

Mathematics Level 5

Chapter 2 — Fractions, decimals and percentages

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Decimals beyond hundredths: place value, comparing and ordering

Chapter 2 · Subtopic 2A · Level 5

interpret, compare and order numbers with more than 2 decimal places, including numbers greater than one, using place value understanding; represent these on a number line — VC2M5N01

From the Level 5 achievement standard: *Students use place value to write and order decimals including decimals greater than one.*

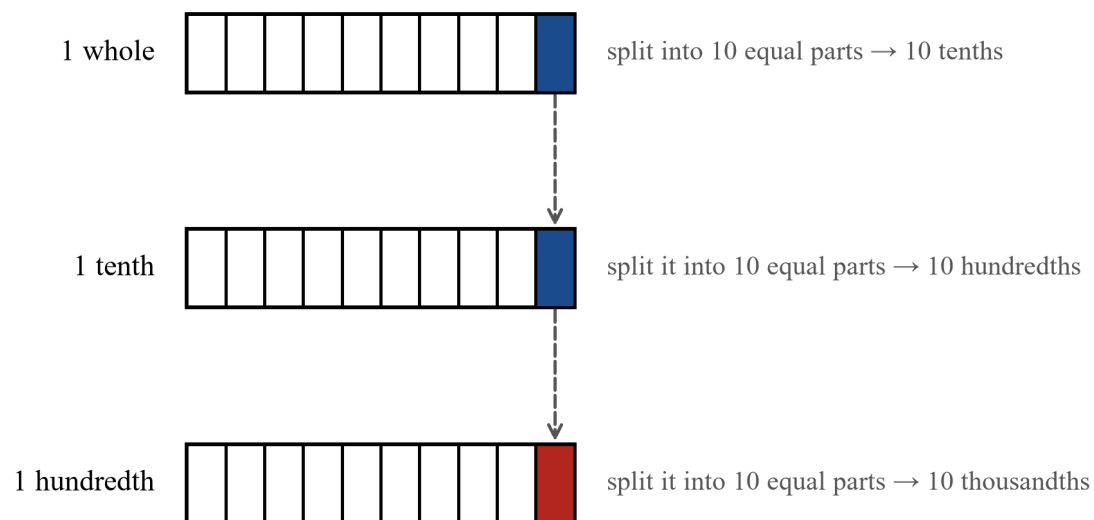
Theory

What you bring to this subtopic

In Level 4 you worked with decimals to tenths and hundredths (VC2M4N01). This subtopic builds on that; it does not re-teach it. At the start you know:

- the first place to the right of the decimal point is the **tenths** place, and 0.1 is 1 out of 10 equal parts of a whole, $\frac{1}{10}$;
- the second place is the **hundredths** place, and 0.01 is 1 out of 100 equal parts of a whole, $\frac{1}{100}$;
- 10 hundredths make 0.1, and 10 tenths make 1.

One more split: thousandths



Each new piece is 10 times smaller than the one before it.

A whole bar split into 10 equal tenths, then one of the tenths split into 10 hundredths, then one of the hundredths split into 10 thousandths, with each new piece shaded and 10 times smaller.

Decimal place value works in tens. Each place is worth 10 times the place on its right, so each place is 10 times smaller than the place on its left.

Split a whole into 10 equal parts and the parts are **tenths**. Split 1 of those tenths into 10 equal parts and the parts are **hundredths**. Split 1 of those hundredths into 10 equal parts and the tiny new parts are **thousandths**. One thousandth is written 0.001, or as the fraction $\frac{1}{1000}$.

Because each new part is made by splitting into 10 again:

- 10 thousandths make 0.01;

- 100 thousandths make 0.1;
- 1000 thousandths make 1.

So thousandths are **10 times smaller** than hundredths, and hundredths are 10 times smaller than tenths. That times-10 relationship between places that sit next to each other is what place value means, and it keeps going as far to the right as you ever need.

Ten-thousandths and beyond

Split a thousandth into 10 and each part is a **ten-thousandth**, 0.0001, the fourth place to the right of the point. There is nothing special about two decimal places — a number can have three places, four places, or more. The pattern never changes: each place is 10 times smaller than the one on its left.

The decimal place-value chart

Tens	Ones		Tenths	Hundredths	Thousandths
10	1		0.1	0.01	0.001

An empty decimal place-value chart with columns for tens, ones, a decimal point, tenths, hundredths and thousandths, and a bottom row saying each column is worth 10, 1, 0.1, 0.01 and 0.001.

A place-value chart gives each digit its own column, with the decimal point between the ones column and the tenths column. The column a digit sits in tells you what the digit is worth. The bottom row of the chart says what 1 in each column is worth.

Tens	Ones		Tenths	Hundredths	Thousandths
3	6	.	0	4	7
30	6		0	0.04	0.007
10	1		0.1	0.01	0.001

36.047

The place-value chart filled with 36.047: digits 3, 6, point, 0, 4, 7, with the value of each digit below: 30, 6, 0, 0.04, 0.007.

Here is 36.047 in the chart.

- The **3** is in the tens column. It is worth 30.

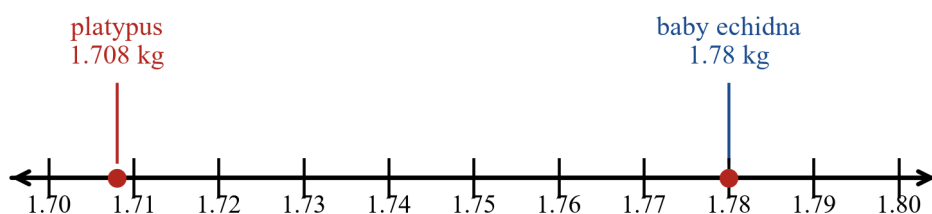
- The **6** is in the ones column. It is worth 6.
- The **0** is in the tenths column. It is worth 0 tenths. This 0 is a **placeholder**: it holds the tenths column open so the 4 and the 7 stay in their places. Without it, 36.047 would become 36.47 — a greater number.
- The **4** is in the hundredths column. It is worth 0.04.
- The **7** is in the thousandths column. It is worth 0.007.

Say 36.047 as *thirty-six and forty-seven thousandths*, or read the digits after the point one at a time: *thirty-six point zero four seven*. Both are correct; the first way says the size of the number more clearly.

To interpret a decimal means to say what it is telling you: which column each digit sits in, and what each digit is worth.

Comparing decimals

To compare two decimals — to say which is greater, which is smaller, or whether they are equal — line up the decimal points and look at the columns from the left. The first column where the digits differ decides it. The number of digits tells you nothing.



1.708 sits between 1.70 and 1.71 — closer to 1.71. 1.78 sits right on the 1.78 tick.

A number line from 1.70 to 1.80 in hundredths. A platypus of mass 1.708 kilograms sits just before 1.71, and a baby echidna of mass 1.78 kilograms sits on the 1.78 tick, clearly to the right of the platypus.

Source: VCAA, Victorian Curriculum F–10 Mathematics 2.0, VC2M5N01.E04, retrieved 29 July 2026.

The Victorian Curriculum gives this example: a baby echidna with a mass of 1.78 kilograms and a platypus with a mass of 1.708 kilograms. *Surely 1.708 is greater, because it has more digits?* No. Line up the points and add a zero to the shorter number — adding zeros after the last digit of a decimal does not change its value, because 78 hundredths is exactly 780 thousandths:

- $1.78 = 1.780$
- $1.708 = 1.708$

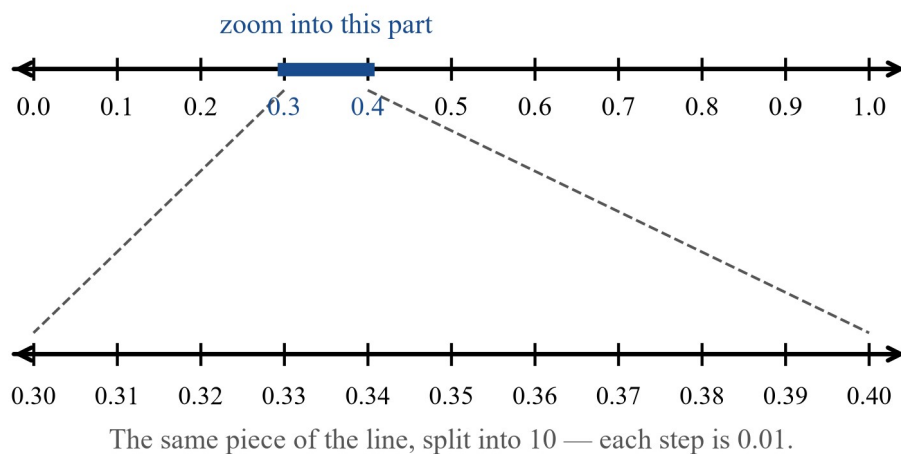
Ones: 1 and 1, equal. Tenths: 7 and 7, equal. Hundredths: **8** against **0** — and 8 hundredths is more. So $1.78 > 1.708$: the baby echidna has the greater mass, even though its numeral is shorter.

The number line agrees. The greater number sits further to the right. 1.708 is squeezed between 1.70 and 1.71, while 1.78 sits a long way further right, on its own tick.

Ordering decimals

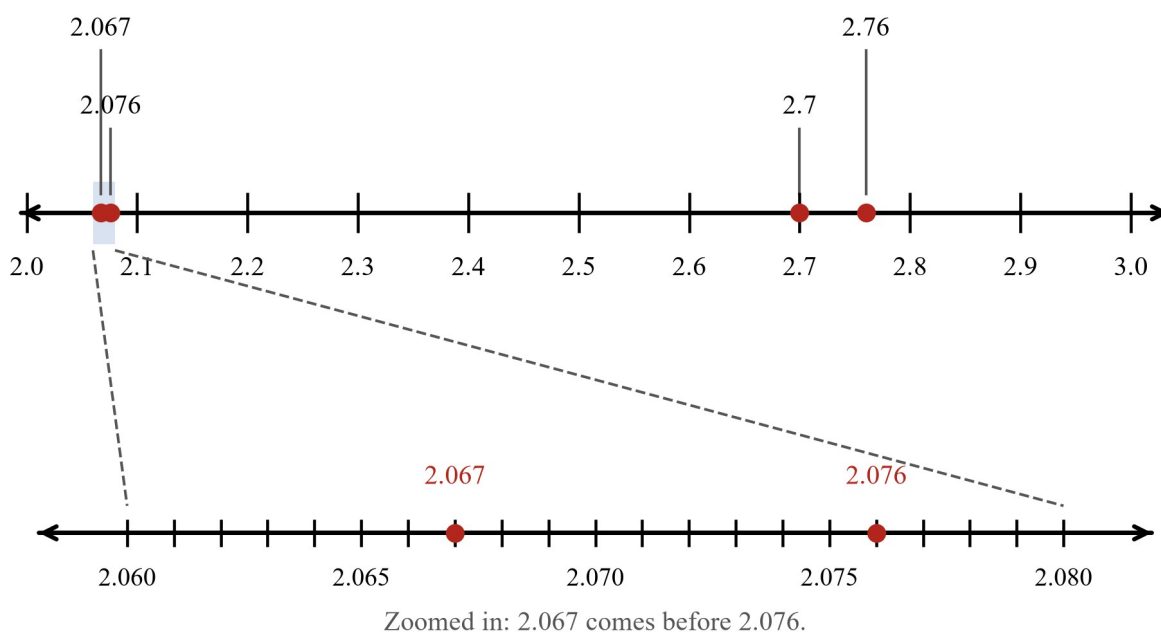
To order decimals, compare them, then write them from smallest to biggest — or biggest to smallest, if the question asks for that. A number line is a good check, because the order of numbers is the same as their order from left to right on the line.

When two numbers are very close together, one scale may not separate them. Zoom in: turn one small gap into its own new line.



A number line from 0 to 1 in tenths with the gap from 0.3 to 0.4 marked in blue, and below it a zoomed number line from 0.30 to 0.40 in hundredths joined to that gap.

You know this move already: the gap from 0.3 to 0.4 in tenths becomes a new line from 0.30 to 0.40 in hundredths. The same move, one place deeper, separates thousandths.



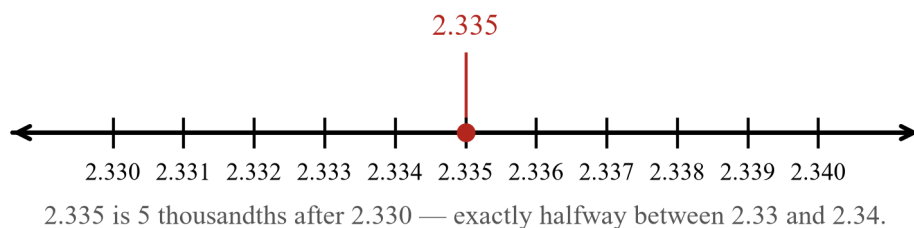
A number line from 2 to 3 in tenths with 2.067, 2.076, 2.7 and 2.76 marked, and a zoom of the part from 2.060 to 2.080 in thousandths showing that 2.067 comes before 2.076.

On the line from 2 to 3, the numbers 2.067 and 2.076 look almost on top of each other. The zoom separates them: 2.067 is 7 thousandths past 2.060, and 2.076 is 6 thousandths further right again. So 2.067 comes before 2.076.

Decimals on a number line

To place a decimal on a number line:

1. Find the two ticks it sits between. For 2.335 those are 2.33 and 2.34.
2. Split that gap in your mind into 10 equal parts — the parts are the next place down, here thousandths.
3. Count that many parts along. 2.335 is 5 thousandths past 2.33.



A number line from 2.330 to 2.340 in thousandths, with 2.335 marked exactly halfway between the two ends.

5 parts out of 10 is half of the gap, so 2.335 sits exactly halfway between 2.33 and 2.34. Every number line in this subtopic is drawn to scale: equal steps are drawn equal, so the picture is an argument, not a guess.

Renaming for mental work

A decimal can be renamed to make some mental work easier. The question $0.6 \div 10$ looks awkward until you rename. 0.6 is 6 tenths. Rename 6 tenths as 60 hundredths — the same amount, named in smaller parts. Dividing 60 hundredths by 10 gives 6 hundredths, which is 0.06. So $0.6 \div 10 = 0.06$. There is no trick in it: 60 hundredths is exactly 6 tenths, just renamed.

Worked examples

Worked example 1 — step by step: read a decimal in a place-value chart

Write 58.604 in a place-value chart, state the value of each digit, and write the number in words.

Working	Comment
Tens 5, ones 8, point, tenths 6, hundredths 0, thousandths 4.	Line up the point first, then one digit per column.
5 is worth 50; 8 is worth 8; 6 is worth 0.6; 0 is worth 0 hundredths; 4 is worth 0.004.	Each column says what its digit is worth.
In words: <i>fifty-eight and six hundred and four thousandths</i> .	0.604 is 604 thousandths.

$\therefore$ 58.604 has 5 tens, 8 ones, 6 tenths, 0 hundredths and 4 thousandths, and the 0 is a placeholder.

Worked example 2 — step by step: rename to divide by 10

Rename to find $0.6 \div 10$.

Working	Comment
$0.6 = 6$ tenths.	The 6 sits in the tenths column.
6 tenths = 60 hundredths.	Split each of the tenths into 10 hundredths.
$60 \text{ hundredths} \div 10 = 6 \text{ hundredths} = 0.06$.	Dividing a count of hundredths by 10 keeps them hundredths.

$\therefore 0.6 \div 10 = 0.06$.

Worked example 3 — step by step: which mass is greater?

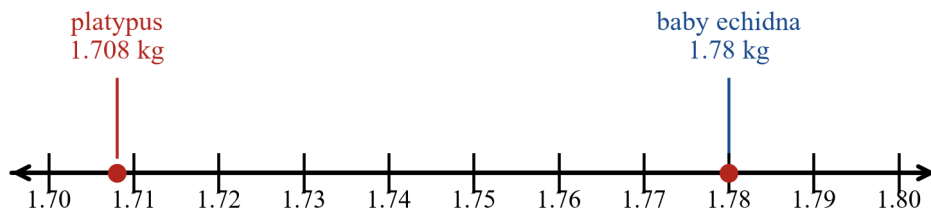
Which has the greater mass: a baby echidna at 1.78 kilograms, or a platypus at 1.708 kilograms?

Working	Comment
Write 1.78 as 1.780.	Adding a final zero does not change the number.
Ones: 1 and 1. Tenths: 7 and 7. Equal so far.	Work from the left.
Hundredths: 8 against 0.	The first different column decides it.

$1.780 > 1.708$, so $1.78 > 1.708$.

More digits does not mean greater.

$\therefore$ The baby echidna has the greater mass.



1.708 sits between 1.70 and 1.71 — closer to 1.71. 1.78 sits right on the 1.78 tick.

A number line from 1.70 to 1.80 in hundredths. A platypus of mass 1.708 kilograms sits just before 1.71, and a baby echidna of mass 1.78 kilograms sits on the 1.78 tick, clearly to the right of the platypus.

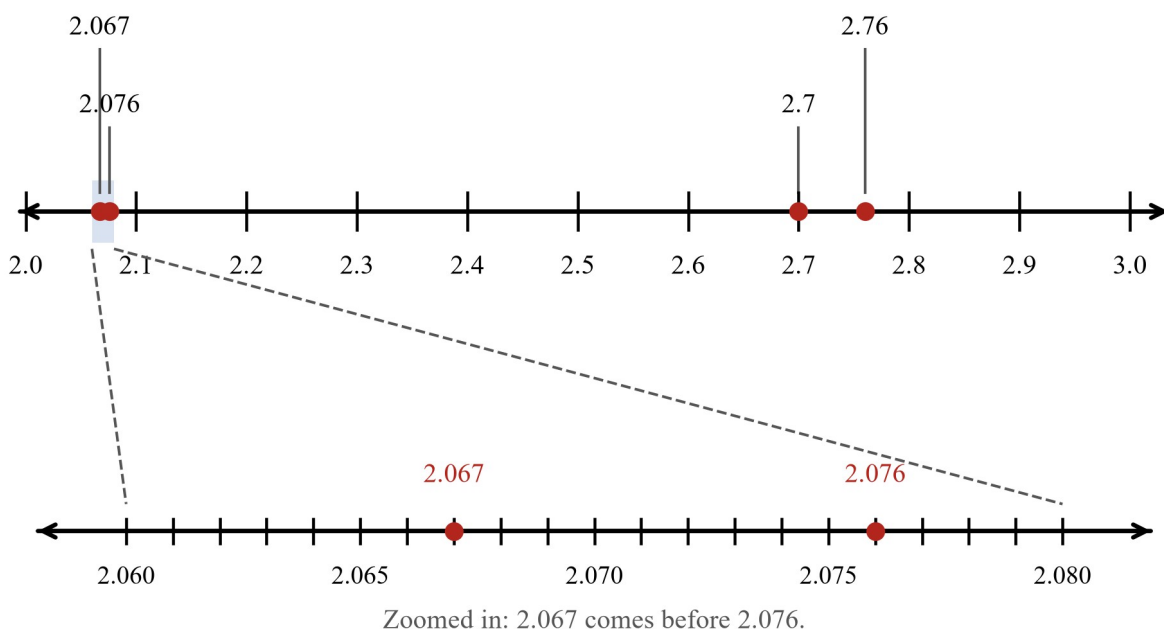
Worked example 4 — on your own: order four decimals

Put these numbers in order from smallest to biggest, and show the order on a number line: 2.7, 2.076, 2.76, 2.067.

Write each with 3 decimal places: 2.700, 2.076, 2.760, 2.067.

Compare from the left. Tenths: the two 0s (2.076, 2.067) come before the two 7s (2.700, 2.760). Within the first pair, hundredths: 6 against 7, so 2.067 comes before 2.076. Within the second pair, 2.700 comes before 2.760.

$\therefore$ In order: $2.067 < 2.076 < 2.7 < 2.76$.



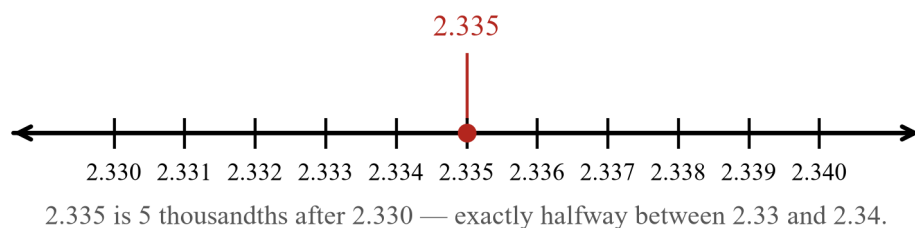
A number line from 2 to 3 in tenths with 2.067, 2.076, 2.7 and 2.76 marked, and a zoom of the part from 2.060 to 2.080 in thousandths showing that 2.067 comes before 2.076.

Worked example 5 — on your own: place 2.335 on a number line

Place 2.335 on a number line and justify your placement.

2.335 sits between 2.33 and 2.34. The gap from 2.33 to 2.34 is 0.01; split it into 10 equal parts and each part is one thousandth. 2.335 is 5 thousandths past 2.33, and 5 parts out of 10 is half the gap.

$\therefore$ 2.335 sits exactly halfway between 2.33 and 2.34.



A number line from 2.330 to 2.340 in thousandths, with 2.335 marked exactly halfway between the two ends.

Practice questions

Q1. Write each number in words.

- a. 4.073
- b. 16.081
- c. 0.009
- d. 3.2409

Q2. State the value of the digit 7 in each number.

- a. 7.205
- b. 3.174
- c. 12.708
- d. 0.437

Q3. Fill in each blank.

- a. $0.1 = \underline{\hspace{1cm}}$ hundredths
- b. $0.01 = \underline{\hspace{1cm}}$ thousandths
- c. 3 tenths = $\underline{\hspace{1cm}}$ hundredths
- d. $0.5 = 50 \underline{\hspace{1cm}}$
- e. 8 hundredths = $\underline{\hspace{1cm}}$ thousandths
- f. 2 ones = $\underline{\hspace{1cm}}$ tenths

Q4. Write the number described.

- a. 6 ones, 2 tenths, 0 hundredths, 5 thousandths
- b. 5 tens, 2 ones, 0 tenths, 0 hundredths, 9 thousandths
- c. 0 ones, 4 hundredths, 1 thousandth

Q5. True or false? If false, say why.

- a. Thousandths are 10 times smaller than hundredths.
- b. 0.30 is 10 times as big as 0.3.
- c. 10 thousandths make 0.01.
- d. In 8.06, the 0 says there are no tenths.

Q6. Put 41.605 into a place-value chart and state the value of each digit.

Q7. Rename, then find the answer.

- a. $0.8 \div 10$

- b. $0.3 \div 10$
c. $2.4 \div 10$

Q8. Write $<$, $>$ or $=$ in each box.

- a. $1.78 \square 1.708$
b. $0.6 \square 0.60$
c. $3.05 \square 3.050$
d. $2.409 \square 2.49$
e. $0.09 \square 0.1$
f. $6.302 \square 6.32$

Q9. Three friends measure their long jumps: Ava 3.205 m, Ben 3.198 m, Chloe 3.215 m.

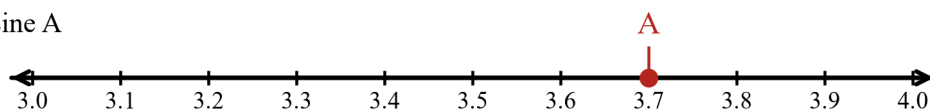
- a. Who had the longest jump?
b. Write the three jumps in order, smallest to biggest.

Q10. Order from smallest to biggest: 8.16, 8.061, 8.6, 8.106.

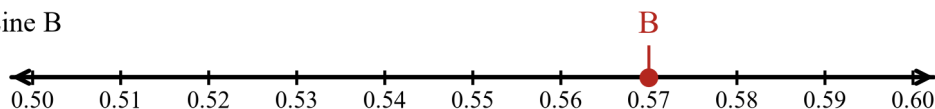
Q11. Order from biggest to smallest: 6.12, 6.1, 6.02, 6.012.

Q12. Read the number marked by each dot.

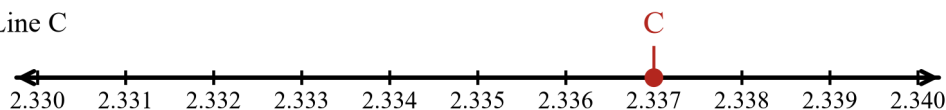
Line A



Line B

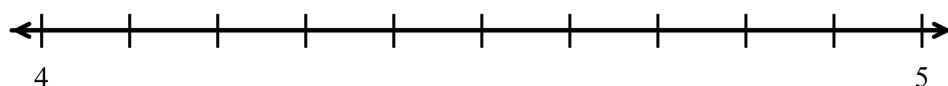


Line C



Three number lines. Line A runs from 3.0 to 4.0 in tenths with a dot on 3.7. Line B runs from 0.50 to 0.60 in hundredths with a dot on 0.57. Line C runs from 2.330 to 2.340 in thousandths with a dot on 2.337.

Q13. Place 4.3, 4.8 and 4.05 on this number line. The ticks between 4 and 5 split each whole into tenths.



The ticks between 4 and 5 split each whole into tenths.

A number line from 4 to 5 marked in tenths; only the ends, 4 and 5, are labelled.

Q14. Write the number exactly halfway between each pair.

- a. 5.06 and 5.07
- b. 0.59 and 0.60
- c. 2.3 and 2.4

Q15. You have digit cards 7, 0, 3, 5 and a decimal point. Use each card once.

- a. Make the biggest number smaller than 10.
- b. Make the smallest number greater than 0.

Q16. Mia says 0.725 must be greater than 0.8 because 0.725 has more digits. Is Mia correct? Use place value to explain.

Q17. Think about the number 7.452.

- a. Which thousandth number is just before it, and which is just after it?
- b. Which two numbers with 2 decimal places does it sit between?

Q18. Count on in thousandths. Fill in the missing numbers.

- a. 1.204, 1.205, _____, _____, 1.208
- b. 0.997, 0.998, 0.999, _____, _____

Answers

- Q1.** a. four and 73 thousandths b. sixteen and 81 thousandths c. nine thousandths d. three point two four zero nine (three and 2409 ten-thousandths).
- Q2.** a. 7 ones, worth 7 b. 7 hundredths, worth 0.07 c. 7 tenths, worth 0.7 d. 7 thousandths, worth 0.007.
- Q3.** a. 10 b. 10 c. 30 d. hundredths e. 80 f. 20.
- Q4.** a. 6.205 b. 52.009 c. 0.041.
- Q5.** a. true b. false — 0.30 is the same number as 0.3 c. true d. true.
- Q6.** Tens 4 (worth 40), ones 1 (worth 1), tenths 6 (worth 0.6), hundredths 0 (worth 0), thousandths 5 (worth 0.005).
- Q7.** a. 0.08 b. 0.03 c. 0.24.
- Q8.** a. $>$ b. $=$ c. $=$ d. $<$ e. $<$ f. $<$.
- Q9.** a. Chloe b. 3.198 m, 3.205 m, 3.215 m.
- Q10.** 8.061, 8.106, 8.16, 8.6.
- Q11.** 6.12, 6.1, 6.02, 6.012.
- Q12.** A = 3.7, B = 0.57, C = 2.337.
- Q13.** 4.3 on the third tick after 4; 4.8 on the 8th tick after 4; 4.05 halfway between 4 and 4.1.
- Q14.** a. 5.065 b. 0.595 c. 2.35.
- Q15.** a. 7.530 b. 0.357.
- Q16.** No — see solution.
- Q17.** a. 7.451 and 7.453 b. 7.45 and 7.46.
- Q18.** a. 1.206, 1.207 b. 1.000 (the same as 1), 1.001.

Fully-worked solutions

Q1.

- a. 4.073: 0.073 is 73 thousandths $\rightarrow$ *four and seventy-three thousandths*, or *four point zero seven three*.
- b. 16.081: 0.081 is 81 thousandths $\rightarrow$ *sixteen and eighty-one thousandths*.
- c. 0.009: *nine thousandths*.
- d. 3.2409: the fourth place is ten-thousandths $\rightarrow$ *three point two four zero nine*, or *three and 2409 ten-thousandths*.

Q2.

- a. In 7.205 the 7 is left of the point, in the ones column: worth 7.
- b. In 3.174 the 7 is in the hundredths column: worth 0.07.
- c. In 12.708 the 7 is in the tenths column: worth 0.7.
- d. In 0.437 the 7 is in the thousandths column: worth 0.007.

Q3.

- a. 0.1 splits into 10 hundredths $\rightarrow$ 10.
- b. 0.01 splits into 10 thousandths $\rightarrow$ 10.
- c. 3 tenths = 3 lots of 10 hundredths = 30 hundredths.
- d. 0.5 is 5 tenths = 50 hundredths.
- e. 8 hundredths = 8 lots of 10 thousandths = 80 thousandths.

- f. Each one is 10 tenths, so 2 ones = 20 tenths.

Q4.

- a. Ones 6, tenths 2, hundredths 0, thousandths 5 $\rightarrow$ 6.205.
- b. Tens 5, ones 2, tenths 0, hundredths 0, thousandths 9 $\rightarrow$ 52.009.
- c. Ones 0, tenths 0, hundredths 4, thousandths 1 $\rightarrow$ 0.041.

Q5.

- a. True — that is the times-10 relationship between places.
- b. False — 0.30 is 30 hundredths, which is exactly 3 tenths, so $0.30 = 0.3$. A final zero adds no size.
- c. True — 10 thousandths = 0.01.
- d. True — the 0 holds the tenths column open.

Q6. In the chart: tens 4, ones 1, point, tenths 6, hundredths 0, thousandths 5. Values: 40, 1, 0.6, 0, 0.005. The 0 hundredths is a placeholder.

Q7.

- a. $0.8 = 8 \text{ tenths} = 80 \text{ hundredths}$; $80 \text{ hundredths} \div 10 = 8 \text{ hundredths} = 0.08$.
- b. $0.3 = 30 \text{ hundredths}$; $30 \text{ hundredths} \div 10 = 3 \text{ hundredths} = 0.03$.
- c. $2.4 = 24 \text{ tenths} = 240 \text{ hundredths}$; $240 \text{ hundredths} \div 10 = 24 \text{ hundredths} = 0.24$.

Q8.

- a. $1.78 = 1.780$; 780 thousandths $>$ 708 thousandths, so $1.78 > 1.708$.
- b. 0.60 is 60 hundredths = 6 tenths = 0.6, so $=$.
- c. 3.050 is 3 and 50 thousandths = 3 and 5 hundredths, so $=$.
- d. $2.49 = 2.490$; 490 thousandths $>$ 409 thousandths, so $2.409 < 2.49$.
- e. $0.1 = 0.100$; 90 thousandths $<$ 100 thousandths, so $0.09 < 0.1$.
- f. $6.32 = 6.320$; 320 thousandths $>$ 302 thousandths, so $6.302 < 6.32$.

Q9.

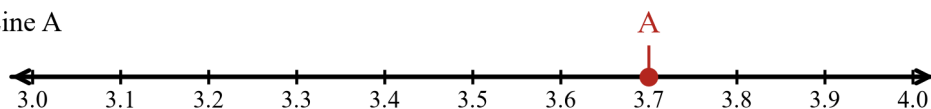
- a. Tenths first: Ben has 1, Ava has 2, Chloe has 2, so Ben had the shortest jump. Ava 3.205 against Chloe 3.215: hundredths 0 against 1, so Chloe had the longest jump.
- b. 3.198 m, then 3.205 m, then 3.215 m.

Q10. Write with 3 places: 8.160, 8.061, 8.600, 8.106. Compare the tenths: 8.061 has 0 tenths, so it comes first. 8.160 and 8.106 both have 1 in the tenths column, so compare the hundredths: 1 against 6 $\rightarrow$ 8.106, then 8.16. 8.600 has 6 tenths, so 8.6 is last. Order: 8.061, 8.106, 8.16, 8.6.

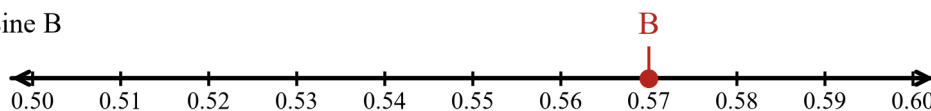
Q11. Write with 3 places: 6.120, 6.100, 6.020, 6.012. Biggest tenths first: 6.120, then 6.100, then 6.020, then 6.012. So 6.12, 6.1, 6.02, 6.012.

Q12.

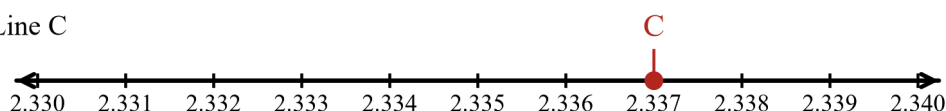
Line A



Line B



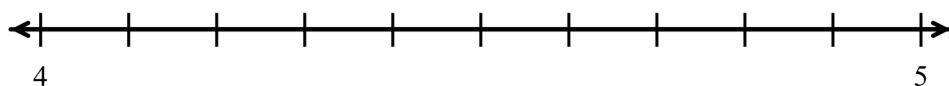
Line C



Three number lines. Line A runs from 3.0 to 4.0 in tenths with a dot on 3.7. Line B runs from 0.50 to 0.60 in hundredths with a dot on 0.57. Line C runs from 2.330 to 2.340 in thousandths with a dot on 2.337.

A: the dot is on the 7th tick after 3.0 $\rightarrow$ 3.7. B: the dot is on the 7th tick after 0.50 $\rightarrow$ 0.57. C: the dot is on the 7th tick after 2.330 $\rightarrow$ 2.337.

Q13.



The ticks between 4 and 5 split each whole into tenths.

A number line from 4 to 5 marked in tenths; only the ends, 4 and 5, are labelled.

4.3 is 3 tenths past 4: the third tick. 4.8 is the 8th tick. 4.05 is 5 hundredths past 4 — half of the first gap — so halfway between the 4 tick and the 4.1 tick.

Q14.

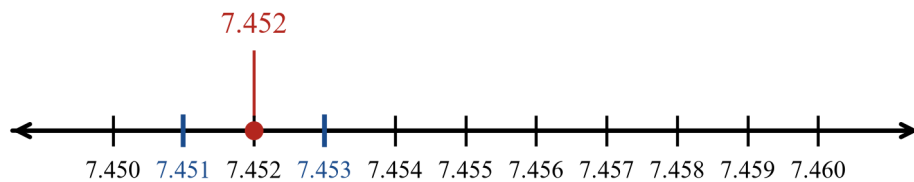
- The gap from 5.06 to 5.07 is 10 thousandths; halfway is 5 thousandths on: 5.065.
- Halfway from 0.59 to 0.60 is 0.595.
- Halfway from 2.3 to 2.4 is 2.35.

Q15.

- Smaller than 10 means the ones digit is the biggest card, 7; then put the rest in biggest-first order: 7.530.
- Greater than 0 but as small as possible: 0 ones, then the smallest digits first after the point: 0.357.

Q16. No. Line up the points: $0.8 = 0.800$. In the tenths column, 8 against 7 — the 8 wins straight away. 0.8 is 800 thousandths and 0.725 is only 725 thousandths, so 0.8 is greater. More digits never makes a decimal greater.

Q17.



The thousandth numbers either side of 7.452 are 7.451 and 7.453.

A number line from 7.450 to 7.460 in thousandths with 7.452 marked, and the ticks 7.451 and 7.453 highlighted.

- Counting in thousandths: 7.451, 7.452, 7.453. Just before is 7.451 and just after is 7.453.
- 7.452 is 2 thousandths past 7.45 and before 7.46, so it sits between 7.45 and 7.46.

Q18.

- Count on one thousandth each step: 1.204, 1.205, 1.206, 1.207, 1.208.
- 0.997, 0.998, 0.999, then one more thousandth makes 1000 thousandths = 1, written 1.000, then 1.001.
Counting in thousandths walks straight over 1.

Comparing and ordering fractions, including mixed numerals

Chapter 2 · Subtopic 2B · Level 5

compare and order common unit fractions with the same and related denominators, including mixed numerals, applying knowledge of factors and multiples; represent these fractions on a number line — VC2M5N03

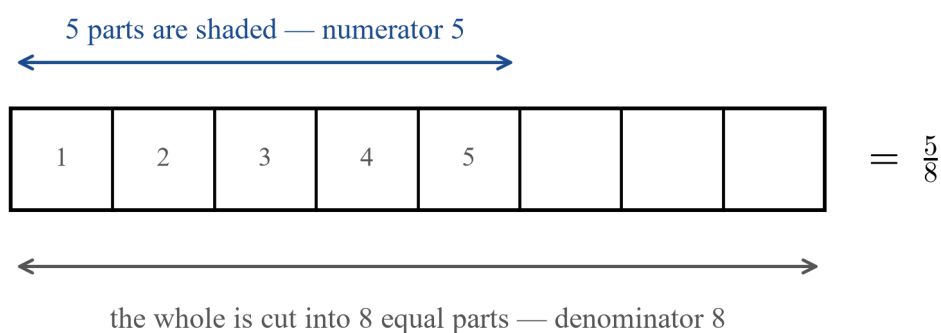
From the Level 5 achievement standard: *Students order and represent, add and subtract fractions with the same or related denominators.*

At Level 4 you already met equivalent fractions and counted along number lines in halves, quarters and thirds (VC2M4N03, VC2M4N04). This section builds on that. In 1A you found factors and multiples — that knowledge is what tells us when two denominators are **related**, and related denominators are the key to comparing and ordering fractions at this level.

Theory

What a fraction names

A fraction names equal parts of one whole. The bottom number is the **denominator**: how many equal parts the whole is cut into. The top number is the **numerator**: how many of those parts we are counting.



A bar cut into 8 equal parts with 5 parts shaded, showing 5 over 8. The top arrow shows the 5 shaded parts, the numerator. The bottom arrow shows the 8 equal parts, the denominator.

Here the whole is cut into 8 equal parts and 5 parts are shaded, so the fraction is $\frac{5}{8}$.

A **unit fraction** has numerator 1 — just one of the equal parts. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{10}$ are all unit fractions.

Same denominators: count the parts

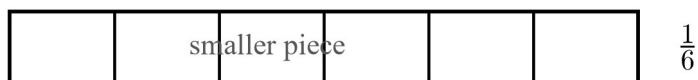
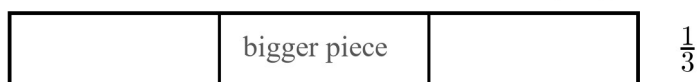
When two fractions have the **same denominator**, their parts are the same size. Then the fraction with more parts is more. On a number line, the fraction that is more sits further from 0.

So $\frac{5}{8} > \frac{3}{8}$: 5 parts out of 8 is more than 3 parts out of 8, just as 5 of anything is more than 3 of the same thing.

Unit fractions: more parts means less in each part

Unit fractions go the other way. Cutting the same whole into more parts makes each part less of the whole. So $\frac{1}{6}$ is less than $\frac{1}{3}$.

the same whole, cut into different numbers of equal parts



Two bars the same length. The top bar is cut into 3 equal parts with 1 part shaded, one-third, and the shaded piece is more of the whole. The bottom bar is cut into 6 equal parts with 1 part shaded, 1 part out of 6, and the shaded piece is less of the whole.

So $\frac{1}{3} > \frac{1}{6}$, even though 6 is more than 3. With unit fractions, the more parts the whole is cut into, the less each part is.

Related denominators: use factors and multiples

Two denominators are **related** when one is a multiple of the other. That is where 1A comes in: 8 is a multiple of 4 ($4 \times 2 = 8$), so 4 and 8 are related denominators. The families we use in this section are:

- **denominators 2, 4 and 8** (4 is a multiple of 2; 8 is a multiple of 4 and of 2)
- **denominators 3 and 6** (6 is a multiple of 3)
- **denominators 5 and 10** (10 is a multiple of 5)

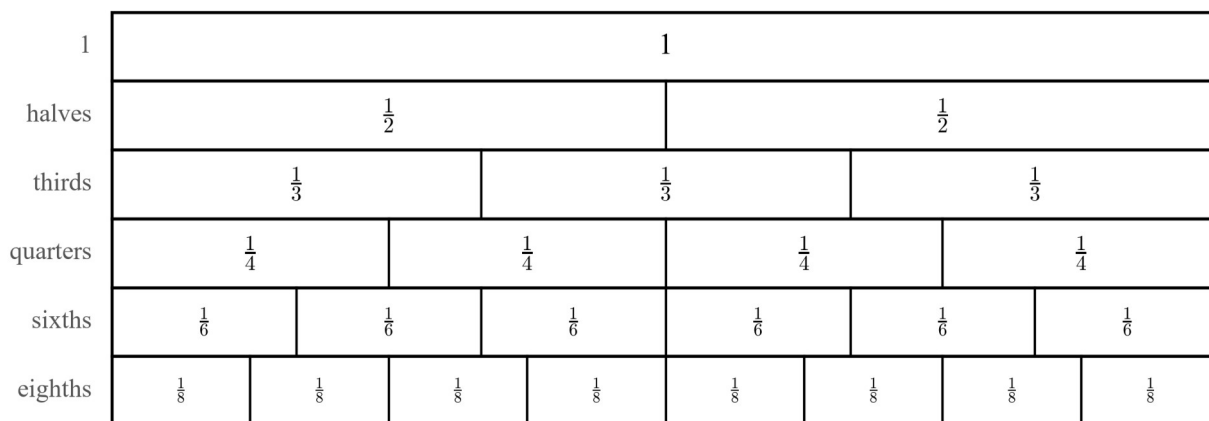
When denominators are related, we can write both fractions with the same denominator, using equivalent fractions.

For example, to compare $\frac{3}{4}$ with $\frac{5}{8}$, we write $\frac{3}{4}$ with denominator 8: $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$. Now the denominators are the same, and $\frac{6}{8} > \frac{5}{8}$.

Denominators that are **not** related — like 3 and 4, or 6 and 8, where one is not a multiple of the other — are not compared in this section.

The fraction wall

A fraction wall is built from strips of paper tape that are all the same length, with each row cutting one whole into equal parts.



A fraction wall with rows for 1 whole, halves, thirds, quarters, and rows of 6 and 8 equal parts. Red lines join the places where pieces line up: one half lines up with two quarters, 3 parts out of 6 and 4 parts out of 8; one quarter lines up with 2 parts out of 8.

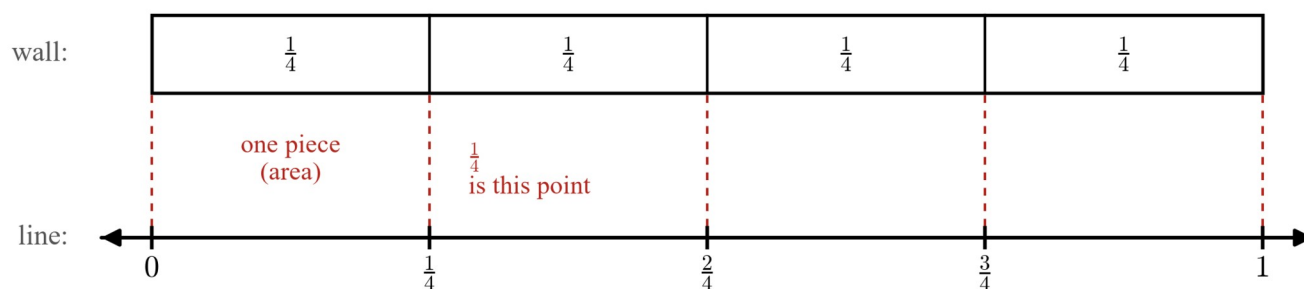
Where the ends of the pieces line up, the fractions are equal. Reading down the red lines:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} \text{ and } \frac{1}{4} = \frac{2}{8}, \text{ and } \frac{1}{3} = \frac{2}{6}.$$

The wall also shows what *not* related means: the ends of the 6 equal parts do not line up with the ends of the 8 equal parts, because 6 and 8 are not related.

The wall and the number line

The fraction wall and the number line show the same fractions in two different ways. On the wall, $\frac{1}{4}$ is a **piece**: the area of one quarter of the whole. On the number line, $\frac{1}{4}$ is a **point**: one quarter of the distance from 0 to 1.

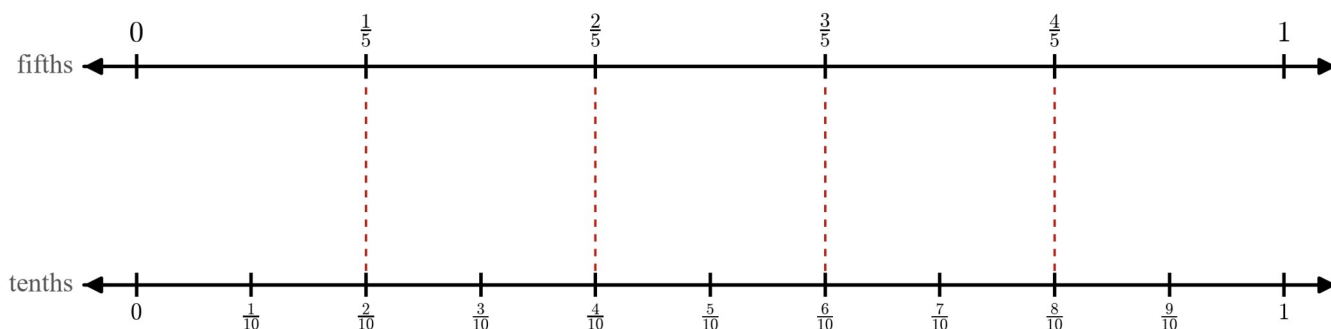


A row of quarters from a fraction wall above a number line from 0 to 1 marked in quarters. Red lines join each wall edge to the matching point on the number line: the first piece on the wall ends at the point one-quarter.

Both models agree about size and order — the wall shows the pieces, the line shows where they sit.

Parallel number lines

When the denominators are related, two number lines, one above the other, make the equal fractions easy to see. The bottom line is cut into 10 equal parts and the top line into 5, because 10 is a multiple of 5.



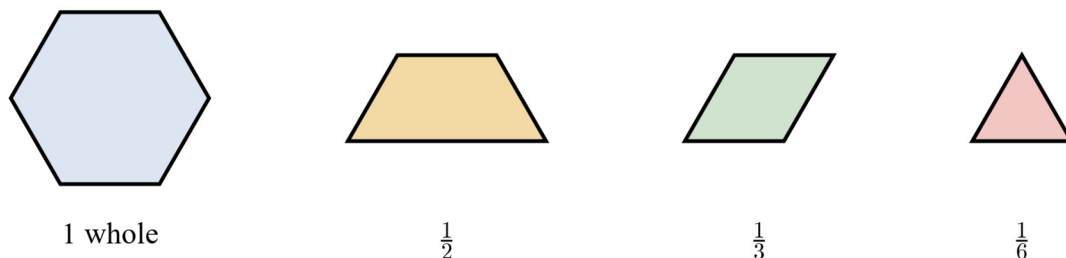
Two parallel number lines from 0 to 1. The top line is marked in 5 equal parts and the bottom line in 10 equal parts. Red lines join each top mark to the matching bottom mark: 1 over 5 with 2 over 10, 2 over 5 with 4 over 10, 3 over 5 with 6 over 10, 4 over 5 with 8 over 10.

$\frac{1}{5}$ is the **same point** as $\frac{2}{10}$, $\frac{2}{5}$ is the same point as $\frac{4}{10}$, and so on. Two names, one point — that is what equivalent fractions mean on a number line.

Pattern blocks

Pattern blocks are another way to see equal fractions. First choose which block is one whole. If the hexagon block is the whole:

when this hexagon block is the whole ...



Pattern blocks. A hexagon labelled 1 whole, then a trapezium labelled $\frac{1}{2}$, a rhombus labelled $\frac{1}{3}$ and a triangle labelled $\frac{1}{6}$.

- 2 trapezium blocks fill the hexagon, so one trapezium is $\frac{1}{2}$
- 3 rhombus blocks fill the hexagon, so one rhombus is $\frac{1}{3}$
- 6 triangles fill the hexagon, so one triangle is $\frac{1}{6}$

The blocks show the related family with denominators 3 and 6: 2 triangles cover a rhombus, so $\frac{1}{3} = \frac{2}{6}$.

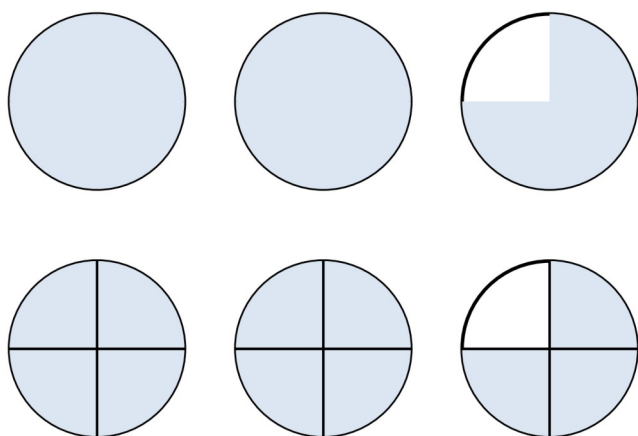
Mixed numerals and improper fractions

A **mixed numeral** is a whole number and a fraction written together: $2\frac{3}{4}$ means 2 wholes and $\frac{3}{4}$ of another whole.

An **improper fraction** is worth more than one whole: its numerator is more than its denominator, so $\frac{11}{4}$ means 11 quarter pieces.

To change a mixed numeral to an improper fraction, count all the parts. Each whole is 4 quarters, so 2 wholes are 8 quarters; add the extra 3 quarters and there are 11 quarters.

To change an improper fraction back to a mixed numeral, divide: $11 \div 4 = 2$ with 3 left over, so the 11 quarters make 2 wholes and 3 quarters.



Top row: two whole circles and a circle with three-quarters shaded, showing two and three-quarters. Bottom row: the same circles cut into quarters with 11 quarters shaded, showing $8 + 3 = 11$ quarters, eleven-quarters.

$2\frac{3}{4} = \frac{11}{4}$. Converting first makes it easy to place mixed numerals and improper fractions on a number line.

Ordering fractions and mixed numerals

To **order** fractions: write them with the same denominator (using related denominators), then compare the numerators. To order mixed numerals: compare the whole-number parts first; if the wholes are equal, compare the fraction parts. A number line always checks the order, because numbers that are less sit closer to 0.

Worked examples

Work through each example, then check it against the number line.

Worked example 1 — same denominators

Which is more, $\frac{3}{8}$ or $\frac{5}{8}$?

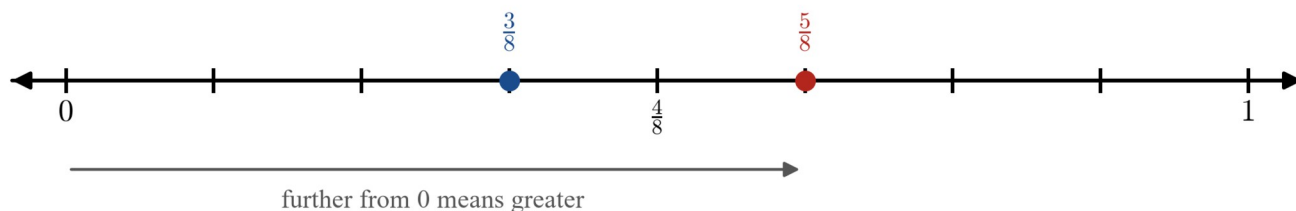
Working

Both fractions have denominator 8.

Comment

Same denominator, so same-size parts.

Working	Comment
$5 > 3$.	$\frac{5}{8}$ has more of the same-size parts.
$\frac{5}{8} > \frac{3}{8}$.	On the line, $\frac{5}{8}$ is further from 0.



A number line from 0 to 1 cut into 8 equal parts, with dots at 3 over 8 and 5 over 8. 5 over 8 is further from 0.

$\therefore \frac{5}{8}$ is more.

Worked example 2 — unit fractions

Which is more, $\frac{1}{4}$ or $\frac{1}{8}$?

Working	Comment
Both are unit fractions.	Each is one equal part of the same whole.
$8 > 4$.	The whole is cut into more parts.
More parts means less in each part.	A quarter is more of the whole than 1 part out of 8.
$\frac{1}{4} > \frac{1}{8}$.	
$\therefore \frac{1}{4}$ is more.	

Worked example 3 — related denominators

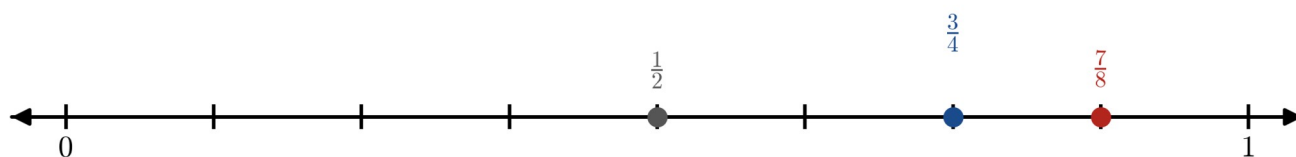
Which is more, $\frac{3}{4}$ or $\frac{5}{8}$?

Working	Comment
8 is a multiple of 4.	4 and 8 are related denominators.
$\frac{3}{4} = \frac{6}{8}$.	Multiply top and bottom by 2: equivalent fractions.
$6 > 5$.	$\frac{6}{8}$ has more of the same-size parts.
$\frac{3}{4} > \frac{5}{8}$.	On the wall, the $\frac{3}{4}$ edge sits past $\frac{5}{8}$.
$\therefore \frac{3}{4}$ is more.	

Worked example 4 — ordering fractions

Put $\frac{1}{2}$, $\frac{7}{8}$ and $\frac{3}{4}$ in order, smallest to greatest.

Working	Comment
8 is a multiple of 2 and of 4.	All three denominators are related to 8.
$\frac{1}{2} = \frac{4}{8}$, $\frac{3}{4} = \frac{6}{8}$.	Equivalent fractions with denominator 8.
$\frac{4}{8}$, $\frac{6}{8}$, $\frac{7}{8}$.	Same denominator now, so compare numerators: $4 < 6 < 7$.
$\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.	Left to right on the number line.



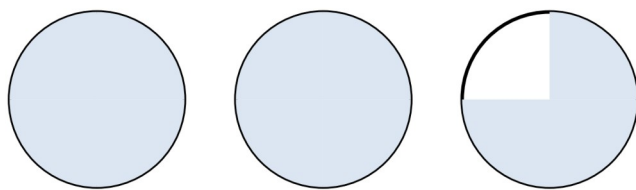
A number line from 0 to 1 cut into 8 equal parts, with dots at 1 over 2, 3 over 4 and 7 over 8, in order from left to right.

$\therefore$ In order: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.

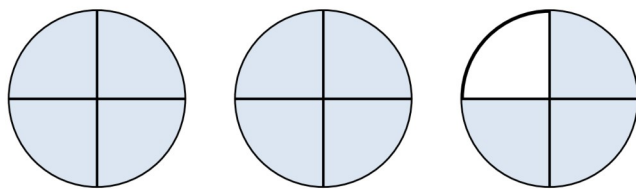
Worked example 5 — mixed numeral to improper fraction and back

Write $2\frac{3}{4}$ as an improper fraction. Then write $\frac{11}{4}$ as a mixed numeral.

Working	Comment
2 wholes = 8 quarters.	Each whole is 4 quarters: $2 \times 4 = 8$.
$8 + 3 = 11$ quarters.	Add the extra 3 quarters.
$2\frac{3}{4} = \frac{11}{4}$.	11 quarter pieces in all.
$11 \div 4 = 2$ with 3 left over.	The 11 quarters make 2 wholes and 3 quarters.
$\frac{11}{4} = 2\frac{3}{4}$.	Converting back gives the same number.



$$2\frac{3}{4} = 2 \text{ wholes and } \frac{3}{4}$$



$$8 + 3 = 11 \text{ quarters} = \frac{11}{4}$$

Top row: two whole circles and a circle with three-quarters shaded, showing two and three-quarters. Bottom row: the same circles cut into quarters with 11 quarters shaded, eleven-quarters.

Worked example 6 — ordering mixed numerals

Put $2\frac{1}{4}$, $1\frac{1}{2}$ and $1\frac{3}{4}$ in order, smallest to greatest.

Working

Comment

$2\frac{1}{4}$ has whole part 2; the others have 1.

Compare the whole-number parts first.

Compare $1\frac{1}{2}$ and $1\frac{3}{4}$.

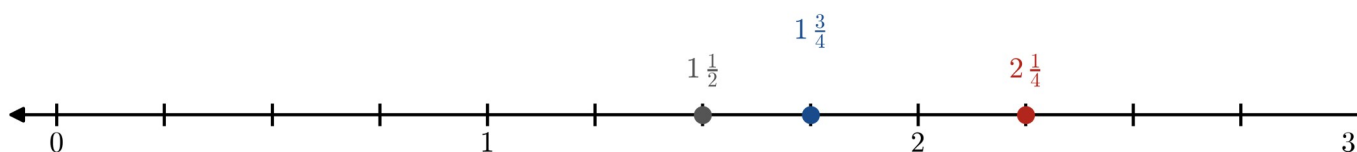
Same whole, so compare the fraction parts.

$\frac{1}{2} = \frac{2}{4}$, and $\frac{2}{4} < \frac{3}{4}$.

Related denominators: quarters.

$1\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{1}{4}$.

Left to right on the number line.



A number line from 0 to 3 marked in quarters, with dots at one and a half, one and three-quarters, and two and a quarter, in order from left to right.

$\therefore$ In order: $1\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{1}{4}$.

Practice questions

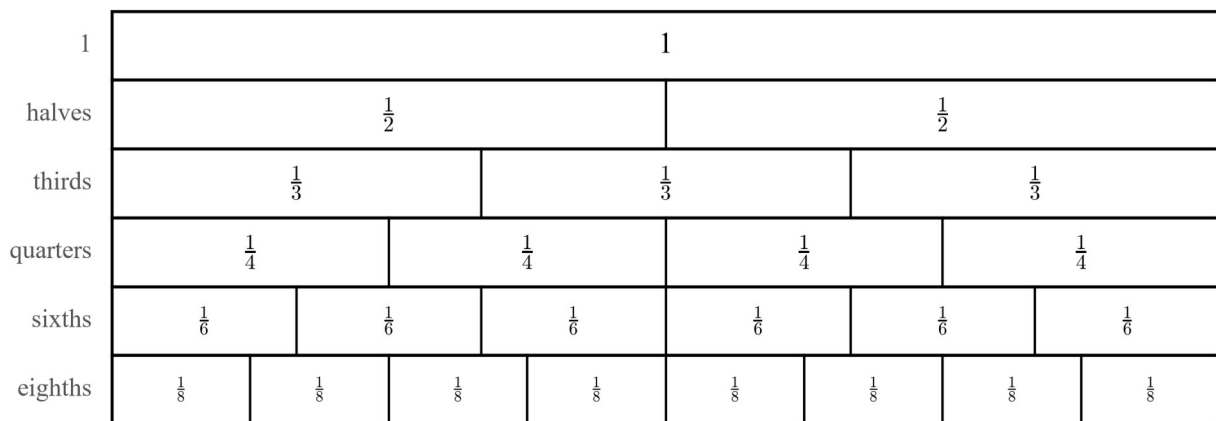
Q1. Fill in $<$, $>$ or $=$.

- a. $\frac{2}{7}$ — $\frac{5}{7}$
- b. $\frac{6}{10}$ — $\frac{9}{10}$
- c. $\frac{5}{6}$ — $\frac{3}{6}$

Q2. Circle the unit fraction that is more.

- a. $\frac{1}{4}$ or $\frac{1}{8}$
- b. $\frac{1}{3}$ or $\frac{1}{6}$
- c. $\frac{1}{5}$ or $\frac{1}{10}$

Q3. Use the fraction wall.

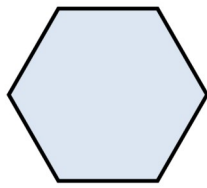


A fraction wall with rows for 1 whole, halves, thirds, quarters, and rows of 6 and 8 equal parts.

- a. Name two fractions on the wall that are equal to $\frac{1}{2}$.
- b. Which is more, $\frac{3}{8}$ or $\frac{1}{4}$? Say how the wall shows it.

Q4. The hexagon block is one whole.

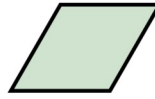
when this hexagon block is the whole ...



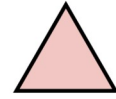
1 whole



$\frac{1}{2}$



$\frac{1}{3}$

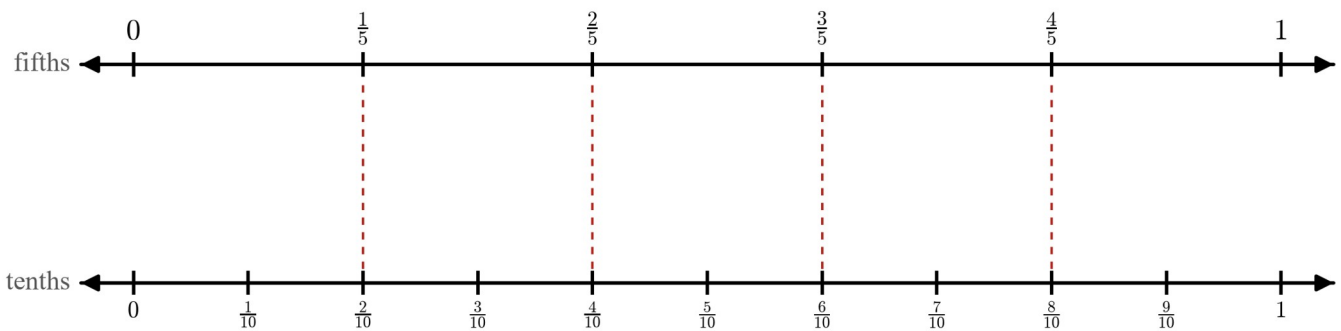


$\frac{1}{6}$

Pattern blocks. A hexagon labelled 1 whole, then a trapezium labelled $\frac{1}{2}$, a rhombus labelled $\frac{1}{3}$ and a triangle labelled $\frac{1}{6}$.

- What fraction of the whole is the trapezium?
- What fraction of the whole is the rhombus?
- What fraction of the whole is the triangle?

Q5. Use the parallel number lines.



Two parallel number lines from 0 to 1. The top line is marked in 5 equal parts and the bottom line in 10 equal parts, with red lines joining equal points.

- $\frac{1}{5} = \frac{?}{10}$
- $\frac{3}{5} = \frac{?}{10}$
- $\frac{8}{10} = \frac{?}{5}$

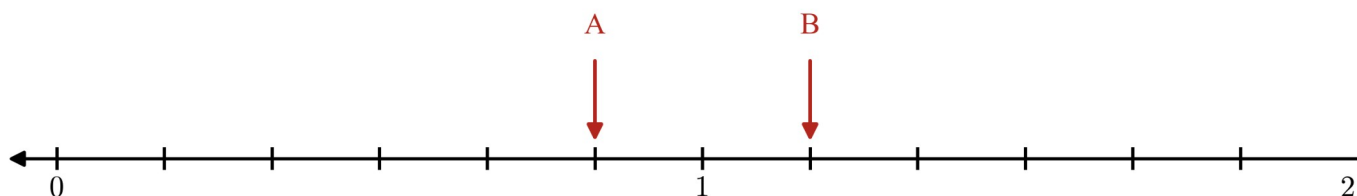
Q6. Write each mixed numeral as an improper fraction.

- $1\frac{1}{2}$
- $2\frac{3}{4}$
- $3\frac{2}{5}$

Q7. Write each improper fraction as a mixed numeral.

- a. $\frac{7}{4}$
- b. $\frac{13}{5}$
- c. $\frac{9}{2}$

Q8. Arrows A and B point to marks on the number line.



A number line from 0 to 2 marked in six equal parts each whole, with arrows A and B. Arrow A points to the fifth mark after 0 and arrow B points to the first mark after 1.

- a. What fraction is arrow A pointing to?
- b. Write the number at arrow B as a mixed numeral.
- c. Write the number at arrow B as an improper fraction.

Q9. Anh has $\frac{5}{8}$ of a chocolate block, and Kai has $\frac{3}{4}$ of a block the same size. Who has more chocolate? Show your working.

Q10. Put these fractions in order, smallest to greatest.

- a. $\frac{3}{4}, \frac{3}{8}, \frac{1}{2}$
- b. $\frac{2}{3}, \frac{1}{6}, \frac{5}{6}$
- c. $\frac{1}{2}, \frac{3}{10}, \frac{4}{5}$

Q11. Put these mixed numerals in order, smallest to greatest. Then draw a number line from 0 to 3 and mark them on it.

$$2\frac{1}{2}, 1\frac{3}{4}, 2\frac{1}{4}$$

Q12. Mia says $\frac{1}{8}$ must be more than $\frac{1}{4}$ because 8 is more than 4. Is Mia correct? Explain your answer.

Answers

Q1. a. $<$ b. $<$ c. $>$

Q2. a. $\frac{1}{4}$ b. $\frac{1}{3}$ c. $\frac{1}{5}$

Q3. a. Any two of $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$. b. $\frac{3}{8}$; $\frac{1}{4} = \frac{2}{8}$, and $\frac{3}{8}$ reaches further along the wall than $\frac{2}{8}$.

Q4. a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{1}{6}$

Q5. a. $\frac{2}{10}$ b. $\frac{6}{10}$ c. $\frac{4}{5}$

Q6. a. $\frac{3}{2}$ b. $\frac{11}{4}$ c. $\frac{17}{5}$

Q7. a. $1\frac{3}{4}$ b. $2\frac{3}{5}$ c. $4\frac{1}{2}$

Q8. a. $\frac{5}{6}$ b. $1\frac{1}{6}$ c. $\frac{7}{6}$

Q9. Kai. $\frac{3}{4} = \frac{6}{8}$, and $\frac{6}{8} > \frac{5}{8}$.

Q10. a. $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$ b. $\frac{1}{6}$, $\frac{2}{3}$, $\frac{5}{6}$ c. $\frac{3}{10}$, $\frac{1}{2}$, $\frac{4}{5}$

Q11. $1\frac{3}{4}$, $2\frac{1}{4}$, $2\frac{1}{2}$.

Q12. No. Cutting the same whole into 8 parts makes each part less than cutting it into 4 parts does, so $\frac{1}{8} < \frac{1}{4}$.

Fully-worked solutions

Q1. Same denominators, so the parts are the same size; compare the numerators. a. $2 < 5$, so $\frac{2}{7} < \frac{5}{7}$. b. $6 < 9$, so $\frac{6}{10} < \frac{9}{10}$. c. $5 > 3$, so $\frac{5}{6} > \frac{3}{6}$.

Q2. Unit fractions: the more parts the whole is cut into, the less each part is. a. $4 < 8$, so $\frac{1}{4}$ is more. b. $3 < 6$, so $\frac{1}{3}$ is more. c. $5 < 10$, so $\frac{1}{5}$ is more.

Q3. a. The red lines on the wall show $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$, so any two of $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ are equal to $\frac{1}{2}$. b. On the wall, $\frac{1}{4} = \frac{2}{8}$. $\frac{3}{8}$ reaches further along the row than $\frac{2}{8}$, so $\frac{3}{8} > \frac{1}{4}$.

Q4. With the hexagon as one whole: a. 2 trapezium blocks fill it, so the trapezium is $\frac{1}{2}$. b. 3 rhombus blocks fill it, so the rhombus is $\frac{1}{3}$. c. 6 triangles fill it, so the triangle is $\frac{1}{6}$.

Q5. 10 is a multiple of 5, so $\frac{1}{5}$ is the same point as $\frac{2}{10}$. a. $\frac{1}{5} = \frac{2}{10}$. b. $\frac{3}{5} = \frac{6}{10}$. c. $\frac{8}{10} = \frac{4}{5}$.

Q6. Count all the parts. a. 1 whole = 2 halves, plus 1 half = 3 halves: $\frac{3}{2}$. b. 2 wholes = 8 quarters, plus 3 quarters = 11 quarters: $\frac{11}{4}$. c. 3 wholes = 15 fifths, plus 2 fifths = 17 fifths: $\frac{17}{5}$.

Q7. Divide, and the remainder is the fraction part. a. $7 \div 4 = 1$ with 3 left over: $1\frac{3}{4}$. b. $13 \div 5 = 2$ with 3 left over: $2\frac{3}{5}$. c. $9 \div 2 = 4$ with 1 left over: $4\frac{1}{2}$.

Q8. The line is marked in six equal parts each whole. a. Arrow A points to the fifth mark after 0: $\frac{5}{6}$. b. Arrow B points to the first mark after 1: $1\frac{1}{6}$. c. 1 whole is 6 of the parts, plus 1 part is 7 parts: $\frac{7}{6}$.

Q9. The denominators 8 and 4 are related, because 8 is a multiple of 4. Write $\frac{3}{4}$ with denominator 8: $\frac{3}{4} = \frac{6}{8}$. Then $\frac{6}{8} > \frac{5}{8}$.

$\therefore$ Kai has more chocolate.

Q10. Write each set with the same denominator, then compare numerators. a. With denominator 8: $\frac{6}{8}, \frac{3}{8}, \frac{4}{8} \rightarrow \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$. b. With denominator 6: $\frac{4}{6}, \frac{1}{6}, \frac{5}{6} \rightarrow \frac{1}{6}, \frac{2}{3}, \frac{5}{6}$. c. With denominator 10: $\frac{5}{10}, \frac{3}{10}, \frac{8}{10} \rightarrow \frac{3}{10}, \frac{1}{2}, \frac{4}{5}$.

Q11. Compare the whole-number parts first: $1\frac{3}{4}$ has whole part 1, so it is smallest. Then compare $2\frac{1}{4}$ and $2\frac{1}{2}$: $\frac{1}{4} < \frac{1}{2}$, so $2\frac{1}{4}$ comes next.

$\therefore$ In order: $1\frac{3}{4}, 2\frac{1}{4}, 2\frac{1}{2}$. On a number line from 0 to 3 marked in quarters they sit at 7 quarters, 9 quarters and 10 quarters, left to right.

Q12. Mia is not correct. Both fractions are unit fractions of the same whole. Cutting the whole into 8 equal parts makes each part less than cutting it into 4 equal parts does, so 1 part out of 8 is less than 1 part out of 4: $\frac{1}{8} < \frac{1}{4}$. With unit fractions, the more parts the whole is cut into, the less each part is.

Adding and subtracting fractions with the same or related denominators

solve problems involving addition and subtraction of fractions with the same or related denominators, using different strategies — VC2M5N05

Level 5 achievement standard (extract): *Students order and represent, add and subtract fractions with the same or related denominators.*

This section draws on VC2M5N05.E01, VC2M5N05.E02 and VC2M5N05.E03. It uses what you learned in Year 4: equivalent fractions (VC2M4N03) and counting and placing halves, quarters and thirds on number lines (VC2M4N04). It comes after 1A (factors and multiples) and 2B (comparing and ordering fractions).

Theory

The two numbers in a fraction

A fraction counts equal parts of one whole. The **bottom number** names the size of each part. The **top number** counts how many of those parts you have.

Look at $\frac{3}{8}$ and $\frac{5}{8}$. The denominators are both 8: each whole is split into 8 equal parts. The top numbers are different: one fraction counts 3 parts and the other counts 5 parts.

Same denominators: count the parts

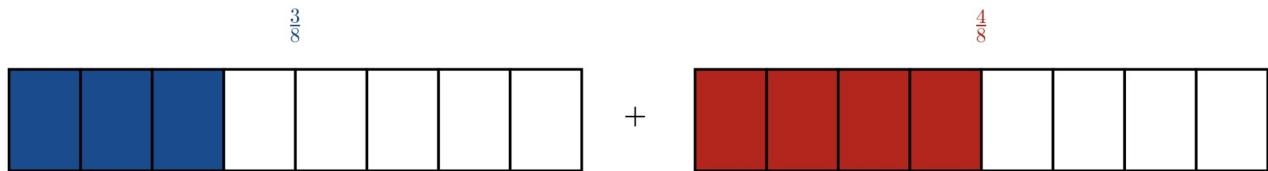
When the denominators are the same, the parts are the same size. So you add or subtract the top numbers, and the bottom number stays the same.

$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$

$$\frac{7}{8} - \frac{4}{8} = \frac{7-4}{8} = \frac{3}{8}$$

The strips show both:

Same denominators: the parts are the same size

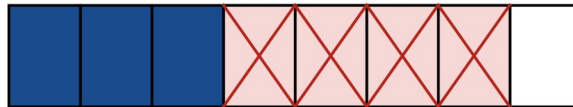


8 equal parts in each whole

$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$



$$\frac{7}{8} - \frac{4}{8} = \frac{7-4}{8} = \frac{3}{8}$$

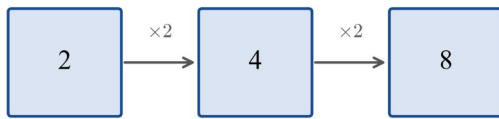


Strips split into 8 equal parts: 3 parts joined with 4 parts make 7 of the 8 parts; 7 parts with 4 crossed out leaves 3 parts.

Related denominators: rename first

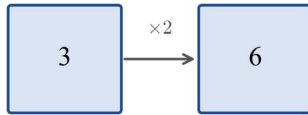
Two denominators are **related** when one is a multiple of the other (from 1A). 4 is a multiple of 2. 8 is a multiple of 4. 6 is a multiple of 3. 10 is a multiple of 5.

Related denominators: one is a multiple of the other



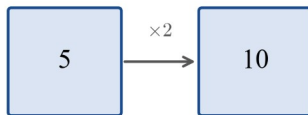
$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

halves, quarters, 8 parts



$$\frac{1}{3} = \frac{2}{6}$$

thirds and 6 parts



$$\frac{1}{5} = \frac{2}{10}$$

fifths and 10 parts

Multiples from 1A: 4 is a multiple of 2, 6 is a multiple of 3, 10 is a multiple of 5.

Three rows of boxes showing related denominators: 2, 4 and 8 with times-2 arrows; 3 and 6; 5 and 10.

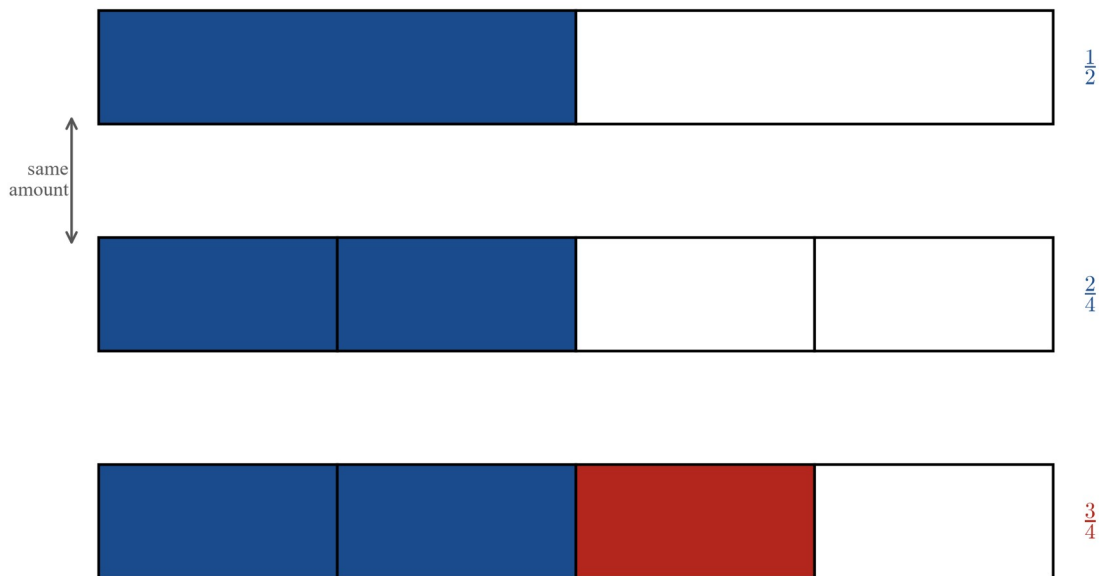
When the denominators are related, rename one fraction as an **equivalent fraction** so that both denominators match. Then add or subtract as usual. The renaming does not change the amount: it makes the parts the same size (Year 4, VC2M4N03).

For example, $\frac{1}{2} + \frac{1}{4}$. 4 is a multiple of 2, so rename the half as quarters: $\frac{1}{2} = \frac{2}{4}$. Now

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

The bars show why this works:

Rename $\frac{1}{2}$ as quarters, then add



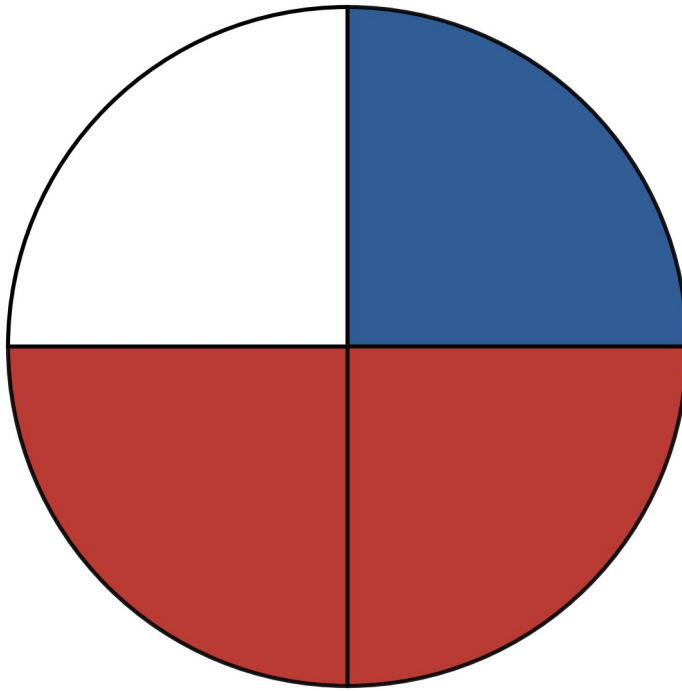
$$\frac{1}{2} + \frac{1}{4} \text{ is the same as } \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

Three bars the same size: half of one bar is the same amount as 2 of 4 parts; 2 of 4 parts plus 1 of 4 parts is 3 of 4 parts.

Different strategies, same answer

The same problem can be solved in more than one way.

- **Bars and strips**, like the ones above.
- **Diagrams of shapes**: parts of a circle or a rectangle.

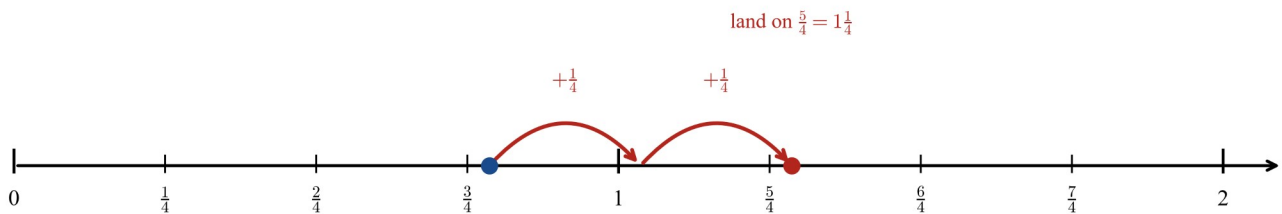


$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

A circle split into 4 equal parts, with 1 part and then 2 more parts shaded, making 3 of the 4 parts.

- **Jumps on a number line:** start at the first fraction and jump on the second one.

$$\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$$



A number line from 0 to 2 in quarters, with a dot on $\frac{3}{4}$ and two quarter-jumps landing on $\frac{5}{4}$.

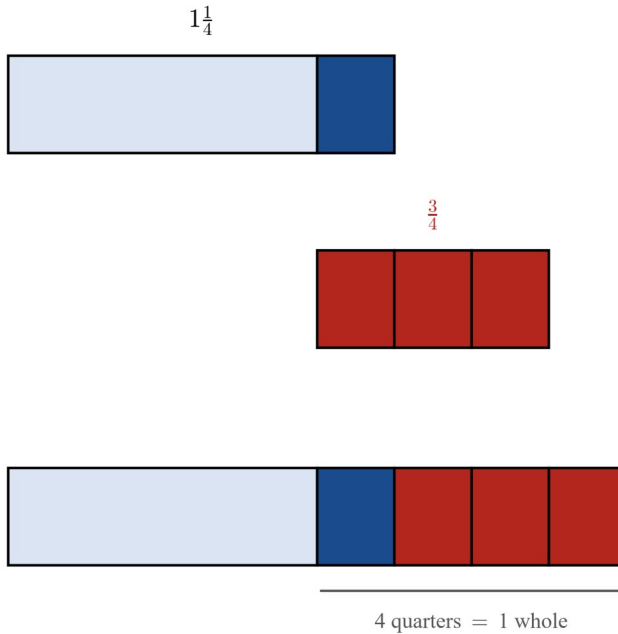
- **Rewriting the number sentence:** rename the fractions first, then count the parts. $\frac{1}{2} + \frac{1}{4}$ is the same as $\frac{2}{4} + \frac{1}{4}$.

Every strategy gives the same answer. If you want, check your work with a second strategy.

Mixed numerals

A **mixed numeral** like $1\frac{1}{4}$ is wholes and a fraction part. To add, keep the wholes and add the fraction parts. If the fraction parts make one whole or more, group them into a new whole.

Add $1\frac{1}{4}$ and $\frac{3}{4}$ with paper strips



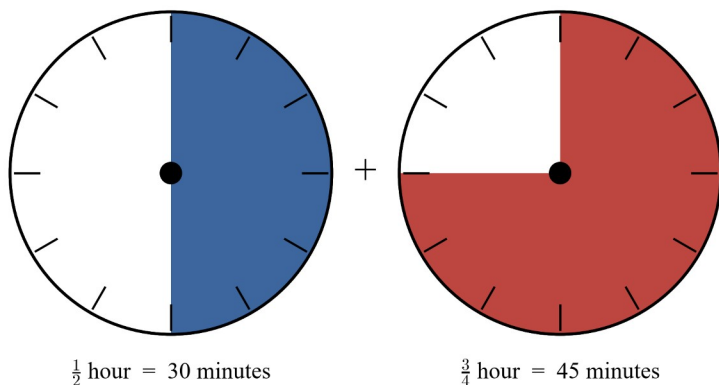
$$1\frac{1}{4} + \frac{3}{4} = 1\frac{4}{4} = 1 + 1 = 2$$

Paper strips: 1 whole and 1 quarter, and 3 quarters; the four quarters together make one more whole, so the total is 2 wholes.

To subtract when the fraction part is too small, rename one whole as fraction parts: $2\frac{1}{4} = 1\frac{5}{4}$. Now there are enough quarters to take away.

Fractions of things you measure

The same maths works with anything you measure. A clock face splits an hour into equal parts, a cup splits a whole cup, and a number line splits a metre.

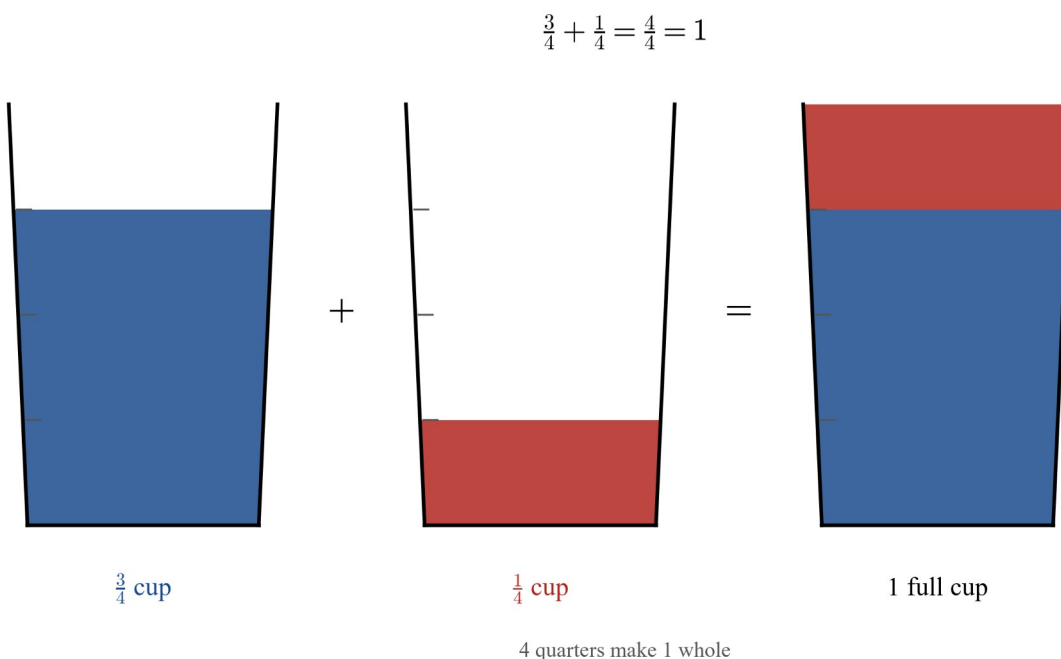


$$\begin{aligned} & \frac{1}{2} + \frac{3}{4} \\ &= \frac{2}{4} + \frac{3}{4} = \frac{5}{4} \\ &= 1\frac{1}{4} \text{ hours} \end{aligned}$$

1 hour and 15 minutes

Rename the half as quarters: the clock is split into 4 equal parts.

Two clock faces: one shows half an hour shaded and one shows three-quarters of an hour shaded; together they make one and a quarter hours.



Three cups: one filled to $\frac{3}{4}$, one filled to $\frac{1}{4}$, and one full cup showing that together they make 1 whole cup.

Worked examples

Example 1 — same denominators (bar model)

Find $\frac{3}{8} + \frac{4}{8}$ and $\frac{7}{8} - \frac{4}{8}$.

Working

The denominators are both 8.

$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

Comment

The parts are the same size, so you can count them.

Add the top numbers: $3 + 4 = 7$. The bottom number stays 8.

Working	Comment
$\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$	Subtract the top numbers: $7 - 4 = 3$. The bottom number stays 8.
$\therefore \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ and $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$.	

Example 2 — related denominators (bars)

Find $\frac{1}{2} + \frac{1}{4}$.

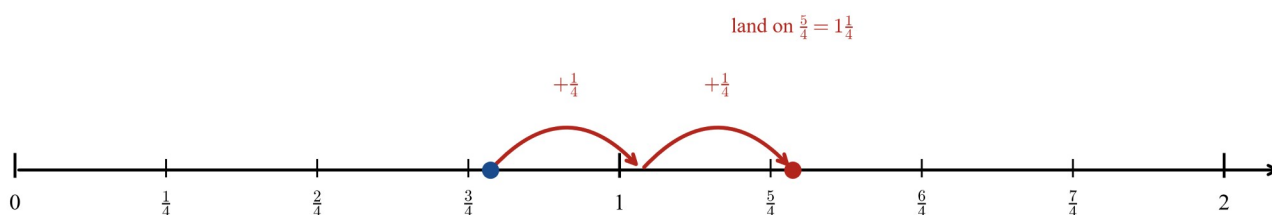
Working	Comment
$\frac{1}{2} = \frac{2}{4}$	4 is a multiple of 2, so rename the half as quarters.
$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$	Now the denominators match, so add the top numbers.
$\therefore \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.	

Example 3 — jumps on a number line

Find $\frac{3}{4} + \frac{2}{4}$.

Start at $\frac{3}{4}$. Jump on $\frac{2}{4}$ as two quarter-jumps: one jump to 1, one more jump to $\frac{5}{4}$.

$$\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$$



A number line from 0 to 2 in quarters, with a dot on $\frac{3}{4}$ and two quarter-jumps landing on $\frac{5}{4}$.

$\frac{5}{4}$ is more than 1 whole: it is $1\frac{1}{4}$.

$$\therefore \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}.$$

Example 4 — time on a clock

Mai practises the flute for $\frac{1}{2}$ an hour in the morning and $\frac{3}{4}$ of an hour in the afternoon. How long does she practise in all?

Working

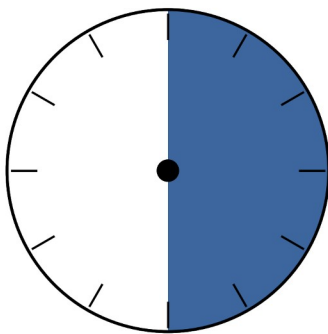
Comment

$$\frac{1}{2} \text{ hour} = \frac{2}{4} \text{ hour}$$

A clock face splits into 4 equal parts.

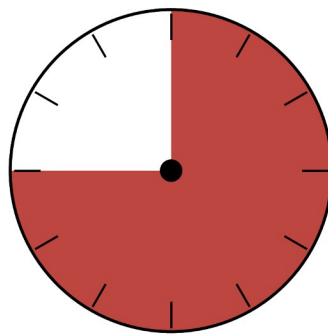
$$\frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$$

The denominators match, so add the top numbers.



$\frac{1}{2}$ hour = 30 minutes

+



$\frac{3}{4}$ hour = 45 minutes

$$\frac{1}{2} + \frac{3}{4}$$

$$= \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$$

$$= 1\frac{1}{4} \text{ hours}$$

1 hour and 15 minutes

Rename the half as quarters: the clock is split into 4 equal parts.

Two clock faces: one shows half an hour shaded and one shows three-quarters of an hour shaded; together they make one and a quarter hours.

$\therefore$ Mai practises for $1\frac{1}{4}$ hours. That is 1 hour and 15 minutes.

Example 5 — subtracting from a mixed numeral (metres)

A ribbon is $2\frac{1}{4}$ metres long. Ben cuts off $\frac{3}{4}$ of a metre. How long is the ribbon now?

Working

Comment

$$2\frac{1}{4} = 1\frac{5}{4}$$

$\frac{1}{4}$ is too small to take away $\frac{3}{4}$, so rename one whole as 4 quarters.

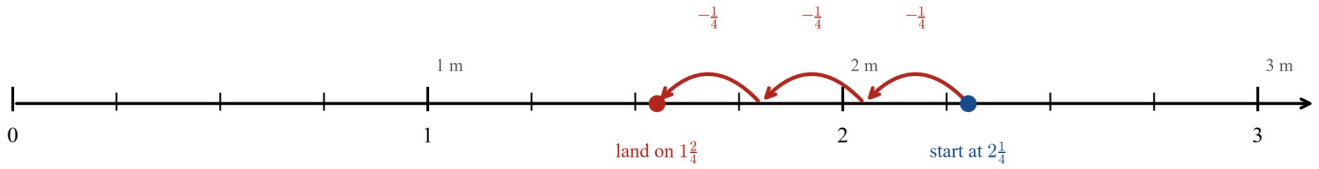
$$1\frac{5}{4} - \frac{3}{4} = 1\frac{2}{4}$$

Now subtract the fraction parts.

$$1\frac{2}{4} = 1\frac{1}{2}$$

$\frac{2}{4}$ is the same amount as $\frac{1}{2}$.

$$2\frac{1}{4} - \frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2} \text{ metres}$$



A number line from 0 to 3 in quarters, starting at 2 and $\frac{1}{4}$ and jumping back three quarter-jumps to land on 1 and $\frac{2}{4}$.

$\therefore$ The ribbon is now $1\frac{1}{2}$ metres long.

Example 6 — fifths and tenths

Find $\frac{1}{5} + \frac{3}{10}$.

Working

$$\frac{1}{5} = \frac{2}{10}$$

$$\frac{2}{10} + \frac{3}{10} = \frac{5}{10}$$

$$\frac{5}{10} = \frac{1}{2}$$

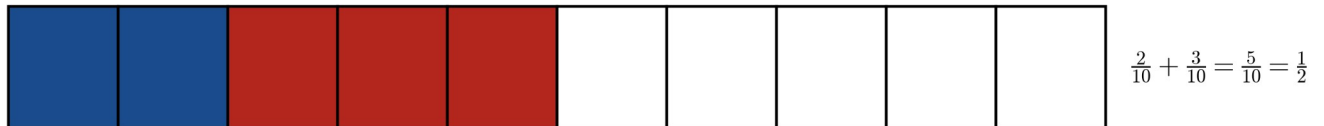
Comment

10 is a multiple of 5, so rename the fifth as tenths.

Now the denominators match, so add the top numbers.

5 of 10 parts is the same amount as one half.

Rename fifths as tenths, then add



Each fifth splits into 2 tenths, so 10 parts cover the same whole.

Two bars the same size: one fifth of the top bar is the same as 2 of 10 parts; 2 of 10 parts plus 3 of 10 parts is 5 of 10 parts, or one half.

$$\therefore \frac{1}{5} + \frac{3}{10} = \frac{1}{2}.$$

Practice questions

Q1. Add.

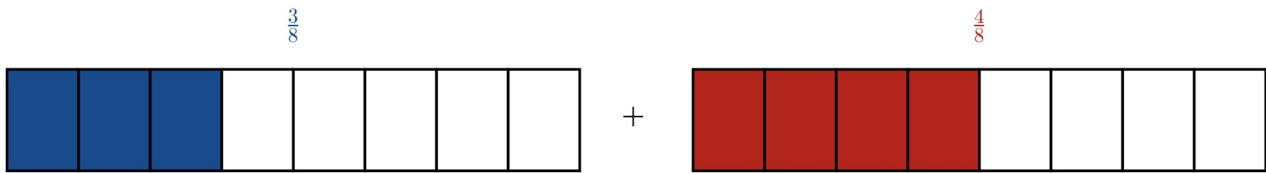
- a. $\frac{1}{6} + \frac{3}{6}$
- b. $\frac{2}{9} + \frac{5}{9}$
- c. $\frac{3}{10} + \frac{4}{10}$

Q2. Subtract.

- a. $\frac{5}{8} - \frac{1}{8}$
- b. $\frac{7}{9} - \frac{4}{9}$
- c. $\frac{9}{10} - \frac{3}{10}$

Q3. Write the two number sentences shown by the strips.

Same denominators: the parts are the same size

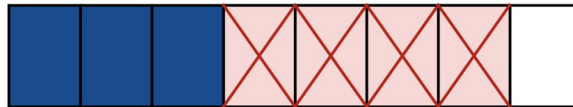


8 equal parts in each whole

$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$



$$\frac{7}{8} - \frac{4}{8} = \frac{7-4}{8} = \frac{3}{8}$$

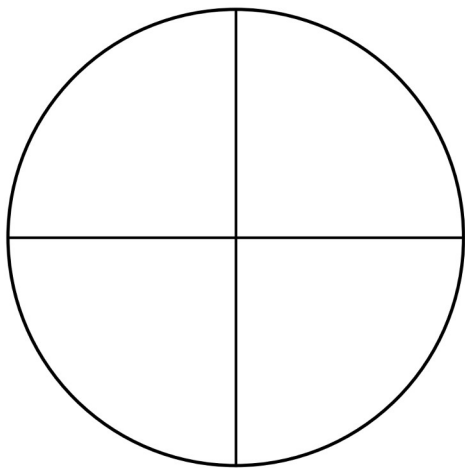


Strips split into 8 equal parts: 3 parts joined with 4 parts make 7 of the 8 parts; 7 parts with 4 crossed out leaves 3 parts.

Q4. Shade the parts to show each answer, using the shapes below.

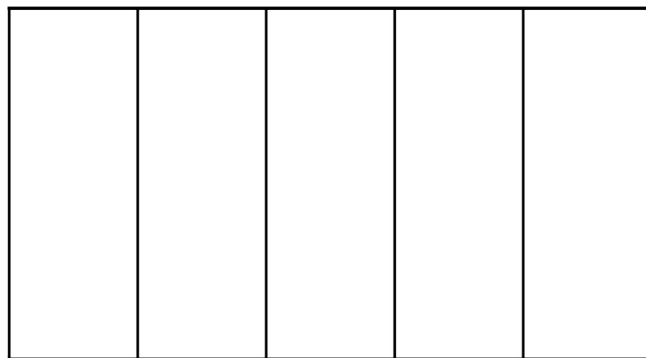
- $\frac{1}{4} + \frac{2}{4}$ on the circle
- $\frac{2}{5} + \frac{2}{5}$ on the strip

a



split into 4 equal parts

b



split into 5 equal parts

A blank circle split into 4 equal parts, and a blank strip split into 5 equal parts, ready to shade.

Q5. Rename, then add.

a. $\frac{1}{2} + \frac{1}{8}$

b. $\frac{1}{3} + \frac{1}{6}$

c. $\frac{1}{5} + \frac{3}{10}$

Q6. Rename, then subtract.

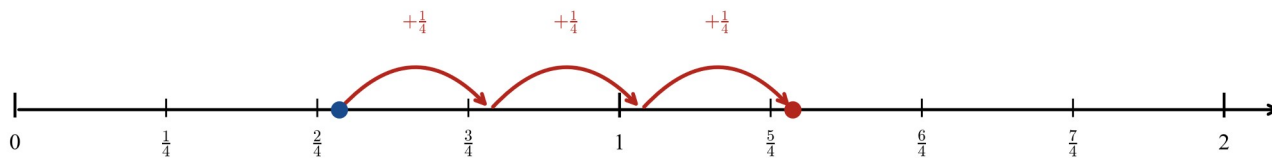
a. $\frac{3}{4} - \frac{1}{2}$

b. $\frac{5}{6} - \frac{1}{3}$

c. $1 - \frac{3}{10}$

Q7. Write the number sentence shown on the number line.

Start at $\frac{1}{2}$. Jump on $\frac{3}{4}$ more.



Write the number sentence: $\frac{1}{2} + \frac{3}{4} = \square$

A number line from 0 to 2 in quarters, with a dot on $\frac{1}{2}$ and three quarter-jumps landing on $\frac{5}{4}$.

Q8. Rewrite each number sentence so the denominators match, then find the answer.

a. $\frac{1}{2} + \frac{3}{8} = \frac{\square}{8} + \frac{3}{8} = \square$

b. $\frac{3}{4} - \frac{1}{2} = \frac{6}{8} - \frac{\square}{8} = \square$

Q9. Add the mixed numerals.

a. $1\frac{1}{4} + 2\frac{2}{4}$

b. $2\frac{1}{3} + 1\frac{1}{3}$

c. $1\frac{2}{5} + 3\frac{1}{5}$

Q10. Add or subtract. Rename where you need to.

a. $1\frac{1}{2} + \frac{3}{4}$

b. $3\frac{1}{4} - \frac{3}{4}$

Q11. Linh practises the piano for $\frac{3}{4}$ of an hour on Monday and $\frac{3}{4}$ of an hour on Tuesday. How many hours does she practise in all?

Q12. A bottle holds $\frac{7}{8}$ of a litre of water. Sam drinks $\frac{3}{8}$ of a litre. How much water is left in the bottle?

Q13. Jack says $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$. Is he right? Explain your answer.

Q14. Find the missing fraction.

a. $\frac{3}{8} + \square = \frac{7}{8}$

b. $\frac{5}{6} - \square = \frac{1}{6}$

c. $\square - \frac{1}{5} = \frac{3}{5}$

Q15. A board is $2\frac{1}{2}$ metres long. Dad cuts off $\frac{3}{4}$ of a metre. How long is the board now?

Q16. Solve $\frac{3}{4} + \frac{1}{8}$ in two different ways. What do you notice?

Q17. A class is painting a mural. They finish $\frac{3}{8}$ of it before lunch and $\frac{4}{8}$ of it after lunch.

- a. What fraction of the mural is painted?
- b. What fraction is left to paint?

Q18. Two fractions whose denominators are both 4 add up to $1\frac{1}{4}$. Find two different pairs that work.

Answers

Q1. a. $\frac{4}{6}$ b. $\frac{7}{9}$ c. $\frac{7}{10}$.

Q2. a. $\frac{4}{8}$, the same as $\frac{1}{2}$ b. $\frac{3}{9}$, the same as $\frac{1}{3}$ c. $\frac{6}{10}$, the same as $\frac{3}{5}$.

Q3. $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ and $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$.

Q4. a. 3 of the 4 parts shaded. b. 4 of the 5 parts shaded.

Q5. a. $\frac{5}{8}$ b. $\frac{3}{6}$, the same as $\frac{1}{2}$ c. $\frac{5}{10}$, the same as $\frac{1}{2}$.

Q6. a. $\frac{1}{4}$ b. $\frac{3}{6}$, the same as $\frac{1}{2}$ c. $\frac{7}{10}$.

Q7. $\frac{1}{2} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$.

Q8. a. $\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$ b. $\frac{6}{8} - \frac{4}{8} = \frac{2}{8}$, the same as $\frac{1}{4}$.

Q9. a. $3\frac{3}{4}$ b. $3\frac{2}{3}$ c. $4\frac{3}{5}$.

Q10. a. $2\frac{1}{4}$ b. $2\frac{1}{2}$.

Q11. $\frac{6}{4} = 1\frac{1}{2}$ hours.

Q12. $\frac{4}{8}$ of a litre, the same as $\frac{1}{2}$ litre.

Q13. No. $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, not $\frac{2}{6}$.

Q14. a. $\frac{4}{8}$ b. $\frac{4}{6}$ c. $\frac{4}{5}$.

Q15. $1\frac{3}{4}$ metres.

Q16. $\frac{7}{8}$, both ways.

Q17. a. $\frac{7}{8}$ b. $\frac{1}{8}$.

Q18. For example, $\frac{1}{4} + \frac{4}{4}$ and $\frac{2}{4} + \frac{3}{4}$.

Worked solutions

Q1. The denominators are the same, so add the top numbers. a. $\frac{1+3}{6} = \frac{4}{6}$ b. $\frac{2+5}{9} = \frac{7}{9}$ c. $\frac{3+4}{10} = \frac{7}{10}$.

Q2. The denominators are the same, so subtract the top numbers. a. $\frac{5-1}{8} = \frac{4}{8}$, and $\frac{4}{8}$ is the same amount as $\frac{1}{2}$. b. $\frac{7-4}{9} = \frac{3}{9}$, the same as $\frac{1}{3}$. c. $\frac{9-3}{10} = \frac{6}{10}$, the same as $\frac{3}{5}$.

Q3. First strip: 3 parts and 4 parts join to make 7 of the 8 parts, so $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$. Second strip: 7 parts with 4 crossed out leaves 3 parts, so $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$.

Q4. a. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$, so shade 3 of the 4 parts of the circle. b. $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$, so shade 4 of the 5 parts of the strip.

Q5. Rename so the denominators match. a. $\frac{1}{2} = \frac{4}{8}$, so $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$. b. $\frac{1}{3} = \frac{2}{6}$, so $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$, the same as $\frac{1}{2}$. c. $\frac{1}{5} = \frac{2}{10}$, so $\frac{2}{10} + \frac{3}{10} = \frac{5}{10}$, the same as $\frac{1}{2}$.

Q6. Rename so the denominators match. a. $\frac{1}{2} = \frac{2}{4}$, so $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$. b. $\frac{1}{3} = \frac{2}{6}$, so $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$, the same as $\frac{1}{2}$. c. $1 = \frac{10}{10}$, so $\frac{10}{10} - \frac{3}{10} = \frac{7}{10}$.

Q7. The dot starts at $\frac{1}{2}$ and makes three quarter-jumps, so the jump on is $\frac{3}{4}$. It lands on $\frac{5}{4} = 1\frac{1}{4}$. The number sentence is $\frac{1}{2} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$. Check by renaming: $\frac{1}{2} = \frac{2}{4}$ and $\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$. ✓

Q8. a. $\frac{1}{2} = \frac{4}{8}$, so $\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$. b. $\frac{1}{2} = \frac{4}{8}$, so $\frac{6}{8} - \frac{4}{8} = \frac{2}{8}$, the same as $\frac{1}{4}$.

Q9. Add the wholes and the fraction parts. a. $1 + 2 = 3$ wholes and $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$, so $3\frac{3}{4}$. b. $2 + 1 = 3$ wholes and $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$, so $3\frac{2}{3}$. c. $1 + 3 = 4$ wholes and $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$, so $4\frac{3}{5}$.

Q10. a. $\frac{1}{2} = \frac{2}{4}$, so $1\frac{2}{4} + \frac{3}{4} = 1\frac{5}{4}$. The $\frac{5}{4}$ is 1 whole and $\frac{1}{4}$, so the answer is $2\frac{1}{4}$. b. Rename $3\frac{1}{4}$ as $2\frac{5}{4}$, then $2\frac{5}{4} - \frac{3}{4} = 2\frac{2}{4}$, the same as $2\frac{1}{2}$.

Q11. $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$. ∴ Linh practises for $1\frac{1}{2}$ hours in all.

Q12. $\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$, the same as $\frac{1}{2}$. ∴ $\frac{1}{2}$ litre of water is left in the bottle.

Q13. No. You can only add top numbers when the parts are the same size. Rename the half as quarters: $\frac{1}{2} = \frac{2}{4}$, so $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$. The bars in Example 2 show that $\frac{3}{4}$ is much more than $\frac{2}{6}$, so Jack's answer is wrong.

Q14. a. $7 - 3 = 4$, so the missing fraction is $\frac{4}{8}$. b. $5 - 1 = 4$, so the missing fraction is $\frac{4}{6}$. c. $3 + 1 = 4$, so the missing fraction is $\frac{4}{5}$.

Q15. $2\frac{1}{2} = 2\frac{2}{4}$. Rename as $1\frac{6}{4}$, then $1\frac{6}{4} - \frac{3}{4} = 1\frac{3}{4}$. $\therefore$ The board is now $1\frac{3}{4}$ metres long.

Q16. Way 1, renaming: $\frac{3}{4} = \frac{6}{8}$, so $\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$. Way 2, a number line split into 8 equal parts: start at $\frac{6}{8}$ and jump on $\frac{1}{8}$ to land on $\frac{7}{8}$. Both ways give $\frac{7}{8}$ — different strategies give the same answer.

Q17. a. $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ of the mural is painted. b. The whole mural is $\frac{8}{8}$, so $\frac{8}{8} - \frac{7}{8} = \frac{1}{8}$ is left to paint.

Q18. $1\frac{1}{4} = \frac{5}{4}$, so the top numbers must add to 5. Two pairs that work: $\frac{1}{4} + \frac{4}{4} = 1\frac{1}{4}$ and $\frac{2}{4} + \frac{3}{4} = 1\frac{1}{4}$.

Percentages, and their fraction and decimal equivalents

Code VC2M5N04 (Victorian Curriculum F–10 Version 2.0, Mathematics, Level 5; authority file `maths_5.json`): “recognise that 100% represents the complete whole and use percentages to describe, represent and compare relative size; connect familiar percentages to their decimal and fraction equivalents”. This section draws on VC2M5N04.E01, .E02, .E03 and .E04.

Theory

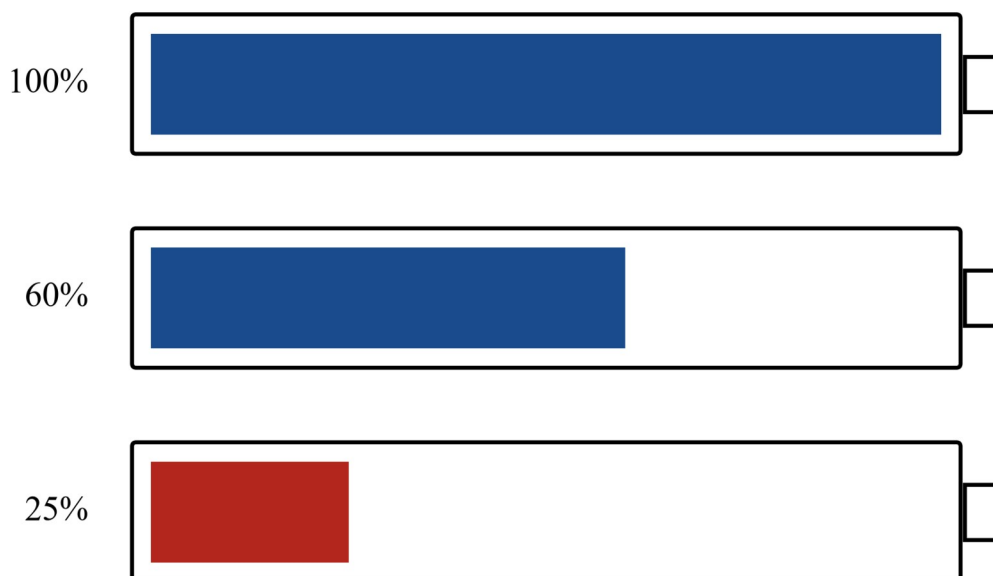
What does % mean?

The sign % is read as *per cent*. Per cent means **out of 100**.

30% means **30 out of 100**. **1%** means **1 out of 100**.

You see percentages in everyday life. When a phone or tablet charges, a bar shows how much charge is left as a percentage.

Battery charge left



The % tells how much of the whole charge is left.

Three battery bars. The top bar is filled all the way and labelled 100%. The middle bar is filled 60 parts out of 100 and labelled 60%. The bottom bar is filled a quarter of the way and labelled 25%.

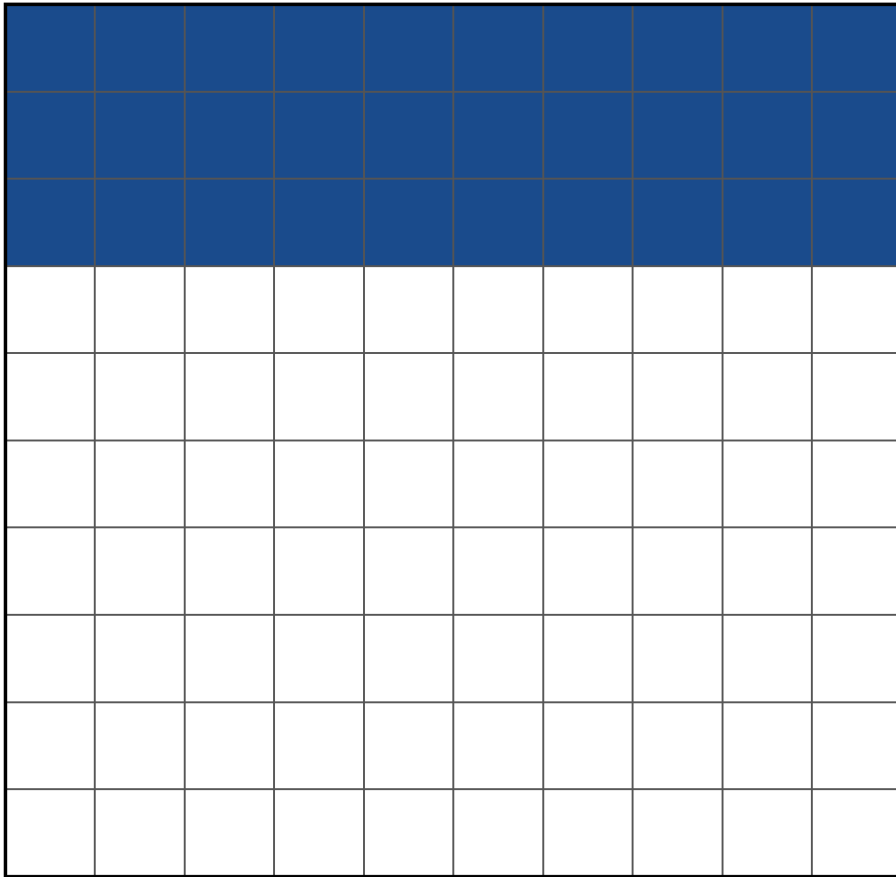
The whole battery is **100%**. That is the complete whole: all of the charge. A bar filled 60 parts out of 100 shows **60%** of the charge left. A bar a quarter full shows **25%** left. A bar with nothing in it shows **0%**: no charge is left.

100% is always the complete whole. A whole grid, a whole bar, a whole bottle, a whole trip: all of it is 100%.

The 100-grid

A good way to see percentages is the **100-grid**: one big square cut into 100 small squares. The whole grid is 100%, so each small square is 1 out of 100, or **1%**.

One grid is the whole: 100 small squares



30 squares shaded out of 100

30% means 30 out of 100

A 10 by 10 grid of 100 small squares with the top three rows shaded: 30 squares out of 100 are shaded, which is 30%.

Count the shaded squares: 30. The shaded part is 30 out of 100, so **30% of the grid is shaded**. The part not shaded is the rest of the grid.

The percentages we know well

Some percentages match fractions and decimals you already know.

Half. Cut the whole into 2 equal parts and shade 1.

Half of the bar is shaded.



=

$\frac{1}{2}$	three names, one amount
0.5	
50%	

A bar with its left half shaded, named three ways: one half, 0.5 and 50%.

Half of the whole is $\frac{1}{2}$ as a fraction and **0.5** as a decimal. As a percentage it is **50%**: 50 out of 100. Three names, one amount.

Quarters. Cut the whole into 4 equal parts.

Bars cut into 4 equal parts



$$\frac{1}{4} = 0.25 = 25\%$$



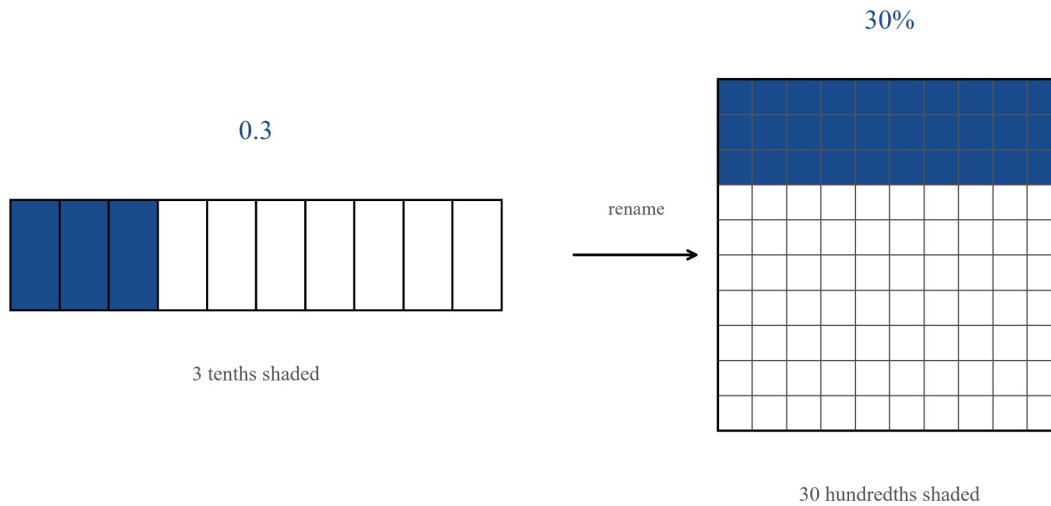
$$\frac{3}{4} = 0.75 = 75\%$$

Two bars cut into 4 equal parts. The top bar has 1 part shaded: one quarter, 0.25, 25%. The bottom bar has 3 parts shaded: three quarters, 0.75, 75%.

One quarter is $\frac{1}{4} = 0.25 = \mathbf{25\%}$. Three quarters is $\frac{3}{4} = 0.75 = \mathbf{75\%}$.

Tenths. Cut the whole into 10 equal parts. Each part is $\frac{1}{10} = 0.1$. Now rename the tenths as hundredths: each of the 10 parts is 10 small squares on the 100-grid, so $0.1 = \mathbf{10\%}$.

The same amount, said two ways



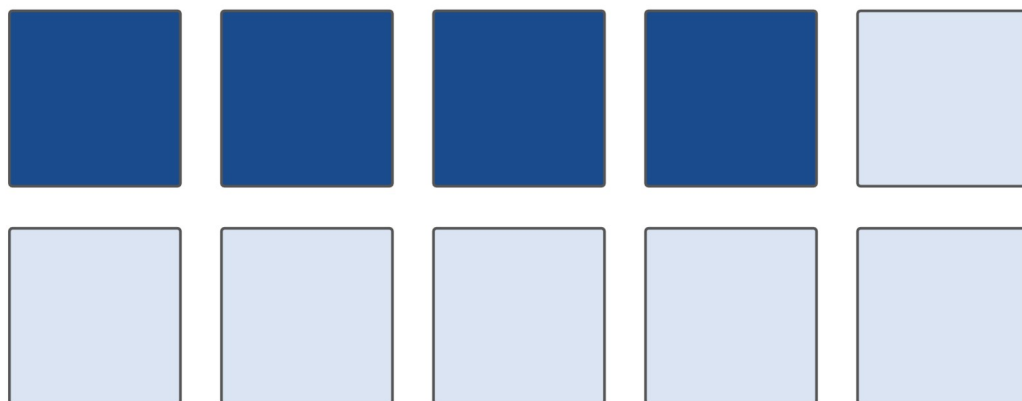
0.3 is 3 out of every 10, which is 30 out of every 100, which is 30%.

Left, a bar cut into 10 equal parts with 3 parts shaded, labelled 0.3. An arrow points right to a 10 by 10 grid with 30 squares shaded, labelled 30%.

0.3 is 3 out of every 10, which is 30 out of every 100, which is **30%**.

A collection works the same way. With 10 blocks, 4 shaded blocks are 4 out of 10.

A collection of 10 blocks



4 out of 10 blocks = 40 out of 100 blocks = 40%

Ten blocks in two rows of five. The first four blocks are shaded dark: 4 out of 10 blocks, which is 40 out of 100, which is 40%.

4 out of 10 blocks is 40 out of 100 blocks, so **40%** of the blocks are shaded.

Money too: \$1 is 100c. 10c is 10 out of those 100 cents, so 10c is 10% of a dollar. 25c is a quarter of a dollar and 25% of a dollar.

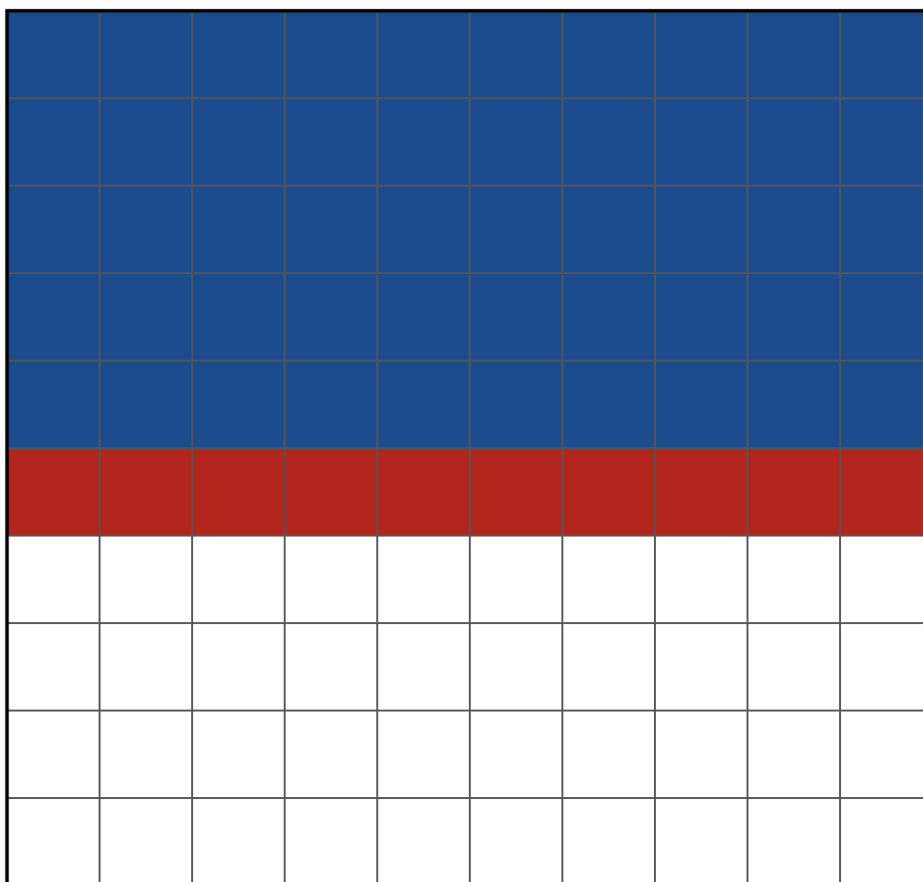
Here is the family to know quickly:

The part	Fraction	Decimal	Percentage
the whole	1	1	100%
three quarters	$\frac{3}{4}$	0.75	75%
seven tenths	$\frac{7}{10}$	0.7	70%
six tenths	$\frac{6}{10}$	0.6	60%
half	$\frac{1}{2}$	0.5	50%
four tenths	$\frac{4}{10}$	0.4	40%
three tenths	$\frac{3}{10}$	0.3	30%
a quarter	$\frac{1}{4}$	0.25	25%
two tenths	$\frac{2}{10}$	0.2	20%
ten hundredths	$\frac{1}{10}$	0.1	10%
nothing	0	0	0%

Reading down the table, the amounts get smaller. Percentages make that easy to see: 75, 70, 60, 50, and so on. The higher percentage is always more of the whole.

You can also build percentages from the ones you know. 60% is 50% and 10% more.

Building 60%



blue: 50% red: 10% more

$$50\% + 10\% = 60\%$$

*A 10 by 10 grid with 50 squares shaded blue and the next 10 squares shaded red, making 60 shaded squares. Below:
 $50\% + 10\% = 60\%$.*

Parts that make 100%

The parts of a whole always add to 100%. If you know one part, you know the other.

the whole = 100%



$$70\% + 30\% = 100\%$$

The two parts combine to make the whole.

A bar standing for the whole. The left part is shaded and labelled 70%, the right part is not shaded and labelled 30%. Above the bar: the whole = 100%. Below: $70\% + 30\% = 100\%$.

70% shaded and 30% not shaded combine to make 100%. Pairs like this always make 100%: 10% and 90%, 20% and 80%, 25% and 75%, 50% and 50%.

To find the missing part, take the known part away from 100%:

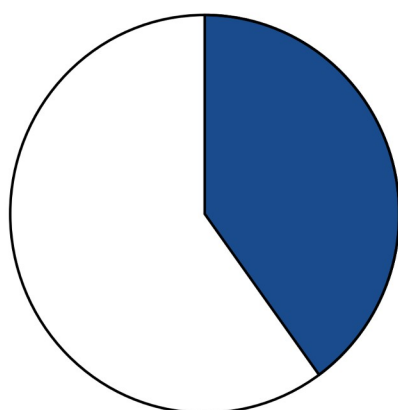
$$100\% - 70\% = 30\%.$$

Describing and comparing with percentages

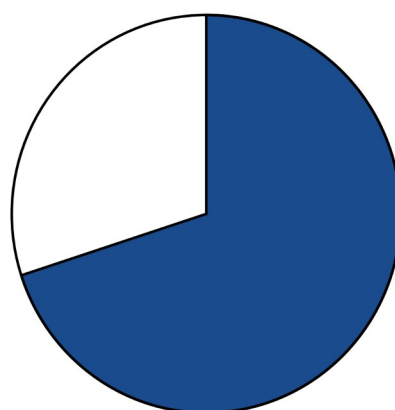
A percentage **describes** how much of the whole you have, using 100 as the whole. “The bottle is half full” and “the bottle is 50% full” say the same thing.

A percentage **compares** relative size. When two wholes are the same size, the higher percentage is more of the whole.

Same-sized wholes: which shows more shaded?



40%



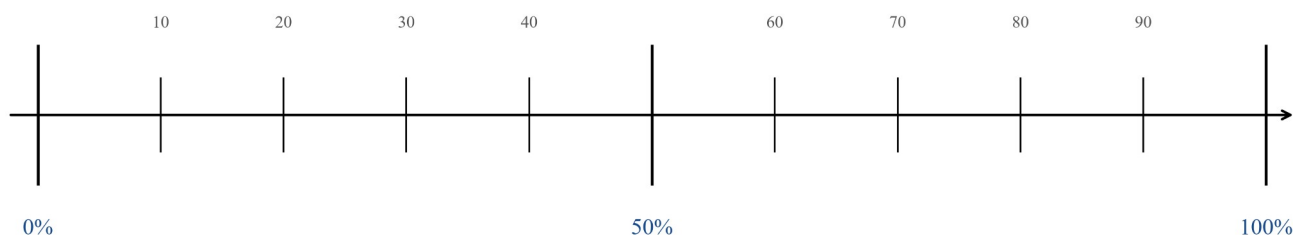
70%

Two circles the same size. The left circle has 40% shaded. The right circle has 70% shaded.

Both circles are the same size. 70% of a circle is more of it than 40%, so the right circle has more shaded.

The **percentage line** puts all of this in order, from no part of it to all of it.

The percentage line: 0% is no part of it, 100% is all of it



A number line from 0% to 100% with a tick at every 10%. The ticks at 0%, 25%, 50%, 75% and 100% are taller and labelled in blue.

0% is no part of it. 100% is all of it. 50% sits in the middle: half. 25% sits halfway between 0% and 50%.

To compare a percentage with a fraction or a decimal, rename first so all three names match, then compare. 0.6 is 60%, and 60% is more than 55%, so 0.6 is more than 55%.

Worked examples

Example 1 — with help: read a 100-grid

A 100-grid has 4 full rows shaded. What percentage is shaded, and what percentage is not shaded?

Working

Each full row is 10 squares, so 4 rows is 40 squares.

40 out of 100 squares shaded = 40%.

$100\% - 40\% = 60\%$ not shaded.

So 40% is shaded and 60% is not shaded.

Comment

10 squares = 10%

% means out of 100

The parts add to 100%

Example 2 — with help: three names for one amount

Write $\frac{3}{4}$ as a decimal and as a percentage.

Working

$$\frac{3}{4} = 0.75$$

$$0.75 = 75 \text{ out of } 100 = 75\%$$

$$\text{So } \frac{3}{4} = 0.75 = 75\%.$$

Comment

Three quarters is 75 hundredths

Hundredths rename straight to %

Example 3 — with help: build a percentage from known ones

Show 60% using 50% and 10%.

Working

50% is half the grid: 5 rows.

10% more is 1 more row.

Comment

Half = 50%

1 row = 10%

$50\% + 10\% = 60\%$.

Check on the grid above

So 60% is 50% and 10% more: 6 rows of the 100-grid.

Example 4 — on your own: the missing part

A battery bar shows 40% of the charge left. What percentage of the charge is used?

100% is the whole charge. $100\% - 40\% = 60\%$. **60% is used.**

Example 5 — on your own: which is more?

Which is more: $\frac{3}{4}$ or 70%?

Rename $\frac{3}{4}$ as 75%. 75% is more than 70%, so $\frac{3}{4}$ **is more than 70%.**

Example 6 — on your own: describe with a fraction

A bottle is 25% full of water. What fraction of the bottle is full?

25% is a quarter, $\frac{1}{4}$. **The bottle is $\frac{1}{4}$ full.**

Practice questions

With help

Q1. A 100-grid has 6 full rows shaded. What percentage is shaded?

Q2. A 100-grid has 55 squares shaded. Write the shaded part as a decimal.

Q3. Write 0.8 as a percentage.

Q4. Write $\frac{1}{2}$ as a percentage.

Q5. Write 25% as a fraction and as a decimal.

Q6. A 100-grid is 20% shaded. What percentage is not shaded?

Q7. Which is more: 30% or 0.3? Or are they the same amount?

Q8. A battery bar shows 75% charge left. What fraction of the charge is left?

On your own

Q9. A 100-grid has 7 full columns shaded. Write the shaded part as a percentage, a decimal and a fraction.

Q10. Rename 0.25 as a percentage and as a fraction.

Q11. Which is more: 0.9 or 85%?

Q12. Which is more: $\frac{1}{4}$ or 30%?

Q13. Put in order from smallest to biggest: 0.4, 45%, $\frac{1}{2}$.

Q14. A bar is 65% shaded. What percentage is not shaded?

Q15. $80\% = 50\% + \square$. What number fills the box?

Q16. On the percentage line, which is closer to 50%: 45% or 60%?

Maths in real life

Q17. Tom's tablet shows 25% charge left. What fraction of the charge is left?

Q18. A download bar shows 90% done. What percentage is still to go?

Q19. In a sale, shop signs show percentages off the price. One sign shows 25% and one shows 50%. (You do not need any prices here.) a. Which sign takes more of the price off? b. Write 50% as a fraction.

Answers

Q1. 60%. **Q2.** 0.55. **Q3.** 80%. **Q4.** 50%. **Q5.** $\frac{1}{4}$, 0.25. **Q6.** 80%. **Q7.** The same amount. **Q8.** $\frac{3}{4}$. **Q9.** 70%, 0.7, $\frac{7}{10}$.
Q10. 25%, $\frac{1}{4}$. **Q11.** 0.9. **Q12.** 30%. **Q13.** 0.4, 45%, $\frac{1}{2}$. **Q14.** 35%. **Q15.** 30%. **Q16.** 45%. **Q17.** $\frac{1}{4}$. **Q18.** 10%. **Q19.** a.
The 50% sign. b. $\frac{1}{2}$.

Worked answers

Q1. Each row is 10 squares, so 6 rows is 60 squares: 60 out of 100. $\therefore$ 60% is shaded.

Q2. 55 out of 100 squares is 55 hundredths. $\therefore$ 0.55.

Q3. 0.8 is 8 tenths, which is 80 out of 100. $\therefore$ 80%.

Q4. Half is 50 out of 100. $\therefore$ 50%.

Q5. 25% is 25 out of 100, a quarter. $\therefore$ $\frac{1}{4}$ and 0.25.

Q6. The parts add to 100%: $100\% - 20\% = 80\%$. $\therefore$ 80% is not shaded.

Q7. 0.3 is 3 tenths, which is 30 out of 100, which is 30%. $\therefore$ they are the same amount.

Q8. 75% is 75 out of 100, three quarters. $\therefore$ $\frac{3}{4}$ of the charge is left.

Q9. Each column is 10 squares, so 7 columns is 70 squares: 70%. As a decimal, 0.7; as a fraction, $\frac{7}{10}$. $\therefore 70\% = 0.7 = \frac{7}{10}$.

Q10. 0.25 is 25 hundredths: 25 out of 100, a quarter. $\therefore$ 25% and $\frac{1}{4}$.

Q11. Rename 0.9 as 90%. 90% is more than 85%. $\therefore$ 0.9 is more.

Q12. Rename $\frac{1}{4}$ as 25%. 25% is less than 30%. $\therefore$ 30% is more.

Q13. Rename all three as percentages: $0.4 = 40\%$ and $\frac{1}{2} = 50\%$. In order: 40%, 45%, 50%. $\therefore$ 0.4, 45%, $\frac{1}{2}$.

Q14. The parts add to 100%: $100\% - 65\% = 35\%$. $\therefore$ 35% is not shaded.

Q15. $50\% + 30\% = 80\%$. $\therefore$ the box is 30.

Q16. 45% is 5 away from 50%. 60% is 10 away from 50%. $\therefore$ 45% is closer.

Q17. 25% is a quarter. $\therefore$ $\frac{1}{4}$ of the charge is left.

Q18. The whole download is 100%: $100\% - 90\% = 10\%$. $\therefore$ 10% is still to go.

Q19. a. 50 is more than 25, so the 50% sign takes more of the price off. b. 50% is half. $\therefore$ a. the 50% sign; b. $\frac{1}{2}$.

Test

Chapter 2 test (T2) — Number: fractions, decimals and percentages

Codes: VC2M5N01 (2A) · VC2M5N03 (2B) · VC2M5N05 (2C) · VC2M5N04 (2D) · **Marks:** 20 · **Guided duration:** 25 min

This is a classroom check, not an exam. There is no time limit; the 25 minutes is a guide (about 1 minute per mark, plus 5 minutes reading). Answer every question, and show your working wherever a question asks you to rename, compare, explain or draw.

Equipment: a ruler, for the number-line questions (Q3 and Q6).

You may use a calculator if you want to, but no question needs one, and no content description in this chapter says to work without one. The marks for renaming, comparing and explaining go to your working, not just to the final answer.

Q1. (2 marks) Write $<$, $>$ or $=$ in each box.

- (a) $5.3 \square 5.30$ (1 mark)
- (b) $2.18 \square 2.108$ (1 mark)

Q2. (1 mark) Write these numbers in order from smallest to biggest: 3.4, 3.046, 3.46.

Q3. (2 marks) Use your ruler to draw a number line from 4.20 to 4.30, with a tick at every hundredth.

- (a) Mark 4.23 on your line. (1 mark)
- (b) Mark 4.275 on your line. (1 mark)

Q4. (1 mark) Which fraction is the greatest? Circle the letter.

- A $\frac{1}{8}$
- B $\frac{1}{3}$
- C $\frac{1}{5}$
- D $\frac{1}{4}$

Q5. (2 marks)

- (a) Which is more, $\frac{2}{3}$ or $\frac{5}{6}$? Show how renaming helps you decide. (1 mark)
- (b) Write these fractions in order from smallest to biggest: $\frac{1}{3}$, $\frac{5}{6}$, $\frac{1}{6}$. (1 mark)

Q6. (2 marks)

- (a) Write $2\frac{1}{4}$ as an improper fraction. (1 mark)
- (b) Use your ruler to draw a number line from 0 to 3, with a tick at every quarter. Mark your improper fraction from part (a) on your line. (1 mark)

Q7. (2 marks) Work these out.

- (a) $\frac{1}{6} + \frac{4}{6}$ (1 mark)

(b) $\frac{7}{8} - \frac{1}{4}$ (1 mark)

Q8. (2 marks) There is $\frac{7}{10}$ of a litre of water in a bottle. Mia pours in another $\frac{1}{5}$ of a litre. How much water is in the bottle now? Show your working. (1 mark for the renaming and the calculation, 1 mark for the answer.)

Q9. (1 mark) Lily says $\frac{1}{2} + \frac{1}{8} = \frac{2}{10}$. Is Lily right? Explain your answer.

Q10. (1 mark) Write 0.7 as a percentage and as a fraction.

Q11. (2 marks)

(a) A 100-grid has 45 of its squares shaded. What percentage of the grid is **not** shaded? (1 mark)

(b) Which is more, $\frac{1}{2}$ or 55%? Explain how you know. (1 mark)

Q12. (2 marks)

(a) Write these in order from smallest to biggest: $\frac{1}{2}$, 20%, 0.75. (1 mark)

(b) A download bar shows 70% done. What percentage is still to go? (1 mark)

Solutions

Worked solutions

Q1. (a) 5.3 is 5 and 3 tenths. 5.30 is 5 and 30 hundredths. 3 tenths is exactly 30 hundredths, so adding a zero after the last digit changes nothing: $5.3 = 5.30$. (b) Line up the points and write 2.18 as 2.180. Ones: 2 and 2. Tenths: 1 and 1. Hundredths: 8 against 0, and 8 hundredths is more. So $2.18 > 2.108$ — more digits does not mean a greater number. Award 1 mark for each correct symbol.

Q2. Write each number with 3 decimal places: 3.400, 3.046, 3.460. Compare from the left. Tenths: 0 comes before 4, so 3.046 is smallest. Then 3.400 against 3.460: hundredths 0 against 6, so 3.400 comes next. $\therefore$ 3.046, 3.4, 3.46.

Q3. The line runs from 4.20 to 4.30 with 10 equal gaps, each one hundredth. (a) 4.23 is 3 hundredths past 4.20, so it sits on the third tick. (b) 4.275 is 5 thousandths past 4.27; 5 parts out of 10 is half the gap, so it sits exactly halfway between the 4.27 tick and the 4.28 tick. Award 1 mark for each point placed in the correct position on a correctly marked line.

Q4. All four are unit fractions of the same whole. The more parts the whole is cut into, the less each part is: $\frac{1}{8}$, $\frac{1}{5}$ and $\frac{1}{4}$ are all less than $\frac{1}{3}$, because 8, 5 and 4 are all more than 3. The answer is **B**.

Q5. (a) 6 is a multiple of 3, so 3 and 6 are related denominators. Rename $\frac{2}{3}$ as sixths: $\frac{2}{3} = \frac{4}{6}$. Now the parts are the same size: $\frac{4}{6} < \frac{5}{6}$, so $\frac{5}{6}$ is more. (b) Rename $\frac{1}{3}$ as $\frac{2}{6}$, then compare the numerators: $1 < 2 < 5$. $\therefore$ In order: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{5}{6}$. Award 1 mark each.

Q6. (a) Each whole is 4 quarters, so 2 wholes are 8 quarters. Add the extra 1 quarter: $8 + 1 = 9$ quarters. $\therefore 2\frac{1}{4} = \frac{9}{4}$. (b) On a line from 0 to 3 marked in quarters, $\frac{9}{4}$ sits on the ninth tick after 0 — the same point as $2\frac{1}{4}$, between 2 and 3. Award 1 mark for the correct point placed correctly, following through the student's answer from part (a).

Q7. (a) The denominators are the same, so the parts are the same size: add the top numbers, $1 + 4 = 5$, and the bottom number stays 6. $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$. (b) 8 is a multiple of 4, so rename $\frac{1}{4}$ as eighths: $\frac{1}{4} = \frac{2}{8}$. Then $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$. Award 1 mark each.

Q8. 10 is a multiple of 5, so rename $\frac{1}{5}$ as tenths: $\frac{1}{5} = \frac{2}{10}$. Now the denominators match, so add the top numbers: $\frac{7}{10} + \frac{2}{10} = \frac{9}{10}$. $\therefore$ There is $\frac{9}{10}$ of a litre of water in the bottle now. Award 1 mark for renaming to matching denominators and adding, and 1 mark for the answer $\frac{9}{10}$ of a litre.

Q9. No. You can only add counts of parts when the parts are the same size. Rename $\frac{1}{2}$ as eighths: $\frac{1}{2} = \frac{4}{8}$. Then $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$, so $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$, not $\frac{2}{10}$. Lily added the top numbers together and the bottom numbers together, which changes the size of the parts. Award 1 mark for a correct “no” with a valid explanation.

Q10. 0.7 is 7 tenths, which is 70 hundredths, which is 70 out of 100, which is 70%. As a fraction, 7 tenths is $\frac{7}{10}$. $\therefore$ $0.7 = 70\% = \frac{7}{10}$. Award 1 mark for both names.

Q11. (a) The whole grid is 100%, and the parts add to 100%: $100\% - 45\% = 55\%$. 55% of the grid is not shaded. (b) Rename $\frac{1}{2}$ as 50%. 55% is more than 50%, so 55% is more. Award 1 mark each.

Q12. (a) Rename so all three names match: $\frac{1}{2} = 50\%$ and $0.75 = 75\%$. In order, 20%, 50%, 75%. $\therefore 20\%, \frac{1}{2}, 0.75$. (b) The whole download is 100%: $100\% - 70\% = 30\%$. 30% is still to go. Award 1 mark each.

Answer key

Question	Answer	Marks	Code
Q1(a)	=	1	VC2M5N01
Q1(b)	>	1	VC2M5N01
Q2	3.046, 3.4, 3.46	1	VC2M5N01
Q3(a)	4.23 marked on the third tick after 4.20	1	VC2M5N01
Q3(b)	4.275 marked halfway between 4.27 and 4.28	1	VC2M5N01
Q4	B ($\frac{1}{3}$)	1	VC2M5N03
Q5(a)	$\frac{5}{6}$, after renaming $\frac{2}{3}$ as $\frac{4}{6}$	1	VC2M5N03
Q5(b)	$\frac{1}{6}, \frac{1}{3}, \frac{5}{6}$	1	VC2M5N03
Q6(a)	$\frac{9}{4}$	1	VC2M5N03
Q6(b)	the part-(a) fraction marked correctly on the quarters line	1	VC2M5N03
Q7(a)	$\frac{5}{6}$	1	VC2M5N05
Q7(b)	$\frac{5}{8}$	1	VC2M5N05
Q8	rename $\frac{1}{5} = \frac{2}{10}$ and add, $\frac{7}{10} + \frac{2}{10}$	1	VC2M5N05
Q8	$\frac{9}{10}$ of a litre	1	VC2M5N05
Q9	no; $\frac{1}{2} = \frac{4}{8}$, so the sum is $\frac{5}{8}$	1	VC2M5N05
Q10	70% and $\frac{7}{10}$	1	VC2M5N04
Q11(a)	55%	1	VC2M5N04
Q11(b)	55%, because $\frac{1}{2} = 50\%$	1	VC2M5N04
Q12(a)	20%, $\frac{1}{2}, 0.75$	1	VC2M5N04
Q12(b)	30%	1	VC2M5N04

Question	Answer	Marks	Code
Total		20	

Marks by content description. VC2M5N01 — 5 marks (Q1–Q3) · VC2M5N03 — 5 marks (Q4–Q6) · VC2M5N05 — 5 marks (Q7–Q9) · VC2M5N04 — 5 marks (Q10–Q12). That is 5 marks per content description, per the §7 marking rule. The only multiple-choice item is Q4, 1 mark of 20 — MCQ used sparingly, where selection is genuinely the skill being tested.

Achievement standard extracts (verbatim from `maths_5.json`): VC2M5N01 — *students use place value to write and order decimals including decimals greater than one.* VC2M5N03 — *Students order and represent, add and subtract fractions with the same or related denominators.* VC2M5N05 — *Students order and represent, add and subtract fractions with the same or related denominators.* VC2M5N04 — *They represent common percentages and connect them to their fraction and decimal equivalents.*