Mathematics Level 10

Chapter 5 — Space · 2 subtopics · 2 CDs

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Deductive reasoning and proof with shapes in the plane

Mathematics 10 · Chapter 5 Space · Subtopic 5A (Level 10) · Extended block

Strand	Content description	Elaboration ids carried	Weight
Space	VC2M10SP01	E01, E02, E03, E04, E05	E

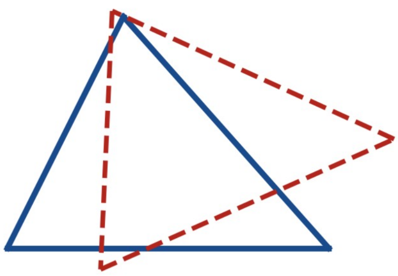
> **VC2M10SP01 (Space, Level 10)** — apply deductive reasoning to formulate proofs involving shapes in the plane and use theorems to solve spatial problems
>
> **Achievement standard extract (Level 10):** Students use deductive reasoning, theorems and algorithms to solve spatial problems.

You need first: the congruence tests for triangles (SSS, SAS, ASA, RHS) and the idea of similar shapes from Level 8 (VC2M8SP01, VC2M8SP02); angle facts for parallel lines and Pythagoras' theorem. Level 9 work on enlargement and similarity (VC2M9SP02) and on spatial problems (VC2M9M03) is rehearsed here, not re-taught. A *theorem* is a statement that has been proved; once proved it may be quoted as a reason in every later proof.

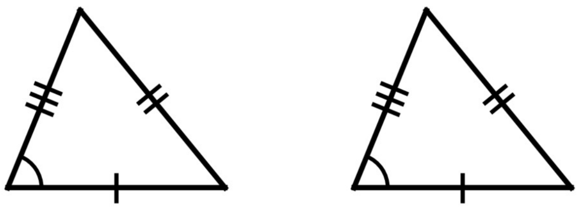
Theory

1. A demonstration is not a proof

Cut two triangles from card so that their three sides match, three sides to three sides. Place one on top of the other and they fit exactly.



A practical demonstration: one triangle placed on top of the other. They match in this drawing.



A proof: the marked parts are given equal, so the triangles must match in every case.

Left panel: two congruent triangles drawn on top of each other, one rotated, matching only in this drawing. Right panel: two separate triangles with matching tick marks on corresponding sides and matching angle arcs.

This is a **practical demonstration**. It checks *these two* triangles, on *your* desk, on *this* day. It says nothing about every other pair of triangles with those sides — including ones nobody has ever drawn. Measuring many drawn triangles with a protractor is the same kind of thing: more cases, still not all cases.

A **proof** is a chain of statements that covers every case at once. Each line of the chain is:

- a **given** (the setup we are told),
- a **definition** (what a word means), or
- a step licensed by a **theorem** that was already proved.

The congruence tests are what replace the scissors: if SSS holds, the triangles *must* fit, so the matching is forced, not observed. That move — from "it looks right here" to "it must be right everywhere" — is the whole of this subtopic.

2. The shape of a proof

Every proof in this section is set out the same way:

- **Given** — the facts we may start from.
- **To show** — the statement the chain must reach.
- **Proof** — one statement per line, each with its reason in brackets.

The reason must name the theorem (or definition, or given) that licenses the step. "Looks equal" and "by symmetry" are not reasons. When you are marked on a proof, the chain is what scores: what is given, what is to be shown, and which theorem licenses each step — not the final answer on its own.

3. Angle facts we may quote

The angle facts below are taken as licensed theorems in this section (they were proved in earlier years). Each is named so that you can quote it as a reason.

- **(A1) Straight line:** angles on a straight line add to 180°.
- **(A2) Point:** angles at a point add to 360°.
- **(A3) Vertically opposite angles** are equal.
- **(A4) Triangle angle sum:** the angles of a triangle add to 180°.
- **(A5) Exterior angle:** an exterior angle of a triangle equals the sum of the two interior angles opposite to it.
- **(A6) Alternate angles:** if two lines are parallel, then alternate angles (the Z-shape) cut by a crossing line are equal; and conversely, if the alternate angles are equal, the lines are parallel.
- **(A7) Corresponding angles:** if two lines are parallel, then corresponding angles (the F-shape) are equal; and conversely, if the corresponding angles are equal, the lines are parallel.
- **(A8) Co-interior angles:** if two lines are parallel, then co-interior angles (the C-shape) add to 180°; and conversely, if they add to 180°, the lines are parallel.

The "conversely" halves matter: (A6)–(A8) are how a proof *shows* two lines are parallel.

4. Congruent triangles (revision)

Two triangles are **congruent** if one fits exactly on the other — same sides, same angles. It is enough to check four of the six matched parts, in the right pattern:

- **SSS** — three sides of one equal three sides of the other.
- **SAS** — two sides and the included angle.
- **ASA** — two angles and the side between them.
- **RHS** — right angle, hypotenuse and one other side.

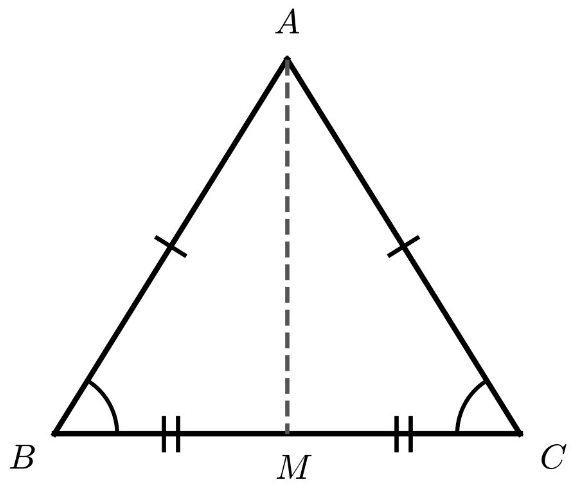
From congruence we may then read off that **all** matched sides and matched angles are equal — that is usually the point of proving the congruence.

Warning: **AAA is not a congruence test.** Three equal angles force the same shape but not the same size (one triangle can be an enlarged copy of the other). AAA belongs to similarity, not congruence.

5. The base angles of an isosceles triangle

A first theorem of our own, proved the way this section will keep proving things: state it, then build the chain.

Theorem 1 (base angles of an isosceles triangle). *If two sides of a triangle are equal, the angles opposite those sides are equal.*



Isosceles triangle ABC with apex A, M the midpoint of base BC; AM dashed, single ticks on the equal sides AB and AC, double ticks on BM and MC, equal angle arcs at B and C.

Given: △ABC with AB = AC; M the midpoint of BC. **To show:** ∠ABC = ∠ACB.

#	Statement	Reason
1	AB = AC	given
2	BM = CM	M is the midpoint of BC
3	AM = AM	common side
4	∴ △ABM ≅ △ACM	SSS (1, 2, 3)
5	∴ ∠ABM = ∠ACM	matched angles of congruent triangles

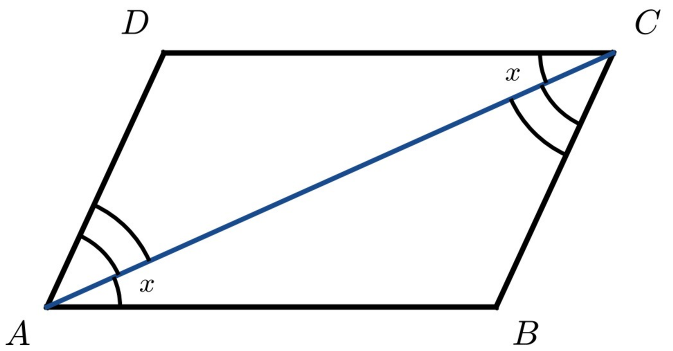
and ∠ABM is ∠ABC, ∠ACM is ∠ACB. ■

The **converse** of a theorem is the statement read backwards: here, *if two angles of a triangle are equal, the sides opposite them are equal*. (Its proof is the same triangle read the other way: in △ABC and △ACB, ∠ABC = ∠ACB is given, BC = CB is common, and ∠ACB = ∠ABC is given, so △ABC ≅ △ACB by ASA.) A converse is a new statement and needs its own proof — it does not come free with the original.

6. Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. A **rectangle** is a parallelogram with a right angle; a **rhombus** is a parallelogram with two adjacent sides equal.

Theorem 2. *In a parallelogram, opposite sides are equal and opposite angles are equal.*



Parallelogram ABCD with diagonal AC drawn; single arcs mark the alternate angles BAC and ACD, double arcs mark the alternate angles BCA and CAD.

Given: parallelogram ABCD (AB ∥ DC, AD ∥ BC); diagonal AC. **To show:** AB = CD, BC = DA, ∠BAD = ∠BCD, ∠B = ∠D.

#	Statement	Reason
1	∠BAC = ∠DCA	alternate angles, AB ∥ DC (A6)
2	∠BCA = ∠DAC	alternate angles, AD ∥ BC (A6)
3	AC = CA	common side
4	∴ △ABC ≅ △CDA	ASA (1, 3, 2)
5	∴ AB = CD, BC = DA, ∠B = ∠D	matched parts of congruent triangles
6	∠BAD = ∠BAC + ∠CAD = ∠DCA + ∠ACB = ∠BCD	1 and 2

■

Two more theorems about diagonals. The first is stated here with its proof set as a question; the second is proved in Example 6.

Theorem 3. *The diagonals of a parallelogram bisect each other.* (To **bisect** means to cut into two equal parts. Proof: Q11.)

Theorem 4 (diagonal test). *If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.* (Proof: Example 6.)

Theorem 5 (side test). *If one pair of opposite sides of a quadrilateral is equal and parallel, the quadrilateral is a parallelogram.* (Proof: Q9.)

7. Similar triangles (revision)

Two triangles are **similar** if one is an enlargement of the other. The tests (Level 8, via the enlargement transformation of Level 9):

- **AA** — two angles of one equal two angles of the other.
- **SSS similarity** — the three sides of one are in the same ratio as the three sides of the other.
- **SAS similarity** — two sides in the same ratio, with the included angle equal.

From similarity, matched sides are in the same ratio — that ratio is the scale factor of the enlargement.

8. The midpoint theorem

Theorem 6 (midpoint theorem). *The segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.*

Read the proof against the figure in Example 4: M and N are the midpoints of sides AB and AC of △ABC.

#	Statement	Reason
1	AM/AB = 1/2 = AN/AC	M, N are midpoints
2	∠MAN = ∠BAC	common angle
3	∴ △AMN ~ △ABC	SAS similarity, ratio 1 : 2 (1, 2)
4	∴ MN = ½BC	matched sides in ratio 1 : 2
5	∴ ∠AMN = ∠ABC	matched angles of similar triangles
6	∴ MN ∥ BC	converse of corresponding angles (A7), 5

■

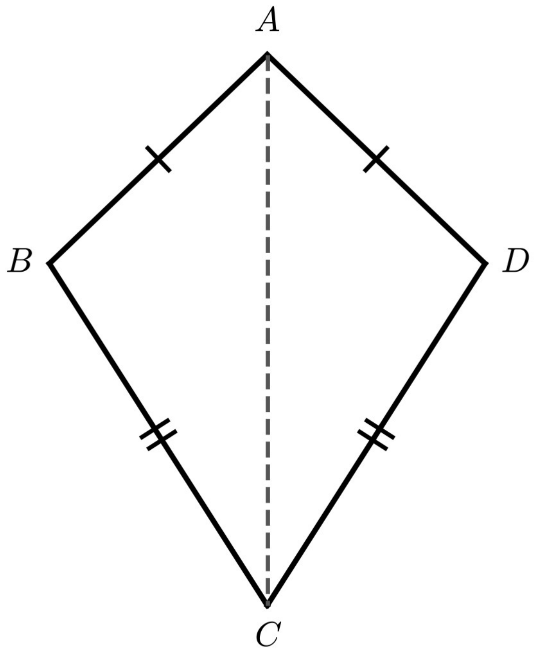
9. Proofs you can see

Some proofs are pictures. The tiling of Q22 (figure below in the practice set) cuts a square of side $a + b$ into four right-angled triangles and one tilted square of side c, and reads Pythagoras' theorem straight off the areas. A picture earns the name *proof* only after we have checked, with theorems, that the pieces fit in **every** case — that the tilted piece really is a square for every right-angled triangle, not just the one drawn. Section 1 again: the picture suggests, the chain compels.

Worked examples

Example 1 — A kite: demonstrating versus proving

A **kite** is a quadrilateral ABCD with AB = AD and CB = CD (two pairs of adjacent equal sides).



Kite ABCD with the diagonal AC dashed; single ticks on the equal sides AB and AD, double ticks on the equal sides CB and CD.

A student cuts out the kite, folds it about AC, and the two halves match. That is a demonstration of this kite (Section 1). The proof below covers every kite.

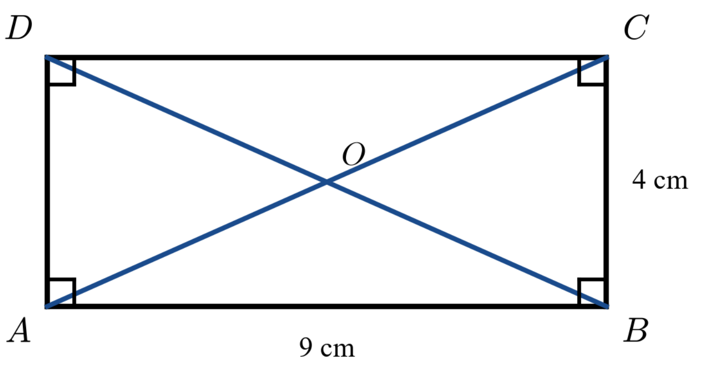
Given: kite ABCD; diagonal AC. **To show:** △ABC ≅ △ADC, and hence AC bisects ∠BAD.

Working	Comment
AB = AD	given (sides of the kite)
CB = CD	given (sides of the kite)
AC = AC	common side
∴ △ABC ≅ △ADC	SSS
∴ ∠BAC = ∠DAC	matched angles of congruent triangles
∴ AC bisects ∠BAD	definition of bisect ■

(The fold the student made is exactly this congruence — the proof is why the fold works.)

Example 2 — The diagonals of a rectangle are equal

Given: rectangle ABCD with diagonals AC and BD; AB = 9 cm, BC = 4 cm; diagonals meeting at O. **To show:** AC = BD; then find OA to two decimal places.

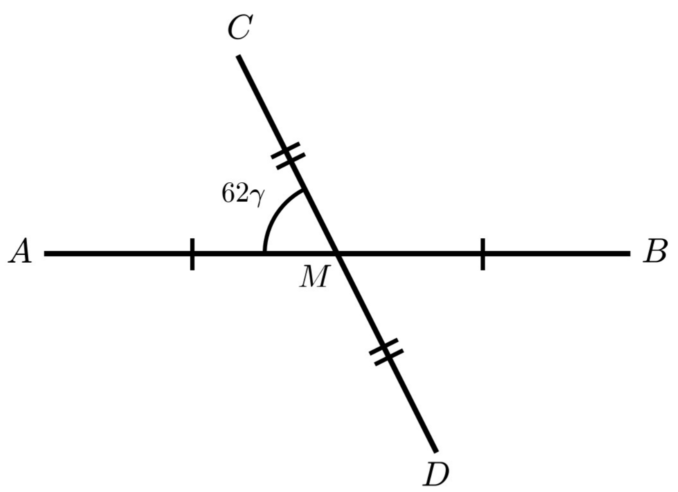


Rectangle ABCD, 9 cm by 4 cm, with both diagonals drawn meeting at O; small square marks at all four corners show the right angles.

Working	Comment
ABCD is a parallelogram with right angles	definition of rectangle
$AB = DC$	Theorem 2
$\angle ABC = \angle DCB = 90°$	right angles of the rectangle
$BC = CB$	common side
$\therefore \triangle ABC \cong \triangle DCB$	SAS
$\therefore AC = DB$	matched sides: the diagonals are equal ■
$AC^2 = 9^2 + 4^2 = 97$	Pythagoras in $\triangle ABC$ (right angle at B)
$AC = \sqrt{97}$ cm	
$OA = \frac{1}{2}AC$	Theorem 3 (diagonals of a parallelogram bisect each other; a rectangle is a parallelogram)
$OA = \sqrt{97}/2 \approx 4.92$ cm	to two decimal places

Example 3 — Two segments with a common midpoint

Segments AB and CD cross at M, and M is the midpoint of **both**. $\angle AMC = 62°$.



Segments AB and CD crossing at their common midpoint M; single ticks on AM and MB, double ticks on CM and MD; the angle AMC marked 62 degrees.

Given: $AM = MB$, $CM = MD$, $\angle AMC = 62°$. **To show:** $\triangle AMC \cong \triangle BMD$, and hence $AC \parallel BD$.

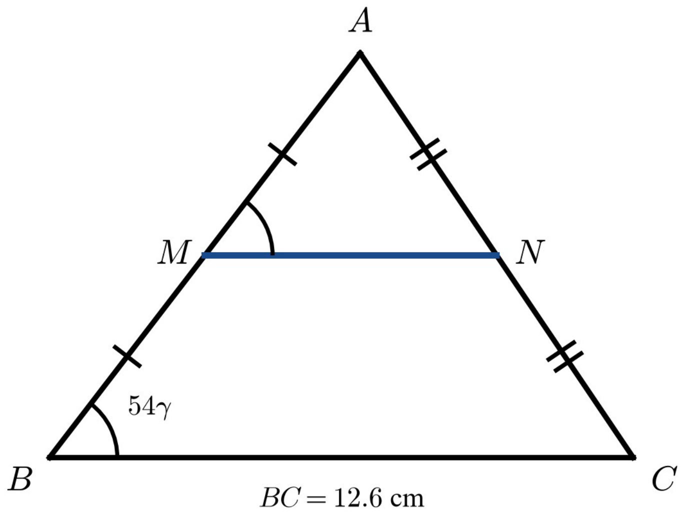
Working	Comment
$AM = BM$	given

Working	Comment
∠AMC = ∠BMD	vertically opposite angles (A3)
CM = DM	given
∴ △AMC ≅ △BMD	SAS
∴ ∠MAC = ∠MBD	matched angles
∴ AC ∥ BD	converse of alternate angles (A6), transversal AB ■

(Also ∠BMD = 62°, and in △AMC the other two angles add to 180° − 62° = 118° by (A4).)

Example 4 — The midpoint theorem at work

In △ABC, M and N are the midpoints of AB and AC. BC = 12.6 cm and ∠ABC = 54°. Find MN and ∠AMN.

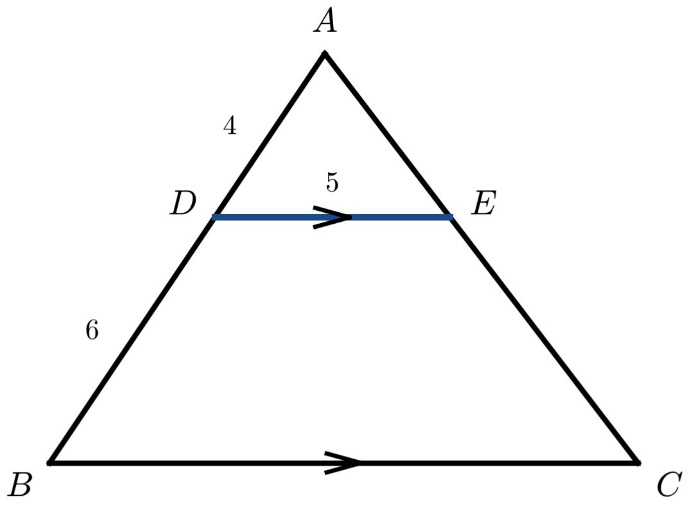


Triangle ABC with M and N the midpoints of AB and AC; MN drawn thick; single ticks on AM and MB, double ticks on AN and NC; 54 degrees marked at B and an equal-angle arc at M.

Working	Comment
MN = ½BC	Theorem 6
MN = 12.⁶⁄₂ = 6.3 cm	
∠AMN = ∠ABC	Theorem 6 (matched angles; MN ∥ BC)
∠AMN = 54°	■

Example 5 — A segment parallel to a side

In △ABC, D lies on AB and E on AC with DE ∥ BC. AD = 4, DB = 6 and DE = 5. Find BC.

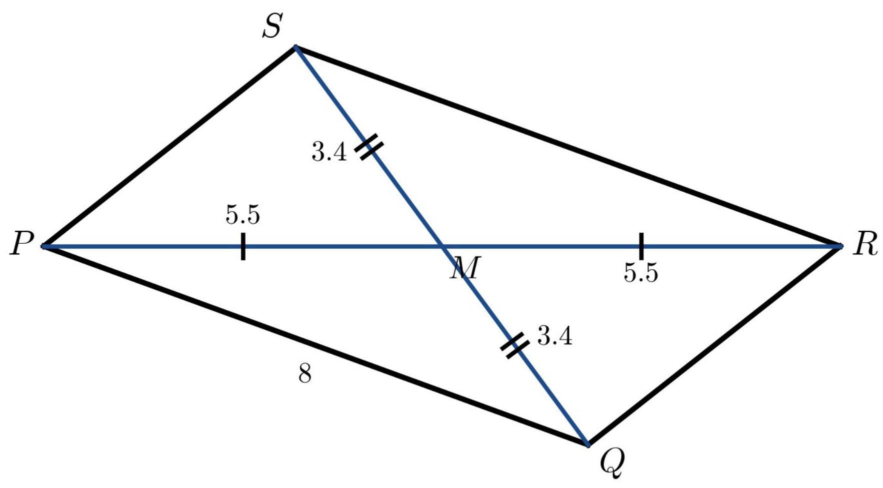


Triangle ABC with DE parallel to BC, D on AB and E on AC; AD labelled 4, DB labelled 6, DE labelled 5; arrow marks show DE parallel to BC.

Working	Comment
∠ADE = ∠ABC	corresponding angles, DE ∥ BC (A7)
∠DAE = ∠BAC	common angle
∴ △ADE ~ △ABC	AA
AD/AB = DE/BC	matched sides in the same ratio
4/10 = 5/BC	AB = AD + DB = 10
BC = 5 × $\frac{10}{4}$ = 12.5	■

Example 6 — When the diagonals bisect each other (Theorem 4)

In quadrilateral PQRS the diagonals PR and QS meet at M, with PM = MR = 5.5 cm, QM = MS = 3.4 cm and PQ = 8 cm.



Quadrilateral PQRS with diagonals PR and QS meeting at M; single ticks on PM and MR, double ticks on QM and MS; PQ labelled 8, the half-diagonals labelled 5.5 and 3.4.

Given: PM = MR, QM = MS. **To show:** PQRS is a parallelogram; find RS.

Working	Comment
PM = RM	given
∠PMQ = ∠RMS	vertically opposite angles (A3)
QM = SM	given
∴ △PMQ ≅ △RMS	SAS
∴ ∠MPQ = ∠MRS and PQ = RS	matched parts
∴ PQ ∥ RS	converse of alternate angles (A6), transversal PR
PM = RM, ∠PMS = ∠RMQ, SM = QM	given; vertically opposite (A3)
∴ △PMS ≅ △RMQ	SAS
∴ PS ∥ RQ	converse of alternate angles (A6), transversal PR
∴ PQRS is a parallelogram	definition: both pairs of opposite sides parallel
RS = PQ = 8 cm	from the first congruence ■

Practice questions

Scaffolded

Q1. Say whether each is a demonstration or a proof, giving the reason in one line. (a) Two cut-out triangles are placed one on top of the other and they match. (b) Two triangles are shown to have two sides and the included angle equal, so by SAS they must match. (c) The angles of five drawn triangles are measured and each set adds to about 180°. (d) A chain of statements uses (A6) and ASA to show that two triangles must match. *[Hint: ask whether the argument covers every case, or only the cases in front of you.]*

Q2. Name the congruence test each list of matched parts gives, or say why it is not a test. (a) Two sides and the included angle. (b) Three sides. (c) Two angles and the side between them. (d) Right angle, hypotenuse and one other side. (e) Three angles.

Q3. An isosceles triangle has apex angle 46°. (a) Find a base angle. *[Hint: Theorem 1, then (A4).]* (b) Find the exterior angle at a base vertex. *[Hint: (A5), or (A1) with your answer to (a).]*

Q4. Two right-angled triangles have hypotenuse 13 cm each, and a pair of legs (the sides other than the hypotenuse) equal at 5 cm each. (a) State the test that makes them congruent. (b) Find the third side of each. (c) Explain in one line why the angles opposite the 5 cm sides are equal.

Q5. In parallelogram PQRS, ∠P = 58°. Find ∠Q, R and ∠S, naming a reason for each. *[Hint: (A8) for the neighbour, Theorem 2 for the opposite angle.]*

Q6. In △ABC, M and N are the midpoints of AB and AC. If BC = 9.4 cm, find MN.

Q7. △DEF is similar to △ABC with scale factor 3. If AB = 4.2 cm and BC = 5 cm, find DE and EF.

Q8. The diagonals of quadrilateral WXYZ meet at P, with WP = PY = 6 cm and XP = PZ = 3.5 cm. (a) Prove △WPX ≅ △YPZ, giving reasons. *[Hint: SAS with a pair of vertically opposite angles.]* (b) Hence state which two sides are equal and which two sides are parallel.

Unscaffolded

Q9. Prove Theorem 5: if one pair of opposite sides of a quadrilateral is equal and parallel, the quadrilateral is a parallelogram. (Draw quadrilateral ABCD with AB ∥ DC and AB = DC, and join AC.)

Q10. Points A and D lie on opposite sides of segment BC, with AB = DB and AC = DC. Prove that ∠BAC = ∠BDC.

Q11. Prove Theorem 3: the diagonals of a parallelogram bisect each other. (In parallelogram ABCD the diagonals AC and BD meet at M; show △ABM ≅ △CDM.)

Q12. Prove that if both pairs of opposite sides of a quadrilateral are equal, the quadrilateral is a parallelogram. (In ABCD, AB = CD and BC = DA; join AC.)

Q13. Prove that the diagonals of a rhombus meet at right angles. (You may quote Theorem 3.)

Q14. The diagonals of quadrilateral ABCD are equal and bisect each other. Prove, in three parts, that ABCD is a rectangle: (a) ABCD is a parallelogram; (b) ∠ABC = 90°; (c) conclude.

Q15. In kite ABCD (AB = AD, CB = CD) the diagonals AC and BD meet at M. Prove that AC ⊥ BD. (Two congruences: first the whole triangles on AC, then the small triangles at M.)

Q16. In △ABC, M and N are the midpoints of AB and AC. If MN = (x + 2) cm and BC = (3x − 4) cm, find x, MN and BC.

Q17. In △ABC, D lies on AB and E on AC with DE ∥ BC. If AD = 6, DB = x, AE = 8 and EC = 12, find x.

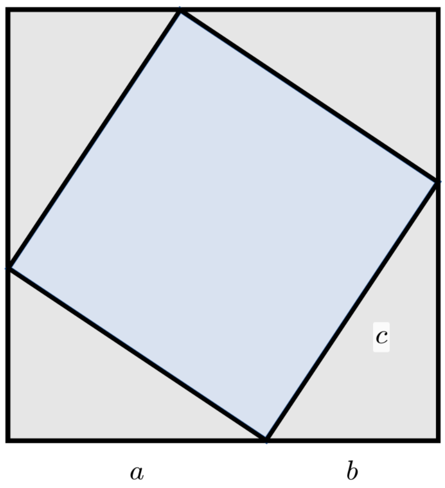
Q18. In trapezium ABCD, AB ∥ DC and the diagonals meet at M. (A **trapezium** here has exactly one pair of parallel sides.) If MA = 12, MB = 9 and MC = 8, show that △MAB ~ △MCD and find MD.

Q19. Trapezium ABCD has AB ∥ DC and AD = BC. Prove that the base angles at A and B are equal: ∠DAB = ∠CBA. (Construction: draw CE parallel to DA, meeting AB at E.)

Q20. In parallelogram ABCD, M is the midpoint of BC. The line DM, extended, meets the line AB, extended past B, at E. Prove △MCD ≅ △MBE, and hence find AE when AB = 7 cm.

Q21. In △ABC, AB = AC and ∠BAC = 40°. D lies on AC with BD = BC. Find ∠ABD.

Q22. The figure shows a square of side $a+b$ cut into four right-angled triangles with legs a and b and hypotenuse c, plus a tilted inner quadrilateral.



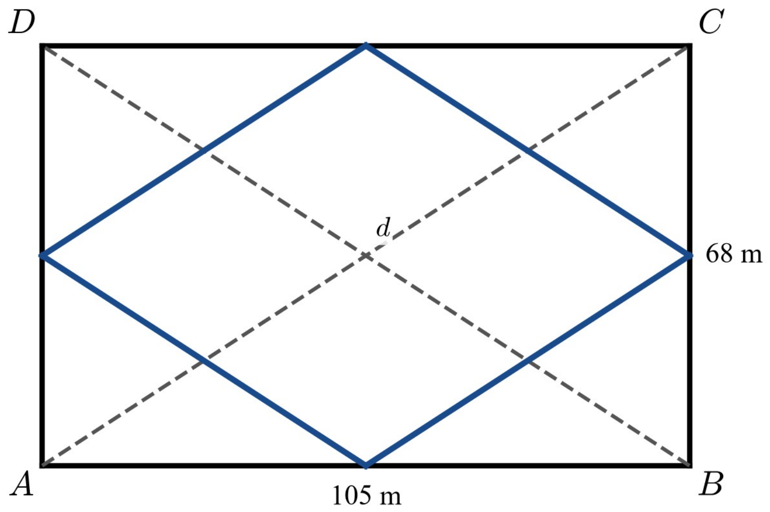
A square of side $a + b$ tiled by four right-angled triangles and one square of side c.

Square of side a plus b tiled by four grey right-angled triangles with legs a and b, leaving a tilted blue inner square of side c.

(a) Explain why the four triangles are congruent.
(b) Prove that the inner quadrilateral is a square.
(c) Hence, by comparing areas, prove that $a^2+b^2=c^2$.

Real context

Q23. A rectangular pitch is marked at 105 m by 68 m — the size FIFA recommends for international football, within the range the Laws of the Game allow. Label the rectangle ABCD, with P, Q, R, S the midpoints of AB, BC, CD, DA.



Rectangular pitch 105 m by 68 m with dashed diagonals and the blue midpoint quadrilateral PQRS drawn inside.

(a) Show that the diagonal AC satisfies $AC^2 = 15\,649$ m², and give AC to one decimal place.
(b) Prove that PQRS is a parallelogram, and hence a rhombus. (Quote Theorem 6 and Example 2.)
(c) Hence find the perimeter of PQRS to one decimal place.

Source: FIFA stadium guidelines, §5.3 *Pitch Dimensions and Surrounding Areas* (recommended field of play 105 m × 68 m), as cited in "Association football pitch", Wikipedia; permitted range for international matches from IFAB, *Laws of the Game*, Law 1. Retrieved 20 August 2026.

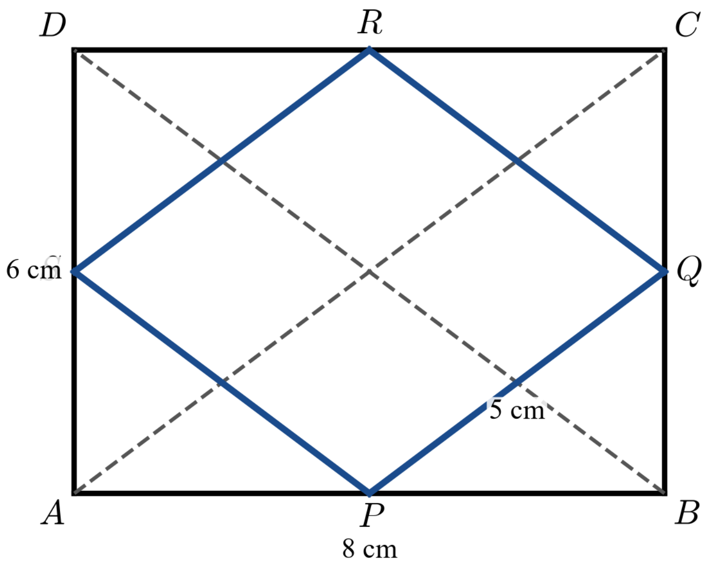
Digital tool task

DIGITAL TOOL — this task assumes dynamic geometry software. Your teacher will nominate the tool. The tool-free path below is taught in class and is what is assessed.

Input. A rectangle ABCD with AB = 8 cm and BC = 6 cm; a point P on AB, Q on BC, R on CD, S on DA (one vertex on each side). **Output.** The perimeter of quadrilateral PQRS as the four points are dragged; the smallest perimeter you can reach.

With the tool. Drag the four points and watch the perimeter. Conjecture the smallest value, and note the shape of PQRS when each point sits at a midpoint.

Tool-free path.



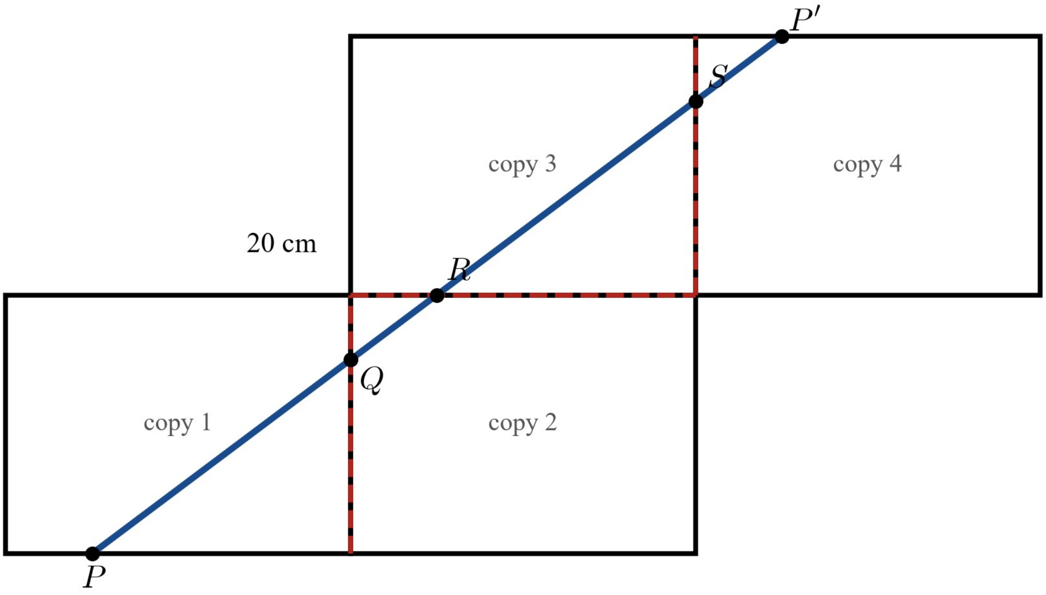
Rectangle 8 cm by 6 cm with dashed diagonals and the blue midpoint rhombus PQRS, each side labelled 5 cm.

(a) *The midpoint case.* With P, Q, R, S the midpoints of the sides: the diagonals of the rectangle are 10 cm (Pythagoras), and each side of PQRS is half a diagonal by Theorem 6, so each side is 5 cm and the perimeter is 20 cm. (PQRS is a rhombus: each side is parallel to, and half of, a diagonal.)

(b) *Another quadrilateral with the same perimeter.* Take P on AB with PB = 6 cm; draw PQ through P parallel to diagonal AC (meeting BC at Q), then QR through Q parallel to BD (meeting CD at R), then RS through R parallel to AC (meeting DA at S), and join SP.

- PQ ∥ SR and QR ∥ PS (construction), so PQRS is a parallelogram by definition — this is the proof the elaboration asks for.
- △BPQ ~ △BAC (AA: corresponding angles, PQ ∥ AC), ratio PB/AB = 6/8, so BQ = (¾) × 6 = 4.5 cm and PQ = (¾) × 10 = 7.5 cm; then QC = 6 − 4.5 = 1.5 cm and △CQR ~ △CBD (AA, QR ∥ BD), ratio QC/BC = 1.⅚ = ¼, so QR = (¼) × 10 = 2.5 cm.
- Perimeter = 2 × (7.5 + 2.5) = 20 cm — the same as the midpoint case.

(c) *Why 20 cm is the smallest.* Reflect the rectangle in the side BC, then the copy in its top side, then that copy in its right side; the four copies form a staircase, and the closed path P → Q → R → S → P unfolds into the single straight segment from P to its third image P′, a horizontal run of 2 × 8 = 16 cm and a vertical rise of 2 × 6 = 12 cm.



Staircase of four reflected copies of the 8 by 6 rectangle; the path P, Q, R, S unfolds to the straight blue segment from P to P', labelled 20 cm, crossing the mirror sides at Q, R and S.

- $PP' = \sqrt{16^2 + 12^2} = 20$ cm, and a straight segment is the shortest path between its ends, so no quadrilateral of this kind can beat 20 cm.
- The midpoint rhombus of (a) reaches 20 cm. ∴ the smallest perimeter is **20 cm**, reached by a whole family of parallelograms whose sides are parallel to the diagonals of the rectangle.

Answers

Q1. (a) demonstration — checks one pair of triangles. (b) proof — SAS covers every case. (c) demonstration — five triangles are not all triangles. (d) proof — the chain covers every case. **Q2.** (a) SAS. (b) SSS. (c) ASA. (d) RHS. (e) not a test — AAA fixes shape, not size. **Q3.** (a) 67°. (b) 113°. **Q4.** (a) RHS. (b) 12 cm. (c) matched angles of congruent triangles. **Q5.** ∠Q = 122°, ∠R = 58°, ∠S = 122°. **Q6.** 4.7 cm. **Q7.** DE = 12.6 cm, EF = 15 cm. **Q8.** (a) WP = YP, ∠WPX = ∠YPZ (vertically opposite), XP = ZP, ∴ SAS. (b) WX = YZ and WX ∥ YZ. **Q9.** ∠BAC = ∠DCA (A6); AC common; AB = CD given; ∴ △ABC ≅ △CDA (SAS); ∴ ∠BCA = ∠DAC; ∴ AD ∥ CB (A6 converse); both pairs parallel ∴ parallelogram. **Q10.** In △ABC and △DBC: AB = DB, AC = DC (given), BC = CB (common) → △ABC ≅ △DBC (SSS); ∴ ∠BAC = ∠BDC. **Q11.** AB ∥ DC → ∠MAB = ∠MCD and ∠MBA = ∠MDC ((A6)); AB = CD (Theorem 2); ∴ △ABM ≅ △CDM (ASA); ∴ AM = CM, BM = DM. **Q12.** △ABC ≅ △CDA (SSS); ∴ ∠BAC = ∠DCA → AB ∥ DC, and ∠BCA = ∠DAC → AD ∥ CB (A6 converse). **Q13.** Diagonals meet at M with AM = CM (Theorem 3); AB = CB (sides); BM common; ∴ △ABM ≅ △CBM (SSS); ∴ ∠AMB = ∠CMB; these add to 180° (A1) ∴ each 90°. **Q14.** (a) Theorem 4. (b) △ABC ≅ △DCB (SSS: AB = DC by Theorem 2, AC = DB given, BC common) → ∠ABC = ∠DCB; these are co-interior for AB ∥ DC (A8) ∴ each 90°. (c) parallelogram with a right angle = rectangle. **Q15.** △ABC ≅ △ADC (SSS) → ∠BAM = ∠DAM; △ABM ≅ △ADM (SAS: AB = AD, ∠BAM = ∠DAM, AM common) → ∠AMB = AMD; sum 180° (A1) → each 90°. **Q16.** x = 8; MN = 10 cm; BC = 20 cm. **Q17.** x = 9. **Q18.** ∠MAB = ∠MCD and ∠MBA = ∠MDC (alternate angles, (A6)) → △MAB ~ △MCD (AA); MA/MC = MB/MD → MD = 6. **Q19.** AECD is a parallelogram (both pairs of opposite sides parallel) → EC = AD = CB → △CEB isosceles → ∠CEB = ∠CBE (Theorem 1); ∠CEB = ∠DAB (A7, EC ∥ AD) → ∠DAB = ∠CBA. **Q20.** ∠MCD = ∠MBE (A6, DC ∥ AE), CM = BM, ∠DMC = ∠EMB (A3) → ASA; ∴ BE = CD = 7; AE = AB + BE = 14 cm. **Q21.** 30°. **Q22.** (a) SAS: legs a and b with the right angle of the square between them. (b) all four sides are c; at each point of the outer square's side, the two acute angles of the triangles add to 90° (A4), so the inner angle is 180° − 90° = 90° (A1) — a square. (c) $(a+b)^2 = c^2 + 4 \times 1/2\, ab \rightarrow a^2 + b^2 = c^2$. **Q23.** (a) $AC^2 = 105^2 + 68^2 = 15\,649$; AC ≈ 125.1 m. (b) PQ and SR are each parallel to AC and half of it (Theorem 6 in △ABC and △CDA) → PQ ∥ SR, PQ = SR → parallelogram (Theorem 5); AC = BD (Example 2) gives QR = ½BD = ½AC = PQ → rhombus. (c) 2 × AC ≈ 250.2 m. **Digital task.** (a) 20 cm. (b) parallelogram by construction; 7.5 cm and 2.5 cm sides; 20 cm. (c) unfolded path is the straight segment PP′ = $\sqrt{(16^2 + 12^2)}$ = 20 cm; straight is shortest; the midpoint rhombus reaches it; smallest perimeter 20 cm.

Fully-worked solutions

Q1. A proof must cover every case; a demonstration checks the cases in front of you. (a) and (c) check particular triangles (cut-outs; five measured drawings) → demonstrations. (b) and (d) quote a congruence test or an angle theorem, which hold for every triangle of that kind → proofs.

Q2. (a) SAS. (b) SSS. (c) ASA. (d) RHS. (e) AAA is not a congruence test: three equal angles allow one triangle to be an enlarged copy of the other, so the triangles need not be the same size.

Q3. (a) Base angles equal (Theorem 1): each = (180° − 46°)⁄2 = 67°. (b) Exterior angle = sum of the two opposite interior angles (A5) = 46° + 67° = 113°. (Check with (A1): 180° − 67° = 113°. ✓)

Q4. (a) RHS — right angle, hypotenuse 13 cm, side 5 cm. (b) Third side = $\sqrt{(13^2 - 5^2)} = \sqrt{144}$ = 12 cm (Pythagoras). (c) The triangles are congruent, and matched angles of congruent triangles are equal.

Q5. ∠Q = 180° − 58° = 122° (co-interior angles, (A8), PQ ∥ SR). ∠R = ∠P = 58° and ∠S = ∠Q = 122° (Theorem 2, opposite angles of a parallelogram).

Q6. MN = ½BC (Theorem 6) = 9.4⁄2 = 4.7 cm.

Q7. Matched sides of similar triangles are in the ratio of the scale factor: DE = 3 × AB = 12.6 cm; EF = 3 × BC = 15 cm.

Q8. (a) WP = YP (given, both 6 cm); ∠WPX = ∠YPZ (vertically opposite, (A3)); XP = ZP (given, both 3.5 cm); ∴ △WPX ≅ △YPZ (SAS). (b) WX = YZ (matched sides), and ∠XWP = ∠ZYP (matched angles) → WX ∥ YZ by the converse of alternate angles (A6), transversal WY.

Q9. Given: ABCD with AB ∥ DC and AB = DC. **To show:** ABCD is a parallelogram. Join AC. ∠BAC = ∠DCA (alternate angles, (A6)). AC = CA (common). AB = CD (given). ∴ △ABC ≅ △CDA (SAS). ∴ ∠BCA = ∠DAC (matched angles). ∴ AD ∥ CB (converse of alternate angles, (A6)). Both pairs of opposite sides are parallel ∴ ABCD is a parallelogram. ■

Q10. In △ABC and △DBC: AB = DB (given); AC = DC (given); BC = CB (common). ∴ △ABC △DBC (SSS). ∴ ∠BAC = ∠BDC (matched angles). ■

Q11. In parallelogram ABCD let the diagonals meet at M. AB ∥ DC → ∠MAB = ∠MCD and ∠MBA = ∠MDC (alternate angles, (A6)). AB = CD (Theorem 2). ∴ △ABM ≅ △CDM (ASA). ∴ AM = CM and BM = DM (matched sides), i.e. each diagonal cuts the other in half: the diagonals bisect each other. ■

Q12. Join AC. AB = CD (given); BC = DA (given); AC = CA (common). ∴ △ABC ≅ △CDA (SSS). ∴ ∠BAC = ∠DCA → AB ∥ DC, and ∠BCA = ∠DAC → AD ∥ CB (converse of alternate angles, (A6)). ∴ ABCD is a parallelogram. ■

Q13. Let the diagonals of rhombus ABCD meet at M. AM = CM (Theorem 3). AB = CB (sides of the rhombus). BM = BM (common). ∴ △ABM ≅ △CBM (SSS). ∴ ∠AMB = ∠CMB (matched angles). ∠AMB + ∠CMB = 180° (angles on a straight line, (A1), since AMC is straight). ∴ ∠AMB = 90°: the diagonals meet at right angles. ■

Q14. (a) The diagonals bisect each other ∴ ABCD is a parallelogram (Theorem 4). (b) AB = DC (Theorem 2); AC = DB (given); BC = CB (common). ∴ △ABC ≅ △DCB (SSS). ∴ ∠ABC = ∠DCB. AB ∥ DC → ∠ABC + ∠DCB = 180° (co-interior angles, (A8)). ∴ ∠ABC = 90°. (c) A parallelogram with a right angle is a rectangle (definition). ■

Q15. Step 1: AB = AD (given); CB = CD (given); AC = AC (common). ∴ △ABC ≅ △ADC (SSS). ∴ ∠BAM = ∠DAM (matched angles). Step 2: AB = AD (given); ∠BAM = ∠DAM (Step 1); AM = AM (common). ∴ △ABM ≅ △ADM (SAS). ∴ ∠AMB = ∠AMD. ∠AMB + ∠AMD = 180° ((A1), BMD is a straight line). ∴ ∠AMB = 90°, i.e. AC ⊥ BD. ■

Q16. MN = ½BC (Theorem 6): x + 2 = (3x − 4)⁄2 → 2x + 4 = 3x − 4 → x = 8. MN = 8 + 2 = 10 cm; BC = 3(8) − 4 = 20 cm. (Check: 10 = 20⁄2 ✓.)

Q17. DE ∥ BC → △ADE ~ △ABC (AA, as in Example 5). ∴ AD/AB = AE/AC: 6⁄(6 + x) = 8⁄20 → 8(6 + x) = 120 → 6 + x = 15 → x = 9.

Q18. AB ∥ DC → MAB = ∠MCD and ∠MBA = ∠MDC (alternate angles, (A6)). ∴ △MAB ~ △MCD (AA). ∴ MA/MC = MB/MD: 12⁄8 = 9⁄MD → MD = 9 × 8⁄12 = 6.

Q19. Draw CE parallel to DA, meeting AB at E. AE ∥ DC (given AB ∥ DC) and AD ∥ EC (construction) → AECD is a parallelogram → EC = AD (Theorem 2). AD = BC (given) → EC = BC → △CEB is isosceles → ∠CEB = ∠CBE (Theorem 1). EC ∥ AD → ∠CEB = ∠DAB (corresponding angles, (A7)). ∠CBE is ∠CBA. ∴ ∠DAB = ∠CBA. □

Q20. DC ∥ AE (AB ∥ DC, E on line AB) → ∠MCD = ∠MBE (alternate angles, (A6)). CM = BM (M midpoint). ∠DMC = ∠EMB (vertically opposite, (A3)). ∴ △MCD ≅ △MBE (ASA). ∴ BE = CD (matched sides) = AB (Theorem 2) = 7 cm. ∴ AE = AB + BE = 7 + 7 = 14 cm. ■

Q21. AB = AC → ∠ABC = ∠ACB = (180° − 40°)⁄2 = 70° (Theorem 1 and (A4)). BD = BC → ∠BDC = ∠BCD = 70° (Theorem 1 in △BDC). ∴ ∠DBC = 180° − 70° − 70° = 40° ((A4)). ∴ ∠ABD = ∠ABC − ∠DBC = 70° − 40° = 30°.

Q22. (a) Each triangle has legs a and b with the right angle of the outer square between them → congruent by SAS. (b) All four sides of the inner quadrilateral are hypotenuses, each c. At each point where two triangles meet on a side of the outer square, their acute angles α and β satisfy $\alpha+\beta=90°$ ((A4) with the right angle), so the inner angle there is 180° − 90° = 90° ((A1)). Four equal sides and right angles → a square. (c) Area of outer square two ways: $(a+b)^2=c^2+4\times 1/2\,ab$. Expanding: $a^2+2ab+b^2=c^2+2ab$ → $a^2+b^2=c^2$. ■ (The picture suggested this; the checks in (a) and (b) are what make it a proof — Section 9.)

Q23. (a) ∠ABC = 90° → $AC^2 = AB^2 + BC^2 = 105^2 + 68^2$ = 11 025 + 4624 = 15 649 m² (Pythagoras). AC = √15 649 ≈ 125.1 m. (b) In △ABC, P and Q are midpoints of AB and BC → PQ ∥ AC and PQ = ½AC (Theorem 6). In △CDA, R and S are midpoints of CD and DA → RS ∥ AC and RS = ½AC. ∴ PQ ∥ RS and PQ = RS → PQRS is a parallelogram (Theorem 5). In △BCD, Q and R are midpoints of BC and CD → QR = ½BD; BD = AC (Example 2) → QR = ½AC = PQ. A parallelogram with two adjacent sides equal is a rhombus. ■ (c) Perimeter = 4 × ½AC = 2 × AC = $2\sqrt{15\,649} \approx 250.2$ m.

Digital task. (a) Diagonal = $\sqrt{(8^2 + 6^2)} = 10$ cm; each midpoint side = ½ × 10 = 5 cm (Theorem 6); perimeter = 20 cm. (b) PQ ∥ SR and QR ∥ PS by construction → parallelogram. △BPQ ~ △BAC (AA, PQ ∥ AC) with ratio PB/AB = ⁶⁄₈ → BQ = (¾) × 6 = 4.5 cm and PQ = (¾) × 10 = 7.5 cm. QC = 6 − 4.5 = 1.5 cm; △CQR ~ △CBD (AA, QR ∥ BD) with ratio QC/BC = 1.⁵⁄₆ = ¼ → QR = 2.5 cm. Perimeter = 2 × (7.5 + 2.5) = 20 cm. (c) Reflections in a side preserve length, so the perimeter equals the unfolded path from P to P′. The run is 2 × 8 = 16 cm and the rise 2 × 6 = 12 cm, so $PP' = \sqrt{(16^2 + 12^2)} = \sqrt{400} = 20$ cm; the straight segment is the shortest path between its ends, so every such quadrilateral has perimeter ≥ 20 cm, and (a) reaches 20 cm. Smallest perimeter = 20 cm, attained by the family of parallelograms whose sides run parallel to the diagonals of the rectangle. ■

Teacher note

- **Prior knowledge rehearsed (D3):** congruence tests and quadrilateral properties from Level 8 (VC2M8SP01, VC2M8SP02) are stated compactly as toolkit (Theory §3–§4, §7); Level 9 codes rehearsed, not re-taught: VC2M9SP02 (enlargement → similarity) and VC2M9M03 (angle properties, Pythagoras in spatial problems).
- **Scope check (D6):** no vectors, no coordinate proofs, no circle theorems, no formal logic beyond step chains. Deductive proof itself is the content of VC2M10SP01 and is fully in scope. Definitions adopted: rectangle = parallelogram with a right angle; rhombus = parallelogram with adjacent sides equal; trapezium = quadrilateral with exactly one pair of parallel sides (exclusive reading); kite defined at first use.
- **Tool classes (D13):** the single digital task assumes the class *dynamic geometry software* only (elaboration E05); no CAS anywhere in 5A. The tool-free path (midpoint rhombus, a second family member proved a parallelogram, reflection/unfolding lower bound) is taught in class and is what is assessed.
- **Assessment:** every 5A item scores the chain — what is given, what is to be shown, which theorem licenses each step — not the conclusion alone (see TOC note on 5A).
- **VCE destinations (§5 forward map):** deductive geometric proof feeds only Specialist Mathematics Units 3&4, Area of Study 1 (Discrete mathematics — Logic and proof). For every other pathway 5A is terminal; its value for those students is the discipline of licensed reasoning, and items should be marked with that in view.

Networks, network diagrams and connectedness

Mathematics 10 · Chapter 5 Space · Subtopic 5B (Level 10) · Extended block

Strand	Content description	Elaboration ids carried	Weight
Space	VC2M10SP02	E01, E02, E03, E04, E05	E

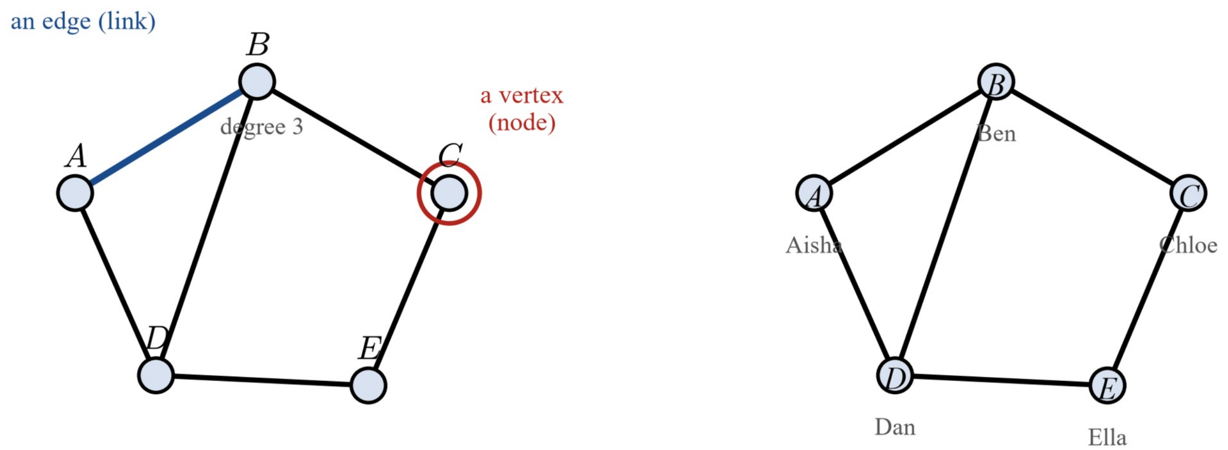
> **Elaboration note:** `VC2M10SP02.E06` (kinship systems as networks) is excluded from this section under the book's content policy Rule 1 (Aboriginal and Torres Strait Islander content excluded; see the front-of-book note). The exclusion is ACCEPTED, not a gap. This section carries E01–E05.
>
> **VC2M10SP02 (Space, Level 10)** — interpret networks and network diagrams used to represent relationships in practical situations and describe connectedness
>
> **Achievement standard extract (Level 10):** Students interpret networks used to represent practical situations and describe connectedness.

You need first: odd and even numbers, and adding and halving whole numbers; experience reading simple diagrams that stand for real things — maps, timetables, flow charts. No earlier network theory is assumed: this subtopic is the entry point to networks, which become a whole Area of Study in General Mathematics.

Theory

1. A network: vertices and edges

A **network** is a diagram of dots joined by lines. The dots are **vertices** (also called **nodes**) and the lines are **edges** (also called **links**). A network says nothing about geometry — the length of a drawn line, and where the dots sit, do not matter. All that matters is *which vertices are joined to which*.



An abstract network: 5 vertices, 6 edges.
The degree of a vertex is the number of edges at it.

One reading of the same network: vertices are people,
an edge means "friends with".

Two panels showing the same five-vertex network. Left: vertices labelled A to E, one edge coloured blue and labelled "an edge (link)", one vertex circled in red and labelled "a vertex (node)", and vertex B marked degree 3. Right: the same network with the vertices read as people Aisha, Ben, Chloe, Dan and Ella, an edge meaning "friends with".

The **degree** of a vertex is the number of edges at it. In the figure, B has degree 3 (edges to A, C and D), and the degrees are A 2, B 3, C 2, D 3, E 2.

In this book we draw at most one edge between any pair of vertices, and we allow a vertex joined to nothing (degree 0).

2. What the vertices and edges stand for

A network is mathematics the moment we say what the vertices and edges *stand for* — that is the interpreting the content description asks for. The one diagram above can be read as:

- **friendship** — vertices are people; an edge means the two people are friends;
- **travel** — vertices are towns; an edge means a road runs directly between them;
- **wiring** — vertices are devices; an edge means a cable joins them.

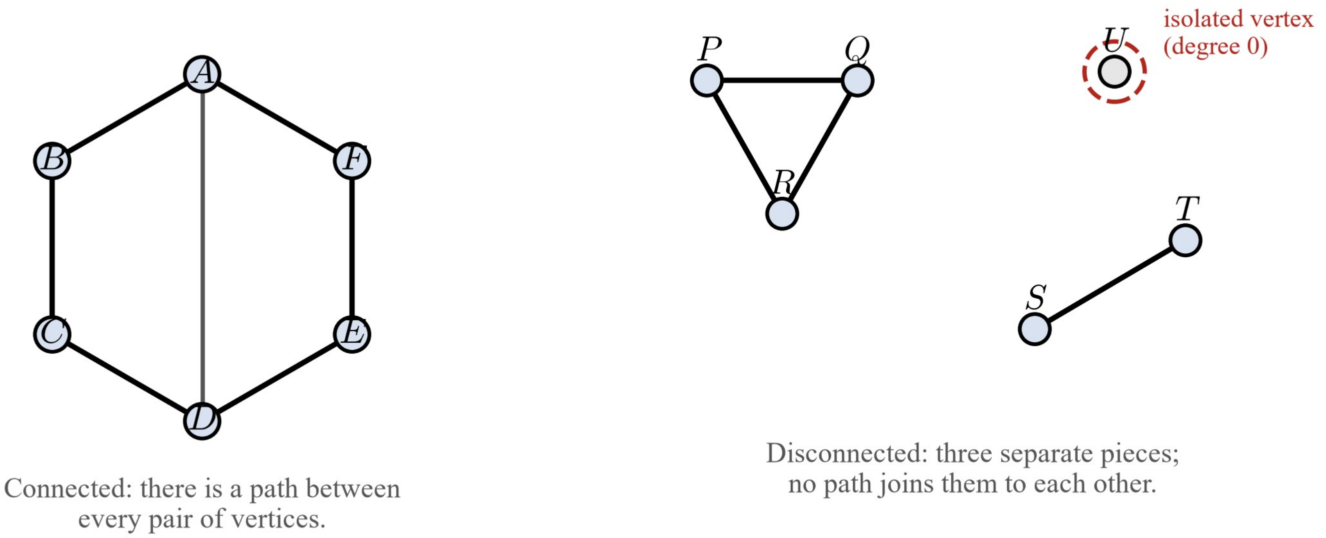
Same dots, same lines, three different situations. The mathematics (degrees, paths, connectedness) works for all three at once, which is exactly why networks are worth studying.

Often a number is written on each edge — a distance, a time, a cost, a capacity. That number is the **weight** of the edge, and a network carrying weights is a **weighted network**. The weight belongs to the edge's real meaning (the road is 6 km long), not to the length of the line on the page.

3. Paths, circuits and connectedness

A **path** is a run of edges, each one starting where the last one ended — a route through the network, written as the vertices it visits in order, like A → B → D. A **circuit** (a **closed walk**) is a path that arrives back where it started.

A network is **connected** if there is a path between every pair of vertices; otherwise it is **disconnected**. A vertex with degree 0 is an **isolated vertex** — no path reaches it.



Two panels. Left: a connected network of six vertices A to F in a hexagon with one extra edge from A to D. Right: a disconnected network in three pieces: triangle P Q R, single edge S T, and isolated vertex U circled in red with degree 0.

To **describe connectedness** is to say which of these the network is, and in how many separate pieces it falls. The right-hand network has three pieces (the triangle, the pair S–T, and the isolated vertex U), so it is disconnected.

4. Counting edges: three results

Result 1 (degree sum). *In any network, the sum of all the degrees equals twice the number of edges.*

Reason: each edge has two ends, so counting the edges at every vertex counts each edge exactly twice. In the figure of Section 1: degrees 2 + 3 + 2 + 3 + 2 = 12 = 2 × 6 edges.

Result 2 (odd vertices come in pairs). *The number of vertices of odd degree is even.*

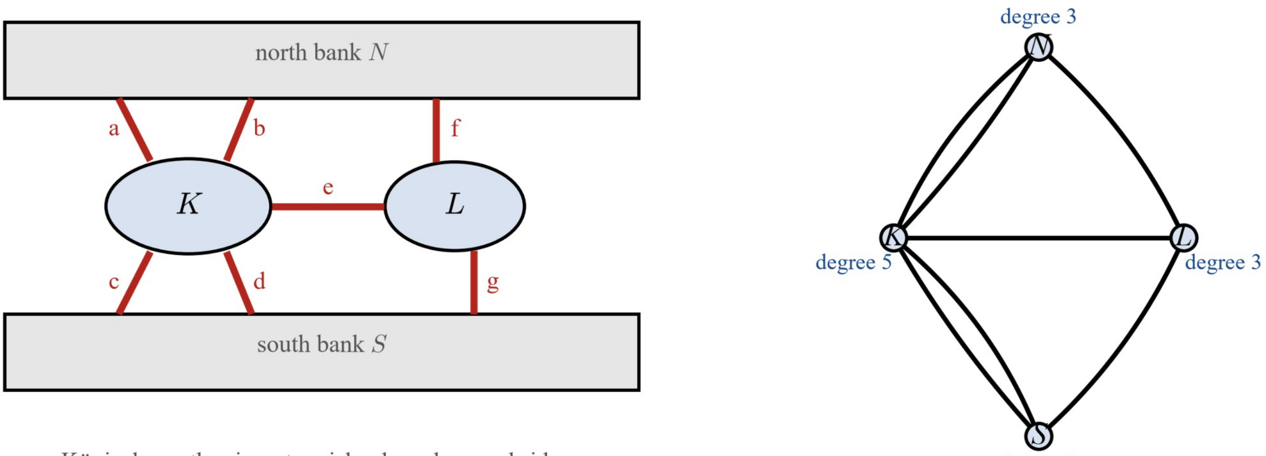
Reason: the degree sum is 2 × E, an even number. The even degrees add to something even, so the odd degrees must add to an even number too — and a list of odd numbers adds to an even number only when the list has an even number of entries. (Q20 asks you to write this out in full.)

Result 3 (all joined to all). *If every pair of n vertices is joined by an edge, the network has n(n − 1)/2 edges.*

Reason: each of the n vertices has an edge to the other n − 1 vertices, giving n(n − 1) ends; by Result 1 that is twice the edge count. Example: 4 vertices all joined to all give 4 × 3/2 = 6 edges.

5. The Seven Bridges of Königsberg

In the old Prussian city of Königsberg the river Pregel ran around two islands, crossed by seven bridges. The townspeople asked whether a walk could start anywhere, cross **each bridge exactly once**, and finish anywhere. Leonhard Euler settled it in 1736 by replacing each piece of land with a vertex and each bridge with an edge — the move that began network mathematics.



Königsberg: the river, two islands and seven bridges (schematic). Could a walk cross each bridge exactly once?

The model: vertices = land, edges = bridges. All four vertices have odd degree.

Two panels. Left: a map of Königsberg with the north bank N, south bank S and two islands K and L in the river, and seven bridges drawn in red and labelled a to g. Right: the network model with vertices K, L, N, S and one edge per bridge; degrees marked 5, 3, 3, 3.

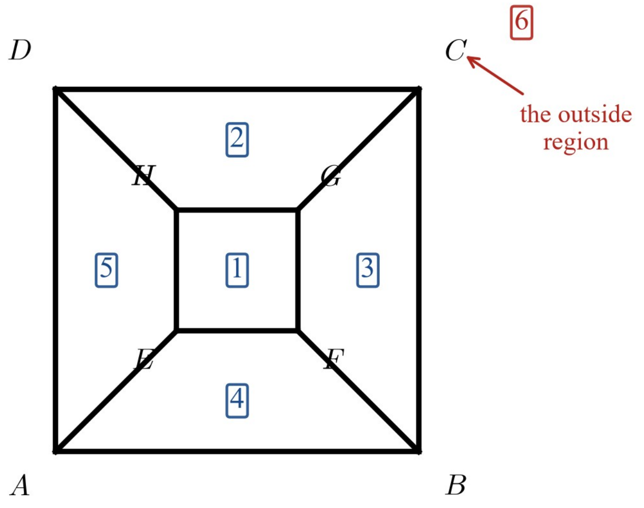
Result 4 (Euler's walk test). *A walk that uses every edge exactly once is possible only when the number of odd vertices is 0 or 2. If it is 0 the walk can start anywhere and close into a circuit; if it is 2 the walk must start at one odd vertex and end at the other.*

Reason for the test: passing *through* a vertex uses two edges (one in, one out), so every vertex other than the start and the finish must have an even degree. The start and the finish may be odd — hence at most two odd vertices, and zero if the walk closes.

In Königsberg the degrees are 5, 3, 3, 3 — **four** odd vertices. The walk is impossible, and no amount of trying was ever going to find it. (For the networks in this book the test works both ways: 0 or 2 odd vertices means such a walk exists. Checking that half is beyond Level 10; you are asked to use the test, not prove it.) A walk using every edge exactly once is now called an **Euler walk**, and a closed one an **Euler circuit**, after Euler.

6. Polyhedra as networks: Euler's formula

A **polyhedron** is a solid with flat polygon faces, straight edges and corner vertices (a cube, a pyramid, a prism). Flatten one onto the page: stretch one face out to become the outside, and the edges become a network drawn without crossings. The cube flattens to a square inside a square:



A cube drawn as a flat network: $V = 8$, $E = 12$,
$F = 6$ faces — the outside counts as a face.

A cube drawn flat as a network: an outer square A B C D and an inner square E F G H with matching corners joined; the six faces numbered 1 to 6, face 6 being the region outside the drawing.

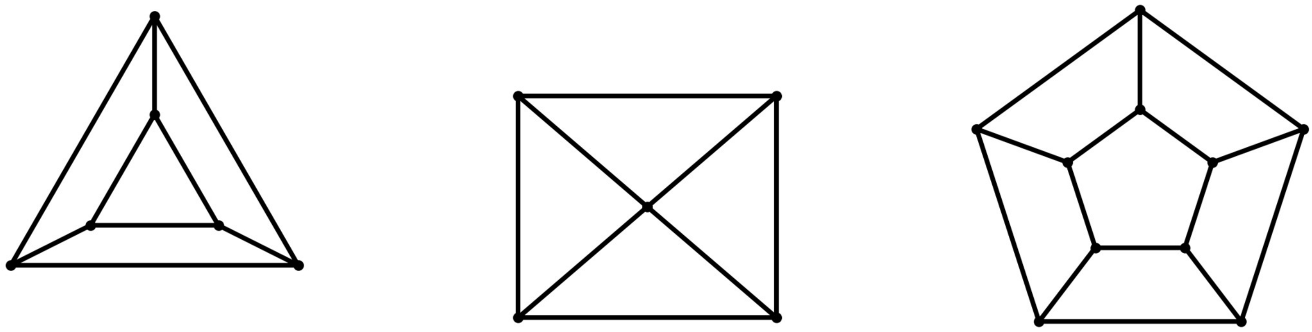
Count on the flattened cube: V = 8 vertices, E = 12 edges, and F = 6 **faces** — the five numbered regions *plus the outside region*, which is the stretched sixth face. Forgetting the outside is the classic error.

Result 5 (Euler's formula for polyhedra). *For any polyhedron, or any connected network drawn without crossings, counting the outside as a face:*

F + V = E + 2.

We do not take it on trust — we check it on real solids, where V, E and F can be counted by hand:

Solid	F	V	E	F + V	E + 2
Cube	6	8	12	14	14 ✓
Tetrahedron (triangular pyramid)	4	4	6	8	8 ✓
Triangular prism	5	6	9	11	11 ✓
Square pyramid	5	5	8	10	10 ✓
Pentagonal prism	7	10	15	17	17 ✓



Triangular prism: $V = 6$, $E = 9$, $F = 5$ Square pyramid: $V = 5$, $E = 8$, $F = 5$ Pentagonal prism: $V = 10$, $E = 15$, $F = 7$

Three polyhedra drawn as flat networks: a triangular prism (V = 6, E = 9, F = 5), a square pyramid (V = 5, E = 8, F = 5) and a pentagonal prism (V = 10, E = 15, F = 7).

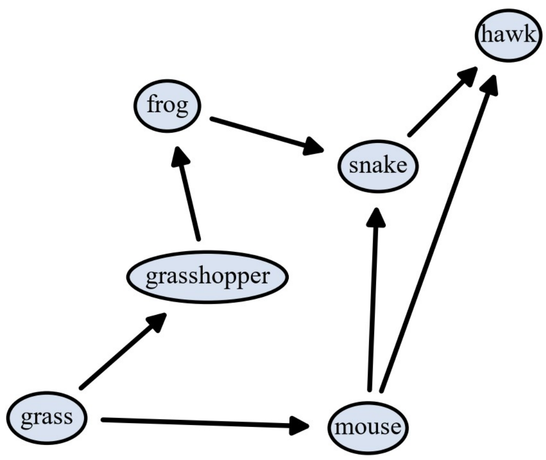
Every row agrees, and the formula then earns its keep in the other direction: count any two of F, V, E and it gives you the third (Example 5).

7. Networks everywhere

Two more readings of networks that you will meet in other subjects and in life.

Electrical and home networks. A house circuit is a network: the breaker is a vertex, each device is a vertex, and an edge joins the breaker to each device on its line. The weights are the currents, and the safety rule is a fact about the weights: the currents on one line add, and must not pass the breaker’s rating. A home computer network (a LAN) is read the same way — the router is the vertex everything joins to, and a device with no working path to the router is, in our language, in a disconnected piece.

Food webs. A food web is a network with **directed** edges: each arrow points from a food to the animal that eats it, so the direction carries the meaning.



A food web: each arrow points from the food to the animal that eats it.

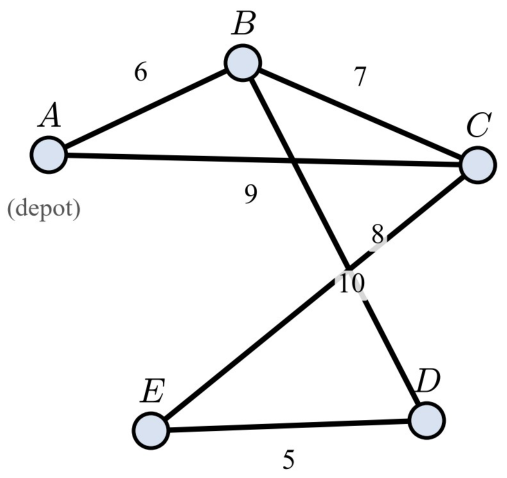
A food web on six vertices: grass, grasshopper, mouse, frog, snake, hawk. Arrows point from each food to the animal that eats it: grass to grasshopper and mouse; grasshopper to frog; frog to snake; mouse to snake and hawk; snake to hawk.

Here grass has two arrows out and none in (a producer), the hawk has none out, and the snake has two in (frog, mouse) and one out. Connectedness still matters: if one animal disappears, the vertices that lose their arrows in are the ones in trouble (Q7).

Worked examples

Example 1 — Reading a delivery network

A courier firm draws its routes between five locations as a weighted network; the weights are distances in km.



Delivery routes between five locations;
each weight is a distance in km.

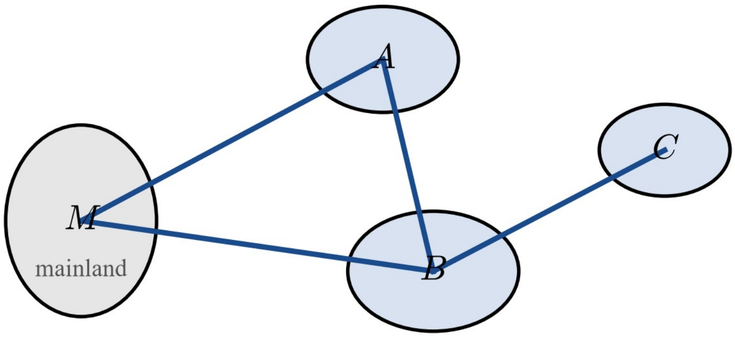
A weighted network on five vertices A to E, A the depot. Edges and weights: A–B 6, A–C 9, B–C 7, B–D 8, C–E 10, D–E 5.

(a) Say what the vertices, edges and weights stand for. (b) Find the degree of each vertex and check Result 1. (c) The driver leaves depot A, calls at B, D, E and C in that order, and returns to A. Find the distance driven.

Working	Comment
(a) vertices = the five locations; edges = direct routes; weight = distance of the route in km	interpreting the diagram
(b) degrees: A 2, B 3, C 3, D 2, E 2	count the edges at each vertex
sum of degrees = 2 + 3 + 3 + 2 + 2 = 12 = 2 × 6	Result 1, with E = 6 edges ✓
(c) A→B→D→E→C→A uses 6 + 8 + 5 + 10 + 9	one weight per edge travelled
distance = 38 km	■

Example 2 — Modelling a ferry network, and describing connectedness

A mainland M and three islands A, B, C are joined by ferry services: M to A, M to B, A to B, and B to C.



Ferry routes: vertices are places,
edges are ferry services.

Four land masses drawn as shaded shapes: the mainland M and islands A, B, C. Blue edges show ferry services M to A, M to B, A to B and B to C.

(a) Describe the connectedness of the network. (b) The M–B service is stopped for a week while the wharf is repaired. Is the network still connected? (c) Which single service, if stopped, would disconnect the network?

Working	Comment
(a) degrees: M 2, A 2, B 3, C 1	one edge per service
every island has a path to the mainland (A direct; A→B? no — B direct; C via B)	read the paths off the diagram
the network is connected, in one piece	describing connectedness
(b) without M–B: routes M–A, A–B, B–C remain	remove the edge, re-read
M→A→B→C still joins all four vertices: still connected	B and C now travel via A
(c) stop B–C: C is left with no edge — isolated, two pieces	an isolated vertex makes a second piece
stop M–A, M–B or A–B instead: all four places still join up through B	still one piece
only stopping B–C disconnects the network	■

Example 3 — Every team plays every team

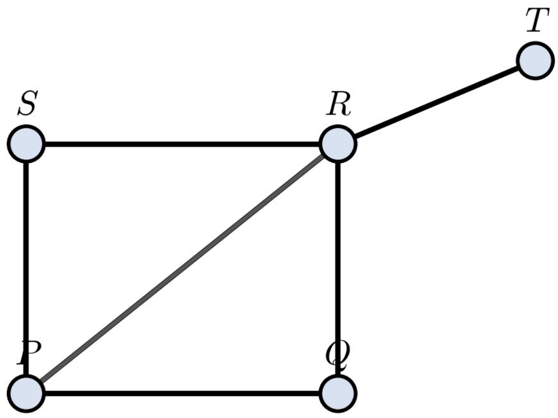
A competition has 10 teams, and each team plays each other team exactly once.

(a) Model the competition as a network, saying what the vertices and edges stand for. (b) Find the number of matches. (c) A smaller group plays each other once for 15 matches in all. How many teams are in the group?

Working	Comment
(a) vertices = teams; an edge = the match between the two teams it joins	interpreting
(b) every pair joined once: $E = 10 \times 9/2$	Result 3 with $n = 10$
$E = 45$ matches	
(c) $n(n-1)/2 = 15$, so $n(n-1) = 30$	Result 3 read backwards
$n = 6$, since $6 \times 5 = 30$	two whole numbers in a row ■

Example 4 — Walking every path once

A park’s paths form the network below; vertices are junctions and edges are paths. A ranger wants to walk along every path exactly once, starting and finishing wherever the paths allow.



Park paths: vertices are junctions,
edges are paths.

A park path network: square P Q R S with a diagonal path P to R and an extra path from R to a gate T outside the square.

Decide whether the walk is possible, and if so give one, naming its start and finish.

Working	Comment
degrees: P 3, Q 2, R 4, S 2, T 1	count at each vertex
odd vertices: P and T — exactly two	Result 4
the walk exists; it must start at one of P, T and finish at the other	Result 4
P → Q → R → S → P → R → T	check as we go
edges used: PQ, QR, RS, SP, PR, RT — all six paths, each once	■

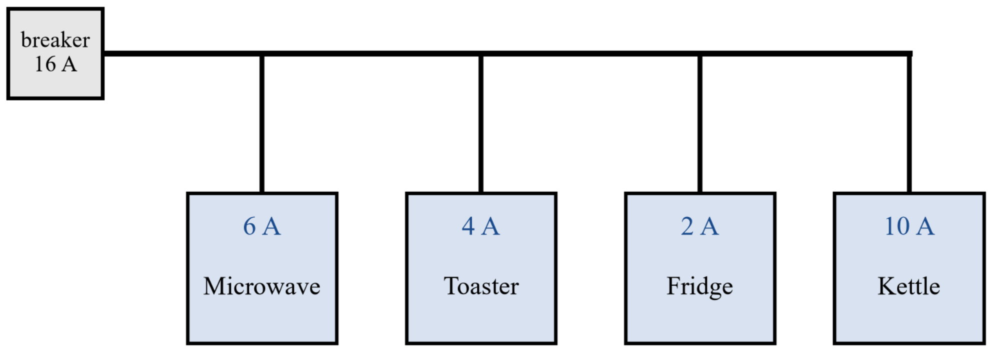
Example 5 — Euler's formula as a tool

(a) A pentagonal prism has V = 10 and E = 15. Use Euler's formula to find F, and check the answer against the solid. (b) A polyhedron has V = 9 and E = 13. Find F. (c) A connected network is drawn without crossings and has no circuit in it, so the only face is the outside. If it has V = 12 vertices, find E.

Working	Comment
(a) F = E − V + 2 = 15 − 10 + 2	rearrange F + V = E + 2
F = 7	
check: 5 rectangle side faces + 2 pentagon ends = 7 ✓	count on the solid, not on the formula
(b) F = 13 − 9 + 2 = 6	same rearrangement
(c) F = 1, so E = V + F − 2 = 12 + 1 − 2	the outside counts as the one face
E = 11	■

Example 6 — A house circuit as a network

One house circuit runs from a breaker rated at 16 A; on it are a microwave (6 A), a toaster (4 A), a fridge (2 A) and a kettle (10 A). The currents of all running devices add along the one line, and the breaker trips if the total passes 16 A.



One house circuit: the currents of all running devices add along the one line.

A house circuit diagram: a breaker rated 16 A on one line with four devices hanging off it, microwave 6 A, toaster 4 A, fridge 2 A and kettle 10 A.

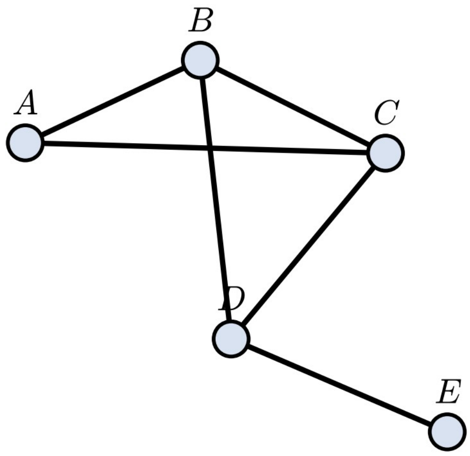
(a) Model the circuit as a network, saying what the vertices, edges and weights stand for. (b) The microwave, toaster and fridge run together. Does the breaker trip? (c) The kettle is switched on as well. What happens? (d) Name one way to run the kettle with the total at or under 16 A.

Working	Comment
(a) vertices = breaker and devices; edge breaker–device = the device is on this line; weight = its current in A	interpreting
(b) $6 + 4 + 2 = 12$ A	currents add
$12 \le 16$: the breaker does not trip	
(c) $12 + 10 = 22$ A	kettle added
$22 > 16$: the breaker trips	
(d) kettle + microwave = $10 + 6 = 16$ A	at the rating, not past it
$16 \le 16$, so kettle and microwave may run together (kettle + toaster + fridge = 16 A also works)	■

Practice questions

Scaffolded

Q1. The figure shows a network on vertices A to E.



A network on five vertices: edges A–B, A–C, B–C, B–D, C–D and D–E.

(a) Count the vertices and the edges.
(b) Find the degree of each vertex.
(c) Check that the degrees add to twice your edge count from (a).
(d) Describe the connectedness of the network. *[Hint: (c) is Result 1; for (d), ask whether a path reaches E.]*

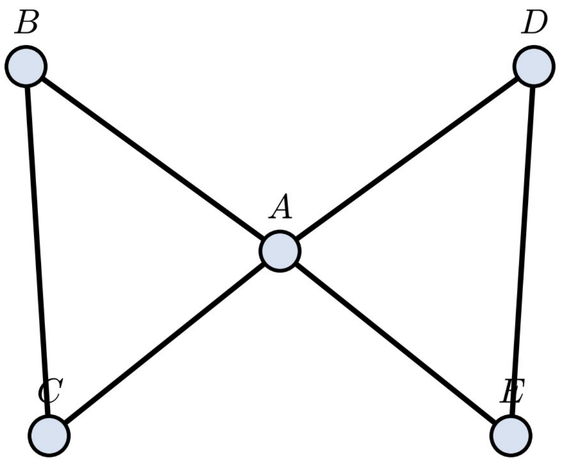
Q2. Four computers A, B, C, D are joined by cables: A–B, A–C, B–C, B–D, C–D. (a) Draw the network, saying what the vertices and edges stand for. (b) Find the degree of each vertex. (c) Is the network connected? *[Hint: for (c), find a path between each pair of vertices, or one that fails.]*

Q3. Describe the connectedness of each network, in one sentence each. (a) Vertices P, Q, R, S with edges P–Q, Q–R, R–S. (b) Vertices P, Q, R, S with edges P–Q and R–S only. (c) The network of (b) with the edge Q–R added. *[Hint: count the separate pieces in each case.]*

Q4. A network has 10 edges and seven vertices, with degrees 3, 3, 4, 4, 2, 2 and one unknown degree x. Find x. *[Hint: Result 1.]*

Q5. A square pyramid has V = 5 and E = 8. Use Euler's formula to find F, then name the faces to check your count. *[Hint: the base is a face too.]*

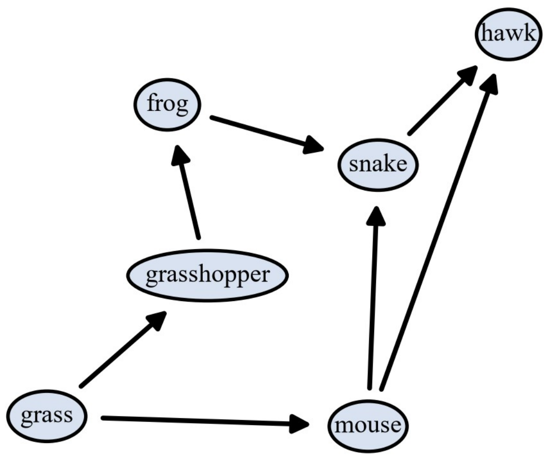
Q6. The figure shows a network made of two triangles sharing the vertex A.



A network of two triangles sharing vertex A: triangle A B C and triangle A D E.

(a) Find the degree of every vertex, and state what Result 4 then says about walking every edge exactly once and returning to the start.
(b) Write out one such walk. *[Hint: all degrees even → an Euler circuit exists.]*

Q7. The figure shows a food web: each arrow points from a food to the animal that eats it.



A food web: each arrow points from the food to the animal that eats it.

A food web on six vertices: grass, grasshopper, mouse, frog, snake, hawk, with arrows grass to grasshopper and mouse; grasshopper to frog; frog to snake; mouse to snake and hawk; snake to hawk.

(a) Say what a vertex stands for, and what an arrow stands for.
(b) How many arrows leave the grass, and how many enter the snake?
(c) If the grasshoppers disappeared from this ecosystem, which one animal loses its only food? *[Hint: for (c), look for a vertex whose single arrow in is from the grasshopper.]*

Q8. Eight players each play every other player once in a round of a chess club. Model this as a network and find the number of games. *[Hint: Result 3 with $n = 8$.]*

Unscaffolded

Q9. A network has five vertices and seven edges. Four of the degrees are 2, 3, 2 and 3. Find the fifth degree.

Q10. At a meeting, each person shakes hands with every other person exactly once, and 28 handshakes are counted. How many people attended? Model the situation as a network in your answer.

Q11. Which of these could be the degree list of a network with five vertices and five edges? For (a), draw one if it is possible. (a) 1, 1, 2, 3, 3 (b) 1, 2, 2, 2, 2

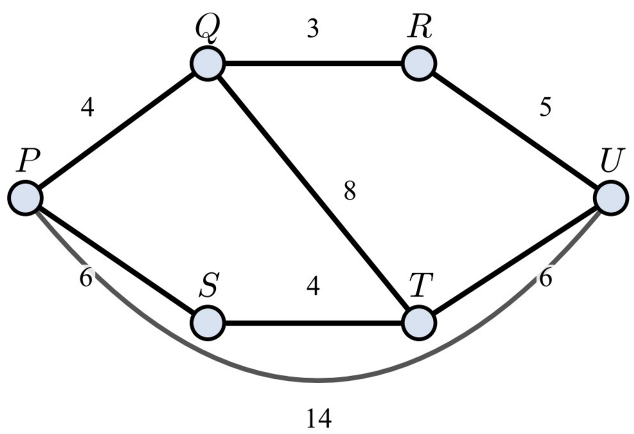
Q12. Back in Königsberg, the town council decides to build one new bridge between any two of the four pieces of land (there may then be two bridges between the same pair). Show that, wherever the new bridge is built, a walk crossing every bridge exactly once becomes possible. Why does the walk still not close into a circuit?

Q13. (a) A hexagonal prism has V = 12 and E = 18. Use Euler's formula to find F. (b) A pyramid on an n-sided base has V = n + 1 and E = 2n. Use Euler's formula to show F = n + 1, and say what those faces are.

Q14. A connected network is drawn without crossings and has V = 14 and E = 21. Counting the outside as a face, find F.

Q15. A connected network drawn without crossings contains no circuit at all, so its only face is the outside. If it has 15 vertices, how many edges does it have?

Q16. The figure shows a road network; the weights are distances in km.



A road network: each weight is a distance in km.

A weighted road network on six vertices P to U. Upper route P–Q 4, Q–R 3, R–U 5; lower route P–S 6, S–T 4, T–U 6; cross road Q–T 8; and a long direct road P–U 14 drawn as a curve below.

(a) Find the shortest route from P to U and its length.
(b) The road R–U is closed by a flood. Find the shortest route from P to U now.
(c) Describe the connectedness of the network while R–U is closed.

Q17. Five students model their friendships as a network: vertices Aisha, Ben, Chloe, Dan, Ella; an edge joins two friends. The edges are Aisha–Ben, Aisha–Chloe, Ben–Chloe, Chloe–Dan, Dan–Ella, Chloe–Ella. (a) Find the degree of each vertex. (b) Chloe moves away. Into how many separate friendship groups does the network fall, and who is in each?

Q18. A food web on a farm edge is described as follows: grass is eaten by the rabbit and the mouse; the rabbit is eaten by the fox and the hawk; the mouse is eaten by the snake; the snake is eaten by the hawk. (a) Model this as a network, saying what the vertices and the directed edges stand for, and count the edges. (b) Name the vertices with no arrow entering them, and the vertices with no arrow leaving them.

Q19. A home network has a router joined directly to a laptop, a printer, a phone, a TV and a console, and also joined to a switch; the switch is joined to two more devices. (a) Model this as a network and count its vertices and edges. (b) Find the degree of the router and of the switch, and check Result 1. (c) Describe the connectedness, and say what happens to it if the switch fails.

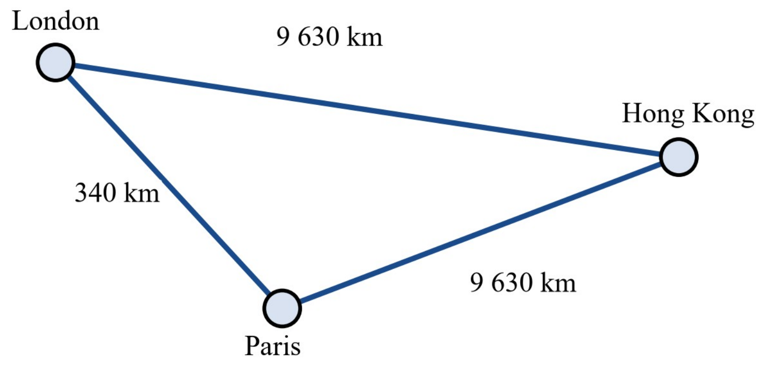
Q20. Prove Result 2: in any network, the number of vertices of odd degree is even. (Use Result 1, and the facts that a sum of even numbers is even, and that a sum of odd numbers is even only when it has an even number of terms.)

Q21. A campsite circuit runs from a breaker rated at 16 A. Plugged into it are devices drawing 10 A, 8 A, 6 A, 4 A and 2 A. List every combination of devices whose currents add to exactly 16 A, and explain why the 10 A and 8 A devices can never run at the same time on this circuit.

Q22. Königsberg again. (a) Explain in two lines why no walk could cross every bridge once and return to its start, even before Euler's test is quoted. (b) Find the smallest number of new bridges that would make such a closed walk possible, and say where they could be built.

Real context

Q23. Direct flights run between London and Paris, between London and Hong Kong, and between Paris and Hong Kong. The great-circle distances, rounded to the nearest 10 km, are: London–Paris 340 km, London–Hong Kong 9 630 km, Paris–Hong Kong 9 630 km.



Air routes between the three cities; weights are great-circle distances rounded to the nearest 10 km.

A triangle network of three cities: London, Paris, Hong Kong. Edge weights: London–Paris 340 km, London–Hong Kong 9 630 km, Paris–Hong Kong 9 630 km.

(a) Model the routes as a network, saying what the vertices, edges and weights stand for, and find the degree of each vertex.
(b) A passenger flies London → Paris → Hong Kong → London, using each route exactly once. Show that this is a circuit in the network sense, and find the total distance flown.
(c) The Paris–Hong Kong route is suspended for a season. Find the shortest way for a passenger to fly from Paris to Hong Kong now, and how much longer it is than the direct route.

Source: city coordinates from the Wikipedia articles "London", "Paris" and "Hong Kong" (London 51.50722° N, 0.12750° W; Paris 48.8567° N, 2.3522° E; Hong Kong 22.250° N, 114.167° E); distances are great-circle distances computed from those coordinates with Earth radius 6371 km, rounded to the nearest 10 km. Retrieved 20 August 2026.

Answers

Q1. (a) $V = 5$, $E = 6$. (b) A 2, B 3, C 3, D 3, E 1. (c) $2 + 3 + 3 + 3 + 1 = 12 = 2 \times 6$ ✓. (d) connected — every vertex has a path to every other (E only through D). **Q2.** (a) vertices = computers, edges = cables. (b) A 2, B 3, C 3, D 2. (c) connected. **Q3.** (a) connected, one piece. (b) disconnected, two pieces. (c) connected again, one piece. **Q4.** $x = 2$. **Q5.** $F = 5$: four triangle side faces and the square base. **Q6.** (a) A 4, B 2, C 2, D 2, E 2 — all even, so an Euler circuit exists. (b) A → B → C → A → D → E → A. **Q7.** (a) vertex = an organism; arrow = "is eaten by" (energy flows along the arrow). (b) two arrows leave the grass; two enter the snake. (c) the frog. **Q8.** $8 \times 7/2 = 28$ games. **Q9.** 4. **Q10.** 8 people. **Q11.** (a) possible — e.g. edges A–B, A–C, A–D, B–C, B–E give degrees 3, 3, 2, 1, 1. (b) impossible: the degrees add to 9, which is odd, against Result 1. **Q12.** All four degrees (5, 3, 3, 3) are odd; a new bridge adds 1 to two of them, making those two even and leaving exactly two odd — so an Euler walk exists (starting at one odd vertex, ending at the other). It cannot close because two odd vertices remain, not zero. **Q13.** (a) $F = 8$. (b) $F = E - V + 2 = 2n - (n + 1) + 2 = n + 1$: the n triangle side faces and the n-sided base. **Q14.** $F = 9$. **Q15.** 14. **Q16.** (a) P → Q → R → U, 12 km. (b) the direct road, 14 km. (c) still connected, in one piece. **Q17.** (a) Aisha 2, Ben 2, Chloe 4, Dan 2, Ella 2. (b) two groups: {Aisha, Ben} and {Dan, Ella}. **Q18.** (a) vertices = the six organisms; directed edge = "is eaten by"; 6 edges. (b) no arrow in: grass. No arrow out: fox, hawk. **Q19.** (a) $V = 9$, $E = 8$. (b) router 6, switch 3; degrees $6 + 3 + 7 \times 1 = 16 = 2 \times 8$ ✓. (c) connected; if the switch fails, the two devices behind it form a disconnected piece. **Q20.** By Result 1 the degree sum is 2E, even. The even degrees contribute an even sum, so the odd degrees contribute an even sum; a sum of odd numbers is even only with an even number of terms. ■ **Q21.** {10, 6}, {10, 4, 2}, {8, 6, 2}. $10 + 8 = 18 > 16$, so those two can never run together. **Q22.** (a) A closed walk through a vertex uses two of its edges, so every degree would have to be even; all four are odd. (b) Two: one bridge between one pair of land masses and one between the other pair makes all four degrees even (e.g. new bridges K–N and S–L). One bridge is not enough — it fixes only two of the four odd degrees. **Q23.** (a) vertices = cities, edges = direct routes, weights = great-circle distances in km; every vertex has degree 2. (b) it starts and finishes at London and uses each edge once — a circuit; 340 + 9 630 + 9 630 = 19 600 km. (c) Paris → London → Hong Kong, 340 + 9 630 = 9 970 km, which is 340 km longer than the direct route.

Fully-worked solutions

Q1. (a) Five dots, six lines: $V = 5$, $E = 6$. (b) A: edges to B, C → 2. B: to A, C, D → 3. C: to A, B, D → 3. D: to B, C, E → 3. E: to D → 1. (c) $2 + 3 + 3 + 3 + 1 = 12$ and $2E = 12$ ✓ (Result 1). (d) Connected: A, B, C, D reach each other inside the triangle-and-tail, and E joins on through D. One piece.

Q2. (a) Vertices = the four computers; an edge = a cable joining two of them. Draw A, B, C as a triangle with D joined to B and C. (b) A 2 (to B, C); B 3 (to A, C, D); C 3 (to A, B, D); D 2 (to B, C). (c) Connected: A↔B direct, A↔C direct, A↔D via B, B↔C direct, B↔D direct, C↔D direct.

Q3. (a) Connected: P→Q→R→S runs through all four vertices, so every pair has a path; one piece. (b) Disconnected: P and Q reach each other and R and S reach each other, but no path joins the pair {P, Q} to the pair {R, S}; two pieces. (c) Connected: Q–R joins the two pieces of (b) into one.

Q4. Result 1: degree sum = $2E = 20$. So $3 + 3 + 4 + 4 + 2 + 2 + x = 20 \rightarrow 18 + x = 20 \rightarrow x = 2$.

Q5. $F = E - V + 2 = 8 - 5 + 2 = 5$. Check on the solid: four triangle side faces and the square base — 5 ✓.

Q6. (a) A lies on both triangles: degree 4; B, C, D, E each lie on one triangle's two sides: degree 2 each. All five degrees are even, so by Result 4 a walk using every edge once can close into an Euler circuit. (b) A → B → C → A → D → E → A: edges AB, BC, CA, AD, DE, EA — all six, each once, back at A.

Q7. (a) A vertex = an organism; an arrow from X to Y = X is eaten by Y (energy flows along the arrow). (b) Out of the grass: to the grasshopper and the mouse — 2. Into the snake: from the frog and the mouse — 2. (c) The frog: its only arrow in is from the grasshopper. (The snake and the hawk keep other arrows in.)

Q8. Vertices = players; an edge = a game. Every pair once is Result 3 with $n = 8$: $E = 8 \times 7/2 = 28$ games.

Q9. Degree sum = $2E = 14$. So the fifth degree is $14 - (2 + 3 + 2 + 3) = 14 - 10 = 4$.

Q10. Vertices = people; an edge = a handshake. Result 3: $n(n - 1)/2 = 28 \rightarrow n(n - 1) = 56 = 8 \times 7 \rightarrow n = 8$ people.

Q11. (a) The degrees add to $1 + 1 + 2 + 3 + 3 = 10 = 2 \times 5$, so Result 1 allows it. It is possible: label the vertices A 3, B 3, C 2, D 1, E 1 and draw A–B, A–C, A–D, B–C, B–E. (b) The degrees add to $1 + 2 + 2 + 2 + 2 = 9$, an odd number; Result 1 says the sum must be 2E, even. Impossible.

Q12. The degrees are K 5, N 3, S 3, L 3 — all four odd. A new bridge joins two land masses and adds 1 to each of their degrees: those two become even, the other two stay odd. Exactly two odd vertices remain, so by Result 4 an Euler walk exists, starting at one of the remaining odd vertices and ending at the other — and this whatever pair the new bridge joins. The walk cannot close into a circuit, because a circuit needs zero odd vertices and two remain.

Q13. (a) $F = E - V + 2 = 18 - 12 + 2 = 8$ (six rectangle side faces and two hexagon ends). (b) $F = E - V + 2 = 2n - (n + 1) + 2 = n + 1$. On the solid: n triangle side faces and one n-sided base — $n + 1$ ✓.

Q14. $F = E - V + 2 = 21 - 14 + 2 = 9$ faces, counting the outside.

Q15. No circuit means the one outside region is the only face: $F = 1$. Then $E = V + F - 2 = 15 + 1 - 2 = 14$ edges.

Q16. (a) Compare the routes: direct 14; P→Q→R→U = 4 + 3 + 5 = 12; P→S→T→U = 6 + 4 + 6 = 16; P→Q→T→U = 4 + 8 + 6 = 18. Shortest: P → Q → R → U, 12 km. (b) With R–U closed the 12 km route is gone; the remaining options are 14, 16 and 18, so the shortest is the direct road, 14 km. (c) Still connected: every pair of vertices still has a path (the direct road alone keeps P and U joined to the rest through the other roads). One piece.

Q17. (a) Aisha 2 (Ben, Chloe); Ben 2 (Aisha, Chloe); Chloe 4 (Aisha, Ben, Dan, Ella); Dan 2 (Chloe, Ella); Ella 2 (Dan, Chloe). (b) Remove Chloe and her four edges: what remains is Aisha–Ben and Dan–Ella. Two separate groups: {Aisha, Ben} and {Dan, Ella}. Chloe was the only vertex joining the two pairs.

Q18. (a) Vertices = grass, rabbit, mouse, fox, snake, hawk; a directed edge X → Y = X is eaten by Y. Edges: grass→rabbit, grass→mouse, rabbit→fox, rabbit→hawk, mouse→snake, snake→hawk — 6 edges. (b) No arrow enters the grass (nothing eats it here — it is the producer). No arrow leaves the fox or the hawk (nothing eats them here).

Q19. (a) Vertices: router, switch and 7 devices = 9. Edges: router to 5 devices (5), router to switch (1), switch to 2 devices (2) — E = 8. (b) Router: 5 + 1 = 6. Switch: 1 + 2 = 3. The seven devices have degree 1 each. Sum = $6 + 3 + 7 = 16 = 2 \times 8$ ✓ (Result 1). (c) Connected: every device has a path to the router, directly or through the switch. If the switch fails (its edges go), the two devices behind it lose their only path — the network falls into three pieces (the router's piece, and the two isolated devices), so it is disconnected.

Q20. Let the network have E edges. By Result 1 the sum of all degrees is 2E, which is even. Split the sum into the even degrees and the odd degrees. A sum of even numbers is even, so the even degrees contribute an even total; the odd degrees must therefore contribute 2E minus an even number, which is even. Each odd degree is an odd number, and a sum of odd numbers is even only when the number of terms is even. Hence the number of odd vertices is even. ■

Q21. Check the combinations: 10 + 6 = 16; 10 + 4 + 2 = 16; 8 + 6 + 2 = 16. No other choice gives 16 (10 + 8 = 18, and adding more only increases it; 8 + 4 + 2 = 14; 6 + 4 + 2 = 12; single devices give less). So the exact-16 combinations are {10, 6}, {10, 4, 2} and {8, 6, 2}. The 10 A and 8 A devices together draw 18 A > 16 A, past the breaker's rating, so they can never run at the same time on this circuit.

Q22. (a) A closed walk passes through each vertex on its way, using two of that vertex's edges each time, so every degree would have to be even — but all four degrees (5, 3, 3, 3) are odd. (b) Two new bridges. One new bridge leaves two odd vertices (Solution Q12), which allows an open walk but not a closed one; a second new bridge between the two remaining odd land masses makes all four degrees even, and then an Euler circuit exists. For example, build one bridge K–N (degrees become K 6, N 4) and one S–L (S 4, L 4): all even, closed walk possible. One bridge can never be enough, since it changes the parity of only two of the four odd vertices.

Q23. (a) Vertices = the three cities; an edge = a direct flight route; weight = the great-circle distance of the route in km. Each city sits on two routes, so every vertex has degree 2. (b) The trip starts at London, returns to London, and uses each of the three edges exactly once — a closed walk using every edge once, i.e. a circuit (an Euler circuit, in fact, since all degrees are even). Total = 340 + 9 630 + 9 630 = 19 600 km. (c) With Paris–Hong Kong suspended, the remaining network is Paris–London and London–Hong Kong, and the only Paris→Hong Kong route is Paris →

London → Hong Kong: 340 + 9 630 = 9 970 km. That is 9 970 − 9 630 = 340 km longer than the direct route — one extra London–Paris leg.

Teacher note

- **Prior knowledge rehearsed (D3):** none re-taught — 5B is the entry point to the whole networks Area of Study (§5). The arithmetic used is adding, doubling/halving and odd/even, all secured well before Level 9; reading diagrams that stand for real things (maps, flow charts) is assumed from the primary years. No Level 9 code is re-taught because none is needed.
- **Scope check (D6):** the section stops at what VC2M10SP02 asks — interpreting networks and describing connectedness. In scope: degrees and the degree sum (Result 1), the odd-vertex parity fact (Result 2), the complete-network count (Result 3), Euler's walk test used as a test (Result 4, proof of the "only if" half given, converse used without proof at this level), Euler's formula F + V = E + 2 verified on real polyhedra (Result 5), weighted and directed readings. Out of scope: Hamilton walks, shortest-path and minimum-connector algorithms, flow, matching and scheduling — all General Mathematics Units 3&4 material, named here only as destination. Definitions adopted at first use: network, vertex/node, edge/link, degree, weight, path, circuit (closed walk), connected/disconnected, isolated vertex, polyhedron, Euler walk/circuit (named for Euler, 1736). `VC2M10SP02.E06` (kinship systems as networks) excluded per Rule 1 — ACCEPTED exclusion, front-of-book note.
- **Tool classes (D13):** none — SP02 is not a digital-tool subtopic, and 5B carries no digital task.
- **Assessment:** every item scores the two verbs of the CD — *interpret* (what the vertices, edges, directions and weights stand for) and *describe connectedness* (how many pieces, which vertices join them) — plus the counting results. A right number with no reading of the diagram scores half.
- **VCE destinations (§5 forward map):** 5B → **GM U3&4 AoS 4 'Discrete mathematics: Networks and decision mathematics'** (graphs and networks, exploring and travelling, trees and minimum connectors, shortest path, flow, matching and scheduling) and **GM U2 AoS 2 'Discrete maths: graphs & networks'**; also **SM U1 AoS 2 'Discrete mathematics'**. Cross-subject (implicit prerequisite 11): VCE Biology and Environmental Science read food webs and metabolic pathways as networks — 5B is the only mathematical treatment of that idea, which is why Q7 and Q18 sit here.

Test

Test T3 · Chapters 4–5 — Measurement and space · Level 10

This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 10 achievement standard, not against a mark on this paper.

Time: 35 minutes — 5 minutes reading time plus 30 minutes writing time (1 minute per mark).

Structure: two parts, as the curriculum itself is split (D13, §7). **Part A is technology-free** (12 marks): no calculators and no digital tools. **Part B is technology-active** (18 marks): a scientific calculator, spreadsheet, graphing tool, dynamic geometry software or a general purpose programming language, as nominated by the teacher. **No CAS.** Answer all questions.

Equipment: ruler, protractor and pair of compasses.

Reasoning: in this paper, reasoning is marked separately from the answer where the content description requires a proof or a stated reason (Q3, Q4c and Q8). On those items a correct answer with no licensing theorem does not score full marks.

Cumulative reach: tests are cumulative (§7.1), so this paper may assume Chapters 1–3. The skills it draws on are used as tools, exactly as the chapters used them: powers of 10 and the exponent laws (1B), the volume formulas for prisms and cylinders rehearsed in 4A from Year 9 (`VC2M9M01`), and the rounding habit of 1A — keep values exact while working, round once at the end. No mark is scored against a Chapter 1–3 content description.

Dated items (R3): none — the real figures on this paper (the heights of Eureka Tower, the stations of the City Loop) are stable facts, not current prices or rates.

Mark schedule — 5 marks per content description the group owns (§7.1); every CD is separately scored:

Subtopic	CD	Part A	Part B	Total	Items
4A	`VC2M10M01`	2	3	**5**	Q1, Q5
4B	`VC2M10M03`	2	3	**5**	Q2, Q7a, Q7b
4C	`VC2M10M04`	0	5	**5**	Q7c, Q7d, Q9
4D	`VC2M10M02`	0	5	**5**	Q6
5A	`VC2M10SP01`	3	2	**5**	Q3, Q8
5B	`VC2M10SP02`	5	0	**5**	Q4
Total		**12**	**18**	**30**	

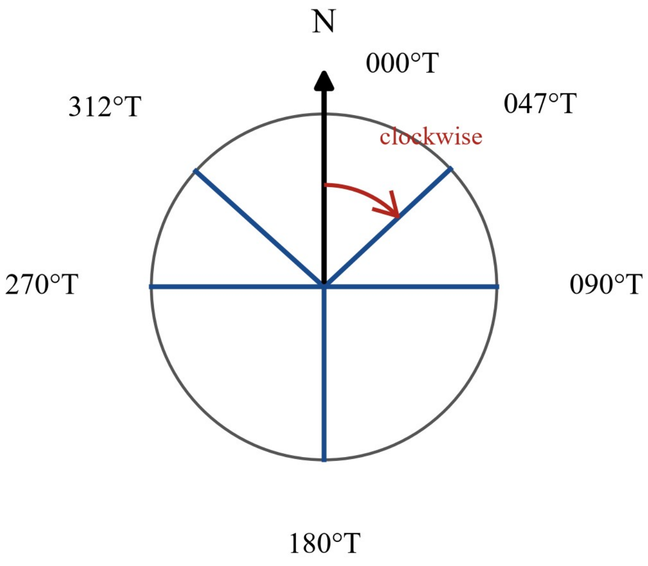
The real data in Q4 and Q7 carries its source line at the point of use (R1). Every other context is a clearly-labelled SCENARIO whose numbers are given in the question (R2).

Part A — technology-free (12 marks)

Q1 (2 marks). SCENARIO — a planter box. A planter box is built as one box, $50\,\text{cm}$ by $30\,\text{cm}$ by $20\,\text{cm}$, with a rectangular cavity $40\,\text{cm}$ by $20\,\text{cm}$ by $15\,\text{cm}$ cut out of the top. a. Find the volume of material in the box. **(1 mark)** b. Find the capacity of the cavity, in litres. **(1 mark)**

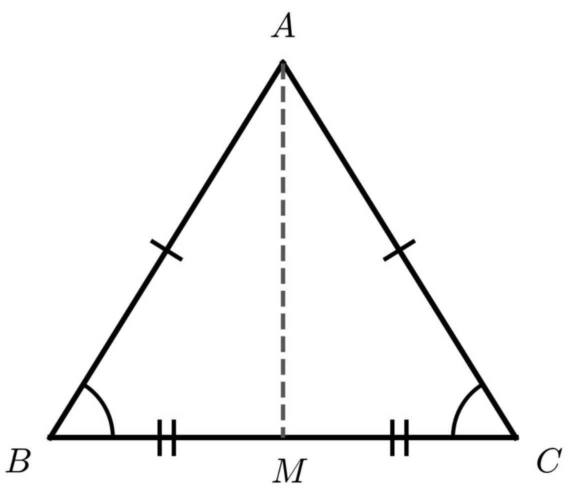
Q2 (2 marks). a. Find the back bearing of `078°T`. **(1 mark)** b. The bearing of a tower from a ship is `304°T`. Which one of the following is the acute angle that the line from the ship to the tower makes with the north line at the ship? **(1 mark)**

A 34 ° B 56 ° C 66 ° D 124 °



A compass circle with a north arrow pointing up and rays drawn clockwise from north, labelled as three-figure bearings 000 degrees T, 047 degrees T, 090 degrees T, 180 degrees T, 270 degrees T and 312 degrees T

Q3 (3 marks). In △ABC, AB = AC and M is the midpoint of BC. Prove that AM ⊥ BC.



Isosceles triangle ABC with apex A, M the midpoint of base BC; AM dashed, single ticks on the equal sides AB and AC, double ticks on BM and MC, equal angle arcs at B and C.

Set the proof out the way 5A does: one statement per line, each with its reason. The marks are for the chain — the congruence and its licensing test, the matched angles, and the closing step.

Q4 (5 marks). Real Victorian context — the City Loop.

The City Loop rail service in central Melbourne calls, in order, at Flinders Street, Parliament, Melbourne Central, Flagstaff and Southern Cross, and then returns to Flinders Street. Model the service as a network with one vertex per station and one edge between consecutive stations on the loop.

a. Draw the network, say what the vertices and edges stand for, and find the degree of each vertex. **(1 mark)**
b. Verify Result 1 of 5B — the degrees add to twice the number of edges — on your network. **(1 mark)**

c. A track-inspection train must travel over every edge of the loop exactly once and finish back at Flinders Street. Is this possible? Name the result that licenses your answer. **(1 mark)**
d. When the network is drawn in the plane without crossings, how many faces does it have, counting the outside as a face? Verify Euler's formula, F + V = E + 2, for this network. **(2 marks)**

Source: station order and loop layout from "City Loop", Wikipedia (the Metro Trains Melbourne City Loop: Flinders Street, Parliament, Melbourne Central, Flagstaff, Southern Cross, returning to Flinders Street). Retrieved 23 August 2026.

Part B — technology-active (18 marks)

Q5 (3 marks). SCENARIO — a monument. A monument is built from a rectangular-prism base, $1.2\,\mathrm{m}$ by $0.8\,\mathrm{m}$ by $0.5\,\mathrm{m}$, with a solid cylinder of radius $0.2\,\mathrm{m}$ and height $1.5\,\mathrm{m}$ standing on top of it. a. Find the volume of the base. **(1 mark)** b. Find the volume of the cylinder, exact in π. **(1 mark)** c. Find the total volume of the monument, in litres, to the nearest litre. **(1 mark)**

Q6 (5 marks). a. On a logarithmic scale calibrated in powers of 10, which one of the following is closest to the value midway between the marks 10 and 100? **(1 mark)**

A 25 B 32 C 50 D 55

b. How many orders of magnitude separate 5 and $50\,000$? **(1 mark)**
c. Earthquake X measures M4.1 on the Richter scale and earthquake Y measures M5.1. Compare (i) the shaking of the ground and (ii) the energy released. **(2 marks)**
d. A sound-level meter reads $35\,\mathrm{dB}$ in a library and $71\,\mathrm{dB}$ beside a machine. How many times more intense is the machine sound than the library sound, to the nearest whole number? **(1 mark)**

Q7 (6 marks). Real Victorian context — Eureka Tower.

Eureka Tower in Southbank, Melbourne, is $297.3\,\mathrm{m}$ tall. Its observation deck, the Skydeck, is $285\,\mathrm{m}$ above the ground. A surveyor stands on flat ground at a point where the base of the tower is $430\,\mathrm{m}$ away, measured along the ground. Ignore the height of the surveyor and the instruments.

a. Find the angle of elevation from the surveyor to the top of the tower, to one decimal place. **(2 marks)**
b. The bearing of the surveyor from the Skydeck is $017°\mathrm{T}$. Find the bearing of the Skydeck from the surveyor. **(1 mark)**
c. The $430\,\mathrm{m}$ distance was recorded to the nearest $10\,\mathrm{m}$. State the lower and upper bounds of the true distance. **(1 mark)**
d. Recalculate the angle of elevation of part (a) using the upper bound of the distance, to one decimal place. Is this angle larger or smaller than the angle of part (a)? **(2 marks)**

Source: building heights from "Eureka Tower", Wikipedia (tower height 297.3 m; the Skydeck on level 88 at 285 m). Retrieved 23 August 2026.

Q8 (2 marks). In △PQR, X is the midpoint of PQ and Y is the midpoint of PR. QR = 18.6 cm and ∠PQR = 43°. a. Find XY. **(1 mark)** b. Find ∠PXY. **(1 mark)** Name the theorem that licenses each answer.

Q9 (2 marks). SCENARIO — a block of land. The length of a block of land is recorded as $86\,\mathrm{m}$, to the nearest metre. a. State the lower and upper bounds of the true length. **(1 mark)** b. Find the greatest possible percentage error of the recording, to one decimal place. **(1 mark)**

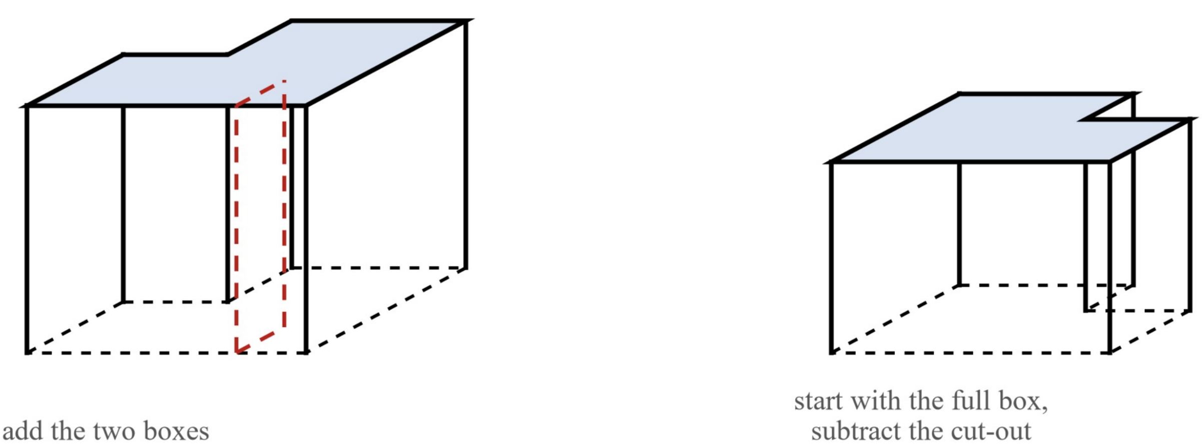
Solutions

Answer key

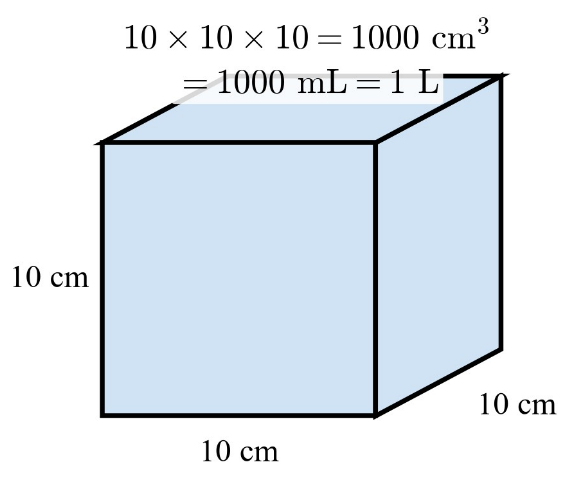
Item	Part	CD	Answer	Marks
Q1a	A	VC2M10M01	$18\,000$ cm³	1
Q1b	A	VC2M10M01	12 L	1
Q2a	A	VC2M10M03	258°T	1
Q2b	A	VC2M10M03	B ($56°$)	1
Q3	A	VC2M10SP01	proof: SSS → matched angles → (A1)	3
Q4a	A	VC2M10SP02	cycle drawing; every degree 2	1
Q4b	A	VC2M10SP02	$10 = 2 \times 5$ ✓	1
Q4c	A	VC2M10SP02	yes — 0 odd vertices (Result 4)	1
Q4d	A	VC2M10SP02	$F = 2$; $2 + 5 = 5 + 2$ ✓	2
Q5a	B	VC2M10M01	0.48 m³	1
Q5b	B	VC2M10M01	0.06π m³	1
Q5c	B	VC2M10M01	668 L	1
Q6a	B	VC2M10M02	B (≈ 32)	1
Q6b	B	VC2M10M02	4	1
Q6c	B	VC2M10M02	(i) 10 times; (ii) about 32 times	2
Q6d	B	VC2M10M02	≈ 3981	1
Q7a	B	VC2M10M03	$34.7°$	2
Q7b	B	VC2M10M03	197°T	1
Q7c	B	VC2M10M04	425 m and 435 m	1
Q7d	B	VC2M10M04	$34.4°$, smaller	2
Q8a	B	VC2M10SP01	9.3 cm	1
Q8b	B	VC2M10SP01	$43°$	1
Q9a	B	VC2M10M04	85.5 m and 86.5 m	1
Q9b	B	VC2M10M04	$\approx 0.6\%$	1
			Total	**30**

Fully-worked solutions

Q1. a Volume of the full box first, then subtract the cavity (4A §3 — subtract a cut-out): $V_{\text{box}} = 50 \times 30 \times 20 = 30\,000$ cm³. $V_{\text{cavity}} = 40 \times 20 \times 15 = 12\,000$ cm³. $V = 30\,000 - 12\,000 = 18\,000$ cm³. b Capacity uses the cavity, not the material: 1000 cm³ $= 1$ L, so $12\,000 \div 1000 = 12$ L. ∴ $18\,000$ cm³ of material; a capacity of 12 L — the cavity is what holds the water, the material is what is left.

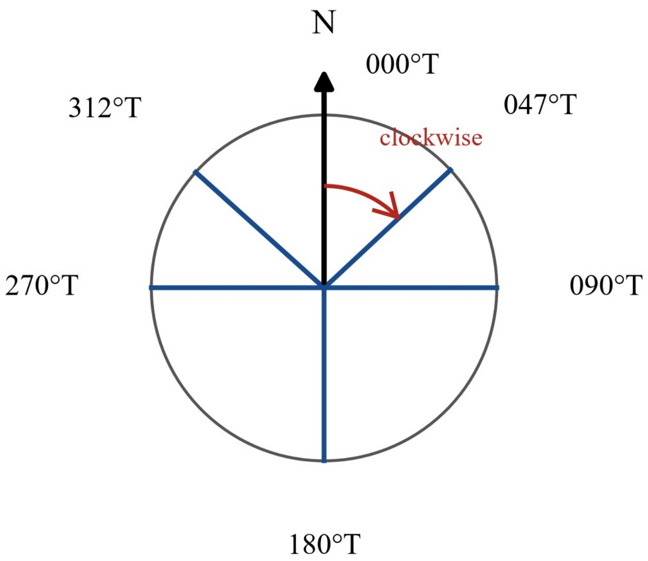


The same L-shaped solid two ways: split into two boxes whose volumes add, and as a full box with a smaller cut-out box subtracted



A cube 10 cm by 10 cm by 10 cm: its volume of 1000 cubic centimetres is 1000 mL, exactly one litre

Q2. a Add $180°$; the sum does not pass $360°$, so no subtraction is needed: $78° + 180° = 258°$. The back bearing is `258°T`. b `304°T` is measured clockwise from north, past `270°T` (west). The acute angle back to the north line is the rest of the full turn: $360° - 304° = 56°$. ∴ `258°T`; and the answer is **B** ($56°$).

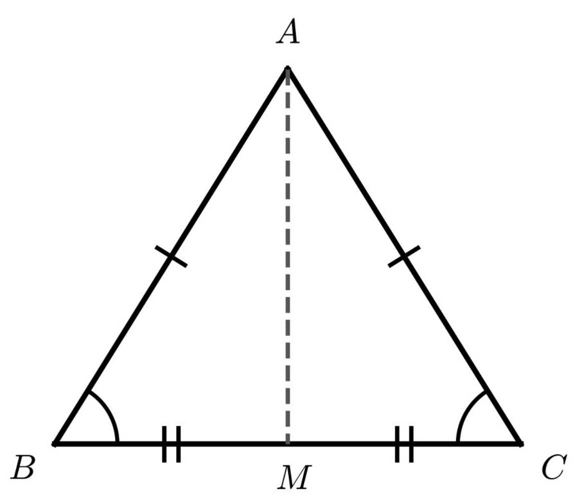


A compass circle with a north arrow pointing up and rays drawn clockwise from north, labelled as three-figure bearings 000 degrees T, 047 degrees T, 090 degrees T, 180 degrees T, 270 degrees T and 312 degrees T

Q3. Given: △ABC with AB = AC; M the midpoint of BC. **To show:** AM ⊥ BC.

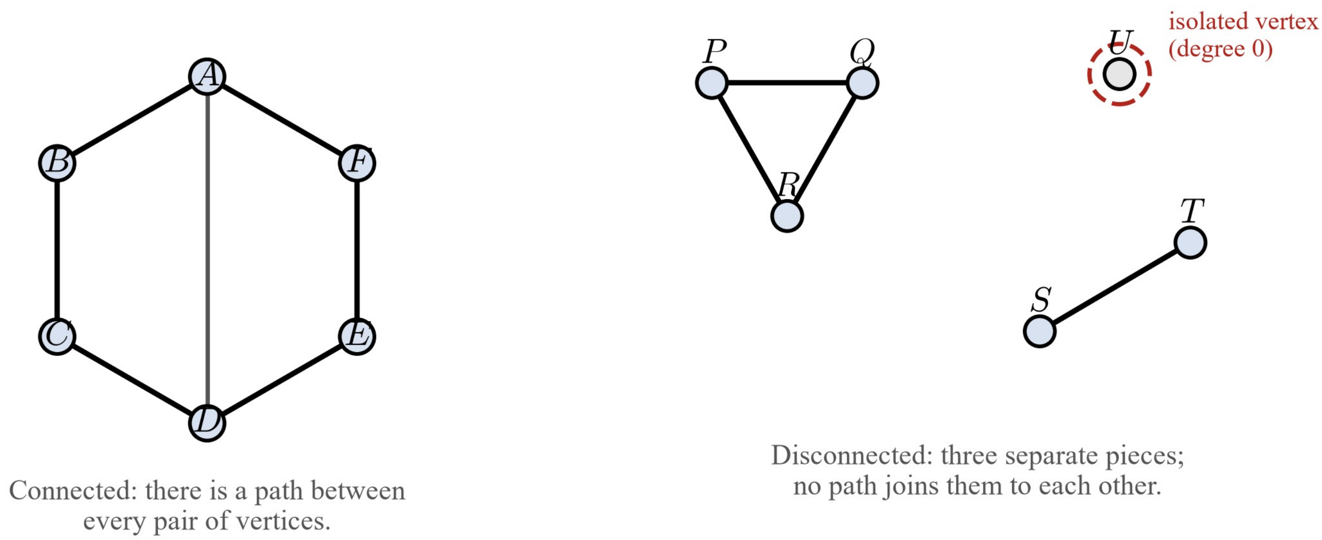
#	Statement	Reason
1	AB = AC	given
2	BM = CM	M is the midpoint of BC
3	AM = AM	common side
4	∴ △ABM ≅ △ACM	SSS (1, 2, 3)
5	∴ ∠AMB = ∠AMC	matched angles of congruent triangles
6	∠AMB + ∠AMC = 180°	angles on a straight line (A1)
7	∴ ∠AMB = ∠AMC = 90°, so AM ⊥ BC	from 5 and 6 ■

Marks: the congruence with its licensing test (lines 1–4) carries one mark, the matched angles (line 5) one, and the closing straight-line step (lines 6–7) one. The conclusion without the chain scores nothing — 5A scores the chain, not the conclusion.



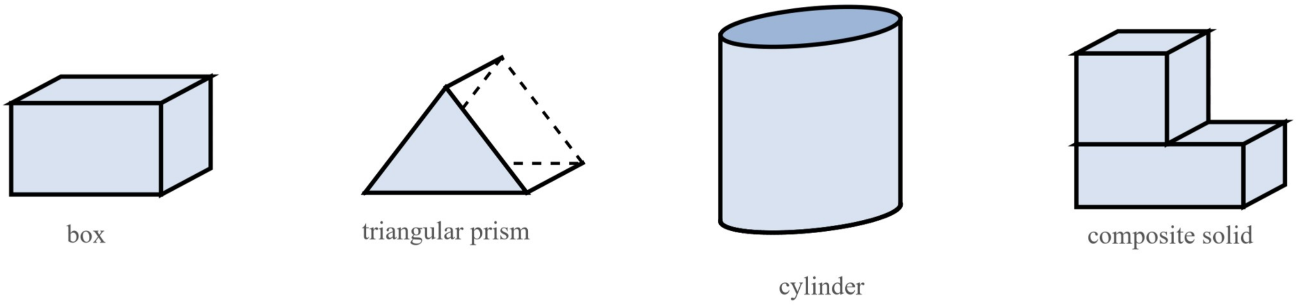
Isosceles triangle ABC with apex A, M the midpoint of base BC; AM dashed, single ticks on the equal sides AB and AC, double ticks on BM and MC, equal angle arcs at B and C.

Q4. a Vertices = the five stations; an edge = the stretch of track between two consecutive stations on the loop. The drawing is a closed loop of five vertices — Flinders Street to Parliament, Parliament to Melbourne Central, Melbourne Central to Flagstaff, Flagstaff to Southern Cross, Southern Cross back to Flinders Street. Every station has exactly two neighbours, so every degree is 2. b Degree sum $=2+2+2+2+2=10$, and $2\times E=2\times 5=10$: the two agree ✓ (Result 1). c Every vertex has degree 2, an even number, so there are **0 odd vertices**. Result 4 (Euler's walk test) says a walk using every edge exactly once is possible when the number of odd vertices is 0 or 2, and with 0 it closes into a circuit. Yes — the train can start at Flinders Street, use every edge once, and finish back there. d The loop encloses one inside region; counting the outside, $F=2$. Check: $F+V=2+5=7$, and $E+2=5+2=7$ ✓ (Result 5, Euler's formula — the outside counts as a face). ∴ degrees all 2; degree sum $10=2\times 5$; yes, by Result 4 with 0 odd vertices; $F=2$ and $F+V=E+2=7$.

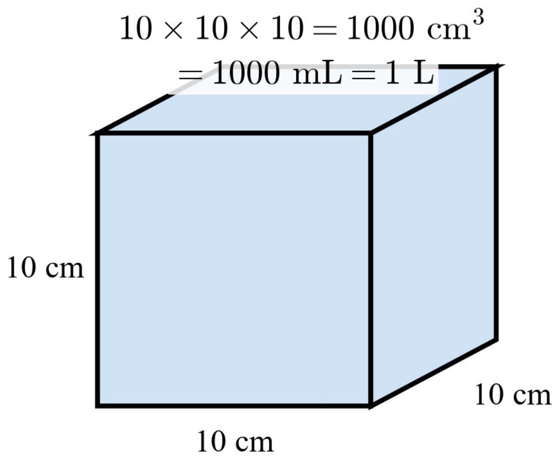


Two panels. Left: a connected network of six vertices A to F in a hexagon with one extra edge from A to D. Right: a disconnected network in three pieces: triangle P Q R, single edge S T, and isolated vertex U circled in red with degree 0.

Q5. a $V=lwh=1.2\times 0.8\times 0.5=0.48$ m³. b $V=\pi r^2 h=\pi\times 0.2^2\times 1.5=0.06\pi$ m³ — leave π exact (1A, 4A). c Add the parts, then round once at the end: $V_{\text{total}}=0.48+0.06\pi=0.66849\ldots$ m³ — keep the calculator value, with π exact to here. 1 m³ = 1000 L, so $0.66849\ldots\times 1000=668.49\ldots$ L, which rounds to 668 L. ∴ 0.48 m³; 0.06π m³; 668 L — add the parts for volume, convert at the end for capacity.

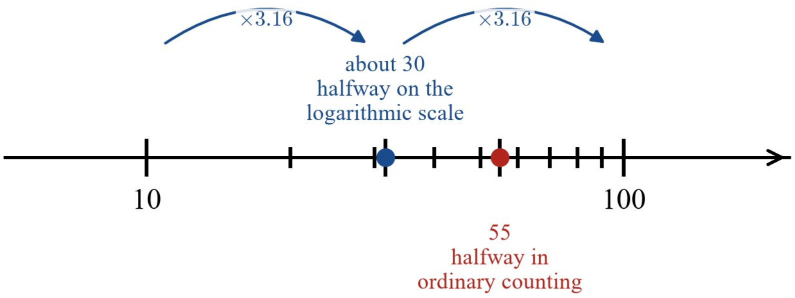


A box, a triangular prism and a cylinder, the parts this subtopic builds with, beside a composite solid made from two boxes joined together

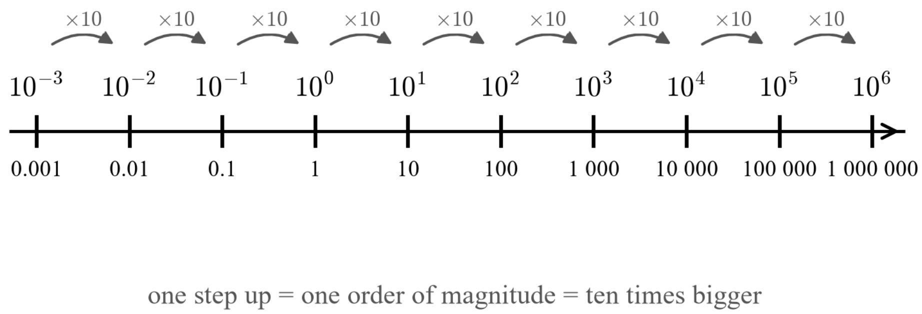


A cube 10 cm by 10 cm by 10 cm: its volume of 1000 cubic centimetres is 1000 mL, exactly one litre

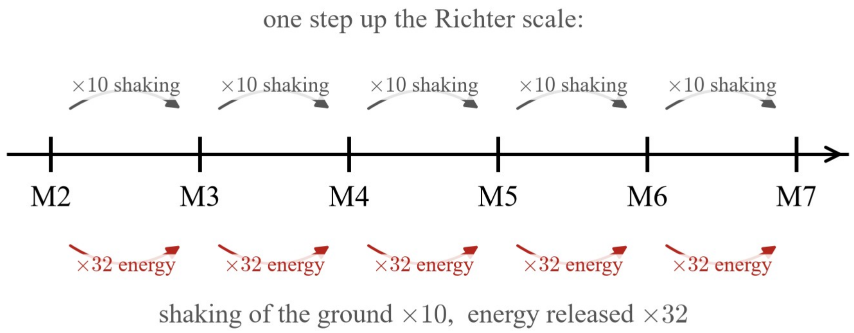
Q6. a The midway mark of a decade sits the same *factor* above the start as below the end: $10 \times f = m$ and $m \times f = 100$, so $m \times m = 10 \times 100 = 1000$ and $m \approx 31.6$. The closest option is 32 — the answer is **B**. (55 is the trap: the midway point in ordinary counting, not on the logarithmic scale — 4D §4.) b Divide to find the factor: $50\,000 \div 5 = 10\,000 = 10^4$. Four orders of magnitude. c The difference is $5.1 - 4.1 = 1$ step on the scale. (i) Each step multiplies the shaking of the ground by 10: Y shakes the ground 10 times as much as X. (ii) Each step multiplies the energy released by about 32: Y releases about 32 times the energy of X. d $71 - 35 = 36$ dB $= 3.6$ steps of 10 dB, and each step multiplies the intensity by 10: factor $= 10^{3.6} \approx 3981$ (calculator). The machine sound is about 3981 times more intense. ∴ **B**; four orders of magnitude; 10 times the shaking and about 32 times the energy; about 3981 times — the same difference on the scale, two different factors in the world.



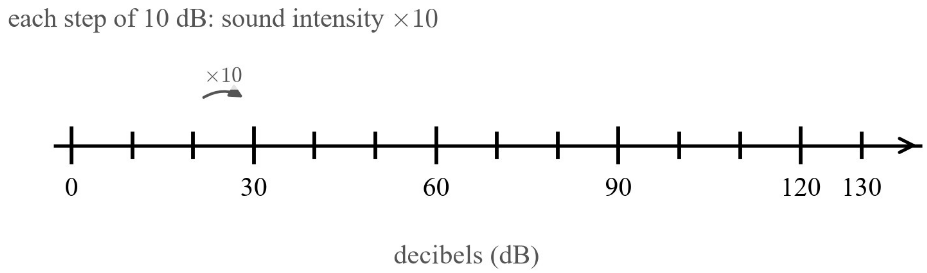
One decade of a logarithmic scale from 10 to 100 with minor marks at 20, 30, 40 and so on: the halfway mark in ordinary counting, 55, sits right of centre, while the halfway mark on the logarithmic scale, about 30, sits at the centre with equal factors of about 3.16 on each side of it



A powers-of-10 ladder from 10 to the power of −3, which is 0.001, up to 10 to the power of 6, which is 1 000 000, with an arrow labelled ×10 between each pair of rungs; one step up is one order of magnitude, ten times bigger

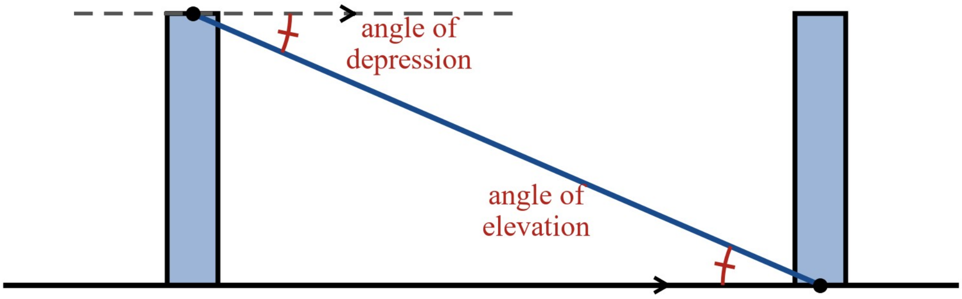


The Richter scale from M2 to M7 as a ladder: each step up multiplies the shaking of the ground by 10 and the energy released by about 32

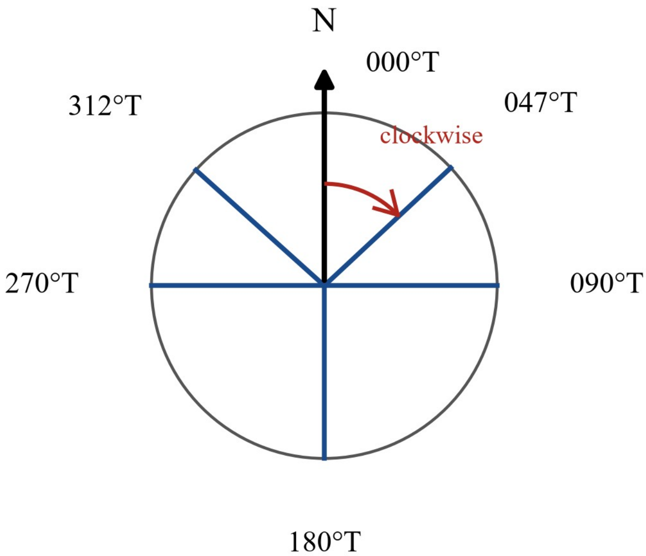


The decibel scale from 0 to 130 decibels with a mark every 10 decibels: each step of 10 decibels multiplies the sound intensity by 10

Q7. a The 430 m along the ground is the adjacent side and the 297.3 m height is the opposite side, so the tangent ratio connects the known parts with the angle: $\tan\theta = \frac{297.3}{430}$. $\theta = \tan^{-1}(0.6914\ldots) \approx 34.7°$ (1 d.p.). b The back bearing adds $180°$ (the sum does not pass $360°$): $17° + 180° = 197°$. The bearing is `197°T`. c Recorded to the nearest 10 m → half-unit $10 \div 2 = 5$ m: lower bound $= 430 - 5 = 425$ m, upper bound $= 430 + 5 = 435$ m. d Use the upper bound 435 m as the adjacent side: $\tan\theta = \frac{297.3}{435}$, so $\theta = \tan^{-1}(0.6834\ldots) \approx 34.4°$ (1 d.p.). $34.4° < 34.7°$: the angle is **smaller**. ∴ $34.7°$; `197°T`; 425–435 m; $34.4°$, smaller — a distance recorded too large pushes the calculated angle down: the error in the measurement changes the accuracy of the result, which is the judgement 4C builds. The model also assumes flat ground and ignores the surveyor's height; either assumption shifts the angle slightly.

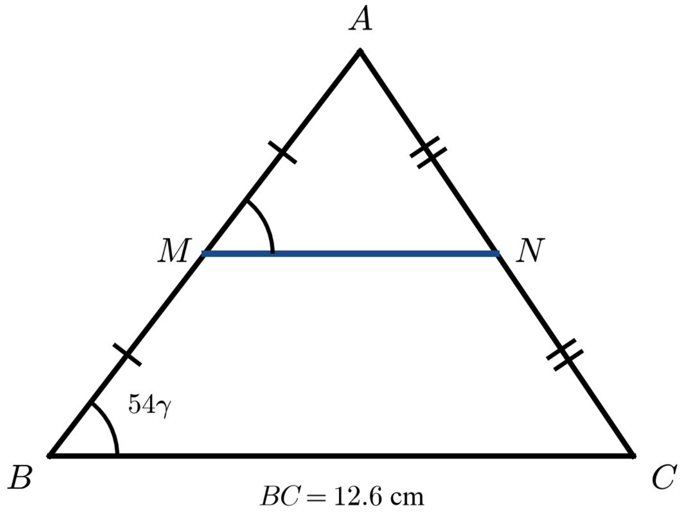


Two vertical towers on flat ground with a line of sight from the top of the left tower down to the base of the right tower: the angle between the line of sight and the horizontal at the top is labelled the angle of depression, and the angle between the line of sight and the horizontal at the base is labelled the angle of elevation; the two horizontal lines are marked parallel, so the two angles are equal



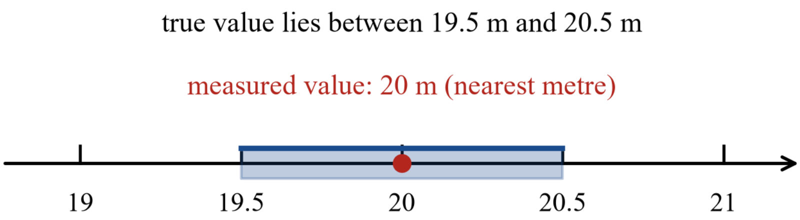
A compass circle with a north arrow pointing up and rays drawn clockwise from north, labelled as three-figure bearings 000 degrees T, 047 degrees T, 090 degrees T, 180 degrees T, 270 degrees T and 312 degrees T

Q8. a XY joins the midpoints of two sides of △PQR, so the midpoint theorem applies: XY = ½QR (Theorem 6) $= 18.6 \div 2 = 9.3$ cm. b Theorem 6 also gives XY ∥ QR. With PQ as the crossing line, ∠PXY = ∠PQR as corresponding angles (A7), so $\angle PXY = 43°$. ∴ $XY = 9.3$ cm (Theorem 6); $\angle PXY = 43°$ (Theorem 6 for the parallelism, then corresponding angles (A7)). Each value without its licensing theorem does not score its mark.



Triangle ABC with M and N the midpoints of AB and AC; MN drawn thick; single ticks on AM and MB, double ticks on AN and NC; 54 degrees marked at B and an equal-angle arc at M.

Q9. a Nearest metre → half-unit $1 \div 2 = 0.5$ m: lower bound $= 86 - 0.5 = 85.5$ m, upper bound $= 86 + 0.5 = 86.5$ m. b The greatest absolute error is 0.5 m, and the recorded value stands in for the unknown true value: greatest percentage error $= 0.5 \div 86 \times 100\% = 0.581\ldots\% \approx 0.6\%$ (1 d.p.). ∴ 85.5–86.5 m; greatest percentage error about 0.6 %.



A number line from 19 to 21 with a shaded band from 19.5 to 20.5 labelled true value lies between 19.5 m and 20.5 m, and a red point at 20 labelled measured value: 20 m (nearest metre).

Marking guide

Criteria are written in the language of the achievement-standard extract each content description carries in the authority file, quoted verbatim beside the items it scores (§7). The reasoning marks named in the paper's front matter are awarded only when the licensing theorem or result is stated.

`VC2M10M01` — 5 marks (Q1, Q5). *"Students solve measurement problems involving surface area and volume of composite objects."* The student solves both composite-object problems: subtracting the cut-out from the full box and converting to capacity with correct units (Q1), and adding a prism and a cylinder with π kept exact until the final rounding (Q5).

`VC2M10M02` — 5 marks (Q6). *"They interpret and use logarithmic scales representing small or large quantities or change in applied contexts."* The student interprets a powers-of-10 scale — the midway mark of a decade and the count of orders of magnitude — and uses two real logarithmic scales, Richter and decibel, to compare small and large quantities.

`VC2M10M03` — 5 marks (Q2, Q7a, Q7b). *"Students apply Pythagoras' theorem and trigonometry to solve practical problems involving right-angled triangles."* The student works with direction (back bearings and the acute angle a bearing makes with north) and applies the tangent ratio to an angle of elevation in a practical right-angled-triangle problem.

`VC2M10M04` — 5 marks (Q7c, Q7d, Q9). *"They identify the impact of measurement errors on the accuracy of results. Students use mathematical modelling to solve practical problems involving direct and inverse proportion and scaling, evaluating and modifying models, and reporting assumptions, methods and findings."* The student states bounds, finds a greatest percentage error, and identifies the impact of the recorded distance's error on the calculated angle — including the direction of the change.

`VC2M10SP01` — 5 marks (Q3, Q8). *"Students use deductive reasoning, theorems and algorithms to solve spatial problems."* The student formulates a complete proof with every step licensed by a named theorem or given (Q3), and uses a named theorem to solve a spatial problem (Q8).

`VC2M10SP02` — 5 marks (Q4). *"They interpret networks used to represent practical situations and describe connectedness."* The student interprets a real network — saying what the vertices and edges stand for — verifies the degree-sum result, applies Euler's walk test to a closed inspection route, and verifies Euler's formula counting the outside as a face.