Physics Units 3 & 4

Chapter 8 — Matter waves and quantisation

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Contents

Theory

By the early twentieth century light was understood to be *both* a wave (interference, diffraction) and a stream of particles called **photons** (the photoelectric effect). In 1924 Louis de Broglie proposed a bold symmetry: if light waves can behave as particles, then particles of **matter** — electrons, protons, atoms — should be able to behave as waves. Every moving particle has an associated **de Broglie wavelength**

$$\lambda = \frac{h}{p} = \frac{h}{mv},$$

where $p = mv$ is the particle's momentum and $h = 6.63 \times 10^{-34}$ J s is Planck's constant. The larger a particle's momentum, the *shorter* its wavelength.

The same relation links a photon's momentum to its wavelength. A photon of wavelength λ carries momentum

$$p = \frac{h}{\lambda},$$

which is just $\lambda = \frac{h}{p}$ rearranged. The single equation $p = \frac{h}{\lambda}$ therefore applies to **both** matter and light: it is the bridge between the particle picture (momentum p) and the wave picture (wavelength λ).

How large is the de Broglie wavelength? For an everyday object the momentum mv is enormous compared with h, so λ is unimaginably small. A 0.15 kg ball thrown at 20 m s^{-1} has $\lambda = \frac{6.63 \times 10^{-34}}{0.15 \times 20} \approx 2 \times 10^{-34}$ m — around twenty powers of ten smaller than an atomic nucleus. No aperture or obstacle is remotely this small, so the wave nature of macroscopic matter is never observed. An **electron**, however, has a tiny mass ($m_e = 9.11 \times 10^{-31}$ kg), so even at modest speeds its wavelength is comparable to the spacing between atoms in a solid ($\sim 10^{-10}$ m). That is exactly the scale at which diffraction becomes significant, so electron waves *can* be detected.

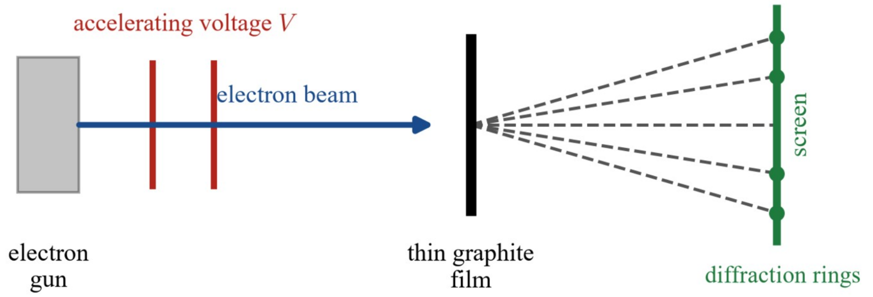
Accelerating an electron through a potential difference. The usual way to give an electron a known, controllable wavelength is to accelerate it from rest through a potential difference V. The work done on the electron, qV, becomes kinetic energy:

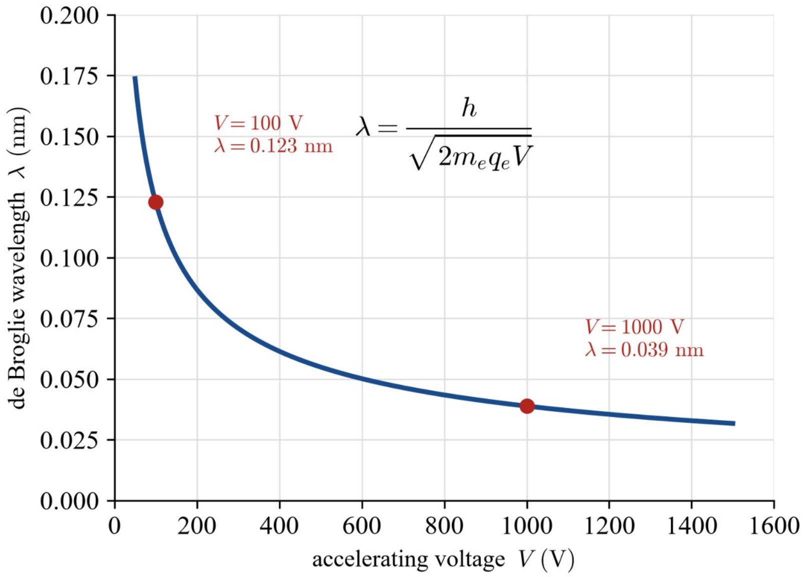
$$qV = 1/2\, mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}}.$$

Its momentum is then $p = mv = \sqrt{2mqV}$, and its de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}.$$

A larger accelerating voltage gives the electron more momentum and therefore a **shorter** wavelength ($\lambda \propto 1/\sqrt{V}$).

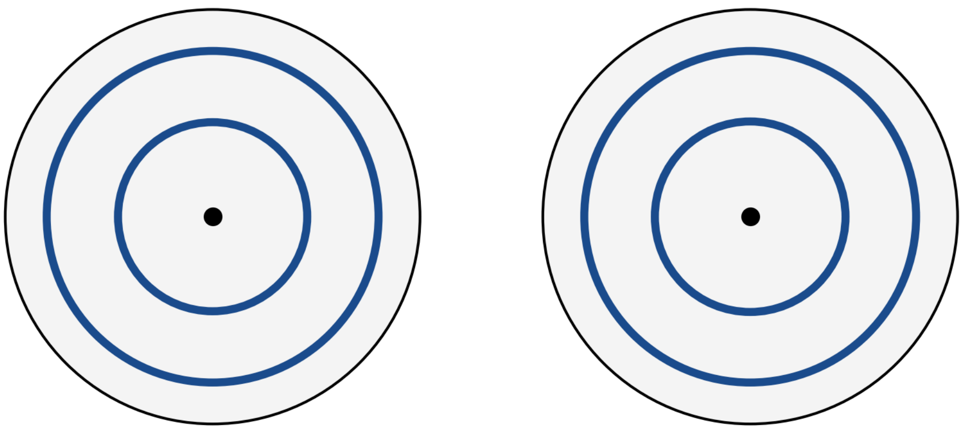




Electron diffraction as evidence for the wave nature of matter. When a narrow beam of electrons is fired at a thin film of graphite (a crystal in which the atoms are regularly spaced), the electrons emerge in a pattern of **concentric bright rings** on a screen beyond the film. Rings are a diffraction pattern — they are produced when a wave spreads around and between the regularly spaced atoms and the spread-out parts recombine constructively in particular directions. A stream of tiny *particles* could not produce such a pattern; a *wave* can. The ring pattern is therefore direct experimental evidence that the electrons are behaving as waves, with the de Broglie wavelength $\lambda = h/p$.

The **size of the rings depends on the wavelength**: the longer the wavelength, the more the wave spreads, and the larger the ring radius (ring radius $\propto \lambda$). Because $\lambda \propto 1/\sqrt{V}$, *decreasing* the accelerating voltage lengthens the wavelength and makes the rings grow; *increasing* the voltage shortens the wavelength and shrinks the rings. Measuring how the rings change as V is varied confirms the $\lambda = h/p$ relationship quantitatively.

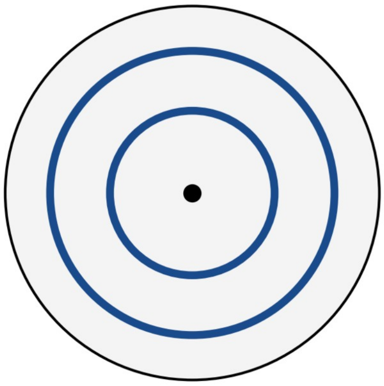
lower voltage $\rightarrow$ longer λ $\rightarrow$ larger rings



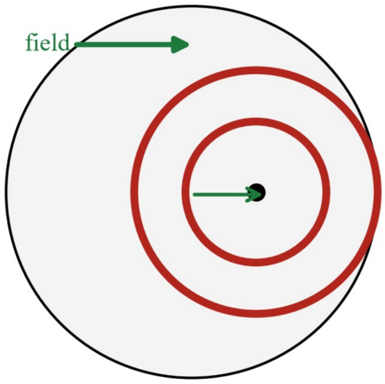
$V = 2.0$ kV, $\lambda = 0.0275$ nm $V = 5.0$ kV, $\lambda = 0.0174$ nm

Distinguishing electron and photon (X-ray) diffraction patterns. X-rays are electromagnetic waves — photons — with wavelengths of about 10^{-10} m, the same order as the de Broglie wavelength of a moderately accelerated electron. When X-rays pass through the same graphite film they produce a ring pattern too. If an X-ray beam and an electron beam have the **same wavelength**, their ring patterns are essentially **identical** — same number of rings, same radii — because diffraction depends only on the wavelength and the spacing of the atoms, not on what is doing the diffracting. This close match is itself powerful evidence that matter genuinely has a wave nature.

To tell the two patterns *apart* you use a property that matter has and light does not: **electric charge and mass**. Electrons are charged, so passing the beam through an electric or magnetic field deflects it and shifts or distorts the electron pattern; X-ray photons are uncharged and are undeflected by the same field. (Electrons also carry momentum enough to be stopped by thin foils and deposit charge on a collector, whereas photons do not.) So the pattern that moves when a field is applied is the electron pattern.



X-rays (photons):
no charge, pattern stays centred



electrons: charged,
pattern shifts in a field

Comparing the momentum of photons and matter of the same wavelength. Because $p = h / \lambda$ holds for both, an electron and a photon that have the **same wavelength** have exactly the **same momentum**. They do *not*, however, carry the same energy. A photon's energy is

$$E = hf = \frac{hc}{\lambda} = pc,$$

whereas a slow (non-relativistic) particle of the same momentum has kinetic energy

$$E_k = 1/2\, m v^2 = \frac{p^2}{2m}.$$

For a wavelength around 10^{-10} m the photon energy works out to be many thousands of electronvolts, while the electron's kinetic energy is only a few hundred electronvolts — the same wavelength and the same momentum, but the photon carries far more energy. The reason is structural: the photon's energy is proportional to its momentum ($E = pc$), while the electron's rises only as the *square* of its (much smaller) speed.

Throughout this subtopic the constants $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s, $m_e = 9.11 \times 10^{-31}$ kg, $q_e = 1.60 \times 10^{-19}$ C and $c = 3.00 \times 10^{8}$ m s^{-1} are used, with $1\,\text{eV} = 1.60 \times 10^{-19}$ J.

Worked examples

Example 1 — scaffolded

An electron moves at a speed of 2.5×10^{6} m s^{-1}. Calculate its momentum and its de Broglie wavelength.

Working	Comment
$m_e = 9.11 \times 10^{-31}$ kg, $v = 2.5 \times 10^{6}$ m s^{-1}.	List the data; the electron mass is a constant.
$p = m_e v = 9.11 \times 10^{-31} \times 2.5 \times 10^{6} = 2.28 \times 10^{-24}$ kg m s .	Momentum $p = mv$.
$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{2.28 \times 10^{-24}} = 2.91 \times 10^{-10}$ m.	de Broglie wavelength $\lambda = h / p$.
This is close to atomic spacing ($\sim 10^{-10}$ m), so the electron will diffract through a crystal.	Interpret the size of the wavelength.

Example 2 — scaffolded

An electron is accelerated from rest through a potential difference of $120\,\mathrm{V}$. Calculate the electron's speed, its momentum, and its de Broglie wavelength.

Working	Comment
Energy gained: $qV = 1/2\,m_e v^2$, so $v=\sqrt{\dfrac{2qV}{m_e}}$.	Work done by the field becomes kinetic energy.
$v=\sqrt{\dfrac{2\times 1.60\times 10^{-19}\times 120}{9.11\times 10^{-31}}}=6.49\times 10^{6}\,\mathrm{m\ s^{-1}}$.	Substitute $q=q_e$, $V=120\,\mathrm{V}$, $m=m_e$.
$p=m_e v=9.11\times 10^{-31}\times 6.49\times 10^{6}=5.91\times 10^{-24}\,\mathrm{kg\ m}$.	Momentum from $p=mv$.
$\lambda=\dfrac{h}{p}=\dfrac{6.63\times 10^{-34}}{5.91\times 10^{-24}}=1.12\times 10^{-10}\,\mathrm{m}$.	de Broglie wavelength.

Example 3 — unscaffolded

An electron and an X-ray photon each have a wavelength of $0.30\,\mathrm{nm}$. Calculate the momentum of each, and compare the photon's energy with the electron's kinetic energy.

Both a photon and a particle obey $p=\dfrac{h}{\lambda}$, so with $\lambda=0.30\,\mathrm{nm}=3.0\times 10^{-10}\,\mathrm{m}$ each has the **same** momentum:

$$p=\frac{h}{\lambda}=\frac{6.63\times 10^{-34}}{3.0\times 10^{-10}}=2.21\times 10^{-24}\,\mathrm{kg\ m\ s^{-1}}.$$

The photon's energy is

$$E=\frac{hc}{\lambda}=pc=2.21\times 10^{-24}\times 3.00\times 10^{8}=6.63\times 10^{-16}\,\mathrm{J}=4.14\times 10^{3}\,\mathrm{eV}.$$

The electron of the same momentum has kinetic energy

$$E_k=\frac{p^2}{2m_e}=\frac{(2.21\times 10^{-24})^2}{2\times 9.11\times 10^{-31}}=2.68\times 10^{-18}\,\mathrm{J}=16.8\,\mathrm{eV}.$$

∴ the electron and photon have the **same momentum** ($2.21\times 10^{-24}\,\mathrm{kg\ m\ \ s^{-1}}$), but the photon's energy ($4.14\,\mathrm{keV}$) is more than two hundred times the electron's kinetic energy ($16.8\,\mathrm{eV}$).

Example 4 — unscaffolded

A tennis ball of mass $57\,\mathrm{g}$ is served at $30\,\mathrm{m\ s^{-1}}$. Calculate its de Broglie wavelength, and explain why the wave nature of the ball is never observed.

The momentum of the ball is

$$p=mv=0.057\times 30=1.71\,\mathrm{kg\ m\ s^{-1}},$$

so its de Broglie wavelength is

$$\lambda=\frac{h}{p}=\frac{6.63\times 10^{-34}}{1.71}=3.88\times 10^{-34}\,\mathrm{m}.$$

Diffraction and interference are only significant when a wave meets a gap or obstacle whose size is comparable to its wavelength. This wavelength is about twenty-four powers of ten smaller than the width of an atom — and smaller still than an atomic nucleus — so the ball never encounters anything close to small enough to make it diffract.

∴ although the ball does have a de Broglie wavelength, $\lambda = 3.88 \times 10^{-34}\,\text{m}$ is so extraordinarily small that its wave behaviour can never be detected.

Practice questions

Question 1 An electron moves at a speed of $4.0 \times 10^{6}\,\text{m s}^{-1}$. (a) Calculate the momentum of the electron. (b) Calculate its de Broglie wavelength. (c) State whether this electron would produce a noticeable diffraction pattern when passed through a crystal, and justify your answer.

Question 2 An electron is accelerated from rest through a potential difference of $240\,\text{V}$. (a) Calculate the speed reached by the electron. (b) Calculate the momentum of the electron. (c) Calculate its de Broglie wavelength.

Question 3 An X-ray photon has a wavelength of $0.24\,\text{nm}$. (a) Calculate the momentum of the photon. (b) Calculate the energy of the photon, in joules. (c) An electron has the same wavelength, $0.24\,\text{nm}$. State the momentum of the electron, and justify your answer without further calculation.

Question 4 In an electron diffraction tube, electrons are fired at a thin graphite film and form a pattern of concentric rings on a screen. (a) State what the ring pattern shows about the nature of the electrons, and name the wave behaviour responsible. (b) Explain, in terms of the de Broglie wavelength, why the electrons diffract on passing through the graphite but a fine stream of sand grains fired at the film would not. (c) The accelerating voltage of the tube is decreased. State and explain what happens to the diameter of the rings.

Question 5 A crystal produces a pattern of diffraction rings when an X-ray beam is passed through it, and a very similar pattern of rings when an electron beam of the same wavelength is passed through it. (a) Explain why the two ring patterns are so similar. (b) Describe one way to determine experimentally which pattern was produced by the electron beam. (c) State how the momentum of the electrons compares with the momentum of the X-ray photons, given that the two beams have the same wavelength, and justify your answer.

Question 6 An electron has a de Broglie wavelength of $0.10\,\text{nm}$. (a) Calculate the momentum of the electron. (b) Calculate the speed of the electron. (c) Calculate the potential difference through which the electron must have been accelerated from rest to reach this wavelength.

Question 7 An electron travels at $1.5 \times 10^{7}\,\text{m s}^{-1}$. Calculate its de Broglie wavelength.

Question 8 A proton of mass $1.67 \times 10^{-27}\,\text{kg}$ moves at $3.0 \times 10^{5}\,\text{m s}^{-1}$. Calculate its de Broglie wavelength. Give your answer to three significant figures.

Question 9 A cricket ball of mass $0.16\,\text{kg}$ is bowled at $40\,\text{m s}^{-1}$. (a) Calculate the de Broglie wavelength of the ball. (b) Explain why the wave nature of the cricket ball is never observed in practice.

Question 10 An electron and a gamma-ray photon each have a wavelength of $2.5 \times 10^{-12}\,\text{m}$. Calculate the momentum of each, and state how the two momenta compare.

Question 11 An electron is accelerated from rest through a potential difference of $350\,\text{V}$. (a) Show that the de Broglie wavelength of the electron is $6.56 \times 10^{-11}\,\text{m}$. (b) An X-ray photon has this same wavelength. State the momentum of the photon, and calculate its energy in both joules and electronvolts.

Question 12 Two electron diffraction photographs are taken of the same graphite film, one with the electrons accelerated through $2.0\,\text{kV}$ and the other through $5.0\,\text{kV}$. (a) State which accelerating voltage produces the larger-diameter rings, and explain your answer in terms of $\lambda = h/p$. (b) Calculate the de Broglie wavelength of the electrons at each voltage, and hence the ratio of the two wavelengths.

Question 13 An electron beam and a beam of X-rays are directed in turn at identical thin metal foils, and each produces a pattern of concentric rings on a screen. A student concludes that “electrons and X-rays are really the same kind of thing”. (a) Explain what the ring pattern produced by the electron beam demonstrates about electrons. (b) Explain how the electron beam and the X-ray beam could be distinguished experimentally, even though they produce very similar ring patterns. (c) Comment on whether the student’s conclusion is justified.

Question 14 A neutron of mass 1.67×10^{-27} kg has a de Broglie wavelength of 1.6×10^{-10} m, comparable to the spacing of atoms in a crystal. (a) Calculate the momentum of the neutron. (b) Calculate the speed of the neutron.

Question 15 An electron and a proton (of mass 1.67×10^{-27} kg) travel at the same speed of $5.0 \times 10^{5}\ \mathrm{m\ s^{-1}}$. (a) Calculate the de Broglie wavelength of each. (b) State which particle has the longer wavelength, and explain your answer in terms of momentum.

Question 16 An electron microscope requires electrons with a de Broglie wavelength of 2.5×10^{-11} m. Calculate the potential difference through which the electrons must be accelerated from rest to achieve this wavelength.

Question 17 An electron and a photon each have a wavelength of 1.4×10^{-10} m. (a) Calculate the momentum of each. (b) Calculate the energy of the photon and the kinetic energy of the electron, in joules. (c) With reference to your results, comment on the statement: "particles and photons of the same wavelength carry the same energy".

Question 18 In an electron diffraction tube, electrons are accelerated from rest through 4.5 kV. Calculate the de Broglie wavelength of the electrons. Give your answer to three significant figures.

Question 19 A student says: "If electrons behave as waves, then a person walking through a doorway should show diffraction too." (a) Calculate the de Broglie wavelength of a 70 kg person walking at $1.5\ \mathrm{m\ s^{-1}}$. (b) Using your answer, explain why diffraction is observed for electrons passing through a crystal but never for a person walking through a doorway.

Question 20 An X-ray photon of wavelength 7.1×10^{-11} m and an electron are found to produce diffraction rings of identical radius when passed in turn through the same crystal. (a) State what this tells you about the de Broglie wavelength of the electron, and justify your answer. (b) Calculate the momentum and the speed of the electron. (c) Calculate the potential difference through which the electron must have been accelerated from rest. (d) Calculate the energy of the X-ray photon and the kinetic energy of the electron, and compare them.

Answers

Question 1 (a) $3.64 \times 10^{-24}\,\text{kg m s}^{-1}$; (b) $1.82 \times 10^{-10}\,\text{m}$; (c) yes — λ is comparable to atomic spacing. **Question 2** (a) $9.18 \times 10^{6}\,\text{m s}^{-1}$; (b) $8.36 \times 10^{-24}\,\text{kg m s}^{-1}$; (c) $7.93 \times 10^{-11}\,\text{m}$. **Question 3** (a) $2.76 \times 10^{-24}\,\text{kg m s}^{-1}$; (b) $8.29 \times 10^{-16}\,\text{J}$; (c) same momentum, $2.76 \times 10^{-24}\,\text{kg m s}^{-1}$ ($p = h/\lambda$ is identical for equal λ). **Question 4** (a) wave-like behaviour, shown by diffraction; (b) electron λ matches atomic spacing, sand-grain λ is vanishingly small; (c) rings grow larger. **Question 5** (a) same wavelength and same atomic spacing give the same diffraction; (b) apply an electric or magnetic field — only the electron pattern shifts; (c) equal momenta ($p = h/\lambda$). **Question 6** (a) $6.63 \times 10^{-24}\,\text{kg m s}^{-1}$; (b) $7.28 \times 10^{6}\,\text{m s}^{-1}$; (c) $151\,\text{V}$. **Question 7** $4.85 \times 10^{-11}\,\text{m}$. **Question 8** $1.32 \times 10^{-12}\,\text{m}$. **Question 9** (a) $1.04 \times 10^{-34}\,\text{m}$; (b) λ is far smaller than any gap, so no diffraction occurs. **Question 10** $2.65 \times 10^{-22}\,\text{kg m s}^{-1}$ each; the momenta are equal. **Question 11** (a) $6.56 \times 10^{-11}\,\text{m}$; (b) $1.01 \times 10^{-23}\,\text{kg m s}^{-1}$, $E = 3.03 \times 10^{-15}\,\text{J} = 1.89 \times 10^{4}\,\text{eV}$. **Question 12** (a) $2.0\,\text{kV}$ gives the larger rings; (b) $\lambda_{2\,\text{kV}} = 2.75 \times 10^{-11}\,\text{m}$, $\lambda_{5\,\text{kV}} = 1.74 \times 10^{-11}\,\text{m}$, ratio 1.58. **Question 13** (a) electrons behave as waves (diffraction); (b) deflect the beams in a field — only the electrons (charged) are deflected; (c) not justified — same wave behaviour, but electrons have mass and charge, photons do not. **Question 14** (a) $4.14 \times 10^{-24}\,\text{kg m s}^{-1}$; (b) $2.48 \times 10^{3}\,\text{m s}^{-1}$. **Question 15** (a) electron $1.46 \times 10^{-9}\,\text{m}$, proton $7.94 \times 10^{-13}\,\text{m}$; (b) the electron, because it has the smaller momentum. **Question 16** $2.41 \times 10^{3}\,\text{V}$. **Question 17** (a) $4.74 \times 10^{-24}\,\text{kg m s}^{-1}$ each; (b) photon $1.42 \times 10^{-15}\,\text{J}$, electron $1.23 \times 10^{-17}\,\text{J}$; (c) false — same momentum but very different energies. **Question 18** $1.83 \times 10^{-11}\,\text{m}$. **Question 19** (a) $6.31 \times 10^{-36}\,\text{m}$; (b) this λ is far smaller than the doorway, so no diffraction; the electron λ matches the crystal spacing. **Question 20** (a) the electron's de Broglie wavelength is also $7.1 \times 10^{-11}\,\text{m}$; (b) $9.34 \times 10^{-24}\,\text{kg m s}^{-1}$, $1.03 \times 10^{7}\,\text{m s}^{-1}$; (c) $299\,\text{V}$; (d) photon $2.80 \times 10^{-15}\,\text{J}$ ($1.75 \times 10^{4}\,\text{eV}$), electron $4.79 \times 10^{-17}\,\text{J}$ ($299\,\text{eV}$) — the photon carries far more energy.

Fully-worked solutions

Question 1 $m_e = 9.11 \times 10^{-31}\,\text{kg}$, $v = 4.0 \times 10^{6}\,\text{m s}^{-1}$. (a) $p = m_e v = 9.11 \times 10^{-31} \times 4.0 \times 10^{6} = 3.64 \times 10^{-24}\,\text{kg m s}^{-1}$. (b) $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{3.64 \times 10^{-24}} = 1.82 \times 10^{-10}\,\text{m}$. (c) Yes. The wavelength $1.82 \times 10^{-10}\,\text{m}$ is of the same order as the spacing between atoms in a crystal ($\sim 10^{-10}\,\text{m}$), and diffraction is significant when the wavelength is comparable to the size of the "gaps". The electron will therefore produce a noticeable ring diffraction pattern.

Question 2 Electron accelerated through $V = 240\,\text{V}$. (a) $qV = 1/2\, m_e v^2$, so $v = \sqrt{\dfrac{2qV}{m_e}} = \sqrt{\dfrac{2 \times 1.60 \times 10^{-19} \times 240}{9.11 \times 10^{-31}}} = 9.18 \times 10^{6}\,\text{m s}^{-1}$. (b) $p = m_e v = 9.11 \times 10^{-31} \times 9.18 \times 10^{6} = 8.36 \times 10^{-24}\,\text{kg m s}^{-1}$. (c) $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{8.36 \times 10^{-24}} = 7.93 \times 10^{-11}\,\text{m}$.

Question 3 X-ray photon, $\lambda = 0.24\,\text{nm} = 2.4 \times 10^{-10}\,\text{m}$. (a) $p = \dfrac{h}{\lambda} = \dfrac{6.63 \times 10^{-34}}{2.4 \times 10^{-10}} = 2.76 \times 10^{-24}\,\text{kg m s}^{-1}$. (b) $E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{2.4 \times 10^{-10}} = 8.29 \times 10^{-16}\,\text{J}$. (c) The electron has the same momentum, $2.76 \times 10^{-24}\,\text{kg m s}^{-1}$. The relation $p = h/\lambda$ applies to both photons and matter, so two objects of the same wavelength must have the same momentum, regardless of what they are.

Question 4 (a) The ring pattern is a **diffraction** pattern, and diffraction is a property of **waves**. The rings therefore show that the electrons behave as waves, with a de Broglie wavelength $\lambda = h/p$. (b) Diffraction is only significant when the wavelength is comparable to the size of the gaps (here, the spacing between atoms in the graphite, $\sim 10^{-10}\,\text{m}$). An accelerated electron has $\lambda \sim 10^{-10}\,\text{m}$, matching that spacing, so it diffracts strongly. A sand grain has an enormous momentum compared with an electron, so its de Broglie wavelength is smaller than the atomic spacing by many powers of ten; it meets no gap remotely close to its wavelength and so shows no diffraction — it simply behaves as a particle. (c) Decreasing the accelerating voltage decreases the electrons' momentum ($p = \sqrt{2 m_e q V}$), and since $\lambda = h / p$ a smaller momentum means a **longer** wavelength. A longer wavelength diffracts through larger angles, so the rings become **larger in diameter**.

Question 5 (a) Diffraction depends only on the wavelength of the incoming wave and the spacing of the atoms in the crystal, not on what the wave is. The X-rays and the electrons have the same wavelength and pass through the same crystal, so they are diffracted through the same angles and form rings of the same radii. (b) Apply an electric or magnetic field to each beam before it reaches the screen. Electrons are charged, so the field exerts a force on them and the electron ring pattern shifts or distorts; X-ray photons are uncharged and are undeflected. The pattern that moves when the field is applied is the one produced by the electrons. (c) The momenta are **equal**. Since $p = h / \lambda$ holds for both a photon and a particle, and the two beams have the same wavelength λ, they must have the same momentum $p = h / \lambda$.

Question 6 Electron, $\lambda = 0.10\,\text{nm} = 1.0 \times 10^{-10}\,\text{m}$. (a) $p = \dfrac{h}{\lambda} = \dfrac{6.63 \times 10^{-34}}{1.0 \times 10^{-10}} = 6.63 \times 10^{-24}\,\text{kg m s}^{-1}$. (b) $v = \dfrac{p}{m_e} = \dfrac{6.63 \times 10^{-24}}{9.11 \times 10^{-31}} = 7.28 \times 10^{6}\,\text{m s}^{-1}$. (c) The accelerating voltage supplies the kinetic energy: $qV = \dfrac{p^2}{2 m_e}$, so $V = \dfrac{p^2}{2 m_e q} = \dfrac{(6.63 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19}} = 151\,\text{V}$.

Question 7 $p = m_e v = 9.11 \times 10^{-31} \times 1.5 \times 10^{7} = 1.37 \times 10^{-23}\,\text{kg m s}^{-1}$, so $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{1.37 \times 10^{-23}} = 4.85 \times 10^{-11}\,\text{m}$.

Question 8 $p = m v = 1.67 \times 10^{-27} \times 3.0 \times 10^{5} = 5.01 \times 10^{-22}\,\text{kg m s}^{-1}$, so $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{5.01 \times 10^{-22}} = 1.32 \times 10^{-12}\,\text{m}$ (to three significant figures).

Question 9 $m = 0.16\,\text{kg}$, $v = 40\,\text{m s}^{-1}$. (a) $p = m v = 0.16 \times 40 = 6.4\,\text{kg m s}^{-1}$, so $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{6.4} = 1.04 \times 10^{-34}\,\text{m}$. (b) A wave diffracts noticeably only when it meets a gap or obstacle of a size comparable to its wavelength. At $1.04 \times 10^{-34}\,\text{m}$ the ball's de Broglie wavelength is smaller than an atomic nucleus by about twenty powers of ten, so there is no object in existence small enough to make the ball diffract. Its wave nature, though real, can never be observed.

Question 10 For both the electron and the photon, $p = \dfrac{h}{\lambda} = \dfrac{6.63 \times 10^{-34}}{2.5 \times 10^{-12}} = 2.65 \times 10^{-22}\,\text{kg m s}^{-1}$. Because $p = h / \lambda$ applies equally to matter and to light, the two momenta are **equal**.

Question 11 Electron accelerated through $V = 350\,\text{V}$. (a) $p = \sqrt{2 m_e q V} = \sqrt{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19} \times 350} = 1.01 \times 10^{-23}\,\text{kg m s}^{-1}$, so

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.01 \times 10^{-23}} = 6.56 \times 10^{-11}\,\text{m}\text{, as required.}$$

(b) The photon has the same wavelength, so it has the same momentum, $p = \dfrac{h}{\lambda} = 1.01 \times 10^{-23}\,\text{kg m s}^{-1}$. Its energy is

$$E = p c = 1.01 \times 10^{-23} \times 3.00 \times 10^{8} = 3.03 \times 10^{-15}\,\text{J} = \frac{3.03 \times 10^{-15}}{1.60 \times 10^{-19}} = 1.89 \times 10^{4}\,\text{eV}.$$

Question 12 (a) The $2.0\,\text{kV}$ setting produces the **larger** rings. A lower accelerating voltage gives the electrons less momentum ($p = \sqrt{2 m_e q V}$), and since $\lambda = h / p$ a smaller momentum means a longer de Broglie wavelength. A longer wavelength diffracts through larger angles, producing larger-diameter rings. (b) $\lambda=\dfrac{h}{\sqrt{2m_e qV}}$. At $2.0\,\text{kV}$: $\lambda_{2\,\text{kV}}=2.75\times10^{-11}\,\text{m}$. At $5.0\,\text{kV}$: $\lambda_{5\,\text{kV}}=1.74\times10^{-11}\,\text{m}$. The ratio is $\dfrac{\lambda_{2\,\text{kV}}}{\lambda_{5\,\text{kV}}}=\sqrt{\dfrac{5.0}{2.0}}=1.58$ (since $\lambda\propto1/\sqrt{V}$).

Question 13 (a) The electron beam produces a diffraction pattern (rings), and diffraction is a wave phenomenon. This demonstrates that electrons — ordinarily thought of as particles — have a wave-like nature, described by the de Broglie wavelength $\lambda=h/p$. (b) Pass each beam through an electric or magnetic field on its way to the screen. The electrons carry charge, so they experience a force and their pattern is deflected; the X-ray photons are uncharged and are unaffected. Alternatively, the electron beam deposits charge on a metal collector and can be stopped by a thin sheet of metal, whereas X-rays penetrate it — either test identifies the electron beam. (c) The conclusion is **not** justified. The similar patterns show that electrons and X-rays *both* exhibit wave behaviour with the same wavelength, but they are not “the same kind of thing”: electrons are particles of matter with mass and electric charge, whereas X-rays are electromagnetic radiation (massless, chargeless photons). The experiment shows a shared wave property, not identical nature.

Question 14 Neutron, $m=1.67\times10^{-27}\,\text{kg}$, $\lambda=1.6\times10^{-10}\,\text{m}$. (a) $p=\dfrac{h}{\lambda}=\dfrac{6.63\times10^{-34}}{1.6\times10^{-10}}=4.14\times10^{-24}\,\text{kg m s}^{-1}$. (b) $v=\dfrac{p}{m}=\dfrac{4.14\times10^{-24}}{1.67\times10^{-27}}=2.48\times10^{3}\,\text{m s}^{-1}$.

Question 15 Same speed $v=5.0\times10^{5}\,\text{m s}^{-1}$. (a) Electron: $p=m_e v=9.11\times10^{-31}\times5.0\times10^{5}=4.555\times10^{-25}\,\text{kg m s}^{-1}$, so $\lambda_e=\dfrac{h}{p}=\dfrac{6.63\times10^{-34}}{4.555\times10^{-25}}=1.46\times10^{-9}\,\text{m}$. Proton: $p=m_p v=1.67\times10^{-27}\times5.0\times10^{5}=8.35\times10^{-22}\,\text{kg m s}^{-1}$, so $\lambda_p=\dfrac{h}{p}=\dfrac{6.63\times10^{-34}}{8.35\times10^{-22}}=7.94\times10^{-13}\,\text{m}$. (b) The **electron** has the longer wavelength. At the same speed the proton, being about 1800 times more massive, has about 1800 times the momentum; since $\lambda=h/p$, the greater momentum gives the shorter wavelength, so the lighter electron has the longer de Broglie wavelength.

Question 16 Required $\lambda=2.5\times10^{-11}\,\text{m}$. First the momentum: $p=\dfrac{h}{\lambda}=\dfrac{6.63\times10^{-34}}{2.5\times10^{-11}}=2.65\times10^{-23}\,\text{kg m s}^{-1}$. The accelerating voltage supplies the kinetic energy $qV=\dfrac{p^2}{2m_e}$, so

$$V=\frac{p^2}{2m_e q}=\frac{(2.65\times10^{-23})^2}{2\times9.11\times10^{-31}\times1.60\times10^{-19}}=2.41\times10^{3}\,\text{V}.$$

Question 17 Electron and photon, $\lambda=1.4\times10^{-10}\,\text{m}$. (a) $p=\dfrac{h}{\lambda}=\dfrac{6.63\times10^{-34}}{1.4\times10^{-10}}=4.74\times10^{-24}\,\text{kg m s}^{-1}$ for each. (b) Photon: $E=pc=4.74\times10^{-24}\times3.00\times10^{8}=1.42\times10^{-15}\,\text{J}$. Electron: $E_k=\dfrac{p^2}{2m_e}=\dfrac{(4.74\times10^{-24})^2}{2\times9.11\times10^{-31}}=1.23\times10^{-17}\,\text{J}$. (c) The statement is false. The electron and photon have the **same momentum** (because they have the same wavelength), but their energies are very different: the photon’s energy is more than a hundred times the electron’s kinetic energy. A photon’s energy is proportional to its momentum ($E=pc$), whereas a slow particle’s kinetic energy depends on the *square* of its speed ($E_k=p^2/2m$), so equal wavelength does not mean equal energy.

Question 18 $V=4.5\,\text{kV}=4500\,\text{V}$. $\lambda=\dfrac{h}{\sqrt{2m_e qV}}=\dfrac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.60\times10^{-19}\times4500}}=1.83\times10^{-11}\,\text{m}$ (to three significant figures).

Question 19 (a) $p = mv = 70 \times 1.5 = 105\,\text{kg m s}^{-1}$, so $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{105} = 6.31 \times 10^{-36}\,\text{m}$. (b) Diffraction is only noticeable when the wavelength is comparable to the width of the gap. The doorway is about $1\,\text{m}$ wide, but the person's de Broglie wavelength, $6.31 \times 10^{-36}\,\text{m}$, is smaller than this by some thirty-six powers of ten, so the spreading is utterly undetectable. An accelerated electron, by contrast, has $\lambda \sim 10^{-10}\,\text{m}$, which matches the spacing of atoms in a crystal, so its diffraction is easily observed. The student is right in principle but wrong in practice: everyday objects have wavelengths far too small to diffract through everyday openings.

Question 20 X-ray photon, $\lambda = 7.1 \times 10^{-11}\,\text{m}$. (a) Because diffraction depends only on wavelength and atomic spacing, rings of identical radius from the same crystal mean the two beams have the same wavelength. The electron's de Broglie wavelength is therefore also $7.1 \times 10^{-11}\,\text{m}$. (b) $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{7.1 \times 10^{-11}} = 9.34 \times 10^{-24}\,\text{kg m s}^{-1}$, and $v = \frac{p}{m_e} = \frac{9.34 \times 10^{-24}}{9.11 \times 10^{-31}} = 1.03 \times 10^{7}\,\text{m s}^{-1}$. (c) $V = \frac{p^2}{2m_e q} = \frac{(9.34 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19}} = 299\,\text{V}$. (d) Photon: $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{7.1 \times 10^{-11}} = 2.80 \times 10^{-15}\,\text{J} = 1.75 \times 10^{4}\,\text{eV}$. Electron: $E_k = \frac{p^2}{2m_e} = 4.79 \times 10^{-17}\,\text{J} = 299\,\text{eV}$. The two have the same wavelength and the same momentum, but the photon's energy is nearly sixty times the electron's kinetic energy. (Note that the electron's kinetic energy in electronvolts, $299\,\text{eV}$, equals the accelerating voltage $299\,\text{V}$, as expected from $E_k = qV$.)

Theory

Quantisation. Classical physics pictured light as a continuous electromagnetic wave and allowed an object to have any energy at all. Two kinds of experiment showed this picture to be incomplete. First, the photoelectric effect and the spectrum of a hot body could only be explained if the energy of light comes in discrete packets, or **quanta**, called **photons**, each carrying

$$E = hf = \frac{hc}{\lambda},$$

where $h = 6.63 \times 10^{-34}$ J s is Planck's constant, f the frequency and λ the wavelength of the light. Second, atoms emit and absorb light only at certain sharp wavelengths. This forced a second, equally radical idea: the energy of the electrons in an atom is also **quantised** — an electron may occupy only certain allowed **energy levels** and nothing in between. The idea of quantisation is the single thread that ties together the modern understanding of both light (as photons) and matter (as atoms with discrete energy levels), and it is the foundation of quantum physics.

Energy levels in an atom. The allowed energies of an atom's electron are drawn as an **energy-level diagram**: a set of horizontal lines, each labelled with its energy. The energies are measured downward from the **ionisation** level (taken as $0\,\mathrm{eV}$, the energy of an electron just free of the atom), so the bound levels are **negative**. The lowest level is the **ground state**; the higher levels are **excited states**. Because energies at the atomic scale are tiny, they are quoted in **electron-volts**:

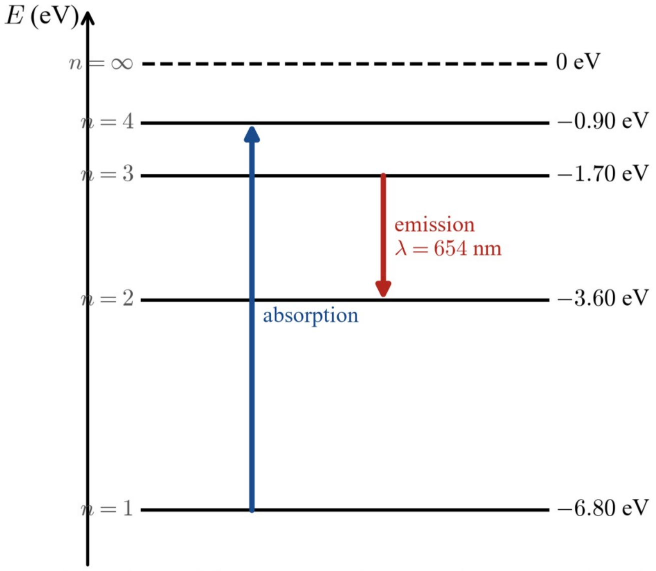
$$1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J},$$

the energy gained by an electron accelerated through $1\,\mathrm{V}$. To move between joules and electron-volts, multiply by 1.60×10^{-19} to go from eV to J, and divide by the same number to go the other way.

Transitions: emission and absorption. An electron can jump between two levels only if it gains or loses exactly the energy difference between them. In **emission**, the electron **falls** from a higher level E_{upper} to a lower level E_{lower} and the atom emits a single photon; in **absorption**, the electron is lifted from a lower to a higher level by absorbing a photon. Either way the photon energy exactly equals the gap between the two states:

$$E_{\text{photon}} = E_{\text{upper}} - E_{\text{lower}} = hf = \frac{hc}{\lambda}.$$

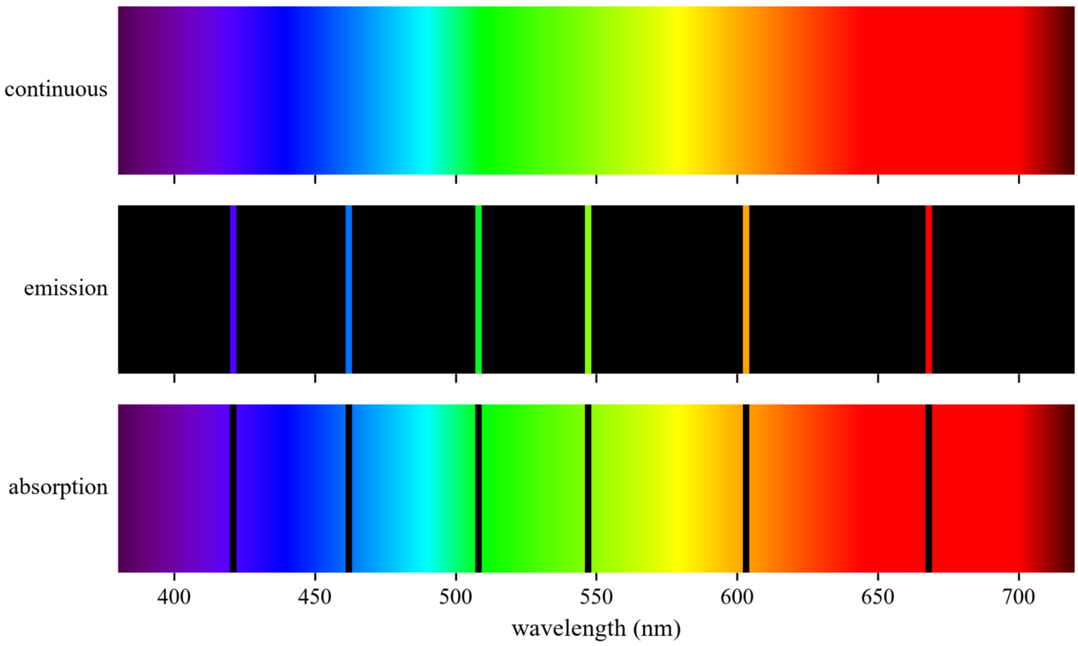
A photon whose energy does *not* match any gap is simply not absorbed — it passes through. This is why the transitions of an atom involve only a fixed set of photon energies, and hence a fixed set of frequencies and wavelengths.



emission: electron falls, photon out; absorption: electron rises, photon in
$E_{\text{photon}} = E_{\text{upper}} - E_{\text{lower}} = hf = hc/\lambda$

Reading an energy-level diagram. To find the wavelength emitted or absorbed in a transition: compute the energy gap $\Delta E = E_{\text{upper}} - E_{\text{lower}}$ (in eV), convert it to joules, then use $\lambda = \dfrac{hc}{\Delta E}$. On the diagram, an emission is drawn as an arrow pointing **down** (electron falling, photon leaving) and an absorption as an arrow pointing **up** (electron rising, photon absorbed). The largest gaps give the most energetic photons — the highest frequency and the **shortest** wavelength; the smallest gaps give the longest wavelength.

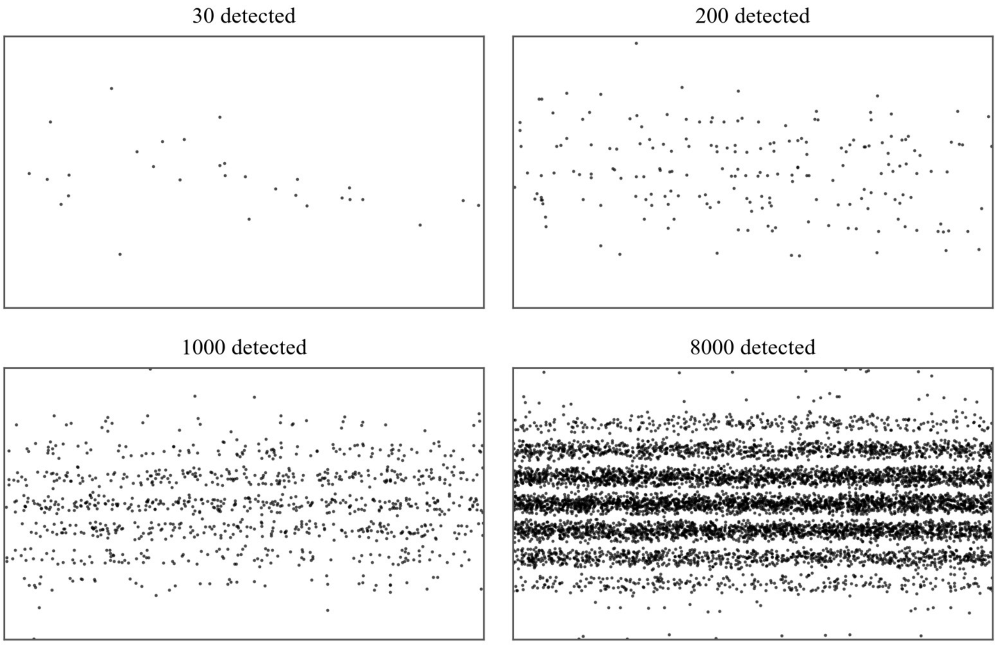
Emission and absorption line spectra. When the light from a source is spread out by a prism or grating, the pattern of wavelengths is its **spectrum**. A hot, dense solid or a filament lamp gives a **continuous spectrum** (an unbroken band of all colours). A hot, low-pressure gas instead gives an **emission line spectrum**: a few bright coloured lines on a dark background, one line for each downward transition available to its atoms. If, on the other hand, white light with a continuous spectrum is passed through a cooler gas, the atoms of the gas absorb exactly those photons whose energies match their transitions, lifting electrons to higher levels. Those wavelengths are removed from the beam, leaving an **absorption line spectrum**: dark lines on a continuous coloured background, at the **same** wavelengths as the bright lines the same gas would emit.



Every element is unique. Because each element has its own distinctive set of energy levels, it has its own distinctive set of transition energies, and therefore its own unique pattern of spectral lines — a "fingerprint" by which it can be identified. This is the basis of **metal vapour lamps**: a sodium vapour lamp glows with the characteristic yellow-orange of sodium's strongest transition, and a mercury vapour lamp with the blue-white mixture of mercury's lines, because the excited atoms in the lamp emit only at their own wavelengths. The same fingerprints let astronomers read the composition of stars from the dark absorption lines in starlight.

Discrete lines are evidence for quantisation. The fact that spectra are sets of sharp lines, rather than continuous bands, is direct evidence that atomic energies are quantised. Sharp lines mean only certain photon energies are emitted; certain photon energies mean only certain energy *differences* exist; and fixed energy differences mean the electron is restricted to a fixed set of allowed energy levels. A classical atom, whose electron could have any energy, would radiate a continuous smear of wavelengths — which is not what is seen.

The dual nature of light and matter. Light behaves as a wave (it diffracts and interferes, as in Young's double-slit experiment, Chapter 7A) yet is emitted and absorbed as particle-like photons. A striking demonstration is the **single-photon double-slit experiment**: light is dimmed until only one photon at a time passes through a double slit. Each photon is detected as a single localised **dot** on the screen — particle-like behaviour. But as thousands of photons are recorded, the dots build up, one by one, into the bright-and-dark **interference fringes** predicted by the wave model. Neither the wave picture (which cannot explain single localised detections) nor the particle picture (which cannot explain fringes) is sufficient on its own: light has a **dual** wave–particle nature.



each particle arrives as one dot; the pattern builds up fringe by fringe

Matter too. When the same experiment is done with **electrons** — sent through a double slit one at a time — exactly the same thing happens: each electron arrives as a single dot, yet the accumulated pattern is an interference pattern. Electrons, which we think of as particles, show wave-like interference, just as photons do. Each electron has an associated wavelength $\lambda = \frac{h}{p}$ (Chapter 8A). The single-photon and single-electron double-slit experiments are together the clearest evidence that both light and matter have a dual wave–particle nature.

Throughout this subtopic the data-sheet constants $h = 6.63 \times 10^{-34}$ J s ($= 4.14 \times 10^{-15}$ eV s), $c = 3.00 \times 10^{8}\ \mathrm{m\ s^{-1}}$ and $1\,\mathrm{eV} = 1.60 \times 10^{-19}$ J are used.

Worked examples

Example 1 — scaffolded

The energy-level diagram above shows the first four energy levels of a hypothetical atom, with the ground state at $-6.80\,\mathrm{eV}$, and levels at $-3.60\,\mathrm{eV}$, $-1.70\,\mathrm{eV}$ and $-0.90\,\mathrm{eV}$. An electron makes a transition from the $-1.70\,\mathrm{eV}$ level to the $-3.60\,\mathrm{eV}$ level. Calculate the energy of the emitted photon (in eV and in J), its frequency, and its wavelength, and state the region of the electromagnetic spectrum in which it lies.

Working	Comment
The electron falls, so a photon is emitted: $\Delta E = E_{\text{upper}} - E_{\text{lower}} = -1.70 - (-3.60) = 1.90\,\text{eV}$.	Photon energy = gap between the two levels.
$\Delta E = 1.90 \times 1.60 \times 10^{-19} = 3.04 \times 10^{-19}\,\text{J}$.	Convert eV to joules ($\times 1.60 \times 10^{-19}$).
$f = \dfrac{E}{h} = \dfrac{3.04 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.59 \times 10^{14}\,\text{Hz}$.	Rearrange $E = hf$.
$\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{3.04 \times 10^{-19}} = 6.54 \times 10^{-7}\,\text{m}$.	Or $\lambda = \dfrac{c}{f}$; equals $654\,\text{nm}$.
$654\,\text{nm}$ is in the **red** part of the visible spectrum.	Visible range $\approx 400 - 700\,\text{nm}$.

Example 2 — scaffolded

In a certain gas, an electron in the ground state at $-5.30\,\text{eV}$ absorbs a photon and is lifted to a level at $-2.85\,\text{eV}$. Calculate the energy (in eV and J) and the wavelength of the absorbed photon. Then state, with a reason, what happens to a photon of energy $2.00\,\text{eV}$ incident on an identical atom in the ground state.

Working	Comment
$\Delta E = E_{\text{upper}} - E_{\text{lower}} = -2.85 - (-5.30) = 2.45\,\text{eV}$.	Energy absorbed = gap between the levels.
$\Delta E = 2.45 \times 1.60 \times 10^{-19} = 3.92 \times 10^{-19}\,\text{J}$.	Convert to joules.
$\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{3.92 \times 10^{-19}} = 5.07 \times 10^{-7}\,\text{m} = 50$.	Wavelength of the absorbed photon.
The $2.00\,\text{eV}$ photon does not match any gap from the ground state, so it is **not absorbed** — it passes straight through the gas.	Absorption needs an exact energy match.

Example 3 — unscaffolded

A metal vapour lamp used for street lighting emits an intense line of wavelength $6.12 \times 10^{-7}\,\text{m}$. Calculate the energy of each photon in both joules and electron-volts, and its frequency, and state the type of spectrum the lamp produces.

The photon energy is

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{6.12 \times 10^{-7}} = 3.25 \times 10^{-19}\,\text{J}.$$

In electron-volts,

$$E = \frac{3.25 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.03\,\text{eV}.$$

The frequency is

$$f = \frac{E}{h} = \frac{3.25 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.90 \times 10^{14}\,\text{Hz}.$$

The lamp contains an excited low-pressure vapour, so the light comes only from the discrete downward transitions of its atoms.

$\therefore$ each photon has energy $3.25 \times 10^{-19}\,\text{J} = 2.03\,\text{eV}$ and frequency $4.90 \times 10^{14}\,\text{Hz}$; the lamp produces an **emission line spectrum**.

Example 4 — unscaffolded

Explain, in terms of energy levels and photons, how a low-pressure gas produces an emission line spectrum, and why the spectrum consists of a set of discrete lines rather than a continuous band of colour.

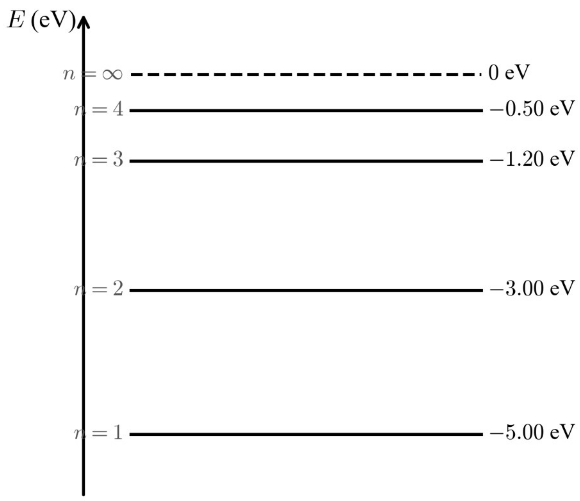
Atoms in the gas are given energy (by an electric current or heating), which raises their electrons from the ground state to higher, excited energy levels. An excited electron is unstable and falls back to a lower level. When it falls, the atom emits a single photon whose energy is exactly the difference between the two levels, $E_{\text{photon}} = E_{\text{upper}} - E_{\text{lower}} = hf$. Each allowed transition therefore produces photons of one particular energy, and hence one particular frequency and wavelength, seen as one bright coloured line.

Because the electron's energy is **quantised**, only certain energy levels exist, so only certain energy differences are possible. The gas can emit photons of only those few energies — not a continuous range — so the spectrum is a set of sharp, discrete lines. A continuous spectrum would require a continuous range of available energies, which a single atom's quantised levels do not provide.

∴ discrete line spectra are a direct consequence of the quantisation of atomic energy levels.

Practice questions

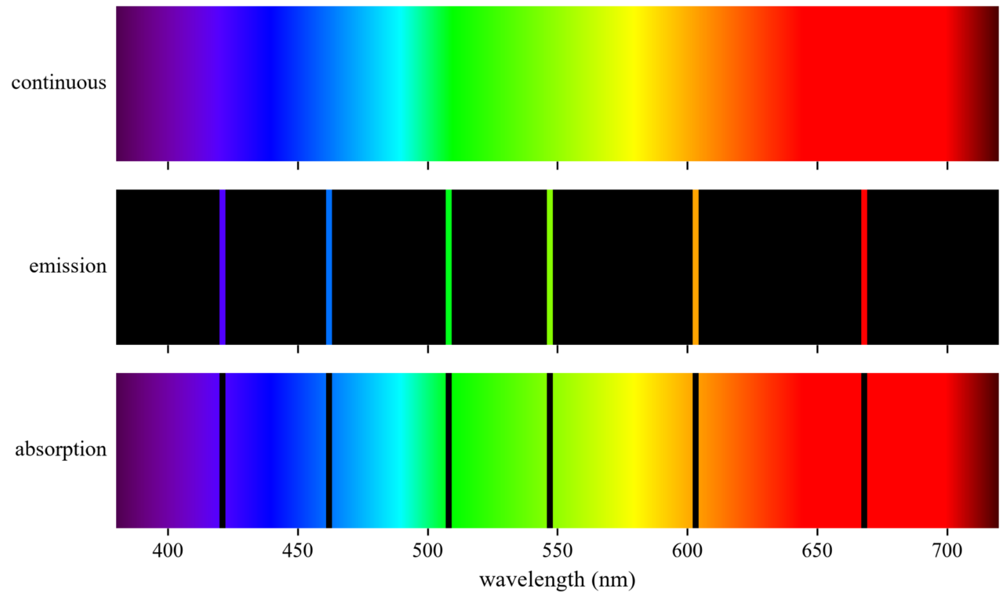
Question 1 The energy-level diagram below shows four levels of an atom (labelled by n). An electron makes a transition from the $n=3$ level to the $n=2$ level.



(a) Calculate the energy of the emitted photon, in eV and in J.
(b) Calculate its wavelength.
(c) State whether the transition is an emission or an absorption, in which direction the transition arrow points on the diagram, and the region of the electromagnetic spectrum in which the photon lies.

Question 2 Use the same energy-level diagram as in Question 1 (levels at $-5.00\,\text{eV}$, $-3.00\,\text{eV}$, $-1.20\,\text{eV}$ and $-0.50\,\text{eV}$). (a) Show that the transition from the $n=4$ level to the $n=2$ level emits a photon of energy 4.00×10^{-19} J. (b) Hence calculate the wavelength of this photon. (c) State the region of the electromagnetic spectrum in which it lies.

Question 3 The figure shows a continuous spectrum, an emission line spectrum and an absorption line spectrum of the same element.

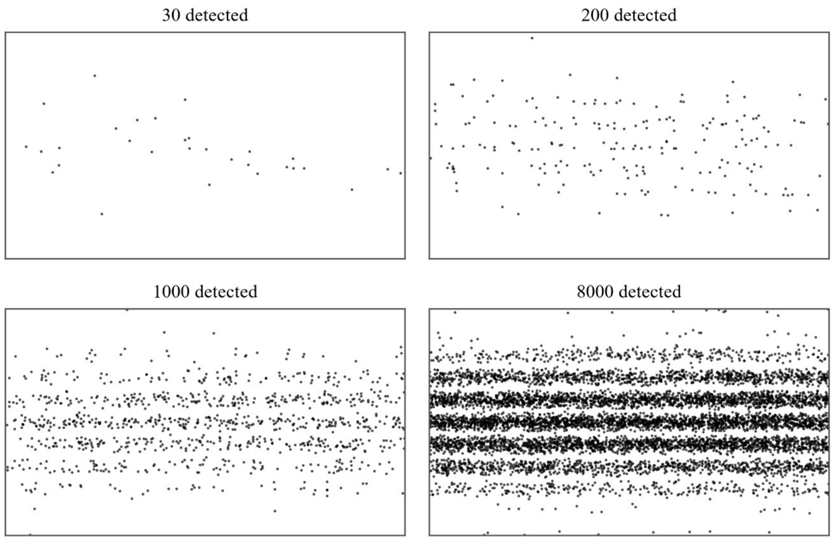


(a) State the difference in appearance between an emission line spectrum and an absorption line spectrum.
(b) Explain how a metal vapour lamp produces an emission line spectrum.
(c) Explain why the dark lines of the absorption spectrum occur at exactly the same wavelengths as the bright lines of the emission spectrum of the same element.

Question 4 A photon has energy $3.10\,\mathrm{eV}$. (a) Convert this energy to joules. (b) A second photon has frequency $5.50\times10^{14}\,\mathrm{Hz}$. Calculate its energy, in J and in eV. (c) Calculate the wavelength of the second photon.

Question 5 A metal vapour lamp emits two visible lines, of wavelength $4.50\times10^{-7}\,\mathrm{m}$ and $6.50\times10^{-7}\,\mathrm{m}$. (a) Calculate the energy, in joules, of a photon of each line. (b) Express each energy in electron-volts. (c) State which line's photons carry more energy, and explain why in terms of wavelength.

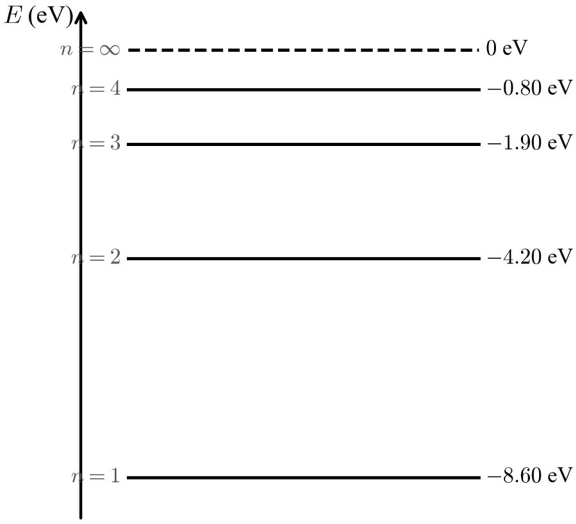
Question 6 In a single-photon double-slit experiment, light is dimmed so that only one photon at a time passes through the slits, and a detector records where each photon lands.



each particle arrives as one dot; the pattern builds up fringe by fringe

(a) Describe what is observed on the screen after a few photons, and after very many photons, have been detected.
(b) Explain how this experiment shows that light has both a particle-like and a wave-like nature.
(c) State what is observed when the experiment is repeated with electrons instead of photons, and what this shows about the nature of matter.

Question 7 An atom has energy levels at $-8.60\,\mathrm{eV}$ ($n=1$), $-4.20\,\mathrm{eV}$ ($n=2$), $-1.90\,\mathrm{eV}$ ($n=3$) and $-0.80\,\mathrm{eV}$ ($n=4$).



Calculate the wavelength of the photon emitted when an electron falls from the $n = 4$ level to the $n = 2$ level, and state the region of the electromagnetic spectrum in which it lies.

Question 8 For the atom of Question 7, state which single downward transition between the four levels shown produces the photon of **longest** wavelength, explain your reasoning, and calculate that wavelength.

Question 9 For the atom of Question 7, exactly one downward transition between the four levels shown produces a photon of visible light. Identify the transition, calculate its wavelength, and justify why it is the only visible line.

Question 10 An atom with the energy levels of Question 7 is in its ground state. It absorbs a photon of energy 7.04×10^{-19} J. Determine the energy of the photon in electron-volts, and identify the level to which the electron is raised.

Question 11 A spectral line has a frequency of 6.50×10^{14} Hz. Calculate the energy of each photon in electron-volts, and the wavelength of the line.

Question 12 Discuss the importance of the idea of quantisation in the development of knowledge about light and in explaining the nature of atoms. Refer to both the quantisation of light and the quantisation of atomic energy.

Question 13 Explain why the existence of discrete line spectra, rather than continuous spectra, is evidence that the energy of the electrons in an atom is quantised. Set out the reasoning as a chain from the observation to the conclusion.

Question 14 Explain why each element produces a unique line spectrum, and describe one practical use that is made of this fact.

Question 15 White light with a continuous spectrum is passed through a cool, low-pressure gas and then examined. Explain, in terms of photons and energy levels, how the resulting absorption line spectrum is produced.

Question 16 A student says: “If photons pass through a double slit one at a time, then each photon has nothing to interfere with, so no interference pattern can possibly form.” Explain what is wrong with this reasoning, and describe what is actually observed.

Question 17 A beam of electrons is fired at a double slit, one electron at a time, and the arrival point of each electron is recorded. Describe what pattern builds up, explain why this result could not be predicted by treating the electrons purely as particles, and state what it shows about the nature of matter.

Question 18 A sodium vapour street lamp glows with a characteristic yellow-orange colour, and a mercury vapour lamp with a blue-white colour. Explain, in terms of atomic energy levels and photons, why each type of lamp has its own characteristic colour.

Question 19 A photon has energy 4.80×10^{-19} J. (a) Express this energy in electron-volts. (b) Calculate the wavelength of the photon, and state whether it lies in the visible range.

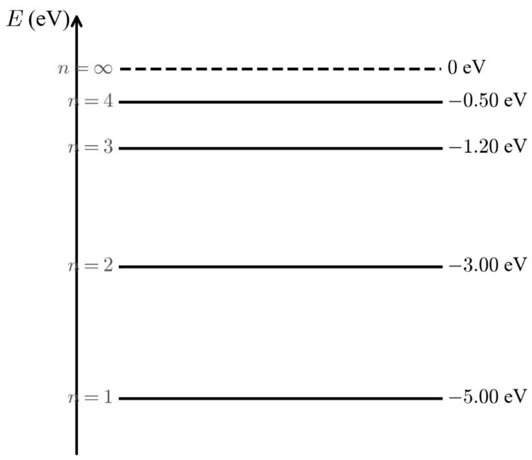
Question 20 An atom has its ground state at -6.80 eV and higher levels at -3.60 eV, -1.70 eV and -0.90 eV (the atom of Worked example 1). An electron in the ground state absorbs a photon and is lifted to the -0.90 eV level. It then returns to the ground state in two steps: first to the -3.60 eV level, then to the ground state. (a) Calculate the wavelength of the absorbed photon. (b) Calculate the wavelength of each of the two emitted photons. (c) Show that the energy of the absorbed photon equals the total energy of the two emitted photons, and state the principle this illustrates.

Answers

Question 1 (a) $1.80\,\text{eV} = 2.88 \times 10^{-19}\,\text{J}$; (b) $6.91 \times 10^{-7}\,\text{m}$ ($691\,\text{nm}$); (c) emission, arrow points down, red (visible). **Question 2** (a) $4.00 \times 10^{-19}\,\text{J}$ ($2.50\,\text{eV}$); (b) $4.97 \times 10^{-7}\,\text{m}$ ($497\,\text{nm}$); (c) blue-green (visible). **Question 3** (a) Emission: bright lines on a dark background; absorption: dark lines on a continuous coloured background. (b) The current excites the vapour's electrons to higher levels; falling back, they emit photons of one definite energy per transition ($E = hf$), giving the set of bright lines. (c) Absorption removes from the beam only photons matching the gaps between the same pairs of levels, so the wavelengths absorbed ($\lambda = hc / \Delta E$) are identical to the wavelengths emitted. **Question 4** (a) $4.96 \times 10^{-19}\,\text{J}$; (b) $3.65 \times 10^{-19}\,\text{J} = 2.28\,\text{eV}$; (c) $5.45 \times 10^{-7}\,\text{m}$ ($545\,\text{nm}$). **Question 5** (a) $4.42 \times 10^{-19}\,\text{J}$ and $3.06 \times 10^{-19}\,\text{J}$; (b) $2.76\,\text{eV}$ and $1.91\,\text{eV}$; (c) the $4.50 \times 10^{-7}\,\text{m}$ line (shorter wavelength ⇒ higher energy). **Question 6** (a) A few scattered dots; then interference fringes built from the dots. (b) Each photon lands at one localised point (particle-like), yet the accumulated dots form interference fringes (wave-like) — light has a dual wave–particle nature. (c) Electrons give the same result: single dots building into an interference pattern, showing that matter too has a wave-like nature. **Question 7** $3.66 \times 10^{-7}\,\text{m}$ ($366\,\text{nm}$), ultraviolet. **Question 8** The $n = 4 \to n = 3$ transition (smallest energy gap): $1.13 \times 10^{-6}\,\text{m}$ ($1130\,\text{nm}$), infrared. **Question 9** The $n = 3 \to n = 2$ transition: $5.40 \times 10^{-7}\,\text{m}$ ($540\,\text{nm}$), green. **Question 10** $4.40\,\text{eV}$; the electron is raised from the ground state to the $n = 2$ level ($-4.20\,\text{eV}$). **Question 11** $2.69\,\text{eV}$; $4.62 \times 10^{-7}\,\text{m}$ ($462\,\text{nm}$). **Question 12** Quantised light (photons of $E = hf$) explains the photoelectric effect and the spectrum of a hot body; quantised electron energy levels in atoms explain atomic stability and discrete line spectra; together they unify the understanding of light and matter and are the foundation of quantum physics. **Question 13** Sharp lines ⇒ definite photon energies ($E = hc / \lambda$) ⇒ a fixed set of energy gaps between levels ⇒ the electron is restricted to a fixed set of allowed levels, i.e. its energy is quantised. **Question 14** Unique set of energy levels ⇒ unique set of transition energies ⇒ unique line pattern; used to identify elements (e.g. metal vapour lamps, composition of stars). **Question 15** The cool gas absorbs only photons whose energies exactly match the gaps between its levels, lifting electrons from lower to higher levels; those wavelengths are removed from the continuous beam, leaving dark lines at the gas's transition wavelengths. **Question 16** A single photon does not need a second photon to interfere with: each photon is described by a wave that passes through both slits and interferes with itself; the dots still land one at a time but accumulate into the same bright-and-dark fringes as an intense beam. **Question 17** An interference pattern builds up from single dots; a pure particle model predicts only two bands; matter has a wave-like nature. **Question 18** Each element has its own set of energy levels, hence its own transition energies and emitted wavelengths; sodium's strongest lines are yellow-orange and mercury's strong lines mix to blue-white. **Question 19** (a) $3.00\,\text{eV}$; (b) $4.14 \times 10^{-7}\,\text{m}$ ($414\,\text{nm}$), yes — violet, within the visible range. **Question 20** (a) $2.11 \times 10^{-7}\,\text{m}$ ($211\,\text{nm}$); (b) $4.60 \times 10^{-7}\,\text{m}$ ($460\,\text{nm}$) and $3.88 \times 10^{-7}\,\text{m}$ ($388\,\text{nm}$); (c) $5.90\,\text{eV} = 2.70\,\text{eV} + 3.20\,\text{eV}$ — conservation of energy.

Fully-worked solutions

Question 1 The electron falls from $n = 3$ ($-1.20\,\text{eV}$) to $n = 2$ ($-3.00\,\text{eV}$).



(a) $\Delta E = -1.20 - (-3.00) = 1.80\,\text{eV} = 1.80 \times 1.60 \times 10^{-19} = 2.88 \times 10^{-19}\,\text{J}$.

(b) $\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{2.88 \times 10^{-19}} = 6.91 \times 10^{-7}\,\text{m}$ (691 nm).

(c) The electron loses energy, so this is an **emission**; the transition arrow points **down** (from $n=3$ to $n=2$). At 691 nm the photon is in the **red** part of the visible spectrum.

Question 2 (a) $\Delta E = E_{\text{upper}} - E_{\text{lower}} = -0.50 - (-3.00) = 2.50\,\text{eV}$. In joules, $\Delta E = 2.50 \times 1.60 \times 10^{-19} = 4.00 \times 10^{-19}\,\text{J}$, as required. (b) $\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{4.00 \times 10^{-19}} = 4.97 \times 10^{-7}\,\text{m}$ (497 nm). (c) 497 nm lies in the **blue-green** part of the visible spectrum.

Question 3 (a) An emission line spectrum is a set of **bright coloured lines on a dark background**; an absorption line spectrum is a set of **dark lines on a continuous coloured (rainbow) background**. (b) A metal vapour lamp contains a low-pressure vapour whose atoms are excited by an electric current. Electrons are lifted to higher energy levels and then fall back to lower levels. Each fall emits a photon of energy equal to the gap between the two levels, $E = hf$, giving light of one definite wavelength. The set of all the downward transitions produces the set of bright lines. (c) The dark absorption lines are produced when photons of exactly the transition energies are absorbed from a continuous beam, lifting electrons up between the **same** pairs of levels that produce emission. Since the energy levels — and therefore the energy gaps — are the same whether the atom is emitting or absorbing, the wavelengths absorbed ($\lambda = hc/\Delta E$) are identical to the wavelengths emitted. The dark lines therefore fall at exactly the positions of the bright lines.

Question 4 (a) $3.10\,\text{eV} = 3.10 \times 1.60 \times 10^{-19} = 4.96 \times 10^{-19}\,\text{J}$. (b) $E = hf = 6.63 \times 10^{-34} \times 5.50 \times 10^{14} = 3.65 \times 10^{-19}\,\text{J}$. In electron-volts, $E = \dfrac{3.65 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.28\,\text{eV}$. (c) $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}}{5.50 \times 10^{14}} = 5.45 \times 10^{-7}\,\text{m}$ (545 nm).

Question 5 (a) For $\lambda = 4.50 \times 10^{-7}\,\text{m}$: $E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{4.50 \times 10^{-7}} = 4.42 \times 10^{-19}\,\text{J}$. For $\lambda = 6.50 \times 10^{-7}\,\text{m}$: $E = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{6.50 \times 10^{-7}} = 3.06 \times 10^{-19}\,\text{J}$. (b) $\dfrac{4.42 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.76\,\text{eV}$ and $\dfrac{3.06 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.91\,\text{eV}$. (c) The $4.50 \times 10^{-7}\,\text{m}$ line carries more energy per photon. Since $E = \dfrac{hc}{\lambda}$, photon energy is inversely proportional to wavelength, so the **shorter** wavelength corresponds to the **higher** energy.

Question 6 (a) After only a few photons, the screen shows a small number of **scattered dots** with no obvious order — each photon lands at a single, apparently random point. After very many photons the dots pile up into a regular pattern of **bright and dark interference fringes**. (b) Each photon is detected at one localised point and delivers its energy all at once, which is **particle-like** behaviour. Yet the pattern that the dots build up is an **interference** pattern, with fringes whose spacing is set by the wavelength — behaviour that only a **wave** can produce. Neither model alone is enough: light shows a **dual** wave–particle nature. (c) When the experiment is repeated with electrons sent one at a time, exactly the same thing is observed: each electron arrives as a single dot, but the dots build up into an interference pattern. This shows that **matter**, like light, also has a wave-like nature and displays wave–particle duality.

Question 7 The electron falls from $n=4$ ($-0.80\,\text{eV}$) to $n=2$ ($-4.20\,\text{eV}$), so

$$\Delta E = -0.80 - (-4.20) = 3.40\,\text{eV} = 3.40 \times 1.60 \times 10^{-19} = 5.44 \times 10^{-19}\,\text{J}.$$

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{5.44 \times 10^{-19}} = 3.66 \times 10^{-7}\,\text{m}\,(366\,\text{nm}).$$

This is shorter than 400 nm, so it lies in the **ultraviolet**.

Question 8 The longest wavelength corresponds to the **smallest** energy gap, because $\lambda = \frac{hc}{\Delta E}$. The smallest gap between the four levels is between the two closest, $n = 4$ ($-0.80\,\mathrm{eV}$) and $n = 3$ ($-1.90\,\mathrm{eV}$):

$$\Delta E = -0.80 - (-1.90) = 1.10\,\mathrm{eV} = 1.76\times10^{-19}\,\mathrm{J},\ \lambda = \frac{6.63\times10^{-34}\times3.00\times10^{8}}{1.76\times10^{-19}} = 1.13\times10^{-6}\,\mathrm{m}.$$

∴ the $n = 4 \to n = 3$ transition gives the longest wavelength, $1.13\times10^{-6}\,\mathrm{m}$ ($1130\,\mathrm{nm}$), in the infrared.

Question 9 A visible photon has a wavelength between about $400\,\mathrm{nm}$ and $700\,\mathrm{nm}$, i.e. an energy between about $1.8\,\mathrm{eV}$ and $3.1\,\mathrm{eV}$. Checking the downward transitions: $n = 4 \to 3$ is $1.10\,\mathrm{eV}$ (infrared, too little energy); $n = 3 \to 2$ is $2.30\,\mathrm{eV}$ (visible); $n = 4 \to 2$ is $3.40\,\mathrm{eV}$, $n = 2 \to 1$ is $4.40\,\mathrm{eV}$ and all transitions ending on $n = 1$ are larger still (ultraviolet, too much energy). Only $n = 3 \to n = 2$ falls in the visible band. Its wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\times3.00\times10^{8}}{2.30\times1.60\times10^{-19}} = 5.40\times10^{-7}\,\mathrm{m}\,(540\,\mathrm{nm}),$$

which is **green**.

Question 10 The absorbed photon energy in electron-volts is

$$E = \frac{7.04\times10^{-19}}{1.60\times10^{-19}} = 4.40\,\mathrm{eV}.$$

Starting from the ground state at $-8.60\,\mathrm{eV}$, the electron is lifted to $-8.60 + 4.40 = -4.20\,\mathrm{eV}$, which is the $n = 2$ level.

∴ the electron is raised from the ground state to the $n = 2$ level.

Question 11 $E = hf = 6.63\times10^{-34}\times6.50\times10^{14} = 4.31\times10^{-19}\,\mathrm{J}$, so

$$E = \frac{4.31\times10^{-19}}{1.60\times10^{-19}} = 2.69\,\mathrm{eV},\ \lambda = \frac{c}{f} = \frac{3.00\times10^{8}}{6.50\times10^{14}} = 4.62\times10^{-7}\,\mathrm{m}\,(462\,\mathrm{nm}).$$

Question 12 Before quantum ideas, light was modelled as a continuous wave and an object was allowed any energy. This classical picture failed to explain several observations — most importantly the discrete line spectra of atoms and the photoelectric effect. Two connected ideas of **quantisation** resolved these failures.

First, the energy of **light** is quantised: light is emitted and absorbed in discrete packets, or photons, each of energy $E = hf$. This explained the photoelectric effect and the spectrum of a hot body, which the wave model could not.

Second, the energy of the **electrons in atoms** is quantised: an electron may occupy only certain allowed energy levels. This explained why atoms are stable and why they emit and absorb only at sharp, characteristic wavelengths — each spectral line corresponds to a transition between two allowed levels, with $E_{\text{photon}} = E_{\text{upper}} - E_{\text{lower}}$.

Together these ideas unified the treatment of light and matter: light, once purely a wave, is also particle-like; matter, once purely particles, has an energy-level structure and (as the double-slit experiments show) wave-like behaviour. Quantisation is therefore the foundation on which the modern understanding of both light and the nature of atoms — and the whole of quantum physics — is built.

Question 13 The reasoning runs as a chain: (1) When atoms are excited, they emit light only at a set of sharp, separate wavelengths — a line spectrum, not a continuous band. (2) A definite wavelength means a definite photon energy, since $E = \frac{hc}{\lambda}$; so the atom emits photons of only a few definite energies. (3) By $E_{\text{photon}} = E_{\text{upper}} - E_{\text{lower}}$, a fixed set of photon energies means a fixed set of energy *differences* between the atom's states. (4) Fixed energy differences are only possible if the electron is restricted to a fixed set of allowed energy levels — that is, if its energy is quantised. If the electron could have any energy, all energy differences would be possible and the spectrum would be a continuous smear. The discrete lines therefore show the energy is quantised.

Question 14 Each element has its own unique arrangement of electron energy levels, which no other element shares. The set of possible transitions between those levels — and hence the set of photon energies $E_{\text{upper}} - E_{\text{lower}}$, frequencies and wavelengths — is therefore unique to that element. The spectrum acts as a "fingerprint" that identifies the element. This is used, for example, to identify the elements present in a gas or flame from its emission spectrum, and to determine the chemical composition of stars from the dark absorption lines in their light.

Question 15 The white light contains photons of every visible energy (a continuous spectrum). As it passes through the cool gas, a photon is absorbed **only** if its energy exactly matches a gap between two of the gas atoms' energy levels; such a photon lifts an electron from a lower to a higher level. Photons of those particular energies are therefore removed from the beam, so the wavelengths $\lambda = hc/\Delta E$ corresponding to the atom's transitions are missing from the transmitted light and appear as **dark lines** against the continuous background. (The absorbed energy is quickly re-emitted, but in all directions and not just forward, so very little of it returns to the original beam — the lines stay dark.)

Question 16 The reasoning is wrong because a single photon does not need a *second* photon to interfere with; in quantum physics each photon is described by a wave (a probability amplitude) that passes through **both** slits and interferes **with itself**. What is actually observed is that each photon is still detected as a single dot at one point, but the *positions* at which the dots land are governed by this interference: over many photons the dots accumulate into the same bright-and-dark interference fringes seen with an intense beam. The interference pattern builds up one photon at a time, so sending photons individually does **not** destroy it.

Question 17 The electrons build up an **interference pattern** — a series of bright and dark bands — even though each electron arrives as a single localised dot and they are sent one at a time. Treated purely as particles, the electrons would be expected to pass through one slit or the other and form just two bands lined up behind the slits, with no fringes. The appearance of an interference pattern, which is a wave phenomenon, shows that electrons — and matter in general — have a **wave-like** nature as well as a particle-like one; that is, matter exhibits wave–particle duality.

Question 18 Both lamps contain an excited low-pressure vapour of a metal, and each emits light only from the downward transitions between the energy levels of **its own** atoms. Sodium and mercury have different energy-level structures, so they have different sets of transition energies and therefore emit different sets of wavelengths. Sodium's strongest transitions emit in the yellow-orange, so a sodium lamp looks yellow-orange; mercury's strong lines lie in the blue and green (and beyond), so their mixture looks blue-white. The characteristic colour of each lamp is thus set by the particular photon energies $E = hf = E_{\text{upper}} - E_{\text{lower}}$ that its atoms are able to emit.

Question 19 (a) $E = \dfrac{4.80 \times 10^{-19}}{1.60 \times 10^{-19}} = 3.00\,\text{eV}$. (b) $\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{4.80 \times 10^{-19}} = 4.14 \times 10^{-7}\,\text{m}$ ($414\,\text{nm}$). Since $414\,\text{nm}$ lies between about $400\,\text{nm}$ and $700\,\text{nm}$, it **is** visible — in the violet.

Question 20 (a) The absorbed photon lifts the electron from the ground state ($-6.80\,\text{eV}$) to the $-0.90\,\text{eV}$ level: $\Delta E = -0.90 - (-6.80) = 5.90\,\text{eV} = 9.44 \times 10^{-19}\,\text{J}$.

$$\lambda_{\text{abs}} = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{9.44 \times 10^{-19}} = 2.11 \times 10^{-7}\,\text{m}\,(211\,\text{nm}).$$

(b) First emission, $-0.90\,\text{eV} \to -3.60\,\text{eV}$: $\Delta E = 2.70\,\text{eV} = 4.32 \times 10^{-19}\,\text{J}$, so

$$\lambda_1 = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{4.32 \times 10^{-19}} = 4.60 \times 10^{-7}\,\text{m}\,(460\,\text{nm}).$$

Second emission, $-3.60\,\text{eV} \to -6.80\,\text{eV}$: $\Delta E = 3.20\,\text{eV} = 5.12 \times 10^{-19}\,\text{J}$, so

$$\lambda_2 = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{5.12 \times 10^{-19}} = 3.88 \times 10^{-7}\,\text{m}\,(388\,\text{nm}).$$

(c) The absorbed photon carries $5.90\,\text{eV}$; the two emitted photons carry $2.70\,\text{eV} + 3.20\,\text{eV} = 5.90\,\text{eV}$ in total. The energy taken in when the electron is raised equals the total energy released as it returns to the ground state.

∴ the absorbed photon energy equals the sum of the two emitted photon energies, illustrating **conservation of energy**.

Topic 4 Test A — Light and matter (multiple choice)

Time allowed: 25 minutes · 20 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used.

Choose the response that is correct or that best answers the question. A correct answer scores 1 mark; an incorrect answer scores 0. Marks are not deducted for incorrect answers. No marks are given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this test are not drawn to scale. The constants $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s, $c = 3.00 \times 10^{8}$ m s^{-1}, $m_e = 9.11 \times 10^{-31}$ kg *and* $q_e = 1.60 \times 10^{-19}$ C *are provided on the formula sheet.*

Question 1

Which one of the following best describes how an electromagnetic wave is produced and how it travels through a vacuum?

A. An accelerating charge produces a changing electric field and an associated changing magnetic field, which propagate together at c.

B. A stationary charge produces a steady electric field that spreads outward at c.

C. A charge moving at constant velocity radiates a wave whose speed depends on its frequency.

D. Only charges oscillating at radio frequencies radiate; higher frequencies travel faster.

Question 2

A microwave source in a vacuum emits radiation of wavelength 3.0 cm. Which one of the following is closest to the frequency of the radiation?

A. 9.0×10^{6} Hz

B. 1.0×10^{10} Hz

C. 1.0×10^{8} Hz

D. 1.0×10^{11} Hz

Question 3

A string 0.80 m long is fixed at both ends and vibrates in its third harmonic. Waves travel along the string at 240 m s^{-1}. Which one of the following is closest to the frequency of vibration?

A. 150 Hz

B. 300 Hz

C. 450 Hz

D. 900 Hz

Question 4

In a double-slit experiment, light of wavelength $640\,\text{nm}$ illuminates two slits $0.25\,\text{mm}$ apart. The interference pattern is observed on a screen $2.0\,\text{m}$ from the slits. Which one of the following is closest to the spacing Δx between adjacent bright fringes?

A. $5.12 \times 10^{-6}\,\text{m}$

B. $2.56 \times 10^{-3}\,\text{m}$

C. $1.02 \times 10^{-2}\,\text{m}$

D. $5.12 \times 10^{-3}\,\text{m}$

Question 5

A beam of waves passes through a gap. Which one of the following best describes when the diffraction (directional spreading) of the beam is most significant?

A. When the wavelength is much smaller than the gap width.

B. When the wavelength is of about the same order of magnitude as the gap width.

C. When the wavelength is much larger than the gap width, so that no waves pass through.

D. Diffraction is the same for every ratio of wavelength to gap width.

Question 6

Light of wavelength $5.30 \times 10^{-7}\,\text{m}$ is incident on a metal surface. Which one of the following is closest to the energy of a single photon of this light?

A. $1.17\,\text{eV}$

B. $3.75\,\text{eV}$

C. $2.35\,\text{eV}$

D. $4.69\,\text{eV}$

Question 7

A metal has a work function of $2.20\,\text{eV}$. Light of frequency $8.0 \times 10^{14}\,\text{Hz}$ strikes the metal. Which one of the following is closest to the maximum kinetic energy of the emitted photoelectrons?

A. $5.51\,\text{eV}$

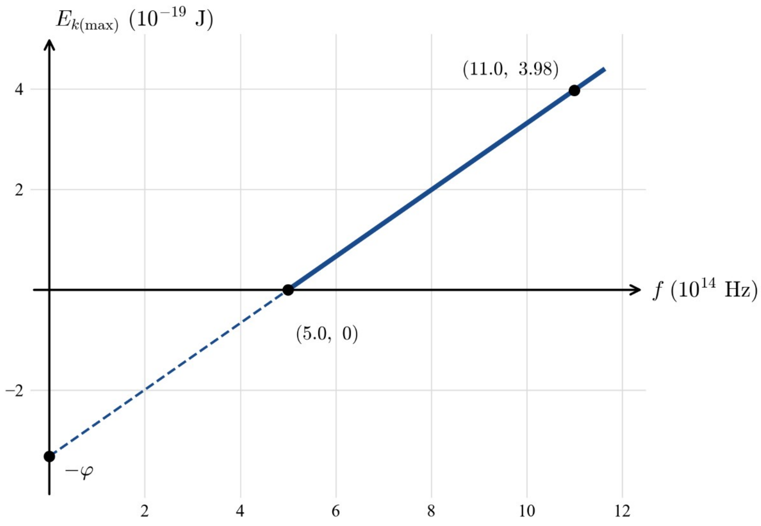
B. $3.31\,\text{eV}$

C. $2.20\,\text{eV}$

D. $1.11\,\text{eV}$

Question 8

The graph shows the maximum kinetic energy of photoelectrons emitted from a metal as a function of the frequency of the incident light.



Which one of the following is closest to the work function of the metal?

A. 3.32×10^{-19} J

B. 3.98×10^{-19} J

C. 7.29×10^{-19} J

D. 6.63×10^{-34} J

Question 9

Monochromatic light causes photoelectrons to be emitted from a metal surface. The intensity of the light is then increased while its frequency is kept constant. Which one of the following best describes the effect on the emitted photoelectrons?

A. Their maximum kinetic energy increases.

B. The stopping voltage increases.

C. The number emitted per second increases, but their maximum kinetic energy is unchanged.

D. Both the number emitted per second and their maximum kinetic energy increase.

Question 10

Which one of the following observations of the photoelectric effect cannot be explained by the wave model of light?

A. The photocurrent increases when the intensity of the light is increased.

B. Brighter light of the same colour produces a larger saturation current.

C. The photoelectrons leave the surface in a range of directions.

D. No electrons are emitted when the frequency is below a threshold value, however intense the light.

Question 11

An electron moves at $2.5 \times 10^{6}\,\mathrm{m\ s^{-1}}$. Which one of the following is closest to its de Broglie wavelength?

A. $2.91 \times 10^{-10}\,\mathrm{m}$

B. $1.59 \times 10^{-13}\,\mathrm{m}$

C. $5.82 \times 10^{-10}\,\mathrm{m}$

D. $1.46 \times 10^{-10}\,\mathrm{m}$

Question 12

A photon and an electron each have a wavelength of $5.0 \times 10^{-11}\,\mathrm{m}$. Which one of the following is closest to the magnitude of the momentum of the electron?

A. $2.65 \times 10^{-23}\,\mathrm{kg\ m\ s^{-1}}$

B. $1.33 \times 10^{-23}\,\mathrm{kg\ m\ s^{-1}}$

C. $3.32 \times 10^{-44}\,\mathrm{kg\ m\ s^{-1}}$

D. $6.63 \times 10^{-24}\,\mathrm{kg\ m\ s^{-1}}$

Question 13

An electron is accelerated from rest through a potential difference of $120\,\mathrm{V}$. Which one of the following is closest to its de Broglie wavelength?

A. $1.59 \times 10^{-10}\,\mathrm{m}$

B. $2.62 \times 10^{-12}\,\mathrm{m}$

C. $7.93 \times 10^{-11}\,\mathrm{m}$

D. $1.12 \times 10^{-10}\,\mathrm{m}$

Question 14

A beam of electrons passing through a thin crystal produces a pattern of concentric rings on a screen. Which one of the following is the best interpretation of this observation?

A. The electrons behave as waves; the rings are produced by diffraction.

B. The electrons collide with atoms in the crystal and scatter into rings.

C. The electrons repel one another, spreading the beam into rings.

D. The crystal fluoresces in rings where the electrons strike it.

Question 15

A beam of electrons and a beam of photons are found to produce diffraction ring patterns of exactly the same size when directed at identical crystals. Which one of the following must be true?

A. The electrons and photons have the same speed.

B. The electrons and photons have the same wavelength.

C. The electrons and photons have the same kinetic energy.

D. The electrons and photons have the same mass.

Question 16

An atom emits a photon when its electron moves from an energy level at $-1.5\,\text{eV}$ to a level at $-3.9\,\text{eV}$. Which one of the following is closest to the wavelength of the emitted photon?

A. $8.29\times10^{-7}\,\text{m}$

B. $3.19\times10^{-7}\,\text{m}$

C. $5.18\times10^{-7}\,\text{m}$

D. $2.30\times10^{-7}\,\text{m}$

Question 17

A cool gas is placed between a source of continuous (white) light and a spectrometer. Which one of the following best describes the spectrum that is observed?

A. A continuous coloured spectrum crossed by dark lines at particular wavelengths.

B. A set of bright coloured lines on a dark background.

C. A continuous coloured spectrum with no lines.

D. A completely dark spectrum, because the gas absorbs all of the light.

Question 18

The energy levels of an atom are $-7.0\,\text{eV}$, $-4.0\,\text{eV}$ and $-1.4\,\text{eV}$. Which one of the following is closest to the longest wavelength of light that the atom can emit in a transition between two of these levels?

A. $2.22\times10^{-7}\,\text{m}$

B. $4.78\times10^{-7}\,\text{m}$

C. $4.14\times10^{-7}\,\text{m}$

D. $7.65\times10^{-26}\,\text{m}$

Question 19

A metal vapour lamp emits a spectral line of wavelength $5.90\times10^{-7}\,\text{m}$. Which one of the following is closest to the energy of the atomic transition responsible for this line?

A. $4.21\,\text{eV}$

B. $3.37\,\text{eV}$

C. $2.11\,\text{eV}$

D. $1.05\,\text{eV}$

Question 20

In a double-slit experiment, single photons are sent through the apparatus one at a time and are detected on a screen. After many photons, an interference pattern gradually builds up from individual spots. Which one of the following is the best interpretation of this result?

A. The photons collide with one another on the way to the screen to form the pattern.

B. Light behaves purely as a wave, so no individual spots should appear.

C. The result proves that light is only a wave and never a particle.

D. Each photon arrives as a localised spot, yet the accumulated pattern shows interference — evidence for the dual nature of light.

End of Topic 4 Test A

Topic 4 Test A — solutions

Question	Answer	Question	Answer
1	A	11	A
2	B	12	B
3	C	13	D
4	D	14	A
5	B	15	B
6	C	16	C
7	D	17	A
8	A	18	B
9	C	19	C
10	D	20	D

Question 1 — A. An electromagnetic wave originates from an *accelerating* charge, which sets up a changing electric field that in turn produces a changing magnetic field; the two fields regenerate each other and propagate together at c. **B** (stationary charge) involves no acceleration and no wave; **C** wrongly ties the speed to the frequency (all EM waves travel at c in a vacuum); **D** repeats the speed-depends-on-frequency error and wrongly restricts radiation to radio frequencies.

Question 2 — B. $f=\frac{c}{\lambda}=\frac{3.00\times10^{8}}{0.030}=1.0\times10^{10}\,\text{Hz}$. **A** ($9.0\times10^{6}$) multiplies c by λ instead of dividing; **C** (1.0×10^{8}) leaves the wavelength in centimetres; **D** (1.0×10^{11}) reads the wavelength as $3.0\,\text{mm}$.

Question 3 — C. With nodes at both ends the third harmonic has $\lambda=\frac{2L}{3}=\frac{2(0.80)}{3}=0.533\,\text{m}$, so $f=\frac{v}{\lambda}=\frac{240}{0.533}=450\,\text{Hz}$. **A** (150) is the fundamental ($n=1$); **B** (300) takes $\lambda=L$; **D** (900) uses $f=\frac{nv}{L}$, dropping the factor of 2.

Question 4 — D. $\Delta x=\frac{\lambda L}{d}=\frac{(640\times10^{-9})(2.0)}{0.25\times10^{-3}}=5.12\times10^{-3}\,\text{m}$. **A** ($5.12\times10^{-6}$) leaves the slit separation in metres as 0.25 (the classic $\times10^{3}$ unit slip); **B** (2.56×10^{-3}) divides by $2d$; **C** (1.02×10^{-2}) uses $2d$ the other way (doubles the result).

Question 5 — B. Diffraction is most pronounced when the wavelength and the gap width are of the same order of magnitude ($\lambda/w\approx1$). **A** is the common misconception (no diffraction unless $\lambda<w$); **C** is false (waves still pass and spread); **D** ignores the λ/w dependence entirely.

Question 6 — C. $E=\frac{hc}{\lambda}=\frac{(6.63\times10^{-34})(3.00\times10^{8})}{5.30\times10^{-7}}=3.75\times10^{-19}\,\text{J}=2.35\,\text{eV}$. **B** (3.75) reads the joule mantissa (3.75×10^{-19} J) as electronvolts; **A** (1.17) uses twice the wavelength; **D** (4.69) uses half the wavelength.

Question 7 — D. $E_{\text{photon}}=hf=(4.14\times10^{-15})(8.0\times10^{14})=3.31\,\text{eV}$, so $E_{k(\max)}=hf-\varphi=3.31-2.20=1.11\,\text{eV}$. **A** (5.51) adds φ instead of subtracting; **B** (3.31) forgets to subtract φ; **C** (2.20) simply quotes the work function.

Question 8 — A. The line is $E_{k(\max)}=hf-\varphi$, so the magnitude of the (negative) vertical intercept is the work function, $\varphi=hf_0=(6.63\times10^{-34})(5.0\times10^{14})=3.32\times10^{-19}$ J (using the threshold $f_0=5.0\times10^{14}\,\text{Hz}$ read from the horizontal intercept). **B** (3.98×10^{-19}) mistakes the plotted E_k at the upper point for φ; **C** (7.29×10^{-19}) uses the upper frequency $11\times10^{14}\,\text{Hz}$ as the threshold; **D** (6.63×10^{-34}) quotes the gradient (Planck’s constant), which has the wrong quantity and units.

Question 9 — C. At fixed frequency each photon still carries the same energy, so $E_{k(\max)} = hf - \varphi$ (and hence the stopping voltage) is unchanged; more intense light simply delivers more photons per second, freeing more electrons per second. **A** and **B** wrongly tie the maximum kinetic energy / stopping voltage to intensity; **D** combines both errors.

Question 10 — D. The existence of a threshold frequency — below which no electrons are emitted no matter how bright the light — cannot be explained if light is a wave delivering energy continuously; it requires the photon model ($E = hf$). **A**, **B** and **C** are all consistent with a wave picture (more energy or a spread of ejection directions), so they do not defeat the wave model.

Question 11 — A. $\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(2.5 \times 10^6)} = 2.91 \times 10^{-10}\,\mathrm{m}$. **B** ($1.59 \times 10^{-13}$) uses the proton mass; **C** (5.82×10^{-10}) uses $p = 1/2\,mv$ (a kinetic-energy confusion); **D** (1.46×10^{-10}) uses $p = 2mv$.

Question 12 — B. $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.0 \times 10^{-11}} = 1.33 \times 10^{-23}\,\mathrm{kg\ m\ s^{-1}}$ — and this is the same for the photon, since $p = h/\lambda$ depends only on wavelength. **A** (2.65×10^{-23}) halves the wavelength; **C** (3.32×10^{-44}) multiplies h by λ; **D** (6.63×10^{-24}) doubles the wavelength.

Question 13 — D. $1/2\,m_e v^2 = q_e V$ gives $v = \sqrt{\frac{2 q_e V}{m_e}} = 6.49 \times 10^6\,\mathrm{m\ s^{-1}}$, so $\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(6.49 \times 10^6)} = 1.12 \times 10^{-10}\,\mathrm{m}$. **A** ($1.59 \times 10^{-10}$) drops the factor 2 in $1/2\,m_e v^2 = q_e V$; **B** (2.62×10^{-12}) uses the proton mass; **C** (7.93×10^{-11}) uses twice the accelerating voltage.

Question 14 — A. Diffraction is a wave phenomenon, so a ring diffraction pattern is direct evidence that the electrons behave as waves (with $\lambda = h/p$ comparable to the atomic spacing). **B** (collisions), **C** (mutual repulsion) and **D** (fluorescence) are all particle-scattering or emission pictures that cannot produce an ordered interference pattern.

Question 15 — B. The size of a diffraction pattern is set by the wavelength (relative to the spacing), and for both photons and electrons $p = h/\lambda$; identical patterns from identical crystals therefore require identical wavelengths. **A**, **C** and **D** are not implied — a photon and an electron of equal wavelength have very different speeds and kinetic energies, and a photon has no mass.

Question 16 — C. $\Delta E = 3.9 - 1.5 = 2.4\,\mathrm{eV} = 3.84 \times 10^{-19}\,\mathrm{J}$, so $\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{3.84 \times 10^{-19}} = 5.18 \times 10^{-7}\,\mathrm{m}$. **A** ($8.29 \times 10^{-7}$) uses the upper level ($1.5\,\mathrm{eV}$) alone; **B** (3.19×10^{-7}) uses the lower level ($3.9\,\mathrm{eV}$) alone; **D** (2.30×10^{-7}) adds the two level energies ($5.4\,\mathrm{eV}$).

Question 17 — A. The cool gas absorbs photons at exactly the wavelengths its atoms can absorb, removing them from the continuous background — an absorption spectrum: a continuous coloured band crossed by dark lines. **B** describes an *emission* spectrum (hot gas alone); **C** ignores the absorption; **D** wrongly assumes total absorption.

Question 18 — B. The longest wavelength comes from the *smallest* energy gap. The gaps are $2.6\,\mathrm{eV}$ (-1.4 to -4.0), $3.0\,\mathrm{eV}$ (-4.0 to -7.0) and $5.6\,\mathrm{eV}$ (-1.4 to -7.0); the smallest is $2.6\,\mathrm{eV}$, giving $\lambda = \frac{hc}{\Delta E} = \frac{1.989 \times 10^{-25}}{4.16 \times 10^{-19}} = 4.78 \times 10^{-7}\,\mathrm{m}$. **A** ($2.22 \times 10^{-7}$) uses the largest gap ($5.6\,\mathrm{eV}$); **C** ($4.14 \times 10^{-7}$) uses the middle gap ($3.0\,\mathrm{eV}$); **D** ($7.65 \times 10^{-26}$) forgets to convert the $2.6\,\mathrm{eV}$ gap to joules.

Question 19 — C. $E = \frac{hc}{\lambda} = \frac{1.989 \times 10^{-25}}{5.90 \times 10^{-7}} = 3.37 \times 10^{-19}\,\mathrm{J} = 2.11\,\mathrm{eV}$. **B** ($3.37$) reads the joule mantissa as electronvolts; **A** (4.21) uses half the wavelength; **D** (1.05) uses twice the wavelength.

Question 20 — D. Each photon is detected as a single localised spot (particle-like), yet the pattern that builds up from many independent single-photon arrivals shows interference (wave-like) — the signature of wave–particle duality. **A**

is impossible (the photons pass one at a time); **B** and **C** ignore either the discrete spots or the interference and so describe only half of the behaviour.

Topic 4 Test B — Light and matter (written responses)

Time allowed: 50 minutes · 40 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used. In questions where more than one mark is available, appropriate working must be shown.

Answer all questions in the spaces provided. Where a final answer has a required unit, give your answer in that unit. Unless otherwise indicated, the diagrams in this test are not drawn to scale.

Question 1 (4 marks)

A string is stretched between two fixed supports and is made to vibrate so that a standing wave forms along it.

a. Explain how a standing wave is formed on a string that is fixed at both ends. 2 marks

b. A string of length $0.75\,\mathrm{m}$, fixed at both ends, vibrates in its third harmonic. Waves travel along the string at $200\,\mathrm{m\ s^{-1}}$. Calculate the frequency of vibration. Give your answer in Hz. 2 marks

Question 2 (6 marks)

a. State the condition, in terms of the wavelength λ and the gap width w, under which the diffraction of a wave is most significant. 1 mark

b. Radio waves of wavelength $3.0\,\mathrm{m}$ and green light of wavelength $5.5\times10^{-7}\,\mathrm{m}$ each pass through a doorway $0.90\,\mathrm{m}$ wide. Explain, with reference to the λ/w ratio, which of the two is diffracted significantly on passing through the doorway and which is not. 2 marks

In a separate experiment, coherent light of wavelength $6.0\times10^{-7}\,\mathrm{m}$ illuminates two narrow slits $0.30\,\mathrm{mm}$ apart. An interference pattern is observed on a screen $1.5\,\mathrm{m}$ from the slits.

c. Show that the spacing between adjacent bright fringes is $3.0\times10^{-3}\,\mathrm{m}$. 1 mark

d. The two slits are then moved closer together. State and explain the effect this has on the fringe spacing. 2 marks

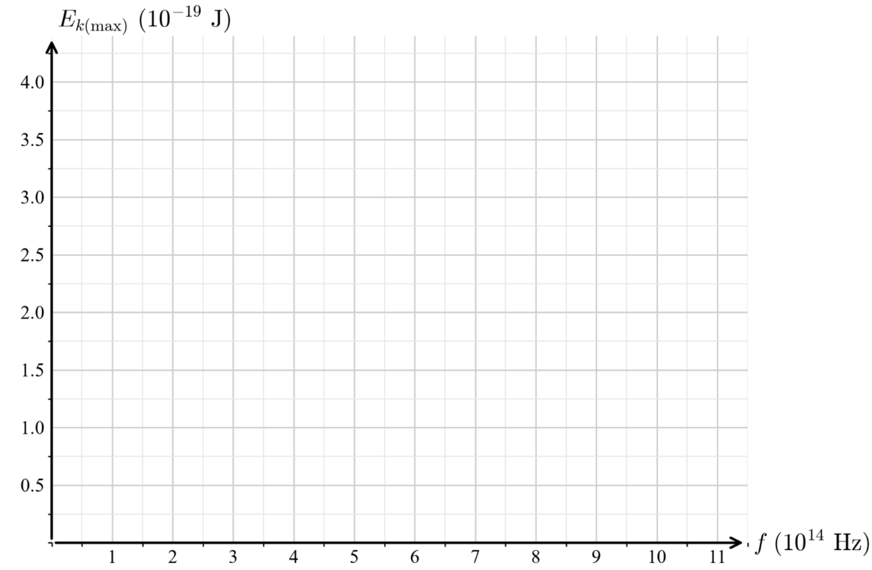
Question 3 (7 marks)

A student shines light of six different frequencies on a metal surface and, for each frequency, measures the maximum kinetic energy $E_{k(\max)}$ of the emitted photoelectrons. The results are shown in the table; each energy has an uncertainty of $\pm0.10\times10^{-19}\,\mathrm{J}$.

$f\,(10^{14}\,\mathrm{Hz})$	5.0	6.0	7.0	8.0	9.0	10.0
$E_{k(\max)}\,(10^{-19}\,\mathrm{J}$	0.40	1.06	1.72	2.39	3.05	3.71

a. State the independent variable and the dependent variable in this investigation. 1 mark

b. On the grid provided, plot $E_{k(\max)}$ against f, including an uncertainty bar on each point, and draw a line of best fit. 3 marks



c. Using two well-separated points on your line of best fit, show that the gradient of the line is approximately 6.6×10^{-34} J s, and state the physical quantity that this gradient represents. 2 marks

d. Use your line of best fit to estimate the threshold frequency f_0 of the metal. Give your answer in Hz. 1 mark

Question 4 (3 marks)

A clean metal surface has a work function of $2.10\,\text{eV}$. Light of wavelength $4.20 \times 10^{-7}\,\text{m}$ is incident on the surface.

a. Calculate the maximum kinetic energy of the emitted photoelectrons. Give your answer in eV. 2 marks

b. The intensity of the incident light is now doubled, with its wavelength unchanged. Using the photon model, explain the effect on the maximum kinetic energy of the emitted photoelectrons. 1 mark

Question 5 (6 marks)

An electron is accelerated from rest through a potential difference of $200\,\text{V}$ in an electron-diffraction tube.

a. Show that the electron reaches a speed of $8.4 \times 10^{6}\,\text{m s}^{-1}$. 2 marks

b. Calculate the de Broglie wavelength of the electron. Give your answer in m. 2 marks

c. The accelerating voltage is then increased. State and justify the effect this has on the de Broglie wavelength of the electron. 1 mark

d. State one experimental observation, in this tube, that would show that the electrons have wave-like properties. 1 mark

Question 6 (4 marks)

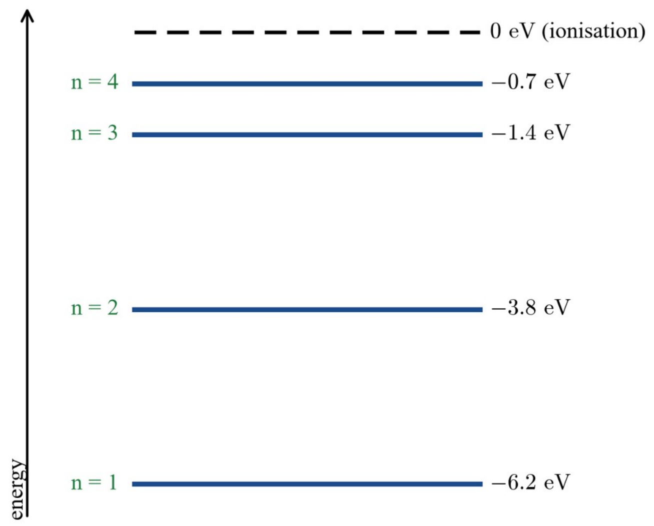
A beam of electrons is directed at a thin crystal and produces a pattern of concentric rings on a screen behind it.

a. Explain how this ring pattern provides evidence that matter has a wave-like nature. 2 marks

b. A beam of X-rays produces a ring pattern of exactly the same size when directed at the same crystal, corresponding to a wavelength of $1.2 \times 10^{-10}\,\text{m}$. Calculate the momentum of the electrons and state how it compares with the momentum of the X-ray photons. Give your answer in kg m s^{-1}. 2 marks

Question 7 (6 marks)

The diagram shows four of the energy levels of an isolated atom, together with the ionisation level at $0\,\text{eV}$.



a. State whether a photon is absorbed or emitted when the electron moves from the $n=1$ level to the $n=3$ level, and calculate the energy of this photon. Give your answer in eV. 2 marks

b. The atom emits a photon when an electron falls from the $n=4$ level to the $n=2$ level. Calculate the wavelength of the emitted photon, and on the diagram draw an arrow representing this transition. Give your answer in m. 2 marks

c. (Circle your answer.) Compared with the photon emitted in the $n=3 \rightarrow n=1$ transition, the photon emitted in the $n=2 \rightarrow n=1$ transition has a wavelength that is: *shorter* / *the same* / *longer*. Justify your answer with reference to the energy-level diagram. No calculations are required. 2 marks

Question 8 (4 marks)

a. Explain, with reference to the quantised energy levels of atoms, why each chemical element produces a line spectrum that is unique to that element. 2 marks

b. Distinguish between the appearance of an emission line spectrum and an absorption line spectrum. 1 mark

c. Electrons are sent one at a time through a double-slit apparatus and, after many electrons, build up an interference pattern from individual detected spots. State what this reveals about the nature of electrons. 1 mark

End of Topic 4 Test B

Topic 4 Test B — solutions

Question 1

(a) A standing wave forms from the **superposition of two travelling waves of the same frequency and amplitude moving in opposite directions** — here the wave sent along the string and the wave reflected back from the fixed end. Where the two waves are always in phase they reinforce (antinodes); where they are always out of phase they cancel (nodes). Because the ends are fixed they cannot move, so a **node occurs at each end**, and only whole numbers of half-wavelengths fit on the string. **(1 mark** superposition of a travelling wave and its reflection; **1 mark** fixed ends force a node at each end / nodes and antinodes at fixed positions.)

(b) With nodes at both ends the third harmonic ($n=3$) has $\lambda=\frac{2L}{n}=\frac{2(0.75)}{3}=0.50\,\text{m}$. Then

$$f=\frac{v}{\lambda}=\frac{200}{0.50}=400\,\text{Hz}.$$

∴ the string vibrates at $400\,\text{Hz}$. **(1 mark** $\lambda=2L/3=0.50\,\text{m}$; **1 mark** $f=v/\lambda=400\,\text{Hz}$.)

Question 2

(a) Diffraction is most significant when the **wavelength is of the same order of magnitude as the gap width**, i.e. $\lambda/w\approx 1$ (equivalently $\lambda\approx w$). **(1 mark.)**

(b) For the radio waves, $\frac{\lambda}{w}=\frac{3.0}{0.90}\approx 3.3$ — of order 1 (indeed larger), so the radio waves **diffract significantly** and spread out through the doorway. For the green light, $\frac{\lambda}{w}=\frac{5.5\times10^{-7}}{0.90}\approx 6\times10^{-7}$ — far smaller than 1, so the light is **not** diffracted appreciably and passes essentially straight through. **(1 mark** radio $\lambda/w\approx 1$ (or larger), so significant diffraction; **1 mark** light $\lambda/w \ll 1$, so negligible diffraction.)

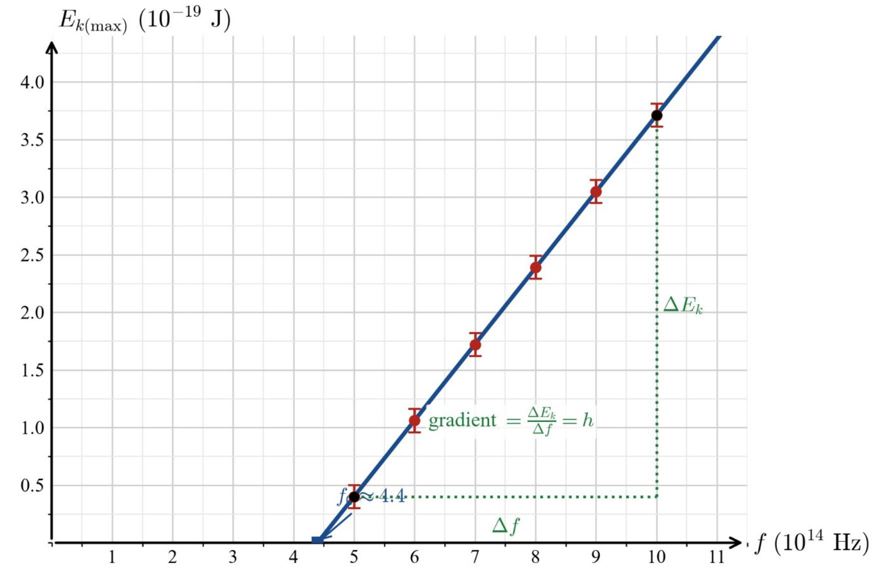
(c) $\Delta x=\frac{\lambda L}{d}=\frac{(6.0\times10^{-7})(1.5)}{0.30\times10^{-3}}=\frac{9.0\times10^{-7}}{3.0\times10^{-4}}=3.0\times10^{-3}\,\text{m}$, as required (with $d=0.30\,\text{mm}=3.0\times10^{-4}\,\text{m}$).
(1 mark: correct substitution, including the mm-to-m conversion, and result.)

(d) The **fringe spacing increases**. From $\Delta x=\frac{\lambda L}{d}$, with λ and L fixed, $\Delta x\propto\frac{1}{d}$; reducing the slit separation d therefore increases Δx, so the bright fringes move **farther apart**. **(1 mark** fringe spacing increases; **1 mark** justified by $\Delta x\propto 1/d$.)

Question 3

(a) The **independent variable** is the frequency f of the incident light (the quantity the student deliberately sets); the **dependent variable** is the maximum kinetic energy $E_{k(\max)}$ of the photoelectrons (the quantity measured in response). **(1 mark: both correct.)**

(b) The completed plot is shown below: the six points with $\pm 0.10\times10^{-19}\,\text{J}$ uncertainty bars and a straight line of best fit that balances the points.



(1 mark points correctly plotted; **1 mark** uncertainty bars drawn from the given $\pm 0.10 \times 10^{-19}$ J; **1 mark** a single straight line of best fit that weighs all points, not forced through the first and last data points.)

(c) Reading two well-separated points **on the line of best fit** — for example $(5.0 \times 10^{14}\,\text{Hz}, 0.40 \times 10^{-19}\,\text{J})$ and $(10.0 \times 10^{14}\,\text{Hz}, 3.71 \times 10^{-19}\,\text{J})$:

$$\text{gradient} = \frac{\Delta E_{k(\max)}}{\Delta f} = \frac{(3.71 - 0.40) \times 10^{-19}}{(10.0 - 5.0) \times 10^{14}} = \frac{3.31 \times 10^{-19}}{5.0 \times 10^{14}} = 6.6 \times 10^{-34}\,\text{J s},$$

as required. Since $E_{k(\max)} = hf - \varphi$, the gradient is **Planck's constant** h. **(1 mark** correct gradient $\approx 6.6 \times 10^{-34}$ J s from two on-line, well-separated points; **1 mark** identifies the gradient as Planck's constant h.)

(d) The threshold frequency is the horizontal-axis intercept of the line, where $E_{k(\max)} = 0$. Extrapolating the line of best fit back gives

$$f_0 \approx 4.4 \times 10^{14}\,\text{Hz}.$$

∴ below about 4.4×10^{14} Hz no photoelectrons are emitted. (As a check, $\varphi = hf_0 = (6.63 \times 10^{-34})(4.4 \times 10^{14}) = 2.9 \times 10^{-19}\,\text{J} \approx 1.8\,\text{eV}$.) **(1 mark: $f_0 \approx 4.4 \times 10^{14}$ Hz read as the f-axis intercept.)**

Question 4

(a) The photon energy is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{4.20 \times 10^{-7}} = 4.74 \times 10^{-19}\,\text{J} = \frac{4.74 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.96\,\text{eV}.$$

Then $E_{k(\max)} = hf - \varphi = 2.96 - 2.10 = 0.86\,\text{eV}$. **(1 mark** photon energy 2.96 eV (or 4.74×10^{-19} J); **1 mark** $E_{k(\max)} = 0.86\,\text{eV}$.)

(b) There is **no change** to the maximum kinetic energy. In the photon model each photoelectron is freed by a **single photon**, and $E_{k(\max)} = hf - \varphi$ depends only on the photon energy (the frequency/wavelength) and the work function. Doubling the intensity at the same wavelength doubles the **number** of photons arriving per second (so more electrons are emitted per second), but each photon still carries the same energy, so the maximum kinetic energy is unchanged. **(1 mark: maximum kinetic energy unchanged because it depends on photon energy, not intensity — a one-photon-per-electron argument.)**

Question 5

(a) All the electrical work becomes kinetic energy: $q_e V = 1/2\, m_e v^2$, so

$$v=\sqrt{\frac{2q_e V}{m_e}}=\sqrt{\frac{2(1.60\times10^{-19})(200)}{9.11\times10^{-31}}}=\sqrt{7.03\times10^{13}}=8.4\times10^{6}\,\text{m s}^{-1},$$

as required. **(1 mark** $q_e V = 1/2\, m_e v^2$; **1 mark** correct value $8.4\times10^{6}\,\text{m s}^{-1}$.)

(b) $p=m_e v=(9.11\times10^{-31})(8.38\times10^{6})=7.64\times10^{-24}\,\text{kg m s}^{-1}$, so

$$\lambda=\frac{h}{p}=\frac{6.63\times10^{-34}}{7.64\times10^{-24}}=8.68\times10^{-11}\,\text{m}.$$

∴ the de Broglie wavelength is $8.68\times10^{-11}\,\text{m}$ (comparable to atomic spacings, which is why diffraction is observed). **(1 mark** $\lambda=h/(m_e v)$; **1 mark** correct value $8.68\times10^{-11}\,\text{m}$.)

(c) The de Broglie wavelength **decreases**. A larger accelerating voltage gives the electron more kinetic energy and hence greater momentum p; since $\lambda=h/p$, increasing p decreases λ. **(1 mark: λ decreases, justified by $\lambda=h/p$ with p increasing.)**

(d) The electrons produce a **diffraction pattern** (concentric rings, or an interference pattern) after passing through the thin crystal — a wave behaviour that particles alone cannot produce. **(1 mark: electron diffraction / interference pattern observed.)**

Question 6

(a) Diffraction (the spreading and formation of a regular ring/interference pattern) is a **wave** phenomenon. The electrons produce such a pattern only because each electron has an associated **de Broglie wavelength** $\lambda=h/p$ that is comparable to the spacing between atoms in the crystal, so the crystal acts as a diffraction grating for the electron waves. A stream of classical particles would simply pass through or scatter randomly and could not build an ordered ring pattern; the pattern is therefore evidence that matter has a wave-like nature. **(1 mark** diffraction/interference is a wave property; **1 mark** electrons have a de Broglie wavelength comparable to the atomic spacing, so a classical-particle model cannot explain the ordered pattern.)

(b) For a wave, $p=\dfrac{h}{\lambda}$, so the electrons' momentum is

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}}{1.2\times10^{-10}}=5.53\times10^{-24}\,\text{kg m s}^{-1}.$$

Because the X-rays produce the same-size pattern they have the **same wavelength**, and $p=h/\lambda$ depends only on wavelength, so the X-ray photons have the **same momentum**, $5.53\times10^{-24}\,\text{kg m s}^{-1}$. **(1 mark** $p=h/\lambda=5.53\times10^{-24}\,\text{kg m s}^{-1}$; **1 mark** the photons have the same momentum (equal wavelength gives equal momentum).)

Question 7

Levels: $n=1$ at $-6.2\,\text{eV}$, $n=2$ at $-3.8\,\text{eV}$, $n=3$ at $-1.4\,\text{eV}$, $n=4$ at $-0.7\,\text{eV}$.

(a) Moving from $n=1$ to $n=3$ takes the electron to a **higher** energy state, so the atom must **absorb** a photon. The photon energy equals the difference between the states:

$$\Delta E=E_3-E_1=(-1.4)-(-6.2)=4.8\,\text{eV}.$$

∴ a photon of energy $4.8\,\text{eV}$ is absorbed. **(1 mark** absorbed (electron moves up); **1 mark** $\Delta E=4.8\,\text{eV}$.)

(b) The $n=4 \to n=2$ transition releases $\Delta E=E_4-E_2=(-0.7)-(-3.8)=3.1\,\text{eV}=4.96\times10^{-19}\,\text{J}$. The emitted wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{4.96 \times 10^{-19}} = 4.01 \times 10^{-7}\,\text{m}.$$

On the diagram the transition is drawn as a **downward** arrow from the $n = 4$ level to the $n = 2$ level (emission). **(1 mark** $\Delta E = 3.1\,\text{eV}$ converted to J and $\lambda = hc/\Delta E = 4.01 \times 10^{-7}\,\text{m}$; **1 mark** downward arrow from $n = 4$ to $n = 2$.)

(c) **Longer.** The $n = 3 \rightarrow n = 1$ transition spans $\Delta E = (-1.4) - (-6.2) = 4.8\,\text{eV}$, whereas $n = 2 \rightarrow n = 1$ spans only $\Delta E = (-3.8) - (-6.2) = 2.4\,\text{eV}$. Since $\lambda = hc/\Delta E$, the **smaller** energy gap of the $n = 2 \rightarrow n = 1$ transition produces the **longer** wavelength. **(1 mark** circles *longer*; **1 mark** justification: smaller energy gap gives a longer wavelength via $\lambda = hc/\Delta E$. The circled option alone scores 0 without the reasoning.)

Question 8

(a) The electrons in an atom can occupy only a **discrete (quantised) set of energy levels**, and this set of levels is **unique to each element**. A photon is emitted or absorbed only when an electron moves between two of these levels, with $E = hf = \frac{hc}{\lambda}$ equal to the energy difference. Because each element has its own particular pattern of level spacings, it can emit or absorb only its own particular set of photon energies — hence a **unique set of spectral lines** that acts like a fingerprint for the element. **(1 mark** atoms have discrete/quantised energy levels unique to each element; **1 mark** transitions between these levels give a unique set of photon energies, hence a unique line pattern.)

(b) An **emission** line spectrum appears as **bright coloured lines on a dark (black) background** (light emitted only at particular wavelengths), whereas an **absorption** line spectrum appears as **dark lines on a continuous coloured background** (those wavelengths removed from otherwise continuous light). **(1 mark: bright lines on dark vs dark lines on a continuous background.)**

(c) Each electron is detected as a single localised spot (particle-like behaviour), yet the interference pattern that builds up from many single electrons is a wave behaviour. The experiment therefore shows that electrons — matter — have a **dual wave–particle nature**. **(1 mark: electrons show both particle-like and wave-like behaviour / wave–particle duality.)**