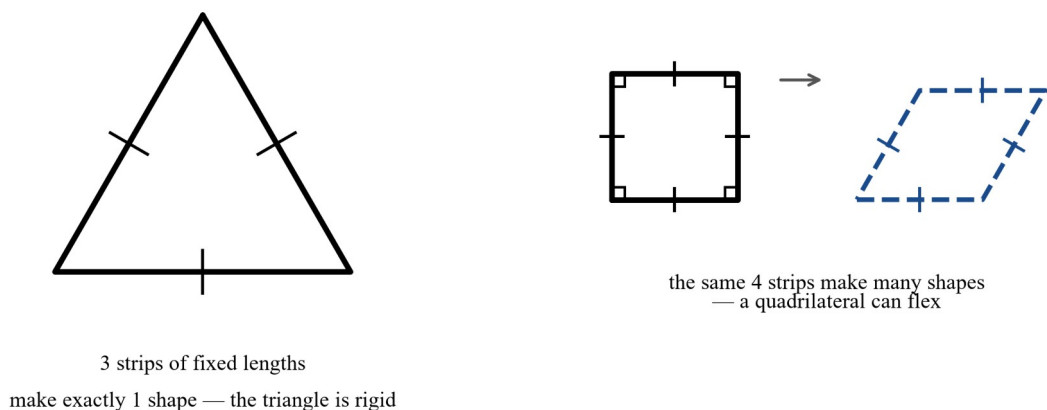


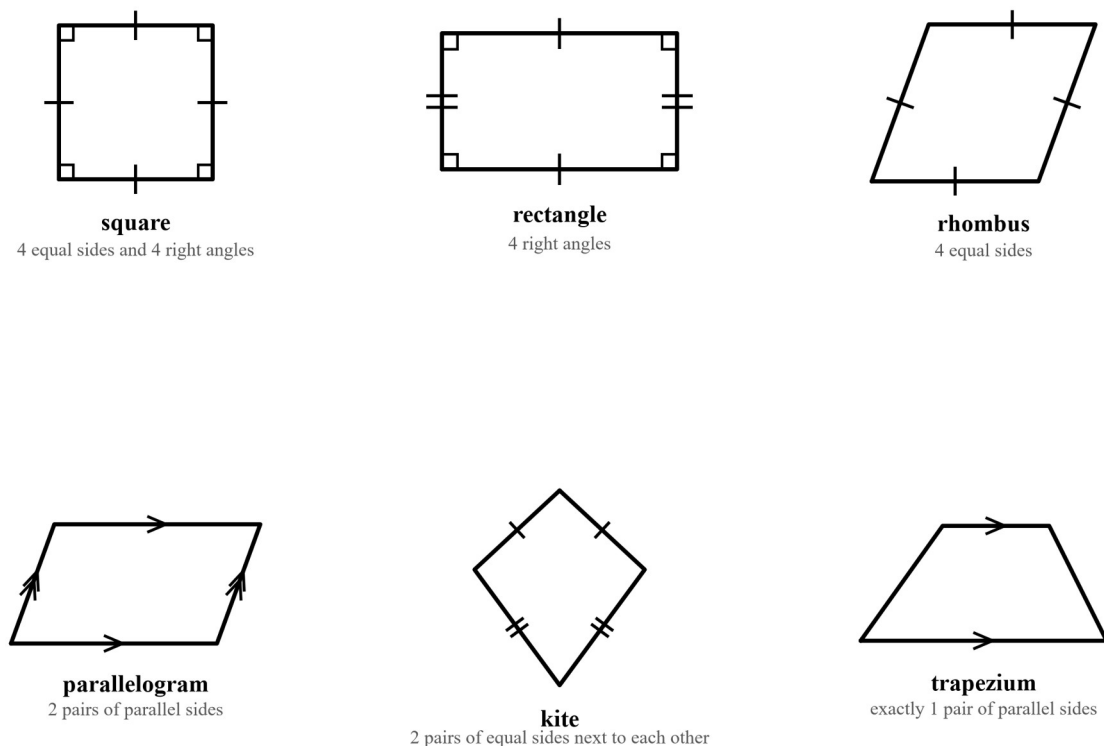
Figure 4. Two panels built from strips of the stated lengths. Left: strips of 3, 4 and 5 join end to end and close into a triangle, captioned ' $3 + 4 = 7$ , and  $7 > 5$ : the strips meet'. Right: a strip of 8 with a strip of 3 and a strip of 4 leaning in from its ends; their free ends fall short, a dotted line marks the gap, and dashed arcs show how far each short strip can reach; the caption reads ' $3 + 4 = 7$ , and  $7 < 8$ : the strips cannot meet'.

**Triangles are rigid; quadrilaterals flex.** Join 3 strips of fixed lengths into a triangle and the shape cannot change (Figure 5, left): there is exactly one triangle on those 3 sides. Four strips of equal length make a square, a leaning rhombus, and every lean in between (Figure 5, right) — a quadrilateral on 4 fixed sides can flex. That is why gates, bridges and roofs carry braces from corner to corner: a brace cuts a quadrilateral into two triangles, and triangles do not flex (Q5).



*Figure 5. Two panels. Left: one triangle built from 3 strips, each with a tick, captioned '3 strips of fixed lengths make exactly 1 shape — the triangle is rigid'. Right: a square drawn in solid lines with a tick on each side and its 4 right angles marked, an arrow pointing to the same 4 strips pushed into a leaning rhombus drawn in dashed lines, captioned 'the same 4 strips make many shapes — a quadrilateral can flex'.*

**The six quadrilaterals.** Figure 6 shows the six families of quadrilaterals this book names, drawn with the marking conventions: ticks for equal sides, arrows for parallel sides, and the small square for right angles. A **square** has 4 equal sides and 4 right angles. A **rectangle** has 4 right angles. A **rhombus** has 4 equal sides. A **parallelogram** has 2 pairs of parallel sides. A **kite** has 2 pairs of equal sides next to each other. A **trapezium** has exactly 1 pair of parallel sides — some books say 'at least one pair'; this book says exactly one.



*Figure 6. The six quadrilateral families in two rows, each drawn with standard markings and its name and rule beneath: square (a tick on each of its 4 equal sides and 4 right-angle marks), rectangle (opposite sides ticked in pairs and 4 right-angle marks), rhombus (a tick on each of its 4 equal sides), parallelogram (one arrow on one pair of*

parallel sides and two arrows on the other), kite (one tick on each of the top pair of equal sides and two ticks on each of the bottom pair), trapezium (an arrow on its one pair of parallel sides).

**How the families sit inside one another.** The names nest (Figure 7). Every square is a rectangle *and* a rhombus; every rectangle and every rhombus is a parallelogram; every parallelogram is a quadrilateral; and a rhombus has 2 pairs of equal sides next to each other, so every rhombus is a special kite too. The boxes say which ‘every’ claims are true; the rest are ‘sometimes’ or ‘never’. A rectangle is a rhombus *sometimes* — when its 4 sides happen to be equal — and a trapezium is *never* a parallelogram under this book’s exactly-one-pair rule. Reading those relationships off the boxes is the reasoning this subtopic asks for.

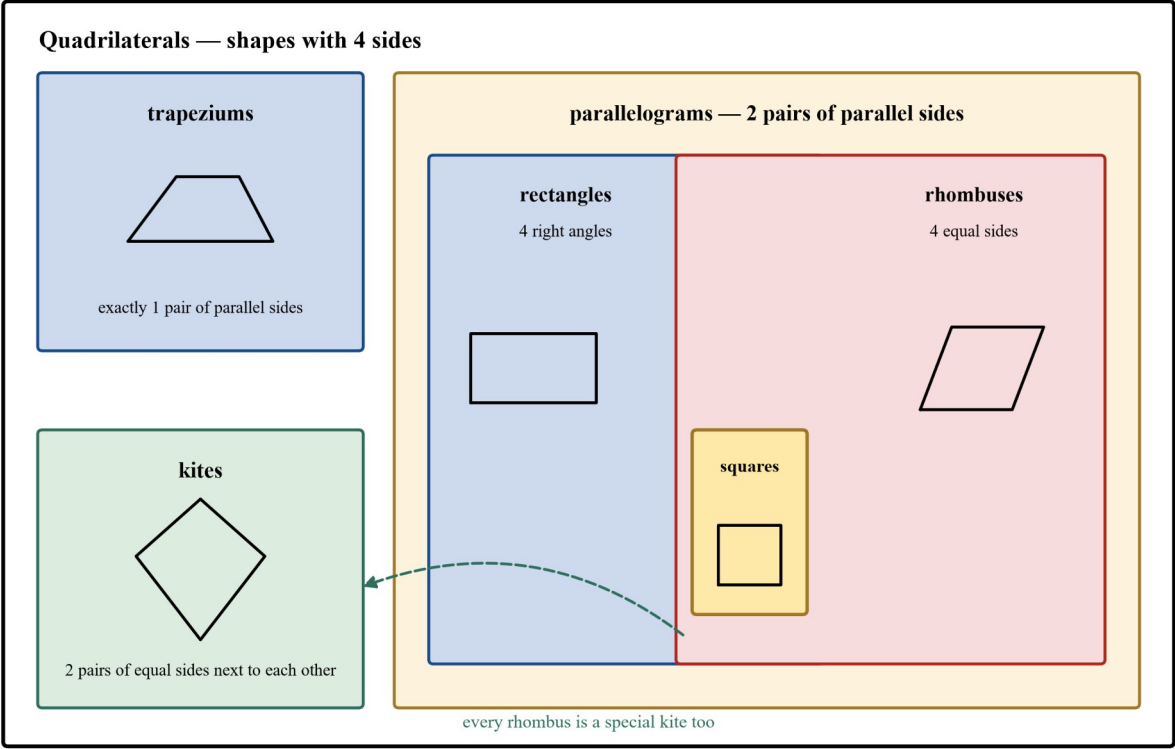


Figure 7. A hierarchy of quadrilaterals drawn as boxes inside boxes: one large box ‘Quadrilaterals — shapes with 4 sides’ holds boxes for trapeziums (exactly 1 pair of parallel sides) and kites (2 pairs of equal sides next to each other), and a large ‘parallelograms’ box holding the rectangles box (4 right angles) and the rhombuses box (4 equal sides) overlapping, with a small squares box in the overlap; a dashed arrow from the rhombuses box to the kites box reads ‘every rhombus is a special kite too’; a sketch of each shape sits in its box.

**Reasoning with angles.** Once a shape’s family is known, its angles follow from the angle facts of 4D and 4E. In a parallelogram, each pair of neighbouring angles sits between two parallel sides, so the pair is co-interior angles (4D) and adds to 180 °; opposite angles of a parallelogram are therefore equal. Worked example 4 puts this to work.

**The words of this subtopic**, with synonyms you may meet:

| Term                                     | Meaning here                                                                | Synonyms and related words |
|------------------------------------------|-----------------------------------------------------------------------------|----------------------------|
| <b>polygon</b>                           | a closed shape in a plane with straight sides                               | —                          |
| <b>scalene / isosceles / equilateral</b> | of a triangle: no equal sides / 2 equal sides / 3 equal sides               | —                          |
| <b>acute / right-angled / obtuse</b>     | of a triangle: all angles under 90 ° / one 90 ° angle / one angle over 90 ° | —                          |
| <b>parallelogram</b>                     | a quadrilateral with 2 pairs of parallel sides                              | —                          |
| <b>rectangle</b>                         | a quadrilateral with 4 right angles                                         | oblong (informal)          |
| <b>rhombus</b>                           | a quadrilateral with 4 equal sides                                          | diamond (informal)         |

| Term             | Meaning here                                                        | Synonyms and related words |
|------------------|---------------------------------------------------------------------|----------------------------|
| <b>square</b>    | a quadrilateral with 4 equal sides and 4 right angles               | —                          |
| <b>kite</b>      | a quadrilateral with 2 pairs of equal sides next to each other      | —                          |
| <b>trapezium</b> | in this book: a quadrilateral with exactly 1 pair of parallel sides | —                          |

## Worked examples

### Example 1 — scaffolded

The triangle in Figure 8 carries a right-angle mark and no ticks. Classify it by its sides, then by its angles.

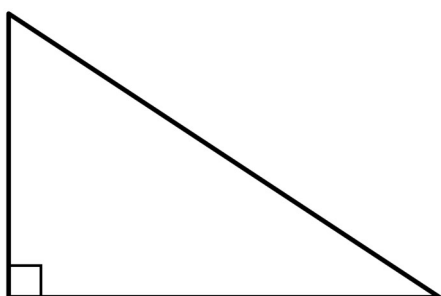


Figure 8. The triangle to classify: a right-angle mark in the corner between its two shorter sides, and no ticks, so no two sides are equal.

| Working                                                                                                         | Comment                                                                                         |
|-----------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| <b>Sides.</b> No sides carry ticks, so no two sides are equal.<br>Sides result: <b>scalene</b> .                | Read the marking, not just how the shape looks.<br>Scalene = no equal sides (Figure 2).         |
| <b>Angles.</b> The small square marks one angle of exactly $90^\circ$ .<br>Angles result: <b>right-angled</b> . | The small square is the right-angle mark (Figure 3).<br>One right angle is enough for the name. |

$\therefore$  the triangle is a right-angled scalene triangle.

### Example 2 — scaffolded

Do the lengths 6 cm, 9 cm and 14 cm close into a triangle? What about 6 cm, 9 cm and 16 cm?

| Working                                                                                                                                            | Comment                                                                                                            |
|----------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| <b>First set.</b> The two shorter lengths are 6 and 9; $6 + 9 = 15$ , and $15 > 14$ .<br>Result: <b>yes</b> , the 3 lengths close into a triangle. | Always compare the two shorter lengths with the longest.<br>The rule asks for ‘more than’, and 15 is more than 14. |
| <b>Second set.</b> Again $6 + 9 = 15$ , but now $15 < 16$ .<br>Result: <b>no</b> — the two short strips cannot reach across the 16 cm strip.       | Same check, new lengths.<br>Like the right-hand panel of Figure 4.                                                 |

$\therefore$  6, 9 and 14 close into a triangle; 6, 9 and 16 cannot.

### Example 3 — unscaffolded

A quadrilateral has both pairs of opposite sides parallel, all 4 angles right angles, and not all 4 sides equal. Give its most special name, then list every family in Figure 7 it belongs to.

The shape has 4 right angles, so it is a rectangle. It is not a rhombus, since not all 4 sides are equal, so the most special name it carries is **rectangle**. Reading the boxes of Figure 7 from the inside out, the rectangles box sits inside the parallelograms box, inside the quadrilaterals box.

$\therefore$  the most special name is rectangle, and the shape also belongs to the parallelograms and the quadrilaterals — three families in all.

### Example 4 — unscaffolded

Figure 9 shows a parallelogram  $ABCD$  with one angle marked  $65^\circ$ . Find  $p$ ,  $q$  and  $r$ , giving a reason for each.

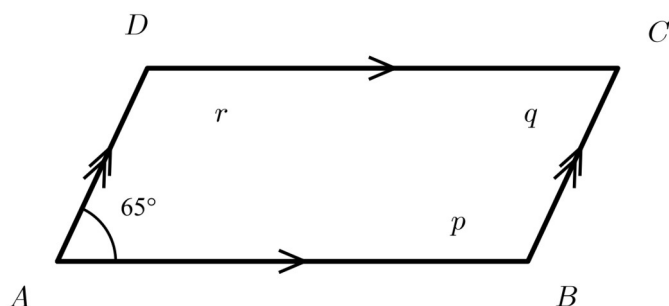


Figure 9. A parallelogram  $ABCD$  with vertices labelled in order around the shape, one arrow on each of the top and bottom sides and two arrows on each of the left and right sides; the angle at  $A$  is marked  $65$  degrees, and the angles at  $B$ ,  $C$  and  $D$  are labelled  $p$ ,  $q$  and  $r$ .

$65^\circ + p = 180^\circ$ , so  $p = 115^\circ$   $\cdot$  co-interior angles add to  $180^\circ$  ( $AD \parallel BC$ , cut by  $AB$  — 4D).  $q = 65^\circ$   $\cdot$  opposite angles of a parallelogram are equal (each is  $180^\circ$  minus  $p$ ).  $r = 115^\circ$   $\cdot$  opposite angles of a parallelogram are equal (each is  $180^\circ$  minus  $65^\circ$ ).

$\therefore p = 115^\circ$ ,  $q = 65^\circ$ ,  $r = 115^\circ$  — and  $65^\circ + 115^\circ + 65^\circ + 115^\circ = 360^\circ$ , the angle sum of a quadrilateral (4E), which checks the answer.

## Practice questions

### Scaffolded

**Q1.** Figure 1 names polygons by their number of sides. Name a polygon with (a) 5 sides, (b) 7 sides, (c) 8 sides, (d) 6 sides.

**Q2.** Figure 2 sorts triangles by their sides. Name the family of a triangle with (a) 3 equal sides, (b) exactly 2 equal sides, (c) no equal sides.

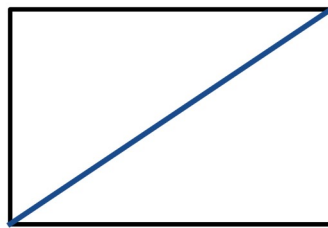
**Q3.** Figure 3 sorts triangles by their angles. Name the family of a triangle with angles (a)  $90^\circ$ ,  $50^\circ$ ,  $40^\circ$ , (b)  $110^\circ$ ,  $35^\circ$ ,  $35^\circ$ , (c)  $70^\circ$ ,  $60^\circ$ ,  $50^\circ$ .

**Q4.** Write true or false for each, with a short reason. (a) An equilateral triangle is also an isosceles triangle. (b) A right-angled triangle can also be an isosceles triangle. (c) A triangle can have two right angles.

**Q5.** Figure 10 shows a gate frame without a brace, and the same frame with one. (a) Explain in one sentence why the unbraced frame can sag. (b) Explain in one sentence why the brace stops it sagging.



no brace: the frame can sag



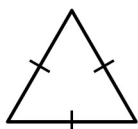
a brace makes triangles: the frame is rigid

Figure 10. Two panels. Left: a rectangular gate frame drawn dashed with the same four bars pushed into a leaning shape drawn solid and an arrow showing the top corner falling, captioned 'no brace: the frame can sag'. Right: the rectangular frame solid with a thick brace from one corner to the opposite corner, captioned 'a brace makes triangles: the frame is rigid'.

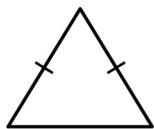
**Q6.** Will 3 strips of these lengths close into a triangle? Answer yes or no in each case. (a) 4 cm, 5 cm, 6 cm (b) 2 cm, 3 cm, 6 cm (c) 5 cm, 5 cm, 9 cm (d) 3 cm, 4 cm, 7 cm

### Un scaffolded

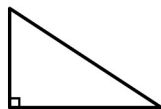
**Q7.** Give each shape A–J in Figure 11 its most special name.



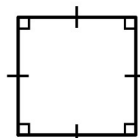
A



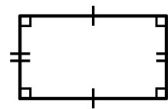
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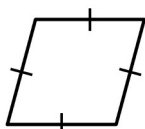
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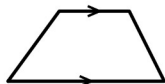
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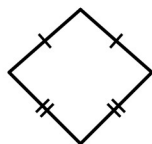
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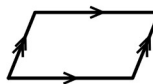
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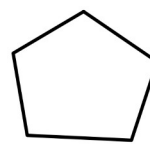
G



H



I



J

Figure 11. Ten shapes labelled A to J, drawn with standard markings: A an equilateral triangle with a tick on each side; B an isosceles triangle with one tick on each of its 2 equal sides; C a right-angled triangle with the right angle marked and no equal sides; D a square with a tick on each side and 4 right-angle marks; E a rectangle with opposite sides ticked in pairs and 4 right-angle marks; F a rhombus with a tick on each side; G a trapezium with an arrow on its one pair of parallel sides; H a kite with one tick on its top pair of equal sides and two ticks on its bottom pair; I a parallelogram with one arrow on one pair of parallel sides and two arrows on the other; J a pentagon with no markings.

**Q8.** Write true or false for each, with a reason from Figure 7. (a) Every square is a rhombus. (b) Every rhombus is a square. (c) Every rectangle is a parallelogram. (d) Every parallelogram is a trapezium. (e) Every rhombus is a kite.

**Q9.** Figure 12 shows a kite with two angles marked. Find  $p$  and  $q$ , giving a reason for each.

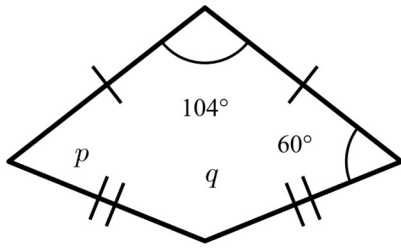


Figure 12. A kite with one tick on each of its two top equal sides and two ticks on each of its two bottom equal sides; the top angle is marked 104 degrees, the right angle 60 degrees, and the left and bottom angles are labelled  $p$  and  $q$ .

- Q10.** Two sides of a triangle measure 7 cm and 3 cm. The third side is a whole number of centimetres. Find the longest and the shortest the third side could be, with a reason.
- Q11.** Write always, sometimes or never for each, with a reason. (a) A rectangle is a square. (b) A parallelogram is a rhombus. (c) A trapezium is a parallelogram. (d) An equilateral triangle is an acute triangle.
- Q12.** ‘Every square is a rectangle’ is true, yet ‘every rectangle is a square’ is false. Explain why, using the features in Figure 6 and the boxes in Figure 7.
-

## Answers

**Q1.** (a) pentagon; (b) heptagon; (c) octagon; (d) hexagon.

**Q2.** (a) equilateral; (b) isosceles; (c) scalene.

**Q3.** (a) right-angled; (b) obtuse; (c) acute.

**Q4.** (a) true — it has 2 equal sides, since 3 equal sides includes 2; (b) true — the two sides either side of the right angle can be equal; (c) false — two right angles would use all  $180^\circ$  of the angle sum (4E), leaving nothing for the third angle.

**Q5.** (a) The frame is a quadrilateral on 4 fixed sides, and such a quadrilateral can flex (Figure 5), so the corners can lean and the frame sags. (b) The brace cuts the frame into two triangles on 3 fixed sides each, and a triangle cannot flex.

**Q6.** (a) yes —  $4 + 5 = 9 > 6$ ; (b) no —  $2 + 3 = 5 < 6$ ; (c) yes —  $5 + 5 = 10 > 9$ ; (d) no —  $3 + 4 = 7$  is not more than 7: the strips lie flat and make no corner.

**Q7.** A equilateral triangle; B isosceles triangle; C right-angled scalene triangle; D square; E rectangle; F rhombus; G trapezium; H kite; I parallelogram; J pentagon.

**Q8.** (a) true — the squares box sits inside the rhombuses box; (b) false — a rhombus without 4 right angles is not a square; (c) true — the rectangles box sits inside the parallelograms box; (d) false — a parallelogram has 2 pairs of parallel sides, but this book's trapezium has exactly 1 pair; (e) true — 4 equal sides give 2 pairs of equal sides next to each other, the kite's feature.

**Q9.**  $p = 60^\circ$  — the kite is symmetric about the line through its top and bottom vertices, so the angle at the left vertex equals the angle at the right vertex.  $q = 360^\circ - 104^\circ - 60^\circ - 60^\circ = 136^\circ$  — the 4 angles of a quadrilateral add to  $360^\circ$  (4E).

**Q10.** Longest 9 cm, shortest 5 cm.

**Q11.** (a) sometimes — when its 4 sides are equal; (b) sometimes — when its 4 sides are equal; (c) never — a parallelogram has 2 pairs of parallel sides, this book's trapezium exactly 1 pair; (d) always — 3 equal sides force 3 equal angles of  $60^\circ$ , each under  $90^\circ$ .

**Q12.** A square has 4 right angles, so it meets the rectangle's feature and sits in the rectangles box; a rectangle needs only 4 right angles, and one whose sides are not all equal sits outside the squares box.

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## Fully-worked solutions

**Q1.** Count the sides and read the name off Figure 1: (a) 5 sides is a pentagon; (b) 7 sides is a heptagon; (c) 8 sides is an octagon; (d) 6 sides is a hexagon.

**Q2.** (a) 3 equal sides is the equilateral family; (b) exactly 2 equal sides is the isosceles family; (c) no equal sides is the scalene family. The ticks in Figure 2 show the marking for each.

**Q3.** (a) One angle is  $90^\circ$ , so right-angled. (b) One angle is greater than  $90^\circ$  ( $110^\circ$ ), so obtuse. (c) All 3 angles are less than  $90^\circ$ , so acute.

**Q4.** (a) True: an isosceles triangle needs 2 equal sides, and an equilateral triangle has 3 equal sides, which includes 2. (b) True: the two sorts are independent — draw the two sides either side of the right angle equal and the triangle is right-angled and isosceles at once. (c) False: the angles of a triangle add to  $180^\circ$  (4E), and two right angles already add to  $180^\circ$ , leaving the third angle with nothing.

**Q5.** (a) The frame is a quadrilateral, and a quadrilateral on 4 fixed sides can flex (Figure 5), so its corners can lean and the frame sags. (b) The brace runs from one corner to the opposite corner and cuts the frame into two triangles, each on 3 fixed sides; a triangle on 3 fixed sides is rigid, so the frame holds its shape.

**Q6.** Add the two shorter lengths and compare with the longest. (a)  $4 + 5 = 9 > 6$ : yes. (b)  $2 + 3 = 5 < 6$ : no. (c)  $5 + 5 = 10 > 9$ : yes. (d)  $3 + 4 = 7$ , equal to the longest and not more than it: resting flat, the two short strips just reach along the 7 cm strip and make no corner — no.

**Q7.** Read the markings. A: 3 equal sides, equilateral triangle. B: 2 equal sides, isosceles triangle. C: a right angle and no equal sides, right-angled scalene triangle. D: 4 equal sides and 4 right angles, square. E: 4 right angles with opposite sides equal but not all 4 equal, rectangle. F: 4 equal sides and no right angles, rhombus. G: exactly 1 pair of parallel sides, trapezium. H: 2 pairs of equal sides next to each other, kite. I: 2 pairs of parallel sides with no right angles and not all sides equal, parallelogram. J: 5 sides, pentagon.

**Q8.** (a) True: in Figure 7 the squares box sits inside the rhombuses box, so every square has 4 equal sides and is a rhombus. (b) False: the rhombuses box is larger than the squares box — a rhombus without 4 right angles (the rhombus in Figure 6) is not a square. (c) True: the rectangles box sits inside the parallelograms box. (d) False: a parallelogram has 2 pairs of parallel sides, but a trapezium in this book has exactly 1 pair, so no parallelogram sits in the trapeziums box. (e) True: a rhombus's 4 equal sides are 2 pairs of equal sides next to each other, the kite's feature — the dashed arrow in Figure 7.

**Q9.** A kite is symmetric about the line joining the vertices where its equal sides meet (the top and bottom vertices in Figure 12), so the left angle equals the right angle:  $p = 60^\circ$ . The angles of a quadrilateral add to  $360^\circ$  (4E), so  $q = 360^\circ - 104^\circ - 60^\circ - 60^\circ = 136^\circ$ . Check:  $104^\circ + 60^\circ + 136^\circ + 60^\circ = 360^\circ$ .

**Q10.** Call the third side  $x$  cm. If  $x$  is the longest side, the 7 and 3 strips must reach across it:  $7 + 3 > x$ , so  $x < 10$ . If 7 stays the longest, 3 and  $x$  must reach across the 7:  $3 + x > 7$ , so  $x > 4$ . The whole numbers between 4 and 10 are 5 to 9, so the third side is at most 9 cm and at least 5 cm.

**Q11.** (a) Sometimes: a rectangle whose 4 sides are equal is a square; one whose sides are not all equal is not. (b) Sometimes: a parallelogram with 4 equal sides is a rhombus; one without 4 equal sides is not. (c) Never: a parallelogram has 2 pairs of parallel sides and a trapezium in this book has exactly 1 pair, so no shape is both. (d) Always: the 3 angles of an equilateral triangle are equal and add to  $180^\circ$  (4E), so each is  $60^\circ$ , less than  $90^\circ$  — the triangle is acute.

**Q12.** The rectangle's only feature is 4 right angles (Figure 6), and a square has 4 right angles, so every square meets the rectangle's feature — in Figure 7 the squares box sits inside the rectangles box, which is what 'every square is a rectangle' says. The squares box is smaller, though: it adds the feature '4 equal sides'. A rectangle whose sides are not all equal — the rectangle sketch in Figure 6 — sits in the rectangles box but outside the squares box, so 'every rectangle is a square' fails. The nesting of the boxes makes one 'every' claim true and its reverse false.

## Teacher note

- **Elaboration coverage.** E01: Figure 5 and Q5 (fixed strips; the rigidity of triangles against the flexibility of quadrilaterals; bracing). E02: Figure 4, WE2, Q6 and Q10 — the question 'can any 3 lengths be used?' is answered by the two-shorter-sums-more-than-the-longest rule, including the flat equality case  $3 + 4 = 7$ . E03: Figures 2–3, WE1, Q1–Q4 and Q7 — all six triangle families with the geometric conventions (ticks, the right-angle square, degree labels). E04: Figures 6–7, WE3, Q8 and Q12 — the six quadrilaterals described and contrasted, and the relationships between them explained by nesting. The 'identify and reason about relationships' verb is carried by Q8–Q12 and WE4.
- **Level boundary (D4.11).** Congruence is Level 8 (VC2M8SP01) and is not tested here; every feature is read from markings placed on the figures, never proved. Figures are generated from real geometry with each claimed feature asserted at generation time (equal sides exactly equal, right angles exactly  $90^\circ$ , labelled angles computed from their vertices, the sheared quadrilateral keeping exactly the same side lengths, the trapezium and kite passing exactly-one-pair / no-parallel-pair checks).
- **Contested term — trapezium.** Exactly one pair of parallel sides, stated at first use in the theory and in the words table, matching 5C; 'at least one pair' is named as the frame some other books use. Q8(d) and Q11(c) are false/never under this frame.
- **Vocabulary ownership (D4).** 5B teaches the triangle and quadrilateral names and their definitions; 5C takes them as given. 5B deliberately leaves regular/concave/convex to 5C.

- **Angle facts.** Reasoned-with facts (triangle angle sum, quadrilateral angle sum, co-interior angles) are cross-referenced to 4D and 4E, per the TOC decision; reasons follow the estate's statement · reason · conclusion form in WE4 and Q9.
- **Assessment (T5).** 5B's share of T5 is 5 marks in the 20-mark Chapter 5 test, with reasoning marked separately; Q9–Q12 are the reasoning-shaped items, and an item worth 5 marks here must carry at least 1 mark for a stated reason.
- **Digital tools (D9).** VC2M7SP02 names none and none are used.
- **Content policy.** Aboriginal and Torres Strait Islander content is excluded (Rule 1); VC2M7SP02 and its elaborations carry none.

## Algorithms that sort and classify shapes

**Content description VC2M7SP04:** design algorithms involving a sequence of steps and decisions that will sort and classify sets of shapes according to their attributes, and describe how the algorithms work

**Achievement standard (extract):** Students classify polygons according to their features and design an algorithm to sort and classify shapes.

### Theory

**What an algorithm is.** An **algorithm** is a set of steps and decisions for sorting or classifying a set of shapes. Run on any shape from the set, it always finishes and gives exactly one result. A **flow chart** is a picture of an algorithm: an oval marks the start and the finish, a box holds each step, a diamond holds each **decision** (a question answered yes or no), and arrows show the **sequence** — the order the steps run in. Figure 1 shows the four symbols on a small algorithm that sorts shapes by their number of sides.

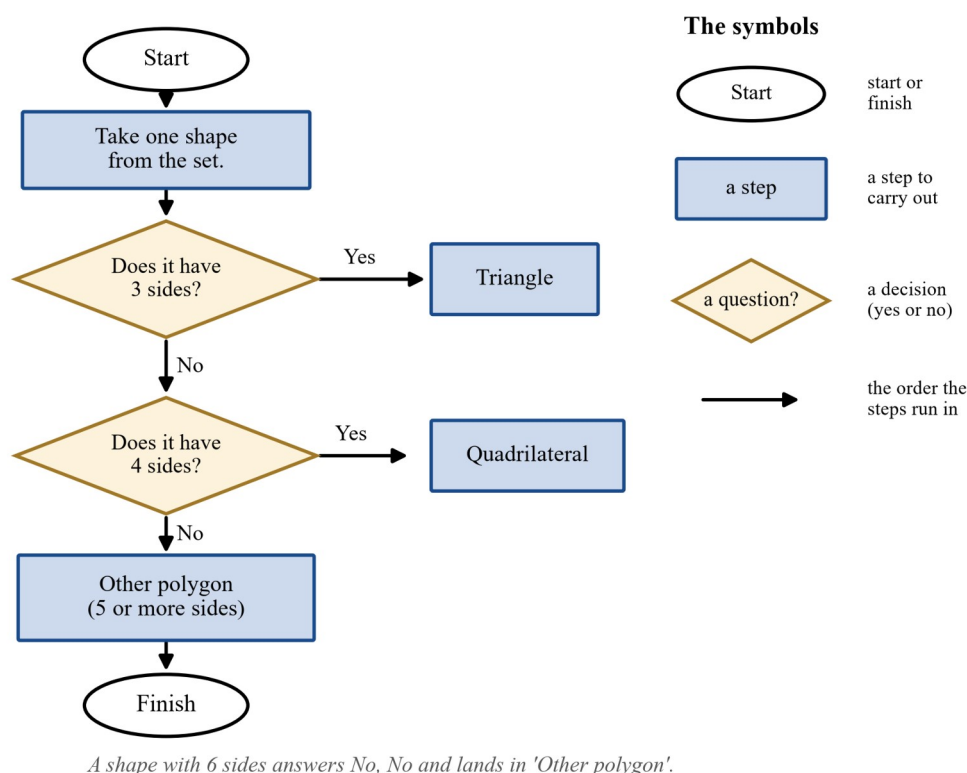


Figure 1. A flow chart that sorts shapes by number of sides — start; take one shape from the set; decision 'does it have 3 sides?' with yes leading to 'triangle'; no leading to the decision 'does it have 4 sides?' with yes leading to 'quadrilateral'; no leading to 'other polygon (5 or more sides)' and then finish — beside a key of the four symbols: an oval for start or finish, a box for a step, a diamond for a decision, and an arrow for the order the steps run in.

To use a flow chart, take one shape, start at the top oval, and follow the arrows, answering every decision about the shape in front of you. Two things must be true of a good chart: every shape in the set reaches a result, and no shape can reach two different results. A chart that sends some shape to the wrong result, to two results, or to no result at all is broken — finding and fixing the break is part of designing algorithms (Worked example 4).

**Sorting triangles.** Subtopic 5B names the six families of triangles, and Figure 2 is the reference strip for them: triangles are sorted by their sides (equilateral, isosceles, scalene) and by their angles (right-angled, acute, obtuse). Equal sides are marked with the same number of ticks, and the small square marks a right angle.

### Triangles sorted by their sides

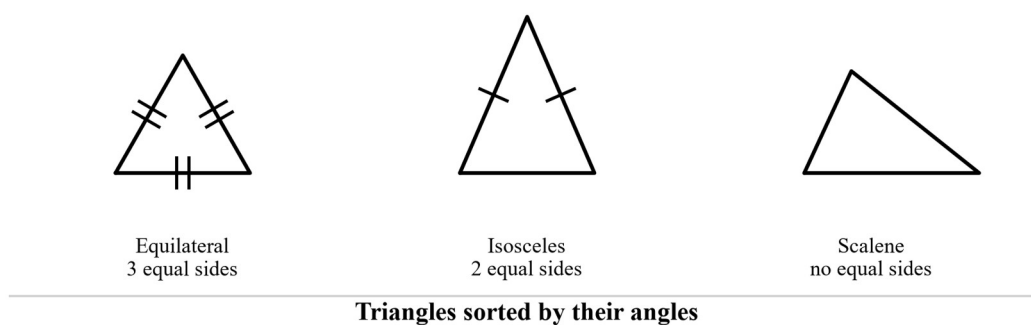
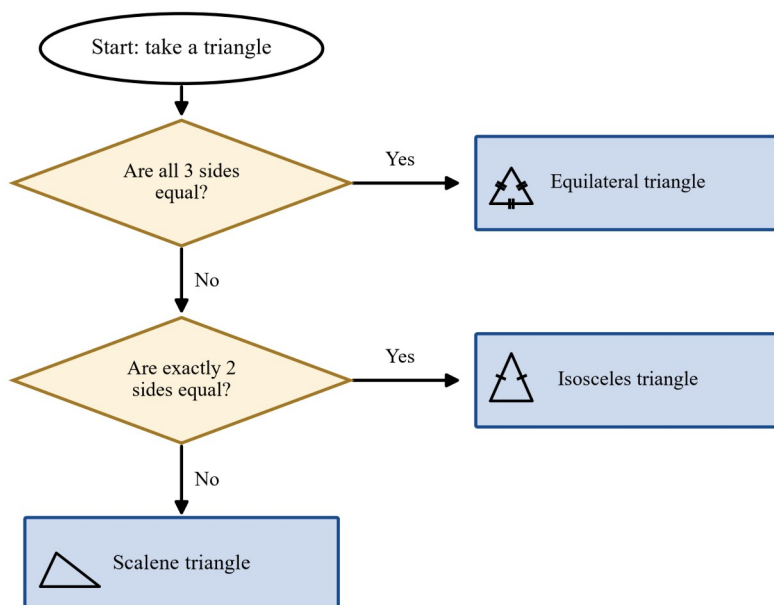


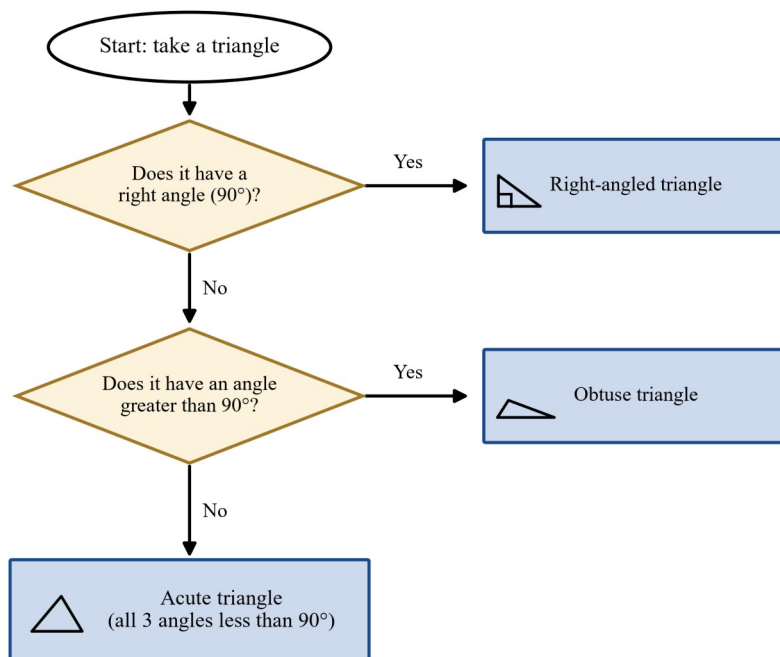
Figure 2. Six triangles in two rows. Top row, sorted by sides: equilateral with two tick marks on each of its 3 equal sides; isosceles with one tick on each of its 2 equal sides; scalene with no equal sides. Bottom row, sorted by angles: right-angled with the right angle marked by a small square; acute with all angles less than  $90^\circ$ ; obtuse with one angle greater than  $90^\circ$ .

Figure 3 is a classification scheme for triangles by their sides, and Figure 4 the matching scheme by their angles. In each chart the decisions are asked in a fixed sequence, and every triangle ends at exactly one result.



Every triangle answers exactly one of the three results.

Figure 3. A flow chart that sorts any triangle by its sides: start; decision 'are all 3 sides equal?' with yes leading to 'equilateral triangle'; no leading to the decision 'are exactly 2 sides equal?' with yes leading to 'isosceles triangle' and no leading to 'scalene triangle'; a small sketch of each kind of triangle sits in its result box.



*A triangle has at most one right angle and at most one obtuse angle, so each triangle reaches exactly one result.*

*Figure 4. A flow chart that sorts any triangle by its angles: start; decision ‘does it have a right angle (90 degrees)?’ with yes leading to ‘right-angled triangle’; no leading to the decision ‘does it have an angle greater than 90 degrees?’ with yes leading to ‘obtuse triangle’ and no leading to ‘acute triangle (all 3 angles less than 90 degrees)’; a small sketch of each kind of triangle sits in its result box.*

The side chart asks ‘all 3’ before ‘exactly 2’ on purpose: an equilateral triangle also has 2 equal sides, so it is a special isosceles triangle as well, and asking ‘all 3’ first gives the shape its most special name. The angle chart works because a triangle has at most one right angle and at most one obtuse angle, so no triangle can reach two results.

**Quadrilaterals: a hierarchy and a flow chart.** The quadrilateral names from 5B fit inside one another, and a **hierarchy** — boxes inside boxes — shows how (Figure 5). A **trapezium** in this book is a quadrilateral with *exactly* one pair of parallel sides; some books say ‘at least one pair’, and this book does not. On that rule, trapeziums sit outside the parallelogram box. A **kite** has two pairs of equal sides next to each other; a rhombus has this feature too, so a rhombus is a special kite, though the figure keeps the kite box separate from the parallelogram box to keep all six names easy to see — the flow chart of Figure 6 sorts it out by giving every shape its most special name.

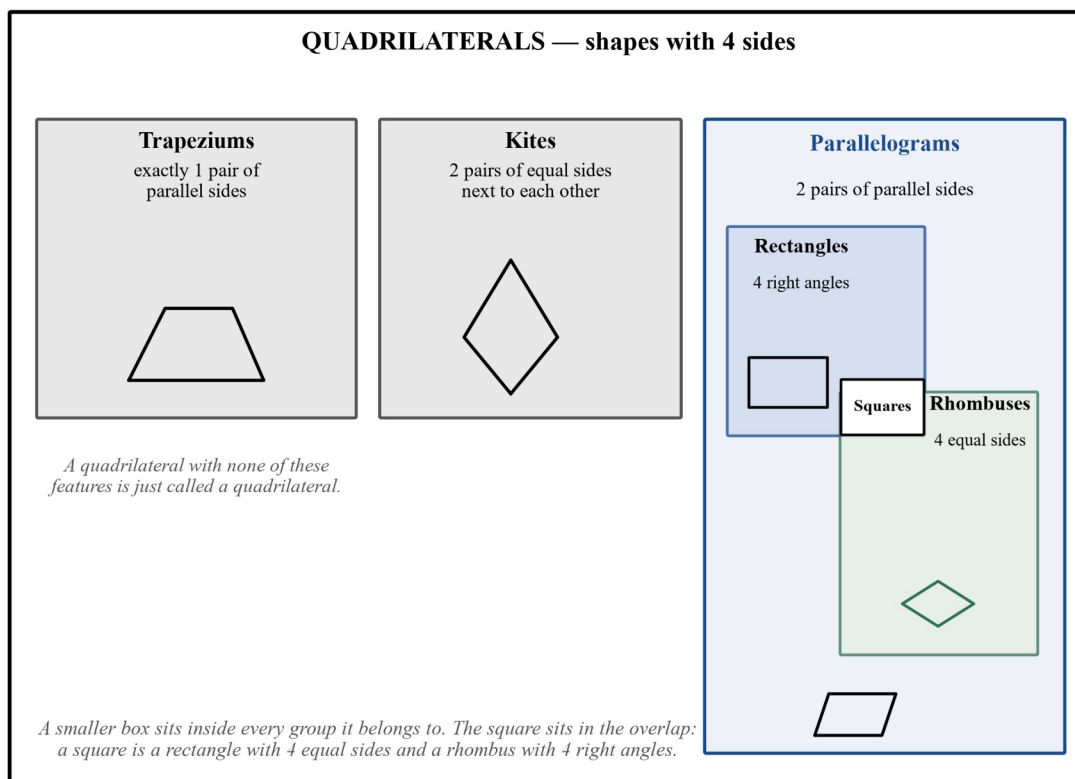
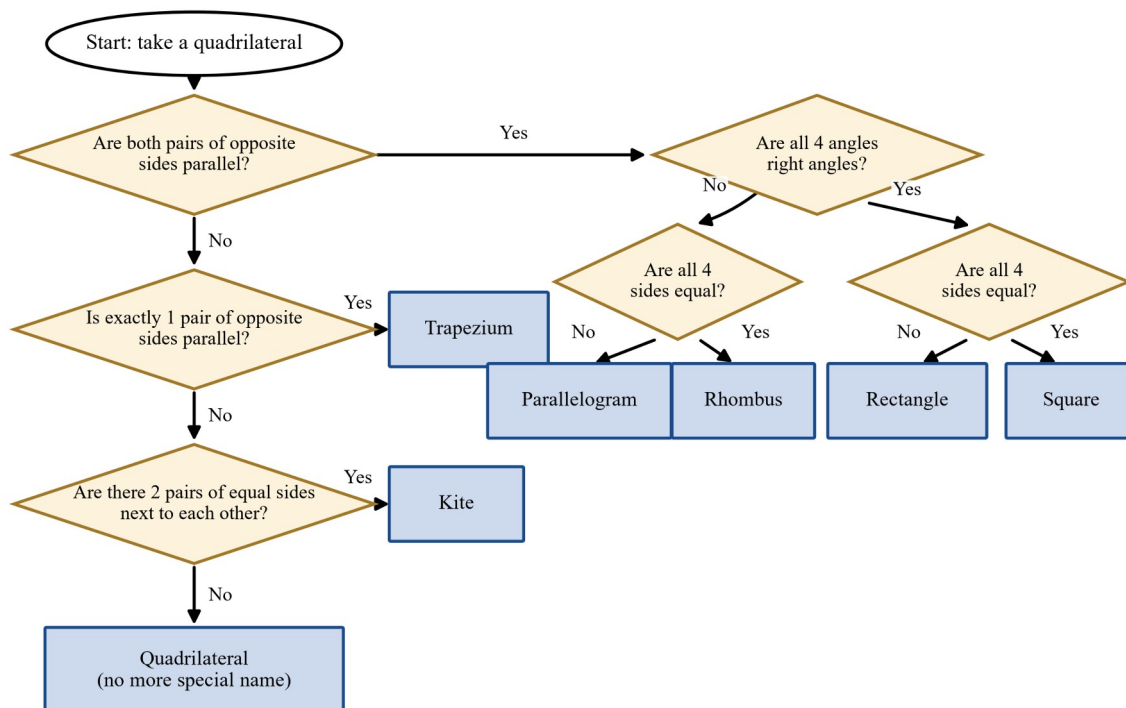


Figure 5. A hierarchy of quadrilaterals drawn as boxes inside boxes: one large box ‘quadrilaterals — shapes with 4 sides’ holds three boxes — trapeziums (exactly 1 pair of parallel sides), kites (2 pairs of equal sides next to each other), and parallelograms (2 pairs of parallel sides). Inside the parallelogram box, the rectangles box (4 right angles) and the rhombuses box (4 equal sides) overlap, and a small ‘squares’ box sits in the overlap. A sketch of each shape sits in its box.

The overlap is the point: a square is a rectangle with 4 equal sides *and* a rhombus with 4 right angles, so the squares box sits inside both. A smaller box sits inside every group it belongs to — which is why a chart that names a shape must give its most special name, and the hierarchy then lists every group the shape belongs to.

Figure 6 turns the hierarchy into an algorithm: a flow chart that gives any quadrilateral its most special name.



The flow chart gives each quadrilateral its most special name. The hierarchy of Figure 5 shows the groups each shape belongs to — a square is also a rectangle and a rhombus.

Figure 6. A decision flow chart that names any quadrilateral: start; ‘are both pairs of opposite sides parallel?’ — yes leads to the parallelogram family: ‘are all 4 angles right angles?’ with yes then ‘are all 4 sides equal?’ giving square (yes) or rectangle (no), and no then ‘are all 4 sides equal?’ giving rhombus (yes) or parallelogram (no); the first decision’s no leads to ‘is exactly 1 pair of opposite sides parallel?’ with yes giving trapezium; no leads to ‘are there 2 pairs of equal sides next to each other?’ with yes giving kite and no giving ‘quadrilateral (no more special name)’.

**All polygons: sorted by number of sides.** The same ideas reach every polygon. Figure 7 sorts a set of polygons by their number of sides, and within each number of sides shows three rows: **regular** (all sides equal and all angles equal), **irregular but convex** (no dent), and **concave** (one dent — an angle greater than  $180^\circ$ ). There is no concave triangle: with only 3 sides, no dent can form.

|                    | Number of sides                                                                   |                                                                                   |                                                                                   |                                                                                   |                                                                                     |                                                                                     |
|--------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|                    | 3<br>triangle                                                                     | 4<br>quadrilateral                                                                | 5<br>pentagon                                                                     | 6<br>hexagon                                                                      | 7<br>heptagon                                                                       | 8<br>octagon                                                                        |
| Regular            |  |  |  |  |  |  |
| Irregular (convex) |  |  |  |  |  |  |
| Concave            | (none — every triangle is convex)                                                 |  |  |  |  |  |

Figure 7. A set of polygons shown in a grid: columns for 3 to 8 sides (triangle, quadrilateral, pentagon, hexagon, heptagon, octagon) and rows for regular, irregular (convex) and concave examples of each, drawn from real geometry; the concave row for triangles reads ‘none — every triangle is convex’.

Figure 8 is the classification scheme the gallery follows: first count the sides and name the polygon by that number, then decide regular or not, and for the not-regular shapes decide concave (a dent) or convex (no dent). A regular polygon is always convex.

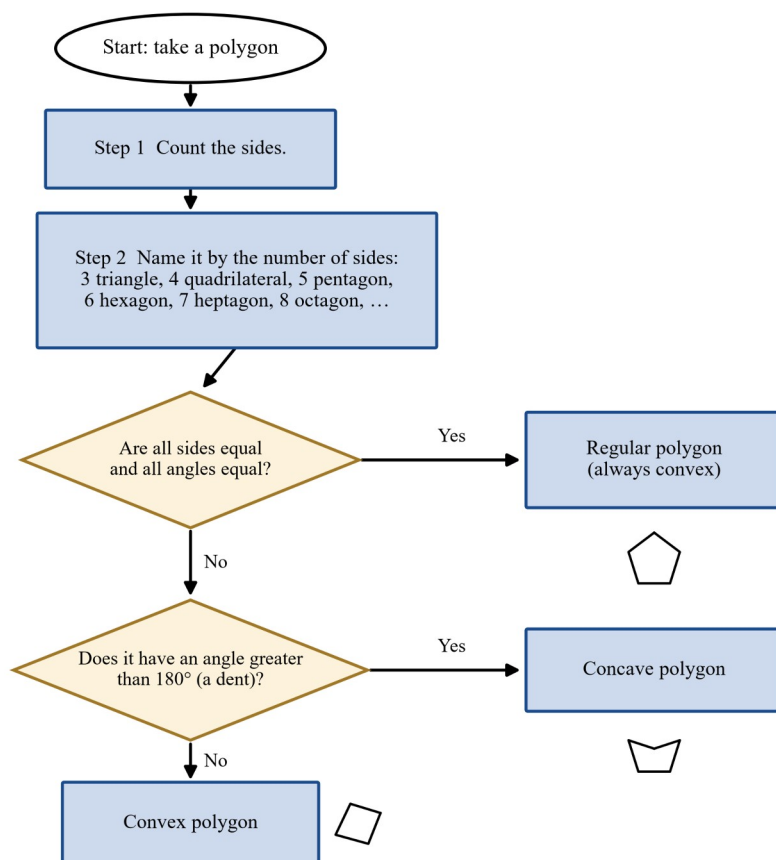


Figure 8. A classification scheme for any polygon: start; step 1 count the sides; step 2 name it by the number of sides (3 triangle, 4 quadrilateral, 5 pentagon, 6 hexagon, 7 heptagon, 8 octagon, and so on); decision ‘are all sides equal and all angles equal?’ with yes leading to ‘regular polygon (always convex)’; no leading to the decision ‘does it have

*an angle greater than 180 degrees (a dent)?' with yes leading to 'concave polygon' and no to 'convex polygon'; a small example shape sits under each result.*

**The words of this subtopic**, with synonyms you may meet:

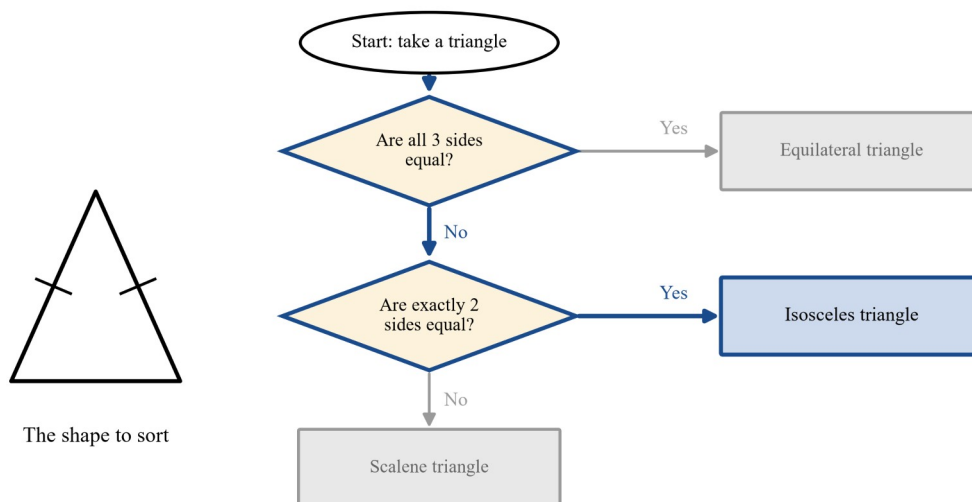
| Term                    | Meaning here                                                                                                         | Synonyms and related words   |
|-------------------------|----------------------------------------------------------------------------------------------------------------------|------------------------------|
| <b>algorithm</b>        | a set of steps and decisions that sorts or classifies shapes and always finishes                                     | method, recipe, set of rules |
| <b>flow chart</b>       | a picture of an algorithm: ovals for start and finish, boxes for steps, diamonds for decisions, arrows for the order | decision diagram             |
| <b>decision</b>         | a yes-or-no question inside an algorithm                                                                             | question, choice             |
| <b>sequence</b>         | the fixed order the steps of an algorithm run in                                                                     | order                        |
| <b>attribute</b>        | a feature of a shape the algorithm checks: number of sides, equal sides, right angles, a dent                        | feature, property            |
| <b>hierarchy</b>        | boxes inside boxes showing which groups of shapes sit inside which                                                   | family tree of shapes        |
| <b>regular</b>          | of a polygon: all sides equal and all angles equal                                                                   | —                            |
| <b>convex / concave</b> | no dent / at least one dent (an angle greater than $180^\circ$ )                                                     | —                            |
| <b>trapezium</b>        | in this book: a quadrilateral with exactly one pair of parallel sides                                                | —                            |
| <b>kite</b>             | a quadrilateral with two pairs of equal sides next to each other                                                     | —                            |

Sorting shapes by their features is the same job Science does with living things: a flow chart of yes–no decisions works exactly like a key that sorts living things into groups. The maths of this subtopic — a sequence of steps and decisions — carries straight across.

## Worked examples

### Example 1 — scaffolded

The triangle in Figure 9 has two sides marked with a single tick. Classify it by its sides using Figure 3, then by its angles using Figure 4.



Trace: 3 equal sides? No. Exactly 2 equal sides? Yes. Result: isosceles triangle.

Figure 9. The triangle to sort, with one tick on each of two equal sides, beside the flow chart of Figure 3 with the path taken drawn in bold: ‘are all 3 sides equal?’ no, then ‘are exactly 2 sides equal?’ yes, reaching ‘isosceles triangle’; the paths not taken are greyed.

| Working                                                                                                                                                              | Comment                                                                            |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| <b>Read the shape.</b> Two sides carry the same tick, so exactly those two sides are equal. The third side has no tick and is a different length.                    | Ticks mean ‘same length’, as in 5B: same ticks, same length.                       |
| <b>Figure 3, decision 1:</b> are all 3 sides equal? <b>No</b> — only two are marked equal.                                                                           | A ‘no’ at the first decision does not mean the triangle has no equal sides at all. |
| <b>Figure 3, decision 2:</b> are exactly 2 sides equal? <b>Yes</b> . Result: <b>isosceles triangle</b> .                                                             | The bold path in Figure 9 is this run.                                             |
| <b>Figure 4, decision 1:</b> does it have a right angle? <b>No</b> — no corner is marked with the small square, and none looks like one.                             | Check the marking, not just how the shape looks.                                   |
| <b>Figure 4, decision 2:</b> does it have an angle greater than $90^\circ$ ? <b>No</b> — all three angles are less than $90^\circ$ . Result: <b>acute triangle</b> . | No right angle and no obtuse angle leaves acute.                                   |

$\therefore$  the shape is an isosceles triangle by its sides and an acute triangle by its angles: an isosceles acute triangle.

### Example 2 — scaffolded

Run a square through the flow chart of Figure 6 to get its most special name, then use the hierarchy of Figure 5 to list every group the square belongs to.

| Working                                                                                                                                                  | Comment                                              |
|----------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| <b>Decision 1:</b> are both pairs of opposite sides parallel? <b>Yes</b> — a square’s opposite sides are parallel.                                       | Yes sends the square to the parallelogram family.    |
| <b>Decision 2:</b> are all 4 angles right angles? <b>Yes</b> .                                                                                           | A square has four $90^\circ$ angles.                 |
| <b>Decision 3:</b> are all 4 sides equal? <b>Yes</b> . Result: <b>square</b> .                                                                           | The chart stops at the most special name.            |
| <b>Figure 5:</b> the squares box sits inside the rectangles box, inside the rhombuses box, inside the parallelograms box, inside the quadrilaterals box. | A smaller box sits inside every group it belongs to. |

$\therefore$  the most special name is square, and the square also belongs to the rectangles, the rhombuses, the parallelograms and the quadrilaterals — five names in all.

### Example 3 — unscaffolded

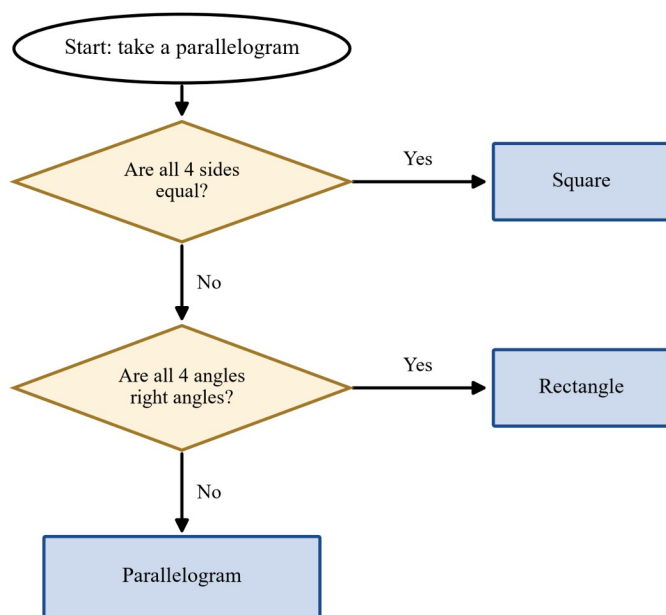
A shape has 5 sides. Its sides are not all equal, and one of its angles is greater than  $180^\circ$  (the shape has a dent, like the 5-sided shape in the bottom row of Figure 7). Classify the shape using the scheme of Figure 8.

Step 1 names the shape by its number of sides: 5 sides gives a **pentagon**. Step 2's first decision asks whether all sides and all angles are equal — they are not, so the shape is **not regular**. The second decision asks whether it has an angle greater than  $180^\circ$  — it does, so the shape is **concave**.

$\therefore$  the shape is a concave, irregular pentagon.

### Example 4 — unscaffolded

Mia designed the flow chart of Figure 10 to name the members of the parallelogram family: parallelograms, rectangles, rhombuses and squares.



*Mia's algorithm for the parallelogram family*

Figure 10. Mia's algorithm for the parallelogram family: start, take a parallelogram; decision 'are all 4 sides equal?' with yes leading to 'square'; no leading to the decision 'are all 4 angles right angles?' with yes leading to 'rectangle' and no leading to 'parallelogram'.

- Run a rhombus that is not a square through Mia's chart and write down the name it reaches.
- Explain what is wrong with the chart, and fix it by adding one decision.
- A rhombus has 4 equal sides, so the first decision is **yes** and the chart writes **square** — the wrong name for a rhombus that is not a square.
- The chart decides 'square' from equal sides alone, but a square needs equal sides *and* right angles. The fix: after 'are all 4 sides equal?' yes, add the decision 'are all 4 angles right angles?' — yes gives square, no gives rhombus. With the fix, a square still reaches square and a rhombus reaches rhombus, and the rest of the chart was already right for the family: a shape without 4 equal sides and without 4 right angles is a parallelogram, and one with 4 right angles but not 4 equal sides is a rectangle.

$\therefore$  Mia's chart misnames a rhombus as a square; one added decision — 'are all 4 angles right angles?' after the yes — puts it right.

## Practice questions

### Scaffolded

**Q1.** Figure 1 shows the four symbols of a flow chart. (a) Which symbol holds a decision? (b) Which symbol shows the order the steps run in? (c) A step to carry out is written in which symbol?

**Q2.** Run the algorithm of Figure 1 on a hexagon. Write down the answer at each decision and the result the hexagon reaches.

**Q3.** Copy and complete this table for a scalene triangle run through the flow chart of Figure 3.

| Decision                   | Answer |
|----------------------------|--------|
| Are all 3 sides equal?     | No     |
| Are exactly 2 sides equal? |        |
| Result                     |        |

**Q4.** Use the hierarchy of Figure 5. Write true or false for each. (a) Every square is a rectangle. (b) Every rectangle is a square. (c) Every rhombus is a parallelogram. (d) Every trapezium is a parallelogram. (e) Every square is a parallelogram.

**Q5.** A polygon has 7 sides. All of its sides are equal, but its angles are not all equal, and it has no dent. Use the scheme of Figure 8. (a) Name the polygon by its number of sides. (b) Is it regular? Give a reason. (c) Is it convex or concave?

**Q6.** A regular hexagon is drawn. (a) How many sides does it have? (b) Use Figure 8 to decide whether it is convex or concave. (c) True or false: a regular polygon can have a dent.

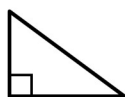
### Un scaffolded

**Q7.** Write the algorithm of Figure 4 as a list of numbered steps in words, without drawing a flow chart.

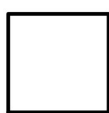
**Q8.** Shape F in Figure 11 is a kite. Run it through the flow chart of Figure 6, writing down the answer at every decision and the result it reaches.



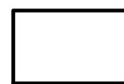
A



B



C



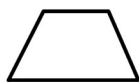
D



E



F



G



H

*Figure 11. Eight shapes labelled A to H for sorting: A an equilateral triangle; B a right-angled triangle with the right angle marked; C a square; D a rectangle; E a rhombus with a tick on each of its 4 equal sides; F a kite; G a trapezium; H a concave pentagon.*

**Q9.** In Figure 5 the small squares box sits inside the rectangles box and inside the rhombuses box at once. In one sentence each, write the two features this says a square has.

- Q10.** Choose one attribute of the shapes in Figure 11, and use it to sort the eight shapes into two groups. State the attribute and list the two groups.
- Q11.** Design a flow chart that sorts the shapes A–H of Figure 11 into three groups: triangles, quadrilaterals and pentagons.
- Q12.** Describe in words how your algorithm of Q11 works: say what it checks, in what order, and where each of the eight shapes ends.
- Q13.** ‘Every convex polygon is regular.’ True or false? Use one shape from Figure 7 to back up your answer.
- Q14.** Tara redraws Figure 3 but writes the first decision as ‘does the triangle have 2 equal sides?’, with yes leading straight to ‘isosceles triangle’ and no leading to a second decision, ‘are all 3 sides equal?’, with yes leading to ‘equilateral triangle’ and no to ‘scalene triangle’. (a) Run an equilateral triangle through Tara’s chart. What result does it reach? (b) Explain why that result is not the equilateral triangle’s most special name. (c) Reword the first decision so the chart gives every triangle its most special name, keeping the order of the decisions as Tara has it.
-

## Answers

**Q1.** (a) the diamond; (b) the arrow; (c) the box.

**Q2.** ‘Does it have 3 sides?’ no. ‘Does it have 4 sides?’ no. Result: other polygon (5 or more sides).

**Q3.** No; scalene triangle.

**Q4.** (a) true; (b) false; (c) true; (d) false; (e) true.

**Q5.** (a) heptagon; (b) no — regular needs all sides equal *and* all angles equal; (c) convex.

**Q6.** (a) 6; (b) convex — a regular polygon is always convex (Figure 8); (c) false.

**Q7.** 1. Take a triangle. 2. If it has a right angle, write ‘right-angled triangle’ and stop. 3. If it has an angle greater than  $90^\circ$ , write ‘obtuse triangle’ and stop. 4. Otherwise write ‘acute triangle’.

**Q8.** Both pairs of opposite sides parallel? No. Exactly 1 pair of opposite sides parallel? No. Two pairs of equal sides next to each other? Yes. Result: kite.

**Q9.** A square has 4 right angles (the rectangle feature) and 4 equal sides (the rhombus feature).

**Q10.** Any attribute that splits the set cleanly; for example ‘has 4 sides’: group 1 C, D, E, F, G; group 2 A, B, H.

**Q11.** Start; ‘does it have 3 sides?’ yes → triangle; no → ‘does it have 4 sides?’ yes → quadrilateral; no → pentagon; finish.

**Q12.** The chart counts sides: 3 sends the shape to triangles, 4 to quadrilaterals, and the rest (5) to pentagons; A and B end at triangles; C, D, E, F, G at quadrilaterals; H at pentagons.

**Q13.** False — any shape in the ‘irregular (convex)’ row of Figure 7, such as the convex quadrilateral, is convex but not regular.

**Q14.** (a) isosceles triangle; (b) an equilateral triangle has 3 equal sides, so ‘2 equal sides’ is true of it, but its most special name is equilateral; (c) ‘are *exactly* 2 sides equal?’ — then an equilateral triangle answers no and falls through to the ‘all 3 sides equal’ decision.

---

## Fully-worked solutions

**Q1.** (a) A decision is a yes-or-no question, and questions sit in the diamond. (b) Arrows join the symbols and show which step runs next — the sequence. (c) Steps to carry out sit in boxes; the ovals hold only start and finish.

**Q2.** A hexagon has 6 sides. ‘Does it have 3 sides?’ no. ‘Does it have 4 sides?’ no. The run leaves the second decision by the no arrow and lands in ‘other polygon (5 or more sides)’ — correct, since 6 is more than 4.

**Q3.** A scalene triangle has no equal sides, so ‘are exactly 2 sides equal?’ is no, and the no arrow leads to the result scalene triangle. The chart names the shape it was given — a good check on any run.

**Q4.** (a) True: the squares box sits inside the rectangles box. (b) False: a rectangle without 4 equal sides is not a square. (c) True: the rhombuses box sits inside the parallelograms box. (d) False: in this book a trapezium has exactly one pair of parallel sides, so it sits outside the parallelograms box. (e) True: the squares box sits inside the parallelograms box (through the rectangles and rhombuses boxes).

**Q5.** (a) 7 sides names it a heptagon (step 2 of Figure 8). (b) Not regular: regular needs all sides equal and all angles equal, and the angles are not all equal. (c) No dent means the second decision is no: convex.

**Q6.** (a) A hexagon has 6 sides. (b) It is regular, and Figure 8 says a regular polygon is always convex, so it is convex. (c) False: a dent is an angle greater than  $180^\circ$ , and the regular box of Figure 8 sits away from the concave box — regular polygons have no dent.

**Q7.** Read straight down the chart, with each yes ending the run: 1. Take a triangle. 2. Does it have a right angle? If yes, write ‘right-angled triangle’ and stop. 3. Does it have an angle greater than  $90^\circ$ ? If yes, write ‘obtuse triangle’ and stop. 4. Write ‘acute triangle’. The numbered list and the flow chart are the same algorithm in two forms.

**Q8.** Shape F is a kite. Decision 1: its opposite sides are not parallel, so no. Decision 2: no pair of opposite sides is parallel, so ‘exactly 1 pair’ is no. Decision 3: the two top sides are equal and the two bottom sides are equal — two pairs of equal sides next to each other — yes. Result: kite. The chart returns the name the shape was given, which checks the run.

**Q9.** Sitting in the rectangles box says a square has 4 right angles; sitting in the rhombuses box says it has 4 equal sides. Together those two features are exactly what a square is.

**Q10.** The attribute must put every shape in one group only. ‘Has 4 sides’ works: C, D, E, F and G have 4 sides; A, B (3 sides) and H (5 sides) do not. Equally good: ‘has a right angle’ gives B, C, D against A, E, F, G, H. An attribute that puts one shape in both groups — say ‘has equal sides’, since C and E both do but so does A — still sorts, as long as the two groups are stated as ‘has it / does not’.

**Q11.** Start; take one shape; decision ‘does it have 3 sides?’ — yes leads to the box ‘triangle’; no leads to the decision ‘does it have 4 sides?’ — yes leads to ‘quadrilateral’, no leads to ‘pentagon’; finish. Three results, and every shape in the set reaches exactly one.

**Q12.** The algorithm checks one attribute — the number of sides — in a fixed sequence of two decisions: 3 sides ends the run at triangle, 4 sides at quadrilateral, and any other count at pentagon. A and B have 3 sides, so both end at triangle; C, D, E, F, G have 4 sides and end at quadrilateral; H has 5 sides and ends at pentagon. No shape can reach two groups, because the yes arrows leave the run.

**Q13.** False. The second row of Figure 7 is labelled ‘irregular (convex)’: every shape in it has no dent, so it is convex, yet none has all sides and all angles equal, so none is regular. The convex quadrilateral in that row is one counter to the claim; a rectangle would be another.

**Q14.** (a) An equilateral triangle has 2 equal sides (it has 3, which includes 2), so Tara’s first decision is yes and the chart writes isosceles triangle. (b) The most special name of the shape is equilateral — all 3 sides equal — and ‘isosceles’ says less about it than that; the chart never asked whether all 3 sides were equal. (c) Reword as ‘are *exactly* 2 sides equal?’. Then an equilateral triangle answers no (3 is not exactly 2) and falls to the next decision, ‘all 3 sides equal?’, which answers yes; isosceles triangles still answer yes at the first decision, and scalene triangles no at both. Wording and order are what make an algorithm sort correctly.

## Teacher note

- **All three elaboration areas delivered.** E01: Figures 2–4 (classification schemes for triangles by sides and by angles, as flow charts using sequences and decisions), WE1, Q2, Q3, Q7, Q14. E02: Figures 5–6 (both a hierarchy *and* a flow chart for quadrilaterals, covering trapeziums, parallelograms, rhombuses, rectangles, squares and kites), WE2, Q4, Q8, Q9. E03: Figures 7–8 (regular/irregular/concave/convex polygons sorted by number of sides), WE3, Q5, Q6, Q13. The design-and-describe verbs of the CD are carried by Q10–Q12 (design a flow chart, then describe how it works) and by WE4/Q14 (debugging decisions). A single flow chart for one of the three areas is not delivery of a CD whose verb is plural.
- **Level boundary (D4.11).** No congruency testing anywhere — that is Level 8 (VC2M8SP04). Features are read from markings (ticks, the right-angle square) placed on the figures, never proved; every figure is generated from real geometry with its claimed features asserted at generation time (equal sides exactly equal, right angles exactly  $90^\circ$ , concave shapes carrying exactly one reflex angle, convex shapes passing a convexity check).
- **Vocabulary ownership (D4).** The triangle and quadrilateral names and their definitions are taught in 5B (VC2M7SP02); 5C takes them as given and builds algorithms on them, per the elaborations’ own division of labour. Figure 2 is a reference strip, not a re-teach.
- **Contested term — trapezium.** This book’s frame, stated at first use in the theory and in the words table: exactly one pair of parallel sides, with the ‘at least one pair’ frame named as the one some other books use. Under this frame trapeziums sit outside the parallelogram box in Figures 5–6, and Q4(d) is false. The kite

definition given (two pairs of equal sides next to each other) makes a rhombus a special kite; that is said once in the theory rather than drawn, keeping Figure 5 legible.

- **Cross-subject parallel (§5.3).** A flow chart of yes–no decisions that sorts a set is structurally the same instrument as a dichotomous key in Science VC2S8U01; the closing paragraph of the theory names the parallel for students, and it can be pointed to when the cohort meets keys in Science.
- **Assessment (T5).** T5 is 20 marks in 25 minutes with 5 marks per subtopic and reasoning marked separately; Q13 and Q14 are the reasoning-shaped items here (counterexample; decision wording and order), and Q12 assesses the CD’s ‘describe how the algorithms work’ verb.
- **Digital tools (D9).** VC2M7SP04 names no digital tools and none are used; algorithms are written as numbered steps and flow charts, never in a named programming language. No CAS.
- **Content policy.** Aboriginal and Torres Strait Islander content is excluded (Rule 1); VC2M7SP04 and its elaborations carry none.

## Transformations of points on the Cartesian plane

**Content description VC2M7SP03:** describe the effect of transformations of a set of points using coordinates in the Cartesian plane, including translations, reflections in an axis, and rotations about the origin

**Achievement standard (extract):** They use coordinates to describe transformations of points in the plane.

### Theory

**What this subtopic is about.** At Level 6 you slid, flipped and turned shapes on squared paper and described what you saw. This subtopic makes the same moves on the **Cartesian plane** and describes each one with **coordinates**. Instead of saying “the shape moved to the right”, you write a rule such as  $(x, y) \rightarrow (x + 4, y + 2)$  that says exactly where every point goes. The three transformations here are **translations** (slides), **reflections in an axis** (flips) and **rotations about the origin** (turns). They all happen on the same plane you used for plotting points in subtopic 3D.

**Recall the plane.** The two axes of the Cartesian plane cross at the point  $(0, 0)$ , called the **origin** and labelled  $O$ . The axes divide the plane into four **quadrants**. Every point has an ordered pair of coordinates  $(x, y)$ : the  $x$ -coordinate says how far left or right, and the  $y$ -coordinate says how far up or down. Figure 1 shows the points  $P(3, 2)$  and  $Q(-4, -3)$ :  $P$  is 3 right and 2 up from  $O$ , while  $Q$  is 4 left and 3 down.

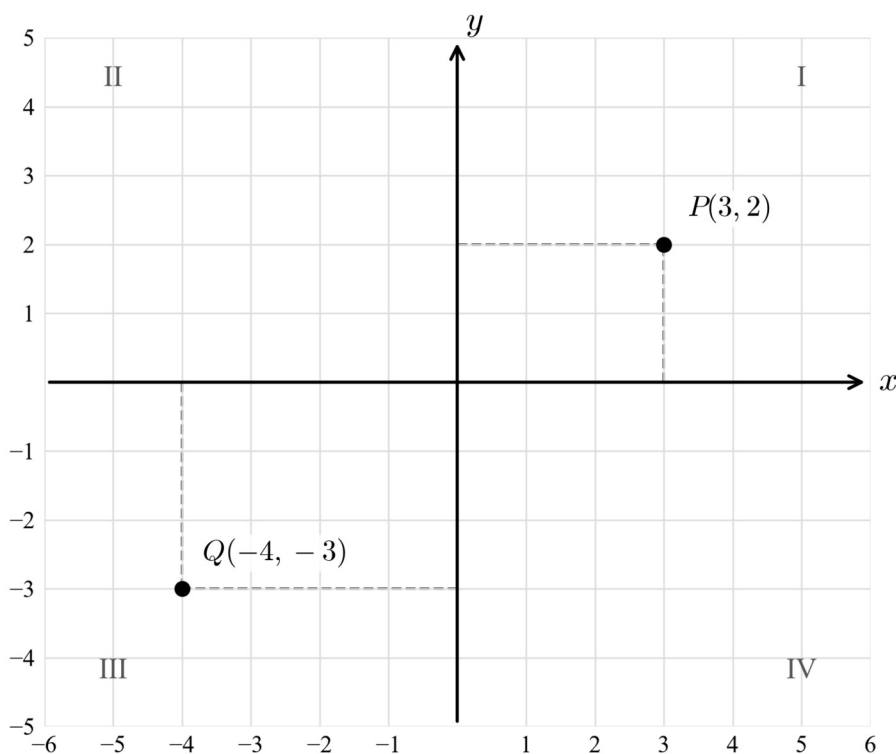


Figure 1. A Cartesian plane with its four quadrants labelled I, II, III and IV, showing the point  $P(3, 2)$  in quadrant I and the point  $Q(-4, -3)$  in quadrant III, each with dashed guide lines to both axes.

**Translations — slides.** A **translation** slides every point the same distance in the same direction. The image is the same size and the same shape as the original, and it faces the same way. To translate a point  $a$  units to the right and  $b$  units up, add  $a$  to the  $x$ -coordinate and add  $b$  to the  $y$ -coordinate:

$$(x, y) \rightarrow (x + a, y + b)$$

A slide to the left or down subtracts instead. Figure 2 translates the triangle with vertices  $A(1, 2)$ ,  $B(4, 2)$  and  $C(2, 5)$  by 5 right and 3 up: every  $x$ -coordinate increases by 5 and every  $y$ -coordinate increases by 3, giving  $A'(6, 5)$ ,  $B'(9, 5)$  and  $C'(7, 8)$ . The apostrophe mark is called **prime**: the image of point  $A$  is named  $A'$ , read “ $A$  prime”.

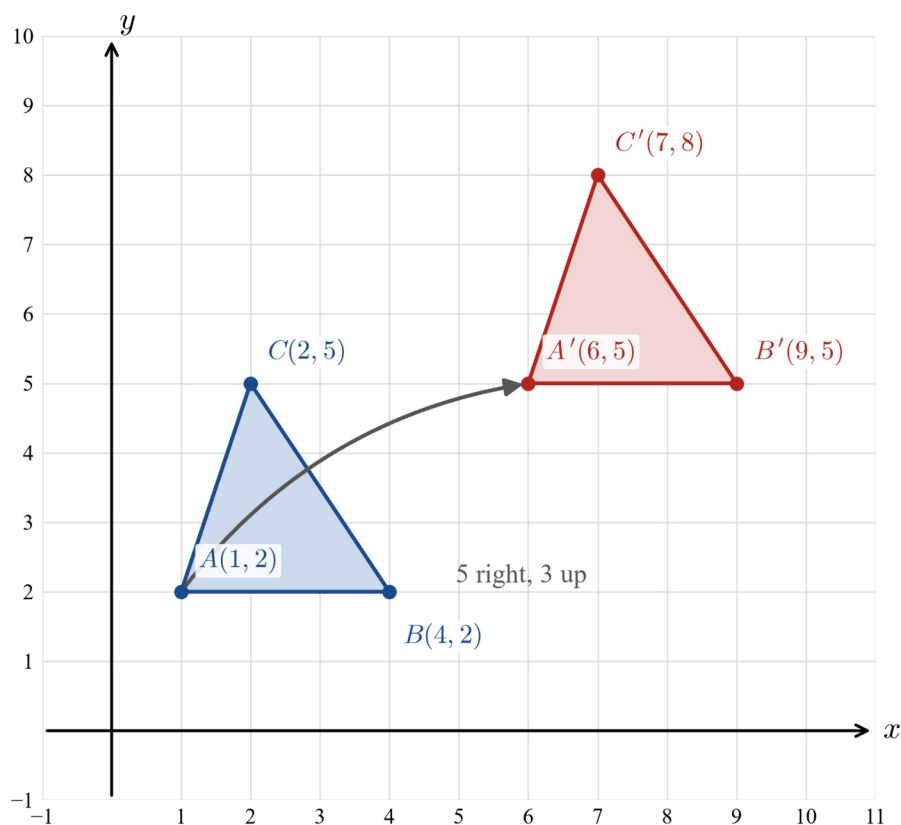


Figure 2. The triangle  $A(1, 2)$ ,  $B(4, 2)$ ,  $C(2, 5)$  translated 5 units right and 3 units up to the image triangle  $A'(6, 5)$ ,  $B'(9, 5)$ ,  $C'(7, 8)$ , with a curved arrow from  $A$  to  $A'$  labelled 5 right, 3 up.

**Reflections — flips.** A **reflection** flips every point over a mirror line. In this subtopic the mirror line is always one of the axes. Each point and its image are the same distance from the mirror line, on opposite sides of it. The image is the same size and the same shape as the original, but it faces the opposite way.

*Reflection in the x-axis.* The x-coordinate stays the same and the y-coordinate flips sign:

$$(x, y) \rightarrow (x, -y)$$

Figure 3 reflects  $S(3, 4)$  in the x-axis. The point  $S$  is 4 units above the mirror line, so  $S'$  is 4 units below it:  $S'(3, -4)$ .

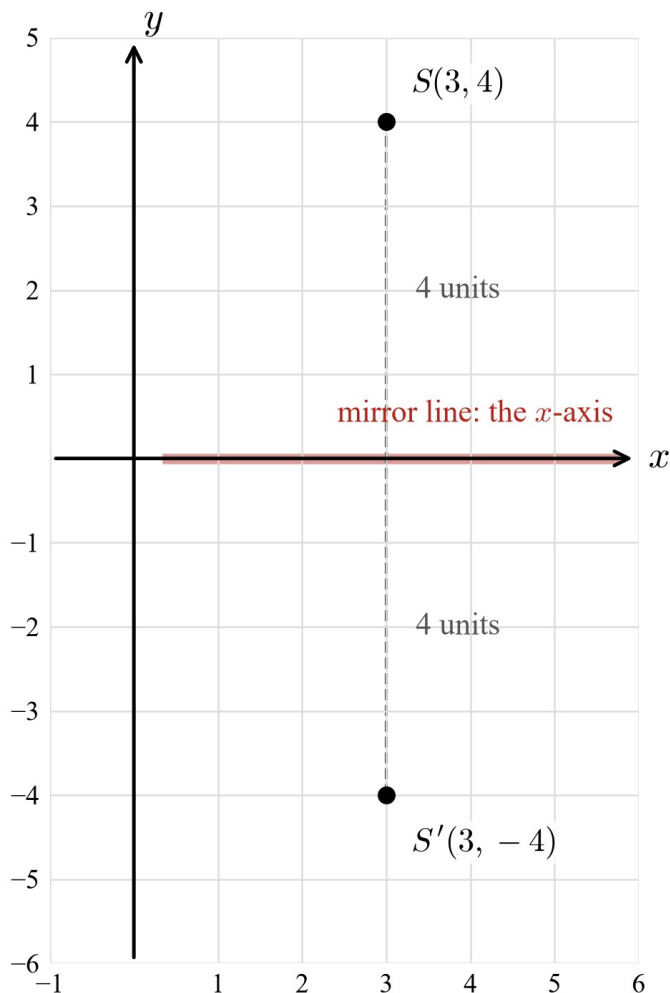


Figure 3. The point  $S(3, 4)$  reflected in the  $x$ -axis, which is marked as the mirror line, to the point  $S'(3, -4)$ ; the dashed join runs perpendicular to the axis, with both points marked as 4 units from it.

Reflection in the  $y$ -axis. The  $y$ -coordinate stays the same and the  $x$ -coordinate flips sign:

$$(x, y) \rightarrow (-x, y)$$

Figure 4 reflects the triangle with vertices  $D(2, 1)$ ,  $E(5, 1)$  and  $F(3, 4)$  in the  $y$ -axis. Each  $x$ -coordinate flips sign and each  $y$ -coordinate is unchanged, giving  $D'(-2, 1)$ ,  $E'(-5, 1)$  and  $F'(-3, 4)$ .

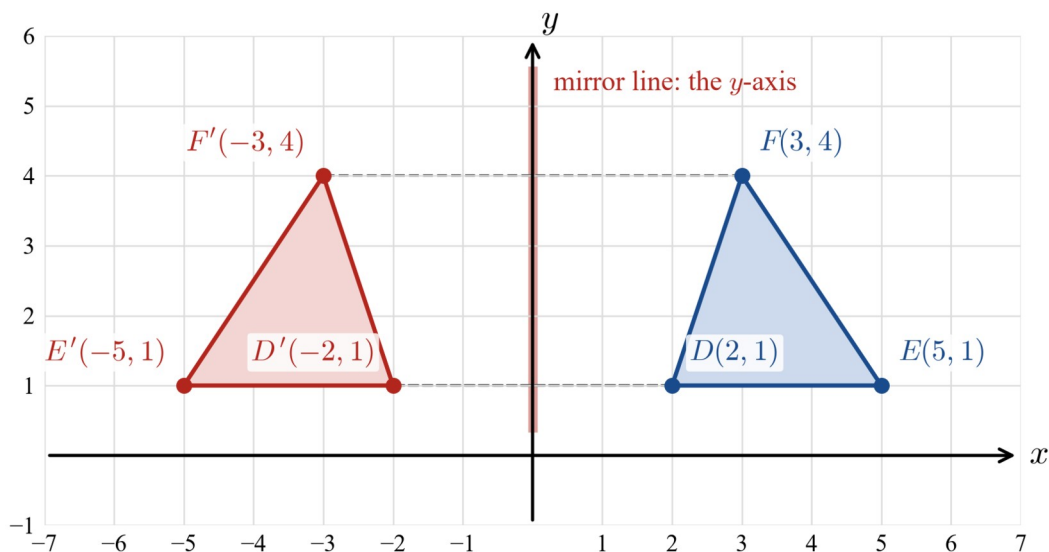


Figure 4. The triangle  $D(2, 1)$ ,  $E(5, 1)$ ,  $F(3, 4)$  reflected in the  $y$ -axis, which is marked as the mirror line, to the image triangle  $D'(-2, 1)$ ,  $E'(-5, 1)$ ,  $F'(-3, 4)$ , with dashed joins from each vertex to its image.

**Rotations — turns.** A **rotation** turns every point about a fixed centre through a given angle in a given direction. In this subtopic the centre is always the origin  $O$ . The two turns to use are a quarter-turn,  $90^\circ$ , and a half-turn,  $180^\circ$ . Clockwise and anticlockwise are the two possible directions; for a half-turn, both directions give the same result. The image is the same size and the same shape as the original.

The rules for rotations about the origin are:

| Rotation about the origin      | Rule                          |
|--------------------------------|-------------------------------|
| $90^\circ$ anticlockwise       | $(x, y) \rightarrow (-y, x)$  |
| $90^\circ$ clockwise           | $(x, y) \rightarrow (y, -x)$  |
| $180^\circ$ (either direction) | $(x, y) \rightarrow (-x, -y)$ |

Figure 5 rotates the triangle with vertices  $U(2, 1)$ ,  $V(5, 1)$  and  $W(2, 3)$  by  $90^\circ$  anticlockwise about  $O$ , using the rule  $(x, y) \rightarrow (-y, x)$ : the coordinates swap roles, and the new  $x$ -coordinate is negative. The image is  $U'(-1, 2)$ ,  $V'(-1, 5)$  and  $W'(-3, 2)$ .

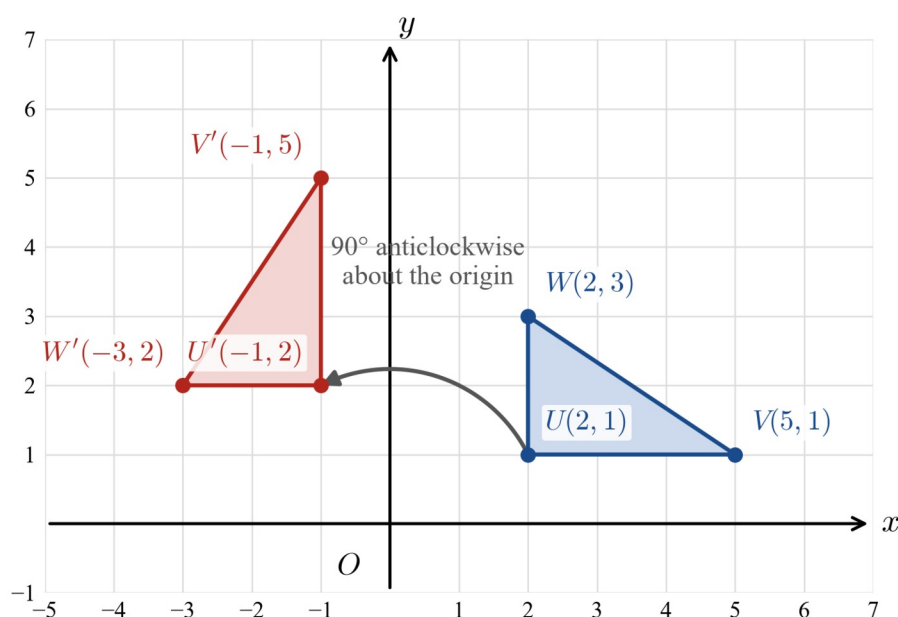


Figure 5. The triangle  $U(2, 1)$ ,  $V(5, 1)$ ,  $W(2, 3)$  rotated 90 degrees anticlockwise about the origin to the image triangle  $U'(-1, 2)$ ,  $V'(-1, 5)$ ,  $W'(-3, 2)$ , with a curved arrow showing the quarter-turn about the origin.

You can check a quarter-turn by counting grid squares instead of measuring the angle:  $U$  is 2 right and 1 up from  $O$ , so a  $90^\circ$  anticlockwise turn puts its image 2 up and 1 left of  $O$ , at  $(-1, 2)$ . The two counts simply swap roles.

Figure 6 rotates the triangle with vertices  $J(1, 1)$ ,  $K(4, 1)$  and  $L(1, 3)$  by  $180^\circ$  about  $O$ , using the rule  $(x, y) \rightarrow (-x, -y)$ : both coordinates flip sign. The image is  $J'(-1, -1)$ ,  $K'(-4, -1)$  and  $L'(-1, -3)$ .

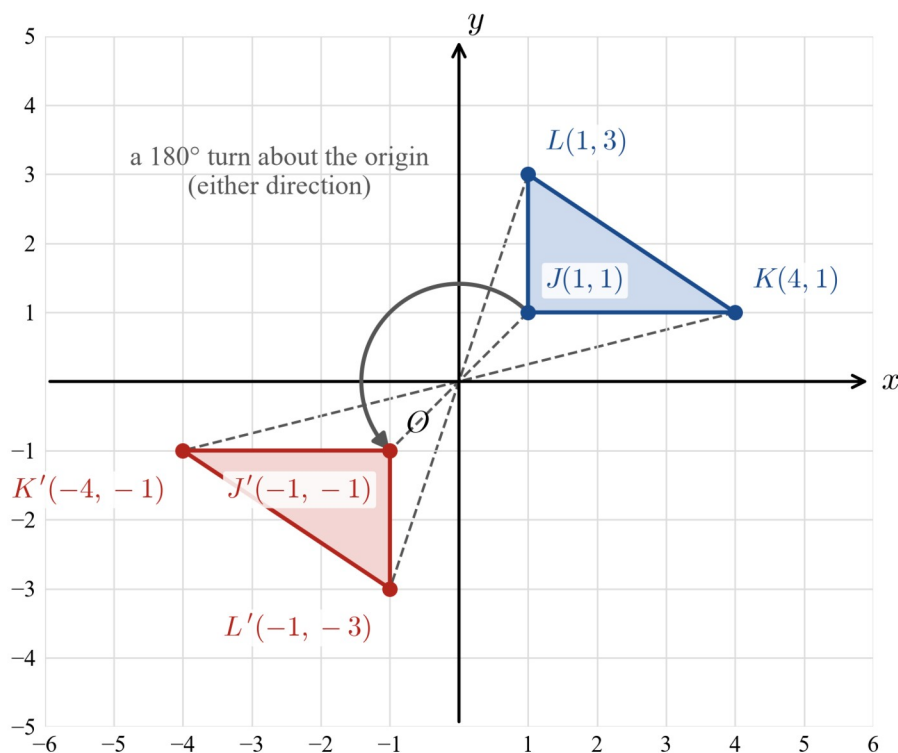


Figure 6. The triangle  $J(1, 1)$ ,  $K(4, 1)$ ,  $L(1, 3)$  rotated 180 degrees about the origin to the image triangle  $J'(-1, -1)$ ,  $K'(-4, -1)$ ,  $L'(-1, -3)$ , with a curved arrow showing the half-turn and dashed joins from each vertex to its image passing through the origin.

Notice in Figure 6 that the join from each vertex to its image passes straight through  $O$ : for a half-turn, the origin sits exactly halfway between every point and its image.

**The same size and the same shape.** All three transformations give an image that is the same size and the same shape as the original. That gives you a check that always works: count the grid squares between matching vertices before and after. If the side lengths match, your image is likely to be correct.

**Two transformations in a row.** Transformations can be applied one after another. Start with the triangle  $A(1, 1)$ ,  $B(4, 1)$ ,  $C(2, 3)$  and reflect it in the  $x$ -axis, then reflect the result in the  $y$ -axis. The first reflection flips the sign of the  $y$ -coordinate and the second flips the sign of the  $x$ -coordinate, so together they flip both signs:  $(x, y) \rightarrow (-x, -y)$ . That is exactly the rule for a  $180^\circ$  rotation about the origin — two different ways to produce the same transformation, with the same final image  $A''(-1, -1)$ ,  $B''(-4, -1)$ ,  $C''(-2, -3)$  (Figure 7). Reflecting twice in the *same* axis is also worth knowing: the second flip undoes the first, and the shape returns to where it started.

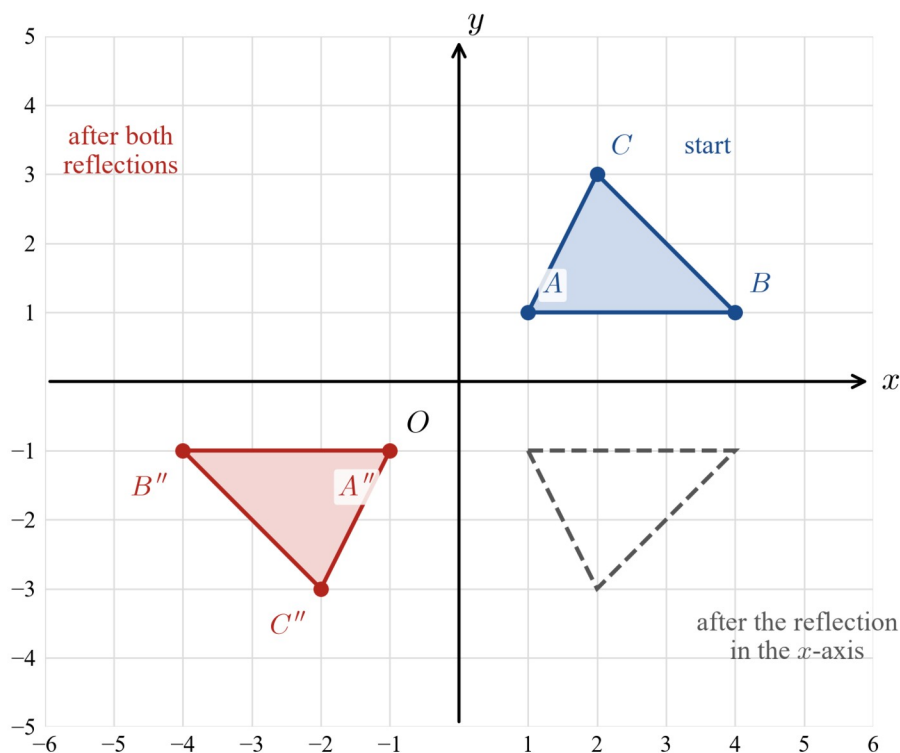


Figure 7. The triangle  $A(1, 1)$ ,  $B(4, 1)$ ,  $C(2, 3)$  first reflected in the  $x$ -axis, shown as a grey dashed outline, and then reflected in the  $y$ -axis to the final red triangle  $A''$ ,  $B''$ ,  $C''$ , which is exactly where a  $180^\circ$ -degree rotation about the origin puts it.

**Patterns from transformations.** Repeating the same transformation builds a pattern. Figure 8 starts with one shape and translates it 4 units to the right, again and again, to make a border pattern: every copy is exactly the same as the one before, just further along. Combinations of translations, reflections and rotations can create many more designs; you can experiment with them using a digital tool.

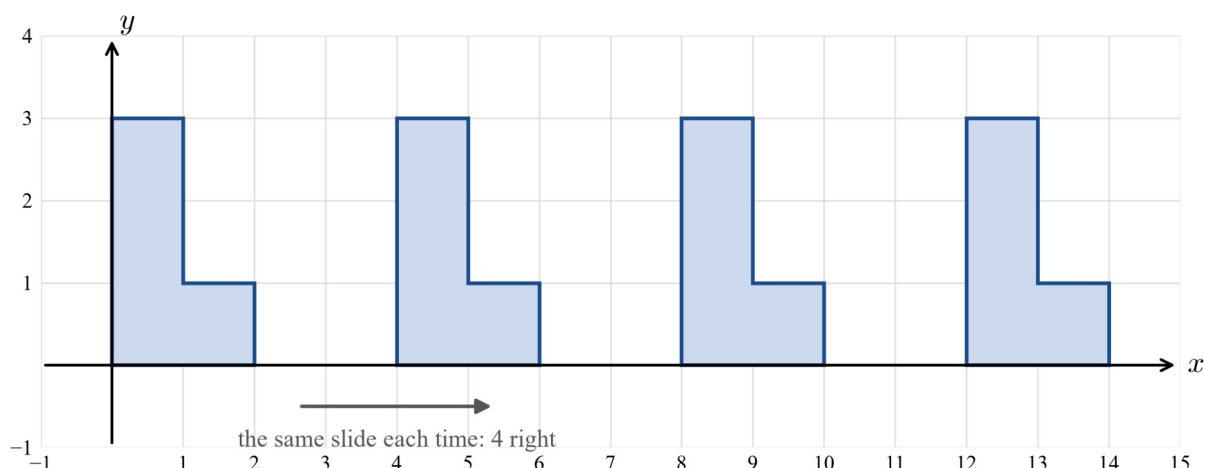


Figure 8. A border pattern made from one starting shape translated 4 units to the right, again and again, with an arrow marking the same slide each time.

### Vocabulary.

| Term           | Meaning here                                                       | Synonyms and related words |
|----------------|--------------------------------------------------------------------|----------------------------|
| transformation | a move applied to every point of a shape or a set of points        | —                          |
| translation    | a slide: every point moves the same distance in the same direction | slide                      |
| reflection     | a flip in a mirror line — in this                                  | flip, mirror image         |

| Term                     | Meaning here                                                        | Synonyms and related words |
|--------------------------|---------------------------------------------------------------------|----------------------------|
|                          | subtopic, one of the axes                                           |                            |
| rotation                 | a turn about a fixed point — in this subtopic, the origin           | turn                       |
| quarter-turn, half-turn  | a turn of $90^\circ$ , a turn of $180^\circ$                        | —                          |
| image                    | the shape or point you get after a transformation                   | —                          |
| vertex (plural vertices) | a corner point of a shape                                           | corner                     |
| prime notation, $A'$     | the name of the image of point $A$ , read “ $A$ prime”              | —                          |
| rule                     | a short statement of what a transformation does to every coordinate | —                          |
| origin                   | the point $(0, 0)$ where the axes cross                             | $O$                        |
| clockwise, anticlockwise | the two directions a rotation can turn                              | —                          |

## Worked examples

### Example 1 — scaffolded

Translate the triangle in Figure 2 by 5 right and 3 up. Write the coordinates of the image.

| Working                                                                                                                                                      | Comment                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| <b>Choose.</b> A translation 5 right and 3 up has the rule $(x, y) \rightarrow (x+5, y+3)$ .                                                                 | The same numbers are added to every vertex: 5 to each $x$ , 3 to each $y$ .                        |
| <b>Solve.</b> $A(1, 2) \rightarrow A'(6, 5)$ ; $B(4, 2) \rightarrow B'(9, 5)$ ; $C(2, 5) \rightarrow C'(7, 8)$ .                                             | $1+5=6$ and $2+3=5$ gives $A'$ ; the other vertices work the same way.                             |
| <b>Check.</b> The side from $A$ to $B$ is 3 units long (from $x=1$ to $x=4$ ); the image side from $A'$ to $B'$ is also 3 units long (from $x=6$ to $x=9$ ). | A translation keeps the same size and the same shape, so matching side lengths support the answer. |

$\therefore$  the image is the triangle  $A'(6, 5)$ ,  $B'(9, 5)$ ,  $C'(7, 8)$ , as shown in Figure 2.

### Example 2 — scaffolded

Reflect the triangle  $D(2, 1)$ ,  $E(5, 1)$ ,  $F(3, 4)$  in the  $y$ -axis (Figure 4). Write the coordinates of the image.

| Working                                                                                                             | Comment                                                                                               |
|---------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| <b>Choose.</b> A reflection in the $y$ -axis has the rule $(x, y) \rightarrow (-x, y)$ .                            | The $y$ -coordinate stays the same; the $x$ -coordinate flips sign.                                   |
| <b>Solve.</b> $D(2, 1) \rightarrow D'(-2, 1)$ ; $E(5, 1) \rightarrow E'(-5, 1)$ ; $F(3, 4) \rightarrow F'(-3, 4)$ . | Flip the sign of each $x$ -coordinate; copy each $y$ -coordinate unchanged.                           |
| <b>Check.</b> $E$ is 5 units to the right of the $y$ -axis, and $E'$ is 5 units to the left of it.                  | A reflection puts each point and its image the same distance from the mirror line, on opposite sides. |

$\therefore$  the image is the triangle  $D'(-2, 1)$ ,  $E'(-5, 1)$ ,  $F'(-3, 4)$ .

### Example 3 — unscaffolded

Rotate the triangle  $U(2, 1)$ ,  $V(5, 1)$ ,  $W(2, 3)$  by  $90^\circ$  anticlockwise about the origin (Figure 5). Write the coordinates of the image.

| Working                                                                                                                                       | Comment                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| A $90^\circ$ anticlockwise rotation about the origin has the rule $(x, y) \rightarrow (-y, x)$ .                                              | The coordinates swap roles, and the new $x$ -coordinate is negative.                      |
| $U(2, 1) \rightarrow U'(-1, 2); V(5, 1) \rightarrow V'(-1, 5);$<br>$W(2, 3) \rightarrow W'(-3, 2)$ .                                          | Apply the rule to each vertex in turn.                                                    |
| Check $U$ : it is 2 right and 1 up from $O$ ; after a quarter-turn anticlockwise it should be 2 up and 1 left of $O$ , which is $(-1, 2)$ . ✓ | Counting right-left and up-down on the grid checks the turn without measuring any angles. |
| $\therefore$ the image is the triangle $U'(-1, 2), V'(-1, 5), W'(-3, 2)$ .                                                                    |                                                                                           |

#### Example 4 — unscaffolded

The triangle  $A(1, 1), B(4, 1), C(2, 3)$  is reflected in the  $x$ -axis, and the result is then reflected in the  $y$ -axis. (a) Find the coordinates of the final image. (b) Name the single transformation that takes the original triangle straight to the final image.

| Working                                                                                                                                                                                                       | Comment                                                                                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| (a) Reflection in the $x$ -axis: $(x, y) \rightarrow (x, -y)$ . So<br>$A(1, 1) \rightarrow (1, -1), B(4, 1) \rightarrow (4, -1),$<br>$C(2, 3) \rightarrow (2, -3)$ .                                          | The first reflection flips the sign of each $y$ -coordinate.                                    |
| Then reflection in the $y$ -axis: $(x, y) \rightarrow (-x, y)$ . The final image is $A''(-1, -1), B''(-4, -1),$<br>$C''(-2, -3)$ .                                                                            | The second reflection flips the sign of each $x$ -coordinate; together, both signs are flipped. |
| (b) Flipping both signs, $(x, y) \rightarrow (-x, -y)$ , is the rule for a $180^\circ$ rotation about the origin.                                                                                             | The half-turn gives exactly the same image as the two reflections (Figure 7).                   |
| $\therefore$ the final image is $A''(-1, -1), B''(-4, -1), C''(-2, -3)$ , and the single transformation is a $180^\circ$ rotation about the origin — two different ways of producing the same transformation. |                                                                                                 |

## Practice questions

### Scaffolded

**Q1.** Read the coordinates of each point in Figure 9. (a)  $A$  (b)  $B$  (c)  $C$  (d)  $D$  (e)  $E$

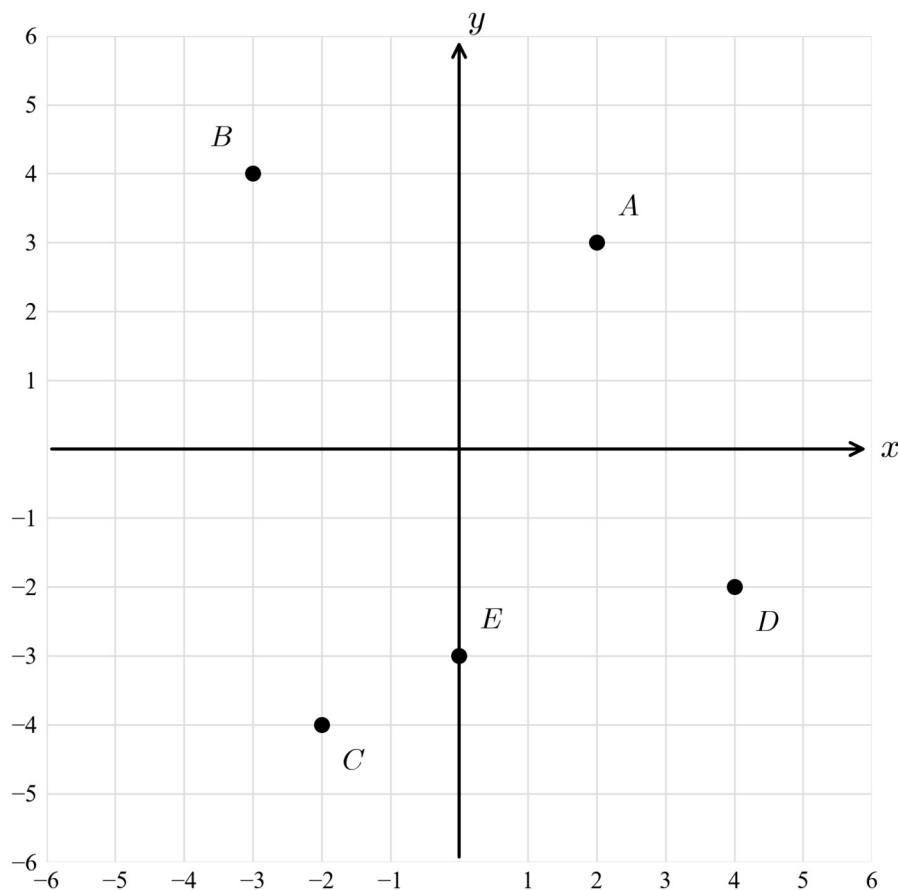


Figure 9. A Cartesian plane showing five labelled points: A in quadrant I, B in quadrant II, C in quadrant III, D in quadrant IV, and E on the y-axis below the origin.

**Q2.** A translation has the rule  $(x, y) \rightarrow (x + 4, y + 2)$ . Find the image of each point. (a)  $(1, 3)$  (b)  $(-2, 5)$  (c)  $(0, -4)$  (d)  $(-3, -1)$

**Q3.** The triangle  $P(-1, 2)$ ,  $Q(3, 2)$ ,  $R(1, 5)$  is translated 3 right and 4 down. (a) Find the coordinates of the image. (b) Write the rule of this translation in the form  $(x, y) \rightarrow (\square, \square)$ .

**Q4.** Figure 10 shows the triangle  $H(2, 1)$ ,  $K(5, 1)$ ,  $M(2, 4)$  and its mirror line. (a) Reflect the triangle in the x-axis and write the coordinates of the image. (b) Which coordinate stays the same for every point and its image? (c) Check your answer by comparing how far  $H$  and  $H'$  are from the mirror line.

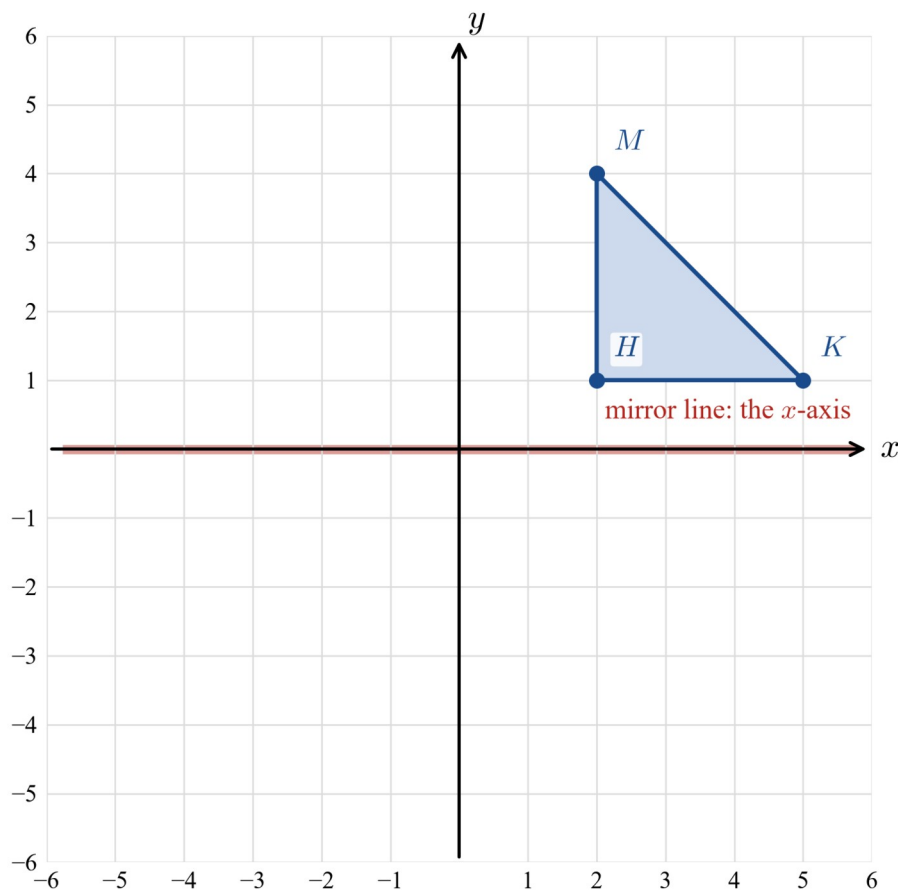


Figure 10. A Cartesian plane showing the triangle  $H(2, 1)$ ,  $K(5, 1)$ ,  $M(2, 4)$  above the  $x$ -axis, with the  $x$ -axis marked as the mirror line.

**Q5.** A reflection in the  $y$ -axis has the rule  $(x, y) \rightarrow (-x, y)$ . Find the image of each point. (a)  $(4, 2)$  (b)  $(-1, 3)$  (c)  $(-5, -2)$  (d)  $(0, 6)$  (e) Write in words what this rule does to the coordinates.

**Q6.** Say whether each statement is true or false. (a) Reflecting  $(3, 5)$  in the  $x$ -axis gives  $(-3, 5)$ . (b) Reflecting  $(3, 5)$  in the  $x$ -axis gives  $(3, -5)$ . (c) Translating  $(-2, 4)$  by 6 right gives  $(4, 4)$ . (d) Rotating  $(2, -7)$  by  $180^\circ$  about the origin gives  $(-2, 7)$ .

### Unscaffolded

**Q7.** Find the image of each point under a  $180^\circ$  rotation about the origin. (a)  $(2, 5)$  (b)  $(-4, 1)$  (c)  $(-3, -6)$  (d)  $(5, -2)$

**Q8.** Figure 11 shows the triangle  $E(3, 1)$ ,  $F(5, 1)$ ,  $G(3, 2)$ . (a) Rotate the triangle  $90^\circ$  anticlockwise about the origin and write the coordinates of the image. (b) Draw the image on a copy of the grid. (c) Write the rule for a  $90^\circ$  anticlockwise rotation about the origin.

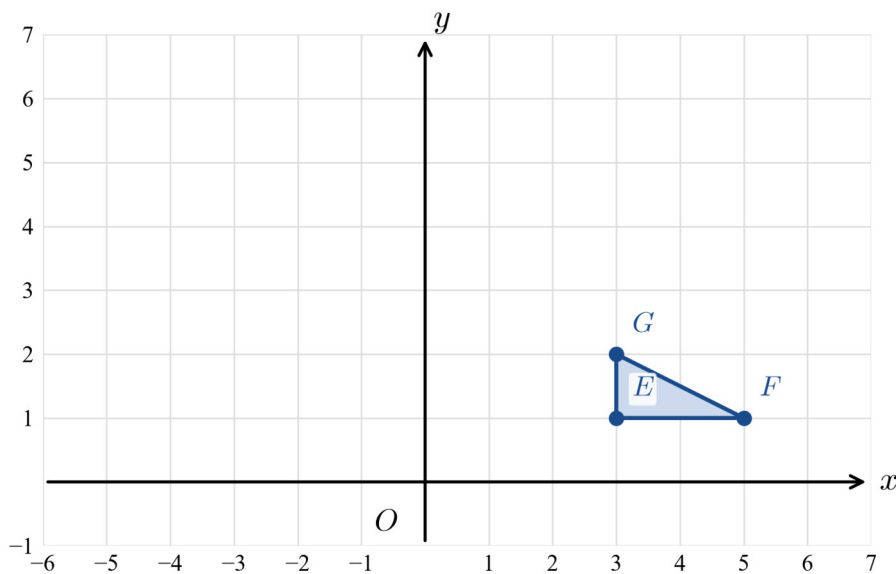


Figure 11. A Cartesian plane showing the triangle  $E(3, 1)$ ,  $F(5, 1)$ ,  $G(3, 2)$  in quadrant I, with the origin labelled  $O$ .

**Q9.** Each rule below describes one of the transformations of this subtopic. Name the transformation. (a)  $(x, y) \rightarrow (x, -y)$  (b)  $(x, y) \rightarrow (-x, y)$  (c)  $(x, y) \rightarrow (-x, -y)$  (d)  $(x, y) \rightarrow (x+4, y+3)$  (e)  $(x, y) \rightarrow (-y, x)$

**Q10.** The point  $(3, 4)$  is reflected in the  $x$ -axis, and the result is then reflected in the  $y$ -axis. (a) Find the coordinates after each step. (b) Name the single transformation that takes  $(3, 4)$  straight to the final image. (c) Would the final image change if the reflection in the  $y$ -axis came first? Check by calculation.

**Q11.** The point  $(2, 3)$  is translated 3 left and 2 up, and the result is then reflected in the  $y$ -axis. (a) Find the coordinates after the translation. (b) Find the coordinates after the reflection. (c) Describe the two steps in words, in the order they happened.

**Q12.** Mia and Sam rotate the point  $P(2, 7)$  by  $180^\circ$  about the origin. Mia says the image is  $(-2, -7)$ . Sam says the image is  $(2, -7)$ . Who is correct? Give two reasons: one using the rule, and one using the distances of the point from the axes.

**Q13.** Figure 12 shows the starting shape of a border pattern. (a) Describe the translation that takes the starting shape to the next copy, and write its rule. (b) Draw the next two copies of the shape on a copy of the grid. (c) Design a starting shape of your own on a grid and use one transformation to build a border pattern. Say which transformation you used.

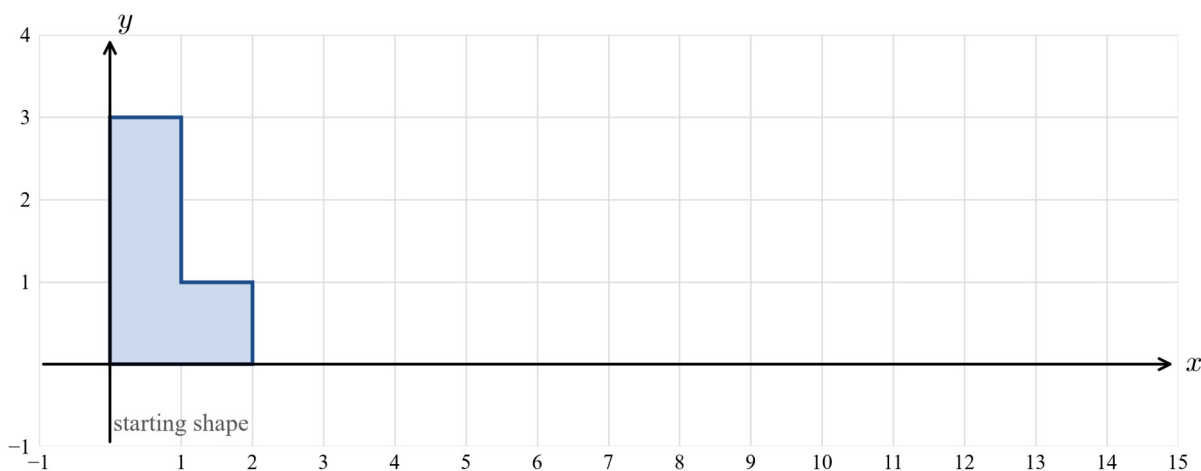


Figure 12. A grid showing one starting shape for a border pattern, a six-sided outline occupying the grid squares from  $x = 0$  to  $x = 2$  and  $y = 0$  to  $y = 3$ , with empty grid space to its right for further copies.

**Q14.** Use a digital tool, such as a graphing app, to draw a shape of your own on the Cartesian plane. Apply one translation, one reflection in an axis, and one rotation about the origin. For each one, describe the transformation and record the coordinates of the image of at least two of your points.



## Answers

**Q1.** (a)  $A(2, 3)$ ; (b)  $B(-3, 4)$ ; (c)  $C(-2, -4)$ ; (d)  $D(4, -2)$ ; (e)  $E(0, -3)$ .

**Q2.** (a)  $(5, 5)$ ; (b)  $(2, 7)$ ; (c)  $(4, -2)$ ; (d)  $(1, 1)$ .

**Q3.** (a)  $P'(2, -2)$ ,  $Q'(6, -2)$ ,  $R'(4, 1)$ ; (b)  $(x, y) \rightarrow (x+3, y-4)$ .

**Q4.** (a)  $H'(2, -1)$ ,  $K'(5, -1)$ ,  $M'(2, -4)$ ; (b) the  $x$ -coordinate; (c)  $H$  is 1 unit above the  $x$ -axis and  $H'$  is 1 unit below it — the same distance on opposite sides.

**Q5.** (a)  $(-4, 2)$ ; (b)  $(1, 3)$ ; (c)  $(5, -2)$ ; (d)  $(0, 6)$ ; (e) the  $x$ -coordinate flips sign and the  $y$ -coordinate stays the same.

**Q6.** (a) false; (b) true; (c) true; (d) true.

**Q7.** (a)  $(-2, -5)$ ; (b)  $(4, -1)$ ; (c)  $(3, 6)$ ; (d)  $(-5, 2)$ .

**Q8.** (a)  $E'(-1, 3)$ ,  $F'(-1, 5)$ ,  $G'(-2, 3)$ ; (b) see fully-worked solutions; (c)  $(x, y) \rightarrow (-y, x)$ .

**Q9.** (a) reflection in the  $x$ -axis; (b) reflection in the  $y$ -axis; (c) a  $180^\circ$  rotation about the origin; (d) a translation 4 right and 3 up; (e) a  $90^\circ$  anticlockwise rotation about the origin.

**Q10.** (a) after the first reflection:  $(3, -4)$ ; after the second:  $(-3, -4)$ ; (b) a  $180^\circ$  rotation about the origin; (c) no — reflecting in the  $y$ -axis first gives  $(-3, 4)$ , then reflecting in the  $x$ -axis gives  $(-3, -4)$ , the same final point.

**Q11.** (a)  $(-1, 5)$ ; (b)  $(1, 5)$ ; (c) a translation 3 left and 2 up, then a reflection in the  $y$ -axis.

**Q12.** Mia. The rule  $(x, y) \rightarrow (-x, -y)$  gives  $(2, 7) \rightarrow (-2, -7)$ ; Sam flipped only the  $y$ -coordinate, which is what a reflection in the  $x$ -axis does. A distance check agrees:  $P$  is 2 right and 7 up from  $O$ , so a half-turn puts its image 2 left and 7 down, at  $(-2, -7)$ .

**Q13.** (a) a translation 4 right, with rule  $(x, y) \rightarrow (x+4, y)$ ; (b) see fully-worked solutions; (c) answers will vary.

**Q14.** Answers will vary; see fully-worked solutions for one possible response.

---

## Fully-worked solutions

**Q1.** Count across from the  $y$ -axis for the  $x$ -coordinate, then up or down from the  $x$ -axis for the  $y$ -coordinate. (a)  $A$  is 2 right and 3 up:  $A(2, 3)$ . (b)  $B$  is 3 left and 4 up:  $B(-3, 4)$ . (c)  $C$  is 2 left and 4 down:  $C(-2, -4)$ . (d)  $D$  is 4 right and 2 down:  $D(4, -2)$ . (e)  $E$  sits on the  $y$ -axis, 3 down:  $E(0, -3)$ .

**Q2.** The rule  $(x, y) \rightarrow (x+4, y+2)$  adds 4 to the  $x$ -coordinate and 2 to the  $y$ -coordinate. (a)  $(1+4, 3+2)=(5, 5)$ . (b)  $(-2+4, 5+2)=(2, 7)$ . (c)  $(0+4, -4+2)=(4, -2)$ . (d)  $(-3+4, -1+2)=(1, 1)$ .

**Q3.** A slide of 3 right and 4 down adds 3 to each  $x$ -coordinate and subtracts 4 from each  $y$ -coordinate. (a)  $P(-1, 2) \rightarrow P'(-1+3, 2-4)=P'(2, -2)$ ;  $Q(3, 2) \rightarrow Q'(3+3, 2-4)=Q'(6, -2)$ ;  $R(1, 5) \rightarrow R'(1+3, 5-4)=R'(4, 1)$ . (b)  $(x, y) \rightarrow (x+3, y-4)$ .

**Q4.** Reflection in the  $x$ -axis:  $(x, y) \rightarrow (x, -y)$ . (a)  $H(2, 1) \rightarrow H'(2, -1)$ ;  $K(5, 1) \rightarrow K'(5, -1)$ ;  $M(2, 4) \rightarrow M'(2, -4)$ . (b) The  $x$ -coordinate stays the same for every point and its image. (c)  $H$  is 1 unit above the mirror line and  $H'$  is 1 unit below it — equal distances on opposite sides, as a reflection requires.

**Q5.** Flip the sign of the  $x$ -coordinate; keep the  $y$ -coordinate. (a)  $(4, 2) \rightarrow (-4, 2)$ . (b)  $(-1, 3) \rightarrow (1, 3)$ . (c)  $(-5, -2) \rightarrow (5, -2)$ . (d)  $(0, 6) \rightarrow (0, 6)$  — a point on the mirror line stays where it is. (e) The  $x$ -coordinate flips sign and the  $y$ -coordinate stays the same.

**Q6.** (a) False — reflecting in the  $x$ -axis keeps the  $x$ -coordinate and flips the  $y$ -coordinate;  $(3, 5) \rightarrow (3, -5)$ . The answer given is a reflection in the  $y$ -axis. (b) True — the  $y$ -coordinate flips sign. (c) True —  $-2 + 6 = 4$ , and the  $y$ -coordinate is unchanged. (d) True — a  $180^\circ$  rotation flips both signs:  $(2, -7) \rightarrow (-2, 7)$ .

**Q7.** A  $180^\circ$  rotation about the origin flips both signs:  $(x, y) \rightarrow (-x, -y)$ . (a)  $(2, 5) \rightarrow (-2, -5)$ . (b)  $(-4, 1) \rightarrow (4, -1)$ . (c)  $(-3, -6) \rightarrow (3, 6)$ . (d)  $(5, -2) \rightarrow (-5, 2)$ .

**Q8.** (a) Use the rule  $(x, y) \rightarrow (-y, x)$ :  $E(3, 1) \rightarrow E'(-1, 3)$ ;  $F(5, 1) \rightarrow F'(-1, 5)$ ;  $G(3, 2) \rightarrow G'(-2, 3)$ . (b) Plot the three image points and join them; the image sits in quadrant II, the same size and the same shape as the original, turned a quarter-turn anticlockwise. (c)  $(x, y) \rightarrow (-y, x)$ .

**Q9.** Read each rule as a change to the coordinates. (a) Only the  $y$ -coordinate flips sign: reflection in the  $x$ -axis. (b) Only the  $x$ -coordinate flips sign: reflection in the  $y$ -axis. (c) Both coordinates flip sign: a  $180^\circ$  rotation about the origin. (d) 4 is added to  $x$  and 3 to  $y$ : a translation 4 right and 3 up. (e) The coordinates swap roles and the new  $x$ -coordinate is negative: a  $90^\circ$  anticlockwise rotation about the origin.

**Q10.** (a) Reflection in the  $x$ -axis flips the sign of  $y$ :  $(3, 4) \rightarrow (3, -4)$ . Reflection in the  $y$ -axis then flips the sign of  $x$ :  $(3, -4) \rightarrow (-3, -4)$ . (b) Both coordinates have flipped sign,  $(3, 4) \rightarrow (-3, -4)$ , which is the rule for a  $180^\circ$  rotation about the origin. (c) In the other order: reflection in the  $y$ -axis gives  $(3, 4) \rightarrow (-3, 4)$ , and reflection in the  $x$ -axis then gives  $(-3, 4) \rightarrow (-3, -4)$ . The final image is the same — the order of the two reflections does not matter.

**Q11.** (a) A translation 3 left and 2 up:  $(2, 3) \rightarrow (2 - 3, 3 + 2) = (-1, 5)$ . (b) Reflection in the  $y$ -axis flips the sign of  $x$ :  $(-1, 5) \rightarrow (1, 5)$ . (c) First a translation 3 left and 2 up, then a reflection in the  $y$ -axis.

**Q12.** Mia is correct. Rule: a  $180^\circ$  rotation about the origin has the rule  $(x, y) \rightarrow (-x, -y)$ , so  $(2, 7) \rightarrow (-2, -7)$ . Sam's answer  $(2, -7)$  flips only the  $y$ -coordinate — that is what a reflection in the  $x$ -axis does, not a rotation. Distances:  $P$  is 2 units to the right of the  $y$ -axis and 7 units above the  $x$ -axis. A half-turn about  $O$  puts the image the same distances on the opposite sides: 2 left of the  $y$ -axis and 7 below the  $x$ -axis, which is  $(-2, -7)$ . Sam's point  $(2, -7)$  is still on the right-hand side, so it cannot be the image of a half-turn.

**Q13.** (a) Each copy starts 4 units to the right of the one before: a translation 4 right, with rule  $(x, y) \rightarrow (x + 4, y)$ . (b) Add 4 and then 8 to every  $x$ -coordinate of the starting shape. The starting vertices  $(0, 0)$ ,  $(2, 0)$ ,  $(2, 1)$ ,  $(1, 1)$ ,  $(1, 3)$ ,  $(0, 3)$  give next copies at  $(4, 0)$ ,  $(6, 0)$ ,  $(6, 1)$ ,  $(5, 1)$ ,  $(5, 3)$ ,  $(4, 3)$  and then at  $(8, 0)$ ,  $(10, 0)$ ,  $(10, 1)$ ,  $(9, 1)$ ,  $(9, 3)$ ,  $(8, 3)$ . (c) Answers will vary; for example, a triangle translated 5 right each time, or a shape reflected in a vertical line and then translated. Accept any design where the same transformation is repeated.

**Q14.** Answers will vary. One possible response: start with the triangle  $(1, 1)$ ,  $(4, 1)$ ,  $(1, 3)$ . Translation 2 right and 3 up, rule  $(x, y) \rightarrow (x + 2, y + 3)$ :  $(1, 1)$  maps to  $(3, 4)$  and  $(4, 1)$  maps to  $(6, 4)$ . Reflection in the  $y$ -axis, rule  $(x, y) \rightarrow (-x, y)$ :  $(1, 1)$  maps to  $(-1, 1)$  and  $(4, 1)$  maps to  $(-4, 1)$ . Rotation  $90^\circ$  anticlockwise about the origin, rule  $(x, y) \rightarrow (-y, x)$ :  $(1, 1)$  maps to  $(-1, 1)$  and  $(4, 1)$  maps to  $(-1, 4)$ . Check each image in the digital tool before recording it.

## Teacher note

- **Ownership and boundaries.** Per the Chapter 5 plan in TOC §3, 5D (VC2M7SP03) owns transformations described by *their effect on coordinates*: translations, reflections **in an axis**, and rotations **about the origin** (rules table, WE1–WE4, Q2–Q12). Reflections in lines other than the axes, rotations about other centres, and 3-D coordinate work are outside this subtopic (D4.17). The word *congruent* is deliberately not used: the image is described as “the same size and the same shape”, with congruence reserved for Level 8 (D4.11).
- **Level 6 recall (D3).** Plotting points with integer coordinates in all four quadrants (VC2M6SP02, VC2M6N01) and sliding, flipping and turning shapes, including combinations of transformations (VC2M6SP03), are assumed. They are recalled in one paragraph and Figure 1, then maintained inside the questions — not re-taught.

- **Elaboration coverage.** E01 is delivered by Q14 (a digital tool used to transform a shape, with the transformations described and recorded). E02 is delivered by the “two transformations in a row” paragraph, WE4 and Q10: two successive reflections in the axes equal a  $180^\circ$  rotation about the origin, and reflecting twice in the same axis returns the shape to its start — different ways to produce the same transformation. E02’s sample of two reflections matching a *translation* needs mirror lines that are not axes, which is outside the scope set above; it is therefore met within scope, as noted. E03 is delivered by the patterns paragraph and Q13(c), with digital tools permitted per D9.
- **Digital tools (D9).** No CAS is used or needed. Where elaborations name digital tools (Q14, optionally Q13), they are named generically (“a digital tool, such as a graphing app”); no specific product is endorsed.
- **Reasonableness habit (P7).** WE1’s check, Q4(c), the grid-counting check in Example 3 and Q12’s distance argument all ask students to check an answer is reasonable before accepting it, per TOC §5.2.
- **Figures.** All twelve figures are generated by `src/gen_5d.py` from the exact coordinates stated in the text; rotations are computed as true rotations about the origin, so image vertices cannot disagree with the rules in the theory.
- **Vocabulary gate.** `src/vocab_gate_5d.py` checks the student-facing text of this section against the ladder at ceiling Yr7, with the curriculum-word exemption built from VC2M7SP03’s content description, achievement-standard extract and elaborations. Gate status: CLEAN.
- **No D11 content.** None of VC2M7SP03.E01–.E03 names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.
- **Forward.** The coordinate rules here are the groundwork for Level 8, where transformations are used to describe and test congruence; no Level 8 content appears in this section.

# Test

## Chapter test T5 — Space

### Assessed content descriptions

VC2M7SP01 — subtopic 5A, Representing three-dimensional objects in two dimensions; VC2M7SP02 — subtopic 5B, Classifying triangles, quadrilaterals and other polygons; VC2M7SP04 — subtopic 5C, Algorithms that sort and classify shapes; VC2M7SP03 — subtopic 5D, Transformations of points on the Cartesian plane

### Marks

**20** — Section A (5A): 5 marks · Section B (5B): 5 marks · Section C (5C): 5 marks · Section D (5D): 5 marks

### Time

20 minutes working time, plus 5 minutes reading time (about 25 minutes in all)

### Equipment

Ruler, protractor and pair of compasses

### Calculators and digital tools

None required. This paper is completed without a calculator, spreadsheet, dynamic geometry software or simulation tool.

This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 7 achievement standard, not against a mark on this paper.

*Answer every question. Show your working, and give a reason where a question asks for one. The marks for each part are shown in brackets.*

## Section A — Representing three-dimensional objects in two dimensions (5A, VC2M7SP01)

**Question 1 [1 mark].** Four students each draw the same cardboard box. Priya draws a net, Marcus draws the plan view, the front view and the side view, Ella draws an isometric drawing, and Zoe draws a perspective drawing. The class wants to make a copy of the box out of cardboard. Whose drawing is the most useful for making the box?

- A Priya's drawing
- B Marcus's drawing
- C Ella's drawing
- D Zoe's drawing

**Question 2 [2 marks].** A designer shows a new building to the public as an isometric drawing.

- a. Give one advantage of using an isometric drawing to show the building. [1 mark]
- b. Give one disadvantage of using an isometric drawing to show the building. [1 mark]

**Question 3 [2 marks].** A net is made of two triangles of the same size and shape and three rectangles.

- a. Name the solid this net folds into. [1 mark]
- b. How many edges does that solid have? [1 mark]

## Section B — Classifying triangles, quadrilaterals and other polygons (5B, VC2M7SP02)

**Question 4 [1 mark].** The angles of a triangle are  $34^\circ$ ,  $56^\circ$  and  $90^\circ$ . Classify the triangle by its angles.

**Question 5 [2 marks].** Three sticks measure 5 cm, 7 cm and 13 cm.

- a. Can the three sticks be joined end to end to form a triangle? [1 mark]
- b. Show the check that supports your answer. [1 mark]

**Question 6 [2 marks].** 'Every parallelogram is a rectangle.'

- a. Is this statement always true, sometimes true or never true? [1 mark]
- b. Give a reason for your answer. [1 mark]

### Section C — Algorithms that sort and classify shapes (5C, VC2M7SP04)

*An algorithm gives any triangle its most special name by its sides. It runs like this.*

*Step 1. Take one triangle. Step 2. Are all 3 sides equal? If yes, write 'equilateral triangle' and stop. Step 3. Are exactly 2 sides equal? If yes, write 'isosceles triangle' and stop. Step 4. Write 'scalene triangle' and stop.*

#### Question 7 [3 marks].

- a. Run the algorithm on a triangle with sides 4 cm, 4 cm and 6 cm. Write the result it reaches. [1 mark]
- b. Run the algorithm on a triangle with sides 5 cm, 5 cm and 5 cm. Write the result it reaches, and the number of the step where it stops. [1 mark]
- c. Can any triangle reach two different results in this algorithm? Explain why or why not. [1 mark]

**Question 8 [2 marks].** Here is a set of shapes: a square, a rectangle and a rhombus that is not a square. Design an algorithm — as numbered steps or as a flow chart — that uses exactly two decisions and gives each shape in the set its most special name.

### Section D — Transformations of points on the Cartesian plane (5D, VC2M7SP03)

**Question 9 [2 marks].** A translation has the rule  $(x, y) \rightarrow (x + 4, y - 3)$ . Find the image of each point.

- a.  $A(1, 5)$  [1 mark]
- b.  $B(-2, 4)$  [1 mark]

**Question 10 [1 mark].** Find the image of the point  $C(3, -7)$  under a reflection in the  $y$ -axis.

**Question 11 [1 mark].** Find the image of the point  $D(-4, 2)$  under a  $180^\circ$  rotation about the origin.

**Question 12 [1 mark].** The point  $E(2, 6)$  is reflected in the  $x$ -axis, and its image is then reflected in the  $y$ -axis. Name the single transformation that takes  $E$  straight to the final image.

**End of test — 20 marks.**

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## Solutions

Fully worked solutions for every question, followed by a compact answer key for self-checking.

### Worked solutions

#### Question 1.

Making the box means cutting a flat pattern from cardboard and folding it. A net is exactly that — a flat pattern that folds up to make the solid, with every face shown flat at its true size. The plan, front and side views and the isometric drawing show true lengths but are pictures of the finished box, not patterns to cut, and a perspective drawing does not even show true lengths. The answer is **A** — Priya's net. **[1 mark]**

#### Question 2.

- a. One advantage: an isometric drawing shows the whole solid in one picture, with true lengths in all three directions — so lengths can be measured from it. **[1 mark]**
- b. One disadvantage: in an isometric drawing, right angles do not look like right angles (a rectangular face is not drawn as a rectangle), so the shape can be harder to picture than a view from one side. **[1 mark]**

*Any advantage or disadvantage of an isometric drawing from the table in 5A earns the mark.*

#### Question 3.

- a. The two triangles of the same size and shape are the matching ends, and the three rectangles join them, so the net folds into a **triangular prism**. **[1 mark]**
- b. Count the edges: each triangle end has 3 edges, and 3 more edges run between the matching corners of the two ends:

$$3 + 3 + 3 = 9$$

The solid has **9 edges**. **[1 mark]**

#### Question 4.

One angle is exactly  $90^\circ$ , and a triangle with one  $90^\circ$  angle is classified by its angles as **right-angled**. (The other two angles,  $34^\circ$  and  $56^\circ$ , are both under  $90^\circ$ , as they must be: the three angles of a triangle add to  $180^\circ$ .) **[1 mark]**

#### Question 5.

- a. **No**, the three sticks cannot form a triangle. **[1 mark]**
- b. The check compares the two shorter lengths with the longest. The two shorter sticks are 5 cm and 7 cm:

$$5 + 7 = 12, \text{ and } 12 < 13$$

The two shorter lengths add to less than the longest length, so the two short sticks cannot reach across the 13 cm stick — no triangle is possible. **[1 mark]**

#### Question 6.

- a. **Sometimes true**. **[1 mark]**
- b. A parallelogram whose 4 angles are right angles is a rectangle, so some parallelograms are rectangles. But a parallelogram can lean without any right angles, and such a parallelogram is not a rectangle — so the statement is true for some parallelograms and not for others. **[1 mark]**

#### Question 7.

- a. Sides 4 cm, 4 cm and 6 cm: not all 3 sides are equal (no at Step 2), and exactly 2 sides are equal (yes at Step 3). The result is **isosceles triangle**. **[1 mark]**

- b. Sides 5 cm, 5 cm and 5 cm: all 3 sides are equal, so the algorithm answers yes at Step 2 and writes **equilateral triangle**. It stops at **Step 2**. [1 mark]
- c. **No**. Every yes answer ends the run — the algorithm stops — so a triangle can only reach the first result it matches. The steps run in a fixed sequence, and Step 4 catches every triangle that answers no to Steps 2 and 3, so every triangle reaches exactly one result. [1 mark]

### Question 8.

One possible algorithm:

*Step 1. Take one shape from the set. Step 2. Are all 4 angles right angles? If no, write 'rhombus' and stop. If yes, go to Step 3. Step 3. Are all 4 sides equal? If yes, write 'square'. If no, write 'rectangle'. Stop.*

Check against the set: the rhombus answers no at Step 2 and reaches 'rhombus'; the square answers yes at both decisions and reaches 'square'; the rectangle answers yes then no and reaches 'rectangle'. Every shape reaches exactly one result, and each result is its most special name.

Marking: [1 mark] for two decisions asked in a sequence that can sort the set; [1 mark] for the square, the rectangle and the rhombus each reaching the correct most special name. Any working design earns full marks — for example, asking 'are all 4 sides equal?' first, with yes leading to a decision on the angles (square or rhombus) and no leading straight to rectangle, or the same algorithm drawn as a flow chart.

### Question 9.

The rule adds 4 to each  $x$ -coordinate and subtracts 3 from each  $y$ -coordinate.

- a.  $A(1, 5) \rightarrow A'(1+4, 5-3) = A'(5, 2)$ . [1 mark]
- b.  $B(-2, 4) \rightarrow B'(-2+4, 4-3) = B'(2, 1)$ . [1 mark]

### Question 10.

A reflection in the  $y$ -axis keeps the  $y$ -coordinate and flips the sign of the  $x$ -coordinate:  $(x, y) \rightarrow (-x, y)$ . So

$$C(3, -7) \rightarrow C'(-3, -7)$$

[1 mark]

### Question 11.

A  $180^\circ$  rotation about the origin flips the sign of both coordinates:  $(x, y) \rightarrow (-x, -y)$ . So

$$D(-4, 2) \rightarrow D'(4, -2)$$

[1 mark]

### Question 12.

Follow the two reflections. Reflection in the  $x$ -axis flips the sign of  $y$ :

$$E(2, 6) \rightarrow (2, -6)$$

Reflection in the  $y$ -axis then flips the sign of  $x$ :

$$(2, -6) \rightarrow (-2, -6)$$

Both coordinates have flipped sign, and  $(x, y) \rightarrow (-x, -y)$  is the rule for a  $180^\circ$  rotation about the origin. The single transformation is a  $180^\circ$  **rotation about the origin**. [1 mark]

## Answer key

| Question | Answer                                                                                                             |
|----------|--------------------------------------------------------------------------------------------------------------------|
| 1        | A — Priya's net                                                                                                    |
| 2a       | One advantage of an isometric drawing; e.g. the whole solid in one picture, with true lengths that can be measured |
| 2b       | One disadvantage; e.g. right angles do not look like right angles                                                  |
| 3a       | Triangular prism                                                                                                   |
| 3b       | 9 edges                                                                                                            |
| 4        | Right-angled                                                                                                       |
| 5a       | No                                                                                                                 |
| 5b       | $5 + 7 = 12$ , and $12 < 13$                                                                                       |
| 6a       | Sometimes true                                                                                                     |
| 6b       | A parallelogram with 4 right angles is a rectangle; a leaning one is not                                           |
| 7a       | Isosceles triangle                                                                                                 |
| 7b       | Equilateral triangle; stops at Step 2                                                                              |
| 7c       | No — every yes ends the run, and Step 4 catches the rest, so each triangle reaches exactly one result              |
| 8        | Any working two-decision algorithm; see worked solution for one example                                            |
| 9a       | $A'(5, 2)$                                                                                                         |
| 9b       | $B'(2, 1)$                                                                                                         |
| 10       | $C'(-3, -7)$                                                                                                       |
| 11       | $D'(4, -2)$                                                                                                        |
| 12       | A $180^\circ$ rotation about the origin                                                                            |

**Total: 20 marks — Section A (5A, VC2M7SP01): 5 · Section B (5B, VC2M7SP02): 5 · Section C (5C, VC2M7SP04): 5 · Section D (5D, VC2M7SP03): 5.**