Mathematics Level 7

Chapter 1 — Whole numbers: integers, powers and square numbers (Number)

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Contents

1A Comparing, ordering, adding and subtracting integers

Content description `VC2M7N08`: compare, order and solve problems involving addition and subtraction of integers

Achievement standard (extract): Students solve problems involving addition and subtraction of integers.

Theory

Integers as directed numbers. The counting numbers $1, 2, 3, \ldots$, together with 0 and the negative numbers $-1, -2, -3, \ldots$, make up the **integers**. Every integer is either positive, negative or zero, and zero itself is neither positive nor negative. The + or − in front of an integer is called its **sign**: $+5$ (usually written just 5) is positive five and -5 is negative five. Because the sign gives each number a direction — this side of zero or that side — integers are also called **directed numbers**. We meet them everywhere: a temperature of $-4\,°C$ is four degrees below zero, and a bank balance of $-\$30$ means the account owes the bank $\$30$.

The number line. At Level 6 the integers were placed on a number line; here that picture does the work. Figure 1 shows the number line: zero sits in the middle, the positive integers run to the right and the negative integers run to the left, and every step of 1 is the same width. The one fact that drives everything else is that **numbers get larger as you move right**. So -4, which sits to the left of 3, is smaller than 3.

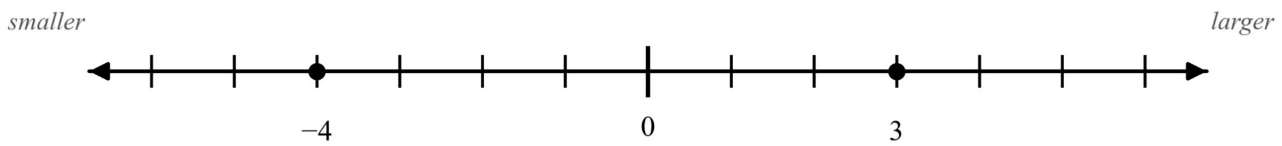


Figure 1. The integer number line from −6 to 6, with −4 and 3 marked; numbers get smaller to the left and larger to the right.

Comparing and ordering with < and >. The sign < means "is less than" and > means "is greater than"; the wide open end always faces the larger number. Because -5 lies to the left of 2 on the number line, we write $-5 < 2$, read as "negative five is less than positive two". A chain such as $-4 < -1 < 0 < 3$ says that each number is smaller than the one after it, which is exactly the left-to-right order on the line. Writing a set of integers **from smallest to largest** is called putting them in **ascending order**, and writing them from largest to smallest is **descending order**. Watch the negative side: -12 lies to the *left* of -8, so $-12 < -8$. The bigger a negative number looks, the smaller it really is.

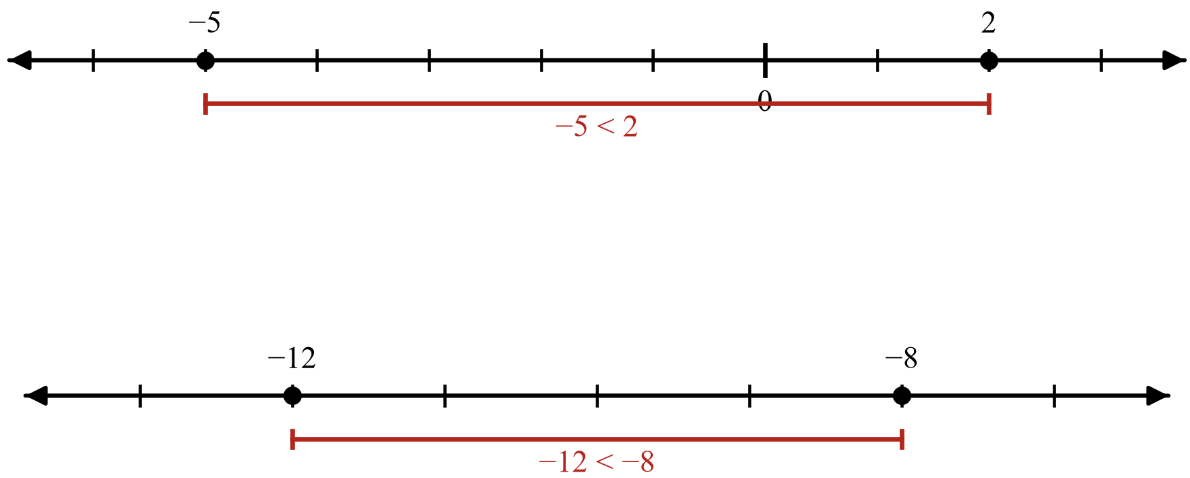


Figure 2. Two number lines used for comparing: on the first, −5 and 2 are marked and a bracket below reads −5 < 2; on the second, −12 and −8 are marked and a bracket below reads −12 < −8.

Sign and magnitude. An integer's **sign** tells which side of zero it is on, and its **magnitude** tells how far from zero it is. The magnitude is the distance from zero, so it is never negative: the magnitude of -7 is 7, the magnitude of 6 is 6, and the magnitude of 0 is 0. Two integers such as 5 and -5 have the same magnitude but opposite signs; they are called **opposites** of each other, sitting the same distance from zero on opposite sides.

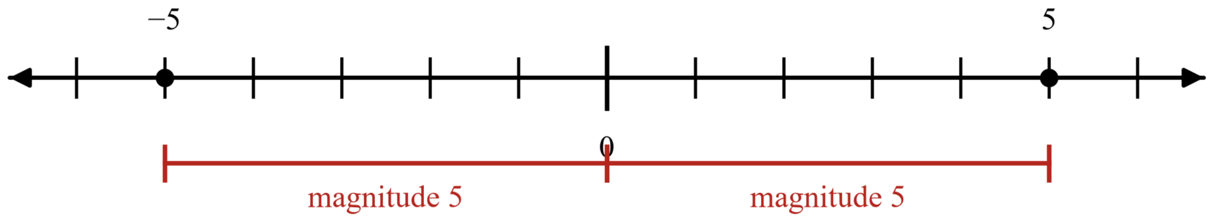


Figure 3. A number line from −6 to 6 with −5 and 5 marked; a bracket below runs 5 steps from −5 to 0 and another 5 steps from 0 to 5, each labelled magnitude 5, so −5 and 5 sit the same distance from zero on opposite sides.

Addition as movement. Adding is moving along the number line: adding a positive integer moves you right, and adding a negative integer moves you left. Starting at -3 and adding 7 moves seven steps right to 4, so $-3+7=4$; starting at -3 and adding -4 moves four steps left to -7, so $-3+(-4)=-7$. Away from the line, two facts give every sum of two integers:

- **Same sign:** add the magnitudes and keep the common sign. $-4+(-6)$ has magnitudes $4+6=10$ and the common sign $-$, so it is -10.
- **Different signs:** subtract the smaller magnitude from the larger, and keep the sign of the integer with the larger magnitude. $-9+5$ has $9-5=4$, and -9 has the larger magnitude, so it is -4.

Adding 0 changes nothing, and an integer added to its opposite gives 0: $-8+8=0$.

Subtraction as movement. Subtracting a positive integer moves you left: $2-6$ starts at 2 and moves six steps left to -4. Subtracting a negative integer moves you right, because taking away a negative is the same as adding its positive opposite:

$$7-(-2)=7+2=9$$

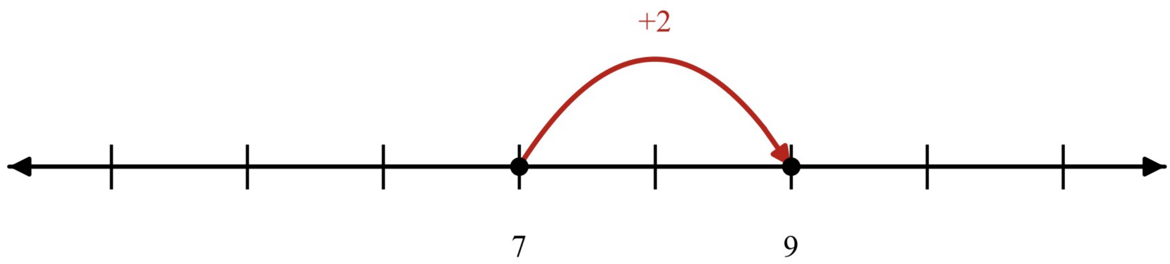


Figure 4. A number line from 4 to 11 with 7 and 9 marked; a red jump labelled +2 runs from 7 to 9, showing that subtracting −2 moves to the right, the same as adding 2.

So $6-9=6+(-9)=-3$ and $6-(-9)=6+9=15$. In any longer string of additions and subtractions, work from left to right, one step at a time.

Difference. The **difference** between two integers is the result of a subtraction — the gap between them on the number line. From -3 to 5 the gap is 8 steps, and the same number comes from subtracting the smaller from the larger: $5-(-3)=5+3=8$. Differences answer everyday questions: how much warmer one day is than another, or how far apart two floors of a building are, when one is below the ground.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
addition	combining two numbers into one	plus, sum, total, “increase by”
subtraction	taking one number away from another	minus, take away, “decrease by”
magnitude	distance of an integer from zero	size
difference	result of a subtraction; gap between two values	“how much more”, “how much less”, change
sign	whether an integer is positive or negative	positive, negative

The same number line returns in subtopic 2A, where it is extended to carry fractions and decimals as well as integers.

Worked examples

Example 1 — scaffolded

Plot -4, 2, -1 and 5 on a number line, then use the line to write the four numbers in ascending order, joined with the < sign.

Working	Comment
Plot the four numbers, as in Figure 5: -4 and -1 sit left of zero; 2 and 5 sit right of zero.	Every step of 1 is the same width, so the picture is true to size.
Reading the plotted numbers from left to right gives -4, -1, 2, 5.	Numbers get larger as you move right, so left to right is smallest to largest.
Write the chain $-4<-1<2<5$.	Each < opens toward the larger number beside it.

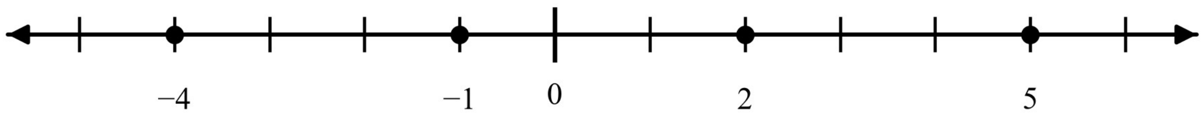


Figure 5. Number line from −5 to 6 with the points −4, −1, 2 and 5 marked.

Example 2 — scaffolded

Use a number line to find each answer: (a) $-3+7$; (b) $2-6$.

Working	Comment
(a) Start at -3. Adding 7 moves seven steps right, landing on 4; so $-3+7=4$ (top line of Figure 6).	Adding a positive integer moves you right.
(b) Start at 2. Subtracting 6 moves six steps left, landing on -4; so $2-6=-4$ (bottom line of Figure 6).	Two steps reach 0; four more steps left land on -4.

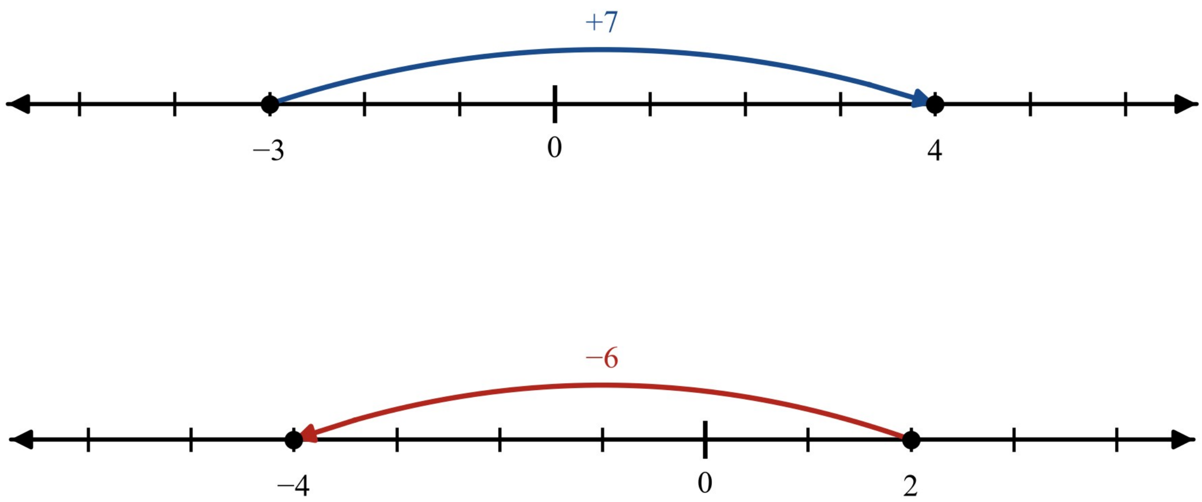


Figure 6. Two number lines: from −3 an arc of $+7$ *lands on* 4*; from* 2 *an arc of* -6 *lands on* -4*.*

Example 3 — unscaffolded

A freezer is kept at $-18\,°C$ and the room it stands in is at $21\,°C$. Find the difference between the two temperatures.

The difference is the gap between -18 and 21 on the number line (Figure 7). From -18 up to 0 is 18 degrees, and from 0 up to 21 is a further 21 degrees, so the gap is $18+21=39$. As a single subtraction of the smaller from the larger:

$$21-(-18)=21+18=39$$

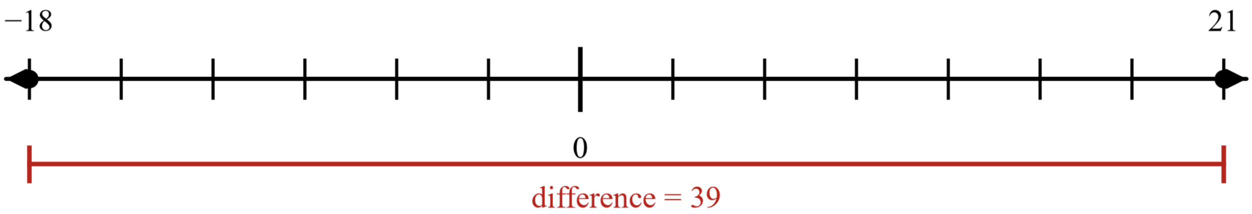


Figure 7. Number line from −18 to 21 in steps of 3; a bracket below the line marks the gap from −18 to 21 labelled "difference = 39".

∴ the difference between the two temperatures is $39\,°C$.

Example 4 — unscaffolded

Evaluate $-8+15-(-4)$.

Work from left to right. The first step, $-8+15$, has different signs: the magnitudes give $15-8=7$, and $+15$ has the larger magnitude, so $-8+15=7$. The next step is $7-(-4)$; subtracting a negative is adding its positive opposite, so $7-(-4)=7+4=11$.

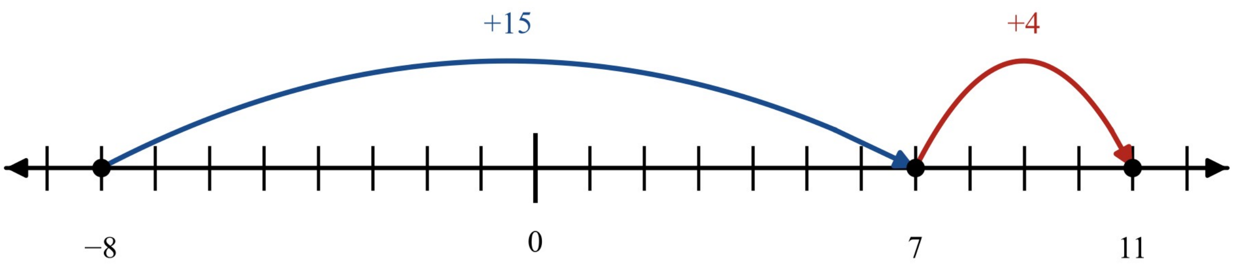


Figure 8. A number line from −9 to 12 with −8, 7 and 11 marked; a blue jump labelled +15 runs from −8 to 7 and a red jump labelled +4 runs from 7 to 11.

$\therefore -8+15-(-4)=11.$

Practice questions

Scaffolded

Q1. A larger number always sits to the right on the number line. Using that fact, replace each blank with < or > to make a true statement. (a) 4 ___ 9 (b) -2 ___ 5 (c) -6 ___ -1 (d) 0 ___ -3 (e) -8 ___ -12 (f) 7 ___ -7

Q2. Write each set of integers in the order asked. (a) ascending: $5, -2, 0, 3, -7$ (b) ascending: $-1, -9, -4, 2, -6$ (c) descending: $4, -3, 8, -8, 0$ (d) descending: $-5, 6, -11, 11, -6$

Q3. Each step on the number line is 1 unit; moving left gives a smaller number, moving right a larger one. Write the integer described. (a) 4 steps left of 0 (b) 6 steps right of 0 (c) 3 steps right of -2 (d) 5 steps left of 2

Q4. Find each sum by moving along a number line. State where you start, which way you move, and where you land.

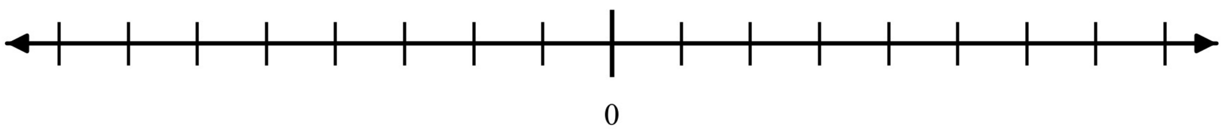


Figure 9. A blank number line from −8 to 8 with a tick at every integer, ready for jumps to be drawn on.

(a) $2+5$
(b) $-3+4$
(c) $-6+2$
(d) $5+(-3)$
(e) $-1+(-4)$
(f) $0+(-7)$

Q5. Find each difference by moving along a number line. Remember that subtracting a negative moves you right.

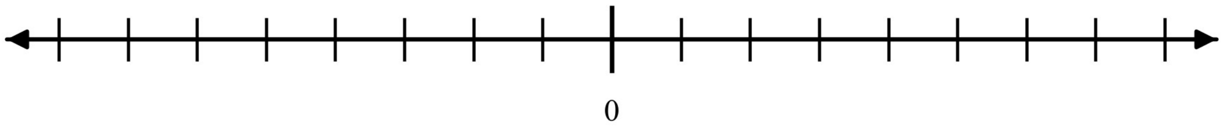


Figure 9. A blank number line from −8 to 8 with a tick at every integer, ready for jumps to be drawn on.

(a) $8-3$
(b) $3-7$
(c) $-2-3$
(d) $-1-(-4)$
(e) $6-(-2)$
(f) $-5-(-5)$

Q6. State the magnitude of each integer, then answer (g) and (h). (a) 9 (b) -6 (c) 0 (d) -14 (e) $+3$ (f) -1 (g) Which has the larger magnitude, -8 or 5? (h) An integer has magnitude 12 and a negative sign. What is the integer?

Unscaffolded

Q7. Calculate: (a) $6+9$; (b) $-4+10$; (c) $-7+(-5)$; (d) $15+(-8)$; (e) $-8+8$; (f) $-12+5$.

Q8. Calculate: (a) $9-4$; (b) $5-12$; (c) $-3-6$; (d) $-8-(-2)$; (e) $7-(-6)$; (f) $-9-(-10)$.

Q9. The number line in Figure 10 has equally spaced ticks, but only two numbers are printed. Write the integer that belongs at each of the letters A, B and C.

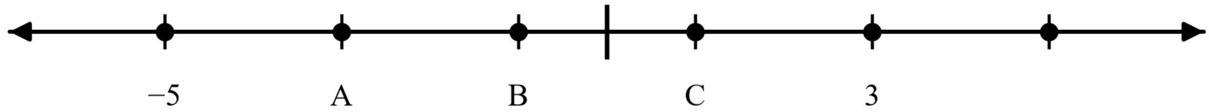


Figure 10. Number line with equally spaced ticks; −5 and 3 are printed and the letters A, B, C mark the ticks between them.

Q10. Overnight temperatures in five towns were $-2\,°C$, $7\,°C$, $-9\,°C$, $0\,°C$ and $-5\,°C$. (a) Write the temperatures from coldest to warmest. (b) How many of the five temperatures were below $0\,°C$?

Q11. State whether each statement is true or false. Correct every false statement. (a) $-6>-9$ (b) The magnitude of -7 is -7. (c) 0 is a positive integer. (d) $-3+5=2$ (e) The difference between -2 and 6 is 4. (f) $5-(-3)=2$

Q12. Evaluate, working from left to right: (a) $-4+9-6$ (b) $7-10+(-2)$ (c) $-3-(-8)-4$ (d) $12+(-5)-(-3)$

Q13. At 6 pm the temperature in a town was $6\,°C$. By midnight it had fallen to $-3\,°C$. (a) Find the fall in temperature between 6 pm and midnight. (b) By 9 am the next day the temperature had risen $11\,°C$ from the midnight value. Find the temperature at 9 am.

Q14. The entrance of a mine is 35 m above sea level, and its lowest tunnel is 180 m below sea level. (a) Write the height of the entrance and the height of the lowest tunnel as integers. (b) Find the difference in height between the entrance and the lowest tunnel.

Q15. Mia's bank account shows $-\$120$, which means she owes the bank $\$120$. (a) She pays $\$350$ into the account. Find the new balance. (b) A bill of $\$95$ is then taken from the account. Find the balance after the bill.

Q16. (a) Write the integers $-7, 3, -1, 9, -5, 0$ in order of magnitude, from smallest magnitude to largest. (b) Which of these integers sits furthest from zero on the number line?

Q17. Find the missing integer in each statement. (a) $-6+?=2$ (b) $?-9=-4$ (c) $7-?=12$ (d) $?+(-13)=-13$

Q18. At 6 am the temperature in a mountain town was $-8\,°C$. By midday it had risen $14\,°C$, and by the next midnight it had fallen $17\,°C$ from the midday value. (a) Find the midday temperature. (b) Find the midnight temperature. (c) Find the difference between the midnight temperature and the 6 am temperature.

Answers

Q1. (a) <; (b) <; (c) <; (d) >; (e) >; (f) >.

Q2. (a) $-7,-2,0,3,5$; (b) $-9,-6,-4,-1,2$; (c) $8,4,0,-3,-8$; (d) $11,6,-5,-6,-11$.

Q3. (a) -4; (b) 6; (c) 1; (d) -3.

Q4. (a) 7; (b) 1; (c) -4; (d) 2; (e) -5; (f) -7.

Q5. (a) 5; (b) -4; (c) -5; (d) 3; (e) 8; (f) 0.

Q6. (a) 9; (b) 6; (c) 0; (d) 14; (e) 3; (f) 1; (g) -8; (h) -12.

Q7. (a) 15; (b) 6; (c) -12; (d) 7; (e) 0; (f) -7.

Q8. (a) 5; (b) -7; (c) -9; (d) -6; (e) 13; (f) 1.

Q9. $A=-3$; $B=-1$; $C=1$.

Q10. (a) $-9°C,-5°C,-2°C,0°C,7°C$; (b) 3.

Q11. (a) true; (b) false — the magnitude of -7 is 7; (c) false — 0 is neither positive nor negative; (d) true; (e) false — the difference is 8; (f) false — $5-(-3)=8$.

Q12. (a) -1; (b) -5; (c) 1; (d) 10.

Q13. (a) $9°C$; (b) $8°C$.

Q14. (a) 35 and -180; (b) 215 m.

Q15. (a) $\$230$; (b) $\$135$.

Q16. (a) $0,-1,3,-5,-7,9$; (b) 9.

Q17. (a) 8; (b) 5; (c) -5; (d) 0.

Q18. (a) $6°C$; (b) $-11°C$; (c) $3°C$.

Fully-worked solutions

Q1. On the number line, the larger number is to the right. 4 is left of 9, so $4<9$; every negative is left of every positive, so $-2<5$ and $7>-7$; on the negative side, -6 is left of -1, so $-6<-1$, and -12 is left of -8, so $-8>-12$; 0 is right of every negative, so $0>-3$.

Q2. (a) Reading the set from most negative to most positive: $-7,-2,0,3,5$. (b) $-9,-6,-4,-1,2$. (c) Descending reads the line from right to left: $8,4,0,-3,-8$. (d) $11,6,-5,-6,-11$.

Q3. (a) 4 steps left of 0 lands on -4. (b) 6 steps right of 0 lands on 6. (c) $-2+3=1$. (d) $2-5=-3$.

Q4. (a) Start at 2, move 5 right: $2+5=7$. (b) Start at -3, move 4 right: $-3+4=1$. (c) Start at -6, move 2 right: $-6+2=-4$. (d) Start at 5, move 3 left: $5+(-3)=2$. (e) Start at -1, move 4 left: $-1+(-4)=-5$. (f) Start at 0, move 7 left: $0+(-7)=-7$.

Q5. (a) Start at 8, move 3 left: $8-3=5$. (b) Start at 3, move 7 left: $3-7=-4$. (c) Start at -2, move 3 left: $-2-3=-5$. (d) Start at -1, move 4 right: $-1-(-4)=3$. (e) Start at 6, move 2 right: $6-(-2)=8$. (f) Start at -5, move 5 right: $-5-(-5)=0$.

Q6. The magnitude is the distance from zero. (a) 9. (b) 6. (c) 0. (d) 14. (e) 3. (f) 1. (g) The magnitude of -8 is 8 and the magnitude of 5 is 5, so -8 has the larger magnitude. (h) Magnitude 12 with a negative sign is -12.

Q7. (a) $6+9=15$. (b) Different signs: $10-4=6$, keep +: 6. (c) Same sign: $7+5=12$, keep $-$: -12. (d) Different signs: $15-8=7$, keep +: 7. (e) An integer added to its opposite: 0. (f) Different signs: $12-5=7$, keep $-$: -7.

Q8. (a) $9-4=5$. (b) $5-12=5+(-12)=-7$. (c) $-3-6=-3+(-6)=-9$. (d) $-8-(-2)=-8+2=-6$. (e) $7-(-6)=7+6=13$. (f) $-9-(-10)=-9+10=1$.

Q9. From -5 to 3 is a gap of 8, crossed in 4 equal steps, so each step is 2. Reading right by twos: -5, A $=-3$, B $=-1$, C $=1$, then 3, and the last tick is 5.

Q10. (a) Coldest to warmest is the left-to-right order on the number line: $-9\,°C, -5\,°C, -2\,°C, 0\,°C, 7\,°C$. (b) The temperatures below $0\,°C$ are $-9\,°C$, $-5\,°C$ and $-2\,°C$: 3 of the five.

Q11. (a) -6 is right of -9, so $-6>-9$ is **true**. (b) **False**: a magnitude is a distance, so the magnitude of -7 is 7. (c) **False**: 0 is neither positive nor negative. (d) $-3+5=2$ is **true**. (e) **False**: the difference between -2 and 6 is $6-(-2)=8$. (f) **False**: $5-(-3)=5+3=8$.

Q12. (a) $-4+9=5$, then $5-6=-1$. (b) $7-10=-3$, then $-3+(-2)=-5$. (c) $-3-(-8)=-3+8=5$, then $5-4=1$. (d) $12+(-5)=7$, then $7-(-3)=7+3=10$.

Q13. (a) The fall is the difference between 6 and -3: $6-(-3)=6+3=9$, a fall of $9\,°C$. (b) Rising $11\,°C$ from $-3\,°C$: $-3+11=8$, so the temperature at 9 am is $8\,°C$.

Q14. (a) 35 m above sea level is 35; 180 m below sea level is -180. (b) The difference is $35-(-180)=35+180=215$, a difference in height of 215 m.

Q15. (a) $-120+350=230$, so the new balance is $\$230$. (b) $230-95=135$, so the balance after the bill is $\$135$.

Q16. (a) The magnitudes are $7, 3, 1, 9, 5, 0$. Smallest magnitude to largest gives $0, -1, 3, -5, -7, 9$. (b) Furthest from zero means largest magnitude, which is 9.

Q17. (a) $-6+?=2$ needs a move of 8 right, so $?=8$ (check: $-6+8=2$). (b) $?=-4+9=5$ (check: $5-9=-4$). (c) $?=7-12=-5$ (check: $7-(-5)=12$). (d) $?=-13+13=0$ (check: $0+(-13)=-13$).

Q18. (a) $-8+14=6$, so midday is $6\,°C$. (b) $6-17=-11$, so midnight is $-11\,°C$. (c) The gap between -8 and -11 is 3 steps (check: $-8-(-11)=-8+11=3$), so the midnight temperature is $3\,°C$ lower than the 6 am temperature; the difference is $3\,°C$.

Expanded notation with powers of 10; products of powers of primes

Content description `VC2M7N02`: represent natural numbers in expanded notation using powers of 10, and as products of powers of prime numbers using exponent notation

Achievement standard (extract): students represent natural numbers in expanded form and as products of prime factors, using exponent notation.

Theory

Powers of 10. Multiplying a number by itself again and again gets tiring to write, so there is a short way of writing it. In 10^3, the 10 is the **base** — the number being multiplied — and the small raised 3 is the **exponent**, which counts how many factors of the base there are:

$$10^3 = 10 \times 10 \times 10 = 1000$$

We read 10^3 as "ten to the power of three", and the whole thing, 10^3, is called a **power**. Watch the trap: 10^3 is *not* $10 \times 3 = 30$. The exponent counts factors of 10; it is not multiplied on.

Figure 1 lines up the powers of 10 three ways: as a power, as a product of 10s, and as a value.

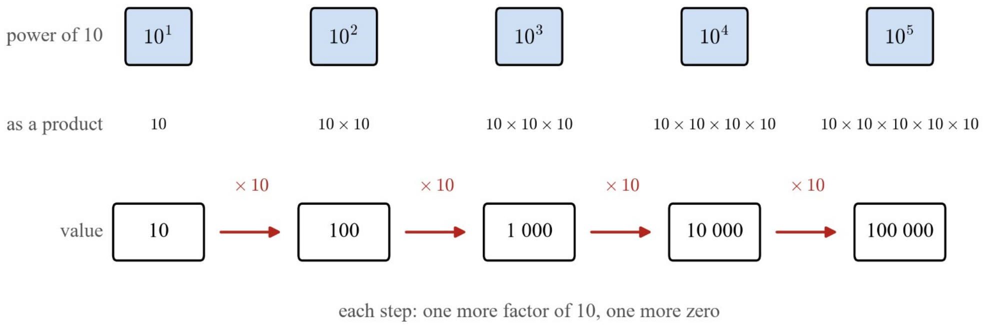


Figure 1. Powers of 10 from 10^1 to 10^5 shown three ways: as a power, as a product of 10s, and as a value; red arrows mark that each value is 10 times the one before, adding one more factor of 10 and one more zero.

Every step to the right multiplies by 10, which adds one more factor of 10 and one more zero. That is why $10^4 = 10\,000$ and $10^5 = 100\,000$: the exponent matches the number of zeros.

Expanded notation. In Level 6 you met prime and composite numbers, and place value out beyond 10 000 (VC2M6N02, VC2M6N04). Place value is what powers of 10 are built on: the tens column is worth 10^1, the hundreds 10^2, the thousands 10^3, and so on. So every natural number can be split into its column values, and each column value written as a digit times a power of 10. Writing the number as the sum of these parts is called **expanded notation**.

thousands	hundreds	tens	ones
10^3	10^2	10^1	1
4	2	6	5
$4 \times 10^3 = 4000$	$2 \times 10^2 = 200$	$6 \times 10^1 = 60$	5

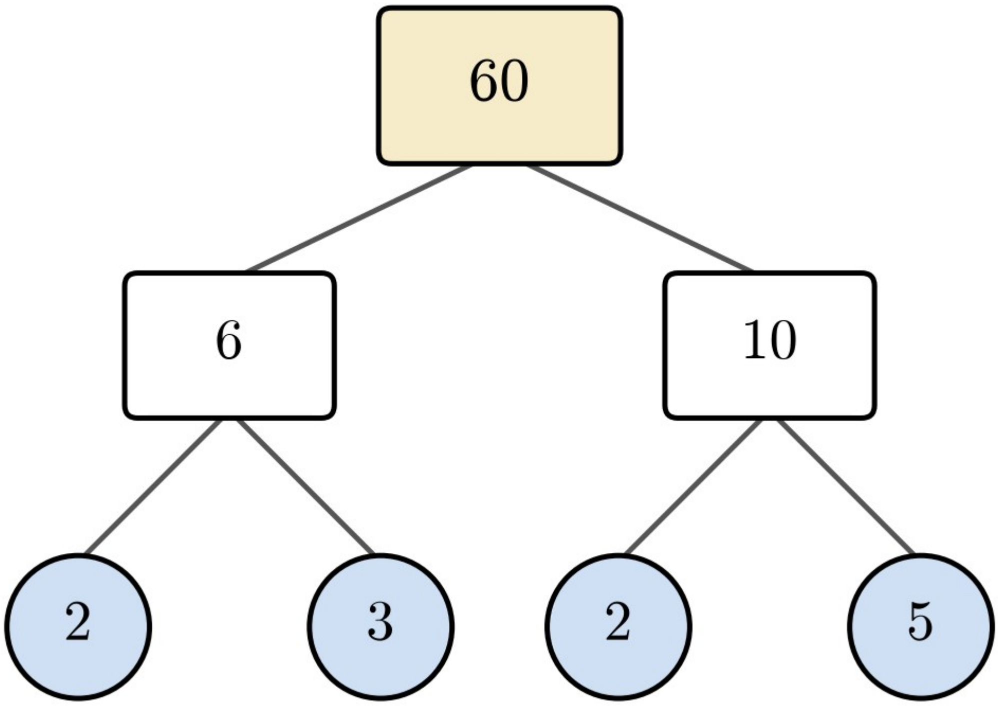
$$4265 = 4 \times 10^3 + 2 \times 10^2 + 6 \times 10^1 + 5$$

expanded notation

Figure 2. A place-value table for 4265 with columns for thousands, hundreds, tens and ones; under each column the digit's value is written as a digit times a power of 10, and the expanded notation $4265 = 4 \times 10^3 + 2 \times 10^2 + 6 \times 10^1 + 5$ is shown below.

A zero digit adds nothing, so its part is simply left out: $5083 = 5 \times 10^3 + 8 \times 10^1 + 3$, with no hundreds part at all. You will see this split step by step in Worked example 1.

Prime factorisation. A prime number has exactly two factors, 1 and itself; a **composite** number is a natural number greater than 1 that is not prime, so it can be split into smaller factors. Writing a composite number as a product of primes only is called its **prime factorisation**. One way is a factor tree: split into any two factors, keep splitting the composite numbers, and stop when every branch ends in a prime.



$$60 = 2 \times 2 \times 3 \times 5$$

$$= 2^2 \times 3 \times 5$$

Figure 3. A factor tree for 60: 60 splits into 6 and 10, then 6 splits into the primes 2 and 3 and 10 splits into the primes 2 and 5; below, 60 = 2 × 2 × 3 × 5 is regrouped as $2^2 \times 3 \times 5$.

The leaves are the prime factors, and multiplying them back gives the number you started with. Equal primes can be grouped with exponent notation: $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$. That is exactly what the content description means by a *product of powers of primes*.

A second way is repeated division: start at the number, divide by primes until 1 is reached, and keep the divisors.

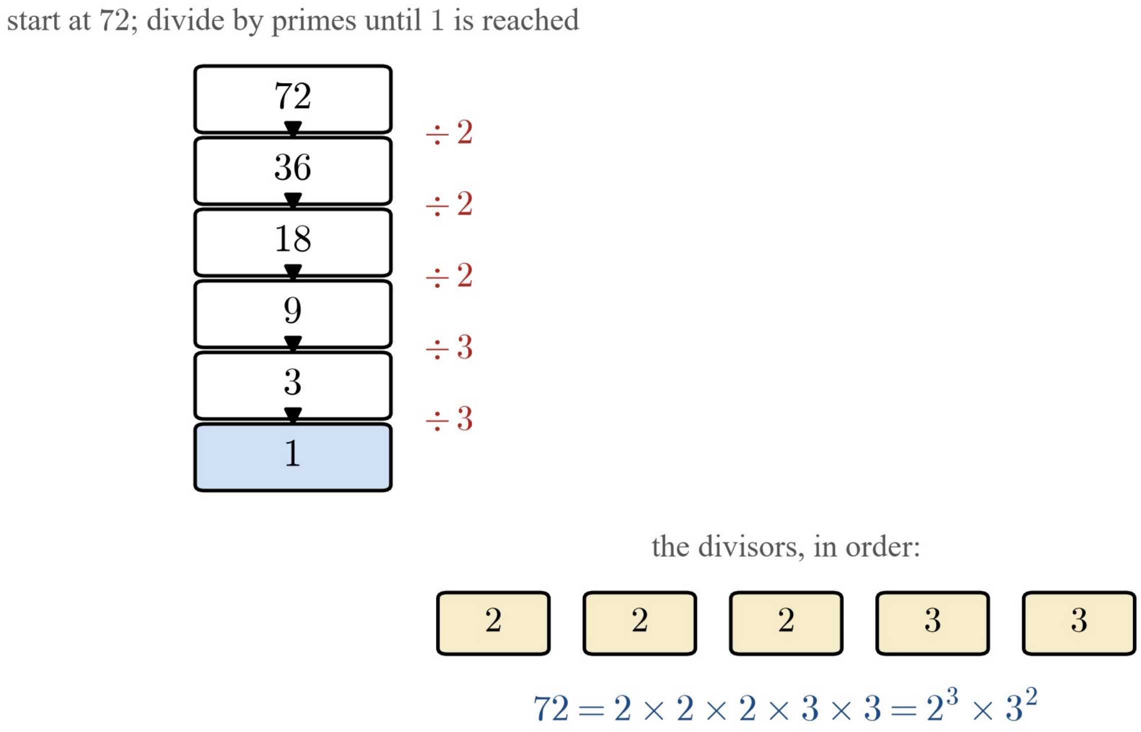


Figure 4. A repeated-division ladder for 72: 72, 36, 18, 9, 3, 1, with the divisors 2, 2, 2, 3, 3 shown beside the steps; below, the divisors are collected as chips and $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ is written in exponent notation.

Both ways give the same prime factors (just possibly in a different order), and both are checked the same way: multiply the primes back and see if you land on the number you started with.

Powers worth knowing. Factorising is much faster when you can spot powers of 2, 3 and 5 straight away. Figure 5 builds each sequence by multiplying by the base at every step, starting from 1.

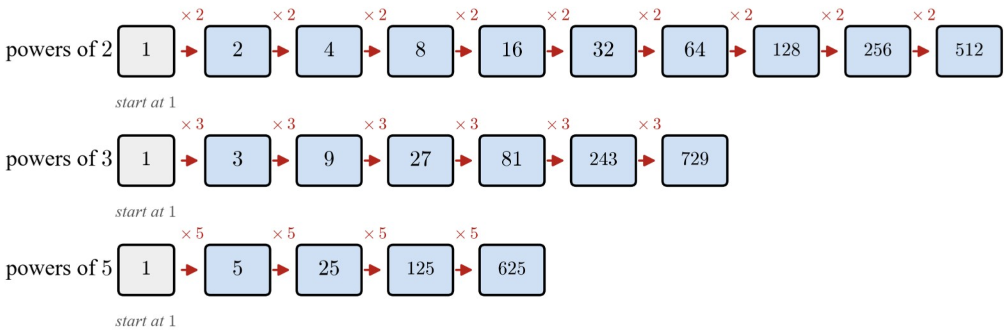


Figure 5. Three rows of chips: powers of 2 from 1 to 512 with red ×2 arrows, powers of 3 from 1 to 729 with red ×3 arrows, and powers of 5 from 1 to 625 with red ×5 arrows.

Knowing $2^4 = 16$, $3^3 = 27$ or $5^3 = 125$ at a glance lets you pull big factors out in one step: $72 = 8 \times 9 = 2^3 \times 3^2$.

HCF and LCM from prime factorisations. Once two numbers are written as products of primes, the **highest common factor (HCF)** and the **lowest common multiple (LCM)** can be read straight off the two rows of factors.

- The **HCF** is built from the factors the two numbers *share*.
- The **LCM** is built from *every* factor needed — each prime as many times as it appears in either number.

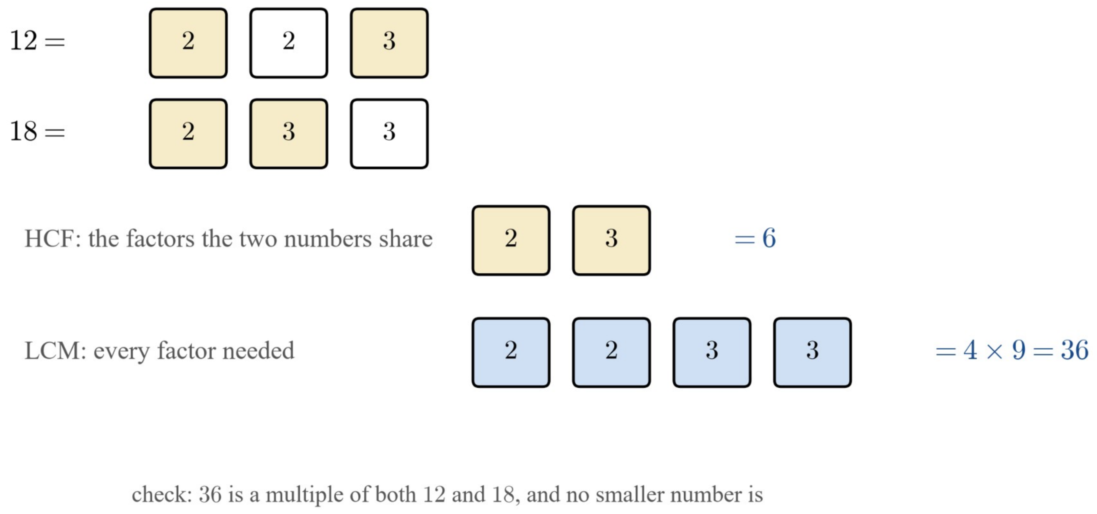


Figure 6. The prime factors of 12 and 18 as rows of chips, with the shared factors 2 and 3 coloured; the HCF row collects the shared factors giving 2 × 3 = 6, and the LCM row collects every factor needed giving 2 × 2 × 3 × 3 = 4 × 9 = 36, with a check that 36 is a multiple of both 12 and 18.

Then multiply out as usual: $2 \times 3 = 6$ for the HCF, and $2 \times 2 \times 3 \times 3 = 36$ for the LCM. Nothing new is invented here — the factorisations do the thinking, and the last step is ordinary multiplication.

Where this leads. The base–exponent language you are setting up here is used again in 1C, where powers meet square numbers and square roots. Prime factorisation and the HCF also come back in 2A, where they are the tool for writing fractions in simplest form.

Word	Meaning
base	the number being multiplied again and again in a power: in 10^3 the base is 10
exponent	the small raised number in a power; it counts the factors of the base: in 10^3 the exponent is 3
expanded notation	writing a number as the sum of its column values, using powers of 10
prime factorisation	writing a composite number as a product of primes only
highest common factor (HCF)	the biggest natural number that is a factor of two or more numbers
lowest common multiple (LCM)	the smallest natural number that is a multiple of two or more numbers

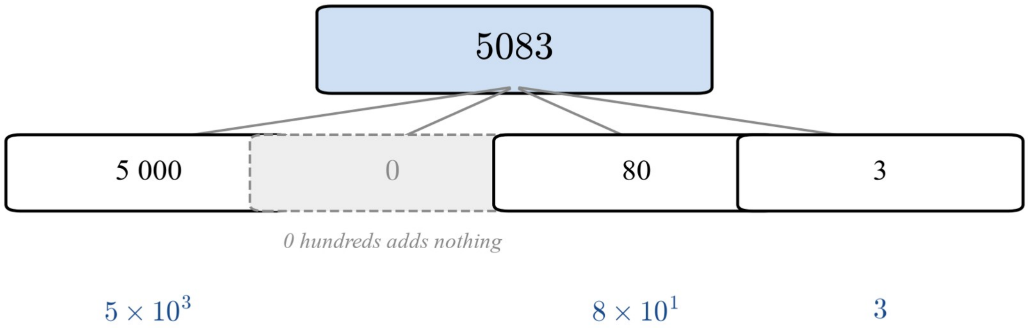
Worked examples

Example 1 (scaffolded)

Write 5083 in expanded notation using powers of 10.

Working	Comment
$5083 = 5000 + 80 + 3$	Read the column values: 5 thousands, 0 hundreds, 8 tens, 3 ones. The zero hundreds adds nothing, so it is left out.
$= 5 \times 10^3 + 8 \times 10^1 + 3$	The thousands column is worth 10^3 and the tens column 10^1; the ones digit stands alone.

Figure 7 shows the same bridge from the number down to its expanded notation.



$$5083 = 5 \times 10^3 + 8 \times 10^1 + 3$$

Figure 7. The number 5083 at the top, joined down to its column values 5000, 80 and 3; the empty hundreds column is shown as a grey 0 that adds nothing, and the expanded notation 5083 = 5 × 10³ + 8 × 10¹ + 3 is written below.

Example 2 (scaffolded)

Write 60 as a product of powers of primes.

Working	Comment
$60 = 6 \times 10$	Split into any two factors; the tree does not care which.
$6 = 2 \times 3$ and $10 = 2 \times 5$	Split each composite number until every branch ends in a prime (the circled leaves).
$60 = 2 \times 2 \times 3 \times 5$	Check by multiplying the primes back: $2 \times 2 \times 3 \times 5 = 60$. ✓
$= 2^2 \times 3 \times 5$	The two 2s are grouped as 2^2; the 3 and 5 appear once each.

Example 3

What number is $7 \times 10^4 + 4 \times 10^2 + 9$?

$7 \times 10^4 = 70\,000$ and $4 \times 10^2 = 400$, so the sum is $70\,000 + 400 + 9 = 70\,409$. The missing 10^3 and 10^1 parts mean those digits are 0 — a handy check when you read a number back off its expanded notation.

Example 4

The canteen buys pies in trays of 12 and sausage rolls in trays of 15. It wants to order the *same* number of pies and sausage rolls, with no leftovers on either tray type. What is the smallest number of each it can order, and how many trays of each is that?

The number ordered must be a multiple of both 12 and 15, and we want the lowest one — the LCM. From the factorisations $12 = 2 \times 2 \times 3$ and $15 = 3 \times 5$, the LCM takes every factor needed: $2 \times 2 \times 3 \times 5 = 60$. So the canteen orders 60 of each: $60 \div 12 = 5$ trays of pies and $60 \div 15 = 4$ trays of rolls.

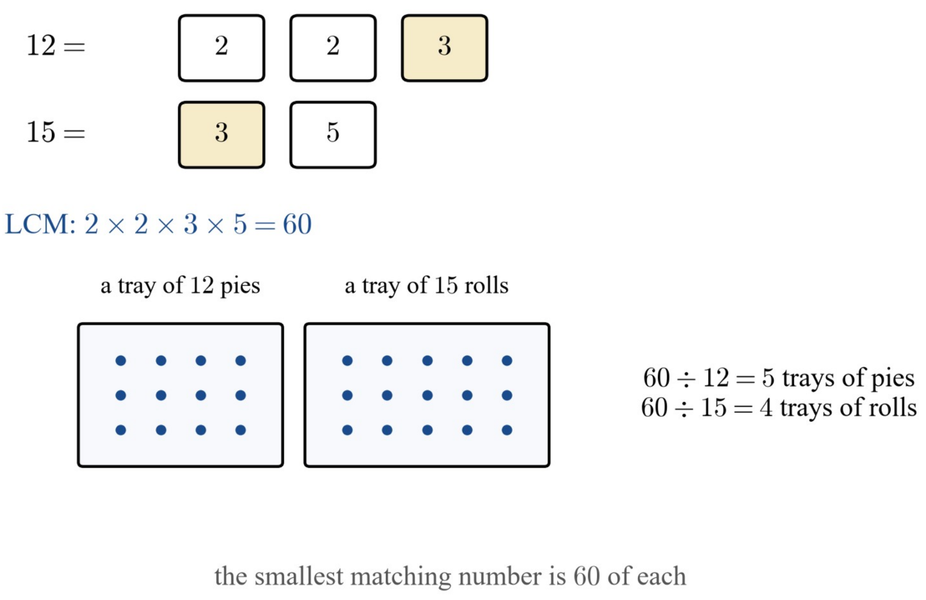


Figure 8. The prime factors of 12 and 15 as rows of chips with the shared 3 coloured, giving LCM 2 × 2 × 3 × 5 = 60; below, a tray of 12 pies drawn as 3 rows of 4 dots and a tray of 15 rolls as 3 rows of 5 dots, with 60 ÷ 12 = 5 trays of pies and 60 ÷ 15 = 4 trays of rolls.

Practice questions

Scaffolded

Q1. Write each power as a product of 10s, then as a value. (a) 10^2 (b) 10^4 (c) 10^1 (d) 10^6

Q2. Fill the gaps in the table.

Power of 10	As a product	Value
10^2		
	$10 \times 10 \times 10 \times 10$	
		$100\,000$

Q3. Write each number in expanded notation using powers of 10, as in Figure 2. (a) 472 (b) 3905 (c) 60 040 (d) 8050 (e) 906 (f) 500 000

Q4. What number is each of these? (a) $3 \times 10^3 + 7 \times 10^2 + 5 \times 10^1$ (b) $6 \times 10^4 + 2 \times 10^2 + 8$ (c) $9 \times 10^3 + 4 \times 10^1$ (d) $2 \times 10^5 + 7$

Q5. Use a factor tree to write each number as a product of powers of primes. (a) 18 (b) 36 (c) 45 (d) 56

Q6. (a) Count on by multiples of 2 to list the powers of 2 from 2 up to 512, then fill the gap: $512 = 2^{\square}$. (b) Do the same for powers of 3 up to 729: $729 = 3^{\square}$. (c) Which is bigger: 2^7 or 5^3? Write both values to show how you know.

Unscaffolded

Q7. Write each of these as a single number. (a) 2^5 (b) 3^4 (c) 5^4 (d) $2^3 \times 3^2$

Q8. Write each product using exponent notation. (a) $10 \times 10 \times 10 \times 10 \times 10$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$ (c) $3 \times 3 \times 5 \times 5 \times 5$ (d) $7 \times 7 \times 7 \times 7$

Q9. The factor tree in Figure 9 has two gaps, A and B. Circles hold prime numbers.

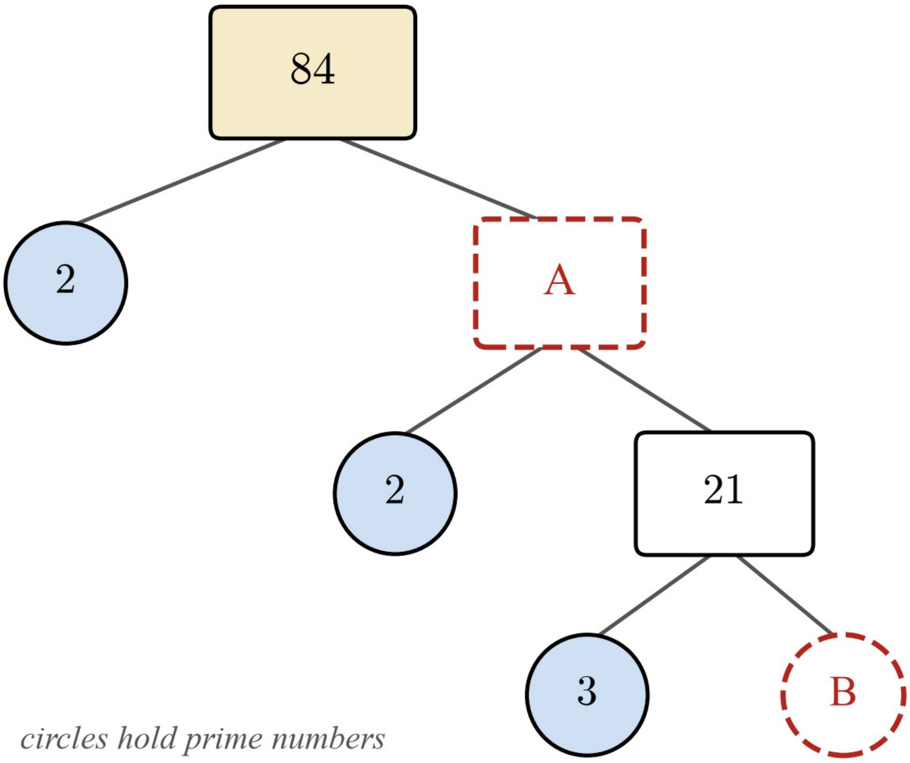


Figure 9. A factor tree for 84 with gaps: 84 splits into the prime 2 and a blank box A; A splits into the prime 2 and 21; 21 splits into the prime 3 and a blank circle B.

(a) Find A and B. (b) Use the leaves of the tree to write 84 as a product of powers of primes.

Q10. Is each number prime, composite, or neither? Give a reason. (a) 29 (b) 51 (c) 91 (d) 1

Q11. Write both numbers as products of primes, then find the HCF. (a) 12 and 20 (b) 18 and 45 (c) 24 and 36 (d) 14 and 25

Q12. Write both numbers as products of primes, then find the LCM. (a) 6 and 10 (b) 8 and 12 (c) 9 and 15 (d) 4 and 9

Q13. True or false? If false, correct the statement. (a) $10^4 = 40$ (b) $48 = 2^4 \times 3$ (c) The HCF of 8 and 9 is 1. (d) The LCM of 4 and 6 is 24.

Q14. Find the mistake and fix it. (a) Priya says $10^5 = 50$. (b) Tom says the LCM of 6 and 8 is 48.

Q15. Two runners start together at the same point on a track. One takes 4 minutes a lap and the other takes 6 minutes a lap, each keeping a steady pace. After how many minutes will they first be back at the start together?

Q16. A teacher has 24 rulers and 36 pencils. She wants to make up packets that are all the same, using every ruler and every pencil. What is the greatest number of packets she can make, and what goes in each?

Q17. The number $4 \times 10^3 + d \times 10^2 + 7$ has a missing hundreds digit d. (a) Find d if the number is 4507. (b) What is the smallest and the biggest number of this form, as d runs from 0 to 9?

Q18. (a) Write $2^3 \times 3^2$ as a single number. (b) Two natural numbers have an LCM of 12. One of them is 4. List every possible value of the other.

Answers

Q1. (a) $10 \times 10 = 100$ (b) $10 \times 10 \times 10 \times 10 = 10\,000$ (c) 10 (d) $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000$

Q2. Row 1: 10×10, 100. Row 2: 10^4, $10\,000$. Row 3: 10^5, $10 \times 10 \times 10 \times 10 \times 10$.

Q3. (a) $4 \times 10^2 + 7 \times 10^1 + 2$ (b) $3 \times 10^3 + 9 \times 10^2 + 5$ (c) $6 \times 10^4 + 4 \times 10^1$ (d) $8 \times 10^3 + 5 \times 10^1$ (e) $9 \times 10^2 + 6$ (f) 5×10^5

Q4. (a) 3750 (b) $60\,208$ (c) 9040 (d) $200\,007$

Q5. (a) 2×3^2 (b) $2^2 \times 3^2$ (c) $3^2 \times 5$ (d) $2^3 \times 7$

Q6. (a) $2, 4, 8, 16, 32, 64, 128, 256, 512$; $512 = 2^9$ (b) $3, 9, 27, 81, 243, 729$; $729 = 3^6$ (c) $2^7 = 128$ and $5^3 = 125$, so 2^7 is bigger.

Q7. (a) 32 (b) 81 (c) 625 (d) $8 \times 9 = 72$

Q8. (a) 10^5 (b) 2^6 (c) $3^2 \times 5^3$ (d) 7^4

Q9. (a) $A = 42$, $B = 7$ (b) $84 = 2^2 \times 3 \times 7$

Q10. (a) prime — only 1 and 29 divide it (b) composite — $51 = 3 \times 17$ (c) composite — $91 = 7 \times 13$ (d) neither — 1 has only one factor, so it is not prime, and it is not composite either

Q11. (a) 4 (b) 9 (c) 12 (d) 1

Q12. (a) 30 (b) 24 (c) 45 (d) 36

Q13. (a) false — $10^4 = 10\,000$ (b) true (c) true (d) false — the LCM of 4 and 6 is 12

Q14. (a) 10^5 means five factors of 10, so $10^5 = 100\,000$; 50 is 10×5, which is not what the exponent says. (b) 48 is *a* common multiple of 6 and 8, but not the lowest: the LCM is 24.

Q15. 12 minutes (the LCM of 4 and 6).

Q16. 12 packets, each with 2 rulers and 3 pencils.

Q17. (a) $d = 5$ (b) smallest 4007 (with $d = 0$); biggest 4907 (with $d = 9$)

Q18. (a) 72 (b) 3, 6 or 12

Fully-worked solutions

Q1. The exponent counts the factors of 10, and each factor adds one zero. (a) $10^2 = 10 \times 10 = 100$. (b) $10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$. (c) $10^1 = 10$ — one factor of 10 is just 10. (d) $10^6 = 1\,000\,000$, a 1 followed by six zeros.

Q2. Row 1: 10^2 is two factors of 10: $10 \times 10 = 100$. Row 2: four factors of 10 is $10^4 = 10\,000$. Row 3: $100\,000$ has five zeros, so it is $10^5 = 10 \times 10 \times 10 \times 10 \times 10$.

Q3. Split into column values, then write each as digit × power of 10, leaving out zero columns. (a) $472 = 400 + 70 + 2 = 4 \times 10^2 + 7 \times 10^1 + 2$. (b) $3905 = 3 \times 10^3 + 9 \times 10^2 + 5$. (c) $60\,040 = 6 \times 10^4 + 4 \times 10^1$. (d) $8050 = 8 \times 10^3 + 5 \times 10^1$. (e) $906 = 9 \times 10^2 + 6$. (f) $500\,000 = 5 \times 10^5$.

Q4. Work out each part, then add; missing powers are zero digits. (a) $3000 + 700 + 50 = 3750$. (b) $60\,000 + 200 + 8 = 60\,208$. (c) $9000 + 40 = 9040$. (d) $200\,000 + 7 = 200\,007$.

Q5. Factor to primes, then group equals. (a) $18 = 2 \times 9 = 2 \times 3 \times 3 = 2 \times 3^2$. (b) $36 = 6 \times 6 = (2 \times 3)(2 \times 3) = 2^2 \times 3^2$. (c) $45 = 9 \times 5 = 3^2 \times 5$. (d) $56 = 8 \times 7 = 2^3 \times 7$.

Q6. (a) Each power of 2 doubles the last: $2, 4, 8, 16, 32, 64, 128, 256, 512$. Counting the factors, $512 = 2^9$. (b) Each power of 3 triples the last: $3, 9, 27, 81, 243, 729$, so $729 = 3^6$. (c) $2^7 = 128$ and $5^3 = 125$; $128 > 125$, so 2^7 is bigger — a good reminder that a bigger base does not always mean a bigger power.

Q7. Multiply out. (a) $2^5 = 32$. (b) $3^4 = 81$. (c) $5^4 = 625$. (d) $2^3 \times 3^2 = 8 \times 9 = 72$.

Q8. Count the equal factors. (a) five 10s: 10^5. (b) six 2s: 2^6. (c) two 3s and three 5s: $3^2 \times 5^3$. (d) four 7s: 7^4.

Q9. (a) $84 = 2 \times A$, so $A = 42$; $21 = 3 \times B$, so $B = 7$, which is prime as the circle demands. (b) The leaves are $2, 2, 3, 7$, so $84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$. Check: $4 \times 3 \times 7 = 84$. ✓

Q10. (a) 29 is prime: no number between 2 and 5 divides it. (b) $51 = 3 \times 17$, composite (the digits add to 6, so 3 divides it). (c) $91 = 7 \times 13$, composite — a famous trap, since it *looks* prime. (d) 1 is neither: a prime has exactly two factors and a composite has more, but 1 has only one factor, itself.

Q11. Compare the prime rows and keep the shared factors. (a) $12 = 2 \times 2 \times 3$, $20 = 2 \times 2 \times 5$; shared $2, 2$ → HCF $= 4$. (b) $18 = 2 \times 3 \times 3$, $45 = 3 \times 3 \times 5$; shared $3, 3$ → HCF $= 9$. (c) $24 = 2 \times 2 \times 2 \times 3$, $36 = 2 \times 2 \times 3 \times 3$; shared $2, 2, 3$ → HCF $= 12$. (d) $14 = 2 \times 7$, $25 = 5 \times 5$; nothing shared → HCF $= 1$.

Q12. Take every factor needed. (a) $6 = 2 \times 3$, $10 = 2 \times 5$; LCM $= 2 \times 3 \times 5 = 30$. (b) $8 = 2^3$, $12 = 2^2 \times 3$; LCM $= 2 \times 2 \times 2 \times 3 = 24$. (c) $9 = 3^2$, $15 = 3 \times 5$; LCM $= 3 \times 3 \times 5 = 45$. (d) $4 = 2^2$, $9 = 3^2$; nothing shared, so LCM $= 4 \times 9 = 36$.

Q13. (a) False: $10^4 = 10\,000$, not 40 — the exponent is not multiplied on. (b) True: $2^4 \times 3 = 16 \times 3 = 48$. (c) True: $8 = 2^3$ and $9 = 3^2$ share no prime, so the HCF is 1. (d) False: 12 is a common multiple of 4 and 6, and no smaller number is, so the LCM is 12, not 24.

Q14. (a) Priya multiplied 10×5. The exponent 5 counts factors of 10: $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$. (b) Tom found *a* common multiple, not the lowest. $6 = 2 \times 3$ and $8 = 2 \times 2 \times 2$, so the LCM is $2 \times 2 \times 2 \times 3 = 24$; check $24 = 6 \times 4 = 8 \times 3$. ✓

Q15. They meet at the start when the minutes are a multiple of both 4 and 6; the first time is the LCM. $4 = 2 \times 2$, $6 = 2 \times 3$, so LCM $= 2 \times 2 \times 3 = 12$ minutes (the first runner has done 3 laps, the second 2).

Q16. Every packet gets the same number of rulers and the same number of pencils, so the number of packets must divide both 24 and 36; the greatest number is the HCF. $24 = 2 \times 2 \times 2 \times 3$, $36 = 2 \times 2 \times 3 \times 3$; shared $2, 2, 3$ → HCF $= 12$ packets, with $24 \div 12 = 2$ rulers and $36 \div 12 = 3$ pencils in each.

Q17. (a) $4507 = 4 \times 10^3 + 5 \times 10^2 + 7$, so $d = 5$. (b) The smallest is with $d = 0$: 4007; the biggest is with $d = 9$: 4907.

Q18. (a) $2^3 \times 3^2 = 8 \times 9 = 72$. (b) The other number must supply the factor 3 (since $4 = 2 \times 2$ has none), and it must not need any prime factor beyond $2 \times 2 \times 3$. Trying the options: LCM of 4 and 3 is 12 ✓; of 4 and $6 = 2 \times 3$ is 12 ✓; of 4 and $12 = 2 \times 2 \times 3$ is 12 ✓. Anything else fails (for example 8 gives LCM 8, and 9 gives LCM 36). So the other number is 3, 6 or 12.

Teacher note

- **Ownership.** This section owns VC2M7N02 and draws on all five elaborations (E01–E05). Primes/composites and place value beyond 10 000 are Level 6 ideas (VC2M6N02, VC2M6N04) and are recalled in a single sentence under D3, not retaught.
- **No exponent laws (D4.6).** Nowhere in this section are powers combined or index laws used — that work belongs to later levels. Every power is evaluated by straight multiplication ($2^3 \times 3^2 = 8 \times 9 = 72$), and

HCF/LCM rows are multiplied out as ordinary products ($2 \times 2 \times 3 \times 3 = 36$). Exponent notation appears only as shorthand for repeated equal factors, which is exactly what VC2M7N02 asks for.

- **Terminology (D12).** The superscript is always the *exponent* (never "index", and never "power" — *power* names the whole expression, e.g. 10^3). Numbers are *natural numbers*, never "counting numbers". *Base* and *exponent* and *composite* are defined at first use in the theory prose and in the vocabulary table.
- **Zero exponents avoided.** The ones column is written as the digit alone (or ×1 in Figure 2); 10^0 is never used, keeping every exponent here a natural number ≥ 1.
- **Elaboration coverage.** E01 → Figure 1, Q1–Q2; E02 → Figures 2 and 7, Example 1, Q3–Q4, Q17; E03 → Figures 3, 4 and 9, Example 2, Q5, Q9; E04 → Figure 5, Q6–Q7; E05 → Figures 6 and 8, Example 4, Q11–Q12, Q15–Q16, Q18. Elaborations are treated as depth signals: e.g. E04's sequences are built in Figure 5 and then used for spotting factors, not memorised as a drill.
- **HCF/GCD naming.** E05 names the greatest common divisor; this book uses the HCF wording familiar from Victorian primary years and lists it as "highest common factor (HCF)" in the vocabulary table. The GCD alias is deliberately left to this note only.
- **Cross-references.** 1C assumes the base/exponent language established here (its prose refers back to 1B for the 2 in 7^2); 2A reuses prime factorisation and the HCF for simplest form. Both forward pointers appear in "Where this leads".
- **Tools.** VC2M7N02 names no digital tools, so there is no calculator or digital strand here (D9); every value is small enough for mental or written multiplication.
- **First Nations content.** None, per Rule 1 of the content policy.
- **Assessment.** This section contributes 5 marks to the Chapter 1 test (T1) per the TOC assessment plan; Q9 and Q14 are the items written at test difficulty.
- **Figures.** All nine figures are generated by `src/gen_1b.py`; every numeric value in them (place columns, tree leaves, division ladders, HCF/LCM chips, tray dots) is computed in-script and asserted against the prose values.

Perfect square numbers and square roots

Content description `VC2M7N01`: describe the relationship between perfect square numbers and square roots, and use squares of numbers and square roots of perfect square numbers to solve problems

Achievement standard (extract): They solve problems involving squares of numbers and square roots of perfect square numbers.

Theory

Squares and the exponent notation. At Level 6 you met the square numbers, and subtopic 1B gave the notation this subtopic uses. To **square** a number means to multiply it by itself: the **square** of 7 is $7 \times 7 = 49$, written 7^2 and read "seven squared". The small raised 2 is called the **exponent**; it says that 7 is used twice as a factor, so 7^2 means 7×7, never 7×2. That is the most common mistake with squares — doubling the number instead of multiplying it by itself: $7^2 = 49$, while $7 \times 2 = 14$ is something quite different. In this subtopic we square the **natural numbers** $1, 2, 3, 4, \ldots$.

Why they are called square numbers. A square number can be drawn as a square. Figure 1 shows the dots of n^2 sitting in n rows of n: one dot makes a 1×1 square, four dots a 2×2 square, nine dots a 3×3 square, and so on. A number whose dots cannot be arranged like this — 20, for instance — is not a square number.

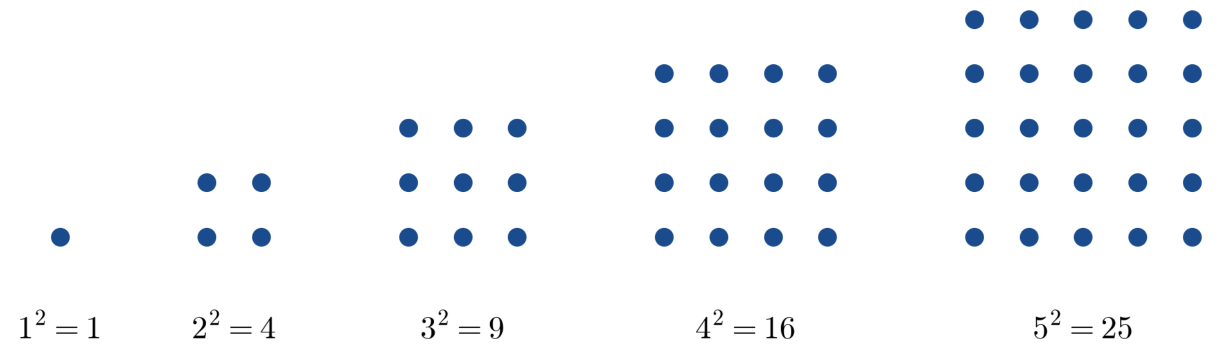


Figure 1. Square dot patterns: one dot for 1 squared; two rows of two for 2 squared = 4; three rows of three for 3 squared = 9; four rows of four for 4 squared = 16; five rows of five for 5 squared = 25.

You are asked to know the squares of the natural numbers from 1 to 20. The table is worth learning, because square roots read straight off it.

n	1	2	3	4	5	6	7	8	9	10
n^2	1	4	9	16	25	36	49	64	81	100

n	11	12	13	14	15	16	17	18	19	20
n^2	121	144	169	196	225	256	289	324	361	400

Square roots undo squares. The **square root** of a number asks the question the other way round: *which natural number, multiplied by itself, gives this?* The answer is written with the sign $\sqrt{\square}$, so $\sqrt{49} = 7$ because $7 \times 7 = 49$, read "the square root of 49 is 7". A **perfect square number** — usually just a **square number** — is the square of a natural number: $1, 4, 9, 16, \ldots, 400$. Every perfect square has a natural number as its square root, and numbers like 20 or 43 do not.

The relationship runs in both directions, and each operation undoes the other (Figure 2). Square 7 and you get 49; take the square root of 49 and you are back at 7:

$$\left(\sqrt{49}\right)^2 = 49 \sqrt{7^2} = 7$$

So every square fact you know is also a square root fact: because $12^2 = 144$, you know $\sqrt{144} = 12$ without any extra work.

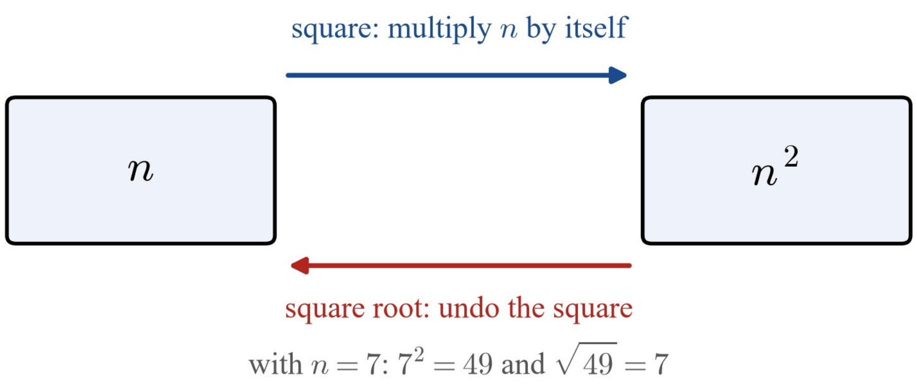


Figure 2. A two-way diagram: an arrow from n to n squared labelled "square: multiply n by itself", and an arrow back from n squared to n labelled "square root: undo the square", with the example 7 squared = 49 and the square root of 49 = 7 underneath.

Where does the square root of a number that is not a perfect square lie? There is no natural number whose square is 43, but the square root of 43 can still be pinned down. Look for the perfect squares either side of 43: they are 36 and 49. Because $36 < 43 < 49$, taking the square root of all three keeps them in the same order:

$$\sqrt{36} < \sqrt{43} < \sqrt{49} \text{ so } 6 < \sqrt{43} < 7$$

so $\sqrt{43}$ lies between the consecutive natural numbers 6 and 7 (Figure 3). The same trick works for any number: $\sqrt{90}$ lies between 9 and 10 because $81 < 90 < 100$.

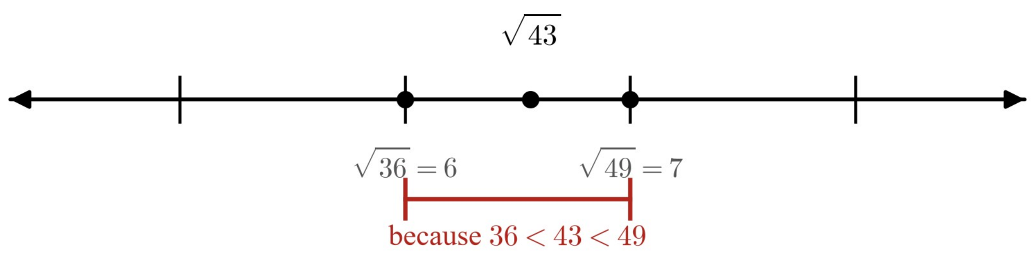


Figure 3. A number line with 6 and 7 marked as the square root of 36 and the square root of 49; the square root of 43 sits between them, because 36 is less than 43 and 43 is less than 49.

Squaring a two-digit number with an area diagram. The area model of Figure 4 turns any two-digit square into easy pieces. Split 43 into $40 + 3$; a square with side 43 then breaks into four parts with areas $40 \times 40 = 1600$, $40 \times 3 = 120$, $3 \times 40 = 120$ and $3 \times 3 = 9$. The whole area is the four parts added together:

$$43^2 = (40 + 3)^2 = 40^2 + 2 \times 40 \times 3 + 3^2 = 1600 + 240 + 9 = 1849$$

The method works for every two-digit number: split it into tens and ones, square each piece, and add the two matching rectangles twice.

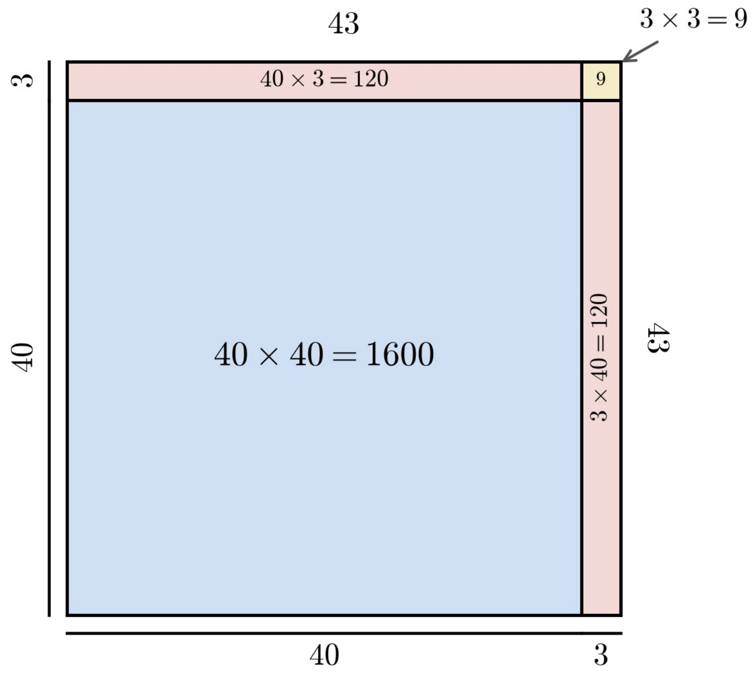


Figure 4. A square of side 43 split into parts 40 and 3 along each side; the four regions are labelled with their areas, 1600, 120, 120 and 9, which add to 1849.

Patterns in the square numbers. Listing the squares and their differences (Figure 6) brings two patterns to the surface. The differences between consecutive squares are $3, 5, 7, 9, 11, \ldots$ — the odd numbers in order. Figure 5 shows why: growing a 4×4 square of dots into a 5×5 square adds a strip of $2 \times 4 + 1 = 9$ new dots along two sides of the old square, and every step up adds the next odd number. The second difference — the difference between those differences — is always 2. That constant second difference is the fingerprint of squaring.

A third pattern makes a quick check. The last digits of $1^2, 2^2, \ldots, 10^2$ are $1, 4, 9, 6, 5, 6, 9, 0, 1, 4$, and the pattern repeats for every larger number: a square can only end in $0, 1, 4, 5, 6$ or 9. A number ending in $2, 3, 7$ or 8 — say 4392 — cannot be a perfect square, no checking needed.

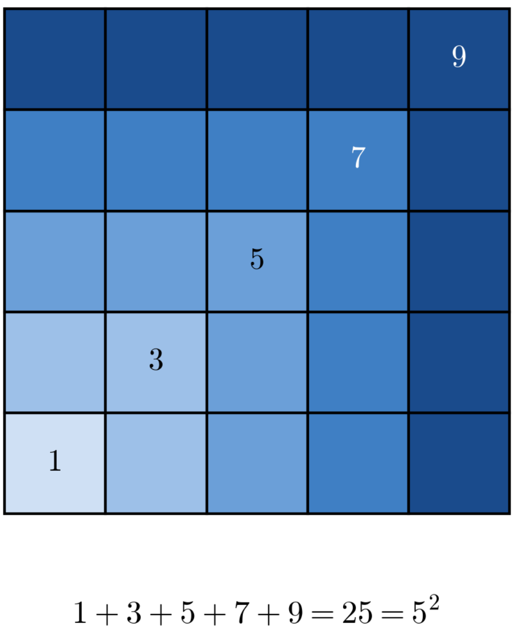


Figure 5. A 5 by 5 grid of cells shaded in strips of 1, 3, 5, 7 and 9 cells, each strip along two sides of the square before it, showing 1 + 3 + 5 + 7 + 9 = 25 = 5 squared.

n	1	2	3	4	5	6	7	8	9	10	
n^2	1	4	9	16	25	36	49	64	81	100	
difference	3	5	7	9	11	13	15	17	19		
2nd difference	2	2	2	2	2	2	2	2			← the same every time

Figure 6. A difference table for the square numbers 1 to 100: the differences between consecutive squares are 3, 5, 7 and so on up to 19, and every second difference is 2.

Squares, roots and side lengths. Because the area of a square is its side squared, a square root turns an area back into a side length: a square floor covered by 81 square tiles has 9 tiles along each side, since $\sqrt{81}=9$, and then $9\times4=36$ tile lengths around the edge. Worked example 4 and Q12 use this link.

The words of this subtopic, with synonyms you may meet:

Term	Meaning here	Synonyms and related words
square (of a number)	the number multiplied by itself; $7^2=49$	“7 squared”
perfect square number	the square of a natural number: $1,4,9,16,\ldots$	square number
square root	the natural number that, multiplied by itself, gives the original; $\sqrt{49}=7$	“$\sqrt{\square}$”
exponent	the small raised number saying how many times the base is used as a factor (from 1B)	the 2 in 7^2
consecutive	following one after the other, as 6 and 7	“in a row”

The two-way link between a square and its square root returns in Chapter 4, wherever an area has to be turned back into a side length.

Worked examples

Example 1 — scaffolded

(a) Write 9^2 as a product and find its value. (b) Find $\sqrt{64}$. (c) Of 20, 25 and 30, which are perfect square numbers?

Working	Comment
(a) $9^2=9\times9=81$	The exponent 2 says to use 9 twice as a factor — not to double it.
(b) $\sqrt{64}=8$, because $8\times8=64$	The square root undoes the square: $8^2=64$.
(c) 25 is a perfect square, because $5^2=25$. 20 and 30 are not: no natural number squared gives either.	Check each number against the table of squares.

Example 2 — scaffolded

Use the area method of Figure 4 to find 43^2.

Working	Comment
Split 43 into $40+3$ and draw the square with side 43, as in Figure 4.	The area of the whole square is 43^2.
Parts: $40\times40=1600$; $40\times3=120$; $3\times40=120$; $3\times3=9$.	Each part is a rectangle, so its area is length × width.
$43^2=1600+120+120+9=1849$	The whole area is the four parts added together.

$\therefore 43^2=1849$.

Example 3 — unscaffolded

Find the two consecutive natural numbers that $\sqrt{43}$ lies between.

The perfect squares either side of 43 are 36 and 49: $6^2=36$ and $7^2=49$. Because $36<43<49$,

$$\sqrt{36}<\sqrt{43}<\sqrt{49}\text{ so }6<\sqrt{43}<7,$$

as Figure 3 shows to scale. $\therefore \sqrt{43}$ lies between 6 and 7.

Example 4 — unscaffolded

A square floor is covered by 81 square tiles of the same size. (a) How many tiles lie along one side? (b) Find the perimeter of the floor in tile lengths.

(a) The floor is a square of tiles, so the number of tiles along a side is the number whose square is 81: $\sqrt{81}=9$, as $9\times9=81$. There are 9 tiles along each side.

(b) A square has four equal sides, so the perimeter is $4\times9=36$ tile lengths, shown in red in Figure 7.

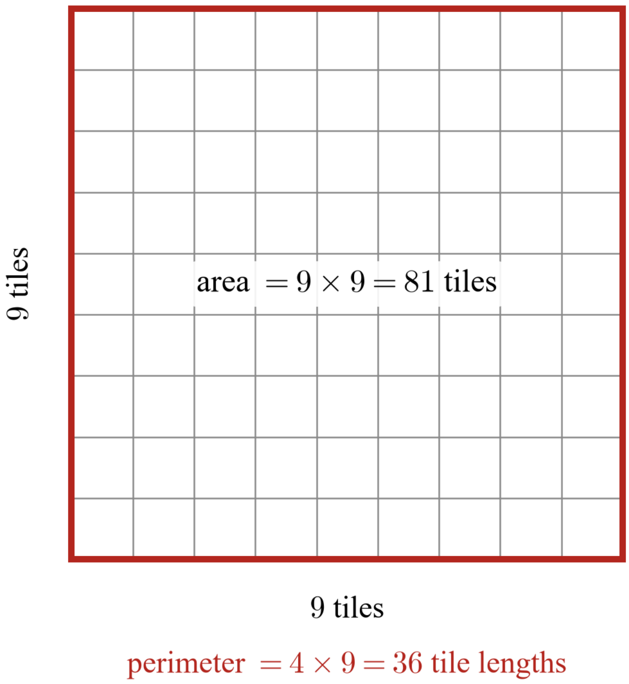


Figure 7. A square floor of nine rows of nine square tiles, 81 tiles in all; each side is 9 tiles long and the perimeter is marked as 4 times 9 = 36 tile lengths.

$\therefore$ the floor has 9 tiles along each side and a perimeter of 36 tile lengths.

Practice questions

Scaffolded

Q1. Write each square as a product and find its value. (a) 5^2 (b) 8^2 (c) 11^2 (d) 15^2 (e) 20^2

Q2. Use the dot patterns of Figure 1. (a) How many dots are in the dot pattern for 6^2? (b) A dot pattern of 49 dots is the square of which natural number? (c) In one sentence, say why square numbers have their name.

Q3. Find the value of each square root. (a) $\sqrt{25}$ (b) $\sqrt{49}$ (c) $\sqrt{81}$ (d) $\sqrt{100}$ (e) $\sqrt{144}$ (f) $\sqrt{196}$

Q4. Fill in the missing number. (a) $7^2=\_\_\_$ (b) $\_\_\_^2=121$ (c) $\sqrt{\_\_\_}=9$ (d) $\left(\sqrt{64}\right)^2=\_\_\_$

Q5. Pick out the perfect square numbers from the list, and write the square root of each one you pick: $16, 24, 36, 50, 64, 90$.

Q6. State whether each statement is true or false. Correct every false statement. (a) $\sqrt{81}=9$ (b) $12^2=24$ (c) $\left(\sqrt{25}\right)^2=25$ (d) $\sqrt{1}=0$ (e) 36 is a perfect square number. (f) 40 is a perfect square number.

Unscaffolded

Q7. Calculate. (a) 6^2+8^2 (b) 13^2-5^2 (c) 4×9^2 (d) $\left(\sqrt{144}\right)^2+1$

Q8. Use the tens-and-ones split of Worked example 2 to find each square. (a) 27^2 (b) 56^2 (c) 73^2

Q9. Find the two consecutive natural numbers each square root lies between. (a) $\sqrt{20}$ (b) $\sqrt{60}$ (c) $\sqrt{90}$ (d) $\sqrt{150}$ (e) $\sqrt{3}$ (f) $\sqrt{400}$

Q10. Use the squares $1, 4, 9, 16, 25, 36, 49, 64, 81, 100$ and Figure 6. (a) The differences between consecutive squares begin $3, 5, 7$. Write the next four differences. (b) What do you notice about these differences? (c) Find the differences between the differences. What do you notice? (d) Use the pattern to find 11^2 and 12^2 without multiplying 11×11 and 12×12.

Q11. (a) Write the last digit of each of $1^2, 2^2, 3^2, \ldots, 10^2$. (b) Which of the digits 0–9 never appear as the last digit of a square number? (c) Can 4392 be a perfect square number? Explain your answer. (d) Can 1369 be a perfect square number? Explain your answer.

Q12. A square floor is covered by square tiles of the same size (Figure 8). (a) The floor uses 144 square tiles. How many tiles lie along one side? (b) Find the perimeter of the floor in tile lengths. (c) A second square floor has a perimeter of 52 tile lengths. How many square tiles cover it?

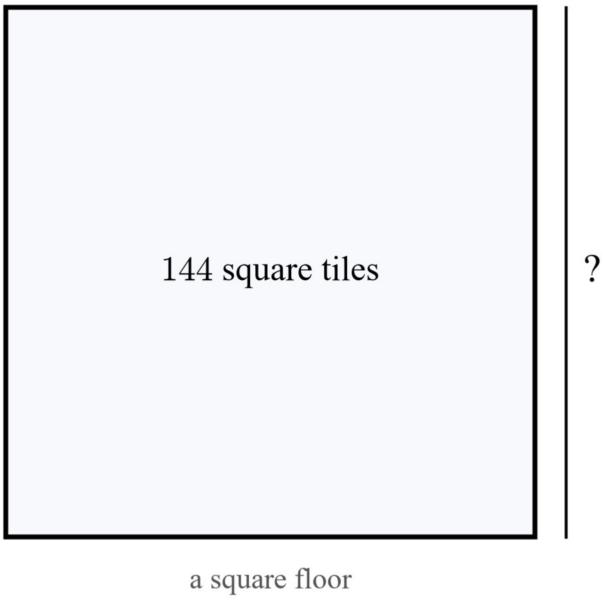


Figure 8. A square floor covered by 144 square tiles, with a question mark on one side standing for the unknown number of tiles along that side.

Q13. A square garden bed has an area of 25 m². (a) Find the length of one side of the bed. (b) Find the perimeter of the bed.

Q14. (a) A calculator gives $27^2 = 729$. Use the squares of 20 and 30 to explain why 729 is a sensible answer. (b) Tom says $\sqrt{80}$ lies between 8 and 9. Is he correct? Explain.

Q15. Find the missing number. (a) $n^2 = 225$ (b) $\sqrt{n} = 13$ (c) $\left(\sqrt{n}\right)^2 = 49$ (d) $\sqrt{n} = n$, for a natural number n.

Q16. Write 4^2, $\sqrt{9}$, 2^2, $\sqrt{100}$ and 3^2 in order, from smallest to largest.

Q17. Chairs for a school concert are set out in a square: the same number of rows as columns. (a) There are 169 chairs. How many rows are there? (b) Five more chairs arrive, making 174 in all. Can 174 chairs be set out in a complete square? Explain.

Q18. (a) Find two perfect square numbers whose sum is 25. (b) Find two perfect square numbers whose difference is 7. (c) Show that 50 can be written as the sum of two perfect square numbers.

Answers

Q1. (a) 25; (b) 64; (c) 121; (d) 225; (e) 400.

Q2. (a) 36; (b) 7; (c) because the dots of a square number can be arranged in a square, n rows of n.

Q3. (a) 5; (b) 7; (c) 9; (d) 10; (e) 12; (f) 14.

Q4. (a) 49; (b) 11; (c) 81; (d) 64.

Q5. 16 (root 4), 36 (root 6), 64 (root 8).

Q6. (a) true; (b) false — $12^2=144$; (c) true; (d) false — $\sqrt{1}=1$; (e) true; (f) false — the squares either side of 40 are 36 and 49.

Q7. (a) 100; (b) 144; (c) 324; (d) 145.

Q8. (a) 729; (b) 3136; (c) 5329.

Q9. (a) 4 and 5; (b) 7 and 8; (c) 9 and 10; (d) 12 and 13; (e) 1 and 2; (f) none — $\sqrt{400}=20$ exactly.

Q10. (a) $11, 13, 15, 17$; (b) they are the odd numbers in order, each 2 more than the last; (c) all 2 — the second difference is constant; (d) 121 and 144.

Q11. (a) $1, 4, 9, 6, 5, 6, 9, 0, 1, 4$; (b) $2, 3, 7, 8$; (c) no — it ends in 2; (d) possibly — it ends in 9 (and in fact $37^2=1369$).

Q12. (a) 12; (b) 48; (c) 169.

Q13. (a) 5 m; (b) 20 m.

Q14. (a) $20<27<30$, so $400<27^2<900$, and 729 sits between 400 and 900; (b) yes — $64<80<81$.

Q15. (a) 15; (b) 169; (c) 49; (d) 1.

Q16. $\sqrt{9}, 2^2, 3^2, \sqrt{100}, 4^2$.

Q17. (a) 13; (b) no — $169<174<196$, and 174 is not a perfect square.

Q18. (a) 9 and 16; (b) 16 and 9; (c) $25+25=50$.

Fully-worked solutions

Q1. Each square is the number used twice as a factor. (a) $5^2=5\times5=25$. (b) $8^2=8\times8=64$. (c) $11^2=11\times11=121$. (d) $15^2=15\times15=225$. (e) $20^2=20\times20=400$.

Q2. (a) The pattern for 6^2 has 6 rows of 6, so 36 dots. (b) $49=7\times7$, so the pattern is 7 rows of 7: the number is 7. (c) They are called square numbers because their dots can be arranged as a square, n rows of n.

Q3. Each square root reads off the table of squares. (a) 5, since $5^2=25$. (b) 7, since $7^2=49$. (c) 9, since $9^2=81$. (d) 10, since $10^2=100$. (e) 12, since $12^2=144$. (f) 14, since $14^2=196$.

Q4. (a) $7^2=7\times7=49$. (b) 11, because $11\times11=121$. (c) 81, because $\sqrt{81}=9$. (d) Squaring undoes the square root, so $\left(\sqrt{64}\right)^2=64$.

Q5. Against the table of squares: $16=4^2$, $36=6^2$ and $64=8^2$ are perfect squares, with square roots 4, 6 and 8. The others are not squares: 24 sits between 16 and 25, 50 between 49 and 64, and 90 between 81 and 100.

Q6. (a) **True**, as $9 \times 9 = 81$. (b) **False**: $12^2 = 12 \times 12 = 144$, not 24 — that is 12×2. (c) **True**: squaring undoes the square root. (d) **False**: $\sqrt{1} = 1$, since $1 \times 1 = 1$. (e) **True**, as $6^2 = 36$. (f) **False**: 40 lies between the squares 36 and 49, so it is not a perfect square.

Q7. (a) $6^2 + 8^2 = 36 + 64 = 100$. (b) $13^2 - 5^2 = 169 - 25 = 144$. (c) $4 \times 9^2 = 4 \times 81 = 324$. (d) $(\sqrt{144})^2 + 1 = 144 + 1 = 145$. (Note that (a) and (b) come out as square numbers themselves: $100 = 10^2$ and $144 = 12^2$.)

Q8. (a) $27 = 20 + 7$: $27^2 = 20^2 + 2 \times 20 \times 7 + 7^2 = 400 + 280 + 49 = 729$. (b) $56 = 50 + 6$: $56^2 = 50^2 + 2 \times 50 \times 6 + 6^2 = 2500 + 600 + 36 = 3136$. (c) $73 = 70 + 3$: $73^2 = 70^2 + 2 \times 70 \times 3 + 3^2 = 4900 + 420 + 9 = 5329$.

Q9. Trap the number between the perfect squares either side of it. (a) $16 < 20 < 25$, so $\sqrt{20}$ is between 4 and 5. (b) $49 < 60 < 64$: between 7 and 8. (c) $81 < 90 < 100$: between 9 and 10. (d) $144 < 150 < 169$: between 12 and 13. (e) $1 < 3 < 4$: between 1 and 2. (f) 400 **is** a perfect square, $\sqrt{400} = 20$, so it does not lie between two consecutive natural numbers at all.

Q10. (a) The differences continue $11, 13, 15, 17$ (check: $36 - 25 = 11$, $49 - 36 = 13$, $64 - 49 = 15$, $81 - 64 = 17$). (b) They are the odd numbers in order — each difference is 2 more than the one before. (c) $5 - 3 = 2$, $7 - 5 = 2$, and so on: every second difference is 2, the constant second difference. (d) The difference after 19 is 21, so $11^2 = 100 + 21 = 121$; the next is 23, so $12^2 = 121 + 23 = 144$.

Q11. (a) $1, 4, 9, 6, 5, 6, 9, 0, 1, 4$. (b) $2, 3, 7$ and 8 never appear. (c) No: 4392 ends in 2, and no square number ends in 2. (d) Possibly: 1369 ends in 9, which a square can end in — the last-digit check cannot rule it out (and in fact $37^2 = 1369$).

Q12. (a) The number of tiles along a side is the number whose square is 144: $\sqrt{144} = 12$. (b) The perimeter is $4 \times 12 = 48$ tile lengths. (c) A perimeter of 52 tile lengths means a side of $52 \div 4 = 13$ tiles, so the floor holds $13^2 = 169$ tiles.

Q13. (a) The side squared is the area, so the side is $\sqrt{25} = 5$ m. (b) The perimeter is $4 \times 5 = 20$ m.

Q14. (a) 27 lies between 20 and 30, so 27^2 lies between $20^2 = 400$ and $30^2 = 900$; 729 is between 400 and 900, so it is a sensible answer. (b) Yes: $8^2 = 64$ and $9^2 = 81$, and $64 < 80 < 81$, so $\sqrt{80}$ lies between 8 and 9.

Q15. (a) $15^2 = 225$, so $n = 15$. (b) If $\sqrt{n} = 13$ then $n = 13^2 = 169$. (c) Squaring undoes the square root, so $n = 49$. (d) $1 \times 1 = 1$, so $\sqrt{1} = 1$ and $n = 1$.

Q16. The five values are $4^2 = 16$, $\sqrt{9} = 3$, $2^2 = 4$, $\sqrt{100} = 10$ and $3^2 = 9$. In order: $3, 4, 9, 10, 16$, so $\sqrt{9} < 2^2 < 3^2 < \sqrt{100} < 4^2$.

Q17. (a) The number of rows is the number whose square is 169: $\sqrt{169} = 13$ rows. (b) No: $13^2 = 169$ and $14^2 = 196$, and 174 sits between them, so 174 is not a perfect square and the chairs cannot make a complete square.

Q18. (a) $9 + 16 = 25$, and $9 = 3^2$, $16 = 4^2$. (b) $16 - 9 = 7$, the difference of the same two squares. (c) $25 + 25 = 50$, and $25 = 5^2$, so 50 is the sum of two perfect squares.

Teacher note

- **Ownership.** This subtopic owns `VC2M7N01` alone: the two-way relationship between perfect squares and square roots, squaring natural numbers (to 20), the area-model square of a two-digit number, locating a square root between consecutive natural numbers, the patterns of E04, and the tile-floor link of E05. Square numbers are recalled from `VC2M6N02` (D3) in one sentence, not re-taught; the exponent notation is recalled from 1B per the Chapter 1 order (TOC §3).

- **The relationship in both directions.** The CD's verb is *describe*, so the inverse link is the centre of the subtopic (theory paragraph 3, Figure 2, WE1, Q4, Q15, Q16), not one fact among many: every square fact is taught as also being a square root fact.
- **No irrational numbers (D4.5).** $\sqrt{43}$ is located between 6 and 7 exactly as `VC2M7N01.E03` invites, and never named irrational — that classification is `VC2M8N01` (Level 8). The prose says there is no *natural* number whose square is 43 and goes no further.
- **No combining of powers (D4.6).** Exponents appear only as the notation for $n \times n$; no exponent law is used or stated anywhere in the subtopic.
- **Terminology (D12).** The superscript is named the **exponent** (never *index*, and not *power*), and the numbers squared are the **natural numbers** (never *counting numbers*).
- **Elaboration coverage (D14).** E01: Figure 1 and the 1–20 table (Q1–Q2). E02: Figure 4, WE2, Q8 — the VCAA $43^2 = 1849$ split is used as the depth signal it is. E03: Figure 3, WE3, Q9. E04: Figures 5–6, Q10–Q11, including the constant second difference. E05: Figures 7–8, WE4, Q12 — the elaboration's 144-tile, 48-length pair is set as a question so the method is used, not copied. The subtopic is not structured as a walk through E01–E05.
- **Perimeter of a square** is assumed from primary years and maintained in WE4 and Q12–Q13 because E05 needs it; it is not taught as content (no Level 7 CD owns it).
- **No digital tools.** `VC2M7N01` names none, so per D9 no calculator strand appears; Q14(a) uses a calculator's output as the thing to be checked, which is the reasonableness habit (P7), not a tool requirement.
- **No First Nations content.** None of `VC2M7N01.E01–.E05` names Aboriginal or Torres Strait Islander knowledges, so nothing is excluded under the content policy for this subtopic.
- **Assessment.** This subtopic contributes 5 marks to T1 (Ch1, 15 marks, 20 min); the item shapes above (two-way relationship, locating a root, the area method, patterns) are what those marks sample.
- **Figures.** All eight figures are generated by `src/gen_1c.py` from the exact values stated in the text: dot counts are n^2, the area diagram is split at $40 + 3$ with true part areas, the root line places $\sqrt{43}$ at its computed value, the difference table is built from the squares themselves, and the floors are true squares.

Test

Mathematics Level 7 · Chapter 1 — Whole numbers: integers, powers and square numbers · Chapter test T1

Marks	15 (5 marks for each subtopic)
Time	20 minutes: 5 minutes reading time, then about 15 minutes working time (1 minute per mark)
Calculators	Not permitted — the content descriptions of Chapter 1 name no digital tools, so every item is to be worked out mentally or in writing
Equipment	None — pencil and paper only

This is a classroom check, not an exam.

Instructions

- Answer every question, and show your working clearly.
- The marks for each question or part are shown in brackets.

Subtopic	Content description	Marks
1A Comparing, ordering, adding and subtracting integers	`VC2M7N08`	5
1B Expanded notation with powers of 10; products of powers of primes	`VC2M7N02`	5
1C Perfect square numbers and square roots	`VC2M7N01`	5
Total		**15**

1A · Comparing, ordering, adding and subtracting integers

Q1. Write the integers 6, -9, -1, 4, -5 in ascending order. (1 mark)

Q2. Copy each statement and replace the blank with < or > to make it true. (1 mark)

(a) -7 ___ 3
(b) -8 ___ -12
(c) 0 ___ -4

Q3. A fridge is kept at $-3\,°C$. Its freezer compartment is $12\,°C$ colder than the fridge. (3 marks)

(a) Find the temperature of the freezer compartment. (1 mark)
(b) The kitchen the fridge stands in is at $21\,°C$. Find the difference between the kitchen temperature and the freezer temperature. (1 mark)
(c) During a power blackout the freezer compartment warms from its temperature in part (a) to $-8\,°C$. Find the rise in temperature. (1 mark)

1B · Expanded notation with powers of 10; products of powers of primes

Q4. Write 7049 in expanded notation using powers of 10. (1 mark)

Q5. (2 marks)

(a) Write 90 as a product of powers of primes. Show your working. (1 mark)
(b) Write $2^3 \times 3 \times 5^2$ as a single number. (1 mark)

Q6. A teacher has 30 pencils and 42 erasers. She makes up prizes that are all the same, using every pencil and every eraser. (2 marks)

(a) What is the greatest number of prizes she can make? (1 mark)
(b) How many pencils and how many erasers go in each prize? (1 mark)

1C · Perfect square numbers and square roots

Q7. (2 marks)

(a) Evaluate 17^2, showing your working. (1 mark)
(b) Use your answer to part (a) to write down $\sqrt{289}$ with no further multiplying. (1 mark)

Q8. Between which two consecutive natural numbers does $\sqrt{70}$ lie? Name the two perfect squares you used. (1 mark)

Q9. A square floor is covered by 225 square tiles of the same size. (2 marks)

(a) How many tiles lie along one side of the floor? (1 mark)
(b) Find the perimeter of the floor in tile lengths. (1 mark)

Solutions

Answer key

Q	Answer	Marks
1	$-9, -5, -1, 4, 6$	1
2	(a) $<$ · (b) $>$ · (c) $>$	1
3	(a) $-15\,°C$ · (b) $36\,°C$ · (c) $7\,°C$	3
4	$7049 = 7 \times 10^3 + 4 \times 10^1 + 9$	1
5	(a) $90 = 2 \times 3^2 \times 5$ · (b) 600	2
6	(a) 6 prizes · (b) 5 pencils and 7 erasers in each	2
7	(a) 289 · (b) 17	2
8	8 and 9 (using 64 and 81)	1
9	(a) 15 tiles · (b) 60 tile lengths	2
	Total	**15**

Fully worked solutions

Q1. The negative integers come first, most negative first: on the number line -9 lies to the left of -5, -5 to the left of -1, and every negative lies to the left of every positive. ∴ ascending order: $-9, -5, -1, 4, 6$.

Q2. The wide open end of the sign always faces the larger number. (a) Every negative is less than every positive: $-7 < 3$. (b) On the negative side, -8 lies to the right of -12 — closer to zero — so $-8 > -12$. (c) 0 lies to the right of every negative, so $0 > -4$. ∴ (a) $<$; (b) $>$; (c) $>$.

Marks: 1 mark for all three correct.

Q3. (a) $12\,°C$ colder means 12 less: $-3 - 12 = -15$. The freezer compartment is at $-15\,°C$. (b) The difference is the gap on the number line, found by subtracting the smaller from the larger: $21 - (-15) = 21 + 15 = 36$. The difference between the two temperatures is $36\,°C$. (c) The rise is the gap from -15 up to -8: $-8 - (-15) = -8 + 15 = 7$. The temperature rose $7\,°C$.

Marks: 1 mark for each part.

Q4. Read off the column values: 7 thousands, 0 hundreds, 4 tens, 9 ones. The zero hundreds adds nothing, so that part is left out. ∴ $7049 = 7 \times 10^3 + 4 \times 10^1 + 9$.

Q5. (a) Split 90 into factors until every branch ends in a prime: $90 = 9 \times 10 = 3 \times 3 \times 2 \times 5$. The two equal factors 3 are grouped as 3^2, so $90 = 2 \times 3^2 \times 5$. Check by multiplying back: $2 \times 9 \times 5 = 90$. ✓ (b) Evaluate each power first: $2^3 = 2 \times 2 \times 2 = 8$ and $5^2 = 5 \times 5 = 25$, so $2^3 \times 3 \times 5^2 = 8 \times 3 \times 25 = 24 \times 25 = 600$. ∴ (a) $90 = 2 \times 3^2 \times 5$; (b) 600.

Marks: 1 mark for each part. In part (a), accept any correct working that reaches the same product of powers of primes, such as repeated division.

Q6. Every prize gets the same number of pencils and the same number of erasers, and nothing is left over, so the number of prizes must divide both 30 and 42; the greatest possible number is the highest common factor (HCF). Write both numbers as products of primes: $30 = 2 \times 3 \times 5$ and $42 = 2 \times 3 \times 7$. The factors the two numbers share are 2 and 3, so the HCF is $2 \times 3 = 6$. (a) The greatest number of prizes is 6. (b) $30 \div 6 = 5$ pencils and $42 \div 6 = 7$ erasers in each prize. Check: $6 \times 5 = 30$ and $6 \times 7 = 42$, so every pencil and every eraser is used. ✓ ∴ (a) 6 prizes; (b) 5 pencils and 7 erasers in each.

Marks: 1 mark for part (a); 1 mark for both numbers in part (b).

Q7. (a) The exponent 2 says to use 17 twice as a factor — not to double it: $17^2 = 17 \times 17 = 289$. (Splitting $17 = 10 + 7$ also works: $10^2 + 2 \times 10 \times 7 + 7^2 = 100 + 140 + 49 = 289$.) (b) The square root undoes the square: because $17^2 = 289$, $\sqrt{289} = 17$ with no further multiplying. ∴ (a) 289; (b) 17.

Marks: 1 mark for each part.

Q8. Trap 70 between the perfect squares either side of it: $8^2 = 64$ and $9^2 = 81$. Because $64 < 70 < 81$, taking square roots keeps the order: $\sqrt{64} < \sqrt{70} < \sqrt{81}$, so $8 < \sqrt{70} < 9$. ∴ $\sqrt{70}$ lies between the consecutive natural numbers 8 and 9, using the perfect squares 64 and 81.

Q9. (a) The tiles form a square, so the number of tiles along one side is the number whose square is 225: $\sqrt{225} = 15$, since $15 \times 15 = 225$. There are 15 tiles along each side. (b) A square has four equal sides, so the perimeter is $4 \times 15 = 60$ tile lengths. ∴ (a) 15 tiles; (b) 60 tile lengths.

Marks: 1 mark for each part.