Physics Units 3 & 4

Chapter 10 — Scientific investigation and data analysis

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Chapter 10 Scientific investigation and data analysis — 10A Measurement, uncertainty and error

Theory

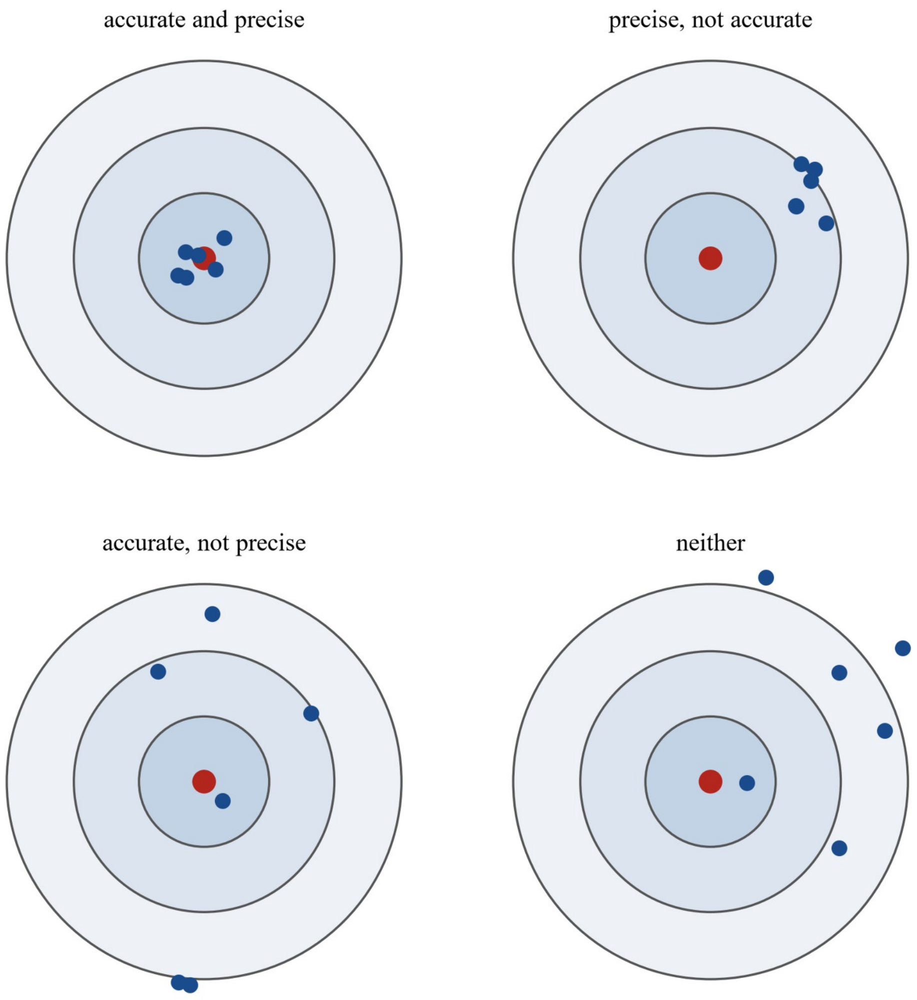
Every measurement in a physics investigation is an *estimate* of a **true value** — the value the quantity actually has. Because no instrument and no observer is perfect, a measured value almost never equals the true value exactly. This subtopic is about describing, in the language the study design prescribes, *how good* a measurement is: how close it is to the true value, how well repeated readings agree, and how the doubt in a result is stated and shown on a graph.

Error and uncertainty are different ideas. An **error** is the difference between a measured value and the true value of the quantity. You can rarely know an error exactly, because you rarely know the true value; instead errors are *classified* by how they behave (below). An **uncertainty** is the range of values within which the true value is expected to lie — it is a statement of the *doubt* attached to a result, and it is written as a ± interval. In short, an error is a single deviation from the true value, while an uncertainty is an interval that quantifies the doubt in a reported result.

Accuracy and precision. These two words are not interchangeable.

- **Accuracy** describes how close a measurement (or the mean of a set of measurements) is to the true value.
- **Precision** describes how close repeated measurements are to *one another* — that is, how small their spread is. Precision says nothing about closeness to the true value.

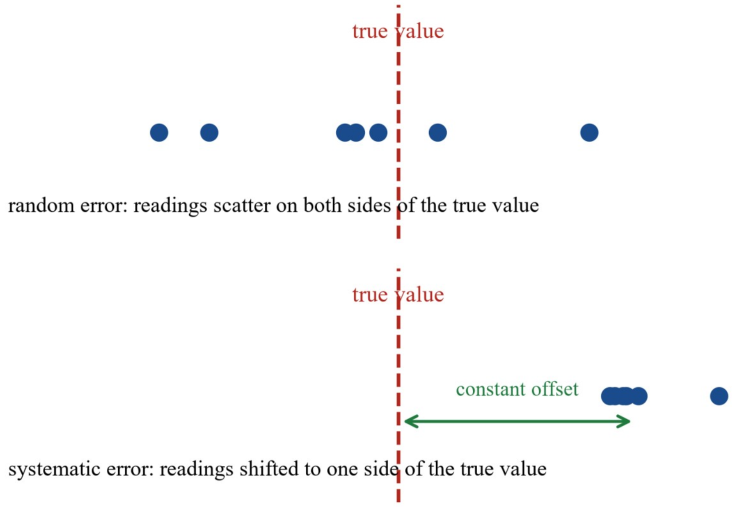
A set of readings can therefore be precise but not accurate, accurate but not precise, both, or neither. The target diagrams below make the distinction concrete: the red centre marks the true value and each blue dot is one measurement.



Tightly grouped darts near the centre are **accurate and precise**; a tight group away from the centre is **precise but not accurate** (a sign of a systematic error); a wide scatter centred on the bullseye is **accurate but not precise** (large random errors); a wide scatter off-centre is **neither**.

Random and systematic errors. Experimental errors are of two kinds, and they affect a result in different ways.

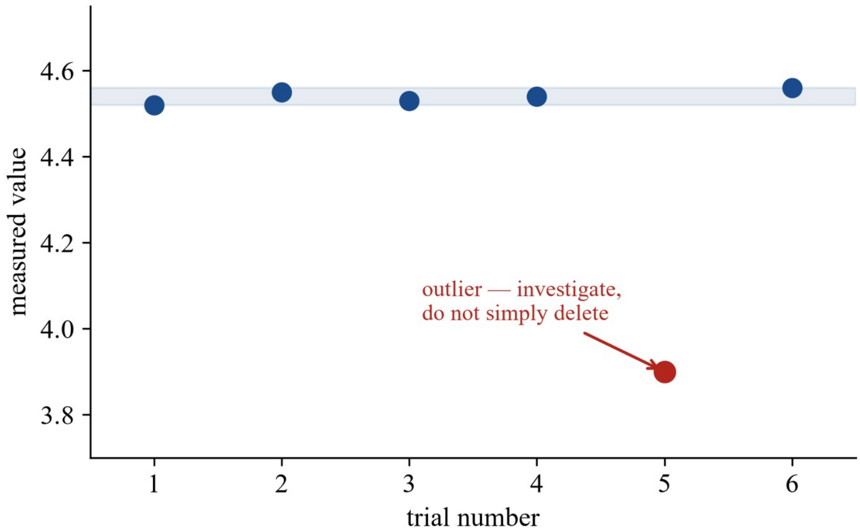
- **Random errors** cause repeated readings to scatter *above and below* the mean by varying, unpredictable amounts. They affect the **precision** (spread) of the data. Typical causes in a physics laboratory are the variation in a person's reaction time when starting and stopping a stopwatch, estimating the last digit of a scale differently each time, small fluctuations in the conditions, and reading an instrument from a slightly different angle on each occasion. The effect of random error on a result is *reduced by taking repeated readings and averaging*.
- **Systematic errors** shift *every* reading by the same amount in the same direction. They affect the **accuracy** of the data but not its precision. Typical causes are a **zero error** (an instrument that does not read zero when the true value is zero — for example a spring balance reading $0.2\,\mathrm{N}$ with nothing attached), a **calibration error** (a wrongly graduated scale), a consistent **parallax** error (always viewing a scale from the same wrong angle), or a method that is consistently flawed (always starting a timer a little late). A systematic error is *not* removed by repeating and averaging; it must be found and corrected — by re-zeroing, re-calibrating, or improving the method.



The two dot plots above show the same idea on a measurement axis: with only random error the readings straddle the true value; with a systematic error the whole group is displaced to one side by a roughly constant offset, no matter how many readings are taken.

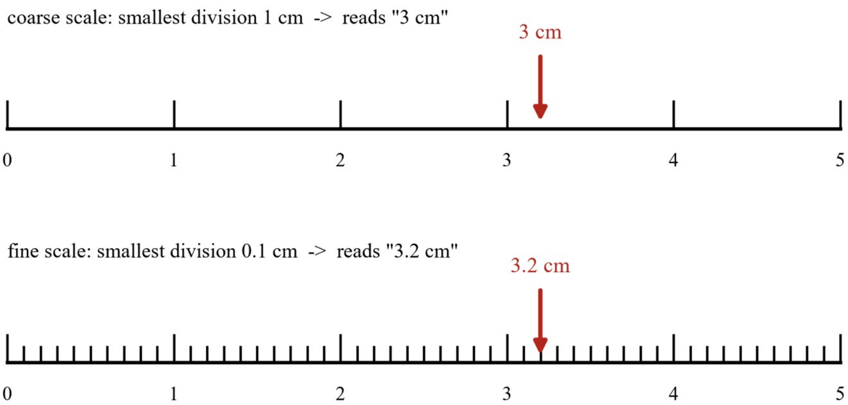
Mistakes are not experimental errors. A **mistake** (or personal error) is a one-off blunder — misreading a scale, miscounting oscillations, writing down the wrong number, or knocking the apparatus. Mistakes are *excluded* from error analysis: they are neither random nor systematic errors. A reading spoilt by a known mistake should be discarded and, where possible, the measurement repeated.

Outliers are analysed, not simply dismissed. An **outlier** is a reading that lies far from the others. An outlier must be *investigated*, not automatically deleted: it may point to a mistake, or it may reveal something real about the experiment. A reading is only excluded once there is a reason to exclude it (for instance, once it is identified as a mistake).



In the run above, five readings cluster together (shaded band) while one near the value 3.9 sits well below the rest. That point is flagged for investigation; it is not crossed out on sight.

Resolution. The **resolution** of an instrument is the smallest change in the measured quantity that it can detect or display. For a scale it is the smallest division; for a digital display it is the increment of the last digit. A finer (smaller) resolution lets an instrument detect smaller changes, and it limits how precisely a single reading can be recorded — and therefore how many significant figures the reading can justify.



Both scales above point to the same place, but the coarse scale (smallest division $1\,\text{cm}$) can only be read as "$3\,\text{cm}$", while the fine scale (smallest division $0.1\,\text{cm}$) resolves the same position as "$3.2\,\text{cm}$".

Repeatability and reproducibility. Both describe how well repeated measurements agree, but under different conditions.

- **Repeatability** is the closeness of agreement between successive measurements of the same quantity carried out under the *same* conditions — the same operator, the same instrument, the same method, the same location, over a short time.
- **Reproducibility** is the closeness of agreement between measurements of the same quantity carried out under *changed* conditions — for example a different operator, a different instrument, a different method, or a different laboratory.

Reproducible results give more confidence than repeatable ones alone: agreement that survives a *change* of operator, instrument or method argues strongly against a hidden systematic error or a flaw peculiar to one method.

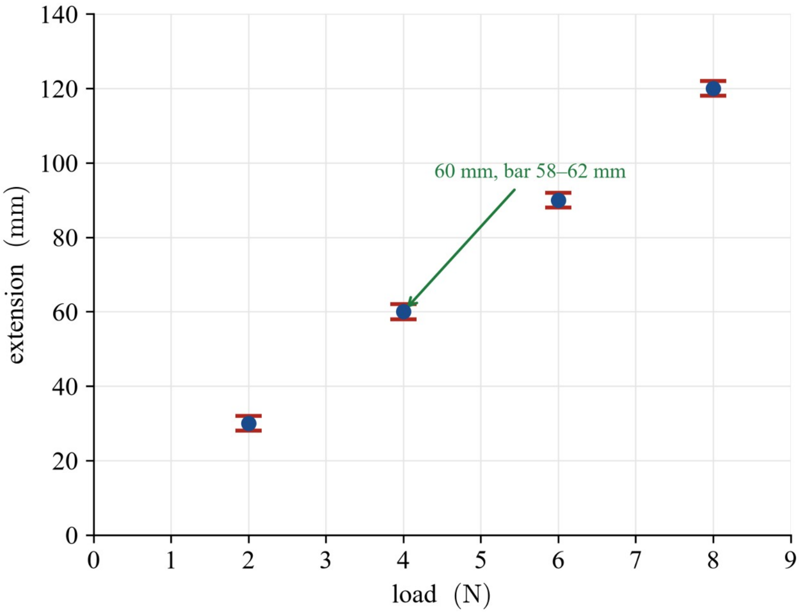
Validity. A measurement or an investigation is **valid** if it actually measures what it is intended to measure. Validity is about whether the method answers the question, and is distinct from the accuracy or precision of individual readings. An investigation loses validity if other variables are not controlled (so the effect measured is not due to the intended cause alone), if the wrong quantity is measured, or if the instrument's resolution is too coarse to detect the effect being studied.

The mean and the "mean ± uncertainty" notation. The best estimate of a quantity from a set of repeated readings is their **mean** (average). The result is reported as a value together with its uncertainty,

$$\text{result} = \text{mean} \pm \text{uncertainty}\,,$$

for example $t = 2.02 \pm 0.01\,\text{s}$, which means the true value is expected to lie between $2.01\,\text{s}$ and $2.03\,\text{s}$. In VCE Physics the numerical uncertainty is *given* — most often it is the resolution of the instrument — and you use it as stated. Determining an uncertainty from the *spread* of a set of measurements is beyond the scope of this course, so you will never be asked to calculate a ± value from how widely the readings scatter.

Uncertainty bars. On a graph, the uncertainty in a plotted value is shown by an **uncertainty bar**. A vertical bar for an uncertainty of $\pm u$ extends a distance u above and u below the point, so its total length is $2u$. Two skills are required, and no more: given a numerical uncertainty you can **draw** the bars, and given a graph you can **read** a value and its uncertainty off a bar.



In the spring graph above each extension was measured with an uncertainty of $\pm 2\,\text{mm}$, so every bar is $4\,\text{mm}$ long. For example, the point at a load of $4.0\,\text{N}$ shows an extension of $60\,\text{mm}$ with a bar reaching from $58\,\text{mm}$ to $62\,\text{mm}$.

Why repeats help. Because random errors are as likely to be positive as negative, they tend to cancel when readings are averaged, so the mean of many readings lies closer to the true value than a typical single reading — and the more repeats are taken, the smaller the effect of random error on the mean. Averaging does **not** reduce a systematic error: a systematic error shifts every reading, and hence the mean itself, by the same amount, so it survives no matter how many repeats are taken.

Worked examples

Example 1 — scaffolded

A student measures the time for a trolley to roll $1.0\,\text{m}$ down a ramp five times, using a stopwatch of resolution $0.01\,\text{s}$, and records 1.52, 1.49, 1.53, 1.50 and $1.18\,\text{s}$. On the fifth run the student stopped the stopwatch before the trolley reached the finish mark, and noted this at the time. The accepted value is $1.50\,\text{s}$. Identify any reading that should not be used and say why, calculate the best estimate of the time, and comment on the accuracy and precision of the valid readings, naming the type of error responsible for their spread.

Working	Comment
Readings: 1.52, 1.49, 1.53, 1.50, $1.18\,\text{s}$ (resolution $0.01\,\text{s}$); accepted value $1.50\,\text{s}$.	List the data and the accepted value.
The $1.18\,\text{s}$ reading lies far below the others — an	Investigate an outlier before acting; a known blunder is a

Working	Comment
outlier. Investigating it, the student had stopped the stopwatch early on that run, so it is a **mistake** (personal error).	mistake, not experimental error.
A mistake is excluded and the trial ideally repeated. Valid readings: 1.52, 1.49, 1.53, 1.50 s.	Mistakes are neither random nor systematic errors.
Mean $=\dfrac{1.52+1.49+1.53+1.50}{4}=\dfrac{6.04}{4}=1.51\text{ s}$.	Best estimate = mean of the valid readings.
Spread of the valid readings is only 1.49–1.53 s (small) ⇒ **precise**; the mean 1.51 s is close to the accepted 1.50 s ⇒ **accurate**.	Precision = agreement of repeats; accuracy = closeness to the true value.
The small scatter about the mean is due to **random error** (reaction-time variation in starting and stopping).	Random error affects precision, not accuracy.

Example 2 — scaffolded

A student measures the extension of a spring under a fixed load of 5.0 N four times with a ruler, obtaining 62, 61, 63 and 62 mm. The ruler's resolution gives each reading an uncertainty of $\pm 1\text{ mm}$. Calculate the mean extension, express the result in mean ± uncertainty notation, state the range in which the true extension is expected to lie, and describe the uncertainty bar that would be drawn for this point.

Working	Comment
Readings 62, 61, 63, 62 mm; stated uncertainty $\pm 1\text{ mm}$ (the ruler's resolution).	The uncertainty is *given* by the instrument, not found from the spread.
Mean $=\dfrac{62+61+63+62}{4}=\dfrac{248}{4}=62\text{ mm}$.	Best estimate.
Result: $62\pm 1\text{ mm}$.	mean ± uncertainty notation.
The true extension is expected to lie between 61 mm and 63 mm.	The ± gives the interval.
Plotted at $(5.0\text{ N}, 62\text{ mm})$, the vertical uncertainty bar runs from 61 mm to 63 mm — a total length $2\times 1=2\text{ mm}$, one division above and one below the point.	A $\pm u$ bar has total length $2u$.

Example 3 — unscaffolded

In a "g by free-fall" experiment the accepted value is 9.81 m s^{-2}. Two groups each report five values (in m s^{-2}):

- Group A: 9.79, 9.82, 9.80, 9.83, 9.81
- Group B: 9.50, 9.52, 9.51, 9.49, 9.50

Discuss the accuracy and precision of each group's results, and identify the kind of error most likely present in Group B's data.

Group A's values agree closely with one another (spread only 9.79–9.83 m s^{-2}), so they are **precise**, and their mean,

$$\acute{a}_{\text{A}}=\frac{9.79+9.82+9.80+9.83+9.81}{5}=\frac{49.05}{5}=9.81\text{ m s}^{-2},$$

equals the accepted value, so they are also **accurate**. Group B's values also agree closely with one another (spread only 9.49–9.52 m s^{-2}), so they too are **precise**; but their mean,

$$\acute{a}_{\text{B}}=\frac{9.50+9.52+9.51+9.49+9.50}{5}=\frac{47.52}{5}=9.504\text{ m s}^{-2},$$

which is $9.50\,\mathrm{m\,s^{-2}}$ to three significant figures, lies well below the accepted $9.81\,\mathrm{m\,s^{-2}}$, so they are **not accurate**. A consistent displacement of every reading in the same direction is the signature of a **systematic error** (for example an uncorrected zero or calibration error, or a timing method that consistently over-reads), which repeating the measurements cannot remove.

∴ Group A is both accurate and precise; Group B is precise but not accurate, its results shifted by a systematic error.

Example 4 — unscaffolded

A group measures the time for a pendulum to complete ten swings five times, using the same stopwatch and the same operator, and the readings scatter by about $\pm 0.3\,\mathrm{s}$ about their mean. State whether this is a test of repeatability or of reproducibility, and explain why averaging the five readings gives a better estimate of the true time than a single reading, stating clearly which type of error this addresses and which it does not.

The five readings are of the same quantity under the *same* conditions — the same stopwatch, the same operator, the same method, over a short time — so this is a test of **repeatability**. (Reproducibility would require *changed* conditions, such as a different operator or a different instrument.) The scatter of about $\pm 0.3\,\mathrm{s}$ is caused by **random error**, chiefly the variation in the operator's reaction time when starting and stopping the watch. Random errors are as likely to make a reading too large as too small, so when the readings are averaged the positive and negative deviations tend to cancel; the mean therefore lies closer to the true time than a typical single reading, and taking more repeats reduces the effect of random error on the mean still further. Averaging does *not* correct a **systematic error**, because such an error shifts every reading — and hence the mean — by the same amount in the same direction.

∴ this is a repeatability test; averaging reduces the effect of random error on the result but leaves any systematic error unchanged.

Practice questions

Question 1 A student measures the diameter of a steel ball five times with digital callipers of resolution $0.01\,\mathrm{mm}$, obtaining 12.44, 12.46, 12.45, 12.44 and $12.46\,\mathrm{mm}$. The accepted value is $12.45\,\mathrm{mm}$. (a) State the resolution of the callipers. (b) Calculate the mean diameter and comment on the precision of the readings. (c) With reference to the accepted value, comment on the accuracy of the result.

Question 2 A student measures the time for a trolley to travel $2.0\,\mathrm{m}$ five times with a hand-operated stopwatch; the readings scatter above and below their mean. A second student later finds that all of the first student's times are $0.20\,\mathrm{s}$ too long, because the watch was always started an instant before the trolley was released. (a) Name the type of error responsible for the scatter of the first student's readings about their mean. (b) Name the type of error responsible for the constant $0.20\,\mathrm{s}$ excess. (c) For each of these two errors, state whether taking more repeats and averaging would reduce its effect on the result.

Question 3 A spring is loaded with $6.0\,\mathrm{N}$ and its extension is measured four times with a ruler, giving 47, 48, 47 and $46\,\mathrm{mm}$. Each reading has an uncertainty of $\pm 1\,\mathrm{mm}$ (the ruler's resolution). (a) Distinguish between the *error* and the *uncertainty* of a single extension reading. (b) Calculate the mean extension and express the result in mean ± uncertainty notation. (c) The result is to be plotted as a point at $(6.0\,\mathrm{N}, 47\,\mathrm{mm})$. State the values at the top and bottom of its vertical uncertainty bar.

Question 4 In a photoelectric experiment a student measures the stopping voltage five times with a digital voltmeter of resolution $0.01\,\mathrm{V}$, recording 1.42, 1.45, 1.44, 1.45 and $0.95\,\mathrm{V}$. (a) Identify the outlier. (b) The student recalls bumping the apparatus while taking the fifth reading. State whether the $0.95\,\mathrm{V}$ reading is best treated as a random error, a systematic error or a mistake, and say what should be done with it. (c) Calculate the best estimate of the stopping voltage from the valid readings.

Question 5 A transformer's secondary voltage is about $12\,\mathrm{V}$. Voltmeter X can be read to the nearest $1\,\mathrm{V}$; voltmeter Y can be read to the nearest $0.1\,\mathrm{V}$. (a) State the resolution of each voltmeter. (b) State which voltmeter has the finer resolution, and explain what this means for the smallest change in voltage it can detect. (c) A student uses voltmeter X to investigate how the secondary voltage changes when one extra turn is added to the secondary coil, a change of about $0.5\,\mathrm{V}$. Explain why this choice threatens the validity of the investigation.

Question 6 In Scenario 1, one student measures the focal length of a lens five times with the same ruler in a single session. In Scenario 2, three different students in three different laboratories each measure the focal length of the same lens with their own equipment, and obtain closely agreeing results. (a) (Circle your answer.) Scenario 1 tests the *repeatability* / *reproducibility* of the measurement. Justify your choice. (b) (Circle your answer.) Scenario 2 tests the *repeatability* / *reproducibility* of the measurement. Justify your choice. (c) Explain why the close agreement found in Scenario 2 gives more confidence in the measured focal length than repeated agreement by a single student would.

Question 7 A student states: "My five readings are all within $0.1\,\mathrm{s}$ of each other, so my result must be accurate." Explain why this reasoning is flawed, using the terms *precision* and *accuracy* correctly.

Question 8 A spring balance reads $0.2\,\mathrm{N}$ when nothing is hanging from it, and a student uses it to measure several forces without noticing. Name the type of error this introduces, state whether repeating the measurements would remove it, and describe its effect on the accuracy and on the precision of the recorded forces.

Question 9 Explain, as a step-by-step chain of reasoning, why averaging a large number of repeated readings reduces the effect of random errors on the final result, but has no effect on a systematic error.

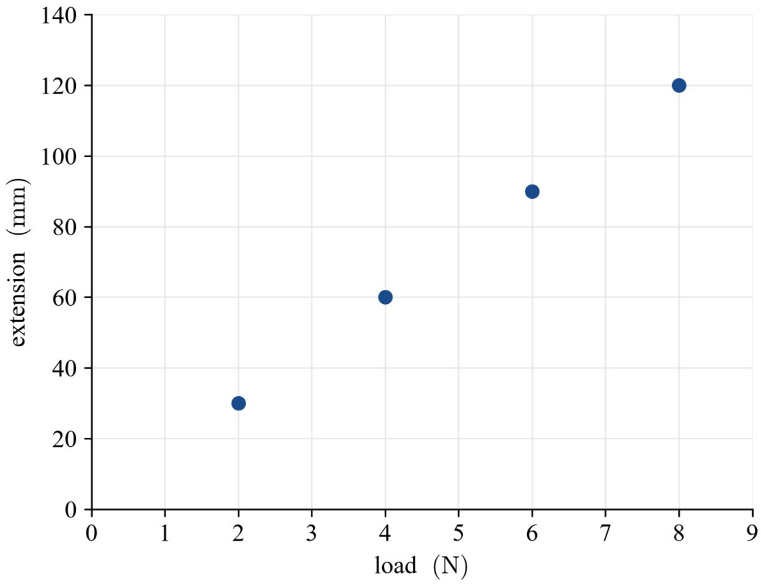
Question 10 In one experiment a student reads a liquid-in-glass thermometer by looking at it from a slightly different angle each time — sometimes from above, sometimes from below. In another experiment a student always reads a ruler while viewing it from one fixed angle to the side. For each case, state whether the parallax produces a random error or a systematic error, and justify your answer.

Question 11 A ball is launched horizontally from a bench six times and the horizontal distance it travels is measured with a tape (resolution $0.01\,\mathrm{m}$): 1.84, 1.88, 1.86, 1.40, 1.87 and $1.85\,\mathrm{m}$. On the fourth launch the student saw the ball clip the edge of the bench as it left. The predicted range is $1.86\,\mathrm{m}$. Identify the outlier, state what the investigation of it shows and hence justify excluding it, calculate the mean range of the remaining readings, and comment on the accuracy of the result.

Question 12 The secondary voltage of a transformer is measured six times with a voltmeter of resolution $0.1\,\mathrm{V}$, giving 24.1, 24.3, 24.2, 24.4, 24.2 and $24.4\,\mathrm{V}$. Each reading has an uncertainty of $\pm 0.1\,\mathrm{V}$. (a) Show that the mean of these readings is $24.3\,\mathrm{V}$. (b) Express the result in mean ± uncertainty notation. (c) State the range within which the true secondary voltage is expected to lie.

Question 13 A student measures the time for a pendulum to complete 25 oscillations five times, recording 46.2, 46.5, 46.3, 46.4 and $46.7\,\mathrm{s}$. Calculate the mean time for 25 oscillations. Give your answer to three significant figures.

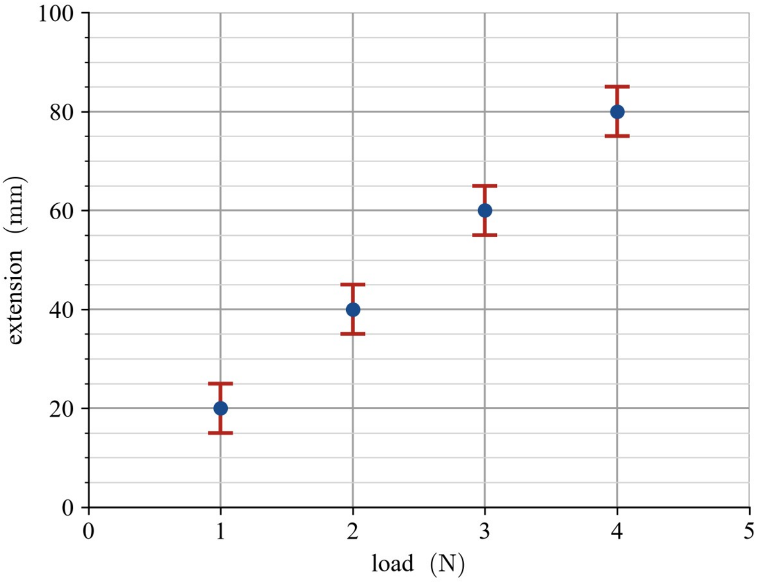
Question 14 Figure (10a_bars_blank) shows four points from a spring experiment, plotted without uncertainty bars, at loads of 2.0, 4.0, 6.0 and $8.0\,\mathrm{N}$ with extensions of 30, 60, 90 and $120\,\mathrm{mm}$. Every extension was measured with an uncertainty of $\pm 2\,\mathrm{mm}$.



(a) Describe how you would add a vertical uncertainty bar to each point.

(b) State the total length of each bar, and the values at the top and bottom of the bar on the point at a load of 2.0 N.

Question 15 Figure (10a_bars_read) shows the extension of a *different* spring at four loads, each point drawn with a vertical uncertainty bar. The smallest division on the extension axis is 5 mm.



(a) Read from the graph the extension, with its uncertainty, at a load of 3.0 N.

(b) State the range of extension values consistent with this point.

Question 16 A student sets out to investigate how the extension of a spring depends on the load placed on it. During the experiment the temperature of the room rises noticeably, and the student also replaces the spring with a stiffer one halfway through the readings. Explain how *each* of these threatens the validity of the investigation, using the idea of controlled variables.

Question 17 Two laboratories measure the resistance of the same resistor, each using its own equipment and its own method. Laboratory 1 reports $47.2 \pm 0.1\,\Omega$ and Laboratory 2 reports $49.8 \pm 0.1\,\Omega$; within each laboratory the repeated readings agree very closely with one another. State what these results show about the repeatability and about the reproducibility of the measurement, explain what the disagreement between the two laboratories implies about at least one of the two methods, and state whether either laboratory could remove the disagreement by taking more repeats.

Question 18 A digital balance displays mass to the nearest 0.001 g; an analogue balance can be read to the nearest 0.1 g. A mass of about 15 g is to be measured. (a) State the resolution of each balance. (b) Explain which balance allows the mass to be recorded to more significant figures, and why simply writing more digits than the instrument's resolution justifies would be misleading.

Question 19 The accepted value of a spring constant is $20.0\ \mathrm{N\ m^{-1}}$. Three groups each report five values (in $\mathrm{N\ m^{-1}}$):

- Group P: 19.8, 20.1, 20.0, 19.9, 20.2
- Group Q: 17.9, 18.1, 18.0, 17.9, 18.1
- Group R: 18.5, 21.6, 19.8, 22.1, 18.0

For each group, state whether the results are accurate, precise, both or neither, and identify the dominant type of error present. Relate each group's pattern to one of the target diagrams in Figure (10a_targets).

Question 20 A student investigates the period of a pendulum at a fixed length, timing a single period six times with a stopwatch of resolution 0.01 s: 2.01, 2.03, 2.02, 2.45, 2.02 and 2.04 s. Midway through the fourth timing the student miscounted the swing. Each valid reading has an uncertainty of ±0.01 s, and the accepted period at this length is 2.00 s. (a) Identify the reading spoilt by the miscount and state how it should be treated. (b) Calculate the mean period from the valid readings and express it in mean ± uncertainty notation. (c) State the range in which the true period is expected to lie, and comment on the accuracy and precision of the result.

Answers

Question 1 (a) $0.01\,\text{mm}$; (b) $12.45\,\text{mm}$, precise (spread only $0.02\,\text{mm}$); (c) accurate — the mean equals the accepted $12.45\,\text{mm}$. **Question 2** (a) random error; (b) systematic error; (c) random: reduced by repeats and averaging; systematic: not reduced. **Question 3** (a) the error is the deviation of the reading from the true extension; the uncertainty is the range in which the true extension is expected to lie; (b) $47 \pm 1\,\text{mm}$; (c) top $48\,\text{mm}$, bottom $46\,\text{mm}$. **Question 4** (a) $0.95\,\text{V}$; (b) a mistake (personal error) — exclude it and, if possible, repeat the reading; (c) $1.44\,\text{V}$. **Question 5** (a) X: $1\,\text{V}$; Y: $0.1\,\text{V}$; (b) Y is finer — it can detect a change as small as $0.1\,\text{V}$; (c) X cannot resolve a $0.5\,\text{V}$ change, so it cannot measure the effect being investigated — the data would not be valid. **Question 6** (a) repeatability; (b) reproducibility; (c) agreement under changed conditions argues against a hidden systematic error or a method-specific flaw. **Question 7** Close agreement means the readings are precise, not necessarily accurate; a systematic error could make every reading precisely wrong. **Question 8** Systematic error (a zero error); repeating would not remove it; it reduces accuracy (every force $0.2\,\text{N}$ too high) but leaves precision unchanged. **Question 9** Random deviations are equally likely to be positive or negative and cancel on averaging, so the mean nears the true value; a systematic error shifts every reading equally and so survives averaging. **Question 10** Varying viewing angle $\rightarrow$ random error; fixed (consistently wrong) viewing angle $\rightarrow$ systematic error. **Question 11** $1.40\,\text{m}$ is the outlier; investigating it shows the ball clipped the bench, so that launch is a mistake (a spoilt reading) and is excluded on that ground; mean of the remaining five $= 1.86\,\text{m}$, equal to the predicted $1.86\,\text{m}$, so the result is accurate. **Question 12** (a) mean $= 24.3\,\text{V}$; (b) $24.3 \pm 0.1\,\text{V}$; (c) $24.2\,\text{V}$ to $24.4\,\text{V}$. **Question 13** $46.4\,\text{s}$. **Question 14** (a) draw a line $2\,\text{mm}$ above and $2\,\text{mm}$ below each point; (b) each bar is $4\,\text{mm}$ long; the bar at $2.0\,\text{N}$ runs from $28\,\text{mm}$ to $32\,\text{mm}$. **Question 15** (a) $60 \pm 5\,\text{mm}$; (b) $55\,\text{mm}$ to $65\,\text{mm}$. **Question 16** Both temperature and the spring are uncontrolled variables, so any change in extension can no longer be attributed to the load alone — the investigation is not valid. **Question 17** Each laboratory's own readings are highly repeatable, but the two disagree by $2.6\,\Omega$ — far more than their stated uncertainties allow — so the measurement is *not* reproducible; at least one method carries a systematic error, and more repeats cannot remove it. **Question 18** (a) $0.001\,\text{g}$ and $0.1\,\text{g}$; (b) the digital balance (finer resolution) allows more significant figures; recording more digits than the resolution justifies overstates the precision and is misleading. **Question 19** P: accurate and precise; Q: precise but not accurate (systematic error); R: accurate but not precise (large random errors). **Question 20** (a) the fourth reading, $2.45\,\text{s}$ — a mistake; exclude it; (b) mean $= 2.024\,\text{s}$, which is $2.02\,\text{s}$ to three significant figures, so $2.02 \pm 0.01\,\text{s}$; (c) $2.01\,\text{s}$ to $2.03\,\text{s}$; the readings are tightly grouped, so the result is precise, but this range does not include the accepted $2.00\,\text{s}$ and every valid reading lies above it, so the result is not accurate — the pattern points to a systematic error.

Fully-worked solutions

Question 1 Callipers reading to $0.01\,\text{mm}$; readings $12.44, 12.46, 12.45, 12.44, 12.46\,\text{mm}$. (a) The resolution is the smallest increment the display can show: $0.01\,\text{mm}$. (b) Mean $= \dfrac{12.44 + 12.46 + 12.45 + 12.44 + 12.46}{5} = \dfrac{62.25}{5} = 12.45\,\text{mm}$. The readings span only $0.02\,\text{mm}$, a very small spread, so they are precise. (c) The mean, $12.45\,\text{mm}$, equals the accepted value $12.45\,\text{mm}$, so the result is accurate as well as precise.

Question 2 (a) The scatter of readings above and below their mean is caused by **random error** (chiefly the variation in the student's reaction time in starting and stopping the watch). (b) A constant excess of $0.20\,\text{s}$ on every reading, always in the same direction, is a **systematic error** (here a consistent flaw in the timing method). (c) Repeating and averaging reduces the effect of the **random** error, because its positive and negative deviations tend to cancel. It does **not** reduce the **systematic** error: every reading is shifted by the same $0.20\,\text{s}$, so the mean is shifted by $0.20\,\text{s}$ too, no matter how many repeats are taken.

Question 3 (a) The **error** of a reading is the difference between that reading and the true extension of the spring — a single (usually unknown) deviation. The **uncertainty**, $\pm 1\,\text{mm}$, is the range within which the true extension is expected to lie — a statement of the doubt in the result. (b) Mean $= \dfrac{47 + 48 + 47 + 46}{4} = \dfrac{188}{4} = 47\,\text{mm}$. With the stated uncertainty the result is $47 \pm 1\,\text{mm}$. (c) The bar extends $1\,\text{mm}$ each way from the point, so its top is at $48\,\text{mm}$ and its bottom is at $46\,\text{mm}$.

Question 4 (a) The outlier is the $0.95\,\text{V}$ reading, which lies far below the tightly grouped values around $1.44\,\text{V}$. (b) Because the student knows the apparatus was bumped, the reading is a **mistake** (a personal error), not a random or systematic error. It should be excluded from the analysis and, if possible, the reading repeated. (c) Best estimate from the four valid readings:

$$\acute{V} = \frac{1.42 + 1.45 + 1.44 + 1.45}{4} = \frac{5.76}{4} = 1.44\,\text{V}.$$

Question 5 (a) Voltmeter X: resolution $1\,\text{V}$. Voltmeter Y: resolution $0.1\,\text{V}$. (b) Voltmeter Y has the finer resolution: its smallest division is $0.1\,\text{V}$, so it can detect a change as small as $0.1\,\text{V}$, whereas voltmeter X can only detect changes of about $1\,\text{V}$ or more. (c) The change being investigated, about $0.5\,\text{V}$ per turn, is smaller than voltmeter X's resolution of $1\,\text{V}$. Voltmeter X therefore cannot reveal the change the experiment is designed to study, so the readings would not measure what the investigation intends — the investigation would not be valid. Voltmeter Y should be used instead.

Question 6 (a) **Repeatability.** The same quantity is measured under the same conditions — the same student, the same ruler, the same method, in one session — which is exactly the definition of a repeatability test. (b) **Reproducibility.** The same quantity is measured under changed conditions — different students, different laboratories and different equipment — which is the definition of a reproducibility test. (c) Repeated agreement by one student could arise even if that student's method or instrument carries a systematic error, because the same error would recur every time. Agreement between different people using different equipment (Scenario 2) is far less likely to be the result of a shared systematic error, so reproducible results give greater confidence that the measured focal length is close to the true value.

Question 7 The five readings agreeing to within $0.1\,\text{s}$ shows only that they are **precise** — that they have a small spread. Precision says nothing about how close the readings are to the true value, which is **accuracy**. If a systematic error is present (for example an uncorrected zero or a consistently mistimed start), every reading can be shifted by the same amount, so the readings are precisely in agreement yet all inaccurate. Close agreement alone therefore does not establish accuracy.

Question 8 The balance has a **zero error**, a form of **systematic error**: it reads $0.2\,\text{N}$ too high for every force because it does not start from zero. Repeating the measurements would **not** remove it, since each repeat is shifted by the same $0.2\,\text{N}$. It reduces the **accuracy** of the recorded forces — each is $0.2\,\text{N}$ larger than the true force — but it does not change the **precision**, because the spread of the readings about their (shifted) mean is unaffected. The remedy is to re-zero the balance (or subtract $0.2\,\text{N}$ from each reading).

Question 9 A random error changes each reading by an unpredictable amount that is as likely to be above the true value as below it. When many readings are averaged, these positive and negative deviations tend to cancel, so the mean lies closer to the true value than a typical single reading; the more readings are averaged, the more complete the cancellation and the smaller the residual effect of random error on the mean. A systematic error, by contrast, shifts *every* reading by the same amount in the same direction. Averaging readings that are all shifted by the same amount gives a mean that is shifted by that same amount, so the systematic error passes straight through the averaging unchanged and cannot be reduced by taking more repeats.

Question 10 In the first experiment the viewing angle changes from reading to reading, so the parallax makes each reading too high or too low unpredictably; this produces a **random error**, scattering the readings about the mean. In the second experiment the student always views from the same wrong angle, so the parallax shifts every reading by the same amount in the same direction; this produces a **systematic error**, displacing all the readings from the true value.

Question 11 The $1.40\,\text{m}$ reading lies far below the tight cluster of the other five (which fall between 1.84 and $1.88\,\text{m}$), so it is an outlier. An outlier is investigated rather than deleted on sight, and the investigation here supplies a cause: the ball clipped the edge of the bench on that launch, so it did not leave the bench horizontally as the method requires. The reading is therefore a **mistake** (a spoilt trial), not a random or systematic error, and it is on that ground — not merely because it is far from the others — that it is excluded. The mean of the remaining five readings is

$$\acute{r} = \frac{1.84 + 1.88 + 1.86 + 1.87 + 1.85}{5} = \frac{9.30}{5} = 1.86\,\text{m}.$$

This equals the predicted range of $1.86\,\mathrm{m}$, so the result is accurate.

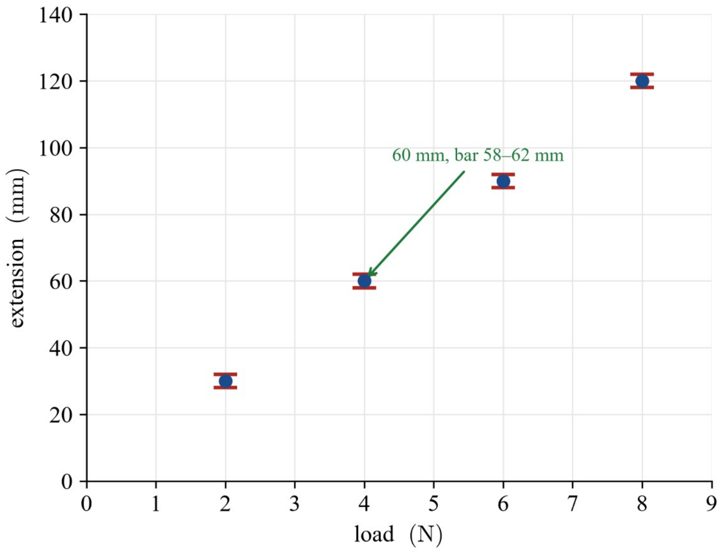
Question 12 (a) Mean $= \frac{24.1+24.3+24.2+24.4+24.2+24.4}{6} = \frac{145.6}{6} = 24.27\,\mathrm{V}$ to four significant figures, which is $24.3\,\mathrm{V}$ to three significant figures, as required. (b) With the stated uncertainty of $\pm 0.1\,\mathrm{V}$, the result is $24.3 \pm 0.1\,\mathrm{V}$. (c) The true secondary voltage is expected to lie between $24.2\,\mathrm{V}$ and $24.4\,\mathrm{V}$.

Question 13

$$\acute{t} = \frac{46.2+46.5+46.3+46.4+46.7}{5} = \frac{232.1}{5} = 46.42\,\mathrm{s},$$

which is $46.4\,\mathrm{s}$ to three significant figures.

Question 14 (a) Each extension carries an uncertainty of $\pm 2\,\mathrm{mm}$, so a vertical uncertainty bar is drawn on each point reaching $2\,\mathrm{mm}$ above the point and $2\,\mathrm{mm}$ below it (with short horizontal caps at the ends). The completed graph looks like Figure (10a_bars).



(b) Each bar has a total length of $2 \times 2 = 4\,\mathrm{mm}$. On the point at a load of $2.0\,\mathrm{N}$ (extension $30\,\mathrm{mm}$) the bar runs from $28\,\mathrm{mm}$ to $32\,\mathrm{mm}$.

Question 15 (a) The point at a load of $3.0\,\mathrm{N}$ sits on the $60\,\mathrm{mm}$ line of the extension axis. The top of its uncertainty bar lies one small division above, on the $65\,\mathrm{mm}$ line, and the bottom lies one small division below, on the $55\,\mathrm{mm}$ line. Each small division is $5\,\mathrm{mm}$, so the bar reaches $5\,\mathrm{mm}$ each way and the extension is $60 \pm 5\,\mathrm{mm}$. (Equivalently, the bar has a total length of $10\,\mathrm{mm}$, and a $\pm u$ bar has total length $2u$, so $u = 5\,\mathrm{mm}$.) (b) The values consistent with this point run from the bottom of the bar to the top: $55\,\mathrm{mm}$ to $65\,\mathrm{mm}$.

Question 16 In this investigation the independent variable is the load and the dependent variable is the extension; for the result to be valid, everything else that could affect the extension must be held constant (a controlled variable). The rise in room temperature is an uncontrolled variable — it can change the stiffness of the spring, so part of any change in extension may be due to temperature rather than load. Replacing the spring with a stiffer one changes the very object being studied partway through, so readings taken before and after are not comparable. In both cases a variable that should have been controlled was not, so a measured change in extension can no longer be attributed to the load alone, and the investigation does not validly test how extension depends on load.

Question 17 Within each laboratory the readings were taken under the *same* conditions and agree very closely, so each laboratory's measurement is highly **repeatable**. (The quoted $\pm 0.1\,\Omega$ is each laboratory's stated instrument uncertainty; it is not calculated from the spread of the readings.) Reproducibility, however, is agreement under *changed* conditions — different equipment and a different method — and here the two results do not agree: $47.2\,\Omega$ and $49.8\,\Omega$ differ by $2.6\,\Omega$, whereas the stated uncertainties allow the true resistance to lie only in 47.1–$47.3\,\Omega$ on Laboratory 1's result and in 49.7–$49.9\,\Omega$ on Laboratory 2's. These ranges do not overlap, so the disagreement is far too large to be explained by the doubt each laboratory has quoted. The measurement is therefore **not reproducible**.

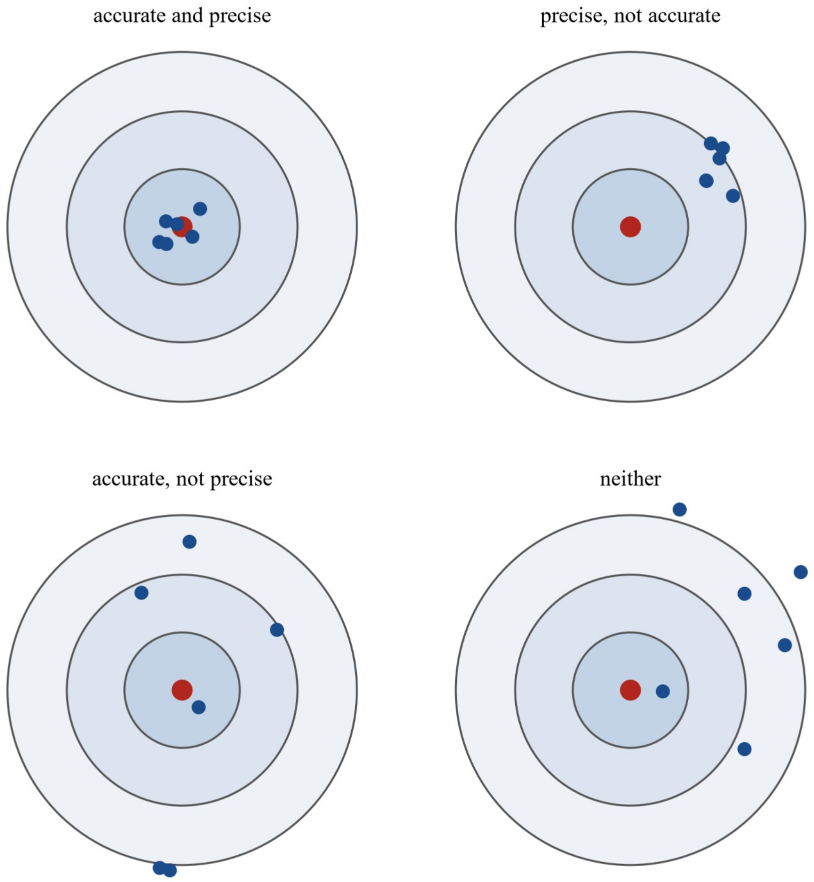
Because random error would show up as scatter *within* each laboratory's repeats — and there is almost none — the discrepancy must come from a **systematic error** in at least one of the two methods (for example an uncorrected zero error on a meter, a wrongly calibrated scale, or lead resistance included in one method but not the other). A systematic error shifts every reading of that method by the same amount, so neither laboratory could remove the disagreement by taking more repeats: averaging more shifted readings simply gives a mean shifted by the same amount. The fault must be traced and corrected — for instance by re-zeroing and re-calibrating each instrument, or by having both laboratories measure the same reference resistor of known value.

High repeatability therefore does not establish that a value is right; it is the failure of *reproducibility* that has exposed the problem here.

Question 18 (a) The digital balance has a resolution of $0.001\,\text{g}$; the analogue balance has a resolution of $0.1\,\text{g}$. (b) For a mass of about $15\,\text{g}$, the digital balance can display the value to the nearest $0.001\,\text{g}$ (for example $15.000\,\text{g}$, five significant figures), whereas the analogue balance can only be read to the nearest $0.1\,\text{g}$ (for example $15.0\,\text{g}$, three significant figures). The digital balance therefore allows more significant figures. Writing more digits than an instrument's resolution justifies — for example quoting $15.03\,\text{g}$ from the analogue balance — is misleading, because it claims a precision the instrument cannot deliver and implies the last figures are meaningful when they are not.

Question 19 Comparing each group's five values with the accepted $20.0\,\text{N m}^{-1}$:

- Group P: mean $= \dfrac{19.8+20.1+20.0+19.9+20.2}{5} = 20.0\,\text{N m}^{-1}$, with a small spread. The values cluster tightly around the accepted value, so the results are **accurate and precise** — random errors are small and there is no evidence of a systematic error. This matches the "accurate and precise" target.
- Group Q: mean $= \dfrac{17.9+18.1+18.0+17.9+18.1}{5} = 18.0\,\text{N m}^{-1}$, with a small spread but well below $20.0\,\text{N m}^{-1}$. The results are **precise but not accurate**, the tell-tale sign of a **systematic error**. This matches the "precise, not accurate" target.
- Group R: mean $= \dfrac{18.5+21.6+19.8+22.1+18.0}{5} = 20.0\,\text{N m}^{-1}$, but the values are widely scattered. The mean is close to the accepted value, so the results are broadly **accurate**, but the large spread makes them **not precise**, indicating large **random errors**. This matches the "accurate, not precise" target.



Question 20 (a) The fourth reading, 2.45 s, was spoilt by the miscounted swing, so it is a **mistake** (a personal error). It is excluded from the analysis; the remaining five readings are valid. (b) Mean of the valid readings:

$$\bar{T} = \frac{2.01 + 2.03 + 2.02 + 2.02 + 2.04}{5} = \frac{10.12}{5} = 2.024\,\text{s}\,,$$

which is 2.02 s to three significant figures — the stopwatch reads to 0.01 s, so a third decimal place is not justified. With the stated uncertainty of ±0.01 s, the period is $T = 2.02 \pm 0.01$ s. (c) The ± interval says the true period is expected to lie between 2.01 s and 2.03 s. The five valid readings are tightly grouped, spanning only 2.01–2.04 s, so the result is **precise**. It is not, however, shown to be **accurate**: the accepted period, 2.00 s, lies *outside* the interval 2.01–2.03 s, so the result disagrees with the accepted value by more than the doubt claimed for it. Every one of the five valid readings is also greater than 2.00 s — a one-sided displacement of the whole set, which is the signature of a **systematic error** (for example the stopwatch consistently stopped a little late, or the pendulum set up slightly longer than the length intended), and not of random error, which would scatter the readings on both sides of the accepted value. Taking more repeats would not correct it; the method or the instrument must be checked. Close agreement between repeats establishes precision; on its own it never establishes accuracy.

Theory

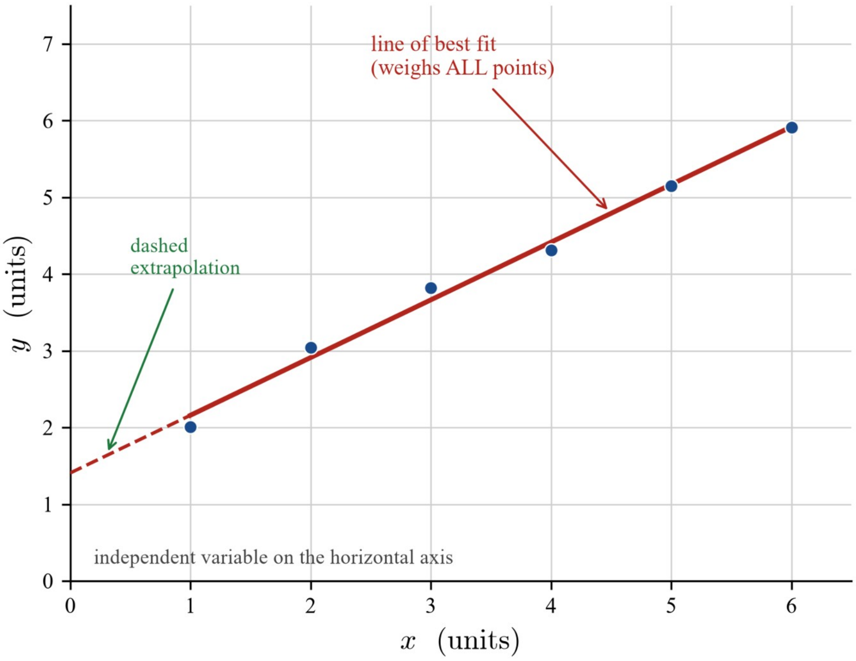
Physics is an empirical science. A relationship between two quantities is established by measuring one of them — the **dependent variable** — while the other — the **independent variable** — is deliberately changed, then displaying the results in a table and a graph. This subtopic sets out the conventions for presenting that data, for drawing a **line of best fit**, for extracting a **gradient**, and for using **linearisation** so that the gradient and the intercepts of a graph carry a clear physical meaning.

Recording data in a table. A results table follows a fixed set of conventions:

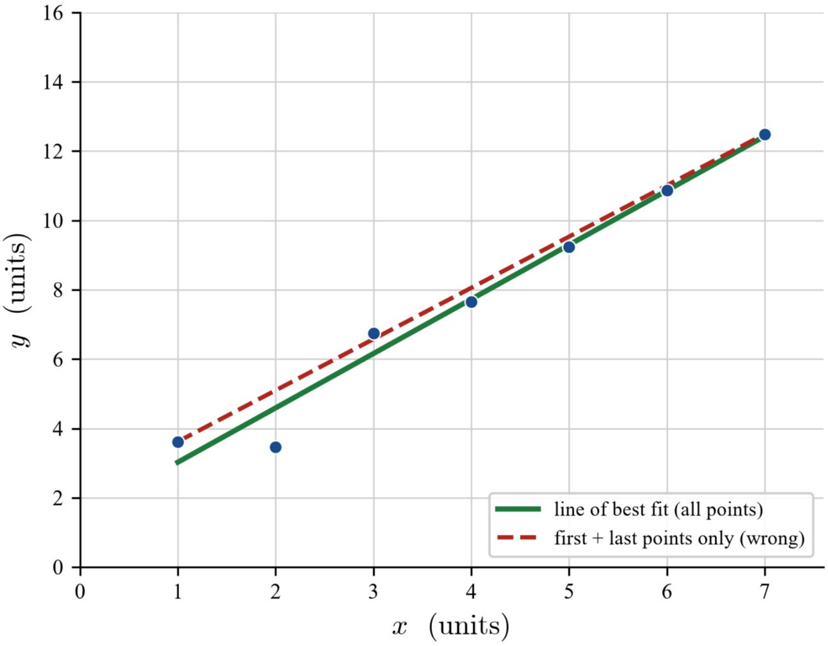
- it has a brief title describing the measurement;
- the **independent variable** occupies the first column, listed in the order it was set;
- each column is headed with the quantity *and its unit*, and any power of ten is shown in the heading so that the entries themselves are plain numbers — for example a column headed $1/r\ (10^{-8}\,\mathrm{m}^{-1})$ contains entries such as 2.5 and 12.5;
- **repeated readings** (replicates) are recorded and then **averaged**, because averaging repeated measurements reduces the effect of random error on the plotted value.

Plotting a graph. The graph of the data also follows fixed conventions:

- the **independent variable is placed on the horizontal axis** and the dependent variable on the vertical axis;
- both axes are labelled with the quantity and its unit;
- the scales are chosen so that the plotted points **fill as much of the grid as possible** — points must not be cramped into one corner;
- the points are plotted accurately, and a single **smooth trend line** is drawn (a straight line of best fit when the relationship is linear);
- any extension of the line beyond the measured data — an **extrapolation** — is drawn as a **dashed** line;
- if the two quantities are **directly proportional**, the line of best fit passes through the **origin** $(0,0)$; the origin is plotted as a *data point* only if it was authentically measured, never merely assumed.



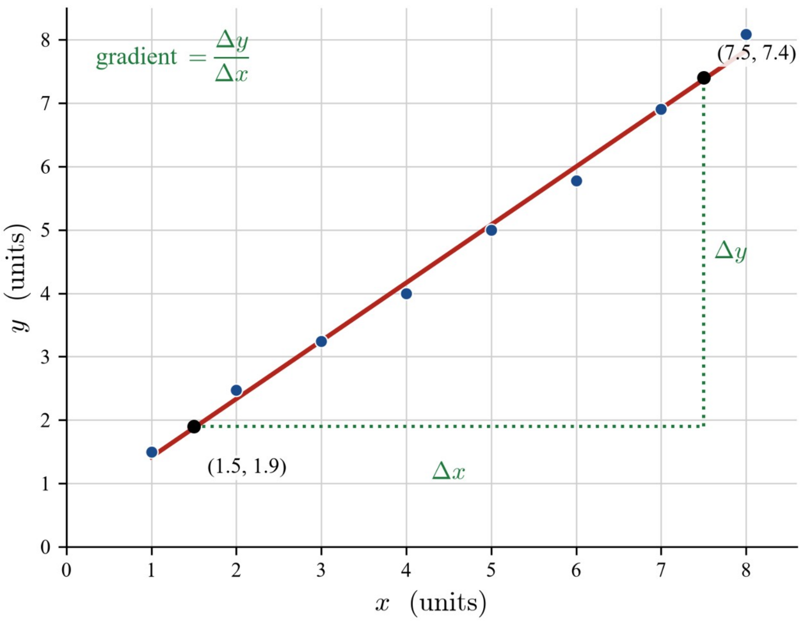
The line of best fit. A straight line of best fit is drawn so that it follows the trend of *all* the plotted points, with the points scattered as evenly as possible above and below it. Every point is weighed. The single most common — and most heavily penalised — error in data analysis is to draw the line **through the first and last data points only**. Those two points scatter about the true line by just as much as any other point, so a line pinned to them throws away all the information carried by the points in between and usually gives the wrong gradient.



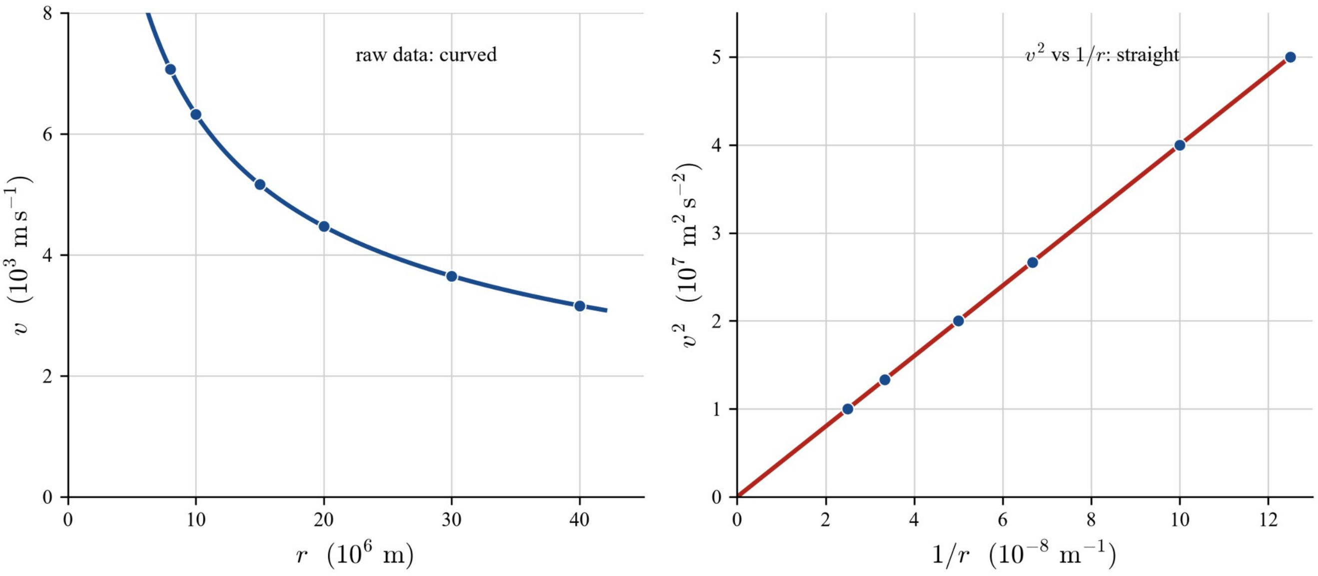
Reading the gradient. The gradient of a straight-line graph is

$$\text{gradient} = \frac{\Delta y}{\Delta x},$$

calculated from **two points that lie ON the line of best fit** and are as far apart as possible (a large Δx reduces the effect of reading error). These are **not** table data points — the data points scatter about the line — they are points read off the drawn line. The gradient is whatever the line gives; it must never be nudged toward a value that is already expected.



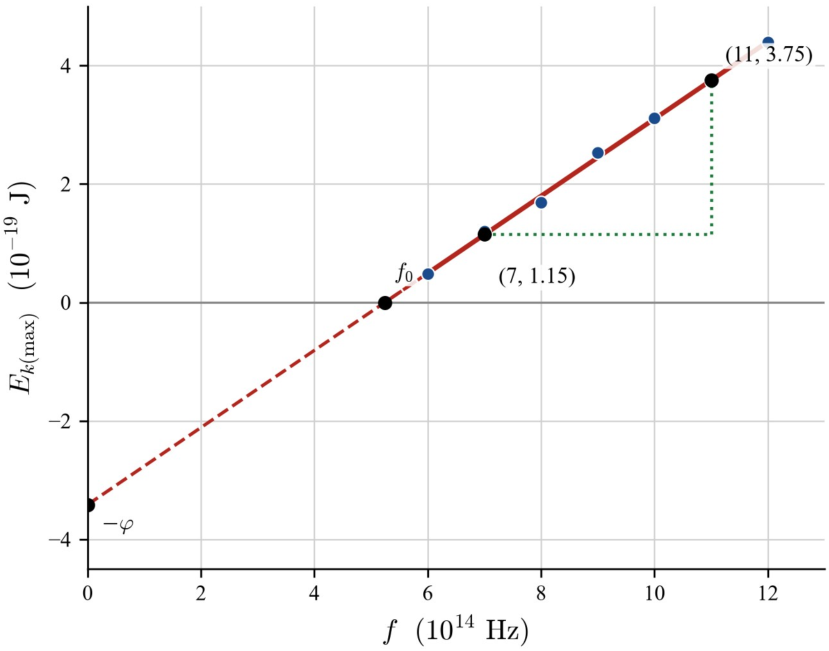
Linearisation. Many physical relationships are not straight lines. By plotting a suitable function of one quantity against a suitable function of the other, the relationship can be turned into a straight line whose gradient — and intercepts — carry physical meaning. Once the data have been transformed to the form $y = mx + c$, the gradient is m and the vertical-axis intercept is c.



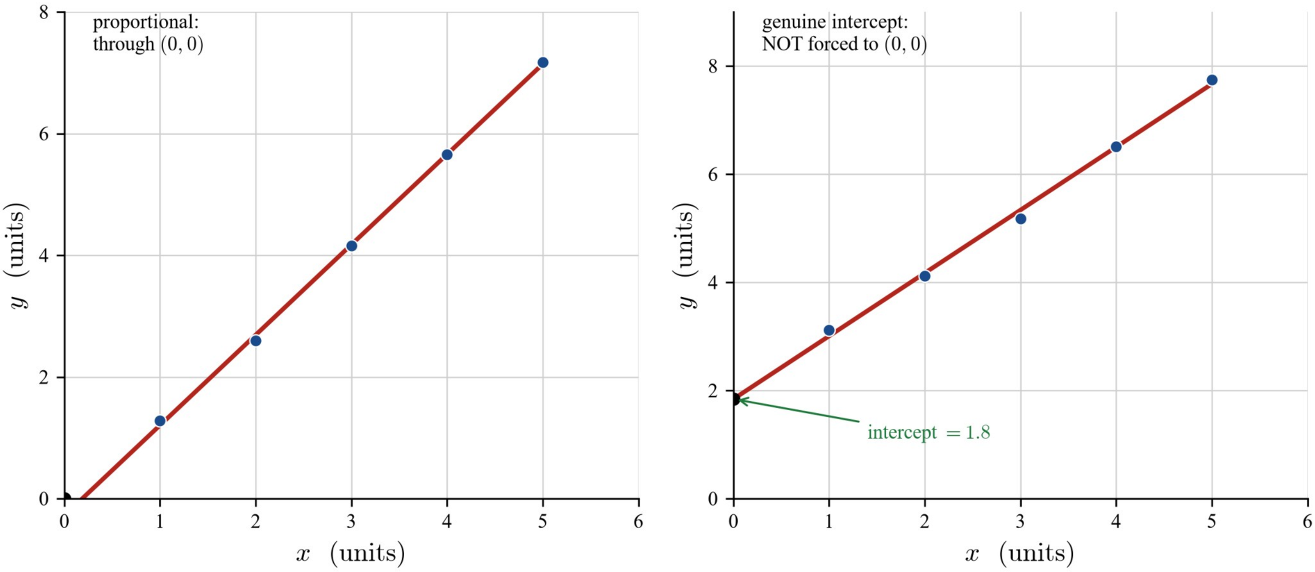
The linearisations used throughout Units 3 & 4 are collected here.

Relationship	Plot to linearise	Gradient represents	Intercept
$E_k = hf - \varphi$	E_k vs f	h	y: $-\varphi$; x: f_0
$v^2 = GM/r$	v^2 vs $1/r$	GM	through origin
$T^2 = \frac{4\pi^2}{GM} r^3$	T^2 vs r^3	$\frac{4\pi^2}{GM}$	through origin
$F = \frac{k q_1 q_2}{r^2}$	F vs $1/r^2$	$k q_1 q_2$	through origin
$E_k = 1/2 m v^2$	E_k vs v^2	$1/2m$	through origin
$E_s = 1/2 k x^2$	E_s vs x^2	$1/2k$	through origin
$\lambda = h/p$	λ vs $1/p$	h	through origin
$E = hc/\lambda$	E vs $1/\lambda$	hc	through origin

Intercepts and extrapolation. The gradient is not the only useful feature of a graph; the intercepts often carry physical meaning too, and are frequently found by **extrapolating** the line (dashed) beyond the measured points to reach an axis. The photoelectric graph of maximum photoelectron kinetic energy against frequency is the standard example: the gradient is Planck's constant h, the vertical intercept is $-\varphi$ (so the work function is φ), and the horizontal intercept is the threshold frequency f_0.



A relationship that is a direct proportion must give a line through the origin. If the fitted line instead cuts an axis away from the origin, that intercept is telling you something physical — an initial value, or a work function — or it is evidence of a systematic error.



Throughout this subtopic the VCAA constant values are used: $h = 6.63 \times 10^{-34}$ J s, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $q_e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, $k = 8.99 \times 10^9$ N m^2 C^{-2} and $c = 3.00 \times 10^8$ m s^{-1}.

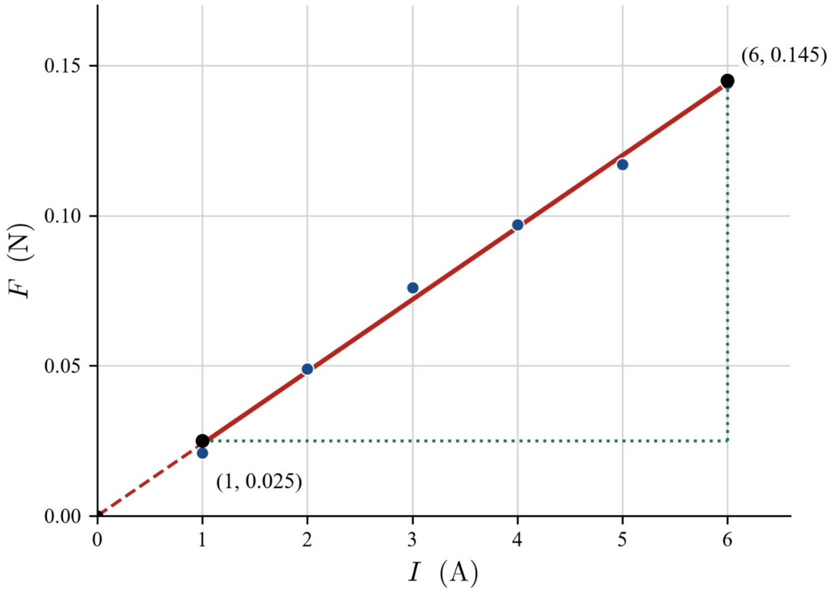
Worked examples

Example 1 — scaffolded

A student passes a current I through a straight horizontal wire of length $l = 0.080$ m placed at right angles to a uniform magnetic field, and measures the force F on the wire for six values of the current, repeating each measurement three times. The averaged results are shown below. Determine, using a graph of F against I, the magnetic field strength B.

Current I (A)	Trial 1 F (N)	Trial 2 F (N)	Trial 3 F (N)	Average F (N)
1.0	0.021	0.017	0.026	0.021
2.0	0.051	0.047	0.049	0.049

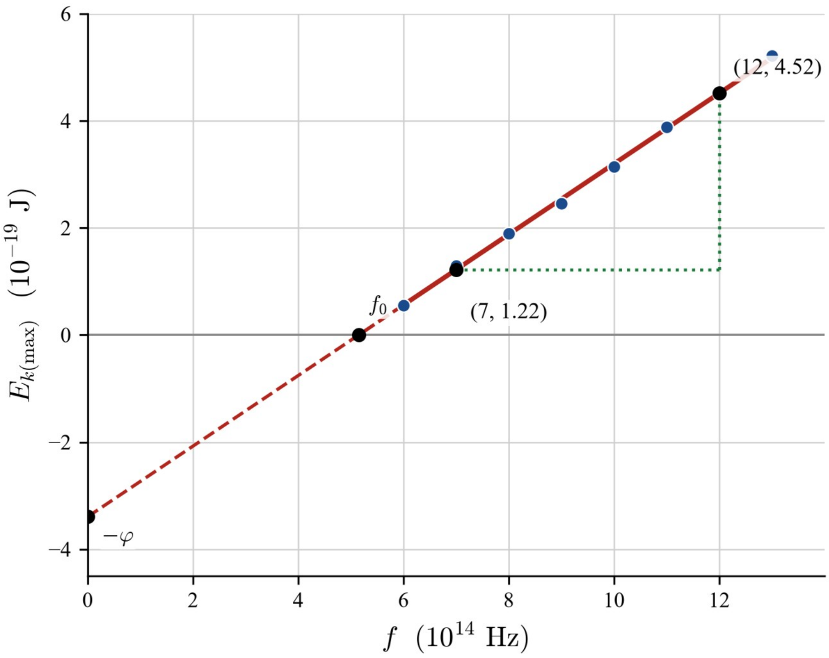
Current I (A)	Trial 1 F (N)	Trial 2 F (N)	Trial 3 F (N)	Average F (N)
3.0	0.078	0.076	0.074	0.076
4.0	0.098	0.093	0.099	0.097
5.0	0.114	0.119	0.118	0.117
6.0	0.138	0.144	0.150	0.144



Working	Comment
I is the independent variable (set by the student) → horizontal axis; averaged F is dependent → vertical axis.	Independent variable always horizontal.
Plot the six averaged points and draw a straight line of best fit that weighs all six, scattered evenly about the line.	Not a line through the first and last points only.
Read two well-separated points ON the line: $(1.0, 0.025)$ and $(6.0, 0.145)$.	Points on the drawn line, not table data points.
$\text{gradient} = \frac{0.145 - 0.025}{6.0 - 1.0} = \frac{0.120}{5.0} = 0.024\,\text{N A}^{-1}$.	$\Delta F / \Delta I$ from the two on-line points.
For a wire, $F = BIl$, so a graph of F against I has gradient Bl: $Bl = 0.024$.	Physical meaning of the gradient.
$B = \frac{0.024}{l} = \frac{0.024}{0.080} = 0.30\,\text{T}$.	Divide the gradient by the known length.

Example 2 — scaffolded

In a photoelectric experiment the maximum kinetic energy E_k of the emitted photoelectrons is measured for a range of frequencies f of the incident light, giving the graph below. Use the graph to determine Planck's constant h, the work function φ of the metal, and the threshold frequency f_0.



Working	Comment
The data follow $E_k = hf - \varphi$, a straight line: gradient h, vertical intercept $-\varphi$, horizontal intercept f_0.	Already linear in f — no transformation needed.
Two points on the line: $(7.0, 1.22)$ and $(12.0, 4.52)$, i.e. f in 10^{14} Hz, E_k in 10^{-19} J.	Well-separated points on the line of best fit.
$h = \frac{(4.52-1.22)\times10^{-19}}{(12.0-7.0)\times10^{14}} = \frac{3.30\times10^{-19}}{5.0\times10^{14}} = 6.6\times10^{-34}$ J s .	Gradient $= \Delta E_k / \Delta f = h$ (accepted value 6.63×10^{-34}).
Extrapolate (dashed) to the vertical axis: intercept $= -3.4\times10^{-19}$ J $= -\varphi$, so $\varphi = 3.4\times10^{-19}$ J $= 2.1$ eV.	The intercept gives the work function.
Extrapolate to the horizontal axis: $f_0 = 5.2\times10^{14}$ Hz.	Threshold frequency = x-intercept.

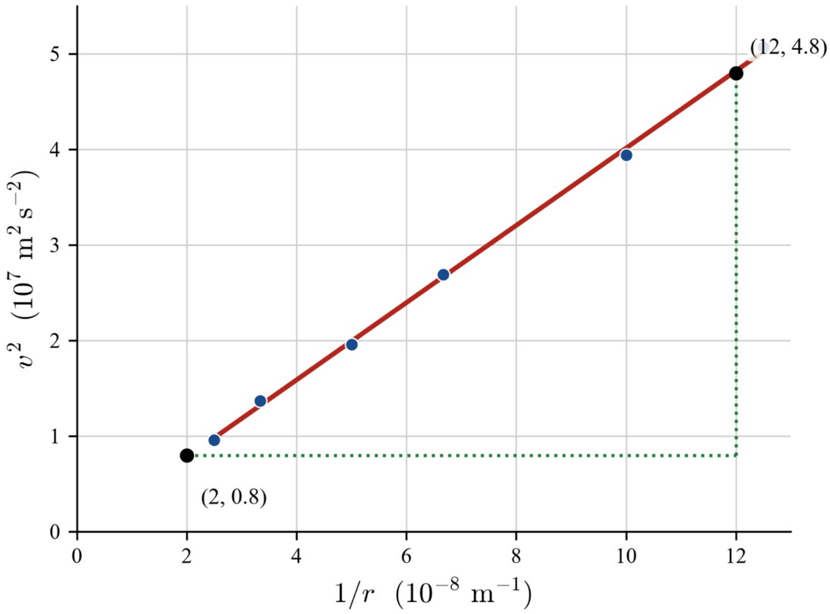
Example 3 — unscaffolded

The orbital speed v of several small satellites of a planet is measured at orbital radii r, and the results are displayed on the graph below. A graph of v against r is a curve. State what should be plotted to obtain a straight line, explain what its gradient represents, and hence determine the mass M of the planet.

Because the satellites move under gravity, $\frac{v^2}{r} = \frac{GM}{r^2}$, which rearranges to

$$v^2 = \frac{GM}{r}.$$

This is not linear in r, but it *is* linear in $\frac{1}{r}$: plotting v^2 against $\frac{1}{r}$ gives a straight line through the origin with gradient GM.



Reading two well-separated points on the line of best fit, $(2.0, 0.8)$ and $(12.0, 4.8)$ — with $\frac{1}{r}$ in $10^{-8}\,\text{m}^{-1}$ and v^2 in $10^7\,\text{m}^2\text{s}^{-2}$ — the gradient is

$$GM = \frac{(4.8 - 0.8) \times 10^7}{(12.0 - 2.0) \times 10^{-8}} = \frac{4.0 \times 10^7}{1.0 \times 10^{-7}} = 4.0 \times 10^{14}\,\text{m}^3\text{s}^{-2}.$$

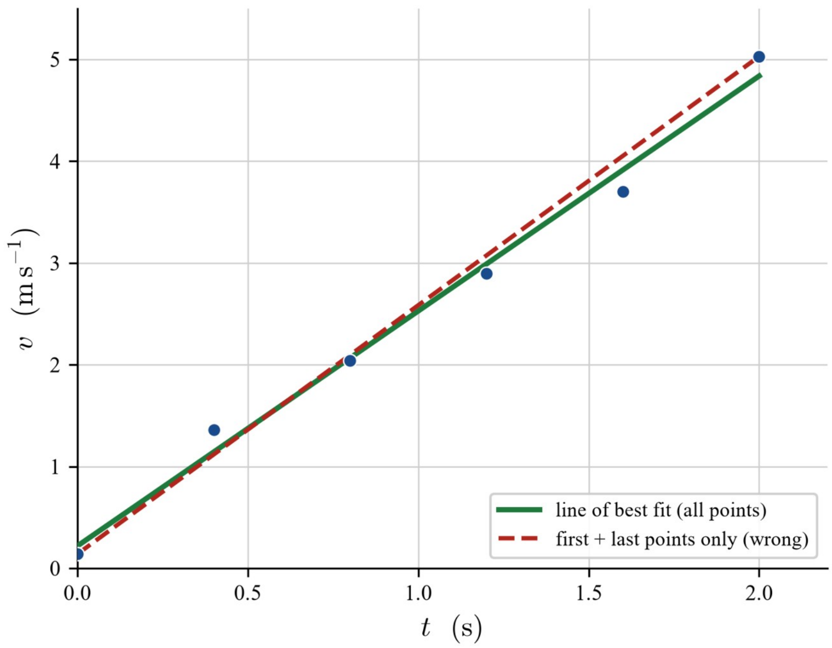
Hence

$$M = \frac{GM}{G} = \frac{4.0 \times 10^{14}}{6.67 \times 10^{-11}} = 6.0 \times 10^{24}\,\text{kg}.$$

∴ plotting v^2 against $1/r$ linearises the data, the gradient is GM, and the planet's mass is $M = 6.0 \times 10^{24}$ kg.

Example 4 — unscaffolded

A trolley accelerates down a ramp. Its velocity v is measured at six times t, giving the data plotted below. A student finds the acceleration by joining the first and last data points (the dashed line). Explain why this is not the correct method, and determine the acceleration and the initial velocity using the line of best fit.



The dashed line uses only two of the six points. Both of those points scatter about the true trend — here the first point happens to lie low and the last point high — so a line pinned to them is too steep and ignores the four points between. The correct line of best fit (solid) weighs all six points, with an even scatter above and below.

Reading two points on the line of best fit, $(0.20, 0.68)$ and $(1.80, 4.38)$:

$$a = \text{gradient} = \frac{4.38 - 0.68}{1.80 - 0.20} = \frac{3.70}{1.60} = 2.3\,\text{m s}^{-2}.$$

The vertical intercept of the best-fit line is the velocity at $t = 0$, i.e. the initial velocity $u = 0.2\,\text{m s}^{-1}$. (Joining the first and last points instead gives a gradient of $2.4\,\text{m s}^{-2}$ — too large.)

∴ weighing all points gives $a = 2.3\,\text{m s}^{-2}$ and $u = 0.2\,\text{m s}^{-1}$; the first-and-last-points shortcut overestimates the acceleration.

Practice questions

Question 1 A straight wire of length $l = 0.12\,\text{m}$ carries a current I at right angles to a uniform magnetic field. The force F on the wire is measured three times at each of five currents; the averaged results are shown.

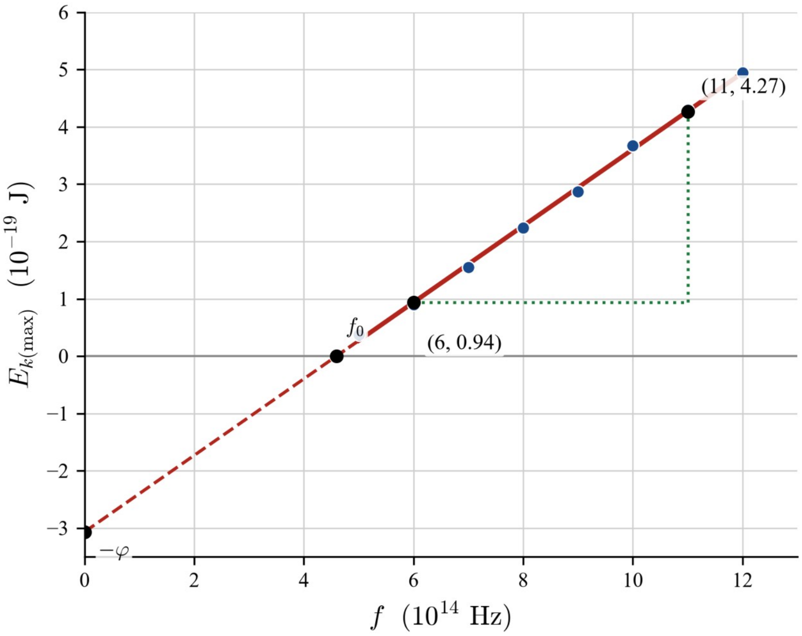
Current I (A)	Trial 1 F (N)	Trial 2 F (N)	Trial 3 F (N)	Average F (N)
1.0	0.054	0.038	0.030	0.041
2.0	0.079	0.094	0.086	0.086
3.0	0.124	0.127	0.128	0.126
4.0	0.174	0.169	0.176	0.173
5.0	0.207	0.210	0.207	0.208

(a) State the independent variable and the axis on which it should be plotted.

(b) State the rule for drawing the line of best fit, and explain why joining the first and last points only would be incorrect.

(c) Using two points on your line of best fit, determine the gradient and hence the magnetic field strength B.

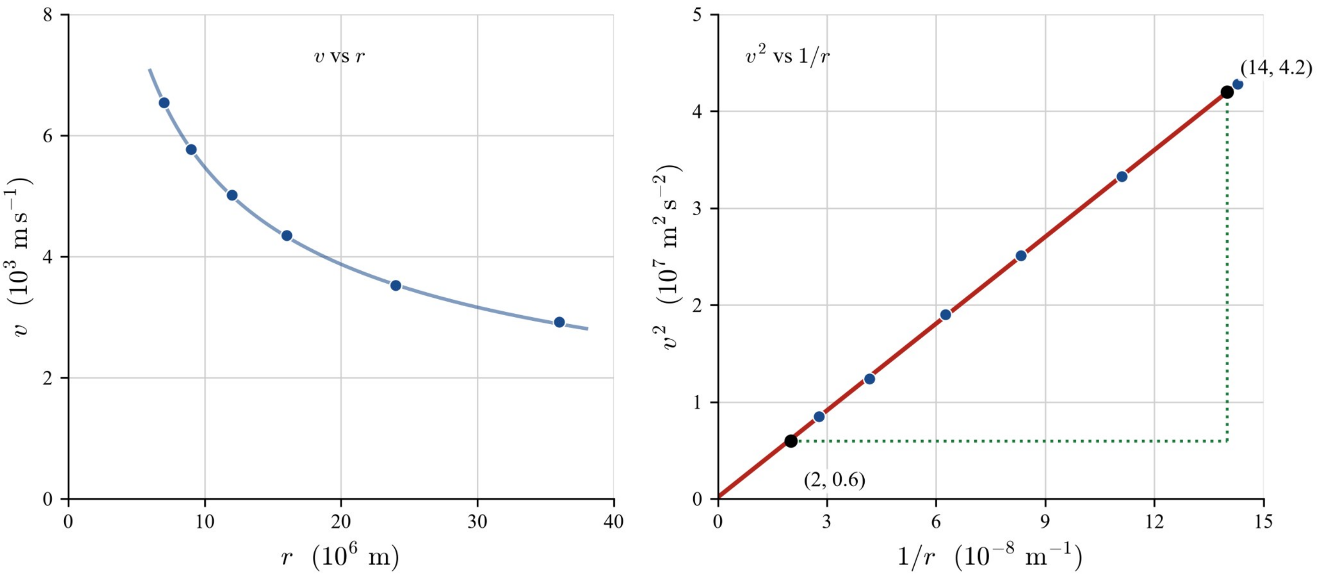
Question 2 The graph below shows the maximum kinetic energy E_k of photoelectrons emitted from a metal against the frequency f of the incident light.



(a) Explain why a graph of E_k against f is a straight line.

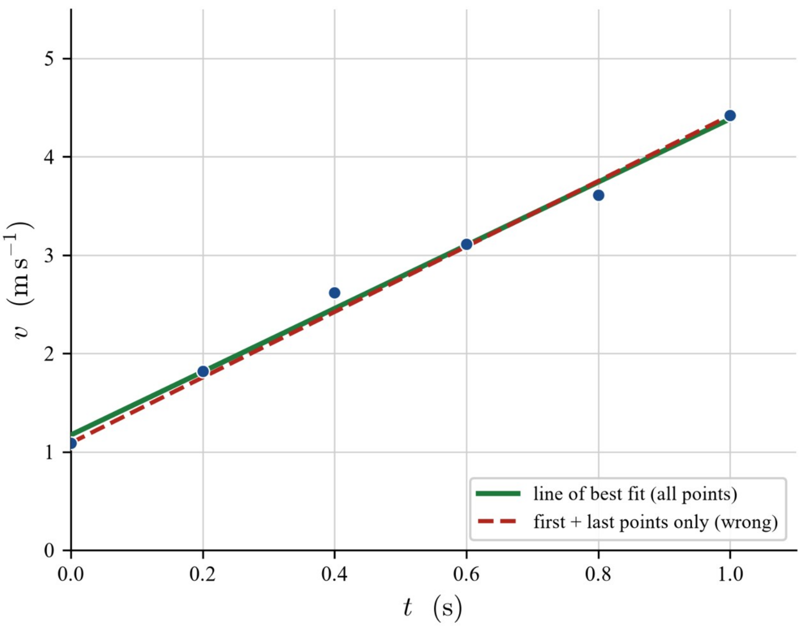
(b) Determine Planck's constant h from the gradient.
(c) Determine the work function φ from the vertical intercept, and state the threshold frequency f_0.

Question 3 The orbital speed v of several moons of a planet is measured at orbital radius r. The left panel shows v against r; the right panel shows the same data replotted.



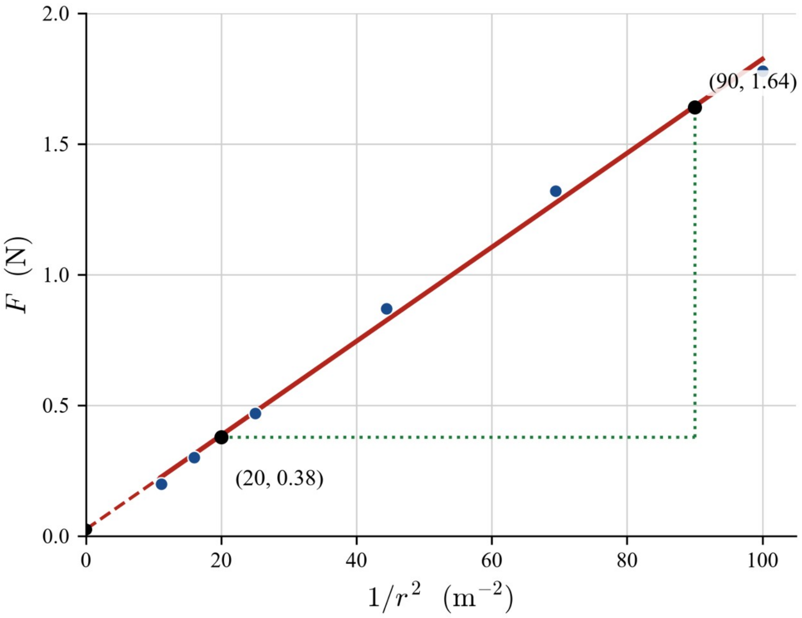
(a) Explain why the graph of v against r is curved rather than straight.
(b) State which quantities have been plotted in the right-hand panel to linearise the data, and what the gradient of that line represents.
(c) Using two points on the line of best fit in the right-hand panel, determine the mass M of the planet.

Question 4 A trolley runs down a ramp; its velocity v is recorded at six times t, plotted below. A student has drawn a line through the first and last points (dashed) and the line of best fit (solid).



(a) State how a line of best fit should be drawn, and explain why the dashed line is not acceptable.
(b) Using two points on the line of best fit, determine the acceleration of the trolley.
(c) State the physical quantity given by the vertical intercept of the line of best fit, and read its value.

Question 5 Two small charged spheres are held a distance r apart. One carries a charge $q_1 = 2.0 \times 10^{-6}$ C. The electric force F between them is measured at several separations, and F is plotted against $1/r^2$.

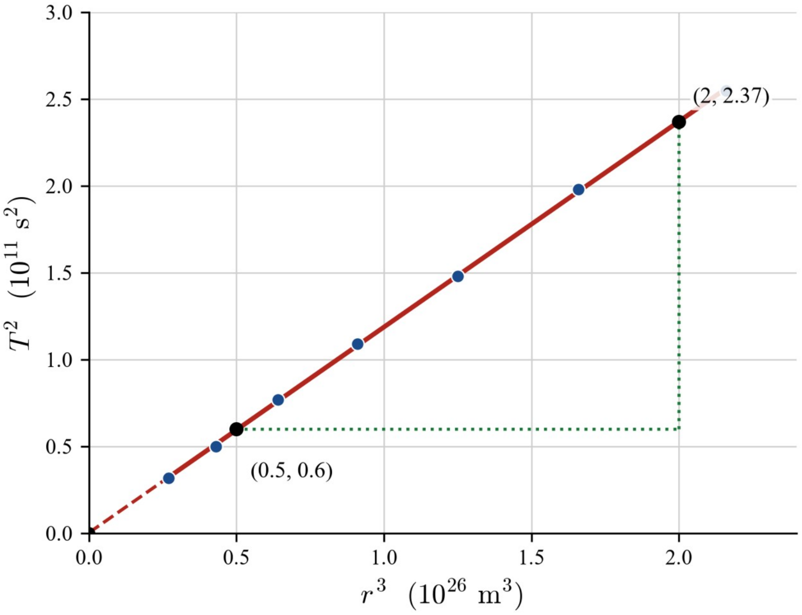


(a) Explain why a graph of F against r is not a straight line, whereas a graph of F against $1/r^2$ is, and state whether it should pass through the origin.

(b) Using two points on the line of best fit, determine the gradient, and hence the charge q_2 on the second sphere.

(c) The student extends the line of best fit back to the origin. Discuss whether the point $(0,0)$ should be plotted as a measured data point.

Question 6 The orbital period T and orbital radius r of several moons of a planet are measured, and T^2 is plotted against r^3.

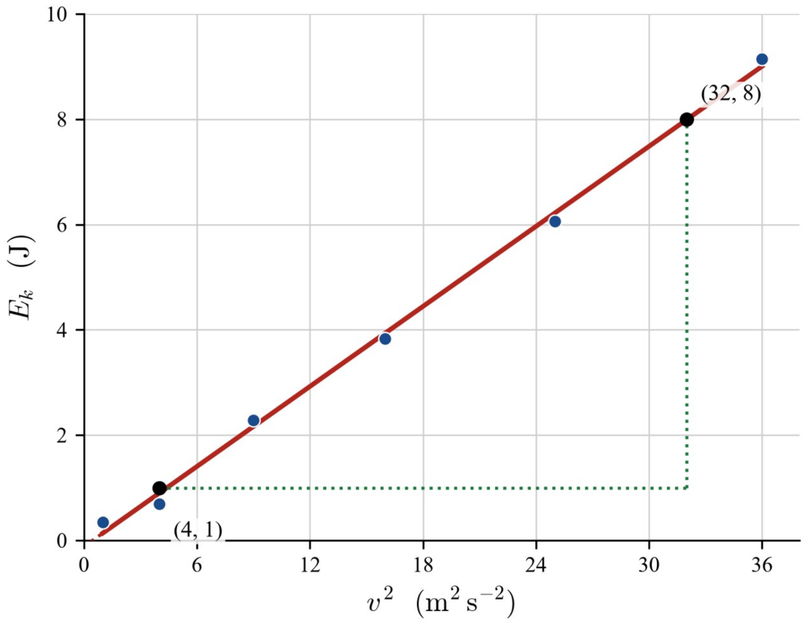


(a) Circular-orbit theory gives $T^2 = \frac{4\pi^2}{GM} r^3$. State why a graph of T^2 against r^3 is a straight line and what its gradient represents.

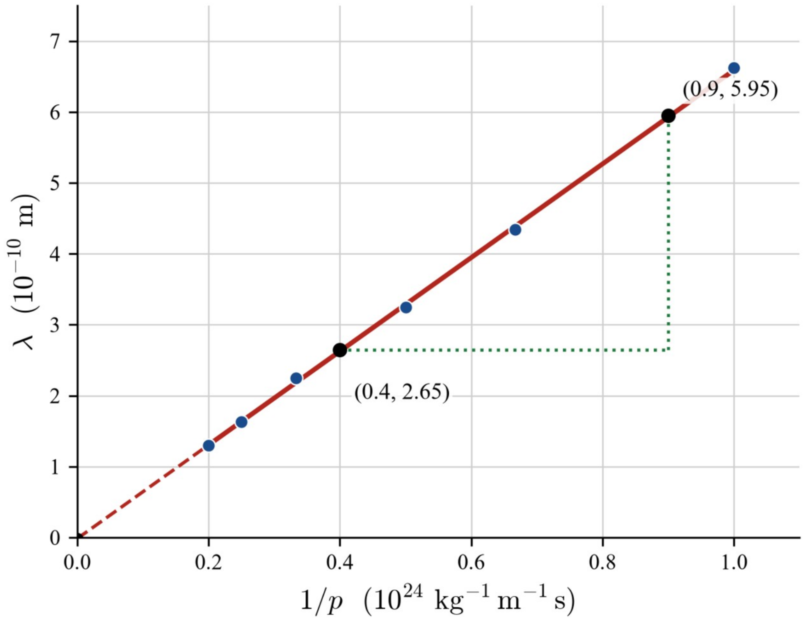
(b) Show that the gradient of the line of best fit is $1.18 \times 10^{-15}\, s^2\, m^{-3}$, and hence determine the mass M of the planet.

(c) Explain why the line of best fit should pass through the origin, and state what a non-zero vertical intercept would indicate.

Question 7 The kinetic energy E_k of a moving object is measured at several speeds v, and E_k is plotted against v^2. Using two well-separated points on the line of best fit, $(4.0, 1.0)$ and $(32.0, 8.0)$ (with v^2 in $m^2 s^{-2}$ and E_k in J), determine the mass of the object.

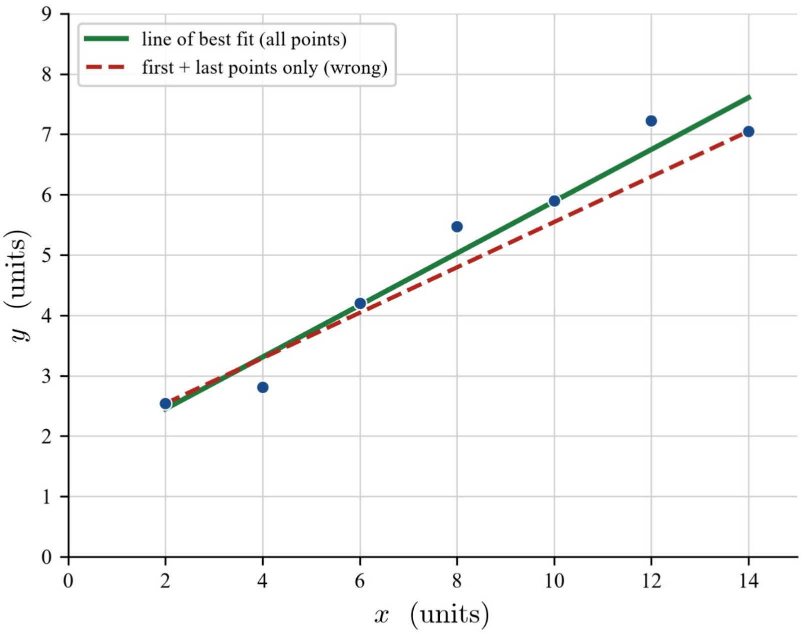


Question 8 The de Broglie wavelength λ of electrons is measured for several momenta p, and λ is plotted against $1/p$, giving a straight line through the origin. Using the two points $(0.4, 2.65)$ and $(0.9, 5.95)$ on the line — with $1/p$ in $10^{24}\,kg^{-1} m^{-1} s$ and λ in 10^{-10} m — determine the value of Planck's constant that this experiment gives.



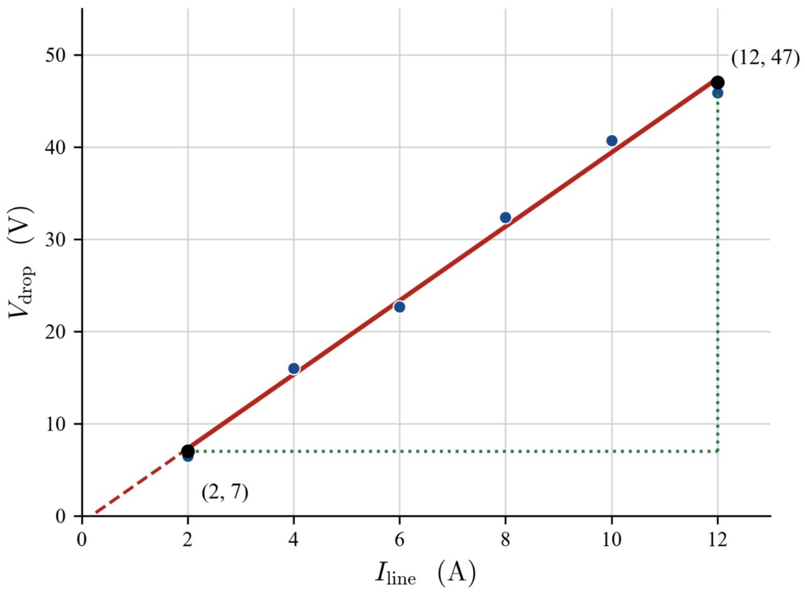
Question 9 For each of the following relationships, state what should be plotted on each axis to obtain a straight-line graph, and what the gradient of that graph would represent. (a) The elastic potential energy of a spring, $E_s = 1/2\,k\,x^2$. (b) The gravitational field strength of a planet, $g = \frac{GM}{r^2}$. (c) The orbital period of a satellite, $T^2 = \frac{4\pi^2}{GM} r^3$. (d) The de Broglie wavelength of a particle, $\lambda = \frac{h}{p}$.

Question 10 A student measures a quantity y against a quantity x and obtains the data below. Line A is the line of best fit; line B joins the first and last data points.



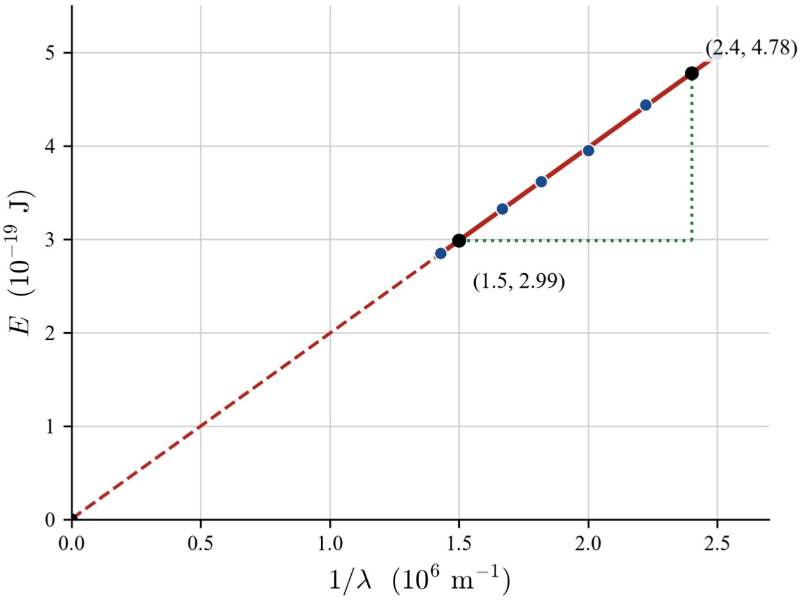
Explain what is wrong with line B, describe how the correct line of best fit (line A) is drawn, and state which of the two gradients — approximately 0.43 or approximately 0.38 — should be quoted as the result.

Question 11 In a transmission-line experiment the voltage drop V_{drop} along a length of cable is measured for several line currents I_{line}, and V_{drop} is plotted against I_{line}.



Using two points on the line of best fit, determine the resistance of the cable. Give your answer to three significant figures.

Question 12 The energy E of photons of several wavelengths λ is measured, and E is plotted against $1/\lambda$, giving a straight line through the origin.



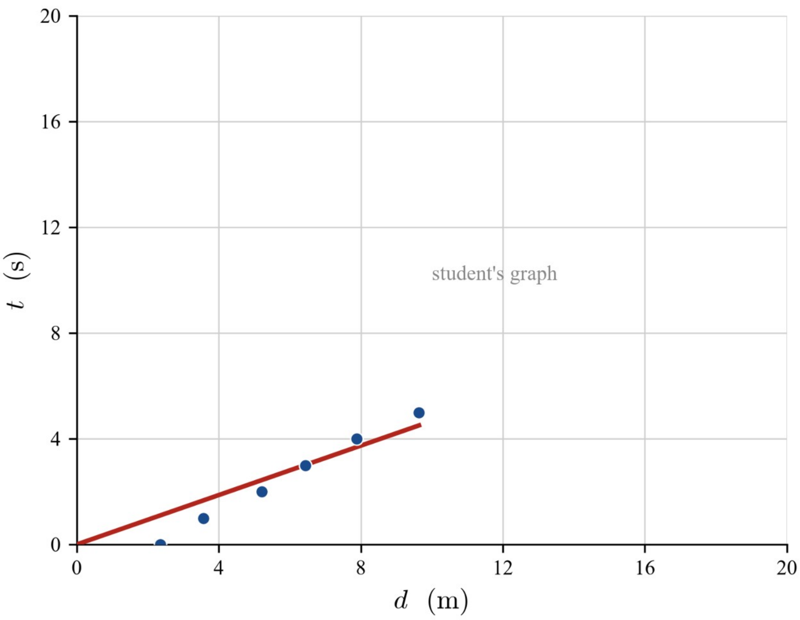
Using two points on the line of best fit, $(1.5, 2.99)$ and $(2.4, 4.78)$ — with $1/\lambda$ in $10^6\ \text{m}^{-1}$ and E in 10^{-19} J — determine the gradient, and hence Planck's constant h (take $c = 3.00 \times 10^8\ \text{m s}^{-1}$).

Question 13 A student presents the following table of results for the extension of a spring under different loads.

Mass	Extension
0.10	12
0.20	25
0.30	39
0.40	52
0.50	64

Identify three faults in the way this table has been set out, and write a corrected heading row that follows the conventions for presenting data.

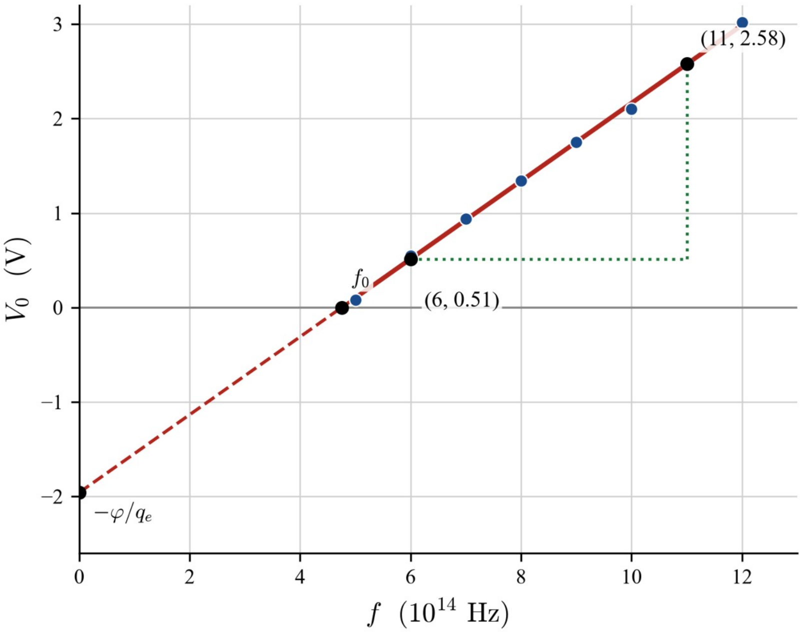
Question 14 A student measured the distance d travelled by a trolley at a series of times t that they chose (so t is the independent variable). The graph the student drew is shown below.



Identify three faults in the way this graph has been drawn, and state the correction for each.

Question 15 Electrons of speed v move in circular paths of radius r in a uniform magnetic field B, where $r = \frac{m_e v}{q_e B}$. A graph of r against v is a straight line through the origin with gradient 1.14×10^{-11} s. Taking $m_e = 9.11 \times 10^{-31}$ kg and $q_e = 1.60 \times 10^{-19}$ C, determine the magnetic field strength B.

Question 16 The stopping voltage V_0 needed to halt the photoelectrons from a metal is measured for several frequencies f of incident light, and V_0 is plotted against f.



The theory gives $V_0 = \frac{h}{q_e} f - \frac{\varphi}{q_e}$. (a) Using two points on the line of best fit, determine the gradient and hence Planck's constant h. (b) State the physical meaning of the vertical and horizontal intercepts, and use the graph to find the work function φ (in eV) and the threshold frequency f_0. (c) Explain why the intercepts must be found by extrapolating the line beyond the measured data.

Question 17 The elastic potential energy E_s stored in a spring is measured for several extensions x, and E_s is plotted against x^2. The graph is a straight line through the origin with gradient 125 N m^{-1}. Given that $E_s = 1/2\,k\,x^2$, determine the spring constant k.

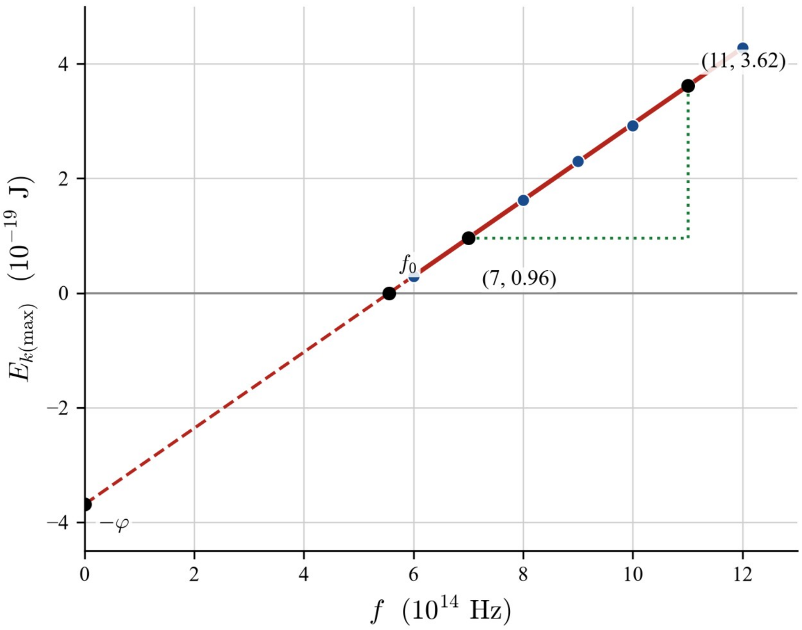
Question 18 Two experiments each produce a straight-line graph. In experiment A the two quantities are directly proportional; in experiment B the graph has a clear positive vertical-axis intercept. (a) For which experiment should the line of best fit be drawn through the origin? Justify your answer. (b) Under what circumstance may the point $(0, 0)$ itself be plotted as a data point? (c) For experiment B, explain why forcing the line through the origin would be incorrect, and give two things the intercept might represent.

Question 19 The gravitational field strength g around a planet is measured at several distances r from its centre, and g is plotted against $1/r^2$. The graph is a straight line through the origin with gradient 4.90×10^{14} m^3 s^{-2}. Given that $g = \frac{GM}{r^2}$ and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, determine the mass M of the planet.

Question 20 A student proposes that the force F between two magnets varies with their separation r according to $F \propto \frac{1}{r^2}$. Describe how the student could test this proposal graphically, state what the graph would have to show to support it, and explain what a *curved* graph, or a straight graph with a *non-zero intercept*, would each indicate.

Question 21 A student investigates the photoelectric effect by shining light of different frequencies f on a metal surface and measuring the maximum kinetic energy E_k of the ejected photoelectrons, repeating each measurement three times. The averaged results are used to plot the graph shown.

Frequency f (10^{14} Hz)	Trial 1 E_k (10^{-19} J)	Trial 2 E_k (10^{-19} J)	Trial 3 E_k (10^{-19} J)	Average E_k (10^{-19} J)
6.0	0.34	0.28	0.25	0.29
7.0	0.93	1.02	0.96	0.97
8.0	1.64	1.68	1.53	1.62
9.0	2.31	2.29	2.30	2.30
10.0	2.92	2.99	2.86	2.92
11.0	3.64	3.59	3.67	3.63
12.0	4.19	4.28	4.37	4.28



(a) Identify the independent and the dependent variable.

(b) State two features that the results table above shows in accordance with the conventions for presenting data.

(c) Using two points on the line of best fit, determine Planck's constant h.

(d) Determine the work function φ and the threshold frequency f_0 from the intercepts.

(e) State whether the line of best fit should pass through the origin, and justify your answer with reference to the photoelectric effect.

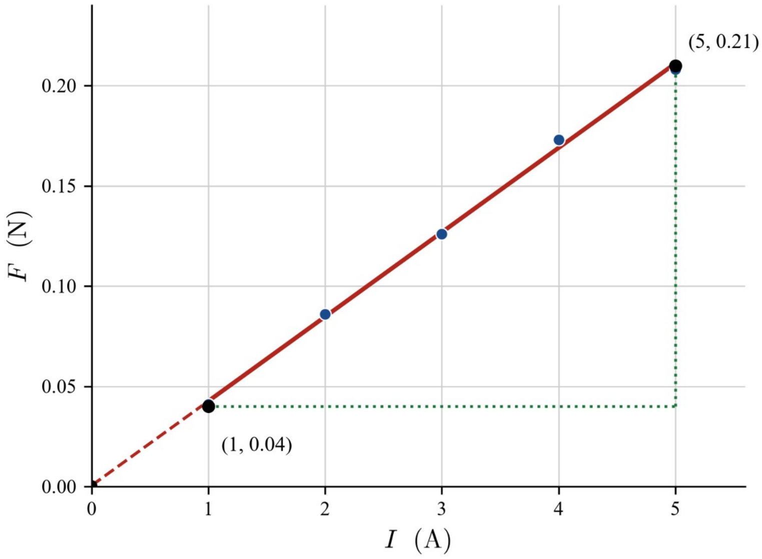
Answers

Question 1 (a) Current I; horizontal axis. (b) Weigh all points, scattered evenly about the line; not through the first and last points only. (c) gradient $= 0.0425\,\mathrm{N\,A^{-1}}$; $B = 0.35\,\mathrm{T}$. **Question 2** (a) $E_k = hf - \varphi$ is linear in f. (b) $h \approx 6.7\times10^{-34}\,\mathrm{J\,s}$. (c) $\varphi \approx 3.1\times10^{-19}\,\mathrm{J} = 1.9\,\mathrm{eV}$; $f_0 \approx 4.6\times10^{14}\,\mathrm{Hz}$. **Question 3** (a) $v = \sqrt{GM/r}$ — an inverse-square-root, not a straight line. (b) v^2 against $1/r$; gradient $= GM$. (c) $M \approx 4.5\times10^{24}\,\mathrm{kg}$. **Question 4** (a) Weigh all points; the dashed line uses only two scattered points. (b) $a \approx 3.2\,\mathrm{m\,s^{-2}}$. (c) vertical intercept = initial velocity $u \approx 1.2\,\mathrm{m\,s^{-1}}$. **Question 5** (a) $F \propto 1/r^2$, not $1/r$; F vs $1/r^2$ is a straight line through the origin. (b) gradient $= 0.018\,\mathrm{N\,m^2} = k q_1 q_2$; $q_2 = 1.0\times10^{-6}\,\mathrm{C}$. (c) No — $(0,0)$ is a theoretical limit ($1/r^2 \to 0$), not a measured point. **Question 6** (a) T^2 is directly proportional to r^3; gradient $= 4\pi^2/GM$. (b) $M \approx 5.0\times10^{26}\,\mathrm{kg}$. (c) $T^2 = 0$ when $r^3 = 0$; a non-zero intercept indicates a systematic error. **Question 7** $m = 0.50\,\mathrm{kg}$. **Question 8** $h \approx 6.6\times10^{-34}\,\mathrm{J\,s}$. **Question 9** (a) E_s vs x^2, gradient $1/2k$. (b) g vs $1/r^2$, gradient GM. (c) T^2 vs r^3, gradient $4\pi^2/GM$. (d) λ vs $1/p$, gradient h. **Question 10** Line B uses only two scattered points and gives the wrong gradient; line A weighs all points. Quote line A: gradient ≈ 0.43. **Question 11** $R_{\text{line}} = 4.00\,\Omega$. **Question 12** gradient $= 1.99\times10^{-25}\,\mathrm{J\,m} = hc$; $h = 6.63\times10^{-34}\,\mathrm{J\,s}$. **Question 13** No units/powers of ten in the headings; no repeated readings or average; no title; independent variable (mass) should still be first — corrected heading below. **Question 14** Independent variable t is on the vertical axis (should be horizontal); scale does not fill the grid; line forced through the origin despite a genuine intercept. **Question 15** $B = 0.50\,\mathrm{T}$. **Question 16** (a) gradient $= 0.414\,\mathrm{V}/(10^{14}\,\mathrm{Hz}) = 4.14\times10^{-15}\,\mathrm{V\,s}$; $h = 6.6\times10^{-34}\,\mathrm{J\,s}$. (b) vertical intercept $= -\varphi/q_e \Rightarrow \varphi \approx 2.0\,\mathrm{eV}$; horizontal intercept $f_0 \approx 4.8\times10^{14}\,\mathrm{Hz}$. (c) the intercepts lie outside the measured range, so the line must be extrapolated. **Question 17** $k = 250\,\mathrm{N\,m^{-1}}$. **Question 18** (a) Experiment A (proportional). (b) Only if $(0,0)$ was authentically measured. (c) The data have a real intercept; it could be a systematic zero error or a genuine physical constant. **Question 19** $M = 7.35\times10^{24}\,\mathrm{kg}$. **Question 20** Plot F against $1/r^2$; a straight line through the origin supports $F \propto 1/r^2$. A curve rejects it; a straight line with a non-zero intercept indicates a systematic error. **Question 21** (a) independent f, dependent E_k. (b) units + powers of ten in the headings, and an average of three trials. (c) $h \approx 6.65\times10^{-34}\,\mathrm{J\,s}$. (d) $\varphi \approx 3.7\times10^{-19}\,\mathrm{J} = 2.3\,\mathrm{eV}$; $f_0 \approx 5.6\times10^{14}\,\mathrm{Hz}$. (e) No — it has a vertical intercept $-\varphi$; electrons are only emitted above f_0.

Fully-worked solutions

Question 1 (a) The current I is the quantity the student sets, so it is the independent variable and goes on the **horizontal** axis; the averaged force is the dependent variable and goes on the vertical axis.

(b) The line of best fit is drawn to follow the trend of **all five** averaged points, with the points scattered as evenly as possible above and below it. Joining only the first and last points is wrong because each of those points carries the same random scatter as any other; a line pinned to them ignores the three points in between and generally gives an inaccurate gradient.



(c) Reading two well-separated points on the line of best fit, $(1.0, 0.04)$ and $(5.0, 0.21)$:

$$\text{gradient} = \frac{0.21 - 0.04}{5.0 - 1.0} = \frac{0.17}{4.0} = 0.0425\,\mathrm{N\,A^{-1}}.$$

For a straight wire $F = BIl$, so the gradient of F against I equals Bl. Hence

$$B = \frac{\text{gradient}}{l} = \frac{0.0425}{0.12} = 0.35\,\mathrm{T}.$$

Question 2 (a) The photoelectric equation is $E_k = hf - \varphi$. With h and φ constant this has the form $y = mx + c$ (a straight line) with E_k as y and f as x, so a graph of E_k against f is a straight line of gradient h and vertical intercept $-\varphi$.

(b) Two points on the line of best fit are $(6.0, 0.94)$ and $(11.0, 4.27)$ (with f in $10^{14}\,\mathrm{Hz}$ and E_k in $10^{-19}\,\mathrm{J}$):

$$h = \frac{(4.27 - 0.94) \times 10^{-19}}{(11.0 - 6.0) \times 10^{14}} = \frac{3.33 \times 10^{-19}}{5.0 \times 10^{14}} = 6.7 \times 10^{-34}\,\mathrm{J\,s},$$

close to the accepted value $6.63 \times 10^{-34}\,\mathrm{J\,s}$.

(c) Extrapolating the line to the vertical axis gives an intercept of about $-3.1 \times 10^{-19}\,\mathrm{J}$. Since the intercept is $-\varphi$,

$$\varphi = 3.1 \times 10^{-19}\,\mathrm{J} = \frac{3.1 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.9\,\mathrm{eV}.$$

The line meets the horizontal axis at the threshold frequency $f_0 \approx 4.6 \times 10^{14}\,\mathrm{Hz}$.

Question 3 (a) For a gravitational orbit $\frac{v^2}{r} = \frac{GM}{r^2}$, so $v = \sqrt{\frac{GM}{r}}$. The speed depends on the inverse square root of r, which is a curve, not a straight line.

(b) Squaring gives $v^2 = \frac{GM}{r} = GM\left(\frac{1}{r}\right)$, which is linear in $\frac{1}{r}$. The right-hand panel plots v^2 against $\frac{1}{r}$; the gradient of that straight line is GM.

(c) Reading two points on the line of best fit, $(2.0, 0.6)$ and $(14.0, 4.2)$ — with $\frac{1}{r}$ in $10^{-8}\,\mathrm{m^{-1}}$ and v^2 in $10^{7}\,\mathrm{m^2\,s^{-2}}$:

$$GM = \frac{(4.2 - 0.6) \times 10^{7}}{(14.0 - 2.0) \times 10^{-8}} = \frac{3.6 \times 10^{7}}{1.2 \times 10^{-7}} = 3.0 \times 10^{14}\,\mathrm{m^3\,s^{-2}},$$

$$M=\frac{3.0\times10^{14}}{6.67\times10^{-11}}=4.5\times10^{24}\,\text{kg}.$$

Question 4 (a) A line of best fit is drawn to follow the trend of all six points, with roughly equal numbers of points above and below it. The dashed line joins only the first and last points; because those two points scatter about the true line (here the first is low and the last is high), the dashed line is too steep and is not a valid line of best fit.

(b) Reading two points on the line of best fit, $(0.10,1.49)$ and $(0.90,4.06)$:

$$a=\frac{4.06-1.49}{0.90-0.10}=\frac{2.57}{0.80}=3.2\,\text{m s}^{-2}.$$

(c) The vertical intercept is the velocity when $t=0$, i.e. the initial velocity of the trolley, $u\approx1.2\,\text{m s}^{-1}$.

Question 5 (a) Coulomb's law gives $F=\frac{k q_1 q_2}{r^2}$: F depends on $\frac{1}{r^2}$, not on r, so a graph of F against r is a curve. Written as $F=k q_1 q_2\left(\frac{1}{r^2}\right)$ the relationship is linear in $\frac{1}{r^2}$, so F against $\frac{1}{r^2}$ is a straight line; because $F\to0$ as $\frac{1}{r^2}\to0$, it passes through the origin.

(b) Two points on the line of best fit are $(20,0.38)$ and $(90,1.64)$ (with $1/r^2$ in m^{-2} and F in N):

$$\text{gradient}=\frac{1.64-0.38}{90-20}=\frac{1.26}{70}=0.018\,\text{N m}^2.$$

The gradient equals $k q_1 q_2$, so

$$q_1 q_2=\frac{0.018}{8.99\times10^9}=2.0\times10^{-12}\,\text{C}^2,\ q_2=\frac{2.0\times10^{-12}}{2.0\times10^{-6}}=1.0\times10^{-6}\,\text{C}.$$

(c) The line *should* be drawn through the origin because the relationship is a direct proportion, but $(0,0)$ should **not** be plotted as a measured data point: it corresponds to infinite separation ($1/r^2=0$), which cannot be measured. It is a theoretical limit that the extrapolated line approaches, not an authentic reading.

Question 6 (a) $T^2=\frac{4\pi^2}{GM}r^3$ has the form $y=mx$ with $y=T^2$ and $x=r^3$, so T^2 is directly proportional to r^3 and the graph is a straight line through the origin with gradient $\frac{4\pi^2}{GM}$.

(b) Two points on the line of best fit are $(0.5,0.60)$ and $(2.0,2.37)$ (with r^3 in $10^{26}\,\text{m}^3$ and T^2 in $10^{11}\,\text{s}^2$):

$$\text{gradient}=\frac{(2.37-0.60)\times10^{11}}{(2.0-0.5)\times10^{26}}=\frac{1.77\times10^{11}}{1.5\times10^{26}}=1.18\times10^{-15}\,\text{s}^2\,\text{m}^{-3}.$$

Since the gradient is $\frac{4\pi^2}{GM}$,

$$M=\frac{4\pi^2}{G\times\text{gradient}}=\frac{4\pi^2}{6.67\times10^{-11}\times1.18\times10^{-15}}=5.0\times10^{26}\,\text{kg}.$$

(c) The relationship is a direct proportion, so when $r^3=0$ then $T^2=0$: the line must pass through the origin. A non-zero vertical intercept would not be physically meaningful here and would signal a systematic error in the timing or radius measurements.

Question 7 The kinetic energy is $E_k=1/2\,m v^2=(1/2\,m)v^2$, so a graph of E_k against v^2 has gradient $1/2\,m$. Reading two points on the line of best fit, $(4.0,1.0)$ and $(32.0,8.0)$:

$$1/2\,m=\frac{8.0-1.0}{32.0-4.0}=\frac{7.0}{28.0}=0.25\,\text{kg}\,,m=2\times0.25=0.50\,\text{kg}\,.$$

Question 8 Since $\lambda=h\left(\frac{1}{p}\right)$, the gradient of λ against $\frac{1}{p}$ is h. Reading the two points $(0.4\,,2.65)$ and $(0.9\,,5.95)$:

$$h=\frac{(5.95-2.65)\times10^{-10}}{(0.9-0.4)\times10^{24}}=\frac{3.30\times10^{-10}}{0.5\times10^{24}}=6.6\times10^{-34}\,\text{J s}\,.$$

Question 9 (a) $E_s=(1/2\,k)\,x^2$: plot E_s (vertical) against x^2 (horizontal); gradient $=1/2\,k$. (b) $g=G\,M\left(\frac{1}{r^2}\right)$: plot g against $\frac{1}{r^2}$; gradient $=G\,M$. (c) $T^2=\frac{4\,\pi^2}{G\,M}r^3$: plot T^2 against r^3; gradient $=\frac{4\,\pi^2}{G\,M}$. (d) $\lambda=h\left(\frac{1}{p}\right)$: plot λ against $\frac{1}{p}$; gradient $=h$.

Question 10 Line B is drawn through only the first and last data points. Those two points scatter about the true trend just as much as any other point — here the first point sits above the trend and the last below it — so a line joining them is too shallow, has the wrong gradient and ignores the five points between. The correct line of best fit, line A, is drawn to follow the trend of **all** the points, with the points scattered as evenly as possible above and below it. The result to quote is the gradient of line A, approximately 0.43; line B's value of about 0.38 is roughly 10 % too low and should not be used.

Question 11 For a resistive cable $V_{\text{drop}}=R_{\text{line}}I_{\text{line}}$, so the gradient of V_{drop} against I_{line} is R_{line}. Reading two points on the line of best fit, $(2.0\,,7.0)$ and $(12.0\,,47.0)$:

$$R_{\text{line}}=\frac{47.0-7.0}{12.0-2.0}=\frac{40.0}{10.0}=4.00\,\Omega\,.$$

Question 12 Since $E=h\,c\left(\frac{1}{\lambda}\right)$, the gradient of E against $\frac{1}{\lambda}$ is $h\,c$. Reading the two points $(1.5\,,2.99)$ and $(2.4\,,4.78)$ — with $1/\lambda$ in $10^6\,\text{m}^{-1}$ and E in 10^{-19} J:

$$h\,c=\frac{(4.78-2.99)\times10^{-19}}{(2.4-1.5)\times10^{6}}=\frac{1.79\times10^{-19}}{0.9\times10^{6}}=1.99\times10^{-25}\,\text{J m}\,.$$

Hence

$$h=\frac{h\,c}{c}=\frac{1.99\times10^{-25}}{3.00\times10^{8}}=6.63\times10^{-34}\,\text{J s}\,.$$

Question 13 Three faults:

1. The column headings give no **units** and no **power of ten** — is the mass in kilograms or grams, and is the extension in millimetres or centimetres?
2. Only a **single reading** is recorded for each load: there are no repeated trials and no **average**, so the effect of random error cannot be reduced or judged.
3. There is **no title** describing what was measured.

A corrected heading row (with three trials averaged) is:

Mass m (kg)	Trial 1 x (mm)	Trial 2 x (mm)	Trial 3 x (mm)	Average x (mm)

Question 14 Three faults, with corrections:

1. The independent variable t has been placed on the **vertical** axis. It should be on the **horizontal** axis, with the dependent variable d on the vertical axis.

2. The **scale does not fill the grid** — every point is squashed into the bottom-left corner. A scale with fewer units per square should be chosen so the points spread across the whole grid.
3. The line has been **forced through the origin** even though the data do not pass through $(0,0)$ — extended backwards, the trend meets an axis away from the origin, so the genuine intercept has been ignored. The line of best fit should follow the data and not be pinned to the origin.

Question 15 For the circular path $r=\frac{m_e v}{q_e B}=\left(\frac{m_e}{q_e B}\right)v$, so the gradient of r against v is $\frac{m_e}{q_e B}$. Rearranging for B:

$$B=\frac{m_e}{q_e \times \text{gradient}}=\frac{9.11\times 10^{-31}}{1.60\times 10^{-19}\times 1.14\times 10^{-11}}=0.50\,\text{T}.$$

Question 16 (a) Two points on the line of best fit are $(6.0,0.51)$ and $(11.0,2.58)$ (with f in 10^{14} Hz and V_0 in V):

$$\text{gradient}=\frac{2.58-0.51}{(11.0-6.0)\times 10^{14}}=\frac{2.07}{5.0\times 10^{14}}=4.14\times 10^{-15}\,\text{V s}.$$

The gradient is $\frac{h}{q_e}$, so

$$h=q_e\times \text{gradient}=1.60\times 10^{-19}\times 4.14\times 10^{-15}=6.6\times 10^{-34}\,\text{J s}.$$

(b) The vertical intercept is $-\frac{\varphi}{q_e}$; the line meets the vertical axis at about $-2.0\,\text{V}$, so $\varphi\approx 2.0\,\text{eV}$. The horizontal intercept is the threshold frequency $f_0\approx 4.8\times 10^{14}$ Hz, the lowest frequency that ejects electrons.

(c) The measured data all lie at frequencies above f_0 and at positive stopping voltages, so neither intercept falls within the measured range. Both are found by extending (extrapolating) the straight line of best fit — drawn dashed — beyond the data until it reaches each axis.

Question 17 With $E_s=1/2\,k\,x^2=(1/2\,k)\,x^2$, the gradient of E_s against x^2 is $1/2\,k$. Therefore

$$k=2\times \text{gradient}=2\times 125=250\,\text{N m}^{-1}.$$

Question 18 (a) The line should be drawn through the origin only in **experiment A**, because directly proportional quantities satisfy $y=m\,x$: when $x=0$, $y=0$, so the line must pass through $(0,0)$.

(b) The point $(0,0)$ may be plotted as a data point only if it was **authentically measured** — for example, if setting the independent variable to zero (zero current, zero load) really was observed to give a zero reading. It must never be added just to force the line through the origin.

(c) In experiment B the data themselves have a genuine vertical intercept, so forcing the line through the origin would misrepresent the measurements and change the gradient. The intercept could represent (i) a **systematic error**, such as a zero-offset in the instrument, or (ii) a **real physical quantity**, such as an initial value (u in a v–t graph) or a work function ($-\varphi$ in an E_k–f graph).

Question 19 For a gravitational field $g=\frac{GM}{r^2}=GM\left(\frac{1}{r^2}\right)$, so the gradient of g against $\frac{1}{r^2}$ is GM. Hence

$$M=\frac{\text{gradient}}{G}=\frac{4.90\times 10^{14}}{6.67\times 10^{-11}}=7.35\times 10^{24}\,\text{kg}.$$

Question 20 The proposal $F\propto\frac{1}{r^2}$ can be tested by measuring F at several separations r and plotting F against $\frac{1}{r^2}$ (not against r). If the proposal is correct the graph is a **straight line through the origin**: straightness confirms that F

depends on $\frac{1}{r^2}$, passing through the origin confirms a direct proportion (no offset), and the gradient gives the constant of proportionality. If instead the F-versus-$\frac{1}{r^2}$ graph is **curved**, then F is not proportional to $\frac{1}{r^2}$ and the proposed power is wrong. If the graph is straight but has a **non-zero intercept**, the proportionality is spoiled by a systematic error (for example a zero-offset in the force sensor) or by an additional constant force acting throughout.

Question 21 (a) The **independent** variable is the frequency f (chosen by the student); the **dependent** variable is the maximum kinetic energy E_k.

(b) The table follows the conventions by (i) heading each column with the quantity, its unit and the **power of ten** (e.g. f in 10^{14} Hz), and (ii) recording **three repeated trials** and their **average**, which reduces the effect of random error on each plotted point.

(c) Reading two well-separated points on the line of best fit, $(7.0, 0.96)$ and $(11.0, 3.62)$ (with f in 10^{14} Hz and E_k in 10^{-19} J):

$$h = \frac{(3.62 - 0.96) \times 10^{-19}}{(11.0 - 7.0) \times 10^{14}} = \frac{2.66 \times 10^{-19}}{4.0 \times 10^{14}} = 6.65 \times 10^{-34}\ \text{J s},$$

close to the accepted value 6.63×10^{-34} J s.

(d) Extrapolating the line to the vertical axis gives an intercept of about $-3.7 \times 10^{-19}\ \text{J} = -\varphi$, so

$$\varphi = 3.7 \times 10^{-19}\ \text{J} = \frac{3.7 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.3\ \text{eV}.$$

The line meets the horizontal axis at the threshold frequency $f_0 \approx 5.6 \times 10^{14}$ Hz.

(e) No. The line has a **negative** vertical intercept, $-\varphi$: photoelectrons are only emitted when the frequency exceeds the threshold f_0, so below f_0 there is no kinetic energy at all and the straight-line relationship $E_k = hf - \varphi$ does not pass through the origin.

Theory

Every scientific investigation begins as a **design**: a plan that turns a broad curiosity into a question that measurements can actually answer. In Unit 4 Area of Study 2 you design and conduct your own **student-designed practical investigation** into an aspect of *fields, motion or light*. It must generate **primary data** — data you collect first-hand — and it must be built around **one continuous independent variable**. The key knowledge of investigation design is examinable on the end-of-year examination, where questions typically supply a short scenario (“stem”) and ask you to draw out its aim, variables, methodology, safety and ethics. This subtopic is about reading such a stem and designing well.

Primary versus secondary data. *Primary data* are measurements the investigator generates directly — the timings, voltages, currents and lengths you record at the bench. *Secondary data* are sourced from someone else’s work (a textbook table, a published dataset, a manufacturer’s specification). The Unit 4 investigation must rest on **primary data**; secondary sources may support the discussion but cannot be the evidence itself.

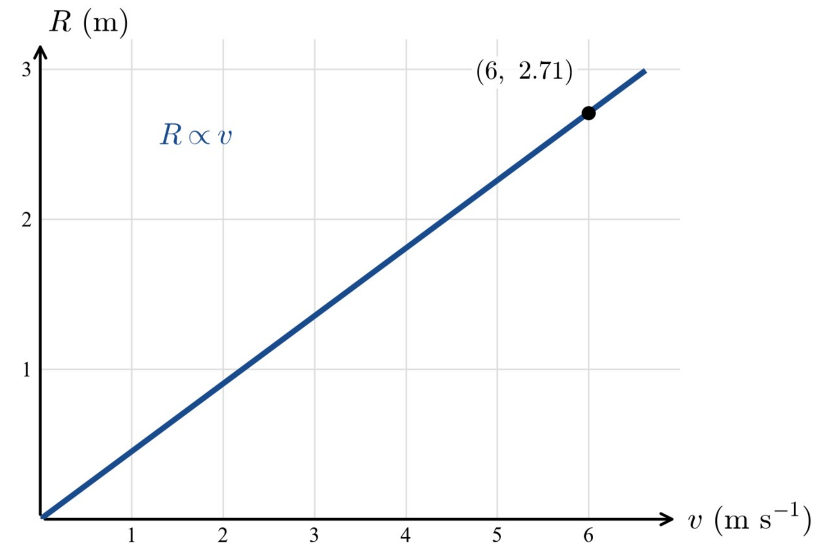
Aim. The **aim** states the purpose of the investigation in one sentence. A good aim names the relationship under study by identifying both variables, for example: *“To investigate how the horizontal range of a projectile depends on its launch speed.”* From the aim a matching **research question** follows directly — *“How does the horizontal range depend on the launch speed?”*

Hypothesis. A **hypothesis** is a *testable, falsifiable* proposed answer to the research question — a statement of the expected relationship between the variables, together with the physics reasoning behind it. It is not a guess and not a definition. A strong hypothesis:

- names the independent and dependent variables and the **direction** of the expected relationship;
- is grounded in physics theory; and
- could in principle be shown **false** by the data.

For example: *“The horizontal range will be proportional to the launch speed, because a ball fired horizontally from a fixed height h falls for a time $t=\sqrt{2h/g}$ that does not depend on the launch speed, so the range is $R=vt=v\sqrt{2h/g}$, i.e. $R\propto v$.”*

Prediction. A **prediction** is a specific, usually quantitative, statement of what will be *observed in a particular case* if the hypothesis is correct — typically an “if … then …” statement. Where a hypothesis proposes the general relationship, a prediction pins it to numbers you can check: *“If the launch speed is doubled, then the range will double.”* Distinguishing the two matters: the hypothesis is the general claim; the prediction is a testable consequence of it.



Scientific methodologies. The study design recognises eight **scientific methodologies** — broad approaches to generating evidence. You should be able to recognise each and choose the one that suits a given aim:

- **controlled experiment** — deliberately change one independent variable, measure the dependent variable, and hold all other relevant variables constant;
- **case study** — an in-depth study of a single activity, event or situation;
- **classification and identification** — arranging phenomena into groups by shared characteristics;
- **fieldwork** — collecting primary data outside the laboratory, in a real-world setting;
- **modelling** — constructing a physical or mathematical representation of a system to explain or predict its behaviour;
- **simulation** — using a computational model to imitate a system and explore its behaviour;
- **literature review** — collating and evaluating secondary data from published sources;
- **product, process or system development** — designing and building an artefact or procedure to meet a need.

A Unit 4 physics investigation with one continuous independent variable and primary data is almost always a **controlled experiment**. Note the difference between a **methodology** (the overall type of approach — a controlled experiment) and the **method** (the specific step-by-step procedure — the apparatus, the quantities set, the measurements taken, the number of repeats).

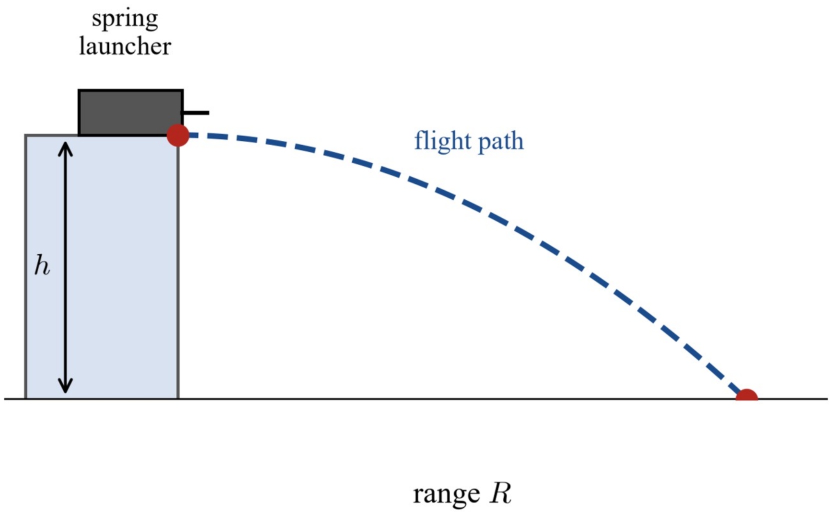
Variables. The design of a controlled experiment turns on three kinds of variable, and correctly classifying them from a scenario is one of the most heavily examined skills in this area — on the 2025 examination fewer than one in ten students gained full marks on the variables table.

- The **independent variable (IV)** is the *one* quantity the investigator deliberately changes. In the Unit 4 investigation it must be **continuous** — able to take any value in a range (a speed, a distance, an angle, an intensity) — not a category or a label.
- The **dependent variable (DV)** is the quantity **measured** in response to the change in the IV.
- The **controlled variables** are the other quantities that could affect the DV and are therefore **held constant** throughout, so that any change in the DV can be attributed to the IV alone.

Three rules follow, and each is a common examination trap:

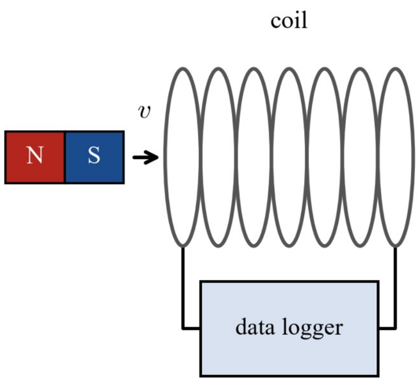
1. **One variable per cell.** State a single physical quantity in each entry, not a list.
2. **A variable is a measurable quantity, never a piece of equipment.** A photogate, a coil, a ramp or an ammeter is *apparatus*, not a variable. The variable is the *quantity* the apparatus sets or measures — "the launch angle", not "the launcher"; "the number of turns on the coil", not "the coil".
3. **Take the variables from the stem.** The independent and dependent variables are named by the aim itself (in "how the **range** depends on the **launch speed**", the DV = range and the IV = launch speed); the controlled variables are the *other* physical quantities in the scenario that would otherwise confound the result.

Any other quantity that is *not* held constant and could affect the DV is an **extraneous** (or **confounding**) variable — a flaw in the design that must be eliminated by adding it to the list of controlled variables.



A worked frame — motion. For *"how the range of a projectile depends on its launch speed"* a spring launcher fires a steel ball horizontally from a bench; the launch speed is set from the spring compression and measured with a pair of photogates, and the range is measured with a tape on the floor. Here the IV is the **launch speed**, the DV is the **horizontal range**, and the controlled variables include the **launch angle**, the **launch height above the floor** and the **mass of the projectile**. The launcher, photogates and tape are equipment, not variables.

A worked frame — fields. For *"how the peak emf induced in a coil depends on the speed of a magnet passing through it"* a bar magnet is moved through a coil connected to a data logger; the peak emf is read from the logger. The IV is the **speed of the magnet**, the DV is the **peak induced emf**, and the controlled variables include the **strength of the magnet**, the **number of turns on the coil** and the **cross-sectional area of the coil**. This design rests on Faraday's law, $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$: a faster magnet produces a greater rate of change of flux, so the hypothesis is that the peak emf is directly proportional to the magnet's speed.



Logbook and authentication. A **logbook** is a dated record, in hard copy or electronic form, kept *throughout* the investigation. Entries are made **contemporaneously** — at the time the work is done — and record the aim, the method and any changes to it, the **raw primary data exactly as measured**, observations, calculations and reflections. The logbook **authenticates** the data as the student's own genuine primary data: it shows the work was actually carried out and was not fabricated or copied. Because it is a permanent record, corrections are made by ruling a **single line** through an entry (never erasing or deleting it), so the full history stays visible and any anomaly can be traced.

Health, safety and ethical guidelines. Before generating data you must carry out a **risk assessment**: identify each hazard, judge its likelihood and consequences, and state the control that reduces the risk. Controls typical of physics investigations include:

- **motion** (projectiles, falling or moving masses): wear safety glasses, keep the flight path and landing zone clear of people, clamp apparatus securely and keep launch energies modest;

- **fields — electricity** (transformers, transmission, motors): use low-voltage supplies, never exceed component ratings, and do not touch live circuits — a step-up transformer can produce a dangerously high secondary voltage;
- **fields — magnetism**: strong neodymium magnets can pinch skin, shatter if allowed to snap together, erase magnetic media and are hazardous near cardiac pacemakers;
- **light** (photoelectric, diffraction): never look into a laser beam or its specular reflection, use the lowest-powered source that works, direct the beam onto a matte screen, and use eye protection with ultraviolet sources.

Ethical guidelines centre on **integrity**: record and report *all* data — including outliers — without fabricating, deleting or "improving" results; acknowledge every source and give proper references; and respect intellectual property. The logbook is itself an ethical instrument, because it makes the data honest and traceable.

Throughout Unit 4 the investigation frame is fixed: **primary data, one continuous independent variable, drawn from fields, motion or light**, and communicated in a scientific poster (maximum 600 words) supported by the logbook.

Worked examples

Example 1 — scaffolded

A student investigates *how the horizontal range of a projectile depends on its launch speed*. A spring launcher fires a steel ball horizontally from a fixed height above the floor; the launch speed is varied and, for each speed, the ball's range along the floor is measured. Classify the variables of this investigation by completing the table, giving one physical quantity per cell.

Variable type	Physical quantity
Independent variable	
Dependent variable	
Controlled variable (1)	
Controlled variable (2)	

Working	Comment
The aim names two quantities: how the **range** depends on the **launch speed**.	The IV and DV always come straight from the aim/stem.
IV = the **launch speed** of the ball.	The one quantity the student deliberately changes; it is continuous.
DV = the **horizontal range** along the floor.	The quantity measured in response to the IV.
Controlled (1) = the **launch angle** (kept horizontal).	A quantity that would change the range if allowed to vary — held constant.
Controlled (2) = the **launch height above the floor**.	Also affects the flight time and hence the range — held constant.
The launcher, photogates and tape are **equipment, not variables**.	A variable is a measurable quantity, never a piece of apparatus.

Example 2 — scaffolded

A student plans an investigation into *how the peak emf induced in a coil depends on the speed of a bar magnet passed through it*. (a) Write a suitable aim. (b) State a hypothesis, with reasoning. (c) At a magnet speed of $0.40\,\mathrm{m\ s^{-1}}$ the peak emf is $0.12\,\mathrm{V}$. Using your hypothesis, predict the peak emf when the magnet speed is $0.80\,\mathrm{m\ s^{-1}}$.

Working	Comment
(a) Aim: *to investigate how the peak emf induced in a*	A one-sentence purpose naming both variables.

Working	Comment
coil depends on the speed of a magnet moving through it.	
(b) By Faraday's law $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, a faster magnet gives a greater rate of change of flux.	Ground the hypothesis in physics theory.
Hypothesis: *the peak induced emf is directly proportional to the speed of the magnet.*	Testable, falsifiable, with a stated direction.
(c) Proportionality means $\varepsilon \propto v$, so doubling v doubles ε: $0.12 \times 2 = 0.24\ \text{V}$.	A prediction is a specific quantitative consequence of the hypothesis.
Prediction: *if the speed is doubled to* $0.80\ \text{m s}^{-1}$*, the peak emf will be* $0.24\ \text{V}$.	"If … then …" form, tied to numbers.

Example 3 — unscaffolded

A student intends to answer the question *"How does the horizontal range of a projectile depend on its launch speed?"* by firing a ball from a launcher and measuring the range for a series of launch speeds. Name the scientific methodology this investigation uses, justify your choice, and distinguish the methodology from the method.

The investigation deliberately changes one independent variable (the launch speed), measures the dependent variable (the range) in response, and holds the other relevant quantities (launch angle, launch height, projectile mass) constant. Generating **primary data** by systematically varying one variable while controlling the rest is the defining feature of a **controlled experiment**, so that is the methodology. It is not fieldwork (the data are generated in a laboratory, not a real-world setting), nor a literature review (the evidence is first-hand, not sourced from publications), nor a simulation (a real ball is fired, not a computational model).

The **methodology** is the *type* of approach — a controlled experiment. The **method** is the specific procedure that carries it out: the spring launcher and photogates used, the set launch speeds, the measurement of range with a floor tape, and the number of repeat trials at each speed.

∴ the methodology is a **controlled experiment**; the method is the detailed step-by-step procedure that implements it.

Example 4 — unscaffolded

An ideal transformer investigation tests *how the output (secondary) voltage depends on the number of turns on the secondary coil*. The primary coil has 300 turns and is driven by a $6.0\ \text{V}$ AC supply. Show that, for a secondary coil of 900 turns, the hypothesis $V_2 \propto N_2$ predicts an output voltage of $18\ \text{V}$, and state the predicted relationship the graph should reveal.

For an ideal transformer the turns and voltages are related by

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} \Rightarrow V_2 = V_1\frac{N_2}{N_1}.$$

With $V_1 = 6.0\ \text{V}$, $N_1 = 300$ and $N_2 = 900$,

$$V_2 = 6.0 \times \frac{900}{300} = 6.0 \times 3 = 18\ \text{V}.$$

Because V_1 and N_1 are held constant (controlled variables), $V_2 = \left(\frac{V_1}{N_1}\right)N_2$, so V_2 is directly proportional to N_2.

∴ the predicted output for 900 turns is $18\ \text{V}$, and a graph of output voltage against number of secondary turns should be a straight line through the origin ($V_2 \propto N_2$).

Practice questions

Question 1 A student investigates *how the horizontal range of a projectile depends on its launch speed*. A ball is fired horizontally from a spring launcher at a fixed height above the floor, and its range is measured for each of several launch speeds. (a) State the independent variable and the dependent variable. (b) State **two** controlled variables for this investigation. (c) A classmate lists "the spring launcher" as a controlled variable. Explain why this is incorrect, and give the physical quantity that should be listed instead.

Question 2 A student plans to investigate *how the peak emf induced in a coil depends on the speed of a magnet dropped through it*. (a) Write a suitable aim for the investigation. (b) State a testable hypothesis, and give the physics reasoning that supports it. (c) At a magnet speed of $0.50\,\mathrm{m\ s^{-1}}$ the peak emf is $0.15\,\mathrm{V}$. Using your hypothesis, predict the peak emf at a speed of $1.0\,\mathrm{m\ s^{-1}}$.

Question 3 A student will investigate *how the range of a projectile depends on its launch speed* by firing a ball in the laboratory and measuring the range at each of several speeds. (a) Name the scientific methodology best suited to this investigation. (b) Justify your choice by reference to how the data are generated. (c) Distinguish between the *methodology* and the *method* of an investigation.

Question 4 A student designs a photoelectric-effect investigation using an ultraviolet lamp and a photocell. (a) Identify **two** health-and-safety hazards of this investigation and state a control for each. (b) State one **ethical** guideline the student must follow when recording and reporting the data. (c) Explain how keeping a logbook helps to authenticate the student's primary data.

Question 5 A student investigates *how the photocurrent from a photocell depends on the intensity of the light striking it*, keeping the frequency of the light and the collecting voltage fixed. Under these conditions the hypothesis is that the photocurrent is directly proportional to the intensity. (a) Complete the variables table, one physical quantity per cell.

Variable type	Physical quantity
Independent variable	
Dependent variable	
Controlled variable (1)	
Controlled variable (2)	

(b) At a certain intensity the photocurrent is $2.0\ \mu\mathrm{A}$. **Show that** the hypothesis predicts a photocurrent of $6.0\ \mu\mathrm{A}$ when the intensity is trebled.

(c) State the shape of the graph of photocurrent against intensity that would support the hypothesis.

Question 6 A student writes: *"My investigation will test whether changing things about a falling magnet changes the electricity in the coil."* (a) Explain why this statement is **not** a suitable hypothesis. (b) Rewrite it as a testable hypothesis for the investigation *"how the peak emf depends on the magnet's speed"*. (c) Explain why the Unit 4 investigation requires the independent variable to be **continuous**, and name a continuous independent variable suitable for this magnet investigation.

Question 7 An ideal transformer investigation tests *how the output (secondary) voltage depends on the number of turns on the secondary coil*. The primary coil and the AC supply are unchanged throughout. Complete the variables table, one physical quantity per cell.

Variable type	Physical quantity
Independent variable	
Dependent variable	
Controlled variable (1)	
Controlled variable (2)	

Question 8 A bar magnet is passed through a coil at a range of speeds and the peak emf is recorded at each speed. Complete the variables table for the investigation *"how the peak induced emf depends on the speed of the magnet"*, one physical quantity per cell.

Variable type	Physical quantity
Independent variable	
Dependent variable	
Controlled variable (1)	
Controlled variable (2)	

Question 9 A student hypothesises that *the horizontal range of a projectile launched at* 45° *is proportional to the square of its launch speed*, on the basis that $R=\frac{v^2\sin 2\theta}{g}$. (a) At a launch speed of $5.0\ \mathrm{m\ s^{-1}}$, use $R=\frac{v^2\sin 2\theta}{g}$ with $\theta=45^\circ$ and $g=9.81\ \mathrm{m\ s^{-2}}$ to predict the range. Give your answer to three significant figures. (b) State, with a reason, what the student's prediction says will happen to the range if the launch speed is tripled.

Question 10 For each of the following investigations, name the most appropriate scientific methodology from the study-design list. (a) A student uses computer software to imitate the motion of a projectile through air and explores how the trajectory changes with launch angle. (b) A student collates and evaluates published measurements of the speed of light made over the last century. (c) A student fires a ball from a launcher in the laboratory and measures its range at a series of launch speeds.

Question 11 Distinguish between a **hypothesis** and a **prediction**, and illustrate the distinction with an example drawn from an investigation into how the induced emf in a coil depends on the speed of a magnet.

Question 12 In a variables table for the projectile investigation of Question 1, a student writes "a photogate" as the independent variable and "the tape measure" as a controlled variable. Identify the error common to both entries, and rewrite the two entries correctly.

Question 13 A student plans to investigate *how the peak emf induced in a coil depends on the speed of a strong neodymium magnet* moving through it. (a) Identify **two** safety hazards specific to using strong neodymium magnets, and state a control for each. (b) The student proposes to discard any measurement that looks "too far" from the others without recording it. Explain why this breaches the ethical guidelines for the investigation.

Question 14 Explain the role of a logbook in a scientific investigation, referring to **three** features that allow it to authenticate the primary data.

Question 15 An ideal transformer investigation uses a primary coil of 400 turns driven by a $9.0\ \mathrm{V}$ AC supply, and varies the number of secondary turns. (a) **Show that** the hypothesis $V_2=V_1\frac{N_2}{N_1}$ predicts an output of $27\ \mathrm{V}$ for a secondary coil of 1200 turns. (b) State the relationship between the output voltage and the number of secondary turns that a graph of the results should reveal, and the shape of that graph.

Question 16 In a controlled experiment, explain — as a reasoning chain — why changing **one** independent variable while holding the other relevant quantities constant allows the investigator to draw a valid conclusion about the cause of any change in the dependent variable.

Question 17 Design an outline for an investigation into *how the photocurrent from a photocell depends on the intensity of the light striking it*. (a) Write a suitable aim. (b) State a testable hypothesis for the investigation, and give the physics reasoning that supports it. (c) State **one** safety precaution appropriate to the light source used.

Question 18 A student proposes to investigate *how the range of a projectile depends on the type of ball used* (tennis ball, golf ball or table-tennis ball). (a) Explain why this investigation does **not** meet the Unit 4 requirement for a **continuous** independent variable. (b) Suggest a change to the independent variable that would meet the requirement, keeping the apparatus essentially the same.

Question 19 A student's completed investigation into *how the range of a projectile depends on its launch speed* is described below. Read it, then answer the questions.

"I set the launcher to six different launch speeds. At each speed I fired the ball three times and recorded the range each time. I kept the launch angle and the launch height the same throughout, and used the same steel ball. I wrote every measurement straight into my logbook as I went."

(a) Identify the independent and dependent variables.

(b) State one reason the student fired the ball three times at each speed.

(c) Explain how recording measurements "straight into the logbook as I went" supports the authentication of the data.

Question 20 A student is to design and conduct an investigation into *how the horizontal range of a projectile fired at* 45° *depends on its launch speed*, generating primary data with a spring launcher and a floor tape. (a) Write an aim and a testable hypothesis for the investigation. (b) Present a complete variables table (independent, dependent and two controlled variables), one physical quantity per cell. (c) State one safety precaution and one reason the student should take repeated measurements at each launch speed. (d) At a launch speed of $4.0\ \mathrm{m\ s^{-1}}$, predict the range (use $R=\frac{v^2\sin 2\theta}{g}$, $\theta=45^{\circ}$, $g=9.81\ \mathrm{m\ s^{-2}}$), and state the shape of the graph of range against launch speed the results should produce.

Answers

Question 1 (a) IV = launch speed; DV = horizontal range. (b) e.g. launch angle; launch height above the floor (also projectile mass). (c) A launcher is equipment, not a variable; the controlled quantity is a measurable one set by the launcher — e.g. the **launch angle** (kept horizontal) or the **launch height above the floor**. **Question 2** (a) To investigate how the peak induced emf depends on the magnet's speed. (b) Peak emf $\propto$ speed, from $\varepsilon = -N\,\Delta\Phi_B/\Delta t$. (c) $0.30\ \text{V}$. **Question 3** (a) Controlled experiment. (b) One IV varied, DV measured, others controlled, primary data. (c) Methodology = type of approach; method = the specific procedure. **Question 4** (a) e.g. UV eye/skin exposure — UV-blocking goggles; electric shock — low-voltage supply. (b) Report all data honestly, including outliers; acknowledge sources. (c) Contemporaneous, dated raw records show the data are genuine and the student's own. **Question 5** (a) IV = light intensity; DV = photocurrent; controlled = frequency of the light, collecting voltage (also cathode material). (b) $2.0 \times 3 = 6.0$ μA. (c) A straight line through the origin. **Question 6** (a) Too vague; no named variables, no direction, not testable. (b) e.g. "The peak emf is directly proportional to the magnet's speed." (c) A continuous IV can take any value in a range, allowing a graph and gradient; e.g. the speed of the magnet. **Question 7** IV = number of secondary turns; DV = output (secondary) voltage; controlled = primary voltage, number of primary turns (also core, AC frequency). **Question 8** IV = speed of the magnet; DV = peak induced emf; controlled = magnet strength, number of coil turns (also coil area). **Question 9** (a) $2.55\ \text{m}$. (b) Range $\propto v^2$, so tripling the speed makes the range $3^2 = 9$ times larger. **Question 10** (a) Simulation. (b) Literature review. (c) Controlled experiment. **Question 11** Hypothesis = general proposed relationship with reasoning; prediction = specific "if … then …" consequence for a particular case. **Question 12** Both entries name equipment, not quantities. IV = launch speed; the controlled entry should be the launch height above the floor — the tape measures the range (the DV), and equipment is never a variable. **Question 13** (a) e.g. pinching/impact when magnets snap together — handle one at a time, keep apart; danger to pacemakers — keep clear of anyone with a pacemaker. (b) Discarding data unrecorded breaches integrity; outliers must be recorded and analysed, not silently dropped. **Question 14** Dated, contemporaneous, raw primary records, kept throughout, corrections struck through not erased — showing the data are genuine and the student's own. **Question 15** (a) $9.0 \times 1200/400 = 27\ \text{V}$. (b) $V_2 \propto N_2$; a straight line through the origin. **Question 16** Holding all other relevant quantities constant means only the IV can cause a change in the DV, so the observed change is attributable to the IV alone. **Question 17** (a) To investigate how the photocurrent depends on light intensity. (b) e.g. *The photocurrent is directly proportional to the intensity of the light*: at fixed frequency a more intense beam delivers more photons per second, so more photoelectrons are released per second and the current is proportionally larger. (c) e.g. do not look into a laser beam / use UV eye protection. **Question 18** (a) "Type of ball" is a category, not a continuous quantity — it cannot take intermediate values or give a gradient. (b) Vary the **launch speed** (continuous) instead. **Question 19** (a) IV = launch speed; DV = horizontal range. (b) To reduce the effect of random errors / to check repeatability. (c) Contemporaneous raw entries show the data are genuine, unaltered and the student's own. **Question 20** (a)/(b) see solution. (c) e.g. safety glasses + clear the flight path; repeats reduce random-error effects. (d) $1.63\ \text{m}$; range $\propto v^2$, so a curve (upward-bending parabola through the origin).

Fully-worked solutions

Question 1 (a) The aim states how the **range** depends on the **launch speed**, so the independent variable is the **launch speed** of the ball and the dependent variable is the **horizontal range** along the floor. (b) Any two quantities that would change the range if allowed to vary, held constant: the **launch angle** (kept horizontal), the **launch height above the floor**, and the **mass of the projectile**. (c) A spring launcher is a piece of **apparatus**, not a physical quantity, so it cannot be a variable. The quantity that must be held constant is a *measurable* one — for example the **launch angle** or the **launch height** — set by the launcher but distinct from it. (A variable is always a measurable quantity, never the equipment used to set or measure it.)

Question 2 (a) *To investigate how the peak emf induced in a coil depends on the speed of a magnet moving through it.* (b) Hypothesis: *the peak induced emf is directly proportional to the speed of the magnet.* Reasoning: by Faraday's law $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, a faster magnet changes the flux through the coil in a shorter time, giving a greater rate of change of flux and hence a greater peak emf; doubling the speed doubles the rate of change of flux. (c) The hypothesis is $\varepsilon \propto v$, so doubling the speed from $0.50\ \text{m s}^{-1}$ to $1.0\ \text{m s}^{-1}$ doubles the peak emf: $\varepsilon = 0.15 \times 2 = 0.30\ \text{V}$.

Question 3 (a) A **controlled experiment**. (b) The data are **primary** (generated first-hand in the laboratory) and are produced by deliberately changing **one** independent variable — the launch speed — while measuring the dependent variable and holding the other relevant quantities constant. Systematic variation of a single variable under controlled conditions is exactly what defines a controlled experiment; it is not fieldwork, a simulation or a literature review. (c) The **methodology** is the overall *type* of approach — here, a controlled experiment. The **method** is the specific *procedure*: the launcher and photogates used, the particular launch speeds set, how the range is measured, and how many repeats are taken at each speed. One is the kind of investigation; the other is how it is actually carried out.

Question 4 (a) Two hazards, each with a control:

- **Ultraviolet light** can damage the eyes and skin — wear UV-blocking goggles and avoid direct exposure to the beam.
- **Electric shock** from the photocell circuit — use a low-voltage supply and do not touch live connections.

(b) An ethical guideline of **integrity**: record and report *all* data honestly, including any outliers, without fabricating or deleting results; and acknowledge all sources used.

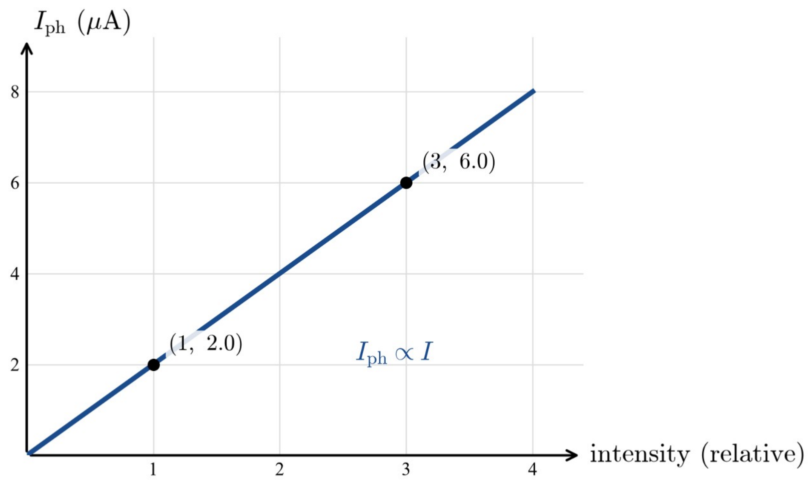
(c) A logbook records the raw primary data **as it is measured**, with dated, contemporaneous entries kept throughout the investigation. Because the record is made at the time and corrections are struck through rather than erased, it demonstrates that the data are genuine, unaltered and the student's own work — this is what *authentication* means.

Question 5 (a)

Variable type	Physical quantity
Independent variable	the intensity of the incident light
Dependent variable	the photocurrent
Controlled variable (1)	the frequency (colour) of the incident light
Controlled variable (2)	the collecting (anode–cathode) voltage

(b) The hypothesis is photocurrent $\propto$ intensity, so trebling the intensity trebles the photocurrent: $2.0 \times 3 = 6.0$ μA, as required.

(c) A graph of photocurrent against intensity that is a **straight line through the origin** would support the hypothesis of direct proportionality.



Question 6 (a) The statement is not testable: it names no specific variables ("things about a falling magnet", "the electricity"), gives no direction for the relationship, and could not be shown false by any particular measurement. A hypothesis must identify the independent and dependent variables and propose a definite, falsifiable relationship. (b) For example: *"The peak emf induced in the coil is directly proportional to the speed of the magnet as it passes through the coil."* (c) A **continuous** independent variable can take any value across a range, so it can be set to many levels and plotted on a numerical axis, giving a graph whose gradient carries physical meaning. A categorical variable could not. A suitable continuous IV here is the **speed of the magnet** as it enters the coil.

Question 7

Variable type	Physical quantity
Independent variable	the number of turns on the secondary coil
Dependent variable	the output (secondary) voltage
Controlled variable (1)	the input (primary) voltage
Controlled variable (2)	the number of turns on the primary coil

(Other acceptable controlled variables: the transformer core and the frequency of the AC supply. The voltmeter and the supply are equipment, not variables.)

Question 8

Variable type	Physical quantity
Independent variable	the speed of the magnet through the coil
Dependent variable	the peak induced emf
Controlled variable (1)	the strength of the magnet
Controlled variable (2)	the number of turns on the coil

(Other acceptable controlled variables: the cross-sectional area of the coil. The coil and the data logger are equipment, not variables.)

Question 9 (a) With $\theta=45^\circ$, $\sin 2\theta=\sin 90^\circ=1$, so $R=\frac{v^2}{g}=\frac{5.0^2}{9.81}=\frac{25}{9.81}=2.55\,\text{m}$ (to three significant figures). (b) The prediction follows from $R\propto v^2$: tripling the launch speed multiplies the range by $3^2=9$, so the range becomes nine times as large.

Question 10 (a) **Simulation** — a computational model imitates the projectile's motion. (b) **Literature review** — published (secondary) data are collated and evaluated. (c) **Controlled experiment** — one IV is varied and the DV measured first-hand under controlled conditions.

Question 11 A **hypothesis** is a general, testable statement of the proposed relationship between the variables, supported by physics reasoning — for example, *"the peak induced emf is directly proportional to the speed of the magnet, because a faster magnet gives a greater rate of change of flux."* A **prediction** is a specific, usually quantitative, consequence of that hypothesis for a particular case, in "if … then …" form — for example, *"if the magnet's speed is doubled, then the peak emf will double."* The hypothesis proposes the relationship; the prediction states a testable outcome that would be observed if the hypothesis holds.

Question 12 The error common to both entries is that they name **equipment, not physical quantities** — a photogate and a tape measure are apparatus. A variable must be a measurable quantity. Corrected: the independent variable is the **launch speed** of the ball (the photogate merely measures it). The tape measure is likewise equipment — and it is the instrument that measures the *dependent* variable, the range — so it cannot be a controlled variable; the controlled quantity intended is the **launch height above the floor**, which is set by the launcher's position and held constant.

Question 13 (a) Two hazards specific to strong neodymium magnets, each with a control:

- They snap together with great force and can **pinch or bruise** fingers, or **shatter** and throw fragments — handle one magnet at a time, keep them apart, and wear safety glasses.
- Their strong field is **hazardous to anyone with a cardiac pacemaker** and can **erase magnetic media** (cards, drives) — keep them well clear of people with medical implants and of magnetic storage.

(b) Silently discarding an unexpected measurement without recording it breaches the ethical guideline of **integrity/honesty**. All data — including outliers — must be recorded in the logbook and then analysed; an anomaly may reveal a genuine effect or a method fault, and deciding in advance to delete inconvenient results biases the conclusion and misrepresents the evidence.

Question 14 A logbook authenticates primary data because it is:

- **contemporaneous and dated** — entries are made at the time the measurements are taken, showing when the work was actually done;
- a **complete raw record** — it holds the measurements exactly as read, together with the method and any changes, so the data are traceable and reproducible; and
- **permanent and transparent** — corrections are ruled through with a single line rather than erased, so nothing can be secretly altered.

Together these features demonstrate that the data are genuine, unaltered and the student's own work.

Question 15 (a) For an ideal transformer $V_2 = V_1 \frac{N_2}{N_1}$. With $V_1 = 9.0\,\mathrm{V}$, $N_1 = 400$ and $N_2 = 1200$,

$$V_2 = 9.0 \times \frac{1200}{400} = 9.0 \times 3 = 27\,\mathrm{V},$$

as required. (b) Since V_1 and N_1 are held constant, $V_2 = \left(\frac{V_1}{N_1}\right) N_2$, so the output voltage is **directly proportional** to the number of secondary turns, $V_2 \propto N_2$. A graph of output voltage against number of secondary turns is therefore a **straight line through the origin**.

Question 16 A valid causal conclusion requires a reasoning chain:

1. The dependent variable could in principle be affected by several quantities.
2. In a controlled experiment all of those quantities except one — the independent variable — are held constant.
3. Therefore the **only** quantity free to cause a change in the dependent variable is the independent variable.
4. So any change observed in the dependent variable must be **due to the independent variable alone**, and the relationship found between them is a valid one.

If instead two quantities were allowed to change together, a change in the DV could not be attributed to either one, and no valid conclusion about cause could be drawn.

Question 17 (a) *To investigate how the photocurrent from a photocell depends on the intensity of the light striking it.* (b) Hypothesis: *the photocurrent is directly proportional to the intensity of the incident light (at fixed frequency and collecting voltage).* Reasoning: at fixed frequency, a greater intensity means more photons strike the cathode each second, so more photoelectrons are released per second and the photocurrent — the rate of charge flow — increases in the same proportion. (c) A suitable precaution: if a laser is used as the light source, never look into the beam or its reflection and direct it onto a matte screen; if an ultraviolet source is used, wear UV eye protection.

Question 18 (a) "Type of ball" is a **categorical** variable — its values (tennis, golf, table-tennis) are distinct labels with no numerical scale. It cannot take intermediate values and cannot be plotted on a continuous numerical axis, so it does not meet the Unit 4 requirement for **one continuous independent variable** and yields no gradient. (b) Keep the same ball and launcher but vary a **continuous** quantity instead — most simply the **launch speed** of the ball (or the launch angle), which can take any value across a range and be plotted numerically.

Question 19 (a) The independent variable is the **launch speed**; the dependent variable is the **horizontal range**. (b) Firing three times at each speed provides **repeat measurements**, which reduce the effect of random errors (by averaging) and allow the repeatability of the results to be checked. (c) Writing each measurement **straight into the logbook as it is taken** makes the record **contemporaneous** and unaltered: it captures the raw primary data at the moment of measurement, which shows the data are genuine and the student's own, and lets any later result be traced back to its original reading.

Question 20 (a) Aim: *to investigate how the horizontal range of a projectile launched at* 45° *depends on its launch speed.* Hypothesis: *the range is directly proportional to the square of the launch speed, because* $R = \frac{v^2 \sin 2\theta}{g}$ *gives* $R \propto v^2$ *at fixed angle.* (b)

Variable type	Physical quantity
Independent variable	the launch speed of the ball
Dependent variable	the horizontal range
Controlled variable (1)	the launch angle (45°)
Controlled variable (2)	the launch height above the floor

(c) Safety: wear safety glasses and keep the flight path and landing area clear of people. Repeats: taking several measurements at each launch speed and averaging reduces the effect of random errors and lets repeatability be checked.

(d) With $\theta = 45^\circ$, $\sin 2\theta = 1$, so $R = \frac{v^2}{g} = \frac{4.0^2}{9.81} = \frac{16}{9.81} = 1.63\,\mathrm{m}$. Because $R \propto v^2$, a graph of range against launch speed is a **curve** (an upward-bending parabola through the origin), not a straight line; plotting range against launch-speed *squared* would give a straight line through the origin.

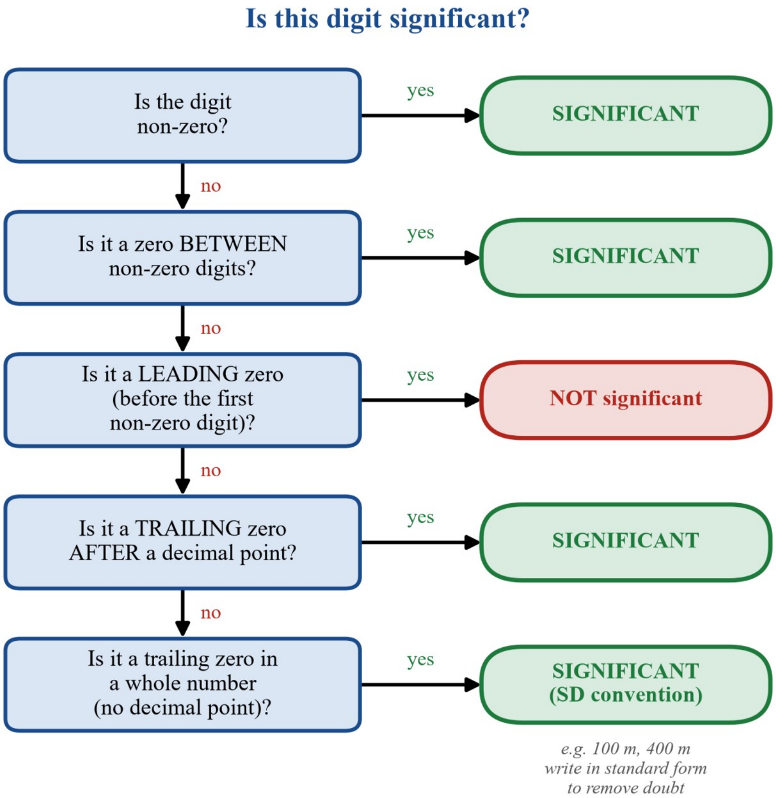
Theory

A scientific investigation is only complete when its ideas are **communicated** clearly and honestly to others. In this study the findings of a student-designed investigation are presented on a **scientific poster**, supported by a **logbook** that records how the data were generated. Good communication follows agreed **conventions**: correct scientific terminology and representations, standard symbols and abbreviations, the right number of **significant figures**, and SI **units of measurement**. This subtopic collects those conventions and the rules for reporting a result so that another person can understand, trust and check it.

Significant figures. The **significant figures** of a measured value are the digits that carry real information about its size and precision. The rules used in this study are:

- every **non-zero** digit is significant;
- a **zero between** non-zero digits is significant (for example 205 has three);
- a **leading zero** — one before the first non-zero digit — is *never* significant; it only fixes the decimal point (for example 0.0480 has three significant figures: the 4, the 8 and the trailing 0);
- a **trailing zero after a decimal point** *is* significant (so 12.00 has four, and 0.0250 has three);
- for a **whole number** the trailing zeros are treated as significant, so $100\,\text{m}$ **has three significant figures** and $8000\,\text{kg}$ has four;
- in **standard form** (scientific notation) every digit of the coefficient is significant, so $3.60 \times 10^{4}\,\text{N}$ has three.

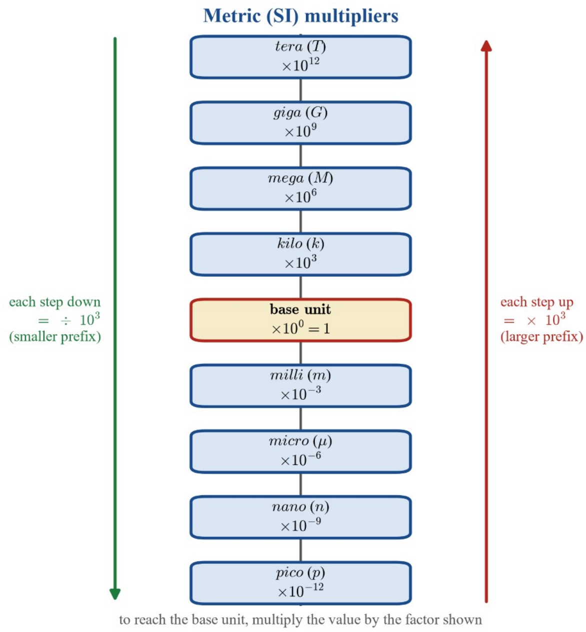
Writing a value in standard form removes any doubt about which zeros count — $1.00 \times 10^{2}\,\text{m}$ is unambiguously three significant figures. The number of significant figures reflects the **precision** of the measurement.



Rounding a calculated result. A calculated answer can never be more precise than the data it comes from. When quantities are multiplied or divided, round the final answer to the **same number of significant figures as the least-**

precise datum used — the answer *matches the least-precise data*. For example, $\frac{100\,\text{m}}{12.00\,\text{s}} = 8.333\ldots$, but because $100\,\text{m}$ carries only three significant figures the speed is written $8.33\,\text{m s}^{-1}$. Keep extra digits during the working and round only at the end.

SI units and metric multipliers. Physical quantities are reported in the **International System of Units (SI)**. The base units used in this course are the **metre** (m), **kilogram** (kg), **second** (s), **ampere** (A) and **kelvin** (K); other units such as the newton (N), joule (J), watt (W), volt (V) and hertz (Hz) are *derived* from these. A **metric multiplier** (prefix) attached to a unit stands for a power of ten:



To convert a prefixed value into the base unit, **multiply by the power of ten** the prefix represents: $655\,\text{nm} = 655 \times 10^{-9}\,\text{m} = 6.55 \times 10^{-7}\,\text{m}$. The ladder above shows the eight prefix multipliers together with the base unit, between milli and kilo: each rung differs from the next by a factor of 10^3, so milli (10^{-3}) and kilo (10^3) themselves differ by 10^6. Two further conversions are worth memorising because they appear often:

$$1\,\text{tonne (t)} = 10^3\,\text{kg}\,,\, 1\,\text{kilowatt hour (kW h)} = 3.6 \times 10^6\,\text{J}\,.$$

The kilowatt hour is an energy: a power of $1\,\text{kW} = 10^3\,\text{W}$ acting for one hour ($3600\,\text{s}$) delivers $10^3 \times 3600 = 3.6 \times 10^6\,\text{J}$. A speed in km h^{-1} is converted to m s^{-1} by writing the kilometres as metres and the hours as seconds: $108\,\text{km h}^{-1} = \frac{108 \times 10^3\,\text{m}}{3600\,\text{s}} = 30.0\,\text{m s}^{-1}$. **Converting every quantity to base SI units before substituting into a formula** is the single most reliable way to avoid calculation errors.

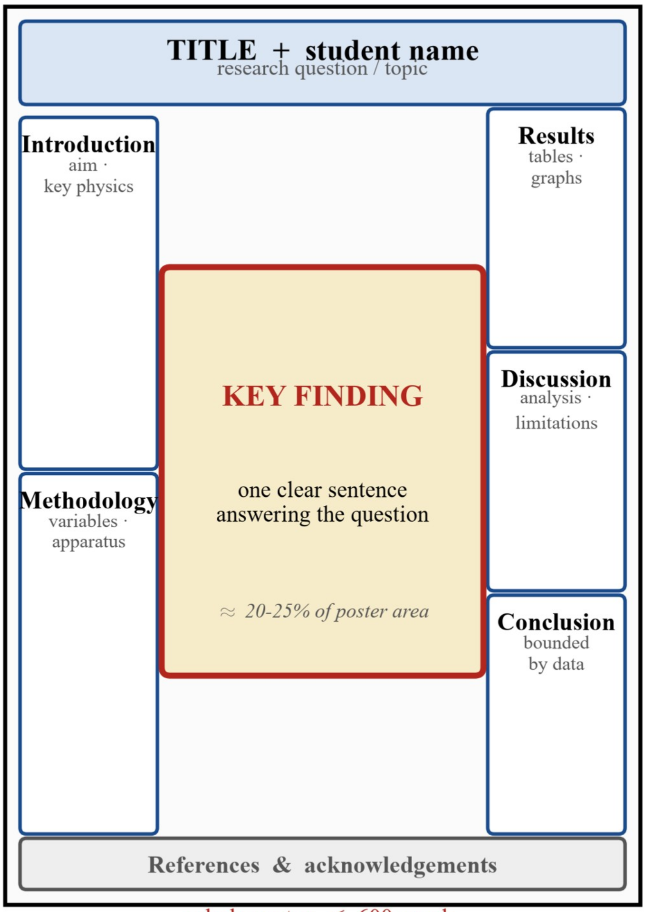
Units in written work. A number without a unit is meaningless. Report a quantity as a value, a thin space and the unit, using the correct notation — $12.0\,\text{m s}^{-1}$, not "12" and not "$12\,\text{m/s}$" mixed with words. Symbols for quantities are italic (v, F, t); symbols for units are upright (m, s, N).

The scientific poster. The investigation is communicated on a single **scientific poster** of **no more than 600 words**, which forces the student to communicate *succinctly*. A poster is set out in a conventional order so a reader can follow it quickly:

- a **title** (and the student's name);
- an **introduction**: the aim, the hypothesis or prediction, and the relevant physics concepts;
- the **methodology and method**: how the data were generated, including the independent, dependent and controlled variables;
- the **results**: tables and graphs of the primary data;
- a **discussion**: what the results mean, patterns and relationships, and the assumptions and limitations;

- a **conclusion**: a statement, bounded by the data, that answers the aim;
- **references and acknowledgements** of any sources used.

At the centre of the poster, occupying about **20–25% of its area**, sits a single, concise statement — about one sentence — of the **key finding** of the investigation. This is the first thing a reader sees and should state the main result plainly.



Key findings and their implications. Communicating a result well means reporting not only the **key finding** but also its **implication**. The *key finding* is a plain statement of *what the data show* — the measured result or relationship that answers the research question. An *implication* goes one step further and states *what that finding means*: how it bears on a practical **application**, what **further investigation** it points to, or how it **supports, extends or challenges an existing model or theory**. For example, the finding *"the stopping voltage increased linearly with the frequency of the incident light"* carries the implication that the data support the particle (photon) model of light and can be used to obtain a value for Planck's constant. Reporting only the finding leaves a reader to guess why it matters; a sound discussion states the implication explicitly — while keeping it **bounded by the data**, never claiming more than the measurements support.

The logbook. Alongside the poster, the student keeps a **logbook**: a dated record, kept in hard copy or electronic form, written *at the time* the work is done. It contains the method as it was actually carried out, all raw measurements, observations, calculations, and any changes or problems. Its main purpose is to **authenticate** the primary data — to provide evidence that the data were genuinely generated by the student. Because entries are contemporaneous and dated, the logbook also lets the work be traced and repeated. The logbook is the *complete, private* record; the poster is the *concise, public* summary that draws on it.

Evidence and opinion. A scientific claim must rest on **evidence** — data and observations that can be measured, repeated and checked — not on **opinion**, which is a personal belief or judgement. "The current decreased as the wire lengthened" is evidence (a measured relationship); "this is the best experiment in the class" is opinion. In a discussion the two must be kept separate, so a reader can see which statements the data actually support.

Conclusions bounded by the data. A conclusion must be **supported by, and limited to, the data collected**. It answers the aim, states the relationship found, and gives the range and conditions over which it holds. It should not **extrapolate** beyond the values tested, generalise to variables that were never changed, or claim more certainty than the measurements allow. Acknowledging the assumptions and limitations of the method is part of an honest conclusion.

Scientific terminology and representations. Communicating clearly means using the **agreed terms and representations** of physics: the **force due to gravity** (F_g) rather than "weight"; the **normal force** (F_N) rather than a "reaction force"; **precision** (closeness of repeated readings) as distinct from **accuracy** (closeness to the true value); and **resolution** for the smallest division an instrument can read. Data are shown in the representation that suits them — a **data table** for recorded readings (independent variable first, units in the column headings), a **line graph** for a continuous relationship (independent variable on the horizontal axis, a line of best fit weighing all points, uncertainty bars where given, extrapolation shown dashed), or a **labelled diagram** for apparatus and vectors. Standard symbols, equations, abbreviations, significant figures and units complete the convention.

Worked examples

Example 1 — scaffolded

State the number of significant figures in each of $100\,\text{m}$, $0.0250\,\text{s}$, $3.60\times10^{4}\,\text{N}$ and $12.00\,\text{s}$. Then a distance of $100\,\text{m}$ is travelled in a time of $12.00\,\text{s}$; calculate the average speed, quoted to the correct number of significant figures.

Working	Comment
$100\,\text{m}$: trailing zeros in a whole number count — **3 s.f.**	Study-design convention; $1.00\times10^{2}\,\text{m}$ makes it explicit.
$0.0250\,\text{s}$: leading zeros do not count, the trailing 0 does — **3 s.f.**	A trailing zero after the decimal point is significant.
$3.60\times10^{4}\,\text{N}$: every coefficient digit counts — **3 s.f.**	In standard form all shown digits are significant.
$12.00\,\text{s}$: all four digits significant — **4 s.f.**	Trailing zeros after the decimal point count.
$v=\dfrac{100\,\text{m}}{12.00\,\text{s}}=8.333\ldots\,\text{m s}^{-1}$.	Average speed = distance ÷ time.
Least-precise datum $100\,\text{m}$ has 3 s.f., so $v=8.33\,\text{m s}^{-1}$.	The answer matches the least-precise data.

Example 2 — scaffolded

Convert each quantity to the base SI unit indicated: (i) $655\,\text{nm}$ to metres; (ii) $2.5\,\text{kW h}$ to joules; (iii) $3.0\,\text{t}$ to kilograms; (iv) $108\,\text{km h}^{-1}$ to m s^{-1}.

Working	Comment
(i) $655\,\text{nm}=655\times10^{-9}\,\text{m}=6.55\times10^{-7}\,\text{m}$.	nano $=10^{-9}$; multiply by the prefix factor.
(ii) $2.5\,\text{kW h}=2.5\times(3.6\times10^{6})\,\text{J}=9.0\times10^{6}\,\text{J}$.	$1\,\text{kW h}=3.6\times10^{6}\,\text{J}$; keep 2 s.f.
(iii) $3.0\,\text{t}=3.0\times10^{3}\,\text{kg}$.	$1\,\text{tonne}=10^{3}\,\text{kg}$.
(iv) $108\,\text{km h}^{-1}=\dfrac{108\times10^{3}\,\text{m}}{3600\,\text{s}}=30.0\,\text{m s}^{-1}$.	Write km as m and h as s; 108 has 3 s.f.

Example 3 — unscaffolded

A trailer of mass $1.8\,\text{t}$ travels along a straight road at $72\,\text{km h}^{-1}$. Calculate the magnitude of its momentum, giving your answer to the correct number of significant figures.

First convert both quantities to base SI units:

$$m = 1.8\,\text{t} = 1.8 \times 10^3\,\text{kg}\,,\; v = 72\,\text{km h}^{-1} = \frac{72 \times 10^3\,\text{m}}{3600\,\text{s}} = 20\,\text{m s}^{-1}.$$

Then

$$p = m\,v = \left(1.8 \times 10^3\right)(20) = 3.6 \times 10^4\,\text{kg m s}^{-1}.$$

Both data are given to two significant figures, so the answer is quoted to two significant figures.

∴ the momentum of the trailer is $3.6 \times 10^4\,\text{kg m s}^{-1}$.

Example 4 — unscaffolded

A student heated $200\,\text{g}$ of water with an immersion heater and recorded its temperature once a minute for ten minutes. In the discussion they wrote: *"The temperature rose steadily as time went on. This shows the heater is the most efficient one available and would boil any volume of water quickly."* Identify which part of the statement is supported by the evidence and which parts are opinion or extend beyond the data, and write an improved conclusion.

The claim *"the temperature rose steadily as time went on"* describes the measured relationship between the two recorded quantities and **is supported by the evidence**. The claim that the heater *"is the most efficient one available"* is **opinion**: no other heater was tested, so the data cannot support a comparison. The claim that it *"would boil any volume of water quickly"* **extends beyond the data**: only one volume ($200\,\text{g}$) was used, over a limited time and temperature range, so the result cannot be generalised. A sound conclusion is bounded by the data actually collected — for example: *"Over the ten minutes measured, the temperature of the* $200\,\text{g}$ *water sample increased steadily with heating time."*

∴ only the first statement is evidence; the comparative and general claims are unsupported, and the conclusion must be limited to the conditions tested.

Practice questions

Question 1 Consider the measured values below. (a) State the number of significant figures in each of (i) $0.0480\,\text{s}$, (ii) $250\,\text{kg}$, (iii) $3.00 \times 10^8\,\text{m s}^{-1}$. (b) A constant force of $4.0 \times 10^2\,\text{N}$ acts on a trolley for $0.0640\,\text{s}$. Calculate the impulse $F\,\Delta t$ on the trolley, giving your answer to the correct number of significant figures. (c) Write the measured charge $96\,500\,\text{C}$ in standard form, to three significant figures.

Question 2 Convert each of the following. (a) Express a current of 45 μA in amperes, in standard form. (b) Express an energy of $6.0\,\text{kW h}$ in joules, to two significant figures. (c) A power station has an output of $1.5\,\text{GW}$. Express this power in watts, and in megawatts.

Question 3 A student is preparing a scientific poster to present a completed investigation. (a) State the maximum number of words allowed on the poster, and name the single item that should occupy the central 20–25% of the poster. (b) List, in a sensible reading order, the main sections the poster should contain. (c) Explain why references must be acknowledged on the poster.

Question 4 A student is keeping records during an investigation. (a) State the main purpose of keeping a logbook. (b) Explain one reason each logbook entry should be dated and written at the time the work is done. (c) Classify each of the following as *evidence-based* or *opinion*: (i) "The spring extended from $12.0\,\text{cm}$ to $15.4\,\text{cm}$ when the mass was added." (ii) "This is the neatest set of results in the class." (iii) "The voltage across the resistor was measured as $3.2\,\text{V}$."

Question 5 A car of mass $1.20\,\text{t}$ travels along a straight road at $90.0\,\text{km h}^{-1}$. (a) Show that the car's speed is $25.0\,\text{m s}^{-1}$. (b) Express the car's mass in kilograms, in standard form. (c) Hence calculate the momentum of the car, giving your answer to three significant figures.

Question 6 A draft report is being corrected for its use of scientific conventions. (a) Rewrite each of the following using correct scientific terminology: (i) "the weight of the $2.0\,\text{kg}$ trolley"; (ii) "the reaction force on the block by the bench"; (iii) "the ball is weightless at the top of the loop". (b) A student measures how the current in a resistor changes as the applied voltage is increased in steps. State which representation — a labelled diagram, a data table, or a line graph — best displays the relationship between the two quantities, and justify your choice. (c) Explain why recording a measured speed as "$v = 12$" is not acceptable in a scientific report.

Question 7 State the number of significant figures in each of the following measured values. (a) $205\,\mathrm{m}$ (b) $0.00730\,\mathrm{s}$ (c) $6.40 \times 10^{-19}\,\mathrm{C}$ (d) $8000\,\mathrm{kg}$ (e) $30.05\,\mathrm{m}$ (f) $0.0009\,\mathrm{A}$

Question 8 A helium–neon laser emits light of wavelength $633\,\mathrm{nm}$. In a school experiment its frequency is measured as $4.74 \times 10^{14}\,\mathrm{Hz}$. Convert the wavelength to metres, then use $v = f\lambda$ to calculate the speed of the light, giving your answer to three significant figures.

Question 9 Rewrite each of the following measured values in standard form, to three significant figures. (a) $100\,\mathrm{m}$ (b) $47\,200\,\mathrm{kg}$ (c) $0.005096\,\mathrm{s}$ (d) $8\,315\,000\,\mathrm{J}$

Question 10 Convert each quantity to the base SI unit shown, expressing each answer in standard form. (a) $2.5\,\mathrm{GHz}$ to Hz (b) 15 μA to A (c) $8.0\,\mathrm{ps}$ to s (d) $340\,\mathrm{MW}$ to W (e) $6.0\,\mathrm{kV}$ to V (f) $90\,\mathrm{nm}$ to m

Question 11 A dishwasher is rated at $1.8\,\mathrm{kW}$ and runs for $2.0\,\mathrm{h}$ in one cycle. Calculate the energy it uses in one cycle in kilowatt hours and in joules (to two significant figures), and, if electricity costs \$0.28 per $\mathrm{kW\,h}$, the cost of running one cycle.

Question 12 A scientific poster is printed at A1 size, $0.594\,\mathrm{m}$ by $0.841\,\mathrm{m}$. (a) Show that the area of the poster is $0.500\,\mathrm{m}^2$ (to three significant figures). (b) The key-finding statement should occupy 20–25% of the poster area. Calculate the range of allowed areas for this central section. (c) A first draft of the poster text contains 750 words. Calculate the percentage by which the word count must be reduced to meet the limit.

Question 13 A satellite is placed in a circular orbit at an altitude of $550\,\mathrm{km}$ above the Earth's surface. Take the radius of the Earth as $R_E = 6.37 \times 10^6\,\mathrm{m}$. (Give each answer to three significant figures.) (a) Calculate the radius of the orbit, measured from the centre of the Earth, in metres. (b) The satellite completes one orbit in $96.0\,\mathrm{min}$. Express this period in seconds.

Question 14 A student investigated how the steady current through a length of nichrome wire depended on the wire's length, over the range $10.0\,\mathrm{cm}$ to $50.0\,\mathrm{cm}$, at a fixed applied voltage. From the data they wrote: *"The current decreased as the wire got longer. This proves that longer wires are always poor conductors, and that nichrome is the worst possible material to use in any circuit."* Evaluate this conclusion. Identify the part that is supported by the evidence and the parts that go beyond the data, and write an improved conclusion that is properly bounded by the data.

Question 15 A discussion section contains the statement: "The measured resistance increased with temperature, which I think makes this the most interesting result of the whole experiment." (a) (Circle your answer.) The claim that the resistance "increased with temperature" is: *evidence* / *opinion*. Justify your choice. (b) (Circle your answer.) The claim that this is "the most interesting result" is: *evidence* / *opinion*. Justify your choice. (c) Explain why a scientific discussion should keep evidence and opinion separate.

Question 16 A student's poster presents a completed investigation, but its central section reads: "We did lots of measurements and the graphs show many interesting trends across all our results." (a) Explain why this central statement does not meet the conventions for the key-finding section of a scientific poster. (b) The investigation found that the period of a pendulum increased as its length increased. Write a single sentence suitable for the central key-finding section. (c) Explain one reason the poster has a strict word limit.

Question 17 During an investigation a student records raw readings, dates and observations directly into their logbook, including a note that one trial was disturbed when the apparatus was bumped. (a) Explain how the logbook helps to authenticate that the primary data are the student's own work. (b) Explain why the note about the disturbed trial should be kept in the logbook rather than erased. (c) Distinguish between the role of the logbook and the role of the poster in communicating the investigation.

Question 18 A report contains several terms that are not used correctly in physics. Rewrite each so that it uses correct scientific terminology, and in each case state briefly why the original is wrong. (a) "The accuracy of the metre ruler is

1 mm.” (b) “Our three repeat readings were 4.90 s, 4.91 s and 4.90 s, which shows the timing was very accurate.” (c) “The block rests on the bench because gravity is cancelled by the reaction force.”

Question 19 A student measured the extension of a spring for a series of applied forces and wants to present the relationship as a graph. (a) State which quantity should be plotted on the horizontal axis, and why. (b) State two conventions that should be followed when drawing the axes and plotting the points. (c) The student draws the line of best fit so that it passes exactly through the first and last data points. Explain why this is not the correct way to draw a line of best fit.

Question 20 A student claims their poster is scientifically convincing “because it looks professional, uses lots of colour, and every section is completely full of text.” Its conclusion states a firm numerical relationship, but the logbook shows only two measurements were taken and one of them was known to be disturbed. Explain, with reference to the conventions of scientific communication and to the idea of evidence, why the poster is not yet scientifically sound, and state two specific changes that would improve it.

Question 21 A student investigates how the extension of a spring depends on the force applied to it. Over the range of forces tested, the extension is found to be directly proportional to the applied force, giving a spring constant of $25\ \text{N m}^{-1}$. (a) State the key finding of this investigation in a single sentence. (b) State one implication of this finding — for an existing model, a practical application, or a further investigation — and explain how an implication differs from a key finding.

Answers

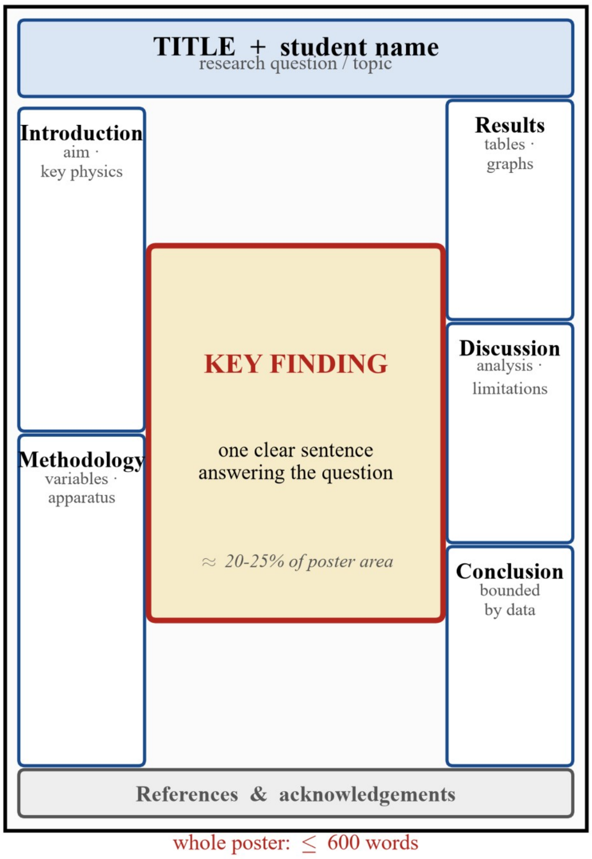
Question 1 (a) 3; 3; 3. (b) $26\,\mathrm{N\ s}$ (i.e. $2.6 \times 10^1\,\mathrm{N\ s}$). (c) $9.65 \times 10^4\,\mathrm{C}$. **Question 2** (a) $4.5 \times 10^{-5}\,\mathrm{A}$; (b) $2.2 \times 10^7\,\mathrm{J}$; (c) $1.5 \times 10^9\,\mathrm{W} = 1.5 \times 10^3\,\mathrm{MW}$ ($1500\,\mathrm{MW}$). **Question 3** (a) 600 words; the one-sentence key finding. (b) Title/name → introduction (aim, physics) → methodology & method → results → discussion → conclusion → references. (c) To credit sources and let a reader check them. **Question 4** (a) To authenticate the primary data as the student's own work. (b) A dated, contemporaneous record is trustworthy. (c) (i) evidence; (ii) opinion; (iii) evidence. **Question 5** (a) $25.0\,\mathrm{m\ s^{-1}}$; (b) $1.20 \times 10^3\,\mathrm{kg}$; (c) $3.00 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$. **Question 6** (a) force due to gravity on the trolley; normal force on the block by the bench; the force due to gravity still acts on the ball (not "weightless"). (b) A line graph. (c) A value is meaningless without its unit. **Question 7** (a) 3; (b) 3; (c) 3; (d) 4; (e) 4; (f) 1. **Question 8** $\lambda = 6.33 \times 10^{-7}\,\mathrm{m}$; $v = 3.00 \times 10^8\,\mathrm{m\ s^{-1}}$. **Question 9** (a) $1.00 \times 10^2\,\mathrm{m}$; (b) $4.72 \times 10^4\,\mathrm{kg}$; (c) $5.10 \times 10^{-3}\,\mathrm{s}$; (d) $8.32 \times 10^6\,\mathrm{J}$. **Question 10** (a) $2.5 \times 10^9\,\mathrm{Hz}$; (b) $1.5 \times 10^{-5}\,\mathrm{A}$; (c) $8.0 \times 10^{-12}\,\mathrm{s}$; (d) $3.4 \times 10^8\,\mathrm{W}$; (e) $6.0 \times 10^3\,\mathrm{V}$; (f) $9.0 \times 10^{-8}\,\mathrm{m}$. **Question 11** $3.6\,\mathrm{kW\ h}$; $1.3 \times 10^7\,\mathrm{J}$; about \$1.01. **Question 12** (a) $0.500\,\mathrm{m^2}$; (b) $0.100\,\mathrm{m^2}$ to $0.125\,\mathrm{m^2}$; (c) $20.0\,\%$. **Question 13** (a) $6.92 \times 10^6\,\mathrm{m}$; (b) $5.76 \times 10^3\,\mathrm{s}$. **Question 14** Only "current decreased as length increased" is supported; "always" and "worst material in any circuit" exceed the data. A bounded conclusion limits the claim to this wire, this voltage and the 10.0–$50.0\,\mathrm{cm}$ range. **Question 15** (a) evidence; (b) opinion; (c) so a reader can tell measured results from personal views. **Question 16** (a) Too vague; states no actual result. (b) e.g. "The pendulum's period increased as its length increased." (c) It forces succinct communication. **Question 17** (a) A dated, contemporaneous record shows the data are the student's own. (b) It preserves an honest record and flags a possible outlier. (c) Logbook = complete private record; poster = concise public summary. **Question 18** (a) resolution, not accuracy; (b) precise, not accurate; (c) force due to gravity F_g balanced by the normal force F_N (not "reaction force"). **Question 19** (a) The applied force (the independent variable) on the horizontal axis. (b) Any two valid conventions (see solution). (c) The line must balance all the points, not just the first and last. **Question 20** Appearance is not evidence; two measurements (one disturbed) cannot support a firm relationship, so the conclusion is not bounded by the data. Improve by taking many more repeats and by repeating or discarding the disturbed trial (and adding uncertainty bars). **Question 21** (a) Key finding: over the range tested, the spring's extension is directly proportional to the applied force (spring constant $25\,\mathrm{N\ m^{-1}}$). (b) An implication (any one): the result is consistent with Hooke's law over this range, so the spring could be used as a calibrated force meter; or it motivates a further investigation of whether proportionality still holds for larger extensions. An implication states what the finding *means* (for a model, application or further work); the key finding states only what the data show.

Fully-worked solutions

Question 1 (a) (i) $0.0480\,\mathrm{s}$: the leading zeros are not significant; the 4, 8 and the trailing 0 are — **3 s.f.** (ii) $250\,\mathrm{kg}$: a whole number, so the trailing zero is treated as significant — **3 s.f.** (iii) $3.00 \times 10^8\,\mathrm{m\ s^{-1}}$: every coefficient digit is significant — **3 s.f.** (b) $F\,\Delta t = (4.0 \times 10^2)(0.0640) = 25.6\,\mathrm{N\ s}$. The least-precise datum, $4.0 \times 10^2\,\mathrm{N}$, has two significant figures, so the impulse is $26\,\mathrm{N\ s}$ ($= 2.6 \times 10^1\,\mathrm{N\ s}$). (c) $96\,500\,\mathrm{C} = 9.65 \times 10^4\,\mathrm{C}$.

Question 2 (a) 45 μA $= 45 \times 10^{-6}\,\mathrm{A} = 4.5 \times 10^{-5}\,\mathrm{A}$. (b) $6.0\,\mathrm{kW\ h} = 6.0 \times (3.6 \times 10^6)\,\mathrm{J} = 2.16 \times 10^7\,\mathrm{J} = 2.2 \times 10^7\,\mathrm{J}$ (2 s.f.). (c) $1.5\,\mathrm{GW} = 1.5 \times 10^9\,\mathrm{W}$. Since $1\,\mathrm{MW} = 10^6\,\mathrm{W}$, this is $\dfrac{1.5 \times 10^9}{10^6} = 1.5 \times 10^3\,\mathrm{MW} = 1500\,\mathrm{MW}$.

Question 3 (a) The poster may contain **no more than 600 words**. The central 20–25% should hold a concise, roughly one-sentence statement of the **key finding** — the main result answering the research question. (b) A sensible order is: **title and student name → introduction** (aim, hypothesis, relevant physics) **→ methodology and method → results** (tables and graphs) **→ discussion → conclusion → references and acknowledgements**, with the key finding displayed centrally.



(c) References acknowledge the sources of ideas, data or apparatus that were not the student's own. This gives proper credit to the original authors, lets a reader trace and check those sources, and clearly separates the student's own contribution from established work — all part of honest scientific communication.

Question 4 (a) A logbook is a dated, contemporaneous record of everything done in the investigation — the method as actually carried out, raw measurements, observations and changes. Its main purpose is to **authenticate the primary data as the student's own work**. (b) Dating each entry and writing it at the time keeps a reliable, ordered record that the data were generated by the student as the work proceeded; entries reconstructed later from memory could be mistaken and could not be trusted as authentic. (c) (i) **evidence** — a measured observation; (ii) **opinion** — a personal judgement not established by data; (iii) **evidence** — a recorded measurement.

Question 5 (a) $v = 90.0\,\text{km h}^{-1} = \dfrac{90.0 \times 10^3\,\text{m}}{3600\,\text{s}} = 25.0\,\text{m s}^{-1}$, as required. (b) $m = 1.20\,\text{t} = 1.20 \times 10^3\,\text{kg}$. (c) $p = mv = (1.20 \times 10^3)(25.0) = 3.00 \times 10^4\,\text{kg m s}^{-1}$ (3 s.f.).

Question 6 (a) (i) "the **force due to gravity** on the $2.0\,\text{kg}$ trolley" — the term "weight" is not used in this study. (ii) "the **normal force** on the block by the bench" — the contact force is the normal force, not a "reaction force". (iii) "the **force due to gravity still acts** on the ball at the top of the loop" — there is no "weightlessness"; the force due to gravity does not vanish. (b) A **line graph** best displays the relationship, because both current and voltage are continuous quantities: a graph shows the trend, allows a line of best fit, and reveals whether the relationship is linear. A data table only lists the readings, and a labelled diagram shows apparatus rather than the relationship. (c) A measured value is meaningless without its unit: "12" could be $12\,\text{m s}^{-1}$, $12\,\text{km h}^{-1}$ or $12\,\text{cm s}^{-1}$. The value must be written with its unit, as $v = 12\,\text{m s}^{-1}$.

Question 7 (a) $205\,\text{m}$ — the zero lies between non-zero digits — **3**. (b) $0.00730\,\text{s}$ — leading zeros not counted, trailing zero counted — **3**. (c) $6.40 \times 10^{-19}\,\text{C}$ — all coefficient digits — **3**. (d) $8000\,\text{kg}$ — whole-number trailing zeros treated as significant — **4**. (e) $30.05\,\text{m}$ — all digits, including the middle zeros — **4**. (f) $0.0009\,\text{A}$ — only the 9 is significant — **1**.

Question 8 $\lambda = 633\,\text{nm} = 633 \times 10^{-9}\,\text{m} = 6.33 \times 10^{-7}\,\text{m}$. $v = f\lambda = (4.74 \times 10^{14})(6.33 \times 10^{-7}) = 3.00 \times 10^8\,\text{m s}^{-1}$ (3 s.f.) — the speed of light c.

Question 9 (a) $100\,\text{m} = 1.00 \times 10^2\,\text{m}$. (b) $47\,200\,\text{kg} = 4.72 \times 10^4\,\text{kg}$. (c) $0.005096\,\text{s} = 5.10 \times 10^{-3}\,\text{s}$ (the fourth digit, 6, rounds the 9 up). (d) $8\,315\,000\,\text{J} = 8.32 \times 10^6\,\text{J}$ (the fourth digit, 5, rounds up).

Question 10 (a) $2.5\,\text{GHz} = 2.5 \times 10^9\,\text{Hz}$. (b) $15\,\mu\text{A} = 1.5 \times 10^{-5}\,\text{A}$. (c) $8.0\,\text{ps} = 8.0 \times 10^{-12}\,\text{s}$. (d) $340\,\text{MW} = 3.4 \times 10^8\,\text{W}$. (e) $6.0\,\text{kV} = 6.0 \times 10^3\,\text{V}$. (f) $90\,\text{nm} = 9.0 \times 10^{-8}\,\text{m}$.

Question 11 Energy: $E = P\,t = (1.8\,\text{kW})(2.0\,\text{h}) = 3.6\,\text{kW h}$. In joules: $E = 3.6 \times (3.6 \times 10^6) = 1.296 \times 10^7\,\text{J} = 1.3 \times 10^7\,\text{J}$ (2 s.f.). Cost: at \$0.28 per kilowatt hour, $3.6 \times 0.28 = 1.008$, so the cost of one cycle is about \$1.01.

Question 12 (a) $A = (0.594)(0.841) = 0.499554\,\text{m}^2 = 0.500\,\text{m}^2$ (3 s.f.), as required. (b) $20\,\%$ of $0.500\,\text{m}^2 = 0.100\,\text{m}^2$; $25\,\%$ of $0.500\,\text{m}^2 = 0.125\,\text{m}^2$. The central key-finding section should have an area between $0.100\,\text{m}^2$ and $0.125\,\text{m}^2$. (c) reduction $= \dfrac{750 - 600}{750} \times 100 = 20.0\,\%$.

Question 13 (a) $r = R_E + h = 6.37 \times 10^6 + 550 \times 10^3 = 6.37 \times 10^6 + 0.550 \times 10^6 = 6.92 \times 10^6\,\text{m}$. (b) $T = 96.0\,\text{min} = 96.0 \times 60\,\text{s} = 5760\,\text{s} = 5.76 \times 10^3\,\text{s}$.

Question 14 The observation *"the current decreased as the wire got longer"* describes the measured relationship between the independent variable (length) and the dependent variable (current), and **is supported by the evidence**. The claim that *"longer wires are always poor conductors"* over-generalises: the data span only $10.0\,\text{cm}$ to $50.0\,\text{cm}$ at one fixed voltage, so nothing can be claimed outside that range, and "always" is not established. The claim that *"nichrome is the worst possible material to use in any circuit"* goes well beyond the data: only one material was tested, so no comparison between materials was made, and "any circuit" invokes conditions never investigated. A conclusion must be **supported by, and limited to, the data collected**. An improved conclusion: *"For this nichrome wire at constant applied voltage, the steady current decreased as the length was increased over the range* $10.0\,\text{cm}$ *to* $50.0\,\text{cm}$*."*

Question 15 (a) **Evidence.** It reports a relationship between resistance and temperature that was actually measured, and could be repeated and checked. (b) **Opinion.** "Most interesting" is a personal judgement; the measurements neither establish nor test it. (c) A discussion must let a reader see which statements are established by the data and which are the writer's personal view. Mixing them makes the reasoning unreliable and can lead a reader to accept an opinion as though it were a measured result.

Question 16 (a) The key finding should be a **concise, specific statement of the result** — about one sentence — that answers the research question. The given text is vague ("lots of measurements", "many interesting trends"), names no variables and states no actual relationship, so it does not communicate the finding. (b) e.g. *"The period of the pendulum increased as its length was increased."* (c) The 600-word limit forces the student to **communicate succinctly**, keeping only the essential information so a reader can grasp the investigation quickly — the whole purpose of a poster.

Question 17 (a) Because the logbook is a **dated, contemporaneous record** made as the work proceeds, it shows the raw readings were generated by the student at particular times during the investigation, rather than copied or invented afterwards; this authenticates the primary data as the student's own. (b) The note records what actually happened while the data were generated. Keeping it preserves an honest record and flags a reading that may be an **outlier** for proper analysis; erasing it would hide relevant information and weaken the authentication of the logbook. (c) The **logbook** is the complete, private, working record of everything done and measured; the **poster** is the concise, public summary that presents only the aim, method, key results, discussion and conclusion. The logbook authenticates and supports the claims made on the poster.

Question 18 (a) Should read *"the **resolution** of the metre ruler is* $1\,\text{mm}$*"*. Resolution is the smallest change an instrument can distinguish (its smallest scale division); **accuracy** is how close a reading is to the true value — a different idea. (b) Should read *"...which shows the timing was very **precise**"*. Closely-agreeing repeat readings show high **precision** (a small spread); **accuracy** would require comparison with an accepted value, which was not done here. (c) Should read *"...the block rests on the bench because the **force due to gravity*** F_g *is balanced by the **normal***

force F_N *on the block by the bench"*. The upward contact force is the normal force, not a "reaction force", and the downward force is the force due to gravity.

Question 19 (a) The **applied force** should be on the horizontal axis, because it is the **independent variable** — the quantity the experimenter chose and controlled. The extension it produces is the dependent variable and goes on the vertical axis. (b) Any two of: label each axis with the quantity **and its unit**; choose scales that spread the data across most of the grid; plot the points accurately; draw **uncertainty bars** where an uncertainty is given; draw a single straight **line of best fit**; show any extrapolation to an intercept as a **dashed** line (and include the origin only if $(0,0)$ is a genuine measured point). (c) A line of best fit should **balance all the plotted points**, with roughly equal numbers scattered on each side. The first and last points are just two ordinary measurements, each with its own uncertainty; forcing the line through them ignores every other point and biases the gradient toward those two readings.

Question 20 A poster is judged on the quality of its **science and communication**, not its appearance: a professional look, colour and full text boxes do not make the content correct or convincing. The conclusion states a firm numerical relationship, yet the logbook shows only **two measurements**, one of them disturbed — effectively a single reliable reading. Two points cannot establish a relationship, so the conclusion is **not supported by the evidence** and is not bounded by the data. Scientific communication requires every claim to rest on sufficient, trustworthy evidence, and conclusions to be limited to what the data support. Two improvements (any two): take **many more repeated measurements** across the range so a reliable trend can be found; **repeat or discard** the disturbed trial and note the reason; present the data with **uncertainty bars** and a line of best fit; and **soften the conclusion** so it matches the range and quality of the data actually collected.

Question 21 (a) The **key finding** is a plain statement of *what the data show*: *"Over the range of forces tested, the extension of the spring was directly proportional to the applied force, with a spring constant of* $25\ \mathrm{N\ m^{-1}}$*."* (b) An **implication** goes further and states *what the finding means*. Any **one** of, for example:

- *existing model:* the result is consistent with **Hooke's law** ($F = k\,x$) for this spring over the range tested, supporting the linear-spring model;
- *application:* because extension is proportional to force, the spring could be used as a **calibrated force meter** (spring balance) within this range;
- *further investigation:* it motivates testing **whether the proportionality still holds beyond the tested range**, where the spring may reach its elastic limit.

A key finding reports only the measured result or relationship; an implication says what that result *means* — for an application, an existing model or theory, or the next investigation — while staying bounded by the data actually collected.