Mathematics Level 5

Chapter 1 — Multiplication, division and factors

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Contents

Factors, multiples and divisibility

> *express natural numbers as products of their factors, recognise multiples and determine if one number is divisible by another* — VC2M5N02

Level 5 achievement standard (extract): *They express natural numbers as products of factors and identify multiples and divisors.*

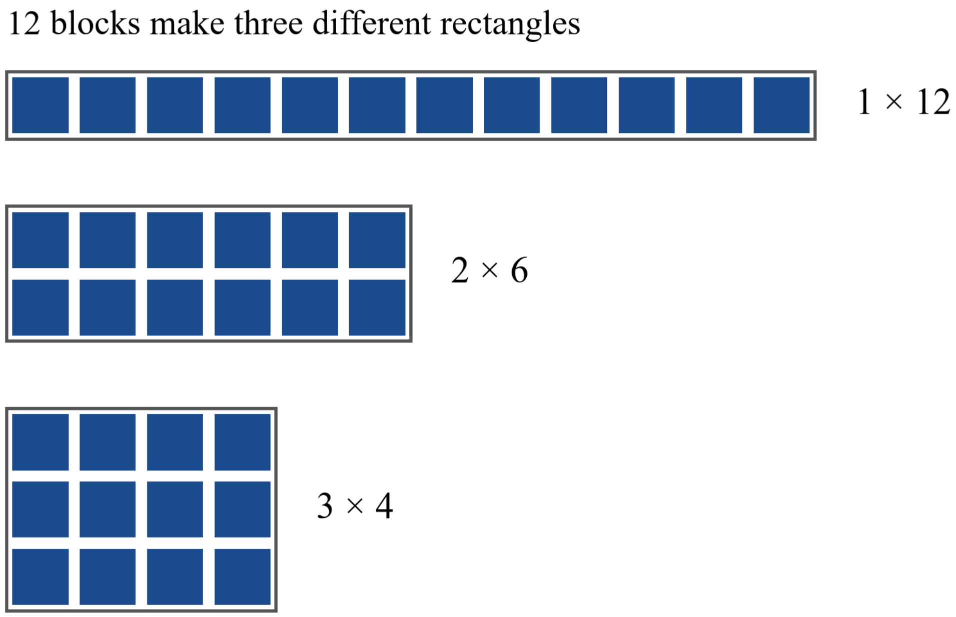
In this section we work with the counting numbers 1, 2, 3, 4, and so on. They are also called the **natural numbers**. You will meet three ideas: factors, multiples and divisibility. They are three ways of looking at the same fact, and your times tables to 10 × 10 from Year 4 do most of the work.

Theory

Factors — the numbers that multiply to make it

A **factor** of a number is a counting number that multiplies with another counting number to make it. Because 3 × 4 = 12, the numbers 3 and 4 are factors of 12. Because 2 × 6 = 12, the numbers 2 and 6 are factors of 12 too.

Blocks make this easy to see. Twelve blocks fit into three different rectangles:



Twelve blocks in three rectangles: one row of twelve, two rows of six, and three rows of four.

The sides of each rectangle give a **factor pair**: 1 and 12, 2 and 6, 3 and 4. Two numbers that multiply to make 12. So the factors of 12 are 1, 2, 3, 4, 6 and 12.

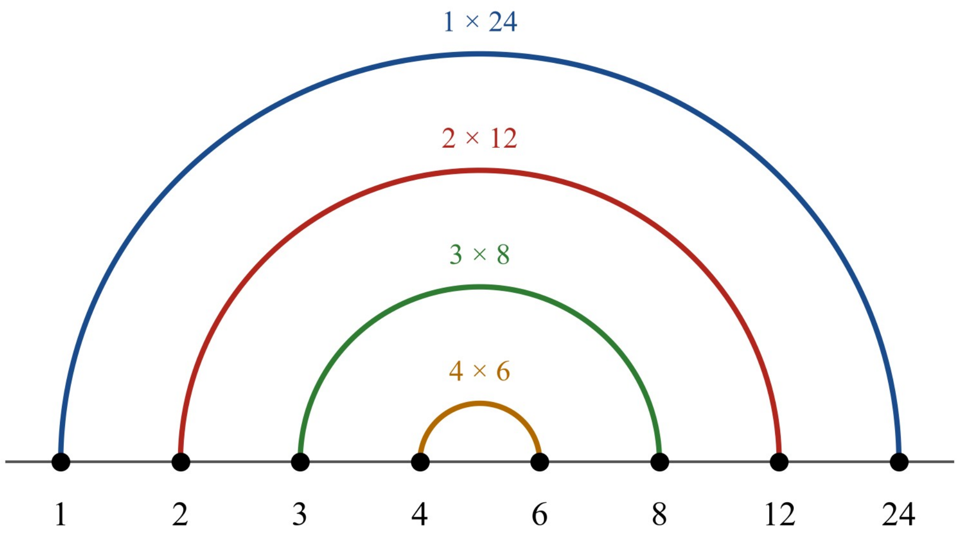
We can write 12 as a **product** of its factors. The product is the answer to a multiplication:

$12 = 1 \times 12$ $12 = 2 \times 6$ $12 = 3 \times 4$

Every counting number has 1 and itself as a factor. For example, 1 × 12 = 12, so 1 and 12 are both factors of 12.

Finding all the factors of a number

To find **all** the factors, start at 1 and work up, writing each factor pair as you find one. Stop when a pair would repeat one you already have. A **factor rainbow** joins each pair with a line so you can see them all. Here is one for 24:



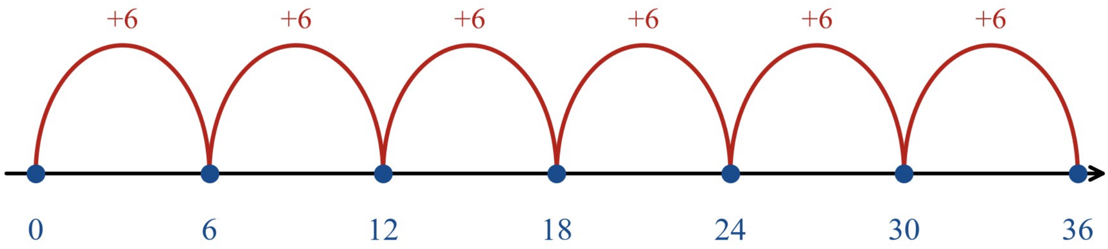
The numbers 1, 2, 3, 4, 6, 8, 12 and 24 in a row, with curved lines joining the factor pairs 1 and 24, 2 and 12, 3 and 8, and 4 and 6.

The pairs are 1 × 24, 2 × 12, 3 × 8 and 4 × 6. Try 5 next: 5 × 4 = 20 and 5 × 5 = 25, so no counting number times 5 makes 24 — 5 is not a factor. The next pair would begin with 6, but 6 is already there (6 × 4), so we stop.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

Multiples — counting on by a number

A **multiple** of a number is a number you reach when you count on by it — a number in its times table. The multiples of 6 are 6, 12, 18, 24, 30, 36, and so on, one jump of 6 at a time:



The multiples of 6 are 6, 12, 18, 24, 30, 36, and so on.

A number line from 0 to 36 with six equal jumps of 6, landing on 6, 12, 18, 24, 30 and 36.

The list of multiples never ends, because you can always add one more 6.

Multiples make patterns on a hundreds chart. Here the multiples of 3 are shaded:

Multiples of 3 are shaded.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A hundreds chart with the numbers 1 to 100, where every multiple of 3 is shaded blue.

The shaded numbers step down and across in the same way every row. Knowing the pattern lets you check a number at a glance: 21 is shaded, and sure enough 21 = 3 × 7.

Divisible — divides exactly, with nothing left over

A number is **divisible** by another number when you can divide it exactly, with nothing left over. 12 ÷ 3 = 4 with nothing left over, so 12 is divisible by 3. 13 ÷ 3 = 4 with 1 left over, so 13 is not divisible by 3.

The three ideas are one fact wearing three faces. The single fact 3 × 4 = 12 says:

One fact	Three ways to say it
3 × 4 = 12	3 and 4 are factors of 12
	12 is a multiple of 3, and a multiple of 4
	12 is divisible by 3, and by 4

So *factor*, *multiple* and *divisible* always travel together. If you know one sentence, you know all three.

Divisibility tests — quick rules for big numbers

For a big number you do not need to try the long division. A **divisibility test** is a quick rule that tells you the answer. Here are the tests we use at this level:

- **By 2:** the last digit is 0, 2, 4, 6 or 8. These are the **even** numbers. The **odd** numbers end in 1, 3, 5, 7 or 9.
- **By 5:** the last digit is 0 or 5.
- **By 10:** the last digit is 0.

Each of these tests only needs the last digit:

test by 2: $\boxed{8}\,\boxed{4}\,\boxed{6}$ > It ends in 6, so it is divisible by 2.

test by 5: $\boxed{3}\,\boxed{7}\,\boxed{5}$ > It ends in 5, so it is divisible by 5.

test by 10: $\boxed{2}\,\boxed{9}\,\boxed{0}$ > It ends in 0, so it is divisible by 10.

Three numbers in digit boxes with the last digit marked in red: 846 ends in 6 so it is divisible by 2, 375 ends in 5 so it is divisible by 5, and 290 ends in 0 so it is divisible by 10.

- **By 3:** add the digits. If the **digit sum** is divisible by 3, the number is divisible by 3.
- **By 9:** add the digits. If the digit sum is divisible by 9, the number is divisible by 9.

The digit-sum test works on numbers of any size. Try it on 89 472:

$$\boxed{8} + \boxed{9} + \boxed{4} + \boxed{7} + \boxed{2} = 30$$

30 is divisible by 3, since 30 = 3 × 10.

So 89 472 is divisible by 3. It is a multiple of 3.

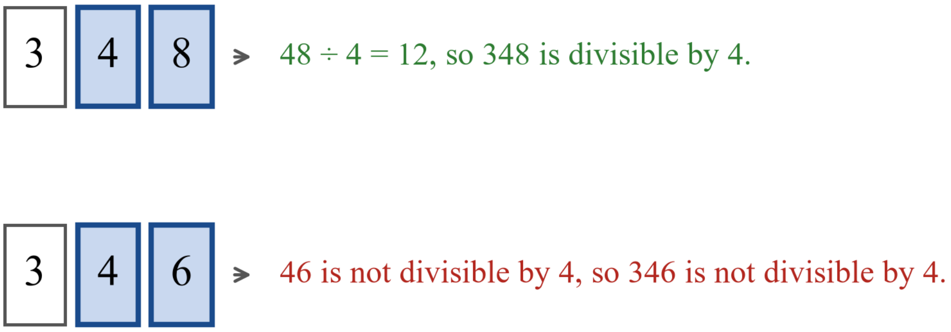
But 30 is not divisible by 9,

so 89 472 is not divisible by 9.

The digits of 89 472 added together: 8 plus 9 plus 4 plus 7 plus 2 equals 30. 30 is divisible by 3, so 89 472 is divisible by 3; 30 is not divisible by 9, so 89 472 is not divisible by 9.

- **By 4:** look at the last two digits. If the number they make is divisible by 4, the whole number is divisible by 4.

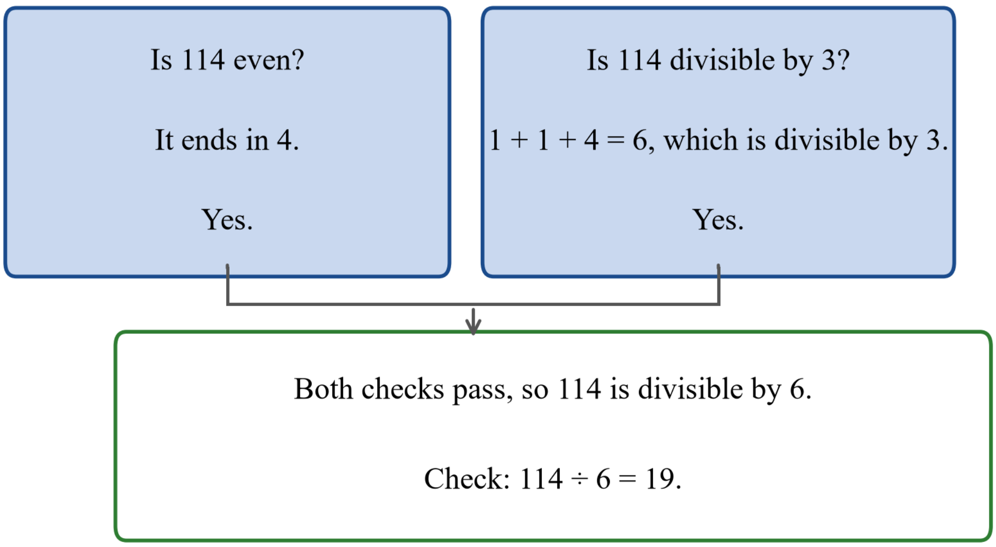
Test by 4: divide the last two digits by 4.



Digit boxes for 348 and 346 with the last two digits marked: 48 divided by 4 is 12, so 348 is divisible by 4; 46 is not divisible by 4, so 346 is not divisible by 4.

- **By 6:** do the test by 2 and the test by 3. If both pass, the number is divisible by 6, because 6 = 2 × 3.

To test by 6, check by 2 and by 3.



Two check boxes for 114: it ends in 4 so it is even, and 1 plus 1 plus 4 is 6 which is divisible by 3, so 114 is divisible by 6; 114 divided by 6 is 19.

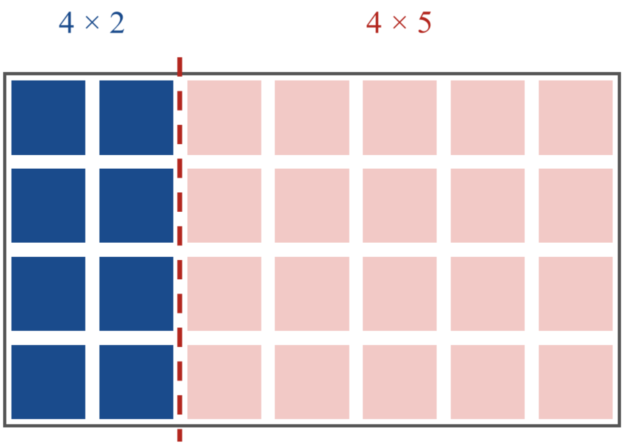
Making multiples by splitting

A multiple can be built by joining two multiples you already know. Because 7 = 2 + 5, every multiple of 7 is the matching multiple of 2 plus the matching multiple of 5:

$3 \times 7 = 3 \times 2 + 3 \times 5 = 6 + 15 = 21$

$4 \times 7 = 4 \times 2 + 4 \times 5 = 8 + 20 = 28$

The picture shows why this must work:



$$4 \times 7 = 4 \times 2 + 4 \times 5 = 8 + 20 = 28$$

Split 7 into 2 and 5: the 4 rows of 7 are 4 rows of 2 and 4 rows of 5.

An array of 4 rows of 7 blocks split into two parts, 4 rows of 2 and 4 rows of 5, showing 4 times 7 equals 4 times 2 plus 4 times 5, which is 8 plus 20, which is 28.

Four rows of 7 is the same as four rows of 2 joined to four rows of 5. This is a handy way to reach the 7 times table from tables you already know, and it is the same splitting idea that makes the tests by 6 and by 4 work.

You will use factors and multiples again in 1D, where you build step-by-step rules for testing numbers, and in 2B, where they help you compare fractions.

Worked examples

Worked example 1 — step by step: find all the factors of 18

Find every factor of 18, and write 18 as a product of two factors in each way.

Working	Comment
$1 \times 18 = 18$	1 and 18 are factors.
$2 \times 9 = 18$ (18 is even)	2 and 9 are factors.
$3 \times 6 = 18$ ($1 + 8 = 9$, divisible by 3)	3 and 6 are factors.
4? $4 \times 4 = 16$, $4 \times 5 = 20$ — no.	4 is not a factor.
5? $5 \times 3 = 15$, $5 \times 4 = 20$ — no.	5 is not a factor.
The next pair would start at 6, already found. Stop.	Every pair is on the list.

∴ The factors of 18 are 1, 2, 3, 6, 9 and 18, and $18 = 1 \times 18 = 2 \times 9 = 3 \times 6$.

Worked example 2 — step by step: test a big number

Without dividing, decide whether 89 472 is divisible by 2, by 3, by 5 and by 9.

Working	Comment
The last digit is 2.	Even, so 89 472 is divisible by 2.
$8 + 9 + 4 + 7 + 2 = 30$, and 30 is divisible by 3.	Divisible by 3.
The last digit is not 0 or 5.	Not divisible by 5.
30 is not divisible by 9.	Not divisible by 9.

∴ 89 472 is divisible by 2 and by 3, and not divisible by 5 or 9. (Check on a calculator if you like: 89 472 ÷ 3 = 29 824 exactly.)

Worked example 3 — step by step: tests by 4 and by 6

Is 156 divisible by 4? Is it divisible by 6?

Working	Comment
Last two digits: 56. 56 ÷ 4 = 14.	Divisible by 4.
156 is even, and 1 + 5 + 6 = 12, divisible by 3.	Both checks pass, so divisible by 6.

∴ 156 is divisible by 4 and by 6. Check: 156 ÷ 4 = 39 and 156 ÷ 6 = 26.

Worked example 4 — on your own: build the 7s by splitting

Use 4 × 7 = 4 × 2 + 4 × 5 to find 6 × 7.

6 × 7 = 6 × 2 + 6 × 5 = 12 + 30 = 42.

∴ 6 × 7 = 42, so 42 is a multiple of 7, and 6 and 7 are factors of 42.

Worked example 5 — on your own: chairs in equal rows

A hall has 36 chairs set in equal rows, with the same number of chairs in every row and nothing left over. List every way this can be done.

Each way is a factor pair of 36: 1 × 36, 2 × 18, 3 × 12, 4 × 9 and 6 × 6.

∴ There are 5 ways: 1 row of 36, 2 rows of 18, 3 rows of 12, 4 rows of 9, and 6 rows of 6.

Practice questions

Q1. Write 16 as a product of two factors in every way, then list all the factors of 16.

Q2. Use the rainbow method to find all the factors of 20.

Q3. Find all the factors of 21.

Q4. Write the first six multiples of each number.

a. 4 b. 7 c. 9

Q5. Which of these numbers are multiples of 5?

35, 53, 60, 65, 56, 105

Q6. Look at the hundreds chart with the multiples of 3 shaded.

Multiples of 3 are shaded.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A hundreds chart with the numbers 1 to 100, where every multiple of 3 is shaded blue.

a. Is 51 a multiple of 3? How can you tell from the chart?
b. Is 74 a multiple of 3?
c. Is 87 a multiple of 3?
d. In your own words, describe the pattern the shaded numbers make.

Q7. True or false? If false, say why.

a. 8 is a factor of 24.
b. 24 is a multiple of 5.
c. 1 is a factor of every counting number.
d. 7 is a factor of 42.
e. 40 is divisible by 3.

Q8. Without dividing, say whether each number is divisible by 2, by 5 and by 10.

a. 460 b. 725 c. 1 008

Q9. Use the digit-sum test. Which of these numbers are divisible by 3?

243, 538, 4 521

Q10. Use the digit-sum test. Which of these numbers are divisible by 9?

891, 456, 729

Q11. Use the last-two-digits test. Which of these numbers are divisible by 4?

236, 518, 1 244

Q12. Use the test by 6. Which of these numbers are divisible by 6?

96, 74, 123

Q13. The number 4□2 (four, a missing digit, two) is divisible by 3. Find every digit that could go in the box.

Q14. Build the 7s by splitting, as in Worked example 4.

a. $5 \times 7 = 5 \times 2 + 5 \times 5 = ?$
b. $8 \times 7 = ?$ (split 7 into 2 and 5)

Q15. Sixteen students line up in equal rows for a photo, with the same number in every row and nothing left over. In how many different ways can they line up?

Q16. A number is divisible by 2 and by 3. By what other single number must it be divisible? Check your answer with an example.

Q17. Mai says 46 must be divisible by 4 because it is even. Is Mai right? Explain.

Q18. Write 30 as a product of two factors in three different ways, then list all the factors of 30.

Answers

Q1. $16 = 1 \times 16 = 2 \times 8 = 4 \times 4$. Factors: 1, 2, 4, 8, 16.

Q2. 1, 2, 4, 5, 10, 20.

Q3. 1, 3, 7, 21.

Q4. a. 4, 8, 12, 16, 20, 24. b. 7, 14, 21, 28, 35, 42. c. 9, 18, 27, 36, 45, 54.

Q5. 35, 60, 65 and 105.

Q6. a. Yes — 51 is shaded. b. No — 74 is not shaded. c. Yes — 87 is shaded. d. The shaded numbers step one place left each new row (any correct description).

Q7. a. True. b. False. c. True. d. True. e. False.

Q8. a. By 2 yes, by 5 yes, by 10 yes. b. By 2 no, by 5 yes, by 10 no. c. By 2 yes, by 5 no, by 10 no.

Q9. 243 and 4 521.

Q10. 891 and 729.

Q11. 236 and 1 244.

Q12. 96 only.

Q13. 0, 3, 6 or 9.

Q14. a. $10 + 25 = 35$. b. $8 \times 2 + 8 \times 5 = 16 + 40 = 56$.

Q15. 3 ways: 1 row of 16, 2 rows of 8, 4 rows of 4.

Q16. 6. Example: 18 is divisible by 2 and 3, and $18 \div 6 = 3$.

Q17. No.

Q18. $30 = 5 \times 6 = 3 \times 10 = 2 \times 15$. Factors: 1, 2, 3, 5, 6, 10, 15, 30.

Step-by-step solutions

Q1. Test from 1 up: 1×16, 2×8, then 4×4. 3 does not work, because $3 \times 5 = 15$ and $3 \times 6 = 18$ sit either side of 16. 5, 6 and 7 do not work either, and after 4×4 the pairs repeat. $\therefore$ $16 = 1 \times 16 = 2 \times 8 = 4 \times 4$, and the factors of 16 are 1, 2, 4, 8, 16.

Q2. Pairs: 1×20, 2×10, 4×5. 3 does not work ($2 + 0 = 2$, not divisible by 3), and after 4×5 the next pair would repeat. $\therefore$ The factors of 20 are 1, 2, 4, 5, 10, 20.

Q3. Pairs: 1×21, 3×7. 2 does not work (21 is odd), 4 and 5 do not work, and after 3×7 the pairs repeat. $\therefore$ The factors of 21 are 1, 3, 7, 21.

Q4. Count on by each number: a. 4, 8, 12, 16, 20, 24. b. 7, 14, 21, 28, 35, 42. c. 9, 18, 27, 36, 45, 54.

Q5. Multiples of 5 end in 0 or 5: 35, 60, 65 and 105 are multiples of 5; 53 and 56 are not.

Q6. a. 51 is shaded on the chart, so 51 is a multiple of 3. (Check: $5 + 1 = 6$, divisible by 3.) b. 74 is not shaded, so it is not a multiple of 3. ($7 + 4 = 11$, not divisible by 3.) c. 87 is shaded, so it is a multiple of 3. ($8 + 7 = 15$, divisible by 3.) d. The shaded numbers step across and down in the same way each row, so every third number is shaded.

Q7. a. True: $8 \times 3 = 24$. b. False: multiples of 5 end in 0 or 5, and 24 ends in 4. c. True: $1 \times 24 = 24$ and $1 \times 7 = 7$ — 1 times any counting number gives that number back, so 1 is a factor of every counting number. d. True: $7 \times 6 = 42$. e. False: $4 + 0 = 4$, which is not divisible by 3.

Q8. a. 460 ends in 0: divisible by 2, by 5 and by 10. b. 725 ends in 5: divisible by 5, not by 2, not by 10. c. 1 008 ends in 8: divisible by 2, not by 5, not by 10.

Q9. 243: $2 + 4 + 3 = 9$, divisible by 3 — yes. 538: $5 + 3 + 8 = 16$, not divisible by 3 — no. 4 521: $4 + 5 + 2 + 1 = 12$, divisible by 3 — yes.

Q10. 891: $8 + 9 + 1 = 18$, divisible by 9 — yes. 456: $4 + 5 + 6 = 15$, not divisible by 9 — no. 729: $7 + 2 + 9 = 18$, divisible by 9 — yes.

Q11. 236: last two digits 36, and $36 \div 4 = 9$ — yes. 518: last two digits 18, and 18 is not divisible by 4 — no. 1 244: last two digits 44, and $44 \div 4 = 11$ — yes.

Q12. 96: even, and $9 + 6 = 15$ is divisible by 3 — yes. 74: even, but $7 + 4 = 11$ is not divisible by 3 — no. 123: odd, so the test by 2 fails straight away — no. $\therefore$ Only 96 is divisible by 6 ($96 \div 6 = 16$).

Q13. The digits must add to a multiple of 3: $4 + 2 = 6$, so the missing digit must add with 6 to give a multiple of 3. That happens for 0 (6), 3 (9), 6 (12) and 9 (15). $\therefore$ The box can hold 0, 3, 6 or 9.

Q14. a. $5 \times 7 = 5 \times 2 + 5 \times 5 = 10 + 25 = 35$. b. $8 \times 7 = 8 \times 2 + 8 \times 5 = 16 + 40 = 56$.

Q15. Each way is a factor pair of 16: 1×16, 2×8 and 4×4. (3, 5, 6 and 7 do not work, and after 4×4 the pairs repeat.) $\therefore$ 3 ways: 1 row of 16, 2 rows of 8, or 4 rows of 4.

Q16. A number divisible by 2 and by 3 is divisible by 6, because $6 = 2 \times 3$. Example: 18 is even and $1 + 8 = 9$ is divisible by 3, and $18 \div 6 = 3$. $\therefore$ The number must be divisible by 6.

Q17. No. The test by 4 uses the last **two** digits: here the last two digits are 46, and 46 is not divisible by 4 (44 and 48 are the multiples of 4 near it). Check: $46 \div 4 = 11$ with 2 left over. Being even only shows 46 is divisible by 2.

Q18. Three products: $30 = 5 \times 6 = 3 \times 10 = 2 \times 15$. The full list of factor pairs is 1×30, 2×15, 3×10 and 5×6, so the factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30.

Multiplying larger numbers by one- and two-digit numbers

Chapter 1 — Multiplication, division and factors. This subtopic owns `VC2M5N06`*.*

> *solve problems involving multiplication of larger numbers by one- or two-digit numbers, choosing efficient mental and written calculation strategies and using digital tools where appropriate; check the reasonableness of answers* — VC2M5N06

Students use their proficiency with multiplication facts and efficient mental and written calculation strategies to multiply large numbers by one- and two-digit numbers and divide by one-digit numbers. — Level 5 achievement standard (extract)

Before you start: your multiplication facts to 10 × 10 (Level 4) and multiplying by 10 and 100 in your head (Level 4) are the tools this subtopic builds on.

Theory

Four ways to multiply

There is more than one good way to work out a multiplication. The skill in this subtopic is **choosing** the way that fits the numbers in front of you. This subtopic teaches four strategies:

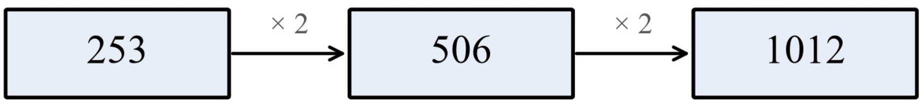
1. **Doubling** — best when the number we multiply by is 2, 4 or 8.
2. **Factors and rearranging** — best when the numbers break into friendly factors like 5 × 2 = 10 or 25 × 4 = 100.
3. **Place-value partitioning** — split the big number into hundreds, tens and ones, multiply each part, then add the parts.
4. **The standard written algorithm** — the column method, the safe choice whenever you multiply by a one- or two-digit number.

Every answer is then checked to see if it makes sense.

Strategy 1 — doubling

To multiply by 4, double, then double again. To multiply by 8, double three times. This works because 4 = 2 × 2 and 8 = 2 × 2 × 2.

$$253 \times 4 = 253 \times 2 \times 2$$



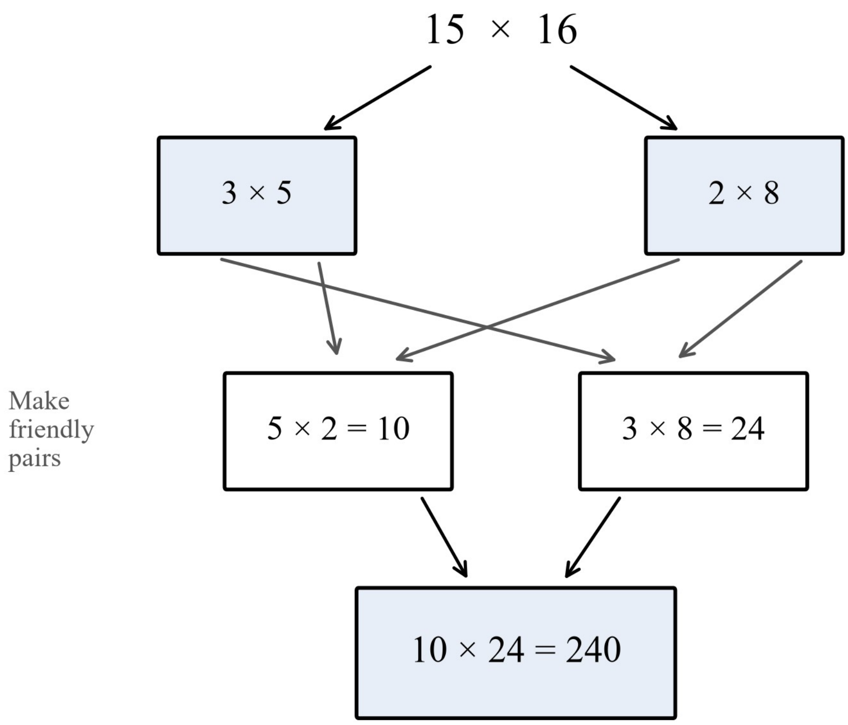
Double, then double again: ×4 is two doublings.

Doubling chain: 253 becomes 506 after the first doubling and 1012 after the second doubling, so 253 × 4 = 1012.

Doubling 253 in your head is easy: 2 × 250 = 500 and 2 × 3 = 6, so 506. Doubling 506 gives 1012. So 253 × 4 = 1012, and no column work was needed.

Strategy 2 — factors and rearranging

If both numbers break into small factors, we may swap the factors into friendly pairs. This uses factors (see 1A) and the fact that we may multiply factors in any order.

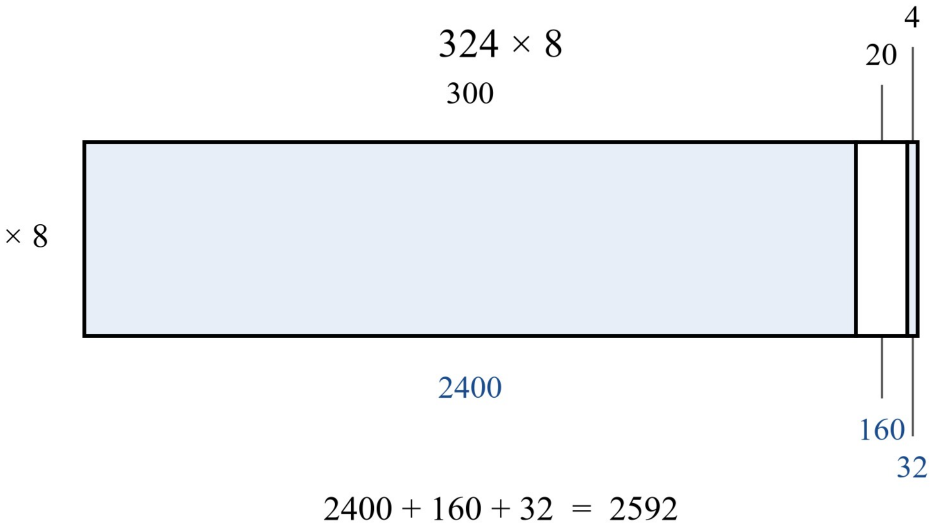


Factor map for 15 × 16: 15 splits into 3 × 5 and 16 splits into 2 × 8; the 5 is paired with the 2 to make 10, and the 3 is paired with the 8 to make 24, so 10 × 24 = 240.

The pair 5 × 2 = 10 is what makes this easy: 10 × 24 is a fact we can write straight down. So 15 × 16 = 240.

Strategy 3 — place-value partitioning

Split the big number into its places, multiply each part, then add the parts. An array shows why this works: the whole rectangle is cut into three smaller rectangles, so the three parts must add up to the whole.



Array for 324 × 8, cut into pieces 300, 20 and 4 wide; the pieces give 2400, 160 and 32, and 2400 + 160 + 32 = 2592.

- 300 × 8 = 2400

- 20 × 8 = 160
- 4 × 8 = 32
- 2400 + 160 + 32 = 2592

So 324 × 8 = 2592. Each part uses a fact we already know (3 × 8, 2 × 8, 4 × 8) with the right number of zeros.

Strategy 4 — the standard written algorithm

The column method works for every one- or two-digit multiplication. Multiply from the ones up, and carry when a step gives more than 9.

628 × 7

$$\begin{array}{rcccc} & & 1 & 5 & \\ & & 6 & 2 & 8 \\ \times & & & & 7 \\ \hline & 4 & 3 & 9 & 6 \end{array}$$

4396

Carry digits are shown in red above the columns.

Column method for 628 × 7 with the carry digits 5 and 1 written above in red; the answer is 4396.

- 8 × 7 = 56: write 6, carry 5 tens.
- 2 × 7 = 14, add the 5 carried: 19: write 9, carry 1 hundred.
- 6 × 7 = 42, add the 1 carried: 43: write 43.

When the number you multiply by has two digits, write one row for the ones and one row for the tens, then add the rows.

342 × 26		3	4	2		
	×		2	6		
	2	0	5	2	342 × 6	
	6	8	4	0	342 × 20	
8892	8	8	9	2	add the two rows	

Multiply by the ones (6), then by the tens (20), then add.

Column method for 342 × 26: the first row is 342 × 6 = 2052, the second row is 342 × 20 = 6840, and the rows add to 8892.

The second row is 342 × 20, not 342 × 2: the 2 sits in the tens place, so it stands for 20. The array above shows the same idea in pieces.

Choosing a strategy

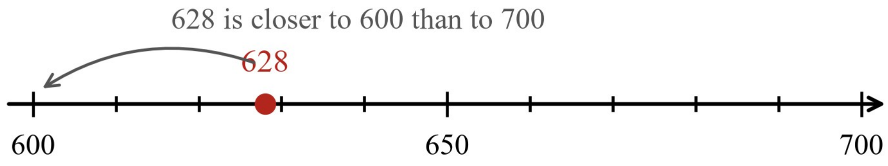
The content description says *choosing* efficient strategies, so the choice is the point. A good rule is to look at the number you multiply by first.

Problem	A good choice	Why
253 × 4	doubling	4 = 2 × 2, and 253 is easy to double
25 × 32	factors and rearranging	32 = 4 × 8, and 25 × 4 = 100
324 × 8	partitioning	3 × 8, 2 × 8 and 4 × 8 are known facts
628 × 7	algorithm	7 is not 2, 4 or 8, and 628 has no friendly factors
342 × 26	algorithm	multiplying by two digits is safest in columns

Two students may make different choices for the same problem, and both may be right. The test of a choice is: did it make the working easier to do *and* easier to check?

Checking that the answer makes sense

Before or after the exact working, round the big number to the nearest hundred and multiply in your head.



$$628 \approx 600, \text{ so } 628 \times 7 \approx 600 \times 7 = 4200$$

Number line from 600 to 700 with 628 marked; 628 sits closer to 600, so 628 × 7 is about 600 × 7 = 4200.

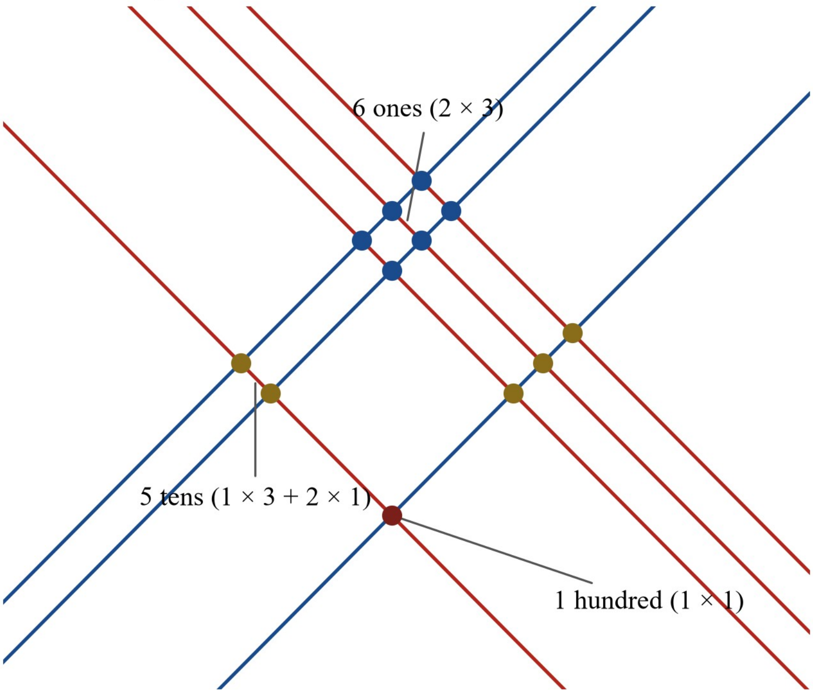
628 is closer to 600 than to 700, so 628 × 7 should be close to 600 × 7 = 4200. An exact answer of 4396 makes sense; an answer of 43 960 or 439 would not. Checking reasonableness like this is part of this subtopic, and 3A takes it further.

Digital tools. Where appropriate, a calculator or another digital tool may check your answer after you have shown a mental or written strategy. The tool checks your working; it does not do the thinking for you.

When a method does not help

Some methods are fine for small numbers and poor for large ones. One visual method draws a line for each digit and counts the places where lines cross. The Victorian Curriculum calls it the Japanese visual method for multiplication.

12 × 13 by the line method: 1 hundred, 5 tens, 6 ones = 156



12 × 13 drawn with lines for the digits; the crossings group into 1 hundred, 5 tens and 6 ones, giving 156.

For 12 × 13 it works: the ones place has 2 × 3 = 6 crossings, the tens place has 1 × 3 + 2 × 1 = 5, and the hundreds place has 1 × 1 = 1, giving 156. But try it for 342 × 26: the picture needs 9 lines crossing 8 lines, so 72 crossings, and one group alone holds 4 × 6 + 2 × 2 = 28 crossings. That is slow and easy to get wrong, which is exactly why the strategy of choice for 342 × 26 is the algorithm. Knowing a strategy's limitations is part of choosing well.

Worked examples

Worked example 1 — doubling

Find 246 × 4.

Working	Comment
2 × 246 = 492	First doubling: 2 × 200 = 400, 2 × 46 = 92.
2 × 492 = 984	Second doubling: 2 × 490 = 980, 2 × 2 = 4.
Check: 246 is close to 250; 250 × 4 = 1000, and 984 is close to 1000. ✓	The answer makes sense.

∴ 246 × 4 = 984.

Worked example 2 — factors and rearranging

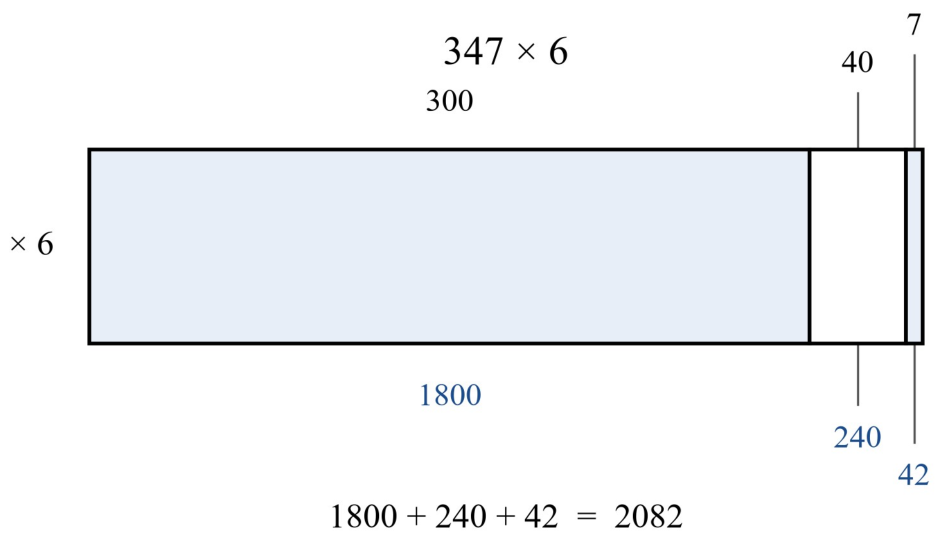
Find 25 × 12.

Working	Comment
12 = 4 × 3	Break 12 into factors that suit 25.
25 × 4 = 100	A friendly pair.
100 × 3 = 300	Easy to write down.

∴ 25 × 12 = 300.

Worked example 3 — partitioning with an array

Find 347 × 6.



Array for 347 × 6, cut into pieces 300, 40 and 7 wide; the pieces give 1800, 240 and 42, and 1800 + 240 + 42 = 2082.

Working	Comment
300 × 6 = 1800	Hundreds part.
40 × 6 = 240	Tens part.
7 × 6 = 42	Ones part.
1800 + 240 + 42 = 2082	Add the three parts.
Check: 347 is close to 350; 350 × 6 = 2100, and 2082 is close to 2100. ✓	The answer makes sense.

∴ 347 × 6 = 2082.

Worked example 4 — the algorithm with two digits

Find 342 × 26, then check it makes sense.

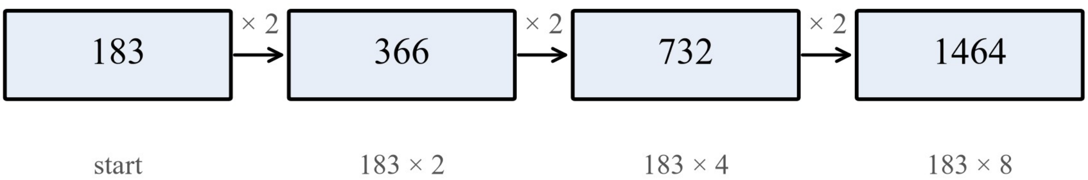
Working	Comment
342 × 6 = 2052	Row 1: multiply by the ones.
342 × 20 = 6840	Row 2: multiply by the tens.
2052 + 6840 = 8892	Add the rows.
Check: 342 ≈ 300 and 26 ≈ 30, so the answer should be close to 300 × 30 = 9000. 8892 is close to 9000. ✓	Reasonable.

∴ 342 × 26 = 8892.

Practice questions

Q1. Use the doubling chain to fill in the gaps.

Each step doubles: ×2, then ×4, then ×8.



Doubling chain starting at 183: doubling gives 366, then 732, then 1464, which are 183 × 2, 183 × 4 and 183 × 8.

a. 183 × 2 = ____
b. 183 × 4 = ____
c. 183 × 8 = ____

Q2. Use doubling to find:

a. 352 × 4
b. 137 × 8

Q3. Break both numbers into factors, then rearrange to find:

a. 25 × 12
b. 14 × 15

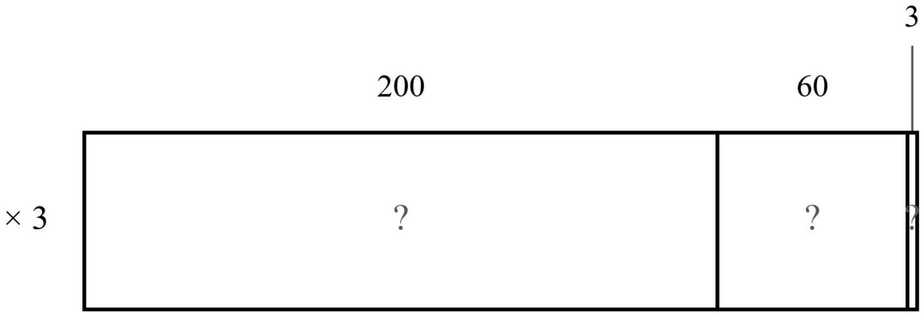
Q4. Fill in the gaps to finish the rearranging: 15 = 3 × 5 and 14 = 2 × 7, so 5 × 2 = ____, 3 × 7 = ____, and 15 × 14 = 10 × ____ = ____.

Q5. Split 436 into its places and find 436 × 7:

400 × 7 = ____ 30 × 7 = ____ 6 × 7 = ____ Total = ____

Q6. The array shows 263 × 3. Write the value of each part, then add the three parts.

The array shows 263 × 3.



Find the value of each region, then add the three parts.

Blank array for 263 × 3, cut into pieces 200, 60 and 3 wide, with a question mark in each piece to fill in.

Q7. Use the standard written algorithm to find:

a. 573 × 4
b. 816 × 5
c. 794 × 6

Q8. Use the standard written algorithm to find:

a. 243 × 15
b. 518 × 24

Q9. Choose a strategy for each problem and say in one sentence why it fits:

a. 25 × 32
b. 253 × 4
c. 417 × 9

Q10. Mia’s class is packing hampers to give away. Each hamper holds 35 items, and the class packs 26 hampers. How many items are packed in total?

Q11. A baker bakes 148 loaves each day. How many loaves does the baker bake in 7 days?

Q12. The Melbourne Cricket Ground can hold up to 100 000 people. If six sold-out shows are held there in a row, how many people is that in total?

Source: Melbourne Cricket Ground, *About the MCG*, mcg.org.au, retrieved 20 August 2026.

Q13. Theo writes 462 × 8 = 2496. Without finding the exact answer, explain how you can tell his answer cannot be right.

Q14. Find 385 × 21 using the algorithm. Then check the answer two ways: first by rounding, then with a calculator or another digital tool.

Q15. Look at the line method for 12 × 13 in the theory above.

a. Explain where the numbers 6, 5 and 1 come from.
b. Explain why the line method is a poor choice for 342 × 26.

Q16. Find the missing number:

a. □ × 8 = 648
b. 25 × □ = 1000

Q17. A train has 8 carriages, and each carriage has 56 seats. Three of these trains run in the morning. How many seats are there in the three trains together?

Q18. Which strategy would you choose for 50 × 24? Find the answer your way, and check it makes sense.

Answers

Q1. a. 366 b. 732 c. 1464.

Q2. a. 1408 b. 1096.

Q3. a. 300 b. 210.

Q4. $5 \times 2 = 10$, $3 \times 7 = 21$, $15 \times 14 = 10 \times 21 = 210$.

Q5. 2800, 210, 42; total 3052.

Q6. 600, 180, 9; total 789.

Q7. a. 2292 b. 4080 c. 4764.

Q8. a. 3645 b. 12 432.

Q9. a. Factors and rearranging ($25 \times 4 = 100$). b. Doubling ($4 = 2 \times 2$). c. The algorithm, or partitioning.

Q10. 910 items.

Q11. 1036 loaves.

Q12. 600 000 people.

Q13. 462 is more than 400, so the answer must be more than $400 \times 8 = 3200$; 2496 is less than 3200.

Q14. 8085; about $400 \times 20 = 8000$, so 8085 makes sense.

Q15. a. The ones place has $2 \times 3 = 6$ crossings, the tens place has $1 \times 3 + 2 \times 1 = 5$, the hundreds place has $1 \times 1 = 1$. b. 342×26 needs 9 lines crossing 8 lines (72 crossings); the busiest group holds 28 crossings, which is too slow and too easy to get wrong.

Q16. a. 81 b. 40.

Q17. 1344 seats.

Q18. 1200. A good choice: $50 \times 24 = 50 \times 2 \times 12 = 100 \times 12 = 1200$.

Fully-worked solutions

Q1. Each arrow doubles: $183 \times 2 = 366$; $366 \times 2 = 732$; $732 \times 2 = 1464$. a. 366. b. 732. c. 1464.

Q2. a. $2 \times 352 = 704$; $2 \times 704 = 1408$. So $352 \times 4 = 1408$. b. $2 \times 137 = 274$; $2 \times 274 = 548$; $2 \times 548 = 1096$. So $137 \times 8 = 1096$.

Q3. a. $12 = 4 \times 3$, so $25 \times 12 = 25 \times 4 \times 3 = 100 \times 3 = 300$. b. $14 = 2 \times 7$ and $15 = 3 \times 5$, so $14 \times 15 = (2 \times 5) \times (7 \times 3) = 10 \times 21 = 210$.

Q4. $5 \times 2 = 10$ and $3 \times 7 = 21$, so $15 \times 14 = 10 \times 21 = 210$.

Q5. $400 \times 7 = 2800$; $30 \times 7 = 210$; $6 \times 7 = 42$. $2800 + 210 + 42 = 3052$. So $436 \times 7 = 3052$.

Q6. $200 \times 3 = 600$; $60 \times 3 = 180$; $3 \times 3 = 9$. $600 + 180 + 9 = 789$. So $263 \times 3 = 789$.

Q7. a. $3 \times 4 = 12$ (write 2, carry 1); $7 \times 4 = 28 + 1 = 29$ (write 9, carry 2); $5 \times 4 = 20 + 2 = 22$. Answer 2292. b. $6 \times 5 = 30$ (write 0, carry 3); $1 \times 5 = 5 + 3 = 8$; $8 \times 5 = 40$. Answer 4080. c. $4 \times 6 = 24$ (write 4, carry 2); $9 \times 6 = 54 + 2 = 56$ (write 6, carry 5); $7 \times 6 = 42 + 5 = 47$. Answer 4764.

Q8. a. $243 \times 5 = 1215$; $243 \times 10 = 2430$; $1215 + 2430 = 3645$. b. $518 \times 4 = 2072$; $518 \times 20 = 10\ 360$; $2072 + 10\ 360 = 12\ 432$.

Q9. a. 25×32: factors and rearranging, because $32 = 4 \times 8$ and $25 \times 4 = 100$, so $25 \times 32 = 100 \times 8 = 800$. b. 253×4: doubling, because $4 = 2 \times 2$: $2 \times 253 = 506$ and $2 \times 506 = 1012$. c. 417×9: the algorithm (or partitioning: $400 \times 9 + 10 \times 9 + 7 \times 9 = 3600 + 90 + 63 = 3753$).

Q10. 35×26. Algorithm: $35 \times 6 = 210$; $35 \times 20 = 700$; $210 + 700 = 910$. Check: 35×26 is close to $35 \times 25 = 875$, so 910 makes sense. $\therefore$ 910 items are packed.

Q11. 148×7. Partitioning: $100 \times 7 = 700$; $40 \times 7 = 280$; $8 \times 7 = 56$; $700 + 280 + 56 = 1036$. $\therefore$ 1036 loaves.

Q12. $100\,000 \times 6 = 600\,000$. $\therefore$ 600 000 people in total.

Q13. 462 is more than 400, so 462×8 must be more than $400 \times 8 = 3200$. 2496 is less than 3200, so it cannot be right. (The exact answer is $462 \times 8 = 3696$.)

Q14. $385 \times 1 = 385$; $385 \times 20 = 7700$; $385 + 7700 = 8085$. Rounding: $385 \approx 400$ and $21 \approx 20$, so the answer should be close to $400 \times 20 = 8000$; 8085 is close, so it makes sense. A calculator gives 8085 too.

Q15. a. Each crossing stands for a pair of digits: the ones place comes from the ones digits, $2 \times 3 = 6$; the tens place from $1 \times 3 + 2 \times 1 = 5$; the hundreds place from $1 \times 1 = 1$. So 156. b. For 342×26 the picture has $3 + 4 + 2 = 9$ lines one way and $2 + 6 = 8$ the other, so $9 \times 8 = 72$ crossings, and the busiest group alone has $4 \times 6 + 2 \times 2 = 28$ crossings to count. Counting that many crossings is slow and easy to get wrong, so the algorithm is the better strategy.

Q16. a. Think $8 \times \square = 648$. Division undoes multiplication, so $648 \div 8 = 81$. b. $1000 \div 25 = 40$, or $25 \times 4 = 100$ so $25 \times 40 = 1000$. $\square = 40$.

Q17. One train: $56 \times 8 = 448$ seats. Three trains: $448 \times 3 = 1344$ seats. $\therefore$ 1344 seats.

Q18. 50×24: factors and rearranging — $50 \times 2 = 100$ and $24 = 2 \times 12$, so $50 \times 24 = 100 \times 12 = 1200$. Check: 50×24 is half of $100 \times 24 = 2400$, and half of 2400 is 1200. ✓

Division, and interpreting the remainder

Code `VC2M5N07` (Victorian Curriculum F–10 Version 2.0, Mathematics, Level 5; authority file `maths_5.json`): “solve problems involving division, choosing efficient mental and written strategies and using digital tools where appropriate; interpret any remainder according to the context and express results as a whole number, decimal or fraction”. This section draws on `VC2M5N07.E01`, `.E02` and `.E03`.

Achievement standard (Level 5, extract): “Students use their proficiency with multiplication facts and efficient mental and written calculation strategies to multiply large numbers by one- and two-digit numbers and divide by one-digit numbers.”

What you need to know

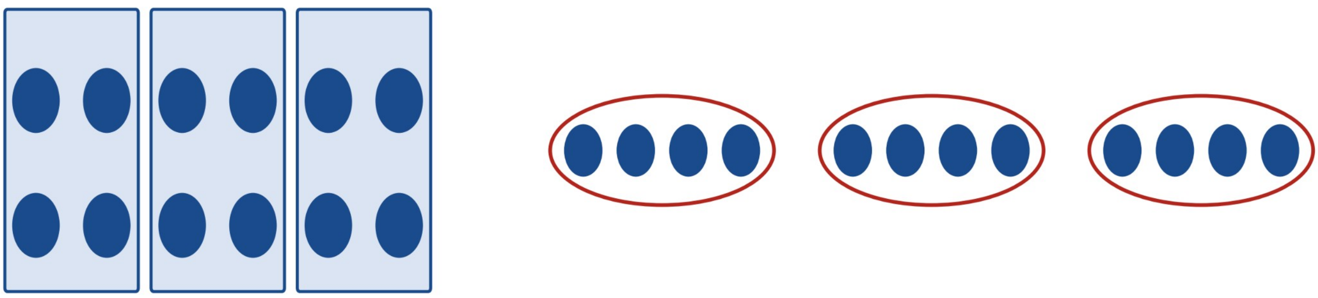
You already know your multiplication facts to 10 × 10 from earlier years, and in 1A you met factors and multiples. Both are tools you will use here. In 4A you will see that multiplication and division are **opposite** operations — each undoes the other — and this section leans on that connection from the very first fact.

Two ways to see division

Division shows up in life in two shapes, and you should be able to read every division both ways (`VC2M5N07.E02`).

- **Sharing:** share 12 into 3 equal groups → 4 in each group. That is 12 ÷ 3 = 4.
- **Grouping:** how many 4s make 12? → 3 groups of 4. That is 12 ÷ 4 = 3.

Sharing: 12 ÷ 3 = 4 Grouping: 12 ÷ 4 = 3



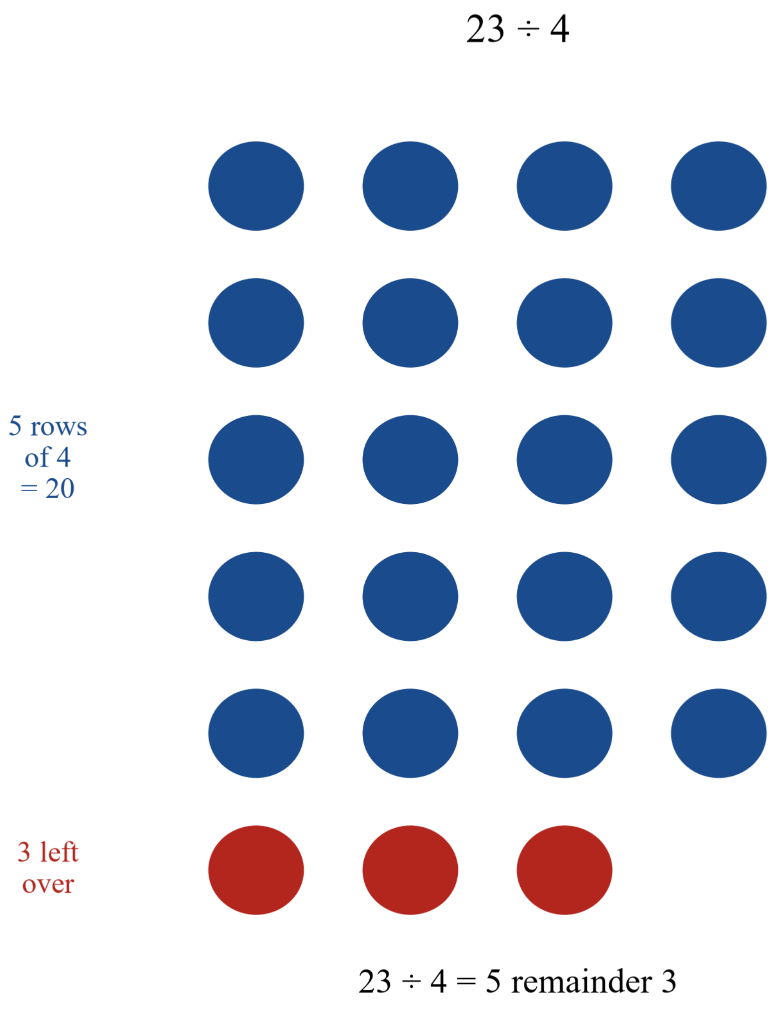
Share 12 into 3 equal groups: 4 in each group. How many 4s make 12? 3 groups of 4.

Two pictures of 12 counters. On the left, 12 counters shared into 3 boxes with 4 in each box. On the right, 12 counters in a row, ringed into 3 groups of 4.

The grouping shape is the faster one for mental work. To find 72 ÷ 9, think *how many nines make 72?* and the fact answers itself: □ × 9 = 72, and □ = 8 because 8 × 9 = 72. You can also read it the sharing way: *share 72 equally 9 ways* gives 8 each way. Both readings are the same fact family, seen from either end.

Remainders

Sharing does not always come out even. Take 23 counters and put them into rows of 4:



23 dots in rows of 4. Five full rows of 4 are blue, and 3 leftover dots are red.

Five full rows use 5 × 4 = 20 counters, and 3 are left over. The left-over amount is called the **remainder**. We write:

23 ÷ 4 = 5 remainder 3

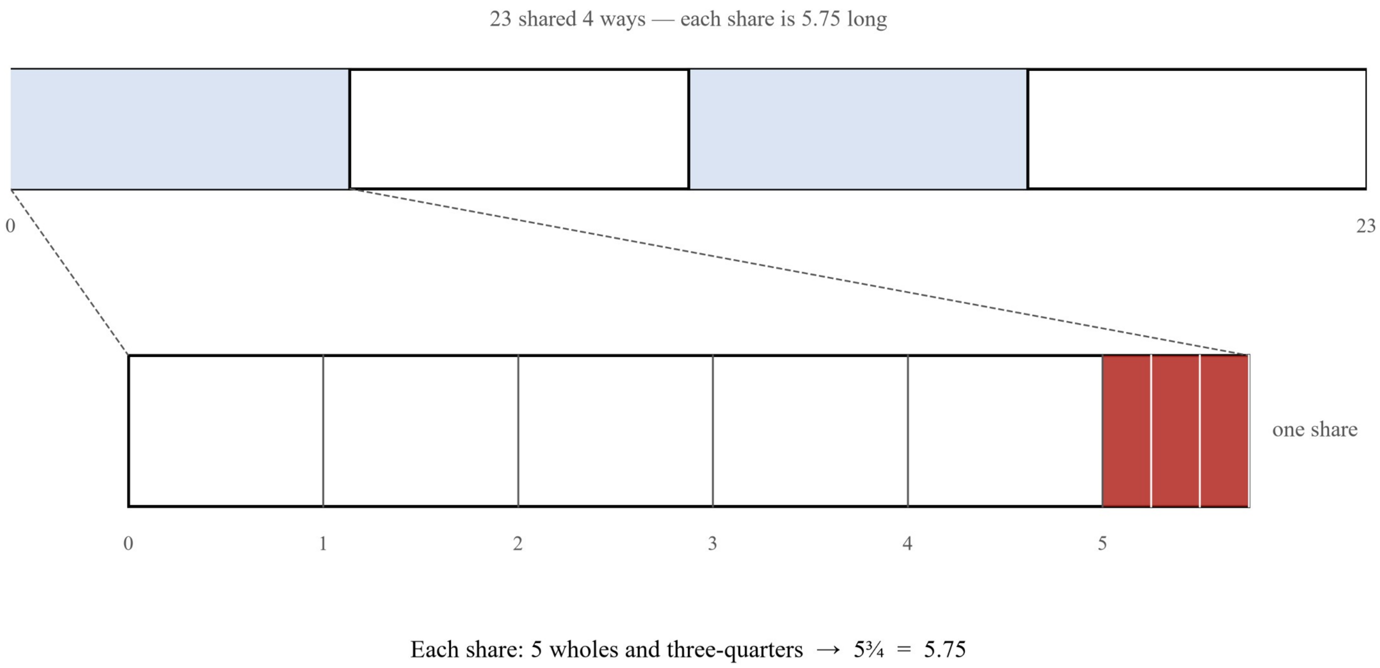
One rule always holds: **the remainder is less than the number we divide by.** If 4 or more were left over, we could build one more full row, so we had not finished dividing. When you check your work, check the remainder first.

The same answer three ways

You need to be able to write a division result three ways: as a **whole number with a remainder**, as a **fraction**, or as a **decimal**. They are the same answer, written for different jobs.

Take 23 ÷ 4 again, read the sharing way this time: share 23 into 4 equal shares. Each share gets 5 wholes, and the 3 leftovers are shared too — each share picks up three-quarters of the next whole. So each share is 5¾, and in decimals that is 5.75.

$$23 \div 4 = 5 \text{ remainder } 3 = 5\tfrac{3}{4} = 5.75$$



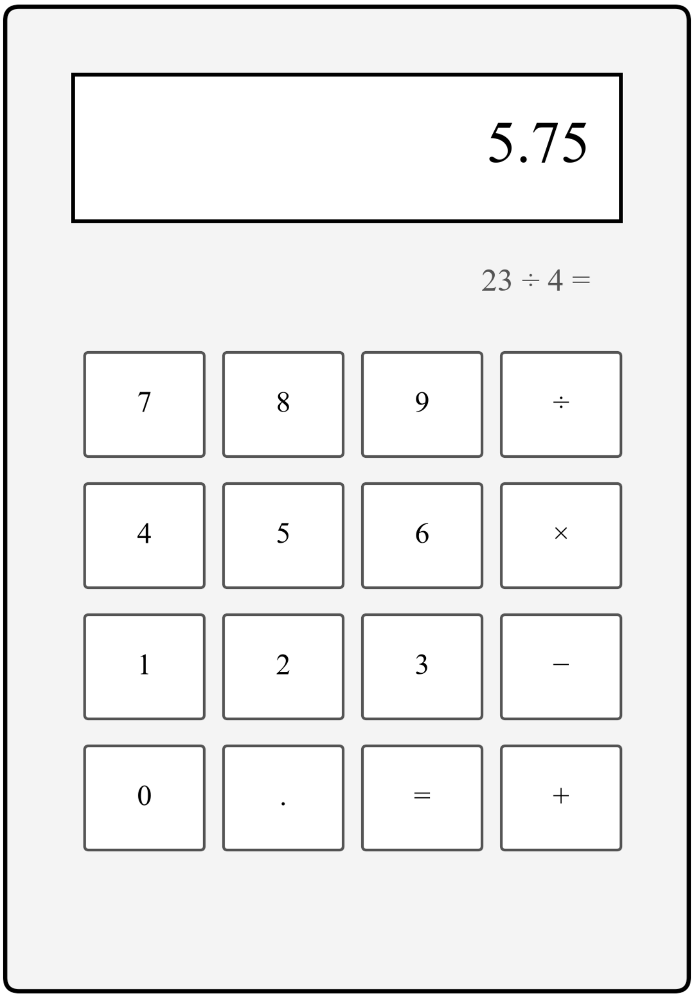
A bar of 23 cut into 4 equal shares. One share is shown large below: 5 whole parts and 3 quarters of the next part, so 23 divided by 4 is 5 remainder 3, which is 5 and three quarters, or 5.75.

So one division, three names:

Whole number and remainder	Fraction	Decimal
23 ÷ 4 = 5 remainder 3	5¾	5.75

The fraction and the decimal come from the same move: the remainder gets shared too. Three leftovers shared 4 ways is three-quarters each, and three-quarters is 0.75.

A calculator will hand you the decimal form straight away:



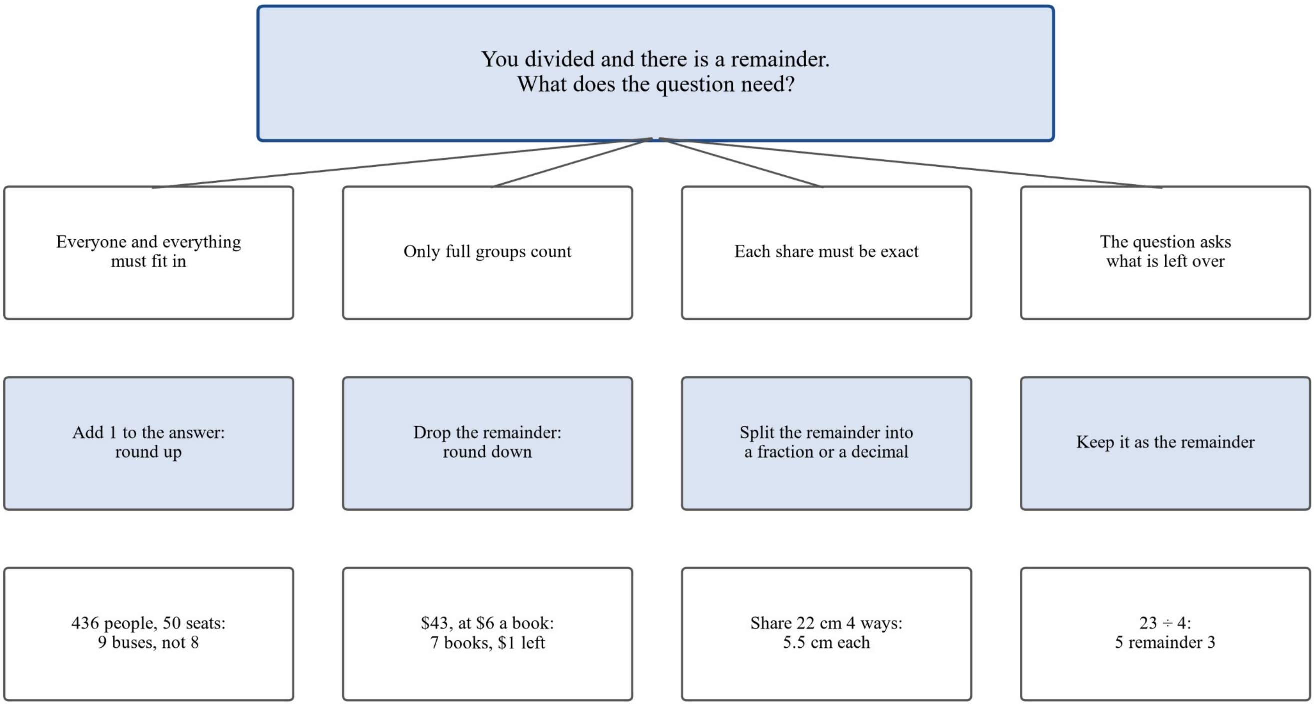
The calculator gives the decimal form.

A calculator. The screen shows 5.75 after pressing 23 divide by 4.

Use a calculator or a spreadsheet as a **check** on your written work, not as a replacement for it. Notice what the calculator does *not* do: it does not tell you what 5.75 means in the story the question came from. That is the next idea, and it is the heart of this section.

What does the question need?

In real problems, the remainder has to be interpreted — that is, you have to decide what to do with it — and the right thing to do depends on the question, not on the numbers. There are four usual decisions.



The same calculation gets different answers in real life, because the question is different.

A chart of 4 choices when there is a remainder: round up, round down, split the remainder, or keep the remainder, each with an example.

- **Round up** when everyone and everything must fit in. Buses, boats, tables, boxes: the leftovers still need a seat or a spot, so you add one more group.
- **Round down** when only full groups count. Money is the classic case: the leftover dollars do not buy another book, so they stay in your pocket.
- **Split the remainder** when each share must be exact. Ribbon, rope, cake, time: the leftover gets cut up and shared, and the answer becomes a fraction or a decimal.
- **Keep the remainder** when the question asks what is left over, or when groups and leftovers both matter: "5 rows of 4 and 3 left over" is itself the answer.

The same calculation can get different answers in real life, because the question is different. Deciding which one is right is called interpreting the remainder, and it is what this section is named for.

Short division by a one-digit number

When the number being divided is big, work place by place, left to right, writing each answer digit above the number. This is **short division**. Work through 857 ÷ 5:

Short division: 857 ÷ 5

	hundreds	tens	ones
	1	7	1
5	8	^{3}5	7

8 hundreds ÷ 5 = 1 hundred, with 3 hundreds left over;

carry the 3 hundreds across as 30 tens, making 35 tens

35 tens ÷ 5 = 7 tens

7 ones ÷ 5 = 1 one, with 2 ones left over

857 ÷ 5 = 171 remainder 2

Check: 171 × 5 = 855, and 855 + 2 = 857

The 2 leftover ones shared 5 ways make 0.4, so the same answer as a decimal is 171.4

The short division of 857 divided by 5 worked out place by place: 171 remainder 2, and as a decimal, 171.4.

Place	Think	Write
Hundreds	8 hundreds ÷ 5 = 1 hundred, remainder 3 hundreds	1 above the 8
Carry	the 3 hundreds become 30 tens; with the 5 that is 35 tens	small 3 by the 5
Tens	35 tens ÷ 5 = 7 tens	7 above the 5
Ones	7 ones ÷ 5 = 1 one, remainder 2	1 above the 7; the 2 is the remainder

So 857 ÷ 5 = 171 remainder 2. Check it with the opposite operation: 171 × 5 = 855, and 855 + 2 = 857. ✓ (That is the 3A habit: estimate first — 850 ÷ 5 = 170 — so an answer near 170 is sensible.)

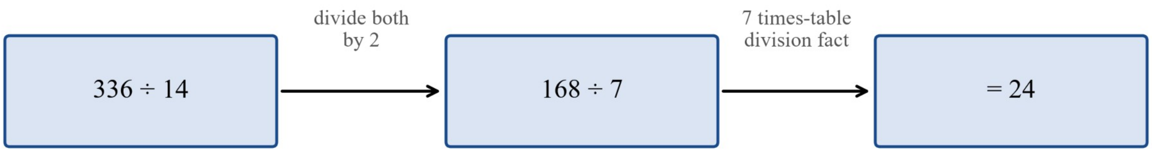
If the story needs an exact share, split the remainder: the 2 leftover ones shared 5 ways make two-fifths each, which is 0.4, so 857 ÷ 5 = 171.4.

Dividing by numbers like 14: equivalent division

Most division at this level is by a one-digit number. A number with two digits only becomes fair game when a strategy turns it into one you know — and the neatest strategy is **equivalent division** (`VC2M5N07.E03`):

If both numbers are divided by the same factor, the answer does not change.

Halving both numbers is the move you will use most. 336 ÷ 14 looks hard; halve both and it is 168 ÷ 7, which is a times-table fact: 168 ÷ 7 = 24. So 336 ÷ 14 = 24 as well.



Halving both numbers does not change the answer.

Check: 24 × 14 = 336

A chain: 336 divided by 14 becomes 168 divided by 7 when both are halved, giving 24.

Why does it work? Picture 336 counters sorted into piles of 14 — the number of piles is the answer we want. Now pair everything up: pair the counters, so 336 counters become 168 pairs, and pair the members of each pile, so each pile of 14 becomes a pile of 7 pairs. Nothing moved between piles, so the number of piles has not changed — it is still the answer — and now it is 168 ÷ 7. Shrinking both numbers by the same factor keeps the same piles. Here the tool is what makes 336 ÷ 14 a one-digit division; in 1A and 4A you will meet the same idea again as factors and opposite operations.

Worked examples

Example 1 — step by step: mental division, then the remainder decides

Sixty-seven students line up for tables. Each table seats 8. How many tables are needed?

Working	Comment
Think: how many 8s make 67? 8 × 8 = 64, and 64 + 8 = 72 is too many.	Use the grouping reading and the 8 times-table.
67 ÷ 8 = 8 remainder 3.	8 full tables seat 64; 3 students are left standing.
The 3 leftover students still need a table, so add one more: 8 + 1 = 9.	Everyone must fit in, so round up.

So **9 tables** are needed. Eight tables would leave 3 students with nowhere to sit.

Example 2 — step by step: short division with a remainder, three ways

Work out 857 ÷ 5 by short division, and give the answer as a whole number with a remainder and as a decimal.

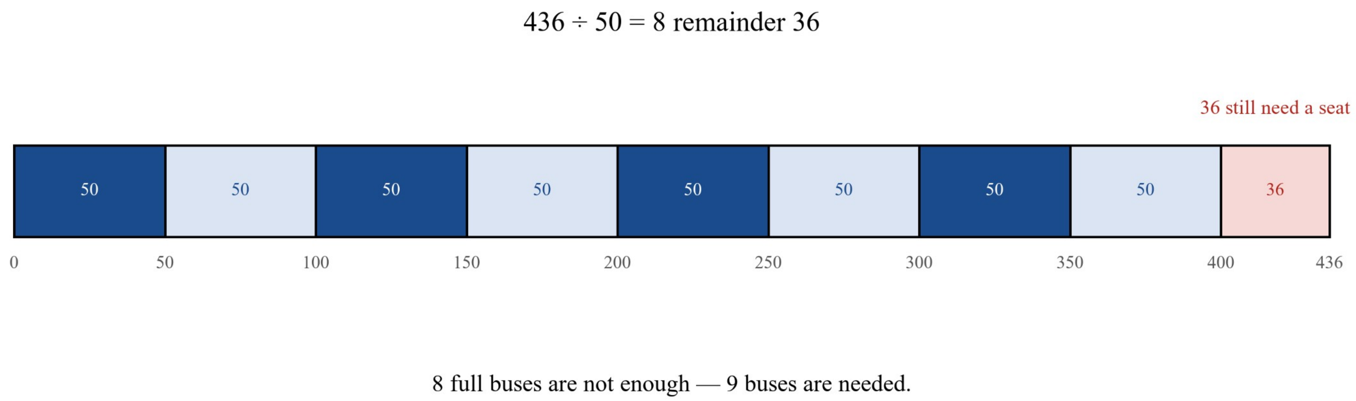
Working	Comment
8 hundreds ÷ 5 = 1 hundred, remainder 3 hundreds. Write 1.	Left to right, place by place.
Carry the 3 hundreds across as 30 tens: with the 5 tens that is 35 tens.	35 tens ÷ 5 = 7 tens. Write 7.
7 ones ÷ 5 = 1 one, remainder 2. Write 1; the remainder is 2.	The remainder 2 is less than 5. ✓
Whole-number form: 857 ÷ 5 = 171 remainder 2.	Check: 171 × 5 = 855, and 855 + 2 = 857.
Split the remainder for the decimal: 2 ones shared 5 ways is two-fifths = 0.4.	857 ÷ 5 = 171.4.

Example 3 — step by step: how many buses? (VCAA’s own problem)

`VC2M5N07.E01` asks: *how many buses are needed if there are 436 passengers and each bus carries 50 people?* Decide whether to round up or down, and justify the choice.

Working	Comment
Count in 50s: 50, 100, 150, 200, 250, 300, 350, 400 — that is 8 buses, carrying 400.	Counting multiples of 50 is an efficient mental strategy.
436 − 400 = 36 people still need a seat. So 436 ÷ 50 = 8 remainder 36.	The remainder 36 is less than 50. ✓
Those 36 people cannot stay home, so one more bus goes: 8 + 1 = 9.	Round **up**, and justify: 8 buses seat only 400 of the 436.

So **9 buses** are needed. The remainder is interpreted by the context: people are not leftover that can be dropped, so rounding down would strand 36 passengers.



A bar of 436 made of 8 blue parts of 50 and a red part of 36, showing 8 full buses and 36 people still needing a seat, so 9 buses.

Example 4 — on your own: one division, three names

A piece of ribbon 22 cm long is shared equally between 4 children. How long is each share? Give the answer as a whole number with a remainder, as a fraction, and as a decimal.

22 ÷ 4: the biggest multiple of 4 below 22 is 20, which is 5 × 4, so 22 ÷ 4 = 5 remainder 2. Each child gets 5 cm, and the 2 leftover centimetres are shared too: 2 cm shared 4 ways is half a centimetre each. So each share is 5½ cm, and as a decimal that is 5.5 cm.

All three names for the one answer: **5 remainder 2 = 5½ = 5.5**. Here the context splits the remainder, because ribbon can be cut exactly.

Example 5 — on your own: equivalent division makes 336 ÷ 14 a one-digit division

Work out 336 ÷ 14 using equivalent division.

Halve both numbers: 336 ÷ 2 = 168 and 14 ÷ 2 = 7, and dividing both by the same factor keeps the answer the same. So 336 ÷ 14 = 168 ÷ 7. Now we divide by a one-digit number: 168 ÷ 7 = 24, because 7 × 20 = 140 and 7 × 4 = 28, and 140 + 28 = 168.

So **336 ÷ 14 = 24**. Check with the opposite operation: 24 × 14 = 336. ✓

Practice questions

Questions with help

Q1. Fill the box: □ × 7 = 42. So 42 ÷ 7 = □.

Q2. Put 53 counters into rows of 6. How many full rows, and how many left over? Write 53 ÷ 6 with a remainder.

Q3. Use short division: 936 ÷ 3.

Q4. Work out 91 ÷ 4. Write the answer with a remainder, then as a decimal.

Q5. Equivalent division: 240 ÷ 12 = 120 ÷ 6. What is the answer? Which factor divided both numbers?

Q6. A farmer packs 82 apples into bags of 9. Every apple must be packed. How many bags are needed?

Q7. A ribbon 45 cm long is shared equally between 4 children. How long is each share, as a decimal?

Q8. Mental division: 240 ÷ 6. (Think: 24 ÷ 6, then count the tens.)

Questions on your own

Q9. 78 ÷ 9, with a remainder.

Q10. Short division: 652 ÷ 4.

Q11. Write 35 ÷ 4 three ways: whole number with remainder, fraction, decimal.

Q12. You have $43. Books cost $6 each. How many books can you buy, and what is left over?

Q13. Use equivalent division: 540 ÷ 18.

Q14. A sticker album holds 7 stickers to a page. You have 218 stickers. How many pages do you need so every sticker has a spot?

Q15. A rope 52 m long is cut into 8 equal pieces. How long is each piece, as a decimal?

Q16. Choose an efficient strategy for 400 ÷ 5 and explain your choice in one sentence.

Q17. Equivalent division, two halvings: 960 ÷ 24. Show each step.

Q18. A calculator shows 58 ÷ 4 = 14.5. In the story "58 biscuits are put on trays of 4", explain in words what the whole 14 and the 0.5 each mean.

Applied

Q19. An excursion takes 127 students. Each mini-bus carries 9 students. How many mini-buses are needed? Justify your rounding choice in one sentence.

Q20. You share 93 cm of ribbon equally between 6 gifts. How long is each piece? Give the answer as a fraction and as a decimal, and say which of the four remainder decisions you used.

Answers

Q1. 6. **Q2.** 8 remainder 5. **Q3.** 312. **Q4.** 22 remainder 3; 22.75. **Q5.** 20; the factor 2 (both numbers halved). **Q6.** 10 bags. **Q7.** 11.25 cm. **Q8.** 40. **Q9.** 8 remainder 6. **Q10.** 163. **Q11.** 8 remainder 3; $8\frac{3}{4}$; 8.75. **Q12.** 7 books, $1 left over. **Q13.** 30. **Q14.** 32 pages. **Q15.** 6.5 m. **Q16.** Any strategy that lands on 80 (for example 40 ÷ 5 = 8, so 400 ÷ 5 = 80). **Q17.** 960 ÷ 24 = 480 ÷ 12 = 240 ÷ 6 = 40. **Q18.** 14 full trays; the 0.5 is half a tray, the 2 leftover biscuits. **Q19.** 15 mini-buses. **Q20.** $15\frac{1}{2}$ cm = 15.5 cm; split the remainder.

Worked answers

Q1. 6 × 7 = 42, so the box is 6, and 42 ÷ 7 = 6.

Q2. 6 × 8 = 48 and 53 − 48 = 5, so 53 ÷ 6 = 8 remainder 5. The remainder 5 is less than 6. ✓

Q3. 9 hundreds ÷ 3 = 3 hundreds; 3 tens ÷ 3 = 1 ten; 6 ones ÷ 3 = 2 ones. So 936 ÷ 3 = 312. Check: 312 × 3 = 936. ✓

Q4. 4 × 22 = 88 and 91 − 88 = 3, so 91 ÷ 4 = 22 remainder 3. Split the remainder: 3 shared 4 ways is three-quarters = 0.75, so the decimal is 22.75.

Q5. 240 ÷ 12 = 20, and 120 ÷ 6 = 20 — the answer is 20. Both numbers were halved, so the factor is 2.

Q6. 9 × 9 = 81 and 82 − 81 = 1, so 82 ÷ 9 = 9 remainder 1. The last apple still needs a bag, so round up: 9 + 1 = 10 bags.

Q7. 45 ÷ 4 = 11 remainder 1, and 1 shared 4 ways is one-quarter = 0.25, so each share is 11.25 cm.

Q8. 24 ÷ 6 = 4, and 240 is 24 tens, so 240 ÷ 6 = 40.

Q9. 9 × 8 = 72 and 78 − 72 = 6, so 78 ÷ 9 = 8 remainder 6.

Q10. 6 hundreds ÷ 4 = 1 hundred, remainder 2 hundreds. Carry the 2 hundreds across as 20 tens; with the 5 tens that is 25 tens. 25 tens ÷ 4 = 6 tens, remainder 1 ten. Carry the 1 ten across as 10 ones; with the 2 ones that is 12 ones. 12 ÷ 4 = 3. So 652 ÷ 4 = 163. Check: 163 × 4 = 652. ✓

Q11. 4 × 8 = 32 and 35 − 32 = 3, so 35 ÷ 4 = 8 remainder 3. Split the remainder: 3 shared 4 ways is three-quarters, so the fraction is $8\frac{3}{4}$ and the decimal is 8.75.

Q12. 6 × 7 = 42 and 43 − 42 = 1, so $43 buys 7 books with $1 left over. Only full groups count — the last dollar buys no book — so round down.

Q13. Halve both: 540 ÷ 18 = 270 ÷ 9 = 30, because 9 × 30 = 270.

Q14. 7 × 31 = 217 and 218 − 217 = 1, so 218 ÷ 7 = 31 remainder 1. The last sticker still needs a page, so round up: 32 pages.

Q15. 52 ÷ 8 = 6 remainder 4, and 4 shared 8 ways is a half, so each piece is 6.5 m.

Q16. For example: 40 ÷ 5 = 8, and 400 is 40 tens, so 400 ÷ 5 = 80 — a mental strategy using place value, faster than writing it out.

Q17. Halve twice: 960 ÷ 24 = 480 ÷ 12 = 240 ÷ 6 = 40. Check: 40 × 24 = 960. ✓

Q18. 14 is the number of full trays (14 × 4 = 56 biscuits), and 0.5 is half a tray — the 2 leftover biscuits fill half of one more tray. The calculator's 14.5 trays is exact for the division, but in the story you would say 14 full trays and 2 biscuits left over.

Q19. 9 × 14 = 126 and 127 − 126 = 1, so 127 ÷ 9 = 14 remainder 1. Every student must go, so round up: 14 + 1 = 15 mini-buses.

Q20. 93 ÷ 6 = 15 remainder 3, and 3 shared 6 ways is a half, so each piece is $15\frac{1}{2}$ cm = 15.5 cm. The remainder was split, because ribbon can be cut exactly.

Algorithms with branching and repetition

> *follow a mathematical algorithm involving branching and repetition (iteration); create and use algorithms involving a sequence of steps and decisions and digital tools to experiment with factors, multiples and divisibility; identify, interpret and describe emerging patterns* — VC2M5N10
>
> *They design and use algorithms to identify and explain patterns in the factors and multiples of numbers.* — Level 5 achievement standard (extract)

An **algorithm** is a set of steps you follow in order to get a job done. In this section the steps **branch** (a yes/no question decides what happens next) and **repeat** (a step is done again and again). You will follow algorithms, create your own — on paper and in a spreadsheet — and look out for patterns in factors and multiples as they appear.

Theory

What is an algorithm?

An algorithm is a list of steps with an order. Follow the steps in that order and the job gets done the same way every time.

Order matters. Think about putting on socks and shoes:

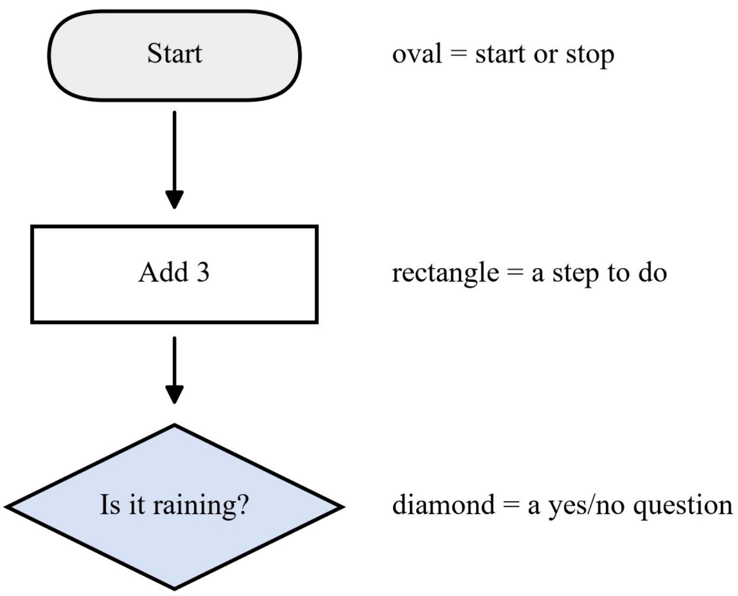
1. Put on your socks.
2. Put on your shoes.

Change the order and it does not work. An algorithm only works when its steps are followed in the right order.

Branching: steps that make a decision

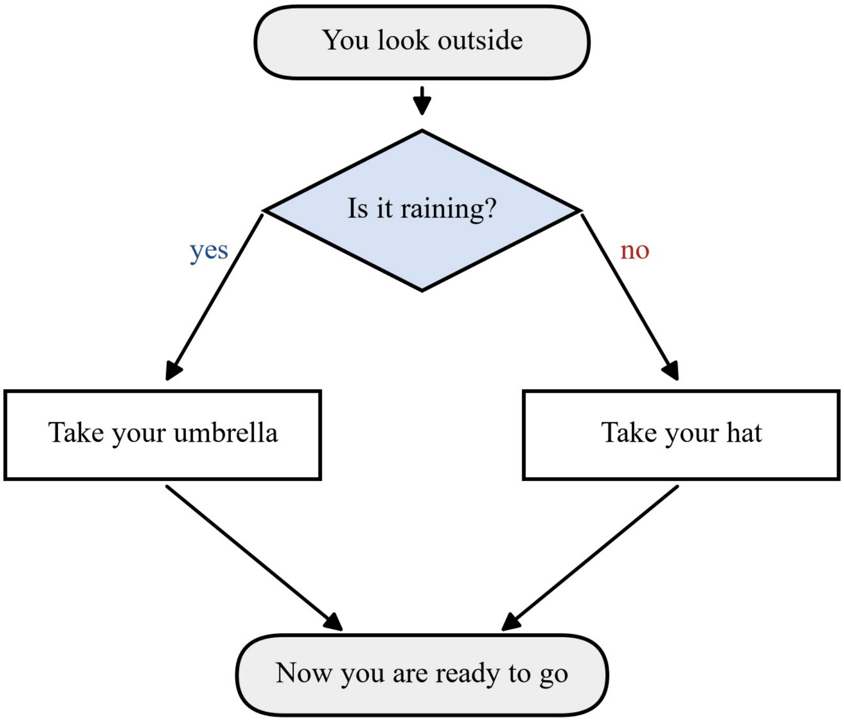
Some algorithms have a question in them. The answer to the question decides which step comes next. This is called **branching**.

A flowchart draws an algorithm with shapes. There are three shapes:



The three flowchart shapes: an oval for start or stop, a rectangle for a step to do, and a diamond for a yes/no question.

Here is an algorithm you might use every morning:



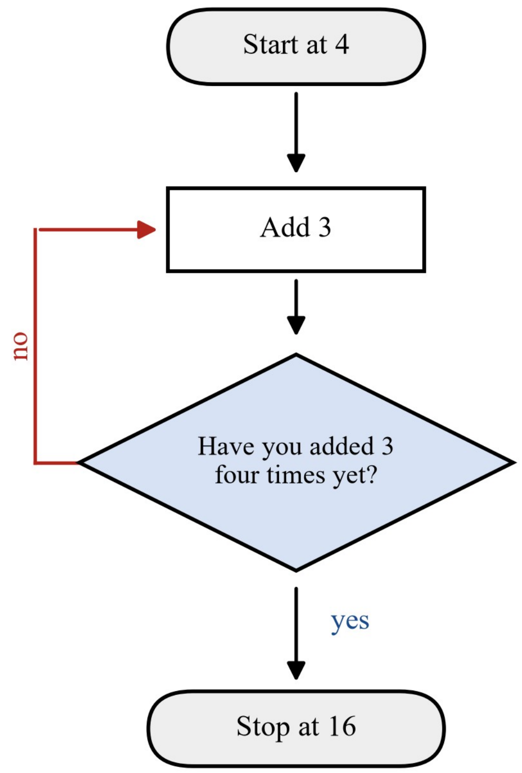
Flowchart: you look outside; is it raining? Yes, take your umbrella; no, take your hat; then you are ready to go.

The diamond is the branch. One question — *Is it raining?* — has two answers, and each answer leads to a different step.

Repetition: doing a step again and again

Algorithms can also **repeat** a step. Doing a step again and again is called **repetition**, and the steps that get repeated are called a **loop**.

This flowchart starts at 4 and adds 3 four times:



After the 1st time: $4 + 3 = 7$

After the 2nd time: $7 + 3 = 10$

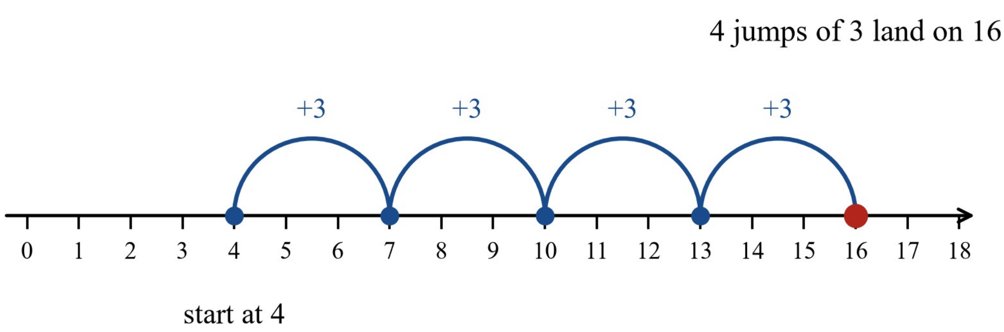
After the 3rd time: $10 + 3 = 13$

After the 4th time: $13 + 3 = 16$

Flowchart with a loop: start at 4; add 3; have you added 3 four times yet? No goes back to add 3; yes stops at 16. The record shows 7, 10, 13 and 16.

The red arrow is the loop. It sends the steps back to *Add 3* until adding 3 has been done four times.

The same repetition drawn on a number line:

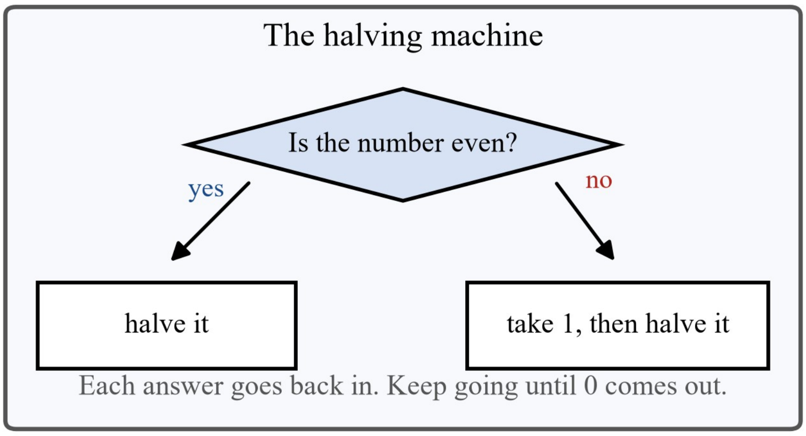


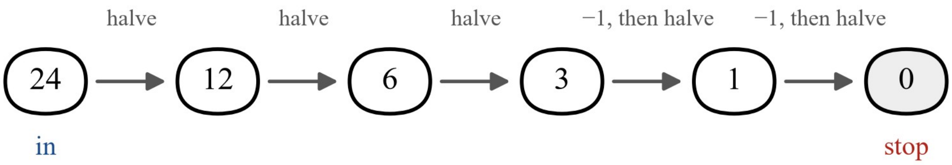
Number line from 0 to 18 with four jumps of 3 from 4, landing on 7, 10, 13 and 16.

Four jumps of 3 from 4 land on 16: 4, 7, 10, 13, 16.

The halving machine

Here is a machine with a branch in it. Put a number in. If the number is even, halve it. If the number is odd, take 1, then halve it. The answer goes back in, and the machine keeps going until 0 comes out.





The halving machine: is the number even? Yes, halve it; no, take 1 then halve it. A trace from 24 gives 24, 12, 6, 3, 1, 0.

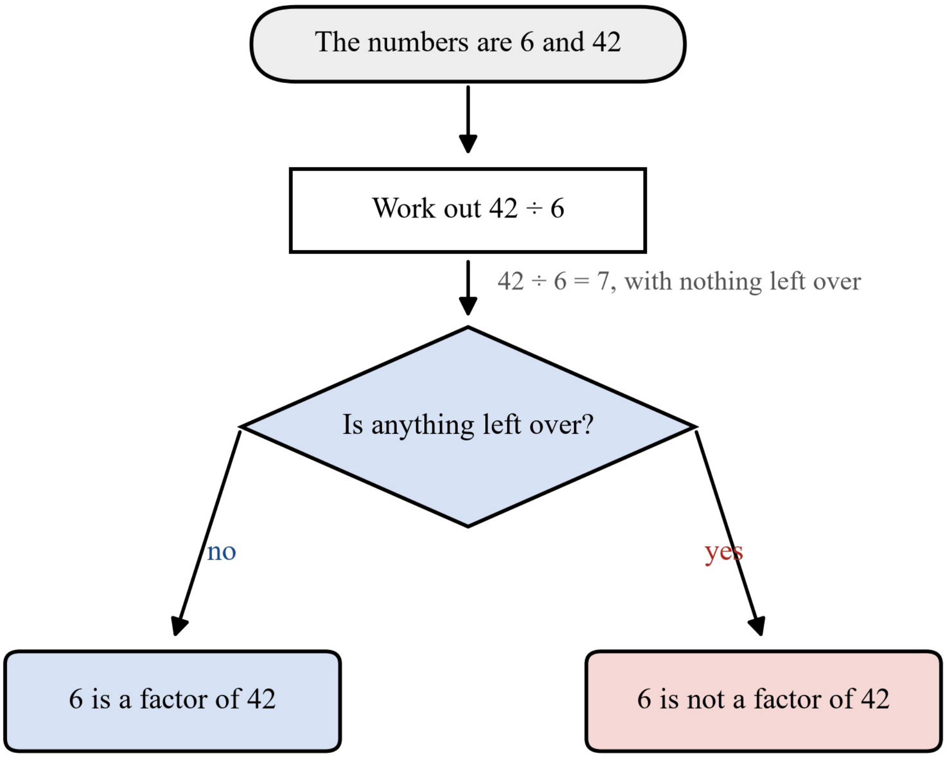
Trace from 24:

- 24 is even → halve → 12
- 12 is even → halve → 6
- 6 is even → halve → 3
- 3 is odd → take 1, then halve → 1
- 1 is odd → take 1, then halve → 0
- 0 comes out, so the machine stops

The branch (*Is the number even?*) is what makes this an algorithm with branching, and the "goes back in" part is the repetition.

A flowchart that tests factors

In 1A you met **factors** and **multiples**: 6 is a factor of 42 because 42 ÷ 6 = 7 with nothing left over, and 42 is a multiple of 6. A flowchart can test this with one branch: divide, then ask *Is anything left over?*



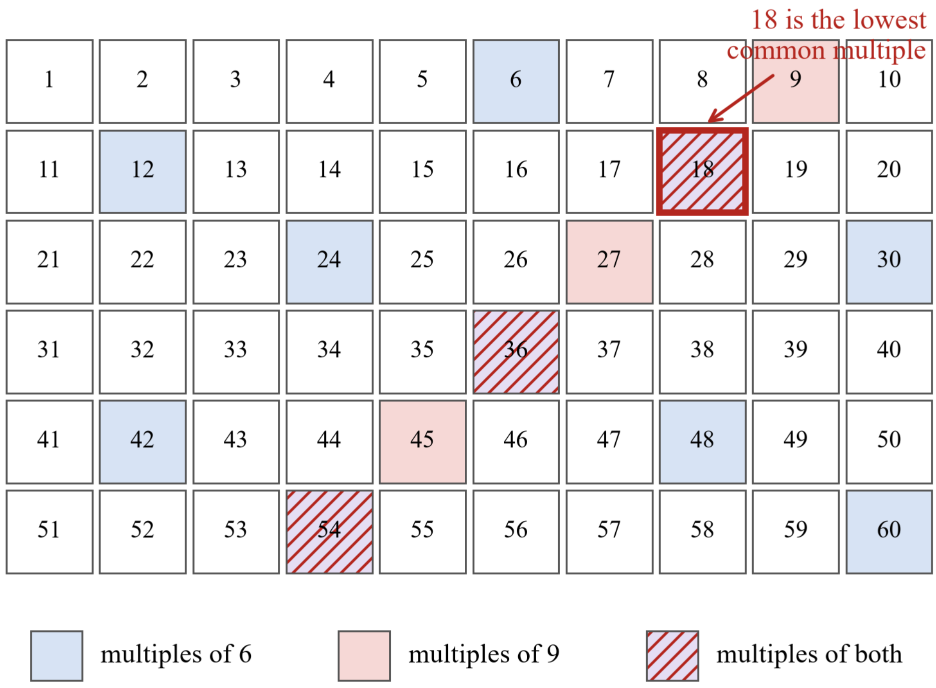
Flowchart: the numbers are 6 and 42; work out 42 ÷ 6 = 7, with nothing left over; is anything left over? No, 6 is a factor of 42; yes, 6 is not a factor of 42.

42 ÷ 6 = 7 with nothing left over, so the flowchart walks down the *no* branch and ends at *6 is a factor of 42*. The division facts you use here are the same fact families as 4A, and reading what is left over is the skill from 1C.

Lowest common multiple and highest common factor

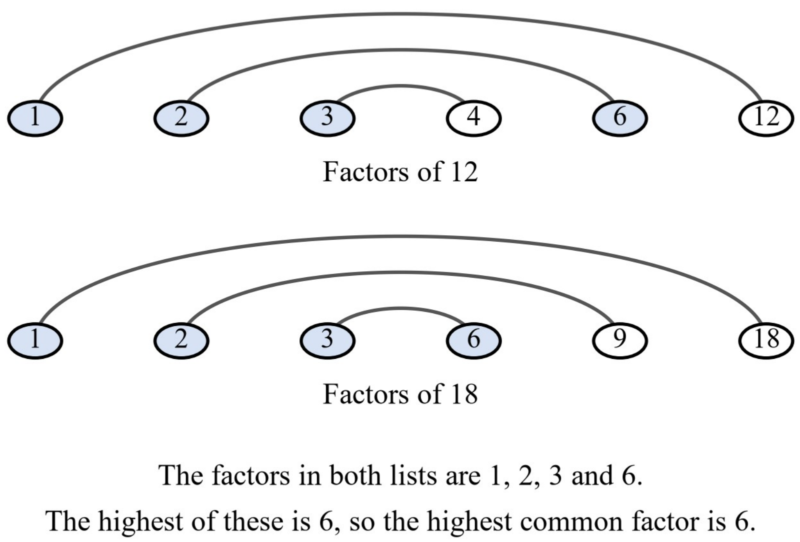
When two lists of multiples share a number, the lowest shared one is the **lowest common multiple (LCM)**. When two lists of factors share a number, the highest shared one is the **highest common factor (HCF)**.

Here are the multiples of 6 and 9 on a grid to 60. The shared cells are 18, 36 and 54, and the first of these, 18, is the LCM:



Grid of the numbers 1 to 60 with the multiples of 6 shaded blue, the multiples of 9 shaded red, and 18, 36 and 54 in both; 18 has a red ring around it as the lowest common multiple.

Here are factor rainbows for 12 and 18. The shared factors are 1, 2, 3 and 6, and the highest of these, 6, is the HCF:



Factor rainbows for 12 and 18; the factors in both lists, 1, 2, 3 and 6, are shaded blue, so the highest common factor is 6.

For three numbers the idea is the same. The multiples of 3, 4 and 5 first share 60, so the LCM is 60. The only factor all three share is 1, so the HCF is 1.

A spreadsheet builds the multiples

Using a digital tool is part of the job in this section, and a spreadsheet is a good one.

Put the starting number in cell C1 — say 4. Column A holds 1, 2, 3, … and the rule in B2 is `= A2 × $C$1`, which means “take the number in A2 and multiply it by whatever is in C1”. **Fill down** copies that rule down column B.

B2	= A2 × C1

	A	B	C
1	1	4	4
2	2	8	
3	3	12	
4	4	16	
5	5	20	
6	6	24	
7	7	28	
8	8	32	

the number you enter

Fill down copies the rule down the column.

A spreadsheet with 4 entered in C1 and the rule = A2 × C1 filled down column B, building 4, 8, 12 and so on to 32.

The dollar signs in `$C$1` keep the rule pointing at C1 as it copies down. Without them the rule would move down to C2, C3, … and the multiples would break.

Now look at column B: 4, 8, 12, 16, 20, 24, 28, 32. Patterns you can see:

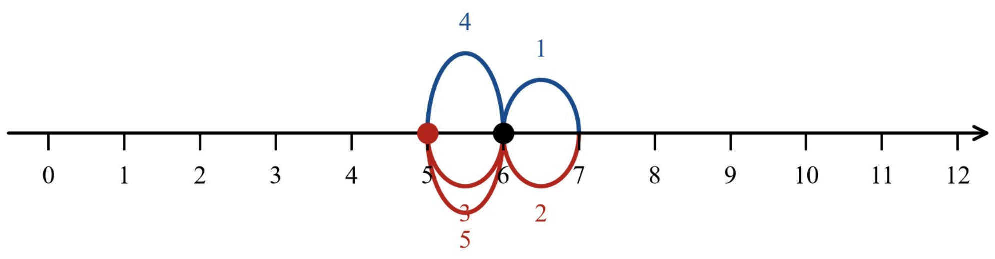
- each number is 4 more than the one above it;
- every number is a multiple of 4;
- the last digits repeat in the same order: 4, 8, 2, 6, 0, then 4, 8, 2, 6, 0, …

Change C1 to any number and the same kinds of patterns appear for that number. That is the experiment: change one cell and watch the pattern.

A random walk with a die

Here is an algorithm where a die makes the branch for you. Start at 6 on a number line. Roll the die: an **even** roll steps you 1 forward, an **odd** roll steps you 1 back.

Rolls: 2, 5, 5, 6, 1 — an even roll steps forward, an odd roll steps back.



The black dot is the start (6). The red dot is the finish.

Number line walk from 6 with rolls 2, 5, 5, 6, 1: forward to 7, back to 6, back to 5, forward to 6, back to 5; the black dot is the start and the red dot is the finish at 5.

Rolls 2, 5, 5, 6, 1 give: 6 → 7 → 6 → 5 → 6 → 5. The walk finishes at 5.

Your rolls will be different, so your walk will be different — but the rules stay the same every time. That is what an algorithm is: the steps stay the same, even when the numbers going in are not.

Where this fits: 1D comes straight after 1A, which teaches factors and multiples, and before 2B. The multiplication and division fact families it uses are the work of 4A, and finding an unknown value in a number sentence is 4B.

Worked examples

Example 1 — with a working table

Follow this algorithm: start at 7; add 5; repeat 4 times. Write down every number you get.

Working	Comment
Start: 7	Write the starting number.
7 + 5 = 12	First time through the loop.
12 + 5 = 17	Second time.
17 + 5 = 22	Third time.
22 + 5 = 27	Fourth time, then stop.

∴ the numbers are 7, 12, 17, 22 and 27.

Example 2 — with a working table

Put 20 into the halving machine. List every number that comes out until it stops.

Working	Comment
20 is even → 20 ÷ 2 = 10	Halve it.
10 is even → 10 ÷ 2 = 5	Halve it.
5 is odd → 5 − 1 = 4, 4 ÷ 2 = 2	Take 1, then halve.
2 is even → 2 ÷ 2 = 1	Halve it.
1 is odd → 1 − 1 = 0, 0 ÷ 2 = 0	Take 1, then halve. 0 comes out, so stop.

∴ the machine gives 10, 5, 2, 1 and 0.

Example 3 — with a working table

Run the factor flowchart with the numbers 7 and 56. Which end box do you reach?

Working	Comment
Work out 56 ÷ 7 = 8	The rectangle step.
Nothing is left over	The diamond question.
Walk down the *no* branch	*Is anything left over?* → no

∴ the flowchart ends at *7 is a factor of 56*.

Example 4 — with a working table

Find the lowest common multiple and the highest common factor of 6 and 9.

Working	Comment
Multiples of 6: 6, 12, 18, 24, …	Count on by 6.
Multiples of 9: 9, 18, 27, …	Count on by 9.
First shared multiple: 18	The LCM.
Factors of 6: 1, 2, 3, 6	Pairs 1 × 6 and 2 × 3.
Factors of 9: 1, 3, 9	Pairs 1 × 9 and 3 × 3.
Shared factors: 1 and 3; highest is 3	The HCF.

∴ the LCM of 6 and 9 is 18, and the HCF is 3.

Example 5 — in one piece

Find the LCM and the HCF of 12 and 18.

Multiples of 12: 12, 24, 36, 48, … Multiples of 18: 18, 36, 54, … The first shared multiple is 36.

Factors of 12: 1, 2, 3, 4, 6, 12. Factors of 18: 1, 2, 3, 6, 9, 18. The shared factors are 1, 2, 3 and 6; the highest is 6.

∴ the LCM is 36 and the HCF is 6.

Example 6 — in one piece

A spreadsheet has 6 in C1, and the rule = `A2 × $C$1` is filled down column B as far as B8. What numbers fill B1 to B8? What would B9 and B10 show?

B1 to B8 hold 6, 12, 18, 24, 30, 36, 42, 48 — the multiples of 6. Filling down two more rows, B9 shows 54 and B10 shows 60.

∴ column B counts on by 6: every cell is a multiple of 6, and the pattern keeps going as far as you fill.

Practice questions

Q1. Follow this algorithm: start at 10; subtract 2; repeat 3 times. Write down the numbers you get.

Q2. Follow this algorithm: start at 5; ×2; repeat 3 times. Write down the numbers you get.

Q3. Put 16 into the halving machine. List every number that comes out until it stops.

Q4. Put 15 into the halving machine. List every number that comes out until it stops.

Q5. Sort these numbers into even and odd: 8, 13, 22, 31, 46.

Q6. Run the factor flowchart with the numbers 4 and 30. Which end box do you reach? Show the division.

Q7. List the multiples of 3 and of 5 up to 30. (a) Which numbers are in both lists? (b) What is the LCM of 3 and 5?

Q8. List the factors of 8 and of 12. (a) Which factors are shared? (b) What is the HCF of 8 and 12?

Q9. Find the LCM and the HCF of 4 and 10.

Q10. Find the LCM and the HCF of 2, 3 and 5.

Q11. A spreadsheet has 8 in C1 and the rule `= A2 × $C$1` filled down column B. (a) What is in B5? (b) What is in B6? (c) Write one thing you notice about column B.

Q12. Create your own algorithm, with one diamond, that tests whether a number is a multiple of 5. Draw it as a flowchart and trace it once with a number of your choice.

Q13. Do the die walk from 6: an even roll steps 1 forward, an odd roll steps 1 back. Roll a die five times (or use the rolls 2, 5, 5, 6, 1) and write your position after each roll.

Q14. A spreadsheet starting at 5 builds 5, 10, 15, 20 down column B. (a) Say the rule in words. (b) What are the next two numbers? (c) What do you notice about the last digits?

Q15. Which numbers come out of the halving machine at 0 after exactly two steps? (There are two answers.)

Q16. One drum beats every 3 seconds and another beats every 4 seconds. They beat together at time 0. When is the first time after 0 that they beat together again?

Answers

Q1. 8, 6, 4. **Q2.** 10, 20, 40. **Q3.** 8, 4, 2, 1, 0. **Q4.** 7, 3, 1, 0. **Q5.** Even: 8, 22, 46; odd: 13, 31. **Q6.** *4 is not a factor of 30* — 30 ÷ 4 = 7 with 2 left over. **Q7.** (a) 15 and 30; (b) 15. **Q8.** (a) 1, 2, 4; (b) 4. **Q9.** LCM 20, HCF 2. **Q10.** LCM 30, HCF 1. **Q11.** (a) 40; (b) 48; (c) each number is 8 more than the one above it (any true pattern). **Q12.** Answers will be different — see the worked solution. **Q13.** Using 2, 5, 5, 6, 1: 7, 6, 5, 6, 5. **Q14.** (a) add 5; (b) 25 and 30; (c) the last digit is always 0 or 5. **Q15.** 2 and 3. **Q16.** 12 seconds.

Worked solutions

Q1. 10 − 2 = 8; 8 − 2 = 6; 6 − 2 = 4. ∴ the numbers are 8, 6 and 4.

Q2. 5 × 2 = 10; 10 × 2 = 20; 20 × 2 = 40. ∴ the numbers are 10, 20 and 40.

Q3. 16 is even → 8; 8 is even → 4; 4 is even → 2; 2 is even → 1; 1 is odd → 0. ∴ the machine gives 8, 4, 2, 1 and 0.

Q4. 15 is odd → 14 ÷ 2 = 7; 7 is odd → 6 ÷ 2 = 3; 3 is odd → 2 ÷ 2 = 1; 1 is odd → 0. ∴ the machine gives 7, 3, 1 and 0.

Q5. Even numbers end in 0, 2, 4, 6 or 8: so 8, 22 and 46 are even, and 13 and 31 are odd.

Q6. 30 ÷ 4 = 7 with 2 left over. Something is left over, so the *yes* branch ends at *4 is not a factor of 30*.

Q7. Multiples of 3 to 30: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. Multiples of 5 to 30: 5, 10, 15, 20, 25, 30. (a) Both lists hold 15 and 30. (b) The lowest of these is 15, so the LCM is 15.

Q8. Factors of 8: 1, 2, 4, 8. Factors of 12: 1, 2, 3, 4, 6, 12. (a) Shared: 1, 2 and 4. (b) The highest shared factor is 4, so the HCF is 4.

Q9. Multiples of 4: 4, 8, 12, 16, 20, … Multiples of 10: 10, 20, … First shared multiple: 20. Factors of 4: 1, 2, 4. Factors of 10: 1, 2, 5, 10. Shared: 1 and 2. ∴ the LCM is 20 and the HCF is 2.

Q10. Multiples of 2: 2, 4, 6, …, 30; multiples of 3: 3, 6, 9, …, 30; multiples of 5: 5, 10, 15, …, 30. The first number in all three lists is 30. The only factor all three share is 1. ∴ the LCM is 30 and the HCF is 1.

Q11. (a) B5 = 5 × 8 = 40. (b) B6 = 6 × 8 = 48. (c) Each number is 8 more than the one above it; every number is a multiple of 8; the last digits repeat 8, 6, 4, 2, 0.

Q12. For example: oval *a number*; rectangle *work out the number ÷ 5*; diamond *is anything left over?*; *no → it is a multiple of 5*; *yes → it is not a multiple of 5*. Trace with 35: 35 ÷ 5 = 7 with nothing left over, so 35 is a multiple of 5.

Q13. Roll 2 (even) → 7; roll 5 (odd) → 6; roll 5 (odd) → 5; roll 6 (even) → 6; roll 1 (odd) → 5. The positions are 7, 6, 5, 6, 5.

Q14. (a) Add 5 each row. (b) 25 and 30. (c) The last digit is always 0 or 5, in the same order again and again.

Q15. To stop after exactly two steps, the second step must be 1 → 0. So the first step must give 1: 2 halves to 1, and 3 gives 1 after taking 1 and halving. ∴ the two numbers are 2 and 3.

Q16. The first drum beats at 3, 6, 9, 12, … seconds; the second at 4, 8, 12, … seconds. The first shared time is 12 — the LCM of 3 and 4. ∴ they next beat together at 12 seconds.

Test

Mathematics Level 5 · Chapter 1 — Multiplication, division and factors · Chapter test T1

Marks	20 (5 marks for each subtopic)
Guidance time	25 minutes: 5 minutes reading time, then about 1 minute per mark
Equipment	None — the 1D items are pencil-and-paper traces of an algorithm

Instructions

- Answer every question, and show your working clearly.
- The marks for each question or part are shown in brackets.

Subtopic	Content description	Marks
1A Factors, multiples and divisibility	`VC2M5N02`	5
1B Multiplying larger numbers by one- and two-digit numbers	`VC2M5N06`	5
1C Division, and interpreting the remainder	`VC2M5N07`	5
1D Algorithms with branching and repetition	`VC2M5N10`	5
Total		**20**

1A · Factors, multiples and divisibility

Q1. The number 28 can be written as a product of two factors in three different ways.

a. Write 28 as a product of two factors in every way. (1 mark)
b. List all the factors of 28. (1 mark)

Q2. Without dividing, decide whether 7362 is divisible by 2, by 3, by 5 and by 9. Show the test you use for each answer. (2 marks)

Q3. The number 2□5 (two, a missing digit, five) is divisible by 9. Find the digit that goes in the box. (1 mark)

1B · Multiplying larger numbers by one- and two-digit numbers

Q4. Find 25 × 24. Choose an efficient strategy, and say in one sentence why it fits these numbers. (2 marks)

Q5. Use the standard written algorithm to find 315 × 23. (2 marks)

Q6. Mia writes 386 × 7 = 2402. Without finding the exact answer, explain how you can tell her answer cannot be right. (1 mark)

1C · Division, and interpreting the remainder

Q7. Work out 79 ÷ 4, and write the answer three ways.

a. as a whole number with a remainder (1 mark)
b. as a fraction, and as a decimal (1 mark)

Q8. 148 students go to camp. Each tent holds 12 students. 148 ÷ 12 = 12 remainder 4. What is the right thing to do with the remainder? (1 mark)

A. Round up: 13 tents are needed. B. Round down: 12 tents are needed. C. Split the remainder: $12\frac{1}{3}$ tents are needed. D. Keep the remainder: 12 tents, with 4 students left over, is the answer.

Q9. Use equivalent division to work out 252 ÷ 12. Show each step. (2 marks)

1D · Algorithms with branching and repetition

Q10. Here is the halving machine from 1D: if the number is even, halve it; if the number is odd, take 1, then halve it. Put the answer back in, and keep going until 0 comes out. Put 27 into the machine, and list every number that comes out until it stops. (2 marks)

Q11. Find the lowest common multiple (LCM) and the highest common factor (HCF) of 6 and 15. Show your lists of multiples and of factors. (2 marks)

Q12. A spreadsheet builds the multiples of 9 in column B: 9, 18, 27, 36, 45, … Write down one pattern you can see in column B. (1 mark)

Solutions

Answer key

Q	Answer	Marks
1	a. 28 = 1 × 28 = 2 × 14 = 4 × 7 · b. 1, 2, 4, 7, 14, 28	2
2	Divisible by 2 and by 3 and by 9; not divisible by 5	2
3	2 (225 = 9 × 25)	1
4	600, with an efficient strategy and a reason that fits	2
5	7245	2
6	386 > 350 and 350 × 7 = 2450, so the answer must be more than 2450; 2402 is less	1
7	a. 19 remainder 3 · b. 19¾ and 19.75	2
8	A	1
9	252 ÷ 12 = 126 ÷ 6 = 21	2
10	13, 6, 3, 1, 0	2
11	LCM 30, HCF 3	2
12	Any true pattern, for example each number is 9 more than the one above it	1
	Total	**20**

Fully worked solutions

Q1. Work up from 1, writing each factor pair. 1 × 28 = 28; 2 × 14 = 28 (28 is even); 3 does not work, because 3 × 9 = 27 and 3 × 10 = 30 sit either side of 28; 4 × 7 = 28; 5 does not work (28 does not end in 0 or 5); the next pair would start at 6, and 6 × 4 repeats a pair, so stop. ∴ a. 28 = 1 × 28 = 2 × 14 = 4 × 7. b. The factors of 28 are 1, 2, 4, 7, 14 and 28.

Q2. By 2: the last digit is 2, so 7362 is even — divisible by 2. By 3: the digit sum is 7 + 3 + 6 + 2 = 18, and 18 is divisible by 3 — divisible by 3. By 5: the last digit is not 0 or 5 — not divisible by 5. By 9: the digit sum is 18, and 18 is divisible by 9 — divisible by 9. ∴ 7362 is divisible by 2, by 3 and by 9, and not by 5. (It is a multiple of 3 and of 9: 7362 = 3 × 2454 = 9 × 818.)

Marks: 1 mark for the four correct judgements; 1 mark for the tests shown (last digit for 2 and 5, digit sum 18 for 3 and 9).

Q3. Use the digit-sum test by 9: 2 + 5 = 7, so the missing digit must add with 7 to make a multiple of 9. 7 + 2 = 9, and the next multiple, 18, would need the digit 11, which is not a digit. ∴ The box holds 2. Check: 225 ÷ 9 = 25 exactly.

Q4. 24 = 4 × 6, and 25 × 4 = 100 is a friendly pair, so use factors and rearranging: 25 × 24 = 25 × 4 × 6 = 100 × 6 = 600. ∴ 25 × 24 = 600. The reason fits because 24 breaks into factors, and one of them pairs with 25 to make 100.

Marks: 1 mark for 600 found by an efficient strategy; 1 mark for a reason that fits the numbers. Other sound choices are accepted, for example 25 × 8 = 200 and 200 × 3 = 600, since 24 = 8 × 3.

Q5. Multiply by the ones, then by the tens, then add the rows:

Working	Comment
315 × 3 = 945	Ones row: 5 × 3 = 15, write 5 carry 1; 1 × 3 = 3, add the

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	1 carried = 4; 3 × 3 = 9.
315 × 20 = 6300	Tens row: the 2 sits in the tens place, so it stands for 20.
945 + 6300 = 7245	Add the rows.

Check: 315 is close to 300 and 23 is close to 20, so the answer should be a little more than 300 × 20 = 6000; 7245 is sensible. ✓ ∴ 315 × 23 = 7245.

Marks: 1 mark for the two correct rows; 1 mark for the correct total.

Q6. 386 is more than 350, and 350 × 7 = 2450, so 386 × 7 must be more than 2450. But 2402 is less than 2450, so Mia's answer cannot be right. (Estimating also works: 386 ≈ 400, and 400 × 7 = 2800, so an answer near 2800 is expected.) ∴ 2402 cannot be right; the exact answer is 2702.

Q7. The biggest multiple of 4 below 79 is 76, because 4 × 19 = 76, and 79 − 76 = 3. So 79 ÷ 4 = 19 remainder 3. For the exact share, split the remainder: 3 shared 4 ways is three-quarters, and three-quarters is 0.75. ∴ a. 19 remainder 3. b. As a fraction, 19¾; as a decimal, 19.75.

Marks: 1 mark for part a; 1 mark for both forms in part b.

Q8. A. Every student needs a tent, and the 4 leftover students still need one, so round up: 12 + 1 = 13 tents. Rounding down would leave 4 students out in the weather, splitting the remainder would mean a third of a tent, and keeping the remainder does not fit everyone in. ∴ A.

Q9. Halve both numbers: 252 ÷ 2 = 126 and 12 ÷ 2 = 6, and dividing both numbers by the same factor keeps the answer the same. So 252 ÷ 12 = 126 ÷ 6, which is a one-digit division: 126 ÷ 6 = 21, because 6 × 20 = 120 and 6 × 1 = 6, and 120 + 6 = 126. ∴ 252 ÷ 12 = 21. Check with the opposite operation: 21 × 12 = 252. ✓

Marks: 1 mark for halving both numbers correctly; 1 mark for 21.

Q10. Follow the branch at every step:

Working	Comment
27 is odd → 27 − 1 = 26, 26 ÷ 2 = 13	Take 1, then halve.
13 is odd → 13 − 1 = 12, 12 ÷ 2 = 6	Take 1, then halve.
6 is even → 6 ÷ 2 = 3	Halve it.
3 is odd → 3 − 1 = 2, 2 ÷ 2 = 1	Take 1, then halve.
1 is odd → 1 − 1 = 0, 0 ÷ 2 = 0	Take 1, then halve. 0 comes out, so stop.

∴ The machine gives 13, 6, 3, 1 and 0.

Marks: 1 mark for the first two outputs (13 and 6); 1 mark for the full list through to 0.

Q11. Multiples of 6: 6, 12, 18, 24, 30, … Multiples of 15: 15, 30, … The first number in both lists is 30. Factors of 6: 1, 2, 3, 6. Factors of 15: 1, 3, 5, 15. The shared factors are 1 and 3, and the highest of these is 3. ∴ The LCM of 6 and 15 is 30, and the HCF is 3.

Marks: 1 mark for the LCM; 1 mark for the HCF.

Q12. Any true pattern earns the mark. For example: each number is 9 more than the one above it; the ones digit goes down by 1 each row (9, 8, 7, 6, 5, …); the digits of each number add to 9 (9, then 1 + 8, 2 + 7, 3 + 6, 4 + 5). ∴ One clear pattern, such as "each number is 9 more than the one above it".