Mathematics Level 6

Chapter 8 — Probability

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Contents

Describing probabilities with fractions, decimals and percentages

Content description `VC2M6P01`: describe probabilities using fractions, decimals and percentages; recognise that probabilities lie on numerical scales of 0–1 or 0%–100%; use estimation to assign probabilities that events occur in a given context, using common fractions, percentages and decimals

In Year 5 you described chance with words — likely, unlikely, certain, impossible. Those words point the right way, but they are rough: two people can hear "likely" and picture two different chances. This subtopic replaces the rough words with numbers. You will describe every probability as a fraction, a decimal and a percentage, place it on a scale from 0 to 1, and use counting and estimation to assign probabilities in everyday situations.

Theory

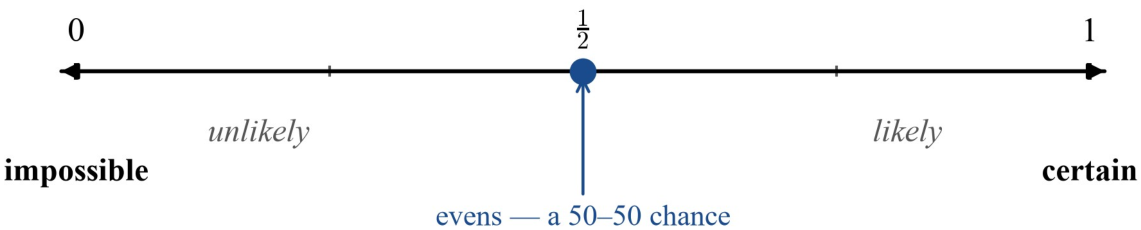
From chance words to numbers. You already know the language of chance. Some things **cannot happen**; some things **will definitely happen**; most things sit somewhere in between. Chance words sort events roughly, but they cannot settle a disagreement — "likely" is not exact. Mathematics does better. The **probability** of an event is a number that measures how likely the event is to happen. Because it is a number, it can be compared, ordered, written in different forms and placed on a scale.

It is written with the letter P in front of the event: P(rolling a 5) means "the probability of rolling a 5".

The probability scale from 0 to 1

Every probability lives on one scale. The scale runs from 0 to 1. Where a probability sits tells you how likely the event is.

The probability scale



Probabilities closer to 0 are less likely to happen; probabilities closer to 1 are more likely.

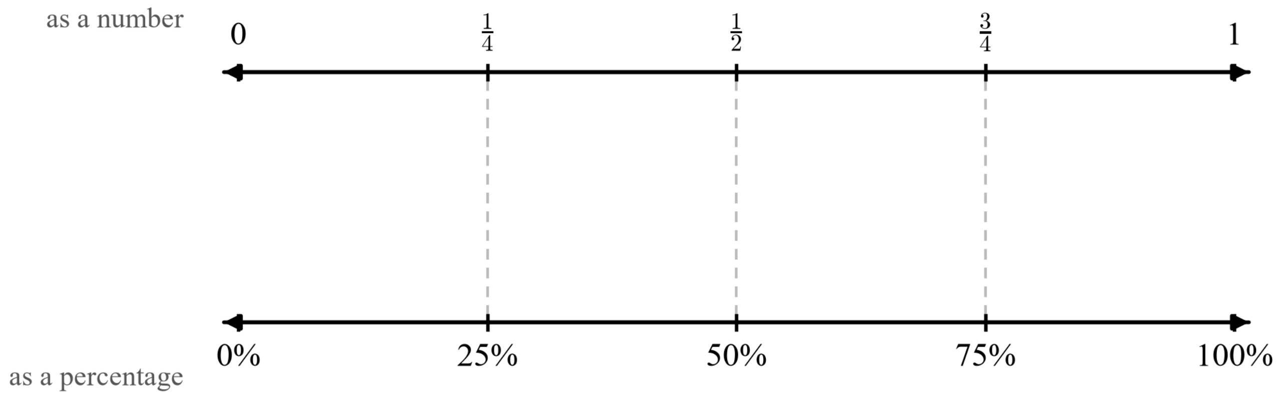
A number line from 0 to 1 labelled the probability scale, with impossible at 0, certain at 1, evens at one half, and the words unlikely and likely over the lower and upper halves

- **0** means the event is **impossible** — it cannot happen. Rolling a 7 on a normal die has probability 0.
- **1** means the event is **certain** — it must happen. Rolling a number less than 7 on a normal die has probability 1.
- $\frac{1}{2}$ is the middle. We call it **evens** — a 50–50 chance, like heads on a fair coin.
- Probabilities closer to **0** are **unlikely**. Probabilities closer to **1** are **likely**.

A probability can never be less than 0 or greater than 1. Nothing can be less likely than impossible, and nothing can be more likely than certain.

The same scale in percentages

The scale from 0 to 1 is the same scale as 0% to 100%. They are two ways to label the same line: 0 is 0%, $\frac{1}{2}$ is 50%, and 1 is 100%.



The two scales say the same thing: 0 means it cannot happen, 1 (or 100%) means it is certain.

Two matching number lines one above the other: the top line marked 0, one quarter, one half, three quarters and 1 as numbers, and the bottom line marked 0%, 25%, 50%, 75% and 100% as percentages, with dashed lines joining the matching marks

Which form you use depends on the situation. Weather forecasts use percentages ("75% chance of rain"). Card and dice games usually use fractions. Decimals sit comfortably with both. The good news is that you already know how to move between all three forms — that is Subtopic 2A all over again.

One probability, three names

A probability can be written as a fraction, a decimal or a percentage — they are three names for the same number.

One probability, three names

$$\frac{3}{4} = 0.75 = 75\%$$

Three parts out of four are shaded — the same chance, written three ways.

A bar cut into 4 equal parts with 3 parts shaded blue, labelled three ways: the fraction three quarters, the decimal 0.75 and the percentage 75%

To change forms:

- **Fraction → decimal:** divide the top number by the bottom number. $\frac{3}{4} = 3 \div 4 = 0.75$.
- **Decimal → percentage:** multiply by 100. $0.75 \times 100 = 75\%$.
- **Percentage → fraction:** write the number over 100, then simplify. $75\% = \frac{75}{100} = \frac{3}{4}$.

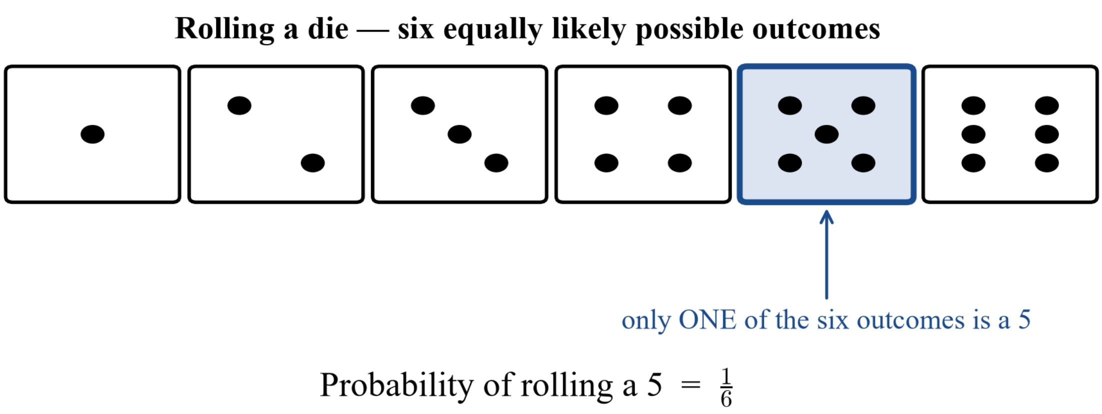
Some probabilities are worth knowing by heart, just like the friendships from 2E:

Fraction	Decimal	Percentage
$\frac{1}{100}$	0.01	1%

Fraction	Decimal	Percentage
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	0.333…	33.3%
$\frac{3}{8}$	0.375	37.5%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

Equally likely outcomes: probability by counting

When all outcomes are equally likely, probability is found by counting. A fair die has 6 faces, and each face is just as likely as every other. The possible outcomes are 1, 2, 3, 4, 5, 6.



Six die faces with dots showing 1, 2, 3, 4, 5 and 6, with the face showing 5 shaded blue, and a label saying only one of the six outcomes is a 5, so the probability of rolling a 5 is one sixth

Only one outcome is a 5, out of 6 possible outcomes, so

$$P(\text{rolling a } 5) = \frac{1}{6}$$

The counting rule is:

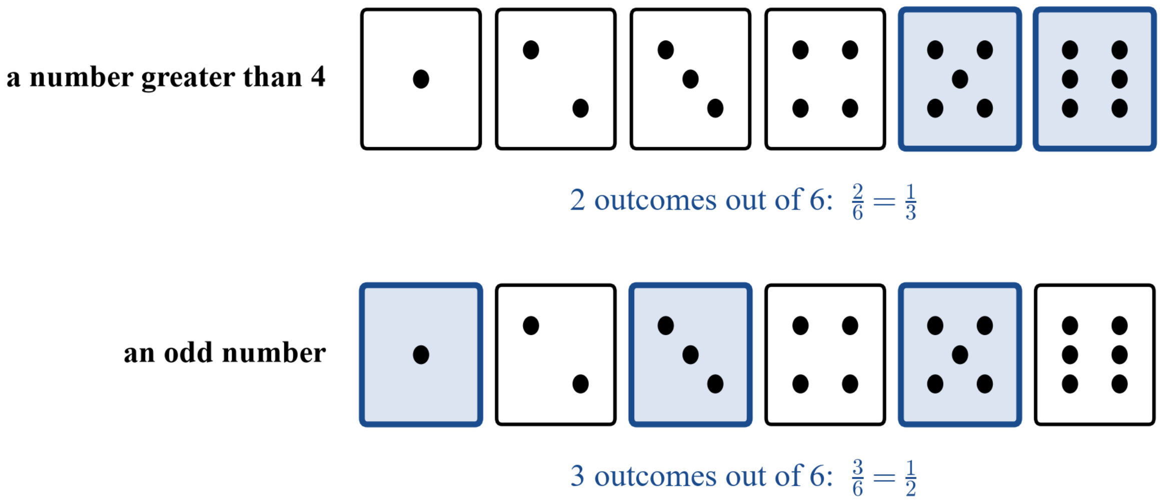
$$\text{probability} = \frac{\text{number of outcomes that fit the event}}{\text{total number of possible outcomes}}$$

The counting rule only works when the outcomes are **equally likely** — when no outcome has more of a chance than another. A fair die, a fair coin and a spinner with equal parts all fit this rule.

Events with more than one outcome

An event can hold several outcomes. "Rolling a number greater than 4" is one event, but it happens in two ways: rolling a 5 or rolling a 6. Count *every* outcome that fits.

Events on a die — count the outcomes that fit



Two rows of six die faces: in the top row the faces 5 and 6 are shaded blue for a number greater than 4, giving 2 outcomes out of 6, and in the bottom row the faces 1, 3 and 5 are shaded blue for an odd number, giving 3 outcomes out of 6

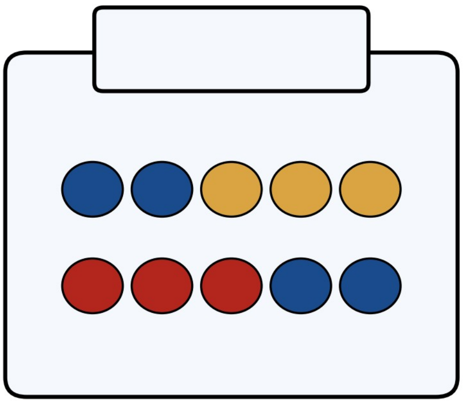
- **Greater than 4:** the outcomes are 5 and 6 — two outcomes fit. $P(\text{greater than } 4)=\frac{2}{6}=\frac{1}{3}$.
- **An odd number:** the outcomes are 1, 3 and 5 — three outcomes fit. $P(\text{odd})=\frac{3}{6}=\frac{1}{2}$.

List the possible outcomes first, tick the ones that fit, then count. That order stops you missing one.

The other side of an event

An event either happens or it does not — there is no third choice. So the probability that it happens, and the probability that it does not happen, must add up to 1.

A jar of 10 counters



A jar with 10 counters, 3 red, 4 blue and 3 yellow, labelled with the probability of blue, 4 out of 10 or 0.4 or 40%, and the probability of red, 3 out of 10 or 0.3 or 30%

This jar holds 10 counters: 3 red, 4 blue, 3 yellow. Every counter is equally likely to be picked.

- $P(\text{blue}) = \frac{4}{10} = 0.4 = 40\%$
- $P(\text{red}) = \frac{3}{10} = 0.3 = 30\%$
- $P(\text{not blue}) = \frac{6}{10} = 0.6 = 60\%$ — the 3 red and 3 yellow counters together.

Check the other side: $0.4 + 0.6 = 1$. It always works, and it gives you a shortcut — if you know a probability, you can find its other side by subtracting from 1:

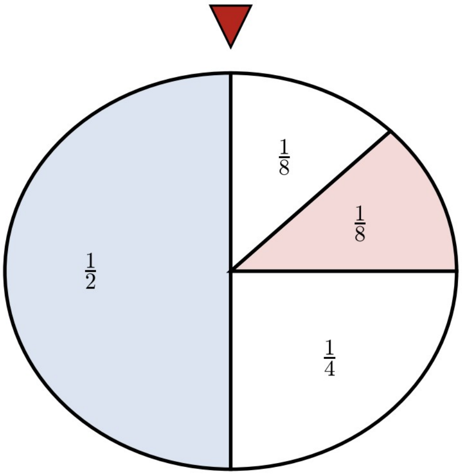
$$P(\text{not blue}) = 1 - P(\text{blue}) = 1 - 0.4 = 0.6$$

This also means all the probabilities of all the possible outcomes, added together, come to exactly 1 (or 100%). That makes a powerful check: if your probabilities do not add to 1, one of them is wrong.

When outcomes are not equally likely

Not every chance situation can be counted. Look at this spinner. Its four parts are different sizes, so landing on the big part is more likely than landing on a small part. The counting rule from before does not work here — there are 4 parts, but the probability of each part is *not* $\frac{1}{4}$.

Not all spinners have equal parts



The sizes tell the probabilities: the blue part is half the spinner, so it has a $\frac{1}{2}$ chance.

A spinner with 4 unequal parts labelled 1/2, 1/4, 1/8 and 1/8, with a red arrow at the top

With an unequal spinner, probability follows the *size* of the part. The part labelled $\frac{1}{2}$ covers half the spinner, so it has probability $\frac{1}{2}$. Each of the two smallest parts covers $\frac{1}{8}$ of the spinner, so each has probability $\frac{1}{8}$. Check: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=\frac{4}{8}+\frac{2}{8}+\frac{1}{8}+\frac{1}{8}=\frac{8}{8}=1$.

When you cannot count equally likely outcomes, you estimate. A weather forecast, a sports result, whether the bus is late — in these situations there are no equally likely outcomes to count. Instead you **assign** a probability by **estimation**: you use what you know to choose a sensible number between 0 and 1. Two rules keep estimates honest:

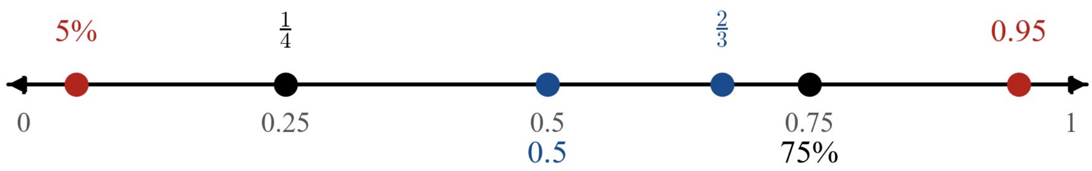
- use **common fractions, decimals and percentages** (like $\frac{3}{4}$, 0.8 or 70%), not fiddly numbers;
- give your **reason** — an estimate with no reason is just a guess.

Different people may assign slightly different probabilities to the same event, and that is fine. What matters is that the number sits in the right part of the scale — an unlikely event gets a number near 0, a likely event gets a number near 1.

Placing probabilities on the scale

To place probabilities on the scale, first write them all in one form. Decimals are usually the easiest to compare.

Placing probabilities on the scale



Each point sits exactly where its number says: $\frac{2}{3}$ is a little short of 0.75, and 5% is close to 0.

A number line from 0 to 1 with dots placed at 5%, one quarter, 0.5, two thirds, 75% and 0.95

Place 5%, $\frac{1}{4}$, 0.5, $\frac{2}{3}$, 75% and 0.95 on the scale:

Probability	As a decimal	Where it sits
5%	0.05	just above 0 — very unlikely
$\frac{1}{4}$	0.25	a quarter of the way along — unlikely
0.5	0.5	in the middle — evens
$\frac{2}{3}$	0.666…	between 0.5 and 0.75 — likely
75%	0.75	three quarters of the way along — likely
0.95	0.95	just below 1 — very likely, but not certain

Notice that only 0 and 1 sit on the ends. Every real chance event sits somewhere in between.

Chance language in real life

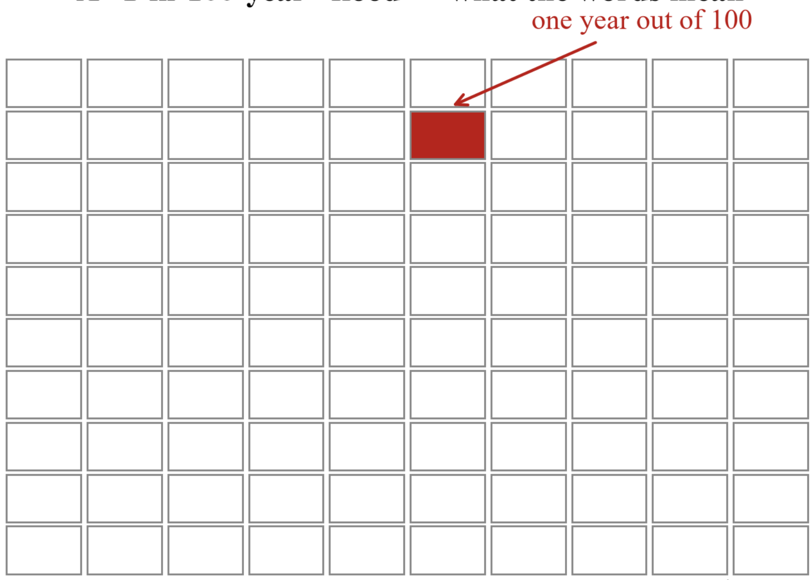
Probability numbers hide inside everyday language. Once you can read them, you can test what people say.

"Lucky." When someone calls a result lucky, they mean its probability was very small — close to 0. Winning a raffle when you hold only a few tickets is lucky; winning when you hold half the tickets is not.

"75% chance of rain." The weather forecast is assigning a probability: $75\%=0.75$, near the certain end of the scale. It means that it rained on about 75 out of 100 days with weather like today's. It does *not* mean rain for 75% of the day. It tells you rain is likely — sensible to take an umbrella — but not certain.

"A 1-in-100-years flood." This one is easy to misread. It does *not* mean a big flood comes once every 100 years, or that after one flood you are safe for 99 years. It means that in *any* year, the probability of a flood that big is $\frac{1}{100}=0.01=1\%$.

A “1-in-100-year” flood — what the words mean



Each square is one year. In ANY year, the chance of a flood that big is $\frac{1}{100} = 0.01 = 1\%$.

The red square can sit anywhere in the grid — two big floods can even happen close together.

A grid of 100 small squares in 10 rows of 10, one square for each of 100 years, with one square shaded red, showing that any one year has a 1-in-100 chance of a big flood

Each year is a fresh 1-in-100 chance. Two big floods can land close together, and a town can also go 200 years without one. Small probabilities over many years add up to real risk — that is why houses are still built with flood rules in mind even in towns that have never flooded.

Words for this subtopic, with what they mean here:

Term	Meaning here
probability	a number from 0 to 1 that measures how likely an event is to happen
outcome	one possible result of a chance experiment, like rolling a 5
possible outcomes	the full list of results that could happen
event	something we ask about — it may hold one outcome or several
equally likely	when no outcome has more of a chance than another
impossible	cannot happen; probability 0
certain	must happen; probability 1
evens	a 50–50 chance; probability $\frac{1}{2}$
unlikely	closer to 0 than to 1
likely	closer to 1 than to 0
estimate	to assign a sensible probability using what you know, when outcomes cannot be counted

In **8B** you will put these numbers to work: you run trials, record what actually happens, and compare your results with the probabilities you assign here.

Worked examples

Example 1 — with help

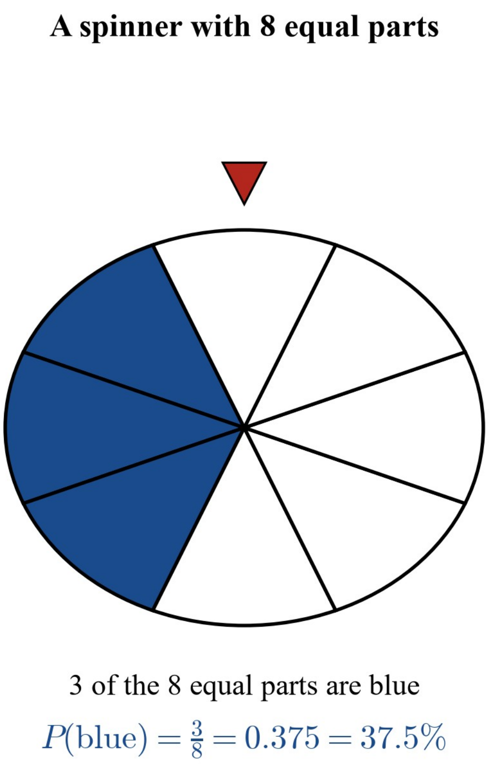
A fair die is rolled once. Find each probability by counting.

Working	Comment
The possible outcomes are 1, 2, 3, 4, 5, 6 — six outcomes, all equally likely.	List the possible outcomes first, then count them.
a. P(rolling a 4) = $\frac{1}{6}$	Only one face is a 4.
b. "Greater than 4" means 5 or 6 — two outcomes fit. P(greater than 4) = $\frac{2}{6} = \frac{1}{3}$	An event can hold several outcomes. Count every one that fits.
c. "Less than 5" means 1, 2, 3 or 4 — four outcomes fit. P(less than 5) = $\frac{4}{6} = \frac{2}{3}$	Simplify the fraction at the end.
d. No face shows 10, so it cannot happen. P(rolling a 10) = 0	An impossible event has probability 0.

All four answers sit between 0 and 1, as every probability must. Notice that nothing was measured — with equally likely outcomes, probability is pure counting.

Example 2 — with help

A spinner is divided into 8 equal parts. Three parts are blue and the rest are white. Find each probability as a fraction, a decimal and a percentage.



A spinner cut into 8 equal parts with a red arrow at the top, 3 of the parts shaded blue, labelled with the probability of blue: three eighths, 0.375 or 37.5%

Working	Comment
8 equal parts, 3 blue. P(blue) = $\frac{3}{8}$	Equal parts means the counting rule works.

Working	Comment
$\frac{3}{8}=3\div 8=0.375$, and $0.375\times 100=37.5\,\%$	Divide top by bottom for the decimal, then × 100 for the percentage.
P(blue) = $\frac{3}{8}$ = 0.375 = 37.5%	Three names for one probability.
Not blue is the other 5 parts: P(not blue) = $\frac{5}{8}$ = 0.625 = 62.5%	The other side of an event is counted the same way.
Check: $37.5\,\%+62.5\,\%=100\,\%$	An event and its other side always add to 1.

Example 3 — with help

A jar holds 3 red counters, 4 blue counters and 3 yellow counters. One counter is picked without looking.

Working	Comment
$3+4+3=10$ counters, all equally likely to be picked.	Find the total first.
a. P(blue) = $\frac{4}{10}=0.4=40\,\%$	Count the blue counters.
b. P(red) = $\frac{3}{10}=0.3=30\,\%$	The yellow counters give the same probability, $\frac{3}{10}$.
c. Not blue means red or yellow: $3+3=6$ counters.	Count them — or subtract: $1-0.4=0.6$.
P(not blue) = $\frac{6}{10}=0.6=60\,\%$	
d. 40% is the biggest probability, so blue is most likely.	To compare, write the probabilities in the same form.
Check: $40\,\%+30\,\%+30\,\%=100\,\%$	All the outcomes together must add to 1.

Example 4 — with help

Place these probabilities on the scale from 0 to 1: $5\,\%$, $\frac{1}{4}$, 0.5, $\frac{2}{3}$, $75\,\%$, 0.95.

Working	Comment
Write them all as decimals: 0.05, 0.25, 0.5, 0.666…, 0.75, 0.95.	One form makes them easy to compare.
0.05 sits just above 0; 0.25 sits a quarter of the way along.	Both are closer to 0 — unlikely events.
0.5 sits exactly in the middle — evens.	Halfway between 0 and 1.
0.666… and 0.75 sit between the middle and 1.	Likely events; $\frac{2}{3}$ is closer to 0.5 than 75% is.
0.95 sits just below 1.	Very likely — but only probability 1 is certain.

Example 5 — on your own

Assign a probability to each event, using a common fraction, decimal or percentage. Say whether you found it by **counting** equally likely outcomes or by **estimating**.

a. A player made 7 of her last 10 shots. She takes another shot.
b. Your friend tosses a fair coin. It lands heads.
c. Dark clouds are moving in, and the radio says rain is likely this afternoon. Event: it rains before you walk home.
d. A bag holds 9 red marbles and 1 blue marble. One marble is picked without looking. Event: blue.

Answers: a. About 0.7 (70%) — estimated from her record; she can still miss. b. $\frac{1}{2}$ (50%) — counted: two equally likely outcomes. c. Any sensible estimate near the likely end, e.g. 0.8 (80%) — estimated from the clouds and the radio report. d. $\frac{1}{10}$ (10%) — counted: 1 blue marble out of 10.

Example 6 — on your own

Read each piece of chance language, then answer.

a. Saturday's weather forecast says there is a "75% chance of rain". What does this tell you about Saturday?
b. A town had a big flood last year. A resident says, "It was a 1-in-100-years flood, so we are safe for the next 99 years." Is the resident right? Explain.
c. Sam holds 3 of the 600 tickets in a school raffle. He says winning would be "a bit lucky". Is that a fair way to describe it? Use the numbers in your answer.

Answers: a. It rained on about 75 out of 100 days with weather like today's — rain is likely but not certain. b. No — in *any* year the probability of a flood that big is $\frac{1}{100}$ = 1%, including next year. Floods can come close together. c. Yes — $P(\text{win})=\frac{3}{600}=\frac{1}{200}=0.005=0.5\%$, which is very close to 0, so winning really is lucky.

Example 7 — on your own

A raffle sells 250 tickets. You hold 10 of them.

a. Find the probability that you win, as a fraction, a decimal and a percentage.
b. Find the probability that you do *not* win.
c. Is winning likely? Use the scale to explain.

Answers: a. $\frac{10}{250}=\frac{4}{100}=0.04=4\%$. b. $1-0.04=0.96=96\%$. c. No — 4% sits close to 0 on the scale, so winning is unlikely.

Example 8 — on your own

A spinner has red, blue and green parts. $P(\text{red})=\frac{1}{2}$ and $P(\text{blue})=0.25$.

a. Find P(green) as a decimal.
b. Write P(green) as a fraction and as a percentage.
c. Describe how you could draw this spinner.

Answers: a. $1-0.5-0.25=0.25$. b. $0.25=\frac{1}{4}=25\%$. c. Divide it into 4 equal parts: 2 red, 1 blue, 1 green.

Practice questions

With help

Q1. Match each chance word with the number that fits it best: *impossible, certain, evens, unlikely, likely* — numbers 0, 0.15, 0.5, 0.8, 1.

Q2. Copy and complete the table. Three ways to name the same probability.

Fraction	Decimal	Percentage
$\frac{1}{2}$		

Fraction	Decimal	Percentage
$\frac{1}{4}$		
$\frac{3}{4}$		
$\frac{1}{10}$		
$\frac{1}{100}$		

Q3. Write each percentage as a fraction in its simplest form: a. 25% b. 50% c. 75% d. 10% e. 1%.

Q4. A fair die is rolled once. Find each probability. a. P(rolling a 2) b. P(rolling an odd number) c. P(rolling a number greater than 4) d. P(rolling a number less than 7) Which of these events is certain?

Q5. A spinner has 5 equal parts, numbered 1 to 5. a. Find P(spinning a 4) as a fraction, a decimal and a percentage. b. Find P(spinning an even number) as a fraction. c. Find P(spinning a number greater than 1) as a fraction.

Q6. A bag holds 2 red, 3 blue and 5 green marbles. One marble is picked without looking. a. Find P(red) as a fraction, a decimal and a percentage. b. Find P(green) as a fraction, a decimal and a percentage. c. Find P(not green) as a fraction.

Q7. Find the probability of the other side of each event. a. P(rain) = 0.3, so P(no rain) = ? b. P(win) = $\frac{1}{4}$, so P(not win) = ? c. P(the bus is late) = 20%, so P(the bus is on time) = ?

Q8. In each pair, say which probability is more likely — or whether they are equal. a. 0.45 or 45% b. $\frac{1}{4}$ or 30% c. 70% or $\frac{3}{4}$

Q9. Order these probabilities from least likely to most likely: $\frac{3}{4}$, $\frac{1}{10}$, 50%, 0.2.

Q10. True or false? Correct each false statement. a. A probability can be greater than 1. b. A probability of 0 means the event cannot happen. c. A probability of 100% means the event is certain. d. A probability can be −0.5.

On your own

Q11. A fair die is rolled once. Find each probability as a fraction in its simplest form. a. P(rolling a number less than 3) b. P(rolling an even number) c. P(rolling a 7) d. P(not rolling a 6)

Q12. A spinner has 8 equal parts: 1 red, 3 blue and 4 green. Find the probability of each colour as a fraction, a decimal and a percentage.

Q13. A bag holds 6 red counters and 4 blue counters. One counter is picked without looking. a. Find P(red) as a fraction, a decimal and a percentage. b. Find P(blue) as a fraction, a decimal and a percentage. c. Find P(not red). d. Check that P(red) and P(not red) add to 1.

Q14. Look at the spinner in Worked example 2 (8 equal parts, 3 blue). True or false? Give a reason for each. a. P(blue) = 37.5%. b. P(not blue) = $\frac{5}{8}$ = 0.625. c. P(blue) is greater than $\frac{1}{2}$.

Q15. Order these probabilities from least likely to most likely: 0.6, $\frac{2}{3}$, 55%, $\frac{1}{2}$.

Q16. A probability scale is marked at 0, 0.25, 0.5, 0.75 and 1. For each probability, say whether it sits exactly on a mark or between which two marks it sits. a. $\frac{1}{4}$ b. 0.7 c. 80% d. $\frac{3}{8}$

Q17. A player made 7 of her last 10 shots. a. Assign a probability to her making her next shot, and explain how you chose it. b. Is she certain to make the next shot? Why not?

Q18. For each event, say whether you would find the probability by **counting** equally likely outcomes or by **estimating**, then assign a probability. a. A fair coin is tossed and lands heads. b. It rains on your birthday next month. c. One counter is picked from a jar holding 19 blue counters and 1 yellow counter, and it is yellow.

Q19. Find the probability of the other side of each event, as a decimal or a percentage. a. P(the bus is late) = 0.15 b. P(win) = 32% c. P(red) = $\frac{3}{8}$

Q20. Look at the unequal spinner in the Theory (parts of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{8}$). a. Which part is most likely? b. What is its probability? c. Find the probability of the smallest part, as a fraction, a decimal and a percentage. d. Find the probability of landing on one of the two smallest parts.

Q21. The weather forecast says there is a "25% chance of snow" tomorrow in the mountains. a. What does the 25% mean? b. Is snow likely tomorrow? Use the scale to explain.

Q22. Write one probability that sits between 0.4 and 0.6, in all three forms (fraction, decimal and percentage).

Maths in real life

Q23. The forecast for the school sports day says there is an 80% chance of rain. a. What does the 80% mean? b. Does it mean it will rain for 80% of the day? Explain. c. Should the school have a backup plan for the day? Use the scale to explain.

Q24. A river had a "1-in-20-year flood" last year. a. Write the probability of a flood that big happening in any year as a fraction, a decimal and a percentage. b. A resident says, "We had one last year, so we are safe for the next 19 years." Is the resident right? Explain.

Q25. A school raffle sells 500 tickets. Mia holds 10 of them. a. Find the probability that Mia wins, as a fraction, a decimal and a percentage. b. Mia's friend says, "You are almost sure to win!" Is the friend right? Use the numbers to explain.

Choosing your strategy

Q26. For each event, say whether you would assign the probability by **counting** equally likely outcomes or by **estimating**. Then assign a probability. a. Rolling an even number on a fair die. b. It rains next Wednesday. c. Picking a blue marble from a bag holding 7 blue marbles and 3 red marbles. d. Your team wins the grand final.

Q27. Design a spinner with $P(\text{red})=\frac{1}{2}$ and $P(\text{blue})=\frac{1}{4}$. a. What must P(green) be? b. Divide your spinner into equal parts and say how many of each colour you need. c. Draw the spinner and label each part with its probability.

Q28. A student says, "This spinner has 3 colours, so the probability of each colour is $\frac{1}{3}$." When is the student right, and when is the student wrong? Explain.

Q29. Make your own. Choose a probability between 0 and 1. Write it in all three forms, say where it sits on the scale, and describe an event that could have that probability.

Answers

Q1. impossible → 0; unlikely → 0.15; evens → 0.5; likely → 0.8; certain → 1.

Q2. $\frac{1}{2}$ = 0.5 = 50%; $\frac{1}{4}$ = 0.25 = 25%; $\frac{3}{4}$ = 0.75 = 75%; $\frac{1}{10}$ = 0.1 = 10%; $\frac{1}{100}$ = 0.01 = 1%.

Q3. a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $\frac{3}{4}$ d. $\frac{1}{10}$ e. $\frac{1}{100}$.

Q4. a. $\frac{1}{6}$ b. $\frac{3}{6}=\frac{1}{2}$ c. $\frac{2}{6}=\frac{1}{3}$ d. $\frac{6}{6}=1$. Part d is certain.

Q5. a. $\frac{1}{5}$ = 0.2 = 20%; b. $\frac{2}{5}$; c. $\frac{4}{5}$.

Q6. a. $\frac{2}{10}=\frac{1}{5}$ = 0.2 = 20%; b. $\frac{5}{10}=\frac{1}{2}$ = 0.5 = 50%; c. $\frac{5}{10}=\frac{1}{2}$.

Q7. a. $1-0.3=0.7$; b. $1-\frac{1}{4}=\frac{3}{4}$; c. $100\%-20\%=80\%$.

Q8. a. equal — both are 0.45; b. 30% — 0.3 is greater than 0.25; c. $\frac{3}{4}$ — 0.75 is greater than 0.7.

Q9. $\frac{1}{10}$, 0.2, 50%, $\frac{3}{4}$.

Q10. a. False — a probability is always between 0 and 1. b. True. c. True. d. False — a probability can never be less than 0.

Q11. a. $\frac{2}{6}=\frac{1}{3}$; b. $\frac{3}{6}=\frac{1}{2}$; c. 0; d. $\frac{5}{6}$.

Q12. red: $\frac{1}{8}$ = 0.125 = 12.5%; blue: $\frac{3}{8}$ = 0.375 = 37.5%; green: $\frac{4}{8}=\frac{1}{2}$ = 0.5 = 50%.

Q13. a. $\frac{6}{10}=\frac{3}{5}$ = 0.6 = 60%; b. $\frac{4}{10}=\frac{2}{5}$ = 0.4 = 40%; c. $\frac{4}{10}$ = 0.4 = 40%; d. $0.6+0.4=1$. ✓

Q14. a. True; b. True; c. False — $\frac{3}{8}$ is less than $\frac{4}{8}=\frac{1}{2}$.

Q15. $\frac{1}{2}$, 55%, 0.6, $\frac{2}{3}$.

Q16. a. exactly on 0.25; b. between 0.5 and 0.75; c. between 0.75 and 1; d. between 0.25 and 0.5.

Q17. a. About 0.7 = 70%, estimated from her record of 7 out of 10. b. No — 0.7 is less than 1, so she can still miss.

Q18. a. counting, $\frac{1}{2}$ = 50%; b. estimating (any reasoned estimate, e.g. about $\frac{1}{4}$ = 25%); c. counting, $\frac{1}{20}$ = 0.05 = 5%.

Q19. a. 0.85; b. 68%; c. $\frac{5}{8}$ = 0.625 = 62.5%.

Q20. a. the $\frac{1}{2}$ part; b. $\frac{1}{2}$; c. $\frac{1}{8}$ = 0.125 = 12.5%; d. $\frac{1}{8}+\frac{1}{8}=\frac{2}{8}=\frac{1}{4}$ = 0.25 = 25%.

Q21. a. It snowed on about 25 out of 100 days with weather like tomorrow's. b. No — 0.25 is closer to 0 than to 1, so snow is unlikely, though still possible.

Q22. Any value between 0.4 and 0.6, e.g. $\frac{1}{2}$ = 0.5 = 50%.

Q23. a. It rained on about 80 out of 100 days with weather like this. b. No — it is about whether rain happens at all, not how long it lasts. c. Yes — 0.8 sits close to 1, so rain is likely and the school should plan for it.

Q24. a. $\frac{1}{20}$ = 0.05 = 5%. b. No — the probability is 5% in *every* year, including next year. Floods can come close together.

Q25. a. $\frac{10}{500}=\frac{2}{100}=\frac{1}{50}$ = 0.02 = 2%. b. No — 2% sits close to 0, so winning is unlikely, not almost sure.

Q26. a. counting, $\frac{3}{6}=\frac{1}{2}$; b. estimating (any reasoned estimate); c. counting, $\frac{7}{10}$ = 0.7 = 70%; d. estimating (any reasoned estimate).

Q27. a. $\frac{1}{4}$; b. 4 equal parts — 2 red, 1 blue, 1 green; c. any drawing with those parts, labelled $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$.

Q28. Right only when the 3 parts are the same size. Wrong when the parts are different sizes — probability follows the size of the part, not the number of colours.

Q29. Any reasonable answer, e.g. $\frac{3}{10}$ = 0.3 = 30%, sitting between 0.25 and 0.5, for picking a red counter from a bag of 3 red and 7 blue.

All the working

Q1. On the scale from 0 to 1, impossible sits at 0 and certain sits at 1. Evens sits in the middle at 0.5. Of the two numbers left, 0.15 is close to 0 and 0.8 is close to 1. ∴ impossible → 0, unlikely → 0.15, evens → 0.5, likely → 0.8, certain → 1.

Q2. Use the friendships: divide the top by the bottom, then × 100. $\frac{1}{2}=1\div 2=0.5$, and $0.5\times 100=50\%$. $\frac{1}{4}=1\div 4=0.25=25\%$. $\frac{3}{4}=3\div 4=0.75=75\%$. $\frac{1}{10}=1\div 10=0.1=10\%$. $\frac{1}{100}=1\div 100=0.01=1\%$. ∴ the completed table is $\frac{1}{2}$ = 0.5 = 50%; $\frac{1}{4}$ = 0.25 = 25%; $\frac{3}{4}$ = 0.75 = 75%; $\frac{1}{10}$ = 0.1 = 10%; $\frac{1}{100}$ = 0.01 = 1%.

Q3. Write each percentage over 100, then simplify: $25\%=\frac{25}{100}=\frac{1}{4}$; $50\%=\frac{50}{100}=\frac{1}{2}$; $75\%=\frac{75}{100}=\frac{3}{4}$; $10\%=\frac{10}{100}=\frac{1}{10}$; $1\%=\frac{1}{100}$. ∴ a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $\frac{3}{4}$ d. $\frac{1}{10}$ e. $\frac{1}{100}$.

Q4. There are 6 possible outcomes, all equally likely. a. One outcome is a 2: $P(2)=\frac{1}{6}$. b. Odd means 1, 3, 5 — three outcomes: $P(\text{odd})=\frac{3}{6}=\frac{1}{2}$. c. Greater than 4 means 5, 6 — two outcomes: $P(\text{greater than } 4)=\frac{2}{6}=\frac{1}{3}$. d. Every face is less than 7 — six outcomes: $P(\text{less than } 7)=\frac{6}{6}=1$. ∴ a. $\frac{1}{6}$ b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. 1; part d is certain because its probability is 1.

Q5. There are 5 equal parts. a. One part is a 4: $P(4)=\frac{1}{5}=1\div5=0.2$, and $0.2\times100=20\%$. b. Even means 2, 4 — two parts: $P(\text{even})=\frac{2}{5}$. c. Greater than 1 means 2, 3, 4, 5 — four parts: $P(\text{greater than }1)=\frac{4}{5}$. ∴ a. $\frac{1}{5}$ = 0.2 = 20%; b. $\frac{2}{5}$; c. $\frac{4}{5}$.

Q6. Total marbles: $2+3+5=10$. a. $P(\text{red})=\frac{2}{10}=\frac{1}{5}=0.2=20\%$. b. $P(\text{green})=\frac{5}{10}=\frac{1}{2}=0.5=50\%$. c. Not green means red or blue: $2+3=5$ marbles, so $P(\text{not green})=\frac{5}{10}=\frac{1}{2}$. ∴ a. $\frac{1}{5}$ = 0.2 = 20%; b. $\frac{1}{2}$ = 0.5 = 50%; c. $\frac{1}{2}$.

Q7. An event and its other side add to 1. a. $P(\text{no rain})=1-0.3=0.7$. b. $P(\text{not win})=1-\frac{1}{4}=\frac{3}{4}$. c. $P(\text{on time})=100\%-20\%=80\%$. ∴ a. 0.7; b. $\frac{3}{4}$; c. 80%.

Q8. Write each pair in one form. a. $0.45=45\%$, so they are equal. b. $\frac{1}{4}=0.25$, and $0.25<0.3$, so 30% is more likely. c. $\frac{3}{4}=0.75$, and $0.75>0.7$, so $\frac{3}{4}$ is more likely. ∴ a. equal; b. 30%; c. $\frac{3}{4}$.

Q9. As decimals: $\frac{3}{4}=0.75$, $\frac{1}{10}=0.1$, $50\%=0.5$, $0.2=0.2$. Order from smallest: $0.1, 0.2, 0.5, 0.75$. ∴ $\frac{1}{10}$, 0.2, 50%, $\frac{3}{4}$.

Q10. a. False — no event can be more likely than certain, so a probability is never greater than 1. b. True — 0 is the impossible end. c. True — 100% = 1, the certain end. d. False — no event can be less likely than impossible, so a probability is never less than 0.

Q11. a. Less than 3 means 1, 2: $P=\frac{2}{6}=\frac{1}{3}$. b. Even means 2, 4, 6: $P=\frac{3}{6}=\frac{1}{2}$. c. No face shows 7, so $P=0$. d. Not 6 means 1, 2, 3, 4, 5 — five outcomes: $P=\frac{5}{6}$. ∴ a. $\frac{1}{3}$; b. $\frac{1}{2}$; c. 0; d. $\frac{5}{6}$.

Q12. There are 8 equal parts. Red: 1 part, $P=\frac{1}{8}=1\div8=0.125=12.5\%$. Blue: 3 parts, $P=\frac{3}{8}=3\div8=0.375=37.5\%$. Green: 4 parts, $P=\frac{4}{8}=\frac{1}{2}=0.5=50\%$. Check: $12.5\%+37.5\%+50\%=100\%$. ∴ red $\frac{1}{8}$ = 0.125 = 12.5%; blue $\frac{3}{8}$ = 0.375 = 37.5%; green $\frac{1}{2}$ = 0.5 = 50%.

Q13. There are 10 counters. a. $P(\text{red})=\frac{6}{10}=\frac{3}{5}=0.6=60\%$. b. $P(\text{blue})=\frac{4}{10}=\frac{2}{5}=0.4=40\%$. c. Not red means blue: $P(\text{not red})=\frac{4}{10}=0.4=40\%$. d. $0.6+0.4=1$. ∴ a. $\frac{3}{5}$ = 0.6 = 60%; b. $\frac{2}{5}$ = 0.4 = 40%; c. 0.4; d. they add to 1. ✓

Q14. The spinner has 8 equal parts, 3 blue. a. $P(\text{blue})=\frac{3}{8}=0.375=37.5\%$ — true. b. Not blue is 5 parts: $\frac{5}{8}=5\div8=0.625$ — true. c. $\frac{3}{8}$ is less than $\frac{4}{8}=\frac{1}{2}$ — false. ∴ a. True; b. True; c. False.

Q15. As decimals: 0.6, $\frac{2}{3} \approx 0.667$, $55\% = 0.55$, $\frac{1}{2} = 0.5$. Order from smallest: $0.5, 0.55, 0.6, 0.667$. ∴ $\frac{1}{2}$, 55%, 0.6, $\frac{2}{3}$.

Q16. Write each as a decimal and compare with the marks. a. $\frac{1}{4} = 0.25$ — exactly on the 0.25 mark. b. 0.7 sits between 0.5 and 0.75. c. $80\% = 0.8$ — between 0.75 and 1. d. $\frac{3}{8} = 0.375$ — between 0.25 and 0.5. ∴ a. on 0.25; b. between 0.5 and 0.75; c. between 0.75 and 1; d. between 0.25 and 0.5.

Q17. a. Her record is 7 out of 10, so estimate $P(\text{making the next shot}) = \frac{7}{10} = 0.7 = 70\%$. This is an estimate — her record suggests it, but does not guarantee it. b. No — 0.7 is less than 1, so she can still miss. Only probability 1 means certain. ∴ a. about 70%, estimated from her record; b. not certain.

Q18. a. A fair coin has 2 equally likely outcomes — count them: $P(\text{heads}) = \frac{1}{2} = 50\%$. b. Weather cannot be counted from equally likely outcomes — estimate it, e.g. about $\frac{1}{4} = 25\%$ (any reasoned estimate is fine). c. The jar holds 20 equally likely counters, 1 of them yellow — count them: $P(\text{yellow}) = \frac{1}{20} = 0.05 = 5\%$. ∴ a. counting, 50%; b. estimating (example answer 25%); c. counting, 5%.

Q19. Subtract from 1. a. $1 - 0.15 = 0.85$. b. $100\% - 32\% = 68\%$. c. $1 - \frac{3}{8} = \frac{5}{8} = 0.625 = 62.5\%$. ∴ a. 0.85; b. 68%; c. $\frac{5}{8}$ = 0.625 = 62.5%.

Q20. The parts are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}$. a. The biggest part is most likely — the $\frac{1}{2}$ part. b. Its probability is $\frac{1}{2}$. c. The smallest parts are $\frac{1}{8}$: $\frac{1}{8} = 1 \div 8 = 0.125 = 12.5\%$. d. One of the two smallest parts: $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4} = 0.25 = 25\%$. ∴ a. the $\frac{1}{2}$ part; b. $\frac{1}{2}$; c. $\frac{1}{8}$ = 0.125 = 12.5%; d. $\frac{1}{4}$ = 25%.

Q21. a. The forecast means that it snowed on about 25 out of 100 days with weather like tomorrow's. b. $25\% = 0.25$ sits closer to 0 than to 1 on the scale, so snow is unlikely — but 0.25 is not 0, so snow is still possible. ∴ a. about 25 out of 100 similar days had snow; b. unlikely but possible.

Q22. Pick any number between 0.4 and 0.6. For example $\frac{1}{2}$: $\frac{1}{2} = 0.5 = 50\%$, and 0.5 sits between 0.4 and 0.6. ∴ example answer: $\frac{1}{2}$ = 0.5 = 50%.

Q23. a. It means that it rained on about 80 out of 100 days with weather like this. b. No — the percentage is about whether rain happens at all, not about how much of the day it lasts. c. $80\% = 0.8$ sits close to 1, so rain is likely. The school should have a backup plan ready — likely is not certain, but it is close enough to plan for. ∴ a. about 80 out of 100 similar days had rain; b. no; c. yes, because rain is likely.

Q24. a. 1-in-20 means $P = \frac{1}{20} = 1 \div 20 = 0.05 = 5\%$. b. The resident is wrong. The probability of a flood that big is 5% in *every* year — last year's flood does not change next year's chance, and floods can come close together. ∴ a. $\frac{1}{20}$ = 0.05 = 5%; b. no — every year carries the same 5% chance.

Q25. a. Mia holds 10 of the 500 tickets: $P(\text{win})=\frac{10}{500}=\frac{2}{100}=\frac{1}{50}=0.02=2\%$. b. The friend is wrong: 2% sits close to 0 on the scale, so winning is unlikely — far from almost sure. ∴ a. $\frac{1}{50}$ = 0.02 = 2%; b. no — 2% is unlikely.

Q26. a. A die has equally likely outcomes — count them: even means 2, 4, 6, so $P=\frac{3}{6}=\frac{1}{2}$. b. Weather cannot be counted — estimate it (any reasoned estimate, e.g. about $\frac{1}{4}$). c. The marbles are equally likely — count them: $P(\text{blue})=\frac{7}{10}=0.7=70\%$. d. A grand final result cannot be counted — estimate it (any reasoned estimate). ∴ a. counting, $\frac{1}{2}$; b. estimating; c. counting, 70%; d. estimating.

Q27. a. All probabilities add to 1: $P(\text{green})=1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$. b. Use 4 equal parts: red needs $\frac{1}{2}=2$ parts, blue needs 1 part, green needs 1 part. c. Draw a spinner with 4 equal parts — 2 red, 1 blue, 1 green — labelled $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$. ∴ a. $\frac{1}{4}$; b. 4 equal parts: 2 red, 1 blue, 1 green.

Q28. The student is right only when the 3 parts are the same size — then each part is $\frac{1}{3}$. The student is wrong when the parts are different sizes: probability follows the size of the part, not the number of colours. A spinner that is half red, a quarter blue and a quarter green has 3 colours, but its probabilities are $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$. ∴ right for equal parts only; wrong otherwise.

Q29. Example answer: choose $\frac{3}{10}$. $\frac{3}{10}=0.3=30\%$, which sits between 0.25 and 0.5 on the scale. An event with this probability: picking a red counter from a bag holding 3 red and 7 blue counters. ∴ answers vary; check that the three forms match and the position on the scale is correct.

What comes next

In **8B**, the last subtopic of this chapter, you put these numbers to the test. You will run trials — spin the spinner, roll the die, toss the coin — record what actually happens, and compare the results with the probabilities you assigned here (`VC2M6P02`). That is where estimation meets evidence.

In Year 7, probability gets its own structure: you will list every possible outcome of two-step experiments, and you will use digital tools to simulate chance experiments (`VC2M7P01`, `VC2M7P02`). Everything in this subtopic — the scale, the three forms, counting and estimating — carries straight across.

Beyond school, probability is one of the threads that runs into senior secondary mathematics: Mathematical Methods and Specialist Mathematics both build on it, in data analysis and in chance at a deeper level. If your path leads through Foundation Mathematics or Further Mathematics instead, probability does not return after this book — so treat this chapter as your chance to make the ideas firm. Either way, the scale from 0 to 1 is yours for life: every forecast, every raffle and every “lucky” moment is a number waiting to be read.

Simulations and the effect of increasing the number of trials

> **Content description `VC2M6P02`:** conduct repeated chance experiments and run simulations with an increasing number of trials using digital tools; compare observations with expected results and discuss the effect on variation of increasing the number of trials

In 8A you put a number on chance: probabilities written as fractions, decimals and percentages, all living on one scale from 0 to 1. This subtopic puts those numbers on trial. Run the experiment, tally what happens, and compare it with the prediction — then do what the curriculum asks: increase the number of trials and watch what happens to the variation.

Theory

What you bring from Year 5 and 8A

A **chance experiment** is anything we run where the result is a matter of luck: tossing a coin, rolling a die, spinning a spinner. Each result it can give is an **outcome**. Tossing a coin has two outcomes — heads or tails. Rolling a die has six: the faces 1 to 6.

When the outcomes are **equally likely**, the probability of one outcome is 1 divided by the number of outcomes. From 8A, on the 0–1 scale:

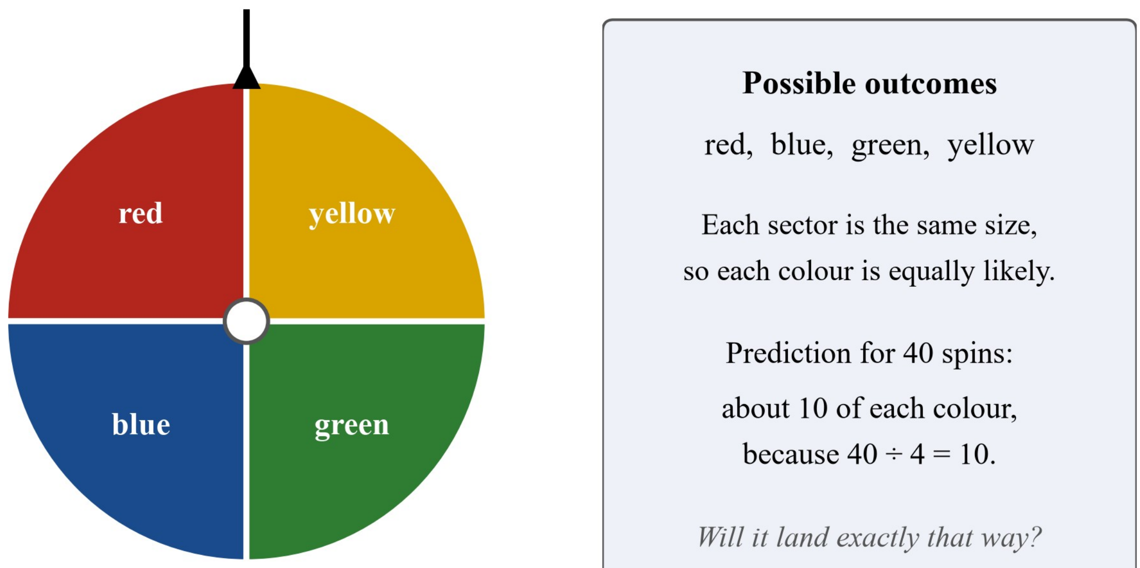
- a fair coin: heads has probability $\frac{1}{2} = 0.5 = 50\,\%$;
- a fair die: a six has probability $\frac{1}{6}$, about 0.167, about $16.7\,\%$;
- the fair spinner below: each colour has probability $\frac{1}{4} = 0.25 = 25\,\%$.

Those numbers are the **prediction**. They tell us what *should* happen over the whole experiment. This subtopic asks the next question: what actually happens when we run the trials, and how close the two agree?

Trials, tallies and the prediction

One run of a chance experiment is a **trial**: one toss, one roll, one spin. A class spun the fair spinner below 40 times and kept a **tally** — counting in fives, with the fifth mark crossing the previous four.

A fair spinner with four equal sectors



A fair spinner with four equal quarters coloured red, blue, green and yellow, beside a card listing the four possible outcomes, each with probability one-quarter, which is 0.25 or 25 per cent

A tally sheet for 40 spins of the fair spinner

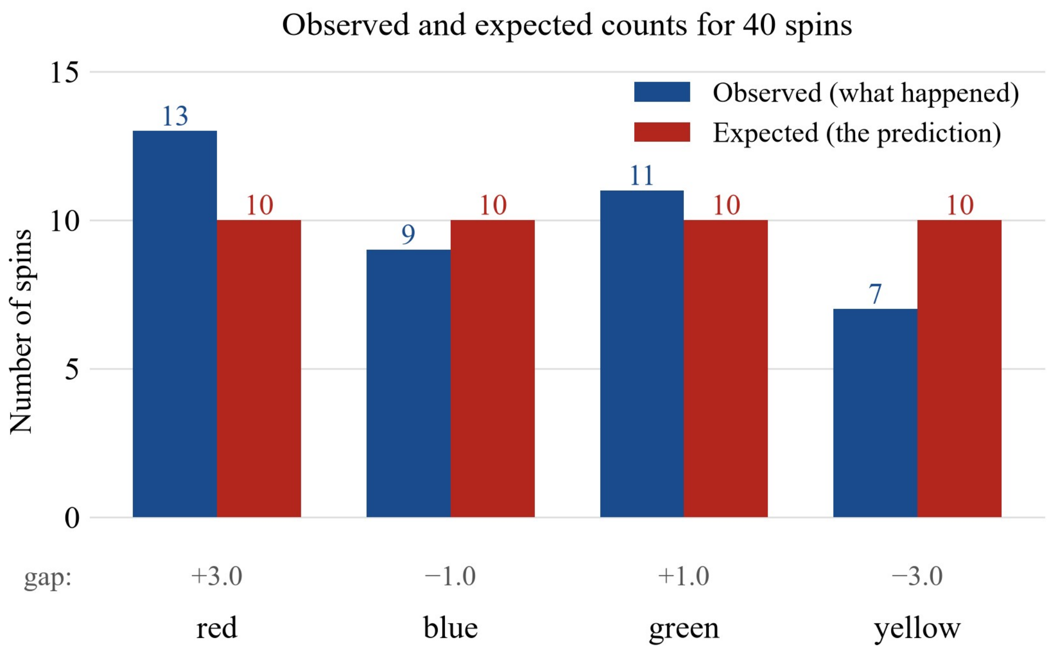
Colour	Tally	Number of spins
red	卌 卌 |||	13
blue	卌 ||||	9
green	卌 卌 |	11
yellow	卌 ||	7
Total		**40**

A tally sheet for 40 spins of the fair spinner: red 13, blue 9, green 11, yellow 7, with the fives crossed in the usual way and a total of 40

The **expected** count for each colour is the prediction: 40 spins, four equally likely colours, so

$40 \times \frac{1}{4} = 10$ spins each.

Check the tally sheet: $13 + 9 + 11 + 7 = 40$ — every spin is counted once, so the frequencies must add to the number of trials. ✓ But no colour landed exactly 10 times. Red ran 3 ahead of the prediction and yellow ran 3 behind. That gap between what happens and what we expect is ordinary in chance work — it has a name: **variation**.



A bar chart comparing what happened with what was expected in 40 spins: red 13 against 10, blue 9 against 10, green 11 against 10, yellow 7 against 10, with the gap under each pair written as +3.0, −1.0, +1.0 and −3.0

With only 40 trials, gaps of 1 to 3 either side of the prediction are exactly what chance produces. The prediction is not a promise about 40 spins; it is a picture of what happens over *many* trials.

Relative frequency

Counts are hard to compare when the number of trials changes — 13 reds in 40 spins and 130 reds in 400 spins are very different stories. So we divide. The **relative frequency** of an outcome is relative frequency = frequency ÷ number of trials.

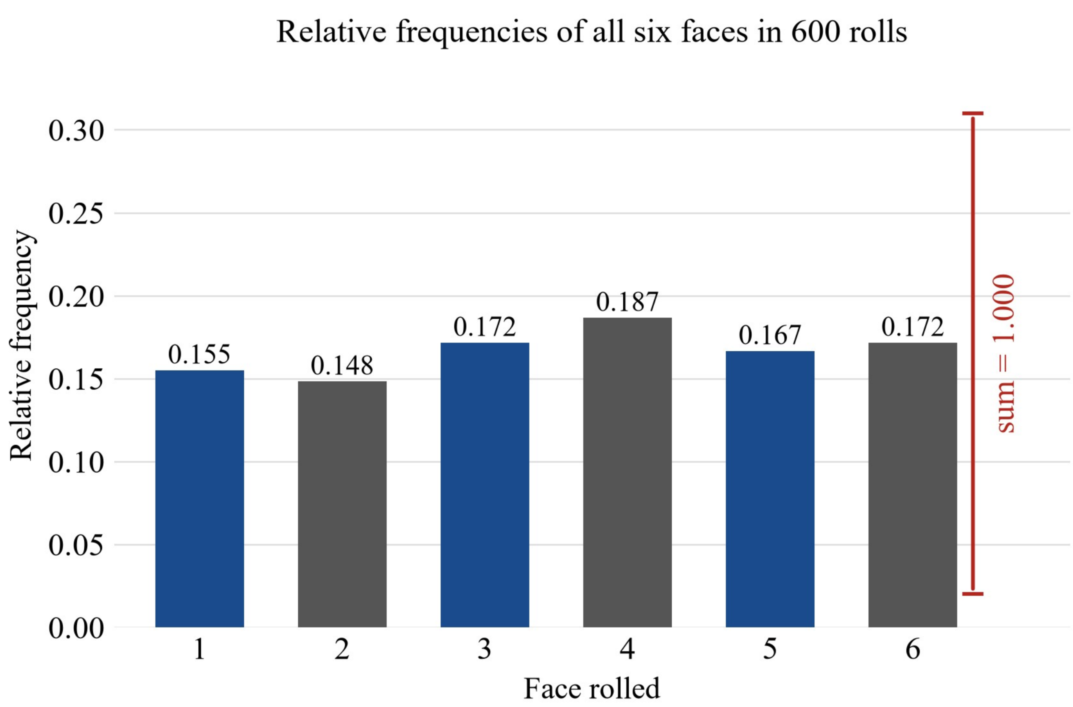
For the 40 spins:

- red: $13 \div 40 = 0.325$
- blue: $9 \div 40 = 0.225$
- green: $11 \div 40 = 0.275$
- yellow: $7 \div 40 = 0.175$

(A handy trick with 40: multiply top and bottom by 25 to get thousandths — $\frac{13}{40} = \frac{325}{1000} = 0.325$.) Relative frequencies sit on the same 0–1 scale as probabilities, so the red result, 0.325, can be held right up against the prediction, 0.25.

Relative frequencies always add to 1

Because every trial gives exactly one outcome, the frequencies add to the number of trials — and the relative frequencies add to 1. For the spinner: $0.325 + 0.225 + 0.275 + 0.175 = 1$. The same rule holds with any number of outcomes. The figure below shows one 600-roll run of a fair die, done in a simulation: six faces, six relative frequencies, and the bars still stack to exactly 1.



A bar chart of the relative frequency of each face in 600 rolls of a fair die: 1 gives 0.155, 2 gives 0.148, 3 gives 0.172, 4 gives 0.187, 5 gives 0.167 and 6 gives 0.172, with a bracket showing the six relative frequencies sum to 1.000

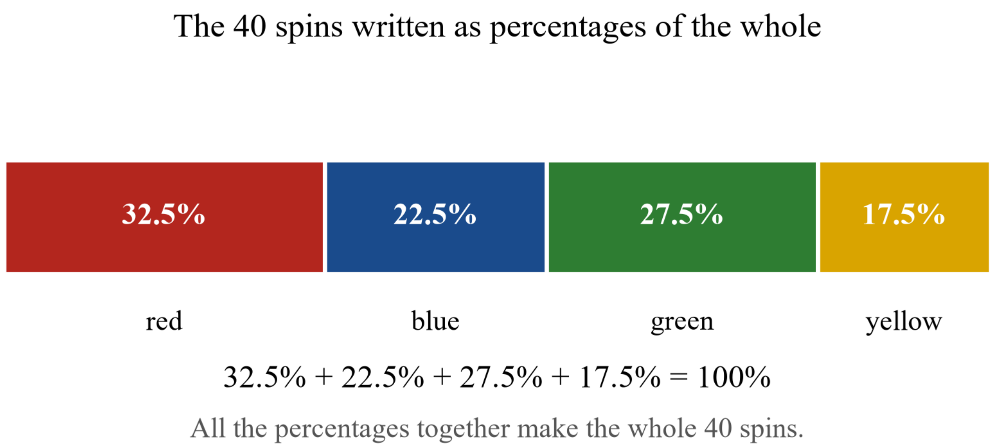
This is a fact worth checking in your own work: if your relative frequencies do not add to 1, a count has gone wrong.

Percentages of the whole

A relative frequency is a share of the whole, so it can be written as a percentage — multiply by 100. The 40 spins become:

- red: $0.325 = 32.5\%$
- blue: $0.225 = 22.5\%$
- green: $0.275 = 27.5\%$
- yellow: $0.175 = 17.5\%$

and because the relative frequencies add to 1, the percentages add to 100%: $32.5\% + 22.5\% + 27.5\% + 17.5\% = 100\%$.

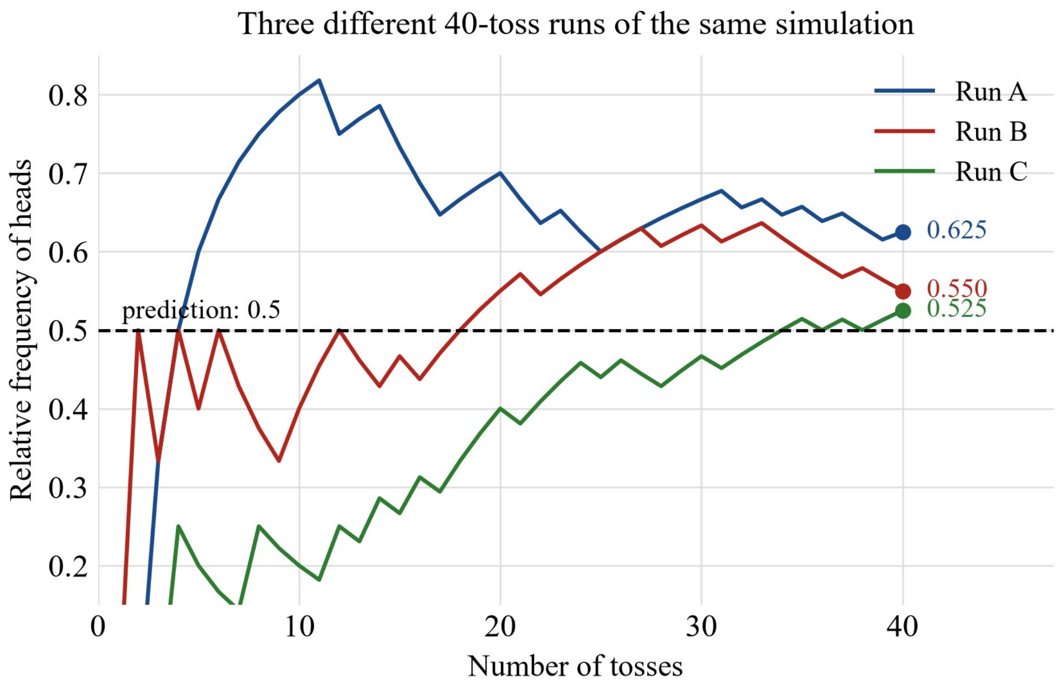


A percentage strip of the 40 spins: a single bar split into red 32.5 per cent, blue 22.5 per cent, green 27.5 per cent and yellow 17.5 per cent, with the four percentages written below adding to 100 per cent

Reading the strip against the prediction — 25% per colour — is another quick way to see the variation: red took 7.5% more of the spins than expected, yellow 7.5% fewer.

Variation between runs

Variation is not just the gap from the prediction; it is also the gap *between* runs of the same experiment. Three students each tossed a fair coin 40 times and drew their running relative frequency of heads:



Three runs of 40 coin tosses drawn as running relative frequency of heads against the prediction line at 0.5: run A ends at 0.625, run B at 0.550 and run C at 0.525

Same coin, same prediction, same number of trials — and three different answers: 25, 22 and 21 heads, giving relative frequencies of 0.625, 0.550 and 0.525. No run landed on 0.5. With 40 trials, that is normal: **the smaller the number of trials, the more variation** you see between one run and the next. The natural question is whether the variation shrinks when the number of trials grows — and that is where digital tools come in.

Simulations with digital tools

Tallying 40 spins by hand is fine; tallying 600 rolls is an afternoon. A **simulation** hands the work to a digital tool: the tool runs the trials, keeps the tally and updates the relative frequencies as it goes. This subtopic requires the digital tool — the large runs below were all produced that way.

One simulation tool's table for rolling a dice many times

Trial	Result	Sixes so far	Relative frequency of a six
1	6	1	1.000
2	6	2	1.000
3	5	2	0.667
4	6	3	0.750
5	1	3	0.600
6	5	3	0.500
7	6	4	0.571
8	2	4	0.500
		⋮	
600	5	103	0.172

The tool does the repeating, the counting and the dividing.

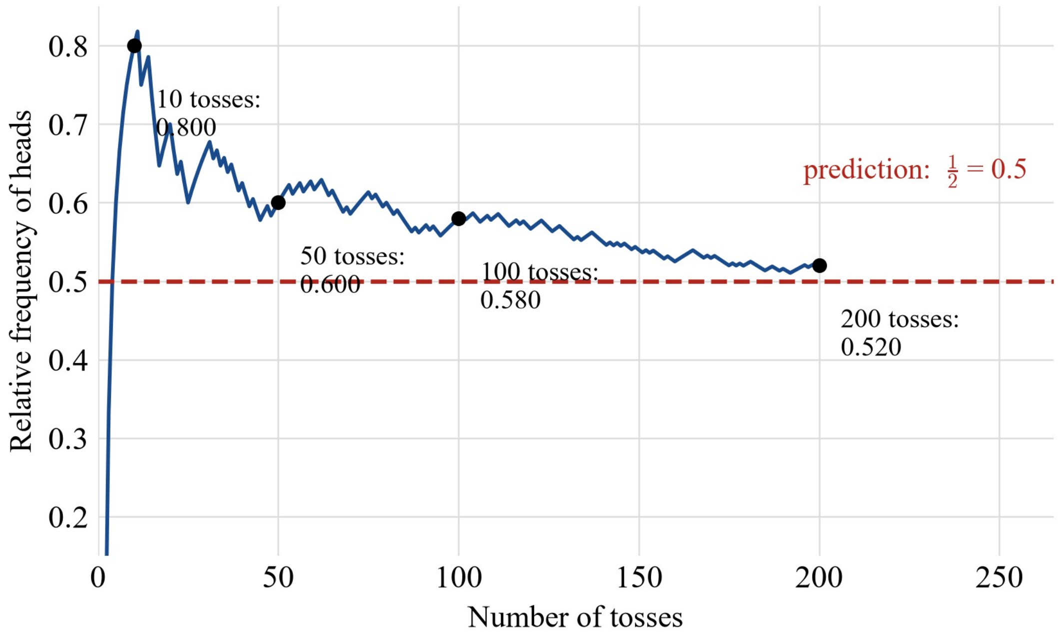
A simulation tool's running table for rolling one fair die: the first eight trials listed one by one with the running count and relative frequency of sixes, then a jump to trial 600, where 103 sixes give a relative frequency of 0.172

Read the table the way the tool builds it. After trial 1 the relative frequency of sixes is $1 \div 1 = 1$. After trial 4 it is $3 \div 4 = 0.75$. After trial 8 it is $4 \div 8 = 0.5$ — half the rolls showing six! Then the table jumps to 600 trials: 103 sixes, a relative frequency of $103 \div 600 = 0.172$ (to 3 decimal places), sitting close to the prediction $\frac{1}{6} \approx 0.167$. The early numbers were not wrong; they were based on too few trials to be trusted.

Increasing the number of trials

The clearest picture of the effect of increasing trials is one long run with the relative frequency drawn as it goes. Here is one run of 200 tosses of a fair coin, done in a simulation.

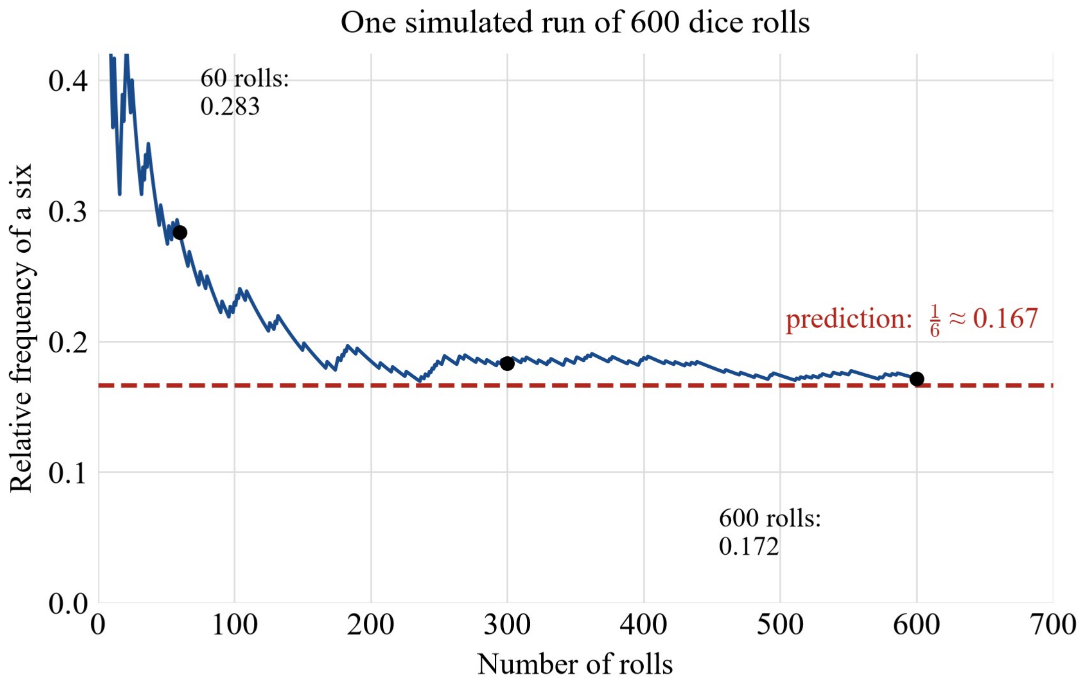
One simulated run of 200 coin tosses



One run of 200 coin tosses in a simulation: the running relative frequency of heads starts at 0.800 after 10 tosses, then 0.600 after 50, 0.580 after 100 and 0.520 after 200, closing on the dashed prediction line at 0.5

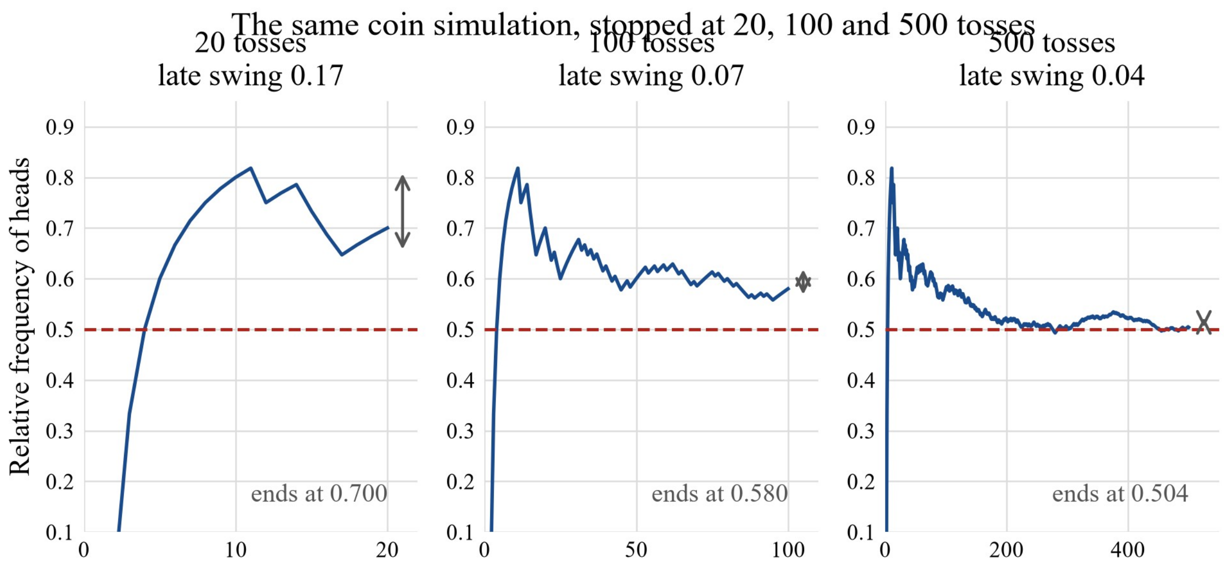
After 10 tosses the relative frequency of heads is 0.800 — a long way from the prediction of 0.5. At 50 tosses it is 0.600; at 100, 0.580; at 200, 0.520. The line does not jump to 0.5; it *settles towards* it, and the swings get smaller as the number of trials grows.

The die run shows the same settling: 600 rolls, with the relative frequency of sixes ending at 0.172 against a prediction of about 0.167.



One run of 600 die rolls in a simulation: the running relative frequency of sixes swings a lot over the first rolls and settles close to the dashed prediction line at 1/6 (about 0.167), ending at 0.172

To see the shrinking variation directly, stop the *same* coin simulation at 20, 100 and 500 tosses and look only at the late part of each run — the swings in the second half:



The same coin simulation stopped at 20, 100 and 500 tosses: the late swing in the relative frequency of heads shrinks from 0.17 to 0.07 to 0.04, while the run ends at 0.700, then 0.580, then 0.504, closer and closer to the prediction 0.5

The late swing falls from 0.17 (20 tosses) to 0.07 (100 tosses) to 0.04 (500 tosses), and the ending creeps from 0.700 to 0.580 to 0.504 — closer to 0.5 at every step. That is the effect of increasing the number of trials, in one sentence: **the variation shrinks, and the results tend towards the prediction.** It is why a fair coin can show 8 heads in 10 tosses without raising an eyebrow, while 80% heads over 1 000 tosses would make anyone check the coin.

Words for this subtopic

Word	Meaning
chance experiment	something we run where the result is a matter of luck — a toss, a roll, a spin
outcome	one possible result of a chance experiment
trial	one run of a chance experiment — one toss, one roll, one spin
tally	counting in fives, with the fifth mark crossing the previous four
frequency	the number of times an outcome happened
expected	the count or share we predict before the trials are run
relative frequency	frequency ÷ number of trials; the share of the trials that gave that outcome
variation	the spread in results — the gaps from the prediction and between runs
simulation	handing the trials to a digital tool, which runs and counts them for us
fair	every outcome equally likely, like the coin, die and spinner here

Worked examples

In the with-help examples the steps are labelled for you; in the on-your-own examples you carry them out yourself. Every example ends with a check.

Example 1 — with help

Use the spinner and the tally sheet for 40 spins. Work out the expected count for each colour, then find the gap between what happened and what was expected for every colour.

Step	Working	Comment
Read	Four equally likely colours; 40 spins. The tally gives red 13, blue 9, green 11, yellow 7.	The tally sheet holds what happened.
Plan	Expected count = $40 \times \frac{1}{4}$ for each colour; gap = tally − expected.	One prediction, four comparisons.
Work out	$40 \times \frac{1}{4} = 10$ each. Gaps: red $13 - 10 = +3$; blue $9 - 10 = -1$; green $11 - 10 = +1$; yellow $7 - 10 = -3$.	A gap above the prediction is +, below is −.
Check	Expected counts: $10 \times 4 = 40$ = number of spins ✓. Gaps: $+3 - 1 + 1 - 3 = 0$ ✓.	The gaps must cancel — what one colour gains, another loses.

Each colour was expected **10 times**; the gaps are **+3, −1, +1, −3** — ordinary variation over 40 spins.

Example 2 — with help

Turn the tally sheet into relative frequencies for all four colours.

Step	Working	Comment
Read	Frequencies 13, 9, 11, 7 over 40 spins.	Relative frequency = frequency ÷ trials.
Plan	Divide each frequency by 40, using the thousandths trick: multiply top and bottom by 25.	$\frac{13}{40}=\frac{325}{1000}$.
Work out	Red $\frac{13}{40}=0.325$; blue $\frac{9}{40}=0.225$; green $\frac{11}{40}=0.275$; yellow $\frac{7}{40}=0.175$.	All four sit on the 0–1 scale.
Check	$0.325+0.225+0.275+0.175=1$ ✓	Relative frequencies always add to 1.

The relative frequencies are **0.325, 0.225, 0.275 and 0.175** — they add to 1.

Example 3 — with help

Write the four relative frequencies of Example 2 as percentages, and check them against the percentage strip figure.

Step	Working	Comment
Read	Relative frequencies 0.325, 0.225, 0.275, 0.175.	A percentage is a share out of 100.
Plan	Multiply each by 100; then add the percentages.	Multiplying by 100 shifts the digits two places.
Work out	Red 32.5%; blue 22.5%; green 27.5%; yellow 17.5%. Sum: $32.5+22.5+27.5+17.5=100$.	The strip figure shows the same four pieces.
Check	The percentages add to 100% ✓, matching the strip's 100%.	Same rule as adding to 1, in percentage dress.

As percentages: **32.5%, 22.5%, 27.5%, 17.5%** — against a prediction of 25% each.

Example 4 — with help

Use the coin-run figure (one 200-toss run). Read the relative frequency of heads at each marked point, and say which is closest to the prediction.

Step	Working	Comment
Read	Marked points: 10 tosses → 0.800; 50 → 0.600; 100 → 0.580; 200 → 0.520. Prediction 0.5.	Four snapshots of one run.
Plan	Find each gap from 0.5; the smallest gap is closest.	Compare like with like.
Work out	Gaps: $0.800-0.5=0.3$; $0.600-0.5=0.1$; $0.580-0.5=0.08$; $0.520-0.5=0.02$. Smallest: 0.02, at 200 tosses.	The gaps shrink as the run grows.
Check	At 200 tosses, 104 heads: $104\div200=0.52$ ✓	The count and the picture agree.

The **200-toss point (0.520)** is closest to the prediction — the longest run has the smallest gap.

Example 5 — on your own

A simulation rolls a fair die 600 times. (a) How many sixes are expected? (b) The run gives 103 sixes. Find the gap from the prediction. (c) Find the relative frequency of sixes to 3 decimal places, and compare it with $\frac{1}{6} \approx 0.167$.

Plan. Expected = $600 \times \frac{1}{6}$. Gap = count − expected. Relative frequency = $103 \div 600$.

Work out. (a) \$600 × = \$ **100 sixes**. (b) \$103 − 100 = \$ **+3**. (c) $103 \div 600 = 0.1716\ldots$, which is **0.172** to 3 decimal places — 0.172 against a prediction of about 0.167, a gap of only 0.005.

Check. $100 \div 600 = 0.1666\ldots$, so the expected count does give the prediction 0.167 ✓ — and 103 is very close to 100 over 600 trials.

Example 6 — on your own

Use the simulation-tool figure. (a) Read the running count of sixes after 8 trials and check the printed relative frequency. (b) Do the same at 600 trials. (c) In one sentence, say why the two relative frequencies are so different.

Plan. Relative frequency = running count ÷ trial number; work out both rows by hand.

Work out. (a) After 8 trials the count is 4: $4 \div 8 = 0.5$ — the table's 0.500 ✓. (b) At 600 trials the count is 103: $103 \div 600 = 0.172$ ✓. (c) Eight trials are too few to trust: the relative frequency swings a lot early on, and settles towards the prediction $\frac{1}{6} \approx 0.167$ only when the number of trials grows.

Check. Both hand computations match the tool's printed values ✓.

Example 7 — on your own

Three runs of 40 coin tosses gave 25, 22 and 21 heads. (a) Find the relative frequency of heads in each run. (b) Put the runs in order, closest to the prediction first.

Plan. Divide each count by 40 (thousandths trick again), then compare the gaps from 0.5.

Work out. (a) Run A: $\frac{25}{40} = 0.625$; run B: $\frac{22}{40} = 0.550$; run C: $\frac{21}{40} = 0.525$. (b) Gaps from 0.5: 0.125, 0.05, 0.025 — so the order is **C, then B, then A**.

Check. The three-runs figure shows exactly this: run C ends lowest of the three, nearest the 0.5 prediction line ✓.

Example 8 — on your own

Use the settling figure (the same coin simulation stopped at 20, 100 and 500 tosses). (a) Read the late swing in each panel. (b) Read each ending. (c) State, in one sentence, the effect of increasing the number of trials.

Plan. Read the two numbers printed in each panel, then describe the trend across the three panels.

Work out. (a) Late swings: 0.17, then 0.07, then 0.04. (b) Endings: 0.700, then 0.580, then 0.504. (c) As the number of trials increases, the variation shrinks and the relative frequency gets closer to the prediction 0.5.

Check. The swings fall at every step (0.17 → 0.07 → 0.04) and the endings move towards 0.5 at every step ✓ — one trend, two ways of seeing it.

Practice questions

With help

Q1. Write the probability of each outcome as a fraction, a decimal and a percentage: (a) heads on a fair coin; (b) red on the fair spinner; (c) a six on a fair die (give the decimal and percentage to 3 significant figures, using "about").

Q2. Find the expected count. (a) Heads in 100 tosses of a fair coin. (b) Sixes in 60 rolls of a fair die. (c) Green in 80 spins of the fair spinner.

Q3. Use the tally sheet. (a) Check that the four frequencies add to the number of spins. (b) Which colour came up most, and which least?

Q4. Use the tally sheet to find the relative frequency of (a) red, (b) blue.

Q5. Write as a percentage: (a) the relative frequency of green, 0.275; (b) the relative frequency of yellow, 0.175.

Q6. The relative frequencies of red, blue and green in the 40 spins are 0.325, 0.225 and 0.275. Use the add-to-1 rule to find the relative frequency of yellow without using the tally.

Q7. Use the coin-run figure. Read the relative frequency of heads at (a) 10 tosses, (b) 100 tosses. (c) Which of the four marked points is closest to the prediction, and what is its gap from 0.5?

Q8. Use the simulation-tool figure. Read the relative frequency of sixes after (a) 4 trials, (b) 8 trials, and check each by dividing the running count by the trial number.

Q9. Use the six-run figure. (a) How many sixes came up in the 600 rolls? (b) What relative frequency does the run end at? (c) The prediction is $\frac{1}{6} \approx 0.167$. How far is the ending from it?

Q10. Use the settling figure. Read the late swing at (a) 20 tosses, (b) 500 tosses. (c) What happens to the late swing as the number of tosses increases?

On your own

Q11. Find the expected count. (a) Tails in 200 tosses of a fair coin. (b) Sixes in 120 rolls of a fair die. (c) Red in 200 spins of the fair spinner.

Q12. Find the relative frequency. (a) 44 heads in 80 tosses. (b) 18 sixes in 72 rolls. (c) 21 reds in 60 spins.

Q13. Write each relative frequency of Q12 as a percentage.

Q14. A three-outcome game spinner (win, lose, replay) is spun many times. The relative frequencies of win and lose are 0.45 and 0.35. Find the relative frequency of replay.

Q15. The expected count for each colour in 40 spins is 10. Write the gap from the prediction for (a) red, 13; (b) yellow, 7; (c) which colour has a gap of -1?

Q16. Three runs of 40 coin tosses gave 25, 22 and 21 heads. Find the relative frequency of heads in run C, then put the three runs in order, closest to the prediction first.

Q17. One coin shows these relative frequencies of heads: 0.700 after 20 tosses, 0.580 after 100, 0.504 after 500. Which of the three would you trust most as an estimate of the coin's true chance of heads? Give the reason in one sentence.

Q18. A simulation rolls a fair die 600 times and gets 103 sixes. (a) Find the expected count of sixes. (b) Find the gap. (c) Find the relative frequency to 3 decimal places and compare it with $\frac{1}{6} \approx 0.167$.

Q19. The 40 spins written as percentages are 32.5%, 22.5% and 27.5% for red, blue and green. Use the add-to-100% rule to find yellow's percentage.

Q20. Use the settling figure. (a) Read the ending of the 100-toss run. (b) Read the ending of the 500-toss run. (c) Describe what happens to the late swing across the three panels.

Maths in real life

Q21. A board-game maker tests the fair spinner with 40 spins and gets the tally sheet's results: 13 reds against an expected 10. A tester says the spinner must be unfair because red came up so often. Using the ideas of expected counts and variation, give one reason *for* the tester's doubt and one reason *against* it, then say what longer run you would order and what you would expect to see in it.

Q22. A club's match coin has been tossed 200 times this season, with 104 heads. One player claims the coin gives heads too often. (a) Find the relative frequency of heads. (b) Find its gap from 0.5. (c) Using the coin-run figure as a guide, say whether 200 tosses is enough to back the claim.

Q23. You buy a second-hand die and roll it 600 times to check it: 103 sixes. (a) What count was expected? (b) Find the relative frequency of sixes to 3 decimal places. (c) Compare it with $\frac{1}{6} \approx 0.167$ and give a one-sentence verdict on the die.

Choosing your strategy

Q24. Work back from a relative frequency. (a) After 200 tosses the relative frequency of heads is 0.45. How many heads came up? (b) After 40 spins the relative frequency of red is 0.35. How many reds came up?

Q25. Two runs of a coin: 30 heads in 50 tosses, and 52 heads in 100 tosses. (a) Find each relative frequency. (b) Which run sits closer to the prediction? (c) Which run would you trust more, and why?

Q26. Digital tool task. Open a simulation tool, set it to toss one fair coin, and run 20 trials; record the relative frequency of heads. Then extend the same run to 200 trials and record it again. (a) Before running, predict how the gap from 0.5 will change. (b) Run it and compare what happens with your prediction. (c) Explain in one sentence why the 200-trial figure is the more trustworthy one.

Q27. Explain to a Year 5 student why tossing a coin ten times and getting 7 heads does not mean the coin is unfair. Use the words *trial* and *variation* in your explanation.

Answers

Q1. (a) $\frac{1}{2}$, 0.5, 50%. (b) $\frac{1}{4}$, 0.25, 25%. (c) $\frac{1}{6}$, about 0.167, about 16.7%. **Q2.** (a) 50. (b) 10. (c) 20. **Q3.** (a) $13+9+11+7=40$ ✓. (b) Most: red (13); least: yellow (7). **Q4.** (a) $13 \div 40=0.325$. (b) $9 \div 40=0.225$. **Q5.** (a) 27.5%. (b) 17.5%. **Q6.** $1-(0.325+0.225+0.275)=1-0.825=0.175$. **Q7.** (a) 0.800. (b) 0.580. (c) The 200-toss point, 0.520, gap 0.02. **Q8.** (a) 0.750; check $3 \div 4=0.75$ ✓. (b) 0.500; check $4 \div 8=0.5$ ✓. **Q9.** (a) 103. (b) 0.172. (c) $0.172-0.167=0.005$ — very close. **Q10.** (a) 0.17. (b) 0.04. (c) It shrinks (0.17 → 0.07 → 0.04). **Q11.** (a) 100. (b) 20. (c) 50. **Q12.** (a) 0.55. (b) 0.25. (c) 0.35. **Q13.** (a) 55%. (b) 25%. (c) 35%. **Q14.** $1-(0.45+0.35)=0.2$. **Q15.** (a) +3. (b) −3. (c) Blue (9 against 10). **Q16.** Run C: $\frac{21}{40}=0.525$; order C (0.525), B (0.550), A (0.625). **Q17.** The 500-toss figure, 0.504 — the more trials, the smaller the variation, so the longer run sits closer to the true chance. **Q18.** (a) 100. (b) +3. (c) 0.172, a gap of 0.005 from 0.167 — close enough to call the die fair. **Q19.** $100\%-82.5\%=17.5\%$. **Q20.** (a) 0.580. (b) 0.504. (c) It shrinks: 0.17 → 0.07 → 0.04. **Q21.** For: red ran 3 ahead of its expected 10, the biggest gap on the sheet. Against: over only 40 spins, gaps of ±3 are ordinary variation — the three-runs figure shows the same coin producing 25, 22 and 21 heads. Longer run: e.g. 400 spins, expecting each colour near 100 and each relative frequency near 0.25. **Q22.** (a) 0.52. (b) 0.02. (c) No — the coin-run figure's 200-toss run ends at 0.520 with a perfectly fair coin, so 0.52 over 200 tosses is ordinary variation, not evidence. **Q23.** (a) 100. (b) 0.172. (c) 0.172 sits 0.005 above 0.167 — within ordinary variation over 600 rolls, so the die shows no sign of being unfair. **Q24.** (a) $0.45 \times 200=90$ heads. (b) $0.35 \times 40=14$ reds. **Q25.** (a) 0.6 and 0.52. (b) The 100-toss run (gap 0.02 against 0.1). (c) The 100-toss run — more trials, smaller variation. **Q26.** (a) Answers will vary — the gap should usually shrink. (b) Answers will vary; compare your two figures with the settling figure's trend. (c) More trials mean smaller variation, so the longer run sits closer to the prediction. **Q27.** Answers will vary — e.g. ten trials are too few to judge: the three-runs figure shows a fair coin giving 25 heads in 40, and the coin-run figure shows 0.800 after 10 tosses; with so few trials the variation is big, so 7 heads in 10 is well within it.

All the working

Q1. (a) Two equally likely outcomes: $\frac{1}{2}=0.5=50\%$. (b) Four equally likely outcomes: $\frac{1}{4}=0.25=25\%$. (c) Six equally likely outcomes: $\frac{1}{6}=0.1666\ldots$, about 0.167 or about 16.7%.

Q2. (a) \$100 × = \$ **50**. (b) \$60 × = \$ **10**. (c) \$80 × = \$ **20**.

Q3. (a) $13+9+11+7=40$ ✓ — matches the 40 spins. (b) Most: **red, 13**; least: **yellow, 7**.

Q4. (a) \$ = = \$ **0.325**. (b) \$ = = \$ **0.225**.

Q5. (a) \$0.275 × 100 = \$ **27.5%**. (b) \$0.175 × 100 = \$ **17.5%**.

Q6. $0.325+0.225+0.275=0.825$; \$1 − 0.825 = \$ **0.175** — the same as $\frac{7}{40}$ on the tally sheet ✓.

Q7. (a) **0.800**. (b) **0.580**. (c) The gaps: 0.3, 0.1, 0.08, 0.02 — closest is the **200-toss point, 0.520, gap 0.02**.

Q8. (a) Table: **0.750**; check: 3 sixes in 4 trials, $3 \div 4=0.75$ ✓. (b) Table: **0.500**; check: 4 sixes in 8 trials, $4 \div 8=0.5$ ✓.

Q9. (a) **103**. (b) **0.172**. (c) \$0.172 − 0.167 = \$ **0.005** — the 600-roll run ends almost on the prediction.

Q10. (a) **0.17**. (b) **0.04**. (c) The late swing **shrinks**: 0.17 → 0.07 → 0.04 as the tosses grow 20 → 100 → 500.

Q11. (a) \$200 × = \$ **100**. (b) \$120 × = \$ **20**. (c) \$200 × = \$ **50**.

Q12. (a) \$44 ÷ 80 = \$ **0.55**. (b) \$18 ÷ 72 = \$ **0.25**. (c) \$21 ÷ 60 = \$ **0.35**.

Q13. (a) **55%**. (b) **25%**. (c) **35%**.

Q14. $0.45+0.35=0.8$; \$1 − 0.8 = \$ **0.2**.

Q15. (a) \$13 − 10 = \$ **+3**. (b) \$7 − 10 = \$ **−3**. (c) Blue: $9-10=-1$.

Q16. Run C: $\frac{21}{40}=\frac{525}{1000}=0.525$. Gaps from 0.5: A 0.125, B 0.05, C 0.025 — order **C, B, A**.

Q17. 0.504 (500 tosses): the variation shrinks as the trials increase, so the longest run gives the most trustworthy picture of the coin.

Q18. (a) \$600 × = \$ **100**. (b) \$103 − 100 = \$ **+3**. (c) \$103 ÷ 600 = 0.1716… ≈ \$ **0.172**; against 0.167 the gap is 0.005.

Q19. $32.5+22.5+27.5=82.5$; \$100 − 82.5 = \$ **17.5%** — the strip figure shows the same piece ✓.

Q20. (a) **0.580**. (b) **0.504**. (c) The late swing shrinks, 0.17 → 0.07 → 0.04, while the endings move towards 0.5.

Q21. For the doubt: red's 13 is the biggest gap on the sheet (+3). Against: over 40 spins such gaps are ordinary variation — three identical fair-coin runs gave 25, 22 and 21 heads. Longer run: e.g. **400 spins**, expecting about 100 per colour and relative frequencies near 0.25; if red stays near 0.325 over 400 spins, *then* doubt the spinner.

Q22. (a) \$104 ÷ 200 = \$ **0.52**. (b) \$0.52 − 0.5 = \$ **0.02**. (c) **Not enough**: the coin-run figure shows a fair coin ending a 200-toss run at exactly 0.520, so the season's figure is ordinary variation.

Q23. (a) \$600 × = \$ **100**. (b) \$103 ÷ 600 ≈ \$ **0.172**. (c) 0.172 against 0.167 is a gap of 0.005 — ordinary variation, so **no sign of an unfair die**.

Q24. (a) Relative frequency × trials: \$0.45 × 200 = \$ **90 heads**. (b) \$0.35 × 40 = \$ **14 reds**.

Q25. (a) $30\div50=0.6$; $52\div100=0.52$. (b) Gaps: 0.1 and 0.02 — the **100-toss run** is closer. (c) The **100-toss run**: more trials, smaller variation.

Q26. (a) Answers will vary; the prediction to hold: the gap from 0.5 should usually shrink by 200 trials. (b) Answers will vary — your two numbers should sit like the settling figure's: big swings early, small swings late. (c) With more trials the variation shrinks, so the 200-trial relative frequency sits closer to the true chance of heads.

Q27. Answers will vary — e.g. ten tosses are only ten **trials**, and with few trials the **variation** is big: the coin-run figure shows a fair coin at 0.800 after 10 tosses, and 7 heads in 10 (0.7) is no stranger than that. A fair coin is judged over hundreds of trials, not ten.

What comes next

That closes Chapter 8, and with it this book. In **8A** you put probabilities on the 0–1 scale as fractions, decimals and percentages, and in **8B** you put those numbers to the test: tallying trials, comparing what happens with the prediction, and running simulations in a digital tool to see the big idea of the chapter — as the number of trials increases, the variation shrinks and the results tend towards the prediction.

Within this book the thread stands on Chapter 2 (the fractions, decimals and percentages that probabilities are written with) and on 7A's data displays, and it shares its habit of mind with 3A and 2C: every time you compare a result with what you expect, and ask how far off it could be, you are using the same thinking 8B makes visible.

In Year 7 the thread continues with new names and longer runs. `VC2M7P01` gives the list of all possible outcomes a name — the **sample space** — and `VC2M7P02` runs simulations with a large number of trials, comparing the prediction with what happens over many runs. Everything there starts from the picture this subtopic built: a prediction on the 0–1 scale, a tally of trials, and variation that shrinks as the trials grow.

Test

Chapter test T8 — Probability

Assessed content descriptions	`VC2M6P01` — subtopic 8A, Describing probabilities with fractions, decimals and percentages; `VC2M6P02` — subtopic 8B, Simulations and the effect of increasing the number of trials
Marks	**10** — Section A (8A): 5 marks · Section B (8B): 5 marks
Time	10 minutes working time, plus 5 minutes reading time (about 15 minutes in all)
Equipment	Simulation tool access for the 8B items; dice and spinners

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 6 achievement standard, not against a mark on this paper.

Answer every question. Show your working where a question asks for it. The marks for each part are shown in brackets.

Section A — Describing probabilities (8A, `VC2M6P01`)

Question 1 [2 marks]. A bag holds 4 red counters, 5 blue counters and 1 yellow counter. One counter is picked without looking.

a. Find P(blue) as a fraction. **[1 mark]**

b. Write P(blue) as a decimal and as a percentage. **[1 mark]**

Question 2 [2 marks]. A spinner has 8 equal parts, numbered 1 to 8. The spinner is spun once.

a. Find the probability of spinning a number greater than 6, as a fraction in its simplest form. **[1 mark]**

b. Find the probability of *not* spinning a number greater than 6. Give your answer as a decimal or a percentage. **[1 mark]**

Question 3 [1 mark]. The probability that the bus is late tomorrow is 0.85. Which word best describes the chance that the bus is late?

- **A** impossible
- **B** unlikely
- **C** evens
- **D** likely

Section B — Simulations and the effect of increasing the number of trials (8B, `VC2M6P02`)

The results in Question 4 came from a simulation tool.

Question 4 [3 marks]. A simulation tool rolls a fair die 300 times.

a. How many sixes are expected? **[1 mark]**

b. The run gives 54 sixes. Find the gap between the observed count and the expected count. **[1 mark]**

c. Find the relative frequency of sixes in the run. **[1 mark]**

Question 5 [2 marks].

a. Roll the die 10 times. Count how many of your rolls show an even number, then find the relative frequency of even numbers in your 10 rolls. **[1 mark]**

b. The prediction for rolling an even number on a fair die is $\frac{1}{2} = 0.5$. Explain why the result of a 300-trial run is likely to sit closer to the prediction than your result from 10 rolls. **[1 mark]**

End of test — 10 marks.

Solutions

Fully worked solutions for every question, followed by a compact answer key for self-checking.

Worked solutions

Question 1.

a. Find the total first: $4+5+1=10$ counters, all equally likely to be picked. Five of them are blue, so count 5 outcomes out of 10:

$$P(\text{blue})=\frac{5}{10}=\frac{1}{2}$$

Either form earns the mark. **[1 mark]**

b. Change the fraction to a decimal by dividing top by bottom, then to a percentage by multiplying by 100:

$$\frac{1}{2}=1\div 2=0.5, 0.5\times 100=50\%$$

P(blue) = 0.5 = 50%. **[1 mark]**

Question 2.

a. The 8 equal parts mean all 8 outcomes are equally likely, so the counting rule works. "Greater than 6" means 7 or 8 — two outcomes fit:

$$P(\text{greater than 6})=\frac{2}{8}=\frac{1}{4}$$

[1 mark]

b. The other side of an event is everything left: the 6 parts numbered 1 to 6. Count them, or subtract from 1:

$$P(\text{not greater than 6})=\frac{6}{8}=\frac{3}{4}=0.75=75\%(\text{or } 1-\frac{1}{4}=\frac{3}{4})$$

A decimal or a percentage earns the mark. **[1 mark]**

Question 3.

Write the probability in words using the scale from 0 to 1: 0.85 sits close to 1, between evens and certain. Events close to 1 are **likely** (0.85 is not 1, so the event is not certain). The answer is **D**. **[1 mark]**

Question 4.

a. A fair die has 6 equally likely outcomes, so the probability of a six is $\frac{1}{6}$. The expected count is the number of trials times that probability:

$$300\times\frac{1}{6}=50\,\text{sixes}$$

[1 mark]

b. Gap = observed count − expected count:

$$54-50=+4$$

The run gave 4 more sixes than expected — a gap above the prediction is positive. **[1 mark]**

c. Relative frequency = frequency ÷ number of trials:

$$54 \div 300 = 0.18$$

Check against the prediction: $\frac{1}{6} \approx 0.167$, and 0.18 sits a little above it — ordinary variation over 300 trials. **[1 mark]**

Question 5.

a. Answers vary with the student's rolls. The mark is awarded for a relative frequency correctly computed from the student's own count: relative frequency = number of even rolls ÷ 10. For example, if 4 of the 10 rolls show an even number, the relative frequency is $4 \div 10 = 0.4$. **[1 mark]**

b. The number of trials matters: **the more trials, the smaller the variation**. With only 10 rolls the relative frequency swings a lot from run to run, while over 300 trials the swings shrink and the result settles towards the prediction of 0.5. So the 300-trial result is the one expected to sit closer to the prediction. Accept any response that names the effect of increasing the number of trials (variation shrinks / results tend towards the prediction). **[1 mark]**

Answer key

Question	Answer
1a	$\frac{5}{10} = \frac{1}{2}$
1b	0.5; 50%
2a	$\frac{2}{8} = \frac{1}{4}$
2b	0.75 or 75% ($\frac{3}{4}$)
3	D — likely
4a	50 sixes
4b	+4
4c	0.18
5a	Varies; relative frequency = even rolls ÷ 10 (e.g. 4 even rolls → 0.4)
5b	More trials → smaller variation, so the result settles closer to the prediction

Total: 10 marks — Section A (8A, `VC2M6P01`): 5 · Section B (8B, `VC2M6P02`): 5.