Mathematics Level 10

Chapter 2 — Equations and inequalities · 6 subtopics · 7 CDs

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Contents

Linear equations, from formulas and with simple algebraic fractions

Mathematics 10 · Chapter 2 Equations and inequalities · Subtopic 2A (Level 10) · Extended block

Strand	Algebra
Content description	`VC2M10A07`, `VC2M10A12`
Elaboration ids carried	`VC2M10A07.E01`; `VC2M10A12.E01`, `VC2M10A12.E02`
Weight	Extended (double allocation)

> **Content descriptions (verbatim):** *solve problems involving linear equations, including those derived from formulas* (`VC2M10A07`); *solve linear equations involving simple algebraic fractions* (`VC2M10A12`)
>
> **Achievement standard (extract):** *They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, algebraically and graphically, with and without the use of digital tools, and justify solutions.*

You need first: clearing an algebraic fraction from an equation from 1C (`VC2M10A03`), and formulas, substitution and transposition from 1E (`VC2M10A05`), on top of Level 9 solving of linear equations (`VC2M9A03`). This subtopic joins those skills and applies them; it does not re-teach them.

Theory

Every equation in this subtopic is **linear**: the pronumeral appears only to the first power. Squared pronumerals belong to 2D, and two pronumerals locked together belong to 2C. The two content descriptions here are one skill — solving a linear equation — practised on a wide range of equations: with brackets, from formulas, and with one or two simple algebraic fractions.

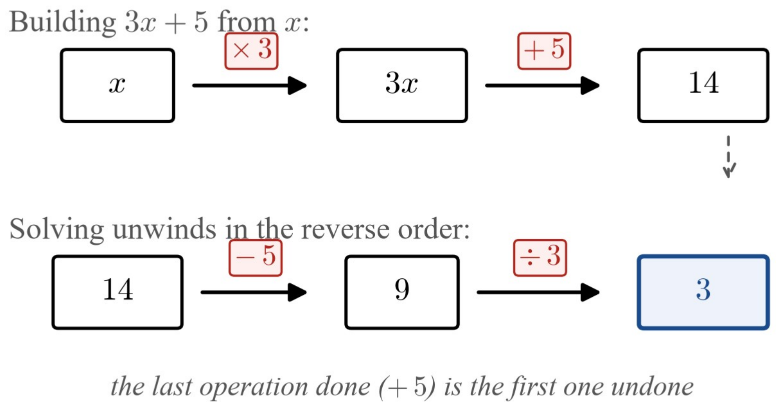
1. The balance principle

An **equation** is a claim that two expressions are two names for the same number. The **balance principle** is the one rule every solve in this subtopic uses:

> doing the **same** operation to **both** sides of a true equation keeps it true.

So solving is a sequence of small true equations, one step per line, each obtained from the last by one operation done to both sides. Writing one step per line is not tidy-up advice; it is the method. A linear equation always ends the same way — $x = $ a number — and the number is then checked by **substitution**: put it back in the original equation and watch both sides come out equal.

2. Unwinding in reverse order



Flow diagram: building three x plus five from x by multiplying by three and then adding five to reach fourteen, and solving by unwinding fourteen, subtracting five to get nine and dividing by three to get three

The expression $3x+5$ is built from x by two operations: multiply by 3, then add 5. Solving $3x+5=14$ unwinds those operations **in the reverse order** — the last one done is the first one undone:

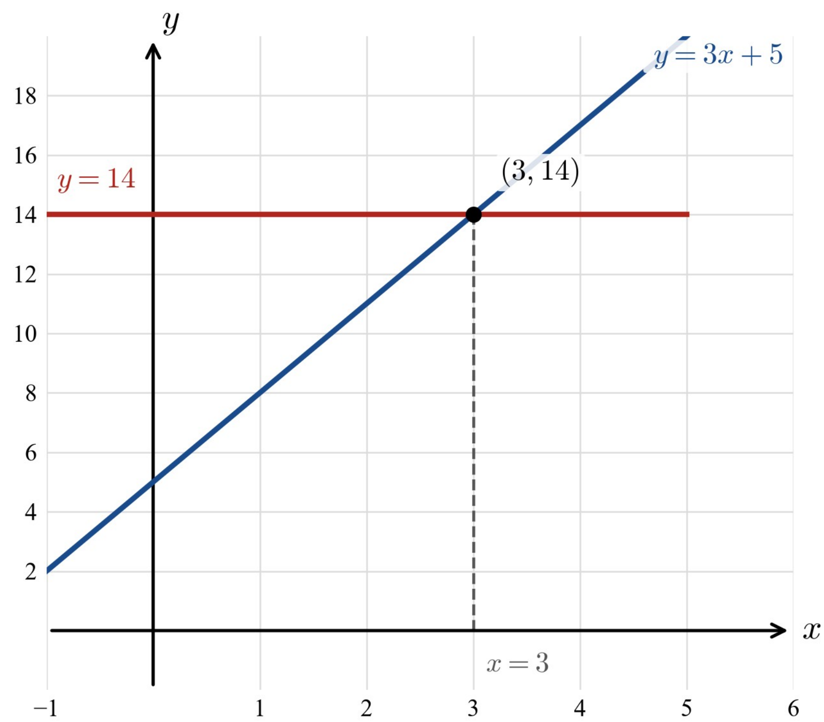
$$3x+5=14$$

$$3x=9$$

$$x=3$$

Subtract 5 from both sides first (undoing the $+5$), then divide both sides by 3 (undoing the $\times 3$). The check: $3\times 3+5=14$. ✓

3. Reading a solution from a graph

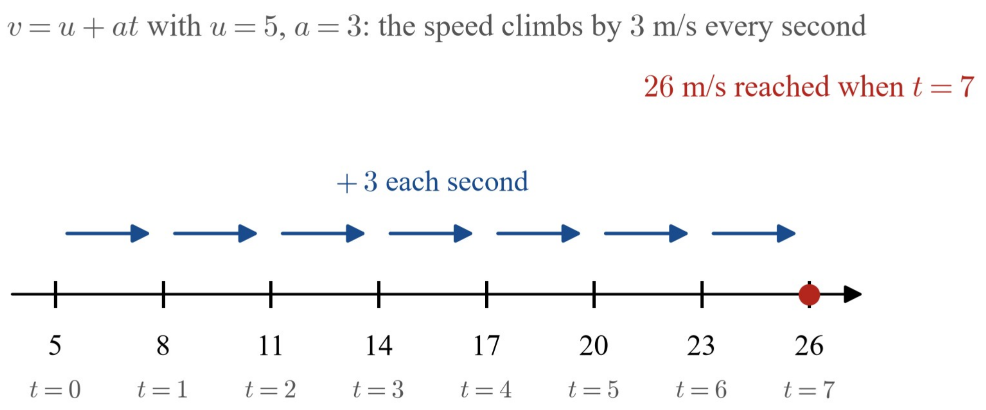


Graph of the line y equals three x plus five and the horizontal line y equals fourteen, crossing at the point three comma fourteen, with a dashed guide down to x equals three on the horizontal axis

The equation $3x+5=14$ asks: *where does the line* $y=3x+5$ *reach height* 14*?* Graphing the line and the horizontal line $y=14$ (a Year 9 skill), the two meet at $(3, 14)$; reading down gives $x=3$ — the same answer the algebra gave. The graph is a picture of the equation and a free check on it.

4. From a formula: substitute, then solve

A **formula** is an equation between letters, such as $v=u+at$. Giving the letters values turns the formula into a linear equation in the one letter that is left unknown.



Ladder diagram for v equals u plus a t with u equals five and a equals three: the values step up by three each second from five at t equals zero to twenty-six at t equals seven, and the target value twenty-six is marked at t equals seven

With $u=5$ and $a=3$, the formula says $v=5+3t$; the diagram steps v up by 3 each second. Setting $v=26$ gives a linear equation in t:

$$26=5+3t$$

$$21=3t$$

$$t=7$$

so 26 is reached after 7 seconds. Substitute first, then solve with the balance principle exactly as in section 2.

5. From a formula: transpose first, then substitute

When the same letter is wanted again and again, 1E's method pays: **transpose** the formula first — rearrange it to make that letter the subject — then substitute. From $v=u+at$, subtract u from both sides and divide both sides by a:

$$v-u=at \Rightarrow t=\frac{v-u}{a}.$$

Now the values $v=26$, $u=5$, $a=3$ give $t=\frac{26-5}{3}=\frac{21}{3}=7$, the answer of section 4 by the other route. Transposing never loses the equation: every step is the balance principle applied to letters.

The same move on $F=ma$, dividing both sides by a, gives $m=\frac{F}{a}$. With $F=84$ and $a=1.2$: $m=\frac{84}{1.2}=70$.

6. Simple algebraic fractions: one fraction

Recall from 1C the **simple algebraic fraction**: the denominator is a number, a single term in one pronumeral, or a linear two-term expression — $\frac{x}{6}$, $\frac{3x-2}{5}$ and $\frac{12}{x}$ are all simple. An equation carrying one of them is solved by one extra first step: **multiply both sides by the denominator.**

Unknown in the numerator, $\frac{x}{6}=5$: multiply both sides by 6 and $x=30$. If other terms are present, move them first: $\frac{x}{4}+3=11$ gives $\frac{x}{4}=8$, then multiply both sides by 4: $x=32$. A bracket on top is handled the same way, because the whole top is one numerator: $\frac{3x-2}{5}=8$ gives $3x-2=40$, so $3x=42$ and $x=14$.

Unknown in the denominator, $\frac{12}{x}=4$: multiply both sides by x to clear it.

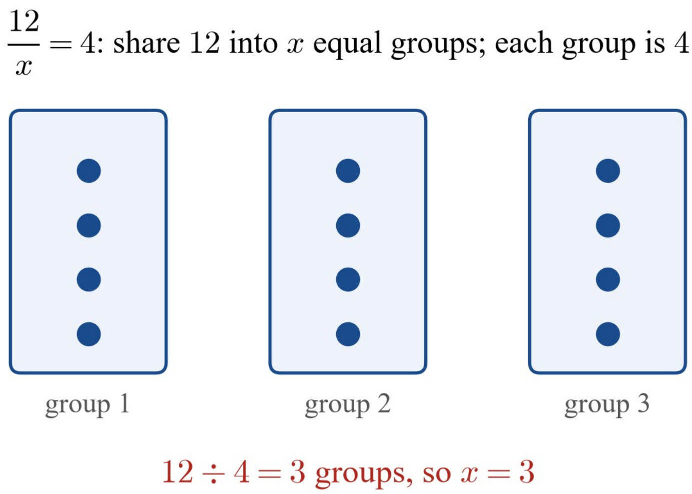


Diagram of twelve dots shared into three equal groups of four dots each, showing that twelve divided into x equal groups of four gives three groups, so twelve over x equals four has solution x equals three

$$\frac{12}{x}=4 \Rightarrow 12=4x \Rightarrow x=3.$$

The picture says why: 12 shared into x equal groups of 4 makes 3 groups. Equations of this shape are the one new step the range adds — after the multiply, they are linear like any other.

7. Two fractions: multiply every term by the LCD

With two fractions, multiply **every term on both sides** by the lowest common denominator (LCD). For $\frac{x}{2}+\frac{x}{3}=15$ the LCD of 2 and 3 is 6:

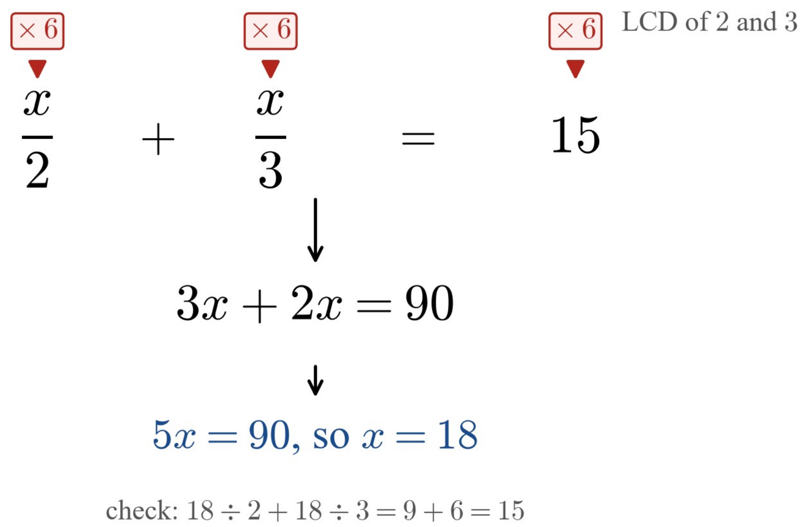


Diagram of the equation x over two plus x over three equals fifteen with a multiply-by-six badge over every term, giving three x plus two x equals ninety, then five x equals ninety and x equals eighteen, with the check eighteen divided by two plus eighteen divided by three equals nine plus six equals fifteen

$$6 \times \frac{x}{2} + 6 \times \frac{x}{3} = 6 \times 15$$

$$3x + 2x = 90$$

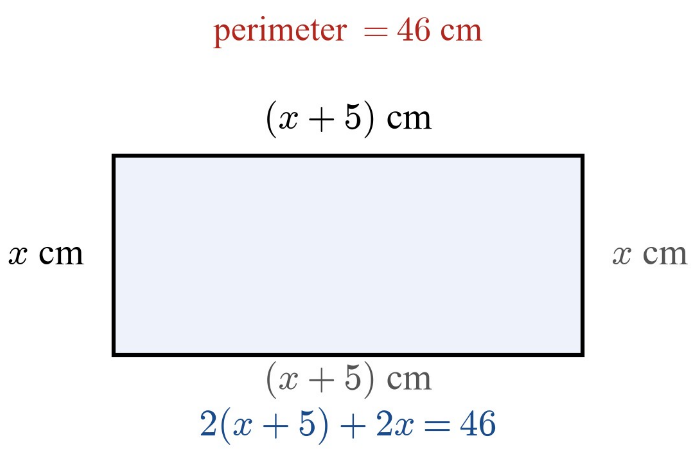
$$5x = 90$$

$$x = 18$$

Each multiply cancels a denominator: $6 \times \frac{x}{2} = 3x$ and $6 \times \frac{x}{3} = 2x$. The classic error is to leave the right side untouched — the balance principle says **every** term, so the 15 becomes 90. Check: $\frac{18}{2} + \frac{18}{3} = 9 + 6 = 15$. ✓

8. From words to an equation — and checking

A word problem is solved in four moves: **let** a letter stand for what the question asks; **translate** each fact into algebra; **solve**; **answer in words** and check by substitution.



Rectangle labelled x centimetres on each short side and x plus five centimetres on each long side, with perimeter forty-six centimetres and the equation two times x plus five plus two x equals forty-six

A rectangle is 5 *cm longer than it is wide, and its perimeter is* 46 *cm. Find the width.* Let the width be x cm, so the length is $(x+5)$ cm. The perimeter adds all four sides:

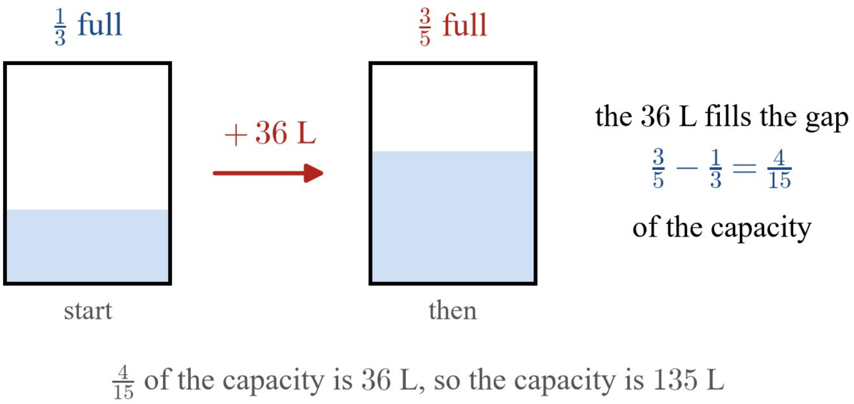
$$2(x+5)+2x=46$$

$$4x+10=46$$

$$4x=36$$

$$x=9$$

The width is 9 cm (and the length 14 cm). Check: $2\times14+2\times9=28+18=46$. ✓



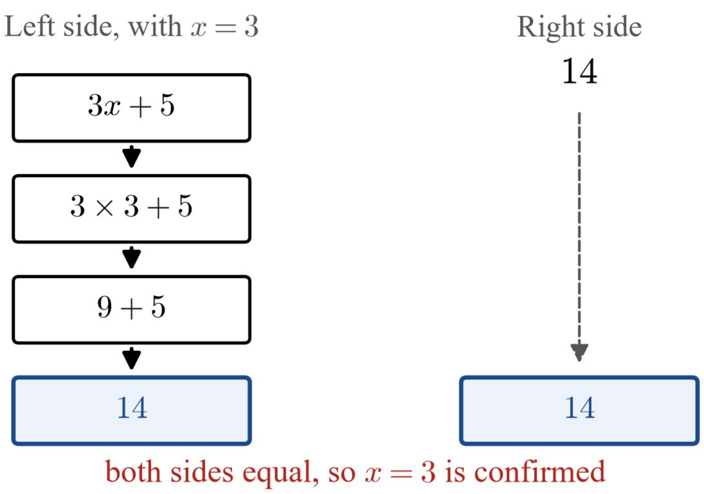
Two tank diagrams: the first one third full, then three fifths full after thirty-six litres is added; the thirty-six litres fills the gap of three fifths minus one third, which is four fifteenths of the capacity, so the capacity is one hundred and thirty-five litres

A tank is one third full. Adding 36 L *makes it three fifths full. Find the capacity.* Let the capacity be C litres. The starting amount plus 36 L equals the new amount:

$$\frac{C}{3}+36=\frac{3C}{5}$$

Multiply every term by the LCD, 15: $5C+540=9C$, so $540=4C$ and $C=135$. The capacity is 135 L. Check: $\frac{135}{3}+36=45+36=81=\frac{3\times135}{5}$. ✓

Checking is not a formality — it is half of "justify solutions" in the achievement standard, and it is named in `VC2M10A12.E01`. The check always substitutes into the **original** equation:



Substitution check for three x plus five equals fourteen: the left side with x equals three becomes three times three plus five, then nine plus five, then fourteen, matching the right side fourteen, so x equals three is confirmed

Worked examples

Example 1 — Brackets and unknowns on both sides

(a) Solve $5(x-2)=2x+11$.

Working	Comment
$5(x-2)=2x+11$	
$5x-10=2x+11$	Expand the bracket.
$3x-10=11$	Subtract $2x$ from both sides.
$3x=21$	Add 10 to both sides.
$x=7$	Divide both sides by 3.
$\therefore x=7$.	Check: $5(7-2)=25$ and $2\times7+11=25$. ✓

(b) Solve $7x-3(2x-1)=5(x+2)-4$.

Working	Comment
$7x-3(2x-1)=5(x+2)-4$	
$7x-6x+3=5x+10-4$	Expand both brackets; $-3\times-1=+3$.
$x+3=5x+6$	Collect like terms on each side.
$3=4x+6$	Subtract x from both sides.
$-3=4x$	Subtract 6 from both sides.
$x=-\frac{3}{4}$	Divide both sides by 4.
$\therefore x=-\frac{3}{4}$.	Check: both sides equal $\frac{9}{4}$. ✓

The solution of a linear equation does not have to be a whole number — the pronumeral is just a number.

Example 2 — From a formula: substitute, then solve

(a) Using $v=u+at$, find t when $v=26$, $u=5$ and $a=3$.

Working	Comment
$v=u+at$	
$26=5+3t$	Substitute the given values.
$21=3t$	Subtract 5 from both sides.
$t=7$	Divide both sides by 3.
$\therefore t=7$.	

(b) Using $A=\frac{1}{2}bh$, find h when $A=42$ and $b=12$.

Working	Comment
$A=\frac{1}{2}bh$	
$42=\frac{1}{2}\times12\times h$	Substitute.
$42=6h$	$\frac{1}{2}\times12=6$.
$h=7$	Divide both sides by 6.
$\therefore h=7$.	Check: $\frac{1}{2}\times12\times7=42$. ✓

Working	Comment

Example 3 — From a formula: transpose first, then substitute

(a) Make t the subject of $v=u+at$, then find t when $v=26$, $u=5$ and $a=3$.

Working	Comment
$v=u+at$	
$v-u=at$	Subtract u from both sides.
$t=\dfrac{v-u}{a}$	Divide both sides by a.
$t=\dfrac{26-5}{3}=\dfrac{21}{3}=7$	Substitute.
$\therefore t=7$ — the answer of Example 2(a), by the other route.	

(b) Make m the subject of $F=ma$, then find m when $F=84$ and $a=1.2$.

Working	Comment
$F=ma$	
$m=\dfrac{F}{a}$	Divide both sides by a.
$m=\dfrac{84}{1.2}=70$	Substitute; $84\div 1.2=70$.
$\therefore m=70$.	

Example 4 — One simple algebraic fraction

(a) Solve $\dfrac{x}{4}+3=11$.

Working	Comment
$\dfrac{x}{4}+3=11$	
$\dfrac{x}{4}=8$	Subtract 3 from both sides.
$x=32$	Multiply both sides by 4.
$\therefore x=32$.	Check: $32\div 4+3=8+3=11$. ✓

(b) Solve $\dfrac{3x-2}{5}=8$.

Working	Comment
$\dfrac{3x-2}{5}=8$	
$3x-2=40$	Multiply both sides by 5; the whole top is one numerator.
$3x=42$	Add 2 to both sides.
$x=14$	Divide both sides by 3.
$\therefore x=14$.	Check: $\dfrac{3\times 14-2}{5}=\dfrac{40}{5}=8$. ✓

(c) Solve $\dfrac{12}{x}=4$.

Working	Comment
$\dfrac{12}{x}=4$	
$12=4x$	Multiply both sides by x.
$x=3$	Divide both sides by 4.
$\therefore x=3$.	Check: $12\div 3=4$. ✓

Example 5 — Two fractions: clear the denominators

(a) Solve $\dfrac{x}{2}+\dfrac{x}{3}=20$.

Working	Comment
$\dfrac{x}{2}+\dfrac{x}{3}=20$	
$3x+2x=120$	Multiply **every term** by the LCD, 6.
$5x=120$	
$x=24$	Divide both sides by 5.
$\therefore x=24$.	Check: $\dfrac{24}{2}+\dfrac{24}{3}=12+8=20$. ✓

(b) Solve $\dfrac{x}{4}-\dfrac{x}{6}=3$.

Working	Comment
$\dfrac{x}{4}-\dfrac{x}{6}=3$	
$3x-2x=36$	Multiply every term by the LCD, 12.
$x=36$	
$\therefore x=36$.	Check: $\dfrac{36}{4}-\dfrac{36}{6}=9-6=3$. ✓

Example 6 — Word problems to equations

(a) A rectangle is 5 cm longer than it is wide and its perimeter is 46 cm. Find the width and the length.

Working	Comment
Let the width be x cm; the length is $(x+5)$ cm.	**Let** and **translate**.
$2(x+5)+2x=46$	Perimeter = all four sides.
$4x+10=46$	Expand and collect.
$4x=36$	Subtract 10 from both sides.
$x=9$	Divide both sides by 4.
$\therefore$ the width is 9 cm and the length is 14 cm.	Check: $2\times 14+2\times 9=46$. ✓

(b) A tank is one third full. Adding 36 L makes it three fifths full. Find the capacity.

Working	Comment
Let the capacity be C litres.	**Let**.
$\dfrac{C}{3}+36=\dfrac{3C}{5}$	Start $+36$ L = then.
$5C+540=9C$	Multiply every term by the LCD, 15.
$540=4C$	Subtract $5C$ from both sides.
$C=135$	Divide both sides by 4.

Working	Comment
$\therefore$ the capacity is 135 L.	Check: $45+36=81=\frac{3\times 135}{5}$. ✓

Practice questions

Scaffolded

Q1. (a) Solve $3(x+4)=27$. First divide both sides by 3. (b) Solve $5x-7=2x+14$. First subtract $2x$ from both sides.

Q2. (a) Solve $2(3x-1)=4(x+3)$. First expand both brackets. (b) Solve $6-2x=3(4-x)$.

Q3. (a) Using $v=u+at$, find t when $v=19$, $u=4$ and $a=3$. (b) Using $P=2(l+w)$, find w when $P=30$ and $l=9$.

Q4. (a) Solve $\frac{x}{7}=4$. Multiply both sides by 7. (b) Solve $\frac{x}{5}-2=6$.

Q5. (a) Solve $\frac{x+3}{4}=5$. The whole top is one numerator. (b) Solve $\frac{2x-1}{3}=7$.

Q6. (a) Solve $\frac{x}{3}+\frac{x}{4}=14$. The LCD of 3 and 4 is 12. (b) Solve $\frac{x}{2}-\frac{x}{5}=9$.

Q7. (a) Solve $\frac{15}{x}=5$. Multiply both sides by x. (b) Solve $\frac{24}{x}=6$.

Q8. Three times a number, minus 7, gives 20. Let the number be n, write an equation, and solve it.

Unscaffolded

Q9. Solve: (a) $4(x-3)=2(x+5)$; (b) $3(2x+1)=5(x-2)$; (c) $7(x+2)=3(2x+9)$.

Q10. Solve: (a) $\frac{2x}{3}=10$; (b) $\frac{3x}{4}+2=11$; (c) $\frac{5x+1}{3}=7$.

Q11. Solve: (a) $\frac{x}{2}+\frac{x}{7}=9$; (b) $\frac{x}{3}-\frac{x}{8}=5$; (c) $\frac{x}{2}+\frac{x}{3}+\frac{x}{6}=12$.

Q12. Solve: (a) $\frac{8}{x}=2$; (b) $\frac{21}{2x+1}=3$; (c) $\frac{10}{x-1}=2$.

Q13. (a) Using $A=\frac{1}{2}bh$, find b when $A=60$ and $h=8$. (b) Twelve identical workbooks cost \$132. Let the price of one be a dollars, write an equation, and find a.

Q14. An investment of \$6,000 earns \$900 simple interest over 3 years. Using $I=Prn$, where P is the amount invested, r is the rate per year (as a decimal) and n is the number of years, find r. Write it also as a percentage.

Q15. The sum of three consecutive whole numbers is 72. Let the smallest be n, write an equation, and find the three numbers.

Q16. I spent a third of my money, then \$14, and had \$26 left. Let my starting amount be x dollars, write an equation, and find x.

Q17. An isosceles triangle has perimeter 32 cm, and its third side is 4 cm shorter than each of the two equal sides. Find the lengths of the three sides.

Q18. Solve $\frac{x+2}{3}+\frac{x-1}{2}=6$.

Q19. Solve $\frac{24}{x}-3=5$.

Q20. Sam translates "three times the sum of x and 2 is equal to x plus 10" as $3x+2=x+10$ and solves it to get $x=4$. (a) Solve Sam's equation to confirm $x=4$. (b) Find Sam's error, write the correct equation, and solve it. (c) Show by substitution that $x=4$ does not satisfy the correct equation.

Q21. A rectangle is 2 cm more than three times as long as it is wide, and its perimeter is 44 cm. Find the width and the length.

Q22. Solve $\frac{x}{3}+5=\frac{x}{2}-1$.

Real context

Q23. For Australian residents in 2025–26, the tax on a taxable income between $18,200 and $45,000 is 16% of the amount over $18,200. (a) Write an equation and find the tax on a taxable income of $36,450. (b) A taxpayer in this bracket paid $3,872 tax. Write an equation and find their taxable income. (c) Show that the most tax payable in this bracket is $4,288, and explain why this confirms your answer to (b) sits in the bracket.

Source: Australian Taxation Office, *Tax rates for Australian residents*, 2025. Retrieved 20 August 2026.

Digital tool task

DIGITAL TOOL — this task assumes a graphing tool. Your teacher will nominate the tool.

Task. Use a graphing tool to solve $5x-7=2x+14$ by finding an intersection.

Input (what to graph):

1. the line $y=5x-7$;
2. the line $y=2x+14$;
3. the point where the two lines meet.

Output (what to expect): the lines meet at $(7,28)$, so $x=7$. Reading the intersection is the graph version of section 3: the solution is where the two sides are equal.

Tool-free path (still assessed): solve $5x-7=2x+14$ algebraically: $3x=21$, so $x=7$; check $5\times7-7=28=2\times7+14$. ✓

Answers

Q1. (a) $x=5$; (b) $x=7$. **Q2.** (a) $x=7$; (b) $x=6$. **Q3.** (a) $t=5$; (b) $w=6$. **Q4.** (a) $x=28$; (b) $x=40$. **Q5.** (a) $x=17$; (b) $x=11$. **Q6.** (a) $x=24$; (b) $x=30$. **Q7.** (a) $x=3$; (b) $x=4$. **Q8.** $n=9$.

Q9. (a) $x=11$; (b) $x=-13$; (c) $x=13$. **Q10.** (a) $x=15$; (b) $x=12$; (c) $x=4$. **Q11.** (a) $x=14$; (b) $x=24$; (c) $x=12$. **Q12.** (a) $x=4$; (b) $x=3$; (c) $x=6$. **Q13.** (a) $b=15$; (b) $a=11$. **Q14.** $r=0.05$, that is 5%. **Q15.** 23, 24, 25. **Q16.** $x=60$. **Q17.** 12 cm, 12 cm and 8 cm. **Q18.** $x=7$. **Q19.** $x=3$. **Q20.** (a) $x=4$; (b) the bracket: $3(x+2)=x+10$, so $x=2$; (c) $3(4+2)=18\neq 14$. **Q21.** width 5 cm, length 17 cm. **Q22.** $x=36$. **Q23.** (a) \$2,920; (b) \$42,400; (c) \$4,288.

Digital tool task. $x=7$.

Fully-worked solutions

Q1. (a) $3(x+4)=27 \Rightarrow x+4=9 \Rightarrow x=5$. (b) $5x-7=2x+14 \Rightarrow 3x=21 \Rightarrow x=7$.

Q2. (a) $6x-2=4x+12 \Rightarrow 2x=14 \Rightarrow x=7$. (b) $6-2x=12-3x \Rightarrow x=6$ (add $3x$, subtract 6).

Q3. (a) $19=4+3t \Rightarrow 15=3t \Rightarrow t=5$. (b) $30=2(9+w) \Rightarrow 15=9+w \Rightarrow w=6$.

Q4. (a) $x=7\times 4=28$. (b) $\frac{x}{5}=8 \Rightarrow x=40$.

Q5. (a) $x+3=20 \Rightarrow x=17$. (b) $2x-1=21 \Rightarrow 2x=22 \Rightarrow x=11$.

Q6. (a) Multiply every term by 12: $4x+3x=168 \Rightarrow 7x=168 \Rightarrow x=24$. (b) Multiply every term by 10: $5x-2x=90 \Rightarrow 3x=90 \Rightarrow x=30$.

Q7. (a) $15=5x \Rightarrow x=3$. (b) $24=6x \Rightarrow x=4$.

Q8. $3n-7=20 \Rightarrow 3n=27 \Rightarrow n=9$.

Q9. (a) $4x-12=2x+10 \Rightarrow 2x=22 \Rightarrow x=11$. (b) $6x+3=5x-10 \Rightarrow x=-13$. (c) $7x+14=6x+27 \Rightarrow x=13$.

Q10. (a) $2x=30 \Rightarrow x=15$. (b) $\frac{3x}{4}=9 \Rightarrow 3x=36 \Rightarrow x=12$. (c) $5x+1=21 \Rightarrow 5x=20 \Rightarrow x=4$.

Q11. (a) Multiply by 14: $7x+2x=126 \Rightarrow 9x=126 \Rightarrow x=14$. (b) Multiply by 24: $8x-3x=120 \Rightarrow 5x=120 \Rightarrow x=24$. (c) Multiply by 6: $3x+2x+x=72 \Rightarrow 6x=72 \Rightarrow x=12$.

Q12. (a) $8=2x \Rightarrow x=4$. (b) $21=3(2x+1) \Rightarrow 7=2x+1 \Rightarrow x=3$. (c) $10=2(x-1) \Rightarrow 5=x-1 \Rightarrow x=6$.

Q13. (a) $60=\frac{1}{2}\times b\times 8=4b \Rightarrow b=15$. (b) $12a=132 \Rightarrow a=11$.

Q14. $900=6\,000\times r\times 3=18\,000r \Rightarrow r=\frac{900}{18\,000}=0.05=5\%$.

Q15. $n+(n+1)+(n+2)=72 \Rightarrow 3n+3=72 \Rightarrow 3n=69 \Rightarrow n=23$: the numbers are 23, 24, 25.

Q16. $x-\frac{x}{3}-14=26 \Rightarrow x-\frac{x}{3}=40$; multiply by 3: $3x-x=120 \Rightarrow 2x=120 \Rightarrow x=60$. Check: $60-20-14=26$. ✓

Q17. Let each equal side be x cm; the third side is $(x-4)$ cm. $x+x+(x-4)=32 \Rightarrow 3x=36 \Rightarrow x=12$: the sides are 12 cm, 12 cm, 8 cm.

Q18. Multiply every term by 6: $2(x+2)+3(x-1)=36 \Rightarrow 2x+4+3x-3=36 \Rightarrow 5x+1=36 \Rightarrow 5x=35 \Rightarrow x=7$. Check: $\frac{9}{3}+\frac{6}{2}=3+3=6$. ✓

Q19. $\frac{24}{x}=8 \Rightarrow 24=8x \Rightarrow x=3$.

Q20. (a) $3x+2=x+10 \Rightarrow 2x=8 \Rightarrow x=4$. (b) "The sum of x and 2" is a bracket: $3(x+2)=x+10 \Rightarrow 3x+6=x+10 \Rightarrow 2x=4 \Rightarrow x=2$. (c) In $3(x+2)=x+10$: left side $3(4+2)=18$, right side $4+10=14$; $18 \neq 14$, so $x=4$ fails the check — the signal that Sam's equation was wrong.

Q21. Let the width be w cm; the length is $(3w+2)$ cm. $2(3w+2)+2w=44 \Rightarrow 6w+4+2w=44 \Rightarrow 8w=40 \Rightarrow w=5$; the length is $3 \times 5+2=17$ cm. Check: $2 \times 17+2 \times 5=44$. ✓

Q22. Multiply every term by 6: $2x+30=3x-6 \Rightarrow 36=x$. Check: $\frac{36}{3}+5=17=\frac{36}{2}-1$. ✓

Q23. (a) Let the tax be T dollars. $T=0.16 \times (36\,450-18\,200)=0.16 \times 18\,250=2920$: the tax is \$2,920. (b) Let the taxable income be I dollars. $0.16 \times (I-18\,200)=3872 \Rightarrow I-18\,200=\frac{3872}{0.16}=24\,200 \Rightarrow I=42\,400$: the taxable income is \$42,400. (c) Most tax in the bracket is at $I=45\,000$: $0.16 \times (45\,000-18\,200)=0.16 \times 26\,800=4288$, so \$4,288. Since \$3,872 is less than \$4,288, an income producing \$3,872 does sit in this bracket — the answer to (b) is consistent.

Digital tool task. The lines $y=5x-7$ and $y=2x+14$ meet at $(7,28)$, so $x=7$. Tool-free: $5x-7=2x+14 \Rightarrow 3x=21 \Rightarrow x=7$; check $5 \times 7-7=28=2 \times 7+14$. ✓

Teacher note

Level 9 code rehearsed (D3): `VC2M9A03` — solving linear equations — is what every solve here becomes once brackets are opened and denominators cleared ($3x=21$, $5x=90$, $4x=36$). It is rehearsed, not re-taught, and is not assessed as a Level 9 outcome. The chapter's own prerequisites do the same: 1C's clearing of an algebraic fraction (`VC2M10A03`) is the first step of every fractioned equation, and 1E's formulas and transposition (`VC2M10A05`) are sections 4–5 verbatim. Downstream, 2B assumes the solving mechanics, 2C assumes the one-equation skills before adding a second unknown, and 2F ranges over these equations in its "have you found them all?" question (§3 prerequisite table).

Paired codes evidenced separately (D8/T2): `VC2M10A07` is evidenced by the formula and word-problem load — sections 4–5 and 8, Examples 2, 3 and 6, Q3, Q8, Q13–Q17, Q20, Q21, Q23 (`VC2M10A07.E01`). `VC2M10A12` is evidenced by the range of fractioned equations and the substitution checks — sections 6–8, Examples 4–6, Q4–Q7, Q10–Q12, Q16, Q18, Q19, Q22 (`.E01`), with the word-problem-as-fractioned-equation items Q16 and Q22 and Example 6(b) carrying `.E02`. Checkpoint items are tagged per code so each CD is separately visible in the coverage join.

Tool classes (D13): scientific calculator for the decimal and percentage arithmetic (Example 3(b), Q14, Q23) and a graphing tool for the digital tool task only (`VC2M10A07`'s "with and without the use of digital tools"). No CAS: the book prints the input and expected output beside the task and assesses the tool-free path.

VCE destinations (§5, reference only): `VC2M10A07` is named in the Units 3 & 4 evidence of Foundation Mathematics, General Mathematics and Mathematical Methods; `VC2M10A12`'s fractioned equations are Mathematical Methods Units 1 & 2 AoS 1 content *(inferred)*. 2A also carries implicit prerequisite 6: transposition of $F=ma$-style formulas is assumed by the sciences in this same year.

Level check (D6): no simultaneous equations (2C), no quadratics (2D), no inequalities (2B), and no $f(x)$ or function language anywhere in this subtopic. The only graph work is reading the Year 9 linear relation $y=3x+5$ against a horizontal line (section 3 and the digital task), which is `VC2M9A03`-era skill, not 3B. $I=Prn$ in Q14 is used purely as a formula to substitute into and transpose; no finance theory is assumed.

Dated data (R3): Q23 carries the 2025–26 resident tax rates ($18,200 threshold, 16 % to $45,000) as at 20 August 2026. Listed here for the chapter refresh register so a reissue can update the figures without re-reading the chapter.

Linear inequalities and their solutions on a number line

Mathematics 10 · Chapter 2 Equations and inequalities · Subtopic 2B (Level 10) · Standard block

Strand	Algebra
Content description	`VC2M10A08`
Elaboration ids carried	`VC2M10A08.E01` `VC2M10A08.E02` (`.E02` is truncated upstream — D12; see Teacher note)
Weight	Standard (single allocation)

> **Content description (verbatim):** *solve linear inequalities and graph their solutions on a number line*
>
> **Achievement standard (extract):** *They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, algebraically and graphically, with and without the use of digital tools, and justify solutions.*

You need first: solving linear equations from Year 9 (`VC2M9A03`). Every balancing move that solved an equation in 2A works here unchanged, with one addition — the direction rule this subtopic teaches.

Theory

1. From equations to inequalities

Subtopic 2A asked *which single value makes the two sides equal?* An inequality asks a different question: *which values make one side bigger than the other?* A **linear inequality** looks exactly like a linear equation with the = replaced by one of four symbols.

$<$	$>$	$\leq$	$\geq$
less than	greater than	less than or equal to	greater than or equal to
$3 < 5$	$8 > 2$	$x \leq 4$	$x \geq -1$

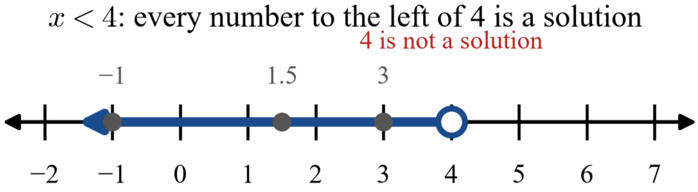
Four panels showing the four inequality symbols with names and an example each: less than, as 3 is less than 5; greater than, as 8 is greater than 2; less than or equal to, as x is less than or equal to 4; greater than or equal to, as x is greater than or equal to negative 1.

Read each symbol aloud as a sentence: $x < 4$ says “x is less than 4”; $x \geq -1$ says “x is greater than or equal to -1”. Questions and word problems usually hide the symbol inside ordinary words, and these translations cover nearly all of them:

Words	Symbol
less than, fewer than	$<$
greater than, more than	$>$
at most, no more than	$\leq$
at least	$\geq$

2. A solution is now a whole set of numbers

Consider $x < 4$. Then 3 works, 0 works, -2 works, 3.9 works, 3.99 works — in fact every number to the left of 4 works, and nothing else does.



Number line from −2 to 7 with an open dot at 4 and a blue arrow pointing left; grey dots mark the sample solutions −1, 1.5 and 3, and the label at 4 says 4 is not a solution. The title reads: x less than 4, every number to the left of 4 is a solution.

So an inequality normally has **infinitely many solutions**, and listing them is impossible. Instead we do two things: we **check** a candidate by substituting it (try $x=3$: $3<4$ is true, so 3 is a solution; try $x=4$: $4<4$ is false, so 4 is not), and we **graph** the whole solution set on a number line (section 5).

3. Solving: adding and subtracting

Solve $x+4<9$.

$x+4<9$

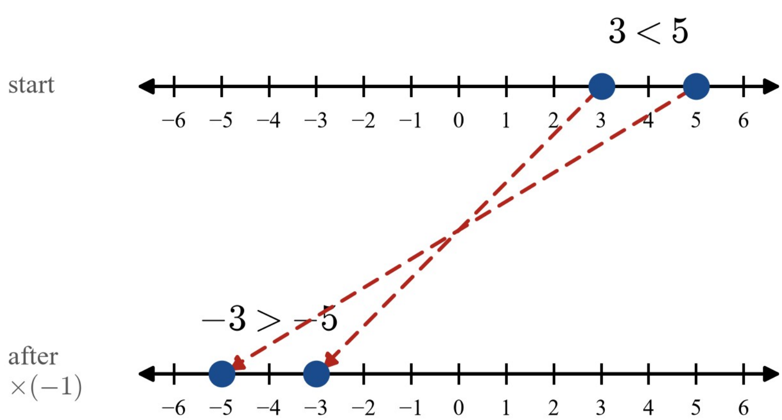
$x<5$ — subtract 4 from both sides.

Adding or subtracting the same number on both sides of an inequality keeps the statement true and never moves the direction of the symbol — exactly as in 2A's equations. Up to here, solving an inequality is solving an equation with a different last symbol.

4. Solving: multiplying and dividing — and the one new rule

Multiplying or dividing both sides by a **positive** number also changes nothing. From $\frac{x}{3}\leq 4$, multiply both sides by 3: $x\leq 12$. The direction stays.

The new rule appears with a **negative** number. Start from the true statement $3<5$ and multiply both sides by -1. The sides become -3 and -5 — and Figure 2B.3 shows what happened on the number line: multiplying by -1 reflects each number in 0, so the order reverses. -3 now sits to the **right** of -5, which means the true statement is $-3>-5$. The symbol had to flip.



Multiplying by −1 reflects each number in 0 — the order reverses.

Two number lines: the top line marks 3 and 5 with the statement 3 is less than 5; the bottom line marks −5 and −3 with the statement −3 is greater than −5; dashed red arrows pass through 0 from each top number to its negative below, showing the reflection. Caption: multiplying by −1 reflects each number in 0, so the order reverses.

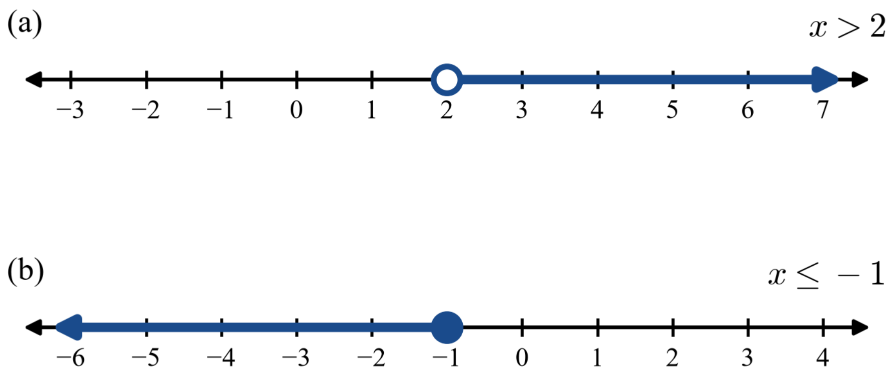
Direction rule. Multiplying or dividing both sides of an inequality by a **positive** number keeps the direction of the symbol. Multiplying or dividing both sides by a **negative** number **reverses** it: < swaps with >, and ≤ swaps with ≥.

For example, from $-2x<10$, divide both sides by -2 **and flip**: $x>-5$. Forgetting the flip is the single most common error with inequalities, and it is the one VCE markers see most — every solution here that divides or multiplies by a negative writes the flip down on purpose, as its own step.

5. Graphing solutions on a number line

The number line shows the whole solution set at once, using three conventions:

- an **open dot** at the endpoint when the endpoint is *not* a solution (< or >);
- a **closed dot** at the endpoint when the endpoint *is* a solution (≤ or ≥);
- an **arrow** from the endpoint in the direction of all the other solutions.



Two number lines: (a) an open dot at 2 with the blue arrow to the right, labelled x greater than 2; (b) a closed dot at −1 with the blue arrow to the left, labelled x less than or equal to −1.

Reading a graph back into a symbol is the same skill in reverse: the dot gives the endpoint, its open or closed fill gives the symbol, and the arrow gives the direction.

> **Beyond the number line — not assessed at Level 10.** When an inequality has **two** unknowns, its solutions are pairs of numbers, and their graph is a shaded region of the Cartesian plane rather than a ray on a line. Graphing regions is a VCE skill (see Teacher note); VCAA's own wording for this level says the solutions are graphed **on a number line**, and that is what this subtopic teaches and assesses. Every question here uses the number line.

6. From words to an inequality

Word problems run on four steps.

1. Choose a letter for the unknown quantity, and say what it counts.
2. Translate the words into an inequality, using the table in section 1.
3. Solve it using sections 3 and 4.
4. Answer the question **in words**, and check the answer fits the situation — a count of people or things must be a whole number, *at least* includes the boundary value, and *fewer than* does not.

Worked examples

Example 1 — Adding, subtracting and positive division, with graphs

(a) Solve $x+4<9$, and graph the solutions on a number line.

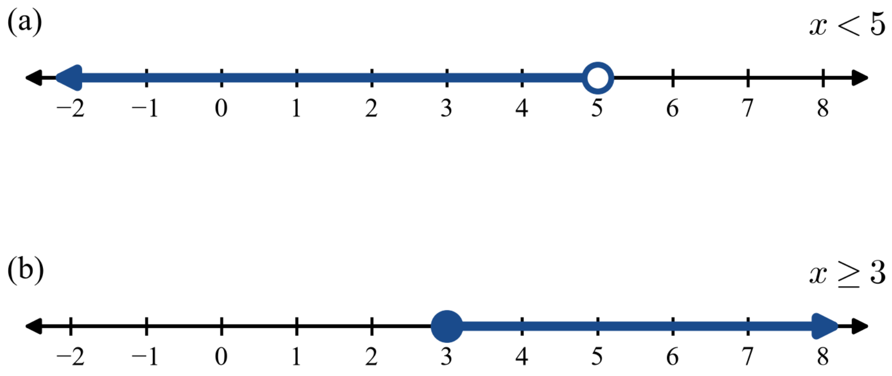
Working	Comment
$x+4<9$	
$x<5$	Subtract 4 from both sides.
$\therefore x<5.$	Check: $x=3$ gives $3+4=7<9$. ✓

Graph: open dot at 5, arrow to the left (Figure 2B.5(a)).

(b) Solve $5x\geq 15$, and graph the solutions on a number line.

Working	Comment
$5x \geq 15$	
$x \geq 3$	Divide both sides by 5 — positive, so the direction stays.
$\therefore x \geq 3.$	Check: $x = 3$ gives $15 \geq 15$, so the endpoint is included. ✓

Graph: closed dot at 3, arrow to the right (Figure 2B.5(b)).



Number-line graphs of Worked Example 1: (a) an open dot at 5 with the blue arrow to the left, labelled x less than 5; (b) a closed dot at 3 with the blue arrow to the right, labelled x greater than or equal to 3.

Example 2 — The direction rule

(a) Solve $-2x < 10$, and graph the solutions on a number line.

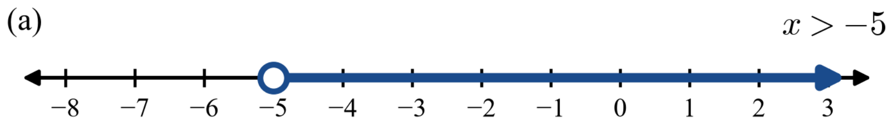
Working	Comment
$-2x < 10$	
$x > -5$	Divide both sides by -2 — negative, so **flip** the symbol.
$\therefore x > -5.$	Check: $x = 0$ gives $0 < 10$ ✓; $x = -6$ gives $12 < 10$, false, as it should be. ✓

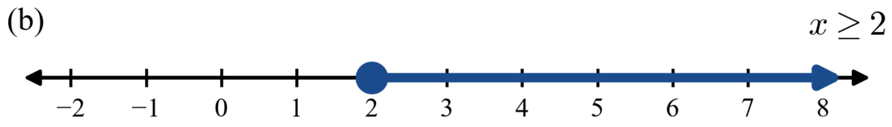
Graph: open dot at -5, arrow to the right (Figure 2B.6(a)).

(b) Solve $8 - 3x \leq 2$, and graph the solutions on a number line.

Working	Comment
$8 - 3x \leq 2$	
$-3x \leq -6$	Subtract 8 from both sides.
$x \geq 2$	Divide both sides by -3 and **flip** the symbol.
$\therefore x \geq 2.$	Check: $x = 2$ gives $8 - 6 = 2 \leq 2$ ✓; $x = 1$ gives $5 \leq 2$, false. ✓

Graph: closed dot at 2, arrow to the right (Figure 2B.6(b)).





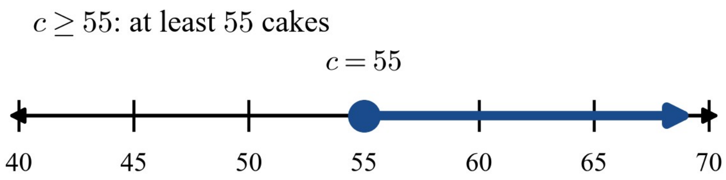
Number-line graphs of Worked Example 2: (a) an open dot at −5 with the blue arrow to the right, labelled x greater than −5; (b) a closed dot at 2 with the blue arrow to the right, labelled x greater than or equal to 2.

Example 3 — A word problem

SCENARIO A school fete stall has \$75 already in the tin, and each cake sold adds \$3. The committee's goal is to raise **at least** \$240. How many cakes must it sell?

Working	Comment
Let c = the number of cakes sold.	Step 1 — choose the letter.
$75+3c \geq 240$	Step 2 — "at least" becomes ≥.
$3c \geq 165$	Subtract 75 from both sides.
$c \geq 55$	Divide both sides by 3 — positive, direction stays.
∴ the stall must sell at least 55 cakes.	Step 4 — answer in words.

Check: $75+3\times 55=240$, exactly the goal, so 55 counts and the dot is closed; selling 56 raises \$243, more than the goal. ✓



Number line of the number of cakes c, from 40 to 70 in steps of 5, with a closed dot at 55 and the blue arrow to the right, showing at least 55 cakes.

Practice questions

Scaffolded

Q1. Check by substituting. (a) Which of 6, 5.5 and -1 are solutions of $x<6$? (b) Which of -2, 0 and -3 are solutions of $x \geq -2$?

Q2. Translate into a symbol and a letter: (a) y is at least 10; (b) n is no more than 12; (c) t is greater than 4; (d) p is at most -1.

Q3. Solve, then graph on a number line. (Hint: add or subtract the same number on both sides.) (a) $x+3<7$ (b) $x-5 \geq 2$

Q4. Solve, then graph. (Hint: divide by a positive number — the direction stays.) (a) $4x \leq 20$ (b) $\frac{x}{2}>3$

Q5. Solve, then graph. (Hint: dividing by a negative number **flips** the symbol.) (a) $-x<4$ (b) $-3x \geq 12$

Q6. Write each number line below as an inequality in x.

(a)

(b)

Two number lines to read as inequalities: (a) an open dot at −2 with the blue arrow to the right; (b) a closed dot at 4 with the blue arrow to the left.

Q7. Solve $5-x>2$, and graph the solutions. (Hint: subtract 5 first, then divide both sides by -1 and flip the symbol.)

Q8. SCENARIO A goods lift carries at most 600 kg, and each box loaded onto it weighs 25 kg. (a) Write an inequality for the number of boxes n. (b) Solve it. (c) State the greatest number of boxes the lift can carry.

Unscaffolded

Q9. Solve and graph: (a) $x-7<2$; (b) $x+8\geq 3$; (c) $6+x\leq 1$.

Q10. Solve and graph: (a) $3x>21$; (b) $\frac{x}{4}\leq 5$; (c) $7x\geq -21$.

Q11. Solve and graph: (a) $-4x<16$; (b) $-\frac{x}{2}\geq 3$; (c) $-5x\leq 40$.

Q12. Solve: (a) $2x+3>11$; (b) $4x-1\leq 15$; (c) $5x+7<2$.

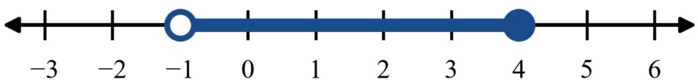
Q13. Solve: (a) $7-2x\geq 1$; (b) $3x+8>5x$; (c) $9-x<4x+4$.

Q14. Solve: (a) $2(x-3)<8$; (b) $3(4-x)\geq 6$.

Q15. Graph on a number line: (a) $x\leq -4$; (b) $x>2$; (c) $x\geq 0$.

Q16. List every **integer** that satisfies: (a) $-2<x\leq 3$; (b) $-3\leq x<3$.

Q17. The solutions of an inequality are shown below. (a) Write them as an inequality in x. (b) List every integer solution.



Number line from −3 to 6 with an open dot at −1, a closed dot at 4, and the part of the line between the two dots drawn in blue.

Q18. (a) State the smallest integer satisfying $x\geq 4.5$. (b) State the largest integer satisfying $x<-1.5$. (c) Solve $4x-3\geq 9$ and state its smallest whole-number solution.

Q19. SCENARIO Priya has saved \$120 towards a \$300 bike and saves \$15 each week. Write and solve an inequality for the number of weeks w until her savings reach at least \$300, and answer in words.

Q20. `SCENARIO` A warehouse trolley weighs 12 kg empty and each crate loaded onto it weighs 18 kg. The lift it must travel in carries at most 450 kg. Find the greatest number of crates that can go on the trolley.

Q21. A student solves $-3x > 12$ and writes $x > -4$. Identify the error, and give the correct solution.

Q22. (a) Starting from $2 < 5$, explain why multiplying both sides of an inequality by -1 reverses its direction. (b) Without solving fully, decide whether the solutions of $5 - x < 1$ are the values greater than 4 or the values less than 4, and give your reason.

Real context

Q23. Victoria's estimated resident population was $7\,121\,900$ at 31 December 2025, having grown by $117\,300$ in the year to that date. Suppose the population kept growing by exactly $117\,300$ people each year.

(a) Write an expression for the population P, n years after December 2025.
(b) Write and solve an inequality for the values of n for which the model predicts a population greater than $8\,000\,000$. Give the boundary to two decimal places.
(c) State the first whole number of years after December 2025 for which the model predicts more than $8\,000\,000$ people.
(d) In which calendar year does the model first predict more than $8\,000\,000$ people?

Source: Australian Bureau of Statistics, *National, state and territory population*, 2026. Retrieved 20 August 2026.

Answers

Q1. (a) 5.5 and -1 are solutions; 6 is not. (b) -2 and 0 are solutions; -3 is not. **Q2.** (a) $y \ge 10$; (b) $n \le 12$; (c) $t > 4$; (d) $p \le -1$. **Q3.** (a) $x < 4$; (b) $x \ge 7$. **Q4.** (a) $x \le 5$; (b) $x > 6$. **Q5.** (a) $x > -4$; (b) $x \le -4$. **Q6.** (a) $x > -2$; (b) $x \le 4$. **Q7.** $x < 3$. **Q8.** (a) $25n \le 600$; (b) $n \le 24$; (c) 24 boxes. **Q9.** (a) $x < 9$; (b) $x \ge -5$; (c) $x \le -5$. **Q10.** (a) $x > 7$; (b) $x \le 20$; (c) $x \ge -3$. **Q11.** (a) $x > -4$; (b) $x \le -6$; (c) $x \ge -8$. **Q12.** (a) $x > 4$; (b) $x \le 4$; (c) $x < -1$. **Q13.** (a) $x \le 3$; (b) $x < 4$; (c) $x > 1$. **Q14.** (a) $x < 7$; (b) $x \le 2$. **Q15.** graphs (see solutions). **Q16.** (a) $-1, 0, 1, 2, 3$; (b) $-3, -2, -1, 0, 1, 2$. **Q17.** (a) $-1 < x \le 4$; (b) $0, 1, 2, 3, 4$. **Q18.** (a) 5; (b) -2; (c) $x \ge 3$, smallest whole-number solution 3. **Q19.** $w \ge 12$ — at least 12 weeks. **Q20.** 24 crates. **Q21.** the symbol was not flipped when dividing by -3; correctly, $x < -4$. **Q22.** (a) see solutions; (b) greater than 4. **Q23.** (a) $P = 7\,121\,900 + 117\,300n$; (b) $n > 7.49$ (exactly, $n > \frac{878\,100}{117\,300}$); (c) 8 years; (d) 2033.

Fully-worked solutions

Q1. (a) $6 < 6$ is false; $5.5 < 6$ is true; $-1 < 6$ is true. So 5.5 and -1 are solutions, and 6 is not.

(b) $-2 \ge -2$ is true; $0 \ge -2$ is true; $-3 \ge -2$ is false. So -2 and 0 are solutions, and -3 is not.

Q2. (a) $y \ge 10$. (b) $n \le 12$. (c) $t > 4$. (d) $p \le -1$.

Q3. (a)

$x + 3 < 7$

$x < 4$ — subtract 3 from both sides.

Graph: open dot at 4, arrow to the left.

$x - 5 \ge 2$

$x \ge 7$ — add 5 to both sides.

Graph: closed dot at 7, arrow to the right.

Q4. (a)

$4x \le 20$

$x \le 5$ — divide both sides by 4.

Graph: closed dot at 5, arrow to the left.

$\frac{x}{2} > 3$

$x > 6$ — multiply both sides by 2.

Graph: open dot at 6, arrow to the right.

Q5. (a)

$-x < 4$

$x > -4$ — divide both sides by -1 and flip.

Graph: open dot at -4, arrow to the right.

$-3x \ge 12$

$x \le -4$ — divide both sides by -3 and flip.

Graph: closed dot at -4, arrow to the left.

Q6. (a) Open dot at -2, arrow right: $x > -2$.

(b) Closed dot at 4, arrow left: $x \leq 4$.

Q7.

$5 - x > 2$

$-x > -3$ — subtract 5 from both sides.

$x < 3$ — divide both sides by -1 and flip.

Graph: open dot at 3, arrow to the left.

Q8. (a) $25n \leq 600$.

$25n \leq 600$

$n \leq 24$ — divide both sides by 25.

(c) The greatest number of boxes is 24.

Q9. (a)

$x - 7 < 2$

$x < 9$ — add 7. Graph: open dot at 9, arrow left.

$x + 8 \geq 3$

$x \geq -5$ — subtract 8. Graph: closed dot at -5, arrow right.

$6 + x \leq 1$

$x \leq -5$ — subtract 6. Graph: closed dot at -5, arrow left.

Q10. (a)

$3x > 21$

$x > 7$ — divide by 3. Graph: open dot at 7, arrow right.

$\frac{x}{4} \leq 5$

$x \leq 20$ — multiply by 4. Graph: closed dot at 20, arrow left.

$7x \geq -21$

$x \geq -3$ — divide by 7. Graph: closed dot at -3, arrow right.

Q11. (a)

$-4x < 16$

$x > -4$ — divide by -4 and flip. Graph: open dot at -4, arrow right.

$-\frac{x}{2} \geq 3$

$x \leq -6$ — multiply by -2 and flip. Graph: closed dot at -6, arrow left.

$-5x \leq 40$

$x \geq -8$ — divide by -5 and flip. Graph: closed dot at -8, arrow right.

Q12. (a)

$2x+3>11$

$2x>8$ — subtract 3.

$x>4$ — divide by 2.

$4x-1 \leq 15$

$4x \leq 16$ — add 1.

$x \leq 4$ — divide by 4.

$5x+7<2$

$5x<-5$ — subtract 7.

$x<-1$ — divide by 5.

Q13. (a)

$7-2x \geq 1$

$-2x \geq -6$ — subtract 7.

$x \leq 3$ — divide by -2 and flip.

$3x+8>5x$

$8>2x$ — subtract $3x$ from both sides.

$4>x$, that is $x<4$ — divide by 2.

$9-x<4x+4$

$5<5x$ — add x and subtract 4 on both sides.

$1<x$, that is $x>1$ — divide by 5.

Q14. (a)

$2(x-3)<8$

$x-3<4$ — divide both sides by 2.

$x<7$ — add 3.

$3(4-x) \geq 6$

$4-x \geq 2$ — divide both sides by 3.

$-x \geq -2$ — subtract 4.

$x \leq 2$ — divide by -1 and flip.

Q15. (a) Closed dot at -4, arrow to the left. (b) Open dot at 2, arrow to the right. (c) Closed dot at 0, arrow to the right.

Q16. (a) The integers strictly greater than -2 and at most 3 are $-1, 0, 1, 2, 3$.

(b) The integers at least -3 and strictly less than 3 are $-3, -2, -1, 0, 1, 2$.

Q17. (a) Open dot at -1 and closed dot at 4 with the line between shaded: $-1 < x \le 4$.

(b) The integers strictly greater than -1 and at most 4 are $0, 1, 2, 3, 4$.

Q18. (a) The integers ≥ 4.5 are $5, 6, 7, \ldots$; the smallest is 5.

(b) The integers < -1.5 are $\ldots, -4, -3, -2$; the largest is -2.

$4x - 3 \ge 9$

$4x \ge 12$ — add 3.

$x \ge 3$ — divide by 4.

The whole numbers ≥ 3 start at 3, so the smallest is 3.

Q19. Let w = the number of weeks. Her savings after w weeks are $120 + 15w$ dollars, and these must reach at least 300:

$120 + 15w \ge 300$

$15w \ge 180$ — subtract 120.

$w \ge 12$ — divide by 15.

∴ Priya must save for at least 12 weeks.

Check: $120 + 15 \times 12 = 300$, exactly the price, so 12 weeks is enough. ✓

Q20. Let n = the number of crates. The loaded trolley weighs $12 + 18n$ kilograms, and this must be at most 450:

$12 + 18n \le 450$

$18n \le 438$ — subtract 12.

$n \le 24\frac{1}{3}$ — divide by 18.

A crate count is a whole number, so the greatest number of crates is 24.

Check: $12 + 18 \times 24 = 444 \le 450$, while 25 crates give $462 > 450$. ✓

Q21. Dividing both sides by -3 must flip the symbol, and the student did not. Correctly:

$-3x > 12$

$x < -4$ — divide by -3 and flip.

Check: $x = -5$ gives $15 > 12$ ✓, while the student's $x = 0$ would give $0 > 12$, false. ✓

Q22. (a) $2 < 5$ is true. Multiplying by -1 reflects both numbers in 0 (Figure 2B.3): 2 becomes -2 and 5 becomes -5, and -2 now lies to the right of -5. So the true statement after the step is $-2 > -5$ — the symbol reversed. The same reflection happens for every pair, which is why the rule holds in general.

(b) $5 - x < 1$ gives $-x < -4$ after subtracting 5; dividing by -1 flips the symbol, so $x > 4$ — the solutions are the values **greater** than 4.

Q23. (a) $P = 7\,121\,900 + 117\,300n$.

(b) Set $P > 8\,000\,000$:

$7\,121\,900 + 117\,300n > 8\,000\,000$

$117\,300\,n > 878\,100$ — subtract $7\,121\,900$.

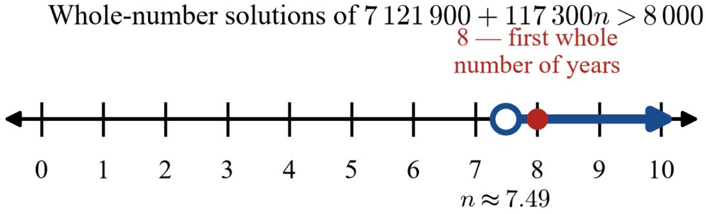
$n > \dfrac{878\,100}{117\,300} \approx 7.49$ — divide by $117\,300$.

(c) The first whole number of years with $n > 7.49$ is 8.

Check: at $n = 7$ the model gives $7\,943\,000 < 8\,000\,000$; at $n = 8$ it gives $8\,060\,300 > 8\,000\,000$. ✓

(d) 8 years after December 2025 is December 2033, so the calendar year is 2033.

Whole-number solutions of $7\,121\,900 + 117\,300n > 8\,000\,000$



Number line of years n from 0 to 10 with an open dot at about 7.49 and the blue arrow to the right; the whole number 8 is marked in red as the first whole number of years.

Teacher note

Level 9 code rehearsed (D3): `VC2M9A03` — solving linear equations. Every solution in this subtopic runs on 2A's balancing moves (expand, collect, divide), rehearsed rather than re-taught, and the direction rule of §4 is the only new content. The symbols <, >, ≤, ≥ and simple one-step inequalities are assumed from earlier years; §1 restates them briefly and §2 supplies the set-of-solutions idea that is new at this level.

Tool classes (D13): none. `VC2M10A08` names no digital tool in its content description or elaborations, so this subtopic carries no banner and no digital task; the number line is a by-hand skill and is assessed by hand.

VCE destinations (§5, reference only): `VC2M10A08` is named in the Units 3 & 4 assumed background of Foundation Mathematics, General Mathematics and Mathematical Methods, with the skill *"Solve a linear inequality, including sign flip when multiplying or dividing by a negative"* — exactly the content assessed here, which is why the direction rule gets its own worked step throughout. **Direction-of-travel trap (D12):** all three skill indexes also attach a *"half-plane region for a linear inequality in two variables"* skill to `VC2M10A08`. That is a VCE expectation reading back onto a Level 10 code; it does not widen this subtopic, and the labelled aside in §5 is the only place regions are mentioned.

Elaboration `VC2M10A08.E02` — carried for linkage and coverage only. It is truncated in VCAA's own export and in the authority file, reading in full exactly:

```
graphing regions corresponding to inequalities in the Cartesian plane (for
example, graphing $2x+3y
```

Per D12 this book does not complete the inequality, does not guess the relation symbol, and does not quote the elaboration as though it were whole. (The quote is set as a code block because the authority's own text ends on an unbalanced dollar sign.) The CD's own wording — "graph their solutions **on a number line**" — sets the assessed skill, and the number line is what this subtopic teaches, practises and assesses.

Level check (D6, R11): no interval notation and no set notation; compound statements appear only as a single bounded range read from one number line (Q17); no inequalities in two variables beyond the labelled aside; no quadratic or absolute-value inequalities; no function or relation terminology; the solving methods are addition/subtraction, multiplication/division and one division by a negative per step — the full Level 10 CD and nothing past it.

Dated data (R3): Q23 carries the ABS estimated resident population for Victoria at 31 December 2025 ($7\,121\,900$) and the annual increase to that date ($117\,300$). Listed here for the chapter's refresh register so a reissue can update the figures without re-reading the chapter.

Simultaneous linear equations, algebraically and graphically

Mathematics 10 · Chapter 2 Equations and inequalities · Subtopic 2C (Level 10) · Extended block

Strand	Algebra
Content description	`VC2M10A09`
Elaboration ids carried	`VC2M10A09.E01` `VC2M10A09.E02` `VC2M10A09.E03` `VC2M10A09.E04` (`VC2M10A09.E05` not delivered — see Teacher note)
Weight	Extended (largest bank code in the book)

> **Content description (verbatim):** *solve simultaneous linear equations, using algebraic and graphical techniques including using digital tools*
>
> **Achievement standard (extract):** *They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, algebraically and graphically, with and without the use of digital tools, and justify solutions.*

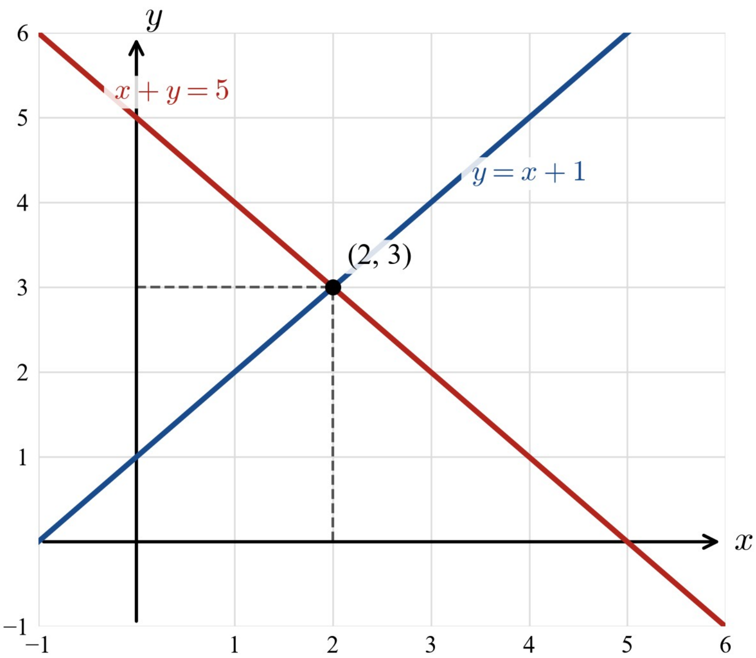
You need first: solving linear equations and sketching straight-line graphs (`VC2M9A03`), and the gradient–intercept form $y=mx+c$ from Level 9. This subtopic rehearses both skills and applies them; it does not re-teach them.

Theory

One equation in two unknowns, such as $x+y=5$, has many solutions. A table of values gives some of them:

x	0	1	2	3	4	5
y	5	4	3	2	1	0

Every pair in that table makes $x+y=5$ true, and there are infinitely many more besides. On a graph, all of them sit on one straight line. So one equation cannot pin the pair down — but a **second** equation in the same two unknowns can. Ask which pairs also make $y=x+1$ true, and the table shrinks fast: $(2,3)$ works, since $3=2+1$, and it is the only pair in the table that does. Two equations that must both be true at once are called **simultaneous equations**, and a pair (x,y) that makes both of them true is their **solution**.



The straight line x + y = 5 falling from left to right and the straight line y = x + 1 rising from left to right, crossing at the single point (2, 3); dashed guides run from the point to 2 on the x-axis and to 3 on the y-axis.

*Figure 2C.1 — The pair $(2,3)$ is the only pair on **both** lines, so it is the only solution of the pair of equations.*

On the graph, the solutions of $x+y=5$ fill one line and the solutions of $y=x+1$ fill the other, so a pair that solves both equations must sit on both lines — at the point where they cross, their **intersection**. That is the whole picture this subtopic builds on:

$$\text{the solution of two simultaneous linear equations} = \text{the coordinates of the intersection of their graphs.}$$

Three methods find that pair: **substitution**, **elimination** and **graphing**. The algebraic methods give exact answers; the graph shows what the answer means.

1. Checking a proposed solution

A pair is a solution only if it makes **both** equations true, so checking takes two substitutions. Is $(3,2)$ a solution of $x+y=5$ and $2x-y=4$?

- First equation: $3+2=5$. ✓
- Second equation: $2\times3-2=4$. ✓

Both true, so $(3,2)$ is a solution. Is $(1,4)$ a solution of $x+y=6$ and $y=x+3$? The second equation holds ($4=1+3$) but the first fails ($1+4=5\neq6$), so $(1,4)$ is **not** a solution. One failed equation is enough to rule a pair out.

This check is also how every answer in this subtopic is justified: substitute the pair back into both original equations.

2. Method 1 — Substitution

When one equation already says what a variable is, like $y=2x-1$, that equation can be **substituted** into the other: everywhere the second equation writes y, write $2x-1$ instead. Two unknowns become one, and a linear equation in one unknown is exactly the skill of 2A.

1. Pick the equation with a variable on its own (or make one, e.g. $y=\ldots$ or $x=\ldots$).
2. Substitute that expression into the **other** equation.
3. Solve the resulting one-variable equation.
4. Substitute back to find the second variable.
5. Check the pair in both original equations.

3. Method 2 — Elimination

Line the two equations up so that matching variables sit under each other. If one variable has the **same coefficient** in both equations, subtracting one equation from the other removes — **eliminates** — that variable, because $3x-3x=0$. If the coefficients are opposite in sign ($+2y$ and $-2y$), **adding** the equations eliminates it instead. What remains is one equation in one variable.

1. Line the equations up, x under x and y under y.
2. Add or subtract so that one variable disappears; if no coefficient already matches, multiply one (or both) equations by a number first so that it does.
3. Solve the one-variable equation.
4. Substitute back to find the other variable.
5. Check the pair in both original equations.

4. Choosing the efficient method

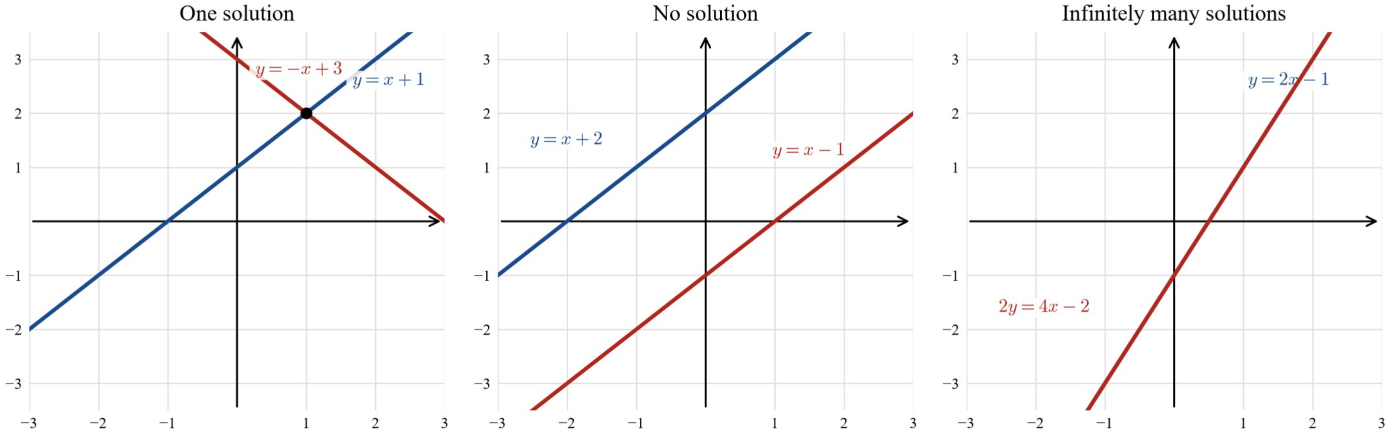
Both methods always work, but one is usually shorter. The judgement:

- **Substitution** is shortest when a variable is already on its own, or nearly so: $y=3x-2$, $x=4y-3$, $y=5-x$. Substituting costs one line.
- **Elimination** is shortest when both equations are lined up in the form $ax+by=c$ and a coefficient already matches, or matches after one multiplication: $3x+4y=17$ and $3x+2y=13$ share the term $3x$, so subtracting finishes the x-problem at once.

When neither is obvious, either choice is fine — the check at the end is what justifies the answer.

5. The graph: one solution, none, or infinitely many

Two straight lines on the same axes do exactly one of three things, and each case reads off as a statement about solutions.



Three panels. Left panel: two lines with different gradients crossing at one marked point, labelled one solution. Middle panel: two parallel lines that never cross, labelled no solution. Right panel: a single line carrying both equation labels, labelled infinitely many solutions.

Figure 2C.2 — The three cases: crossing lines give one solution; parallel lines give none; one line carrying both equations gives infinitely many.

- **The lines cross at one point.** One intersection, one solution — the usual case, and the case in Figure 2C.1.
- **The lines are parallel.** Parallel lines have the same gradient and never cross, so there is **no solution**: no pair sits on both lines. Algebra announces this as an impossible statement such as $8=10$.
- **The lines are the same line**, written two ways. Every point of the line is on both lines, so there are **infinitely many solutions**. Algebra announces this as a statement that is always true, such as $6=6$.

6. Comparison and break-even problems

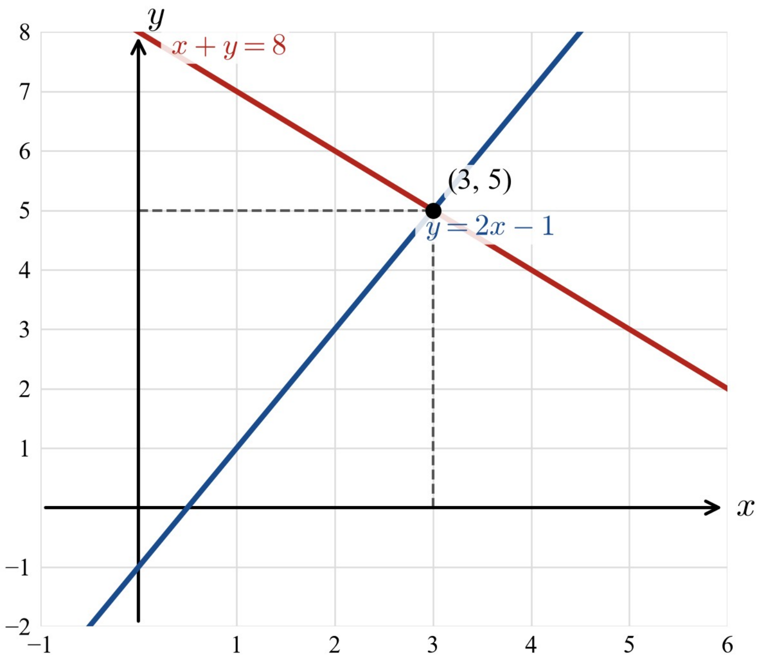
Two simultaneous equations often arrive as two competing rules for the same quantity — two quotes for the same job, two plans for the same service. Each rule is a line; the intersection is the **break-even point**, the case where both options cost the same. On one side of that point the first option is cheaper, on the other side the second is — and the graph shows which is which at a glance, because the cheaper option is simply the lower line. Reading a comparison off the graph in everyday language — *for jobs under five hours, the second quote is cheaper* — is exactly what the intersection means in context.

Worked examples

Example 1 — Substitution

(a) Solve $y=2x-1$ and $x+y=8$.

Working	Comment
$x+(2x-1)=8$	Substitute $2x-1$ for y in the second equation.
$3x-1=8$	Collect the x-terms.
$3x=9$	Add 1 to both sides.
$x=3$	Divide both sides by 3.
$y=2\times3-1=5$	Substitute back into $y=2x-1$.
$\therefore x=3,\ y=5.$	Check: $3+5=8$. ✓



The line y = 2x − 1 rising steeply and the line x + y = 8 falling from left to right, crossing at the marked point (3, 5) with dashed guides to 3 on the x-axis and 5 on the y-axis.

Figure 2C.3 — The graphs of the two equations in Example 1(a) cross at $(3, 5)$*, the algebraic solution.*

(b) Solve $x = 4y - 3$ and $2x - y = 8$.

Working	Comment
$2(4y-3) - y = 8$	Substitute $4y-3$ for x in the second equation.
$8y - 6 - y = 8$	Expand the bracket.
$7y - 6 = 8$	Collect the y-terms.
$7y = 14$	Add 6 to both sides.
$y = 2$	Divide both sides by 7.
$x = 4 \times 2 - 3 = 5$	Substitute back into $x = 4y - 3$.
$\therefore x = 5,\ y = 2.$	Check: $2 \times 5 - 2 = 8$. ✓

The variable on its own does not have to be y — substituting for x works the same way.

Example 2 — Elimination

(a) Solve $3x + 2y = 16$ and $x + 2y = 8$.

Working	Comment
$3x + 2y = 16$	
$x + 2y = 8$	Both equations carry the term $+2y$.
$(3x - x) + (2y - 2y) = 16 - 8$	Subtract the second equation from the first.
$2x = 8$	The y-terms eliminate.
$x = 4$	
$4 + 2y = 8$, so $y = 2$	Substitute into the shorter equation.
$\therefore x = 4,\ y = 2.$	Check: $3 \times 4 + 2 \times 2 = 16$. ✓

(b) Solve $2x + 3y = 13$ and $3x - y = 3$.

Working	Comment
$3x - y = 3$	No coefficient matches yet.
$9x - 3y = 9$	Multiply that equation by 3, to make $-3y$ against $+3y$

Working	Comment
	.
$(2x+9x)+(3y-3y)=13+9$	Add the first equation to the new one.
$11x=22$	The y-terms eliminate.
$x=2$	
$3\times 2-y=3$, so $y=3$	Substitute into $3x-y=3$.
$\therefore x=2,\ y=3.$	Check: $2\times 2+3\times 3=13$. ✓

Multiplying an equation by a number does not change its solutions, so the new equation is safe to use.

Example 3 — Choosing the efficient method

(a) Solve $y=3x-2$ and $2x+y=13$.

Working	Comment
$2x+(3x-2)=13$	y is already on its own: substitution is the short path.
$5x-2=13$	
$5x=15$	
$x=3$	
$y=3\times 3-2=7$	
$\therefore x=3,\ y=7.$	Check: $2\times 3+7=13$. ✓

(b) Solve $3x+4y=17$ and $3x+2y=13$.

Working	Comment
$(3x-3x)+(4y-2y)=17-13$	Both equations carry $3x$: elimination is the short path.
$2y=4$	
$y=2$	
$3x+4=13$, so $x=3$	Substitute into $3x+2y=13$.
$\therefore x=3,\ y=2.$	Check: $3\times 3+4\times 2=17$. ✓

Elimination here took two lines; substitution would have meant rearranging first. The method is a choice, and the choice is part of the skill.

Example 4 — No solution, and infinitely many

(a) Solve $2x+y=4$ and $4x+2y=10$.

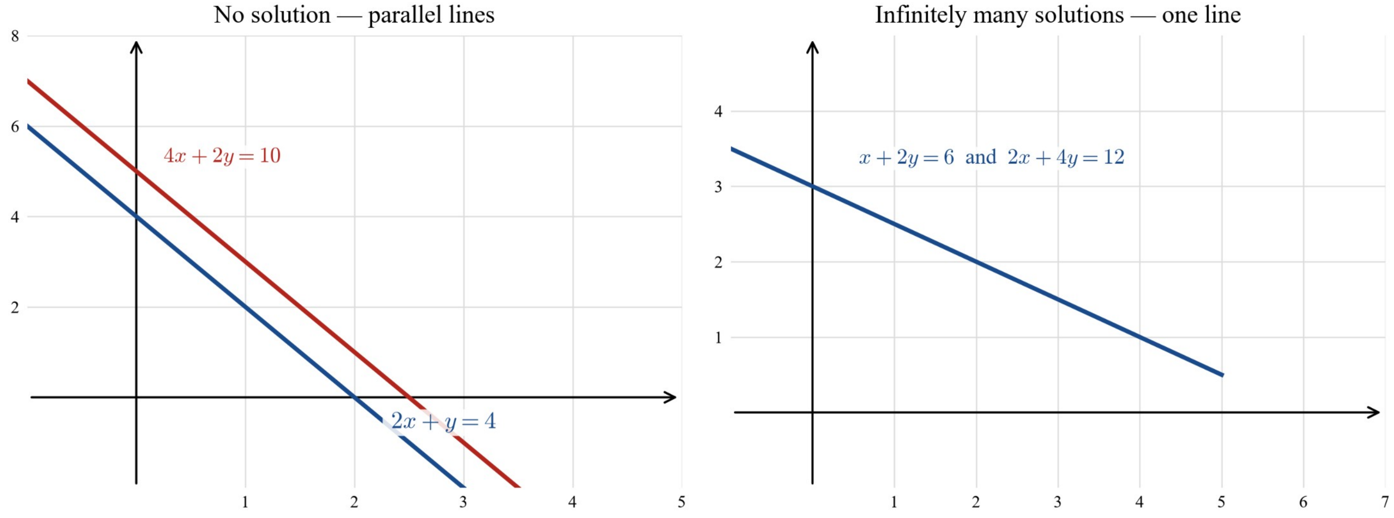
Working	Comment
$4x+2y=8$	Double the first equation.
$4x+2y=10$	The second equation.
$8=10$	Subtract: impossible.

The left-hand sides are identical and the right-hand sides are not — no pair (x, y) can make both true. There is **no solution**. The graph shows why: $y=-2x+4$ and $y=-2x+5$ are **parallel** lines with the same gradient -2, and parallel lines never cross.

(b) Solve $x+2y=6$ and $2x+4y=12$.

Working	Comment
$2x+4y=12$	Double the first equation.
$2x+4y=12$	The second equation — identical.
$12=12$	Always true.

The two equations are the same line written two ways: both rearrange to $y = 3 - \frac{x}{2}$. Every point of that line solves both equations, so there are **infinitely many solutions**.



Left panel: the parallel lines 2x + y = 4 and 4x + 2y = 10, both falling with gradient −2 and never crossing, so there is no solution. Right panel: one falling line carrying both labels x + 2y = 6 and 2x + 4y = 12, so every point on it is a solution.

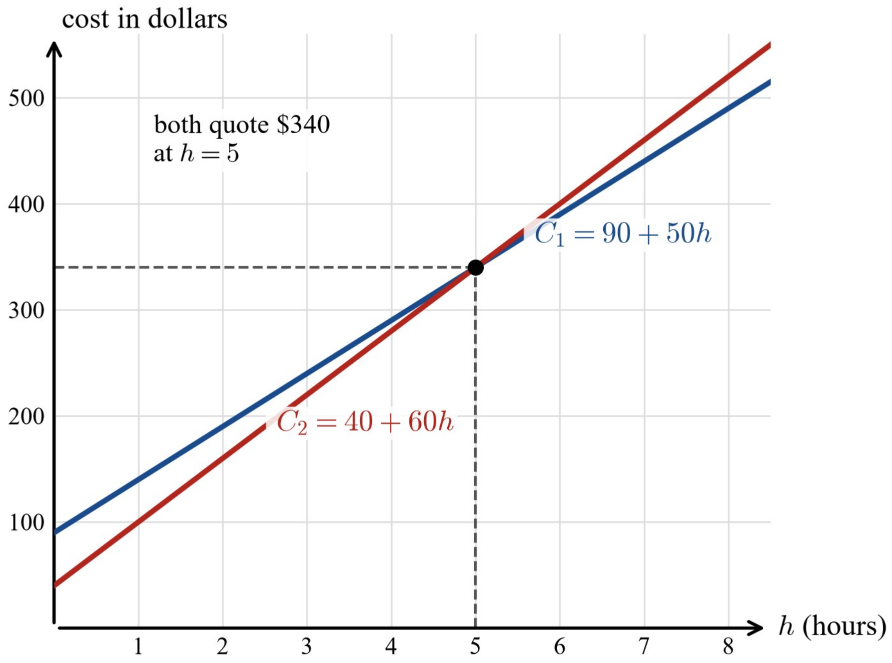
Figure 2C.4 — Parallel lines never cross (no solution); one line carrying both equations has every point in common (infinitely many solutions).

Example 5 — Break-even: comparing two quotes

SCENARIO Two electricians quote for a job. Electrician A charges a 90 call-out fee plus 50 per hour. Electrician B charges a 40 call-out fee plus 60 per hour. For how many hours of work do the two quotes come to the same amount, and what is that amount?

Let h be the number of hours and C the cost in dollars.

Working	Comment
$C = 90 + 50h$ and $C = 40 + 60h$	One line per quote: fee plus hourly rate.
$90 + 50h = 40 + 60h$	Same cost: set the rules equal.
$90 + 50h - 50h = 40 + 60h - 50h$	Subtract $50h$ from both sides.
$90 = 40 + 10h$	
$50 = 10h$	Subtract 40 from both sides.
$h = 5$	
$C = 90 + 50 \times 5 = 340$	Substitute back.
$\therefore$ both quotes are $\$340$ at $h = 5$ hours.	Check: $40 + 60 \times 5 = 340$. ✓



Cost in dollars against hours of work for the two quotes: the line C1 = 90 + 50h starting higher but rising more slowly, and the line C2 = 40 + 60h starting lower but rising faster; the lines cross at the marked point h = 5, cost 340 dollars, with dashed guides to both axes.

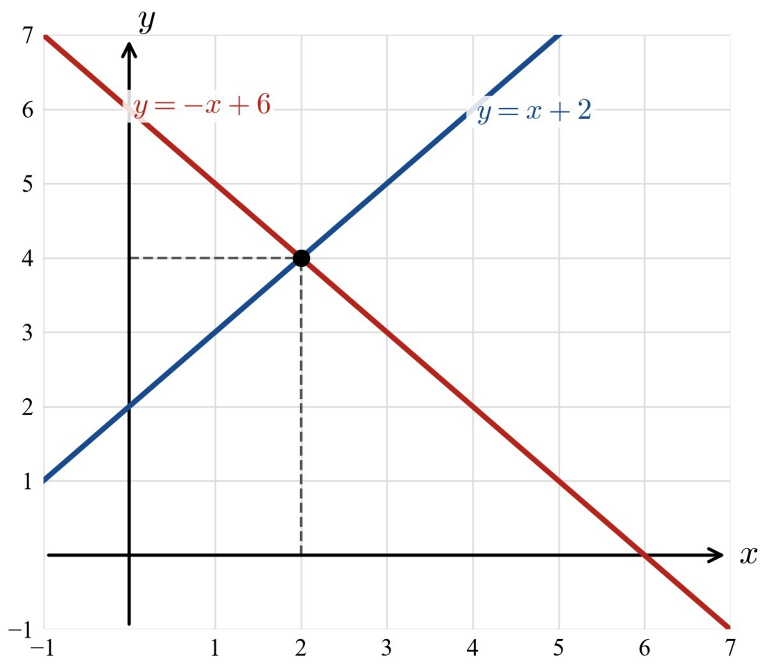
Figure 2C.5 — The break-even point of the two quotes. To the left of $h=5$ *the red line (B) is lower; to the right, the blue line (A) is lower.*

The graph turns the answer into advice: for a job of **under** 5 hours, B is the cheaper quote (its line is lower), and for **over** 5 hours, A is cheaper. At exactly 5 hours there is nothing to choose.

Example 6 — Solving graphically

Solve $y=x+2$ and $y=-x+6$ by graphing, and check the answer by substitution.

Working	Comment
Draw $y=x+2$: intercept 2, gradient 1.	Gradient–intercept sketching from `VC2M9A03`.
Draw $y=-x+6$: intercept 6, gradient -1.	Same axes, same scale.
The lines cross at $(2,4)$.	Read the intersection.
$\therefore x=2,\ y=4$.	The coordinates of the intersection are the solution.
Check: $4=2+2$ ✓ and $4=-2+6$ ✓	Both equations hold.



The line y = x + 2 rising and the line y = −x + 6 falling, crossing at the marked point (2, 4) with dashed guides to 2 on the x-axis and 4 on the y-axis.

Figure 2C.6 — The intersection of $y=x+2$ *and* $y=-x+6$ *reads as* $(2,4)$.

Graphing shows the solution rather than computing it, and it is the method that makes the three cases of Figure 2C.2 visible. Its limit is precision: an intersection that does not land on grid lines can only be read approximately, and that is when the algebraic methods take over.

Practice questions

Scaffolded

Q1. Check by substitution, as in Theory §1. (a) Is $(3,2)$ a solution of $x+y=5$ and $2x-y=4$? (b) Is $(1,4)$ a solution of $x+y=6$ and $y=x+3$?

Q2. Solve by substitution — the first equation already gives y. (a) $y=x+1$ and $x+y=7$. (b) $y=3x$ and $2x+y=10$.

Q3. Solve by elimination — a coefficient already matches, so add or subtract straight away. (a) $x+y=9$ and $x-y=3$. (b) $2x+y=11$ and $2x-y=5$.

Q4. Solve by elimination — adding the equations eliminates y at once. (a) $x+2y=11$ and $3x-2y=9$. (b) $2x+3y=18$ and $x-3y=0$.

Q5. Look at Figure 2C.6 from Example 6. (a) Read the intersection of the two lines. (b) Check by substitution that the pair you read makes both equations $y=x+2$ and $y=-x+6$ true.

Q6. Without solving, state how many solutions each pair has, and say which case of Figure 2C.2 it is. (a) $y=2x+1$ and $y=2x-3$. (b) $y=3x$ and $y=-x+8$. (c) $x+y=4$ and $2x+2y=8$.

Q7. `SCENARIO` A school prints its newsletter with one of two services. Service A charges a 120 set-up fee plus 3 per copy. Service B charges 4 per copy with no set-up fee. Let n be the number of copies and C the cost in dollars, so $C=120+3n$ for A and $C=4n$ for B. (a) Copy the table and complete it.

n	50	100	150
cost with A			
cost with B			

(b) Which service is cheaper for 50 copies? For 150 copies? (c) Solve $120+3n=4n$ and say what the answer means for the two services.

Q8. (a) Show by substitution that $(4,1)$ makes both $2x+3y=11$ and $5x-2y=18$ true. (b) Now solve the same pair by elimination and confirm that you get the same pair.

Unscaffolded

Q9. Solve by substitution: (a) $y=x-4$ and $3x+y=16$; (b) $y=2x+3$ and $4x-y=1$; (c) $x+y=7$ and $2x+3y=17$.

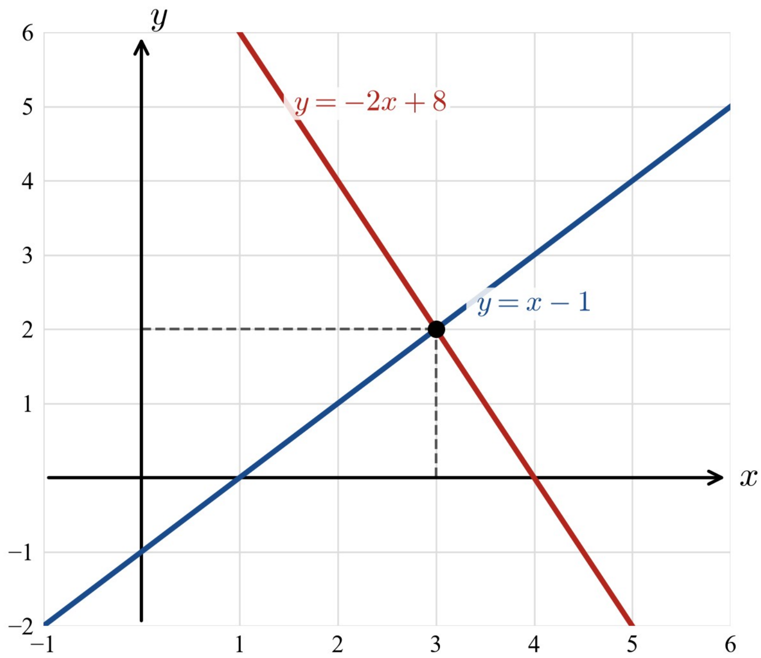
Q10. Solve by elimination: (a) $x+y=11$ and $x-y=5$; (b) $3x+y=10$ and $3x-y=2$; (c) $2x+3y=12$ and $2x+y=8$.

Q11. Solve by either method: (a) $2x+y=7$ and $3x-2y=0$; (b) $3x+4y=10$ and $5x-2y=8$; (c) $4x-3y=1$ and $2x+5y=7$.

Q12. First choose the more efficient method, then solve. (a) $y=4x-1$ and $x+y=9$; (b) $3x+5y=24$ and $3x+2y=15$; (c) $x-2y=1$ and $3x-y=8$.

Q13. State whether each pair has one solution, no solution, or infinitely many. If there is one solution, find it. (a) $2x+y=4$ and $2x+y=9$; (b) $3x-y=2$ and $6x-2y=4$; (c) $y=3x-1$ and $y=x+5$.

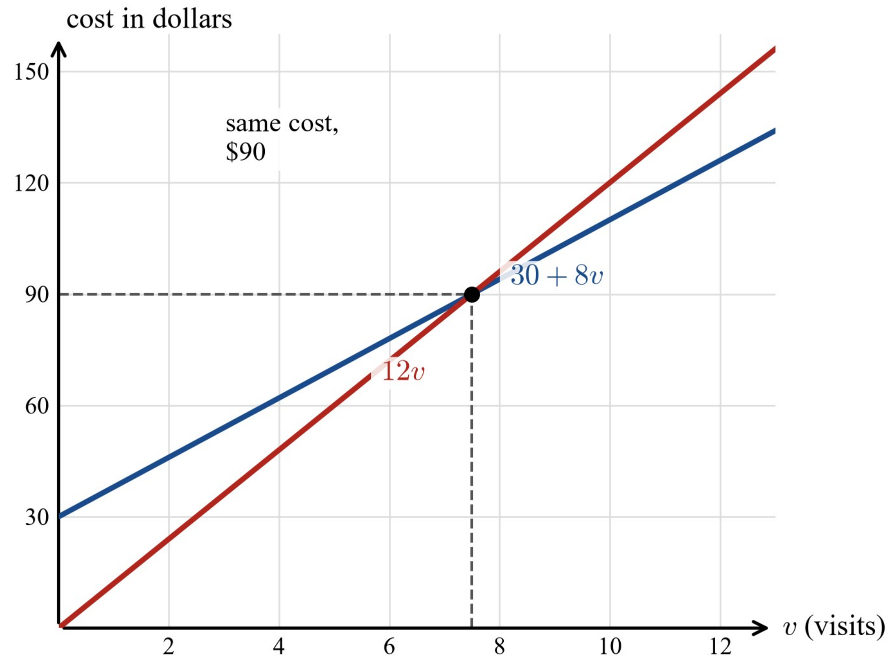
Q14. The figure shows the lines $y=x-1$ and $y=-2x+8$. (a) Read the coordinates of their intersection. (b) Check by substitution that the pair is the solution of both equations.



The line y = x − 1 rising gently and the line y = −2x + 8 falling steeply, crossing at the marked point (3, 2) with dashed guides to 3 on the x-axis and 2 on the y-axis.

Figure 2C.7 — The graph for Q14: read the intersection, then check it algebraically.

Q15. `SCENARIO` A gym offers two membership plans. Plan A costs a 30 joining fee plus 8 per visit. Plan B costs 12 per visit with no joining fee. (a) Write a cost rule for each plan, using v for the number of visits and C for the cost in dollars. (b) Solve the two rules simultaneously to find when the plans cost the same. (c) A member expects at least 8 visits a term. Which plan is cheaper for them?



Cost in dollars against number of visits for two gym plans: the line 30 + 8v starting at 30 and rising slowly, and the line 12v starting at the origin and rising faster; the lines cross at the marked point 7.5 visits, cost 90 dollars, with dashed guides to both axes.

Figure 2C.8 — The two plans of Q15 cost the same at 7.5 *visits; beyond that point the line for Plan A is lower.*

Q16. SCENARIO A club orders trophies. Option A costs a 400 set-up fee plus 3 per trophy. Option B costs 8 per trophy with no set-up fee. Find the number of trophies for which the two options cost the same, and say which option is cheaper beyond that number.

Q17. (a) Two numbers have a sum of 31 and a difference of 7. Find them. (b) A rectangle has a perimeter of 46 cm, and its length is 5 cm more than its width. Find the length and the width.

Q18. Solve $3x - y = 10$ and $x + 2y = 1$. (One of the variables will be negative.)

Q19. Solve: (a) $3x + 2y = 16$ and $5x - 4y = 12$; (b) $4x + 3y = 10$ and $2x - 5y = -8$.

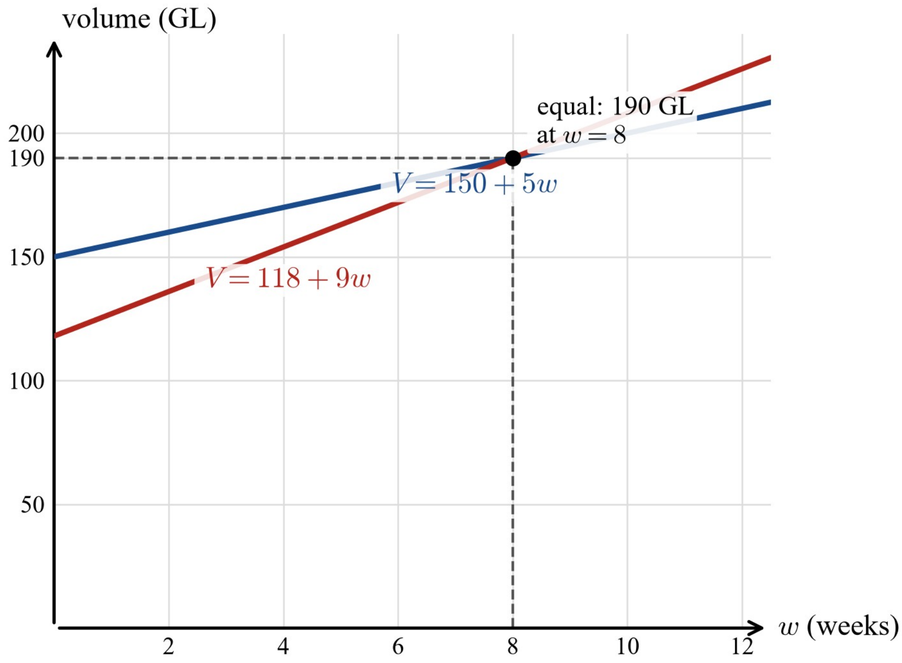
Q20. SCENARIO Rider A sets out along a bike path at 20 km/h. Half an hour later, Rider B leaves the same start along the same path at 30 km/h. Let t be the time in hours since Rider A started and d the distance in km from the start, so $d = 20t$ for Rider A and $d = 30(t - 0.5)$ for Rider B. (a) Solve the two equations simultaneously. (b) Say what your answer means about when and where Rider B catches Rider A.

Q21. Two angles add to $180°$. The larger angle is $30°$ more than twice the smaller. Find both angles.

Q22. Solve: (a) $2y - x = 7$ and $3x + 2y = 11$; (b) $x + 3y = 5$ and $2x - y = 4$. (Not every answer is a whole number.)

Real context

Q23. In a wet winter, water flows into Melbourne's Cardinia Reservoir and Upper Yarra Reservoir. Suppose that at the start of the rains the Cardinia Reservoir holds 150 GL and gains 5 GL a week, while the Upper Yarra Reservoir holds 118 GL and gains 9 GL a week, so that w weeks later their volumes are $V = 150 + 5w$ and $V = 118 + 9w$ gigalitres. (a) Solve the two equations simultaneously to find when the two reservoirs hold the same volume. (b) State that volume. (c) The Cardinia Reservoir holds about 287 GL full and the Upper Yarra about 201 GL full. Is the volume from (b) under both capacities, so that the model stays sensible that far?



Volume in gigalitres against weeks: the line V = 150 + 5w starting at 150 and the line V = 118 + 9w starting at 118 and rising faster; the lines cross at the marked point 8 weeks, 190 gigalitres, with dashed guides to both axes.

Figure 2C.9 — The two reservoir volumes of Q23 become equal at 8 *weeks,* 190 *GL.*

Source: Melbourne Water, *Cardinia Reservoir* and *Upper Yarra Reservoir*, 2026. Retrieved 20 August 2026.

Digital tool task

DIGITAL TOOL — this task assumes a graphing tool. Your teacher will nominate the tool.

Task. Use a graphing tool to solve $y = 2x - 3$ and $y = -x + 4$ by finding the intersection.

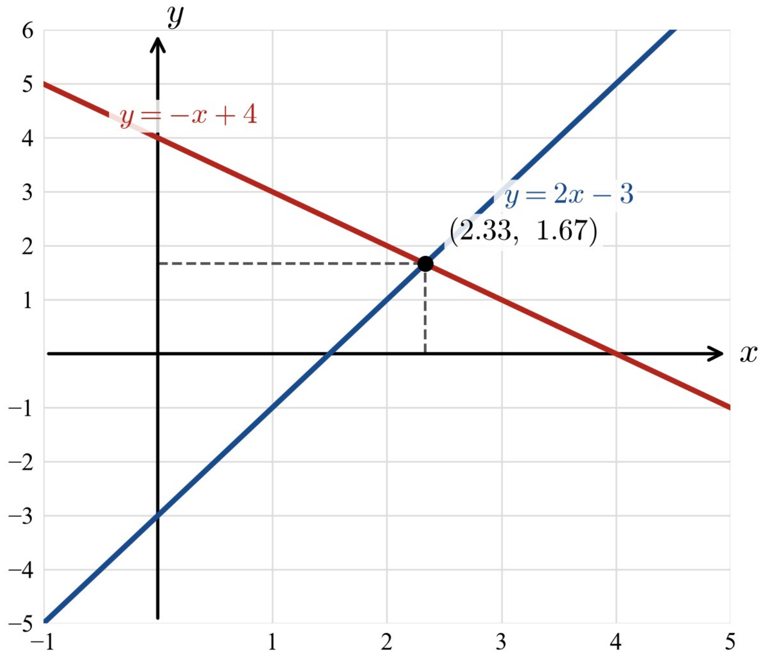
Input (what to graph):

1. the line $y = 2x - 3$;
2. the line $y = -x + 4$;
3. the point where the two lines meet, and its coordinates.

Output (what to expect): the tool reports the intersection as about $(2.33, 1.67)$ — the exact answer is $\left(\frac{7}{3}, \frac{5}{3}\right)$, and the tool rounds it. Because the point does not land on grid lines, this pair of equations is one a graph can only ever read approximately: exactly the case named at the end of Example 6.

Tool-free path (still assessed): solve the pair by substitution.

Working	Comment
$2x - 3 = -x + 4$	Both equations equal y: set them equal.
$3x - 3 = 4$	Add x to both sides.
$3x = 7$	Add 3.
$x = \frac{7}{3} \approx 2.33$	
$y = -\frac{7}{3} + 4 = \frac{5}{3} \approx 1.67$	Substitute into $y = -x + 4$.



The line y = 2x − 3 rising steeply and the line y = −x + 4 falling gently, crossing between grid lines at the marked point about (2.33, 1.67), with dashed guides to both axes.

Figure 2C.10 — The intersection of the digital task does not land on grid lines, so the graph reads it only approximately.

Answers

Q1. (a) yes — $3+2=5$ and $2\times3-2=4$; (b) no — $y=x+3$ holds but $1+4=5\neq6$. **Q2.** (a) $x=3,\ y=4$; (b) $x=2,\ y=6$. **Q3.** (a) $x=6,\ y=3$; (b) $x=4,\ y=3$. **Q4.** (a) $x=5,\ y=3$; (b) $x=6,\ y=2$. **Q5.** (a) $(2,4)$; (b) $4=2+2$ ✓ and $4=-2+6$ ✓. **Q6.** (a) no solution — parallel lines, same gradient 2; (b) one solution, $(2,6)$; (c) infinitely many — the second equation is the first doubled. **Q7.** (a) A: 270, 420, 570; B: 200, 400, 600; (b) B for 50 copies, A for 150 copies; (c) $n=120$ — the two services cost the same, $\$480$, at 120 copies; below that, B is cheaper; above, A. **Q8.** (a) $2\times4+3\times1=11$ ✓ and $5\times4-2\times1=18$ ✓; (b) $x=4,\ y=1$. **Q9.** (a) $x=5,\ y=1$; (b) $x=2,\ y=7$; (c) $x=4,\ y=3$. **Q10.** (a) $x=8,\ y=3$; (b) $x=2,\ y=4$; (c) $x=3,\ y=2$. **Q11.** (a) $x=2,\ y=3$; (b) $x=2,\ y=1$; (c) $x=1,\ y=1$. **Q12.** (a) substitution — $x=2,\ y=7$; (b) elimination — $x=3,\ y=3$; (c) elimination (after one multiplication) — $x=3,\ y=1$. **Q13.** (a) no solution; (b) infinitely many; (c) one solution, $x=3,\ y=8$. **Q14.** (a) $(3,2)$; (b) $2=3-1$ ✓ and $2=-2\times3+8$ ✓. **Q15.** (a) $C=30+8v$ and $C=12v$; (b) $v=7.5$ visits, cost $\$90$; (c) Plan A. **Q16.** $n=80$ trophies; beyond 80, Option A is cheaper. **Q17.** (a) 19 and 12; (b) length 14 cm, width 9 cm. **Q18.** $x=3,\ y=-1$. **Q19.** (a) $x=4,\ y=2$; (b) $x=1,\ y=2$. **Q20.** (a) $t=1.5,\ d=30$; (b) Rider B catches Rider A 1.5 hours after Rider A started, 30 km from the start. **Q21.** $130°$ and $50°$. **Q22.** (a) $x=1,\ y=4$; (b) $x=\frac{17}{7},\ y=\frac{6}{7}$. **Q23.** (a) $w=8$ weeks; (b) 190 GL; (c) yes — $190<201<287$, so both are under capacity. **Digital tool task:** $x=\frac{7}{3}\approx2.33,\ y=\frac{5}{3}\approx1.67$.

Fully-worked solutions

Q1. (a) $3+2=5$ ✓ and $2\times3-2=4$ ✓ — both hold, so $(3,2)$ **is** a solution.

(b) $1+4=5\neq6$ — the first equation fails, so $(1,4)$ is **not** a solution (even though $4=1+3$ holds).

Q2. (a)

$x+(x+1)=7$

$2x+1=7$

$2x=6$

$x=3$

$y=3+1=4$

$2x+3x=10$

$5x=10$

$x=2$

$y=3\times2=6$

Q3. (a) Adding: $(x+x)+(y-y)=9+3$, so $2x=12$, $x=6$; then $6+y=9$, $y=3$.

(b) Adding: $4x=16$, so $x=4$; then $8+y=11$, $y=3$.

Q4. (a) Adding: $4x=20$, so $x=5$; then $5+2y=11$, $2y=6$, $y=3$.

(b) Adding: $3x=18$, so $x=6$; then $6-3y=0$, $y=2$.

Q5. (a) The lines in Figure 2C.6 cross at $(2,4)$.

(b) $4=2+2$ ✓ and $4=-2+6$ ✓ — the pair makes both equations true.

Q6. (a) Both lines have gradient 2 and different intercepts, so they are parallel: **no solution**.

(b) The gradients 3 and -1 differ, so the lines cross once: **one solution**. Solving, $3x = -x + 8$ gives $4x = 8$, $x = 2$, $y = 6$ — the solution is $(2, 6)$.

(c) Doubling $x + y = 4$ gives $2x + 2y = 8$, the second equation — the same line twice: **infinitely many solutions**.

Q7. (a)

n	50	100	150
cost with A	270	420	570
cost with B	200	400	600

(b) At 50 copies B is cheaper ($\$200$ against $\$270$); at 150 copies A is cheaper ($\$570$ against $\$600$).

$120 + 3n = 4n$

$120 = 4n - 3n$

$n = 120$

At 120 copies both services cost $4 \times 120 = \$480$ — the break-even point. Below 120 copies B is cheaper; above, A.

Q8. (a) $2 \times 4 + 3 \times 1 = 8 + 3 = 11$ ✓ and $5 \times 4 - 2 \times 1 = 20 - 2 = 18$ ✓.

(b) Double the first equation: $4x + 6y = 22$. Triple the second: $15x - 6y = 54$. Add: $19x = 76$, so $x = 4$; then $8 + 3y = 11$, $y = 1$ — the same pair as the check, $(4, 1)$.

Q9. (a)

$3x + (x - 4) = 16$

$4x - 4 = 16$

$4x = 20$

$x = 5$

$y = 5 - 4 = 1$

$4x - (2x + 3) = 1$

$2x - 3 = 1$

$2x = 4$

$x = 2$

$y = 2 \times 2 + 3 = 7$

(c) From the first equation, $y = 7 - x$. Then

$2x + 3(7 - x) = 17$

$2x + 21 - 3x = 17$

$-x = -4$

$x = 4$

$y = 7 - 4 = 3$

Q10. (a) Adding: $2x = 16$, $x = 8$; then $8 + y = 11$, $y = 3$.

(b) Adding: $6x = 12$, $x = 2$; then $6 + y = 10$, $y = 4$.

(c) Subtracting: $2y=4$, $y=2$; then $2x+2=8$, $x=3$.

Q11. (a) From the first equation, $y=7-2x$. Then $3x-2(7-2x)=0$, so $3x-14+4x=0$, $7x=14$, $x=2$ and $y=7-4=3$.

(b) Double the second equation: $10x-4y=16$. Add to the first: $13x=26$, $x=2$; then $6+4y=10$, $y=1$.

(c) Double the second equation: $4x+10y=14$. Subtract the first equation: $(4x-4x)+(10y+3y)=14-1$, so $13y=13$, $y=1$; then $2x+5=7$, $x=1$.

Q12. (a) y is on its own, so substitute: $x+4x-1=9$, $5x=10$, $x=2$; $y=4\times2-1=7$.

(b) Both equations carry $3x$, so subtract: $3y=9$, $y=3$; then $3x+6=15$, $x=3$.

(c) Double the second equation: $6x-2y=16$. Subtract the first: $5x=15$, $x=3$; then $3-2y=1$, $y=1$.

Q13. (a) The left-hand sides are identical and the right-hand sides differ, so the lines $y=-2x+4$ and $y=-2x+9$ are parallel: **no solution**.

(b) Doubling the first equation gives the second, so both are the line $y=3x-2$: **infinitely many solutions**.

(c) $3x-1=x+5$, so $2x=6$, $x=3$ and $y=8$: **one solution**, $(3,8)$.

Q14. (a) The intersection reads as $(3,2)$.

(b) $2=3-1$ ✓ and $2=-2\times3+8=2$ ✓ — the pair solves both equations.

Q15. (a) Plan A: $C=30+8v$. Plan B: $C=12v$.

$30+8v=12v$

$30=4v$

$v=7.5$

and $C=12\times7.5=90$: the plans cost the same at 7.5 visits, $\$90$.

(c) For 8 or more visits, Plan A’s line is the lower one (Figure 2C.8): $30+8\times8=94$ against $12\times8=96$. Plan A is cheaper.

Q16. Set $400+3n=8n$, so $400=5n$ and $n=80$. At 80 trophies both options cost $8\times80=\$640$; beyond 80, Option A is cheaper (its 3-per-trophy line rises more slowly).

Q17. (a) Let the numbers be x and y:

$x+y=31$

$x-y=7$

Adding: $2x=38$, $x=19$; then $19+y=31$, $y=12$. The numbers are 19 and 12.

(b) Let the length be l cm and the width w cm:

$2l+2w=46$

$l=w+5$

Substitute: $2(w+5)+2w=46$, so $4w+10=46$, $4w=36$, $w=9$; and $l=9+5=14$. The rectangle is 14 cm by 9 cm.

Q18. From the first equation, $y=3x-10$. Then

$x+2(3x-10)=1$

$x+6x-20=1$

$7x=21$

$x=3$

$y=3\times3-10=-1$

Check: $3\times3-(-1)=10$ ✓ and $3+2\times(-1)=1$ ✓.

Q19. (a) Double the first equation: $6x+4y=32$. Add to the second: $11x=44$, $x=4$; then $12+2y=16$, $y=2$.

(b) Double the second equation: $4x-10y=-16$. Subtract it from the first: $(4x-4x)+(3y+10y)=10+16$, so $13y=26$, $y=2$; then $2x-10=-8$, $x=1$.

Q20. (a)

$20t=30(t-0.5)$

$20t=30t-15$

$15=10t$

$t=1.5$

$d=20\times1.5=30$

(b) Rider B catches Rider A 1.5 hours after Rider A set out, 30 km from the start — that is, 1 hour after Rider B set out.

Q21. Let the larger angle be a and the smaller b:

$a+b=180$

$a=2b+30$

Substitute: $2b+30+b=180$, so $3b=150$, $b=50$; and $a=2\times50+30=130$. The angles are $130°$ and $50°$.

Q22. (a) From the first equation, $x=2y-7$. Then

$3(2y-7)+2y=11$

$6y-21+2y=11$

$8y=32$

$y=4$

$x=2\times4-7=1$

(b) From the second equation, $y=2x-4$. Then

$x+3(2x-4)=5$

$x+6x-12=5$

$7x=17$

$x=\frac{17}{7}$

$y=2\times\frac{17}{7}-4=\frac{34}{7}-\frac{28}{7}=\frac{6}{7}$

A solution does not have to be a whole number — it only has to make both equations true, and $\left(\frac{17}{7},\frac{6}{7}\right)$ does.

Q23. (a)

$150+5w=118+9w$

$150-118=9w-5w$

$32=4w$

$w=8$

(b) $V=150+5\times 8=190$ GL (check: $118+9\times 8=190$ ✓).

(c) Yes: 190 GL is under the Upper Yarra's 201 GL and the Cardinia's 287 GL, so eight weeks into this pattern both reservoirs are still within their capacities.

Digital tool task. The tool reports the intersection of $y=2x-3$ and $y=-x+4$ as about $(2.33,1.67)$, matching the exact pair $\left(\frac{7}{3},\frac{5}{3}\right)$.

Tool-free path:

$2x-3=-x+4$

$3x=7$

$x=\frac{7}{3}\approx 2.33$

$y=-\frac{7}{3}+4=\frac{5}{3}\approx 1.67$

Teacher note

Level 9 code rehearsed (D3): `VC2M9A03` — sketching linear graphs and solving linear equations — does the graphical method (every line in Example 6, Q5 and Q14 is sketched by gradient and intercept) and closes every algebraic method once it has reduced the system to one equation in one unknown. It is rehearsed, not re-taught, and is not assessed as a Level 9 outcome. Downstream in this book, 2F ranges over these systems alongside the linear, quadratic and exponential equations of the chapter, and 3A's gradients of parallel lines make the no-solution case of Figure 2C.2 algebraic.

Tool classes (D13): a graphing tool for the digital tool task only (`VC2M10A09` names digital tools). No CAS: the book limits digital work to graphing and reading intersections, prints the input and expected output beside the task, and assesses the tool-free path — here substitution, since both equations are already equal to y. The digital task uses a non-lattice intersection on purpose, so students see what a graph can and cannot read exactly.

VCE destinations (§5, reference only): `VC2M10A09` is named in the Units 3 & 4 assumed background of Foundation Mathematics, General Mathematics and Mathematical Methods, with skills including *"solve simultaneous linear equations using substitution"*, *"...using elimination"*, *"choose the more efficient algebraic method for a given system"* and *"connect the algebraic solution of a simultaneous system to the coordinates of the graphs' intersection"* — Example 3 and Example 6 are those skills at Year 10. FM U3&4 solve break-even problems with simultaneous equations: Example 5 and Q7 are that in a workplace context.

Elaboration `VC2M10A09.E05` is not delivered. It invites the instructive game *Weme* of the Warlpiri Peoples of central Australia as a context for solving simultaneous equations. This book excludes Aboriginal and Torres Strait

Islander content (see the front-of-book note); the solving skills the elaboration exercises are delivered in full through the contexts of `VC2M10A09.E01–.E04`.

Level check (D6): every system in this subtopic is a pair of **linear** equations in two unknowns; no non-linear system and no three-unknown system appears, and the algebra stays at the two taught methods. The graphical method reads intersections only, and the three cases of Figure 2C.2 are stated without proof-theoretic language (no *consistent/inconsistent* vocabulary). Fractional answers appear (Q22, the digital task) and are justified by substitution, never by reading a graph. Every solution is justified by checking both original equations, per the achievement standard's *justify solutions*.

Dated data (R3): Q23 carries the full capacities of the Cardinia Reservoir (about 287 GL) and the Upper Yarra Reservoir (about 201 GL) from Melbourne Water; the starting volumes and weekly gains are a supposed scenario on those real capacities. Listed here for the chapter refresh register so a reissue can update the figures without re-reading the chapter.

Quadratic equations by a range of strategies, including the null factor law

Mathematics 10 · Chapter 2 Equations and inequalities · Subtopic 2D (Level 10) · Extended block

Strand	Algebra
Content description	VC2M10A13
Elaboration ids carried	VC2M10A13.E01 VC2M10A13.E02 VC2M10A13.E03
Weight	Extended

> **Content description (verbatim):** *solve simple quadratic equations using a range of strategies, including null factor law*
>
> **Achievement standard (extract):** *They solve problems involving linear equations and inequalities, quadratic equations and pairs of simultaneous linear equations and related graphs, algebraically and graphically, with and without the use of digital tools, and justify solutions.*

You need first: factorisation of quadratics, including the product-and-sum method, from 1D (VC2M10A04) — every strategy in this subtopic ends in factorisation or undoes it — and the graphing of quadratics and the null factor law for monic quadratics with integer roots from Year 9 (VC2M9A05). This subtopic rehearses those skills, extends them to the full range of strategies, and does not re-teach them.

Theory

So far in this chapter the unknown has appeared only to the first power: $3x+5=14$ in 2A. A **quadratic equation** lets the unknown be squared — and with the square comes a new idea, that an equation can have two solutions, one, or none at all. This subtopic builds four strategies for solving them — factorising with the null factor law, square roots, completing the square, and the quadratic formula — and ends with a way to see, before solving, how many solutions an equation has.

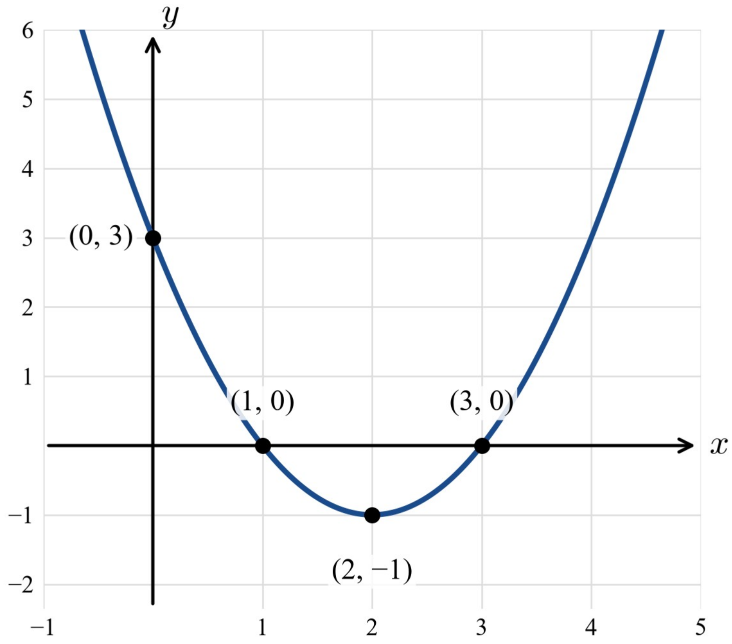
1. What a quadratic equation is

A **quadratic equation** is an equation that can be written in the form

$$ax^2+bx+c=0,$$

where a, b and c are numbers and $a\neq 0$. The form $ax^2+bx+c=0$ is called **standard form**. If a were 0 the x^2 term would vanish and the equation would be linear, so $a\neq 0$ is what makes the equation quadratic. The terms may need collecting and moving before the equation is in standard form: $x^2+7x=3x+12$ becomes $x^2+4x-12=0$.

A solution of a quadratic equation is a number that makes the equation true when it is substituted for the unknown. The graph of $y=ax^2+bx+c$ is a parabola (1D, and VC2M9A05 in Year 9), and substituting $y=0$ gives the equation itself — so **the solutions of $ax^2+bx+c=0$ are the x-coordinates of the points where the parabola $y=ax^2+bx+c$ crosses the x-axis.**



Graph of y = x squared minus 4x plus 3: a parabola crossing the x-axis at (1, 0) and (3, 0), with turning point (2, −1) and y-intercept (0, 3) marked

The figure shows the equation $x^2 - 4x + 3 = 0$: the parabola $y = x^2 - 4x + 3$ crosses the x-axis at $x = 1$ and $x = 3$, and sure enough $1^2 - 4 \times 1 + 3 = 0$ and $3^2 - 4 \times 3 + 3 = 0$. The equation has **two** solutions.

A parabola is a smooth bowl, and a straight line such as the x-axis can meet it at most twice — it cuts in, or touches at the bottom, or misses entirely. **So a quadratic equation has at most two solutions**, and the graph shows the three possibilities: two solutions, exactly one solution, or no real solutions at all. Subtopic 7 returns to this with a way to tell the cases apart without graphing.

2. The null factor law

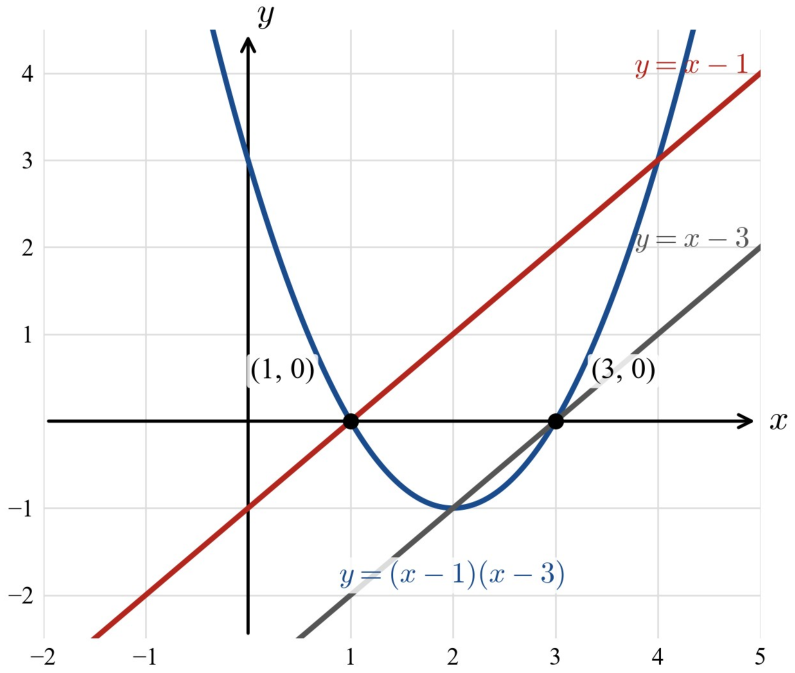
The first strategy rests on one fact about numbers.

The null factor law. If two numbers multiply to zero, then at least one of them is zero:

$$\text{if } A \times B = 0\text{, then } A = 0 \text{ or } B = 0.$$

The reason is short: multiplying by a non-zero number always gives a non-zero answer, so if the product is 0, one of the factors must have been 0. The law says nothing about *which* factor is zero — hence the *or*, and hence the two solutions a quadratic can have.

In Year 9 (`VC2M9A05`) the law solved monic quadratics with integer roots. Here it applies to any equation whose left side is a product of factors and whose right side is 0 — including factors such as $(2x - 1)$, where the unknown has a coefficient.



Graphs of y = (x − 1)(x − 3), y = x − 1 and y = x − 3 on the same axes: the parabola crosses the x-axis at (1, 0) and (3, 0), exactly where the factor lines y = x − 1 and y = x − 3 cross it

The figure makes the law visible. The product $(x-1)(x-3)$ is zero exactly where a factor is zero: the red line $y=x-1$ is zero at $x=1$, the grey line $y=x-3$ is zero at $x=3$, and the parabola $y=(x-1)(x-3)$ is zero at both — and nowhere else, because a product of non-zero numbers is non-zero.

The method is: get one side to 0, factorise the other, then let the law split the quadratic into two linear equations.

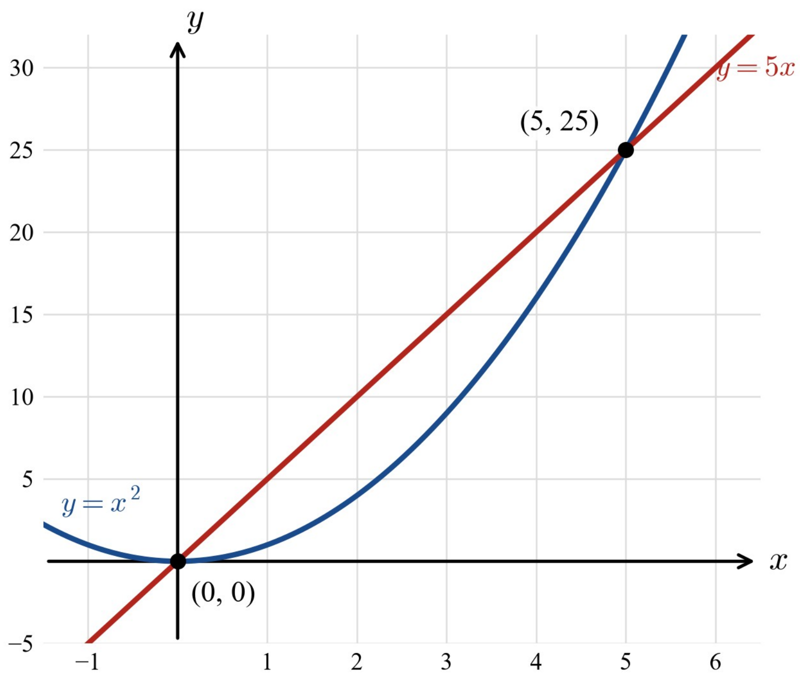
3. Strategy 1 — factorising and the null factor law

Steps.

1. Collect like terms and move everything to one side, so the equation reads (expression) $=0$.
2. Factorise the expression, choosing two integers with the required product and sum where needed (1D).
3. Apply the null factor law: set each factor equal to 0.
4. Solve each linear equation.
5. Check by substituting each solution back into the *original* equation.

Look for a **common factor first**: $x^2-9x=0$ factorises to $x(x-9)=0$, and the law gives $x=0$ or $x=9$. The difference of perfect squares is a factorisation too: $x^2-81=(x-9)(x+9)$.

Never divide both sides by the unknown. From $x^2=9x$, dividing by x gives $x=9$ — and the solution $x=0$ is lost, because dividing by x silently assumes $x\neq0$. Factorising instead, $x^2-9x=x(x-9)=0$, keeps both solutions.



Graphs of y = x squared and y = 5x meeting at (0, 0) and (5, 25): the equation x squared = 5x has two solutions, and dividing both sides by x would lose the one at (0, 0)

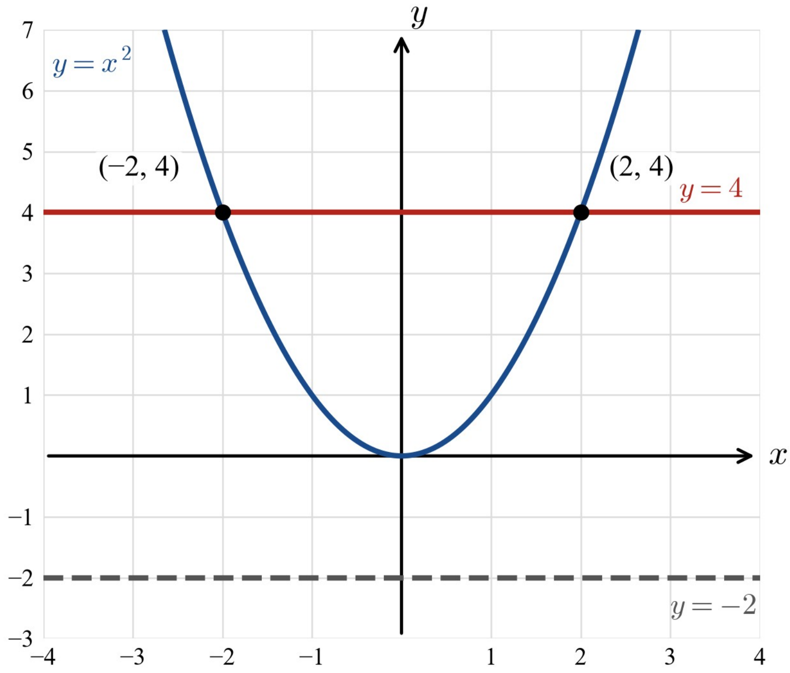
The figure shows why: $y = x^2$ and $y = 5x$ meet at **two** points, $(0, 0)$ and $(5, 25)$, so $x^2 = 5x$ has two solutions, $x = 0$ and $x = 5$. Dividing by x keeps only the second meeting.

4. Strategy 2 — square roots

Some quadratics need no factorising. If the equation can be written $x^2 = k$, take square roots of both sides — remembering that both a positive and a negative number square to the same answer:

- if $k > 0$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$ — **two solutions**, written $x = \pm\sqrt{k}$;
- if $k = 0$, then $x = 0$ — **one solution**;
- if $k < 0$, then **there are no real solutions**, because no real number squares to a negative.

So $x^2 = 25$ gives $x = \pm 5$, while $x^2 = -2$ has no real solutions. When k is not a perfect square, leave the answer in exact surd form: $x^2 = 7$ gives $x = \pm\sqrt{7}$.



Graph of y = x squared with the horizontal lines y = 4 and y = −2: the line y = 4 cuts the parabola at (−2, 4) and (2, 4), giving the two solutions of x squared = 4, while the line y = −2 never meets the parabola, so x squared = −2 has no real solutions

The figure reads both facts off the graph: the height $y=4$ is reached twice, at $x=-2$ and $x=2$, and the height $y=-2$ is never reached.

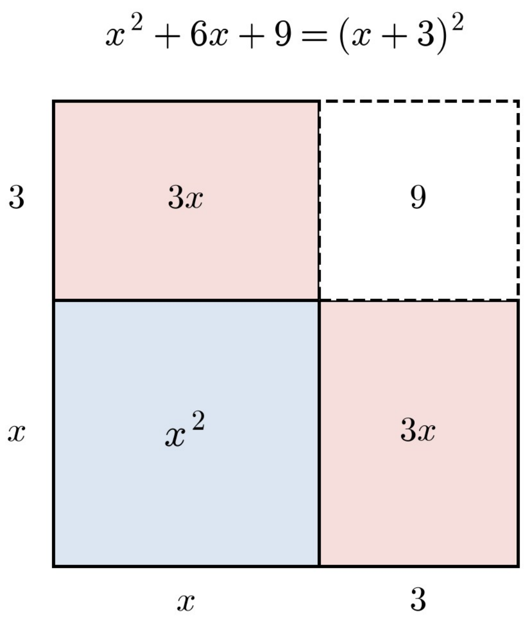
The same idea solves a perfect square equal to a number: from $(x+p)^2=k$, square roots give $x+p=\pm\sqrt{k}$, then a linear step finishes. This is exactly the last step of the next strategy.

5. Strategy 3 — completing the square

Completing the square (1D) rewrites a quadratic so that Strategy 2 applies. The key fact, for a monic quadratic, is

$$x^2+bx=(x+b/2)^2-(b/2)^2,$$

because expanding the right side returns x^2+bx: the square gives $x^2+bx+(b/2)^2$, and subtracting $(b/2)^2$ removes it again.



Area model for completing the square on x squared + 6x: a square of area x squared and two strips of area 3x leave a missing 3 by 3 corner; adding 9 completes a square of side x + 3, so x squared + 6x + 9 = (x + 3) squared

The figure shows the case $b=6$ as areas: the two $3x$ strips are the $6x$ split in half, and the dashed 3×3 corner is the $(6/2)^2=9$ that completes the square.

Steps, for $x^2+6x+2=0$:

1. Move the constant: $x^2+6x=-2$.
2. Complete the square: $(x+3)^2-9=-2$, so $(x+3)^2=7$.
3. Square roots (Strategy 2): $x+3=\pm\sqrt{7}$.
4. Solve: $x=-3\pm\sqrt{7}$.

Completing the square works on *every* monic quadratic, factorisable or not — it is the strategy to reach for when no product-and-sum pair exists. For a non-monic quadratic such as $2x^2+5x-4=0$, the quadratic formula below is the quicker path in this book.

6. Strategy 4 — the quadratic formula

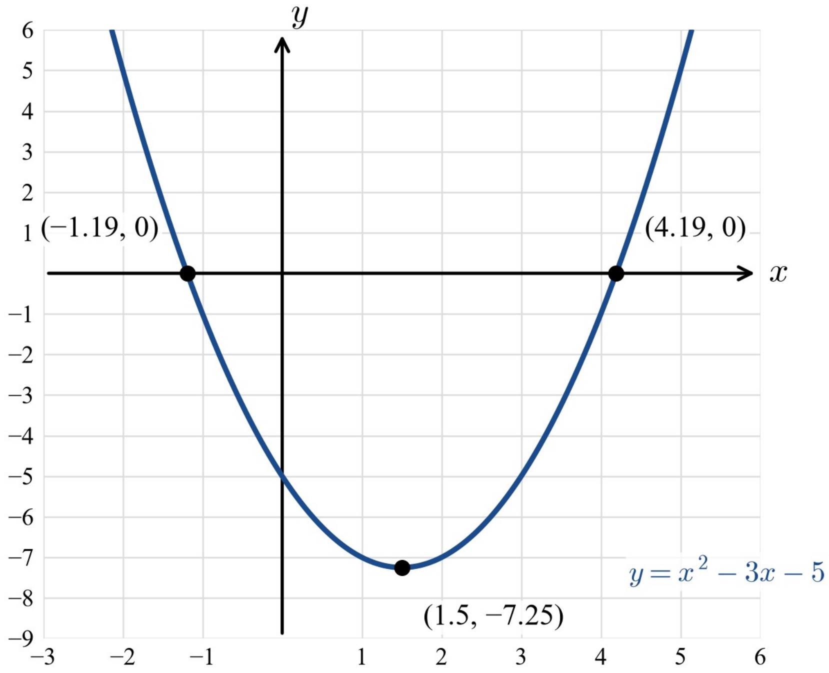
For any quadratic equation $ax^2+bx+c=0$ with $a\neq 0$, the solutions are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

In this book the formula is a tool: it is stated, used, and checked by substitution, and its derivation (which is completing the square on ax^2+bx+c) is left to later study.

Steps.

1. Write the equation in standard form.
2. Read off a, b and c, keeping their signs.
3. Substitute into the formula.
4. Simplify, leaving the answer in exact surd form; give decimals only when the question asks.
5. Check, by substitution or against a graph.



Graph of y = x squared minus 3x minus 5 crossing the x-axis at (−1.19, 0) and (4.19, 0) with turning point (1.5, −7.25): the two x-intercepts are the two values the quadratic formula gives for x squared − 3x − 5 = 0

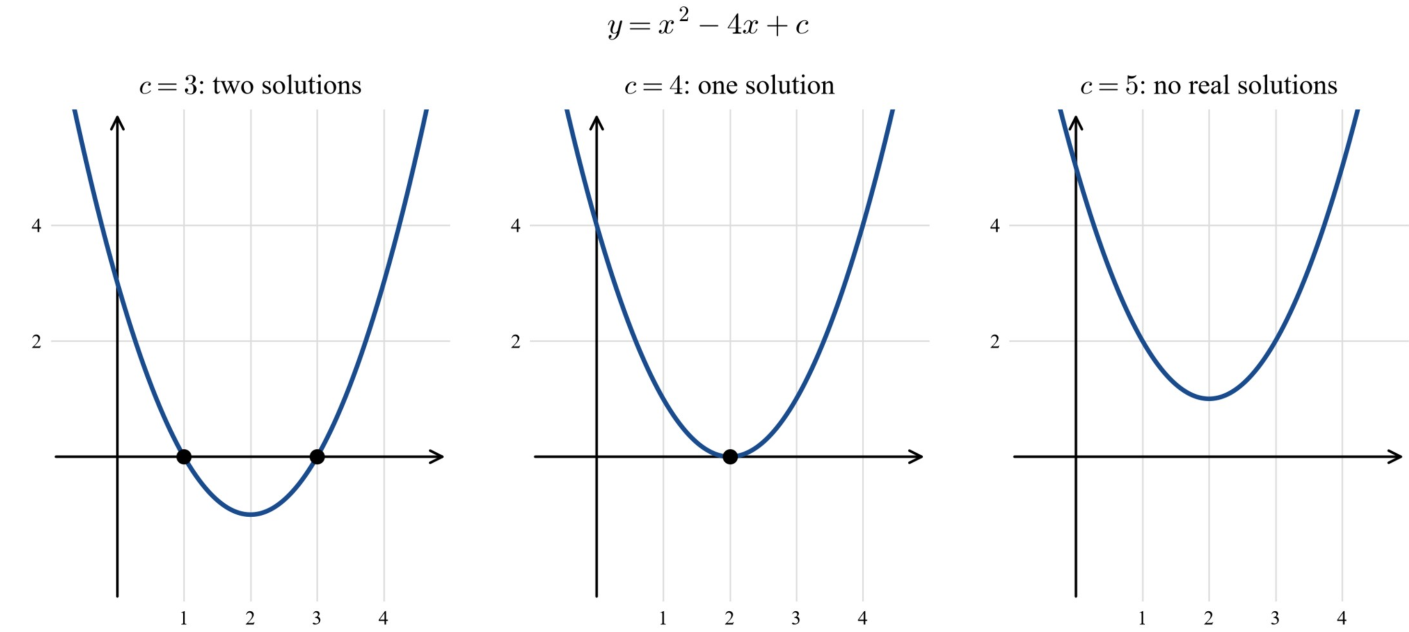
The figure checks the tool against the graph for $x^2 - 3x - 5 = 0$: with $a = 1$, $b = -3$, $c = -5$ the formula gives $x = \dfrac{3 \pm \sqrt{29}}{2}$, that is $x \approx -1.19$ or $x \approx 4.19$ — the two x-intercepts marked on the graph.

7. The discriminant and the number of solutions

The expression under the square root in the formula, $b^2 - 4ac$, is called the **discriminant**. Its **value** decides how many real solutions the equation has, because it decides whether the $\pm\sqrt{b^2 - 4ac}$ in the formula produces two values, one, or none.

Explore the family $x^2 - 4x + c = 0$ by changing only c:

Equation	Discriminant $b^2 - 4ac$	Solutions	Graph of $y = x^2 - 4x + c$
$x^2 - 4x + 3 = 0$	$16 - 12 = 4$	two: $x = 1$, $x = 3$	cuts the x-axis twice
$x^2 - 4x + 4 = 0$	$16 - 16 = 0$	one: $x = 2$	touches the x-axis once
$x^2 - 4x + 5 = 0$	$16 - 20 = -4$	no real solutions	never meets the x-axis



Three graphs of y = x squared minus 4x plus c side by side: for c = 3 the parabola cuts the x-axis at 1 and 3 (two solutions); for c = 4 it touches the x-axis at 2 (one solution); for c = 5 it sits above the x-axis entirely (no real solutions)

The pattern the table and figure show holds for every quadratic equation $ax^2+bx+c=0$:

- **discriminant >0:** two real solutions — the parabola cuts the x-axis twice;
- **discriminant $=0$:** one real solution — the parabola touches the x-axis at its turning point;
- **discriminant <0:** no real solutions — the parabola never meets the x-axis.

Graphically, the discriminant's value counts x-intercepts; algebraically, a negative discriminant means the formula would take a square root of a negative number, which no real number supplies. The digital tool task at the end of this subtopic explores this link, graphically and algebraically, on screen.

8. Choosing a strategy

All four strategies are correct on every equation they apply to; the skill is choosing the short road.

- **Already a product equal to zero** — use the null factor law straight away.
- **No x term, or a perfect square equal to a number** — use square roots.
- **A clean product-and-sum pair, or a common factor, or a difference of squares** — factorise.
- **Monic, but no factor pair in sight** — complete the square.
- **Non-monic, or nothing else appeals** — the quadratic formula always works.

Whichever strategy is used, the solutions are checked the same way: substitute them back into the original equation, or read them off the graph. And whatever the working, there are at most two solutions to find (section 1).

Worked examples

Example 1 — The null factor law

(a) Solve $(x-4)(x+7)=0$.

Working	Comment
$(x-4)(x+7)=0$	The left side is already a product; the right side is 0.
$x-4=0$ or $x+7=0$	Null factor law.
$x=4$ or $x=-7$	Solve each linear equation.
$\therefore x=4$ or $x=-7$.	Check: $(4-4)(4+7)=0\times 11=0$. ✓

Working	Comment
	$(-7-4)(-7+7)=(-11)\times 0=0$. ✓

(b) Solve $3(x+2)(x-5)=0$.

Working	Comment
$3(x+2)(x-5)=0$	
$(x+2)(x-5)=0$	The factor 3 is never 0, so the product of the other two factors must be 0.
$x+2=0$ or $x-5=0$	Null factor law.
$x=-2$ or $x=5$	
$\therefore x=-2$ or $x=5$.	Check: $3(-2+2)(-2-5)=3\times 0\times(-7)=0$. ✓ $3(5+2)(5-5)=3\times 7\times 0=0$. ✓

(c) Solve $(2x-1)(x+6)=0$.

Working	Comment
$(2x-1)(x+6)=0$	
$2x-1=0$ or $x+6=0$	Null factor law — the factor $(2x-1)$ is allowed.
$x=\frac{1}{2}$ or $x=-6$	
$\therefore x=\frac{1}{2}$ or $x=-6$.	Check: $(2\times\frac{1}{2}-1)(\frac{1}{2}+6)=0\times\frac{13}{2}=0$. ✓ $(2\times(-6)-1)(-6+6)=(-13)\times 0=0$. ✓

The law does not mind what the factors look like — a coefficient inside a factor just makes the linear equation one step longer.

Example 2 — Factorising first

(a) Solve $x^2-5x+6=0$.

Working	Comment
$x^2-5x+6=0$	Seek two numbers with product 6 and sum -5: they are -2 and -3.
$(x-2)(x-3)=0$	Factorise.
$x-2=0$ or $x-3=0$	Null factor law.
$x=2$ or $x=3$	
$\therefore x=2$ or $x=3$.	Check: $2^2-5\times 2+6=4-10+6=0$. ✓ $3^2-5\times 3+6=9-15+6=0$. ✓

(b) Solve $x^2+7x=3x+12$.

Working	Comment
$x^2+7x=3x+12$	
$x^2+7x-3x-12=0$	Move every term to the left side.
$x^2+4x-12=0$	Collect like terms — the equation is now in standard form.
$(x+6)(x-2)=0$	Product -12, sum 4: the numbers are 6 and -2.
$x+6=0$ or $x-2=0$	Null factor law.
$x=-6$ or $x=2$	
$\therefore x=-6$ or $x=2$.	Check: $(-6)^2+7\times(-6)=36-42=-6$ and

Working	Comment
	$3\times(-6)+12=-6$. ✓ $2^2+7\times2=18$ and $3\times2+12=18$. ✓

The null factor law only works on an equation whose right side is 0 — collecting terms first is not a formality, it is what makes the law available.

Example 3 — Difference of squares, common factors and square roots

(a) Solve $x^2-81=0$.

Working	Comment
$x^2-81=0$	
$(x-9)(x+9)=0$	Difference of perfect squares.
$x=9$ or $x=-9$	Null factor law.
$\therefore x=\pm9$.	Check: $9^2-81=0$. ✓ $(-9)^2-81=0$. ✓

(b) Solve $3x^2=27x$.

Working	Comment
$3x^2=27x$	
$3x^2-27x=0$	Move everything to one side — do **not** divide by x.
$3x(x-9)=0$	Common factor $3x$.
$3x=0$ or $x-9=0$	Null factor law.
$x=0$ or $x=9$	
$\therefore x=0$ or $x=9$.	Check: $3\times0^2=0=27\times0$. ✓ $3\times9^2=243=27\times9$. ✓

(c) Solve $x^2=25$, and solve $x^2=7$ leaving the answers in exact form.

Working	Comment
$x^2=25$	Square roots, Strategy 2.
$x=\pm5$	Both 5 and -5 square to 25.
$x^2=7$	7 is not a perfect square.
$x=\pm\sqrt{7}$	Exact surd form — no decimal is needed or wanted.
$\therefore x=\pm5$ for the first; $x=\pm\sqrt{7}$ for the second.	Check: $(\pm5)^2=25$. ✓ $(\pm\sqrt{7})^2=7$. ✓

Example 4 — Completing the square

(a) Solve $x^2+6x+2=0$, leaving the answers in exact form.

Working	Comment
$x^2+6x+2=0$	
$x^2+6x=-2$	Move the constant.
$(x+3)^2-9=-2$	Complete the square: half of 6 is 3, and $3^2=9$.
$(x+3)^2=7$	Add 9 to both sides.
$x+3=\pm\sqrt{7}$	Square roots.
$x=-3\pm\sqrt{7}$	Subtract 3.
$\therefore x=-3+\sqrt{7}$ or $x=-3-\sqrt{7}$ (about -0.35 or -5.65).	Check: $(-3+\sqrt{7})^2+6(-3+\sqrt{7})+2=16-6\sqrt{7}-18+6\sqrt{7}+2$. ✓

(b) Solve $x^2-4x-1=0$, leaving the answers in exact form.

Working	Comment
$x^2-4x-1=0$	
$x^2-4x=1$	Move the constant.
$(x-2)^2-4=1$	Half of -4 is -2, and $(-2)^2=4$.
$(x-2)^2=5$	
$x-2=\pm\sqrt{5}$	Square roots.
$x=2\pm\sqrt{5}$	
$\therefore x=2+\sqrt{5}$ or $x=2-\sqrt{5}$ (about 4.24 or -0.24).	Check: $(2+\sqrt{5})^2-4(2+\sqrt{5})-1=9+4\sqrt{5}-8-4\sqrt{5}-1=0$. ✓

Example 5 — The quadratic formula

(a) Solve $x^2-3x-5=0$, correct to 2 decimal places.

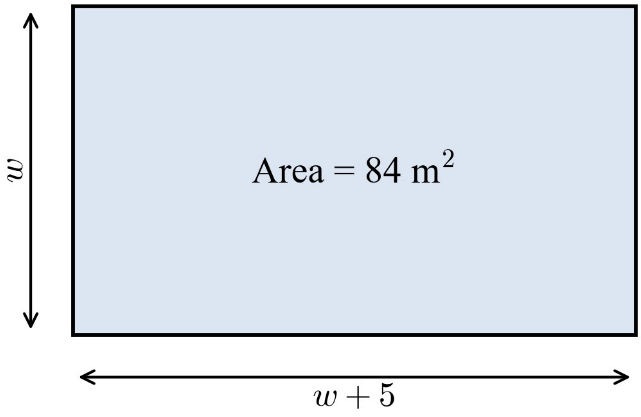
Working	Comment
$x^2-3x-5=0$	Standard form already; $a=1$, $b=-3$, $c=-5$.
$x=\dfrac{-(-3)\pm\sqrt{(-3)^2-4\times1\times(-5)}}{2\times1}$	Substitute into the formula.
$x=\dfrac{3\pm\sqrt{9+20}}{2}$	
$x=\dfrac{3\pm\sqrt{29}}{2}$	Exact form first.
$x\approx\dfrac{3+5.4498}{2}$ or $x\approx\dfrac{3-5.4498}{2}$	$\sqrt{29}\approx5.4498$.
$\therefore x\approx4.19$ or $x\approx-1.19$.	Check by graph: these are the intercepts in the figure of section 6. ✓

(b) Solve $2x^2+5x-4=0$, correct to 2 decimal places.

Working	Comment
$2x^2+5x-4=0$	$a=2$, $b=5$, $c=-4$ — signs included.
$x=\dfrac{-5\pm\sqrt{5^2-4\times2\times(-4)}}{2\times2}$	Substitute.
$x=\dfrac{-5\pm\sqrt{25+32}}{4}$	
$x=\dfrac{-5\pm\sqrt{57}}{4}$	Exact form.
$x\approx\dfrac{-5+7.5498}{4}$ or $x\approx\dfrac{-5-7.5498}{4}$	$\sqrt{57}\approx7.5498$.
$\therefore x\approx0.64$ or $x\approx-3.14$.	Check: $2\times0.64^2+5\times0.64-4\approx0.82+3.2-4=0.02\approx0$, up to rounding. ✓

Example 6 — A courtyard to pave

`SCENARIO` A rectangular courtyard is to be paved so that its length is 5 m more than its width, and the paving must cover 84 m². Find the width and the length of the courtyard.



Rectangle labelled with width w and length w + 5, with area 84 square metres written inside

Working	Comment
Let the width be w m; the length is $(w+5)$ m.	Name the unknown; the figure records the data.
$w(w+5)=84$	Area = length × width.
$w^2+5w-84=0$	Expand and collect into standard form.
$(w+12)(w-7)=0$	Product -84, sum 5: the numbers are 12 and -7.
$w=-12$ or $w=7$	Null factor law.
$w=7$, rejecting $w=-12$	A width is a length — it cannot be negative.
∴ the width is 7 m and the length is $7+5=12$ m.	Check: $7\times12=84$. ✓

Quadratic worded problems often give two solutions of the equation but only one answer to the question — the context does the rejecting.

Practice questions

Scaffolded

Q1. Use the null factor law to solve: (a) $(x-3)(x+8)=0$; (b) $(x+5)(x-5)=0$; (c) $(2x+7)(x-1)=0$.

Q2. Solve $x^2-7x+12=0$. (First find two numbers with product 12 and sum -7.)

Q3. Solve by square roots: (a) $x^2-25=0$; (b) $x^2=5$, leaving the answer in exact form.

Q4. Solve $x^2+8x=9$. (First move the 9 across, then factorise.)

Q5. Solve $x^2+4x-3=0$ by completing the square, leaving the answer in exact form. (Half of 4 is 2.)

Q6. Solve $x^2+3x-3=0$ with the quadratic formula, leaving the answer in exact form.

Q7. Without solving completely, use the discriminant to say how many real solutions each equation has: (a) $x^2-8x+16=0$; (b) $x^2-2x+6=0$. Then solve (a) — what does a discriminant of 0 give?

Q8. `SCENARIO` A rectangular garden bed has a length 4 m more than its width and an area of 60 m². (a) Write an equation in w, the width in metres. (b) Solve it and state the dimensions of the bed.

Unscaffolded

Q9. Solve by factorising: (a) $x^2-7x-18=0$; (b) $3x^2+10x-8=0$; (c) $x^2+5x-6=0$.

Q10. Solve by factorising: (a) $x^2-8x+15=0$; (b) $x^2+x-20=0$; (c) $x^2+6x+9=0$.

Q11. Collect like terms into standard form, then solve: (a) $x^2=-7x+18$; (b) $x^2-4=7x-16$; (c) $x^2+5=4x+50$.

Q12. Solve by square roots, leaving exact forms where needed: (a) $x^2=13$; (b) $x^2+9=0$; (c) $4x^2=100$.

Q13. Solve by taking out a common factor: (a) $x^2-6x=0$; (b) $x^2=9x$; (c) $x^2+4x=0$.

Q14. Solve by completing the square, leaving the answers in exact form: (a) $x^2+8x+5=0$; (b) $x^2-6x+3=0$; (c) $x^2+2x-4=0$.

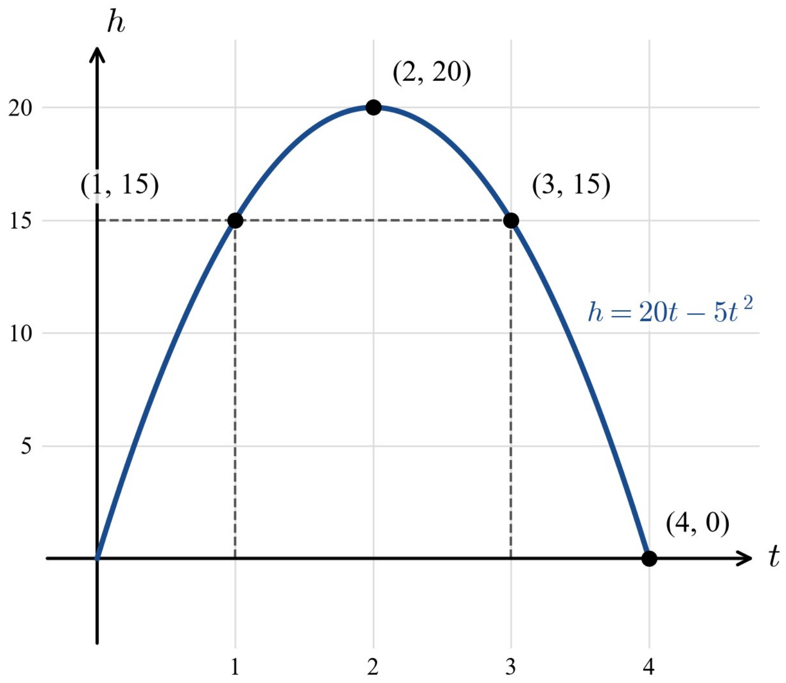
Q15. Solve with the quadratic formula, leaving the answers in exact form: (a) $x^2-5x-3=0$; (b) $2x^2-3x-4=0$; (c) $x^2+5x+3=0$.

Q16. Solve $x^2-3x-7=0$ with the quadratic formula, correct to 1 decimal place.

Q17. Use the discriminant to say how many real solutions each equation has, then find them where they exist: (a) $x^2-10x+25=0$; (b) $x^2+2x+5=0$; (c) $x^2-x-1=0$.

Q18. Find the two values of k for which $x^2+kx+9=0$ has exactly one real solution.

Q19. SCENARIO A ball is thrown straight up from ground level. Its height h metres after t seconds is modelled by $h=20t-5t^2$, until it lands.



Graph of h = 20t − 5t squared from t = 0 to t = 4, marking the points (1, 15), (3, 15) and (4, 0) and the highest point (2, 20)

(a) After how many seconds does the ball land? (b) The ball is at a height of 15 m twice. Find both times. (c) By completing the square, write h in the form $A-B(t-p)^2$ and hence find the maximum height and when it occurs.

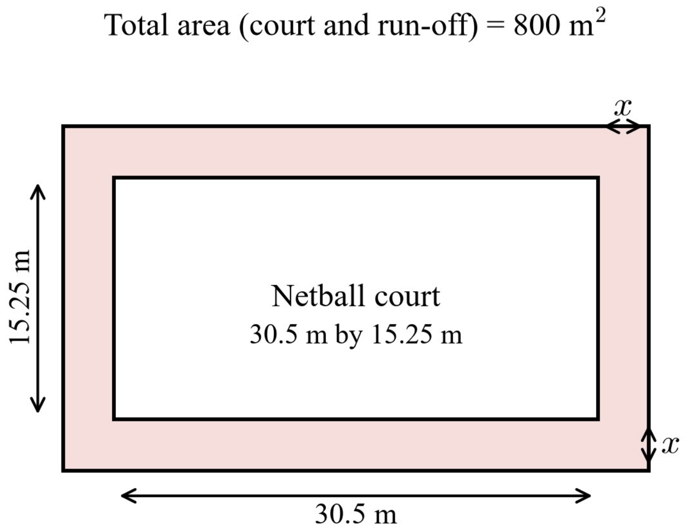
Q20. Choose the quickest strategy for each, and solve: (a) $(x-3)(x+11)=0$; (b) $x^2=17$; (c) $x^2-6x+2=0$.

Q21. SCENARIO A photograph measuring 30 cm by 20 cm is mounted with a border of uniform width x cm all round. The total area of the mounted photograph is 704 cm². Find the width of the border.

Q22. The product of two consecutive positive integers is 132. Find them. (Let the smaller be n.)

Real context

Q23. A netball court measures 30.5 m by 15.25 m. A club plans a flat run-off zone of uniform width x m around all four sides of the court, and the whole rectangle — court and run-off — is to have an area of 800 m².



Plan view of a netball court 30.5 metres by 15.25 metres, surrounded on all sides by a shaded run-off zone of width x metres; the total area of court and run-off is 800 square metres

(a) Show that x satisfies $4x^2 + 91.5x - 334.875 = 0$. (b) Solve this equation with the quadratic formula, correct to 1 decimal place, saying why you reject one of the values.
(b) Hence state the overall dimensions of the court and run-off, correct to 1 decimal place.

Source: Netball Australia, *World Netball — Rules of Netball 2024*, 2024. Retrieved 20 August 2026.

Digital tool task

DIGITAL TOOL — this task assumes a graphing tool. Your teacher will nominate the tool.

Task. Explore the link between the discriminant and the number of real solutions, graphically and algebraically, for the family $x^2 - 4x + c = 0$.

Input (what to enter):

1. graph the three curves $y = x^2 - 4x + 3$, $y = x^2 - 4x + 4$ and $y = x^2 - 4x + 5$ on the same axes;
2. for each curve, read off the x-intercepts (the tool may call them roots or zeros);
3. for each curve, compute the discriminant $b^2 - 4ac$ with $a = 1$, $b = -4$ and $c = 3, 4, 5$ — a scientific calculator is enough for this step.

Output (what to expect): $y = x^2 - 4x + 3$ cuts the axis at $x = 1$ and $x = 3$ and its discriminant is $16 - 12 = 4 > 0$; $y = x^2 - 4x + 4$ touches the axis at $x = 2$ only and its discriminant is $16 - 16 = 0$; $y = x^2 - 4x + 5$ never meets the axis and its discriminant is $16 - 20 = -4 < 0$. In every case the number of x-intercepts matches the sign of the discriminant: positive — two, zero — one, negative — none.

Tool-free path (still assessed): find the solutions of each equation by hand.

- $x^2 - 4x + 3 = (x-1)(x-3) = 0$, so $x = 1$ or $x = 3$.
- $x^2 - 4x + 4 = (x-2)^2 = 0$, so $x = 2$ — one solution.
- $x^2 - 4x + 5 = (x-2)^2 + 1$, and $(x-2)^2 + 1 = 0$ would give $(x-2)^2 = -1$, which has no real solutions.

Answers

Q1. (a) $x=3$ or $x=-8$; (b) $x=\pm 5$; (c) $x=-\frac{7}{2}$ or $x=1$. **Q2.** $x=3$ or $x=4$. **Q3.** (a) $x=\pm 5$; (b) $x=\pm\sqrt{5}$. **Q4.** $x=1$ or $x=-9$. **Q5.** $x=-2\pm\sqrt{7}$. **Q6.** $x=\frac{-3\pm\sqrt{21}}{2}$. **Q7.** (a) discriminant 0 — one solution, $x=4$; (b) discriminant -20 — no real solutions. **Q8.** (a) $w(w+4)=60$, i.e. $w^2+4w-60=0$; (b) $w=6$, so the bed is 6 m by 10 m. **Q9.** (a) $x=9$ or $x=-2$; (b) $x=\frac{2}{3}$ or $x=-4$; (c) $x=1$ or $x=-6$. **Q10.** (a) $x=3$ or $x=5$; (b) $x=-5$ or $x=4$; (c) $x=-3$ (one solution). **Q11.** (a) $x=-9$ or $x=2$; (b) $x=3$ or $x=4$; (c) $x=-5$ or $x=9$. **Q12.** (a) $x=\pm\sqrt{13}$; (b) no real solutions; (c) $x=\pm 5$. **Q13.** (a) $x=0$ or $x=6$; (b) $x=0$ or $x=9$; (c) $x=0$ or $x=-4$. **Q14.** (a) $x=-4\pm\sqrt{11}$; (b) $x=3\pm\sqrt{6}$; (c) $x=-1\pm\sqrt{5}$. **Q15.** (a) $x=\frac{5\pm\sqrt{37}}{2}$; (b) $x=\frac{3\pm\sqrt{41}}{4}$; (c) $x=\frac{-5\pm\sqrt{13}}{2}$. **Q16.** $x\approx 4.5$ or $x\approx -1.5$. **Q17.** (a) one, $x=5$; (b) none; (c) two, $x=\frac{1\pm\sqrt{5}}{2}$. **Q18.** $k=6$ or $k=-6$. **Q19.** (a) 4 seconds; (b) $t=1$ and $t=3$; (c) $h=20-5(t-2)^2$, maximum height 20 m at $t=2$ s. **Q20.** (a) $x=3$ or $x=-11$ (null factor law); (b) $x=\pm\sqrt{17}$ (square roots); (c) $x=3\pm\sqrt{7}$ (completing the square). **Q21.** 1 cm. **Q22.** 11 and 12. **Q23.** (b) $x\approx 3.2$; (c) about 36.9 m by 21.7 m. **Digital tool task:** intercepts 1 and 3; 2 only; none — matching discriminants $4, 0, -4$.

Fully-worked solutions

Q1. (a) $x-3=0$ or $x+8=0$, so $x=3$ or $x=-8$.

(b) $x+5=0$ or $x-5=0$, so $x=-5$ or $x=5$, i.e. $x=\pm 5$.

(c) $2x+7=0$ or $x-1=0$, so $x=-\frac{7}{2}$ or $x=1$.

Q2. $x^2-7x+12=0$

$(x-3)(x-4)=0$

$x-3=0$ or $x-4=0$

$x=3$ or $x=4$

Q3. (a) $x^2=25$

$x=\pm 5$

(b) $x=\pm\sqrt{5}$

Q4. $x^2+8x=9$

$x^2+8x-9=0$

$(x+9)(x-1)=0$

$x=-9$ or $x=1$

Q5. $x^2+4x-3=0$

$(x+2)^2-4-3=0$

$(x+2)^2=7$

$x+2=\pm\sqrt{7}$

$x = -2 \pm \sqrt{7}$

Q6. $a = 1$, $b = 3$, $c = -3$.

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-3)}}{2}$$

$$x = \frac{-3 \pm \sqrt{21}}{2}$$

Q7. (a) $(-8)^2 - 4 \times 16 = 64 - 64 = 0$: one real solution. In fact $x^2 - 8x + 16 = (x - 4)^2 = 0$, so $x = 4$.

(b) $(-2)^2 - 4 \times 6 = 4 - 24 = -20 < 0$: no real solutions.

Q8. (a) $w(w + 4) = 60$, that is $w^2 + 4w - 60 = 0$.

(b) $(w + 10)(w - 6) = 0$

$w = -10$ or $w = 6$

$w = 6$, rejecting -10. The bed is 6 m wide and 10 m long. Check: $6 \times 10 = 60$. ✓

Q9. (a) $(x - 9)(x + 2) = 0$, so $x = 9$ or $x = -2$.

(b) $(3x - 2)(x + 4) = 0$, so $x = \frac{2}{3}$ or $x = -4$.

(c) $(x - 1)(x + 6) = 0$, so $x = 1$ or $x = -6$.

Q10. (a) $(x - 3)(x - 5) = 0$, so $x = 3$ or $x = 5$.

(b) $(x + 5)(x - 4) = 0$, so $x = -5$ or $x = 4$.

(c) $(x + 3)^2 = 0$, so $x = -3$ — the only solution.

Q11. (a) $x^2 + 7x - 18 = 0$

$(x + 9)(x - 2) = 0$

$x = -9$ or $x = 2$

(b) $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3$ or $x = 4$

(c) $x^2 - 4x - 45 = 0$

$(x + 5)(x - 9) = 0$

$x = -5$ or $x = 9$

Q12. (a) $x = \pm\sqrt{13}$.

(b) $x^2 = -9$ has no real solutions — no real number squares to a negative.

(c) $x^2 = 25$, so $x = \pm 5$.

Q13. (a) $x(x - 6) = 0$, so $x = 0$ or $x = 6$.

(b) $x^2 - 9x = 0$, so $x(x-9) = 0$, so $x = 0$ or $x = 9$.

(c) $x(x+4) = 0$, so $x = 0$ or $x = -4$.

Q14. (a) $(x+4)^2 - 16 + 5 = 0$

$(x+4)^2 = 11$

$x = -4 \pm \sqrt{11}$

(b) $(x-3)^2 - 9 + 3 = 0$

$(x-3)^2 = 6$

$x = 3 \pm \sqrt{6}$

(c) $(x+1)^2 - 1 - 4 = 0$

$(x+1)^2 = 5$

$x = -1 \pm \sqrt{5}$

Q15. (a) $a = 1$, $b = -5$, $c = -3$:

$x = \dfrac{5 \pm \sqrt{25+12}}{2} = \dfrac{5 \pm \sqrt{37}}{2}$

(b) $a = 2$, $b = -3$, $c = -4$:

$x = \dfrac{3 \pm \sqrt{9+32}}{4} = \dfrac{3 \pm \sqrt{41}}{4}$

(c) $a = 1$, $b = 5$, $c = 3$:

$x = \dfrac{-5 \pm \sqrt{25-12}}{2} = \dfrac{-5 \pm \sqrt{13}}{2}$

Q16. $a = 1$, $b = -3$, $c = -7$:

$x = \dfrac{3 \pm \sqrt{9+28}}{2} = \dfrac{3 \pm \sqrt{37}}{2}$

$\sqrt{37} \approx 6.0828$, so $x \approx \dfrac{9.0828}{2} \approx 4.5$ or $x \approx \dfrac{-3.0828}{2} \approx -1.5$.

Q17. (a) $(-10)^2 - 4 \times 25 = 0$: one real solution; $(x-5)^2 = 0$ gives $x = 5$.

(b) $2^2 - 4 \times 5 = -16 < 0$: no real solutions.

(c) $(-1)^2 - 4 \times (-1) = 5 > 0$: two real solutions; $x = \dfrac{1 \pm \sqrt{5}}{2}$.

Q18. Exactly one real solution means the discriminant is 0:

$k^2 - 4 \times 9 = 0$

$k^2 = 36$

$k = 6$ or $k = -6$

(Check: $x^2+6x+9=(x+3)^2$ and $x^2-6x+9=(x-3)^2$ — both perfect squares.)

Q19. (a) Landing means $h=0$:

$20t-5t^2=0$

$5t(4-t)=0$

$t=0$ or $t=4$

$t=0$ is the launch, so the ball lands after 4 seconds.

(b) $20t-5t^2=15$

$t^2-4t+3=0$ (divide by -5)

$(t-1)(t-3)=0$

$t=1$ or $t=3$ — going up, then coming down.

(c) $h=-5(t^2-4t)=-5((t-2)^2-4)=20-5(t-2)^2$.

Since $-5(t-2)^2\leq 0$, the greatest value of h is 20, at $t=2$: the maximum height is 20 m, after 2 s.

Q20. (a) Null factor law, already factorised: $x=3$ or $x=-11$.

(b) Square roots: $x=\pm\sqrt{17}$.

(c) Completing the square: $(x-3)^2-9+2=0$, so $(x-3)^2=7$, so $x=3\pm\sqrt{7}$.

Q21. $(30+2x)(20+2x)=704$

$600+100x+4x^2=704$

$4x^2+100x-104=0$

$x^2+25x-26=0$

$(x+26)(x-1)=0$

$x=-26$ or $x=1$

$x=1$, rejecting -26: the border is 1 cm wide. Check: $32\times 22=704$. ✓

Q22. $n(n+1)=132$

$n^2+n-132=0$

$(n+12)(n-11)=0$

$n=-12$ or $n=11$

$n=11$, since the integers are positive. The integers are 11 and 12. Check: $11\times 12=132$. ✓

Q23. (a) The overall rectangle is $(30.5+2x)$ m by $(15.25+2x)$ m, so

$(30.5+2x)(15.25+2x)=800$

$465.125+61x+30.5x+4x^2=800$

$4x^2+91.5x-334.875=0$

(b) $a=4$, $b=91.5$, $c=-334.875$:

$$x=\frac{-91.5\pm\sqrt{91.5^2-4\times4\times(-334.875)}}{2\times4}$$

$$x=\frac{-91.5\pm\sqrt{8372.25+5358}}{8}=\frac{-91.5\pm\sqrt{13\,730.25}}{8}$$

$\sqrt{13\,730.25}\approx117.18$, so $x\approx\dfrac{25.68}{8}\approx3.2$ or $x\approx\dfrac{-208.68}{8}\approx-26.1$.

The negative value would make the run-off a negative width, so $x\approx3.2$ m.

(c) Overall: $30.5+2\times3.21\approx36.9$ m by $15.25+2\times3.21\approx21.7$ m.

Digital tool task. The three graphs cut, touch and miss the x-axis as the table of section 7 predicts: two intercepts at 1 and 3 when the discriminant is 4; one at 2 when it is 0; none when it is -4.

Tool-free: $(x-1)(x-3)=0$ gives $x=1$ or $x=3$; $(x-2)^2=0$ gives $x=2$; $(x-2)^2+1=0$ gives $(x-2)^2=-1$, which has no real solutions.

Teacher note

Level 9 code rehearsed (D3): `VC2M9A05` — graphing quadratics and the null factor law for monic quadratics with integer roots — is the launch pad every strategy here extends: the law keeps its Year 9 statement but gains non-monic factors (Example 1(c)), and the graphing rehearsed in Year 9 carries the whole of section 7. The in-book dependency is 1D (`VC2M10A04`): every strategy in `VC2M10A13.E01` — grouping like terms, completing the square, the null factor law — is factorisation from 1D, and Q21–Q23 assess it under load. Downstream, 3B assumes 2D.

Tool classes (D13): a graphing tool for the digital tool task only (`VC2M10A13.E02`, `.E03`), and a scientific calculator for the decimal evaluations in Examples 4–5, Q16 and Q23. No CAS: the elaborations name digital exploration of the discriminant's link to solutions, and the book limits digital work to that exploration, prints the input and expected output beside the task, and assesses the tool-free path.

VCE destinations (§5, reference only): `VC2M10A13` is named in none of the Units 3 & 4 evidence of Foundation Mathematics, General Mathematics or Mathematical Methods — an artefact of what was mapped, not an absence: quadratic equations are Mathematical Methods Units 1 & 2 AoS 1 'Functions and graphs' content *(inferred)*, and completing the square and the discriminant return in Methods Units 3 & 4 as solving and analysis tools.

Level check (D6): the quadratic formula is used as a stated tool and is not derived — its derivation by completing the square on ax^2+bx+c is not taught here. The discriminant is named and its value used to count real solutions, as `.E03` requires, without the case-analysis proof. There is no $f(x)$ notation, no domain or range, no technical use of *function* or *relation*, and no complex numbers — a negative discriminant gives 'no real solutions'. Exact surd forms are preferred; decimals appear only where a question asks for them.

Dated data (R3): Q23 carries the court dimensions (30.5 m by 15.25 m) from the standing rules of netball. Listed here for the chapter refresh register; the dimensions change only with a rules revision, but the retrieval date should be refreshed at reissue.

Simple exponential equations

Mathematics 10 · Chapter 2 Equations and inequalities · Subtopic 2E (Level 10) · Standard block

Strand	Algebra
Content description	`VC2M10A14`
Elaboration ids carried	`VC2M10A14.E01` `VC2M10A14.E02` `VC2M10A14.E03` `VC2M10A14.E04` (`VC2M10A14.E05` not delivered — see Teacher note)
Weight	Standard (single allocation)

> **Content description (verbatim):** *solve simple exponential equations*
>
> **Achievement standard (extract):** *They represent linear, quadratic and exponential functions numerically, graphically and algebraically, and use them to model situations and solve practical problems.*

You need first: the exponent laws with integer and zero exponents from Year 9 (`VC2M9A01`), and solving linear equations (`VC2M9A03`). This subtopic rehearses those skills and applies them; it does not re-teach them.

Theory

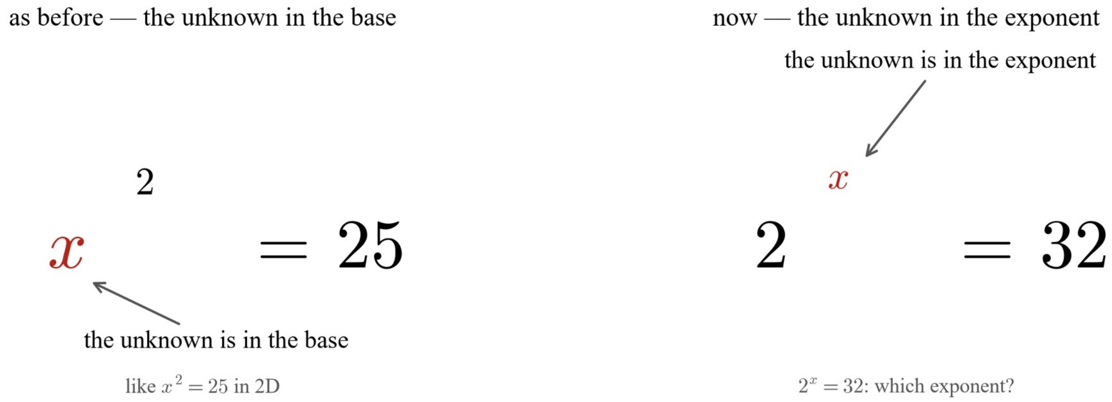
So far in this chapter the unknown has sat in the base of the expression, or on its own: $3x+5=14$ in 2A, $x^2=25$ in 2D. An exponential equation turns this around: **the unknown is in the exponent.** The questions it answers are the ones growth and decay keep asking — *when will the population reach …?*, *when will the amount shrink to …?*

1. What an exponential equation is

An **exponential equation** is an equation in which the unknown appears in the exponent of a power:

$$2^x=32,\, 3^{2x}=81,\, 5\times 2^t=160.$$

The unknown is still just a number — and, as the worked examples will show, that number can be a fraction or a negative number, not only a whole number.



Two equations side by side. Left: x squared equals 25, with the unknown x sitting in the base. Right: 2 to the power of x equals 32, with the unknown x sitting in the exponent, so the question is how many 2s multiplied together give 32.

2. Same base, one exponent

The whole method rests on one fact. Take a base a with $a>0$ and $a\neq 1$. Each step up in the exponent multiplies by a again, so the outputs never repeat: when $a>1$ they keep growing, and when $0<a<1$ they keep shrinking. Hence

$$\text{if } a^x=a^y\text{, then } x=y.$$

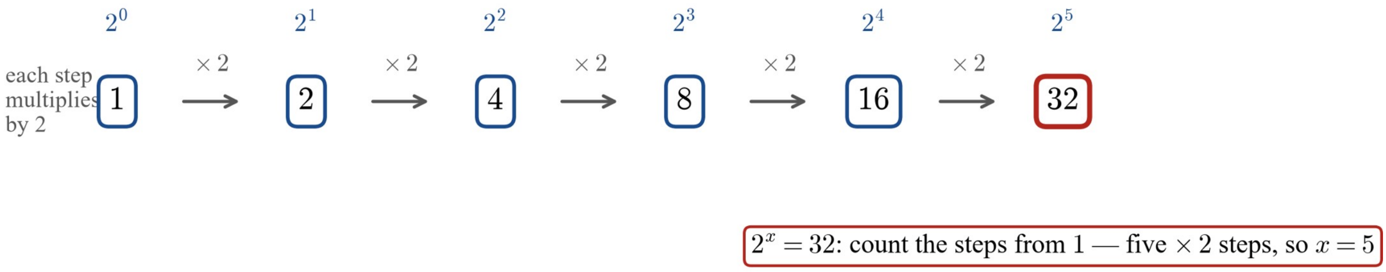
One exponent gives one output, and one output can only have come from one exponent.

The same picture gives one more fact: a^x is **always positive**. So an equation such as $2^x = -4$ or $2^x = 0$ has **no solution** — no power of 2 is ever negative or zero.

This gives the method, in five steps.

1. Write each side as a power of the **same base**.
2. Use the exponent laws to simplify each side to a single power.
3. Drop the matching bases and **equate the exponents**.
4. Solve the resulting equation — it is linear, so the skills of 2A finish the job.
5. Check by substituting your answer back into the original equation.

A common mistake is to divide instead: from $2^x = 32$, the answer is **not** $32 \div 2$. The unknown is in the exponent, so the real question is *"how many 2s multiplied together give 32?"*



A chain of powers of 2, each box found by multiplying the last by 2: 1, 2, 4, 8, 16 and 32, with the exponents 0 to 5 above the boxes; the last box is marked in red, since 2 to the power of x equals 32 asks which exponent gives 32, and the answer is 5.

The method works whenever both sides can be written with one common base. When they cannot — $2^x = 10$ is the standing example — the graph and trial methods of section 5 take over.

3. Rewriting the base with the exponent laws

Recall from Year 9 (`VC2M9A01`) the exponent laws, here with the rewrites they produce:

- $(a^m)^n = a^{mn}$: so $4^x = (2^2)^x = 2^{2x}$, $8^x = 2^{3x}$, $9^x = 3^{2x}$, $25^x = 5^{2x}$ and $27^x = 3^{3x}$.
- $a^{-n} = \dfrac{1}{a^n}$: so $\left(\dfrac{1}{2}\right)^x = \dfrac{1}{2^x} = 2^{-x}$, and similarly $\left(\dfrac{1}{3}\right)^x = 3^{-x}$.
- $a^0 = 1$ for any base a: so when the other side is 1, write it as a power of the common base, for example $1 = 6^0$.

These rewrites are the engine of the method: they turn two different-looking bases into one.

rewriting each side with one base

as given		one base
$4^x = (2^2)^x$	$\longrightarrow$	2^{2x}
$8^x = (2^3)^x$	$\longrightarrow$	2^{3x}
$9^x = (3^2)^x$	$\longrightarrow$	3^{2x}
$25^x = (5^2)^x$	$\longrightarrow$	5^{2x}
$27^x = (3^3)^x$	$\longrightarrow$	3^{3x}
$\left(\frac{1}{2}\right)^x = \frac{1}{2^x}$	$\longrightarrow$	2^{-x}
1	$\longrightarrow$	6^0

$$(a^m)^n = a^{mn}, \quad a^{-n} = \frac{1}{a^n}, \quad a^0 = 1$$

A table of rewrites with one common base: 4 to the power of x becomes 2 to the power of 2x, 8 to the power of x becomes 2 to the power of 3x, 9 to the power of x becomes 3 to the power of 2x, 25 to the power of x becomes 5 to the power of 2x, 27 to the power of x becomes 3 to the power of 3x, one half to the power of x becomes 2 to the power of negative x, and 1 becomes 6 to the power of 0.

4. Tables with a constant ratio are exponential

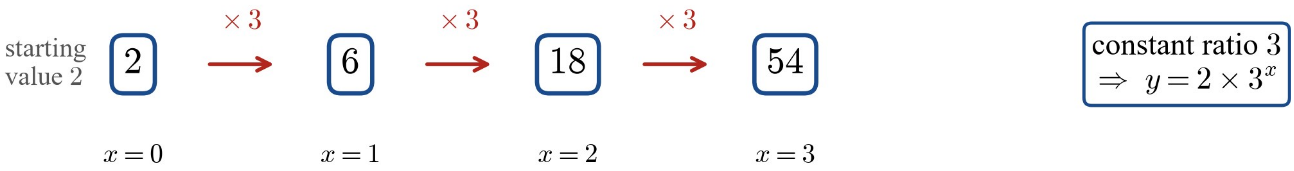
Look at this table of values.

x	0	1	2	3
y	2	6	18	54

Each y-value is 3 times the one before it: $\frac{6}{2}=3$, $\frac{18}{6}=3$, $\frac{54}{18}=3$. When the ratio of consecutive y-values is **constant**, each step multiplies by the same number, and the pattern is **exponential**. The starting value (at $x=0$) is 2 and the constant ratio is 3, so the rule is

$$y=2\times 3^x.$$

In general: a starting value a and a constant ratio b give the rule $y=a\times b^x$.



The table values 2, 6, 18 and 54 in a row of boxes with times-3 arrows between them, showing the constant ratio; the starting value is 2, so the rule is y equals 2 times 3 to the power of x.

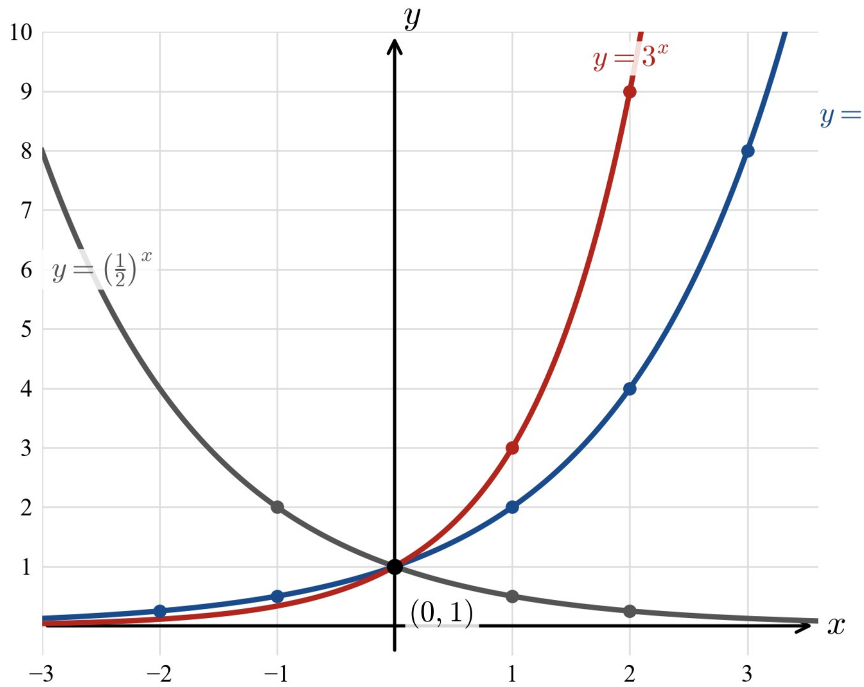
- When $b > 1$, each step multiplies by more than 1 and the values **grow**.
- When $0 < b < 1$, each step multiplies by less than 1 and the values **decay**.

This is how an exponential model is found from data: check the ratios first, then read off a and b. Once the rule is known, questions like *"when does y reach ...?"* become exponential equations.

5. Reading solutions from a graph

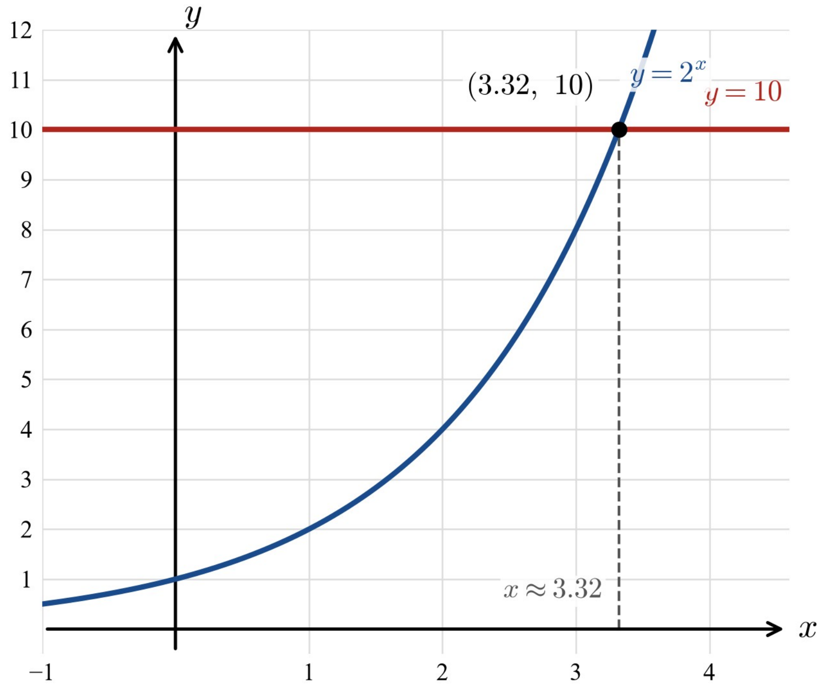
The table for $y = 2^x$ runs:

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Three curves on the same axes: y equals 2 to the power of x, y equals 3 to the power of x and y equals one half to the power of x. All three pass through the point (0, 1); the first two grow, with the larger base growing faster, and the third decays.

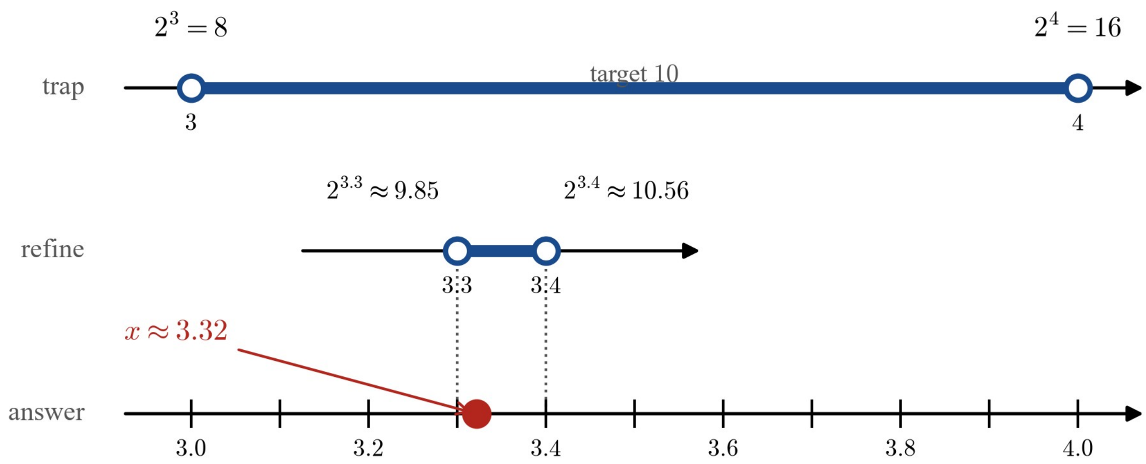
Now consider $2^x = 10$. No whole number of 2s multiplied together gives 10, so there is no neat whole-number answer. On the graph, 2^x is the height of the curve $y = 2^x$, so the solution of $2^x = 10$ is the x-coordinate of the point where the curve reaches height 10 — that is, where it meets the horizontal line $y = 10$.



The curve y equals 2 to the power of x and the horizontal line y equals 10 on the same axes, meeting at the marked point (3.32, 10) with dashed guides to about 3.32 on the x-axis and 10 on the y-axis; the solution of 2 to the power of x equals 10 is about 3.32.

When the solution is not a whole number, trap it, then refine:

- **Trap between whole numbers.** $2^3 = 8$ and $2^4 = 16$, and $8 < 10 < 16$, so the solution lies between 3 and 4.
- **Refine by guess-check-and-refine.** Try values in between, one decimal place at a time, keeping the trial that sits just below the target and the one just above it. Subtopic 2F takes this method up in full; here it is used to read and check graphs.



Three number lines, one above the other. The top line traps the solution of 2 to the power of x equals 10 between 3 and 4, since 2 to the power of 3 is 8 and 2 to the power of 4 is 16. The middle line refines the trap to between 3.3 and 3.4, since 2 to the power of 3.3 is about 9.85 and 2 to the power of 3.4 is about 10.56. The bottom line marks the answer, x about 3.32.

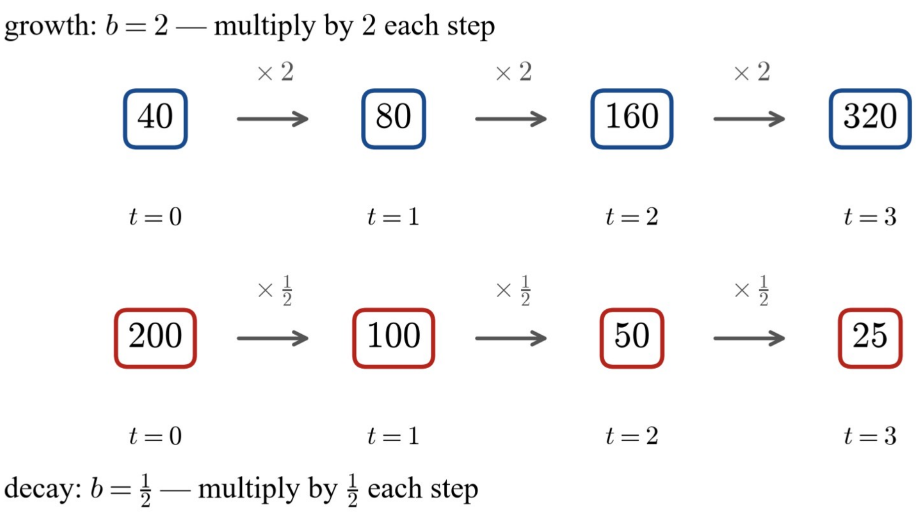
6. Growth and decay models

A quantity that is multiplied by the same factor in every equal step of time is modelled by

$$P = P_0 \times b^t,$$

where P_0 is the starting amount, b is the factor per step, and t is the number of steps.

- $b > 1$ is **growth**: doubling each hour means $b = 2$; growing by 10% each year means multiplying by 1.1, so $b = 1.1$; growing by 1.6% means $b = 1.016$.
- $0 < b < 1$ is **decay**: halving each hour means $b = \frac{1}{2}$.



Two rows of boxes. Top row, growth with b equals 2: 40, 80, 160 and 320, each step multiplying by 2. Bottom row, decay with b equals one half: 200, 100, 50 and 25, each step multiplying by one half.

Every model question is one of two types: **evaluate** (substitute a given t and find P) or **solve** (set P equal to a target and find t). The second type is exactly the exponential equation of this subtopic.

Worked examples

Example 1 — Equating the exponents

(a) Solve $5^x = 125$.

Working	Comment
$5^x = 125$	
$5^x = 5^3$	Write 125 as a power of 5.
$x = 3$	Same base, so equate the exponents.
$\therefore x = 3.$	Check: $5^3 = 125$. ✓

(b) Solve $3^{2x} = 243$.

Working	Comment
$3^{2x} = 243$	
$3^{2x} = 3^5$	$243 = 3^5$.
$2x = 5$	Equate the exponents.
$x = \frac{5}{2}$	Solve the linear equation.
$\therefore x = \frac{5}{2}.$	Check: when $x = \frac{5}{2}$, the exponent $2x$ is 5, so the left side is $3^5 = 243$. ✓

The solution of an exponential equation does not have to be a whole number. There is no need to evaluate a power like $3^{5/2}$ — check by substituting into the exponent, as above.

Example 2 — Coefficients, different bases and negative answers

(a) Solve $4 \times 2^x = 128$.

Working	Comment
$4 \times 2^x = 128$	
$2^x = 32$	Divide both sides by the coefficient 4.
$2^x = 2^5$	$32 = 2^5$.
$x = 5$	Equate the exponents.
$\therefore x = 5$.	Check: $4 \times 2^5 = 4 \times 32 = 128$. ✓

(b) Solve $9^x = 27$.

Working	Comment
$9^x = 27$	Neither side is a power of the other — find a common base.
$(3^2)^x = 3^3$	$9 = 3^2$ and $27 = 3^3$.
$3^{2x} = 3^3$	$(a^m)^n = a^{mn}$.
$2x = 3$	Equate the exponents.
$x = \dfrac{3}{2}$	
$\therefore x = \dfrac{3}{2}$.	Check: when $x = \dfrac{3}{2}$, the exponent $2x$ is 3, so the left side is $3^3 = 27$. ✓

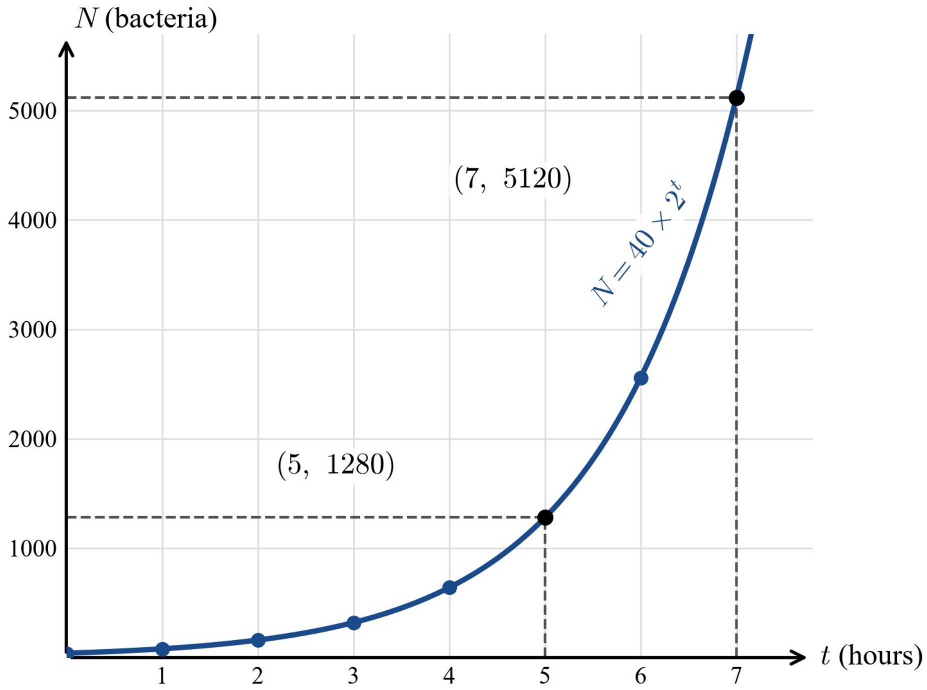
(c) Solve $\left(\dfrac{1}{2}\right)^x = 32$.

Working	Comment
$\left(\dfrac{1}{2}\right)^x = 32$	
$2^{-x} = 32$	$\left(\dfrac{1}{2}\right)^x = \dfrac{1}{2^x} = 2^{-x}$.
$2^{-x} = 2^5$	$32 = 2^5$.
$-x = 5$	Equate the exponents.
$x = -5$	
$\therefore x = -5$.	Check: $\left(\dfrac{1}{2}\right)^{-5} = 2^5 = 32$. ✓

Solutions can be fractions and they can be negative — the unknown is just a number.

Example 3 — A growth model

`SCENARIO` A colony of bacteria starts with 40 bacteria and doubles every hour. The number of bacteria t hours after the start is modelled by $N = 40 \times 2^t$. (a) How many bacteria are there after 5 hours? (b) When will the colony first reach 5120 bacteria?



The growth curve N equals 40 times 2 to the power of t for t from 0 to 7 hours, drawn through the computed points 40, 80, 160, 320, 640, 1280, 2560 and 5120; the points (5, 1280) and (7, 5120) are marked with dashed guides for parts (a) and (b).

Working	Comment
(a) $N = 40 \times 2^t$ with $t = 5$.	A given time — evaluate.
$N = 40 \times 2^5$	Substitute $t = 5$.
$N = 40 \times 32$	
$N = 1280$	
$\therefore$ there are 1280 bacteria after 5 hours.	
(b) $40 \times 2^t = 5120$	A given number — set up an equation in t.
$2^t = 128$	Divide both sides by 40.
$2^t = 2^7$	$128 = 2^7$.
$t = 7$	Equate the exponents.
$\therefore$ the colony first reaches 5120 bacteria after 7 hours.	

Practice questions

Scaffolded

Q1. (a) Solve $3^x = 27$. First write 27 as a power of 3. (b) Solve $2^x = 32$. First write 32 as a power of 2.

Q2. (a) Solve $6^x = 216$. (Hint: $216 = 6 \times 6 \times 6$.) (b) Solve $10^x = 10\,000$.

Q3. Solve $2^{x+1} = 16$. (Hint: write 16 as a power of 2, equate the exponents, then solve the equation $x + 1 = \ldots$)

Q4. Solve $3 \times 2^x = 96$. (Hint: first divide both sides by 3.)

Q5. Solve $4^x = 64$. (Hint: write 4^x as $(2^2)^x = 2^{2x}$, and write 64 as a power of 2.)

Q6. The table shows an exponential pattern.

x	0	1	2	3
y	3	6	12	24

(a) Divide each y-value by the one before it. What do you notice? (b) Write the rule in the form $y = a \times b^x$. (c) Use the rule to find y when $x = 5$.

Q7. The table shows an exponential pattern.

x	0	1	2	3
y	400	200	100	50

(a) Show that the ratio of consecutive y-values is constant, and state it. (b) Write the rule in the form $y = a \times b^x$. (c) Use the rule to find x when $y = 25$.

Q8. The number of insects in a colony is modelled by $N = 20 \times 3^t$, where t is the number of weeks after the study begins. (a) Find N when $t = 0$ and when $t = 2$. (b) Solve $20 \times 3^t = 1620$ and say what your answer means.

Unscaffolded

Q9. Solve: (a) $7^x = 49$; (b) $2^x = 128$; (c) $6^x = 1$.

Q10. Solve: (a) $4^x = 64$; (b) $8^x = 4$; (c) $25^x = 125$.

Q11. Solve: (a) $5^{x+1} = 625$; (b) $3^{x-2} = 81$; (c) $2^{3x} = 64$.

Q12. Solve: (a) $5 \times 2^x = 160$; (b) $3 \times 5^x = 375$; (c) $7 \times 2^x = 448$.

Q13. Solve: (a) $2^{x+1} = 32$; (b) $3^{2x} = 81$; (c) $4^{x+1} = 8^x$.

Q14. Solve: (a) $\left(\frac{1}{2}\right)^x = 8$; (b) $\left(\frac{1}{5}\right)^x = 25$; (c) $2^{-x} = 16$.

Q15. The table shows an exponential pattern.

x	0	1	2	3
y	5	15	45	135

(a) Show that the ratio of consecutive y-values is constant, and state it. (b) Write the rule $y = a \times b^x$. (c) Find y when $x = 6$.

Q16. The table shows an exponential pattern.

x	0	1	2	3
y	1000	200	40	8

(a) Show that the ratio of consecutive y-values is constant, and state it. (b) Write the rule $y = a \times b^x$. (c) Find x when $y = 1.6$.

Q17. The exponential pattern $y = a \times b^x$, with $b > 0$, passes through $(0, 7)$ and $(2, 28)$. Find a and b, and hence find y when $x = 5$.

Q18. `SCENARIO` A town has a population of 800, growing by 10% each year. The population t years from now is modelled by $P = 800 \times 1.1^t$. (a) Find the population after 2 years. (b) Use guess-check-and-refine, correct to 1 decimal place, to find when the population first reaches 1200.

Q19. `SCENARIO` A patient is given a $200\,\text{mg}$ dose of a medicine. Each hour, the body removes half of the medicine that is present, so that $M = 200 \times \left(\frac{1}{2}\right)^t$ milligrams remain after t hours. Find how long it takes until only $25\,\text{mg}$ remain.

Q20. Consider the pattern $y = 5^x$. (a) Complete a table of values for $x = 0, 1, 2, 3$. (b) Sketch the graph of $y = 5^x$ from your table, and use your graph to estimate the solution of $5^x = 60$.

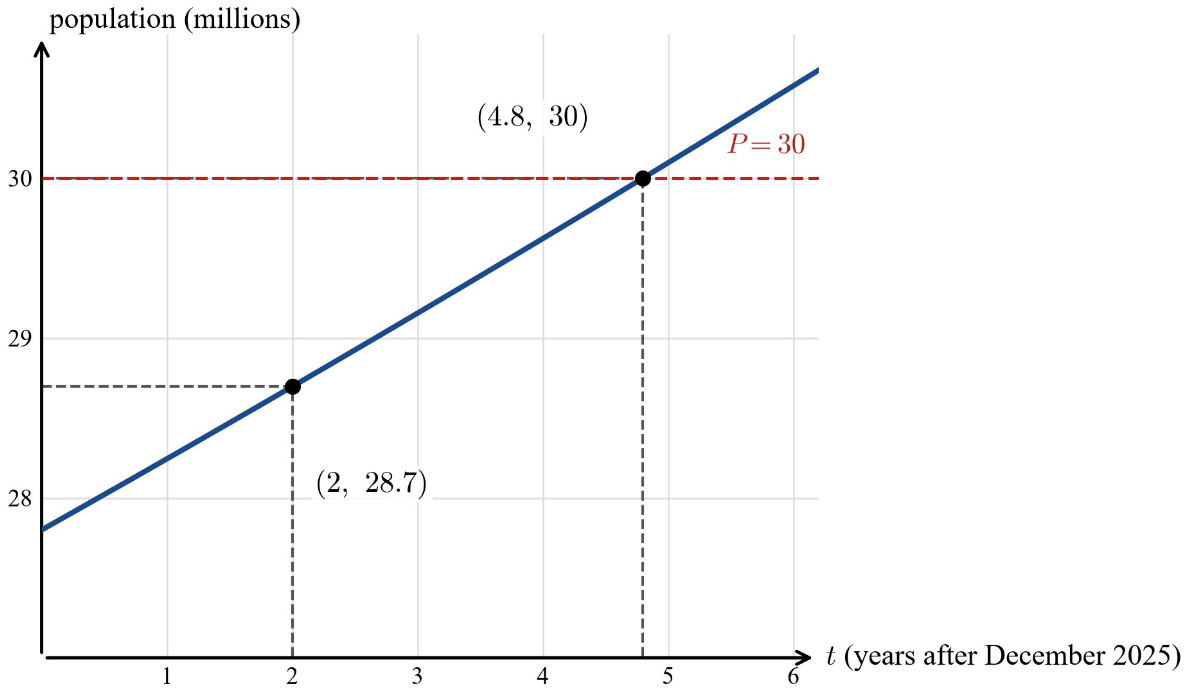
Q21. (a) Show that the solution of $2^x = 50$ lies between 5 and 6. (b) Use guess-check-and-refine to find the solution correct to 1 decimal place.

Q22. `SCENARIO` A colony of little penguins numbers 2000. After protection work begins, the colony grows by a factor of 1.5 each year, so that t years after the work begins the population is modelled by $P = 2000 \times 1.5^t$. (a) Find the population after 3 years. (b) Find the first whole number of years after the work begins at which the population exceeds $10\,000$.

Real context

Q23. Australia's estimated resident population was 27.8 million as at December 2025, and in the year to March 2025 the population grew by about $1.6\,\%$. Suppose the population continues to grow by $1.6\,\%$ each year, so that t years after December 2025 it is modelled by $P = 27.8 \times 1.016^t$ million. (a) Use the model to estimate Australia's population in December 2027, correct to 1 decimal place. (b) Use guess-check-and-refine, correct to 1 decimal place, to find when the model first predicts a population of 30 million. (c) In which calendar year does the model first predict 30 million?

Source: Australian Bureau of Statistics, *National, state and territory population*, 2025. Retrieved 19 August 2026.



The curve P equals 27.8 times 1.016 to the power of t, the model for Australia's population in millions, for t from 0 to 6 years after December 2025; the point (2, 28.7) is marked with dashed guides, and the curve meets the dashed line P equals 30 at about t equals 4.8.

Digital tool task

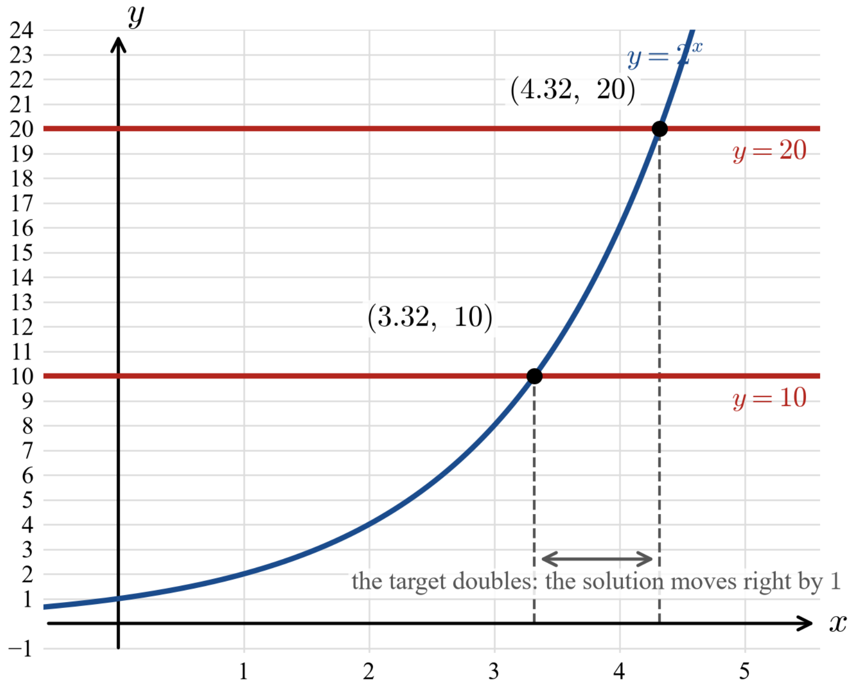
DIGITAL TOOL — this task assumes a graphing tool. Your teacher will nominate the tool.

Task. Use a graphing tool to solve $2^x = 10$ and $2^x = 20$ by finding intersections.

Input (what to graph):

1. the curve $y = 2^x$;
2. the horizontal line $y = 10$;
3. the point where the curve and the line meet;
4. repeat steps 2–3 with the line $y = 20$.

Output (what to expect): the intersections are about $(3.32, 10)$ and $(4.32, 20)$, so $x \approx 3.32$ for $2^x = 10$ and $x \approx 4.32$ for $2^x = 20$. Notice that 20 is double 10, and the solution has moved right by exactly 1: since $2^{x+1} = 2 \times 2^x$, doubling the target adds 1 to the exponent.



The curve y equals 2 to the power of x with the horizontal lines y equals 10 and y equals 20, meeting it at the marked points (3.32, 10) and (4.32, 20) with dashed guides to the axes; an arrow shows the solution moving right by 1 when the target doubles.

Tool-free path (still assessed): solve $2^x = 10$ by guess-check-and-refine.

- $2^3 = 8$ and $2^4 = 16$, so the solution lies between 3 and 4.
- $2^{3.3} \approx 9.85$ and $2^{3.4} \approx 10.56$, so it lies between 3.3 and 3.4.
- $2^{3.32} \approx 9.99$ and $2^{3.33} \approx 10.06$, so $x \approx 3.32$.

Answers

Q1. (a) $x=3$; (b) $x=5$. **Q2.** (a) $x=3$; (b) $x=4$. **Q3.** $x=3$. **Q4.** $x=5$. **Q5.** $x=3$. **Q6.** (a) the ratio is constant, 2; (b) $y=3\times 2^x$; (c) $y=96$. **Q7.** (a) the ratio is constant, $\frac{1}{2}$; (b) $y=400\times\left(\frac{1}{2}\right)^x$; (c) $x=4$. **Q8.** (a) $N=20$ and $N=180$; (b) $t=4$, so the colony reaches 1620 insects 4 weeks after the study begins. **Q9.** (a) $x=2$; (b) $x=7$; (c) $x=0$. **Q10.** (a) $x=3$; (b) $x=\frac{2}{3}$; (c) $x=\frac{3}{2}$. **Q11.** (a) $x=3$; (b) $x=6$; (c) $x=2$. **Q12.** (a) $x=5$; (b) $x=3$; (c) $x=6$. **Q13.** (a) $x=4$; (b) $x=2$; (c) $x=2$. **Q14.** (a) $x=-3$; (b) $x=-2$; (c) $x=-4$. **Q15.** (a) the ratio is constant, 3; (b) $y=5\times 3^x$; (c) $y=3645$. **Q16.** (a) the ratio is constant, $\frac{1}{5}$; (b) $y=1000\times\left(\frac{1}{5}\right)^x$; (c) $x=4$. **Q17.** $a=7$, $b=2$; $y=224$. **Q18.** (a) 968; (b) $t\approx 4.3$ years. **Q19.** 3 hours. **Q20.** (a) $1,5,25,125$; (b) $x\approx 2.5$ (accept 2.4–2.7 from a sketch). **Q21.** (b) $x\approx 5.6$. **Q22.** (a) 6750; (b) 4 years. **Q23.** (a) 28.7 million; (b) $t\approx 4.8$ years after December 2025; (c) 2030. **Digital tool task:** $x\approx 3.32$; $x\approx 4.32$.

Fully-worked solutions

Q1. (a)

$3^x=27$

$3^x=3^3$

$x=3$

$2^x=32$

$2^x=2^5$

$x=5$

Q2. (a)

$6^x=216$

$6^x=6^3$

$x=3$

$10^x=10\,000$

$10^x=10^4$

$x=4$

Q3. $2^{x+1}=16$

$2^{x+1}=2^4$

$x+1=4$

$x=3$

Q4. $3\times 2^x=96$

$2^x=32$

$2^x=2^5$

$x=5$

Q5. $4^x=64$

$2^{2x}=2^6$

$2x=6$

$x=3$

Q6. (a) $\frac{6}{3}=2$, $\frac{12}{6}=2$, $\frac{24}{12}=2$ — the ratio of consecutive y-values is constant, and it is 2.

(b) The starting value is 3 and the ratio is 2, so the rule is

$y=3\times 2^x$.

(c) When $x=5$:

$y=3\times 2^5$

$y=3\times 32$

$y=96$

Q7. (a) $\frac{200}{400}=\frac{1}{2}$, $\frac{100}{200}=\frac{1}{2}$, $\frac{50}{100}=\frac{1}{2}$ — the ratio is constant, and it is $\frac{1}{2}$.

(b) The starting value is 400 and the ratio is $\frac{1}{2}$, so the rule is

$y=400\times\left(\frac{1}{2}\right)^x$.

(c) Set $y=25$:

$400\times\left(\frac{1}{2}\right)^x=25$

$\left(\frac{1}{2}\right)^x=\frac{25}{400}$

$\left(\frac{1}{2}\right)^x=\frac{1}{16}$

$\left(\frac{1}{2}\right)^x=\left(\frac{1}{2}\right)^4$

$x=4$

Q8. (a) When $t=0$: $N=20\times 3^0=20\times 1=20$.

When $t=2$: $N=20\times 3^2=20\times 9=180$.

$20\times 3^t=1620$

$3^t=81$

$3^t=3^4$

$t = 4$

$\therefore$ the colony reaches 1620 insects 4 weeks after the study begins.

Q9. (a)

$7^x = 49$

$7^x = 7^2$

$x = 2$

$2^x = 128$

$2^x = 2^7$

$x = 7$

$6^x = 1$

$6^x = 6^0$

$x = 0$

Q10. (a)

$4^x = 64$

$2^{2x} = 2^6$

$2x = 6$

$x = 3$

$8^x = 4$

$2^{3x} = 2^2$

$3x = 2$

$x = \frac{2}{3}$

$25^x = 125$

$5^{2x} = 5^3$

$2x = 3$

$x = \frac{3}{2}$

Q11. (a)

$5^{x+1} = 625$

$5^{x+1} = 5^4$

$x + 1 = 4$

$x = 3$

$3^{x-2}=81$

$3^{x-2}=3^4$

$x-2=4$

$x=6$

$2^{3x}=64$

$2^{3x}=2^6$

$3x=6$

$x=2$

Q12. (a)

$5\times 2^x=160$

$2^x=32$

$2^x=2^5$

$x=5$

$3\times 5^x=375$

$5^x=125$

$5^x=5^3$

$x=3$

$7\times 2^x=448$

$2^x=64$

$2^x=2^6$

$x=6$

Q13. (a)

$2^{x+1}=32$

$2^{x+1}=2^5$

$x+1=5$

$x=4$

$3^{2x}=81$

$3^{2x}=3^4$

$2x=4$

$x=2$

$4^{x+1}=8^x$

$2^{2(x+1)}=2^{3x}$

$2^{2x+2}=2^{3x}$

$2x+2=3x$

$x=2$

Q14. (a)

$\left(\frac{1}{2}\right)^x=8$

$2^{-x}=2^3$

$-x=3$

$x=-3$

$\left(\frac{1}{5}\right)^x=25$

$5^{-x}=5^2$

$-x=2$

$x=-2$

$2^{-x}=16$

$2^{-x}=2^4$

$-x=4$

$x=-4$

Q15. (a) $\frac{15}{5}=3$, $\frac{45}{15}=3$, $\frac{135}{45}=3$ — the ratio is constant, and it is 3.

(b) The starting value is 5 and the ratio is 3, so the rule is

$y=5\times 3^x$.

(c) When $x=6$:

$y=5\times 3^6$

$y=5\times 729$

$y=3645$

Q16. (a) $\frac{200}{1000}=\frac{1}{5}$, $\frac{40}{200}=\frac{1}{5}$, $\frac{8}{40}=\frac{1}{5}$ — the ratio is constant, and it is $\frac{1}{5}$.

(b) The starting value is 1000 and the ratio is $\frac{1}{5}$, so the rule is

$y=1000\times\left(\frac{1}{5}\right)^x$.

(c) Set $y=1.6$:

$1000 \times \left(\frac{1}{5}\right)^x = 1.6$

$\left(\frac{1}{5}\right)^x = \frac{1.6}{1000}$

$\left(\frac{1}{5}\right)^x = \frac{1}{625}$

$\left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^4$

$x = 4$

Q17. The pattern passes through $(0, 7)$, so when $x = 0$, $y = 7$:

$a \times b^0 = 7$

$a = 7$

It also passes through $(2, 28)$:

$7 \times b^2 = 28$

$b^2 = 4$

$b = 2$, since $b > 0$.

The rule is $y = 7 \times 2^x$. When $x = 5$:

$y = 7 \times 2^5$

$y = 7 \times 32$

$y = 224$

Q18. (a)

$P = 800 \times 1.1^2$

$P = 800 \times 1.21$

$P = 968$

$\therefore$ the population after 2 years is 968.

(b) Set $P = 1200$:

$800 \times 1.1^t = 1200$

$1.1^t = 1.5$

No common base will work here, so use guess-check-and-refine.

$t = 4$: $800 \times 1.1^4 = 1171.28$ — below 1200.

$t = 5$: $800 \times 1.1^5 = 1288.41$ — above 1200.

So the solution lies between 4 and 5. Refine:

$t = 4.2$: $800 \times 1.1^{4.2} = 1193.82$ — still below 1200.

$t = 4.3$: $800 \times 1.1^{4.3} = 1205.25$ — above 1200.

$\therefore$ the population first reaches 1200 after about 4.3 years.

Q19. Set $M = 25$:

$200 \times \left(\frac{1}{2}\right)^t = 25$

$\left(\frac{1}{2}\right)^t = \frac{25}{200}$

$\left(\frac{1}{2}\right)^t = \frac{1}{8}$

$\left(\frac{1}{2}\right)^t = \left(\frac{1}{2}\right)^3$

$t = 3$

$\therefore$ it takes 3 hours until only 25 mg remain.

Q20. (a)

x	0	1	2	3
y	1	5	25	125

(b) The graph passes through $(0,1)$, $(1,5)$, $(2,25)$ and $(3,125)$ and climbs faster and faster. Since $5^2 = 25 < 60 < 125 = 5^3$, the solution of $5^x = 60$ lies between 2 and 3; reading from the sketch, $x \approx 2.5$.

Q21. (a) $2^5 = 32$ and $2^6 = 64$, and $32 < 50 < 64$, so the solution lies between 5 and 6.

(b) Refine in tenths:

$x = 5.6$: $2^{5.6} = 48.50$ — below 50.

$x = 5.7$: $2^{5.7} = 51.98$ — above 50.

The solution lies between 5.6 and 5.7. Refine once more to fix the rounding:

$x = 5.64$: $2^{5.64} = 49.87$ — below 50.

$x = 5.65$: $2^{5.65} = 50.22$ — above 50.

The solution lies between 5.64 and 5.65, so to 1 decimal place,

$x \approx 5.6$.

Q22. (a)

$P = 2000 \times 1.5^3$

$P = 2000 \times 3.375$

$P = 6750$

$\therefore$ the population after 3 years is 6750.

(b) Set $P > 10\,000$:

$2000 \times 1.5^t > 10\,000$

$1.5^t > 5$

Trial: $1.5^3 = 3.375$ — not yet above 5.

Trial: $1.5^4 = 5.0625$ — above 5.

$\therefore$ the population first exceeds $10\,000$ after 4 whole years.

Q23. (a) December 2027 is $t = 2$:

$P = 27.8 \times 1.016^2$

$P = 27.8 \times 1.032256$

$P = 28.6967\ldots \approx 28.7$

$\therefore$ the model estimates 28.7 million people in December 2027.

(b) Set $P = 30$:

$27.8 \times 1.016^t = 30$

$1.016^t = \dfrac{30}{27.8} = 1.0791\ldots$

$t = 4$: $27.8 \times 1.016^4 = 29.62\ldots$ — below 30.

$t = 5$: $27.8 \times 1.016^5 = 30.10\ldots$ — above 30.

The solution lies between 4 and 5. Refine:

$t = 4.7$: $27.8 \times 1.016^{4.7} = 29.95\ldots$ — below 30.

$t = 4.8$: $27.8 \times 1.016^{4.8} = 30.00\ldots$ — at 30.

$\therefore$ the model first predicts 30 million after about 4.8 years.

(c) 4.8 years after December 2025 falls late in the year 2030.

$\therefore$ the model first predicts 30 million in the calendar year 2030.

Digital tool task. With the curve $y = 2^x$ and the line $y = 10$, the intersection is about $(3.32, 10)$, so $x \approx 3.32$. With the line $y = 20$, the intersection is about $(4.32, 20)$, so $x \approx 4.32$.

Tool-free check for $2^x = 10$:

$2^3 = 8$ and $2^4 = 16$, so $3 < x < 4$.

$2^{3.3} = 9.85\ldots$ and $2^{3.4} = 10.56\ldots$, so $3.3 < x < 3.4$.

$2^{3.32} = 9.99\ldots$ and $2^{3.33} = 10.06\ldots$, so $x \approx 3.32$.

Teacher note

Level 9 code rehearsed (D3): VC2M9A01 — the exponent laws with integer and zero exponents — does every base rewrite in this subtopic ($4^x = 2^{2x}$, $\left(\frac{1}{2}\right)^x = 2^{-x}$, $a^0 = 1$), and VC2M9A03 — solving linear equations — closes every one of them once the exponents are equated ($2x = 5$, $x + 1 = 4$). Both are rehearsed, not re-taught, and are not assessed as Level 9 outcomes. Downstream in this book, 2F ranges over these equations, and 3B's exponential gallery, 3C's compound interest and 4D's logarithmic scales all assume 2E (§3 prerequisite table).

Tool classes (D13): scientific calculator (guess-check-and-refine trials in Q18, Q21, Q23 and in the digital task's tool-free path) and a graphing tool for the digital tool task only (VC2M10A14.E03, .E04). No CAS: .E04 names symbolic manipulation functionality, but the book limits digital work to graphing and numerical exploration, prints the input and expected output beside the task, and assesses the tool-free path.

VCE destinations (§5, reference only): VC2M10A14 is named in none of the Units 3 & 4 evidence of Foundation Mathematics, General Mathematics or Mathematical Methods — an artefact of what was mapped, not an absence: exponential equations are Mathematical Methods Units 1 & 2 AoS 1 'Functions and graphs' content *(inferred)*. 2E also carries implicit prerequisite 5: with 3C, it carries the school's exponential load — the doubling time and half-life named by VC2M10A15.E03, the half-life arithmetic VC2 Science VC2S10U06 assumes in the same year, and the population and decay models of VCE Biology, Physics and Environmental Science.

Elaboration VC2M10A14.E05 is not delivered. It invites Aboriginal and Torres Strait Islander ranger groups' feral-animal programs as the context for formulating exponential equations. This book excludes Aboriginal and Torres Strait Islander content (see the front-of-book note); the population-growth modelling the elaboration exercises is delivered in full through the contexts of VC2M10A14.E01–.E04.

Level check (D6): no logarithms, no log laws and no `log` operator appear anywhere in this subtopic; there is no $f(x)$ notation, and the words *function* and *relation* are not used technically beyond VCAA's own wording in the quoted material. Solutions are found only by like bases, tables of values, graphs and guess-check-and-refine; quadratic-form substitutions such as $4^x - 3 \times 2^x + 2 = 0$ are not in scope. Fractional and negative exponents appear only as answers — students check them by substituting into the exponent, never by evaluating a fractional power.

Dated data (R3): Q23 carries the ABS population as at December 2025 and the growth figure for the year to March 2025. Listed here for the chapter refresh register so a reissue can update the figures without re-reading the chapter.

Solving graphically and by guess-check-and-refine — and whether every solution has been found

Mathematics 10 · Chapter 2 Equations and inequalities · Subtopic 2F · Standard block

Strand	Algebra
Content description	`VC2M10A16`
Elaboration ids carried	`VC2M10A16.E01` `VC2M10A16.E02`
Weight	Standard (single allocation)

> **Content description (verbatim):** *solve equations graphically or using systematic numerical guess-check-and-refine with digital tools, with consideration of whether all solutions have been found*
>
> **Achievement standard (extract):** *They make and test conjectures involving functions and relations using digital tools.*

You need first: Subtopic 2C (`VC2M10A09`, solving linear equations), Subtopic 2D (`VC2M10A13`, quadratic equations) and Subtopic 2E (`VC2M10A14`, exponential equations); from Year 9, `VC2M9A03` (linear graphs and equations) and `VC2M9A05` (quadratic graphs). This subtopic rehearses those ideas — it does not re-teach them.

Theory

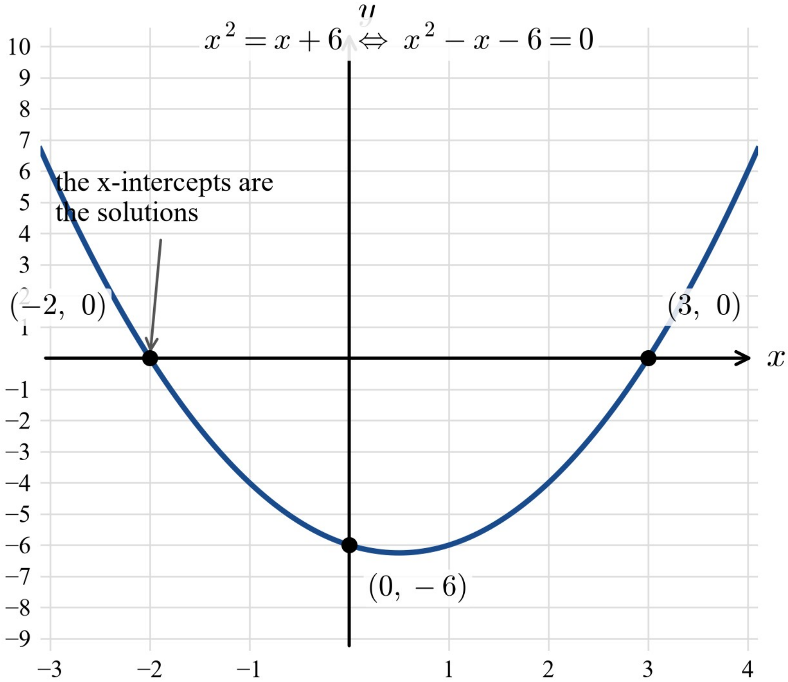
Every equation in this chapter so far came with a method that finished it: undoing steps for linear equations (2C), the null factor law for quadratics (2D), and the same thinking applied to exponential equations (2E). This subtopic closes the chapter with the two questions that remain when no method will finish the job: *what is the solution, to the accuracy you need?* — and *have you found them all?*

1. An equation is two graphs

Every equation has a left-hand side and a right-hand side, and each side is a rule whose value depends on x. The equation $x^2 = x + 6$ asks: *for which input x do the two rules give the same output?*

Graph both rules: $y = x^2$ is a parabola (2D) and $y = x + 6$ is a straight line. The solutions of the equation are exactly the x-values where the two graphs cross — at a crossing, both rules give the same y for the same x.

The same idea has a second form. Move everything to one side: $x^2 = x + 6$ is the same equation as $x^2 - x - 6 = 0$. Its solutions are the x-intercepts of the single graph $y = x^2 - x - 6$. Two graphs crossing, or one graph crossing the x-axis — use whichever form suits the problem.



The parabola y equals x squared minus x minus 6 crossing the x-axis at x equals negative 2 and x equals 3; these two x-intercepts are the two solutions of x squared minus x minus 6 equals 0, the one-graph form of x squared equals x plus 6.

2. Graphical solutions are approximations

A solution read from a graph is only as good as the grid. If the grid lines are 1 apart, readings closer than a tenth are rare, and hand-drawn graphs are rougher still. Report readings honestly: write $x \approx 2.6$, not $x = 2.6$.

Substitution is the check. Put the reading back into both sides of the original equation:

- if the two sides agree, or agree very closely, the reading is reliable;
- if they do not, the reading is wrong — go back and read again.

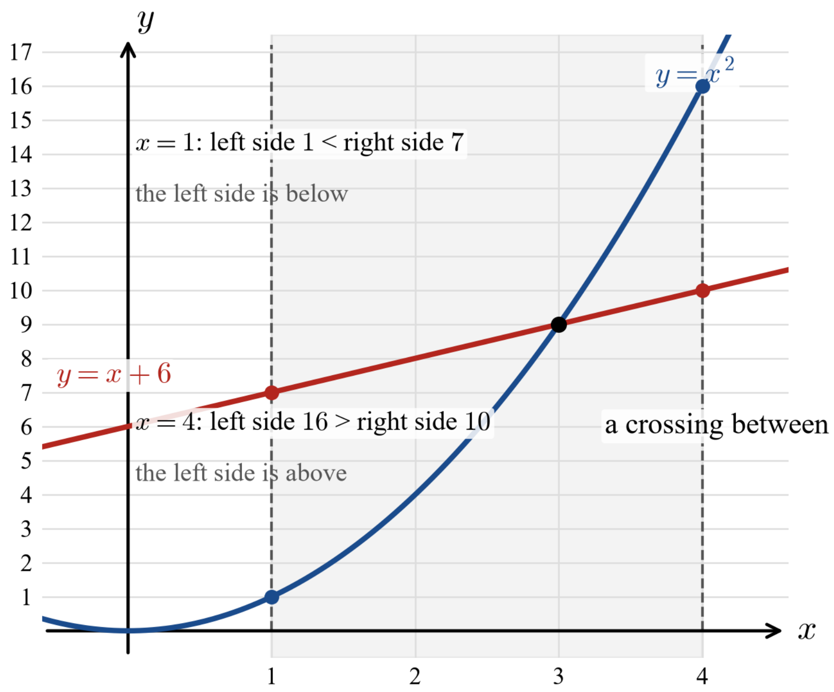
3. Guess-check-and-refine

The same thinking works with no graph at all. Guess a value of x. Substitute it into both sides. Compare: which side is bigger? Use what the comparison tells you to choose a better next guess.

That method is **guess-check-and-refine**. The guesses are not random — each check shows which direction to move in, and how large the next step should be. Keep the working in a table so the record is clear. A scientific calculator is enough; a spreadsheet can do the checking for you once the columns are set up.

4. Refining an interval — zooming in

One fact does all the work. If the two graphs are unbroken, and at $x = a$ the left-hand side is above the right-hand side while at $x = b$ the right-hand side is above the left, then the graphs cross somewhere between a and b: the difference (left − right) changed sign, so it must have passed through zero.



The parabola y equals x squared and the line y equals x plus 6. At x equals 1 the left side is below the right side, and at x equals 4 the left side is above the right side, so the two unbroken graphs must cross somewhere between 1 and 4.

Refine by testing tenths inside the interval, then hundredths:

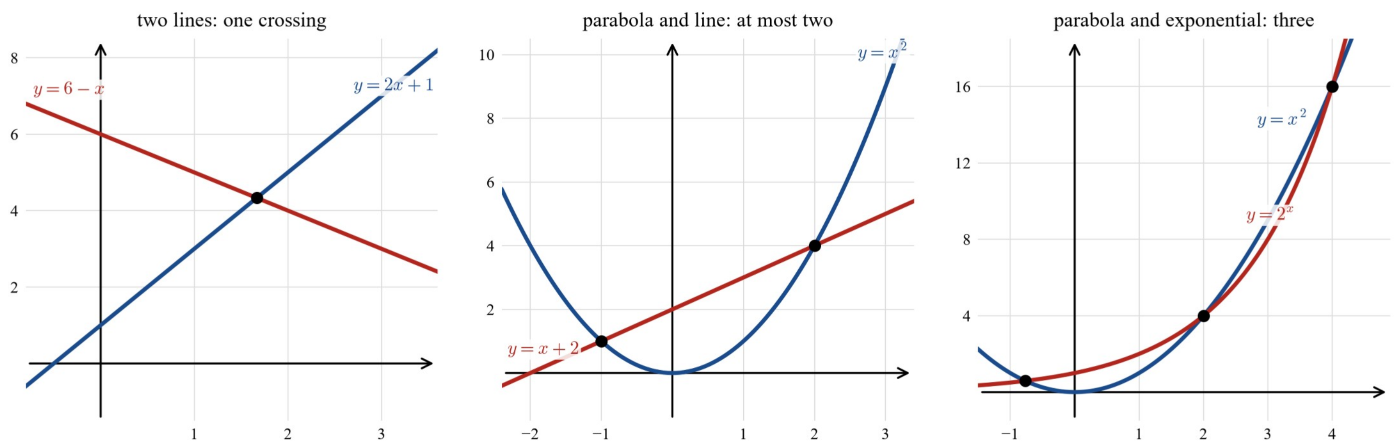
- each pass buys roughly one more correct digit;
- to round the final digit correctly, test the midpoint. If the crossing lies between 2.64 and 2.65, test 2.645: the solution rounds to 2.64 if it lies between 2.64 and 2.645, and to 2.65 if it lies between 2.645 and 2.65.

Zooming with a graphing tool is the same idea drawn: each zoom traps the crossing in a narrower window, so its x-value can be read to one more digit.

5. Have you found them all?

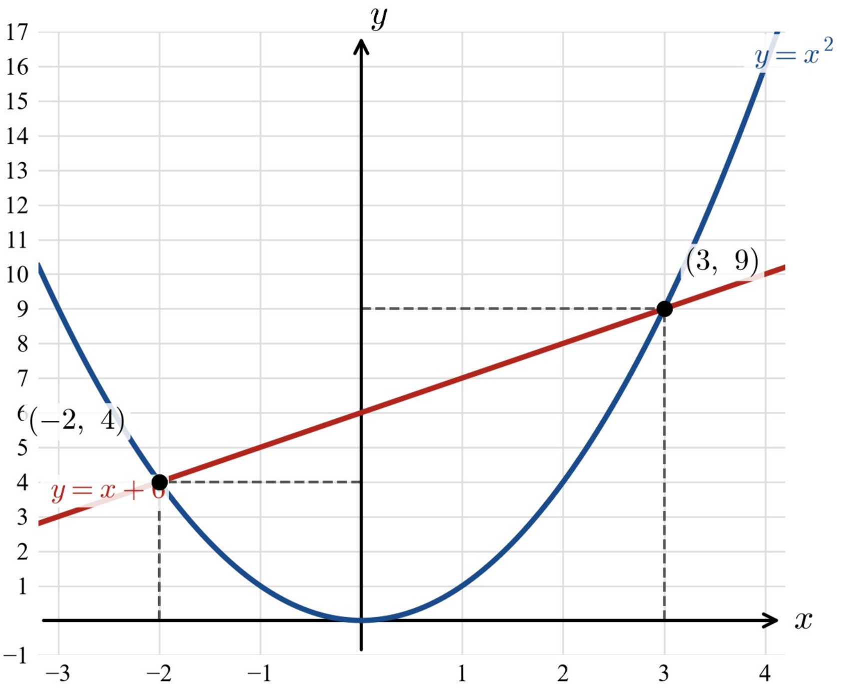
Guess-check-and-refine finds one solution at a time, and a table stops wherever you stop feeding it values — so it is easy to find one solution and miss the others. Before finishing, ask how many solutions to expect. Shape tells you:

- two straight lines that are not parallel cross once, so a linear equation has one solution (2C);
- a parabola and a straight line cross at most twice, so a quadratic equation has at most two solutions (2D);
- other pairs of graphs can meet more often — Example 3 shows a parabola and an exponential graph crossing three times.



Three graphs side by side showing how shape tells you how many solutions to expect: two straight lines crossing once, a parabola and a straight line crossing twice, and a parabola and an exponential graph crossing three times.

A good check is to evaluate (left side − right side) at several test values — between the solutions you have found, and beyond them. Every sign change means a crossing to expect. Then give a reasoned case — a *consideration*, not a proof — that your list is complete.



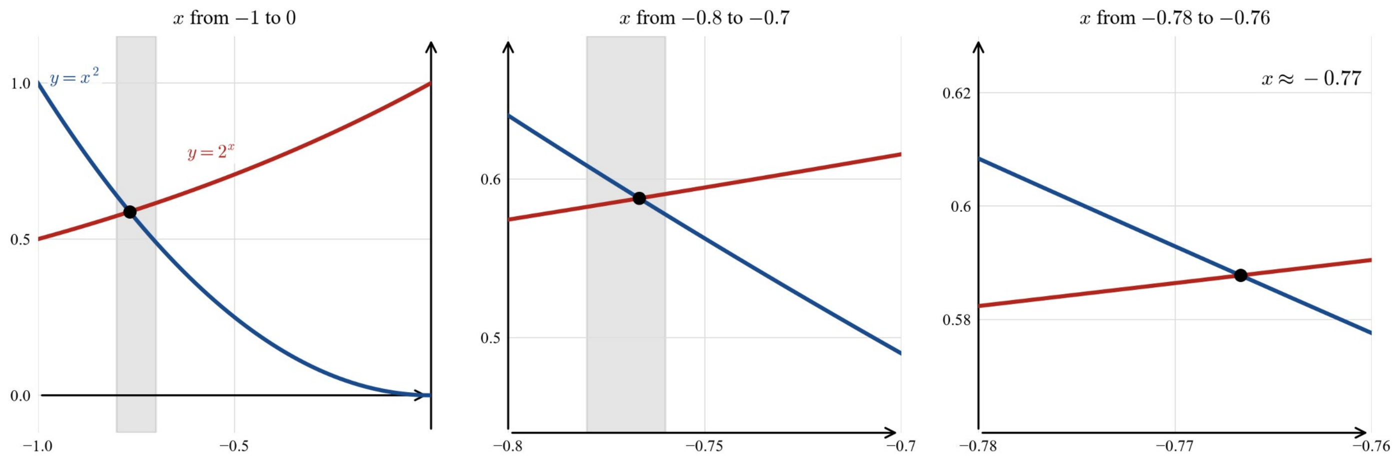
The parabola y equals x squared and the straight line y equals x plus 6, drawn for x from negative 3 to 4, crossing at the marked points (negative 2, 4) and (3, 9); the x-values of the crossings are the solutions of x squared equals x plus 6.

Worked examples

Example 1 — Solving graphically, and checking by substitution

Solve $x^2 = x + 6$ graphically. Check each solution and say whether all solutions have been found.

Working	Comment
Graph $y = x^2$ and $y = x + 6$. A table of values draws them:	
x: −3, −2, −1, 0, 1, 2, 3, 4	
x^2: 9, 4, 1, 0, 1, 4, 9, 16	the parabola
$x + 6$: 3, 4, 5, 6, 7, 8, 9, 10	the line
The graphs cross at (−2, 4) and (3, 9).	read the crossings
$x = -2$: $(-2)^2 = 4$ and $-2 + 6 = 4$	check the first reading
Both sides are 4, so $x = -2$ works.	✓
$x = 3$: $3^2 = 9$ and $3 + 6 = 9$	check the second reading
Both sides are 9, so $x = 3$ works.	✓
$\therefore x = -2$ or $x = 3$.	
A parabola and a straight line cross at most twice.	from 2D
∴ both crossings are found — there are no further solutions.	the list is complete

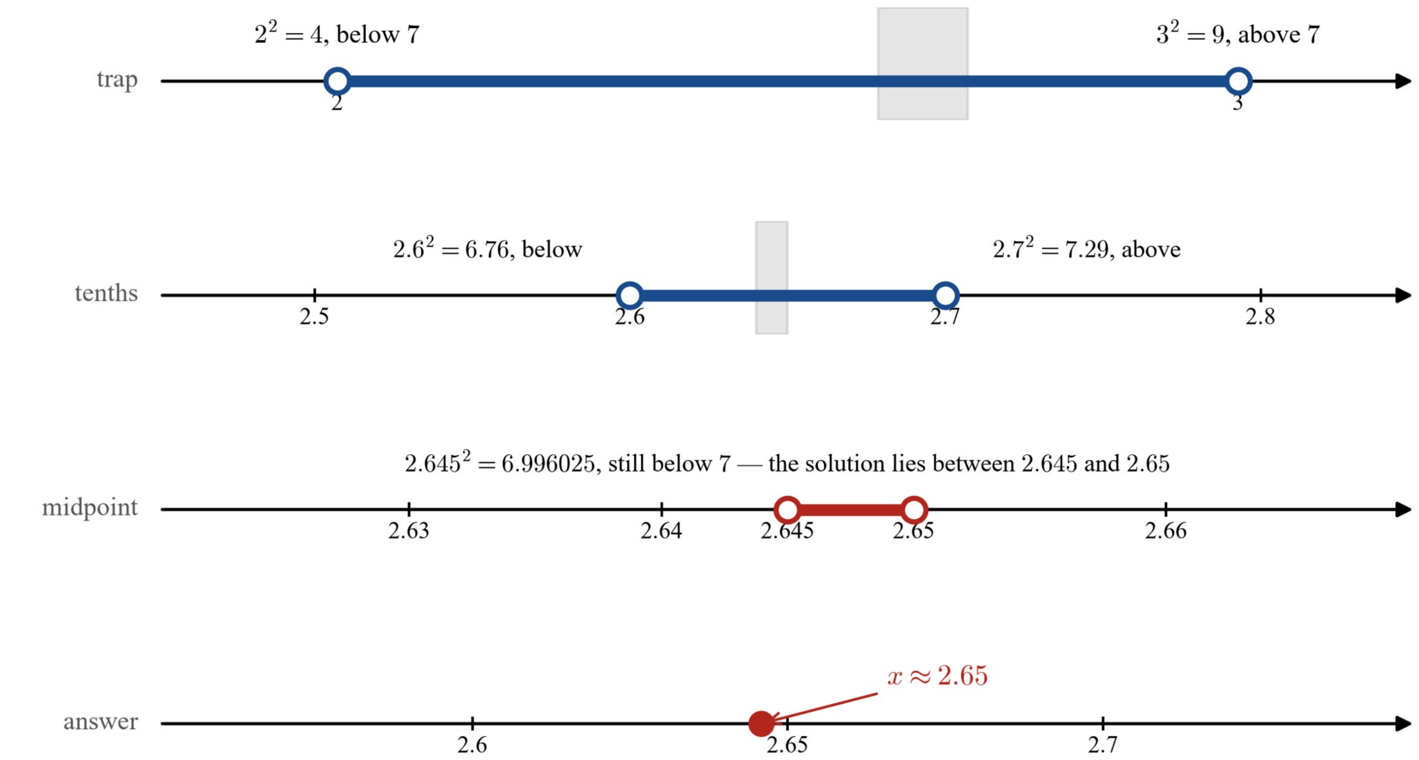


Three zoom windows on the crossing of y equals x squared and y equals 2 to the power of x for negative x: x from negative 1 to 0, then negative 0.8 to negative 0.7, then negative 0.78 to negative 0.76; each window narrows the crossing, which reads x about negative 0.77.

Example 2 — Guess-check-and-refine with a table

Find the positive solution of $x^2 = 7$ correct to 2 decimal places.

Working	Comment
$x = 2$: $2^2 = 4$, below 7	first guess
$x = 3$: $3^2 = 9$, above 7	the solution is between 2 and 3
$x = 2.6$: $2.6^2 = 6.76$, below 7	refine in tenths
$x = 2.7$: $2.7^2 = 7.29$, above 7	between 2.6 and 2.7
$x = 2.64$: $2.64^2 = 6.9696$, below 7	refine in hundredths
$x = 2.65$: $2.65^2 = 7.0225$, above 7	between 2.64 and 2.65
Midpoint: $x = 2.645$: $2.645^2 = 6.996025$, below 7	decides the rounding
The solution lies between 2.645 and 2.65.	so it rounds up
$\therefore x \approx 2.65$ (2 d.p.).	
Exact value: $x = \sqrt{7}$ (1A); here $\sqrt{7} \approx 2.65$.	numerical methods give decimals



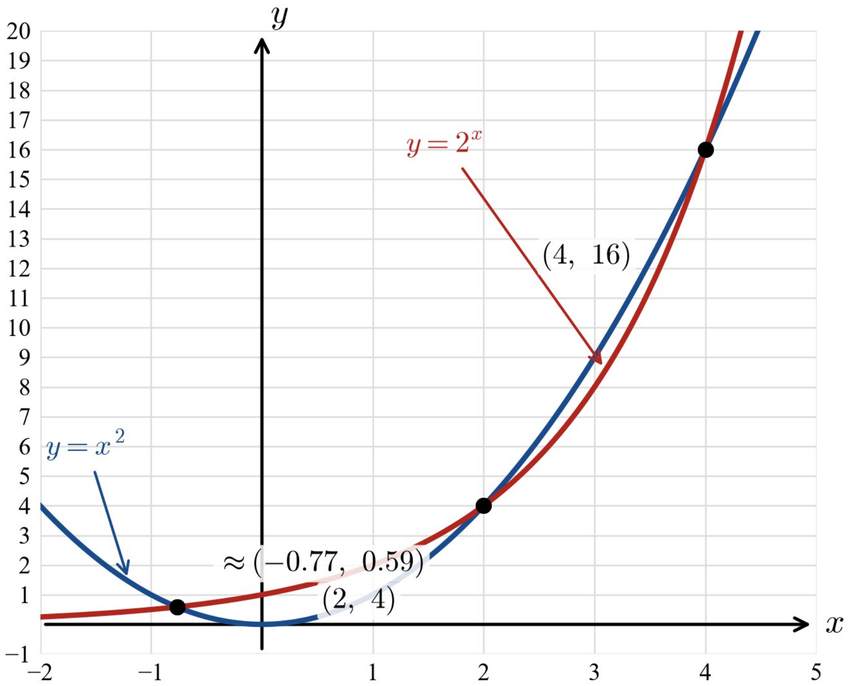
Four number lines, one above the other. The first traps the positive solution of x squared equals 7 between 2 and 3, since 2 squared is 4 and 3 squared is 9. The second refines the trap to between 2.6 and 2.7. The third tests the midpoint 2.645, whose square is still below 7, so the solution lies between 2.645 and 2.65. The fourth marks the answer, x about 2.65.

Example 3 — Finding every solution of $x^2 = 2^x$

Find all solutions of $x^2 = 2^x$, and consider whether every solution has been found.

Working	Comment
Try integers first. $x = 2$: $2^2 = 4$ and $2^2 = 4$.	an exact hit
$x = 4$: $4^2 = 16$ and $2^4 = 16$.	another exact hit
Compare the two sides at nearby values:	look for sign changes
$x = -1$: $x^2 = 1$, $2^x = 0.5 \rightarrow x^2$ above	
$x = 0$: $x^2 = 0$, $2^x = 1 \rightarrow 2^x$ above	the sides switch roles
$\therefore$ a third solution lies between -1 and 0.	sign change means crossing
Refine: $x = -0.8$: $0.64 - 0.574 = +0.066$, above	left − right, in tenths
$x = -0.7$: $0.49 - 0.616 = -0.126$, below	between -0.8 and -0.7
$x = -0.77$: $0.5929 - 0.5864 = +0.0065$, above	hundredths
$x = -0.76$: $0.5776 - 0.5905 = -0.0129$, below	between -0.77 and -0.76
Midpoint: $x = -0.765$: $0.5852 - 0.5885 = -0.0032$, below	decides the rounding
The solution lies between -0.77 and -0.765.	so it rounds to -0.77
$\therefore x \approx -0.77$ (2 d.p.).	
Check the count: evaluate $x^2 - 2^x$ at test values.	
$x = -2$: $+3.75$, $x = 1$: -1, $x = 3$: $+1$, $x = 5$: -7, $x = 10$: -924	
The sign changes three times — near -0.77, at 2 and at 4 — and after $x = 4$ it stays negative, with 2^x pulling away fast.	three crossings, no more

Working	Comment
∴ the solutions are $x \approx -0.77$, $x = 2$ and $x = 4$.	
No method taught in this book solves $x^2 = 2^x$ exactly — this equation has to be handled numerically.	a reason numerical methods matter



The graphs y equals x squared and y equals 2 to the power of x, for x from negative 2 to 5 and y from negative 1 to 20, meeting three times: near (negative 0.77, 0.59), at (2, 4) and at (4, 16).

Practice questions

Scaffolded

Q1. Solve $x^2 - 2 = 2x + 1$ graphically.

(a) Draw $y = x^2 - 2$ and $y = 2x + 1$ for x from -2 to 4, using a table of values.
(b) Read off the solutions, giving each crossing point as a coordinate.
(c) Check both solutions by substitution.
(d) Explain why there are no further solutions.

Q2. The graph of $y = x^2 - 9$ can solve $x^2 - 9 = 0$.

(a) Copy and complete a table of values for $y = x^2 - 9$, x from -3 to 3.
(b) Use the x-intercepts to solve $x^2 - 9 = 0$.
(c) Check both solutions by substitution.

Q3. The table below compares $y = x^2$ and $y = 3x - 1$.

x	−1	0	1	2	3	4
x^2	1	0	1	4	9	16
$3x - 1$	−4	−1	2	5	8	11

(a) Between which two consecutive integers does the first crossing lie? How does the table show it?
(b) Between which two consecutive integers does the second crossing lie? How does the table show it?

Q4. Find the positive solution of $x^2 = 5$ correct to 1 decimal place.

(a) Show that the solution lies between 2 and 3.
(b) Refine the interval to one decimal place: test 2.2 and 2.3.

(c) Test the midpoint to decide whether the answer rounds to 2.2 or to 2.3.

Q5. Find the solution of $2^x = 7$ correct to 1 decimal place.

(a) Show that the solution lies between 2 and 3.
(b) Refine: use a calculator to test 2.8 and 2.9.
(c) Test the midpoint to decide the rounded answer.

Q6. The graph of $y = x^2 - 4x + 1$ crosses the x-axis twice.

(a) Copy and complete the table of values for x from 0 to 4.

x	0	1	2	3	4
$x^2 - 4x + 1$					

(b) Between which consecutive integers do the two x-intercepts lie?
(c) The values in your table show that the axis of symmetry is $x = 2$. Explain how.

Q7. State whether each statement is true or false, with a reason.

(a) A solution read from a graph is always exact.
(b) In guess-check-and-refine, each new guess should use what the last check showed.
(c) If two unbroken graphs have the left side above at $x = 1$ and the right side above at $x = 2$, then a solution lies between 1 and 2.
(d) The equation $x^2 = 2^x$ has exactly two solutions.

Q8. Without solving, state how many solutions each equation has, with a reason.

(a) $3x + 2 = 11$
(b) $x^2 = 25$
(c) $2^x = 32$
(d) $x^2 = -4$

Unscaffolded

Q9. Solve $x^2 = 4x - 3$ graphically: draw both graphs for x from -1 to 5 using tables of values, read off the solutions, check each by substitution, and explain why you have found them all.

Q10. Solve $x^2 + x = 10$ by guess-check-and-refine.

(a) Show that the positive solution lies between 2 and 3, then find it correct to 1 decimal place, showing the midpoint test.
(b) The equation has a negative solution as well. Find it correct to 1 decimal place.
(c) How do you know there are exactly two solutions?

Q11. Solve $2^x = 3x$.

(a) Explain why there is no solution with $x \leq 0$.
(b) Show that one solution lies between 0 and 1 and another between 3 and 4.
(c) Find both solutions correct to 1 decimal place.
(d) Explain why you expect no further solutions with $x > 4$.

Q12. The line $y = 5 - x$ and the parabola $y = x^2 - 3$ cross twice.

(a) Show that the x-values of the crossings satisfy $x^2 + x - 8 = 0$.
(b) Find both solutions correct to 1 decimal place by guess-check-and-refine.
(c) Check each solution by substitution into $x^2 - 3$ and $5 - x$, and comment on how close the two sides are.

Q13. Consider the equation $x^2 = 3^x$.

(a) Conjecture: before doing any calculation, state how many solutions you expect and why.

(b) Test your conjecture using a table of values or a graph.
(c) Find the solution correct to 2 decimal places, showing the midpoint test.
(d) Was your conjecture correct? Explain.

Q14. Without solving, state how many solutions each equation has, with a reason.

(a) $x + 4 = 9$
(b) $(x - 3)^2 = 0$
(c) $x^2 + 4 = 0$
(d) $2^x = 100$

Q15. The equation $x^2 - x = 5$ has two solutions.

(a) Show that the positive solution lies between 2 and 3, then find it correct to 2 decimal places, showing the midpoint test.
(b) Find the negative solution correct to 1 decimal place.

Q16. `SCENARIO` A Year 10 class runs a market stall. They spend \$40 on materials, plus \$3 to make each item. To sell more items they must lower the price: to sell n items, the price has to be $\$(12 - 0.1n)$ per item, so the money taken in from sales is $\$n(12 - 0.1n)$.

(a) Write expressions for the total cost C and the money taken in R when n items are made and sold.
(b) Show that the break-even values of n — where the money taken in equals the cost — satisfy $n^2 - 90n + 400 = 0$.
(c) Find both break-even values of n correct to 1 decimal place, using a graph or guess-check-and-refine.
(d) For which whole numbers of items does the stall make a profit?
(e) Explain, using the shapes of the two graphs, why there are two break-even values rather than one.

Q17. A student solves $x^2 = x + 2$ graphically and reads the solutions as $x = 1.6$ and $x = -1.0$.

(a) Check each reading by substitution. Which one fails?
(b) Find the correct value of the reading that failed.
(c) What does this tell you about solutions read from hand-drawn graphs?

Q18. Jordan solves $x^2 + 2x = 8$ by trying positive numbers only, finds $x = 2$, and writes: "There is only one solution."

(a) Verify that $x = 2$ works.
(b) Find the solution Jordan missed.
(c) Explain how drawing the graph would have made the second solution hard to miss.

Q19. Solve $3 \times 2^x = 50$ correct to 1 decimal place by guess-check-and-refine, recording each refinement in a table.

Q20. The parabola $y = x^2 - 6x + 7$ crosses the x-axis twice.

(a) Copy and complete the table of values for x from 0 to 6.

x	0	1	2	3	4	5	6
$x^2 - 6x + 7$							

(b) Between which consecutive integers do the two x-intercepts lie?
(c) Find the smaller root correct to 1 decimal place, showing the midpoint test.
(d) Use symmetry to find the larger root correct to 1 decimal place without further testing.

Q21. The equation $x^2 = 9$ can be solved three ways.

(a) Solve it exactly by an inverse operation.
(b) Solve it graphically: sketch $y = x^2$ and $y = 9$ and read the crossings.
(c) Find the positive solution by guess-check-and-refine, starting with $x = 2$.

(d) Which of the three methods give exact values? When are graphical and numerical methods the only option?

Q22. Solve $2^x = 5x$.

(a) Explain why there is no solution with $x \le 0$.
(b) Find both positive solutions correct to 2 decimal places, showing the midpoint test for each.
(c) Explain why you expect no further solutions.

Real context

Q23. Victoria's population was 6,981,400 as at 30 June 2024, having grown by 165,100 people (2.4%) in the year to that date. Two simplified models project the population t years after 30 June 2024:

- **Model L (linear):** add 165,100 each year — $P = 6{,}981{,}400 + 165{,}100t$.
- **Model E (exponential):** grow 2% each year — $P = 6{,}981{,}400 \times 1.02^t$.

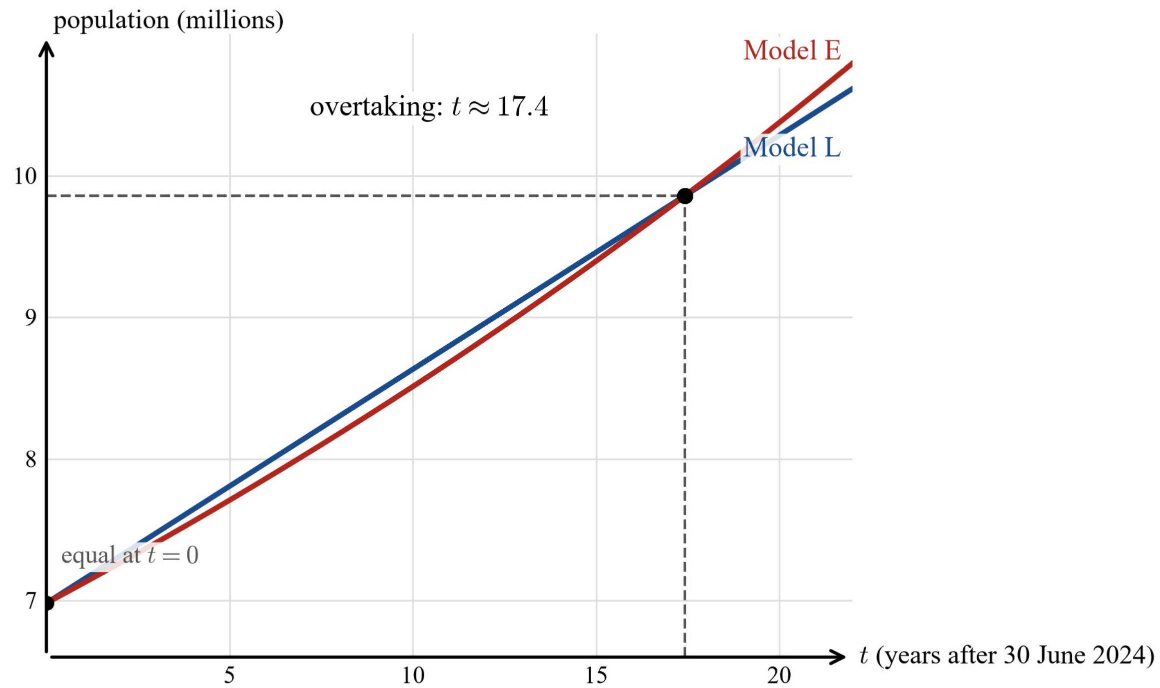
Both models are simplifying assumptions made for this question, not official projections.

(a) Copy and complete the table, rounding Model E to the nearest thousand.

t (years)	0	5	10	15	20
Model L	6,981,400				
Model E	6,981,400				

(b) Show that at t = 17 Model L is still ahead, and that at t = 18 Model E has gone past it.
(c) Find when Model E overtakes Model L, correct to 1 decimal place, showing the midpoint test.
(d) The two models are also equal at t = 0. Explain why your answer to (c) is not the only time the models agree — and why part (c) asked when E *overtakes* L, not when they are equal.
(e) After overtaking, does Model E stay ahead? Test t = 25 and use the shapes of linear and exponential growth to explain.

Source: Australian Bureau of Statistics, *National, state and territory population, Jun 2024*, 2024. Retrieved 19 August 2026. (abs.gov.au/statistics/people/population/national-and-territory-population/jun-2024)



Two population projections from 30 June 2024: the straight line, Model L, adds 165,100 people each year, while the slowly curving line, Model E, grows by 2% each year. They are equal at t equals 0, and Model E overtakes Model L at about t equals 17.4 years.

Digital tool task

DIGITAL TOOL — this task assumes a graphing tool. Your teacher will nominate the tool.

Task. Use a graphing tool to find all solutions of $x^2 = 2^x$, then use zoom to refine the negative solution.

Input (what to enter):

1. Enter the two rules $y = x^2$ and $y = 2^x$.
2. Set the window: x from −2 to 5, y from −1 to 20.
3. Use the intersection (or trace) feature to read each crossing.
4. Zoom in on the negative crossing and read it again.

Output (what to expect): intersections at (2, 4) and (4, 16), and a third crossing near (−0.77, 0.59). Zooming narrows the negative crossing to $x \approx -0.767$ ($y \approx 0.588$) — matching Example 3's table method.

Tool-free path (still assessed): the table refinement of Example 3 finds the negative solution to 2 decimal places ($x \approx -0.77$) without any graphing tool. A spreadsheet or scientific calculator can also build the table of values.

Answers

Q1 $x = -1$, $x = 3$; crossings $(-1, -1)$ and $(3, 7)$; both check; a parabola and a line cross at most twice. **Q2** (a) 0, −5, −8, −9, −8, −5, 0; (b) $x = -3$, $x = 3$; (c) $(\pm 3)^2 - 9 = 0$ ✓. **Q3** (a) Between 0 and 1 — x^2 goes from above $(0 > -1)$ to below $(1 < 2)$; (b) between 2 and 3 — x^2 goes from below $(4 < 5)$ to above $(9 > 8)$. **Q4** Between 2 and 3; between 2.2 and 2.3; midpoint 2.25 gives $5.0625 > 5$, so $x \approx 2.2$. **Q5** Between 2 and 3; between 2.8 and 2.9; midpoint 2.85 gives 2^2.85 $\approx 7.21 > 7$, so $x \approx 2.8$. **Q6** (a) 1, −2, −3, −2, 1; (b) between 0 and 1, and between 3 and 4; (c) the values mirror each other about $x = 2$. **Q7** (a) F — readings are limited by the grid; check by substitution; (b) T — that is what makes it systematic; (c) T — sign change on unbroken graphs; (d) F — three solutions (Example 3). **Q8** (a) one — linear, $x = 3$; (b) two — $x = \pm 5$; (c) one — $x = 5$, exponential crosses $y = 32$ once; (d) none — $x^2 \geq 0$. **Q9** $x = 1$, $x = 3$. **Q10** (a) $x \approx 2.7$; (b) $x \approx -3.7$; (c) it is a quadratic equation — at most two solutions. **Q11** (a) $2^x > 0$ always but $3x \leq 0$ for $x \leq 0$; (b) crossings by sign change; (c) $x \approx 0.5$, $x \approx 3.3$; (d) at $x = 10$, $2^x = 1024$ against $3x = 30$ and pulling away; no sign change after $x = 4$. **Q12** $x \approx 2.4$, $x \approx -3.4$; sides agree closely, not exactly — the answers are rounded. **Q13** One solution; $x \approx -0.69$. **Q14** (a) one — linear; (b) one — a square is zero only when $x - 3 = 0$, the graphs touch once; (c) none — $x^2 + 4 \geq 4$; (d) one — between 6 and 7 since $2^6 = 64 < 100 < 128 = 2^7$, and the exponential crosses once. **Q15** (a) $x \approx 2.79$; (b) $x \approx -1.8$. **Q16** (a) $C = 40 + 3n$, $R = n(12 - 0.1n)$; (c) $n \approx 4.7$, $n \approx 85.3$; (d) $n = 5$ to $n = 85$; (e) at first the money taken grows faster than the cost; later the falling price drags it down while the cost keeps rising — the parabola meets the line twice. **Q17** (a) $x = 1.6$ fails $(2.56 \neq 3.6)$; $x = -1.0$ works; (b) $x = 2$; (c) hand-drawn readings are approximate — substitution exposes a bad reading. **Q18** (a) $4 + 4 = 8$ ✓; (b) $x = -4$; (c) the parabola crosses the x-axis twice. **Q19** $x \approx 4.1$. **Q20** (a) 7, 2, −1, −2, −1, 2, 7; (b) between 1 and 2, and between 4 and 5; (c) $x \approx 1.6$; (d) $x \approx 4.4$. **Q21** (a) $x = \pm 3$; (d) the inverse-operation method gives exact values; graphical and numerical methods are the only option when no algebraic method applies, e.g. $x^2 = 2^x$. **Q22** $x \approx 0.24$, $x \approx 4.49$; no solution for $x \leq 0$ since $2^x > 0 \geq 5x$; no further crossings — at $x = 10$, $2^x = 1024$ against $5x = 50$ and pulling away. **Q23** (a) L: 7,806,900; 8,632,400; 9,457,900; 10,283,400. E: 7,708,000; 8,510,000; 9,396,000; 10,374,000; (b) at $t = 17$, L leads by 12,455; at $t = 18$, E leads by 17,958; (c) $t \approx 17.4$; (d) they also agree at $t = 0$, the shared starting point — overtaking means crossing from behind; (e) at $t = 25$, E leads by 344,827; exponential growth (a percentage) outgrows linear growth (a fixed amount), so E is expected to stay ahead. **Digital tool task** Intersections $(-0.77, 0.59)$, $(2, 4)$, $(4, 16)$; zoom gives $x \approx -0.767$.

Fully-worked solutions

Q1.

(a) Table of values:

x	−2	−1	0	1	2	3	4
$x^2 - 2$	2	−1	−2	−1	2	7	14
$2x + 1$	−3	−1	1	3	5	7	9

(b) The graphs cross at $(-1, -1)$ and $(3, 7)$. $\therefore x = -1$ or $x = 3$.

(c) $x = -1$: $(-1)^2 - 2 = -1$ and $2(-1) + 1 = -1$ ✓ $x = 3$: $3^2 - 2 = 7$ and $2(3) + 1 = 7$ ✓

(d) A parabola and a straight line cross at most twice (2D), so both solutions are found.

Q2.

(a) $y = x^2 - 9$: 0, −5, −8, −9, −8, −5, 0 for $x = -3, -2, -1, 0, 1, 2, 3$.

(b) The graph crosses the x-axis at $x = -3$ and $x = 3$. $\therefore x = -3$ or $x = 3$.

(c) $(-3)^2 - 9 = 0$ ✓ and $3^2 - 9 = 0$ ✓

Q3.

(a) At $x = 0$: $x^2 = 0$ is above $3x - 1 = -1$. At $x = 1$: $x^2 = 1$ is below $3x - 1 = 2$. The sides switch roles, so the first crossing lies between 0 and 1.

(b) At $x = 2$: $x^2 = 4$ is below $3x - 1 = 5$. At $x = 3$: $x^2 = 9$ is above $3x - 1 = 8$. The sides switch back, so the second crossing lies between 2 and 3.

Q4.

(a) $x = 2$: $2^2 = 4$, below 5. $x = 3$: $3^2 = 9$, above 5. $\therefore$ the solution lies between 2 and 3.

(b) $x = 2.2$: $2.2^2 = 4.84$, below 5. $x = 2.3$: $2.3^2 = 5.29$, above 5. ∴ between 2.2 and 2.3.
(c) Midpoint $x = 2.25$: $2.25^2 = 5.0625$, above 5. The solution lies between 2.2 and 2.25, so it rounds down. ∴ $x \approx 2.2$ (1 d.p.).

Q5.

(a) $x = 2$: $2^2 = 4$, below 7. $x = 3$: $2^3 = 8$, above 7. ∴ the solution lies between 2 and 3.
(b) $x = 2.8$: $2^{2.8} \approx 6.96$, below 7. $x = 2.9$: $2^{2.9} \approx 7.46$, above 7. ∴ between 2.8 and 2.9.
(c) Midpoint $x = 2.85$: $2^{2.85} \approx 7.21$, above 7. The solution lies between 2.8 and 2.85, so it rounds down. ∴ $x \approx 2.8$ (1 d.p.).

Q6.

(a) $x^2 - 4x + 1$: 1, −2, −3, −2, 1 for $x = 0, 1, 2, 3, 4$.
(b) The sign changes between 0 and 1, and between 3 and 4 — the x-intercepts lie there.
(c) The y-values read the same forwards and backwards: 1, −2, −3, −2, 1. The mirror line is the middle value, $x = 2$.

Q7.

(a) False. A reading is limited by the spacing of the grid; check every reading by substitution.
(b) True. Each check shows which way to move — that is what makes the method systematic.
(c) True. On unbroken graphs, a sign change means a crossing between.
(d) False. Example 3 found three solutions: $x \approx -0.77$, $x = 2$ and $x = 4$.

Q8.

(a) One — a linear equation: two non-parallel lines cross once. In fact $x = 3$.
(b) Two — $x = 5$ and $x = -5$; a parabola meets a horizontal line twice.
(c) One — $x = 5$; the exponential graph rises and crosses $y = 32$ once.
(d) None — x^2 is never negative, so it can never equal −4.

Q9.

Table of values:

x	−1	0	1	2	3	4	5
x^2	1	0	1	4	9	16	25
$4x - 3$	−7	−3	1	5	9	13	17

The graphs cross at (1, 1) and (3, 9). ∴ $x = 1$ or $x = 3$. Check: $x = 1$: $1^2 = 1$ and $4(1) - 3 = 1$ ✓ $x = 3$: $3^2 = 9$ and $4(3) - 3 = 9$ ✓ A parabola and a line cross at most twice, so both solutions are found.

Q10.

(a) $x = 2$: $2^2 + 2 = 6$, below 10. $x = 3$: $3^2 + 3 = 12$, above 10. ∴ between 2 and 3. $x = 2.7$: $2.7^2 + 2.7 = 9.99$, below 10. $x = 2.8$: $2.8^2 + 2.8 = 10.64$, above 10. ∴ between 2.7 and 2.8. Midpoint $x = 2.75$: $2.75^2 + 2.75 = 10.3125$, above 10. The solution lies between 2.7 and 2.75, so it rounds down. ∴ $x \approx 2.7$ (1 d.p.).
(b) $x = -3.7$: $(-3.7)^2 + (-3.7) = 9.99$, below 10. $x = -3.8$: $(-3.8)^2 + (-3.8) = 10.64$, above 10. ∴ between −3.8 and −3.7. Midpoint $x = -3.75$: $(-3.75)^2 + (-3.75) = 10.3125$, above 10. The solution lies between −3.75 and −3.7, so it rounds to −3.7. ∴ $x \approx -3.7$ (1 d.p.).
(c) $x^2 + x = 10$ rearranges to $x^2 + x - 10 = 0$, a quadratic equation — at most two solutions (2D), and two have been found.

Q11.

(a) For $x \le 0$, $3x \le 0$ but $2^x > 0$ always, so the two sides can never be equal.

(b) $x = 0$: $2^0 = 1$ against $3(0) = 0$ — 2^x above. $x = 1$: 2 against 3 — $3x$ above. ∴ a crossing between 0 and 1. $x = 3$: 8 against 9 — $3x$ above. $x = 4$: 16 against 12 — 2^x above. ∴ a crossing between 3 and 4.

(c) First crossing: $x = 0.4$: $2^{0.4} \approx 1.32$ against 1.2, above. $x = 0.5$: $2^{0.5} \approx 1.41$ against 1.5, below. ∴ between 0.4 and 0.5. Midpoint $x = 0.45$: $2^{0.45} \approx 1.37$ against 1.35, above. ∴ the solution rounds to 0.5. $x \approx 0.5$. Second crossing: $x = 3.3$: $2^{3.3} \approx 9.85$ against 9.9, below. $x = 3.4$: $2^{3.4} \approx 10.56$ against 10.2, above. ∴ between 3.3 and 3.4. Midpoint $x = 3.35$: $2^{3.35} \approx 10.20$ against 10.05, above. ∴ the solution rounds to 3.3. $x \approx 3.3$.

(d) At $x = 10$, $2^x = 1024$ against $3x = 30$, and the exponential pulls further away each year. With no sign change after $x = 4$, no further crossings are expected.

Q12.

(a) At a crossing, $x^2 - 3 = 5 - x$. $x^2 - 3 - 5 + x = 0$ $x^2 + x - 8 = 0$ ∴

(b) Positive solution: $x = 2.3$: $2.3^2 - 3 = 2.29$ against $5 - 2.3 = 2.7$ — left below. $x = 2.4$: $2.4^2 - 3 = 2.76$ against $5 - 2.4 = 2.6$ — left above. ∴ between 2.3 and 2.4. Midpoint $x = 2.35$: $2.35^2 - 3 = 2.5225$ against 2.65 — left below. ∴ the solution rounds to 2.4. $x \approx 2.4$. Negative solution: $x = -3.3$: $(-3.3)^2 - 3 = 7.89$ against $5 - (-3.3) = 8.3$ — left below. $x = -3.4$: $(-3.4)^2 - 3 = 8.56$ against 8.4 — left above. ∴ between -3.4 and -3.3. Midpoint $x = -3.35$: $(-3.35)^2 - 3 = 8.2225$ against 8.35 — left below. ∴ the solution rounds to -3.4. $x \approx -3.4$.

(c) $x = 2.4$: sides 2.76 and 2.6 — close, not equal, because 2.4 is rounded. $x = -3.4$: sides 8.56 and 8.4 — close for the same reason. The near-misses confirm the readings without making them exact.

Q13.

(a) Answers vary — e.g. "one, since 3^x grows faster than x^2" or "two, like $x^2 = 2^x$ had a positive pair".

(b) $x = -1$: $x^2 = 1$ against $3^{-1} \approx 0.33$ — x^2 above. $x = 0$: 0 against 1 — 3^x above. ∴ a crossing between -1 and 0. $x = 0$: 0 against 1; $x = 1$: 1 against 3; $x = 2$: 4 against 9; $x = 3$: 9 against 27 — 3^x stays above and the gap widens, so no crossing is expected for $x \geq 0$.

(c) $x = -0.69$: $(-0.69)^2 = 0.4761$ against $3^{-0.69} \approx 0.4686$ — x^2 above (difference +0.0075). $x = -0.68$: $(-0.68)^2 = 0.4624$ against $3^{-0.68} \approx 0.4738$ — below (difference -0.0114). ∴ between -0.69 and -0.68. Midpoint $x = -0.685$: $(-0.685)^2 \approx 0.4692$ against $3^{-0.685} \approx 0.4712$ — below. The solution lies between -0.69 and -0.685, so it rounds to -0.69. ∴ $x \approx -0.69$ (2 d.p.).

(d) Depends on the conjecture — the equation has exactly one solution.

Q14.

(a) One — linear: $x = 5$.

(b) One — a square equals zero only when $x - 3 = 0$; graphically, the parabola touches the x-axis once, at $x = 3$.

(c) None — $x^2 + 4 \geq 4$, so it never reaches 0.

(d) One — $2^6 = 64 < 100 < 128 = 2^7$, so the crossing lies between 6 and 7; the exponential rises and crosses once.

Q15.

(a) $x = 2$: $2^2 - 2 = 2$, below 5. $x = 3$: $3^2 - 3 = 6$, above 5. ∴ between 2 and 3. $x = 2.7$: $2.7^2 - 2.7 = 4.59$, below 5. $x = 2.8$: $2.8^2 - 2.8 = 5.04$, above 5. ∴ between 2.7 and 2.8. $x = 2.79$: $2.79^2 - 2.79 = 4.9941$, below 5. $x = 2.80$: 5.04, above 5. ∴ between 2.79 and 2.80. Midpoint $x = 2.795$: $2.795^2 - 2.795 = 5.017025$, above 5. The solution lies between 2.79 and 2.795, so it rounds down. ∴ $x \approx 2.79$ (2 d.p.).

(b) $x = -1.7$: $(-1.7)^2 - (-1.7) = 4.59$, below 5. $x = -1.8$: $(-1.8)^2 - (-1.8) = 5.04$, above 5. ∴ between -1.8 and -1.7. Midpoint $x = -1.75$: $(-1.75)^2 - (-1.75) = 4.8125$, below 5. The solution lies between -1.8 and -1.75, so it rounds to -1.8. ∴ $x \approx -1.8$ (1 d.p.).

Q16.

(a) $C = 40 + 3n$ and $R = n(12 - 0.1n) = 12n - 0.1n^2$.

(b) Break-even means $R = C$: $12n - 0.1n^2 = 40 + 3n$ $9n - 0.1n^2 = 40$ $90n - n^2 = 400$ (multiplying by 10) $n^2 - 90n + 400 = 0$ ∴

(c) Smaller root: $n = 4.6$: $R = 53.084$ against $C = 53.8$ — R below. $n = 4.7$: $R = 54.191$ against $C = 54.1$ — R above. ∴ between 4.6 and 4.7. Midpoint $n = 4.65$: $R \approx 53.638$ against $C = 53.95$ — R below. ∴ $n \approx 4.7$. Larger root: $n = 85.3$: $R = 295.991$ against $C = 295.9$ — R above. $n = 85.4$: $R = 295.484$ against $C = 296.2$ — R below. ∴ between 85.3 and 85.4. Midpoint $n = 85.35$: $R \approx 295.738$ against $C = 296.05$ — R below. ∴ $n \approx 85.3$. ∴ break-even at $n \approx 4.7$ and $n \approx 85.3$.

(d) Profit when $R > C$, i.e. between the roots: $n = 5, 6, \ldots, 85$. Check: $n = 5$: $R = 57.5$, $C = 55$ — profit. $n = 85$: $R = 297.5$, $C = 295$ — profit. $n = 86$: $R = 292.4$, $C = 298$ — loss.

(e) The money taken in forms a parabola (it rises, peaks, then falls as the price drops), while the cost is a straight line rising steadily — so the parabola meets the line twice: once going up, once coming down.

Q17.

(a) $x = 1.6$: left side $1.6^2 = 2.56$; right side $1.6 + 2 = 3.6$. Not equal — the reading fails. $x = -1.0$: left side $(-1.0)^2 = 1$; right side $-1.0 + 2 = 1$. Equal — the reading works.

(b) The second crossing of $y = x^2$ and $y = x + 2$ is at $x = 2$: $2^2 = 4$ and $2 + 2 = 4$ ✓

(c) Readings from hand-drawn graphs are approximate. Substitution exposes a bad reading — always check.

Q18.

(a) $x = 2$: $2^2 + 2(2) = 4 + 4 = 8$ ✓

(b) $x = -4$: $(-4)^2 + 2(-4) = 16 - 8 = 8$ ✓

(c) $y = x^2 + 2x - 8$ is a parabola that crosses the x-axis twice; a sketch shows both crossings, so positive-only guessing could not hide one of them.

Q19.

$x = 4$: $3 \times 2^4 = 48$, below 50. $x = 5$: $3 \times 2^5 = 96$, above 50. ∴ between 4 and 5. $x = 4.05$: $3 \times 2^{4.05} \approx 49.69$, below 50. $x = 4.1$: $3 \times 2^{4.1} \approx 51.45$, above 50. ∴ between 4.05 and 4.1. Midpoint $x = 4.055$: $3 \times 2^{4.055} \approx 49.86$, below 50. The solution lies between 4.055 and 4.1, so it rounds to 4.1. ∴ $x \approx 4.1$ (1 d.p.).

Q20.

(a) $x^2 - 6x + 7$: 7, 2, −1, −2, −1, 2, 7 for $x = 0, 1, 2, 3, 4, 5, 6$.

(b) Sign changes between 1 and 2, and between 4 and 5 — the x-intercepts lie there.

(c) $x = 1.5$: $1.5^2 - 6(1.5) + 7 = 0.25$, above 0. $x = 1.6$: $1.6^2 - 6(1.6) + 7 = -0.04$, below 0. ∴ between 1.5 and 1.6. Midpoint $x = 1.55$: $1.55^2 - 6(1.55) + 7 = 0.1025$, above 0. The solution lies between 1.55 and 1.6, so it rounds to 1.6. ∴ the smaller root is $x \approx 1.6$ (1 d.p.).

(d) The values mirror about $x = 3$, so the two roots sit symmetrically about $x = 3$ and add to 6. Larger root $\approx 6 - 1.6 = 4.4$. ∴ $x \approx 4.4$ (1 d.p.).

Q21.

(a) $x^2 = 9 \rightarrow x = \pm\sqrt{9} = \pm 3$.

(b) $y = x^2$ and $y = 9$ cross at $(-3, 9)$ and $(3, 9)$: readings $x = -3$, $x = 3$.

(c) $x = 2$: $2^2 = 4$, below 9. $x = 3$: $3^2 = 9$ — an exact hit. ∴ $x = 3$; by symmetry $x = -3$ also.

(d) The inverse-operation method gives the exact values ±3; a graphical reading or a lucky guess can also land on them here. Graphical and numerical methods are the only option when no algebraic method applies — e.g. $x^2 = 2^x$, which no method in this book solves exactly.

Q22.

(a) For $x \leq 0$, $5x \leq 0$ but $2^x > 0$ always, so the two sides can never be equal.

(b) Small root: $x = 0.2$: $2^{0.2} \approx 1.149$ against $5(0.2) = 1.0$ — above. $x = 0.3$: $2^{0.3} \approx 1.231$ against 1.5 — below. ∴ between 0.2 and 0.3. $x = 0.23$: $2^{0.23} \approx 1.1728$ against 1.15 — above. $x = 0.24$: $2^{0.24} \approx 1.1810$ against 1.20 — below. ∴ between 0.23 and 0.24. Midpoint $x = 0.235$: $2^{0.235} \approx 1.1769$ against 1.175 — above. The solution rounds to 0.24. ∴ $x \approx 0.24$. Large root: $x = 4.4$: $2^{4.4} \approx 21.11$ against 22 — below. $x = 4.5$: $2^{4.5} \approx 22.63$ against 22.5 — above. ∴ between 4.4 and 4.5. $x = 4.48$: $2^{4.48} \approx 22.316$ against 22.4 — below. $x = 4.49$: $2^{4.49} \approx 22.471$ against 22.45 — above. ∴ between 4.48 and 4.49. Midpoint $x = 4.485$: $2^{4.485} \approx 22.393$ against 22.425 — below. The solution rounds to 4.49. ∴ $x \approx 4.49$.

(c) At $x = 10$, $2^x = 1024$ against $5x = 50$, and the exponential pulls further away each year; with no sign change after $x = 4.49$, no further crossings are expected.

Q23.

(a) Model L adds 165,100 each step; Model E multiplies by 1.02 each year:

t	0	5	10	15	20
L	6,981,400	7,806,900	8,632,400	9,457,900	10,283,400
E	6,981,400	7,708,000	8,510,000	9,396,000	10,374,000

(b) $t = 17$: $L = 6{,}981{,}400 + 17 \times 165{,}100 = 9{,}788{,}100$; $E = 6{,}981{,}400 \times 1.02^{17} \approx 9{,}775{,}645$. L leads by 12,455. $t = 18$: $L = 9{,}953{,}200$; $E \approx 9{,}971{,}158$. E leads by 17,958.

(c) $t = 17.4$: $L = 9{,}854{,}140$; $E \approx 9{,}853{,}386$ — L ahead by about 754. $t = 17.5$: $L = 9{,}870{,}650$; $E \approx 9{,}872{,}918$ — E ahead by about 2,268. Midpoint $t = 17.45$: $L = 9{,}862{,}395$; $E \approx 9{,}863{,}147$ — E ahead by about 752. The crossing lies between 17.4 and 17.45, so it rounds down. $\therefore$ E overtakes L at $t \approx 17.4$ years — during the eighteenth year after 30 June 2024.

(d) The models also agree at $t = 0$, where both start from 6,981,400 — that equality is the shared starting value, not an overtaking. Part (c) asked when E crosses *from behind*, which happens once, at $t \approx 17.4$.

(e) $t = 25$: $L = 11{,}108{,}900$; $E \approx 11{,}453{,}727$ — E leads by 344,827, and the lead is growing. Model E adds a percentage of an ever-larger population each year while Model L adds a fixed amount, so E is expected to stay ahead. (This is a consideration based on the models' shapes, not a proof.)

Digital tool task. With $y = x^2$ and $y = 2^x$ in the window x: −2 to 5, y: −1 to 20, the intersection feature gives (2, 4), (4, 16) and a crossing near (−0.77, 0.59). Zooming on the negative crossing reads $x \approx -0.767$, $y \approx 0.588$ — consistent with Example 3's table refinement.

Teacher note

Level 9 codes rehearsed (D3): `VC2M9A03` (linear graphs and linear equations) and `VC2M9A05` (quadratic graphs and the null factor law) are rehearsed throughout — in the shape arguments of Theory part 5 and in the sufficiency checks of Q1, Q9 and Q10 — but not re-taught.

Tool classes (D13): a graphing tool (zoom and intersection features) for the Digital tool task; a spreadsheet or scientific calculator for tables of values in guess-check-and-refine. `VC2M10A16` names digital tools in the content description itself, so both tool classes appear with expected input and output; no CAS is used or assumed. The tool-free path (the table refinement of Examples 2 and 3) is taught first and remains assessed.

VCE destinations (§5, reference only): `VC2M10A16` is not named in any Units 3&4 content-description index — an artefact of the mapping. Subtopic 2F's numerical refinement is the ancestor of the bisection-style reasoning that Methods Units 3 & 4 use inside their calculus outcomes (inferred). The absence is a mapping artefact, not a signal that the skill drops out.

Level check (D6): no f(x) notation; no technical use of "function" or "relation" in student-facing prose (the achievement-standard extract carries the curriculum's own wording and is quoted verbatim); no logarithm laws — equations like $2^x = 7$ stay numerical; $x^2 = 2^x$ is solved numerically only, per the TOC note that it "cannot be solved by any method taught in this book"; sufficiency is presented as a reasoned consideration, never a proof; bisection is not named as a VCE method.

Position (D9): deliberately last in Chapter 2 — its whole point is the question "have you found them all?", which ranges over the equation types taught in 2C, 2D and 2E. It is a subtopic about sufficiency of solution, not a digital-tools appendix to 2E.