Mathematics Level 6

Chapter 1 — Integers and number properties

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Prime, composite, square and triangular numbers

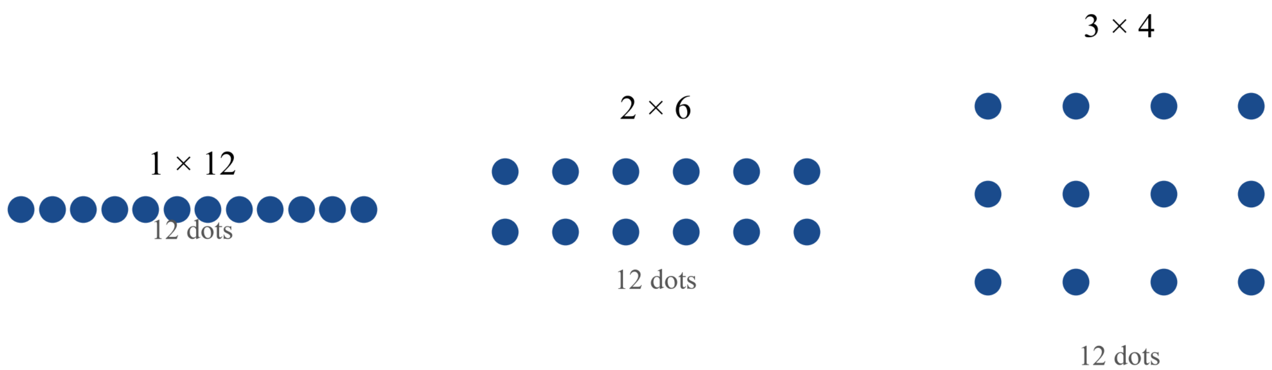
> **Content description `VC2M6N02`:** identify and describe the properties of prime, composite, square and triangular numbers and use these properties to solve problems and simplify calculations

In Year 5 you learned to split counting numbers into factors. This subtopic sorts every counting number into families using that idea — the primes, with exactly two factors, and the composite numbers, with more — and adds two older families you may have met before: the square numbers and the triangular numbers. The point is not just to name the families. It is to use what you know about them: to test numbers quickly, to spot patterns, and to turn hard multiplications into easy ones.

Theory

What you bring from Year 5: factors

A **factor** of a number is a counting number that divides it exactly, leaving no remainder. Factors come in **factor pairs** — pairs that multiply to give the number. The picture below shows 12 dots rearranged as rectangles, one rectangle for each factor pair of 12.



Twelve dots arranged as three rectangles: one row of twelve, two rows of six, and three rows of four, one rectangle for each factor pair of 12

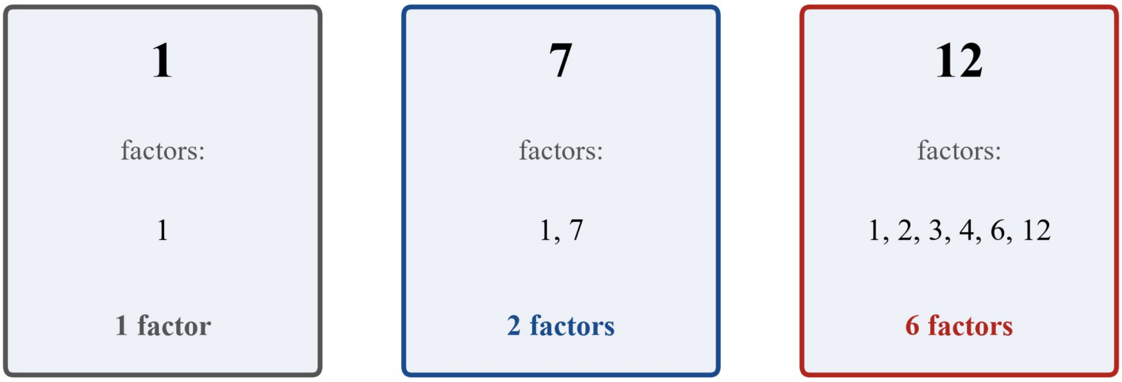
Reading off the rectangles: $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$. So the factors of 12 are 1, 2, 3, 4, 6 and 12 — six factors in all. Listing factor pairs in order, from $1 \times$ up, is the reliable way to find every factor without missing any.

Prime and composite numbers

Count the factors, and every counting number above 1 falls into one of two families.

- A **prime number** (or **prime**) has exactly two factors: 1 and itself.
- A **composite number** (or **composite**) has more than two factors.

The three cards below count the factors of 1, 7 and 12.



Three cards listing factors: 1 has the single factor 1; 7 has the two factors 1 and 7; 12 has the six factors 1, 2, 3, 4, 6 and 12

- 7 has exactly two factors, so 7 is **prime**.
- 12 has six factors, so 12 is **composite**.
- 1 has only **one** factor — itself. A prime needs exactly two factors, and a composite needs more than two, so **1 is not prime and not composite**. This follows straight from the definition: the definition counts factors, and 1 does not have two of them.

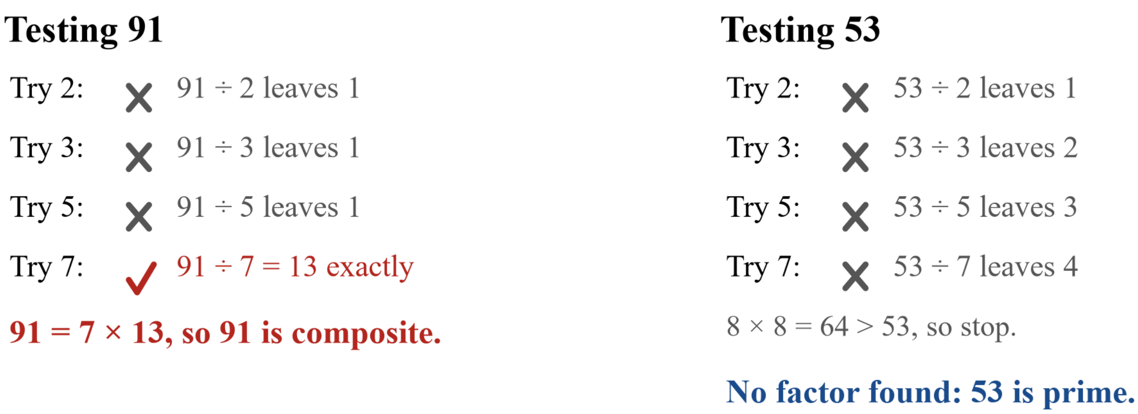
One more fact worth knowing: **2 is the only even prime.** Every other even number has 1, itself and 2 as factors — at least three — so every other even number is composite.

Testing a number by division

To test whether a number is prime, divide it by 2, 3, 5, 7, … in turn and watch the remainders. If a division leaves no remainder, you have found a factor and the number is composite. If no division up to the stopping point below leaves no remainder, the number is prime.

How far do you need to test? Think about 53. If 53 were composite, it would have a factor pair — and if *both* numbers in the pair were 8 or more, the product would be $8 \times 8 = 64$ or more, which 53 is not. So one number in the pair must be less than 8: testing 2, 3, 5 and 7 is enough for any number below 64.

The picture below runs the tests for 91 and 53.

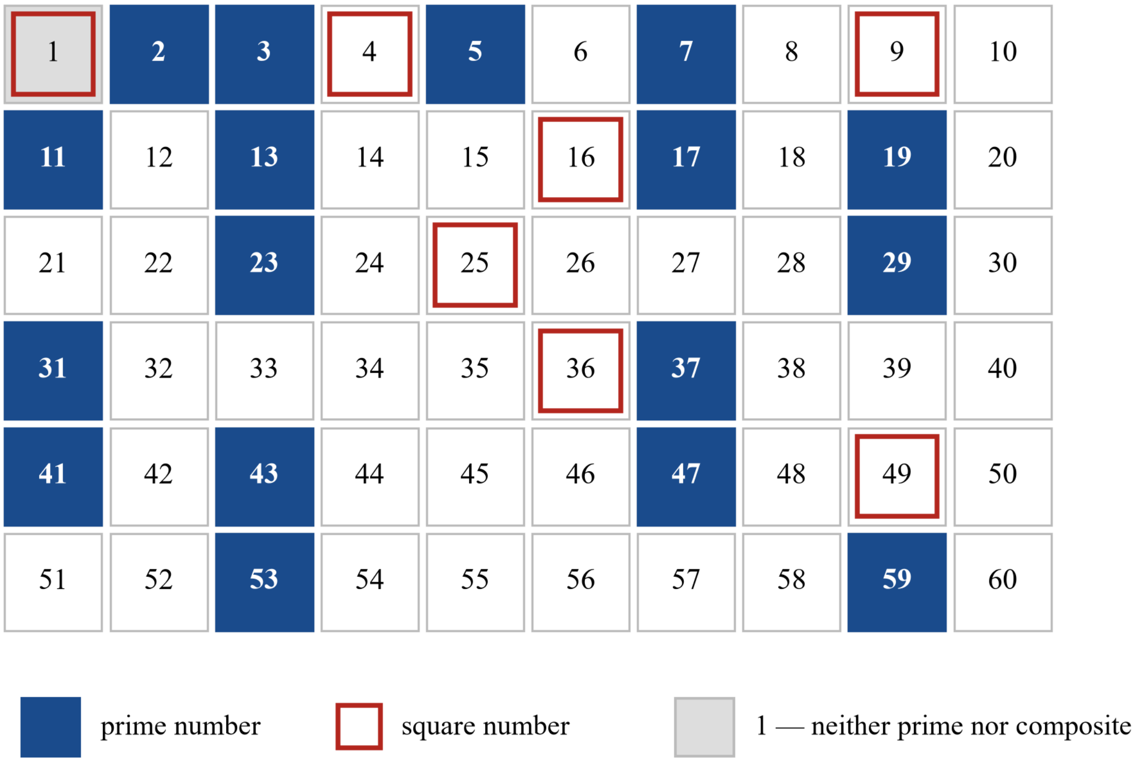


Division tests of 91 and 53 by 2, 3, 5 and 7: 91 divides exactly by 7 giving 13, so 91 is composite; 53 leaves a remainder every time, and 8 times 8 is 64 which is more than 53, so 53 is prime

- 91 is odd (2 fails), its digits $9+1=10$ do not divide by 3 (3 fails), and it does not end in 0 or 5 (5 fails). But $91 \div 7 = 13$ exactly, so $91 = 7 \times 13$: **composite**.

- 53 fails 2, 3, 5 and 7 — every division leaves a remainder — and $8 \times 8 = 64$ is more than 53, so the search stops: **prime**.

The chart below shades the primes blue and rings the square numbers red, for the numbers 1 to 60.



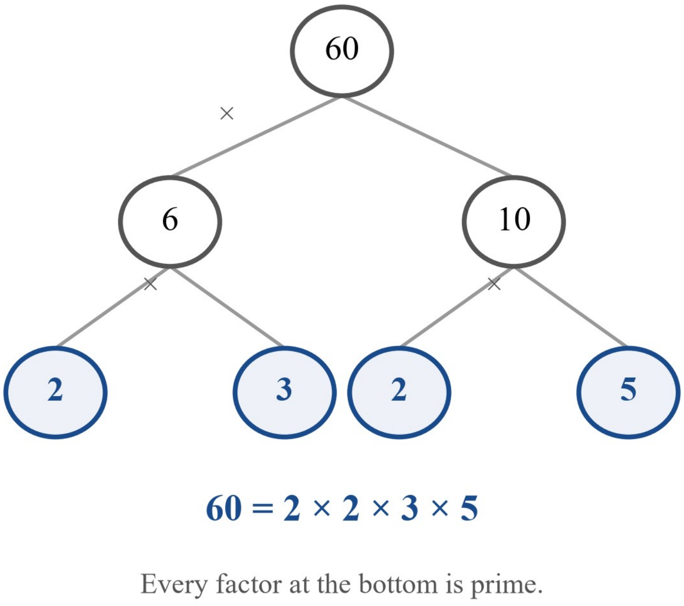
A chart of the numbers 1 to 60 in rows of ten, with prime numbers shaded blue, square numbers ringed in red, and 1 shaded grey because it is not prime and not composite

Things to notice in the chart:

- 1 is shaded grey — not prime and not composite, as the definition says.
- 2 is the only blue even number; after it, every blue number is odd.
- The primes thin out: there are four primes in the first row after 1, but only two in the last row (53 and 59).
- Except for 2 and 5, every prime ends in 1, 3, 7 or 9 — numbers ending in an even digit or in 5 divide by 2 or 5. (The reverse is not true: 21, 27, 39, 49 and 51 end in 1, 3, 7 or 9 but are composite, so the ending only gives a quick “no”, never a quick “yes”.)

Every composite as a product of primes

Keep splitting a composite number into smaller factors, and in the end you must reach factors that cannot be split any further — primes. A **factor tree** shows the splitting. The tree below splits 60 through 6×10.

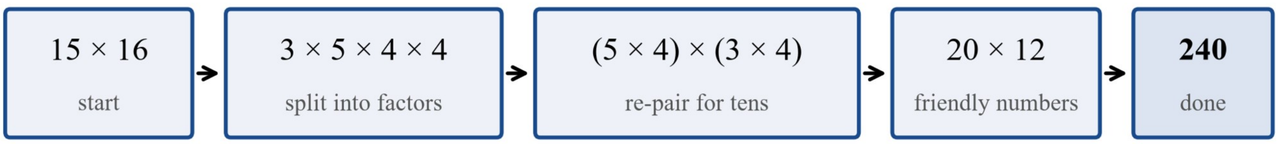


A factor tree splitting 60 into 6 and 10, then 6 into the primes 2 and 3 and 10 into the primes 2 and 5, with the product 60 = 2 times 2 times 3 times 5 written below

The bottom row is all prime: $60 = 2 \times 2 \times 3 \times 5$. These are the **prime factors** of 60. A useful property: it does not matter how you split. Start with 4×15 instead of 6×10 and the bottom row is still 2, 2, 3, 5, in some order — every composite number breaks into the same primes whatever path the tree takes.

Using factors to simplify calculations

These properties are not just for naming families — you can *use* them, and the nicest use is turning an ugly multiplication into a friendly one. Split each number into factors, then regroup the factors to build tens and hundreds. The flow below does this for 15×16.

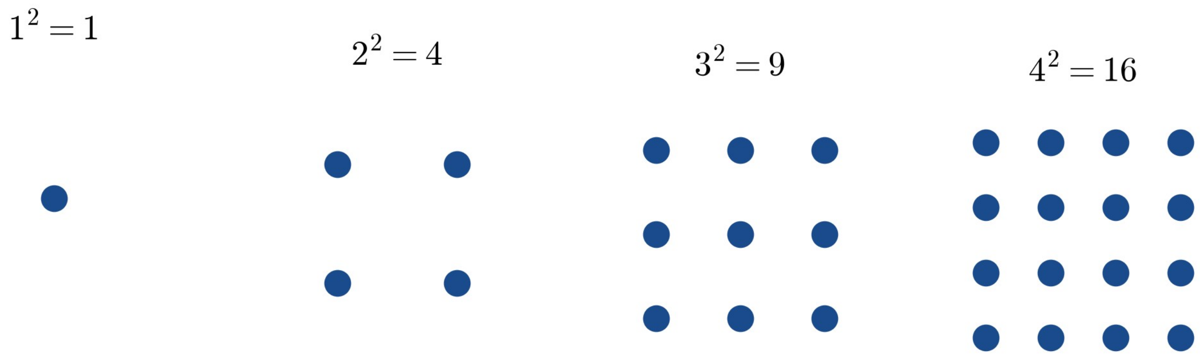


A flow of five boxes: 15 times 16 is split into the factors 3 times 5 times 4 times 4, regrouped as (5 times 4) times (3 times 4), which is 20 times 12, giving 240

15×16 becomes $3 \times 5 \times 4 \times 4$; pairing $5 \times 4 = 20$ and $3 \times 4 = 12$ gives $20 \times 12 = 240$ — two friendly numbers instead of one hard multiplication. The same trick shines with three numbers: $25 \times 13 \times 4$ is $\left(25 \times 4\right) \times 13 = 100 \times 13 = 1300$, because 25 and 4 were waiting to make a hundred.

Square numbers

A **square number** (or **square**) is a number times itself. Dots make the name obvious: the dots below sit in perfect squares.



Dot squares showing 1 squared equals 1, 2 squared equals 4, 3 squared equals 9, and 4 squared equals 16

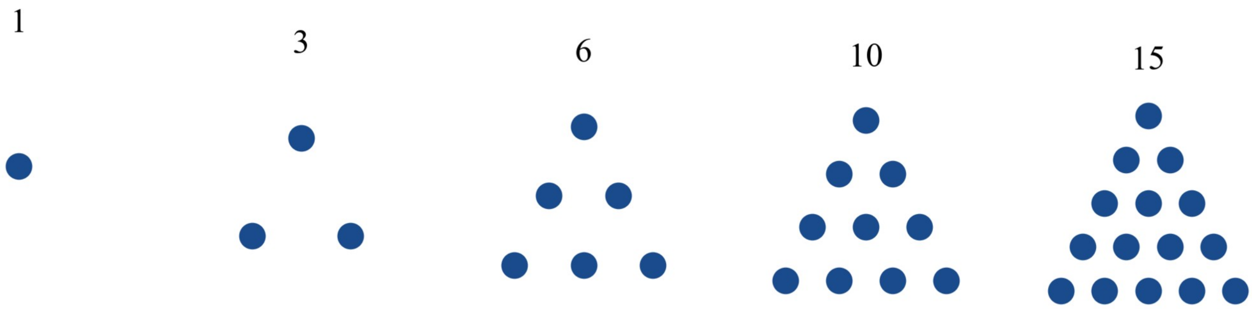
We write 3×3 as 3^2, said "3 squared", and read $3^2 = 9$. For now the square is the only power written this way. The squares to know by heart:

Number	1	2	3	4	5	6	7	8	9	10	11	12
Its square	1	4	9	16	25	36	49	64	81	100	121	144

The gaps between squares grow by 2 each time: $4 - 1 = 3$, $9 - 4 = 5$, $16 - 9 = 7$, $25 - 16 = 9$. Each new square adds the next odd number — the new dots sit along two sides of the old square. So adding the first few odd numbers always lands on a square: $1 + 3 + 5 + 7 = 16 = 4^2$.

Triangular numbers

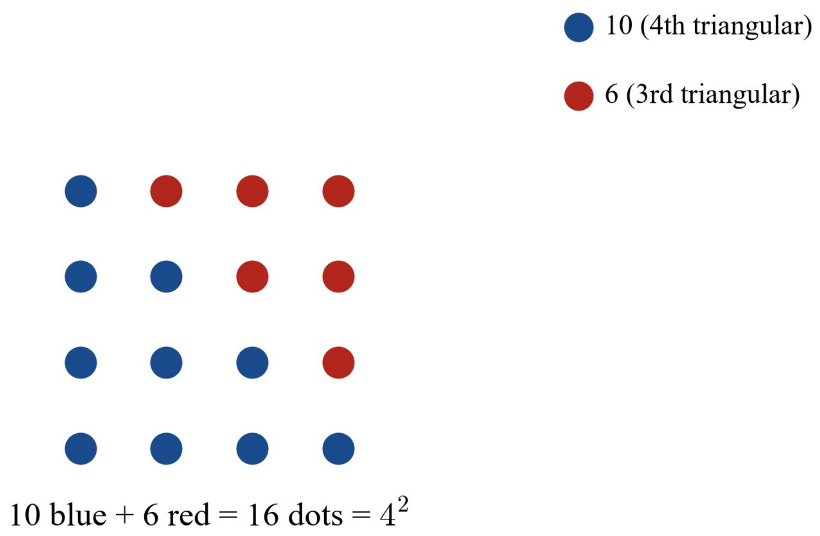
Stack dots in rows of 1, 2, 3, … and the rows form triangles. The counts are the **triangular numbers**.



Dot triangles showing the triangular numbers 1, 3, 6, 10 and 15, each with one more row than the last

The list begins 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. The rule for growing the list: each new triangular number adds one more than the last one did — add 2, then 3, then 4, then 5 — because each new triangle has one more row of dots.

Now look at the square of 16 dots again, cut along a diagonal: one side holds the 4th triangular number (10 dots) and the other holds the 3rd (6 dots).



A four-by-four square of 16 dots cut along a diagonal into a blue triangle of 10 dots and a red triangle of 6 dots, showing that two triangular numbers in a row make the square number 16

$10+6=16=4^2$: **two triangular numbers in a row make a square number.** Check the next pair yourself: $15+21=36=6^2$. It keeps working — and it means some numbers belong to both families at once. In the list up to 60, **1 and 36 are both square and triangular.**

Looking for patterns in a spreadsheet

A spreadsheet is a good place to hunt for patterns about factors, because it lists and counts for you. The sheet below lists the numbers you can build from the first three primes — 2, 3 and 5 — and counts the factors of each.

	A	B	C
	Number	**Its factors**	**How many**
1	**1**	1	1
2	**2**	1, 2	2
3	**3**	1, 3	2
4	**4**	1, 2, 4	**3**
5	**5**	1, 5	2
6	6	1, 2, 3, 6	4
7	8	1, 2, 4, 8	4
8	**9**	1, 3, 9	**3**
9	10	1, 2, 5, 10	4
10	15	1, 3, 5, 15	4
11	**25**	1, 5, 25	**3**
12	27	1, 3, 9, 27	4

The highlighted numbers have up to 3 factors. The ones with exactly 3 — 4, 9, 25 — are all squares of primes.

A spreadsheet listing the numbers 1, 2, 3, 4, 5, 6, 8, 9, 10, 15, 25 and 27 with their factors, highlighting the numbers with up to three factors, and noting that the ones with exactly three factors, 4, 9 and 25, are squares of primes

The highlighted numbers have up to 3 factors. Look at the ones with **exactly** 3 factors: 4, 9 and 25. Something is going on — those are exactly the squares of the primes 2, 3 and 5. That observation is a **conjecture**: a guess stated clearly enough to be tested. Test it: 49 is the square of the prime 7, and its factors are 1, 7, 49 — exactly 3, so 49 fits the conjecture. But 36 is a square with 9 factors, because 36 is not the square of a *prime*. So the sharpened conjecture is: *the square of a prime has exactly 3 factors* — and you can see why, because the only factors of 7×7 are 1, 7 and 49.

Sorting numbers into families

When a question asks which family a number is in, the definition always works: count the factors. Two properties are worth carrying out of this subtopic.

- **A square greater than 1 is never prime.** A square is some number times itself, so besides 1 and itself it also has that number as a factor — at least three factors, which makes it composite. (36 has 1, 6 and 36 already.)
- **The families overlap in surprises.** 2 is prime *and* even; 1 and 36 are square *and* triangular; every square is made of prime factors that come in pairs. Knowing the families turns facts like these into shortcuts.

Words for this subtopic, with what they mean here:

Term	Meaning here
factor	a counting number that divides another exactly, leaving no remainder
factor pair	two factors that multiply to give the number
prime number	a counting number with exactly two factors: 1 and itself
composite number	a counting number with more than two factors
square number	a number times itself, like $9 = 3 \times 3$
squared, 3^2	the way of writing 3×3; the small 2 says "times itself"
triangular number	a count of dots that stacks into rows of 1, 2, 3, … , like 10
factor tree	a splitting of a number into smaller and smaller factors until all are prime
prime factors	the primes at the bottom of a factor tree, whose product is the number
conjecture	a pattern stated clearly enough to be tested

Worked examples

In the with-help examples the thinking is laid out line by line; in the on-your-own examples you carry it. Every example uses a property of one of the four families — and most of them check the answer a second way.

Example 1 — with help

List the factor pairs of 30, write out all its factors, and say whether 30 is prime or composite.

Working	Comment
Factor pairs of 30: 1×30, 2×15, 3×10, 5×6.	Work from $1 \times$ upwards; stop when a pair would repeat.
The factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 — eight factors.	Read the pairs back as one list, smallest to largest.
Eight factors is more than two, so 30 is **composite**.	The definition decides: two factors → prime; more than two → composite.

Check: 30 is even and greater than 2, and 2 is the only even prime — so 30 had to be composite before we even listed the factors. ✓

Example 2 — with help

Use division to test whether 91 and 53 are prime. (The picture in the Theory shows these same tests.)

Working	Comment
91: odd, so 2 fails. $9+1=10$, not divisible by 3, so 3 fails. Does not end in 0 or 5, so 5 fails. $91 \div 7=13$ exactly.	A clean division means a factor found.
$91=7 \times 13$, so 91 is **composite**.	Two factors besides 1 and 91 — more than two factors in all.
53: $53 \div 2$, $53 \div 3$, $53 \div 5$, $53 \div 7$ all leave remainders.	53 is odd; $5+3=8$; ends in 3; $53 \div 7=7$ remainder 4.
$8 \times 8=64$ is more than 53, so the search stops: 53 is **prime**.	If both factors were 8 or more, the number would be 64 or more.

Check: the chart of 1 to 60 shades 53 blue and leaves 91 not shaded — the same verdict as the division tests. ✓

Example 3 — with help

Use the definition to sort 1, 2 and 9 into prime, composite, or not in either family.

Working	Comment
1: the only factor is 1 — one factor.	Count factors; that is the whole definition.
1 is **not prime and not composite**. A prime needs exactly two factors, and 1 has only one.	This is exactly why the definition says “exactly two”: it puts 1 outside both families.
2: factors 1, 2 — two factors, so 2 is **prime**.	The only even prime.
9: factors 1, 3, 9 — three factors, so 9 is **composite**.	$9=3 \times 3$; odd numbers can be composite too.

Check: on the chart, 1 is grey, 2 is blue, and 9 is not shaded and ringed — 1 is not in either family, 2 is prime, and 9 is composite and a square. ✓

Example 4 — with help

Two students split 60 into prime factors, one starting with 6×10 and one starting with 4×15. Show that they reach the same primes.

Working	Comment
First tree: $60=6 \times 10=(2 \times 3) \times (2 \times 5)$, giving 2, 2, 3, 5.	Split until nothing left at the bottom can be split.
Second tree: $60=4 \times 15=(2 \times 2) \times (3 \times 5)$, giving 2, 2, 3, 5.	A different first split, the same bottom row.
Both trees give $60=2 \times 2 \times 3 \times 5$.	The primes of a number are fixed — the tree is just a path to them.

Check: $2 \times 2 \times 3 \times 5=4 \times 3 \times 5=12 \times 5=60$. ✓ Multiplying the primes back must always return the number you started with.

Example 5 — on your own

Use the factors of each number to simplify the calculations. (a) 12×25. (b) 14×15.

Split into factors. (a) $12 \times 25=(4 \times 3) \times 25$. (b) $14 \times 15=(2 \times 7) \times (3 \times 5)$.

Regroup for tens. (a) $3 \times (4 \times 25)=3 \times 100=300$. (b) $(2 \times 5) \times (7 \times 3)=10 \times 21=210$.

Check. (a) 12×25 is 12 lots of a quarter of 100, which is 3 lots of 100 — 300. ✓ (b) $14 \times 15=14 \times 10+14 \times 5=140+70=210$. ✓ The friendly-path answer and a different method agree.

Example 6 — on your own

(a) Write 7^2 and 12^2 as a number times itself and find each value.

(b) Without adding them one by one, find $1+3+5+7+9+11+13$.

(a) $7^2 = 7 \times 7 = 49$, and $12^2 = 12 \times 12 = 144$. Both values sit in the squares table.

(b) The sum of the first few odd numbers is a square: $1+3+5+7=16=4^2$ added the first 4 odds. Here there are 7 odds, so the sum is $7^2 = 49$.

Check. Pair the odds around the middle: $1+13=14$, $3+11=14$, $5+9=14$, and 7 left over — $3 \times 14 + 7 = 42 + 7 = 49$. ✓

Example 7 — on your own

A hall is set up with chairs in a triangle: 1 chair in the top row, 2 in the next, and so on, down to 10 chairs in the bottom row. (a) How many chairs are set up? (b) The teacher asks for exactly 50 chairs in a triangle like this. Is it possible?

(a) The count is the 10th triangular number: $1+2+3+\cdots+10$. Pair the ends: $1+10=11$, $2+9=11$, and so on — five pairs of 11, giving 55 chairs.

(b) The triangular numbers near 50 are 45 (9 rows) and 55 (10 rows). The list jumps straight over 50, so **no** — a triangle of this shape cannot hold exactly 50 chairs. The teacher chooses 45 or 55.

Check. Grow the list by its rule: 45, then $45+10=55$. 50 never appears. ✓

Example 8 — on your own (digital-tool path)

(Digital tool.) Open a spreadsheet and, as in the picture in the Theory, list the numbers built from the primes 2, 3 and 5 — 1, 2, 3, 4, 5, 6, 8, 9, 10, 15, 25, 27 — with their factors and the count of factors. (a) Which of these numbers have exactly 3 factors? (b) State a conjecture about them and test it on 49 and on 36.

(a) Exactly 3 factors: 4 (1, 2, 4), 9 (1, 3, 9) and 25 (1, 5, 25).

(b) Conjecture: *a number with exactly 3 factors is the square of a prime.* Test 49: its factors are 1, 7, 49 — exactly 3, and $49 = 7^2$ with 7 prime: it fits. Test 36: it is a square but not of a prime, and it has 9 factors — it shows the conjecture really needs the word *prime*, because plain squares can have many factors. Why the conjecture is true: the factors of 7×7 can only be 1, 7 and 49.

Check. The spreadsheet count and the hand list agree for 4, 9, 25, 49 and 36. ✓

Practice questions

With help

Q1. List the factor pairs, then all the factors, of each number. (a) 18. (b) 28.

Q2. Sort these numbers into prime, composite, or not in either family: 1, 2, 9, 17, 25, 31, 46. Give the one-line reason for 1.

Q3. Use the chart of 1 to 60 in the Theory. (a) List the primes between 40 and 60. (b) Write down the only even prime. (c) Why is 1 shaded grey?

Q4. True or false? If false, give a number that shows it. (a) Every even number is composite. (b) Every prime is odd. (c) A counting number with exactly two factors is prime. (d) Every composite number has at least three factors.

Q5. Test by dividing by 2, 3, 5 and 7. Which of 39, 43 and 49 are prime? For each composite, write the factor the test finds.

Q6. Draw a factor tree for 36, then write 36 as a product of its prime factors.

Q7. Simplify $4 \times 17 \times 25$ by regrouping the factors.

Q8. Find each square. (a) 6^2. (b) 9^2. (c) 12^2.

Q9. The squares run 1, 4, 9, 16, … Write the next two squares two ways: as a number times itself, and by adding the next odd number.

Q10. The triangular numbers run 1, 3, 6, 10, … Write the next two, and the rule that grows the list.

On your own

Q11. List all the factors of 48. Is 48 prime or composite, and how does the list decide it?

Q12. Prime or composite? Test 51, 57 and 61 by division, saying which divisions you try and why you may stop there.

Q13. Riddle: *I am a prime between 30 and 50. My two digits add to 5. What am I?*

Q14. Find the two composite numbers below 20 that have exactly six factors.

Q15. Simplify by splitting into factors and regrouping. (a) $25 \times 13 \times 4$. (b) $5 \times 23 \times 2$.

Q16. Which of 54, 64, 84 and 100 are square numbers? Say how you know 84 is not, without listing any factors.

Q17. Use the odd-number pattern $1+3+5+7=16$. (a) $1+3+5+7+9$. (b) $1+3+5+7+9+11$.

Q18. Is 50 a triangular number? Explain using the list.

Q19. Which numbers up to 60 are both square and triangular?

Q20. A supermarket stacks cans in a triangle: 1 can in the top row, and each row below holds one more can than the row above. (a) How many cans are in a triangle with 7 rows? (b) A triangle with more rows uses exactly 36 cans. How many rows does it have? (c) Name another family 36 belongs to.

Q21. Simplify 15×12 by splitting both numbers into factors.

Q22. Explain, using the definition of a prime, why a square number greater than 1 can never be prime.

Maths in real life

Q23. In eastern North America, the periodical cicada — an insect — spends most of its life underground, then whole swarms come out together every 13 or every 17 years: both prime numbers. An animal that eats cicadas might come out on its own cycle of a few years. (a) Cicadas on a 13-year cycle come out in years 13, 26, 39, … and a 4-year animal appears in years 4, 8, 12, … List both lists to 60 and find the first year they meet. (b) Do the same for cicadas on a 12-year cycle and the same 4-year animal. (c) Which cycle keeps the cicadas safer, and why do primes help?

Source: Wikipedia, *Periodical cicadas*. Retrieved 20 August 2026.

Q24. A square floor of tiles in a school hall is 9 tiles by 9 tiles. The school extends it to 10 tiles by 10 tiles. How many new tiles are laid? Connect your answer to the gap pattern between squares.

Q25. Five team captains line up, and each captain shakes hands once with each of the others. (a) Count the handshakes by adding $4+3+2+1$. What family is the answer in? (b) How many handshakes with six captains? (c) Explain why this counting always gives triangular numbers.

Choosing your strategy

Q26. For each, choose how you would work it out — chart, head, or paper — and give the answer. (a) Is 83 prime? (b) 25×44. (c) 12^2.

Q27. Find the flaw. Mei tests 51: it is odd, so 2 fails, and it does not end in 0 or 5, so 5 fails. “51 is prime,” she says. What did she miss?

Q28. Always, sometimes, or never? Give an example each way to back your answer. (a) A square number is even. (b) A triangular number is also a square number. (c) The sum of two primes is composite.

Q29. Make your own. Write a number riddle that uses at least two of the four families (prime, composite, square, triangular) and has exactly one answer below 60. Swap with a partner and solve each other's.

Answers

Q1. (a) pairs 1×18, 2×9, 3×6; factors 1, 2, 3, 6, 9, 18 — six; (b) pairs 1×28, 2×14, 4×7; factors 1, 2, 4, 7, 14, 28 — six. **Q2.** prime: 2, 17, 31; composite: 9, 25, 46; not in either family: 1 — it has only one factor. **Q3.** (a) 41, 43, 47, 53, 59; (b) 2; (c) 1 has one factor, so it is not prime and not composite. **Q4.** (a) false — 2 is even and prime; (b) false — 2 again; (c) true; (d) true. **Q5.** 39 composite (3×13); 43 prime; 49 composite (7×7). **Q6.** $36 = 2 \times 2 \times 3 \times 3$. **Q7.** 1700. **Q8.** (a) 36; (b) 81; (c) 144. **Q9.** 25 (5×5, and $16 + 9$); 36 (6×6, and $25 + 11$). **Q10.** 15 and 21; each step adds one more than the step before. **Q11.** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 — ten factors, so composite. **Q12.** 51 composite (3×17); 57 composite (3×19); 61 prime. **Q13.** 41. **Q14.** 12 and 18. **Q15.** (a) 1300; (b) 230. **Q16.** 64 (8^2) and 100 (10^2); 84 sits between $81 = 9^2$ and $100 = 10^2$, and no counting number squares to it. **Q17.** (a) $25 = 5^2$; (b) $36 = 6^2$. **Q18.** no — the list runs 45, then 55, jumping over 50. **Q19.** 1 and 36. **Q20.** (a) 28; (b) 8 rows; (c) square — $36 = 6^2$. **Q21.** 180. **Q22.** a square has 1, itself, and its side as factors — at least three — so it is composite. **Q23.** (a) 52; (b) they meet every 12 years: 12, 24, 36, 48; (c) the prime cycles — a prime has no small factors, so small cycles do not often line up with it. **Q24.** 19 new tiles; the gaps between squares are the odd numbers, and $81 + 19 = 100$. **Q25.** (a) 10 — triangular; (b) 15; (c) each new captain adds one handshake per captain already there. **Q26.** (a) paper — test 2, 3, 5, 7: prime; (b) head — $25 \times 4 \times 11 = 1100$; (c) head — 144, from the squares table. **Q27.** she never tested 3: $5 + 1 = 6$ divides by 3, and $51 = 3 \times 17$. **Q28.** (a) sometimes — 4 is, 9 is not; (b) sometimes — 36 is, 10 is not; (c) sometimes — $3 + 5 = 8$ is, $2 + 3 = 5$ is not. **Q29.** sample: *I am below 60. I am square and triangular.* — 36.

All the working

Q1. (a) Pairs from $1 \times$ up: 1×18, 2×9, 3×6; the next pair would repeat. Factors: 1, 2, 3, 6, 9, 18 — six factors. (b) 1×28, 2×14, 4×7. Factors: 1, 2, 4, 7, 14, 28 — six factors.

Q2. 2, 17, 31 have exactly two factors: **prime**. 9 $(1, 3, 9)$, 25 $(1, 5, 25)$ and 46 $(1, 2, 23, 46)$ have more: **composite**. 1 has a single factor, so it is **not in either family** — the definition counts factors, and 1 has only one.

Q3. (a) Reading the blue cells between 40 and 60: **41, 43, 47, 53, 59**. (b) **2** — the only even prime. (c) 1 has only one factor; a prime needs exactly two and a composite more than two, so 1 is **not in either family**, and the chart shades it grey to say so.

Q4. (a) **False** — 2 is even and prime. (b) **False** — 2 is a prime that is not odd. (c) **True** — that is the definition of a prime. (d) **True** — "more than two factors" means at least three: 1, the number itself, and one more.

Q5. All three are odd (2 fails) and not one ends in 0 or 5 (5 fails), so the work is in 3 and 7. 39: $3 + 9 = 12$ divides by 3, and $39 \div 3 = 13$ — **composite**, 3×13. 43: $43 \div 3$ and $43 \div 7$ leave remainders, and $8 \times 8 = 64 > 43$ — **prime**. 49: $49 \div 7 = 7$ — **composite**, 7×7.

Q6. One tree: $36 = 6 \times 6 = (2 \times 3) \times (2 \times 3)$. Product of primes: $36 = 2 \times 2 \times 3 \times 3$. Check: $2 \times 2 \times 3 \times 3 = 4 \times 9 = 36$. ✓ (Any tree ends at the same four primes.)

Q7. Regroup 25 with the 4: $(4 \times 25) \times 17 = 100 \times 17 = 1700$.

Q8. (a) $6^2 = 6 \times 6 = 36$. (b) $9^2 = 9 \times 9 = 81$. (c) $12^2 = 12 \times 12 = 144$.

Q9. Next square: $5 \times 5 = 25$, and by the gap pattern $16 + 9 = 25$. The one after: $6 \times 6 = 36$, and $25 + 11 = 36$. The odd numbers added run 3, 5, 7, 9, 11, …

Q10. $10 + 5 = 15$, then $15 + 6 = 21$. Rule: each new triangular number adds one more than the previous step did (add 2, add 3, add 4, …), because the new row of the triangle has one more dot.

Q11. Pairs: 1×48, 2×24, 3×16, 4×12, 6×8. Factors: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 — ten factors. More than two, so **composite**.

Q12. All three are below 64, and $8 \times 8 = 64$, so testing 2, 3, 5, 7 is enough for each. 51: odd, and $5 + 1 = 6$ divides by 3 — $51 \div 3 = 17$, so **composite**. 57: $5 + 7 = 12$ divides by 3 — $57 \div 3 = 19$, **composite**. 61: $61 \div 3$ leaves remainder 1, $61 \div 7$ leaves remainder 5 (2 and 5 fail at a glance) — no factor found, so **prime**.

Q13. Numbers between 30 and 50 whose digits add to 5: 32, 41, 50. 32 is even and 50 is even — composite. 41 fails the tests 2, 3, 5, 7 ($7 \times 7 = 49 > 41$) — prime. The answer is **41**.

Q14. Count factors below 20: 12 has 1, 2, 3, 4, 6, 12 — six. 18 has 1, 2, 3, 6, 9, 18 — six. (16 has only five: 1, 2, 4, 8, 16.) The two numbers are **12 and 18**.

Q15. (a) $25 \times 13 \times 4 = (25 \times 4) \times 13 = 100 \times 13 = 1300$. (b) $5 \times 23 \times 2 = (5 \times 2) \times 23 = 10 \times 23 = 230$.

Q16. $64 = 8^2$ and $100 = 10^2$ are squares. 54 sits between $49 = 7^2$ and $64 = 8^2$, and 84 sits between $81 = 9^2$ and $100 = 10^2$; between two squares in a row there is no room for another, so not one of them is a square. For 84: the square of a counting number jumps from 81 straight to 100, so nothing squares to 84.

Q17. (a) Five odds: $5^2 = 25$. Check: $16 + 9 = 25$. ✓ (b) Six odds: $6^2 = 36$. Check: $25 + 11 = 36$. ✓

Q18. The triangular numbers run …, 36, 45, 55, … The step from 45 adds 10, landing on 55, so 50 is skipped: **no**, 50 is not triangular.

Q19. Squares to 60: 1, 4, 9, 16, 25, 36, 49. Triangulars to 60: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. Both lists hold **1 and 36**.

Q20. (a) $1 + 2 + \cdots + 7 = 28$ cans (the 7th triangular number). (b) Keep going: $28 + 8 = 36$ — **8 rows**. (c) 36 is also **square**: $36 = 6^2$ — the same overlap as Q19.

Q21. $15 \times 12 = (3 \times 5) \times (4 \times 3)$. Regroup: $(5 \times 4) \times (3 \times 3) = 20 \times 9 = 180$. Check: $15 \times 12 = 15 \times 10 + 15 \times 2 = 150 + 30 = 180$. ✓

Q22. A square greater than 1 is some counting number times itself — say $6 \times 6 = 36$. Then it has at least the three factors 1, 6 and 36. A prime has exactly two factors, so a square greater than 1 has too many to be prime; it is composite. (The argument is the same for every square: 1, the number itself, and the number that was squared.)

Q23. (a) 13-cycle: 13, 26, 39, 52. 4-year animal: 4, 8, 12, …, 48, 52, 56. First meeting: year **52**. (b) 12-cycle: 12, 24, 36, 48 — the 4-year list hits every one of them: they meet **every 12 years**. (c) The **prime cycles** keep the cicadas safer. 12 has the small factors 2, 3, 4 and 6, so animals on those cycles line up with the cicadas often; 13 and 17 have no small factors, so a 2-to-6-year animal does not often meet them (as in (a), not until year 52). That is the property of primes doing real work.

Q24. Before: $9 \times 9 = 81$ tiles. After: $10 \times 10 = 100$ tiles. New tiles: $100 - 81 = 19$. The gap pattern between squares says the step from 9^2 to 10^2 adds the next odd number after 17, which is 19 — and the new tiles do form that border: a row of 9, a column of 9, and 1 corner tile, $9 + 9 + 1 = 19$. ✓

Q25. (a) $4 + 3 + 2 + 1 = 10$ handshakes — the 4th **triangular number**. (b) Six captains: $5 + 4 + 3 + 2 + 1 = 15$ — the 5th triangular number. (c) The newest captain shakes hands once with each captain already there: the 2nd adds 1, the 3rd adds 2, the 4th adds 3 — exactly the rows of a triangular stack of dots.

Q26. (a) **Paper**: run the division tests — 83 is odd; $8 + 3 = 11$; not ending in 0 or 5; $83 \div 7 = 11$ remainder 6. $10 \times 10 = 100 > 83$, so the tests stop there: **prime**. (b) **Head**: $25 \times 44 = (25 \times 4) \times 11 = 100 \times 11 = 1100$. (c) **Head**: $12^2 = 144$, straight from the squares table.

Q27. Mei tested 2 and 5 but skipped 3 (and 7). The digit test gives it away: $5 + 1 = 6$, which divides by 3, so $51 \div 3 = 17$ — 51 is 3×17, **composite**. Being odd and not ending in 0 or 5 only says 2 and 5 are not factors; it never says a number is prime.

Q28. (a) **Sometimes.** 4 and 16 are even squares; 9 and 25 are odd squares. (Squares of even numbers are even, squares of odd numbers are odd.) (b) **Sometimes.** 36 is both; 10 is triangular but not square, and 4 is square but not triangular. (c) **Sometimes.** $3 + 5 = 8$, composite; but $2 + 3 = 5$, prime — 2, the odd one out among primes, makes the "always" fail.

Q29. Sample riddle: *I am below 60. I am a square and a triangular number. I am more than 1.* Answer: 36. Check for a good riddle: the clues must pin down exactly one number — here the only square-and-triangular numbers below 60 are 1 and 36, and “more than 1” leaves 36.

What comes next

That opens Chapter 1. In **1A** you sorted the counting numbers into families — primes with exactly two factors, composite numbers with more, 1 not in either family — and added the square and triangular families, then put all four to work: testing numbers by division, breaking composite numbers into their prime factors, and regrouping factors to make hard multiplications easy.

Next, **1B** leaves the families and puts numbers on a line and on a grid: integers on the number line and the Cartesian plane (`VC2M6N01`). The primes do not stay away for long — in Chapter 2, common multiples become the key to comparing and combining fractions in 2A and 2B.

In Year 7 the families come back with new tools: squares and square roots (`VC2M7N01`), and products of prime factors written using powers, like $60 = 2^2 \times 3 \times 5$ (`VC2M7N02`) — the factor trees of 1A are the ground that notation stands on.

Integers on the number line and the Cartesian plane

Content description `VC2M6N01`: recognise situations, including financial contexts, that use integers; locate and represent integers on a number line and as coordinates on the Cartesian plane

*In Year 5 you placed numbers on a number line and found points on a grid using two coordinates. This subtopic stretches both ideas at once: the number line grows a whole new half — the numbers below zero — and the grid grows from one quarter into four. The new numbers are the **integers**, and they turn up in any situation where something can be above or below a starting point: money made or lost, floors above or below the ground, heights above or below the sea, temperatures above or below freezing. The single big idea is that the sign tells you **which side of zero** you are on, and the number tells you **how far**.*

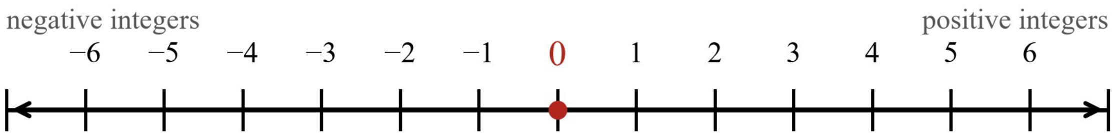
Theory

What you bring from Year 5: number lines and a grid

A number line is a straight line on which every number sits at its own place, in order, with equal steps between the counting numbers. In Year 5 you placed whole numbers and decimals on number lines, and you located points on a square grid by counting across and then up from the corner. Both skills come back here — the number line is about to grow, and the grid is about to grow with it.

The number line grows both ways

The counting numbers sit to the **right** of zero, each one step from the next. Nothing stops the line from continuing to the **left** of zero with the same equal steps. The numbers on the left are the **negative integers**, written with the sign −: −1, −2, −3, … The numbers on the right are the **positive integers**, which may carry a plus sign: +1, +2, +3, … (though the plus is usually left off). The **integers** are all of these together with zero: …, −3, −2, −1, 0, 1, 2, 3, …



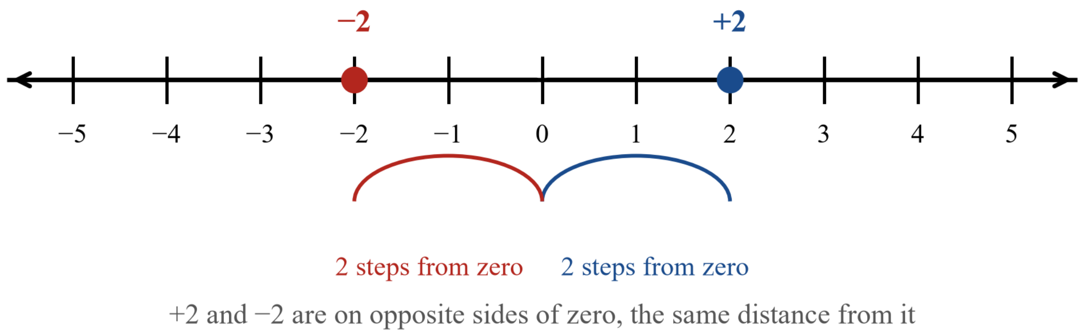
A horizontal number line drawn to scale from −6 to 6 with an arrowhead at each end, zero marked with a red dot, the negative integers labelled on the left and the positive integers on the right

Two reading rules that never fail:

- **Left means smaller.** Any number on the line is smaller than every number to its right, and greater than every number to its left. This works below zero too: −5 is to the left of −2, so −5 is smaller than −2.
- **Zero is the divider.** Zero is an integer, but it is not positive and not negative. It is the starting point that both sides are measured from.

Opposites

Look at +2 and −2 on the line. They are on **opposite sides** of zero, and each is the **same distance** — 2 steps — from it. Pairs like this are called **opposites**: the opposite of +2 is −2, and the opposite of −2 is +2.

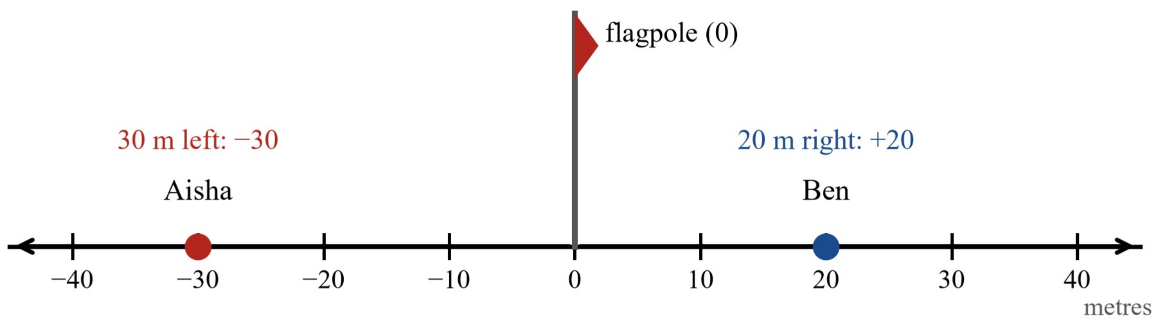


A number line from −5 to 5 with −2 marked in red and +2 marked in blue, a red curve and a blue curve under the line, each covering 2 steps to zero, labelled "2 steps from zero", showing that +2 and −2 are on opposite sides of zero and the same distance from it

The opposite of a number is the same distance from zero, on the other side. So the opposite of +5 is −5, the opposite of −7 is +7, and — the quiet special case — the opposite of 0 is 0, because zero has no other side to move to.

The sign shows the direction from zero

Here is the idea the whole subtopic stands on: **the sign is a direction, measured from a zero that we choose.** Pick a starting point and call it 0. Decide that right is positive. Then left is negative, and every position gets an integer.



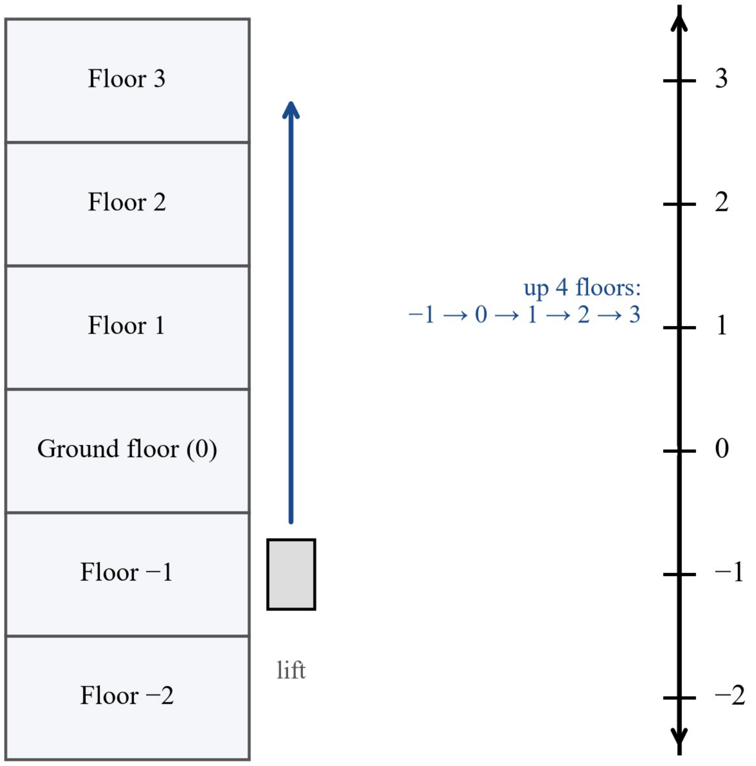
A horizontal number line in metres with a flagpole marked as 0, Aisha standing 30 metres to the left labelled −30 in red, and Ben standing 20 metres to the right labelled +20 in blue

With the flagpole as zero, Aisha, 30 m to the left, is at **−30**, and Ben, 20 m to the right, is at **+20**. The 30 and the 20 say *how far*; the signs say *which way*. Change the direction rule and the signs swap: if we had called left positive instead, Aisha would be at +30 and Ben at −20. The positions have not moved — only the frame has. That is why a sign never means anything until zero and the positive direction are fixed.

Vertical number lines

Nothing says a number line must lie down. Stand it up and the same rules hold, with **up** positive and **down** negative.

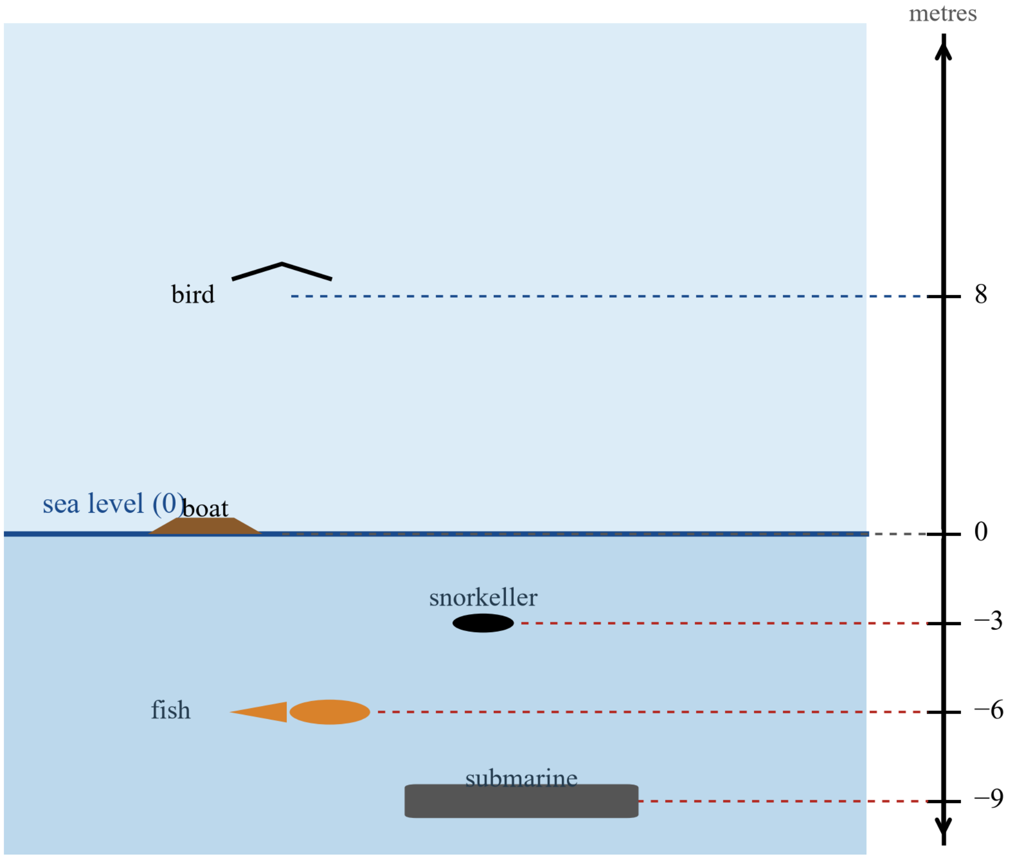
A lift is a vertical number line. Call the ground floor 0; floors above are positive and floors below are negative.



A building drawn as six stacked floors labelled Floor 3, Floor 2, Floor 1, Ground floor (0), Floor −1 and Floor −2, with a lift arrow rising from Floor −1 to Floor 3 labelled "up 4 floors: −1 → 0 → 1 → 2 → 3", beside a matching vertical number line from −2 to 3 with arrowheads at both ends

Starting at Floor −1 and riding up, the lift passes −1 → 0 → 1 → 2 → 3: four floors up, counted straight off the line. The floors below the ground are labelled −1 and −2, and the integers keep the order honest: −2 is below −1, which is below 0.

The sea works the same way, with **sea level** as zero.

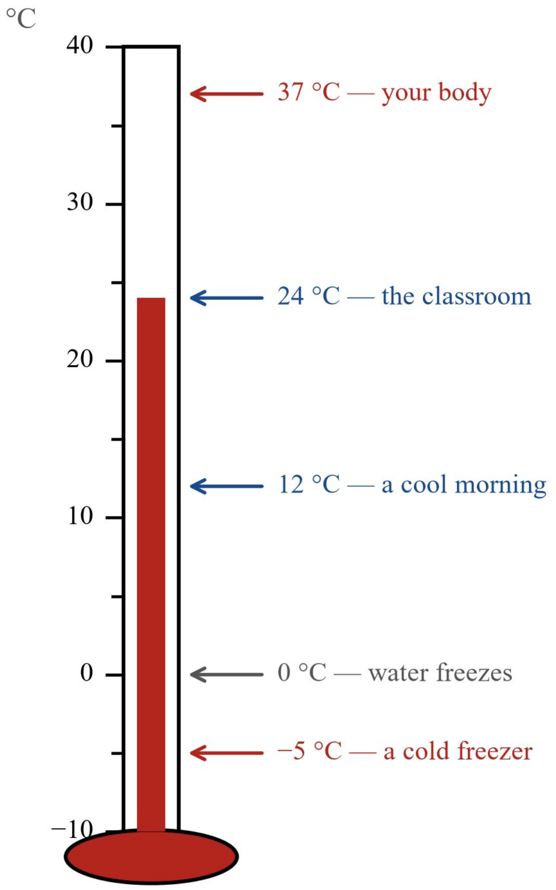


A scene with the sky above and the sea below a blue sea-level line marked 0: a bird at 8 metres, a boat at 0, a snorkeller at −3 metres, a fish at −6 metres and a submarine at −9 metres, each joined by a dashed line to a vertical number line in metres running from −9 to 8 with arrowheads at both ends

The bird is at +8 m, the boat at 0, the snorkeller at −3 m, the fish at −6 m and the submarine at −9 m. Reading top to bottom is reading largest to smallest: +8, 0, −3, −6, −9. Going down the line, the numbers get smaller the whole way — there is no moment where the rule changes at the water's edge.

A thermometer is a vertical number line

A thermometer models a vertical number line in degrees Celsius, with 0 °C — the temperature at which water freezes — as its zero.

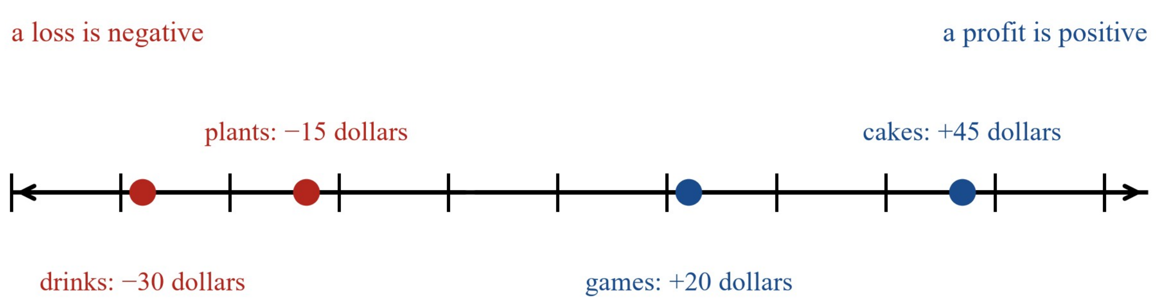


A thermometer drawn as a vertical number line scaled from −10 °C to 40 °C with a red column, marked readings at −5 °C "a cold freezer", 0 °C "water freezes", 12 °C "a cool morning", 24 °C "the classroom" and 37 °C "your body"

Readings above zero are positive temperatures; readings below zero are negative temperatures. A freezer at −5 °C is 5 degrees **below** zero; the classroom at 24 °C is 24 degrees **above** it. The colder something is, the smaller its integer: −5 °C is smaller than 0 °C, which is smaller than 12 °C.

Profit and loss

Money is the most useful home of the integers. When a stall **makes** money, the result is a **profit**, written positive; when it **loses** money, the result is a **loss**, written negative; and when takings match costs exactly, the result is 0.

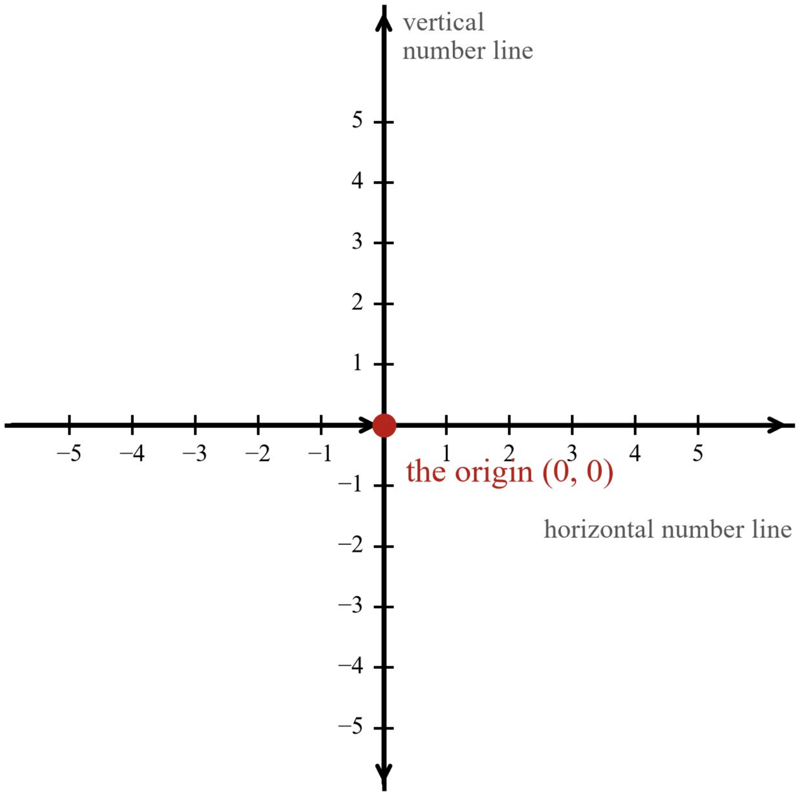


A horizontal number line of dollars with four stalls of a class fete marked on it: drinks at −30 dollars and plants at −15 dollars in red on the left, games at +20 dollars and cakes at +45 dollars in blue on the right, with "a loss is negative" at the left end and "a profit is positive" at the right end

The cakes stall made a $45 profit, +45; the games stall a $20 profit, +20; the plants stall a $15 loss, −15; and the drinks stall a $30 loss, −30. The further right a result sits, the better the stall did; the further left it sits, the smaller its result. The drinks stall's −30 is the smallest result on the line — the poorest result of the four — even though "30" on its own looks greater than "20". With integers, the sign is part of the size.

Two number lines cross: the Cartesian plane

Now let the grid grow. Take a horizontal number line and a vertical number line and cross them **at their zeros**. The pair of lines is the **Cartesian plane**, and the point where they cross is the **origin**, written (0, 0). Each of the two lines is called an **axis** (plural **axes**): the horizontal axis and the vertical axis.

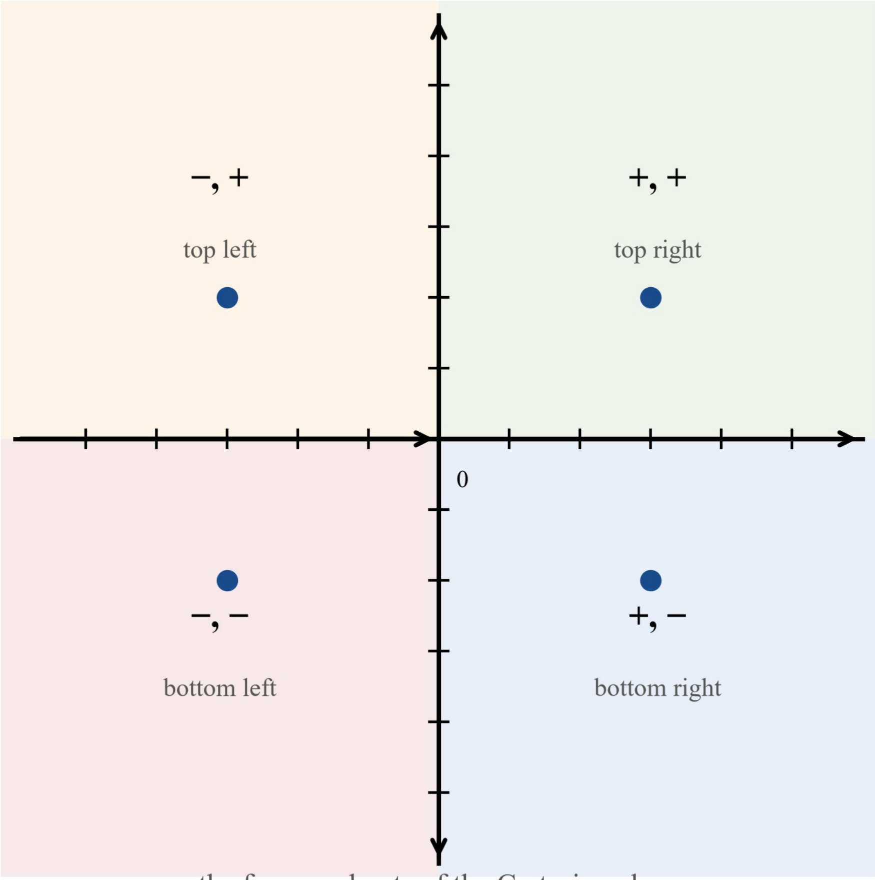


A horizontal number line and a vertical number line, each scaled from −5 to 5 with arrowheads at both ends, crossing at their common zero, which is marked with a red dot labelled "the origin (0, 0)"

A point on the plane is located by an **ordered pair** of integers — **coordinates** — written in brackets with the horizontal move first and the vertical move second: (across, up). Order matters, which is why the pair is *ordered*: (2, 5) and (5, 2) are different points.

The four quadrants

The two crossing lines cut the plane into four quarters called **quadrants**. Because each axis carries negatives as well as positives, every quadrant has its own pair of signs.



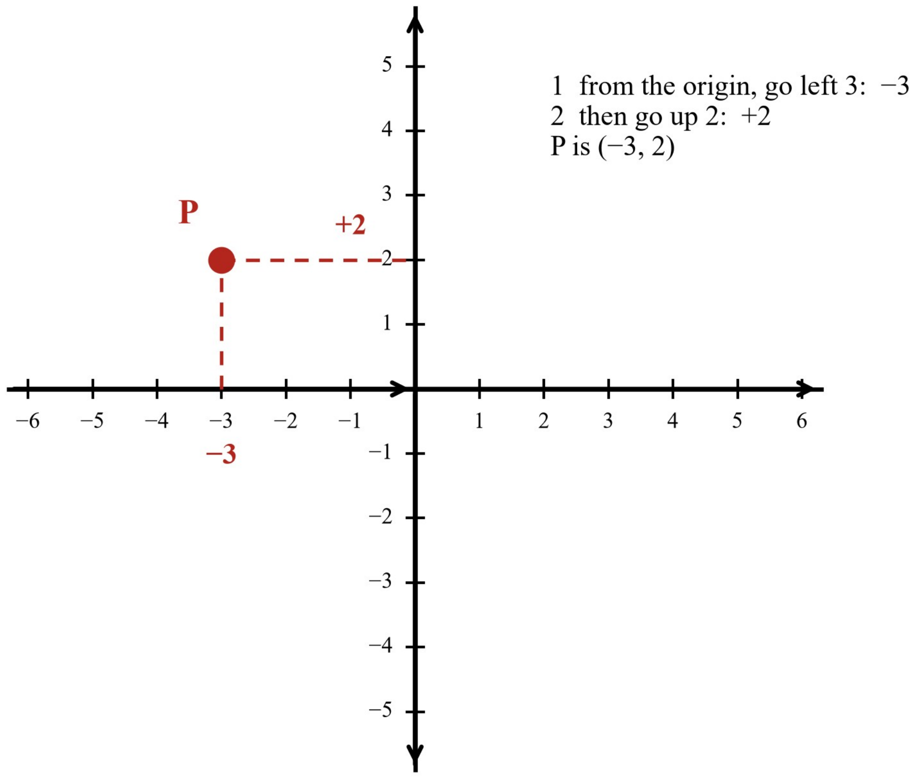
the four quadrants of the Cartesian plane

The Cartesian plane shaded in four colours, one per quadrant, each quadrant showing its sign pair and a sample point: top right "+, +", top left "−, +", bottom left "−, −", bottom right "+, −", captioned "the four quadrants of the Cartesian plane"

Starting top right and moving **anti-clockwise**, the sign pairs are (+, +), (−, +), (−, −) and (+, −). Naming the quadrant is therefore a quick check on any point: the signs of its coordinates say which quarter it lives in.

Reading and plotting points

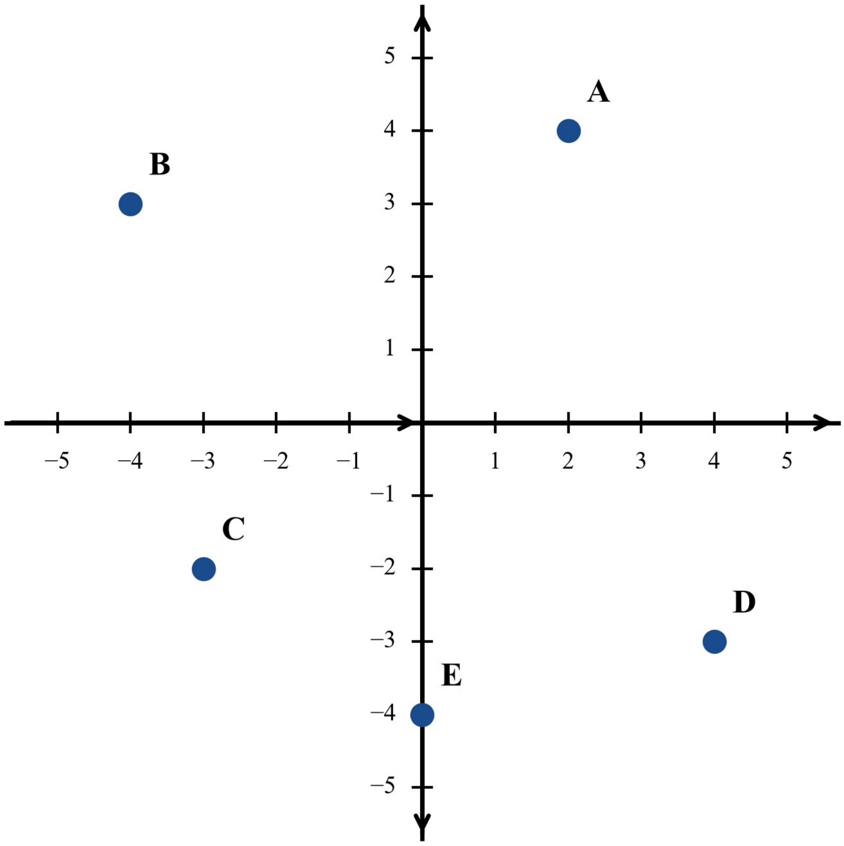
To **read** a point, walk back to each axis. From the origin, how far across, then how far up or down?



The Cartesian plane with a red point P at (−3, 2) joined by dashed guide lines to −3 on the horizontal axis and +2 on the vertical axis, with the steps "1 from the origin, go left 3: −3; 2 then go up 2: +2; P is (−3, 2)"

P sits 3 steps left of the origin, then 2 steps up: P is (−3, +2), usually written (−3, 2). Its signs (−, +) put it in the top-left quadrant.

To **plot** a point, do the walk in reverse: start at the origin, move across by the first coordinate (left if it is negative), then up or down by the second.

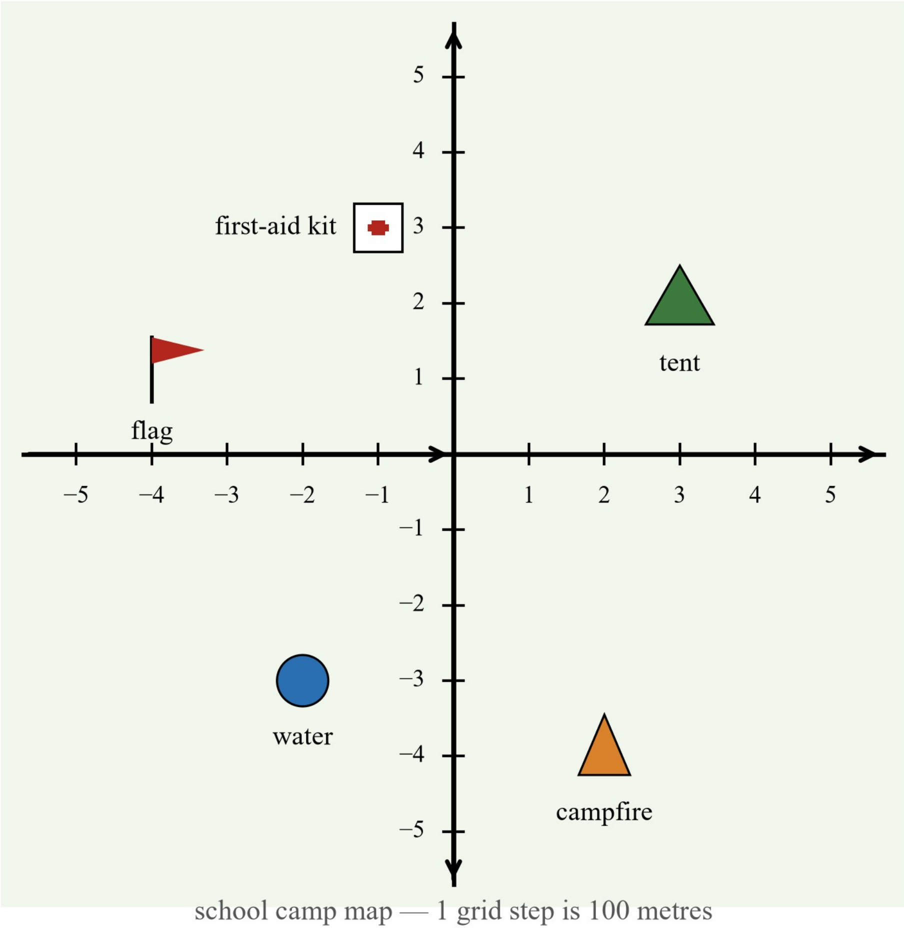


The Cartesian plane with five plotted points: A at (2, 4), B at (−4, 3), C at (−3, −2), D at (4, −3) and E at (0, −4)

A (2, 4) is top right; B (−4, 3) top left; C (−3, −2) bottom left; D (4, −3) bottom right. And E (0, −4) is the trap worth knowing: its first coordinate is 0, so it never leaves the vertical axis. **A point on an axis is not in any quadrant.**

A map on the plane

Once every point has coordinates, a whole map fits inside the plane. On this school-camp map, the origin is the flagpole at the centre of the ground, and one grid step is 100 metres.



school camp map — 1 grid step is 100 metres

A school-camp map on the Cartesian plane scaled from −5 to 5 on both axes: a tent at (3, 2), a flag at (−4, 1), a first-aid kit at (−1, 3), a water point at (−2, −3) and a campfire at (2, −4), captioned "school camp map — 1 grid step is 100 metres"

The tent is at (3, 2), the flag at (−4, 1), the first-aid kit at (−1, 3), the water at (−2, −3) and the campfire at (2, −4). Coordinates make directions short: "meet at (−2, −3)" is a complete set of walking orders from the flagpole — 2 steps left, 3 steps down — with nothing left to argue about.

Three checks for any situation

When a question gives you a situation and asks for the integer, run the same three checks every time.

- **Where is zero?** The ground floor, sea level, freezing, no profit and no loss, the flagpole. The zero is chosen, and everything is measured from it.
- **Which way is positive?** Right, up, profit, above. The other way is negative.
- **How far?** The number part, counted in steps from zero.

Words for this subtopic, with what they mean here:

Term	Meaning here
integer	any of the counting numbers, their opposites below zero, and zero itself
positive integer	an integer to the right of (or above) zero, like +3
negative integer	an integer to the left of (or below) zero, written with the sign −, like −3
zero	the starting point both sides are measured from; not positive and not negative
opposites	two integers the same distance from zero on opposite sides, like +2 and −2

Term	Meaning here
profit / loss	money made (positive) or lost (negative)
Cartesian plane	a horizontal and a vertical number line crossed at their zeros
origin	the point where the two number lines cross, (0, 0)
coordinates	an ordered pair (across, up) locating a point; the first number is written first
quadrant	one of the four quarters of the plane, named by the signs of its points
axis / axes	one of the two number lines that cross to make the Cartesian plane: the horizontal axis and the vertical axis
sea level	the height of the sea's surface, used as zero when heights are measured above and below it

Worked examples

In the with-help examples the thinking is laid out line by line; in the on-your-own examples you carry it. Every example locates, represents or orders integers; adding and subtracting them is Year 7's job.

Example 1 — with help

Mark −4, 1, −6 and 5 on a number line, then write them in order from smallest to largest.

Working	Comment
−4 and −6 sit left of zero: −6 at six steps left, −4 at four. 1 and 5 sit right of zero.	Negative first: left of zero. The greater the number after the sign −, the further left.
Left to right on the line: −6, −4, 1, 5.	Reading the line from the left end is reading smallest to largest.
Order: −6, −4, 1, 5.	Check on the line: each number sits to the right of the one before it. ✓

Example 2 — with help

A lift starts at Floor −1 and travels up to Floor 3. List the floors it passes in order, and say how many floors it moves.

Working	Comment
Floors passed: −1 → 0 → 1 → 2 → 3.	Count straight up the vertical number line — the same trip the lift figure shows.
Jumps between floors: 4.	Count the steps, not the floor labels: five labels, four jumps.
The lift moves up 4 floors.	Check: from −1 to 0 is 1 floor, from 0 to 3 is 3 more — 1 and 3 make 4. ✓

Example 3 — with help

A class runs a fete. The cakes stall costs $55 to set up and takes $100 on the day. The drinks stall costs $80 and takes $50. Write each stall's result as an integer, and say which result is further from zero.

Working	Comment
Cakes: takings $100, costs $55 — $45 made. Profit: **+45**.	Takings above costs is a profit, positive.
Drinks: takings $50, costs $80 — $30 lost. Loss: **−30**.	Takings below costs is a loss, negative.
Distances from zero: 45 steps and 30 steps. The cakes stall's result is further from zero.	Distance from zero ignores the sign: +45 sits 45 steps right, −30 sits 30 steps left.

Example 4 — with help

Read the point P in the figure in the Theory, and name its quadrant.

Working	Comment
From the origin, P is 3 steps left: first coordinate −3.	Across first. Left is negative.
Then 2 steps up: second coordinate +2.	Up second. Up is positive.
P is (−3, 2). Signs (−, +): the **top-left quadrant**.	The sign pair names the quarter, exactly as in the quadrants figure.

Example 5 — on your own

Write the opposite of each number: +5, −7, 12, 0.

The opposite of **+5** is **−5**; of **−7** is **+7**; of **12** (that is, +12) is **−12**; and of **0** is **0**.

Check. Each pair must sit the same number of steps from zero on opposite sides: +5 and −5 both 5 steps out, −7 and +7 both 7, 12 and −12 both 12, and 0 meets its opposite at zero itself. ✓

Example 6 — on your own

For the school concert, the stage is chosen as zero and right is positive. Priya waits 25 m to the right of the stage; Tom waits 40 m to the left. Write each position as an integer, and say who is further from the stage.

Right is positive, so Priya is at **+25** and Tom at **−40**. Distance from the stage ignores the sign: 25 steps against 40, so **Tom** is further from the stage.

Check. The signs point opposite ways (+ and −) and the numbers give the distances — exactly the flagpole picture with a new zero. ✓

Example 7 — on your own

Plot (−2, 4), (5, 1), (−4, −3), (2, −5) and (0, 3) on the Cartesian plane, and name the quadrant of each point, or say when it is in no quadrant.

Start at the origin each time; across first, then up or down. (−2, 4): left 2, up 4 — signs (−, +), **top left**. (5, 1): right 5, up 1 — (+, +), **top right**. (−4, −3): left 4, down 3 — (−, −), **bottom left**. (2, −5): right 2, down 5 — (+, −), **bottom right**. (0, 3): no move across, up 3 — it lands **on the vertical axis, in no quadrant**.

Check. All four sign pairs (+, +), (−, +), (−, −), (+, −) appear once each, one per quadrant — the fifth point tests the rule for a point on an axis. ✓

Example 8 — on your own

Use the school-camp map. (a) Write the coordinates of the campfire and the first-aid kit. (b) Which object is at (−4, 1)? (c) Describe the walk from the flagpole to the tent in steps.

(a) Campfire: right 2, down 4 — **(2, −4)**. First-aid kit: left 1, up 3 — **(−1, 3)**. **(b)** Left 4, up 1 is the **flag**. **(c)** The tent is at (3, 2): from the flagpole, **3 steps right and 2 steps up** — 300 m across and 200 m up, since one step is 100 m.

Check. Reading each plotted object back gives the pair used to place it — the map and the pairs agree. ✓

Practice questions

With help

Q1. Write each description as an integer. (a) 4 above zero. (b) 7 below zero. (c) Zero itself.

Q2. Use the number line in the Theory. Mark −4, 1, −6 and 5 on a copy, then write them in order from smallest to largest.

Q3. True or false? Correct the false ones. (a) −4 is to the left of −1. (b) 2 is to the left of −2. (c) 0 is to the right of −6. (d) −6 is to the right of −3.

Q4. Write the opposite of each number. (a) −8. (b) +6. (c) 9. (d) 0.

Q5. Fill each box with > or <. (a) −3 ☐ 2. (b) −5 ☐ −7. (c) 0 ☐ −9. (d) −4 ☐ 4.

Q6. Use the lift figure. (a) Write the floor two below the ground floor as an integer. (b) A lift travels up from Floor −2 to Floor 2. List, in order, the floors it passes.

Q7. Use the thermometer figure. (a) Write down the coldest marked reading. (b) Write down the hottest. (c) Counting along the scale, how many degrees is the classroom reading above the cool-morning reading?

Q8. Write each fete result as an integer. (a) A profit of $12. (b) A loss of $8. (c) Takings exactly match costs.

Q9. Use the figure of the point P. Write P's coordinates and name its quadrant.

Q10. Use the quadrants figure. Write the sign pair for (a) the top-left quadrant; (b) the bottom-right quadrant.

On your own

Q11. Write these integers in order from smallest to largest: −12, 5, −3, 0, 11, −8.

Q12. List every integer (a) between −3 and 1; (b) between −5 and −3; (c) between 0 and 2.

Q13. From the list −9, 4, −4, 9, 0, pick the two pairs of opposites.

Q14. Always, sometimes or never? (a) The opposite of a number is negative. (b) The opposite of a negative integer is positive. (c) A number and its opposite are the same distance from zero. (d) The opposite of a positive integer is greater than the opposite of a negative integer.

Q15. Use the sea-level figure. (a) Write the five heights in order from highest to lowest. (b) Which of the five is the greatest distance from sea level?

Q16. A delivery robot moves along a painted number line on a warehouse floor, starting at 0, with right positive. (a) It rolls 5 steps left. What integer is its position? (b) It then rolls 2 steps right. What integer now? (c) From there, how many steps right would reach +4?

Q17. At a market, the fruit stall is chosen as zero and left is negative. The bread stall is 15 m to the right; the flower stall is 20 m to the left. (a) Write each stall's position as an integer. (b) Which stall is further from the fruit stall?

Q18. Use the fete figure. (a) Which stall made the largest profit, and which the largest loss? Write both results as integers. (b) Order the four results from smallest to largest. (c) How many dollars from zero is the games stall's result?

Q19. Plot (−5, 2), (3, 4), (−2, −4), (5, −1) and (0, 3). Name the quadrant of each point, or say when it lies on an axis.

Q20. Name the quadrant of a point whose coordinates are (a) both positive; (b) negative then positive; (c) both negative; (d) positive then negative.

Q21. (a) Write the ordered pair for "4 left and 2 down" from the origin. (b) Point B in the plotting figure is (−4, 3). Write the point whose coordinates are the opposites of B's, and give the letter of that point in the figure.

Q22. Use the school-camp map. (a) Write the coordinates of the water. (b) Which object is at (2, −4)? (c) Name the quadrant the tent is in. (d) Which object lies in the bottom-left quadrant?

Maths in real life

Q23. The summit of Mount Kosciuszko is Australia's highest mountain, 2228 m above sea level. The bed of Kati Thanda–Lake Eyre is Australia's lowest point, 15 m below sea level. (a) Write both heights as integers, taking sea level as zero. (b) Sketch a vertical number line with a scale of 500 m per step from −500 to 2500 and mark both integers on it. (c) Which of the two is further from sea level?

Source: Wikipedia, *Mount Kosciuszko*; Wikipedia, *Lake Eyre*. Retrieved 20 August 2026.

Q24. On a cold Victorian winter morning the thermometer outside the school hall read −2 °C at 7 am. By 1 pm the same thermometer read 9 °C. (a) Write both readings as integers. (b) Counting along the number line, how many degrees did the temperature climb between morning and afternoon? (c) Mark both readings on a copy of the thermometer scale in the Theory.

Q25. A city car park has three levels below the ground and five above. The ground level is 0. (a) Write the level two below the ground as an integer. (b) A car enters at level −3 and the lift carries it to level 4. Counting level by level, how many levels does it rise? (c) List the negative integers in this car park.

Choosing your strategy

Q26. Without drawing, decide which is greater: −17 or −7. Give your reason in one sentence about the number line.

Q27. Find the flaw. Sam writes "−8 is greater than −3, because 8 is greater than 3." Explain Sam's mistake, and write the true statement.

Q28. Always, sometimes or never? Give an example each way. (a) The opposite of an integer is negative. (b) A positive integer is greater than a negative integer. (c) Two opposite integers are the same distance from zero. (d) A point whose first coordinate is negative is in the top-left quadrant.

Q29. Make your own. Choose a spot (a tree, a door, a goal post) as zero and decide which way is positive. Write the integers for two positions, one each side of zero. Swap with a partner and draw each other's number line.

Answers

Q1. (a) +4; (b) −7; (c) 0. **Q2.** −6, −4, 1, 5. **Q3.** (a) true; (b) false — 2 is right of −2; (c) true; (d) false — −6 is left of −3. **Q4.** (a) +8; (b) −6; (c) −9; (d) 0. **Q5.** (a) <; (b) >; (c) >; (d) <. **Q6.** (a) −2; (b) −2, −1, 0, 1, 2. **Q7.** (a) −5 °C; (b) 37 °C; (c) 12 degrees. **Q8.** (a) +12; (b) −8; (c) 0. **Q9.** (−3, 2), top-left quadrant. **Q10.** (a) −, +; (b) +, −. **Q11.** −12, −8, −3, 0, 5, 11. **Q12.** (a) −2, −1, 0; (b) −4; (c) 1. **Q13.** −9 and 9; 4 and −4. **Q14.** (a) sometimes; (b) always; (c) always; (d) never. **Q15.** (a) +8, 0, −3, −6, −9; (b) the submarine, −9 m. **Q16.** (a) −5; (b) −3; (c) 7 steps. **Q17.** (a) bread +15, flowers −20; (b) the flower stall. **Q18.** (a) cakes +45; drinks −30; (b) −30, −15, +20, +45; (c) 20 dollars. **Q19.** (−5, 2) top left; (3, 4) top right; (−2, −4) bottom left; (5, −1) bottom right; (0, 3) on the vertical axis. **Q20.** (a) top right; (b) top left; (c) bottom left; (d) bottom right. **Q21.** (a) (−4, −2); (b) (4, −3), point D. **Q22.** (a) (−2, −3); (b) the campfire; (c) top right; (d) the water. **Q23.** (a) +2228 and −15; (b) −15 just below 0, +2228 between 2000 and 2500; (c) Mount Kosciuszko. **Q24.** (a) −2 and +9; (b) 11 degrees; (c) marks at −2 and 9. **Q25.** (a) −2; (b) 7 levels; (c) −1, −2, −3. **Q26.** −7 — it sits to the right of −17. **Q27.** Sam compared the distances from zero and forgot the sides; −8 < −3. **Q28.** (a) sometimes; (b) always; (c) always; (d) sometimes. **Q29.** sample: door is 0, right positive: the bin at 3 m left is −3, the tap at 5 m right is +5.

All the working

Q1. (a) Above zero is positive: **+4**. (b) Below zero is negative: **−7**. (c) Zero is zero: **0** — not positive and not negative.

Q2. On the line, −6 is left of −4, which is left of 1, which is left of 5: **−6, −4, 1, 5**. Left to right is smallest to largest.

Q3. (a) **True** — −4 is further left than −1. (b) **False** — 2 is to the **right** of −2. (c) **True** — 0 is right of every negative integer. (d) **False** — −6 is to the **left** of −3, so it is smaller.

Q4. Opposites swap sides and keep the distance: (a) **+8**; (b) **−6**; (c) **−9**; (d) **0**.

Q5. (a) −3 < 2 — negatives sit left of positives. (b) −5 > −7 — −5 is to the right of −7. (c) 0 > −9 — zero is right of every negative. (d) −4 < 4 — left of zero is smaller than right of zero.

Q6. (a) Two below the ground floor is **−2**. (b) Counting up the line from −2 to 2: **−2, −1, 0, 1, 2** — four jumps between five floor labels.

Q7. (a) The lowest mark is the freezer: **−5 °C**. (b) The highest is your body: **37 °C**. (c) From 12 up to 24 is **12 degrees**, counted along the scale.

Q8. (a) Profit is positive: **+12**. (b) Loss is negative: **−8**. (c) Takings match costs exactly: **0**.

Q9. Across: 3 left gives −3; up: 2 gives +2. P is **(−3, 2)**; signs (−, +) put it in the **top-left quadrant**.

Q10. (a) Top left: **−, +**. (b) Bottom right: **+, −**.

Q11. Left to right on the imagined line: **−12, −8, −3, 0, 5, 11**. Because 12 and 8 are greater than 3, −12 and −8 sit to the left of −3.

Q12. (a) Between −3 and 1: **−2, −1, 0**. (b) Between −5 and −3: only **−4**. (c) Between 0 and 2: only **1**.

Q13. Same distance, opposite sides: **−9 and 9**, and **4 and −4**. The 0 is its own opposite and pairs with nothing here.

Q14. (a) **Sometimes** — the opposite of +6 is −6 (negative), but the opposite of −6 is +6 (not), and the opposite of 0 is 0 (not). (b) **Always** — flipping a left-of-zero number lands it right of zero. (c) **Always** — same distance is the definition of opposites. (d) **Never** — the opposite of a positive integer is negative, and the opposite of a negative integer is positive, and a negative is never greater than a positive: the opposite of +3 is −3, the opposite of −5 is +5, and −3 < +5.

Q15. (a) Top to bottom: **+8, 0, −3, −6, −9**. (b) The greatest distance from 0 is the **submarine at −9 m** — 9 steps out, against the bird's 8.

Q16. (a) 5 steps left of 0: **−5**. (b) 2 steps right from −5: **−3**. (c) From −3 to +4: 3 steps to zero and 4 more — **7 steps**.

Q17. (a) Bread, 15 m right: **+15**. Flowers, 20 m left: **−20**. (b) The **flower stall** — 20 steps from zero against 15.

Q18. (a) Largest profit **cakes, +45**; largest loss **drinks, −30**. (b) Smallest to largest is left to right: **−30, −15, +20, +45**. (c) +20 sits **20 dollars** from zero.

Q19. (−5, 2): (−, +) **top left**. (3, 4): (+, +) **top right**. (−2, −4): (−, −) **bottom left**. (5, −1): (+, −) **bottom right**. (0, 3): first coordinate 0 — **on the vertical axis, no quadrant**.

Q20. (a) (+, +) **top right**. (b) (−, +) **top left**. (c) (−, −) **bottom left**. (d) (+, −) **bottom right**.

Q21. (a) 4 left, 2 down: **(−4, −2)**. (b) Opposites of (−4, 3) are (4, −3) — that is point **D**.

Q22. (a) Water: **(−2, −3)**. (b) (2, −4) is the **campfire**. (c) The tent (3, 2) is (+, +): **top right**. (d) Bottom left (−, −) holds the **water**.

Q23. (a) Above sea level positive: **+2228**; below, negative: **−15**. (b) On a scale of 500: −15 sits a hair below the 0 mark; 2228 sits between the 2000 and 2500 marks, a little under halfway between them. (c) **Mount Kosciuszko** — 2228 steps from zero against 15.

Q24. (a) Morning **−2**, afternoon **+9**. (b) From −2 up to 0 is 2 degrees, from 0 up to 9 is 9 — counting on, **11 degrees**. (c) −2 just below the 0 mark, 9 just below the 10 mark.

Q25. (a) Two below the ground: **−2**. (b) −3 → −2 → −1 → 0 → 1 → 2 → 3 → 4: **7 levels**. (c) The below-ground levels: **−1, −2, −3**.

Q26. −7 is greater. On the number line −7 sits to the right of −17; left means smaller, whatever the number parts look like.

Q27. Sam compared how far each number is from zero and ignored which side. 8 steps left is further out than 3 steps left, but further left is **smaller**: **−8 < −3**. The true statement is "−8 is smaller than −3".

Q28. (a) **Sometimes.** The opposite of +4 is −4 (negative); the opposite of −4 is +4 (not); the opposite of 0 is 0 (not). (b) **Always.** Every positive integer sits right of zero and every negative left, so any positive beats any negative: +1 > −100. (c) **Always.** Same distance, opposite sides, is what opposites are. (d) **Sometimes.** (−2, 5) is top left, but (−2, −5) is bottom left — a negative first coordinate only fixes the left half, not the quarter.

Q29. Sample: the door is 0 and right is positive: the bin 3 m left is **−3**, the tap 5 m right is **+5**. A good swap has the zero and the positive direction stated, or the integers cannot be drawn.

What comes next

That closes Chapter 1. In **1B** the number line grew a left half and the grid grew into four quarters: the integers, located and ordered around a chosen zero; the sign as direction from zero; profit and loss, floors, sea level and thermometers as vertical and horizontal number lines; and the Cartesian plane with its origin, ordered pairs and four quadrants.

In this book the plane comes straight back in **6B**, where ordered pairs in all four quadrants become the language for describing how a point's coordinates change when the point moves — 6B stands on this subtopic's quadrants and refers back here. Chapter 2 opens with **2A**, where the number line you extended today carries halves, thirds and quarters.

In Year 7 the integers themselves come back with new tools: adding and subtracting them (`VC2M7N08`) and placing negative fractions and decimals on the line (`VC2M7N03`). Everything there starts from the picture this subtopic built — a zero, a direction, and a distance.

Test

Mathematics Level 6 · Chapter 1 — Integers and number properties · Chapter test T1

Marks	10 (5 marks for each subtopic)
Guidance time	About 15 minutes: 5 minutes reading time, then about 1 minute per mark
Equipment	None — no calculator and no digital tool for this paper

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 6 achievement standard, not against a mark on this paper.

Instructions

- Answer every question, and show your working clearly.
- The marks for each question or part are shown in brackets.
- No calculator and no digital tool may be used.

Subtopic	Content description	Marks
1A Prime, composite, square and triangular numbers	`VC2M6N02`	5
1B Integers on the number line and the Cartesian plane	`VC2M6N01`	5
Total		**10**

1A · Prime, composite, square and triangular numbers

Q1. Look at the numbers 1, 23, 33 and 58.

a. Sort them into prime, composite, or not in either family. (1 mark)
b. Use the definition of a prime to explain why 1 is not a prime number. (1 mark)

Q2. Write 72 as a product of its prime factors. (1 mark)

Q3. Use factors and regrouping to simplify $4 \times 19 \times 5$ — without doing a hard multiplication first. Show the regrouping, and give the answer. (1 mark)

Q4. Which of these numbers is both a square number and a triangular number? (1 mark)

A. 16 B. 21 C. 36 D. 45

1B · Integers on the number line and the Cartesian plane

Q5. Write each description as an integer. (1 mark)

a. A temperature of 8 °C below zero
b. A profit of $15
c. A floor 3 below the ground floor, with the ground floor labelled 0

Q6. Without drawing, decide which integer is smaller: −12 or −4. Give a one-line reason about the number line. (1 mark)

Q7. At the class market, the gardening club’s stall takes in $60 and costs $85 to run. The art club’s stall takes in $90 and costs $35. Write each club’s result as an integer. (1 mark)

Q8. Point P is 3 steps left and 4 steps down from the origin of the Cartesian plane.

a. Write the ordered pair (across first, then up or down) for P. (1 mark)
b. Name the quadrant P sits in. Point Q is (5, −2): name the quadrant Q sits in. (1 mark)

Solutions

Answer key

Q	Answer	Marks
1	a. prime: 23; composite: 33, 58; not in either family: 1 · b. 1 has only one factor; a prime has exactly two	2
2	$72 = 2 \times 2 \times 2 \times 3 \times 3$	1
3	$(4 \times 5) \times 19 = 20 \times 19 = 380$	1
4	C (36)	1
5	a. −8 · b. +15 · c. −3	1
6	−12 — it sits further left on the number line	1
7	Gardening club −25; art club +55	1
8	a. (−3, −4) · b. P bottom-left quadrant; Q bottom-right quadrant	2
	Total	**10**

Fully worked solutions

Q1. Count the factors of each number — that is what the definitions do.

Number	Factors	Family
23	1, 23 — two factors	prime
33	1, 3, 11, 33 — $33 = 3 \times 11$	composite
58	1, 2, 29, 58 — $58 = 2 \times 29$	composite
1	1 — one factor	not in either family

a. Prime: **23**. Composite: **33, 58**. Not in either family: **1**.
b. A prime has **exactly two** factors. The number 1 has only **one** factor — itself — so it does not fit the definition, and 1 is not prime. (It is not composite either: a composite needs more than two factors.)

Marks: 1 mark for all four numbers sorted correctly; 1 mark for the reason — 1 has only one factor, a prime needs exactly two.

Q2. Split 72 into smaller factors until everything is prime. One path: $72 = 8 \times 9$, then $8 = 2 \times 2 \times 2$ and $9 = 3 \times 3$. So $72 = 2 \times 2 \times 2 \times 3 \times 3$. Check: $2 \times 2 \times 2 = 8$, $3 \times 3 = 9$, and $8 \times 9 = 72$. ✓ Any other factor tree for 72 ends at the same five primes.

Q3. Look for factors that pair into a friendly number: 4 and 5 make 20. Regroup: $4 \times 19 \times 5 = \left(4 \times 5\right) \times 19 = 20 \times 19$. Then, since $2 \times 19 = 38$, $20 \times 19 = 380$. Check a different way: $4 \times 19 \times 5 = 4 \times 95 = 380$. ✓ The regrouping turned the calculation into a multiplication by 20 instead of a hard one.

Q4. Test each number against both families.

- 16: $16 = 4 \times 4$, a square — but the triangular list 1, 3, 6, 10, 15, 21, … jumps over 16, so it is not triangular.
- 21: $21 = 1 + 2 + 3 + 4 + 5 + 6$, triangular — but 21 sits between $4^2 = 16$ and $5^2 = 25$, so it is not a square.
- 36: $36 = 6 \times 6$, a square; and $36 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$, the 8th triangular number. **36 is in both families.**
- 45: triangular ($1 + 2 + \cdots + 9$) but not a square — it sits between $6^2 = 36$ and $7^2 = 49$.

The answer is **C**.

Q5. Below zero, below costs and below the starting point are negative; profit is positive. a. 8 °C below zero: **−8**. b. A profit of $15: **+15**. c. Three floors below the ground floor (0): **−3**.

Marks: 1 mark for all three integers correct, with the right signs.

Q6. Both integers are negative, so both sit left of zero. On the number line, −12 is 12 steps left of zero and −4 is 4 steps left — −12 is further left, and left means smaller. So **−12 is smaller**: $-12 < -4$. The number parts (12 and 4) compare the wrong way round; the number line decides.

Q7. Compare takings with costs for each club.

- Gardening club: takings $60, costs $85 — costs are $25 more than takings ($85 - 60 = 25$), a loss of $25: **−25**.
- Art club: takings $90, costs $35 — takings are $55 more than costs ($90 - 35 = 55$), a profit of $55: **+55**.

Marks: 1 mark for both integers with the correct signs.

Q8. a. From the origin, across first: 3 steps left gives first coordinate −3. Then up or down: 4 steps down gives second coordinate −4. P is **(−3, −4)**. b. P's signs are (−, −), which is the **bottom-left quadrant**. Q is (5, −2): signs (+, −), the **bottom-right quadrant**.

Marks: 1 mark for the ordered pair; 1 mark for both quadrants.