# Mathematics Level 6

## Chapter 6 — Space



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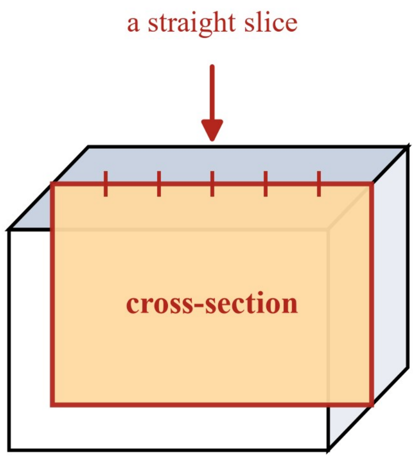
### Contents

## Cross-sections and right prisms

*compare the parallel cross-sections of objects and recognise their relationships to right prisms —* VC2M6SP01

### Theory

**What a cross-section is.** Take a solid and slice it straight through, the way you slice a loaf. The new flat face you can see on the cut is called a **cross-section** of the solid.




Slice a foam block straight through.

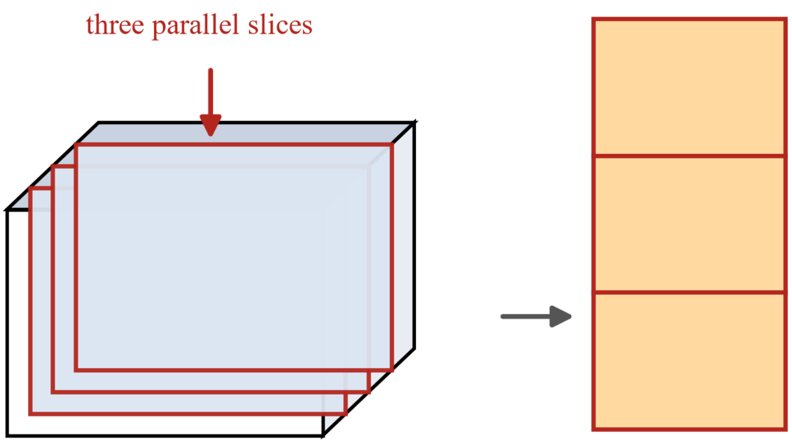
The new flat face you can see

is called the cross-section.

*A foam block with a straight slice shown part-way through; the new flat face made by the slice is highlighted and labelled cross-section.*

Every solid has cross-sections, and the cross-section is a flat shape: slice a foam block and the face you see is a rectangle; slice it a different way and you may see a different shape. The cross-section belongs to **one particular cut**, so when you talk about a cross-section you should always be able to say which cut made it.

**Parallel cross-sections.** Now make several slices of the same solid, all in the same direction, so the cuts never meet. Cuts like that are **parallel** to each other, and the cross-sections they make are called **parallel cross-sections**.




The slices are parallel, and each cross-section is the same rectangle.

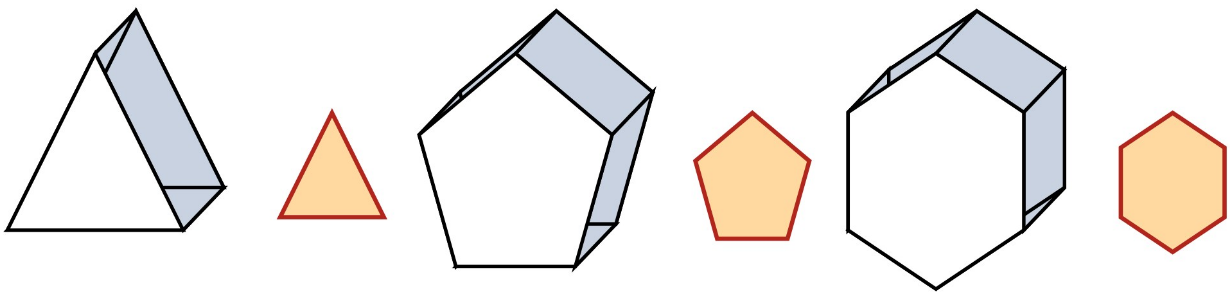
*A rectangular prism with three parallel slices marked; taken out, the three cross-sections are three identical rectangles.*

For a rectangular prism, every slice parallel to the end gives the same rectangle, the same shape and the same size, no matter where you cut along the prism. That fact is what a right prism is.

**Right prisms.** The Victorian Curriculum gives this definition (elaboration VC2M6SP01.E04): a **right prism** is an object whose parallel cross-sections, perpendicular to the base of the prism, are the same shape and size. The definition does not say whether the cross-section must be straight-sided, so this subtopic adds that frame: in this book, the cross-section of a right prism is a straight-sided shape. The cylinder, later in this subtopic, is exactly the object that makes that extra part necessary.

A right prism is named after the shape of its cross-section.

**A prism is named after the shape of its cross-section.**



**triangular prism:** cross-section is a triangle

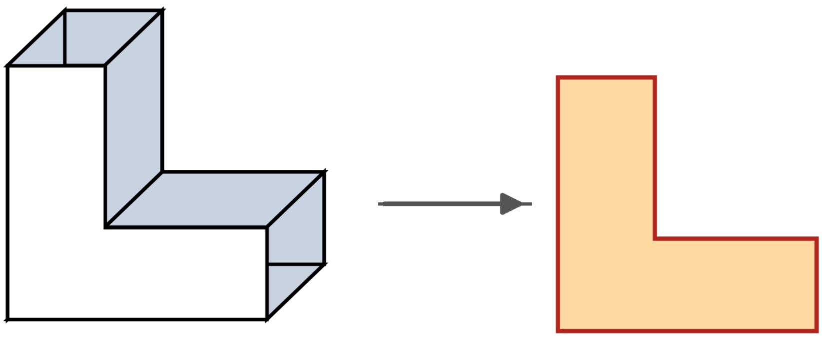
**pentagon prism:** cross-section is a pentagon

**hexagonal prism:** cross-section is a hexagon

*A triangular prism shown next to a triangle, a pentagon prism next to a pentagon, and a hexagonal prism next to a hexagon: each prism is named after the shape of its cross-section.*

A prism whose cross-section is a triangle is a **triangular prism**; a pentagon gives a **pentagon prism**; a hexagon gives a **hexagonal prism**. The cross-section does not have to be a shape you meet often.

**The cross-section does not have to be a usual shape.**



A solid with an L-shaped cross-section is still a prism.

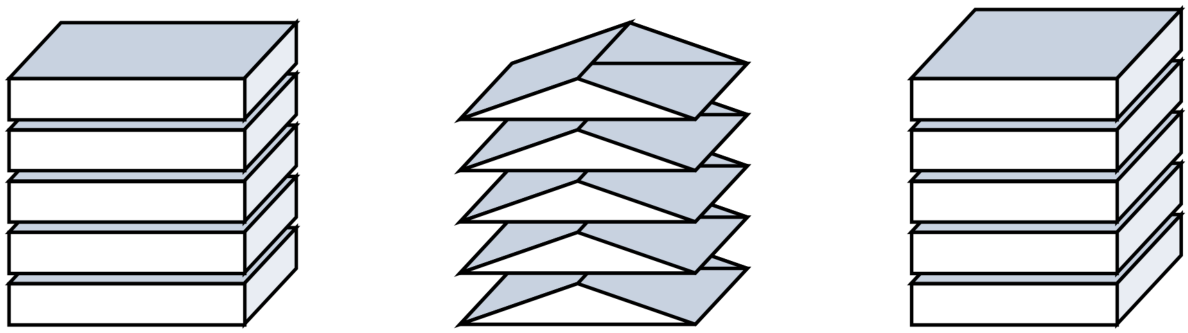
Its cross-section: any straight-sided shape works.

*A solid whose cross-section is an L-shape, shown next to that L-shape: the cross-section of a prism can be any straight-sided shape.*

The solid above has the same L-shape all the way through, so it is a right prism too. Any straight-sided cross-section, kept the same shape and size, makes one.

**A prism is a stack.** Because the cross-section never changes, you can think of any right prism as a **stack** of its cross-section, like a stack of identical slices.

**The same cross-section, stacked all the way through.**



A rectangular prism is a stack of the same rectangles.

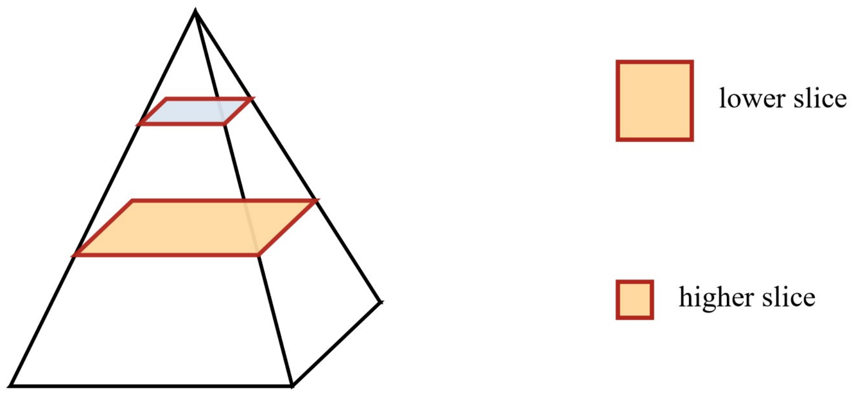
A triangular prism is a stack of the same triangles.

A cube is a stack of the same squares.

*Three stacks: five identical rectangles stacked make a rectangular prism; five identical triangles stacked make a triangular prism; five identical squares stacked make a cube.*

A rectangular prism is a stack of the same rectangles. A triangular prism is a stack of the same triangles. A cube is a stack of the same squares. If someone tells you the cross-section of a right prism, you know the whole solid: the same shape, stacked all the way through.

**Not every object is a prism.** The way to test an object is to compare its parallel cross-sections. They must be the **same shape and the same size**.

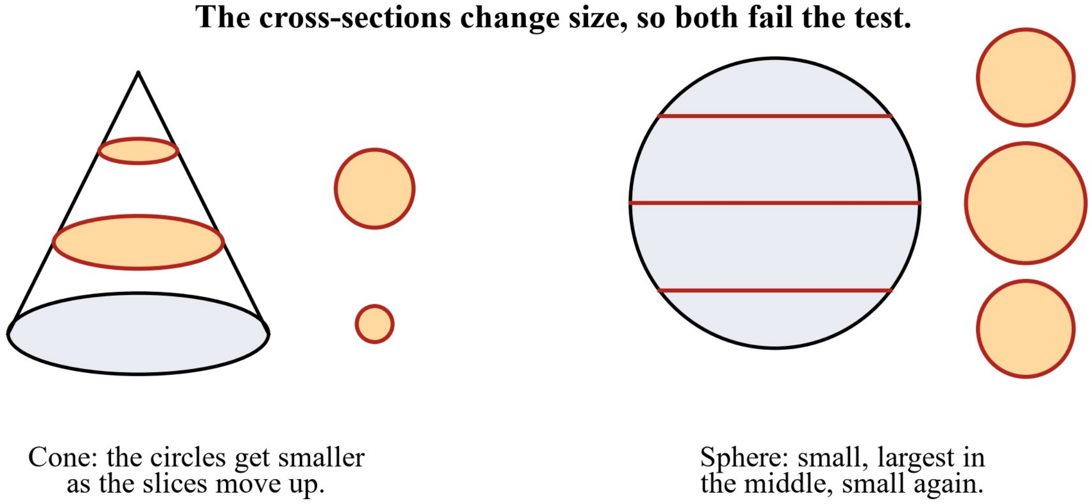



Both cross-sections are squares, but the higher one is smaller. Not a prism.

*A square pyramid sliced twice parallel to the base; the lower cross-section is a larger square and the higher cross-section is a smaller square, so a pyramid is not a prism.*

A square pyramid fails the test. Its parallel cross-sections are all squares, so the shape is right, but the higher you slice, the smaller the square. Same shape is not enough: the size must stay the same too.

**The cross-sections change size, so both fail the test.**

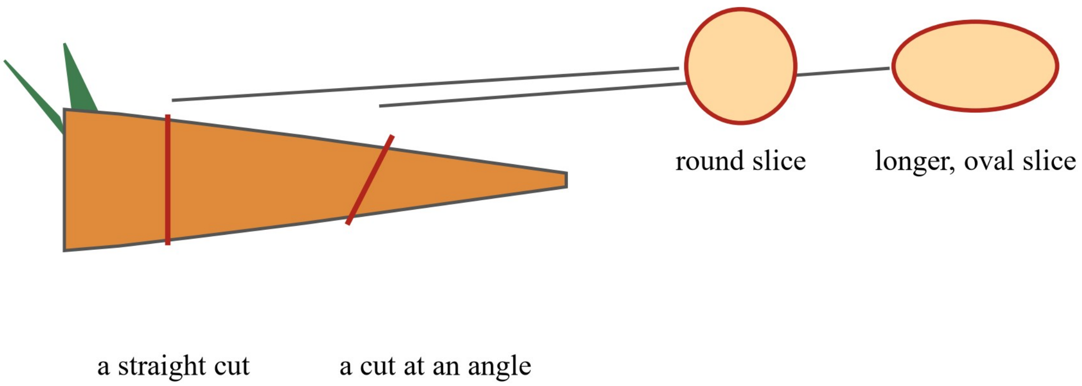



*A cone whose circular cross-sections get smaller towards the top, and a sphere whose circular cross-sections are small, largest in the middle, and small again; both fail the prism test.*

A cone and a sphere fail for the same kind of reason: their cross-sections change size. A cone's circles shrink as the slices move up. A sphere's circles are small near the top, largest through the middle, and small again at the bottom.

**The cut decides the shape.** The cross-section you get also depends on the direction of the cut. Slice a carrot straight across and the cross-section is round; slice at an angle and the cross-section is longer and oval.

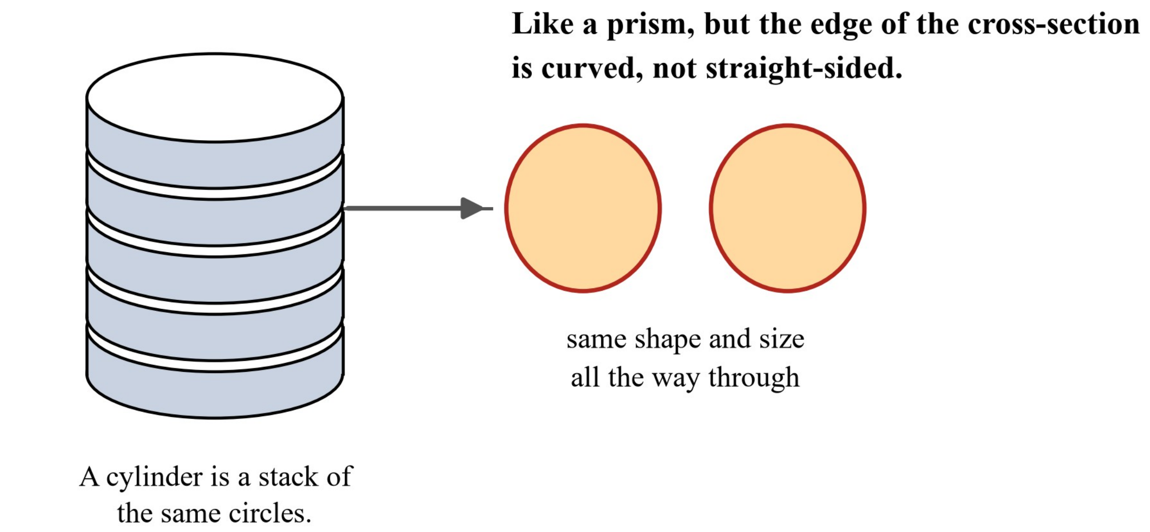
**Same carrot, different cuts: different cross-sections.**




*A carrot cut straight across gives a round slice, while a cut at an angle gives a longer, oval slice.*

So when you compare cross-sections, compare cuts made the same way: that is why the test for a prism uses **parallel** cross-sections.

**The cylinder: like a prism, not a prism.** A cylinder passes half of the test.

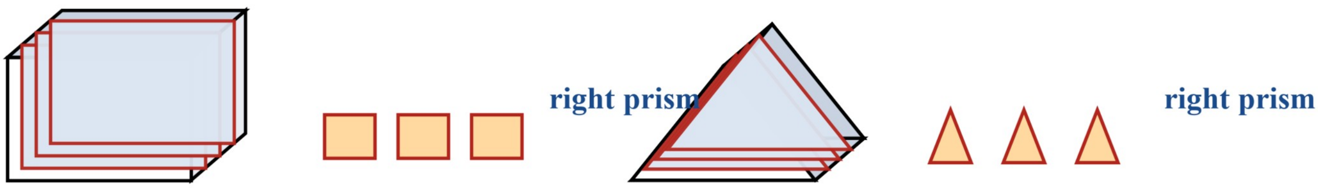



*A cylinder drawn as a stack of same-sized circles, like a stack of coins, next to two identical circles: the cross-section is the same all the way through, but its edge is curved.*

Its parallel cross-sections are the same circle, the same shape and the same size, all the way through, so a cylinder behaves like a stack in exactly the way a right prism does. But the edge of the circle is curved, not straight-sided. Under the frame this subtopic uses, a cylinder is **not** a right prism; its relationship to right prisms is that it shares the stacking property while having a curved cross-section.

**Comparing objects.** Putting the test to work on four objects:

**Compare the parallel cross-sections to decide.**




Rectangular prism: the same rectangle every time.

Triangular prism: the same triangle every time.


Square pyramid: squares, but they get smaller.

Cylinder: the same circle, but the edge is curved.

*Four objects compared: a rectangular prism and a triangular prism have the same cross-section all the way through and are right prisms; a square pyramid's square cross-sections get smaller, and a cylinder's cross-section is a curved circle, so both fail the prism test.*

The two prisms keep the same cross-section all the way through. The pyramid's cross-sections shrink, and the cylinder's cross-section is curved. Comparing parallel cross-sections is how you tell prisms from the rest.

**Words for this subtopic**, with what they mean here:

| Term | Meaning here |
|---|---|
| **cross-section** | the flat face you see when you slice straight through a solid |
| **parallel cross-sections** | cross-sections made by cuts in the same direction, which never meet |
| **right prism** | a solid whose parallel cross-sections are the same straight-sided shape and size |
| **base** | the face the cross-sections are parallel to; the shape of the base is the shape of the stack |
| **stack** | a way to picture a right prism: its cross-section repeated all the way through |
| **straight-sided shape** | a flat shape whose edges are straight lines, like a triangle, square or L-shape |

## Worked examples

### *Example 1 — with help*

Every parallel cross-section of a solid is the same hexagon. Is the solid a right prism? If so, what is its name?

| Working | Comment |
|---|---|
| Same shape? Yes: every cross-section is a hexagon. | The test looks at shape first. |
| Same size? Yes: every cross-section is the same hexagon. | Shape alone is not enough; the size must stay the same too. |
| Straight-sided? Yes: a hexagon has straight edges. | The frame for this subtopic. |
| The solid is a right prism, a **hexagonal prism**. | A prism is named after its cross-section. |

### *Example 2 — with help*

Mia slices a square pyramid twice, parallel to the base. Both cross-sections are squares, so Mia says the pyramid must be a right prism. What has she missed?

| Working | Comment |
|---|---|
| The cross-sections are both squares: same shape. | Mia is right about the shape. |
| The higher square is smaller than the lower one. | The sizes are not the same. |
| A right prism needs the same shape **and** the same size. | The test has two parts. |
| The pyramid is **not** a right prism. | Same shape is not enough. |

### *Example 3 — on your own*

A solid has the same circle as its parallel cross-section, all the way through. Describe the solid's relationship to right prisms.

The solid is a cylinder. Like a right prism, its parallel cross-sections are the same shape and the same size all the way through, so it can be pictured as a stack of identical slices. Unlike a right prism, its cross-section is a circle with a curved edge, not a straight-sided shape. So under the frame this subtopic uses, the cylinder is like a prism but is not one.

### *Example 4 — on your own*

Ana makes parallel slices of a mystery solid, from the bottom up: a small circle, then larger circles, then the largest circle through the middle, then smaller circles again. What could the solid be, and could it be a right prism?

The cross-sections are circles that change size, growing to the middle and shrinking again, which is what a sphere does. So the solid could be a sphere (or something sphere-like, such as an orange). It cannot be a right prism, for two reasons: the cross-sections change size, and a circle is curved, not straight-sided.

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## Practice questions

### *With help*

**Q1.** A block of playdough is sliced straight through. What is the flat face you can see on the cut called?

**Q2.** True or false? Fix each false sentence. (a) A right prism has parallel cross-sections that are the same shape and the same size. (b) A triangular prism is a stack of the same rectangles. (c) The parallel cross-sections of a square pyramid are all the same size.

**Q3.** Name the prism from its cross-section. (a) triangle (b) pentagon (c) square (d) hexagon

**Q4.** Fill in the gaps. (a) A cube is a stack of the same _____. (b) A triangular prism is a stack of the same _____. (c) A rectangular prism is a stack of the same _____.

**Q5.** A carrot is cut two ways: straight across, and at an angle. Match each cut to its cross-section: round slice; longer, oval slice.

**Q6.** Write *prism* or *not a prism* for each solid. (a) cuboid (b) cone (c) cube (d) sphere (e) triangular prism (f) square pyramid

### *On your own*

**Q7.** Every parallel cross-section of a solid is the same pentagon. (a) Is the solid a right prism? (b) What is its name?

**Q8.** Using cross-sections, explain why a square pyramid is not a right prism.

**Q9.** The parallel cross-sections of a cylinder are all the same circle. Explain how a cylinder is *like* a right prism, and why this subtopic does not call it one.

**Q10.** Parallel slices of a solid, from the bottom up, are: a small circle, a larger circle, the largest circle, a smaller circle, a small circle. What does that pattern tell you about the solid? Could it be a right prism?

**Q11.** A solid has the same L-shape as its cross-section all the way through. Is it a right prism? Give a reason using the definition.

**Q12.** Mia says a cube is not a prism, because "a prism has to look like a tent, with triangle ends". Use the definition of a right prism to explain why Mia is wrong.

**Q13.** An orange is roughly a sphere. Predict the cross-section you would see if you cut it (a) straight through the middle, and (b) near the top. Which cross-section is larger? Is the orange a right prism?

**Q14.** Copy and complete the table for the four solids below, then use it to say what makes a right prism.

| Solid | Shape of parallel cross-sections | Same size all the way through? | Right prism? |
|---|---|---|---|
| rectangular prism | | | |
| triangular prism | | | |
| square pyramid | | | |
| cylinder | | | |

## Answers

**Q1.** A cross-section. **Q2.** (a) True. (b) False: a triangular prism is a stack of the same triangles. (c) False: the higher you slice, the smaller the square. **Q3.** (a) triangular prism; (b) pentagon prism; (c) square prism (a cube is one too, when the stack is the same height all round); (d) hexagonal prism. **Q4.** (a) squares; (b) triangles; (c) rectangles. **Q5.** Straight across gives the round slice; the cut at an angle gives the longer, oval slice. **Q6.** (a) prism; (b) not a prism; (c) prism; (d) not a prism; (e) prism; (f) not a prism. **Q7.** (a) Yes. (b) A pentagon prism. **Q8.** Its parallel cross-sections are squares, but they get smaller higher up, so they are not the same size all the way through. **Q9.** Like a prism, its parallel cross-sections are the same shape and size all the way through, so it stacks the same way; unlike a prism, the cross-section is a circle with a curved edge, not straight-sided. **Q10.** The solid is sphere-like; no, its cross-sections change size and are curved. **Q11.** Yes: the parallel cross-sections are the same straight-sided shape (an L-shape) and the same size all the way through. **Q12.** A cube is a stack of the same squares, so its parallel cross-sections are the same shape and size; the definition never says the cross-section must be a triangle. **Q13.** (a) a circle, the largest one; (b) a smaller circle; the middle one is larger; no, a sphere's cross-sections change size and are curved. **Q14.** rectangle / yes / yes; triangle / yes / yes; square / no (they shrink) / no; circle / yes / no (the edge is curved).

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## Worked solutions

**Q1.** Slicing straight through a solid shows a flat face, and that face is called a cross-section.

**Q2.** (a) True: same shape and same size is exactly the definition used here. (b) False: the cross-section of a triangular prism is a triangle, so it is a stack of the same triangles. (c) False: slicing a square pyramid higher gives a smaller square, so the sizes are not the same.

**Q3.** A prism is named after its cross-section: (a) triangle gives a triangular prism; (b) pentagon gives a pentagon prism; (c) square gives a square prism; (d) hexagon gives a hexagonal prism.

**Q4.** Picture each solid as a stack: (a) a cube stacks the same squares; (b) a triangular prism stacks the same triangles; (c) a rectangular prism stacks the same rectangles.

**Q5.** The straight cut is parallel to the round end, so it gives the round slice. The cut at an angle is a different direction, so it gives the longer, oval slice. Same carrot, different cuts, different cross-sections.

**Q6.** (a) A cuboid keeps the same rectangle all the way through: prism. (b) A cone's circles shrink: not a prism. (c) A cube keeps the same square all the way through: prism. (d) A sphere's circles change size: not a prism. (e) A triangular prism keeps the same triangle all the way through: prism. (f) A square pyramid's squares shrink: not a prism.

**Q7.** (a) The parallel cross-sections are the same shape (a pentagon), the same size, and straight-sided, so yes, it is a right prism. (b) It is named after its cross-section: a pentagon prism.

**Q8.** A right prism needs parallel cross-sections that are the same shape **and** the same size. A square pyramid's parallel cross-sections are all squares, so the shape is right, but the higher the slice, the smaller the square. The size changes, so a square pyramid is not a right prism.

**Q9.** A cylinder's parallel cross-sections are the same circle, the same shape and the same size, all the way through, so it has the stacking property of a right prism and can be pictured as a stack of identical coins. But the edge of the circle is curved, not straight-sided, and the frame this subtopic uses asks for a straight-sided cross-section. So a cylinder is like a right prism without being one.

**Q10.** Circles that grow to the largest one in the middle and shrink again are the cross-sections of a sphere (or a sphere-like solid). It could not be a right prism: the cross-sections are not the same size, and a circle is curved, not straight-sided.

**Q11.** Yes. The parallel cross-sections are all the same L-shape, which is straight-sided, and the same size all the way through, so the solid fits the definition of a right prism. The definition does not require the cross-section to be a familiar shape.

**Q12.** Slice a cube parallel to any face and every cross-section is the same square, the same shape and the same size, so a cube is a right prism: it is a stack of the same squares. The definition says nothing about looking like a tent; a triangular prism is only one kind of prism, named after its own cross-section.

**Q13.** (a) Straight through the middle, the cross-section is a circle, and it is the largest circle the orange gives. (b) Near the top, the cross-section is a smaller circle. The middle cross-section is larger. The orange is not a right prism: its cross-sections change size, and they are circles with curved edges.

**Q14.** Completed table:

| Solid | Shape of parallel cross-sections | Same size all the way through? | Right prism? |
|---|---|---|---|
| rectangular prism | rectangle | yes | yes |
| triangular prism | triangle | yes | yes |
| square pyramid | square | no: the squares get smaller | no |
| cylinder | circle | yes | no: the edge is curved |

What makes a right prism: parallel cross-sections that are the same straight-sided shape and the same size, all the way through. The pyramid fails on size; the cylinder fails on being straight-sided.

## The four quadrants of the Cartesian plane

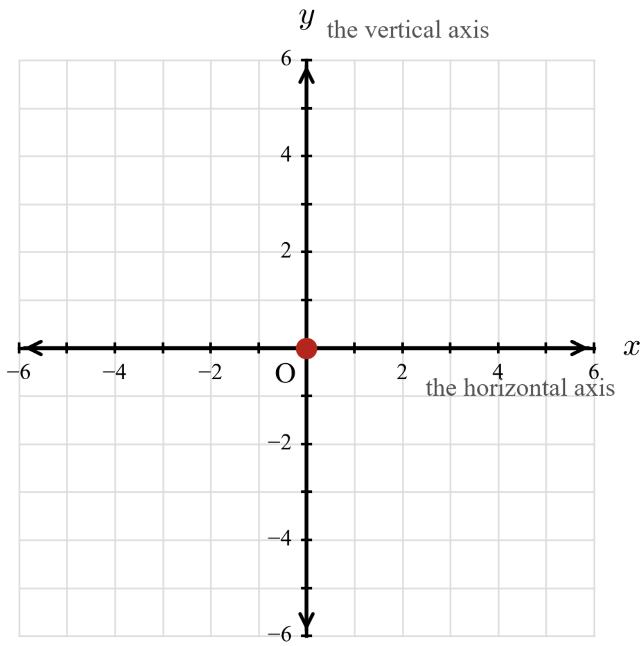
> **Content description VC2M6SP02:** locate points in the 4 quadrants of the Cartesian plane; describe changes to the coordinates when a point is moved to a different position in the plane

*In subtopic 1B you placed integers on a plane. This subtopic turns that plane into a tool: every point on it is named by two numbers — and when the point moves, the numbers tell the story of the move.*

### Theory

#### *Two number lines crossing at zero*

The **Cartesian plane** is made from two number lines that cross at right angles: a horizontal one named x, and a vertical one named y. The two number lines are called the **axes** of the plane. They cross at 0 on both lines. That crossing point is the **origin**, written O, with coordinates (0, 0).




*A Cartesian plane with its four quadrants; both number lines count from −6 to 6 in steps of 1, with labels every 2. The horizontal and vertical number lines cross at the origin, marked with a red dot at (0, 0) and labelled O.*

The origin is **fixed**. Every position on the plane is described from O — how far left or right, then how far up or down — always from O, never from another point. That is the whole idea of the plane: a picture of position, measured from a fixed origin.

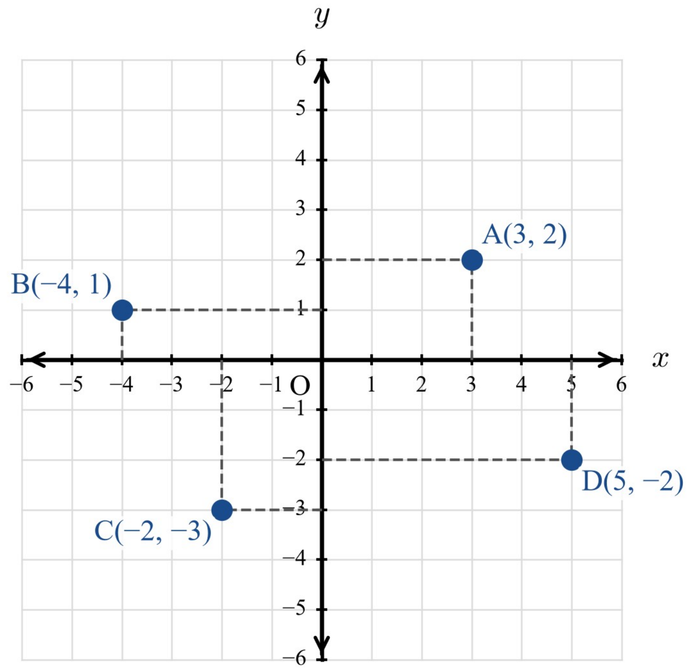
The two axes cut the plane into four parts, the **quadrants**. As in 1B, the signs of the two coordinates tell you which part a point is in:

| Part of the plane | signs of (first, second) |
|---|---|
| top right | (+, +) |
| top left | (−, +) |
| bottom left | (−, −) |
| bottom right | (+, −) |

The quadrants are also numbered I, II, III and IV, starting at the top right and turning anti-clockwise.

### *Reading and writing an ordered pair*

A position on the plane is written as two coordinates in brackets, **horizontal first, vertical second**: (first, second). To plot A(3, 2): start at O, count 3 right, then 2 up. To plot B(−4, 1): count 4 left, then 1 up. The dashed guide lines in the figure show the two counts for each point.




*A Cartesian plane from −6 to 6 with four blue points and dashed guide lines to both number lines: A(3, 2) in the top right, B(−4, 1) in the top left, C(−2, −3) in the bottom left and D(5, −2) in the bottom right.*

**To plot a point:**

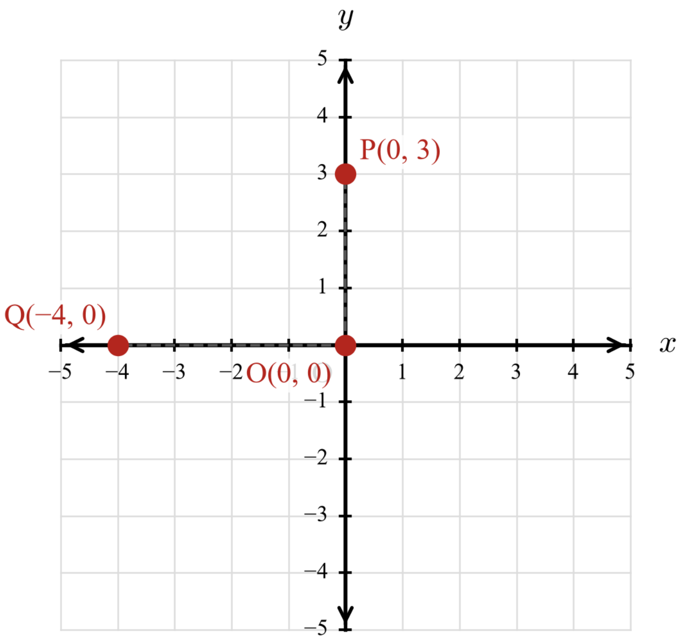
1. Start at the origin O.
2. First number: move right if it is positive, left if it is negative.
3. Second number: move up if it is positive, down if it is negative.
4. Mark the point and write its name.

**To read a point**, work the steps the other way round: a guide line down or up to the horizontal number line gives the first number; a guide line left or right to the vertical number line gives the second.

The order matters. (3, 2) is 3 right and 2 up; (2, 3) is 2 right and 3 up — a different point. Swapping the numbers swaps the point, which is why the pair is written in a fixed order.

### *Points on the axes*

A point whose first coordinate is 0 makes no left–right move, so it sits **on the vertical number line**, the one named y. A point whose second coordinate is 0 sits **on the horizontal number line**, the one named x. Points on the number lines are not inside the four quadrants.



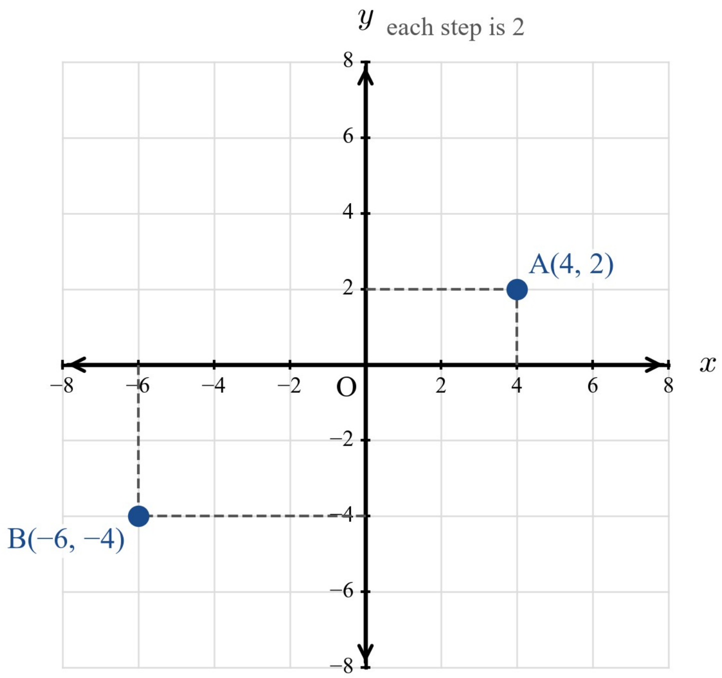

*A Cartesian plane from −5 to 5 with three red points: P(0, 3) on the vertical number line, Q(−4, 0) on the horizontal number line, and O(0, 0) where the two number lines cross.*

P(0, 3) is on the vertical number line, Q(−4, 0) is on the horizontal number line, and O(0, 0) is on both at once. All three sit on the number lines; they are not inside the four quadrants.

### *Choosing a scale*

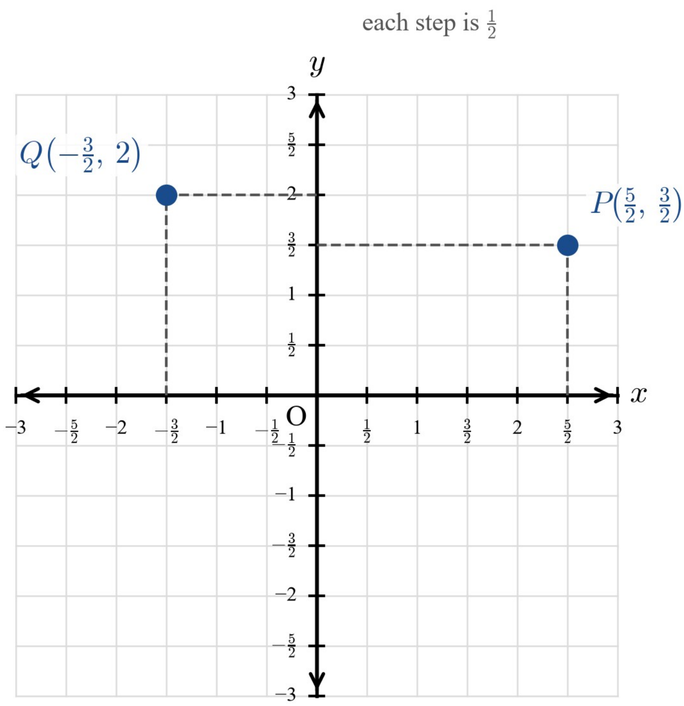
The number lines of the plane can count in any step you choose. The step is the **scale** of that number line, and the right scale depends on your purpose.

Counting in 1s is fine for small numbers, but when the numbers are large, a larger step keeps the plane readable. On the plane below each step is 2: A(4, 2) is two steps right and one step up, and B(−6, −4) is three steps left and two steps down.



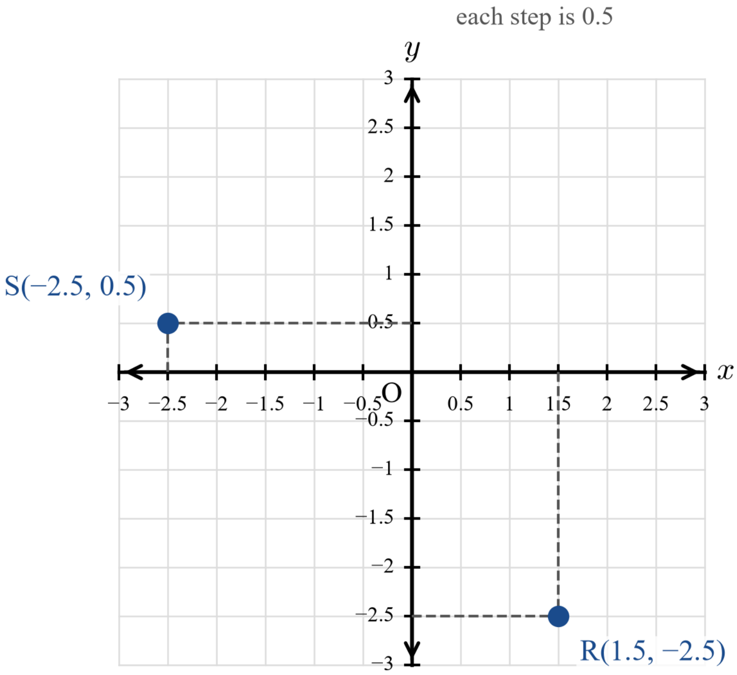

*A Cartesian plane with both number lines counting in steps of 2 from −8 to 8, every tick labelled. Blue points at A(4, 2) and B(−6, −4) with dashed guide lines to both number lines.*

Whole-number steps are not enough when the points sit between whole numbers. Then the number lines count in halves, and the ticks can be labelled as fractions.



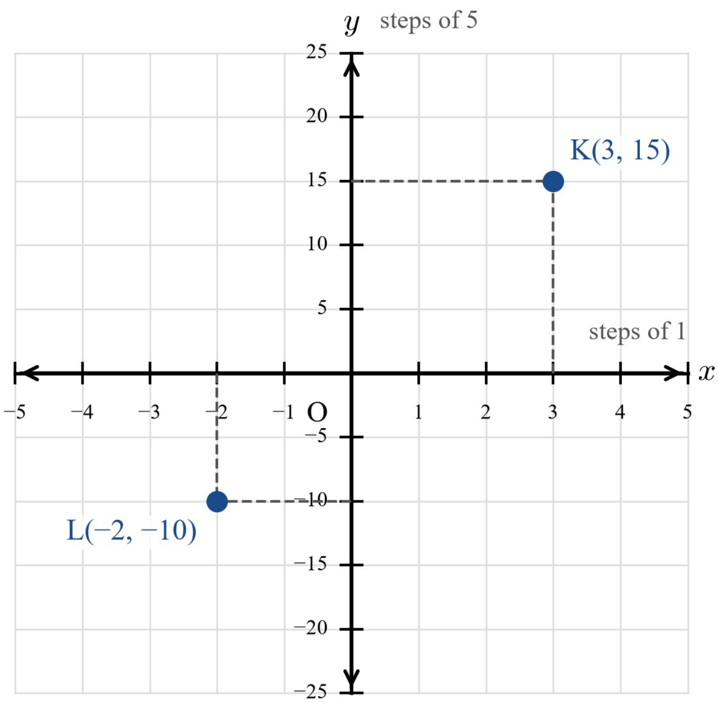

*A Cartesian plane with both number lines counting in halves from −3 to 3, every tick labelled as a fraction. Blue points at P(5/2, 3/2) and Q(−3/2, 2) with dashed guide lines; the caption notes each step is one half.*

The very same plane can be labelled in decimals instead of fractions, because $\frac{1}{2}$ and 0.5 are two names for the same number.




*A Cartesian plane from −3 to 3 counting in steps of 0.5, every tick labelled as a decimal. Blue points at R(1.5, −2.5) and S(−2.5, 0.5) with dashed guide lines; the caption notes each step is 0.5.*

Finally, the two number lines do not have to use the same scale. Where the first numbers are small and the second numbers are large, count the horizontal one in 1s and the vertical one in 5s. K(3, 15) is 3 right and three steps of 5 up.



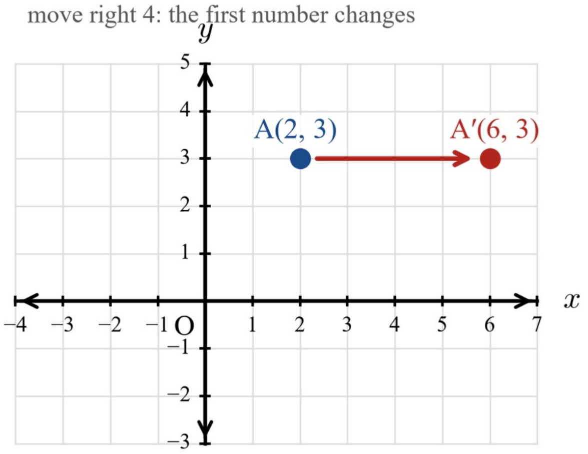

*A Cartesian plane whose horizontal number line counts in steps of 1 from −5 to 5 and whose vertical number line counts in steps of 5 from −25 to 25, every tick labelled. Blue points at K(3, 15) and L(−2, −10) with dashed guide lines.*

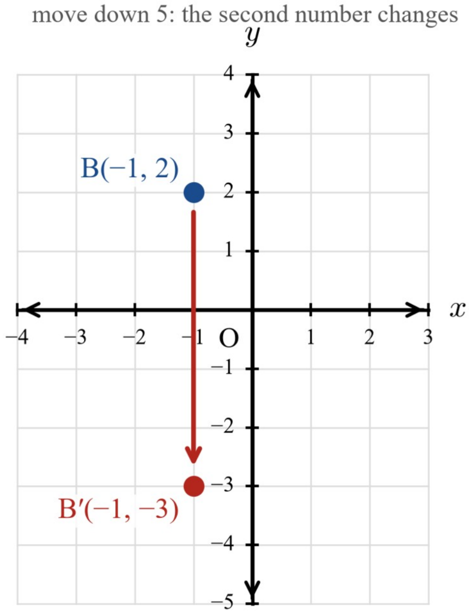
Whatever scale you choose, **every tick is labelled**, so the reader sees the scale instead of guessing it. Look at the biggest and smallest numbers you must plot, then pick a step that fits them. You will make exactly this choice in science when you draw a graph from data — the plane is the same tool.

### ***Moving a point***

When a point moves, the coordinates change in a very regular way:

- a move **right or left** changes the **first** coordinate;
- a move **up or down** changes the **second** coordinate.

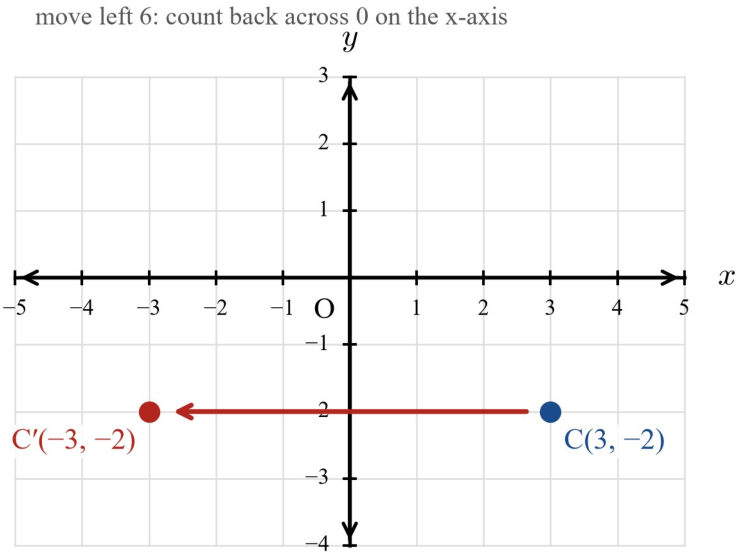



*Two planes side by side. Left: A(2, 3) moves 4 right to A'(6, 3), and only the first number changes. Right: B(−1, 2) moves 5 down to B'(−1, −3), and only the second number changes.*

A(2, 3) moves 4 right: count on 3, 4, 5, 6 along the horizontal number line, so the first coordinate changes from 2 to 6 and the second stays 3 — A′(6, 3). B(−1, 2) moves 5 down: count down 1, 0, −1, −2, −3, so the second coordinate changes from 2 to −3 and the first stays −1 — B′(−1, −3).

A move can cross from one side of the plane to the other, and counting takes you straight across 0.

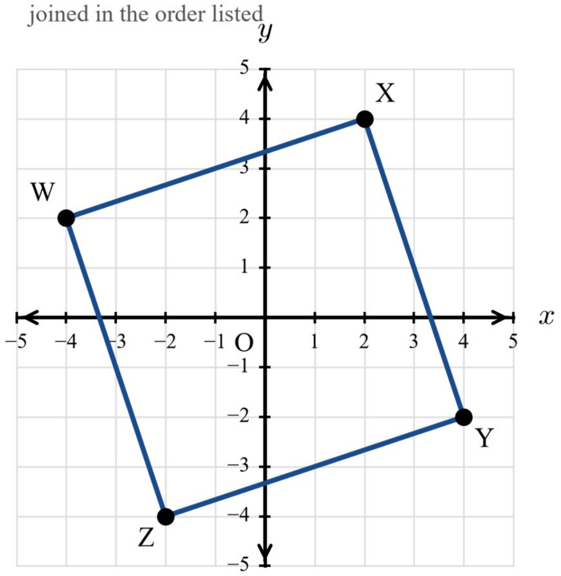


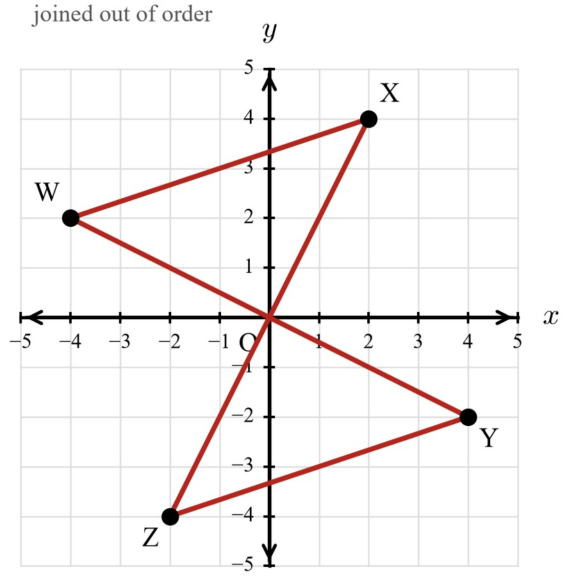

*A Cartesian plane where C(3, −2) moves 6 left, counting back across 0 on the horizontal number line, to C'(−3, −2).*

C(3, −2) moves 6 left: count back 2, 1, 0, −1, −2, −3, landing at C′(−3, −2). The point began in the bottom right and finished in the bottom left — crossing the vertical number line flipped the sign of its first coordinate. To describe the change, say what happened to each coordinate: *the first coordinate changed from 3 to −3; the second stayed −2.*

### *Drawing lines and polygons*

A list of coordinates can be joined into a shape: plot each point, join them **in the order listed**, and join the last point back to the first to close the shape.




*Two planes carrying the same four points W(−4, 2), X(2, 4), Y(4, −2) and Z(−2, −4). Left: joined in the order listed, W to X to Y to Z and back to W, they form a clean polygon. Right: joined out of order, W to X to Z to Y and back to W, two edges cross and the shape is not the polygon asked for.*

W(−4, 2), X(2, 4), Y(4, −2), Z(−2, −4) joined in the order listed make a clean polygon. The same four points joined out of order give crossing edges — a different shape, not the one asked for. The order in the list is part of the shape.

**Words for this subtopic**, with what they mean here:

| Term | Meaning here |
|---|---|
| **Cartesian plane** | the plane formed by two number lines crossing at right angles |
| **origin** | the fixed point where the axes cross, O, at (0, 0) |
| **coordinates** | the two numbers (first, second) that locate a point; horizontal first |
| **quadrants** | the four parts the axes cut the plane into |
| **axes** | the two number lines of the plane; the horizontal one is named x, the vertical one y |
| **scale** | the size of each step along a number line, chosen to suit the points being plotted |

In subtopic 6C you will use this same plane to transform whole shapes; at Level 7 the plane becomes the place where patterns and relationships are plotted.

## Worked examples

### *Example 1 — with help*

Plot P(−3, 4) and Q(5, −1) on a Cartesian plane, and say which of the four quadrants each point is in.

| Working | Comment |
|---|---|
| P: start at O, count 3 left, then 4 up. Mark P. | First number horizontal, second vertical. |

| Working | Comment |
|---|---|
| P is left and up, so its signs are (−, +): the top left of the four quadrants. | The signs tell you where the point is. |
| Q: start at O, count 5 right, then 1 down. Mark Q. | Same two steps. |
| Q is right and down, signs (+, −): the bottom right of the four quadrants. | The signs tell you where the point is. |

### *Example 2 — with help*

M(−2, 3) moves 5 right and 4 down to M′. Find the coordinates of M′ and describe what changed.

| Working | Comment |
|---|---|
| First coordinate: count 5 right from −2: −1, 0, 1, 2, 3. | A right–left move changes the first number. |
| Second coordinate: count 4 down from 3: 2, 1, 0, −1. | An up–down move changes the second number. |
| M′ is (3, −1). | Both counts together give the new point. |
| The first coordinate changed from −2 to 3; the second changed from 3 to −1. The point left the top left and landed in the bottom right. | Describe each coordinate in turn. |

### *Example 3 — on your own*

The points E(−3, 1), F(2, 3), G(3, −2) and H(−2, −4) are to be joined in the order E → F → G → H and back to E, closing a polygon. Plot and join them in that order. A classmate joins E → F → H → G → E instead. Explain in one sentence what goes wrong.

The order listed gives a clean closed shape. The out-of-order joining runs the edge F → H straight across the edge G → E, so two edges cross and the shape is not the polygon asked for — the order in the list is part of the shape.

### *Example 4 — on your own*

A game places a treasure at (2.5, −1.5) and a trap at (−1, 2). Choose a scale for the axes, say why it fits, and plot both points.

Every coordinate (2.5, 1.5, 1, 2) is a multiple of 0.5, so counting each number line in steps of 0.5 puts every point on a tick — a scale of 1 would leave the treasure between ticks. Labelling the ticks in decimals (or in halves, the same numbers as fractions) and plotting gives the treasure at five steps right and three steps down, the trap at two steps left and four steps up.

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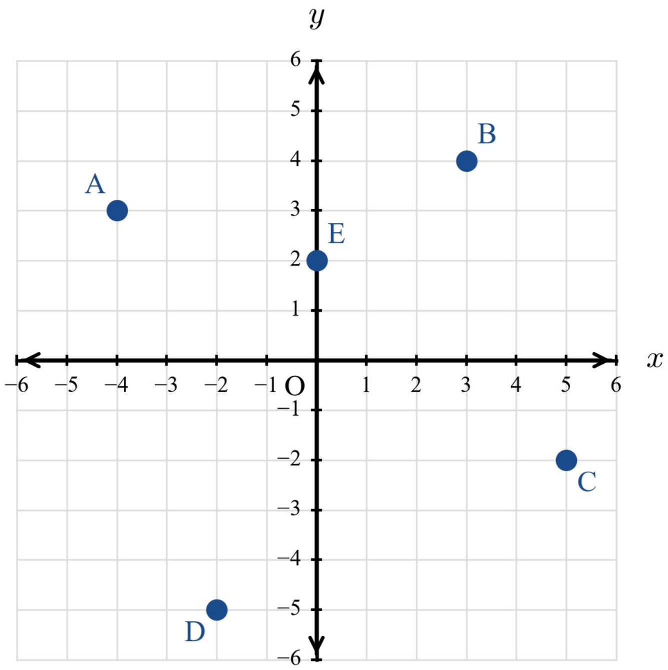
## Practice questions

### *With help*

**Q1.** The figure shows five points. Write the coordinates of each, and say which of the four quadrants it is in — or which of the axes it is on.

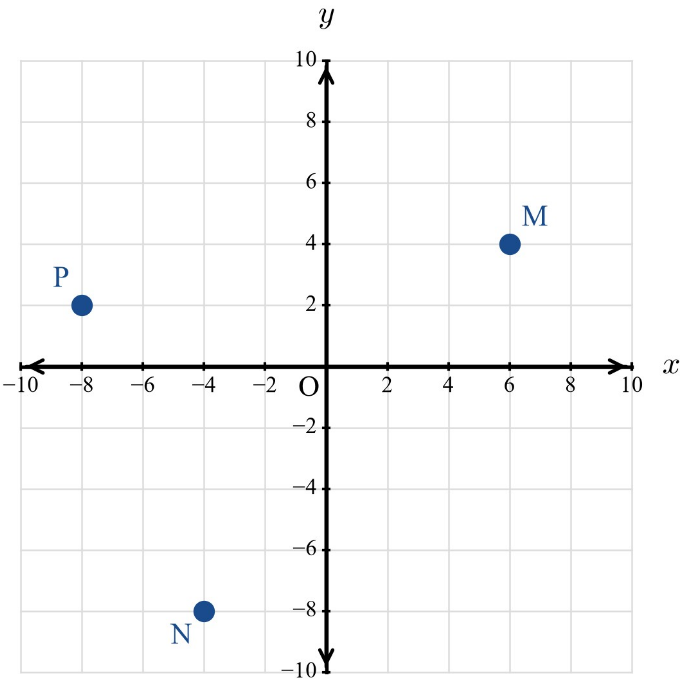



*A Cartesian plane from −6 to 6 with five blue points: A(−4, 3) in the top left, B(3, 4) in the top right, C(5, −2) in the bottom right, D(−2, −5) in the bottom left and E(0, 2) on the vertical number line.*

**Q2.** Plot these points on a Cartesian plane counting in 1s: G(2, 5), H(−5, 2), I(−1, −4), J(4, −3).

**Q3.** True or false? Give a reason in one sentence. (a) The point (0, −4) is on the horizontal number line. (b) The origin is inside all four quadrants. (c) A point whose first coordinate is negative and whose second coordinate is positive sits in the top left.

**Q4.** Both number lines on this plane count in 2s. Write the coordinates of M, N and P.




*A Cartesian plane with both number lines counting in steps of 2 from −10 to 10, with three blue points: M(6, 4), N(−4, −8) and P(−8, 2).*

**Q5.** Start at A(1, 2). Move 3 right to A′, then move 5 down to A″. Write the coordinates of A′ and A″, counting across 0 where you need to, and say which coordinate changed at each move.

**Q6.** A plane counts in halves. (a) Plot the point $\left(\frac{5}{2}, -\frac{3}{2}\right)$. (b) Write the same point with decimal labels.

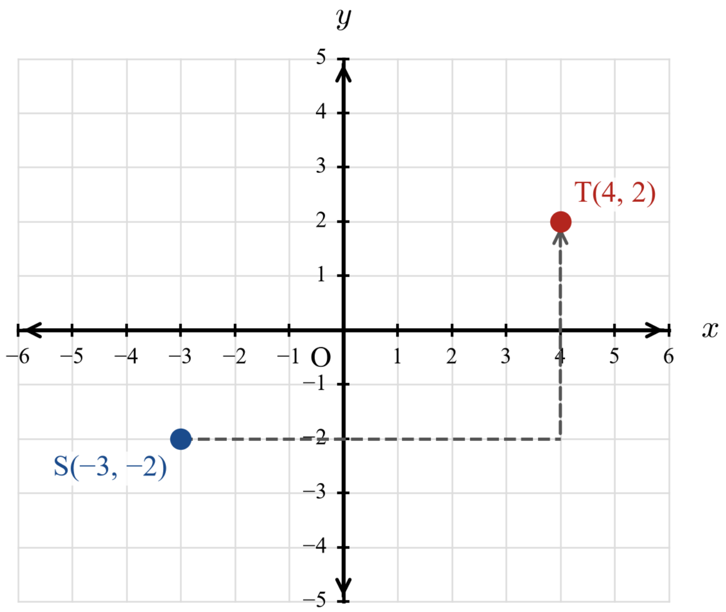
### *On your own*

**Q7.** Write each position as a pair of coordinates. (a) From O, 4 left and 3 down. (b) From O, 6 right and no move up or down. Where does this point sit?

**Q8.** Without plotting, say which of the four quadrants — or which of the axes — each point is in or on: (a) (7, 2); (b) (−7, 2); (c) (−7, −2); (d) (7, −2); (e) (0, 7).

**Q9.** You must plot (10, 5), (−5, 10) and (15, −5). Which scale — steps of 1, 2 or 5 — is the best choice for both number lines? Give your reason.

**Q10.** The dashed path runs from S to T in two moves. Describe each move in words, and say how each coordinate changed.




*A Cartesian plane from −6 to 6 and −5 to 5 with S(−3, −2) and T(4, 2). A dashed path runs 7 right from S to (4, −2), then 4 up to T.*

**Q11.** N(−1, −2) moves 4 right and 5 up. Find the new coordinates, counting across 0 where you need to.

**Q12.** Three corners of a rectangle are (−2, 1), (4, 1) and (4, −3). Find the coordinates of the fourth corner.

**Q13.** Describe the single move in each case. (a) From (2, −4) to (2, 3). (b) From (−5, 1) to (1, 1).

**Q14.** Dana says that when a point moves from the top right of the plane to the bottom left, both of its coordinates must change. Is she right? Explain.

---

## Answers

**Q1.** A(−4, 3) top left; B(3, 4) top right; C(5, −2) bottom right; D(−2, −5) bottom left; E(0, 2) on the vertical number line, not inside the four quadrants. **Q2.** plotted: G top right, H top left, I bottom left, J bottom right. **Q3.** (a) false — first coordinate 0 puts it on the vertical number line; (b) false — the origin sits on both number lines, not inside the four quadrants; (c) true. **Q4.** M(6, 4), N(−4, −8), P(−8, 2). **Q5.** A′(4, 2), first coordinate changed; A″(4, −3), second coordinate changed. **Q6.** (a) plotted five steps right and three steps down on the halves scale; (b) (2.5, −1.5). **Q7.** (a) (−4, −3); (b) (6, 0), on the horizontal number line. **Q8.** (a) top right; (b) top left; (c) bottom left; (d) bottom right; (e) on the vertical number line. **Q9.** steps of 5 — every coordinate lands on a tick and the plane stays small. **Q10.** right 7 (first coordinate −3 to 4), then up 4 (second coordinate −2 to 2). **Q11.** (3, 3). **Q12.** (−2, −3). **Q13.** (a) up 7; (b) right 6. **Q14.** yes — top right is (+, +) and bottom left is (−, −), so both signs, and both coordinates, change.

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## Worked solutions

**Q1.** Read each point with guide lines: A(−4, 3) is left and up, top left; B(3, 4) is right and up, top right; C(5, −2) is right and down, bottom right; D(−2, −5) is left and down, bottom left; E(0, 2) makes no left–right move, so it sits on the vertical number line and is not inside the four quadrants.

**Q2.** G(2, 5): 2 right, 5 up — top right. H(−5, 2): 5 left, 2 up — top left. I(−1, −4): 1 left, 4 down — bottom left. J(4, −3): 4 right, 3 down — bottom right.

**Q3.** (a) False: the first coordinate is 0, so there is no left–right move and the point sits on the vertical number line. (b) False: the origin is on both number lines at once, and a point on the number lines is not inside the four quadrants. (c) True: left and up is (−, +), the top left.

**Q4.** Count ticks of 2: M is 3 ticks right and 2 up: (6, 4). N is 2 ticks left and 4 down: (−4, −8). P is 4 ticks left and 1 up: (−8, 2).

**Q5.** A(1, 2) moves 3 right: count on 2, 3, 4, so A′(4, 2) — the first coordinate changed, the second stayed 2. A′ moves 5 down: count down 1, 0, −1, −2, −3, so A″(4, −3) — the second coordinate changed, the first stayed 4.

**Q6.** (a) On a halves scale, $\frac{5}{2}$ is five half-steps right and $-\frac{3}{2}$ is three half-steps down; the point sits in the bottom right. (b) The same ticks read in decimals are 2.5 and −1.5, so the point is (2.5, −1.5).

**Q7.** (a) 4 left and 3 down is (−4, −3). (b) 6 right and no up–down move is (6, 0); a second coordinate of 0 puts the point on the horizontal number line.

**Q8.** Read the signs: (a) (+, +) top right; (b) (−, +) top left; (c) (−, −) bottom left; (d) (+, −) bottom right; (e) first coordinate 0, so on the vertical number line, not inside the four quadrants.

**Q9.** Steps of 5. Every coordinate (10, 5, −5, 15) is a multiple of 5, so all six numbers land exactly on ticks, and each number line needs only three ticks each way. Steps of 1 would need fifteen ticks a side, and steps of 2 would never land the odd numbers 5, 15 and −5 on a tick.

**Q10.** First move: 7 right — the first coordinate counts from −3 across 0 to 4, and the second stays −2. Second move: 4 up — the second coordinate counts from −2 to 2, and the first stays 4. So the path is “right 7, then up 4”, taking S(−3, −2) to T(4, 2).

**Q11.** Right 4 from −1: count 0, 1, 2, 3 — the first coordinate is 3. Up 5 from −2: count −1, 0, 1, 2, 3 — the second coordinate is 3. The new point is (3, 3).

**Q12.** The corners (−2, 1) and (4, 1) share a horizontal edge; (4, 1) and (4, −3) share a vertical edge. The fourth corner sits under (−2, 1) and across from (4, −3): first coordinate −2, second coordinate −3, so (−2, −3).

**Q13.** (a) The first coordinate stays 2 and the second counts from −4 up to 3: count −3, −2, −1, 0, 1, 2, 3 — up 7. (b) The second coordinate stays 1 and the first counts from −5 to 1: count −4, −3, −2, −1, 0, 1 — right 6.

**Q14.** Yes. A point in the top right has signs (+, +); a point in the bottom left has signs (−, −). The first coordinate goes from positive to negative, so it changes; the second does the same. If only one coordinate changed, the point would land in the top left or the bottom right instead — so Dana is right, both coordinates must change.

## Combining transformations: tessellations and geometric patterns

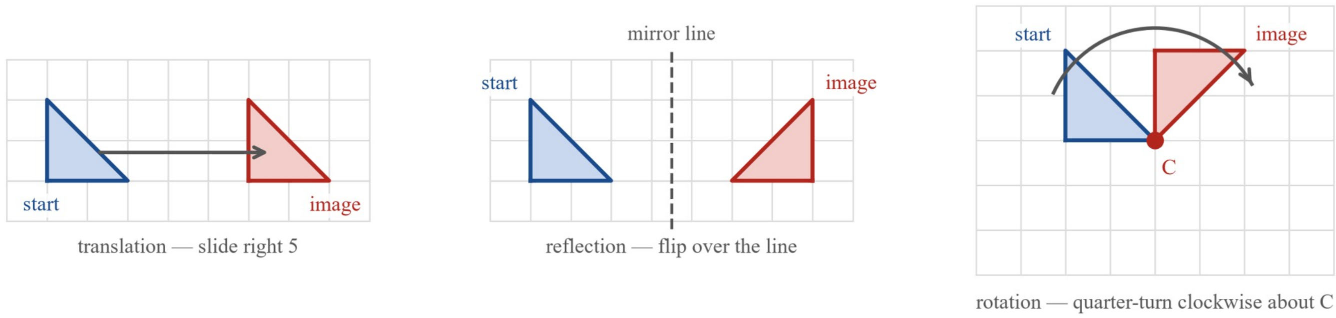
> **Content description VC2M6SP03:** recognise and use combinations of transformations to create tessellations and other geometric patterns, using dynamic geometry software where appropriate

*At Level 5 you learned the three moves that change a shape's position without changing the shape itself: slides, flips and turns. This subtopic joins those moves together into combinations — and puts the combinations to work making tessellations, border patterns, badges and designs.*

### Theory

#### *Three moves you already know*

The three transformations of Level 5 do all the work in this subtopic, so they are worth one clean revision. A **translation** slides every point of a shape the same distance in the same direction. A **reflection** flips a shape over a **mirror line**: each corner of the image sits the same distance from the line as its corner, on the other side. A **rotation** turns a shape about a fixed point; a **quarter-turn** is 90° and a **half-turn** is 180°. The shape you finish with is the **image** of the shape you started with.

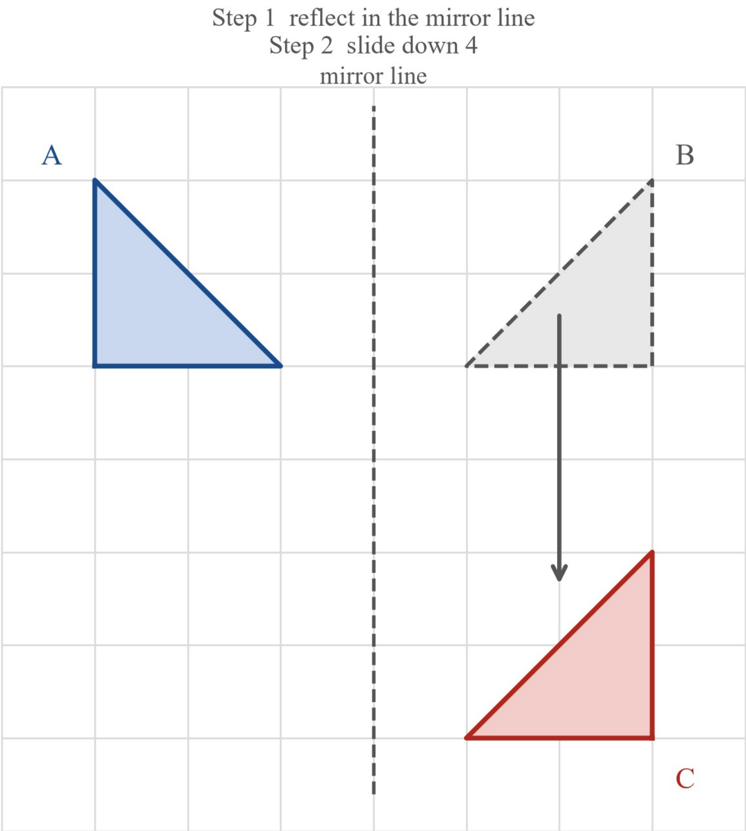



*Three panels on grid paper. Left: a blue triangle and its red image after a translation, an arrow labelled slide right 5. Middle: a blue triangle and its red image on either side of a dashed mirror line, a reflection. Right: a blue triangle and its red image after a quarter-turn clockwise about the red point C, with a curved arrow showing the turn.*

Not one of the three moves changes the size of a shape or the shape itself — the image is a perfect copy of the object in a new place, turned or flipped or both. That fact is what makes the patterns in this subtopic work.

#### *One shape, two or more moves*

A **combination of transformations** is two or more of the three moves done one after the other, each move acting on the whole shape. Follow one combination in the figure below: first a reflection, then a translation.

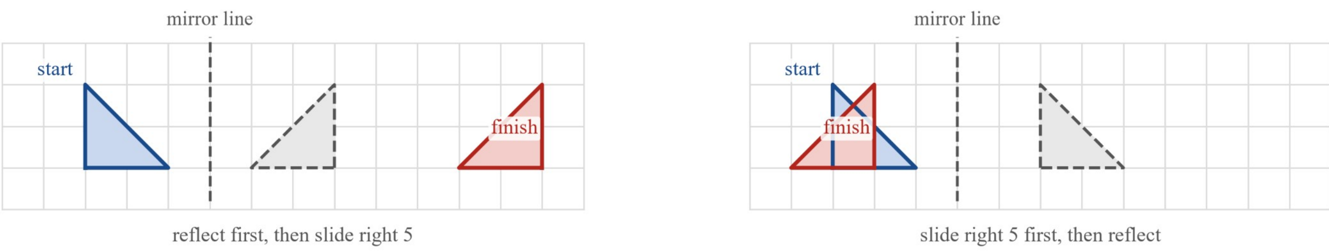



*Grid paper from 0 to 7 in both directions. The blue triangle A with corners (0, 4), (0, 6) and (2, 4) is reflected in the dashed mirror line x = 3 to give the grey triangle B with corners (6, 4), (6, 6) and (4, 4); B is then slid down 4 to give the red triangle C with corners (6, 0), (6, 2) and (4, 0).*

Track one corner and the method is easy to hold on to. The corner of A at (0, 4) is 3 squares from the mirror line, so its image in B is 3 squares the other side, at (6, 4); the slide down 4 then carries it to (6, 0). Every corner does the same two moves, in the same order. The shape after the first move is the shape the second move acts on — never skip the in-between shape when you work out a combination.

### *Order can matter*

The same two moves, done in the other order, can land the shape somewhere different. The figure below uses the same start triangle, the same mirror line and the same slide — only the order changes.




*Two panels on identical grid paper with a dashed mirror line at x = 3 and a start triangle with corners (0, 1), (0, 3) and (2, 1). Left panel: reflected first, then slid right 5, the finish triangle has corners (11, 1), (11, 3) and (9, 1). Right panel: slid right 5 first, then reflected, the finish triangle has corners (1, 1), (1, 3) and (−1, 1). The two finishes are in different places.*

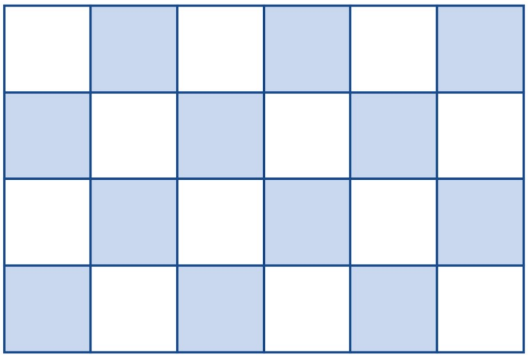
Reflect-then-slide finishes on the right of the grid; slide-then-reflect finishes on the left, back across the mirror line. **The order of a combination is part of the combination.** (Two translations are the exception you can trust: a slide right 2 then down 2 lands exactly where down 2 then right 2 lands. Once a reflection or a rotation joins the combination, always expect order to matter until you have checked.)

Repeating a combination again and again is how most of the patterns in this subtopic are made — and repeating transformations inside a pattern, over and over within themselves, is how **fractals** are made: shapes that hold copies of themselves inside themselves.

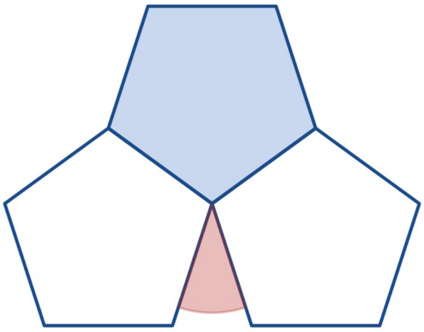
### *Tessellations: no gaps, no overlaps*

A **tessellation** is a pattern of shapes that covers the plane with **no gaps and no overlaps**. Pavers in a courtyard, tiles on a floor and bricks in a wall are tessellations you have stood on. The left panel below is the simplest one there is; the right panel is a shape that fails.

regular pentagons do not tessellate



squares: no gaps, no overlaps — it tessellates



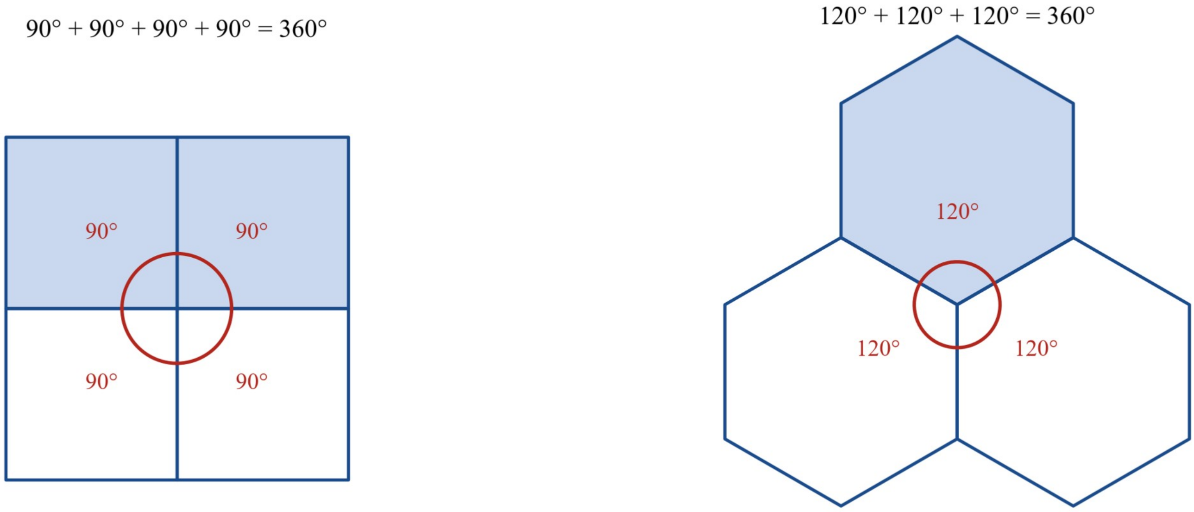

pentagons: the corners do not fill the point

*Left panel: a 6 by 4 array of squares in two alternating shades, edges meeting exactly, captioned squares tessellate. Right panel: three regular pentagons meeting at one point, with a red wedge-shaped gap between two of them labelled a 36 degree gap, captioned regular pentagons do not tessellate.*

Squares tessellate: every edge of every square lies exactly along an edge of the square next to it, and the pattern can grow as far as you like. Regular pentagons do not: three pentagons at a point leave a gap, and a fourth would overlap. The gap in the figure is exactly 36° — and the next subheading shows where that number comes from.

### *The 360° test*

Look at any point of a tessellation where corners of tiles meet. Going once around that point is a full turn, 360°, and the tile corners at the point must fill it exactly. So a shape will tessellate on its own only if its corner angle fits into 360° a whole number of times — that is the **360° test**.



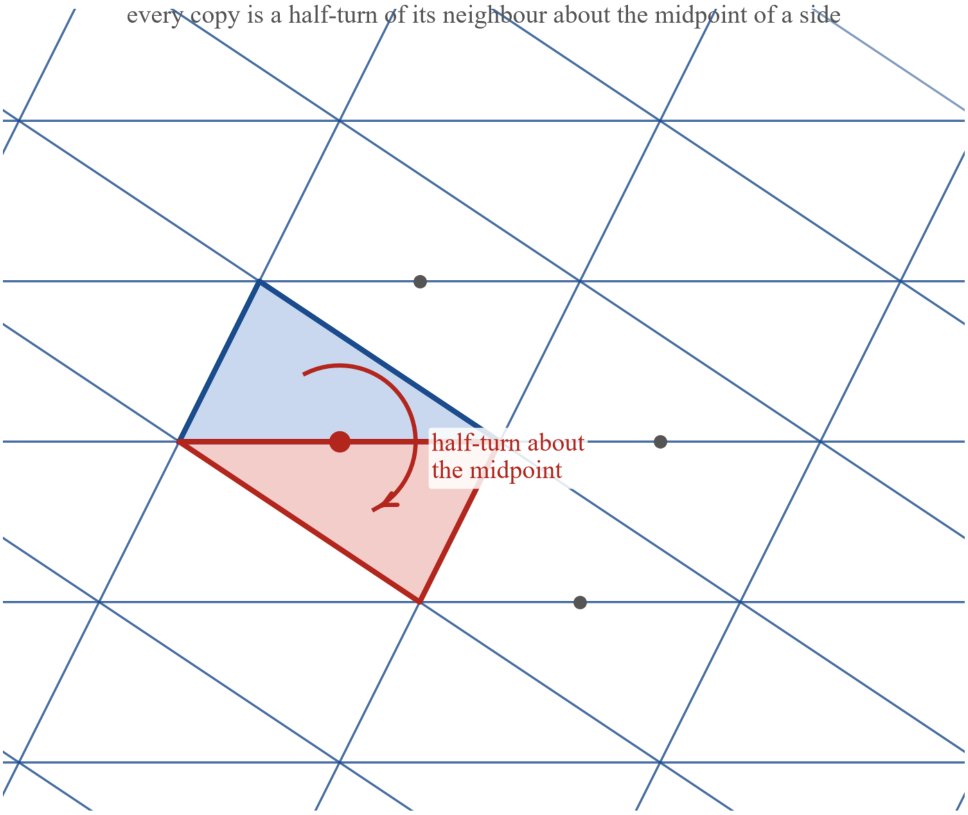

*Left panel: four unit squares meeting at one point, each corner marked 90 degrees, with the sum 90 + 90 + 90 + 90 = 360 degrees. Right panel: three regular hexagons meeting at one point, each corner marked 120 degrees, with the sum 120 + 120 + 120 = 360 degrees.*

- **Squares:** the corner angle is 90°, and $360° \div 90° = 4$ — four squares at every point, as in the figure. ✓
- **Regular hexagons:** the corner angle is 120°, and $360° \div 120° = 3$ — three at every point. ✓
- **Regular pentagons:** the corner angle is 108°. Three give $3 \times 108° = 324°$, leaving the 36° gap you saw; four would give $4 \times 108° = 432°$, which is too many. No whole number works, so regular pentagons do not tessellate on their own. ✗

The test also finds tessellations made from **two** shapes. The corner angle of a regular octagon is 135°, and on its own the octagon fails the test. But $135° + 135° + 90° = 360°$: two octagons and one square fit exactly around a point — and repeating that meeting point across the plane gives a true tessellation of octagons with a square in every gap between them. Combining shapes is not a fix for a bad tile; it is a design choice of its own.

### *Do all triangles tessellate?*

One of the questions a tessellation explorer asks first is: *do all triangles tessellate, or only some?* The answer is a surprise: **every triangle will tessellate** — any triangle at all, whatever its three sides. The tool that makes it true is the **half-turn**, and the point turned about is the **midpoint** of a side, the point halfway along the side.



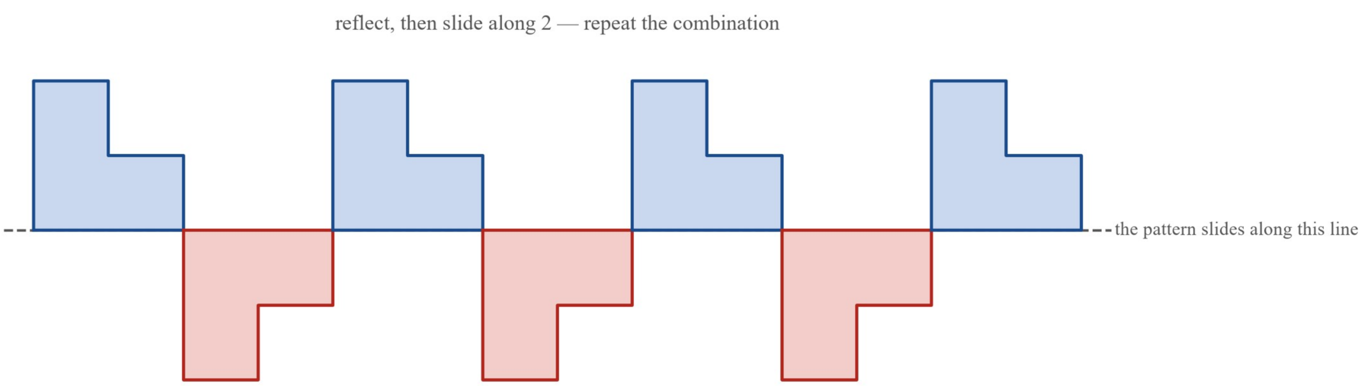

*A tessellation of the plane by one triangle with corners (0, 0), (4, 0) and (1, 2) and its copies. The blue triangle and the red triangle beside it share the side from (0, 0) to (4, 0); a curved arrow shows the half-turn about the midpoint (2, 0) that carries the blue triangle onto the red one. Grey dots mark other midpoints where the same half-turns happen.*

A half-turn about the midpoint of a side carries that side exactly onto itself — the two ends swap, and every point between them lands on the side. So the new triangle shares the **whole** side with the first one: no gap along that side, no overlap. Do the same half-turn at every side of every copy and the pattern grows to fill the plane, as in the figure. The squares, hexagons and pentagons of the last two figures were regular shapes with equal corners; this method never asks what the angles are, which is why it works for every triangle.

### *Building patterns with combinations*

Tessellations are one kind of geometric pattern. Here are three more patterns, each made by a combination of transformations.

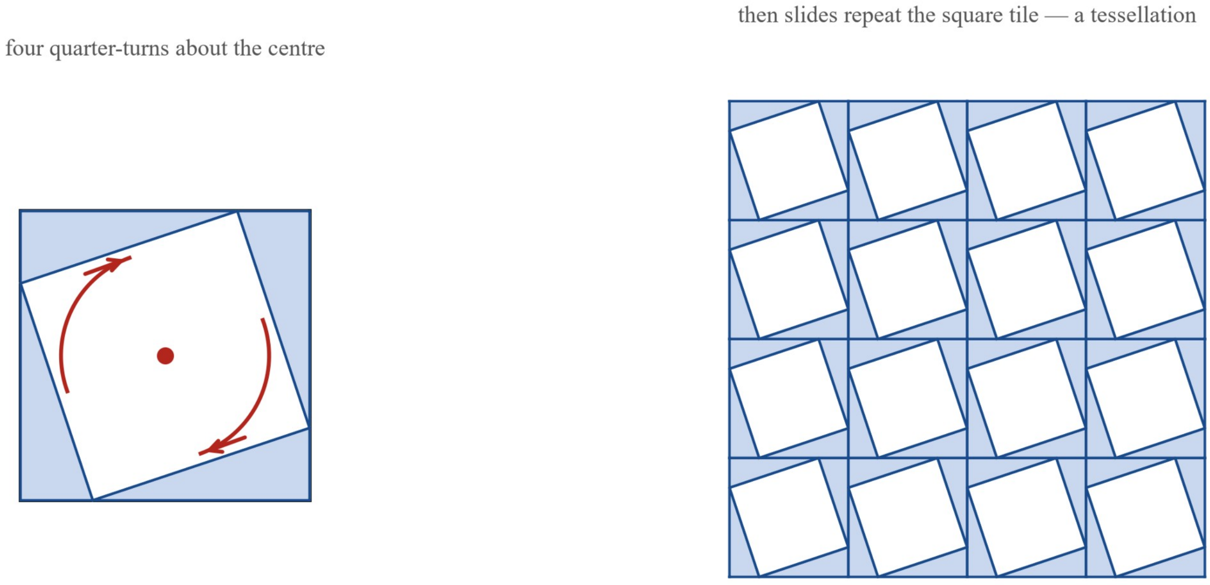
**A border pattern.** Reflect, then slide along, then repeat:




*A horizontal border of seven step-shaped tiles along a dashed horizontal line: tiles 1, 3, 5 and 7 sit above the line in blue, tiles 2, 4 and 6 are their reflections below the line in red, each tile 2 squares to the right of the one before.*

The combination is a reflection in the dashed line followed by a translation 2 squares along it; doing that combination again and again runs the border as far as you like. Borders like this edge fabric, wrapping paper and page decorations.

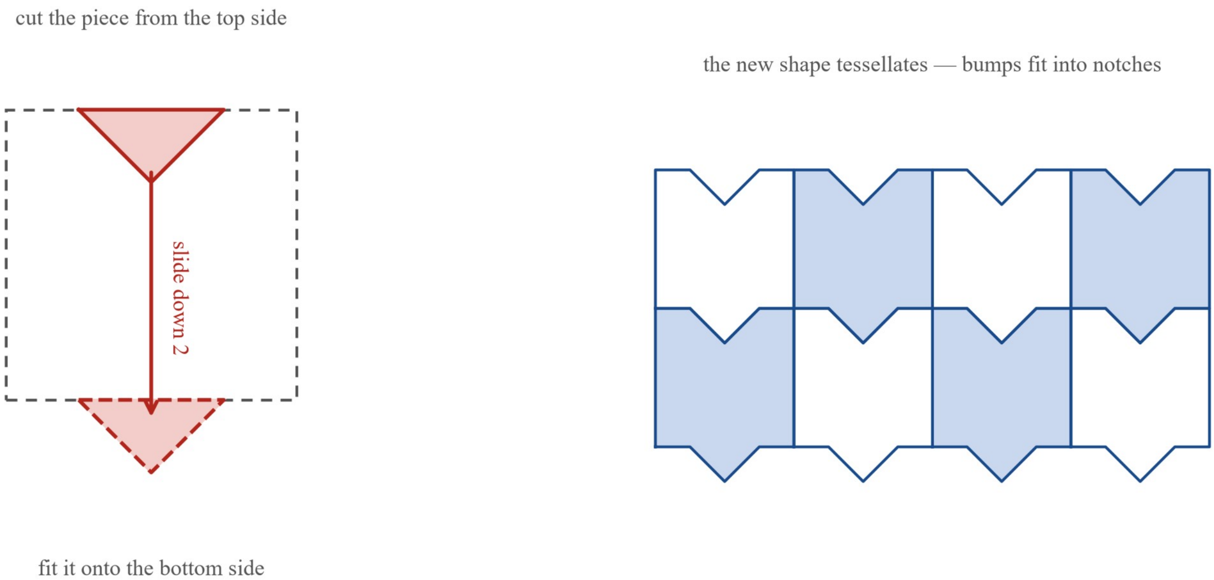
**A turning pattern.** Quarter-turns build one tile; translations repeat it:




*Left panel: one square divided into four turning corner blades around a tilted inner square, with curved arrows showing quarter-turns about the centre of the square. Right panel: a 4 by 4 array of the same square tile, the tiles meeting edge to edge with no gaps, a tessellation.*

Inside each square, each blade is a quarter-turn of the blade before it about the centre — four quarter-turns and you are back where you started. Between squares, plain translations repeat the whole tile. The pattern turns and slides at once.

**An interlocking pattern.** Cut a piece from one side of a square, translate it to the opposite side, and the new shape will tessellate:



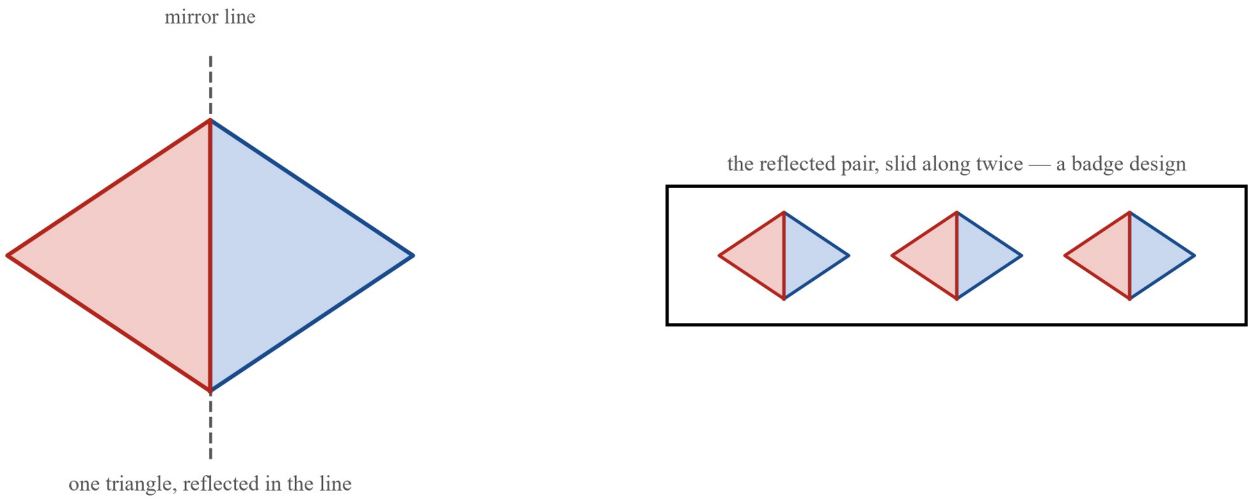

*Left panel: a dashed square with a triangular piece cut from its top side and an arrow showing the piece slid down 2 to sit on the bottom side. Right panel: the new shape, a square with a notch on top and a bump below, in a 4 by 2 array where every bump fills the notch of the tile below it.*

The bump on the bottom is the exact piece the notch on top lost, so a tile above always fills the notch of the tile below; the left and right sides are still straight. Shapes made this way — a piece moved from one side to the opposite side — always tessellate, and they are how artists draw interlocking lizards, birds and fish instead of squares.

A class with **dynamic geometry software** — screen tools that slide, flip and turn shapes exactly — can build every pattern in this subtopic on screen: make one shape, apply the combination, repeat, and drag to check there are no gaps. The combinations are the same on screen as on paper; the software is a tool, not a different maths.

### *A badge from one triangle*

Designing a logo or a badge is the same work worn as art: start with one shape, choose transformations, and describe every move so someone else could rebuild the design exactly.

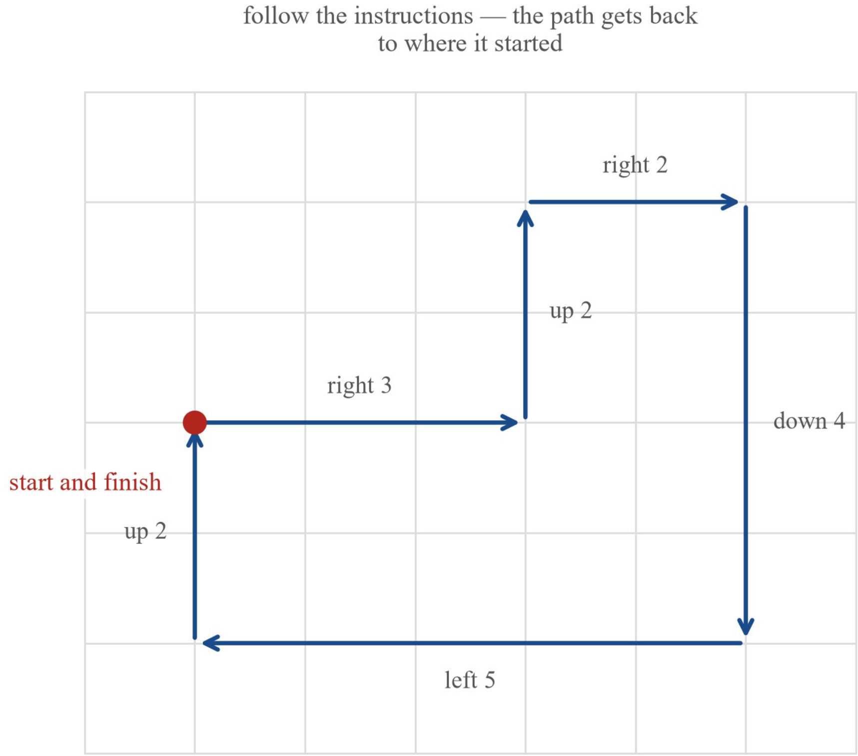



*Left panel: a blue triangle with corners (0, 0), (0, 2) and (1.5, 1) beside its red reflection in a vertical mirror line, the pair forming a bowtie shape. Right panel: a rectangular badge frame holding three copies of the bowtie, each 4 squares to the right of the last.*

One reflection makes the bowtie from the triangle; translations 4 squares to the right place the copies. Two kinds of transformation, one design — and a description anyone can follow: *reflect in the line, then slide right 4, then slide right 4 again.*

### *Instructions that get back to the start*

A combination of transformations can also be written as a list of instructions — an **algorithm**. A robot that follows *right 3, up 2, right 2, down 4, left 5, up 2* moves a shape's starting point along the path below.




*Grid paper with an arrowed closed path starting and finishing at the red point (1, 1): right 3 to (4, 1), up 2 to (4, 3), right 2 to (6, 3), down 4 to (6, −1), left 5 to (1, −1), up 2 back to (1, 1). Each leg is labelled with its instruction.*

The path gets back to where it started, and you can check that without walking: the rights, $3+2=5$, balance the lefts, 5; the ups, $2+2=4$, balance the downs, 4. Left–right and up–down both net to zero, so the last point is the first point. Writing instructions that transform a shape and return it to its start — and checking the balance of moves — is transforming with combinations, one step at a time.

### *Words for this subtopic*

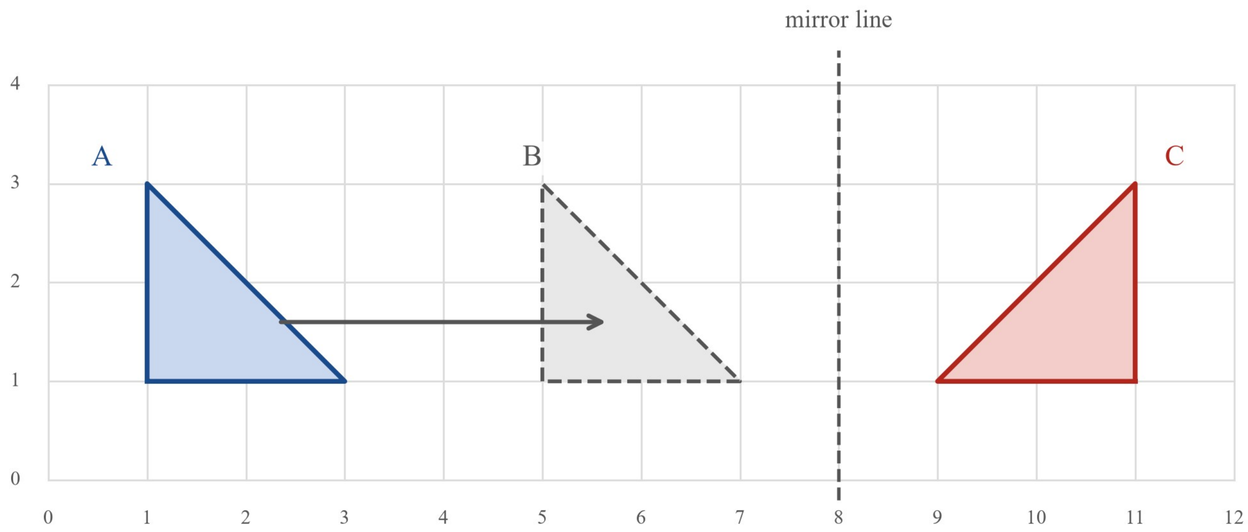
| Word | Meaning |
|---|---|
| combination of transformations | two or more of the three moves done one after the other |
| translation | a slide — every point of the shape moves the same distance, the same way |
| reflection | a flip over a mirror line |
| rotation | a turn about a fixed point |
| mirror line | the line a reflection flips over |
| image | the shape you finish with after a transformation |
| tessellation | a pattern of shapes covering the plane with no gaps and no overlaps |
| the 360° test | tiles fit exactly around a point when their corners there add to 360° |
| midpoint | the point halfway along a side |
| algorithm | a list of instructions in order, one step after another |

## Worked examples

*In the with-help examples the steps are labelled for you; in the on-your-own examples you carry them out yourself. Every example ends with a check.*

### *Example 1 — with help*

Triangle A has corners at (1, 1), (1, 3) and (3, 1). It is slid right 4 to make B, and B is reflected in the mirror line x = 8 to make C. Find the corners of B and of C.



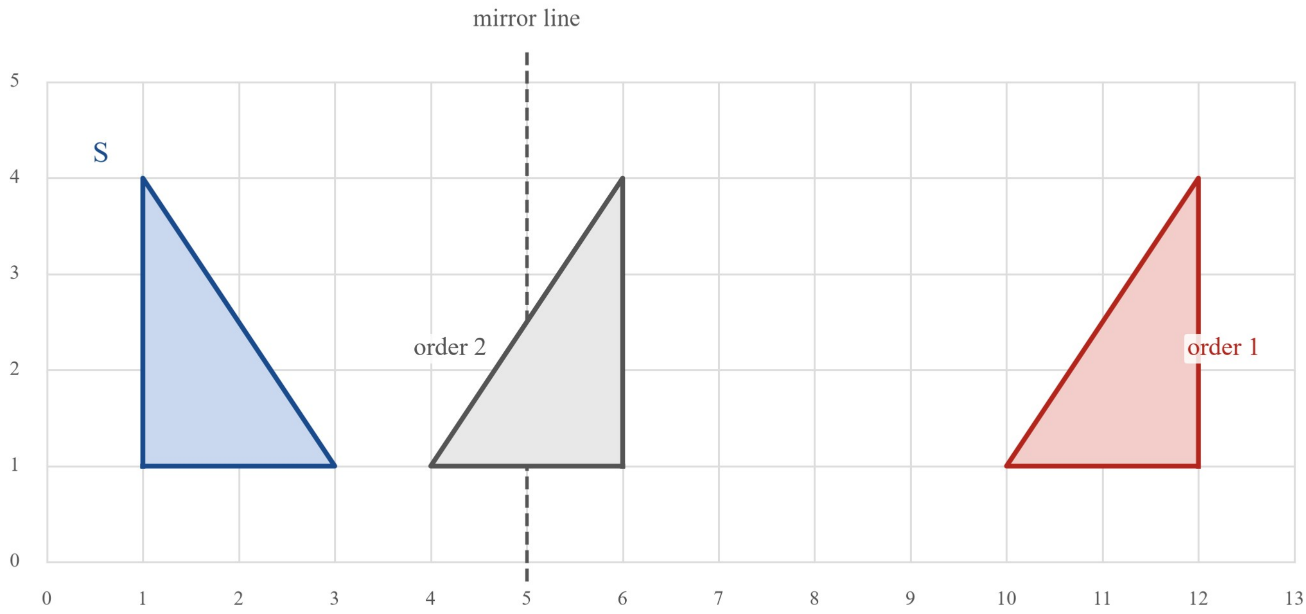

*Numbered grid paper from (0, 0) to (12, 4). The blue triangle A has corners (1, 1), (1, 3) and (3, 1); the grey dashed triangle B is A slid right 4; a dashed mirror line stands at x = 8; the red triangle C is the reflection of B.*

| Step | Working | Comment |
|---|---|---|
| **Read** | A: (1, 1), (1, 3), (3, 1). Move 1: slide right 4. Move 2: reflect in x = 8. | Two moves, one after the other — a combination. |
| **Plan** | Slide: add 4 to every first coordinate. Reflect: a corner d squares from the line lands d squares the other side. | Do move 1 completely, then move 2. |
| **Work out** | B: (5, 1), (5, 3), (7, 1). Distances to x = 8 are 3, 3, 1, so C: (11, 1), (11, 3), (9, 1). | The first coordinate changes in both moves; the second never does. |
| **Check** | The corners of C sit 3, 3, 1 from the line on the far side — matching B’s 3, 3, 1. ✓ | A reflection keeps distances to the mirror line. |

B has corners **(5, 1), (5, 3), (7, 1)** and C has corners **(11, 1), (11, 3), (9, 1)** — as printed in the figure.

### *Example 2 — with help*

Triangle S has corners (1, 1), (1, 4) and (3, 1). The two moves are: reflect in the mirror line x = 5, and slide right 3. Do them in both orders. Do the two orders finish in the same place?




*Numbered grid paper from (0, 0) to (13, 5) with a dashed mirror line at x = 5. The blue triangle S has corners (1, 1), (1, 4) and (3, 1); the red triangle order 1 has corners (10, 1), (12, 1) and (12, 4); the grey triangle order 2 has corners (4, 1), (6, 1) and (6, 4).*

| Step | Working | Comment |
|---|---|---|
| **Read** | S: (1, 1), (1, 4), (3, 1). Moves: reflect in x = 5; slide right 3. | Same moves, two orders. |
| **Plan** | Order 1: reflect, then slide. Order 2: slide, then reflect. Work each corner both ways. | Keep the two workings side by side. |
| **Work out** | Order 1: reflect gives (9, 1), (9, 4), (7, 1); slide gives (12, 1), (12, 4), (10, 1). Order 2: slide gives (4, 1), (4, 4), (6, 1); reflect gives (6, 1), (6, 4), (4, 1). | Reflecting in x = 5: first coordinates become 10 − what they were. |
| **Check** | Both finishes have the same shape as S, turned the same way over — but they stand in different places. ✓ | Same moves, different order, different finish. |

The two orders finish in **different places** — for reflections and rotations, the order of a combination is part of the combination.

### *Example 3 — with help*

Four tiles are offered for a floor: squares (corner 90°), equilateral triangles (corner 60°), regular hexagons (corner 120°) and regular pentagons (corner 108°). Use the 360° test to decide which of them tessellate on their own — and then test regular octagons (corner 135°) with a square as a partner.

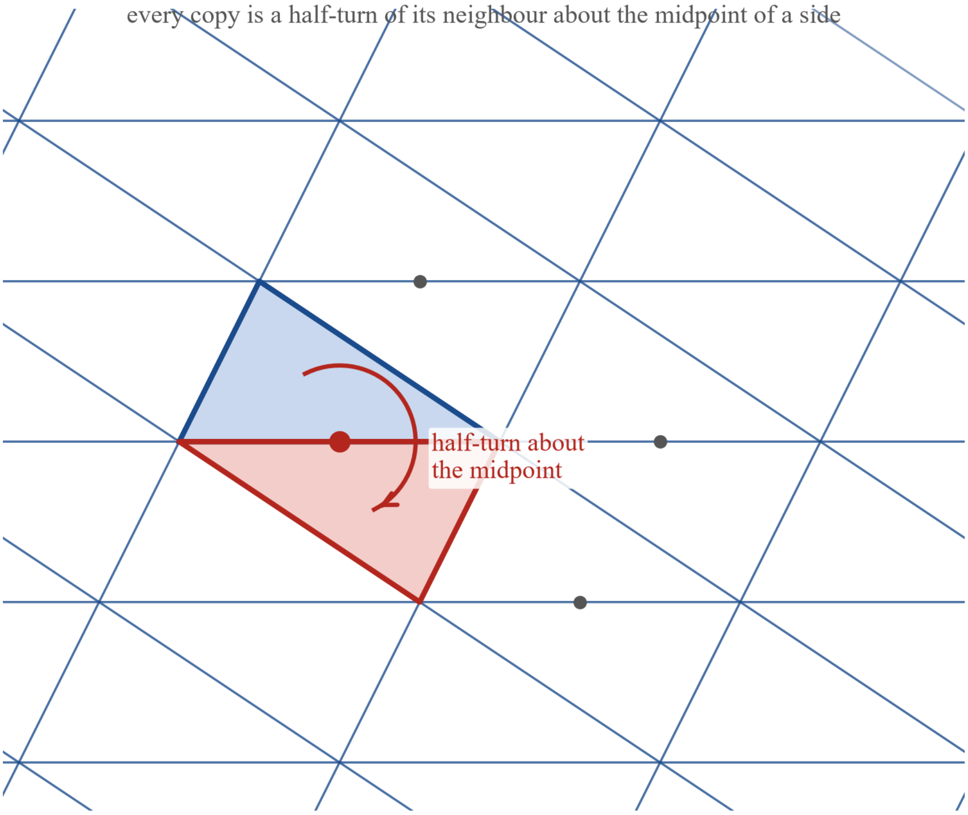
| Step | Working | Comment |
|---|---|---|
| **Read** | Corner angles: 90°, 60°, 120°, 108°, and octagon 135°. | The test looks at one meeting point. |
| **Plan** | A shape will tessellate on its own when 360° ÷ corner angle is a whole number. | Corners must fill the full turn exactly. |
| **Work out** | $360° \div 90° = 4$ ✓; $360° \div 60° = 6$ ✓; $360° \div 120° = 3$ ✓; $360° \div 108°$ is not a whole number | The pentagon leaves 36° or overlaps by 72°. |

| Step | Working | Comment |
|---|---|---|
| | ✗. Octagon: $135° + 135° + 90° = 360°$ ✓ with a square. | |
| **Check** | $4 \times 90° = 360°$, $6 \times 60° = 360°$, $3 \times 120° = 360°$ — all fill the turn; $3 \times 108° = 324°$ does not. ✓ | Adding back is the check. |

Squares, equilateral triangles and regular hexagons tessellate on their own; regular pentagons do not. Regular octagons do not on their own, but **two octagons plus one square** fit exactly around every point — a tessellation of two shapes.

### *Example 4 — with help*

The blue triangle in the figure has corners (0, 0), (4, 0) and (1, 2). The red triangle below it is a half-turn of the blue one about the midpoint of the shared side. Find the corners of the red triangle, and say why the two triangles leave no gap along the shared side.




*The blue triangle with corners (0, 0), (4, 0) and (1, 2) above the side from (0, 0) to (4, 0), and the red triangle with corners (4, 0), (0, 0) and (3, −2) below it; a curved arrow shows the half-turn about the midpoint (2, 0).*

| Step | Working | Comment |
|---|---|---|
| **Read** | Blue: (0, 0), (4, 0), (1, 2). Turn: half-turn about (2, 0), the midpoint of the side from (0, 0) to (4, 0). | The midpoint is halfway: (2, 0). |
| **Plan** | A half-turn about (2, 0) sends a corner (x, y) to (4 − x, −y). Apply it to all three corners. | Both coordinates change. |
| **Work out** | (0, 0) → (4, 0); (4, 0) → (0, 0); (1, 2) → (3, −2). Red: (4, 0), (0, 0), (3, −2). | The two ends of the shared side swap. |
| **Check** | The red triangle's side from (4, 0) to (0, 0) is the blue triangle's side from | A half-turn about a midpoint carries the side onto itself. |

| Step | Working | Comment |
| --- | --- | --- |
| | (0, 0) to (4, 0) — the whole side, exactly. ✓ | |

The red corners are **(4, 0), (0, 0), (3, −2)**. There is no gap along the shared side because a half-turn about its midpoint carries the side **onto itself** — every point of the side lands on the side. Repeating the half-turns at every side builds the whole tessellation in the figure, for any triangle.

### *Example 5 — on your own*

Use the border pattern figure (seven tiles, numbered 1 to 7 from the left). (a) Name the two transformations in the combination that makes each tile from the one before it. (b) Name the single transformation that carries tile 1 straight onto tile 3. (c) Describe how to make tile 8 from tile 7.

**Plan.** Odd tiles sit above the dashed line, even tiles below it; every tile stands 2 squares to the right of the one before.

**Work out.** (a) A **reflection** in the dashed line, then a **translation** 2 squares along it. (b) Tile 3 is tile 1 slid **4 squares along the line** — a single translation. (c) Reflect tile 7 in the dashed line and slide the image 2 squares to the right.

**Check.** Tile 1 to tile 3 by two of the combinations: right 2 twice gives right 4 ✓ — and tile 3 sits above the line like tile 1, as two reflections should leave it.

### *Example 6 — on your own*

Use the pinwheel figure. (a) Inside one square, each blade is made from the blade before it by one transformation. Name it, including its size and its centre. (b) Name the transformation that repeats the whole square tile across the pattern. (c) Could the four blades inside one square be made by reflections instead? How can you tell from the figure?

**Plan.** Watch which way the blades lean as you go around the square, and which way the copies move between squares.

**Work out.** (a) A **quarter-turn (90°) about the centre of the square**, four times. (b) **Translations** — slides right and up, repeating the square. (c) No: every blade leans the same way around the centre, like steps on a spiral. A reflection would lean the other way, so reflections cannot make one blade from the next.

**Check.** Four quarter-turns make a full turn: $4 \times 90° = 360°$, so the fourth blade turns back onto the first — the square closes up exactly. ✓

### *Example 7 — on your own*

Use the interlocking figure. (a) Describe the translation that carries the cut piece from the top side of the square to the bottom side. (b) Explain, in two or three sentences, why the new shape will tessellate.

**Plan.** Compare the piece's old and new positions; then look at what the bump and the notch do when tiles stand one above the other.

**Work out.** (a) **Down 2** — the side length of the square; left–right the piece does not move. (b) The bump on the bottom of a tile is the exact piece the notch on top lost, so the tile above always fills the notch of the tile below with no gap and no overlap. The left and right sides are still straight lines of length 2, so columns stand side by side exactly. Slides right 2 and up 2 then repeat the tile across the plane.

**Check.** What the shape gains below equals what it lost above, so every tile still covers the same amount of the plane as the square did — sixteen tiles cover what sixteen squares covered. ✓

### *Example 8 — on your own*

Use the badge figure. The badge is built from the one blue triangle. List every transformation used, in order, and write the description as an instruction list a classmate could follow to rebuild the badge.

**Plan.** First the bowtie is made (one move), then the bowtie is copied (more moves). Count the copies in the frame.

**Work out.** The red half of the bowtie is the blue triangle **reflected in the mirror line** — one reflection. The second bowtie is the first slid **right 4**, and the third is the second slid **right 4** again — two translations. Instruction list: *draw the triangle; reflect it in the line to make a bowtie; slide the bowtie right 4 and draw it; slide right 4 again and draw it.*

**Check.** Three bowties need the original plus two copies — two translations, as listed; and each bowtie needs one reflection. Nothing else was used. ✓

## Practice questions

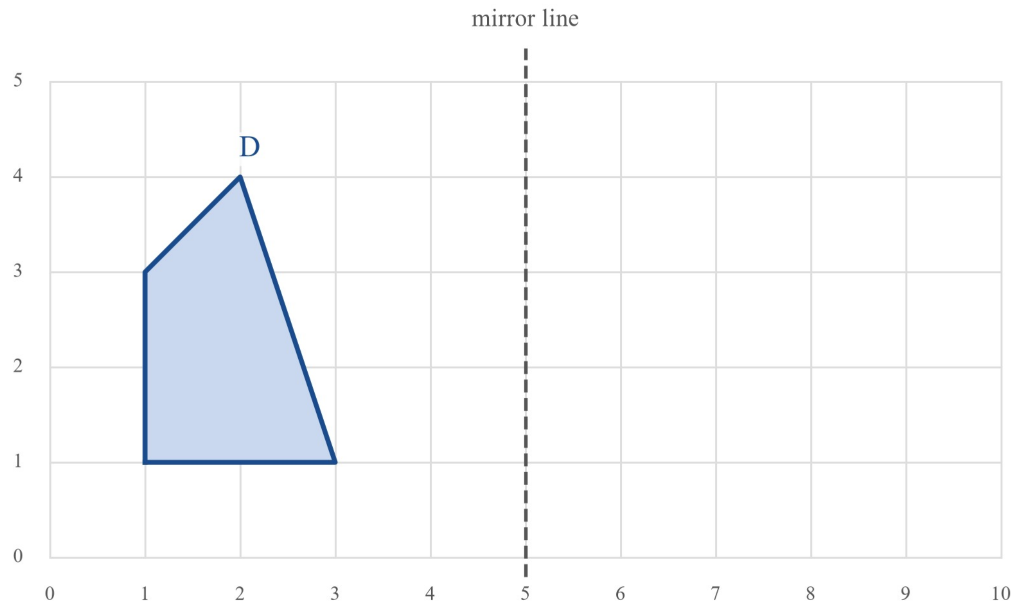
### *With help*

**Q1.** Use the three-move revision figure. Name the transformation shown in each panel.

**Q2.** Use the translation panel of that figure, where the slide is right 5. If the slide had been right 6 instead, where would the corner at (1, 1) have landed?

**Q3.** Use the reflection panel, where the mirror line is x = 4.5. How far is the corner at (3, 1) from the mirror line, and how far is its image corner from the line?

**Q4.** Use the rotation panel, a quarter-turn clockwise about C at (4, 3). The corner at (2, 3) lands at (4, 5). Where does the corner at (2, 5) land?




**Q5.**

Reflect D in the mirror line, then slide the image left 3. Write the corners of the final shape.

**Q6.** Use the order figure. Write the three corners of the finish triangle in each panel. What does the difference between the two finishes tell you?

**Q7.** At a point of a tessellation, three tiles meet with corner angles 90°, 120° and one more. Find the third corner angle.

**Q8.** Use the pentagon figure and the corner angle 108°. Show with the 360° test why regular pentagons do not tessellate on their own.

**Q9.** An equilateral triangle's corner angle is 60°. How many meet at each point of its tessellation, and which calculation says so?

**Q10.** Use the robot figure. The robot follows *right 3, up 2, right 2, down 4* and stops. Where is it? Which two instructions would finish the loop back at the start?

### *On your own*

**Q11.** Use the Example 1 figure. Write the corners of B, then predict the corners of C before you look at them in the figure.

**Q12.** Use the Example 1 figure again. The red triangle C is now slid down 3. Write the corners of its new image.

**Q13.** Use the Example 2 figure. (a) Write the corners of the order-2 finish triangle. (b) Which order leaves the finish triangle closer to the mirror line?

**Q14.** A corner of a shape ends at (9, 2) after this combination: reflect in the mirror line x = 5, then slide right 6. Where was the corner before the slide, and where was it before the reflection?

**Q15.** Four tiles meet at a point with corner angles 90°, 90° and 60°, and one more. Find the fourth corner angle.

**Q16.** A regular hexagon (120°) and two squares (90° each) meet at a point. (a) How much of the 360° turn is still unfilled? (b) Which tile's corner angle fills it exactly?

**Q17.** Use the any-triangle figure. (a) The blue and red triangles share the side from (0, 0) to (4, 0). Write the midpoint of that side. (b) The red triangle is the half-turn of the blue one about that midpoint. Check it by finding the image of the corner (1, 2).

**Q18.** Use the border pattern figure. (a) Name the two transformations in the combination that makes each tile from the one before. (b) Name the single transformation carrying tile 1 onto tile 3. (c) Describe how to make tile 8 from tile 7.

**Q19.** Use the pinwheel figure. (a) Name the turn that makes each blade from the one before it, inside one square. (b) Name the transformation that repeats the square tile across the pattern. (c) How many of those turns bring a blade back onto itself?

**Q20.** Use the interlocking figure. (a) Describe the translation that carries the cut piece from the top side to the bottom side. (b) In two or three sentences, explain why the new shape will tessellate.

**Q21.** Use the badge figure. (a) Name the transformation that makes the red half of the bowtie from the blue half. (b) Name the transformation that carries the first bowtie onto the second. (c) List every transformation used to build the whole badge from the one triangle.

**Q22.** On grid paper, draw a shape of your own with corners on grid points. Choose a combination of two transformations, apply it to your shape, then apply it once more. Describe, move by move, what you made.

### *Maths in real life*

**Q23.** A courtyard is paved with square pavers. At a point where four pavers meet, what do the four corner angles add to, and why does that mean squares will always pave a flat courtyard with no gaps?

**Q24.** A ribbon design repeats one shape by reflecting it and sliding it along the ribbon, exactly like the border pattern figure. Name the two transformations the printer used, and name the single transformation that carries every second shape onto the next one like it.

**Q25.** A club wants a badge made from one shape using at least two different transformations, like the badge figure. Sketch one idea on grid paper and list the transformations you used, in order.

### *Choosing your strategy*

**Q26.** Explain, using the 360° test, why a regular hexagon will tessellate on its own but a regular pentagon will not. Which single fact about the corner angles decides it?

**Q27.** Mia says that reflecting a shape and then sliding it always lands it in the same place as sliding it and then reflecting it. Use the order figure to show she is wrong.

**Q28.** A robot's instructions are *right 4, up 3, left 4*. What single instruction gets it back to its start? Check by balancing the moves.

**Q29.** A friend prints a pattern and calls it a tessellation. What two things do you check before you agree, and what calculation at a meeting point settles it?

## Answers

**Q1.** Translation; reflection; rotation. **Q2.** (7, 1). **Q3.** 1.5 squares each side — equal distances. **Q4.** (6, 5). **Q5.** (6, 1), (6, 3), (5, 4), (4, 1). **Q6.** Left panel (11, 1), (11, 3), (9, 1); right panel (1, 1), (1, 3), (−1, 1); the order of a combination matters. **Q7.** 150°. **Q8.** 3 × 108° = 324° (a 36° gap) and 4 × 108° = 432° (overlap) — no whole number of 108s fills 360°. **Q9.** Six; $360° \div 60° = 6$. **Q10.** At (6, −1); *left 5, up 2*. **Q11.** B: (5, 1), (5, 3), (7, 1); C: (11, 1), (11, 3), (9, 1). **Q12.** (11, −2), (11, 0), (9, −2). **Q13.** (a) (6, 1), (6, 4), (4, 1); (b) order 2. **Q14.** Before the slide (3, 2); before the reflection (7, 2). **Q15.** 120°. **Q16.** (a) 60°; (b) an equilateral triangle. **Q17.** (a) (2, 0); (b) (1, 2) → (3, −2) — the red corner in the figure. **Q18.** (a) reflection in the dashed line, then translation 2 along it; (b) translation right 4; (c) reflect tile 7 in the dashed line, slide the image right 2. **Q19.** (a) a quarter-turn (90°) about the centre of the square; (b) translations; (c) four. **Q20.** (a) down 2; (b) the bump below is the exact piece the notch above lost, and the left–right sides stay straight. **Q21.** (a) reflection in the mirror line; (b) translation right 4; (c) one reflection, then two translations (right 4, right 4). **Q22.** Answers will vary. **Q23.** 360°; four 90° corners fill the turn exactly, at every point. **Q24.** Reflection and translation; translation (two slides' worth). **Q25.** Answers will vary. **Q26.** $360° \div 120° = 3$ is a whole number; $360° \div 108°$ is not. **Q27.** The two finishes stand in different places (right of the grid vs left of it) — same moves, different order. **Q28.** *Down 3*; rights balance lefts (4 = 4), so only the up 3 is left to undo. **Q29.** No gaps and no overlaps; the corner angles at each meeting point add to 360°.

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## All the working

**Q1.** Panel 1 slides the shape — translation. Panel 2 flips it over the dashed line — reflection. Panel 3 turns it about C — rotation.

**Q2.** The slide adds 6 to the first coordinate: \$(1 + 6, 1) = \$ **(7, 1)**.

**Q3.** (3, 1) is $4.5 - 3 = 1.5$ squares from the line; its image (6, 1) is \$6 − 4.5 = \$ **1.5 squares** the other side — a reflection keeps the distance.

**Q4.** The corner (2, 5) is 2 left and 2 up from C; a quarter-turn clockwise about C carries it to 2 up and 2 right of C: \$(4 + 2, 3 + 2) = \$ **(6, 5)** — as printed in the figure.

**Q5.** Reflect in x = 5 (first coordinates become 10 − what they were): (9, 1), (9, 3), (8, 4), (7, 1). Slide left 3: **(6, 1), (6, 3), (5, 4), (4, 1)**.

**Q6.** Left (reflect then slide): reflect gives (6, 1), (6, 3), (4, 1); slide right 5 gives **(11, 1), (11, 3), (9, 1)**. Right (slide then reflect): slide gives (5, 1), (5, 3), (7, 1); reflect gives **(1, 1), (1, 3), (−1, 1)**. The finishes differ, so **the order of a combination matters**.

**Q7.** \$360° − 90° − 120° = \$ **150°**.

**Q8.** Three pentagon corners: $3 \times 108° = 324°$, leaving a 36° gap; four: $4 \times 108° = 432°$, which would overlap. No whole number of 108s makes 360°, so **regular pentagons do not tessellate on their own**.

**Q9.** \$360° ÷ 60° = \$ **6** triangles at each point — so equilateral triangles tessellate.

**Q10.** (1, 1) → right 3 (4, 1) → up 2 (4, 3) → right 2 (6, 3) → down 4 **(6, −1)**. Home needs **left 5** (to (1, −1)) and **up 2** (to (1, 1)).

**Q11.** Slide right 4 adds 4 to each first coordinate: B **(5, 1), (5, 3), (7, 1)**. B's corners sit 3, 3, 1 from x = 8, so C is 3, 3, 1 beyond it: **(11, 1), (11, 3), (9, 1)** — matching the figure.

**Q12.** Slide down 3 takes 3 from each second coordinate: **(11, −2), (11, 0), (9, −2)**.

**Q13.** (a) Slide S right 3: (4, 1), (4, 4), (6, 1); reflect in x = 5: **(6, 1), (6, 4), (4, 1)**. (b) Order 2's finish runs from x = 4 to 6, crossing the line, while order 1's runs from 10 to 12 — **order 2** is closer.

**Q14.** Undo the slide: $(9 - 6, 2) = (3, 2)$. Undo the reflection in x = 5: \$(10 − 3, 2) = \$ **(7, 2)**. So before the slide (3, 2); before the reflection (7, 2).

**Q15.** $360^\circ - 90^\circ - 90^\circ - 60^\circ =$ **120°**.

**Q16.** (a) $120^\circ + 90^\circ + 90^\circ = 300^\circ$; unfilled $360^\circ - 300^\circ =$ **60°**. (b) A corner of **60° — an equilateral triangle** — fills it exactly.

**Q17.** (a) Halfway from (0, 0) to (4, 0): **(2, 0)**. (b) The half-turn about (2, 0) sends (x, y) to (4 − x, −y): (1, 2) → **(3, −2)** — the red triangle's third corner in the figure. ✓

**Q18.** (a) **Reflection** in the dashed line, then **translation** 2 squares along it. (b) Tile 3 is tile 1 moved 4 along the line, same side up: **translation right 4**. (c) **Reflect** tile 7 in the dashed line and **slide the image right 2**.

**Q19.** (a) A **quarter-turn (90°) about the centre of the square**. (b) **Translations** right and up. (c) $360^\circ \div 90^\circ =$ **4** turns.

**Q20.** (a) **Down 2** (the square's side length), with no left–right move. (b) The bump added below is the exact piece the notch above lost, so each tile fills the notch of the tile below — no gap, no overlap — and the straight left and right sides let columns stand side by side; slides right 2 and up 2 repeat it across the plane.

**Q21.** (a) **Reflection** in the mirror line. (b) **Translation right 4**. (c) **One reflection** (to make the bowtie) and **two translations** (right 4, then right 4 again) to place the three bowties.

**Q22.** Answers will vary. A correct attempt draws a shape with corners on grid points, names a combination of two transformations, applies it twice, and describes each move so the drawing could be rebuilt from the words.

**Q23.** Four paver corners at the point: $4 \times 90^\circ =$ **360°** — a full turn, exactly filled. The same four-right-angle meeting happens at every point of the paving, which is why squares always tessellate a flat courtyard.

**Q24.** The printer used a **reflection** and a **translation**, repeated. Every second shape faces the same way, so it is carried onto the next one like it by a **translation** alone (two slides' worth).

**Q25.** Answers will vary. A correct badge names at least two different transformations in order — for example, reflect the shape in a line, then slide the pair right 3 twice — and the sketch matches the list.

**Q26.** Hexagon: $360^\circ \div 120^\circ = 3$, a whole number, so three hexagons fill every point exactly. Pentagon: $360^\circ \div 108^\circ$ is not a whole number (3 leave a gap, 4 overlap). The deciding fact: **whether 360° ÷ corner angle is a whole number**.

**Q27.** In the figure the same start triangle, mirror line and slide give a finish on the right of the grid one way (corners (11, 1), (11, 3), (9, 1)) and a finish on the left the other way (corners (1, 1), (1, 3), (−1, 1)). Same moves, different order, different place — **Mia's claim fails**.

**Q28.** Rights $= 4$, lefts $= 4$ — they balance. Ups $= 3$, downs $= 0$ — so the one instruction is ***down 3***. Check: rights − lefts $= 0$, ups − downs $= 3 - 3 = 0$; the path closes.

**Q29.** Check **no gaps and no overlaps** in the whole pattern; then pick a meeting point and check the **corner angles there add to 360°** (for example $90^\circ + 90^\circ + 90^\circ + 90^\circ = 360^\circ$). Both holding is what a tessellation is.

### What comes next

That closes Chapter 6. Next in this book, **7A** (`VC2M6ST01`) takes the Statistics strand up with comparing data sets by mode, range and shape. The work of this subtopic itself returns at Level 7: `VC2M7SP03` transforms whole sets of points on the plane with coordinate rules, and `VC2M7A05` plots relationships there — the plane you read in 6B and the combinations of transformations you built here are exactly what those subtopics lean on.

## Test

**Chapter test T6 — Space**

*Mathematics Level 6 · Chapter 6 · 6A Cross-sections and right prisms · 6B The four quadrants of the Cartesian plane · 6C Combining transformations: tessellations and geometric patterns*

**Codes assessed:** `VC2M6SP01` · `VC2M6SP02` · `VC2M6SP03`

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 6 achievement standard, not against a mark on this paper.

Name: ______________________________ Date: ______________

This is a classroom check, not an exam. Attempt every question, and show your working where the question asks for it.

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|---|---|
| **Marks** | 15 (Section A: 5 · Section B: 5 · Section C: 5) |
| **Guidance time** | 20 minutes |
| **Timing** | Guidance is about 1 minute per mark plus 5 minutes reading. There is no strict time limit; your teacher will let you finish. |
| **Equipment** | Ruler and grid paper. You will also need a pencil. |
| **Calculators** | No calculator is needed for any question on this paper. |

### Section A — Cross-sections and right prisms (5 marks)

**Q1.** (a) Which one of these objects is a right prism? (1 mark)

A. A cone

B. A cube

C. A sphere

D. A square pyramid

(b) Hana makes two parallel slices through a square pyramid, both slices parallel to the base. Both cross-sections are squares, so Hana says the pyramid must be a right prism. Use the definition of a right prism to say what Hana has missed. (1 mark)

(c) Every parallel cross-section of a solid is the same pentagon.

i. Is the solid a right prism? Give a reason from the definition. (1 mark)

ii. Name the solid. (1 mark)

(d) The parallel cross-sections of a cylinder are all the same circle, the same shape and the same size all the way through. Give one reason this book does not call the cylinder a right prism. (1 mark)

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### Section B — The four quadrants of the Cartesian plane (5 marks)

**Q2.** (a) On grid paper, draw a Cartesian plane with a horizontal axis and a vertical axis, each counting from −5 to 5 in steps of 1. Mark the origin and label it O. Then plot the points A(−4, 2) and B(3, −4). (2 marks)

(b) Which of the four quadrants is A in, and which of the four quadrants is B in? (1 mark)

(c) The point A moves 6 right, then 3 down, to a new point A′.

i. Find the coordinates of A′, counting across 0 where you need to. (1 mark)

ii. Describe how each coordinate changed from A to A′. (1 mark)

## Section C — Combining transformations: tessellations and geometric patterns (5 marks)

**Q3.** (a) A pattern of shapes covers the plane with no gaps and no overlaps. What is the maths name for a pattern like this? (1 mark)

(b) Triangle T has corners at (1, 1), (1, 3) and (3, 1). It is reflected in the mirror line $x = 5$ to make T′, and T′ is then translated down 2 to make T″. Find the corners of T′ and the corners of T″. (2 marks)

(c) At one point of a tessellation, corners of four tiles meet. Three of the corner angles are 90°, 90° and 120°. Find the fourth corner angle. Show your working. (1 mark)

(d) Zoe reflects a shape in a mirror line, then translates the image right 4. Luca translates the same shape right 4 first, then reflects the image in the same mirror line. Must their two images finish in the same place? Answer yes or no, and give a reason. (1 mark)

**End of test — total 15 marks.**

## Solutions

Teacher-facing. Test T6 is a classroom check, not an exam: a 20-minute guidance time (about 1 minute per mark plus 5 minutes reading), with the §7 equipment line for T6 stated on the paper — ruler and grid paper — and no calculator needed anywhere. All items are authored fresh for this paper, as §7 requires (D6): the bank holds only 3 MCQs for each Space code, so there is no meaningful bank to draw on. Item types follow §7: short-answer and worked-response throughout, with multiple choice used sparingly — 1 mark of 15, Q1(a), four options A–D, where selection — classifying an object against the prism test — is genuinely the skill being tested. Scope discipline observed (D9): nothing is computed except angle sums at one meeting point of a tessellation — no volume, no surface area, no circle measurement, no π — and moves in Q2 and Q3 are described by counting along an axis, counting across 0 where needed, never by adding or subtracting negatives. The dynamic geometry software permission in `VC2M6SP03` is not exercised on this paper. No real-world data is used, so no source lines appear.

### Coverage and mark map

| CD | Subtopic | Questions | Marks |
|---|---|---|---|
| `VC2M6SP01` | 6A Cross-sections and right prisms | Q1 | 5 |
| `VC2M6SP02` | 6B The four quadrants of the Cartesian plane | Q2 | 5 |
| `VC2M6SP03` | 6C Combining transformations: tessellations and geometric patterns | Q3 | 5 |
| | **Total** | | **15** |

The §7 marking rule of 5 marks per subtopic is met exactly. The paper assesses all three Space content descriptions, each against its own CD: `VC2M6SP01` (comparing parallel cross-sections and recognising right prisms), `VC2M6SP02` (locating points in the four quadrants — both its clauses, locating and describing how coordinates change when a point moves) and `VC2M6SP03` (combinations of transformations creating tessellations and geometric patterns).

### Answer key

| Q | Part | Answer | Marks |
|---|---|---|---|
| 1 | (a) | B — a cube | 1 |
| 1 | (b) | The cross-sections are not the same size | 1 |
| 1 | (c) | i. Yes — same shape and same size, straight-sided · ii. Pentagonal prism | 2 |
| 1 | (d) | A circle has a curved edge, not straight sides | 1 |
| 2 | (a) | Plane drawn with origin O; A(−4, 2) and B(3, −4) plotted | 2 |
| 2 | (b) | A: top left · B: bottom right | 1 |
| 2 | (c) | i. A′(2, −1) · ii. first changed from −4 to 2; second changed from 2 to −1 | 2 |
| 3 | (a) | Tessellation | 1 |
| 3 | (b) | T′: (9, 1), (9, 3), (7, 1) · T′′: (9, −1), (9, 1), (7, −1) | 2 |

| Q | Part | Answer | Marks |
|---|---|---|---|
| 3 | (c) | 60° | 1 |
| 3 | (d) | No — with a reflection in the combination, order can matter | 1 |
| | | **Total** | **15** |

## Worked solutions

**Q1.** (a) Test each object against the definition: a right prism has parallel cross-sections that are the same shape and the same size all the way through, and the cross-section is straight-sided. A cube's parallel cross-sections are all the same square, so a cube is a right prism (a stack of the same squares). A cone's cross-sections shrink, a sphere's cross-sections change size, and a square pyramid's square cross-sections get smaller higher up — none of them keeps the same size. The answer is **B**.

(b) A right prism needs parallel cross-sections that are the same shape **and** the same size. Hana's two cross-sections are both squares, so the shape passes — but slicing a square pyramid higher gives a smaller square, so the sizes are not the same. Hana checked the shape and missed the size; the pyramid is not a right prism.

(c)

i. Yes. Every parallel cross-section is the same pentagon — the same shape and the same size all the way through — and a pentagon is a straight-sided shape, so the solid fits the definition of a right prism. ii. A prism is named after the shape of its cross-section: a **pentagonal prism**.

(d) Under the frame this book uses, the cross-section of a right prism is a straight-sided shape. A circle has a curved edge, so the cylinder is not a right prism — even though, like a prism, it keeps the same cross-section all the way through.

*Marks: 1 for (a), 1 for (b), 1 for each of (c)i and (c)ii, 1 for (d). In (b) the mark needs the size point — "the squares are not the same size" or "the higher slice is smaller". In (d) accept any statement of the curved-edge / not-straight-sided reason.*

**Q2.** (a) A correctly drawn plane has a horizontal axis and a vertical axis that cross at right angles at O, each counting from −5 to 5 in steps of 1, with the origin marked and labelled O. To plot A(−4, 2): start at O, count 4 left, then 2 up. To plot B(3, −4): start at O, count 3 right, then 4 down.

(b) A is left and up, signs (−, +): the **top left** quadrant. B is right and down, signs (+, −): the **bottom right** quadrant. (Quadrant II and quadrant IV respectively are also accepted.)

(c)

i. A right–left move changes the first coordinate: count 6 right from −4 — −3, −2, −1, 0, 1, 2 — landing at 2. An up–down move changes the second coordinate: count 3 down from 2 — 1, 0, −1 — landing at −1. So **A′ is (2, −1)**. ii. The first coordinate changed from −4 to 2; the second coordinate changed from 2 to −1. The point began in the top left and finished in the bottom right.

*Marks: in (a), 1 for the plane (both axes numbered −5 to 5, crossing at the origin labelled O) and 1 for both points plotted correctly — one slip in one point earns 0 on the plotting mark. In (c), 1 for A'(2, −1) and 1 for describing both coordinate changes; counting across 0 must be shown or stated for full credit in (c)i.*

**Q3.** (a) A pattern of shapes that covers the plane with no gaps and no overlaps is a **tessellation**.

(b) Reflect in the mirror line x = 5: a corner d squares from the line lands d squares on the other side, and the second coordinate does not change. (1, 1) is 4 squares from the line, so its image is (9, 1); (1, 3) is 4 squares from the line, so its image is (9, 3); (3, 1) is 2 squares from the line, so its image is (7, 1). So **T′ has corners (9, 1), (9, 3) and (7, 1)**. Then translate down 2: count each second coordinate down 2 — 1 counts down to 0, −1; 3 counts down to 2, 1. So **T″ has corners (9, −1), (9, 1) and (7, −1)**.

(c) The corner angles at one meeting point of a tessellation fill a full turn, 360°. The three given angles: 90° + 90° + 120° = 300°. The fourth angle: 360° − 300° = **60°**.

(d) **No.** Once a reflection joins a combination, the order of the moves can matter: the same two moves, done in a different order, can finish in different places. Zoe's image is reflected first and then carried right 4; Luca's image is carried right 4 first and then reflected back across the mirror line, so it lands somewhere Zoe's does not. (Order does not matter only when both moves are translations.)

*Marks: in (b), 1 for the three corners of T′ and 1 for the three corners of T″ — accept corners listed in any order. In (c), 1 for 60° with the subtraction shown. In (d), 1 for "no" with a reason that names order; a bare yes/no earns 0.*