Mathematics Level 5

Chapter 8 — Probability

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Contents

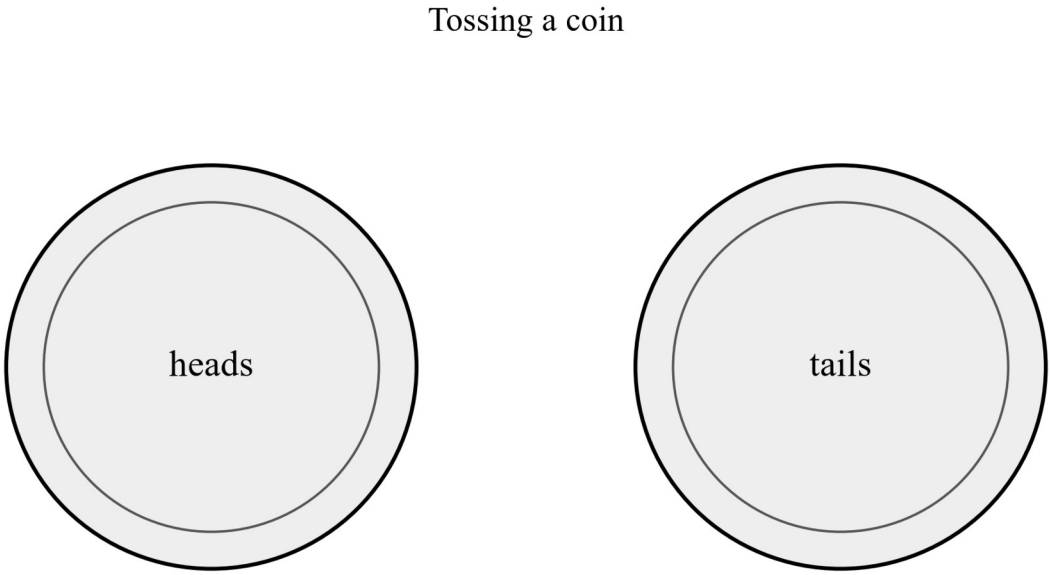
Listing outcomes: equally likely and not equally likely

Code `VC2M5P01` (Victorian Curriculum F–10 Version 2.0, Mathematics, Level 5; authority file `maths_5.json`): "list the possible outcomes of chance experiments involving equally likely outcomes and compare to those that are not equally likely". This section draws on `VC2M5P01.E01`, `.E02`, `.E03`, `.E04`, `.E05` and `.E06`.

Theory

What is a chance experiment?

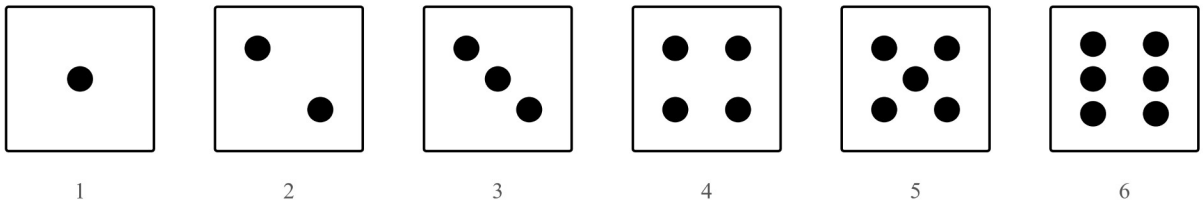
A **chance experiment** is a thing you do where you know what *could* happen, but you cannot know what *will* happen before you do it. Tossing a coin is a chance experiment. So is rolling a dice, or drawing a card without looking. Each possible result of a chance experiment is called an **outcome**.



Two coins side by side. The left coin is labelled heads and the right coin is labelled tails. Caption: 2 outcomes, heads or tails, and they are equally likely.

Tossing a coin has **2 outcomes**: heads or tails.

Rolling a fair die



6 outcomes: 1, 2, 3, 4, 5, 6. Each number is equally likely.

Six faces of a fair die in a row, showing 1, 2, 3, 4, 5 and 6 pips. Caption: 6 outcomes, and each number is equally likely.

Rolling a fair die has **6 outcomes**: 1, 2, 3, 4, 5 or 6.

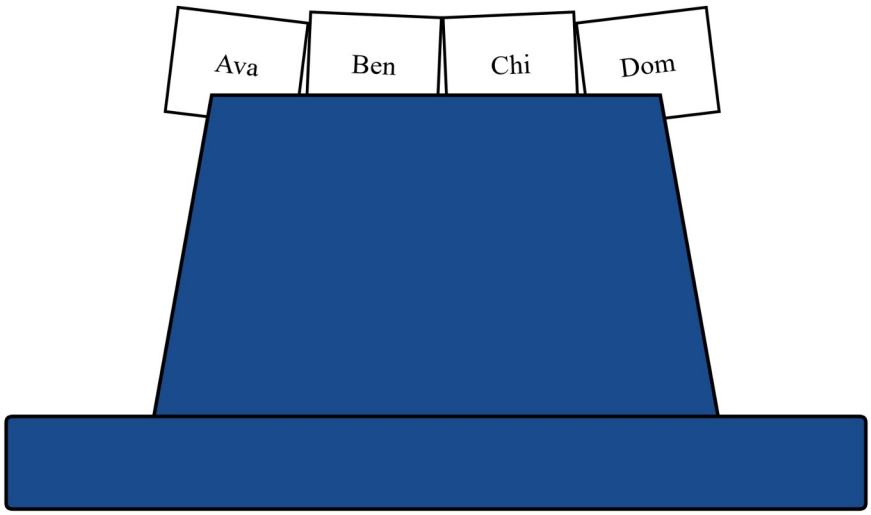
The first job in any chance experiment is to **list the outcomes**.

Equally likely outcomes

Outcomes are **equally likely** when each one has the same chance of happening.

Heads and tails are equally likely on a coin. Each of the six numbers is equally likely on a fair die.

Names in a hat



4 names, 4 outcomes. Each name is equally likely to be drawn.

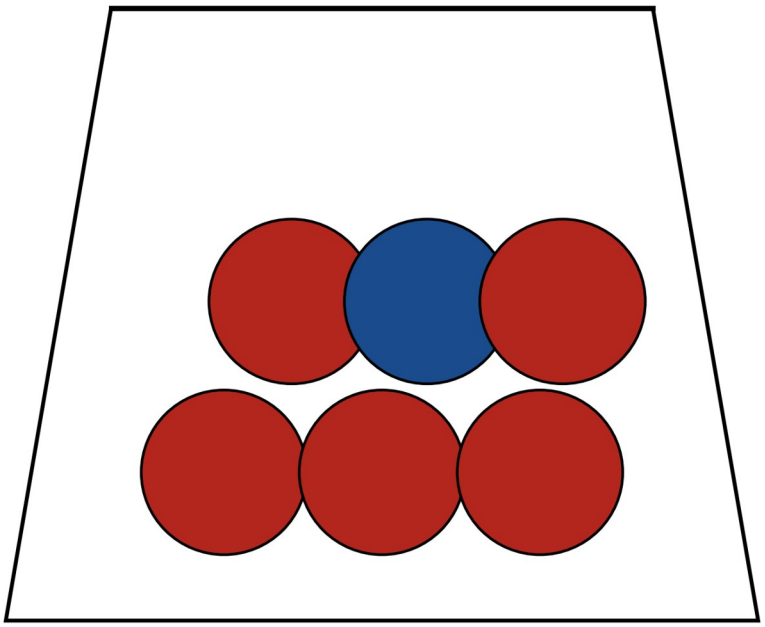
A blue hat with four folded name papers poking out of the top, labelled Ava, Ben, Chi and Dom. Caption: 4 names, 4 outcomes, and each name is equally likely to be drawn.

Put the names Ava, Ben, Chi and Dom in a hat and draw one without looking. There are 4 outcomes, one for each name, and each name is equally likely to be drawn. A chance experiment like this, where all the outcomes are equally likely, is called a **random** chance experiment.

Not equally likely outcomes

Not every chance experiment is like that. Sometimes one outcome has a better chance than another.

Drawing a counter from a bag



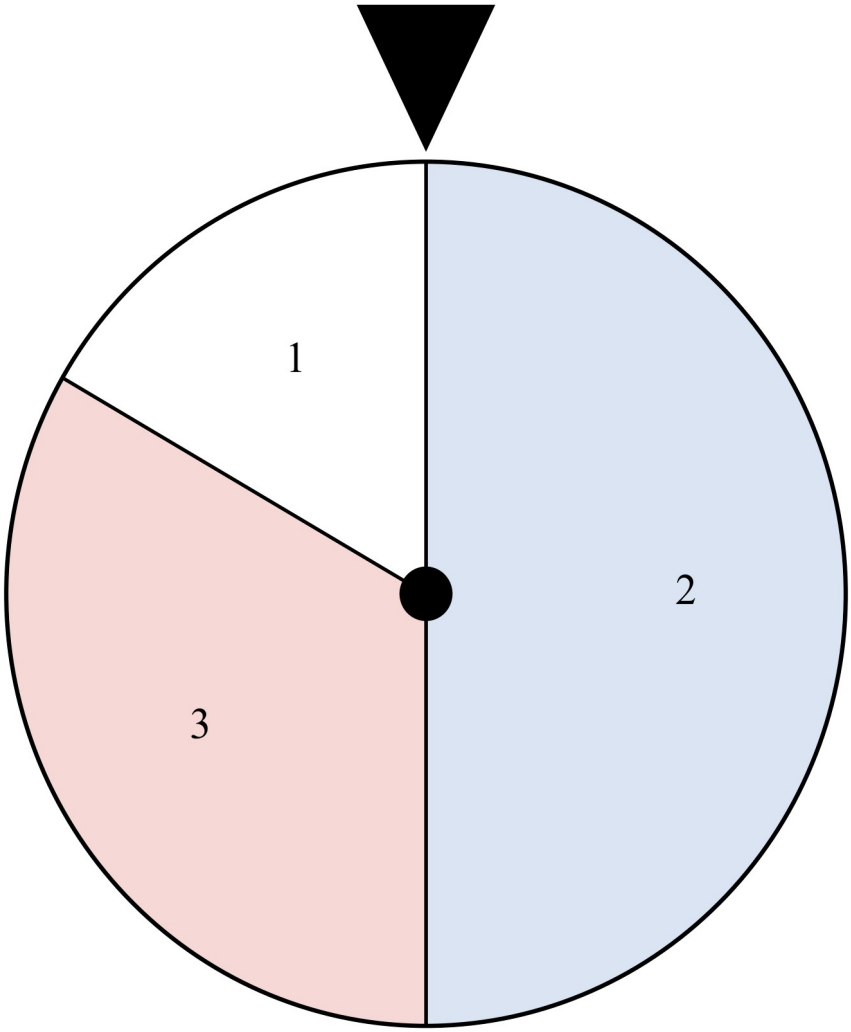
5 red counters and 1 blue counter

Drawing red is more likely than drawing blue.

An open bag holding 6 counters: 5 red counters and 1 blue counter. Caption: drawing red is more likely than drawing blue.

This bag holds 5 red counters and 1 blue counter. Drawing a counter has **2 outcomes**: red or blue. But the two outcomes are **not equally likely**. Each of the 6 counters is just as easy to draw as any other, and 5 of the 6 are red, so drawing red is **more likely**. Drawing blue is **less likely**.

A spinner with unequal regions



Region 2 is the biggest, so 2 is most likely.

Region 1 is the smallest, so 1 is least likely.

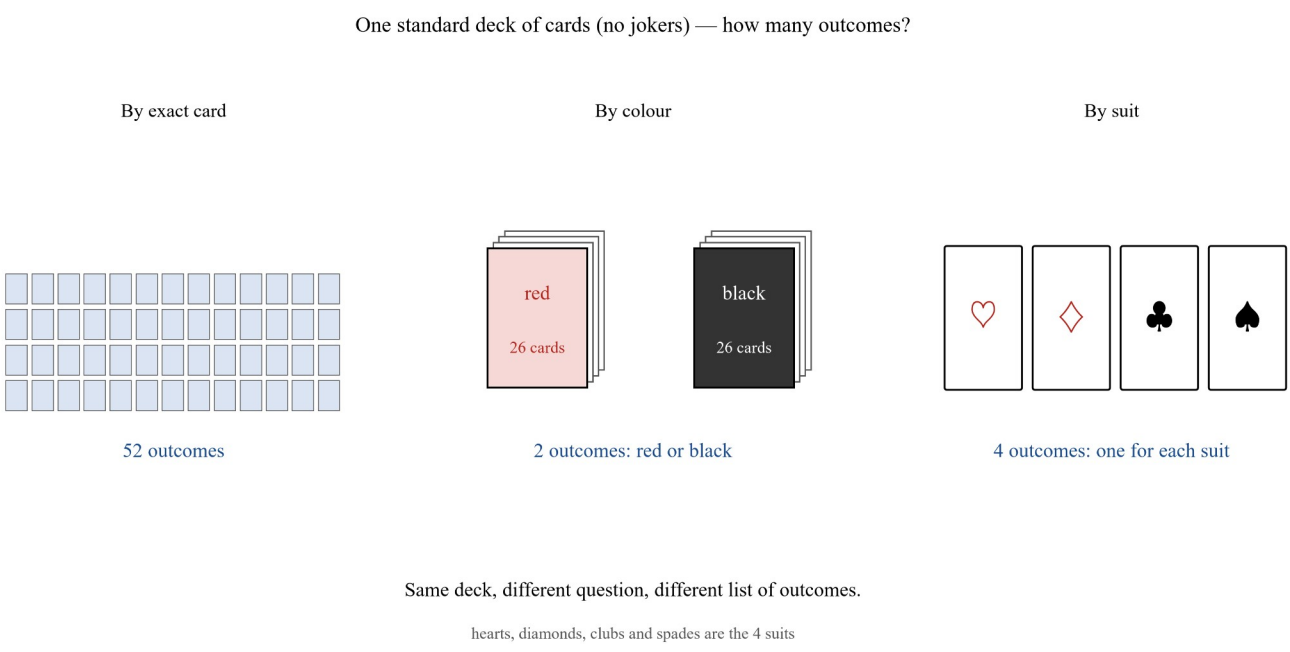
A spinner with a pointer and 3 regions of different sizes, labelled 1, 2 and 3. Region 2 is half of the spinner, region 3 is a third and region 1 is the smallest of the three. Captions: region 2 is the biggest, so 2 is most likely; region 1 is the smallest, so 1 is least likely.

This spinner has 3 outcomes: 1, 2 or 3. The regions are not the same size. Region 2 takes up the most of the spinner, so landing on 2 is **most likely**. Region 1 takes up the least, so landing on 1 is **least likely**. The outcomes are not equally likely.

To compare chances, first list the outcomes, then ask: does each outcome have the same chance, or not?

How many outcomes? It depends what you ask

One chance experiment can have different lists of outcomes, depending on what you are asking about.



One standard deck of cards shown three ways. Left, 52 card backs in 13 columns of 4, labelled by exact card, 52 outcomes. Middle, a red pile and a black pile of 26 cards each, labelled by colour, 2 outcomes, red or black. Right, one card for each suit, hearts, diamonds, clubs and spades, labelled by suit, 4 outcomes. Caption: same deck, different question, different list of outcomes.

Draw one card from a standard deck of cards (52 cards, no jokers).

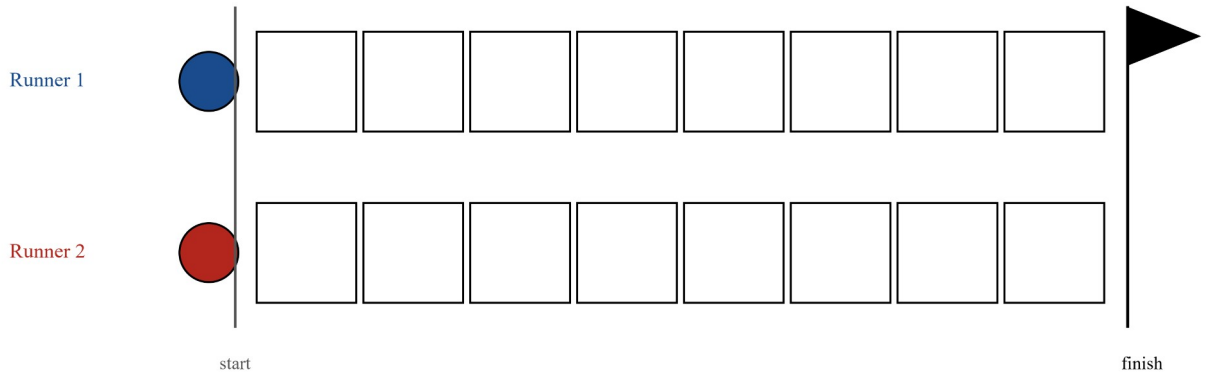
- Ask about the **exact card**: there are **52 outcomes**.
- Ask about the **colour**: there are **2 outcomes**, red or black.
- Ask about the **suit**: there are **4 outcomes**, hearts, diamonds, clubs or spades.

Same deck, same draw. The list of outcomes changes with the question you ask.

Is the race fair?

A game is **fair** when each player has the same chance of winning. To check a game, list the outcomes and count how many give each player a win.

The race game



Take turns to roll 2 dice and work out the difference.

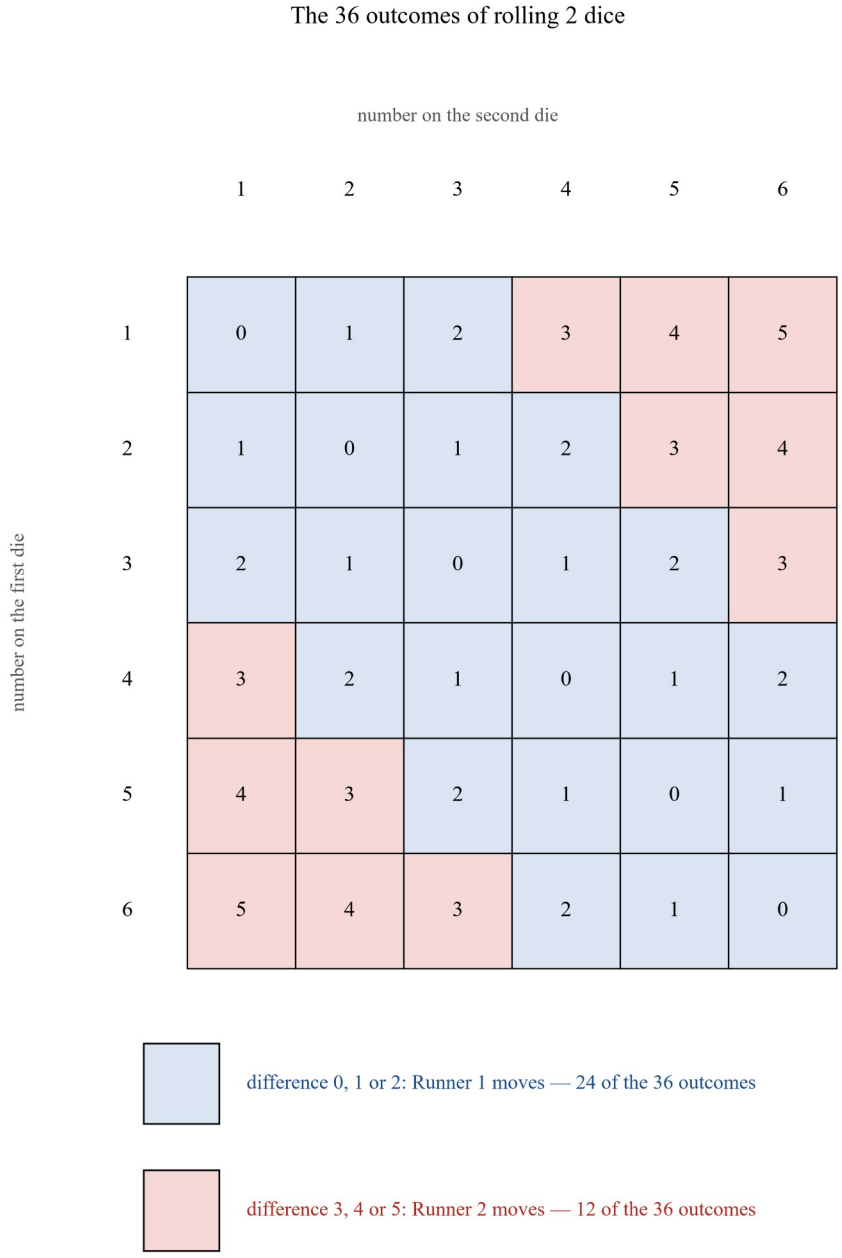
Runner 1 moves a square if the difference is 0, 1 or 2.

Runner 2 moves a square if the difference is 3, 4 or 5.

A race track with 2 lanes of 8 squares, a start line and a finish flag. A blue Runner 1 token and a red Runner 2 token sit at the start. Captions: take turns to roll 2 dice and work out the difference; Runner 1 moves a square if the difference is 0, 1 or 2; Runner 2 moves a square if the difference is 3, 4 or 5.

Two players race. Take turns to roll 2 dice and work out the difference between the numbers. Runner 1 moves a square if the difference is 0, 1 or 2. Runner 2 moves a square if the difference is 3, 4 or 5.

At first look the game seems fair: each runner moves on three differences. But are the differences equally likely? To find out, list **all** the outcomes of rolling 2 dice. The first die has 6 faces and so does the second, so there are $6 \times 6 = 36$ outcomes. The grid shows every one, with its difference.



A 6 by 6 grid of all 36 outcomes of rolling 2 dice. Each cell shows the difference between the two dice. Cells with difference 0, 1 or 2 are shaded blue; cells with difference 3, 4 or 5 are shaded red. Legend: difference 0, 1 or 2 means Runner 1 moves, 24 of the 36 outcomes; difference 3, 4 or 5 means Runner 2 moves, 12 of the 36 outcomes.

Count the blue cells: Runner 1 moves on **24** of the 36 outcomes. Count the red cells: Runner 2 moves on only **12**. Runner 1 moves twice as often, so the game is **not fair**.

The grid answers three good questions:

- *Is this game fair?* No. Runner 1 has the better chance.
- *Are some differences more likely than others?* Yes. A difference of 1 comes up in 10 cells, but a difference of 5 comes up in only 2 cells.
- *How can you work that out?* By listing all 36 outcomes and counting.

Making a game fair

A game with equally likely outcomes is fair when each player wins on the same number of them.

Rolling one die: you win on an even number (2, 4, 6) and your friend wins on an odd number (1, 3, 5). Each player wins on 3 of the 6 equally likely outcomes, so the game is fair.

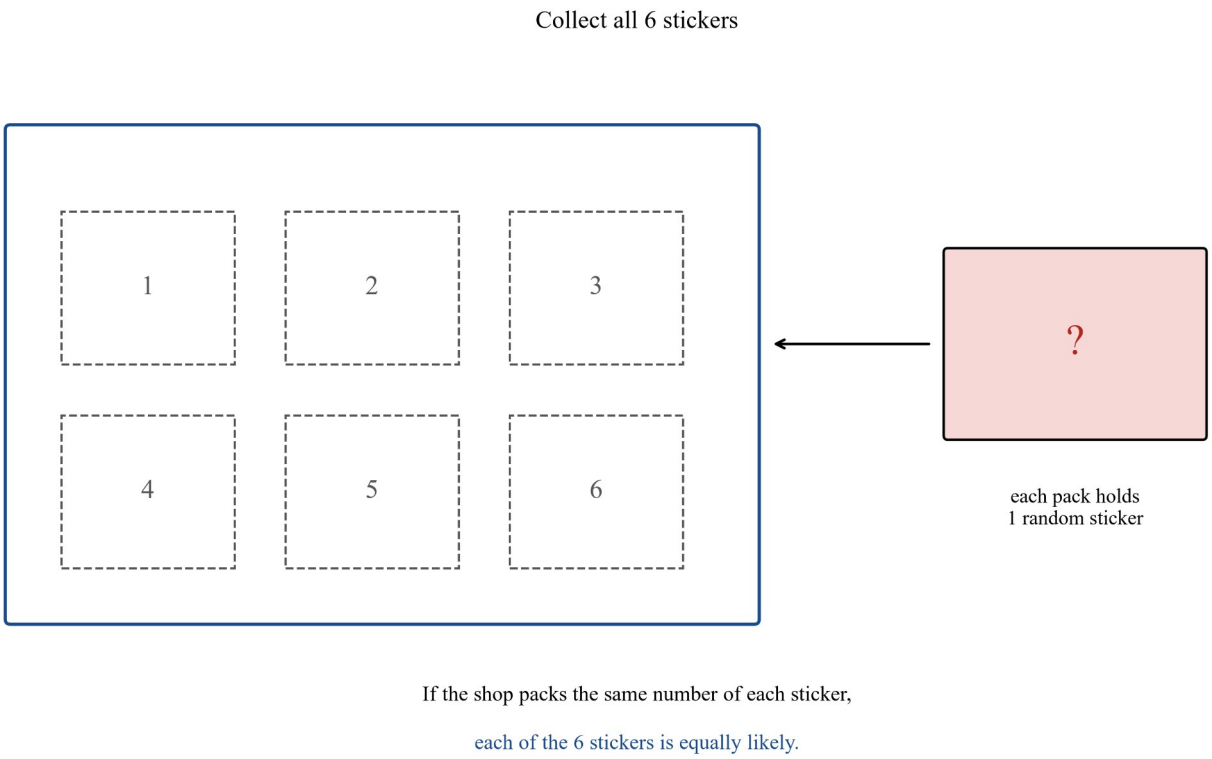
The race can be fixed the same way. Each runner needs half of the 36 cells: 18 each. One fair rule is this:

- Runner 1 moves on a difference of **0, 1 or 5**: that is 6 + 10 + 2 = 18 cells.
- Runner 2 moves on a difference of **2, 3 or 4**: that is 8 + 6 + 4 = 18 cells.

Check it on the grid above. With those rules, both runners move on 18 of the 36 outcomes, and the race is fair.

Chance in real life

Supermarkets and shops often run promotions where each pack holds one random sticker, and you collect the set.



A sticker album with 6 empty dashed slots numbered 1 to 6, and one sealed pack marked with a question mark, with an arrow into the album. Caption: if the shop packs the same number of each sticker, each of the 6 stickers is equally likely.

If the shop packs the same number of each sticker, then each sticker is equally likely to be in your pack. If the shop packs lots of some stickers and only a few of others, the rare ones are less likely, and the outcomes are not equally likely. Seeing the difference is exactly the skill of this section: list the outcomes, then compare their chances.

Worked examples

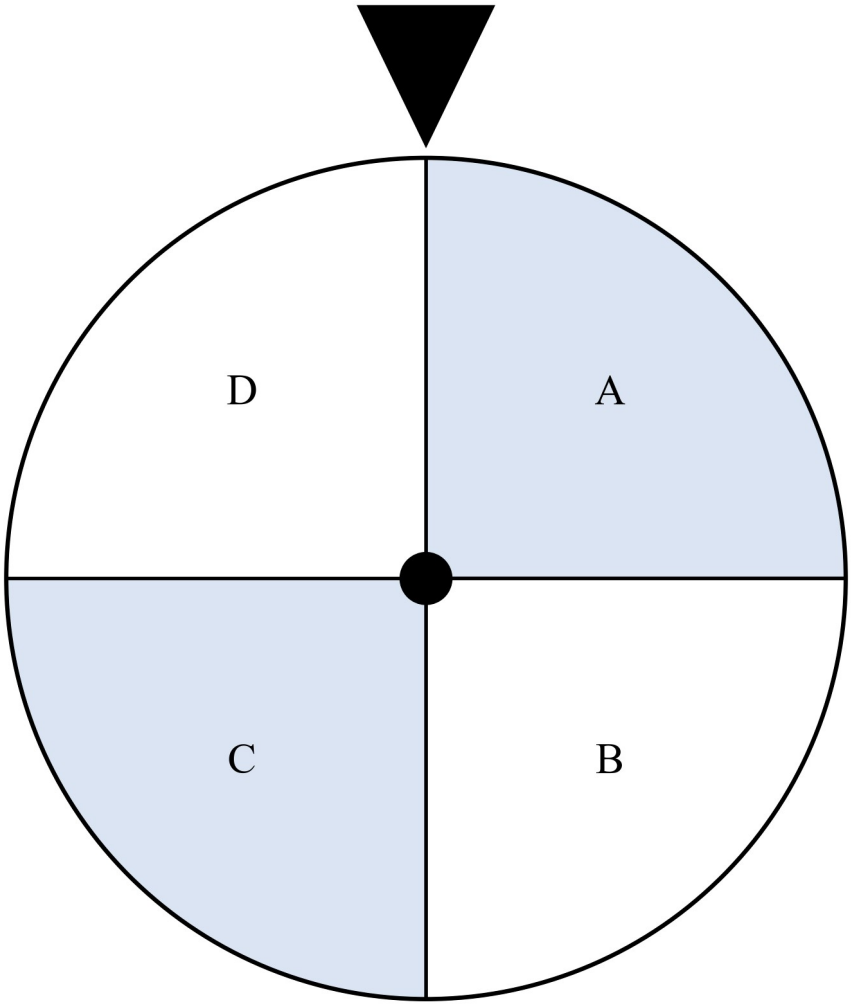
Example 1 — with help: list the outcomes

A spinner has 4 equal regions labelled A, B, C and D. List the possible outcomes. Are they equally likely?

Working	Comment
The pointer can stop on A, B, C or D.	List every result
4 outcomes	Count the list
The regions are the same size, so the outcomes are equally likely.	Same size means same chance

So the outcomes are A, B, C and D, and they are equally likely.

A spinner with equal regions



4 equal regions: 4 equally likely outcomes

A spinner with a pointer and 4 equal regions labelled A, B, C and D. Caption: 4 equal regions, 4 equally likely outcomes.

Example 2 — with help: two questions, one deck

Maya draws one card from a standard deck (no jokers). How many outcomes are there if she is interested in (a) the colour? (b) the suit?

Working	Comment
(a) Red or black: 2 outcomes	Ask about the colour
(b) Hearts, diamonds, clubs or spades: 4 outcomes	Ask about the suit
The deck is the same; the question changed the list.	Outcomes depend on what you ask

Example 3 — with help: which is more likely?

A bag holds 5 red counters and 2 blue counters. One counter is drawn without looking. Are the two colours equally likely? Which colour is more likely?

Working	Comment
Outcomes: red or blue	List first
5 red counters, 2 blue counters	Count each kind
5 is more than 2, so red is more likely. The colours are not equally likely.	More counters means a better chance

Example 4 — on your own: a coin

List the outcomes of tossing one coin. Are they equally likely?

Heads or tails: **2 outcomes, equally likely.**

Example 5 — on your own: names in a hat

The names of all 12 students in a class are placed in a hat and one is drawn. How many outcomes are there? Is the draw random?

12 outcomes (one per name). Each name is equally likely, so **yes, it is a random draw.**

Example 6 — on your own: is it fair?

Noah and Priya play a game with a fair die. Noah wins if the die shows 1, 2 or 3. Priya wins if it shows 4, 5 or 6. Is the game fair?

Noah wins on 3 of the 6 equally likely outcomes and Priya wins on the other 3. **Yes, the game is fair.**

Practice questions

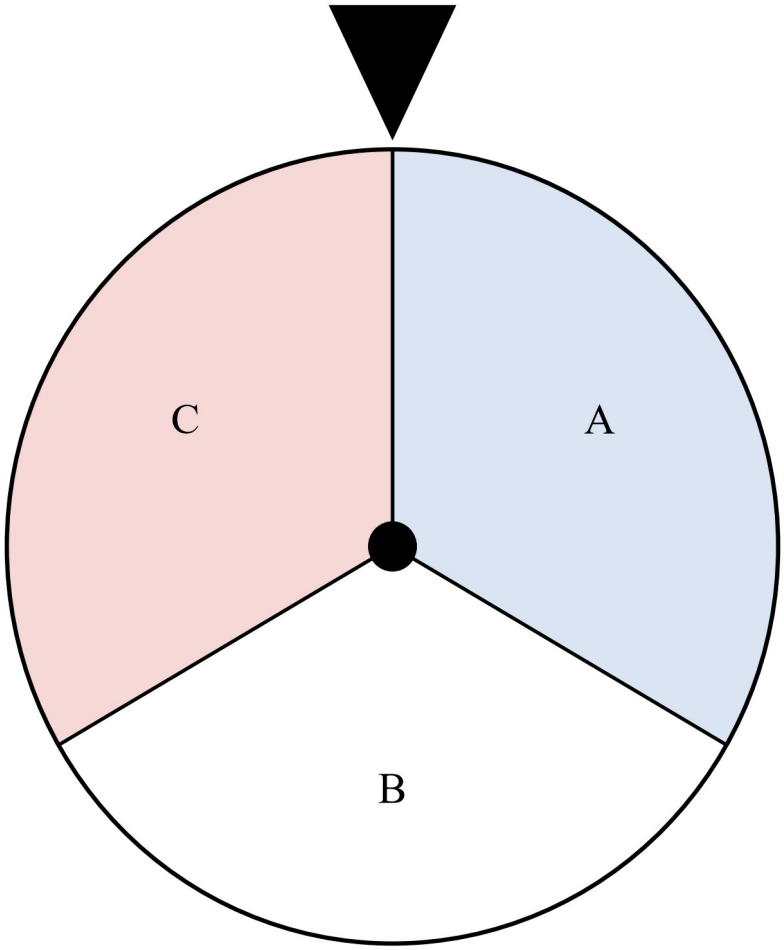
With help

Q1. Toss a coin. List the outcomes. Are they equally likely?

Q2. Roll a fair die. How many outcomes are there? Are they equally likely?

Q3. The spinner below has 3 equal regions. List the outcomes. Are they equally likely?

The spinner for Q3

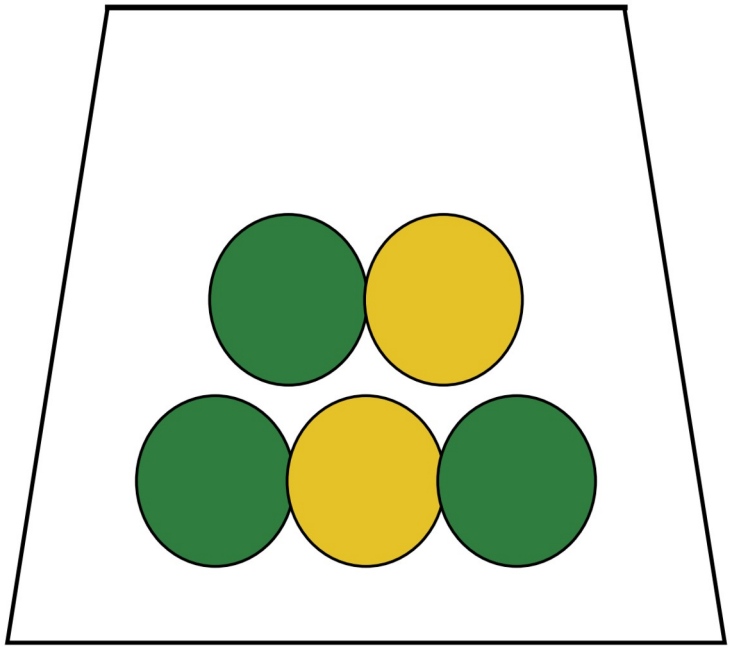


The 3 regions are equal.

A spinner with a pointer and 3 equal regions labelled A, B and C. Caption: the 3 regions are equal.

Q4. The bag below holds 3 green counters and 2 yellow counters. One counter is drawn without looking. Which colour is more likely? Are the colours equally likely?

The bag for Q4



3 green counters and 2 yellow counters

An open bag holding 5 counters: 3 green counters and 2 yellow counters.

Q5. One card is drawn from a standard deck (no jokers). How many outcomes are there if you are interested in: a. the exact card? b. the colour? c. the suit?

Q6. A hat holds the names of 15 students. One name is drawn. a. How many outcomes are there? b. Is each name equally likely to be drawn?

Q7. Which of these chance experiments have equally likely outcomes? a. tossing a coin b. drawing a counter from a bag holding 6 red and 2 blue counters c. rolling a fair die

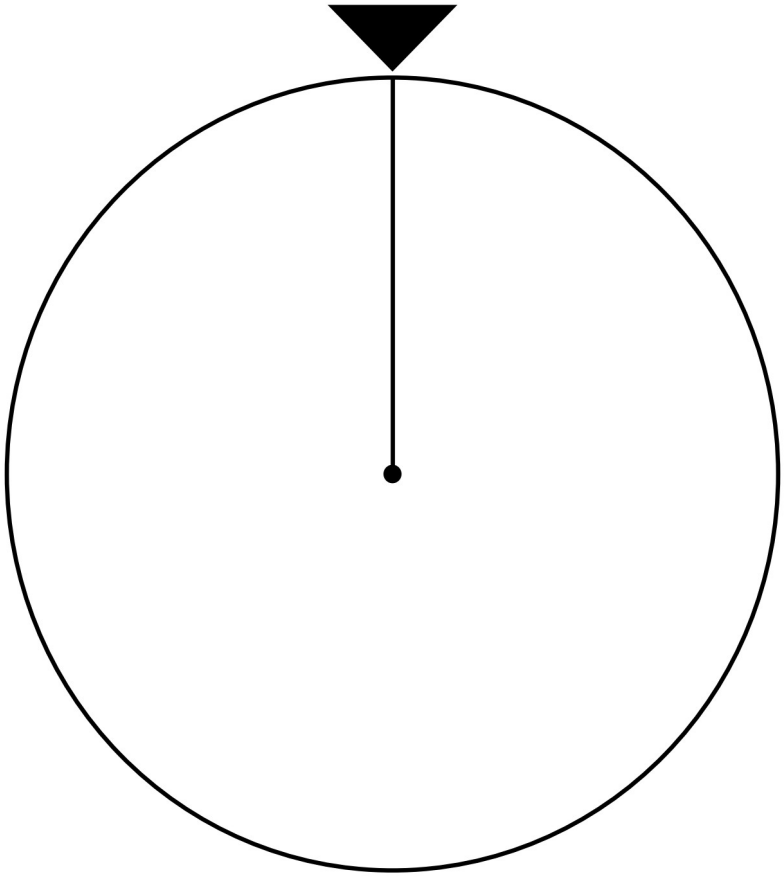
Q8. Two players roll a fair die. Player 1 wins on 1, 2 or 3. Player 2 wins on 4, 5 or 6. Is the game fair? Explain.

On your own

Q9. A fair die is rolled. Which is more likely: rolling an even number, or rolling a 6? Explain your answer.

Q10. The spinner below has one line drawn already. Draw one more straight line so the spinner is split into 2 equal parts and a two-player game on it would be fair.

The spinner for Q10



One line is drawn already.

Draw one more straight line to split the spinner into 2 equal parts.

A spinner with a pointer and one straight line drawn from the centre to the top edge. Caption: draw one more straight line to split the spinner into 2 equal parts.

Q11. A bag holds 4 red counters and 1 blue counter. How many blue counters must you add so that drawing red and drawing blue are equally likely?

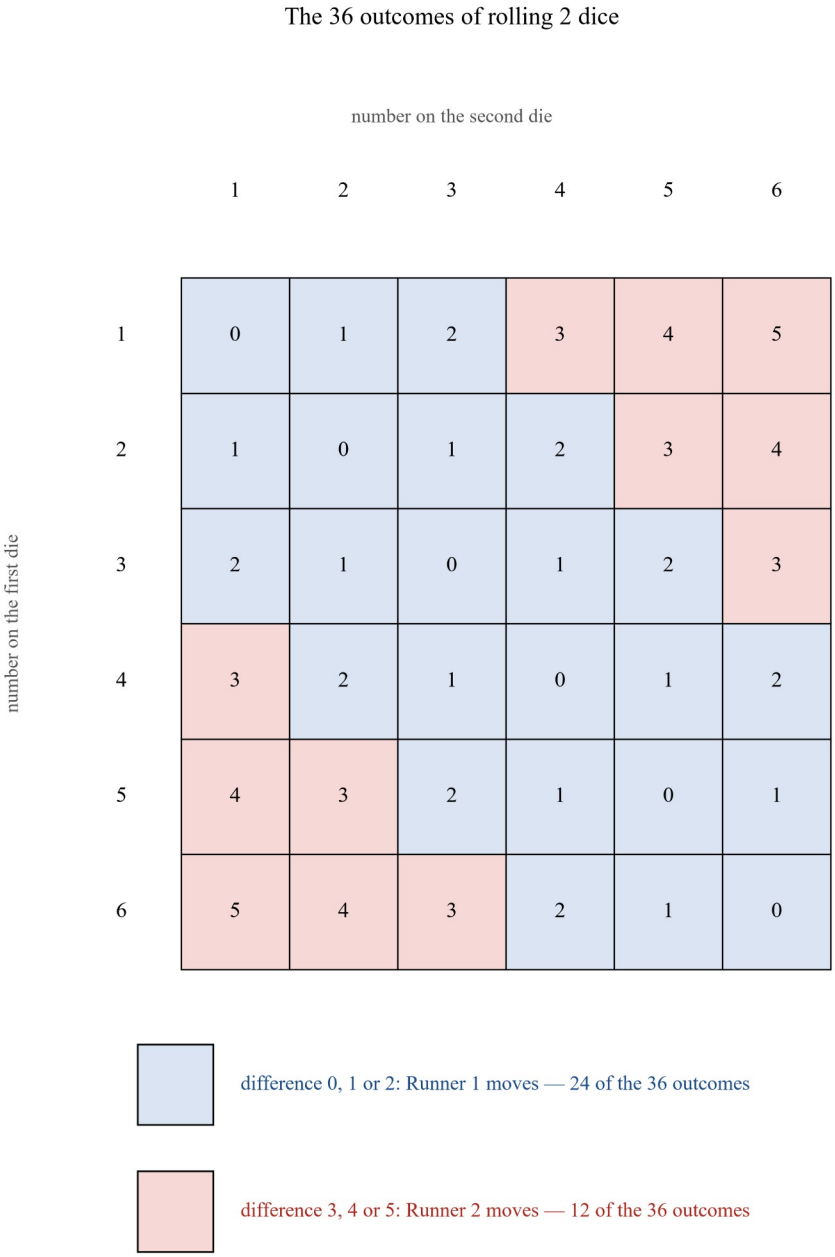
Q12. Use the spinner with unequal regions from the theory (regions 1, 2 and 3). a. Are the outcomes equally likely? b. Which number is most likely? c. Which number is least likely?

Q13. Lara rolls 2 dice and adds the two numbers. The sums run from 2 to 12. Is each sum equally likely? Explain. (Think: how many ways make 2? How many make 7?)

Q14. One card is drawn from a standard deck (no jokers). You are only interested in whether the card is a heart. How many outcomes are you interested in? What are they?

Q15. Design a fair game for 2 players using one fair die. Say what each player wins on, and explain why your game is fair.

Q16. In the race game from the theory, which single difference is most likely to come up? Use the grid of 36 outcomes to explain.



A 6 by 6 grid of all 36 outcomes of rolling 2 dice. Each cell shows the difference between the two dice. Cells with difference 0, 1 or 2 are shaded blue; cells with difference 3, 4 or 5 are shaded red. Legend: difference 0, 1 or 2 means Runner 1 moves, 24 of the 36 outcomes; difference 3, 4 or 5 means Runner 2 moves, 12 of the 36 outcomes.

Maths in real life

Q17. In supermarket promotions, each pack holds one random sticker, and there are 6 stickers to collect. If the shop packs the same number of each sticker, is each sticker equally likely to be in your pack? The shop then packs lots of 4 of the stickers and only a few of the other 2. Are the 6 stickers still equally likely? Which stickers are less likely?

Q18. Two friends play rock, paper, scissors. Each player throws rock, paper or scissors. a. List the possible throws for one player. b. Is the game fair for the two players? Explain.

Q19. At a sports day, two teams choose ends with a coin toss: Team A wins the toss if it lands heads, Team B if it lands tails. Is this fair? Explain.

Answers

Q1. Heads or tails; yes, equally likely. **Q2.** 6 outcomes; yes, equally likely. **Q3.** A, B, C; yes, equally likely. **Q4.** Green; no, not equally likely. **Q5.** a. 52. b. 2. c. 4. **Q6.** a. 15. b. Yes. **Q7.** a and c. **Q8.** Yes: each player wins on 3 of the 6 equally likely outcomes. **Q9.** Rolling an even number: 3 outcomes (2, 4, 6) against 1 outcome (6). **Q10.** A straight line from the centre to the edge, opposite the first line. **Q11.** 3 blue counters. **Q12.** a. No. b. 2. c. 1. **Q13.** No: 7 can be made in more ways than 2. **Q14.** 2 outcomes: a heart, or not a heart. **Q15.** Any split of the 6 outcomes into two groups of 3, for example 1, 2, 3 against 4, 5, 6. **Q16.** A difference of 1: it fills 10 cells, more than any other difference. **Q17.** Yes at first; no after the change; the 2 rare stickers are less likely. **Q18.** a. Rock, paper or scissors. b. Yes: both players have the same 3 throws. **Q19.** Yes: heads and tails are equally likely.

Worked answers

Q1. A coin lands heads or tails, and each side has the same chance. ∴ The outcomes are heads and tails, equally likely.

Q2. A fair die shows 1, 2, 3, 4, 5 or 6, and each face has the same chance. ∴ 6 outcomes, equally likely.

Q3. The pointer stops on A, B or C, and the regions are the same size. ∴ 3 outcomes, equally likely.

Q4. Each of the 5 counters is just as easy to draw as any other, and 3 of the 5 are green. ∴ Green is more likely, and the colours are not equally likely.

Q5. a. Every card is different: 52 outcomes. b. Red or black: 2 outcomes. c. Hearts, diamonds, clubs or spades: 4 outcomes.

Q6. a. One outcome per name: 15. b. One name is drawn without looking, so each name has the same chance: yes.

Q7. a. Heads and tails are equally likely: yes. b. 6 red against 2 blue is not equal: no. c. All 6 faces are equally likely: yes. ∴ a and c.

Q8. Each of the 6 faces is equally likely; Player 1 wins on 3 faces and Player 2 on the other 3. ∴ The game is fair.

Q9. Even means 2, 4 or 6: 3 outcomes. Rolling a 6 is 1 outcome. 3 outcomes against 1. ∴ Rolling an even number is more likely.

Q10. The two parts must be the same size. The first line runs from the centre to the edge; a second line from the centre to the edge straight opposite it splits the circle into two equal halves. ∴ Draw the second line opposite the first.

Q11. Drawing red and drawing blue are equally likely when the bag holds the same number of each: 4 red needs 4 blue. $4 - 1 = 3$. ∴ Add 3 blue counters.

Q12. a. The regions are different sizes: no. b. Region 2 is the biggest: 2 is most likely. c. Region 1 is the smallest: 1 is least likely.

Q13. A sum of 2 comes only from 1 + 1: one way. A sum of 7 comes from 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2 or 6 + 1: six ways. ∴ Not equally likely: 7 is more likely than 2.

Q14. The card is a heart, or it is not. ∴ 2 outcomes: heart, not a heart.

Q15. Example: Player 1 wins on 1, 2 or 3; Player 2 wins on 4, 5 or 6. Each player wins on 3 of the 6 equally likely outcomes. ∴ The game is fair. (Any two groups of 3 faces each is fair.)

Q16. Count the cells for each difference on the grid: 0 comes up 6 times, 1 comes up 10 times, 2 comes up 8, 3 comes up 6, 4 comes up 4 and 5 comes up 2. ∴ A difference of 1 is most likely.

Q17. With the same number of each sticker packed, each of the 6 is equally likely. After the change, 2 stickers are packed in smaller numbers. ∴ No longer equally likely: the 2 rare stickers are less likely.

Q18. a. One player can throw rock, paper or scissors: 3 outcomes. b. Both players have the same 3 throws, each equally likely. ∴ The game is fair.

Q19. Heads and tails are equally likely, and each team wins on one of them. ∴ The coin toss is fair.

Repeated chance experiments and frequency

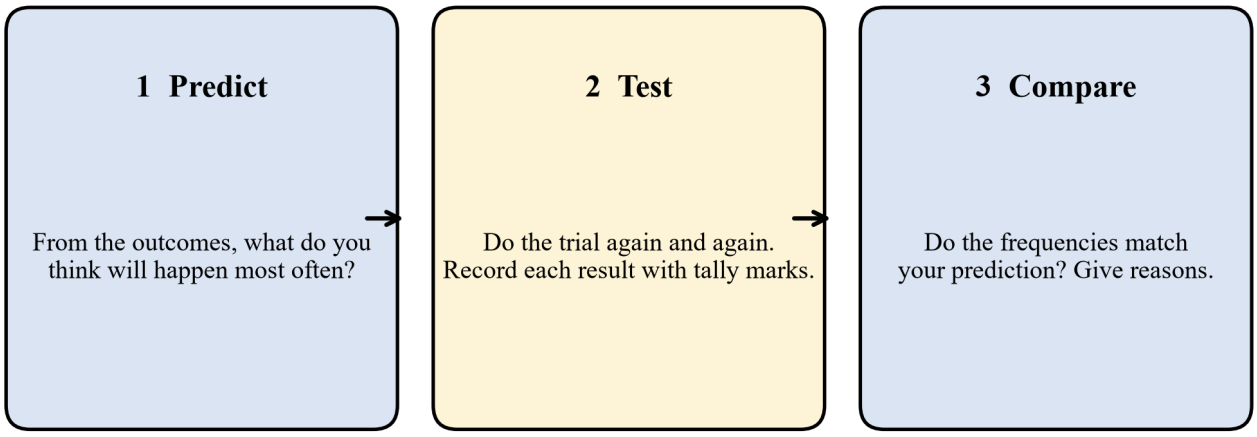
Code VC2M5P02 (Victorian Curriculum F–10 Version 2.0, Mathematics, Level 5; authority file `maths_5.json`): "conduct repeated chance experiments, including those with and without equally likely outcomes, and observe and record the results; use frequency to compare outcomes and estimate their likelihoods". This section draws on `VC2M5P02.E01`, `.E02`, `.E03`, `.E04` and `.E05`. `VC2M5P02.E06` is excluded under CONTENT_POLICY.md Rule 1.

Theory

The method: predict, test, compare

A **chance experiment** is something you can do again and again, where you cannot say for certain what the result will be before you do it. Flipping a coin, spinning a spinner and drawing a card are all chance experiments. One go of the experiment is called a **trial**. The things that can happen are the **outcomes** — you listed these in 8A.

In this section we use the same three steps for every chance experiment.



The same three steps for every chance experiment in this section

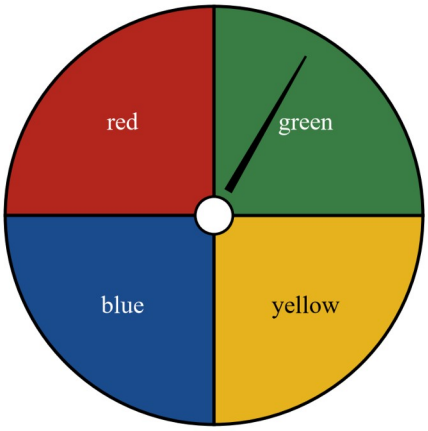
Three steps joined by arrows. Step 1, Predict: from the outcomes, what do you think will happen most often? Step 2, Test: do the trial again and again and record each result with tally marks. Step 3, Compare: do the frequencies match your prediction? Give reasons.

At the end, the recorded results tell us what actually happened. We use them to compare the outcomes and to estimate how likely each one is.

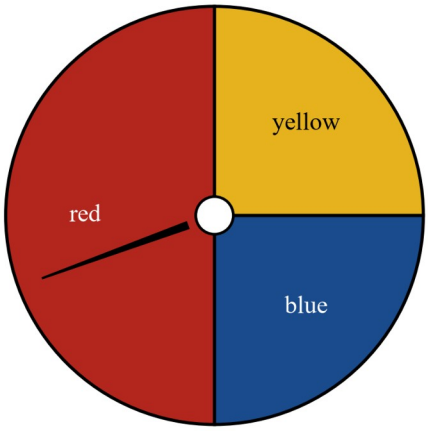
Equally likely outcomes, and outcomes that are not

Flip a fair coin. The outcomes are **heads** and **tails**, and both have the same chance of coming up: they are **equally likely**. Roll a fair die and the six numbers are equally likely too, because each number is on exactly one face.

Spinners work the same way. Equal parts give equally likely outcomes; unequal parts do not.



Spinner A: 4 equal parts



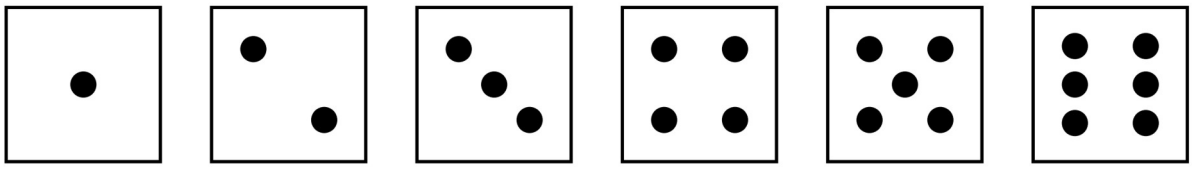
Spinner B: parts not equal

Two spinners. Spinner A is cut into 4 equal parts coloured red, blue, yellow and green, so each colour covers one quarter of the circle. Spinner B has parts that are not equal: red covers half of the circle, blue covers one quarter and yellow covers one quarter.

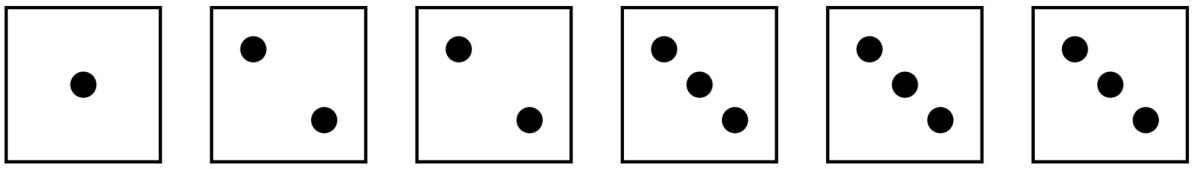
On **Spinner A** the four parts are equal, so red, blue, yellow and green are equally likely. On **Spinner B** the parts are not equal. Red covers half of the circle, more than blue or yellow, so the outcomes are **not equally likely**: we predict red will come up most often.

Dice and bags can be unequal too.

A fair die: each number is on one face

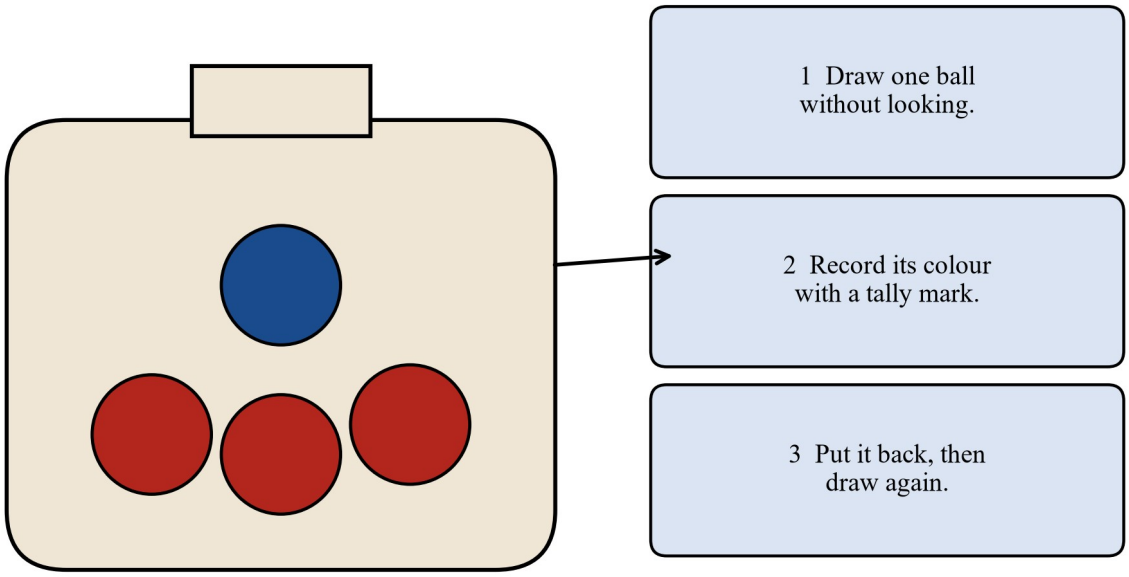


A special die: 2 and 3 are on more than one face



Two dice shown as their six faces. Top row: a fair die with faces 1, 2, 3, 4, 5, 6, where each number is on one face. Bottom row: a special die with faces 1, 2, 2, 3, 3, 3, where 2 is on two faces and 3 is on three faces.

The **fair die** has each number on one face, so 1, 2 and 3 are equally likely. The **special die** has 3 on three of its six faces and 1 on only one face, so the outcomes are not equally likely: 3 should come up most often.

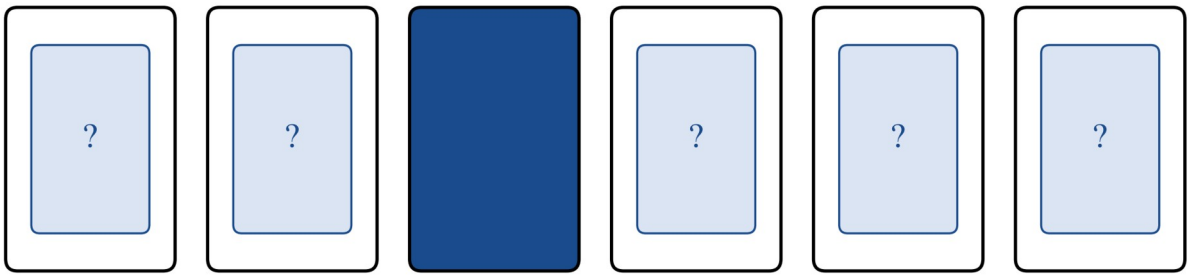


3 red balls, 1 blue ball

A bag holding 3 red balls and 1 blue ball, with the rule in three steps: 1, draw one ball without looking; 2, record its colour with a tally mark; 3, put it back, then draw again.

The bag has 3 red balls and only 1 blue ball, so red and blue are **not** equally likely. Drawing a red ball should happen more often. Putting the ball back each time keeps the bag the same for every trial.

6 cards in a pile: 3 blue, 2 red, 1 yellow



blue

Shuffle, turn over one card, record the colour, put it back, shuffle again

A pile of 6 cards spread out: 3 blue, 2 red, 1 yellow. Five cards show their backs with a question mark and one card is turned over, showing blue. Below: shuffle, turn over one card, record the colour, put it back, shuffle again.

The pile has 3 blue cards, 2 red cards and 1 yellow card. The three colours are not equally likely: blue should be turned over most often and yellow least often.

Recording results: tally and frequency

Predictions are only predictions. To find out what really happens, we **conduct** the experiment: do the trial again and again and record every result with a tally mark. Counting the tally marks for one outcome gives a number. That number is called the **frequency** of the outcome.

The frequency of an outcome is the number of times it happens.

A class flipped a coin 20 times and recorded the results.

Coin flips: 20 trials

Outcome	Tally	Frequency
Heads		11 out of 20
Tails		9 out of 20

Heads: 11 of the 20 flips, that is $\frac{11}{20}$. Tails: 9 of the 20 flips, that is $\frac{9}{20}$.

Tally chart of 20 coin flips. Heads shows two bundles of five tally marks and one extra mark, with frequency 11 out of 20. Tails shows one bundle of five and four extra marks, with frequency 9 out of 20. Below the chart: heads is 11 of the 20 flips, the fraction 11 over 20, and tails is 9 of the 20 flips, the fraction 9 over 20.

Heads happened 11 times, so the frequency of heads is **11**. We can also say heads came up **11 out of 20** flips, or write it as the fraction $\frac{11}{20}$ of the flips. Tails came up 9 out of 20 flips: frequency 9, or $\frac{9}{20}$. Three ways to say the same thing — a count, an "out of", and a fraction.

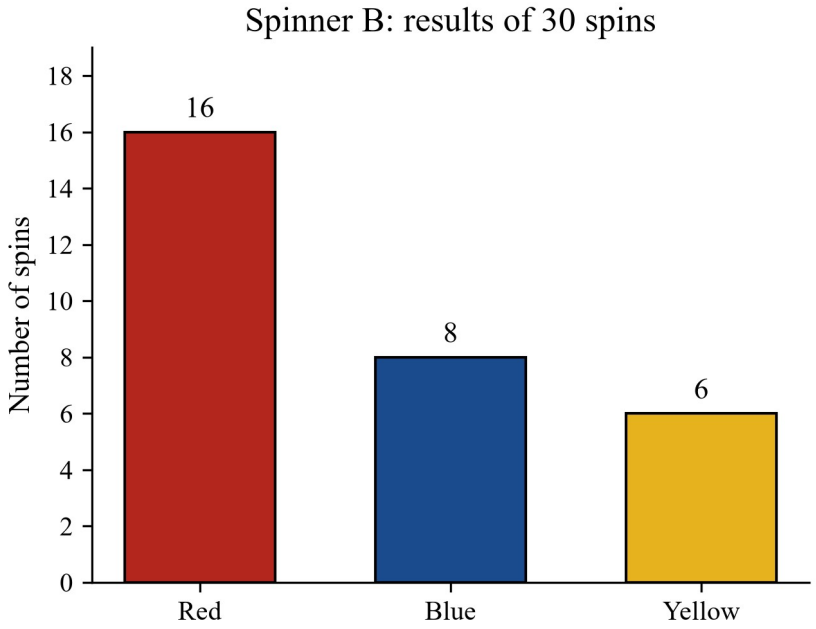
Here is another class experiment: 30 spins of Spinner B, recorded with tally marks.

Spinner B: 30 spins

Colour	Tally	Frequency
Red	𝍸 𝍸 𝍸 |	16 out of 30
Blue	𝍸 |||	8 out of 30
Yellow	𝍸 |	6 out of 30

Tally chart of 30 spins of Spinner B. Red shows three bundles of five tally marks and one extra mark, with frequency 16 out of 30. Blue shows one bundle of five and three extra marks, with frequency 8 out of 30. Yellow shows one bundle of five and one extra mark, with frequency 6 out of 30.

Red came up 16 out of 30 spins, blue 8 out of 30 and yellow 6 out of 30. The same frequencies can be drawn as a bar graph, so they are easier to compare at a glance.



Bar graph titled "Spinner B: results of 30 spins". The red bar is 16 high, the blue bar is 8 high and the yellow bar is 6 high. The numbers 16, 8 and 6 are written above their bars.

Or the same results can be recorded in a **spreadsheet**, a digital tool that counts and adds for you.

	A	B	C	D
1	**Colour**	**Tally**	**Frequency**	**Fraction**
2	Red	𝍸 𝍸 𝍸 |	16	16/30
3	Blue	𝍸 |||	8	8/30
4	Yellow	𝍸 |	6	6/30
5	**Total**		**30**	
6				
7	The total cell shows 30. The spreadsheet worked it out with =SUM(C2:C4)			

A spreadsheet with columns A Colour, B Tally, C Frequency and D Fraction. Row 2: Red, 16 tally marks, frequency 16, fraction 16/30. Row 3: Blue, 8 tally marks, frequency 8, fraction 8/30. Row 4: Yellow, 6 tally marks, frequency 6, fraction 6/30. Row 5: Total, frequency 30. Below: the total cell shows 30, worked out with =SUM(C2:C4).

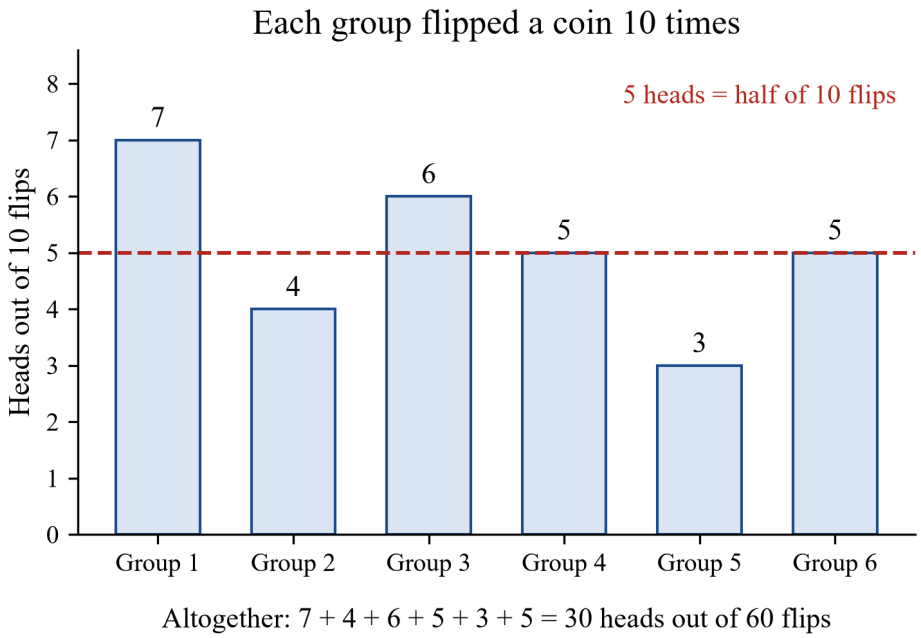
The frequency column shows 16, 8 and 6, and the fraction column writes each frequency out of the 30 spins. The total of the frequencies is 30 — every spin is counted once.

Using frequency to compare and estimate

Frequencies let us **compare** outcomes. In the 30 spins, red's frequency (16) is more than blue's (8), and blue's is more than yellow's (6). Red came up most often.

Frequencies also let us **estimate** how likely each outcome is. The outcome with the higher frequency happened more often, so we estimate that it is the more likely outcome. Red's 16 out of 30 is more than half the spins, so we estimate red is more likely than blue and yellow put together — and that matches the spinner, where red covers half of the circle.

One small set of trials can vary a lot. Six groups each flipped a coin 10 times.



Altogether: 7 + 4 + 6 + 5 + 3 + 5 = 30 heads out of 60 flips

Bar graph titled "Each group flipped a coin 10 times". Six bars show the heads in each group: Group 1 has 7, Group 2 has 4, Group 3 has 6, Group 4 has 5, Group 5 has 3 and Group 6 has 5. A dashed line at 5 is labelled "5 heads = half of 10 flips". Below: altogether, 7 + 4 + 6 + 5 + 3 + 5 = 30 heads out of 60 flips.

The groups got 7, 4, 6, 5, 3 and 5 heads. With only 10 flips each, the results vary — one group even got 3 out of 10.

But put all six groups together and the class conducted 60 flips: 30 heads out of 60, the fraction $\frac{30}{60}$, which is $\frac{1}{2}$.

Heads and tails came up equally often, so we estimate they are about equally likely — just what a fair coin should do.

That is the whole method: **predict** from the outcomes, **test** with trials, **record** with tally marks, then **compare** frequencies to estimate likelihoods.

Worked examples

Example 1 — with help: list the outcomes, then predict

Look at the pile of 6 cards: 3 blue, 2 red, 1 yellow. List the outcomes of turning one card over, and say whether they are equally likely.

Working	Comment
Outcomes: blue, red, yellow.	The three colours that can be turned over
3 blue, 2 red, 1 yellow — the counts are not the same.	Equal counts would give equally likely outcomes
Blue should come up most often, yellow least often.	More cards of a colour means more likely

So the outcomes are blue, red and yellow, and they are **not** equally likely.

Example 2 — with help: read frequencies three ways

Look at the tally chart of 20 coin flips. Write the frequency of heads as a count, as an "out of", and as a fraction.

Working	Comment
Heads: two bundles of five and one more = 11.	Count the tally marks
11 out of 20 flips were heads.	The count out of the trials
$\frac{11}{20}$ of the flips were heads.	The fraction of the trials

So the frequency of heads is 11: that is 11 out of 20, or $\frac{11}{20}$.

Example 3 — with help: compare frequencies and estimate

Look at the results of 30 spins of Spinner B. Compare the frequencies and estimate which outcome is the most likely.

Working	Comment
Red 16, blue 8, yellow 6.	Read the frequencies
16 is more than 8, and 8 is more than 6.	Compare the counts
Red came up most often, so estimate red is the most likely.	Higher frequency, more likely outcome

So red is the most likely outcome — and that matches the spinner, where red covers half of the circle.

Example 4 — on your own: the special die

The special die (faces 1, 2, 2, 3, 3, 3) was rolled 24 times. The frequencies were: 1 → 3, 2 → 9, 3 → 12. Write the frequency of 3 as a fraction of the rolls, and say which outcome you estimate is the most likely. Give a reason from the faces.

3 was rolled 12 times out of 24: $\frac{12}{24}$, which is $\frac{1}{2}$ of the rolls. Its frequency is the biggest, so **3 is estimated as the most likely outcome**. Reason: 3 is on three of the six faces, more faces than any other number.

Example 5 — on your own: the bag of balls

A bag holds 3 red balls and 1 blue ball. A ball was drawn, recorded and put back, 20 times. Red came up 14 times and blue 6 times. Write the frequency of red as an "out of" and as a fraction, and compare the likelihood of red and blue.

Red came up **14 out of 20** draws, the fraction $\frac{14}{20}$. Blue came up 6 out of 20. 14 is more than 6, so **red is estimated as the more likely colour** — matching the bag, which has 3 red balls and only 1 blue.

Example 6 — on your own: the whole class results

Six groups each flipped a coin 10 times and got 7, 4, 6, 5, 3 and 5 heads. How many flips did the class conduct altogether, and how many heads? Write the heads as a fraction of the flips and estimate the likelihood of heads.

Altogether: 6 × 10 = 60 flips, and 7 + 4 + 6 + 5 + 3 + 5 = 30 heads. That is 30 out of 60, the fraction $\frac{30}{60} = \frac{1}{2}$. **Heads came up about half the time, so heads and tails are estimated as about equally likely** — a fair coin.

Practice questions

With help

Q1. A coin is flipped once. List the possible outcomes. Are they equally likely?

Q2. Look at the tally chart of 20 coin flips. How many times did tails come up? Write your answer as an "out of 20".

Q3. Look at the tally chart of 20 coin flips. Write the frequency of heads as a fraction of the flips.

Q4. List the outcomes of spinning Spinner A. Are they equally likely? Give a reason.

Q5. Look at Spinner B. Which colour covers the biggest part of the circle? Predict which colour will come up most often.

Q6. Look at the tally chart of 30 spins of Spinner B. Which colour came up most often? Write its frequency as an "out of 30".

Q7. The special die has faces 1, 2, 2, 3, 3, 3. List the outcomes of one roll. Which number is on the most faces?

Q8. A bag holds 3 red balls and 1 blue ball. Predict which colour will come up more often in 20 draws. Give a reason.

On your own

Q9. Look at the tally chart of 20 coin flips. Compare the frequency of heads and the frequency of tails. What do you estimate about their likelihood?

Q10. Look at the results of 30 spins of Spinner B. Write the frequency of blue as a fraction. Is blue's frequency more than yellow's, or less?

Q11. Look at the bar graph of 30 spins of Spinner B. How many more times did red come up than yellow?

Q12. The special die was rolled 24 times: 1 → 3, 2 → 9, 3 → 12. Write the frequency of 2 as a fraction of the rolls. Which outcome do you estimate is the most likely?

Q13. In 20 draws from the bag, red came up 14 times and blue 6 times. Write the frequency of red as a fraction. Which colour do you estimate is the more likely?

Q14. A pile holds 3 blue cards, 2 red cards and 1 yellow card. List the outcomes of turning one card over, and put them in order from most likely to least likely.

Q15. Look at the bar graph of six groups flipping a coin 10 times each. Which group got the most heads? Which group got exactly half of its flips as heads?

Q16. Put together, the six groups got 30 heads out of 60 flips. Write this as a fraction. What do you estimate about the likelihood of heads and tails?

Maths in real life

Q17. At a school fete, a game uses Spinner B: you win when the spinner lands on red. Use the 30-spin results to say whether you would expect to win often. Give a reason.

Q18. A cricket club uses a coin to decide who bats first. Over a season the coin was flipped 60 times and landed heads 30 times. Write the heads as a fraction of the flips. Does the coin seem fair? Explain.

Q19. A friend has a bag of red and blue marbles but will not show you inside. You draw a marble, record its colour and put it back, 20 times: red comes up 14 times and blue 6 times. What do you guess about the marbles in the bag? Give a reason.

Answers

Q1. Heads, tails; equally likely. **Q2.** 9 out of 20. **Q3.** $\frac{11}{20}$. **Q4.** Red, blue, yellow, green; equally likely, the four parts are equal. **Q5.** Red; predict red most often. **Q6.** Red; 16 out of 30. **Q7.** 1, 2, 3; the number 3. **Q8.** Red; 3 red balls against 1 blue. **Q9.** About equally likely. **Q10.** $\frac{8}{30}$; more than yellow's. **Q11.** 10 more times. **Q12.** $\frac{9}{24}$; 3. **Q13.** $\frac{14}{20}$; red. **Q14.** Blue, red, yellow. **Q15.** Group 1; Group 4. **Q16.** $\frac{30}{60} = \frac{1}{2}$; about equally likely. **Q17.** Yes, expect to win more often than not; red came up 16 out of 30. **Q18.** $\frac{30}{60} = \frac{1}{2}$; yes, it seems fair. **Q19.** More red than blue marbles; red came up 14 out of 20.

Worked answers

Q1. A coin has two outcomes: heads and tails. A fair coin helps both outcomes the same, so they are equally likely. ∴ heads, tails; equally likely.

Q2. The tally for tails is one bundle of five and four more: 9. ∴ 9 out of 20 flips were tails.

Q3. Heads came up 11 times out of 20 flips. ∴ $\frac{11}{20}$ of the flips were heads.

Q4. The outcomes are red, blue, yellow and green. Spinner A is cut into 4 equal parts, so no colour is helped more than another. ∴ equally likely.

Q5. Red covers half of Spinner B, the biggest part. ∴ predict red will come up most often.

Q6. Red's frequency is 16, blue's 8 and yellow's 6: red came up most often. ∴ red, 16 out of 30.

Q7. The numbers that can be rolled are 1, 2 and 3. The face 3 is on three faces, more than 2 (two faces) or 1 (one face). ∴ outcomes 1, 2, 3; the number 3 is on the most faces.

Q8. The bag has 3 red balls and 1 blue ball, so a red draw should happen more often. ∴ predict red, because there are more red balls.

Q9. Heads 11, tails 9: the frequencies are close, and 11 out of 20 is near half. ∴ estimate heads and tails as about equally likely.

Q10. Blue came up 8 times out of 30: $\frac{8}{30}$. Yellow's frequency is 6, and 8 is more than 6. ∴ blue's frequency is more than yellow's.

Q11. Red 16, yellow 6: $16 - 6 = 10$. ∴ red came up 10 more times than yellow.

Q12. 2 came up 9 times out of 24: $\frac{9}{24}$. The biggest frequency is 3's (12), so estimate 3 as the most likely. ∴ $\frac{9}{24}$; 3.

Q13. Red came up 14 times out of 20: $\frac{14}{20}$. 14 is more than 6, so estimate red as the more likely. ∴ $\frac{14}{20}$; red.

Q14. The outcomes are blue, red and yellow. The pile has 3 blue, 2 red and 1 yellow, so more cards means more likely. ∴ blue, red, yellow.

Q15. Group 1's bar is the tallest at 7 heads. Group 4 got 5 heads out of 10, exactly half. ∴ Group 1; Group 4.

Q16. 30 heads out of 60 flips is $\frac{30}{60} = \frac{1}{2}$: heads about half the time. ∴ estimate heads and tails as about equally likely.

Q17. Red came up 16 out of 30 spins — more than half — and red covers half of the spinner. ∴ yes, expect to win more often than not.

Q18. 30 heads out of 60 flips is $\frac{30}{60} = \frac{1}{2}$, so heads and tails came up equally often. ∴ yes, the coin seems fair.

Q19. Red came up 14 out of 20 draws, much more often than blue's 6. Frequencies this different suggest the bag is not equal. ∴ guess more red marbles than blue in the bag.

Test

Chapter test T8 — Probability: outcomes and frequency

Codes: VC2M5P01, VC2M5P02 · **Total: 10 marks**

This is a classroom check, not an exam. The guidance duration is **15 minutes**: about 1 minute per mark, plus 5 minutes reading.

Name: _____ Date: _____

You will need: paper and a pencil. No other equipment is needed — the chance experiments in this chapter were carried out in class, and their results are given in Section B.

Calculators: you do not need a calculator for any question in this test.

Attempt every question. Show your working.

Section A — Listing outcomes (VC2M5P01) — 5 marks

A **chance experiment** is an activity where you know what *could* happen, but you cannot know what *will* happen before you do it. Each possible result is called an **outcome**. Outcomes are **equally likely** when each one has the same chance of happening.

Q1. (1 mark) Which of these chance experiments has equally likely outcomes?

A Drawing a counter from a bag that holds 5 red counters and 1 blue counter

B Tossing a fair coin

C Spinning a spinner with one big region and two small regions

D Rolling a die with faces 1, 2, 2, 3, 3, 3

Q2. (2 marks) One card is drawn from a standard deck of cards (52 cards, no jokers).

(a) You ask about the **colour** of the card. How many outcomes are you interested in? What are they?

(b) You ask about the **suit** of the card. How many outcomes are you interested in? What are they?

Q3. (2 marks) Ivy and Jack play a game with a fair die. Ivy wins if the die shows 1, 2, 3 or 4. Jack wins if the die shows 5 or 6. A game is **fair** when each player has the same chance of winning.

Is the game fair? Explain your answer.

Section B — Repeated chance experiments and frequency (VC2M5P02) — 5 marks

Each go of a chance experiment is called a **trial**. The experiments below were carried out in class, one trial at a time, and their results are given here. The **frequency** of an outcome is the number of times it happens.

Q4. (2 marks) A spinner has 3 regions: red covers half of the circle, blue covers one quarter and yellow covers one quarter. The class spun the spinner 40 times.

Colour	Frequency
Red	21
Blue	10
Yellow	9

(a) Write the frequency of red as an “out of 40”.

(b) Use the frequencies to estimate which colour is most likely to come up. Give a reason.

Q5. (2 marks) A special die has faces 1, 2, 2, 3, 3, 3. The class rolled the die 30 times. The frequencies were: $1 \to 4$, $2 \to 9$, $3 \to 17$.

(a) Write the frequency of 3 as a fraction of the rolls.

(b) Which outcome do you estimate is the most likely? Give a reason from the faces of the die.

Q6. (1 mark) A class flipped a coin 20 times. Heads came up 12 times. Every flip is counted once, so the frequencies of heads and tails add to 20.

How many times did tails come up? Write the frequency of tails as an "out of 20".

Total: 10 marks

Solutions

Teacher-facing. T8 is a classroom check, not an exam: guidance duration 15 minutes (about 1 minute per mark, plus 5 minutes reading). Per §7, trials are conducted in class, not in the test; Section B supplies the class results, so a student absent on experiment days still has a written path to `VC2M5P02`. No "no calculator" instruction appears, since no Level 5 CD says "without a calculator" (D10); in practice no question needs one. There is no numerical probability anywhere on the paper — fractions, decimals or percentages as likelihoods, and a 0–1 scale, are Level 6 (`VC2M6P01`); frequencies here are counts, "out of" statements and fractions only. No simulation with an increasing number of trials (`VC2M6P02`). `VC2M5P02.E06` is excluded under CONTENT_POLICY.md Rule 1, so no First Nations context appears.

Answer key

Q	Answer	Marks
Q1	B	1
Q2	(a) 2 outcomes: red, black. (b) 4 outcomes: hearts, diamonds, clubs, spades	2
Q3	Not fair: Ivy wins on 4 of the 6 equally likely outcomes, Jack on 2	2
Q4	(a) 21 out of 40. (b) Red — it has the greatest frequency	2
Q5	(a) $\frac{17}{30}$. (b) 3 — it is on the most faces	2
Q6	8 out of 20	1
	Total	**10**

Worked solutions

Q1 (`VC2M5P01`, 1 mark). Tossing a fair coin has 2 outcomes, heads and tails, and each side has the same chance: the outcomes are equally likely. **The answer is B.** Check why the others are not equally likely:

- A: 5 red counters against 1 blue — drawing red is more likely.
- C: the regions are different sizes, so the outcomes do not have the same chance.
- D: 3 is on three faces, but 1 is on only one face — 3 is more likely.

Q2 (`VC2M5P01`, 2 marks).

(a) Ask about the colour: every card in the deck is red or black. ∴ **2 outcomes: red and black.**

(b) Ask about the suit: every card is hearts, diamonds, clubs or spades. ∴ **4 outcomes: hearts, diamonds, clubs and spades.**

The deck is the same in both parts; the question you ask changes the list of outcomes.

Marking: 1 mark for each part. The mark requires the correct number *and* the list.

Q3 (`VC2M5P01`, 2 marks). The die is fair, so the 6 outcomes 1, 2, 3, 4, 5, 6 are equally likely. Count the winning outcomes for each player:

- Ivy wins on 1, 2, 3 or 4: **4 outcomes**.
- Jack wins on 5 or 6: **2 outcomes**.

4 is more than 2, so Ivy has the better chance. ∴ **The game is not fair.**

Marking: 1 mark for the correct judgement (not fair); 1 mark for an explanation that counts the equally likely outcomes for each player. A judgement without a count scores 1 mark only.

Q4 (VC2M5P02, 2 marks).

(a) Red came up 21 times in the 40 spins. ∴ The frequency of red is **21 out of 40**.

(b) The frequencies are red 21, blue 10, yellow 9. 21 is the greatest frequency — red came up most often. ∴ **Estimate red as the most likely colour.** (This matches the spinner itself, where red covers half of the circle.)

Marking: (a) 1 mark for "21 out of 40"; (b) 1 mark for red with a reason that refers to the frequencies, for example "red has the greatest frequency" or "red came up most often".

Q5 (VC2M5P02, 2 marks).

(a) 3 came up 17 times out of the 30 rolls. ∴ The frequency of 3 is $\frac{17}{30}$ of the rolls.

(b) The die has 6 faces. The number 3 is on 3 faces, the number 2 is on 2 faces and the number 1 is on only 1 face. More faces means a better chance. ∴ **Estimate 3 as the most likely outcome.** Check against the results: 3 also has the greatest frequency (17).

Marking: (a) 1 mark for $\frac{17}{30}$; (b) 1 mark for 3 with a reason from the faces of the die, as the question asks.

Q6 (VC2M5P02, 1 mark). Every flip is counted once, so the frequencies of heads and tails add to the 20 flips. Heads came up 12 times: 20 − 12 = 8. ∴ Tails came up **8 out of 20** times.

Marks by content description

CD	Subtopic	Items	Marks
VC2M5P01	8A Listing outcomes: equally likely and not equally likely	Q1 (1), Q2 (2), Q3 (2)	5
VC2M5P02	8B Repeated chance experiments and frequency	Q4 (2), Q5 (2), Q6 (1)	5
Total		**6 items**	**10**

Each CD carries exactly 5 marks, per the §7 marking rule. Multiple choice is 1 mark of 10, used sparingly and only where selection is the skill being tested (picking the experiment with equally likely outcomes), with four options A–D; every other item is short-answer or worked-response. Both CDs of Chapter 8 are assessed, as §7's coverage requirement demands.