Mathematics Level 7

Chapter 3 — Algebra: expressions, formulas, equations and relationships

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Contents

Variables and everyday formulas; substituting to find an unknown

Content description `VC2M7A01`: recognise and use variables to represent everyday formulas algebraically and substitute values into formulas to determine an unknown

Achievement standard (extract): Students use algebraic expressions to represent situations, describe the relationships between variables from authentic data and substitute values into formulas to determine unknown values.

Theory

A variable is a letter standing for a number that can vary. In the AFL scoring formula $p = 6g + b$, the letters g and b are variables: the number of goals and the number of behinds change from match to match, so the total points p change too. The 6 is not a variable — every goal is worth 6 points, in every match, always. A **formula** is a rule that links variables, and writing the rule with letters is what the curriculum calls representing a situation algebraically. Figure 1 names each part of the AFL formula.

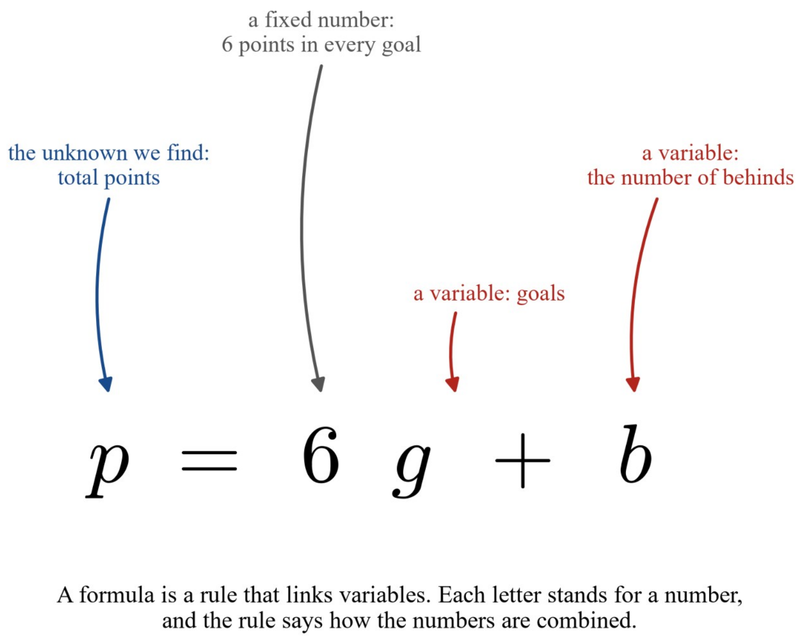

Figure 1. The formula p = 6g + b written large, with arrows labelling p as the unknown we find (total points), 6 as a fixed number (6 points in every goal), g as a variable (goals) and b as a variable (the number of behinds).

Substituting means replacing each letter with its number, then calculating. The order of operations still applies: multiply and divide before you add and subtract. Every substitution in this section follows the same five steps, shown in Figure 2 for the score 12 goals and 9 behinds.

1 Write the formula $p = 6g + b$

2 Say what each letter stands for $g = 12$ goals, $b = 9$ behinds

3 Substitute the numbers $p = 6 \times 12 + 9$

4 Calculate (order of operations) $p = 72 + 9 = 81$

5 Write the answer with its unit 81 points

Figure 2. The five-step substitution routine in boxes joined by arrows: 1 write the formula, p = 6g + b; 2 say what each letter stands for, g = 12 goals and b = 9 behinds; 3 substitute the numbers, p = 6 × 12 + 9; 4 calculate using the order of operations, p = 72 + 9 = 81; 5 write the answer with its unit, 81 points.

Formulas you will meet in this section. Every formula here is a real one, taken from the curriculum or from a published source; none are invented.

Formula	What it gives	Source
$A = l \times w$	area of a rectangle (cm^2) from length l and width w in cm	VCAA, VC2M7A01 elaboration E01, retrieved 20 August 2026
$p = 6g + b$	AFL score in points from goals g and behinds b	VCAA, VC2M7A01 elaboration E01, retrieved 20 August 2026
$P = 2l + 2w$	perimeter of a rectangle (cm)	the perimeter rule from primary school
$V = l \times w \times h$	volume of a rectangular prism (cm^3) from length, width and height in cm	the volume rule from primary school
$d = m \div V$	density (g/cm^3) from mass m in g and volume V in cm^3	VCAA, VC2M7A01 elaboration E03, retrieved 20 August 2026
$W = b + 1.5 \times h \times r$	weekly wage in dollars from base wage b, overtime hours h at 1.5 times the hourly rate r	VCAA, VC2M7A01 elaboration E03, retrieved 20 August 2026
$\text{MHR} = 220 - \text{age}$	maximum heart rate in beats per minute from age in years	"Maximum heart rate", Wikipedia (English), retrieved 20 August 2026; the rule is credited there to Haskell and Fox (2007)

Units travel with the numbers. If l and w are in cm, the area A comes out in cm^2 and the perimeter P in cm (Figure 3); a volume built from cm comes out in cm^3 (Figure 4). Write the unit on the answer line every time.

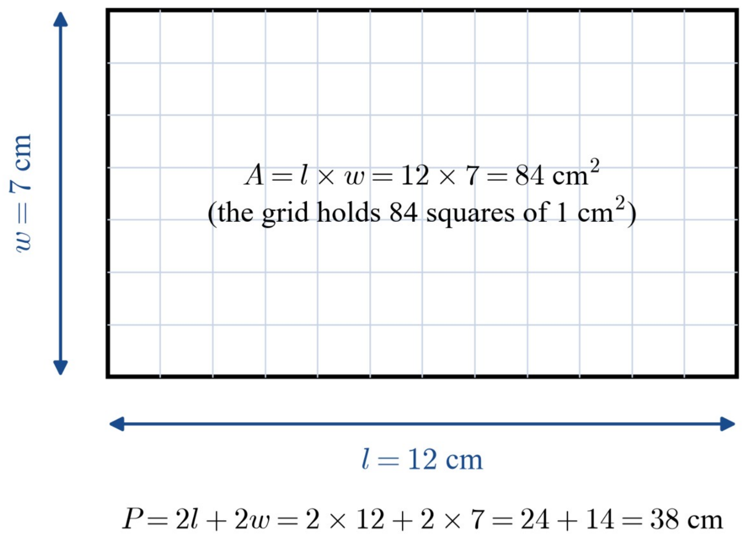

Figure 3. A 12 cm by 7 cm rectangle drawn on a square grid, with the working A = 12 × 7 = 84 cm² and P = 2 × 12 + 2 × 7 = 38 cm written beneath it.

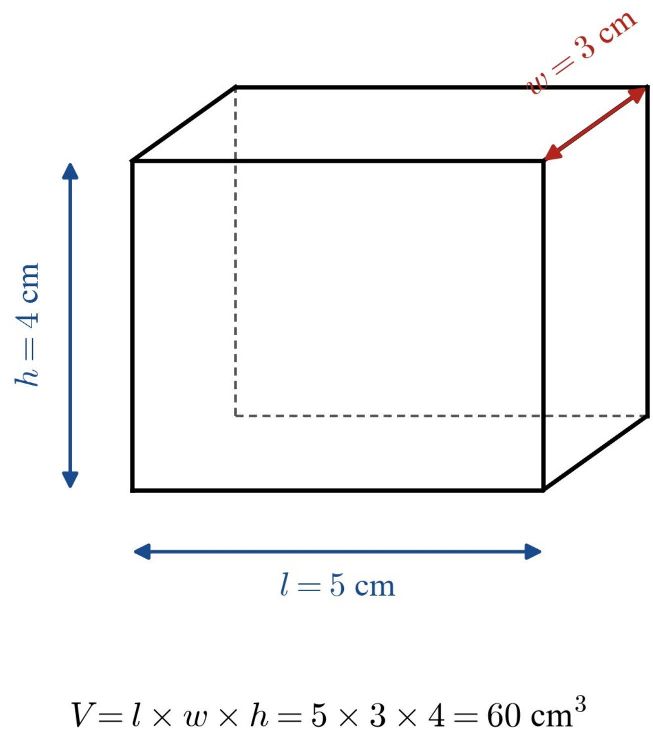

Figure 4. A rectangular prism 5 cm long, 3 cm wide and 4 cm high, with hidden edges dashed, and the working V = l × w × h = 5 × 3 × 4 = 60 cm³ beneath it.

Density compares mass with volume. A block of a substance denser than water (more than 1.0 g/cm³) sinks in water; a less dense block floats. Heart rate works the same way as a formula from another source: $220 - \text{age}$ estimates the fastest rate (in beats per minute) a person should reach, and moderate exercise aims at a target zone of 65% to 85% of that maximum (percentages are Topic 2E).

What substitution can and cannot do. Substitution finds the one missing output when every input is known — that is all this section does. If the output is known and an input is missing (say the score and the behinds are known and the goals are wanted), that is solving an equation, which is 3C. Rearranging a formula itself is Level 8 work (VC2M8A01). Formulas that link speed, distance and time are kept for 3F.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
variable	a letter standing for a number that can vary	pronumeral
formula	a rule that links variables	rule
substitute	replace a letter with its number	put in, replace
unknown	the value we are finding	missing value
fixed number	a number that never changes within the formula, like the 6 in $p=6g+b$	—
perimeter	the distance all the way around a shape	boundary
density	the mass in each cm³ of a substance, $d=m\div V$	how packed a substance is
maximum heart rate (MHR)	the fastest rate, in beats per minute, a person should reach	—
target heart rate	the zone aimed at during moderate exercise	training zone
base wage	pay for ordinary hours, before overtime	—
overtime	hours beyond the ordinary week, paid at a higher rate	—

Worked examples

Example 1 — scaffolded

A team kicked 12 goals and 9 behinds in a match. Use $p=6g+b$ to find the team's score.

Source: the scoring rule (a goal worth 6 points, a behind worth 1 point) is VCAA's, from VC2M7A01 elaboration E01, retrieved 20 August 2026.

Working	Comment
$p=6g+b$	Write the formula.
$g=12$ goals, $b=9$ behinds.	Say what each letter stands for.
$p=6\times12+9$	Substitute the numbers.
$p=72+9=81$	Multiply before adding (order of operations).

∴ the team scored 81 points (Figure 5).

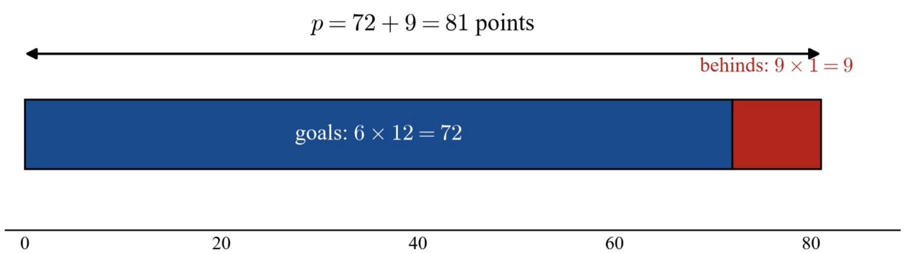

Figure 5. A stacked bar showing 72 points from 12 goals in blue and 9 points from 9 behinds in red, totalling 81 points.

Example 2 — scaffolded

A student is 13 years old. Use $\text{MHR} = 220 - \text{age}$ to find the student's maximum heart rate, and the target zone for moderate exercise (65% to 85% of the maximum), to the nearest whole beat.

Source: "Maximum heart rate", Wikipedia (English), retrieved 20 August 2026; the 220 − age rule is credited there to Haskell and Fox (2007), and 65–85% is the aerobic target given there. The rule is an estimate for a whole population, not medical advice.

Working	Comment
$\text{MHR} = 220 - 13 = 207$ beats per minute.	Substitute the age.
Lower end: $207 \times 0.65 = 134.55 \approx 135$ beats per minute.	65% of the maximum (Topic 2E).
Upper end: $207 \times 0.85 = 175.95 \approx 176$ beats per minute.	85% of the maximum.

∴ the maximum heart rate is 207 beats per minute, and the target zone is about 135–176 beats per minute (Figure 6).

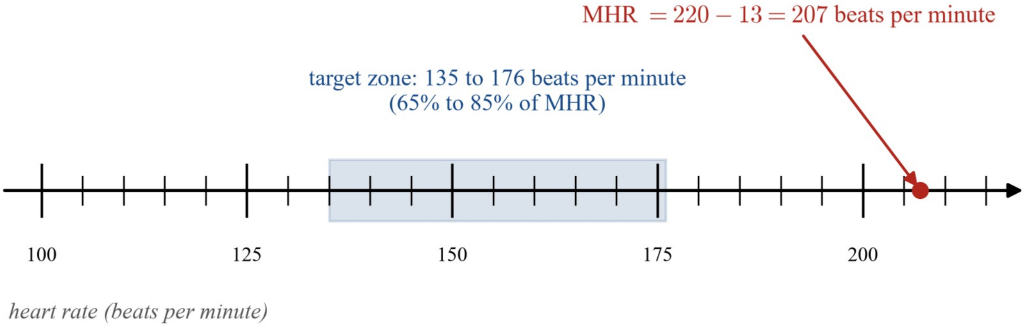

Figure 6. A number line from 100 to 215 beats per minute with the target zone 135–176 beats per minute shaded and a red dot at 207 beats per minute labelled MHR = 220 − 13 = 207 beats per minute.

Example 3 — unscaffolded

A block of a substance has mass $m = 54$ g and volume $V = 20$ cm³. Use $d = m \div V$ to find its density, and say whether the block sinks or floats in water (water is 1.0 g/cm³).

$d = 54 \div 20 = 2.7$ g/cm³. Because $2.7 > 1.0$, the block is denser than water.

∴ the density is 2.7 g/cm³ and the block sinks (Figure 7).

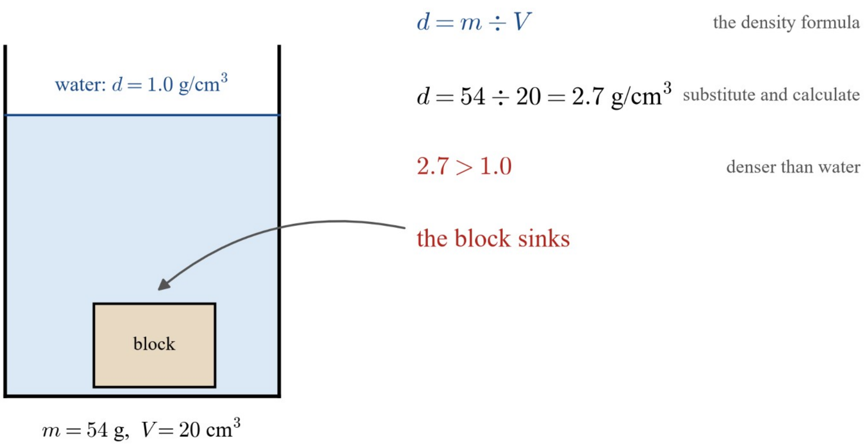

Figure 7. A beaker of water with a block sunk to the bottom, beside the working d = m ÷ V = 54 ÷ 20 = 2.7 g/cm³ and the comparison 2.7 > 1.0, so the block sinks.

Example 4 — unscaffolded

A worker's week (practice figures, not a real award): base wage $b = \$934.80$, and 4 overtime hours at 1.5 times an hourly rate of $r = \$24.60$. Use $W = b + 1.5 \times h \times r$ to find the weekly wage W.

The formula is VCAA's (VC2M7A01 elaboration E03, retrieved 20 August 2026); the dollar values are practice figures. Current real rates are published by Fair Work.

$W = 934.80 + 1.5 \times 4 \times 24.60 = 934.80 + 147.60 = 1082.40$.

∴ the weekly wage is \$1082.40 — the 4 overtime hours add \$147.60 to the base wage (Figure 8).

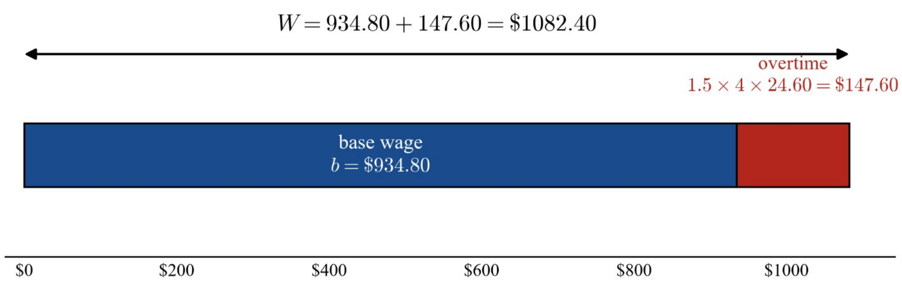

Figure 8. A stacked bar showing the base wage \$934.80 in blue and the overtime pay \$147.60 in red, totalling \$1082.40.

Practice questions

Scaffolded

Q1. For each formula, list the variables and the unit the answer is measured in. (a) $A = l \times w$, with l and w in cm. (b) $p = 6g + b$. (c) $d = m \div V$, with m in g and V in cm³. (d) $V = l \times w \times h$, with lengths in cm.

Q2. Use $p = 6g + b$ to find the score when: (a) $g = 4$, $b = 7$. (b) $g = 10$, $b = 3$. (c) $g = 7$, $b = 7$. (d) $g = 15$, $b = 11$.

Q3. Use $A = l \times w$ and $P = 2l + 2w$ to find the area and the perimeter of rectangles with: (a) $l = 9$ cm, $w = 4$ cm. (b) $l = 12$ cm, $w = 7$ cm. (c) $l = 15$ cm, $w = 6$ cm. (d) $l = 8$ cm, $w = 8$ cm.

Q4. Use $V = l \times w \times h$ to find the volume of rectangular prisms with: (a) $l = 5$ cm, $w = 3$ cm, $h = 4$ cm. (b) $l = 10$ cm, $w = 4$ cm, $h = 3$ cm. (c) $l = 7$ cm, $w = 5$ cm, $h = 4$ cm.

Q5. Use $d = m \div V$ to find each density, and say whether the block sinks or floats in water (1.0 g/cm³). (a) $m = 78$ g, $V = 10$ cm³. (b) $m = 8$ g, $V = 10$ cm³. (c) $m = 12$ g, $V = 12$ cm³. (d) $m = 36$ g, $V = 20$ cm³.

Q6. Use $\mathrm{MHR} = 220 - \mathrm{age}$. (a) Find the maximum heart rate for people aged 11, 15, 30 and 50. (b) Find the target zone (65% to 85% of the maximum) for a 20-year-old.

Unscaffolded

Q7. A worker's week (practice figures): base wage $b = \$817.00$ and 6 overtime hours at 1.5 times an hourly rate of $r = \$21.50$. Use $W = b + 1.5 \times h \times r$ to find W.

Q8. Say whether each statement is true or false. If false, correct it. (a) In $p = 6g + b$, the 6 is a variable. (b) Substituting $g = 3$ and $b = 2$ into $p = 6g + b$ gives $p = 20$. (c) A rectangle with $l = 6$ cm and $w = 4$ cm has area $A = 10$ cm². (d) In $d = m \div V$, doubling m while V stays the same doubles d.

Q9. A farmer fences a rectangular paddock 12 m long and 8 m wide, leaving a 3 m gate unfenced. How many metres of fencing are needed?

Q10. Team A kicked 9 goals and 8 behinds; Team B kicked 10 goals and 1 behind. Which team won, and by how many points?

Q11. Mia substituted $g=5$ and $b=3$ into $p=6g+b$ and wrote $p=5+6\times3=23$. Explain her mistake and find the correct score.

Q12. The tank in Figure 9 is a rectangular prism. Find its volume.

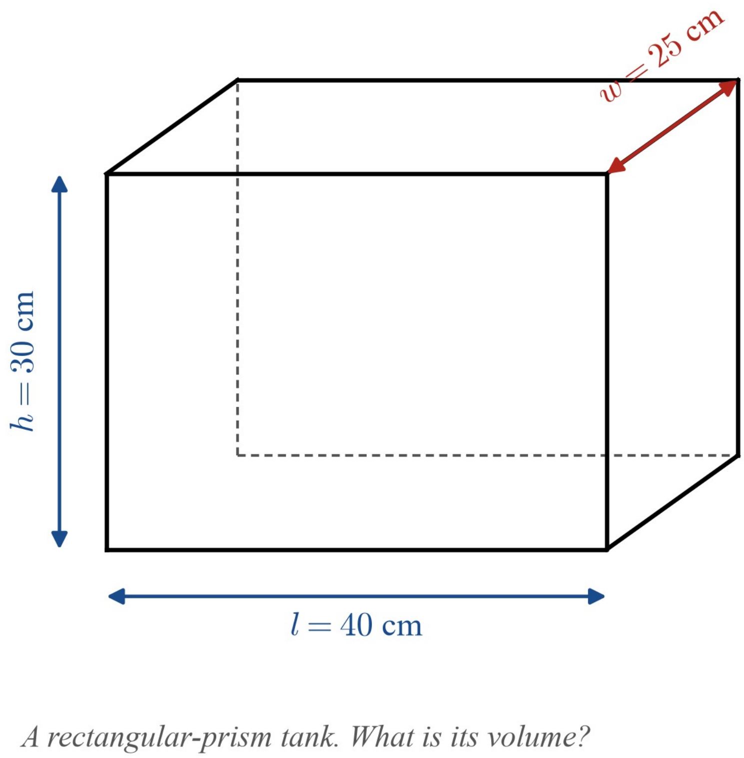

Figure 9. A rectangular-prism tank labelled l = 40 cm along the front, w = 25 cm along the depth and h = 30 cm for the height, asking what its volume is.

Q13. Use $\text{MHR}=220-\text{age}$. How much higher is the maximum heart rate of a 12-year-old than that of a 40-year-old?

Q14. A worker's base wage is $b=\$817.00$ and overtime is paid at 1.5 times an hourly rate of $r=\$21.50$ (practice figures). Use $W=b+1.5\times h\times r$ to find W when: (a) $h=2$. (b) $h=8$. (c) How much more does the 8-overtime-hour week pay than the 2-overtime-hour week?

Q15. Figure 10 shows two samples with their masses and volumes. Which sample has the greater density?

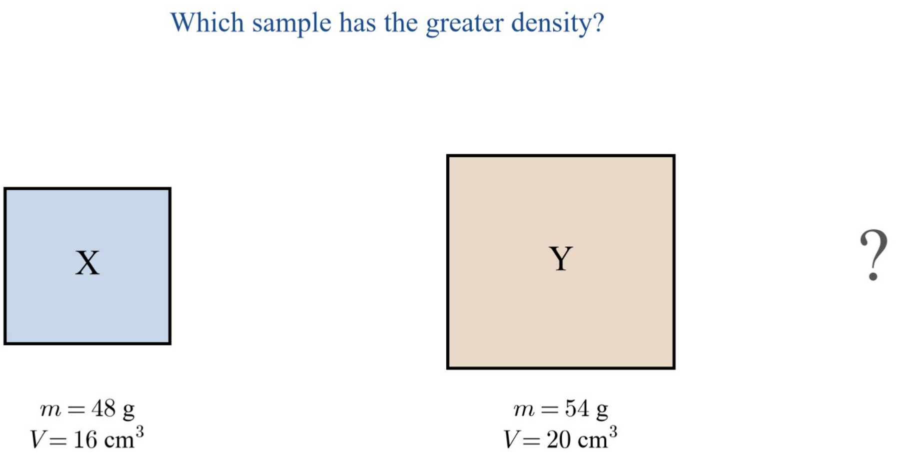

Figure 10. Two blocks, X with m = 48 g and V = 16 cm³ and Y with m = 54 g and V = 20 cm³, asking which sample has the greater density.

Q16. A pizza shop's pricing rule is $C=12+3n$, where n is the number of pizzas and C is the total cost in dollars. (a) Find the cost of 4 pizzas. (b) Find the cost of 7 pizzas. (c) Explain in words what the 12 and the 3 mean in the shop's pricing.

Q17. Dana watched a match and wrote down that a team scored 115 points from 9 goals and 7 behinds. Without calculating exactly at first, is that reasonable? Check with $p=6g+b$ and explain.

Answers

Q1. (a) l, w; cm². (b) g, b; points. (c) m, V; g/cm³. (d) l, w, h; cm³.

Q2. (a) 31 points. (b) 63 points. (c) 49 points. (d) 101 points.

Q3. (a) $A=36$ cm², $P=26$ cm. (b) $A=84$ cm², $P=38$ cm. (c) $A=90$ cm², $P=42$ cm. (d) $A=64$ cm², $P=32$ cm.

Q4. (a) 60 cm³. (b) 120 cm³. (c) 140 cm³.

Q5. (a) 7.8 g/cm³, sinks. (b) 0.8 g/cm³, floats. (c) 1.0 g/cm³, neither — it hovers, as dense as water. (d) 1.8 g/cm³, sinks.

Q6. (a) 209, 205, 190 and 170 beats per minute. (b) 130–170 beats per minute.

Q7. $1010.50.

Q8. (a) False — the 6 is a fixed number; g and b are the variables. (b) True ($6\times3+2=20$). (c) False — $A=6\times4=24$ cm² (10 is $l+w$, not $l\times w$). (d) True.

Q9. 37 m.

Q10. Team A, by 1 point (62 to 61).

Q11. Mia swapped the letters: she computed $g+6b$ instead of $6g+b$. Correctly, $p=6\times5+3=33$ points.

Q12. 30 000 cm³ (which is 30 L).

Q13. 28 beats per minute.

Q14. (a) $881.50. (b) $1075.00. (c) $193.50.

Q15. Sample X (3.0 g/cm³ against 2.7 g/cm³).

Q16. (a) $24. (b) $33. (c) The 12 is a fixed charge (the delivery fee) paid even for no pizzas, and the 3 is the price added per pizza.

Q17. No — 9 goals alone give only 54 points and 10 goals would give 60, so 115 is far too big. The check: $p=6\times9+7=61$ points.

Fully-worked solutions

Q1. The variables are the letters that can vary; the unit follows from what the formula measures. (a) l and w; area is in cm². (b) g and b; the score is in points. (c) m and V; density is in g/cm³. (d) l, w and h; volume is in cm³. (The 6 in (b) is fixed, not a variable.)

Q2. (a) $p=6\times4+7=24+7=31$ points. (b) $p=6\times10+3=60+3=63$ points. (c) $p=6\times7+7=42+7=49$ points. (d) $p=6\times15+11=90+11=101$ points. Multiply before adding each time.

Q3. (a) $A=9\times4=36$ cm²; $P=2\times9+2\times4=18+8=26$ cm. (b) $A=12\times7=84$ cm²; $P=2\times12+2\times7=24+14=38$ cm. (c) $A=15\times6=90$ cm²; $P=2\times15+2\times6=30+12=42$ cm. (d) $A=8\times8=64$ cm²; $P=2\times8+2\times8=32$ cm.

Q4. (a) $V=5\times3\times4=60$ cm³. (b) $V=10\times4\times3=120$ cm³. (c) $V=7\times5\times4=140$ cm³.

Q5. (a) $d=78\div10=7.8$ g/cm³; $7.8>1.0$, so it sinks. (b) $d=8\div10=0.8$ g/cm³; $0.8<1.0$, so it floats. (c) $d=12\div12=1.0$ g/cm³; equal to water, so it neither sinks nor floats. (d) $d=36\div20=1.8$ g/cm³; it sinks.

Q6. (a) Age 11: $220-11=209$ beats per minute; age 15: $220-15=205$ beats per minute; age 30: $220-30=190$ beats per minute; age 50: $220-50=170$ beats per minute. (b) MHR $=220-20=200$ beats per minute; $200\times0.65=130$ beats per minute and $200\times0.85=170$ beats per minute, so the zone is 130–170 beats per minute.

Q7. $W = 817.00 + 1.5 \times 6 \times 21.50 = 817.00 + 9 \times 21.50 = 817.00 + 193.50 = 1010.50$, so $W = \$1010.50$.

Q8. (a) False: the 6 never changes, so it is a fixed number; only g and b vary. (b) True: $6 \times 3 + 2 = 18 + 2 = 20$. (c) False: $A = l \times w = 6 \times 4 = 24$ cm²; 10 cm² would come from adding $l + w$ instead of multiplying. (d) True: $d = m \div V$, so with V fixed, twice the mass gives twice the density.

Q9. The whole perimeter is $P = 2 \times 12 + 2 \times 8 = 24 + 16 = 40$ m; the 3 m gate needs no fencing, so $40 - 3 = 37$ m of fencing.

Q10. Team A: $p = 6 \times 9 + 8 = 54 + 8 = 62$ points. Team B: $p = 6 \times 10 + 1 = 61$ points. Team A won by $62 - 61 = 1$ point — more goals does not always mean more points.

Q11. The formula multiplies the goals by 6: $6g$ means $6 \times g$. Mia used the 6 with the behinds instead, computing $5 + 6 \times 3 = 23$. Correctly, $p = 6 \times 5 + 3 = 30 + 3 = 33$ points.

Q12. $V = l \times w \times h = 40 \times 25 \times 30$. Now $40 \times 25 = 1000$ and $1000 \times 30 = 30\,000$, so $V = 30\,000$ cm³. (Since 1000 cm³ = 1 L, the tank holds 30 L.)

Q13. Age 12: $220 - 12 = 208$ beats per minute; age 40: $220 - 40 = 180$ beats per minute; the difference is $208 - 180 = 28$ beats per minute.

Q14. (a) $W = 817.00 + 1.5 \times 2 \times 21.50 = 817.00 + 64.50 = 881.50$. (b) $W = 817.00 + 1.5 \times 8 \times 21.50 = 817.00 + 258.00 = 1075.00$. (c) $1075.00 - 881.50 = 193.50$. Each extra overtime hour adds $1.5 \times 21.50 = 32.25$, and 6 extra hours give $6 \times 32.25 = 193.50$ — the same difference.

Q15. X: $d = 48 \div 16 = 3.0$ g/cm³. Y: $d = 54 \div 20 = 2.7$ g/cm³. X has the greater density, even though Y has more mass — density compares mass with volume, so the smaller, denser block wins.

Q16. (a) $C = 12 + 3 \times 4 = 12 + 12 = 24$, so \$24. (b) $C = 12 + 3 \times 7 = 12 + 21 = 33$, so \$33. (c) The 12 is charged no matter how many pizzas are ordered — it is the fixed delivery fee — and each pizza adds \$3, so 3 is the price per pizza.

Q17. A reasonableness check first: 9 goals at 6 points each give 54 points, and even 10 goals would give only 60, so a score of 115 cannot be right. The exact check confirms it: $p = 6 \times 9 + 7 = 54 + 7 = 61$ points, not 115. (Dana's 115 is exactly $12 \times 9 + 7$ — the 6 points per goal was doubled.)

Teacher notes

- **Level boundary (P5).** This subtopic teaches substitution only. Questions where an input must be found from a known output (solving an equation) belong to 3C, and transposing formulas is Level 8 (`VC2M8A01`); the boundary is named for students in the theory. Speed, distance and time formulas are reserved for 3F (D4.8). Implied multiplication ($6g = 6 \times g$) is used and named here but formalised in 3B.
- **Digital tools (D9).** `VC2M7A01` names no digital tools, so every calculation in this section is by hand; no CAS or spreadsheet appears.
- **First Nations exclusion (CONTENT_POLICY.md Rule 1, recorded 2026-08-20).** `VC2M7A01.E04` invites applying everyday formulas to contexts on Country/Place. Aboriginal and Torres Strait Islander content is excluded from this book as accuracy cannot be guaranteed; this subtopic therefore teaches the content description on other contexts, and the elaboration is recorded as an accepted exclusion rather than a gap. The front matter carries the book-wide notice.
- **Sourcing (§6).** The AFL scoring rule and the wage and density formulas are VCAA's own (VC2M7A01 elaborations E01 and E03, retrieved 20 August 2026); $A = l \times w$ is E01's. The $220 - \text{age}$ maximum-heart-rate rule and the 65–85% moderate-exercise target come from "Maximum heart rate", Wikipedia (English), retrieved 20 August 2026, which credits the rule to Haskell and Fox (2007); the AIS and CDC pages were unreachable at authoring, and the rule should be taught as a population estimate, not medical advice. Wage dollar values are transparently fictional practice figures (the Fair Work Ombudsman site was unreachable at authoring); direct students to Fair Work for real current rates. Perimeter and prism-volume rules are recalled from primary measurement, not sourced as new facts.

- **Assessment.** 3A is assessed in chapter test T3 (5 marks). The reasonableness item (Q17) mirrors the achievement standard's habit of checking substituted values against the situation.
- **Cross-curriculum and cross-references.** Density $d = m \div V$ is the same relationship students meet in science at this level; the heart-rate work draws on percentages from 2E, and Q6(b) and Example 2 can supply the zone values if 2E has not yet been taught.

The associative, commutative and distributive laws; writing algebraic expressions

Content description `VC2M7A02`: apply the associative, commutative and distributive laws to aid mental and written computation, and formulate algebraic expressions using constants, variables, operations and brackets

Achievement standard (extract): Students use algebraic expressions to represent situations, describe the relationships between variables from authentic data and substitute values into formulas to determine unknown values.

Theory

What this subtopic is about. Subtopic 3A brought letters into the maths: a **variable** is a letter standing for a number, and substituting a value for it turns an expression into a number. This subtopic steps back and looks at the rules the numbers were already following. Arithmetic obeys three laws — the **commutative**, **associative** and **distributive** laws — and naming them turns the shortcuts you already use into tools you can choose on purpose. The same laws then do a second job: they explain the writing rules of algebra, from why $x+x+x$ is written $3x$, to the difference between $3x+4$ and $3(x+4)$.

The commutative law. Adding two numbers gives the same answer whichever order they go in: $a+b=b+a$. Multiplying works the same way: $a\times b=b\times a$. Figure 1 shows both on dots: $4+3$ and $3+4$ count the same 7 dots, and an array of 3 rows of 5 holds the same 15 dots as 5 rows of 3 turned on its side. The law says you may **swap the order**, and a swap is worth making when the new order is easier — like starting $37+63$ because it makes 100.

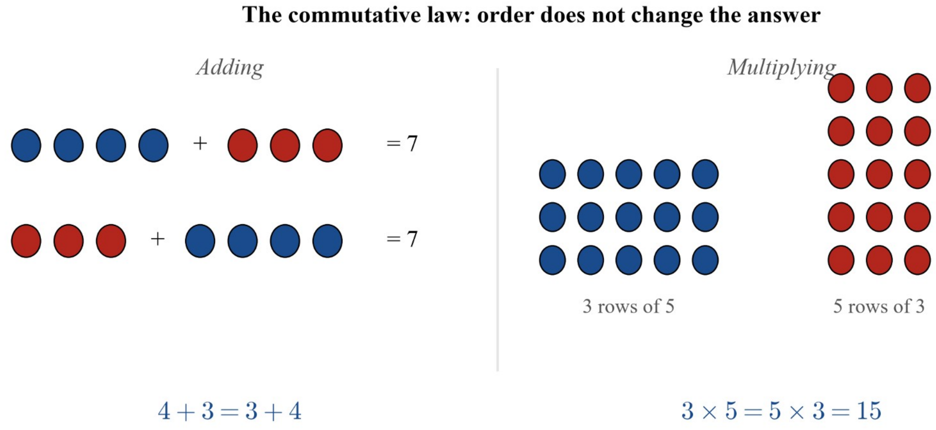

Figure 1. Two rows of dots showing 4 plus 3 equals 7 and 3 plus 4 equals 7; below, two arrays of dots, 3 rows of 5 and 5 rows of 3, both holding 15 dots, with the line 3 times 5 equals 5 times 3 equals 15.

Subtraction and division are **not** commutative (Figure 2). $7-3=4$, but $3-7$ is not 4 — you cannot take 7 counters from 3. And $12\div4=3$, but $4\div12$ is not 3. For subtraction and division, the order is part of the meaning, so the order stays as the question wrote it.

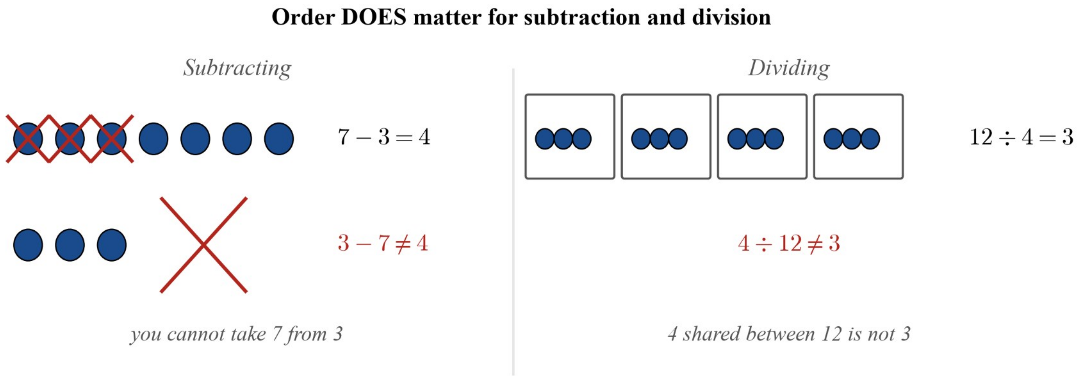

Figure 2. Four panels: 7 minus 3 equals 4 shown with 7 dots where 3 are crossed out; 3 minus 7 marked as not equal to 4 because you cannot take 7 from 3; 12 divided by 4 equals 3 shown as 12 dots in 3 equal groups of 4; and 4 divided by 12 marked as not equal to 3.

The associative law. When three numbers are added, the brackets — the choice of which pair to do first — do not change the answer: $(a+b)+c=a+(b+c)$. Figure 3 shows the same nine counters grouped two ways: $(2+3)+4=5+4=9$ and $2+(3+4)=2+7=9$. Multiplying is associative too: $(2\times3)\times4=6\times4=24$ and $2\times(3\times4)=2\times12=24$. Subtracting is not: $(10-6)-2=2$ but $10-(6-2)=6$, so in a subtraction the brackets are not free to move.

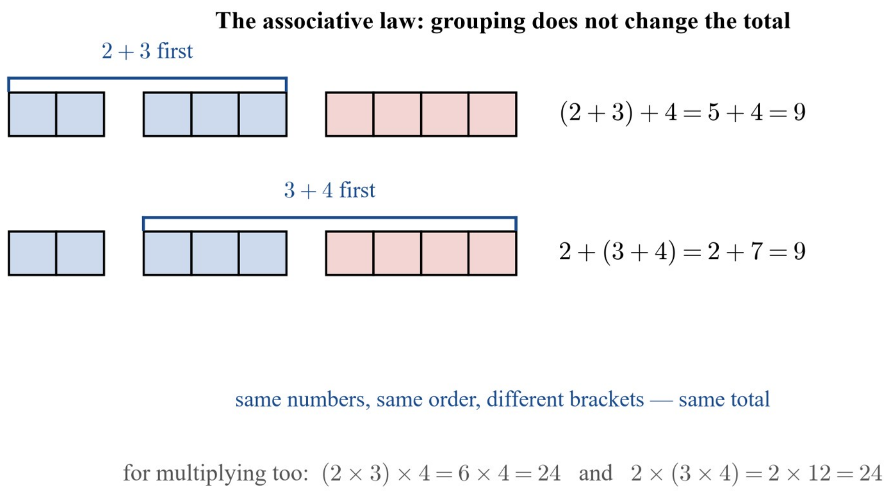

Figure 3. The same nine counters grouped two ways: (2 plus 3) plus 4 gives 5 plus 4 equals 9, and 2 plus (3 plus 4) gives 2 plus 7 equals 9; below, the matching multiplication lines (2 times 3) times 4 equals 6 times 4 equals 24 and 2 times (3 times 4) equals 2 times 12 equals 24.

Using the two laws together. They are at their best in pairs. To add $64+37+36+63$ mentally, the commutative law lets you swap the order of the numbers and the associative law lets you group them into pairs: $(64+36)+(37+63)=100+100=200$. To find $2\times13\times5$, group the friendly pair first: $(2\times5)\times13=10\times13=130$. Subtopic 2D already put these same moves to work on fractions and decimals; here the moves get their names, and next they get written with letters.

The distributive law. Multiplying a number by a sum gives the same answer as multiplying it by each part of the sum and adding the results: $a\times(b+c)=a\times b+a\times c$. Figure 4 draws this as an area. A rectangle 6 high and 27 wide is split into a piece of width 20 and a piece of width 7, so $6\times27=6\times(20+7)=6\times20+6\times7=120+42=162$. The law works in both directions, and it works over subtraction too:

$8 \times 47 = 8 \times (50 - 3) = 8 \times 50 - 8 \times 3 = 400 - 24 = 376$. It is the strongest of the three laws for mental work, because it turns any hard number into a round number plus a small correction.

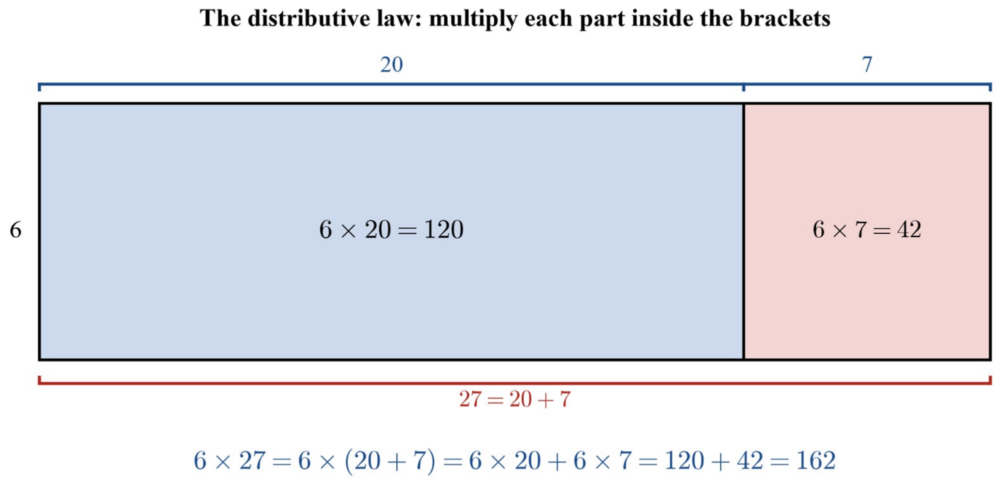

Figure 4. A rectangle 6 high and 27 wide, split into a part 20 wide labelled 6 times 20 equals 120 and a part 7 wide labelled 6 times 7 equals 42, with the line 6 times 27 equals 6 times (20 plus 7) equals 6 times 20 plus 6 times 7 equals 120 plus 42 equals 162.

Checking with an estimate. Subtopic 2B's rounding habit checks every answer: round first, compute, then compare. 6×38 should sit near $6 \times 40 = 240$ (Figure 5). The exact answer, 228, is close to 240, so it is reasonable; an answer like 2280 or 28 would fail the check before anyone looks at the last digit.

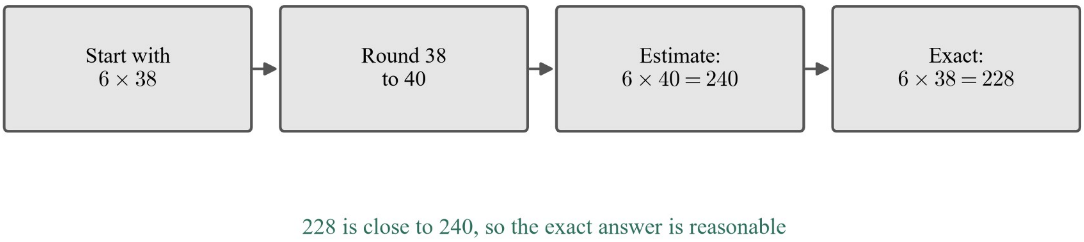

Figure 5. A row of boxes: 6 times 38, then round 38 to 40, then estimate 6 times 40 equals 240, then exact 6 times 38 equals 228, with the note that 228 is close to 240, so the exact answer is reasonable.

From arithmetic to algebra. Repeated addition is multiplication, and letters obey the same rule. Just as $7 + 7 + 7 = 3 \times 7$, three lots of x is $x + x + x = 3 \times x$ (Figure 6). Algebra then drops the × sign: $3 \times x$ is written $3x$. That is **implied multiplication** — the times sign is understood, not written. Figure 7 collects the writing rules: the number goes first ($x \times 7$ is written $7x$), a factor of 1 disappears ($1 \times m$ is just m), and repeated addition becomes multiplication ($y + y$ is $2y$).

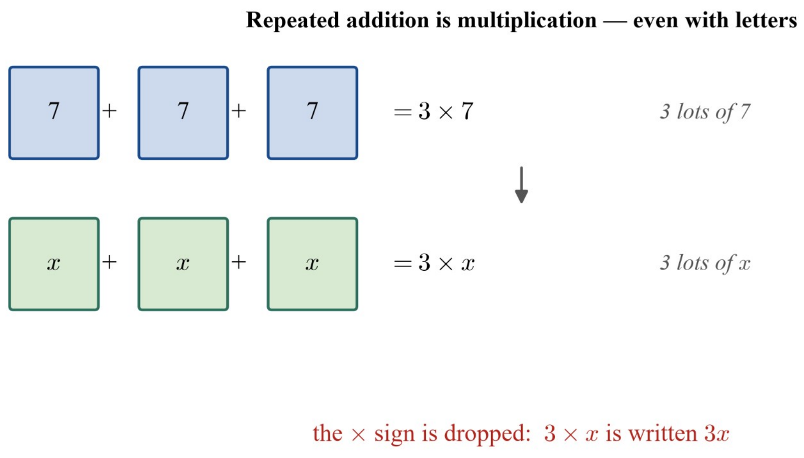

Figure 6. Two rows of tiles: the top row shows 7 plus 7 plus 7 as three tiles of 7, equal to 3 times 7; the bottom row shows x plus x plus x as three tiles of x, equal to 3 times x, with the note that the times sign is dropped and 3 times x is written 3x.

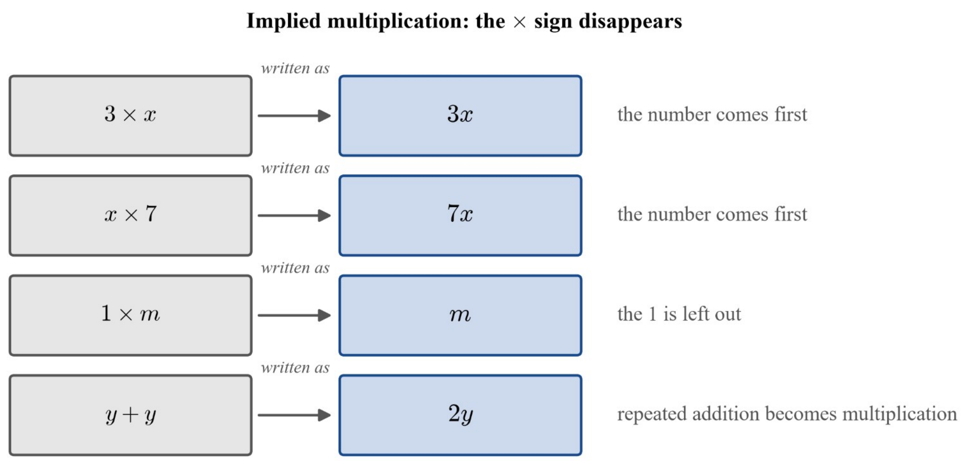

Figure 7. Four lines of implied-multiplication rules: 3 times x is written 3x with the number first; x times 7 is written 7x; 1 times m is written m because the 1 disappears; and y plus y is written 2y.

In an expression like $3x+4$, the numbers 3 and 4 are **constants** — they never change — while x is the **variable**, the letter that can stand for many different numbers. The pieces joined by + and − signs, here $3x$ and 4, are the **terms** of the expression.

Brackets change everything. The expressions $3x+4$ and $3(x+4)$ look almost the same, but they are not the same. Figure 8 sends the same input through two machines: one multiplies by 3 first and then adds 4; the other adds 4 first and then multiplies by 3. Starting with $x=2$, the first machine ends at 10 and the second ends at 18.

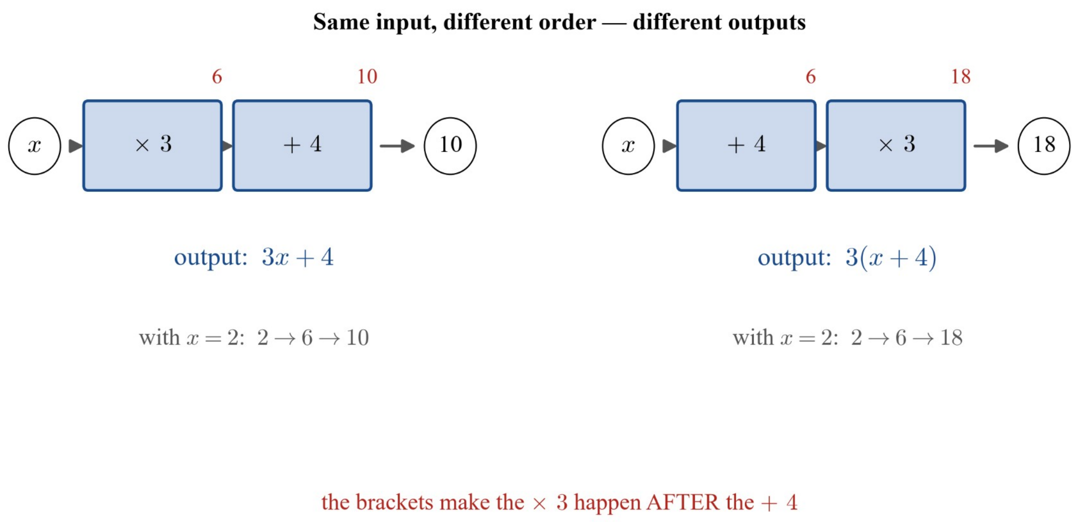

Figure 8. Two machines: machine A has a times-3 step then a plus-4 step and outputs 3x plus 4, with the trace x equals 2 going 2 to 6 to 10; machine B has a plus-4 step then a times-3 step and outputs 3(x plus 4), with the trace 2 to 6 to 18; the note says the brackets make the times 3 happen after the plus 4.

In $3x+4$, only the x is multiplied by 3. In $3(x+4)$, the 3 multiplies *everything inside the brackets* — three lots of x **and** three lots of 4. That is exactly what the distributive law says: $3(x+4)=3x+12$, which is $12-4=8$ more than $3x+4$. Figure 9 checks the two expressions for $x=1,2,3,4,5$: the second is 8 more than the first every time, so the two are never equal. When brackets appear, the order of operations says the bracket is done first — the machines show why.

Comparing $3x+4$ with $3(x+4)$

x	$3x+4$	$3(x+4)$
1	7	15
2	10	18
3	13	21
4	16	24
5	19	27

$3(x+4)$ is 8 more every time

the difference is always 8 — the two expressions are never equal

Figure 9. A table of values for x from 1 to 5: 3x plus 4 gives 7, 10, 13, 16, 19 and 3(x plus 4) gives 15, 18, 21, 24, 27, with a bracket down the right noting that 3(x plus 4) is 8 more than 3x plus 4 every time.

Writing expressions from words. Translating a sentence into symbols is the reverse of reading one (Figure 10). The order of the words matters: "subtract 4 from n" is $n-4$, not $4-n$. Division is written with a fraction bar: "divide n by 4" is $\frac{n}{4}$. And where the sentence says "then", brackets keep the earlier step together: "add 4 to n, then multiply the result by 3" is $3(n+4)$ — the brackets hold the adding until the multiplying happens.

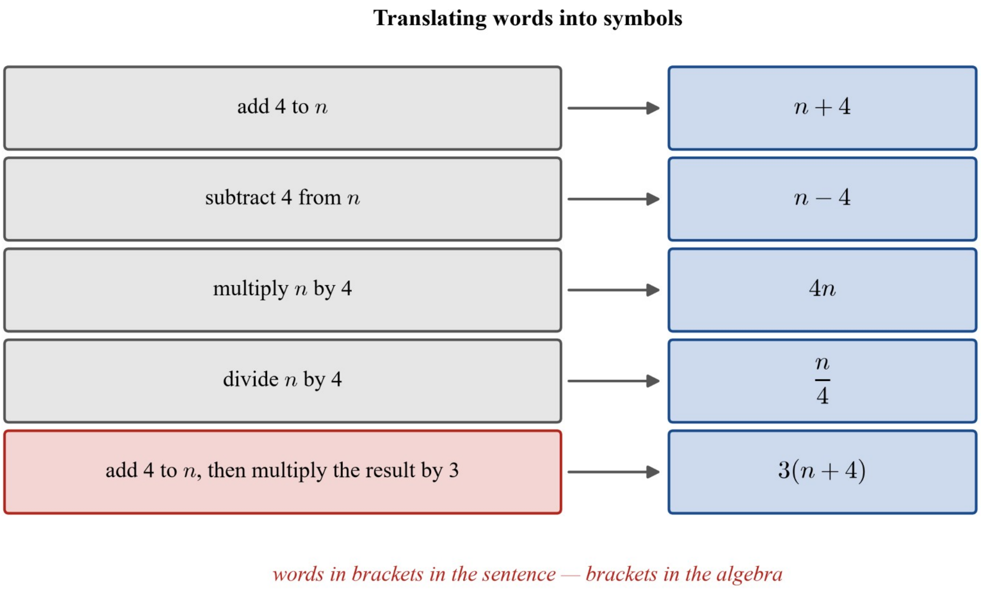

Figure 10. Five lines translating words into algebra: add 4 to n gives n plus 4; subtract 4 from n gives n minus 4; multiply n by 4 gives 4n; divide n by 4 gives n over 4; and a highlighted line, add 4 to n then multiply the result by 3, gives 3(n plus 4), with the note that words in brackets in the sentence get brackets in the algebra.

Think-of-a-number chains. A string of instructions becomes a chain of steps (Figure 11): “think of a number, add 6, multiply the result by 2, subtract 8” is written $2(n+6)-8$. Following the chain with $n=5$ gives $5 \rightarrow 11 \rightarrow 22 \rightarrow 14$, the same answer you get by substituting 5 for n, as practised in 3A.

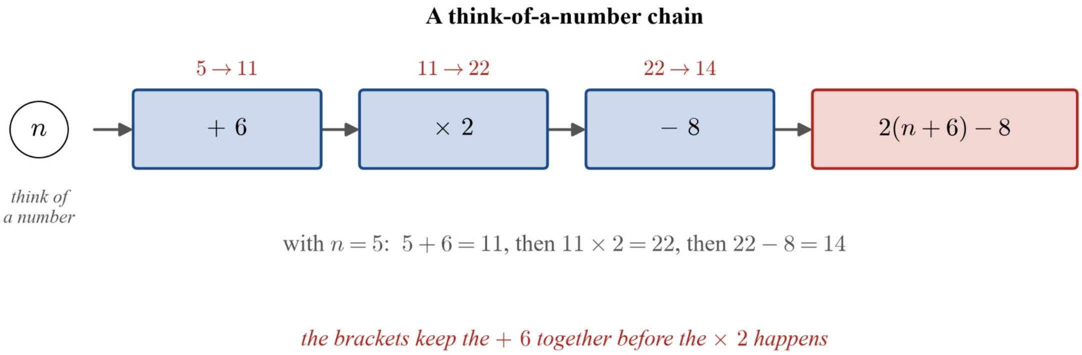

Figure 11. A chain of steps: a number n passes through add 6, then times 2, then subtract 8, making the expression 2(n plus 6) minus 8, with the worked trace for n equals 5 going 5 to 11 to 22 to 14.

A variable changes. The whole point of a variable is that its value can change, and the expression changes with it. Feed $n=0,1,2,3,4,5$ into $2n+1$ and the values are $1,3,5,7,9,11$ — the odd numbers, all from one expression (Figure 12). Each time n goes up by 1, the value goes up by 2. Seeing how the values move in a table like this one is where subtopic 3D begins.

The value changes as the variable changes: $2n+1$

n	$2n+1$
0	1
1	3
2	5
3	7
4	9
5	11

each time n goes up by 1, the value goes up by 2

$1, 3, 5, 7, 9, 11$ — *the odd numbers, all from one expression*

Figure 12. A table of values of 2n plus 1 for n from 0 to 5, giving 1, 3, 5, 7, 9, 11, with arrows down the rows and the notes that each time n goes up by 1 the value goes up by 2, and that one expression produces the odd numbers.

The words of this subtopic, with the writing rules they carry:

Term	Meaning here	Synonyms and related words
commutative law	the order does not change the answer: $a+b=b+a$ and $a\times b=b\times a$	swapping the order
associative law	the grouping does not change the answer: $(a+b)+c=a+(b+c)$	choosing which pair to do first
distributive law	multiplying a sum means multiplying each part: $a\times(b+c)=a\times b+a\times c$	multiplying each part inside the brackets
constant	a number in an expression that never changes, like the 3 and the 4 in $3x+4$	fixed number
variable	a letter standing for a number that can change, like the x in $3x+4$	letter
term	one of the pieces of an expression joined by + and − signs: $3x+4$ has two terms	part
implied multiplication	writing $3x$ to mean $3\times x$; the times sign is understood	$3x$ means 3 times x
expression	numbers, variables, operations and brackets written together, like $3x+4$ or $3(n+4)$	algebraic phrase
estimate	a rounded answer used to check the size of the exact one (2B)	rough answer
natural numbers	the numbers $1, 2, 3, 4, \ldots$ used for counting	counting numbers

This subtopic leans on the substituting of 3A and the estimation habit of 2B, and it uses the reordering and grouping already seen on fractions in 2D. Ahead, 3C solves equations built from these expressions, and 3D turns tables like Figure 12 into graphs.

Worked examples

Example 1 — scaffolded

Find $64+37+36+63$ mentally, naming the law behind each move.

Working	Comment
Look for pairs that make a round number. 64 pairs with 36, and 37 pairs with 63.	$4+6$ and $7+3$ each end in 0, so both pairs make a whole hundred.
Swap the order (commutative law). $64+36+37+63$	Putting each pair side by side is the swap the law allows.
Group (associative law). $(64+36)+(37+63)$	The brackets say which pair to add first.
Solve. $100+100=200$.	Two whole hundreds — no column work needed.

$\therefore$ $64+37+36+63=200$.

Example 2 — scaffolded

Use the distributive law to find 7×42 mentally, then check that the answer is reasonable.

Working	Comment
Split the second number into a round number plus a small part. $42=40+2$, so $7\times42=7\times(40+2)$.	40 is the round number; 2 is the rest.
Multiply each part (distributive law). $7\times40+7\times2=280+14$	The 7 multiplies each part inside the brackets.
Solve. $280+14=294$.	Two easy products, then one easy sum.
Check with an estimate. 7×42 sits near $7\times40=280$, and 294 is close to 280.	The estimate checks the size of the answer before the last digit.

$\therefore$ $7\times42=294$.

Example 3 — unscaffolded

(a) Copy and complete a table of values for $3x+4$ and $3(x+4)$ using $x=1,2,5$.
(b) Which expression gives the bigger values, and by how much?
(c) Use the distributive law to explain why the gap is always the same.

Working	Comment
(a) $x=1$: $3\times1+4=7$ and $3\times(1+4)=15$. $x=2$: 10 and 18. $x=5$: 19 and 27.	Brackets first: in the second expression, add before multiplying.
(b) $3(x+4)$ is bigger by 8 each time: $15-7=8$, $18-10=8$, $27-19=8$.	Three values of x all show the same gap.
(c) By the distributive law, $3(x+4)$ is three lots of x and three lots of 4, so it equals $3x+12$. That is $12-4=8$ more than $3x+4$, whatever x is.	The bracket makes the 3 multiply the 4 as well.

$\therefore$ the two expressions are never equal: $3(x+4)$ is always 8 more than $3x+4$.

Example 4 — unscaffolded

"Think of a number. Add 6. Multiply the result by 2. Subtract 8." (a) Write the instructions as one expression, using n for the number. (b) Find the result when the number is 5. (c) Write, in words, the instructions that produce the expression $4(n-2)$.

Working	Comment
(a) $2(n+6)-8$	The brackets hold the "add 6" step together before the

Working	Comment
	doubling.
(b) $n=5$: $5+6=11$, then $11\times 2=22$, then $22-8=14$.	Follow the chain step by step, or substitute 5 into the expression.
(c) "Think of a number, subtract 2, then multiply the result by 4."	Read the expression back into words: the bracket step happens first.

$\therefore$ the chain is $2(n+6)-8$, the result for 5 is 14, and $4(n-2)$ means "subtract 2, then multiply the result by 4".

Practice questions

Scaffolded

Q1. Name the law — commutative, associative or distributive — that makes each line true. (a) $9+14=14+9$ (b) $5\times(20+3)=5\times 20+5\times 3$ (c) $(7+3)+8=7+(3+8)$ (d) $6\times 15=6\times 10+6\times 5$

Q2. Find each answer by reordering or regrouping first. Show the step you used. (a) $74+59+26$ (b) $17+45+83+55$ (c) $4\times 19\times 25$

Q3. Use the distributive law to find each product mentally. Show the split you used. (a) 6×34 (b) 9×23 (c) 5×78

Q4. True or false? If a line is false, say what the two sides actually are. (a) $15-7=7-15$ (b) $(3\times 4)\times 5=3\times(4\times 5)$ (c) $20\div 4=4\div 20$ (d) $8+(12+2)=(8+12)+2$

Q5. Write each expression using implied multiplication. (a) $y+y+y+y$ (b) $m+m$ (c) $1\times k$ (d) $5\times p$ (e) $a+a+a+b+b$

Q6. Match each instruction to its expression. (i) add 4 to n (ii) subtract 4 from n (iii) multiply n by 4 (iv) divide n by 4 (v) add 4 to n, then multiply the result by 3

A. $4n$ B. $3(n+4)$ C. $n-4$ D. $n+4$ E. $\frac{n}{4}$

Unscaffolded

Q7. Copy and complete the table for $2n+5$ and $2(n+5)$ using $n=1,2,3,4$.

n	1	2	3	4
$2n+5$				
$2(n+5)$				

How much bigger is $2(n+5)$ each time? Use the distributive law to explain the gap.

Q8. (a) Copy and complete the table for $20-3n$ using $n=1,2,3,4,5,6$.

n	1	2	3	4	5	6
$20-3n$						

(b) In one sentence, describe what happens to the value each time n goes up by 1.

Q9. Write an expression for each description, using brackets where they are needed. (a) Add 4 to m, then multiply the result by 5. (b) Multiply m by 5, then add 4. (c) In one sentence, explain the difference between your two expressions.

Q10. "Think of a number. Multiply it by 4. Add 10." (a) Write the instructions as one expression, using n for the number. (b) Find the result when the number is 3. (c) Ruby says the instructions "add 10, then multiply the result by 4 " give the same answers. Test her claim using your number from (b) and say whether she is right.

Q11. Estimate first, then find the exact answer, and say whether it is reasonable. (a) $58+193+42$ (b) 7×39

Q12. Noah finds 6×47 and writes 242. Use an estimate to explain why his answer cannot be right, then use the distributive law to find the correct answer.

Q13. Mia says $3x+4$ and $3(x+4)$ must give the same values "because both contain a 3 and a 4". (a) Show, using $x=2$, that Mia is wrong. (b) Is there a natural number x that makes the two expressions equal? Explain your answer.

Q14. A school buys n boxes of pencils, and each box holds 12 pencils. (a) Write an expression for the total number of pencils. (b) The school hands out 30 pencils to each of c classes. Write an expression for the number of pencils handed out. (c) Write an expression for the number of pencils left. (d) Find the number of pencils left when $n=8$ and $c=3$.

Answers

Q1. (a) commutative; (b) distributive; (c) associative; (d) distributive.

Q2. (a) 159; (b) 200; (c) 1900.

Q3. (a) 204; (b) 207; (c) 390.

Q4. (a) false — $15-7=8$ but $7-15$ is not 8; (b) true — both sides are 60; (c) false — $20\div 4=5$ but $4\div 20$ is not 5; (d) true — both sides are 22.

Q5. (a) $4y$; (b) $2m$; (c) k; (d) $5p$; (e) $3a+2b$.

Q6. (i) D ($n+4$); (ii) C ($n-4$); (iii) A ($4n$); (iv) E ($\frac{n}{4}$); (v) B ($3(n+4)$).

Q7. $2n+5$: $7, 9, 11, 13$; $2(n+5)$: $12, 14, 16, 18$; bigger by 5 every time, because $2(n+5)=2n+10$ and $10-5=5$.

Q8. (a) $17, 14, 11, 8, 5, 2$; (b) each time n goes up by 1, the value goes down by 3.

Q9. (a) $5(m+4)$; (b) $5m+4$; (c) $5(m+4)$ multiplies the 4 too: it is $5m+20$, which is always 16 more than $5m+4$.

Q10. (a) $4n+10$; (b) 22; (c) no — with $n=3$, Ruby's version gives $4\times(3+10)=52$, not 22.

Q11. (a) about $60+190+40=290$; exact 293 — close, so reasonable; (b) about $7\times 40=280$; exact 273 — close, so reasonable.

Q12. 6×47 is near $6\times 50=300$, and 242 is about 60 short, so it cannot be right; the correct answer is $6\times(40+7)=240+42=282$.

Q13. (a) $x=2$: $3\times 2+4=10$ but $3\times(2+4)=18$, so the values are different; (b) no — $3(x+4)=3x+12$, which is always 8 more than $3x+4$.

Q14. (a) $12n$; (b) $30c$; (c) $12n-30c$; (d) $12\times 8-30\times 3=96-90=6$.

Fully-worked solutions

Q1. (a) The two numbers swap places and the sum is unchanged — commutative law. (b) The 5 multiplies each part inside the brackets — distributive law. (c) The brackets move to group a different pair — associative law. (d) The 6 multiplies each part of $10+5$ — distributive law.

Q2. (a) Reorder and group: $(74+26)+59=100+59=159$. (b) Two friendly pairs: $(17+83)+(45+55)=100+100=200$. (c) Group 4 with 25 first: $(4\times 25)\times 19=100\times 19=1900$.

Q3. (a) $6\times 34=6\times(30+4)=180+24=204$. (b) $9\times 23=9\times(20+3)=180+27=207$. (c) $5\times 78=5\times(80-2)=400-10=390$. The split works over subtraction too.

Q4. (a) False: $15-7=8$, but taking 15 from 7 does not give 8 — order matters in subtraction. (b) True: both sides group the same three factors, and multiplying gives 60 either way. (c) False: $20\div 4=5$, but 4 divided into 20 parts is not 5 — order matters in division. (d) True: both sides add the same three numbers, and each gives 22.

Q5. (a) Four lots of y: $4y$. (b) Two lots of m: $2m$. (c) One lot of k is just k. (d) $5\times p$ is written $5p$, number first. (e) Three lots of a and two lots of b: $3a+2b$.

Q6. (i) "add 4 to n" is $n+4$ (D). (ii) "subtract 4 from n" starts at n and takes 4 away: $n-4$ (C). (iii) "multiply n by 4 " is $4n$ (A). (iv) "divide n by 4" is written with a fraction bar: $\frac{n}{4}$ (E). (v) The word "then" means the adding is finished before the multiplying starts, so the sum goes in brackets: $3(n+4)$ (B).

Q7. $2n+5$: $2\times1+5=7$, 9, 11, 13. $2(n+5)$: $2\times(1+5)=12$, 14, 16, 18. The second expression is 5 bigger every time. By the distributive law, $2(n+5)$ is two lots of n and two lots of 5, so it equals $2n+10$; that is $10-5=5$ more than $2n+5$, whatever n is.

Q8. (a) $20-3\times1=17$; then 14, 11, 8, 5, 2. (b) Each time n goes up by 1, the value goes down by 3, because one more lot of 3 is taken from 20.

Q9. (a) The adding happens first, so it sits inside brackets: $5(m+4)$. (b) The multiplying happens first and the 4 is added after: $5m+4$. (c) In $5(m+4)$ the 5 multiplies the 4 as well, giving $5m+20$; that is 16 more than $5m+4$, whatever m is.

Q10. (a) Multiply first, then add: $4n+10$. (b) $n=3$: $4\times3=12$, then $12+10=22$. (c) Ruby's version is $4(n+10)$. With $n=3$ it gives $4\times13=52$, which is not 22, so her claim is wrong — the brackets change which step happens first.

Q11. (a) Round to the nearest ten: about $60+190+40=290$. Exact: group 58 with 42: $(58+42)+193=100+193=293$. 293 is close to 290, so the answer is reasonable. (b) Round 39 to 40: about $7\times40=280$. Exact: $7\times39=7\times(40-1)=280-7=273$. 273 is close to 280, so the answer is reasonable.

Q12. Round 47 to 50: 6×47 is near $6\times50=300$. Noah's 242 is about 60 below that, so it cannot be right. By the distributive law, $6\times47=6\times(40+7)=6\times40+6\times7=240+42=282$, which sits close to the estimate of 300.

Q13. (a) With $x=2$: $3\times2+4=6+4=10$, but $3\times(2+4)=3\times6=18$. The values are different, so Mia is wrong. (b) No. By the distributive law, $3(x+4)=3x+12$, and $3x+12$ is 8 more than $3x+4$ for every value of x — the gap never closes, so no natural number makes them equal.

Q14. (a) n boxes of 12 pencils: $12n$. (b) 30 pencils to each of c classes: $30c$. (c) Left over means subtract what was handed out: $12n-30c$. (d) Substitute $n=8$ and $c=3$: $12\times8-30\times3=96-90=6$ pencils left.

Teacher note

- **Ownership and boundaries.** 3A (`VC2M7A01`) owns substitution into everyday formulas — it is recalled here (WE4(b), Q14(d)), not re-taught. 2D (`VC2M7N06`) owns the commutative and associative properties applied to fractions and positive rationals; this section states the three laws *by name* and applies them to whole-number computation and to algebra, and cross-references 2D rather than repeating it. 3D (`VC2M7A05`) owns tables of values and plotting; Figure 12 and Q8 deliberately stop at "how the values move" and hand the graphing forward. Per D4.14, expanding and factorising are Level 8 (`VC2M8A01`): the single distributive reading in WE3(c) and Q13(b) — "$3(x+4)$ is three lots of x and three lots of 4" — is the law itself applied, not an expanding drill; no question asks students to expand or factorise as a technique.
- **Elaborations delivered.** E01 (simplifying calculations): WE1, Q2. E02 (forming simple estimates for combined operations): Figure 5, WE2, Q11, Q12. E03 (generalising arithmetic to algebra, implied multiplication, the $3x+4$ vs $3(x+4)$ distinction): Figures 6–9, WE3, Q5, Q7, Q9, Q13 — both named elements of E03 are taught explicitly and assessed, per the Chapter 3 note. E04 (formulating expressions, translating words to symbols, think-of-a-number): Figures 10–11, WE4, Q6, Q9, Q10. E05 (variable as something that can change): Figure 12 and Q8 deliver the mathematical half; 3D takes the relationship between variables further.
- **E05 exclusion (recorded ACCEPTED).** The second half of `VC2M7A02.E05` — the application to processes on Country/Place and Aboriginal and Torres Strait Islander storytelling — is excluded under CONTENT_POLICY Rule 1 (no Aboriginal and Torres Strait Islander content in this build). Recorded ACCEPTED for the coverage audit.
- **Terminology (D12).** VCAA's wording is used throughout: the three *laws* (not "properties"), and *natural numbers* where counting numbers are meant (Q13(b)); the vocabulary table carries both.
- **Reasonableness habit (P7).** WE2 and Q11 ask students to estimate and then judge the exact answer against it; Q12 uses an estimate to reject a wrong answer, per TOC §5.2.

- **Figures.** All twelve figures are generated by `src/gen_3b.py` from the exact values stated in the text; tables, machines and area models compute their labels from those values.
- **Digital tools.** The CD does not require a tool; no software is named and no CAS is used.
- **Contexts.** All contexts are transparently fictional (machines, number chains, pencil boxes), so no real data is asserted and no source lines are required under SOURCE_USE.

Solving one-variable linear equations with natural number solutions

Content description `VC2M7A03`: solve one-variable linear equations of increasing complexity with natural number solutions; verify equation solutions by substitution

Elaboration `VC2M7A03.E01`: recognising that solving an equation is a process of determining a value that makes the equation true; and using substitution to determine whether a given number is a solution to an equation or not

Elaboration `VC2M7A03.E02`: solving equations using concrete materials, the balance model and backtracking, explaining the process

Elaboration `VC2M7A03.E03`: solving linear equations such as $3x+7=19$ algebraically, and verifying the solution by substitution

Achievement standard (extract): They solve linear equations with natural number solutions and verify their solutions through substitution.

Theory

What an equation says. In subtopic 3B we built expressions such as $3x+7$ out of numbers, **variables** and operations. An **equation** joins two expressions with an equals sign, for example

$$3x+7=19.$$

The expression on the left of the equals sign is the **left-hand side** (LHS) and the expression on the right is the **right-hand side** (RHS). The equals sign says the two sides have the same value. Here x is a variable standing for one unknown number. An equation like this is neither obviously true nor obviously false — it is true for some values of x and false for the rest. **Solving** the equation means determining the value of x that makes the equation true, and that value is called the **solution**. In this subtopic every equation has a solution that is a **natural number**, that is one of the numbers $1,2,3,4,\ldots$.

One way to find the solution is to try numbers one at a time. Figure 1 tests $x=1,2,3,4$ in the equation $2x+1=9$. Only $x=4$ makes the equation true, so the solution is $x=4$.

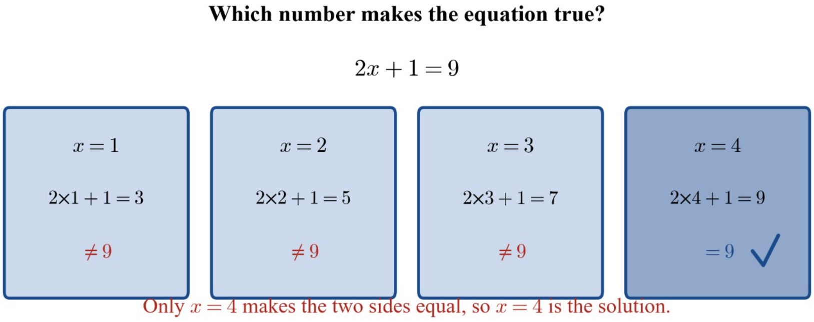

Figure 1. The equation 2x + 1 = 9 tested with the values x = 1, 2, 3 and 4; the first three trials give 3, 5 and 7 and are marked with a cross, while x = 4 gives 9 and is marked with a tick.

Testing a number by substitution. The method of replacing the variable with a number and working out the value is called **substitution**, which we practised with formulas in subtopic 3A. Substitution is also how we decide whether a given number is a solution to an equation: substitute the number for x, work out each side, and compare. If the two sides come out equal, the number is a solution; if not, it is not.

Figure 2 tests $x=3$ and $x=4$ in the equation $3x+2=11$.

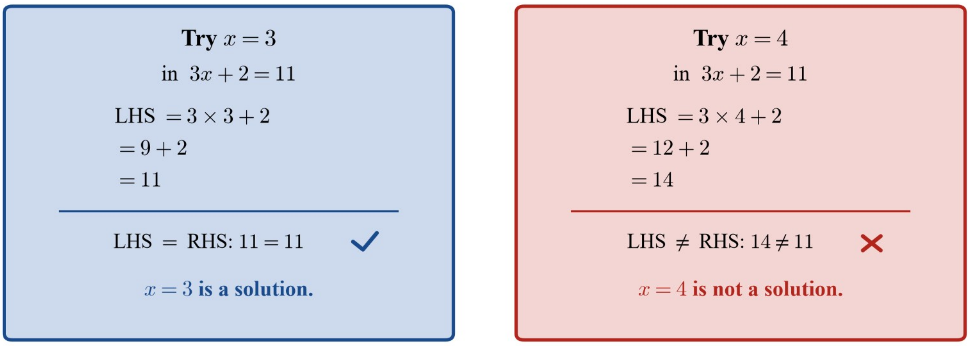

Figure 2. Two side-by-side panels testing the equation 3x + 2 = 11; the left panel substitutes x = 3 to get 11 and is marked with a tick, and the right panel substitutes x = 4 to get 14 and is marked with a cross.

Every solution we find in this subtopic will finish with exactly this check — substitute the answer back into the original equation and confirm that both sides match. Keep this habit: a solution that has not been checked is only a guess.

A solution with concrete materials. Imagine a covered card hiding an unknown number of counters, together with some loose counters. Figure 3 shows one covered card labelled x and five loose counters balancing nine counters on the other side. The card must hide four counters, because four counters plus five counters makes nine. We have just solved the equation $x+5=9$ by reasoning about what the card must hold.

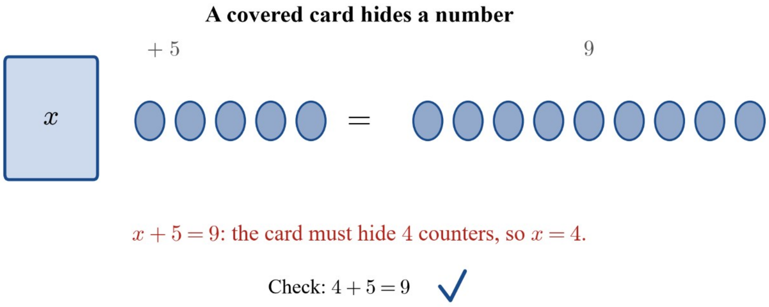

Figure 3. A covered card labelled x together with five loose counters shown on the left of an equals sign, with nine counters on the right, modelling the equation x + 5 = 9.

Concrete materials let us see why a solution works, but drawing cards and counters quickly gets slow. Two models carry the same ideas onto paper: the balance model and backtracking.

The balance model. Picture a set of scales. When the two pans hold equal total weights the beam is level — that is what the equals sign means. Bags labelled x stand for the unknown number and counters stand for known amounts. Figure 4 shows the equation $2x+3=11$ on a balance: two bags and three counters on the left pan balance eleven counters on the right pan.

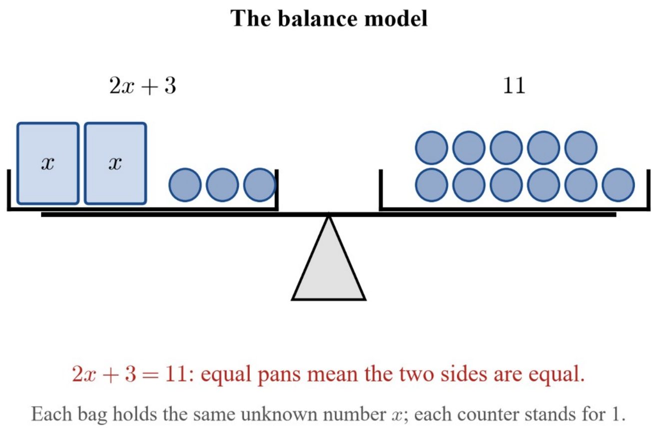

Figure 4. A balance with two bags labelled x and three counters on the left pan, and eleven counters on the right pan, modelling the equation 2x + 3 = 11.

Solving on the balance. The golden rule of the balance is: whatever you do to one side, do the same to the other side, and the pans stay level. To solve $2x+3=11$, remove three counters from each pan, leaving two bags against eight counters. Then split each pan into two equal groups: one bag against four counters. So $x=4$. Figure 5 shows both steps.

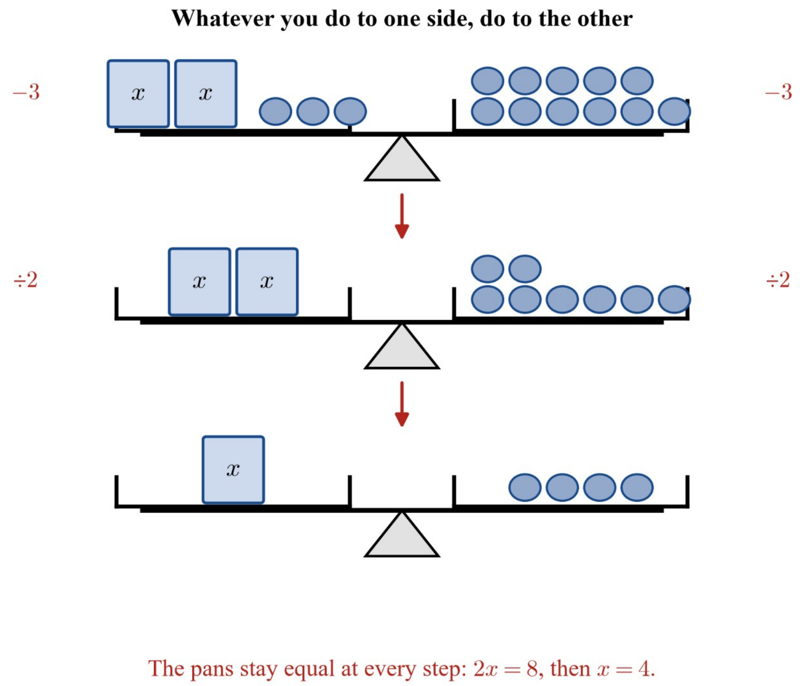

Figure 5. The balance model solving 2x + 3 = 11 in two steps: three counters removed from each pan leaves 2x against 8, then each side is divided by 2 leaving x against 4.

The balance model explains why each step is allowed, but it is not always the quickest way to write the working. Backtracking often is.

Backtracking: undoing in reverse order. Look at $3x+2=11$ as a chain of operations done to x: first x is multiplied by 3, then 2 is added, giving 11. To find x, walk back along the chain in reverse order and undo each operation. The operation that undoes another is called its **inverse operation**: subtraction undoes addition, and division undoes multiplication. Working backwards from 11: subtract 2 to get 9, then divide by 3 to get $x=3$. Figure 6 shows the forward chain and the backward journey.

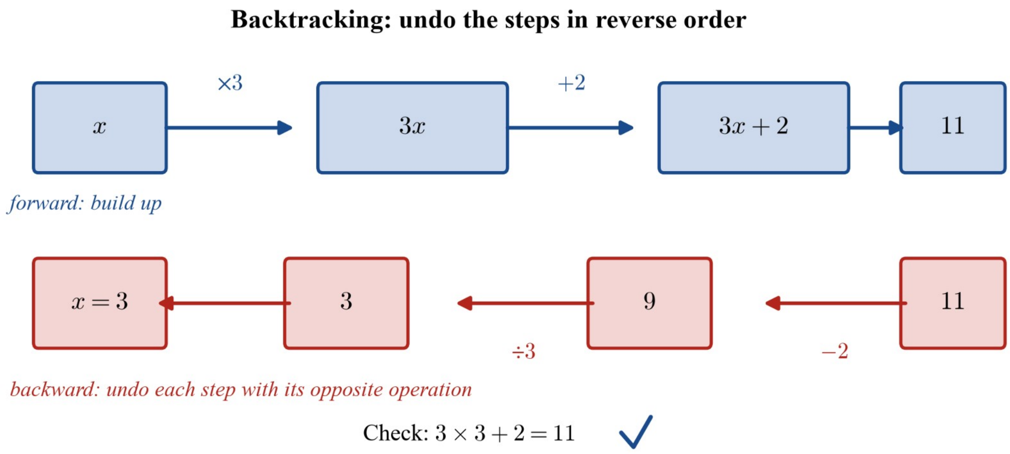

Figure 6. The equation 3x + 2 = 11 as a flow of operations; forward from x the operations are multiply by 3 then add 2, and backward from 11 the inverse operations are subtract 2 then divide by 3, giving x = 3.

Backtracking works whenever one side of the equation is a chain of operations on x and the other side is a number. Check the answer by substitution: $3 \times 3 + 2 = 11$.

Solving algebraically. Once you can see the steps, you can write them as algebra. The algebraic method applies the balance rule — the same operation to both sides — and records it as a column of working, undoing one operation per line. The last line names the solution, and the final step is always the substitution check.

Figure 7 solves $4x + 3 = 27$ this way.

Solving on paper: one inverse operation per line

$$
\begin{aligned}
4x + 3 &= 27 \\
4x + 3 - 3 &= 27 - 3 && \text{subtract 3 from both sides} \\
4x &= 24 \\
4x \div 4 &= 24 \div 4 && \text{divide both sides by 4} \\
x &= 6
\end{aligned}
$$

Check: $4 \times 6 + 3 = 24 + 3 = 27$ ✓ substitution verifies the solution

Figure 7. The equation 4x + 3 = 27 solved algebraically: subtracting 3 from both sides gives 4x = 24, then dividing both sides by 4 gives x = 6.

Notice the pattern: undo the operations in reverse order, one per line, keeping both sides balanced, and check at the end.

Equations with brackets. The brackets in an equation such as $2(x+4) = 18$ mark a group: two copies of the group $(x+4)$ make 18. At this level we solve by dividing both sides by 2 first, which splits the group:

$$x + 4 = 9,$$

and then subtracting 4 from both sides gives $x = 5$. Figure 8 shows the two groups before they are split.

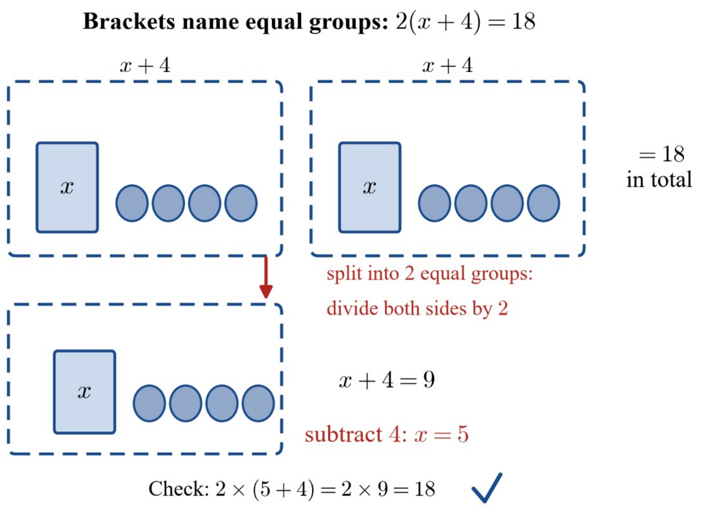

Figure 8. Two identical dashed groups, each containing a bag labelled x and four counters, together making 18, modelling the equation 2(x + 4) = 18.

Checking: $2(5+4)=2\times 9=18$. Dividing first keeps the group together and the numbers small; later, at Level 8, you will learn a second method for bracket equations.

The variable on both sides. Sometimes x appears on both sides of the equation, as in $5x+2=3x+10$. The idea is unchanged: use inverse operations on both sides until x stands alone on one side. Subtracting $3x$ from both sides moves all the x's to the left:

$$2x+2=10,$$

then subtracting 2 and dividing by 2 gives $x=4$. Figure 9 shows the bags on both sides and the first move.

The variable on both sides: $5x+2=3x+10$

$$
\begin{aligned}
5x+2 &= 3x+10 && \\
5x-3x+2 &= 3x-3x+10 && \text{subtract } 3x \text{ from both sides}\\
2x+2 &= 10 && \\
2x &= 8 && \text{subtract 2 from both sides}\\
x &= 4 && \text{divide both sides by 2}
\end{aligned}
$$

Check: $5\times 4+2=22$ and $3\times 4+10=22$ ✓ both sides agree

Figure 9. The equation 5x + 2 = 3x + 10 shown as bags and counters on both sides of an equals sign, with the first step of subtracting 3x from both sides labelled.

Checking: LHS $=5\times 4+2=22$ and RHS $=3\times 4+10=22$, so the two sides agree.

Natural number solutions only. In this subtopic every equation has been chosen so that its solution is a natural number. If your working ever gives a fraction or a negative number as the solution, check your steps — one of them has gone wrong. At Level 8 the story grows: solutions may become fractions and negative numbers, and equations connect to graphs and inequalities.

Vocabulary of this subtopic.

Term	Meaning here	Synonyms
equation	a statement that two expressions are equal, written with an equals sign,	—

Term	Meaning here	Synonyms
	such as $3x+7=19$	
variable	a letter, here x, standing for the unknown number	pronumeral, unknown
left-hand side	the expression before the equals sign	LHS
right-hand side	the expression after the equals sign	RHS
solution	the value of the variable that makes the equation true	answer
substitution	replacing the variable with a number to work out a value or check an answer	—
inverse operation	the operation that undoes another: subtraction undoes addition, division undoes multiplication	opposite operation
balance model	a picture of an equation as scales, where doing the same thing to both sides keeps the pans level	scales model
backtracking	solving by undoing the operations in reverse order	working backwards

Subtopic 3D builds on this one. There, rules written as equations such as $y=3x+2$ are used to complete tables of values and plot points on a graph — and every entry in those tables comes from the same skill you now have: substituting a number and working out the value.

Worked examples

Example 1 — scaffolded (balance model). Solve $x+6=15$, and check your answer.

Working	Comment
$x+6=15$	Write the equation.
$x+6-6=15-6$	Remove 6 from each side, like taking six counters off each pan (Figure 5).
$x=9$	The x now stands alone.
Check: $9+6=15$	Substitute $x=9$; both sides agree.

$\therefore x=9$.

Example 2 — scaffolded (backtracking). Solve $3x+5=20$, and check your answer.

Working	Comment
$3x+5=20$	Write the equation. Forward: x is multiplied by 3, then 5 is added.
$20-5=15$	Undo the last operation first: subtract 5.
$15\div 3=5$	Undo the multiplication: divide by 3.
Check: $3\times 5+5=15+5=20$	Substitute $x=5$; both sides agree.

$\therefore x=5$.

Example 3 — unscaffolded (algebraic). Solve $3x+7=19$, and check your answer.

Working	Comment
$3x+7=19$	Write the equation.
$3x+7-7=19-7$	Subtract 7 from both sides.
$3x=12$	Simplify both sides.
$3x\div3=12\div3$	Divide both sides by 3.
$x=4$	The x now stands alone.
Check: $3\times4+7=12+7=19$	Substitute $x=4$; both sides agree.

$\therefore x=4$.

Example 4 — unscaffolded (variable on both sides). Solve $7x+3=4x+18$, and check your answer.

Working	Comment
$7x+3=4x+18$	Write the equation.
$7x-4x+3=4x-4x+18$	Subtract $4x$ from both sides.
$3x+3=18$	Simplify both sides.
$3x+3-3=18-3$	Subtract 3 from both sides.
$3x=15$	Simplify both sides.
$3x\div3=15\div3$	Divide both sides by 3.
$x=5$	The x now stands alone.
Check: LHS $=7\times5+3=35+3=38$; RHS $=4\times5+18=20+18=38$	Substitute $x=5$ into both sides; they agree.

$\therefore x=5$.

Practice questions

Scaffolded

Q1. Use substitution to decide whether each given number is a solution to its equation. Show your substitution and answer *yes* or *no*.

- (a) $x+8=15$, with $x=7$
- (b) $4x=36$, with $x=9$
- (c) $2x+3=14$, with $x=5$
- (d) $x\div5=6$, with $x=30$

Q2. Solve each equation by inspection (that is, work out the answer in your head), then check your answer by substitution.

- (a) $x+7=12$
- (b) $x-9=14$
- (c) $6x=42$

- (d) $x \div 4 = 8$

Q3. Solve each equation, showing one step of working, and check your answer by substitution.

- (a) $x + 12 = 31$
- (b) $x - 7 = 18$
- (c) $7x = 56$
- (d) $x \div 6 = 9$

Q4. Solve each two-step equation, showing your working, and check your answer by substitution.

- (a) $2x + 3 = 17$
- (b) $5x - 4 = 21$
- (c) $x \div 3 + 6 = 10$
- (d) $4x + 8 = 32$

Q5. Consider the equation $5x + 3 = 28$.

- (a) Copy and complete the forward chain: $x \xrightarrow{\times 5} \xrightarrow{+3} = 28$.
- (b) Use backtracking to solve the equation. Write each undoing step.
- (c) Check your answer by substitution.

Q6. The balance in Figure 10 is level: three bags labelled x and four counters balance nineteen counters.

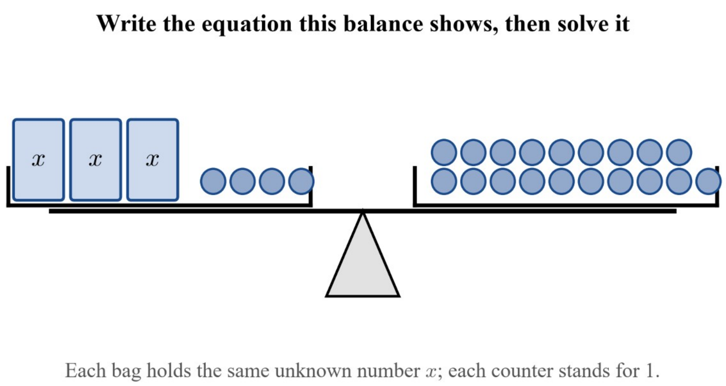

Figure 10. A level balance with three bags labelled x and four loose counters on the left pan, and nineteen counters on the right pan.

- (a) Write the equation shown by the balance.

- (b) Solve the equation to find the number of counters in each bag.
- (c) Check your answer by substitution.

Q7. Consider the equation $4x+3=35$.

- (a) Without solving, decide which number is closest to the solution: 5, 8 or 12. Explain your estimate.
- (b) Solve the equation, showing your working.
- (c) Check your answer by substitution, and say whether it matches your estimate.

Unscaffolded

Q8. Solve each equation and check your answer by substitution.

- (a) $6x-7=41$
- (b) $x\div 5+9=16$
- (c) $3x+14=35$
- (d) $9x-5=40$

Q9. Solve each equation with brackets by dividing first. Check your answers.

- (a) $3(x+2)=21$
- (b) $4(x+1)=36$
- (c) $5(x-3)=30$

Q10. Solve each equation where the variable appears on both sides. Check your answers.

- (a) $7x=4x+15$
- (b) $6x+1=4x+9$
- (c) $9x-2=7x+10$

Q11. Zoe solved the equation $2x+5=17$ and wrote down $x=7$.

- (a) Use substitution to show that Zoe's answer is not a solution.
- (b) Solve the equation correctly, and check your answer.

Q12. In Figure 11, the same shape always stands for the same number, and all numbers are natural numbers.

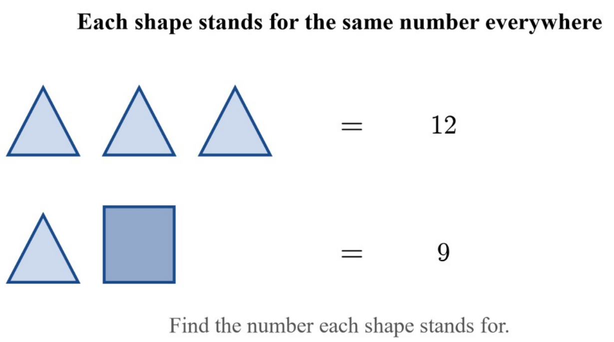

Figure 11. Two statements made with shapes: three triangles equal 12, and one triangle plus one square equals 9.

- (a) Find the value of the triangle, showing your reasoning.
- (b) Find the value of the square, showing your reasoning.
- (c) Check both values by substitution.

Q13. Write an equation for each situation, solve it, and check your answer in the context of the question.

- (a) Three identical bags each hold x kilograms of sand. Together with a 2-kilogram weight they balance a 17-kilogram weight. How many kilograms of sand are in each bag?
- (b) Liam saves the same amount of money each week. After 6 weeks he has saved $84. How much does he save each week?
- (c) Sixty pencils are shared equally among n students, and each student receives 4 pencils. How many students are there?

Q14.

- (a) Write your own two-step equation of the form $ax+b=c$ that has the solution $x=6$, and solve it to show that it works.
- (b) I think of a number, double it, then add 7. The result is 25. Write an equation and find my number. Check your answer.

Answers

Q1. (a) yes (b) yes (c) no (d) yes

Q2. (a) $x=5$ (b) $x=23$ (c) $x=7$ (d) $x=32$

Q3. (a) $x=19$ (b) $x=25$ (c) $x=8$ (d) $x=54$

Q4. (a) $x=7$ (b) $x=5$ (c) $x=12$ (d) $x=6$

Q5. (a) $5x$, then $5x+3$ (b) $x=5$ (c) check below

Q6. (a) $3x+4=19$ (b) $x=5$ (c) check below

Q7. (a) 8 (b) $x=8$ (c) check below

Q8. (a) $x=8$ (b) $x=35$ (c) $x=7$ (d) $x=5$

Q9. (a) $x=5$ (b) $x=8$ (c) $x=9$

Q10. (a) $x=5$ (b) $x=4$ (c) $x=6$

Q11. (a) check below (b) $x=6$

Q12. (a) triangle $=4$ (b) square $=5$ (c) check below

Q13. (a) $x=5$ kg (b) \$14 per week (c) $n=15$ students

Q14. (a) example: $3x+2=20$ (b) $x=9$

Fully-worked solutions

Q1. Substitute the given number for x and compare the two sides.

- (a) $7+8=15$, and $15=15$, so $x=7$ is a solution. **Yes.**
- (b) $4\times 9=36$, and $36=36$, so $x=9$ is a solution. **Yes.**
- (c) $2\times 5+3=10+3=13$, but $13\neq 14$, so $x=5$ is not a solution. **No.**
- (d) $30\div 5=6$, and $6=6$, so $x=30$ is a solution. **Yes.**

Q2.

- (a) $x+7=12$, so $x=5$. Check: $5+7=12$.
- (b) $x-9=14$, so $x=14+9=23$. Check: $23-9=14$.
- (c) $6x=42$, so $x=42\div 6=7$. Check: $6\times 7=42$.
- (d) $x\div 4=8$, so $x=8\times 4=32$. Check: $32\div 4=8$.

Q3.

-

- (a) $x+12=31$. Subtract 12 from both sides: $x=19$. Check: $19+12=31$.
- (b) $x-7=18$, so add 7 to both sides: $x=25$. Check: $25-7=18$.
- (c) $7x=56$, so divide both sides by 7: $x=8$. Check: $7\times 8=56$.
- (d) $x\div 6=9$, so multiply both sides by 6: $x=54$. Check: $54\div 6=9$.

Q4.

- (a) $2x+3=17$. Subtract 3: $2x=14$. Divide by 2: $x=7$. Check: $2\times 7+3=14+3=17$.
- (b) $5x-4=21$. Add 4: $5x=25$. Divide by 5: $x=5$. Check: $5\times 5-4=25-4=21$.
- (c) $x\div 3+6=10$. Subtract 6: $x\div 3=4$. Multiply by 3: $x=12$. Check: $12\div 3+6=4+6=10$.
- (d) $4x+8=32$. Subtract 8: $4x=24$. Divide by 4: $x=6$. Check: $4\times 6+8=24+8=32$.

Q5.

- (a) Forward chain: $x \xrightarrow{\times 5} 5x \xrightarrow{+3} 5x+3=28$.
- (b) Backwards from 28: $28-3=25$ (undo $+3$), then $25\div 5=5$ (undo $\times 5$), so $x=5$.
- (c) Check: $5\times 5+3=25+3=28$.

Q6.

- (a) Three bags and four counters balance nineteen counters: $3x+4=19$.
- (b) Subtract 4: $3x=15$. Divide by 3: $x=5$. Each bag holds 5 counters.
- (c) Check: $3\times 5+4=15+4=19$.

Q7.

- (a) $4\times 8+3=32+3=35$ already, so 8 is the closest estimate. (For 5: $4\times 5+3=23$, too small; for 12: $4\times 12+3=51$, too big.)
- (b) $4x+3=35$. Subtract 3: $4x=32$. Divide by 4: $x=8$.
- (c) Check: $4\times 8+3=32+3=35$, and the solution $x=8$ matches the estimate from part (a).

Q8.

- (a) $6x-7=41$. Add 7: $6x=48$. Divide by 6: $x=8$. Check: $6\times 8-7=48-7=41$.
-

- (b) $x \div 5 + 9 = 16$. Subtract 9: $x \div 5 = 7$. Multiply by 5: $x = 35$. Check: $35 \div 5 + 9 = 7 + 9 = 16$.
- (c) $3x + 14 = 35$. Subtract 14: $3x = 21$. Divide by 3: $x = 7$. Check: $3 \times 7 + 14 = 21 + 14 = 35$.
- (d) $9x - 5 = 40$. Add 5: $9x = 45$. Divide by 9: $x = 5$. Check: $9 \times 5 - 5 = 45 - 5 = 40$.

Q9. Divide both sides by the number outside the brackets first.

- (a) $3(x+2) = 21$. Divide by 3: $x + 2 = 7$. Subtract 2: $x = 5$. Check: $3(5+2) = 3 \times 7 = 21$.
- (b) $4(x+1) = 36$. Divide by 4: $x + 1 = 9$. Subtract 1: $x = 8$. Check: $4(8+1) = 4 \times 9 = 36$.
- (c) $5(x-3) = 30$. Divide by 5: $x - 3 = 6$. Add 3: $x = 9$. Check: $5(9-3) = 5 \times 6 = 30$.

Q10. Collect the variable on one side first.

- (a) $7x = 4x + 15$. Subtract $4x$: $3x = 15$. Divide by 3: $x = 5$. Check: LHS $= 7 \times 5 = 35$; RHS $= 4 \times 5 + 15 = 35$.
- (b) $6x + 1 = 4x + 9$. Subtract $4x$: $2x + 1 = 9$. Subtract 1: $2x = 8$. Divide by 2: $x = 4$. Check: LHS $= 6 \times 4 + 1 = 25$; RHS $= 4 \times 4 + 9 = 25$.
- (c) $9x - 2 = 7x + 10$. Subtract $7x$: $2x - 2 = 10$. Add 2: $2x = 12$. Divide by 2: $x = 6$. Check: LHS $= 9 \times 6 - 2 = 52$; RHS $= 7 \times 6 + 10 = 52$.

Q11.

- (a) Substitute $x = 7$: $2 \times 7 + 5 = 14 + 5 = 19$, but $19 \neq 17$, so $x = 7$ is not the solution.
- (b) $2x + 5 = 17$. Subtract 5: $2x = 12$. Divide by 2: $x = 6$. Check: $2 \times 6 + 5 = 12 + 5 = 17$.

Q12.

- (a) Three triangles make 12, so one triangle is $12 \div 3 = 4$.
- (b) Triangle + square $= 9$, so square $= 9 - 4 = 5$.
- (c) Check: $4 + 4 + 4 = 12$, and $4 + 5 = 9$. Both statements hold.

Q13.

- (a) Equation: $3x + 2 = 17$. Subtract 2: $3x = 15$. Divide by 3: $x = 5$. Check: $3 \times 5 + 2 = 17$. Each bag holds 5 kilograms of sand.
- (b) Equation: $6x = 84$. Divide by 6: $x = 14$. Check: $6 \times 14 = 84$. Liam saves $14 each week.
- (c) Equation: $4n = 60$. Divide by 4: $n = 15$. Check: $4 \times 15 = 60$. There are 15 students.

Q14.

-
  (a) Example: choose $a=3$ and $b=2$. With $x=6$, $c=3\times 6+2=20$, so $3x+2=20$ has the solution $x=6$. Check: $3\times 6+2=20$. (Any equation $ax+b=c$ with natural numbers and $a\times 6+b=c$ earns the mark.)
-
  (b) Let the number be x. Equation: $2x+7=25$. Subtract 7: $2x=18$. Divide by 2: $x=9$. Check: $2\times 9+7=18+7=25$.

Teacher note

- **Ownership and boundaries.** This subtopic delivers VC2M7A03 in full: solving one-variable linear equations of increasing complexity (one-step, two-step, brackets, variable on both sides) with **natural number solutions only** (D4.2), and verification by substitution. Fractions, negative numbers, inequalities and graphical methods belong to VC2M8A02 at Level 8 and are deliberately absent here. Per D4.14, bracket equations are solved by **dividing first** — expanding or factorising brackets is not taught at this level, and no transposing ("change side, change sign") shortcut is shown; every step is the same operation applied to both sides.
- **Verification is examined.** Per the TOC's assessment decision for Chapter 3 (T3), items award a mark for the substitution check. Every worked example and every fully-worked solution above ends with one; students should expect the same demand in the MCQ set.
- **Elaboration coverage.** E01 (equation as "determining the value that makes the equation true"; substitution to test a given number) is carried by Figures 1–2 and Q1. E02 (concrete materials, balance model, backtracking, explaining the process) is carried by Figures 3–6, Q5 and Q6. E03 (algebraic solving with verification; its own example $3x+7=19$) is carried by Figures 7–9, Example 3, and the remainder of the exercise.
- **Recall, do not reteach (D3).** Level 6 work on unknowns in numerical equations (VC2M6A01–A03) is assumed knowledge; one-step equations in Q2–Q3 are the recall pathway. The four operations and their inverse relationships are recalled from 1A.
- **Digital tools (D9).** VC2M7A03 names no digital tools, so none are used in this subtopic; verification is done by hand through substitution, which is also the examined skill.
- **Terminology (D12, contested terms).** "Natural numbers" is used throughout, never "counting numbers", consistent with 1C. The contested-term convention for natural numbers (whether 0 is included) does not affect this subtopic: no solution in the exercise set is 0, so no definition beyond "$1,2,3,4,\ldots$" is required. "Variable" is defined at first use; the vocabulary table lists "pronumeral" and "unknown" as synonyms, in line with the curriculum's terminology.
- **Contexts and sourcing.** All worded contexts (Q6, Q13) are transparently generic fictional scenarios — bags of counters, sand, pocket money, pencils — making no real-world data claims, so no source or date lines are required under the source-use policy.
- **Figures.** All eleven figures were generated by `src/gen_3c.py` (run in this tree) in the house style of `src/mmdiag.py`, drawing ticks and crosses rather than relying on special glyphs.
- **Reasonableness (P7).** Q7 asks for an estimate before solving and a comparison afterwards, satisfying the chapter's reasonableness requirement.
- **Cross-references.** Substitution was introduced with formulas in 3A and practised there; expressions and the meaning of brackets were built in 3B. The closing paragraph points forward to 3D, where equations act as rules for tables of values.

Tables of values from patterns and rules; plotting on the Cartesian plane

Content description `VC2M7A05`: generate tables of values from visually changing patterns or the rule of a function; describe and plot these relationships on the Cartesian plane

Achievement standard (extract): Students create tables of values relating to algebraic expressions and formulas, and describe how the values change.

Theory

What this subtopic is about. At Level 6 you wrote rules for number patterns and plotted points on the Cartesian plane, including points with negative coordinates. This subtopic joins those two skills: you take a **rule** — or a pattern you can see growing — and use it to build a **table of values**, then you plot the pairs from the table as points on the plane and describe what the points show.

The plane, recalled. The two axes of the Cartesian plane are number lines that cross at the **origin**, labelled O. They divide the plane into four **quadrants**, numbered I, II, III and IV. Every point has **coordinates** (x, y): the x-coordinate says how far left or right of O, and the y-coordinate says how far up or down. Figure 1 shows one point in each quadrant: $A(3, 2)$ is 3 right and 2 up; $B(-2, 4)$ is 2 left and 4 up; $C(-3, -2)$ is 3 left and 2 down; $D(4, -3)$ is 4 right and 3 down.

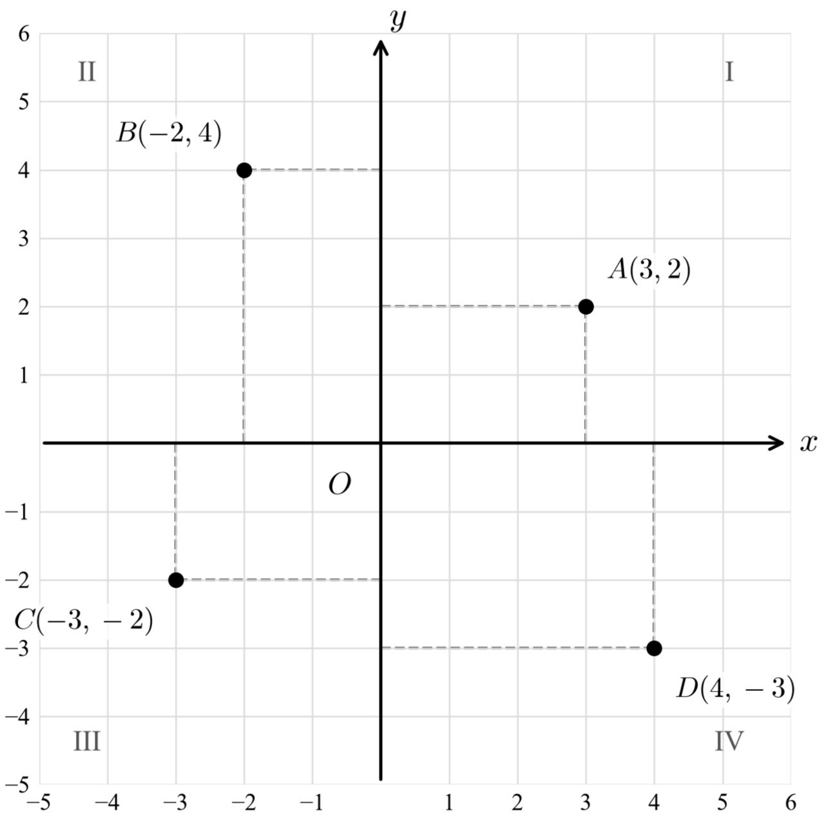

Figure 1. A Cartesian plane with its four quadrants labelled I, II, III and IV, showing one point in each quadrant: A(3, 2) in quadrant I, B(−2, 4) in quadrant II, C(−3, −2) in quadrant III and D(4, −3) in quadrant IV, each point joined to both axes by dashed guide lines, with the origin labelled O.

Rules: inputs and outputs. A **rule** tells you how to turn one number into another. The number you start with is the **input**, and the number you end with is the **output**. A **function machine** is a picture of a rule as steps that a number travels through. Figure 2 shows the rule "multiply by 2, then add 3" with input 4: the first step doubles 4 to get 8, and the second step adds 3 to get 11, so the output is 11.

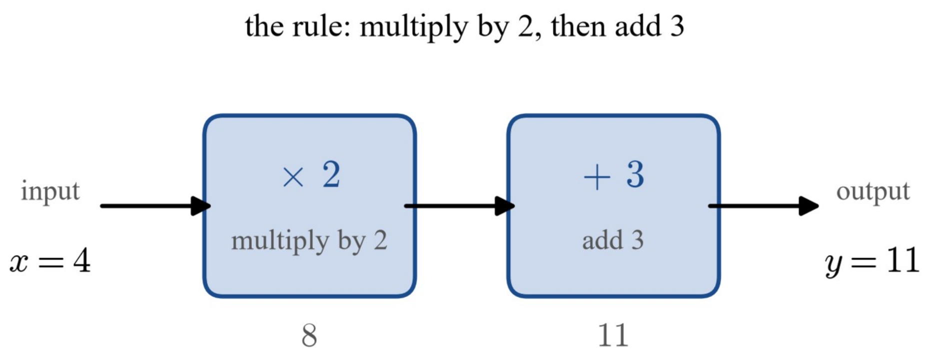

Figure 2. A function machine for the rule "multiply by 2, then add 3": the input x = 4 enters a first box labelled multiply by 2, which gives 8, then a second box labelled add 3, which gives the output y = 11.

Tables of values. A **table of values** lists several input–output pairs from one rule. Feeding $0, 1, 2, 3, 4$ through the machine of Figure 2 gives:

input x	0	1	2	3	4
output y	3	5	7	9	11

Each column of the table is an **ordered pair** (input, output): $(0, 3)$, $(1, 5)$, $(2, 7)$, $(3, 9)$, $(4, 11)$. Always describe how the values change: each time the input goes up by 1, the output goes up by 2 — the "× 2" step of the rule shows up in the table as the same jump of 2 every time.

Plotting the pairs. Plot each ordered pair with the input along the x-axis and the output along the y-axis. Figure 3 shows the five points from the table above. All five points lie on a **straight line** — for many simple rules the plotted points line up like this, and noticing it is part of describing the relationship.

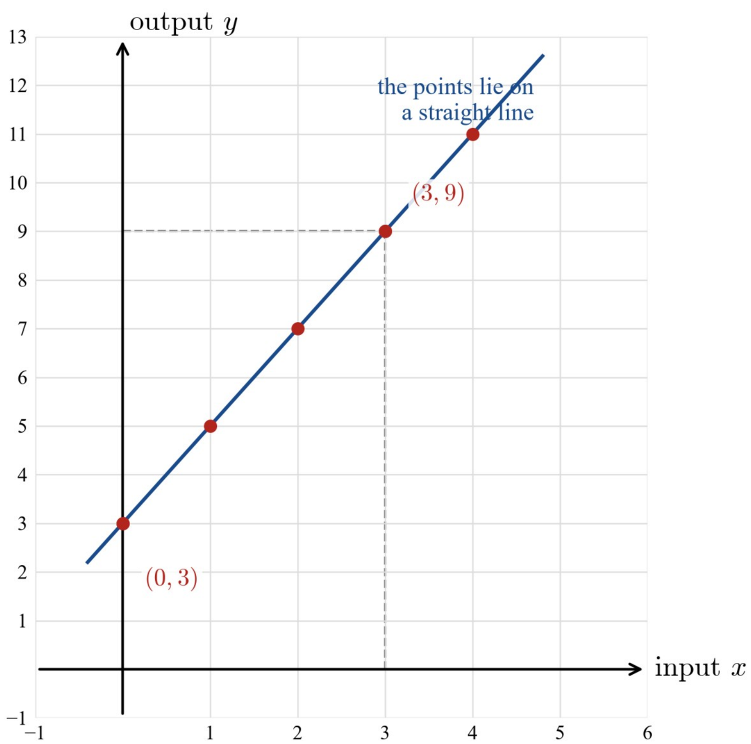

Figure 3. The ordered pairs (0, 3), (1, 5), (2, 7), (3, 9) and (4, 11) from the rule y = 2x + 3 plotted on a Cartesian plane, with the input along the x-axis and the output along the y-axis; the five points lie on a straight line, and dashed guide lines mark the point (3, 9).

The rule in symbols. If the input is x and the output is y, the rule of Figure 2 can be written $y=2x+3$, read "the output is 2 times the input, plus 3". With the rule written like this, you can find the output for *any* input without building the whole table: input 10 gives $y=2\times10+3=23$.

Growing patterns. Rules also come from patterns you can see. Figure 4 shows square tables pushed together in a row, with one seat on each side of every table and one seat at each end of the row. Counting the seats:

tables t	1	2	3	4	5
seats s	4	6	8	10	12

Each extra table adds 2 seats — one on top and one underneath — while the two end seats stay where they are. The rule in words is "double the number of tables and add 2"; in symbols it is $s=2t+2$. Plotting the pairs (Figure 5) again gives points on a straight line, and the way the pattern grows — 2 more seats for every table — is the same as the jump of 2 in the table.

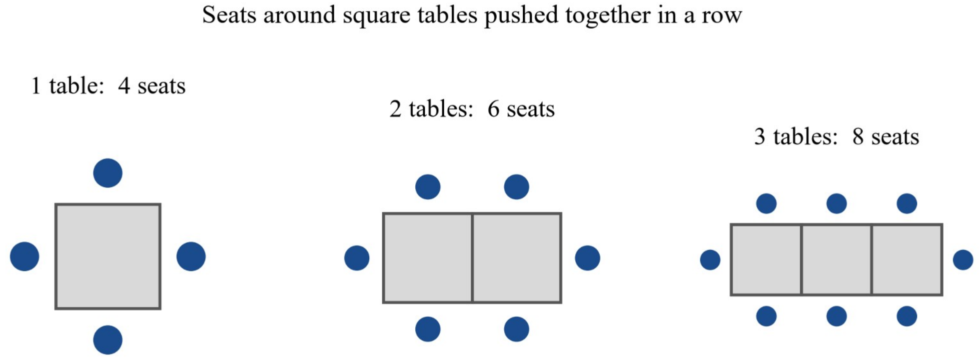

Figure 4. Square tables pushed together in a row with seats around them: 1 table has 4 seats, 2 tables have 6 seats and 3 tables have 8 seats — one seat on each side of every table and one seat at each end of the row.

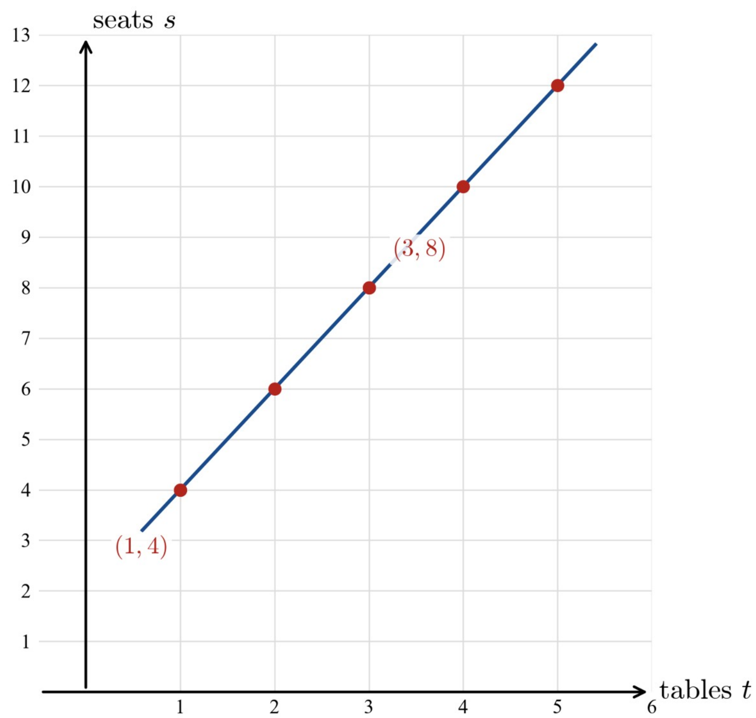

Figure 5. The pairs (1, 4), (2, 6), (3, 8), (4, 10) and (5, 12) from the tables-and-seats rule s = 2t + 2 plotted with tables t along the x-axis and seats s along the y-axis; the points lie on a straight line, with the points (1, 4) and (3, 8) labelled.

Negative inputs. Inputs can be negative numbers too. The rule $y=3-x$ with inputs $-2, -1, 0, 1, 2, 3, 4, 5$ gives outputs $5, 4, 3, 2, 1, 0, -1, -2$. Figure 6 plots them: the points still lie on a straight line, but now the outputs go *down* by 1 each time the input goes up by 1, and the line runs downhill from left to right, passing through quadrants II, I and IV.

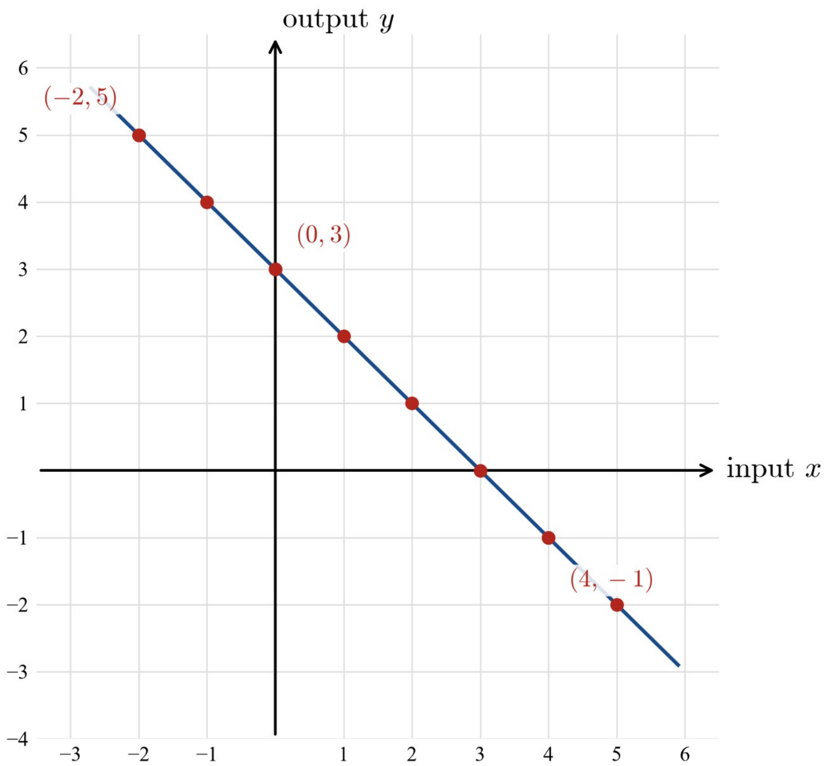

Figure 6. The pairs from the rule y = 3 − x for inputs −2 to 5 plotted on a Cartesian plane; the points lie on a straight line that falls from left to right, passing through quadrant II, quadrant I and quadrant IV, with the points (−2, 5), (0, 3) and (4, −1) labelled.

Not every relationship is a straight line. Figure 7 shows a pattern of square tiles: figure n is an $n \times n$ square of tiles, so the number of tiles is $n \times n = n^2$:

figure n	1	2	3	4	5
tiles	1	4	9	16	25

The growth is not the same each time here: as n goes up by 1, the tiles go up by 3, then 5, then 7, then 9 — the jump itself keeps getting bigger. Plotting the pairs (Figure 8) shows points that do **not** lie on a straight line: they bend upwards along a **curve**. Describing the shape of the plot — a straight line or a curve, rising or falling — is how you describe the relationship in terms of shape.

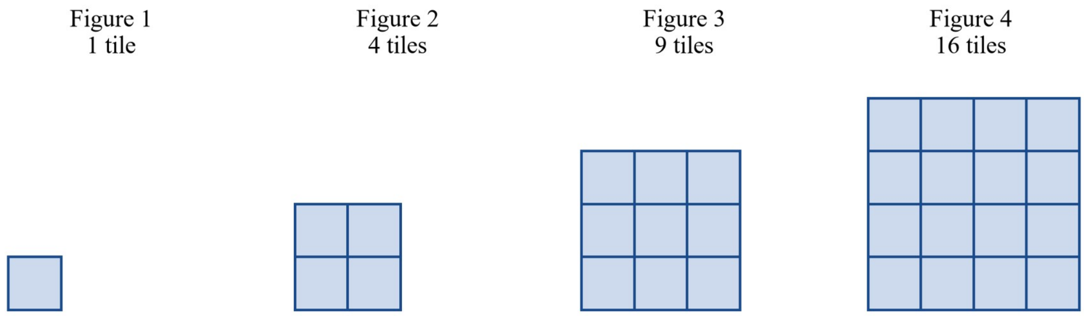

Figure 7. A growing pattern of square tiles: figure 1 is 1 tile, figure 2 is a 2 by 2 square of 4 tiles, figure 3 is a 3 by 3 square of 9 tiles and figure 4 is a 4 by 4 square of 16 tiles.

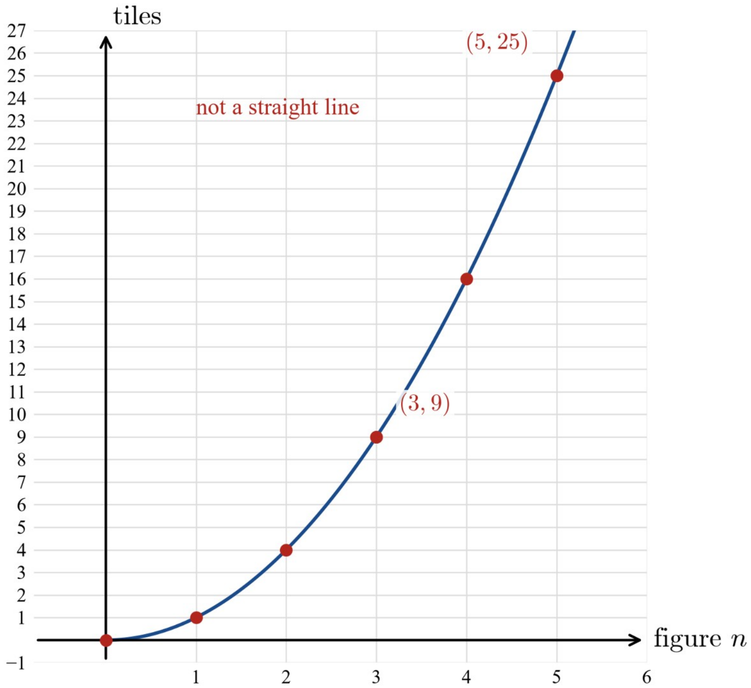

Figure 8. The pairs (0, 0), (1, 1), (2, 4), (3, 9), (4, 16) and (5, 25) from the square-tile pattern plotted with the figure number n along the x-axis and the number of tiles along the y-axis; the points do not lie on a straight line but bend upwards along a curve, with the points (3, 9) and (5, 25) labelled.

Checking answers make sense. When you build a table from a rule, check that the jumps between outputs match the rule — a jump that breaks the pattern means an arithmetic slip. When you plot, check each point against the table before you accept the picture: a point that sits away from the other points is a sign to redo that one value.

Vocabulary.

Term	Meaning here	Synonyms and related words
rule	a statement of how to turn the input into the output	—
input	the number you start with	—
output	the number the rule gives you	—
function machine	a picture of a rule as steps a number travels through	—
table of values	a table of input–output pairs from one rule	—
ordered pair	the pair (input, output) taken from one column of a table	—
coordinates, (x, y)	the pair of numbers that says where a point is on the plane	—
origin, O	the point $(0, 0)$ where the axes cross	—
quadrant	one of the four parts the axes divide the plane into	—
straight line	the shape the points lie on for rules like $y = 2x + 3$	—
curve	a bent shape, such as the plot of the square-number pattern	—
relationship	how the output changes as the input changes	—

Worked examples

Example 1 — scaffolded

Use the rule "multiply by 3, then subtract 1". (a) Build a table of values for inputs $0,1,2,3,4$. (b) Plot the ordered pairs and describe the shape they make. (c) Describe how the output changes as the input goes up by 1.

Working	Comment
Build. Input 0: $3\times0-1=-1$. Input 1: $3\times1-1=2$. Input 2: $3\times2-1=5$. Input 3: $3\times3-1=8$. Input 4: $3\times4-1=11$.	Do the multiply step first and the subtract step second, for every input.
Table. Outputs $-1,2,5,8,11$.	Each column gives an ordered pair: $(0,-1)$, $(1,2)$, $(2,5)$, $(3,8)$, $(4,11)$.
Plot. The five points lie on a straight line.	Rules of the form "multiply, then add or subtract a number" always line up like this.
Describe. The output goes up by 3 each time the input goes up by 1.	The jump of 3 is the "$\times3$" step of the rule, showing up in the table.

$\therefore$ the table has outputs $-1,2,5,8,11$; the points lie on a straight line; and the output rises by 3 for every rise of 1 in the input.

Example 2 — scaffolded

Use the tables-and-seats pattern of Figure 4. (a) Check the number of seats for 3 tables by counting top, bottom and end seats. (b) Write the rule in symbols. (c) Use the rule to find the number of seats for 10 tables, and check your answer by extending the table.

Working	Comment
Count. 3 tables: 3 seats on top, 3 underneath and 2 at the ends, so $3+3+2=8$ seats.	This matches the $t=3$ column of the table of values in the theory.
Rule. $s=2t+2$.	Two seats per table (top and bottom), plus the two end seats.
Use. $t=10$: $s=2\times10+2=22$.	The rule gives the output for any input, without drawing 10 tables.
Check. Extend the table by 2s: $t=6,7,8,9,10$ gives $s=14,16,18,20,22$.	Two different ways of finding the answer agree, which supports it.

$\therefore$ ten tables pushed together in a row seat 22 people.

Example 3 — unscaffolded

The rule is $y=3-x$. (a) Build a table of values for inputs $-2,-1,0,1,2,3,4,5$. (b) Plot the ordered pairs on the Cartesian plane. (c) Describe the relationship: the shape of the plot, and how the output changes as the input goes up by 1.

Working	Comment
$x=-2$: $y=3-(-2)=5$; then $4,3,2,1,0,-1,-2$ for $x=-1,0,1,2,3,4,5$.	Subtracting a negative input adds: $3-(-2)=3+2=5$.
The points lie on a straight line falling from left to right (Figure 6).	Negative inputs put points in quadrant II; negative outputs put points in quadrant IV.
Each time the input goes up by 1, the output goes down by 1.	The "$-x$" part of the rule makes the outputs fall.

$\therefore$ the table has outputs $5,4,3,2,1,0,-1,-2$; the points lie on a straight line that falls as the input rises.

Example 4 — unscaffolded

Use the square-tile pattern of Figure 7. (a) How many tiles are in figure 7? (b) Describe how the number of tiles grows as the figure number goes up by 1. (c) Do the plotted points of this pattern lie on a straight line? How can you tell from the table alone?

Working	Comment
(a) Figure 7 is a 7×7 square of tiles: $7\times7=49$ tiles.	The number of tiles in figure n is $n\times n=n^2$.
(b) The tiles go up by 3, then 5, then 7, then 9, … — the jump grows by 2 each time.	Unlike the tables-and-seats pattern, the jump is not the same every time.
(c) No. The jumps between outputs are not all the same, so the points cannot line up; Figure 8 shows them bending upwards along a curve.	A straight-line plot needs the same jump in the output for the same jump in the input.

∴ figure 7 has 49 tiles, and because the growth is not the same each time, the points do not lie on a straight line.

Practice questions

Scaffolded

Q1. A function machine has the rule “add 4”. Build a table of values for inputs $0,1,2,3,4,5$, and list the ordered pairs.

Q2. Use the rule “multiply by 3”. (a) Build a table of values for inputs $0,1,2,3,4$. (b) Describe how the output changes as the input goes up by 1. (c) Use the rule to find the output for input 7.

Q3. The rule is “multiply by 2, then subtract 1”. (a) Build a table of values for inputs $0,1,2,3,4$. (b) Plot the ordered pairs on the Cartesian plane. (c) Do the points lie on a straight line?

Q4. Figure 9 shows a growing pattern of tiles. (a) How many tiles are in each of figures 1, 2 and 3? (b) Build a table of values showing the number of tiles in figures 1 to 5. (c) Describe how the pattern grows. (d) How many tiles are in figure 10?

A growing pattern of tiles

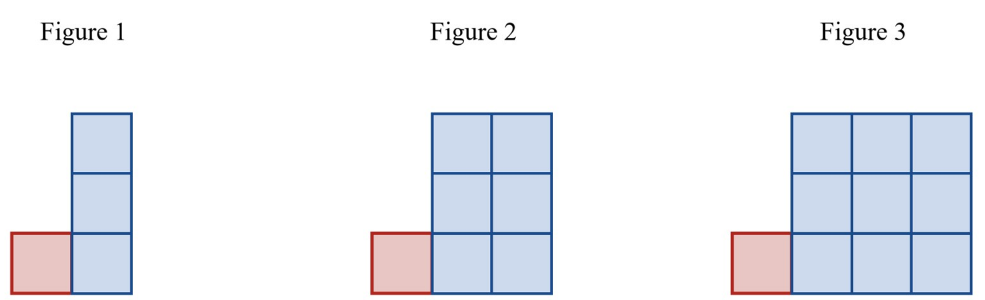

Figure 9. A growing pattern of tiles: figure 1 is one red tile with one column of three blue tiles, figure 2 is one red tile with two columns of three blue tiles, and figure 3 is one red tile with three columns of three blue tiles.

Q5. Plot these points on the Cartesian plane, and name the quadrant each one lies in, or the axis it sits on: $A(2,4)$, $B(-3,4)$, $C(-3,-2)$, $D(5,-1)$, $E(0,-3)$.

Q6. Say whether each statement is true or false. (a) For the rule $y=2x+3$, input 3 gives output 9. (b) The ordered pair from input 0 and output 3 is $(3,0)$. (c) For the rule $y=3-x$, input -2 gives output 5. (d) The points from a table of values for $y=x^2$ lie on a straight line.

Unscaffolded

Q7. The rule is “multiply by 4, then add 1”. Build a table of values for inputs $-2, -1, 0, 1, 2$, plot the ordered pairs, and describe the shape of the plot and how the output changes as the input goes up by 1.

Q8. A sunflower is 5 cm tall and grows by 2 cm each week. (a) Build a table of values showing the height for weeks $0, 1, 2, 3, 4, 5$. (b) Write the rule in symbols, using w for weeks and h for height in cm. (c) Use the rule to find the height at week 10, and check by extending the table. (d) Describe the relationship in the context of the situation.

Q9. A table of values has inputs $1, 2, 3, 4, 5$ and outputs $6, 8, 10, 12, 14$. (a) Write the rule in words and in symbols. (b) Use the rule to find the output for input 12.

Q10. A row of triangles is built from matches, where each new triangle shares one side with the one before: 1 triangle takes 3 matches, 2 triangles take 5, and 3 triangles take 7. (a) Build a table of values for 1 to 6 triangles. (b) Write the rule in symbols, using t for triangles and m for matches. (c) How many matches does a row of 10 triangles take? (d) Describe how the pattern grows, in the context of the situation.

Q11. A pattern of dots arranged in triangles gives the numbers $1, 3, 6, 10$. (a) Write the next two numbers of the pattern. (b) Build a table of values for figures 1 to 6. (c) Plot the pairs. Do the points lie on a straight line? How can you tell from the table alone?

Q12. Zara and Tom build a table for the rule “multiply by 5, then subtract 1”. Zara says input 6 gives output 29. Tom says output 31. Who is correct, and what might the other person have done?

Q13. The rule is “start at 30, then subtract the input times itself”: $y = 30 - x^2$. (a) Build a table of values for inputs $0, 1, 2, 3, 4, 5$. (b) Plot the pairs. Is the shape a straight line or a curve? (c) Describe how the output changes as the input goes up by 1.

Q14. Make up a rule of your own of the form “multiply by a number, then add or subtract a number”. Build a table of at least five values, plot the pairs on the Cartesian plane, and describe the relationship: how the output changes, and the shape the points make. Check one plotted point against your table.

Answers

Q1. Outputs $4, 5, 6, 7, 8, 9$; pairs $(0, 4)$, $(1, 5)$, $(2, 6)$, $(3, 7)$, $(4, 8)$, $(5, 9)$.

Q2. (a) Outputs $0, 3, 6, 9, 12$; (b) the output goes up by 3 each time the input goes up by 1; (c) $3 \times 7 = 21$.

Q3. (a) Outputs $-1, 1, 3, 5, 7$; (b) plot the pairs $(0, -1)$, $(1, 1)$, $(2, 3)$, $(3, 5)$, $(4, 7)$; (c) yes.

Q4. (a) 4, 7, 10; (b) $4, 7, 10, 13, 16$; (c) each figure adds one more column of 3 tiles, so the tiles go up by 3 each time; (d) 31.

Q5. A quadrant I; B quadrant II; C quadrant III; D quadrant IV; E sits on the y-axis.

Q6. (a) true; (b) false; (c) true; (d) false.

Q7. Outputs $-7, -3, 1, 5, 9$; the points lie on a straight line; the output goes up by 4 each time the input goes up by 1.

Q8. (a) $5, 7, 9, 11, 13, 15$; (b) $h = 2w + 5$; (c) 25 cm; (d) the sunflower starts at 5 cm and grows by 2 cm every week, so the points lie on a straight line.

Q9. (a) "double the input and add 4", $y = 2x + 4$; (b) 28.

Q10. (a) $3, 5, 7, 9, 11, 13$; (b) $m = 2t + 1$; (c) 21; (d) each new triangle shares one side with the last, so it adds only 2 more matches.

Q11. (a) 15 and 21; (b) $1, 3, 6, 10, 15, 21$; (c) no — the jumps between outputs are $2, 3, 4, 5, 6$, which are not all the same.

Q12. Zara: $5 \times 6 - 1 = 29$. Tom most likely added the 1 instead of subtracting it: $5 \times 6 + 1 = 31$.

Q13. (a) $30, 29, 26, 21, 14, 5$; (b) a curve; (c) the output goes down by 1, then 3, then 5, then 7, then 9 — it falls faster and faster.

Q14. Answers will vary; see fully-worked solutions for one possible response.

Fully-worked solutions

Q1. The rule adds 4 to each input: $0 + 4 = 4$, $1 + 4 = 5$, $2 + 4 = 6$, $3 + 4 = 7$, $4 + 4 = 8$, $5 + 4 = 9$. The ordered pairs are $(0, 4)$, $(1, 5)$, $(2, 6)$, $(3, 7)$, $(4, 8)$, $(5, 9)$. Check: the output goes up by 1 when the input goes up by 1, matching the "add 4" rule's steady jump.

Q2. (a) $3 \times 0 = 0$, $3 \times 1 = 3$, $3 \times 2 = 6$, $3 \times 3 = 9$, $3 \times 4 = 12$. (b) The output goes up by 3 each time the input goes up by 1. (c) $3 \times 7 = 21$. Check by extending the table: $12, 15, 18, 21$ for inputs $5, 6, 7$.

Q3. (a) $2 \times 0 - 1 = -1$; $2 \times 1 - 1 = 1$; $2 \times 2 - 1 = 3$; $2 \times 3 - 1 = 5$; $2 \times 4 - 1 = 7$. (b) Plot $(0, -1)$, $(1, 1)$, $(2, 3)$, $(3, 5)$, $(4, 7)$ with the input along the x-axis and the output along the y-axis. (c) Yes — the points lie on a straight line, and the table shows the same jump of 2 each time.

Q4. (a) Counting the tiles: figure 1 has 4, figure 2 has 7, figure 3 has 10. (b) Each new figure adds one column of 3 tiles: $4, 7, 10, 13, 16$ for figures 1 to 5. (c) The pattern grows by 3 tiles each time — one more column of three blue tiles, with the red tile staying. (d) Figure 10: start at 4 and add 3 nine times, $4 + 3 \times 9 = 31$. Check with the rule in symbols, tiles $= 3n + 1$: $3 \times 10 + 1 = 31$. ✓

Q5. $A(2, 4)$: 2 right, 4 up — quadrant I. $B(-3, 4)$: 3 left, 4 up — quadrant II. $C(-3, -2)$: 3 left, 2 down — quadrant III. $D(5, -1)$: 5 right, 1 down — quadrant IV. $E(0, -3)$: x-coordinate 0, so it sits on the y-axis, 3 below O.

Q6. (a) True — $2\times3+3=9$. (b) False — the input comes first: the pair is $(0,3)$. (c) True — $3-(-2)=3+2=5$. (d) False — the jumps between outputs grow ($1,3,5,7,\ldots$), so the points bend along a curve (Figure 8).

Q7. $4\times(-2)+1=-7$; $4\times(-1)+1=-3$; $4\times0+1=1$; $4\times1+1=5$; $4\times2+1=9$. Plot $(-2,-7)$, $(-1,-3)$, $(0,1)$, $(1,5)$, $(2,9)$ with the input along the x-axis and the output along the y-axis: the points lie on a straight line, and the output goes up by 4 each time the input goes up by 1 — the "$\times4$" step of the rule.

Q8. (a) Week 0: 5 cm; then add 2 each week: $5,7,9,11,13,15$. (b) $h=2w+5$. (c) $h=2\times10+5=25$ cm. Check by extending the table by 2s to week 10: $17,19,21,23,25$. ✓ (d) The sunflower starts at 5 cm and grows by the same 2 cm every week, so the heights rise steadily and the plotted points lie on a straight line.

Q9. (a) The outputs go up by 2 each time, and doubling each input gives $2,4,6,8,10$ — four short of the outputs — so the rule is "double the input and add 4": $y=2x+4$. Check: $2\times5+4=14$. ✓ (b) $2\times12+4=28$.

Q10. (a) $3,5,7,9,11,13$ for $t=1$ to 6. (b) Each triangle after the first adds 2 matches, and one triangle takes 3: $m=2t+1$. (c) $m=2\times10+1=21$ matches. Check by extending the table: $15,17,19,21$ for $t=7,8,9,10$. ✓ (d) Every new triangle shares one side with the triangle before it, so it needs only 2 extra matches instead of 3.

Q11. (a) The jumps are 2, then 3, then 4, so the next jumps are 5 and 6: $10+5=15$ and $15+6=21$. (b) $1,3,6,10,15,21$ for figures 1 to 6. (c) Plot the pairs: the points bend upwards, not a straight line. From the table alone: the jumps $2,3,4,5,6$ are not all the same, and a straight line needs the same jump every time.

Q12. Zara is correct. The rule says multiply first, then subtract: $5\times6=30$, then $30-1=29$. Tom's 31 is $30+1$, so he most likely added the 1 instead of subtracting it. Check with a smaller input: input 2 gives $5\times2-1=9$, not 11 — adding would give the wrong answer there too.

Q13. (a) $30-0\times0=30$; $30-1=29$; $30-4=26$; $30-9=21$; $30-16=14$; $30-25=5$. (b) The points bend downwards along a curve, not a straight line. (c) The output falls by 1, then 3, then 5, then 7, then 9 — it falls faster and faster as the input goes up.

Q14. Answers will vary. One possible response: rule "multiply by 2, then subtract 3". Table for inputs 0 to 4: $-3,-1,1,3,5$. Plot $(0,-3)$, $(1,-1)$, $(2,1)$, $(3,3)$, $(4,5)$: the points lie on a straight line, and the output goes up by 2 each time the input goes up by 1. Check one point against the table: $(3,3)$ — $2\times3-3=3$. ✓

Teacher note

- **Ownership and boundaries (D4.9).** 3D generates tables of values from rules and from visually growing patterns, plots the pairs, and may observe that the points lie on a straight line (`VC2M7A05.E01`). It does not find a gradient, an intercept or an equation of a line: the words *gradient*, *slope*, *intercept*, *y-intercept*, *x-intercept* and the form $y=mx+c$ do not appear in student-facing text, and rules in symbols (such as $y=2x+3$) are presented only as function rules read back in words, per E02. E02's circumference example is deliberately not used: $C=\pi d$ is established later, in 4C (D4.1), so this section uses same-level contexts (tables and seats, matches, plant growth) instead.
- **Level 6 recall (D3).** Number patterns and their rules (`VC2M6A01`), coordinates (`VC2M6N01`) and plotting points in all four quadrants (`VC2M6SP02`) are assumed. They are recalled in one paragraph and Figure 1, and maintained inside the questions — not re-taught. The plane here is the same plane 5D transforms points on.
- **Elaboration coverage.** E01: WE1, WE3, Q3, Q6, Q7 (plotting from tables and recognising the straight line). E02: the "rule in symbols" paragraph, WE2, WE4 and Q8–Q13 (variables, a general rule, and the output for any input). E03: function machines (Figure 2, Q1–Q3), ordered pairs, and describing the graph in terms of shape (WE4, Q11, Q13). E04: diagrams forming linear growth patterns, representing them in tables, and describing the growth in context (Figure 4, WE2, Q4, Q8, Q10).
- **Digital tools (D9).** `VC2M7A05` names no digital tools, so none are required; plotting is by hand on squared paper, and no product is named in student-facing text.
- **Reasonableness habit (P7).** The check rows of WE1–WE2, Q3(c)'s line test, Q12's error analysis and Q14's point-against-table check all ask students to check an answer makes sense before accepting it, per TOC §5.2.

- **Figures.** All nine figures are generated by `src/gen_3d.py` from the exact values stated in the text; plotted points are computed from the same rules as the tables, so a figure cannot disagree with the prose.
- **Vocabulary gate.** `src/vocab_gate_3d.py` checks the student-facing text of this section against the ladder at ceiling Yr7, with the curriculum-word exemption built from `VC2M7A05`'s content description, achievement-standard extract and elaborations, plus the small set of geometric subject terms already established at Level 6 (coordinates, origin, quadrant, axis), each defined at first use. Gate status: CLEAN.
- **No D11 content.** None of `VC2M7A05.E01–.E04` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.
- **Forward.** Gradient, intercepts and equations of lines are Level 9 work and do not appear here; the tables and plots of this subtopic are the ground those ideas grow from.

Reading and describing graphs of relationships built from authentic data

Content description `VC2M7A04`: investigate, interpret and describe relationships between variables represented in graphs of functions developed from authentic data

Achievement standard (extract): Students use algebraic expressions to represent situations, describe the relationships between variables from authentic data and substitute values into formulas to determine unknown values.

Theory

What this subtopic is about. A graph is a picture of a **relationship**: it shows how one **variable** (a quantity that can change) moves as another variable moves. Subtopic 3D teaches the building side — turning tables of data into plots. Here the graphs are already drawn, and the skills are **reading** them: look up values, watch the line rise and fall, **describe** in words what is happening, and **interpret** what that says about the real situation behind the data. Some graphs in this subtopic are built from **authentic data** — real measurements with a named source — and some are drawn from everyday situations, like a walk to school.

The parts of a graph. Figure 1 is built from real temperature data and labels the parts every graph has. The **title** says what the graph shows. The **vertical axis** carries one variable — here the temperature in degrees Celsius (°C) — and the **horizontal axis** carries the other, here the month. The **scale** is the counting the axes use; this one runs from 10 to 35 in fives. A **point** on the line is one pair of values: the red point says that in July the mean maximum temperature at Mildura Airport is 15.5 °C.

Source: Bureau of Meteorology, "Climate statistics for Australian locations", station 076031 (Mildura Airport), monthly means, all years of record 1946–2026; retrieved 20 August 2026.

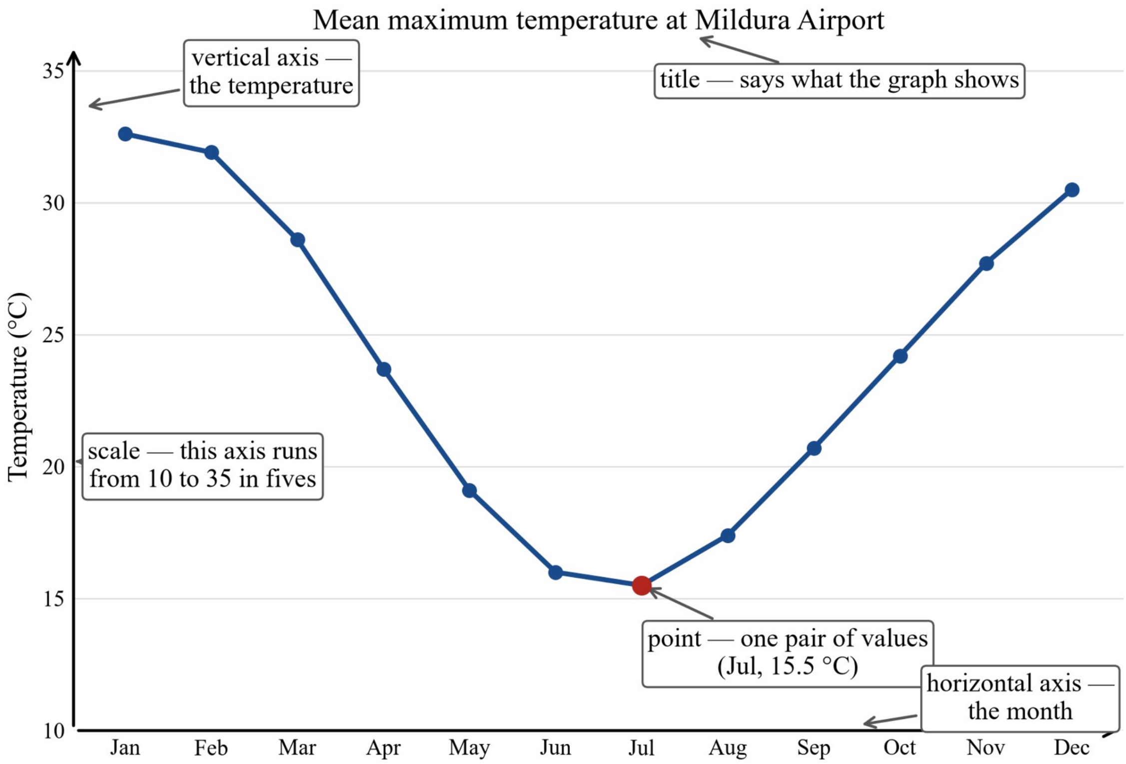

Figure 1. A line graph of mean maximum temperature at Mildura Airport with five parts labelled by callouts: the title; the vertical axis, which carries the temperature in degrees Celsius; the horizontal axis, which carries the month; the scale, running from 10 to 35 in fives; and one point marked in red at July, 15.5 degrees Celsius.

Rising, falling and horizontal. Reading left to right, a line that climbs says the vertical variable is increasing as we move along the horizontal axis; a line that falls says it is decreasing. A **horizontal segment** — a flat part — says the vertical quantity is not changing while the horizontal one moves on. On a **travel graph** (distance from home against time), a horizontal segment means the person has stopped: the time keeps growing, but the distance from home stays the same.

Slope in words. The **slope** of a line is its tilt. Where a line is steeper, the vertical quantity changes faster; where it is gentler, the change is slower. In this subtopic slope is described in words only — steeper, gentler, faster, slower, stopped, returning — and is never turned into a number.

Two lines, one frame. When two lines share a frame, the graph invites a comparison: which line is higher, how wide is the gap between them, and do they rise and fall together? Figure 5 pairs the mean maximum with the mean minimum temperature at the same place, so the gap between the two lines is the day-to-night temperature swing for each month.

The words of this subtopic.

Word	Meaning
variable	a quantity that can change, like the month or the temperature
relationship	how one variable moves as the other moves
interpret	work out what the graph is saying about the real situation
describe	say in words what the line does: rises, falls, or stays level
slope	the tilt of a line; steeper means a faster change
horizontal segment	a flat part of a graph, where the vertical quantity does not change
scale	the counting an axis uses, like “in fives” or “in tens”
authentic data	real measurements, with a named source
travel graph	a graph of distance from home against time

Worked examples

Example 1 — scaffolded

Figure 2 shows the mean maximum temperature at Mildura Airport, month by month. (a) Which month is hottest, and what is the mean maximum temperature that month? (b) Which month is coolest, and what is the mean maximum temperature that month? (c) Describe what happens to the mean maximum temperature from February to July, and from August to January.

Working	Comment
Read the top. The highest point is January, at 32.6 °C.	The highest point of the line is the hottest month.
Read the bottom. The lowest point is July, at 15.5 °C.	The lowest point of the line is the coolest month.
February to July: the line falls. Each month’s point sits below the one before it, from 31.9 °C down to 15.5 °C.	A falling line says the temperature decreases month by month.
August to January: the line rises. The points climb from 17.4 °C back up to 32.6 °C.	A rising line says the temperature increases again.

∴ The hottest month is January (32.6 °C) and the coolest is July (15.5 °C); the mean maximum temperature falls from February to July and rises from August to January — one cycle over the year, peaking in the Australian summer.

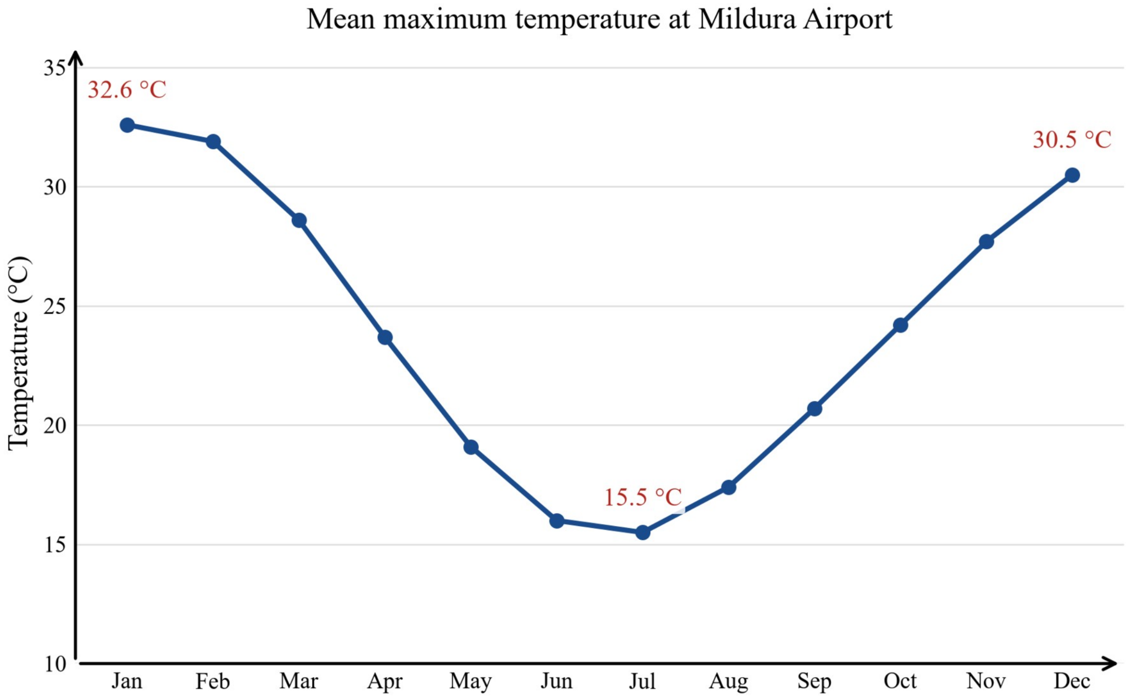

Figure 2. A line graph of mean maximum temperature at Mildura Airport for each month; the line starts high at 32.6 degrees Celsius in January, falls to a low of 15.5 degrees Celsius in July, and climbs back to 30.5 degrees Celsius in December, with those three values labelled in red.

Example 2 — scaffolded

Figure 3 is a travel graph for one student's day. (a) How far from home is the school? (b) What is the student doing during the horizontal segment B, and for how long? (c) Which is faster — the walk to school (A) or the walk home (C)? How does the graph show it?

Working	Comment
Read the height of B. The horizontal segment sits at 1.5 km, so the school is 1.5 km from home.	While the student is at school, the distance from home never changes.
Time the flat part. B runs from 20 minutes to 150 minutes: 150 − 20 = 130 minutes at school.	A horizontal segment means stopped; the time axis says for how long.
Compare A and C. A covers 1.5 km in 20 minutes; C covers the same 1.5 km in 165 − 150 = 15 minutes, and C is the steeper line.	The same distance in less time means faster — on the graph, the steeper line.

∴ The school is 1.5 km from home; the student stays at school for 130 minutes; and the walk home is faster, because its line is steeper (1.5 km in 15 minutes, not 20).

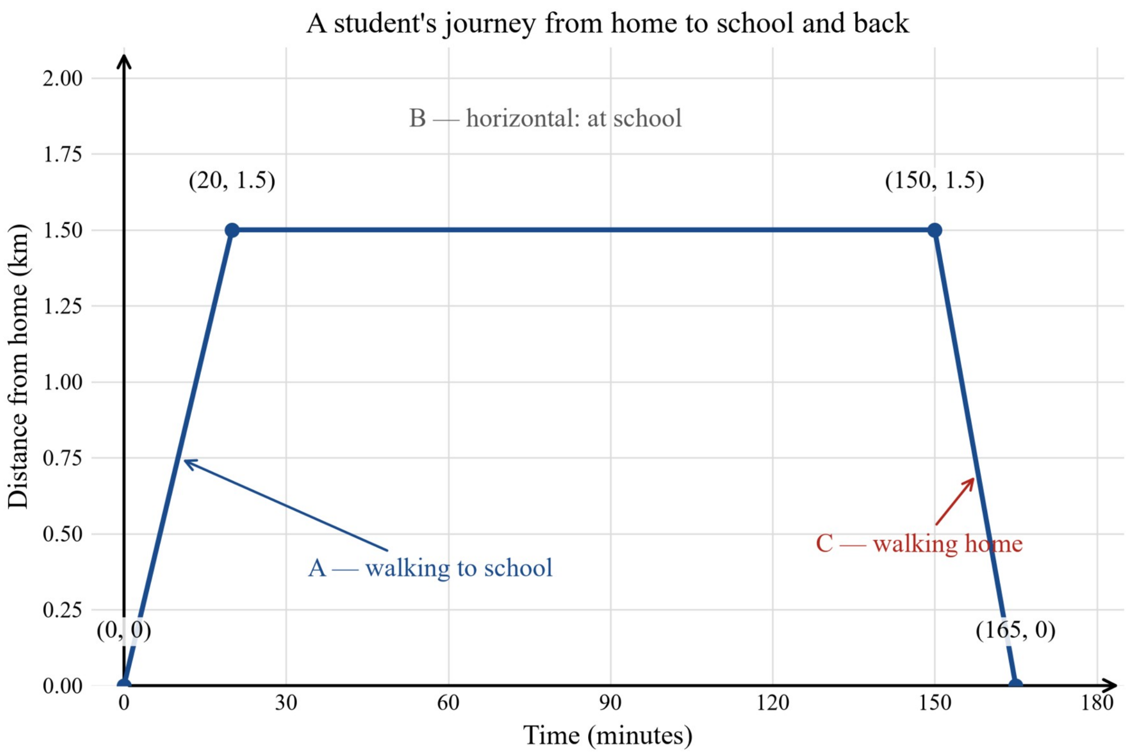

Figure 3. A travel graph of one student's day: the line rises from (0, 0) to (20, 1.5), labelled A, walking to school; runs horizontal at 1.5 km from 20 to 150 minutes, labelled B, at school; then falls to (165, 0), labelled C, walking home.

Example 3 — unscaffolded

Water is poured into the bottle of Figure 4 at a steady flow, and the graph shows the height of the water against the volume poured in. (a) Explain in words why the graph is two straight parts, with the second part steeper than the first. (b) Read the height of the water when 600 mL has been poured.

Working	Comment
(a) The bottom is wide, so each 100 mL poured lifts the water only a little: the height rises slowly, a gentler slope. The neck is narrow, so the same 100 mL lifts the water much more: the height rises faster, a steeper slope.	The slope records how fast the height grows; the bottle's shape changes that speed at 480 mL, where the water reaches the neck.
(b) 600 mL is past the neck point (480 mL, height 8 cm). The extra 600 − 480 = 120 mL fills the narrow neck, where 30 mL gives 1 cm of height, so 120 mL gives 4 cm: 8 + 4 = 12 cm.	Check against the bottle picture: the red arrow marks a water height of 12 cm.

∴ The graph bends steeper where the bottle narrows, and at 600 mL the water stands 12 cm high.

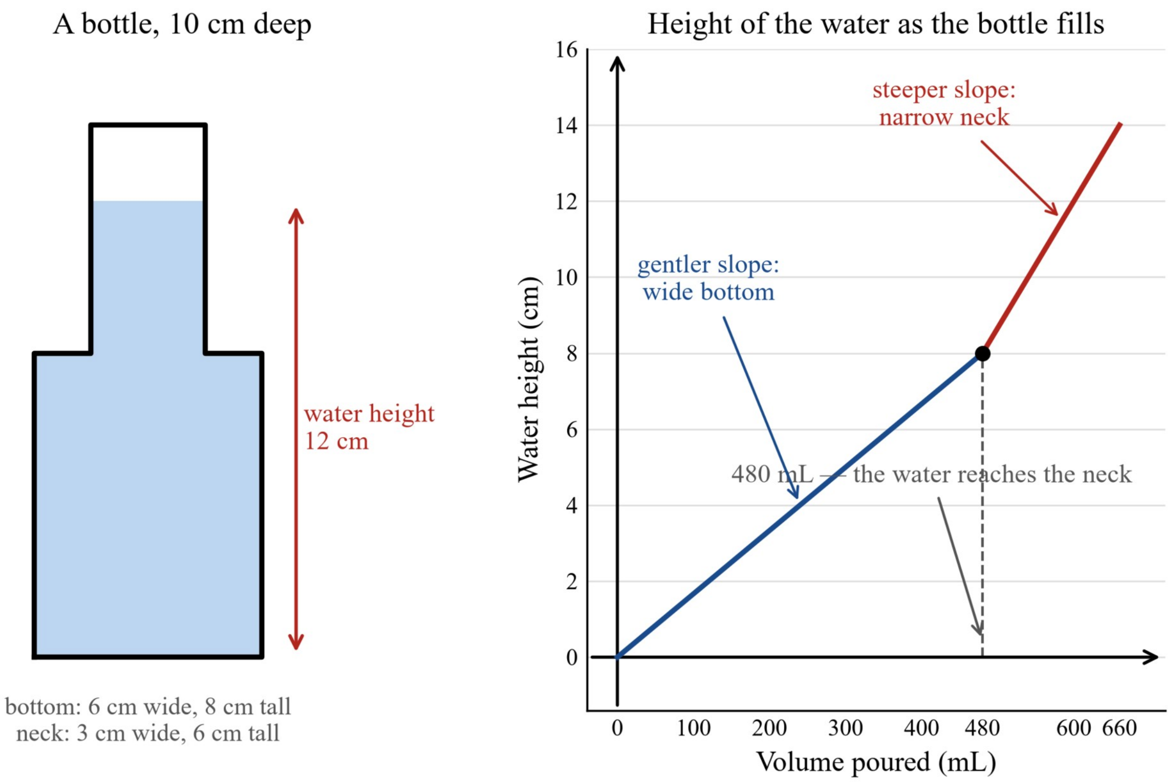

Figure 4. Left: the cross-section of a bottle 10 cm deep with a wide bottom (6 cm wide, 8 cm tall) and a narrow neck (3 cm wide, 6 cm tall), filled with water to a height of 12 cm. Right: a graph of water height against volume poured, rising with a gentler slope from (0, 0) to (480, 8), then with a steeper slope from (480, 8) to (660, 14), with the bend at 480 mL marked.

Example 4 — unscaffolded

Figure 5 shows the mean maximum and the mean minimum temperature at Mildura Airport on one frame. (a) Read the mean maximum and the mean minimum temperature for July. (b) Find the gap between them for July. (c) Which month has the biggest gap? What does that say about summer and winter days at Mildura?

Working	Comment
(a) July: mean maximum 15.5 °C, mean minimum 4.4 °C.	One vertical line through July meets each line once.
(b) 15.5 − 4.4 = 11.1 °C.	The gap between the two lines is the day-to-night swing.
(c) January's gap is 32.6 − 16.9 = 15.7 °C, the widest on the graph; June's is the narrowest at 16.0 − 5.2 = 10.8 °C.	Compare the gaps by eye first, then check the two likely months with subtraction.

∴ In July the day-to-night swing is 11.1 °C; the biggest swing is January's 15.7 °C, so at Mildura the gap between day and night temperatures is bigger in summer than in winter.

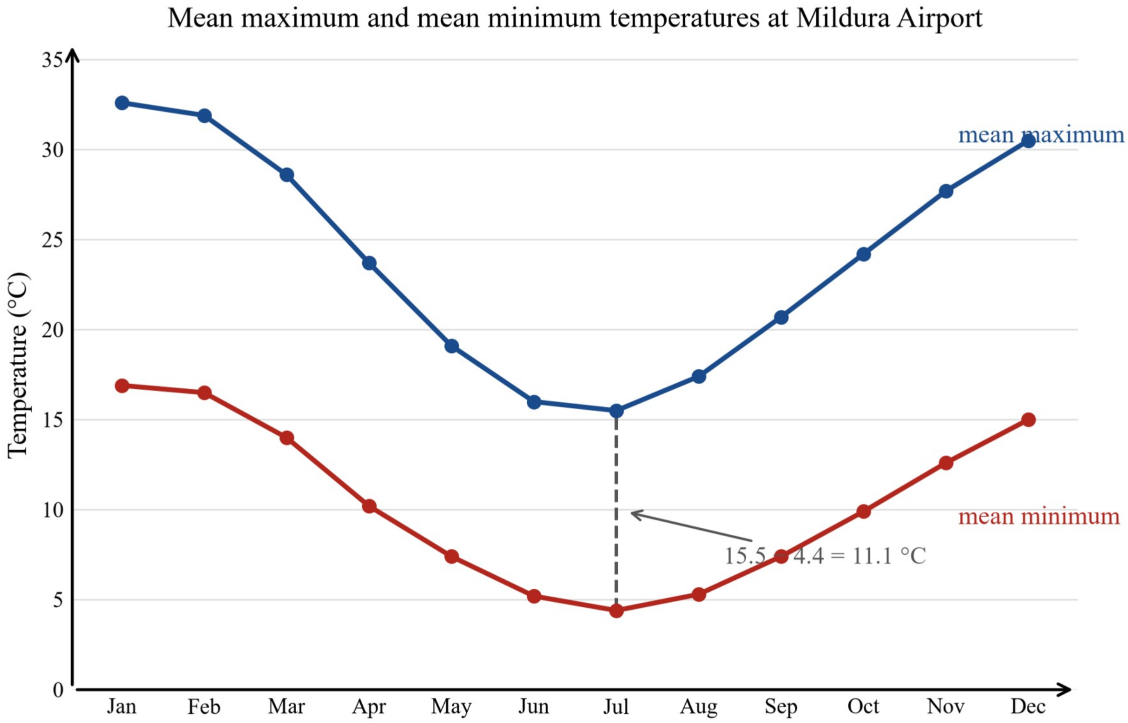

Figure 5. Two lines on one frame for Mildura Airport: the mean maximum temperature in blue and the mean minimum in red, both highest in January and lowest in July; a dashed vertical line joins the July pair, and the gap is labelled 15.5 minus 4.4 equals 11.1 degrees Celsius.

Figures for the practice questions. The remaining five figures are used by the questions below: temperature and rainfall together (Figure 6), two journeys to one school (Figure 7), a longer journey to tell as a story (Figure 8), one household's electricity use (Figure 9), and four small travel graphs to match with stories (Figure 10).

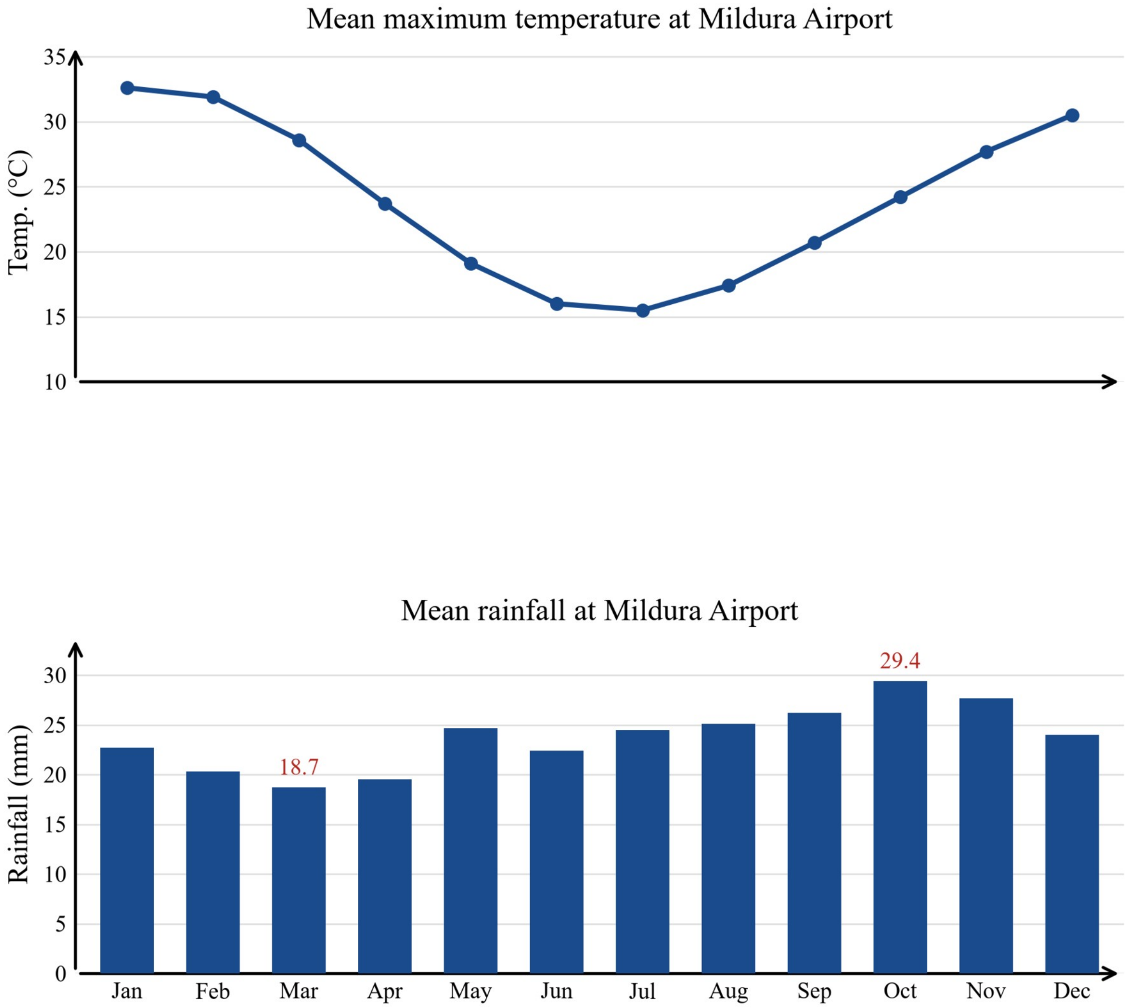

Figure 6. Two panels for Mildura Airport sharing the months on the horizontal axis: the top panel is a line graph of the mean maximum temperature, highest in January and lowest in July; the bottom panel is a bar chart of the mean rainfall, about even across the year with the tallest bar in October (29.4 mm) and the shortest in March (18.7 mm), both labelled in red.

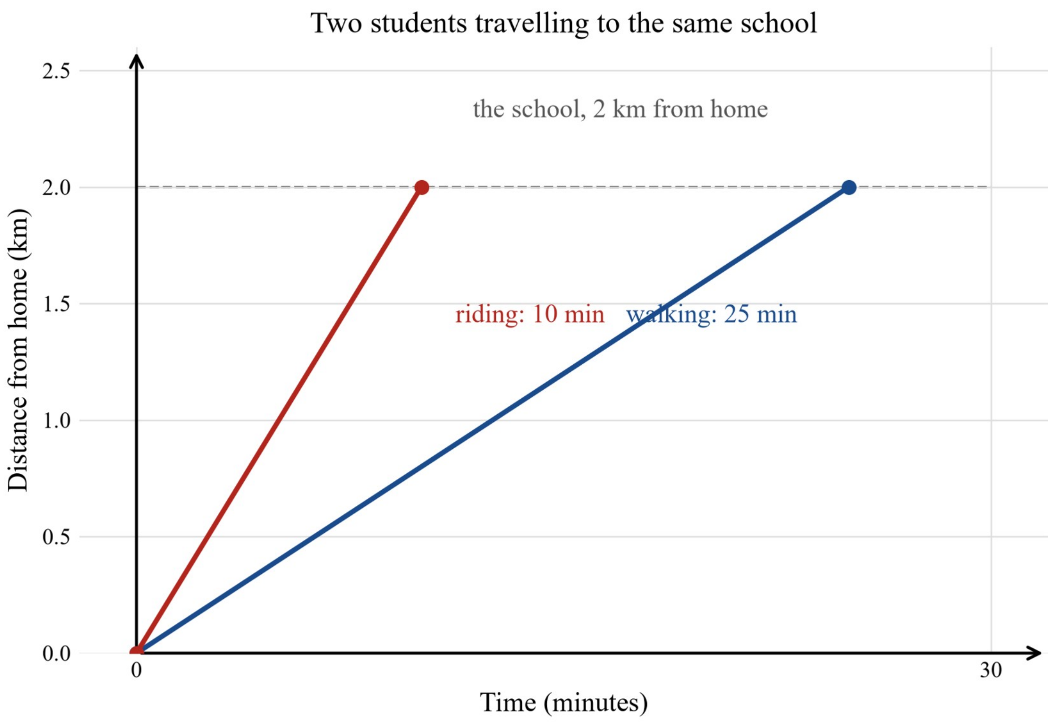

Figure 7. Two lines from (0, 0) to the same height of 2 km, marked “the school, 2 km from home”: a steep red line reaching it in 10 minutes, labelled riding, and a gentler blue line reaching it in 25 minutes, labelled walking.

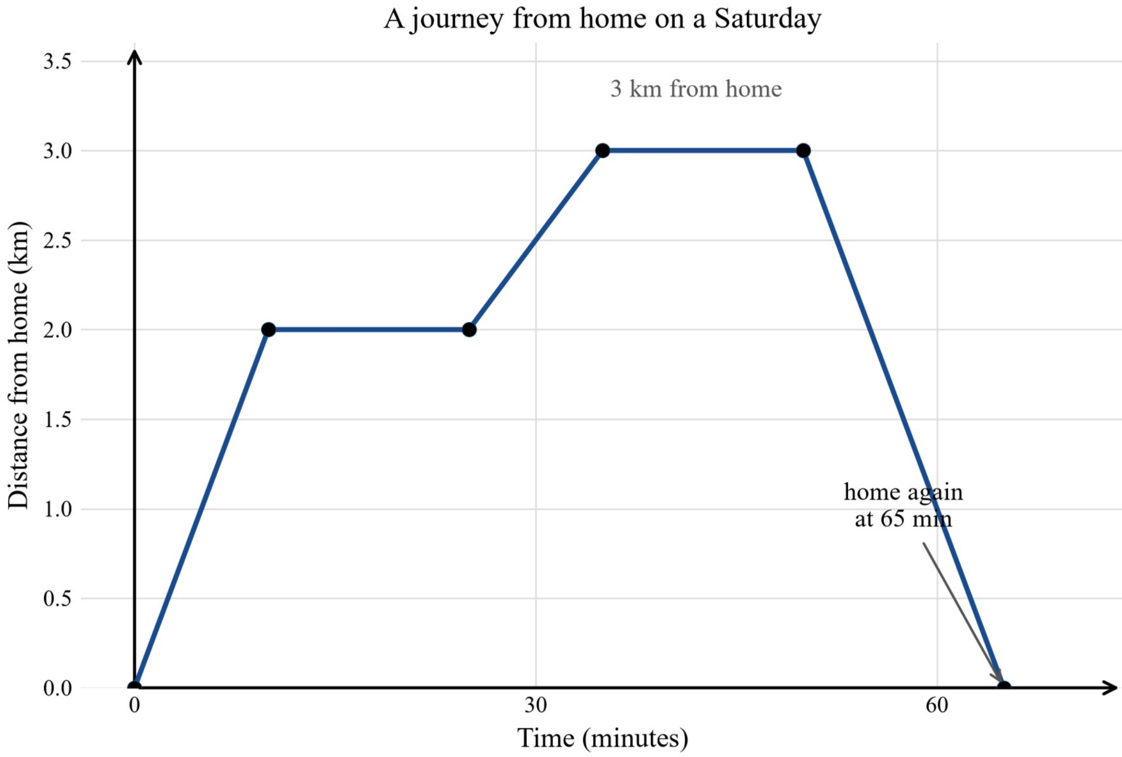

Figure 8. A travel graph of a journey from home on a Saturday: the line rises from (0, 0) to (10, 2), runs horizontal to (25, 2), rises to (35, 3), runs horizontal to (50, 3), labelled "3 km from home", and falls to (65, 0), labelled "home again at 65 min".

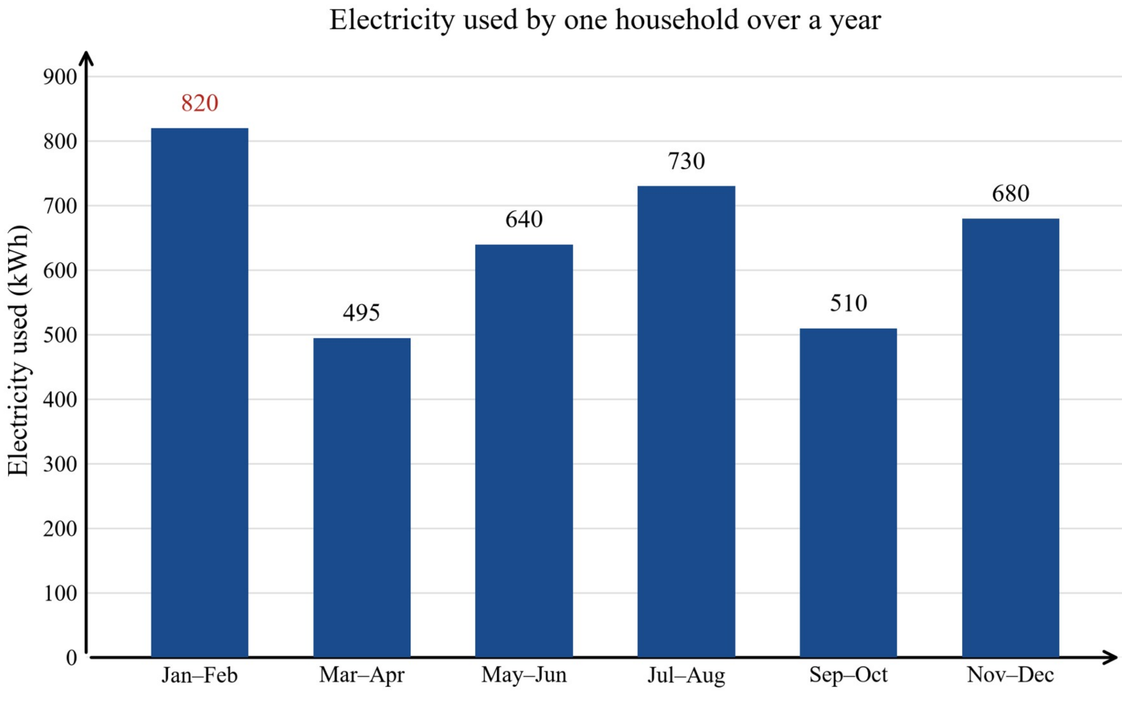

Figure 9. A bar chart of the electricity used by one household over a year, in six two-month blocks: Jan–Feb 820 kWh (the tallest, in red), Mar–Apr 495, May–Jun 640, Jul–Aug 730, Sep–Oct 510, Nov–Dec 680.

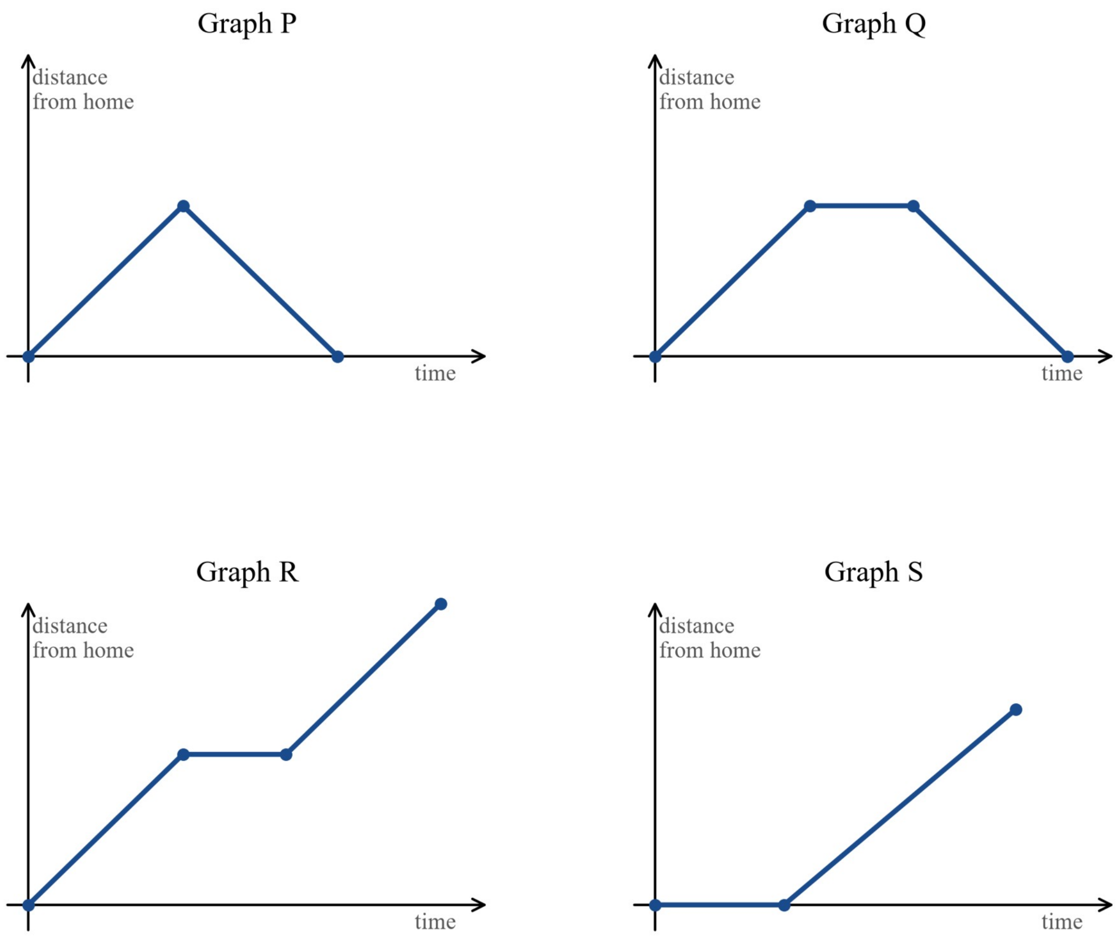

Figure 10. Four small travel graphs of distance from home against time: graph P rises to a peak and falls straight back to the axis; graph Q rises, runs horizontal, then falls back to the axis; graph R rises, runs horizontal, then rises again at the same slope; graph S runs horizontal along the axis and then rises.

Practice questions

Scaffolded

Q1. Figure 1 labels five parts of a graph. Name the part that: (a) says what the graph shows; (b) carries the months; (c) carries the temperatures; (d) tells you that the vertical axis counts in fives; (e) gives one pair of values, (Jul, 15.5 °C).

Q2. Use Figure 2. (a) Read the mean maximum temperature for December. (b) In which months is the mean maximum above 30 °C? (c) Between which two months side by side does the mean maximum fall the most? (d) In one sentence, describe the shape of the graph across the whole year.

Q3. Use the travel graph of Figure 3. (a) How long does the student take to walk to school? (b) For how long does the graph say the student is at school? (c) How does the graph show that the walk home is faster than the walk there? (d) At how many minutes after leaving home is the student home again?

Unscaffolded

Q4. Figure 7 shows two students travelling from home to the same school, one walking and one riding. Write three things the graph tells you about their journeys, comparing the two lines.

Q5. Use Figure 4. (a) Water is poured into the bottle at a steady flow. Why does the height graph have a gentler slope first and a steeper slope after 480 mL? (b) Read the height of the water when 600 mL has been poured. (c) Imagine a bottle that is the same width all the way up, like a tin. Describe in words what the height graph would look like for that bottle.

Q6. Use the travel graph of Figure 8. (a) How far from home is the first stop, and how long does the person stay there? (b) What is the greatest distance from home on this journey? (c) When is the person home again? (d) The first 10 minutes cover 2 km, and the trip home covers 3 km in 15 minutes. Is one of these parts faster, or are they the same pace? Justify your answer with the graph.

Q7. Use Figure 5. (a) Read the mean minimum temperature for January. (b) Find the gap between the mean maximum and the mean minimum for January. (c) A gardener says: “At Mildura, the gap between the day and night temperatures is about the same in every month.” Do the graphs support this? Use two months in your answer.

Q8. Figure 6 shows the temperature and the rainfall at Mildura Airport in two panels. A friend looks at them and claims: “Mildura gets most of its rain in the hot months.” Do the graphs support the claim? Quote at least two rainfall values in your answer.

Q9. Figure 9 shows the electricity used by one household over a year, in two-month blocks. The bars are measured in kWh, the unit an electricity meter counts in. (a) Which block used the most electricity, and which used the least? (b) Find the change in use from May–June to July–August, and from July–August to September–October, saying for each whether it is an increase or a decrease. (c) The tall bars line up with the hottest and the coldest months of Figure 2. Suggest in one sentence why this household uses more electricity in January–February and in July–August. (d) Over the whole year, does the use simply rise, simply fall, or neither? Explain.

Q10. Figure 10 shows four travel graphs, P, Q, R and S. Match each story below to one graph, and give a reason for each match. (i) “I waited at home for a while, then walked away.” (ii) “I walked to the letterbox and straight back, at the same pace both ways.” (iii) “I walked to the park, stayed for a while, then walked home.” (iv) “I walked for a while, stopped to rest, then kept walking at the same pace and did not come back.”

Q11. Sketch a travel graph for Mia’s trip to school: she walks from home for 5 minutes to her bus stop, 0.5 km away; she waits 10 minutes; then the bus takes 15 minutes to cover the last 4 km to school. List the four points your sketch must pass through, and say which segment is steeper — the walk or the bus.

Q12. A website claims: “The mean maximum temperature at Mildura falls every month of the year.” Use Figure 2 to explain why this cannot be right.

Q13. True or false? Give a reason for each. The statements are about travel graphs in general. (a) A horizontal segment on a travel graph means the person is moving slowly. (b) On a travel graph, a steeper line means a faster change in distance from home. (c) On a travel graph, a falling line means the person is heading back towards home.

Q14. Write the story of Figure 8 as a short paragraph: where the person goes, how long they stay at each stop, and how the trip home compares with the rest of the day.

Answers

Q1. (a) the title; (b) the horizontal axis; (c) the vertical axis; (d) the scale; (e) the point (the red one).

Q2. (a) 30.5 °C; (b) January, February and December; (c) March to April, a fall of 4.9 °C (28.6 → 23.7 °C); (d) sample: the line falls from a summer high in January to a winter low in July, then rises back to a summer high — one cycle over the year.

Q3. (a) 20 minutes; (b) 130 minutes (150 − 20); (c) C is steeper: the same 1.5 km in 15 minutes instead of 20; (d) 165 minutes.

Q4. Sample: both students cover the same 2 km, because both lines end at the same height; the rider takes 10 minutes and the walker 25 minutes; the rider's line is steeper, so the rider is faster.

Q5. (a) The wide bottom means each 100 mL lifts the height only a little (gentler slope); in the narrow neck the same 100 mL lifts the height much more (steeper slope). (b) 12 cm. (c) One straight line with the same slope all the way up, because the width — and so the speed the height grows — never changes.

Q6. (a) 2 km from home, staying 15 minutes (from 10 to 25 minutes); (b) 3 km; (c) at 65 minutes; (d) the same pace — each part covers 1 km every 5 minutes, and the two lines have the same tilt.

Q7. (a) 16.9 °C; (b) 32.6 − 16.9 = 15.7 °C; (c) no — the gap is 15.7 °C in January but only 10.8 °C in June (16.0 − 5.2), so the gap changes through the year.

Q8. No — the rainfall bars are about even, between 18.7 and 29.4 mm; the wettest month is October (29.4 mm), a spring month, and the driest is March (18.7 mm), a summer month, so the rain does not peak in the hot months.

Q9. (a) most: January–February, 820 kWh; least: March–April, 495 kWh; (b) 640 → 730 is an increase of 90 kWh; 730 → 510 is a decrease of 220 kWh; (c) sample: heating and cooling — the hot summer days and the cold winter nights both call for more electricity; (d) neither — the bars rise and fall through the year, following the seasons.

Q10. (i) S; (ii) P; (iii) Q; (iv) R.

Q11. (0, 0), (5, 0.5), (15, 0.5), (30, 4.5); the bus segment is steeper than the walk.

Q12. The line falls only from January to July; from July to December it rises, so "falls every month" is false.

Q13. (a) false — a horizontal segment means the distance from home is not changing, so the person is stopped, not moving slowly; (b) true — steeper means the distance changes faster; (c) true — a falling line means the distance from home is getting smaller.

Q14. Sample: the person leaves home and walks away for 10 minutes, stopping 2 km out; they stay there 15 minutes; then they walk on for 10 minutes to a spot 3 km from home and stay 15 minutes; finally they head straight home, covering the 3 km in 15 minutes at the same pace as the first part, and are home again 65 minutes after leaving.

Fully-worked solutions

Q1. (a) The title — "Mean maximum temperature at Mildura Airport" — says what the graph shows. (b) The months run along the horizontal axis. (c) The temperatures run up the vertical axis. (d) The scale: the vertical axis counts from 10 to 35 in fives. (e) The point: the red point is the pair (Jul, 15.5 °C).

Q2. (a) The December point sits at 30.5 °C. (b) Above 30 °C: January (32.6 °C), February (31.9 °C) and December (30.5 °C). (c) The falls are: Jan→Feb 0.7 °C, Feb→Mar 3.3 °C, Mar→Apr 4.9 °C, Apr→May 4.6 °C, May→Jun 3.1 °C, Jun→Jul 0.5 °C; the biggest fall is March to April. (d) The graph falls to a winter low and rises back to a summer high: one cycle over the year.

Q3. (a) A runs from 0 to 20 minutes. (b) B runs from 20 to 150 minutes: 150 − 20 = 130 minutes. (c) C covers the same 1.5 km in 15 minutes (150 to 165) that A covers in 20, and C's line is steeper. (d) The line meets the time axis again at 165 minutes.

Q4. Both lines end at the same height, 2 km, so both students travel the same distance to the same school. The red line reaches 2 km in 10 minutes and the blue line in 25 minutes, so the rider is faster — and the red line is the steeper one, as "faster" should look on a travel graph.

Q5. (a) In the wide bottom the height grows slowly, so the slope is gentle; once the water passes 480 mL and fills the narrow neck, the same volume poured lifts the height faster, so the slope turns steeper. (b) Above 600 mL the graph reads a height of 12 cm. (c) With one width all the way up, the height would grow at the same speed the whole time: the graph would be one straight line, with no bend.

Q6. (a) The first horizontal segment sits at 2 km and runs from 10 to 25 minutes, so the first stop is 2 km from home and lasts 25 − 10 = 15 minutes. (b) The highest point of the graph is 3 km. (c) The line returns to 0 at 65 minutes. (d) First part: 2 km in 10 minutes is 1 km every 5 minutes; trip home: 3 km in 15 minutes is also 1 km every 5 minutes — the same pace, and the graph shows it as two lines with the same tilt.

Q7. (a) The red line at January reads 16.9 °C. (b) 32.6 − 16.9 = 15.7 °C. (c) The claim fails: January's gap is 15.7 °C while June's gap is 16.0 − 5.2 = 10.8 °C, almost 5 °C smaller. The gap widens and narrows through the year, so it is not about the same every month.

Q8. The claim fails. The top panel says the hot months are December to February, but the bottom panel says those months get 24.0, 22.7 and 20.3 mm of rain — around the driest of the year. The wettest month is October (29.4 mm), in spring. The rainfall bars stay about even (18.7 to 29.4 mm), so the graphs do not show most rain in the hot months.

Q9. (a) The tallest bar is January–February at 820 kWh; the shortest is March–April at 495 kWh. (b) May–June to July–August: 730 − 640 = 90, an increase of 90 kWh. July–August to September–October: 730 − 510 = 220, a decrease of 220 kWh. (c) January–February is the hot summer and July–August the cold winter at Mildura, so cooling and heating push the use up at both ends of the year. (d) Neither: the bars go down (into autumn), up (into winter), down (into spring) and up (into summer) — the use rises and falls with the seasons instead of simply rising or falling.

Q10. (i) S — the line starts flat at 0 (waiting at home) and only then rises. (ii) P — the line rises and falls straight away with no flat part in between: out to the letterbox and straight back, with equal slopes for equal pace. (iii) Q — a rise, then a horizontal segment (staying at the park), then a fall back to 0 (home). (iv) R — a rise, a horizontal segment (resting), then a rise again at the same slope (same pace), never returning to 0.

Q11. Walk: (0, 0) to (5, 0.5). Wait: horizontal from (5, 0.5) to (15, 0.5). Bus: (15, 0.5) to (30, 4.5), since 0.5 + 4 = 4.5 km and 15 + 15 = 30 minutes. The bus segment is steeper: it covers 4 km in 15 minutes while the walk covers only 0.5 km in 5 minutes.

Q12. From January to July the line does fall (32.6 °C down to 15.5 °C), but from July to December it rises (15.5 °C up to 30.5 °C). A graph that falls and then rises cannot "fall every month", so the claim is false — it gets the first half of the year right and the second half wrong.

Q13. (a) False. On a horizontal segment the distance from home does not change while the time grows, so the person is stopped; a slow mover would still have a rising (gentle) line. (b) True — a steeper line covers more distance in the same time, so the distance from home changes faster. (c) True — a falling line means the distance from home is getting smaller, so the person is heading back towards home.

Q14. Sample paragraph: On Saturday morning a person leaves home and walks for 10 minutes to a spot 2 km away — perhaps a friend's place or a shop — and stays there for 15 minutes. They then walk on for 10 minutes to a place 3 km from home, where they stay 15 minutes. Finally they head straight home, covering the 3 km in 15 minutes at the same pace as the first part of the morning, arriving home 65 minutes after they left. The trip home is the longest single part of the journey, but its line has the same tilt as the first part because the pace is the same.

Teacher note

- **Ownership and the boundary with 3D.** Per the Chapter 3 plan in TOC §3, 3D owns building tables and plotting points; 3E owns the verbs *investigate, interpret and describe* applied to graphs that are already drawn. Items ask what the graph says, never what its rule is: no equation of a line, no intercepts (D4.9). Slope is described in words only — steeper, gentler, faster, slower, stopped, returning — with no gradient computed and no rates such as km/h (D4.8, P4; rates are Level 8). WE3's "30 mL gives 1 cm" is a volume-to-height read from the bottle's shape, not a rate over time.

- **Time on axes (D4.18).** Travel graphs (E02) require time on an axis; axes use minutes from leaving home, which is the curriculum-driven minimum. No clock-reading or time conversion is taught or assessed here.
- **Authentic data and sourcing (§6).** Figures 1, 2, 5 and 6 plot real Bureau of Meteorology climate statistics for station 076031 (Mildura Airport), monthly means over all years of record 1946–2026, retrieved 20 August 2026; the source line is printed under Figure 1 and no plotted value is adjusted from the source. Figures 3, 4 and 7–10 are transparently generic teaching settings — no real student, school, bottle or household is asserted — so, as in 2D, they carry no source line.
- **E04 not delivered (Rule 1).** Elaboration `VC2M7A04.E04` (First Nations water management) is excluded because Aboriginal and Torres Strait Islander content is excluded under CONTENT_POLICY Rule 1. The remaining elaborations are delivered: E01 (temperature over a year, electricity usage) in WE1, WE4, Q9; E02 (travel graphs, slope in words, horizontal segments) in WE2, Q3, Q4, Q6, Q11, Q13; E03 (telling the story, pouring) in WE3, Q5, Q14.
- **Reasonableness (P7).** Q8, Q12 and Q13 ask students to test a claim against the graph and judge it, per TOC §5.2; Q2(c) and Q7(c) compare gaps rather than computing with them.
- **Southern-hemisphere seasons.** January is the hottest month and July the coolest; WE1's "Australian summer" line and Q9(c)'s heating/cooling reasoning depend on it.
- **Vocabulary gate.** Student-facing text sits at or below Yr7 (ceiling rank 7); the curriculum family (variable, relationship, interpret, describe, authentic, slope, horizontal, segment, axis) is defined in the words table at first use.
- **Figures.** All ten figures are generated by `src/gen_3e.py` from the exact values stated in this text; point labels, gaps and bar values are computed from the data, never placed by eye.

Varying a formula systematically with digital tools

Content description `VC2M7A06`: manipulate formulas involving several variables using digital tools, and describe the effect of systematic variation in the values of the variables

Achievement standard (extract): Students create tables of values relating to algebraic expressions and formulas, and describe how the values change.

Theory

What we bring from 3A. In 3A we met formulas like $V = l \times w \times h$ and learnt to substitute: replace each letter with a number, then work out the result. A letter in a formula stands for a value that can change, so it is called a **variable**. The formula $V = l \times w \times h$ has three variables on the right — l, w and h — and once we choose values for them, V is decided.

Varying one thing at a time. If two variables change at once, we cannot see which change caused what. So the plan in this subtopic is: **fix** all but one variable, and vary the last one in equal, planned steps — $h = 1, 2, 3, 4, 5, 6$ cm, say. That is **systematic variation**: because the steps are planned and equal, and everything else is held still, we can describe exactly what that one change does to the result. Figure 1 shows the plan for the prism formula with l and w fixed at 4 cm.

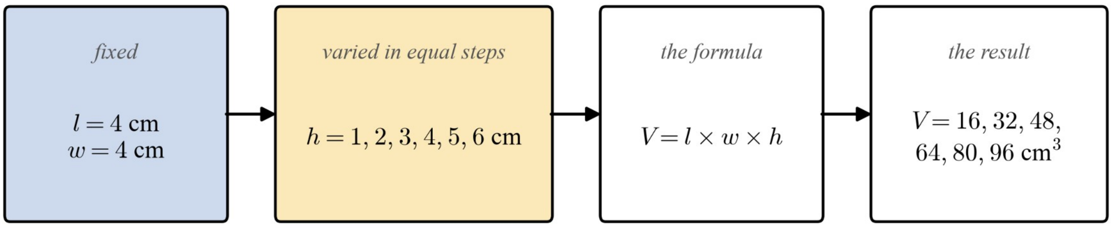

Figure 1. A flow of four boxes joined by arrows. The first box, shaded blue, holds the fixed inputs l = 4 cm and w = 4 cm. The second box, shaded amber, holds the varied input h = 1, 2, 3, 4, 5, 6 cm. The third box holds the formula V = l × w × h. The fourth box holds the results V = 16, 32, 48, 64, 80, 96 cm³.

The digital tool does the repeating. This subtopic is a *doing* subtopic: a digital tool — a spreadsheet — is part of the method, not an extra. Values sit in **cells** named by column letter and row number, and a formula finds its inputs by **cell references**: $=B1*B2*A5$ means "multiply the value in B1 by the value in B2, then by the value in A5". Type the formula once, copy it down the column, and every result is worked out for you. Change one input cell and every result updates at once — Worked example 2 shows exactly this.

Describing the effect. The achievement standard asks us to *describe how the values change*. A good description names the step and the response: "each extra 1 cm of height adds 16 cm³", or "doubling the width doubles every volume". In the questions you will meet three kinds of story: steps that **add** the same amount each time, steps that **double** (or three-times) the result, and steps where the result **gets smaller** as the varied variable grows.

The real formulas we use. The volume of a rectangular prism, $V = l \times w \times h$; the distance formula $d = v \times t$, where v is the average speed in km/h and t is the time in h; density = mass ÷ volume; and the American Heart Association's heart-rate rule, maximum heart rate $= 220 -$ age, with a target heart rate a chosen percentage of that maximum. *Source: American Heart Association, "Target Heart Rates", retrieved 20 August 2026.*

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
variable	a letter standing for a value that can change	l, w, h, t, v

Term	Meaning here	Synonyms and related words
fixed value	a variable we choose to leave alone during a sweep	held constant
systematic variation	changing one variable in equal, planned steps	a fair sweep
cell	one box of a spreadsheet, named like B2	—
cell reference	the name a formula uses to find an input	B1, A5
effect	how the results respond to the varied variable	the pattern of change

Worked examples

Example 1 — scaffolded

A rectangular prism has $l = w = 4$ cm (Figure 2). Sweep the height through $h = 1, 2, 3, 4, 5, 6$ cm, build the table of values for V, and describe the effect of changing h.

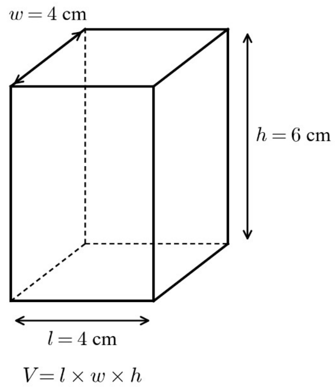

Figure 2. A rectangular prism drawn in 3-D with hidden edges dashed, labelled l = 4 cm along the front bottom edge, w = 4 cm along the sloping depth edge and h = 6 cm on the right-hand vertical edge, with the formula V = l × w × h below.

Working	Comment
$V = l \times w \times h$ with $l = w = 4$ cm fixed.	Two variables fixed; only h will vary.
$h = 1$: $V = 4 \times 4 \times 1 = 16$ cm³.	The same substitution skill as 3A.
$h = 2, 3, 4, 5, 6$: $V = 32, 48, 64, 80, 96$ cm³.	Equal steps in, so the pattern out is easy to read.
Each extra 1 cm of height adds 16 cm³; doubling h from 2 to 4 doubles V from 32 to 64 cm³.	The description the achievement standard asks for.

height h (cm)	1	2	3	4	5	6
volume V (cm³)	16	32	48	64	80	96

Figure 3 draws the same table as a bar chart: the bars climb by the same amount each step, because the height grows by the same amount each step.

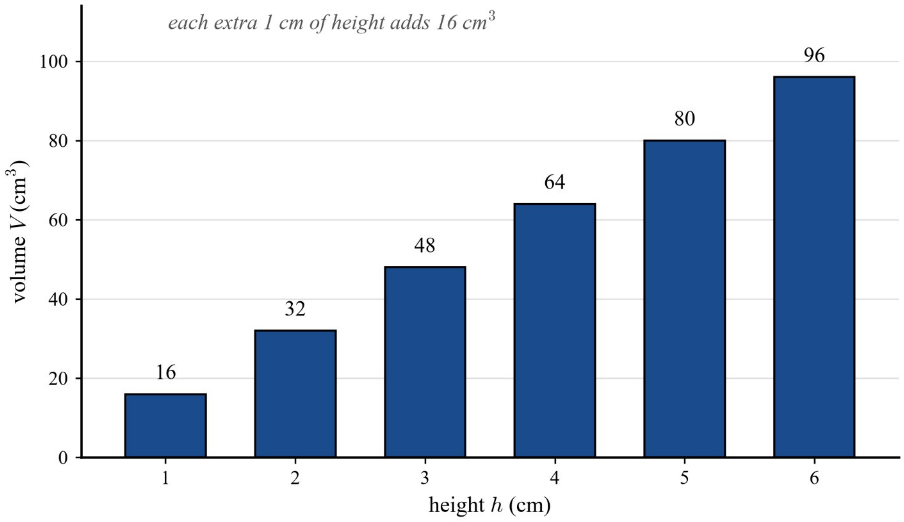

Figure 3. Bar chart of volume against height for the prism of Worked example 1: bars of 16, 32, 48, 64, 80 and 96 cm³ for heights 1 to 6 cm, climbing in equal steps of 16 cm³, with the note "each extra 1 cm of height adds 16 cm³".

Example 2 — scaffolded

Now let the spreadsheet do Example 1. Set up the sheet of Figure 4: B1 and B2 hold the fixed length and width, A5:A10 hold the heights, and B5 holds the formula $=B1*B2*A5$, copied down to B10. Then change **one** cell — B2, the width — from 4 to 8, and describe the effect.

Working	Comment
Type 4 into B1 and B2; type $1,2,3,4,5,6$ into A5:A10.	The fixed inputs and the varied variable, laid out in cells.
In B5 type $=B1*B2*A5$ and copy it down to B10.	One formula produces the whole table: $16,32,48,64,80,96$ cm³ — Example 1 again.
Change B2 from 4 to 8 (Figure 5). The column becomes $32,64,96,128,160,192$ cm³.	The formula was never touched; only one input cell changed, and every result updated.
Doubling the width doubled **every** volume.	One sentence, and the effect is described.

B5	=B1*B2*A5

	A	B
1	length l (cm)	4
2	width w (cm)	4
3		
4	height h (cm)	volume V (cm^3)
5	1	=B1*B2*A5
6	2	32
7	3	48
8	4	64
9	5	80
10	6	96

Figure 4. A spreadsheet with a formula bar showing cell B5 holding =B1B2A5. Row 1 labels B1 as the length l = 4 cm and row 2 labels B2 as the width w = 4 cm, both shaded amber as the cells the formula refers to. Row 4 heads

the table “height h (cm)” and “volume V (cm³)”; rows 5 to 10 hold heights 1 to 6 beside volumes 16, 32, 48, 64, 80 and 96, with B5 selected.

B5	=B1*B2*A5

	A	B
1	length l (cm)	4
2	width w (cm)	8
3		
4	height h (cm)	volume V (cm^3)
5	1	32
6	2	64
7	3	96
8	4	128
9	5	160
10	6	192

Figure 5. The same spreadsheet after one change: cell B2, shaded red and selected, now holds 8, and the volume column has updated to 32, 64, 96, 128, 160 and 192 while the formula =B1B2A5 in the formula bar is unchanged.

Example 3 — unscaffolded

The American Heart Association says a person’s maximum heart rate is about $220 -$ age (in beats per minute), and that exercising at a chosen percentage of that maximum — 50%, 60%, 70%, 80% — gives a target heart rate. Use the formula twice, sweeping a different variable each time, and describe each effect. (*Source: American Heart Association, “Target Heart Rates”, retrieved 20 August 2026.*)

Sweep A — age fixed at 40 years, percentage varied 50% → 80% in steps of 10%. The maximum is $220-40=180$ beats per minute, so the targets are $50\% \times 180=90$, $60\% \times 180=108$, $70\% \times 180=126$ and $80\% \times 180=144$ beats per minute (Figure 6). Each extra 10% of the maximum adds 18 beats per minute to the target.

Sweep B — percentage fixed at 70%, age varied 10 → 60 years in steps of 10. The targets are $147, 140, 133, 126, 119, 112$ beats per minute (Figure 7): for example age 40 gives $70\% \times (220-40)=70\% \times 180=126$. Each extra 10 years of age removes 7 beats per minute from the target.

Same formula, two sweeps, two different stories — which story you get depends on which variable you choose to vary.

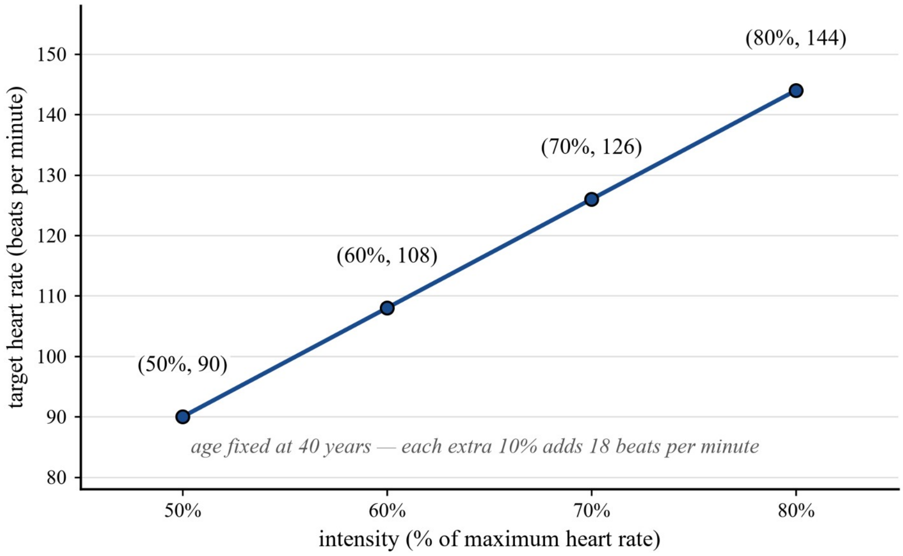

Figure 6. Line graph of target heart rate against the percentage of maximum heart rate with age fixed at 40 years: points (50%, 90), (60%, 108), (70%, 126) and (80%, 144) on a rising straight line, with the note "age fixed at 40 years — each extra 10% adds 18 beats per minute".

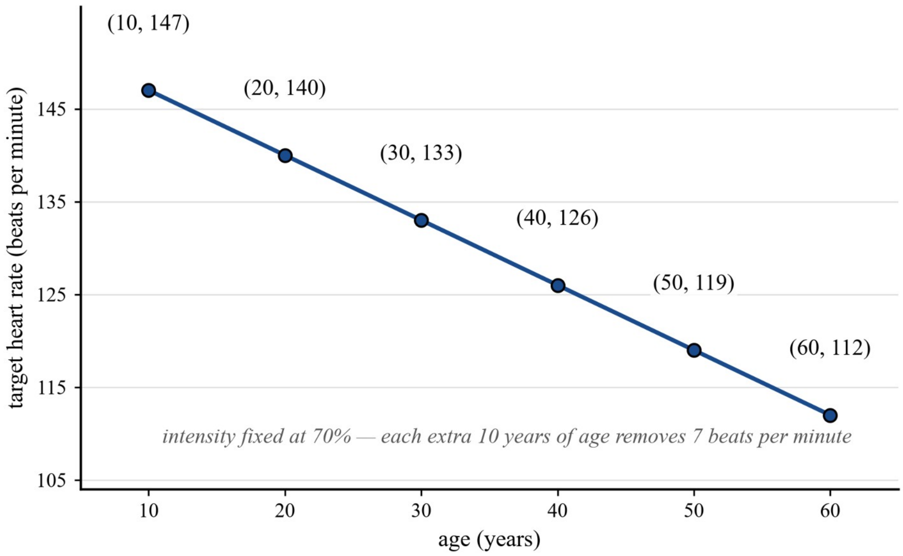

Figure 7. Line graph of target heart rate against age with the percentage fixed at 70%: points (10, 147), (20, 140), (30, 133), (40, 126), (50, 119) and (60, 112) on a falling straight line, with the note "intensity fixed at 70% — each extra 10 years of age removes 7 beats per minute".

Example 4 — unscaffolded

A car travels at an average speed of 60 km/h and a second car at 100 km/h. Use $d = v \times t$ to sweep the time $t = 1, 2, 3, 4, 5$ h for **both** speeds, and compare the two trips.

Working	Comment
$v = 60$: $d = 60, 120, 180, 240, 300$ km.	Each extra hour adds 60 km.
$v = 100$: $d = 100, 200, 300, 400, 500$ km.	Each extra hour adds 100 km.
At $t = 3$ h: 180 km against 300 km.	The same formula with one variable — the speed — fixed at two different values.
The gap is $40, 80, 120, 160, 200$ km: it grows by 40 km every hour.	A comparison is also a description of an effect.

Figure 8 shows the two tables side by side and Figure 9 draws them: the points of each sweep sit on a straight line through zero, and the faster car's line rises more quickly. (We read the pattern from the table — gradients and line equations belong to later years.)

time t (h)	distance at 60 km/h (km)	distance at 100 km/h (km)
1	60	100
2	120	200
3	180	300
4	240	400
5	300	500

Figure 8. A three-column table. The first column lists time t = 1 to 5 h; the second gives the distance at 60 km/h as 60, 120, 180, 240 and 300 km; the third gives the distance at 100 km/h as 100, 200, 300, 400 and 500 km.

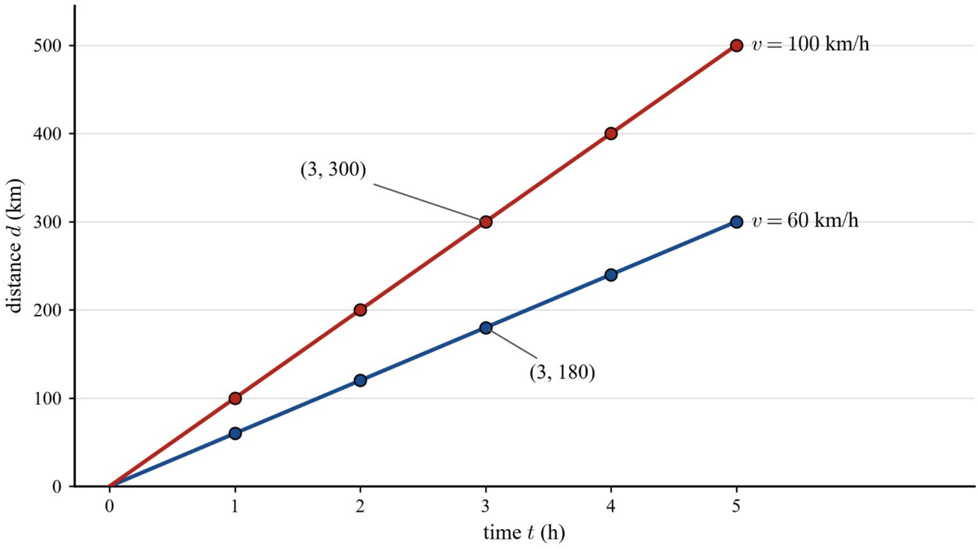

Figure 9. Line graph of distance against time with two sweeps marked as points: the line v = 60 km/h through (1, 60) to (5, 300) and the steeper line v = 100 km/h through (1, 100) to (5, 500), with the comparison points (3, 180) and (3, 300) labelled with leader lines.

Practice questions

Scaffolded

Q1. Replace each blank with the correct word: *variable, fixed, systematically, cell reference*. (a) A letter in a formula that stands for a value that can change is a _____. (b) In a systematic sweep we change one variable and keep the others _____. (c) Changing a variable in equal, planned steps is varying it _____. (d) In the formula $=B1*B2*A5$, the names B1, B2 and A5 are _____s.

Q2. A prism has $l=w=3$ cm. Sweep $h=1,2,3,4,5$ cm and build the table of values for $V=l\times w\times h$. Describe the effect of changing h.

Q3. True or false? (a) In a systematic sweep we may change as many variables as we like at once. (b) In a spreadsheet, changing one input cell updates every cell whose formula refers to it. (c) If every step of a sweep adds the same amount to the result, the results climb in equal steps.

Q4. Figure 10 shows a spreadsheet in which B2 holds the formula $=45*A2$, copied down the column. (a) What value does cell B2 show? (b) Write the values shown in B3, B4 and B5. (c) The formula is copied down to row 7, where A7 holds 6. What value does B7 show? (d) Describe the effect of increasing t. (e) This table is a sweep of $d=v\times t$. What is the speed v of this trip?

B2	=45*A2

	A	B
1	time t (h)	distance d (km)
2	1	=45*A2
3	2	90
4	3	135
5	4	180

*Figure 10. A spreadsheet with a formula bar showing cell B2 holding =45*A2. Row 1 heads the columns "time t (h)" and "distance d (km)"; rows 2 to 5 hold times 1 to 4 beside the formula in B2 (selected) and the values 90, 135 and 180.*

Q5. Use $d = v \times t$ with $v = 80$ km/h fixed. Sweep $t = 1, 2, 3, 4$ h, build the table, and describe the effect of changing t in two different ways.

Q6. Priya is 13. Using the American Heart Association rule of Worked example 3: (a) find her maximum heart rate; (b) find her target heart rate at 60% of the maximum; (c) find her target heart rate at 80% of the maximum; (d) her training zone is 50% to 85% of the maximum, that is 103.5 to 175.95 beats per minute. Is a heart rate of 170 beats per minute inside her zone?

Q7. A prism has $l = w = 5$ cm. Sweep $h = 2, 4, 6, 8$ cm (steps of 2 cm this time). (a) Build the table of values for V. (b) Describe the effect of each 2 cm step. (c) What happens to V when h doubles from 2 to 4?

Q8. Density is mass ÷ volume. A 120 g sample of a liquid is poured into cylinders of different volumes: $V = 10, 20, 30, 40, 60$ cm³. Build the table of density values and describe the effect of increasing V. (This sweep gets smaller — not every effect grows.)

Unscaffolded

Q9. Lena's holiday job pays $24 an hour for the first 6 h of a day, and $36 an hour for each overtime hour after that. Her day's pay is therefore $144 plus $36 for each overtime hour. Sweep the overtime hours $k = 0, 1, 2, 3$ to build the table of possible pays, and describe the effect of each overtime hour.

Q10. A rectangle has length 8 cm fixed and width w. The area is $A = 8w$ (that is, $8 \times w$). Sweep $w = 3, 6, 9, 12$ cm, build the table, and describe what happens to A when w goes from 3 to 9.

Q11. Tom used a spreadsheet to find the distance travelled at 60 km/h in 3 h, and his tool showed 2160 km. (a) Without doing the exact calculation, explain why 2160 km cannot be sensible. (b) Tom's formula bar showed $= 60 * 36$. What did he type by mistake? (c) Write the correct distance.

Q12. Mia swept a prism's volume "all at once": her first prism was $5 \times 4 \times 4 = 80$ cm³ and her second was $5 \times 6 \times 8 = 240$ cm³, and she wrote "changing the width and the height made the volume three times as big". Explain why her sweep cannot show that, and what she should have done instead.

Q13. Use the table of Worked example 4 (Figure 8). How much further ahead is the 100 km/h car than the 60 km/h car at $t = 1$ h and at $t = 5$ h? Describe how the gap changes hour by hour.

Q14. At 80% of the maximum, find the target heart rate of a 15-year-old and of a 45-year-old. What is the difference, and what does it tell you about age and target heart rate at the same percentage?

Q15. In the sheet of Worked example 2 (Figure 4), suppose instead that B1 — the length — is changed from 4 to 12 while B2 stays 4. Without re-drawing the sheet, write the new volume column and describe the effect.

Q16. Complete the sentence: "To vary a formula systematically, I change _____ variable(s) in equal _____, keep the other variables _____, and then I can describe the _____ of that one change."

Q17. Your own sweep. Choose one formula from this subtopic. Fix values for all but one variable, vary that one in five equal steps, and use a digital tool to build the table of values. Write the table and one sentence describing the effect.

Q18. A cube has volume $V = s \times s \times s$, where s is the edge length (Figure 11). (a) Build the table for $s = 2, 4, 6$ cm. (b) What happens to V when s doubles from 2 to 4? Check with your table. (c) Explain why, using the three factors in the formula.

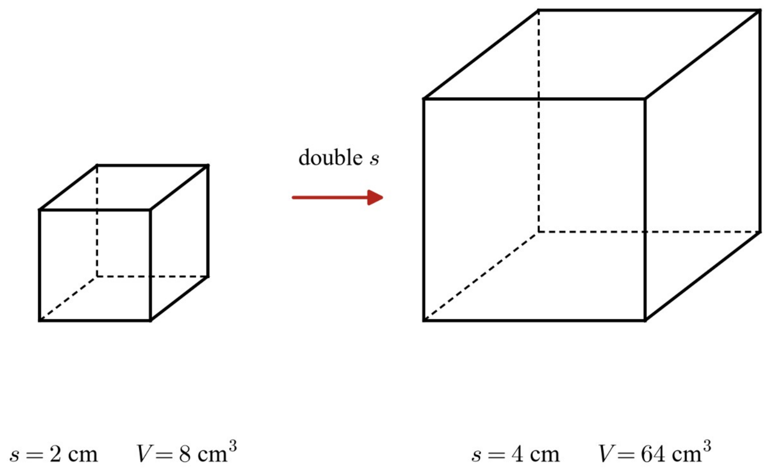

Figure 11. Two cubes drawn to the same scale in 3-D with hidden edges dashed: a small cube captioned s = 2 cm, V = 8 cm³ and a larger cube captioned s = 4 cm, V = 64 cm³, with a red arrow between them labelled "double s".

Answers

Q1. (a) variable (b) fixed (c) systematically (d) cell reference

Q2. $V = 9, 18, 27, 36, 45$ cm³; each extra 1 cm of height adds 9 cm³.

Q3. (a) False (b) True (c) True

Q4. (a) 45 (b) $90, 135, 180$ (c) 270 (d) each extra hour adds 45 km (e) 45 km/h

Q5. $d = 80, 160, 240, 320$ km; each extra hour adds 80 km, and doubling t doubles d.

Q6. (a) 207 beats per minute (b) 124.2 (c) 165.6 (d) yes — 170 sits between 103.5 and 175.95.

Q7. (a) $V = 50, 100, 150, 200$ cm³ (b) each 2 cm step adds 50 cm³ (c) V doubles (50 → 100 cm³).

Q8. $12, 6, 4, 3, 2$ g/cm³; as the volume grows the density shrinks — doubling V halves the density.

Q9. $144, 180, 216, 252$; each overtime hour adds $36.

Q10. $A = 24, 48, 72, 96$ cm²; three-timing w (3 → 9) makes A three times as big (24 → 72).

Q11. (a) 60 km each hour for 3 h is about 180 km; 2160 km is more than ten times too big. (b) He typed 36 where the time 3 should be. (c) 180 km.

Q12. She changed two variables at once, so no one change can be blamed for the 240 cm³; a systematic sweep changes one variable at a time.

Q13. 40 km at $t = 1$ h; 200 km at $t = 5$ h; the gap grows by 40 km every hour.

Q14. 164 and 140 beats per minute; a difference of 24 — at the same percentage, older age gives a lower target.

Q15. $48, 96, 144, 192, 240, 288$ cm³; three-timing the length makes every volume three times as big.

Q16. one; steps; fixed; effect.

Q17. Answers vary; a sample is in the solutions.

Q18. (a) $V = 8, 64, 216$ cm³ (b) V becomes 8 times as big (8 → 64) (c) doubling s doubles each of the three factors, so V doubles three times: $2 \times 2 \times 2 = 8$.

Fully-worked solutions

Q1. (a) A **variable** — the letter's value can vary. (b) **fixed** — holding the others still is what makes the sweep fair. (c) **systematically** — equal, planned steps. (d) **cell reference** — the names the formula uses to find its inputs.

Q2. $V = 3 \times 3 \times h = 9h$: $h = 1 \rightarrow 9$; $2 \rightarrow 18$; $3 \rightarrow 27$; $4 \rightarrow 36$; $5 \rightarrow 45$ cm³. The effect: each extra 1 cm of height adds 9 cm³.

Q3. (a) **False** — change two things at once and you cannot tell which did what (see Q12). (b) **True** — that is the whole point of cell references (Figure 5). (c) **True** — equal steps in give equal steps out (Figure 3).

Q4. (a) B2 $= 45 \times 1 = 45$. (b) B3 $= 45 \times 2 = 90$; B4 $= 135$; B5 $= 180$. (c) B7 $= 45 \times 6 = 270$. (d) Each extra hour adds 45 km. (e) The formula is $d = v \times t$ with $v = 45$, so the speed is 45 km/h.

Q5. $d = 80t$: $80, 160, 240, 320$ km. Effect one way: each extra hour adds 80 km. Effect a second way: doubling t (say 1 → 2) doubles d (80 → 160).

Q6. (a) $220 - 13 = 207$ beats per minute. (b) $60\% \times 207 = 124.2$. (c) $80\% \times 207 = 165.6$. (d) The zone runs from $50\% \times 207 = 103.5$ to $85\% \times 207 = 175.95$ beats per minute, and 170 sits inside it: yes.

Q7. (a) $V = 5 \times 5 \times h = 25h$: $h = 2 \to 50$; $4 \to 100$; $6 \to 150$; $8 \to 200$ cm³. (b) Each 2 cm step adds 50 cm³. (c) $h = 2 \to 4$ sends $V = 50 \to 100$ cm³: doubling h doubles V.

Q8. density $= 120 \div V$: $120 \div 10 = 12$; $\div 20 = 6$; $\div 30 = 4$; $\div 40 = 3$; $\div 60 = 2$ g/cm³. Effect: the bigger the volume the same mass is spread through, the smaller the density — doubling V ($10 \to 20$) halves the density ($12 \to 6$), and again ($20 \to 40$) halves it ($6 \to 3$).

Q9. $k = 0$: 144; $k = 1$: $144 + 36 = 180$; $k = 2$: 216; $k = 3$: 252. Effect: each overtime hour adds \$36 to the day's pay.

Q10. $A = 8w$: $w = 3 \to 24$; $6 \to 48$; $9 \to 72$; $12 \to 96$ cm². From $w = 3$ to $w = 9$ the width is three times as big, and A goes $24 \to 72$: three times as big too.

Q11. (a) At 60 km/h each hour contributes about 60 km, so 3 h is about 180 km — 2160 km would be more than ten times that, which is not sensible for a three-hour trip. (b) $60 \times 36 = 2160$: he typed the time as 36 instead of 3. (c) $60 \times 3 = 180$ km — and 180 km matches the about-180 km we expected in (a).

Q12. Between her two prisms, w changed ($4 \to 6$) *and* h changed ($4 \to 8$). The volume did grow three times ($80 \to 240$), but with two changes at once we cannot say how much each one did — the doubling of h alone would already have doubled V. A systematic sweep holds everything but one variable fixed, so she should have changed w first (sweep h fixed), described that effect, then changed h.

Q13. At $t = 1$: $100 - 60 = 40$ km; at $t = 5$: $500 - 300 = 200$ km. The gaps are $40, 80, 120, 160, 200$ km: the faster car pulls 40 km further ahead every hour.

Q14. Age 15: maximum $220 - 15 = 205$, target $80\% \times 205 = 164$. Age 45: maximum $220 - 45 = 175$, target $80\% \times 175 = 140$. Difference 24 beats per minute: at the same percentage, the older person's target is lower (matching the falling line of Figure 7).

Q15. The length goes $4 \to 12$, three times as big, so every volume is three times the Example 2 column: $3 \times 16 = 48$, then $96, 144, 192, 240, 288$ cm³. Effect: three-timing the length makes every volume three times as big.

Q16. one; steps; fixed; effect.

Q17. Sample answer using $d = v \times t$ with $v = 70$ km/h fixed and $t = 1, 2, 3, 4, 5$ h: $d = 70, 140, 210, 280, 350$ km. "Each extra hour of travel adds 70 km to the distance." Any correct table with one variable fixed and equal steps, plus a true description, is acceptable.

Q18. (a) $s = 2$: $2 \times 2 \times 2 = 8$; $s = 4$: 64; $s = 6$: 216 cm³. (b) $8 \to 64$: V becomes 8 times as big (and $64 \to 216$ is not a doubling story — $2 \to 6$ makes s three times as big and V 27 times as big). (c) V multiplies three factors of s; doubling s doubles each factor, so V is doubled three times: $2 \times 2 \times 2 = 8$.

Teacher notes

- **The digital tool is compulsory (D9).** Every sweep in this subtopic is meant to be run in a spreadsheet; the section reads fine on paper, but the class activity should use the tool. Tools are kept generic (no product names or menu paths), and no CAS is used — the spreadsheet's plain `=B1*B2*A5`-style formulas are the level of this year.
- **Level boundaries.** Speed appears only as a value substituted into $d = v \times t$ (rates are Level 8, D4.8); Figure 9's lines are read for their pattern only — no gradient, no $y = mx + c$ (D4.9); no transposing of formulas anywhere (P5).
- **Sources (§6).** The heart-rate rule (maximum $= 220 -$ age; target a percentage of the maximum) is cited to the American Heart Association, retrieved 20 August 2026; it is a teaching estimate and real hearts vary — say so if a student asks. The wage rates in Q9 are illustrative; the $1.5\times$ overtime multiplier is the common "time and a half" convention. The density context cross-references Science (mass ÷ volume) without teaching it.
- **Reasonableness (P7).** Q11 is the section's "is this answer sensible?" item; it pairs naturally with a class discussion of checking a tool's output against a mental estimate.

- **Error analysis.** Q12 targets the defining habit of systematic variation — one variable at a time — by showing a two-changes-at-once “sweep” that proves nothing.
- **Cross-references.** Substitution from 3A; volume of prisms recurs in 4B; density is used as a decreasing-effect context only.
- **Figures.** All eleven figures are computed, not hand-drawn: `src/gen_3f.py` regenerates them and asserts every value transcribed above.
- **Vocabulary.** Student-facing text passes the Yr7 gate: run `src/vocab_gate_3f.py` → CLEAN at ceiling Yr7.

Test

Chapter test T3 — Algebra: expressions, formulas, equations and relationships

Assessed content descriptions	`VC2M7A01` — subtopic 3A, Variables and everyday formulas; substituting to find an unknown · `VC2M7A02` — subtopic 3B, The associative, commutative and distributive laws; writing algebraic expressions · `VC2M7A03` — subtopic 3C, Solving one-variable linear equations with natural number solutions · `VC2M7A05` — subtopic 3D, Tables of values from patterns and rules; plotting on the Cartesian plane · `VC2M7A04` — subtopic 3E, Reading and describing graphs of relationships built from authentic data · `VC2M7A06` — subtopic 3F, Varying a formula systematically with digital tools
Marks	**30** — Section A (3A): 5 marks · Section B (3B): 5 marks · Section C (3C): 5 marks · Section D (3D): 5 marks · Section E (3E): 5 marks · Section F (3F): 5 marks
Time	35 minutes in all: 5 minutes reading time, then 30 minutes working time (about 1 minute per mark)
Equipment	Spreadsheet or graphing software for the Section F (3F) items; no other equipment needed

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 7 achievement standard, not against a mark on this paper.

Answer every question. Show your working where a question asks for it. The marks for each part are shown in brackets.

No digital tools — scientific calculators, spreadsheets or other software — are permitted in Sections A to E: these content descriptions do not name digital tools. Spreadsheet or graphing software is required for Section F: `VC2M7A06` *names digital tools.*

Section A — Variables and everyday formulas; substituting to find an unknown (3A, `VC2M7A01`)

Question 1 [2 marks]. A team's score is found with the formula $p = 6g + b$, where g is the number of goals and b is the number of behinds.

Source: the scoring rule (a goal worth 6 points, a behind worth 1 point) is VCAA's, from VC2M7A01 elaboration E01, retrieved 20 August 2026.

a. Find the score of a team that kicked 8 goals and 5 behinds. **[1 mark]**

b. Noah watched a match and wrote down that a team scored 100 points from 14 goals and 2 behinds. Is that reasonable? Check with $p = 6g + b$ and explain your answer. **[1 mark]**

Question 2 [2 marks]. A worker's weekly wage is found with the formula $W = b + 1.5 \times h \times r$, where b is the base wage in dollars, h is the number of overtime hours and r is the hourly rate in dollars. One week, $b = 680$, $h = 4$ and $r = 20$.

The formula is VCAA's (VC2M7A01 elaboration E03, retrieved 20 August 2026); the dollar values are practice figures. Current real rates are published by Fair Work.

Substitute the values into the formula and find W. **[2 marks]**

Question 3 [1 mark]. A block has mass $m = 45$ g and volume $V = 15$ cm³. Use $d = m \div V$ to find the block's density, and say whether it sinks or floats in water (water is 1.0 g/cm³).

Source: the formula $d = m \div V$ is VCAA's, from VC2M7A01 elaboration E03, retrieved 20 August 2026.

Section B — The associative, commutative and distributive laws; writing algebraic expressions (3B, `VC2M7A02`)

Question 4 [1 mark]. Which expression means "add 6 to n, then multiply the result by 2"?

- **A** $2n+6$
- **B** $2(n+6)$
- **C** $6n+2$
- **D** $n+12$

Question 5 [2 marks]. Find each answer mentally, by reordering, regrouping or splitting first. Show the step you used, and name the law behind it.

a. $37+58+63$ **[1 mark]**

b. 6×29 **[1 mark]**

Question 6 [2 marks].

a. Write an expression for "subtract 5 from n, then multiply the result by 4". Use brackets where they are needed. **[1 mark]**

b. Find the value of your expression when $n=9$. **[1 mark]**

Section C — Solving one-variable linear equations with natural number solutions (3C, `VC2M7A03`)

Solving and verifying: `VC2M7A03` requires solutions to be verified by substitution. A mark is awarded for the check in Questions 7 and 8; a correct answer with no check does not score full marks.

Question 7 [2 marks]. Solve $5x+4=29$. Show your working, then check your solution by substitution.

a. Solve the equation. **[1 mark]**

b. Check your solution by substitution. **[1 mark]**

Question 8 [3 marks]. A number is multiplied by 3, and then 7 is added. The result is 28. Use x for the number.

a. Write an equation for this. **[1 mark]**

b. Solve the equation, showing your working. **[1 mark]**

c. Check your solution by substitution. **[1 mark]**

Section D — Tables of values from patterns and rules; plotting on the Cartesian plane (3D, `VC2M7A05`)

Question 9 [5 marks]. The rule is $y=3x+1$, where x is the input and y is the output.

a. Copy and complete the table of values for inputs $0,1,2,3,4$. **[2 marks]**

input x	0	1	2	3	4
output y					

b. Describe how the output changes each time the input goes up by 1. **[1 mark]**

c. If the ordered pairs from the table were plotted on the Cartesian plane, would the points lie on a straight line or on a curve? How can you tell from the table alone? **[1 mark]**

d. Use the rule to find the output when the input is 10. **[1 mark]**

Section E — Reading and describing graphs of relationships built from authentic data (3E, `VC2M7A04`)

Question 10 [3 marks]. The table shows the mean maximum temperature at Mildura Airport for January to August. The line graph of these values rises and falls across the months.

Month	January	February	March	April	May	June	July	August
Mean maximum temperature (°C)	32.6	31.9	28.6	23.7	19.1	16.0	15.5	17.4

Source: Bureau of Meteorology, "Climate statistics for Australian locations", station 076031 (Mildura Airport), monthly means, all years of record 1946–2026; retrieved 20 August 2026.

a. Which month in the table is the hottest, and what is its mean maximum temperature? **[1 mark]**

b. Describe what happens to the mean maximum temperature from February to July. **[1 mark]**

c. A website claims: "From January to August, the mean maximum temperature at Mildura falls every month." Is the claim true? Use two values from the table to justify your answer. **[1 mark]**

Question 11 [2 marks]. These questions are about travel graphs — graphs of distance from home against time.

a. What does a horizontal segment on a travel graph tell you about the person? **[1 mark]**

b. One segment of a travel graph covers 3 km in 30 minutes. A later segment covers 3 km in 20 minutes. Which part of the journey is faster, and how does the graph show it? **[1 mark]**

Section F — Varying a formula systematically with digital tools (3F, `VC2M7A06`)

The content description for 3F names digital tools, so these items are spreadsheet items. Use spreadsheet or graphing software to set up each sweep and check your values.

Question 12 [3 marks]. Priya sets up a spreadsheet to sweep the distance formula $d = v \times t$, where v is the average speed in km/h and t is the time in hours. Cell B1 holds 70. Cells A4 to A7 hold the times 1, 2, 3, 4. Cell B4 holds the formula `=B1*A4`, and Priya copies it down to B7.

	A	B
1		70
2		
3		
4	1	`=B1*A4`
5	2	
6	3	
7	4	

a. What values do cells B4 and B5 show? **[1 mark]**

b. Write the values shown in B6 and B7. **[1 mark]**

c. Describe the effect of increasing t. **[1 mark]**

Question 13 [2 marks]. Zoe sweeps the volume formula $V = l \times w \times h$. Her first prism has $l = 5$ cm, $w = 2$ cm and $h = 3$ cm, so $V = 30$ cm³. Her second prism has $l = 5$ cm, $w = 4$ cm and $h = 6$ cm, so $V = 120$ cm³. Zoe writes: "doubling the height doubled the volume."

a. Explain why Zoe's sweep cannot show the effect of doubling the height. **[1 mark]**

b. What should Zoe hold fixed, and what should she change, to test the effect of doubling h? **[1 mark]**

End of test — 30 marks.

Solutions

Fully worked solutions for every question, followed by a compact answer key for self-checking.

Worked solutions

Question 1.

a. Substitute $g = 8$ and $b = 5$ into $p = 6g + b$, and multiply before adding:

$$p = 6 \times 8 + 5 = 48 + 5 = 53$$

The team scored 53 points. **[1 mark]**

b. Estimate first: 14 goals alone give $14 \times 6 = 84$ points, so the score must be a little above 84 — 100 is too high to be right. The exact check confirms it:

$$p = 6 \times 14 + 2 = 84 + 2 = 86$$

No, it is not reasonable: the true score is 86 points, not 100. **[1 mark]**

Question 2.

Substitute $b = 680$, $h = 4$ and $r = 20$ into $W = b + 1.5 \times h \times r$. Multiply before adding (order of operations):

$$W = 680 + 1.5 \times 4 \times 20 = 680 + 6 \times 20 = 680 + 120 = 800$$

One mark for the correct substitution, one mark for the value of W: the weekly wage is **$800**. **[2 marks]**

Question 3.

Substitute $m = 45$ g and $V = 15$ cm³ into $d = m \div V$:

$$d = 45 \div 15 = 3.0\,\text{g/cm}^3$$

Because $3.0 > 1.0$, the block is denser than water, so it **sinks**. **[1 mark]**

Question 4.

The word “then” means the adding is finished before the multiplying starts, so the sum goes in brackets: “add 6 to n” gives $n + 6$, and multiplying that result by 2 gives $2(n + 6)$. Option A multiplies first and adds 6 afterwards; option C multiplies n by 6; option D only adds. The answer is **B**. **[1 mark]**

Question 5.

a. Swap the order (commutative law) and group the pair that makes a hundred (associative law):

$$(37 + 63) + 58 = 100 + 58 = 158$$

Accept any correct reordering that names a law. **[1 mark]**

b. Split 29 into a round number plus a small part, and multiply each part (distributive law):

$$6 \times 29 = 6 \times (30 - 1) = 6 \times 30 - 6 \times 1 = 180 - 6 = 174$$

Accept $6 \times (20 + 9) = 120 + 54 = 174$ with the law named. **[1 mark]**

Question 6.

a. The subtracting happens first, so it sits inside brackets: $4(n - 5)$. **[1 mark]**

b. Substitute $n = 9$ and work out the bracket first:

$$4 \times (9 - 5) = 4 \times 4 = 16$$

[1 mark]

Question 7.

a. Undo the operations in reverse order, keeping both sides balanced:

$$5x+4=29$$

$$5x+4-4=29-4$$

$$5x=25$$

$$5x\div 5=25\div 5$$

$$x=5$$

[1 mark]

b. Check by substituting $x=5$ into the original equation:

$$\text{LHS}=5\times 5+4=25+4=29=\text{RHS}$$

Both sides agree, so $x=5$ is the solution. **[1 mark]**

Question 8.

a. "Multiplied by 3, then 7 added, result 28" is written:

$$3x+7=28$$

[1 mark]

b. Subtract 7 from both sides, then divide both sides by 3:

$$3x+7-7=28-7$$

$$3x=21$$

$$3x\div 3=21\div 3$$

$$x=7$$

[1 mark]

c. Check by substituting $x=7$:

$$3\times 7+7=21+7=28$$

Both sides agree, so $x=7$ is the solution — the number is 7. **[1 mark]**

Question 9.

a. Substitute each input into $y=3x+1$: $x=0$ gives $3\times 0+1=1$; $x=1$ gives 4; $x=2$ gives 7; $x=3$ gives 10; $x=4$ gives 13.

input x	0	1	2	3	4
output y	1	4	7	10	13

One mark for the first three outputs, one mark for the last two. **[2 marks]**

b. Each time the input goes up by 1, the output goes up by 3 — the "× 3" step of the rule shows up in the table as the same jump of 3 every time. **[1 mark]**

c. A straight line. The jumps between the outputs are all the same (3 each time), and a straight-line plot needs the same jump in the output for the same jump in the input. **[1 mark]**

d. Substitute $x = 10$:

$$y = 3 \times 10 + 1 = 30 + 1 = 31$$

The output is 31. (Check by extending the table in jumps of 3: 16, 19, 22, 25, 28, 31 for inputs 5 to 10 — the same answer.) **[1 mark]**

Question 10.

a. Read the top of the table: the highest value is 32.6 °C, in **January**. **[1 mark]**

b. From February to July the line falls: the mean maximum temperature decreases every month, from 31.9 °C down to 15.5 °C. **[1 mark]**

c. The claim is false. The table falls from January to July, but from July to August the mean maximum temperature rises again — from 15.5 °C to 17.4 °C. A series that falls and then rises cannot "fall every month". **[1 mark]**

Question 11.

a. On a horizontal segment the distance from home does not change while the time grows, so the person is **stopped**. **[1 mark]**

b. The segment covering 3 km in 20 minutes is the faster part. Both segments cover the same distance, but 20 minutes is less time — on the graph, that segment is the steeper one. **[1 mark]**

Question 12.

a. The formula in B4 multiplies B1 by A4: $70 \times 1 = 70$. Copied down one row, B5 is $70 \times 2 = 140$. So **B4 shows 70** and **B5 shows 140**. **[1 mark]**

b. B6 is $70 \times 3 = 210$; B7 is $70 \times 4 = 280$. **[1 mark]**

c. Each extra hour of travel adds 70 km to the distance. (Equally acceptable: doubling t doubles d; the values climb in equal steps of 70 km.) **[1 mark]**

Question 13.

a. Zoe changed two variables at once: the width went from 2 cm to 4 cm while the height went from 3 cm to 6 cm. With two changes together, no one change can be blamed for the growth from 30 cm³ to 120 cm³ — the volume actually became four times as big, not two. A systematic sweep changes one variable at a time. **[1 mark]**

b. Zoe should hold the length and the width fixed (say $l = 5$ cm and $w = 2$ cm) and change only the height — then any change in V is the height's effect. **[1 mark]**

Answer key

Question	Answer
1a	53 points
1b	No — 14 goals alone give 84 points; the score is 86, not 100
2	$800
3	3.0 g/cm³; sinks
4	B — $2(n+6)$
5a	158
5b	174
6a	$4(n-5)$
6b	16
7a	$x = 5$

Question	Answer
7b	Check: $5 \times 5 + 4 = 25 + 4 = 29$
8a	$3x + 7 = 28$
8b	$x = 7$
8c	Check: $3 \times 7 + 7 = 21 + 7 = 28$
9a	1, 4, 7, 10, 13
9b	The output goes up by 3 each time the input goes up by 1
9c	A straight line — the jumps between outputs are all the same
9d	31
10a	January, 32.6 °C
10b	It falls every month, from 31.9 °C to 15.5 °C
10c	No — it rises from July (15.5 °C) to August (17.4 °C)
11a	The person is stopped — the distance from home is not changing
11b	The 20-minute segment — it is steeper, same distance in less time
12a	B4: 70; B5: 140
12b	B6: 210; B7: 280
12c	Each extra hour adds 70 km
13a	Two variables changed at once, so no one change can be blamed
13b	Hold l and w fixed; change only h

Total: 30 marks — Section A (3A, `VC2M7A01`): 5 · Section B (3B, `VC2M7A02`): 5 · Section C (3C, `VC2M7A03`): 5 · Section D (3D, `VC2M7A05`): 5 · Section E (3E, `VC2M7A04`): 5 · Section F (3F, `VC2M7A06`): 5.