Mathematics Level 7

Chapter 2 — Rational number, percentage and ratio (Number)

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Contents

Equivalent representations of rational numbers; positives and negatives on a number line

Content description `VC2M7N03`: find equivalent representations of rational numbers and represent positive and negative rational numbers and mixed numbers on a number line

Achievement standard (extract): Students choose between equivalent representations of rational numbers and percentages to assist in calculations and make simple estimates to judge the reasonableness of results.

Theory

One number, many names. A **rational number** is any number that can be written as a fraction of two whole numbers, with the bottom number not zero. The numbers $\frac{3}{4}$, $-\frac{5}{2}$, 0.75, $28\,\%$ and 6 (which is $\frac{6}{1}$) are all rational numbers. At Level 6 you worked with fractions, and subtopic 1A placed the integers on the number line. This subtopic joins those two ideas together: it looks at the different names the one rational number can be written with, and it places positive and negative rational numbers on the number line.

Equivalent fractions. Two fractions are **equivalent** when they name the same number. Multiplying the top and bottom of a fraction by the same number changes the size of the parts but not the amount shown, because $\frac{2}{2}$, $\frac{3}{3}$ and $\frac{4}{4}$ are each just another name for 1. So $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{6}{9}$ all name the one number: each time, the top and bottom were multiplied by 2, then by 3.

$$\frac{2}{3} = \frac{2\times 2}{3\times 2} = \frac{4}{6}\ \frac{2}{3} = \frac{2\times 3}{3\times 3} = \frac{6}{9}$$

Figure 1 shows why the three fractions are the same amount: on each strip, the shading stops at exactly the same place. To rename a fraction, look for **multiples** of its denominator: 6 and 9 are multiples of 3, so thirds can be renamed as sixths and as ninths. When two fractions must share a denominator, look for a **common multiple** of the two denominators: 12 is a common multiple of 3 and 4, so $\frac{2}{3}$ and $\frac{3}{4}$ can both be written with denominator 12, as $\frac{8}{12}$ and $\frac{9}{12}$.

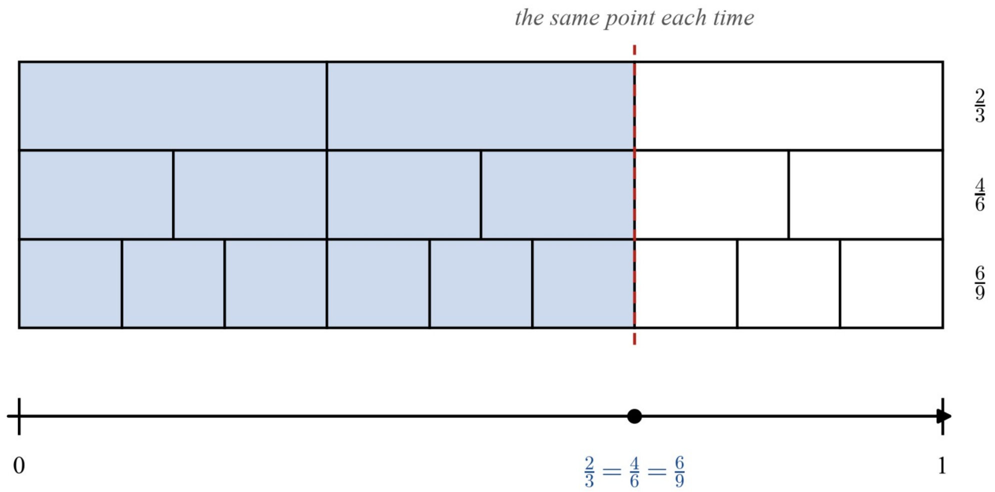


Figure 1. Three equal strips: the first is split into thirds with two parts shaded, the second into sixths with four parts shaded and the third into ninths with six parts shaded; a dashed line falls at the same point of each strip, and a number line from zero to one underneath marks the point as two thirds, four sixths and six ninths.

Simplest form. Dividing the top and bottom of a fraction by the same number gives an equivalent fraction with smaller numbers. Dividing by a **common factor** — a whole number that divides both the top and the bottom exactly — keeps the value unchanged. A fraction is in its **simplest form** when no common factor greater than 1 is left. For example, 6 is a common factor of 18 and 24, and dividing top and bottom by 6 gives

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

and 3 and 4 share no common factor greater than 1, so $\frac{3}{4}$ is the simplest form. You can divide by common factors one step at a time (by 2, then by 3), or in one step using the highest common factor. The prime factorisation from subtopic 1B finds it quickly: $18 = 2 \times 3 \times 3$ and $24 = 2 \times 2 \times 2 \times 3$, and the factors they share give $2 \times 3 = 6$.

Fractions, decimals and percentages. These are three different ways of writing rational numbers, and the one number can move between them.

- **Fraction to decimal:** divide the top by the bottom. $\frac{7}{25} = 7 \div 25 = 0.28$.
- **Decimal to percentage:** multiply by 100, because "per cent" means "out of one hundred". $0.28 \times 100 = 28\,\%$.
- **Percentage to fraction:** write the number over 100, then simplify. $28\,\% = \frac{28}{100} = \frac{7}{25}$, dividing top and bottom by 4.

So the one rational number has three names: $\frac{7}{25} = 0.28 = 28\,\%$. Figure 2 shows the same amount shaded on two different grids, and Figure 3 sets out the three moves. A fast shortcut: when the bottom of a fraction goes into 100, scale it up to one hundredths. Since $25 \times 4 = 100$,

$$\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 28\,\%$$

Going the other way, $45\,\% = \frac{45}{100} = \frac{9}{20}$, and $9 \div 20 = 0.45$.

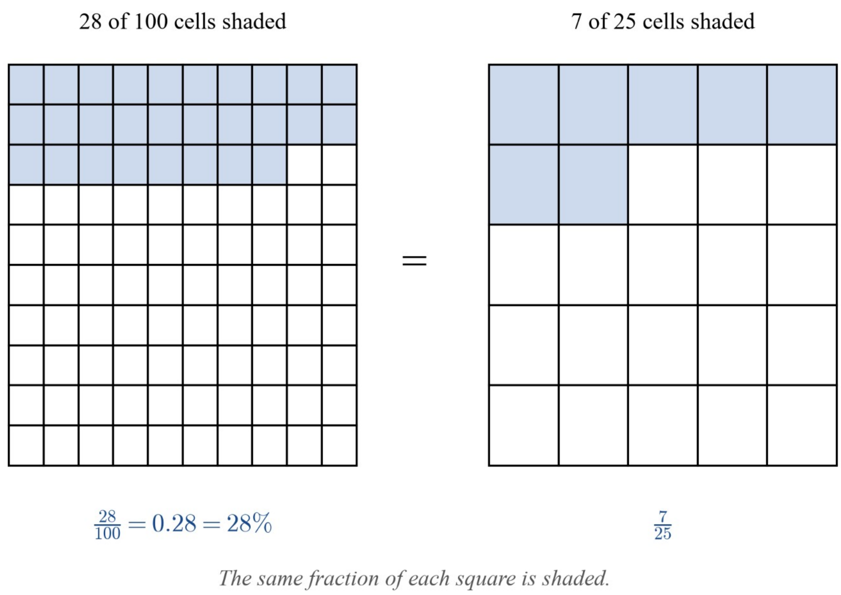


Figure 2. Two equal squares: the first is a ten by ten grid with 28 of its 100 cells shaded, and the second is a five by five grid with 7 of its 25 cells shaded; the same fraction of each square is shaded, so twenty-eight hundredths equals seven twenty-fifths.

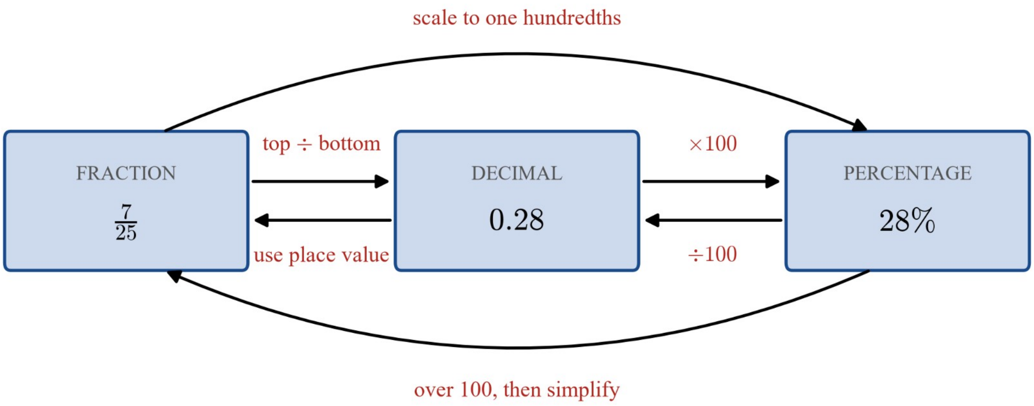


Figure 3. Three boxes in a row labelled fraction, decimal and percentage, holding seven twenty-fifths, zero point two eight and twenty-eight per cent; arrows between them are labelled top divided by bottom, use place value, times one hundred, divide by one hundred, scale to one hundredths, and over one hundred then simplify.

Some equivalences are worth remembering:

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50 %
$\frac{1}{4}$	0.25	25 %
$\frac{3}{4}$	0.75	75 %
$\frac{1}{5}$	0.2	20 %
$\frac{2}{5}$	0.4	40 %
$\frac{1}{8}$	0.125	12.5 %
$\frac{3}{8}$	0.375	37.5 %
$\frac{1}{10}$	0.1	10 %
$\frac{1}{20}$	0.05	5 %
$\frac{1}{25}$	0.04	4 %
$\frac{1}{50}$	0.02	2 %

Subtopic 2E takes percentages further; here a percentage is simply another name for the same number.

Fractions on the number line. The number line of subtopic 1A carries fractions and decimals as well as integers. Between any two whole numbers, the gap can be split into equal parts, and the denominator tells you how many parts each gap is split into. Figure 4 shows the line from 0 to 2 split into quarters: $\frac{1}{4}$ sits one quarter-step right of 0, and $\frac{3}{4}$ sits three quarter-steps right of 0. Keep counting past 1: $\frac{5}{4}$ is five quarter-steps from 0, landing at the very point called

$1\frac{1}{4}$ — a **mixed number** (also called a mixed numeral), made of a whole number and a fraction. Likewise $\frac{7}{4} = 1\frac{3}{4}$. Every fraction greater than 1 can be written as a mixed number by counting how many wholes it contains.

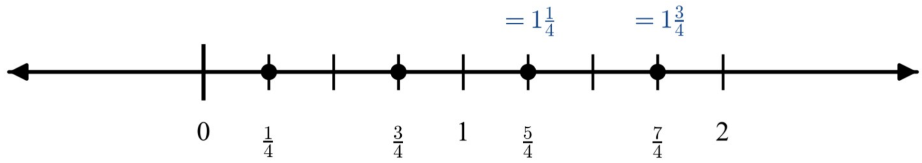


Figure 4. A number line from zero to two with every gap split into quarters; dots mark one quarter, three quarters, five quarters and seven quarters, and the last two dots are also labelled one and one quarter and one and three quarters.

Negatives and opposites. The negative side of the line mirrors the positive side. $-\frac{3}{4}$ sits three quarter-steps to the *left* of zero, and $-1\frac{3}{4}$ sits three quarter-steps left of -1. The pair $\frac{3}{4}$ and $-\frac{3}{4}$ are **opposites**: the same distance from zero, on opposite sides of it (Figure 5). As with integers, numbers get smaller as you move left, so on the negative side $-1\frac{3}{4} < -\frac{3}{4}$ — the number closer to zero is the larger one.

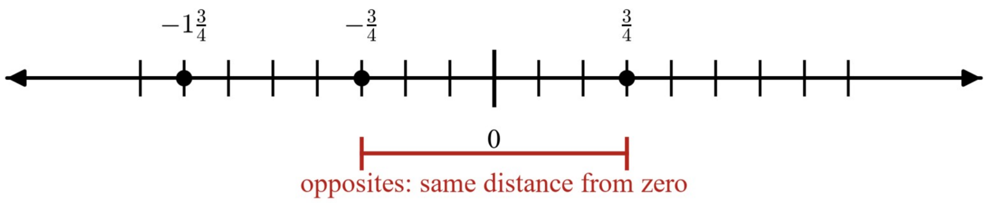


Figure 5. A number line from minus two to two with every gap split into quarters; dots mark minus one and three quarters, minus three quarters and three quarters, and a bracket from minus three quarters to three quarters shows the two opposites sit the same distance from zero.

Any interval of the line. A number line does not have to sit symmetrically about zero. It might run from $-1\frac{1}{2}$ to $2\frac{1}{4}$, or give a closer look at just the gap between 1 and 2, with no zero in sight (Figure 6). To read a line like this: find two labelled numbers, count the equal steps between them, work out the size of one step, then count along from a label. Subtopic 2B looks closely at the numbers that sit between two whole numbers; here the line simply carries whatever rational numbers the situation needs.

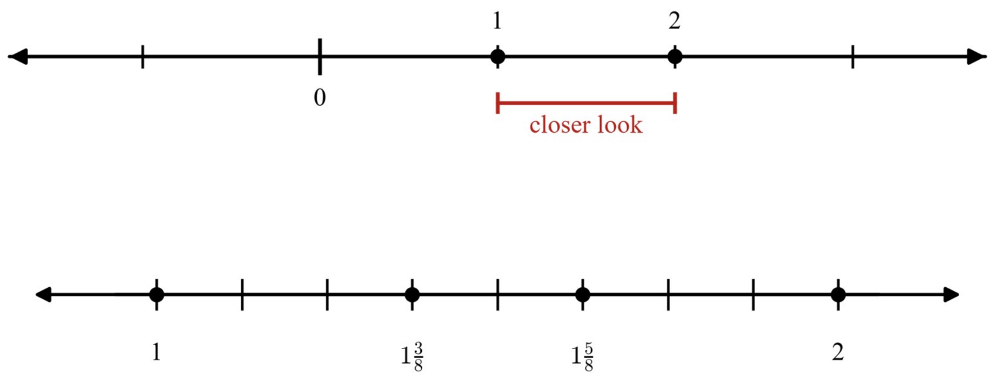


Figure 6. A number line from minus one to three with a bracket over the gap between one and two; below, that gap is shown again split into eighths, with dots marking one and three eighths and one and five eighths.

Comparing numbers in different forms. To compare or order rational numbers written in different forms, first write them all the one way — usually as decimals. To compare -0.6 and $-\frac{5}{8}$, write $-\frac{5}{8} = -0.625$. On the negative side the number closer to zero is larger: -0.6 is closer to zero than -0.625, so $-0.6 > -\frac{5}{8}$.

Choose the form that suits the job. One name for a number often makes a calculation easier than another. To find $37.5\,\%$ of 56, the decimal 0.375 is awkward to use, but the table says $37.5\,\% = \frac{3}{8}$, and $\frac{3}{8}$ of $56 = 3 \times 7 = 21$ is mental work. A quick estimate checks the answer is reasonable: $37.5\,\%$ sits between $25\,\%$ and $50\,\%$, so the answer should sit between 14 and 28 — and 21 does. Subtopic 2B builds this estimating habit further.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
rational number	a number that can be written as a fraction of two whole numbers, the bottom one not zero	a fraction, decimal or percentage value
equivalent	naming the same number	equal in value
simplest form	a fraction with no common factor greater than 1 left in its top and bottom	fully cancelled
common factor	a whole number that divides two or more numbers exactly	common divisor
numerator	the top number of a fraction	top
denominator	the bottom number of a fraction	bottom
mixed number	a whole number plus a fraction, such as $1\frac{3}{4}$	mixed numeral
opposites	two numbers the same distance from zero on opposite sides, like $\frac{3}{4}$ and $-\frac{3}{4}$	one is the negative of the other
interval	the numbers between two given values on the line	section of the line

Ahead, subtopics 2C and 2D put the four operations to work on fractions and decimals, and 2E works with percentages in depth. Everything rests on moving between the forms fluently, which is what this subtopic builds.

Worked examples

Example 1 — scaffolded

Write $\frac{18}{24}$ in its simplest form, then write two equivalent fractions with larger numbers.

Working	Comment
Find a common factor. $18 = 2 \times 3 \times 3$ and $24 = 2 \times 2 \times 2 \times 3$, so the highest common factor is $2 \times 3 = 6$ (1B).	Dividing top and bottom by the same number keeps the value.
Divide. $\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$.	3 and 4 share no common factor greater than 1, so this is the simplest form (Figure 7).
Scale up. $\frac{3}{4} = \frac{6}{8}$ (multiply by 2) and $\frac{3}{4} = \frac{15}{20}$ (multiply by 5).	Multiplying top and bottom by the same number is multiplying by 1.

$\therefore$ the simplest form is $\frac{3}{4}$, and two larger equivalents are $\frac{6}{8}$ and $\frac{15}{20}$.

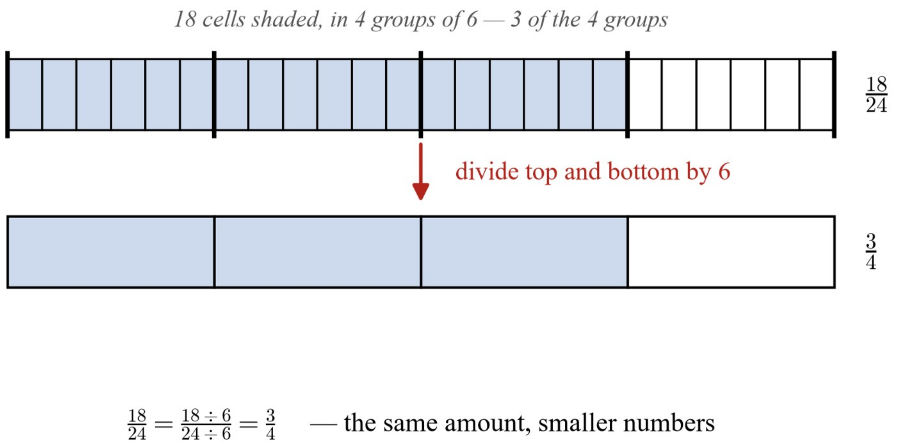


Figure 7. Two equal bars: the top bar is split into 24 cells with 18 shaded and grouped in blocks of six, and the bottom bar is split into four cells with three shaded; both bars show the same amount, so eighteen twenty-fourths equals three quarters.

Example 2 — scaffolded

(a) Write $\frac{7}{25}$ as a decimal and as a percentage.

(b) Use the form you find easiest to find $28\,\%$ of 50.

Working	Comment
(a) **Divide.** $\frac{7}{25} = 7 \div 25 = 0.28$.	Top divided by bottom turns a fraction into a decimal.
Convert. $0.28 \times 100 = 28\,\%$.	Check by scaling: $25 \times 4 = 100$, so $\frac{7}{25} = \frac{28}{100} = 28\,\%$.
(b) **Choose a form.** $28\,\% = \frac{28}{100}$, so $28\,\%$ of 50 is $\frac{28 \times 50}{100} = \frac{1400}{100} = 14$.	$0.28 \times 50 = 14$ gives the same answer; pick whichever form computes easily.
Estimate. $25\,\%$ of 50 is 12.5, and $28\,\%$ is a little more,	A quick estimate checks the size of the answer.

so 14 is reasonable.

$\therefore\ \frac{7}{25} = 0.28 = 28\%$, and 28% of 50 is 14.

Example 3 — unscaffolded

Plot $-1\frac{3}{4}$, $-\frac{1}{2}$, $\frac{3}{4}$ and $1\frac{1}{2}$ on a number line, then write the four numbers in ascending order joined with <.

Split each gap between whole numbers into quarters, so every tick is one quarter-step (Figure 8). Then $-1\frac{3}{4}$ sits three quarter-steps left of -1, $-\frac{1}{2} = -\frac{2}{4}$ sits two quarter-steps left of 0, $\frac{3}{4}$ sits three quarter-steps right of 0, and $1\frac{1}{2} = 1\frac{2}{4}$ sits two quarter-steps right of 1. Reading the plotted numbers from left to right:

$$-1\frac{3}{4} < -\frac{1}{2} < \frac{3}{4} < 1\frac{1}{2}$$

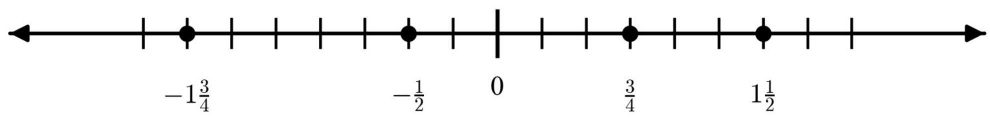


Figure 8. A number line from minus two to two with every gap split into quarters; dots mark minus one and three quarters, minus one half, three quarters and one and one half.

Example 4 — unscaffolded

The number line in Figure 9 runs from $-1\frac{1}{2}$ to $2\frac{1}{4}$ with equally spaced ticks. Only $-1\frac{1}{2}$, 0 and $2\frac{1}{4}$ are printed. Write the value at each of the letters P, Q and R as a fraction or a mixed number.

From 0 to $2\frac{1}{4}$ is 9 ticks, and $2\frac{1}{4} = \frac{9}{4}$, so each tick is one quarter. The line is not symmetrical about zero — there is more of it on the positive side — but the counting works the same way from any printed label.

- **P** is 3 ticks right of $-1\frac{1}{2}$: counting by quarters, $-1\frac{1}{4}$, -1, $-\frac{3}{4}$. So **P** $= -\frac{3}{4}$.
- **Q** is 2 ticks right of 0: **Q** $= \frac{2}{4} = \frac{1}{2}$.
- **R** is 3 ticks left of $2\frac{1}{4}$: counting back by quarters, 2, $1\frac{3}{4}$, $1\frac{1}{2}$. So **R** $= 1\frac{1}{2}$.

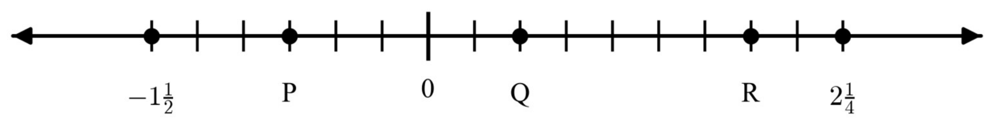


Figure 9. A number line from minus one and one half to two and one quarter with equally spaced ticks; only minus one and one half, zero and two and one quarter are printed, and the letters P, Q and R mark three of the ticks in between.

Practice questions

Scaffolded

Q1. Complete each statement by finding the missing number. (a) $\frac{2}{3} = \frac{\square}{9}$ (b) $\frac{3}{5} = \frac{12}{\square}$ (c) $\frac{4}{7} = \frac{\square}{21}$ (d) $\frac{5}{8} = \frac{15}{\square}$

Q2. Write each fraction in its simplest form. (a) $\frac{6}{8}$ (b) $\frac{10}{12}$ (c) $\frac{9}{24}$ (d) $\frac{14}{21}$ (e) $\frac{25}{40}$ (f) $\frac{18}{27}$

Q3. Write each fraction as a decimal, then as a percentage. (a) $\frac{3}{5}$ (b) $\frac{7}{10}$ (c) $\frac{1}{8}$ (d) $\frac{9}{20}$

Q4. Write each number as a fraction in its simplest form. (a) 0.4 (b) 0.55 (c) 36% (d) 62.5%

Q5. Choose a fraction form from the table above, then find each value mentally. (a) 50% of 26 (b) 25% of 44 (c) 12.5% of 32 (d) 20% of 35

Q6. Each number line in Figure 10 runs from 0 to 1 and is split into equal parts. Write the fraction at each letter.

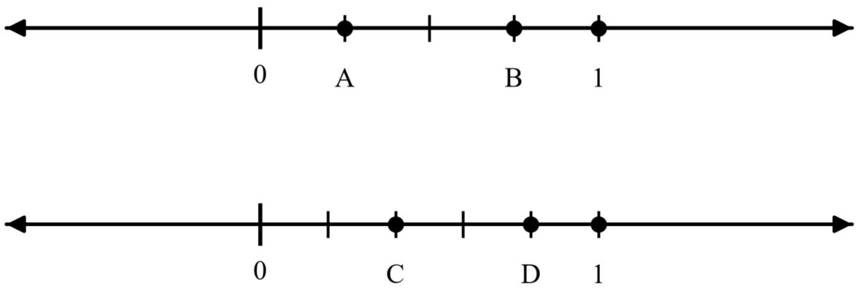


Figure 10. Two number lines from zero to one: the first is split into quarters with the letters A and B marking two of the ticks, and the second is split into fifths with the letters C and D marking two of the ticks.

Unscaffolded

Q7. Replace each blank with <, > or =. (Hint: write both numbers the same way first.) (a) 0.6 ___ $\frac{5}{8}$ (b) $\frac{3}{4}$ ___ 70% (c) -0.5 ___ $-\frac{3}{8}$ (d) $\frac{1}{8}$ ___ 0.12 (e) $-1\frac{1}{4}$ ___ -1.3 (f) 45% ___ $\frac{9}{20}$

Q8. Write the numbers 0.32, $\frac{3}{8}$, 30% and 0.35 in ascending order.

Q9. The number line in Figure 11 runs from $-2\frac{1}{3}$ to $1\frac{1}{3}$ with equally spaced ticks. Only $-2\frac{1}{3}$, 0 and $1\frac{1}{3}$ are printed. Write the value at each of the letters A, B and C as a fraction or a mixed number.

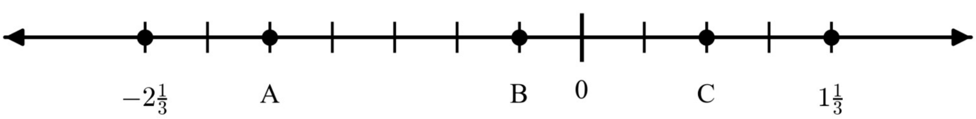


Figure 11. A number line from minus two and one third to one and one third with equally spaced ticks; only minus two and one third, zero and one and one third are printed, and the letters A, B and C mark three of the ticks in between.

Q10. Convert between mixed numbers and fractions. (a) Write $2\frac{3}{8}$ as a fraction. (b) Write $\frac{11}{5}$ as a mixed number. (c) Write $-1\frac{5}{8}$ as a fraction. (d) Write $-\frac{9}{4}$ as a mixed number.

Q11. Opposites and distance from zero. (a) Write the opposite of $\frac{5}{8}$. (b) Write the opposite of $-1\frac{1}{4}$. (c) Which is closer to zero, $-\frac{7}{8}$ or $\frac{3}{4}$? (d) A number sits $1\frac{1}{4}$ units to the left of zero on the number line. What is the number?

Q12. Five cold rooms at a food warehouse are kept at $-2\frac{1}{4}°C$, $-\frac{1}{2}°C$, $-1\frac{3}{4}°C$, $-3°C$ and $-1°C$. (a) Write the temperatures from coldest to warmest. (b) How many of the cold rooms are colder than $-1\frac{1}{2}°C$?

Q13. Choose the form that makes each calculation easy, then find the value. Show the swap you used. (a) 37.5% of 56 (b) 4% of 75 (c) 0.75×36

Q14. Without calculating exactly, decide which of 24, 12 or 6 is closest to 37.5% of 33. Explain your choice.

Q15. State whether each statement is true or false. Correct every false statement. (a) $0.6>\frac{5}{8}$ (b) $-\frac{3}{4}$ lies to the right of -1 on the number line. (c) $\frac{3}{5}=60\%$ (d) The simplest form of $\frac{12}{18}$ is $\frac{4}{6}$. (e) $-1\frac{1}{4}<-1\frac{1}{2}$ (f) The opposite of $-\frac{3}{5}$ is $\frac{3}{5}$.

Answers

Q1. (a) 6; (b) 20; (c) 12; (d) 24.

Q2. (a) $\frac{3}{4}$; (b) $\frac{5}{6}$; (c) $\frac{3}{8}$; (d) $\frac{2}{3}$; (e) $\frac{5}{8}$; (f) $\frac{2}{3}$.

Q3. (a) 0.6, 60%; (b) 0.7, 70%; (c) 0.125, 12.5%; (d) 0.45, 45%.

Q4. (a) $\frac{2}{5}$; (b) $\frac{11}{20}$; (c) $\frac{9}{25}$; (d) $\frac{5}{8}$.

Q5. (a) 13; (b) 11; (c) 4; (d) 7.

Q6. A $=\frac{1}{4}$; B $=\frac{3}{4}$; C $=\frac{2}{5}$; D $=\frac{4}{5}$.

Q7. (a) $<$; (b) $>$; (c) $<$; (d) $>$; (e) $>$; (f) $=$.

Q8. $30\%, 0.32, 0.35, \frac{3}{8}$.

Q9. A $=-1\frac{2}{3}$; B $=-\frac{1}{3}$; C $=\frac{2}{3}$.

Q10. (a) $\frac{19}{8}$; (b) $2\frac{1}{5}$; (c) $-\frac{13}{8}$; (d) $-2\frac{1}{4}$.

Q11. (a) $-\frac{5}{8}$; (b) $1\frac{1}{4}$; (c) $\frac{3}{4}$; (d) $-1\frac{1}{4}$.

Q12. (a) $-3\,°C, -2\frac{1}{4}\,°C, -1\frac{3}{4}\,°C, -1\,°C, -\frac{1}{2}\,°C$; (b) 3.

Q13. (a) 21; (b) 3; (c) 27.

Q14. 12.

Q15. (a) false — $0.6<\frac{5}{8}$; (b) true; (c) true; (d) false — the simplest form is $\frac{2}{3}$; (e) false — $-1\frac{1}{4}>-1\frac{1}{2}$; (f) true.

Fully-worked solutions

Q1. Multiply or divide top and bottom by the same number. (a) $3\times3=9$, so multiply by 3: $\frac{2}{3}=\frac{6}{9}$. (b) $3\times4=12$, so multiply by 4: $\frac{3}{5}=\frac{12}{20}$. (c) $7\times3=21$, so multiply by 3: $\frac{4}{7}=\frac{12}{21}$. (d) $5\times3=15$, so multiply by 3: $\frac{5}{8}=\frac{15}{24}$.

Q2. Divide top and bottom by the highest common factor. (a) $\frac{6}{8}=\frac{3}{4}$ (divide by 2). (b) $\frac{10}{12}=\frac{5}{6}$ (divide by 2). (c) $\frac{9}{24}=\frac{3}{8}$ (divide by 3). (d) $\frac{14}{21}=\frac{2}{3}$ (divide by 7). (e) $\frac{25}{40}=\frac{5}{8}$ (divide by 5). (f) $\frac{18}{27}=\frac{2}{3}$ (divide by 9).

Q3. Divide top by bottom for the decimal, then multiply by 100 for the percentage. (a) $3\div5=0.6=60\%$. (b) $7\div10=0.7=70\%$. (c) $1\div8=0.125=12.5\%$. (d) $9\div20=0.45=45\%$.

Q4. Use place value, or write over 100, then simplify. (a) $0.4=\frac{4}{10}=\frac{2}{5}$. (b) $0.55=\frac{55}{100}=\frac{11}{20}$. (c) $36\%=\frac{36}{100}=\frac{9}{25}$. (d) $62.5\%=\frac{62.5}{100}=\frac{625}{1000}=\frac{5}{8}$.

Q5. Swap the percentage for a fraction, then divide. (a) $50\%=\frac{1}{2}$, and $\frac{1}{2}$ of $26=13$. (b) $25\%=\frac{1}{4}$, and $\frac{1}{4}$ of $44=11$. (c) $12.5\%=\frac{1}{8}$, and $\frac{1}{8}$ of $32=4$. (d) $20\%=\frac{1}{5}$, and $\frac{1}{5}$ of $35=7$.

Q6. The denominator is the number of equal parts between 0 and 1, and the numerator counts the steps from 0. The first line is split into quarters: A is 1 step, so A $=\frac{1}{4}$, and B is 3 steps, so B $=\frac{3}{4}$. The second line is split into fifths: C is 2 steps, so C $=\frac{2}{5}$, and D is 4 steps, so D $=\frac{4}{5}$.

Q7. Write both numbers as decimals, then compare. (a) $\frac{5}{8}=0.625$, and $0.6<0.625$, so $0.6<\frac{5}{8}$. (b) $\frac{3}{4}=0.75$, and $70\%=0.7$, so $\frac{3}{4}>70\%$. (c) $-\frac{3}{8}=-0.375$; on the negative side the number closer to zero is larger, so $-0.5<-0.375$. (d) $\frac{1}{8}=0.125$, and $0.125>0.12$. (e) $-1\frac{1}{4}=-1.25$; -1.25 is closer to zero than -1.3, so $-1\frac{1}{4}>-1.3$. (f) $\frac{9}{20}=\frac{45}{100}=45\%$, so $45\%=\frac{9}{20}$.

Q8. As decimals: $30\%=0.3$, $\frac{3}{8}=0.375$. Reading $0.3, 0.32, 0.35, 0.375$ from smallest to largest gives $30\%, 0.32, 0.35, \frac{3}{8}$.

Q9. From 0 to $1\frac{1}{3}=\frac{4}{3}$ is 4 equal steps, so each step is one third. Counting from the printed labels: A is 2 steps right of $-2\frac{1}{3}$, landing at $-1\frac{2}{3}$; B is 1 step left of 0, so B $=-\frac{1}{3}$; C is 2 steps right of 0, so C $=\frac{2}{3}$.

Q10. (a) $2\frac{3}{8}$ is 2 wholes and $\frac{3}{8}$: $2\times 8+3=19$, so $2\frac{3}{8}=\frac{19}{8}$. (b) $11\div 5=2$ remainder 1, so $\frac{11}{5}=2\frac{1}{5}$. (c) $-1\frac{5}{8}$ is the opposite of $1\frac{5}{8}=\frac{13}{8}$, so $-1\frac{5}{8}=-\frac{13}{8}$. (d) $9\div 4=2$ remainder 1, so $\frac{9}{4}=2\frac{1}{4}$, and $-\frac{9}{4}=-2\frac{1}{4}$.

Q11. Opposites sit the same distance from zero on the other side. (a) $-\frac{5}{8}$. (b) $1\frac{1}{4}$. (c) $\frac{3}{4}=\frac{6}{8}$, and $\frac{6}{8}$ is closer to zero than $\frac{7}{8}$, so $\frac{3}{4}$ is closer to zero than $-\frac{7}{8}$. (d) Moving left of zero gives a negative number: $-1\frac{1}{4}$.

Q12. (a) Coldest to warmest is left to right on the number line: $-3\,°C, -2\frac{1}{4}\,°C, -1\frac{3}{4}\,°C, -1\,°C, -\frac{1}{2}\,°C$. (b) $-1\frac{1}{2}=-1.5$. The rooms colder than $-1.5\,°C$ are $-3\,°C$, $-2\frac{1}{4}\,°C$ and $-1\frac{3}{4}\,°C$ (since $-1.75<-1.5$): 3 rooms.

Q13. (a) $37.5\%=\frac{3}{8}$, and $\frac{3}{8}$ of $56=3\times 7=21$. (b) $4\%=\frac{4}{100}=\frac{1}{25}$, and $\frac{1}{25}$ of $75=3$. (c) $0.75=\frac{3}{4}$, and $\frac{3}{4}$ of $36=3\times 9=27$.

Q14. $37.5\% = \frac{3}{8}$, and 33 is close to 32. $\frac{3}{8}$ of $32 = 12$, so 37.5% of 33 is a little more than 12 — the closest value is 12 (it is not as large as 24, which would need about 75%, and 6 would only be about 18%).

Q15. (a) **False**: $\frac{5}{8} = 0.625$, and $0.6 < 0.625$. (b) **True**: $-\frac{3}{4}$ is closer to zero than -1, so it lies to the right of -1. (c) **True**: $\frac{3}{5} = 0.6 = 60\%$. (d) **False**: $\frac{4}{6}$ is equivalent to $\frac{12}{18}$ but not in simplest form; dividing by 6 gives $\frac{12}{18} = \frac{2}{3}$. (e) **False**: $-1\frac{1}{4} = -1.25$ is closer to zero than -1.5, so $-1\frac{1}{4} > -1\frac{1}{2}$. (f) **True**: opposites sit the same distance from zero on opposite sides.

Teacher note

- **Ownership and the boundary with 1A, 2C, 2D and 2E.** Per the Chapter 2 plan in TOC §3, this subtopic (`VC2M7N03`) owns **equivalent representations** of rational numbers (fraction ⇄ decimal ⇄ percentage, simplest form) and **locating and ordering** positive and negative rational numbers and mixed numbers on a number line. 1A owns the integer number line and the comparison symbols; 2C and 2D own the four operations; 2E owns percentages of quantities. Negatives here are **located and ordered only** (D4.4): no operation on a negative number appears in this subtopic, and the counting in WE4 and Q9 moves along the line rather than computing with negatives.
- **Level 6 recall (D3).** Fraction equivalence and ordering (`VC2M6N03`, `VC2M6N05`) and integers on the number line (`VC2M6N01`) are assumed. They are recalled in one sentence each in the theory and maintained inside the questions, not taught as new content.
- **1B's prime factorisation, reused.** The highest-common-factor route to simplest form cross-references 1B's prime factorisation (TOC §3, Chapter 1 note), and WE1 shows it finding the HCF of 18 and 24 in one step.
- **E04 intervals are drawn to scale.** The asymmetric-interval work (`VC2M7N03.E04`) is assessable, so Figures 6, 9 and 11 are generated to scale from the exact values stated (TOC §3, Chapter 2 note); Figures 4, 5, 8 and 10 carry the symmetric cases.
- **No classification of decimal expansions (D4.13).** Every conversion in this subtopic terminates; no recurring decimal, no overbar notation, and no use of the words *terminating* or *recurring*. That classification is `VC2M8N03` (Level 8).
- **Percentages as names, not yet quantities.** Percentages appear only as another representation of the same number (E03, achievement-standard extract). The one small "percentage of" calculation (WE2(b), Q5, Q13) exists to deliver the extract's "choose between equivalent representations … to assist in calculations"; the full skill belongs to 2E (`VC2M7N07`).
- **Reasonableness habit (P7).** WE2's estimate step and Q14 deliver the achievement-standard extract's "make simple estimates to judge the reasonableness of results", per TOC §5.2. 2B takes the judgement further.
- **No real data.** All contexts (the food warehouse cold rooms, the values themselves) are generic and transparently fictional, so no §6 source lines are required.
- **Digital tools (D9).** The CD and its elaborations name manipulatives, number lines and diagrams, not digital tools; the work is done by hand with accurately drawn lines. No CAS.
- **Figures.** All eleven figures are generated by `src/gen_2a.py` from the exact values stated in the text; every number line is to scale, so a figure cannot disagree with the arithmetic it shows.
- **No D11 content.** None of `VC2M7N03.E01–.E04` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

Rounding to a given accuracy; estimating to check reasonableness

Content description VC2M7N04: round decimals to a given accuracy appropriate to the context and use appropriate rounding and estimation to check the reasonableness of computations

Achievement standard (extract): Students choose between equivalent representations of rational numbers and percentages to assist in calculations and make simple estimates to judge the reasonableness of results.

Theory

Between two whole numbers. A decimal that is not itself a whole number always sits between two **consecutive integers** — two whole numbers that follow one another, like 3 and 4. The whole-number part tells you where: 3.72 is 3 wholes and a bit more, so it sits between 3 and 4. The set of numbers between two such integers is called an **interval**. In Figure 1, 1.25 sits in the interval between 1 and 2, 3.72 sits in the interval between 3 and 4, and 4.6 sits in the interval between 4 and 5. Knowing the interval a number sits in is the first step in rounding, because rounding means choosing the nearest endpoint of that interval.

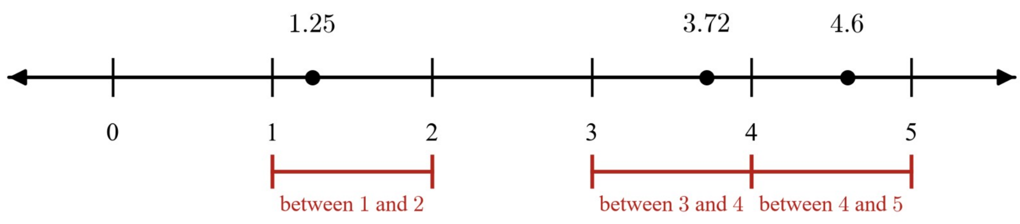


Figure 1. A number line from zero to five with dots marking one point two five, three point seven two and four point six; a bracket under the gap holding each dot marks the interval between the two whole numbers it sits between.

At Level 6 you rounded whole numbers to the nearest 10, 100 and 1000 and used estimates to check answers. This subtopic does the same jobs with decimals, and then asks a question that matters in real life: *how much accuracy does this job actually need?*

Rounding to the nearest whole number. To round a decimal to the nearest whole number, decide which endpoint of its interval it is closer to. Figure 2 zooms in on the interval from 3 to 4. The number 3.28 is closer to 3, so it rounds down to 3. The number 3.72 is closer to 4, so it rounds up to 4. The halfway point 3.5 is the same distance from both ends, so it belongs to neither side more than the other; everyone follows the same rule for it — a number exactly halfway rounds up. So 3.5 rounds to 4, and 12.5 rounds to 13.

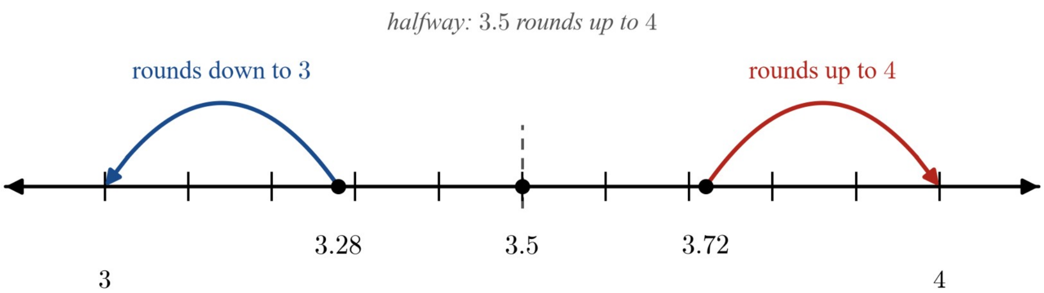


Figure 2. A number line from three to four with ticks at each tenth and a dashed line at the halfway point three point five; arcs show three point two eight rounding down to three and three point seven two rounding up to four.

Rounding to a given number of decimal places. The same idea works on smaller intervals. To round to **one decimal place**, find the nearest tenth: the interval for 3.72 runs from 3.7 to 3.8, its halfway point is 3.75, and 3.72 is closer to 3.7, so it rounds to 3.7. Figure 3 shows 3.72 rounding to 3.7 and 3.78 rounding to 3.8. To round to **two decimal places**, find the nearest hundredth: 3.724 rounds to 3.72 because 3.724 is closer to 3.72 than to 3.73.

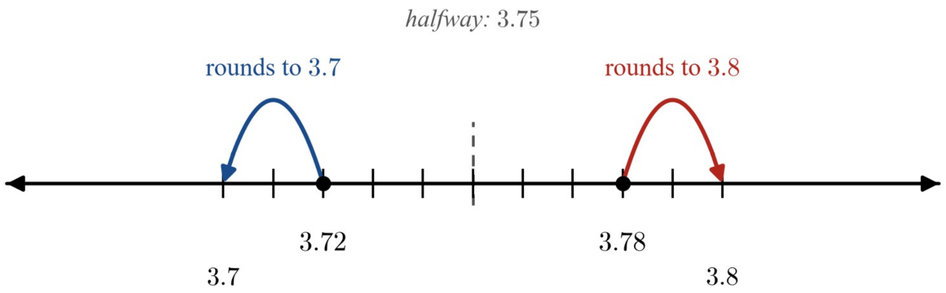


Figure 3. A number line from three point seven to three point eight with ticks at each hundredth and a dashed line at the halfway point three point seven five; arcs show three point seven two rounding down to three point seven and three point seven eight rounding up to three point eight.

The rounding method. Instead of drawing a line every time, use the digits.

1. Find the last digit you want to keep.
2. Look at the digit next door to its right.
3. If that digit is 5 or more, round up. If it is less than 5, leave the last digit as it is.
4. Drop every digit after that.

Figure 4 runs the one number 18.4675 through all three rounds. Rounding to two decimal places, the last digit kept is the 6 and the digit next door is 7, so 18.4675 rounds up to 18.47. Rounding to one decimal place, the digit next door to the 4 is 6, so 18.4675 rounds up to 18.5. Rounding to the nearest whole number, the digit next door to the 8 is 4, which is less than 5, so 18.4675 stays at 18.

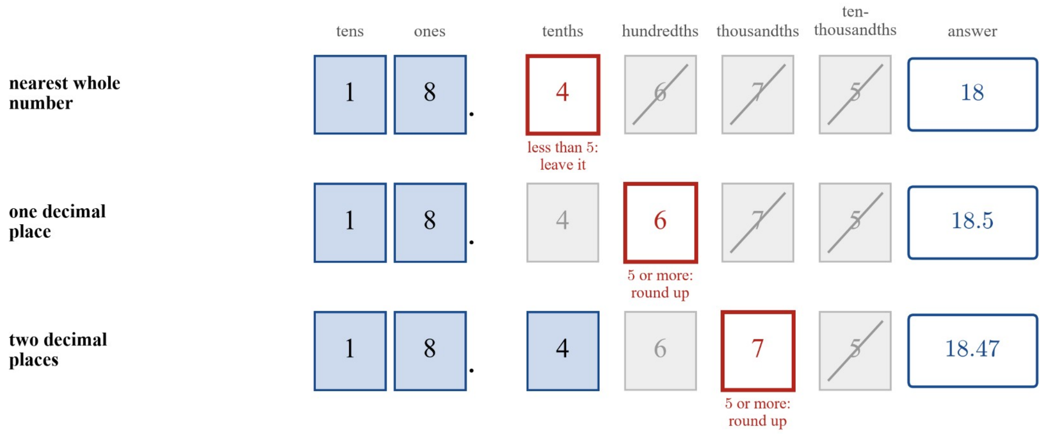


Figure 4. The digits of eighteen point four six seven five in place-value columns; three rows show the rounding to the nearest whole number, to one decimal place and to two decimal places, with the kept digits shaded, the next-door digit marked, the dropped digits crossed out, and the answers eighteen, eighteen point five and eighteen point four seven.

Two things trip people up. First, when a number rounds up past a whole number, keep the zero: 6.99 rounds to 7.0 to one decimal place, not 7 — the zero shows the answer was rounded to one decimal place. Second, drop the digits after the rounding point; never round twice. To round 2.348 to one decimal place, look only at the next-door digit 4, giving 2.3 — not at the 8 further along.

Rounding to the nearest 10, 100 or 1000. The same method works left of the decimal point. In $3{,}847$, rounding to the nearest hundred keeps the 8 in the hundreds column; the digit next door is 4, which is less than 5, so $3{,}847$ rounds to $3{,}800$. Rounding to the nearest ten keeps the 4; the next-door digit is 7, so $3{,}847$ rounds to $3{,}850$. Money rounds the same way: a budget item of $\$2{,}450$ rounds up to $\$2{,}500$ to the nearest hundred dollars, because the halfway rule rounds 450 up.

Estimate first, then check. An **estimate** is a quick, rough answer — close to the exact one, found with numbers that are easy to work with. To estimate a calculation, first round every number to a nearby value you can work with in your head, then calculate.

- $4.87 + 3.21$: round to $5 + 3$, so the estimate is 8. The exact answer is 8.08 — close to 8, so it is reasonable (Figure 5).
- $6 \times \$4.95$: round $\$4.95$ to $\$5$, so the estimate is $6 \times \$5 = \$30$. The exact answer is $\$29.70$, close to $\$30$ (Figure 6).

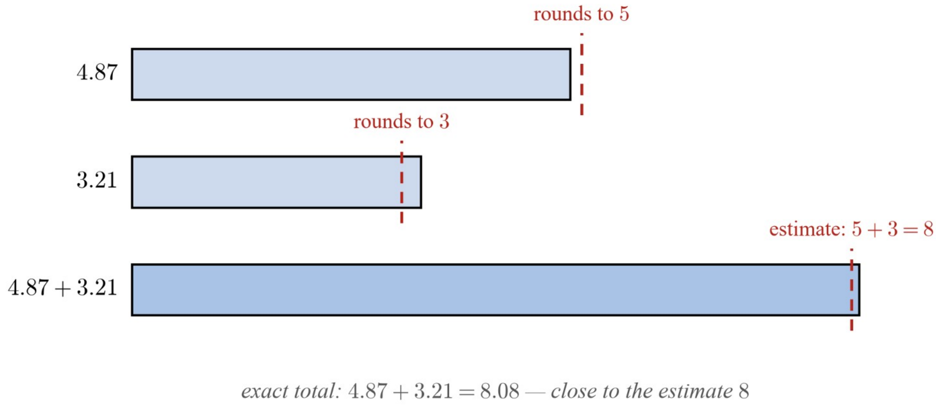


Figure 5. Three bars: one four point eight seven long with a mark at five, one three point two one long with a mark at three, and a joined bar eight point zero eight long with a mark at eight; the estimate five plus three equals eight sits close to the exact total eight point zero eight.

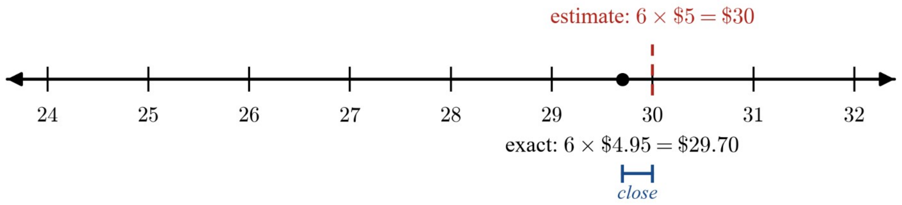


Figure 6. A number line from twenty-four to thirty-two dollars with the exact answer six times four dollars and ninety-five cents marked at twenty-nine dollars and seventy cents and the estimate six times five dollars marked at thirty dollars.

The estimate is the check that stands between you and a wrong answer. If your exact answer sits nowhere near your estimate, something has gone wrong. This matters most with calculators, where one slip of a key or a decimal point produces a number that looks fine until you compare it with the estimate. Suppose a calculator shows 1538.6 for 31.4×4.9. Rounding gives $30 \times 5 = 150$, and 1538.6 is about ten times too big — the decimal point has slipped one place, and the true answer is 153.86 (Figure 7). Subtopics 2C and 2D use this check on every longer calculation.

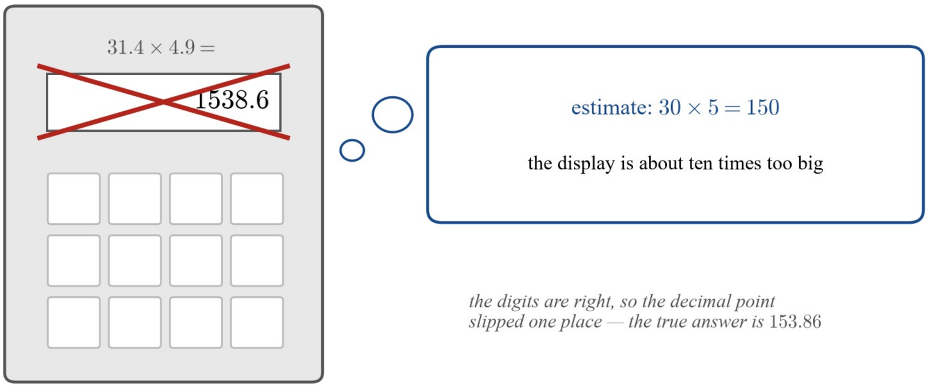


Figure 7. A calculator with the keys three one point four times four point nine and a display reading one five three eight point six, marked wrong with a cross; a thought bubble shows the estimate thirty times five equals one hundred and fifty, so the display is about ten times too big.

When every number in a sum is rounded up, the estimate comes out a little **high**; when every number is rounded down, it comes out a little **low**. Either way the estimate does its job, because it tells you roughly where the exact answer must land.

How accurate is accurate enough? Rounding throws away information, and the context decides how much you can afford to throw away. Different jobs round to different accuracies (Figure 8). A budget for renovating a house can round every item to the nearest $\$ 100$, because the point is to see whether the job can be paid for, not to pay the bills. A weather report rounds the maximum temperature to the nearest degree. But a length measured for a piece of carpet rounds to the nearest centimetre, and a sum of money on a receipt is exact to the cent. Rounding a carpet length to the nearest metre could leave a gap; rounding a renovation budget to the nearest dollar wastes effort without telling you anything new.

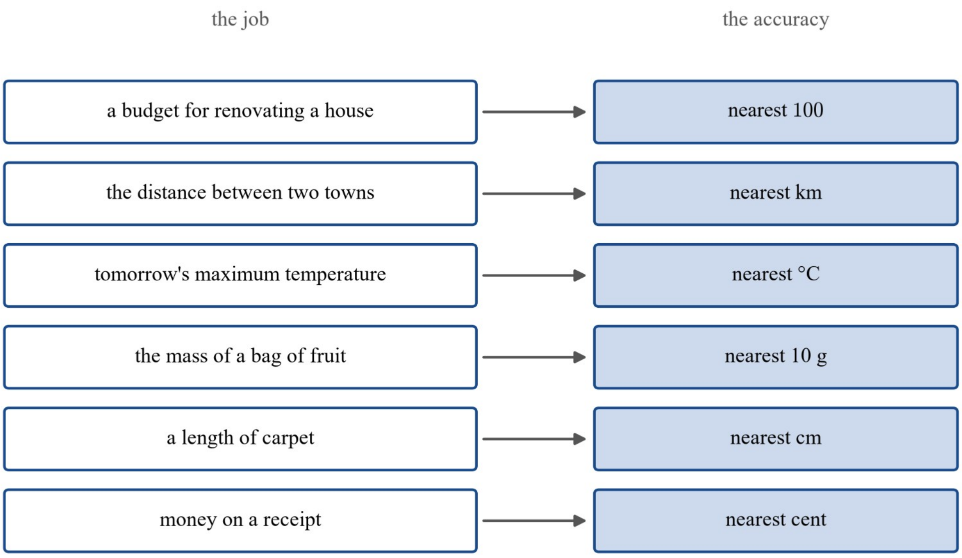


Figure 8. Six rows pairing a job with the accuracy it deserves: renovating a house budget to the nearest hundred dollars, the distance between towns to the nearest kilometre, the maximum temperature to the nearest degree, the mass of a bag of fruit to the nearest ten grams, a length of carpet to the nearest centimetre and money on a receipt to the nearest cent.

Sometimes rounding to the nearest is the wrong direction. When there must be **enough** — paint for a wall, money for a shopping trip, cans for a drink stall — rounding down leaves you short, so you round up instead. Worked Example 4 makes this decision with tins of paint, and the questions return to it with shopping and budgets. The habit to take from this subtopic is the three questions to ask before you round:

1. What is the number used for?

2. Is a rounded number good enough for that purpose?
3. Should the number be rounded to the nearest value, or up so there is enough?

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
consecutive integers	whole numbers that follow one another, like 3 and 4	neighbouring whole numbers
interval	the numbers between two given values on the line	the gap between
round	replace a number with a nearby value that is easier to use	round off
nearest whole number	the closest integer, with halfway values rounding up	nearest integer
decimal place	a position to the right of the decimal point: tenths, hundredths, …	—
estimate	a quick, rough answer used to judge a calculation	approximation
accurate / accuracy	how close a number is to the true value; how much detail it keeps	exactness
reasonable	an answer that makes sense — near the estimate, and right for the context	sensible
round up	choose the higher value, so there is always enough	over-estimate (informal)

Ahead, subtopics 2C and 2D use the estimating habit every time a calculation gets longer, 2E checks percentage answers the same way, and 3A uses it to spot a silly answer when a formula is worked out. Everything rests on deciding how much accuracy the job needs, which is what this subtopic builds.

Worked examples

Example 1 — scaffolded

For each of 6.38, 6.72 and 5.5: state the two consecutive integers it sits between, then round it to the nearest whole number.

Working	Comment
6.38**:** the whole-number part is 6, so 6.38 sits between 6 and 7.	The interval comes from the whole-number part.
6.38 is less than the halfway point 6.5, so it is closer to 6.	Compare with halfway to choose a side.
6.72**:** sits between 6 and 7; $6.72 > 6.5$, so it is closer to 7.	Past halfway rounds up.
5.5**:** sits between 5 and 6; 5.5 is exactly halfway, so it rounds up.	Halfway values round up (Figure 9).

∴ 6.38 rounds to 6, 6.72 rounds to 7, and 5.5 rounds to 6.

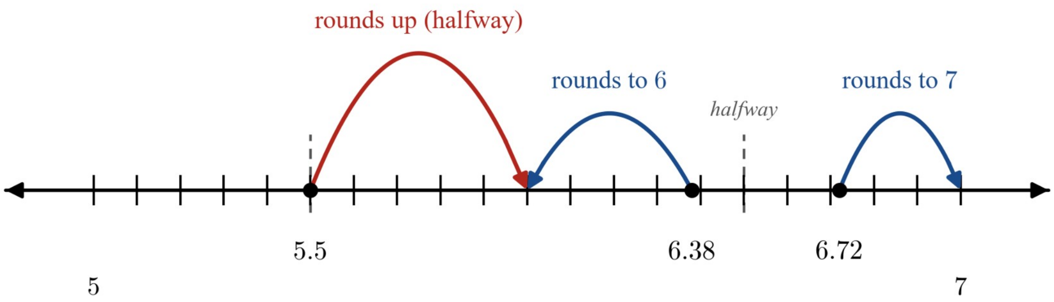


Figure 9. A number line from five to seven with ticks at each tenth and dashed lines at the halfway points five point five and six point five; arcs show five point five rounding up to six, six point three eight rounding down to six and six point seven two rounding up to seven.

Example 2 — scaffolded

Round 18.4675 (a) to the nearest whole number, (b) to one decimal place, and (c) to two decimal places.

Working	Comment
(a) **Keep the ones.** The last digit kept is 8; the digit next door is 4.	4 is less than 5, so the 8 stays.
$18.4675 \to 18$.	Drop every digit after the ones.
(b) **Keep the tenths.** The last digit kept is 4; the digit next door is 6.	6 is 5 or more, so round up: $4 \to 5$.
$18.4675 \to 18.5$.	Drop the digits after the tenths.
(c) **Keep the hundredths.** The last digit kept is 6; the digit next door is 7.	7 is 5 or more, so round up: $6 \to 7$ (Figure 4).
$18.4675 \to 18.47$.	Drop the last two digits.

∴ 18.4675 is 18 (nearest whole), 18.5 (one decimal place) and 18.47 (two decimal places).

Example 3 — unscaffolded

Estimate $5.83+12.06+3.92$ by rounding each number to the nearest whole number, then find the exact sum and check that it is reasonable.

Rounding each number to the nearest whole: $5.83 \to 6$, $12.06 \to 12$ and $3.92 \to 4$, so the estimate is $6+12+4=22$.

The exact sum is $5.83+12.06+3.92=21.81$.

Since 21.81 is close to the estimate 22, the exact answer is reasonable.

∴ the exact sum is 21.81, and the estimate of 22 checks it.

Example 4 — unscaffolded

Khai is painting his bedroom. The walls need exactly 1.89 litres of paint, and the paint is sold in 1 L and 4 L cans. How many cans should Khai buy? Why is rounding to the nearest litre not the end of the story?

Rounding 1.89 to the nearest whole number gives 2, since $1.89>1.5$. Two 1 L cans hold 2 L in total, which is more than 1.89 L, so Khai buys **two 1 L cans** (Figure 10). A single 4 L can would also cover the walls, but it holds 2.11 L more than needed, so two small cans are the better choice.

The rounding worked here because it rounded **up** — 2 L is enough. If the walls had needed 2.3 L, rounding to the nearest litre would still say 2, but 2 L is *not* enough: the wall would run out of paint. With only 1 L and 4 L cans, the choices would be three 1 L cans (3 L — enough) or one 4 L can. When there must be enough, always check that the rounded amount is not less than the exact amount.

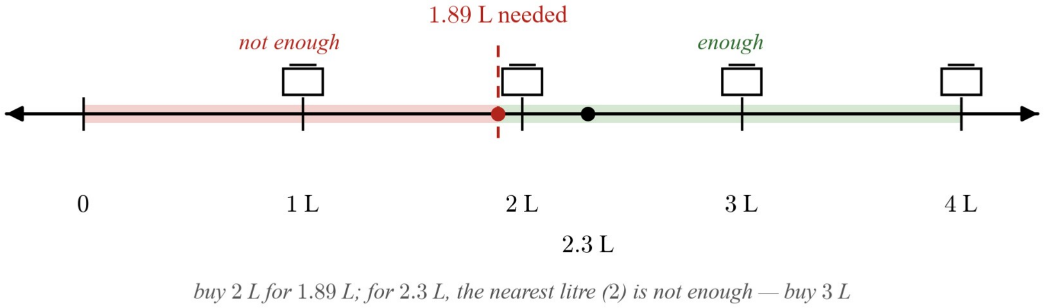


Figure 10. A number line from zero to four litres with a tick and can symbol at each whole litre; dots mark one point eight nine and two point three, and the region below one point eight nine is shaded and labelled not enough, while the region above it is labelled enough.

Practice questions

Scaffolded

Q1. Write the two consecutive integers each number sits between. (a) 4.63 (b) 0.87 (c) 12.05 (d) 9.99

Q2. Round each number to the nearest whole number. (a) 7.3 (b) 12.5 (c) 0.49 (d) 18.86

Q3. Round each number to one decimal place. (a) 5.24 (b) 8.37 (c) 2.95 (d) 6.99

Q4. Round each number to two decimal places. (a) 3.456 (b) 7.891 (c) 4.008 (d) 5.555

Q5. Round $3,847$ to (a) the nearest 10 (b) the nearest 100 (c) the nearest 1000. (d) Round $\$6,952$ to the nearest $\$100$.

Q6. Round each number to the nearest whole number first, then find an estimate. After that, find the exact answer and check it is close to your estimate. (a) $4.87+3.21$ (b) $6.12+2.95$ (c) $5\times\$2.96$

Unscaffolded

Q7. Copy and complete the table.

Number	Nearest whole	One decimal place	Two decimal places
8.0463			
0.9725			

Q8. Jess is painting her room. The walls need exactly 2.3 L of paint, and paint is sold only in 1 L cans and 4 L cans. (a) Jess rounds 2.3 to the nearest whole number and says she needs 2 L. Explain why 2 L is not enough. (b) What is the smallest amount of paint Jess can buy? (c) How much paint will be left over after the job?

Q9. A family is budgeting for renovating their kitchen. The three quotes are shown in Figure 11.

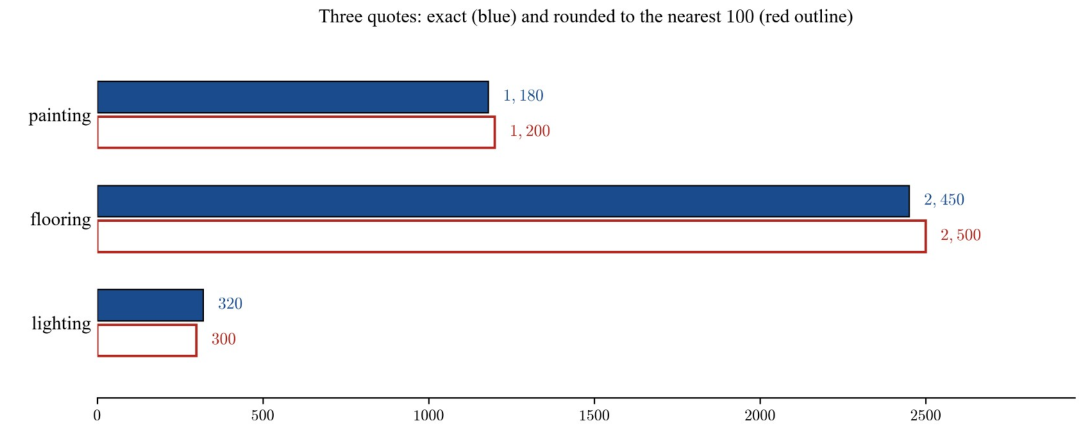


Figure 11. A bar chart of three renovation quotes: painting one thousand one hundred and eighty dollars, flooring two thousand four hundred and fifty dollars, and lighting three hundred and twenty dollars, with each exact bar drawn next to its rounded-to-the-nearest-hundred value.

(a) Round each quote to the nearest $\$100$ and use the rounded numbers to estimate the total.
(b) Find the exact total of the three quotes.
(c) The family has saved $\$4{,}000$. The estimate from part (a) says the work will cost about $\$4{,}000$. Explain whether the estimate alone is enough to decide if the savings will cover the work.

Q10. Tom enters 31.4×4.9 into a calculator, and the display shows 1538.6. (a) Round each number to the nearest whole and estimate the product. (b) Explain why the calculator display must be wrong. (c) The exact answer is 153.86 . Compare it with the display and describe the slip Tom most likely made.

Q11. Four pieces of timber are joined end to end. Their lengths are 1.38 m, 2.05 m, 0.97 m and 1.61 m (Figure 12).

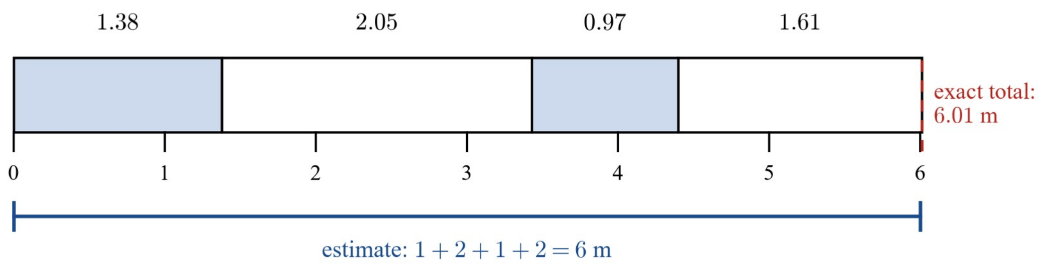


Figure 12. Four bars joined end to end, one point three eight metres, two point zero five metres, zero point nine seven metres and one point six one metres long, with a bracket underneath labelled estimate: one plus two plus one plus two equals six metres.

(a) Round each length to the nearest metre and estimate the total length.
(b) Find the exact total length.
(c) A builder needs at least 6 m of timber in total. Is there enough?

Q12. Each statement below rounds a number. Say whether the accuracy is right for the purpose. If it is not, say what you would round to instead, and why. (a) The school fair raised $\$1{,}287$. The newsletter reports that it raised “about $\$1{,}300$”. (b) A cake recipe needs 0.25 kg of flour. Jo measures out 0.3 kg. (c) The weather report says tomorrow’s maximum temperature will be $19\,°C$, from a forecast of $18.6\,°C$. (d) A bag of apples labelled “about 3 kg” actually holds 2.984 kg.

Q13. State whether each statement is true or false. Correct every false statement. (a) 4.65 rounds to 4.7 to one decimal place. (b) 7.996 rounds to 8.0 to one decimal place. (c) 9.95 rounds to 9 to the nearest whole number. (d) $3{,}050$ rounds to $3{,}000$ to the nearest hundred. (e) If every number in a sum is rounded up, the estimate is greater than the exact total.

Q14. Without finding exact answers, say whether you agree with each student and give your reason. (a) Rani says "5.9×4.2 is a bit more than 24." (b) Ben says "$0.48 + 0.51$ is about 1."

Q15. Mia is buying three items: $\$3.95$, $\$2.18$ and $\$1.97$. (a) Round each price to the nearest dollar and estimate the total cost. (b) Find the exact total cost. (c) Mia has a $\$10$ note. Does she have enough money? (d) Mia rounds each price **up** to the next whole dollar: $\$4$, $\$3$ and $\$2$, totalling $\$9$. Explain why this tells her for certain that $\$10$ is enough, without needing the exact total.

Answers

Q1. (a) 4 and 5; (b) 0 and 1; (c) 12 and 13; (d) 9 and 10.

Q2. (a) 7; (b) 13; (c) 0; (d) 19.

Q3. (a) 5.2; (b) 8.4; (c) 3.0; (d) 7.0.

Q4. (a) 3.46; (b) 7.89; (c) 4.01; (d) 5.56.

Q5. (a) $3,850$; (b) $3,800$; (c) $4,000$; (d) $\$7,000$.

Q6. (a) estimate 8, exact 8.08; (b) estimate 9, exact 9.07; (c) estimate $\$15$, exact $\$14.80$.

Q7. 8.0463: 8, 8.0, 8.05. 0.9725: 1, 1.0, 0.97.

Q8. (a) $2 < 2.3$, so 2 L leaves 0.3 L of wall unpainted; (b) three 1 L cans, 3 L; (c) 0.7 L.

Q9. (a) $\$1,200 + \$2,500 + \$300 = \$4,000$; (b) $\$3,950$; (c) see solutions.

Q10. (a) $30 \times 5 = 150$; (b) see solutions; (c) the decimal point slipped one place, making the display ten times too big.

Q11. (a) $1 + 2 + 1 + 2 = 6$ m; (b) 6.01 m; (c) yes.

Q12. (a) right; (b) not right — 0.25 kg needs two decimal places; (c) right; (d) right.

Q13. (a) true; (b) true; (c) false — 9.95 rounds to 10; (d) false — $3,050$ rounds to $3,100$; (e) true.

Q14. (a) agree; (b) agree — see solutions for reasons.

Q15. (a) $\$8$; (b) $\$8.10$; (c) yes; (d) see solutions.

Fully-worked solutions

Q1. The whole-number part gives the lower integer; the next integer is one more. (a) 4.63 sits between 4 and 5. (b) 0.87 sits between 0 and 1. (c) 12.05 sits between 12 and 13. (d) 9.99 sits between 9 and 10 — close to the top end, but still between them.

Q2. Compare each number with the halfway point of its interval. (a) $7.3 < 7.5$, so 7.3 rounds to 7. (b) 12.5 is exactly halfway, so it rounds up to 13. (c) $0.49 < 0.5$, so 0.49 rounds to 0. (d) $18.86 > 18.5$, so 18.86 rounds to 19.

Q3. Look at the hundredths digit (the digit next door to the tenths). (a) The next-door digit is 4, so $5.24 \to 5.2$. (b) The next-door digit is 7, so $8.37 \to 8.4$. (c) The next-door digit is 5, so round up: $2.95 \to 3.0$ — keep the zero to show one decimal place. (d) The next-door digit is 9, so $6.99 \to 7.0$ — the 9 rounds up past the whole number.

Q4. Look at the thousandths digit (the digit next door to the hundredths). (a) The next-door digit is 6, so $3.456 \to 3.46$. (b) The next-door digit is 1, so $7.891 \to 7.89$. (c) The next-door digit is 8, so $4.008 \to 4.01$ — keep both digits after the point. (d) The next-door digit is 5, so $5.555 \to 5.56$.

Q5. Use the digit next door to the place you are rounding to. (a) Nearest 10: the next-door digit is 7, so $3,847 \to 3,850$. (b) Nearest 100: the next-door digit is 4, so $3,847 \to 3,800$. (c) Nearest 1000: the next-door digit is 8, so $3,847 \to 4,000$. (d) $\$6,952$ to the nearest $\$100$: the next-door digit is 5, so round up to $\$7,000$.

Q6. Round first, add or multiply the rounded numbers, then find the exact answer and compare. (a) $4.87 \to 5$ and $3.21 \to 3$: estimate $5 + 3 = 8$. Exact: $4.87 + 3.21 = 8.08$, close to 8. (b) $6.12 \to 6$ and $2.95 \to 3$: estimate $6 + 3 = 9$. Exact: $6.12 + 2.95 = 9.07$, close to 9. (c) $\$2.96 \to \$3$: estimate $5 \times \$3 = \$15$. Exact: $5 \times \$2.96 = \$14.80$, close to $\$15$.

Q7. Work down the columns: nearest whole, then one decimal place, then two. For 8.0463: the next-door digit to the ones is 4, so 8; next door to the tenths is 4, so 8.0 (keep the zero); next door to the hundredths is 6, so 8.05. For

0.9725: the next-door digit to the ones is 9, so 1; next door to the tenths is 7, so 1.0; next door to the hundredths is 2, so 0.97.

Q8. (a) Rounding 2.3 to the nearest whole number gives 2, but $2<2.3$: with 2 L, 0.3 L of wall would be left unpainted. Rounding to the nearest is not the right move when there must be enough. (b) With 1 L and 4 L cans, the amounts Jess can buy are 1, 2, 3, 4, … litres. The smallest amount that is at least 2.3 L is 3 L — **three** 1 **L cans**. (One 4 L can is also enough, but it is more paint than she needs.) (c) $3-2.3=0.7$ L left over.

Q9. (a) $\$1,180 \rightarrow \$1,200$ (next-door digit 8), $\$2,450 \rightarrow \$2,500$ (exactly halfway rounds up), $\$320 \rightarrow \$300$ (next-door digit 2). Estimate: $\$1,200+\$2,500+\$300=\$4,000$. (b) $\$1,180+\$2,450+\$320=\$3,950$. (c) No — the estimate alone is not quite enough here. It says "about $\$4,000$", and the savings are exactly $\$4,000$, so the estimate cannot say which side of the savings the true cost lands on. The exact total, $\$3,950$, is $\$50$ less than the savings, so the savings do cover the work. An estimate is good for planning; when the estimate lands this close to the money available, the exact total is needed.

Q10. (a) 31.4 → 31 or 30 (either is a sensible round choice; use 30 for mental work) and 4.9 → 5: estimate $30\times5=150$. (b) The display shows 1538.6, but the estimate says the answer must be near 150 — about ten times smaller. A number that far from the estimate cannot be right. (c) 1538.6 is exactly 10 times 153.86: the digits are the right ones, so Tom most likely slipped the decimal point one place when entering a number — entering 314 for 31.4, for instance.

Q11. (a) 1.38 → 1, 2.05 → 2, 0.97 → 1, 1.61 → 2: estimate $1+2+1+2=6$ m. (b) $1.38+2.05+0.97+1.61=6.01$ m. (c) $6.01>6$, so yes — there is enough, with 0.01 m to spare. The estimate of 6 m already hinted the total would sit near 6, and the exact total confirms it.

Q12. (a) **Right.** For reporting roughly how much the fair raised, the nearest $\$100$ is the right accuracy — the exact dollars are not the point of the report. (b) **Not right.** The recipe says 0.25 kg, which is given to two decimal places. Measuring 0.3 kg adds 0.05 kg — one-fifth more flour than the recipe asks for. Measure to two decimal places (0.25 kg) instead. (c) **Right.** A maximum temperature to the nearest degree is all a weather report needs; nobody chooses different clothes for $18.6\,°C$ and $19\,°C$. (d) **Right.** For a bag of fruit sold as one item, "about 3 kg" is the right accuracy — the nearest 10 g would tell the buyer nothing useful. (If the apples were priced by the kilogram, the exact mass would matter, and the scale's reading to the nearest gram would be used.)

Q13. (a) **True**: the next-door digit is 5, so 4.65 rounds up to 4.7. (b) **True**: 7.996 — the next-door digit to the tenths is 9, so the 9 rounds up, carrying past the whole number: 8.0. (c) **False**: $9.95>9.5$, so 9.95 rounds to 10, not 9. (d) **False**: in 3,050, the next-door digit to the hundreds is 5, so round up: 3,100. (e) **True**: every part of the sum is made bigger, so the rounded total must be bigger than the exact total.

Q14. (a) **Agree.** Round 5.9 → 6 and 4.2 → 4: the estimate is $6\times4=24$. Both numbers are close to the ones used, so the true product sits near 24 — a little more, in fact, since 5.9 is close to 6 and 4.2 is a little more than 4. (b) **Agree.** 0.48 → 0.5 and 0.51 → 0.5: the estimate is $0.5+0.5=1$. The exact sum is 0.99, so "about 1" is right.

Q15. (a) $\$3.95 \rightarrow \$4$, $\$2.18 \rightarrow \$2$, $\$1.97 \rightarrow \$2$: estimate $\$4+\$2+\$2=\$8$. (b) $\$3.95+\$2.18+\$1.97=\$8.10$. (c) Yes: $\$8.10$ is less than $\$10$. (d) Rounding every price **up** makes every item cost more than it really does, so $\$9$ is more than the true total. Since even this too-large total fits under $\$10$, the real total must fit too. Rounding up like this is a sure way to check you have enough money.

Teacher note

- **Ownership and the boundary with 2A, 2C, 2D and 2E.** Per the Chapter 2 plan in TOC §3, this subtopic (`VC2M7N04`) owns **rounding decimals to a given accuracy appropriate to the context** (nearest whole number, decimal places, nearest 10/100/1000 for estimation) and **estimation to check the reasonableness of computations**. It owns the interval-between-consecutive-integers idea and the judgement about how much accuracy a context deserves — TOC §3 names the paint-tin and renovation-budget framing of the elaborations as "exactly the point", so WE4, Q8 and Q9 deliver them directly, and a subtopic that only rounds to two decimal places on command would not have delivered this CD. 2A owns locating and ordering on the number line (recalled here, not re-taught); 2C and 2D own the four operations (the exact calculations in the questions are maintenance of Level 6 decimal arithmetic, not new technique); 2E owns percentages of quantities.

- **Level 6 recall (D3).** Rounding and estimation to check reasonableness, and decimal place value (`VC2M6N04`, `VC2M6N06`), are assumed. They are recalled in one sentence at the top of the theory and maintained inside the questions, not taught as new content.
- **P3 — significant figures are deliberately absent.** TOC §5.2 records that the term "significant figures" appears nowhere in the VC2 Mathematics or Science F–10 exports until Level 9. This book rounds to **decimal places** (`VC2M7N04.E02`) and does not teach significant figures; there is no CD for it and adding it would be inflation. The Level 9–10 books and the Science band book carry it deliberately.
- **The judgement without the vocabulary (§5.3).** `VC2M7N04.E03` asks whether the accuracy of a rounding is suitable for context and purpose; VC2 Science Levels 7–8 asks about sources of error (`VC2S8I06`). This subtopic teaches the *judgement* — enough or not, and how much accuracy the purpose deserves — without the words "uncertainty" or "significant figures", which have no Level 7 Mathematics CD. TOC §5.3 records the deliberate split.
- **P7 — estimation as a standing habit.** `VC2M7N04` is the only CD in the book that owns "check the reasonableness of computations". 2C and 2D cross-reference this subtopic as their standing check (a decimal-point slip makes an answer ten or a hundred times too big, which Figure 7 and Q10 catch), and every later chapter carries at least one reasonableness item tracing back here.
- **No negatives (D4.4).** Negatives appear in Chapter 2 only in 2A, located and ordered; every number in this subtopic is positive.
- **No classification of decimal expansions (D4.13).** No recurring decimals, no overbar notation; every decimal here is given or terminates naturally.
- **No rates, no time content (D4.8, D4.18).** All contexts are lengths, masses, money and temperatures; none uses speed, and none uses elapsed time or timetables.
- **No digital tools (D9).** The CD and its elaborations name no digital tools, so the calculator appears here only as an object whose *output* is checked by hand (Q10); students do not use one. No CAS.
- **No real data.** All contexts (Khai's bedroom, Jess's room, the family kitchen, Mia's shopping) are generic and transparently fictional, so no §6 source lines are required. Money is in Australian dollars; GST is not relevant to these contexts.
- **Figures.** All twelve figures are generated by `src/gen_2b.py` from the exact values stated in the text; every number line is to scale, so a figure cannot disagree with the arithmetic it shows.
- **No D11 content.** None of `VC2M7N04.E01–.E03` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

Multiplying and dividing fractions and decimals

Content description `VC2M7N05`: multiply and divide fractions and decimals using efficient mental and written strategies, and digital tools

Elaboration `VC2M7N05.E01`: investigating multiplication of fractions and decimals, using strategies including patterning and multiplication as repeated addition, with both concrete materials and digital tools, and identifying the processes for division as the inverse of multiplication

Achievement standard (extract): They use all 4 operations in calculations involving positive fractions and decimals, choosing efficient mental and written calculation strategies.

Theory

What this subtopic is about. Subtopic 2C teaches the *methods* for multiplying and dividing fractions and decimals. The methods live here; choosing which operation fits a problem is 2D's job. The work assumes Level 6 skills — equivalent fractions, simplifying, adding and subtracting fractions, and multiplying and dividing by 10, 100 and 1000 — which are recalled where needed, not re-taught. Every number in this subtopic is positive, and the standing habit is: mental or written strategy first, digital tool second as a check.

Multiplication as repeated addition. Four jumps of 0.6 make 4×0.6, exactly as four stacks of 6 make 4×6. Figure 1 shows the jumps on a number line: $0.6+0.6+0.6+0.6=2.4$. The same picture works for fractions: $5 \times \frac{1}{4}$ is five quarters added together, which is $\frac{5}{4}=1\frac{1}{4}$. So a whole number times a fraction multiplies the numerator and keeps the denominator: $5 \times \frac{1}{4}=\frac{5}{4}$.

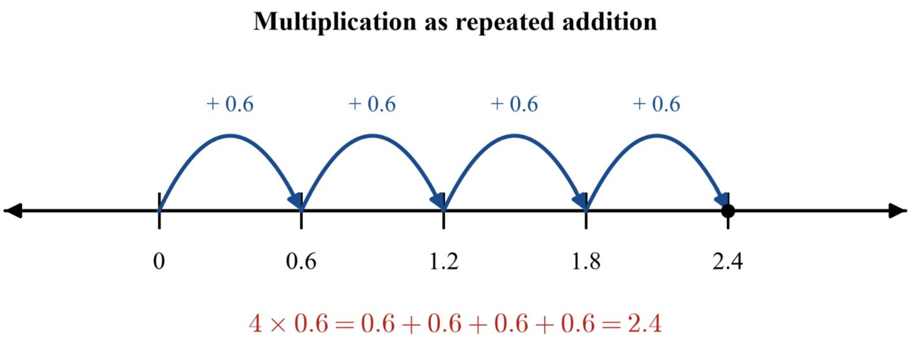


Figure 1. A number line from 0 to 2.4 with a tick every 0.6; four blue arrowed jumps of 0.6 land on 0.6, 1.2, 1.8 and 2.4, each jump labelled plus 0.6; underneath, 4 times 0.6 equals 0.6 plus 0.6 plus 0.6 plus 0.6, which equals 2.4.

Fraction times fraction: the area model. $\frac{2}{3} \times \frac{3}{4}$ reads as "$\frac{2}{3}$ **of** $\frac{3}{4}$". In Figure 2 a unit square is cut into 4 columns and 3 rows, making 12 equal cells. Shading 3 of the 4 columns shows $\frac{3}{4}$; taking 2 of the 3 rows of that shading picks out the darker cells. The darker cells are 6 of the 12, so $\frac{2}{3} \times \frac{3}{4}=\frac{6}{12}=\frac{1}{2}$. The picture is the rule: multiply the numerators, multiply the denominators, then simplify.

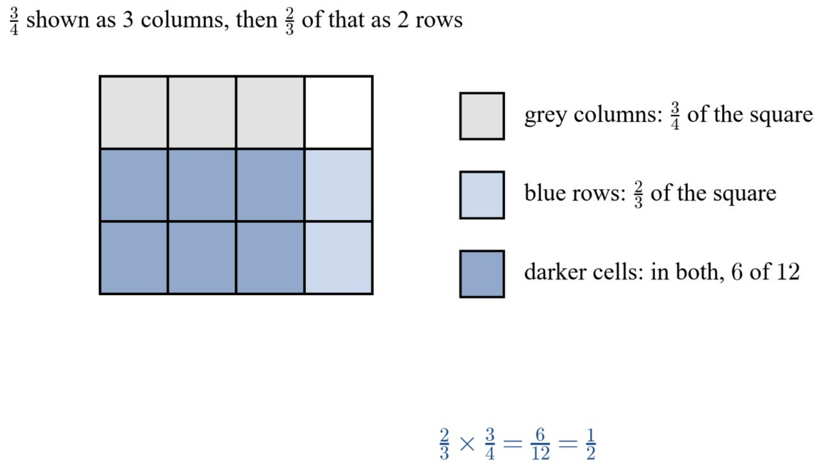


Figure 2. A unit square cut into 4 columns and 3 rows: 3 columns shaded grey show three quarters, 2 rows shaded blue show two thirds, and the darker cells in both are 6 of the 12; a legend reads grey columns three quarters of the square, blue rows two thirds of the square, darker cells in both 6 of 12; underneath, two thirds times three quarters equals six over twelve, which equals one half.

Simplify before you multiply. Cancelling common factors *before* multiplying keeps the numbers small. In Figure 3, $\frac{2}{3} \times \frac{9}{10}$: the 3 and the 9 share a factor of 3 (becoming 1 and 3), and the 2 and the 10 share a factor of 2 (becoming 1 and 5). The product is $\frac{1}{1} \times \frac{3}{5} = \frac{3}{5}$, with no simplifying left to do at the end.

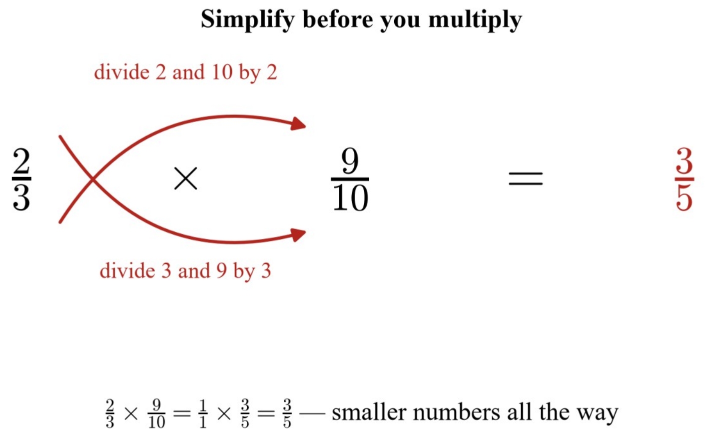
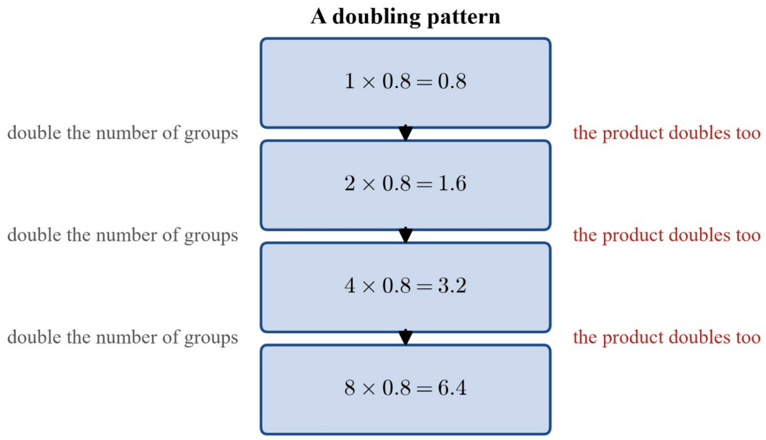


Figure 3. Two thirds times nine tenths equals three fifths, with two red arcs crossing between the fractions: the lower arc joins 3 and 9 and is labelled divide 3 and 9 by 3, the upper arc joins 2 and 10 and is labelled divide 2 and 10 by 2; underneath, two thirds times nine tenths equals one over one times three fifths, which equals three fifths, with smaller numbers all the way.

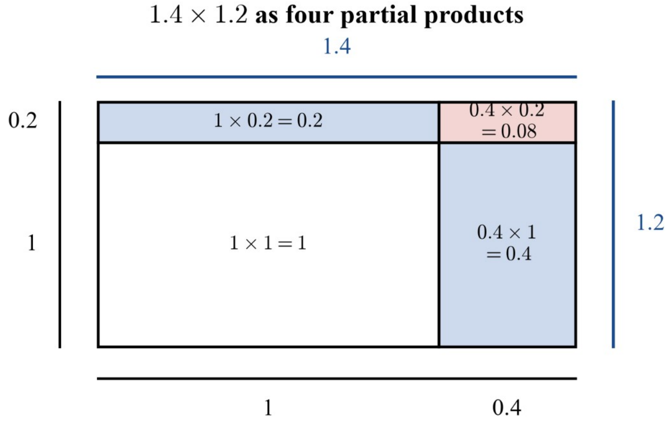
Patterning as a strategy. Patterns turn one known fact into many. In Figure 4 the number of groups doubles each step — $1, 2, 4, 8$ groups of 0.8 — and the product doubles with it: $0.8, 1.6, 3.2, 6.4$. Knowing $4 \times 0.8 = 3.2$ gives $8 \times 0.8 = 6.4$ for free. Place value gives patterns too: from $8 \times 8 = 64$, $8 \times 0.8 = 6.4$ and $8 \times 0.08 = 0.64$, because dividing one factor by 10 divides the product by 10.



The number of groups doubles each time — so the product doubles too.

Figure 4. Four boxes stacked with arrows between them: 1 times 0.8 equals 0.8, then 2 times 0.8 equals 1.6, then 4 times 0.8 equals 3.2, then 8 times 0.8 equals 6.4; beside each arrow, double the number of groups on the left and the product doubles too on the right.

Decimal times decimal: partial products. The area model of Figure 2 stretches to decimals. To find 1.4×1.2, split 1.4 into $1 + 0.4$ and 1.2 into $1 + 0.2$; the rectangle in Figure 5 then breaks into four easy pieces — $1 \times 1 = 1$, $0.4 \times 1 = 0.4$, $1 \times 0.2 = 0.2$ and $0.4 \times 0.2 = 0.08$ — and the pieces add to $1 + 0.4 + 0.2 + 0.08 = 1.68$.



$1 + 0.4 + 0.2 + 0.08 = 1.68$, so $1.4 \times 1.2 = 1.68$

Figure 5. A rectangle 1.4 wide and 1.2 high, drawn to scale and cut into four parts labelled 1 times 1 equals 1, 0.4 times 1 equals 0.4, 1 times 0.2 equals 0.2, and 0.4 times 0.2 equals 0.08; the parts add to 1.68, so 1.4 times 1.2 equals 1.68.

Counting decimal places. A short cut avoids drawing the rectangle every time: multiply as whole numbers, then count places. In Figure 6, 0.25×0.4 becomes $25 \times 4 = 100$; the factors have 2 and 1 decimal places, so the answer needs $2 + 1 = 3$: counting back three places from the right of 100 gives 0.100, which is 0.1. The count works because every piece of the partial-products picture carries its own decimal places.

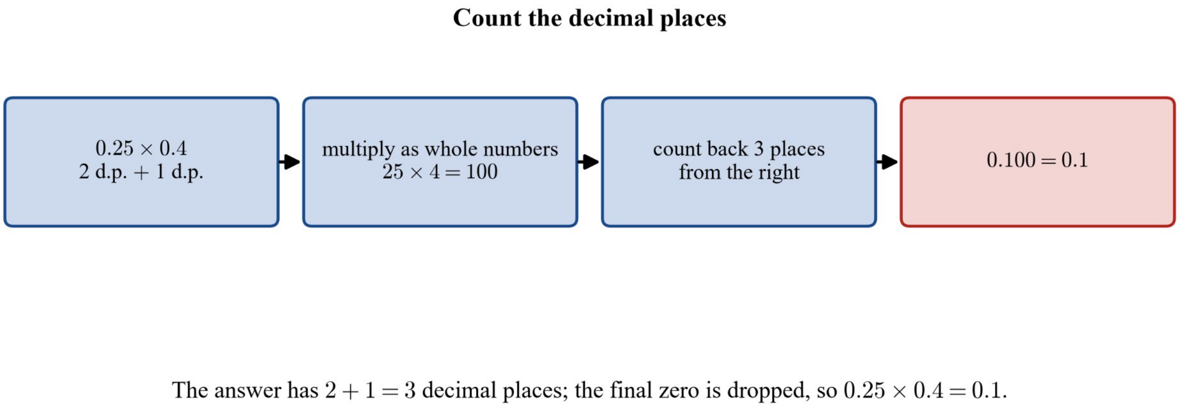


Figure 6. A flow of four boxes joined by arrows: 0.25 times 0.4, noted 2 decimal places plus 1 decimal place; multiply as whole numbers, 25 times 4 equals 100; count back 3 places from the right; and 0.100 equals 0.1; underneath, the answer has 2 plus 1 equals 3 decimal places, and the final zero is dropped, so 0.25 times 0.4 equals 0.1.

Division undoes multiplication. Division asks for a missing factor. From $\frac{3}{4} \times 4 = 3$, two division facts follow (Figure 7): $3 \div 4 = \frac{3}{4}$ (divide 3 into 4 equal shares) and $3 \div \frac{3}{4} = 4$ (how many three-quarters fit into 3). The three numbers always travel as a family: one multiplication fact, two division facts.

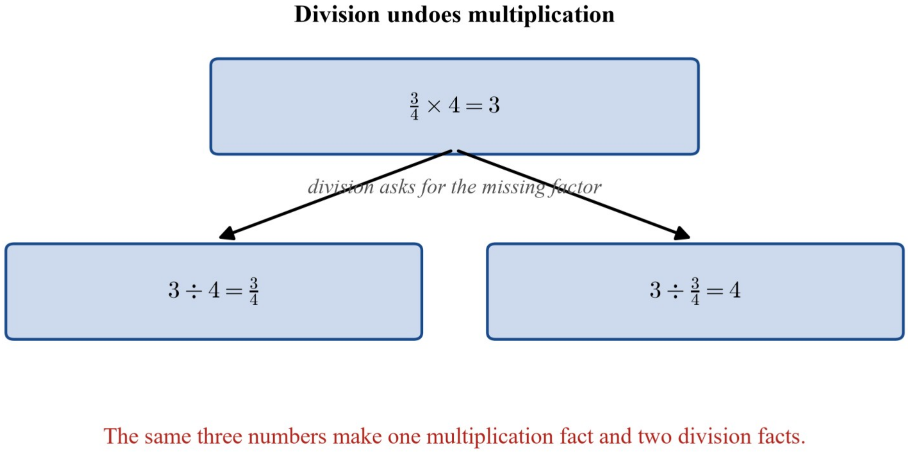


Figure 7. A top box reading three quarters times 4 equals 3 branches down to two boxes, 3 divided by 4 equals three quarters and 3 divided by three quarters equals 4, with the note division asks for the missing factor; underneath, the same three numbers make one multiplication fact and two division facts.

How many groups fit? $3 \div \frac{3}{4}$ counts groups. Figure 8 draws 3 wholes as 12 quarters; the quarters bundle into groups of 3, and $12 \div 3 = 4$ bundles, so $3 \div \frac{3}{4} = 4$. Dividing by a fraction smaller than 1 gives an answer *larger* than the starting number — more than one group fits into each whole.

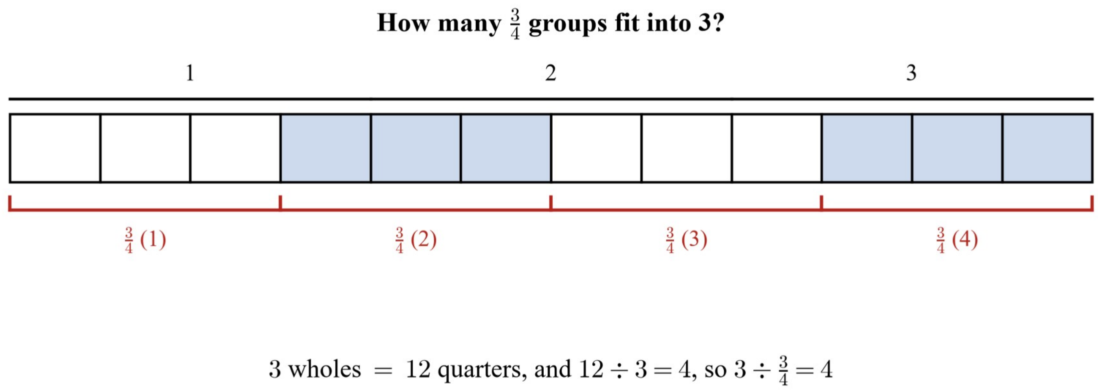


Figure 8. Three wholes drawn as a row of 12 quarter cells, grouped by red brackets into four groups of three quarters, labelled three quarters (1), (2), (3) and (4); underneath, 3 wholes equals 12 quarters, and 12 divided by 3 equals 4, so 3 divided by three quarters equals 4.

Dividing by a fraction: use the reciprocal. Two numbers are **reciprocals** of each other when their product is 1. Flipping a fraction gives its reciprocal: the reciprocal of $\dfrac{2}{3}$ is $\dfrac{3}{2}$, and of $\dfrac{1}{4}$ is 4; decimals have them too, since $1.25 \times 0.8 = 1$ (Figure 9). The rule: dividing by a number is the same as multiplying by its reciprocal. So $3 \div \dfrac{3}{4} = 3 \times \dfrac{4}{3} = \dfrac{12}{3} = 4$, agreeing with Figure 8.

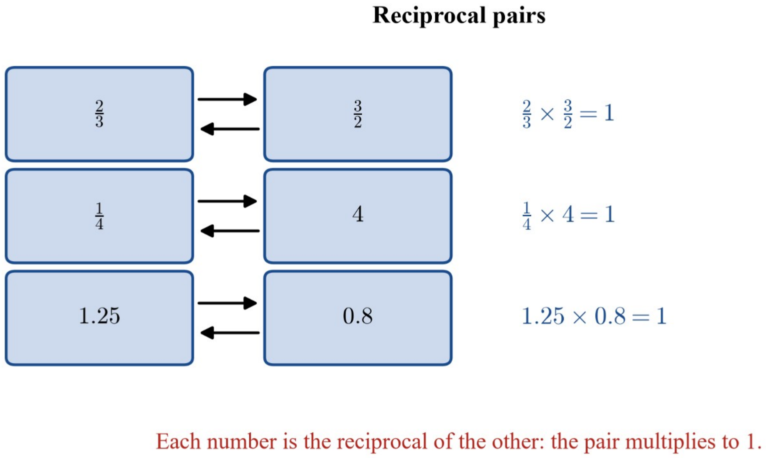


Figure 9. Three pairs of boxes joined by two-way arrows: two thirds with three halves, one quarter with 4, and 1.25 with 0.8; beside each pair, the product equals 1: two thirds times three halves equals 1, one quarter times 4 equals 1, 1.25 times 0.8 equals 1; underneath, each number is the reciprocal of the other: the pair multiplies to 1.

Dividing by a decimal: make the divisor whole. For $5.4 \div 0.6$, multiply *both* numbers by 10: the question becomes $54 \div 6 = 9$ (Figure 10). The answer does not change, because multiplying both numbers by 10 is like multiplying top and bottom of a fraction by 10 — the Level 6 equivalence idea. Choose the power of 10 that makes the divisor a whole number: for $5.4 \div 0.06$ multiply both by 100, giving $540 \div 6 = 90$.

Make the divisor a whole number

$$\frac{5.4}{0.6} \xrightarrow[\times 10]{\times 10} \frac{54}{6} = 9$$

$5.4 \div 0.6 = 54 \div 6 = 9$ — multiplying both numbers by 10 does not change the answer.

Figure 10. The fraction 5.4 over 0.6 with two red arrows labelled times 10 leading to the fraction 54 over 6, which equals 9; underneath, 5.4 divided by 0.6 equals 54 divided by 6, which equals 9, because multiplying both numbers by 10 does not change the answer.

Checking with a digital tool. A calculator checks a strategy; it does not replace one. Work mentally or on paper first, then compare — and estimate first (subtopic 2B) so a slip of ten times shows up at once. Remember that a display drops trailing zeros: for 0.25×0.4 it shows 0.1, the same number as your written 0.100.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
product	the answer to a multiplication	result of multiplying
factor	one of the numbers being multiplied	the numbers in a product
divisor	the number you divide by	the size of each group
reciprocal	a number's flip: $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals, because their product is 1	the number turned upside down
numerator / denominator	the top / bottom number of a fraction	how many parts / how many equal parts the whole is cut into
simplify	write a fraction with the smallest whole numbers that show the same amount	cancel, reduce
decimal places	the digits to the right of the decimal point	d.p.
mixed number	a whole number plus a fraction, like $1\frac{1}{4}$	mixed numeral

Ahead, 2D puts these methods to work in problems and chooses between them, 2E builds percentages on the same decimal skills, and 2F uses fractions in ratio.

Worked examples

Example 1 — scaffolded

Find $\frac{5}{6} \times \frac{3}{5}$.

Working	Comment
Look for common factors. The 5 on top and the 5 on the bottom cancel to 1; the 3 and the 6 share a factor of 3, becoming 1 and 2.	Simplify before you multiply (Figure 3): the numbers stay small.
Multiply across. $\frac{1}{2}\times\frac{1}{1}=\frac{1}{2}$.	Multiply the numerators and multiply the denominators.
Check without cancelling. $\frac{5\times 3}{6\times 5}=\frac{15}{30}=\frac{1}{2}$. Same answer.	Both routes must agree — a built-in check.

$\therefore \frac{5}{6}\times\frac{3}{5}=\frac{1}{2}$.

Example 2 — scaffolded

Ava buys 1.5 m of fabric at $\$3.20$ a metre. What does the fabric cost?

Working	Comment
Set up. Cost $=3.20\times 1.5$.	The price of 1 m times the number of metres.
Multiply as whole numbers. $320\times 15=4800$.	Ignore the points for now (Figure 6).
Count decimal places. $2+1=3$ places: $4.800=4.80$.	Two places in 3.20, one in 1.5.
Check by estimating. $3\times 1.5=4.5$, and $\$4.80$ is close to $\$4.50$.	The estimate (2B) says the point is in the right place.

$\therefore$ the fabric costs $\$4.80$.

Example 3 — unscaffolded

The canteen makes 6 L of cordial and fills bottles that each hold $\frac{3}{4}$ L. How many bottles does it fill?

Working	Comment
Bottles $=6\div\frac{3}{4}$.	How many three-quarter groups fit into 6 (Figure 8).
Multiply by the reciprocal: $6\times\frac{4}{3}=\frac{24}{3}=8$.	Dividing by a fraction is multiplying by its reciprocal (Figure 9).
Check. $8\times\frac{3}{4}=\frac{24}{4}=6$ L.	The multiplication undoes the division (Figure 7).

$\therefore$ the canteen fills 8 bottles.

Example 4 — unscaffolded

A 5.4 kg bag of rice is repacked into bags of 0.6 kg. How many small bags are filled?

Working	Comment
Bags $=5.4\div 0.6$.	How many 0.6 kg groups fit into 5.4 kg.
Multiply both numbers by 10: $54\div 6=9$.	Make the divisor a whole number (Figure 10); the answer does not change.
Check. $9\times 0.6=5.4$ kg.	The inverse multiplication returns the starting amount.

$\therefore$ 9 small bags are filled.

Practice questions

Scaffolded

Q1. Multiplication as repeated addition (Figure 1). (a) Find 3×0.7 by writing it as a sum of three 0.7s. (b) Find $5 \times \frac{1}{4}$ and write the answer as a mixed number.

Q2. Use the area-model idea of Figure 2 to find $\frac{1}{2} \times \frac{3}{5}$. (a) A grid with 5 columns and 2 rows has how many equal cells? (b) Shading $\frac{3}{5}$ as columns and taking $\frac{1}{2}$ of the rows picks out how many cells? (c) Write the product as a single fraction in its simplest form.

Q3. Find each product, simplifying before you multiply where you can. (a) $\frac{1}{4} \times \frac{2}{3}$ (b) $\frac{3}{4} \times \frac{2}{3}$ (c) $\frac{3}{4} \times \frac{2}{5}$

Q4. A doubling pattern (Figure 4). Given $1 \times 1.5 = 1.5$, use the pattern — double the number of groups, double the product — to write down 2×1.5, 4×1.5 and 8×1.5.

Q5. Multiply as whole numbers, then count the decimal places (Figure 6). (a) 4×2.3 (b) 0.6×0.8 (c) 0.9×0.5 (d) 0.25×0.4

Q6. Division undoes multiplication (Figure 7). From each multiplication fact, write the two division facts. (a) $0.7 \times 6 = 4.2$ (b) $\frac{3}{5} \times 5 = 3$

Unscaffolded

Q7. Find each product, simplifying first where you can. (a) $\frac{3}{5} \times \frac{7}{8}$ (b) $2\frac{1}{4} \times \frac{2}{3}$ (c) 0.9×0.6 (d) 0.8×0.7

Q8. Find each answer. (a) $2 \div \frac{1}{3}$ (b) $4 \div \frac{2}{3}$ (c) $3 \div \frac{3}{4}$ (d) $4 \div \frac{2}{5}$

Q9. Find each answer. (a) $4 \div 5$ (b) $4 \div 0.5$ (c) $0.9 \div 0.3$ (d) $5.4 \div 0.6$

Q10. (a) Write the reciprocal of $\frac{5}{6}$ as a mixed number. (b) Use the reciprocal to find $4 \div \frac{4}{3}$.

Q11. Mia calculates 0.7×0.7 and writes 4.9. Explain her mistake, and find the correct answer.

Q12. A rope is 9 m long. How many pieces of length 0.75 m can be cut from it? Check your answer with a multiplication.

Q13. Estimate first, then check with a calculator, and say whether the display agrees with your estimate. (a) $1.44 \div 1.2$ (b) 0.375×2.4

Q14. (a) A ribbon 17.2 cm long is cut into 8 equal pieces. How long is each piece? (b) Identical books 2.15 cm thick stand in a row 43 cm long. How many books are in the row?

Answers

Q1. (a) 2.1; (b) $1\frac{1}{4}$.

Q2. (a) 10 cells; (b) 3 cells; (c) $\frac{3}{10}$.

Q3. (a) $\frac{1}{6}$; (b) $\frac{1}{2}$; (c) $\frac{3}{10}$.

Q4. 1.5, 3, 6, 12.

Q5. (a) 9.2; (b) 0.48; (c) 0.45; (d) 0.1.

Q6. (a) $4.2 \div 6 = 0.7$ and $4.2 \div 0.7 = 6$; (b) $3 \div 5 = \frac{3}{5}$ and $3 \div \frac{3}{5} = 5$.

Q7. (a) $\frac{21}{40}$; (b) $1\frac{1}{2}$; (c) 0.54; (d) 0.56.

Q8. (a) 6; (b) 6; (c) 4; (d) 10.

Q9. (a) 0.8; (b) 8; (c) 3; (d) 9.

Q10. (a) $1\frac{1}{5}$; (b) 3.

Q11. 0.49.

Q12. 12 pieces.

Q13. (a) 1.2; (b) 0.9.

Q14. (a) 2.15 cm; (b) 20 books.

Fully-worked solutions

Q1. (a) $3 \times 0.7 = 0.7 + 0.7 + 0.7 = 2.1$. (b) $5 \times \frac{1}{4} = \frac{5}{4} = 1\frac{1}{4}$.

Q2. (a) $5 \times 2 = 10$ equal cells. (b) $\frac{3}{5}$ shades 3 of the 5 columns; taking 1 of the 2 rows picks out 3 cells. (c) 3 of the 10 cells: $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$, already in its simplest form.

Q3. (a) Cancel the 2 and the 4: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. (b) Cancel the 3s and the 2 with the 4: $\frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$. (c) Cancel the 2 and the 4: $\frac{3}{2} \times \frac{1}{5} = \frac{3}{10}$.

Q4. $1 \times 1.5 = 1.5$; doubling the groups doubles the product each time: $2 \times 1.5 = 3$, $4 \times 1.5 = 6$, $8 \times 1.5 = 12$.

Q5. (a) $4 \times 23 = 92$, one decimal place: 9.2. (b) $6 \times 8 = 48$, two places: 0.48. (c) $9 \times 5 = 45$, two places: 0.45. (d) $25 \times 4 = 100$, three places: $0.100 = 0.1$.

Q6. (a) $4.2 \div 6 = 0.7$ and $4.2 \div 0.7 = 6$. (b) $3 \div 5 = \frac{3}{5}$ and $3 \div \frac{3}{5} = 5$.

Q7. (a) No common factors: $\frac{3\times7}{5\times8}=\frac{21}{40}$. (b) $2\frac{1}{4}=\frac{9}{4}$; cancel the 2 and the 4: $\frac{9}{2}\times\frac{1}{3}=\frac{9}{6}=1\frac{1}{2}$. (c) $9\times6=54$, two places: 0.54. (d) $8\times7=56$, two places: 0.56.

Q8. (a) $2\times3=6$. (b) $4\times\frac{3}{2}=6$. (c) $3\times\frac{4}{3}=4$. (d) $4\times\frac{5}{2}=10$.

Q9. (a) $4\div5=0.8$ — four shared five ways. (b) Multiply both by 10: $40\div5=8$. (c) Multiply both by 10: $9\div3=3$. (d) Multiply both by 10: $54\div6=9$.

Q10. (a) Flip: $\frac{6}{5}=1\frac{1}{5}$. (b) $4\div\frac{4}{3}=4\times\frac{3}{4}=3$.

Q11. The factors have 1 decimal place each, so the product needs $1+1=2$ places: $7\times7=49$ becomes 0.49, not 4.9. Mia counted only one decimal place. An estimate agrees: 0.7 is less than 1, so the product must be less than 0.7 — and 4.9 is not.

Q12. $9\div0.75$: multiply both by 100: $900\div75=12$ pieces. Check: $12\times0.75=9$ m.

Q13. (a) Estimate: 1.44 is a little more than 1.2, so the answer is a little more than 1; the calculator shows 1.2 — agrees. (b) Estimate: $0.4\times2.5=1$, so the answer is near 1; the calculator shows 0.9 — agrees.

Q14. (a) $17.2\div8=2.15$ cm each. (b) $43\div2.15$: multiply both by 100: $4300\div215=20$ books.

Teacher note

- **Ownership and the boundary with 2D.** Per the Chapter 2 plan in TOC §3, this subtopic (`VC2M7N05`) owns the *methods* of multiplying and dividing fractions and decimals; 2D (`VC2M7N06`) owns problem solving across all four operations, the choice of strategy, and the commutative and associative properties applied to fractions. Accordingly, 2C never names those properties and never asks students to choose an operation — every question here states or implies the operation, and 2D's questions reuse these methods without re-teaching them.
- **Level 6 recall (D3).** Equivalent fractions, simplifying, fraction $+$ / $-$, decimal place value, $\times$ / $\div$ by powers of 10, and fraction-of-a-quantity are assumed; each is recalled in a line where used (Figures 2, 8, 10), not re-taught.
- **E01 delivery.** Multiplication as repeated addition (Figure 1), patterning (Figure 4), concrete materials (area model Figure 2, quarter-strip Figure 8), digital tools (the checking paragraph, Q13), and division as the inverse of multiplication (Figures 7 and 10) each appear explicitly.
- **Positive rationals only (D4.4).** Every number in this subtopic is positive; no negatives appear anywhere in 2C.
- **Digital tools (D9).** The calculator enters as a check on a mental or written strategy, never as the method; the trailing-zero display note (0.100 shows as 0.1) pre-empts the common mismatch with written work. No CAS.
- **Contexts, generically framed.** Fabric, cordial, rice, ribbon and books are transparently generic settings; no real data is asserted, so no source lines are required under §6.
- **Figures.** All ten figures are generated by `src/gen_2c.py` from the exact values stated in the text; grids, strips and boxes compute their labels from those values.
- **No D11 content.** None of `VC2M7N05.E01` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

The four operations with positive rational numbers

Content description `VC2M7N06`: use the 4 operations with positive rational numbers, including fractions and decimals, to solve problems using efficient mental and written calculation strategies

Achievement standard (extract): They use all 4 operations in calculations involving positive fractions and decimals, choosing efficient mental and written calculation strategies.

Theory

What this subtopic is about. Subtopic 2C teaches the *methods* for multiplying and dividing fractions and decimals, and adding and subtracting fractions comes from Level 6. This subtopic puts all four operations to work on **problems**. The new skill is the choosing: which operation fits the situation, which way of writing the numbers suits the job, and which strategy keeps the calculation short. Every number in this subtopic is a **positive rational number** — a number greater than zero that can be written as a fraction, such as $\frac{3}{4}$, 0.6, $12.5\,\%$ or $5=\frac{5}{1}$.

Choose the representation first. The one number can wear many names: $\frac{1}{8}=0.125=12.5\,\%=\frac{125}{1000}$. Figure 1 shows one of these names as a picture: one part of a bar split into eight equal parts. Picking the name that fits the other numbers in a problem can turn a hard calculation into a mental one. For example, $12.5\,\%$ of 96 is found more easily as $\frac{1}{8}$ of 96, because $96 \div 8 = 12$ is a known fact. Worked example 1 makes this swap.

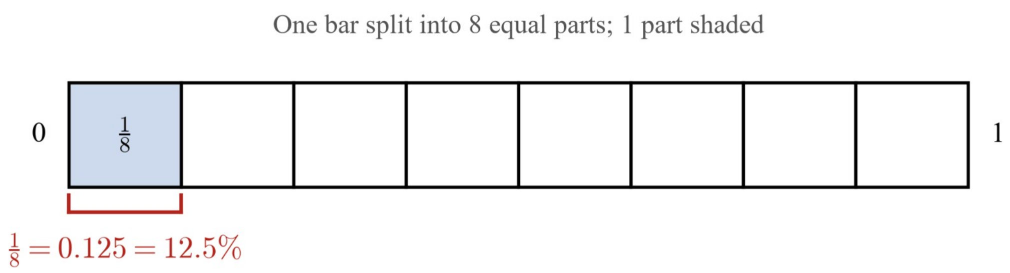


Figure 1. A bar split into eight equal parts with the first part shaded; a bracket under the shaded part marks one eighth, equal to 0.125, equal to 12.5 per cent.

Some swaps are worth knowing off by heart:

Percentage	Decimal	Fraction
$10\,\%$	0.1	$\frac{1}{10}$
$12.5\,\%$	0.125	$\frac{1}{8}$
$20\,\%$	0.2	$\frac{1}{5}$
$25\,\%$	0.25	$\frac{1}{4}$
$50\,\%$	0.5	$\frac{1}{2}$
$75\,\%$	0.75	$\frac{3}{4}$

Subtopic 2E works with percentages in depth; here they appear only as another name for the same number, chosen when they make the computation easier.

Efficient addition and subtraction. Three strategies do most of the work.

Reorder and group. The **commutative property** says the order of adding does not matter ($a+b=b+a$), and the **associative property** says the grouping does not matter ($(a+b)+c=a+(b+c)$). Together they let you hunt for pairs that make whole numbers. Figure 2 shows the idea on the sum $2.75+3.6+1.25+4.4$: the terms are reordered so that 2.75 sits beside 1.25 and 3.6 beside 4.4, because each pair makes a whole number. The total is $4+8=12$, found with no column addition at all.

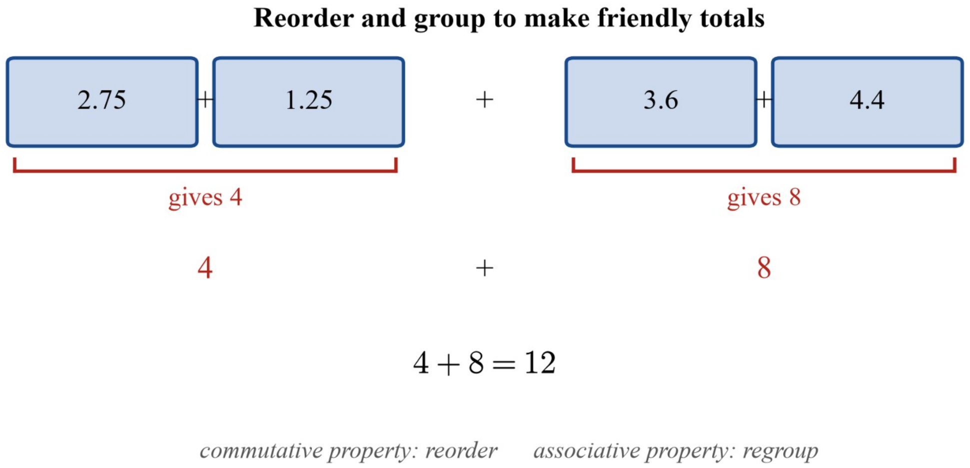


Figure 2. The sum 2.75 plus 3.6 plus 1.25 plus 4.4 reordered so that 2.75 and 1.25 are grouped together and 3.6 and 4.4 are grouped together; the first pair makes 4 and the second makes 8, and the total is 12.

Partition. Split one number into parts that suit the other. To find $6.4+3.8$, split 3.8 into $3.6+0.2$: then $6.4+3.6=10$, and $10+0.2=10.2$. The same idea works for subtraction: to find $7.2-2.65$, split 2.65 into $2.2+0.45$; then $7.2-2.2=5$ and $5-0.45=4.55$.

Rectangular arrays for fraction sums. A grid of equal cells can show what two fractions add to. Figure 3 uses a grid with the denominators as its dimensions — 5 columns for fifths and 4 rows for quarters, so 20 equal cells in all. In that grid, $\frac{3}{5}$ covers 3 of the 5 columns (12 cells) and $\frac{1}{4}$ covers 1 of the 4 rows (5 cells). Adding the counts gives $12+5=17$ cells out of 20, so $\frac{3}{5}+\frac{1}{4}=\frac{17}{20}$. This is the same answer the common-denominator method gives: $\frac{12}{20}+\frac{5}{20}=\frac{17}{20}$.

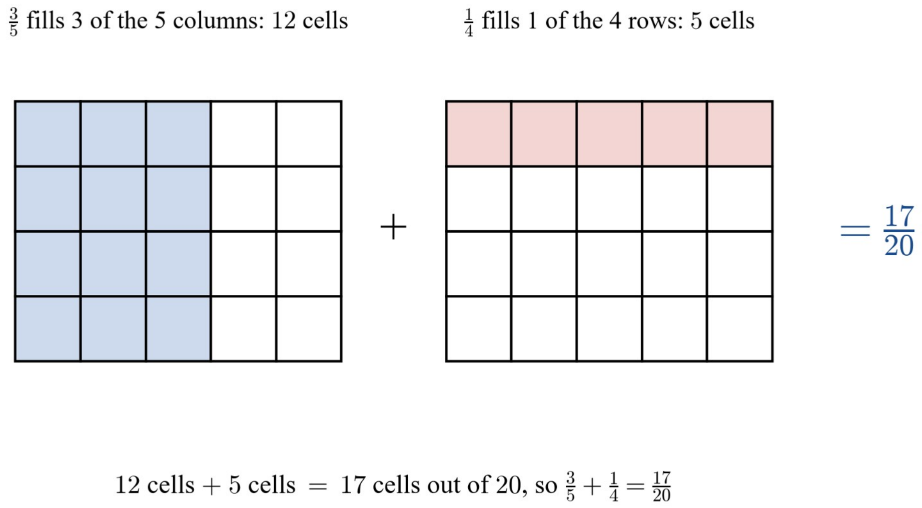


Figure 3. Two copies of the same grid of 5 columns and 4 rows, 20 equal cells in each: in the first grid 3 columns are shaded, showing three fifths as 12 cells; in the second grid 1 row is shaded, showing one quarter as 5 cells; the counts add to 17 cells, so three fifths plus one quarter equals seventeen twentieths.

Subtraction of fractions uses a common denominator, as at Level 6: $\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$. Algebra tiles are another model for these sums, working the same way as the array in Figure 3.

Efficient multiplication and division. The same habit — look at the numbers before you calculate — pays off here too.

Swap the order. The commutative property works for multiplication as well: $\frac{2}{3}$ of $\frac{1}{2}$ is the same amount as $\frac{1}{2}$ of $\frac{2}{3}$. Figure 4 shows both on unit squares, and the blue part is the same size each time. Half of two-thirds is the easier one to see: half of $\frac{2}{3}$ is $\frac{1}{3}$, so both products equal $\frac{1}{3}$.

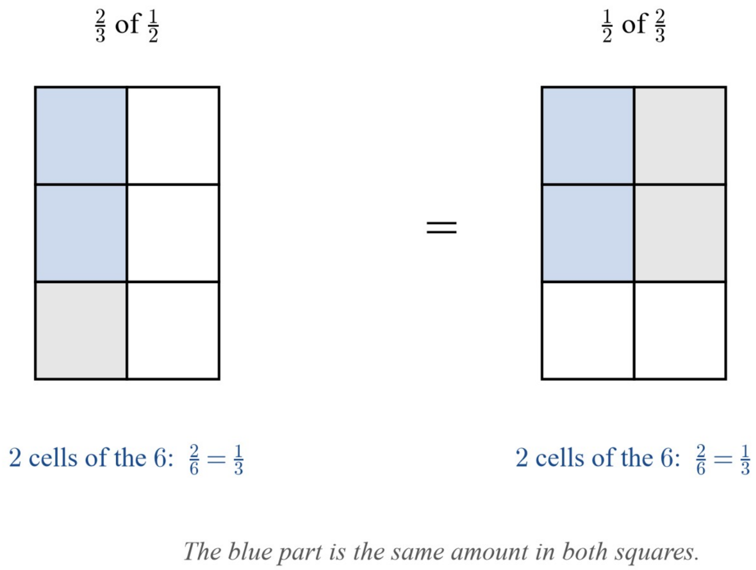


Figure 4. Two equal unit squares split into 2 columns and 3 rows: the left square is labelled two thirds of one half, the right square one half of two thirds; in each square the same 2 cells of the 6 are shaded blue, showing that both products equal two sixths, which is one third.

Group friendly factors. The associative property lets you choose which pair to multiply first: $2.5 \times 3.7 \times 4 = (2.5 \times 4) \times 3.7 = 10 \times 3.7 = 37$, because 2.5 and 4 make a friendly pair.

Place value and known facts. One multiplication fact carries a whole family with it. Figure 5 starts from $7 \times 3 = 21$: dividing one factor by 10 divides the product by 10, giving $7 \times 0.3 = 2.1$ and $7 \times 0.03 = 0.21$. Division runs the other way: from $21 \div 3 = 7$, dividing the divisor by 10 gives ten times as many groups, so $21 \div 0.3 = 70$.

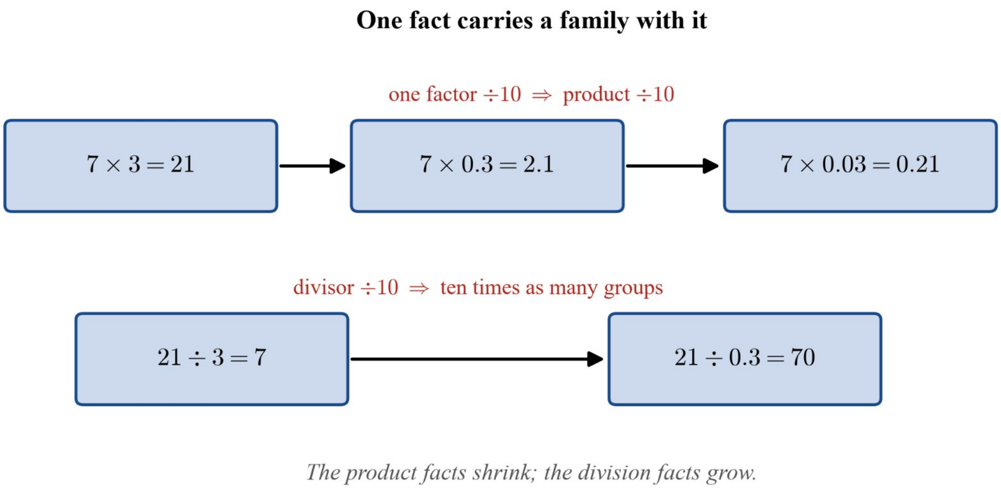


Figure 5. Two rows of number facts: the top row runs from 7 times 3 equals 21 to 7 times 0.3 equals 2.1 to 7 times 0.03 equals 0.21, with arrows noting that dividing one factor by 10 divides the product by 10; the bottom row runs from 21 divided by 3 equals 7 to 21 divided by 0.3 equals 70, with an arrow noting that dividing the divisor by 10 gives ten times as many groups.

Halve and double. Multiplying by 0.5 is halving; multiplying by 0.25 is halving twice; multiplying by 0.125 is halving three times. So 0.125×36 is $36 \to 18 \to 9 \to 4.5$. The halving links back to the swaps table: 12.5 % of an amount is half of half of half of it.

Fraction walls. A fraction wall stacks rows of the same total length, each row split into a different number of equal parts (Figure 6). Longer cell means larger fraction, and rows line up where fractions are equal — the $\frac{1}{2}$ mark sits above $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ and $\frac{5}{10}$. Walls also help compose answers: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ is 4 eighths, 2 eighths and 1 eighth, which is $\frac{7}{8}$ of a row.

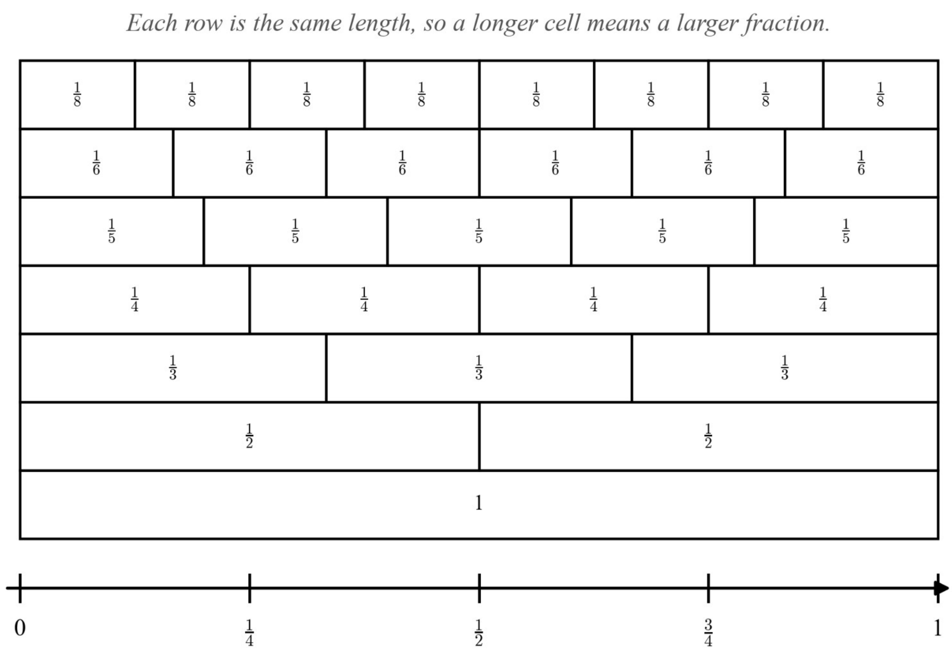


Figure 6. A fraction wall: seven rows of equal total length; the bottom row is the whole and the rows above are split into 2, 3, 4, 5, 6 and 8 equal parts, each cell labelled with its fraction, with a number line from 0 to 1 underneath.

Which operation? Many problems are decided before any calculation happens — by reading what the situation is about. Figure 7 sorts the four operations by the job they do: joining amounts is addition, taking away or finding the difference is subtraction, equal groups or a fraction *of* an amount is multiplication, and sharing or finding how many fit is division.

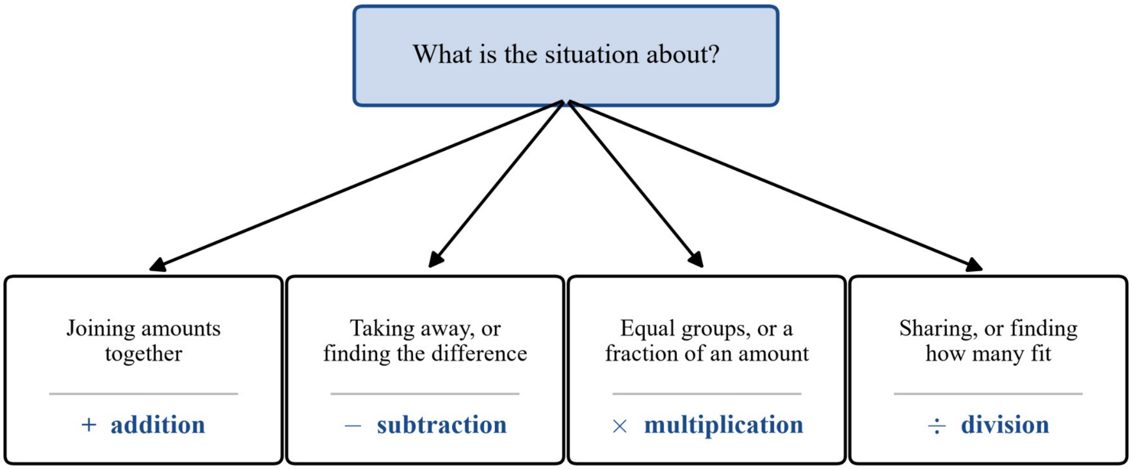


Figure 7. A decision diagram: a top box asks what the situation is about, with four branches down to boxes reading joining amounts together for addition, taking away or finding the difference for subtraction, equal groups or a fraction of an amount for multiplication, and sharing or finding how many fit for division.

Is the answer reasonable? Estimation (subtopic 2B) is the standing check on every result: round first, compute second, then ask whether the exact answer sits near the estimate. For instance, 4.9×5.1 is close to $5 \times 5 = 25$, so an answer of 249.9 cannot be right — it is ten times too big. Worked example 4 and Questions 10 and 11 practise this habit.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
rational number	a number that can be written as a fraction of two numbers, the second not zero	fraction, decimal or percentage form of the one value
positive	greater than zero	above zero
representation	the way a number is written: as a fraction, a decimal or a percentage	form, name
efficient strategy	a way of calculating that uses the numbers' features to keep the work short	shortcut, clever method
commutative property	order does not change the answer: $a+b=b+a$ and $a \times b = b \times a$	swapping the order
associative property	grouping does not change the answer: $(a+b)+c=a+(b+c)$	changing the brackets
partitioning	splitting a number into parts that are easier to work with	breaking apart
regrouping	reordering and grouping terms so pairs make friendly totals	making pairs
rectangular array	a grid of equal cells used to show fraction sums	grid picture

Term	Meaning here	Synonyms and related words
fraction wall	rows of equal total length, each split into a different number of equal parts	fraction strips
difference	the result of a subtraction; how much one number is above another	gap

This subtopic leans on the methods of 2C and the estimation habit of 2B, and it assumes the Level 6 work on adding and subtracting fractions. Ahead, 2E takes the percentage work further, 2F builds ratios on the same fraction skills, and 2G uses all of it in modelling.

Worked examples

Example 1 — scaffolded

The canteen orders 96 bottles of juice for the sports day. It sells $12.5\,\%$ of them before lunch. How many bottles are sold before lunch?

Working	Comment
Choose. The question asks for $12.5\,\%$ of 96. From the swaps table, $12.5\,\% = \frac{1}{8}$.	Choose the representation first (Figure 1): the fraction turns a percentage job into a division by 8.
Solve. $\frac{1}{8}$ of $96 = 96 \div 8 = 12$.	$96 \div 8$ is a known fact — that is why the fraction form was chosen.
Check. $10\,\%$ of 96 is 9.6, and $12.5\,\%$ is more than $10\,\%$, so the answer should be more than 9.6. 12 is.	A quick estimate (2B) checks the answer is reasonable.

$\therefore$ 12 bottles are sold before lunch.

Example 2 — scaffolded

Find the total mass of one order for the canteen: 2.75 kg of flour, 3.6 kg of apples, 1.25 kg of butter and 4.4 kg of sugar.

Working	Comment
Look for friendly pairs. 2.75 pairs with 1.25 ($0.75 + 0.25$ makes a whole), and 3.6 pairs with 4.4 ($0.6 + 0.4$ makes a whole).	Hunt for decimal parts that add to 1 before adding anything.
Reorder and group (Figure 2). $(2.75 + 1.25) + (3.6 + 4.4)$	The commutative property allows the reordering; the associative property allows the grouping.
Solve. $4 + 8 = 12$.	Each pair made a whole number, so the total is found mentally.

$\therefore$ the total mass of the order is 12 kg.

Example 3 — unscaffolded

The school garden has an area of 24 m². Half of the garden is vegetables, and $\frac{2}{3}$ of the vegetable area is carrots. What area is planted with carrots?

Working	Comment
Carrots $= \frac{2}{3}$ of $\frac{1}{2}$ of 24.	Two "of" steps mean two multiplications.

Working	Comment
Swap the order (commutative property): $\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$ (Figure 4).	Half of two-thirds is the easier picture: $\frac{1}{3}$ of the whole garden is carrots.
$\frac{1}{3}$ of $24 = 24 \div 3 = 8$.	Check the other way round: $\frac{1}{2}$ of $24 = 12$, then $\frac{2}{3}$ of $12 = 8$. Same answer.

∴ the area planted with carrots is 8 m².

Example 4 — unscaffolded

The school office keeps its electricity bills for the first three terms of the year (Figure 8): Term 1 $\$268.75$, Term 2 $\$344.25$, Term 3 $\$291.60$. (a) Find the total cost over the three terms. (b) Find the change from Term 1 to Term 2, and from Term 2 to Term 3, saying whether each change is an increase or a decrease. (c) Use estimation to check that your total in (a) is reasonable.

Working	Comment
(a) $268.75 + 344.25 + 291.60$. Group the friendly pair first: $(268.75 + 344.25) + 291.60 = 613.00 + 291.60 = 904.60$.	$0.75 + 0.25 = 1.00$, so the first two bills make a whole-dollar total.
(b) Term 1 to Term 2: $344.25 - 268.75 = 75.50$, an increase. Term 2 to Term 3: $344.25 - 291.60 = 52.65$, a decrease.	The change between two amounts is found by subtraction, whichever way it goes.
(c) Round to the nearest ten: $270 + 340 + 290 = 900$. The exact total 904.60 is close to 900, so it is reasonable.	Estimation checks the size of the answer, not its last digit.

∴ the total is $\$904.60$; the bill increased by $\$75.50$ from Term 1 to Term 2 and decreased by $\$52.65$ from Term 2 to Term 3.

Figure 8. A bar chart of the electricity bills for three school terms: 268.75 dollars, 344.25 dollars and 291.60 dollars, with an arrow between the first two bar tops marked plus 75.50 dollars and an arrow between the last two marked minus 52.65 dollars.

Practice questions
Scaffolded

Q1. One number, three names. (a) Write 0.125 as a fraction in its simplest form. (b) Write $\frac{1}{8}$ as a decimal and as a percentage. (c) Which form would you choose to find $12.5\,\%$ of 48 mentally? Find the value.

Q2. Find each total by regrouping into friendly pairs first. Show your regrouping. (a) $1.65+4.3+3.35+5.7$ (b) $2\frac{5}{8}+1\frac{1}{4}+\frac{3}{8}$

Q3. Partition one number to find each answer. Show the split you used. (a) $6.4+3.8$ (b) $7.2-2.65$

Q4. Use the rectangular array of Figure 3 to find $\frac{3}{5}+\frac{1}{4}$. (a) How many equal cells does the whole array have? (b) How many cells show $\frac{3}{5}$? How many show $\frac{1}{4}$? (c) Write the sum as one fraction.

Q5. Find $\frac{3}{4}$ of $\frac{2}{5}$ of 40 in two ways. (a) Take $\frac{2}{5}$ of 40 first, then $\frac{3}{4}$ of the result. (b) Swap the order (commutative property): take $\frac{3}{4}$ of 40 first, then $\frac{2}{5}$ of the result. (c) What do you notice about the two answers?

Q6. Use place value and the known fact $6\times7=42$ to complete each one. (a) $6\times0.7=\square$ (b) $0.6\times0.7=\square$ (c) $42\div0.7=\square$

Unscaffolded

Q7. Find each value efficiently, and name the property or strategy you used. (a) $2.5\times3.7\times4$ (b) $\frac{5}{6}$ of 18 (c) $7.68-2.9$ (d) $3\div\frac{3}{4}$

Q8. (a) Find 0.5×36, 0.25×36 and 0.125×36. (b) In one sentence, explain the pattern in your three answers.

Q9. For each situation, name the operation that fits, then find the answer. (i) A 2.4 m ribbon is cut into pieces of 0.3 m. How many pieces are there? (ii) One batch of biscuits needs 0.75 kg of flour. How much flour do 3 batches need? (iii) One jug holds 1.25 L of water and another holds 0.85 L. How much water altogether? (iv) A full 2 L bottle has 0.6 L poured out. How much is left?

Q10. Leo finds 4.9×5.1 and writes 249.9. Without doing the multiplication again, explain why Leo's answer cannot be right, and give a reasonable estimate for the true value.

Q11. A household's electricity bills for three account cycles were $\$287.60$, $\$312.15$ and $\$265.40$. (a) Find the total of the three bills. (b) Find the change from cycle 1 to cycle 2, and from cycle 2 to cycle 3, saying whether each is an increase or a decrease. (c) Use estimation to check that your total in (a) is reasonable.

Q12. On a three-day hike, a group eats 0.65 kg of food on day 1, 0.8 kg on day 2 and 0.55 kg on day 3. (a) Find the total mass of food eaten. (b) If $\frac{1}{4}$ of the food is protein, find the mass of protein eaten.

Q13. Use the fraction wall of Figure 6. (a) Find $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$. (b) Find $1-\frac{3}{8}$. (c) Which is larger, $\frac{2}{3}$ or $\frac{5}{8}$? How does the wall show it?

Q14. A new suburb is planned: $\frac{2}{5}$ of its land is parkland, $\frac{1}{4}$ is set aside for manufacturing and retail, and the rest is housing. (a) What fraction of the land is parkland or is set aside for manufacturing and retail? (b) What fraction of the land is housing? (c) The suburb covers 60 km² in all. Find the area set aside as parkland.

Answers

Q1. (a) $\frac{1}{8}$; (b) 0.125, 12.5%; (c) $\frac{1}{8}$ of $48=6$.

Q2. (a) 15; (b) $4\frac{1}{4}$.

Q3. (a) 10.2; (b) 4.55.

Q4. (a) 20 cells; (b) 12 cells and 5 cells; (c) $\frac{17}{20}$.

Q5. (a) 12; (b) 12; (c) the two ways give the same answer — the order of the two "of" steps does not matter.

Q6. (a) 4.2; (b) 0.42; (c) 60.

Q7. (a) 37; (b) 15; (c) 4.78; (d) 4.

Q8. (a) 18, 9, 4.5; (b) each answer is half of the one before it.

Q9. (i) division, 8 pieces; (ii) multiplication, 2.25 kg; (iii) addition, 2.1 L; (iv) subtraction, 1.4 L.

Q10. 4.9×5.1 is close to $5\times5=25$, and 249.9 is about ten times too big; a reasonable estimate is 25 (the exact value is 24.99).

Q11. (a) $\$865.15$; (b) increase of $\$24.55$, then decrease of $\$46.75$; (c) $290+310+265=865$, close to $\$865.15$.

Q12. (a) 2 kg; (b) 0.5 kg.

Q13. (a) $\frac{7}{8}$; (b) $\frac{5}{8}$; (c) $\frac{2}{3}$ — its cell on the wall is longer than the $\frac{5}{8}$ cell.

Q14. (a) $\frac{13}{20}$; (b) $\frac{7}{20}$; (c) 24 km².

Fully-worked solutions

Q1. (a) $0.125=\frac{125}{1000}$, and dividing top and bottom by 125 gives $\frac{1}{8}$. (b) $\frac{1}{8}=1\div8=0.125$, and $0.125=\frac{12.5}{100}=12.5\%$. (c) The fraction suits mental work: 12.5% of $48=\frac{1}{8}$ of $48=48\div8=6$.

Q2. (a) Reorder and group: $(1.65+3.35)+(4.3+5.7)=5+10=15$. (b) Group the fractions with matching denominators: $2\frac{5}{8}+\frac{3}{8}=2\frac{8}{8}=3$, then $3+1\frac{1}{4}=4\frac{1}{4}$.

Q3. (a) Split $3.8=3.6+0.2$: $6.4+3.6=10$, then $10+0.2=10.2$. (b) Split $2.65=2.2+0.45$: $7.2-2.2=5$, then $5-0.45=4.55$.

Q4. The array has 5 columns and 4 rows. (a) $5\times4=20$ equal cells. (b) $\frac{3}{5}$ fills 3 of the 5 columns: $3\times4=12$ cells. $\frac{1}{4}$ fills 1 of the 4 rows: 5 cells. (c) $12+5=17$ cells out of 20, so $\frac{3}{5}+\frac{1}{4}=\frac{17}{20}$.

Q5. (a) $\frac{2}{5}$ of $40 = 40 \div 5 \times 2 = 16$; then $\frac{3}{4}$ of $16 = 16 \div 4 \times 3 = 12$. (b) Swapping the order: $\frac{3}{4}$ of $40 = 30$; then $\frac{2}{5}$ of $30 = 30 \div 5 \times 2 = 12$. (c) Both ways give 12: multiplication is commutative, so the order of the two "of" steps does not change the result.

Q6. One factor divided by 10 divides the product by 10: (a) $6 \times 0.7 = 4.2$; (b) both factors divided by 10 divides the product by 100: $0.6 \times 0.7 = 0.42$; (c) division runs the other way — the divisor 0.7 is one tenth of 7, so ten times as many groups fit: $42 \div 0.7 = 60$.

Q7. (a) Associative property: $(2.5 \times 4) \times 3.7 = 10 \times 3.7 = 37$. (b) $\frac{5}{6}$ of $18 = 18 \div 6 \times 5 = 3 \times 5 = 15$. (c) Partition $2.9 = 3 - 0.1$: $7.68 - 3 = 4.68$, then add 0.1 back: 4.78. (d) $3 \div \frac{3}{4}$ asks how many three-quarters fit into 3: four of them, so 4.

Q8. (a) $0.5 \times 36 = 18$; $0.25 \times 36 = 9$; $0.125 \times 36 = 4.5$. (b) Each multiplier is half of the one before ($0.5, 0.25, 0.125$), so each answer is half of the one before — the answers are 36 halved once, twice and three times.

Q9. (i) Division — how many 0.3 m pieces fit into 2.4 m: $2.4 \div 0.3 = 8$ pieces. (ii) Multiplication — three equal groups of 0.75 kg: $3 \times 0.75 = 2.25$ kg. (iii) Addition — joining two amounts: $1.25 + 0.85 = 2.1$ L. (iv) Subtraction — taking away from a full amount: $2 - 0.6 = 1.4$ L.

Q10. Round the factors: $4.9 \approx 5$ and $5.1 \approx 5$, so the product is close to $5 \times 5 = 25$. Leo's 249.9 is about ten times larger than that, so it cannot be right — he likely dropped the decimal point in 4.9 and multiplied 49×5.1 instead. A reasonable estimate is 25 (the exact value is 24.99).

Q11. (a) $287.60 + 312.15 + 265.40 = 599.75 + 265.40 = \$865.15$. (b) Cycle 1 to cycle 2: $312.15 - 287.60 = 24.55$, an increase of $\$24.55$. Cycle 2 to cycle 3: $312.15 - 265.40 = 46.75$, a decrease of $\$46.75$. (c) Round to the nearest ten: $290 + 310 + 265 = 865$, which is close to 865.15, so the total is reasonable.

Q12. (a) $0.65 + 0.8 + 0.55$. Group the pair that makes a whole: $(0.65 + 0.55) + 0.8 = 1.2 + 0.8 = 2$ kg. (b) $\frac{1}{4}$ of $2 = 2 \div 4 = 0.5$ kg of protein.

Q13. (a) On the wall, $\frac{1}{2}$ lines up with 4 eighths and $\frac{1}{4}$ with 2 eighths, so the sum is $4 + 2 + 1 = 7$ eighths: $\frac{7}{8}$. (b) The whole row is 8 eighths; removing 3 leaves 5: $1 - \frac{3}{8} = \frac{5}{8}$. (c) $\frac{2}{3}$ is larger. On the wall, the $\frac{2}{3}$ cell reaches past the $\frac{5}{8}$ cell: $\frac{2}{3} = \frac{16}{24}$ and $\frac{5}{8} = \frac{15}{24}$, so $\frac{2}{3}$ is one twenty-fourth ahead.

Q14. (a) $\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$. (b) The rest: $1 - \frac{13}{20} = \frac{7}{20}$ is housing. (c) Parkland $= \frac{2}{5}$ of $60 = 60 \div 5 \times 2 = 24$ km².

Teacher note

- **Ownership and the boundary with 2C.** Per the Chapter 2 plan in TOC §3, 2C (`VC2M7N05`) owns the *methods* of multiplying and dividing fractions and decimals; this subtopic (`VC2M7N06`) owns **problem solving across all four operations**, the choice of an efficient strategy, and the commutative and associative properties applied to fractions (WE1–WE3, Q2, Q5, Q7). Neither absorbs the other: questions here *use* 2C's methods without re-teaching them.
- **Level 6 recall (D3).** Adding and subtracting fractions with common denominators (`VC2M6N03`, `VC2M6N05`) and decimal place value (`VC2M6N04`) are assumed. They are recalled in one line each (Figure 3's common-denominator check, the subtraction example in the theory) and maintained inside the questions, not taught as new content.

- **Positive rationals only (D4.4).** Every number in this subtopic is positive; no negatives appear. Chapter 2's negatives belong to 2A's number line and are located and ordered there, not operated on.
- **No percentage increase or decrease (D4.7).** The energy-account context of `VC2M7N06.E06` is delivered as **dollar differences** (WE4, Q11): an increase of $\$75.50$ is a subtraction, never "add 10%". Percentage change is `VC2M8N05` (Level 8) and does not appear.
- **Efficient representation (E02, E06).** Figure 1 and the swaps table deliver the $12.5\% \leftrightarrow \frac{1}{8} \leftrightarrow 0.125$ choice; the percentages appear only as another name for the same number, with the depth left to 2E (`VC2M7N07`).
- **E06's contexts, generically framed.** Energy accounts (WE4, Q11), food intake (Q12) and land use (Q14) all use transparently generic settings; no real data is asserted, so no source lines are required. Real-data versions would need §6 sourcing.
- **Digital tools (D9).** The CD does not require a tool; E01 and E04 name digital tools and calculators among the representations, so their use is permitted where appropriate. No CAS.
- **Reasonableness habit (P7).** WE4(c), Q10 and Q11(c) ask whether an answer is reasonable before or instead of asking for the answer, per TOC §5.2.
- **Figures.** All eight figures are generated by `src/gen_2d.py` from the exact values stated in the text; grids, walls and charts compute their labels from those values.
- **No D11 content.** None of `VC2M7N06.E01–.E06` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

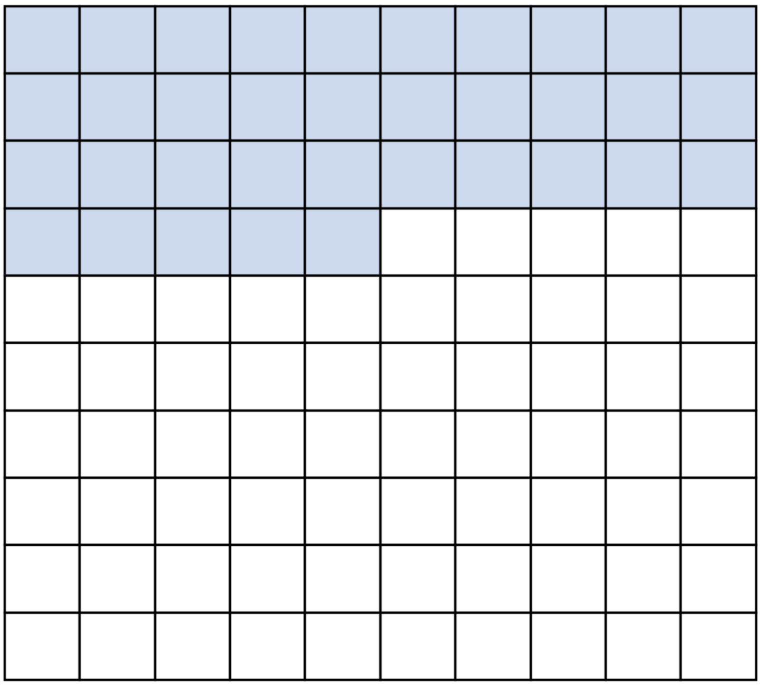
Percentages of quantities; one quantity as a percentage of another

Content description `VC2M7N07`: find percentages of quantities and express one quantity as a percentage of another, with and without digital tools

Achievement standard (extract): They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.

Theory

What a percentage is. Per cent means "out of 100", and a **percentage** is a number written that way: $35\,\%$ means 35 out of 100, the same number as the fraction $\dfrac{35}{100}$ and the decimal 0.35 (the three names for one number, from subtopic 2A). Figure 1 shows $35\,\%$ as a picture: 35 of the 100 small squares are shaded. This subtopic does two jobs with percentages. **Job 1:** find a percentage *of* a quantity. **Job 2:** express one quantity *as a percentage of* another. Both jobs are done without a calculator and with one, because the content description asks for both.



35 of the 100 squares are shaded: $\frac{35}{100} = 35\%$

Figure 1. A hundred grid of 10 by 10 small squares with 35 of the squares shaded, showing 35 out of 100, equal to 35 per cent.

Job 1 without a calculator: build from blocks. A few percentages are quick to find of any quantity, so they are useful **building blocks**: $50\,\%$ (half of the quantity), $25\,\%$ (a quarter), $10\,\%$ (divide by 10), $5\,\%$ (half of $10\,\%$) and $1\,\%$ (divide by 100). Figure 2 splits the quantity 180 into ten equal blocks: each block is $10\,\%$ of 180, which is $180 \div 10 = 18$, and every other percentage of 180 can be built from blocks of this size.

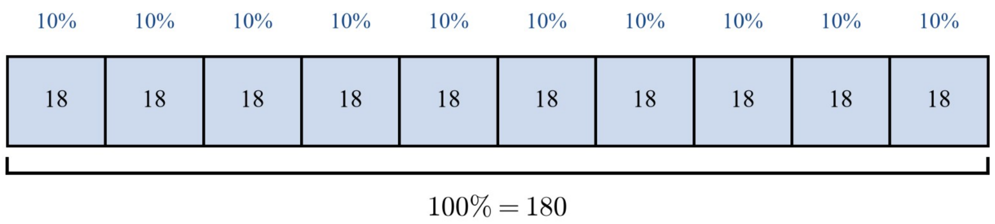


Figure 2. A bar standing for the quantity 180 split into ten equal blocks; every block is labelled 18 with 10% written above it, and a bracket underneath marks 100% = 180.

Figure 3 builds 65 % of 180 from the blocks. Six blocks give 60 %: $6 \times 18 = 108$. Half a block gives 5 %: $18 \div 2 = 9$. Putting the parts together, 65 % of 180 is $108 + 9 = 117$.

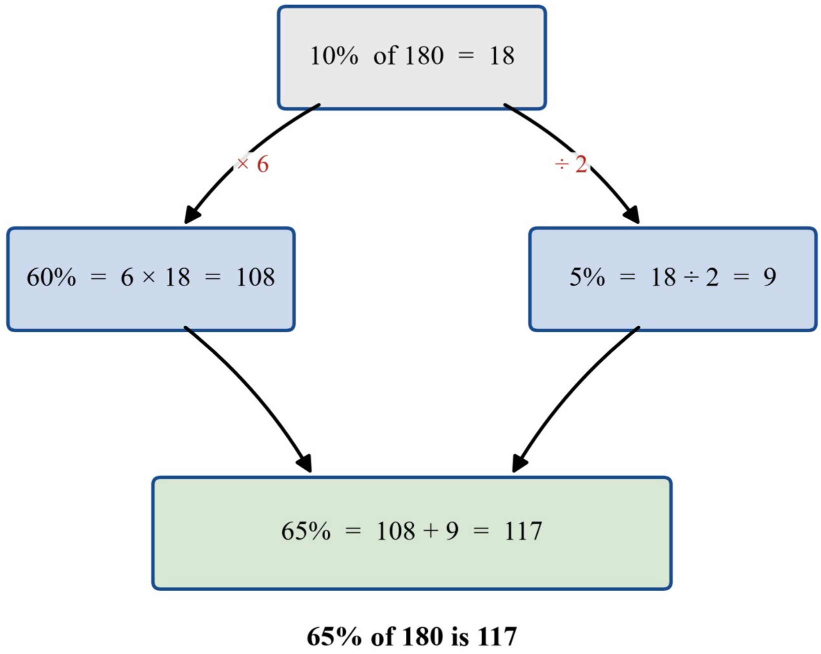


Figure 3. A build-up diagram: at the top, 10% of 180 = 18; a left arrow marked times 6 leads to 60% = 6 × 18 = 108, a right arrow marked divided by 2 leads to 5% = 18 ÷ 2 = 9, and both arrows join into 65% = 108 + 9 = 117.

The blocks of any quantity are worth knowing by heart:

Percentage	How to find it	Of 180
50 %	half	90
25 %	a quarter	45
10 %	divide by 10	18
5 %	half of 10 %	9
1 %	divide by 100	1.8

Job 1 without a calculator: swap to a fraction. When the percentage is a friendly fraction, swap it for the fraction and divide. In Figure 4, $25\,\% = \frac{1}{4}$, so 25 % of 92 is $92 \div 4 = 23$: each of the four equal parts of the bar holds 23. The swaps worth knowing are $50\,\% = \frac{1}{2}$, $25\,\% = \frac{1}{4}$, $75\,\% = \frac{3}{4}$, $20\,\% = \frac{1}{5}$, $12.5\,\% = \frac{1}{8}$ and $10\,\% = \frac{1}{10}$ — subtopic 2D practises choosing the name that makes a calculation easy, and here that choice does real work.

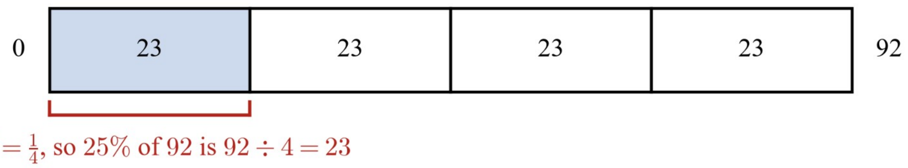


Figure 4. A bar standing for 92 split into four equal parts of 23; the first part is shaded, and a bracket underneath it marks 25% = 1/4, so 25% of 92 is 92 ÷ 4 = 23.

Job 2 without a calculator: scale to out of 100. To express one quantity as a percentage of another, first write the comparison as a fraction — the part over the whole — then scale the fraction until the bottom is 100. In Figure 5, 18 of the 24 students in a class walk to school. Dividing top and bottom by 6 gives $\frac{3}{4}$, and multiplying top and bottom by 25 gives $\frac{75}{100}$: the walkers are $75\,\%$ of the class.

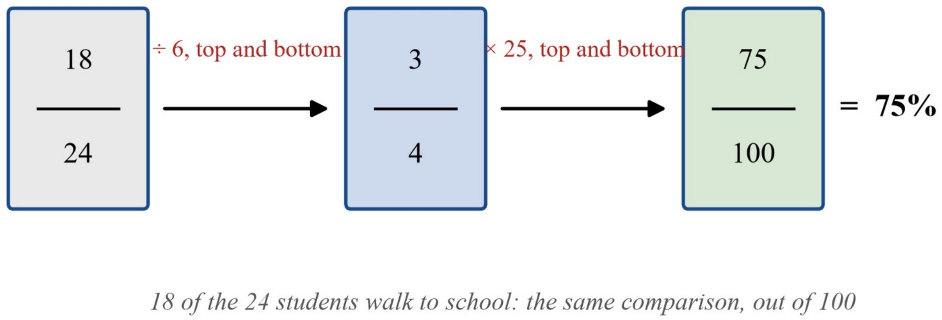


Figure 5. Three fraction boxes joined by arrows: 18/24, then divide top and bottom by 6 to get 3/4, then multiply top and bottom by 25 to get 75/100, equal to 75%; the caption says 18 of the 24 students walk to school.

Job 2 without a calculator: divide when 100 is out of reach. Sometimes the bottom number does not scale to 100 by a whole number. For 7 of 24 students, divide the part by the whole: $7 \div 24 = 0.291666\ldots$, and multiplying by 100 (which moves the point two places) gives $29.1666\ldots\%$, or $29.2\,\%$ to one decimal place (rounding as in 2B). Figure 6 shows the same comparison on a double number line: 7 sits a little past the $25\,\%$ mark on the way from 0 to 24, so $29.2\,\%$ is a sensible reading. A calculator does the division in one go, which is the calculator version of Job 2 below.

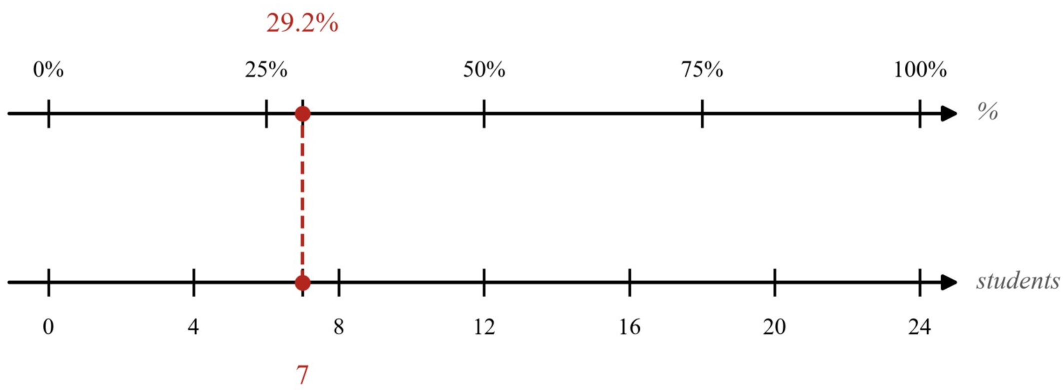


Figure 6. A double number line: the lower line runs from 0 to 24 students with ticks every 4, the upper line runs from 0% to 100% with ticks every 25%; a dashed red line joins 7 on the lower line to 29.2% on the upper line, just past the 25% mark.

With a digital tool. Job 1 with a calculator: write the percentage as a decimal and multiply. 37 % of 86 is $0.37 \times 86 = 31.82$ (Figure 7). Estimate first, as in 2B: 10 % of 86 is 8.6, so 40 % is about 34.4, and the answer should land a little below that — 31.82 does. Some calculators also have a % key that makes the swap for you, and a spreadsheet does the same multiplication in a cell; the decimal method works on every tool. Job 2 with a calculator is the division above: part ÷ whole, then × 100.

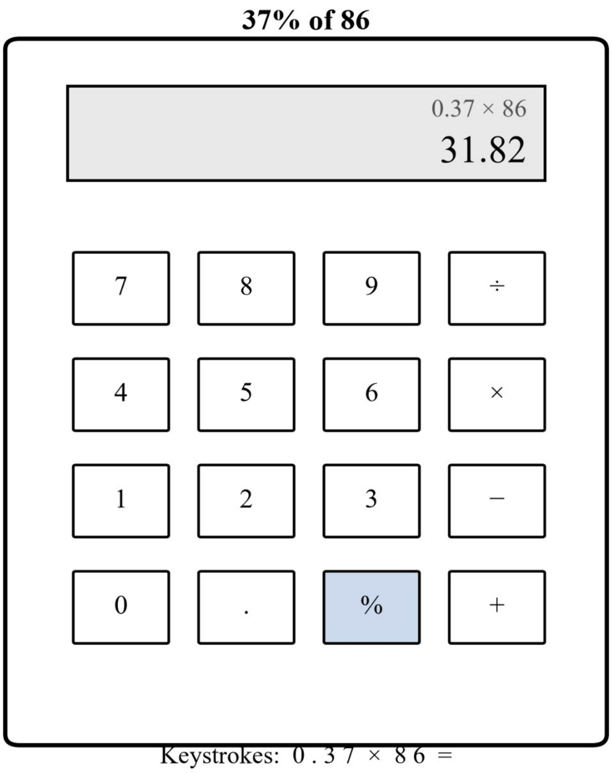


Figure 7. A drawn calculator titled 37% of 86; the display shows 0.37 × 86 with the result 31.82, the percent key is shaded, and the keystrokes 0.37 × 86 = are written below.

The two jobs are the same three numbers. Figure 8 holds one comparison three ways: 30 % of 150 is 45, and 45 expressed as a percentage of 150 is 30 %. Doing Job 1 and then Job 2 returns you to where you started, which gives every answer a built-in check: after finding a percentage of a quantity, express your result as a percentage of the whole and see whether the starting percentage comes back.

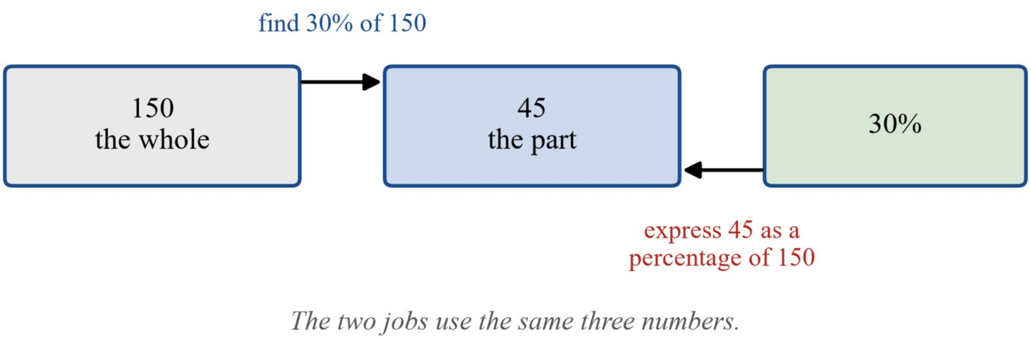


Figure 8. Three boxes holding 150 the whole, 45 the part, and 30%; a top arrow labelled find 30% of 150 runs from 150 to 45, and a return arrow labelled express 45 as a percentage of 150 runs from 30% back to 45.

Percentages greater than 100. When the quantity being compared is *larger* than the whole it is compared with, the percentage is greater than 100. In Figure 9 a recipe asks for 220 g of flour and Ben puts in 385 g: $\frac{385}{220} = 1.75$, so Ben's flour is 175 % of the recipe's flour. That is a comparison — "one and three-quarter times as much" — not a change: nothing has grown by 175 %, and percentages that describe increases and decreases belong to Level 8. Here, 175 % simply compares two quantities, exactly as 75 % did in Figure 5.

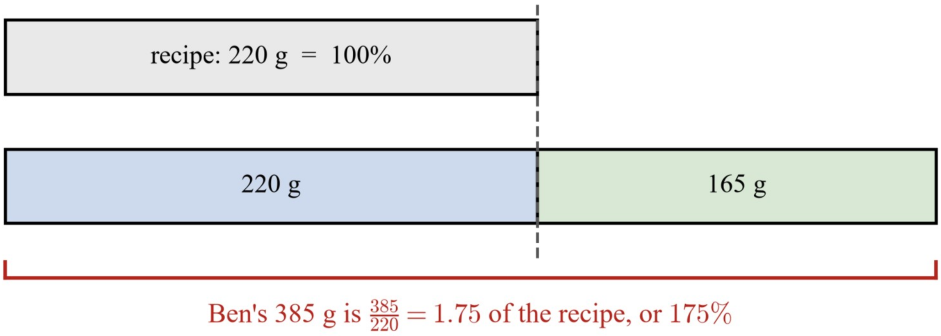


Figure 9. Two bars on the same scale: the top bar is the recipe's 220 g, marked 100%; the lower bar is Ben's 385 g, split by a dashed line at the 100% mark into 220 g and 165 g, with a bracket underneath showing that 385 g is 385/220 = 1.75 of the recipe, or 175%.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
percentage	a number written out of 100; the sign %	percent, rate
per cent	"out of 100": 35 % means 35 out of 100	percent
quantity	an amount; how many or how much	amount, number
whole	the total amount that 100 % stands for	total, all of it
part	the share of the whole being found or compared	share, piece
building block	a simple percentage found quickly: 50 %, 25 %, 10 %, 5 %, 1 %	friendly percentage
estimate	a quick, close answer used to check the size of the real one	rough check
digital tools	a scientific calculator or a spreadsheet	calculator

Percentages lean on the rest of Chapter 2: the three names for one number come from 2A, rounding and estimating from 2B, and the fraction and decimal calculation from 2C and 2D. Ahead, 2F writes comparisons like these as ratios, and 2G puts both jobs of this subtopic to work inside its modelling cycle.

Worked examples

Example 1 — scaffolded

The school library takes delivery of 180 new books, and 65 % of them are fiction. How many fiction books are there? (No calculator.)

Working	Comment
10 % of 180 is $180 \div 10 = 18$ (Figure 2).	The 10 % block is the building block everything else comes from.
$60\,\% = 6 \times 18 = 108$.	Six blocks of 18.

Working	Comment
$5\% = 18 \div 2 = 9$.	Half of one block.
$65\% = 108 + 9 = 117$ (Figure 3).	Put the parts together.

$\therefore$ there are 117 fiction books.

Example 2 — scaffolded

At training, a netball player makes 22 of her 25 goal attempts. Express the goals made as a percentage of the attempts. (No calculator.)

Working	Comment
The comparison is $\frac{22}{25}$: part over whole.	The part (goals made) goes on top; the whole (attempts) goes on the bottom.
$\frac{22}{25} = \frac{22 \times 4}{25 \times 4} = \frac{88}{100}$.	Scale top and bottom by the same number until the bottom is 100; here $\times 4$ does it.
$\frac{88}{100} = 88\%$.	A fraction out of 100 *is* the percentage.

$\therefore$ the player makes 88% of her goal attempts.

Example 3 — unscaffolded

Hannah earns $\$95$ from a weekend job and puts 62% of it into a savings account. How much does she save? Use a calculator, and check the answer is sensible before you accept it.

Write 62% as the decimal 0.62, then multiply: $0.62 \times 95 = 58.9$, so the saving is $\$58.90$. **Check by estimate:** 10% of 95 is 9.50, so 60% is $6 \times 9.50 = \$57$; the answer should be a little more than $\$57$, and $\$58.90$ is. The answer also sits between 50% of 95 ($\$47.50$) and the whole ($\$95$), where 62% must sit.

$\therefore$ Hannah saves $\$58.90$.

Example 4 — unscaffolded

In a class survey, 16 of the 22 students say they ride to school at least once a week. Express 16 as a percentage of 22, to one decimal place, and say in one sentence what the percentage means. (Calculator.)

Divide the part by the whole, then multiply by 100: $16 \div 22 = 0.727272\ldots$, so the percentage is $72.7272\ldots\%$, which rounds to 72.7%. **Interpret:** a little more than seven out of every ten students in the class ride to school at least once a week. The double number line of Figure 6 shows how to read such a line: on one drawn for this survey, 16 of 22 would sit between the 50% and 100% marks, close to 75%, exactly where 72.7% belongs.

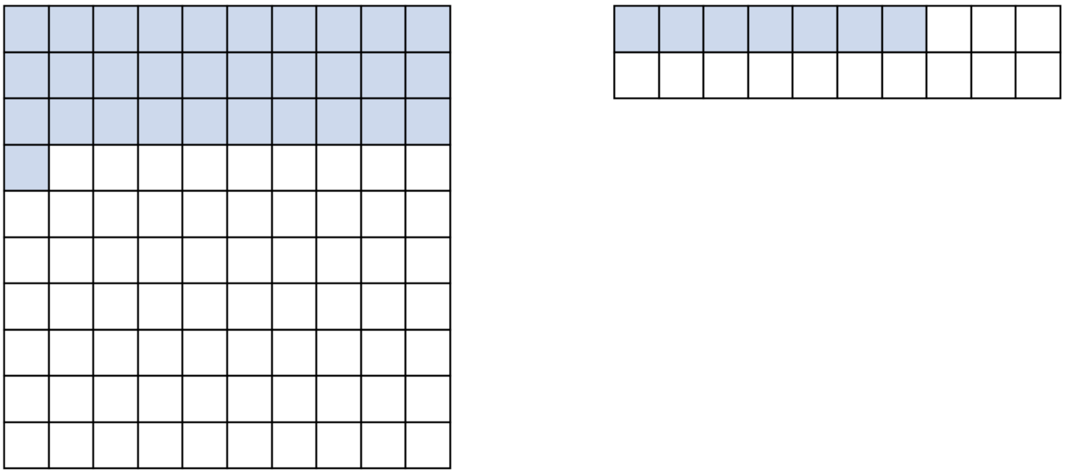
$\therefore$ 72.7% of the class ride to school at least once a week.

Practice questions

Scaffolded

Q1–Q6 are the calculator-free strand of this subtopic: do them with jottings only.

Q1. Figure 10 shows two grids. For each grid, write the shaded area (a) as a fraction out of 100, and (b) as a percentage.



Grid A Grid B

Figure 10. Two grids: Grid A is a hundred grid of 10 by 10 squares with 31 squares shaded; Grid B is a grid of 20 squares in two rows of ten with 7 squares shaded.

Q2. Copy and complete this table of building blocks for the quantity 480, then use it to find 35 % of 480.

Percentage	10 %	5 %	50 %	35 %
Of 480				

Q3. Swap to a fraction, then find, without a calculator: (a) 25 % of 92; (b) 75 % of 28; (c) 12.5 % of 56; (d) 20 % of 65.

Q4. Express, without a calculator, by scaling to a bottom of 100: (a) 17 out of 20; (b) 13 out of 25; (c) 9 out of 50; (d) 33 out of 44.

Q5. The bottom does not reach 100 in one step: simplify the fraction first, then scale. Without a calculator, express: (a) 14 out of 35; (b) 33 out of 55.

Q6. (a) Find 40 % of 150 without a calculator. (b) Now express your answer from (a) as a percentage of 150. (c) In one sentence, say what you notice, and connect it to Figure 8.

Unscaffolded

A scientific calculator is allowed except where a question is marked (no calculator).

Q7. Find, with a calculator: (a) 62 % of 350 g; (b) 4.5 % of $ 260; (c) 17.5 % of 800 mL.

Q8. Express as a percentage, to one decimal place: (a) 17 out of 22; (b) 9 out of 14; (c) 13 out of 16.

Q9. (no calculator) Priya finds 60 % of 50 and gets 300. Without redoing the calculation, explain why her answer cannot be right, then find a sensible value.

Q10. (no calculator) A drink recipe uses 250 mL of soda water. Jack pours 450 mL. Express Jack's soda water as a percentage of the recipe's amount.

Q11. There are 320 students in Year 7, and 55 % of them travel by school bus. (a) Find the number of bus students. (b) Of the bus students, 44 enter through the main gate. Express 44 as a percentage of the bus students. (c) Express 44 as a percentage of *all* 320 students, exactly. Why are your two percentages different?

Q12. (no calculator) In a class survey of 30 students, 18 walk to school, 9 ride and 3 are driven. (a) Express the walkers as a percentage of the class. (b) Express the riders as a percentage of the class. (c) A student says "half the class rides or is driven". Is this correct? Give a reason.

Q13. A fete committee fills a jar with 280 raffle tickets. By 2 pm, 77 tickets have sold. (a) Express the tickets sold as a percentage of the jar. (b) The committee's target is to sell $90\,\%$ of the tickets. How many tickets is that? (c) How many more tickets must sell to reach the target?

Q14. (no calculator) Sam finds $25\,\%$ of 120 and writes 48. Check Sam's answer by expressing 48 as a percentage of 120. What do you notice, and what should the answer be?

Answers

Q1. (a) Grid A $\frac{31}{100}$, Grid B $\frac{35}{100}$; (b) 31% and 35%.

Q2. $10\% = 48$, $5\% = 24$, $50\% = 240$; $35\% = 3 \times 48 + 24 = 168$.

Q3. (a) 23; (b) 21; (c) 7; (d) 13.

Q4. (a) 85%; (b) 52%; (c) 18%; (d) 75%.

Q5. (a) $\frac{14}{35} = \frac{2}{5} = \frac{40}{100} = 40\%$; (b) $\frac{33}{55} = \frac{3}{5} = \frac{60}{100} = 60\%$.

Q6. (a) 60; (b) 40%; (c) the second job undid the first — the same three numbers, read the other way (Figure 8).

Q7. (a) 217 g; (b) $\$11.70$; (c) 140 mL.

Q8. (a) 77.3%; (b) 64.3%; (c) 81.3%.

Q9. 60% of 50 must sit between 25 (50%) and 50 (100%), so 300 is impossible; the value is 30.

Q10. 180%.

Q11. (a) 176; (b) 25%; (c) 13.75%; the same count is a percentage of a different whole, so the percentage changes.

Q12. (a) 60%; (b) 30%; (c) no — riders and driven make 12 of 30, which is 40%, not 50%.

Q13. (a) 27.5%; (b) 252; (c) 175.

Q14. 48 is 40% of 120, not 25%, so Sam's answer is wrong; 25% of 120 is 30.

Fully-worked solutions

Q1. Grid A shades 31 of the 100 squares, so it shows $\frac{31}{100} = 31\%$. Grid B shades 7 of 20 squares; multiplying top and bottom by 5 gives $\frac{35}{100}$, so 35%.

Q2. 10% of 480 is $480 \div 10 = 48$; 5% is half of that, 24; 50% is half of 480, 240. Then $35\% = 30\% + 5\% = 3 \times 48 + 24 = 144 + 24 = 168$.

Q3. (a) $25\% = \frac{1}{4}$: $92 \div 4 = 23$. (b) $75\% = \frac{3}{4}$: $28 \div 4 = 7$, $3 \times 7 = 21$. (c) $12.5\% = \frac{1}{8}$: $56 \div 8 = 7$. (d) $20\% = \frac{1}{5}$: $65 \div 5 = 13$.

Q4. (a) $\frac{17}{20} = \frac{85}{100} = 85\%$. (b) $\frac{13}{25} = \frac{52}{100} = 52\%$. (c) $\frac{9}{50} = \frac{18}{100} = 18\%$. (d) $\frac{33}{44} = \frac{3}{4} = \frac{75}{100} = 75\%$ (divide top and bottom by 11, then $\times 25$).

Q5. (a) Divide top and bottom by 7: $\frac{14}{35} = \frac{2}{5}$; then $\times 20$ top and bottom: $\frac{40}{100} = 40\%$. (b) Divide by 11: $\frac{33}{55} = \frac{3}{5}$; then $\times 20$: $\frac{60}{100} = 60\%$.

Q6. (a) $10\,\%$ of 150 is 15, so $40\,\%$ is $4 \times 15 = 60$. (b) $\frac{60}{150}$: divide top and bottom by 3 to get $\frac{20}{50}$, then double to $\frac{40}{100} = 40\,\%$. (c) Job 2 undid Job 1: the three numbers 150, 60 and $40\,\%$ are one comparison read in two directions (Figure 8), so running both jobs returns the starting percentage.

Q7. (a) $0.62 \times 350 = 217$, so 217 g. (b) $0.045 \times 260 = 11.7$, so $\$11.70$. (c) $0.175 \times 800 = 140$, so 140 mL.

Q8. (a) $17 \div 22 = 0.772727\ldots$, so $77.3\,\%$. (b) $9 \div 14 = 0.642857\ldots$, so $64.3\,\%$. (c) $13 \div 16 = 0.8125$, so $81.25\,\%$, which rounds to $81.3\,\%$ at one decimal place.

Q9. $100\,\%$ of 50 is 50, so no percentage of 50 can ever give 300: $60\,\%$ of 50 must sit between 25 (the $50\,\%$ block) and 50 (the whole). A sensible value: $10\,\%$ of 50 is 5, so $60\,\%$ is $6 \times 5 = 30$.

Q10. The comparison is $\frac{450}{250}$; divide top and bottom by 50 to get $\frac{9}{5}$, then $\times 20$ top and bottom gives $\frac{180}{100} = 180\,\%$. Jack poured $180\,\%$ of the recipe's soda water — a percentage above $100\,\%$ because he poured more than the recipe asks for.

Q11. (a) $10\,\%$ of 320 is 32, so $50\,\%$ is 160 and $5\,\%$ is 16: $55\,\%$ is $160 + 16 = 176$ students. (b) $\frac{44}{176} = \frac{1}{4} = 25\,\%$ (divide top and bottom by 44). (c) $\frac{44}{320}$: divide top and bottom by 32 to get $\frac{1.375}{10} = 0.1375$, so $13.75\,\%$. The count 44 is the same, but a percentage always says "out of *which* whole": out of the bus students it is $25\,\%$, out of the whole year level it is $13.75\,\%$.

Q12. (a) $\frac{18}{30} = \frac{6}{10} = \frac{60}{100} = 60\,\%$ (divide top and bottom by 3, then multiply by 10). (b) $\frac{9}{30} = \frac{3}{10} = \frac{30}{100} = 30\,\%$. (c) Riders and driven make $9 + 3 = 12$ students: $\frac{12}{30} = \frac{40}{100} = 40\,\%$, and $40\,\% \neq 50\,\%$, so the claim is not correct — half the class would be 15 students, and 12 is fewer.

Q13. (a) $77 \div 280 = 0.275$, so $27.5\,\%$. (b) $10\,\%$ of 280 is 28, so $90\,\%$ is $9 \times 28 = 252$ tickets. (c) $252 - 77 = 175$ more tickets.

Q14. Express 48 as a percentage of 120: $\frac{48}{120} = \frac{2}{5} = 40\,\%$ (divide top and bottom by 24). If 48 were $25\,\%$ of 120, this check would return $25\,\%$; it returns $40\,\%$, so Sam's number is not $25\,\%$ of 120. The correct answer: $25\,\%$ of 120 is $120 \div 4 = 30$ — which is exactly what the built-in check of Figure 8 is for.

Teacher note

- **Two strands (D9).** `VC2M7N07` requires the work "with and without digital tools": the calculator-free strand runs through the theory (blocks, fraction swaps, scaling), WE1–WE2 and Q1–Q6, and the calculator strand through the decimal-multiplication and divide-by-whole methods, WE3–WE4 and the unmarked questions. T2 draws 2E's 5 marks from both strands, and its calculator-free items come from the former.
- **Level boundary (D4.7).** Percentage increase, decrease and percentage error are Level 8 (`VC2M8N05`) and do not appear. Figure 9's $175\,\%$ is a *comparison* ("one and three-quarter times as much"), never a change; the theory states the boundary so the figure cannot be misread as increase work. Money items ask for a percentage *of* an amount only.
- **Scope is the two jobs.** Finding a percentage of a quantity and expressing one quantity as a percentage of another are the whole of the CD; the built-in check (Figure 8, Q6, Q14) makes students run one job inside the other. `VC2M7N07.E01`'s authentic-problem signal is delivered through realistic school, sport, recipe and survey settings, transparently fictional per §6 (no real data is asserted, so no source lines are required; a real-data version would need §6 sourcing).

- **Recall, not re-teach (D3).** Equivalent fractions and scaling (assumed from `VC2M6N03`) carry Job 2; rounding to one decimal place is maintained from 2B (`VC2M7N04`), not re-taught; the three-names-for-one-number idea is recalled from 2A in one line.
- **Thin CD, full subtopic (D13, D14).** The bank holds 6 MCQs against `VC2M7N07`, so the practice set is almost entirely originally authored; the single elaboration is used as a depth signal (authentic contexts) without structuring the subtopic as a walk through it.
- **Chapter links.** 2D leaves the depth of percentage work here; 2G consumes both jobs (its Figures 3 and 4) inside the modelling cycle; 2F follows with the same comparisons written as ratios. Task I2 also leans on this subtopic's two jobs.
- **Reasonableness habit (P7).** Q9 asks whether an answer is reasonable before the answer is accepted, and WE3 builds the estimate into the calculator strand.
- **Figures.** All ten figures are generated by `src/gen_2e.py` from the exact values stated in the text; the double number line and the over-100 bars place every mark at its true scaled position.
- **No D11 content.** The CD's single elaboration names no Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

Ratios: part–part and part–whole

Content description VC2M7N09: recognise, represent and solve problems involving ratios

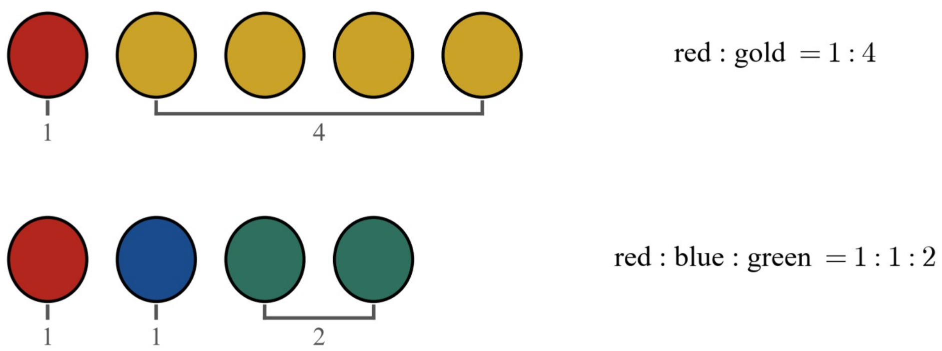
Achievement standard (extract): They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.

Theory

What this subtopic is about. A **ratio** compares two or more amounts by saying how many of one there are for every lot of another. A fruit bowl holding 6 apples and 4 oranges shows the ratio $6:4$, read aloud as "six to four". Ratios can compare counts, lengths, dollars or millilitres — anything that can be counted or measured. This subtopic teaches three habits: writing a ratio from a picture or situation, deciding whether a question wants a **part–part** comparison or a **part–whole** comparison, and using equal parts to share amounts and solve problems.

Ratios compare two or more groups. Figure 1 shows ratios built from beads, like beads threaded on a string. The top string has 1 red bead for every 4 gold beads, so red : gold $= 1:4$. The bottom string has 1 red, 1 blue and 2 green beads, so red : blue : green $= 1:1:2$. A ratio can compare any number of groups, and the order of the numbers matters: red : gold $= 1:4$ is not the same ratio as gold : red $= 4:1$.

Two bead strings and the ratios they show



Each ratio compares the numbers of beads in two or more groups.

Figure 1. Two bead strings and the ratios they show: the top string has 1 red bead then 4 gold beads, labelled red to gold equals 1 to 4; the bottom string has 1 red, 1 blue and 2 green beads, labelled red to blue to green equals 1 to 1 to 2; a caption notes that each ratio compares the numbers of beads in two or more groups.

One picture, two readings: part–part and part–whole. The same set of counters can be read two different ways. Figure 2 shows 1 red counter and 4 white counters, 5 counters in all.

- The *part–part* reading compares one group with another group: red : white $= 1:4$.
- The *part–whole* reading compares one group with the total: red is $\frac{1}{5}$ of the counters.

Telling these two readings apart is the key skill of this subtopic — it is where ratio problems are won or lost. Before calculating anything, decide: is this question comparing a part with a part, or a part with the whole?

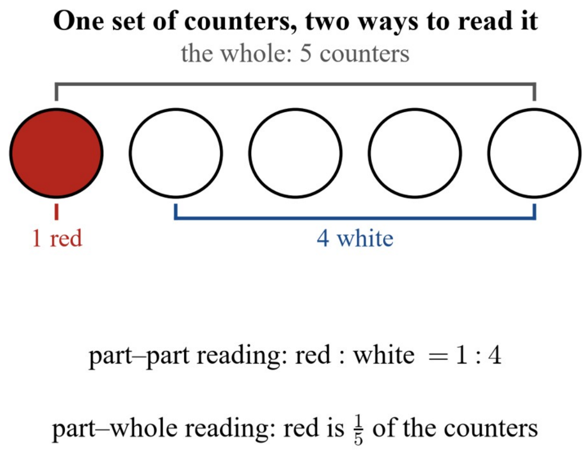


Figure 2. One set of 5 counters, 1 red and 4 white, read two ways: a part–part reading, red to white equals 1 to 4, and a part–whole reading, red is one fifth of the counters.

Tape diagrams. A tape diagram draws each group as a row of equal part-boxes, so both readings can be seen at once. In Figure 3 the red tape has 2 equal parts and the blue tape has 3, all the same size, so red : blue $= 2 : 3$. The same picture says red is $\dfrac{2}{3}$ of blue, and blue is $\dfrac{3}{2}$ — that is, $1\dfrac{1}{2}$ times — red. Figure 4 joins the tapes into one whole of 5 equal parts: red is $\dfrac{2}{5}$ of the whole and blue is $\dfrac{3}{5}$ of the whole. Writing the part–whole reading as a fraction hands the problem over to the fraction methods of 2D.

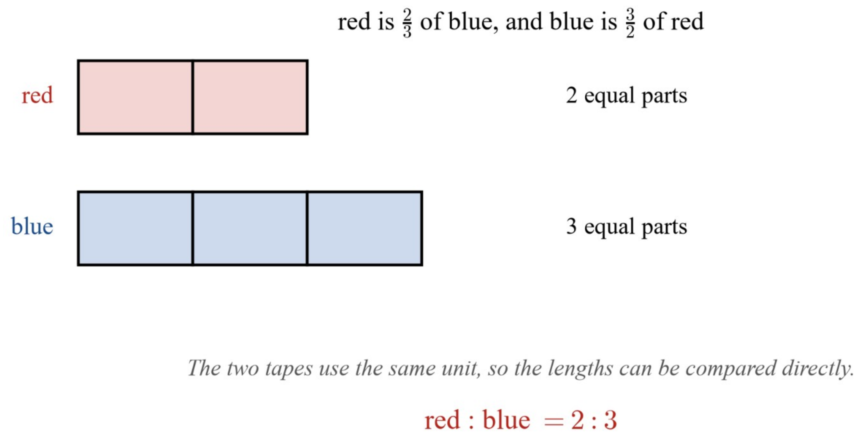


Figure 3. Two tape diagrams using the same-size parts: a red tape of 2 equal parts and a blue tape of 3 equal parts, labelled red to blue equals 2 to 3, with the notes that red is two thirds of blue and blue is three halves of red.

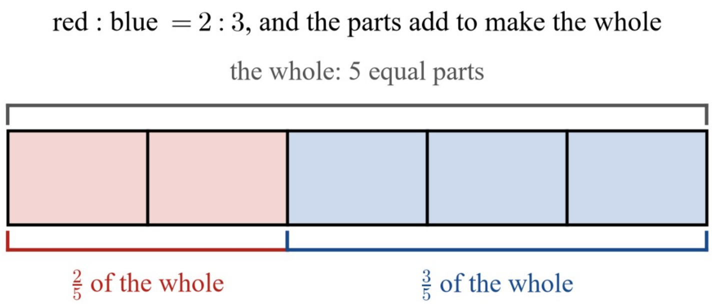


Figure 4. One tape of 5 equal parts, the first 2 red and the last 3 blue, labelled red to blue equals 2 to 3 with the parts adding to make the whole; brackets mark red as two fifths of the whole and blue as three fifths of the whole.

Sharing a total in a given ratio. When a total is shared in a ratio, the ratio numbers count the equal parts. Figure 5 shares $\$20$ between Alex and Sam in the ratio $2:3$. There are $2+3=5$ equal parts in all, and $\$20 \div 5 = \$4$ in each part, so Alex gets $2 \times \$4 = \$8$ and Sam gets $3 \times \$4 = \$12$. The check: $\$8 + \$12 = \$20$, the whole amount shared.

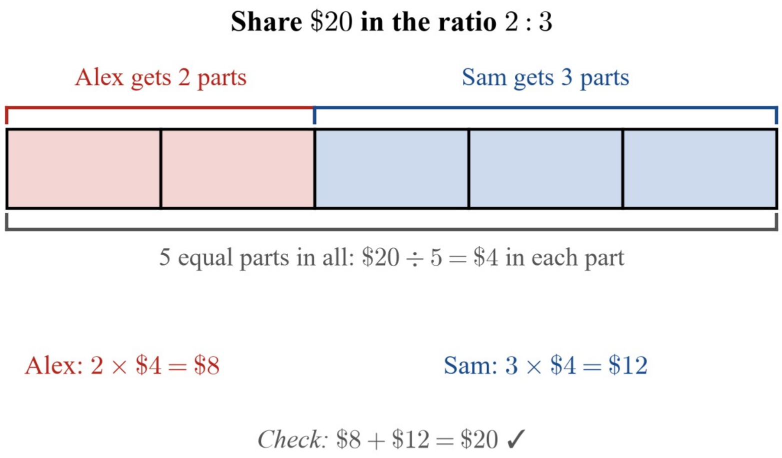


Figure 5. A tape of 5 equal parts sharing 20 dollars in the ratio 2 to 3: Alex gets 2 parts and Sam gets 3 parts, each part is 20 divided by 5 equals 4 dollars, so Alex gets 8 dollars and Sam gets 12 dollars, checked by 8 plus 12 equals 20.

Equivalent ratios and simplest form. Dividing (or multiplying) both numbers of a ratio by the same number gives an **equivalent ratio** — the same comparison written with different numbers, exactly like equivalent fractions from 2D. Figure 6 shows the same split counted two ways: $6:4$, and dividing both numbers by 2, the simpler $3:2$. A ratio is in its **simplest form** when no whole number greater than 1 divides both of its numbers, so $6:4=3:2$ and $3:2$ cannot be simplified further.

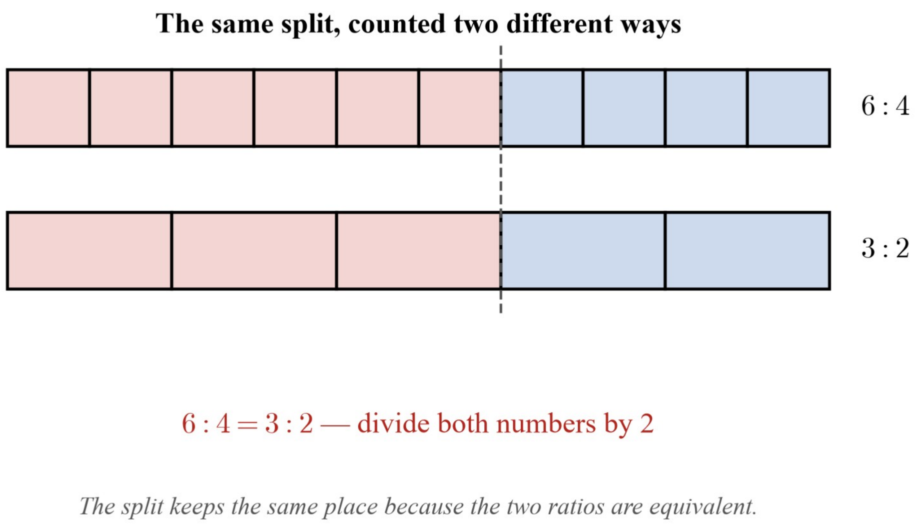


Figure 6. Two bars showing the same split counted two different ways: the top bar as 6 red parts to 4 blue parts, the bottom bar as 3 red parts to 2 blue parts, with 6 to 4 equals 3 to 2 because both numbers are divided by 2.

Ratios with three groups. A ratio can compare three or more groups. Figure 7 shows a paint mix in the ratio white : red : blue $=1:1:2$: there are $1+1+2=4$ equal parts in all, so white is $\frac{1}{4}$ of the mix, red is $\frac{1}{4}$ and blue is $\frac{2}{4}=\frac{1}{2}$ of the mix. The sharing method still works: count the parts in all, find what one part holds, then multiply.

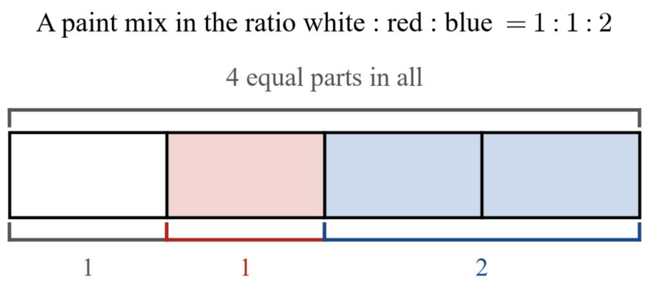


Figure 7. A paint-mix tape in the ratio white to red to blue equals 1 to 1 to 2, showing 4 equal parts in all: 1 white part, 1 red part and 2 blue parts.

Mixtures: the part–whole reading as a decimal. Mixtures are where the part–whole reading earns its keep. Figure 8 shows 500 mL of drink mixed in the ratio $1:4$ (concentrate : water). The 5 equal parts hold $500 \div 5 = 100$ mL each, so the drink is 100 mL of concentrate and 400 mL of water. As fractions of the whole drink, concentrate is $\frac{1}{5} = 0.2$ and water is $\frac{4}{5} = 0.8$ — the decimal names come straight from 2D's fraction–decimal swaps.

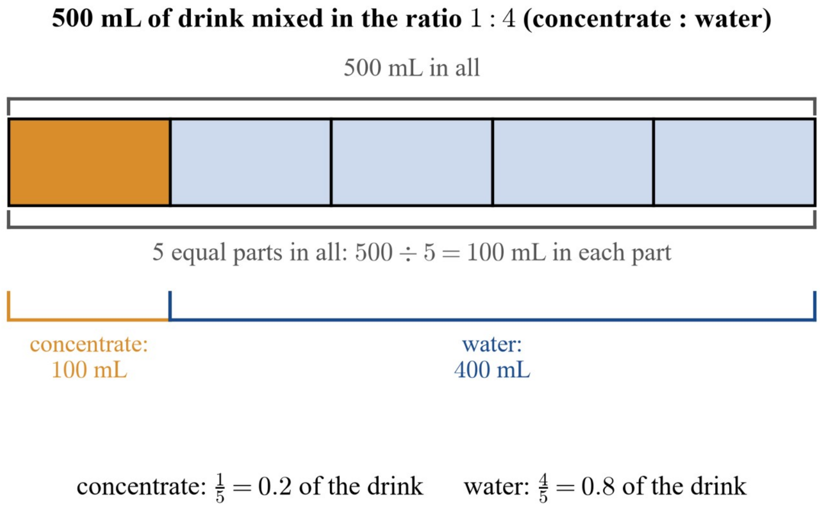


Figure 8. A tape of 5 equal parts showing 500 millilitres of drink mixed in the ratio 1 to 4, concentrate to water: each part is 500 divided by 5 equals 100 millilitres, giving 100 millilitres of concentrate, one fifth or 0.2 of the drink, and 400 millilitres of water, four fifths or 0.8 of the drink.

Scale: a ratio between a drawing and real life. A plan or map gives its **scale** as a ratio of a length on the drawing to the matching length in real life, the same unit on both sides. Figure 9 shows a scale of $1:100$: every 1 cm on the drawing stands for 100 cm in real life. Subtopic 5A meets this same notation again on maps and plans; the method of scaling a length up or down taught here is the one 5A builds on.

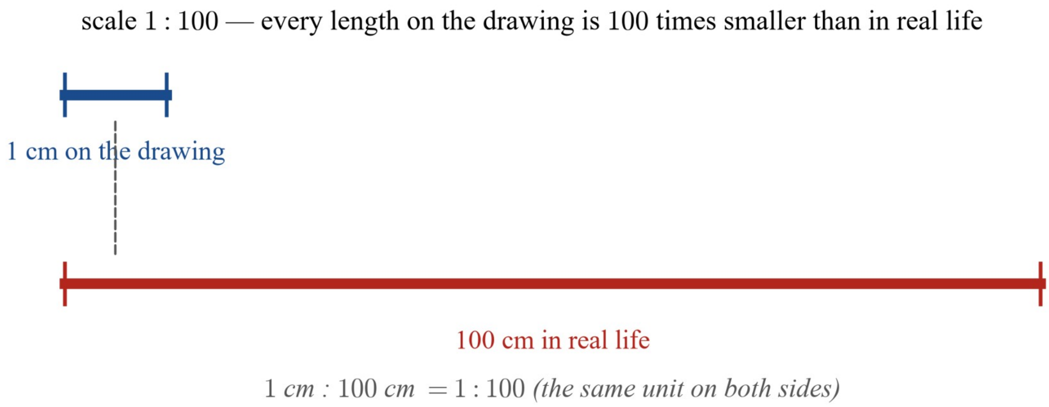


Figure 9. A scale of 1 to 100: a short line labelled 1 centimetre on the drawing matched to a long line labelled 100 centimetres in real life, so every length on the drawing is 100 times smaller than in real life.

Which reading is the question asking for? Most ratio errors come from answering the wrong question. Figure 10 sorts the three jobs a ratio question can set: comparing one part with another (write a part–part ratio), finding a part of the whole (write a fraction), and sharing a total in a given ratio (split the total into equal parts first). The worked examples and practice questions each sit under one of these three branches.

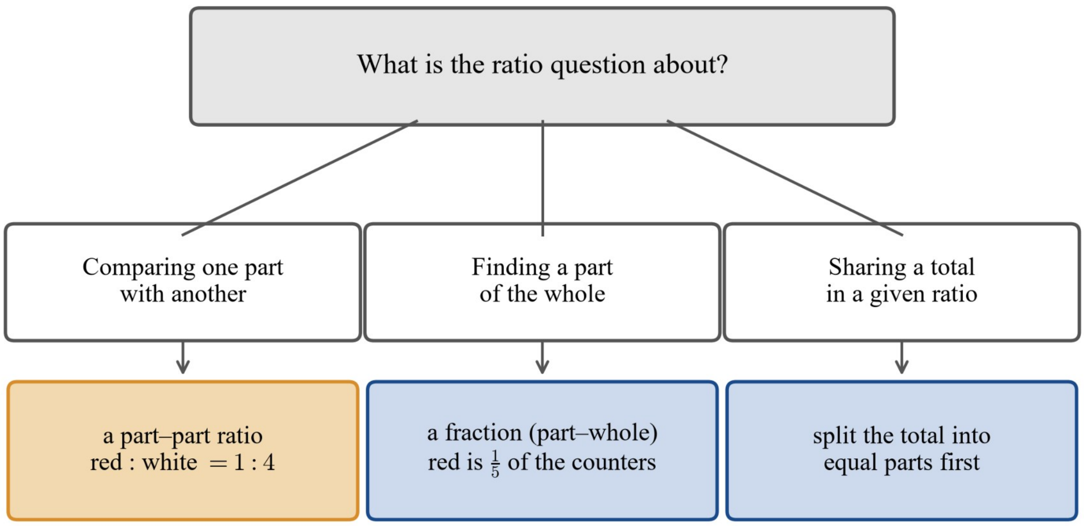


Figure 10. A decision diagram asking what the ratio question is about, with three branches: comparing one part with another leads to a part–part ratio, finding a part of the whole leads to a fraction, and sharing a total in a given ratio leads to splitting the total into equal parts first.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
ratio	a comparison of two or more amounts: how many of one for every lot of another	comparison, the : notation
part–part comparison	one group compared with another group, written as a ratio	group-to-group comparison
part–whole comparison	one group compared with the total, written as a fraction	fraction of the whole
equivalent ratios	the same comparison written with different numbers, like $6:4$ and $3:2$	equal ratios

Term	Meaning here	Synonyms and related words
simplest form	a ratio whose numbers have no whole-number factor greater than 1 in common	fully simplified
tape diagram	a bar of equal part-boxes used to picture a ratio	bar model
scale	the ratio of a drawing length to the matching real-life length	map ratio

This subtopic leans on 2D's fraction skills (simplifying, fraction–decimal swaps, fractions of amounts) and on 2B's checking habit. Ahead, 2E takes the part–whole idea into percentages, 2G uses ratios in modelling, and 5A meets the scale notation of Figure 9 again on maps and plans.

Worked examples

Example 1 — scaffolded

A fruit bowl holds 6 apples and 4 oranges. (a) Write the part–part ratio of apples to oranges in its simplest form. (b) Write the part–whole comparison: what fraction of the fruit are apples?

Working	Comment
Write the ratio. apples : oranges $= 6:4$.	The order of the words fixes the order of the numbers: apples first.
Simplify. Divide both numbers by 2: $6:4 = 3:2$.	Dividing both numbers by the same number keeps the same comparison (Figure 6).
Part–whole. Fruit in all: $6+4=10$; apples are $\frac{6}{10}=\frac{3}{5}$ of the fruit.	A part–whole comparison is a fraction: the group over the total (Figure 4).

∴ apples : oranges $= 3:2$, and apples are $\frac{3}{5}$ of the fruit.

Example 2 — scaffolded

A necklace is threaded with 20 beads in the ratio red : gold $= 1:4$, like the top string in Figure 1. How many beads of each colour are there, and what fraction of the beads are red?

Working	Comment
Count the parts. $1+4=5$ equal parts in all.	The ratio numbers count the equal parts.
One part. $20 \div 5 = 4$ beads in each part.	Sharing the total across the equal parts is a division (Figure 5).
Each colour. Red: $1 \times 4 = 4$ beads; gold: $4 \times 4 = 16$ beads.	Multiply each ratio number by the size of one part.
Part–whole. Red is $\frac{1}{5}$ of the beads.	The same reading Figure 2 shows; check $4+16=20$, the whole necklace.

∴ the necklace has 4 red and 16 gold beads, and red beads are $\frac{1}{5}$ of the whole.

Example 3 — unscaffolded

Alex and Sam share $\$20$ in the ratio $2:3$. How much does each person get?

Working	Comment
Parts in all: $2+3=5$.	The ratio numbers count equal parts of the total.
One part: $\$20 \div 5 = \$4$.	Divide the total by the number of parts.
Alex: $2 \times \$4 = \$8$. Sam: $3 \times \$4 = \$12$.	Check: $\$8+\$12=\$20$, the whole amount.

∴ Alex gets $\$8$ and Sam gets $\$12$.

Example 4 — unscaffolded

A bottle holds 500 mL of drink mixed in the ratio $1:4$ (concentrate : water). Find the volume of concentrate and of water, and write each as a fraction and a decimal of the whole drink.

Working	Comment
Parts in all: $1+4=5$; one part: $500 \div 5 = 100$ mL.	The sharing method of Example 3, with millilitres.
Concentrate: $1 \times 100 = 100$ mL; water: $4 \times 100 = 400$ mL.	Check: $100+400=500$ mL, the whole bottle.
Concentrate is $\frac{1}{5}=0.2$ of the drink; water is $\frac{4}{5}=0.8$ of the drink.	The part–whole reading, as a fraction and as a decimal (Figure 8).

∴ the bottle holds 100 mL of concentrate and 400 mL of water; concentrate is 0.2 and water is 0.8 of the drink.

Practice questions

Scaffolded

Q1. A box holds 8 red counters and 12 blue counters. (a) Write the part–part ratio red : blue in its simplest form. (b) Write the fraction of the counters that are red (the part–whole reading).

Q2. A drink is mixed in the ratio cordial : water $=1:6$. (a) How many equal parts are there in all? (b) What fraction of the drink is cordial? What fraction is water?

Q3. A grid of 5 equal squares has 2 squares shaded and 3 left unshaded. (a) Write the part–part ratio shaded : unshaded. (b) Write the fraction of the grid that is shaded.

Q4. Share each total in the given ratio, using the method of Figure 5. (a) $\$30$ in the ratio $1:2$ (b) 24 lollies in the ratio $3:5$

Q5. A bottle holds 600 mL of drink mixed in the ratio $1:5$ (concentrate : water). Find the volume of concentrate and the volume of water.

Q6. A string of 24 beads is made in the ratio red : blue : green $=1:1:2$, like the bottom string in Figure 1. How many beads of each colour are there?

Unscaffolded

Q7. Write each ratio in its simplest form. (a) $6:8$ (b) $15:10$ (c) $12:18$ (d) $4:10:2$

Q8. A class has 12 girls and 15 boys. (a) Write the ratio girls : boys in its simplest form. (b) What fraction of the class are girls?

Q9. Share $\$75$ in the ratio $2:3$.

Q10. A jar of counters has red : blue $=3:5$. There are 12 red counters. How many blue counters are there, and how many counters are there in all?

Q11. Drink A is mixed in the ratio concentrate : water $=1:4$. Drink B is mixed in the ratio concentrate : water $=2:9$. Which drink has the stronger taste of concentrate? Show the part–whole fractions you compare.

Q12. A plan is drawn at a scale of $1:50$. A wall is 3 m long in real life. How long is the wall on the plan, in centimetres? (Remember: the same unit on both sides of the ratio.)

Q13. A muffin mix uses flour : sugar : oats $= 2:3:4$. (a) One batch uses 900 g of mix altogether. Find the mass of the flour, the sugar and the oats. (b) A half batch uses 450 g of mix. Find the mass of the flour, the sugar and the oats.

Q14. Mia shares $\$48$ between herself and her brother in the ratio $1:3$, and says she gets $\$16$ and her brother gets $\$36$. Without redoing the sharing, explain why Mia's answer cannot be right, then find the correct amounts.

Answers

Q1. (a) $2:3$; (b) $\frac{8}{20}=\frac{2}{5}$.

Q2. (a) 7; (b) cordial $\frac{1}{7}$, water $\frac{6}{7}$.

Q3. (a) $2:3$; (b) $\frac{2}{5}$.

Q4. (a) $\$10$ and $\$20$; (b) 9 lollies and 15 lollies.

Q5. 100 mL of concentrate and 500 mL of water.

Q6. 6 red, 6 blue and 12 green beads.

Q7. (a) $3:4$; (b) $3:2$; (c) $2:3$; (d) $2:5:1$.

Q8. (a) $4:5$; (b) $\frac{12}{27}=\frac{4}{9}$.

Q9. $\$30$ and $\$45$.

Q10. 20 blue counters; 32 counters in all.

Q11. Drink A: $\frac{1}{5}=\frac{11}{55}$; Drink B: $\frac{2}{11}=\frac{10}{55}$; Drink A has the stronger taste.

Q12. 6 cm.

Q13. (a) 200 g of flour, 300 g of sugar, 400 g of oats; (b) 100 g of flour, 150 g of sugar, 200 g of oats.

Q14. $\$16+\$36=\$52$, not $\$48$, so the answer cannot be right; the correct amounts are $\$12$ and $\$36$.

Fully-worked solutions

Q1. (a) Divide both numbers by 4: $8:12=2:3$. (b) Counters in all: $8+12=20$; red is $\frac{8}{20}=\frac{2}{5}$ of the counters.

Q2. (a) $1+6=7$ equal parts. (b) Cordial is 1 of the 7 parts, so $\frac{1}{7}$; water is 6 of the 7 parts, so $\frac{6}{7}$.

Q3. (a) shaded : unshaded $=2:3$. (b) Squares in all: $2+3=5$; shaded is $\frac{2}{5}$ of the grid.

Q4. (a) $1+2=3$ parts; $\$30\div3=\$10$ a part; the shares are $1\times\$10=\$10$ and $2\times\$10=\$20$. (b) $3+5=8$ parts; $24\div8=3$ lollies a part; the shares are $3\times3=9$ and $5\times3=15$ lollies.

Q5. $1+5=6$ parts; $600\div6=100$ mL a part; concentrate $1\times100=100$ mL; water $5\times100=500$ mL. Check: $100+500=600$ mL.

Q6. $1+1+2=4$ parts; $24\div4=6$ beads a part; red $1\times6=6$, blue $1\times6=6$, green $2\times6=12$. Check: $6+6+12=24$ beads.

Q7. (a) Divide by 2: $6:8=3:4$. (b) Divide by 5: $15:10=3:2$. (c) Divide by 6: $12:18=2:3$. (d) Divide by 2: $4:10:2=2:5:1$.

Q8. (a) Divide both numbers by 3: girls : boys $=4:5$. (b) Students in all: $12+15=27$; girls are $\frac{12}{27}=\frac{4}{9}$ of the class.

Q9. $2+3=5$ parts; $\$75\div5=\$15$ a part; the shares are $2\times\$15=\$30$ and $3\times\$15=\$45$. Check: $\$30+\$45=\$75$
.

Q10. Red is 3 parts and holds 12 counters, so one part is $12\div3=4$ counters. Blue is 5 parts: $5\times4=20$ blue counters. In all: $12+20=32$ counters.

Q11. Compare the part–whole fractions of concentrate. Drink A: $\frac{1}{5}=\frac{11}{55}$. Drink B: $\frac{2}{11}=\frac{10}{55}$. Since $\frac{11}{55}>\frac{10}{55}$, Drink A has the stronger taste of concentrate.

Q12. Work in centimetres: 3 m $=300$ cm. The scale $1:50$ means the drawing length is 50 times smaller: $300\div50=6$ cm on the plan.

Q13. (a) $2+3+4=9$ parts; $900\div9=100$ g a part; flour $2\times100=200$ g, sugar $3\times100=300$ g, oats $4\times100=400$ g. (b) $450\div9=50$ g a part; flour 100 g, sugar 150 g, oats 200 g.

Q14. A check on the total: $\$16+\$36=\$52$, which is more than the $\$48$ shared, so Mia's answer cannot be right. Correctly: $1+3=4$ parts; $\$48\div4=\$12$ a part; Mia gets $1\times\$12=\$12$ and her brother gets $3\times\$12=\$36$. Check: $\$12+\$36=\$48$.

Teacher note

- **Ownership and the part–part/part–whole split.** Per the Chapter 2 plan in TOC §3, this subtopic (`VC2M7N09`) names the part–part versus part–whole distinction explicitly at first use (Figures 2–4, the words table, Figure 10), because that distinction is where ratio problems are lost. Equivalent-ratio simplification leans on 2D's equivalent fractions and is not re-taught from scratch; percentages stay with 2E, which takes the part–whole idea further.
- **Rates excluded (D4.8).** Rates and speed (e.g. km/h, $/kg) are Level 8 and do not appear here; every comparison in this subtopic is a ratio between like quantities or a part–whole fraction.
- **Positive rationals only (D4.4).** Every number in this subtopic is positive; no negatives appear.
- **Scale notation (P6).** Figure 9 introduces the $1:100$ notation with the same-unit convention; 5A cross-references this subtopic for the notation, and the scaling technique (multiply or divide both sides by the same number) remains 2F's and 4F's to teach.
- **Elaboration coverage.** E01 → Figures 1–2, WE2, Q6 (beads and counters; $1:4$ and $1:1:2$); E02 → Figures 3–4, WE1, Q1–Q3, Q8 (fractions for ratio problems; part–part and part–whole; the $1:2$ → 3-parts idea generalised); E03 → Figure 5, WE3, Q4, Q9 (sharing $\$20$ in $2:3$ verbatim in WE3); E04 → Figure 8, WE4, Q5, Q11 (500 mL at $1:4$, with the 0.2/0.8 decimal readings and interpretation in context).
- **Reasonableness (P7).** Q14 asks whether an answer is reasonable before asking for the correct one, per TOC §5.2; WE1–WE4 and the sharing solutions model the add-back check.
- **Contexts are transparently fictional.** Fruit bowls, bead necklaces, cordial, paint mixes and plan scales are generic settings; no real data is asserted, so no source lines are required under SOURCE_USE.
- **Digital tools (D9).** E01 names diagrams and physical or virtual materials (counters, beads), so virtual manipulatives are permitted where appropriate. No CAS.
- **Figures.** All ten figures are generated by `src/gen_2f.py` from the exact values stated in the text; tapes, brackets and labels compute from those values.
- **No D11 content.** None of `VC2M7N09.E01–.E04` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.

Modelling with rational numbers and percentages, including best buys

Content description `VC2M7N10`: use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts such as 'best buys'; formulate problems, choosing representations and efficient calculation strategies, designing algorithms and using digital tools as appropriate; interpret and communicate solutions in terms of the situation, justifying choices made about the representation

Achievement standard (extract): They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.

Theory

What a model is. When maths is used on a real situation, the numbers stand for something: floors of a building, metres above or below the sea, dollars in a money box, millilitres in a bottle. A **model** is the maths we choose to stand for the situation — a number line, a diagram, a table, a bar chart or a set of steps. **Modelling** is the whole job: turn the situation into maths, do the maths, then turn the answer back into words about the situation. A calculation only counts as modelling once the answer has been said back in terms of the situation it came from.

The four steps. This subtopic runs the same four steps every time, in a cycle (Figure 1):

1. **Formulate** the problem — say exactly what question is being asked, and pick out the information that matters.
2. **Choose** a representation and an efficient calculation strategy.
3. **Solve —** carry out the calculation.
4. **Interpret and communicate** — say what the answer means in the real situation, and justify the choice of representation.

If the answer does not make sense in the situation, the cycle is run again from step 1 with a better model.

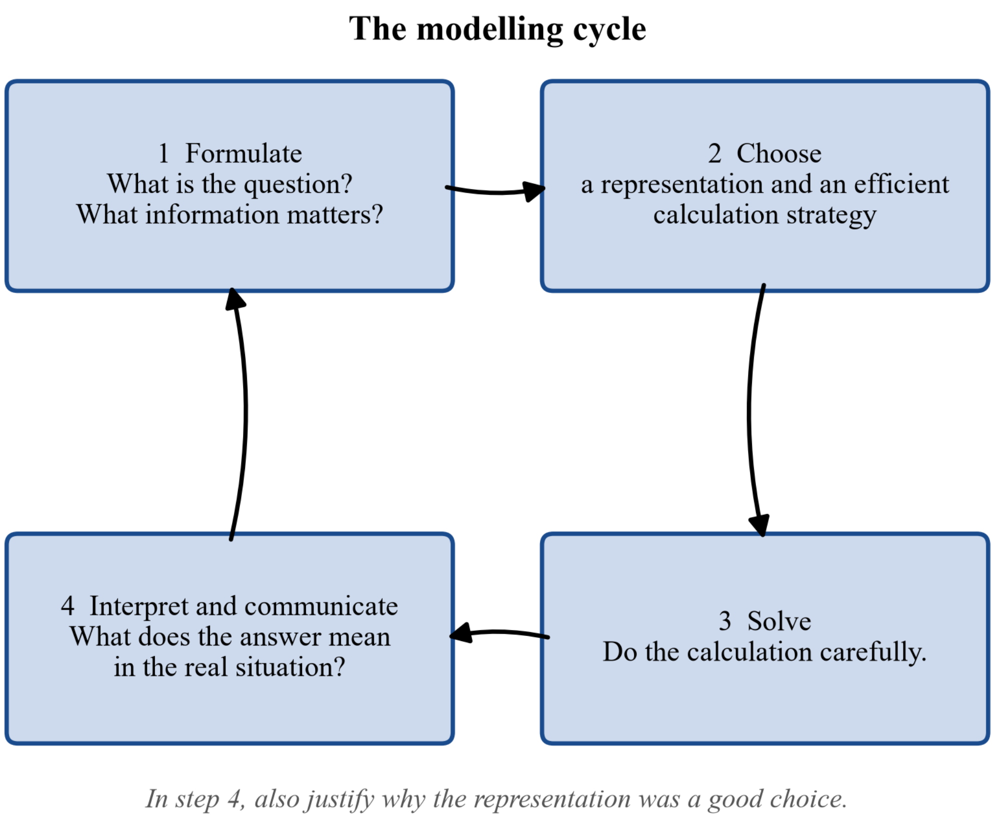


Figure 1. The modelling cycle: step 1 formulate, step 2 choose a representation and a strategy, step 3 solve, step 4 interpret and communicate, with arrows joining the steps in a loop and a note that step 4 also justifies the choice of representation.

Additive models: up and down. The integers from subtopic 1A do the work here. In a building, the ground floor is 0; floors above it are positive and floors below it are negative, so going down is subtraction and going up is addition. Heights work the same way with sea level as zero: above sea level is positive, below is negative. Figure 2 shows one journey: a lift starts on floor 3, goes down 6 floors to $3-6=-3$, then up 5 floors to $-3+5=2$. The model is the integer sum $3-6+5=2$, and the answer is read back into the building: the lift finishes on floor 2, two floors above the ground floor.

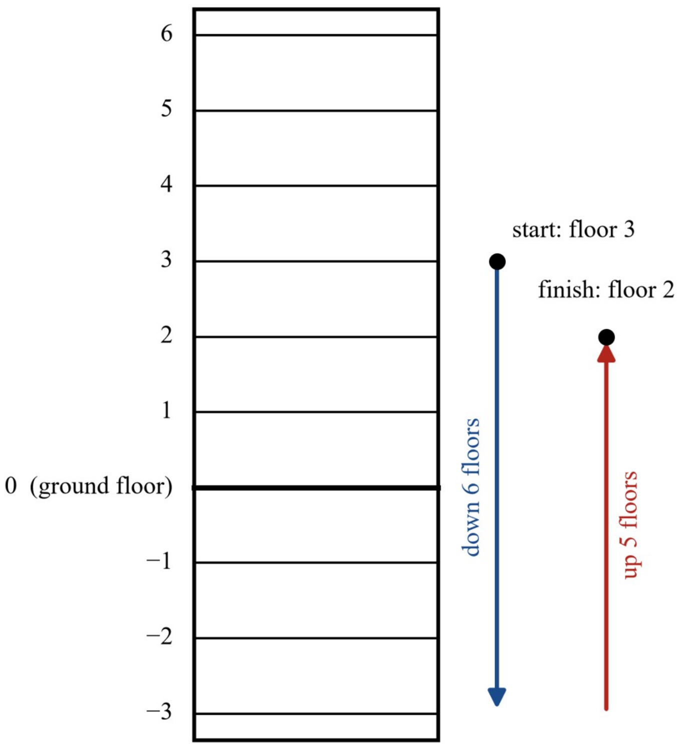


Figure 2. A building with floors −3 to 6 and the ground floor marked 0; a lift journey from floor 3 down 6 floors to −3, then up 5 floors to floor 2, drawn as two arrowed lines beside the building.

Percentage models. Subtopic 2E gives the two percentage jobs used here: find a percentage *of* a quantity, and write one quantity *as a percentage of* another. A bar split into parts is a useful representation. In Figure 3 the whole bar stands for all 200 students in Year 7; 45 % of them, 90 students, are shaded, and the rest, 55 %, is 110 students. The numbers come from 10 % of 200 = 20, so 40 % is 80 and 5 % is 10, giving 80 + 10 = 90. The answer "90 students" is a count of people, not a bare number — saying that is the interpret step.

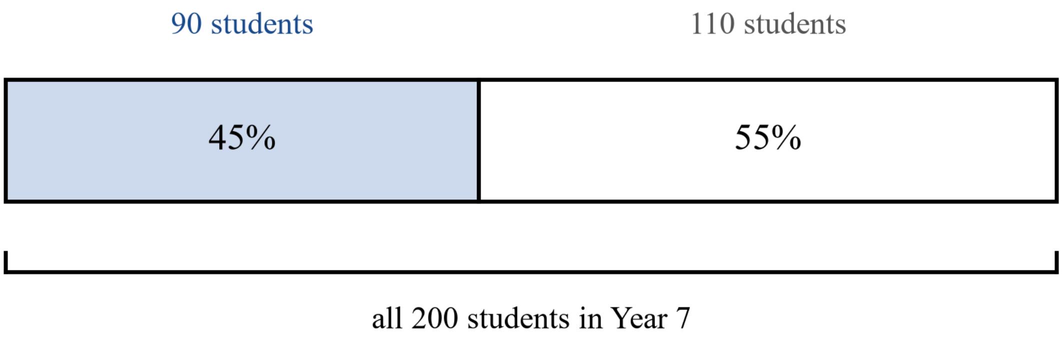


Figure 3. A bar split into two parts: 45% (90 students) shaded and 55% (110 students) unshaded, with a bracket underneath marking all 200 students in Year 7.

Money models: profit and loss. A stall or an event has money **spent** to set it up and money **taken** on the day. If more is taken than spent, the difference is a **profit**; if less is taken than spent, the difference is a **loss**. Writing the profit or loss as a percentage of the money spent is the second percentage job above — one quantity as a percentage of another. Figure 4 shows a stall that spent \$160 and took \$400: the profit is 400 − 160 = \$240, and 240 ÷ 160 = 1.5, so the profit is 150 % of the money spent.

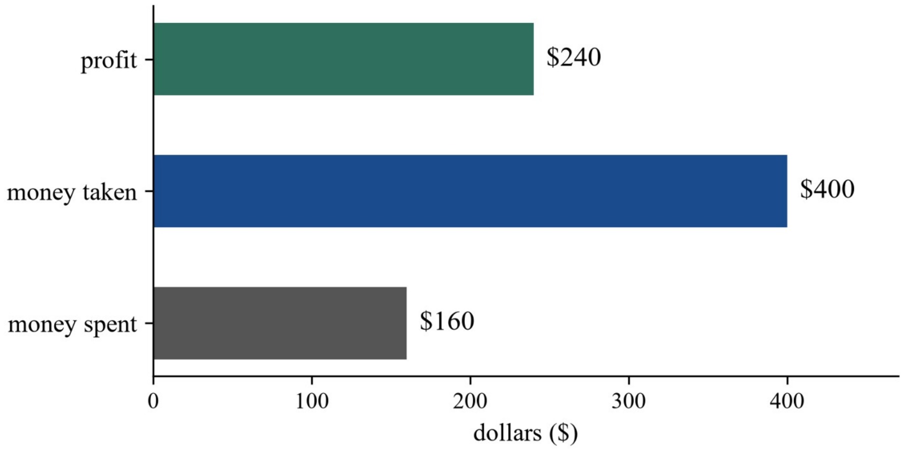


Figure 4. Bar chart with three bars: money spent $160, money taken $400 and profit $240.

Best buys. In this subtopic, the **better buy** is the option with the lower cost for the same amount — its **unit price**. Unit prices are found by division: cost per 100 g, per 100 mL, per litre or per item. Options can only be compared like with like, so sizes are first written in the same unit (2 L $=2000$ mL, a conversion assumed from Level 6). Figure 5 compares two bottles of the same juice: 600 mL for $\$2.10$ and 2 L for $\$6.00$. The cost per 100 mL is $2.10\div 6=\$0.35$ for the small bottle and $6.00\div 20=\$0.30$ for the large one, so the 2 L bottle is the better buy. Worked example 2 shows this comparison in full.

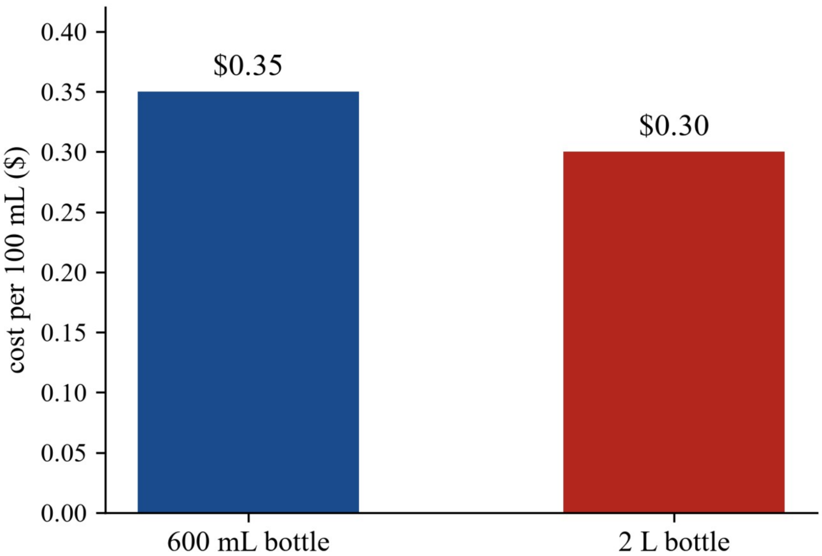


Figure 5. Bar chart of the cost per 100 mL of two bottles of juice: the 600 mL bottle at $0.35 and the 2 L bottle at $0.30, so the 2 L bottle is the better buy.

Algorithms and loop structures. An **algorithm** is a set of steps for doing a calculation that always finishes and gives an answer. A **loop** is a part of an algorithm that sends the run back to an earlier step, so the same steps repeat. Figure 6 is a flow chart for an algorithm that adds the counting numbers from 1 up to a chosen number n: with $n=4$ the loop runs four times and total becomes $1+2+3+4=10$. The same few steps add the numbers up to any n, which is exactly why a loop is useful — and a spreadsheet or a computer will run the loop as often as asked, where a digital tool is appropriate. Worked example 4 runs this algorithm by hand.

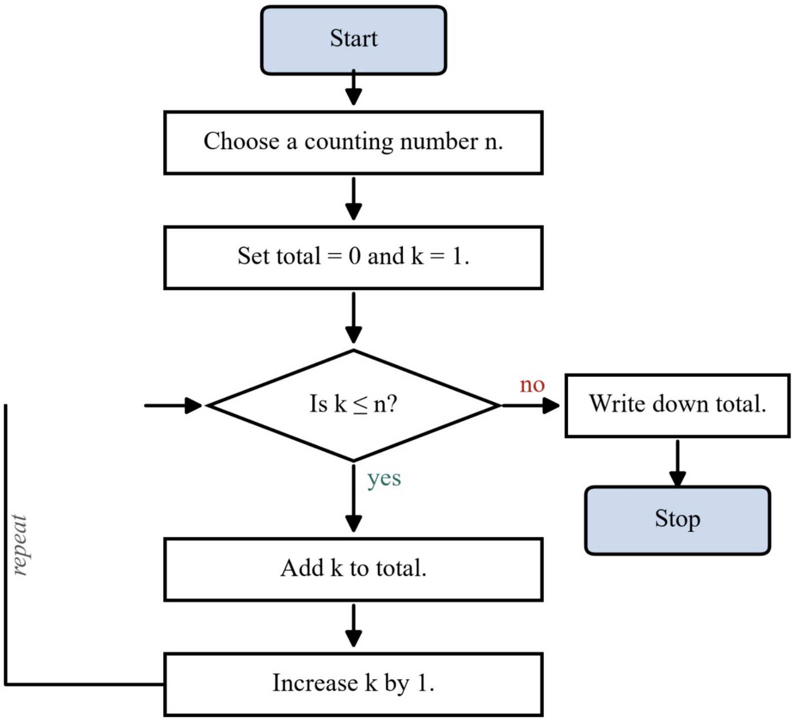


Figure 6. Flow chart of the loop algorithm: start; choose a counting number n; set total = 0 and k = 1; decision "Is k ≤ n?"; yes adds k to total, increases k by 1 and repeats; no writes down total and stops.

Geometric patterns with digital tools. Some models are pictures rather than numbers. A honeycomb pattern is built by repeating one hexagon, and dynamic geometry software draws each new hexagon exactly on the edges of the last. Figure 7 shows the start of one such pattern: Figure 1 has 1 hexagon, Figure 2 has 3, Figure 3 has 5, and each new figure adds 2 more. The pattern has a rule — hexagons $= 2 \times$ (figure number) $- 1$ — and the rule is the model: it answers questions about figures that have not been drawn.

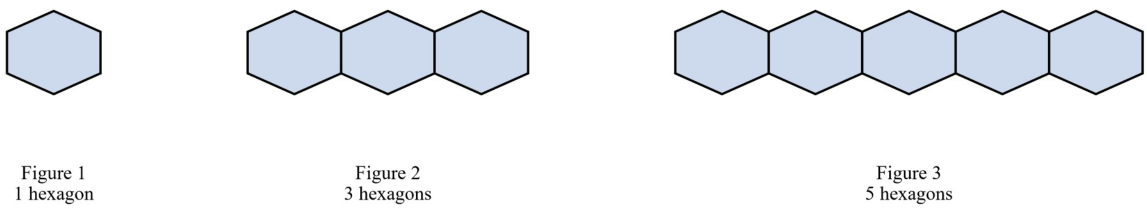


Figure 7. A honeycomb pattern in three growing figures of 1, 3 and 5 hexagons, each new figure adding two hexagons to the row.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
model, modelling	the maths chosen to stand for a	representation, picture of the

Term	Meaning here	Synonyms and related words
	situation; the four-step job of using it	situation
formulate	say the situation as a maths question and pick out the information that matters	set up, write as a number sentence
representation	the number line, diagram, table, bar chart or steps chosen in step 2	picture, diagram, chart
unit price	the cost of one fixed amount: per 100 g, per 100 mL, per litre, per item	cost per unit
better buy	the option with the lower unit price, unless the situation says otherwise	best value
profit	money taken minus money spent, when the result is positive	gain
loss	the gap when money taken is less than money spent	shortfall
algorithm	a set of steps for a calculation that always finishes	recipe, method, flow chart
loop	the part of an algorithm that repeats earlier steps	repeat

Modelling leans on the arithmetic of the rest of Chapter 2: percentages come from 2E, fraction and decimal calculation from 2C and 2D, and the integers from 1A. The ratio work of 2F and the modelling task I2 use the same four steps in their own situations.

Worked examples

Example 1 — scaffolded

A lift in a tall building starts on floor 3. It travels down 6 floors, then up 5 floors. The ground floor is 0, and floors below the ground are negative. Which floor does the lift finish on?

Working	Comment
Formulate. The question is "which floor does the lift finish on?" The information that matters: start 3, down 6, up 5, ground $=0$.	Floors are integers (1A): up is addition, down is subtraction.
Choose. Model the journey as one sum: $3-6+5$. A sketch of the building (Figure 2) is the representation.	The sketch keeps the below-ground floors visible, so a negative answer cannot be missed.
Solve. $3-6=-3$, then $-3+5=2$.	Work left to right, one step at a time.
Interpret. The lift finishes on floor 2.	Say the answer back into the building: two floors above the ground floor. It passed through -3 on the way, which the building allows, so the answer is sensible.

$\therefore$ the lift finishes on floor 2.

Example 2 — scaffolded

The canteen buys the same juice in two bottle sizes: 600 mL for $\$2.10$, and 2 L for $\$6.00$. Which size is the better buy?

Working	Comment
Formulate. The question is "which bottle costs less for	The two prices cannot be compared as they stand — the

Working	Comment
the same amount of juice?" Information: 600 mL for $\$2.10$; 2 L for $\$6.00$.	bottles hold different amounts.
Choose. Write both sizes in mL: 2 L $=2000$ mL. Then find each cost per 100 mL.	Like with like: the unit price (cost per 100 mL) makes the options comparable. The L-to-mL step is unit conversion from Level 6.
Solve. Small bottle: $2.10 \div 6 = \$0.35$ per 100 mL. Large bottle: $6.00 \div 20 = \$0.30$ per 100 mL.	600 mL holds six lots of 100 mL; 2000 mL holds twenty.
Interpret. The 2 L bottle is the better buy: it costs 5 c less for every 100 mL (Figure 5).	Justifying the representation: prices per 100 mL give a fair comparison, because each is the price of the same amount.

∴ the 2 L bottle is the better buy.

Example 3 — unscaffolded

The Year 7 cake stall spends $\$160$ on ingredients, trays and packaging, and takes $\$400$ in sales on the day. (a) Did the stall make a profit or a loss? Find the amount. (b) Write the profit or loss as a percentage of the money spent. (c) In one sentence, say what your percentage means for the stall.

Working	Comment
(a) Money taken − money spent $=400-160=\$240$. The result is positive, so it is a **profit** of $\$240$.	A loss would show as money taken less than money spent.
(b) $240 \div 160 = 1.5 = 150\%$. The profit is 150% of the money spent.	This is one quantity written as a percentage of another (2E).
(c) For every $\$100$ the stall spent, it made $\$150$ more than it spent.	The percentage is said back in terms of the stall, not left as 1.5.

∴ the stall made a profit of $\$240$, which is 150% of the money spent.

Example 4 — unscaffolded

Run the algorithm of Figure 6 with $n=6$. Keep a table of each pass of the loop, and write down the number the algorithm gives at the end.

Set total $=0$ and $k=1$. Each pass adds k to total, then increases k by 1; the loop runs while $k \le 6$.

Pass	k added	total after the pass
1	1	1
2	2	3
3	3	6
4	4	10
5	5	15
6	6	21

After pass 6, k becomes 7, which is not ≤ 6, so the loop ends and the algorithm writes down 21. **Interpret:** the algorithm has added the six counting numbers, $1+2+3+4+5+6=21$ — the same sum the loop built one step at a time.

∴ with $n=6$ the algorithm gives 21.

Practice questions

Scaffolded

Q1. The four steps of the modelling cycle are listed out of order: (i) solve; (ii) formulate the problem; (iii) interpret and communicate the answer; (iv) choose a representation and a calculation strategy. (a) Write the steps in the order the cycle runs them. (b) Name two representations a model could use.

Q2. A lift starts on floor 2. It travels down 5 floors, then up 7 floors. The ground floor is 0 and floors below the ground are negative. (a) Write the journey as one number sentence. (b) Find the floor the lift finishes on. (c) How many floors did the lift travel in total? (d) In one sentence, say where the lift finishes, in terms of the building.

Q3. The top of a coastal cliff is 40 m above sea level, and the sea floor at the base of the cliff is 25 m below sea level (Figure 8). (a) Write the two heights as integers, with sea level as 0. (b) Find the difference in height between the cliff top and the sea floor. (c) In one sentence, interpret your answer in terms of the cliff.

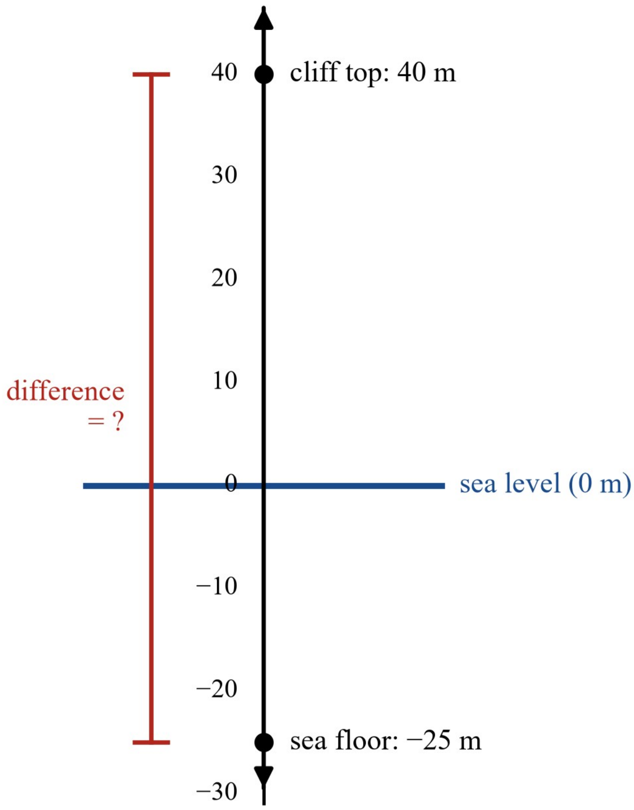


Figure 8. A vertical number line with sea level marked 0 m; the cliff top is a point at 40 m and the sea floor a point at −25 m, with a bracket between them marking the difference to find.

Q4. There are 240 students in Year 7 at a school, and 35% of them take part in the school concert. Find the number of students who take part.

Q5. In a vote about a new school uniform, 27 of the 60 students in Year 7 voted ‘yes’. Write the ‘yes’ vote as a percentage of the year level.

Q6. The canteen sells the same breakfast cereal in two box sizes: 375 g for $\$4.50$ and 550 g for $\$6.05$. (a) Find the cost per 100 g of each box. (b) Which box is the better buy?

Unscaffolded

Q7. Apple juice is sold as a 1.5 L bottle for $\$3.60$ and as a 750 mL bottle for $\$2.25$. (a) Write 1.5 L in mL. (b) Find the cost per 100 mL of each bottle. (c) Which bottle is the better buy, and by how much per 100 mL?

Q8. In Australia, the GST (Goods and Services Tax) is 10% of the price of an item. Find the GST on a bicycle priced at $\$850$.

Q9. The student council runs a drink stall at the sports day. It spends $\$40$ on cups and fruit and takes $\$34$ in sales. (a) Did the stall make a profit or a loss? Find the amount. (b) Write the profit or loss as a percentage of the money spent.

Q10. Use the algorithm of Figure 6. (a) Copy and complete this table for a run with $n=4$.

Pass	k added	total after the pass
1	1	
2		3
3	3	
4		

(b) Run the algorithm with $n=10$ and write down the answer.

(c) The algorithm gives 5050 when $n=100$. In one sentence, say why the loop makes this possible without writing 100 new steps.

Q11. The honeycomb pattern of Figure 7 grows by 2 hexagons each figure: 1, 3, 5, and so on. (a) How many hexagons are in Figure 4? (b) How many hexagons are in Figure 10? (c) Could any figure of this pattern have exactly 50 hexagons? Give a reason.

Q12. Sam finds the cost per 100 g of a 500 g bag of flour priced at $\$1.80$, and gets $\$3.60$ per 100 g. Without redoing the calculation, explain why Sam's answer cannot be right, then find a sensible value.

Q13. The canteen needs at least 8 L of apple juice for the sports day. The store sells the juice in 2 L bottles for $\$5.00$ and in 3 L bottles for $\$8.40$; bottles cannot be split. (a) Find the cost per litre of each bottle size. (b) Which size is the better buy? (c) List the sensible ways to buy at least 8 L, cost each one, and find the cheapest. (d) In one sentence, say why the unit price alone was not enough to choose the cheapest way.

Q14. A car park has floors -3 to 4, with the ground floor 0. A car enters at the ground floor, drives up 3 floors to park, and later the driver walks down 6 floors to the exit lift. (a) Write the journey as one number sentence and find the floor of the exit lift. (b) How many floors did the car and the driver travel in total? (c) A second car parks on floor -1 and its driver takes a lift up 5 floors to the shops. Which floor are the shops on?

Answers

Q1. (a) (ii), (iv), (i), (iii); (b) any two of: number line, diagram or sketch, table, bar chart, list of steps.

Q2. (a) $2-5+7$; (b) 4; (c) 12 floors; (d) the lift finishes on floor 4, four floors above the ground floor.

Q3. (a) 40 and -25; (b) 65 m; (c) the cliff top is 65 m higher than the sea floor at its base.

Q4. 84 students.

Q5. 45%.

Q6. (a) $\$1.20$ per 100 g and $\$1.10$ per 100 g; (b) the 550 g box.

Q7. (a) 1500 mL; (b) $\$0.24$ and $\$0.30$ per 100 mL; (c) the 1.5 L bottle, by 6 c per 100 mL.

Q8. $\$85$.

Q9. (a) a loss of $\$6$; (b) 15%.

Q10. (a) totals $1,3,6,10$ (k added $1,2,3,4$); (b) 55; (c) the loop repeats the same two steps 100 times, so no new steps are needed for a bigger n.

Q11. (a) 7; (b) 19; (c) no — the counts are $1,3,5,7,\ldots$, all odd, and 50 is even.

Q12. 100 g is part of 500 g, so its cost must be less than $\$1.80$; a sensible value is $1.80\div 5=\$0.36$ per 100 g.

Q13. (a) $\$2.50$ and $\$2.80$ per litre; (b) the 2 L bottle; (c) 4×2 L $=\$20.00$ is cheapest (others: 2×3 L $+1\times 2$ L $=\$21.80$; 3×3 L $=\$25.20$; 1×3 L $+3\times 2$ L $=\$23.40$); (d) bottles come whole, so the cheapest plan must be checked against the total cost of reaching at least 8 L, not just the unit price.

Q14. (a) $0+3-6=-3$, floor -3; (b) 9 floors; (c) floor 4.

Fully-worked solutions

Q1. (a) The cycle of Figure 1 runs formulate → choose → solve → interpret and communicate, so the order is (ii), (iv), (i), (iii). (b) A model may be drawn or tabulated: a number line, a sketch of the situation, a table, a bar chart or a set of steps all serve as representations.

Q2. (a) Down is subtraction and up is addition, so the journey is $2-5+7$. (b) Work left to right: $2-5=-3$, then $-3+7=4$; the lift finishes on floor 4. (c) Distance travelled ignores direction: $5+7=12$ floors. (d) Floor 4 is four floors above the ground floor, so the lift ends above ground even though it passed below it.

Q3. (a) With sea level 0: cliff top 40, sea floor -25. (b) The difference is $40-(-25)=40+25=65$, so 65 m. (c) Read back into the cliff: its top stands 65 m above the sea floor at its base.

Q4. 10% of 240 is 24, so 30% is 72 and 5% is 12; $72+12=84$. So 84 students take part — a count of people, not a bare 84.

Q5. $\frac{27}{60}=\frac{45}{100}=45\%$: the 'yes' vote is 45% of the year level.

Q6. (a) 375 g holds 3.75 lots of 100 g: $4.50\div 3.75=\$1.20$ per 100 g. 550 g holds 5.5 lots: $6.05\div 5.5=\$1.10$ per 100 g. (b) $\$1.10<\$1.20$, so the 550 g box is the better buy.

Q7. (a) 1.5 L $=1500$ mL. (b) 1500 mL holds fifteen lots of 100 mL: $3.60\div 15=\$0.24$; 750 mL holds 7.5 lots: $2.25\div 7.5=\$0.30$. (c) The 1.5 L bottle is the better buy, cheaper by $0.30-0.24=\$0.06$, or 6 c, per 100 mL.

Q8. 10% of $\$850$ is $850\div 10=\$85$. The GST is $\$85$.

Q9. (a) Money taken is less than money spent: $40-34=\$6$, a **loss** of $\$6$. (b) $6\div40=0.15=15\%$: the loss is 15% of the money spent.

Q10. (a) Pass 1 adds 1: total 1. Pass 2 adds 2: total 3. Pass 3 adds 3: total 6. Pass 4 adds 4: total 10; then $k=5>4$ and the algorithm writes down 10. (b) With $n=10$ the loop adds $1+2+3+4+5+6+7+8+9+10=55$. (c) The loop sends the run back to the same two steps, so $n=100$ needs no new ideas — the steps repeat 100 times and give 5050.

Q11. (a) Figure 4 has $5+2=7$ hexagons. (b) Figure 10 has $2\times10-1=19$ hexagons. (c) The counts run $1,3,5,7,\ldots$ — each is $2\times$ (figure number) -1, an odd number — and 50 is even, so no figure has exactly 50 hexagons.

Q12. 100 g is only part of the 500 g bag, so 100 g must cost *less* than the whole bag's $\$1.80$; $\$3.60$ is more than the bag itself, so it cannot be right. A sensible value: 500 g holds five lots of 100 g, so $1.80\div5=\$0.36$ per 100 g.

Q13. (a) 2 L bottle: $5.00\div2=\$2.50$ per litre. 3 L bottle: $8.40\div3=\$2.80$ per litre. (b) The 2 L bottle is the better buy. (c) Ways to reach at least 8 L: 4×2 L gives 8 L for $4\times5.00=\$20.00$; 2×3 L $+1\times2$ L gives 8 L for $2\times8.40+5.00=\$21.80$; 1×3 L $+3\times2$ L gives 9 L for $8.40+3\times5.00=\$23.40$; 3×3 L gives 9 L for $3\times8.40=\$25.20$. The cheapest is 4×2 L at $\$20.00$. (d) Bottles are sold whole, so plans overshoot 8 L by different amounts — the total cost of each whole-bottle plan, not the unit price alone, decides the cheapest.

Q14. (a) $0+3-6=-3$: the exit lift is on floor -3, the lowest car-park floor. (b) $3+6=9$ floors in total. (c) $-1+5=4$: the shops are on floor 4.

Teacher note

- **Modelling cycle (D8).** Every worked example and the modelling questions run the full cycle — formulate, choose, solve, interpret and communicate with the choice of representation justified. An item answered with a bare number has not delivered `VC2M7N10`; the (c)/(d) interpret parts carry that intent in the practice set. T2 awards 2G's 5 marks on this basis, and task I2 builds on the same cycle with a real context.
- **Algorithm (D8).** The loop algorithm of Figure 6 is the `VC2M7N10.E04` vehicle, written as numbered steps and a flow chart, not in a named programming language. Q10 asks students to run it by hand and to see why the loop generalises.
- **Digital tools (D9).** Permitted where appropriate, not compulsory: a spreadsheet runs the loop algorithm at $n=100$, and dynamic geometry software builds the honeycomb of Figure 7. No CAS.
- **Unit conversion (P1).** The best-buy items (WE2, Q7, Q13) convert L ↔ mL and work with 100 g lots; conversion is assumed from `VC2M6M01` and is maintained inside these items, not taught as new content.
- **Level boundaries (D4).** Percentages appear only as a percentage *of* a quantity and one quantity *as a percentage of* another (`VC2M7N07`); there is no percentage increase or decrease here (Level 8). GST in Q8 is therefore "10% of the price", not "add 10%". Integers are used additively only, per `VC2M7N08`.
- **`VC2M7N10.E06` is not taught in this subtopic**: Aboriginal and Torres Strait Islander content is excluded from ClassBasics books by the content policy; the mathematics of proportional comparison is delivered through the other elaborations instead.

Test

Chapter 2 test (T2) — Number: rational number, percentage and ratio

Codes: `VC2M7N03` (2A) · `VC2M7N04` (2B) · `VC2M7N05` (2C) · `VC2M7N06` (2D) · `VC2M7N07` (2E) · `VC2M7N09` (2F) · `VC2M7N10` (2G) · **Marks:** 35 · **Guided duration:** 40 min (35 min working, plus 5 minutes reading)

This is a classroom check, not an exam. There is no strict time limit; the 40 minutes is a guide (about 1 minute per mark, plus 5 minutes reading). If your teacher splits the paper into two sittings, the first sitting is Q1–Q12 (subtopics 2A–2D) and the second is Q13–Q20 (subtopics 2E–2G). Answer every question, and show your working wherever a question asks you to show, explain, compare, check or draw.

Equipment: a ruler, for the number-line question (Q2).

Calculators: you may use a scientific calculator, except on questions marked **(no calculator)**. Those questions come from subtopic 2E, whose content description requires the work both with and without digital tools. No question needs a calculator — every calculation here can be done by hand — and the marks go to your working, not just to the final answer.

Q1. (2 marks)

(a) Write $\frac{3}{8}$ as a decimal and as a percentage. (1 mark)

(b) Write 55% as a fraction in its simplest form. (1 mark)

Q2. (2 marks) Use your ruler to draw a number line from -1 to 2, with a tick at every quarter.

(a) Mark $-\frac{3}{4}$ on your line. (1 mark)

(b) Mark $\frac{7}{4}$ on your line. (1 mark)

Q3. (1 mark) Write <, > or = in the box.

$-0.85 \square -\frac{5}{6}$

Q4. (2 marks)

(a) Round 6.95 to one decimal place. (1 mark)

(b) Round 8.5 to the nearest whole number. (1 mark)

Q5. (1 mark) Round each number to the nearest whole number first, then estimate the sum $4.92+3.87$. Show your rounded numbers.

Q6. (2 marks) The drink stall at the sports day needs exactly 3.1 L of juice. Juice is sold only in 1 L bottles.

(a) Rounding 3.1 to the nearest whole number gives 3. Explain why 3 L is not enough. (1 mark)

(b) What is the smallest number of bottles the stall can buy, and how much juice will be left over? (1 mark)

Q7. (2 marks) Find each product. Simplify before you multiply where you can.

(a) $\frac{2}{3}\times\frac{3}{8}$ (1 mark)

(b) 0.35×0.6 (1 mark)

Q8. (1 mark) Find $5\div\frac{5}{6}$.

Q9. (2 marks) A 3.6 kg bag of flour is repacked into bags of 0.4 kg.

(a) How many small bags are filled? (1 mark)

(b) Check your answer with a multiplication. (1 mark)

Q10. (1 mark) A student wants to find 37.5% of 48 mentally. Which form of 37.5% makes the calculation easiest? Circle the letter.

A 0.375

B $\frac{3}{8}$

C $\frac{37.5}{100}$

D $\frac{75}{200}$

Q11. (2 marks) Find this total by regrouping into friendly pairs first, and show your regrouping: $1.35+4.7+2.65+5.3$. (1 mark for the regrouping, 1 mark for the total.)

Q12. (2 marks) In a class of 32 students, half of the class want pizza at the end-of-term party, and $\frac{3}{4}$ of those students want cheese pizza.

(a) What fraction of the class want cheese pizza? (1 mark)

(b) How many students is this? (1 mark)

Q13. (2 marks) (no calculator) Find 35% of 240. Show the percentage blocks you build from the 10% block. (1 mark for the blocks, 1 mark for the answer.)

Q14. (1 mark) (no calculator) In a survey of 25 students, 16 said they walk to school. Express 16 as a percentage of 25.

Q15. (2 marks)

(a) There are 340 students at a school, and 55% of them walk to school. Find the number of students who walk. (1 mark)

(b) In a different survey, 11 of the 16 students said they read every day. Express 11 as a percentage of 16, to one decimal place. (1 mark)

Q16. (2 marks) A box holds 15 red counters and 10 blue counters.

(a) Write the part–part ratio red : blue in its simplest form. (1 mark)

(b) Write the fraction of the counters that are red (the part–whole reading). (1 mark)

Q17. (2 marks) Zoe and Luca share $\$40$ in the ratio $3:5$. How much does each person get? (1 mark for the parts method, 1 mark for both amounts.)

Q18. (1 mark) A drink is mixed in the ratio concentrate : water $=2:3$. What fraction of the drink is concentrate?

Q19. (2 marks) The canteen sells the same juice in two bottle sizes: 500 mL for $\$2.00$, and 2 L for $\$6.40$.

(a) Find the cost per 100 mL of each bottle. (1 mark)

(b) Which bottle is the better buy? In one sentence, explain why comparing the cost per 100 mL is a fair way to decide. (1 mark)

Q20. (3 marks) The student council runs a cake stall. It spends $\$120$ on ingredients and takes $\$270$ in sales.

(a) Did the stall make a profit or a loss? Find the amount, and write the calculation that models it. (1 mark)

(b) Express the profit or loss as a percentage of the money spent. (1 mark)

(c) In one sentence, say what your percentage means for the cake stall. (1 mark)

Solutions

Worked solutions

Q1. (a) Divide the top by the bottom: $3 \div 8 = 0.375$. Multiply by 100 to move to a percentage: $0.375 = 37.5\,\%$. (b) Write over 100, then simplify: $55\,\% = \frac{55}{100}$; dividing top and bottom by 5 gives $\frac{11}{20}$. Award 1 mark for each part.

Q2. The line from -1 to 2 has 3 whole-number gaps, and each gap is split into quarters, so ticks run every $\frac{1}{4}$. The interval is not symmetrical about zero — there is more of the line on the positive side — but the counting works the same way from any label. (a) $-\frac{3}{4}$ sits one tick to the right of -1, the same point as three quarter-steps left of 0. (b) $\frac{7}{4} = 1\frac{3}{4}$ sits three ticks to the right of 1. Award 1 mark for each point placed in the correct position on a correctly drawn line with every gap split into quarters.

Q3. Write both numbers the same way: $-\frac{5}{6} = -0.8333\ldots$. On the negative side, the number further from zero is the smaller one. -0.85 is further from zero than $-0.8333\ldots$, so $-0.85 < -\frac{5}{6}$. Award 1 mark for <.

Q4. (a) The last digit kept is the 9 in the tenths; the digit next door is 5, so round up: $6.95 \rightarrow 7.0$ — the zero is kept to show one decimal place. (b) 8.5 is exactly halfway between 8 and 9, and halfway values round up: $8.5 \rightarrow 9$. Award 1 mark each.

Q5. Round each number to the nearest whole: $4.92 \rightarrow 5$ (next-door digit 9) and $3.87 \rightarrow 4$ (next-door digit 8). The estimate is $5 + 4 = 9$. Award 1 mark for an estimate of 9 with the rounded numbers shown.

Q6. (a) $3 < 3.1$: with 3 L, the stall would be 0.1 L short of the juice it needs. Rounding to the nearest is not the right move when there must be enough. (b) The smallest whole number of litres that is at least 3.1 is 4 — four 1 L bottles. Left over: $4 - 3.1 = 0.9$ L. Award 1 mark for the explanation in (a), and 1 mark for 4 bottles and 0.9 L left over in (b).

Q7. (a) The 3 on top and the 3 on the bottom cancel to 1: $\frac{2}{3} \times \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$. Multiplying across and then simplifying gives the same answer: $\frac{6}{24} = \frac{1}{4}$. (b) Multiply as whole numbers: $35 \times 6 = 210$. The factors have 2 and 1 decimal places, so the product needs $2 + 1 = 3$: counting back three places gives 0.210, which is 0.21. Award 1 mark each.

Q8. Dividing by a fraction is the same as multiplying by its reciprocal: $5 \div \frac{5}{6} = 5 \times \frac{6}{5} = \frac{30}{5} = 6$. Equivalently, count groups: 6 lots of $\frac{5}{6}$ fit into 5. Award 1 mark for 6.

Q9. (a) Bags $= 3.6 \div 0.4$. Multiply both numbers by 10 to make the divisor a whole number: $36 \div 4 = 9$ bags. (b) Check with the inverse multiplication: $9 \times 0.4 = 3.6$ kg, the starting amount, so the answer is confirmed. Award 1 mark for 9 bags and 1 mark for a correct checking multiplication.

Q10. All four options name the one number, so the choice is about which name computes easily. 48 divides cleanly by 8: $\frac{3}{8}$ of 48 is $48 \div 8 \times 3 = 6 \times 3 = 18$, found mentally. The other forms would need a written multiplication of 48 by 0.375, $\frac{37.5}{100}$ or $\frac{75}{200}$. The answer is **B** — choosing the representation that makes the computation efficient is exactly the skill being tested. Award 1 mark for B.

Q11. Look for decimal parts that add to 1: 1.35 pairs with 2.65 ($0.35+0.65=1$), and 4.7 pairs with 5.3 ($0.7+0.3=1$). Reorder and group: $(1.35+2.65)+(4.7+5.3)=4+10=14$. Award 1 mark for the friendly-pair regrouping shown, and 1 mark for the total 14.

Q12. (a) Two "of" steps mean two multiplications: $\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$ of the class want cheese pizza. (b) $\frac{3}{8}$ of 32 is $32\div 8\times 3=4\times 3=12$ students. Award 1 mark each.

Q13. Build from the 10 % block: 10 % of 240 is $240\div 10=24$. Then $30\%=3\times 24=72$ and $5\%=24\div 2=12$. Put the parts together: $35\%=30\%+5\%=72+12=84$. Award 1 mark for the building blocks shown and 1 mark for 84.

Q14. The comparison is $\frac{16}{25}$: part over whole. Multiply top and bottom by 4 to reach a bottom of 100:

$\frac{16}{25}=\frac{64}{100}=64\%$. Award 1 mark for 64 %.

Q15. (a) Write the percentage as a decimal and multiply: $55\%=0.55$, and $0.55\times 340=187$, so 187 students walk. A quick check: 50 % of 340 is 170 and 5 % is 17, and $170+17=187$. (b) Divide the part by the whole, then multiply by 100: $11\div 16=0.6875$, so the percentage is 68.75 %, which rounds to 68.8 % to one decimal place. Award 1 mark each.

Q16. (a) Divide both numbers by the same number until no common factor greater than 1 is left: divide by 5 to get red : blue $=3:2$. (b) Counters in all: $15+10=25$. Red is $\frac{15}{25}=\frac{3}{5}$ of the counters — the part–whole reading. Award 1 mark each.

Q17. The ratio numbers count equal parts. Parts in all: $3+5=8$. One part: $\$40\div 8=\$5$. Zoe: $3\times\$5=\$15$. Luca: $5\times\$5=\$25$. Check: $\$15+\$25=\$40$, the whole amount shared. Award 1 mark for the parts method (8 parts, $5 a part) and 1 mark for both amounts.

Q18. Parts in all: $2+3=5$. Concentrate is 2 of the 5 equal parts, so concentrate is $\frac{2}{5}$ of the drink. Award 1 mark for $\frac{2}{5}$.

Q19. (a) **Formulate and choose.** The two prices cannot be compared as they stand — the bottles hold different amounts — so write both sizes in the same unit (2 L = 2000 mL) and find each cost per 100 mL. **Solve.** The 500 mL bottle holds five lots of 100 mL: $2.00\div 5=\$0.40$ per 100 mL. The 2 L bottle holds twenty lots: $6.40\div 20=\$0.32$ per 100 mL. (b) **Interpret and communicate.** $\$0.32<\$0.40$, so the 2 L bottle is the better buy. Comparing the cost per 100 mL is fair because each number is then the price of the same amount of juice — like with like. Award 1 mark for both unit prices in (a), and 1 mark for the correct choice with a fair-comparison reason in (b).

Q20. (a) **Formulate and solve.** The calculation that models the situation is money taken − money spent: $270-120=150$. The result is positive, so the stall made a **profit** of $150. (b) One quantity as a percentage of another (2E): $\frac{150}{120}=150\div 120=1.25=125\%$. The profit is 125 % of the money spent. (c) **Interpret and communicate.** For every $100 the stall spent, it made $125 more than it spent. Award 1 mark each; part (c) needs the percentage said back in terms of the stall — a bare 1.25 or 125 % without the sentence does not score the mark.

Answer key

Question	Answer	Marks	Code
Q1(a)	0.375 and 37.5 %	1	VC2M7N03
Q1(b)	$\frac{11}{20}$	1	VC2M7N03
Q2(a)	$-\frac{3}{4}$ marked one tick right of -1	1	VC2M7N03

Question	Answer	Marks	Code
Q2(b)	$\frac{7}{4}$ marked three ticks right of 1	1	VC2M7N03
Q3	$<$	1	VC2M7N03
Q4(a)	7.0	1	VC2M7N04
Q4(b)	9	1	VC2M7N04
Q5	$5+4=9$	1	VC2M7N04
Q6(a)	$3<3.1$, so 3 L leaves the stall short	1	VC2M7N04
Q6(b)	4 bottles; 0.9 L left over	1	VC2M7N04
Q7(a)	$\frac{1}{4}$	1	VC2M7N05
Q7(b)	0.21	1	VC2M7N05
Q8	6	1	VC2M7N05
Q9(a)	9 bags	1	VC2M7N05
Q9(b)	$9\times 0.4=3.6$ kg	1	VC2M7N05
Q10	B ($\frac{3}{8}$)	1	VC2M7N06
Q11	$(1.35+2.65)+(4.7+5.3)=$	2	VC2M7N06
Q12(a)	$\frac{3}{8}$	1	VC2M7N06
Q12(b)	12 students	1	VC2M7N06
Q13	$10\%=24$, $30\%=72$, $5\%=12$; 84	2	VC2M7N07
Q14	64%	1	VC2M7N07
Q15(a)	187 students	1	VC2M7N07
Q15(b)	68.8%	1	VC2M7N07
Q16(a)	$3:2$	1	VC2M7N09
Q16(b)	$\frac{3}{5}$	1	VC2M7N09
Q17	Zoe $\$15$, Luca $\$25$	2	VC2M7N09
Q18	$\frac{2}{5}$	1	VC2M7N09
Q19(a)	$\$0.40$ and $\$0.32$ per 100 mL	1	VC2M7N10
Q19(b)	the 2 L bottle, with a fair-comparison reason	1	VC2M7N10
Q20(a)	profit of $\$150$; $270-120=150$	1	VC2M7N10
Q20(b)	125%	1	VC2M7N10
Q20(c)	e.g. for every $\$100$ spent, $\$125$ made	1	VC2M7N10
Total		**35**	

Marks by content description. `VC2M7N03` — 5 marks (Q1–Q3) · `VC2M7N04` — 5 marks (Q4–Q6) · `VC2M7N05` — 5 marks (Q7–Q9) · `VC2M7N06` — 5 marks (Q10–Q12) · `VC2M7N07` — 5 marks (Q13–Q15; Q13 and Q14 are the calculator-free items, Q15 the calculator-strand item, per `VC2M7N07`'s "with and without digital tools") · `VC2M7N09` — 5 marks (Q16–Q18) · `VC2M7N10` — 5 marks (Q19–Q20). That is 5 marks per content description, per the §7 marking rule, and 2G's marks are awarded across the full modelling cycle (formulate → choose → solve → interpret and communicate, with the representation justified), per D8. The only multiple-choice item is Q10, 1 mark of 35 — MCQ used sparingly, where selection genuinely is the skill being tested.

Achievement standard extracts (verbatim from `maths_7.json`): `VC2M7N03` — *Students choose between equivalent representations of rational numbers and percentages to assist in calculations and make simple estimates to judge the reasonableness of results.* `VC2M7N04` — *Students choose between equivalent representations of rational numbers and percentages to assist in calculations and make simple estimates to judge the reasonableness of results.* (same extract) `VC2M7N05` — *They use all 4 operations in calculations involving positive fractions and decimals, choosing efficient mental and written calculation strategies.* `VC2M7N06` — *They use all 4 operations in calculations involving positive fractions and decimals, choosing efficient mental and written calculation strategies.* (same extract) `VC2M7N07` — *They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.* `VC2M7N09` — *They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.* (same extract) `VC2M7N10` — *They use mathematical modelling to solve practical problems involving rational numbers, percentages and ratios in spatial, financial and other applied contexts, justifying choices of representation.* (same extract)