

Mathematics Level 10

Chapter 3 — Graphs, models and algorithms · 4 subtopics · 4 CDs

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Gradients of parallel and perpendicular lines

Mathematics 10 · Chapter 3 Graphs, models and algorithms · Subtopic 3A (Level 10) · Standard block

Strand	Algebra
Content description	VC2M10A10
Elaboration ids carried	VC2M10A10.E01 VC2M10A10.E02
Weight	Standard (single allocation)

Content description (verbatim): *solve problems involving gradients of parallel and perpendicular lines*

Achievement standard (extract): *They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, algebraically and graphically, with and without the use of digital tools, and justify solutions.*

You need first: gradient as rise over run and the gradient formula between two points, from VC2M9A04 (Year 9), and reading a straight-line graph as $y = mx + c$, from VC2M9A03. This subtopic rehearses both and builds on them; it does not re-teach them, and they are not assessed as Level 9 outcomes here.

Theory

In Year 9 you drew straight lines and measured how steep each one is. This subtopic asks the next question: what does the gradient tell you about **two** lines — about how they sit relative to each other? There are two results, one for parallel lines and one for perpendicular lines, and in this book each one is used **both ways**: the fact itself, and its converse. The converses are what most problems actually need.

1. What gradient remembers

The **gradient** of a straight line measures its steepness and direction. It is

$$m = \frac{\text{rise}}{\text{run}},$$

the vertical change divided by the horizontal change as you move along the line. Between two points (x_1, y_1) and (x_2, y_2) on the line,

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Four cases to keep straight:

- a line climbing from left to right has **positive** gradient;
- a line falling from left to right has **negative** gradient;
- a horizontal line has gradient 0 (the rise is always 0);
- a vertical line has **undefined** gradient (the run is 0, and division by 0 has no meaning).

The equation $y = mx + c$ draws a straight line with gradient m ; the number c is the **y-intercept**, the y -value where the line crosses the y -axis. All of this is Year 9 work (VC2M9A04, VC2M9A03) and it runs underneath everything below.

2. Parallel lines have the same gradient

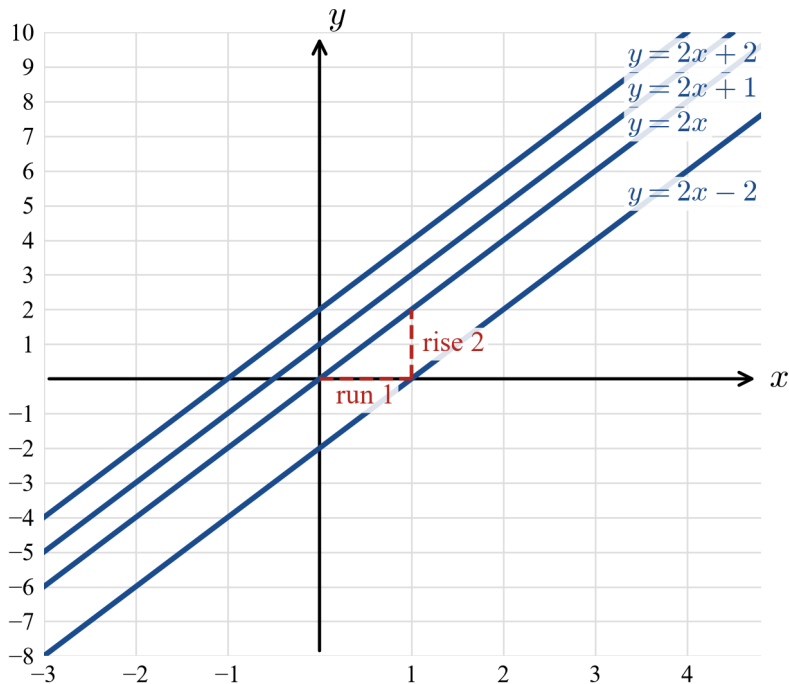
Parallel lines point in the same direction: one is a slide of the other. Slide a line without turning it and its steepness cannot change, so:

Parallel lines have the same gradient. If two lines are parallel, then $m_1 = m_2$.

The fact that runs this subtopic is the **converse**, used just as often:

If two lines have the same gradient, then they are parallel.

Same gradient means same steepness and same direction, so the lines can never meet — they are parallel. In Figure 3A.1, every line has gradient 2: for a run of 1, each rises 2. The gradient triangle on $y = 2x$ reads run 1, rise 2; slide that triangle onto any of the other three lines and it reads exactly the same.



Four parallel lines of gradient 2 on a coordinate grid, y equals $2x$ minus 2, y equals $2x$, y equals $2x$ plus 1 and y equals $2x$ plus 2, with a dashed gradient triangle on y equals $2x$ showing a run of 1 and a rise of 2.

The four lines have different y -intercepts — that is what keeps them apart — but the same gradient, and that is what makes them parallel.

One exception to keep in mind: two vertical lines are parallel to each other, but neither has a gradient at all. The equal-gradient rule applies to lines that have gradients.

3. Perpendicular lines: gradients multiply to -1

Perpendicular lines meet at right angles. Their gradients are linked by multiplication:

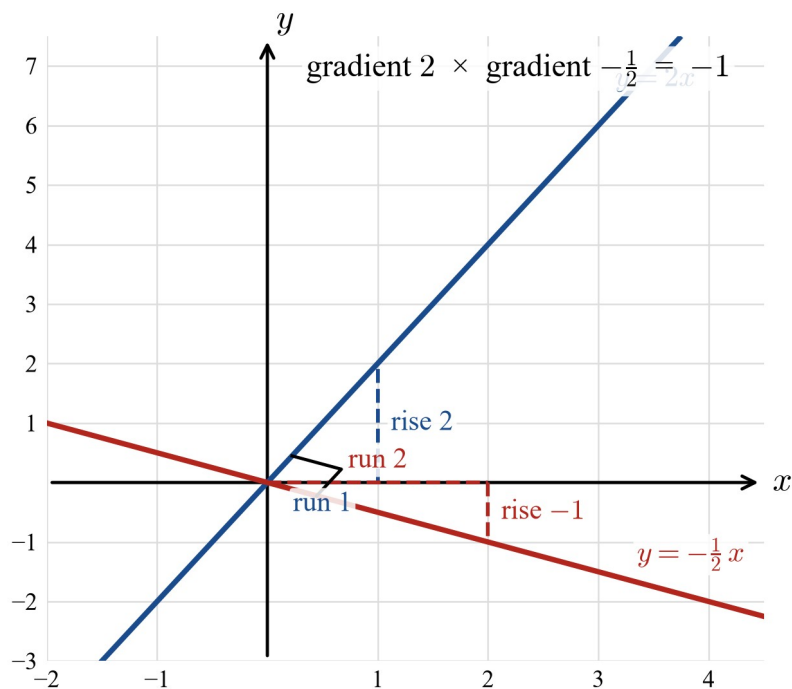
The product of the gradients of two perpendicular lines is -1 . If two lines with gradients m_1 and m_2 are perpendicular, then $m_1 \times m_2 = -1$.

Again, the working converse goes with it:

If the product of the gradients of two lines is -1 , then the lines are perpendicular.

Figure 3A.2 shows why the rule looks the way it does. The blue line has gradient 2: run 1, rise 2. Rotate that triangle a quarter-turn about the origin and it sits on the red line: the old run becomes the new rise and the old rise becomes the new run — but one of them now points the opposite way, so the sign changes. Run 1, rise 2 turns into run 2, rise -1 :

gradient 2 turns into gradient $-\frac{1}{2}$, and $2 \times \left(-\frac{1}{2}\right) = -1$.



Two perpendicular lines through the origin on a coordinate grid, a blue line with gradient 2 and a red line with gradient minus one half, each with a gradient triangle; the run of one is the rise of the other and the sign changes.

So the perpendicular gradient has a shortcut: **flip the fraction and change the sign**. The **reciprocal** of a number is 1 divided by that number — for a fraction, flip it, so the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ — and the perpendicular gradient is the reciprocal with the sign changed. That is why it is sometimes called the negative reciprocal.

gradient m

perpendicular gradient $-\frac{1}{m}$

2	→	$-\frac{1}{2}$
$\frac{3}{4}$	→	$-\frac{4}{3}$
-5	→	$\frac{1}{5}$
1	→	-1

flip the fraction and change the sign

A matching diagram pairing each gradient with its perpendicular gradient: 2 with minus one half, three quarters with minus four thirds, minus 5 with one fifth, and 1 with minus 1.

Gradient m

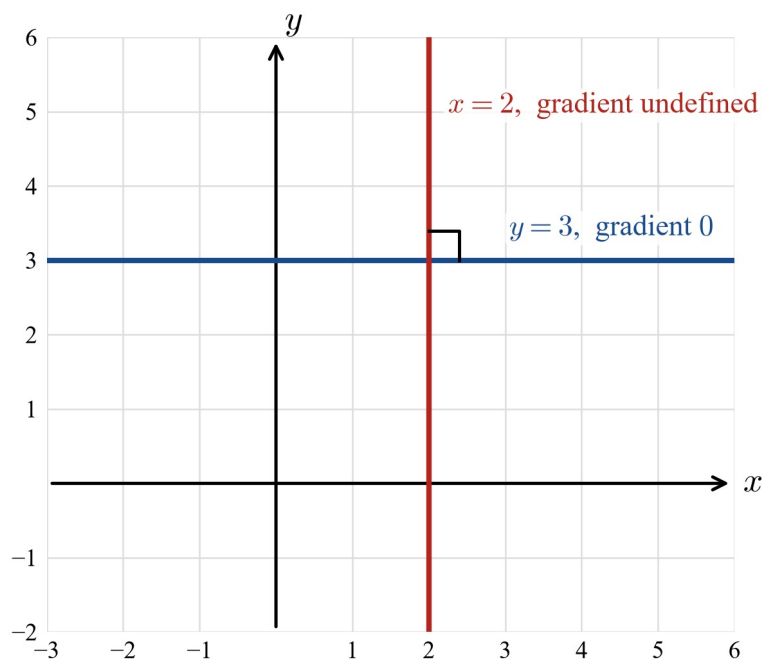
Perpendicular gradient

2	$-\frac{1}{2}$
$\frac{3}{4}$	$-\frac{4}{3}$
-5	$\frac{1}{5}$
1	-1

Gradient m	Perpendicular gradient
$-\frac{2}{3}$	$\frac{3}{2}$

Check each row by multiplying the two gradients — every row gives -1 .

Special case. The rule needs both gradients to exist. A horizontal line and a vertical line are perpendicular (Figure 3A.4), but one has gradient 0 and the other has none at all, so “ $m_1 \times m_2 = -1$ ” does not apply. Handle horizontal and vertical lines separately: the line perpendicular to a horizontal line is vertical, and the line perpendicular to a vertical line is horizontal.



The horizontal line y equals 3 and the vertical line x equals 2 meeting at a right angle on a coordinate grid; the horizontal line has gradient 0 and the vertical line has no defined gradient.

4. Using the results with equations

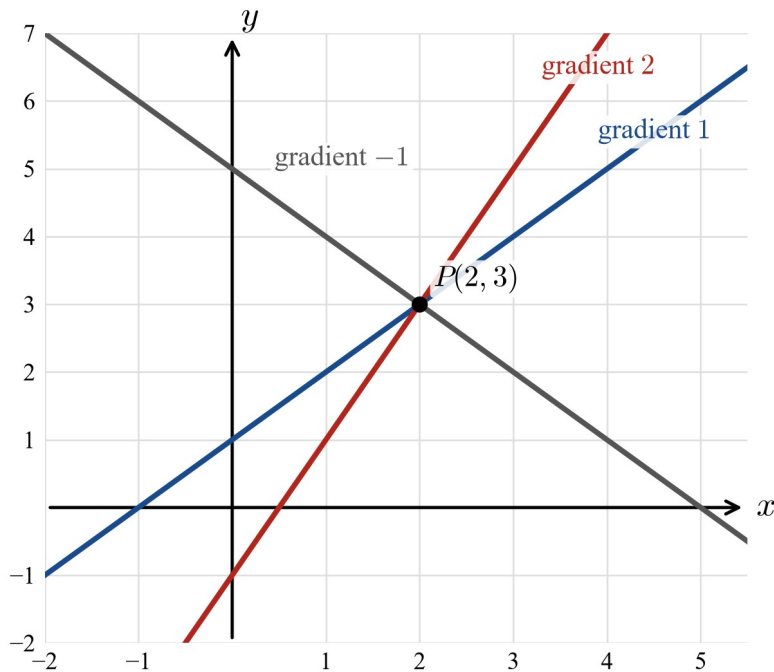
Everything so far combines with $y = mx + c$. Three moves do almost all the work:

Read the gradient. In $y = mx + c$, the gradient is the number multiplying x . If the equation is not in that form, rearrange it first: $2y = 8x + 5$ becomes $y = 4x + \frac{5}{2}$, gradient 4; $y - 3 = 5x$ becomes $y = 5x + 3$, gradient 5.

Parallel through a point. The new line takes the **same gradient**; only the y -intercept changes. Substitute the point into $y = mx + c$ to find the new c .

Perpendicular through a point. The new line takes the **negative reciprocal** of the gradient; then substitute the point to find c , exactly as above.

Figure 3A.5 shows why both numbers matter: infinitely many lines pass through one point, and the gradient chooses the direction. The gradient and the intercept together fix the line.



Three lines through the one point $P(2, 3)$ on a coordinate grid, with gradients 1, 2 and minus 1.

5. Testing shapes with gradients

The converse results turn gradients into shape tests. The sides of a shape drawn on a grid are line segments, and each side has a gradient found from its two endpoints.

- **Parallelogram test.** If both pairs of opposite sides have equal gradients, the opposite sides are parallel (converse from §2), so the shape is a parallelogram.
- **Rectangle test.** If the shape is a parallelogram and the gradients of two adjacent sides multiply to -1 , the sides meet at right angles (converse from §3), so the parallelogram is a rectangle.
- **Right angle in a triangle.** If the gradients of two sides meeting at a vertex multiply to -1 , the triangle is right-angled at that vertex.

One caution when computing side gradients: subtract the coordinates in the **same order** top and bottom of the fraction, $\frac{y_2 - y_1}{x_2 - x_1}$. Reversing one order but not the other flips the sign of the gradient, and a flipped sign breaks both tests.

Worked Example 3 runs both tests on one shape.

Worked examples

Example 1 — Parallel lines

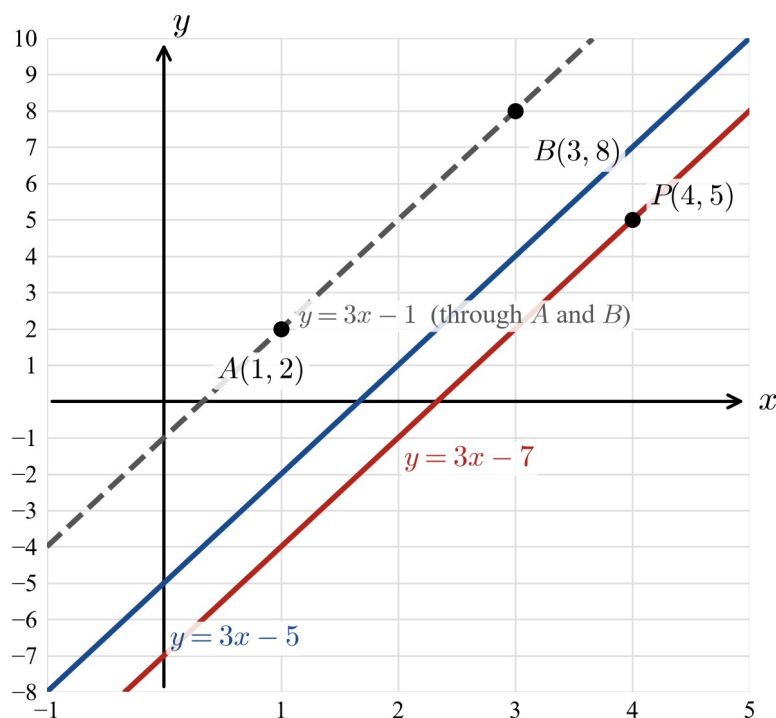
(a) Show that the line $L_1: y = 3x - 5$ is parallel to the line L_2 through $A(1, 2)$ and $B(3, 8)$.

Working	Comment
$m_1 = 3$	Read the gradient of L_1 from $y = mx + c$.
$m_2 = \frac{8 - 2}{3 - 1}$	Gradient formula between A and B .
$m_2 = \frac{6}{2} = 3$	
$m_1 = m_2 = 3$	The gradients are equal.
$\therefore L_1$ is parallel to L_2 .	Same gradient, so parallel.

(b) Find the equation of the line through $P(4, 5)$ parallel to L_1 .

Working	Comment
$m = 3$	Parallel to L_1 , so the same gradient.
$y = 3x + c$	Start from $y = mx + c$; c is still unknown.
$5 = 3 \times 4 + c$	The line passes through $(4, 5)$: substitute $x = 4$, $y = 5$.
$5 = 12 + c$	
$c = -7$	
$\therefore y = 3x - 7$	Check: when $x = 4$, $y = 3 \times 4 - 7 = 5$. ✓

Figure 3A.6 shows both parts: the dashed line through A and B and the red line through P are both slides of $y = 3x - 5$.



Three parallel lines of gradient 3 on a coordinate grid: y equals $3x$ minus 5, the dashed line through $A(1, 2)$ and $B(3, 8)$, and y equals $3x$ minus 7 through $P(4, 5)$.

Example 2 — Perpendicular lines

(a) Find the gradient of the line perpendicular to $y = -\frac{2}{5}x + 1$.

Working	Comment
$m = -\frac{2}{5}$	Read the gradient from $y = mx + c$.
$m_{\perp} = -1 \div \left(-\frac{2}{5}\right)$	Perpendicular gradients multiply to -1 .
$m_{\perp} = \frac{5}{2}$	Flip the fraction and change the sign.

(b) Find the equation of the line through $Q(4, 5)$ perpendicular to $y = 4x + 1$.

Working	Comment
$m = 4$	Gradient of the given line.
$m_{\perp} = -\frac{1}{4}$	Flip and change sign; check $4 \times \left(-\frac{1}{4}\right) = -1$.

$$y = -\frac{1}{4}x + c$$

Start from $y = mx + c$.

$$5 = -\frac{1}{4} \times 4 + c$$

Substitute $(4, 5)$.

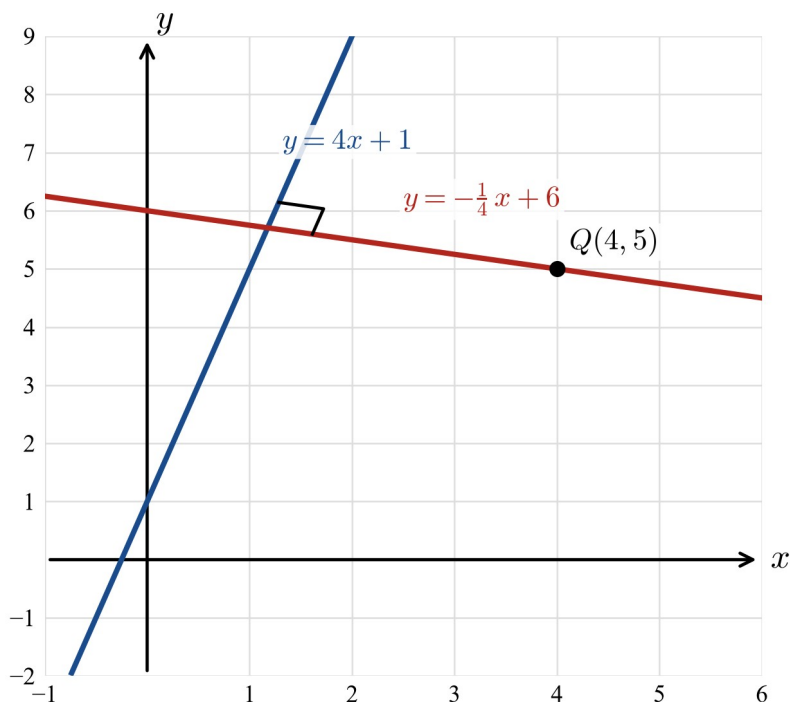
$$5 = -1 + c$$

$$c = 6$$

$$\therefore y = -\frac{1}{4}x + 6.$$

Check: when $x = 4$, $y = -1 + 6 = 5$. ✓

Figure 3A.7 shows part (b): the steep line and the shallow line meet at a right angle. A steep line always pairs with a shallow one — their gradients multiply to -1 .



Two perpendicular lines on a coordinate grid, y equals $4x$ plus 1 and y equals minus one quarter x plus 6 , meeting at a marked right angle, with the point $Q(4, 5)$ on the second line.

Example 3 — Testing a shape

Show that $A(0, 0)$, $B(4, 2)$, $C(2, 6)$ and $D(-2, 4)$, taken in order, are the vertices of a rectangle.

$$m_{AB} = \frac{2-0}{4-0} = \frac{1}{2}$$

Gradient of each side, from its endpoints.

$$m_{BC} = \frac{6-2}{2-4} = -2$$

$$m_{CD} = \frac{4-6}{-2-2} = \frac{1}{2}$$

$$m_{DA} = \frac{0-4}{0-(-2)} = -2$$

$$m_{AB} = m_{CD} \text{ and } m_{BC} = m_{DA}$$

Both pairs of opposite sides have equal gradients.

$\therefore ABCD$ is a parallelogram.

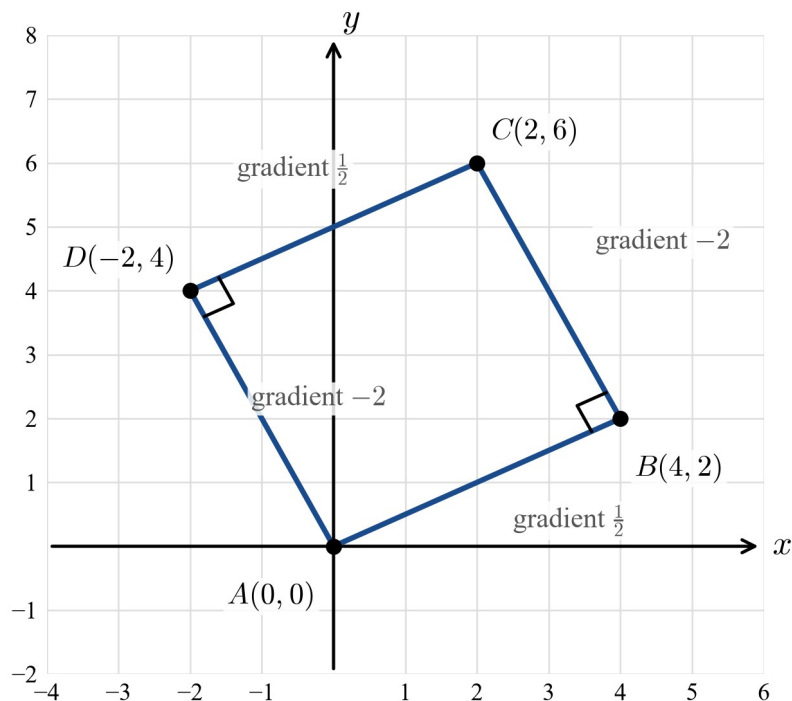
Equal gradient $\Rightarrow$ parallel sides (the converse).

$$m_{AB} \times m_{BC} = \frac{1}{2} \times (-2) = -1$$

Test two adjacent sides.

$\therefore ABCD$ is a rectangle.

A parallelogram with one right angle has four right angles.



The rectangle $ABCD$ with vertices $A(0, 0)$, $B(4, 2)$, $C(2, 6)$ and $D(\text{minus } 2, 4)$; side gradients are labelled one half, minus 2, one half and minus 2, with the right angles marked at B and D .

Notice the order of the argument: gradients first, then the two converses — same gradient gives the parallelogram, product -1 gives the right angles. Neither test alone would be enough.

Practice questions

Scaffolded

Q1. Find the gradient of each line. (Hint: gradient = rise $\div$ run, or use $m = \frac{y_2 - y_1}{x_2 - x_1}$.) (a) The line through $(0, 1)$ and $(1, 5)$. (b) The line through $(0, 7)$ and $(2, 1)$. (c) The line through $(0, 0)$ and $(4, 2)$. (d) The line through $(1, 3)$ and $(5, 3)$.

Q2. Write down the gradient of the line parallel to each line. (Hint: parallel lines have the same gradient.) (a) $y = 5x + 1$; (b) $y = -\frac{2}{3}x + 4$; (c) $y = 4x$; (d) $y = -x + 6$.

Q3. Write down the gradient of the line perpendicular to each line. (Hint: flip the fraction and change the sign.) (a) $y = 3x - 2$; (b) $y = -2x$; (c) $y = \frac{4}{5}x + 1$; (d) $y = -\frac{3}{7}x$.

Q4. Which pairs of lines are parallel? (Hint: compare the gradients.) A: $y = 2x + 3$ and $y = 2x - 7$; B: $y = 5x$ and $y = -5x$; C: $y = \frac{1}{3}x + 2$ and $y = \frac{1}{3}x - 4$; D: $y = 4x - 1$ and $y = \frac{1}{4}x + 1$.

Q5. Which pairs of gradients belong to perpendicular lines? (Hint: multiply the pair.) (a) 2 and $-\frac{1}{2}$; (b) 3 and $\frac{1}{3}$; (c) -4 and $\frac{1}{4}$; (d) $\frac{3}{2}$ and $-\frac{2}{3}$.

Q6. Find the equation of the line through $(2, 7)$ parallel to $y = 3x - 1$. (Hint: first write down the gradient, then substitute the point into $y = 3x + c$ to find c .)

Q7. Find the equation of the line through $(3, 8)$ perpendicular to $y = -\frac{1}{2}x + 4$. (Hint: first write down the perpendicular gradient, then substitute the point to find c .)

Q8. The line ℓ has equation $y = -4$. (a) Write down the gradient of any line parallel to ℓ . (Hint: ℓ is horizontal.) (b) A line perpendicular to ℓ is vertical. What is its gradient? (c) Write the equation of one line perpendicular to ℓ .

Unscaffolded

Q9. Find the gradient of the line parallel to each line. (a) $y = -6x + 1$; (b) $y = \frac{3}{4}x - 5$; (c) $2y = 8x + 5$; (d) $y - 3 = 5x$.

Q10. For each gradient, write down (i) the gradient of a parallel line and (ii) the gradient of a perpendicular line. (a) 6; (b) $-\frac{3}{2}$; (c) 1; (d) $-\frac{5}{8}$.

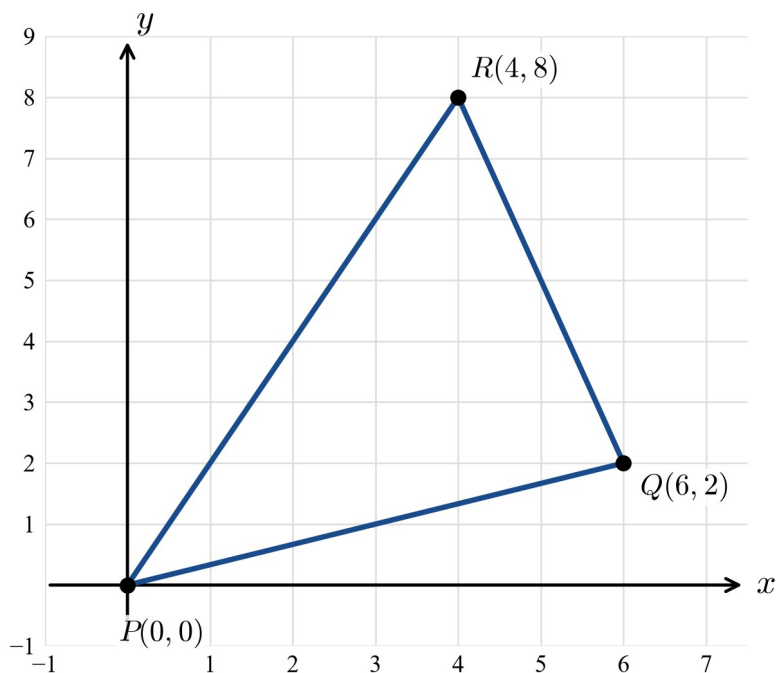
Q11. $A(2, 5)$ and $B(6, 13)$. (a) Find the gradient of AB . (b) Find the equation of the line through $(0, -1)$ parallel to AB . (c) Write down the gradient of the line perpendicular to AB .

Q12. State whether each pair of lines is parallel, perpendicular or neither. (a) $y = 3x + 2$ and $6x - 2y = 5$; (b) $y = \frac{2}{3}x - 1$ and $2x + 3y = 7$; (c) $y = -4x + 6$ and $x - 4y = 2$.

Q13. Find the equation of the line through $(-3, 4)$ parallel to $y = -2x + 6$.

Q14. Find the equation of the line through $(5, -3)$ perpendicular to $y = \frac{1}{3}x + 2$.

Q15. $P(0, 0)$, $Q(6, 2)$ and $R(4, 8)$. (a) Find the gradients of PQ and QR . (b) Hence explain why triangle PQR is right-angled, and state at which vertex the right angle sits.



The triangle PQR with vertices $P(0, 0)$, $Q(6, 2)$ and $R(4, 8)$ drawn on a coordinate grid.

Q16. Show that $A(-2, 4)$, $B(2, 6)$, $C(4, 2)$ and $D(0, 0)$, taken in order, are the vertices of a rectangle.

Q17. Find k in each case. (a) The line through $(1, 2)$ and $(3, k)$ is parallel to $y = 4x + 5$. (b) The line through $(0, 3)$ and $(k, 7)$ is parallel to a line of gradient 2. (c) The line through $(1, 2)$ and $(5, k)$ is perpendicular to a line of gradient 2.

Q18. The line L has equation $y = \frac{3}{2}x - 4$. (a) Find the equation of the line parallel to L through $(3, 1)$. (b) Find the equation of the line perpendicular to L through $(0, 4)$.

Q19. Show that $A(-3, 1)$, $B(1, 4)$, $C(4, 0)$ and $D(0, -3)$, taken in order, form a square. (Hint: gradients show the right angles; side lengths show the equal sides.)

Q20. Find the equation of the line that passes through the midpoint of the segment joining $(1, 2)$ and $(5, 4)$ and is perpendicular to that segment.

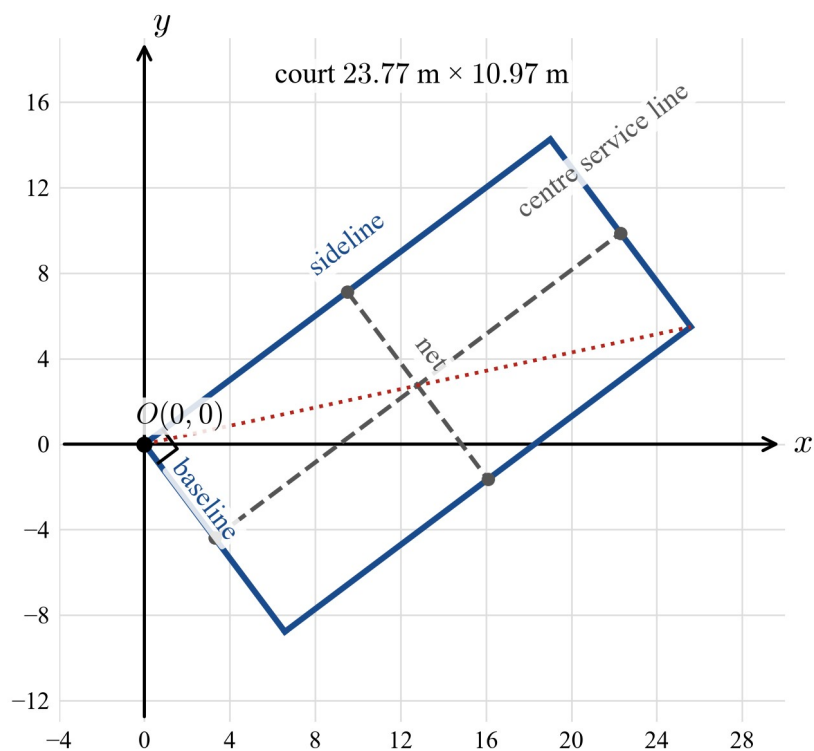
Q21. $A(2, 5)$, $B(k, 9)$, $C(0, 0)$ and $D(3, 6)$. (a) If AB is parallel to CD , find k . (b) If AB is perpendicular to a line of gradient 3, find k .

Q22. The line L_1 has equation $y = 2x + 1$. The line L_2 passes through $(0, -4)$ and is perpendicular to L_1 . (a) Find the equation of L_2 . (b) L_2 meets the x -axis at A . Find the coordinates of A . (c) The line L_3 passes through A and is parallel to L_1 . Find the equation of L_3 . (d) Explain why L_1 and L_3 never meet.

Real context

Q23. A tennis court is a rectangle 23.77 m long and 10.97 m wide for doubles. A designer draws the court on a coordinate grid in metres, with one corner O at the origin and one long side — a **sideline** — lying on a line of gradient $\frac{3}{4}$.

- State the gradient of the opposite sideline. Give a reason.
- State the gradient of each short side — a **baseline**. Give a reason.
- The net crosses the middle of the court, parallel to the baselines. State its gradient.
- The centre service line runs down the middle of the court, parallel to the sidelines. State its gradient.
- Find the length of the diagonal of the court from O to the opposite corner, correct to 2 decimal places.



A tennis court drawn as a tilted rectangle on a coordinate grid with one corner at the origin O ; the sidelines, baselines, net and centre service line are labelled and a dotted diagonal joins O to the opposite corner.

Source: International Tennis Federation, *Rules of Tennis* (court 23.77 m long, 10.97 m wide for doubles), as cited in ‘Tennis court’, Wikipedia. Retrieved 20 August 2026.

Answers

Q1. (a) 4; (b) -3 ; (c) $\frac{1}{2}$; (d) 0. **Q2.** (a) 5; (b) $-\frac{2}{3}$; (c) 4; (d) -1 . **Q3.** (a) $-\frac{1}{3}$; (b) $\frac{1}{2}$; (c) $-\frac{5}{4}$; (d) $\frac{7}{3}$. **Q4.** A and C.
Q5. (a), (c) and (d). **Q6.** $y = 3x + 1$. **Q7.** $y = 2x + 2$. **Q8.** (a) 0; (b) undefined; (c) any line $x = k$, for example $x = 3$.
Q9. (a) -6 ; (b) $\frac{3}{4}$; (c) 4; (d) 5. **Q10.** (a) 6 and $-\frac{1}{6}$; (b) $-\frac{3}{2}$ and $\frac{2}{3}$; (c) 1 and -1 ; (d) $-\frac{5}{8}$ and $\frac{8}{5}$. **Q11.** (a) 2; (b)
 $y = 2x - 1$; (c) $-\frac{1}{2}$. **Q12.** (a) parallel; (b) neither; (c) perpendicular. **Q13.** $y = -2x - 2$. **Q14.** $y = -3x + 12$. **Q15.** (a)
 $\frac{1}{3}$ and -3 ; (b) the product is -1 , so the right angle is at Q. **Q16.** side gradients $\frac{1}{2}$, -2 , $\frac{1}{2}$, -2 : opposite sides
parallel, adjacent sides perpendicular. **Q17.** (a) $k = 10$; (b) $k = 2$; (c) $k = 0$. **Q18.** (a) $y = \frac{3}{2}x - \frac{7}{2}$; (b) $y = -\frac{2}{3}x + 4$.
Q19. side gradients $\frac{3}{4}$, $-\frac{4}{3}$, $\frac{3}{4}$, $-\frac{4}{3}$, and all four sides have length 5. **Q20.** $y = -2x + 9$. **Q21.** (a) $k = 4$; (b) $k = -10$.
Q22. (a) $y = -\frac{1}{2}x - 4$; (b) $A(-8, 0)$; (c) $y = 2x + 16$; (d) same gradient, different intercepts — parallel lines never
meet. **Q23.** (a) $\frac{3}{4}$, opposite sides of a rectangle are parallel; (b) $-\frac{4}{3}$, a baseline is perpendicular to a sideline; (c) $-\frac{4}{3}$;
(d) $\frac{3}{4}$; (e) 26.18 m.

Fully-worked solutions

Q1. (a)

$$m = \frac{5-1}{1-0} = \frac{4}{1} = 4$$

$$m = \frac{1-7}{2-0} = \frac{-6}{2} = -3$$

$$m = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$$

$$m = \frac{3-3}{5-1} = \frac{0}{4} = 0$$

Q2. Parallel lines have the same gradient, so read each gradient from $y = mx + c$. (a) 5; (b) $-\frac{2}{3}$; (c) 4; (d) -1 .

Q3. Perpendicular gradients multiply to -1 : flip the fraction and change the sign. (a) $-\frac{1}{3}$; (b) $\frac{1}{2}$; (c) $-\frac{5}{4}$; (d) $\frac{7}{3}$.

Q4. Compare gradients. A: both 2 — parallel. B: 5 and -5 — different, not parallel. C: both $\frac{1}{3}$ — parallel. D: 4 and
 $\frac{1}{4}$ — different, not parallel. The parallel pairs are A and C.

Q5. Multiply each pair. (a) $2 \times \left(-\frac{1}{2}\right) = -1$ ✓; (b) $3 \times \frac{1}{3} = 1$ ✗ (the product must be -1 , not 1); (c) $-4 \times \frac{1}{4} = -1$ ✓;
(d) $\frac{3}{2} \times \left(-\frac{2}{3}\right) = -1$ ✓. The perpendicular pairs are (a), (c) and (d).

Q6. Parallel to $y = 3x - 1$, so $m = 3$.

$$y = 3x + c$$

$$7 = 3 \times 2 + c$$

$$7 = 6 + c$$

$$c = 1$$

$$y = 3x + 1$$

Q7. Perpendicular to gradient $-\frac{1}{2}$, so the new gradient is 2.

$$y = 2x + c$$

$$8 = 2 \times 3 + c$$

$$8 = 6 + c$$

$$c = 2$$

$$y = 2x + 2$$

Q8. (a) ℓ is horizontal, so its gradient is 0, and any parallel line also has gradient 0.

(b) A line perpendicular to a horizontal line is vertical, and a vertical line has undefined gradient.

(c) Any vertical line is perpendicular to ℓ : for example, $x = 3$.

Q9. Parallel gradients are equal; rearrange first where needed. (a) -6 . (b) $\frac{3}{4}$. (c) $2y = 8x + 5$ gives $y = 4x + \frac{5}{2}$, gradient 4. (d) $y - 3 = 5x$ gives $y = 5x + 3$, gradient 5.

Q10. Parallel: the same gradient. Perpendicular: flip the fraction and change the sign. (a) 6 and $-\frac{1}{6}$; (b) $-\frac{3}{2}$ and $\frac{2}{3}$;

(c) 1 and -1 ; (d) $-\frac{5}{8}$ and $\frac{8}{5}$.

Q11. (a)

$$m = \frac{13 - 5}{6 - 2} = \frac{8}{4} = 2$$

(b) Parallel, so $m = 2$, and the line passes through $(0, -1)$ — that is the y -intercept, so $c = -1$:

$$y = 2x - 1$$

(c) Perpendicular to gradient 2: $-\frac{1}{2}$.

Q12. Find both gradients first.

(a) $y = 3x + 2$ has gradient 3; $6x - 2y = 5$ rearranges to $2y = 6x - 5$, so $y = 3x - \frac{5}{2}$, gradient 3. Same gradient, so the lines are **parallel**.

(b) $y = \frac{2}{3}x - 1$ has gradient $\frac{2}{3}$; $2x + 3y = 7$ rearranges to $3y = -2x + 7$, so $y = -\frac{2}{3}x + \frac{7}{3}$, gradient $-\frac{2}{3}$. The gradients differ, and $\frac{2}{3} \times \left(-\frac{2}{3}\right) = -\frac{4}{9} \neq -1$, so the lines are **neither** parallel nor perpendicular.

(c) $y = -4x + 6$ has gradient -4 ; $x - 4y = 2$ rearranges to $4y = x - 2$, so $y = \frac{1}{4}x - \frac{1}{2}$, gradient $\frac{1}{4}$.

$-4 \times \frac{1}{4} = -1$, so the lines are **perpendicular**.

Q13. Parallel to $y = -2x + 6$, so $m = -2$.

$$y = -2x + c$$

$$4 = -2 \times (-3) + c$$

$$4 = 6 + c$$

$$c = -2$$

$$y = -2x - 2$$

Q14. Perpendicular to gradient $\frac{1}{3}$, so the new gradient is -3 .

$$y = -3x + c$$

$$-3 = -3 \times 5 + c$$

$$-3 = -15 + c$$

$$c = 12$$

$$y = -3x + 12$$

Q15. (a)

$$m_{PQ} = \frac{2-0}{6-0} = \frac{2}{6} = \frac{1}{3}$$

$$m_{QR} = \frac{8-2}{4-6} = \frac{6}{-2} = -3$$

$$m_{PQ} \times m_{QR} = \frac{1}{3} \times (-3) = -1$$

so PQ is perpendicular to QR and triangle PQR is right-angled, with the right angle at Q .

Q16.

$$m_{AB} = \frac{6-4}{2-(-2)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{2-6}{4-2} = \frac{-4}{2} = -2$$

$$m_{CD} = \frac{0-2}{0-4} = \frac{-2}{-4} = \frac{1}{2}$$

$$m_{DA} = \frac{4-0}{-2-0} = \frac{4}{-2} = -2$$

$m_{AB} = m_{CD}$ and $m_{BC} = m_{DA}$, so both pairs of opposite sides are parallel and $ABCD$ is a parallelogram. Adjacent sides:

$$\frac{1}{2} \times (-2) = -1,$$

so the sides meet at right angles and the parallelogram is a rectangle.

Q17. (a) Parallel to $y = 4x + 5$ means gradient 4:

$$\frac{k-2}{3-1} = 4$$

$$\frac{k-2}{2} = 4$$

$$k-2=8$$

$$k=10$$

(b) Gradient 2:

$$\frac{7-3}{k-0} = 2$$

$$\frac{4}{k} = 2$$

$$k=2$$

(c) Perpendicular to gradient 2 means gradient $-\frac{1}{2}$:

$$\frac{k-2}{5-1} = -\frac{1}{2}$$

$$\frac{k-2}{4} = -\frac{1}{2}$$

$$k-2=-2$$

$$k=0$$

Q18. (a) Parallel, so $m = \frac{3}{2}$:

$$y = \frac{3}{2}x + c$$

$$1 = \frac{3}{2} \times 3 + c$$

$$1 = \frac{9}{2} + c$$

$$c = 1 - \frac{9}{2} = -\frac{7}{2}$$

$$y = \frac{3}{2}x - \frac{7}{2}$$

(b) Perpendicular to gradient $\frac{3}{2}$, so the new gradient is $-\frac{2}{3}$, and the line passes through $(0, 4)$, so $c = 4$:

$$y = -\frac{2}{3}x + 4$$

Q19. Side gradients:

$$m_{AB} = \frac{4-1}{1-(-3)} = \frac{3}{4}$$

$$m_{BC} = \frac{0-4}{4-1} = -\frac{4}{3}$$

$$m_{CD} = \frac{-3-0}{0-4} = \frac{3}{4}$$

$$m_{DA} = \frac{1-(-3)}{-3-0} = -\frac{4}{3}$$

$m_{AB} = m_{CD}$ and $m_{BC} = m_{DA}$, so $ABCD$ is a parallelogram, and $\frac{3}{4} \times \left(-\frac{4}{3}\right) = -1$, so its angles are right angles: it is a rectangle. Side lengths:

$$AB = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ and } BC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5,$$

and likewise $CD = 5$ and $DA = 5$. A rectangle with all four sides equal is a square.

Q20. Midpoint of $(1, 2)$ and $(5, 4)$:

$$\left(\frac{1+5}{2}, \frac{2+4}{2}\right) = (3, 3)$$

Gradient of the segment:

$$m = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$$

so the perpendicular gradient is -2 .

$$y = -2x + c$$

$$3 = -2 \times 3 + c$$

$$3 = -6 + c$$

$$c = 9$$

$$y = -2x + 9$$

Q21. First $m_{CD} = \frac{6-0}{3-0} = 2$, and $m_{AB} = \frac{9-5}{k-2} = \frac{4}{k-2}$.

(a) Parallel, so $m_{AB} = m_{CD}$:

$$\frac{4}{k-2} = 2$$

$$4 = 2(k-2)$$

$$k - 2 = 2$$

$$k = 4$$

(b) Perpendicular to gradient 3, so $m_{AB} = -\frac{1}{3}$:

$$\frac{4}{k-2} = -\frac{1}{3}$$

$$12 = -(k-2)$$

$$k-2 = -12$$

$$k = -10$$

Q22. (a) Perpendicular to gradient 2, so L_2 has gradient $-\frac{1}{2}$, and it passes through $(0, -4)$, so $c = -4$:

$$y = -\frac{1}{2}x - 4$$

(b) On the x -axis, $y = 0$:

$$0 = -\frac{1}{2}x - 4$$

$$\frac{1}{2}x = -4$$

$$x = -8$$

so $A(-8, 0)$.

(c) Parallel to L_1 , so $m = 2$, and L_3 passes through $(-8, 0)$:

$$0 = 2 \times (-8) + c$$

$$0 = -16 + c$$

$$c = 16$$

$$y = 2x + 16$$

(d) L_1 and L_3 both have gradient 2, with different y -intercepts (1 and 16), so they are parallel. Parallel lines never meet.

Q23. (a) The opposite sideline is parallel to the first one — opposite sides of a rectangle are parallel — so its gradient is also $\frac{3}{4}$.

(b) Each baseline is perpendicular to the sideline, so its gradient is the negative reciprocal of $\frac{3}{4}$: $-\frac{4}{3}$. (Check:

$$\frac{3}{4} \times \left(-\frac{4}{3}\right) = -1.)$$

(c) The net is parallel to the baselines, so its gradient is $-\frac{4}{3}$.

(d) The centre service line is parallel to the sidelines, so its gradient is $\frac{3}{4}$.

(e) The diagonal is the hypotenuse of a right triangle with sides 23.77 m and 10.97 m:

$$d = \sqrt{23.77^2 + 10.97^2}$$

$$d = \sqrt{565.0129 + 120.3409}$$

$$d = \sqrt{685.3538}$$

$$d \approx 26.18$$

The diagonal is 26.18 m, correct to 2 decimal places.

Teacher note

Level 9 code rehearsed (D3): VC2M9A04 does the heavy lifting — gradient as rise over run and the gradient formula between two points open §1 and support every gradient computation in the subtopic, and the midpoint and distance formulas from the same code close Q20 and Q19 respectively. VC2M9A03 — straight-line graphs and reading $y = mx + c$ — supports §4. Both are rehearsed, not re-taught, and are not assessed as Level 9 outcomes. Downstream in this book, 6B’s line of good fit reads gradients and intercepts exactly the way this subtopic reads them (§3 prerequisite table).

Tool classes (D13): VC2M10A10 and both of its elaborations name no digital tool, so no DIGITAL TOOL banner appears in this subtopic (R8). Every result here is exact-fraction work done by hand; a graphing tool may be used to check drawn lines, but no question requires one.

VCE destinations (§5, reference only): VC2M10A10 is named in none of the Units 3 & 4 evidence of Foundation Mathematics, General Mathematics or Mathematical Methods; its destination is Methods’ coordinate geometry and, later, the normal to a curve in Methods Units 3 & 4 Calculus (*inferred*) — real, but stated as inference. The load-bearing point is the TOC §3 note: the product of perpendicular gradients being -1 is stated **and used both ways**, since the converse is what problems actually need — §3 states the fact and its converse side by side, and Q15, Q16, Q19 and the shape tests of §5 lean on the converse.

Level check (D6): no $f(x)$ notation, and the words *function* and *relation* are not used technically; no vectors, and no three-dimensional distance formula — Q19 uses only the Year 9 distance formula from VC2M9A04. Lines through a point are found by substituting into $y = mx + c$ throughout — the point-gradient form is not introduced, since VC2M10A10 does not name it.

Dated data (R3): none — Q23’s tennis-court dimensions are rule-defined constants, not time-sensitive figures, so there is nothing to list in the chapter refresh register for this subtopic.

Algebraic and graphical forms of quadratic, reciprocal, circle and exponential relations

Mathematics 10 · Chapter 3 Graphs, models and algorithms · Subtopic 3B (Level 10) · Extended block

Strand	Algebra
Content description	VC2M10A11
Elaboration ids carried	VC2M10A11.E01 VC2M10A11.E02 VC2M10A11.E03 VC2M10A11.E04
Weight	Extended (single allocation)

Content description (verbatim): *explore the connection between algebraic and graphical representations of relations such as simple quadratic, reciprocal, circle and exponential, using digital tools as appropriate*

Achievement standard (extract): *They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems.*

You need first: factorisation and completing the square from 1D and 2D (they produce the three forms of a quadratic used here), Pythagoras' theorem for the circle rule, and the graph of $y = a^x$ from 2E. This subtopic takes those algebraic forms and reads their graphs; it does not re-teach the algebra.

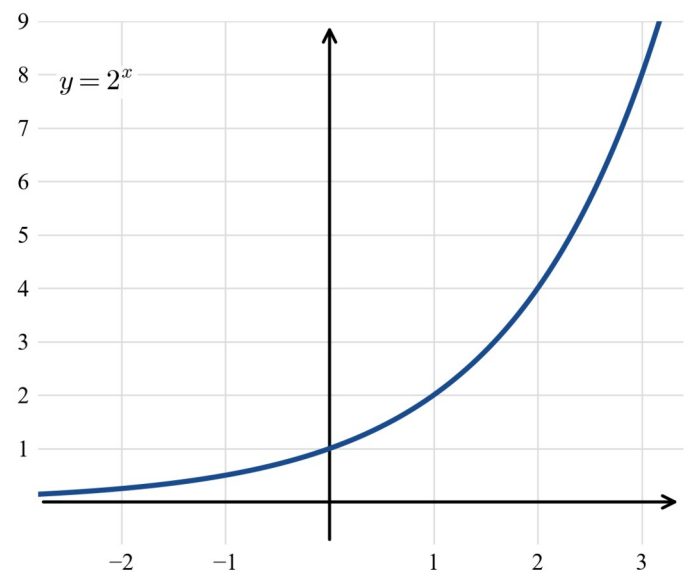
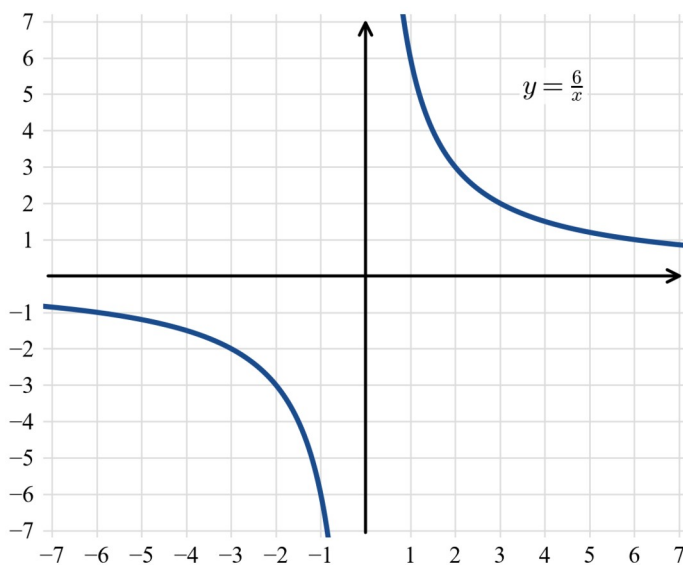
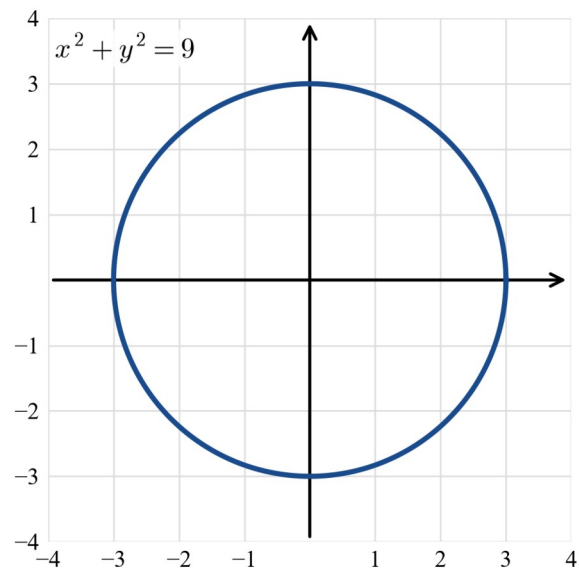
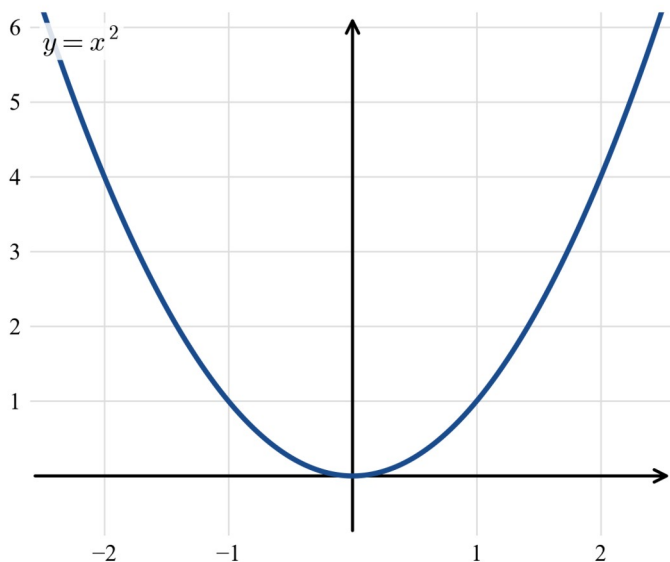
Theory

Every relation in this subtopic has two faces: an equation you can write down, and a picture you can draw. The point of the subtopic is that each face tells you the other one. Look at the equation and you should see the shape; look at the shape and you should be able to write the equation. That two-way reading is what *sketching* means in this book: drawing the right shape with its important features — intercepts, turning points, centres and radii, guide lines — marked, without plotting every single point.

1. Relations: one rule, two views

Relation. The Victorian Curriculum 2.0 uses the word *relation* but does not define it. In this book: a **relation** is any rule connecting numbers x and y — in practice, any equation in x and y — and its **graph** is the set of all points (x, y) that make the equation true.

This subtopic is a gallery of four families of relations. One member of each family is drawn below, and each family has a signature shape.



Gallery of the four families: the parabola $y = x^2$, the circle $x^2 + y^2 = 9$, the two branches of $y = \frac{6}{x}$ over x , and the growth curve $y = 2^x$.

- **Quadratic** — $y = x^2$. The shape is a **parabola**, a symmetric U (or an upside-down U).
- **Circle** — $x^2 + y^2 = 9$. The shape is a circle; here centred at the origin with radius 3.
- **Reciprocal** — $y = \frac{6}{x}$. The graph breaks into two separate **branches**, one in the top-right **quadrant** and one in the bottom-left. (The axes cut the plane into four quadrants, numbered I to IV starting top-right and moving anticlockwise.)
- **Exponential** — $y = 2^x$. The growth curve from 2E: it passes through $(0, 1)$ and doubles every step to the right.

The same two faces work for every member of a family: change the numbers in the equation and the shape moves, stretches or flips, but it is still recognisably the same shape. Sections 2–6 give the rules for exactly how it moves.

2. The quadratic family: three forms

A quadratic can be written in three forms, and each form hands you one feature of the parabola for free. (Moving between the forms is the algebra of 1D and 2D; here we only *read* them.)

- **General form** $y = ax^2 + bx + c$: the y -intercept is c , because at $x = 0$ the first two terms vanish.
- **Factorised form** $y = a(x - p)(x - q)$: the x -intercepts are p and q , because a product is zero only when one of its factors is zero.

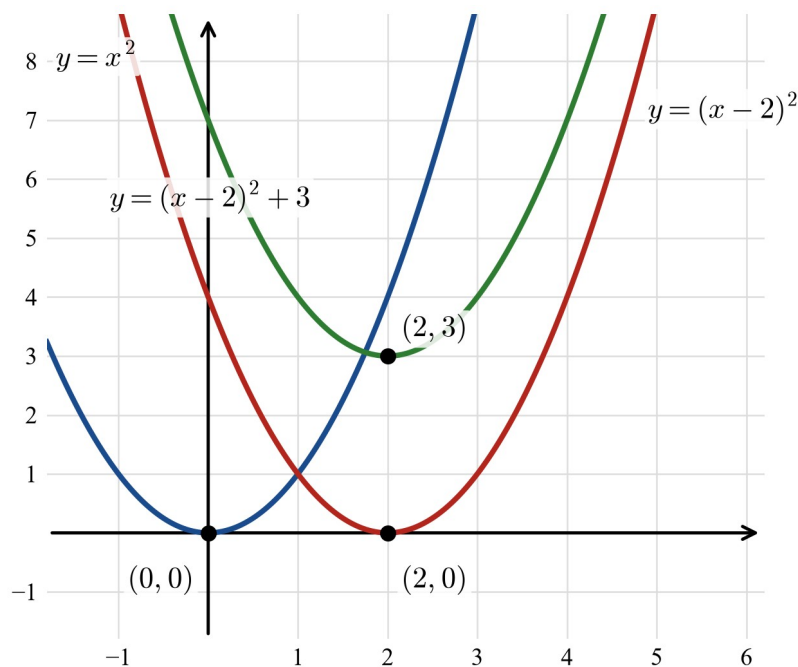
- **Turning-point form** $y = a(x - h)^2 + k$: the turning point is (h, k) .

In every form, a says which way the parabola opens and how steep it is: $a > 0$ opens up, $a < 0$ opens down.

The x -intercepts are the points where the graph cuts the x -axis (where $y = 0$); the y -intercept is where it cuts the y -axis (where $x = 0$). The **axis of symmetry** is the vertical mirror line through the turning point — it sits halfway between the x -intercepts when they exist — and the turning point lies on it. So one feature leads to the next: roots give the axis, the axis gives the turning point's x -value, and substituting gives its y -value.

3. Transforming the parabola $y = x^2$

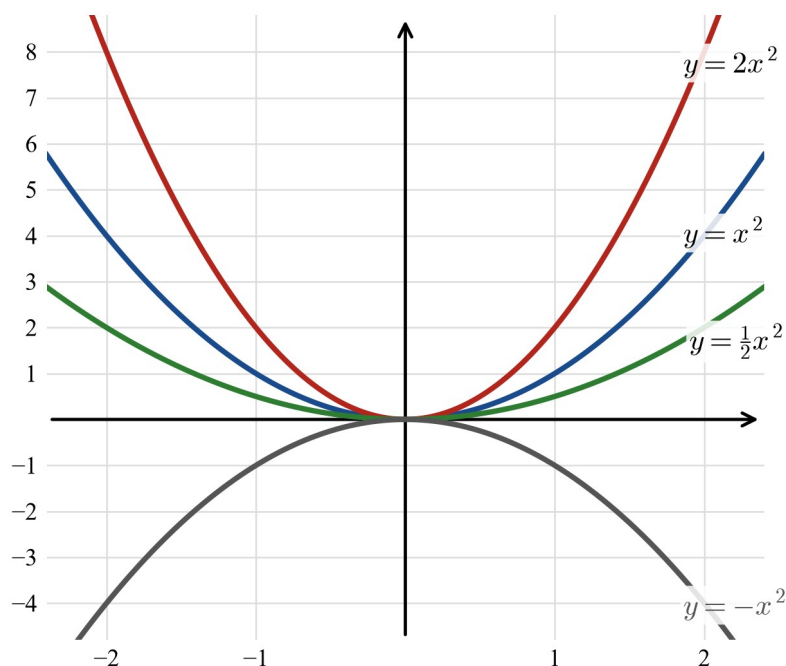
Every parabola in the quadratic family is $y = x^2$ that has been moved, stretched or flipped. Start with the three **translations** (slides).



The parabola $y = x^2$ moved 2 to the right, then 3 up: turning points at $(0, 0)$, $(2, 0)$ and $(2, 3)$.

Replacing x with $x - 2$ slides the graph 2 to the right: the turning point moves from $(0, 0)$ to $(2, 0)$. Then adding 3 slides it 3 up: the turning point lands on $(2, 3)$. The shape never changes — only the position. Adding a negative number slides left or down by its size.

Now the **stretches** and the **reflection**.



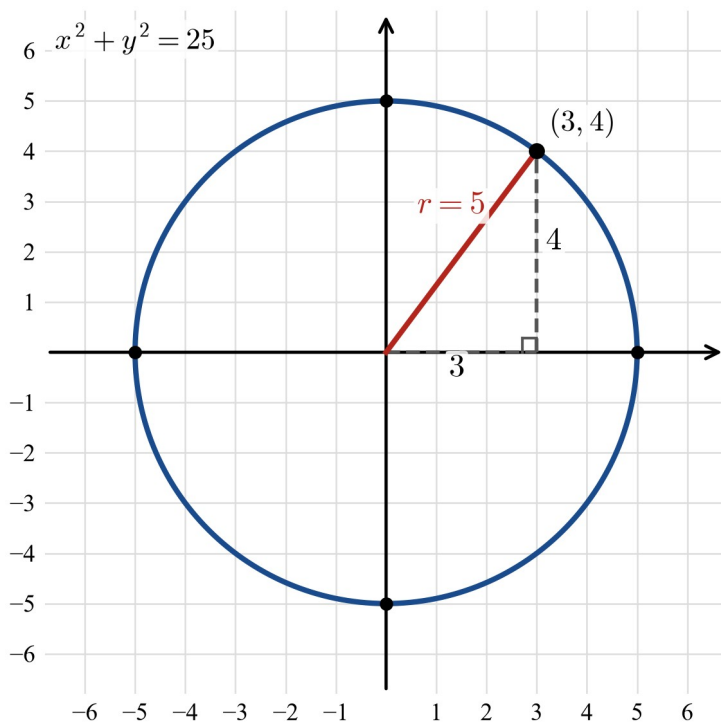
The parabolas $y = x^2$, $y = 2x^2$, $y = \text{half } x^2$ and $y = -x^2$ on one set of axes, showing a narrower, a wider and an upside-down shape.

Multiplying the right-hand side by 2 gives $y = 2x^2$: every height doubles and the parabola is **narrower**. Multiplying by $\frac{1}{2}$ gives $y = \frac{1}{2}x^2$: every height halves and the parabola is **wider**. Multiplying by -1 gives $y = -x^2$: every height changes sign and the parabola is **reflected in the x-axis**, opening down.

Putting the moves together, $y = a(x - h)^2 + k$ is exactly the record of what was done to $y = x^2$: slide h across, stretch by a factor of a (flip as well when a is negative), then slide k up. That is why the turning point ends up at (h, k) — the translation story and the turning-point form are the same fact in two languages.

4. The circle family

A circle is the set of all points a fixed distance r (the **radius**) from a fixed point (the **centre**). Its equation is Pythagoras' theorem in disguise. Take a circle centred at the origin and any point (x, y) on it: the horizontal leg of the right-angled triangle is x , the vertical leg is y , and the hypotenuse is r .



The circle $x^2 + y^2 = 25$ with the right-angled triangle from $(0, 0)$ to $(3, 4)$: sides 3 and 4, hypotenuse $r = 5$.

Pythagoras gives $x^2 + y^2 = r^2$. On the circle shown, $(3, 4)$ works because $3^2 + 4^2 = 9 + 16 = 25 = 5^2$, and so do the axis points $(5, 0)$, $(0, 5)$, $(-5, 0)$ and $(0, -5)$.

Moving the centre to (h, k) just shifts the two legs to $x - h$ and $y - k$:

$$(x - h)^2 + (y - k)^2 = r^2.$$

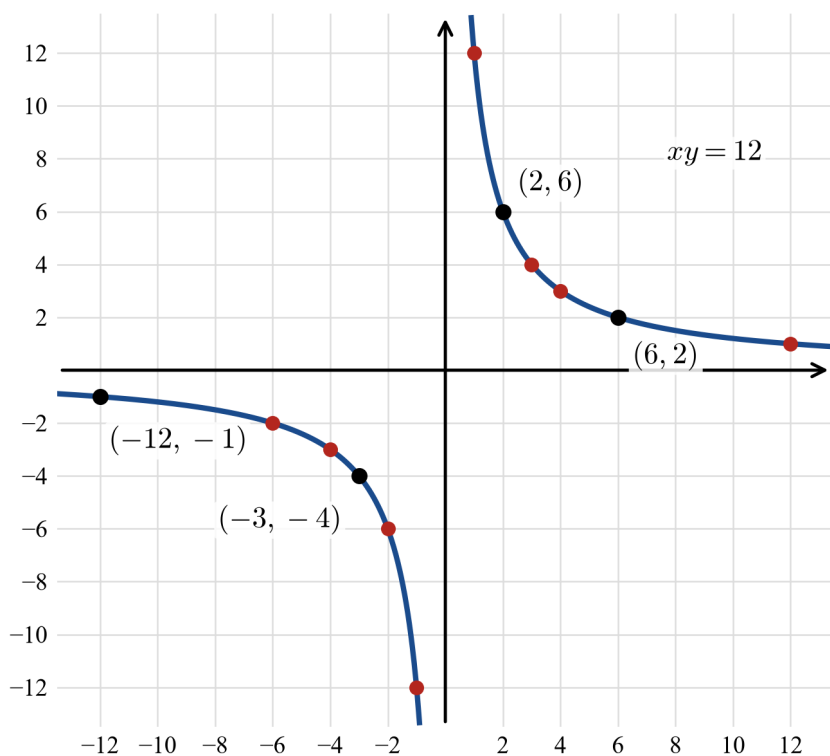
Reading the centre means flipping the signs inside the brackets: $(x - 3)^2$ goes with $h = 3$, and $(y + 2)^2$ goes with $k = -2$. Notice that a circle is not given by a “ $y = \dots$ ” rule: for most x -values there are two y -values (one on the top arc, one on the bottom). That is exactly why the curriculum speaks of *relations* here rather than demanding a single output for each input.

5. The reciprocal family: $x y = k$

The reciprocal family is the set of relations where **the product of x and y is a fixed constant k** :

$$x y = k, \text{ equivalently } y = \frac{k}{x}.$$

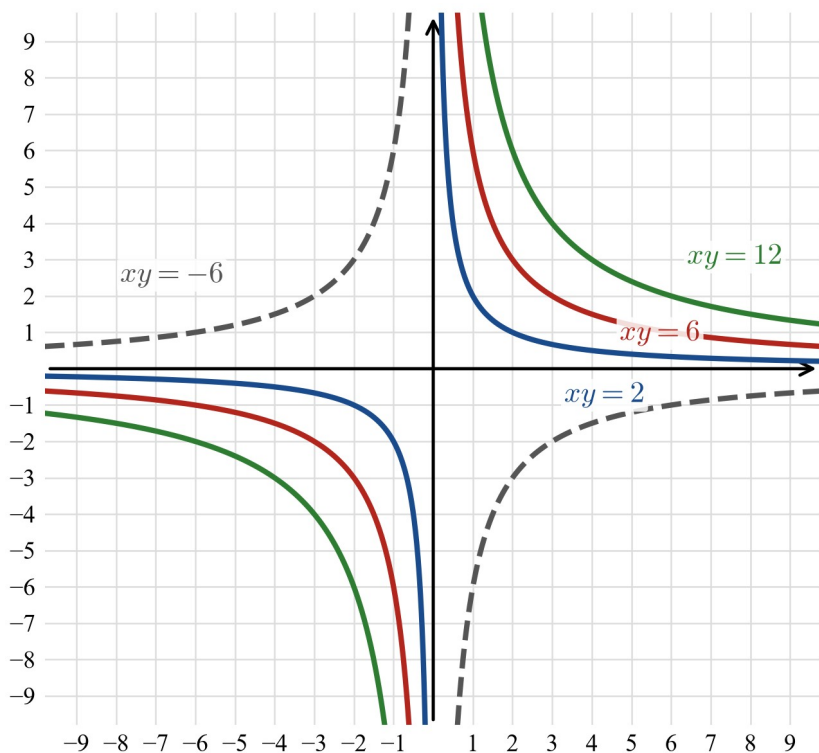
Take $k = 12$. Every pair of numbers that multiplies to 12 is a point on the graph — the factor pairs of 12 sit exactly on the curve.



The hyperbola $xy = 12$ with its factor-pair points marked, including $(2, 6)$, $(6, 2)$, $(-3, -4)$ and $(-12, -1)$.

Two things are worth noticing. First, there is **no point with** $x=0$: $0 \times y=0$, never 12, so the graph never touches the y -axis (and by the same argument never touches the x -axis). The branches get closer and closer to the axes but never reach them. Second, because a negative times a negative is positive, the factor pairs with both entries negative — $(-3, -4)$, $(-12, -1)$ — also multiply to 12: that is the bottom-left branch. This two-branched curve is called a **hyperbola**, and because its two never-touch lines are the two axes at right angles, the name for this particular shape is a **rectangular hyperbola**.

Now let k vary and the whole family appears.

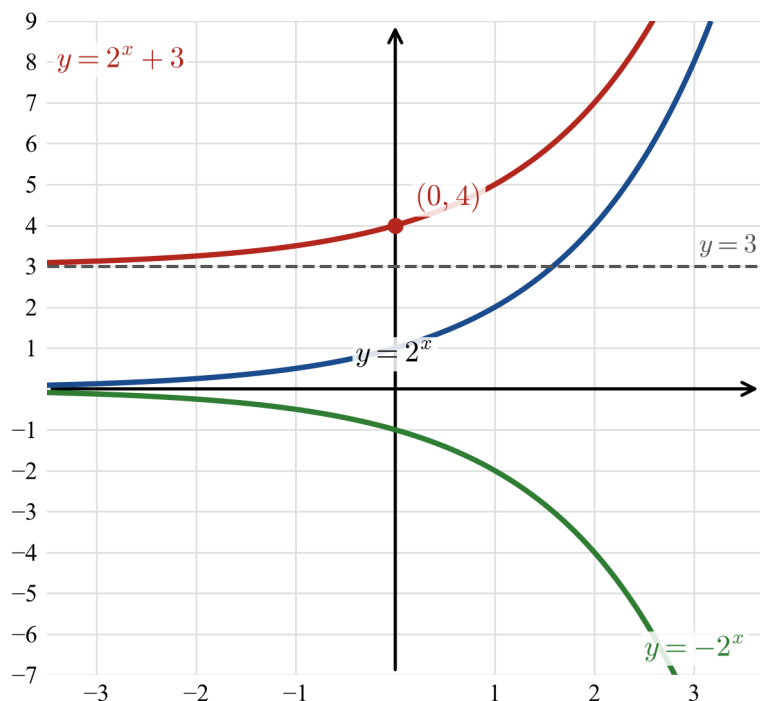


The family $xy = k$: the hyperbolas $xy = 2$, $xy = 6$ and $xy = 12$ in the first and third quadrants, and $xy = -6$ (dashed) in the second and fourth.

The bigger k is, the further the branches sit from the origin — $xy = 12$ passes through $(6, 2)$ while $xy = 2$ can only manage $(2, 1)$. And when k is **negative** the product must come from one positive and one negative coordinate, so the branches move to the top-left and bottom-right quadrants instead.

6. The exponential family

From 2E, $y = 2^x$ passes through $(0, 1)$, doubles every step to the right, and flattens towards the x -axis on the left without ever reaching it. The same transformations as for the parabola apply.



The curve $y = 2^x$, the same curve moved up 3 to $y = 2^x + 3$ with its guide line $y = 3$, and the reflection $y = -2^x$.

Adding 3 slides the whole curve up: the y -intercept rises from $(0, 1)$ to $(0, 4)$, and the line the curve flattens towards rises from $y = 0$ to $y = 3$ — we call that dashed line a **guide line**, and it is the first thing to draw. Reflecting in the x -axis gives $y = -2^x$: the y -intercept becomes $(0, -1)$ and the curve falls instead of grows, flattening towards the x -axis from below.

Stretches work too, though for the exponential a stretch can be rewritten as a change of base: $2 \times 2^x = 2^{x+1}$, so doubling an exponential is the same as sliding it one step to the left. The family to recognise at this level is $y = a^x$ moved up or down and reflected: $y = a^x + c$ and $y = -a^x$.

7. Reading the connection both ways

The whole subtopic in one table:

Family	Rule you see	Shape you draw	Features to mark
Quadratic	$y = a(x - p)(x - q)$, $y = a(x - h)^2 + k$ or $y = ax^2 + bx + c$	parabola	x - and y -intercepts, turning point, direction of opening
Circle	$(x - h)^2 + (y - k)^2 = r^2$	circle	centre (h, k) , radius r , axis crossings
Reciprocal	$xy = k$ (that is $y = \frac{k}{x}$)	rectangular hyperbola	sign of k (which quadrants), a few factor-pair points
Exponential	$y = a^x + c$ or $y = -a^x$	growth or decay curve	y -intercept, guide line, direction

Algebra to graph: name the family, read the features off the form, draw the shape through them. Graph to algebra: name the family from the shape, read the features off the grid (intercepts, centre and radius, factor pairs, intercept and guide line), and rebuild the rule. Worked example 6 is exactly that second reading.

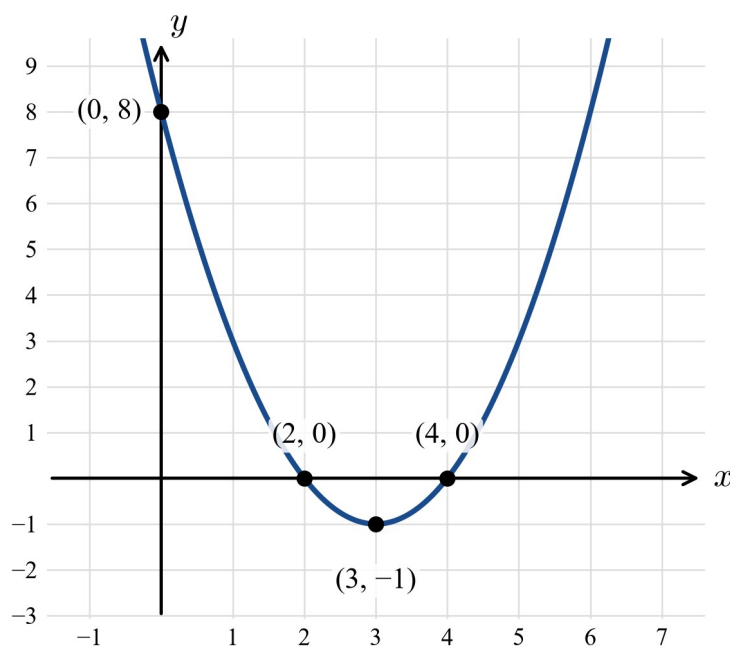
Worked examples

Example 1 — Sketching a parabola from general form

Sketch $y = x^2 - 6x + 8$, marking the intercepts and the turning point.

Working	Comment
$x^2 - 6x + 8 = (x - 2)(x - 4)$	factorise: two numbers with product 8 and sum -6 are -2 and -4
$x = 2, x = 4$	x -intercepts: a factor is zero at each
$y = 8$ at $x = 0$	y -intercept is the constant term
axis: $x = \frac{2+4}{2} = 3$	axis of symmetry sits halfway between the roots
$y = 3^2 - 6 \times 3 + 8 = 9 - 18 + 8 = -1$	substitute the axis into the rule
$\therefore$ turning point $(3, -1)$, opens up since $a = 1 > 0$	all features found

Draw the smooth U through the three marked points:



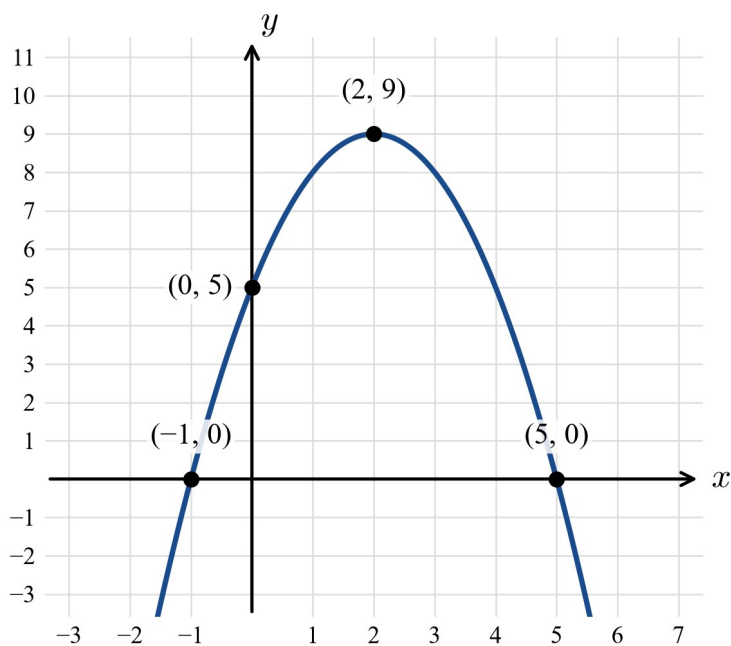
The parabola $y = x^2 - 6x + 8$, opening up, cutting the x -axis at 2 and 4, the y -axis at 8, with turning point $(3, -1)$.

Example 2 — Sketching from turning-point form, with the transformation story

Sketch $y = -(x - 2)^2 + 9$.

Working	Comment
turning point $(2, 9)$; $a = -1 < 0 \therefore$ opens down	read straight off the form
$-(x - 2)^2 + 9 = 0 \rightarrow (x - 2)^2 = 9$	x -intercepts: set $y = 0$
$x - 2 = 3$ or $x - 2 = -3$	the square roots of 9 are 3 and -3
$x = 5$ or $x = -1$	the two x -intercepts
$y = -(0 - 2)^2 + 9 = -4 + 9 = 5$	y -intercept: set $x = 0$

The transformation story: start with $y = x^2$, reflect in the x -axis (the minus sign), slide 2 right and 9 up. The turning point's journey is $(0, 0) \rightarrow (2, 0) \rightarrow (2, 9)$.



The upside-down parabola $y = -(x - 2)^2 + 9$ with turning point $(2, 9)$, x -intercepts -1 and 5 , and y -intercept 5 .

Example 3 — Sketching circles

Sketch (a) $x^2 + y^2 = 25$ and (b) $(x - 3)^2 + (y + 2)^2 = 16$.

Working

(a) centre $(0, 0)$; $r^2 = 25 \therefore r = 5$

axis crossings $(5, 0), (-5, 0), (0, 5), (0, -5)$

check $(3, 4)$: $9 + 16 = 25 \checkmark$

(b) centre $(3, -2)$ — signs flipped; $r^2 = 16 \therefore r = 4$

from the centre: $(7, -2), (-1, -2), (3, 2), (3, -6)$

Comment

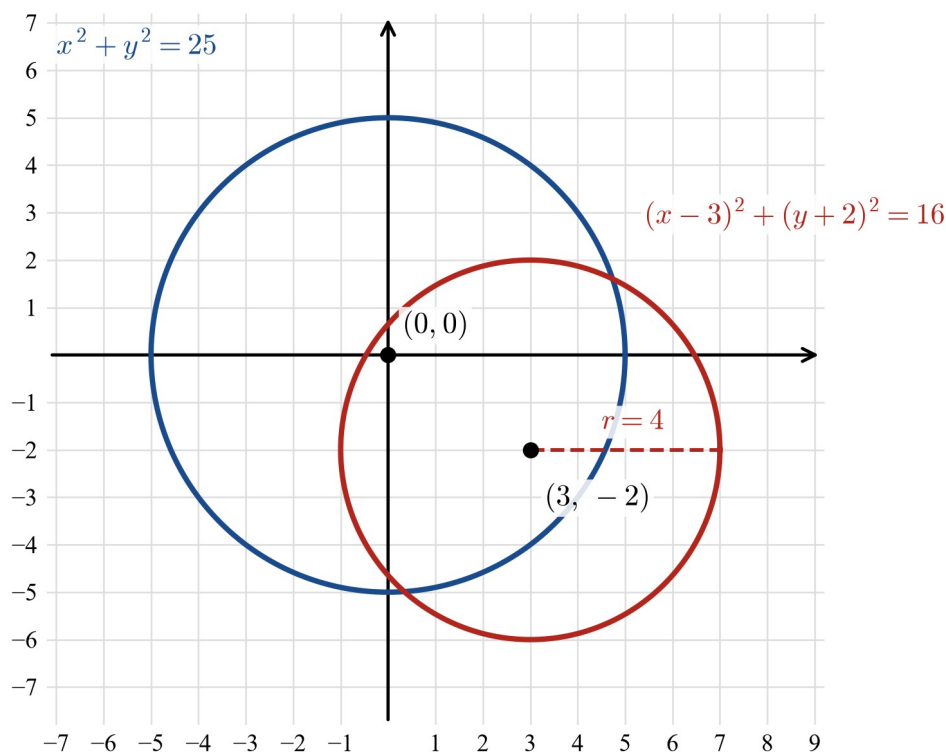
read off centre and radius

centre $\pm$ radius along each axis

a Pythagorean triple sits on the circle

$(y + 2)^2$ means $k = -2$

centre $\pm$ radius across and up/down



Two circles: $x^2 + y^2 = 25$ centred at $(0, 0)$ with radius 5, and $(x - 3)^2 + (y + 2)^2 = 16$ centred at $(3, -2)$ with radius 4.

Example 4 — A reciprocal relation through a given point

A rectangular hyperbola $xy = k$ passes through $(3, 8)$. Find k and describe the graph.

Working

$$k = 3 \times 8 = 24$$

$(1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1)$ factor pairs of 24 all lie on the curve

$(-1, -24), (-2, -12), (-3, -8), \dots$

no point with $x = 0$, since $0 \times y = 0 \neq 24$

Comment

substitute the point into $xy = k$

factor pairs of 24 all lie on the curve

negative $\times$ negative gives $+24$: the second branch

the branches never touch the axes

$\therefore$ the graph is the hyperbola $xy = 24$: two branches, in the top-right and bottom-left quadrants because $k > 0$, passing through every factor pair of 24 — including the given point $(3, 8)$.

Example 5 — Transforming an exponential graph

Sketch $y = 2^x - 4$, and on the same axes the reflection $y = -2^x$.

Working

guide line: $y = -4$ (the x -axis slid down 4)

$$x = 0: y = 1 - 4 = -3$$

$$x = 2: y = 4 - 4 = 0$$

$$x = 3: y = 8 - 4 = 4$$

$$y = -2^x: (0, -1), (1, -2), (2, -4)$$

Comment

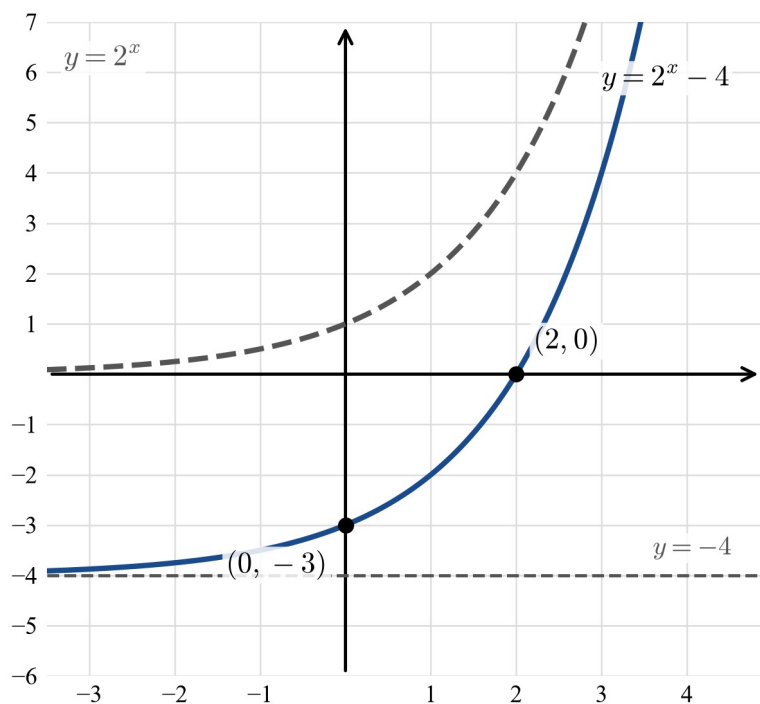
draw this dashed, first

y -intercept $(0, -3)$

x -intercept $(2, 0)$

one more point to fix the steep part

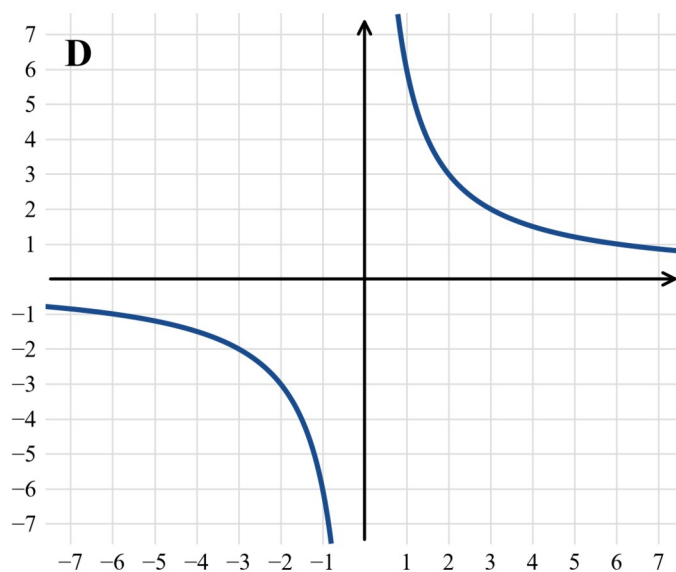
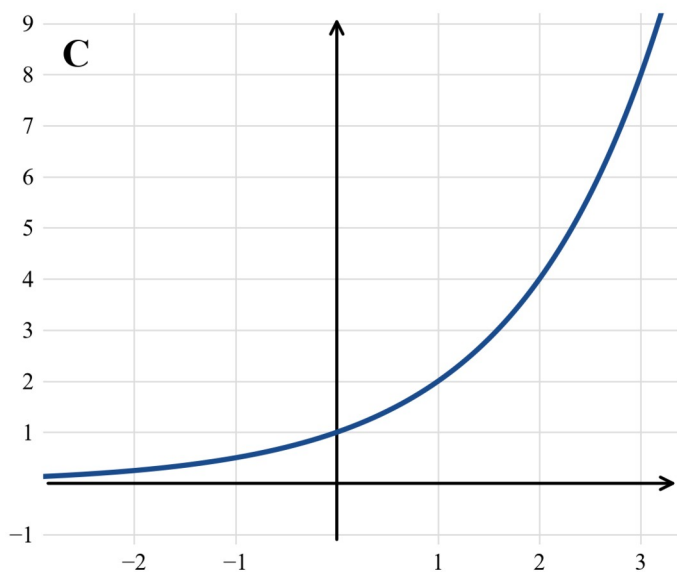
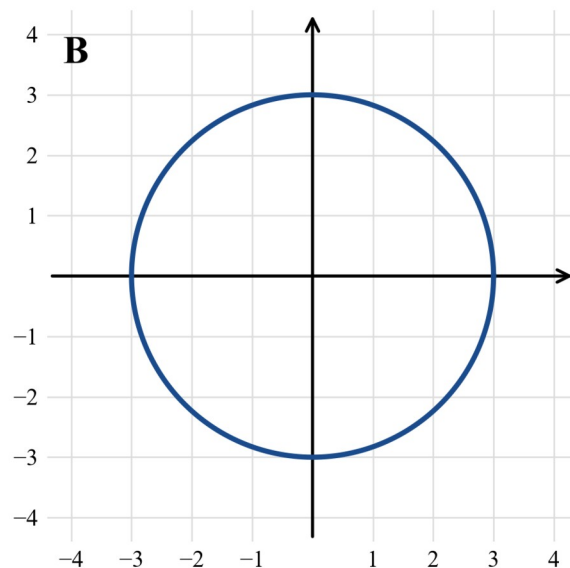
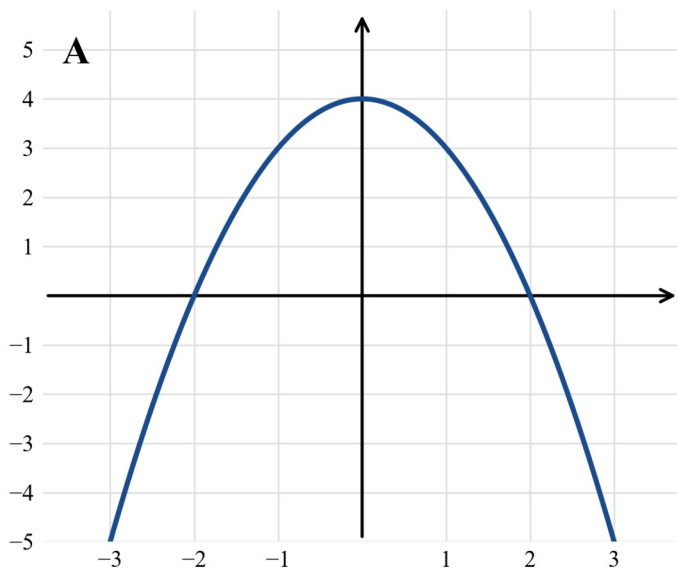
the reflection falls, flattening towards the x -axis



The curve $y = 2^x - 4$ with guide line $y = -4$, y -intercept $(0, -3)$ and x -intercept $(2, 0)$, beside the dashed $y = 2^x$.

Example 6 — From graph to algebra

Each graph below matches one of these rules: $y = 2^x$, $y = \frac{6}{x}$, $y = -x^2 + 4$, $x^2 + y^2 = 9$. One rule matches no graph:
 $y = 2x + 1$.



Four unmarked graphs labelled A to D: an upside-down parabola, a circle, a growth curve, and a two-branched hyperbola.

Panel	Reading	Rule
A	upside-down parabola, turning point $(0, 4)$, x -intercepts -2 and 2	$y = -x^2 + 4$
B	circle centred at the origin, radius 3	$x^2 + y^2 = 9$
C	growth curve through $(0, 1)$, doubling: $(1, 2)$, $(2, 4)$, $(3, 8)$	$y = 2^x$
D	two branches; check $(2, 3)$: $2 \times 3 = 6$	$y = \frac{6}{x}$

The leftover rule $y = 2x + 1$ is a straight line — no panel shows a line, so it matches nothing.

Practice questions

Scaffolded

Q1. $y = (x - 2)(x - 6)$. Write down (a) the x -intercepts, (b) the y -intercept, (c) the axis of symmetry, (d) the turning point, then (e) sketch the parabola.

Q2. $y = (x - 3)^2 + 1$. Write down (a) the turning point, (b) which way the parabola opens, (c) the y -intercept, then (d) explain in one sentence why this parabola has no x -intercepts, and (e) sketch it.

Q3. $x^2 + y^2 = 49$. (a) Write down the centre and the radius. (b) Show that $(6, 4)$ is **not** on the circle. (c) Write down the four points where the circle crosses the axes.

Q4. $(x - 1)^2 + (y - 4)^2 = 9$. Write down the centre and the radius, then sketch the circle, marking the four points reached from the centre by moving 3 left, right, up and down.

Q5. Complete the table for $y = \frac{10}{x}$, then plot the points and join them with two smooth branches.

x	-5	-2	1	2	5	10
y						

Q6. A rectangular hyperbola $xy = k$ passes through $(4, 3)$. (a) Find k . (b) Is $(-2, -6)$ on the graph? Show the check.

Q7. (a) Complete the table for $y = 3^x + 2$ and sketch the curve. (b) Write down the y -intercept. (c) Which horizontal line does the curve flatten towards on the left?

x	0	1	2
y			

Q8. (a) Complete the table for $y = -5^x$ and sketch the curve. (b) Describe in one sentence how the graph sits compared with $y = 5^x$.

x	0	1	2
y			

Unscaffolded

Q9. Sketch $y = x^2 + 2x - 8$, marking the intercepts and the turning point.

Q10. Sketch $y = -x^2 + 2x + 8$, marking the intercepts and the turning point.

Q11. A parabola has turning point $(-3, -2)$ and passes through $(0, 16)$. Find its rule in turning-point form.

Q12. Write down the turning point of each parabola: (a) $y = (x + 4)^2 - 1$; (b) $y = -3x^2$; (c) $y = (x - 5)^2$; (d) $y = x^2 + 6$.

Q13. A circle centred at the origin passes through $(6, 8)$. (a) Find its equation. (b) Is $(-10, 0)$ on the circle?

Q14. $(x + 5)^2 + (y - 2)^2 = 36$. (a) Write down the centre and the radius. (b) Is the point $(-2, 6)$ inside, on, or outside the circle? Show the working that decides it.

Q15. Sketch $y = -\frac{8}{x}$ using a table of at least four points on each branch. Which quadrants do the branches lie in, and why?

Q16. On the same axes, sketch $xy = 3$ and $xy = 12$ using three factor-pair points each. Which hyperbola sits further from the origin, and which single point on each shows it?

Q17. A rectangular hyperbola $xy = k$ passes through $(-3, 5)$. (a) Find k . (b) Find y when $x = 5$, and when $x = -15$.

Q18. Sketch $y = 4 - 2^x$, marking the y -intercept, the x -intercept and the guide line.

Q19. Explain why $y = \left(\frac{1}{3}\right)^x$ is the same rule as $y = 3^{-x}$, and hence sketch it, marking the y -intercept and the guide line.

Q20. A parabola cuts the x -axis at -1 and 5 and the y -axis at -10 . (a) Find its rule in factorised form. (b) Find the turning point.

Q21. Without drawing, match each rule to its description: (i) $x^2 + y^2 = 36$; (ii) $y = \frac{5}{x}$; (iii) $y = -x^2$; (iv) $y = 2^x + 5$; (v) $xy = -2$.

- (a) a hyperbola with branches in the top-left and bottom-right quadrants
- (b) a parabola with turning point $(0, 0)$ opening down
- (c) a circle centred at the origin with radius 6
- (d) a growth curve flattening towards the line $y = 5$
- (e) a hyperbola with branches in the top-right and bottom-left quadrants

Q22. Find the point where the graphs of $y = x^2$ and $y = \frac{8}{x}$ meet.

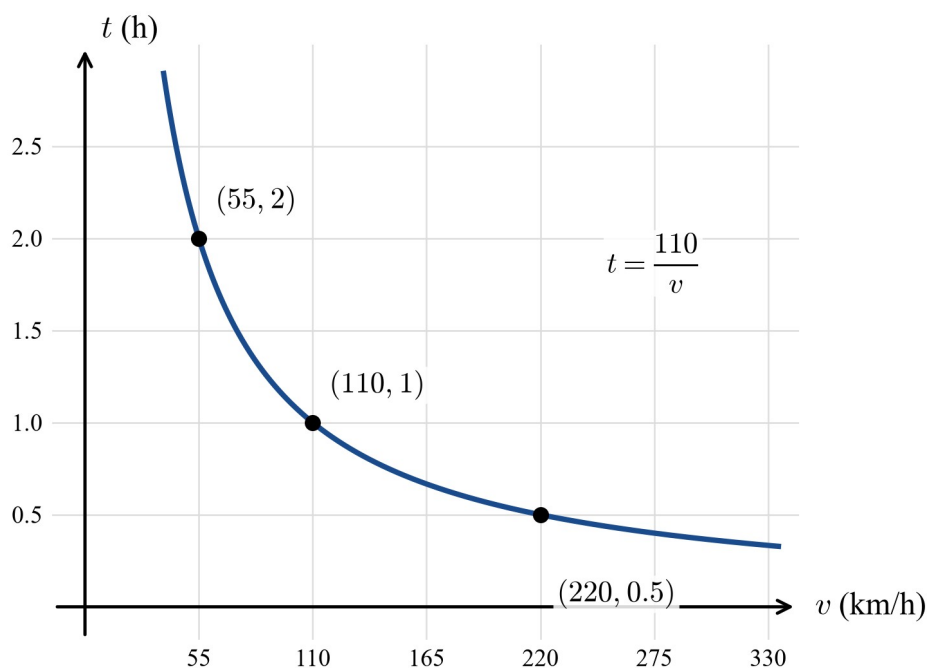
Real context

Q23. A small aircraft flies the straight-line distance of 110 km from Melbourne to Ballarat at an average speed of v km/h. The flight time is t hours, where $t = \frac{110}{v}$.

(a) Complete the table.

v (km/h)	55	110	220
t (h)			

- (b) Sketch t against v and name the family the graph belongs to.
 (c) What average speed makes the flight take 40 minutes?



The curve $t = 110$ over v for the flight, falling from $(55, 2)$ through $(110, 1)$ to $(220, 0.5)$.

Source: Wikipedia, *Ballarat* (infobox: 110 km NW of Melbourne), 2026. Retrieved 20 August 2026.

Digital tool task

DIGITAL TOOL — this task assumes a graphing tool. Your teacher will nominate the tool.

T1. Enter the rules $xy=1$, $xy=4$, $xy=9$ and $xy=-4$ into the tool (as $y=1/x$, $y=4/x$, $y=9/x$, $y=-4/x$) and display all four on the one screen.

- Sketch what you see, labelling each branch with its rule.
- For $xy=4$ and $xy=9$, use the tool to find the point on each branch in the first quadrant where the two coordinates are equal. Check each by multiplying the coordinates.
- Explain why $xy=-4$ has no such point.
- Describe what happens to the branches as k grows from 1 to 4 to 9.

Then check the tool by hand: without it, build a four-point table for $xy=4$ and for $xy=9$ and confirm the tool's curves pass through your points. The hand table is what an exam expects; the tool is what makes the family visible.

Answers

1. (a) $x=2$, $x=6$. (b) 12. (c) $x=4$. (d) $(4, -4)$. (e) U opening up through those points. 2. (a) $(3, 1)$. (b) up. (c) 10. (d) The turning point $(3, 1)$ is above the x -axis and the parabola opens up, so it never reaches the axis. (e) U with turning point $(3, 1)$, y -intercept 10. 3. (a) centre $(0, 0)$, $r=7$. (b) $6^2 + 4^2 = 36 + 16 = 52 \neq 49$. (c) $(7, 0)$, $(-7, 0)$, $(0, 7)$, $(0, -7)$. 4. centre $(1, 4)$, $r=3$; extra points $(-2, 4)$, $(4, 4)$, $(1, 7)$, $(1, 1)$. 5. $y = -2, -5, 10, 5, 2, 1$ (in the table's order). 6. (a) $k=12$. (b) $(-2) \times (-6) = 12$ ✓ yes. 7. (a) $y=3, 5, 11$. (b) $(0, 3)$. (c) $y=2$. 8. (a) $y = -1, -5, -25$. (b) It is $y=5^x$ reflected in the x -axis: it falls instead of grows. 9. x -intercepts -4 and 2 ; y -intercept -8 ; turning point $(-1, -9)$; opens up. 10. x -intercepts -2 and 4 ; y -intercept 8 ; turning point $(1, 9)$; opens down. 11. $y=2(x+3)^2-2$. 12. (a) $(-4, -1)$. (b) $(0, 0)$. (c) $(5, 0)$. (d) $(0, 6)$. 13. (a) $x^2 + y^2 = 100$. (b) yes: $(-10)^2 + 0^2 = 100$. 14. (a) centre $(-5, 2)$, $r=6$. (b) inside: $(-2+5)^2 + (6-2)^2 = 9 + 16 = 25 < 36$. 15. e.g. $(1, -8)$, $(2, -4)$, $(4, -2)$, $(8, -1)$ and $(-1, 8)$, $(-2, 4)$, $(-4, 2)$, $(-8, 1)$; branches in the top-left and bottom-right quadrants because $k = -8 < 0$, so x and y must have opposite signs. 16. $xy=12$ is further out; compare $(2, 6)$ on $xy=12$ with $(1, 3)$ on $xy=3$. 17. (a) $k = -15$. (b) $y = -3$; $y = 1$. 18. y -intercept $(0, 3)$; x -intercept $(2, 0)$; guide line $y = 4$; the curve falls. 19. $\left(\frac{1}{3}\right)^x = \frac{1}{3^x} = 3^{-x}$; y -intercept $(0, 1)$, guide line $y = 0$, decay curve. 20. (a) $y = 2(x+1)(x-5)$. (b) $(2, -18)$. 21. (i)–(c); (ii)–(e); (iii)–(b); (iv)–(d); (v)–(a). 22. $(2, 4)$. 23. (a) 2, 1, 0.5. (b) the reciprocal family (a hyperbola branch). (c) 165 km/h. T1. (b) $(2, 2)$ since $2 \times 2 = 4$; $(3, 3)$ since $3 \times 3 = 9$. (c) equal coordinates need $x \times x = -4$, and no number squared is negative. (d) the branches move further from the origin. Hand tables: $xy=4$: $(1, 4)$, $(2, 2)$, $(4, 1)$, $(-2, -2)$; $xy=9$: $(1, 9)$, $(3, 3)$, $(9, 1)$, $(-3, -3)$.

Fully-worked solutions

Q1. (a) A factor is zero at $x=2$ and at $x=6$. (b) $y = (0-2)(0-6) = (-2)(-6) = 12$. (c) Halfway between the roots: $x = \frac{2+6}{2} = 4$. (d) $y = (4-2)(4-6) = 2 \times (-2) = -4 \therefore$ turning point $(4, -4)$. (e) U opening up through $(2, 0)$, $(6, 0)$, $(0, 12)$, $(4, -4)$.

Q2. (a) $(3, 1)$ from the form. (b) $a=1 > 0 \therefore$ up. (c) $y = (0-3)^2 + 1 = 9 + 1 = 10$. (d) The lowest point is $(3, 1)$, above the x -axis, and the parabola only goes up from there, so y is never 0. (e) A U sitting entirely above the x -axis, touching $y=1$ at $x=3$.

Q3. (a) Centre $(0, 0)$; $r^2 = 49 \therefore r = 7$. (b) $6^2 + 4^2 = 36 + 16 = 52$, and $52 \neq 49$, so $(6, 4)$ is not on the circle (it lies outside it). (c) $(7, 0)$, $(-7, 0)$, $(0, 7)$, $(0, -7)$.

Q4. $(x-1)^2$ gives $h=1$; $(y-4)^2$ gives $k=4$; $r^2=9 \therefore r=3$. From the centre: left $(-2, 4)$, right $(4, 4)$, up $(1, 7)$, down $(1, 1)$; draw the circle through them.

Q5. $y = \frac{10}{-5} = -2$; $\frac{10}{-2} = -5$; $\frac{10}{1} = 10$; $\frac{10}{2} = 5$; $\frac{10}{5} = 2$; $\frac{10}{10} = 1$. The positive pairs give the top-right branch, the negative pairs the bottom-left branch.

Q6. (a) $k = 4 \times 3 = 12$. (b) $(-2) \times (-6) = 12 = k \therefore$ yes, $(-2, -6)$ is on the graph — a point of the bottom-left branch.

Q7. (a) $3^0 + 2 = 1 + 2 = 3$; $3^1 + 2 = 5$; $3^2 + 2 = 9 + 2 = 11$. (b) $(0, 3)$. (c) As x goes left, 3^x shrinks towards 0, so the curve flattens towards $y=2$.

Q8. (a) $-5^0 = -1$; $-5^1 = -5$; $-5^2 = -25$. (b) Every height of $y=5^x$ has its sign changed, so the graph is the reflection of $y=5^x$ in the x -axis.

Q9. Factorise: $x^2 + 2x - 8 = (x + 4)(x - 2) \therefore$ x -intercepts -4 and 2 . y -intercept -8 . Axis $x = \frac{-4+2}{2} = -1$;

$y = (-1)^2 + 2(-1) - 8 = 1 - 2 - 8 = -9 \therefore$ turning point $(-1, -9)$; opens up.

Q10. Factorise: $-x^2 + 2x + 8 = -(x^2 - 2x - 8) = -(x + 2)(x - 4) \therefore$ x -intercepts -2 and 4 . y -intercept 8 . Axis $x = \frac{-2+4}{2} = 1$; $y = -1 + 2 + 8 = 9 \therefore$ turning point $(1, 9)$; opens down since $a = -1$.

Q11. Turning-point form starts $y = a(x + 3)^2 - 2$. Substitute $(0, 16)$: $a(0 + 3)^2 - 2 = 16 \rightarrow 9a = 18 \rightarrow a = 2 \therefore$
 $y = 2(x + 3)^2 - 2$.

Q12. Read (h, k) with signs flipped: (a) $(-4, -1)$; (b) $(0, 0)$ since $-3x^2 = -3(x - 0)^2 + 0$; (c) $(5, 0)$; (d) $(0, 6)$ since $x^2 + 6 = (x - 0)^2 + 6$.

Q13. (a) $r^2 = 6^2 + 8^2 = 36 + 64 = 100 \therefore x^2 + y^2 = 100$. (b) $(-10)^2 + 0^2 = 100 \checkmark$ on the circle.

Q14. (a) Centre $(-5, 2)$; $r = 6$. (b) Put $(-2, 6)$ into the left side: $(-2 + 5)^2 + (6 - 2)^2 = 3^2 + 4^2 = 9 + 16 = 25$. Since $25 < 36$ the point is closer to the centre than $r \therefore$ **inside** the circle.

Q15. $k = -8 < 0 \therefore$ one coordinate positive and one negative: the top-left and bottom-right quadrants. Top-left: $(-1, 8), (-2, 4), (-4, 2), (-8, 1)$. Bottom-right: $(1, -8), (2, -4), (4, -2), (8, -1)$. Two smooth branches, never touching the axes.

Q16. $xy = 3$: $(1, 3), (3, 1)$ and $(-1, -3), (-3, -1)$. $xy = 12$: $(2, 6), (6, 2)$ and $(-2, -6), (-6, -2)$. $xy = 12$ sits further from the origin — seen at once by comparing $(2, 6)$ with $(1, 3)$: the bigger the constant product, the bigger the coordinates must be.

Q17. (a) $k = (-3) \times 5 = -15$. (b) $y = \frac{-15}{5} = -3$; $y = \frac{-15}{-15} = 1$.

Q18. $y = 4 - 2^x$ is $y = -2^x$ slid up 4: guide line $y = 4$. $x = 0$: $y = 4 - 1 = 3 \therefore (0, 3)$. $x = 2$: $y = 4 - 4 = 0 \therefore (2, 0)$. The curve falls from near the guide line on the left, through those points, steeply down on the right.

Q19. $\left(\frac{1}{3}\right)^x = \frac{1^x}{3^x} = \frac{1}{3^x} = 3^{-x}$, using the negative-exponent law from 2E. $\therefore$ it is a decay curve: y -intercept $(0, 1)$ (any base to the power 0 is 1), guide line $y = 0$, falling towards it on the right.

Q20. (a) Factorised form starts $y = a(x + 1)(x - 5)$. Substitute $(0, -10)$: $a(1)(-5) = -10 \rightarrow -5a = -10 \rightarrow a = 2 \therefore y = 2(x + 1)(x - 5)$. (b) Axis $x = \frac{-1+5}{2} = 2$; $y = 2(2 + 1)(2 - 5) = 2 \times 3 \times (-3) = -18 \therefore$ turning point $(2, -18)$.

Q21. (i) circle, $r^2 = 36 \therefore r = 6 \rightarrow$ (c). (ii) $k = 5 > 0 \rightarrow$ top-right and bottom-left branches $\rightarrow$ (e). (iii) $a = -1$, turning point $(0, 0) \rightarrow$ (b). (iv) growth curve slid up 5 $\rightarrow$ (d). (v) $k = -2 < 0 \rightarrow$ (a).

Q22. At a meeting point $x^2 = \frac{8}{x} \rightarrow$ multiply both sides by x : $x^3 = 8 \rightarrow x = 2$. Then $y = 2^2 = 4$ (check: $\frac{8}{2} = 4 \checkmark$). $\therefore$ the point is $(2, 4)$.

Q23. (a) $t = \frac{110}{55} = 2$; $\frac{110}{110} = 1$; $\frac{110}{220} = 0.5$. (b) The graph falls like one branch of a hyperbola: the product $v \times t = 110$ is fixed, so it is the reciprocal family. (c) 40 minutes $= \frac{40}{60} = \frac{2}{3}$ h. $v = \frac{110}{2/3} = 110 \times \frac{3}{2} = 165$ km/h.

T1. (b) Equal coordinates mean $x \times x = k$: for $k = 4$, $x = 2 \rightarrow (2, 2)$; for $k = 9$, $x = 3 \rightarrow (3, 3)$. (c) For $k = -4$ we would need $x \times x = -4$, and no number multiplied by itself is negative, so no such point exists. (d) As k grows, every

branch moves further from the origin. Hand check — $x y = 4$: $(1, 4), (2, 2), (4, 1), (-2, -2)$; $x y = 9$: $(1, 9), (3, 3), (9, 1), (-3, -3)$; the tool's curves pass through all of them.

Teacher note

- **Level 9 codes rehearsed (D3):** VC2M9A05 (graphing quadratics and the zero-product idea behind factorised form), VC2M9A07 (varying parameters and watching the graph — the entire transformation story of sections 3 and 6), VC2M9M03 (Pythagoras' theorem, the engine of section 4). All are applied here, not re-taught.
- **Tool classes (D13):** the digital task names only the tool class “graphing tool”; no product is nominated and no CAS is assumed. The task ends with the tool-free path (hand tables), which is what the paper-and-pencil items assess.
- **VCE destinations (§5):** this subtopic is the early home of the Methods gallery: *sketch a parabola given an algebraic form (general or vertex), sketch a circle given centre and radius, recognise the graph of $x y = k$ as a rectangular hyperbola, sketch transformed exponential graphs, and apply transformations (translation, reflection, vertical stretch) to the graph of $y = x^2$* . In VCE language the guide line becomes the asymptote and the turning point of $y = a(x - h)^2 + k$ becomes the vertex; neither term is introduced here (see level check).
- **Level check (D6):** no $f(x)$ notation, no domain or range as named objects, and the word “asymptote” is absent by design — “the line the curve flattens towards / never reaches” carries the idea at Level 10. “Function” appears only inside the verbatim achievement-standard extract.
- **Contested terms (Rule 1):** *relation* is defined at first use with its frame stated (the curriculum uses the word without defining it); *hyperbola, rectangular hyperbola, quadrant, intercept* and *guide line* are defined where they first appear.
- **First Nations content (Rule 1):** VC2M10A11 carries no Aboriginal and Torres Strait Islander elaborations, so nothing is excluded from this subtopic under the exclusion rule.
- **Dated data (R3):** Q23 rests on a fixed straight-line geographic distance (Melbourne–Ballarat, 110 km), not on a time-varying statistic.

Modelling growth, decay and inverse proportion — and building the compound interest formula

Mathematics 10 · Chapter 3 Graphs, models and algorithms · Subtopic 3C (Level 10) · Extended block

Strand	Algebra
Content description	VC2M10A15
Elaboration ids carried	VC2M10A15.E01 VC2M10A15.E02 VC2M10A15.E03 VC2M10A15.E04 (VC2M10A15.E05 and VC2M10A15.E06 not delivered — see Teacher note)
Weight	Extended

Content description (verbatim): *use mathematical modelling to solve applied problems involving inverse proportion, growth and decay, including in financial contexts to establish the compound interest formula as repeated applications of simple interest; formulate problems, choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings*

Achievement standard (extract): *Students use mathematical modelling to solve problems involving growth and decay in financial and other applied situations, applying linear, quadratic and exponential functions as appropriate, and solve related equations, numerically and graphically.*

You need first: direct proportion and rates from Year 9 (VC2M9M05), simple interest from Year 9 (VC2M9A06), the exponential equations of 2E, and the graphs of the reciprocal and exponential relations from 3B. This subtopic applies all four; it does not re-teach them.

Theory

Chapters 2 and 3 have built three families of rules — linear, quadratic and exponential — and the skills of solving and graphing them. This subtopic turns that around and asks the modelling question: *given a situation, which family does it belong to, and what does the answer mean?* Three situations carry the chapter: two quantities in **inverse proportion**, a quantity that **grows or decays by a constant percentage** each step, and — the pay-off of the whole chapter — money earning **compound interest**, built here as repeated simple interest.

1. Inverse proportion — when the product is constant

Recall from Year 9 (VC2M9M05) that v and t are in **direct proportion** when the *ratio* $\frac{v}{t}$ is constant. **Inverse proportion** is the mirror picture: two quantities are in inverse proportion when their **product** is constant. Double one and the other halves; triple one and the other becomes a third.

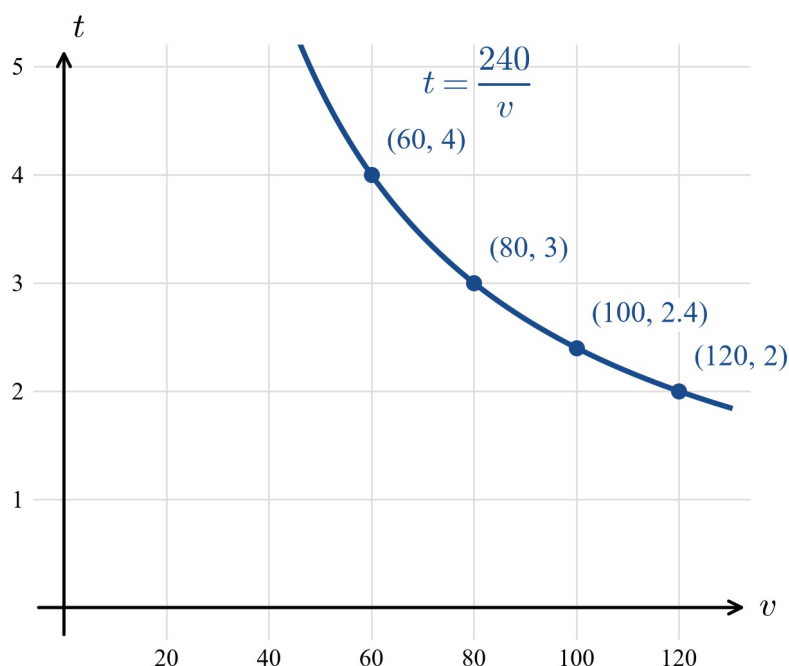
The standing example is a fixed trip. At speed v km/h, a trip of 240 km takes t hours, where

$$v \times t = 240, \text{ so } t = \frac{240}{v}.$$

The product $v \times t$ is the constant 240, so t is in inverse proportion to v .

Speed v (km/h)	60	80	100	120
Time t (h)	4	3	2.4	2

Double the speed from 60 to 120 km/h and the time halves from 4 h to 2 h. At 100 km/h the time is 2.4 h, which is 2 h 24 min — the 0.4 of an hour is $0.4 \times 60 = 24$ minutes. Figure 3C.1 graphs the whole pattern.



Graph of $t = 240/v$ for a 240 km trip: a falling curve through the points (60, 4), (80, 3), (100, 2.4) and (120, 2), never touching either axis.

The graph is a falling curve that never touches either axis: no speed, however large, makes the time zero, and no time, however long, makes the speed zero. Compare with 3B, where this curve appears as the reciprocal relation $y = \frac{k}{x}$.

A second physical example of inverse proportion is brightness. The brightness B of a light source measured at distance d is in inverse proportion to the *square* of the distance:

$$B = \frac{k}{d^2},$$

for some constant k . If $B = 45$ units at $d = 2$ m, then $k = B \times d^2 = 45 \times 4 = 180$, so $B = \frac{180}{d^2}$. At $d = 3$ m the brightness is $\frac{180}{9} = 20$ units; at $d = 6$ m it is $\frac{180}{36} = 5$ units. Trebling the distance (from 2 m to 6 m) divides the brightness by 9 — not by 3. That is the signature of inverse-square proportion.

2. Choosing a model — differences and ratios

The diagnostic of this subtopic, and the one to carry into every later mathematics course, is read straight off a table of values with equally spaced x -values.

- **Constant first differences** (each y increases or decreases by the same *amount*) mean the pattern is **linear**.
- **Constant second differences** (the differences of the differences are constant) mean the pattern is **quadratic**.
- **A constant ratio** (each y is the same *multiple* of the one before) means the pattern is **exponential**.

Three tables show the three signatures side by side.

Table A — linear.

x	0	1	2	3	4
y	5	9	13	17	21

The first differences are $9 - 5 = 4$, $13 - 9 = 4$, $17 - 13 = 4$, $21 - 17 = 4$: constant, so linear, with rule $y = 5 + 4x$ (start 5, add 4 each step).

Table B — quadratic.

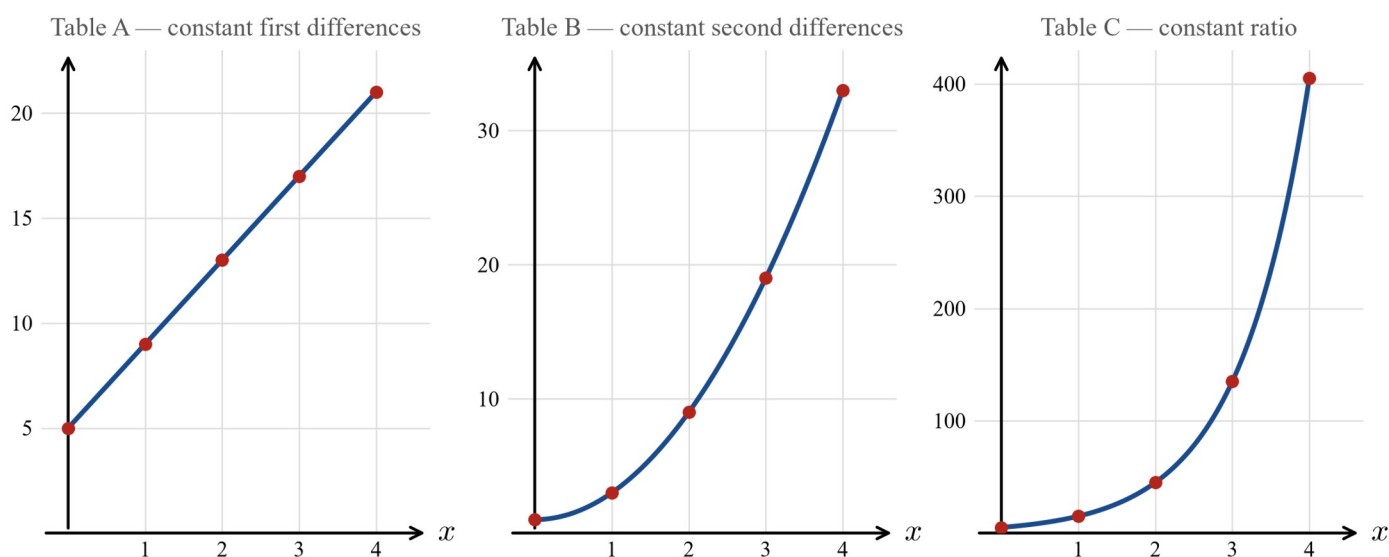
x	0	1	2	3	4
y	1	3	9	19	33

The first differences are 2, 6, 10, 14 — not constant, so not linear. The second differences are $6 - 2 = 4$, $10 - 6 = 4$, $14 - 10 = 4$: constant, so quadratic. (Here the rule is $y = 2x^2 + 1$.)

Table C — exponential.

x	0	1	2	3	4
y	5	15	45	135	405

The first differences 10, 30, 90, 270 are not constant, and neither are the second differences 20, 60, 180. But the *ratios* are: $\frac{15}{5} = 3$, $\frac{45}{15} = 3$, $\frac{135}{45} = 3$, $\frac{405}{135} = 3$. Constant ratio, so exponential, with rule $y = 5 \times 3^x$ (start 5, multiply by 3 each step).



Three graphs side by side of the tables above: Table A rises as a straight line through (0, 5) to (4, 21); Table B rises as a curve through (0, 1) to (4, 33); Table C rises as a steep curve through (0, 5) to (4, 405).

The three pictures (Figure 3C.2) look alike for small x — all three climb — and part apart sharply afterwards. That is why the table, not the picture, is the tool for choosing a model: **check the differences first, then the ratios.**

3. Growth and decay by a constant percentage — doubling time and half-life

A quantity that grows by the same **percentage** each step is multiplied by the same factor each step, so it is exponential. Growth by r per step means multiplying by $1 + r$; decay by r means multiplying by $1 - r$.

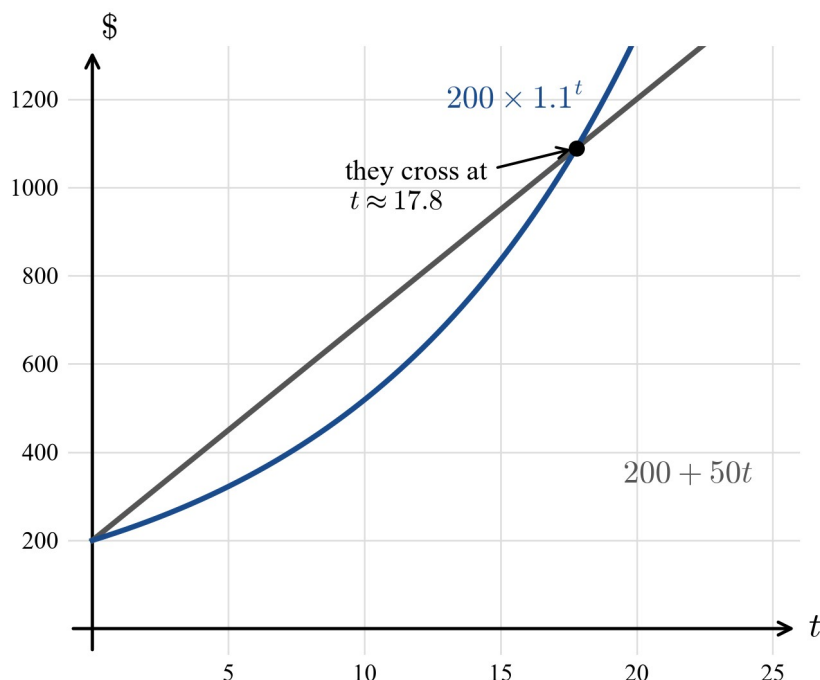
- Growing by 5% each year: multiply by 1.05.
- Growing by 20% each year: multiply by 1.2.
- Losing 15% of its value each year: multiply by $1 - 0.15 = 0.85$.

Contrast this with growth by a constant **amount**. Start with \$200.

	now	after 1 year	after 2 years	after 3 years
add \$50 each year (linear)	200	250	300	350
grow by 5% each year	200	210	220.50	231.53

(exponential)

Early on, the linear option is ahead — adding \$50 beats 5% of a small balance. The exponential rule 200×1.05^t only catches up with $200 + 50t$ after many years. At a faster growth rate the crossover is much earlier: 200×1.1^t passes $200 + 50t$ between $t = 17$ and $t = 18$ (Figure 3C.3). **This is the defining fact of exponential growth: it always overtakes linear growth in the end.**

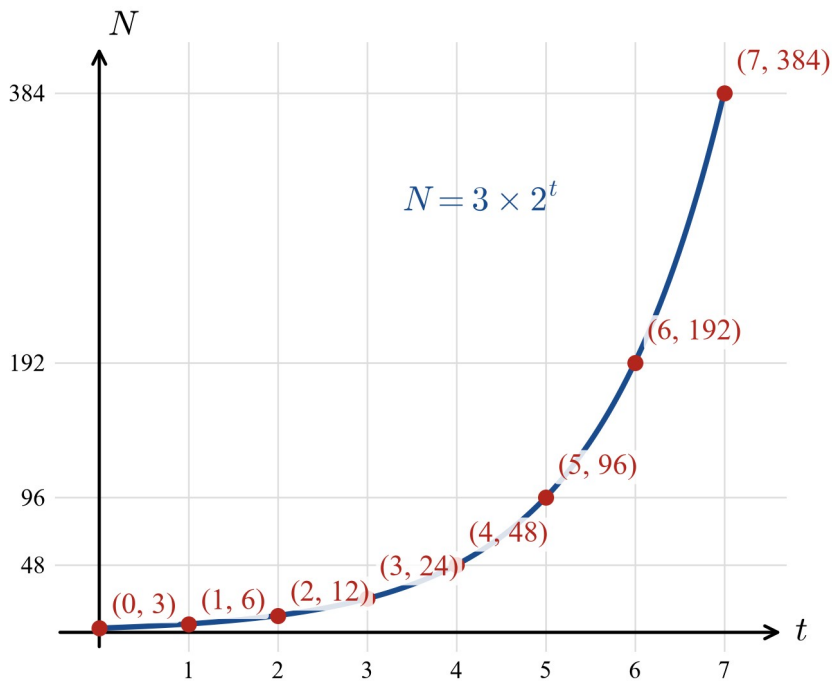


Graph of $200 + 50t$ as a straight line and 200 times 1.1 to the power t as a curve, both starting at $(0, 200)$; the curve starts below the line, meets it at about $t = 17.8$, and then climbs away above it.

Doubling. When a quantity doubles in every equal step, the factor is 2. A patch of pond weed covering 3 m^2 that doubles each week is modelled by

$$N = 3 \times 2^t,$$

where t is the number of weeks. The table runs $3, 6, 12, 24, 48, 96, 192, \dots$ — and the *time* between doublings is always the same: one week, whether from 3 to 6 or from 96 to 192. That constant **doubling time** is the growth-side signature of an exponential pattern (Figure 3C.4).

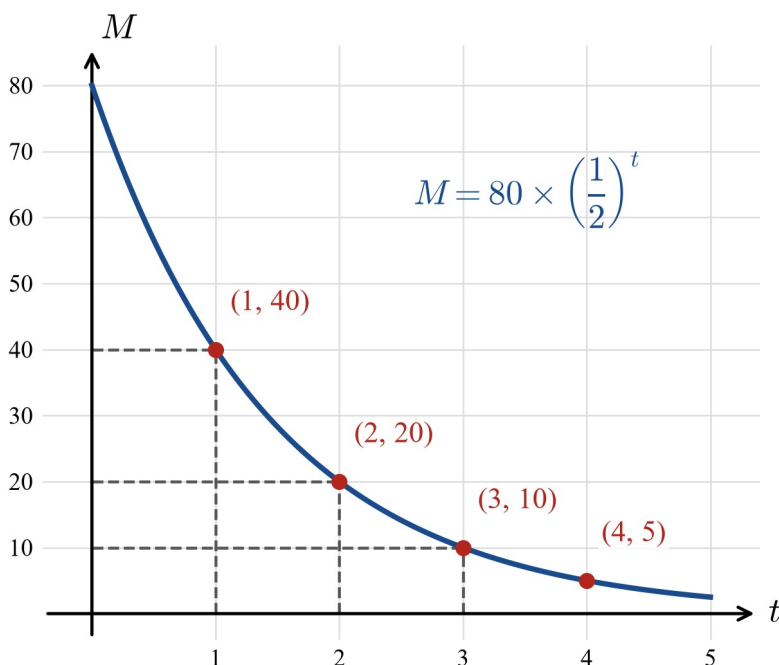


Graph of $N = 3$ times 2 to the power t through the points $(0, 3)$, $(1, 6)$, $(2, 12)$, $(3, 24)$, $(4, 48)$, $(5, 96)$, $(6, 192)$ and $(7, 384)$; each one-week step doubles the height.

Half-life. Decay by a constant percentage has the mirror signature: a constant **half-life**, the time in which a quantity halves (Figure 3C.5). A sample of 80 g that halves every 6 hours is modelled by

$$M = 80 \times \left(\frac{1}{2}\right)^t,$$

where t counts 6-hour periods: 80, 40, 20, 10, 5, ... Every 6 hours, whatever is left is halved — the next 6 hours halve it again.



Graph of $M = 80$ times one-half to the power t : a falling curve through $(0, 80)$, with dashed guides showing $(1, 40)$, $(2, 20)$, $(3, 10)$ and the point $(4, 5)$; each equal time step halves the height.

When the answer is not a whole number of steps, trap and refine exactly as in 2E: to find when 3×2^t first exceeds 100, note $3 \times 2^5 = 96 < 100$ and $3 \times 2^6 = 192 > 100$, so the crossing lies between $t = 5$ and $t = 6$ — the patch first exceeds 100 m² during week 6.

4. Building the compound interest formula as repeated simple interest

Recall simple interest from Year 9 (VC2M9A06): interest I on a **principal** P — the amount first put in — at rate r per period for n periods is

$$I = Prn.$$

\$2000 at 4% per annum simple interest for 3 years earns $I = 2000 \times 0.04 \times 3 = \240 , so the balance after 3 years is \$2240. Simple interest pays the *same* \$80 every year, because it is always 4% of the original \$2000.

Banks do not work that way. Each year's interest is added to the balance, and the next year's interest is paid on the **new** balance — interest earns interest. That is **compound interest**, and the whole formula falls out of repeating simple interest one year at a time.

Year 1. The balance is \$2000; the year's interest is simple interest on \$2000: $2000 \times 0.04 = \$80.00$. New balance $2000 + 80 = \$2080.00$.

Year 2. Now the principal for the year is \$2080; the interest is $2080 \times 0.04 = \$83.20$. New balance $2080 + 83.20 = \$2163.20$.

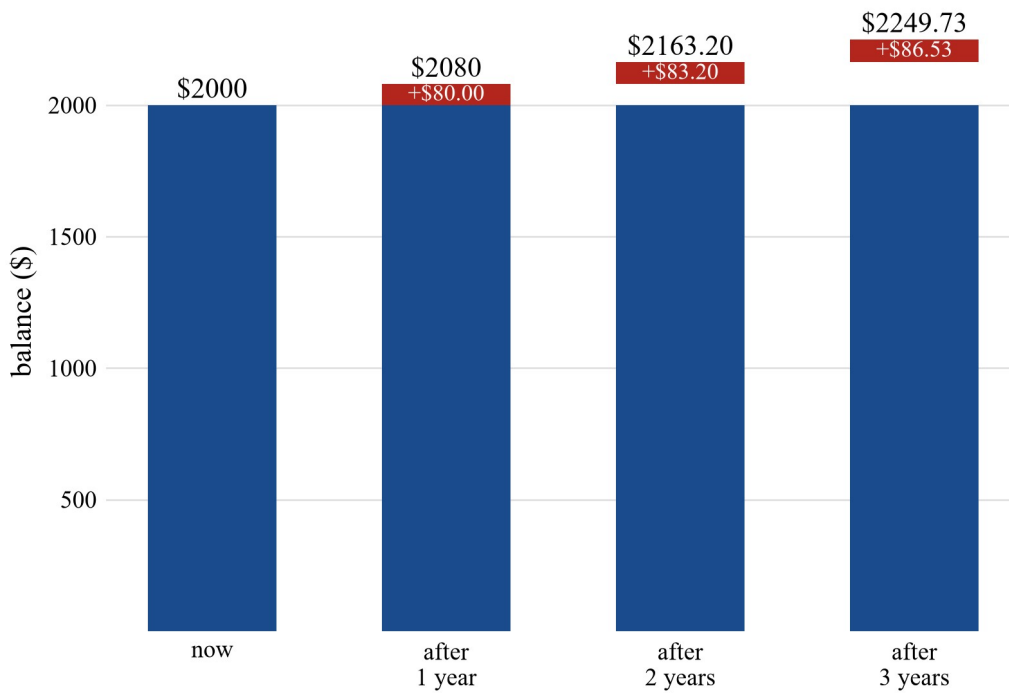
Year 3. The principal is \$2163.20; the interest is $2163.20 \times 0.04 = \$86.528$. New balance $2163.20 + 86.528 = \$2249.728$, which rounds to \$2249.73.

	principal for the year	interest (4 % of that principal)	balance at year's end
Year 1	\$2000.00	\$80.00	\$2080.00
Year 2	\$2080.00	\$83.20	\$2163.20
Year 3	\$2163.20	\$86.53	\$2249.73

Figure 3C.6 splits each balance into the principal below and that year's interest on top — the interest grows because each year it is 4% of a larger balance. Each year's balance is 1.04 times the one before — adding 4% *is* multiplying by 1.04 — so the balances are a constant-ratio table, and the pattern is exponential:

$$2000 \times 1.04, 2000 \times 1.04^2, 2000 \times 1.04^3.$$

Each year's interest is 4% of that year's balance — so it grows



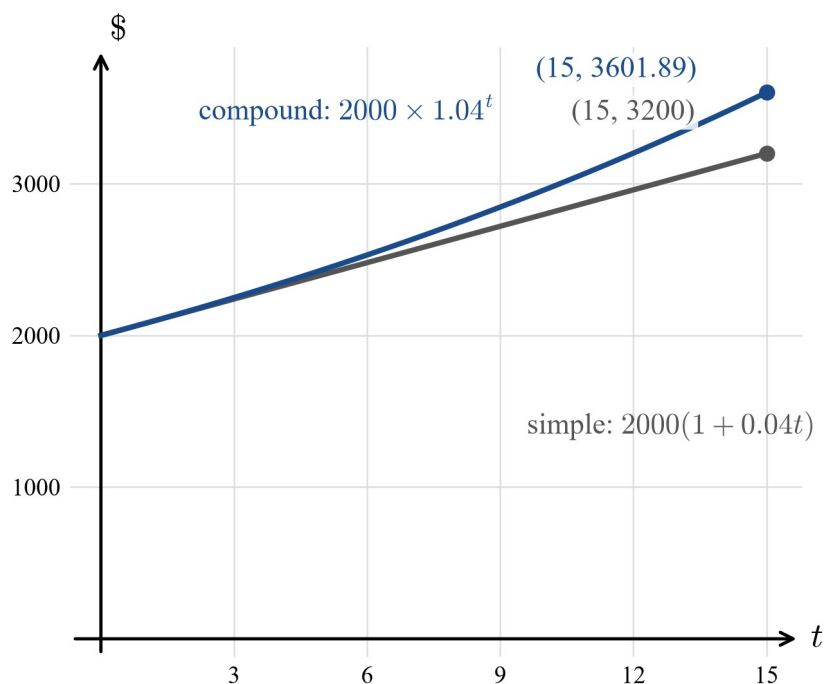
Bar chart of the four balances 2000, 2080, 2163.20 and 2249.73, each bar split into the principal in blue and that year's interest in red: the red segments grow, 80.00 then 83.20 then 86.53, because each year's interest is 4% of a larger balance.

Writing the starting amount as P and the rate per period as r , the same three multiplications give the **compound interest formula**:

$$A = P(1 + r)^n,$$

where A is the balance after n periods. It was not handed down; it is exactly “ P , multiplied by $(1 + r)$, once per period, n times”. For the example: $A = 2000 \times 1.04^3 = \2249.73 — against \$2240 under simple interest. The extra \$9.73 is the interest that interest earned.

Over longer times the gap stops being small (Figure 3C.7). At 4% for 15 years, simple interest gives $2000(1 + 0.04 \times 15) = \3200 , while compound gives $2000 \times 1.04^{15} \approx \3601.89 .



Graph of the simple-interest line $2000(1 + 0.04t)$ and the compound-interest curve 2000 times 1.04 to the power t from $t = 0$ to 15 ; the two start together at $(0, 2000)$ and part steadily, ending at $(15, 3200)$ and $(15, 3601.89)$.

A fall in value by a constant percentage each year is **depreciation** — the decay side of the same formula. A van worth \$24 000 that loses 15 % of its value each year is worth, after 3 years,

$$24\,000 \times (1 - 0.15)^3 = 24\,000 \times 0.85^3 = 24\,000 \times 0.614125 = \$14\,739.00.$$

The formula is identical with r replaced by $-r$: $A = P(1 - r)^n$.

5. The modelling cycle — formulate, solve, interpret, evaluate, report

Every problem in this subtopic, and the extended task X1 at its end, runs the same five-step cycle, which is exactly what VC2M10A15 names.

1. **Formulate.** State the variables and the assumptions, and choose the model — linear, quadratic, exponential or inverse proportion — using the §2 diagnostic.
2. **Solve.** Do the mathematics: evaluate a rule, or solve the related equation, numerically (trap and refine) or graphically (read an intersection, as in 3B).
3. **Interpret.** Read the answer back into the situation, with units and rounding that suit it — $t \approx 5.4$ years means *during the sixth year*, and a balance rounds to the cent.
4. **Evaluate and modify.** Test the model against the situation: does the constant rate really hold? A constant-percentage depreciation assumes the van loses the same *share* of its value every year, forever — in practice old vans stop falling in value, so the model may need modifying after some years.
5. **Report.** Communicate the assumptions, the method and the findings to the reader.

The worked examples run this cycle in full; the practice items run steps 1–4 with the model partly chosen for you; X1 runs all five with data you supply.

Worked examples

Example 1 — Inverse proportion: a trip and a light

A family drives a fixed 240 km trip at an average speed of v km/h, taking t hours.

- (a) Find t when $v = 60$.

Working	Comment
$v \times t = 240$	The product is constant: inverse proportion.
$t = \frac{240}{v}$	Solve for t .
$t = \frac{240}{60} = 4$	Substitute $v = 60$.
$\therefore$ the trip takes 4 h.	Check: $60 \times 4 = 240$. ✓

(b) The trip takes 3 h. Find v .

Working	Comment
$v \times 3 = 240$	The product is still 240.
$v = \frac{240}{3} = 80$	Divide by 3.
$\therefore$ the average speed is 80 km/h.	Check: $80 \times 3 = 240$. ✓

(c) Find t when $v = 100$, in hours and minutes.

Working	Comment
$t = \frac{240}{100} = 2.4$	Substitute $v = 100$.
$0.4 \text{ h} = 0.4 \times 60 = 24 \text{ min}$	Convert the part of an hour.
$\therefore$ the trip takes 2 h 24 min.	

(d) A light measures 45 units of brightness at 2 m. Using $B = \frac{k}{d^2}$, find the brightness at 6 m.

Working	Comment
$k = B \times d^2 = 45 \times 2^2 = 180$	Find the constant from the given pair.
$B = \frac{180}{6^2} = \frac{180}{36} = 5$	Substitute $d = 6$.
$\therefore$ the brightness at 6 m is 5 units.	The distance trebled, so B divides by 9. ✓

Example 2 — Choosing the model from a table

Classify each table as linear, quadratic or exponential, and give its rule.

x	0	1	2	3
y	7	10	13	16

Working			Comment	
first differences: 3, 3, 3			Constant first differences.	
$\therefore$ linear, with rule $y = 7 + 3x$.			Start 7, add 3 each step.	
x	0	1	2	3
y	2	3	6	11

Working	Comment
first differences: 1, 3, 5	Not constant, so not linear.
second differences: 2, 2	Constant second differences.
$\therefore$ quadratic.	Here the rule is $y = x^2 + 2$.

Working	Comment			
x	0	1	2	3
y	4	12	36	108

Working	Comment
ratios: $\frac{12}{4}=3, \frac{36}{12}=3, \frac{108}{36}=3$	Constant ratio.
$\therefore$ exponential, with rule $y = 4 \times 3^x$.	Start 4, multiply by 3 each step.

Example 3 — Growth and doubling time: the pond weed

SCENARIO A patch of pond weed covers 3 m^2 and doubles in area each week, so $N = 3 \times 2^t$.

(a) Find the area after 6 weeks.

Working	Comment
$N = 3 \times 2^6$	Substitute $t = 6$.
$N = 3 \times 64 = 192$	$2^6 = 64$.
$\therefore$ the patch covers 192 m^2 .	

(b) When does the patch cover 96 m^2 ?

Working	Comment
$3 \times 2^t = 96$	Set $N = 96$.
$2^t = 32$	Divide both sides by 3.
$2^t = 2^5$, so $t = 5$	$32 = 2^5$; equate the exponents (2E).
$\therefore$ the patch covers 96 m^2 after 5 weeks.	Check: $3 \times 32 = 96$. ✓

(c) In which week does the area first exceed 100 m^2 ?

Working	Comment
$3 \times 2^5 = 96 < 100$	At $t = 5$ the patch is still under 100 m^2 .
$3 \times 2^6 = 192 > 100$	At $t = 6$ it is over.
$\therefore$ the area first exceeds 100 m^2 during week 6.	Trap between whole steps, as in 2E.

The doubling time is 1 week: from any size, one more week doubles it.

Example 4 — Half-life

SCENARIO A sample of 80 g of a radioactive material halves in mass every 6 hours.

(a) Find the mass after 18 hours.

Working	Comment
$18 \div 6 = 3$ half-lives	Count the halving steps.
$80 \rightarrow 40 \rightarrow 20 \rightarrow 10$	Halve three times.
$\therefore$ the mass after 18 h is 10 g .	Equally: $80 \times \left(\frac{1}{2}\right)^3 = 10$. ✓

(b) When is the mass 5 g ?

Working	Comment
$80 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5$ is 4 halvings	Keep halving to 5.
$4 \times 6 = 24$	Four half-lives of 6 h.

Working	Comment
---------	---------

∴ the mass is 5 g after 24 h.

(c) Find the mass after 30 hours.

Working	Comment
---------	---------

$30 \div 6 = 5$ half-lives

$$80 \times \left(\frac{1}{2}\right)^5 = \frac{80}{32} = 2.5$$

∴ the mass after 30 h is 2.5 g.

Example 5 — Building compound interest: investing \$2000 at 4.0 % p.a. for 3 years

\$2000 is invested at 4.0 % per annum. Find the balance after 3 years (a) under simple interest, (b) under compound interest, and (c) compare.

Working	Comment
---------	---------

$$I = Prn = 2000 \times 0.04 \times 3 = 240$$

Simple interest from Year 9 (VC2M9A06).

$$A = 2000 + 240 = 2240$$

∴ the simple-interest balance is \$2240.00.

(b) Compound interest, one year at a time — each year is simple interest on that year's balance.

Working	Comment
---------	---------

Year 1: $2000 \times 0.04 = 80.00$; balance 2080.00

Interest on \$2000.

Year 2: $2080 \times 0.04 = 83.20$; balance 2163.20

Interest on \$2080.

Year 3: $2163.20 \times 0.04 = 86.528$; balance 2249.728

Interest on \$2163.20.

∴ the compound balance is \$2249.73 (to the cent).

(c) $2249.73 - 2240.00 = 9.73$: compound interest pays \$9.73 more — the interest that interest earned. The shortcut $A = 2000 \times 1.04^3 = 2249.728$ gives the same balance, confirming the formula built in §4.

Example 6 — The formula at work: investing, doubling and depreciation

(a) Find the balance when \$5000 is invested at 5 % p.a., compounded yearly, for 4 years.

Working	Comment
---------	---------

$$A = P(1 + r)^n = 5000 \times 1.05^4$$

$P = 5000, r = 0.05, n = 4.$

$$1.05^4 = 1.21550625$$

$$A = 5000 \times 1.21550625 = 6077.53125$$

∴ the balance is \$6077.53.

(b) At 5 % p.a. compounded yearly, about how long does an investment take to double?

Working	Comment
---------	---------

Need $1.05^n = 2.$

The doubling question is an exponential equation.

$$1.05^{14} \approx 1.98 \text{ and } 1.05^{15} \approx 2.08$$

Trap between whole years.

∴ the money doubles in about 14.2 years — after 14 years it has not quite doubled, and the first whole year by which it has doubled is 15.

Refine by trial if needed: $1.05^{14.2} \approx 2.00.$

(c) A van worth \$24 000 depreciates by 15 % per year. Find its value after 3 years.

Working	Comment
---------	---------

$$A = P(1 - r)^n = 24\,000 \times 0.85^3$$

Decay by 15 % means multiply by 0.85.

$$0.85^3 = 0.614125$$

$$A = 24\,000 \times 0.614125 = 14\,739$$

∴ the van is worth \$14 739.00 after 3 years.

Practice questions

Scaffolded

Q1. y is in inverse proportion to x , and $y = 12$ when $x = 5$. (a) Find the constant k in $y = \frac{k}{x}$. (b) Hence find y when $x = 4$. (c) Find x when $y = 3$.

Q2. A fixed trip of 360 km is driven at an average speed of v km/h, taking t hours; at 90 km/h the trip takes 4 h. (a) Write t in terms of v . (b) Find t when $v = 120$. (c) For how long does the trip take at 80 km/h?

Q3. The table shows a linear pattern.

x	0	1	2	3
y	11	8	5	2

(a) Show that the first differences are constant, and give their value. (b) Write the rule for y in terms of x . (c) Hence find y when $x = 5$.

Q4. The table shows a decay pattern.

x	0	1	2	3
y	200	100	50	25

(a) Show that the ratio of consecutive y -values is constant, and give its value. (b) Write the rule for y in terms of x . (c) Hence find y when $x = 6$.

Q5. Build the compound balance year by year: \$300 at 10% p.a. for 2 years. Year 1 interest is $300 \times 0.10 = \$30$; find the year-2 interest on the new balance, and the final balance.

Q6. A bike worth \$450 depreciates by 20% per year. (a) Write the multiplier per year. (b) Find the bike's value after 2 years.

Q7. A sample of 64 g halves every 3 hours. (a) List the masses after 3, 6 and 9 hours. (b) What is the half-life? (c) Find the mass after 9 hours as one calculation.

Q8. Build the compound balance year by year: \$1000 at 3% p.a. for 2 years. (a) Show that the year-1 balance is \$1030.00. (b) Find the year-2 interest and the final balance.

Un scaffolded

Q9. y is in inverse proportion to x , and $y = 8$ when $x = 6$. (a) Find y when $x = 16$. (b) Find x when $y = 2$.

Q10. The brightness of a lamp is $B = \frac{k}{d^2}$ units at distance d metres, and $B = 90$ at $d = 2$. (a) Find B at $d = 3$. (b) Find B at $d = 6$. (c) Explain, using the rule, why trebling the distance divides the brightness by 9.

Q11. Classify each table as linear, quadratic or exponential, using differences and ratios.

x	0	1	2	3
y	2	7	12	17
x	0	1	2	3
y	1	4	9	16
x	0	1	2	3
y	3	12	48	192

Q12. A table of values runs 3, 4, 7, 12 for $x=0, 1, 2, 3$. (a) Classify the pattern. (b) Given that the rule is $y = x^2 + 3$, predict y when $x=5$.

Q13. A population of 400 grows by 20 % each year. (a) Write the multiplier per year. (b) Find the population after 3 years, to the nearest whole number.

Q14. SCENARIO A dose of 500 mg leaves the body at 30 % per hour, so each hour 70 % remains. Find the amount left after 3 hours.

Q15. Bacteria in a dish number $N = 50 \times 2^{t/4}$ after t hours — the population doubles every 4 hours. (a) Find N at $t = 12$. (b) When does N reach 1600?

Q16. A sample of 200 g has a half-life of 5 days. (a) Find the mass after 20 days. (b) After how many days is the mass 25 g?

Q17. Find the balance when \$800 is invested at 4.5 % p.a., compounded yearly, for 3 years. Give your answer to the cent.

Q18. \$5000 is invested for 2 years at 3 % p.a. Find the final balance under (a) simple interest and (b) compound interest, and (c) the difference between them.

Q19. A machine worth \$18 000 depreciates by 12 % per year. Find its value after 3 years, to the cent.

Q20. At 4 % p.a. compounded yearly, about how long does an investment take to double? Trap the answer between whole years, then refine to one decimal place.

Q21. A population is modelled by $P = 200 \times 1.5^t$. (a) Between which two whole values of t does P lie between 1000 and 1500? (b) Use trial to find t for which $P \approx 1200$, to one decimal place.

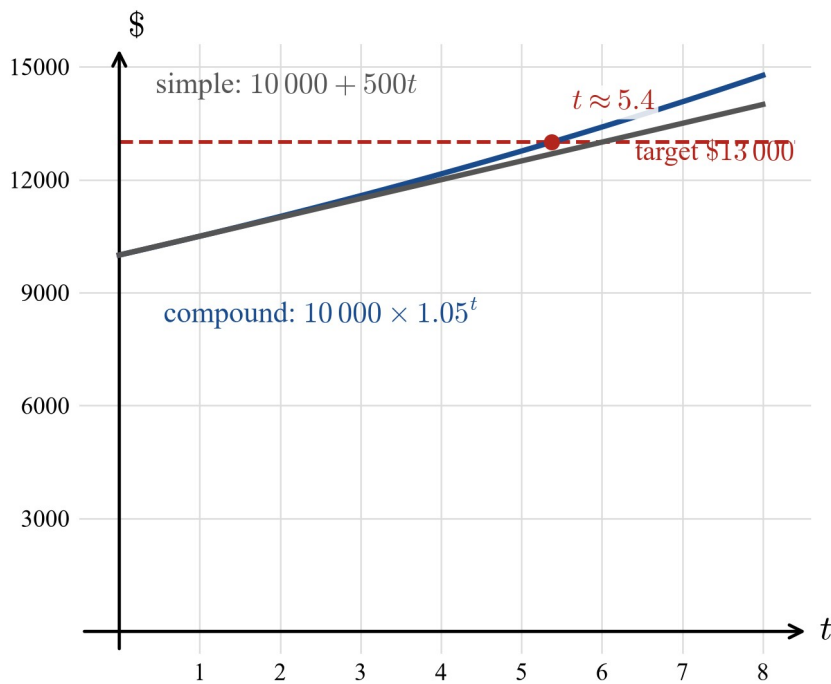
Q22. SCENARIO After the rains stop, a dam loses 10 % of its water each week by evaporation, from 4000 kL: $V = 4000 \times 0.9^t$, with t in weeks. (a) Find V after 3 weeks. (b) In which week does V first fall below 2000 kL? (c) Evaluate the model: name one assumption, and say how you would modify the model if rain fell in week 5.

Real context

Q23. The Reserve Bank of Australia's one-year term deposit rate for deposits of \$10 000 was 5.00 % p.a. as at July 2026. Suppose \$10 000 is invested at that rate, compounded yearly.

- Find the balance after 3 years.
- Find what simple interest at 5.00 % p.a. would give over 3 years, and the difference between the two balances.
- How long does the investment take to reach \$13 000? Trap between whole years, then refine to one decimal place. Check your answer against Figure 3C.8.
- Evaluate.** The RBA's HISTORICAL one-year term deposit rates — trend and comparison only — were 4.50 % p.a. in July 2024, 4.20 % p.a. in December 2024, 3.60 % p.a. in July 2025 and 5.00 % p.a. in July 2026. Comment on the assumption that the rate stays at 5.00 % for the whole period, and describe how the model would be modified to use each year's own rate.

Source: Reserve Bank of Australia, *Statistical Table F4 — Retail Deposit and Investment Rates* (one-year term deposit rate for \$10 000, as at July 2026; historical rates 2024–2025), 2026, published 10 August 2026. Retrieved 20 August 2026.



Graph of the compound balance 10 000 times 1.05 to the power t and the simple balance $10\,000 + 500t$ from $t = 0$ to 8 years, with a dashed horizontal line at the 13 000 dollar target; the compound curve meets the target at about $t = 5.4$.

Extended task X1 — Mathematical modelling (20 rubric marks)

Brief. Choose **one** option and run the full modelling cycle — formulate, solve, interpret, evaluate and modify, report — on data you generate or source yourself. Collect at least four pairs of values, record how they were obtained, and report your assumptions, methods and findings in one page or one slide deck. This task is teacher-marked against the rubric below and is not counted in the test totals.

Option A — growth, decay or inverse proportion (VC2M10A15). Choose a situation in which a quantity grows or decays by a roughly constant percentage per step (a population, a resale value, a cooling temperature), or two quantities in inverse proportion (a trip's time and speed, a light's brightness and distance). Formulate a model, choosing linear, quadratic or exponential as the §2 diagnostic directs; solve two questions — one *evaluate* question and one *solve* question; interpret both in the situation; evaluate the model against a data point you did not use to build it, and modify it if the fit is poor; report.

Option B — scaling and proportion (VC2M10M04). Choose a situation that draws on 4C's proportional reasoning — a map's scale, a recipe scaled up, a photo enlarged — and measure how the modelled values compare with measured ones, including the size of any errors. Formulate the proportional model, solve, interpret, evaluate against measurement, modify, and report.

Planning frame.

Step	Record
Formulate	Situation; variables and units; assumptions (what is held constant); model type and rule; where the data comes from
Solve	The two questions and the full working
Interpret	Each answer read back into the situation, with units and sensible rounding
Evaluate	Model checked against a spare data point; limitations named; model modified where needed
Report	One page or deck for a stated reader, carrying

Step	Record
	assumptions, method and findings

Data-quality checklist. Before solving, check: at least four pairs of values; units recorded with every value; the source or measuring method written down; a spread of values wide enough to show the pattern; outliers noticed and explained; and, for sourced data, the date it applies (*as at* month and year).

Rubric — 5 criteria, 4 marks each.

Criterion	4 marks	2 marks	0–1 marks
Formulate	Variables, assumptions and model type all stated; rule fits the data	Model chosen with some assumptions missing	No workable model
Solve	Mathematics correct and fully shown for both questions	One question correct, or working gaps	Mathematics not correct
Interpret	Both answers read back into the situation with units and sensible rounding	Answers given but not interpreted	No interpretation
Evaluate	Model tested against a spare point, limitations named, model modified where needed	Limitations named but not tested	No evaluation
Report	Assumptions, method and findings communicated clearly to the stated reader	Findings given, method partly shown	Not communicated

EXEMPLAR — practice, not the task. *Supplied data:* a van is bought for \$ 32 000; its trade-in value is recorded as \$ 27 200 after 1 year, \$ 23 120 after 2 years and \$ 19 652 after 3 years.

Formulate. The ratios are $\frac{27\,200}{32\,000} = 0.85$, $\frac{23\,120}{27\,200} = 0.85$, $\frac{19\,652}{23\,120} = 0.85$ — a constant ratio, so exponential decay:

$V = 32\,000 \times 0.85^t$, assuming the van keeps losing 15 % of its value each year.

Solve. Evaluate: $V(5) = 32\,000 \times 0.85^5 = 32\,000 \times 0.4437053125 = 14\,198.57$, so \$ 14 198.57. Solve: $V = 16\,000$ (half the purchase price) needs $0.85^t = 0.5$; $0.85^4 \approx 0.522$ and $0.85^5 \approx 0.444$, so the value halves between $t = 4$ and $t = 5$, about $t \approx 4.3$ years.

Interpret. On this model the van is worth about \$ 14 198.60 after 5 years, and it halves in value in about 4.3 years — during its fifth year.

Evaluate. The model fits the three recorded values exactly (constant ratio 0.85), but it assumes the same 15 % fall forever; in practice old vans stop falling in value, so after about 8–10 years the model would be modified to hold a floor value.

Report. “A van bought for \$ 32 000 loses about 15 % of its value each year; the exponential model $V = 32\,000 \times 0.85^t$ matches the recorded trade-in values, predicts about \$ 14 200 after five years, and is reliable only while the percentage fall holds.”

Answers

Q1. (a) $k = 60$; (b) $y = 15$; (c) $x = 20$. **Q2.** (a) $t = \frac{360}{v}$; (b) 3 h; (c) 4.5 h. **Q3.** (a) first differences are constant, -3 ; (b) $y = 11 - 3x$; (c) $y = -4$. **Q4.** (a) the ratio is constant, $\frac{1}{2}$; (b) $y = 200 \times \left(\frac{1}{2}\right)^x$; (c) $y = 3.125$. **Q5.** year-2 interest \$33.00; balance \$363.00. **Q6.** (a) 0.8; (b) \$288.00. **Q7.** (a) 32, 16, 8 g; (b) 3 h; (c) $64 \times \left(\frac{1}{2}\right)^3 = 8$ g. **Q8.** (a) $1000 \times 0.03 = 30$, balance 1030.00; (b) interest \$30.90, balance \$1060.90.

Q9. (a) $y = 3$; (b) $x = 24$. **Q10.** (a) $B = 40$; (b) $B = 10$; (c) $d \rightarrow 3d$ gives $d^2 \rightarrow 9d^2$, so B divides by 9. **Q11.** (a) linear (first differences 5); (b) quadratic (second differences 2); (c) exponential (ratio 4). **Q12.** (a) quadratic (first differences 1, 3, 5; second differences 2, 2); (b) $y = 28$. **Q13.** (a) 1.2; (b) $691.2 \approx 691$. **Q14.** 171.5 mg. **Q15.** (a) 400; (b) $t = 20$ h. **Q16.** (a) 12.5 g; (b) 15 days. **Q17.** \$912.93. **Q18.** (a) \$5300.00; (b) \$5304.50; (c) \$4.50. **Q19.** \$12 266.50. **Q20.** between 17 and 18 years; ≈ 17.7 years (first whole year: 18). **Q21.** (a) between $t = 4$ and $t = 5$; (b) $t \approx 4.4$. **Q22.** (a) 2916 kL; (b) week 7; (c) answers vary — the model assumes no rain and a constant 10% loss; after rain in week 5, add the inflow and restart the decay from the new V . **Q23.** (a) \$11 576.25; (b) \$11 500.00, difference \$76.25; (c) ≈ 5.4 years; (d) answers vary — the rates moved from 4.50% to 5.00% over 2024–2026, so a constant rate is only an assumption; modify by applying each year's own rate, one multiplication per year.

Fully-worked solutions

Q1. (a) $k = x \times y = 5 \times 12 = 60$.

(b) $y = \frac{60}{4} = 15$.

(c) $x = \frac{60}{3} = 20$.

Q2. (a) $v \times t = 360$, so $t = \frac{360}{v}$.

(b) $t = \frac{360}{120} = 3$ h.

(c) $t = \frac{360}{80} = 4.5$ h.

Q3. (a) $8 - 11 = -3$, $5 - 8 = -3$, $2 - 5 = -3$: constant, -3 , so linear.

(b) Start 11, subtract 3 each step: $y = 11 - 3x$.

(c) $y = 11 - 3 \times 5 = 11 - 15 = -4$.

Q4. (a) $\frac{100}{200} = \frac{1}{2}$, $\frac{50}{100} = \frac{1}{2}$, $\frac{25}{50} = \frac{1}{2}$: constant ratio $\frac{1}{2}$, so exponential decay.

(b) $y = 200 \times \left(\frac{1}{2}\right)^x$.

(c) $y = 200 \times \left(\frac{1}{2}\right)^6 = \frac{200}{64} = 3.125$.

Q5. Year 1: interest $300 \times 0.10 = 30$; balance 330.00. Year 2: interest $330 \times 0.10 = 33$; balance $330 + 33 = \$363.00$.

Q6. (a) Losing 20% leaves 80%: multiplier 0.8.

(b) $450 \times 0.8^2 = 450 \times 0.64 = \288.00 .

Q7. (a) $64 \rightarrow 32 \rightarrow 16 \rightarrow 8$ g.

(b) The half-life is 3 h.

(c) $9 \div 3 = 3$ half-lives: $64 \times \left(\frac{1}{2}\right)^3 = 8$ g.

Q8. (a) Year-1 interest $1000 \times 0.03 = 30$; balance $1000 + 30 = \$1030.00$.

(b) Year-2 interest $1030 \times 0.03 = 30.90$; balance $1030 + 30.90 = \$1060.90$.

Q9. $k = 6 \times 8 = 48$, so $y = \frac{48}{x}$.

(a) $y = \frac{48}{16} = 3$.

(b) $x = \frac{48}{2} = 24$.

Q10. $k = B \times d^2 = 90 \times 4 = 360$, so $B = \frac{360}{d^2}$.

(a) $B = \frac{360}{9} = 40$ units.

(b) $B = \frac{360}{36} = 10$ units.

(c) Trebling d multiplies d^2 by $3^2 = 9$, and $B = \frac{360}{d^2}$ divides by 9: from 90 at 2 m to 10 at 6 m.

Q11. (a) First differences 5, 5, 5: constant, so **linear**.

(b) First differences 3, 5, 7; second differences 2, 2: constant, so **quadratic**. (The values are the square numbers 1, 4, 9, 16.)

(c) Ratios 4, 4, 4: constant, so **exponential**.

Q12. (a) First differences 1, 3, 5; second differences 2, 2: constant second differences, so quadratic.

(b) $y = 5^2 + 3 = 25 + 3 = 28$.

Q13. (a) Growing by 20 % multiplies by 1.2.

(b) $400 \times 1.2^3 = 400 \times 1.728 = 691.2 \approx 691$.

Q14. $500 \times 0.7^3 = 500 \times 0.343 = 171.5$ mg.

Q15. (a) $N = 50 \times 2^{12/4} = 50 \times 2^3 = 50 \times 8 = 400$.

(b) $50 \times 2^{t/4} = 1600$, so $2^{t/4} = 32 = 2^5$, so $\frac{t}{4} = 5$ and $t = 20$ h.

Q16. (a) $20 \div 5 = 4$ half-lives: $200 \times \left(\frac{1}{2}\right)^4 = \frac{200}{16} = 12.5$ g.

(b) $200 \rightarrow 100 \rightarrow 50 \rightarrow 25$ is 3 half-lives: $3 \times 5 = 15$ days.

Q17. $A = 800 \times 1.045^3 = 800 \times 1.141166125 = 912.9329$, so \$ 912.93.

Q18. (a) $I = 5000 \times 0.03 \times 2 = 300$; balance \$ 5300.00.

(b) $A = 5000 \times 1.03^2 = 5000 \times 1.0609 = 5304.50$; balance \$ 5304.50.

(c) $5304.50 - 5300.00 = \$ 4.50$.

Q19. $A = 18\,000 \times 0.88^3 = 18\,000 \times 0.681472 = 12\,266.496$, so \$ 12 266.50.

Q20. Need $1.04^n = 2$. $1.04^{17} \approx 1.948$ and $1.04^{18} \approx 2.026$, so the doubling time lies between 17 and 18 years.

Refining: $1.04^{17.6} \approx 1.994$ and $1.04^{17.7} \approx 2.002$, so $n \approx 17.7$ years; the first whole year by which the money has doubled is 18.

Q21. (a) $P(3) = 200 \times 1.5^3 = 675$; $P(4) = 200 \times 1.5^4 = 1012.5$; $P(5) = 200 \times 1.5^5 = 1518.75$. So P lies between 1000 and 1500 for t between 4 and 5.

(b) Try $t = 4.4$: $P = 200 \times 1.5^{4.4} \approx 200 \times 5.954 = 1190.8$. Try $t = 4.5$: $P \approx 200 \times 6.200 = 1240.1$. So 1200 lies between, nearer the $t = 4.4$ trial; refining, $t = 4.42$ gives $P \approx 1200.5$. To one decimal place, $t \approx 4.4$.

Q22. (a) $V = 4000 \times 0.9^3 = 4000 \times 0.729 = 2916$ kL.

(b) $V(6) = 4000 \times 0.9^6 = 2125.764$ kL, still above 2000; $V(7) = 4000 \times 0.9^7 = 1913.1876$ kL, below. So V first falls below 2000 kL during week 7.

(c) The model assumes no inflow and a constant 10 % loss each week — both idealisations. If rain fell in week 5, the model would be modified by adding the inflow to $V(5)$ and then applying the 10 % weekly loss from the new balance.

Q23. (a) $A = 10\,000 \times 1.05^3 = 10\,000 \times 1.157625 = \$ 11\,576.25$.

(b) Simple: $10\,000 + 3 \times 500 = \$ 11\,500.00$. Difference: $11\,576.25 - 11\,500.00 = \$ 76.25$.

(c) Need $10\,000 \times 1.05^t = 13\,000$, i.e. $1.05^t = 1.3$. $1.05^5 \approx 1.276$ and $1.05^6 \approx 1.340$: between 5 and 6 years. Refining: $1.05^{5.3} \approx 1.295$ and $1.05^{5.4} \approx 1.301$, so $t \approx 5.4$ years — matching the intersection in Figure 3C.8.

(d) The historical rates (4.50 %, 4.20 %, 3.60 %, then 5.00 %) moved by more than a percentage point over 2024–2026, so holding 5.00 % fixed is an assumption, not a fact — useful for comparison, approximate for prediction. The model would be modified to apply each year's own rate: multiply by 1.045, then 1.042, then 1.036, then 1.05, one multiplication per year instead of one power.

Teacher note

Level 9 code rehearsed (D3): VC2M9A06 — simple interest, rehearsed in §4 and Example 5 — and VC2M9M05 — direct proportion and rates, the contrast half of §1. Within the book, 3C depends on 2E (exponential equations: every “solve” question here is one) and 3B (the reciprocal and exponential graphs that Figures 3C.1 and 3C.4 sit on).

Tool classes (D13): scientific calculator only (the trials of Example 6(b), Q20, Q21 and Q23, and the powers throughout). VC2M10A15 names no digital tool, so no tool banner is required; the graphical solves read intersections that 3B's sketching already makes readable.

VCE destinations (§5, reference only): this is the clearest single Level-10-to-VCE line in the book — General Mathematics Units 3 & 4 Area of Study 2, *Recursion and financial modelling*, and Foundation Mathematics Units 3 & 4 Area of Study 3, *Financial and consumer mathematics*. The compound interest derivation of §4 is precisely the repeat-the-step rule $A_{n+1} = A_n(1+r)$ that General Mathematics formalises under that title, and the §2 diagnostic (constant first differences → linear, constant second differences → quadratic, constant ratio → exponential) is the diagnostic students carry into that course. The section also leans on the book's implicit prerequisites 3 (proportional reasoning, behind §1's inverse-square brightness) and 5 (growth and decay, doubling time and half-life, behind §3).

Elaborations VC2M10A15 .E05 and VC2M10A15 .E06 are not delivered. They invite carbon dating of Aboriginal and Torres Strait Islander artefacts and modelling native animal populations on Country/Place. This book excludes Aboriginal and Torres Strait Islander content (see the front-of-book note); the mathematics those contexts carry — exponential decay with half-life, and population modelling — is delivered in full through the contexts of VC2M10A15 .E01–.E04 (§3, Examples 3–4, Q13–Q16, Q21).

Level check (D6): no logarithms, no log laws and no $\log$ operator appear anywhere in this section: doubling and halving times are found by trap-and-refine (Example 6(b), Q20, Q23(c)) and by reading graphs (Figure 3C.8), exactly as the Level 10 boundary allows. No $f(x)$ notation; no technical use of functions or relations beyond the 3B gallery language. The compound interest formula is built as repeated simple interest (§4), as the content description itself requires — never handed down.

Dated data (R3): Q23 and Figure 3C.8 carry the RBA’s one-year term deposit rate of 5.00 % p.a. as at July 2026 (Table F4, published 10 August 2026), within the book’s data window; the cash rate target stood at 4.35 % p.a. at the 11 August 2026 decision (Reserve Bank of Australia, *Monetary Policy Decision Number 2026-19*, 2026, retrieved 20 August 2026). The 2024–2025 rates quoted in Q23(d) are outside the window and are labelled and used as HISTORICAL — trend and comparison only, never as current rates.

Algorithms that use data structures

Mathematics 10 · Chapter 3 Graphs, models and algorithms · Subtopic 3D (Level 10) · Extended block

Strand	Content description	Elaboration ids carried	Weight
Algebra	VC2M10A06	E01, E02	E

VC2M10A06 (Algebra, Level 10) — implement algorithms that use data structures using pseudocode or a general purpose programming language

Achievement standard extract (Level 10): Students can design and implement simple algorithms using pseudocode or other general purpose programming language.

You need first: Level 9 algorithms and pseudocode — a sequence of steps, IF/THEN branches, FOR loops and tracing line by line (VC2M9SP03); moving sets of points in the plane by coordinates — translations, reflections in the axes and rotations about the origin (VC2M7SP03); and enlargement from the origin (VC2M9SP02). Substitution from 1E (VC2M10A05) is the engine of every trace: a variable keeps its current value until a line assigns a new one. A **data structure** is a way of organising data so that an algorithm can work through it — this section uses one-dimensional arrays (lists with positions), two-dimensional arrays (grids of rows and columns) and pointers (variables that store positions).

Theory

1. The pseudocode we use (revision)

An **algorithm** is a finite list of steps that finishes in a result. The same algorithm can be written as pseudocode on paper or typed in a general purpose programming language; the lines below are the conventions this section keeps.

```
x ← 5           assignment: x now stores 5
OUTPUT x        print the current value of x
IF x > 3 THEN    a branch: the test is decided with current values
    OUTPUT "big"
ELSE
    OUTPUT "small"
ENDIF
FOR i ← 0 TO 3   i takes 0, then 1, then 2, then 3
    ...
NEXT i
WHILE x > 0       repeat for as long as the test is true
    ...
ENDWHILE
```

Indented lines belong to the loop or branch above them. One step per line, in order, is what makes an algorithm an algorithm.

2. One-dimensional arrays — values live at positions

An **array** stores several values under one name, each at a position called an **index**. We count indices from 0. With $A = [6, 3, 8, 2]$:

- $A[0] = 6, A[1] = 3, A[2] = 8, A[3] = 2$;
- $\text{LENGTH}(A) = 4$;
- the last legal index is $\text{LENGTH}(A) - 1 = 3$ — $A[4]$ does not exist. That off-by-one step is the classic array mistake, and it is examined in Q10.

Reading $A[i]$ is substitution (the 1E skill): replace $A[i]$ by the value stored at index i . The standard walk through a whole array is

```

FOR i ← 0 TO LENGTH(A) - 1
    ... A[i] ...
NEXT i

```

3. Tracing — run it on paper

To **trace** an algorithm is to run it by hand: keep the current value of every variable, and update it one line of pseudocode at a time, never skipping ahead. A small table — one column per variable, one row per pass of the loop — keeps a trace honest. Worked example 1 shows a full one.

4. Two-dimensional arrays — rows and columns

A **two-dimensional array** is an array whose entries are themselves arrays: a grid. $P[r][c]$ names the entry in row r , column c — row first, column second, each counted from 0.

	$c = 0$	$c = 1$	$c = 2$	$c = 3$
$r = 0$	6	8	7	9
$r = 1$	5	6	6	7
$r = 2$	8	9	8	8

$P[1][2] = 6$ — row 1, column 2

A grid of boxes in three rows and four columns; row labels $r = 0, 1, 2$ at the left and column labels $c = 0$ to $c = 3$ above; row 0 holds 6, 8, 7, 9, row 1 holds 5, 6, 6, 7 and row 2 holds 8, 9, 8, 8; the box in row 1, column 2 is shaded and an arrow labels it $P[1][2] = 6$.

The picture is the array P with $P[1][2] = 6$ picked out. To walk a whole grid, nest the loops: the inner loop runs across one row, the outer loop moves down the rows.

```

FOR r ← 0 TO 2
    FOR c ← 0 TO 3
        ... P[r][c] ...
    NEXT c
NEXT r

```

5. Transformations of point sets, as rules (revision)

Level 7 moved shapes by moving their points (VC2M7SP03), and Level 9 added enlargement (VC2M9SP02). A shape is a set of points; on a computer a set of points is data — a list of coordinate pairs. So every transformation you already know is an algorithm: for each point, apply the rule and store the image.

- **Reflection in the x-axis:** $(x, y) \rightarrow (x, -y)$.
- **Reflection in the y-axis:** $(x, y) \rightarrow (-x, y)$.
- **Translation by (a, b) :** $(x, y) \rightarrow (x + a, y + b)$.
- **Rotation 90° anticlockwise about the origin:** $(x, y) \rightarrow (-y, x)$.
- **Rotation 180° about the origin:** $(x, y) \rightarrow (-x, -y)$.
- **Enlargement from the origin, factor k :** $(x, y) \rightarrow (k \times x, k \times y)$.

6. The five transformations as 2×2 arrays

Every rule in Section 5 has the shape $\text{new}x = a \times x + b \times y$, $\text{new}y = c \times x + d \times y$. Store the four numbers in a two-dimensional array T and apply T to one point with two lines of pseudocode:

```

newx ← T[0][0] × x + T[0][1] × y
newy ← T[1][0] × x + T[1][1] × y

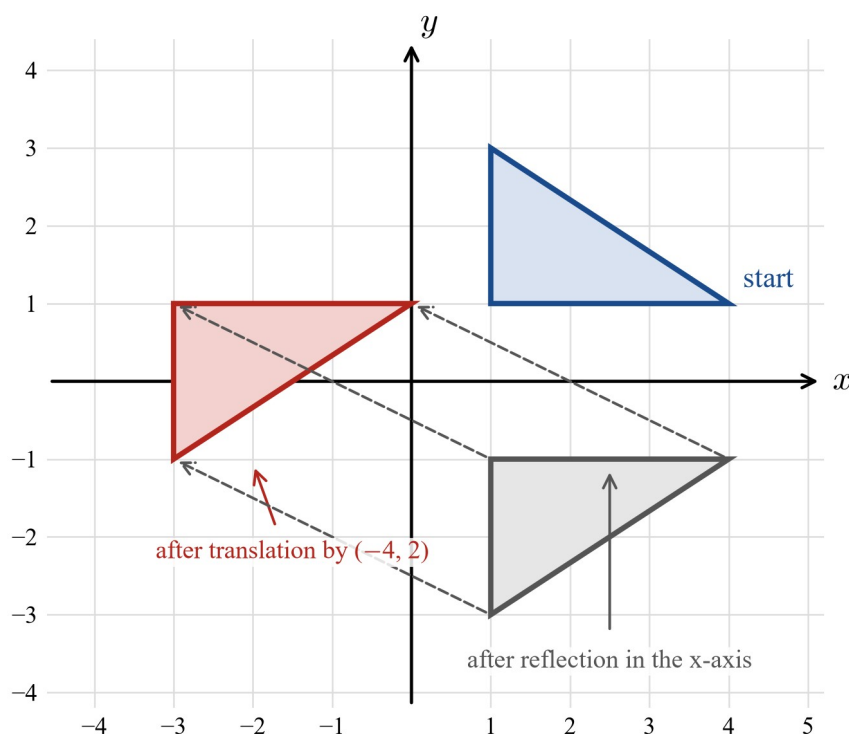
```

Transformation	T	T sends (2, 3) to
reflection in the x-axis	[[1, 0], [0, -1]]	(2, -3)
reflection in the y-axis	[[−1, 0], [0, 1]]	(−2, 3)
rotation 90° anticlockwise	[[0, −1], [1, 0]]	(−3, 2)
rotation 180°	[[−1, 0], [0, −1]]	(−2, −3)
rotation 270° anticlockwise	[[0, 1], [−1, 0]]	(3, −2)
enlargement from the origin, factor k	[[k, 0], [0, k]]	(2k, 3k)

Check the rotation row on (2, 3): $\text{newx} = 0 \times 2 + (-1) \times 3 = -3$ and $\text{newy} = 1 \times 2 + 0 \times 3 = 2$ — the point (−3, 2), exactly the rule $(x, y) \rightarrow (-y, x)$. The elaboration’s word *matrices* is honoured here in the only way this level allows: the 2×2 array is a **data structure inside an algorithm** — four stored numbers and two lines of substitution per point. We never combine two arrays into one, never undo an array, and never use vectors; that is matrix algebra, it belongs to later study, and nothing in this section needs it.

7. Sequences of transformations — and order

A **sequence** of transformations runs one rule, then runs the next on the result. The picture follows the triangle (1, 1), (4, 1), (1, 3) through reflection in the x-axis, then translation by (−4, 2).

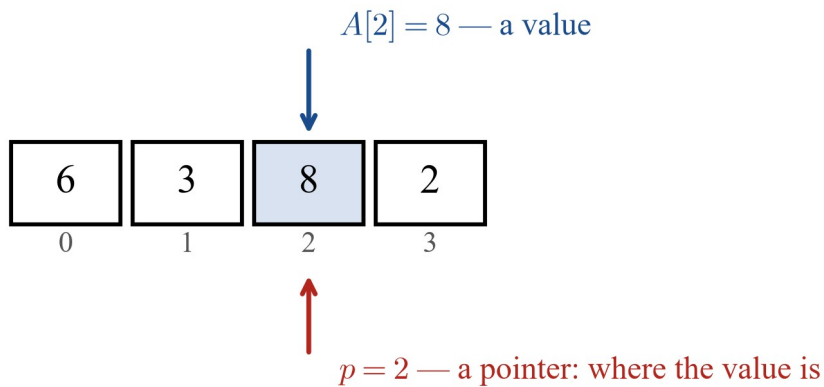


Three triangles on one set of axes: the blue original with vertices (1, 1), (4, 1), (1, 3); the grey image after reflection in the x-axis at (1, −1), (4, −1), (1, −3); and the red final image after translation by (−4, 2) at (−3, 1), (0, 1), (−3, −1), with dashed arrows joining matching grey and red vertices.

Point by point: (1, 1) → (1, −1) → (−3, 1); (4, 1) → (4, −1) → (0, 1); (1, 3) → (1, −3) → (−3, −1). As an algorithm this is two passes over the point list, the second starting from the first’s output. Whether the *order* of two steps matters depends on the steps: usually it does (Worked example 5), and pleasingly, sometimes it does not (Q15).

8. Pointers — storing a position, not a value

A **pointer** is a variable whose value is an index. The pointer and the value it points at are different things:



Four boxes holding 6, 3, 8, 2 with indices 0, 1, 2, 3 below; a blue arrow from above points at the third box, labelled $A[2] = 8$, a value; a red arrow from below points at the same box, labelled $p = 2$, a pointer: where the value is.

With $A = [6, 3, 8, 2]$ and $p = 2$, the line `OUTPUT A[p]` prints 8. The *value* is 8; the *pointer* is 2. Algorithms use pointers whenever the next place to look is itself data — when one cell stores the index of the cell to read next.

The clearest picture at this level is a list kept in two parallel arrays: `data[]` holds the values and `next[]` holds pointers, with one more variable `head` storing the index to start at. The special value `-1` in `next[]` means “nothing after this” — a **sentinel** that ends the walk.

```
p ← head
WHILE p ≠ -1
    OUTPUT data[p]
    p ← next[p]
ENDWHILE
```

Worked example 6 reads a real list this way. Notice what the pointer buys: the values can be visited in any order the `next[]` array encodes, not just index order.

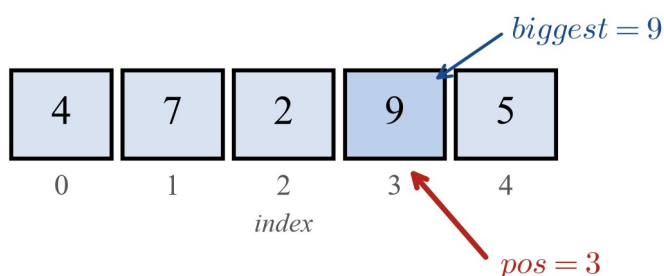
Worked examples

Example 1 — The biggest value and where it is

Trace the algorithm below on $A = [4, 7, 2, 9, 5]$.

```
biggest ← A[0]
pos ← 0
FOR i ← 1 TO LENGTH(A) - 1
    IF A[i] > biggest THEN
        biggest ← A[i]
        pos ← i
    ENDIF
NEXT i
OUTPUT biggest, pos
```

$$A = [4, 7, 2, 9, 5]$$



Five boxes holding 4, 7, 2, 9, 5 with indices 0 to 4 below; a red arrow points up at the box holding 9, labelled $pos = 3$, and a blue arrow points at the same box, labelled $biggest = 9$.

Working	Comment
$biggest = 4, pos = 0$	initialise from index 0, not from nothing
$i = 1: 7 > 4 \rightarrow biggest = 7, pos = 1$	the branch fires
$i = 2: 2 > 7?$ no	nothing changes
$i = 3: 9 > 7 \rightarrow biggest = 9, pos = 3$	the branch fires
$i = 4: 5 > 9?$ no	nothing changes
$\therefore$ OUTPUT gives 9, 3	the value, and the pointer to it ■

The pair ($biggest, pos$) is the point of the algorithm: a value alone is often useless without the position it came from.

Example 2 — Row totals of a grid

The grid of Section 4, $P = [[6, 8, 7, 9], [5, 6, 6, 7], [8, 9, 8, 8]]$, records points scored. Find each row total and the index of the biggest row.

```

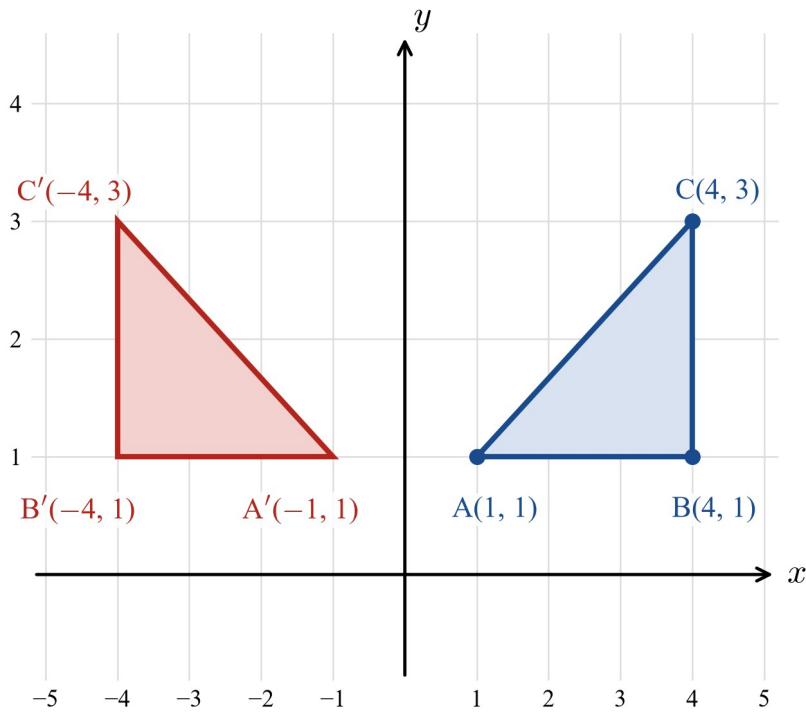
best ← 0
bestrow ← 0
FOR r ← 0 TO 2
  total ← 0
  FOR c ← 0 TO 3
    total ← total + P[r][c]
  NEXT c
  IF total > best THEN
    best ← total
    bestrow ← r
  ENDIF
NEXT r

```

Working	Comment
$r = 0: 6 + 8 + 7 + 9 = 30 \rightarrow best = 30, bestrow = 0$	the inner loop walks across row 0
$r = 1: 5 + 6 + 6 + 7 = 24 \rightarrow 24 > 30?$ no	best unchanged
$r = 2: 8 + 9 + 8 + 8 = 33 \rightarrow best = 33, bestrow = 2$	
$\therefore$ row totals 30, 24, 33; biggest row is index 2 with 33	■

Example 3 — Reflection in the y-axis, by array

Apply $T = [[-1, 0], [0, 1]]$ to the triangle (1, 1), (4, 1), (4, 3), point by point.



Reflection in the y-axis: each x changes sign

A blue triangle with vertices $A(1, 1)$, $B(4, 1)$, $C(4, 3)$ to the right of the y-axis and its red mirror image $A'(-1, 1)$, $B'(-4, 1)$, $C'(-4, 3)$ to the left. Caption: reflection in the y-axis, each x changes sign.

Working

Comment

$(1, 1)$: $\text{newx} = -1 \times 1 + 0 \times 1 = -1$; $\text{newy} = 0 \times 1 + 1 \times 1 = 1$
 $\rightarrow (-1, 1)$

substitute into the two lines

$(4, 1)$: $\text{newx} = -4$, $\text{newy} = 1 \rightarrow (-4, 1)$

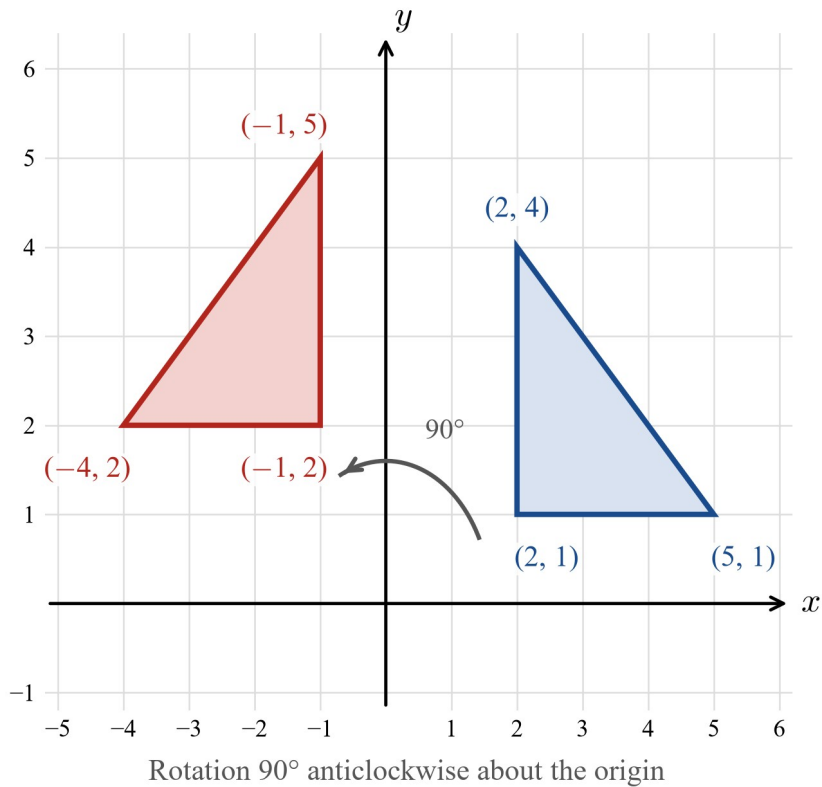
$(4, 3)$: $\text{newx} = -4$, $\text{newy} = 3 \rightarrow (-4, 3)$

$\therefore$ image $(-1, 1)$, $(-4, 1)$, $(-4, 3)$

reflection in the y-axis, as the table in Section 6 promised ■

Example 4 — Rotation 90° anticlockwise, by array

Apply $T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ to the triangle $(2, 1)$, $(5, 1)$, $(2, 4)$.



A blue triangle $(2, 1)$, $(5, 1)$, $(2, 4)$ in the first quadrant and its red image $(-1, 2)$, $(-1, 5)$, $(-4, 2)$ in the second quadrant, with a grey arc and arrowhead at the origin marking the 90 degree turn. Caption: rotation 90 degrees anticlockwise about the origin.

Working

Comment

$(2, 1)$: $\text{newx} = 0 \times 2 + (-1) \times 1 = -1$; $\text{newy} = 1 \times 2 + 0 \times 1 = 2 \rightarrow (-1, 2)$

$(5, 1)$: $\text{newx} = -1$, $\text{newy} = 5 \rightarrow (-1, 5)$

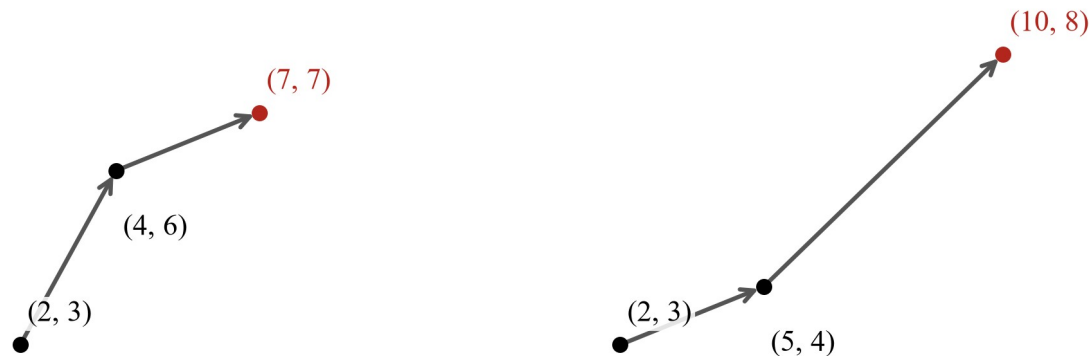
$(2, 4)$: $\text{newx} = -4$, $\text{newy} = 2 \rightarrow (-4, 2)$

$\therefore$ image $(-1, 2)$, $(-1, 5)$, $(-4, 2)$

matches the rule $(x, y) \rightarrow (-y, x)$ at all three points ■

Example 5 — Order matters

Start from $(2, 3)$. Run enlargement factor 2, then translation by $(3, 1)$; then the same two steps in the reverse order.



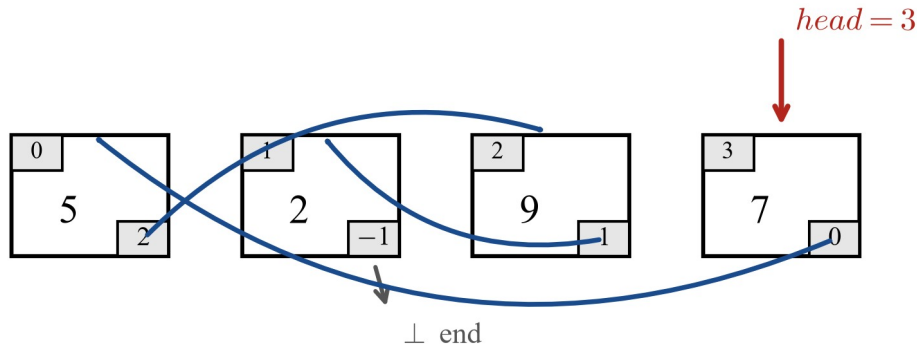
Same start, same two transformations, different ends

Two panels with the same start point (2, 3). Left panel: enlarge to (4, 6), then translate to (7, 7). Right panel: translate to (5, 4), then enlarge to (10, 8). Caption: same start, same two transformations, different ends.

Working	Comment
enlarge first: $(2, 3) \rightarrow (4, 6) \rightarrow (7, 7)$	double, then add
translate first: $(2, 3) \rightarrow (5, 4) \rightarrow (10, 8)$	add, then double — the added part gets doubled too
$\therefore (7, 7) \neq (10, 8)$	the order of the steps is part of the algorithm ■

Example 6 — Reading a list through pointers

data = [5, 2, 9, 7], next = [2, -1, 1, 0], head = 3. Trace the walk of Section 8 and total the values visited.



visit order: 7, 5, 9, 2 — total = 23

each box: index (top-left) · value (centre) · next (bottom-right)

Four boxes indexed 0 to 3 holding the values 5, 2, 9, 7, each with a small next cell at the bottom right holding 2, -1, 1, 0; blue arrows run from each next cell to the top of the box it points at — 3 to 0, 0 to 2, 2 to 1 — and a grey arrow from the -1 cell marks the end; a red arrow labels box 3 with head = 3. Caption: each box shows index top-left, value centre, next bottom-right.

Working	Comment
$p = 3$: OUTPUT data[3] = 7; $p \leftarrow \text{next}[3] = 0$	start at head

$p = 0$: OUTPUT 5; $p \leftarrow \text{next}[0] = 2$
 $p = 2$: OUTPUT 9; $p \leftarrow \text{next}[2] = 1$
 $p = 1$: OUTPUT 2; $p \leftarrow \text{next}[1] = -1$
 $p = -1$: stop
 $\therefore$ visit order 7, 5, 9, 2; total = 23

the sentinel ends the walk

the same four values, in an order only next[] knows ■

Practice questions

Scaffolded

Q1. Write the output of

```

vals ← [2, 4, 7, 5]
FOR i ← 0 TO LENGTH(vals) - 1
    OUTPUT vals[i] + 1
NEXT i
  
```

[Hint: i takes 0, 1, 2, 3 in turn.]

Q2. $A = [2, 5, 1, 6, 3]$. The loop $\text{total} \leftarrow 0$, then FOR $i \leftarrow 0$ TO $\text{LENGTH}(A) - 1$ with $\text{total} \leftarrow \text{total} + A[i]$. List the value of total after each pass, and its final value. *[Hint: keep a running column: 2, then 2 + 5, and so on.]*

Q3. Copy the algorithm of Example 1 with $>$ replaced by $<$ and biggest renamed smallest, and run it on $A = [7, 4, 9, 2, 8]$. What does it output? *[Hint: initialise from index 0, exactly as before.]*

Q4. With $T = [[5, 2, 9], [1, 8, 4], [7, 3, 6]]$, find: (a) $T[0][2]$; (b) $T[2][1]$; (c) the total of row 1. *[Hint: row first, column second, counting from 0.]*

Q5. Reflect the points $(2, 3)$, $(-1, 4)$, $(5, -2)$ in the x-axis. *[Hint: $(x, y) \rightarrow (x, -y)$.]*

Q6. Apply $T = [[-1, 0], [0, 1]]$ to the points $(3, 2)$, $(-2, 5)$, $(0, 4)$. (a) Find the three image points. (b) Name the transformation.

Q7. Rotate the points $(4, 1)$, $(-2, 3)$, $(0, -5)$ by 180° about the origin.

Q8. $\text{data} = [3, 6, 1, 8]$, $\text{next} = [1, 3, -1, 2]$, $\text{head} = 0$. List, in order, the values the walk of Section 8 outputs.

Un scaffolded

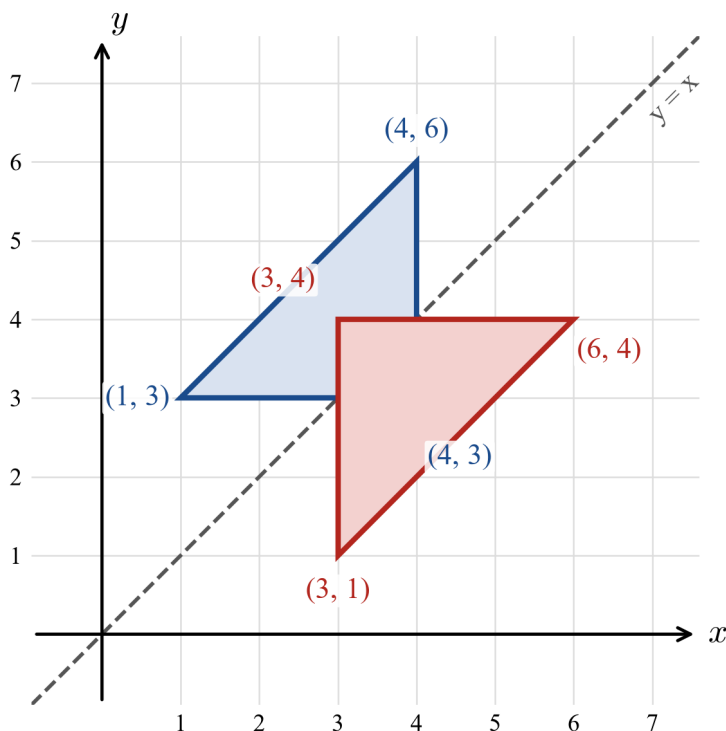
Q9. An algorithm counts the values above 50 in $A = [35, 72, 18, 64, 50, 91]$ and remembers the last index at which a value above 50 occurs. What two numbers does it output? (Careful: 50 itself is not above 50.)

Q10. A student writes FOR $i \leftarrow 0$ TO $\text{LENGTH}(A)$ to walk $A = [8, 3, 6, 1]$. (a) List the values of i the loop takes. (b) Which of them has no cell in A ? (c) Give the correct upper bound.

Q11. For $G = [[4, 9, 2], [7, 1, 6], [3, 8, 5]]$, find the total of each row and the total of each column.

Q12. Enlarge the triangle $(1, 2)$, $(3, 2)$, $(3, 5)$ by factor 2 from the origin, then reflect the result in the x-axis. Give the final vertices.

Q13. Apply $T = [[0, 1], [1, 0]]$ to the triangle $(1, 3)$, $(4, 3)$, $(4, 6)$.



A blue triangle $(1, 3)$, $(4, 3)$, $(4, 6)$ and its red image $(3, 1)$, $(3, 4)$, $(6, 4)$, with the dashed line $y = x$ running between them.

- (a) Find the three image points. (b) Say in words what T does to every point. (c) Name the transformation the picture shows.

Q14. Rotate $(3, -1)$ by 90° anticlockwise about the origin, then translate by $(-4, 2)$.

Q15. On the point $(3, 1)$, show that enlargement factor 2 from the origin and rotation 90° anticlockwise about the origin give the same result in either order, and say in one line why.

Q16. The values $[4, 9, 1, 6]$ are stored at indices 0 to 3. Choose head and a next[] array (with exactly one -1) so that the walk of Section 8 outputs 1, 6, 4, 9.

Q17. data = $[6, 4, 8, 3]$, next = $[3, -1, 0, 1]$. (a) Trace the walk from head = 2 and total the values visited. (b) Trace from head = 0 and total. (c) Which value is never visited in part (b)?

Q18. A swimming carnival stores points in a grid: rows are the four houses (row 0 to row 3) and columns the four events — freestyle, breaststroke, backstroke, relay — in that order. $S = [[8, 5, 6, 7], [6, 7, 4, 8], [7, 4, 8, 5], [5, 8, 7, 3]]$
 (a) Find each house total and the winning house. (b) Find each event total and the event with the biggest total.

Q19. Reflect $(2, 3)$ in the y-axis, then translate by $(5, 0)$.

Q20. data = $[2, 5, 7]$, next = $[2, 0, 0]$, head = 1. (a) Explain in one sentence why the walk of Section 8 never ends with this next[]. (b) Change exactly one entry of next[] so that the walk visits all three values once and stops; give the new walk and its total.

Q21. Translate the triangle $(1, 1)$, $(4, 1)$, $(1, 3)$ by $(-5, 2)$, then reflect the result in the y-axis. Give the final vertices.

Q22. A rotation of 90° anticlockwise about the origin, followed by one translation, sends $(1, 2)$ to $(-4, 1)$. Find the translation, and check your answer on $(3, -2)$.

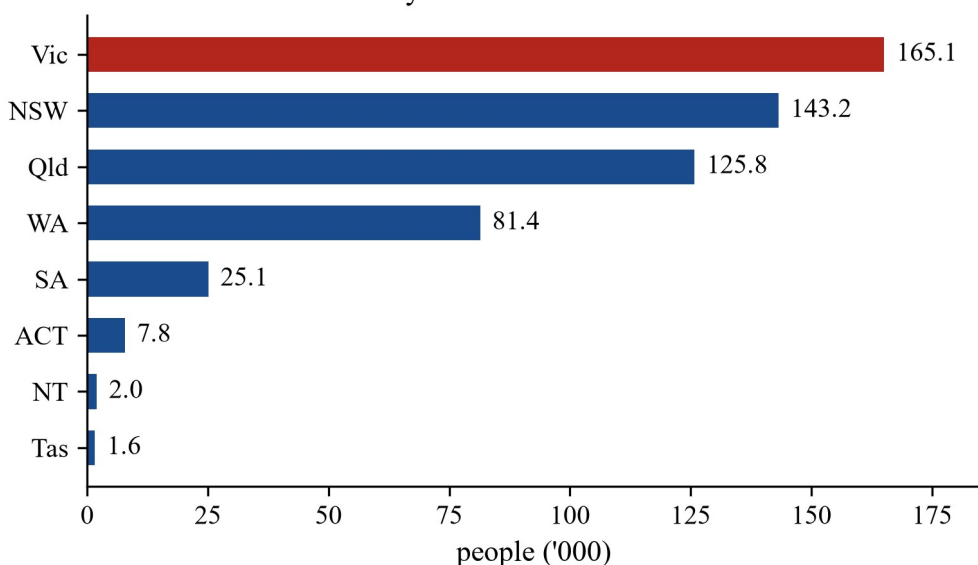
Real context

Q23. The table gives the estimated resident population of each Australian state and territory a year apart, in thousands. Store the two columns as parallel arrays pop23[] and pop24[], indexed 0 to 7 in the table's order.

	Jun 2023 ('000)	Jun 2024 ('000)
NSW	8341.2	8484.4

	Jun 2023 ('000)	Jun 2024 ('000)
Vic	6816.3	6981.4
Qld	5460.5	5586.3
SA	1852.9	1878.0
WA	2883.8	2965.2
Tas	573.8	575.4
NT	253.1	255.1
ACT	466.3	474.1

Population change by state and territory,
year to 30 June 2024



Bar chart of population change by state and territory for the year to 30 June 2024, in thousands of people: Vic 165.1 (bar highlighted), NSW 143.2, Qld 125.8, WA 81.4, SA 25.1, ACT 7.8, NT 2.0, Tas 1.6.

- Write pseudocode that builds a third array `change[]` with `change[i] = pop24[i] - pop23[i]`, then finds and outputs the index of the biggest change (adapt Example 1).
- Run your algorithm on the table: list the eight changes, and give the index and value of the biggest.
- Add the eight changes. Say in words what the total means.
- The biggest *percentage* increase is not the state with the biggest change. Check WA (81.4 on 2883.8) against Vic (165.1 on 6816.3) to one decimal place, and say in one line why the two questions give different winners.

(The eight rows sum to 4.9 ('000) less than the published national total — the difference is other territories. The table is what the algorithm sees.)

Source: Australian Bureau of Statistics, *National, state and territory population, June 2024*, 2024. Retrieved 20 August 2026.

Digital tool task

DIGITAL TOOL — this task assumes a general purpose programming language. Your teacher will nominate the tool. Pseudocode is the paper form of the same algorithm; the tool-free path below is taught in class and is what is assessed.

Input. The triangle $[(2, 1), (5, 1), (2, 4)]$ as a list of points; the rotation array $T = [[0, -1], [1, 0]]$; the translation $(5, -2)$. **Output.** The three image points after rotation then translation, printed by the program — and, on paper, found by hand.

With the tool. Implement the two lines of Section 6 as a small routine, loop over the point list, add the translation, and print each image.

Tool-free path. Rotate first, by the rule $(x, y) \rightarrow (-y, x)$: $(2, 1) \rightarrow (-1, 2)$; $(5, 1) \rightarrow (-1, 5)$; $(2, 4) \rightarrow (-4, 2)$. Then add $(5, -2)$: $(-1, 2) \rightarrow (4, 0)$; $(-1, 5) \rightarrow (4, 3)$; $(-4, 2) \rightarrow (1, 0)$. $\therefore$ the images are $(4, 0)$, $(4, 3)$, $(1, 0)$ — exactly what a correct program prints, because the program is the same two substitutions. ■

Answers

Q1. 3, 5, 8, 6. **Q2.** 2, 7, 8, 14, 17; final value 17. **Q3.** smallest = 2, pos = 3. **Q4.** (a) 9. (b) 3. (c) 13. **Q5.** (2, -3), (-1, -4), (5, 2). **Q6.** (a) (-3, 2), (2, 5), (0, 4). (b) reflection in the y-axis. **Q7.** (-4, -1), (2, -3), (0, 5). **Q8.** 3, 6, 8, 1. **Q9.** count = 3; last index = 5. **Q10.** (a) 0, 1, 2, 3, 4. (b) $i = 4$ — $A[4]$ does not exist. (c) $\text{LENGTH}(A) - 1 = 3$. **Q11.** rows: 15, 14, 16; columns: 14, 18, 13. **Q12.** (2, -4), (6, -4), (6, -10). **Q13.** (a) (3, 1), (3, 4), (6, 4). (b) it swaps the coordinates: $(x, y) \rightarrow (y, x)$. (c) reflection in the line $y = x$. **Q14.** (-3, 5). **Q15.** both orders give (-2, 6); each step doubles the distance from O and turns about the same centre O. **Q16.** head = 2; next = [1, -1, 3, 0]. **Q17.** (a) 8, 6, 3, 4; total 21. (b) 6, 3, 4; total 13. (c) 8. **Q18.** (a) 26, 25, 24, 23 — row 0 wins. (b) 26, 24, 25, 23 — freestyle (column 0) is biggest. **Q19.** (3, 3). **Q20.** (a) from head = 1 the walk goes $1 \rightarrow 0 \rightarrow 2 \rightarrow 0 \rightarrow 2 \rightarrow \dots$ and never meets -1. (b) next[2] $\leftarrow -1$; walk 5, 2, 7; total 14. **Q21.** (4, 3), (1, 3), (4, 5). **Q22.** translation (-2, 0); check: $(3, -2) \rightarrow (2, 3) \rightarrow (0, 3)$. ✓ **Q23.** (a) FOR $i \leftarrow 0$ TO 7 with $\text{change}[i] \leftarrow \text{pop24}[i] - \text{pop23}[i]$, then the Example 1 walk on $\text{change}[]$. (b) changes 143.2, 165.1, 125.8, 25.1, 81.4, 1.6, 2.0, 7.8; biggest at index 1 (Vic), 165.1 ('000). (c) 552.0 ('000) — about 552 000 people added across the states and territories in the year. (d) $\text{WA} \approx 2.8\%$, $\text{Vic} \approx 2.4\%$; biggest change and fastest growth are different questions. **Digital task.** (4, 0), (4, 3), (1, 0).

Fully-worked solutions

Q1. $i = 0: 2 + 1 = 3$; $i = 1: 4 + 1 = 5$; $i = 2: 7 + 1 = 8$; $i = 3: 5 + 1 = 6$. The output is 3, 5, 8, 6.

Q2. After each pass: $2; 2 + 5 = 7$; $7 + 1 = 8$; $8 + 6 = 14$; $14 + 3 = 17$. Final value 17.

Q3. Start smallest = 7, pos = 0. $i = 1: 4 < 7 \rightarrow \text{smallest} = 4$, pos = 1. $i = 2: 9 < 4?$ no. $i = 3: 2 < 4 \rightarrow \text{smallest} = 2$, pos = 3. $i = 4: 8 < 2?$ no. Output 2, 3.

Q4. (a) row 0, column 2 $\rightarrow 9$. (b) row 2, column 1 $\rightarrow 3$. (c) $1 + 8 + 4 = 13$.

Q5. Change the sign of y: (2, -3), (-1, -4), (5, 2).

Q6. (a) $\text{newx} = -x$ and $\text{newy} = y$: (-3, 2), (2, 5), (0, 4). (b) every x changes sign: reflection in the y-axis.

Q7. $(x, y) \rightarrow (-x, -y)$: (-4, -1), (2, -3), (0, 5).

Q8. $p = 0 \rightarrow \text{OUTPUT } 3$; $p \leftarrow 1 \rightarrow \text{OUTPUT } 6$; $p \leftarrow 3 \rightarrow \text{OUTPUT } 8$; $p \leftarrow 2 \rightarrow \text{OUTPUT } 1$; $p \leftarrow -1$ stops. Order: 3, 6, 8, 1.

Q9. Above 50: 72 (index 1), 64 (index 3), 91 (index 5); 50 itself fails the test. count = 3 and the last such index is 5.

Q10. (a) $i = 0, 1, 2, 3, 4$. (b) $i = 4$: the last legal index is $\text{LENGTH}(A) - 1 = 3$, so $A[4]$ does not exist. (c) FOR $i \leftarrow 0$ TO $\text{LENGTH}(A) - 1$.

Q11. Rows: $4 + 9 + 2 = 15$; $7 + 1 + 6 = 14$; $3 + 8 + 5 = 16$. Columns: $4 + 7 + 3 = 14$; $9 + 1 + 8 = 18$; $2 + 6 + 5 = 13$.

Q12. Enlarge: (2, 4), (6, 4), (6, 10). Reflect in the x-axis: (2, -4), (6, -4), (6, -10).

Q13. (a) $\text{newx} = y$ and $\text{newy} = x$: (3, 1), (3, 4), (6, 4). (b) T swaps the two coordinates. (c) reflection in the line $y = x$ — the picture shows each point and its image the same distance either side of the dashed line.

Q14. $(3, -1) \rightarrow (1, 3)$ by $(x, y) \rightarrow (-y, x)$; then $(1, 3) + (-4, 2) = (-3, 5)$.

Q15. Enlarge then rotate: $(3, 1) \rightarrow (6, 2) \rightarrow (-2, 6)$. Rotate then enlarge: $(3, 1) \rightarrow (-1, 3) \rightarrow (-2, 6)$. Same end, because both steps act about the same centre O — doubling and turning a quarter-turn commute.

Q16. The visit order 1, 6, 4, 9 uses indices 2, 3, 0, 1. So head = 2, next[2] = 3, next[3] = 0, next[0] = 1 and next[1] = -1: next = [1, -1, 3, 0].

Q17. (a) $p = 2 \rightarrow 8$; $p \leftarrow \text{next}[2] = 0 \rightarrow 6$; $p \leftarrow \text{next}[0] = 3 \rightarrow 3$; $p \leftarrow \text{next}[3] = 1 \rightarrow 4$; $p \leftarrow -1$ stops. Order 8, 6, 3, 4; total 21. (b) $p = 0 \rightarrow 6$; $p \leftarrow 3 \rightarrow 3$; $p \leftarrow 1 \rightarrow 4$; $p \leftarrow -1$ stops. Order 6, 3, 4; total 13. (c) index 2 — the value 8 — is never reached from head = 0.

Q18. (a) Row totals: $8 + 5 + 6 + 7 = 26$; $6 + 7 + 4 + 8 = 25$; $7 + 4 + 8 + 5 = 24$; $5 + 8 + 7 + 3 = 23$. Row 0 wins with 26. (b) Column totals: freestyle $8 + 6 + 7 + 5 = 26$; breaststroke $5 + 7 + 4 + 8 = 24$; backstroke $6 + 4 + 8 + 7 = 25$; relay $7 + 8 + 5 + 3 = 23$. Freestyle is the biggest event.

Q19. $(2, 3) \rightarrow (-2, 3) \rightarrow (3, 3)$.

Q20. (a) `next[]` sends $1 \rightarrow 0$, $0 \rightarrow 2$ and $2 \rightarrow 0$, so the walk circles $0 \rightarrow 2 \rightarrow 0 \rightarrow 2 \dots$ and never meets the sentinel -1 . (b) Set `next[2]  $\leftarrow -1$` . Then `head = 1` gives 5, 2, 7 and stops; total 14.

Q21. Translate: $(-4, 3)$, $(-1, 3)$, $(-4, 5)$. Reflect in the y-axis (change the sign of x): $(4, 3)$, $(1, 3)$, $(4, 5)$.

Q22. Rotation gives $(1, 2) \rightarrow (-2, 1)$. To reach $(-4, 1)$ the translation must be $(-4 - (-2), 1 - 1) = (-2, 0)$. Check on $(3, -2)$: rotation gives $(2, 3)$, then $(2, 3) + (-2, 0) = (0, 3)$. ✓

Q23. (a) FOR `i  $\leftarrow$  0 TO 7`: `change[i]  $\leftarrow$  pop24[i] - pop23[i]`; NEXT `i`; then the biggest-value walk of Example 1 run on `change[]`. (b) NSW 143.2; Vic 165.1; Qld 125.8; SA 25.1; WA 81.4; Tas 1.6; NT 2.0; ACT 7.8. The biggest is 165.1 at index 1 (Vic). (c) $143.2 + 165.1 + 125.8 + 25.1 + 81.4 + 1.6 + 2.0 + 7.8 = 552.0$ ('000): the states and territories together grew by about 552 000 people in the year. (d) WA: $81.4/_{2883.8} \approx 2.8\%$; Vic: $165.1/_{6816.3} \approx 2.4\%$. Vic added the most people; WA grew fastest for its size. “Biggest change” and “fastest growth” ask different questions.

Digital task. Rotation by $(x, y) \rightarrow (-y, x)$: $(-1, 2)$, $(-1, 5)$, $(-4, 2)$. Translation by $(5, -2)$: $(4, 0)$, $(4, 3)$, $(1, 0)$. A correct program prints exactly these, because the program is the same substitutions.

Teacher note

- **Prior knowledge rehearsed (D3):** VC2M9SP03 (pseudocode, loops, tracing) is revised briefly in Theory §1–§3; VC2M7SP03 (moving point sets by coordinates) and VC2M9SP02 (enlargement) are quoted as rules in §5, not re-taught; substitution from 1E (VC2M10A05) is what every trace executes.
- **Scope check (D6):** the 2×2 array of §6 is a data structure inside an algorithm — four stored numbers and two lines of substitution per point. No combining two arrays into one, no undoing an array, no vectors; sequences are at most two steps; pointers stay at parallel arrays with a -1 sentinel. Nothing here is VCE matrix content.
- **Tool classes (D13):** the CD names the tool class itself — pseudocode or a general purpose programming language. The single digital task assumes only that class, shows its input and output, and its tool-free path (the same substitutions on paper) is taught and assessed; pseudocode needs no tool at all. No CAS anywhere in 3D.
- **Assessment:** 3D sits in the T2 window (Chapters 2–3). Traces are marked line by line — the table of current values is the evidence, not the final number alone.
- **VCE destinations (§5 forward map):** VC2M10A06 is named by MM U3&4 AoS 2 — “Use a 2×2 matrix (a 2D array) to apply a single linear transformation to a point” and “Recognise the role of a pointer in traversing or modifying a data structure”. Stated honestly: no VCE mathematics study has an algorithms-and-data-structures Area of Study; the nearest home is SM Units 1&2 and SM U3&4 AoS 1 ‘Discrete mathematics’; for every other pathway 3D is terminal, and algorithmic thinking otherwise reaches VCE embedded inside other outcomes.

Test T2 — Equations, graphs, models and algorithms (Level 10)

Mathematics 10 · Chapter 3 Graphs, models and algorithms · Topic test T2 (Level 10) · Chapters 2–3 · 55 marks · 60 minutes

This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 10 achievement standard, not against a mark on this paper.

Covers: Chapter 2 Equations and inequalities (2A–2F) and Chapter 3 Graphs, models and algorithms (3A–3D). **CDs scored:** the eleven content descriptions of Chapters 2 and 3, at 5 marks each:

CD code	Subtopic	Marks	Items
VC2M10A07	2A	5	Q2 (4), Q9(a) (1)
VC2M10A12	2A	5	Q3 (3), Q9(b) (2)
VC2M10A08	2B	5	Q1(a) (1), Q5 (4)
VC2M10A09	2C	5	Q6 (5)
VC2M10A13	2D	5	Q1(b) (1), Q4 (4)
VC2M10A14	2E	5	Q11 (5)
VC2M10A16	2F	5	Q7 (2), Q13 (3)
VC2M10A10	3A	5	Q1(c) (1), Q8 (4)
VC2M10A11	3B	5	Q12 (5)
VC2M10A15	3C	5	Q10 (5)
VC2M10A06	3D	5	Q14 (5)

The two codes of subtopic 2A are scored separately: VC2M10A07 items are tagged VC2M10A07 and VC2M10A12 items are tagged VC2M10A12; no item carries both codes.

Timing. 60 minutes for the whole test: Parts A and B together.

Equipment and calculator rules.

- **Part A — technology-free.** No calculator and no digital tool. Give exact values unless a question asks for a decimal answer. This part is collected before Part B is issued.
- **Part B — technology-active.** You may use the technology your teacher nominates: a scientific calculator, a spreadsheet, a graphing tool, dynamic geometry software or a general purpose programming language. **No CAS.** Everything in Part B can also be done by hand: a hand-calculated, hand-drawn response is a complete response.

Answer every question. Show your working clearly wherever marks are allocated to method — one step per line. Every word on this test was introduced in Chapter 2 or Chapter 3.

Part A — Technology-free (25 marks)

No calculator. No digital tools. Exact values unless a question says otherwise.

Q1. (3 marks) Multiple choice. For each part, choose A, B, C or D.

(a) VC2M10A08 The solution of the inequality $-2x \geq 6$ is

A $x \geq -3$

B $x \leq -3$

C $x \geq 3$

D $x \leq 3$

(b) VC2M10A13 Which equation has solutions $x = -2$ and $x = 5$?

A $x^2 - 3x - 10 = 0$

B $x^2 + 3x - 10 = 0$

C $x^2 - 10x - 3 = 0$

D $x^2 + 10x - 3 = 0$

(c) VC2M10A10 The gradient of a line perpendicular to $y = 5x - 2$ is

A 5

B -5

C $\frac{1}{5}$

D $-\frac{1}{5}$

Q2. VC2M10A07 (4 marks) Solve each equation, showing every step.

(a) $4(x - 3) = 2x + 10$

(b) The formula $v = u + at$ connects the quantities v , u , a and t . Solve the formula for t when $v = 23$, $u = 5$ and $a = 4.5$.

Q3. VC2M10A12 (3 marks) Solve each equation, showing every step.

(a) $\frac{x}{5} + 3 = 7$

(b) $\frac{3x - 2}{4} = \frac{x + 3}{2}$

Q4. VC2M10A13 (4 marks)

(a) Solve $x^2 - 7x + 12 = 0$ by factorising, and name the law that lets you read the solutions from your factors.

(b) Solve $x^2 + 6x + 2 = 0$, giving your answers in exact surd form. You may use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Q5. VC2M10A08 (4 marks) Solve each inequality, and graph its solutions on a number line.

(a) $5x - 7 \leq 18$

(b) $-3x < 12$ — show clearly the step where the inequality symbol flips.

Q6. VC2M10A09 (5 marks) As at 31 December 2025, Victoria's estimated resident population was 7.12 million people and was growing by about 117 000 people each year. New South Wales' population was 8.64 million people and was growing by about 105 000 people each year. (Figures rounded from the Australian Bureau of Statistics; the source line is below.) Model both populations as growing linearly: with t the number of years after 31 December 2025 and populations in millions, Victoria is modelled by

$$V = 7.12 + 0.117t.$$

(a) Write the corresponding equation for the New South Wales population N .

- (b) Solve the pair of simultaneous equations to find when the model predicts the two populations are equal. Show every step.
- (c) In which calendar year does the model predict Victoria's population passes that of New South Wales?
- (d) Give one reason why a constant-growth model cannot be trusted this far ahead.

Source: Australian Bureau of Statistics, *National, state and territory population* (estimated resident populations as at 31 December 2025: Victoria 7 121 900, growth 117 300; New South Wales 8 641 100, growth 104 600; figures in the question rounded), 2026. Retrieved 23 August 2026.

Q7. VC2M10A16 (2 marks) Min solves the equation $x^2 = 2^x$ and finds two solutions, $x = 2$ and $x = 4$.

- (a) Verify that $x = 4$ is a solution.
- (b) Evaluate both sides of the equation at $x = -1$ and at $x = 0$, and use your results to explain whether Min has found every solution.

Total for Part A: ☐ out of 25 marks

Part B — Technology-active (30 marks)

You may use the technology your teacher nominates: a scientific calculator, a spreadsheet, a graphing tool, dynamic geometry software or a general purpose programming language. No CAS. Show your working — technology finds values; your working shows where they came from.

Q8. VC2M10A10 (4 marks) The line L passes through $A(2, 1)$ and $B(6, 9)$.

- (a) Find the gradient of L .
- (b) Find the equation of the line parallel to L that passes through $(0, -3)$.
- (c) Find the equation of the line perpendicular to L that passes through $(4, 0)$.

Q9. (3 marks)

- (a) VC2M10A07 A train travels at a constant average speed of 80 km/h, so the distance d km it covers in t hours is given by $d = 80t$. Solve this equation for t when $d = 220$, and give the time in hours and minutes.
- (b) VC2M10A12 Solve $\frac{2x-1}{4} + \frac{x+2}{3} = 3$.

Q10. VC2M10A15 (5 marks) The Reserve Bank of Australia's one-year term deposit rate for deposits of \$10 000 was 5.00 % p.a. as at July 2026. Suppose \$10 000 is invested at that rate, compounded yearly.

- (a) Treating the first year as simple interest, find the interest earned in year 1 and the balance at the end of year 1.
- (b) In year 2 the interest is paid on the new balance. Find the interest earned in year 2 and the balance at the end of year 2.
- (c) Each year's balance is found by multiplying the previous balance by the same number. State that number, and complete the compound interest formula for this investment: $A = 10\,000 \times \dots^n$, where n is the number of years.
- (d) Use your formula to find the balance after 6 years. Give your answer to the cent.
- (e) Find the balance after 6 years under simple interest at 5.00 % p.a., and hence state how much more the compound interest investment earns over the 6 years.

Source: Reserve Bank of Australia, *Statistical Table F4 — Retail Deposit and Investment Rates* (one-year term deposit rate for \$10 000, as at July 2026), 2026, published 10 August 2026. Retrieved 20 August 2026.

Q11. VC2M10A14 (5 marks)

- (a) Solve $5^x = 125$.
- (b) Solve $4^x = 8^{x-2}$. (Hint: write both sides as powers of 2.)
- (c) Solve $3^x = 50$ by guess-check-and-refine, correct to 1 decimal place. Record each trial.

Q12. VC2M10A11 (5 marks)

- (a) Consider the relation $y = x^2 - 4x + 3$.
- (i) Factorise $x^2 - 4x + 3$, and hence find the x -intercepts of its graph.
- (ii) Find the turning point of the graph.
- (iii) Sketch the graph on grid paper, showing the x -intercepts, the y -intercept and the turning point.
- (b) A circle has equation $x^2 + y^2 = 16$. State its centre and its radius.
- (c) State the equations of the two asymptotes of the graph of $y = \frac{8}{x}$.

Q13. VC2M10A16 (3 marks) Consider the equation $2^x = 3x + 1$.

- (a) Show that $x = 0$ is a solution.
- (b) A second solution lies between $x = 3$ and $x = 4$. Use systematic guess-check-and-refine, recording each trial in a table, to find this solution correct to 1 decimal place.

Q14. VC2M10A06 (5 marks) The two-dimensional array $P = \begin{bmatrix} 1, 2 \\ 3, 1 \\ 2, 4 \end{bmatrix}$ stores three points: for the point at index i , the entry $P[i][0]$ is its x -coordinate and $P[i][1]$ is its y -coordinate. Indices count from 0. The two-dimensional array $T = \begin{bmatrix} 0, -1 \\ 1, 0 \end{bmatrix}$ is applied to every point by this algorithm:

```
FOR i ← 0 TO 2
  x ← P[i][0]
  y ← P[i][1]
  newx ← T[0][0] × x + T[0][1] × y
  newy ← T[1][0] × x + T[1][1] × y
  OUTPUT newx, newy
NEXT i
```

You may trace the algorithm by hand, or run it in the programming language your teacher nominates.

- (a) Trace the first pass of the loop, for the point $(1, 2)$: substitute into the `newx` and `newy` lines, showing each step, and state the first pair the algorithm outputs.
- (b) Write the other two pairs the algorithm outputs.
- (c) Name the geometric transformation this algorithm applies to every point.

Total for Part B: □ out of 30 marks

Total for Test T2: □ out of 55 marks

Solutions

Teacher-facing. This is a school-designed classroom test, not a VCAA instrument and not an examination (§7, D19). Marks exist for a school's own reporting; achievement in the Victorian Curriculum is reported against the Level 10 achievement standard. Score method marks for working shown, one step per line (R5); accept any correct method, including methods different from the one shown. For Q5 and Q12(a)(iii) the student's own drawing is the answer; the descriptions below state what a correct drawing shows.

Achievement-standard extracts. The mark scheme is written in the language of the achievement-standard extracts carried in the authority file (`maths_10.json`), quoted verbatim here and tagged to each scored item below:

AS-1 — *They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, algebraically and graphically, with and without the use of digital tools, and justify solutions.* — VC2M10A07, VC2M10A08, VC2M10A09, VC2M10A10, VC2M10A12, VC2M10A13 (shared wording; the six CDs are still scored separately, §7)

AS-2 — *They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems.* — VC2M10A11, VC2M10A14

AS-3 — *Students use mathematical modelling to solve problems involving growth and decay in financial and other applied situations, applying linear, quadratic and exponential functions as appropriate, and solve related equations, numerically and graphically.* — VC2M10A15

AS-4 — *They make and test conjectures involving functions and relations using digital tools.* — VC2M10A16

AS-5 — *Students can design and implement simple algorithms using pseudocode or other general purpose programming language.* — VC2M10A06

Coverage. Every CD of Chapters 2 and 3 scores 5 marks (11 CDs, 55 marks). The 2A pair is scored per code, never on one shared item (D8, §7.1): VC2M10A07 — Q2 (4) + Q9(a) (1) = 5; VC2M10A12 — Q3 (3) + Q9(b) (2) = 5.

Answers

Final values only.

Q1. (a) B; (b) A; (c) D. **Q2.** (a) $x = 11$; (b) $t = 4$. **Q3.** (a) $x = 20$; (b) $x = 8$. **Q4.** (a) $x = 3$ or $x = 4$; (b) $x = -3 + \sqrt{7}$ or $x = -3 - \sqrt{7}$. **Q5.** (a) $x \leq 5$; (b) $x > -4$. **Q6.** (a) $N = 8.64 + 0.105t$; (b) $t = \frac{380}{3} \approx 126.7$ years; (c) during 2152; (d) any reasoned critique of constant growth. **Q7.** (a) $4^2 = 16 = 2^4$; (b) there is a further solution between -1 and 0 ; Min has not found every solution. **Q8.** (a) 2; (b) $y = 2x - 3$; (c) $y = -\frac{1}{2}x + 2$. **Q9.** (a) $t = 2.75$ h = 2 h 45 min; (b) $x = 3.1$. **Q10.** (a) \$500.00; \$10 500.00; (b) \$525.00; \$11 025.00; (c) 1.05, $A = 10\,000 \times 1.05^n$; (d) \$13 400.96; (e) \$13 000.00; \$400.96 more. **Q11.** (a) $x = 3$; (b) $x = 6$; (c) $x \approx 3.6$. **Q12.** (a) $(x - 1)(x - 3)$; $(1, 0)$ and $(3, 0)$; $(2, -1)$; (b) centre $(0, 0)$, radius 4; (c) $x = 0$ and $y = 0$. **Q13.** (a) $2^0 = 1 = 3 \times 0 + 1$; (b) $x \approx 3.5$. **Q14.** (a) $(-2, 1)$; (b) $(-1, 3)$ and $(-4, 2)$; (c) rotation 90° anticlockwise about the origin.

Fully-worked solutions

Q1. VC2M10A08, VC2M10A13, VC2M10A10 — AS-1. 1 mark each; no method required.

- (a) **B.** Divide both sides by -2 and flip the symbol: $x \leq \frac{6}{-2} = -3$. Option A is the error of not flipping; C and D divide without handling the sign.
- (b) **A.** $(x + 2)(x - 5) = x^2 - 3x - 10$, whose roots are -2 and 5 . Option B expands $(x - 2)(x + 5)$ — roots 2 and -5 , the signs swapped.
- (c) **D.** Perpendicular gradients multiply to -1 : $m = -\frac{1}{5}$, since $5 \times \left(-\frac{1}{5}\right) = -1$.

Q2. VC2M10A07 — AS-1. 4 marks.

- (a) 2 marks: 1 for expanding and collecting, 1 for the correct value.

$$4(x - 3) = 2x + 10$$

$$4x - 12 = 2x + 10$$

$$2x = 22$$

$$x = 11$$

Check: $4(11 - 3) = 32$ and $2 \times 11 + 10 = 32$. ✓

- (b) 2 marks: 1 for substituting and rearranging, 1 for the correct value. A linear equation derived from a formula, solved for the number t .

$$v = u + at$$

$$23 = 5 + 4.5t$$

$$18 = 4.5t$$

$$t = 4$$

Q3. VC2M10A12 — AS-1. 3 marks.

- (a) 1 mark.

$$\frac{x}{5} + 3 = 7$$

$$\frac{x}{5} = 4$$

$$x = 20$$

(b) 2 marks: 1 for clearing the fractions, 1 for the correct value.

$$\frac{3x-2}{4} = \frac{x+3}{2}$$

$$2(3x-2) = 4(x+3)$$

$$6x-4 = 4x+12$$

$$2x = 16$$

$$x = 8$$

$$\text{Check: } \frac{3 \times 8 - 2}{4} = \frac{22}{4} = 5.5 \text{ and } \frac{8+3}{2} = 5.5. \checkmark$$

Q4. VC2M10A13 — AS-1. 4 marks.

(a) 2 marks: 1 for the factorisation, 1 for both solutions with the null factor law named.

$$x^2 - 7x + 12 = (x-3)(x-4)$$

$$(x-3)(x-4) = 0$$

$x-3=0$ or $x-4=0$ — the **null factor law**: a product is zero only when a factor is zero.

$$x=3 \text{ or } x=4$$

(b) 2 marks: 1 for the correct substitution ($b^2 - 4ac = 28$), 1 for the exact surd answers. With $a=1$, $b=6$, $c=2$:

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$x = \frac{-6 \pm \sqrt{28}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{7}}{2}$$

$$x = -3 \pm \sqrt{7}$$

The exact answers are $x = -3 + \sqrt{7}$ and $x = -3 - \sqrt{7}$; a decimal approximation is not required and earns no credit here (Part A is exact-value work).

Q5. VC2M10A08 — AS-1. 4 marks; the number line is the assessed representation (2B, D12).

(a) 2 marks: 1 for the solution, 1 for the number line.

$$5x - 7 \leq 18$$

$$5x \leq 25$$

$$x \leq 5$$

Number line: a **closed dot** at 5 (since 5 itself is a solution) with the blue arrow pointing **left**, covering every number less than 5.

(b) 2 marks: 1 for dividing by -3 **with the symbol flipped**, 1 for the number line.

$$-3x < 12$$

$x > -4$ — dividing both sides by the negative number -3 flips the symbol.

Number line: an **open dot** at -4 (since -4 is not a solution) with the blue arrow pointing **right**.

Q6. VC2M10A09 — AS-1. 5 marks; the real-context multi-part question of Part A (§7.1), on ABS data.

(a) 1 mark. Growth of 105 000 people per year is 0.105 million per year:

$$N = 8.64 + 0.105t$$

(b) 2 marks: 1 for equating the two expressions, 1 for the correct value. Solve the pair algebraically:

$$7.12 + 0.117t = 8.64 + 0.105t$$

$$0.117t - 0.105t = 8.64 - 7.12$$

$$0.012t = 1.52$$

$$t = \frac{1.52}{0.012} = \frac{1520}{12} = \frac{380}{3} \approx 126.7$$

The model predicts the populations are equal about 126.7 years after 31 December 2025.

(c) 1 mark. $2025 + 126.7 \approx 2151.7$: the crossing falls **during the year 2152**. Accept 2152; accept “about 127 years after 2025” with the year named.

(d) 1 mark. Any reasoned critique of the constant-growth assumption over more than a century — for example: births, deaths and migration change with policy and circumstance, so neither growth rate stays fixed for 127 years; or the linear model ignores that NSW’s growth rate is itself falling (1.2% and declining in recent ABS releases). The judgement that earns the mark is that the *assumption*, not the algebra, is what cannot be trusted.

Q7. VC2M10A16 — AS-4. 2 marks; the consideration of whether every solution has been found, technology-free.

(a) 1 mark.

$$4^2 = 16 \text{ and } 2^4 = 16, \text{ so both sides are equal: } x = 4 \text{ is a solution. } \checkmark$$

(b) 1 mark. At $x = -1$: $x^2 = 1$ but $2^x = 0.5$ — here x^2 is above. At $x = 0$: $x^2 = 0$ but $2^x = 1$ — here 2^x is above. The two sides switch which is the greater between $x = -1$ and $x = 0$, so their graphs cross in between: there is a third solution, and Min has **not** found every solution. (It lies near $x \approx -0.77$, though finding it is not required.)

Q8. VC2M10A10 — AS-1. 4 marks.

(a) 1 mark.

$$m = \frac{9-1}{6-2} = \frac{8}{4} = 2$$

(b) 1 mark. Parallel lines have the same gradient, so $m = 2$, and the line passes through $(0, -3)$ — its y -intercept — so $c = -3$:

$$y = 2x - 3$$

(c) 2 marks: 1 for the perpendicular gradient, 1 for the equation. Perpendicular gradients multiply to -1 : the new gradient is $-\frac{1}{2}$.

$$y = -\frac{1}{2}x + c$$

$$0 = -\frac{1}{2} \times 4 + c$$

$$0 = -2 + c$$

$$c = 2$$

$$\therefore y = -\frac{1}{2}x + 2. \text{ Check: when } x = 4, y = -2 + 2 = 0. \checkmark$$

Q9. VC2M10A07 (1), VC2M10A12 (2) — AS-1. 3 marks; the 2A pair stays separately tagged (D8).

(a) 1 mark. A linear equation derived from the formula $d = 80t$:

$$220 = 80t$$

$$t = \frac{220}{80} = 2.75$$

2.75 h is 2 h 45 min.

(b) 2 marks: 1 for clearing the fractions, 1 for the correct value. Multiply every term by 12:

$$3(2x - 1) + 4(x + 2) = 36$$

$$6x - 3 + 4x + 8 = 36$$

$$10x + 5 = 36$$

$$10x = 31$$

$$x = 3.1$$

$$\text{Check: } \frac{2 \times 3.1 - 1}{4} + \frac{3.1 + 2}{3} = \frac{5.2}{4} + \frac{5.1}{3} = 1.3 + 1.7 = 3. \checkmark$$

Q10. VC2M10A15 — AS-3. 5 marks; the real-context multi-part question of Part B (§7.1), building the compound interest formula as repeated simple interest — what the CD itself names. The rate is the RBA's published one-year term deposit rate for \$10 000, as at July 2026.

(a) 1 mark. Year 1 is simple interest on the principal:

$$I = 10\,000 \times 0.05 = \$500.00$$

Balance at the end of year 1: $10\,000 + 500 = \$10\,500.00$.

(b) 1 mark. Year 2's principal is the new balance — interest is paid on interest:

$$I = 10\,500 \times 0.05 = \$525.00$$

Balance at the end of year 2: $10\,500 + 525 = \$11\,025.00$.

(c) 1 mark. Each balance is 1.05 times the one before (adding 5% is multiplying by 1.05), so

$$A = 10\,000 \times 1.05^n.$$

(d) 1 mark.

$$A = 10\,000 \times 1.05^6 = 10\,000 \times 1.340095640625 = 13\,400.956\dots$$

∴ the balance after 6 years is \$13 400.96.

(e) 1 mark. Simple interest pays the same \$500 every year:

$I = 10\,000 \times 0.05 \times 6 = \$3\,000.00$, so the simple balance is \$13 000.00.

$13\,400.96 - 13\,000.00 = 400.96$: compound interest earns \$400.96 **more** — the interest that interest earned.

Q11. VC2M10A14 — AS-2. 5 marks.

(a) 1 mark.

$$5^x = 125 = 5^3$$

$$x = 3$$

(b) 2 marks: 1 for writing both sides as powers of 2, 1 for the solution.

$$4^x = 8^{x-2}$$

$$(2^2)^x = (2^3)^{x-2}$$

$$2^{2x} = 2^{3x-6}$$

$2x = 3x - 6$ — equal bases, so the exponents are equal.

$$x = 6$$

Check: $4^6 = 4096$ and $8^4 = 4096$. ✓

(c) 2 marks: 1 for trapping between whole numbers and refining by tenths, 1 for the correct value to 1 decimal place. No common base works, so use guess-check-and-refine (2E):

trial	value	check against 50
$x = 3$	$3^3 = 27$	too low
$x = 4$	$3^4 = 81$	too high — the solution lies between 3 and 4
$x = 3.5$	$3^{3.5} \approx 46.77$	too low
$x = 3.6$	$3^{3.6} \approx 52.20$	too high — between 3.5 and 3.6
$x = 3.55$	$3^{3.55} \approx 49.41$	too low — between 3.55 and 3.6

The solution is closer to 3.6 than to 3.5: $x \approx 3.6$ to 1 decimal place. (Accept any correctly recorded trial table narrowing to a 0.1-wide interval containing the solution.)

Q12. VC2M10A11 — AS-2. 5 marks; the gallery of 3B sampled across its families.

(a) 3 marks: 1 each for (i), (ii) and (iii).

(b) $x^2 - 4x + 3 = (x - 1)(x - 3)$, so the graph meets the x -axis where $x = 1$ or $x = 3$: the x -intercepts are $(1, 0)$ and $(3, 0)$.

(ii) The turning point sits midway between the intercepts: $x = \frac{1+3}{2} = 2$, and $y = 2^2 - 4 \times 2 + 3 = 4 - 8 + 3 = -1$.

The turning point is $(2, -1)$.

(iii) A correct sketch is a parabola opening upwards through $(1, 0)$, $(3, 0)$ and the y -intercept $(0, 3)$, with its lowest point at $(2, -1)$.

(b) 1 mark. $x^2 + y^2 = 16$ is a circle with centre $(0, 0)$ and radius $\sqrt{16} = 4$.

- (c) 1 mark. The rectangular hyperbola $y = \frac{8}{x}$ approaches but never reaches the two axes: asymptotes $x=0$ and $y=0$.

Q13. VC2M10A16 — AS-4. 3 marks; systematic numerical guess-check-and-refine with the trials recorded.

- (a) 1 mark.

$2^0 = 1$ and $3 \times 0 + 1 = 1$ — both sides are equal, so $x=0$ is a solution. ✓

- (b) 2 marks: 1 for narrowing the trap, 1 for the value to 1 decimal place.

trial	2^x	$3x + 1$	comparison
$x = 3$	8	10	2^x below
$x = 4$	16	13	2^x above — the crossing lies between 3 and 4
$x = 3.5$	≈ 11.31	11.5	below
$x = 3.6$	≈ 12.13	11.8	above — between 3.5 and 3.6, closer to 3.5

$\therefore x \approx 3.5$ to 1 decimal place. (The exact crossing is about 3.54.)

Q14. VC2M10A06 — AS-5. 5 marks; a two-dimensional array transforming a set of points (VC2M10A06.E01), traced by hand or run in the nominated language — both score full marks.

- (a) 2 marks: 1 for the two substitutions, 1 for the output pair. With $i=0$: $x = P[0][0] = 1$ and $y = P[0][1] = 2$, so

$$\text{newx} = T[0][0] \times x + T[0][1] \times y = 0 \times 1 + (-1) \times 2 = -2$$

$$\text{newy} = T[1][0] \times x + T[1][1] \times y = 1 \times 1 + 0 \times 2 = 1$$

The first output is the pair $(-2, 1)$.

- (b) 2 marks: 1 each. The loop continues with $i=1$ and $i=2$:

$$i=1: x=3, y=1: \text{newx} = 0 \times 3 + (-1) \times 1 = -1, \text{newy} = 1 \times 3 + 0 \times 1 = 3 \text{ — the pair } (-1, 3).$$

$$i=2: x=2, y=4: \text{newx} = 0 \times 2 + (-1) \times 4 = -4, \text{newy} = 1 \times 2 + 0 \times 4 = 2 \text{ — the pair } (-4, 2).$$

- (c) 1 mark. Each point (x, y) maps to $(-y, x)$ — the rule of a **rotation of 90° anticlockwise about the origin** (3D §6 carries exactly this array for it).

Record the totals: Part A □/25 · Part B □/30 · Test T2 □/55