# Mathematics Level 5

## Chapter 4 — Inverse operations and unknown values



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## Multiplication and division as inverse operations; families of number facts

**Code `VC2M5A01`** (Victorian Curriculum F–10 Version 2.0, Mathematics, Level 5; authority file `maths_5.json`): "recognise and explain the connection between multiplication and division as inverse operations and use this to develop families of number facts". This section draws on `VC2M5A01.E01`, `.E02`, `.E03` and `.E04`.

### Theory

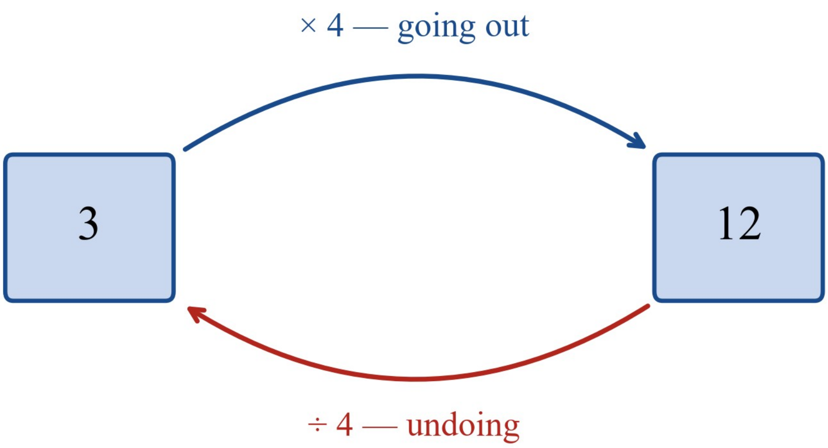
#### *What you bring to this section*

You already know your multiplication facts to 10 × 10, and the division facts that go with them, from Year 4. This section does not re-drill those facts. It does something new with them: it looks at how multiplication and division are **connected**, and it uses that connection to build **families of number facts**.

A quick warm-up, all facts you know already: 3 × 4 = 12, 7 × 8 = 56, 9 × 6 = 54. Keep these in mind — they will do the work in this section.

#### *Operations that undo each other*

Multiplication and division are **inverse operations**. That means each one **undoes** the other.




Start with 3. Multiply by 4 to reach 12. Divide by 4 and you are back at 3.

*The numbers 3 and 12 with two curved arrows between them. The top arrow runs from 3 to 12 and is labelled "× 4 — going out". The bottom arrow runs from 12 back to 3 and is labelled "÷ 4 — undoing". The words below say: start with 3, multiply by 4 to reach 12, divide by 4 and you are back at 3.*

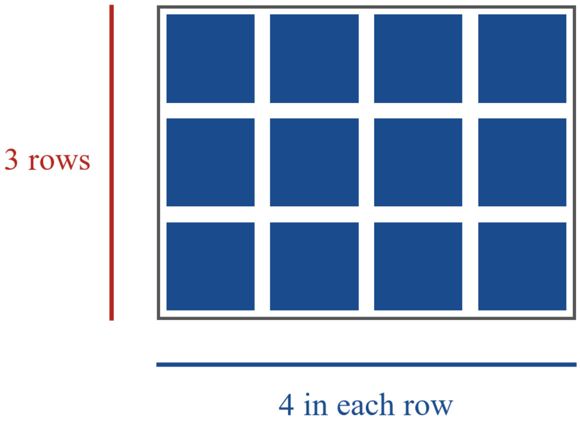
Start with 3. Multiply by 4 and you reach 12. Now divide by 4 and you are back at 3. The division undid the multiplication.

It works from the other end too. Start with 12. Divide by 4 and you reach 3. Multiply by 4 and you are back at 12. The multiplication undid the division.

That is the whole connection: one operation goes out, the inverse brings you back, and you land where you started.

#### *One array, two readings*

One picture of tiles can be read as multiplication **or** as division.




3 × 4 = 12 — 3 rows of 4 tiles make 12 tiles.

12 ÷ 3 = 4 — 12 tiles split into 3 equal rows give 4 in each row.

*An array of 12 tiles in 3 rows with 4 tiles in each row. A red line at the left is labelled "3 rows" and a blue line below is labelled "4 in each row". Below the array: "3 × 4 = 12 — 3 rows of 4 tiles make 12 tiles" and "12 ÷ 3 = 4 — 12 tiles split into 3 equal rows give 4 in each row".*

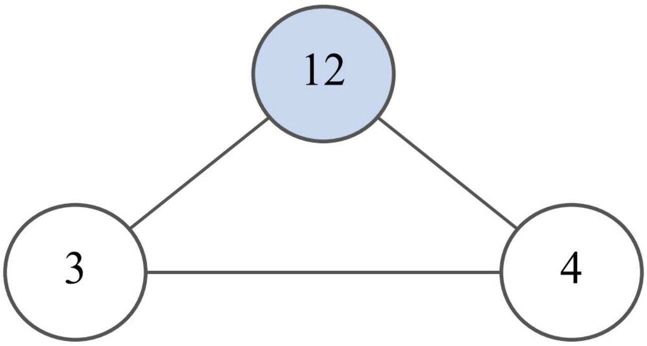
Read as multiplication: 3 rows of 4 tiles make 12 tiles, so 3 × 4 = 12.

Read as division: 12 tiles split into 3 equal rows give 4 in each row, so 12 ÷ 3 = 4.

Same tiles, same picture, two different questions. Multiplication **builds** the array; division **splits it up**. Because they are inverse operations, every array gives you both a multiplication fact and a division fact.

### *Families of number facts*

Because one array reads both ways, every fact brings a whole **family of number facts** with it. The family is built from three numbers: the total at the top, and the two numbers that build it.




$3 \times 4 = 12$ $4 \times 3 = 12$

$12 \div 3 = 4$ $12 \div 4 = 3$

Three numbers make a family of four facts.

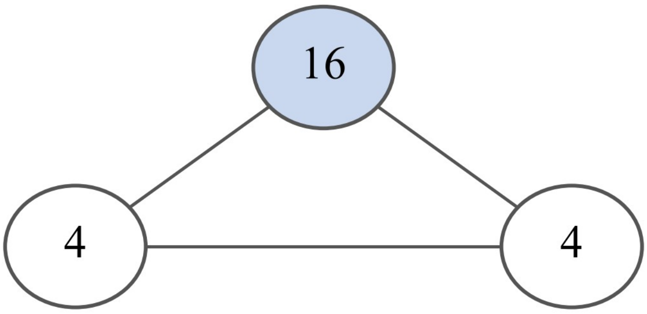
*A fact-family triangle. The number 12 sits in a shaded circle at the top, and 3 and 4 sit in circles at the bottom corners, all joined by lines. Around the triangle are the four facts of the family: 3 × 4 = 12 and 4 × 3 = 12 in blue, 12 ÷ 3 = 4 and 12 ÷ 4 = 3 in red.*

The family of 3, 4 and 12 has four facts:

$3 \times 4 = 12$ $4 \times 3 = 12$ $12 \div 3 = 4$ $12 \div 4 = 3$

Two multiplication facts and two division facts, all from the same three numbers. Know any one of the four and you know the other three. That is what makes families so useful: one fact you remember gives you three more for free.

Sometimes a family is short. When the two bottom numbers are the same, the second multiplication fact repeats the first, and the second division fact repeats the first as well.




$4 \times 4 = 16$ $16 \div 4 = 4$

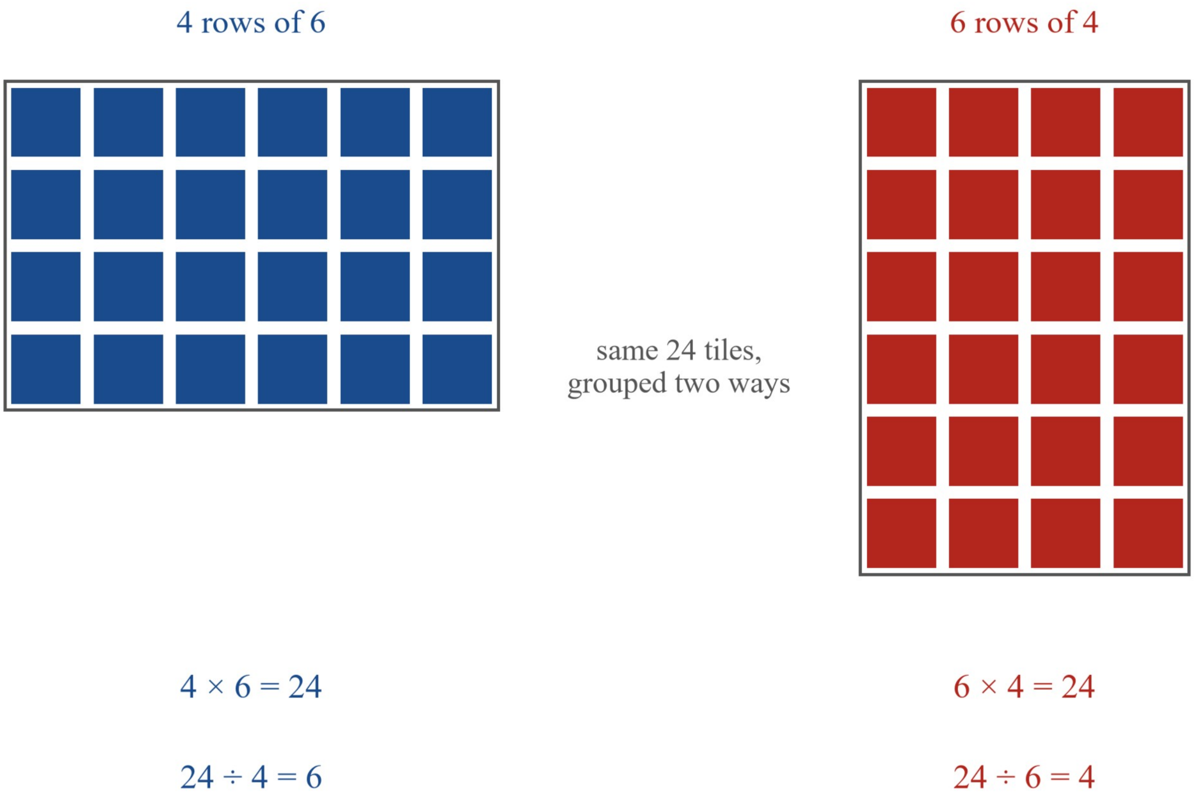
The two corner numbers are the same, so the family has only two different facts.

*A fact-family triangle with 16 in the shaded top circle and 4 in both bottom corners. Below: the two facts 4 × 4 = 16 and 16 ÷ 4 = 4, and the words "the two corner numbers are the same, so the family has only two different facts".*

Here the family has only two different facts: $4 \times 4 = 16$ and $16 \div 4 = 4$. It is still a family — just a short one.

### *Same total, different groupings*

The same total can be grouped in different ways, and each grouping gives its own pair of multiplication and division facts.




*Two arrays of the same 24 tiles, grouped two ways. Left, a blue array of 4 rows of 6 with its facts $4 \times 6 = 24$ and $24 \div 4 = 6$. Right, a red array of 6 rows of 4 with its facts $6 \times 4 = 24$ and $24 \div 6 = 4$. Between them: "same 24 tiles, grouped two ways".*

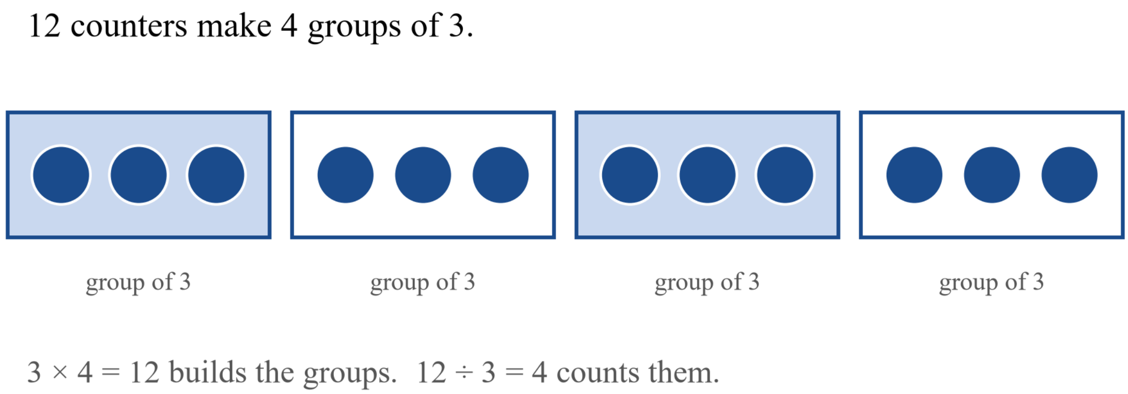
The blue grouping, 4 rows of 6, gives: $4 \times 6 = 24$ and $24 \div 4 = 6$.

The red grouping, 6 rows of 4, gives: $6 \times 4 = 24$ and $24 \div 6 = 4$.

How are the groupings different? The rows and columns trade places: what was a row in one grouping is a column in the other. How are they connected? The total is the same 24 tiles, so all four facts hang together. Knowing $4 \times 6 = 24$ tells you $24 \div 6 = 4$ straight away: if 4 rows of 6 make 24, then 24 tiles laid out in rows of 6 must give 4 rows.

### *Division asks "how many groups?"*

Here is another way to see the connection: division **counts groups**.




*12 counters drawn as four boxed groups of 3, each box labelled "group of 3". Above: "12 counters make 4 groups of 3". Below: "3 × 4 = 12 builds the groups. 12 ÷ 3 = 4 counts them".*

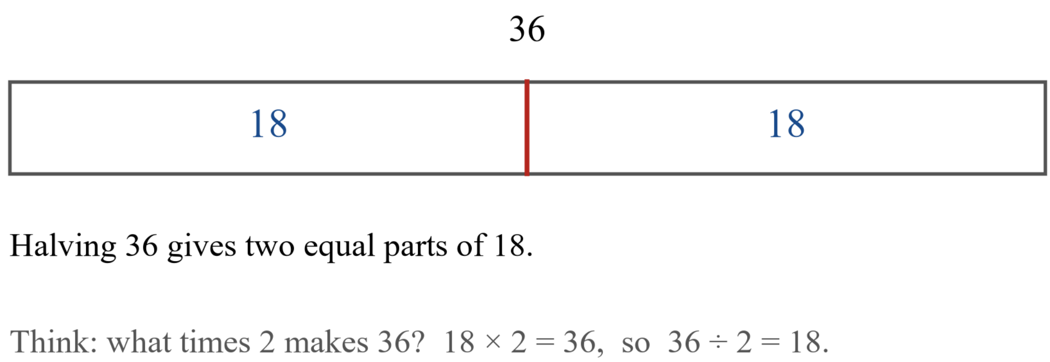
Twelve counters sorted into groups of 3 make 4 groups. That is 12 ÷ 3 = 4.

Now look at the multiplication hiding inside it. Each group holds 3, and there are 4 groups, so 3 × 4 = 12. The division counted the groups; the multiplication rebuilt the total.

So any division fact can be turned into a multiplication question. 12 ÷ 3 = □ is really asking: 3 × □ = 12? The box □ stands for the missing number — the number we have to find. Turning the question around like this is using the inverse.

### *Halving is division by 2*

**Halving** means dividing by 2, and the inverse makes it easy. To halve 36, ask: what times 2 makes 36?

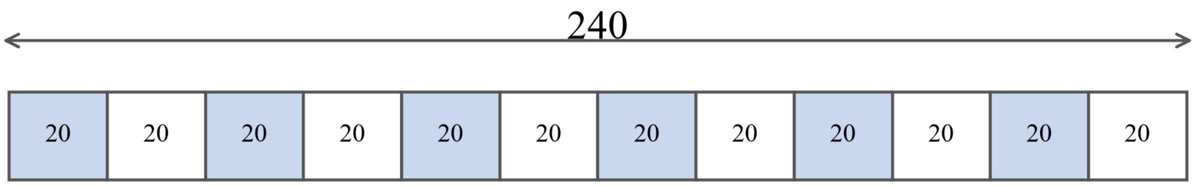



*A bar standing for 36, cut by a red line into two equal parts of 18. Above the bar: 36. Below: "Halving 36 gives two equal parts of 18" and "Think: what times 2 makes 36? 18 × 2 = 36, so 36 ÷ 2 = 18".*

18 × 2 = 36, so 36 ÷ 2 = 18. Halving 36 gives two equal parts of 18.

### *The inverse makes division of large numbers easier*

The same trick works when the numbers get large. What is 240 ÷ 20?




How many chunks of 20 make 240? Count them: 12.

Think: $20 \times \square = 240$

$20 \times 10 = 200$ and $20 \times 2 = 40$, so $20 \times 12 = 240$. So $240 \div 20 = 12$.

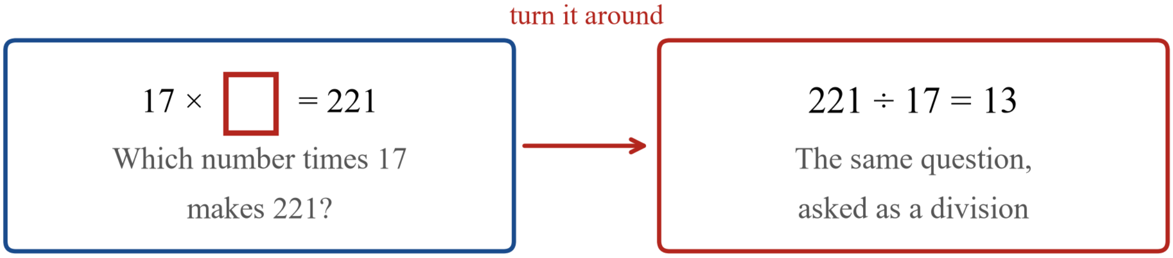
*A long bar standing for 240, divided into 12 equal chunks each labelled 20. Below the bar: "Think: 20 × [empty box] = 240", where the empty box stands for the number of chunks of 20 in 240.*

Picture 240 as a bar made of chunks of 20. The question $240 \div 20$ asks how many chunks there are. Turn it around with the inverse: $20 \times \square = 240$. Now it is a multiplication question. $2 \times 12 = 24$, so $20 \times 12 = 240$ — the box is 12.

$240 \div 20 = 12$. The inverse turned a big division into a multiplication you could see.

### ***The inverse finds missing numbers — and checks your work***

Sometimes a multiplication has a missing number: $17 \times \square = 221$. You may not know that one straight away. Turn it around with the inverse: the box is $221 \div 17$.




Check with the inverse: $13 \times 17 = 221$. ✓

Division found the missing number. Multiplication checks it.

*Two panels joined by an arrow labelled "turn it around". Left panel: "17 × [empty box] = 221 — which number times 17 makes 221?" Right panel: "221 ÷ 17 = 13 — the same question, asked as a division". Below, in green: "Check with the inverse: 13 × 17 = 221".*

$221 \div 17 = 13$, so the missing number is 13.

And the inverse gives you a **check** as well. Division found 13, so check by multiplying: $13 \times 17 = 221$. ✓ Out with one operation, back with the other — when the check lands on the number you started with, the answer is right.

### *Where this leads*

In **4B** you will use this connection, and the properties of numbers and operations, to find unknown values in equations like $15 \div 3 = 30 \div \square$. And in **1D**, earlier in the book, algorithms experiment with factors, multiples and divisibility — the inverse connection you have just met is what makes those experiments worth running.

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## Worked examples

### *Example 1 — with help: write a fact family*

Write the family of facts for 3 rows of 5.

| Working | Comment |
|---|---|
| 3 rows of 5 is 15, so $3 \times 5 = 15$. | Build the total first |
| $5 \times 3 = 15$ | The same array read the other way |
| $15 \div 3 = 5$ | Division splits the array up |
| $15 \div 5 = 3$ | The other division fact |

So the family of 3, 5 and 15 is: $3 \times 5 = 15$, $5 \times 3 = 15$, $15 \div 3 = 5$, $15 \div 5 = 3$.

### *Example 2 — with help: division by the inverse*

Find $56 \div 8$ by turning it into a multiplication question.

| Working | Comment |
|---|---|
| $56 \div 8 = \square$ is asking: $8 \times \square = 56$. | Turn the division around |
| $8 \times 7 = 56$ | A fact you know |
| So $56 \div 8 = 7$. | The box is 7 |

### *Example 3 — with help: dividing large numbers*

Find $240 \div 20$ using the inverse.

| Working | Comment |
|---|---|
| Think: $20 \times \square = 240$. | How many 20s make 240? |
| $2 \times 12 = 24$, so $20 \times 12 = 240$. | 24 tens |
| So $240 \div 20 = 12$. | The inverse did the work |

### *Example 4 — on your own: a family from one fact*

Write the whole family of facts that comes with $6 \times 4 = 24$.

The family of 6, 4 and 24 is: $6 \times 4 = 24$, $4 \times 6 = 24$, $24 \div 6 = 4$, $24 \div 4 = 6$.

### *Example 5 — on your own: a missing number, with a check*

Find the missing number: $17 \times \square = 221$. Then check your answer.

Turn it around: the box is $221 \div 17 = 13$. Check: $13 \times 17 = 221$. ✓ **The missing number is 13.**

### *Example 6 — on your own: halving with a check*

Find $78 \div 2$ by halving, and check with the inverse.

Think: what times 2 makes 78? $30 \times 2 = 60$ and $9 \times 2 = 18$, and $60 + 18 = 78$, so $39 \times 2 = 78$. Check: $39 \times 2 = 78$. ✓ **$78 \div 2 = 39$.**

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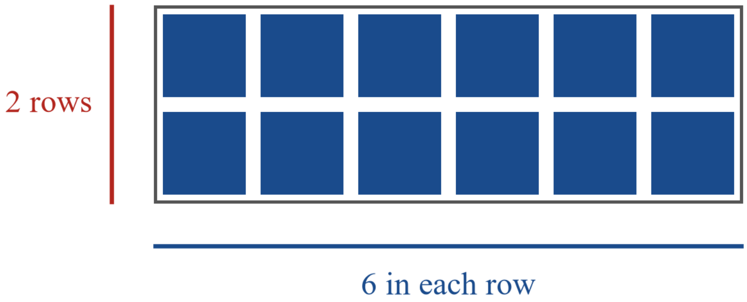
## Practice questions

### *With help*

**Q1.** Start with 6. Multiply by 5, then divide by 5. What number do you land on? What does this tell you about × and ÷?

**Q2.** The array below shows 2 rows of 6 tiles. Write the four facts of its family.




One array, four facts: $2 \times 6 = 12$ and its family.

*An array of 12 tiles in 2 rows with 6 tiles in each row. A red line at the left is labelled "2 rows" and a blue line below is labelled "6 in each row". Below the array: "One array, four facts: 2 × 6 = 12 and its family".*

**Q3.** Write the family of facts for the numbers 7, 8 and 56.

**Q4.** $9 \times 3 = 27$. Write the two division facts in its family.

**Q5.** Find $42 \div 6$ by thinking: $6 \times \square = 42$.

**Q6.** Find $63 \div 9$ the same way.

**Q7.** Halve 46. (Think: what times 2 makes 46?)

**Q8.** 24 tiles in 3 rows of 8 give the facts $3 \times 8 = 24$ and $24 \div 3 = 8$. Write the other two facts of this family.

### *On your own*

**Q9.** Write the whole family of facts for $9 \times 4 = 36$.

**Q10.** $48 \div 6 = 8$ is one fact of a family. Write the other three.

**Q11.** Find $72 \div 8$ by turning it into a multiplication question.

**Q12.** Find the missing number: $\square \times 14 = 84$.

**Q13.** Find $360 \div 60$ using the inverse.

**Q14.** Find $150 \div 15$ using the inverse.

**Q15.** Sam says $168 \div 12 = 13$. Use the inverse operation to check. Is Sam right? If not, find the right answer.

**Q16.** Halve 96, then check your answer by doubling it.

### *Maths in real life*

**Q17.** A bakery tray holds muffins in 4 rows of 6. a. Write a multiplication fact for the tray. b. Write the two division facts of its family.

**Q18.** A teacher shares 45 cards equally into 5 piles. How many cards are in each pile? Write the division fact and the multiplication fact that checks it.

**Q19.** For the school fete, 240 pencils are packed into boxes that each hold 20. How many boxes are packed? Use the inverse to find out.

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## Answers

**Q1.** 6; dividing by 5 undoes multiplying by 5. **Q2.** 2 × 6 = 12, 6 × 2 = 12, 12 ÷ 2 = 6, 12 ÷ 6 = 2. **Q3.** 7 × 8 = 56, 8 × 7 = 56, 56 ÷ 7 = 8, 56 ÷ 8 = 7. **Q4.** 27 ÷ 9 = 3, 27 ÷ 3 = 9. **Q5.** 7. **Q6.** 7. **Q7.** 23. **Q8.** 8 × 3 = 24, 24 ÷ 8 = 3. **Q9.** 9 × 4 = 36, 4 × 9 = 36, 36 ÷ 9 = 4, 36 ÷ 4 = 9. **Q10.** 6 × 8 = 48, 8 × 6 = 48, 48 ÷ 8 = 6. **Q11.** 9. **Q12.** 6. **Q13.** 6. **Q14.** 10. **Q15.** No; 12 × 13 = 156, not 168; the right answer is 14. **Q16.** 48; check 48 × 2 = 96. **Q17.** a. 4 × 6 = 24. b. 24 ÷ 4 = 6, 24 ÷ 6 = 4. **Q18.** 9; 45 ÷ 5 = 9, checked by 9 × 5 = 45. **Q19.** 12 boxes.

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## Worked answers

**Q1.** 6 × 5 = 30, then 30 ÷ 5 = 6. You land on the number you started with. ∴ dividing by 5 undoes multiplying by 5: × and ÷ are inverse operations.

**Q2.** The array is 2 rows of 6, so 2 × 6 = 12 and 6 × 2 = 12; splitting it up, 12 ÷ 2 = 6 and 12 ÷ 6 = 2. ∴ the family is 2 × 6 = 12, 6 × 2 = 12, 12 ÷ 2 = 6, 12 ÷ 6 = 2.

**Q3.** The total is 56, built by 7 and 8. ∴ 7 × 8 = 56, 8 × 7 = 56, 56 ÷ 7 = 8, 56 ÷ 8 = 7.

**Q4.** The family of 9, 3 and 27 also has 27 ÷ 9 = 3 and 27 ÷ 3 = 9. ∴ the two division facts are 27 ÷ 9 = 3 and 27 ÷ 3 = 9.

**Q5.** Think: 6 × □ = 42. 6 × 7 = 42. ∴ 42 ÷ 6 = 7.

**Q6.** Think: 9 × □ = 63. 9 × 7 = 63. ∴ 63 ÷ 9 = 7.

**Q7.** Think: what times 2 makes 46? 23 × 2 = 46. ∴ half of 46 is 23.

**Q8.** The same three numbers 3, 8 and 24 also give 8 × 3 = 24 and 24 ÷ 8 = 3. ∴ the other two facts are 8 × 3 = 24 and 24 ÷ 8 = 3.

**Q9.** The total is 36, built by 9 and 4. ∴ 9 × 4 = 36, 4 × 9 = 36, 36 ÷ 9 = 4, 36 ÷ 4 = 9.

**Q10.** The family's numbers are 6, 8 and 48. ∴ the other three facts are 6 × 8 = 48, 8 × 6 = 48, 48 ÷ 8 = 6.

**Q11.** Think: 8 × □ = 72. 8 × 9 = 72. ∴ 72 ÷ 8 = 9.

**Q12.** Turn it around: the box is 84 ÷ 14. 14 × 6 = 84. ∴ the missing number is 6.

**Q13.** Think: 60 × □ = 360. 6 × 6 = 36, so 60 × 6 = 360. ∴ 360 ÷ 60 = 6.

**Q14.** Think: 15 × □ = 150. 15 × 10 = 150. ∴ 150 ÷ 15 = 10.

**Q15.** Check with the inverse: 12 × 13 = 156, which is not 168, so Sam is wrong. Try 14: 12 × 14 = 168. ∴ Sam is not right; 168 ÷ 12 = 14.

**Q16.** Think: what times 2 makes 96? 48 × 2 = 96. Check by doubling: 48 × 2 = 96. ✓ ∴ half of 96 is 48.

**Q17.** a. 4 rows of 6 muffins is 4 × 6 = 24. b. Splitting the tray up: 24 ÷ 4 = 6 and 24 ÷ 6 = 4. ∴ a. 4 × 6 = 24; b. 24 ÷ 4 = 6 and 24 ÷ 6 = 4.

**Q18.** 45 cards shared into 5 equal piles is 45 ÷ 5. Think: 5 × □ = 45; 5 × 9 = 45. Check: 9 × 5 = 45. ✓ ∴ there are 9 cards in each pile.

**Q19.** The question 240 ÷ 20 asks how many 20s make 240. Think: 20 × □ = 240; 20 × 12 = 240. ∴ 12 boxes are packed.

## Finding unknown values in multiplication and division equations

**Code `VC2M5A02`** (Victorian Curriculum F–10 Version 2.0, Mathematics, Level 5; authority file `maths_5.json`): “find unknown values in numerical equations involving multiplication and division using the properties of numbers and operations”. This section draws on `VC2M5A02.E01`, `.E02`, `.E03`, `.E04` and `.E05`.

**Achievement standard (Level 5, extract):** “Students apply properties of numbers and operations to find unknown values in numerical equations involving multiplication and division.”

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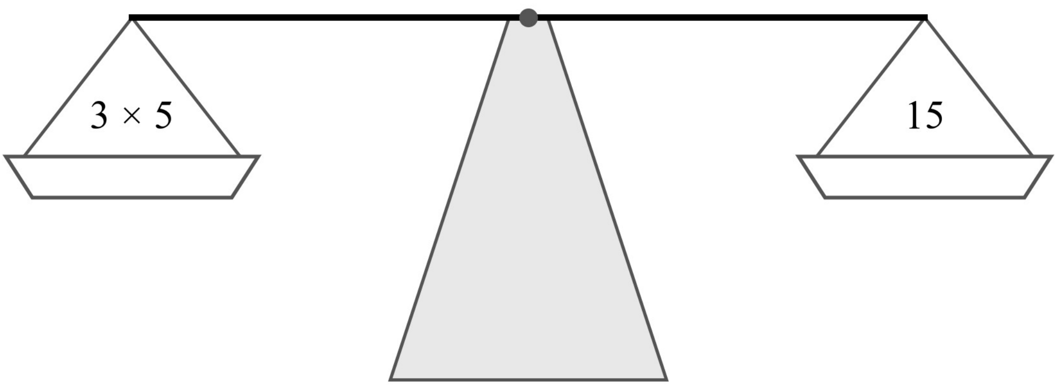
### Theory

#### *The equals sign is a balance*

A number sentence with an = in it is called an **equation**. An equation says one thing: **the two sides have the same value**. The equals sign is not an instruction to work something out. It says the left side and the right side match, like a balance sitting level.

The equals sign says: both sides have the same value.




Balanced: 3 × 5 and 15 are the same value.

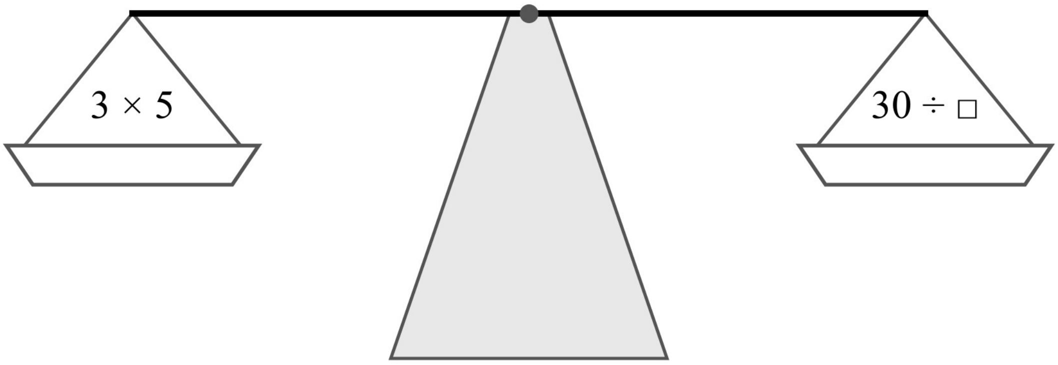
$3 \times 5 = 15$

*A level balance scale. The left pan carries the sentence 3 × 5 and the right pan carries the number 15, so the beam sits level. Below: 3 × 5 = 15.*

$3 \times 5$ has the value 15, and the right side is 15. Both sides have the same value, so the balance is level and $3 \times 5 = 15$ is a true equation.

Sometimes a number in an equation is hidden. The hidden number is called the **unknown**, and we mark it with a shape such as □ or △.

One number is hidden. The balance still tells us it is 2.




$$3 \times 5 = 30 \div \square$$

$\square = 2$, because $3 \times 5 = 15$ and $30 \div 2 = 15$.

*A level balance scale. The left pan carries $3 \times 5$ and the right pan carries $30 \div \square$ with a hidden number. Below: $3 \times 5 = 30 \div \square$, and $\square = 2$, because $3 \times 5 = 15$ and $30 \div 2 = 15$.*

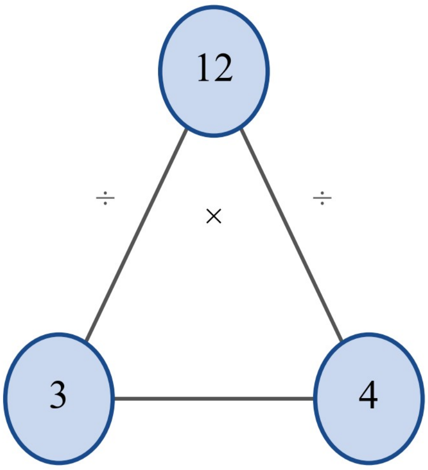
The left side is $3 \times 5 = 15$. So the right side, $30 \div \square$, must also have the value 15. Which number makes $30 \div \square$ equal to 15? We know $30 \div 2 = 15$, so **$\square = 2$**. The balance tells us the unknown.

That is the job of this whole section: **use what you know about numbers and operations to find the number that keeps the balance level.**

### *Fact families: each operation undoes the other*

In 4A you met the fact family. Multiplication and division are **inverse** operations: each one undoes the other. Every fact family holds four facts.

Use the family to find the unknown




One family of facts

$3 \times 4 = 12$

$4 \times 3 = 12$

$12 \div 3 = 4$

$12 \div 4 = 3$

$\square \times 4 = 12 \quad \longrightarrow \quad$ think $12 \div 4 = 3$
$\square = 3$

*A fact-family triangle with 12 on top and 3 and 4 below, next to the four facts 3 × 4 = 12, 4 × 3 = 12, 12 ÷ 3 = 4 and 12 ÷ 4 = 3. A card shows □ × 4 = 12 solved by the inverse fact 12 ÷ 4 = 3, so □ = 3.*

Say the unknown sits in a multiplication: $\square \times 4 = 12$. The inverse fact undoes it: $12 \div 4 = 3$. So $\square$ **= 3**.

The rule to remember: **when the unknown is hiding in a multiplication, undo it with division; when it is hiding in a division, undo it with multiplication.**

### *Two sides with one value: equivalent sentences*

When two sentences have the same value, they can be joined with an equals sign. Sentences with the same value are called **equivalent** (equal in value).

Sentences with the same value are equivalent.

$4 \times 9 = 36$

$72 \div 2 = 36$

Both sentences have the value 36.

$4 \times 9 = 72 \div 2$

Two equal values can be joined with =.

*Two cards, 4 × 9 = 36 and 72 ÷ 2 = 36, each with the value 36, joined below into the one equation 4 × 9 = 72 ÷ 2.*

$4 \times 9 = 36$ and $72 \div 2 = 36$. Both sides have the value 36, so they can be joined:

$4 \times 9 = 72 \div 2$.

Now hide the number 2: $4 \times 9 = 72 \div \square$. The left side is 36, so $72 \div \square$ must also be 36. The fact family gives $\square = 72 \div 36 = 2$. So **$\square$ = 2**, and checking: $72 \div 2 = 36$. ✔

### *True or false? Work out both sides*

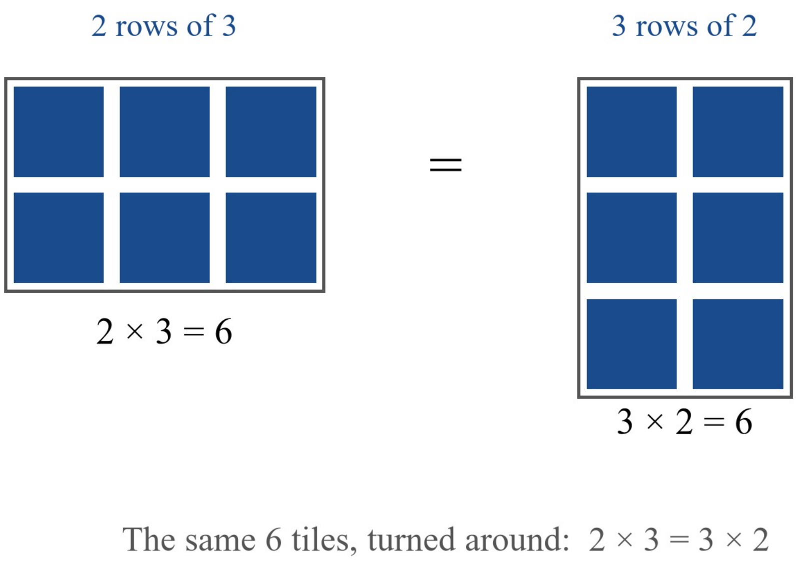
An equation is **true** when both sides really do have the same value, and **false** when they do not. To decide, work out each side, then compare the two values.

Is $15 \div 3 = 30 \div 6$ true? Left side: $15 \div 3 = 5$. Right side: $30 \div 6 = 5$. Both sides have the value 5, so the equation is **true**.

Is $6 \times 4 = 20 \div 2$ true? Left side: 24. Right side: 10. Not the same value, so **false**.

### *Swapping in multiplication: the commutative property*

A **property** is a rule about numbers that is always true. Arrays show one such property of multiplication.

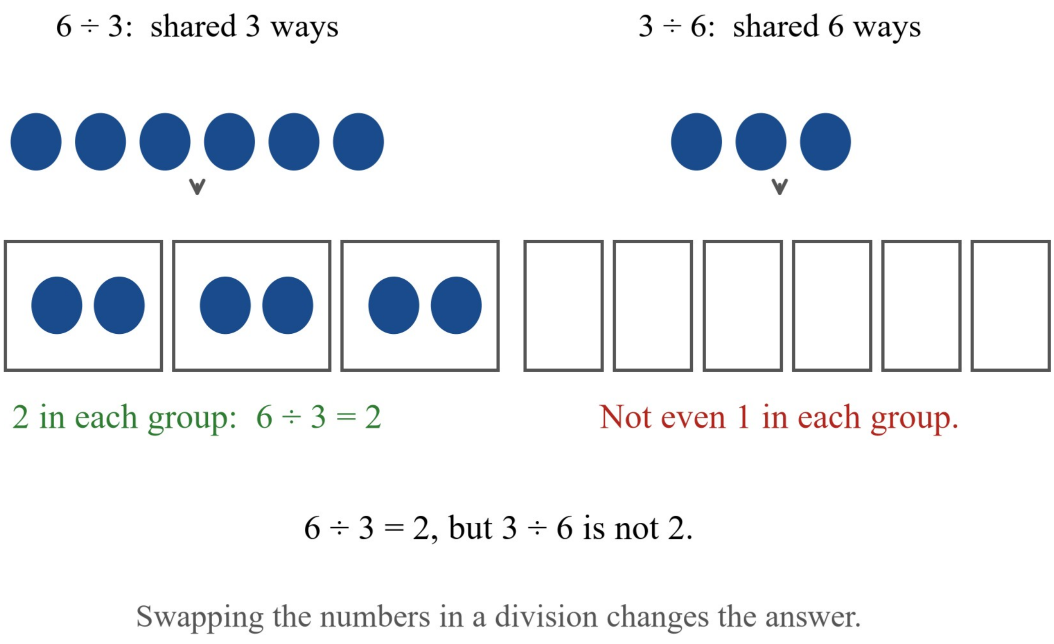



*Two arrays of six tiles: 2 rows of 3 on the left ($2 \times 3 = 6$) and 3 rows of 2 on the right ($3 \times 2 = 6$). The same six tiles turned around, so $2 \times 3 = 3 \times 2$.*

2 rows of 3 is the same six tiles as 3 rows of 2, so $2 \times 3 = 3 \times 2$. This property is called the **commutative property** of multiplication: the two numbers may be swapped and the answer to the multiplication (the **product**) is unchanged.

It finds unknowns quickly: $\square \times 9 = 9 \times 4$. Swapping the right side gives $4 \times 9$, so $\square \times 9 = 4 \times 9$ and **$\square$ = 4**.

### *Swapping in division: it does not work*

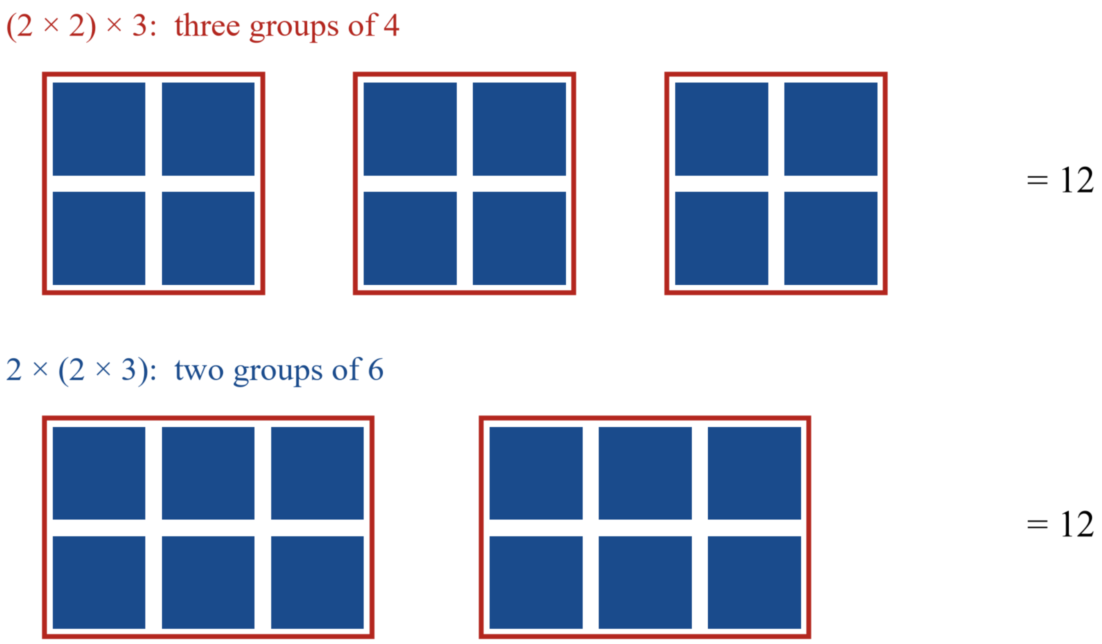



*Left: six counters shared three ways gives two in each group, 6 ÷ 3 = 2. Right: three counters shared six ways gives not even one in each group. Below: 6 ÷ 3 = 2 but 3 ÷ 6 is not 2, so swapping the numbers in a division changes the answer.*

6 ÷ 3 = 2, but 3 ÷ 6 is not 2: sharing 3 counters six ways does not give even 1 each. **Division is not commutative.** Swapping the numbers changes the answer. So when an unknown sits in a division, keep careful track of which number is which.

### *Grouping in multiplication: the associative property*

When three numbers are all multiplied, the way they are grouped does not change the product.




Same 12 tiles, grouped two ways — the total stays 12.

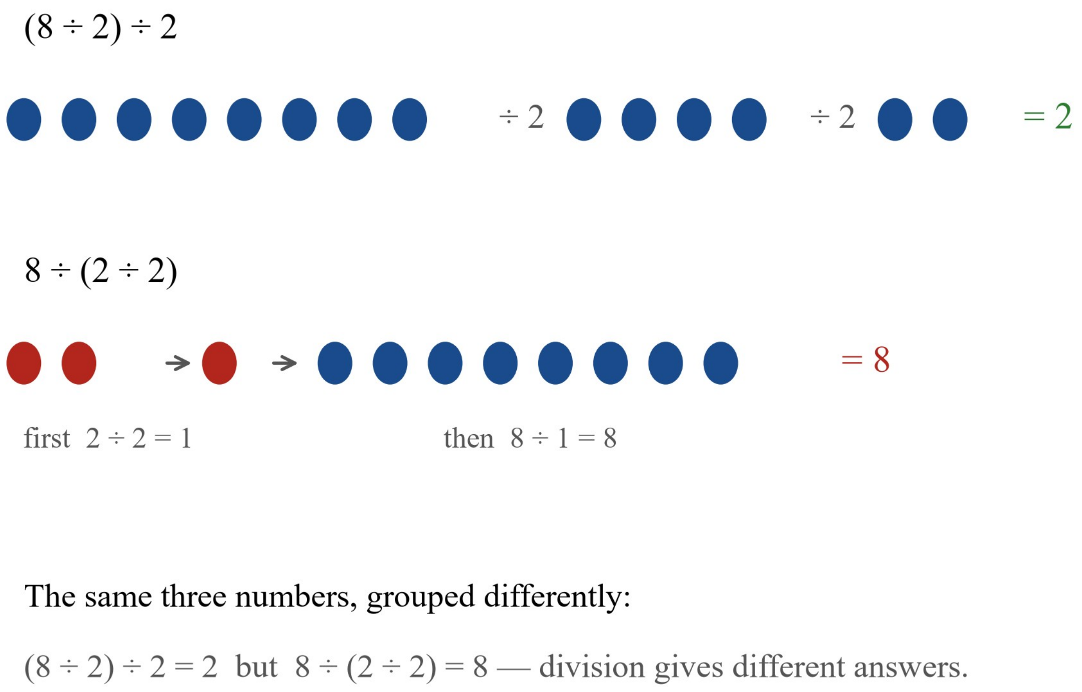
*Top: (2 × 2) × 3 drawn as three groups of four tiles, which is 12. Bottom: 2 × (2 × 3) drawn as two groups of six tiles, which is 12. The same twelve tiles grouped two ways keep the total 12.*

$(2 \times 2) \times 3 = 4 \times 3 = 12$, and $2 \times (2 \times 3) = 2 \times 6 = 12$. Same answer. This is the **associative property** of multiplication: when everything is multiplied, the numbers may be regrouped and the product is unchanged. The ( ) marks just show which part to work out first.

With the commutative property as well, a string of multiplications can be worked in any order and any grouping: $2 \times 2 \times 3 = 12$ however it is arranged.

It finds unknowns too: $(2 \times 5) \times 3 = 2 \times (5 \times \triangle)$. Regrouping the left side gives $2 \times (5 \times 3)$, so $\triangle$ **= 3**.

### *Grouping in division: it does not work*

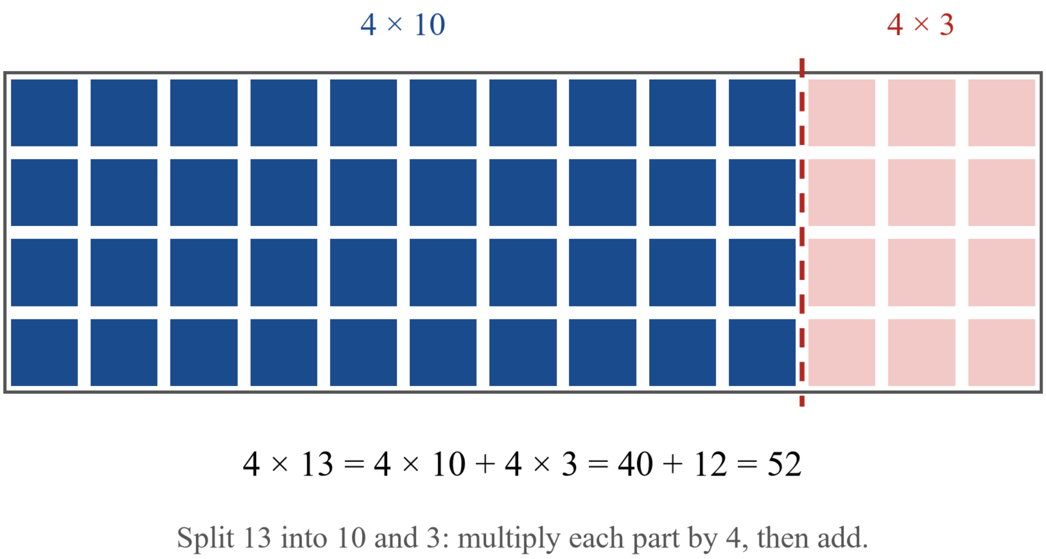



*Top: (8 ÷ 2) ÷ 2 shown as eight dots halved to four dots, then halved to two dots, which is 2. Bottom: 8 ÷ (2 ÷ 2) shown as first 2 ÷ 2 = 1, then 8 ÷ 1 = 8, which is 8. The same three numbers grouped in a different way give different answers.*

$(8 \div 2) \div 2 = 4 \div 2 = 2$, but $8 \div (2 \div 2) = 8 \div 1 = 8$. The same three numbers, grouped in a different way, give different answers. **Division is not associative.** So the grouping in a division must never be changed on the sly.

### *Splitting a number: the distributive property*

A large multiplication can be split into two smaller ones by splitting one of the numbers. Split 13 into 10 and 3:



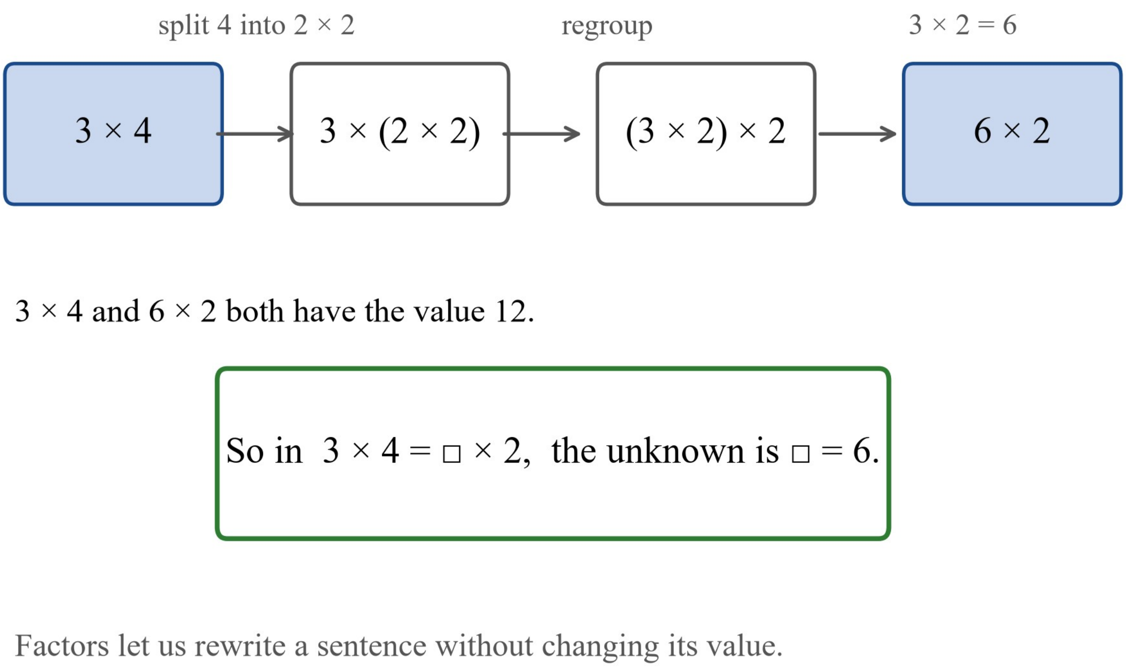

*A 4 by 13 array of tiles split by a dashed line into a 4 by 10 part and a 4 by 3 part. Below: 4 × 13 = 4 × 10 + 4 × 3 = 40 + 12 = 52.*

$4 \times 13 = 4 \times 10 + 4 \times 3 = 40 + 12 = 52$. The 4 is handed out over the two parts, 10 and 3. This is the **distributive property**, and it finds unknowns inside a split:

$5 \times 12 = 5 \times 10 + 5 \times \triangle$. The left side is 60, and $5 \times 10 = 50$, so $5 \times \triangle$ must be $60 - 50 = 10$. Then $\triangle = 2$. (Of course: 12 splits into 10 and 2.)

### *Rewriting with factors*

From 1A, a number can be replaced by its factors. $4 = 2 \times 2$, so:




*A chain of cards rewriting 3 × 4: split 4 into 2 × 2 to get 3 × (2 × 2), regroup to (3 × 2) × 2, then 6 × 2. A closing card says: in 3 × 4 = □ × 2 the unknown is □ = 6.*

$3 \times 4 = 3 \times (2 \times 2)$, replacing 4 with $2 \times 2$. Regrouping gives $(3 \times 2) \times 2 = 6 \times 2$. Both sides of the chain have the value 12, so in $3 \times 4 = \square \times 2$ the unknown is $\square$ **= 6**.

Notice that every step of the chain keeps the value the same. A fixed set of steps like this is a little **algorithm**: follow it and it always works. Section 1D builds algorithms for exactly this kind of experimenting with factors and multiples.

### *Always check*

When you have found an unknown, put the number back into the equation and look at both sides. For $\square = 2$ in $3 \times 5 = 30 \div \square$: left side 15, right side $30 \div 2 = 15$. The balance stays level. ✔ Keep up the 3A habit as well: stop and ask whether the answer is sensible.

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## Worked examples

**Worked example 1 — step by step: undo with the inverse**

Find the unknown in each equation.

a. $\square \times 7 = 42$
b. $45 \div \triangle = 5$

| Working | Comment |
|---|---|
| $\square \times 7 = 42 \rightarrow$ think $42 \div 7$ | Undo the multiplication with division |
| $42 \div 7 = 6$, so $\square = 6$ | Check: $6 \times 7 = 42$ ✔ |
| $45 \div \triangle = 5 \rightarrow$ think $45 \div 5$ | The fact family puts the unknown on its own |
| $45 \div 5 = 9$, so $\triangle = 9$ | Check: $45 \div 9 = 5$ ✔ |

**Worked example 2 — step by step: join two equal values**

It is known that $4 \times 9 = 36$ and $72 \div 2 = 36$. Join them into one equation, then find □ in $4 \times 9 = 72 \div □$.

| Working | Comment |
|---|---|
| Both sentences have the value 36. | Same value → they may be joined with = |
| $4 \times 9 = 72 \div 2$ | Equivalent sentences make a true equation |
| So in $4 \times 9 = 72 \div □$, $□ = 2$ | Because $72 \div 2 = 36$ |

Check: left side $4 \times 9 = 36$; right side $72 \div 2 = 36$. Level. ✔

**Worked example 3 — step by step: true or false?**

Decide whether each equation is true or false.

a. $18 \div 3 = 36 \div 6$
b. $7 \times 5 = 70 \div 2$
c. $6 \times 9 = 100 \div 2$

| Working | Comment |
|---|---|
| a. $18 \div 3 = 6$ and $36 \div 6 = 6$ | Same value → true |
| b. $7 \times 5 = 35$ and $70 \div 2 = 35$ | Same value → true |
| c. $6 \times 9 = 54$ but $100 \div 2 = 50$ | Different values → false |

Work out **both** sides every time. An equation that looks right can still be false.

**Worked example 4 — on your own: swap and regroup**

Find the unknown.

a. $□ \times 9 = 9 \times 4$

b. $(2 \times 5) \times 3 = 2 \times (5 \times △)$

c. $4 \times 2 \times 5 = □ \times 5$

d. Swap the right side: $9 \times 4 = 4 \times 9$, so **□ = 4**.

e. Regroup the left side: $(2 \times 5) \times 3 = 2 \times (5 \times 3)$, so **△ = 3**.

f. Work $4 \times 2$ first: $4 \times 2 \times 5 = 8 \times 5$, so **□ = 8**. (Check: both sides are 40.)

**Worked example 5 — on your own: split with the distributive property**

Find △ in $5 \times 12 = 5 \times 10 + 5 \times △$.

The left side is $5 \times 12 = 60$. The right side starts at $5 \times 10 = 50$, so $5 \times △$ must be $60 - 50 = 10$, which gives **△ = 2**. It matches the split $12 = 10 + 2$.

**Worked example 6 — on your own: rewrite with factors**

Find the unknown.

a. $4 \times 6 = □ \times 3$

b. $2 \times 15 = □ \times 5$

c. Replace 6 with $2 \times 3$: $4 \times 6 = 4 \times (2 \times 3) = (4 \times 2) \times 3 = 8 \times 3$, so **□ = 8**.

d. Replace 15 with $3 \times 5$: $2 \times 15 = 2 \times (3 \times 5) = (2 \times 3) \times 5 = 6 \times 5$, so **□ = 6**.

## Practice questions

### *With help*

**Q1.** Find the unknown. a. $\square \times 6 = 36$ b. $7 \times \triangle = 63$ c. $\square \times 8 = 64$ d. $\triangle \div 5 = 9$

**Q2.** Find the unknown. a. $24 \div \square = 4$ b. $56 \div \triangle = 7$ c. $\square \div 3 = 4$ d. $36 \div \triangle = 36$

**Q3.** $2 \times 9$ and $36 \div \square$ have the same value. Find $\square$.

**Q4.** True or false? a. $12 \div 4 = 30 \div 10$ b. $6 \times 6 = 72 \div 2$ c. $9 \times 4 = 40 \div 2$

**Q5.** Use the commutative property to find the unknown. a. $\square \times 7 = 7 \times 9$ b. $8 \times 4 = 4 \times \square$ c. $12 \times 3 = 3 \times \square$

**Q6.** Use the associative property to find the unknown: $(4 \times 2) \times 3 = 4 \times (2 \times \square)$.

**Q7.** Use the distributive property to find the unknown: $6 \times 14 = 6 \times 10 + 6 \times \square$.

**Q8.** Use factors to find the unknown: $3 \times 8 = \square \times 4$.

### *On your own*

**Q9.** True or false? a. $8 \times 4 = 64 \div 2$ b. $27 \div 3 = 81 \div 9$ c. $3 \times 9 = 56 \div 2$

**Q10.** Find the unknown. a. $3 \times 4 = \square \times 2$ b. $2 \times 6 = 3 \times \square$

**Q11.** Work out each side, and notice what the grouping does. a. $(18 \div 3) \div 3$ b. $18 \div (3 \div 3)$

**Q12.** Use the distributive property to find the unknown. a. $7 \times 11 = 7 \times 10 + 7 \times \square$ b. $9 \times 15 = 9 \times 10 + 9 \times \square$

**Q13.** Use factors to find the unknown. a. $3 \times 12 = \square \times 6$ b. $2 \times 25 = \square \times 5$

**Q14.** True or false? a. $8 \div 4 = 4 \div 8$ b. $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ c. $(8 \div 2) \div 2 = 8 \div (2 \div 2)$

**Q15.** A hall is set out with 8 chairs in every row. There are 56 chairs altogether. The equation $\square \times 8 = 56$ hides the number of rows. Find $\square$.

**Q16.** Ivy has 72 stickers and shares them equally over 9 tables, so $72 \div 9 = \triangle$. Find $\triangle$.

### *Maths in real life*

**Q17.** Mai says $6 \times 6 = 72 \div 2$ is true. Zac says it is false. Who is right, and how do you know?

**Q18.** Make your own true equation with a multiplication on one side and a division on the other (like $5 \times 6 = 60 \div 2$). Then hide one number behind a $\square$ and give the equation to a classmate to solve.

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## Answers

**Q1.** a. 6. b. 9. c. 8. d. 45. **Q2.** a. 6. b. 8. c. 12. d. 1. **Q3.** 2. **Q4.** a. True. b. True. c. False. **Q5.** a. 9. b. 8. c. 12. **Q6.** 3. **Q7.** 4. **Q8.** 6. **Q9.** a. True. b. True. c. False. **Q10.** a. 6. b. 4. **Q11.** a. 2. b. 18. **Q12.** a. 1. b. 5. **Q13.** a. 6. b. 10. **Q14.** a. False. b. True. c. False. **Q15.** 7 rows. **Q16.** 8. **Q17.** Mai is right. **Q18.** Answers will vary; check that both sides have the same value.

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## Worked answers

**Q1.** a. $36 \div 6 = 6$. b. $63 \div 7 = 9$. c. $64 \div 8 = 8$. d. $9 \times 5 = 45$. ∴ a. $\square = 6$; b. $\triangle = 9$; c. $\square = 8$; d. $\triangle = 45$.

**Q2.** a. $24 \div 4 = 6$. b. $56 \div 7 = 8$. c. $4 \times 3 = 12$. d. Only 1 works: $36 \div 1 = 36$. ∴ a. $\square = 6$; b. $\triangle = 8$; c. $\square = 12$; d. $\triangle = 1$.

**Q3.** $2 \times 9 = 18$, so $36 \div \square = 18$, and $36 \div 18 = 2$. ∴ $\square = 2$.

**Q4.** a. $12 \div 4 = 3$ and $30 \div 10 = 3$: true. b. $6 \times 6 = 36$ and $72 \div 2 = 36$: true. c. $9 \times 4 = 36$ but $40 \div 2 = 20$: false.

**Q5.** a. Swap the right side: $7 \times 9 = 9 \times 7$, so $\square = 9$. b. $8 \times 4 = 4 \times 8$, so $\square = 8$. c. $12 \times 3 = 3 \times 12$, so $\square = 12$.

**Q6.** Regrouping does not change the product: $(4 \times 2) \times 3 = 4 \times (2 \times 3)$. ∴ $\square = 3$.

**Q7.** 14 splits into 10 and 4, so $6 \times 14 = 6 \times 10 + 6 \times 4$. ∴ $\square = 4$. (Check: $60 + 24 = 84 = 6 \times 14$.)

**Q8.** Replace 8 with $2 \times 4$: $3 \times 8 = 3 \times (2 \times 4) = (3 \times 2) \times 4 = 6 \times 4$. ∴ $\square = 6$.

**Q9.** a. $8 \times 4 = 32$ and $64 \div 2 = 32$: true. b. $27 \div 3 = 9$ and $81 \div 9 = 9$: true. c. $3 \times 9 = 27$ but $56 \div 2 = 28$: false.

**Q10.** a. $3 \times 4 = 12$ and $6 \times 2 = 12$, so $\square = 6$. b. $2 \times 6 = 12$ and $3 \times 4 = 12$, so $\square = 4$.

**Q11.** a. $(18 \div 3) \div 3 = 6 \div 3 = 2$. b. $18 \div (3 \div 3) = 18 \div 1 = 18$. The same three numbers give different answers because division is not associative.

**Q12.** a. $7 \times 11 = 77$ and $7 \times 10 = 70$, so $7 \times \square = 77 - 70 = 7$ and $\square = 1$. b. $9 \times 15 = 135$ and $9 \times 10 = 90$, so $9 \times \square = 135 - 90 = 45$ and $\square = 5$.

**Q13.** a. Replace 12 with $2 \times 6$: $3 \times 12 = (3 \times 2) \times 6 = 6 \times 6$, so $\square = 6$. b. Replace 25 with $5 \times 5$: $2 \times 25 = (2 \times 5) \times 5 = 10 \times 5$, so $\square = 10$.

**Q14.** a. $8 \div 4 = 2$ but $4 \div 8$ is not 2: false. b. Both sides are 24: true. c. $(8 \div 2) \div 2 = 2$ but $8 \div (2 \div 2) = 8$: false.

**Q15.** $\square \times 8 = 56$, and $7 \times 8 = 56$. ∴ $\square = 7$: there are 7 rows.

**Q16.** $72 \div 9 = 8$. ∴ $\triangle = 8$ stickers on each table.

**Q17.** $6 \times 6 = 36$ and $72 \div 2 = 36$. Both sides have the same value, so the equation is true. ∴ Mai is right.

**Q18.** For example, $5 \times 6 = 60 \div 2$ is true because both sides are 30; hiding the 2 gives $5 \times 6 = 60 \div \square$ with $\square = 2$. Any pair of equal values joined with = is fine.

## Test

**Mathematics Level 5 — Chapter test T4: Inverse operations and unknown values**

*Read each question carefully, and show your working.*

**Marks:** 10

**Time:** Aim for about 15 minutes: about 5 minutes to read through the paper, then about 1 minute per mark. This is a classroom check, not an exam.

**Things you can use:** a pencil and paper. Nothing else is needed for this test.

**Calculators:** you may use a calculator if you want one. No question in this test needs it.

**Q1.** Circle the letter of the right answer. **[1 mark]**

8 × 3 = 24. Which fact belongs to the same **family of number facts**?

| A | B | C | D |
|---|---|---|---|
| 24 ÷ 8 = 3 | 24 ÷ 4 = 6 | 3 × 6 = 18 | 8 × 4 = 32 |

**Q2.** Inverse operations. Multiplication and division are **inverse operations**: each one undoes the other. **[4 marks]**

a. Write all four facts in the family of number facts for 5, 6 and 30. **[2 marks]**

b. Find 54 ÷ 6 by turning it into a multiplication question. **[1 mark]**

c. Use the inverse to find 240 ÷ 12. **[1 mark]**

**Q3.** Finding unknown values. A hidden number is called the **unknown**, and it is marked with a shape such as □ or △. An **equation** says that the two sides have the same value. **[5 marks]**

a. Find the unknown in each equation. **[2 marks]**

(i) □ × 7 = 63

(ii) 48 ÷ △ = 6

b. Is this equation true or false? Work out both sides. **[1 mark]**

6 × 5 = 60 ÷ 2

c. The **commutative property** of multiplication says the numbers may be swapped and the answer (the **product**) is unchanged. Use it to find □. **[1 mark]**

□ × 8 = 8 × 9

d. The **distributive property** splits one number into two parts, and hands the other number out over both parts. Use it to find □. **[1 mark]**

4 × 12 = 4 × 10 + 4 × □

## Solutions

### Marking guide

**Administration (per TOC §7).** This is a classroom check, not an exam. The guidance duration is 15 minutes: about 5 minutes reading plus about 1 minute per mark. No equipment is required (the §7 table row for T4 is "—"). **Calculator status follows D10**: no Level 5 CD says "without a calculator", and D10 forbids a "no calculator" instruction on any §7 paper, so calculators are permitted on this paper — but no question needs one. Every item assessing `VC2M5A01` or `VC2M5A02` asks for the strategy (the inverse, the number properties), which is what makes keying it in useless (§7).

**Marks by item and CD.** Only Chapter 4 CDs are assessed (§4). Each of the two subtopics carries exactly 5 marks (§7 marking rule: 5 marks per subtopic).

| Item | CD code | Subtopic | Type | Marks |
|---|---|---|---|---|
| Q1 | `VC2M5A01` | 4A | MCQ (A–D) | 1 |
| Q2 | `VC2M5A01` | 4A | Short answer / worked response | 4 |
| Q3 | `VC2M5A02` | 4B | Short answer / worked response | 5 |
| | | | **Total** | **10** |

MCQ total: 1 of 10 marks — sparingly per §7, used only because recognising a family member is genuinely a selection skill.

**Rubrics, by CD.** The achievement-standard extract is quoted verbatim from `maths_5.json`; `VC2M5A01` and `VC2M5A02` share one extract. Rubrics score the strategy as well as the answer (§7).

**`VC2M5A01` (4A) — 5 marks (Q1 1, Q2 4).** *"Students apply properties of numbers and operations to find unknown values in numerical equations involving multiplication and division."* Observable actions:

- recognises the family fact that comes with 8 × 3 = 24 — A, 24 ÷ 8 = 3 (1 mark);
- writes all four facts built from the three numbers 5, 6 and 30 — 5 × 6 = 30, 6 × 5 = 30, 30 ÷ 5 = 6, 30 ÷ 6 = 5 (2 marks: 2 for all four facts, 1 for three correct facts);
- turns a division into a multiplication question using the inverse, and finds the answer — 6 × □ = 54; 54 ÷ 6 = 9 (1 mark);
- uses the inverse to divide a larger number — 12 × □ = 240; 240 ÷ 12 = 20 (1 mark).

**`VC2M5A02` (4B) — 5 marks (Q3 5).** *"Students apply properties of numbers and operations to find unknown values in numerical equations involving multiplication and division."* Observable actions:

- undoes a multiplication unknown with the inverse — □ = 9 (1 mark);
- undoes a division unknown with the inverse — △ = 8 (1 mark);
- judges an equation by working out both sides — true, both sides are 30 (1 mark);
- uses the commutative property of multiplication to find the unknown — □ = 9 (1 mark);
- uses the distributive property to find the unknown inside a split — □ = 2 (1 mark).

**MCQ key.** Q1 **A**. One mark for the correct letter; no part marks.

**General marking notes.**

- Q2a: all four facts are needed for full marks. Accept the two multiplication facts in either order and the two division facts in either order.
- Q2b and Q2c: the mark requires the inverse step shown or stated (6 × □ = 54 in b; 12 × □ = 240 in c) as well as the correct answer. A correct answer with no inverse step earns the mark only if the working shown is a valid strategy.

- Q3a: accept either the division move ($63 \div 7$, $48 \div 6$) or a stated fact-family reason, with a correct unknown. A check by multiplying back is encouraged but not required for the mark.
- Q3b: both sides must be worked out — $6 \times 5 = 30$ and $60 \div 2 = 30$ — with the judgement “true”. A bare “true” with no values earns 0.
- Q3c: $\square = 9$. The intended route is the swap ($8 \times 9 = 9 \times 8$, so $\square = 9$). A correct answer reached by working $8 \times 9 = 72$ and solving $\square \times 8 = 72$ also earns the mark.
- Q3d: $\square = 2$. Accept either route: values ($4 \times 12 = 48$, $4 \times 10 = 40$, and $48 - 40 = 8$ gives $4 \times 2 = 8$) or the split (12 splits into 10 and 2).

## Answers

**Q1.** A.

**Q2.** a. $5 \times 6 = 30$, $6 \times 5 = 30$, $30 \div 5 = 6$, $30 \div 6 = 5$ b. 9 c. 20.

**Q3.** a. (i) $\square = 9$ (ii) $\triangle = 8$ b. True (both sides are 30) c. $\square = 9$ d. $\square = 2$.

## All the working

**Q1.** The family of 8, 3 and 24 is built from those three numbers only: $8 \times 3 = 24$, $3 \times 8 = 24$, $24 \div 8 = 3$ and $24 \div 3 = 8$. Option A, $24 \div 8 = 3$, is one of those facts. Options B, C and D are true facts, but they use other numbers, so they belong to other families. $\therefore$ A.

**Q2.** a. The family is built from 5, 6 and 30, with 30 as the total. The two multiplication facts are $5 \times 6 = 30$ and $6 \times 5 = 30$; the two division facts are $30 \div 5 = 6$ and $30 \div 6 = 5$. $\therefore$ the four facts are $5 \times 6 = 30$, $6 \times 5 = 30$, $30 \div 5 = 6$ and $30 \div 6 = 5$.

b. Turn the division around with the inverse: $54 \div 6 = \square$ is really asking $6 \times \square = 54$. $6 \times 9 = 54$, so the box is 9. $\therefore$ $54 \div 6 = 9$.

c. Think: $12 \times \square = 240$. $12 \times 2 = 24$, so $12 \times 20 = 240$ — the box is 20. Check with the inverse: $20 \times 12 = 240$ ✓ $\therefore$ $240 \div 12 = 20$.

**Q3.** a. (i) $\square \times 7 = 63$. Undo the multiplication with division: the box is $63 \div 7 = 9$. Check: $9 \times 7 = 63$ ✓ (ii) $48 \div \triangle = 6$. The fact family puts the unknown on its own: $\triangle$ is $48 \div 6 = 8$. Check: $48 \div 8 = 6$ ✓ $\therefore$ (i) $\square = 9$ (ii) $\triangle = 8$.

b. Left side: $6 \times 5 = 30$. Right side: $60 \div 2 = 30$. Both sides have the value 30, so the equation is true. $\therefore$ True.

c. Swapping does not change the product: $8 \times 9 = 9 \times 8$. So $\square \times 8 = 8 \times 9 = 9 \times 8$, and $\square = 9$. (Check: $9 \times 8 = 72$ and $8 \times 9 = 72$ ✓) $\therefore$ $\square = 9$.

d. Left side: $4 \times 12 = 48$. The right side starts at $4 \times 10 = 40$, so $4 \times \square$ must be $48 - 40 = 8$. $4 \times 2 = 8$, so $\square = 2$. The same answer comes from the split: 12 splits into 10 and 2. $\therefore$ $\square = 2$.