Mathematics Level 7

Chapter 4 — Measurement: area, volume, circles, angles and ratio

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Contents

Areas of rectangles, triangles and parallelograms

Content description `VC2M7M01`: establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem-solving

Achievement standard (extract): They establish and use formulas for the areas of triangles and parallelograms and the volumes of rectangular and triangular prisms to solve problems.

Theory

What this subtopic is about. The **area** of a shape is the amount of flat surface it covers, counted in square units. At Level 6 you found the area of rectangles; this subtopic **establishes** the area formulas — that is, it shows *why* each one works — for rectangles, triangles and parallelograms, and then puts them to work on problems. The rectangle formula is the starting point, and the other two are built from it.

Rectangles: counting squares (Level 6 recall). Area is a count of unit squares. Figure 1 shows a rectangle 6 cm long and 4 cm wide on a 1 cm grid: it holds 4 rows of 6 squares, so $6 \times 4 = 24$ squares, each of area 1 cm². The area is 24 cm². Counting squares always agrees with length times width, which is the Level 6 formula: $A = l\,w$.

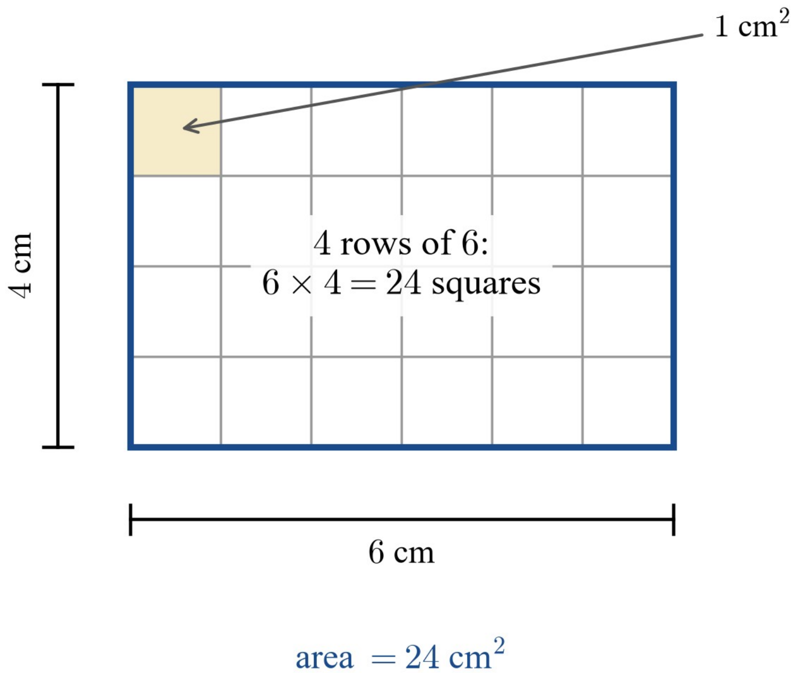
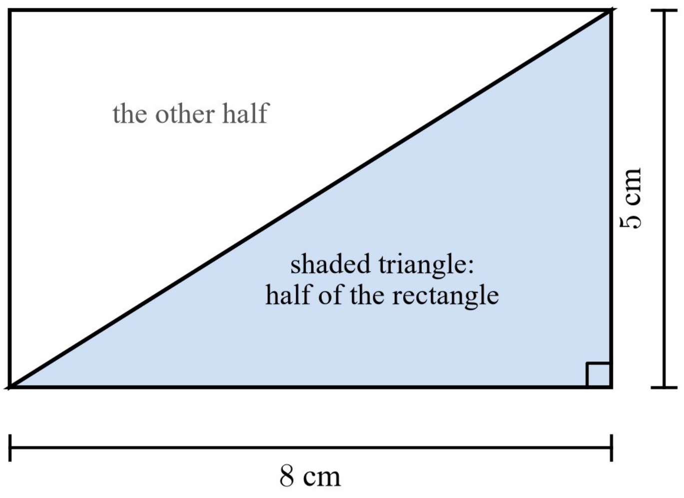


Figure 1. A rectangle 6 centimetres long and 4 centimetres wide drawn on a 1 centimetre grid, holding 4 rows of 6 squares, 24 squares in all; one corner square is shaded and labelled 1 square centimetre; below, 4 rows of 6 gives 24 squares, so the area is 24 square centimetres.

Triangles: half of a rectangle. A diagonal cuts a rectangle into two equal triangles, so a triangle with a right angle is exactly half of the rectangle around it (Figure 2). The rectangle is $8 \times 5 = 40$ cm², so the shaded triangle is $40 \div 2 = 20$ cm². The two sides of the rectangle that frame the triangle are called the **base** b (the side the triangle sits on) and the **perpendicular height** h — the distance from the base to the top corner, measured at right angles to the base. The small square mark on a figure always shows where the height meets the base at right angles.



$$\text{rectangle } = 8 \times 5 = 40 \text{ cm}^2, \text{ so triangle } = 40 \div 2 = 20 \text{ cm}^2$$

Figure 2. An 8 centimetre by 5 centimetre rectangle cut by its diagonal into two equal triangles; the lower triangle is shaded and marked half of the rectangle, the upper one marked the other half; below, the rectangle is 8 times 5 equals 40 square centimetres, so the triangle is 40 divided by 2 equals 20 square centimetres.

The same idea works for *any* triangle, not just ones with a right angle. In Figure 3 the triangle has base 10 cm and height 6 cm. The dashed line straight down from the top corner splits the surrounding 10×6 rectangle into a 4×6 rectangle and a 6×6 rectangle, and each sloping side of the triangle is a diagonal of its smaller rectangle. The triangle holds half of each: half of 24 is 12 cm², half of 36 is 18 cm², and $12 + 18 = 30$ cm². That is exactly $\frac{1}{2} \times 10 \times 6 = 30$ cm². So for every triangle:

$$A = \frac{1}{2} b h$$

A triangle is half of an appropriate rectangle — that is the fact the formula records.

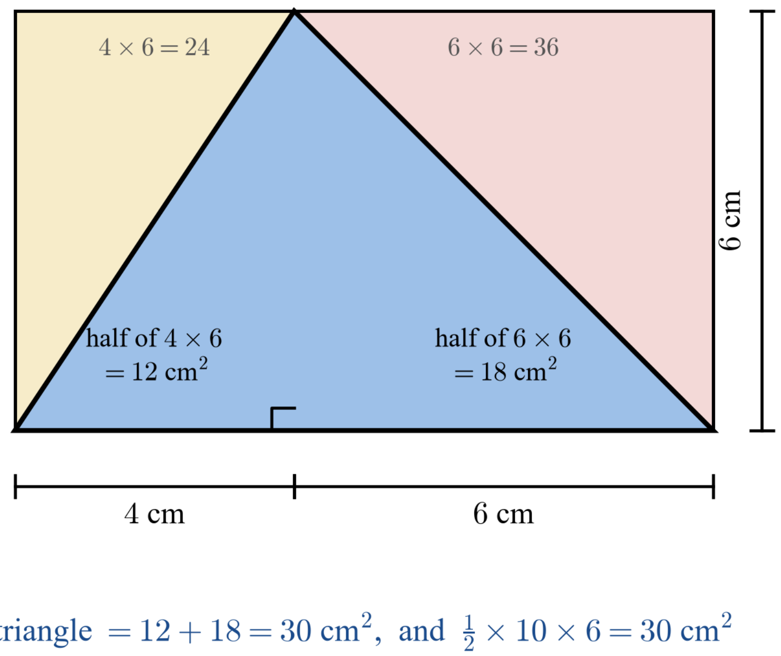


Figure 3. A triangle with base 10 centimetres and height 6 centimetres drawn inside a 10 centimetre by 6 centimetre rectangle; a dashed line straight down from the top corner splits the rectangle into a 4 by 6 rectangle and a 6 by 6 rectangle, and the triangle holds half of each, 12 plus 18 equals 30 square centimetres, which matches one half times 10 times 6 equals 30 square centimetres.

Same base, same height, same area. Because only the base and the height enter the formula, two triangles with the same base and the same height have the same area, even if they look different. Figure 4 shows three triangles on one 10 cm base with every top corner on a line 6 cm above it: each has area $\frac{1}{2} \times 10 \times 6 = 30$ cm². Slanting the sides changes the shape but not the area.

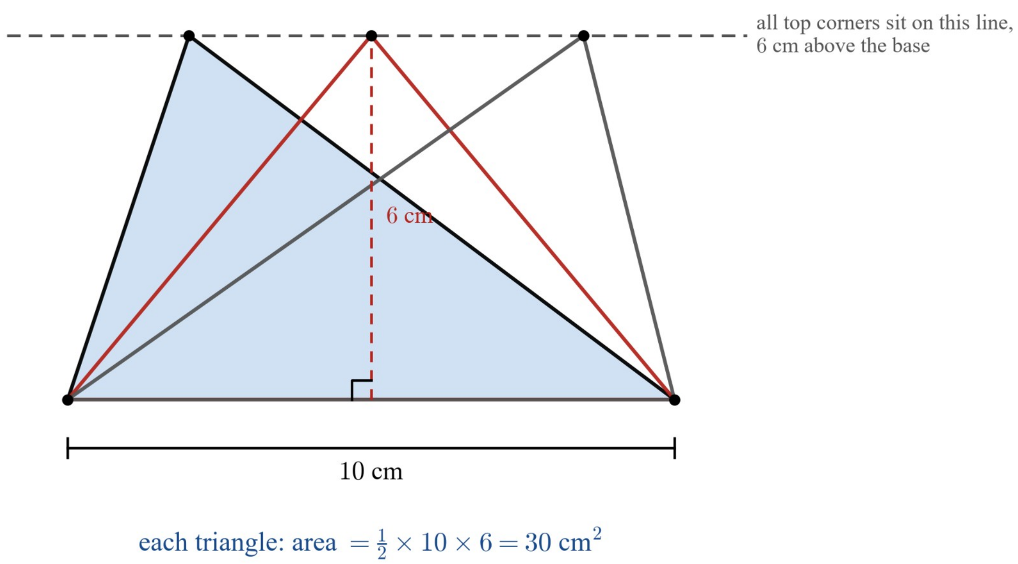


Figure 4. Three triangles drawn on the same base of 10 centimetres with their top corners all on a dashed line 6 centimetres above the base; the middle triangle shows its height of 6 centimetres as a dashed line meeting the base at a right angle, shown by the small square mark; each triangle has area one half times 10 times 6 equals 30 square centimetres.

Parallelograms: cut and slide into a rectangle. A **parallelogram** is a four-sided shape with both pairs of opposite sides parallel. Figure 5 shows how its area works: cut straight down the height and slide the triangle piece across, and the parallelogram becomes a rectangle made of the same pieces. Sliding does not change the area, so the parallelogram has the same area as the rectangle: base times height.

$$A = b\,h$$

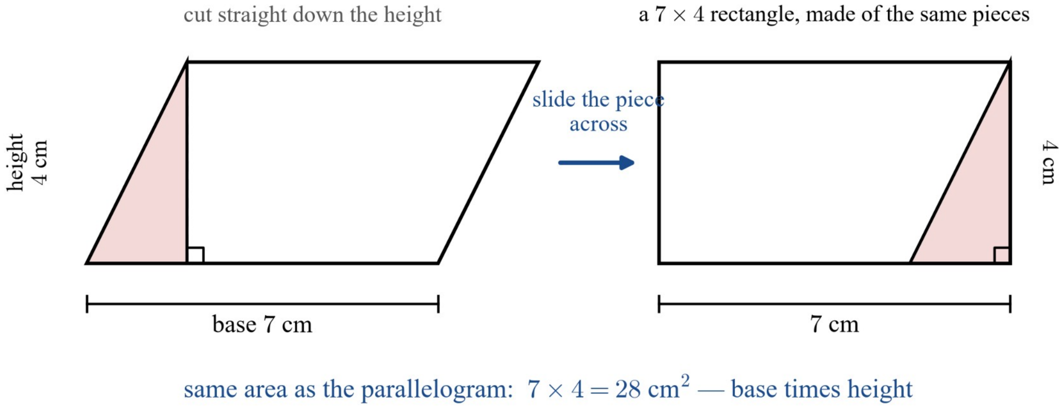


Figure 5. Two panels: the left shows a parallelogram with base 7 centimetres and height 4 centimetres, with a triangle piece shaded where a cut runs straight down the height; an arrow between the panels says slide the piece across; the right panel shows the same pieces making a 7 by 4 rectangle, so the area is 28 square centimetres, base times height.

The height, not the sloping side. The height of a parallelogram is the distance *straight across* between the base and the opposite side, meeting the base at right angles — not the length of the sloping side. In Figure 6 the base is 9 cm, the height is 4 cm and the sloping side is 5 cm. The area is $9 \times 4 = 36$ cm². Using the sloping side ($9 \times 5 = 45$) is the most common mistake with this formula; the sloping side is always longer than the height, so it always gives an answer that is too big.

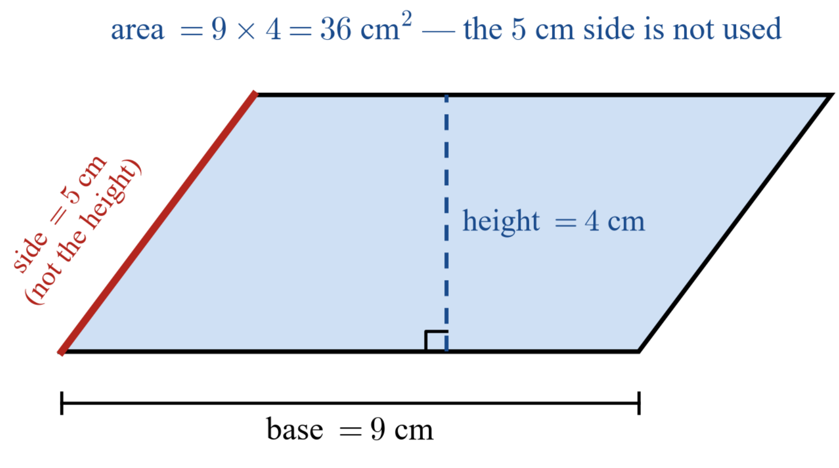


Figure 6. A parallelogram with base 9 centimetres; a dashed height of 4 centimetres meets the base at a right angle, shown by the small square mark; the left sloping side is drawn thick and red and labelled 5 centimetres, not the height; above, the area is 9 times 4 equals 36 square centimetres, and the 5 centimetre side is not used.

The formulas of this subtopic.

Shape	Formula	What the letters mean
rectangle	$A = l\,w$	l = length, w = width
triangle	$A = \frac{1}{2}b\,h$	b = base, h = perpendicular height
parallelogram	$A = b\,h$	b = base, h = perpendicular height

In every formula the base and the height (or length and width) must be in the **same unit** before you substitute. That is Level 6 conversion, recalled here, not new content: 10 mm make 1 cm and 100 cm make 1 m, so 650 mm is 65 cm. The area then comes out in square units: centimetres give cm², metres give m².

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
area	the amount of flat surface a shape covers, counted in square units	size of a surface, coverage
square unit	a square with sides of 1 unit, used to count area (1 cm², 1 m²)	unit square
base	the side a shape is measured from; any side chosen to sit on	bottom side
perpendicular height	the distance from the base to the opposite corner or side, measured at right angles to the base	height measured straight up, not along a slope
length, width	the two measurements across a rectangle, at right angles to each other	width is also called breadth
parallelogram	a four-sided shape with both pairs of opposite sides parallel	a slanted rectangle
formula	a rule in letters and symbols that says how to calculate something	rule
composite shape	a shape built from simpler shapes; its area is the areas of the parts added or taken away	combined shape

This subtopic establishes the three area formulas and uses them on surfaces. Ahead, 4B stacks these same areas up to build the volumes of prisms, and 4F compares areas in ratio problems.

Worked examples

Example 1 — scaffolded

The school grounds team is laying turf on a triangular corner of the courtyard. The triangle has base 2.4 m and perpendicular height 1.5 m. What area of turf is needed?

Working	Comment
Choose. A triangle, so $A=\frac{1}{2}bh$.	The height is given at right angles to the base, so both numbers can go straight in.
Substitute and solve. $A=\frac{1}{2}\times 2.4\times 1.5=\frac{1}{2}\times 3.6=1.8$.	Multiply first, then halve. (Halving first works too — same answer.)
Check. Round the base: $2.4\approx 2$. Then $\frac{1}{2}\times 2\times 1.5=1.5$, so the answer should be a little more than 1.5. 1.8 is.	An estimate first, exact value second: the check is part of the working.

∴ the area of turf needed is 1.8 m².

Example 2 — scaffolded

A metal sheet is cut into a parallelogram with base 650 mm, height 40 cm and sloping side 50 cm. Find its area in cm².

Working	Comment
Convert. 650 mm is 65 cm, because 10 mm make 1 cm.	Lengths must be in the same unit before substituting (Level 6 conversion).
Choose. A parallelogram, so $A = b\,h$. The sloping side of 50 cm is not the height, so it is not used (Figure 6).	The height meets the base at right angles; the 50 cm side is extra information.
Substitute and solve. $A = 65 \times 40 = 2600$.	$65 \times 4 = 260$, so $65 \times 40 = 2600$.

∴ the area of the sheet is 2600 cm².

Example 3 — unscaffolded

A triangle has area 36 cm² and base 9 cm. Find its perpendicular height.

Working	Comment
The formula says $\frac{1}{2} \times 9 \times h = 36$, where h is the height.	Substitute what is known; the height is the unknown.
Double both sides: $9 \times h = 72$.	The base-times-height rectangle is twice the triangle (Figure 3).
$h = 72 \div 9 = 8$.	Undo the multiplication with division.
Check. $\frac{1}{2} \times 9 \times 8 = \frac{1}{2} \times 72 = 36$. ✓	Substituting back gives the stated area, so the height is right.

∴ the perpendicular height is 8 cm.

Example 4 — unscaffolded

The end wall of the school's gardening shed is a rectangle with a triangle on top (Figure 7). The rectangle is 4.2 m wide and 2.5 m high; the triangle has base 4.2 m and height 1.4 m. The whole wall needs one coat of paint. What area must the paint cover?

Working	Comment
Rectangle: $4.2 \times 2.5 = 10.5$.	$4.2 \times 2 = 8.4$ and $4.2 \times 0.5 = 2.1$; together 10.5.
Triangle: $\frac{1}{2} \times 4.2 \times 1.4 = 2.1 \times 1.4 = 2.94$.	Halving the base first keeps the numbers small.
Total: $10.5 + 2.94 = 13.44$.	A composite shape: the parts' areas add.

∴ the paint must cover 13.44 m² of wall.

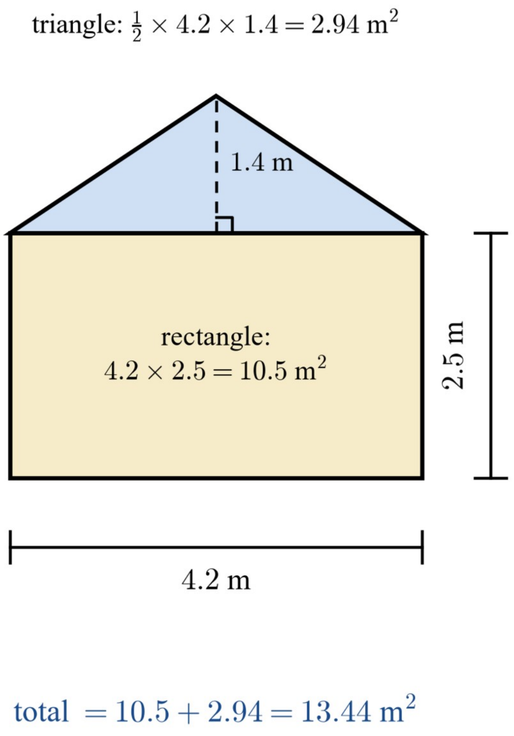


Figure 7. The end wall of a shed: a rectangle 4.2 metres wide and 2.5 metres high with a triangle on top whose base is 4.2 metres and whose height is 1.4 metres, shown as a dashed line meeting the top of the rectangle at a right angle; the rectangle is 4.2 times 2.5 equals 10.5 square metres, the triangle is one half times 4.2 times 1.4 equals 2.94 square metres, and the total is 13.44 square metres.

Practice questions

Scaffolded

Q1. Use Figure 1. (a) How many 1 cm² squares does the 6 cm by 4 cm rectangle hold? (b) Write the multiplication that gives the same count. (c) Use the same idea to find the area of a rectangle 9 cm long and 7 cm wide.

Q2. Find the area of each triangle using $A = \frac{1}{2}bh$.

	Base	Perpendicular height
(a)	8 cm	5 cm
(b)	10 cm	6 cm
(c)	12 cm	3.5 cm

Q3. Find the area of each parallelogram using $A = bh$. Watch for extra information.

	Base	Height	Sloping side
(a)	7 cm	4 cm	—
(b)	9 cm	4 cm	5 cm
(c)	12 cm	6 cm	7 cm

Q4. Use Figure 4. (a) What two measurements do all three triangles share? (b) Find the area of each triangle. (c) In one sentence, explain why the three areas are equal even though the triangles look different.

Q5. Convert to the unit asked. (Level 6 skill, kept in practice here.) (a) 650 mm = ? cm (b) 3 m = ? cm (c) 250 cm = ? m

Q6. Use the triangle on the 1 cm grid in Figure 8. (a) Complete the rectangle around the triangle and find its area. (b) The triangle is half of that rectangle. Find its area. (c) Check your answer with $A = \frac{1}{2} b h$, reading the base and height from the grid.

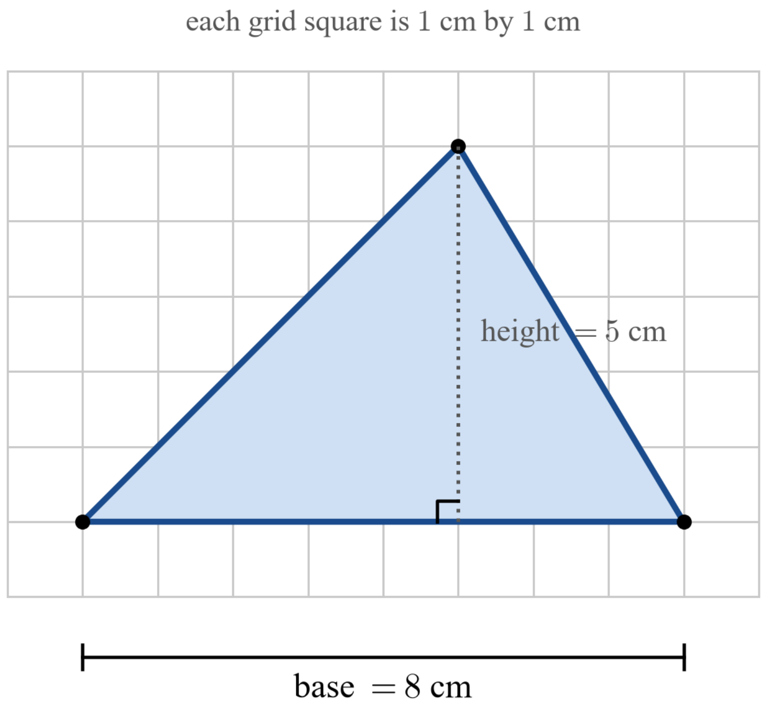


Figure 8. A triangle drawn on a 1 centimetre grid with its base 8 centimetres along a grid line and its top corner 5 centimetres above the base, the height shown as a dotted line meeting the base at a right angle; each grid square is 1 centimetre by 1 centimetre.

Unscaffolded

Q7. A rectangular block of land is 24 m by 18 m (Figure 9). A triangular vegetable garden runs along one 18 m side, reaching 5 m into the block. (a) Find the area of the whole block. (b) Find the area of the vegetable garden. (c) The rest of the block is lawn. Find the lawn area.

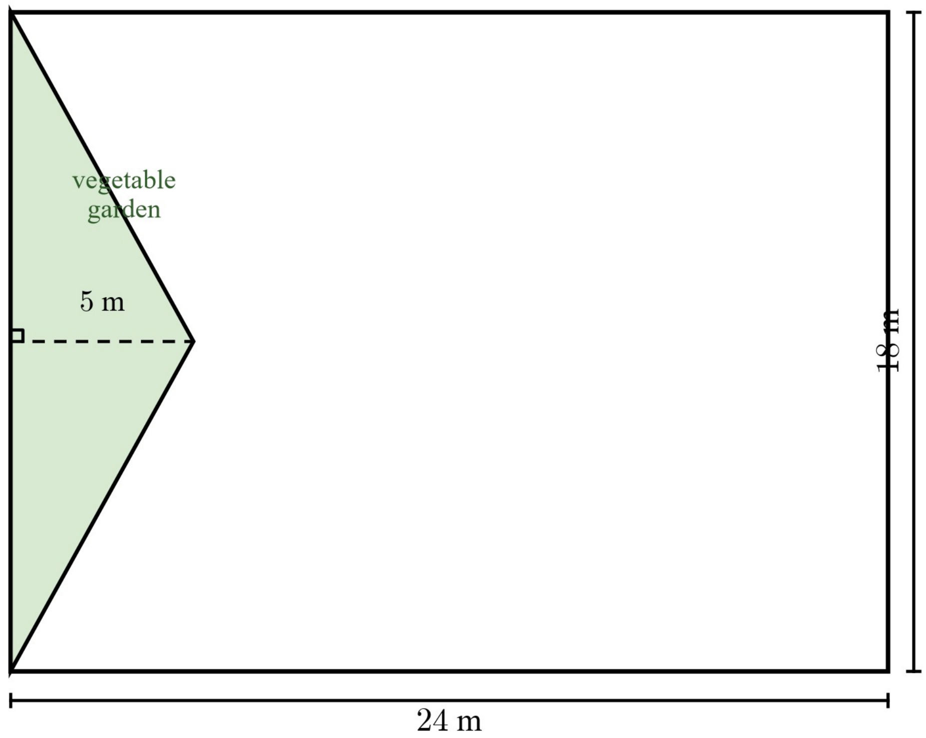


Figure 9. A rectangular block of land 24 metres by 18 metres; along the left 18 metre side a triangular vegetable garden reaches 5 metres into the block, the 5 metre height shown as a dashed line meeting the side at a right angle.

Q8. A parallelogram has area 84 cm² and base 12 cm. Find its height.

Q9. A triangle has area 30 cm² and base 10 cm. Find its perpendicular height.

Q10. A parallelogram has base 2 m and height 45 cm. Find its area (a) in cm²; (b) in m².

Q11. A rectangular paved area is 8 m by 5 m. A triangular garden bed with base 3 m and height 2 m is cut out of one corner and left unpaved. Find the paved area.

Q12. Mia uses Figure 6 and writes area $= 9 \times 5 = 45$ cm². (a) Explain Mia's mistake. (b) Find the correct area.

Q13. Ben estimates the area of a triangle with base 4.9 m and height 10.2 m as "about 50 m²". Without calculating exactly, explain why Ben's estimate cannot be right, and give a better one.

Q14. A parallelogram and a triangle have the same area. The parallelogram has base 8 cm and height 5 cm. The triangle has base 10 cm. Find the triangle's perpendicular height.

Answers

Q1. (a) 24 squares; (b) $6 \times 4 = 24$; (c) $9 \times 7 = 63$ cm².

Q2. (a) 20 cm²; (b) 30 cm²; (c) 21 cm².

Q3. (a) 28 cm²; (b) 36 cm²; (c) 72 cm² — the sloping side is not used in any of them.

Q4. (a) the base (10 cm) and the height (6 cm); (b) 30 cm² each; (c) the area of a triangle depends only on its base and height, and all three share both.

Q5. (a) 65 cm; (b) 300 cm; (c) 2.5 m.

Q6. (a) $8 \times 5 = 40$ cm²; (b) $40 \div 2 = 20$ cm²; (c) $\frac{1}{2} \times 8 \times 5 = 20$ cm² ✓.

Q7. (a) 432 m²; (b) 45 m²; (c) 387 m².

Q8. 7 cm.

Q9. 6 cm.

Q10. (a) 9000 cm²; (b) 0.9 m².

Q11. 37 m².

Q12. (a) she used the 5 cm sloping side instead of the 4 cm height; (b) 36 cm².

Q13. $4.9 \times 10.2 \approx 5 \times 10 = 50$ is the *rectangle*; the triangle is half of that, about 25 m². Ben forgot the half.

Q14. 8 cm.

Fully-worked solutions

Q1. (a) The grid shows 4 rows of 6 squares: 24 squares of 1 cm² each. (b) $6 \times 4 = 24$ — the count agrees with length times width. (c) $9 \times 7 = 63$, so the area is 63 cm².

Q2. (a) $\frac{1}{2} \times 8 \times 5 = \frac{1}{2} \times 40 = 20$ cm². (b) $\frac{1}{2} \times 10 \times 6 = \frac{1}{2} \times 60 = 30$ cm². (c) $\frac{1}{2} \times 12 \times 3.5 = \frac{1}{2} \times 42 = 21$ cm².

Q3. (a) $7 \times 4 = 28$ cm². (b) $9 \times 4 = 36$ cm² — the 5 cm sloping side is not the height, so it is ignored. (c) $12 \times 6 = 72$ cm² — again the sloping side (7 cm) plays no part.

Q4. (a) All three sit on the same 10 cm base, and every top corner lies on the line 6 cm above it, so all three have height 6 cm. (b) Each: $\frac{1}{2} \times 10 \times 6 = 30$ cm². (c) Only the base and the height enter the formula, and the three triangles share both, so their areas are equal however slanted their sides are.

Q5. (a) $650 \div 10 = 65$ cm. (b) $3 \times 100 = 300$ cm. (c) $250 \div 100 = 2.5$ m.

Q6. (a) The surrounding rectangle is 8 cm by 5 cm: $8 \times 5 = 40$ cm². (b) The triangle is half of it: $40 \div 2 = 20$ cm². (c) From the grid, base $= 8$ cm and height $= 5$ cm, so $A = \frac{1}{2} \times 8 \times 5 = 20$ cm² — the same answer.

Q7. (a) $24 \times 18 = 432$ m². (b) The garden's base is the 18 m side and its height is 5 m: $\frac{1}{2} \times 18 \times 5 = \frac{1}{2} \times 90 = 45$ m². (c) $432 - 45 = 387$ m² of lawn.

Q8. $A = bh$ says $12 \times h = 84$, so $h = 84 \div 12 = 7$ cm. Check: $12 \times 7 = 84$ ✓.

Q9. $\frac{1}{2} \times 10 \times h = 30$, so $10 \times h = 60$ and $h = 60 \div 10 = 6$ cm. Check: $\frac{1}{2} \times 10 \times 6 = 30$ ✓.

Q10. (a) 2 m $= 200$ cm, so $A = 200 \times 45 = 9000$ cm². (b) 45 cm $= 0.45$ m, so $A = 2 \times 0.45 = 0.9$ m². (Both are the same area: 0.9 m² $= 9000$ cm², since 1 m² is $10{,}000$ cm².)

Q11. Rectangle: $8 \times 5 = 40$ m². Triangle: $\frac{1}{2} \times 3 \times 2 = 3$ m². Paved area: $40 - 3 = 37$ m².

Q12. (a) Mia multiplied the base by the 5 cm *sloping side*. The formula needs the perpendicular height, the 4 cm dashed line that meets the base at right angles. (b) $9 \times 4 = 36$ cm². (As Figure 6 notes, the sloping-side answer is always too big.)

Q13. Rounding gives $4.9 \approx 5$ and $10.2 \approx 10$, so $b \times h \approx 50$ — but that product is the area of the rectangle around the triangle. The triangle is half of that rectangle, so a good estimate is $50 \div 2 = 25$ m². (The exact area is $\frac{1}{2} \times 4.9 \times 10.2 = 24.99$ m², close to 25.)

Q14. Parallelogram: $8 \times 5 = 40$ cm². The triangle has the same area, so $\frac{1}{2} \times 10 \times h = 40$, giving $10 \times h = 80$ and $h = 8$ cm. Check: $\frac{1}{2} \times 10 \times 8 = 40$ ✓.

Teacher note

- **Establish before use (Chapter 4 plan).** `VC2M7M01` requires the formulas to be established, not stated and drilled. The rectangle is grounded in the unit-square count (Figure 1); the triangle is derived as half of an appropriate rectangle (`VC2M7M01.E02`, Figures 2–3) and the parallelogram by cut-and-slide onto a rectangle (Figure 5). Use then follows in WE1–WE4 and the questions (`VC2M7M01.E03`), including same-base-same-height reasoning (Figure 4, `VC2M7M01.E01`'s build-on-the-rectangle idea running through all three).
- **Level 6 recall (D3).** The rectangle formula (`VC2M6M02`) is recalled in one line with the grid picture and maintained, not re-taught. Metric conversion (`VC2M6M01`) is used in WE2, Q5 and Q10 and maintained per **P1** — flagged here rather than taught as content, as the Chapter 4 note requires.
- **No Pythagoras (D4.10).** Heights are given directly wherever a sloping side appears (Figure 6, Q3, Q12); sloping sides serve only as distractors. No length is ever computed from two others.
- **No rates (D4.8).** Every application asks for an area only. There are no cost-per-m², coverage-rate or paint-per-hour items; rates belong to Level 8 (`VC2M8M05`).
- **Reasonableness (P7).** WE1 carries an estimate check inside the working and Q13 asks whether an answer is reasonable *before* any exact calculation — the chapter's standing-habit item.
- **Measurements used, not taught (P8).** All lengths arrive as stated numbers on figures; no item requires reading an instrument or recording to a precision, which VC2 Science owns.
- **Back-solving stays numerical.** WE3, Q8, Q9 and Q14 solve $b\,h = A$-shaped numerical equations with one unknown (Level 6, `VC2M6A02`); literal transposition of formulas is Level 8 (`VC2M8A01`, P5) and does not appear.
- **Terminology (D12).** *Perpendicular height* and *base* follow VCAA usage; the height-versus- sloping-side distinction is defined at first use and marked with right-angle symbols on every figure, per CONTESTED_TERMS Rule 1.
- **Figures.** All nine figures are generated by `src/gen_4a.py` from the exact values stated in the text; grids, brackets and area labels are computed from those values, nothing drawn by eye.
- **Contexts.** The courtyard, shed wall and block of land are transparently fictional school settings with no real-data claims, so no §6 source lines are required.
- **No D11 content.** None of `VC2M7M01.E01–.E03` names Aboriginal or Torres Strait Islander knowledges, so nothing is included under the content policy (Rule 1).

Volume of right prisms

Content description `VC2M7M02`: solve problems involving the volume of right prisms including rectangular and triangular prisms, using established formulas and appropriate units

Achievement standard (extract): They establish and use formulas for the areas of triangles and parallelograms and the volumes of rectangular and triangular prisms to solve problems.

Theory

What volume measures. The **volume** of a solid tells you how much space it takes up. At Level 6 you measured volume by counting **unit cubes**: a cube that is 1 cm along each edge has a volume of 1 cubic centimetre, written 1 cm³, and bigger solids were measured by the number of these cubes that fill them with no gaps and no overlaps. The same idea works with any unit: 1 mm³ is a cube 1 mm along each edge, and 1 m³ is a cube 1 m along each edge. This subtopic turns the counting into formulas.

Rectangular prisms from unit cubes. Figure 1 shows a rectangular prism built from 1 cm cubes: 4 cubes long, 3 cubes wide and 2 cubes high. The bottom layer is a 4×3 block of 12 cubes, and there are 2 layers, so the prism is built from $2 \times 12 = 24$ cubes and its volume is 24 cm³. The counting is also a multiplication: $4 \times 3 \times 2 = 24$.

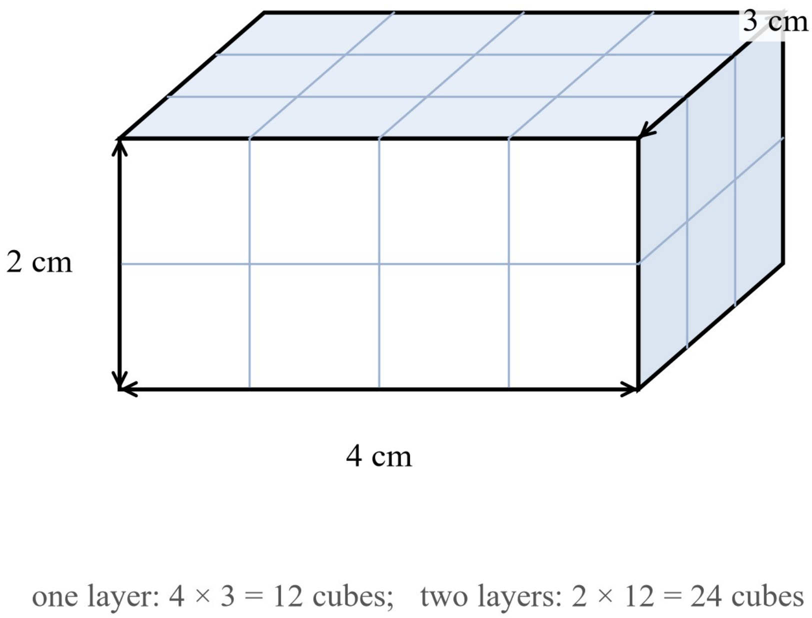


Figure 1. A rectangular prism built from 1 cm unit cubes, 4 cubes long, 3 cubes wide and 2 cubes high; one layer of 12 cubes with 2 layers gives 24 cubes.

Every prism: area of base times height. At Level 6 you also met the **cross-section** of a solid, the shape you get when you slice it. A **prism** is a solid whose cross-section is the same shape and size at every slice taken parallel to its ends: Figure 2 shows the idea, because stacking identical slices builds a prism. A **right prism** is one whose slices stack straight up, so the joining edges meet the end faces at right angles. Since every slice is the same, the volume of any right prism is the area of its **base** (one end face) multiplied by its **height**, the distance between the two ends measured at right angles to them:

$$V = (\text{area of base}) \times \text{height}$$

For the rectangular prism in Figure 1 this says the same thing twice: the base is $4 \times 3 = 12$ cm² and the height is 2 cm, so $V = 12 \times 2 = 24$ cm³, the count from before.

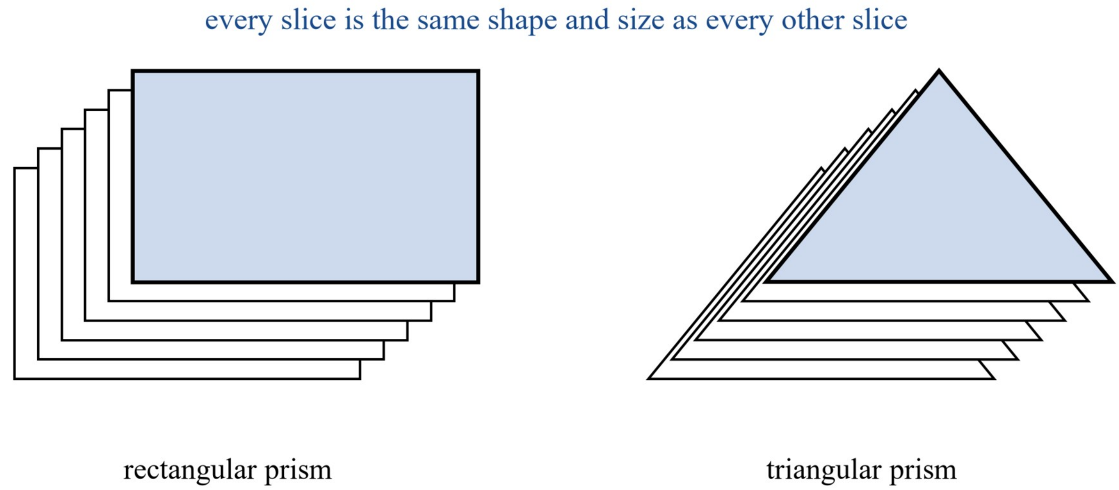


Figure 2. Two stacks of identical slices, a rectangle repeated to build a rectangular prism and a triangle repeated to build a triangular prism; every slice is the same shape and size as every other slice.

The formulas you will use most are the special cases of this one rule:

Prism	Base	Volume formula
rectangular prism	rectangle, length l, width w	$V = l \times w \times h$
triangular prism	triangle, base b, height t	$V = \frac{1}{2} \times b \times t \times h$
any right prism	any shape, area A	$V = A \times h$

Triangular prisms. For a triangular prism, the base is a triangle, and the triangle's area is the one established in subtopic 4A: $\frac{1}{2}$ × base × height. In Figure 3 the triangular end has base 6 cm and height 4 cm, so its area is $\frac{1}{2} \times 6 \times 4 = 12$ cm², and the prism is 9 cm long, so $V = 12 \times 9 = 108$ cm³. The 4 cm is the height *of the triangle*, drawn at right angles to the 6 cm base; the 9 cm is the length of the prism. Two different heights, so take care to use the right one.

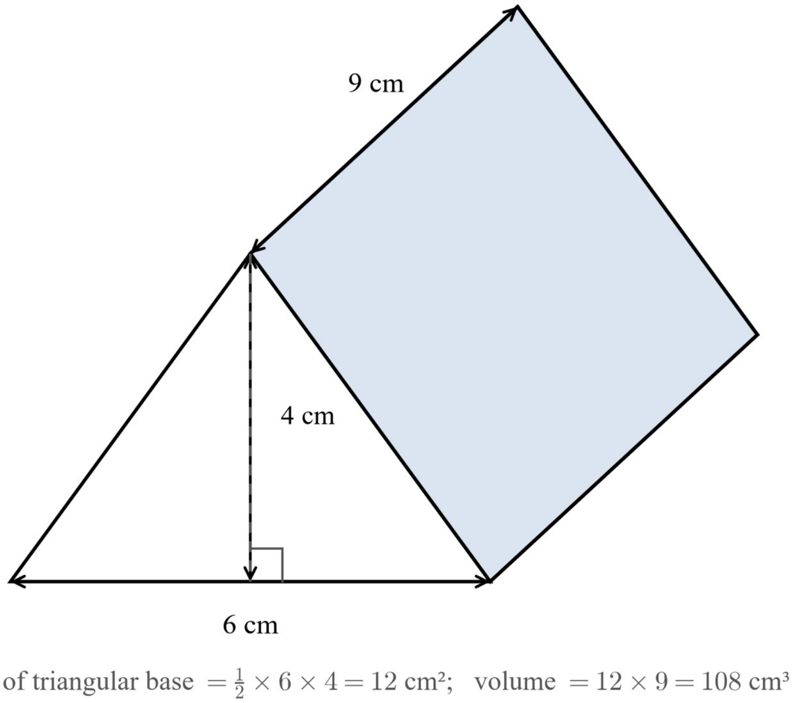


Figure 3. A triangular prism with triangular end of base 6 cm and triangle height 4 cm marked with a dashed altitude meeting the base at right angles, and prism length 9 cm; the volume is 108 cm³.

The high-rise connection. A tower whose floors are all the same shape and size is itself a prism: every horizontal slice is one floor's shape. Figure 4 shows a tower of 12 floors, each 3 m from floor to ceiling, so its height is $12 \times 3 = 36$ m, and each floor measures 20 m by 17 m, an area of $20 \times 17 = 340$ m². The volume is therefore $340 \times 36 = 12\,240$ m³. Read another way, one floor's slice holds $340 \times 3 = 1020$ m³ and 12 identical floors hold $12 \times 1020 = 12\,240$ m³ — the area of the floor space times the height, with the number of floors building the height.

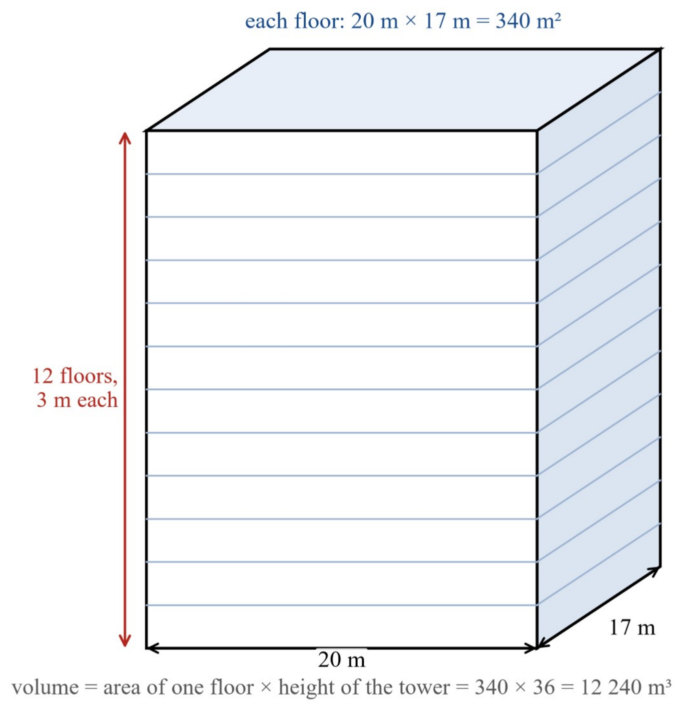


Figure 4. A tower block drawn as a rectangular prism of 12 floors, 3 m each, with floor lines on the faces; each floor is 20 m by 17 m, an area of 340 m², and the volume is 340 × 36 = 12 240 m³.

Choosing units, and moving between them. Volume is only a useful answer with the right unit attached: an eraser is measured in cm³, a bedroom in m³, and a small bead in mm³. The units convert the way lengths do, but in three directions at once (Figure 5): because 1 cm = 10 mm, a cm³ cube holds $10 \times 10 \times 10 = 1000$ mm³, and because 1 m = 100 cm, a m³ cube holds $100 \times 100 \times 100 = 1\,000\,000$ cm³.

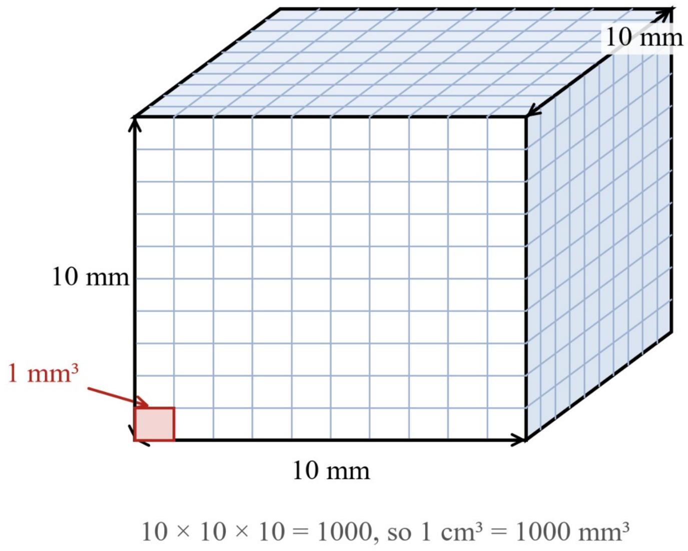


Figure 5. A cube 10 mm along each edge, drawn as a 10 by 10 by 10 array of millimetre cubes with one 1 mm³ cube marked in red; 10 × 10 × 10 = 1000, so 1 cm³ = 1000 mm³.

Predicting with a tool. If you have dynamic geometry software, build a prism in it, change one edge, and predict the new volume from the formula before the software shows it. Predicting first and checking after is the spatial reasoning this subtopic builds.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
volume	the space a solid takes up, counted in cubic units	cubic measure, space inside a solid
prism	a solid with the same cross-section at every slice parallel to its ends	solid with uniform cross-section
right prism	a prism whose joining edges meet the end faces at right angles	upright prism
base	the end face of a prism whose area goes into the formula	cross-section, end face
height	distance between the two end faces, at right angles to them	length, depth
unit cube	a cube 1 unit along each edge, used to measure volume	1 cm³ cube

Subtopic 4A established the triangle and parallelogram area formulas this subtopic uses, and 5A looks again at how prisms are drawn on paper.

Worked examples

Example 1 — scaffolded

Find the volume of the box in Figure 6.

Working	Comment
The base is a 12 cm by 7 cm rectangle: area $= 12 \times 7 = 84$ cm².	Area of a rectangle, from 4A.

Working	Comment
$V = 84 \times 5 = 420$, so the volume is 420 cm³.	Volume = area of base × height.
Check: $12 \times 7 \times 5 = 420$. ✓	Length × width × height gives the same answer.

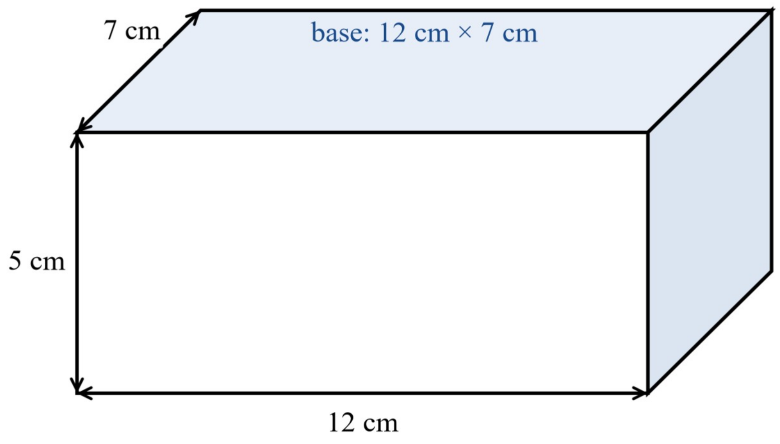


Figure 6. A rectangular box 12 cm long, 7 cm wide and 5 cm high, with the 12 cm by 7 cm base shaded on top.

Example 2 — scaffolded

A tent is a triangular prism, as in Figure 7. The triangular end is 2.4 m wide and 1.5 m high, and the tent is 2 m long. Find the volume of space inside the tent.

Working	Comment
Triangular end: area $= \frac{1}{2} \times 2.4 \times 1.5 = 1.8$ m².	Triangle area from 4A.
$V = 1.8 \times 2 = 3.6$, so the tent holds 3.6 m³ of space.	Base area × length of the prism.

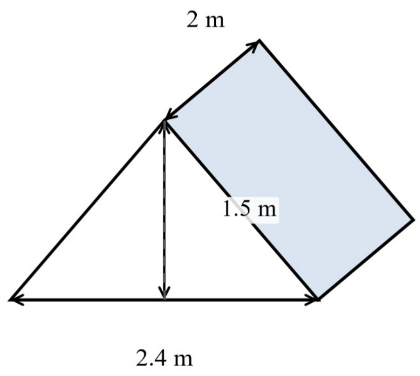


Figure 7. A tent drawn as a triangular prism: the triangular end is 2.4 m wide and 1.5 m high, marked with a dashed altitude, and the tent is 2 m long.

Example 3 — unscaffolded

A tower block has 12 identical floors. Each floor is 3 m from floor to ceiling and has a floor area of 340 m². Treating the tower as a prism, find its volume.

The tower's height is $12 \times 3 = 36$ m. With base area 340 m²,

$$V = 340 \times 36 = 12\,240$$

∴ the volume of the tower is 12 240 m³.

Example 4 — unscaffolded

A rectangular tray is 300 mm long, 200 mm wide and 45 mm deep. Find its volume in cm³.

Work in centimetres from the start: 300 mm = 30 cm, 200 mm = 20 cm and 45 mm = 4.5 cm. Then

$$V = 30 \times 20 \times 4.5 = 600 \times 4.5 = 2700$$

so the volume is 2700 cm³. A quick check: $30 \times 20 \times 5 = 3000$ cm³ is close to 2700 cm³, so the answer is sensible.

Practice questions

Scaffolded

Q1. Each solid in Figure 8 is built from cubes of volume 1 cm³. Count the cubes (use layers: cubes in one layer times number of layers) and write the volume of each solid.

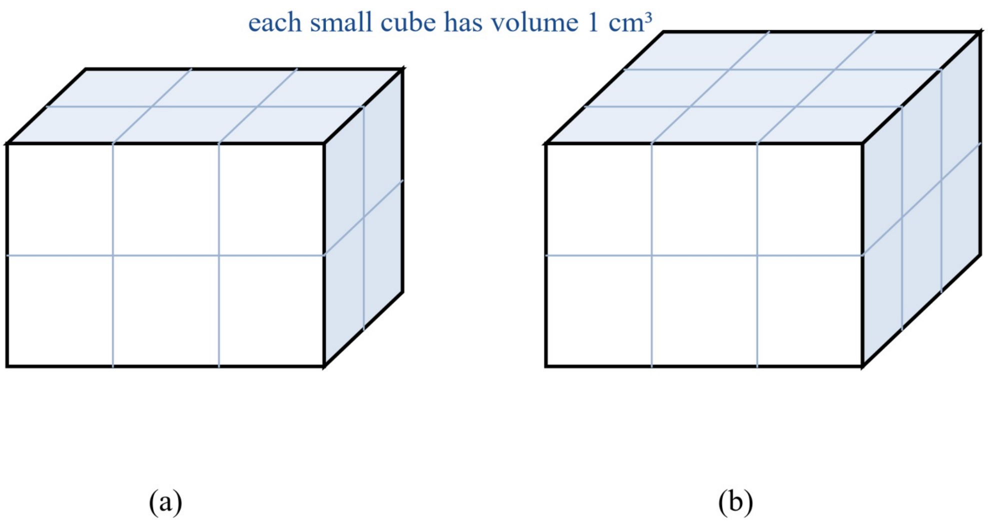


Figure 8. Two solids built from 1 cm³ cubes: (a) 3 cubes long, 2 cubes wide and 2 cubes high; (b) 3 cubes long, 3 cubes wide and 2 cubes high.

Q2. Use $V = l \times w \times h$ to find the volume of each rectangular prism. (a) $l = 9$ cm, $w = 4$ cm, $h = 5$ cm (b) $l = 7$ cm, $w = 6$ cm, $h = 10$ cm (c) $l = 8$ m, $w = 3$ m, $h = 2.5$ m (d) $l = 12$ cm, $w = 5$ cm, $h = 4$ cm

Q3. Complete the table using V = area of base × height.

	Area of base	Height	Volume
(a)	35 cm²	4 cm	
(b)	24 cm²	6 cm	
(c)		5 cm	90 cm³
(d)	50 m²		200 m³

Q4. Use $V = \frac{1}{2} \times b \times t \times h$ to find the volume of each triangular prism. (a) triangle base 8 cm, triangle height 5 cm, prism length 12 cm (b) triangle base 6 cm, triangle height 4 cm, prism length 10 cm (c) triangle base 3 m, triangle height 2 m, prism length 7 m

Q5. Choose the more sensible unit (mm^3, cm^3 or m^3) for the volume of each object. (a) an eraser (b) a lunch box (c) a grain of rice (d) a bedroom (e) a swimming pool

Q6. Convert, using $1\ cm^3 = 1000\ mm^3$ and $1\ m^3 = 1\,000\,000\ cm^3$. (a) $5\ cm^3$ to mm^3 (b) $3\ m^3$ to cm^3 (c) $8000\ mm^3$ to cm^3 (d) $2\,500\,000\ cm^3$ to m^3

Unscaffolded

Q7. Find the volume of the rectangular prism in Figure 9.

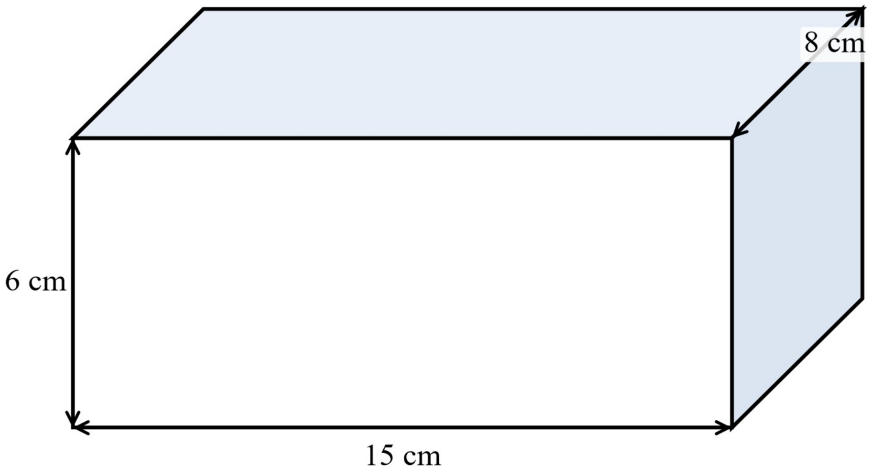


Figure 9. A rectangular prism 15 cm long, 8 cm wide and 6 cm high.

Q8. A garden bed is a triangular prism 3 m long. Its triangular end is 1.2 m wide and 0.5 m high. Find the volume of soil it holds when filled level with the top.

Q9. The rectangular fish tank in Figure 10 has a base 50 cm by 40 cm and a volume of $48\,000\ cm^3$. Find its height h.

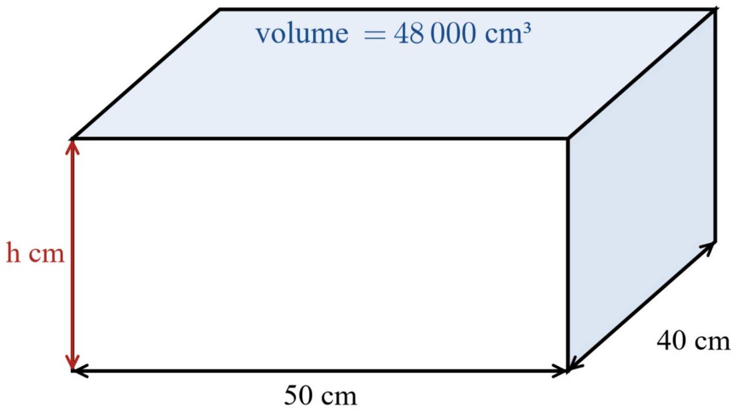


Figure 10. A rectangular fish tank with base 50 cm by 40 cm and unknown height h cm, with volume 48 000 cm³.

Q10. Mia says a tank 60 cm long, 30 cm wide and 40 cm high has a volume of $7200\ cm^3$. Without doing the full multiplication, explain why her answer cannot be right, then find the correct volume.

Q11. (a) A tower has 18 identical floors, each 3 m from floor to ceiling, and each floor has area 425 m^2. Find the volume of the tower. (b) A second tower has the same volume, with 15 identical floors each 3 m high. Find the area of one of its floors.

Q12. A box measures 120 mm by 80 mm by 50 mm. Find its volume in cm^3.

Q13. Prism A is 8 cm by 5 cm by 3 cm. Prism B is 6 cm by 4 cm by 5 cm. Before calculating, predict which prism has the greater volume, and say why. Then calculate both volumes and compare with your prediction.

Q14. A chocolate box is a rectangular prism 20 cm long, 12 cm wide and 4 cm high. (a) Find its volume. (b) A second box has the same base and is 8 cm high. Find its volume. (c) What happened to the volume when the height was doubled?

Q15. A wedge of foam is a triangular prism 2.5 m long. Its triangular end has base 1.6 m and height 0.8 m. Find the volume of the wedge.

Q16. (a) Find the volume of a cube with edge 7 cm. (b) A second cube has edge 14 cm. Find its volume, and find how many times greater it is than the volume in (a).

Q17. A rectangular prism has volume 2400 cm^3. Its base is 20 cm by 10 cm. Find its height.

Q18. A pool is a rectangular prism 25 m long and 10 m wide, with the same depth of 0.8 m everywhere. (a) Find the volume of the pool. (b) Explain why your answer is sensible by comparing it with the volume the pool would have if it were 1 m deep.

Answers

Q1. (a) 12 cm³; (b) 18 cm³.

Q2. (a) 180 cm³; (b) 420 cm³; (c) 60 m³; (d) 240 cm³.

Q3. (a) 140 cm³; (b) 144 cm³; (c) 18 cm²; (d) 4 m.

Q4. (a) 240 cm³; (b) 120 cm³; (c) 21 m³.

Q5. (a) cm³; (b) cm³; (c) mm³; (d) m³; (e) m³.

Q6. (a) 5000 mm³; (b) 3 000 000 cm³; (c) 8 cm³; (d) 2.5 m³.

Q7. 720 cm³.

Q8. 0.9 m³.

Q9. 24 cm.

Q10. Her answer is too small: the base alone is $60 \times 30 = 1800$ cm², so 7200 cm³ would be a tank only 4 cm high; the correct volume is 72 000 cm³.

Q11. (a) 22 950 m³; (b) 510 m².

Q12. 480 cm³.

Q13. Both volumes are 120 cm³; the prisms hold the same amount of space.

Q14. (a) 960 cm³; (b) 1920 cm³; (c) the volume doubled as well.

Q15. 1.6 m³.

Q16. (a) 343 cm³; (b) 2744 cm³, which is 8 times greater.

Q17. 12 cm.

Q18. (a) 200 m³; (b) at 1 m deep the pool would hold 250 m³, and 0.8 m is a little less than 1 m, so 200 m³ (a little less than 250 m³) is sensible.

Fully-worked solutions

Q1. (a) One layer is $3 \times 2 = 6$ cubes and there are 2 layers: $2 \times 6 = 12$, so the volume is 12 cm³. (b) One layer is $3 \times 3 = 9$ cubes and there are 2 layers: $2 \times 9 = 18$, so the volume is 18 cm³.

Q2. (a) $9 \times 4 \times 5 = 180$ cm³. (b) $7 \times 6 \times 10 = 420$ cm³. (c) $8 \times 3 \times 2.5 = 60$ m³. (d) $12 \times 5 \times 4 = 240$ cm³.

Q3. (a) $35 \times 4 = 140$ cm³. (b) $24 \times 6 = 144$ cm³. (c) $90 \div 5 = 18$, so the base area is 18 cm². (d) $200 \div 50 = 4$, so the height is 4 m.

Q4. (a) Triangle area $\frac{1}{2} \times 8 \times 5 = 20$ cm²; $V = 20 \times 12 = 240$ cm³. (b) Triangle area $\frac{1}{2} \times 6 \times 4 = 12$ cm²; $V = 12 \times 10 = 120$ cm³. (c) Triangle area $\frac{1}{2} \times 3 \times 2 = 3$ m²; $V = 3 \times 7 = 21$ m³.

Q5. (a) An eraser is a few cm along each edge: cm³. (b) A lunch box too: cm³. (c) A grain of rice is measured in mm: mm³. (d) A bedroom is measured in metres: m³. (e) A pool too: m³.

Q6. (a) $5 \times 1000 = 5000$ mm³. (b) $3 \times 1\,000\,000 = 3\,000\,000$ cm³. (c) $8000 \div 1000 = 8$ cm³. (d) $2\,500\,000 \div 1\,000\,000 = 2.5$ m³.

Q7. $V = 15 \times 8 \times 6$. First $15 \times 8 = 120$, then $120 \times 6 = 720$, so the volume is 720 cm³.

Q8. Triangle area $= \frac{1}{2} \times 1.2 \times 0.5 = 0.3$ m². $V = 0.3 \times 3 = 0.9$, so the garden bed holds 0.9 m³ of soil.

Q9. The base area is $50 \times 40 = 2000$ cm². The height is $48\,000 \div 2000 = 24$, so $h = 24$ cm. Check: $2000 \times 24 = 48\,000$. ✓

Q10. The base is $60 \times 30 = 1800$ cm². A volume of 7200 cm³ would mean a height of $7200 \div 1800 = 4$ cm, far less than the 40 cm the tank has, so Mia's answer cannot be right. The true volume is $1800 \times 40 = 72\,000$ cm³. (Mia missed a factor of 10.)

Q11. (a) Height $= 18 \times 3 = 54$ m; $V = 425 \times 54 = 22\,950$ m³. (b) Height $= 15 \times 3 = 45$ m; floor area $= 22\,950 \div 45 = 510$ m².

Q12. In centimetres: 12 cm by 8 cm by 5 cm. $V = 12 \times 8 \times 5 = 480$ cm³.

Q13. A prediction might say A, because 8 and 5 are its two biggest edges. Calculating: A is $8 \times 5 \times 3 = 120$ cm³ and B is $6 \times 4 \times 5 = 120$ cm³. The volumes are equal, so neither prism holds more space than the other.

Q14. (a) $V = 20 \times 12 \times 4 = 960$ cm³. (b) $V = 20 \times 12 \times 8 = 1920$ cm³. (c) Doubling the height doubled the volume: $1920 = 2 \times 960$, exactly what $V = \text{base area} \times h$ says.

Q15. Triangle area $= \frac{1}{2} \times 1.6 \times 0.8 = 0.64$ m². $V = 0.64 \times 2.5 = 1.6$, so the volume is 1.6 m³.

Q16. (a) $7 \times 7 \times 7 = 343$ cm³. (b) $14 \times 14 \times 14 = 2744$ cm³; $2744 \div 343 = 8$, so the second cube holds 8 times the volume of the first.

Q17. Base area $= 20 \times 10 = 200$ cm²; height $= 2400 \div 200 = 12$ cm.

Q18. (a) $V = 25 \times 10 \times 0.8 = 250 \times 0.8 = 200$ m³. (b) At 1 m deep the volume would be $25 \times 10 \times 1 = 250$ m³; the real pool is 0.8 m deep, a little less, and 200 m³ is a little less than 250 m³, so the answer is sensible.

Teacher note

- **Ownership and level split.** 4B owns `VC2M7M02`: the volumes of right prisms via the established formulas, including the base-area × height connection (E01, E02) and the floor-area × floors connection (E03). Rectangular and triangular prisms only; cylinders are not Level 7 work (circle area is `VC2M8M03`, D4.1), and nothing here finds an edge by Pythagoras' theorem (D4.10) — every triangle height is given, never derived from a slant edge.
- **4A dependency.** The triangle area formula is *established* in 4A (`VC2M7M01`) and used here without re-derivation; rectangle area is Level 6 (`VC2M6M02`).
- **Level 6 recall (D3).** Cross-sections and their relationship to right prisms (`VC2M6SP01`) are recalled in one sentence (theory, Figure 2), and metric unit conversion (`VC2M6M01`) is maintained inside WE4, Q6 and Q12 rather than taught — per §5.2 **P1**, this is the single most consequential assumption in the book, so it is deliberately exercised here and flagged in this note.
- **No capacity (D4.12).** The mL/cm³ equivalence and capacity are Level 8 (`VC2M8M02`). No question asks how much liquid anything holds; the fish-tank items measure the space itself in cm³.
- **No formula rearranging (P5).** Missing-dimension items (Q3c–d, Q9, Q17) are solved numerically by division ("what times 2000 gives 48 000?"), not by transposing; rearranging formulas is `VC2M8A01`.
- **Digital tools (D9).** The CD names no tool, so none is required; E04 names dynamic geometry software, and the optional "Predicting with a tool" paragraph delivers the spatial-reasoning-and-prediction exploration it suggests. No CAS.
- **Reasonableness habit (P7).** WE4, Q10, Q13 and Q18(b) ask for a prediction or a sensibleness check before or beside the answer.

- **Measurement (P8).** The subtopic uses given measurements; it does not teach measuring.
- **Contexts.** All contexts (box, tent, tower, tank, tray, garden bed, pool) are transparently generic teaching devices; no real building, business or place is asserted, so no source lines are required (§6).
- **Figures.** All ten figures are generated by `src/gen_4b.py` from the exact values stated in the prose, in one shared oblique projection (same scale on every axis, depth at half scale), so drawn prisms carry the stated length ratios; hidden edges are dashed and every dimension arrow repeats a value used in the questions.

π and the circumference, radius and diameter of a circle

Content description `VC2M7M03`: describe the relationship between π and the circumference, radius and diameter of a circle

Elaboration `VC2M7M03.E01`: recognising the features of circles and their relationships to one another; for example, labelling the parts of a circle including centre, radius, diameter and circumference and using one of radius, diameter or circumference to determine the measure of the other 2; and understanding that the diameter of a circle is twice the radius, or that the radius is the circumference divided by 2π

Elaboration `VC2M7M03.E02`: comparing the circumference of circles in relation to their radius and diameter with materials and measuring, to establish measurement formulas; for example, using a compass to draw several circles, then using string to approximate the circumference, comparing the length of string to the diameter of the circle

Elaboration `VC2M7M03.E03`: investigating π as the constant in the proportional relationship between the circumference of a circle and its diameter, and historical approximations from different civilisations, including Egypt, Babylon, Greece, India and China

Elaboration `VC2M7M03.E04` names applications of circles in the everyday life of Aboriginal and Torres Strait Islander Peoples. It is not delivered in this book — see the teacher note and `CONTENT_POLICY.md` Rule 1.

Achievement standard (extract): They describe the relationships between the radius, diameter and circumference of a circle.

Theory

What this subtopic is about. Wheels, plates, coins, pools and the centre circle of a sports ground are all circles, and every one of them hides the same surprising fact: the distance around the circle is a little more than 3 times the distance across it, for *every* circle, whatever its size. This subtopic names the parts of a circle, measures that "three-and-a-bit" for real circles, and gives it its name: π.

The parts of a circle. A circle is the set of all points that sit the same distance from a fixed point, the **centre**, labelled O in Figure 1. A line from the centre to the rim, and its length, is the **radius**, r. A line straight across the circle, through the centre and from rim to rim, is the **diameter**, d. The distance all the way around the circle is the **circumference**, C — the circumference is the perimeter of a circle, and like any length it is measured in cm, m and so on. The arrowhead on the blue rim in Figure 1 shows the direction you would walk to go once around.

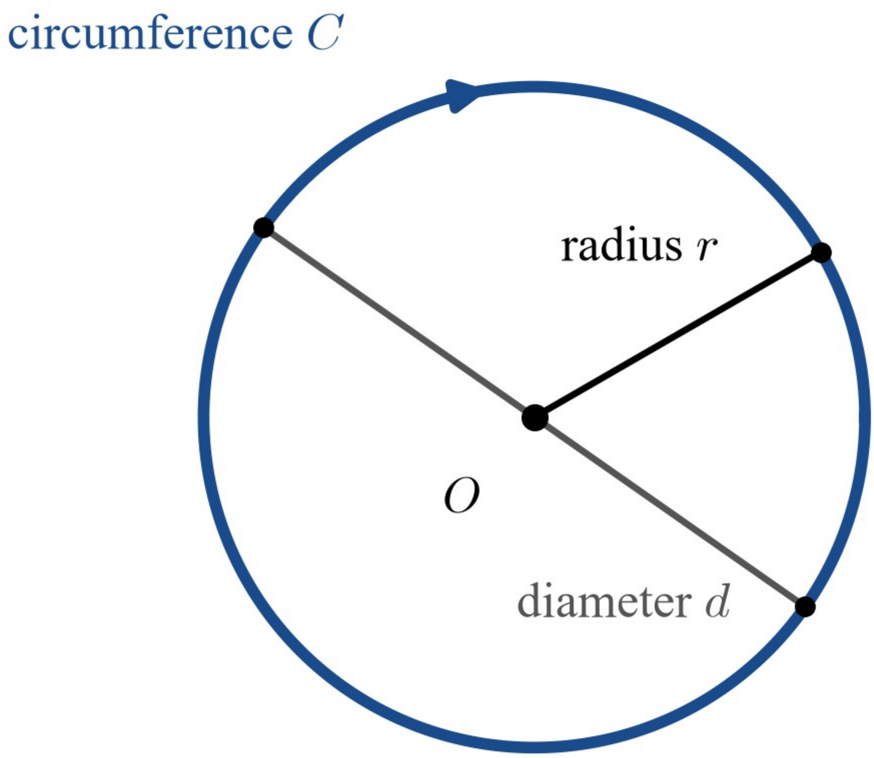


Figure 1. A circle with its four parts labelled: the centre O, a radius r drawn from the centre to the rim, a diameter d drawn through the centre from rim to rim, and the circumference C as the blue rim with an arrow showing the way around.

Radius and diameter. Every radius of one circle has the same length, because every point of the rim sits the same distance from the centre (left of Figure 2). A diameter is two radii end to end (right of Figure 2), so for any circle

$$d = 2r \text{ and } r = d \div 2$$

A circle with radius 4 cm has diameter 8 cm, and a circle with diameter 10 cm has radius 5 cm.

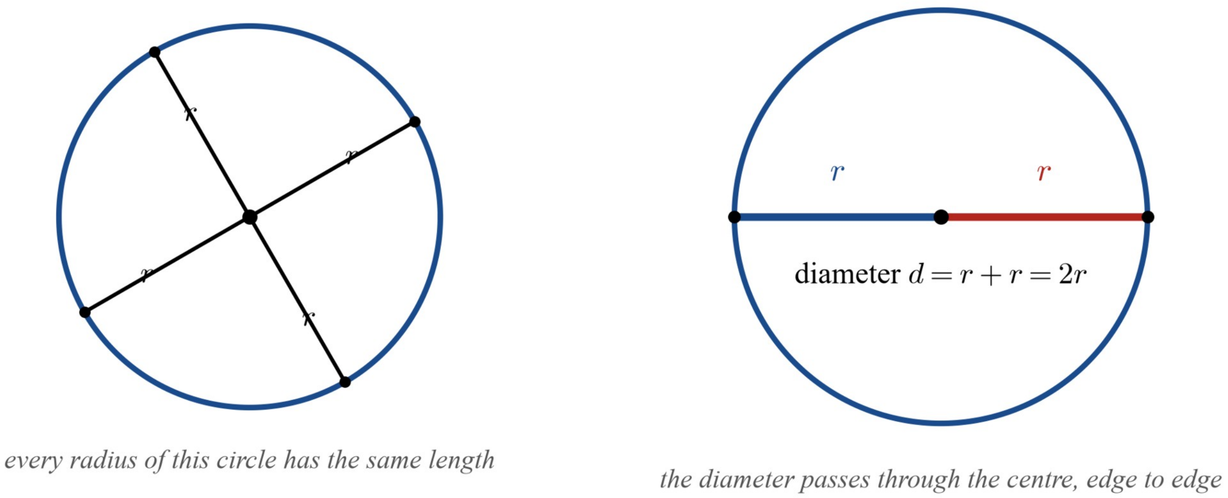


Figure 2. Left: a circle with four radii drawn, each labelled r, all the same length. Right: the same circle with a horizontal diameter drawn as a blue radius plus a red radius, labelled r and r, with d = r + r = 2r.

Measuring around a circle. You cannot lay a straight ruler around a rim, so measure the circumference with string, as in Figure 3: wrap the string once around the rim, mark it, then straighten it along a ruler. A class did this for four circles drawn with a pair of compasses, measuring each diameter and each string length to the nearest millimetre:

circle	diameter d (cm)	string length C (cm)	$C \div d$
A	4.0	12.7	3.18
B	6.0	18.6	3.10
C	8.0	25.3	3.16
D	10.0	31.2	3.12

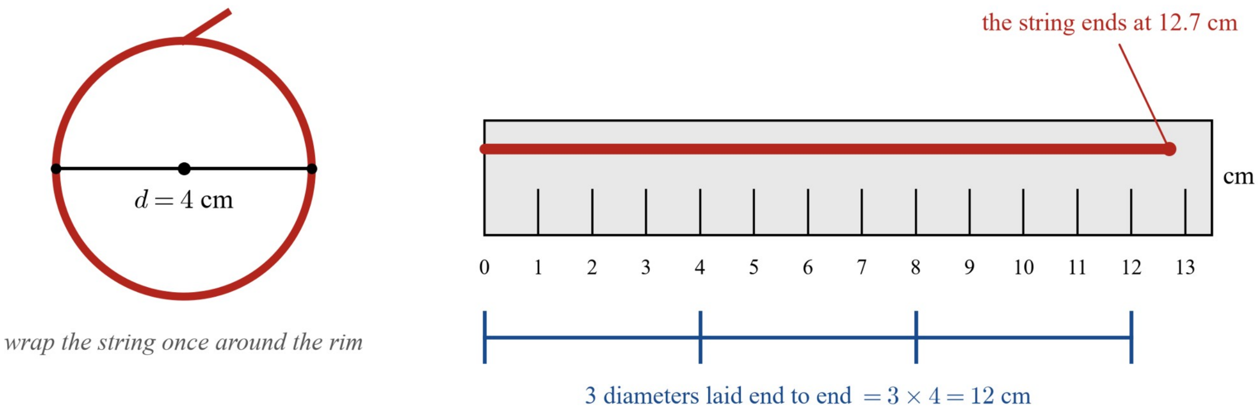


Figure 3. Left: a circle of diameter 4 cm with string wrapped once around its rim. Right: the same string straightened along a centimetre ruler, ending at 12.7 cm, just past three diameters laid end to end, which reach 12 cm.

The string is always a little longer than 3 diameters laid end to end. The last column says the same thing in numbers: $C \div d$ is never exactly the same — string and ruler measurements are never perfect — but every answer sits near 3.1. Repeat the investigation with any circles you like, from a coin to a round table, and the same pattern appears.

The number π. The pattern is the discovery: for *every* circle, $C \div d$ gives the same number. That number has its own name and symbol, the Greek letter π, said "pi". From the measurements above, π is a little more than 3.1; using better tools,

$$\pi \approx 3.14159$$

and in this book we work with $\pi \approx 3.14$. Figure 4 plots the four measured pairs (d, C): they cluster around one straight line through the origin, the line $C = \pi d$. A straight line through the origin means the circumference is proportional to the diameter — double the diameter and the circumference doubles, triple it and the circumference triples; π is the constant in that proportional relationship.

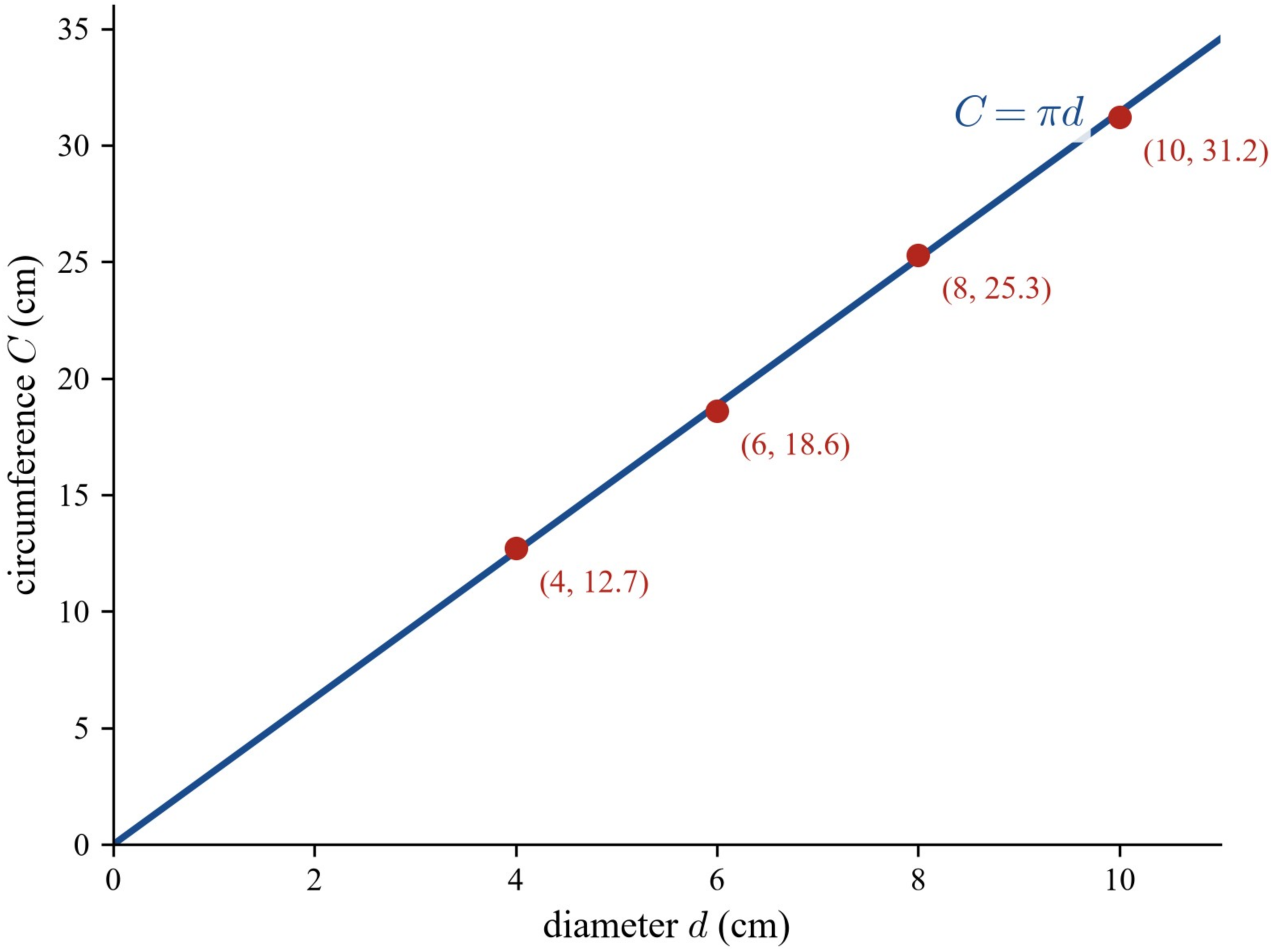


Figure 4. A graph of circumference C against diameter d. The four measured points (4, 12.7), (6, 18.6), (8, 25.3) and (10, 31.2) sit close to the straight line C = πd through the origin.

The relationship as a formula. The investigation has established the relationship, so it can now be used. For any circle,

$$C = \pi \times d = \pi d \text{ and, since } d = 2r, C = 2 \times \pi \times r = 2\pi r$$

and, working backwards, division undoes multiplication,

$$d = C \div \pi \text{ and } r = d \div 2$$

So knowing any one of r, d and C fixes the other two. In Examples 1–4 and the questions, use $\pi \approx 3.14$ unless the question says otherwise.

π through history. People have measured circles for thousands of years, and many civilisations found their own approximations for π. The table and Figure 5 place five of them on one number line; the later the value, the closer it sits to π.

who	where and when	their value	as a decimal
an Egyptian writer	Rhind Papyrus, about 1650 BC	$\frac{256}{81}$	≈ 3.160
Babylonian writers	more than 3500 years ago	$\frac{25}{8}$	3.125

who	where and when	their value	as a decimal
Archimedes	Greece, 287–212 BC	between $\frac{223}{71}$ and $\frac{22}{7}$	between about 3.141 and 3.143
Aryabhata	India, AD 499	$\frac{62832}{20000}$	3.1416
Zu Chongzhi	China, AD 430–501	$\frac{355}{113}$	≈ 3.14159

Source: MacTutor History of Mathematics, University of St Andrews, "Pi through the ages" and "Aryabhata I"; retrieved 20 August 2026.

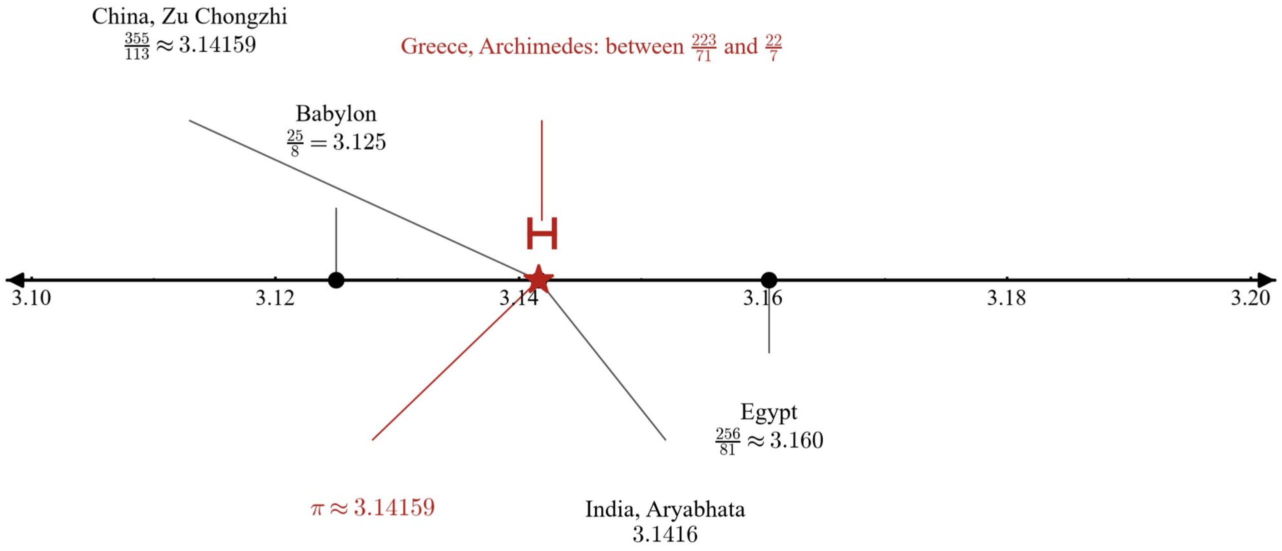


Figure 5. A number line from 3.10 to 3.20 with each historical value marked at its true position: Babylon 3.125, Egypt about 3.160, Archimedes' bounds as a short interval around 3.142, Aryabhata 3.1416, Zu Chongzhi about 3.14159, and π itself marked with a star at about 3.14159.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
centre	the fixed point the same distance from every point of the rim	middle, O
radius	a line from the centre to the rim, and its length	r, half a diameter
diameter	a line through the centre from rim to rim, and its length	d, the distance straight across
circumference	the distance all the way around a circle	C, the perimeter of a circle
π **(pi)**	the number $C \div d$ gives for every circle	about 3.14

Ratio from subtopic 2F returns here in a new form: $C \div d$ is one ratio that never changes, whatever the circle.

Worked examples

Example 1 — scaffolded

Find the circumference of the circle in Figure 6, giving your answer to one decimal place. Use $\pi \approx 3.14$.

Working	Comment
$d = 12$ cm	read from the figure.
$C = \pi \times d \approx 3.14 \times 12$	the circumference is π times the diameter.
$3.14 \times 12 = 31.4 + 6.28 = 37.68$	split 12 into $10 + 2$.
$C \approx 37.7$ cm	one decimal place.

∴ the circumference is about 37.7 cm.

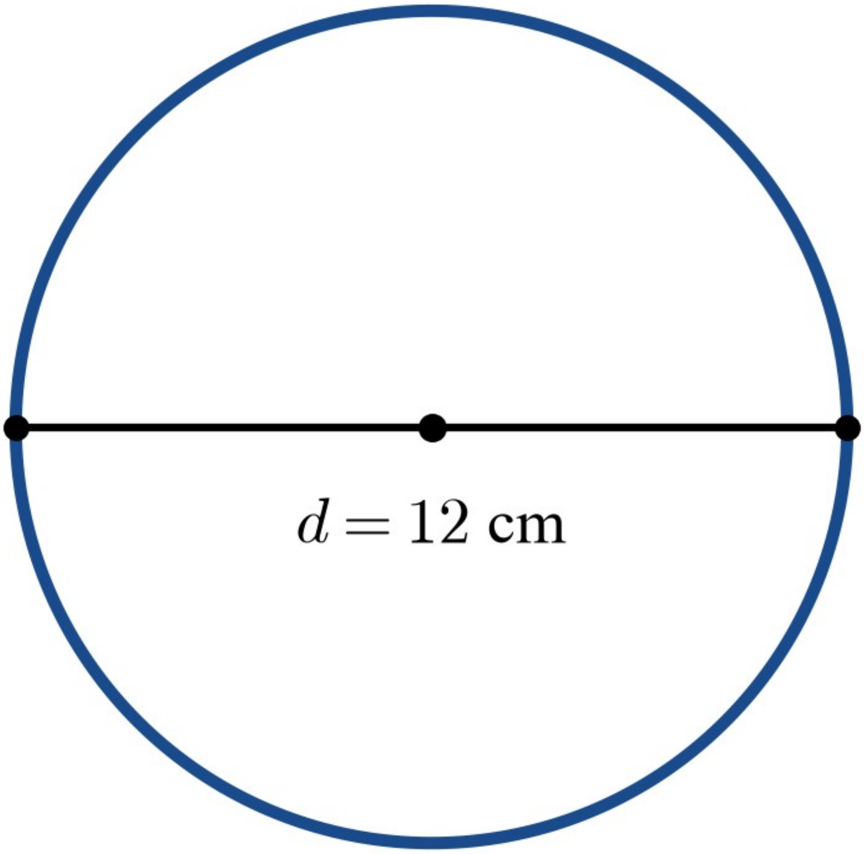


Figure 6. A circle with its diameter drawn and labelled d = 12 cm.

Example 2 — scaffolded

A circular garden bed has radius 5.5 m (Figure 7). Find its circumference to one decimal place.

Working	Comment
$d = 2 \times r = 2 \times 5.5 = 11$ m	the diameter is twice the radius.
$C = \pi \times d \approx 3.14 \times 11$	use $C = \pi d$.
$3.14 \times 11 = 34.54$	
$C \approx 34.5$ m	one decimal place.

∴ the garden bed is about 34.5 m around.

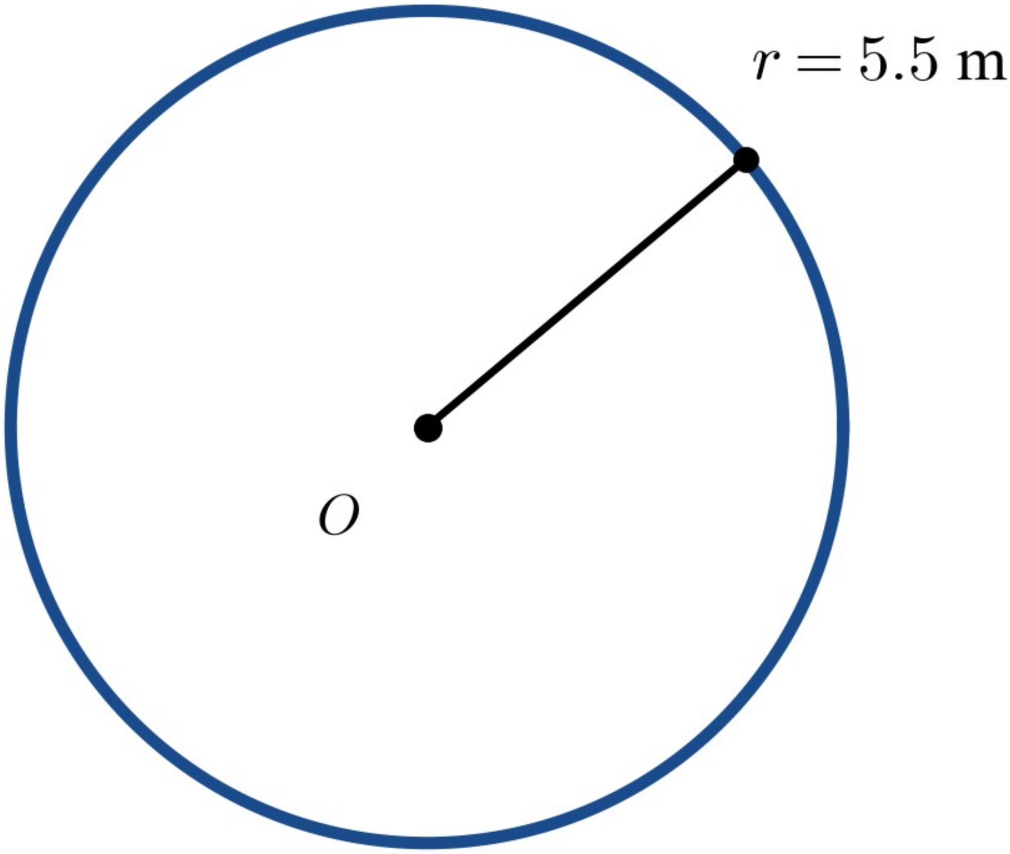


Figure 7. A circle with centre O and a radius drawn and labelled r = 5.5 m.

Example 3 — unscaffolded

A string of fairy lights runs once around the rim of a circular pool and is 37.68 m long. Find the diameter and the radius of the pool.

The string is the circumference, so $C = 37.68$ m. Working backwards, division undoes the multiplication by π:

$$d = C \div \pi \approx 37.68 \div 3.14 = 12$$

because $3.14 \times 12 = 37.68$ exactly. Then $r = d \div 2 = 12 \div 2 = 6$.

$\therefore$ the pool has diameter 12 m and radius 6 m.

Example 4 — unscaffolded

On a football (soccer) ground, the centre circle has radius 9.15 m. Find (a) its diameter and (b) the distance around it, to one decimal place.

Source: The IFAB, Laws of the Game, Law 1 — The Field of Play; retrieved 20 August 2026.

(a) $d = 2 \times 9.15 = 18.3$ m.

(b) $C = \pi \times d \approx 3.14 \times 18.3 = 57.462$, so $C \approx 57.5$ m.

$\therefore$ the centre circle is 18.3 m across and about 57.5 m around.

Practice questions

Scaffolded

Q1. Copy each sentence and complete it with one of: *centre*, *radius*, *diameter*, *circumference*. (a) The distance from the centre of a circle to its rim is called the ___. (b) The distance all the way around a circle is called the ___. (c) A line from one side of a circle to the other, passing through the centre, is called the ___. (d) The point that sits the same distance from every point of the rim is called the ___.

Q2. Complete the table.

radius r	3 cm	8 cm	5.5 cm	2.4 cm			
diameter d					14 cm	9 cm	31 cm

Q3. Use $\pi \approx 3.14$ to complete the table.

diameter d	10 cm	20 cm	4 cm	6.5 cm
circumference C				

Q4. A group repeated the string investigation with three more circles. By hand, find $C \div d$ for each, to two decimal places, then say what you notice. (a) $d = 3.0$ cm, $C = 9.6$ cm (b) $d = 5.0$ cm, $C = 15.5$ cm (c) $d = 7.0$ cm, $C = 22.2$ cm

Q5. Circle A has diameter 5 cm and circle B has diameter 10 cm (Figure 8). (a) Find the circumference of A. (b) Find the circumference of B. (c) Complete the sentence: "When the diameter of a circle doubles, its circumference ___."

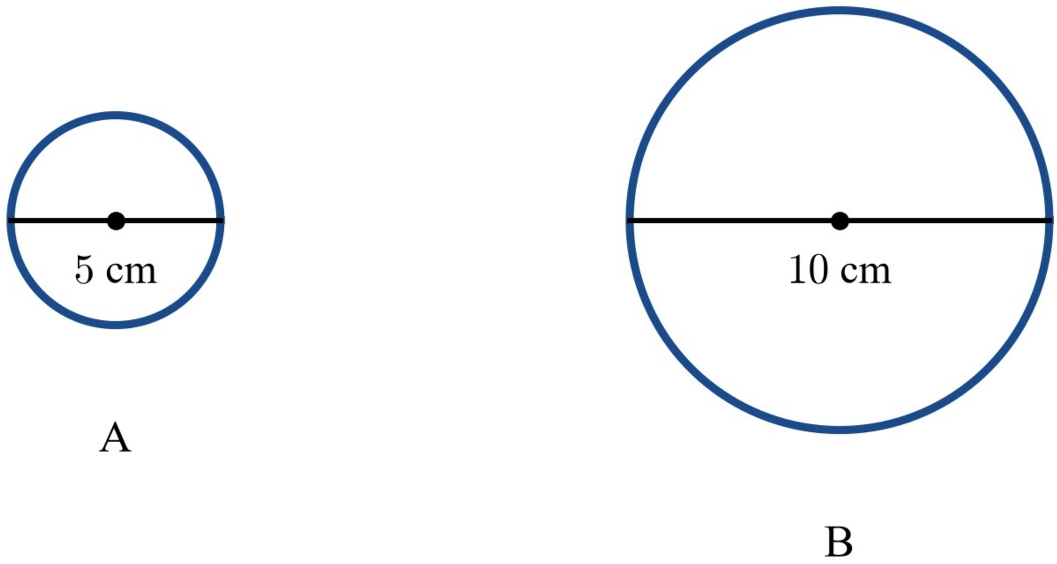


Figure 8. Two circles to scale: circle A with diameter 5 cm and circle B, twice as wide, with diameter 10 cm.

Q6. Work backwards to find the diameter of each circle. Use $\pi \approx 3.14$. (a) $C = 31.4$ cm (b) $C = 62.8$ cm (c) $C = 18.84$ cm (d) $C = 94.2$ cm; for this one, give the radius too.

Unscaffolded

Q7. The Laws of Cricket say the men's ball must measure between 22.4 cm and 22.9 cm around (Figure 9). (a) Find the smallest diameter the ball may have, to one decimal place. (b) Find the largest diameter the ball may have, to one decimal place. (c) A ball has diameter 7.5 cm. Is it allowed? Show working to support your answer.

Source: Marylebone Cricket Club, Laws of Cricket, Law 4 — The Ball; retrieved 20 August 2026.

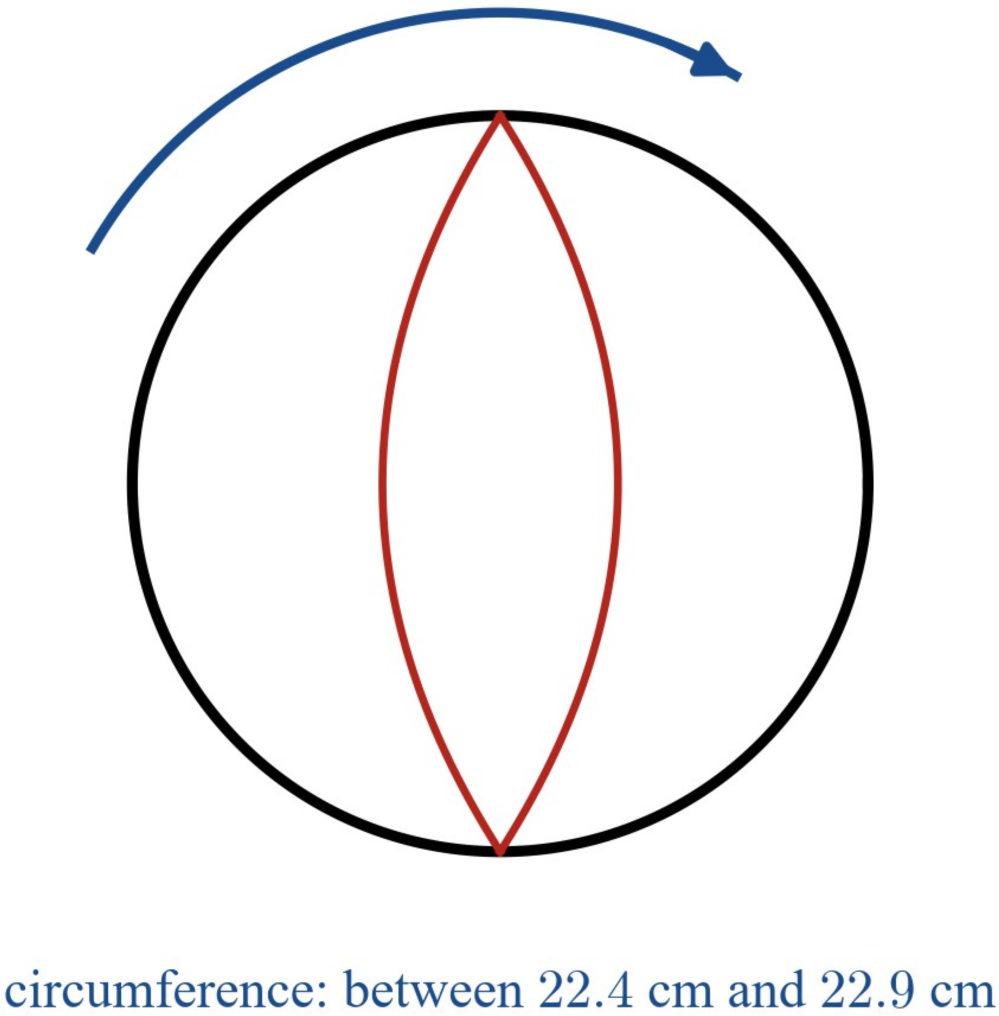


Figure 9. A cricket ball with an arrow running round its widest part, labelled "circumference: between 22.4 cm and 22.9 cm".

Q8. The centre circle of a football ground has radius 9.15 m. (a) Find its diameter. (b) Find its circumference to one decimal place.

Q9. Ben measures a circular plate: the diameter is 26 cm. He says the circumference is 40.8 cm. (a) Without doing the full calculation, explain how you know Ben's answer cannot be right. (b) Find the correct circumference to one decimal place, and suggest what Ben did wrong.

Q10. A wheel has diameter 60 cm and rolls along flat ground (Figure 10). (a) How far does the wheel travel in one full turn? Give your answer in cm. (b) How far, in metres, does it travel in 50 turns?

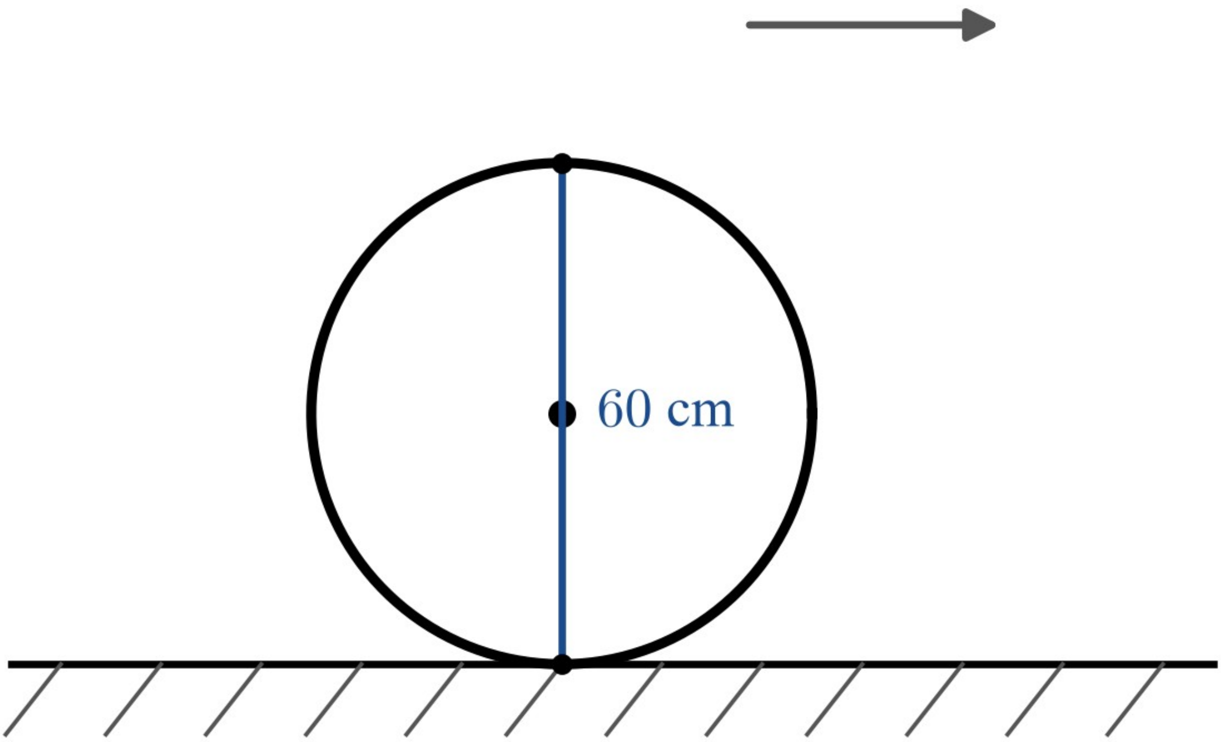


Figure 10. A wheel of diameter 60 cm resting on hatched flat ground, with an arrow showing the rolling direction.

Q11. Say whether each statement is true or false. Correct every false statement. (a) The diameter of a circle is twice its radius. (b) π is exactly 3.14. (c) The circumference of a circle is π × radius. (d) Two circles with the same circumference have the same diameter. (e) A circle with diameter 10 cm has circumference about 31.4 cm.

Q12. Zu Chongzhi's value for π was $\frac{355}{113}$. (a) Use long division to show that $355 \div 113 = 3.14$ to two decimal places. (b) A circle has diameter 113 cm. What circumference does Zu Chongzhi's value give for it? (No long multiplication is needed.)

Q13. A ribbon 40 cm long is bent into a circle with no overlap. Find the diameter of the circle to one decimal place.

Q14. A circular fountain has radius 2.5 m, and a fence is to be built right around it (Figure 11). (a) Find the circumference of the fountain. (b) Fence panels are sold in 2 m lengths, and a whole number of panels must be bought. How many panels are needed to reach all the way around?

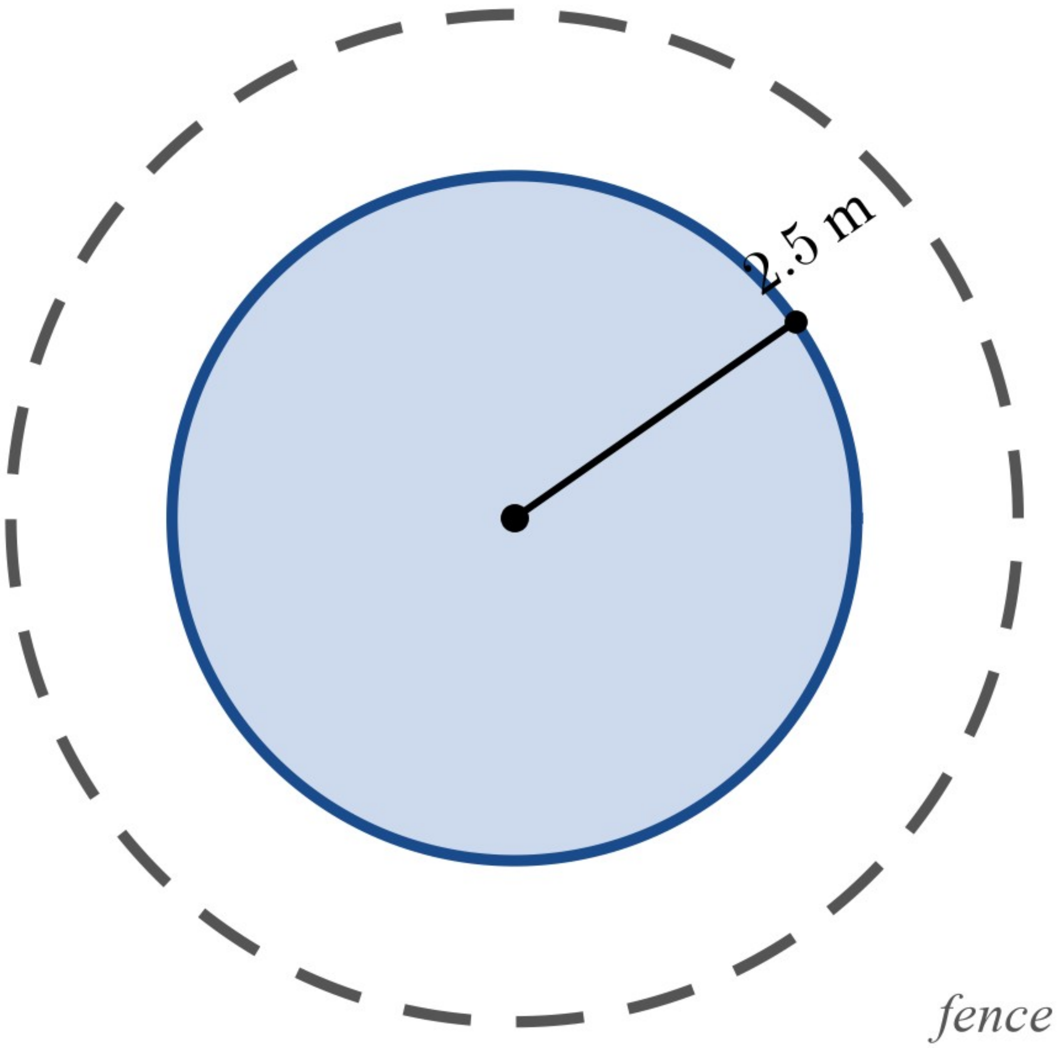


Figure 11. A circular fountain of radius 2.5 m drawn inside a dashed circle that marks the fence line.

Answers

Q1. (a) radius; (b) circumference; (c) diameter; (d) centre.

Q2. Top row, left to right: 6 cm, 16 cm, 11 cm, 4.8 cm; bottom row: 7 cm, 4.5 cm, 15.5 cm.

Q3. 31.4 cm, 62.8 cm, 12.56 cm, 20.41 cm.

Q4. (a) 3.20; (b) 3.10; (c) 3.17. Every answer is close to 3.1, and so close to π — the same pattern as the class table in the theory.

Q5. (a) 15.7 cm; (b) 31.4 cm; (c) doubles.

Q6. (a) 10 cm; (b) 20 cm; (c) 6 cm; (d) $d = 30$ cm, $r = 15$ cm.

Q7. (a) 7.1 cm; (b) 7.3 cm; (c) not allowed — its circumference would be 23.55 cm, more than 22.9 cm.

Q8. (a) 18.3 m; (b) 57.5 m.

Q9. (a) C must be a little more than $3 \times 26 = 78$ cm, and 40.8 is not; (b) 81.6 cm; Ben multiplied π by the radius 13 cm instead of the diameter.

Q10. (a) 188.4 cm; (b) 94.2 m.

Q11. (a) true; (b) false — 3.14 is only an approximation to π; (c) false — $C = \pi \times$ diameter (or $2 \times \pi \times$ radius); (d) true; (e) true.

Q12. (a) $355 \div 113 \approx 3.1415\ldots$, which is 3.14 to two decimal places; (b) 355 cm.

Q13. 12.7 cm.

Q14. (a) 15.7 m; (b) 8 panels.

Fully-worked solutions

Q1. (a) centre-to-rim is the **radius**. (b) around the circle is the **circumference**. (c) rim to rim through the centre is the **diameter**. (d) the fixed point is the **centre**.

Q2. $d = 2r$: $3 \rightarrow 6$ cm, $8 \rightarrow 16$ cm, $5.5 \rightarrow 11$ cm, $2.4 \rightarrow 4.8$ cm. $r = d \div 2$: $14 \rightarrow 7$ cm, $9 \rightarrow 4.5$ cm, $31 \rightarrow 15.5$ cm.

Q3. $C = 3.14 \times d$: $3.14 \times 10 = 31.4$ cm; $3.14 \times 20 = 62.8$ cm; $3.14 \times 4 = 12.56$ cm; $3.14 \times 6.5 = 20.41$ cm.

Q4. (a) $9.6 \div 3.0 = 3.20$. (b) $15.5 \div 5.0 = 3.10$. (c) $22.2 \div 7.0 = 3.17$ (to two decimal places). All three sit near 3.1: the ratio $C \div d$ keeps landing on the same number, π, for every circle.

Q5. (a) $C = 3.14 \times 5 = 15.7$ cm. (b) $C = 3.14 \times 10 = 31.4$ cm. (c) doubles — the circumference is proportional to the diameter, so B's circumference is exactly twice A's.

Q6. $d = C \div 3.14$: (a) $31.4 \div 3.14 = 10$ cm. (b) $62.8 \div 3.14 = 20$ cm. (c) $18.84 \div 3.14 = 6$ cm. (d) $94.2 \div 3.14 = 30$ cm, and $r = 30 \div 2 = 15$ cm.

Q7. (a) $d = C \div \pi \approx 22.4 \div 3.14$: $3.14 \times 7 = 21.98$, leaving 0.42, and $0.42 \div 3.14 \approx 0.13$, so $d \approx 7.13$, that is 7.1 cm. (b) $22.9 \div 3.14$: $3.14 \times 7 = 21.98$, leaving 0.92, and $0.92 \div 3.14 \approx 0.29$, so $d \approx 7.29$, that is 7.3 cm. (c) A ball of diameter 7.5 cm would have $C = 3.14 \times 7.5 = 23.55$ cm, which is more than the allowed 22.9 cm, so the ball is **not** allowed.

Q8. (a) $d = 2 \times 9.15 = 18.3$ m. (b) $C = 3.14 \times 18.3 = 57.462$, so $C \approx 57.5$ m.

Q9. (a) The circumference is a little more than 3 diameters, and $3 \times 26 = 78$ cm, so any answer below 78 cm cannot be right — 40.8 cm is far too small. (b) $C = 3.14 \times 26 = 81.64 \approx 81.6$ cm. Ben's 40.8 cm comes from 3.14×13: he multiplied π by the radius $r = 26 \div 2 = 13$ cm instead of the diameter.

Q10. (a) One turn lays the circumference along the ground: $C = 3.14 \times 60 = 188.4$ cm. (b) 50 turns give $188.4 \times 50 = 9420$ cm $= 94.2$ m.

Q11. (a) **true**. (b) **false**: we use 3.14 for π, but 3.14 is only an approximation — Figure 5 shows closer values such as 3.14159. (c) **false**: $C = \pi \times d$; using the radius it is $2 \times \pi \times r$. (d) **true**: $d = C \div \pi$, so equal circumferences give equal diameters. (e) **true**: $3.14 \times 10 = 31.4$ cm.

Q12. (a) $113 \times 3 = 339$, leaving $355 - 339 = 16$; bring down a zero: $160 \div 113 = 1$ leaving 47; bring down a zero: $470 \div 113 = 4$ leaving 18. So $355 \div 113 = 3.141\ldots = 3.14$ to two decimal places. (b) $C = \frac{355}{113} \times 113 = 355$ cm — the 113s cancel, which is why this fraction is so handy.

Q13. The ribbon is the circumference: $d = C \div \pi \approx 40 \div 3.14$. $3.14 \times 12 = 37.68$, leaving 2.32, and $2.32 \div 3.14 \approx 0.74$, so $d \approx 12.74$, that is 12.7 cm.

Q14. (a) $d = 2 \times 2.5 = 5$ m, so $C = 3.14 \times 5 = 15.7$ m. (b) $15.7 \div 2 = 7.85$, and 7 panels would reach only 14 m, so 8 panels are needed.

Teacher note

- **Ownership and the D4 boundary.** Per the Chapter 4 plan and D4.1, this subtopic *establishes* the relationship between π, C, r and d through the string measurement investigation (E02) before any formula is used; $C = \pi d$ appears only as the result of that investigation, never as a fact to be drilled first. Circle area is Level 8 (`VC2M8M03`) and does not appear, and π is investigated as a constant of proportionality (E03) **without being classified** — no mention of irrational numbers anywhere (D4.5).
- **E04 excluded (Rule 1).** `VC2M7M03.E04` (applications of circles in the everyday life of Aboriginal and Torres Strait Islander Peoples) is not delivered, per `CONTENT_POLICY.md` Rule 1; the mathematics of the CD is taught without that context. The matching §2A recorded-exclusion entry belongs in this build's `TOC.md`, which this authoring job is instructed not to touch — flag to the series editor when the coverage audit runs.
- **Digital tools (D9).** `VC2M7M03` names no digital tools, so the whole subtopic runs without a calculator: every computation is a hand multiplication or division by 3.14, doubling as maintenance of 2C's decimal methods. No CAS.
- **Unit conversion (P1).** Maintained once, not taught: Q10(b) converts 9420 cm to 94.2 m. The chapter note assigns conversion maintenance to 4A, 4B and 4F; 4C's single use is deliberately light.
- **Reasonableness (P7).** Q9 is this subtopic's reasonableness item — the estimate "C is a little more than 3 diameters" is asked for before any calculation.
- **Real data, sourced.** Cricket ball circumference range (MCC Laws of Cricket, Law 4), centre-circle radius 9.15 m (The IFAB, Laws of the Game, Law 1), and the five historical approximations (MacTutor, University of St Andrews) each carry a source line with retrieval date 20 August 2026. All other contexts (pool, garden bed, plate, ribbon, fountain, wheel) are transparently generic and assert no real-world fact.
- **Figures.** All eleven figures are generated by `src/gen_4c.py` from the exact values stated in the text: the scatter (Figure 4) plots the class measurement table itself, the history line (Figure 5) places every recorded value at its true numeric position, and every circle is a true circle.
- **Originality.** Authored from the CD and first principles; no sentence, context or figure is shared with `_build7_8\` or any other build.

Parallel lines and a transversal: corresponding, alternate and co-interior angles

VC2M7M04 — identify corresponding, alternate and co-interior relationships between angles formed when parallel lines are crossed by a transversal; use them to solve problems and explain reasons

Students apply knowledge of angle relationships and the sum of angles in a triangle to solve problems, giving reasons. (achievement standard, extract)

Theory

Parallel lines are lines in the same plane that never meet, however far they are extended. The matching arrow marks on two lines show that they are parallel. A **transversal** is a line that crosses two or more other lines. When a transversal crosses two parallel lines, it makes eight angles. In Figure 1 they are lettered a to h.

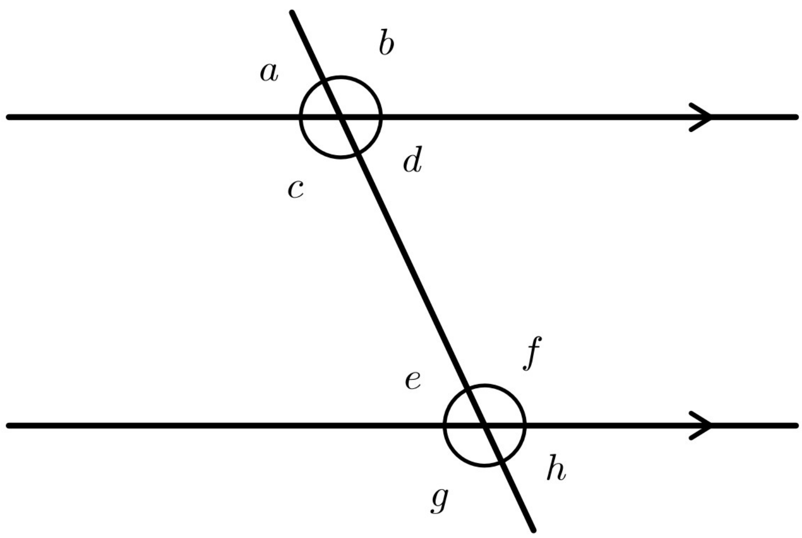


Figure 1. Two horizontal parallel lines crossed by one transversal, making eight angles lettered a to d at the top crossing and e to h at the bottom crossing.

Three pairs of these angles follow rules. The rules are true **only because the lines are parallel**.

Corresponding angles sit in matching corners at the two crossings: the same side of the transversal and the same side of each parallel line. They trace an **F** shape. Corresponding angles are equal.

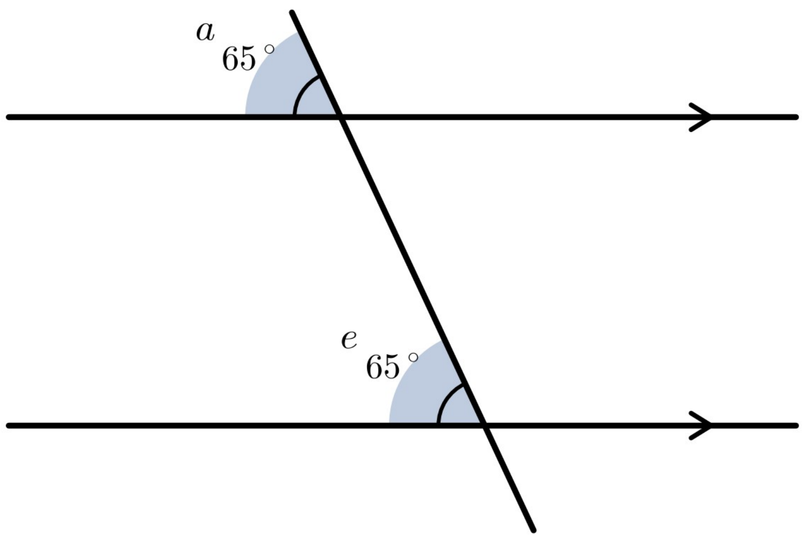


Figure 2. The same two parallel lines and transversal, with the matching top-left corners at each crossing shaded: angle a above and angle e below, both 65 degrees, a corresponding pair.

In Figure 2, a and e are corresponding angles, so $a = e = 65^{\circ}$.

Alternate angles sit between the parallel lines, on opposite sides of the transversal. They trace a **Z** shape. Alternate angles are equal.

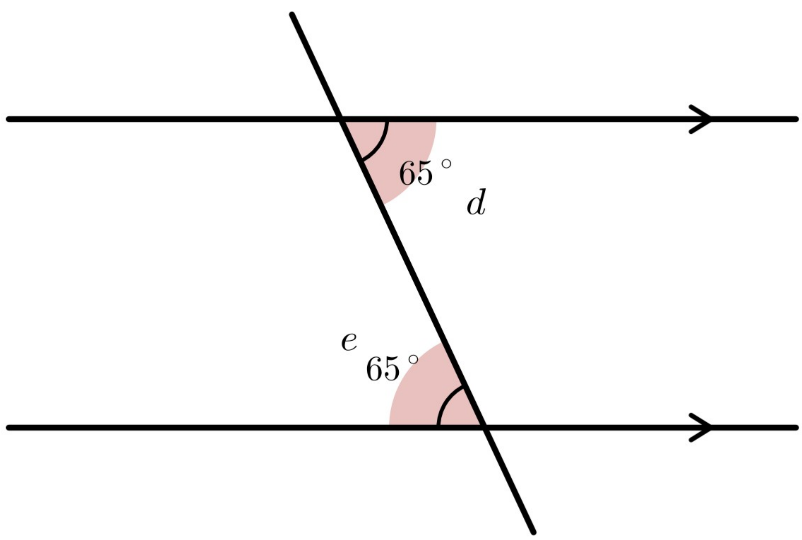


Figure 3. The parallel lines and transversal with angle d (below the top line, right of the transversal) and angle e (above the bottom line, left of the transversal) shaded, both 65 degrees, an alternate pair forming a Z shape.

In Figure 3, d and e are alternate angles, so $d = e = 65^{\circ}$.

Co-interior angles sit between the parallel lines, on the same side of the transversal. They trace a **C** shape (or a **U** shape). Co-interior angles add to 180°; two angles that add to 180° are called **supplementary**.

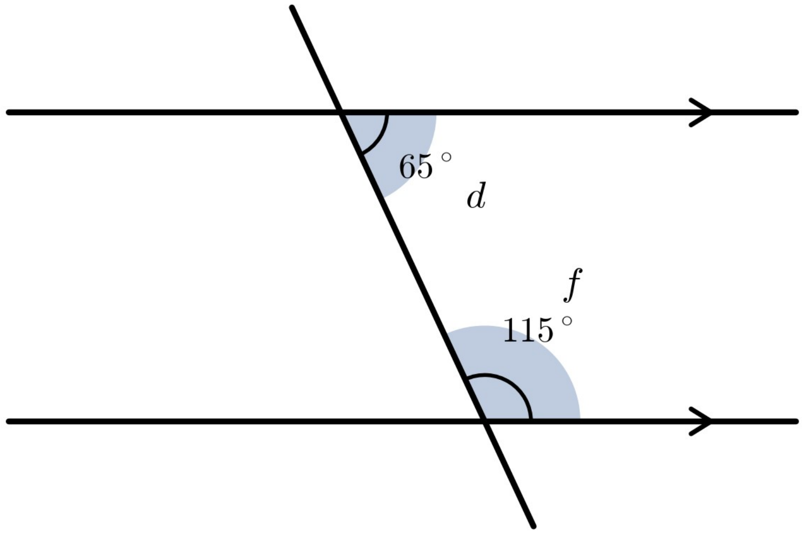


Figure 4. The parallel lines and transversal with angle d (65 degrees) and angle f (115 degrees) shaded, both between the lines on the right of the transversal; 65 degrees plus 115 degrees equals 180 degrees.

In Figure 4, d and f are co-interior angles: $d+f=65^{\circ}+115^{\circ}=180^{\circ}$.

The rules need parallel lines. In Figure 5 the two lines are *not* parallel, so the matching corners are not equal ($73^{\circ}\neq 65^{\circ}$) and no rule applies. Always check the arrow marks (or the question's wording) before you use a rule.

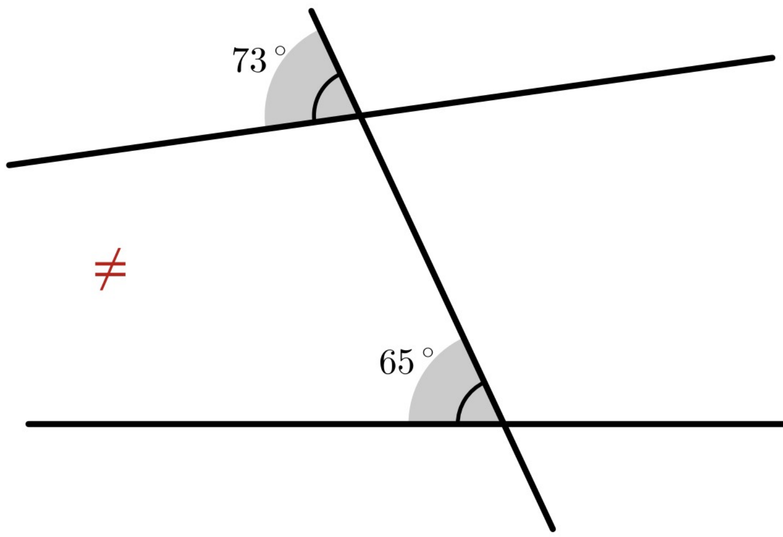
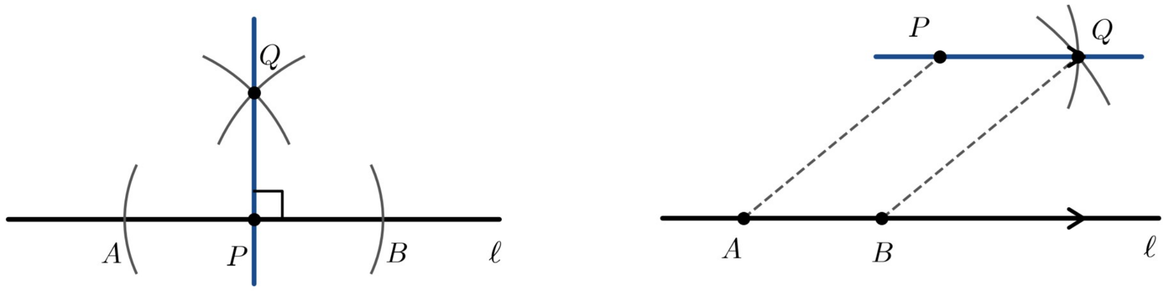


Figure 5. Two lines that are not parallel, crossed by a transversal; the matching corners measure 73 degrees and 65 degrees, so they are not equal and the lines are not parallel.

Why the rules work. Take corresponding angles as the basic fact. Then the other two rules follow from angle facts you already know. In Figure 1: $e=a$ (corresponding angles) and $a=d$ (vertically opposite angles are equal), $\therefore d=e$ — alternate angles are equal. Also $a+c=180^{\circ}$ (angles on a straight line) and $a=e$, $\therefore c+e=180^{\circ}$ — co-interior angles add to 180°.

Drawing parallel and perpendicular lines. With a ruler and compasses (or set squares) you can construct these lines exactly, as in Figure 6.

- (a) **Perpendicular through a point P on a line ℓ:** with the compass point on P, cut equal arcs across ℓ at A and B; from A and from B, with a wider radius, draw arcs crossing at Q; the line PQ is perpendicular to ℓ.
- (b) **Parallel through a point P not on ℓ:** choose points A and B on ℓ and join A to P; with the compass point on B and radius AP draw an arc, and with the compass point on P and radius AB draw an arc crossing it at Q; the line PQ is parallel to ℓ.



(a) perpendicular through P

(b) parallel through P

Figure 6. Two constructions with ruler and compasses: (a) a perpendicular line through point P on line l, built from arcs centred at P, A and B meeting at Q, with a small square mark at P; (b) a parallel line through point P above line l, built by copying the distance from A to P at B and the distance from A to B at P, meeting at Q.

Checking with software. In dynamic geometry software you can draw two parallel lines and a transversal, measure all eight angles, and drag the transversal. The angle sizes change as you drag, but corresponding angles stay equal, alternate angles stay equal, and co-interior angles keep adding to 180° — for as long as the two lines stay parallel.

The words of this subtopic, with synonyms you may meet

Word	Meaning	Watch for
parallel lines	lines in the same plane that never meet	arrow marks on the lines
transversal	a line that crosses two or more lines	“the crossing line”
corresponding angles	matching corners at the two crossings	an F shape
alternate angles	between the lines, opposite sides of the transversal	a Z shape
co-interior angles	between the lines, same side of the transversal	a C or U shape
supplementary	two angles that add to 180°	co-interior angles are supplementary
exterior angle	the angle between one side of a triangle and the next side extended	outside the triangle
perpendicular	meeting at a right angle (90°)	a small square mark

Worked examples

Example 1 (scaffolded) — *Figure 7. The two lines are parallel and $b = 118^\circ$. Find the other seven angles, with a reason for each.*

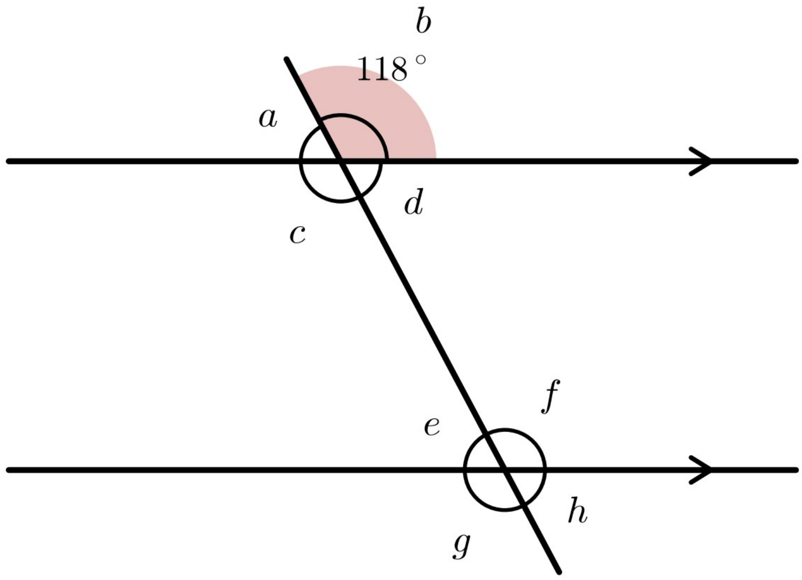


Figure 7. Two parallel lines crossed by a transversal; the top-right angle b is marked 118 degrees and the other seven angles are lettered a, c, d at the top crossing and e, f, g, h at the bottom crossing.

Statement	Reason
$a=180^{\circ}-118^{\circ}=62^{\circ}$	angles on a straight line: $a+b=180^{\circ}$
$c=118^{\circ}$	vertically opposite angles are equal: $c=b$
$d=62^{\circ}$	vertically opposite angles are equal: $d=a$
$e=62^{\circ}$	corresponding angles are equal: $e=a$ (parallel lines)
$f=118^{\circ}$	corresponding angles are equal: $f=b$ (parallel lines)
$g=118^{\circ}$	corresponding angles are equal: $g=c$ (parallel lines)
$h=62^{\circ}$	corresponding angles are equal: $h=d$ (parallel lines)

Every statement carries its reason — that is the shape of a good geometry answer: *statement, reason, conclusion*.

Example 2 (scaffolded) — *Figure 8. A scissor lift: the base and the platform stay parallel as the lift opens, and the two straight arms cross at X. Each arm makes an angle of 58° with the base. (a) Find the angle between an arm and the platform. (b) Find x, the angle between the arms at the top of the crossing.*

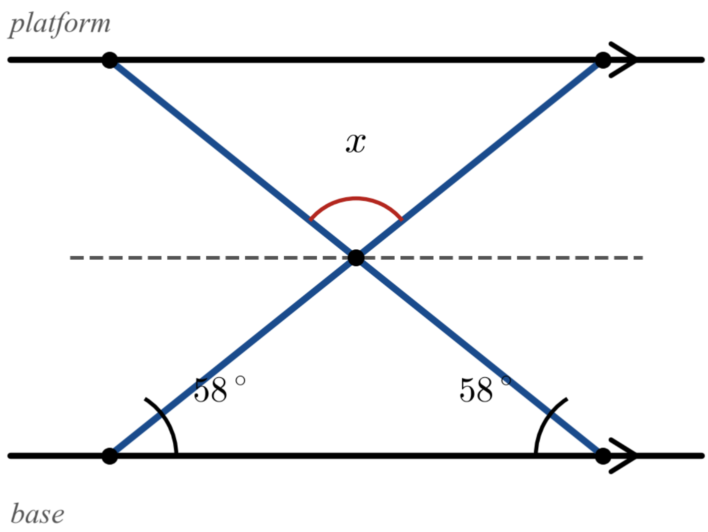


Figure 8. A scissor lift drawn as two parallel horizontal lines, the base below and the platform above, with two straight arms crossing at X; each arm makes 58 degrees with the base, a dashed line through X is parallel to the base, and x is the angle between the arms above X.

Statement	Reason
(a) the arm is a transversal of the parallel base and platform	given
the angle between the arm and the platform $=58^\circ$	alternate angles are equal (parallel lines)
(b) the dashed line through X is parallel to the base	construction
the dashed line makes 58° with each arm	alternate angles with the base angles, and vertically opposite angles
$58^\circ + x + 58^\circ = 180^\circ$	angles on a straight line at X
$x = 180^\circ - 116^\circ = 64^\circ$	conclusion

Example 3 — *Figure 9. In triangle ABC, side BC is extended to D, making the exterior angle e at C. Without using the angle sum of a triangle, show that e equals the sum of the angles at A and B, and find e.*

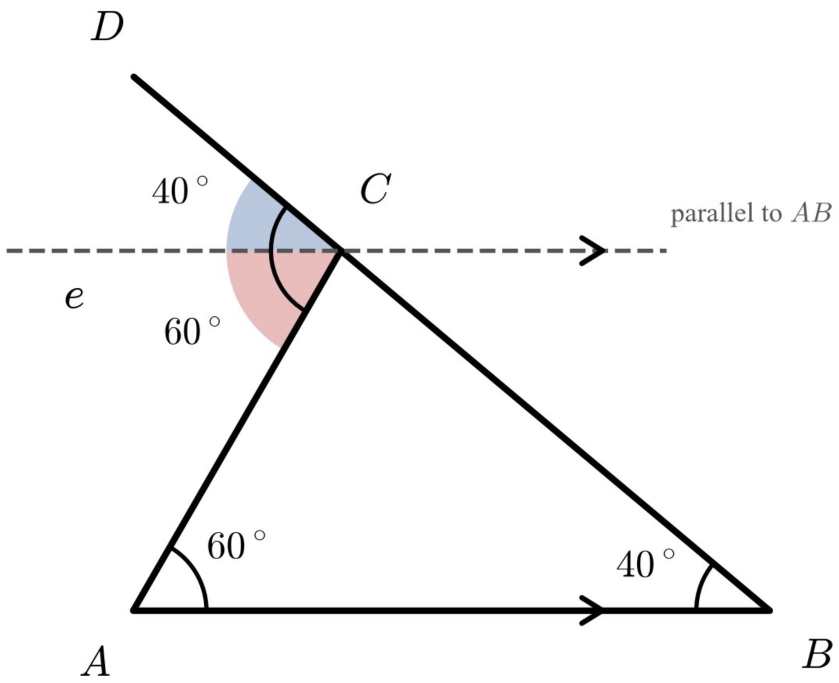


Figure 9. Triangle ABC with side BC extended past C to D; a dashed line through C is parallel to AB, and the exterior angle e at C is split by this dashed line into a 40 degree part and a 60 degree part, matching the angles at B and A.

Draw the dashed line through C parallel to AB.

- The part of e between CD and the dashed line $= 40^\circ$ — corresponding angles with the angle at B.
- The part of e between the dashed line and $CA = 60^\circ$ — alternate angles with the angle at A.

$\therefore e = 40^\circ + 60^\circ = 100^\circ$: **the exterior angle of a triangle equals the sum of the two interior angles on the other side of it.** (The fact that the three angles of a triangle add to 180° is proved in 4E; here we did not need it.)

Example 4 — *Figure 10. Three parallel lines are crossed by a path that kinks at the middle line. Find x and y.*

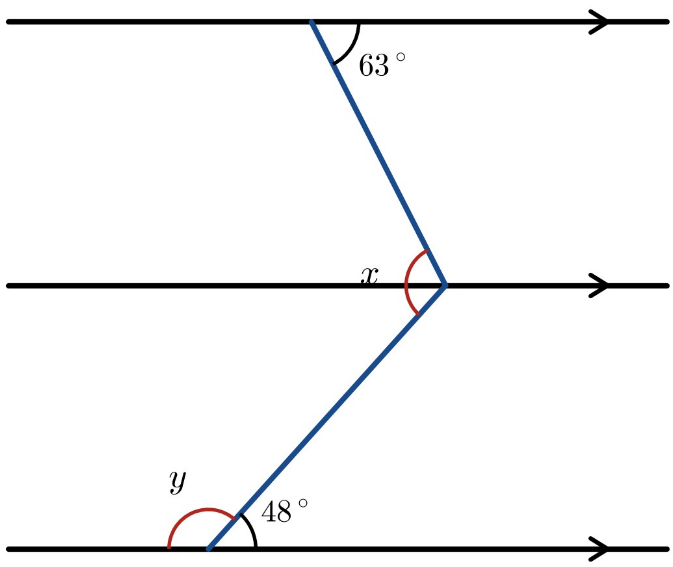


Figure 10. Three parallel horizontal lines crossed by a path that kinks at the middle line; the path makes 63 degrees with the top line and 48 degrees with the bottom line, x is the angle of the kink and y is the angle between the lower part of the path and the bottom line on the left.

The middle line through the kink splits x into two parts.

- Upper part $= 63^{\circ}$ — alternate angles with the top line (parallel lines).
- Lower part $= 48^{\circ}$ — alternate angles with the bottom line (parallel lines).

$\therefore x = 63^{\circ} + 48^{\circ} = 111^{\circ}$. $y = 180^{\circ} - 48^{\circ} = 132^{\circ}$ — angles on a straight line.

Practice questions

Scaffolded

Q1. Figure 1 shows two parallel lines crossed by a transversal. Name the relationship between each pair of angles (corresponding, alternate or co-interior) and state the rule for it.

- (a) a and e
- (b) d and e
- (c) c and e
- (d) b and f
- (e) d and f
- (f) c and g

Q2. In Figure 1 the lines are parallel. Find the unknown angle and give the rule you used.

-

- (a) $a = 47^\circ$; find e.
- (b) $c = 112^\circ$; find f.
- (c) $c = 95^\circ$; find e.
- (d) $d = 61^\circ$; find f.
- (e) $b = 133^\circ$; find f.
- (f) $h = 54^\circ$; find d.

Q3. In Figure 1 the lines are parallel and $e = 123^\circ$. Find the other seven angles, with a reason for each.

Q4. True or false? If false, correct the statement.

- (a) Corresponding angles are always equal.
- (b) Alternate angles are on the same side of the transversal.
- (c) When the lines are parallel, co-interior angles add to 180°.
- (d) Vertically opposite angles are equal.
- (e) If two co-interior angles between parallel lines are equal, each of them is 90°.
- (f) Alternate angles always add to 180°.

Q5. Figure 1 shows parallel lines. Give the reason for each statement.

- (a) $a = e$
- (b) $d = e$
- (c) $c + e = 180^\circ$
- (d) $b = c$
- (e) $a + b = 180^\circ$
- (f) $b = f$

Q6. These steps build the parallel line of Figure 6(b), but they are in the wrong order. Rewrite them in the correct order.

- A. Draw the line through P and Q: this line is parallel to ℓ.
- B. With the compass point on B and radius AP draw an arc; with the compass point on P and radius AB draw an arc crossing it; label the crossing Q.

- C. Choose two points A and B on ℓ, and join A to P.
- D. Mark a point P not on the line ℓ.

Unscaffolded

Q7. *Figure 11.* The two rails of a track are parallel, and a sleeper crosses them. The marked angle is 72°.

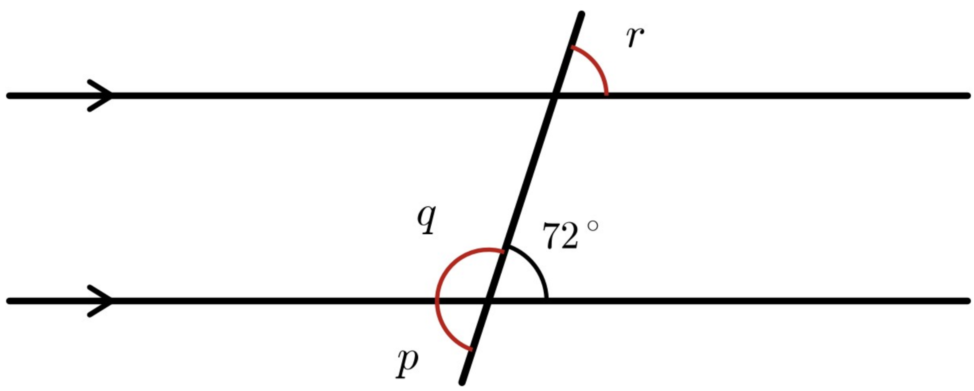


Figure 11. Two parallel rails crossed by a sleeper; the angle between the sleeper and the lower rail on the right is 72 degrees, p is the vertically opposite angle below the lower rail, q is the angle between the sleeper and the lower rail on the left, and r is the matching angle at the upper rail.

- (a) First estimate: is q bigger or smaller than 90°? About how big?
- (b) Find p, with a reason.
- (c) Find q, with a reason. (Hint: q and the 72° angle sit on one straight line.)
- (d) Find r, with a reason.
- (e) Compare your answer to (c) with your estimate. Is it reasonable?

Q8. A scissor lift like Figure 8 has equal base angles, and the angle between the arms at the top of the crossing is 76°. Find the angle each arm makes with the base.

Q9. *Figure 12.* In each case, decide whether the two lines are parallel, and give a reason.

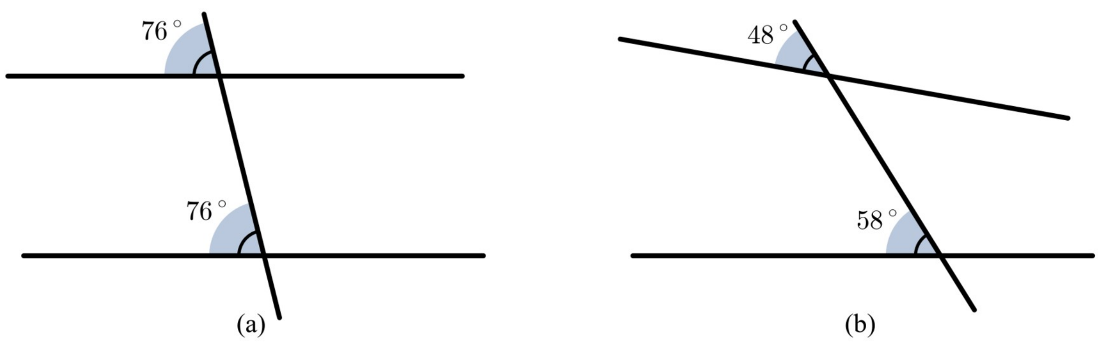


Figure 12. Two crossings of a transversal: in (a) the matching corners both measure 76 degrees, so the lines are parallel; in (b) the matching corners measure 58 degrees and 48 degrees, so the lines are not parallel.

Q10. $ABCD$ is a parallelogram, so $AB \parallel DC$ and $AD \parallel BC$. Given $\angle A = 68^\circ$, find $\angle B$, $\angle C$ and $\angle D$, with reasons.

Q11. Use the result of Example 3 (the exterior angle of a triangle equals the sum of the two interior angles on the other side of it).

- (a) The two interior angles of a triangle are 38° and 77°. Find the exterior angle at the third vertex.
- (b) An exterior angle of a triangle is 124° and one of the two opposite interior angles is 52°. Find the other.

Q12. Two co-interior angles between parallel lines are x° and $(x+40)^\circ$.

- (a) Explain why x must be less than 90.
- (b) Find x and the two angles, then check that your answer agrees with (a).

Q13. Using Figure 1, explain why $c+e=180^\circ$ — that is, why co-interior angles add to 180° when the lines are parallel. (Hint: use a.)

Q14. *Figure 13.* Two parallel lines are joined by a path that kinks at K. Find x.

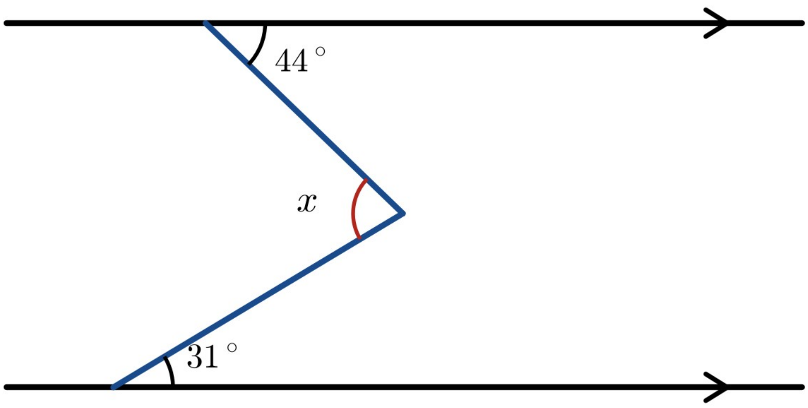


Figure 13. Two parallel lines joined by a path that kinks at K; the path makes 44 degrees with the top line and 31 degrees with the bottom line, and x is the angle of the kink at K.

Q15. The platform of a cherry picker stays parallel to the body of the truck. The lifting arm makes an angle of 62° with the body.

- (a) Find the angle between the arm and the platform, with a reason.
- (b) Find the co-interior partner of that angle on the platform side, with a reason.

Answers

Q1. (a) corresponding — equal (b) alternate — equal (c) co-interior — add to 180° (d) corresponding — equal (e) co-interior — add to 180° (f) corresponding — equal

Q2. (a) $e = 47^\circ$ (corresponding) (b) $f = 112^\circ$ (alternate) (c) $e = 85^\circ$ (co-interior) (d) $f = 119^\circ$ (co-interior) (e) $f = 133^\circ$ (corresponding) (f) $d = 54^\circ$ (corresponding)

Q3. $a = 123^\circ$ (corresponding to e); $b = 57^\circ$ (straight line with a); $c = 57^\circ$ (vertically opposite b); $d = 123^\circ$ (vertically opposite a); $f = 57^\circ$ (straight line with e); $g = 57^\circ$ (vertically opposite f); $h = 123^\circ$ (vertically opposite e)

Q4. (a) false — only when the lines are parallel (b) false — opposite sides (c) true (d) true (e) true (f) false — they are equal; they add to 180° only when each is 90°

Q5. (a) corresponding angles, parallel lines (b) alternate angles, parallel lines (c) co-interior angles, parallel lines (d) vertically opposite angles (e) angles on a straight line (f) corresponding angles, parallel lines

Q6. D, C, B, A

Q7. (a) q looks obtuse, about 110° (any estimate between 95° and 125° is fine) (b) $p = 72^\circ$ (vertically opposite) (c) $q = 108^\circ$ (angles on a straight line) (d) $r = 72^\circ$ (corresponding) (e) 108° is close to the estimate ✓

Q8. 52°

Q9. (a) parallel — corresponding angles are equal ($76^\circ = 76^\circ$) (b) not parallel — corresponding angles are not equal ($58^\circ \neq 48^\circ$)

Q10. $\angle B = 112^\circ$, $\angle C = 68^\circ$, $\angle D = 112^\circ$

Q11. (a) 115° (b) 72°

Q12. (a) the two angles add to 180° and $x + 40 > x$, so x is the smaller and must be under 90 (b) $x = 70$; angles 70° and 110°; $70 < 90$ ✓

Q13. $a = e$ (corresponding) and $a + c = 180^\circ$ (straight line) $\Rightarrow c + e = 180^\circ$

Q14. $x = 75^\circ$

Q15. (a) 62° (alternate angles) (b) 118° (co-interior angles)

Fully-worked solutions

Q1. (a) a and e are in matching corners: **corresponding angles**, which are equal. (b) d and e are between the lines on opposite sides of the transversal: **alternate angles**, which are equal. (c) c and e are between the lines on the same side: **co-interior angles**, which add to 180°. (d) b and f: matching corners: **corresponding**, equal. (e) d and f: same side, between the lines: **co-interior**, add to 180°. (f) c and g: matching corners: **corresponding**, equal.

Q2. (a) $e = a = 47^\circ$ — corresponding angles. (b) $f = c = 112^\circ$ — alternate angles. (c) $e = 180^\circ - 95^\circ = 85^\circ$ — co-interior angles add to 180°. (d) $f = 180^\circ - 61^\circ = 119^\circ$ — co-interior angles. (e) $f = b = 133^\circ$ — corresponding angles. (f) $d = h = 54^\circ$ — corresponding angles.

Q3. Bottom crossing: $f = 180^\circ - 123^\circ = 57^\circ$ (angles on a straight line), $g = f = 57^\circ$ and $h = e = 123^\circ$ (vertically opposite angles are equal). Top crossing: $a = e = 123^\circ$ (corresponding angles are equal, parallel lines), $b = 180^\circ - 123^\circ = 57^\circ$ (angles on a straight line), $c = b = 57^\circ$ and $d = a = 123^\circ$ (vertically opposite angles are equal).

Q4. (a) False. Corresponding angles are equal *when the lines are parallel* (Figure 5 shows they are not equal otherwise). (b) False. Alternate angles are on *opposite* sides of the transversal. (c) True. (d) True. (e) True: they add to

180° and are equal, so each is $180^\circ \div 2 = 90^\circ$. (f) False. Alternate angles are *equal*; they add to 180° only in the special case where each is 90°.

Q5. (a) Corresponding angles are equal (parallel lines). (b) Alternate angles are equal (parallel lines). (c) Co-interior angles add to 180° (parallel lines). (d) Vertically opposite angles are equal. (e) Angles on a straight line add to 180°. (f) Corresponding angles are equal (parallel lines).

Q6. D, C, B, A. Mark P (D); choose A, B on ℓ and join A to P (C); copy the two distances with arcs to find Q (B); draw PQ (A).

Q7. (a) q opens wider than a right angle, so it is obtuse — about 110° is a good estimate. (b) p is vertically opposite the 72° angle, $\therefore p = 72^\circ$. (c) q and the 72° angle lie on the straight lower rail, $\therefore q = 180^\circ - 72^\circ = 108^\circ$. (d) r is corresponding to the 72° angle (parallel rails), $\therefore r = 72^\circ$. (e) 108° is near the estimate of 110°, so the answer is reasonable. ✓

Q8. Draw the dashed line through the crossing, parallel to the base (as in Figure 8). As in Example 2, the dashed line makes the base angle β with each arm (alternate angles, and vertically opposite angles). The three angles at the crossing above the dashed line — β, the 76° angle, and β — lie on a straight line: $\beta + 76^\circ + \beta = 180^\circ$. $\therefore 2\beta = 104^\circ$ and $\beta = 52^\circ$. Each arm makes 52° with the base.

Q9. (a) The marked angles are corresponding angles and they are equal ($76^\circ = 76^\circ$), $\therefore$ the lines **are parallel**. (b) The marked angles are corresponding angles but $58^\circ \neq 48^\circ$, $\therefore$ the lines are **not parallel**.

Q10. $AD \parallel BC$ with transversal AB: $\angle A + \angle B = 180^\circ$ (co-interior angles), $\therefore \angle B = 180^\circ - 68^\circ = 112^\circ$. $AB \parallel DC$ with transversal BC: $\angle B + \angle C = 180^\circ$, $\therefore \angle C = 68^\circ$. $AD \parallel BC$ with transversal DC: $\angle C + \angle D = 180^\circ$, $\therefore \angle D = 112^\circ$. (Opposite angles of a parallelogram are equal — which the co-interior rule has just shown.)

Q11. (a) exterior angle $= 38^\circ + 77^\circ = 115^\circ$. (b) other interior angle $= 124^\circ - 52^\circ = 72^\circ$.

Q12. (a) Co-interior angles add to 180°. Since $(x+40)$ is the bigger angle, x is the smaller of the two, and the smaller of two angles that add to 180° must be under 90. (b) $x + (x+40) = 180 \Rightarrow 2x = 140 \Rightarrow x = 70$. The angles are 70° and 110°: they add to 180° ✓ and $70 < 90$ ✓, agreeing with (a).

Q13. $a = e$ — corresponding angles are equal (parallel lines). $a + c = 180^\circ$ — angles on a straight line. Replacing a with e: $c + e = 180^\circ$. $\therefore$ co-interior angles add to 180°.

Q14. Draw a dashed line through K parallel to both lines. It splits x into two parts: the upper part is alternate to the 44° angle ($= 44^\circ$) and the lower part is alternate to the 31° angle ($= 31^\circ$). $\therefore x = 44^\circ + 31^\circ = 75^\circ$.

Q15. (a) The arm is a transversal of the parallel body and platform, $\therefore$ the angle between the arm and the platform $= 62^\circ$ (alternate angles). (b) The co-interior partner adds to 180°: $180^\circ - 62^\circ = 118^\circ$ (co-interior angles, parallel lines).

Teacher note

- **P2 / D3 recall, not re-teach.** The Level 6 angle facts (angles on a straight line add to 180°, angles at a point add to 360°, vertically opposite angles are equal) are recalled in Example 1's reason column the first time they are needed; they are not re-taught here.
- **Ownership boundary with 4E.** 4D owns the three parallel-line rules and the exterior-angle result, reached *through* parallel lines (Example 3). The angle sum of a triangle and the $180(n-2)$ polygon rule belong to 4E and are not used or stated as facts here; Example 3 flags this for students.
- **House form for geometry with reasons.** Every answer states its reason — *statement, reason, conclusion* — modelled from Example 1 onward. In the T4 chapter test, reasoning carries its own mark, separate from the answer.
- **D12 terminology.** "Co-interior" exactly as VCAA writes it; the section never uses the older synonyms.

- **D9 / E02.** Software is named only in general terms ("dynamic geometry software"), no CAS, no product names; the drag test is the suggested check that the three rules hold exactly when the lines stay parallel.
- **E01, E03 contexts.** Constructions with ruler and compasses (Figure 6, Q6) cover E01; scissor lifts (Example 2, Q8) and the cherry picker (Q15) cover E03.
- **P7 estimation habit.** Q7(a)/(e) and Q12(a)/(b) each ask for an estimate or a size check before or after the exact work.
- **D4 boundaries.** The only equation solved is the natural-number one in Q12; no item needs more than two linked steps, matching the chapter-test rule.

The angle sum of a triangle and of other polygons

Content description VC2M7M05: demonstrate that the interior angle sum of a triangle in the plane is 180° and apply this to determine the interior angle sum of other shapes and the size of unknown angles

Elaboration VC2M7M05.E01: using concrete materials to demonstrate that the sum of the interior angles of a triangle is 180°; for example, using paper triangles and tearing to demonstrate that the interior angles when combined form 180°

Elaboration VC2M7M05.E02: using decomposition and the angle sum of a triangle to generalise the interior angle sum of an *n*-sided polygon, as $180(n-2)=180n-360$

Achievement standard (extract): They apply knowledge of angle relationships and the sum of angles in a triangle to solve problems, giving reasons.

Theory

What this subtopic is about. Subtopic 4E shows that the three angles of any triangle add to $180°$ — first with torn paper, then with a reason — and then splits every other shape into triangles to find its angle sum. The pattern of splits grows into the rule $180(n-2)=180n-360$ for an *n*-sided polygon, and the sums are used to find unknown angles. This is geometry with reasons: every answer carries its reason, in the form *statement · reason · conclusion*. The Level 6 facts — angles on a straight line add to $180°$, angles at a point add to $360°$ — are recalled where needed, not re-taught.

The torn-corner demonstration. Take any triangle cut from paper. In Figure 1 the three corners are shaded and then torn off, and the three torn pieces are placed together at one point on a straight line. They fit exactly — no gap, no overlap — and fill the straight angle. Angles on a straight line add to $180°$, so the three angles of the triangle add to $180°$. Try it with a sharp triangle, a wide triangle and a right-angled triangle: the corners always fill the straight angle.

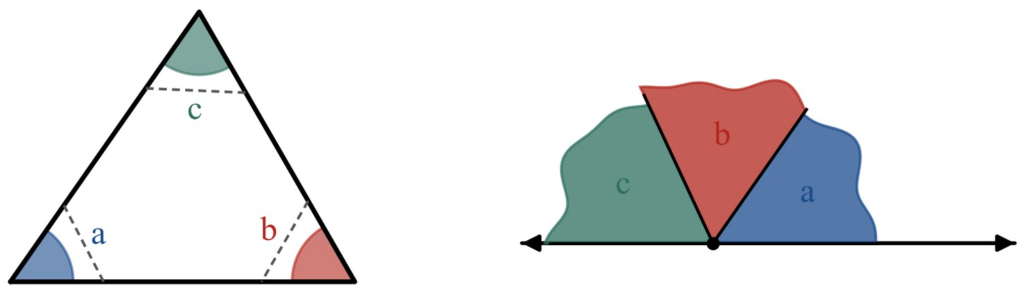


Figure 1. Left: a triangle with its three corners shaded blue, red and green and labelled a, b and c, with dashed tear lines across each corner. Right: the three torn corners placed together at one point on a straight line, c then b then a, filling the straight angle with no gap and no overlap.

Why it must be so. The tearing shows that the fact is true; a reason shows why it must be. In Figure 2 the line through C is drawn parallel to AB. From 4D, alternate angles between parallel lines are equal, so the angle between CA and the line on the left equals a, and the angle between CB and the line on the right equals b. At C those two angles and c sit together on a straight line: $a+c+b=180°$ · angles on a straight line add to $180°$ · the three angles of any triangle add to $180°$.

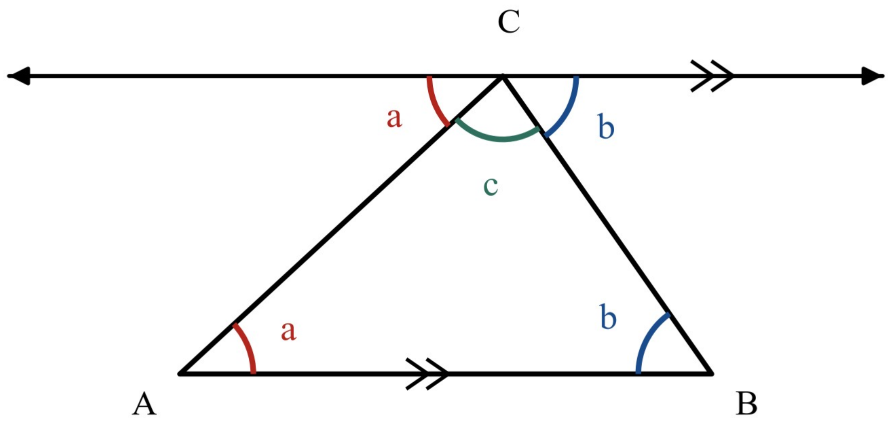


Figure 2. Triangle ABC with a line through C parallel to the base AB, the two parallel lines marked with matching arrow ticks. The angle a at A and the angle b at B are marked, and at C the equal alternate angles a and b are marked on either side of the angle c, so a, c and b together sit on the straight line through C.

Finding an unknown angle. The habit for every question in this subtopic is the same: write the angle fact as a statement, give the reason, then conclude. In Figure 3 the marked angles are $48°$ and $67°$, and the third angle is x. The statement: $48°+67°+x=180°$. The reason: the angles of a triangle add to $180°$. The conclusion: $x=180°-115°=65°$.

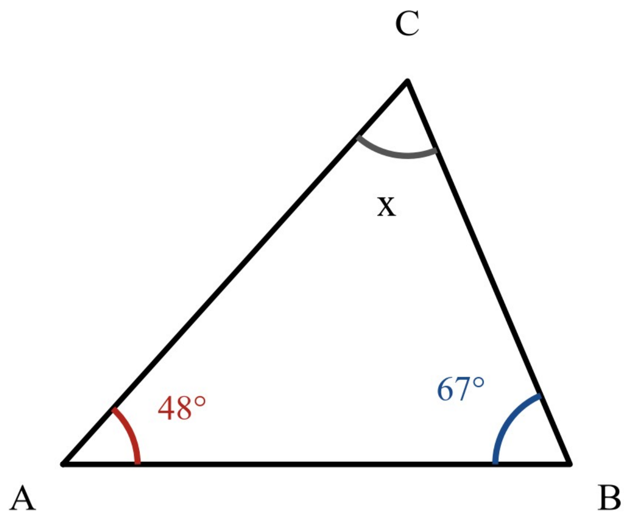


Figure 3. Triangle ABC with the angle at A marked 48 degrees, the angle at B marked 67 degrees, and the angle at C marked x.

Other shapes: split into triangles. A quadrilateral is not a triangle, but it splits into two. In Figure 4 the diagonal from A to C cuts $ABCD$ into triangle 1 and triangle 2. The six triangle angles are exactly the four quadrilateral angles — the two parts at A make the whole corner at A, and the same at C — so the quadrilateral's angle sum is $2\times180°=360°$ · a quadrilateral splits into 2 triangles · its angles add to $360°$. Check with the marked angles: $85°+105°+95°+75°=360°$.

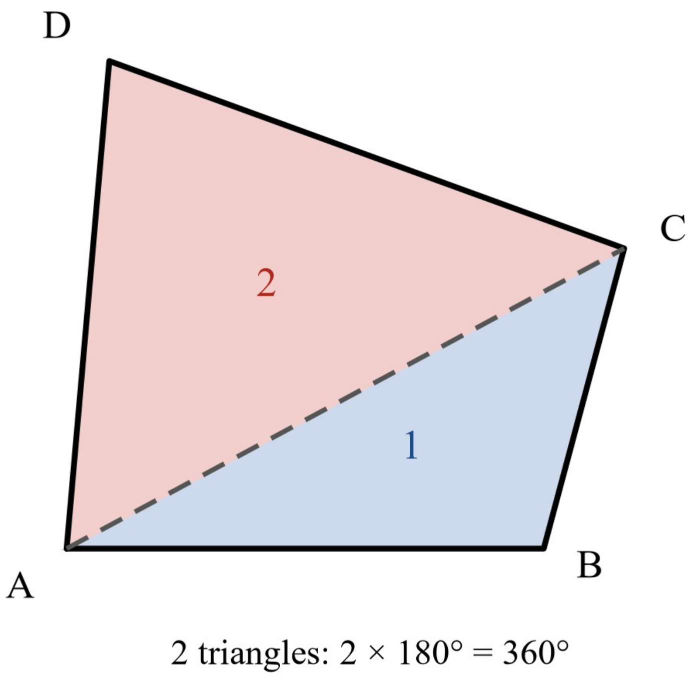


Figure 4. Quadrilateral ABCD with the diagonal from A to C drawn dashed, splitting it into triangle 1 (A, B, C, shaded blue) and triangle 2 (A, C, D, shaded red); underneath, 2 triangles: 2 times 180 degrees equals 360 degrees.

More sides, more triangles. The same split works for any polygon. In Figure 5, three diagonals from one vertex of a hexagon make 4 triangles, so the hexagon's angle sum is $4 \times 180° = 720°$. The pattern is in the table: each new side adds one more triangle.

Shape	Triangles	Angle sum
triangle	1	$180°$
quadrilateral	2	$360°$
pentagon	3	$540°$
hexagon	4	$720°$

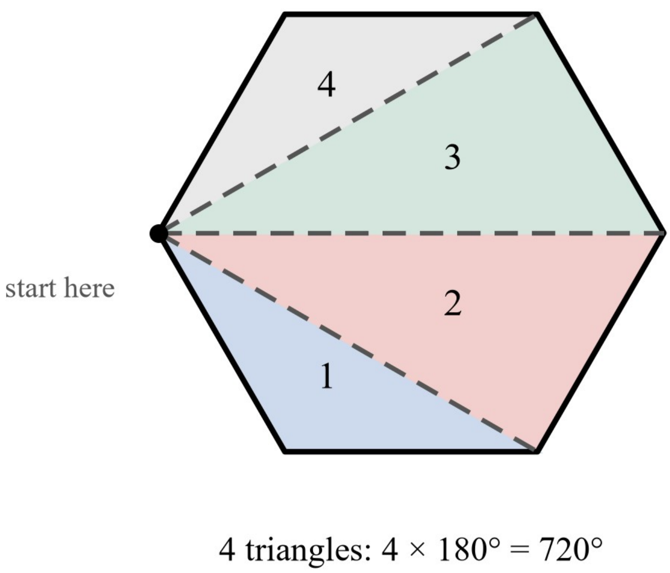
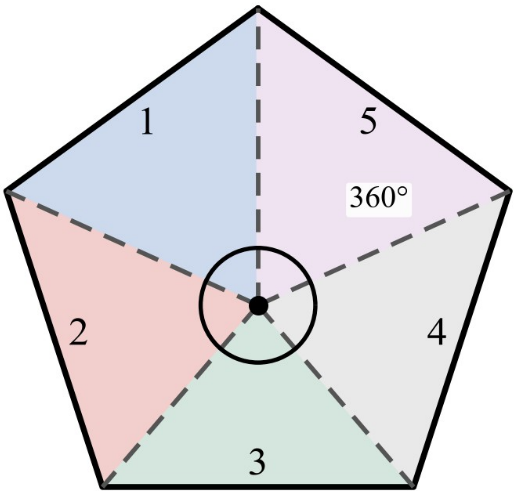


Figure 5. A hexagon with three dashed lines from one vertex, marked start here, to the three far vertices, splitting it into triangles 1, 2, 3 and 4; underneath, 4 triangles: 4 times 180 degrees equals 720 degrees.

The rule for any polygon. An *n*-sided polygon splits from one vertex into $n-2$ triangles, so its interior angle sum is $180(n-2)$. Figure 6 counts the same angles a second way: lines from a point inside a pentagon to the five vertices make 5 triangles, giving $5\times180°=900°$, and then the $360°$ of angles around the inside point — which are not angles of the pentagon — is taken away: $900°-360°=540°$. For an *n*-sided polygon this count is $180n-360$. The two counts cut up the same angles, so $180(n-2)=180n-360$: for a hexagon, $180\times4=720$ and $180\times6-360=720$.



5 triangles: 5 × 180° = 900°, then 900° − 360° = 540°

Figure 6. A pentagon with dashed lines from a point inside to all five vertices, making triangles 1 to 5; a small circle at the point is labelled 360 degrees; underneath, 5 triangles: 5 times 180 degrees equals 900 degrees, then 900 degrees minus 360 degrees equals 540 degrees.

Regular polygons. A regular polygon has equal sides and equal angles, so every angle gets an equal share of the sum. In Figure 7 the regular pentagon's sum is $540°$, and each angle is $540°\div5=108°$ · a regular pentagon has 5 equal angles · each angle is $108°$.

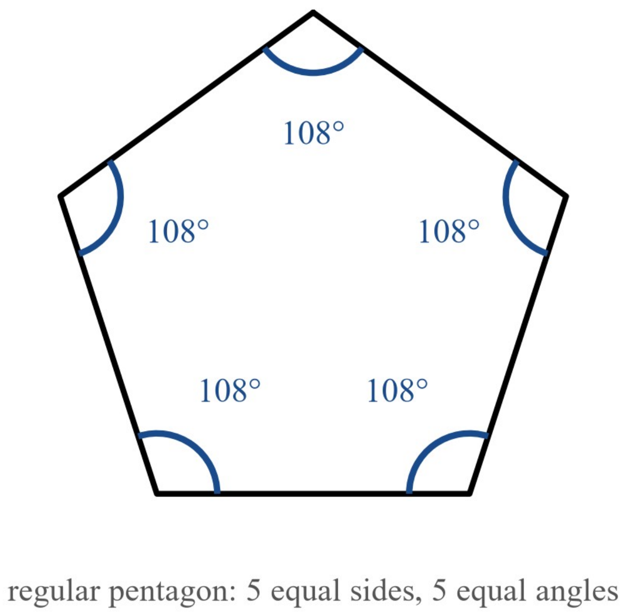


Figure 7. A regular pentagon with every angle marked 108 degrees; underneath, regular pentagon: 5 equal sides, 5 equal angles.

Unknown angles in other shapes. The habit from Figure 3 carries over, with the bigger sum. In Figure 8 the marked angles of the quadrilateral are $95°$, $110°$ and $58°$, and the fourth is x. The statement: $95° + 110° + 58° + x = 360°$. The reason: a quadrilateral's angles add to $360°$ (two triangles). The conclusion: $x = 360° - 263° = 97°$.

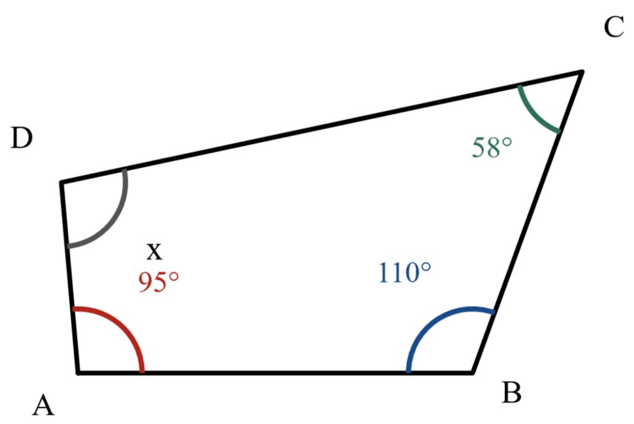


Figure 8. Quadrilateral ABCD with the angle at A marked 95 degrees, at B marked 110 degrees, at C marked 58 degrees, and at D marked x.

Worked examples

Example 1 — scaffolded

Find x in the triangle of Figure 3.

Working	Comment
Statement. $48° + 67° + x = 180°$.	The three angles of the triangle add to $180°$.
Add the known angles. $48° + 67° = 115°$, so	Collect the numbers first.

Working	Comment
$115° + x = 180°$.	
Conclusion. $x = 180° - 115° = 65°$.	Take the known angles from $180°$.
Check. $48° + 67° + 65° = 180°$.	The three angles must fill the sum exactly.

∴ $x = 65°$ · angles in a triangle add to $180°$ · the third angle is $65°$.

Example 2 — scaffolded

Find the interior angle sum of a hexagon by splitting it into triangles (Figure 5).

Working	Comment
Count the triangles. Diagonals from one vertex make 4 triangles.	Each new side beyond the third adds one triangle.
Add the triangle sums. $4 \times 180° = 720°$.	Every triangle contributes $180°$.
Check with the rule. $180(n-2)$ with $n = 6$: $180 \times 4 = 720°$.	The table and the rule agree.

∴ a hexagon's angles add to $720°$ · it splits into 4 triangles · the sum is $4 \times 180° = 720°$.

Example 3 — unscaffolded

Find x in the quadrilateral of Figure 8.

Working	Comment
$95° + 110° + 58° + x = 360°$.	A quadrilateral splits into 2 triangles: $2 \times 180° = 360°$.
$263° + x = 360°$, so $x = 360° - 263° = 97°$.	Take the three known angles from $360°$.
Check. $95° + 110° + 58° + 97° = 360°$.	The four angles fill the sum exactly.

∴ $x = 97°$ · a quadrilateral's angles add to $360°$ · the fourth angle is $97°$.

Example 4 — unscaffolded

Find the size of each angle of a regular pentagon (Figure 7).

Working	Comment
Angle sum: $180(5-2) = 180 \times 3 = 540°$.	A pentagon splits into 3 triangles.
Each angle: $540° \div 5 = 108°$.	A regular polygon's angles share the sum equally.
Check. $5 \times 108° = 540°$.	Five equal angles must return the sum.

∴ each angle is $108°$ · the pentagon's sum of $540°$ is shared by 5 equal angles · each is $108°$.

Practice questions

Scaffolded

Q1. The torn-corner demonstration (Figure 1). (a) The three torn corners placed at one point on a straight line fill what angle, so the three angles of the triangle add to how many degrees? (b) Two corners of a paper triangle measure $55°$ and $60°$. Use the demonstration to find the third corner.

Q2. Find x in each triangle of Figure 9, giving the reason with each answer.

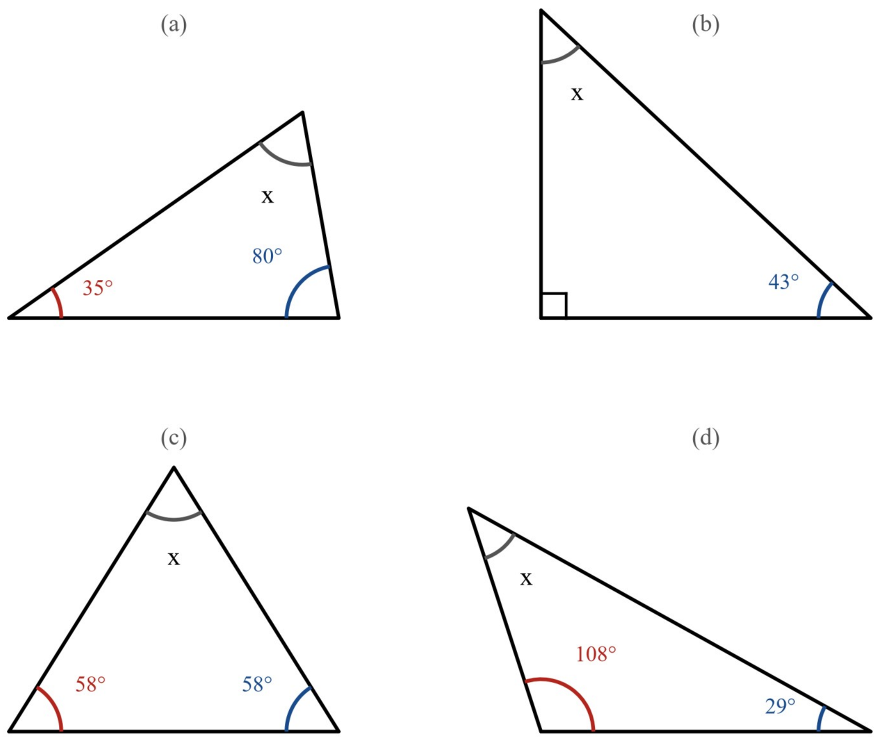


Figure 9. Four triangles labelled (a) to (d), each with two angles marked and the third marked x: (a) 35 degrees and 80 degrees; (b) a right angle and 43 degrees; (c) 58 degrees and 58 degrees; (d) 108 degrees and 29 degrees.

Q3. The quadrilateral of Figure 4. (a) How many triangles does the diagonal make? (b) Each triangle's angles add to how many degrees? (c) Hence write down the angle sum of the quadrilateral. (d) Check by adding the marked angles: $85° + 105° + 95° + 75°$.

Q4. Splitting a hexagon (Figure 5). (a) Diagonals from one vertex split the hexagon into how many triangles? (b) Write its angle sum as a number of $180°$s, then as a single number. (c) The same split of a 7-sided shape makes 5 triangles. What is its angle sum?

Q5. The pattern in the table, and the rule (Figure 6). (a) An *n*-sided polygon splits from one vertex into how many triangles? (b) Write its angle sum as 180 times that number of triangles. (c) The inside-point count gives $180n - 360$. Check that both forms give $720°$ when $n = 6$.

Unscaffolded

Q6. Find x in each shape of Figure 10, giving the reason with each answer.

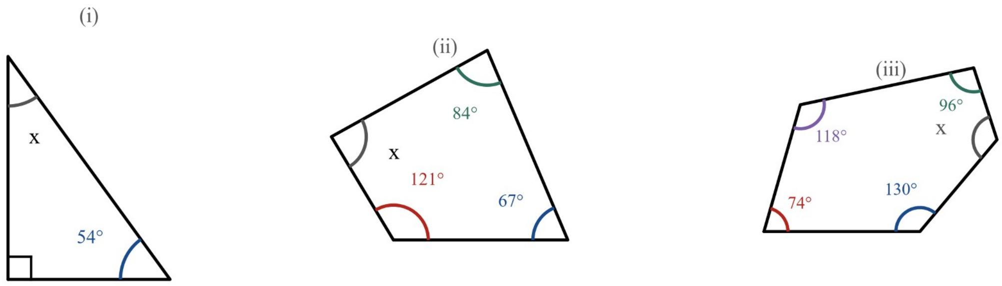


Figure 10. Three shapes labelled (i) to (iii), each with one angle marked x: (i) a right-angled triangle with 54 degrees; (ii) a quadrilateral with 121, 67 and 84 degrees; (iii) a pentagon with 74, 130, 96 and 118 degrees.

Q7. Three angles of a quadrilateral are $102°$, $87°$ and $64°$. Find the fourth angle, giving your reason.

Q8. Four angles of a pentagon are $100°$, $115°$, $92°$ and $128°$. Find the fifth angle, giving your reason.

Q9. A regular hexagon. (a) Find its interior angle sum. (b) Find the size of each angle.

Q10. A regular 10-sided polygon. (a) Find its interior angle sum. (b) Find the size of each angle.

Q11. (a) Use $180(n-2)$ to find the interior angle sum of a 12-sided polygon. (b) A polygon's interior angles add to $1980°$. How many sides does it have?

Q12. Answer each part with a reason. (a) Can a triangle have two right angles? (b) Can a quadrilateral have four obtuse angles? (c) True or false: a polygon whose interior angles add to $900°$ has 7 sides.

Answers

Q1. (a) A straight angle, so $180°$; (b) $65°$ · the corners fill a straight angle · the third corner is $180° - 55° - 60° = 65°$.

Q2. (a) $65°$; (b) $47°$; (c) $64°$; (d) $43°$ · the angles of a triangle add to $180°$ · x is what is left after the marked angles.

Q3. (a) 2; (b) $180°$; (c) $360°$; (d) $360°$ · the quadrilateral splits into 2 triangles · the two counts agree.

Q4. (a) 4; (b) $4 \times 180° = 720°$; (c) $5 \times 180° = 900°$ · each shape splits into that many triangles · the sum is that many $180°$s.

Q5. (a) $n - 2$; (b) $180(n - 2)$; (c) $180(6 - 2) = 720°$ and $180 \times 6 - 360 = 720°$ · both counts cut up the same angles · the two forms agree.

Q6. (i) $36°$ · the angles of a triangle add to $180°$; (ii) $88°$ · a quadrilateral's angles add to $360°$; (iii) $122°$ · a pentagon's angles add to $540°$.

Q7. $107°$ · a quadrilateral's angles add to $360°$ · $360° - 253° = 107°$.

Q8. $105°$ · a pentagon's angles add to $540°$ · $540° - 435° = 105°$.

Q9. (a) $720°$ · a hexagon splits into 4 triangles; (b) $120°$ · 6 equal angles share the sum.

Q10. (a) $1440°$ · a 10-sided polygon splits into 8 triangles; (b) $144°$ · 10 equal angles share the sum.

Q11. (a) $1800°$; (b) 13 sides · $1980° \div 180° = 11$ triangles · $11 + 2 = 13$ sides.

Q12. (a) No · two right angles already add to $180°$, leaving $0°$ for the third angle; (b) no · four angles each bigger than $90°$ add to more than $360°$; (c) true · $900° \div 180° = 5$ triangles, and $5 + 2 = 7$ sides.

Fully-worked solutions

Q1. (a) The three corners fill a straight angle · angles on a straight line add to $180°$ · the three angles of the triangle add to $180°$. (b) $55° + 60° + x = 180°$ · the torn corners fill a straight angle · $x = 180° - 115° = 65°$.

Q2. (a) $35° + 80° + x = 180°$ · angles in a triangle add to $180°$ · $x = 65°$. (b) $90° + 43° + x = 180°$ · the square mark is a right angle and the three angles add to $180°$ · $x = 47°$. (c) $58° + 58° + x = 180°$ · angles in a triangle add to $180°$ · $x = 64°$. (d) $108° + 29° + x = 180°$ · angles in a triangle add to $180°$ · $x = 43°$.

Q3. (a) 2 triangles. (b) $180°$ each · angle sum of a triangle. (c) $2 \times 180° = 360°$ · the quadrilateral splits into 2 triangles · its angles add to $360°$. (d) $85° + 105° + 95° + 75° = 360°$ · the marked angles must fill the same sum · the two counts agree.

Q4. (a) 4 triangles. (b) $4 \times 180° = 720°$ · the hexagon splits into 4 triangles · its angles add to $720°$. (c) $5 \times 180° = 900°$ · a 7-sided shape splits into 5 triangles · its angles add to $900°$.

Q5. (a) $n - 2$ triangles (the table: 3 sides gives 1, 4 gives 2, and so on). (b) $180(n - 2)$. (c) $180(6 - 2) = 180 \times 4 = 720°$, and $180 \times 6 - 360 = 1080° - 360° = 720°$ · both counts cut up the same angles of the hexagon · the two forms give the same sum.

Q6. (i) $90° + 54° + x = 180°$ · the square mark is a right angle and a triangle's angles add to $180°$ · $x = 36°$. (ii) $121° + 67° + 84° + x = 360°$ · a quadrilateral splits into 2 triangles, so its angles add to $360°$ · $x = 360° - 272° = 88°$. (iii) $74° + 130° + 96° + 118° + x = 540°$ · a pentagon splits into 3 triangles, so its angles add to $540°$ · $x = 540° - 418° = 122°$.

Q7. $102° + 87° + 64° + x = 360°$ · a quadrilateral's angles add to $360°$ (2 triangles) · $x = 360° - 253° = 107°$.

Q8. $100° + 115° + 92° + 128° + x = 540°$ · a pentagon's angles add to $540°$ (3 triangles) · $x = 540° - 435° = 105°$.

Q9. (a) $180(6-2) = 720°$ · a hexagon splits into 4 triangles · the sum is $720°$. (b) $720° \div 6 = 120°$ · a regular hexagon has 6 equal angles · each is $120°$.

Q10. (a) $180(10-2) = 180 \times 8 = 1440°$ · a 10-sided polygon splits into 8 triangles · the sum is $1440°$. (b) $1440° \div 10 = 144°$ · the 10 angles are equal · each is $144°$.

Q11. (a) $180(12-2) = 180 \times 10 = 1800°$ · a 12-sided polygon splits into 10 triangles · the sum is $1800°$. (b) $1980° \div 180° = 11$, so the polygon splits into 11 triangles · the number of triangles is $n-2$ · $n = 11 + 2 = 13$ sides.

Q12. (a) No · $90° + 90° = 180°$ already, so the third angle would be $0°$ · a triangle cannot have two right angles. (b) No · each obtuse angle is bigger than $90°$, so four of them add to more than $360°$ · but a quadrilateral's angles add to exactly $360°$. (c) True · $900° \div 180° = 5$ triangles, and $5 + 2 = 7$ · a polygon with angle sum $900°$ has 7 sides.

Teacher note

- **Ownership and the boundary with 4D.** Per the Chapter 4 plan in TOC §3, this subtopic (`VC2M7M05`) owns the triangle angle sum and the $180(n-2)$ polygon generalisation, derived by decomposition, not asserted; 4D (`VC2M7M04`) owns the parallel-line relationships and the exterior-angle result. Figure 2 *uses* alternate angles (taught in 4D, which precedes 4E) only to explain the sum the tearing demonstrates; it teaches no parallel-line fact.
- **Reason-giving (T4).** 4D and 4E are the book's first sustained deductive geometry, and every answer and solution here states its reason in the estate's standard form (statement · reason · conclusion). Per §7, items carry at least 1 mark for the stated reason; a correct number with no reason does not score full marks. The Answers section repeats each reason in short form so the habit is visible even in the answer key.
- **Level 6 recall (P2, D3).** Angles on a straight line and at a point (`VC2M6M04`) are recalled in one sentence each (Figure 1, Figure 2, Figure 6) and are not re-taught.
- **E01 delivery.** The torn paper triangle of Figure 1 is the elaboration's concrete material; the three corners filling a straight angle is the demonstration, and Q1 makes the student run it.
- **E02 delivery.** Decomposition from one vertex (Figures 4–5, the table) gives $180(n-2)$; decomposition from an interior point (Figure 6) gives $180n - 360$. The equality of the two forms is shown as two counts of the same angles — never by expanding brackets, which is outside Level 7 (D4).
- **D4 boundaries.** No Pythagoras, no equations of lines, no expanding or factorising; the pronumeral n enters only as a counting number inside the two given forms.
- **Figures.** All ten figures are generated by `src/gen_4e.py` from the exact values stated in the text; every marked angle is computed from its stated value and asserted at build time, so the diagrams measure what they claim (§6).
- **No D11 content.** None of `VC2M7M05`'s elaborations names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

Modelling with ratios of lengths, areas and volumes

Content description `VC2M7M06`: use mathematical modelling to solve practical problems involving ratios of lengths, areas and volumes; formulate problems, interpret and communicate solutions in terms of the situation, justifying choices made about the representation

Achievement standard (extract): They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.

Theory

What a model is. When maths is used on a real situation, the numbers stand for something: shovels of sand, millilitres of oil, centimetres of a rectangle. A **model** is the maths we choose to stand for the situation — here, usually a ratio, a bar split into parts, or a labelled sketch. **Modelling** is the whole job: turn the situation into maths, do the maths, then turn the answer back into words about the situation. A calculation only counts as modelling once the answer has been said back in terms of the situation it came from.

The four steps. This subtopic runs the same four steps every time, in a cycle (Figure 1):

1. **Formulate** the problem — say exactly what question is being asked, and pick out the information that matters.
2. **Choose** a representation and a calculation plan.
3. **Solve** — carry out the calculation.
4. **Interpret and communicate** — say what the answer means in the real situation, and justify the choice of representation.

If the answer does not make sense in the situation, the cycle is run again from step 1 with a better model.

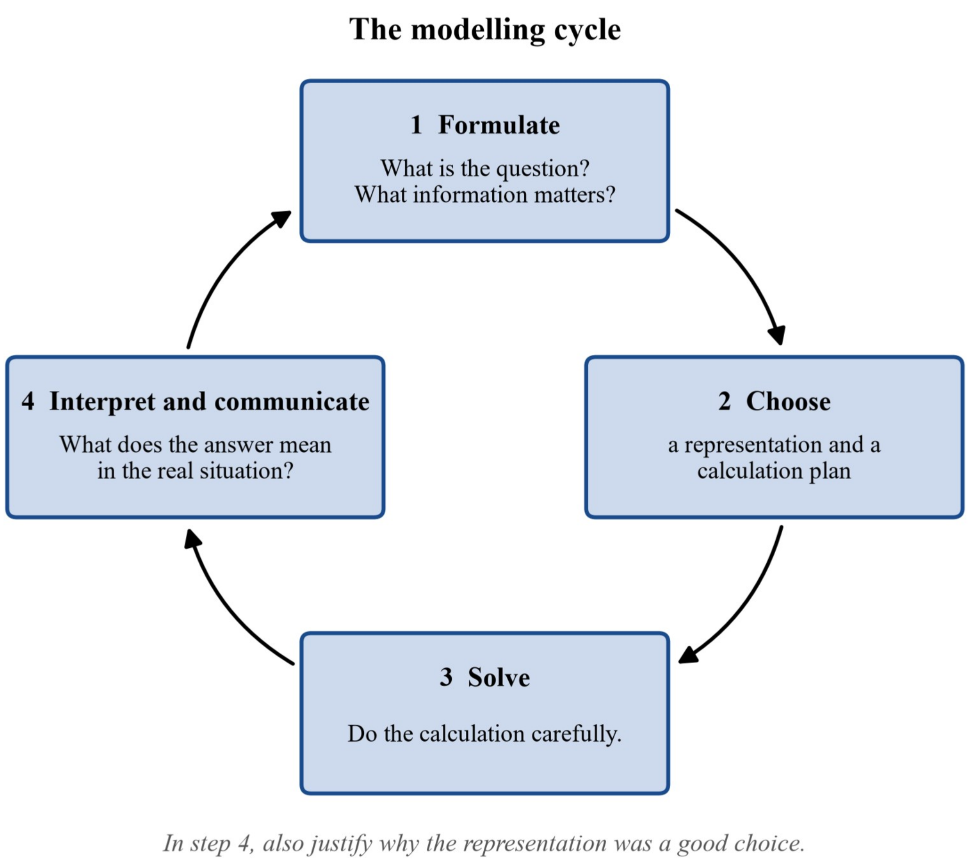


Figure 1. The modelling cycle: step 1 formulate, step 2 choose a representation and a calculation plan, step 3 solve, step 4 interpret and communicate, with arrows joining the steps in a loop and a note that step 4 also justifies the choice of representation.

Part–part and part–whole. A ratio compares one part with another — a **part–part** view. A concrete mix written cement:sand:gravel = 1:2:3 says: for every 1 part of cement there are 2 parts of sand and 3 parts of gravel. The same bars can be read a second way. The whole mix is $1+2+3=6$ parts, so cement is 1 of the 6 parts — $\frac{1}{6}$ of the whole mix — and sand is $\frac{2}{6}$ of it. That is the **part–whole** view, written as fractions. Both views are the same bars read two ways (Figure 2): the ratio 1:2:3 compares the parts with each other, and the fractions $\frac{1}{6}, \frac{2}{6}$ and $\frac{3}{6}$ compare each part with the whole.

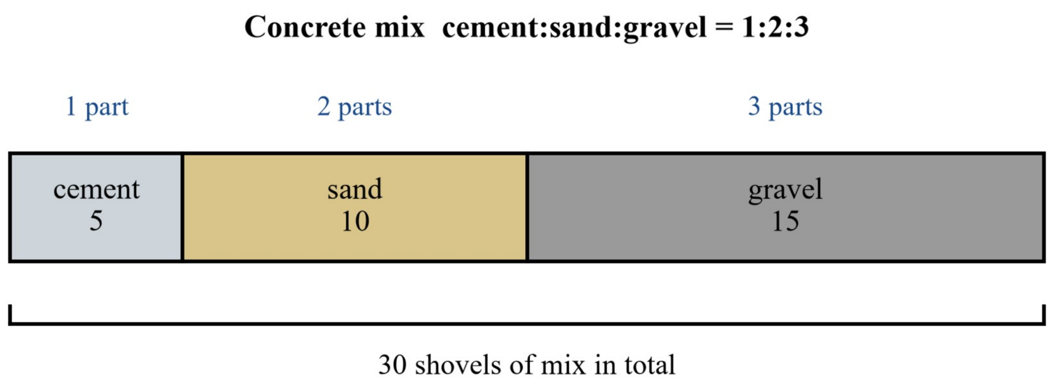


Figure 2. A bar for 30 shovels of concrete mix split into parts in the ratio 1:2:3 — cement 5 shovels, sand 10 shovels, gravel 15 shovels — with 1 part, 2 parts and 3 parts marked above and a bracket underneath marking 30 shovels of mix in total.

Mixes in millilitres and grams. The same model works for any mix. A salad dressing made at oil:vinegar = 3:1 is $3+1=4$ parts in all, so oil is $\frac{3}{4}$ of the dressing (Figure 3). A recipe is a ratio too: when the whole mix is given, the parts say how much of each item goes in. Whether the parts are counted in shovels, millilitres or grams, the calculation is the same — find one part, then find each share.

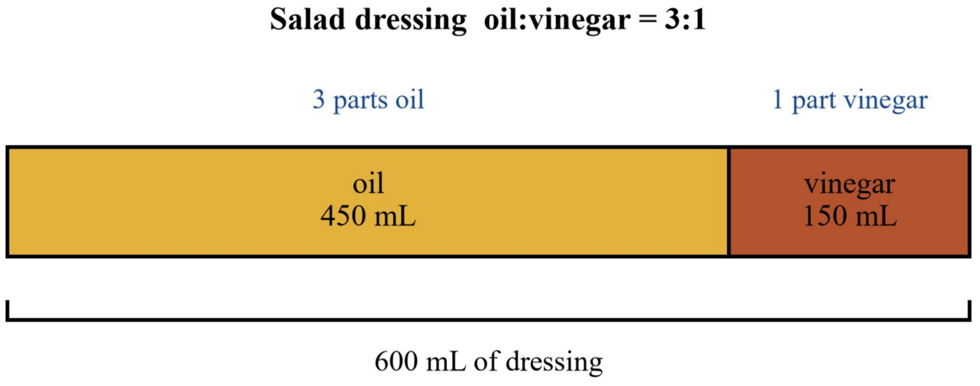


Figure 3. A bar for 600 mL of salad dressing split in the ratio 3:1 — oil 450 mL and vinegar 150 mL — with 3 parts oil and 1 part vinegar marked above and a bracket underneath marking 600 mL of dressing.

The golden ratio in design. Some ratios appear in design. A rectangle whose length:width is 8:5 is close to the **golden ratio**, a ratio of about 8:5 that many people like the look of in art and design. Whether a shape looks best is a matter of opinion, not a mathematical fact — the mathematics is only the ratio itself. Figure 4 draws the three rectangles so the sizes match the ratios, and they can be compared by eye: 1:1, 8:5 and 2:1.

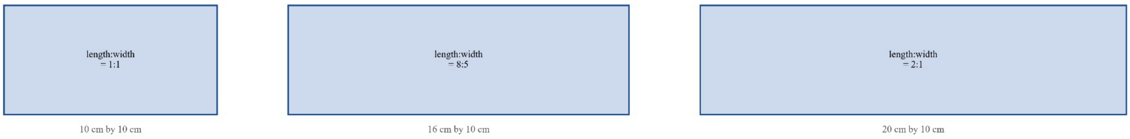


Figure 4. Three rectangles drawn with the same width of 10 cm and lengths that match the ratios: 10 cm by 10 cm labelled length:width = 1:1, 16 cm by 10 cm labelled length:width = 8:5, and 20 cm by 10 cm labelled length:width = 2:1.

Colour mixes and strength. Ratios model colour too. Mixing blue and yellow paint makes green, and the ratio blue:yellow sets the **strength** of each colour in the mix: more blue in the ratio gives a deeper, bluer green (Figure 5). Mix A at 1:1, Mix B at 2:1 and Mix C at 3:1 make three different greens. Two mixes with the same ratio make the same green: 4:2 is the same ratio as 2:1, so a 4:2 mix would match Mix B exactly.

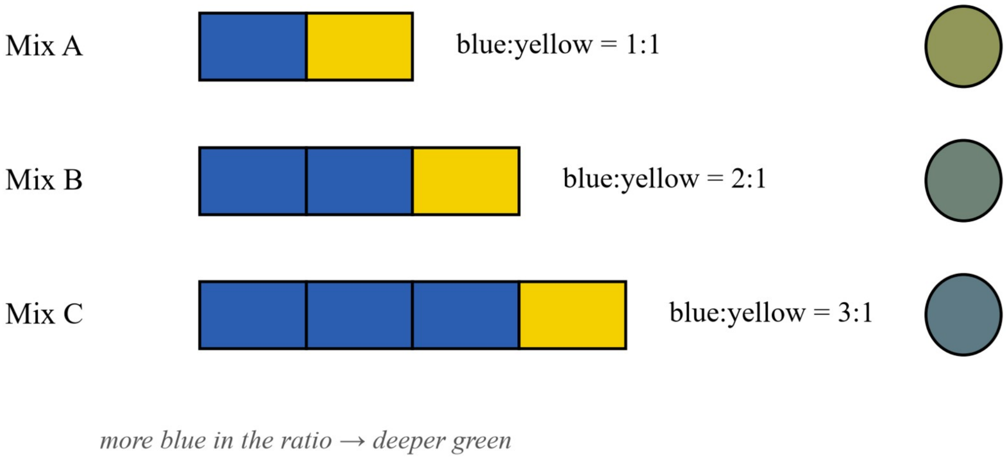


Figure 5. Three rows of blue and yellow squares showing paint mixes — Mix A blue:yellow = 1:1, Mix B blue:yellow = 2:1, Mix C blue:yellow = 3:1 — each row ending in a circle of the green it makes, with a note that more blue in the ratio gives a deeper green.

Lengths, areas and volumes. When a shape is copied bigger and every length is multiplied by the same number, the area and the volume grow by more than that number. Figure 6 shows a 15 cm × 10 cm card copied at 3:1: each length is ×3, so the copy is 45 cm × 30 cm. The area goes from $15 \times 10 = 150$ cm² to $45 \times 30 = 1350$ cm² — that is ×9, not ×3, because $3 \times 3 = 9$, and the copy holds nine cards' worth of area. For a solid, every length ×3 makes the volume ×27, because $3 \times 3 \times 3 = 27$: a cube of side 10 cm holds 1000 cm³, and a cube of side 30 cm holds 27000 cm³ (Figure 7).

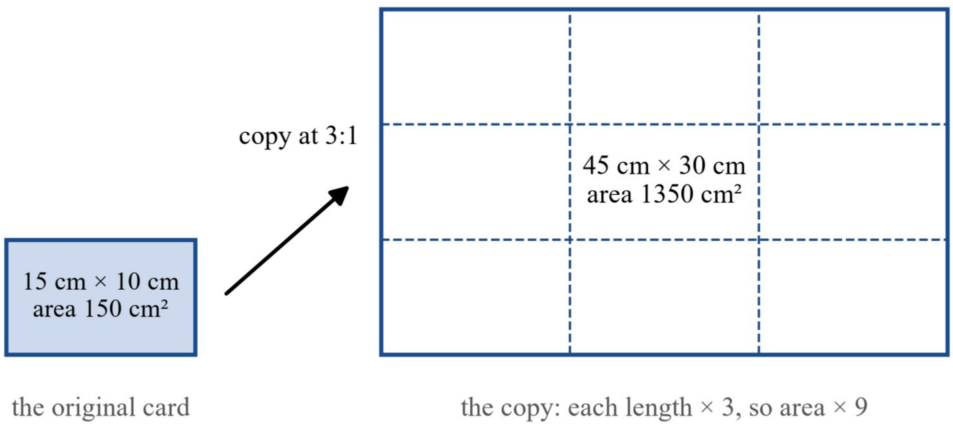


Figure 6. A 15 cm by 10 cm card of area 150 cm² next to its copy at 3:1, 45 cm by 30 cm with area 1350 cm², the copy cut into nine card-sized pieces by dashed lines to show why each length times 3 gives area times 9.

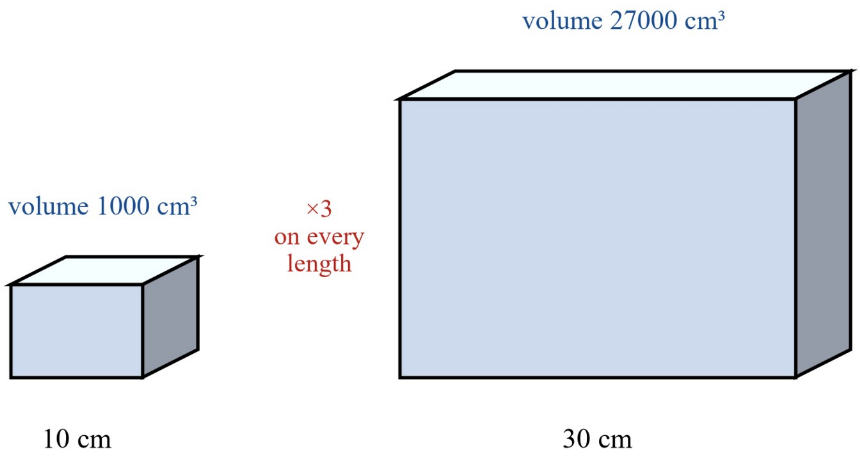


Figure 7. Two cubes drawn so the bigger cube is three times as tall as the smaller: a cube of side 10 cm with volume 1000 cm³, and a cube of side 30 cm with volume 27000 cm³, with a note that each length is times 3 so the volume is times 27.

The pattern holds for every copy in which all lengths grow by the same number:

Every length ×	area ×	volume ×
2	4	8
3	9	27
4	16	64

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
model, modelling	the maths chosen to stand for a situation; the four-step job of using it	representation, picture of the situation
formulate	say the situation as a maths question and pick out the information that	set up, write as a number sentence

Term	Meaning here	Synonyms and related words
	matters	
representation	the ratio, bar, sketch or table chosen in step 2	picture, diagram, chart
ratio	a part–part comparison, written like 3:1	comparison
parts	the equal shares a ratio splits a mix into	shares
part–part	reading the bars as parts compared with each other	ratio view
part–whole	reading one part as a fraction of the whole mix	fraction view
strength	how much of one colour a mix has, set by the ratio	depth of colour
golden ratio	the ratio of about 8:5 that many people like in design	a pleasing rectangle

This subtopic starts from the ratio ideas of 2F and runs them through the modelling cycle shared with 2G. The unit conversions inside Q7 and Q13 (litres to millilitres) are assumed from Level 6 and are used the same way in 4A and 4B. Task I2 uses the same four steps in a context of its own.

Worked examples

Example 1 — scaffolded

A builder mixes concrete at cement:sand:gravel = 1:2:3. The job needs 30 shovels of mix in total. How many shovels of cement, of sand and of gravel does the builder use?

Working	Comment
Formulate. The question is "how many shovels of each material are in 30 shovels of mix?" Information that matters: ratio 1:2:3; total 30 shovels.	The ratio is part–part; the 30 shovels are the whole.
Choose. Draw the mix as a bar of $1+2+3=6$ equal parts (Figure 2) and find one part first.	A bar of parts shows every share and the whole at once, so no share can be missed.
Solve. One part $=30\div 6=5$ shovels. Cement $=1\times 5=5$; sand $=2\times 5=10$; gravel $=3\times 5=15$ shovels.	Check: $5+10+15=30$ shovels, the total the job needs.
Interpret. The builder uses 5 shovels of cement, 10 of sand and 15 of gravel.	Said back in shovels of material. Part–whole check: cement is $\frac{1}{6}$ of 30, which is 5.

∴ 5 shovels of cement, 10 of sand and 15 of gravel.

Example 2 — scaffolded

A salad dressing is made at oil:vinegar = 3:1. A cook makes 600 mL of dressing. How much oil and how much vinegar is in it?

Working	Comment
Formulate. The question is "how much oil and how much vinegar is in 600 mL of dressing?" Information that matters: ratio 3:1; whole 600 mL.	Oil and vinegar are the parts; 600 mL is the whole.

Working	Comment
Choose. Draw the dressing as a bar of $3+1=4$ equal parts (Figure 3); find one part, then each share.	The same bar representation as Example 1 — a bar of parts suits any mix.
Solve. One part $=600\div 4=150$ mL. Oil $=3\times 150=450$ mL; vinegar $=1\times 150=150$ mL.	Check: $450+150=600$ mL.
Interpret. The 600 mL of dressing is 450 mL of oil and 150 mL of vinegar.	Part–whole check: oil is $\frac{3}{4}$ of the dressing, and $\frac{3}{4}$ of 600 is 450.

∴ 450 mL of oil and 150 mL of vinegar.

Example 3 — unscaffolded

A card is 15 cm long and 10 cm wide. A copy of the card is made at 3:1, so every length on the copy is ×3. (a) Find the length and the width of the copy. (b) Find the area of the card and of the copy, and compare them. (c) The card needs 20 mL of paint to cover it. How much paint does the copy need?

Working	Comment
(a) Length $=15\times 3=45$ cm; width $=10\times 3=30$ cm.	Every length is ×3 — both the length and the width.
(b) Card: $15\times 10=150$ cm². Copy: $45\times 30=1350$ cm². $1350\div 150=9$.	The area is ×9, not ×3, because $3\times 3=9$ (Figure 6).
(c) Paint covers area, so the copy needs $20\times 9=180$ mL.	The copy holds nine cards' worth of area, so nine lots of 20 mL.

∴ the copy is 45 cm × 30 cm, its area is 9 times the card's, and it needs 180 mL of paint.

Example 4 — unscaffolded

A cube-shaped box with sides of 10 cm holds 200 g of lollies. A bigger cube-shaped box has sides of 30 cm, and the lollies are packed the same way in both boxes. (a) How many times bigger is each length of the big box? (b) Find the volume of each box. (c) How many grams of lollies does the big box hold?

Working	Comment
(a) $30\div 10=3$: each length is ×3.	The lengths are in the ratio 3:1.
(b) Small: $10\times 10\times 10=1000$ cm³. Big: $30\times 30\times 30=27000$ cm³.	$27000\div 1000=27$: the volume is ×27, because $3\times 3\times 3=27$ (Figure 7).
(c) Same packing, so the mass follows the volume: $200\times 27=5400$ g.	The fact that both boxes are packed the same way is what allows this step.

∴ the big box holds 5400 g of lollies — 27 times as much, not 3 times.

Practice questions

Scaffolded

Q1. Say whether each comparison is part–part (a ratio) or part–whole (a fraction). (a) A drink is made with 1 part syrup to 4 parts water. (b) 3 of every 10 students in a survey walk to school. (c) A concrete mix is cement:sand:gravel = 1:2:3.

Q2. A builder mixes concrete at cement:sand:gravel = 1:2:3 and makes 48 shovels of mix in total. (a) How many equal parts is the mix split into? (b) Find the shovels of cement, of sand and of gravel. (c) Check your three answers against the 48 shovels.

Q3. Green paint is mixed at blue:yellow = 2:3. A painter makes 500 mL of the green. (a) Find the volume of blue paint and of yellow paint used. (b) The painter has a 300 mL tin of yellow paint. Is it enough for this mix?

Q4. A designer draws a rectangle at the golden ratio, length:width = 8:5. The width is 15 cm. Find the length.

Q5. A biscuit recipe uses flour:butter:sugar = 3:2:1 (Figure 8). The mixture made has a mass of 900 g in total. Find the mass of the flour, the butter and the sugar.

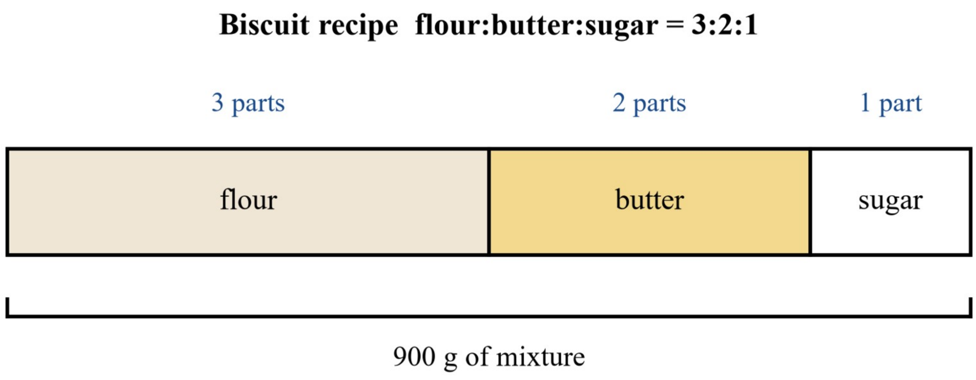


Figure 8. A bar for 900 g of biscuit mixture split in the ratio 3:2:1 into flour, butter and sugar, with 3 parts, 2 parts and 1 part marked above and a bracket underneath marking 900 g of mixture.

Q6. A model maker builds a model of a table so that, for every length, model length:real length = 1:20. (a) A length on the model is 5 cm. Find the real length, in cm. (b) The real table is 300 cm long. Find the length on the model. (c) In one sentence, say why the same ratio 1:20 must be used for every length.

Unscaffolded

Q7. A salad dressing is made at oil:vinegar = 3:1. A jar holds 1 L of the dressing. (a) Write 1 L in mL. (b) Find the volume of oil and of vinegar in the jar. (c) How much more oil than vinegar is in the jar?

Q8. A painter paints a panel 3 m long and 2 m wide for the school hall. The school likes it and asks for a copy at a bigger size on a wall (Figure 9). The copy keeps the same shape, and its width is 6 m. (a) Find the length of the copy. (b) Find the area of the panel and of the copy, and compare them. (c) One tin of paint covers 6 m². How many tins are needed for the copy?

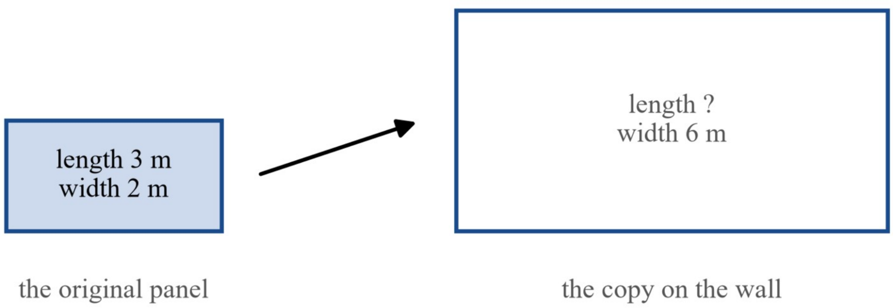


Figure 9. A 3 m by 2 m panel next to a bigger copy of the same shape on a wall whose width is 6 m and whose length is marked with a question mark.

Q9. Two boxes are cuboids (Figure 10). The first box is 20 cm long, 25 cm wide and 30 cm high. The second box is 40 cm long, 50 cm wide and 60 cm high. (a) Find the volume of each box. (b) How many times bigger is the volume of the second box than the first? (c) Each length of the second box is ×2. Explain why the volume is ×8, not ×2.

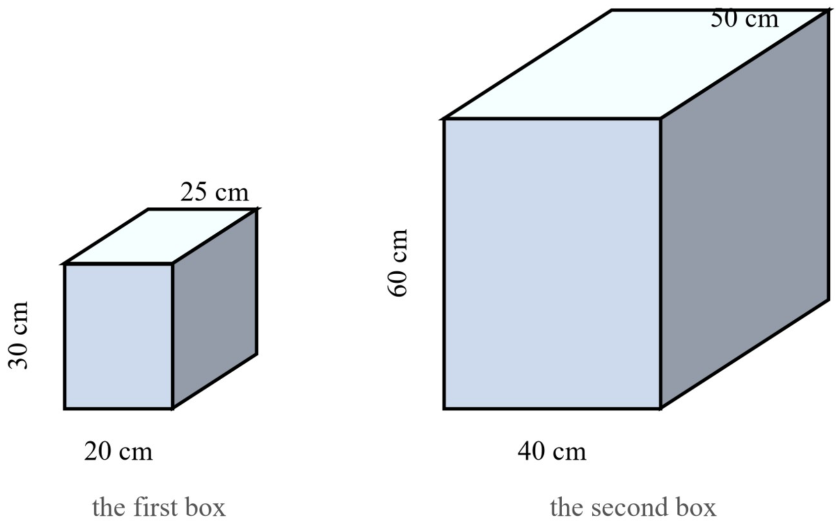


Figure 10. Two cuboid boxes drawn with their dimensions labelled: the first box 20 cm by 25 cm by 30 cm, the second box 40 cm by 50 cm by 60 cm.

Q10. Four green paint mixes are given as blue:yellow — Mix A 1:1, Mix B 2:1, Mix C 3:1 and Mix D 4:2 (Figure 5 shows A, B and C). (a) Which of Mix A, Mix B and Mix C makes the deepest, bluest green? (b) Which mix makes the same green as Mix B? (c) Find the volume of blue paint and of yellow paint needed to make 600 mL of Mix C.

Q11. Mai says: "If I copy a rectangle at 4:1, the area is ×4." Show that Mai is wrong using an example rectangle of your own, and say how many times bigger the area really is.

Q12. Figure 4 shows three rectangles. (a) Write length:width of each rectangle as a ratio. (b) Which rectangle is at the golden ratio? (c) A school banner at the golden ratio has a width of 25 cm. Find its length.

Q13. A plant food is mixed with water at plant food:water = 1:9 before it is used on a garden. (a) A gardener makes 20 L of the mix. Find the volume of plant food and of water in it. (b) The plant food is sold in 500 mL bottles at $3.20 each. How many bottles are needed for the 20 L of mix, and what do they cost? (c) In one sentence, justify your choice of representation for part (a).

Q14. Two cube-shaped boxes have sides of 5 cm and 15 cm. (a) Write the ratio of the sides, big:small, in simplest form. (b) Find the volume of each cube. (c) How many times bigger is the volume of the big cube? (d) How many of the small cubes would fit inside the big cube?

Answers

Q1. (a) part–part; (b) part–whole; (c) part–part.

Q2. (a) 6; (b) cement 8, sand 16, gravel 24 shovels; (c) $8+16+24=48$, the total made.

Q3. (a) blue 200 mL, yellow 300 mL; (b) yes — exactly enough, the mix needs 300 mL of yellow.

Q4. 24 cm.

Q5. flour 450 g, butter 300 g, sugar 150 g.

Q6. (a) 100 cm; (b) 15 cm; (c) so the model keeps the same shape as the real table.

Q7. (a) 1000 mL; (b) oil 750 mL, vinegar 250 mL; (c) 500 mL more oil.

Q8. (a) 9 m; (b) 6 m² and 54 m² — the copy's area is ×9; (c) 9 tins.

Q9. (a) 15000 cm³ and 120000 cm³; (b) ×8; (c) the volume multiplies the three lengths: $2\times2\times2=8$.

Q10. (a) Mix C; (b) Mix D; (c) blue 450 mL, yellow 150 mL.

Q11. ×16 — for example a 1 cm × 2 cm card (area 2 cm²) copies at 4:1 to 4 cm × 8 cm (area 32 cm²), and $32\div2=16$.

Q12. (a) 1:1, 8:5, 2:1; (b) the middle rectangle, 16 cm by 10 cm; (c) 40 cm.

Q13. (a) plant food 2 L, water 18 L; (b) 4 bottles, $\$12.80$; (c) any sensible reason — e.g. a bar of 10 equal parts shows the 1 part of plant food and the 9 parts of water inside the 20 L whole.

Q14. (a) 3:1; (b) 125 cm³ and 3375 cm³; (c) ×27; (d) 27 small cubes.

Fully-worked solutions

Q1. (a) Syrup and water are two parts compared with each other — part–part. (b) The 3 walkers are compared with all 10 students, so one part is compared with the whole — part–whole, the fraction $\frac{3}{10}$. (c) Cement, sand and gravel are parts compared with each other — part–part; the whole mix of $1+2+3=6$ parts only appears when a total is given.

Q2. (a) The ratio 1:2:3 splits the mix into $1+2+3=6$ equal parts. (b) One part is $48\div6=8$ shovels: cement $1\times8=8$, sand $2\times8=16$, gravel $3\times8=24$ shovels. (c) $8+16+24=48$ shovels, which matches the total made, so the answer is sensible.

Q3. (a) The ratio 2:3 is $2+3=5$ parts; one part is $500\div5=100$ mL. Blue $=2\times100=200$ mL; yellow $=3\times100=300$ mL. (b) The mix needs 300 mL of yellow and the tin holds exactly 300 mL — enough, with none to spare.

Q4. In the ratio 8:5 the width is 5 parts, so one part is $15\div5=3$ cm; the length is $8\times3=24$ cm.

Q5. The recipe is $3+2+1=6$ parts; one part is $900\div6=150$ g. Flour $=3\times150=450$ g; butter $=2\times150=300$ g; sugar $=1\times150=150$ g. Check: $450+300+150=900$ g.

Q6. (a) The real length is ×20: $5\times20=100$ cm. (b) The model length is ÷20: $300\div20=15$ cm. (c) Using 1:20 for every length keeps the model the same shape as the real table; if different lengths used different ratios, the model would look stretched.

Q7. (a) 1 L $=1000$ mL. (b) The ratio 3:1 is $3+1=4$ parts; one part is $1000\div4=250$ mL. Oil $=3\times250=750$ mL; vinegar $=1\times250=250$ mL. (c) $750-250=500$ mL more oil than vinegar.

Q8. (a) The width goes 2 m → 6 m, which is ×3, so the length is $3\times3=9$ m. (b) Panel: $3\times2=6$ m². Copy: $9\times6=54$ m². $54\div6=9$: the copy's area is ×9, because $3\times3=9$. (c) Each tin covers 6 m², so the copy needs $54\div6=9$ tins.

Q9. (a) First box: $20 \times 25 \times 30 = 15000$ cm³. Second box: $40 \times 50 \times 60 = 120000$ cm³. (b) $120000 \div 15000 = 8$: the second box holds ×8 the volume. (c) Volume multiplies the three lengths together, so each length ×2 gives $2 \times 2 \times 2 = 8$ times the volume.

Q10. (a) Mix C — the most blue for each part of yellow, so the deepest, bluest green. (b) Mix D: 4:2 is the same ratio as 2:1, so it makes the same green as Mix B. (c) Mix C is $3 + 1 = 4$ parts; one part is $600 \div 4 = 150$ mL. Blue $= 3 \times 150 = 450$ mL; yellow $= 1 \times 150 = 150$ mL.

Q11. Take a 1 cm × 2 cm card: area $1 \times 2 = 2$ cm². Copied at 4:1 it is 4 cm × 8 cm: area $4 \times 8 = 32$ cm². $32 \div 2 = 16$, so the area is ×16, not ×4 — every length ×4 gives area $4 \times 4 = 16$. Mai's rule would only work for lengths.

Q12. (a) 10 cm by 10 cm gives 10:10 = 1:1; 16 cm by 10 cm gives 16:10 = 8:5; 20 cm by 10 cm gives 20:10 = 2:1. (b) The middle rectangle, 16 cm by 10 cm, is at 8:5. (c) The width is 5 parts: one part is $25 \div 5 = 5$ cm, so the length is $8 \times 5 = 40$ cm.

Q13. (a) The ratio 1:9 is $1 + 9 = 10$ parts; one part is $20 \div 10 = 2$ L. Plant food $= 1 \times 2 = 2$ L; water $= 9 \times 2 = 18$ L. (b) The plant food needed is 2 L $= 2000$ mL, which is $2000 \div 500 = 4$ bottles; $4 \times 3.20 = \$12.80$. (c) A bar of 10 equal parts is a good representation because it shows the 1 part of plant food and the 9 parts of water inside the 20 L whole, so one part is found by a single division.

Q14. (a) $15:5$ = 3:1. (b) Small: $5 \times 5 \times 5 = 125$ cm³. Big: $15 \times 15 \times 15 = 3375$ cm³. (c) $3375 \div 125 = 27$: ×27, because $3 \times 3 \times 3 = 27$. (d) The big cube holds 27 of the small cubes — the same number as the volume ratio, since each row of 3 cubes stacks 3 high and 3 deep.

Teacher note

- **Modelling cycle (D8).** Every worked example and the modelling questions run the full cycle — formulate, choose, solve, interpret and communicate with the choice of representation justified. An item answered with a bare number has not delivered `VC2M7M06`; the interpret and justify parts (e.g. Q6(c), Q9(c), Q13(c)) carry that intent in the practice set. T4 awards 4F's 5 marks on this basis, and task I2 builds on the same cycle alongside 2G.
- **Elaborations.** `VC2M7M06.E01` is the part–part/part–whole pair in the theory and Q1; `.E02` supplies the concrete mixes (WE1, Q2), the golden ratio in design (Figure 4, Q4, Q12) and the salad dressing (WE2, Q7); `.E03` supplies the primary-colour mixing and strength of colours (Figure 5, Q3, Q10). The capacity and mass mixes (dressing, biscuits, plant food) keep the ratios of `.E02` across mL and g.
- **Unit conversion (P1).** Q7 and Q13 convert L to mL inside the items (1 L = 1000 mL, 2 L = 2000 mL); conversion is assumed from `VC2M6M01` and is maintained inside these items, not taught as new content. The same assumption runs through 4A and 4B.
- **Level boundaries (D4).** No similarity or congruence language is used: the area ×9 and volume ×27 results are computed from the given lengths (3×3, $3 \times 3 \times 3$) and read off a table, not quoted as a scale-factor rule. There is no capacity↔volume equivalence (Level 8): the boxes of Q9/Q14 stay in cm³ and the mixes stay in mL, never equated. No rates, no percentage increase or decrease; powers appear only as evaluated squares and cubes.
- **`VC2M7M06.E04` is not taught in this subtopic**: Aboriginal and Torres Strait Islander content is excluded from ClassBasics books by the content policy; the mathematics of ratio modelling is delivered through the other elaborations instead.
- **Digital tools (D9).** `VC2M7M06` names none; none are required for this subtopic. No CAS.

Test

Chapter test T4 — Measurement: area, volume, circles, angles and ratio

Mathematics 7 (Level 7) · Victorian Curriculum 2.0

Assessed content descriptions	`VC2M7M01` — subtopic 4A, Areas of rectangles, triangles and parallelograms; `VC2M7M02` — subtopic 4B, Volume of right prisms; `VC2M7M03` — subtopic 4C, π and the circumference, radius and diameter of a circle; `VC2M7M04` — subtopic 4D, Parallel lines and a transversal: corresponding, alternate and co-interior angles; `VC2M7M05` — subtopic 4E, The angle sum of a triangle and of other polygons; `VC2M7M06` — subtopic 4F, Modelling with ratios of lengths, areas and volumes
Marks	**30** — Section A (4A): 5 marks · Section B (4B): 5 marks · Section C (4C): 5 marks · Section D (4D): 5 marks · Section E (4E): 5 marks · Section F (4F): 5 marks
Time	30 minutes working time, plus 5 minutes reading time (about 35 minutes in all)
Equipment	Ruler, protractor and pair of compasses (you may not need every item)
Calculators	A scientific calculator is not needed: none of the assessed content descriptions names a digital tool, and every number in this test is chosen to be worked by hand

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 7 achievement standard, not against a mark on this paper.

Answer every question. Show your working where a question asks for it. The marks for each part are shown in brackets.

Section A — Areas of rectangles, triangles and parallelograms (4A, `VC2M7M01`)

Question 1 [5 marks].

a. A triangle has base 12 cm and perpendicular height 8 cm. Find its area. **[1 mark]**

b. A parallelogram has base 11 cm, perpendicular height 6 cm and sloping side 8 cm. Find its area. **[1 mark]**

c. A basketball court is a rectangle. Under the rules used in Australia, it is 28 m long and 15 m wide. Find the area of the court. **[1 mark]**

Source: FIBA, Official Basketball Rules 2024, Article 2 — Playing court; retrieved 23 August 2026.

d. A school banner is shaped like a rectangle with a triangle on top. The rectangle is 4 m wide and 3 m high. The triangle has base 4 m and height 1.2 m. Find the total area of the banner. **[2 marks]**

Section B — Volume of right prisms (4B, `VC2M7M02`)

Question 2 [5 marks].

a. Which is the most sensible unit for the volume of a brick? **[1 mark]**

- **A** mm³
- **B** m³
- **C** cm³
- **D** m²

b. A box is 150 mm long, 100 mm wide and 60 mm high. Find its volume in cm³. **[2 marks]**

c. A prism has a triangular end. The triangle has base 16 cm and perpendicular height 10 cm, and the prism is 50 cm long. Find the volume of the prism. **[1 mark]**

d. Sam finds the volume of a box 20 cm long, 10 cm wide and 5 cm high by adding the three edges: 20 + 10 + 5 = 35 cm^3. Explain why adding the edges cannot give a volume, and find the correct volume. **[1 mark]**

Section C — π, circumference, radius and diameter (4C, `VC2M7M03`)

Use π ≈ 3.14 for every part of this section.

Question 3 [5 marks].

a. A circle has radius 3.5 cm. Find its diameter. **[1 mark]**

b. In Australian rules football, a centre circle of diameter 3 m is marked at the centre of the ground. Find the circumference of the centre circle, to one decimal place. **[1 mark]**

Source: AFL (Australian Football League), Australian Football Laws of the Game; retrieved 23 August 2026.

c. A string 47.1 cm long is wrapped once around the rim of a circular tin, with no overlap. Find the diameter of the tin. **[1 mark]**

d. A wheel has diameter 50 cm.

  (i) How far does the wheel travel in one full turn? Give your answer in cm. **[1 mark]**

  (ii) How far does the wheel travel in 20 turns? Give your answer in metres. **[1 mark]**

In Sections D and E, marks are awarded for stated reasons. A correct number with no reason does not score full marks.

Section D — Parallel lines and a transversal (4D, `VC2M7M04`)

Question 4 [5 marks]. Two parallel lines are crossed by a transversal. One of the eight angles made is 74°.

a. Find the size of the angle corresponding to the 74° angle. Give a reason. **[2 marks]**

b. Find the size of the co-interior angle to the 74° angle. Give a reason. **[2 marks]**

c. Two co-interior angles between two parallel lines are x° and (x + 34)°. Find x. **[1 mark]**

Section E — Angle sums of triangles and polygons (4E, `VC2M7M05`)

Question 5 [5 marks].

a. Two angles of a triangle are 53° and 58°. Find the third angle. Give a reason. **[2 marks]**

b. Three angles of a quadrilateral are 95°, 108° and 64°. Find the fourth angle. Give a reason. **[2 marks]**

c. Find the interior angle sum of an 8-sided polygon. **[1 mark]**

Section F — Modelling with ratios of lengths, areas and volumes (4F, `VC2M7M06`)

Question 6 [5 marks]. An art club is making green paint for a poster. The green is mixed at blue:yellow = 3:2.

a. The club needs 750 mL of green paint. Find the volume of blue paint and the volume of yellow paint. Show your working. **[2 marks]**

b. In one sentence, state what your answer to part (a) means for the art club. In one sentence, justify the representation you used in part (a). **[2 marks]**

c. The poster design is a rectangle 20 cm long and 12 cm wide. A copy is made at 2:1, so every length on the copy is ×2. State how many times bigger the area of the copy is than the area of the design, and give a reason. (You do not need to work out the two areas.) **[1 mark]**

End of test — 30 marks.

Solutions

Fully worked solutions for every question, then marking notes and a compact answer key for self-checking.

Worked solutions

Question 1.

a. A triangle is half of a rectangle with the same base and height, so $A = \frac{1}{2} b h$:

$$A = \frac{1}{2} \times 12 \times 8 = \frac{1}{2} \times 96 = 48\,\text{cm}^2$$

[1 mark]

b. A parallelogram uses the perpendicular height, not the sloping side. The 8 cm side is extra information and is not used:

$$A = b h = 11 \times 6 = 66\,\text{cm}^2$$

[1 mark]

c. The court is a rectangle, so $A = l w$:

$$A = 28 \times 15 = 420\,\text{m}^2$$

[1 mark]

d. The banner is a composite shape: the areas of the two parts add.

Rectangle: $4 \times 3 = 12$ m². Triangle: $\frac{1}{2} \times 4 \times 1.2 = 2 \times 1.2 = 2.4$ m².

(1 mark for the two part-areas.) Total:

$$12 + 2.4 = 14.4\,\text{m}^2$$

(1 mark for the total.) **[2 marks]**

Question 2.

a. A brick is measured in centimetres along each edge, so its volume is measured in cm³. mm³ is too small and m³ is too big; m² is a unit of area, not volume. The answer is **C**. **[1 mark]**

b. Convert to centimetres first, because the answer is asked in cm³: 150 mm $= 15$ cm, 100 mm $= 10$ cm, 60 mm $= 6$ cm (1 mark). Then

$$V = l \times w \times h = 15 \times 10 \times 6 = 900\,\text{cm}^3$$

(1 mark.) **[2 marks]**

c. The volume of any right prism is the area of its base times its height. Triangular end: $\frac{1}{2} \times 16 \times 10 = 80$ cm².

Then

$$V = 80 \times 50 = 4000\,\text{cm}^3$$

[1 mark]

d. Volume counts the unit cubes that fill the box; it is found by multiplying the three dimensions, not adding them. Adding the three edges gives a length, not a count of cubes. Correct working:

$$V = 20 \times 10 \times 5 = 1000\,\text{cm}^3$$

The mark needs both a valid explanation and the correct volume. **[1 mark]**

Question 3.

a. The diameter is twice the radius:

$$d = 2r = 2 \times 3.5 = 7\,\text{cm}$$

[1 mark]

b. The circumference is π times the diameter:

$$C = \pi d \approx 3.14 \times 3 = 9.42$$

To one decimal place, $C \approx 9.4$ m. (Check: the circumference is a little more than 3 diameters, and 9.4 is a little more than 9.) **[1 mark]**

c. The string is the circumference, so $C = 47.1$ cm. Working backwards, division undoes the multiplication by π :

$$d = C \div \pi \approx 47.1 \div 3.14 = 15\,\text{cm}$$

because $3.14 \times 15 = 47.1$ exactly. **[1 mark]**

d.

(i) One full turn lays one circumference along the ground:

$$C = \pi d \approx 3.14 \times 50 = 157\,\text{cm}$$

[1 mark]

(ii) Twenty turns travel 20 times as far:

$$157 \times 20 = 3140\,\text{cm} = 31.4\,\text{m}$$

[1 mark]

Question 4.

a. $74°$ (1 mark) — corresponding angles are equal (parallel lines) (1 mark).

b. $180° - 74° = 106°$ (1 mark) — co-interior angles add to $180°$ (parallel lines) (1 mark).

c. Co-interior angles between parallel lines add to $180°$, so the reason must appear in the working:

$$x + (x + 34) = 180$$

$$2x = 180 - 34 = 146$$

$$x = 73$$

Check: $73° + 107° = 180°$. ✓ **[1 mark]**

Question 5.

a. Statement: $53° + 58° + x = 180°$. Reason: the angles of a triangle add to $180°$ (1 mark for the reason). Conclusion:

$$x = 180° - 111° = 69°$$

(1 mark.) **[2 marks]**

b. Statement: $95° + 108° + 64° + x = 360°$. Reason: a quadrilateral splits into 2 triangles, so its angles add to $360°$ (1 mark for the reason). Conclusion:

$$x = 360° - 267° = 93°$$

(1 mark.) **[2 marks]**

c. An n-sided polygon splits from one vertex into $n-2$ triangles, so its interior angle sum is $180(n-2)$:

$$180 \times (8-2) = 180 \times 6 = 1080°$$

[1 mark]

Question 6.

a. The ratio 3:2 splits the paint into $3+2=5$ equal parts, and 750 mL is the whole. One part is $750 \div 5 = 150$ mL (1 mark for the representation and the one-part value). Then

$$\text{blue} = 3 \times 150 = 450\,\text{mL}, \text{yellow} = 2 \times 150 = 300\,\text{mL}$$

Check: $450 + 300 = 750$ mL. ✓ (1 mark for both volumes.) **[2 marks]**

b. Interpretation (1 mark): the club mixes 450 mL of blue paint with 300 mL of yellow paint to make the 750 mL of green the poster needs. Justification (1 mark): any sensible justification of the chosen representation — for example, a bar of 5 equal parts shows the two parts and the whole at once, so one division finds one part and no share is missed. (A part–whole reading also earns the mark: blue is $\frac{3}{5}$ of 750 mL and yellow is $\frac{2}{5}$ of 750 mL, because the fractions follow straight from the same bar.) **[2 marks]**

c. The area is $\times 4$ (1 mark). Reason: the area multiplies two lengths together, and each length is $\times 2$, so the area is $2 \times 2 = 4$ times as big. (Working the areas out also earns the mark: $20 \times 12 = 240$ cm², $40 \times 24 = 960$ cm², and $960 \div 240 = 4$.) **[1 mark]**

Marking notes

- Per §7, reasoning is marked separately from the answer in T4. Q4(a), Q4(b), Q5(a) and Q5(b) each carry 1 mark for the stated reason; a correct number with no reason does not score full marks. Reasons may be in the student's own words if they name the correct angle fact.
- Q6 marks the full modelling cycle of `VC2M7M06` (D8): a bare pair of numbers with no representation, interpretation or justification scores at most 2 of the 5 marks.
- Equivalent correct methods are accepted throughout (for example, halving a number before multiplying in Q1a and Q1d, or using $\frac{3}{5}$ of 750 in Q6a).

Answer key

Question	Answer
1a	48 cm²
1b	66 cm² (the 8 cm sloping side is not used)
1c	420 m²
1d	12 m² + 2.4 m² = 14.4 m²
2a	C — cm³
2b	15 cm × 10 cm × 6 cm = 900 cm³
2c	4000 cm³
2d	Volume is found by multiplying the three dimensions; 1000 cm³
3a	7 cm

Question	Answer
3b	9.4 m
3c	15 cm
3d	(i) 157 cm; (ii) 31.4 m
4a	$74°$ — corresponding angles are equal (parallel lines)
4b	$106°$ — co-interior angles add to $180°$ (parallel lines)
4c	$x = 73$
5a	$69°$ — angles in a triangle add to $180°$
5b	$93°$ — a quadrilateral's angles add to $360°$ (2 triangles)
5c	$1080°$
6a	blue 450 mL, yellow 300 mL
6b	Interpretation and justification in terms of the situation (see worked solution)
6c	$\times 4$ — each length is $\times 2$ and $2 \times 2 = 4$

Total: 30 marks — Section A (4A, VC2M7M01): 5 · Section B (4B, VC2M7M02): 5 · Section C (4C, VC2M7M03): 5 · Section D (4D, VC2M7M04): 5 · Section E (4E, VC2M7M05): 5 · Section F (4F, VC2M7M06): 5.