Mathematics Level 10

Chapter 4 — Measurement · 4 subtopics · 4 CDs

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Contents

Surface area and volume of composite objects

Mathematics 10 · Chapter 4 Measurement · Subtopic 4A (Level 10) · Standard block

Strand	Measurement
Content description	`VC2M10M01`
Elaboration ids carried	`VC2M10M01.E01` `VC2M10M01.E02` `VC2M10M01.E03`
Weight	Standard

> **Content description (verbatim):** *solve problems involving the surface area and volume of composite objects using appropriate units*
>
> **Achievement standard (extract):** *Students solve measurement problems involving surface area and volume of composite objects.*

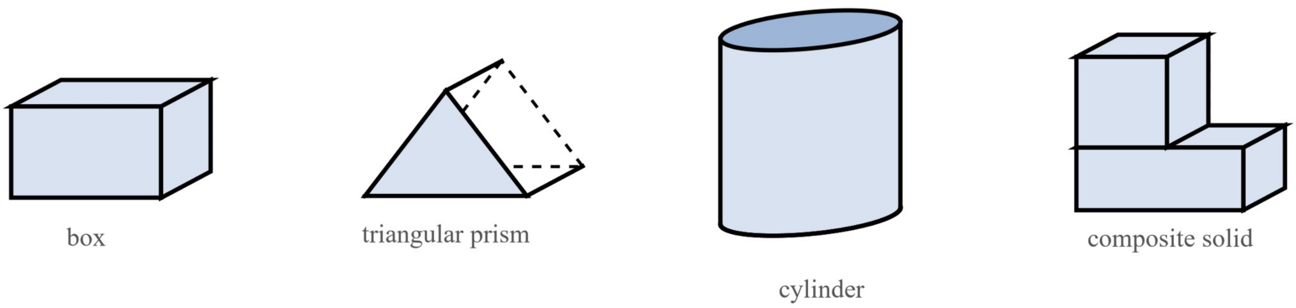
You need first: the volume and surface area of right prisms and cylinders from Year 9 (`VC2M9M01`), and the approximation habits of 1A (`VC2M10N01.E02`): keep π exact while you work, round once at the end, and remember that every approximation you make compounds through the parts of a composite object.

Theory

Everything in this subtopic is built on one move: **a composite object is solved one part at a time.** Split the object into parts you already know, measure each part with a formula you already own, then combine the parts carefully. Volume combines by adding (or by subtracting a cut-out). Surface area combines by counting **exposed faces only** — and that is where the care is needed.

1. Composite objects

A **composite object** (or **composite solid**) is a solid built by joining simpler solids together. In this subtopic, following the curriculum, the simpler solids are **right prisms and cylinders** only. A **right prism** is a solid with a flat cross-section that is the same shape and size all the way through, with the sides at right angles to that cross-section — a box and a cylinder are both right prisms, and so is a triangular prism.



A box, a triangular prism and a cylinder, the parts this subtopic builds with, beside a composite solid made from two boxes joined together

The solid on the right is composite: two boxes joined face to face. Every object in this subtopic is built like that, or by cutting a piece out of a simpler solid.

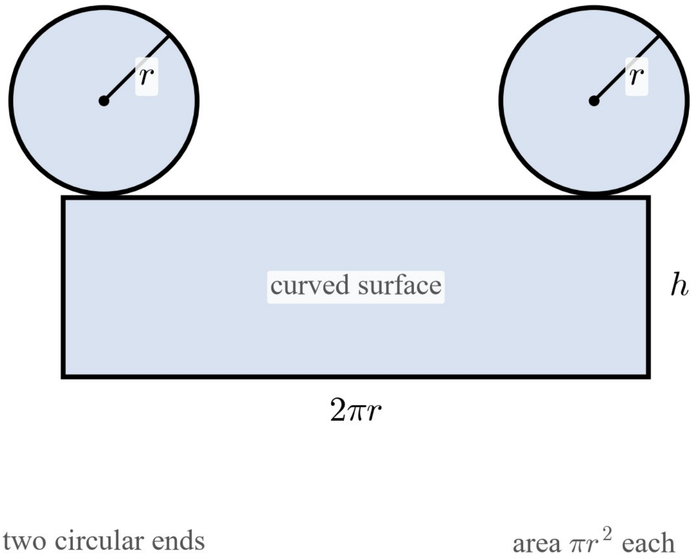
2. The formulas we build on

Every right prism or cylinder with cross-sectional area A and height (or length) h has

$$V = A \times h.$$

For the two prisms we use most often:

- **Box** with length l, width w and height h: $V = l\,w\,h$ and $\mathrm{SA} = 2(l\,w + l\,h + w\,h)$.
- **Cylinder** with radius r and height h: $V = \pi\,r^2 h$; the curved surface unrolls to a rectangle $2\pi\,r$ wide and h high, so the total surface area is $\mathrm{SA} = 2\pi\,r^2 + 2\pi\,r\,h$ (two circular ends plus the curved surface).

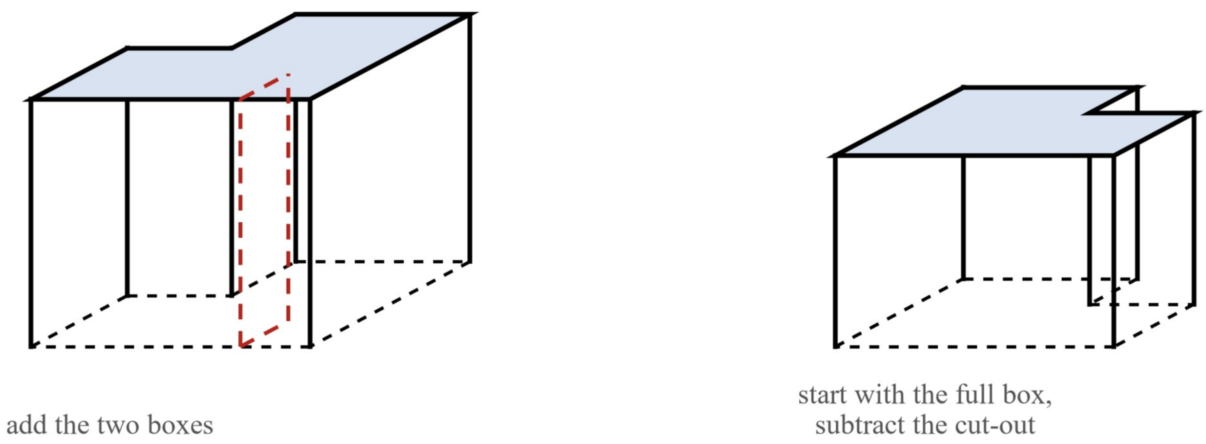


The net of a cylinder: two circular ends of radius r, each with area πr squared, and the curved surface unrolled to a rectangle 2πr wide and h high

3. Volume of a composite solid: add the parts, or subtract a cut-out

Volume is the space inside, so it behaves the way you expect:

- **Add** when the solid is built from parts: $V = V_1 + V_2 + \cdots$
- **Subtract** when the solid is a bigger solid with a piece removed: $V = V_{\text{whole}} - V_{\text{cut-out}}$. Holes, notches and hollow interiors all work this way.



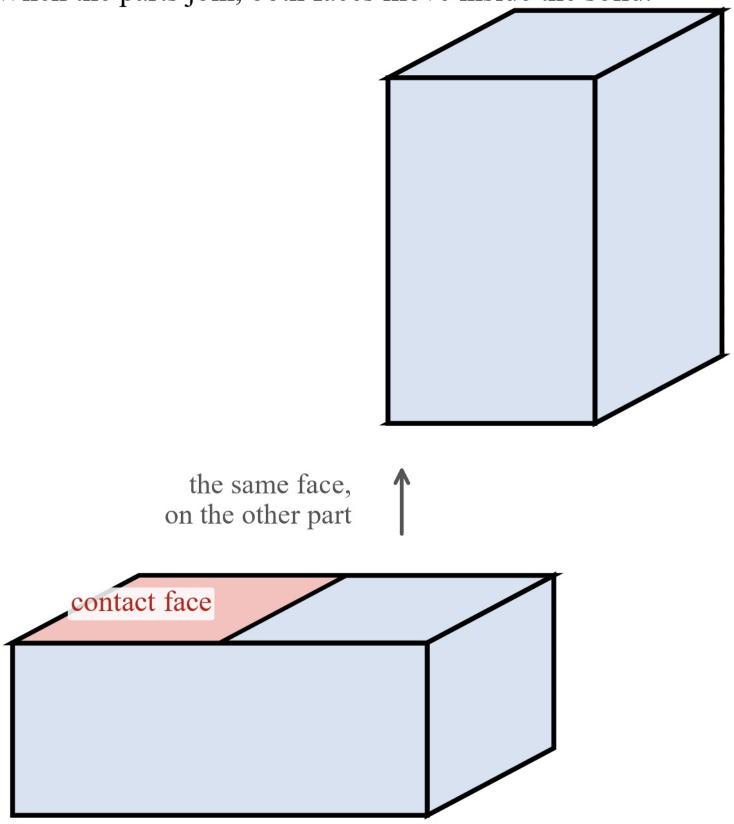
The same L-shaped solid two ways: split into two boxes whose volumes add, and as a full box with a smaller cut-out box subtracted

Both views give the same volume — choose whichever split makes the parts easiest to measure.

4. Surface area of a composite solid: exposed faces only

Surface area is the total area of the **outside**. When two parts join, the two faces that touch disappear **inside** the solid, so they are not counted.

When the parts join, both faces move inside the solid.



Exploded view of two boxes about to join: the shaded contact face on top of the lower box and the same face on the bottom of the upper box both move inside the solid

The reliable routine is:

- find the total surface area of **each part separately**;
- add them;
- **subtract twice the area of each contact face** — once for each part's hidden face.

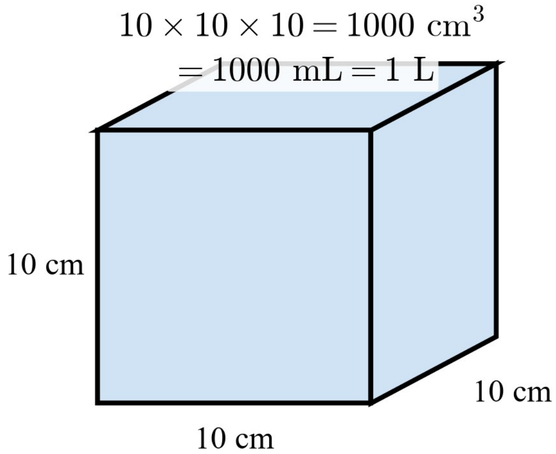
The same idea handles holes: drilling a hole **removes** two circles of surface (the openings) and **adds** the curved surface of the tube inside.

5. Units and capacity

Work in the units that suit the object, and state them in your answer. The capacity facts we use:

- $1\,\text{cm}^3 = 1\,\text{mL}$ and $1000\,\text{cm}^3 = 1\,\text{L}$;
- $1\,\text{m}^3 = 1000\,\text{L} = 1\,\text{kL}$.

A fact worth knowing by heart connects rainfall to capacity: **1 mm of rain on 1 m² of roof delivers 1 L of water.** It follows from the cube below: a layer $1\,\text{m}$ by $1\,\text{m}$ by $1\,\text{mm}$ is $1000\,\text{cm}^3$.



A cube 10 cm by 10 cm by 10 cm: its volume of 1000 cubic centimetres is 1000 mL, exactly one litre

So a roof of area $A\,\text{m}^2$ catching d mm of rain collects $A \times d$ litres, before losses.

6. Estimating and modelling

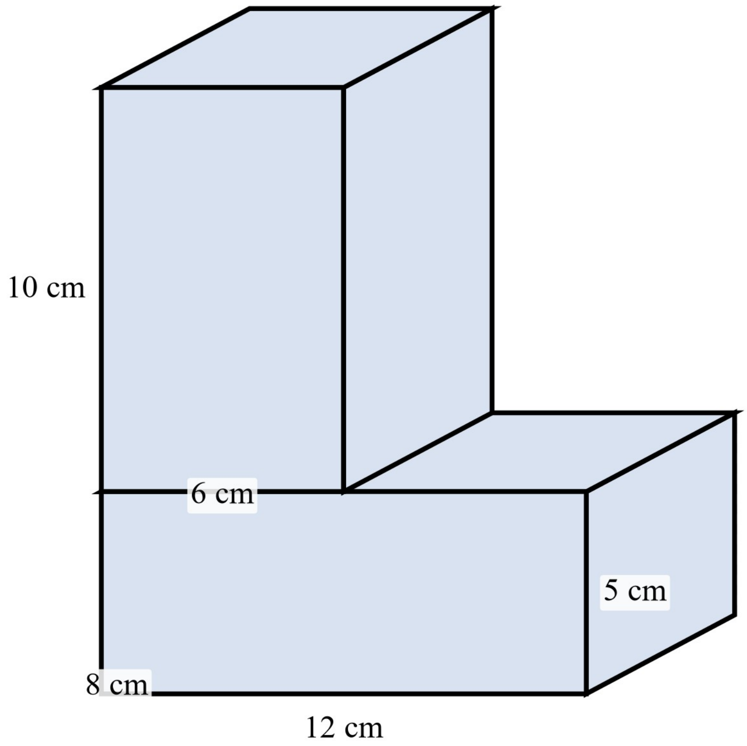
Before calculating exactly, **estimate** by rounding the measurements to one significant figure and using $\pi \approx 3$. An estimate tells you the size your exact answer should be, and catches calculator slips.

Problems in this subtopic are solved as a modelling cycle: **set up** a model (choose the solids, choose the units, state your assumptions), **solve** it, **interpret** the answer back in the situation (units, rounding, does it make sense?), then **check** the model against reality — the assumptions are part of the answer. And as 1A established: keep π and other exact values unrounded while you work, then round **once**, at the end, round half up.

Worked examples

Example 1 — A composite solid from two boxes

The solid shown is made from a box $6\,\text{cm}$ by $8\,\text{cm}$ by $10\,\text{cm}$ standing on the left end of a box $12\,\text{cm}$ by $8\,\text{cm}$ by $5\,\text{cm}$, with the back faces flush. Find (a) the volume and (b) the total surface area of the solid.

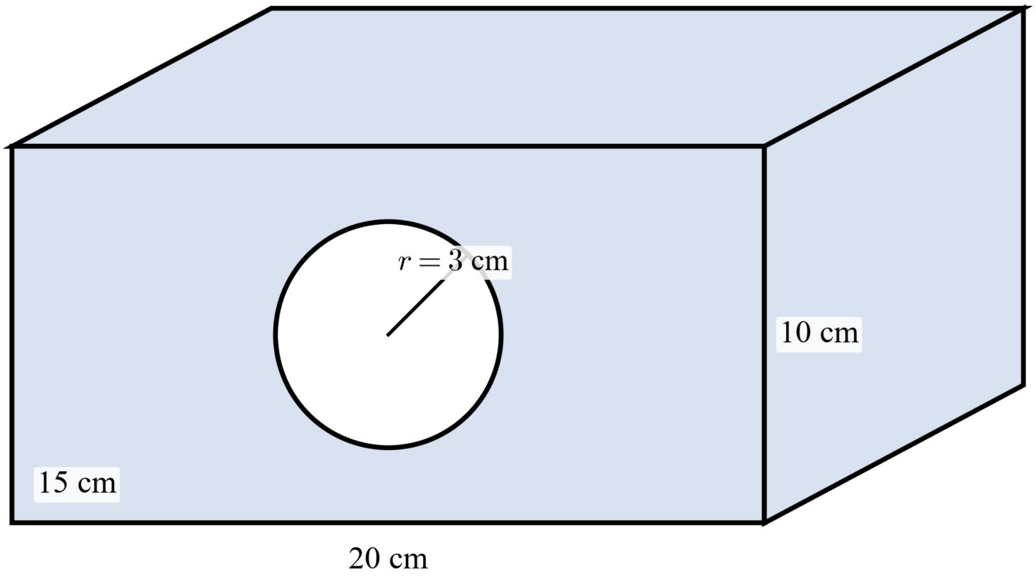


The step block of Example 1: a box 6 cm by 8 cm by 10 cm standing on the left end of a box 12 cm by 8 cm by 5 cm

Working	Comment
(a) $V_{\text{lower}} = 12 \times 8 \times 5 = 480$	Volume of the lower box, in cm³.
$V_{\text{upper}} = 6 \times 8 \times 10 = 480$	Volume of the upper box.
$V = 480 + 480 = 960$	Add the parts.
$\therefore$ the volume is $960\,\text{cm}^3$.	
(b) $\text{SA}_{\text{lower}} = 2(12 \times 8 + 12 \times 5 + 8 \times 5) = 392$	Total surface area of the lower box.
$\text{SA}_{\text{upper}} = 2(6 \times 8 + 6 \times 10 + 8 \times 10) = 376$	Total surface area of the upper box.
contact face $= 6 \times 8 = 48$	The upper box sits on a 6×8 patch of the lower box.
$\text{SA} = 392 + 376 - 2 \times 48 = 672$	Subtract the contact face twice.
$\therefore$ the total surface area is $672\,\text{cm}^2$.	

Example 2 — A block with a cylindrical hole

A block $20\,\text{cm}$ long, $15\,\text{cm}$ deep and $10\,\text{cm}$ high has a cylindrical hole of radius $3\,\text{cm}$ drilled straight through the $15\,\text{cm}$ depth. Find (a) the volume and (b) the total surface area of the block with the hole, correct to 1 decimal place.

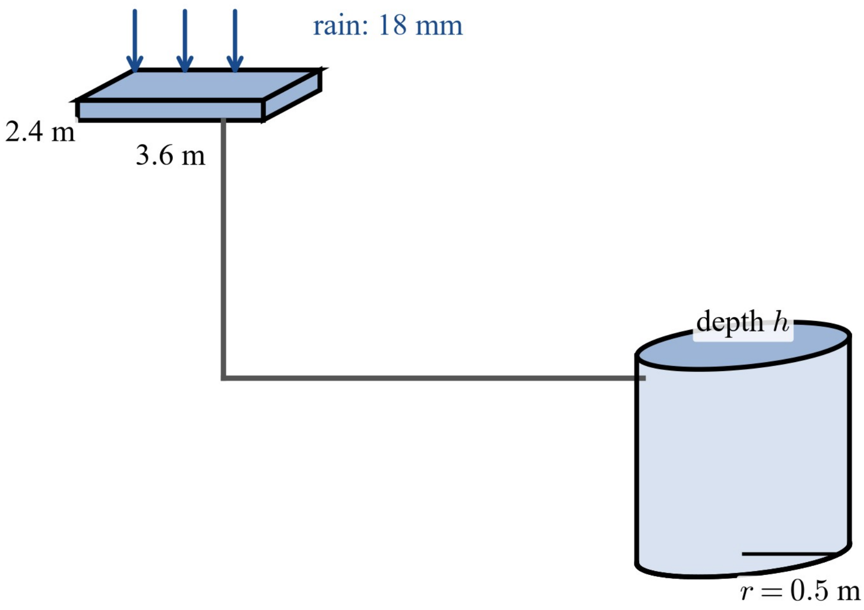


The block of Example 2, 20 cm by 15 cm by 10 cm, with a cylindrical hole of radius 3 cm drilled through the 15 cm depth

Working	Comment
(a) $V_{\text{box}} = 20 \times 15 \times 10 = 3000$	Volume of the whole block.
$V_{\text{hole}} = \pi \times 3^2 \times 15 = 135\pi$	The hole is a cylinder of length 15.
$V = 3000 - 135\pi = 2575.88\ldots \approx 2575.9$	Subtract the cut-out; round once at the end.
$\therefore$ the volume is about $2575.9\,\text{cm}^3$.	
(b) $\text{SA}_{\text{box}} = 2(20 \times 15 + 20 \times 10 + 15 \times 10) = 1300$	Whole block before drilling.
openings removed $= 2 \times \pi \times 3^2 = 18\pi$	Two circles disappear from the faces.
tube added $= 2\pi \times 3 \times 15 = 90\pi$	The curved surface of the hole is new outside.
$\text{SA} = 1300 - 18\pi + 90\pi = 1300 + 72\pi = 1526.19\ldots \approx 15$	A hole can **increase** surface area.
$\therefore$ the surface area is about $1526.2\,\text{cm}^2$.	

Example 3 — Rainfall on a roof, into a tank

`SCENARIO` A flat roof measures $3.6\,\text{m}$ by $2.4\,\text{m}$. In one storm, $18\,\text{mm}$ of rain falls, and every litre that lands on the roof runs into an empty cylindrical tank of radius $0.5\,\text{m}$. (a) Find the volume of water collected, in litres. (b) Find the depth of water in the tank, correct to 2 decimal places. (c) Estimate the volume collected and compare. (d) State one assumption in the model and how it would change the answer.



all water from the roof runs into the empty tank (model)

The roof of Example 3, 3.6 m by 2.4 m, catching 18 mm of rain, with a pipe running down to an empty cylindrical tank of radius 0.5 m

Working	Comment
(a) $V = 3.6 \times 2.4 \times 0.018 = 0.15552\,\text{m}^3$	$18\,\text{mm} = 0.018\,\text{m}$; model the water as a thin box.
$V = 0.15552 \times 1000 = 155.52 \approx 155.5\,\text{L}$	$1\,\text{m}^3 = 1000\,\text{L}$.
(b) $h = \dfrac{0.15552}{\pi \times 0.5^2} = \dfrac{0.15552}{0.25\pi} = 0.19801\ldots \approx 0.20$	Depth = volume ÷ base area.
∴ the water stands about $0.20\,\text{m}$ deep.	
(c) $4 \times 2 \times 0.02 = 0.16\,\text{m}^3 = 160\,\text{L}$	One-significant-figure estimate.
$160\,\text{L}$ is close to $155.5\,\text{L}$, so the exact answer is the right size.	Interpret.
(d) The model assumes **all** rain lands on the roof and runs in.	Check the model.
Splash and evaporation losses would make the real volume smaller; a sloped roof would catch more than its footprint, making it larger.	Assumptions change the answer — say how.

Practice questions

Scaffolded

Q1. A solid is made from a box $10\,\text{cm}$ by $6\,\text{cm}$ by $4\,\text{cm}$ with a cube of side $4\,\text{cm}$ fixed on top. (a) Find the volume of each part. (b) Hence find the volume of the solid. (c) The contact face is a square of side $4\,\text{cm}$; find the total surface area of the solid.

Q2. A right prism has an L-shaped cross-section: a $10\,\text{cm}$ by $8\,\text{cm}$ rectangle with a $4\,\text{cm}$ by $3\,\text{cm}$ notch removed from one corner. The prism is $12\,\text{cm}$ long. (a) Find the area of the cross-section. (b) Hence find the volume of the prism.

Q3. A water tank is a box $1.2\,\mathrm{m}$ by $0.8\,\mathrm{m}$ by $0.5\,\mathrm{m}$ topped by a cylinder of radius $0.3\,\mathrm{m}$ and height $0.6\,\mathrm{m}$. (a) Find the volume of the box. (b) Find the volume of the cylinder, correct to 3 decimal places. (c) Hence find the total volume correct to 2 decimal places, and the capacity to the nearest litre.

Q4. A block $30\,\mathrm{cm}$ by $20\,\mathrm{cm}$ by $12\,\mathrm{cm}$ has a notch $10\,\mathrm{cm}$ by $8\,\mathrm{cm}$ cut out of one corner, running the full $12\,\mathrm{cm}$ height. (a) Find the volume removed. (b) Find the volume left.

Q5. Convert: (a) $2500\,\mathrm{cm}^3$ to mL and to L; (b) $3.4\,\mathrm{m}^3$ to L and to kL; (c) $750\,\mathrm{mL}$ to cm^3; (d) $12\,500\,\mathrm{L}$ to m^3.

Q6. A cylinder has radius $0.48\,\mathrm{m}$ and height $1.1\,\mathrm{m}$. (a) Estimate its volume using $r \approx 0.5\,\mathrm{m}$, $h \approx 1\,\mathrm{m}$ and $\pi \approx 3$. (b) Find the exact volume, correct to 2 decimal places. (c) Is the estimate an underestimate or an overestimate?

Q7. A flat roof has area $40\,\mathrm{m}^2$. (a) How many litres of water fall on it in a storm delivering $25\,\mathrm{mm}$ of rain? (b) The water runs into an empty $2\,\mathrm{kL}$ tank; what fraction of the tank is filled? (c) How many millimetres of rain would fill the tank from empty?

Q8. Two identical cylinders, each of radius $5\,\mathrm{cm}$ and height $10\,\mathrm{cm}$, are glued end to end to form one longer cylinder. (a) Find the volume of the solid, correct to 1 decimal place. (b) Find the total surface area of one cylinder, correct to 1 decimal place. (c) Find the total surface area of the joined solid, correct to 1 decimal place, explaining which faces disappear.

Unscaffolded

Q9. A right prism has a T-shaped cross-section: a bar $18\,\mathrm{cm}$ by $6\,\mathrm{cm}$ on top of a stem $6\,\mathrm{cm}$ wide and $10\,\mathrm{cm}$ high. The prism is $5\,\mathrm{cm}$ deep. Find (a) the volume and (b) the total surface area of the prism.

Q10. `SCENARIO` A house slab needs a concrete footing in the shape of a rectangular frame: outer edges $6.0\,\mathrm{m}$ by $4.0\,\mathrm{m}$, inner opening $5.2\,\mathrm{m}$ by $3.2\,\mathrm{m}$, and depth $0.15\,\mathrm{m}$. Find the volume of concrete in the footing, in m^3.

Q11. A block $40\,\mathrm{cm}$ long, $20\,\mathrm{cm}$ wide and $20\,\mathrm{cm}$ high has two rectangular holes, each $12\,\mathrm{cm}$ by $15\,\mathrm{cm}$, cut straight through the $20\,\mathrm{cm}$ width. Find the volume of the block that remains.

Q12. A cylindrical tank must hold $5000\,\mathrm{L}$. The radius is fixed at $0.9\,\mathrm{m}$ by the space available. Find the minimum height of the tank, correct to 2 decimal places.

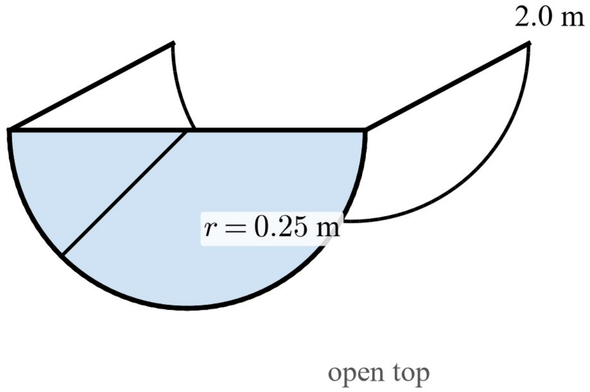
Q13. `SCENARIO` A school fete orders 40 tubs of ice cream, each tub a box $20\,\mathrm{cm}$ by $15\,\mathrm{cm}$ by $12\,\mathrm{cm}$. The hire freezer has an inside space $1.5\,\mathrm{m}$ by $1.0\,\mathrm{m}$ by $0.8\,\mathrm{m}$. By comparing volumes, decide whether the freezer can hold the ice cream, and give one practical reason the tubs might still not fit.

Q14. Two metre-cubes are stacked one on top of the other on flat ground, forming a pillar. The exposed faces (every face except the bottom one and the hidden contact faces) are to be painted. Paint covers $8\,\mathrm{m}^2$ per litre. Find the volume of paint needed, correct to 2 decimal places.

Q15. A glass aquarium has outside measurements $60\,\mathrm{cm}$ by $30\,\mathrm{cm}$ by $40\,\mathrm{cm}$ high, with glass $0.5\,\mathrm{cm}$ thick and an open top. Find its capacity in litres, correct to 1 decimal place.

Q16. Two cylindrical tanks are for sale: tank P has radius $0.5\,\mathrm{m}$ and height $1.6\,\mathrm{m}$; tank Q has radius $1.0\,\mathrm{m}$ and height $0.4\,\mathrm{m}$. Which tank holds more water? Justify with calculations, giving each capacity to the nearest litre.

Q17. A rain gutter is a half-cylinder, radius $0.25\,\mathrm{m}$ and length $2.0\,\mathrm{m}$, open at the top. Find the volume of water it holds when full, to the nearest litre.

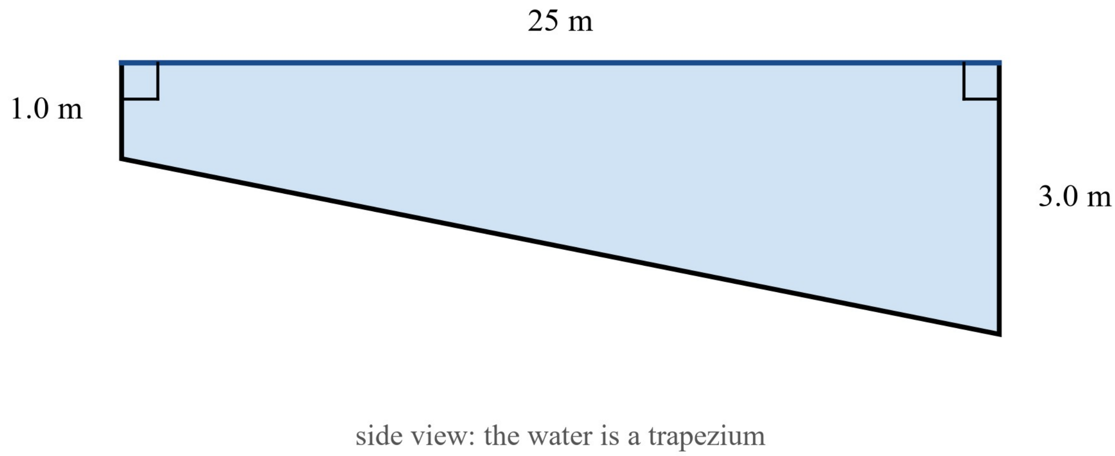


The half-cylinder trough of Q17: radius 0.25 m, length 2.0 m, open at the top

Q18. `SCENARIO` A wheelchair ramp is a triangular prism: the cross-section is a right-angled triangle with horizontal side $4.0\,\mathrm{m}$ and height $0.5\,\mathrm{m}$, and the ramp is $1.2\,\mathrm{m}$ wide. Find the volume of concrete in the ramp.

Q19. A cube of side $10\,\mathrm{cm}$ has a cylindrical hole of radius $2\,\mathrm{cm}$ drilled through it. Find (a) the volume and (b) the total surface area of the drilled cube, each correct to 1 decimal place.

Q20. A pool is $25\,\mathrm{m}$ long and $10\,\mathrm{m}$ wide. The depth increases from $1.0\,\mathrm{m}$ at the shallow end to $3.0\,\mathrm{m}$ at the deep end, the floor sloping in a straight line. Find the volume of water in the full pool, in kL.



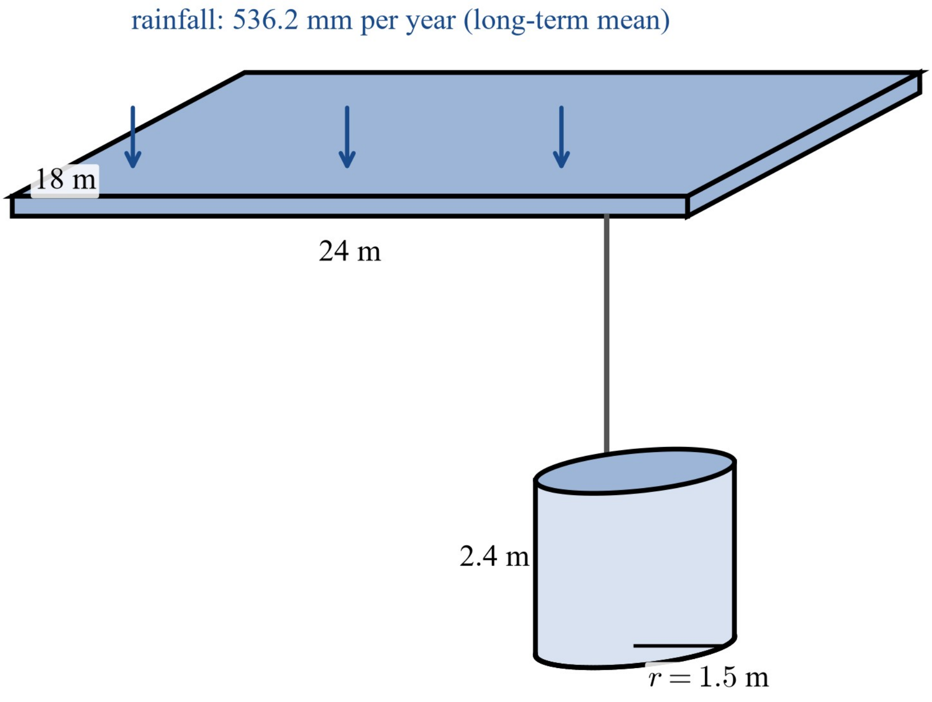
Side view of the pool of Q20: the water is a trapezium 25 m long with parallel depths 1.0 m and 3.0 m

Q21. A farm silo is a box $2.4\,\mathrm{m}$ by $2.4\,\mathrm{m}$ by $0.5\,\mathrm{m}$ high, topped by a cylinder of radius $1.2\,\mathrm{m}$ and height $2.5\,\mathrm{m}$. Find the total volume of the silo, correct to 2 decimal places.

Q22. A set of three concrete steps is $60\,\mathrm{cm}$ wide; each tread is $25\,\mathrm{cm}$ deep, and the risers stand $18\,\mathrm{cm}$, $36\,\mathrm{cm}$ and $54\,\mathrm{cm}$ above the ground. Find the volume of concrete in the steps, in m^3.

Real context

Q23. The roof of a school hall has a footprint of $24\,\mathrm{m}$ by $18\,\mathrm{m}$, and all of its stormwater runs to a single cylindrical tank of radius $1.5\,\mathrm{m}$ and height $2.4\,\mathrm{m}$. The long-term mean annual rainfall at Melbourne Airport is $536.2\,\mathrm{mm}$. (a) Modelling the roof as a flat rectangle, find the volume of rain falling on it in a year, in kL correct to 1 decimal place. (b) Find the capacity of the tank, in kL correct to 1 decimal place. (c) Hence find what percentage of the year's rain one full tank represents, correct to 1 decimal place. (d) State two assumptions in the model and whether each makes the true figure larger or smaller.



school hall roof (footprint modelled as a flat rectangle) and the tank

The school hall roof of Q23, modelled as a flat rectangle 24 m by 18 m catching 536.2 mm of rain per year, with a pipe down to a cylindrical tank 2.4 m high and 1.5 m in radius

Source: Australian Government Bureau of Meteorology, *Climate statistics for Melbourne Airport (station 086282)*, 2026. Retrieved 20 August 2026. (www.bom.gov.au/climate/averages/tables/cw_086282.shtml)

Answers

Q1. (a) $240\,\text{cm}^3$ and $64\,\text{cm}^3$; (b) $304\,\text{cm}^3$; (c) $312\,\text{cm}^2$. **Q2.** (a) $68\,\text{cm}^2$; (b) $816\,\text{cm}^3$. **Q3.** (a) $0.48\,\text{m}^3$; (b) $0.170\,\text{m}^3$; (c) $0.65\,\text{m}^3$, $650\,\text{L}$. **Q4.** (a) $960\,\text{cm}^3$; (b) $6240\,\text{cm}^3$. **Q5.** (a) $2500\,\text{mL} = 2.5\,\text{L}$; (b) $3400\,\text{L} = 3.4\,\text{kL}$; (c) $750\,\text{cm}^3$; (d) $12.5\,\text{m}^3$. **Q6.** (a) $0.75\,\text{m}^3$; (b) $0.80\,\text{m}^3$; (c) underestimate. **Q7.** (a) $1000\,\text{L}$; (b) one half; (c) $50\,\text{mm}$. **Q8.** (a) $1570.8\,\text{cm}^3$; (b) $471.2\,\text{cm}^2$; (c) $785.4\,\text{cm}^2$. **Q9.** (a) $840\,\text{cm}^3$; (b) $676\,\text{cm}^2$. **Q10.** $1.104\,\text{m}^3$. **Q11.** $8800\,\text{cm}^3$. **Q12.** $1.96\,\text{m}$. **Q13.** yes — $0.144\,\text{m}^3 < 1.2\,\text{m}^3$ (packing gaps aside). **Q14.** $1.13\,\text{L}$. **Q15.** $67.6\,\text{L}$. **Q16.** neither — both hold $1257\,\text{L}$. **Q17.** $196\,\text{L}$. **Q18.** $1.2\,\text{m}^3$. **Q19.** (a) $874.3\,\text{cm}^3$; (b) $700.5\,\text{cm}^2$. **Q20.** $500\,\text{kL}$. **Q21.** $14.19\,\text{m}^3$. **Q22.** $0.162\,\text{m}^3$. **Q23.** (a) $231.6\,\text{kL}$; (b) $17.0\,\text{kL}$; (c) $7.3\,\%$; (d) see solution.

Fully-worked solutions

Q1. (a) Box: $10 \times 6 \times 4 = 240$; cube: $4^3 = 64$, both in cm³.

(b) $V = 240 + 64 = 304\,\text{cm}^3$.

(c) $\text{SA}_{\text{box}} = 2(10 \times 6 + 10 \times 4 + 6 \times 4) = 248$; $\text{SA}_{\text{cube}} = 6 \times 4^2 = 96$; contact face $4 \times 4 = 16$.

$\text{SA} = 248 + 96 - 2 \times 16 = 312\,\text{cm}^2$.

Q2. (a) $A = 10 \times 8 - 4 \times 3 = 80 - 12 = 68\,\text{cm}^2$.

(b) $V = 68 \times 12 = 816\,\text{cm}^3$.

Q3. (a) $V_{\text{box}} = 1.2 \times 0.8 \times 0.5 = 0.48\,\text{m}^3$.

(b) $V_{\text{cyl}} = \pi \times 0.3^2 \times 0.6 = 0.054\pi = 0.16964\ldots \approx 0.170\,\text{m}^3$.

(c) $V = 0.48 + 0.054\pi = 0.64964\ldots \approx 0.65\,\text{m}^3$; capacity $= 0.64964\ldots \times 1000 = 649.64\ldots \approx 650\,\text{L}$.

Q4. (a) $10 \times 8 \times 12 = 960\,\text{cm}^3$.

(b) $30 \times 20 \times 12 - 960 = 7200 - 960 = 6240\,\text{cm}^3$.

Q5. (a) $2500\,\text{cm}^3 = 2500\,\text{mL} = 2.5\,\text{L}$. (b) $3.4\,\text{m}^3 = 3400\,\text{L} = 3.4\,\text{kL}$. (c) $750\,\text{mL} = 750\,\text{cm}^3$. (d) $12\,500\,\text{L} = 12.5\,\text{m}^3$.

Q6. (a) $0.5^2 \times 3 \times 1 = 0.75\,\text{m}^3$.

(b) $V = \pi \times 0.48^2 \times 1.1 = 0.25344\pi = 0.79620\ldots \approx 0.80\,\text{m}^3$.

(c) Underestimate: $\pi \approx 3$ and $h \approx 1$ both round down more than r rounds up.

Q7. (a) $40 \times 25 = 1000\,\text{L}$ (1 mm on 1 m² gives 1 L).

(b) $1000\,\text{L} = 1\,\text{kL}$, which is one half of the $2\,\text{kL}$ tank.

(c) $2000 \div 40 = 50\,\text{mm}$.

Q8. (a) $V = 2 \times \pi \times 5^2 \times 10 = 500\pi = 1570.79\ldots \approx 1570.8\,\text{cm}^3$.

(b) $\text{SA} = 2\pi \times 5^2 + 2\pi \times 5 \times 10 = 50\pi + 100\pi = 150\pi = 471.23\ldots \approx 471.2\,\text{cm}^2$.

(c) Joining hides one circular end of each cylinder: subtract $2 \times \pi \times 5^2 = 50\pi$.

$\text{SA} = 2 \times 150\pi - 50\pi = 250\pi = 785.39\ldots \approx 785.4\,\text{cm}^2$.

Q9. (a) Cross-section: $18 \times 6 + 6 \times 10 = 108 + 60 = 168\,\text{cm}^2$; $V = 168 \times 5 = 840\,\text{cm}^3$.

(b) Perimeter of the T: $18 + 6 + 6 + 10 + 6 + 10 + 6 + 6 = 68\,\text{cm}$; $\text{SA} = 2 \times 168 + 68 \times 5 = 336 + 340 = 676\,\text{cm}^2$.

Q10. Frame area $= 6.0 \times 4.0 - 5.2 \times 3.2 = 24 - 16.64 = 7.36\,\mathrm{m}^2$; $V = 7.36 \times 0.15 = 1.104\,\mathrm{m}^3$.

Q11. $V = 40 \times 20 \times 20 - 2 \times (12 \times 15 \times 20) = 16\,000 - 7200 = 8800\,\mathrm{cm}^3$.

Q12. $5000\,\mathrm{L} = 5\,\mathrm{m}^3$; $h = \dfrac{5}{\pi \times 0.9^2} = \dfrac{5}{0.81\pi} = 1.96487\ldots \approx 1.96\,\mathrm{m}$.

Q13. One tub: $20 \times 15 \times 12 = 3600\,\mathrm{cm}^3 = 0.0036\,\mathrm{m}^3$; forty tubs: $40 \times 0.0036 = 0.144\,\mathrm{m}^3$. Freezer: $1.5 \times 1.0 \times 0.8 = 1.2\,\mathrm{m}^3$.

$0.144 < 1.2$, so by volume the ice cream fits with room to spare — but boxes do not pack a space perfectly, so the tubs must actually be arranged to fit.

Q14. Top cube shows 5 faces, bottom cube shows 4 side faces: $5 + 4 = 9\,\mathrm{m}^2$.

Paint $= 9 \div 8 = 1.125 \approx 1.13\,\mathrm{L}$ (round half up).

Q15. Inside: $60 - 1 = 59$ long, $30 - 1 = 29$ wide, $40 - 0.5 = 39.5$ deep (open top loses glass once).

$V = 59 \times 29 \times 39.5 = 67\,584.5\,\mathrm{cm}^3$; capacity $= 67.5845 \approx 67.6\,\mathrm{L}$.

Q16. P: $\pi \times 0.5^2 \times 1.6 = 0.4\pi\,\mathrm{m}^3$; Q: $\pi \times 1.0^2 \times 0.4 = 0.4\pi\,\mathrm{m}^3$.

Both are $0.4\pi \times 1000 = 1256.6\ldots \approx 1257\,\mathrm{L}$: neither holds more — the double radius of Q exactly cancels its quarter height.

Q17. $V = \dfrac{1}{2} \times \pi \times 0.25^2 \times 2.0 = 0.0625\pi = 0.19634\ldots\,\mathrm{m}^3$; $0.19634\ldots \times 1000 = 196.34\ldots \approx 196\,\mathrm{L}$.

Q18. Cross-section $= \dfrac{1}{2} \times 4.0 \times 0.5 = 1.0\,\mathrm{m}^2$; $V = 1.0 \times 1.2 = 1.2\,\mathrm{m}^3$.

Q19. (a) $V = 10^3 - \pi \times 2^2 \times 10 = 1000 - 40\pi = 874.33\ldots \approx 874.3\,\mathrm{cm}^3$.

(b) $\mathrm{SA} = 6 \times 10^2 - 2 \times \pi \times 2^2 + 2\pi \times 2 \times 10 = 600 - 8\pi + 40\pi = 600 + 32\pi = 700.53\ldots \approx 700.5\,\mathrm{cm}^2$.

Q20. Side section is a trapezium with parallel sides 1.0 and 3.0 and width 25: $A = \dfrac{1}{2}(1.0 + 3.0) \times 25 = 50\,\mathrm{m}^2$; $V = 50 \times 10 = 500\,\mathrm{m}^3 = 500\,\mathrm{kL}$.

Q21. $V = 2.4 \times 2.4 \times 0.5 + \pi \times 1.2^2 \times 2.5 = 2.88 + 3.6\pi = 14.1898\ldots \approx 14.19\,\mathrm{m}^3$.

Q22. Slice the steps into three boxes 60 by 25: heights 18, 36, 54.

$V = 60 \times 25 \times (18 + 36 + 54) = 1500 \times 108 = 162\,000\,\mathrm{cm}^3 = 0.162\,\mathrm{m}^3$.

Q23. (a) $V = 24 \times 18 \times 0.5362 = 432 \times 0.5362 = 231.6384 \approx 231.6\,\mathrm{kL}$ ($536.2\,\mathrm{mm} = 0.5362\,\mathrm{m}$; $1\,\mathrm{m}^3 = 1\,\mathrm{kL}$).

(b) $V_{\text{tank}} = \pi \times 1.5^2 \times 2.4 = 5.4\pi = 16.964\ldots \approx 17.0\,\mathrm{kL}$.

(c) $\dfrac{16.964\ldots}{231.6384} \times 100 = 7.323\ldots \approx 7.3\,\%$.

(d) Any two of: no losses to splash, evaporation or first-flush diversion (true capture smaller); the roof is modelled flat, while a pitched roof has more surface than its footprint (true catch larger); the tank overflows in big storms, so one tank cannot store the year's rain in practice (the percentage is a comparison, not a promise).

Teacher note

Level 9 code rehearsed (D3): `VC2M9M01` — volume and surface area of right prisms and cylinders. 4A cannot add or subtract parts it cannot measure; both skills are rehearsed here, not re-taught, and are not assessed as Level 9 outcomes. 4C (`VC2M10M04`) applies scaling and measurement error to the solids measured here.

1A dependence: `VC2M10N01.E02` — approximation compounds. Composite measurement is where that habit earns its keep: students keep π exact until the last line and round once, round half up, as 1A establishes. Q6 and Example 3(c) assess the estimating half of `VC2M10M01.E02` directly.

Tool classes (D13): scientific calculator with a π key. `VC2M10M01` names no digital tool, so there is no digital tool task and no tool banner in this subtopic. No CAS.

VCE destinations (§5, reference only): `VC2M10M01` is named by Foundation Mathematics Units 3 & 4 (Area of Study 4, 'Space and measurement') and by General Mathematics as assumed knowledge — *"Compute the volume of a composite solid from right prisms and cylinders"* and *"Apply mathematical modelling to a real surface-area or volume problem (e.g. roof catchment, freezer hire)"*. Mathematical Methods and Specialist Mathematics have no measurement Area of Study, so 4A ends at FM and GM.

Level check (D6): every part in this subtopic is a right prism or a cylinder, exactly as `VC2M10M01.E01` prescribes — no spheres, cones or pyramids appear, even as cut-outs. Surface area is always assembled from exposed faces with the doubled contact-face subtraction; the subtraction move (holes and cut-outs) is assessed in Example 2 and Q11, Q15 and Q19.

Pythagoras’ theorem and trigonometry in practice: bearings, elevation and depression

Mathematics 10 · Chapter 4 Measurement · Subtopic 4B · Extended block

Strand	Measurement
Content description	`VC2M10M03`
Elaboration ids carried	`VC2M10M03.E01` `VC2M10M03.E02` `VC2M10M03.E03` `VC2M10M03.E04` (`VC2M10M03.E05` not delivered — see Teacher note)
Weight	Extended (double allocation)

> **Content description (verbatim):** *solve practical problems by applying Pythagoras’ theorem and trigonometry to right-angled triangles, including problems involving direction and angles of elevation and depression*
>
> **Achievement standard (extract):** *Students apply Pythagoras’ theorem and trigonometry to solve practical problems involving right-angled triangles.*

You need first: Pythagoras’ theorem, the three trigonometric ratios, and right-angled triangles from Year 9 (`VC2M9M03`), and the fact that the sine, cosine and tangent ratios of an angle are fixed by the angle alone, whatever the size of the triangle (`VC2M9SP01`). This subtopic rehearses those skills and applies them; it does not re-teach them.

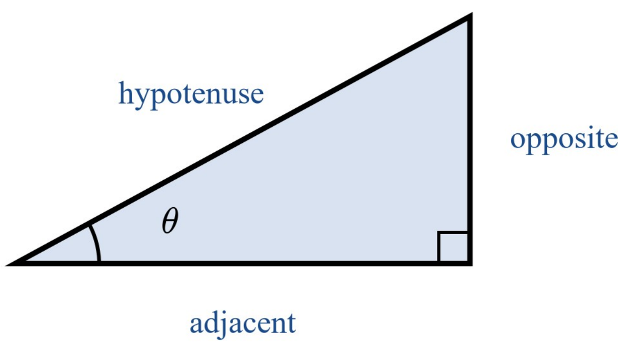
Theory

Everything in this subtopic is built on one habit: **find the right-angled triangle, then choose the tool that connects the parts you know to the part you want.** Pythagoras’ theorem connects three sides. Trigonometry connects an angle with two sides. Bearings and angles of elevation and depression are ways of describing where a line points; each one gives you an angle you can put into a right-angled triangle.

1. The sides of a right-angled triangle

In a right-angled triangle, the sides are named relative to a chosen angle, here called θ:

- the **hypotenuse** is the side opposite the right angle; it is always the longest side;
- the **opposite** side is the side opposite θ;
- the **adjacent** side is the side next to θ that is not the hypotenuse.



A right-angled triangle with the angle theta marked at the lower-left vertex: the side opposite the right angle is labelled the hypotenuse, the side facing theta is labelled the opposite side, and the side next to theta is labelled the adjacent side

2. Pythagoras' theorem

For any right-angled triangle with hypotenuse c and shorter sides a and b:

$$c^2 = a^2 + b^2.$$

Used one way, it finds the hypotenuse: **square the two shorter sides, add, then take the square root.** Used the other way, it finds a shorter side: **square the hypotenuse, subtract the square of the other shorter side, then take the square root.**

Pythagoras' theorem uses lengths only — it needs no angles. Reach for it when you know two sides and want the third.

3. The trigonometric ratios

For an acute angle θ in a right-angled triangle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

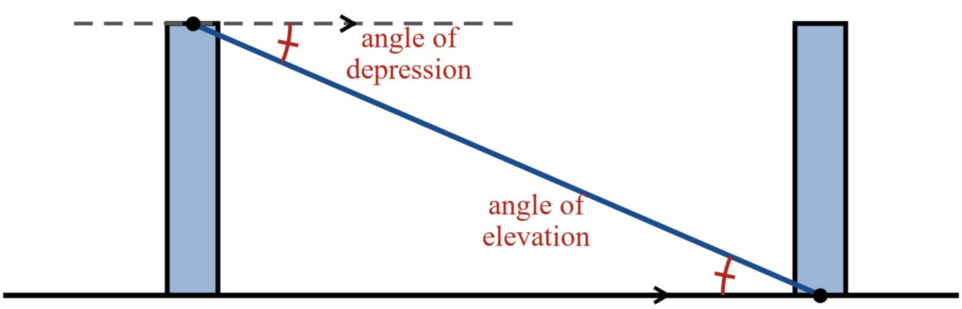
To find a **side**, choose the ratio that links the two sides you know with the side you want, then rearrange. To find an **angle**, choose the ratio whose two sides you know, then use the inverse button on your calculator: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

Work in **degrees** throughout this subtopic — check your calculator is in degree mode. Round only at the end of a calculation, and state the rounding you give.

4. Angles of elevation and angles of depression

Both are measured **from the horizontal**.

- The **angle of elevation** of an object is the angle you look **up** from the horizontal to see it.
- The **angle of depression** of an object is the angle you look **down** from the horizontal to see it.



Two vertical towers on flat ground with a line of sight from the top of the left tower down to the base of the right tower: the angle between the line of sight and the horizontal at the top is labelled the angle of depression, and the *angle between the line of sight and the horizontal at the base is labelled the angle of elevation; the two horizontal lines are marked parallel, so the two angles are equal*

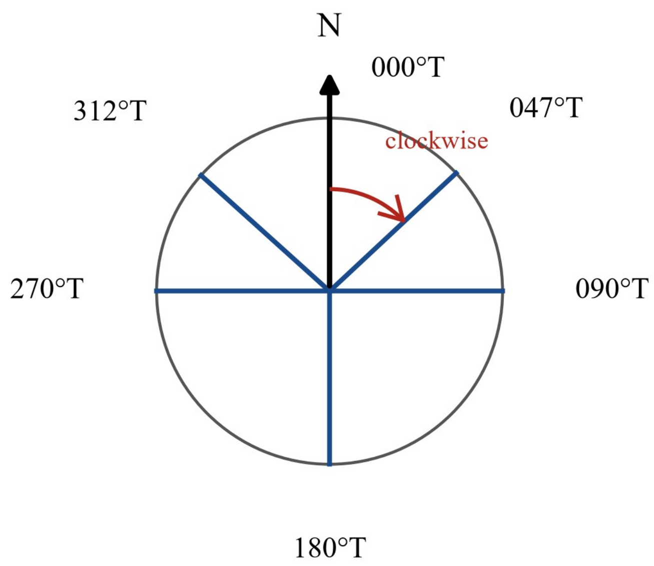
The two horizontal lines are parallel, so the **angle of depression from the top equals the angle of elevation from the bottom** — they are alternate angles. This fact turns most depression problems into ordinary right-angled triangle problems.

5. Bearings

A **bearing** describes direction. In this book, and in Victorian schools and VCE, bearings are **three-figure true bearings**:

- measured **clockwise from north** (true north);
- written as **three figures** followed by $°\mathrm{T}$, with leading zeros where needed.

North is $000°\mathrm{T}$, east is $090°\mathrm{T}$, south is $180°\mathrm{T}$ and west is $270°\mathrm{T}$. A bearing of $047°\mathrm{T}$ is $47°$ clockwise from north; a bearing of $312°\mathrm{T}$ is $48°$ short of a full turn back to north.



A compass circle with a north arrow pointing up and rays drawn clockwise from north, labelled as three-figure bearings 000 degrees T, 047 degrees T, 090 degrees T, 180 degrees T, 270 degrees T and 312 degrees T

The **back bearing** — the bearing for the return trip along the same straight line — is found by adding $180°$, or subtracting $180°$ if adding would pass $360°$. For example, the back bearing of $047°\mathrm{T}$ is $47°+180°=227°\mathrm{T}$. The back bearing of $250°\mathrm{T}$ is $250°-180°=070°\mathrm{T}$.

6. Three-dimensional problems become two-dimensional ones

A three-dimensional problem is solved by **decomposing** it: finding right-angled triangles hiding inside the solid and solving them one at a time.

Two moves cover most cases:

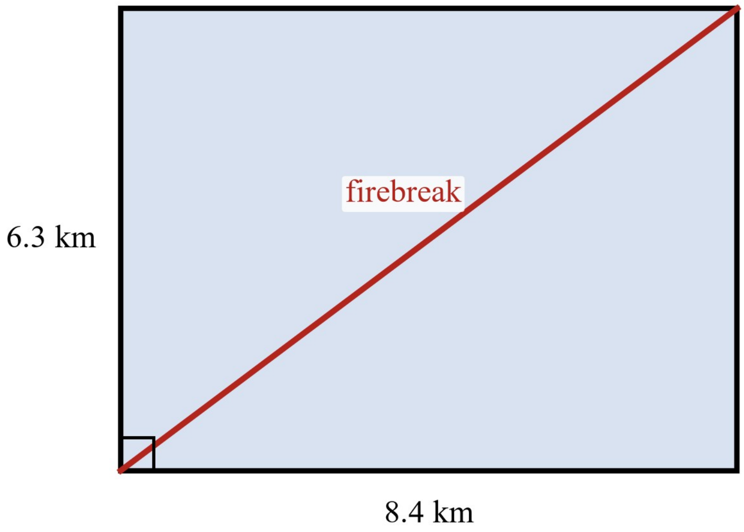
- **The diagonal of a rectangle.** A box of length l and width w has a base diagonal of $\sqrt{l^2+w^2}$ — one application of Pythagoras' theorem.
- **The longest length inside a box.** The diagonal found above, together with the height h of the box, forms a second right-angled triangle inside the box. A second application of Pythagoras' theorem gives the longest straight length from one corner to the opposite corner. The angle that this length makes with the base is found with trigonometry in that same second triangle.

Draw the flat triangle you are working in, separately from the solid, before you calculate. Every part of this subtopic stays inside right-angled triangles: the three-dimensional picture is only ever solved as a sequence of two-dimensional ones.

Worked examples

Example 1 — Pythagoras' theorem in two dimensions

`SCENARIO` A rectangular block of land is $8.4\,\mathrm{km}$ long and $6.3\,\mathrm{km}$ wide. A firebreak runs straight from one corner to the opposite corner. Find the length of the firebreak.



A rectangle 8.4 km long and 6.3 km wide with its diagonal drawn and labelled as the firebreak, and a right angle marked at the lower-left corner

Working	Comment
The firebreak is the diagonal of the rectangle, so it is the hypotenuse of a right-angled triangle with shorter sides 8.4 and 6.3.	Draw the triangle first.
$c^2 = 8.4^2 + 6.3^2$	Pythagoras' theorem, hypotenuse c.
$c^2 = 70.56 + 39.69$	Square each shorter side.
$c^2 = 110.25$	Add.
$c = \sqrt{110.25}$	Take the square root.
$c = 10.5$	
$\therefore$ the firebreak is $10.5\,\mathrm{km}$ long.	

(The sides 6.3, 8.4, 10.5 are the 3–4–5 triangle multiplied by 2.1 — a useful check.)

Example 2 — Finding sides, then finding an angle

(a) A right-angled triangle has hypotenuse $17\,\mathrm{cm}$ and one angle of $28°$. Find the side opposite the $28°$ angle and the side adjacent to it, each correct to 1 decimal place.

Working	Comment
$\sin 28° = \dfrac{\text{opp}}{17}$	Opposite and hypotenuse → sine.
$\text{opp} = 17 \sin 28°$	Rearrange for the opposite side.

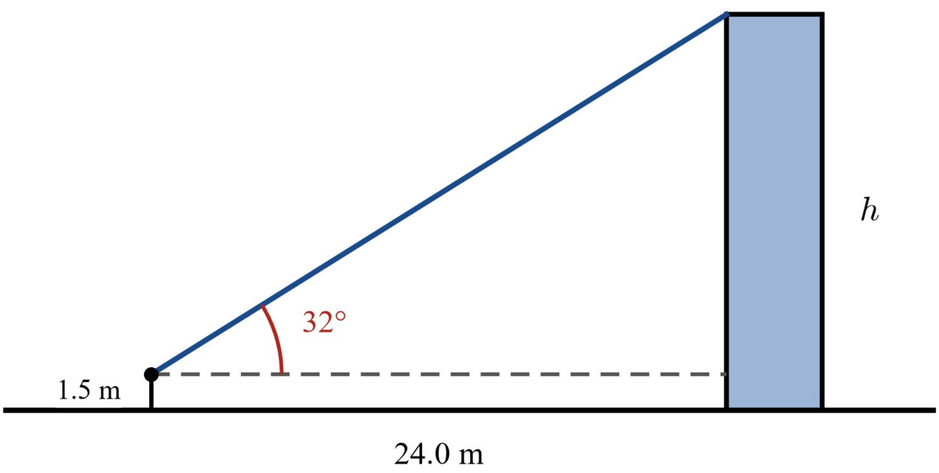
Working	Comment
$\text{opp} = 7.98\ldots \approx 8.0$	Correct to 1 decimal place.
$\cos 28° = \dfrac{\text{adj}}{17}$	Adjacent and hypotenuse → cosine.
$\text{adj} = 17\cos 28°$	Rearrange for the adjacent side.
$\text{adj} = 15.01\ldots \approx 15.0$	Correct to 1 decimal place.
∴ opposite side $\approx 8.0\,\text{cm}$, adjacent side $\approx 15.0\,\text{cm}$.	

(b) A ladder $4.2\,\text{m}$ long has its foot $1.3\,\text{m}$ from the base of a vertical wall. Find the angle the ladder makes with the ground, correct to 1 decimal place.

Working	Comment
The wall is vertical, so the ladder, wall and ground form a right-angled triangle; the ladder is the hypotenuse.	Sketch it.
$\cos\theta = \dfrac{1.3}{4.2}$	The angle at the ground touches the adjacent side (1.3) and the hypotenuse (4.2) → cosine.
$\theta = \cos^{-1}\left(\dfrac{1.3}{4.2}\right)$	Inverse cosine finds the angle.
$\theta = \cos^{-1}(0.3095\ldots)$	Evaluate the fraction first.
$\theta = 71.96\ldots \approx 72.0°$	Correct to 1 decimal place.
∴ the ladder makes an angle of about $72.0°$ with the ground.	

Example 3 — Angle of elevation

`SCENARIO` A student stands on flat ground $24.0\,\text{m}$ from the base of a water tank. She measures the angle of elevation from her eye to the top of the tank as $32°$, using a clinometer — a simple hand-held device for measuring angles of sight. Her eye is $1.5\,\text{m}$ above the ground. Find the height of the water tank, correct to 1 decimal place.

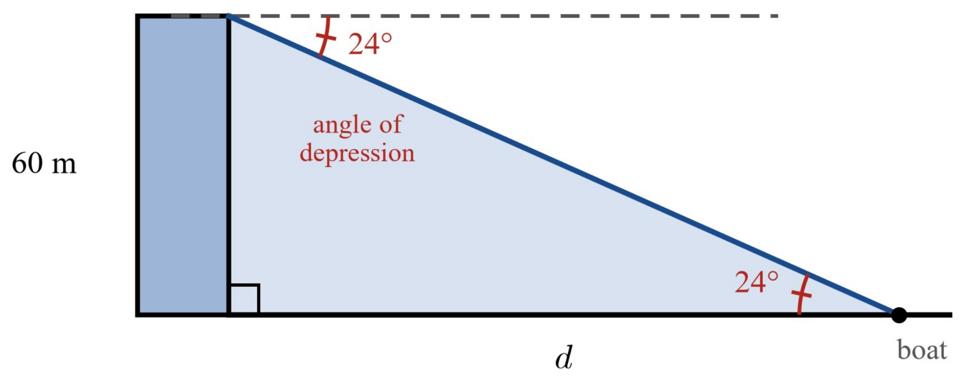


A vertical water tank of height h on flat ground, with an observer 24.0 m away whose eye is 1.5 m above the ground; a dashed horizontal line runs from her eye to the tank, the line of sight runs from her eye to the top of the tank, and the angle of elevation of 32 degrees is marked between them

Working	Comment
Work from her eye level: the triangle has adjacent side $24.0\,\mathrm{m}$, angle $32\,°$, and unknown height x above eye level.	The angle of elevation is measured from the horizontal through her eye.
$\tan 32\,° = \dfrac{x}{24.0}$	Opposite and adjacent → tangent.
$x = 24.0 \times \tan 32\,°$	Rearrange for x.
$x = 14.99\ldots$	Evaluate.
$\text{height} = x + 1.5$	Add the eye height back on.
$\text{height} = 14.99\ldots + 1.5$	Substitute the value of x.
$\text{height} = 16.49\ldots \approx 16.5$	Correct to 1 decimal place.
∴ the water tank is about $16.5\,\mathrm{m}$ high.	

Example 4 — Angle of depression

`SCENARIO` A ranger stands at the top of a vertical cliff $60\,\mathrm{m}$ above sea level. She sights a boat on the sea with an angle of depression of $24\,°$. Find the horizontal distance from the base of the cliff to the boat, correct to 1 decimal place.



A vertical cliff 60 m above sea level and a boat on the sea: at the top of the cliff the angle of depression of 24 degrees is marked between the horizontal and the line of sight down to the boat, and the equal angle of 24 degrees is marked at the boat between the sea-level horizontal and the line of sight; the right-angled triangle formed is shaded

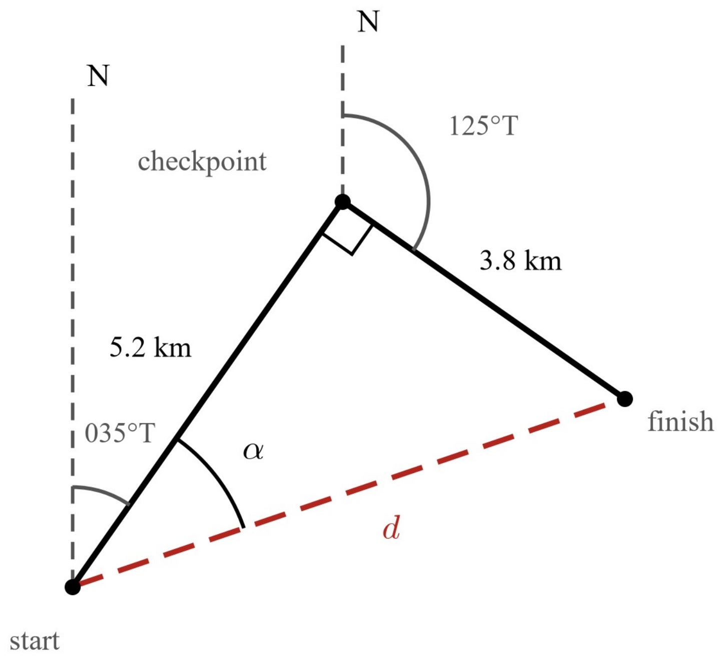
Working	Comment
The angle of depression is $24\,°$ from the horizontal at the top.	Given.
The horizontal at the top and the sea level are parallel, so the angle at the boat is also $24\,°$ (alternate angles).	The key move for depression problems.
The triangle has opposite side $60\,\mathrm{m}$, adjacent side d, and angle $24\,°$ at the boat.	Sketch the triangle.
$\tan 24\,° = \dfrac{60}{d}$	Opposite and adjacent → tangent.
$d = \dfrac{60}{\tan 24\,°}$	Rearrange.

Working	Comment
$d = \dfrac{60}{0.4452\ldots}$	Evaluate $\tan 24°$.
$d = 134.76\ldots \approx 134.8$	Correct to 1 decimal place.

$\therefore$ the boat is about $134.8\,\mathrm{m}$ from the base of the cliff.

Example 5 — Bearings and right-angled trigonometry

`SCENARIO` On an orienteering course, a runner leaves the start on a bearing of $035°\mathrm{T}$ and runs $5.2\,\mathrm{km}$ to a checkpoint. From the checkpoint she runs $3.8\,\mathrm{km}$ on a bearing of $125°\mathrm{T}$ to the finish. Find (a) the straight-line distance from the start to the finish, and (b) the bearing of the finish from the start, correct to the nearest degree.



An orienteering map with dashed north lines at the start and at the checkpoint: the first leg of 5.2 km runs on a bearing of 035 degrees T and the second leg of 3.8 km runs on a bearing of 125 degrees T, meeting at right angles; the straight line from the start to the finish is labelled d and the angle alpha is marked at the start between the first leg and that line

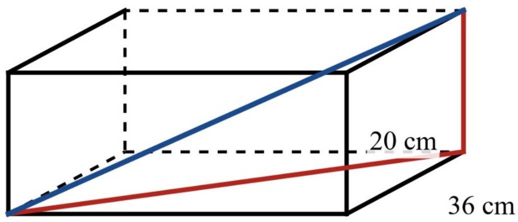
Working	Comment
At the checkpoint, the back bearing of the first leg is $35° + 180° = 215°$.	Direction back towards the start.
The second leg runs on $125°$, so the turn between the legs is $215° - 125° = 90°$.	The two legs meet at right angles.
The start, checkpoint and finish form a right-angled triangle with legs 5.2 and 3.8.	Bearing differences of $90°$ give right angles.
(a) $d^2 = 5.2^2 + 3.8^2$	Pythagoras' theorem.
$d^2 = 27.04 + 14.44$	Square each leg.
$d^2 = 41.48$	Add.
$d = \sqrt{41.48}$	Take the square root.
$d = 6.44\ldots \approx 6.4$	Correct to 1 decimal place.

$\therefore$ the finish is about $6.4\,\mathrm{km}$ from the start.

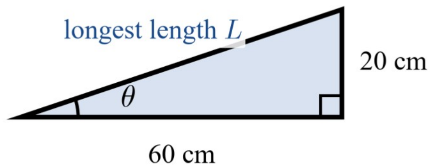
Working	Comment
(b) $\tan \alpha = \frac{3.8}{5.2}$, where α is the angle at the start.	Opposite 3.8, adjacent 5.2.
$\alpha = \tan^{-1}(0.7308\ldots)$	
$\alpha = 36.16\ldots°$	
$\text{bearing} = 35° + 36.16\ldots°$	The line to the finish lies clockwise from the first leg.
$\text{bearing} = 71.16\ldots°$	
$\therefore$ the bearing of the finish from the start is `071°T`.	Three figures, to the nearest degree.

Example 6 — Decomposing a three-dimensional problem

`SCENARIO` A rectangular carton is $48\,\text{cm}$ long, $36\,\text{cm}$ wide and $20\,\text{cm}$ high. (a) Find the diagonal of the base of the carton. (b) Find the longest straight length that fits inside the carton. (c) Find the angle that this longest length makes with the base, correct to 1 decimal place. (d) A rod $65\,\text{cm}$ long must lie flat inside the carton. Will it fit?



the carton



the second triangle, drawn flat

A rectangular carton 48 cm long, 36 cm wide and 20 cm high with the diagonal of its base and the longest length from a bottom corner to the opposite top corner drawn in; beside it, the right-angled triangle formed by the base diagonal of 60 cm, the vertical edge of 20 cm and the longest length L is drawn flat, with the angle theta marked at the base

Working	Comment
(a) The base is a rectangle 48×36, so its diagonal b satisfies $b^2 = 48^2 + 36^2$.	First flat triangle: the base.
$b^2 = 2304 + 1296$	Square each side of the base.
$b^2 = 3600$	Add.
$b = 60$	The base diagonal is $60\,\text{cm}$.
(b) The base diagonal, a vertical edge of $20\,\text{cm}$, and the longest length L form a second right-angled triangle.	Second flat triangle: inside the carton.
$L^2 = 60^2 + 20^2$	Pythagoras' theorem again.
$L^2 = 3600 + 400$	Square each side.
$L^2 = 4000$	Add.
$L = \sqrt{4000}$	Take the square root.
$L = 63.24\ldots \approx 63.2$	Correct to 1 decimal place.
$\therefore$ the longest length that fits is about $63.2\,\text{cm}$.	
(c) $\tan \theta = \frac{20}{60}$, where θ is the angle between the longest length and the base.	In the second triangle: opposite 20, adjacent 60.

Working	Comment
$\theta = \tan^{-1}\left(\frac{1}{3}\right)$	
$\theta = 18.43\ldots° \approx 18.4°$	Correct to 1 decimal place.
$\therefore$ the longest length makes an angle of about $18.4°$ with the base.	
(d) $65 > 63.2$, so the $65\,\text{cm}$ rod **will not** fit flat inside the carton.	Compare with the longest internal length.

Notice the shape of the method: no three-dimensional formula is used. The solid is solved as **two two-dimensional right-angled triangles**, one after the other. Reversing part (d) — finding the smallest carton that holds a rod of a given length — uses exactly the same two triangles run backwards.

Practice questions

Scaffolded

Q1. A rectangular park is $90\,\text{m}$ long and $40\,\text{m}$ wide. A path runs straight from one corner to the opposite corner. (a) Draw the right-angled triangle formed by the length, the width and the path. (b) Write down the Pythagoras equation for the path. (c) Find the length of the path, correct to 1 decimal place.

Q2. In a right-angled triangle, one angle is $34°$ and the side adjacent to it is $22\,\text{m}$. (a) Which ratio links the opposite side, the adjacent side and the angle? (b) Find the opposite side, correct to 1 decimal place. (c) Find the hypotenuse, correct to 1 decimal place.

Q3. In a right-angled triangle, one angle is $23°$ and the side adjacent to it is $60\,\text{m}$. (a) Write the equation linking the opposite side to the $60\,\text{m}$ side. (b) Find the opposite side, correct to 1 decimal place. (c) Find the hypotenuse, correct to 1 decimal place.

Q4. `SCENARIO` A surveyor stands on flat ground $40\,\text{m}$ from the base of a flagpole. The angle of elevation from her eye to the top of the flagpole is $27°$, and her eye is $1.6\,\text{m}$ above the ground. (a) Draw the triangle from her eye to the top of the flagpole. (b) Find the height of the flagpole above her eye level, correct to 1 decimal place. (c) Hence find the total height of the flagpole, correct to 1 decimal place.

Q5. `SCENARIO` From the top of a vertical tower $45\,\text{m}$ high, the angle of depression of a car on flat ground is $18°$. (a) Explain why the angle of elevation from the car to the top of the tower is also $18°$. (b) Draw the right-angled triangle. (c) Find the distance of the car from the base of the tower, correct to 1 decimal place.

Q6. Point B is $6.5\,\text{km}$ from point A on a bearing of $305°\text{T}$. (a) Draw the diagram, with a north line at A. (b) The bearing $305°\text{T}$ is $55°$ west of north. Write the two right-angled triangle equations for the north and west components. (c) Find how far north and how far west of A the point B is, each correct to 1 decimal place.

Q7. (a) Write the bearings of north, east, south and west as three-figure bearings. (b) From a point P, draw and label the bearings $058°\text{T}$ and $215°\text{T}$. (c) The bearing of X from Y is $047°\text{T}$. Find the bearing of Y from X.

Q8. A rectangular box is $12\,\text{cm}$ long, $9\,\text{cm}$ wide and $8\,\text{cm}$ high. (a) Find the diagonal of the base of the box. (b) Find the longest straight length that fits inside the box, correct to 1 decimal place. (c) Find the angle that this length makes with the base, correct to 1 decimal place.

Unscaffolded

Q9. A ladder $5.0\,\text{m}$ long leans against a vertical wall with its foot $1.8\,\text{m}$ from the base of the wall. Find (a) how high up the wall the ladder reaches, and (b) the angle the ladder makes with the ground, each correct to 1 decimal place.

Q10. `SCENARIO` A ship sails $14\,\mathrm{km}$ on a bearing of `060°T`, then $9\,\mathrm{km}$ on a bearing of `150°T`. Find (a) the distance of the ship from its start, and (b) the bearing of the ship from its start, correct to 1 decimal place and to the nearest degree respectively.

Q11. From a point on flat ground $85\,\mathrm{m}$ from the base of a tower, the angle of elevation of the top of the tower is $19°$. Find the height of the tower, correct to 1 decimal place.

Q12. From the top of a building $120\,\mathrm{m}$ high, the angle of depression of a car on flat ground is $34°$. Find the distance of the car from the base of the building, correct to 1 decimal place.

Q13. From a point on flat ground, the angle of elevation of the top of a cliff is $24°$. After walking $60\,\mathrm{m}$ straight towards the cliff, the angle of elevation is $35°$. Find the height of the cliff, correct to 1 decimal place.

Q14. Two buildings face each other on flat ground. From a window $18\,\mathrm{m}$ above the ground in the first building, the angle of depression of the base of the second building is $31.0°$ and the angle of elevation of its top is $22°$. Find (a) the distance between the buildings, and (b) the height of the second building, each correct to 1 decimal place.

Q15. `SCENARIO` A ferry sails $12\,\mathrm{km}$ on a bearing of `140°T`, then $16\,\mathrm{km}$ on a bearing of `230°T`. Find (a) the distance of the ferry from its start, and (b) the bearing of the ferry from its start, correct to 1 decimal place and to the nearest degree respectively.

Q16. A rectangular box is $30\,\mathrm{cm}$ long, $20\,\mathrm{cm}$ wide and $25\,\mathrm{cm}$ high. (a) Find the longest straight length that fits inside the box, correct to 1 decimal place. (b) Will a ruler $40\,\mathrm{cm}$ long fit flat inside the box? (c) Find the angle this longest length makes with the base, correct to 1 decimal place.

Q17. `SCENARIO` A square-based tent has a base $3.0\,\mathrm{m}$ by $3.0\,\mathrm{m}$, and its apex is $2.0\,\mathrm{m}$ above the centre of the base. Find (a) the distance from the centre of the base to a corner, (b) the length of a strut from the apex to a corner, and (c) the angle the strut makes with the ground, each correct to 2 decimal places for (a) and (b) and 1 decimal place for (c).

Q18. `SCENARIO` A wire supports a vertical pole $15\,\mathrm{m}$ high, running from the top of the pole to an anchor in flat ground $8\,\mathrm{m}$ from the base of the pole. Find (a) the length of the wire, and (b) the angle the wire makes with the ground, correct to 1 decimal place.

Q19. `SCENARIO` A coastguard stands at the top of a vertical cliff $75\,\mathrm{m}$ above sea level. The angle of depression of a boat due east of the cliff is $18°$, and the angle of depression of a second boat due south of the cliff is $13°$. Find (a) the distance from the base of the cliff to each boat, each correct to 1 decimal place, (b) the distance between the two boats, correct to 1 decimal place, and (c) the bearing of the southern boat from the eastern boat, to the nearest degree.

Q20. `SCENARIO` From a point P on flat ground, the bearing of the base of a tower is `055°T` and the angle of elevation of the top of the tower is $41°$. The tower is $35\,\mathrm{m}$ high. Find (a) the distance from P to the base of the tower, correct to 1 decimal place, and (b) the bearing of P from the tower.

Q21. `SCENARIO` A kite is flown on a straight string $40\,\mathrm{m}$ long. The string makes an angle of $52°$ with the horizontal ground. Find (a) the height of the kite above the point where the string is held, and (b) the horizontal distance of the kite from that point, each correct to 1 decimal place. Ignore the height of the person's hand.

Q22. A rectangular room is $5\,\mathrm{m}$ long, $4\,\mathrm{m}$ wide and $3\,\mathrm{m}$ high. (a) Find the diagonal of the floor. (b) Find the longest straight length that fits inside the room, correct to 2 decimal places. (c) Find the angle this length makes with the floor, correct to 1 decimal place.

Real context

Q23. The wall of the Thomson Dam in Victoria is $165\,\mathrm{m}$ high, measured from the base of the wall to the top. A surveyor stands on flat ground downstream of the wall and measures the angle of elevation from the ground to the top of the wall as $12°$. (a) Find how far the surveyor is from the base of the dam wall, correct to 1 decimal place. (b) She then walks $100\,\mathrm{m}$ closer to the wall. Find the new angle of elevation, correct to 1 decimal place.

Source: Melbourne Water, *Thomson Reservoir*, 2026. Retrieved 19 August 2026. (melbournewater.com.au/water-and-environment/water-management/water-storage-reservoirs/thomson-reservoir)

Digital tool task

> **DIGITAL TOOL — this task assumes dynamic geometry software. Your teacher will nominate the tool.**

Task. Model a flagpole $12\,\mathrm{m}$ high standing on flat ground, with an observer at a point O on the ground.

Input (what to construct):

1. a horizontal line (the ground) and a point B on it;
2. a vertical segment from B, 12 units high, ending at T (the top of the flagpole);
3. a point O on the ground line, and the segment OT (the line of sight);
4. the angle between the ground line and OT at O, and the length OB, both measured by the software.

Output (what to expect): dragging O changes the distance OB and the angle together. With $OB=20\,\mathrm{m}$ the angle reads about $31.0°$. Dragging O until the angle reads $35°$ gives $OB\approx 17.1\,\mathrm{m}$.

Tool-free path (still assessed): verify both readings by hand.

- When $OB=20$: $\tan\theta=\frac{12}{20}$, giving $\theta=\tan^{-1}(0.6)$. Hence $\theta\approx 31.0°$.
- When $\theta=35°$: $OB=\frac{12}{\tan 35°}$. Hence $OB\approx 17.1\,\mathrm{m}$.

Extend the model, if your teacher asks, by adding the line of sight from T down to a point on the ground beyond O, and compare the angle of depression at T with the angle of elevation at O.

Answers

Q1. 98.5 m. **Q2.** (b) 14.8 m; (c) 26.5 m. **Q3.** (b) 25.5 m; (c) 65.2 m. **Q4.** (b) 20.4 m; (c) 22.0 m. **Q5.** 138.5 m. **Q6.** 3.7 km north, 5.3 km west. **Q7.** (a) north 000°T, east 090°T, south 180°T, west 270°T; (c) 227°T. **Q8.** (a) 15 cm; (b) 17 cm; (c) 28.1°. **Q9.** (a) 4.7 m; (b) 68.9°. **Q10.** (a) 16.6 km; (b) 093°T. **Q11.** 29.3 m. **Q12.** 177.9 m. **Q13.** 73.4 m. **Q14.** (a) 30.0 m; (b) 30.1 m. **Q15.** (a) 20.0 km; (b) 193°T. **Q16.** (a) 43.9 cm; (b) yes; (c) 34.7°. **Q17.** (a) 2.12 m; (b) 2.92 m; (c) 43.3°. **Q18.** (a) 17 m; (b) 61.9°. **Q19.** (a) 230.8 m and 324.9 m; (b) 398.5 m; (c) 215°T. **Q20.** (a) 40.3 m; (b) 235°T. **Q21.** (a) 31.5 m; (b) 24.6 m. **Q22.** (a) $\sqrt{41} \approx 6.40$ m; (b) $\sqrt{50} \approx 7.07$ m; (c) 25.1°. **Q23.** (a) 776.3 m; (b) 13.7°. **Digital tool task:** 31.0°; 17.1 m.

Fully-worked solutions

Q1. The path is the hypotenuse of a right-angled triangle with shorter sides 90 and 40.

$d^2 = 90^2 + 40^2$

$d^2 = 8100 + 1600$

$d^2 = 9700$

$d = \sqrt{9700}$

$d = 98.48\ldots$

∴ the path is about 98.5 m long.

Q2. The angle, the adjacent side and the opposite side are linked by the tangent.

$\tan 34° = \dfrac{\text{opp}}{22}$

$\text{opp} = 22 \tan 34°$

$\text{opp} = 14.83\ldots \approx 14.8\text{ m}$

The adjacent side and the hypotenuse are linked by the cosine.

$\cos 34° = \dfrac{22}{\text{hypotenuse}}$

$\text{hypotenuse} = \dfrac{22}{\cos 34°}$

$\text{hypotenuse} = 26.53\ldots \approx 26.5\text{ m}$

Q3. $\tan 23° = \dfrac{\text{opp}}{60}$

$\text{opp} = 60 \tan 23°$

$\text{opp} = 25.46\ldots \approx 25.5\text{ m}$

$\cos 23° = \dfrac{60}{\text{hypotenuse}}$

$\text{hypotenuse} = \dfrac{60}{\cos 23°}$

$\text{hypotenuse} = 65.18\ldots \approx 65.2\text{ m}$

Q4. Above eye level:

$\tan 27° = \frac{x}{40}$

$x = 40 \tan 27°$

$x = 20.38\ldots \approx 20.4\,\text{m}$

Total height:

$\text{height} = 20.38\ldots + 1.6$

$\text{height} = 21.98\ldots \approx 22.0\,\text{m}$

Q5. The horizontal at the top of the tower and the flat ground are parallel, so the angle of elevation from the car equals the angle of depression, $18°$ (alternate angles).

$\tan 18° = \frac{45}{d}$

$d = \frac{45}{\tan 18°}$

$d = 138.49\ldots \approx 138.5\,\text{m}$

Q6. The bearing $\texttt{305°T}$ is $360° - 305° = 55°$ west of north.

North component: $6.5 \cos 55° = 3.72\ldots$, so about $3.7\,\text{km}$ north.

West component: $6.5 \sin 55° = 5.32\ldots$, so about $5.3\,\text{km}$ west.

Q7. (a) North $\texttt{000°T}$, east $\texttt{090°T}$, south $\texttt{180°T}$, west $\texttt{270°T}$. (b) $\texttt{058°T}$ is drawn $58°$ clockwise from north; $\texttt{215°T}$ is drawn $35°$ past south ($215° - 180° = 35°$). (c) Back bearing: $47° + 180° = 227°$, so the bearing of Y from X is $\texttt{227°T}$.

Q8. Base diagonal:

$b^2 = 12^2 + 9^2$

$b^2 = 144 + 81$

$b^2 = 225$

$b = 15\,\text{cm}$

Longest length:

$L^2 = 15^2 + 8^2$

$L^2 = 225 + 64$

$L^2 = 289$

$L = 17\,\text{cm}$

Angle with the base:

$\tan\theta = \frac{8}{15}$

$\theta = \tan^{-1}(0.5333\ldots)$

$\theta = 28.07\ldots \approx 28.1°$

Q9. Height:

$h^2 = 5.0^2 - 1.8^2$

$h^2 = 25 - 3.24$

$h^2 = 21.76$

$h = \sqrt{21.76}$

$h = 4.66\ldots \approx 4.7\,\text{m}$

Angle with the ground:

$\cos\theta = \dfrac{1.8}{5.0}$

$\cos\theta = 0.36$

$\theta = \cos^{-1}(0.36)$

$\theta = 68.89\ldots \approx 68.9°$

Q10. At the turning point, the back bearing of the first leg is $60° + 180° = 240°$, and the second leg runs on $150°$, so the angle between the legs is $240° - 150° = 90°$.

Distance:

$d^2 = 14^2 + 9^2$

$d^2 = 196 + 81$

$d^2 = 277$

$d = \sqrt{277}$

$d = 16.64\ldots \approx 16.6\,\text{km}$

Angle at the start:

$\tan\alpha = \dfrac{9}{14}$

$\alpha = \tan^{-1}(0.6429\ldots)$

$\alpha = 32.73\ldots°$

Bearing:

$\text{bearing} = 60° + 32.73\ldots°$

$\text{bearing} = 92.73\ldots°$

$\therefore$ the bearing is $093°\text{T}$.

Q11. $\tan 19° = \dfrac{h}{85}$

$h = 85\tan 19°$

$h = 29.26\ldots \approx 29.3\,\text{m}$

Q12. The angle of elevation from the car is $34\,°$ (alternate angles).

$\tan 34\,° = \dfrac{120}{d}$

$d = \dfrac{120}{\tan 34\,°}$

$d = 177.90\ldots \approx 177.9\,\text{m}$

Q13. Let the height be h and the first distance from the cliff be d.

From the first position:

$\tan 24\,° = \dfrac{h}{d}$

$d = \dfrac{h}{\tan 24\,°}$

From the second position:

$\tan 35\,° = \dfrac{h}{d - 60}$

$d - 60 = \dfrac{h}{\tan 35\,°}$

Subtracting the second equation from the first:

$60 = \dfrac{h}{\tan 24\,°} - \dfrac{h}{\tan 35\,°}$

$60 = h(2.2460\ldots - 1.4281\ldots)$

$60 = 0.8179\ldots \times h$

$h = \dfrac{60}{0.8179\ldots}$

$h = 73.35\ldots \approx 73.4\,\text{m}$

Q14. (a) The horizontal at the window and the flat ground are parallel, so the angle of elevation from the base of the second building to the window is also $31.0\,°$ (alternate angles).

$\tan 31.0\,° = \dfrac{18}{d}$

$d = \dfrac{18}{\tan 31.0\,°}$

$d = 29.95\ldots \approx 30.0\,\text{m}$

(b) Height above the window:

$\tan 22\,° = \dfrac{x}{29.95\ldots}$

$x = 29.95\ldots \times \tan 22\,°$

$x = 12.10\ldots\text{ m}$

Height of the second building:

$\text{height} = 18 + 12.10\ldots$

$\text{height} = 30.10\ldots \approx 30.1\text{ m}$

Q15. At the turning point, the back bearing of the first leg is $140° + 180° = 320°$, and the second leg runs on $230°$, so the angle between the legs is $320° - 230° = 90°$.

Distance:

$d^2 = 12^2 + 16^2$

$d^2 = 144 + 256$

$d^2 = 400$

$d = 20.0\text{ km}$

Angle at the start:

$\tan\alpha = \dfrac{16}{12}$

$\alpha = \tan^{-1}(1.3333\ldots)$

$\alpha = 53.13\ldots°$

Bearing:

$\text{bearing} = 140° + 53.13\ldots°$

$\text{bearing} = 193.13\ldots°$

$\therefore$ the bearing is $193°\text{T}$.

Q16. Base diagonal:

$b^2 = 30^2 + 20^2$

$b^2 = 900 + 400$

$b^2 = 1300$

$b = \sqrt{1300}$

$b = 36.05\ldots\text{ cm}$

Longest length:

$L^2 = b^2 + 25^2$

$L^2 = 1300 + 625$

$L^2 = 1925$

$L = \sqrt{1925}$

$L = 43.87\ldots \approx 43.9\text{ cm}$

$43.9 > 40$, so the ruler **will** fit flat inside the box.

Angle with the base:

$\tan\theta=\dfrac{25}{36.05\ldots}$

$\tan\theta=0.6935\ldots$

$\theta=\tan^{-1}(0.6935\ldots)$

$\theta=34.73\ldots\approx 34.7\,°$

Q17. Distance from centre to corner:

$r^2=1.5^2+1.5^2$

$r^2=2.25+2.25$

$r^2=4.5$

$r=\sqrt{4.5}$

$r=2.1213\ldots\approx 2.12\,\text{m}$

Strut length:

$s^2=r^2+2.0^2$

$s^2=4.5+4$

$s^2=8.5$

$s=\sqrt{8.5}$

$s=2.915\ldots\approx 2.92\,\text{m}$

Angle with the ground:

$\tan\theta=\dfrac{2.0}{2.121\ldots}$

$\tan\theta=0.9428\ldots$

$\theta=\tan^{-1}(0.9428\ldots)$

$\theta=43.31\ldots\approx 43.3\,°$

Q18. Wire length:

$w^2=15^2+8^2$

$w^2=225+64$

$w^2=289$

$w=17\,\text{m}$

Angle with the ground:

$\tan\theta=\dfrac{15}{8}$

$\tan\theta=1.875$

$\theta = \tan^{-1}(1.875)$

$\theta = 61.92\ldots \approx 61.9°$

Q19. Eastern boat:

$\tan 18° = \frac{75}{d_1}$

$d_1 = \frac{75}{\tan 18°}$

$d_1 = 230.82\ldots \approx 230.8\,\text{m}$

Southern boat:

$\tan 13° = \frac{75}{d_2}$

$d_2 = \frac{75}{\tan 13°}$

$d_2 = 324.86\ldots \approx 324.9\,\text{m}$

The two boats and the base of the cliff form a right-angled triangle (east and south are at right angles), so Pythagoras' theorem gives the distance between the boats:

$\text{distance}^2 = 230.82\ldots^2 + 324.86\ldots^2$

$\text{distance}^2 = 53280.7\ldots + 105534.4\ldots$

$\text{distance}^2 = 158815.2\ldots$

$\text{distance} = \sqrt{158815.2\ldots}$

$\text{distance} = 398.51\ldots \approx 398.5\,\text{m}$

From the eastern boat, the southern boat lies to the west and to the south. The angle west of south:

$\tan \phi = \frac{230.82\ldots}{324.86\ldots}$

$\phi = \tan^{-1}(0.7105\ldots)$

$\phi = 35.39\ldots°$

Bearing:

$\text{bearing} = 180° + 35.39\ldots°$

$\text{bearing} = 215.39\ldots°$

∴ the bearing is 215°T.

Q20. Distance:

$\tan 41° = \frac{35}{d}$

$d = \frac{35}{\tan 41°}$

$d = 40.26\ldots \approx 40.3\,\text{m}$

Back bearing: $55° + 180° = 235°$, so the bearing of P from the tower is $235°\text{T}$.

Q21. Height:

$$\sin 52° = \frac{h}{40}$$

$h = 40\sin 52°$

$h = 31.52\ldots \approx 31.5\,\text{m}$

Horizontal distance:

$$\cos 52° = \frac{x}{40}$$

$x = 40\cos 52°$

$x = 24.62\ldots \approx 24.6\,\text{m}$

Q22. Floor diagonal:

$f^2 = 5^2 + 4^2$

$f^2 = 25 + 16$

$f^2 = 41$

$f = \sqrt{41} \approx 6.40\,\text{m}$

Longest length:

$L^2 = 41 + 3^2$

$L^2 = 41 + 9$

$L^2 = 50$

$L = \sqrt{50} \approx 7.07\,\text{m}$

Angle with the floor:

$$\tan\theta = \frac{3}{\sqrt{41}}$$

$\tan\theta = 0.4685\ldots$

$\theta = \tan^{-1}(0.4685\ldots)$

$\theta = 25.10\ldots \approx 25.1°$

Q23. (a)

$$\tan 12° = \frac{165}{d}$$

$$d = \frac{165}{\tan 12°}$$

$d = 776.26\ldots \approx 776.3\,\text{m}$

(b) New distance:

$$d_{\text{new}} = 776.26\ldots - 100$$

$$d_{\text{new}} = 676.26\ldots\,\text{m}$$

New angle:

$$\tan\theta = \frac{165}{676.26\ldots}$$

$$\tan\theta = 0.2440\ldots$$

$$\theta = \tan^{-1}(0.2440\ldots)$$

$$\theta = 13.71\ldots \approx 13.7°$$

Digital tool task. With $OB = 20$:

$$\tan\theta = \frac{12}{20}$$

$$\tan\theta = 0.6$$

$$\theta = \tan^{-1}(0.6)$$

$$\theta = 30.96\ldots \approx 31.0°$$

With $\theta = 35°$:

$$OB = \frac{12}{\tan 35°}$$

$$OB = \frac{12}{0.7002\ldots}$$

$$OB = 17.13\ldots \approx 17.1\,\text{m}$$

Teacher note

Level 9 code rehearsed (D3): `VC2M9M03` — spatial problems with angle properties, scale, similarity, ratio, Pythagoras' theorem and right-angled trigonometry — and `VC2M9SP01` — the constancy of the sine, cosine and tangent ratios from similarity. 4B cannot use a bearing without SOH-CAH-TOA; both skills are rehearsed here, not re-taught, and are not assessed as Level 9 outcomes. 4C (`VC2M10M04`) applies scaling and measurement error to the lengths and angles measured here.

Tool classes (D13): scientific calculator (all trigonometry; degree mode), and dynamic geometry software for the digital tool task only (`VC2M10M03.E04`). No CAS. The tool-free path for the digital task is printed beside it and is what is assessed.

VCE destinations (§5, reference only): `VC2M10M03` is named in the Units 3 & 4 assumed background of Foundation Mathematics (AoS 4 'Space and measurement'), General Mathematics (assumed knowledge) and Mathematical Methods, with the skills *"Apply right-angled trigonometry to a bearing/navigation problem"*, *"Decompose a 3D problem into 2D right-triangle problems and apply Pythagoras"* and *"Use an angle of elevation (e.g. measured by clinometer) and a horizontal distance to compute the height"*. MM and SM have no measurement Area of Study, so the destinations are FM and GM. This is the most transferable subtopic in the book.

Elaboration `VC2M10M03.E05` is not delivered. It invites Aboriginal and Torres Strait Islander navigation, design and surveying contexts. This book excludes Aboriginal and Torres Strait Islander content (see the front-of-book note);

the geometric and spatial reasoning the elaboration names is delivered in full through the bearing, surveying and design contexts of `VC2M10M03.E01–.E04`.

Level check (D6): every triangle in this subtopic is right-angled and every angle is acute. The sine and cosine rules, trigonometry beyond 90° and any three-dimensional distance formula are not used; three-dimensional problems are solved only as sequences of two-dimensional right-angled triangles, as `VC2M10M03.E02` describes.

Modelling with direct and inverse proportion, scaling, and measurement error

Mathematics 10 · Chapter 4 Measurement · Subtopic 4C · Extended block

Strand	Measurement
Content description	`VC2M10M04`
Elaboration ids carried	`VC2M10M04.E01` `VC2M10M04.E02` `VC2M10M04.E03` `VC2M10M04.E04` `VC2M10M04.E05`
Weight	Extended (double allocation)

> **Content description (verbatim):** *use mathematical modelling to solve practical problems involving direct and inverse proportion and scaling of objects; formulate problems and interpret solutions in terms of the situation, including the impact of measurement errors on the accuracy of results; evaluate and modify models as necessary, and report assumptions, methods and findings*
>
> **Achievement standard (extract):** *They identify the impact of measurement errors on the accuracy of results. Students use mathematical modelling to solve practical problems involving direct and inverse proportion and scaling, evaluating and modifying models, and reporting assumptions, methods and findings.*

You need first: rates, ratio and direct proportion from Year 9 (`VC2M9M05`), percentage error from Year 9 (`VC2M9M04`), enlargement and scale from similarity (`VC2M9SP02`, `VC2M9M03`), and the measurement skills of Subtopics 4A and 4B of this chapter. This subtopic rehearses those skills and builds them into complete mathematical models; it does not re-teach them.

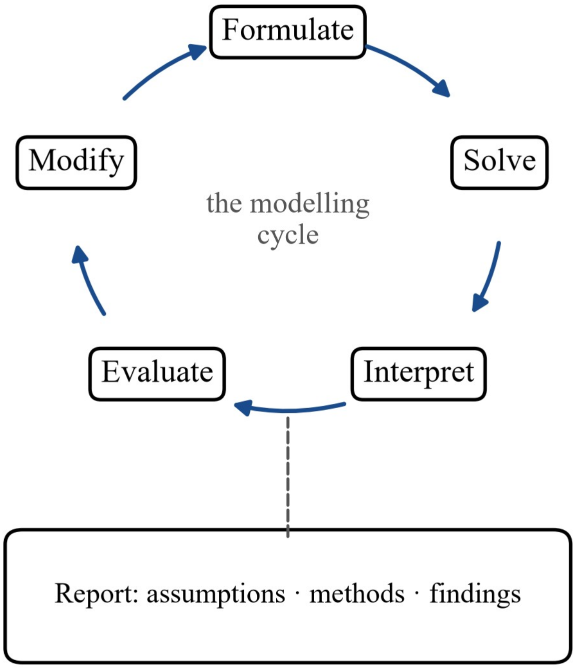
Theory

Everything in this subtopic follows one loop: **turn the situation into a model, use the model, then check the answer against the situation.** The models are familiar — direct proportion, inverse proportion and scale — and the checking is the new habit: how an error in a measurement shows up in the answer the model gives.

1. The modelling cycle

The content description names the steps, and they form a cycle:

- **Formulate** — turn the situation into mathematics: choose the rule (a rate, a proportion, a scale) and write the equation. State what you are assuming.
- **Solve** — use the model to calculate what is asked.
- **Interpret** — say what the number means in terms of the situation, with the right units.
- **Evaluate** — compare the answer with reality: is it sensible, and how far out could the measurements put it?
- **Modify** — if the model does not match reality, change it and solve again.
- **Report** — give the assumptions, the methods and the findings.



A flow chart of five boxes in a ring, Formulate, Solve, Interpret, Evaluate and Modify, joined by arrows running clockwise back to the start, with the words the modelling cycle in the middle; below the ring, a box reading Report: assumptions, methods and findings is joined to the ring by a dashed line.

The cycle never ends at *solve*. A model that has not been checked against the situation is a calculation, not a model.

2. Direct proportion

Two quantities are in **direct proportion** when one is a constant multiple of the other: double one and the other doubles. Writing the quantities as y and x:

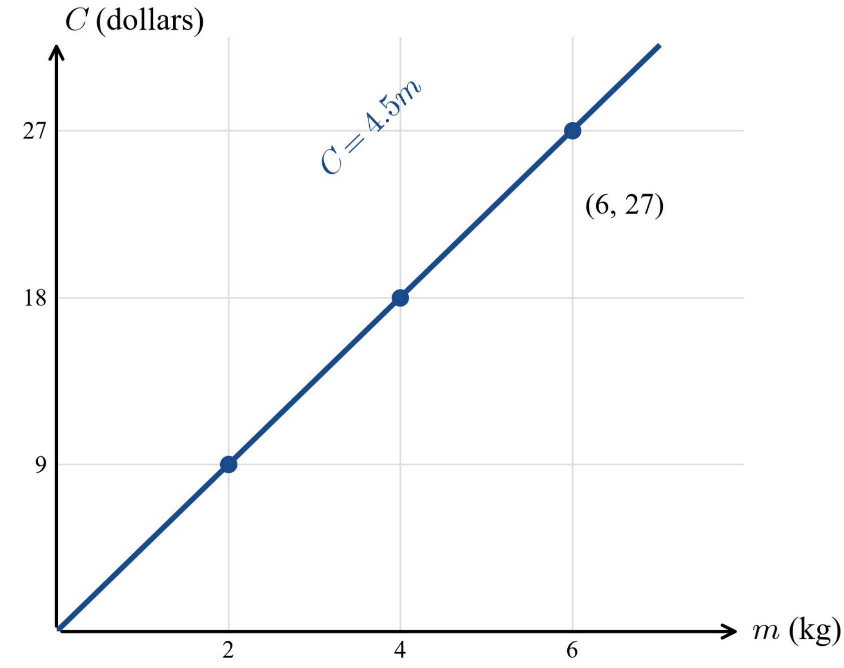
$$y = k\,x$$

where k is the constant multiple, found from any pair of values by $k = y \div x$. The graph of $y = k\,x$ is a **straight line through the origin**, and k is its steepness.

Apples cost $\$4.50$ per kilogram. The cost C is in direct proportion to the mass m:

m (kg)	2	4	6
C (dollars)	9	18	27

Here $k = C \div m = 4.5$, so the model is $C = 4.5\,m$.



A straight blue line through the origin on axes labelled m in kilograms and C in dollars, passing through the marked points (2, 9), (4, 18) and (6, 27); the line is labelled C equals 4.5 m.

Rates are direct proportion: $7.2\,\mathrm{L}$ per $100\,\mathrm{km}$, $\$\,4.50$ per kg, $12\,\mathrm{m}^2$ per litre of paint. If a rate is constant, the quantity and the rate's other variable are in direct proportion.

3. Inverse proportion

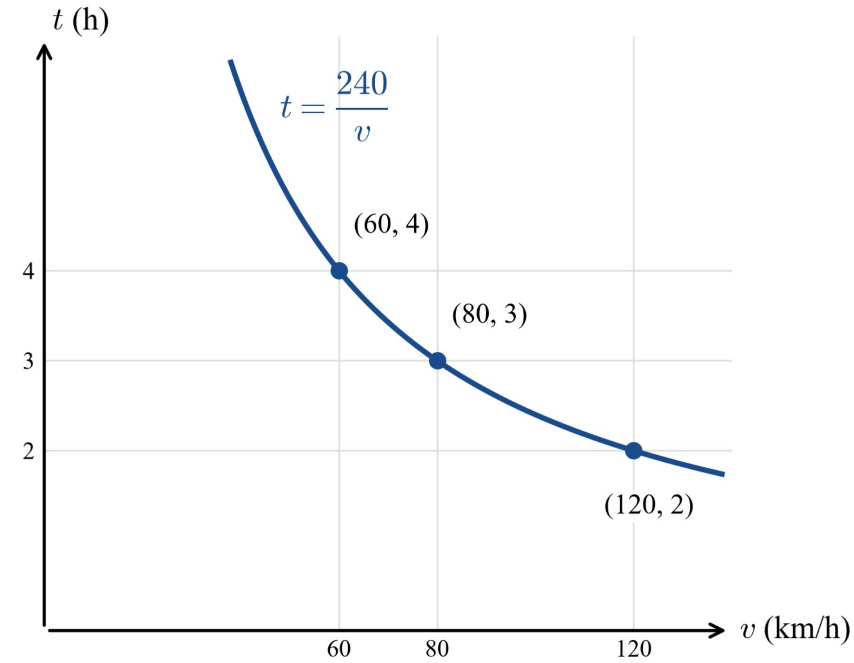
Two quantities are in **inverse proportion** when their product is constant: double one and the other halves. Writing the quantities as y and x:

$$y=\frac{k}{x}\ \text{that is,}\ x\,y=k$$

The graph is a curve that never touches either axis — as one quantity grows, the other shrinks towards zero but never reaches it.

A journey of $240\,\mathrm{km}$ at average speed v takes time t, where $t=\frac{240}{v}$. The product $t\,v=240$ is constant:

v (km/h)	60	80	100	120
t (h)	4	3	2.4	2



A blue curve falling from left to right on axes labelled v in kilometres per hour and t in hours, marked with the points (60, 4), (80, 3) and (120, 2); the curve is labelled t equals 240 divided by v.

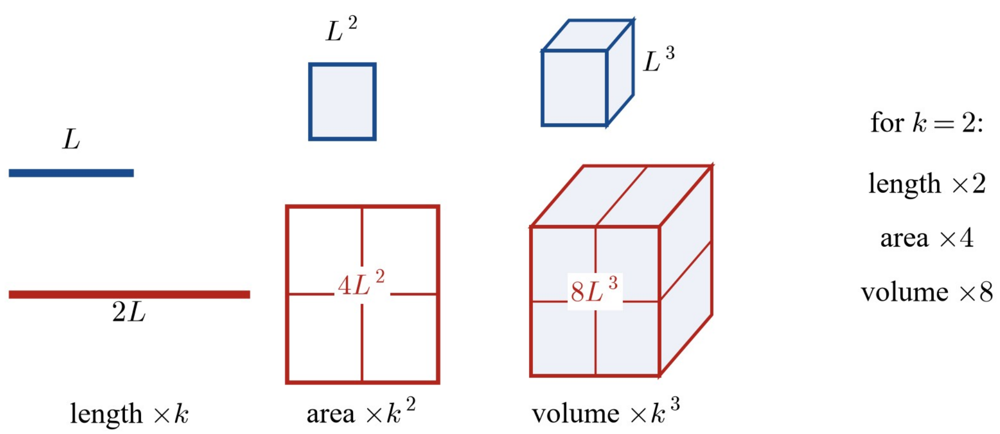
Crews and jobs work the same way: more workers means fewer hours, with the product (the total worker-hours the job needs) constant. **Check before you model:** direct proportion and inverse proportion are easy to confuse. Ask *double one, does the other double (direct) or halve (inverse)?*

4. Scaling objects: length, area and volume

An object scaled by **scale factor** k has every length multiplied by k. Areas and volumes do not grow by k:

- every **length** is multiplied by k;
- every **area** is multiplied by k^2;
- every **volume** is multiplied by k^3.

For $k = 2$: lengths $\times 2$, areas $\times 4$ (the square tiles into four copies of the original), volumes $\times 8$ (the cube packs with eight copies of the original).



Three pairs of shapes comparing an object with its scale factor 2 enlargement: a segment of length L beside one of length 2L, labelled length times k; a square of area L squared beside a square tiled into four copies, labelled area times k squared; a small cube beside a cube built from eight copies, labelled volume times k cubed; a side note reads for k equals 2: length times 2, area times 4, volume times 8.

This is the enlargement idea from Year 9 (`VC2M9SP02`) applied to solids. The most common error in scaling is using k where k^2 or k^3 belongs — a model at scale factor 20 needs 400 times the area of material and 8000 times the volume, not 20 times.

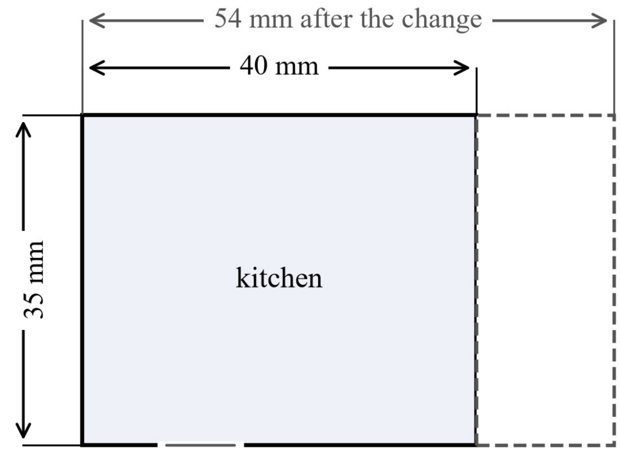
5. Plans and elevations

A **plan** is a drawing of a building or room seen from above; an **elevation** is a drawing of one face seen straight on. Both are drawn to a **scale**, and the scale is what converts drawing lengths to real lengths.

At a scale of $1:100$, one unit on the drawing stands for 100 of the same units in real life. A wall drawn $40\,\mathrm{mm}$ long is

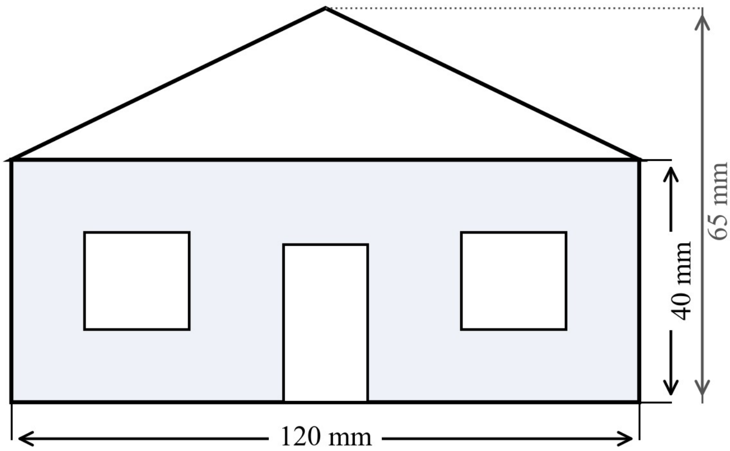
$$40\,\mathrm{mm} \times 100 = 4000\,\mathrm{mm} = 4.0\,\mathrm{m}$$

long in real life — multiply by the scale factor to leave the drawing, and remember $1000\,\mathrm{mm} = 1\,\mathrm{m}$. Going the other way, divide: a real length of $5.4\,\mathrm{m} = 5400\,\mathrm{mm}$ is drawn $5400 \div 100 = 54\,\mathrm{mm}$ long.



Scale 1:100 — dimensions as drawn on the plan

A floor plan of a kitchen drawn to a scale of 1 to 100: a rectangle 40 millimetres by 35 millimetres with a door gap in the lower wall, and a dashed extension of the rectangle out to 54 millimetres labelled 54 mm after the change; dimension arrows show 40 mm, 35 mm and 54 mm as drawn on the plan.



Scale 1:100 — front elevation, dimensions as drawn

A front elevation of a hall drawn to a scale of 1 to 100: a rectangle 120 millimetres wide and 40 millimetres high with a door and two windows, topped by a triangular roof whose apex is 65 millimetres above the base; dimension arrows show 120 mm, 40 mm and 65 mm as drawn.

Areas on plans scale by the **square** of the scale factor, exactly as in §4: at $1:100$, $1\,\text{mm}^2$ on the drawing stands for $100^2 = 10\,000\,\text{mm}^2$ of real floor.

6. Estimating a scale factor from one linear dimension

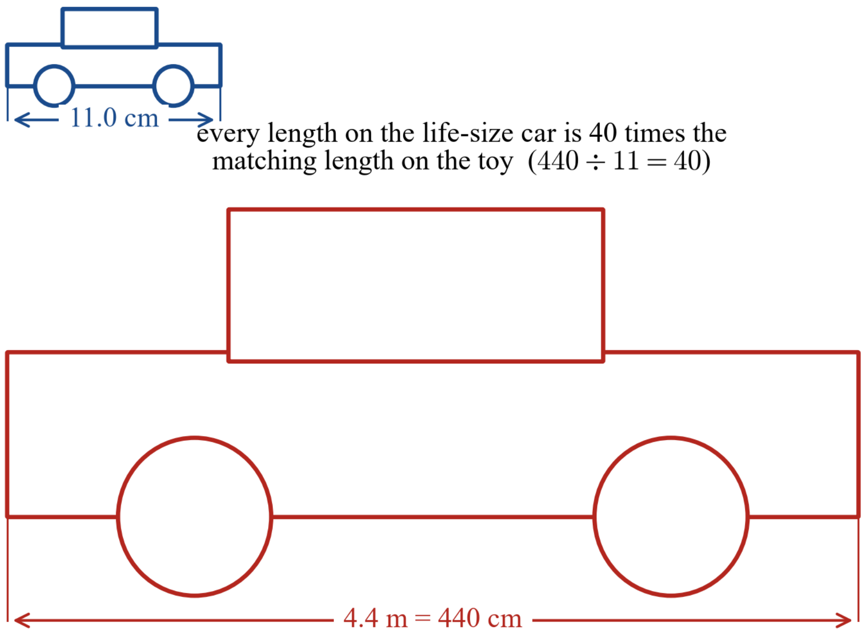
Often a model or a picture gives no scale, but one **linear dimension** of the real object is known or typical. Then the scale factor can be estimated from that one matching pair of lengths:

$$k = \frac{\text{life-size length}}{\text{model length}}$$

A toy car is $11.0\,\text{cm}$ long; a typical life-size car is $4.4\,\text{m} = 440\,\text{cm}$ long. The estimated scale factor is

$$k = 440 \div 11.0 = 40$$

and every other length on the toy, multiplied by 40, estimates the matching real length.



A small blue toy car above a large red life-size car, each with a dimension arrow beneath it: the toy labelled 11.0 cm and the life-size car labelled 4.4 m equals 440 cm; between them, the text every length on the life-size car is 40 times the matching length on the toy, 440 divided by 11 equals 40.

This is an *estimate*: the toy may not be a true scale model, and a typical car is not *this* car. State that assumption when you report the findings.

7. Measurement error, bounds, and how errors grow

No measurement is exact. The **error** of a measurement is

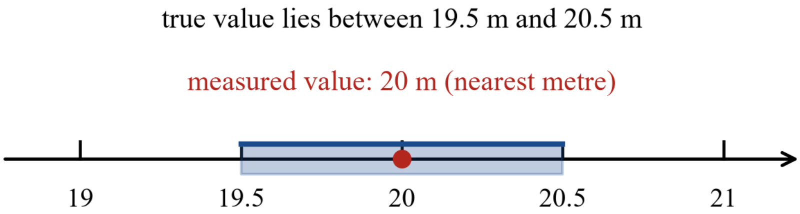
$$\text{error} = \text{measured value} - \text{true value}$$

and the **percentage error** is the error written as a percentage of the true value:

$$\text{percentage error} = \frac{\text{error}}{\text{true value}} \times 100\,\%$$

A side measured as $20.4\,\text{m}$ when it is truly $20.0\,\text{m}$ has error $0.4\,\text{m}$ and percentage error $\dfrac{0.4}{20.0} \times 100\,\% = 2\,\%$.

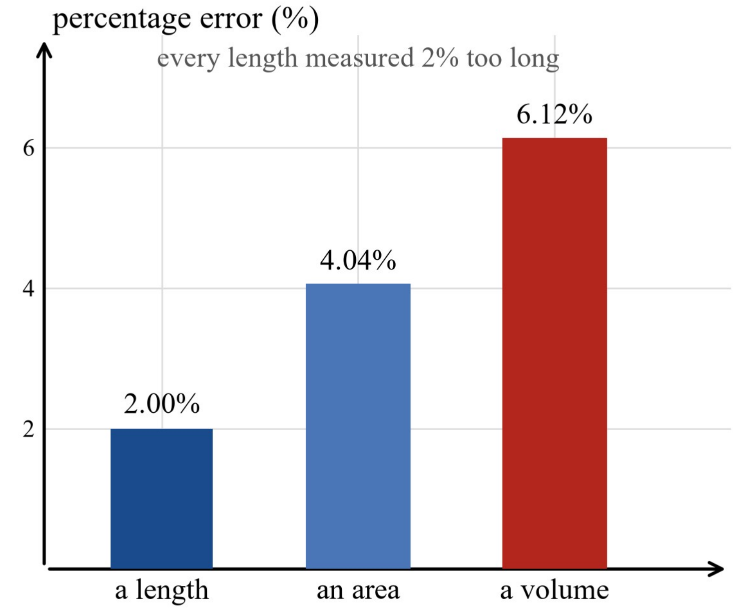
Bounds. A measurement recorded to the nearest metre can be out by up to half a metre either way: $20\,\text{m}$ to the nearest metre means the true value lies between $19.5\,\text{m}$ and $20.5\,\text{m}$. That interval is the **bounds** of the measurement.



A number line from 19 to 21 with a shaded band from 19.5 to 20.5 labelled true value lies between 19.5 m and 20.5 m, and a red point at 20 labelled measured value: 20 m (nearest metre).

How errors grow. When a length with a small percentage error is used to calculate an area or a volume, the percentage error grows. If every length is $2\,\%$ too long, each length is multiplied by 1.02, so areas are multiplied by

$1.02^2 = 1.0404$ (about $4.04\,\%$ too big) and volumes by $1.02^3 \approx 1.0612$ (about $6.12\,\%$ too big). Roughly, the percentage error doubles for areas and triples for volumes — the same k, k^2, k^3 reasoning as §4, applied to the error.



A bar chart with the percentage error on the vertical axis and three bars: a length 2.00 per cent, an area 4.04 per cent and a volume 6.12 per cent, under the heading every length measured 2 per cent too long.

Bias. If an instrument is faulty — a tape that has stretched, a scale that was never zeroed — every reading is wrong *in the same direction*, and repeating the measurement does not fix it. A finding affected in this way is **biased**: it is pushed to one side of the true value, and the error grows into areas and volumes as above.

Worked examples

Example 1 — Direct proportion through the full cycle

`SCENARIO` A car's handbook claims a fuel use of $7.2\,\mathrm{L}$ per $100\,\mathrm{km}$ on country roads. A driver plans a $350\,\mathrm{km}$ country trip.

Working	Comment
Formulate. Fuel F is in direct proportion to distance d: $F = k\,d$ with $k = 7.2 \div 100 = 0.072$.	The claimed rate is the constant multiple. Assumption: the rate holds for the whole trip.
$F = 0.072\,d$	The model.
Solve. $F = 0.072 \times 350$	Substitute $d = 350$.
$F = 25.2$	
Interpret. The model predicts $25.2\,\mathrm{L}$ for the trip.	Units: litres of fuel.
Evaluate. The trip actually used $27.1\,\mathrm{L}$.	Reality check.
$\text{error} = 25.2 - 27.1 = -1.9$	The prediction was $1.9\,\mathrm{L}$ under the actual use.
$\text{percentage error} = \dfrac{-1.9}{27.1} \times 100\,\% \approx -7.0\,\%$	Correct to 1 decimal place.
Modify. New rate: $27.1 \div 350 \times 100 \approx 7.7\,\mathrm{L}$ per $100\,\mathrm{km}$.	The driver's own rate, correct to 1 decimal place.
$F = 0.0774\ldots \times 600 \approx 46.5$	The modified model predicts about $46.5\,\mathrm{L}$ for $600\,\mathrm{km}$.
Report. The handbook model under-predicted by about $7\,\%$; the modified model uses the driver's measured rate of $7.7\,\mathrm{L}$ per $100\,\mathrm{km}$. Assumptions: constant rate,	Assumptions, methods, findings.

Working	Comment
similar country driving.	

Example 2 — Inverse proportion: crews and hours

`SCENARIO` A painting job takes a crew of 6 workers 5 hours.

Working	Comment
The job needs $6 \times 5 = 30$ worker-hours.	The constant product k.
$t = \dfrac{30}{n}$, where n is the number of workers.	Inverse proportion: more workers, fewer hours.
$t = \dfrac{30}{4} = 7.5$	A crew of 4 takes 7.5 hours.
Modify. One worker can only work half the time, so the crew counts as 3.5 workers.	The situation changed; the model is re-used, not rebuilt.
$t = \dfrac{30}{3.5} = 8.57\ldots \approx 8.6$	Correct to 1 decimal place.
$\therefore$ a crew of 4 takes $7.5\,\mathrm{h}$; the reduced crew takes about $8.6\,\mathrm{h}$.	

(Evaluate: 3.5 workers for 8.6 hours is 30.1 worker-hours — the small excess is rounding.)

Example 3 — Scaling a printed prototype

`SCENARIO` A water tank is prototyped on a 3D printer at a scale of $1:20$. The printed model is a cylinder of radius $4\,\mathrm{cm}$ and height $7.5\,\mathrm{cm}$.

Working	Comment
Real radius $= 4 \times 20 = 80\,\mathrm{cm}$; real height $= 7.5 \times 20 = 150\,\mathrm{cm}$.	Lengths $\times\, k$, with $k = 20$.
Model volume $= \pi \times 4^2 \times 7.5 = 120\,\pi \approx 377\,\mathrm{cm}^3$.	Cylinder volume $\pi\, r^2 h$.
Real volume $= \pi \times 80^2 \times 150 = 960\,000\,\pi\,\mathrm{cm}^3$.	Same formula on the real tank.
$960\,000\,\mathrm{cm}^3 = 960\,\pi\,\mathrm{L} \approx 3015.9\,\mathrm{L}$	$1000\,\mathrm{cm}^3 = 1\,\mathrm{L}$; correct to 1 decimal place.
Check: $120\,\pi \times 20^3 = 120\,\pi \times 8000 = 960\,000\,\pi$.	Volumes $\times\, k^3$ — the two answers agree.
Model curved surface $= 2\,\pi \times 4 \times 7.5 = 60\,\pi \approx 188.5\,\mathrm{cm}^2$.	The surface to be coated on the model.
Real curved surface $= 2\,\pi \times 80 \times 150 = 24\,000\,\pi\,\mathrm{cm}^2 = 2.4\,\pi\,\mathrm{m}^2 \approx 7.54\,\mathrm{m}^2$.	Areas $\times\, k^2 = 400$; $10\,000\,\mathrm{cm}^2 = 1\,\mathrm{m}^2$.
$\therefore$ the real tank holds about $3015.9\,\mathrm{L}$, and about $7.54\,\mathrm{m}^2$ of surface must be coated.	

Example 4 — Reading a plan and an elevation

`SCENARIO` A kitchen plan at a scale of $1:100$ shows a rectangle $40\,\mathrm{mm}$ by $35\,\mathrm{mm}$. The owner wants a bench $4.5\,\mathrm{m}$ long and a fridge $0.7\,\mathrm{m}$ wide along the $40\,\mathrm{mm}$ wall.

Working	Comment
Real room: $40 \times 100 = 4000\,\mathrm{mm} = 4.0\,\mathrm{m}$ by $35 \times 100 = 3500\,\mathrm{mm} = 3.5\,\mathrm{m}$.	Drawing lengths $\times\, 100$.
$\text{area} = 4.0 \times 3.5 = 14.0\,\mathrm{m}^2$	
Bench and fridge need $4.5 + 0.7 = 5.2\,\mathrm{m}$; the wall is	$5.2 > 4.0$ — they do not fit.

Working	Comment
$4.0\,\text{m}$.	
Modify. Extend the room to $5.4\,\text{m}$: on the plan, $5400 \div 100 = 54\,\text{mm}$.	Real length $\div 100$ to return to the drawing.
$\text{new area} = 5.4 \times 3.5 = 18.9\,\text{m}^2$	
$\text{gain} = 18.9 - 14.0 = 4.9\,\text{m}^2$	
$\therefore$ as drawn the bench and fridge do not fit; extending the room to $54\,\text{mm}$ on the plan adds $4.9\,\text{m}^2$ and fits them.	Interpret in terms of the plan.

The front elevation of the same house, at $1:100$, is $120\,\text{mm}$ wide with walls $40\,\text{mm}$ high and a roof apex $65\,\text{mm}$ above the base. In real life: width $12.0\,\text{m}$, wall height $4.0\,\text{m}$, apex $6.5\,\text{m}$ above the base.

Example 5 — Estimating a scale factor from one dimension

`SCENARIO` A toy car is $11.0\,\text{cm}$ long. A typical life-size car is $4.4\,\text{m}$ long. The toy's width is $4.6\,\text{cm}$ and its height is $3.7\,\text{cm}$.

Working	Comment
$k = 440 \div 11.0 = 40$	One matching linear dimension gives the scale factor ($4.4\,\text{m} = 440\,\text{cm}$).
$\text{width} \approx 4.6 \times 40 = 184\,\text{cm} = 1.84\,\text{m}$	Every length $\times\, k$.
$\text{height} \approx 3.7 \times 40 = 148\,\text{cm} = 1.48\,\text{m}$	
Evaluate. Real cars are typically about $1.8\,\text{m}$ wide and $1.5\,\text{m}$ high — the estimates are sensible.	Check against the situation.
$\therefore$ the life-size car is estimated at about $1.84\,\text{m}$ wide and $1.48\,\text{m}$ high, assuming the toy is a true scale model.	Report the assumption.

Example 6 — Error, bounds and bias in a finding

`SCENARIO` A student measures one side of a square paddock as $20.4\,\text{m}$; a survey later shows the true side is $20.0\,\text{m}$.

Working	Comment
$\text{error} = 20.4 - 20.0 = 0.4\,\text{m}$	Measured $-$ true.
$\text{percentage error} = \dfrac{0.4}{20.0} \times 100\,\% = 2\,\%$	
Area from the measurement: $20.4^2 = 416.16\,\text{m}^2$.	The student's finding.
True area: $20.0^2 = 400\,\text{m}^2$.	
$\text{error in area} = 416.16 - 400 = 16.16\,\text{m}^2$	
$\text{percentage error} = \dfrac{16.16}{400} \times 100\,\% = 4.04\,\%$	About double the $2\,\%$ length error.
Bounds. Had the side been recorded as $20\,\text{m}$ to the nearest metre, the true side lies between $19.5\,\text{m}$ and $20.5\,\text{m}$.	Half a metre either way.
$19.5^2 = 380.25$ and $20.5^2 = 420.25$	The true area lies between $380.25\,\text{m}^2$ and $420.25\,\text{m}^2$.
$\therefore$ a $2\,\%$ error in the side became a $4.04\,\%$ error in the area, and a reading to the nearest metre leaves the area uncertain by $40\,\text{m}^2$.	Impact of measurement error on the accuracy of the finding.

If the student's tape had *stretched*, every reading would be wrong in the same direction and the area finding would be biased — repeating the measurement would not help.

Practice questions

Scaffolded

Q1. Paint covers $12\,\text{m}^2$ per litre. (a) Explain why the area A that can be painted is in direct proportion to the litres L of paint. (b) Write L in terms of A. (c) Find the litres needed for a wall of area $66\,\text{m}^2$.

Q2. A printing job takes one printer 24 hours, so the job needs 24 printer-hours and the time t and number of printers n are in inverse proportion: $t=\dfrac{24}{n}$. (a) Find the time for 8 printers. (b) Find the time for 3 printers. (c) How many printers finish the job in 6 hours?

Q3. A photo $9\,\text{cm}$ by $6\,\text{cm}$ is enlarged by scale factor 3. (a) Find the dimensions of the enlargement. (b) Find the area of the original and of the enlargement. (c) Confirm that the area grew by k^2.

Q4. A cube has side $5\,\text{cm}$. It is enlarged by scale factor 2. (a) Find the surface area of the original cube and of the enlargement. (b) Find the volume of each. (c) Confirm the ratios k^2 and k^3.

Q5. A plan is drawn at a scale of $1:200$. A wall is $22\,\text{mm}$ long on the plan. Find the real length of the wall in metres.

Q6. A part $900\,\text{mm}$ long is drawn at a scale of $1:50$. Find its length on the drawing, in millimetres.

Q7. A rod is measured as $48.6\,\text{cm}$; its true length is $48\,\text{cm}$. (a) Find the error. (b) Find the percentage error.

Q8. A fence length is recorded as $25\,\text{m}$, to the nearest metre. Write down the bounds of the true length.

Unscaffolded

Q9. A car uses fuel at $9.6\,\text{L}$ per $100\,\text{km}$ on the highway. (a) Find the fuel for a $250\,\text{km}$ highway trip. (b) The tank holds $60\,\text{L}$; find how far the car can travel on a full tank at this rate.

Q10. `SCENARIO` A job takes 5 volunteers 4 hours. (a) Find the time a crew of 8 would take. (b) The job must be done in 1.5 hours; find the smallest crew that achieves this.

Q11. A map has a scale of $1:25\,000$. Two towns are $8.4\,\text{cm}$ apart on the map. Find the real distance between them, in kilometres.

Q12. On the same $1:25\,000$ map, a lake covers $4.8\,\text{cm}^2$. (a) Show that $1\,\text{cm}^2$ on the map stands for 6.25 hectares of real area. (b) Find the real area of the lake in hectares. ($1\,\text{ha}=10\,000\,\text{m}^2$.)

Q13. A model pond at a scale of $1:15$ holds $2.4\,\text{L}$. Find the volume of water the full-size pond holds, in litres.

Q14. `SCENARIO` A model of a statue is made of the same metal as the real statue, with scale factor 4 from model to real. The model has mass $2.5\,\text{kg}$. Find the mass of the real statue.

Q15. `SCENARIO` Escalators in Australian buildings must meet the National Construction Code and the Australian Standard AS 1735 for lifts and escalators. An escalator rises $4.0\,\text{m}$ between two floors at an angle of $30\,°$ to the horizontal. (a) Find the sloping length of the escalator. (b) Find its horizontal run (the horizontal distance it covers), correct to 1 decimal place. (c) The architect allows $7.0\,\text{m}$ of horizontal space; does your answer to (b) fit the allowance?

Q16. A bedroom plan at $1:100$ shows a rectangle $36\,\text{mm}$ by $28\,\text{mm}$. A bed $2.1\,\text{m}$ long and $1.7\,\text{m}$ wide is placed lengthwise along the longer wall, and the owner wants $0.6\,\text{m}$ of clear space beside the bed. Does the bed fit as the owner wants?

Q17. A model train is $18\,\text{cm}$ long; the real locomotive is $21.6\,\text{m}$ long. (a) Find the scale factor. (b) The real locomotive's wheelbase — the distance between its front and back wheels — is $9.0\,\text{m}$. Find the wheelbase on the model, in centimetres.

Q18. `SCENARIO` A paddock is $60\,\text{m}$ by $40\,\text{m}$. A faulty tape records it as $60.6\,\text{m}$ by $40.4\,\text{m}$. (a) Find the area the faulty tape gives. (b) Find the error in the area, and the percentage error correct to 2 decimal places.

Q19. The side of a square is recorded as $14\,\text{m}$, to the nearest metre. Find the bounds of the true area of the square.

Q20. A trip of $360\,\text{km}$ at average speed v takes $t=\dfrac{360}{v}$ hours. (a) Find t when $v=80$. (b) Find t when $v=100$. (c) Find the time saved by travelling at $100\,\text{km/h}$ instead of $80\,\text{km/h}$, in minutes.

Q21. `SCENARIO` A film studio builds a model tree. Their first estimate had the model at $12\,\text{kg}$; when built, it is $9.5\,\text{kg}$. The full-size tree for the set will be 240 times the mass of the model. (a) Find the error and the percentage error (correct to 1 decimal place) in the estimate of the model. (b) Find the estimated and the actual mass of the full-size tree. (c) Comment on the error the estimate would have caused.

Q22. A stretched tape measure reads every length $2\,\%$ too *small*, so a circle of true radius $3.06\,\text{m}$ is recorded as $3.0\,\text{m}$. (a) Find the area calculated from the recorded radius, correct to 2 decimal places. (b) Find the true area, correct to 2 decimal places. (c) Show that the true area is $4.04\,\%$ larger than the calculated one, and name this effect.

Real context

Q23. The Royal Australian Mint's 2-dollar coin is $20.5\,\text{mm}$ in diameter and has a mass of 6.6 grams. A council wants a giant 2-dollar coin for a town square, scaled up by a factor of 100 and made solid from the same metal. (a) Find the diameter of the giant coin, in metres. (b) Find its mass, in tonnes. (c) Evaluate the plan: state one assumption and one practical problem the mass creates.

Source: Royal Australian Mint, *About our coins* (factsheet), 2025. Retrieved 20 August 2026. (ramint.gov.au/sites/default/files/2025-04/factsheets-about-australian-coins.pdf)

Answers

Q1. (b) $L = \frac{A}{12}$; (c) 5.5 L. **Q2.** (a) 3 h; (b) 8 h; (c) 4 printers. **Q3.** (a) 27 cm by 18 cm; (b) 54 cm^2, 486 cm^2; (c) $486 \div 54 = 9 = 3^2$. **Q4.** (a) 150 cm^2, 600 cm^2; (b) 125 cm^3, 1000 cm^3; (c) ratios 4 and 8. **Q5.** 4.4 m. **Q6.** 18 mm. **Q7.** (a) 0.6 cm; (b) 1.25 %. **Q8.** between 24.5 m and 25.5 m. **Q9.** (a) 24 L; (b) 625 km. **Q10.** (a) 2.5 h; (b) 14 people. **Q11.** 2.1 km. **Q12.** (b) 30 hectares. **Q13.** 8100 L. **Q14.** 160 kg. **Q15.** (a) 8.0 m; (b) 6.9 m; (c) yes, $6.9 < 7.0$. **Q16.** yes — $2.1 \le 3.6$ and $1.7 + 0.6 = 2.3 \le 2.8$. **Q17.** (a) 120; (b) 7.5 cm. **Q18.** (a) 2448.24 m^2; (b) 48.24 m^2, 2.01 %. **Q19.** between 182.25 m^2 and 210.25 m^2. **Q20.** (a) 4.5 h; (b) 3.6 h; (c) 54 minutes. **Q21.** (a) 2.5 kg, 26.3 %; (b) 2880 kg and 2280 kg; (c) the estimate is 600 kg out — the same 26.3 %. **Q22.** (a) 28.27 m^2; (b) 29.42 m^2; (c) 4.04 % — bias. **Q23.** (a) 2.05 m; (b) 6.6 t.

Fully-worked solutions

Q1. (a) The area is 12 times the litres, a constant multiple, so A and L are in direct proportion.

$$L = \frac{A}{12}$$

$$L = \frac{66}{12} = 5.5$$

$\therefore$ 5.5 L of paint are needed.

Q2. (a) $t = \frac{24}{8} = 3$

$\therefore$ 3 h.

(b) $t = \frac{24}{3} = 8$

$\therefore$ 8 h.

(c) $n = \frac{24}{6} = 4$

$\therefore$ 4 printers.

Q3. (a) $9 \times 3 = 27$ and $6 \times 3 = 18$: 27 cm by 18 cm.

(b) $9 \times 6 = 54\ \text{cm}^2$; $27 \times 18 = 486\ \text{cm}^2$.

(c) $486 \div 54 = 9 = 3^2$ — areas grow by k^2.

Q4. (a) $6 \times 5^2 = 150\ \text{cm}^2$; $6 \times 10^2 = 600\ \text{cm}^2$.

(b) $5^3 = 125\ \text{cm}^3$; $10^3 = 1000\ \text{cm}^3$.

(c) $600 \div 150 = 4 = 2^2$ and $1000 \div 125 = 8 = 2^3$.

Q5. $22 \times 200 = 4400$ mm

$4400\ \text{mm} = 4.4\ \text{m}$

$\therefore$ the wall is 4.4 m long.

Q6. $900 \div 50 = 18$

$\therefore$ $18\,\mathrm{mm}$ on the drawing.

Q7. (a) error $= 48.6 - 48 = 0.6\,\mathrm{cm}$

(b) $\dfrac{0.6}{48} \times 100\,\% = 1.25\,\%$

Q8. To the nearest metre: between $25 - 0.5 = 24.5\,\mathrm{m}$ and $25 + 0.5 = 25.5\,\mathrm{m}$.

Q9. (a) $F = 0.096 \times 250 = 24$

$\therefore$ $24\,\mathrm{L}$.

(b) $60 \div 0.096 = 625$

$\therefore$ $625\,\mathrm{km}$ on a full tank.

Q10. (a) The job needs $5 \times 4 = 20$ person-hours; $t = \dfrac{20}{8} = 2.5$

$\therefore$ $2.5\,\mathrm{h}$.

(b) $n = \dfrac{20}{1.5} = 13.33\ldots$

13 people are not quite enough, so the smallest crew is 14 people.

Q11. $8.4 \times 25\,000 = 210\,000\,\mathrm{cm}$

$210\,000\,\mathrm{cm} = 2100\,\mathrm{m} = 2.1\,\mathrm{km}$

Q12. (a) $1\,\mathrm{cm}$ on the map is $25\,000\,\mathrm{cm} = 250\,\mathrm{m}$, so $1\,\mathrm{cm}^2$ stands for $250^2 = 62\,500\,\mathrm{m}^2 = 6.25\,\mathrm{ha}$.

(b) $4.8 \times 6.25 = 30$

$\therefore$ 30 hectares.

Q13. Volumes grow by $15^3 = 3375$.

$2.4 \times 3375 = 8100$

$\therefore$ the full-size pond holds $8100\,\mathrm{L}$.

Q14. Same metal, so mass grows with volume: $\times\, 4^3 = 64$.

$2.5 \times 64 = 160$

$\therefore$ $160\,\mathrm{kg}$.

Q15. (a) The rise is opposite the $30\,°$ angle and the sloping length is the hypotenuse: $\sin 30\,° = \dfrac{4.0}{\text{slope}}$, so

$\text{slope} = \dfrac{4.0}{\sin 30\,°} = \dfrac{4.0}{0.5} = 8.0$

$\therefore$ $8.0\,\mathrm{m}$.

(b) $\tan 30\,° = \dfrac{4.0}{\text{run}}$, so $\text{run} = \dfrac{4.0}{\tan 30\,°} = 4\sqrt{3} = 6.928\ldots \approx 6.9$

$\therefore$ about $6.9\,\mathrm{m}$.

(c) $6.9\,\mathrm{m} < 7.0\,\mathrm{m}$ — yes, the run fits the architect's allowance.

Q16. Real room: $3.6\,\text{m}$ by $2.8\,\text{m}$.

Along the long wall: $2.1 \le 3.6$ — the bed's length fits.

Beside the bed: $1.7 + 0.6 = 2.3 \le 2.8$ — the bed and the clear space fit.

∴ yes, the bed fits as the owner wants.

Q17. (a) $k = 2160 \div 18 = 120$ ($21.6\,\text{m} = 2160\,\text{cm}$).

(b) $900 \div 120 = 7.5$

∴ $7.5\,\text{cm}$ on the model.

Q18. (a) $60.6 \times 40.4 = 2448.24$

∴ $2448.24\,\text{m}^2$.

(b) True area $60 \times 40 = 2400\,\text{m}^2$; error $2448.24 - 2400 = 48.24\,\text{m}^2$.

$\dfrac{48.24}{2400} \times 100\,\% = 2.01\,\%$

Q19. The true side lies between $13.5\,\text{m}$ and $14.5\,\text{m}$.

$13.5^2 = 182.25$ and $14.5^2 = 210.25$

∴ the true area lies between $182.25\,\text{m}^2$ and $210.25\,\text{m}^2$.

Q20. (a) $t = \dfrac{360}{80} = 4.5$

∴ $4.5\,\text{h}$.

(b) $t = \dfrac{360}{100} = 3.6$

∴ $3.6\,\text{h}$.

(c) $4.5 - 3.6 = 0.9\,\text{h} = 0.9 \times 60 = 54$

∴ 54 minutes saved.

Q21. (a) $\text{error} = 12 - 9.5 = 2.5\,\text{kg}$

$\dfrac{2.5}{9.5} \times 100\,\% = 26.31\ldots \approx 26.3\,\%$

(b) Estimate: $12 \times 240 = 2880\,\text{kg}$; actual: $9.5 \times 240 = 2280\,\text{kg}$.

(c) The full-size estimate would have been $2880 - 2280 = 600\,\text{kg}$ too high — the same $26.3\,\%$, because scaling multiplies both masses by 240.

Q22. (a) $\pi \times 3.0^2 = 9\pi = 28.274\ldots \approx 28.27$

∴ $28.27\,\text{m}^2$.

(b) $\pi \times 3.06^2 = 9.3636\pi = 29.420\ldots \approx 29.42$

∴ $29.42\,\text{m}^2$.

(c) $\frac{29.42\ldots}{28.27\ldots} = \frac{9.3636}{9} = 1.0404$, so the true area is $4.04\,\%$ larger. Every reading is wrong in the same direction, so this is **bias**: the calculated area is always too small, and the $2\,\%$ length error became a $4.04\,\%$ area error.

Q23. (a) $20.5 \times 100 = 2050\,\text{mm} = 2.05\,\text{m}$

(b) Mass grows with volume: $\times\, 100^3 = 1\,000\,000$.

$6.6 \times 1\,000\,000 = 6\,600\,000\,\text{g} = 6600\,\text{kg} = 6.6\,\text{t}$

(c) Assumption: the giant coin is solid and made of the same metal throughout. A $6.6\,\text{t}$ coin — about the mass of two elephants — cannot sit on ordinary ground; it needs a concrete footing, and it cannot be moved or lifted once placed. (Any sensible evaluation of the mass is accepted.)

Teacher note

Level 9 codes rehearsed (D3): `VC2M9M05` — direct proportion, rates, ratio and scale; `VC2M9M04` — error and percentage error; `VC2M9SP02` — enlargement; `VC2M9M03` — similarity and scale factors. 4C builds these into complete modelling cycles; it does not re-teach them, and they are not assessed here as Level 9 outcomes. In-book prerequisites are 4A and 4B.

Tool classes (D13): scientific calculator only. `VC2M10M04` is not a digital-tool content description, so this subtopic carries no digital-tool banner and no digital tool task; the modelling cycle is assessed on paper and by hand.

VCE destinations (§5, reference only): `VC2M10M04` is named in Foundation Mathematics Units 3 & 4 (Area of Study 4 'Space and measurement'), is assumed knowledge in General Mathematics Units 3 & 4, and appears in Mathematical Methods Units 3 & 4 (Area of Study 1) with the skills *"Apply scale factors to length, area and volume when scaling 3D objects"*, *"Use a scale to convert plan/elevation drawing distances to real-world measurements"* and *"Identify how measurement error propagates and may bias a quantitative finding"*.

Elaboration `VC2M10M04.E05`: delivered through Q15, which names the real National Construction Code and the real Australian Standard AS 1735 (lifts and escalators) rather than a fictitious code (R2). No Aboriginal or Torres Strait Islander content is used anywhere in this subtopic (R10).

Level check (D6): no rate-of-change tools and no formal error-propagation formula — error growth is ratio reasoning only (1.02, 1.02^2, 1.02^3), which is the k, k^2, k^3 scaling idea applied to a $2\,\%$ factor. Percentage error is defined at first use as error over true value times 100%; nothing from the D6 prohibited list appears.

Assessment link (§7.3): the extended modelling task X1, hosted by 3C, assesses the `VC2M10A15` and `VC2M10M04` modelling cycles together, drawing on 2E, 3B and this subtopic. Students' X1 reports should show the same six steps named in §1.

Logarithmic scales in applied contexts

Mathematics 10 · Chapter 4 Measurement · Subtopic 4D (Level 10) · Extended block

Strand	Measurement
Content description	`VC2M10M02`
Elaboration ids carried	`VC2M10M02.E01` `VC2M10M02.E02` `VC2M10M02.E03` (`VC2M10M02.E04` not delivered — see Teacher note)
Weight	Extended (double allocation)

> **Content description (verbatim):** *interpret and use logarithmic scales in applied contexts involving small and large quantities and change*
>
> **Achievement standard (extract):** *They interpret and use logarithmic scales representing small or large quantities or change in applied contexts.*

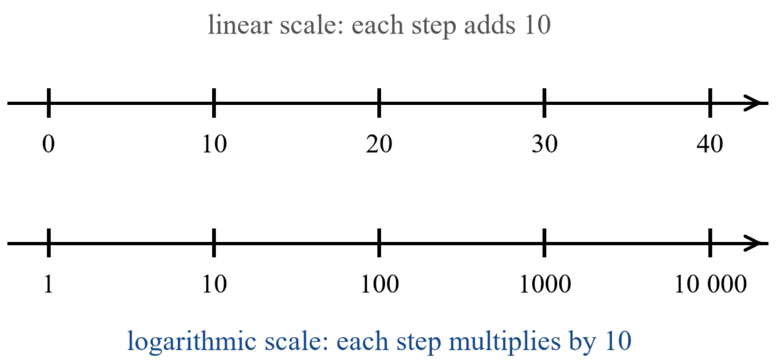
You need first: simple exponential equations (2E, `VC2M10A14`) and growth models built on a constant ratio (3C, `VC2M10A15`); and from Year 9, scientific notation (`VC2M9M02`) and the exponent laws (`VC2M9A01`). This subtopic rehearses those skills and applies them; it does not re-teach them. A logarithmic scale is read against powers of 10 and inverted, by hand, with an exponential relation — no logarithm laws are used anywhere in this subtopic.

Theory

Every scale you have used so far — a ruler, a number line, the axis of a graph — spaces its numbers by **adding** the same amount each step. Some quantities refuse to behave on such a scale: an earthquake that shakes the ground ten times harder, a sound ten times more intense, a colony that doubles every hour. This subtopic builds the other kind of scale, the one that spaces its numbers by **multiplying**, and teaches you to read it, use it, and say when it is the right choice.

1. Two ways to make a scale

Here are two scales, drawn side by side with exactly the same spacing between marks.



Two horizontal scales drawn with equal spacing: the linear scale 0, 10, 20, 30, 40, where each step adds 10, and the logarithmic scale 1, 10, 100, 1000, 10 000, where each step multiplies by 10

The top scale **adds 10** at every step: $0, 10, 20, 30, 40$. This is a **linear scale** — equal distances mean equal *differences*.

The bottom scale **multiplies by 10** at every step: $1, 10, 100, 1000, 10\,000$. This is a **logarithmic scale** — equal distances mean equal *factors*. The distance from 1 to 10 is the same as the distance from 10 to 100, because both steps multiply by 10, even though one step adds 9 and the other adds 90.

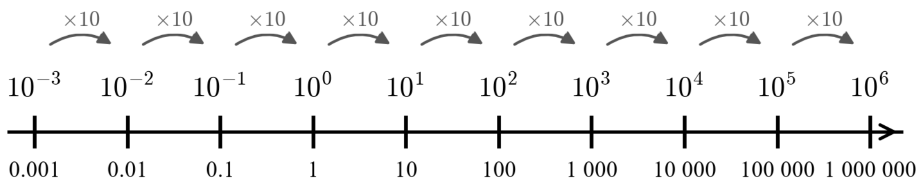
The whole subtopic rests on that one contrast: **on a linear scale, equal steps add equal amounts; on a logarithmic scale, equal steps multiply by equal factors.**

2. Powers of 10 and orders of magnitude

The bottom scale above is calibrated in powers of 10, so powers of 10 are the language of the whole topic. Recall from Year 9 (`VC2M9M02`, `VC2M9A01`):

Power	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}
Value	0.001	0.01	0.1	1	10	100	1000	10 000	100 000	1 000 00(

Each step to the right multiplies by 10; each step to the left divides by 10.



one step up = one order of magnitude = ten times bigger

A powers-of-10 ladder from 10 to the power of −3, which is 0.001, up to 10 to the power of 6, which is 1 000 000, with an arrow labelled ×10 between each pair of rungs; one step up is one order of magnitude, ten times bigger

That step has a name. When one quantity is ten times another, it is **one order of magnitude** bigger. When it is $100 = 10 \times 10$ times bigger, it is two orders of magnitude bigger; when it is 1000 times bigger, three. So the **order of magnitude** of a change counts the powers of 10 in the factor:

- $30 \rightarrow 30\,000$ is $\times 1000 = 10^{3}$: **three** orders of magnitude;
- $500 \rightarrow 5$ is $\div 100 = 10^{-2}$: **two** orders of magnitude smaller.

Scientific notation makes the count automatic. Write each quantity as $a \times 10^{n}$ with $1 \leq a < 10$: then $30\,000 = 3 \times 10^{4}$ and $30 = 3 \times 10^{1}$, and the difference of the exponents, $4 - 1 = 3$, is the number of orders of magnitude between them. (Subtracting the exponents is the Year 9 exponent laws, doing exactly the work a division does: $\dfrac{3 \times 10^{4}}{3 \times 10^{1}} = 10^{3}$.)

3. What a logarithmic scale is

A **logarithmic scale** is a scale calibrated in terms of order of magnitude — each equal step is a fixed multiplying factor, most often a power of 10, but sometimes another fixed factor such as doubling:

- a **powers-of-10 scale**: $1, 10, 100, 1000, \ldots$, each step $\times 10$;
- a **doubling scale**: $1, 2, 4, 8, 16, \ldots$, each step $\times 2$.

Both are logarithmic scales, because both space their marks by equal factors instead of equal differences. A quantity placed on such a scale sits at the mark that says *how many times bigger* it is than the starting mark — its position records a factor, not an amount.

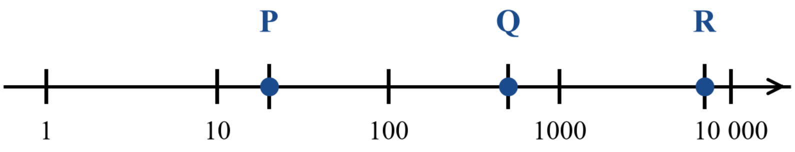
Notice what this does for small and large quantities at once. On the powers-of-10 ladder, 0.001, 1 and $1\,000\,000$ all sit in one picture, three steps left and six steps right of 1. A linear scale cannot do that: it has no room for the tiny values once the large ones arrive.

4. Reading a logarithmic axis

Reading a logarithmic axis is a two-step skill: find the **decade** (which powers of 10 the point lies between), then judge how far through the decade it sits. Distances inside a decade follow a fixed pattern, worth learning as a reading aid:

A point this far through the decade N to $10\,N$	is at about
$30\,\%$ of the way	$2\,N$
$48\,\%$ of the way (near the middle)	$3\,N$
$60\,\%$ of the way	$4\,N$
$70\,\%$ of the way	$5\,N$
$85\,\%$ of the way	$7\,N$
$90\,\%$ of the way	$8\,N$

The pattern to remember is: **the early part of a decade does the most work** — the halfway mark of the decade is already at $3\,N$, not $5\,N$.



A logarithmic axis running from 1 to 10 000 with three points marked: P just after 10 at about 20, Q between 100 and 1000 at about 500, and R just before 10 000 at about 7000

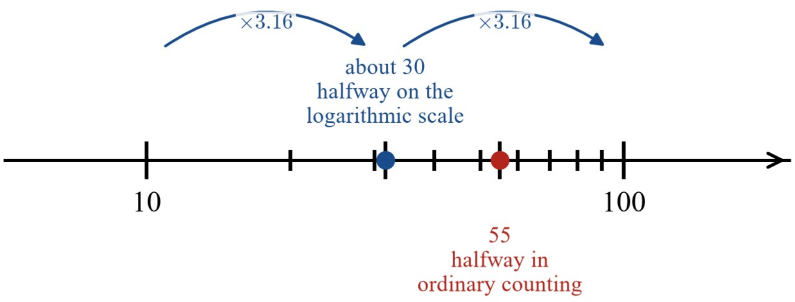
Read the three marked points.

- **P** lies between 10 and 100, about $30\,\%$ into the decade: $P \approx 2 \times 10 = 20$.
- **Q** lies between 100 and 1000, about $70\,\%$ into the decade: $Q \approx 5 \times 100 = 500$.
- **R** lies between 1000 and 10 000, about $85\,\%$ into the decade: $R \approx 7 \times 1000 = 7000$.

The same pattern explains the most common misreading of a logarithmic scale — the **halfway point of a decade**. In ordinary counting, halfway from 10 to 100 is 55. On the logarithmic scale, the halfway mark is the value m that sits the same *factor* above 10 as it sits below 100:

$$10 \times f = m \text{ and } m \times f = 100\,.$$

Multiplying the two equations gives $m \times m = 10 \times 100 = 1000$, so $m \times m = 1000$ and $m \approx 31.6$ — about 30, not 55.



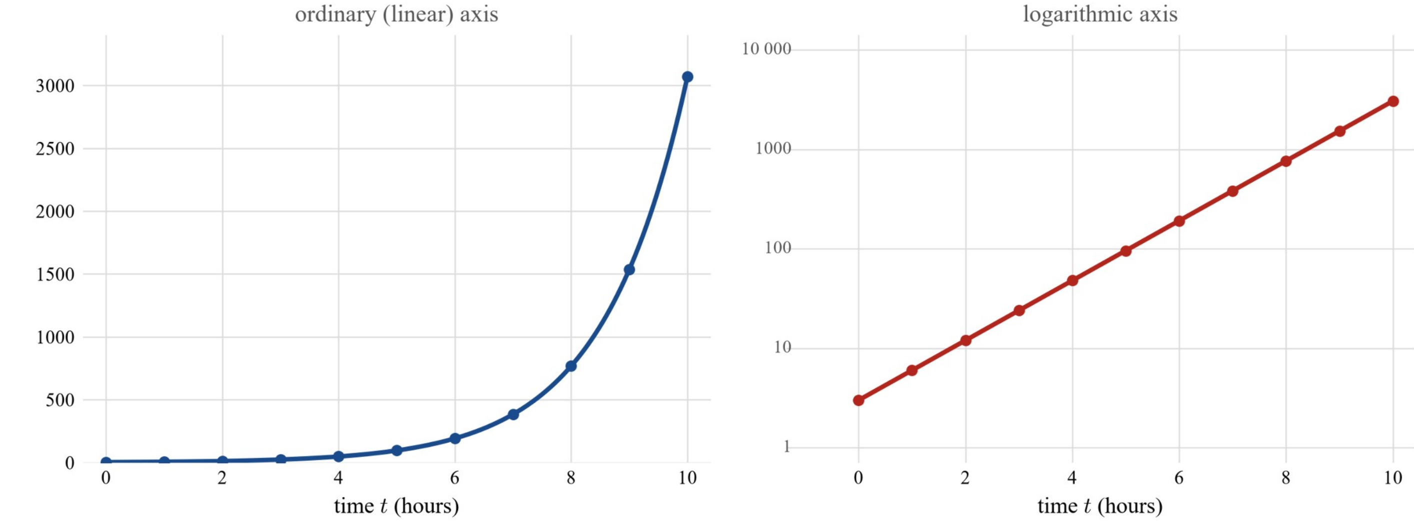
One decade of a logarithmic scale from 10 to 100 with minor marks at 20, 30, 40 and so on: the halfway mark in ordinary counting, 55, sits right of centre, while the halfway mark on the logarithmic scale, about 30, sits at the centre with equal factors of about 3.16 on each side of it

Each half of the decade multiplies by the same factor, about 3.16: $10 \times 3.16 \approx 31.6$ and $31.6 \times 3.16 \approx 100$. On any decade of any logarithmic scale, the middle mark is about 3.16 times the start of the decade — which is why the small numbers look crowded together at the right-hand end of each decade.

5. Logarithmic scales on graphs, and when to choose them

The same idea turns a graph into a logarithmic graph when its axis is calibrated in orders of magnitude. Watch what that does to exponential growth. A colony with $N = 3 \times 2^t$ doubles every hour (3C):

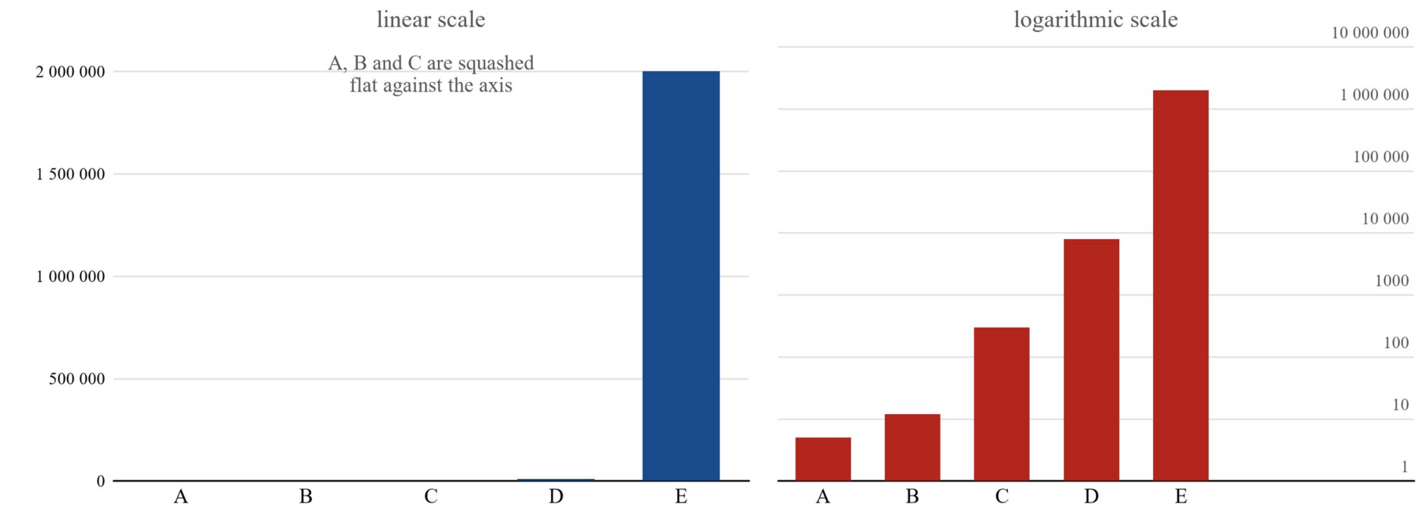
t	0	2	4	6	8	10
N	3	12	48	192	768	3072



The same growth data, N equals 3 times 2 to the power of t, drawn on two panels: on an ordinary linear axis the points climb in a curve that steepens away, while on a logarithmic axis, marked 1, 10, 100, 1000, 10 000, the same points climb in equal steps along a straight line

On the ordinary axis the points climb in a curve that steepens away and leaves the early values squashed against the axis. On the logarithmic axis, **equal steps of time are equal factors, and equal factors are equal distances** — so exponential growth draws a **straight line**. That is the signature: a straight line on a logarithmic axis means a constant multiplying factor (constant percentage growth), and charts of percentage change use logarithmic scales for exactly this reason.

The second reason to choose a logarithmic scale is a **wide range of values**. Plot 5, 12, 300, 8000 and $2\,000\,000$ on one linear axis and the first three vanish:



The same five values, 5, 12, 300, 8000 and 2 000 000, drawn as bars on two panels: on the linear scale the bars for 5, 12 and 300 are squashed flat against the axis under the bar for 2 000 000, while on the logarithmic scale all five bars rise clearly and can be read

On the logarithmic scale every bar can be read, and what the eye compares is *factors* — bar E is about two and a half decades above bar C, meaning a few hundred times taller.

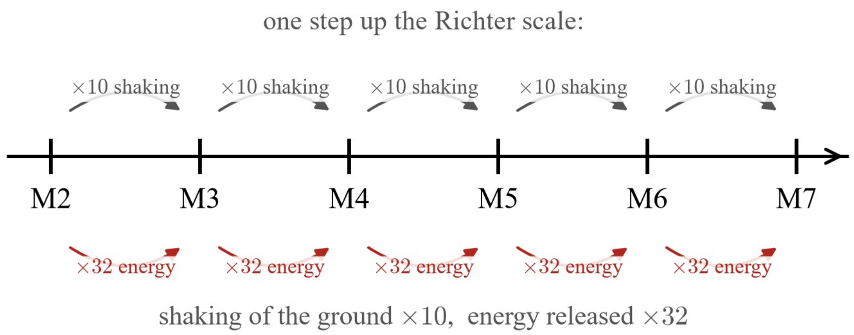
When is a logarithmic scale appropriate, and when not?

- **Appropriate:** values span many orders of magnitude; the pattern of interest is a constant factor or percentage change (exponential growth or decay); small and large quantities must be compared on one chart.
- **Not appropriate:** values are all of much the same size, so a linear scale loses nothing; the question is about *differences* (how many more, not how many times); or the reader expects equal steps to mean equal amounts — a logarithmic scale in front of such a reader misleads.

That last point is the judgement the achievement standard assesses: a scale is a choice, and the choice changes what the reader sees.

6. Logarithmic scales in the real world

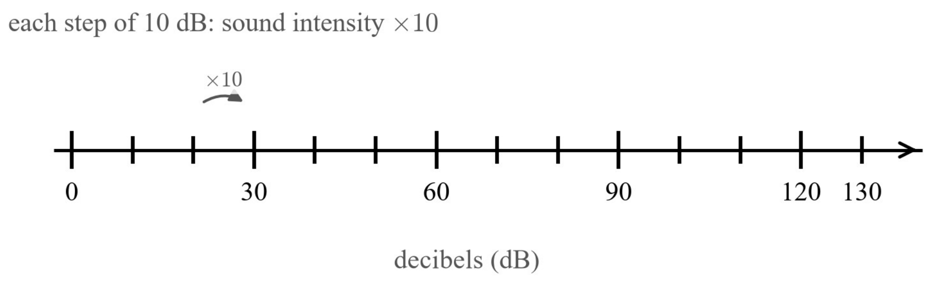
The Richter scale for earthquakes. Each step of 1 on the Richter scale means the shaking of the ground is multiplied by 10 — and the energy released is multiplied by about 32.



The Richter scale from M2 to M7 as a ladder: each step up multiplies the shaking of the ground by 10 and the energy released by about 32

So an M6 earthquake shakes the ground $10 \times 10 = 100$ times more than an M4, and releases about $32 \times 32 = 1024$, roughly 1000 times, the energy. The scale compresses an enormous range of energies into small, comparable numbers — which is precisely what a logarithmic scale is for.

The decibel scale for sound. Each step of 10 dB multiplies the intensity of the sound by 10.



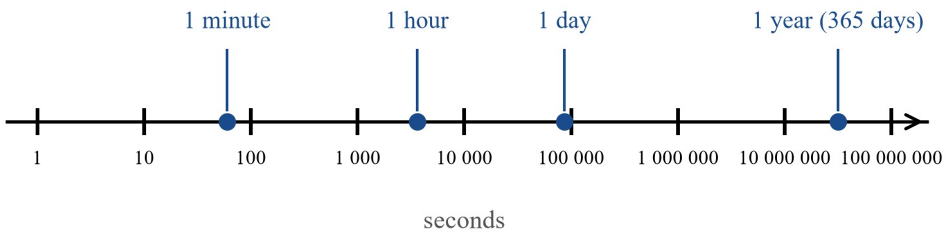
The decibel scale from 0 to 130 decibels with a mark every 10 decibels: each step of 10 decibels multiplies the sound intensity by 10

A 60 dB conversation is 1000 times more intense than a 30 dB whisper — three steps of 10 dB, each $\times 10$. And 130 dB, the threshold of pain, is 10^{13} times the intensity of 0 dB, the threshold of hearing: thirteen orders of magnitude on one scale a person can read.

Sensitivity scales: doubling. Camera film speed (ISO) is a doubling scale: ISO 100, 200, 400, 800, 1600, … — each step doubles the sensitivity. Doubling scales are logarithmic scales with factor 2 instead of factor 10: equal steps, equal factors.

The pH scale for acidity. Each step of 1 on the pH scale is a factor of 10 in acidity: a soil of pH 5 is 100 times more acidic than a soil of pH 7. The scale lets a chemist compare quantities that differ by orders of magnitude using small whole numbers.

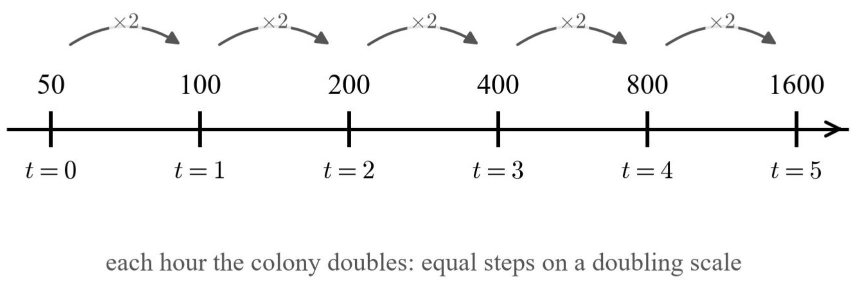
Timescales. How long is a million seconds? On a linear axis of seconds, a minute and a year cannot share one picture. On a logarithmic time scale they sit together, each at its computed position:



A logarithmic time scale in seconds from 1 to 100 000 000, with 1 minute at 60, 1 hour at 3600, 1 day at 86 400 and 1 year at about 31 500 000 marked at their computed positions between the powers of 10

One minute (60 s) sits between 10 and 100; one hour (3600 s) between 1000 and 10 000; one day (86 400 s) just before 100 000; one year (about 31.5 million seconds) between 10 million and 100 million. Four familiar times, spread across seven orders of magnitude, in one readable line.

The spread of microorganisms. A colony of bacteria that doubles every hour grows on a doubling scale: $50, 100, 200, 400, 800, 1600, \ldots$



A doubling scale from 50 to 1600 marked at t equals 0 to t equals 5 hours, with an arrow labelled ×2 over each hour's step: each hour the colony doubles, so the steps are equal on a doubling scale

Five hours after 50, the colony is at 1600 — five equal steps of $\times 2$. This is why outbreaks and infections are described in terms of *doubling time*: on a logarithmic scale, the whole outbreak is a straight climb in equal steps, and the steepness of that climb is the doubling time itself.

Worked examples

Example 1 — Orders of magnitude

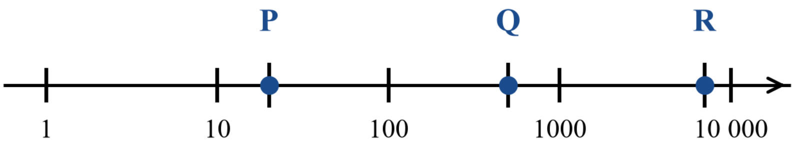
How many orders of magnitude separate each pair? (a) 30 and 30 000; (b) \$250 and \$2 500 000; (c) 0.03 and 3000.

Working	Comment
(a) $\dfrac{30\,000}{30} = 1000$	Divide to find the factor between the two.
$1000 = 10^3$	Count the powers of 10 in the factor.
$\therefore$ 30 000 is 3 orders of magnitude greater than 30.	

Working	Comment
(b) $\dfrac{2\,500\,000}{250}=10\,000=10^4$	One division does the whole job.
$\therefore$ $\$2\,500\,000$ is 4 orders of magnitude greater than $\$250$.	
(c) $\dfrac{3000}{0.03}=100\,000=10^5$	Works the same way with small quantities.
$\therefore$ 3000 is 5 orders of magnitude greater than 0.03.	

Example 2 — Reading points on a logarithmic axis

Read the values of P, Q and R on the logarithmic axis below.



A logarithmic axis running from 1 to 10 000 with three points marked: P between 10 and 100, Q between 100 and 1000, and R between 1000 and 10 000

Working	Comment
P: between 10 and 100, about $30\,\%$ in.	Find the decade first.
$P\approx 2\times 10=20$	$30\,\%$ into a decade is about $\times 2$.
Q: between 100 and 1000, about $70\,\%$ in.	
$Q\approx 5\times 100=500$	$70\,\%$ into a decade is about $\times 5$.
R: between 1000 and $10\,000$, about $85\,\%$ in.	
$R\approx 7\times 1000=7000$	$85\,\%$ into a decade is about $\times 7$.
$\therefore$ $P\approx 20$, $Q\approx 500$, $R\approx 7000$.	Check: the order P, Q, R matches left-to-right on the axis. ✓

Readings from a logarithmic axis are estimates — the skill is the decade and the factor, not false precision.

Example 3 — The Richter scale

Compare an M6 earthquake with an M4 earthquake: (a) the shaking of the ground; (b) the energy released.

Working	Comment
(a) $6-4=2$ steps of 1.	Count steps on the scale.
$10\times 10=100$	Each step multiplies shaking by 10.
$\therefore$ the M6 shakes the ground 100 times more than the M4.	
(b) $32\times 32=1024$	Each step multiplies energy by about 32.
≈ 1000	1024 is about 1000 to 1 significant figure.
$\therefore$ the M6 releases about 1000 times the energy of the M4.	Two steps on one scale, two different factors.

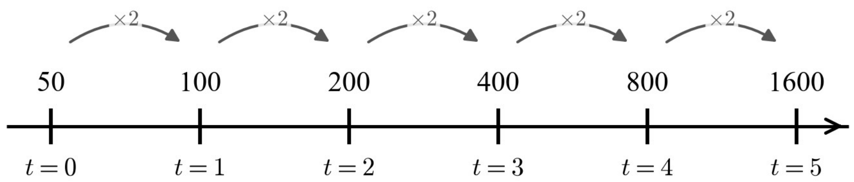
Example 4 — The decibel scale

SCENARIO A sound-level meter reads 30 dB in a library and 60 dB in a workshop. How many times more intense is the workshop sound?

Working	Comment
$60-30=30$ dB $=3$ steps of 10 dB.	Steps of 10 dB on the scale.
$10\times10\times10=1000$	Each step multiplies intensity by 10.
∴ the workshop sound is 1000 times more intense.	A “difference” of 30 on the scale is a factor of 1000 in the world.

Example 5 — A doubling scale

SCENARIO A bacteria colony starts at 50 and doubles every hour. When does it first reach 1600?



each hour the colony doubles: equal steps on a doubling scale

A doubling scale from 50 to 1600 marked at t equals 0 to t equals 5 hours, with an arrow labelled ×2 over each hour’s step

Working	Comment
$\dfrac{1600}{50}=32$	The factor between start and target.
$32=2\times2\times2\times2\times2=2^5$	Write the factor as doublings (2E).
$t=5$	Five doublings, one per hour.
∴ the colony first reaches 1600 after 5 hours.	Check on the ladder: $50,100,200,400,800,1600$. ✓

Example 6 — Growth on a logarithmic axis

SCENARIO $\$5000$ is invested at $6\,\%$ p.a., compounded yearly, so the value after t years is $V=5000\times1.06^t$.

t	0	1	2	3	4	5
V ($)	5000	5300	5618	5955.08	6312.38	6691.13

(a) Show that the value grows by a constant ratio, and explain what the graph looks like on a logarithmic axis. (b) Estimate, to the nearest year, when the investment first doubles.

Working	Comment
(a) $\dfrac{5300}{5000}=\dfrac{5618}{5300}=\cdots=1.06$	Growing by $6\,\%$ means multiplying by 1.06 each year.
∴ equal steps of time are equal factors, so on a logarithmic axis the values climb in **equal steps** — a straight line.	The signature of constant percentage growth (§5).
(b) Doubling needs a factor of 2: find t with $1.06^t=2$.	Set up the exponential relation (2E).
$1.06^{11}\approx1.90$ — below 2.	Trap between whole years.
$1.06^{12}\approx2.01$ — above 2.	

Working	Comment
∴ the investment first doubles in about 12 years.	On a logarithmic axis, every doubling takes the same distance — the next doubling also takes about 12 years.

Practice questions

Scaffolded

Q1. Write as a single power of 10: (a) $10 \times 10 \times 10 \times 10$; (b) $100\,000$; (c) 0.01.

Q2. How many orders of magnitude separate: (a) 50 and $50\,000$; (b) 4 and $4\,000\,000$? (Hint: divide, then count powers of 10.)

Q3. On a logarithmic scale, which power of 10 is each value closest to? (a) 8; (b) 250; (c) 900. (Hint: compare factors — 250 is $\times 2.5$ above 100 but $\times 4$ below 1000.)

Q4. A doubling scale starts at 3. List the next five marks.

Q5. On the Richter scale, how many times more shaking does an M5 earthquake produce than an M3? (Hint: two steps of $\times 10$.)

Q6. A sound rises from 40 dB to 70 dB. How many times more intense is it? (Hint: three steps of 10 dB.)

Q7. One hour is 3600 seconds. Write 3600 in scientific notation, and say which two powers of 10 it lies between on a logarithmic time scale.

Q8. Camera film speed is a doubling scale (ISO 100, 200, 400, …). (a) What is the speed four doublings above ISO 100? (b) How many doublings is it from ISO 100 to ISO 6400?

Unscaffolded

Q9. How many orders of magnitude separate each pair? (a) 700 and $7\,000\,000$; (b) 0.05 and 500; (c) 25 and $250\,000$.

Q10. Write in scientific notation: (a) 0.0001; (b) $45\,000$; (c) 0.007.

Q11. A logarithmic axis runs from 1 to $10\,000$. Describe where each value would sit: (a) 3; (b) 40; (c) 600; (d) 8000.

Q12. Compare an M5 earthquake with an M2 earthquake: (a) the shaking of the ground; (b) the energy released, to 2 significant figures. (c) An earthquake releases about 1000 times the energy of an M2 earthquake. What is its magnitude? (Hint: $32 \times 32 \approx 1000$.)

Q13. (a) How many times more intense is a 70 dB sound than a 40 dB sound? (b) The threshold of pain is about 130 dB and the threshold of hearing is 0 dB. How many times more intense is a sound at the threshold of pain?

Q14. On the ISO doubling scale: (a) what speed is three doublings above ISO 200? (b) How many doublings separate ISO 100 from ISO 6400?

Q15. `SCENARIO` A bacteria colony of 5 doubles every hour. (a) How large is it after 6 hours? (b) In which hour does it first exceed 1000?

Q16. On a logarithmic scale, the halfway mark between 100 and $10\,000$ is the value m with the same factor f on each side: $100 \times f = m$ and $m \times f = 10\,000$. Show that $m = 1000$.

Q17. `SCENARIO` $\$2000$ is invested at 5% p.a., compounded yearly, so $V = 2000 \times 1.05^t$. (a) Find the value after 3 years. (b) Explain why the values plot as equal steps on a logarithmic axis. (c) Estimate, to the nearest year, when the investment first doubles.

Q18. The colony $N = 3 \times 2^t$ is drawn on the logarithmic axis of the two-panel figure in §5. (a) Read the value of N at $t = 8$ from the table in §5. (b) Verify it by calculation. (c) Find the first whole hour at which N exceeds 5000.

Q19. Write these four times in increasing order: 10 hours; $100\,000$ seconds; 1 day; 1 year. (Work in seconds.)

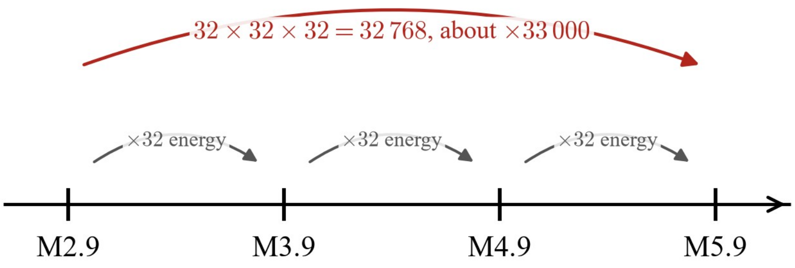
Q20. A garden soil measures pH 7 and a bog measures pH 4. How many times more acidic is the bog?

Q21. `SCENARIO` A health report charts weekly case counts of an illness over five weeks: $12, 15, 140, 900, 6000$. Which scale — linear or logarithmic — would you choose for the chart, and why? Give one reason the other scale would mislead.

Q22. `SCENARIO` A share portfolio grows by $20\,\%$ each year from $\$1000$, so $V = 1000 \times 1.2^t$. (a) Find the value after 4 years. (b) Explain why the graph is a straight line on a logarithmic axis. (c) Find the first whole year at which the value is at least 5 times the starting value.

Real context

Q23. The earthquake of 22 September 2021 near Mansfield, Victoria, was recorded at magnitude 5.9 — the largest earthquake recorded in the state. (a) Compare its shaking of the ground with that of a magnitude 3.9 earthquake. (b) Compare the energy released, to 1 significant figure. (c) An earthquake releases about $33\,000$ times the energy of a magnitude 2.9 earthquake. Use the energy ladder below to find its magnitude.



An energy ladder from M2.9 to M5.9 in three steps: each step multiplies the energy by 32, and 32 times 32 times 32 is 32 768, about 33 000 times in total

Source: Geoscience Australia, *Earthquakes@GA*, 2021. Retrieved 20 August 2026.

Answers

Q1. (a) 10^4; (b) 10^5; (c) 10^{-2}. **Q2.** (a) 3; (b) 6. **Q3.** (a) 10; (b) 100; (c) 1000. **Q4.** $6, 12, 24, 48, 96$. **Q5.** 100 times. **Q6.** 1000 times. **Q7.** 3.6×10^3; between 10^3 and 10^4. **Q8.** (a) ISO 1600; (b) 6. **Q9.** (a) 4; (b) 4; (c) 4. **Q10.** (a) 10^{-4}; (b) 4.5×10^4; (c) 7×10^{-3}. **Q11.** (a) about 48 % of the way from 1 to 10 (near the middle); (b) about 60 % of the way from 10 to 100; (c) about 78 % of the way from 100 to 1000; (d) about 90 % of the way from 1000 to 10 000. **Q12.** (a) 1000 times; (b) 32 768, about 33 000 times; (c) M4. **Q13.** (a) 1000 times; (b) 10^{13} times. **Q14.** (a) ISO 1600; (b) 6. **Q15.** (a) 320; (b) the 8th hour. **Q16.** $m = 1000$. **Q17.** (a) $\$2315.25$; (b) the ratio of consecutive values is the constant 1.05; (c) about 14 years (it doubles during the 15th year). **Q18.** (a) 768; (b) $3 \times 2^8 = 768$; (c) $t = 11$. **Q19.** 10 hours < 1 day < 100 000 seconds < 1 year. **Q20.** 1000 times. **Q21.** logarithmic; reasons vary. **Q22.** (a) $\$2073.60$; (b) constant ratio 1.2; (c) $t = 9$. **Q23.** (a) 100 times more shaking; (b) about 1000 times the energy; (c) M5.9.

Fully-worked solutions

Q1. (a) $10 \times 10 \times 10 \times 10 = 10^4$. (b) $100\,000 = 10^5$. (c) $0.01 = \dfrac{1}{100} = 10^{-2}$.

Q2. (a) $\dfrac{50\,000}{50} = 1000 = 10^3$: 3 orders of magnitude.

(b) $\dfrac{4\,000\,000}{4} = 1\,000\,000 = 10^6$: 6 orders of magnitude.

Q3. (a) 8 is $\times 8$ above 1 but only $\times 1.25$ below 10: closest to 10.

(b) 250 is $\times 2.5$ above 100 but $\times 4$ below 1000: closest to 100.

(c) 900 is $\times 9$ above 100 but only $\times 1.1$ below 1000: closest to 1000.

Q4. $3 \times 2 = 6$, then $12, 24, 48, 96$.

Q5. Two steps of $\times 10$: $10 \times 10 = 100$ times more shaking.

Q6. Three steps of 10 dB: $10 \times 10 \times 10 = 1000$ times more intense.

Q7. $3600 = 3.6 \times 10^3$, so it lies between $10^3 = 1000$ and $10^4 = 10\,000$.

Q8. (a) $100 \times 2^4 = 1600$: ISO 1600.

(b) $\dfrac{6400}{100} = 64 = 2^6$: 6 doublings.

Q9. (a) $\dfrac{7\,000\,000}{700} = 10\,000 = 10^4$: 4 orders.

(b) $\dfrac{500}{0.05} = 10\,000 = 10^4$: 4 orders.

(c) $\dfrac{250\,000}{25} = 10\,000 = 10^4$: 4 orders.

Q10. (a) $0.0001 = 10^{-4}$. (b) $45\,000 = 4.5 \times 10^4$. (c) $0.007 = 7 \times 10^{-3}$.

Q11. (a) 3 sits near the middle of the decade 1–10 (a factor of 3 is about 48 % of the way). (b) 40 sits about 60 % of the way through 10–100. (c) 600 sits about 78 % of the way through 100–1000. (d) 8000 sits about 90 % of the way through 1000–10 000.

Q12. (a) $5 - 2 = 3$ steps: $10 \times 10 \times 10 = 1000$ times more shaking.

(b) $32 \times 32 \times 32 = 32\,768 \approx 33\,000$ times the energy.

(c) $1000 \approx 32 \times 32$: two energy steps, so the magnitude is $2 + 2 = $ M4.

Q13. (a) $70 - 40 = 30$ dB $= 3$ steps: $10^3 = 1000$ times.

(b) $130 - 0 = 130$ dB $= 13$ steps of 10 dB: 10^{13} times more intense.

Q14. (a) $200 \times 2^3 = 1600$: ISO 1600.

(b) $\dfrac{6400}{100} = 64 = 2^6$: 6 doublings.

Q15. (a) $5 \times 2^6 = 320$.

(b) $5 \times 2^7 = 640 < 1000 < 1280 = 5 \times 2^8$: it first exceeds 1000 in the 8th hour.

Q16. Multiply the two equations: $m \times m = 100 \times 10\,000 = 1\,000\,000$, so $m \times m = 1000 \times 1000$ and $m = 1000$. (The factor each side is $f = 10$.)

Q17. (a) $V = 2000 \times 1.05^3 = 2000 \times 1.157625 = 2315.25$: $\$2315.25$.

(b) Each year multiplies the value by the same factor, 1.05, so equal steps of time give equal factors — hence equal steps on a logarithmic axis.

(c) $1.05^{14} \approx 1.98$ — below 2; $1.05^{15} \approx 2.08$ — above 2. The value doubles during the 15th year, so to the nearest year, about 14 years.

Q18. (a) From the table, $N = 768$ at $t = 8$.

(b) $3 \times 2^8 = 3 \times 256 = 768$. ✓

(c) $3 \times 2^{10} = 3072 < 5000 < 6144 = 3 \times 2^{11}$: the first whole hour is $t = 11$.

Q19. In seconds: 10 hours $= 36\,000$; 1 day $= 86\,400$; $100\,000$; 1 year $\approx 31\,500\,000$. So 10 hours < 1 day < $100\,000$ seconds < 1 year.

Q20. $7 - 4 = 3$ steps of 1: $10 \times 10 \times 10 = 1000$ times more acidic.

Q21. Choose the **logarithmic** scale: the counts span from 12 to 6000, nearly three orders of magnitude, and on a linear scale the first two weeks would be squashed flat against the axis while the story — roughly tenfold rises week to week at the peak — reads as a straight climb on the logarithmic scale. The linear scale would mislead by hiding the early weeks entirely; (conversely, a logarithmic scale would mislead a reader who takes equal steps to mean equal numbers of cases).

Q22. (a) $V = 1000 \times 1.2^4 = 1000 \times 2.0736 = 2073.60$: $\$2073.60$.

(b) The ratio of consecutive yearly values is the constant 1.2, so equal steps of time are equal factors — equal distances on a logarithmic axis, a straight line.

(c) Need $1.2^t \ge 5$: $1.2^8 \approx 4.30$ — not yet; $1.2^9 \approx 5.16$ — yes. $\therefore t = 9$.

Q23. (a) $5.9 - 3.9 = 2.0$: two steps of $\times 10$ shaking, so $10 \times 10 = 100$ times more shaking.

(b) Two energy steps: $32 \times 32 = 1024 \approx 1000$ times the energy.

(c) $33\,000 \approx 32\,768 = 32 \times 32 \times 32$: three energy steps, so the magnitude is $2.9 + 3 = $ M5.9 — the Mansfield earthquake itself.

Teacher note

Level 9 code rehearsed (D3): `VC2M9M02` — scientific notation — writes every quantity the orders-of-magnitude work compares, and `VC2M9A01` — the exponent laws — licenses the step $\frac{3 \times 10^4}{3 \times 10^1} = 10^3$. Both are rehearsed, not re-taught, and are not assessed as Level 9 outcomes. The in-book prerequisites are 2E (`VC2M10A14`), whose exponential relation $a b^{t} = $ target inverts every doubling-scale question here, and 3C (`VC2M10A15`), whose constant-ratio growth is what a logarithmic axis straightens.

Tool classes (D13): none — `VC2M10M02` names no digital tool in its content description or elaborations, so this subtopic carries no tool banner; all reading and interpreting is by hand. A scientific calculator may check the multiplying chains ($32 \times 32 \times 32$), and the tool-free path is the assessed one throughout.

VCE destinations (§5, reference only): 4D is in no Units 3 & 4 index *(inferred)*: logarithmic scales feed Mathematical Methods Units 1 & 2, where the scale is formalised, and General Mathematics Units 3 & 4 AoS 1 ‘Data analysis’, where transforming data to orders of magnitude turns an association into a straight line. 4D also carries implicit prerequisite 4: VCE Chemistry’s pH scale and VCE Physics’ decibel scale and logarithmic-axis spectrum charts assume the student already knows what a logarithmic scale is, and VC2 Science `VC2S10U06` and `VC2S10U13` assume order-of-magnitude reasoning in the same year. **If 4D is taught as “here is the Richter scale” and no more, the concept is never delivered anywhere in a student’s schooling.**

Elaboration `VC2M10M02.E04` is not delivered. It invites the Madjedbebe dig in the Northern Territory as a context for interpreting orders of magnitude. This book excludes Aboriginal and Torres Strait Islander content (see the front-of-book note); the order-of-magnitude and timescale reasoning the elaboration exercises is delivered in full through the contexts of `VC2M10M02.E01– .E03`.

Level check (D6): no logarithm laws, no `log` operator and no algebraic manipulation of logarithms appear anywhere in this subtopic (D7); the scale is built and read entirely with powers of 10 and equal factors, and inverted by hand through the exponential relation ($2^t = 32$, $1.06^t = 2$) by matching powers or trapping between whole years — the skills of 2E. The words *logarithmic scale*, *order of magnitude* and *decade* are defined at first use and used consistently.

Dated data (R3): Q23’s Mansfield earthquake (22 September 2021, magnitude 5.9) is a historical event used for comparison, listed here for the chapter refresh register with its Geoscience Australia source line. All other contexts are scenarios or stipulated scale definitions and carry no dated figures.