Mathematics Level 10

Chapter 1 — Number and algebraic manipulation · 5 subtopics · 6 CDs

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Contents

Exact values and what repeated approximation costs

Mathematics 10 · Chapter 1 Number and algebraic manipulation · Subtopic 1A · Standard block

Strand	Number
Content description	`VC2M10N01`
Elaboration ids carried	`VC2M10N01.E01` `VC2M10N01.E02`
Weight	Standard (single allocation)

> **Content description (verbatim):** *recognise the effect of using approximations of real numbers in repeated calculations and compare the results when using exact representations*
>
> **Achievement standard (extract):** *Students recognise the effect of approximations of real numbers in repeated calculations.*

You need first: the real number system and irrational numbers from Year 9 (`VC2M9N01`), absolute, relative and percentage error (`VC2M9M04`), and simple interest (`VC2M9A06`). This subtopic rehearses those skills and builds on them; it does not re-teach them.

Theory

This subtopic answers one question: **what does an approximate answer cost you when the calculation does not stop?** Every approximation throws some digits away. One small error can look harmless — until the same approximate value is used again and again, and the error is carried into every step that follows.

1. Exact values and approximations

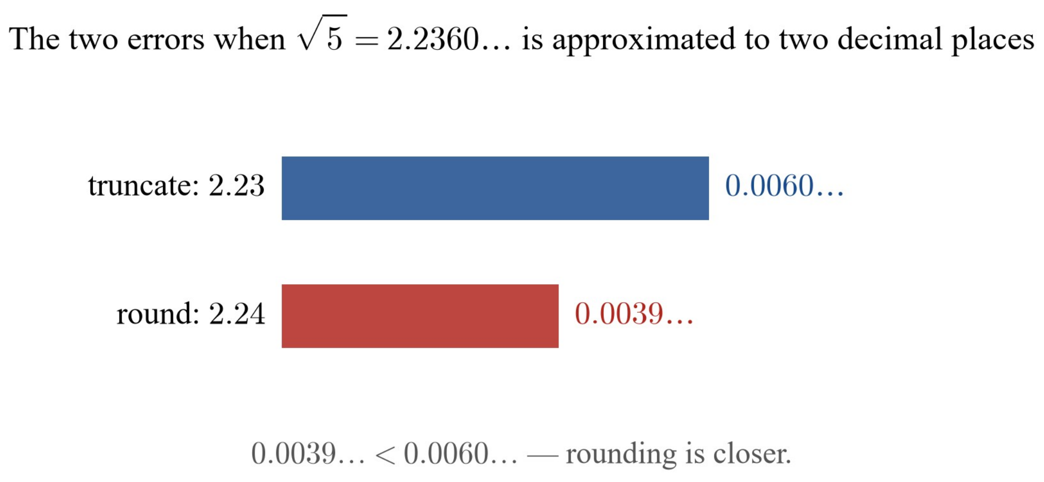
Recall from Year 9 (`VC2M9N01`):

- A **rational number** can be written as a fraction $\frac{p}{q}$ where p and q are integers and $q \neq 0$. Its decimal form either **terminates** ($\frac{3}{8} = 0.375$) or **repeats** ($\frac{1}{3} = 0.333\ldots$).
- An **irrational number**, such as $\sqrt{2}$, $\sqrt{7}$ or π, cannot be written as a fraction. Its decimal form never terminates and never repeats.

An **exact representation** keeps the full value: $\sqrt{2}$, 5π, $\frac{1}{3}$. An **approximation** replaces the exact value with a nearby value that is easier to work with, usually a terminating decimal: $\sqrt{2} \approx 1.41$, $\pi \approx 3.14$, $\frac{1}{3} \approx 0.33$.

Every approximation has an error. From Year 9 (`VC2M9M04`):

- the **absolute error** is $|\text{approximate value} - \text{exact value}|$;
- the **percentage error** is $\dfrac{\text{absolute error}}{\text{exact value}} \times 100\,\%$.



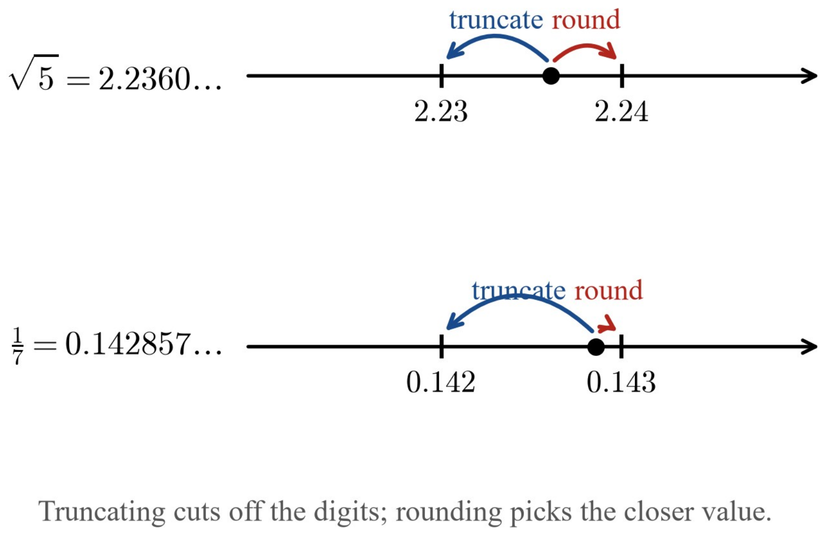
Bar chart of the two errors when the square root of 5 is approximated to two decimal places: the truncation error of 0.0060 continuing is longer than the rounding error of 0.0039 continuing.

2. Rounding and truncation

There are two common ways to shorten a decimal.

Rounding to n decimal places keeps the n-place decimal that is closest to the exact value. The Victorian Curriculum does not fix a tie-breaking rule for rounding, so this book states its own: **round half up** — when the next digit is exactly 5, round up. (For example, 0.625 rounds to 0.63 to two decimal places.)

Truncation is the other method. The curriculum uses the word *truncate* without defining it; in this subtopic, to **truncate** to n decimal places means to cut off every digit after the nth place, with no adjustment. Every question here is answerable on that definition alone.



Two number lines. The point for the square root of 5, 2.2360 continuing, sits between 2.23 and 2.24, with an arrow to 2.23 labelled truncate and an arrow to 2.24 labelled round. The point for one seventh, 0.142857 continuing, sits between 0.142 and 0.143, with an arrow to 0.142 labelled truncate and an arrow to 0.143 labelled round.

For a positive number:

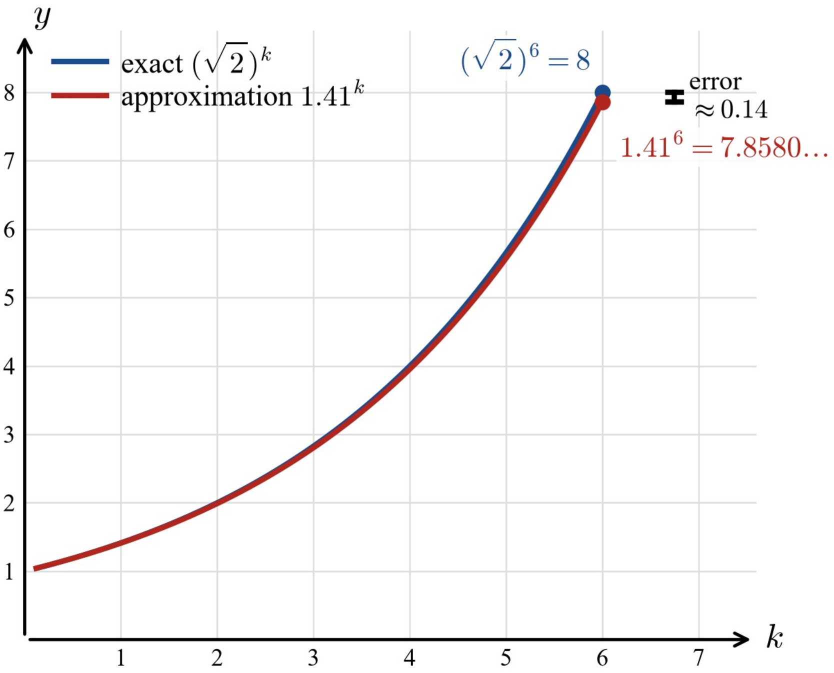
- truncating always gives a value **below** the exact value — the error is one-sided;
- rounding gives the closer value, since it picks the nearer of the two neighbouring n-place decimals.

The size of the error is bounded by the place you cut at. When a positive number is rounded to n decimal places, the absolute error is at most half a unit in the nth place; when it is truncated, the absolute error is less than one full unit in the nth place.

3. Repeated calculations carry the error forward

A single rounding error is small. The problem starts when a calculation is **repeated**.

- **Repeated addition.** Adding the same approximate value again and again adds its error again and again. Using 1.41 for $\sqrt{2}$ three times gives 4.23, while $3\sqrt{2}=4.2426\ldots$ — the error has grown to about 0.0126.
- **Repeated multiplication.** Raising an approximate value to a power makes the error grow faster still. $(\sqrt{2})^6=8$ exactly, but $1.41^6=7.8580\ldots$, an error of about 0.14.
- **Large counts.** A tiny error per item becomes a large error over millions of items. The error multiplies by the count.

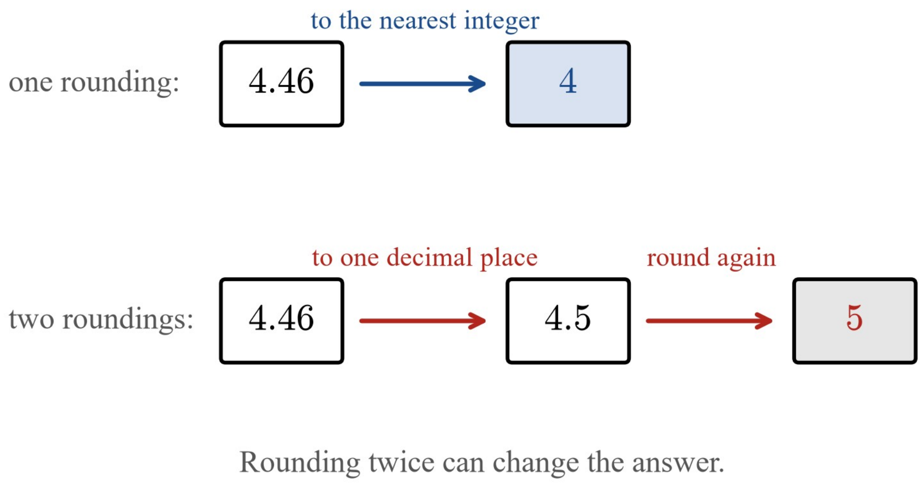


Graph against the power k, from 1 to 6, of the exact values of the square root of 2 to the power k and of the approximate values 1.41 to the power k; at k equals 6 the gap between 8 and 7.8580 continuing is marked as an error of about 0.14.

The deeper point is that **rounding early throws digits away permanently**. No later step can bring them back. Two people who start with the same shape, the same formulas and different rounding habits can finish with different final answers.

4. The habit this subtopic builds

- **Carry exact forms through.** Keep surds, multiples of π and fractions as they are. A **surd** is an exact value written with a root sign, such as $\sqrt{2}$ or $5\sqrt{3}$. On a calculator, use the stored value rather than a written-down approximation.
- **Round once, at the end**, and state the rounding you give — for example, “correct to one decimal place”.
- **When an exact value is asked for, leave it exact**: $49\pi\,\text{cm}^2$ or $5\sqrt{2}\,\text{cm}$. For an irrational number, no decimal is the exact value.
- **Never round a rounded value.** 4.46 is 4 to the nearest integer — but rounding it first to one decimal place gives 4.5, and 4.5 rounds to 5. Rounding twice can change the answer.



Two rows of value chips: 4.46 rounded once to the nearest integer gives 4, while 4.46 rounded to one decimal place gives 4.5 and rounding again gives 5.

Worked examples

Example 1 — Truncating versus rounding

For each number, truncate and round to the stated number of decimal places, find the absolute error of each, and say which is closer to the exact value:

(a) $\sqrt{5}$ to two decimal places;

(b) $\dfrac{1}{7}$ to three decimal places.

Working	Comment
(a) $\sqrt{5} = 2.2360\ldots$	Write more places than you need.
Truncate: 2.23.	Cut off after the second decimal place.
Round: 2.24.	The next digit is 6, so round up.
Truncation error: $2.2360\ldots - 2.23 = 0.0060\ldots$	The truncated value sits below the exact value.
Rounding error: $2.24 - 2.2360\ldots = 0.0039\ldots$	
$\therefore$ rounding is closer: $0.0039\ldots < 0.0060\ldots$	
(b) $\dfrac{1}{7} = 0.142857\ldots$	The decimal repeats.
Truncate: 0.142.	Cut off after the third decimal place.
Round: 0.143.	The next digit is 8, so round up.
Truncation error: $0.142857\ldots - 0.142 = 0.000857\ldots$	Again below the exact value.
Rounding error: $0.143 - 0.142857\ldots = 0.000143\ldots$	
$\therefore$ rounding is closer: $0.000143\ldots < 0.000857\ldots$	

In both cases truncation sits below the exact value and rounding gives the closer answer.

Example 2 — Rounding early, rounding late

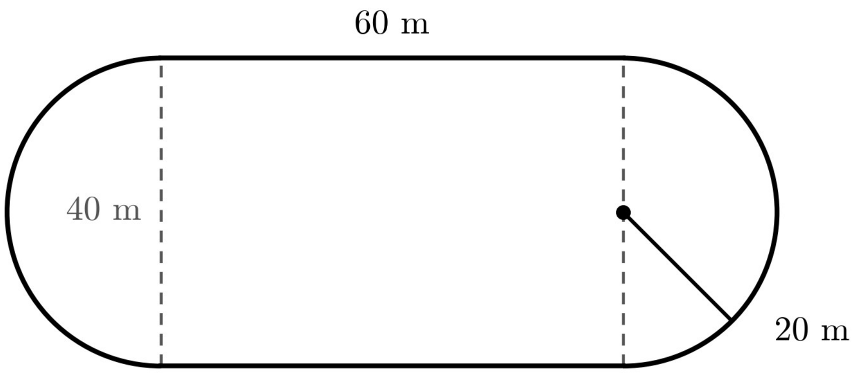
`SCENARIO` A running track is built from a rectangular section $60\,\mathrm{m}$ long and $40\,\mathrm{m}$ wide, with the two $40\,\mathrm{m}$ ends replaced by semicircles. The two semicircles together make one full circle of radius $20\,\mathrm{m}$. Find the perimeter and the area of the track, giving each

(a) exactly, in terms of π;

(b) correct to one decimal place, rounding only at the end;

(c) using $\pi \approx 3.14$ from the start.

Compare the final answers in (b) and (c).



The two semicircles together make one full circle of radius 20 m.

Running track made from two straight sides of 60 m and two semicircular ends; the width is 40 m and the radius of each semicircle is 20 m.

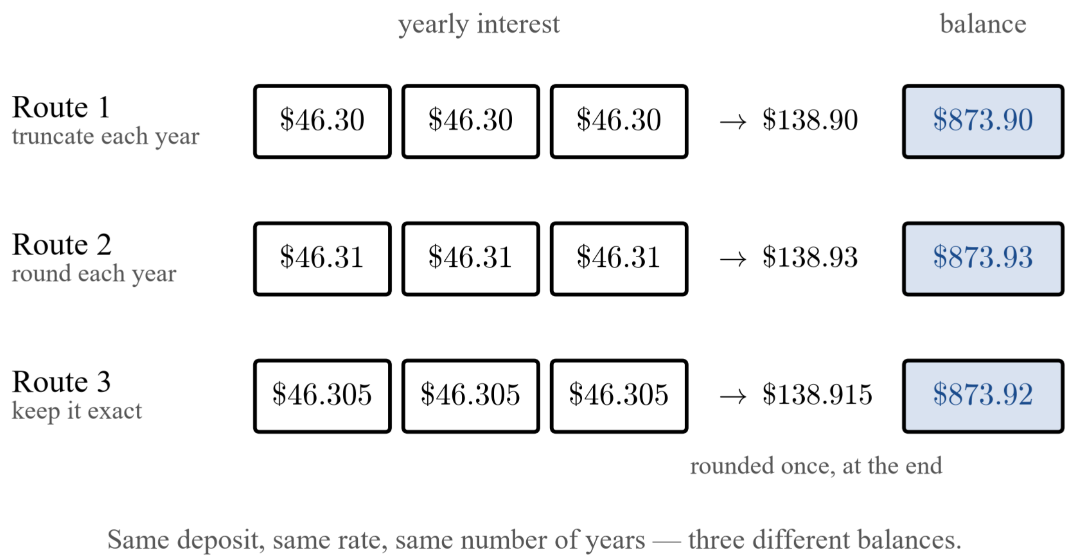
Working	Comment
Perimeter: two straights of $60\,\mathrm{m}$, plus a full circle of radius $20\,\mathrm{m}$.	Sketch the shape first.
$P = 2 \times 60 + 2\pi \times 20 = 120 + 40\pi\,\mathrm{m}$	(a) Exact perimeter.
Area: rectangle 60×40, plus a full circle of radius $20\,\mathrm{m}$.	
$A = 60 \times 40 + \pi \times 20^2 = 2400 + 400\pi\,\mathrm{m}^2$	(a) Exact area.
(b) $P = 120 + 40\pi = 245.66\ldots \approx 245.7\,\mathrm{m}$	Keep π exact; round once, at the end.
(b) $A = 2400 + 400\pi = 3656.63\ldots \approx 3656.6\,\mathrm{m}^2$	Same habit.
(c) Replace π with 3.14 before calculating.	The extra digits are thrown away here and never return.
(c) $P = 120 + 40 \times 3.14 = 120 + 125.6 = 245.6\,\mathrm{m}$	
(c) $A = 2400 + 400 \times 3.14 = 2400 + 1256 = 3656.0\,\mathrm{m}^2$	
$\therefore$ the final answers differ: $245.7\,\mathrm{m}$ against $245.6\,\mathrm{m}$, and $3656.6\,\mathrm{m}^2$ against $3656.0\,\mathrm{m}^2$.	Same shape, same formulas — different results, purely from when the rounding happened.

Example 3 — Interest that is not an exact number of cents

`SCENARIO` A savings account pays simple interest of $6.3\,\%$ per year. A customer deposits $\$735$ and leaves it untouched for three years. With simple interest, the same amount of interest is added each year. Compare three ways of recording the interest in cents.

Working	Comment
Interest for one year: $735 \times 0.063 = \$46.305$.	Not an exact number of cents.
Route 1 — truncate each year: $\$46.30$ per year.	Cut off the third decimal place.
Total: $3 \times 46.30 = \$138.90$.	
Balance: $735 + 138.90 = \$873.90$.	
Route 2 — round each year: $\$46.31$ per year.	Round half up.

Working	Comment
Total: $3 \times 46.31 = \$\,138.93$.	
Balance: $735 + 138.93 = \$\,873.93$.	
Route 3 — keep the exact value, round once at the end.	Carry $\$\,46.305$ through.
Total: $3 \times 46.305 = \$\,138.915 \approx \$\,138.92$.	Round only when the total is final.
Balance: $735 + 138.92 = \$\,873.92$.	
$\therefore$ the three balances are $\$\,873.90$, $\$\,873.92$ and $\$\,873.93$.	Same deposit, same rate, same number of years — three different answers, because of when the rounding happened.



Three rows of dollar chips for the three routes: 46.30 dollars each year totals 138.90 dollars and a balance of 873.90 dollars; 46.31 dollars each year totals 138.93 dollars and 873.93 dollars; the exact 46.305 dollars each year gives 138.915 dollars, rounded once at the end to 138.92 dollars and a balance of 873.92 dollars.

Practice questions

Scaffolded

Q1. (a) State the two possible forms of the decimal expansion of a rational number. (b) Classify each number as rational or irrational: $\sqrt{9}$; $\sqrt{7}$; π; $\frac{22}{7}$; 0.375; $0.375375\ldots$ (the block 375 repeats). (c) Write 0.375 as a fraction in lowest terms.

Q2. For each of $\sqrt{3}$, π, $\frac{5}{8}$ and $\sqrt{10}$: (a) state whether it is rational or irrational; (b) give its decimal value correct to two decimal places; (c) identify which of the four has an exact terminating decimal, and state it.

Q3. For each number, find (i) the truncated value and (ii) the rounded value at the stated number of decimal places: (a) π to three decimal places; (b) $\frac{5}{3}$ to two decimal places; (c) $\sqrt{2}$ to four decimal places. (d) In which case do truncating and rounding give the same result? Explain why.

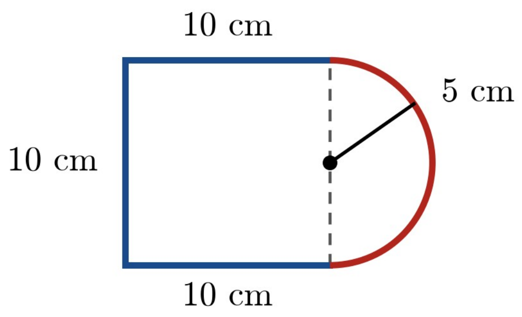
Q4. Three approximations to π are 3.14, $\frac{22}{7}$ and 3.142. For each one: (a) find the absolute error, correct to five decimal places; (b) find the percentage error, correct to four decimal places; (c) state which of the three is closest to π.

Q5. A circle has radius $7\,\text{cm}$. (a) Find the exact area of the circle, in terms of π. (b) Find the area correct to one decimal place, using the π key on your calculator. (c) A student uses $\pi \approx 3.14$ instead. Find the absolute error in the student's area, correct to three decimal places.

Q6. The value $\sqrt{2}$ is added to itself three times. (a) Write the exact total in surd form. (b) Write the exact total as a decimal, correct to four decimal places. (c) Using $\sqrt{2} \approx 1.41$ each time gives $3 \times 1.41 = 4.23$. Find the absolute error, correct to four decimal places.

Q7. `SCENARIO` A bank account pays simple interest of $6.2\,\%$ per year. A customer deposits $\$411$ for three years. (a) Show that the interest for one year is $\$25.482$, and explain why this is not an exact number of cents. (b) Find the total interest over three years if the yearly interest is rounded to the nearest cent first. (c) Find the total interest over three years if the exact yearly interest is used and the total is rounded to the nearest cent only at the end. (d) Which total is larger, and by how much?

Q8. A shape is made from a square of side $10\,\text{cm}$ with a semicircle attached to one side, so the semicircle has radius $5\,\text{cm}$. The perimeter is made of three sides of the square and the curved arc only.



The perimeter is three sides of the square and the arc only.

Shape made from a square of side 10 cm with a semicircle of radius 5 cm attached to one side; the perimeter is the three outer sides of the square and the arc only.

(a) Write the exact perimeter of the shape, in terms of π.

(b) Find the perimeter correct to one decimal place.

Unscaffolded

Q9. Consider $\sqrt{11}$. (a) Truncate it to three decimal places. (b) Round it to three decimal places. (c) Find the absolute error of each, correct to eight decimal places. (d) State which is closer to the exact value.

Q10. For $\frac{7}{11}$, find the truncated value and the rounded value to four decimal places, and find the absolute error of each, correct to eight decimal places.

Q11. Find the exact area of a circle of radius $3.5\,\text{cm}$, and then the area correct to two decimal places.

Q12. A running track has two straights of $100\,\text{m}$ and two semicircular ends of radius $30\,\text{m}$. (a) Write the exact perimeter of the track, in terms of π. (b) Find the perimeter correct to one decimal place, rounding only at the end. (c) Find the perimeter if $\pi \approx 3.14$ is used from the start. (d) Compare your answers to (b) and (c).

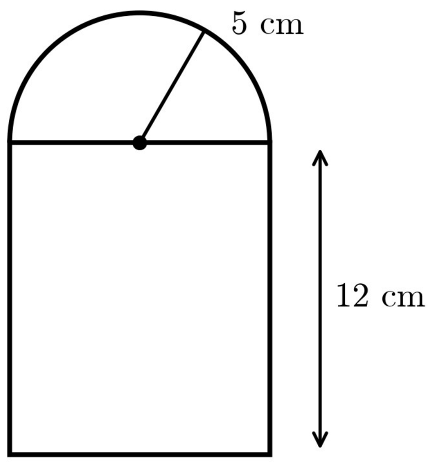
Q13. The areas of a circle of radius $5\,\text{m}$ and a circle of radius $50\,\text{m}$ are both calculated using $\pi \approx 3.14$. Without calculating each error separately, explain why the error for the larger circle is exactly 100 times the error for the smaller one.

Q14. (a) Evaluate $\left(\sqrt{3}\right)^4$ exactly. (b) Using $\sqrt{3} \approx 1.73$, evaluate 1.73^4, correct to four decimal places. (c) Find the absolute error, correct to four decimal places.

Q15. The exact value of $(\sqrt{2})^6$ is 8. (a) Evaluate 1.41^6, correct to four decimal places, and find the absolute error. (b) Evaluate 1.4142^6, correct to four decimal places, and find the absolute error. (c) What do the two errors tell you about the number of digits worth carrying through a repeated calculation?

Q16. Repeat the three routes of Example 3 ($\$735$ at 6.3% per year simple interest) for **six** years instead of three. Give the final balance under each route, and compare the spread between the highest and lowest balances with the spread after three years.

Q17. A capsule-shaped solid is made from a cylinder of radius $5\,\text{cm}$ and height $12\,\text{cm}$ with a hemisphere of radius $5\,\text{cm}$ on top.



A cylinder of height 12 cm with a hemisphere of radius 5 cm on top.

Capsule shape made from a cylinder of radius 5 cm and height 12 cm with a hemisphere of radius 5 cm on top.

(a) Show that the exact surface area is $195\pi\,\text{cm}^2$, and find it correct to one decimal place.

(b) Show that the exact volume is $\dfrac{1150\pi}{3}\,\text{cm}^3$, and find it correct to one decimal place.

(c) Using $\pi \approx 3.14$ from the start, estimate the volume and find the absolute error of the estimate, correct to two decimal places.

Q18. `SCENARIO` A surveyor measures the circumference of a circular lake of diameter $40\,\text{km}$ and records $125.6\,\text{km}$. (a) Show that the surveyor's calculation used $\pi \approx 3.14$. (b) Find the exact circumference, in terms of π, and as a decimal correct to two decimal places. (c) Find the absolute error in the surveyor's value, in km, and convert it to metres, correct to the nearest metre.

Q19. A student writes: "$\dfrac{22}{7}$ is a fraction, and $\pi = \dfrac{22}{7}$, so π is a rational number." Identify the error in this reasoning.

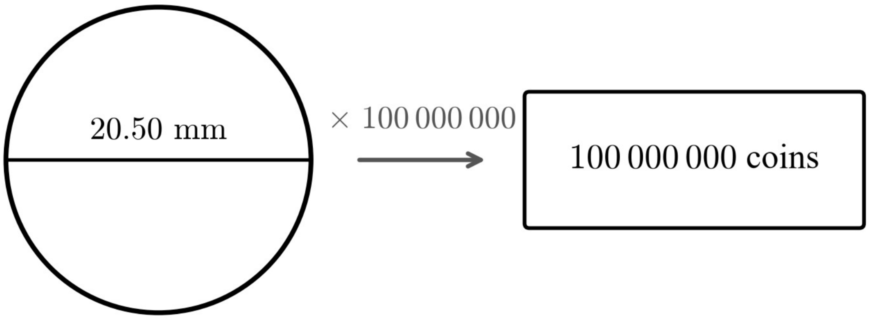
Q20. Evaluate each expression exactly. Leave your answers exact — no decimals. (a) $(2\sqrt{5})^2$ (b) $\sqrt{8} \times \sqrt{2}$ (c) $3\pi + 2\pi$

Q21. The area of a circle is $50\pi\,\text{cm}^2$. (a) Show that the radius is exactly $5\sqrt{2}\,\text{cm}$. (b) Write the exact circumference of the circle in terms of $\sqrt{2}$ and π. (c) Using $r \approx 7.07$ and $\pi \approx 3.14$, find the approximate circumference. (d) Find the absolute error of the approximation in (c), correct to four decimal places.

Q22. `SCENARIO` A factory cuts circular lids of radius $4\,\text{cm}$ from sheet metal, and an order is for $10\,000$ lids. (a) Find the exact total area of the $10\,000$ lids, in terms of π. (b) Find this total area correct to two decimal places. (c) An estimator uses $\pi \approx 3.14$ to find the total area. Find the estimate. (d) Is the estimate an underestimate or an overestimate, and by how much, correct to two decimal places?

Real context

Q23. The Australian two-dollar coin is circular, with a diameter of $20.50\,\text{mm}$. The Royal Australian Mint has produced more than 15 billion circulating coins since February 1966. Suppose one production run strikes $100\,000\,000$ two-dollar coins.



A tiny error per coin, repeated 100 000 000 times, becomes a large error.

Circular two-dollar coin face with diameter 20.50 mm, and a box for the 100 000 000 coins in the production run; a small error per coin becomes a large error when repeated.

(a) Find the exact face area of one coin, in terms of π.
(b) Find the face area using the π key on your calculator, correct to two decimal places.
(c) A designer estimates the face area using $\pi \approx 3.14$. Find this estimate, correct to two decimal places.
(d) Using the unrounded values from (b) and (c), find the total shortfall in face area across the whole run of $100\,000\,000$ coins, in square metres. Comment on the size of the shortfall.

Source: Royal Australian Mint, *Two Dollars*, 2025. Retrieved 19 August 2026. (royalaustralianmint.com.au/collect/national-coin-collection/two-dollars)

Source: Royal Australian Mint, *Circulating coins*, 2025. Retrieved 19 August 2026. (royalaustralianmint.com.au/circulating-coins)

Answers

Q1. (a) terminates or repeats; (b) $\sqrt{9}$ rational, $\sqrt{7}$ irrational, π irrational, $\frac{22}{7}$ rational, 0.375 rational, $0.375375\ldots$ rational; (c) $\frac{3}{8}$. **Q2.** (a) $\sqrt{3}$ irrational, π irrational, $\frac{5}{8}$ rational, $\sqrt{10}$ irrational; (b) 1.73, 3.14, 0.63, 3.16; (c) $\frac{5}{8}=0.625$ exactly. **Q3.** (a) 3.141, 3.142; (b) 1.66, 1.67; (c) 1.4142, 1.4142; (d) case (c), because the fifth decimal digit of $\sqrt{2}$ is 1, which is less than 5. **Q4.** (a) 0.00159, 0.00126, 0.00041; (b) $0.0507\,\%$, $0.0402\,\%$, $0.0130\,\%$; (c) 3.142. **Q5.** (a) $49\pi\,\mathrm{cm}^2$; (b) $153.9\,\mathrm{cm}^2$; (c) $0.078\,\mathrm{cm}^2$. **Q6.** (a) $3\sqrt{2}$; (b) 4.2426; (c) 0.0126. **Q7.** (b) $\$76.44$; (c) $\$76.45$; (d) (c) is larger by $\$0.01$. **Q8.** (a) $30+5\pi\,\mathrm{cm}$; (b) $45.7\,\mathrm{cm}$. **Q9.** (a) 3.316; (b) 3.317; (c) 0.00062479, 0.00037521; (d) the rounded value. **Q10.** 0.6363, 0.6364; errors 0.00006364, 0.00003636. **Q11.** $12.25\pi\,\mathrm{cm}^2$; $38.48\,\mathrm{cm}^2$. **Q12.** (a) $200+60\pi\,\mathrm{m}$; (b) $388.5\,\mathrm{m}$; (c) $388.4\,\mathrm{m}$; (d) different final answers from the same shape. **Q13.** The error is $(\pi-3.14)\times r^2$; since $50^2=100\times 5^2$, the error is 100 times as large. **Q14.** (a) 9; (b) 8.9575; (c) 0.0425. **Q15.** (a) 7.8580, error 0.1420; (b) 7.9995, error 0.0005; (c) carrying more digits through shrinks the final error sharply. **Q16.** Truncate: $\$1012.80$; round each year: $\$1012.86$; exact then round once: $\$1012.83$; the spread doubles from $\$0.03$ to $\$0.06$. **Q17.** (a) $612.6\,\mathrm{cm}^2$; (b) $1204.3\,\mathrm{cm}^3$; (c) $1203.67\,\mathrm{cm}^3$, error $0.61\,\mathrm{cm}^3$. **Q18.** (b) $40\pi\,\mathrm{km}$, $125.66\,\mathrm{km}$; (c) $0.0637\,\mathrm{km}$, about $64\,\mathrm{m}$. **Q19.** $\pi\neq\frac{22}{7}$; $\frac{22}{7}$ is only an approximation, with error about 0.00126, and π is irrational. **Q20.** (a) 20; (b) 4; (c) 5π. **Q21.** (b) $10\sqrt{2}\pi\,\mathrm{cm}$; (c) $44.3996\,\mathrm{cm}$; (d) $0.0292\,\mathrm{cm}$. **Q22.** (a) $160\,000\pi\,\mathrm{cm}^2$; (b) $502\,654.82\,\mathrm{cm}^2$; (c) $502\,400\,\mathrm{cm}^2$; (d) underestimate by $254.82\,\mathrm{cm}^2$. **Q23.** (a) $105.0625\pi\,\mathrm{mm}^2$; (b) $330.06\,\mathrm{mm}^2$; (c) $329.90\,\mathrm{mm}^2$; (d) about $16.7\,\mathrm{m}^2$.

Fully-worked solutions

Q1. (a) The decimal expansion of a rational number either terminates or repeats.

(b) $\sqrt{9}=3$, so it is rational. $\sqrt{7}$ is irrational — 7 is not a perfect square. π is irrational. $\frac{22}{7}$ is a fraction of two integers, so it is rational (it is an approximation to π, not π itself). 0.375 terminates, so it is rational. $0.375375\ldots$ repeats, so it is rational.

(c) $0.375=\frac{375}{1000}=\frac{3}{8}$.

Q2. (a) $\sqrt{3}$ is irrational, π is irrational, $\frac{5}{8}$ is rational, $\sqrt{10}$ is irrational.

(b) $\sqrt{3}=1.7320\ldots\approx 1.73$

$\pi=3.1415\ldots\approx 3.14$

$\frac{5}{8}=0.625\approx 0.63$ (the next digit is exactly 5, so round half up)

$\sqrt{10}=3.1622\ldots\approx 3.16$

(c) $\frac{5}{8}$ has the exact terminating decimal 0.625; the value 0.63 is an approximation.

Q3. (a) $\pi=3.14159\ldots$ Truncate to three decimal places: 3.141. Round: the fourth digit is 5, so round up: 3.142.

(b) $\frac{5}{3}=1.6666\ldots$ Truncate to two decimal places: 1.66. Round: the third digit is 6, so round up: 1.67.

(c) $\sqrt{2} = 1.41421\ldots$ Truncate to four decimal places: 1.4142. Round: the fifth digit is 1, so the fourth digit stays: 1.4142.

(d) In case (c) the two agree, because the digit after the fourth decimal place is $1 < 5$, so rounding keeps the fourth digit exactly as truncation does.

Q4. For each approximation, absolute error $= |\text{approximation} - \pi|$ and percentage error $= \dfrac{\text{absolute error}}{\pi} \times 100\,\%$.

For 3.14:

$|3.14 - \pi| = \pi - 3.14 = 0.00159265\ldots \approx 0.00159$

$\dfrac{0.00159265\ldots}{\pi} \times 100\,\% = 0.05069\ldots\% \approx 0.0507\,\%$

For $\dfrac{22}{7}$:

$|\dfrac{22}{7} - \pi| = \dfrac{22}{7} - \pi = 0.00126449\ldots \approx 0.00126$

$\dfrac{0.00126449\ldots}{\pi} \times 100\,\% = 0.04024\ldots\% \approx 0.0402\,\%$

For 3.142:

$|3.142 - \pi| = 3.142 - \pi = 0.00040735\ldots \approx 0.00041$

$\dfrac{0.00040735\ldots}{\pi} \times 100\,\% = 0.01296\ldots\% \approx 0.0130\,\%$

$\therefore$ 3.142 is closest to π, since it has the smallest error.

Q5. (a) $A = \pi r^2$

$A = \pi \times 7^2$

$A = 49\pi\,\text{cm}^2$

(b) $A = 49\pi = 153.9380\ldots$

$A \approx 153.9\,\text{cm}^2$ (correct to one decimal place)

(c) Student's area: $49 \times 3.14 = 153.86\,\text{cm}^2$

Error: $49\pi - 153.86 = 153.9380\ldots - 153.86 = 0.0780\ldots$

$\therefore$ the absolute error is about $0.078\,\text{cm}^2$.

Q6. (a) $\sqrt{2} + \sqrt{2} + \sqrt{2} = 3\sqrt{2}$

(b) $3\sqrt{2} = 4.24264\ldots \approx 4.2426$

(c) Error: $3\sqrt{2} - 4.23 = 4.24264\ldots - 4.23 = 0.01264\ldots$

$\therefore$ the absolute error is about 0.0126.

Q7. (a) Interest for one year $= 411 \times 0.062 = \$25.482$. This is not an exact number of cents, since it has three decimal places, and the third decimal place — two thousandths of a dollar — is smaller than one cent, so it cannot be recorded as a whole number of cents.

(b) Round each year: $\$25.48$ per year.

$3 \times 25.48 = \$76.44$

(c) Exact total: $3 \times 25.482 = \$76.446$

$\$76.446 \approx \$76.45$ (nearest cent)

(d) The total in (c) is larger, by $\$0.01$.

Q8. (a) Perimeter = three sides of the square + the semicircular arc.

Arc $= \frac{1}{2} \times 2\pi r = \pi \times 5 = 5\pi$

$P = 3 \times 10 + 5\pi = 30 + 5\pi\,\text{cm}$

(b) $P = 30 + 5\pi = 45.7079\ldots$

$P \approx 45.7\,\text{cm}$ (correct to one decimal place)

Q9. $\sqrt{11} = 3.31662479\ldots$

(a) Truncate to three decimal places: 3.316.

(b) Round to three decimal places: the fourth digit is 6, so 3.317.

(c) Truncation error: $3.31662479\ldots - 3.316 = 0.00062479$

Rounding error: $3.317 - 3.31662479\ldots = 0.00037521$

(d) The rounded value is closer, since $0.00037521 < 0.00062479$.

Q10. $\frac{7}{11} = 0.63636363\ldots$

Truncate to four decimal places: 0.6363.

Round to four decimal places: the fifth digit is 6, so 0.6364.

Truncation error: $0.63636363\ldots - 0.6363 = 0.00006364$ (to eight decimal places).

Rounding error: $0.6364 - 0.63636363\ldots = 0.00003636$ (to eight decimal places).

Q11. $A = \pi r^2$

$A = \pi \times 3.5^2$

$A = 12.25\pi\,\text{cm}^2$ exactly

$A = 12.25\pi = 38.4845\ldots \approx 38.48\,\text{cm}^2$ (correct to two decimal places)

Q12. (a) Perimeter = two straights + one full circle of radius $30\,\text{m}$.

$P = 2 \times 100 + 2\pi \times 30 = 200 + 60\pi\,\text{m}$

(b) $P = 200 + 60\pi = 388.4955\ldots \approx 388.5\,\text{m}$ (correct to one decimal place)

(c) $P = 200 + 60 \times 3.14 = 200 + 188.4 = 388.4\,\text{m}$

(d) The two final answers differ — $388.5\,\text{m}$ against $388.4\,\text{m}$ — even though the shape and the formula are the same. Rounding early cost $0.1\,\text{m}$ in the final answer.

Q13. For a circle of radius r, the calculated area is $3.14r^2$ and the exact area is πr^2, so the error is $(\pi - 3.14)r^2$.

For $r = 5$: error $= (\pi - 3.14) \times 5^2 = 25(\pi - 3.14)$.

For $r = 50$: error $= (\pi - 3.14) \times 50^2 = 2500(\pi - 3.14)$.

$2500 = 100 \times 25$, so the error for the larger circle is exactly 100 times the error for the smaller one. The factor $\pi - 3.14$ is the same each time; it is r^2 that scales the error.

Q14. (a) $(\sqrt{3})^4 = ((\sqrt{3})^2)^2 = 3^2 = 9$

(b) $1.73^2 = 2.9929$

$1.73^4 = 2.9929^2 = 8.95745041 \approx 8.9575$

(c) Error: $9 - 8.95745041 = 0.04254959 \approx 0.0425$

$\therefore$ rounding $\sqrt{3}$ to two decimal places before taking the fourth power costs about 0.0425 in the final answer.

Q15. (a) $1.41^6 = 7.85804797\ldots \approx 7.8580$

Error: $8 - 7.85804797\ldots = 0.14195\ldots \approx 0.1420$

(b) $1.4142^6 = 7.99953969\ldots \approx 7.9995$

Error: $8 - 7.99953969\ldots = 0.00046031\ldots \approx 0.0005$

(c) Two extra digits carried through shrink the final error from about 0.14 to about 0.0005 — a factor of about 300. In a repeated calculation, the digits you carry are worth keeping.

Q16. Interest for one year: $\$46.305$ (as in Example 3).

Route 1 — truncate each year: $6 \times 46.30 = \$277.80$

Balance: $735 + 277.80 = \$1012.80$

Route 2 — round each year: $6 \times 46.31 = \$277.86$

Balance: $735 + 277.86 = \$1012.86$

Route 3 — exact, round once: $6 \times 46.305 = \$277.83$

Balance: $735 + 277.83 = \$1012.83$

$\therefore$ the balances are $\$1012.80$, $\$1012.83$ and $\$1012.86$. The spread after six years is $\$0.06$, double the spread of $\$0.03$ after three years: the more times the calculation repeats, the further apart the routes drift.

Q17. (a) Surface area = curved surface of the cylinder + base of the cylinder + curved surface of the hemisphere.

$= 2\pi r h + \pi r^2 + 2\pi r^2$

$= 2\pi \times 5 \times 12 + \pi \times 5^2 + 2\pi \times 5^2$

$= 120\pi + 25\pi + 50\pi$

$= 195\pi \text{ cm}^2$

$195\pi = 612.6105\ldots \approx 612.6 \text{ cm}^2$ (correct to one decimal place)

(b) Volume = volume of the cylinder + volume of the hemisphere.

$= \pi r^2 h + \frac{2}{3}\pi r^3$

$=\pi\times5^2\times12+\frac{2}{3}\pi\times5^3$

$=300\pi+\frac{250\pi}{3}$

$=\frac{900\pi+250\pi}{3}=\frac{1150\pi}{3}\,\text{cm}^3$

$\frac{1150\pi}{3}=1204.2771\ldots\approx1204.3\,\text{cm}^3$ (correct to one decimal place)

(c) Estimate: $3.14\times300+\frac{2}{3}\times3.14\times125=942+261.6666\ldots=1203.6666\ldots$

Exact value: $\frac{1150\pi}{3}=1204.2771\ldots$

Error: $1204.2771\ldots-1203.6666\ldots=0.6105\ldots\approx0.61\,\text{cm}^3$

Q18. (a) Circumference $=\pi d$, so the value of π used is $125.6\div40=3.14$.

(b) Exact circumference: $C=40\pi\,\text{km}$

$C=40\pi=125.6637\ldots\approx125.66\,\text{km}$ (correct to two decimal places)

(c) Error: $40\pi-125.6=125.6637\ldots-125.6=0.0637\ldots\,\text{km}$

$0.0637\ldots\,\text{km}=63.7\ldots\,\text{m}\approx64\,\text{m}$

$\therefore$ the surveyor's value is short by about $0.0637\,\text{km}$, or about $64\,\text{m}$.

Q19. The claim $\pi=\frac{22}{7}$ is false. $\frac{22}{7}$ is a rational number — a fraction of two integers — but it is only an approximation to π, with absolute error about 0.00126. The number π itself is irrational: it cannot be written as a fraction of integers, and its decimal expansion never terminates and never repeats. An approximation being rational does not make the value it approximates rational.

Q20. (a) $(2\sqrt{5})^2=2^2\times(\sqrt{5})^2=4\times5=20$

(b) $\sqrt{8}\times\sqrt{2}=\sqrt{16}=4$

(c) $3\pi+2\pi=5\pi$

Q21. (a) $\pi r^2=50\pi$

$r^2=50$

$r=\sqrt{50}=\sqrt{25\times2}=5\sqrt{2}\,\text{cm}$

(b) $C=2\pi r=2\pi\times5\sqrt{2}=10\sqrt{2}\pi\,\text{cm}$

(c) $C\approx2\times3.14\times7.07=44.3996\,\text{cm}$

(d) Exact: $10\sqrt{2}\pi=44.4288\ldots\,\text{cm}$

Error: $44.4288\ldots-44.3996=0.0292\ldots$

$\therefore$ the absolute error is about $0.0292\,\text{cm}$.

Q22. (a) Area of one lid: $\pi\times4^2=16\pi\,\text{cm}^2$

Total: $10\,000 \times 16\pi = 160\,000\pi\,\text{cm}^2$

(b) $160\,000\pi = 502\,654.8245\ldots \approx 502\,654.82\,\text{cm}^2$

(c) Estimate: $10\,000 \times 16 \times 3.14 = 502\,400\,\text{cm}^2$

(d) $3.14 < \pi$, so the estimate sits below the true value: it is an underestimate.

Shortfall: $160\,000\pi - 502\,400 = 502\,654.8245\ldots - 502\,400 = 254.8245\ldots \approx 254.82\,\text{cm}^2$

Q23. (a) Radius: $20.50 \div 2 = 10.25\,\text{mm}$

$A = \pi \times 10.25^2 = 105.0625\pi\,\text{mm}^2$

(b) $A = 105.0625\pi = 330.0635\ldots \approx 330.06\,\text{mm}^2$ (correct to two decimal places)

(c) Estimate: $105.0625 \times 3.14 = 329.89625 \approx 329.90\,\text{mm}^2$ (correct to two decimal places)

(d) Shortfall per coin: $105.0625\pi - 329.89625 = 0.16732\ldots\,\text{mm}^2$

Across $100\,000\,000$ coins: $0.16732\ldots \times 100\,000\,000 = 16\,732\,816.7\ldots\,\text{mm}^2$

$16\,732\,816.7\ldots\,\text{mm}^2 = 16.7328\ldots\,\text{m}^2 \approx 16.7\,\text{m}^2$

$\therefore$ the shortfall is about $16.7\,\text{m}^2$ — the area of a small room. An error of less than $0.2\,\text{mm}^2$ per coin becomes a square-metre-scale error once it is repeated 100 million times.

Teacher note

Level 9 codes rehearsed (D3): `VC2M9N01` — the real number system and irrationals (Q1, Q2, Q19); `VC2M9M04` — absolute, relative and percentage error (Q4, and every error calculation in the subtopic); `VC2M9A06` — modelling change including simple interest (Example 3, Q7, Q16). All three are rehearsed, not re-taught, and are not assessed as Level 9 outcomes.

Tool classes (D13): `VC2M10N01` and both its elaborations name no digital tool, so no `DIGITAL TOOL` banner appears in this subtopic (R8). A scientific calculator is assumed for the decimal arithmetic; the exact surd and π forms — the point of the subtopic — need no tool at all.

VCE destinations (§5, reference only): the Units 3 & 4 skill indexes of Foundation Mathematics (AoS 1 'Algebra, number and structure'), General Mathematics (assumed knowledge) and Mathematical Methods (AoS 2 'Algebra, number and structure') all name `VC2M10N01`, with the skill *"Investigate how approximation errors compound across multiple calculations"*. Specialist Mathematics Unit 1 'Algebra, number and structure' proves irrationality and requires exact answers in surd or π form (inferred) — 1A is where that discipline is built.

Level check (D6): interest here is simple interest only, with the same amount added each year; the formula for compound interest is not built (that is 3C, `VC2M10A15`). No function notation, no logarithms, no VCE content. Surd work uses only the Year 9 exponent laws already assumed (`VC2M9A01`).

Common factors, and algebraic products and quotients under the exponent laws

Mathematics 10 · Chapter 1 Number and algebraic manipulation · Subtopic 1B · Extended block

Strand	Algebra
Content description	`VC2M10A01`, `VC2M10A02`
Elaboration ids carried	`VC2M10A01.E01` `VC2M10A01.E02` `VC2M10A02.E01`
Weight	Extended (double allocation)

> **Content description (verbatim):** *factorise algebraic expressions by taking out a common algebraic factor* (`VC2M10A01`)
>
> **Content description (verbatim):** *simplify algebraic products and quotients using exponent laws* (`VC2M10A02`)
>
> **Achievement standard (extract):** *Students substitute into formulas, find unknown values, manipulate linear and quadratic algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital tools.*

You need first: the exponent laws from Year 9 (`VC2M9A01`), and expanding and factorising simple algebraic expressions (`VC2M9A02`). This subtopic rehearses those skills and builds on them; it does not re-teach them.

Theory

This subtopic trains two moves that undo each other, and two ways of handling products and quotients. **Factorising** takes a common factor out of an expression; **expanding** puts it back. And the **exponent laws**, which you met with numerical bases in Year 9, apply just as well when the base is a pronumeral. Every result used below is stated here before it is used.

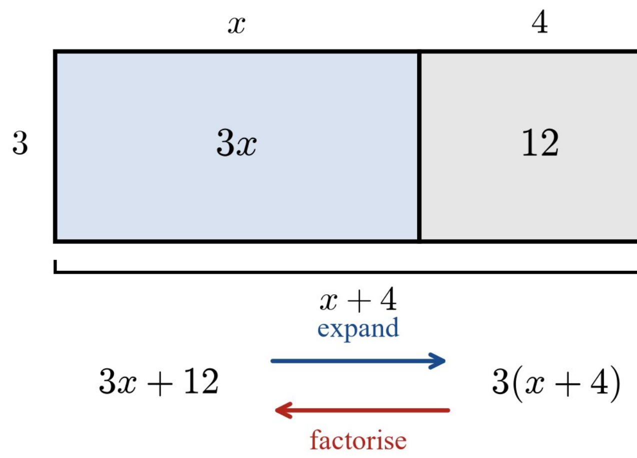
1. Expanding and factorising are opposite moves

The **distributive law** says that for any numbers or pronumerals a, b and c:

$$a(b+c)=ab+ac$$

Read left to right, the law **expands** a product into a sum of terms. Read right to left, it **factorises** a sum of terms into a product. To factorise an expression means to write it as a product of its factors; to expand means to write a product as a sum of single terms.

The area model makes the two directions visible. The rectangle below has height 3 and total width $x+4$, split into two parts.



Rectangle of height 3 split into two parts of width x and 4 with areas 3x and 12; arrows show expanding 3x + 12 into 3(x + 4) and factorising 3(x + 4) back into 3x + 12.

Adding the two parts gives the total area: $3x+12$. Taking height times total width gives the same area: $3(x+4)$. Since both count the same region,

$$3x+12=3(x+4)$$

which is exactly the distributive law with $a=3$, $b=x$ and $c=4$.

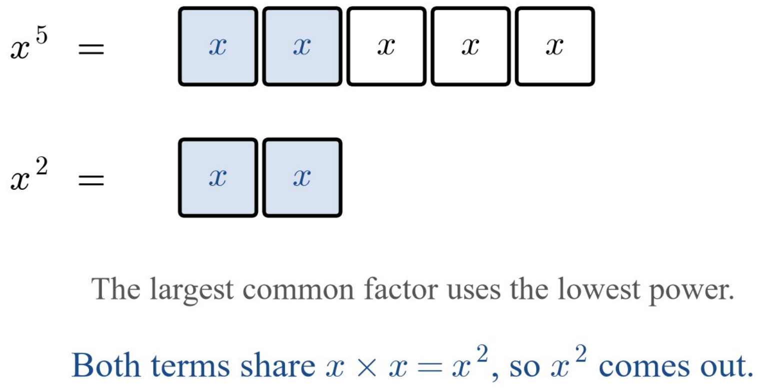
Because the two moves undo each other, **every factorisation can be checked by expanding** the bracketed form and comparing with the original expression. If the expansion does not return the original, the factorisation is wrong. This is the relationship between factorisation and expansion named in `VC2M10A01.E02`, and it is the checking habit this subtopic builds.

2. The highest common factor of two or more terms

To factorise, take out the **largest** factor that every term shares — the **highest common factor** of the terms. Find it in two parts:

- **Coefficients:** the largest whole number that divides every coefficient.
- **Pronumerals:** every pronumeral that appears in **every** term, taken at the **lowest power** that appears in any term.

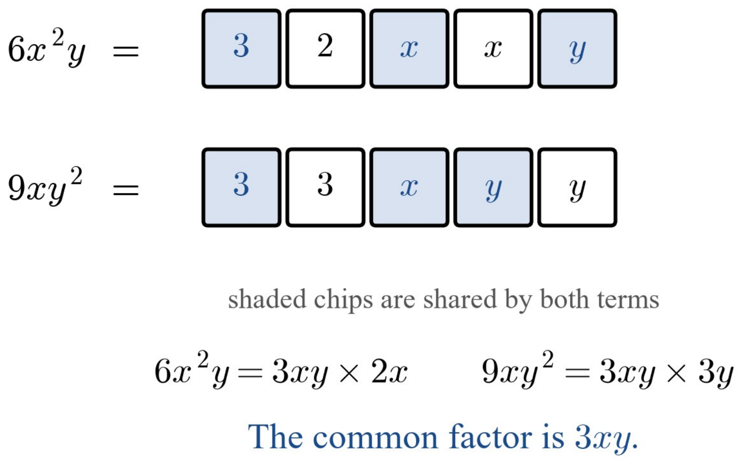
The lowest-power rule has a simple reason. A factor can only be taken out if every term can supply it. The two bars below write x^5 and x^2 as repeated factors of x.



Chips showing x to the power 5 as five factors of x and x squared as two factors of x, with the two shared factors of x shaded in both bars.

Both terms can supply $x \times x = x^2$, and the shorter term has nothing more to give. The extra three factors of x in x^5 stay inside the bracket. So the largest common factor of x^5 and x^2 is x^2 — the lowest power. A pronumeral that is missing from any term supplies nothing, and contributes no factor at all.

The same idea with coefficients and two pronumerals at once:



Factor chips of 6x squared y as 3, 2, x, x, y and of 9xy squared as 3, 3, x, y, y, with the shared chips 3, x and y shaded; the common factor is 3xy.

The shaded chips — one 3, one x, one y — are all the two terms share, so the highest common factor is $3 \times x \times y = 3xy$. Dividing each term by $3xy$ leaves what goes inside the bracket:

$$6x^2y + 9xy^2 = 3xy(2x + 3y)$$

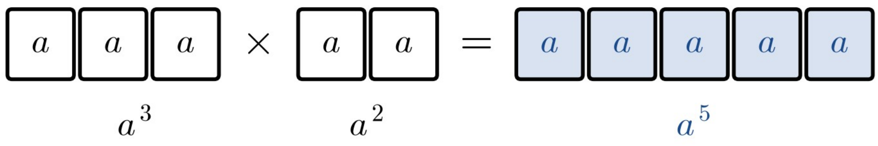
That division is itself an application of the exponent laws (`VC2M10A01.E01`): $6x^2y \div 3xy = 2x$, because $6 \div 3 = 2$, $x^2 \div x = x$ and $y \div y = 1$; likewise $9xy^2 \div 3xy = 3y$. Factorising by a common algebraic factor and the exponent laws are one connected skill, and this subtopic treats them that way.

3. The exponent laws, recalled from Year 9

For a nonzero base a and whole numbers m and n, the three laws you used with numbers in Year 9 (`VC2M9A01`) are:

- **Product law:** $a^m \times a^n = a^{m+n}$

$$a^3 \times a^2 = (a \times a \times a) \times (a \times a) = a^5$$

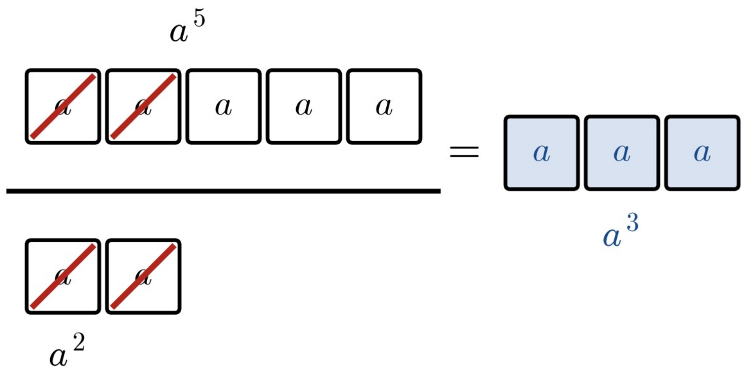


Count the factors: $3 + 2 = 5$, so the exponents add.

Three chips labelled a multiplied by two chips labelled a equal five chips labelled a, showing a cubed times a squared equals a to the power 5.

Counting the factors of a shows why the exponents add: three factors times two more factors is five factors in all.

- **Quotient law:** $a^m \div a^n = a^{m-n}$

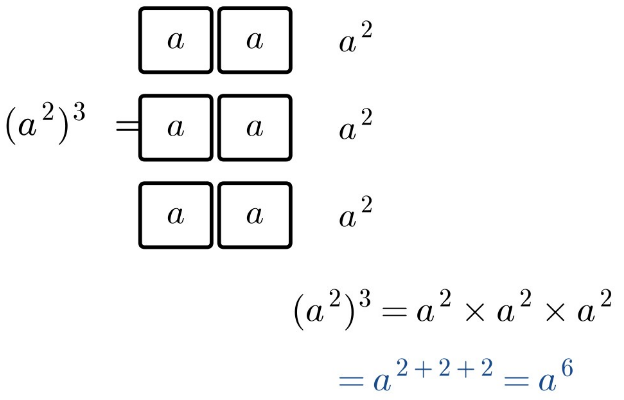


$\dfrac{a^5}{a^2} = a^{5-2} = a^3$ — cancel two a's from the top and the bottom.

Five chips labelled a above a fraction bar and two chips labelled a below it, with two matching pairs crossed out, leaving three chips labelled a, so a to the power 5 divided by a squared equals a cubed.

Every a below cancels one a above, so the exponents subtract.

- **Power of a power:** $(a^m)^n = a^{mn}$



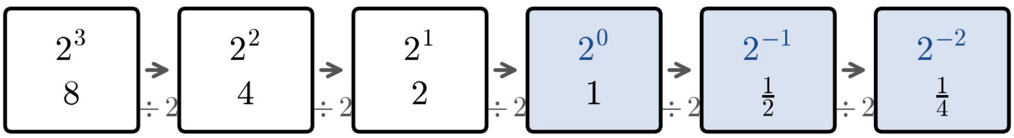
Three rows of two chips labelled a, each row labelled a squared, showing that (a squared) cubed is a squared times a squared times a squared, equal to a to the power 6.

Raising a^2 to the power 3 means three copies of a^2 multiplied together — six factors of a — so the exponents multiply.

Each law applies **only to the same base**. In a product or quotient with several pronumerals, handle the numerical coefficients separately and apply the laws to each base on its own.

4. Zero and negative exponents

Follow a descending chain of powers of 2, dividing by 2 at every step:



Each step divides by 2 — the pattern keeps going past $2^0 = 1$ into negative exponents.

Chain of boxes from 2 cubed equals 8 to 2 squared equals 4, 2 to the 1 equals 2, 2 to the 0 equals 1, 2 to the power minus 1 equals one half and 2 to the power minus 2 equals one quarter, with an arrow labelled divided by 2 between each pair.

The pattern does not stop at 2^0. Dividing 1 by 2 gives $\frac{1}{2}$, and dividing again gives $\frac{1}{4}$. For the quotient law to keep working, the exponents must keep subtracting too, which forces the meanings:

- $a^0 = 1$ for any nonzero a, because $a^m \div a^m = a^{m-m} = a^0$ on one hand, and plainly equals 1 on the other;
- $a^{-n} = \frac{1}{a^n}$ — a **negative exponent** means the reciprocal of the positive power, and, reading the rule the other way, $\frac{1}{a^n} = a^{-n}$.

So a quotient such as $\frac{a^2}{a^5}$ is $a^{2-5}=a^{-3}=\frac{1}{a^3}$. An expression is written with **positive exponents** when no negative exponent remains in the answer. The two forms, a^{-3} and $\frac{1}{a^3}$, are the same number written two ways (`VC2M10A02.E01`).

5. Products and quotients of algebraic terms

To simplify an algebraic **product**, set it out in one fixed order: numerical coefficients multiplied first, then each pronumeral base handled with the product law. For example,

$$(3x^2y)(4xy^5)=3\times4\times x^{2+1}\times y^{1+5}=12x^3y^6$$

To simplify an algebraic **quotient**, divide the coefficients first, then apply the quotient law to each base. For example,

$$\frac{10a^5b^3}{2a^2b^7}=\frac{10}{2}\times a^{5-2}\times b^{3-7}=5a^3b^{-4}=\frac{5a^3}{b^4}$$

Unless a question says otherwise, give answers with positive exponents in this subtopic.

Worked examples

Example 1 — Factorising with a numerical common factor

Factorise, then check each answer by expanding:

(a) $12x+18$;

(b) $14y-21$.

Working	Comment
(a) Coefficients 12 and 18: the HCF is 6.	Start with the numbers.
$12x+18=6(2x+3)$	$12x\div6=2x$ and $18\div6=3$.
Check: $6(2x+3)=12x+18$.	Expanding returns the original.
(b) Coefficients 14 and 21: the HCF is 7.	
$14y-21=7(2y-3)$	$14y\div7=2y$ and $-21\div7=-3$.
Check: $7(2y-3)=14y-21$.	The minus sign is carried into the bracket.
$\therefore$ $12x+18=6(2x+3)$ and $14y-21=7(2y-3)$, both checked.	

Example 2 — A common factor that includes a power of a pronumeral

Factorise $9x^4-6x^2$.

Working	Comment
Coefficients 9 and 6: the HCF is 3.	Numbers first.
Both terms contain x; the lowest power is x^2.	The extra factors of x stay inside the bracket.
Common factor: $3x^2$.	
$9x^4\div3x^2=3x^2$ and $6x^2\div3x^2=2$.	Divide each term by the common factor.
$9x^4-6x^2=3x^2(3x^2-2)$	
Check: $3x^2(3x^2-2)=9x^4-6x^2$.	Expand to verify.
$\therefore$ $9x^4-6x^2=3x^2(3x^2-2)$.	

Example 3 — A common factor in two pronumerals

Factorise $15a^3b+10a^2b^2$, and check by expanding.

Working	Comment
Coefficients 15 and 10: the HCF is 5.	
a appears in both terms: lowest power a^2.	
b appears in both terms: lowest power b.	
Common factor: $5a^2b$.	
$15a^3b\div 5a^2b=3a$ and $10a^2b^2\div 5a^2b=2b$.	
$15a^3b+10a^2b^2=5a^2b(3a+2b)$	
Check: $5a^2b(3a+2b)=15a^3b+10a^2b^2$.	Expand to verify.
$\therefore\ 15a^3b+10a^2b^2=5a^2b(3a+2b)$, checked.	

Example 4 — A product of two algebraic terms

Simplify $(4x^2y^3)(5x^4y)$.

Working	Comment
$(4x^2y^3)(5x^4y)$	
$=4\times 5\times x^2\times x^4\times y^3\times y$	Coefficients together, like bases together.
$=20\times x^{2+4}\times y^{3+1}$	Product law on each base; $y=y^1$.
$=20x^6y^4$	
$\therefore\ (4x^2y^3)(5x^4y)=20x^6y^4$.	

Example 5 — A quotient, with a negative exponent along the way

Simplify $\dfrac{24a^5b^2}{6a^2b^6}$, giving the answer with negative and with positive exponents.

Working	Comment
$\dfrac{24a^5b^2}{6a^2b^6}$	
$=\dfrac{24}{6}\times a^{5-2}\times b^{2-6}$	Quotient law on each base.
$=4a^3b^{-4}$	Negative-exponent form.
$=\dfrac{4a^3}{b^4}$	$b^{-4}=\dfrac{1}{b^4}$: positive-exponent form.
$\therefore\ 4a^3b^{-4}=\dfrac{4a^3}{b^4}$ — the same value, two forms.	

Example 6 — Several laws in one expression

Simplify $\dfrac{(2a^3)^4\times a^{-7}}{a^5}$.

Working	Comment
$(2a^3)^4=2^4\times a^{3\times 4}=16a^{12}$	Power of a power; the coefficient is raised too.
$16a^{12}\times a^{-7}=16a^{12+(-7)}=16a^5$	Product law.

Working	Comment
$16a^5 \div a^5 = 16a^{5-5} = 16a^0$	Quotient law.
$16a^0 = 16 \times 1 = 16$	$a^0 = 1$ for nonzero a.
$\therefore$ the expression equals 16 for every nonzero value of a.	The pronumeral disappears entirely.

Practice questions

Scaffolded

Q1. Copy and complete each factorisation by filling the bracket: (a) $8x + 12 = 4()$ (b) $15y + 10 = 5()$ (c) $9m - 6 = 3()$

Q2. Factorise, taking out the highest common factor: (a) $6x + 9$ (b) $20y - 15$ (c) $14a + 21b$ (d) Check your answer to (a) by expanding it.

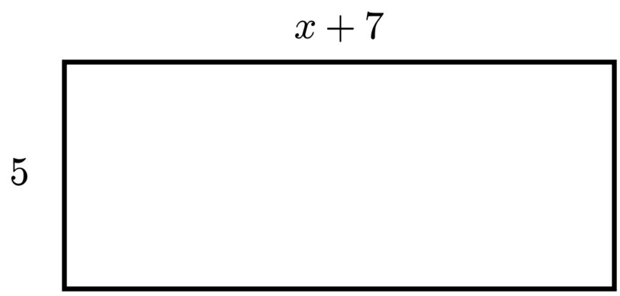
Q3. Factorise: (a) $x^3 + x^2$ (b) $5y^4 - 10y^2$ (c) $8m^5 + 12m^3$

Q4. Simplify each product: (a) $x^2 \times x^5$ (b) $3a \times 7a^3$ (c) $(2m^2 n)(5mn^4)$ (d) $(y^3)^4$

Q5. Simplify each quotient: (a) $a^9 \div a^4$ (b) $\dfrac{12x^6}{3x^2}$ (c) $\dfrac{20p^7q^3}{5p^2q^3}$ (d) $b^8 \div b^8$

Q6. Write each expression with positive exponents: (a) x^{-3} (b) $5a^{-2}$ (c) $\dfrac{7}{m^{-4}}$ (d) $n^4 \times n^{-6}$

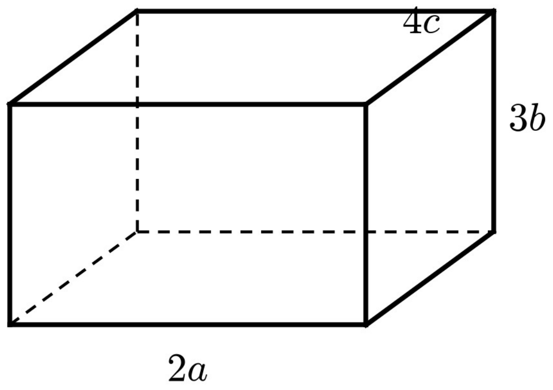
Q7. A rectangle has width 5 and length $x + 7$.



Rectangle with width 5 and length x + 7.

(a) Write the area of the rectangle as a product.
(b) Expand your answer to write the area as a sum of two terms.

Q8. A box has edges of length $2a$, $3b$ and $4c$.



Box drawn in oblique projection with edges labelled 2a along the front, 3b for the height and 4c for the depth; hidden edges are dashed.

(a) Write the volume of the box as a product, and simplify it to a single term.
(b) Every edge of the box is doubled. Write the new volume as a single term.
(c) How many times larger is the new volume than the original? Show the quotient that gives your answer.

Unscaffolded

Q9. Factorise $18x^3+24x^2-6x$.

Q10. Factorise $12a^2b^3-8a^3b^2+20a^2b^2$.

Q11. Factorise: (a) $24x^4+16x^3$ (b) $35m^5n-14m^3n^2$ (c) $9p^6-12p^4+15p^2$

Q12. A student factorises $6x^2+9x$ as $3x(2x+9)$. (a) Expand $3x(2x+9)$ and explain why the student's factorisation is wrong. (b) Write the correct factorisation of $6x^2+9x$.

Q13. Simplify $(4a^2b)^3$.

Q14. Simplify $5x^4\times 2x^{-6}$, giving your answer with positive exponents.

Q15. Simplify $\frac{18m^3}{6m^7}$, giving your answer with positive exponents.

Q16. Simplify $\frac{(2x^3)^2\times x^{-4}}{x}$.

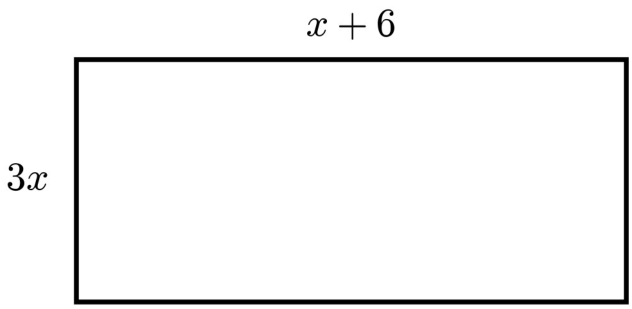
Q17. Simplify $\frac{(6a^4b^{-2})^2}{3a^3b^{-5}}$, giving your answer with positive exponents.

Q18. The same moves work on numbers alone. (a) Show that $2^{15}+2^{12}=2^{12}\times 9$, and hence evaluate $2^{15}+2^{12}$. (b) Write $5^{30}-5^{28}$ as $k\times 5^{28}$ for some whole number k, and state k.

Q19. A cube has an edge of length $4x^2$. (a) Show that the volume of the cube is $64x^6$. (b) Show that the total surface area of the cube is $96x^4$.

Q20. Suppose $3^p=10$. Without finding p, evaluate: (a) 3^{2p} (b) 3^{p+1} (c) 3^{p-2}

Q21. A rectangle has width $3x$ and length $x+6$.



Rectangle with width 3x and length x + 6.

(a) Write the area of the rectangle as a product.
(b) Expand your answer to (a).
(c) Substitute $x = 4$ into both forms of the area and confirm they agree.

Q22. Copy and complete each statement by filling the missing number: (a) $x^5 \times x^{\square} = x^{12}$ (b) $(a^4)^{\square} = a^{20}$ (c) $y^{10} \div y^{\square} = y^4$ (d) $(2m^2)^3 = \square m^6$

Real context

Q23. The Thomson Reservoir, in the catchment of the Thomson River east of Melbourne, is the largest storage in Victoria's water supply system. Its catchment has an area of $48\,700$ hectares, the average yearly rainfall over the catchment is $1\,021$ mm, and the reservoir's full storage is $1\,068\,000$ ML. Since 1997 the reservoir has received, on average, about 182 billion litres of inflow each year.

(a) Show that 1 mm of rain falling on 1 hectare of ground is a volume of 10^4 L. (Use 1 hectare = $10^4\,\text{m}^2$, $1\,\text{mm} = 10^{-3}\,\text{m}$ and $1\,\text{m}^3 = 10^3$ L.)
(b) Write the catchment area as 4.87×10^4 hectares and the rainfall as 1.021×10^3 mm. Hence find the total volume of rain falling on the catchment in a year, in litres, correct to three significant figures.
(c) The full storage is $1\,068\,000$ ML. Write this in litres, and find the ratio of the yearly rainfall total from (b) to the full storage, correct to two decimal places. Comment on the size of the ratio.
(d) The actual average inflow, about 1.82×10^{11} L per year, is well below the rainfall total from (b). Give a reason why not all of the rain reaches the reservoir.

Source: Melbourne Water, *Thomson Reservoir*, 2026. Retrieved 20 August 2026. (melbournewater.com.au/water-and-environment/water-management/water-storage-reservoirs/thomson-reservoir)

Answers

Q1. (a) $4(2x+3)$; (b) $5(3y+2)$; (c) $3(3m-2)$. **Q2.** (a) $3(2x+3)$; (b) $5(4y-3)$; (c) $7(2a+3b)$; (d) $3(2x+3)=6x+9$. **Q3.** (a) $x^2(x+1)$; (b) $5y^2(y^2-2)$; (c) $4m^3(2m^2+3)$. **Q4.** (a) x^7; (b) $21a^4$; (c) $10m^3n^5$; (d) y^{12}. **Q5.** (a) a^5; (b) $4x^4$; (c) $4p^5$; (d) 1. **Q6.** (a) $\frac{1}{x^3}$; (b) $\frac{5}{a^2}$; (c) $7m^4$; (d) $\frac{1}{n^2}$. **Q7.** (a) $5(x+7)$; (b) $5x+35$. **Q8.** (a) $(2a)(3b)(4c)=24abc$; (b) $(4a)(6b)(8c)=192abc$; (c) 8 times, since $192abc \div 24abc=8$. **Q9.** $6x(3x^2+4x-1)$. **Q10.** $4a^2b^2(3b-2a+5)$. **Q11.** (a) $8x^3(3x+2)$; (b) $7m^3n(5m^2-2n)$; (c) $3p^2(3p^4-4p^2+5)$. **Q12.** (a) $3x(2x+9)=6x^2+27x \neq 6x^2+9x$; (b) $3x(2x+3)$. **Q13.** $64a^6b^3$. **Q14.** $\frac{10}{x^2}$. **Q15.** $\frac{3}{m^4}$. **Q16.** $4x$. **Q17.** $12a^5b$. **Q18.** (a) $2^{12}\times 9=36\,864$; (b) $k=24$. **Q19.** (a) $(4x^2)^3=64x^6$; (b) $6\times(4x^2)^2=96x^4$. **Q20.** (a) 100; (b) 30; (c) $\frac{10}{9}$. **Q21.** (a) $3x(x+6)$; (b) $3x^2+18x$; (c) both give 120. **Q22.** (a) 7; (b) 5; (c) 6; (d) 8. **Q23.** (a) $10^4\times 10^{-3}=10$ m³, and $10\times 10^3=10^4$ L; (b) 4.97×10^{11} L; (c) 1.068×10^{12} L; ratio 0.47 — the yearly rain is about half the reservoir's full storage; (d) rain is lost to evaporation and to plants and soil before it can run into the reservoir.

Fully-worked solutions

Q1. (a) $8x\div 4=2x$ and $12\div 4=3$, so $8x+12=4(2x+3)$.

(b) $15y\div 5=3y$ and $10\div 5=2$, so $15y+10=5(3y+2)$.

(c) $9m\div 3=3m$ and $-6\div 3=-2$, so $9m-6=3(3m-2)$.

Q2. (a) The HCF of 6 and 9 is 3.

$6x+9=3(2x+3)$

(b) The HCF of 20 and 15 is 5.

$20y-15=5(4y-3)$

(c) The HCF of 14 and 21 is 7; there is no pronumeral common to both terms.

$14a+21b=7(2a+3b)$

(d) Check of (a): $3(2x+3)=3\times 2x+3\times 3=6x+9$. Expanding returns the original, so the factorisation is correct.

Q3. (a) The lowest power of x in both terms is x^2.

$x^3+x^2=x^2(x+1)$

(b) The HCF of 5 and 10 is 5; the lowest power of y is y^2.

$5y^4-10y^2=5y^2(y^2-2)$

(c) The HCF of 8 and 12 is 4; the lowest power of m is m^3.

$8m^5+12m^3=4m^3(2m^2+3)$

Q4. (a) $x^2\times x^5=x^{2+5}=x^7$

(b) $3a\times 7a^3=3\times 7\times a^{1+3}=21a^4$

(c) $\left(2m^2n\right)\left(5mn^4\right)=2\times5\times m^{2+1}\times n^{1+4}=10m^3n^5$

(d) $\left(y^3\right)^4=y^{3\times4}=y^{12}$

Q5. (a) $a^9\div a^4=a^{9-4}=a^5$

(b) $\dfrac{12x^6}{3x^2}=\dfrac{12}{3}\times x^{6-2}=4x^4$

(c) $\dfrac{20p^7q^3}{5p^2q^3}=\dfrac{20}{5}\times p^{7-2}\times q^{3-3}=4p^5q^0=4p^5$

(d) $b^8\div b^8=b^{8-8}=b^0=1$

Q6. (a) $x^{-3}=\dfrac{1}{x^3}$

(b) $5a^{-2}=5\times\dfrac{1}{a^2}=\dfrac{5}{a^2}$ — only the a carries the negative exponent.

(c) $\dfrac{7}{m^{-4}}=7\times\dfrac{1}{m^{-4}}=7\times m^4=7m^4$

(d) $n^4\times n^{-6}=n^{4-6}=n^{-2}=\dfrac{1}{n^2}$

Q7. (a) Area = width × length:

$A=5\left(x+7\right)$

(b) $A=5\times x+5\times7=5x+35$

Q8. (a) Volume = the three edges multiplied:

$V=\left(2a\right)\left(3b\right)\left(4c\right)$

$V=2\times3\times4\times a\times b\times c=24abc$

(b) The doubled edges are $4a$, $6b$ and $8c$.

$V=\left(4a\right)\left(6b\right)\left(8c\right)=4\times6\times8\times abc=192abc$

(c) $\dfrac{192abc}{24abc}=\dfrac{192}{24}=8$

∴ the new volume is 8 times the original. Doubling every edge multiplies the volume by $2\times2\times2=8$.

Q9. Coefficients 18, 24, 6: the HCF is 6; the lowest power of x is x.

$18x^3+24x^2-6x=6x\left(3x^2+4x-1\right)$

Check: $6x\times3x^2=18x^3$, $6x\times4x=24x^2$, $6x\times\left(-1\right)=-6x$.

Q10. Coefficients 12, 8, 20: the HCF is 4. The lowest power of a is a^2; the lowest power of b is b^2.

$12a^2b^3-8a^3b^2+20a^2b^2=4a^2b^2\left(3b-2a+5\right)$

Q11. (a) HCF of 24 and 16 is 8; lowest power of x is x^3.

$24x^4+16x^3=8x^3\left(3x+2\right)$

(b) HCF of 35 and 14 is 7; lowest powers are m^3 and n.

$35m^5n - 14m^3n^2 = 7m^3n(5m^2 - 2n)$

(c) HCF of 9, 12 and 15 is 3; lowest power of p is p^2.

$9p^6 - 12p^4 + 15p^2 = 3p^2(3p^4 - 4p^2 + 5)$

Q12. (a) $3x(2x+9) = 3x \times 2x + 3x \times 9 = 6x^2 + 27x$

This is not $6x^2 + 9x$, so the bracket is wrong: the student divided $6x^2$ by $3x$ correctly but left the 9 untouched instead of dividing $9x$ by $3x$.

(b) $6x^2 \div 3x = 2x$ and $9x \div 3x = 3$.

$6x^2 + 9x = 3x(2x+3)$

Q13. $(4a^2b)^3 = 4^3 \times (a^2)^3 \times b^3 = 64a^6b^3$

Q14. $5x^4 \times 2x^{-6} = 5 \times 2 \times x^{4+(-6)} = 10x^{-2} = \dfrac{10}{x^2}$

Q15. $\dfrac{18m^3}{6m^7} = \dfrac{18}{6} \times m^{3-7} = 3m^{-4} = \dfrac{3}{m^4}$

Q16. $(2x^3)^2 = 4x^6$

$4x^6 \times x^{-4} = 4x^2$

$\dfrac{4x^2}{x} = 4x^{2-1} = 4x$

Q17. $(6a^4b^{-2})^2 = 36a^8b^{-4}$

$\dfrac{36a^8b^{-4}}{3a^3b^{-5}} = \dfrac{36}{3} \times a^{8-3} \times b^{-4-(-5)} = 12a^5b^1 = 12a^5b$

The answer already has positive exponents.

Q18. (a) $2^{15} + 2^{12} = 2^{12} \times 2^3 + 2^{12} \times 1 = 2^{12}(2^3+1) = 2^{12} \times 9$

$2^{12} \times 9 = 4096 \times 9 = 36\,864$

(b) $5^{30} - 5^{28} = 5^{28} \times 5^2 - 5^{28} \times 1 = 5^{28}(5^2 - 1) = 5^{28} \times 24$

$\therefore k = 24$.

Q19. (a) Volume of a cube = edge^3:

$V = (4x^2)^3 = 4^3 \times (x^2)^3 = 64x^6$

(b) A cube has 6 faces, each a square of edge $4x^2$:

$A = 6 \times (4x^2)^2 = 6 \times 16x^4 = 96x^4$

Q20. (a) $3^{2p} = (3^p)^2 = 10^2 = 100$

(b) $3^{p+1} = 3^p \times 3^1 = 10 \times 3 = 30$

(c) $3^{p-2} = 3^p \div 3^2 = 10 \div 9 = \frac{10}{9}$

Q21. (a) $A = 3x(x+6)$

(b) $A = 3x \times x + 3x \times 6 = 3x^2 + 18x$

(c) Product form: $3 \times 4 \times (4+6) = 12 \times 10 = 120$

Expanded form: $3 \times 4^2 + 18 \times 4 = 48 + 72 = 120$

∴ both forms give 120 — the same area, as they must be.

Q22. (a) The exponents add: $5 + \square = 12$, so the missing exponent is 7.

(b) The exponents multiply: $4 \times \square = 20$, so the missing exponent is 5.

(c) The exponents subtract: $10 - \square = 4$, so the missing exponent is 6.

(d) $(2m^2)^3 = 2^3 \times m^6 = 8m^6$, so the missing coefficient is 8.

Q23. (a) Volume = area × depth:

$V = 10^4\,\text{m}^2 \times 10^{-3}\,\text{m} = 10^{4+(-3)}\,\text{m}^3 = 10\,\text{m}^3$

$10\,\text{m}^3 = 10 \times 10^3\,\text{L} = 10^4\,\text{L}$

∴ 1 mm of rain on 1 hectare is 10^4 L.

(b) Total volume = catchment area × rainfall, in the units of (a):

$V = 48\,700 \times 1\,021 \times 10^4\,\text{L}$

$V = 49\,722\,700 \times 10^4\,\text{L} = 497\,227\,000\,000\,\text{L}$

$V = 4.97227 \times 10^{11}\,\text{L} \approx 4.97 \times 10^{11}\,\text{L}$ (three significant figures)

(c) $1\,068\,000\,\text{ML} = 1\,068\,000 \times 10^6\,\text{L} = 1.068 \times 10^{12}\,\text{L}$

Ratio $= \frac{4.97227 \times 10^{11}}{1.068 \times 10^{12}} = 0.4655\ldots \approx 0.47$

∴ the rain falling on the catchment in a year is about half of the reservoir's full storage — a large ratio, which is why the Thomson can fill despite the losses in (d).

(d) Much of the rain never reaches the reservoir: it evaporates, is taken up by plants and soil, or soaks into ground that does not drain to the reservoir. Only the runoff that reaches the Thomson River and its tributaries flows in — about 1.82×10^{11} L per year, roughly $37\,\%$ of the rainfall total.

Teacher note

Level 9 codes rehearsed (D3): `VC2M9A01` — the exponent laws (recalled in Theory §3–4 and applied throughout); `VC2M9A02` — expanding and factorising simple algebraic expressions (Theory §1, the checking habit in every factorisation). Both are rehearsed, not re-taught, and are not assessed as Level 9 outcomes.

Tool classes (D13): neither `VC2M10A01` nor `VC2M10A02` names a digital tool, so no `DIGITAL TOOL` banner appears in this subtopic. All manipulation is by hand; a scientific calculator is optional for the arithmetic of Q18 and Q23.

VCE destinations (§5, reference only): the Mathematical Methods Units 3 & 4 skill index names `VC2M10A01` and `VC2M10A02` as assumed F–10 background, including the skill *"Simplify a product or quotient that includes negative integer exponents"* (`VC2M10A02`); Foundation Mathematics Units 3 & 4 (AoS 1) and General Mathematics Units 3 & 4 list Chapter 1 skills as assumed knowledge; Mathematical Methods Units 1 & 2 assume all of Chapter 1 (inferred).

Level check (D6): negative exponents appear only as `VC2M10A02.E01` requires — simplifying products and quotients, and moving between negative- and positive-exponent forms. No exponential equations are solved (that is 2E), no algebraic fractions are added or subtracted (that is 1C), and no binomial expansions or quadratic factorisations are attempted (that is 1D). Q20 substitutes into exponent-law forms; it does not solve for p.

Delivering two codes (D8): 1B delivers `VC2M10A01` (Theory §1–2, Examples 1–3, Q1–Q3, Q7, Q9–Q12, Q21) and `VC2M10A02` (Theory §3–5, Examples 4–6, Q4–Q6, Q8, Q13–Q20, Q22, Q23) as separate skills with separate evidence; T1 items are tagged `VC2M10A01` or `VC2M10A02` accordingly (§7.1).

The four operations on simple algebraic fractions

Mathematics 10 · Chapter 1 Number and algebraic manipulation · Subtopic 1C (Level 10) · Extended block

Strand	Algebra
Content description	`VC2M10A03`
Elaboration ids carried	`VC2M10A03.E01` `VC2M10A03.E02`
Weight	Extended (double allocation)

> **Content description (verbatim):** *apply the 4 operations to simple algebraic fractions with numerical or single variable denominators*
>
> **Achievement standard (extract):** *Students substitute into formulas, find unknown values, manipulate linear and quadratic algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital tools.*

You need first: common factors and the exponent laws for algebraic products and quotients from 1B (`VC2M10A01`, `VC2M10A02`), and the arithmetic of numerical fractions — common denominators, multiplying and dividing fractions — from earlier years. This subtopic applies those skills to fractions whose numerator or denominator contains a pronumeral; it does not re-teach them.

Theory

An **algebraic fraction** is a fraction whose numerator, denominator or both contain a pronumeral. The four operations — addition, subtraction, multiplication and division — work on algebraic fractions exactly as they work on numerical fractions. The only new idea is that a denominator can now contain a pronumeral, and the exponent laws from 1B do the simplifying.

> The Victorian Curriculum uses the term *simple algebraic fractions* but does not define it. This subtopic therefore treats an algebraic fraction as *simple* when its denominator is a number, a single term in one pronumeral (such as $4x$ or x^2) or a linear two-term expression in one pronumeral (such as $x+3$). Every question in this subtopic is answerable on that definition alone.

1. The one rule that never breaks

A denominator can never be zero. With numerical fractions this rule is invisible; with algebraic fractions it restricts what the pronumeral may stand for.

- In $\frac{5}{x}$, the pronumeral cannot be 0: $x \neq 0$.
- In $\frac{3}{x+2}$, the denominator is zero when $x = -2$, so $x \neq -2$.

State these restrictions where a question asks for them. They matter again in 2A, where fractions appear inside equations.

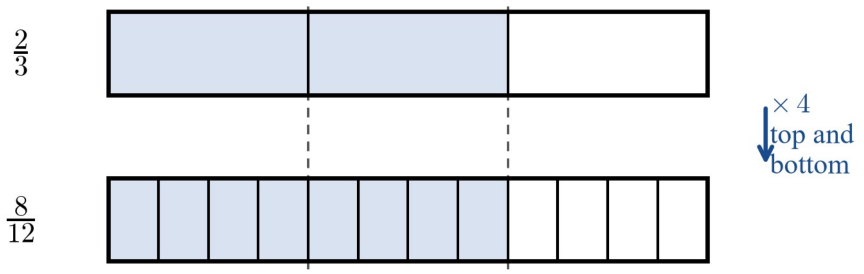
2. Equivalent fractions: building up and cancelling

Multiplying the numerator and the denominator by the same non-zero expression does not change the value of a fraction:

$$\frac{a}{b} = \frac{a \times c}{b \times c}.$$

Used one way, this **builds a fraction up** to a denominator you need:

$$\frac{3}{x} = \frac{3 \times (x+2)}{x \times (x+2)} = \frac{3(x+2)}{x(x+2)}.$$



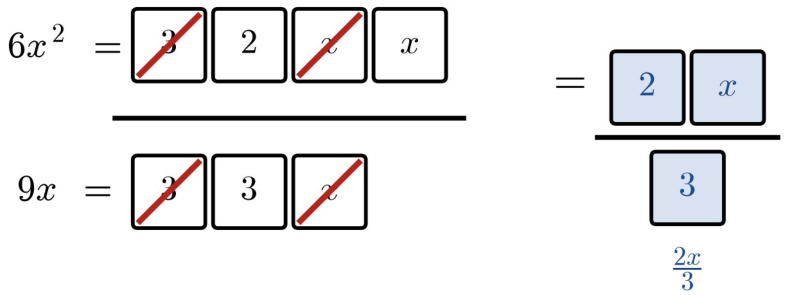
Multiply the numerator and the denominator by 4 — the shaded amount does not change.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Two equal bars. The top bar is split into three equal parts with two parts shaded to show two thirds. The bottom bar is the same length split into twelve equal parts with eight parts shaded, the same shaded amount, eight of the twelve parts, because the numerator and the denominator are both multiplied by four.

Used the other way, it **cancels** a common factor of the numerator and denominator:

$$\frac{6x^2}{9x} = \frac{2x}{3} \text{(divide top and bottom by } 3x\text{).}$$

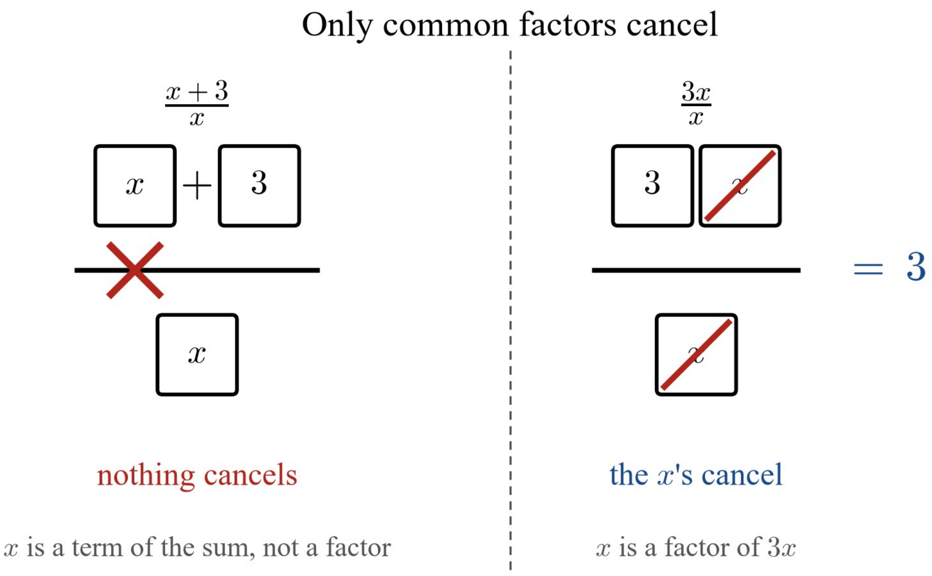


Crossed-out chips are the common factor $3x$: divide top and bottom by it.

$$\frac{6x^2}{9x} = \frac{2x}{3}$$

Factor chips showing six x squared as 3, 2, x, x above a fraction bar and nine x as 3, 3, x below it, with the shared factors 3 and x crossed out on the top and the bottom, leaving two x over three.

Only common **factors** cancel. A term inside a sum is not a factor: in $\frac{x+3}{x}$ the x in the numerator is part of a sum, not a factor of the whole numerator, so nothing cancels. This is the most common false move in the subtopic — check for it every time you simplify.

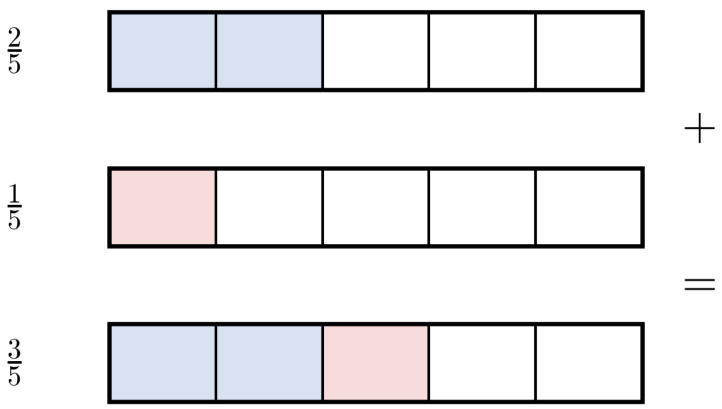


Two fractions side by side under the heading only common factors cancel. Left: the fraction with numerator x + 3 and denominator x, marked with a red cross because the x on top is a term of a sum, not a factor, so nothing cancels. Right: the fraction three x over x, where the factor x is crossed out on the top and the bottom to leave three.

3. Addition and subtraction: find the common denominator

With the **same denominator**, add or subtract the numerators and keep the denominator:

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c},\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}.$$



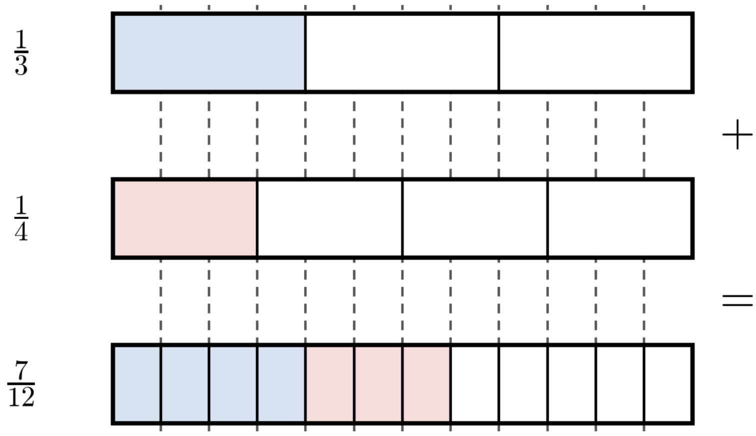
Same denominator: add the numerators, keep the denominator.

Three equal bars each split into five parts. Two parts shaded plus one part shaded makes three parts shaded, so two fifths plus one fifth equals three fifths, and the denominator stays the same.

With **different denominators**, first rewrite each fraction over the **lowest common denominator (LCD)** — the smallest expression that every denominator divides into — then combine. Three cases turn up:

- **Numerical denominators:** the LCD of 3 and 4 is 12.
- **Single-variable denominators:** the LCD of x and x^2 is x^2, since x^2 is a multiple of x (exponent laws: $x \times x = x^2$). The LCD of $2x$ and $3x$ is $6x$.

- **Denominators with no common factor:** when two denominators share no factor, their LCD is their product. The LCD of x and $x+2$ is $x(x+2)$.



Rewrite both fractions over the lowest common denominator, 12.

$$\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$$

Three equal bars. The first is split into thirds with one part shaded, the second into quarters with one part shaded, and the third into twelve equal parts with seven parts shaded: four parts from the first bar and three parts from the second, seven of the twelve parts in all.

Two warnings carry through every exercise:

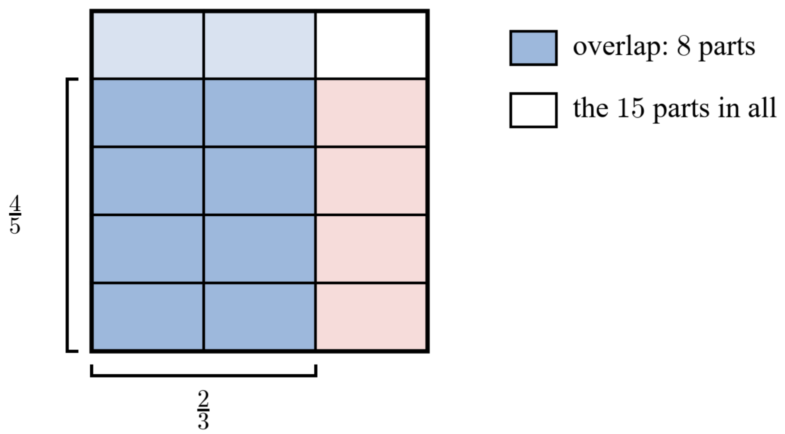
- **Keep the numerator in brackets** when you rewrite it over the LCD. In $\frac{3x-1}{4}-\frac{x+2}{3}$ the minus sign belongs to the **whole** second numerator, not just to x.
- **Simplify the final answer.** Combine like terms in the numerator, then cancel any common factor of the whole numerator and the denominator.

4. Multiplication: multiply across, then cancel

To multiply two fractions, multiply the numerators and multiply the denominators:

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}.$$

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$



Multiply the numerators; multiply the denominators.

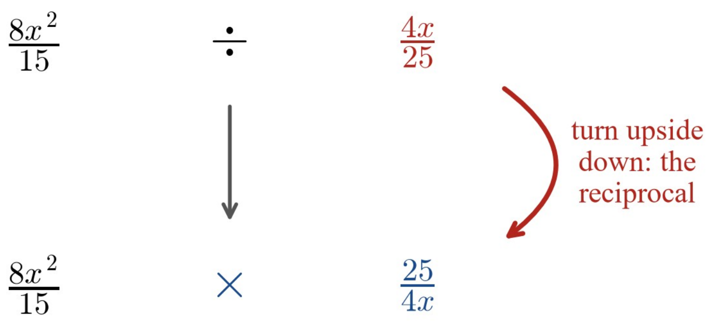
A rectangle cut into a grid of three columns and five rows, fifteen small parts in all. Two of the three columns are shaded one way for two thirds and four of the five rows another way for four fifths, and the eight parts where the two shadings overlap are the product, eight fifteenths.

Then cancel any common factor of a numerator and a denominator. You may cancel **before** you multiply (across the fractions) or **after** — the result is the same, and cancelling first keeps the expressions smaller. The exponent laws from 1B do the work on the pronumerals: for example $x \times x^2 = x^3$, and $x^3 \div x^2 = x$.

5. Division: multiply by the reciprocal

The **reciprocal** of a fraction is the fraction turned upside down: the reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$. To divide by a fraction, multiply by its reciprocal:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$



To divide by a fraction, multiply by its reciprocal.

Two rows. The top row is eight x squared over fifteen divided by four x over twenty five. The bottom row shows the same division rewritten with the second fraction turned upside down to its reciprocal, twenty five over four x, so the division becomes a multiplication.

Then cancel and simplify as in part 4. The divisor itself cannot be zero: dividing by $\frac{4x}{25}$, for instance, carries the restriction $x \neq 0$.

The four operations at a glance:

Operation	The move
Addition / subtraction	Rewrite over a common denominator, then combine the numerators.
Multiplication	Multiply across, then cancel common factors.
Division	Multiply by the reciprocal of the divisor, then cancel.

Worked examples

Example 1 — Addition with numerical denominators

Express $\frac{2x+1}{3}+\frac{x-2}{4}$ as a single fraction in simplest form.

Working	Comment
The denominators are 3 and 4; the LCD is 12.	The smallest number both denominators divide into.
$\frac{2x+1}{3}=\frac{4(2x+1)}{12}$	Multiply top and bottom by 4.
$\frac{x-2}{4}=\frac{3(x-2)}{12}$	Multiply top and bottom by 3.
Sum $=\frac{4(2x+1)+3(x-2)}{12}$	Add the numerators over the common denominator.
$=\frac{8x+4+3x-6}{12}$	Expand the brackets.
$=\frac{11x-2}{12}$	Collect like terms.
$\therefore$ the sum is $\frac{11x-2}{12}$.	$11x-2$ shares no factor with 12; this is simplest form.

Example 2 — Subtraction, and the minus sign

Express $\frac{3x-1}{4}-\frac{x+2}{3}$ as a single fraction in simplest form.

Working	Comment
The LCD of 4 and 3 is 12.	
$=\frac{3(3x-1)}{12}-\frac{4(x+2)}{12}$	Build each fraction up to the LCD.
$=\frac{3(3x-1)-4(x+2)}{12}$	Keep the second numerator in brackets: the minus sign applies to all of it.
$=\frac{9x-3-4x-8}{12}$	Expand: $-4\times x=-4x$ and $-4\times 2=-8$.
$=\frac{5x-11}{12}$	Collect like terms.
$\therefore$ the difference is $\frac{5x-11}{12}$.	No common factor remains.

Example 3 — Single-variable denominators

Express each as a single fraction in simplest form: (a) $\frac{7}{2x}+\frac{5}{2x}$; (b) $\frac{5}{x}-\frac{2}{x^2}$.

Working	Comment
(a) The denominators are already the same.	Both fractions are over $2x$.
$=\frac{7+5}{2x}$	Add the numerators, keep the denominator.
$=\frac{12}{2x}$	
$=\frac{6}{x}$	Divide top and bottom by 2.
$\therefore$ (a) $\frac{7}{2x}+\frac{5}{2x}=\frac{6}{x}$.	
(b) The LCD of x and x^2 is x^2.	x^2 is a multiple of x.
$\frac{5}{x}=\frac{5x}{x^2}$	Multiply top and bottom by x.
$=\frac{5x-2}{x^2}$	Subtract the numerators over x^2.
$\therefore$ (b) $\frac{5}{x}-\frac{2}{x^2}=\frac{5x-2}{x^2}$.	No common factor. In both parts, $x\neq 0$.

Example 4 — Multiplication

Express $\frac{4x}{9}\times\frac{3}{2x^2}$ in simplest form.

Working	Comment
$=\frac{4x\times 3}{9\times 2x^2}$	Multiply the numerators and the denominators.
$=\frac{12x}{18x^2}$	
$=\frac{2}{3x}$	Divide top and bottom by $6x$: $12\div 6=2$, $18\div 6=3$, and $x\div x^2=\frac{1}{x}$ by the exponent laws.
$\therefore$ the product is $\frac{2}{3x}$.	$x\neq 0$.

You could also cancel the 3 with the 9, and one x with the x^2, before multiplying — the answer is the same.

Example 5 — Division

Express $\frac{8x^2}{15}\div\frac{4x}{25}$ in simplest form.

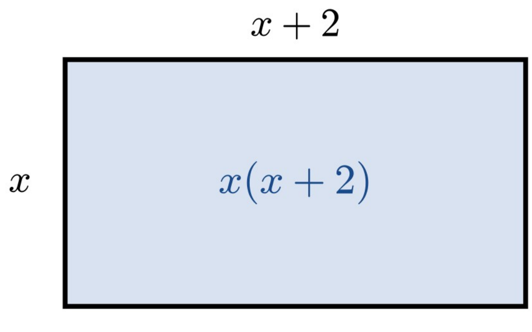
Working	Comment
The reciprocal of $\frac{4x}{25}$ is $\frac{25}{4x}$.	Turn the divisor upside down.

Working	Comment
$= \dfrac{8x^2}{15} \times \dfrac{25}{4x}$	Division becomes multiplication by the reciprocal.
$= \dfrac{8x^2 \times 25}{15 \times 4x}$	Multiply across.
$= \dfrac{200x^2}{60x}$	
$= \dfrac{10x}{3}$	Divide top and bottom by $20x$.
$\therefore$ the quotient is $\dfrac{10x}{3}$.	$x \neq 0$, since the divisor cannot be zero.

Example 6 — Different single-variable denominators

Express $\dfrac{3}{x+2} - \dfrac{2}{x}$ as a single fraction in simplest form.

Working	Comment
The denominators $x+2$ and x share no factor, so the LCD is their product, $x(x+2)$.	Rule from Theory §3.
$\dfrac{3}{x+2} = \dfrac{3x}{x(x+2)}$	Multiply top and bottom by x.
$\dfrac{2}{x} = \dfrac{2(x+2)}{x(x+2)}$	Multiply top and bottom by $x+2$.
$= \dfrac{3x - 2(x+2)}{x(x+2)}$	Subtract; keep the second numerator in brackets.
$= \dfrac{3x - 2x - 4}{x(x+2)}$	Expand.
$= \dfrac{x-4}{x(x+2)}$	Collect like terms.
$\therefore$ the difference is $\dfrac{x-4}{x(x+2)}$.	Restrictions: $x \neq 0$ and $x \neq -2$.



No common factor: the LCD of x and $x+2$ is their product.

A rectangle with sides x and x + 2. Its area, x times x + 2, is the lowest common denominator of fractions with denominators x and x + 2, because the two denominators share no common factor.

Practice questions

Scaffolded

Q1. Express $\frac{x+1}{2}+\frac{x-3}{5}$ as a single fraction. (a) What is the LCD of 2 and 5? (b) Rewrite each fraction over the LCD. (c) Add the numerators and simplify.

Q2. Express $\frac{2x+3}{4}-\frac{x-1}{3}$ as a single fraction. (a) What is the LCD? (b) Rewrite each fraction over the LCD, keeping the second numerator in brackets. (c) Expand and collect like terms.

Q3. (a) Express $\frac{3}{x}+\frac{4}{x}$ as a single fraction. (b) Express $\frac{9}{2x}-\frac{1}{2x}$ as a single fraction, then simplify your answer by cancelling.

Q4. Express $\frac{2}{x}+\frac{5}{x^2}$ as a single fraction. (a) What is the LCD of x and x^2? (b) Rewrite $\frac{2}{x}$ over the LCD. (c) Add the numerators.

Q5. Express $\frac{x+2}{3}+\frac{1}{x}$ as a single fraction. (a) What is the LCD of 3 and x? (b) Rewrite each fraction over the LCD. (c) Expand the numerator.

Q6. Express $\frac{3x+4}{x+2}-\frac{2}{x+2}$ as a single fraction. (a) The denominators already match — what do you do with the numerators? (b) Subtract, keeping brackets. (c) Can the answer be simplified further?

Q7. Express $\frac{2x}{3}\times\frac{9}{4x^2}$ in simplest form. (a) Multiply the numerators and the denominators. (b) What is the highest common factor of $18x$ and $12x^2$? (c) Cancel it.

Q8. Express $\frac{5x}{6}\div\frac{10x^2}{9}$ in simplest form. (a) What is the reciprocal of $\frac{10x^2}{9}$? (b) Rewrite the division as a multiplication. (c) Multiply and cancel.

Unscaffolded

Write each expression as a single fraction in simplest form. Where a denominator contains the pronumeral, remember that it cannot take any value that makes that denominator zero.

Q9. $\frac{3x}{7}+\frac{2x}{7}$

Q10. $\frac{5}{3x}-\frac{2}{3x}$

Q11. $\frac{x-2}{3}+\frac{2x+1}{6}$

Q12. $\frac{3x-2}{5}-\frac{x+1}{2}$

Q13. $\frac{4}{x}+\frac{3}{x^2}$

Q14. $\dfrac{x+3}{4}+\dfrac{2}{x}$

Q15. $\dfrac{4x-1}{x+3}-\dfrac{x-4}{x+3}$

Q16. $\dfrac{12x^3}{25}\times\dfrac{5}{6x^2}$

Q17. $\dfrac{9x}{14}\div\dfrac{3x^2}{7}$

Q18. $\dfrac{3}{x+4}-\dfrac{1}{x}$

Q19. $\dfrac{2}{x+1}+\dfrac{3}{x+2}$

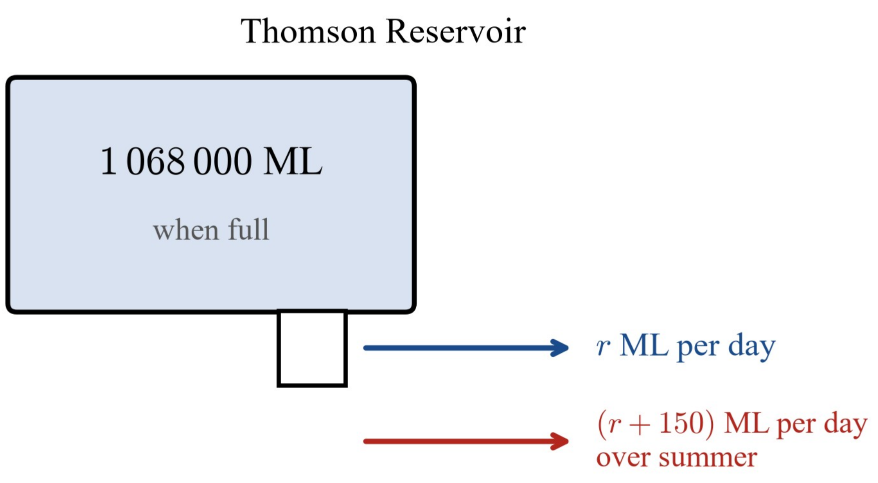
Q20. $\dfrac{5}{x-2}-\dfrac{2}{x+3}$

Q21. $\dfrac{1}{2}+\dfrac{1}{x}+\dfrac{1}{2x}$

Q22. $\dfrac{2x}{3}-\dfrac{x-1}{2}+\dfrac{1}{6}$

Real context

Q23. The Thomson Reservoir in Victoria holds $1\,068\,000$ megalitres (ML) of water when full (1 ML is one million litres). Water is released from the reservoir at a steady rate of r ML per day.



The Thomson Reservoir drawn as a tank holding 1 068 000 ML of water when full, with a release pipe letting water out at r ML per day, and a second arrow for the summer release rate of (r + 150) ML per day.

(a) Write a fraction for the number of days the full reservoir would last.

(b) Over summer, demand rises and the release rate increases by 150 ML per day, to $(r+150)$ ML per day. Write a fraction for the number of days the full reservoir would now last.

(c) Write an expression for how many **fewer** days the reservoir lasts at the increased rate, and express it as a single fraction.

(d) Use your fraction from (c) to find how many fewer days the reservoir lasts when $r = 600$. Check your answer by evaluating your two fractions from (a) and (b) at $r = 600$.

Source: Melbourne Water, *Thomson Reservoir*, 2026. Retrieved 19 August 2026. (melbournewater.com.au/water-and-environment/water-management/water-storage-reservoirs/thomson-reservoir)

Answers

Q1. $\frac{7x-1}{10}$. **Q2.** $\frac{2x+13}{12}$. **Q3.** (a) $\frac{7}{x}$; (b) $\frac{4}{x}$. **Q4.** $\frac{2x+5}{x^2}$. **Q5.** $\frac{x^2+2x+3}{3x}$. **Q6.** $\frac{3x+2}{x+2}$. **Q7.** $\frac{3}{2x}$. **Q8.** $\frac{3}{4x}$. **Q9.** $\frac{5x}{7}$. **Q10.** $\frac{1}{x}$. **Q11.** $\frac{4x-3}{6}$. **Q12.** $\frac{x-9}{10}$. **Q13.** $\frac{4x+3}{x^2}$. **Q14.** $\frac{x^2+3x+8}{4x}$. **Q15.** $\frac{3x+3}{x+3}$, i.e. $\frac{3(x+1)}{x+3}$. **Q16.** $\frac{2x}{5}$. **Q17.** $\frac{3}{2x}$. **Q18.** $\frac{2(x-2)}{x(x+4)}$. **Q19.** $\frac{5x+7}{(x+1)(x+2)}$. **Q20.** $\frac{3x+19}{(x-2)(x+3)}$. **Q21.** $\frac{x+3}{2x}$. **Q22.** $\frac{x+4}{6}$. **Q23.** (a) $\frac{1\,068\,000}{r}$ days; (b) $\frac{1\,068\,000}{r+150}$ days; (c) $\frac{160\,200\,000}{r(r+150)}$ days; (d) 356 days.

Fully-worked solutions

Q1.

LCD of 2 and 5: 10.

$\frac{x+1}{2}+\frac{x-3}{5}=\frac{5(x+1)}{10}+\frac{2(x-3)}{10}$

$=\frac{5(x+1)+2(x-3)}{10}$

$=\frac{5x+5+2x-6}{10}$

$=\frac{7x-1}{10}$

$\therefore$ the sum is $\frac{7x-1}{10}$.

Q2.

LCD of 4 and 3: 12.

$\frac{2x+3}{4}-\frac{x-1}{3}=\frac{3(2x+3)}{12}-\frac{4(x-1)}{12}$

$=\frac{3(2x+3)-4(x-1)}{12}$

$=\frac{6x+9-4x+4}{12}$

$=\frac{2x+13}{12}$

$\therefore$ the difference is $\frac{2x+13}{12}$.

Q3.

(a) The denominators match:

$$\frac{3}{x}+\frac{4}{x}=\frac{3+4}{x}=\frac{7}{x}$$

(b) The denominators match:

$$\frac{9}{2x}-\frac{1}{2x}=\frac{8}{2x}$$

Divide top and bottom by 2:

$$=\frac{4}{x}$$

Q4.

LCD of x and x^2: x^2.

$$\frac{2}{x}+\frac{5}{x^2}=\frac{2x}{x^2}+\frac{5}{x^2}$$

$$=\frac{2x+5}{x^2}$$

$\therefore$ the sum is $\frac{2x+5}{x^2}$.

Q5.

LCD of 3 and x: $3x$.

$$\frac{x+2}{3}+\frac{1}{x}=\frac{x(x+2)}{3x}+\frac{3}{3x}$$

$$=\frac{x(x+2)+3}{3x}$$

$$=\frac{x^2+2x+3}{3x}$$

$\therefore$ the sum is $\frac{x^2+2x+3}{3x}$.

Q6.

The denominators match, so subtract the numerators:

$$\frac{3x+4}{x+2}-\frac{2}{x+2}=\frac{(3x+4)-2}{x+2}$$

$$=\frac{3x+2}{x+2}$$

$3x+2$ has no factor in common with $x+2$, so this is simplest form. The restriction is $x\neq-2$.

Q7.

$$\frac{2x}{3}\times\frac{9}{4x^2}=\frac{2x\times 9}{3\times 4x^2}$$

$$=\frac{18x}{12x^2}$$

Divide top and bottom by $6x$:

$$=\frac{3}{2x}$$

$\therefore$ the product is $\frac{3}{2x}$, with $x \neq 0$.

Q8.

The reciprocal of $\frac{10x^2}{9}$ is $\frac{9}{10x^2}$.

$$\frac{5x}{6} \div \frac{10x^2}{9} = \frac{5x}{6} \times \frac{9}{10x^2}$$

$$=\frac{45x}{60x^2}$$

Divide top and bottom by $15x$:

$$=\frac{3}{4x}$$

$\therefore$ the quotient is $\frac{3}{4x}$, with $x \neq 0$.

Q9.

The denominators match:

$$\frac{3x}{7} + \frac{2x}{7} = \frac{3x+2x}{7} = \frac{5x}{7}$$

Q10.

The denominators match:

$$\frac{5}{3x} - \frac{2}{3x} = \frac{3}{3x}$$

Divide top and bottom by 3:

$$=\frac{1}{x}$$

Q11.

LCD of 3 and 6: 6.

$$\frac{x-2}{3} + \frac{2x+1}{6} = \frac{2(x-2)}{6} + \frac{2x+1}{6}$$

$$=\frac{2(x-2)+(2x+1)}{6}$$

$=\frac{2x-4+2x+1}{6}$

$=\frac{4x-3}{6}$

Q12.

LCD of 5 and 2: 10.

$\frac{3x-2}{5}-\frac{x+1}{2}=\frac{2(3x-2)}{10}-\frac{5(x+1)}{10}$

$=\frac{2(3x-2)-5(x+1)}{10}$

$=\frac{6x-4-5x-5}{10}$

$=\frac{x-9}{10}$

Q13.

LCD of x and x^2: x^2.

$\frac{4}{x}+\frac{3}{x^2}=\frac{4x}{x^2}+\frac{3}{x^2}$

$=\frac{4x+3}{x^2}$

Q14.

LCD of 4 and x: $4x$.

$\frac{x+3}{4}+\frac{2}{x}=\frac{x(x+3)}{4x}+\frac{8}{4x}$

$=\frac{x(x+3)+8}{4x}$

$=\frac{x^2+3x+8}{4x}$

x^2+3x+8 has no factor in common with $4x$, so this is simplest form.

Q15.

The denominators match, so subtract the numerators:

$\frac{4x-1}{x+3}-\frac{x-4}{x+3}=\frac{(4x-1)-(x-4)}{x+3}$

$=\frac{4x-1-x+4}{x+3}$

$=\frac{3x+3}{x+3}$

$$=\frac{3(x+1)}{x+3}$$

Nothing cancels — $x+1$ is not a factor of the denominator. The restriction is $x \neq -3$.

Q16.

$$\frac{12x^3}{25} \times \frac{5}{6x^2} = \frac{12x^3 \times 5}{25 \times 6x^2}$$

$$=\frac{60x^3}{150x^2}$$

Divide top and bottom by $30x^2$:

$$=\frac{2x}{5}$$

∴ the product is $\frac{2x}{5}$.

Q17.

The reciprocal of $\frac{3x^2}{7}$ is $\frac{7}{3x^2}$.

$$\frac{9x}{14} \div \frac{3x^2}{7} = \frac{9x}{14} \times \frac{7}{3x^2}$$

$$=\frac{63x}{42x^2}$$

Divide top and bottom by $21x$:

$$=\frac{3}{2x}$$

∴ the quotient is $\frac{3}{2x}$, with $x \neq 0$.

Q18.

The denominators $x+4$ and x share no factor, so the LCD is $x(x+4)$.

$$\frac{3}{x+4} - \frac{1}{x} = \frac{3x}{x(x+4)} - \frac{x+4}{x(x+4)}$$

$$=\frac{3x-(x+4)}{x(x+4)}$$

$$=\frac{2x-4}{x(x+4)}$$

$$=\frac{2(x-2)}{x(x+4)}$$

Restrictions: $x \neq 0$ and $x \neq -4$.

Q19.

The denominators share no factor, so the LCD is $(x+1)(x+2)$.

$$\frac{2}{x+1}+\frac{3}{x+2}=\frac{2(x+2)}{(x+1)(x+2)}+\frac{3(x+1)}{(x+1)(x+2)}$$

$$=\frac{2(x+2)+3(x+1)}{(x+1)(x+2)}$$

$$=\frac{2x+4+3x+3}{(x+1)(x+2)}$$

$$=\frac{5x+7}{(x+1)(x+2)}$$

Restrictions: $x\neq-1$ and $x\neq-2$.

Q20.

The denominators share no factor, so the LCD is $(x-2)(x+3)$.

$$\frac{5}{x-2}-\frac{2}{x+3}=\frac{5(x+3)}{(x-2)(x+3)}-\frac{2(x-2)}{(x-2)(x+3)}$$

$$=\frac{5(x+3)-2(x-2)}{(x-2)(x+3)}$$

$$=\frac{5x+15-2x+4}{(x-2)(x+3)}$$

$$=\frac{3x+19}{(x-2)(x+3)}$$

Restrictions: $x\neq2$ and $x\neq-3$.

Q21.

The LCD of 2, x and $2x$ is $2x$.

$$\frac{1}{2}+\frac{1}{x}+\frac{1}{2x}=\frac{x}{2x}+\frac{2}{2x}+\frac{1}{2x}$$

$$=\frac{x+2+1}{2x}$$

$$=\frac{x+3}{2x}$$

Q22.

The LCD of 3, 2 and 6 is 6.

$$\frac{2x}{3}-\frac{x-1}{2}+\frac{1}{6}=\frac{4x}{6}-\frac{3(x-1)}{6}+\frac{1}{6}$$

$$=\frac{4x-3(x-1)+1}{6}$$

$=\dfrac{4x-3x+3+1}{6}$

$=\dfrac{x+4}{6}$

Q23.

(a) Days = amount of water ÷ rate of release:

$\dfrac{1\,068\,000}{r}$ days

(b) At the increased rate:

$\dfrac{1\,068\,000}{r+150}$ days

(c) Fewer days = (days at the old rate) − (days at the new rate):

$\dfrac{1\,068\,000}{r}-\dfrac{1\,068\,000}{r+150}$

The LCD is $r(r+150)$:

$=\dfrac{1\,068\,000(r+150)}{r(r+150)}-\dfrac{1\,068\,000r}{r(r+150)}$

$=\dfrac{1\,068\,000(r+150)-1\,068\,000r}{r(r+150)}$

$=\dfrac{1\,068\,000\times150}{r(r+150)}$

$=\dfrac{160\,200\,000}{r(r+150)}$ days

(d) Substitute $r=600$:

$\dfrac{160\,200\,000}{600\times750}=\dfrac{160\,200\,000}{450\,000}=356$

Check with parts (a) and (b):

At $r=600$: $\dfrac{1\,068\,000}{600}=1780$ days, and $\dfrac{1\,068\,000}{750}=1424$ days.

$1780-1424=356$. ✓

∴ the reservoir lasts 356 fewer days at the increased release rate.

Teacher note

Level 9 code rehearsed (D3): `VC2M9A01` — the exponent laws with integer and zero exponents. 1C's simplification of products and quotients (`VC2M10A03.E02`) is those laws applied to fractions: $x\div x^2=\dfrac{1}{x}$ is the integer-exponent law in action. The skill is rehearsed here, not re-taught, and is not assessed as a Level 9 outcome. Within this book the subtopic depends on 1B (`VC2M10A01`, `VC2M10A02`), per the prerequisite table in TOC §3. Downstream, 2A

(`VC2M10A12`) clears these same fractions from linear equations, and 1D’s factorising later lets harder fractions be simplified further.

Tool classes (D13): none. `VC2M10A03` names no digital tool; every manipulation here is by-hand, and no calculator step is required. No CAS.

VCE destinations (§5, reference only): `VC2M10A03` is named in the Mathematical Methods Units 3 & 4 assumed F–10 background (AoS 2 ‘Algebra, number and structure’), with the skill *“Add or subtract simple algebraic fractions”*; MM Units 1 & 2 assumes all of Chapter 1 from week one and never re-teaches it *(inferred — the Units 1 & 2 study design carries no VC2 mapping)*. TOC §5 records: *“A student who arrives at MM Unit 1 without 1C and 1E is behind on day one and does not recover.”*

Level check (D6): denominators throughout are numerical or single-variable — a single term or a linear two-term expression. No quadratic denominator appears, no factorising of quadratics is required (that is 1D’s), no equation containing a fraction is solved here (that is 2A’s), and no partial-fraction or higher-order rational work is attempted. Nothing on the D6 prohibited list is touched.

Expanding binomial products and factorising quadratics by a range of strategies

Mathematics 10 · Chapter 1 Number and algebraic manipulation · Subtopic 1D (Level 10) · Extended block

Strand	Algebra
Content description	`VC2M10A04`
Elaboration ids carried	`VC2M10A04.E01` `VC2M10A04.E02` `VC2M10A04.E03`
Weight	Extended (double allocation)

> **Content description (verbatim):** *expand binomial products and factorise monic quadratic expressions using a variety of strategies*
>
> **Achievement standard (extract):** *Students substitute into formulas, find unknown values, manipulate linear and quadratic algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital tools.*

You need first: common factors from 1B (`VC2M10A01`) — taking out a common factor must be automatic, including a common **binomial** factor — the exponent laws from 1B (`VC2M10A02`), and expanding brackets by the distributive law from Level 9 (`VC2M9A02`). This subtopic builds those skills into a full toolkit for expanding binomial products and factorising quadratics; it does not re-teach them.

Theory

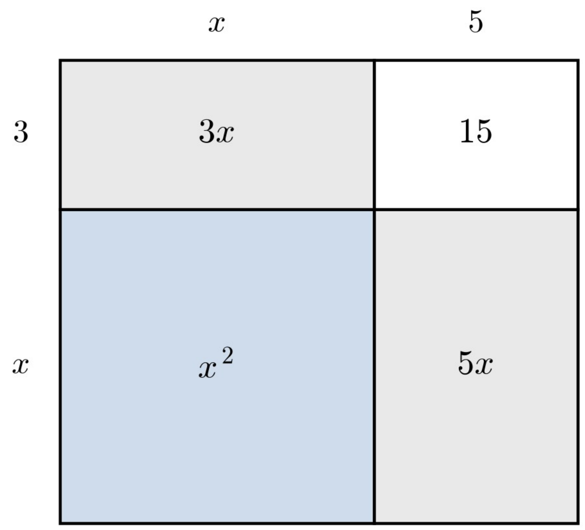
Expanding and factorising are the same skill run in opposite directions. Expanding turns brackets into a sum of terms; factorising turns a sum of terms back into brackets. Every method in this subtopic is one of the two moves used in both directions, and every answer — here and in the exercises — can be checked by running the other move.

1. Expanding binomial products: every term times every term

A **binomial** is an expression with two terms, such as $x+5$ or $x-3$. A **binomial product** is two brackets multiplied together, and the distributive law says every term in the first bracket multiplies every term in the second. For $(x+5)(x+3)$:

$$(x+5)(x+3)=x\times x+x\times 3+5\times x+5\times 3=x^2+3x+5x+15=x^2+8x+15.$$

The area model shows why the four products are exactly the right ones: the four regions fill the whole rectangle, so their areas add up to the total area.



$$(x+5)(x+3) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$$

Area model: a rectangle with side lengths x plus 5 across the top and x plus 3 down the side, split into four regions labelled x squared, 5x, 3x and 15, showing (x + 5)(x + 3) = x² + 8x + 15

In general, for any numbers a and b:

$$(x+a)(x+b) = x^2 + (a+b)x + a \times b.$$

Read off the pattern: the middle term comes from the **sum** $a+b$, and the constant term comes from the **product** $a \times b$. Keep that sum-and-product pair in mind — section 3 runs this result backwards. The rule works for any binomials, including ones whose x-term is not just x: in $(2x+1)(x-4)$ each of $2x$ and 1 still multiplies each of x and -4.

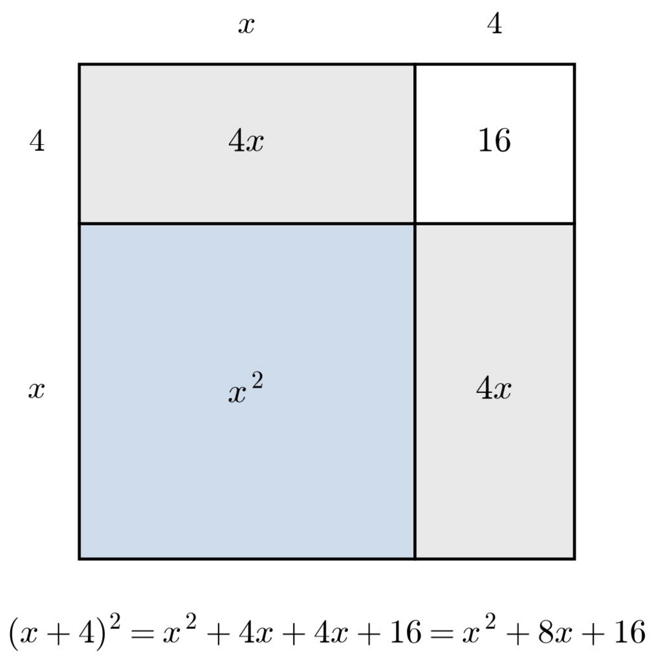
2. Two patterns worth knowing: perfect squares and the difference of two squares

Two special products come up so often that they are worth learning as patterns. Both are just section 1 with particular values of a and b, and both run backwards to factorise.

Perfect squares. Expanding $(x+a)^2 = (x+a)(x+a)$ gives $a+a=2a$ in the middle and $a \times a = a^2$ at the end:

$$(x+a)^2 = x^2 + 2ax + a^2, (x-a)^2 = x^2 - 2ax + a^2.$$

The middle term is **twice** the bracket's number times x — the figure splits $(x+4)^2$ into x^2, two $4x$ strips and the 4×4 corner. The classic error is writing $(x+4)^2 = x^2 + 16$ and losing the middle term $8x$; the two strips are where it lives.

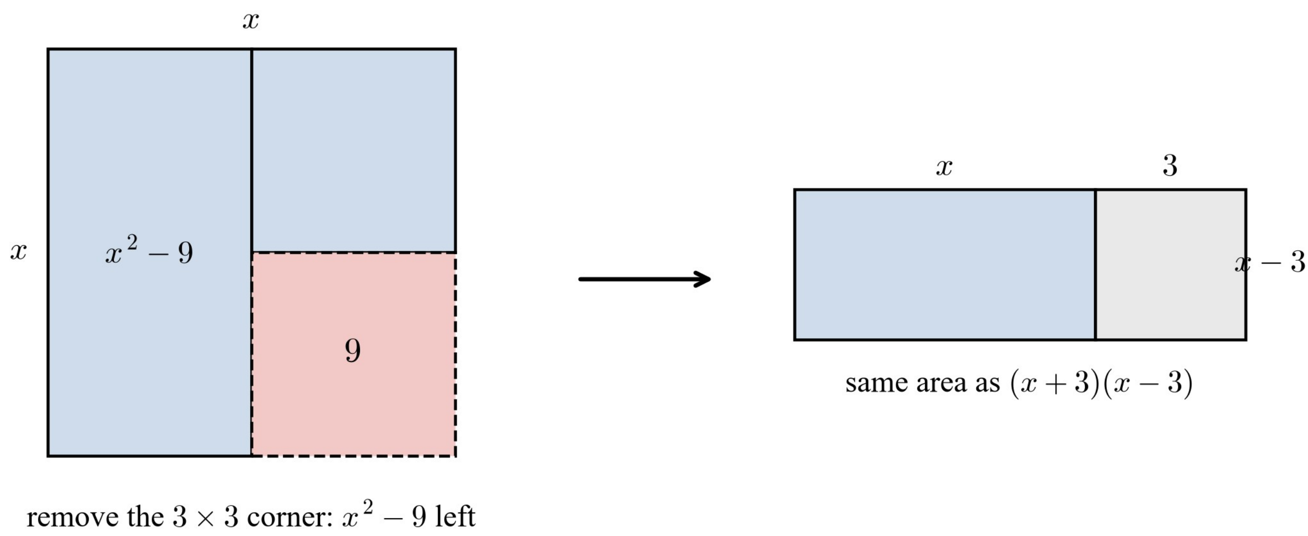


Area model of a perfect square: a square of side x plus 4 split into an x squared region, two 4x strips and a 16 corner square, showing (x + 4)² = x² + 8x + 16

Difference of two squares. In $(x-a)(x+a)$ the middle terms $+ax$ and $-ax$ cancel:

$$(x-a)(x+a)=x^2-a^2.$$

The figure shows it with areas: cutting an $a\times a$ corner from an $x\times x$ square and rearranging what is left gives a rectangle a longer and a narrower.



Two panels: on the left, a square of side x with a 3 by 3 corner removed, leaving area x² − 9; on the right, the remaining pieces rearranged as a rectangle of length x plus 3 and width x minus 3

Run backwards, these factorise at a glance: $x^2+8x+16=(x+4)^2$ because $16=4^2$ and $8=2\times4$, and $x^2-36=(x-6)(x+6)$ because both terms are squares.

3. Factorising monic trinomials: find the sum-and-product pair

A **monic** quadratic is one whose x^2 term is just x^2. To factorise x^2+bx+c, read section 1's result backwards: find two numbers p and q with

$$p+q=b \text{ and } p\times q=c \text{, then } x^2+bx+c=(x+p)(x+q).$$

List factor pairs of c and pick the pair whose sum is b. The signs of b and c tell you what to expect before you start:

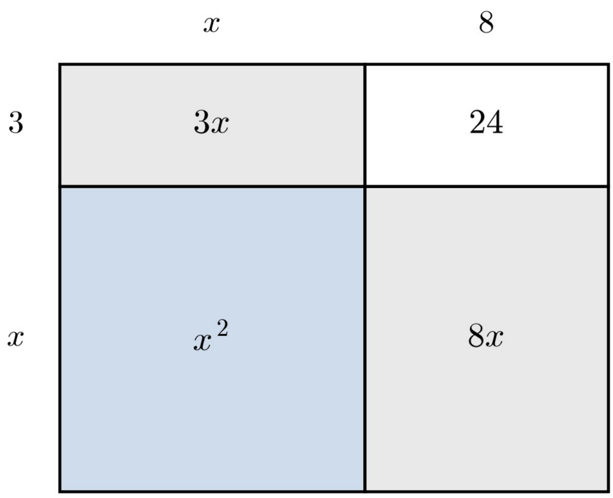
- c positive and b positive: both numbers positive — $x^2+7x+12=(x+3)(x+4)$.
- c positive and b negative: both numbers negative — $x^2-7x+12=(x-3)(x-4)$.
- c negative: one positive, one negative, and the larger one (ignoring signs) carries the sign of b — $x^2+x-12=(x+4)(x-3)$ and $x^2-x-12=(x-4)(x+3)$.

How the signs of b and c fix the signs of p and q ($x^2+bx+c=(x+p)(x+q)$)

$c>0$ and $b>0$ both p and q positive $x^2+7x+12=(x+3)(x+4)$	$c>0$ and $b<0$ both p and q negative $x^2-7x+12=(x-3)(x-4)$
$c<0$ and $b>0$ opposite signs; larger one positive $x^2+x-12=(x+4)(x-3)$	$c<0$ and $b<0$ opposite signs; larger one negative $x^2-x-12=(x-4)(x+3)$

The four sign cases for factorising x² + bx + c, each with a worked example: both factors positive, both negative, and the two mixed-sign cases

The area model works here too: $x^2+11x+24$ fills a rectangle of sides $x+8$ and $x+3$, because $8+3=11$ and $8\times3=24$.



$$3 + 8 = 11 \quad 3 \times 8 = 24$$

$$x^2 + 11x + 24 = (x + 3)(x + 8)$$

Area model: a rectangle of side lengths x plus 8 and x plus 3 split into regions x squared, 8x, 3x and 24, showing x² + 11x + 24 = (x + 3)(x + 8)

Always check by expanding your factors back out. If no pair of whole numbers works, the trinomial is not factorisable by this method — section 5 handles those.

4. Common factors first, and grouping in pairs

Before anything else, **look for a common factor**. Taking out common factors is the skill from 1B, and it applies here unchanged — including when the common factor is a binomial:

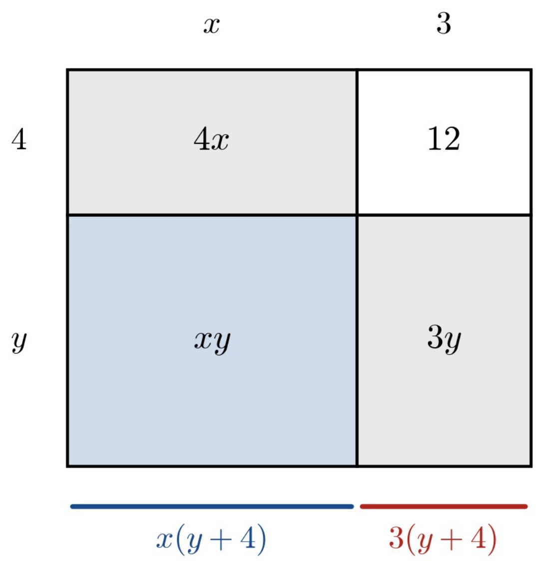
$$5(x-2)+x(x-2)=(x-2)(5+x),$$

because both halves contain the whole bracket $(x-2)$.

With **four terms**, group in pairs: split the expression into two pairs, take out the common factor of each pair, and a common binomial factor appears. For $xy+4x+3y+12$:

$$xy+4x+3y+12=x(y+4)+3(y+4)=(y+4)(x+3).$$

The area model shows the grouping: the top row shares a height y, the bottom row a height 4, and the two columns have widths x and 3.



$$xy + 4x + 3y + 12 = x(y+4) + 3(y+4) = (y+4)(x+3)$$

Area model of grouping in pairs: a rectangle of width x plus 3 and height y plus 4 split into four regions xy, 4x, 3y and 12; the rows share factors y and 4, the columns share factors x and 3

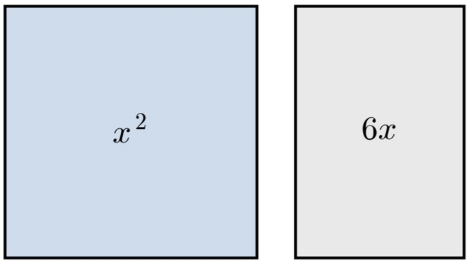
If the first grouping does not produce a common binomial factor, try pairing the terms a different way. As ever, check by expanding.

5. Completing the square

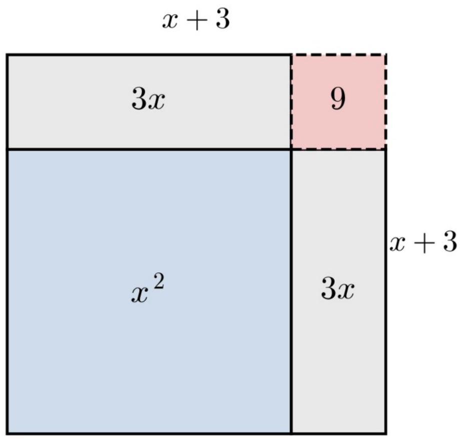
The perfect square of section 2 says $(x+\frac{b}{2})^2 = x^2 + bx + (\frac{b}{2})^2$. Read backwards, $x^2 + bx$ is almost the square $(x+\frac{b}{2})^2$ — it is missing the corner $(\frac{b}{2})^2$. Adding c and keeping the balance gives the method of **completing the square**:

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2.$$

The figure shows $x^2 + 6x$: the $6x$ is cut into two $3x$ strips folded onto two sides of the x^2 square, and the missing 3×3 corner is the -9.



the pieces of $x^2 + 6x$



$(x+3)^2 = x^2 + 6x + 9$, so $x^2 + 6x = (x+3)^2 - 9$

Area model of completing the square: the region x² + 6x rearranged as two 3x strips folded onto an x by x square, forming a square of side x plus 3 missing a 3 by 3 corner, so x² + 6x = (x + 3)² − 9

Completing the square has two uses.

Use 1 — factorising. Once the expression is a square minus a square, the difference of two squares factorises it, even when section 3's pair does not exist in whole numbers. For example $x^2+6x+5=(x+3)^2-9+5=(x+3)^2-4$, and $4=2^2$, so $(x+3)^2-2^2=(x+3-2)(x+3+2)=(x+1)(x+5)$.

Use 2 — solving equations. It also solves quadratics exactly, using one fact about square roots, stated before use:

> **Square-root fact:** if $X^2=k$ with $k\geq 0$, then $X=\sqrt{k}$ or $X=-\sqrt{k}$, written $X=\pm\sqrt{k}$.

For example $x^2-4x+1=0$ becomes $(x-2)^2-4+1=0$, so $(x-2)^2=3$, so $x-2=\pm\sqrt{3}$, so $x=2\pm\sqrt{3}$. The answers are exact: $2+\sqrt{3}$ and $2-\sqrt{3}$, never rounded decimals.

6. Choosing a strategy

With several methods available, the order of checks matters more than any single method. Work through the chart: common factor first, then count the terms, then match a pattern. Whatever the path, the last step is the same — expand your answer and check it returns the original expression.

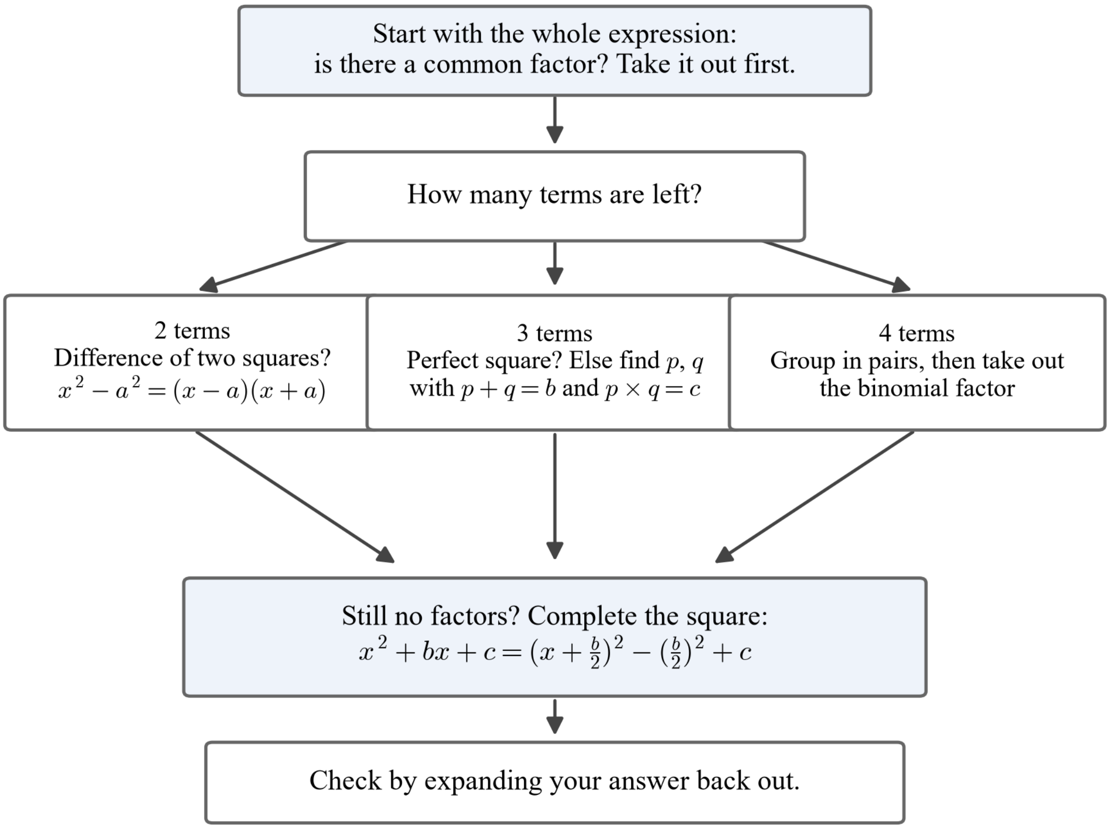


Chart of the strategy order: start with the whole expression and take out any common factor; then count the terms left — two terms, try the difference of two squares; three terms, try a perfect square or find the sum-and-product pair; four terms, group in pairs; if no factors appear, complete the square; always check by expanding the answer back out

Worked examples

Example 1 — Expanding binomial products

Expand $(x+5)(x-3)$ and $(2x+1)(x-4)$.

Working	Comment
$(x+5)(x-3)$	Two binomials: every term times every term.
$=x^2-3x+5x-15$	$x\times x$, $x\times(-3)$, $5\times x$, $5\times(-3)$.
$=x^2+2x-15$	Collect like terms.
$(2x+1)(x-4)$	Same rule when the x-term is not just x.
$=2x^2-8x+x-4$	$2x\times x$, $2x\times(-4)$, $1\times x$, $1\times(-4)$.
$=2x^2-7x-4$	Collect like terms.
$\therefore\ (x+5)(x-3)=x^2+2x-15$ and $(2x+1)(x-4)=2x^2-7x-4$.	Check: expanding the answers returns the brackets.

Example 2 — The two patterns: perfect square and difference of two squares

Expand $(x+4)^2$ and $(x-6)(x+6)$.

Working	Comment
$(x+4)^2=x^2+2\times4\times x+4^2$	Perfect square: middle term is twice the number times x.
$=x^2+8x+16$	Do not drop the middle term.
$(x-6)(x+6)=x^2-6^2$	Difference of two squares: the middle terms cancel.

Working	Comment
$=x^2-36$	Square the number.
$\therefore\ (x+4)^2=x^2+8x+16$ and $(x-6)(x+6)=x^2-36$.	Both patterns run backwards to factorise.

Example 3 — Factorising monic trinomials

Factorise $x^2+11x+24$ and $x^2-3x-18$.

Working	Comment
$x^2+11x+24$: need $p+q=11$, $p\times q=24$.	Both signs positive, so try positive pairs.
$3+8=11$ and $3\times 8=24$	The pair is $3,8$.
$\therefore\ x^2+11x+24=(x+3)(x+8)$	Check by expanding.
$x^2-3x-18$: need $p+q=-3$, $p\times q=-18$.	Constant negative: one number of each sign.
$-6+3=-3$ and $-6\times 3=-18$	The larger number carries the minus.
$\therefore\ x^2-3x-18=(x-6)(x+3)$	Check by expanding.

Example 4 — A common binomial factor, and grouping in pairs

Factorise $5(x-2)+x(x-2)$ and $xy+4x+3y+12$.

Working	Comment
$5(x-2)+x(x-2)$	Both halves contain the bracket $(x-2)$.
$=(x-2)(5+x)$	Take out the common binomial factor.
$=(x-2)(x+5)$	Tidy the second bracket.
$xy+4x+3y+12$	Four terms: group in pairs.
$=x(y+4)+3(y+4)$	x out of the first pair, 3 out of the second.
$=(y+4)(x+3)$	Take out the common binomial factor.
$\therefore\ 5(x-2)+x(x-2)=(x-2)(x+5)$ and $xy+4x+3y+12=(x+3)(y+4)$.	Expanding each answer returns the original.

Example 5 — Completing the square to factorise

Factorise x^2+6x+5 by completing the square.

Working	Comment
$x^2+6x+5=(x+3)^2-9+5$	Half of 6 is 3; subtract $3^2=9$ to keep the balance.
$=(x+3)^2-4$	A square minus a square.
$=(x+3-2)(x+3+2)$	Difference of two squares, since $4=2^2$.
$=(x+1)(x+5)$	Tidy each bracket.
$\therefore\ x^2+6x+5=(x+1)(x+5)$.	Check: $1+5=6$ and $1\times 5=5$, matching section 3.

Example 6 — Completing the square to solve an equation

Solve $x^2-4x+1=0$ exactly.

Working	Comment
$x^2-4x+1=(x-2)^2-4+1$	Half of -4 is -2; subtract $2^2=4$.
$(x-2)^2-3=0$	The equation, completed.
$(x-2)^2=3$	Move the constant across.

Working	Comment
$x-2=\pm\sqrt{3}$	Square-root fact.
$x=2\pm\sqrt{3}$	Add 2 to both sides.
$\therefore\ x=2+\sqrt{3}$ or $x=2-\sqrt{3}$.	Exact form; two solutions, as a quadratic allows.

Practice questions

Scaffolded

Q1. Expand $(x+3)(x+7)$. (a) Write out the four products $x\times x$, $x\times 7$, $3\times x$, 3×7. (b) Collect like terms.

Q2. Expand $(x-6)(x+4)$. (a) Write out the four products, keeping the signs with each. (b) Collect like terms.

Q3. Expand $(x+6)^2$ using the perfect square pattern. What is twice the number, and what is the number squared?

Q4. Expand $(x+9)(x-9)$. Which pattern from Theory section 2 applies, and what is 9^2?

Q5. Factorise $x^2+9x+18$. (a) Find two numbers with sum 9 and product 18. (b) Write the factors. (c) Check by expanding.

Q6. Factorise $x^2-10x+25$. (a) Is 25 a square, and is 10 twice its root? (b) Write the expression as a perfect square.

Q7. Factorise x^2-49. Name the pattern that applies, and write the two factors.

Q8. (a) Factorise $x(y+4)+3(y+4)$ by taking out the common binomial factor. (b) Factorise $2x+8+xy+4y$ by grouping in pairs.

Unscaffolded

Q9. Expand $(x+4)(x+8)$.

Q10. Expand $(x-9)(x+3)$.

Q11. Expand $(x-6)(x-5)$.

Q12. Expand $(x-7)^2$.

Q13. Expand $(x+11)(x-11)$.

Q14. Factorise $x^2+13x+36$.

Q15. Factorise $x^2-12x+27$.

Q16. Factorise $x^2+3x-28$.

Q17. Factorise $x^2-2x-35$.

Q18. Factorise $x^2+16x+64$.

Q19. Factorise x^2-121.

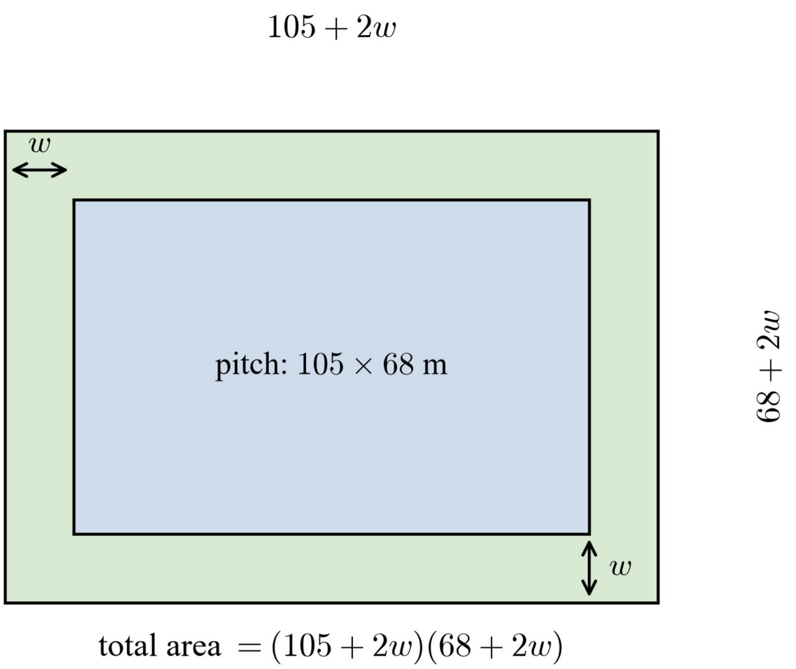
Q20. Factorise $xy+2x+3y+6$.

Q21. Factorise $x^2+10x+17$ completely, by completing the square. Give your answer in exact form.

Q22. Solve $x^2+6x+2=0$ by completing the square. Give your answers in exact form.

Real context

Q23. The recommended field of play for an international football match is a pitch 105 m long and 68 m wide. Grounds keep a rectangular run-off strip of the same width w (in metres) on all four sides of the pitch.



Plan view of a 105 metre by 68 metre pitch inside a run-off strip of width w on all four sides; the overall length is 105 plus 2w and the overall width is 68 plus 2w

(a) Explain why the overall rectangle is $(105+2w)$ m long and $(68+2w)$ m wide, and expand $(105+2w)(68+2w)$ to show the total area is $4w^2+346w+7140$ m².

(b) The pitch itself has area $105\times 68=7140$ m². Write an expression for the area of the run-off strip alone (total area minus pitch area) and simplify it.

(c) Evaluate the strip's area when $w=3$ m.

Source: IFAB, *Laws of the Game*, Law 1 — The Field of Play (field of play for international matches: length 100–110 m, width 64–75 m); the recommended 105 m × 68 m field of play as cited in "Association football pitch", Wikipedia. Retrieved 20 August 2026.

Answers

Q1. $x^2+10x+21$. **Q2.** $x^2-2x-24$. **Q3.** $x^2+12x+36$. **Q4.** x^2-81. **Q5.** $(x+3)(x+6)$. **Q6.** $(x-5)^2$. **Q7.** $(x-7)(x+7)$. **Q8.** (a) $(y+4)(x+3)$; (b) $(x+4)(y+2)$. **Q9.** $x^2+12x+32$. **Q10.** $x^2-6x-27$. **Q11.** $x^2-11x+30$. **Q12.** $x^2-14x+49$. **Q13.** x^2-121. **Q14.** $(x+4)(x+9)$. **Q15.** $(x-3)(x-9)$. **Q16.** $(x+7)(x-4)$. **Q17.** $(x-7)(x+5)$. **Q18.** $(x+8)^2$. **Q19.** $(x-11)(x+11)$. **Q20.** $(x+3)(y+2)$. **Q21.** $(x+5+2\sqrt{2})(x+5-2\sqrt{2})$. **Q22.** $x=-3+\sqrt{7}$ or $x=-3-\sqrt{7}$. **Q23.** (a) $4w^2+346w+7140$ m²; (b) $4w^2+346w$ m²; (c) 1074 m².

Fully-worked solutions

Q1.

$(x+3)(x+7)=x^2+7x+3x+21$

$=x^2+10x+21$

$\therefore\ (x+3)(x+7)=x^2+10x+21$.

Q2.

$(x-6)(x+4)=x^2+4x-6x-24$

$=x^2-2x-24$

$\therefore\ (x-6)(x+4)=x^2-2x-24$.

Q3.

$(x+6)^2=x^2+2\times 6\times x+6^2$

$=x^2+12x+36$

$\therefore\ (x+6)^2=x^2+12x+36$.

Q4.

$(x+9)(x-9)=x^2-9^2$

$=x^2-81$

$\therefore\ (x+9)(x-9)=x^2-81$.

Q5.

Need $p+q=9$ and $p\times q=18$: the pair is $3,6$.

$\therefore\ x^2+9x+18=(x+3)(x+6)$.

Check: $(x+3)(x+6)=x^2+6x+3x+18=x^2+9x+18$. ✓

Q6.

$25=5^2$ and $2\times 5=10$, so this is a perfect square.

$\therefore\ x^2-10x+25=(x-5)^2$.

Q7.

$49=7^2$: difference of two squares.

$\therefore x^2-49=(x-7)(x+7)$.

Q8.

(a) $x(y+4)+3(y+4)=(y+4)(x+3)$ — the bracket $(y+4)$ is the common factor.

(b) $2x+8+xy+4y=2(x+4)+y(x+4)$

$=(x+4)(2+y)$

$=(x+4)(y+2)$

$\therefore 2x+8+xy+4y=(x+4)(y+2)$.

Q9.

$(x+4)(x+8)=x^2+8x+4x+32$

$=x^2+12x+32$

$\therefore (x+4)(x+8)=x^2+12x+32$.

Q10.

$(x-9)(x+3)=x^2+3x-9x-27$

$=x^2-6x-27$

$\therefore (x-9)(x+3)=x^2-6x-27$.

Q11.

$(x-6)(x-5)=x^2-5x-6x+30$

$=x^2-11x+30$

$\therefore (x-6)(x-5)=x^2-11x+30$.

Q12.

$(x-7)^2=x^2-2\times7\times x+7^2$

$=x^2-14x+49$

$\therefore (x-7)^2=x^2-14x+49$.

Q13.

$(x+11)(x-11)=x^2-11^2$

$=x^2-121$

$\therefore (x+11)(x-11)=x^2-121$.

Q14.

Need $p+q=13$ and $p\times q=36$: the pair is $4,9$.

$\therefore x^2+13x+36=(x+4)(x+9)$.

Q15.

Need $p+q=-12$ and $p\times q=27$: the pair is $-3,-9$.

$\therefore x^2-12x+27=(x-3)(x-9)$.

Q16.

Need $p+q=3$ and $p\times q=-28$: the pair is $7,-4$.

$\therefore x^2+3x-28=(x+7)(x-4)$.

Q17.

Need $p+q=-2$ and $p\times q=-35$: the pair is $-7,5$.

$\therefore x^2-2x-35=(x-7)(x+5)$.

Q18.

$64=8^2$ and $2\times 8=16$: a perfect square.

$\therefore x^2+16x+64=(x+8)^2$.

Q19.

$121=11^2$: difference of two squares.

$\therefore x^2-121=(x-11)(x+11)$.

Q20.

$xy+2x+3y+6=x(y+2)+3(y+2)$

$=(y+2)(x+3)$

$\therefore xy+2x+3y+6=(x+3)(y+2)$.

Q21.

$x^2+10x+17=(x+5)^2-25+17$

$=(x+5)^2-8$

$=(x+5)^2-(2\sqrt{2})^2$, since $8=(2\sqrt{2})^2$

$=(x+5+2\sqrt{2})(x+5-2\sqrt{2})$

$\therefore x^2+10x+17=(x+5+2\sqrt{2})(x+5-2\sqrt{2})$.

Q22.

$x^2+6x+2=(x+3)^2-9+2=(x+3)^2-7$

$(x+3)^2-7=0$

$(x+3)^2=7$

$x+3=\pm\sqrt{7}$

$x=-3\pm\sqrt{7}$

$\therefore\ x = -3 + \sqrt{7}$ or $x = -3 - \sqrt{7}$.

Q23.

(a) The strip adds w to **each** end of the length and of the width, so the overall rectangle is $(105 + 2w)$ m by $(68 + 2w)$ m.

$(105 + 2w)(68 + 2w) = 105 \times 68 + 105 \times 2w + 2w \times 68 + 2w \times 2w$

$= 7140 + 210w + 136w + 4w^2$

$= 4w^2 + 346w + 7140$

$\therefore$ the total area is $4w^2 + 346w + 7140$ m².

(b) Strip area = total − pitch $= (4w^2 + 346w + 7140) - 7140$

$= 4w^2 + 346w$

$\therefore$ the strip's area is $4w^2 + 346w$ m².

(c) At $w = 3$: $4 \times 3^2 + 346 \times 3 = 36 + 1038$

$= 1074$

$\therefore$ the strip's area is 1074 m². Check: overall $111 \times 74 = 8214$ m², and $8214 - 7140 = 1074$ m². ✓

Teacher note

Level 9 code rehearsed (D3): `VC2M9A02` — expanding and factorising monic quadratics. 1D takes that Level 9 skill and extends it into the Level 10 range of strategies (`VC2M10A04.E01–.E03`): grouping in pairs, binomial common factors, and completing the square, including exact-form solutions with square roots. The Level 9 skill is rehearsed here, not re-taught, and is not assessed as a Level 9 outcome. Within this book the subtopic depends on 1B (`VC2M10A01`, `VC2M10A02`) per the prerequisite table in TOC §3; downstream, 2D and 3B depend on 1D's factorising.

Tool classes (D13): none. `VC2M10A04` names no digital tool; every manipulation here is by-hand. The achievement standard's "with and without the use of digital tools" is honoured by encouraging a check-by-expanding pass with a CAS or calculator where available, but no question requires one.

VCE destinations (§5, reference only): completing the square is the engine behind the turning point of a parabola in Mathematical Methods Units 1–4, and the difference of two squares and grouping reappear throughout Methods and Specialist algebra. The quadratic formula is *derived* from completing the square in VCE — that derivation is deliberately not taught here (see Level check), so students arrive having used the method on numbers, ready for the general derivation.

Level check (D6): the quadratic formula is never derived and never named as a method; the discriminant is not introduced as a classifier; no function notation $f(x)$ is used; factorising is restricted to monic quadratics per the content description (non-monic **expansion** appears, as the description's "expand binomial products" is unrestricted). Completing the square, grouping in pairs, and the perfect-square / difference-of-squares identities are all named in `VC2M10A04`'s own elaborations and are taught in full, per TOC §3.

Substituting into formulas, and changing the subject

Mathematics 10 · Chapter 1 Number and algebraic manipulation · Subtopic 1E · Standard block

Strand	Algebra
Content description	`VC2M10A05`
Elaboration ids carried	`VC2M10A05.E01` `VC2M10A05.E02`
Weight	Standard (single allocation)

> **Content description (verbatim):** *substitute values into formulas to determine an unknown and rearrange formulas to solve for a particular term*
>
> **Achievement standard (extract):** *Students substitute into formulas, find unknown values, manipulate linear and quadratic algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital tools.*

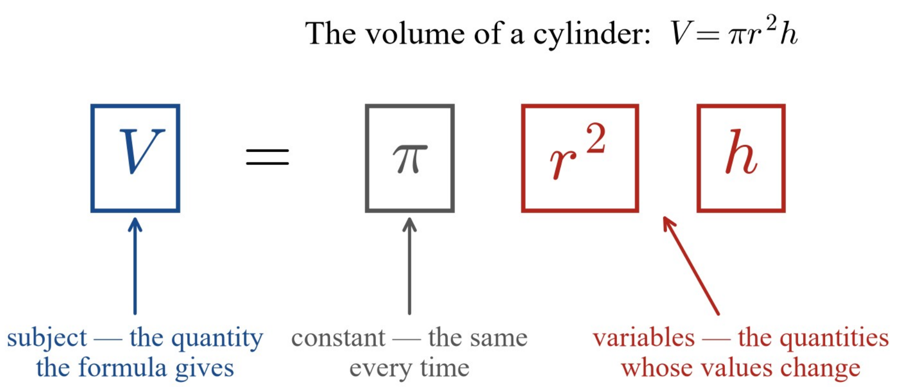
You need first: solving linear equations (`VC2M9A03`), simple interest (`VC2M9A06`, used in Q8), exact surd and π forms from Subtopic 1A (`VC2M10N01`), and the measurement formulas for circles, triangles, cylinders, cones and spheres rehearsed since Year 7. This subtopic rehearses those skills and builds on them; it does not re-teach them.

Theory

This subtopic answers one question: **once a formula is written down, how do you make it give you the quantity you actually need?** A formula usually hands you one quantity (its **subject**) when the others are known. Substituting finds the subject; changing the subject rebuilds the formula so it hands you a different quantity instead.

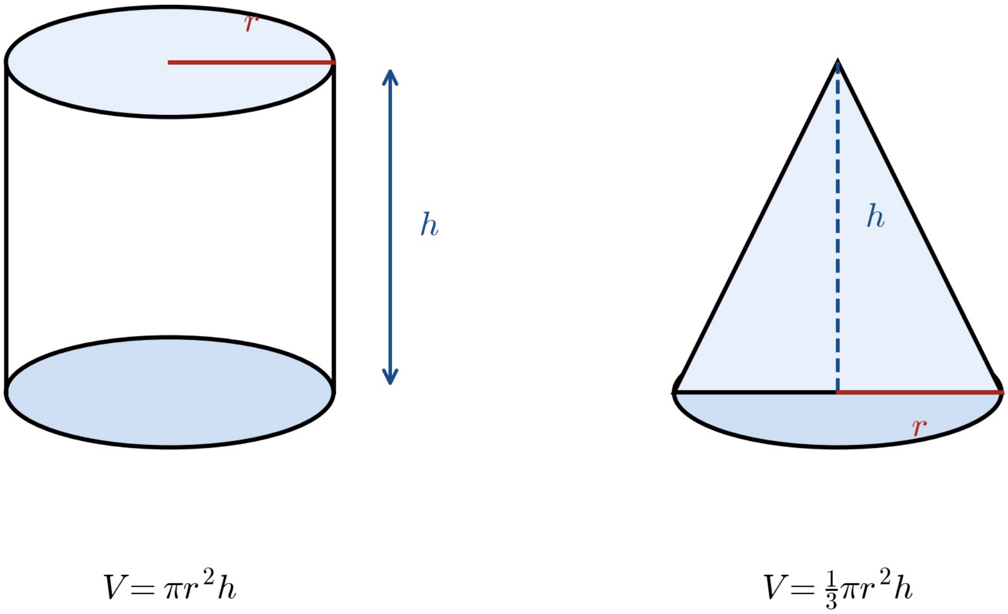
1. Formulas and substitution

A **formula** is a rule that connects quantities. The quantity on its own at the left is the **subject**; the other letters are **variables**, the quantities whose values change; and a number such as π that is the same in every case is a **constant**.



The formula $V = \pi r^2 h$ with each part boxed and labelled: V the subject, π a constant, r and h the variables.

Two formulas used often in this subtopic, for the solids shown, are $V = \pi r^2 h$ (cylinder) and $V = \frac{1}{3}\pi r^2 h$ (cone).



A cylinder and a cone, each labelled with radius r and height h, under the formulas V = πr²h and V = ⅓πr²h.

To **substitute** means to replace each variable with its value, then calculate. Keep the order of operations, and put a value in brackets whenever it is negative. For a cylinder of radius $3\,\text{cm}$ and height $5\,\text{cm}$:

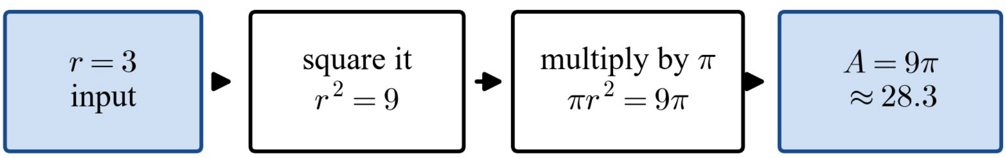
$V = \pi \times 3^2 \times 5$

$V = 45\pi$

$V \approx 141.4\,\text{cm}^3$ (correct to one decimal place)

Substitution follows the order of operations, one step at a time. For $A = \pi r^2$ with $r = 3$: square first, then multiply by π.

Substituting $r = 3$ into $A = \pi r^2$: each operation is done in order

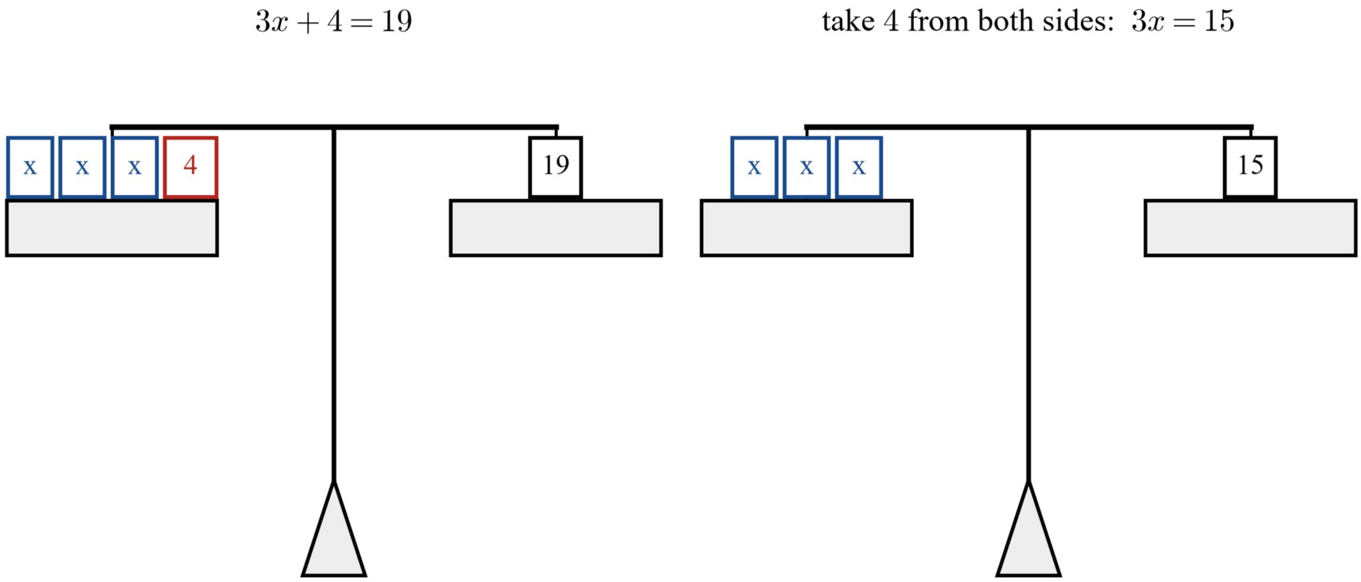


The flow follows the order of operations: square first, then multiply by π.

Flow diagram substituting r = 3 into A = πr²: input 3, square to get 9, multiply by π to get 9π ≈ 28.3.

2. Substituting to find an unknown

Often the subject is known and a different quantity is not. Substituting the known values then produces an **equation**, and solving that equation (a Level 9 skill, `VC2M9A03`) gives the unknown. The balance model says what it always says: do the same to both sides.



Two balance scales: the first shows 3x + 4 = 19 with three x-boxes and a 4-box against a 19-box; the second shows 3x = 15 after 4 is removed from both sides.

For $3x+4=19$, one step per line:

$3x+4=19$

$3x=15$ (subtract 4 from both sides)

$x=5$ (divide both sides by 3)

The same method works when the unknown sits inside a formula: substitute first, then solve.

3. Changing the subject

To **change the subject** means to rebuild a formula so that a different letter stands alone on the left. The balance logic is unchanged — the only new thing is that the other letters are treated as numbers. From $v=u+at$, make a the subject:

$v=u+at$

$v-u=at$ (subtract u from both sides)

$\frac{v-u}{t}=a$ (divide both sides by t)

$a=\frac{v-u}{t}$ (rewrite with the subject on the left)

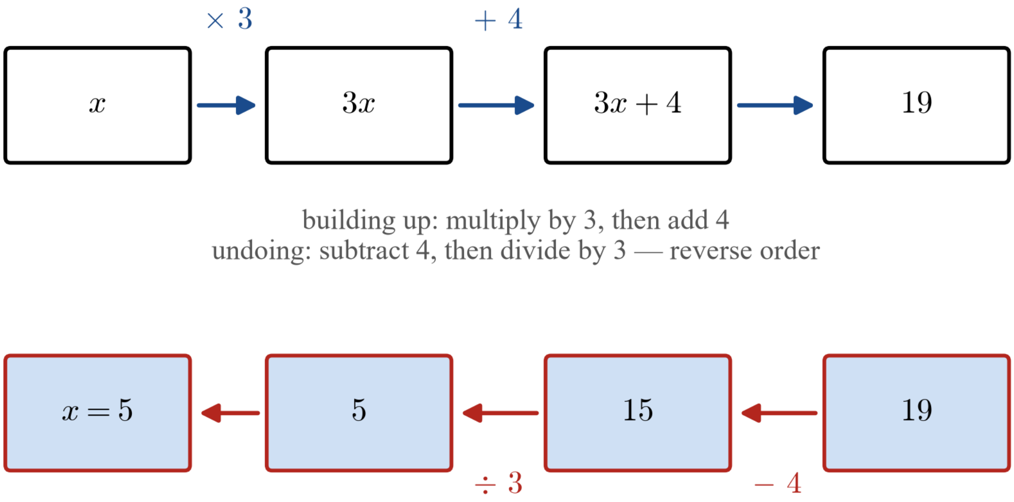
Every letter except the new subject is treated exactly as a number would be. A good habit is to check a rearranged formula by substituting numbers into both versions.

4. Choosing the undoing step

Each step of a rearrangement **undoes** one operation. Undo in the reverse of the order the operations were applied.

Operation on the quantity	Undoing step
add a	subtract a
subtract a	add a
multiply by a (with $a\neq 0$)	divide by a
divide by a	multiply by a

Operation on the quantity	Undoing step
square (quantity positive)	take the positive square root
cube	take the cube root



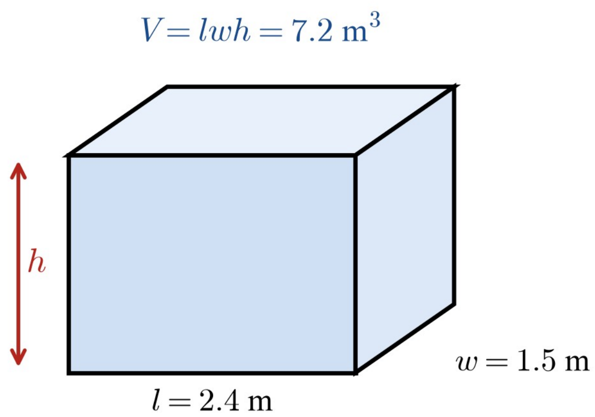
Forward chain x, 3x, 3x + 4, 19 built by multiplying by 3 then adding 4; backward chain 19, 15, 5, x = 5 undoing by subtracting 4 then dividing by 3.

For $3x+4=19$ the building-up order is “multiply by 3, then add 4”, so the undoing order is “subtract 4, then divide by 3”. Where a square is undone, a length or radius is positive, so the positive root is taken; a cube is undone by the cube root either way. Keep exact forms (such as $\frac{3V}{4\pi}$) until the final step, and round once, at the end.

Worked examples

Example 1 — Finding an unknown by substituting then solving

`SCENARIO` A rectangular water tank holds $7.2\,\mathrm{m}^3$ of water. The tank is $2.4\,\mathrm{m}$ long and $1.5\,\mathrm{m}$ wide. Use $V=lwh$ to find the depth h of the water.

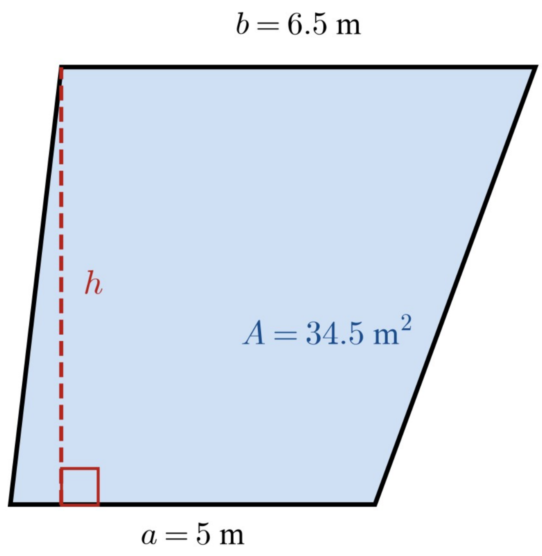


A rectangular tank labelled length l = 2.4 m, width w = 1.5 m and unknown height h, holding V = 7.2 m³ of water.

Working	Comment
$V = l\,w\,h$	Write the formula.
$7.2 = 2.4 \times 1.5 \times h$	Substitute the known values.
$7.2 = 3.6\,h$	Multiply 2.4×1.5 first.
$h = 7.2 \div 3.6$	Divide both sides by 3.6.
$h = 2$	
$\therefore$ the water is $2\,\text{m}$ deep.	Check: $2.4 \times 1.5 \times 2 = 7.2$ ✓

Example 2 — Changing the subject, then using the new formula

`SCENARIO` A trapezium garden bed has parallel sides $a = 5\,\text{m}$ and $b = 6.5\,\text{m}$ and area $34.5\,\text{m}^2$. First make h the subject of $A = \frac{1}{2}(a+b)h$, then find the height.

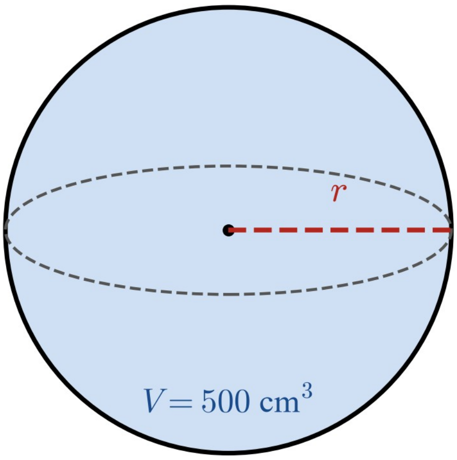


A trapezium with parallel sides a = 5 m and b = 6.5 m, height h drawn as a dashed perpendicular, and area A = 34.5 m².

Working	Comment
$A = \frac{1}{2}(a+b)h$	Write the formula.
$2A = (a+b)h$	Multiply both sides by 2.
$\frac{2A}{a+b} = h$	Divide both sides by $(a+b)$.
$h = \frac{2A}{a+b}$	Rewrite with the subject on the left.
$h = \frac{2 \times 34.5}{5 + 6.5}$	Substitute.
$h = \frac{69}{11.5}$	Evaluate top and bottom.
$h = 6$	
$\therefore$ the garden bed is $6\,\text{m}$ high.	Check: $\frac{1}{2}(5+6.5) \times 6 = 34.5$ ✓

Example 3 — The radius of a sphere with a given volume

A sphere has volume $500\,\text{cm}^3$. Make r the subject of $V = \frac{4}{3}\pi r^3$, then find r correct to one decimal place.



A sphere with centre marked, radius r drawn as a dashed line, and volume V = 500 cm³.

Working	Comment
$V = \frac{4}{3}\pi r^3$	Write the formula.
$3V = 4\pi r^3$	Multiply both sides by 3.
$\frac{3V}{4\pi} = r^3$	Divide both sides by 4π.
$r = \sqrt[3]{\frac{3V}{4\pi}}$	Take the cube root of both sides; r is positive.
$r^3 = \frac{3\times 500}{4\pi} = \frac{375}{\pi} = 119.366\ldots$	Substitute $V = 500$.
$r = 4.9237\ldots$	Cube root.
$\therefore r \approx 4.9\,\text{cm}$ (to one decimal place).	Check: $\frac{4}{3}\pi(4.9237\ldots)^3 = 500$ ✓

Practice questions

Scaffolded

Q1. Substitute the given values and evaluate: (a) $T = a + 4b$, with $a = 3$ and $b = 5$; (b) $A = \pi r^2$, with $r = 4$ (leave your answer in terms of π); (c) $E = \frac{1}{2}mv^2$, with $m = 6$ and $v = 5$.

Q2. The volume of a rectangular prism is $V = lwh$. Given $V = 8.4\,\text{m}^3$, $l = 2\,\text{m}$ and $w = 1.5\,\text{m}$, substitute into the formula and solve the resulting equation for h.

Q3. The perimeter of a rectangle is $P = 2(l + w)$. Given $P = 34\,\text{m}$ and $l = 10.5\,\text{m}$, find w. (Divide both sides by 2 first.)

Q4. The area of a triangle is $A = \frac{1}{2}bh$. Given $A = 42\,\text{cm}^2$ and $h = 7\,\text{cm}$, find b.

Q5. Use $v = u + at$ with $u = -3$, $a = 2$ and $t = 4.5$. Substitute the negative value in brackets and find v.

Q6. The formula $F = \frac{9}{5}C + 32$ converts Celsius to Fahrenheit. Find F when $C = 40$.

Q7. Average speed is $s = \frac{d}{t}$. Find s in km/h when $d = 126\,\text{km}$ and $t = 3.5\,\text{h}$.

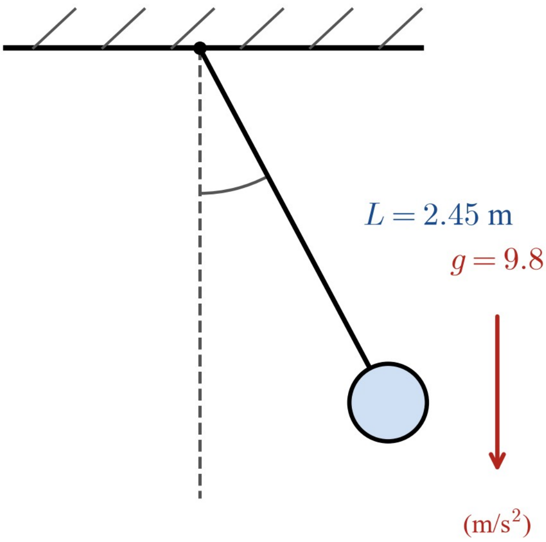
Q8. Simple interest is $I = \frac{Prt}{100}$, where r is the percentage rate per year. An investment of $\$1500$ earns $\$270$ interest over 3 years. Find the rate r per year.

Unscaffolded

Q9. Use $v = u + at$ with $u = -2.5$, $a = 1.6$ and $t = 4$ to find v.

Q10. Find the exact area of a circle of radius $2.5\,\text{cm}$, in terms of π, and then the area correct to one decimal place.

Q11. The period of a pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$ seconds, where L is the length in metres and $g = 9.8$. Find T exactly, and correct to two decimal places, when $L = 2.45\,\text{m}$.



A pendulum of length L = 2.45 m hanging from a fixed support, swinging beside the vertical, with g = 9.8 m/s² shown as a downward arrow.

Q12. Make x the subject of $ax - b = cx + d$, assuming $a \neq c$.

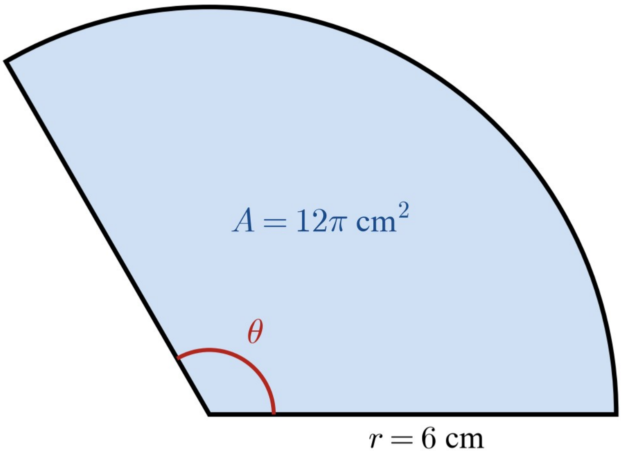
Q13. The area of a trapezium is $A = \frac{1}{2}(a + b)h$. Make a the subject, then find a when $A = 30$, $b = 7$ and $h = 4$.

Q14. Prices in Australia include a 10% goods and services tax (GST), so the price P and the pre-tax price E are related by $P = 1.1E$. (a) A jacket sells for $\$385$. Find the pre-tax price E. (b) A pair of shoes has a pre-tax price of $\$120$. Find the price P.

Q15. A car uses $c = 7.5$ litres of fuel per $100\,\text{km}$, so the fuel F in litres for a trip of d km is $F = \frac{dc}{100}$. (a) Find the fuel used for a $350\,\text{km}$ trip. (b) The car's tank holds 42 litres of usable fuel. Find the distance the car can travel on it.

Q16. A sphere has volume $288\pi\,\text{cm}^3$. Show that its radius is exactly $6\,\text{cm}$.

Q17. The area of a sector of angle θ (in degrees) is $A=\frac{\theta}{360}\pi r^2$. A sector of radius $6\,\text{cm}$ has area $12\pi\,\text{cm}^2$. Find θ.



A sector of a circle with radius r = 6 cm, angle θ at the centre, and area A = 12π cm².

Q18. Use $v^2=u^2+2as$ with $u=3$, $v=7$ and $a=4$ to find s.

Q19. A cone has volume $12\pi\,\text{cm}^3$ and radius $2\,\text{cm}$. Use $V=\frac{1}{3}\pi r^2 h$ (the cone in the figure in section 1) to find its height h.

Q20. A ring between two circles with the same centre, outer radius R and inner radius r, has area $A=\pi\left(R^2-r^2\right)$. Given $A=20\pi\,\text{cm}^2$ and $r=2\,\text{cm}$, find R exactly, as a surd in simplest form.

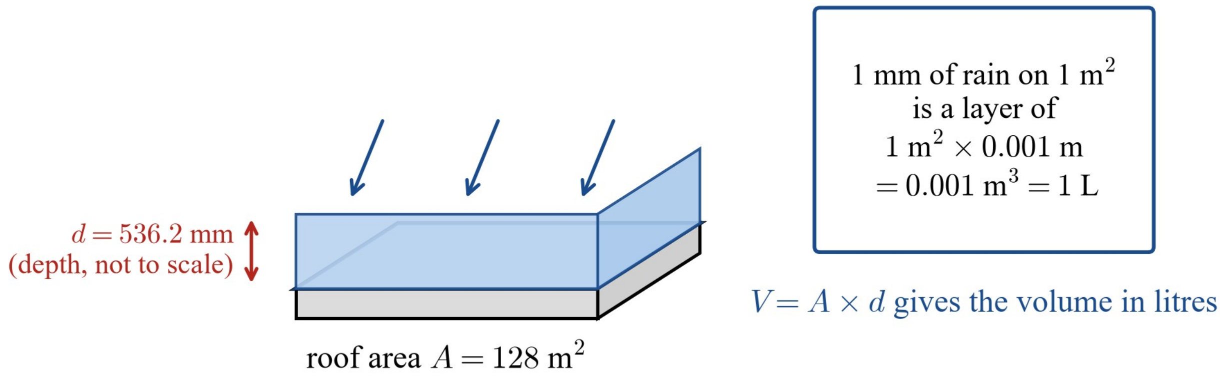
Q21. The formula $F=\frac{9}{5}C+32$ converts Celsius to Fahrenheit. (a) Find F when $C=25$. (b) Make C the subject, and hence find C when $F=14$.

Q22. The variables x and y are related by $y=kx^2$. (a) Given $y=45$ when $x=3$, find k. (b) Make x the subject (take x positive), and hence find x when $y=180$.

Real context

Q23. Rain falling on a roof can be collected in a tank. The mean annual rainfall at Melbourne Airport is $536.2\,\text{mm}$, and $1\,\text{mm}$ of rain on $1\,\text{m}^2$ of roof is $1\,\text{L}$ of water, so the volume collected in litres is $V=A\times d$, where A is the roof area in m^2 and d is the rainfall depth in mm.

Rainfall on a flat roof: volume $=$ area $\times$ depth



Rain falling on a flat roof of area A = 128 m² to a depth d = 536.2 mm, with an inset showing that 1 mm of rain on 1 m² is 1 L.

(a) Explain why $V = A \times d$ gives the volume in litres when d is measured in mm.

(b) A house has a flat roof of area $128\,\mathrm{m}^2$. Find the volume of rain it collects in a mean year, in litres and in kilolitres correct to one decimal place.

(c) Make A the subject of $V = A \times d$, and hence find the roof area needed to collect $50\,000\,\mathrm{L}$ in a mean year, correct to one decimal place.

(d) A second house has a roof area of $95\,\mathrm{m}^2$. Find the volume it collects in a mean year, in kilolitres.

Source: Bureau of Meteorology, *Climate statistics for Australian locations — Melbourne Airport (station 086282)*, 2026. Retrieved 20 August 2026. (bom.gov.au/climate/averages/tables/cw_086282.shtml)

Answers

Q1. (a) 23; (b) 16π; (c) 75. **Q2.** $2.8\,\text{m}$. **Q3.** $6.5\,\text{m}$. **Q4.** $12\,\text{cm}$. **Q5.** 6. **Q6.** 104. **Q7.** $36\,\text{km/h}$. **Q8.** $6\,\%$ per year. **Q9.** 3.9. **Q10.** $6.25\pi\,\text{cm}^2$; $19.6\,\text{cm}^2$. **Q11.** π s; 3.14 s. **Q12.** $x=\dfrac{b+d}{a-c}$. **Q13.** $a=\dfrac{2A-bh}{h}$; $a=8$. **Q14.** (a) $\$350$; (b) $\$132$. **Q15.** (a) $26.25\,\text{L}$; (b) $560\,\text{km}$. **Q16.** $r=6\,\text{cm}$. **Q17.** $120\,°$. **Q18.** 5. **Q19.** $9\,\text{cm}$. **Q20.** $2\sqrt{6}\,\text{cm}$. **Q21.** (a) 77; (b) -10. **Q22.** (a) 5; (b) 6. **Q23.** (b) $68\,633.6\,\text{L}=68.6\,\text{kL}$; (c) $93.2\,\text{m}^2$; (d) $50.939\,\text{kL}$.

Fully-worked solutions

Q1. (a) $T=3+4\times 5$

$T=3+20$

$T=23$

(b) $A=\pi\times 4^2$

$A=16\pi$

(c) $E=\dfrac{1}{2}\times 6\times 5^2$

$E=\dfrac{1}{2}\times 6\times 25$

$E=75$

Q2. $8.4=2\times 1.5\times h$

$8.4=3h$

$h=8.4\div 3$

$h=2.8\,\text{m}$

Q3. $34=2(10.5+w)$

$17=10.5+w$ (divide both sides by 2)

$w=6.5\,\text{m}$

Q4. $42=\dfrac{1}{2}\times b\times 7$

$84=7b$ (multiply both sides by 2)

$b=12\,\text{cm}$

Q5. $v=(-3)+2\times 4.5$

$v=-3+9$

$v=6$

Q6. $F=\dfrac{9}{5}\times 40+32$

$F=72+32$

$F = 104$

Q7. $s = 126 \div 3.5$

$s = 36$ km/h

Q8. $270 = \dfrac{1500 \times r \times 3}{100}$

$270 = 45r$

$r = 6$, so the rate is 6% per year.

Q9. $v = (-2.5) + 1.6 \times 4$

$v = -2.5 + 6.4$

$v = 3.9$

Q10. $A = \pi \times 2.5^2$

$A = 6.25\pi\,\text{cm}^2$ exactly

$A = 6.25\pi = 19.6349\ldots \approx 19.6\,\text{cm}^2$ (correct to one decimal place)

Q11. $\dfrac{L}{g} = \dfrac{2.45}{9.8} = \dfrac{1}{4}$

$T = 2\pi\sqrt{\dfrac{1}{4}} = 2\pi \times \dfrac{1}{2} = \pi$ s exactly

$T = \pi = 3.1415\ldots \approx 3.14$ s (correct to two decimal places)

Q12. $ax - b = cx + d$

$ax - cx = b + d$ (add b and subtract cx on both sides)

$(a - c)x = b + d$

$x = \dfrac{b + d}{a - c}$ (divide by $a - c$; this is allowed since $a \neq c$)

Q13. $A = \dfrac{1}{2}(a + b)h$

$2A = (a + b)h$

$\dfrac{2A}{h} = a + b$

$a = \dfrac{2A}{h} - b$, which is $a = \dfrac{2A - bh}{h}$.

Substitute: $a = \dfrac{2 \times 30 - 7 \times 4}{4} = \dfrac{60 - 28}{4} = \dfrac{32}{4} = 8$

Q14. (a) $385 = 1.1E$

$E = 385 \div 1.1$

$E = \$350$

(b) $P = 1.1 \times 120$

$P = \$132$

Q15. (a) $F = \dfrac{350 \times 7.5}{100}$

$F = \dfrac{2625}{100} = 26.25\,\text{L}$

(b) $42 = \dfrac{d \times 7.5}{100}$

$4200 = 7.5\,d$

$d = 560\,\text{km}$

Q16. $288\,\pi = \dfrac{4}{3}\pi\,r^3$

$288 = \dfrac{4}{3}r^3$ (divide both sides by π)

$r^3 = 288 \times \dfrac{3}{4} = 216$

$r = \sqrt[3]{216} = 6\,\text{cm}$, as required.

Q17. $12\,\pi = \dfrac{\theta}{360} \times \pi \times 6^2$

$12\,\pi = \dfrac{\theta}{360} \times 36\,\pi$

$12 = \dfrac{36\,\theta}{360} = \dfrac{\theta}{10}$ (divide both sides by π)

$\theta = 120\,°$

Q18. $7^2 = 3^2 + 2 \times 4 \times s$

$49 = 9 + 8\,s$

$40 = 8\,s$

$s = 5$

Q19. $12\,\pi = \dfrac{1}{3} \times \pi \times 2^2 \times h$

$12\,\pi = \dfrac{4\,\pi}{3}h$

$h = 12\,\pi \times \dfrac{3}{4\,\pi} = 9\,\text{cm}$

Q20. $20\,\pi = \pi\left(R^2 - 2^2\right)$

$20 = R^2 - 4$ (divide both sides by π)

$R^2 = 24$

$R = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}\,\text{cm}$ (R is positive)

Q21. (a) $F = \dfrac{9}{5} \times 25 + 32$

$F = 45 + 32$

$F = 77$

(b) $F = \dfrac{9}{5}C + 32$

$F - 32 = \dfrac{9}{5}C$

$C = \dfrac{5}{9}(F - 32)$

$C = \dfrac{5}{9}(14 - 32) = \dfrac{5}{9} \times (-18) = -10$

Q22. (a) $45 = k \times 3^2$

$45 = 9k$

$k = 5$

(b) $y = 5x^2$

$x^2 = \dfrac{y}{5}$

$x = \sqrt{\dfrac{y}{5}}$ (x positive)

$x = \sqrt{\dfrac{180}{5}} = \sqrt{36} = 6$

Q23. (a) A depth of d mm is $\dfrac{d}{1000}$ m, so the volume over $A\,\text{m}^2$ is $A \times \dfrac{d}{1000}\,\text{m}^3$. One cubic metre is $1000\,\text{L}$, so the volume in litres is $A \times \dfrac{d}{1000} \times 1000 = A \times d$.

(b) $V = 128 \times 536.2$

$V = 68\,633.6\,\text{L}$

$V = 68.6336\,\text{kL} \approx 68.6\,\text{kL}$ (correct to one decimal place)

(c) $V = A \times d$

$A = \dfrac{V}{d}$

$A = \dfrac{50\,000}{536.2} = 93.2487\ldots \approx 93.2\,\text{m}^2$

(d) $V = 95 \times 536.2$

$V = 50\,939\,\text{L} = 50.939\,\text{kL}$

Teacher note

Level 9 codes rehearsed (D3): `VC2M9A03` — solving linear equations (every item that substitutes then solves, from Example 1 through Q9–Q22); `VC2M9A06` — simple interest (Q8). Both are rehearsed, not re-taught, and are not assessed as Level 9 outcomes.

Tool classes (D13): `VC2M10A05` and both its elaborations name no digital tool, so no `DIGITAL TOOL` banner appears in this subtopic (R8). A scientific calculator is assumed for the π, root and cube-root arithmetic; the rearrangements themselves need no tool at all.

VCE destinations (§5, reference only): the Units 3 & 4 skill indexes of Foundation Mathematics, General Mathematics and Mathematical Methods all name `VC2M10A05`, with the skill *"Rearrange a formula to make a specified variable the subject"*. VCE Physics likewise assumes transposition (`VC2S10U17`, e.g. rearranging $F = ma$) — 1E is one short subtopic carrying a cross-subject and cross-study dependency, and it must not be compressed because it looks like Year 9 revision.

Level check (D6): rearrangements here are linear in the targeted quantity, or undo a square or cube by a single root with the positive-root convention for lengths; no logarithms, no function notation, no VCE content. Quadratic-expression manipulation and factorisation named in the achievement standard extract are carried by 1B–1D, not here.

Test T1 — Number and algebraic manipulation (Level 10)

Mathematics 10 · Chapter 1 Number and algebraic manipulation · Topic test T1 (Level 10) · Chapter 1 · 30 marks · 35 minutes

> This is a school-designed paper. It is not a VCAA instrument. Achievement in the Victorian Curriculum is reported against the Level 10 achievement standard, not against a mark on this paper.

Covers: Chapter 1 Number and algebraic manipulation (1A–1E). **CDs scored:** the six content descriptions of Chapter 1, at 5 marks each:

CD code	Subtopic	Marks	Items
`VC2M10N01`	1A	5	Q11(c) (1), Q11(d) (1), Q12(c) (2), Q12(d) (1)
`VC2M10A01`	1B	5	Q2 (3), Q7 (1), Q13 (1)
`VC2M10A02`	1B	5	Q1 (1), Q3 (2), Q8 (2)
`VC2M10A03`	1C	5	Q11(a) (1), Q11(b) (2), Q14 (2)
`VC2M10A04`	1D	5	Q4 (2), Q9 (1), Q10 (1), Q15 (1)
`VC2M10A05`	1E	5	Q5 (1), Q6 (1), Q12(a) (1), Q12(b) (1), Q12(d) (1)

The two codes of subtopic 1B are scored separately: `VC2M10A01` items are tagged `VC2M10A01` and `VC2M10A02` items are tagged `VC2M10A02`; no item carries both codes.

Timing. 35 minutes for the whole test: Parts A and B together.

Equipment and calculator rules.

- **Part A — technology-free.** No calculator and no digital tool. Give exact values unless a question asks for a decimal answer. This part is collected before Part B is issued.
- **Part B — technology-active.** You may use the technology your teacher nominates: a scientific calculator, a spreadsheet, a graphing tool, dynamic geometry software or a general purpose programming language. **No CAS.** Everything in Part B can also be done by hand: a hand-calculated response is a complete response.

Answer every question. Show your working clearly wherever marks are allocated to method — one step per line. Every word on this test was introduced in Chapter 1.

Part A — Technology-free (20 marks)

No calculator. No digital tools. Exact values unless a question says otherwise.

Q1. `VC2M10A02` (1 mark) Multiple choice. Choose A, B, C or D. The simplest form of $(3x^2)^3 \div 9x^4$ is

A $3x^2$

B $3x^3$

C $9x^2$

D $27x^2$

Q2. `VC2M10A01` (3 marks) Factorise each expression by taking out the highest common factor.

(a) $15a^2b + 10ab^2$

(b) $8x^3-20x^2+4x$

(c) $-12m^2n+18mn^2-6mn$

Q3. `VC2M10A02` (2 marks) Simplify each expression. Write your answers using positive exponents only.

(a) $3x^{-2}y^4\times 5x^5y^{-1}$

(b) $6a^4b\div 2a^6b^5$

Q4. `VC2M10A04` (2 marks)

(a) Expand $(2x-3)(x+5)$.

(b) Factorise $x^2+8x+12$ by completing the square.

Q5. `VC2M10A05` (1 mark) Make t the subject of $v=u+at$.

Q6. `VC2M10A05` (1 mark) The area of a trapezium is $A=\frac{1}{2}(a+b)h$. Make h the subject.

Q7. `VC2M10A01` (1 mark) Factorise completely: $24p^3q^2-36p^2q^3$.

Q8. `VC2M10A02` (2 marks) Simplify.

(a) $(4m^2n)^3\div 8m^5n^6$

(b) $(5x^2)^2\times 2x^{-3}$

Q9. `VC2M10A04` (1 mark) Factorise by grouping in pairs: $x^3+3x^2+5x+15$.

Q10. `VC2M10A04` (1 mark) Factorise: $7x(x-4)+3(x-4)$.

Q11. (5 marks) The Yarra River is approximately 242 km long and flows through Melbourne. `SCENARIO` A paddler whose speed in still water is v km/h paddles the whole 242 km upstream against a current of 2 km/h, then paddles the whole 242 km back downstream with the current.

(a) `VC2M10A03` Write, as algebraic fractions in v, the time taken for (i) the upstream trip and
(ii) the downstream trip.
(b) `VC2M10A03` Find how much longer the upstream trip takes than the downstream trip. Give your answer as a single fraction in v.

(c) `VC2M10N01` When $v=22$, find the exact value of your fraction from part (b), in simplest form.

(d) `VC2M10N01` Ana converts the exact fraction from part (c) to minutes. Ben rounds the fraction to 1 decimal place first, then converts to minutes. By how many minutes do their answers differ?

Source: Melbourne Water, *Yarra River* (length 242 km), 2026. Retrieved 23 August 2026. (melbournewater.com.au)

Total for Part A: □ out of 20 marks

Part B — Technology-active (10 marks)

You may use the technology your teacher nominates: a scientific calculator, a spreadsheet, a graphing tool, dynamic geometry software or a general purpose programming language. No CAS. Show your working — technology finds values; your working shows where they came from.

Q12. (6 marks) `SCENARIO` A Victorian school installs a cylindrical rainwater tank beside its hall. The tank has radius 1.1 m and height 1.9 m. The volume of a cylinder of radius r and height h is $V=\pi r^2h$.

(a) VC2M10A05 Substitute into the formula and find the exact volume of the tank, as a multiple of π.

(b) VC2M10A05 Use the π key on your calculator to find the volume correct to 2 decimal places.

(c) VC2M10N01 The maintenance guide rounds the tank's volume to 7.2 m³, and the school uses the guide to calculate the water held by 6 full tanks as 6×7.2. Find the exact volume of water in 6 full tanks, as a multiple of π, and as a decimal correct to 2 decimal places. Hence find how much less the guide's answer is than the exact volume, correct to 2 decimal places.

(d) The tank is filled by rain from the hall roof, a rectangle 24 m by 13 m. The mean annual rainfall at Melbourne Airport is 536.2 mm, a depth of 0.5362 m, and the volume of rain collected is $V = A \times d$, where A is the roof area in m² and d is the rainfall depth in m. Assume all rain that falls on the roof is collected.

(i) VC2M10A05 Find the volume of rain the roof collects in a mean year.

(ii) VC2M10N01 Hence find the number of complete tanks of water the roof fills in a mean year. The exact quotient is not a whole number — give the number of complete tanks.

Source: Bureau of Meteorology, *Climate statistics for Australian locations — Melbourne Airport (station 086282)*, 2026. Retrieved 20 August 2026. (bom.gov.au/climate/averages/tables/cw_086282.shtml)

Q13. VC2M10A01 (1 mark) Factorise completely: $18x^3y - 27x^2y^2$.

Q14. VC2M10A03 (2 marks) Express $\frac{3}{x-2} - \frac{2}{x+1}$ as a single fraction in simplest form.

Q15. VC2M10A04 (1 mark) Multiple choice. Choose A, B, C or D. The factorised form of $x^2 - 6x + 9$ is

A $(x-3)^2$

B $(x+3)^2$

C $(x-3)(x+3)$

D $(x-9)(x+1)$

Total for Part B: □ out of 10 marks

Total for Test T1: □ out of 30 marks

Solutions

Teacher-facing. This is a school-designed classroom test, not a VCAA instrument and not an examination (§7, D19). Marks exist for a school's own reporting; achievement in the Victorian Curriculum is reported against the Level 10 achievement standard. Score method marks for working shown, one step per line (R5); accept any correct method, including methods different from the one shown. Follow-through marks are available in Q11(d) and Q12(d)(ii) where a later part uses an earlier result.

Achievement-standard extracts. The mark scheme is written in the language of the achievement-standard extracts carried in the authority file (`maths_10.json`), quoted verbatim here and tagged to each scored item below:

> **AS-1** — *students recognise the effect of approximations of real numbers in repeated calculations* — `VC2M10N01`
>
> **AS-2** — *Students substitute into formulas, find unknown values, manipulate linear and quadratic algebraic expressions, expand binomial expressions and factorise monic and simple non-monic quadratic expressions, with and without the use of digital tools.* — `VC2M10A01`, `VC2M10A02`, `VC2M10A03`, `VC2M10A04`, `VC2M10A05` (shared wording; the five CDs are still scored separately, §7)

Coverage. Every CD of Chapter 1 scores 5 marks (6 CDs, 30 marks). The 1B pair is scored per code, never on one shared item (D8, §7.1): `VC2M10A01` — Q2 (3) + Q7 (1) + Q13 (1) = 5; `VC2M10A02` — Q1 (1) + Q3 (2) + Q8 (2) = 5.

Answers

Final values only.

Q1. A. **Q2.** (a) $5ab(3a+2b)$; (b) $4x(2x^2-5x+1)$; (c) $-6mn(2m-3n+1)$. **Q3.** (a) $15x^3y^3$; (b) $\frac{3}{a^2b^4}$. **Q4.** (a) $2x^2+7x-15$; (b) $(x+2)(x+6)$. **Q5.** $t=\frac{v-u}{a}$. **Q6.** $h=\frac{2A}{a+b}$. **Q7.** $12p^2q^2(2p-3q)$. **Q8.** (a) $\frac{8m}{n^3}$; (b) $50x$. **Q9.** $(x+3)(x^2+5)$. **Q10.** $(x-4)(7x+3)$. **Q11.** (a) $\frac{242}{v-2}$ hours, $\frac{242}{v+2}$ hours; (b) $\frac{968}{v^2-4}$; (c) $\frac{121}{60}$ hours; (d) 1 minute. **Q12.** (a) 2.299π m³; (b) 7.22 m³; (c) 13.794π m³ ≈ 43.34 m³, the guide's answer is 0.14 m³ less; (d) (i) 167.2944 m³; (ii) 23 complete tanks. **Q13.** $9x^2y(2x-3y)$. **Q14.** $\frac{x+7}{(x-2)(x+1)}$. **Q15.** A.

Fully-worked solutions

Q1. `VC2M10A02` — AS-2. 1 mark; no method required.

A. Power of a power, then the quotient law:

$(3x^2)^3=27x^6$

$27x^6\div 9x^4=3x^{6-4}=3x^2$

Option B slips the exponent subtraction; C divides the coefficient by 3 instead of 9; D subtracts the exponents but forgets to divide the coefficient.

Q2. `VC2M10A01` — AS-2. 3 marks; 1 mark each.

(a) Highest common factor $5ab$:

$15a^2b+10ab^2=5ab(3a+2b)$

(b) Highest common factor $4x$:

$8x^3-20x^2+4x=4x(2x^2-5x+1)$

(The last term is $+1$, since $4x\times 1=4x$ — a missing constant term is the common error here.)

(c) Highest common factor $-6mn$ — taking out the negative factor leaves the first term inside the brackets positive:

$-12m^2n+18mn^2-6mn=-6mn(2m-3n+1)$

Q3. `VC2M10A02` — AS-2. 2 marks; 1 mark each.

(a) Multiply the coefficients and add the exponents of like bases:

$3x^{-2}y^4\times 5x^5y^{-1}=15x^{-2+5}y^{4-1}$

$=15x^3y^3$

(b) Divide the coefficients and subtract the exponents; the bases with negative exponents move to the denominator:

$\frac{6a^4b}{2a^6b^5}=\frac{3}{a^2b^4}$

Q4. `VC2M10A04` — AS-2. 2 marks; 1 mark each.

(a) Every term in the first bracket multiplies every term in the second:

$(2x-3)(x+5)=2x^2+10x-3x-15$

$=2x^2+7x-15$

(b) Completing the square (1D): half of 8 is 4, so

$x^2+8x+12=(x+4)^2-16+12$

$=(x+4)^2-4$

and the difference of two squares finishes the factorisation:

$(x+4)^2-2^2=(x+4-2)(x+4+2)$

$=(x+2)(x+6)$

Q5. `VC2M10A05` — AS-2. 1 mark.

$v=u+at$

$v-u=at$

$\therefore t=\frac{v-u}{a}$

Q6. `VC2M10A05` — AS-2. 1 mark.

$A=\frac{1}{2}(a+b)h$

$2A=(a+b)h$

$\therefore h=\frac{2A}{a+b}$

Q7. `VC2M10A01` — AS-2. 1 mark. Highest common factor $12p^2q^2$:

$24p^3q^2-36p^2q^3=12p^2q^2(2p-3q)$

Q8. `VC2M10A02` — AS-2. 2 marks; 1 mark each.

(a) $(4m^2n)^3=64m^6n^3$

$\frac{64m^6n^3}{8m^5n^6}=\frac{8m}{n^3}$

(b) $(5x^2)^2=25x^4$

$25x^4\times 2x^{-3}=50x^{4-3}=50x$

Q9. `VC2M10A04` — AS-2. 1 mark. Group in pairs (1D):

$x^3+3x^2+5x+15=x^2(x+3)+5(x+3)$

$=(x+3)(x^2+5)$

Q10. `VC2M10A04` — AS-2. 1 mark. The binomial $(x-4)$ is the common factor:

$7x(x-4)+3(x-4)=(x-4)(7x+3)$

Q11. `VC2M10A03` (a), (b) — AS-2; `VC2M10N01` (c), (d) — AS-1. 5 marks; the real-context multi-part question of Part A (§7.1), on the Yarra River.

(a) 1 mark. Against the current the paddler's speed is $v-2$ km/h; with the current it is $v+2$ km/h. Time is distance divided by speed:

(b) upstream: $\dfrac{242}{v-2}$ hours

(ii) downstream: $\dfrac{242}{v+2}$ hours

(b) 2 marks: 1 for the common denominator, 1 for the correct numerator. Subtract the downstream time from the upstream time:

$$\frac{242}{v-2}-\frac{242}{v+2}$$

$$=\frac{242(v+2)}{(v-2)(v+2)}-\frac{242(v-2)}{(v-2)(v+2)}$$

$$=\frac{242v+484-242v+484}{(v-2)(v+2)}$$

$=\dfrac{968}{v^2-4}$ — the denominator is a difference of two squares.

(c) 1 mark. With $v=22$: $v^2-4=484-4=480$, so

$\dfrac{968}{480}=\dfrac{121}{60}$ hours — exact, dividing top and bottom by 8.

(d) 1 mark. Ana converts the exact value:

$\dfrac{121}{60}\times 60=121$ minutes

Ben rounds first: $\dfrac{121}{60}=2.0166\ldots\approx 2.0$ hours to 1 decimal place, and $2.0\times 60=120$ minutes. Their answers differ by 1 **minute** — rounding before the final step changed the result (1A). Follow-through: award the mark for a correct comparison of the student's own part (c) fraction with its rounded value.

Q12. `VC2M10A05` (a), (b), (d)(i) — AS-2; `VC2M10N01` (c), (d)(ii) — AS-1. 6 marks; the real-context multi-part question of Part B (§7.1), on the Bureau of Meteorology rainfall figure 1E Q23 carries.

(a) 1 mark. Substitute $r=1.1$ and $h=1.9$ into $V=\pi r^2 h$, keeping the exact value:

$V=\pi\times 1.1^2\times 1.9$

$V=\pi\times 1.21\times 1.9$

$V=2.299\pi$ m³

(b) 1 mark. With the π key:

$2.299\pi=7.2225\ldots$

$\therefore V\approx 7.22$ m³ (correct to 2 decimal places)

(c) 2 marks: 1 for the exact volume of 6 tanks, 1 for the difference. The guide's figure for 6 tanks is $6\times 7.2=43.2$ m³. The exact volume is

$$6\times 2.299\pi=13.794\pi \text{ m}^3$$

$$13.794\pi=43.3351\ldots\approx 43.34 \text{ m}^3 \text{ (two decimal places)}$$

The guide's answer is less by

$$43.3351\ldots-43.2=0.1351\ldots\approx 0.14 \text{ m}^3$$

Rounding the tank volume once, early, cost 0.14 m³ on the printed total — the effect of approximation in a repeated calculation (1A).

(d)

(i) 1 mark. The roof area is

$$A=24\times 13=312 \text{ m}^2$$

$$V=A\times d=312\times 0.5362$$

$$V=167.2944 \text{ m}^3$$

(ii) 1 mark. Divide the annual volume by the exact tank volume:

$$167.2944\div 2.299\pi=23.16\ldots$$

The exact quotient is not a whole number, so the roof fills 23 **complete tanks**, with water to spare. (With the guide's rounded 7.2 m³ the quotient is $167.2944\div 7.2=23.24\ldots$ — still 23; here the rounding does not change the count.) Follow-through: award the mark for a correct whole-number reading of the student's own part (a) volume.

Q13. `VC2M10A01` — AS-2. 1 mark. Highest common factor $9x^2y$:

$$18x^3y-27x^2y^2=9x^2y(2x-3y)$$

Q14. `VC2M10A03` — AS-2. 2 marks: 1 for the numerators over the common denominator, 1 for the single fraction in simplest form.

$$\frac{3}{x-2}-\frac{2}{x+1}=\frac{3(x+1)}{(x-2)(x+1)}-\frac{2(x-2)}{(x-2)(x+1)}$$

$$=\frac{3x+3-2x+4}{(x-2)(x+1)}$$

$$=\frac{x+7}{(x-2)(x+1)}$$

The numerator $x+7$ shares no factor with the denominator, so this is simplest form.

Q15. `VC2M10A04` — AS-2. 1 mark; no method required.

A. $(x-3)^2=x^2-6x+9$ — a perfect square, since -3 and -3 multiply to 9 and add to -6. Option B expands to x^2+6x+9, option C to x^2-9, and option D to x^2-8x-9.

Record the totals: Part A □/20 · Part B □/10 · Test T1 □/30