

Mathematics Level 6

Chapter 4 — Patterns, equations and algorithms

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Growing patterns and the rules that generate them

Content description VC2M6A01: recognise and use rules that generate visually growing patterns and number patterns involving rational numbers

*Patterns are one of the oldest ideas in maths: the tiles on a floor, the rows of seats at the cinema, the way a savings tin fills up. This subtopic starts at the very beginning — what a pattern is, and what a rule is — because the rule is the heart of it. A few terms let you **see** a pattern, but only the rule tells you **every** term: term 10, term 100, the one you have not drawn yet. You will meet patterns made of tiles and dots, patterns made of decimals and fractions, and patterns a calculator can help you find — and in every one the job is the same: find the rule, say it in words, and put it to work.*

Theory

What is a growing pattern?

A **pattern** is a list of pictures or numbers, in order, that a single rule makes. Each picture or number in the pattern is called a **term**, and its place in the pattern is its **term number**: the 1st term, the 2nd term, the 3rd term, and so on.

Look at the number pattern 2, 5, 8, 11, ... The terms are 2, 5, 8 and 11; the 1st term is 2 and the 4th term is 11. The change from each term to the next is the same every time — add 3 — so the rule is *start at 2 and add 3 each time*, and the pattern can go on for ever: 14, 17, 20, ...

A **growing pattern** is a pattern in which each new term is built from the one before by doing the same thing every time. Most growing patterns get larger, and the same ideas work for patterns that shrink. Two shapes of rule will do most of the work in this subtopic:

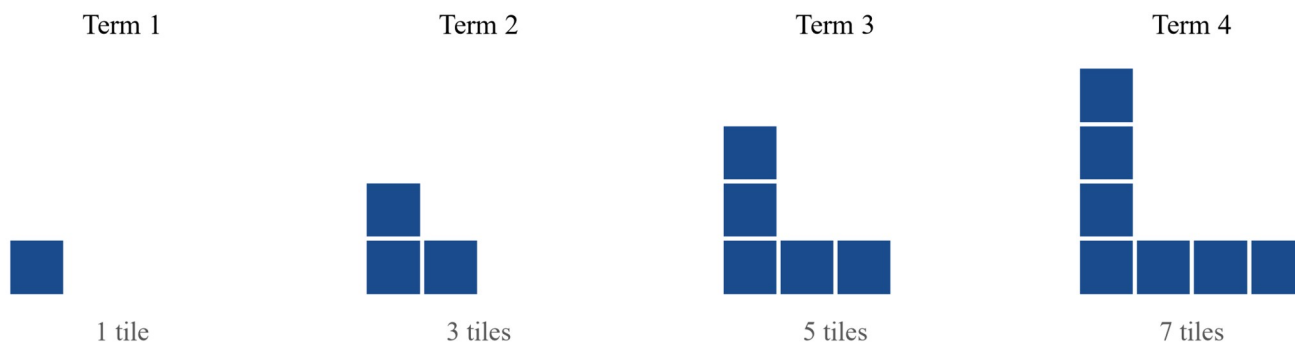
- a **next-term rule** tells you how to get each term from the one before: *start at 2 and add 3 each time*;
- an **any-term rule** tells you how to build a term straight from its term number: *multiply the term number by 3 and take away 1*.

Check the any-term rule on the pattern above: term 1 is $3 \times 1 - 1 = 2$; term 4 is $3 \times 4 - 1 = 11$. Both agree with the list. The any-term rule is the more powerful of the two — it can jump straight to term 100 without writing the ninety-nine terms before it.

Patterns you can see

Some growing patterns are pictures first and numbers second. The tiles below grow an L-shape: each new term adds one tile to the upright arm and one tile to the flat arm.

A growing pattern of tiles — each term adds 2 tiles

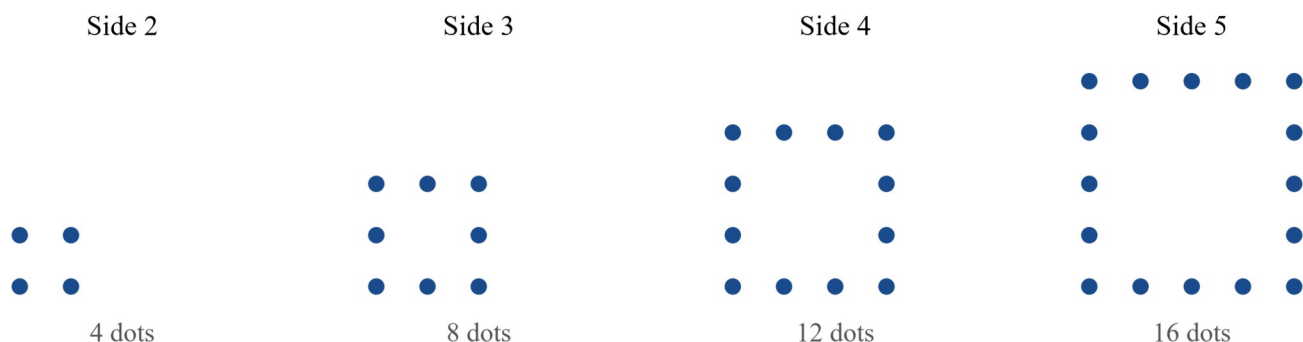


Four L-shapes of blue tiles: term 1 has 1 tile, term 2 has 3 tiles, term 3 has 5 tiles and term 4 has 7 tiles — each term adds 2 tiles.

Count the tiles in each picture: 1, 3, 5, 7. The numbers grow by 2 each time — exactly what the picture shows, because every new term adds two tiles.

The dots below sit on the border of growing squares. Count them: 4, 8, 12, 16.

Dots on the border of a growing square — each term adds 4 dots



Four squares of blue dots with dots on the border only: side 2 has 4 dots, side 3 has 8 dots, side 4 has 12 dots and side 5 has 16 dots — each new square adds 4 dots.

Here the numbers grow by 4 each time, and again the picture says why: when the square grows by one dot per side, four new dots join the border — one past each corner.

That is the habit worth building with visual patterns: **count the terms, watch how the numbers change, and look back at the picture to see why they change by exactly that much.** A rule you can see in the picture is a rule you can trust.

Saying the rule in words

The tile pattern in a table, with two rules that make it:

The tile pattern in a table

Term number	Number of tiles
1	1
2	3
3	5
4	7
5	9

Each term is 2 more than the term before.

Rule 1: start at 1 and add 2 each time.

Rule 2: multiply the term number by 2, then take away 1.

A table of the tile pattern: term numbers 1 to 5 with 1, 3, 5, 7 and 9 tiles, and two rules in words below it — start at 1 and add 2 each time; multiply the term number by 2 and take away 1.

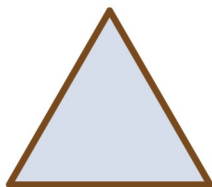
- **Rule 1 (a next-term rule):** start at 1 and add 2 each time.
- **Rule 2 (an any-term rule):** multiply the term number by 2, then take away 1.

Rule 2 checks on every row: term 3 is $2 \times 3 - 1 = 5$; term 5 is $2 \times 5 - 1 = 9$. Now use it to jump: term 10 is $2 \times 10 - 1 = 19$, and term 20 is $2 \times 20 - 1 = 39$ — found in one step, with no long list.

The matchstick pattern below grows a row of triangles that share sides.

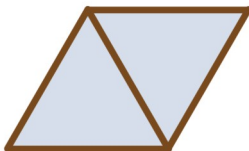
Matchstick triangles — each new triangle adds 2 sticks

1 triangle



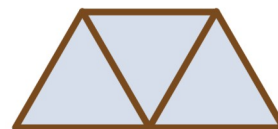
3 sticks

2 triangles



5 sticks

3 triangles



7 sticks

Three pictures of triangles built from matchsticks: 1 triangle uses 3 sticks, 2 triangles use 5 sticks and 3 triangles use 7 sticks — each new triangle adds 2 sticks.

One triangle takes 3 sticks; two take 5; three take 7. The counts grow by 2, and the any-term rule in words is: **multiply the number of triangles by 2, then add 1** — the added 1 is the very first stick, the left side of the first triangle, which every new triangle builds past. Check: 3 triangles take $2 \times 3 + 1 = 7$ sticks. ✓

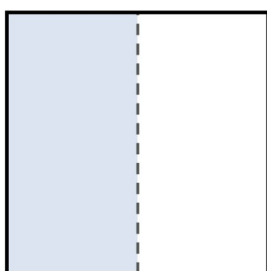
A rule in words has a job to do: it must be clear enough that **someone else can build the same pattern with different materials** — toothpicks instead of matchsticks, drawn squares instead of tiles — and land on exactly the same numbers. If your rule needs a pointing finger to work, it is not finished yet.

Doubling patterns

Not every growing pattern adds. Fold a sheet of paper in half, then in half again, and count the regions:

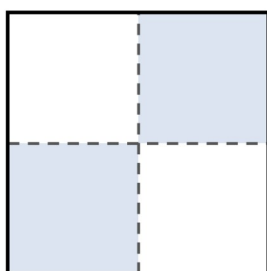
Folding a sheet of paper — each fold doubles the regions

1 fold



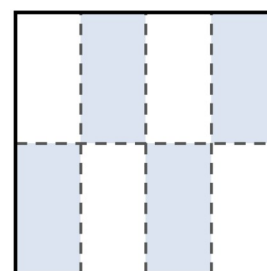
2 regions

2 folds



4 regions

3 folds



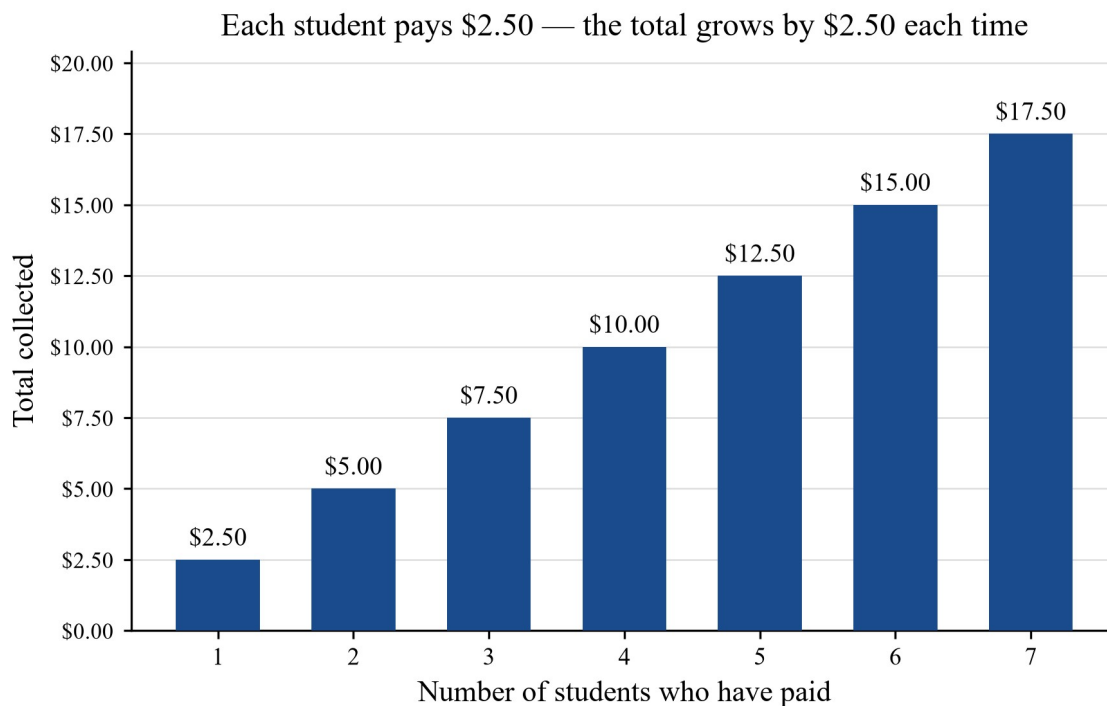
8 regions

Three pictures of a folded sheet of paper: 1 fold makes 2 regions, 2 folds make 4 regions and 3 folds make 8 regions — each fold doubles the regions.

One fold makes 2 regions, two folds make 4, three folds make 8. The change is not adding the same amount — the gaps are 2, then 4 — and the rule in everyday language is simply: **each fold doubles the regions**. So four folds make 16 regions and five folds make 32. Six folds make $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ regions: start with one sheet, and double six times. Doubling and halving are rules too — multiplying rules — and they make patterns that grow (or shrink) faster than any adding pattern.

Number patterns with decimals

Patterns do not have to be made of counting numbers. When every student in a class pays \$2.50 for an incursion, the total collected grows in a pattern:

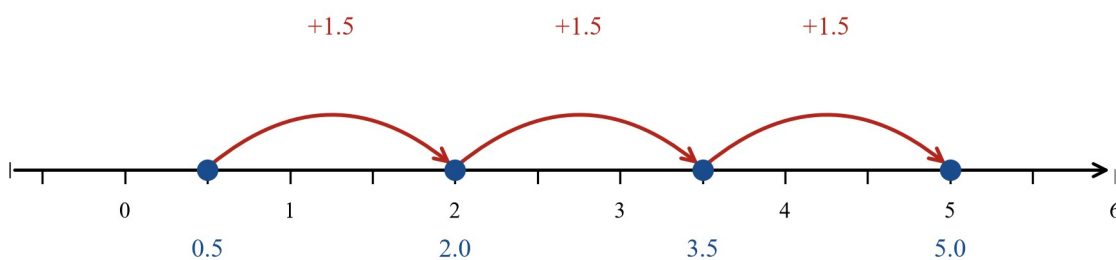


A bar graph of the total money collected as students pay \$2.50 each: from 1 student and \$2.50 up to 7 students and \$17.50 — the total grows by \$2.50 each time.

2.50, 5.00, 7.50, 10.00, 12.50, 15.00, 17.50, ... — each new student adds \$2.50, and the any-term rule is *multiply the number of students by \$2.50*.

The number line below shows a growing decimal pattern, adding 1.5 each time.

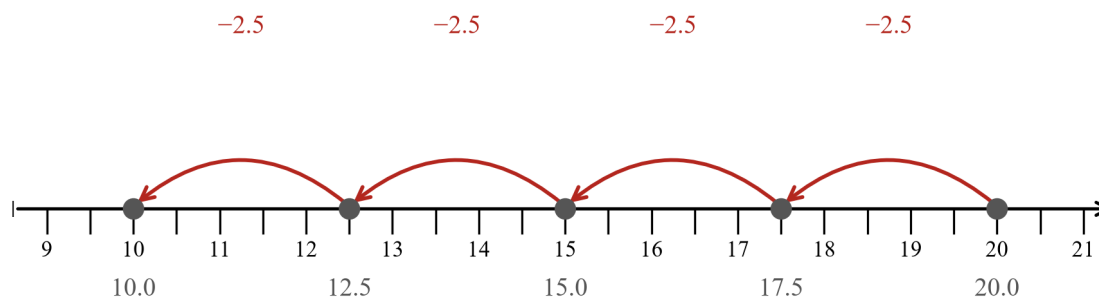
A growing pattern: 0.5, 2, 3.5, 5, ... add 1.5 each time



A number line with dots on 0.5, 2, 3.5 and 5 and red jumps of 1.5 between them — a growing decimal pattern.

And patterns can shrink in the same way. The number line below takes away 2.5 at every step.

A shrinking pattern: 20, 17.5, 15, 12.5, 10, ... take away 2.5 each time



A number line with dots on 20, 17.5, 15, 12.5 and 10 and red jumps back of 2.5 between them — a shrinking decimal pattern.

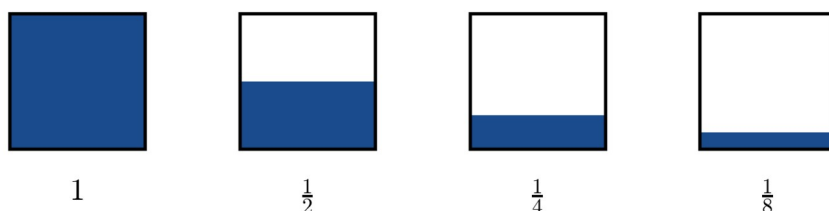
20, 17.5, 15, 12.5, 10, ... The rule is *start at 20 and take away 2.5 each time*, and the pattern steps down the line, step by step, just as the growing one steps up. A shrinking pattern is still a growing-pattern question: the rule still says exactly what changes from term to term.

Number patterns with fractions

Decimals are not the only rational numbers that make patterns. The squares below are shaded 1, then $\frac{1}{2}$, then $\frac{1}{4}$, then $\frac{1}{8}$:

Number patterns with fractions

A shrinking fraction pattern



take half of the last term each time

A growing fraction pattern



add $\frac{1}{2}$ each time

Two fraction patterns: squares shaded 1, $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ with the note “take half of the last term each time”, and bars of $\frac{1}{2}$, 1, $1\frac{1}{2}$ and 2 with the note “add $\frac{1}{2}$ each time”.

Each term is **half of the term before** — a halving rule, and a shrinking pattern. The bars below it grow the other way: $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, ... — *add $\frac{1}{2}$ each time*. Same idea as the decimal patterns, with fractions doing the stepping. When a pattern’s terms are fractions or decimals, nothing about the method changes: watch the change between terms, say the rule in words, and use it.

Patterns a calculator can find

(Digital tool.) A calculator or a spreadsheet is a good partner for hunting patterns, because it can work out many examples quickly and the pattern shows up in the answers. Try these two families:

Patterns a calculator can help you find

Dividing by 9
Multiplying by 11

Try it	Display
$1 \div 9$	0.111...
$2 \div 9$	0.222...
$3 \div 9$	0.333...
$4 \div 9$	0.444...

The repeating digit goes up by 1 each time.

Try it	Display
111×11	1221
222×11	2442
333×11	3663

Each answer is 1221 more than the one before.

Two calculator tables: dividing 1, 2, 3 and 4 by 9 gives 0.111..., 0.222..., 0.333... and 0.444..., and 111, 222 and 333 times 11 gives 1221, 2442 and 3663 — patterns to predict and then check.

Dividing by 9: $1 \div 9 = 0.111...$, $2 \div 9 = 0.222...$, $3 \div 9 = 0.333...$ — the repeating digit climbs by 1 each time, so you can predict $4 \div 9 = 0.444...$ before the calculator says it. Multiplying by 11: $111 \times 11 = 1221$, $222 \times 11 = 2442$, $333 \times 11 = 3663$ — each answer is 1221 more than the one before. The calculator's job is to give you a clean list of examples; **your** job is still to spot the rule and say it in words.

How to find a rule

Finding a rule is a method, not a guess. For any pattern:

1. **Write out the terms**, in order, with their term numbers.
2. **Look at the change** from each term to the next. Is it the same change every time? Then a next-term rule is *add (or take away) that amount*. Is each term double, or half, of the one before? Say so.
3. **Check the rule on every term you have**. A rule that works for three of four terms is not a rule for the pattern.
4. **Use the rule** to make the next term — or to jump to a far one.

One warning before you trust a pattern's first few terms: the same opening can hide two different rules. The start 2, 4, ... might be *add 2* (giving 2, 4, 6, 8, ...) or it might be *double* (giving 2, 4, 8, 16, ...). A few terms are a clue; the stated rule is what makes a pattern certain. That is exactly why this subtopic asks you to **say the rule in words** — and why two people who write different rules for the same three terms are not both right about the pattern, only about the clue.

Words for this subtopic, with what they mean here:

Term	Meaning here
pattern	a list of pictures or numbers, in order, that a single rule makes
term	each picture or number in a pattern
term number	the place of a term in the pattern: 1st, 2nd, 3rd, ...
growing pattern	a pattern whose terms change in size by the same rule at every step — usually getting larger, sometimes

Term	Meaning here
	shrinking
rule	words that say how the pattern is made, clearly enough that someone else can make it too
next-term rule	a rule that builds each term from the one before, like <i>start at 1 and add 2 each time</i>
any-term rule	a rule that builds a term straight from its term number, like <i>multiply the term number by 2, then take away 1</i>
rational numbers	the family of numbers that takes in the counting numbers, the fractions and the decimals

Worked examples

In the with-help examples the thinking is laid out line by line; in the on-your-own examples you carry it. Every example finds a rule, checks it, and then uses it.

Example 1 — with help

The tile pattern runs 1, 3, 5, 7, ... (the first figure in the Theory). Find the change between terms, write both rules in words, and use the any-term rule to find the number of tiles in term 10.

Working	Comment
Changes: $3 - 1 = 2$, $5 - 3 = 2$, $7 - 5 = 2$.	The same change every time — add 2.
Next-term rule: <i>start at 1 and add 2 each time.</i>	Step 1 and 2 of the method.
Any-term rule: <i>multiply the term number by 2, then take away 1.</i>	Check: term 4 gives $2 \times 4 - 1 = 7$. ✓
Term 10: $2 \times 10 - 1 = 19$ tiles.	The any-term rule jumps straight there.

Check by stepping: term 4 is 7, so term 5 is 9, term 6 is 11, ..., term 10 is $7 + 2 \times 6 = 19$. Both paths agree. ✓

Example 2 — with help

The matchstick triangles use 3, 5, 7 sticks for 1, 2, 3 triangles. Write the rule in words, check it on all three pictures, and use it for 6 triangles.

Working	Comment
Changes: $5 - 3 = 2$ and $7 - 5 = 2$.	Each new triangle adds 2 sticks.
Rule: <i>multiply the number of triangles by 2, then add 1.</i>	The added 1 is the first stick of all.
Check: 1 triangle $2 \times 1 + 1 = 3$ ✓; 2 triangles $2 \times 2 + 1 = 5$ ✓; 3 triangles $2 \times 3 + 1 = 7$ ✓.	The rule must work on every term, not just the last one.
6 triangles: $2 \times 6 + 1 = 13$ sticks.	The rule does the building.

Check by picture: 4 triangles take 9 sticks and 5 take 11, so 6 take 13. ✓ And a partner with toothpicks, given only the rule, builds the same row — that is what a rule in words is for.

Example 3 — with help

A shrinking decimal pattern runs 20, 17.5, 15, 12.5, 10, ... Write the rule, find the next two terms, and say which term 12.5 is.

Working	Comment
Change: $20 - 17.5 = 2.5$, $17.5 - 15 = 2.5$, and so on.	The same change every step — take away 2.5.
Rule: <i>start at 20 and take away 2.5 each time.</i>	A shrinking pattern, same method.

Working	Comment
Next two terms: $10 - 2.5 = 7.5$, then $7.5 - 2.5 = 5$.	The pattern keeps stepping down.
12.5 is the 4th term: 20, 17.5, 15, 12.5.	Count along the list.

Check on the number line in the Theory: the dots sit on 20, 17.5, 15, 12.5 and 10, with equal jumps back of 2.5. ✓

Example 4 — with help

Folding a sheet gives 2, 4, 8 regions for 1, 2, 3 folds. Write the rule in everyday language and find the regions for 6 folds.

Working	Comment
Changes: $4 - 2 = 2$, $8 - 4 = 4$ — not the same.	No adding rule fits this pattern.
But $2 \times 2 = 4$ and $4 \times 2 = 8$: each term is double the one before.	Look for a multiplying rule when the gaps grow.
Rule: <i>each fold doubles the regions</i> .	Everyday language is exactly right here.
6 folds: $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ regions.	Double six times, starting from one sheet.

Check by stepping: 4 folds give 16 and 5 folds give 32, so 6 give 64. ✓

Example 5 — on your own

The border-dot pattern runs 4, 8, 12, 16, ... for sides 2, 3, 4, 5. Write an any-term rule in words and use it for a square of side 10.

The change is 4 each time. Any-term rule: *multiply the side by 4, then take away 4*. Check on side 5: $4 \times 5 - 4 = 16$ ✓ — the four corners get counted by four sides, so take them away once. Side 10: $4 \times 10 - 4 = 36$ dots.

Example 6 — on your own

Write the next two terms and the rule in words for each fraction pattern. (a) $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots$ (b) $1, \frac{1}{2}, \frac{1}{4}, \dots$

(a) The change is $\frac{1}{2}$ each time: rule *start at $\frac{1}{2}$ and add $\frac{1}{2}$ each time*. Next terms: $2\frac{1}{2}$, then 3.

(b) Each term is half of the one before: rule *take half of the last term each time*. Next terms: $\frac{1}{8}$, then $\frac{1}{16}$.

Check (b): half of 1 is $\frac{1}{2}$, half of $\frac{1}{2}$ is $\frac{1}{4}$ — the rule works on every step shown. ✓

Example 7 — on your own

Every student pays \$2.50 for an incursion. The total collected runs 2.50, 5.00, 7.50, ... How much is collected when 12 students have paid, and how many students have paid when the total is \$42.50?

Rule: *multiply the number of students by \$2.50*. Twelve students: $12 \times \$2.50 = \30.00 . For \$42.50: count up by 2.50s — $42.50 \div 2.50 = 17$, so 17 students. Check: $17 \times \$2.50 = \42.50 . ✓

Example 8 — on your own (digital-tool path)

(Digital tool.) Use a calculator. (a) The pattern $1 \div 9, 2 \div 9, 3 \div 9, \dots$ gives 0.111..., 0.222..., 0.333... Predict $8 \div 9$, then check it. (b) The pattern $111 \times 11, 222 \times 11, 333 \times 11$ gives 1221, 2442, 3663. Predict 444×11 , then check it.

(a) The repeating digit climbs by 1, so $8 \div 9$ should be 0.888... — the calculator agrees. ✓

(b) Each answer is 1221 more than the last, so 444×11 should be $3663 + 1221 = 4884$ — the calculator agrees. ✓
The rule, in words: *the answer grows by 1221 each time, because the number being multiplied grows by 111 and $111 \times 11 = 1221$.*

Practice questions

With help

Q1. Write the next two terms and the change between terms. (a) 4, 7, 10, 13, ... (b) 30, 26, 22, ... (c) 0.6, 1.2, 1.8, ...
(d) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \dots$

Q2. Use the tile pattern (1, 3, 5, 7, ...). (a) How many tiles are in term 5? (b) Which term has 11 tiles? (c) Write the change between terms.

Q3. True or false? If false, give an example from this subtopic that shows it. (a) In the tile pattern, each term is 2 more than the term before. (b) In the border-dot pattern, the counts grow by 4 each time. (c) Every growing pattern grows by the same amount each time.

Q4. Use the growing decimal number line (0.5, 2, 3.5, 5, ...). (a) What is the next term after 5? (b) Is 7 a term of this pattern? How do you know? (c) Write the rule in words.

Q5. Use the fold pattern (2, 4, 8, ...). (a) How many regions do 4 folds make? (b) How many regions do 5 folds make? (c) Mia says 6 folds make 12 regions. What has she done, and what is the right count?

Q6. Use the matchstick triangles (3, 5, 7, ...). (a) How many sticks do 4 triangles take? (b) How many sticks do 5 triangles take? (c) Check the rule *multiply the number of triangles by 2, then add 1* on the 3-triangle picture.

Q7. The incursion totals run 2.50, 5.00, 7.50, ... (a) Write the next two terms after \$17.50. (b) What is the total when 10 students have paid? (c) The total is \$37.50. How many students have paid?

Q8. The shrinking fraction pattern runs $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ Write the next two terms and the rule in words.

Q9. Use the table of the tile pattern. (a) Use the rule *multiply the term number by 2, then take away 1* to find the tiles in term 8. (b) Which term has 21 tiles? (c) Check the rule on term 5 by counting along the list 1, 3, 5, 7, 9.

Q10. (*Digital tool.*) Enter $100 \div 99$, $101 \div 99$ and $102 \div 99$ on a calculator. What do you notice? Predict $105 \div 99$, then check it.

On your own

Q11. Write the next three terms and the rule in words. (a) 5, 9, 13, ... (b) 1, 2, 4, 8, ... (c) 9.5, 9, 8.5, ... (d) $2\frac{1}{2}, 3, 3\frac{1}{2}, \dots$

Q12. The border-dot pattern runs 4, 8, 12, 16, ... for sides 2, 3, 4, 5. (a) How many dots sit on the border of the side-6 square? Of the side-10 square? (b) Write an any-term rule in words. (c) Can a square in this pattern have exactly 30 border dots? Explain.

Q13. A pattern begins 2, 4, ... Two rules fit the clue so far: *add 2* and *double*. (a) Write the next three terms under each rule. (b) What does this tell you about why a pattern needs a stated rule?

Q14. The shrinking decimal pattern runs 20, 17.5, 15, 12.5, 10, ... (a) What is the next term after 10? (b) Which term is 12.5? (c) Will the pattern ever land on exactly 4? Explain.

Q15. Build the pattern from the rule; write the first five terms. (a) Start at 2 and add 3 each time. (b) Multiply the term number by 3, then add 1. (c) Start at 1 and double.

Q16. Write the next two terms and the rule in words. (a) $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, \dots$ (b) $4, 2, 1, \frac{1}{2}, \dots$

Q17. A savings tin holds \$5.60, then \$8.35, then \$11.10, growing in a pattern. (a) What is the change between terms? (b) Write the next two amounts. (c) Write the rule in words.

Q18. Which of these rules make the pattern 3, 8, 13, 18 true? (a) Add 5 each time. (b) Multiply the term number by 5, then take away 2. (c) Multiply the term number by 3. Then use rule (b) to find term 10.

Q19. The fold pattern runs 2, 4, 8, 16, ... (a) How many regions do 8 folds make? (b) Can folding a sheet ever give exactly 300 regions? Explain.

Q20. Explain, using the picture, why the tile pattern grows by exactly 2 each time.

Q21. A row of matchstick **squares** shares sides: 1 square takes 4 sticks, 2 squares take 7, 3 squares take 10. (a) How many sticks do 4 squares take? (b) Write the rule in words. (c) Can you build a row using exactly 30 sticks? Explain.

Q22. The pattern 1, 3, 6, 10, ... grows, but not by the same amount each time. (a) Write the changes between terms. (b) Write the next two terms. (c) Where else in this book have you met these numbers?

Maths in real life

Q23. A class plans an incursion that costs \$2.50 per student. The teacher lists the total collected after each student pays. (a) Write the totals after the first 8 students. (b) Write the any-term rule in words. (c) The class has 28 students. What is the total when every student has paid?

Q24. A small cinema has 14 seats in row 1, and each row behind it has 2 more seats than the row in front. (a) How many seats are in row 5? (b) Write an any-term rule in words. (c) Which row has exactly 30 seats?

Q25. A runner's training plan starts at 1.5 km in week 1 and adds 0.5 km each week. (a) How far does the runner go in week 6? (b) In which week does the distance first go above 5 km?

Choosing your strategy

Q26. Find the flaw. Tara sees the start 2, 4, 8 and writes the rule *add 2*. Which term shows her rule is wrong, and what rule fits all three terms?

Q27. Always, sometimes, or never? Back your answer with an example each way. (a) A growing pattern adds the same amount each time. (b) A doubling pattern that starts above zero grows. (c) A pattern made by adding the same amount each time shrinks.

Q28. Make your own. Build a growing pattern with materials of your choice — matches, tiles, drawn dots, folds of paper. Write the number pattern it makes, then write the rule in words and hand the rule (not the pattern) to a partner. Your partner builds the pattern with different materials. Do the two patterns match, term for term?

Answers

Q1. (a) 16, 19; add 3. (b) 18, 14; take away 4. (c) 2.4, 3; add 0.6. (d) $1, 1\frac{1}{4}$; add $\frac{1}{4}$. **Q2.** (a) 9; (b) term 6; (c) add 2.
Q3. (a) true; (b) true; (c) false — the fold pattern doubles. **Q4.** (a) 6.5; (b) no — the terms run 6.5, then 8, so 7 is skipped; (c) start at 0.5 and add 1.5 each time. **Q5.** (a) 16; (b) 32; (c) she multiplied the folds by 2; 64. **Q6.** (a) 9; (b) 11; (c) $2 \times 3 + 1 = 7$ ✓. **Q7.** (a) \$20.00, \$22.50; (b) \$25.00; (c) 15. **Q8.** $\frac{1}{16}, \frac{1}{32}$; take half of the last term each time.
Q9. (a) 15; (b) term 11; (c) the list gives 9 and the rule gives $2 \times 5 - 1 = 9$. **Q10.** The two repeating digits climb by 1 each time (1.0101..., 1.0202..., 1.0303...); $105 \div 99 = 1.0606...$ **Q11.** (a) 17, 21, 25; add 4. (b) 16, 32, 64; double. (c) 8, 7.5, 7; take away 0.5. (d) $4, 4\frac{1}{2}, 5$; add $\frac{1}{2}$. **Q12.** (a) 20; 36; (b) multiply the side by 4, then take away 4; (c) no — $30 + 4 = 34$, and 34 is not 4 times a counting number. **Q13.** (a) add 2: 6, 8, 10; double: 8, 16, 32; (b) the same opening terms can hide different rules, so the rule is what makes the pattern certain. **Q14.** (a) 7.5; (b) the 4th term; (c) no — the terms end in .0 or .5 (... , 5, 2.5, 0), so 4 is skipped. **Q15.** (a) 2, 5, 8, 11, 14; (b) 4, 7, 10, 13, 16; (c) 1, 2, 4, 8, 16. **Q16.** (a) $1\frac{2}{3}, 2$; add $\frac{1}{3}$. (b) $\frac{1}{4}, \frac{1}{8}$; take half of the last term each time. **Q17.** (a) \$2.75; (b) \$13.85, \$16.60; (c) start at \$5.60 and add \$2.75 each time. **Q18.** (a) yes; (b) yes; (c) no; term 10 is 48. **Q19.** (a) 256; (b) no — the counts run ..., 128, 256, 512, and 300 is not there. **Q20.** Each new term adds one tile to the upright arm and one to the flat arm — 2 tiles. **Q21.** (a) 13; (b) multiply the number of squares by 3, then add 1; (c) no — $30 - 1 = 29$, and 29 is not 3 times a counting number. **Q22.** (a) +2, +3, +4; (b) 15, 21; (c) the triangular numbers of 1A. **Q23.** (a) \$2.50, \$5.00, \$7.50, \$10.00, \$12.50, \$15.00, \$17.50, \$20.00; (b) multiply the number of students by \$2.50; (c) \$70.00. **Q24.** (a) 22; (b) multiply the row number by 2, then add 12; (c) row 9. **Q25.** (a) 4 km; (b) week 9. **Q26.** The 3rd term: $4 + 2 = 6$, not 8; the rule *double* fits 2, 4, 8. **Q27.** (a) sometimes — the tile pattern does, the fold pattern does not; (b) always — 3, 6, 12, ... only grows; (c) never — adding makes a pattern grow. **Q28.** Sample: toothpick squares, 3, 7, 11, 15 — rule *start at 3 and add 4 each time*; a partner using drawn dots lands on the same counts.

All the working

Q1. (a) $13 + 3 = 16$, $16 + 3 = 19$; the change is add 3. (b) $22 - 4 = 18$, $18 - 4 = 14$; take away 4. (c) $1.8 + 0.6 = 2.4$, $2.4 + 0.6 = 3$; add 0.6. (d) $\frac{3}{4} + \frac{1}{4} = 1$, then $1\frac{1}{4}$; add $\frac{1}{4}$.
Q2. (a) $7 + 2 = 9$ tiles. (b) $9 + 2 = 11$, so term 6. (c) Each term is 2 more than the one before.
Q3. (a) True: $1 + 2 = 3$, $3 + 2 = 5$, $5 + 2 = 7$. (b) True: $4 + 4 = 8$, $8 + 4 = 12$, $12 + 4 = 16$. (c) False: the fold pattern 2, 4, 8 doubles — the gaps are 2, then 4, not the same amount.
Q4. (a) $5 + 1.5 = 6.5$. (b) No. The terms go 6.5, then $6.5 + 1.5 = 8$ — 7 is skipped, and every later term is past it. (c) Start at 0.5 and add 1.5 each time.
Q5. (a) $8 \times 2 = 16$ regions. (b) $16 \times 2 = 32$. (c) Mia used *multiply the folds by 2* ($6 \times 2 = 12$) instead of doubling the regions each fold. Working on: 4 folds 16, 5 folds 32, 6 folds 64.
Q6. (a) $7 + 2 = 9$ sticks. (b) $9 + 2 = 11$. (c) 3 triangles: $2 \times 3 + 1 = 7$ — the picture shows 7. ✓
Q7. (a) $17.50 + 2.50 = \$20.00$, then \$22.50. (b) $10 \times \$2.50 = \25.00 . (c) $37.50 \div 2.50 = 15$ students. Check: $15 \times \$2.50 = \37.50 . ✓
Q8. Half of $\frac{1}{8}$ is $\frac{1}{16}$; half of that is $\frac{1}{32}$. Rule: take half of the last term each time.
Q9. (a) $2 \times 8 - 1 = 15$ tiles. (b) $21 + 1 = 22$, and $22 \div 2 = 11$: term 11. (c) Counting along, term 5 is 9; the rule gives $2 \times 5 - 1 = 9$. ✓

Q10. $100 \div 99 = 1.0101\dots$, $101 \div 99 = 1.0202\dots$, $102 \div 99 = 1.0303\dots$ — the two repeating digits climb by 1 each time. Predict $105 \div 99 = 1.0606\dots$, and the calculator agrees. ✓

Q11. (a) $13 + 4 = 17$, 21, 25; add 4 each time. (b) $8 \times 2 = 16$, 32, 64; double each time. (c) $8.5 - 0.5 = 8$, 7.5, 7; take away 0.5 each time. (d) $3\frac{1}{2} + \frac{1}{2} = 4$, then $4\frac{1}{2}$, 5; add $\frac{1}{2}$ each time.

Q12. (a) Side 6: $16 + 4 = 20$ dots. Side 10: $4 \times 10 - 4 = 36$. (b) *Multiply the side by 4, then take away 4* — check side 4: $4 \times 4 - 4 = 12$ ✓. (c) A side- s square has $4 \times s - 4$ dots; for 30 we would need $4 \times s = 34$, and 34 is not 4 times a counting number — so no.

Q13. (a) Add 2: 6, 8, 10. Double: 8, 16, 32. (b) Two different rules agree on the clue 2, 4 but disagree from term 3 on — so a few terms are only a clue, and the stated rule is what fixes the pattern.

Q14. (a) $10 - 2.5 = 7.5$. (b) Counting along — 20, 17.5, 15, 12.5 — it is the 4th term. (c) No. Starting at 20 and taking away 2.5 keeps the ends at .0 or .5 (... , 5, 2.5, 0), and 4 never appears.

Q15. (a) 2, 5, 8, 11, 14. (b) Term 1: $3 \times 1 + 1 = 4$; then 7, 10, 13, 16. (c) 1, 2, 4, 8, 16.

Q16. (a) $1\frac{1}{3} + \frac{1}{3} = 1\frac{2}{3}$, then 2; add $\frac{1}{3}$ each time. (b) Half of $\frac{1}{2}$ is $\frac{1}{4}$, then $\frac{1}{8}$; take half of the last term each time.

Q17. (a) $8.35 - 5.60 = 2.75$ (and $11.10 - 8.35 = 2.75$). (b) $11.10 + 2.75 = \$13.85$, then $\$16.60$. (c) Start at $\$5.60$ and add $\$2.75$ each time.

Q18. (a) $3 + 5 = 8$, $8 + 5 = 13$, $13 + 5 = 18$ — yes. (b) Term 1: $5 \times 1 - 2 = 3$; term 2: $5 \times 2 - 2 = 8$; term 3: 13; term 4: 18 — yes. (c) $3 \times 2 = 6 \neq 8$ — no. Term 10 by (b): $5 \times 10 - 2 = 48$.

Q19. (a) Keep doubling: 16 (4 folds), 32 (5), 64 (6), 128 (7), 256 (8). (b) The counts are 2, 4, 8, 16, 32, 64, 128, 256, 512, ... — 300 is not in the list and the pattern never lands on it, so no.

Q20. Each new term puts one extra tile on the upright arm and one on the flat arm of the L — two new tiles — so the counts go up by exactly 2 every time. The picture *is* the reason.

Q21. (a) $10 + 3 = 13$ sticks. (b) Each new square adds 3 sticks, and the first square needs one extra: *multiply the number of squares by 3, then add 1*. Check 3 squares: $3 \times 3 + 1 = 10$ ✓. (c) 30 sticks would need $3 \times s + 1 = 30$, so $3 \times s = 29$ — and 29 is not 3 times a counting number, so no.

Q22. (a) $3 - 1 = 2$, $6 - 3 = 3$, $10 - 6 = 4$: the changes run +2, +3, +4. (b) $10 + 5 = 15$, then $15 + 6 = 21$. (c) These are the triangular numbers from 1A — the dot triangles of Chapter 1. (Even the changes make a pattern: they grow by 1 each time.)

Q23. (a) 2.50, 5.00, 7.50, 10.00, 12.50, 15.00, 17.50, 20.00. (b) *Multiply the number of students by \$2.50*. (c) $28 \times \$2.50 = \70.00 .

Q24. (a) Row 5: $14 + 2 + 2 + 2 + 2 = 22$ seats. (b) *Multiply the row number by 2, then add 12* — check row 1: $2 \times 1 + 12 = 14$ ✓. (c) $30 - 12 = 18$, $18 \div 2 = 9$: row 9. Check: $2 \times 9 + 12 = 30$ ✓.

Q25. (a) Week 6: $1.5 + 0.5 \times 5 = 4$ km. (b) The distances run 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5 — week 8 is exactly 5 km, so the first week above 5 km is week 9 (5.5 km).

Q26. Her rule makes the 3rd term $4 + 2 = 6$, but the pattern shows 8 — the 3rd term shows the flaw. The rule *double* fits all three: $2 \times 2 = 4$, $4 \times 2 = 8$. ✓

Q27. (a) Sometimes. The tile pattern adds 2 each time; the fold pattern doubles instead. (b) Always. Doubling a number above zero gives a larger number, so 3, 6, 12, 24, ... only grows. (c) Never. Adding the same amount each time makes every term larger than the one before — the pattern grows, it cannot shrink.

Q28. Sample: build squares from toothpicks sharing sides — 4, 7, 10, 13 — hmm, that is Q21's pattern; choose another, say a triangle of dots adding a row each time: 3, 7, 11, 15 with rule *start at 3 and add 4 each time*. A partner

given only the rule builds 3, 7, 11, 15 with drawn dots or tiles — the same counts, term for term. If the two lists ever disagree, the rule was not clear enough: tighten the words and try again.

What comes next

That completes 4A. You learned to read a growing pattern — tiles, dots, matchsticks, folds — as a list of terms, to watch the change between terms, and to say the rule in words two ways: a next-term rule and an any-term rule. The same method worked on number patterns made of decimals and fractions, on growing patterns and shrinking ones, and a calculator or spreadsheet helped hunt patterns in long lists of examples.

Next, **4B** turns the same thinking to equations: number sentences with brackets and mixed operations, where the unknown is a number to be found (VC2M6A02). **4C** writes rules as step-by-step algorithms that generate whole sets of numbers (VC2M6A03) — the any-term rules of 4A are exactly the kind of steps those algorithms run.

In Year 7 the tables of values you built here become pictures: VC2M7A05 plots the terms of a changing pattern on the Cartesian plane, so the rule you learned to say in words becomes a line you can see. The table comes with you; the plotting is Year 7's.

Unknown values in equations with brackets and mixed operations

find unknown values in numerical equations involving brackets and combinations of arithmetic operations, using the properties of numbers and operations — VC2M6A02

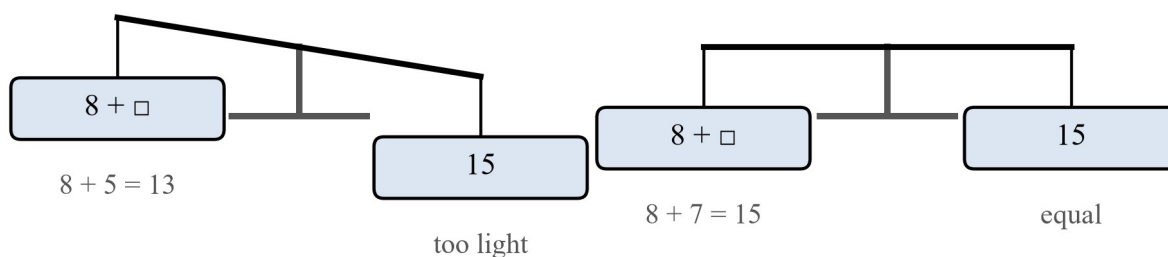
Theory

The equals sign means “is the same as”. A number sentence like $8 + \square = 15$ is called an **equation**. The $\square$ hides a number we do not know yet, called the **unknown**. The equals sign does not mean “the answer comes next”. It says the left side and the right side have the **same value**. Finding the unknown means finding the number that makes the equation **true** — the number that makes both sides equal.

An equation is true when both sides are equal

$$\square = 5$$

$$\square = 7$$



not balanced

$8 + \square = 15$ is true

Two balances: with $\square = 5$ the beam tilts, because $8 + 5 = 13$ is not the same as 15 ; with $\square = 7$ the beam is level, because $8 + 7 = 15$.

An equation works like a balance. With $\square = 5$, the left side is $8 + 5 = 13$, not the same as 15 , so the sentence is **false** and the beam tilts. With $\square = 7$, both sides are 15 , the beam is level, and $8 + \square = 15$ is **true**.

Why we need an agreed order. Look at $4 + 3 \times 5$. It has two operations, and they can be done in two orders — and the two orders give two different answers.

One number sentence, two different answers

$$4 + 3 \times 5 = ?$$

$$4 + 3 = 7 \text{ first}$$

then $7 \times 5 = 35$

Add first

$$3 \times 5 = 15 \text{ first}$$

then $4 + 15 = 19$

Multiply first

$$35 \neq 19$$

The same sentence needs one agreed answer.

The sentence $4 + 3 \times 5$ worked two ways: adding first gives $7 \times 5 = 35$, multiplying first gives $4 + 15 = 19$; $35 \neq 19$, so the same sentence needs one agreed answer.

If everyone could pick their own order, one number sentence would have two answers, and nobody could rely on it. So mathematicians made an **agreement**: a shared set of rules for which operation to do first. The rules are not a law of nature — they are an agreement, like which side of the path we walk on. What matters is that everyone uses the same rules, so every number sentence has just one answer.

The agreed order. In a number sentence with more than one operation:

1. **Brackets** first — work out what is inside the brackets.
2. **Multiplication and division** next, from left to right. They are on the same level, so do them in the order they appear, reading from left to right.
3. **Addition and subtraction** last, from left to right. They are on the same level too.

$$40 \div 2 \times (4 + 6) = \square$$

$$40 \div 2 \times (4 + 6) \longrightarrow (4 + 6) = 10$$

Step 1 brackets first



$$40 \div 2 \times 10 \longrightarrow 40 \div 2 = 20$$

Step 2 $\div$ and $\times$ are equal — left to right



$$20 \times 10 \longrightarrow 20 \times 10 = 200$$

Step 3 finish the $\times$



$$\boxed{200} \longrightarrow \square = 200$$

done

Step by step, $40 \div 2 \times (4 + 6)$ becomes $40 \div 2 \times 10$ (brackets first), then 20×10 ($\div$ and $\times$ are equal, left to right), then 200.

In the example above, the brackets are done first: $4 + 6 = 10$. Then $\div$ and $\times$ are on the same level, so we read left to right: $40 \div 2 = 20$, then $20 \times 10 = 200$. So $\square = 200$.

Brackets choose the order. Brackets let us say “do this part first”, even if it would not normally come first. The same numbers and the same operations give different answers when the brackets sit in different places.

Brackets tell us which part to do first

$$() \quad (4 + 3) \times 5 = 7 \times 5 = 35$$

the brackets say: do the addition first

$$() \quad 4 + (3 \times 5) = 4 + 15 = 19$$

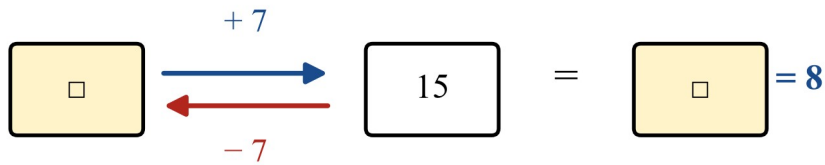
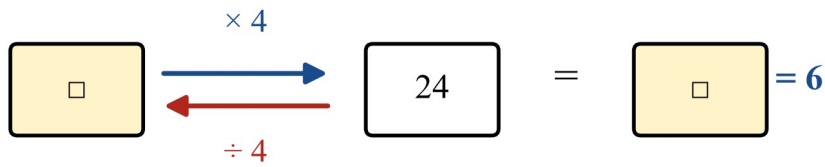
the brackets say: do the multiplication first

Brackets in two places: $(4 + 3) \times 5 = 7 \times 5 = 35$, while $4 + (3 \times 5) = 4 + 15 = 19$.

With brackets around the addition, $(4 + 3) \times 5 = 7 \times 5 = 35$. With brackets around the multiplication, $4 + (3 \times 5) = 4 + 15 = 19$. That is exactly why brackets are useful: they let us write down exactly what we mean. Without brackets, the agreed order decides for us.

Finding the unknown by undoing. When the unknown has one operation done to it, we undo that operation with its **inverse** — the operation that undoes it. Subtraction undoes addition, and division undoes multiplication.

Inverse operations undo each other



Undoing with the inverse operation finds the unknown.

Two undoing diagrams: $\square \times 4 = 24$ is undone by $24 \div 4$, giving $\square = 6$; $\square + 7 = 15$ is undone by $15 - 7$, giving $\square = 8$.

For $\square \times 4 = 24$, ask “what number times 4 makes 24?” Undo with division: $24 \div 4 = 6$, so $\square = 6$. For $\square + 7 = 15$, undo with subtraction: $15 - 7 = 8$, so $\square = 8$.

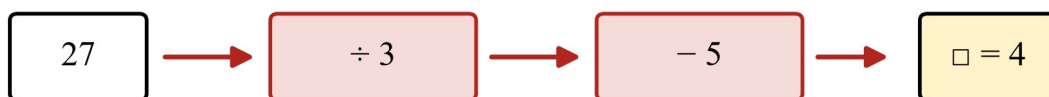
Two-step sentences: undo in reverse order. In $(\square + 5) \times 3 = 27$, the equation does two things to the unknown: it adds 5, then it multiplies by 3. To find the unknown, work back and undo each step — the **last** step done is the **first** step undone.

$$(\square + 5) \times 3 = 27$$

forward:



undo:



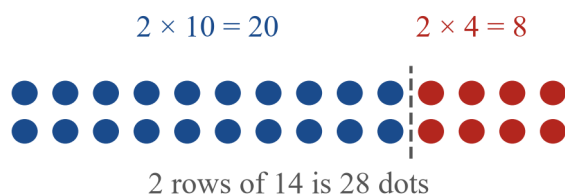
check: $(4 + 5) \times 3 = 9 \times 3 = 27 \checkmark$

Forward, $\square$ goes through $+ 5$ then $\times 3$ to reach 27. Working back, 27 goes through $\div 3$ to reach 9, then $- 5$ to reach $\square = 4$. Check: $(4 + 5) \times 3 = 9 \times 3 = 27$.

Working back: $27 \div 3 = 9$, then $9 - 5 = 4$, so $\square = 4$. Check: $(4 + 5) \times 3 = 9 \times 3 = 27$. $\checkmark$ You can also use number facts: 3 times what makes 27? 9. So $\square + 5 = 9$, and $\square = 4$. Both paths agree — and checking by putting 4 back in proves it.

Equivalent number sentences. Two number sentences are **equivalent** if they have the same value. Using properties of numbers, we can write one sentence as another that means the same thing.

Equivalent number sentences have the same value



$$2 \times (10 + 4) = 2 \times 10 + 2 \times 4$$

$$28 = 20 + 8 \quad \checkmark$$

same dots, same value — the sentences are equivalent

An array of 2 rows of 14 dots, split into 2 rows of 10 and 2 rows of 4: $2 \times (10 + 4) = 2 \times 10 + 2 \times 4$, and $28 = 20 + 8$.

The array shows why $2 \times (10 + 4) = 2 \times 10 + 2 \times 4$. Two rows of 14 is 28 dots. Splitting each row into 10 and 4 gives $2 \times 10 = 20$ blue dots and $2 \times 4 = 8$ red dots — still 28. Multiplying by the whole is the same as multiplying each part and adding. Both sides are 28, so the sentences are equivalent.

Pairs of unknowns. Some equations have two unknowns. Then there is often more than one pair of numbers that makes the equation true, and the job is to find the pairs.

$$3 \times \triangle + \square = 12$$

which natural numbers for $\triangle$ and $\square$ make it true?

$\triangle$	$\square$	check: $3 \times \triangle + \square$
1	9	$3 \times 1 + 9 = 12 \quad \checkmark$
2	6	$3 \times 2 + 6 = 12 \quad \checkmark$
3	3	$3 \times 3 + 3 = 12 \quad \checkmark$

$\triangle = 4$ would need $\square = 0$ — not a natural number, so the pairs stop here.

A table of the natural number pairs that make $3 \times \triangle + \square = 12$ true: $\triangle = 1, \square = 9$; $\triangle = 2, \square = 6$; $\triangle = 3, \square = 3$. $\triangle = 4$ would need $\square = 0$, which is not a natural number.

For $3 \times \triangle + \square = 12$, choose a natural number for $\triangle$ and work out what $\square$ must be: if $\triangle = 1$ then $\square = 9$; if $\triangle = 2$ then $\square = 6$; if $\triangle = 3$ then $\square = 3$. If $\triangle = 4$ then $\square$ would have to be 0, which is not a natural number, so the pairs stop there. Choosing values for one unknown in order, and working out the other, finds **every** pair without missing any. (Here, **natural numbers** are the counting numbers 1, 2, 3, 4, ...)

Words for this subtopic, with what they mean here:

Term	Meaning here
equation	a number sentence whose two sides have the same value
unknown	the hidden number in an equation, shown here as $\square$ or $\triangle$
true or false	an equation is true when both sides really are equal, and false when they are not
brackets	the marks $()$ that show which part of a sentence to work out first
order of operations	the agreed rules for which operation to do first
inverse operation	the operation that undoes another: $-$ undoes $+$, and $\div$ undoes $\times$
equivalent	having the same value
natural numbers	the counting numbers $1, 2, 3, 4, \dots$

Worked examples

Example 1 — with help

Find the unknown: $5 \times (\square + 3) = 35$.

Working	Comment
$5 \times 7 = 35$, so $\square + 3 = 7$.	The equation says 5 times something is 35. Undo the $\times 5$ with $\div 5$: $35 \div 5 = 7$.
$\square = 7 - 3 = 4$.	Undo the $+ 3$ with $- 3$. The last step done is the first step undone.
Check: $5 \times (4 + 3) = 5 \times 7 = 35$. ✓	Put 4 back into the equation. Both sides are 35, so the equation is true.

Example 2 — with help

Find the unknown: $54 \div 6 + \square \times 3 = 21$.

Working	Comment
$54 \div 6 = 9$, so the sentence reads $9 + \square \times 3 = 21$.	Division comes before addition in the agreed order, so work it out first.
$\square \times 3 = 21 - 9 = 12$.	Undo the $+ 9$ with $- 9$.
$\square = 12 \div 3 = 4$.	Undo the $\times 3$ with $\div 3$.
Check: $54 \div 6 + 4 \times 3 = 9 + 12 = 21$. ✓	Both sides are 21, so $\square = 4$ makes the equation true.

Example 3 — on your own

Priya buys 4 muffins at \$3 each and 2 bottles of juice, all at the same price. She pays \$22 altogether. Write an equation and find the price of one bottle of juice.

$4 \times \$3 + 2 \times \square = \22 , so $\$12 + 2 \times \square = \22 . Then $2 \times \square = \$22 - \$12 = \$10$, and $\square = \$10 \div 2 = \5 .

Check: $4 \times \$3 + 2 \times \$5 = \$12 + \$10 = \$22$. ✓ One bottle of juice costs \$5.

Example 4 — on your own

Find **every** pair of natural numbers that makes this equation true: $\square \times 4 + \triangle = 18$.

Choose values for $\square$ in order and work out $\triangle$:

- $\square=1: \triangle=18-4=14$
- $\square=2: \triangle=18-8=10$
- $\square=3: \triangle=18-12=6$
- $\square=4: \triangle=18-16=2$

If $\square=5$, then $\triangle$ would be $18-20$, which is less than 0 — not a natural number — so the pairs stop there. Four pairs work: $(1, 14)$, $(2, 10)$, $(3, 6)$ and $(4, 2)$. Check one: $3 \times 4 + 6 = 12 + 6 = 18$. ✓

Practice questions

With help

Q1. Work out each sentence using the agreed order. (a) $6 + 4 \times 3$ (b) $(6 + 4) \times 3$ (c) $20 - 12 \div 4$ (d) $(20 - 12) \div 4$

Q2. Work out. (a) $5 \times (9 - 6)$ (b) $36 \div (3 + 6)$ (c) $(18 + 2) \div 5$ (d) $7 \times (4 + 8)$

Q3. Work out $3 + 6 \times 2$ using the agreed order, then work out $(3 + 6) \times 2$. The answers are different. Explain, in your own words, why an agreed order of operations is needed.

Q4. Find the unknown by undoing. (a) $\square \times 6 = 42$ (b) $56 \div \square = 8$ (c) $\square - 15 = 27$ (d) $9 + \square = 31$

Q5. Find the unknown. Work out what you can first, then undo one step at a time. (a) $3 \times \square + 4 = 19$ (b) $40 - 5 \times \square = 15$ (c) $(\square + 2) \times 6 = 30$ (d) $4 \times (\square - 3) = 24$

Q6. Work out. (a) $24 \div 4 \times 3$ (b) $24 \div (4 \times 3)$ The answers are different. Which rule makes them different?

On your own

Q7. Work out. (a) $7 + 6 \times 4$ (b) $45 \div 9 + 8$ (c) $100 - 6 \times 12$ (d) $48 \div 6 \div 2$ (e) $(14 + 6) \times 5$ (f) $14 + 6 \times 5$

Q8. Mia says $20 - 8 + 4 = 8$. Leo says $20 - 8 + 4 = 16$. Who is right? Explain what the other person did wrong.

Q9. Find the unknown. (a) $(18 - \triangle) \div 5 = 2$ (b) $\triangle \times 3 + \triangle \times 5 = 48$ (Hint: use a property of numbers to write the left side as one lot of $\triangle$.)

Q10. Noah has \$15 saved and puts away \$8 each week. His savings can be written as $\$15 + \$8 \times \square$, where $\square$ is the number of weeks. After how many weeks does Noah have \$63?

Q11. Add one pair of brackets to make each sentence true. (a) $4 + 6 \times 2 = 20$ (b) $36 \div 4 + 2 = 6$ (c) $5 \times 8 - 3 = 25$

Q12. Equivalent sentences. (a) $3 \times (5 + 7) = 3 \times 5 + 3 \times \square$. Find $\square$, and check that both sides have the same value. (b) Is $(8 + 4) \div 2$ equivalent to $8 \div 2 + 4 \div 2$? Work out both sides to find out.

Q13. Zara worked out $2 + 3 \times 4$ and got 20. What did she do? What is the correct answer under the agreed order?

Q14. Find **every** pair of natural numbers that makes this equation true: $5 \times \triangle + \square = 24$. Show your checking, and explain how you know your list is complete.

Answers

Q1. (a) 18; (b) 30; (c) 17; (d) 2. **Q2.** (a) 15; (b) 4; (c) 4; (d) 84. **Q3.** 15 and 18; without brackets or an agreed order, one sentence would have two answers, so people could not rely on it. **Q4.** (a) 7; (b) 7; (c) 42; (d) 22. **Q5.** (a) 5; (b) 5; (c) 3; (d) 9. **Q6.** (a) 18; (b) 2; the brackets in (b) make the $\times$ come first; without them, $\div$ and $\times$ are done left to right. **Q7.** (a) 31; (b) 13; (c) 28; (d) 4; (e) 100; (f) 44. **Q8.** Leo; 16. **Q9.** (a) 8; (b) 6. **Q10.** 6 weeks. **Q11.** (a) $(4+6) \times 2 = 20$; (b) $36 \div (4+2) = 6$; (c) $5 \times (8-3) = 25$. **Q12.** (a) 7; (b) yes — both sides are 6. **Q13.** She added first: $(2+3) \times 4 = 20$; the correct answer is 14. **Q14.** $(\triangle, \square)$: $(1, 19)$, $(2, 14)$, $(3, 9)$, $(4, 4)$ — four pairs.

Worked solutions

Q1. (a) $4 \times 3 = 12$ first, then $6 + 12 = 18$. (b) Brackets first: $6 + 4 = 10$, then $10 \times 3 = 30$. (c) $12 \div 4 = 3$ first, then $20 - 3 = 17$. (d) Brackets first: $20 - 12 = 8$, then $8 \div 4 = 2$.

Q2. (a) $9 - 6 = 3$, then $5 \times 3 = 15$. (b) $3 + 6 = 9$, then $36 \div 9 = 4$. (c) $18 + 2 = 20$, then $20 \div 5 = 4$. (d) $4 + 8 = 12$, then $7 \times 12 = 84$.

Q3. $3 + 6 \times 2$: multiply first, $6 \times 2 = 12$, then $3 + 12 = 15$. $(3 + 6) \times 2$: brackets first, $9 \times 2 = 18$. The two sentences give different answers, and without brackets it would not be clear which was meant. The agreed order (and brackets when we mean something else) makes sure every sentence has just one answer, so everyone gets the same result.

Q4. (a) $42 \div 6 = 7$. (b) $56 \div 8 = 7$. (c) $27 + 15 = 42$. (d) $31 - 9 = 22$.

Q5. (a) $3 \times \square = 19 - 4 = 15$, so $\square = 15 \div 3 = 5$. (b) $5 \times \square = 40 - 15 = 25$, so $\square = 25 \div 5 = 5$. (c) $\square + 2 = 30 \div 6 = 5$, so $\square = 5 - 2 = 3$. (d) $\square - 3 = 24 \div 4 = 6$, so $\square = 6 + 3 = 9$.

Q6. (a) $\div$ and $\times$ are on the same level, left to right: $24 \div 4 = 6$, then $6 \times 3 = 18$. (b) Brackets first: $4 \times 3 = 12$, then $24 \div 12 = 2$. The brackets make the multiplication come first; without them, the left-to-right rule puts the division first.

Q7. (a) $6 \times 4 = 24$, then $7 + 24 = 31$. (b) $45 \div 9 = 5$, then $5 + 8 = 13$. (c) $6 \times 12 = 72$, then $100 - 72 = 28$. (d) Left to right: $48 \div 6 = 8$, then $8 \div 2 = 4$. (e) $14 + 6 = 20$, then $20 \times 5 = 100$. (f) $6 \times 5 = 30$, then $14 + 30 = 44$.

Q8. Leo is right. $+$ and $-$ are on the same level, done left to right: $20 - 8 = 12$, then $12 + 4 = 16$. Mia added $8 + 4$ first, as if the addition came before the subtraction — it does not.

Q9. (a) $18 - \triangle = 2 \times 5 = 10$, so $\triangle = 18 - 10 = 8$. Check: $(18 - 8) \div 5 = 10 \div 5 = 2$. ✓ (b) $\triangle \times 3 + \triangle \times 5$ is three lots of $\triangle$ plus five lots of $\triangle$, which is eight lots of $\triangle$: $\triangle \times (3 + 5) = \triangle \times 8$. So $\triangle \times 8 = 48$ and $\triangle = 48 \div 8 = 6$. Check: $6 \times 3 + 6 \times 5 = 18 + 30 = 48$. ✓

Q10. $\$15 + \$8 \times \square = \$63$, so $\$8 \times \square = \$63 - \$15 = \48 , and $\square = 48 \div 8 = 6$. After 6 weeks Noah has \$63. Check: $15 + 8 \times 6 = 15 + 48 = 63$. ✓

Q11. (a) $(4 + 6) \times 2 = 10 \times 2 = 20$. ✓ (b) $36 \div (4 + 2) = 36 \div 6 = 6$. ✓ (c) $5 \times (8 - 3) = 5 \times 5 = 25$. ✓

Q12. (a) Left side: $3 \times (5 + 7) = 3 \times 12 = 36$. Right side: $3 \times 5 + 3 \times \square = 15 + 3 \times \square$. For 36 we need $3 \times \square = 36 - 15 = 21$, so $\square = 7$. Check: $15 + 21 = 36$. ✓ ($\square$ is 7, the same number as inside the brackets — multiplying the whole is the same as multiplying each part and adding.) (b) $(8 + 4) \div 2 = 12 \div 2 = 6$, and $8 \div 2 + 4 \div 2 = 4 + 2 = 6$. Both sides are 6, so yes, the sentences are equivalent.

Q13. Zara added first: $2 + 3 = 5$, then $5 \times 4 = 20$ — that is really $(2 + 3) \times 4$. Under the agreed order, multiplication comes first: $3 \times 4 = 12$, then $2 + 12 = 14$.

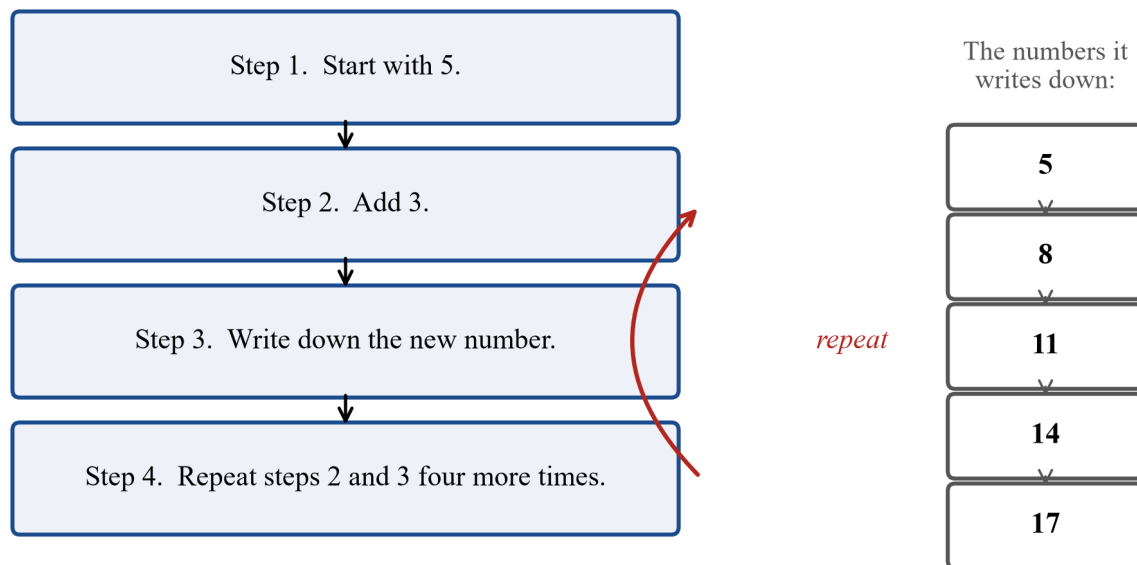
Q14. Choose $\triangle$ in order and work out $\square = 24 - 5 \times \triangle$: $\triangle = 1 \rightarrow \square = 19$; $\triangle = 2 \rightarrow \square = 14$; $\triangle = 3 \rightarrow \square = 9$; $\triangle = 4 \rightarrow \square = 4$. If $\triangle = 5$ then $\square$ would be $24 - 25$, less than 0, so no natural number works and the list is complete. Four pairs: $(1, 19)$, $(2, 14)$, $(3, 9)$, $(4, 4)$. Check one: $5 \times 2 + 14 = 10 + 14 = 24$. ✓ Every value of $\triangle$ was tried in order, so not one was missed.

Algorithms that generate sets of numbers

design and use algorithms involving a sequence of steps and decisions that use rules to generate sets of numbers; identify, interpret and explain emerging patterns — VC2M6A03

Theory

An **algorithm** is a list of steps that gets a job done. You have followed and written algorithms since Level 5: steps in an exact order, with **repetition** (do some steps again) and **decisions** (a question whose answer sends you down one path or another). In this subtopic the job is making numbers. An algorithm **generates** a list of numbers when its steps say where to start and what rule to do again and again.

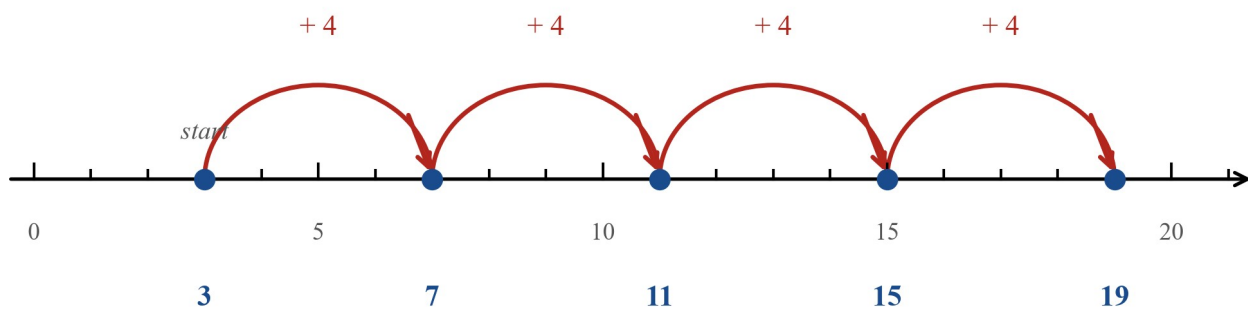


An algorithm is a list of steps that gets a job done — here the job is making a sequence of numbers.

An algorithm as four numbered steps: start with 5; add 3; write down the new number; repeat steps 2 and 3 four more times, with a red arrow looping back for the repeat, and the numbers it writes down beside it: 5, 8, 11, 14, 17.

Follow the steps by hand. Step 1 says start with 5. Step 2 says add 3, giving 8, and step 3 says write it down. Step 4 sends you back to step 2 — that is the repetition — so you add 3 again and again: $8 + 3 = 11$, $11 + 3 = 14$, $14 + 3 = 17$. The numbers written down are 5, 8, 11, 14, 17. The list of numbers an algorithm writes down, **in the order they come out**, is called a **sequence**, and each number in it is a **term**.

The same rule on a number line. Adding the same amount each step shows up as jumps of the same size.

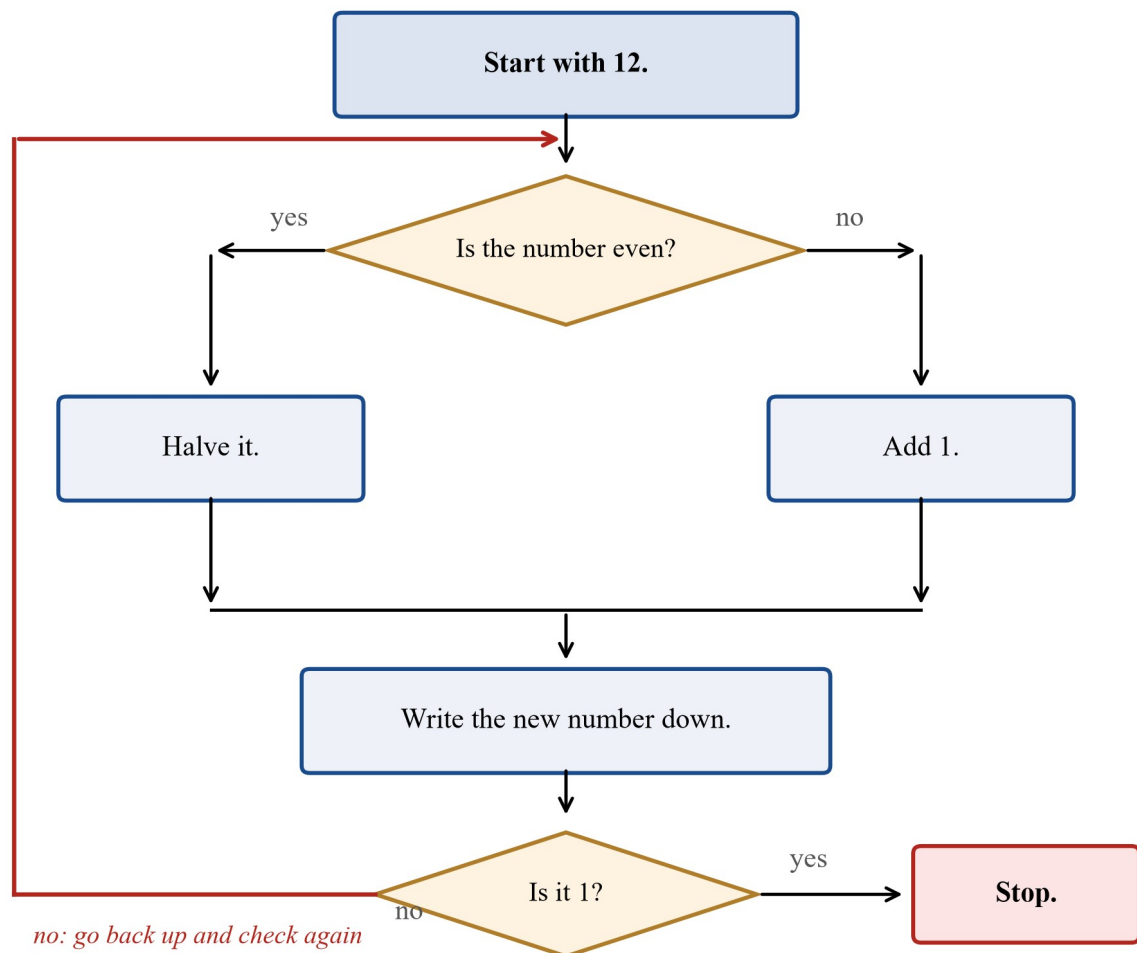


Start at 3 and add 4 each step — the jumps are all the same size.

A number line to scale from 0 to 20 with the points 3, 7, 11, 15 and 19 marked in blue and equal red curved arrows labelled “+ 4” between them.

Start at 3 and add 4 each step: the landings are 3, 7, 11, 15, 19. Every jump is 4 long, because the rule adds 4 every time. A rule that **adds** the same amount gives equal jumps — the sequence grows by the same amount at every step.

Decisions split the path — that is a branch. Some steps are not “do this” but “ask this”. A **decision** is a question with two answers (yes or no), and each answer gives a different next step. The split is called a **branch**, and an algorithm with a decision in it chooses its path from the numbers it meets.



A decision splits an algorithm into two paths — that split is called a branch.

A flow diagram of a branching algorithm: start with 12; the decision “is the number even?” sends yes to “halve it” and no to “add 1”; both paths join at “write the new number down”; the decision “is it 1?” sends yes to stop and no back up to be checked again.

Read the diagram from the top. The number starts at 12. The first decision asks *is it even?* 12 is even, so the yes path halves it to 6. The new number goes back up and is checked again. When the number is odd, like 3, the no path adds 1 instead. The last decision — *is it 1?* — decides between going around again and stopping. One algorithm, many possible paths: the numbers choose the route.

Tracing means following with a record. To **trace** an algorithm is to follow it step by step while writing down what happens at each pass, so nothing is done in your head and nothing can be hidden. A table is the cleanest way.

Tracing the branching algorithm, starting at 12

Step	Number	Even?	Decision	Next number
1	12	yes	halve it	6
2	6	yes	halve it	3
3	3	no	add 1	4
4	4	yes	halve it	2
5	2	yes	halve it	1 — stop

Each row follows one trip around the loop. The numbers written down are 12, 6, 3, 4, 2, 1.

A trace table for the branching algorithm starting at 12: step 1, number 12, even yes, halve it, next number 6; step 2, 6, yes, halve it, 3; step 3, 3, no, add 1, 4; step 4, 4, yes, halve it, 2; step 5, 2, yes, halve it, 1, stop.

The trace shows the sequence 12, 6, 3, 4, 2, 1. Look at its shape before reaching for any answer: it falls, falls, **rises**, then falls to 1. A rule with a branch can make patterns like that — not just steady growing or shrinking — and this topic asks you to notice, read and explain them. That is the next idea.

Identify, interpret, explain. Three different jobs, in order:

1. **Identify** the pattern — say what you notice. *“The numbers get smaller most of the time, but 3 to 4 is a rise.”*
2. **Interpret** it — say what it means, or what the rule will do next. *“After every rise the number is even, so the next step must be a halving.”*
3. **Explain** it — say why it must happen, from the rule itself. *“Halving an even number makes it smaller, and adding 1 to an odd number makes it even, so a rise can never last two steps.”*

A number on its own is not any of the three. The pattern claim, the reading of it, and the reason for it are three separate sentences, and each one can be checked against the rule.

Sets: the algorithm can write down a collection. Sometimes the job is not a sequence in order of coming out but a **set** — a collection of numbers where order does not matter. A set is written inside brackets shaped like $\{\}$ — for example $\{4, 8, 12\}$ — and each number is written once. An algorithm makes a set by checking numbers one at a time and writing down the ones that pass the test.

The set the algorithm writes down: multiples of 4

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

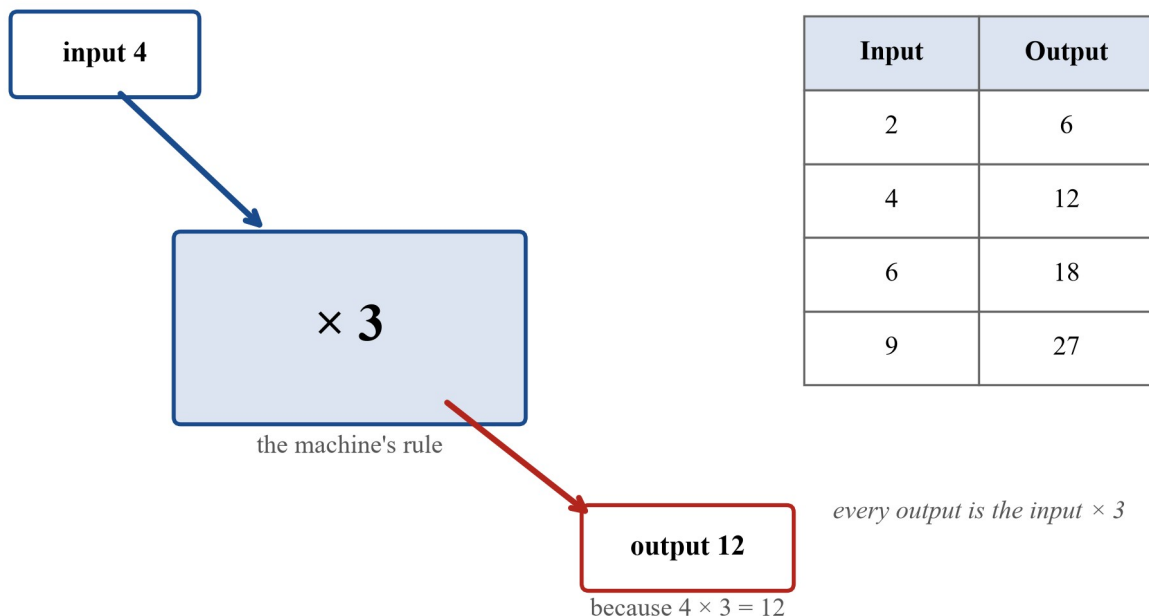
Check each number from 1 to 30: if it is $4 \times$ a whole number, write it into the set.

The set is $\{4, 8, 12, 16, 20, 24, 28\}$.

A grid of the numbers 1 to 30 with the multiples of 4 shaded red: 4, 8, 12, 16, 20, 24, 28, and the set written below as $\{4, 8, 12, 16, 20, 24, 28\}$.

The test here is *is the number 4 times a whole number?* Checking 1, 2, 3, ..., 30 in order, the numbers that pass are 4, 8, 12, 16, 20, 24, 28 — the **multiples** of 4 up to 30. The set is $\{4, 8, 12, 16, 20, 24, 28\}$. Written as a set, the order on the page does not matter; what matters is which numbers passed the test.

Input and output machines. An algorithm whose rule acts on one number at a time can be drawn as a **machine**: an **input** goes in, the rule acts on it, an **output** comes out.

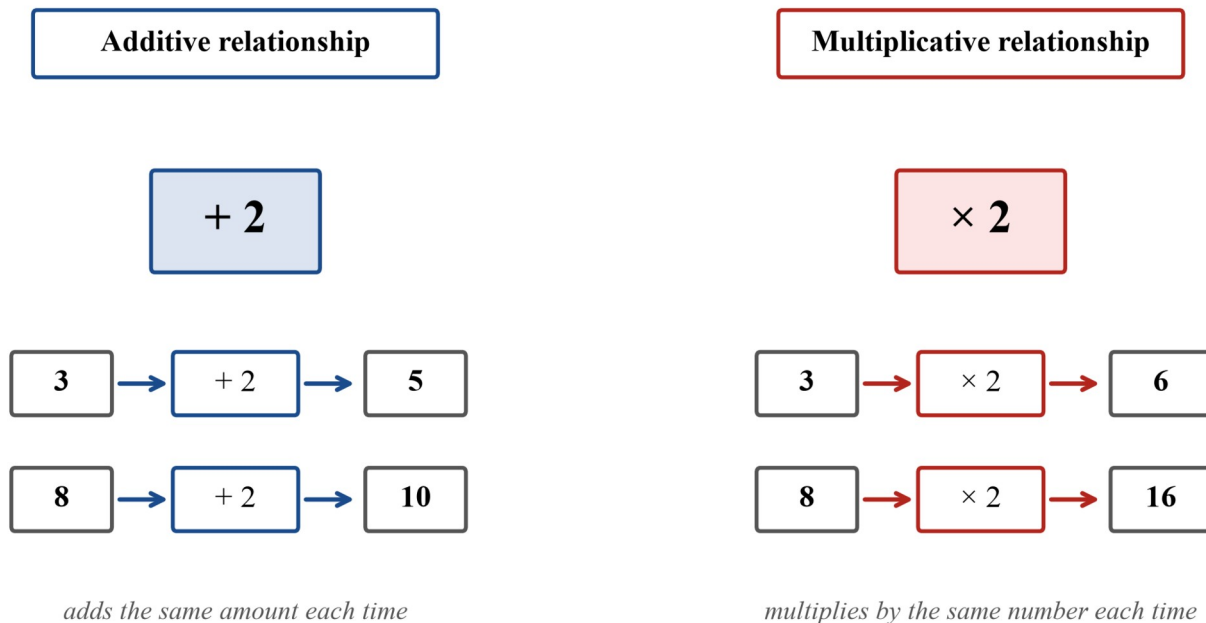


One number goes in, the rule acts on it, one number comes out.

A machine labelled " $\times 3$ " with input 4 going in and output 12 coming out, beside a table of pairs: input 2 output 6, input 4 output 12, input 6 output 18, input 9 output 27.

The machine's rule is $\times 3$. Put in 4 and $4 \times 3 = 12$ comes out. Every input–output pair follows the same rule, so a table of pairs is a picture of the algorithm at work.

Additive and multiplicative relationships. Two machines can look the same but behave in very different ways. Compare adding 2 with multiplying by 2:

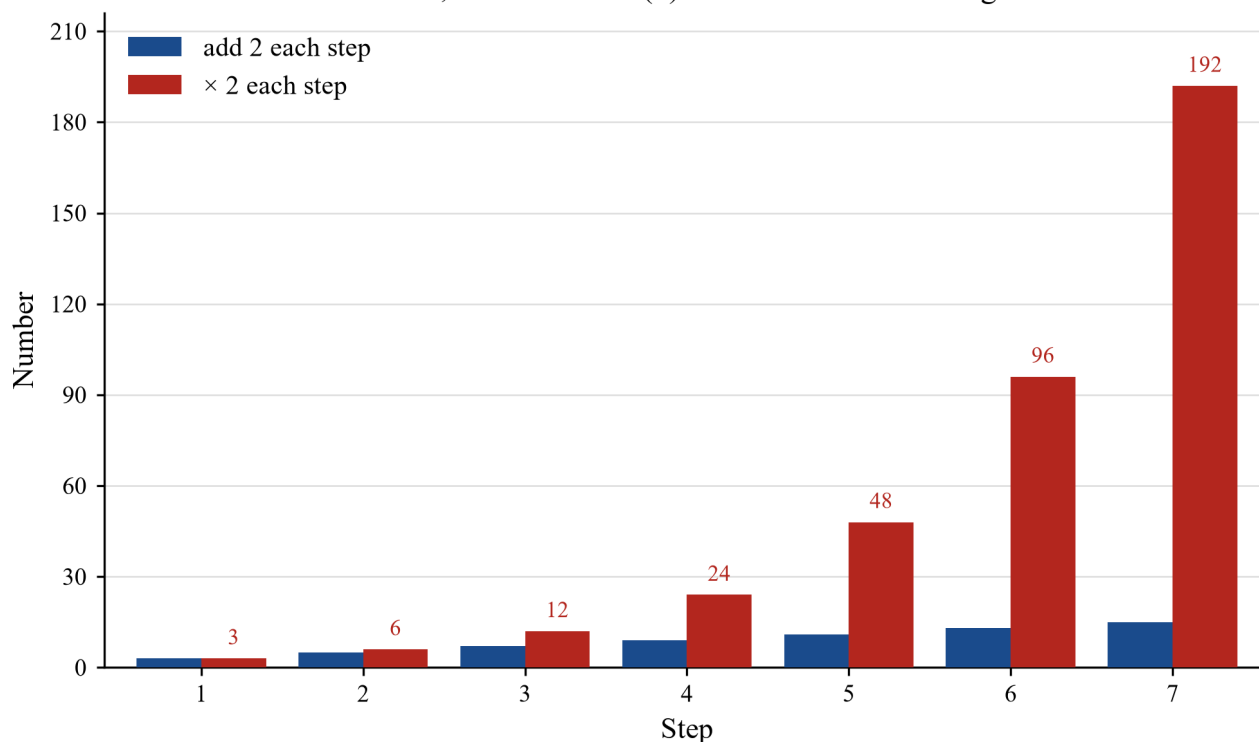


Two machines side by side. The left machine, “+ 2”, turns 3 into 5 and 8 into 10: an additive relationship, adding the same amount each time. The right machine, “ $\times 2$ ”, turns 3 into 6 and 8 into 16: a multiplicative relationship, multiplying by the same number each time.

The + 2 machine is an **additive** relationship: the output is always 2 more than the input, whatever the input is. The $\times 2$ machine is a **multiplicative** relationship: the output is always 2 times the input. You can tell which is which from the pairs. Under + 2, the **difference** between input and output is always 2 ($5 - 3 = 2$, $10 - 8 = 2$). Under $\times 2$, the difference keeps changing ($6 - 3 = 3$ but $16 - 8 = 8$), while the **times** relationship stays the same. Same start, same-looking rule — and the two behave in different ways. That difference is exactly what shows up in the patterns the machines generate.

How fast does it grow? Run the two machines on their own outputs: start at 3, and feed each new output back in.

Two rules, the same start (3) — how fast does each grow?

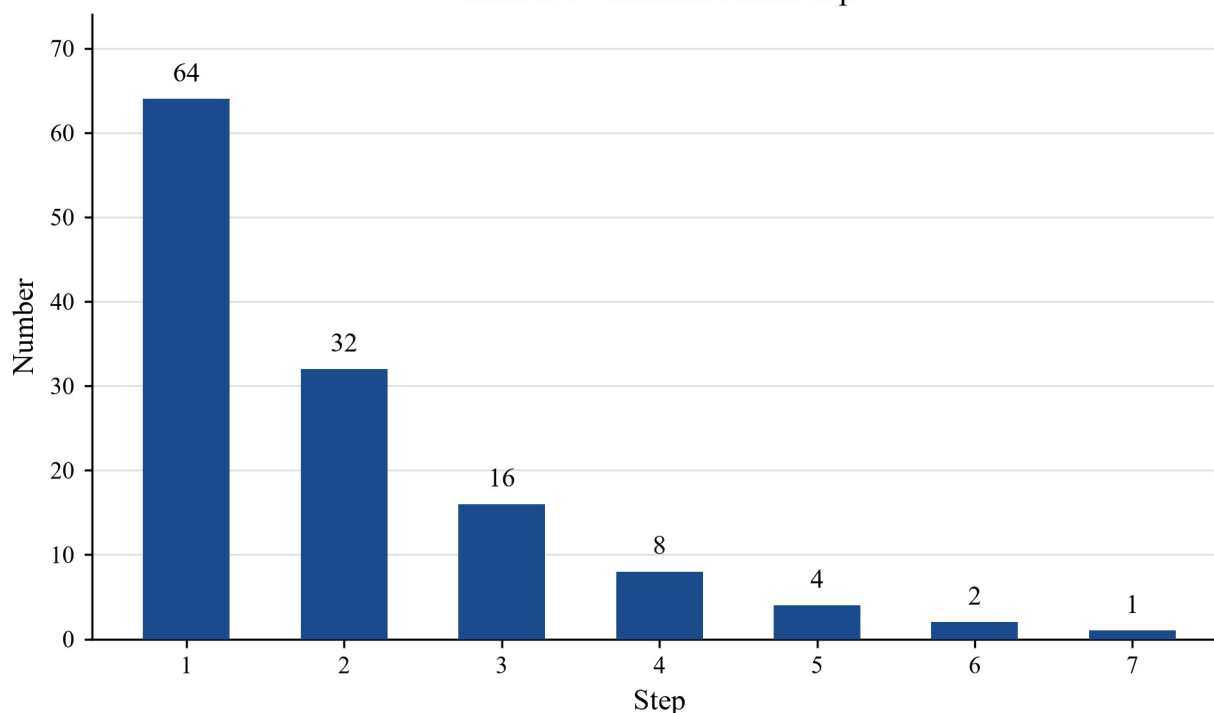


A column graph over steps 1 to 7: “add 2 each step” gives 3, 5, 7, 9, 11, 13, 15 in blue, while “ $\times 2$ each step” gives 3, 6, 12, 24, 48, 96, 192 in red, the red bars climbing away from the blue.

Add 2 gives 3, 5, 7, 9, 11, 13, 15 — steady, like the equal jumps on the number line. Multiply by 2 gives 3, 6, 12, 24, 48, 96, 192. At step 1 the two are level; from step 2 the $\times 2$ sequence is ahead, and the gap gets wider every step. **Explain:** adding 2 puts on the same 2 each time, but doubling puts on as much as the number already is (12 becomes 24 by putting on 12), and that amount keeps growing. That is why multiplicative growth outruns additive growth — the emerging pattern in the graph is not a coincidence, it is the rule made visible.

Patterns can shrink too. Halving each step from 64:

Start at 64 and halve each step



Each number is half the one before it — the pattern shrinks, and the drops get smaller each time.

A column graph of the sequence 64, 32, 16, 8, 4, 2, 1 over steps 1 to 7, the bars shrinking, with each drop half the one before.

The sequence is 64, 32, 16, 8, 4, 2, 1. It shrinks, and the shrinking itself shrinks: the first drop is $64 - 32 = 32$, the next is $32 - 16 = 16$, then 8, 4, 2, 1 — each drop half the drop before. **Identify** (the bars get shorter), **interpret** (the drops get smaller too, so the sequence gets closer and closer to zero without jumping past it), **explain** (halving a smaller number removes less). If you kept halving past 1 you would meet the fractions $\frac{1}{2}, \frac{1}{4}, \dots$ — sequences do not have to stop at whole numbers.

Sequences with rational numbers. A rule can add fractions or decimals just as well as whole numbers. Start at $\frac{1}{4}$ and add $\frac{1}{4}$ each step: $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, \dots$ **Identify:** the terms climb a quarter at a time. **Interpret:** every fourth term lands on a whole number. **Explain:** four quarters make one whole, so four steps of $\frac{1}{4}$ add up to 1. The pattern comes straight from the rule and from what a quarter is.

A digital tool does the repetition for you. A spreadsheet repeats a rule down a column as many times as you like — that is the repetition step of the algorithm, done by the machine part, while you write the rule. The rule is still yours.

	A	B
1	Step	Number
2	1	3
3	2	6
4	3	12
5	4	24
6	5	48
7	6	96
8	7	192

Column B's rule:
start at 3, then each
number is $2 \times$ the
one above it.
Fill the rule down.

The spreadsheet repeats the rule for you — the machine part.

A spreadsheet with column A headed “Step” and column B headed “Number”: B starts at 3 and each cell below is 2 times the one above, giving 3, 6, 12, 24, 48, 96, 192, with a note that the rule is filled down the column.

Column B starts at 3, and each number below is $2 \times$ the one above: 3, 6, 12, 24, 48, 96, 192. Filling the rule down seven rows is seven passes around the loop, done in a moment. That is what makes digital tools useful here (as the elaborations suggest): long sequences and honest pattern-spotting, because no copying mistake creeps in. The thinking — the rule, and the identify–interpret–explain of the pattern — stays with you.

Words for this subtopic, with what they mean here:

Term	Meaning here
algorithm	a list of steps, in an exact order, that gets a job done
generate	to make; the algorithm generates the numbers its rule produces
sequence	numbers in the order they come out of the rule
term	one number in a sequence
set	a collection of numbers where order does not matter, written in brackets like $\{2, 4, 6\}$
rule	the operation the algorithm does again and again, like “add 3” or “ $\times 2$ ”
decision	a yes–no question inside an algorithm that chooses the next step
branch	the split into two paths at a decision
trace	to follow an algorithm step by step with a written record
input / output	the number that goes into a machine, and the number that comes out
additive relationship	a rule that adds the same amount every time, like $+2$
multiplicative relationship	a rule that multiplies by the same number every time, like $\times 2$

Term	Meaning here
multiple	a number that is a whole number of times another: 12 is a multiple of 4

Worked examples

Example 1 — with help

Follow the algorithm and write the sequence. Then identify, interpret and explain the pattern.

Step 1. Start with 5. *Step 2.* Add 3. *Step 3.* Write down the new number. *Step 4.* Repeat steps 2 and 3 four more times.

Working	Comment
Start: 5.	Step 1 gives the first term.
$5 + 3 = 8$; $8 + 3 = 11$; $11 + 3 = 14$; $14 + 3 = 17$.	Step 4 says do “add 3 and write down” four more times — the repetition.
Sequence: 5, 8, 11, 14, 17.	Five terms: the start plus four more.
Identify: each term is 3 more than the one before.	The three jobs, in order, each checked against the rule.
Interpret: the next term would be $17 + 3 = 20$. Explain: the rule adds 3 on every pass, so the gap is always 3.	

Example 2 — with help

A machine’s rule is $\times 3$. Complete the table, then say which kind of relationship it is and how you know.

Working	Comment
$2 \times 3 = 6$; $4 \times 3 = 12$; $6 \times 3 = 18$; $9 \times 3 = 27$.	The rule acts on each input: output = input $\times 3$.
Pairs: (2, 6), (4, 12), (6, 18), (9, 27).	Every output is 3 times its input.
Multiplicative. The times link is the same for every pair, while the differences change: $6 - 2 = 4$ but $12 - 4 = 8$.	An additive rule would keep the same difference; this rule keeps multiplying by the same number.

Example 3 — with help

Trace the branching algorithm from the Theory — *even: halve it; odd: add 1; stop at 1* — starting at 12. Write the sequence and explain its shape.

Working	Comment
12 even $\rightarrow 6$; 6 even $\rightarrow 3$; 3 odd $\rightarrow 4$; 4 even $\rightarrow 2$; 2 even $\rightarrow 1$; stop.	Each pass: ask the decision, do the path it names, write the number.
Sequence: 12, 6, 3, 4, 2, 1.	The start plus five more terms, then the stop.
Identify: it falls, falls, rises once, then falls to 1.	The branch is why the pattern is not a straight climb or slide.
Interpret: the single rise is where 3 was odd. Explain: odd numbers get 1 added (a rise), even numbers get halved (a fall), and adding 1 to an odd number makes an even one — so every rise is followed by a fall.	

Example 4 — with help

An algorithm checks each number from 1 to 30: *is it a multiple of 4?* If yes, write it into the set. Write the set, then identify a pattern in it and explain it.

Working	Comment
$4 \times 1 = 4$ up to $4 \times 7 = 28$; $4 \times 8 = 32$ is past 30.	The test passes exactly at the multiples of 4.
Set: {4, 8, 12, 16, 20, 24, 28}.	Seven numbers; each written once; order on the page does not matter.

Working	Comment
Identify: read in size order, the gap is 4 each time. Explain: consecutive multiples of 4 differ by one more lot of 4.	A set has patterns too — list it in size order to see them.

Example 5 — on your own

Start at 64 and halve each step until you reach 1. Write the sequence. Identify, interpret and explain the pattern in the drops.

Halving gives 64, 32, 16, 8, 4, 2, 1. The drops are $64 - 32 = 32$, then 16, 8, 4, 2, 1: each drop is **half the drop before**. Interpret: the sequence shrinks by smaller and smaller amounts, crowding in towards zero. Explain: halving a number removes half of it, and half of a smaller number is a smaller removal.

Example 6 — on your own (digital-tool path)

In a spreadsheet, column B starts at 3 and each number is $2 \times$ the one above. Fill seven rows, write the sequence, and explain why it grows faster and faster.

The rows give 3, 6, 12, 24, 48, 96, 192. Each step puts on as much as the number already is — doubling 12 puts on 12, doubling 96 puts on 96 — so the amount added grows with the sequence itself. That is why the later steps climb so much harder than the first ones: a multiplicative rule adds more each time, not the same.

Example 7 — on your own

Two algorithms start at 3. One adds 2 each step; the other multiplies by 2 each step. Write both sequences to step 7 and find the first step where the $\times 2$ sequence is larger. Explain, from the rules, why the gap keeps widening.

Add 2: 3, 5, 7, 9, 11, 13, 15. Multiply by 2: 3, 6, 12, 24, 48, 96, 192. At step 1 they are equal; at step 2 the $\times 2$ sequence is already ahead ($6 > 5$), and from there the gap widens every step (7 vs 12, 9 vs 24, ...). **Why:** the additive rule puts on a fixed 2 each step, while the doubling rule puts on the whole current number each step — and that number keeps growing. A growing addition beats a fixed addition every time.

Example 8 — on your own

Design an algorithm with a **decision** that generates the set of odd numbers below 20, and trace it far enough to show the first four numbers of the set.

One design: *Step 1.* Start with 1. *Step 2.* Write the number into the set. *Step 3.* Is the number 19? Yes: stop. No: add 2 and go back to step 2. Tracing: 1 is written, $1 \neq 19$ so add 2; 3 is written, add 2; 5 is written, add 2; 7 is written. First four: 1, 3, 5, 7, and running to the stop gives $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$. The decision (*is it 19?*) is what stops the loop at the right place — a repeating algorithm without a stopping decision would never stop. (Checking *is it odd?* for every number 1–20 and writing the passes is another correct design.)

Practice questions

With help

Q1. Follow the algorithm: start with 7; add 6; write down the new number; repeat the adding and writing three more times. Write the first five terms, then identify the pattern in one sentence.

Q2. A machine's rule is $+5$. Find the outputs for inputs 4, 9, 12, 20. Is the relationship additive or multiplicative, and what stays the same for every pair?

Q3. A machine's rule is $\times 4$. Find the outputs for inputs 3, 5, 7, 9. What stays the same for every pair?

Q4. A machine gives these pairs: (2, 6), (5, 15), (9, 27). Find its rule, and say whether the relationship is additive or multiplicative with a checking sentence.

Q5. A machine gives these pairs: $(4, 7)$, $(10, 13)$, $(21, 24)$. Find its rule, and check it on all three pairs.

Q6. Trace the branching algorithm *even: halve it; odd: add 1; stop at 1* starting at 9, using a trace table like the one in the Theory. Write the sequence.

Q7. An algorithm checks each number from 1 to 40 and writes down every multiple of 5 into a set. Write the set.

Q8. On a number line, start at 2 and jump $+5$ four times. List the five landings, and say what the jumps look like on the line.

Q9. Start at $\frac{1}{2}$ and add $\frac{1}{2}$ each step. Write the first six terms, and interpret the pattern: which terms are whole numbers, and why do they land where they do?

Q10. Start at 32 and halve each step until you reach 1. Write the sequence and identify the pattern in the drops.

On your own

Q11. Design an algorithm (steps and a decision) that generates the set $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$. Write the steps clearly, then trace the first three passes.

Q12. Trace the branching algorithm *even: halve it; odd: add 1; stop at 1* starting at 15. Write the sequence, and explain why the first step is a rise.

Q13. A machine gives these pairs: $(3, 11)$, $(6, 20)$, $(9, 29)$. Its rule does two things, in order. Find the rule and check it on all three pairs.

Q14. Two algorithms start at 1: one adds 5 each step, one multiplies by 2 each step. Write both sequences to step 6. At which step does the $\times 2$ sequence first become larger? Explain from the rules why it stays larger after that.

Q15. In a spreadsheet, column B starts at 4 and each number is 7 more than the one above. Fill eight rows and write the sequence. Identify a pattern in the **last digits** of the terms, and explain it.

Q16. Start at 0.6 and add 0.3 each step. Write the first six terms. Interpret the pattern: when do the terms pass whole numbers?

Q17. An algorithm checks each number from 1 to 50: *is it a multiple of 6?* Write the set it makes, and explain the gap between the numbers of the set.

Q18. Explain, in your own words, why a rule that adds the same amount each step gives equal jumps on a number line, while a rule that multiplies by the same number each step does not. Use the sequences 3, 7, 11, 15, 19 and 3, 6, 12, 24, 48 in your answer.

Q19. Design an algorithm with a decision that generates the set $\{1, 2, 4, 8, 16\}$. Write the steps, and say what the decision is for.

Q20. Always, sometimes or never? "*Halving three times from an even starting number gives whole numbers all the way.*" Give one example each way to back your answer.

Q21. A sequence begins 2, 6, 18, 54. Identify the rule, write the next two terms, and explain why this sequence grows even faster than the $\times 2$ sequence from 3.

Q22. Make your own. Write an algorithm with a repetition **and** a decision. Trace it, write the sequence or set it generates, and give one identify–interpret–explain trio for a pattern in it.

Q23. A sequence comes out of a machine: 4, 9, 14, 19, 24. Find the rule, say whether it is additive or multiplicative with a reason, and write the next two terms.

Q24. An algorithm checks each number from 1 to 30 with two tests: *is it a multiple of 2?* and *is it NOT a multiple of 4?* A number is written into the set only if it passes both tests. Write the set, and describe the pattern in it.

Q25. Start at 5 and take away $\frac{1}{2}$ each step. Write the first six terms, and interpret the pattern: which terms are whole numbers?

Q26. Find the mistake. Tom traced *even: halve it; odd: add 1; stop at 1* starting at 14 and wrote 14, 7, 8, 4, 2, 0. What should the last term be, and why?

Answers

Q1. 7, 13, 19, 25, 31; each term is 6 more than the one before. **Q2.** 9, 14, 17, 25; additive — the difference is always 5. **Q3.** 12, 20, 28, 36; the output is always 4 times the input. **Q4.** $\times 3$; multiplicative — $2 \times 3 = 6$, $5 \times 3 = 15$, $9 \times 3 = 27$ (the differences, 4, 10, 18, are not the same). **Q5.** $+3$; $4 + 3 = 7$, $10 + 3 = 13$, $21 + 3 = 24$. **Q6.** 9, 10, 5, 6, 3, 4, 2, 1. **Q7.** {5, 10, 15, 20, 25, 30, 35, 40}. **Q8.** 2, 7, 12, 17, 22; four equal jumps of 5, like the +4 line in the Theory. **Q9.** $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$; every second term is a whole number, because two halves make 1. **Q10.** 32, 16, 8, 4, 2, 1; the drops are 16, 8, 4, 2, 1 — each half the one before. **Q11.** e.g. *start with 2; write it; is it 20?* yes: stop; no: add 2 and go back — first three passes write 2, 4, 6. **Q12.** 15, 16, 8, 4, 2, 1; 15 is odd, so the first decision sends it down the “add 1” path. **Q13.** $\times 3$ then $+2$; $3 \times 3 + 2 = 11$, $6 \times 3 + 2 = 20$, $9 \times 3 + 2 = 29$. **Q14.** Add 5: 1, 6, 11, 16, 21, 26; $\times 2$: 1, 2, 4, 8, 16, 32; first larger at step 6 ($32 > 26$; at step 5, $16 < 21$). **Q15.** 4, 11, 18, 25, 32, 39, 46, 53; last digits run 4, 1, 8, 5, 2, 9, 6, 3 — adding 7 moves the last digit along that cycle. **Q16.** 0.6, 0.9, 1.2, 1.5, 1.8, 2.1; the terms pass 1 at 1.2 and pass 2 at 2.1. **Q17.** {6, 12, 18, 24, 30, 36, 42, 48}; the gap is 6, one more lot of 6 each time. **Q18.** Adding the same amount puts on a fixed jump, so the jumps match; multiplying adds as much as the number already is, so each jump is larger than the last. **Q19.** e.g. *start with 1; write it; is it 16?* yes: stop; no: $\times 2$ and go back — the decision stops the loop after 16. **Q20.** Sometimes: $8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ works; $12 \rightarrow 6 \rightarrow 3 \rightarrow 1\frac{1}{2}$ does not. **Q21.** $\times 3$; next terms 162 and 486; tripling puts on **two** more lots of the number each step, while doubling puts on one. **Q22.** Answers vary; a correct one has ordered steps, a yes–no decision, a true trace, and a pattern named, read and reasoned from the rule. **Q23.** $+5$; additive, because the gap is always 5; next terms 29 and 34. **Q24.** {2, 6, 10, 14, 18, 22, 26, 30}; the gap is 4 — two more than a multiple of 4 each time. **Q25.** $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, 2\frac{1}{2}$; every second term is a whole number, because two steps of $\frac{1}{2}$ take away 1. **Q26.** The last term should be 1: halving 2 gives 1, and 1 is written down before the stop test; 0 would need $2 - 2$, which is not the rule.

Worked solutions

Q1. Start 7; add 6 each time: 7, 13, 19, 25, 31. Identify: each term is 6 more than the one before — the rule adds 6 on every pass, so that is what the gaps must be.

Q2. $4 + 5 = 9$; $9 + 5 = 14$; $12 + 5 = 17$; $20 + 5 = 25$. Additive: for every pair the difference is 5 ($9 - 4 = 5$, $14 - 9 = 5$, $17 - 12 = 5$, $25 - 20 = 5$).

Q3. $3 \times 4 = 12$; $5 \times 4 = 20$; $7 \times 4 = 28$; $9 \times 4 = 36$. Multiplicative: every output is 4 times its input, while the differences change ($12 - 3 = 9$ but $20 - 5 = 15$).

Q4. $6 \div 2 = 3$, $15 \div 5 = 3$, $27 \div 9 = 3$: the rule is $\times 3$, multiplicative. Checking the additive idea fails: the differences are 4, 10, 18 — not the same — so an add-the-same-amount rule ($+\square$) cannot fit.

Q5. $7 - 4 = 3$, $13 - 10 = 3$, $24 - 21 = 3$: the rule is $+3$. Check: $4 + 3 = 7$ ✓; $10 + 3 = 13$ ✓; $21 + 3 = 24$ ✓.

Q6. 9 odd $\rightarrow$ add 1: 10. 10 even $\rightarrow$ halve: 5. 5 odd $\rightarrow$ 6. 6 even $\rightarrow$ 3. 3 odd $\rightarrow$ 4. 4 even $\rightarrow$ 2. 2 even $\rightarrow$ 1, stop. Sequence: 9, 10, 5, 6, 3, 4, 2, 1. (Trace table: one row per pass, columns number / even? / decision / next number.)

Q7. The multiples of 5 up to 40: 5, 10, 15, 20, 25, 30, 35, 40. Set: {5, 10, 15, 20, 25, 30, 35, 40}.

Q8. 2, then $+5$ jumps: 7, 12, 17, 22. Landings 2, 7, 12, 17, 22 — five marks with four equal jumps of 5 between them, exactly like the $+4$ line in the Theory.

Q9. $\frac{1}{2}$, $\frac{1}{2} + \frac{1}{2} = 1$, $1 + \frac{1}{2} = 1\frac{1}{2}$, 2, $2 + \frac{1}{2} = 2\frac{1}{2}$, 3. Terms: $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3. Every second term is a whole number: two steps of $\frac{1}{2}$ add 1, so the whole numbers land at terms 2, 4, 6.

Q10. 32, 16, 8, 4, 2, 1. Drops: $32 - 16 = 16$, $16 - 8 = 8$, $8 - 4 = 4$, $4 - 2 = 2$, $2 - 1 = 1$ — each drop half the drop before, because halving removes half of what is there.

Q11. One design: *Step 1.* Start with 2. *Step 2.* Write the number into the set. *Step 3.* Is it 20? Yes: stop. No: add 2 and go back to step 2. Trace of the first three passes: write 2 (not 20, add 2); write 4 (add 2); write 6. The decision is the

stopping test; running on gives the full set of even numbers to 20. (Checking *is it even?* on 1–20 and writing the passes is equally correct.)

Q12. 15 is odd, so the first decision takes the “add 1” path: 16 — that is the rise. Then $16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$, all halvings, and stop. Sequence: 15, 16, 8, 4, 2, 1. The first step is a rise because 15 is odd, and the odd path adds.

Q13. Try $\times 3$ first: $3 \times 3 = 9$, but the output is 11 — two more. Check the rule “ $\times 3$, then $+2$ ”: $3 \times 3 + 2 = 11 \checkmark$; $6 \times 3 + 2 = 20 \checkmark$; $9 \times 3 + 2 = 29 \checkmark$. All three pairs fit, so the machine multiplies by 3 and then adds 2.

Q14. Add 5: 1, 6, 11, 16, 21, 26. Multiply by 2: 1, 2, 4, 8, 16, 32. Step by step: $1=1$, $6>2$, $11>4$, $16>8$, $21>16$, and at step 6, $26<32$ — first larger at **step 6**. After that it stays larger: the $\times 2$ step adds the whole current number (at least 32 from then on), while the add rule still adds only 5, so the gap can only widen.

Q15. 4, then $+7$ down the column: 4, 11, 18, 25, 32, 39, 46, 53. Last digits: 4, 1, 8, 5, 2, 9, 6, 3. Identify: each last digit is 3 less than the one before, wrapping round when it goes below 0 (so 1 is followed by 8, since $1 + 10 - 3 = 8$). Explain: adding 7 is adding 10 and taking away 3, so the last digit drops by 3 each row, with one more ten carried when it wraps.

Q16. 0.6, 0.9, 1.2, 1.5, 1.8, 2.1. The terms climb three-tenths at a time; they pass the whole number 1 at the third term (1.2) and pass 2 at term 6 (2.1), because $0.6 + 0.3 + 0.3 = 1.2$ and three more steps add 0.9.

Q17. Multiples of 6 up to 50: $6 \times 1 = 6$ through $6 \times 8 = 48$ ($6 \times 9 = 54$ is too big). Set: {6, 12, 18, 24, 30, 36, 42, 48}. The gap is 6: each member is one more lot of 6 than the last.

Q18. 3, 7, 11, 15, 19 comes from adding 4: every jump puts on the same 4, so on a number line all the jumps are the same length. 3, 6, 12, 24, 48 comes from $\times 2$: doubling 3 puts on 3, doubling 6 puts on 6, doubling 12 puts on 12 — the amount put on is the number itself, and it grows, so the jumps get longer and longer. Adding the same amount gives equal jumps; multiplying by the same number gives growing jumps.

Q19. One design: *Step 1.* Start with 1. *Step 2.* Write the number into the set. *Step 3.* Is it 16? Yes: stop. No: $\times 2$ and go back to step 2. Trace: 1, 2, 4, 8, 16, stop. The decision is the stopping test — without it the doubling would run past 16 and never stop.

Q20. Sometimes. Example where it works: start 8: $8 \rightarrow 4 \rightarrow 2 \rightarrow 1$, all whole. Example where it fails: start 12: $12 \rightarrow 6 \rightarrow 3 \rightarrow 1\frac{1}{2}$, and $1\frac{1}{2}$ is not a whole number (halving the odd number 3 leaves a half). Whether it works depends on the start, so “always” and “never” are both wrong.

Q21. $6 \div 2 = 3$, $18 \div 6 = 3$, $54 \div 18 = 3$: the rule is $\times 3$. Next terms: $54 \times 3 = 162$, $162 \times 3 = 486$. It grows faster than the $\times 2$ sequence because tripling puts on **two** more lots of the current number each step (tripling 54 adds 108), while doubling puts on only one lot of it.

Q22. Answers vary. A full-mark attempt shows: steps in an exact order; a repetition (steps done again); a yes–no decision; a trace that follows the steps truthfully; the sequence or set written down; and three sentences — a pattern named (identify), read (interpret), and reasoned from the rule (explain). For example: *start with 100; is it more than 1? yes: write it and halve it, go back; no: stop* generates 100, 50, 25, $12\frac{1}{2}$, $6\frac{1}{4}$, $3\frac{1}{8}$, ... down towards 1, and the identify–interpret–explain trio is the shrinking–drops one from the Theory.

Q23. The gaps: $9 - 4 = 5$, $14 - 9 = 5$, $19 - 14 = 5$, $24 - 19 = 5$ — always 5, so the rule is $+5$ and the relationship is additive (a multiplicative rule would give growing gaps). Next terms: $24 + 5 = 29$ and $29 + 5 = 34$.

Q24. Pass both tests means: even, but not a multiple of 4. Checking the evens up to 30: 2 yes, 4 no, 6 yes, 8 no, 10 yes, 12 no, 14 yes, 16 no, 18 yes, 20 no, 22 yes, 24 no, 26 yes, 28 no, 30 yes. Set: {2, 6, 10, 14, 18, 22, 26, 30}. Pattern: the gap is 4 each time — each member is 2 more than a multiple of 4 ($4 \times 0 + 2$, $4 \times 1 + 2$, ...), which is exactly what “two more than a multiple of 4” looks like.

Q25. 5, $4\frac{1}{2}$, 4, $3\frac{1}{2}$, 3, $2\frac{1}{2}$. Every second term is a whole number: two steps of $\frac{1}{2}$ take away 1, so the whole numbers come at terms 1, 3, 5 and the halves at terms 2, 4, 6.

Q26. Halving 2 gives 1, not 0 — the rule halves, it does not take away 2. And 1 is written down as the last term, because the stop test happens after the number is written. The true trace is 14, 7, 8, 4, 2, 1: $14 \rightarrow 7$ (halve), $7 \rightarrow 8$ (add 1), $8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ (halve, halve, halve), stop at 1.

Test

Mathematics Level 6 — Chapter test T4: Patterns, equations and algorithms

Chapter 4 · Algebra · Victorian Curriculum 2.0

Codes assessed: VC2M6A01 (4A Growing patterns and the rules that generate them) · VC2M6A02 (4B Unknown values in equations with brackets and mixed operations) · VC2M6A03 (4C Algorithms that generate sets of numbers)

Name: _____ Date: _____

This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 6 achievement standard, not against a mark on this paper.

Marks: 15

Time: Aim for about 20 minutes: about 5 minutes to read through the paper, then about 1 minute per mark. This is a classroom check, not an exam. There is no strict time limit, and your teacher will let you finish.

Things you can use: grid paper (for Question 1) and a pencil.

Calculators: you may use a calculator if you want one. No question in this test needs one.

Attempt every question, and show your working.

Q1. Growing patterns. Each number or picture in a pattern is a **term**, and the words that say how the pattern is made are its **rule**. [5 marks]

(a) Circle the letter of the right answer. [1 mark]

The number pattern 0.5, 1.5, 2.5, 3.5, ... is made by one of these rules. Which one?

A	B	C	D
Add 1 each time	Add 1.5 each time	Double each time	Multiply the term number by 4

(b) A tile pattern grows on grid paper. Term 1 is 1 tile. Term 2 is a cross: 1 tile in the centre, with 1 tile on each of its 4 edges (5 tiles altogether). Term 3 grows each arm of the cross by 1 tile (9 tiles altogether). [2 marks]

(c) On grid paper, draw term 4, and write how many tiles it has. [1 mark]

(ii) Write the next-term rule in words. [1 mark]

(iii) An **any-term rule** builds a term straight from its term number. The any-term rule of the cross pattern is *multiply the term number by 4, then take away 3*. [2 marks]

(iv) Check the rule on term 3. [1 mark]

(v) Use the rule to find the number of tiles in term 10. [1 mark]

Q2. Equations and the order of operations. An **equation** is a number sentence whose two sides have the same value. The hidden number, shown here as $\square$ or $\triangle$, is the **unknown**. [5 marks]

(a) Work out each sentence using the agreed **order of operations**: brackets first, then $\times$ and $\div$ from left to right, then $+$ and $-$ from left to right. [2 marks]

(b) $8 + 6 \times 3$ [1 mark]

(ii) $(8 + 6) \times 3$ [1 mark]

(b) Parts (a)(i) and (a)(ii) use the same numbers and the same operations, and they give different answers. Explain, in one sentence, why an agreed order of operations is needed. [1 mark]

(c) Find the unknown. Work back and undo each step. [1 mark]

$$(\square + 4) \times 6 = 54$$

- (d) Find **every** pair of natural numbers that makes this equation true. (The **natural numbers** are the counting numbers 1, 2, 3, 4, ...) [1 mark]

$$4 \times \triangle + \square = 10$$

Q3. Algorithms. An **algorithm** is a list of steps, in an exact order, that gets a job done. A **decision** is a yes–no question inside an algorithm that chooses the next step. [5 marks]

- (a) This branching algorithm works on one number at a time: *if the number is even, halve it; if it is odd, add 1; stop when you reach 1*. To **trace** an algorithm is to follow it step by step with a written record. Complete the trace of the algorithm starting at 24, and write the **sequence** it generates — the list of numbers written down, in the order they come out. [2 marks]

Pass	Number	Even or odd?	Step taken	Next number
1	24	even	halve it	12
2	12	even	halve it	6
3				
4				
5				
6				

- (i) Complete the four empty rows of the trace table. [1 mark]
- (ii) Write the full sequence. [1 mark]
- (b) A **machine** is an algorithm whose rule acts on one number at a time: an **input** goes in, and an **output** comes out. This machine's rule is $\times 6$. [2 marks]
- (c) Find the outputs for the inputs 3, 7 and 12. [1 mark]
- (ii) Is this an **additive** relationship (the same amount added each time) or a **multiplicative** relationship (multiplied by the same number each time)? What stays the same for every pair? [1 mark]
- (iii) An algorithm checks each number from 1 to 40: *is it a multiple of 8?* The numbers that pass are written into a **set** — a collection of numbers where order does not matter, and each number is written once. Write the set, and identify and explain the pattern in it. [1 mark]

Total: 15 marks

Solutions

Teacher-facing. Test T4 is a classroom check, not an exam: a 20-minute guidance duration (about 1 minute per mark plus 5 minutes reading), grid paper permitted (the §7 table row for T4), used in Q1(b)(i). No Chapter 4 content description says “without a calculator” (D8) — digital tools are a permission only in this chapter — so calculators are permitted, though no question needs one. All numbers in this paper are natural numbers except the tenths in Q1(a); nothing is negative, there are no pronumerals (unknowns are $\square$ and $\triangle$ only), no multiplying or dividing fractions, and no index notation (D9).

Marking guide

Administration (per TOC §7). Guidance duration 20 minutes: about 5 minutes reading plus about 1 minute per mark. Equipment: grid paper (verbatim from the §7 table row for T4), needed only for Q1(b)(i). Calculator status follows D8: no Chapter 4 CD says “without a calculator”, and the digital tool is named only in elaborations in this chapter (VC2M6A01 . E02; VC2M6A03’s elaborations), so it is a permission — calculators are permitted on this paper, but no question needs one, since every item asks for the rule, the order, the undoing or the trace, which is what makes keying it in useless.

Marks by item and CD. Only Chapter 4 CDs are assessed (§4). Each of the three subtopics carries exactly 5 marks (§7 marking rule: 5 marks per subtopic).

Item	CD code	Subtopic	Type	Marks
Q1	VC2M6A01	4A	MCQ (A–D) + short answer / worked response	5
Q2	VC2M6A02	4B	Short answer / worked response	5
Q3	VC2M6A03	4C	Short answer / worked response	5
Total				15

MCQ total: 1 mark of 15 — sparingly per §7, used only because recognising the rule that generates a given list of terms is genuinely a selection skill.

Rubrics, by CD. The achievement-standard extract for each CD is quoted verbatim from `maths_6.json`. Rubrics score the method and the explanation as well as the answer (§7).

VC2M6A01 (4A) — 5 marks (Q1 5). “*They identify and explain rules used to create growing patterns.*” Observable actions:

- recognises the rule that generates 0.5, 1.5, 2.5, 3.5, ... from four options — A, add 1 each time (1 mark);
- draws term 4 of the cross pattern on grid paper and counts it — 13 tiles (1 mark);
- states the next-term rule in words — start at 1 and add 4 each time (1 mark);
- checks the any-term rule on term 3 — $4 \times 3 - 3 = 9$, matching the 9 tiles (1 mark);
- uses the any-term rule to jump to term 10 — 37 tiles (1 mark).

VC2M6A02 (4B) — 5 marks (Q2 5). “*Students find unknown values in numerical equations involving combinations of arithmetic operations.*” Observable actions:

- applies the agreed order with $\times$ before $+$ — 26 (1 mark);
- applies the agreed order with brackets first — 42 (1 mark);
- explains why an agreed order of operations is needed — so every number sentence has just one answer (1 mark);
- finds the unknown in a two-step equation with brackets by undoing — $\square = 5$ (1 mark);
- finds every pair of natural-number unknowns — $(\triangle, \square) = (1, 6)$ and $(2, 2)$ (1 mark).

VC2M6A03 (4C) — 5 marks (Q3 5). “*They design and use algorithms to generate sets of numbers, using a rule.*” Observable actions:

- completes the trace of a branching algorithm — rows for $6 \rightarrow 3$, $3 \rightarrow 4$, $4 \rightarrow 2$ and $2 \rightarrow 1$ (1 mark);
- writes the sequence the algorithm generates — 24, 12, 6, 3, 4, 2, 1 (1 mark);
- runs a machine’s rule on three inputs — 18, 42, 72 (1 mark);
- classifies the relationship and says what stays the same — multiplicative; the output is always 6 times the input (1 mark);

- writes the generated set and identifies and explains the pattern in it — $\{8, 16, 24, 32, 40\}$; the gap is 8, one more lot of 8 each time (1 mark).

MCQ key. Q1(a) A. One mark for the correct letter; no part marks.

General marking notes.

- Q1(a): A. Checking each option against the list is the intended method; see the worked solution.
- Q1(b)(i): the mark requires the drawing and the count — a cross of 13 tiles: the centre tile with four arms of length 3, in any orientation. A count of 13 with no drawing earns 0.
- Q1(b)(ii): accept *start at 1 and add 4 each time* or *add 4 each time*.
- Q1(c)(i): the mark requires substituting term 3 ($4 \times 3 - 3 = 9$) and stating that it agrees with the 9 tiles of term 3.
- Q1(c)(ii): 37. The intended route is substituting term 10 into the rule; a correct count-on from a given term (adding 4 each time) also earns the mark.
- Q2(a): each part needs the correct first step shown ($6 \times 3 = 18$ in (i); $8 + 6 = 14$ in (ii)) as well as the final value.
- Q2(b): the mark is for the idea that, without a shared agreement, the same number sentence could give different answers — the agreement gives every sentence just one answer. One clear sentence is enough.
- Q2(c): working is required. Accept undoing in reverse order ($54 \div 6 = 9$, then $9 - 4 = 5$) or a number-fact route ($6 \times 9 = 54$, so $\square + 4 = 9$). A bare answer with no working earns 0. A check by substituting back is encouraged but not required for the mark.
- Q2(d): both pairs are needed for the mark. Accept any clear layout — a list or a table — and accept any indication that $\triangle = 3$ fails ($10 - 12$ is less than 0, not a natural number).
- Q3(a)(i): each completed row must show the decision taken (halve it / add 1) and the next number; all four rows correct for the mark. Accept consistent follow-through from an incorrect earlier row.
- Q3(a)(ii): the sequence must start at 24 and end at 1 — 24, 12, 6, 3, 4, 2,
 1. Accept consistent follow-through.
- Q3(b)(i): all three outputs (18, 42, 72) for the mark.
- Q3(b)(ii): the mark needs the classification (multiplicative) and what stays the same — the output is always 6 times the input. Accept “the times link is always 6” or equivalent wording.
- Q3(c): the mark needs the complete set, the pattern identified and the pattern explained — gap of 8, each member one more lot of 8. Accept a list without set brackets.

Answers

Q1. (a) A. (b) (i) 13 tiles; (ii) start at 1 and add 4 each time. (c) (i) $4 \times 3 - 3 = 9$, which matches the 9 tiles of term 3; (ii) 37 tiles.

Q2. (a) (i) 26; (ii) 42. (b) Without an agreed order, the same number sentence could have different answers; the agreement gives every sentence just one answer. (c) $\square = 5$. (d) $(\triangle, \square) = (1, 6)$ and $(2, 2)$ — two pairs.

Q3. (a) (i) Row 3: 6, even, halve it, 3; row 4: 3, odd, add 1, 4; row 5: 4, even, halve it, 2; row 6: 2, even, halve it, 1. (ii) 24, 12, 6, 3, 4, 2, 1. (b) (i) 18, 42, 72; (ii) multiplicative — the output is always 6 times the input. (c) {8, 16, 24, 32, 40}; the gap is 8, because each member is one more lot of 8.

All the working

Q1. (a) Check each rule against the list 0.5, 1.5, 2.5, 3.5. Option A: $0.5 + 1 = 1.5$, $1.5 + 1 = 2.5$, $2.5 + 1 = 3.5$ — it works on every step. Option B fails at the first step: $0.5 + 1.5 = 2$, not 1.5. Option C fails: doubling 0.5 gives 1, not 1.5. Option D fails: $4 \times 1 = 4$, not 0.5. $\therefore$ A.

(b)

(i) Each term grows every arm of the cross by 1 tile. Term 3 has arms of length 2, so term 4 has arms of length 3: the centre tile plus $4 \times 3 = 12$ arm tiles, $1 + 12 = 13$ tiles altogether. The drawing on grid paper is a centre tile with 3 tiles stretching out in each of the 4 directions. $\therefore$ 13 tiles.

(ii) The counts run 1, 5, 9, and $5 - 1 = 4$ and $9 - 5 = 4$ — the same change every time. $\therefore$ The next-term rule is *start at 1 and add 4 each time*. Check: $9 + 4 = 13$, the count from part (i). $\checkmark$

(iii)

(i) Term 3 by the rule: $4 \times 3 - 3 = 12 - 3 = 9$, and term 3 in the pattern has 9 tiles — the rule checks. $\therefore 4 \times 3 - 3 = 9 \checkmark$

(iv) Term 10 by the rule: $4 \times 10 - 3 = 40 - 3 = 37$. $\therefore$ 37 tiles.

Q2. (a) (i) Under the agreed order, $\times$ comes before $+$: $6 \times 3 = 18$ first, then $8 + 18 = 26$. $\therefore$ 26.

(ii) Brackets first: $8 + 6 = 14$, then $14 \times 3 = 42$. $\therefore$ 42.

(b) The two parts of (a) show the same numbers and operations giving two different answers (26 and 42), depending on which operation is done first. Without a shared agreement about the order, one number sentence could have more than one answer, and nobody could rely on it — the agreement makes sure every sentence has just one answer. $\therefore$ An agreed order gives every number sentence exactly one answer.

(c) The equation does two things to $\square$: adds 4, then multiplies by 6. Undo in reverse order — the last step done is the first step undone. Undo the $\times 6$: $54 \div 6 = 9$, so $\square + 4 = 9$. Undo the $+ 4$: $9 - 4 = 5$. Check: $(5 + 4) \times 6 = 9 \times 6 = 54 \checkmark \therefore \square = 5$.

(d) Choose values for $\triangle$ in order and work out $\square = 10 - 4 \times \triangle$. $\triangle = 1$: $\square = 10 - 4 = 6$. $\triangle = 2$: $\square = 10 - 8 = 2$. $\triangle = 3$: $\square$ would be $10 - 12$, which is less than 0 and not a natural number, so the pairs stop there. Check: $4 \times 1 + 6 = 4 + 6 = 10 \checkmark$ and $4 \times 2 + 2 = 8 + 2 = 10 \checkmark \therefore$ Two pairs: $(\triangle, \square) = (1, 6)$ and $(2, 2)$.

Q3. (a) (i) Follow the decision at each pass. 6 is even, so halve it: 3. 3 is odd, so add 1: 4. 4 is even, so halve it: 2. 2 is even, so halve it: 1 — and the algorithm stops. The completed rows:

Pass	Number	Even or odd?	Step taken	Next number
3	6	even	halve it	3
4	3	odd	add 1	4
5	4	even	halve it	2
6	2	even	halve it	1

(ii) Reading the Number column and the final next number, in order: $\therefore$ 24, 12, 6, 3, 4, 2, 1.

(b)

(i) The rule acts on each input: $3 \times 6 = 18$, $7 \times 6 = 42$, $12 \times 6 = 72$. $\therefore$ 18, 42, 72.

- (ii) Every output is 6 times its input, while the differences keep changing ($18 - 3 = 15$ but $42 - 7 = 35$) — so the times link, not the difference, is what stays the same. $\therefore$ Multiplicative; the output is always 6 times the input.
- (iii) The multiples of 8 up to 40 are $8 \times 1 = 8$, $8 \times 2 = 16$, $8 \times 3 = 24$, $8 \times 4 = 32$ and $8 \times 5 = 40$ ($8 \times 6 = 48$ is past 40). $\therefore$ The set is $\{8, 16, 24, 32, 40\}$. Read in size order, the gap between members is 8 each time — identify. And that must happen because each member is one more lot of 8 than the one before — explain.