Physics Units 3 & 4

Chapter 7 — Light as a wave and as a particle

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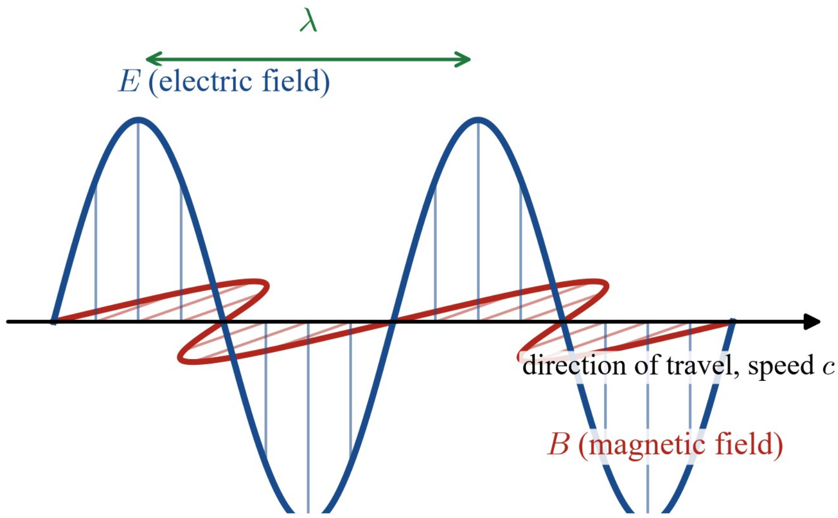
Contents

Theory

Light as an electromagnetic wave. A stationary electric charge is surrounded by a steady electric field, and a charge moving at constant velocity also produces a steady magnetic field. When a charge **accelerates**, however, its electric field changes with time. A changing electric field generates a changing magnetic field, and that changing magnetic field in turn regenerates a changing electric field a little further on. The two fields continually re-create each other and the disturbance spreads outward from the accelerating charge as an **electromagnetic (EM) wave**. Light is such a wave: an oscillating electric field accompanied by an oscillating magnetic field.

An EM wave is **transverse**. The electric field E and the magnetic field B both oscillate at right angles to the direction in which the wave travels, and they are perpendicular to each other. No material medium is needed — the wave is a self-sustaining oscillation of the fields themselves, so light travels through a vacuum.



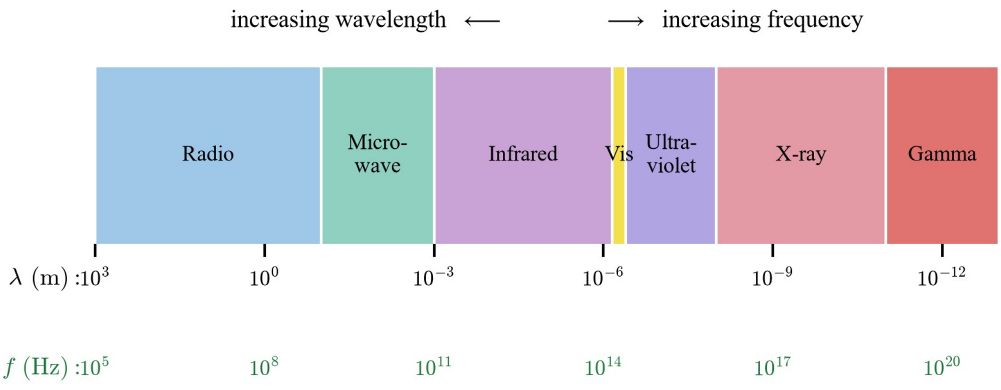
All electromagnetic waves travel at the same speed in a vacuum. Radio waves, microwaves, infrared, visible light, ultraviolet, X-rays and gamma rays are all electromagnetic waves; they differ only in frequency and wavelength. In a vacuum every one of them travels at the same speed,

$$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}.$$

What distinguishes one part of the spectrum from another is the **frequency** f (in Hz) and the **wavelength** λ (in m), related to the speed by the wave equation

$$v = f\lambda, \text{ and for any EM wave in a vacuum } c = f\lambda.$$

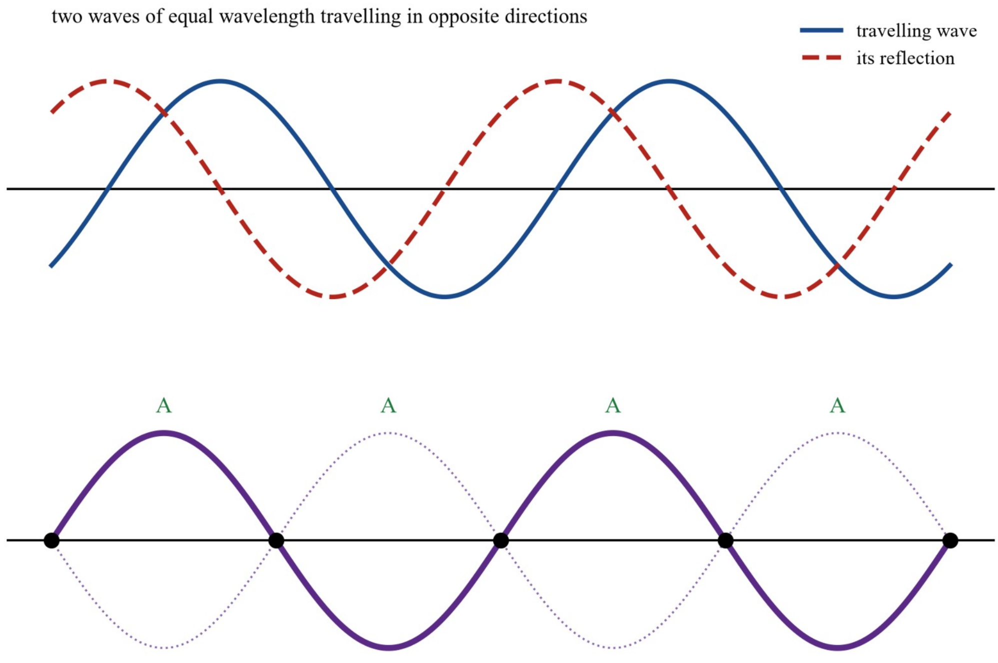
Because c is fixed, a higher frequency always means a shorter wavelength: gamma rays have the highest frequencies and shortest wavelengths, radio waves the lowest frequencies and longest wavelengths.



all travel at $c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$ in vacuum; $c = f\lambda$

Superposition. When two waves overlap in the same region, the resultant disturbance at each point is the **sum** of the individual displacements — the *principle of superposition*. Where the two waves arrive **in phase** (crest on crest) they reinforce: **constructive** interference. Where they arrive **out of phase** (crest on trough) they cancel: **destructive** interference. This single idea underlies both standing waves and the double-slit pattern below.

Standing waves. A **standing** (or stationary) wave is produced when a travelling wave and its reflection — two waves of the same frequency and amplitude travelling in opposite directions — superpose. At certain fixed points the two waves are always out of phase and cancel completely; these permanently stationary points are **nodes**. Half-way between the nodes the two waves are always in phase and the medium oscillates with maximum amplitude; these points are **antinodes**. Unlike a travelling wave, the pattern does not move along — the nodes and antinodes stay in fixed positions, which is why it is called *standing*.



resultant standing wave: nodes (dots) stay fixed; antinodes (A) oscillate

The study of standing waves here is restricted to those with a **node at each end**, as occurs on a string or wire held fixed at both ends (and for the electric field of an EM standing wave set up between two reflectors). A whole number of half-wavelengths must fit exactly into the length L:

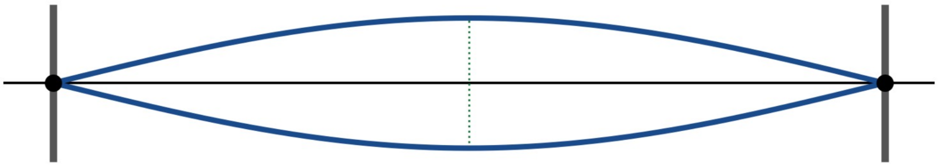
$$L = n\frac{\lambda_n}{2}, n = 1, 2, 3, \ldots$$

so the allowed (resonant) wavelengths and frequencies are

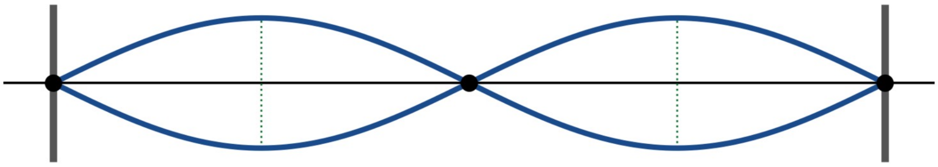
$$\lambda_n = \frac{2L}{n}, f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1.$$

The lowest frequency ($n = 1$) is the **fundamental** or first harmonic; the higher modes are the harmonics. A mode with harmonic number n has n antinodes and $n+1$ nodes, and the distance between adjacent nodes (or adjacent antinodes) is $\frac{\lambda}{2}$.

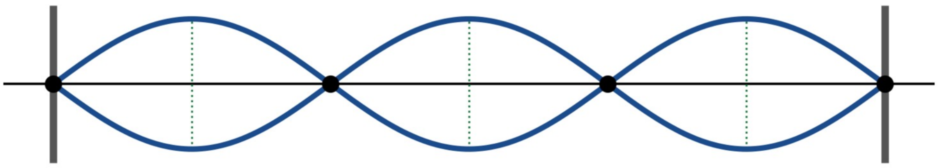
$n = 1$: fundamental (1st harmonic) $\lambda_1 = \frac{2L}{1}$ (1 antinode, 2 nodes)



$n = 2$: 2nd harmonic $\lambda_2 = \frac{2L}{2}$ (2 antinodes, 3 nodes)



$n = 3$: 3rd harmonic $\lambda_3 = \frac{2L}{3}$ (3 antinodes, 4 nodes)

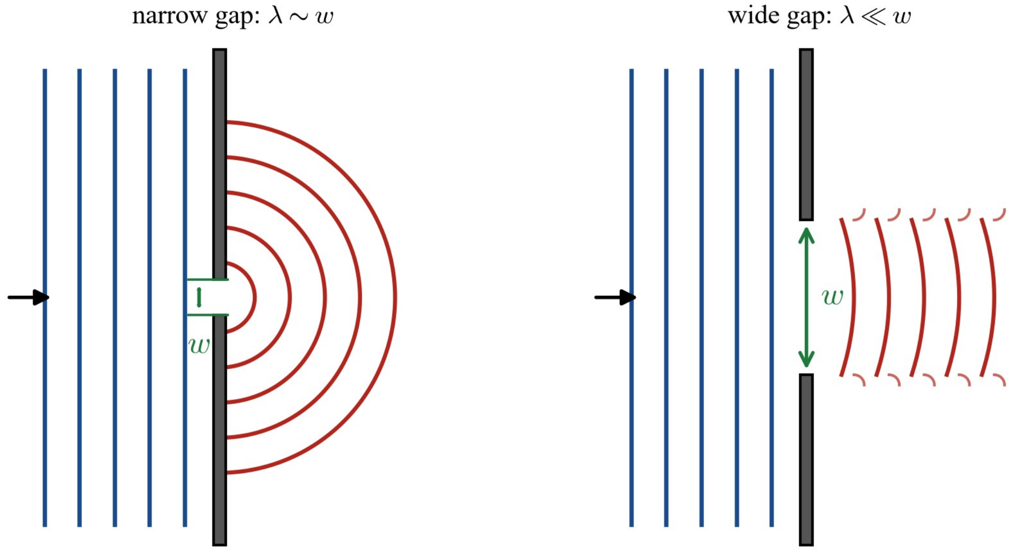


Diffraction. When a wave passes through a gap or around an obstacle it spreads out into the region beyond — this spreading is called **diffraction**. How much the wave spreads depends on the size of the gap or obstacle, of width w, *relative to* the wavelength λ — that is, on the ratio

$$\frac{\lambda}{w}.$$

Diffraction is **significant** (the spreading is large and easily observed) when $\frac{\lambda}{w}$ is of the order of 1, i.e. when the wavelength and the gap width are of the same order of magnitude. When λ is much smaller than w ($\lambda / w \ll 1$) the wave passes through almost straight, with only slight spreading at the edges.

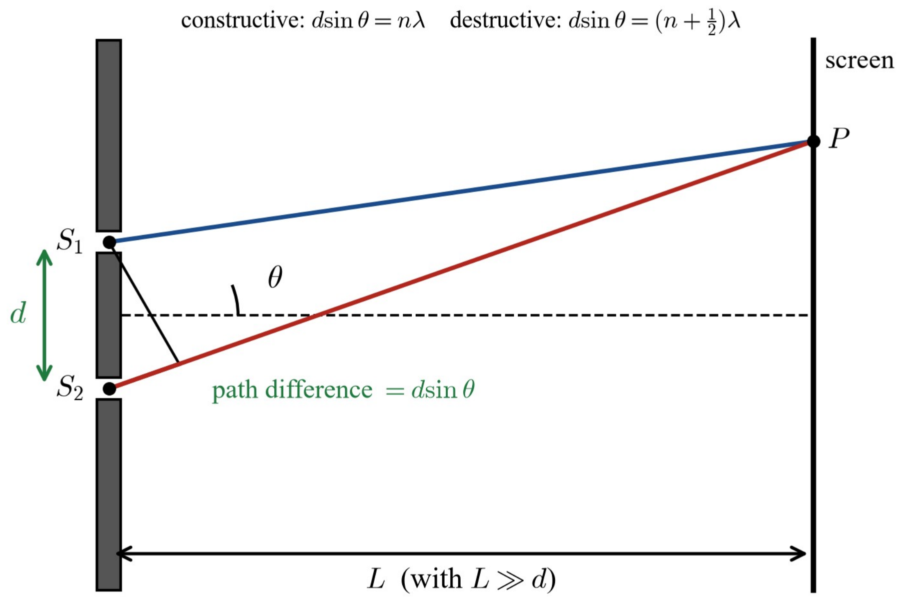
A common misconception is that diffraction "switches off" as soon as the wavelength is smaller than the gap. This is **wrong**: a wave diffracts at *every* gap, for all wavelengths — what changes is only *how much* it spreads. As λ / w decreases the spreading becomes less and less pronounced, but it never disappears.



Diffraction sets a limit on **imaging**. To resolve detail of size w using a wave, the wavelength must be small compared with that detail; if λ is comparable to or larger than the detail, diffraction blurs it out and the detail cannot be seen. This is why an optical microscope using visible light (wavelength $\approx 500\,\mathrm{nm}$) cannot resolve structures much smaller than the wavelength of light, and why finer detail requires radiation of shorter wavelength (ultraviolet, X-rays) or the very short wavelengths associated with electrons in an electron microscope.

Young's double-slit experiment. When light from a single source passes through two narrow, closely spaced slits, a pattern of alternating bright and dark bands — **interference fringes** — is seen on a screen beyond. This is direct evidence for the **wave** nature of light: only waves can superpose to produce points of cancellation (dark fringes) as well as reinforcement (bright fringes). A stream of particles would simply produce two bright bands, one behind each slit.

For a steady pattern the light reaching the two slits must be **coherent** — of a single wavelength and with a constant phase relationship. Illuminating both slits with light from one source guarantees this; the two slits then behave as two synchronised sources.



Light reaching a point P on the screen has travelled slightly further from one slit than from the other. This extra distance is the **path difference**. Whether P is bright or dark depends on the path difference measured in wavelengths:

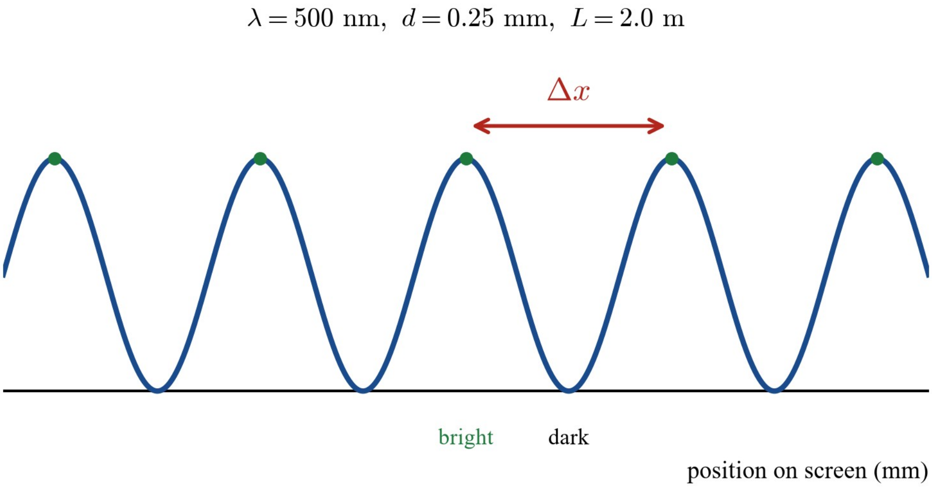
- **constructive** interference (a **bright** fringe) where the path difference is a whole number of wavelengths, $n\lambda$, with $n=0,1,2,\ldots$;

- **destructive** interference (a **dark** fringe) where the path difference is an odd number of half-wavelengths, $(n+1/2)\lambda$, with $n=0,1,2,\ldots$

The central point, equidistant from both slits, has zero path difference and is always bright (the central maximum, $n=0$). Moving outward, the bright fringes are evenly spaced. When the slit-to-screen distance L is much greater than the slit separation d ($L \gg d$), the spacing between adjacent bright (or adjacent dark) fringes is

$$\Delta x=\frac{\lambda L}{d}.$$

The n-th bright fringe lies a distance $x_n = n\,\Delta x$ from the centre. This relation shows at a glance how the pattern responds to the apparatus: the fringes spread out (larger Δx) for a longer wavelength or a more distant screen, and crowd together (smaller Δx) for more widely separated slits.



Throughout this subtopic the speed of every electromagnetic wave in a vacuum is $c=3.00\times10^{8}\,\mathrm{m\ s^{-1}}$.

Worked examples

Example 1 — scaffolded

Red light has a wavelength of $630\,\mathrm{nm}$ in a vacuum. (a) Calculate its frequency. (b) A microwave oven produces microwaves of frequency $2.45\,\mathrm{GHz}$; calculate their wavelength in a vacuum. (c) State how the speeds of the two waves compare.

Working	Comment
Both are EM waves, so $c=f\lambda$ with $c=3.00\times10^{8}\,\mathrm{m\ s^{-1}}$; convert prefixes to SI.	Identify the model and the constant.
(a) $\lambda=630\,\mathrm{nm}=630\times10^{-9}\,\mathrm{m}$.	$1\,\mathrm{nm}=10^{-9}\,\mathrm{m}$.
$f=\frac{c}{\lambda}=\frac{3.00\times10^{8}}{630\times10^{-9}}=4.76\times10^{14}\,\mathrm{Hz}$.	Rearrange $c=f\lambda$ for f.
(b) $f=2.45\,\mathrm{GHz}=2.45\times10^{9}\,\mathrm{Hz}$.	$1\,\mathrm{GHz}=10^{9}\,\mathrm{Hz}$.
$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}}{2.45\times10^{9}}=0.122\,\mathrm{m}$.	Rearrange $c=f\lambda$ for λ.
(c) Both travel at $c=3.00\times10^{8}\,\mathrm{m\ s^{-1}}$ in a vacuum — the **same** speed.	All EM waves travel at c in a vacuum.

Example 2 — scaffolded

A string of length $0.80\,\mathrm{m}$ is fixed at both ends. Transverse waves travel along it at $240\,\mathrm{m\,s^{-1}}$. (a) Calculate the wavelength and frequency of the fundamental standing wave. (b) Calculate the frequency of the third harmonic, and state the number of nodes and antinodes it has.

Working	Comment
Fixed at both ends $\Rightarrow$ a node at each end; $L = n\dfrac{\lambda_n}{2}$, $\lambda_n = \dfrac{2L}{n}$.	Standing wave, nodes at both ends.
(a) Fundamental $n=1$: $\lambda_1 = 2L = 2(0.80) = 1.60\,\mathrm{m}$.	Longest wavelength that fits.
$f_1 = \dfrac{v}{\lambda_1} = \dfrac{240}{1.60} = 150\,\mathrm{Hz}$.	Wave equation $v = f\lambda$.
(b) Third harmonic $n=3$: $f_3 = 3f_1 = 3(150) = 450\,\mathrm{Hz}$.	$f_n = nf_1$.
($\lambda_3 = \dfrac{2L}{3} = 0.533\,\mathrm{m}$, as a check: $f_3 = \dfrac{240}{0.533} = 450\,\mathrm{Hz}$.)	Consistent with $v = f\lambda$.
Antinodes $= n = 3$; nodes $= n+1 = 4$.	A mode with harmonic number n has n antinodes.

Example 3 — unscaffolded

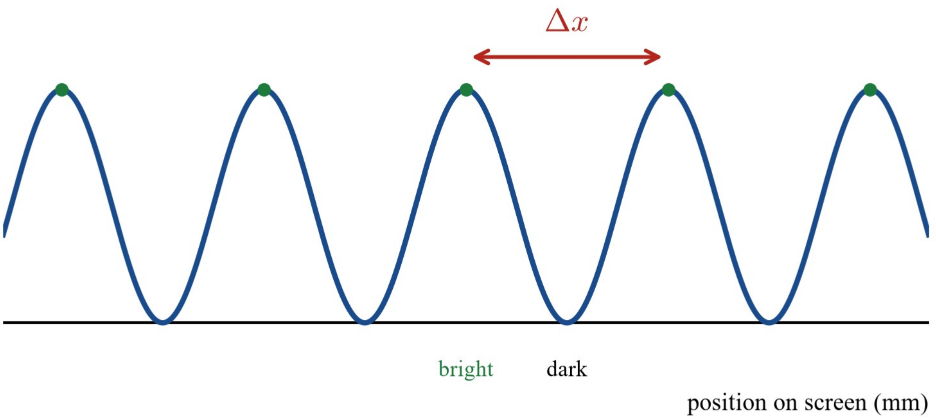
Coherent light of wavelength $546\,\mathrm{nm}$ falls on two narrow slits $0.25\,\mathrm{mm}$ apart. An interference pattern is observed on a screen $1.8\,\mathrm{m}$ from the slits. Calculate the fringe spacing.

The slit-to-screen distance is far greater than the slit separation ($L \gg d$), so the fringe spacing is

$$\Delta x = \frac{\lambda L}{d}.$$

Converting to SI, $\lambda = 546\times10^{-9}\,\mathrm{m}$ and $d = 0.25\,\mathrm{mm} = 0.25\times10^{-3}\,\mathrm{m}$, so

$$\Delta x = \frac{(546\times10^{-9})(1.8)}{0.25\times10^{-3}} = 3.93\times10^{-3}\,\mathrm{m}.$$



$\therefore$ the fringe spacing is $3.93\times10^{-3}\,\mathrm{m}$ (about $3.93\,\mathrm{mm}$).

Example 4 — unscaffolded

In a double-slit experiment using light of wavelength $600\,\mathrm{nm}$, a point P on the screen is such that light from the two slits reaches it with a path difference of $1.50\ \mu\mathrm{m}$. Determine whether P is a bright or a dark fringe, and state the corresponding order.

Express the path difference as a multiple of the wavelength. In SI, the path difference is 1.50×10^{-6} m (1.50 μm) and $\lambda = 600 \times 10^{-9}$ m, so

$$\frac{\text{path difference}}{\lambda} = \frac{1.50 \times 10^{-6}}{600 \times 10^{-9}} = 2.5.$$

The path difference is $2.5\,\lambda = (2 + 1/2)\,\lambda$, an odd number of half-wavelengths. This is the condition $(n + 1/2)\,\lambda$ for **destructive** interference, with $n = 2$.

$\therefore$ P is a **dark** fringe, corresponding to $n = 2$.

Practice questions

Question 1 X-rays used in medical imaging have a frequency of 3.0×10^{18} Hz. (a) Calculate the wavelength of these X-rays in a vacuum. (b) A radio wave has a wavelength of 2.5 m. Calculate its frequency. (c) State which of the two waves travels faster in a vacuum, and justify your answer.

Question 2 A guitar string of length 0.65 m is fixed at both ends. Waves travel along the string at $420\ \text{m s}^{-1}$. (a) Calculate the wavelength of the fundamental (first-harmonic) standing wave. (b) Calculate the frequency of the fundamental. (c) Calculate the frequency of the third harmonic, and state the number of nodes in that mode.

Question 3 Coherent light of wavelength 480 nm passes through two narrow slits 0.30 mm apart. An interference pattern forms on a screen 2.0 m from the slits. (a) Calculate the fringe spacing Δx. (b) Calculate the distance from the central bright fringe to the third bright fringe. (c) State and explain what happens to the fringe spacing if the slit separation is increased.

Question 4 A wave of wavelength λ passes through a gap of width w. (a) State the ratio that determines how much the wave spreads out by diffraction. (b) State the condition on this ratio for the diffraction to be significant. (c) Two gaps, X (width 2.0 mm) and Y (width 0.50 mm), are each illuminated by the same sound wave of wavelength 1.0 mm. State which gap produces the greater spreading, and justify your answer using the ratio.

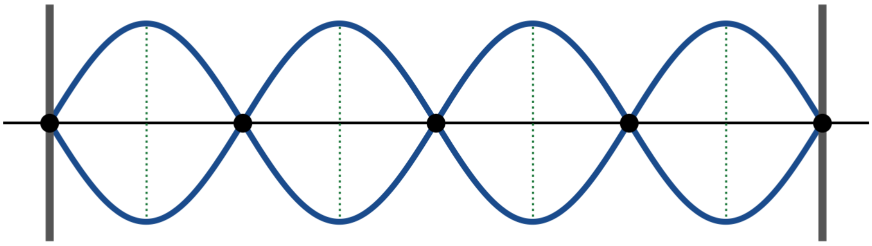
Question 5 In a double-slit experiment the slits are 0.20 mm apart and the screen is 1.5 m from the slits. The bright fringes on the screen are measured to be 4.2 mm apart. (a) Calculate the wavelength of the light used. (b) Give your answer to part (a) in nanometres. (c) The screen is now moved to 3.0 m from the slits. Calculate the new fringe spacing.

Question 6 Microwaves of frequency 1.50×10^{10} Hz are directed at a flat metal reflector. The incident wave and its reflection superpose to form a standing wave in the region between the source and the reflector, with a node at the reflector. (a) Explain how the superposition of the incident wave and its reflection produces a standing wave. (b) Calculate the wavelength of the microwaves. (c) A small detector is moved along the standing wave. State what the distance between adjacent nodes equals in terms of the wavelength, and calculate its value.

Question 7 A helium–neon laser emits red light of wavelength 633 nm in a vacuum. Calculate its frequency.

Question 8 Describe how an accelerating electric charge produces an electromagnetic wave, and state two properties shared by all electromagnetic waves.

Question 9 The diagram shows a standing wave on a string of length 1.2 m that is fixed at both ends.



string length $= 1.2$ m, fixed at both ends

(a) State the harmonic number of this standing wave and the number of nodes and antinodes shown.
(b) Calculate the wavelength of this standing wave.
(c) The wave speed along the string is $180\,\text{m s}^{-1}$. Calculate the frequency of this standing wave.

Question 10 A metal wire fixed at both ends has a fundamental frequency of $256\,\text{Hz}$. The speed of waves along the wire is $320\,\text{m s}^{-1}$. (a) Calculate the length of the wire. (b) Calculate the frequency of the second harmonic.

Question 11 Coherent light of wavelength $589\,\text{nm}$ produces an interference pattern with a fringe spacing of $2.95\,\text{mm}$ on a screen $2.4\,\text{m}$ from the slits. Calculate the slit separation. Give your answer in millimetres.

Question 12 In a double-slit experiment, light of wavelength $632\,\text{nm}$ illuminates two slits $0.35\,\text{mm}$ apart, with a screen $2.1\,\text{m}$ away. (a) Show that the fringe spacing is $3.79\times10^{-3}\,\text{m}$. (b) Hence calculate the distance from the central bright fringe to the fifth dark fringe.

Question 13 In a Young's double-slit experiment, light of wavelength $500\,\text{nm}$ is used. At a point Q on the screen, light from the two slits arrives with a path difference of 2.0 µm. (a) Calculate the path difference as a multiple of the wavelength. (b) State whether Q is a bright or a dark fringe, and justify your answer using the path-difference condition.

Question 14 A student claims: "Diffraction only happens when the wavelength of a wave is larger than the width of the gap it passes through. If the wavelength is smaller than the gap, there is no diffraction at all." Explain what is wrong with this statement, and state correctly the condition under which diffraction is significant. Refer to the ratio λ / w in your answer.

Question 15 Explain why an optical microscope using visible light (wavelength $\approx 500\,\text{nm}$) cannot resolve the detail of an individual atom (diameter $\approx 0.1\,\text{nm}$), and state one change to the apparatus that would allow finer detail to be resolved.

Question 16 A double-slit interference pattern is observed on a screen. State and explain the effect on the fringe spacing Δx of each of the following separate changes. (a) The light is replaced by light of a longer wavelength. (b) The screen is moved further from the slits. (c) Slits with a smaller separation are used.

Question 17 (a) Explain why the interference pattern seen in Young's double-slit experiment is evidence for the wave-like nature of light. (b) Explain why the two slits must be illuminated by light from a single source for a stable interference pattern to be observed.

Question 18 In an experiment, microwaves are reflected between two parallel metal plates, forming a standing wave with a node at each plate. A detector moved between the plates finds adjacent nodes $1.4\,\text{cm}$ apart. (a) Calculate the wavelength of the microwaves. (b) Calculate their frequency. (c) The plates are $8.4\,\text{cm}$ apart. Determine the harmonic number n (for which $L = n\lambda/2$) of the standing wave set up between them.

Question 19 Two slits $0.20\,\text{mm}$ apart are illuminated with coherent light of wavelength $600\,\text{nm}$. On the screen the distance from the central bright fringe to the third bright fringe is $9.0\,\text{mm}$. Calculate the distance from the slits to the screen.

Question 20 A string fixed at both ends has a fundamental frequency of $110\,\mathrm{Hz}$. (a) Calculate the frequencies of the next two harmonics. (b) A standing wave is observed on the string at a frequency of $440\,\mathrm{Hz}$. Determine which harmonic this is, and state the number of antinodes.

Question 21 A double-slit experiment is performed first with blue light ($\lambda = 450\,\mathrm{nm}$) and then, using the same apparatus, with red light ($\lambda = 660\,\mathrm{nm}$). (a) Without calculating, state which colour produces the wider fringe spacing, and justify your answer. (b) The slit separation is $0.25\,\mathrm{mm}$ and the screen is $2.2\,\mathrm{m}$ from the slits. Calculate the fringe spacing for the red light. Give your answer to three significant figures. (c) Calculate how much wider, in millimetres, the red fringe spacing is than the blue fringe spacing.

Answers

Question 1 (a) 1.0×10^{-10} m; (b) 1.2×10^{8} Hz; (c) same speed — all EM waves travel at c in a vacuum. **Question 2** (a) 1.30 m; (b) 323 Hz; (c) 969 Hz, 4 nodes. **Question 3** (a) 3.20×10^{-3} m; (b) 9.60×10^{-3} m; (c) Δx decreases (fringes closer together). **Question 4** (a) λ / w; (b) significant when λ / w is of order 1; (c) gap Y (larger λ / w). **Question 5** (a) 5.6×10^{-7} m; (b) 560 nm; (c) 8.4×10^{-3} m. **Question 6** (b) 0.020 m; (c) $\lambda / 2 = 0.010$ m. **Question 7** 4.74×10^{14} Hz. **Question 8** Accelerating charge → changing E field → changing B field → self-propagating wave; shared properties: transverse, and travel at c in a vacuum. **Question 9** (a) $n=4$, 5 nodes and 4 antinodes; (b) 0.60 m; (c) 300 Hz. **Question 10** (a) 0.625 m; (b) 512 Hz. **Question 11** 4.79×10^{-4} m $= 0.479$ mm. **Question 12** (a) 3.79×10^{-3} m; (b) 1.71×10^{-2} m. **Question 13** (a) 4λ; (b) bright ($n=4$, constructive). **Question 14** Diffraction occurs for all wavelengths; it is significant only when λ / w is of order 1. **Question 15** λ is far larger than the atom, so diffraction blurs the detail; use shorter-wavelength radiation (e.g. electrons or X-rays). **Question 16** (a) increases; (b) increases; (c) increases. **Question 17** (a) fringes (including dark fringes) require superposition — a wave property; (b) coherence needs a constant phase relationship. **Question 18** (a) 0.028 m; (b) 1.07×10^{10} Hz; (c) $n=6$. **Question 19** 1.00 m. **Question 20** (a) 220 Hz and 330 Hz; (b) fourth harmonic, 4 antinodes. **Question 21** (a) red (longer λ); (b) 5.81×10^{-3} m; (c) 1.85 mm.

Fully-worked solutions

Question 1 All EM waves in a vacuum satisfy $c = f\lambda$ with $c = 3.00 \times 10^{8}\ \text{m s}^{-1}$. (a) $\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}}{3.0 \times 10^{18}} = 1.0 \times 10^{-10}$ m. (b) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{2.5} = 1.2 \times 10^{8}$ Hz. (c) Neither is faster: **all** electromagnetic waves travel at the same speed $c = 3.00 \times 10^{8}\ \text{m s}^{-1}$ in a vacuum, regardless of frequency or wavelength.

Question 2 Fixed at both ends ⇒ a node at each end, so $\lambda_n = \frac{2L}{n}$ with $L = 0.65$ m and $v = 420\ \text{m s}^{-1}$. (a) Fundamental $n=1$: $\lambda_1 = 2L = 2(0.65) = 1.30$ m. (b) $f_1 = \frac{v}{\lambda_1} = \frac{420}{1.30} = 323$ Hz. (c) Third harmonic $n=3$: $f_3 = 3f_1 = 3(323) = 969$ Hz; a mode with $n=3$ has $n+1=4$ nodes.

Question 3 $\lambda = 480 \times 10^{-9}$ m, $d = 0.30 \times 10^{-3}$ m, $L = 2.0$ m (with $L \gg d$). (a) $\Delta x = \frac{\lambda L}{d} = \frac{(480 \times 10^{-9})(2.0)}{0.30 \times 10^{-3}} = 3.20 \times 10^{-3}$ m. (b) The third bright fringe is at $x_3 = 3\Delta x = 3(3.20 \times 10^{-3}) = 9.60 \times 10^{-3}$ m from the centre. (c) In $\Delta x = \frac{\lambda L}{d}$, Δx is inversely proportional to d. Increasing the slit separation d therefore **decreases** the fringe spacing — the fringes move closer together.

Question 4 (a) The spreading is governed by the ratio $\frac{\lambda}{w}$ of the wavelength to the gap width. (b) Diffraction is significant when $\frac{\lambda}{w}$ is of the order of 1 — that is, when the wavelength and the gap width are of the same order of magnitude (roughly $\lambda \gtrsim w$). (c) For gap X, $\frac{\lambda}{w} = \frac{1.0}{2.0} = 0.5$; for gap Y, $\frac{\lambda}{w} = \frac{1.0}{0.50} = 2.0$. Gap Y has the larger ratio, so **gap Y** produces the greater spreading.

Question 5 $d = 0.20 \times 10^{-3}$ m, $L = 1.5$ m, $\Delta x = 4.2 \times 10^{-3}$ m. (a) Rearranging $\Delta x = \frac{\lambda L}{d}$ gives $\lambda = \frac{\Delta x\, d}{L} = \frac{(4.2 \times 10^{-3})(0.20 \times 10^{-3})}{1.5} = 5.6 \times 10^{-7}$ m. (b) 5.6×10^{-7} m $= 560 \times 10^{-9}$ m $= 560$ nm. (c) $\Delta x \propto L$, and L has doubled from 1.5 m to 3.0 m, so Δx doubles: $\Delta x = \frac{(5.6 \times 10^{-7})(3.0)}{0.20 \times 10^{-3}} = 8.4 \times 10^{-3}$ m.

Question 6 $f=1.50\times10^{10}\,\text{Hz}$, $c=3.00\times10^{8}\,\text{m s}^{-1}$. (a) The wave reflected from the metal travels back through the incident wave, so two waves of the same frequency and amplitude move through the region in opposite directions. By superposition they add at every point. At certain fixed points they are permanently out of phase and cancel — these are the **nodes** (one lies at the reflector, where the field is forced to zero); half-way between, they are permanently in phase and reinforce — the **antinodes**. The pattern of nodes and antinodes does not move, so it is a standing wave. (b) $\lambda=\frac{c}{f}=\frac{3.00\times10^{8}}{1.50\times10^{10}}=0.020\,\text{m}$. (c) Adjacent nodes are separated by half a wavelength: $\frac{\lambda}{2}=\frac{0.020}{2}=0.010\,\text{m}$.

Question 7 For an EM wave in a vacuum, $c=f\lambda$, so

$$f=\frac{c}{\lambda}=\frac{3.00\times10^{8}}{633\times10^{-9}}=4.74\times10^{14}\,\text{Hz}.$$

Question 8 When an electric charge **accelerates**, the electric field around it changes with time. A changing electric field produces a changing magnetic field, and that changing magnetic field in turn produces a further changing electric field. The two fields continually regenerate each other and the disturbance propagates outward from the charge as an electromagnetic wave. Two properties shared by all EM waves: they are **transverse** (the electric and magnetic fields oscillate at right angles to the direction of travel and to each other), and they all travel at the **same speed** $c=3.00\times10^{8}\,\text{m s}^{-1}$ in a vacuum.

Question 9 $L=1.2\,\text{m}$, fixed at both ends. (a) The pattern has 4 antinodes (loops), so it is the **fourth harmonic**, $n=4$; it has 4 antinodes and $n+1=5$ nodes. (b) $\lambda=\frac{2L}{n}=\frac{2(1.2)}{4}=0.60\,\text{m}$. (c) $f=\frac{v}{\lambda}=\frac{180}{0.60}=300\,\text{Hz}$.

Question 10 Fixed at both ends: $f_1=\frac{v}{2L}$, with $f_1=256\,\text{Hz}$, $v=320\,\text{m s}^{-1}$. (a) $L=\frac{v}{2f_1}=\frac{320}{2(256)}=0.625\,\text{m}$. (b) Second harmonic: $f_2=2f_1=2(256)=512\,\text{Hz}$.

Question 11 Rearranging $\Delta x=\frac{\lambda L}{d}$ for the slit separation,

$$d=\frac{\lambda L}{\Delta x}=\frac{(589\times10^{-9})(2.4)}{2.95\times10^{-3}}=4.79\times10^{-4}\,\text{m}=0.479\,\text{mm}.$$

Question 12 $\lambda=632\times10^{-9}\,\text{m}$, $d=0.35\times10^{-3}\,\text{m}$, $L=2.1\,\text{m}$. (a) $\Delta x=\frac{\lambda L}{d}=\frac{(632\times10^{-9})(2.1)}{0.35\times10^{-3}}=3.79\times10^{-3}\,\text{m}$, as required. (b) Dark fringes lie half-way between the bright fringes. The first dark fringe is $0.5\,\Delta x$ from the centre, the second is $1.5\,\Delta x$, and in general the k-th dark fringe is at $(k-0.5)\Delta x$. The fifth dark fringe ($k=5$) is therefore at

$$x=4.5\,\Delta x=4.5(3.79\times10^{-3})=1.71\times10^{-2}\,\text{m}.$$

Question 13 $\lambda=500\times10^{-9}\,\text{m}$, path difference $=2.0\times10^{-6}\,\text{m}$. (a) $\frac{\text{path difference}}{\lambda}=\frac{2.0\times10^{-6}}{500\times10^{-9}}=4.0$, so the path difference is 4λ. (b) The path difference is a whole number of wavelengths, $n\lambda$ with $n=4$. This is the condition for **constructive** interference, so Q is a **bright** fringe (the fourth-order maximum).

Question 14 The statement is wrong to claim that diffraction stops when the wavelength is smaller than the gap. Diffraction — the spreading of a wave beyond a gap — occurs for **every** wavelength passing through **any** gap; it is never switched off. What actually changes is *how much* the wave spreads, and this is governed by the ratio $\frac{\lambda}{w}$. The spreading is **significant** (large enough to be obvious) only when $\frac{\lambda}{w}$ is of the order of 1 — when λ and w are of the same order of magnitude. When λ is much smaller than w ($\lambda/w\ll1$) the wave still diffracts, but the spreading is so slight that it is hard to notice. So diffraction becomes *less pronounced* as λ/w decreases; it does not cease once $\lambda<w$.

Question 15 To resolve detail of a given size, the wavelength used must be small compared with that detail; diffraction blurs out any feature whose size is comparable to or smaller than the wavelength. Visible light has $\lambda \approx 500\,\mathrm{nm}$, which is about 5000 times larger than an atom ($\approx 0.1\,\mathrm{nm}$). The ratio λ / w for atomic detail is therefore enormous ($\gg 1$), so diffraction completely blurs out structure that small and the atom cannot be resolved. Resolving finer detail requires radiation of much shorter wavelength — for example ultraviolet or X-rays, or the very short wavelengths of electrons used in an electron microscope.

Question 16 The fringe spacing is $\Delta x = \frac{\lambda L}{d}$. (a) Longer wavelength: $\Delta x \propto \lambda$, so Δx **increases**. (b) Screen further away: $\Delta x \propto L$, so Δx **increases**. (c) Smaller slit separation: $\Delta x \propto \frac{1}{d}$, so reducing d **increases** Δx.

Question 17 (a) The pattern consists of alternating bright and dark fringes. The bright fringes occur where light from the two slits arrives in phase (path difference $n\lambda$) and reinforces, and the dark fringes occur where it arrives out of phase (path difference $(n + 1/2)\lambda$) and cancels. Cancellation to give dark regions is possible only if light is a wave that can superpose; particles arriving at the screen could not cancel one another to leave darkness. The existence of the dark fringes is therefore direct evidence for the wave nature of light. (b) A stable pattern requires the two slits to behave as **coherent** sources — emitting the same wavelength with a constant phase difference between them. Illuminating both slits with light from a single source ensures this, because any fluctuation in the source affects both slits together, so their phase relationship stays fixed. If the two slits were lit by independent sources, the phase difference between them would vary rapidly and randomly, the positions of the bright and dark fringes would shift about too quickly to see, and the pattern would average out to uniform illumination.

Question 18 $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$; adjacent nodes are $\frac{\lambda}{2}$ apart. (a) $\frac{\lambda}{2} = 1.4\,\mathrm{cm}$, so $\lambda = 2.8\,\mathrm{cm} = 0.028\,\mathrm{m}$. (b) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{0.028} = 1.07 \times 10^{10}\,\mathrm{Hz}$. (c) With a node at each plate, $L = n\frac{\lambda}{2}$, so

$$n = \frac{2L}{\lambda} = \frac{2(8.4 \times 10^{-2})}{0.028} = 6.$$

The sixth harmonic ($n = 6$) is set up between the plates.

Question 19 $d = 0.20 \times 10^{-3}\,\mathrm{m}$, $\lambda = 600 \times 10^{-9}\,\mathrm{m}$. The third bright fringe is at $x_3 = 3\Delta x = 9.0\,\mathrm{mm}$, so $\Delta x = \frac{9.0 \times 10^{-3}}{3} = 3.0 \times 10^{-3}\,\mathrm{m}$. Rearranging $\Delta x = \frac{\lambda L}{d}$ for L,

$$L = \frac{\Delta x\, d}{\lambda} = \frac{(3.0 \times 10^{-3})(0.20 \times 10^{-3})}{600 \times 10^{-9}} = 1.00\,\mathrm{m}.$$

Question 20 Fixed at both ends: $f_n = n f_1$ with $f_1 = 110\,\mathrm{Hz}$. (a) Second harmonic $f_2 = 2f_1 = 220\,\mathrm{Hz}$; third harmonic $f_3 = 3f_1 = 330\,\mathrm{Hz}$. (b) $\frac{440}{110} = 4$, so $440\,\mathrm{Hz}$ is the **fourth harmonic** ($n = 4$). A mode with $n = 4$ has 4 antinodes.

Question 21 Same apparatus: $d = 0.25 \times 10^{-3}\,\mathrm{m}$, $L = 2.2\,\mathrm{m}$, and $\Delta x = \frac{\lambda L}{d}$. (a) Since $\Delta x \propto \lambda$, the longer wavelength gives the wider spacing: **red** light produces the wider fringe spacing. (b) $\Delta x_{\text{red}} = \frac{(660 \times 10^{-9})(2.2)}{0.25 \times 10^{-3}} = 5.81 \times 10^{-3}\,\mathrm{m}$ (to three significant figures). (c) $\Delta x_{\text{blue}} = \frac{(450 \times 10^{-9})(2.2)}{0.25 \times 10^{-3}} = 3.96 \times 10^{-3}\,\mathrm{m}$, so the red fringes are wider by

$$\Delta x_{\text{red}} - \Delta x_{\text{blue}} = (5.81 - 3.96) \times 10^{-3} = 1.85 \times 10^{-3}\,\mathrm{m} = 1.85\,\mathrm{mm}.$$

Theory

Chapter 7A treats light as a wave — it diffracts, and it forms interference patterns. This subtopic looks at experiments that the wave model **cannot** explain, and that force us to treat light as a stream of particle-like energy packets called **photons**.

The photon and its energy. In the particle model, light of frequency f is carried by photons, each an indivisible packet of energy

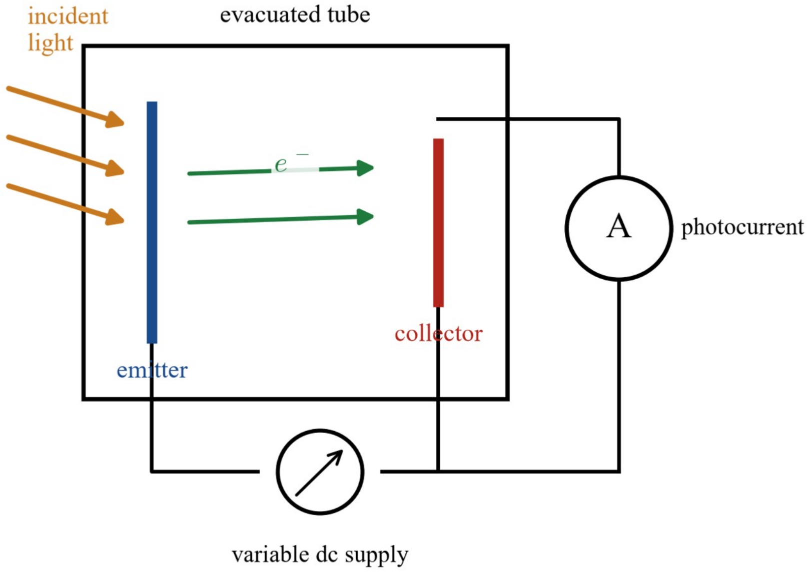
$$E = hf = \frac{hc}{\lambda},$$

where $h = 6.63 \times 10^{-34}$ J s is Planck's constant, $c = 3.00 \times 10^{8}$ m s^{-1} and λ is the wavelength ($f = \frac{c}{\lambda}$). A brighter beam of a single colour is made of *more photons per second*, but every photon still carries the same energy hf. The energy of a single photon is tiny in joules, so it is often quoted in **electron-volts**: one electron-volt is the energy an electron gains when accelerated through a potential difference of 1 V,

$$1\,\text{eV} = q_e \times 1\,\text{V} = 1.60 \times 10^{-19}\,\text{J}.$$

Working in electron-volts, the same photon energy is $E = hf$ with $h = 4.14 \times 10^{-15}$ eV s. To convert an energy in joules to electron-volts, divide by 1.60×10^{-19}; to go the other way, multiply by 1.60×10^{-19}.

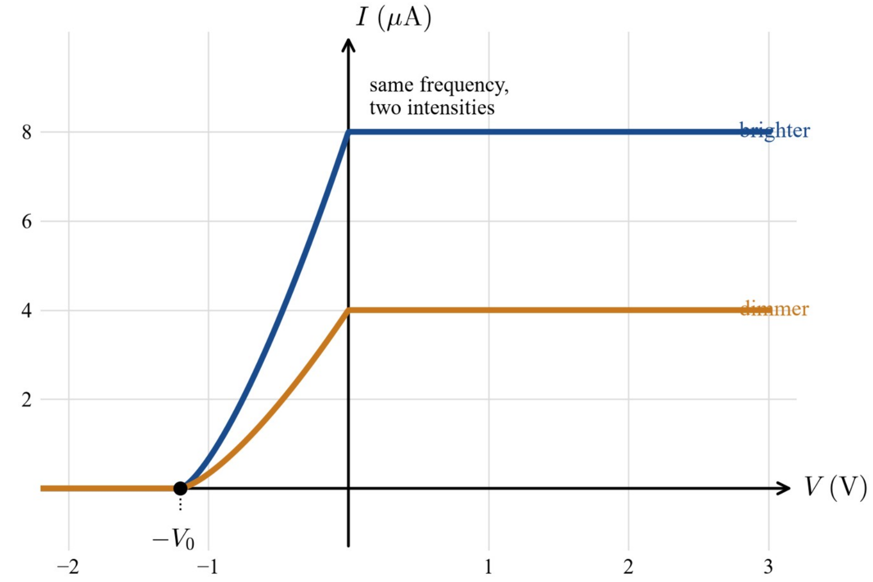
The photoelectric effect. When light of a high enough frequency strikes a clean metal surface, electrons are ejected from the metal. The ejected electrons are called **photoelectrons**. They are studied in an evacuated **photocell**: light falls on an emitter electrode (the photocathode), and the freed electrons cross the vacuum to a collector electrode. A variable direct-current supply sets the potential difference between the electrodes and an ammeter measures the resulting **photocurrent** — the rate at which photoelectrons reach the collector.



Photocurrent versus electrode potential. Suppose the emitter is illuminated with light of one fixed frequency and the collector potential is varied. When the collector is made positive it attracts the photoelectrons, and once it is positive enough *every* emitted electron is collected: the photocurrent levels off at a maximum called the **saturation current**. When the collector is made negative it *retards* the electrons; only those with enough kinetic energy still reach it, so the photocurrent falls. At a particular negative potential — the **stopping voltage** V_0 — even the fastest photoelectrons are turned back and the photocurrent drops to zero. The work done against the retarding field to stop the fastest electron equals that electron's maximum kinetic energy:

$$q_e V_0 = E_{k(\max)}.$$

The graph below shows the photocurrent for the **same frequency** at two different intensities. Increasing the intensity raises the saturation current (more photons per second eject more electrons per second), but the curves cut the voltage axis at the **same stopping voltage** — the maximum kinetic energy of the photoelectrons does not depend on intensity.



Maximum kinetic energy, work function and threshold frequency. Each photoelectron is freed by a single photon. Some of the photon's energy hf is used to release the electron from the metal, and the rest appears as the electron's kinetic energy. The minimum energy needed to free an electron from a particular metal is its **work function** φ. The most energetic photoelectrons are those that needed only φ to escape, so their maximum kinetic energy is

$$E_{k(\max)} = hf - \varphi.$$

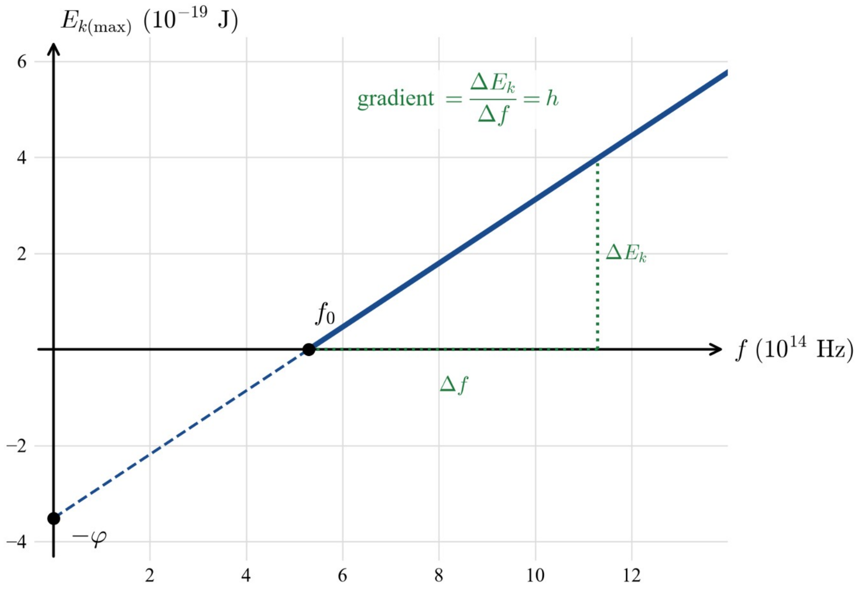
Emission is only possible when a single photon carries at least the work function, $hf \geq \varphi$. The lowest frequency that can eject electrons is the **threshold frequency**

$$f_0 = \frac{\varphi}{h},$$

below which no photoelectrons are emitted, however intense the light. The corresponding **threshold wavelength** is $\lambda_0 = \frac{hc}{\varphi} = \frac{c}{f_0}$; light of longer wavelength than λ_0 cannot cause emission.

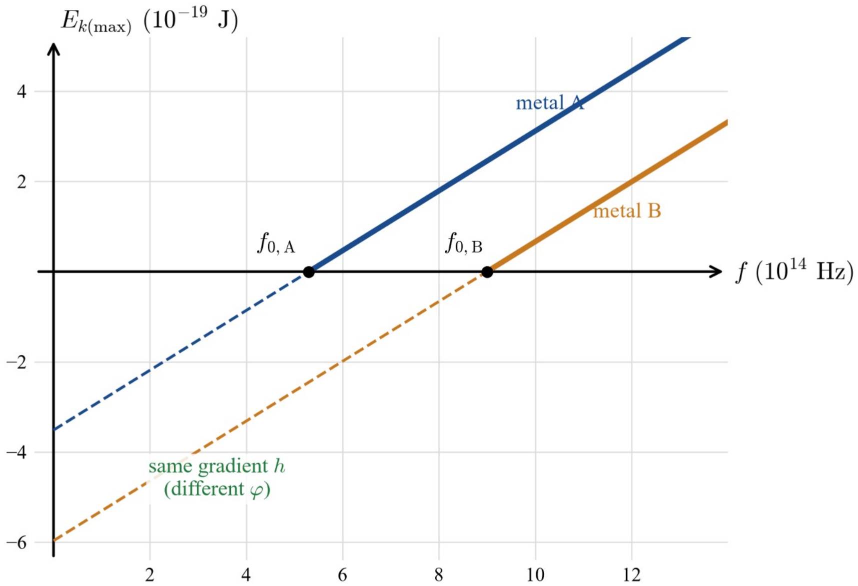
The kinetic-energy–frequency graph. Rearranging $E_{k(\max)} = hf - \varphi$ shows that a graph of maximum kinetic energy against frequency is a **straight line**:

- its **gradient** is Planck's constant h (the same for every metal);
- its intercept on the frequency axis is the **threshold frequency** f_0;
- its intercept on the energy axis (extended back) is $-\varphi$.



Reading these features off the line is a standard way to *measure* h and φ. For example, the line above is for a metal (call it metal A) with threshold frequency $f_0 = 5.30 \times 10^{14}$ Hz. Its work function is $\varphi = hf_0 = 6.63 \times 10^{-34} \times 5.30 \times 10^{14} = 3.51 \times 10^{-19}\,\text{J} = 2.20\,\text{eV}$. If the axes are drawn with E_k in joules and f in hertz, the gradient equals $h = 6.63 \times 10^{-34}$ J s; if E_k is plotted in electron-volts, the gradient equals 4.14×10^{-15} eV s.

Because the gradient is h for every metal, the lines for different metals are **parallel**. A metal with a larger work function has a higher threshold frequency, so its line sits further to the right.



Effect of intensity. Provided the frequency is above the threshold, the **intensity** controls only *how many* photoelectrons are emitted each second. Doubling the intensity of light of a fixed frequency doubles the number of photons per second, hence doubles the number of photoelectrons per second and doubles the saturation current — but it leaves $E_{k(\max)}$ and the stopping voltage unchanged. Below the threshold frequency, *no* intensity produces emission. A subtle case arises when the **power** delivered to the surface is held fixed while the frequency is changed: because each photon carries energy hf, the number of photons per second is $R = \dfrac{P}{hf} = \dfrac{P\lambda}{hc}$, so a longer wavelength (lower frequency) at the same power means *more* photons per second — and a larger saturation current — even though each photoelectron then emerges with less kinetic energy.

Failure of the wave model. The wave model of light cannot account for these results:

- The wave model says the energy of a light wave depends on its **intensity** and is delivered continuously, so a bright enough beam of *any* frequency should eventually free electrons. In fact there is a sharp **threshold frequency** below which nothing is emitted at any intensity.
- The wave model predicts that a more intense beam should eject **faster** electrons. In fact $E_{k(\max)}$ depends on **frequency**, not intensity.
- The wave model predicts a measurable **time delay** while a dim wave slowly deposits enough energy. In fact emission is effectively **instantaneous**, even for very faint light.

The photon model explains all three: light arrives as photons of energy hf; one electron absorbs one photon; emission needs $hf \geq \varphi$ (hence a threshold frequency); the surplus $hf - \varphi$ sets the kinetic energy (which grows with frequency, not intensity); and a single absorption event is essentially instantaneous.

Throughout this subtopic the VCAA constants $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s, $c = 3.00 \times 10^{8}$ m s^{-1}, $q_e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg and $1\ \text{eV} = 1.60 \times 10^{-19}$ J are used.

Worked examples

Example 1 — scaffolded

Violet light has a wavelength of $420\ \text{nm}$. Calculate its frequency and the energy of one of its photons, giving the energy in both joules and electron-volts.

Working	Comment
$\lambda = 420\ \text{nm} = 420 \times 10^{-9}$ m; a photon carries $E = hf = \dfrac{hc}{\lambda}$.	List the data and the photon-energy relation.
$f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^{8}}{420 \times 10^{-9}} = 7.14 \times 10^{14}$ Hz.	Frequency from $f = \dfrac{c}{\lambda}$.
$E = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{420 \times 10^{-9}} = 4.74 \times 10^{-19}$ J.	Using $h = 6.63 \times 10^{-34}$ J s.
$E = \dfrac{4.74 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.96\ \text{eV}$.	Convert joules to electron-volts by dividing by 1.60×10^{-19}.

Example 2 — scaffolded

A clean metal surface has a work function of $2.30\ \text{eV}$. It is illuminated with light of frequency $8.0 \times 10^{14}\ \text{Hz}$. Calculate the threshold frequency of the metal, the maximum kinetic energy of the photoelectrons (in electron-volts and joules), the stopping voltage, and the maximum speed of the photoelectrons.

Working	Comment
$\varphi = 2.30\ \text{eV}$, $f = 8.0 \times 10^{14}$ Hz; use $h = 4.14 \times 10^{-15}$ eV s.	Work in electron-volts.
$f_0 = \dfrac{\varphi}{h} = \dfrac{2.30}{4.14 \times 10^{-15}} = 5.56 \times 10^{14}$ Hz.	Threshold frequency; since $8.0 \times 10^{14} > f_0$, electrons are emitted.
$hf = (4.14 \times 10^{-15})(8.0 \times 10^{14}) = 3.312\ \text{eV}$; $E_{k(\max)} = hf - \varphi = 3.312 - 2.30 = 1.01\ \text{eV}$.	$E_{k(\max)} = hf - \varphi$.
$E_{k(\max)} = 1.01 \times 1.60 \times 10^{-19} = 1.62 \times 10^{-19}$ J.	Convert to joules.
$q_e V_0 = E_{k(\max)} \Rightarrow V_0 = \dfrac{1.62 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.01\ \text{V}$.	Stopping voltage; numerically equals $E_{k(\max)}$ in eV.

Working	Comment
$1/2\, m_e v^2 = E_{k(\max)} \Rightarrow v = \sqrt{\dfrac{2(1.62\times 10^{-19})}{9.11\times 10^{-31}}} = 5.96\times 10^5$ 1	Maximum speed from the maximum kinetic energy.

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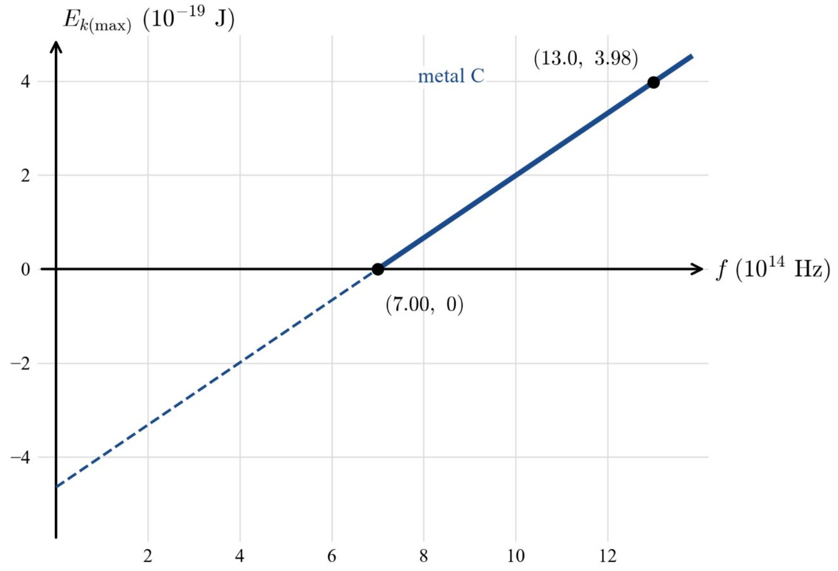
Example 3 — unscaffolded

The graph shows the maximum kinetic energy of photoelectrons emitted from a metal (metal C) as the frequency of the incident light is varied. Use the graph to determine Planck's constant, the work function of metal C (in joules and electron-volts), and the threshold frequency.



The data lie on the straight line $E_{k(\max)} = hf - \varphi$, so its gradient is Planck's constant. Using the two marked points on the line — the frequency-axis intercept $(7.00\times 10^{14}\,\text{Hz}, 0)$ and $(13.0\times 10^{14}\,\text{Hz}, 3.98\times 10^{-19}\,\text{J})$:

$$h = \text{gradient} = \frac{\Delta E_k}{\Delta f} = \frac{3.98\times 10^{-19} - 0}{(13.0 - 7.00)\times 10^{14}} = \frac{3.98\times 10^{-19}}{6.00\times 10^{14}} = 6.63\times 10^{-34}\,\text{J s}.$$

The threshold frequency is the frequency-axis intercept, $f_0 = 7.00\times 10^{14}\,\text{Hz}$. The work function is

$$\varphi = hf_0 = (6.63\times 10^{-34})(7.00\times 10^{14}) = 4.64\times 10^{-19}\,\text{J} = \frac{4.64\times 10^{-19}}{1.60\times 10^{-19}} = 2.90\,\text{eV}.$$

(Equivalently, extending the line back to the energy axis gives an intercept of $-\varphi = -4.64\times 10^{-19}\,\text{J}$.)

$\therefore$ $h = 6.63\times 10^{-34}$ J s, $\varphi = 4.64\times 10^{-19}\,\text{J} = 2.90\,\text{eV}$, and $f_0 = 7.00\times 10^{14}\,\text{Hz}$.

Example 4 — unscaffolded

In a photocell, light of frequency 7.00×10^{14} Hz falls on a metal surface of work function $2.10\,\text{eV}$. Calculate the maximum kinetic energy of the photoelectrons and the stopping voltage. The intensity of the light is then doubled, with the frequency unchanged; state what happens to the saturation photocurrent and to the stopping voltage.

Working in electron-volts with $h = 4.14\times 10^{-15}$ eV s,

$$hf = (4.14\times 10^{-15})(7.00\times 10^{14}) = 2.898\,\text{eV},$$

$$E_{k(\max)} = hf - \varphi = 2.898 - 2.10 = 0.798\,\text{eV} = 0.798\times 1.60\times 10^{-19} = 1.28\times 10^{-19}\,\text{J}.$$

The stopping voltage satisfies $q_e V_0 = E_{k(\max)}$, so

$$V_0 = \frac{1.28 \times 10^{-19}}{1.60 \times 10^{-19}} = 0.798\ \text{V}.$$

Doubling the intensity at the same frequency sends twice as many photons onto the surface each second, so twice as many photoelectrons are released each second and the **saturation photocurrent doubles**. The maximum kinetic energy depends only on the frequency (through $hf - \varphi$), which is unchanged, so the **stopping voltage is unchanged** at $0.798\ \text{V}$.

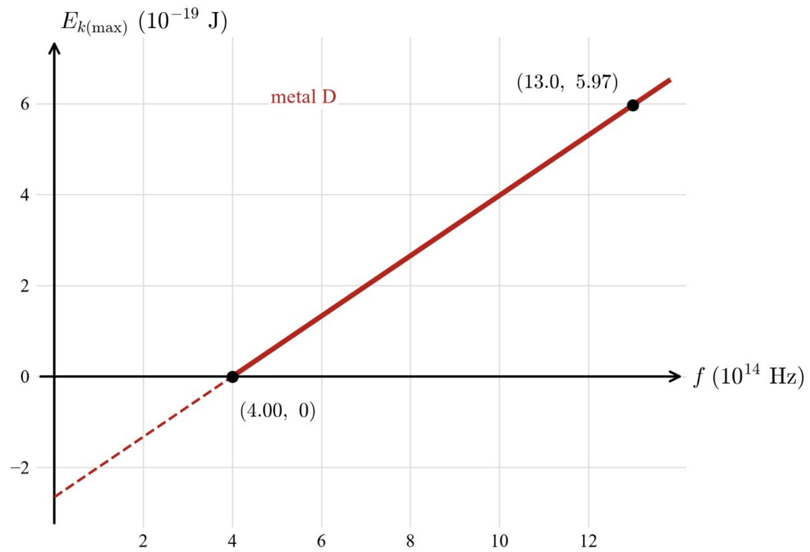
$\therefore\ E_{k(\max)} = 0.798\ \text{eV}$ and $V_0 = 0.798\ \text{V}$; doubling the intensity doubles the saturation current but leaves the stopping voltage unchanged.

Practice questions

Question 1 Green light has a wavelength of $520\ \text{nm}$. (a) Calculate the frequency of the light. (b) Calculate the energy of one photon in joules. (c) Express the photon energy in electron-volts.

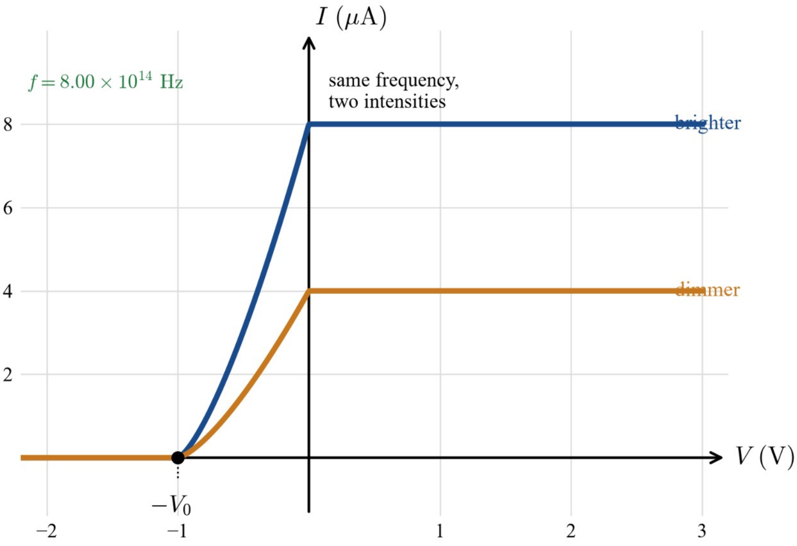
Question 2 A metal has a work function of $2.28\ \text{eV}$. Light of frequency $9.0 \times 10^{14}\ \text{Hz}$ is shone onto it. (a) Calculate the threshold frequency of the metal and state whether photoelectrons are emitted. (b) Calculate the maximum kinetic energy of the photoelectrons, in electron-volts and in joules. (c) Calculate the stopping voltage.

Question 3 The graph shows the maximum kinetic energy of photoelectrons emitted from a metal (metal D) as a function of the frequency of the incident light.



(a) State the threshold frequency of metal D.
(b) Using the two marked points, determine the gradient of the line and state the physical quantity it represents.
(c) Determine the work function of metal D, in joules and in electron-volts.

Question 4 The graph shows the photocurrent from a photocell as the collector potential is varied, for light of frequency $8.00 \times 10^{14}\ \text{Hz}$ at two different intensities.



(a) Read the stopping voltage from the graph and hence calculate the maximum kinetic energy of the photoelectrons, in joules and in electron-volts.
(b) Calculate the work function of the metal, in electron-volts.
(c) The two curves reach zero at the same stopping voltage but have different saturation currents. Explain why.

Question 5 A clean metal surface is illuminated with monochromatic light of wavelength $400\,\mathrm{nm}$. The wavelength is then changed to $600\,\mathrm{nm}$ while the **power** delivered to the surface is kept the same. Both wavelengths are above the threshold for emission. (a) Calculate the energy of a $400\,\mathrm{nm}$ photon and of a $600\,\mathrm{nm}$ photon, in joules and in electron-volts. (b) State and explain what happens to the maximum kinetic energy of the photoelectrons. (c) State and explain what happens to the saturation photocurrent, and calculate the factor by which the number of photoelectrons emitted per second changes.

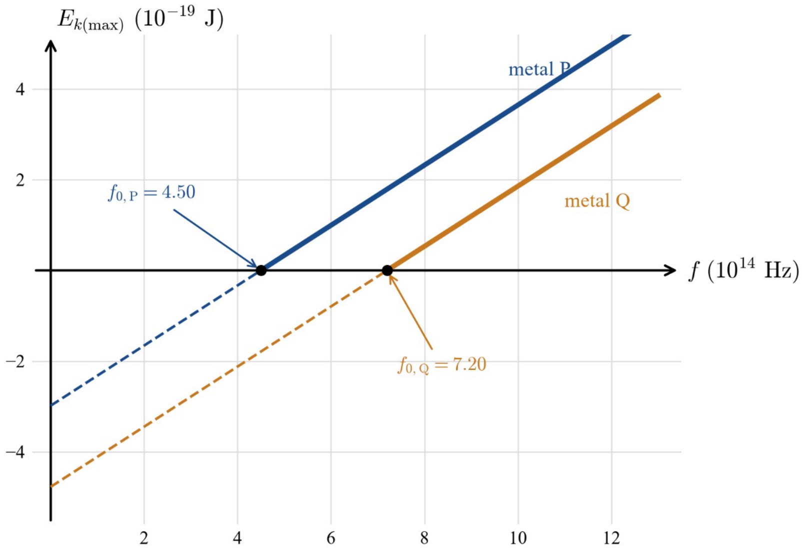
Question 6 The photoelectric effect cannot be explained by the wave model of light. For each observation, state what the wave model predicts and explain how the photon model accounts for the observation. (a) No photoelectrons are emitted if the frequency of the light is below a threshold frequency, no matter how intense the light. (b) The maximum kinetic energy of the photoelectrons depends on the frequency of the light but not on its intensity. (c) Photoelectrons are emitted almost instantaneously even when the light is very dim.

Question 7 Ultraviolet light has a wavelength of $250\,\mathrm{nm}$. Calculate the energy of one of its photons in joules and in electron-volts. Give each answer to three significant figures.

Question 8 A laser emits $5.0\,\mathrm{mW}$ of light of wavelength $650\,\mathrm{nm}$. Calculate the number of photons the laser emits each second.

Question 9 A metal has a work function of $2.50\,\mathrm{eV}$. Calculate the threshold frequency and the threshold wavelength of the metal.

Question 10 The graph shows lines of maximum photoelectron kinetic energy against frequency for two metals, P and Q.



(a) The two lines are parallel. State the quantity their common gradient represents.
(b) State, with a reason, which metal has the larger work function.
(c) Determine the work function of each metal, in electron-volts.
(d) Light of frequency 1.00×10^{15} Hz is shone on each metal. Calculate the maximum kinetic energy of the photoelectrons from each, in joules.

Question 11 Light of wavelength 380 nm strikes a metal of work function 1.95 eV. (a) Calculate the maximum kinetic energy of the photoelectrons, in joules and in electron-volts. (b) Calculate the stopping voltage. (c) Calculate the maximum speed of the photoelectrons.

Question 12 A metal has a work function of 2.16 eV. (a) Show that its threshold wavelength is 5.76×10^{-7} m (that is, 576 nm). (b) Light of wavelength 500 nm is shone on the metal. Calculate the maximum kinetic energy of the photoelectrons, in joules and in electron-volts. (c) Light of wavelength 600 nm is then shone on the metal. State, with a reason, whether photoelectrons are emitted.

Question 13 A photocathode is illuminated by monochromatic light whose wavelength is reduced from 500 nm to 400 nm while the **power** delivered to the surface is held constant. Both wavelengths are above the threshold for emission. State and explain what happens to (a) the maximum kinetic energy of the photoelectrons and (b) the number of photoelectrons emitted per second, calculating the factor by which the number changes.

Question 14 A photon of light has energy 3.10×10^{-19} J. (a) Express this energy in electron-volts. (b) Calculate the frequency of the light. (c) Calculate the wavelength of the light and state the region of the visible spectrum in which it lies.

Question 15 In a photoelectric experiment, the stopping voltage for the emitted photoelectrons is measured to be 1.75 V. (a) State the maximum kinetic energy of the photoelectrons, in electron-volts. (b) Hence express the maximum kinetic energy in joules. (c) Calculate the maximum speed of the photoelectrons.

Question 16 In a photoelectric experiment the maximum kinetic energy of the photoelectrons was measured at two frequencies of incident light: at $f = 5.00 \times 10^{14}$ Hz, $E_{k(\max)} = 1.40 \times 10^{-19}$ J; at $f = 11.0 \times 10^{14}$ Hz, $E_{k(\max)} = 5.38 \times 10^{-19}$ J. Determine Planck's constant, the work function of the metal (in joules and electron-volts), and the threshold frequency.

Question 17 Monochromatic light of a single frequency, above the threshold, falls on a photocathode. The intensity of the light is then tripled, with the frequency unchanged. State, with a justification for each, what happens to (a) the saturation photocurrent, (b) the stopping voltage, and (c) the maximum kinetic energy of the photoelectrons.

Question 18 Red light of wavelength 680 nm and blue light of wavelength 450 nm, of equal intensity, are shone in turn on the same metal, which has a work function of 1.85 eV. Determine which colour, if either, causes photoemission, and compare the maximum kinetic energies of any photoelectrons produced.

Question 19 Light of frequency $1.20 \times 10^{15}\,\text{Hz}$ ejects photoelectrons from a metal of work function $2.40\,\text{eV}$. Calculate the maximum kinetic energy of the photoelectrons in electron-volts and in joules, and the stopping voltage.

Question 20 Photoelectrons are emitted from a surface with a maximum kinetic energy of $3.0\,\text{eV}$. Calculate the maximum speed of the photoelectrons.

Question 21 In an experiment, a student measures the stopping voltage V_0 for photoelectrons emitted from a metal at several frequencies f and plots V_0 against f, obtaining a straight line. (a) Show that the gradient of a graph of V_0 against f is equal to $\frac{h}{q_e}$. (b) The measured gradient is $4.10 \times 10^{-15}\,\text{V s}$. Determine a value for Planck's constant from this experiment. (c) The line meets the frequency axis at $5.8 \times 10^{14}\,\text{Hz}$. Determine the work function of the metal in electron-volts. (d) State one feature of photoelectric results that the wave model of light cannot explain.

Answers

Question 1 (a) 5.77×10^{14} Hz; (b) 3.82×10^{-19} J; (c) 2.39 eV. **Question 2** (a) $f_0=5.51\times10^{14}$ Hz; yes ($9.0\times10^{14}>f_0$); (b) $1.45\text{ eV}=2.31\times10^{-19}$ J; (c) 1.45 V. **Question 3** (a) 4.00×10^{14} Hz; (b) 6.63×10^{-34} J s — Planck's constant; (c) $2.65\times10^{-19}\text{ J}=1.66$ eV. **Question 4** (a) $V_0=1.00$ V, $E_{k(\max)}=1.60\times10^{-19}\text{ J}=1.00$ eV; (b) 2.31 eV; (c) more intense light delivers more photons per second, so a larger saturation current, but the stopping voltage depends only on the (unchanged) frequency. **Question 5** (a) 400 nm: $4.97\times10^{-19}\text{ J}=3.11$ eV; 600 nm: $3.31\times10^{-19}\text{ J}=2.07$ eV; (b) $E_{k(\max)}$ decreases; (c) saturation current increases, by a factor of 1.5. **Question 6** (a)–(c) See solutions (threshold frequency, frequency-not-intensity, instantaneous emission). **Question 7** $7.96\times10^{-19}\text{ J}=4.97$ eV. **Question 8** 1.63×10^{16} photons per second. **Question 9** $f_0=6.03\times10^{14}$ Hz; $\lambda_0=4.97\times10^{-7}$ m (497 nm). **Question 10** (a) Planck's constant h; (b) metal Q (higher threshold frequency); (c) $\varphi_P=1.86$ eV, $\varphi_Q=2.98$ eV; (d) $E_{k,P}=3.65\times10^{-19}$ J, $E_{k,Q}=1.86\times10^{-19}$ J. **Question 11** (a) $2.11\times10^{-19}\text{ J}=1.32$ eV; (b) 1.32 V; (c) $6.81\times10^{5}\text{ m s}^{-1}$. **Question 12** (a) 5.76×10^{-7} m; (b) $5.22\times10^{-20}\text{ J}=0.326$ eV; (c) no — $600\text{ nm}>\lambda_0$, so $hf<\varphi$. **Question 13** (a) $E_{k(\max)}$ increases; (b) number per second decreases, by a factor of 0.8. **Question 14** (a) 1.94 eV; (b) 4.68×10^{14} Hz; (c) 6.42×10^{-7} m (642 nm), in the red region of the visible spectrum. **Question 15** (a) 1.75 eV; (b) 2.80×10^{-19} J; (c) $7.84\times10^{5}\text{ m s}^{-1}$. **Question 16** $h=6.63\times10^{-34}$ J s; $\varphi=1.91\times10^{-19}\text{ J}=1.20$ eV; $f_0=2.89\times10^{14}$ Hz. **Question 17** (a) triples; (b) unchanged; (c) unchanged. **Question 18** Red: no emission ($680\text{ nm}>\lambda_0=672\text{ nm}$); blue: emission, $E_{k(\max)}=1.46\times10^{-19}\text{ J}=0.913$ eV. **Question 19** $2.57\text{ eV}=4.11\times10^{-19}$ J; $V_0=2.57$ V. **Question 20** $1.03\times10^{6}\text{ m s}^{-1}$. **Question 21** (a) proof; (b) $h=6.56\times10^{-34}$ J s; (c) $\varphi=2.38$ eV; (d) e.g. the threshold frequency.

Fully-worked solutions

Question 1 $\lambda=520\text{ nm}=520\times10^{-9}$ m. (a) $f=\dfrac{c}{\lambda}=\dfrac{3.00\times10^{8}}{520\times10^{-9}}=5.77\times10^{14}$ Hz. (b) $E=\dfrac{hc}{\lambda}=\dfrac{(6.63\times10^{-34})(3.00\times10^{8})}{520\times10^{-9}}=3.82\times10^{-19}$ J. (c) $E=\dfrac{3.82\times10^{-19}}{1.60\times10^{-19}}=2.39$ eV.

Question 2 $\varphi=2.28$ eV, $f=9.0\times10^{14}$ Hz, $h=4.14\times10^{-15}$ eV s. (a) $f_0=\dfrac{\varphi}{h}=\dfrac{2.28}{4.14\times10^{-15}}=5.51\times10^{14}$ Hz. Since $9.0\times10^{14}\text{ Hz}>f_0$, photoelectrons **are** emitted. (b) $hf=(4.14\times10^{-15})(9.0\times10^{14})=3.726$ eV, so $E_{k(\max)}=hf-\varphi=3.726-2.28=1.45\text{ eV}=1.45\times1.60\times10^{-19}=2.31\times10^{-19}$ J. (c) $q_eV_0=E_{k(\max)}\Rightarrow V_0=\dfrac{2.31\times10^{-19}}{1.60\times10^{-19}}=1.45$ V.

Question 3 (a) The line meets the frequency axis at $f_0=4.00\times10^{14}$ Hz. (b) Using the marked points $(4.00\times10^{14}\text{ Hz},0)$ and $(13.0\times10^{14}\text{ Hz},5.97\times10^{-19}\text{ J})$,

$$\text{gradient}=\frac{5.97\times10^{-19}-0}{(13.0-4.00)\times10^{14}}=\frac{5.97\times10^{-19}}{9.00\times10^{14}}=6.63\times10^{-34}\text{ J s}.$$

This gradient is **Planck's constant** h. (c) $\varphi=hf_0=(6.63\times10^{-34})(4.00\times10^{14})=2.65\times10^{-19}\text{ J}=\dfrac{2.65\times10^{-19}}{1.60\times10^{-19}}=1.66$ eV.

Question 4 The two curves are for the same frequency, $f=8.00\times10^{14}$ Hz. (a) Both curves reach zero at $V_0=1.00$ V. The maximum kinetic energy is $E_{k(\max)}=q_eV_0=(1.60\times10^{-19})(1.00)=1.60\times10^{-19}\text{ J}=1.00$ eV. (b) $E_{k(\max)}=hf-\varphi$, so $\varphi=hf-E_{k(\max)}$. With $h=4.14\times10^{-15}$ eV s, $hf=(4.14\times10^{-15})(8.00\times10^{14})=3.312$ eV, giving $\varphi = 3.312 - 1.00 = 2.31\,\text{eV}$. (c) The two curves are for the same frequency but different intensities. The stopping voltage is set by $E_{k(\max)} = hf - \varphi$, which depends only on the frequency; both curves therefore share the same stopping voltage. The more intense beam delivers more photons per second, so it releases more photoelectrons per second and reaches a higher saturation current.

Question 5 (a) $E = \frac{hc}{\lambda}$. For $400\,\text{nm}$: $E_1 = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{400\times10^{-9}} = 4.97\times10^{-19}\,\text{J} = 3.11\,\text{eV}$. For $600\,\text{nm}$: $E_2 = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{600\times10^{-9}} = 3.31\times10^{-19}\,\text{J} = 2.07\,\text{eV}$. (b) Going from $400\,\text{nm}$ to $600\,\text{nm}$ lowers the frequency, so each photon carries less energy. Since $E_{k(\max)} = hf - \varphi$, the maximum kinetic energy **decreases** (and the stopping voltage falls). (c) At constant power P, the number of photons per second is $R = \frac{P}{hf} = \frac{P\lambda}{hc}$, which is proportional to λ. Increasing the wavelength from $400\,\text{nm}$ to $600\,\text{nm}$ therefore multiplies the photon rate by $\frac{600}{400} = 1.5$. Each photon above threshold frees one photoelectron, so the number of photoelectrons per second — and hence the saturation photocurrent — **increases by a factor of** 1.5, even though each electron emerges with less energy.

Question 6 (a) *Wave model:* a wave delivers energy continuously and its energy depends on intensity, so a bright enough beam of any frequency should, given time, supply an electron with enough energy to escape — the wave model predicts emission at every frequency. *Observation:* below the threshold frequency, nothing is emitted at any intensity. *Photon model:* light arrives as photons of energy hf, and one electron absorbs one photon; if $hf < \varphi$ (that is, $f < f_0 = \varphi / h$) a single photon cannot supply the work function, so no electron escapes. A more intense beam merely sends more photons, each still of energy $hf < \varphi$, so still none is emitted. (b) *Wave model:* a more intense wave carries more energy, so it should eject electrons with greater kinetic energy — the wave model predicts $E_{k(\max)}$ to rise with intensity. *Observation:* $E_{k(\max)}$ rises with frequency and is independent of intensity. *Photon model:* each electron gains the energy of a single photon; the most energetic escape with $E_{k(\max)} = hf - \varphi$, which grows with f but not with the number of photons. Greater intensity means more photons per second (more electrons), not more energy per electron. (c) *Wave model:* a dim wave spreads its small power over the whole surface, so an electron would need a measurable time to accumulate enough energy — the wave model predicts a time delay before emission at low intensity. *Observation:* emission begins almost immediately even for very dim light. *Photon model:* a single photon with $hf \geq \varphi$ is absorbed by a single electron in one step, so emission occurs as soon as the first suitable photon arrives, however few photons per second there are.

Question 7 $\lambda = 250\,\text{nm} = 250\times10^{-9}\,\text{m}$.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{250\times10^{-9}} = 7.96\times10^{-19}\,\text{J} = \frac{7.96\times10^{-19}}{1.60\times10^{-19}} = 4.97\,\text{eV}.$$

Question 8 Each photon carries $E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{650\times10^{-9}} = 3.06\times10^{-19}\,\text{J}$. The laser emits energy at $P = 5.0\times10^{-3}\,\text{W} = 5.0\times10^{-3}\,\text{J s}^{-1}$, so the number of photons per second is

$$R = \frac{P}{E} = \frac{5.0\times10^{-3}}{3.06\times10^{-19}} = 1.63\times10^{16}\,\text{photons per second}.$$

Question 9 $\varphi = 2.50\,\text{eV} = 2.50\times1.60\times10^{-19} = 4.00\times10^{-19}\,\text{J}$.

$$f_0 = \frac{\varphi}{h} = \frac{4.00\times10^{-19}}{6.63\times10^{-34}} = 6.03\times10^{14}\,\text{Hz},\ \lambda_0 = \frac{hc}{\varphi} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{4.00\times10^{-19}} = 4.97\times10^{-7}\,\text{m}\,(497\,\text{nm}).$$

Question 10 (a) The common gradient represents **Planck's constant** h. (b) Metal **Q** has the larger work function: its line crosses the frequency axis at a higher threshold frequency, and $\varphi = hf_0$, so a higher f_0 means a larger φ. (c) Reading the frequency-axis intercepts, $f_{0,P} = 4.50 \times 10^{14}$ Hz and $f_{0,Q} = 7.20 \times 10^{14}$ Hz, so

$$\varphi_P = hf_{0,P} = (6.63 \times 10^{-34})(4.50 \times 10^{14}) = 2.98 \times 10^{-19}\,\text{J} = 1.86\,\text{eV},$$

$$\varphi_Q = hf_{0,Q} = (6.63 \times 10^{-34})(7.20 \times 10^{14}) = 4.77 \times 10^{-19}\,\text{J} = 2.98\,\text{eV}.$$

(d) At $f = 1.00 \times 10^{15}$ Hz, $hf = (6.63 \times 10^{-34})(1.00 \times 10^{15}) = 6.63 \times 10^{-19}$ J, so

$$E_{k,P} = hf - \varphi_P = 6.63 \times 10^{-19} - 2.98 \times 10^{-19} = 3.65 \times 10^{-19}\,\text{J},$$

$$E_{k,Q} = hf - \varphi_Q = 6.63 \times 10^{-19} - 4.77 \times 10^{-19} = 1.86 \times 10^{-19}\,\text{J}.$$

Question 11 $\lambda = 380\,\text{nm}$, $\varphi = 1.95\,\text{eV} = 3.12 \times 10^{-19}$ J. (a) $E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{380 \times 10^{-9}} = 5.23 \times 10^{-19}$ J, so $E_{k(\max)} = E - \varphi = 5.23 \times 10^{-19} - 3.12 \times 10^{-19} = 2.11 \times 10^{-19}\,\text{J} = \frac{2.11 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.32\,\text{eV}$. (b) $V_0 = \frac{E_{k(\max)}}{q_e} = \frac{2.11 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.32\,\text{V}$. (c) $1/2\, m_e v^2 = E_{k(\max)} \Rightarrow v = \sqrt{\frac{2(2.11 \times 10^{-19})}{9.11 \times 10^{-31}}} = 6.81 \times 10^5\,\text{m s}^{-1}$.

Question 12 $\varphi = 2.16\,\text{eV} = 2.16 \times 1.60 \times 10^{-19} = 3.456 \times 10^{-19}$ J. (a) $\lambda_0 = \frac{hc}{\varphi} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{3.456 \times 10^{-19}} = 5.76 \times 10^{-7}\,\text{m}$ (576 nm), as required. (b) At $\lambda = 500\,\text{nm}$ (which is shorter than λ_0, so emission occurs), $E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{500 \times 10^{-9}} = 3.98 \times 10^{-19}$ J, so

$$E_{k(\max)} = E - \varphi = 3.98 \times 10^{-19} - 3.456 \times 10^{-19} = 5.22 \times 10^{-20}\,\text{J} = \frac{5.22 \times 10^{-20}}{1.60 \times 10^{-19}} = 0.326\,\text{eV}.$$

(c) At $\lambda = 600\,\text{nm}$, the wavelength is **longer** than the threshold wavelength $\lambda_0 = 576\,\text{nm}$, so the photon energy is less than the work function ($hf < \varphi$). No photoelectrons are emitted.

Question 13 (a) Reducing the wavelength from $500\,\text{nm}$ to $400\,\text{nm}$ raises the frequency, so each photon carries more energy. Since $E_{k(\max)} = hf - \varphi$, the maximum kinetic energy **increases**. (b) At constant power the photon rate is $R = \frac{P\lambda}{hc}$, proportional to λ. Reducing the wavelength from $500\,\text{nm}$ to $400\,\text{nm}$ multiplies the photon rate by $\frac{400}{500} = 0.8$. Because each above-threshold photon frees one photoelectron, the number of photoelectrons emitted per second **falls to** 0.8 of its former value (a $20\,\%$ decrease), even though each electron now carries more energy.

Question 14 $E = 3.10 \times 10^{-19}$ J. (a) $E = \frac{3.10 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.94\,\text{eV}$. (b) $E = hf$, so $f = \frac{E}{h} = \frac{3.10 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.68 \times 10^{14}$ Hz. (c) $\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{3.10 \times 10^{-19}} = 6.42 \times 10^{-7}\,\text{m}$ (642 nm), which lies in the red region of the visible spectrum.

Question 15 $V_0 = 1.75\,\text{V}$. (a) Since $q_e V_0 = E_{k(\max)}$, and one electron-volt is the energy an electron gains moving through $1\,\text{V}$, $E_{k(\max)} = 1.75\,\text{eV}$. (b) $E_{k(\max)} = 1.75 \times 1.60 \times 10^{-19} = 2.80 \times 10^{-19}$ J. (c) $1/2\, m_e v^2 = E_{k(\max)} \Rightarrow v = \sqrt{\frac{2(2.80 \times 10^{-19})}{9.11 \times 10^{-31}}} = 7.84 \times 10^5\,\text{m s}^{-1}$.

Question 16 The data satisfy $E_{k(\max)} = hf - \varphi$, so a graph of $E_{k(\max)}$ against f has gradient h and vertical-axis intercept $-\varphi$. Using the two points,

$$h = \frac{\Delta E_k}{\Delta f} = \frac{5.38\times10^{-19} - 1.40\times10^{-19}}{(11.0-5.00)\times10^{14}} = \frac{3.98\times10^{-19}}{6.00\times10^{14}} = 6.63\times10^{-34}\,\text{J s}.$$

The work function follows from either point; using the first,

$$\varphi = hf - E_{k(\max)} = (6.63\times10^{-34})(5.00\times10^{14}) - 1.40\times10^{-19} = 3.315\times10^{-19} - 1.40\times10^{-19} = 1.91\times10^{-19}\,\text{J},$$

which is $\dfrac{1.91\times10^{-19}}{1.60\times10^{-19}} = 1.20\,\text{eV}$. The threshold frequency is

$$f_0 = \frac{\varphi}{h} = \frac{1.91\times10^{-19}}{6.63\times10^{-34}} = 2.89\times10^{14}\,\text{Hz}.$$

Question 17 The frequency is unchanged, so: (a) The saturation photocurrent **triples**. Intensity fixes the number of photons per second; tripling the intensity triples the photons per second and hence the photoelectrons collected per second. (b) The stopping voltage is **unchanged**. It is set by $q_e V_0 = E_{k(\max)} = hf - \varphi$, which depends only on the (unchanged) frequency. (c) The maximum kinetic energy is **unchanged**, for the same reason: $E_{k(\max)} = hf - \varphi$ depends on frequency, not intensity.

Question 18 $\varphi = 1.85\,\text{eV} = 1.85\times1.60\times10^{-19} = 2.96\times10^{-19}\,\text{J}$. The threshold wavelength is $\lambda_0 = \dfrac{hc}{\varphi} = \dfrac{(6.63\times10^{-34})(3.00\times10^{8})}{2.96\times10^{-19}} = 6.72\times10^{-7}$ m (672 nm). Red light (680 nm) has a **longer** wavelength than λ_0: its photon energy is $E_{\text{red}} = \dfrac{hc}{\lambda} = 2.92\times10^{-19}\,\text{J}$, which is less than $\varphi = 2.96\times10^{-19}\,\text{J}$, so **red light causes no emission** (however intense it is). Blue light (450 nm) has a shorter wavelength than λ_0: its photon energy is $E_{\text{blue}} = \dfrac{hc}{\lambda} = 4.42\times10^{-19}\,\text{J}$, which exceeds φ, so **blue light does eject photoelectrons**, with

$$E_{k(\max)} = E_{\text{blue}} - \varphi = 4.42\times10^{-19} - 2.96\times10^{-19} = 1.46\times10^{-19}\,\text{J} = 0.913\,\text{eV}.$$

So only the blue light causes emission; the red light — despite equal intensity — produces none.

Question 19 $\varphi = 2.40\,\text{eV}$, $f = 1.20\times10^{15}\,\text{Hz}$, $h = 4.14\times10^{-15}\,\text{eV s}$.

$$hf = (4.14\times10^{-15})(1.20\times10^{15}) = 4.968\,\text{eV},\ E_{k(\max)} = hf - \varphi = 4.968 - 2.40 = 2.57\,\text{eV}.$$

In joules, $E_{k(\max)} = 2.57\times1.60\times10^{-19} = 4.11\times10^{-19}\,\text{J}$, and the stopping voltage is $V_0 = \dfrac{E_{k(\max)}}{q_e} = 2.57\,\text{V}$.

Question 20 $E_{k(\max)} = 3.0\,\text{eV} = 3.0\times1.60\times10^{-19} = 4.80\times10^{-19}\,\text{J}$.

$$1/2\, m_e v^2 = E_{k(\max)} \Rightarrow v = \sqrt{\frac{2(4.80\times10^{-19})}{9.11\times10^{-31}}} = 1.03\times10^{6}\,\text{m s}^{-1}.$$

Question 21 (a) The stopping voltage satisfies $q_e V_0 = E_{k(\max)} = hf - \varphi$. Dividing by q_e,

$$V_0 = \frac{h}{q_e} f - \frac{\varphi}{q_e}.$$

This is a straight line of V_0 against f with gradient $\frac{h}{q_e}$, as required. (b) $h = \text{gradient} \times q_e = (4.10 \times 10^{-15})(1.60 \times 10^{-19}) = 6.56 \times 10^{-34}$ J s. (c) At the frequency-axis intercept $V_0 = 0$, so $\frac{h}{q_e} f_0 = \frac{\varphi}{q_e}$, giving $\varphi = h f_0$. Expressed in electron-volts, $\varphi = \text{gradient} \times f_0 = (4.10 \times 10^{-15})(5.8 \times 10^{14}) = 2.38\,\text{eV}$. (d) Any one of: the existence of a threshold frequency; the fact that $E_{k(\max)}$ depends on frequency but not intensity; or the essentially instantaneous emission even at very low intensity.