Physics Units 3 & 4

Chapter 6 — Transmission of electricity

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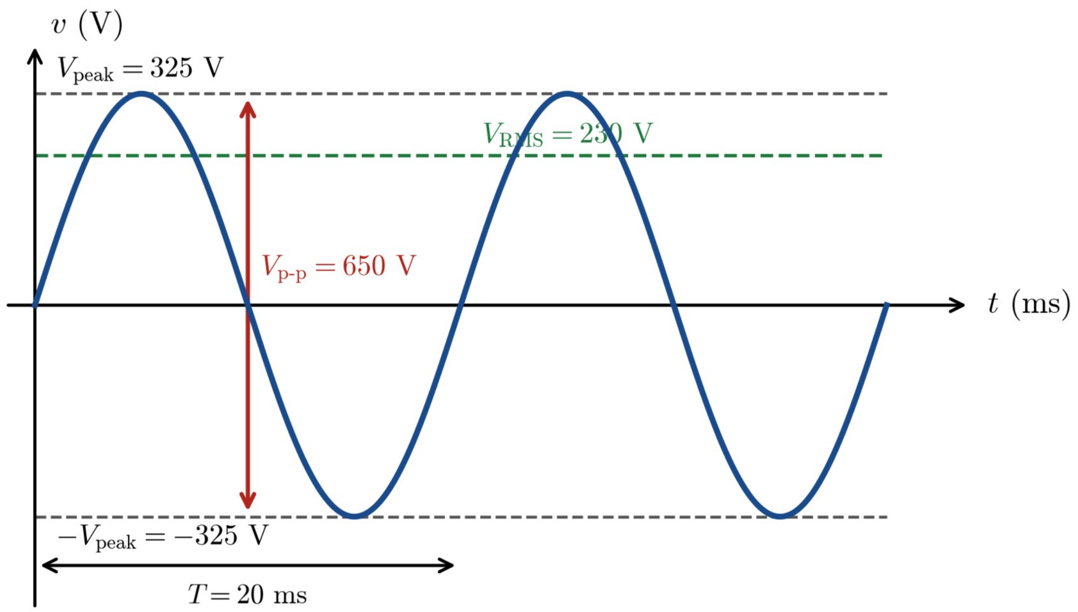
Contents

Theory

When a loop rotates uniformly in a constant magnetic field it generates an **alternating** voltage — a voltage that varies **sinusoidally** with time, driving a current that reverses direction every half turn. The waveform is described by a small set of quantities. The **peak voltage** V_{peak} (the *amplitude*) is the greatest voltage reached in either direction; the **peak-to-peak voltage** $V_{\text{p-p}}$ is the full swing from the positive peak to the negative peak, so

$$V_{\text{p-p}} = 2V_{\text{peak}}, I_{\text{p-p}} = 2I_{\text{peak}}.$$

The **period** T is the time for one complete cycle and the **frequency** $f = \frac{1}{T}$ is the number of cycles each second (in Hz). These features are read directly off a voltage–time graph.



Why a simple average is not useful. Over one complete cycle a sinusoidal voltage spends as much time positive as negative, so its ordinary time-average is **zero**. Yet an alternating current clearly delivers energy — it heats a toaster element and lights a globe. To describe how much power an AC supply actually delivers, we need a measure of the *effective* size of the alternating voltage and current, not their average.

The root-mean-square (rms) value — a DC-equivalent. The effective measure is the **root-mean-square** value. The rms value of an alternating voltage is defined as the value of the **constant DC voltage that would develop the same average power in a resistive component**. In other words, if a resistor dissipates a certain average power when connected to an AC supply, then V_{RMS} is the steady DC voltage that, applied to the *same* resistor, would dissipate that **same** power. The rms current I_{RMS} is defined in the same way. This is the physical meaning to hold on to: **rms voltage is the DC-equivalent voltage for power**.

For a sinusoidal waveform the rms value is smaller than the peak value by a factor of $\sqrt{2}$:

$$V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}}, I_{\text{RMS}} = \frac{I_{\text{peak}}}{\sqrt{2}}.$$

The $\sqrt{2}$ arises because power depends on the **square** of the voltage. As the sinusoid rises and falls, the quantity v^2 swings between zero and V_{peak}^2, and averaged over a complete cycle its mean is exactly **half** of V_{peak}^2. Taking the square **root** of the **mean** of the **square** (root-mean-square) therefore gives $V_{\text{peak}} / \sqrt{2}$.

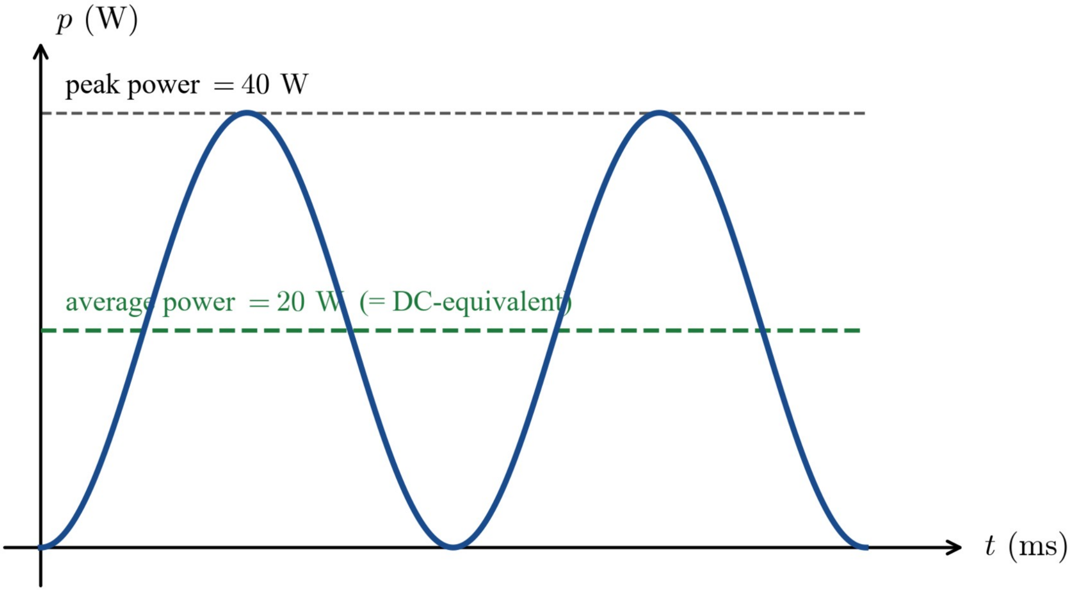
Average power in an AC circuit. Because V_{RMS} and I_{RMS} are the DC-equivalent values, the ordinary power relationships apply to a resistive component provided rms values are used throughout:

$$P = V_{\text{RMS}} I_{\text{RMS}} = \frac{V_{\text{RMS}}^2}{R} = I_{\text{RMS}}^2 R .$$

This P is the **average** power dissipated. It is *not* equal to $V_{\text{peak}} I_{\text{peak}}$. Substituting the $\sqrt{2}$ relationships shows how the two are connected:

$$P = V_{\text{RMS}} I_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}} \cdot \frac{I_{\text{peak}}}{\sqrt{2}} = \frac{V_{\text{peak}} I_{\text{peak}}}{2} = 1/2\, P_{\text{peak}} ,$$

where $P_{\text{peak}} = V_{\text{peak}} I_{\text{peak}}$ is the greatest **instantaneous** power. The average power delivered by a sinusoidal supply is exactly **half** the peak instantaneous power.



The figure above shows the instantaneous power $p = \frac{v^2}{R}$ for a resistor on a sinusoidal supply. The power is always positive (the resistor heats up on both halves of the cycle) and rises and falls between zero and the peak. For a peak instantaneous power of 40 W the average power is 20 W — and a steady DC supply delivering that same 20 W is precisely the DC-equivalent that the rms value represents.

Domestic AC. The Australian mains supply is quoted as 230 V — this is the **rms** voltage, the DC-equivalent for power. Its peak voltage is $V_{\text{peak}} = 230\sqrt{2} \approx 325$ V and its peak-to-peak voltage about 650 V. The power ratings printed on appliances (a 2400 W kettle, a 60 W globe) are the **average** powers drawn at the rms mains voltage, and the currents quoted for AC circuits are rms currents. Whenever a single number is given for an AC voltage or current without qualification, it is the rms value.

Throughout this subtopic the components are treated as **ohmic resistors**, for which $V = IR$ holds at every instant, so the peak, rms and instantaneous voltages and currents are in the same ratio R.

Worked examples

Example 1 — scaffolded

An alternating voltage across a resistor has a peak value of 325 V. Calculate (i) the peak-to-peak voltage, and (ii) the rms voltage.

Working	Comment
Sinusoidal AC: $V_{\text{peak}} = 325\,\text{V}$.	Identify the given peak (amplitude).
$V_{\text{p-p}} = 2\,V_{\text{peak}} = 2 \times 325 = 650\,\text{V}$.	Peak-to-peak is the full swing, twice the peak.
$V_{\text{RMS}} = \dfrac{V_{\text{peak}}}{\sqrt{2}} = \dfrac{325}{\sqrt{2}} = 230\,\text{V}$.	The rms (DC-equivalent) value is the peak divided by $\sqrt{2}$.
This is the Australian mains supply: $230\,\text{V}$ rms, $325\,\text{V}$ peak.	The quoted mains voltage is always the rms value.

Example 2 — scaffolded

A heating element of resistance $48\,\Omega$ is connected to an AC supply of rms voltage $240\,\text{V}$. Calculate (a) the rms current in the element, (b) the average power dissipated, and (c) the constant DC voltage that would dissipate the same power in the element.

Working	Comment
$V_{\text{RMS}} = 240\,\text{V}$, $R = 48\,\Omega$.	List the data; the element is an ohmic resistor.
$I_{\text{RMS}} = \dfrac{V_{\text{RMS}}}{R} = \dfrac{240}{48} = 5.00\,\text{A}$.	Use rms values in $V = I\,R$.
$P = V_{\text{RMS}}\,I_{\text{RMS}} = 240 \times 5.00 = 1.20 \times 10^{3}\,\text{W}$.	Average power, from the rms voltage and current.
Same power from DC: $V_{\text{DC}} = \sqrt{P\,R} = \sqrt{1200 \times 48} = 240\,\text{V}$.	The DC-equivalent voltage equals V_{RMS} — by definition.

Example 3 — unscaffolded

A resistor of $20\,\Omega$ carries an alternating current with a peak value of $3.0\,\text{A}$. Calculate the average power dissipated in the resistor.

The average power must be found from the **rms** current, not the peak current. The rms current is

$$I_{\text{RMS}} = \frac{I_{\text{peak}}}{\sqrt{2}} = \frac{3.0}{\sqrt{2}} = 2.12\,\text{A}.$$

The average power dissipated in the resistor is then

$$P = I_{\text{RMS}}^{2}\,R = \left(\frac{3.0}{\sqrt{2}}\right)^{2} \times 20 = 4.5 \times 20 = 90\,\text{W},$$

using the unrounded rms current. (Equivalently, the peak instantaneous power is $I_{\text{peak}}^{2}\,R = 3.0^{2} \times 20 = 180\,\text{W}$, and the average is exactly half of this.)

∴ the average power dissipated is $90\,\text{W}$.

Example 4 — unscaffolded

The voltage across a $15\,\Omega$ resistor varies sinusoidally with a peak-to-peak value of $48\,\text{V}$. Calculate the rms voltage, the rms current, and the average power dissipated.

The peak voltage is half the peak-to-peak value, $V_{\text{peak}} = \dfrac{48}{2} = 24\,\text{V}$, so the rms voltage is

$$V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}} = \frac{24}{\sqrt{2}} = 17.0\,\text{V}.$$

The rms current follows from $V = I\,R$ using rms values:

$$I_{\mathrm{RMS}} = \frac{V_{\mathrm{RMS}}}{R} = \frac{17.0}{15} = 1.13\,\mathrm{A}\,,$$

and the average power dissipated is

$$P = \frac{V_{\mathrm{RMS}}^2}{R} = \frac{\left(24/\sqrt{2}\right)^2}{15} = \frac{288}{15} = 19.2\,\mathrm{W}\,,$$

using the unrounded rms voltage.

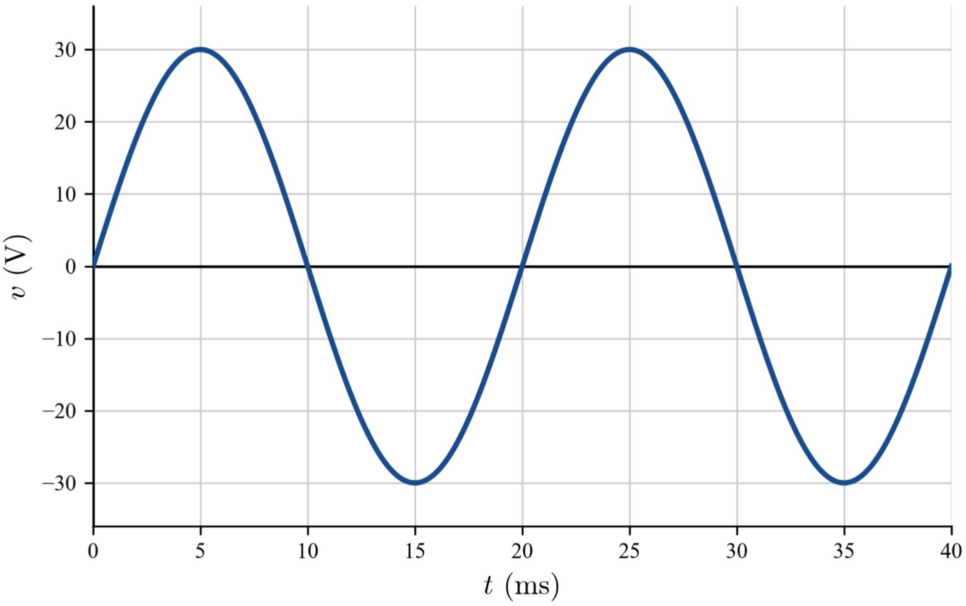
∴ the rms voltage is $17.0\,\mathrm{V}$, the rms current is $1.13\,\mathrm{A}$, and the average power is $19.2\,\mathrm{W}$.

Practice questions

Question 1 An AC generator produces a sinusoidal output with a peak voltage of $170\,\mathrm{V}$. (a) State the peak-to-peak voltage of the output. (b) Calculate the rms voltage of the output. (c) The generator is connected across a $60\,\Omega$ resistor. Calculate the rms current in the resistor.

Question 2 A resistor is connected to an AC supply that has an rms voltage of $18\,\mathrm{V}$. (a) Explain what is meant by the statement that the supply has an rms voltage of $18\,\mathrm{V}$, with reference to a constant DC voltage. No calculations are required. (b) The resistor has a resistance of $9.0\,\Omega$. Calculate the average power dissipated in it. (c) Calculate the peak voltage of the supply.

Question 3 The voltage across a $20\,\Omega$ resistor varies sinusoidally with time as shown in the graph below.



(a) From the graph, state the peak voltage across the resistor.
(b) Calculate the rms voltage.
(c) Calculate the average power dissipated in the resistor.

Question 4 A household toaster is rated at $1200\,\mathrm{W}$ and operates from the $230\,\mathrm{V}$ rms mains supply. (a) Calculate the rms current drawn by the toaster. (b) Calculate the peak current drawn by the toaster. (c) Calculate the resistance of the toaster's heating element, treating it as an ohmic resistor.

Question 5 An alternating current in a $10\,\Omega$ resistor has a peak value of $4.0\,\mathrm{A}$. (a) Calculate the rms current. (b) Calculate the average power dissipated in the resistor. (c) Calculate the maximum (peak) instantaneous power dissipated, and state how it compares with the average power.

Question 6 Two identical resistors are connected to power supplies. Resistor P is connected to a constant DC supply of $20\,\mathrm{V}$; resistor Q is connected to an AC supply with a peak voltage of $20\,\mathrm{V}$. (a) (Circle your answer.) The resistor connected to the AC supply dissipates *more* / *the same* / *less* average power than the resistor connected to the DC supply. Justify your choice. No calculations are required. (b) Calculate the rms voltage of the AC supply. (c) The DC supply voltage is now adjusted so that both resistors dissipate the same average power. State the new DC voltage.

Question 7 The voltage across a $33\,\Omega$ resistor varies sinusoidally with a peak-to-peak value of 34 V. Calculate the average power dissipated in the resistor. Give your answer to three significant figures.

Question 8 An electric kettle is rated at 2400 W and operates from a 240 V rms supply. Calculate the rms current and the peak current drawn by the kettle.

Question 9 Show that a sinusoidal current with a peak value of 6.0 A has an rms value of 4.24 A. Hence calculate the average power this current dissipates in a $12\,\Omega$ resistor.

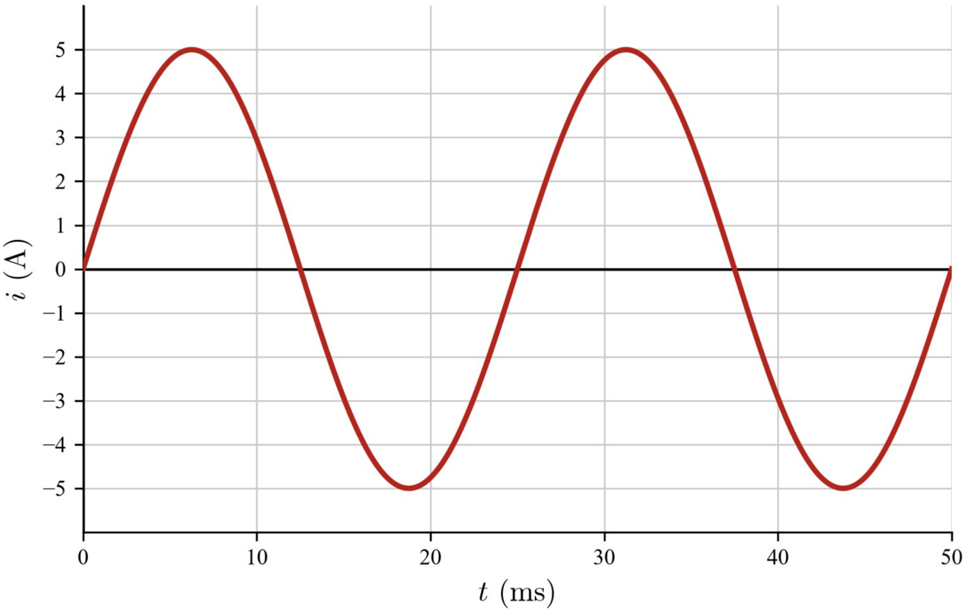
Question 10 A student measures the peak voltage of the AC mains as 325 V but notices that the supply is labelled "230 V". Explain, with reference to the power dissipated in a resistive component, why the mains is rated at 230 V rather than 325 V.

Question 11 An alternating voltage with a peak value of 12 V is applied across a $6.0\,\Omega$ resistor. Calculate the peak instantaneous power and the average power dissipated in the resistor.

Question 12 A $20\,\Omega$ resistor dissipates an average power of 80 W when connected to a sinusoidal AC supply. Calculate the rms voltage of the supply, the peak voltage of the supply, and the constant DC voltage that would dissipate the same power in the resistor.

Question 13 Explain why the average power delivered to a resistor by a sinusoidal AC supply is not equal to $V_{\text{peak}} \times I_{\text{peak}}$, and state the factor by which $V_{\text{peak}} \times I_{\text{peak}}$ must be multiplied to obtain the average power.

Question 14 The current through a $6.0\,\Omega$ resistor varies sinusoidally with time as shown in the graph below.



Calculate the rms current and the average power dissipated in the resistor.

Question 15 Two identical heating elements are used. Element X is connected to a steady DC voltage of 50 V. Element Y is connected to an AC supply whose peak voltage is 50 V. Compare the average power dissipated in the two elements, and explain your reasoning.

Question 16 A 60 W incandescent globe is designed to operate at its rated power from the 230 V rms mains supply. Calculate the rms current in the globe, the resistance of its filament at operating temperature, and the peak voltage across the globe.

Question 17 A resistor on a sinusoidal AC supply has a peak voltage of 40 V and a peak current of 2.5 A. A student calculates the average power dissipated as $P = V_{\text{peak}} I_{\text{peak}} = 40 \times 2.5 = 100\text{ W}$. (a) Explain what is wrong with the student's method. (b) Calculate the correct average power dissipated in the resistor. (c) Verify your answer to (b) by calculating the rms voltage and the rms current and using $P = V_{\text{RMS}} I_{\text{RMS}}$.

Question 18 An electric radiator has a heating element of resistance $46\,\Omega$ connected to the $230\,\text{V}$ rms mains supply. Calculate the rms current in the element, the average power dissipated, and the peak instantaneous power dissipated.

Answers

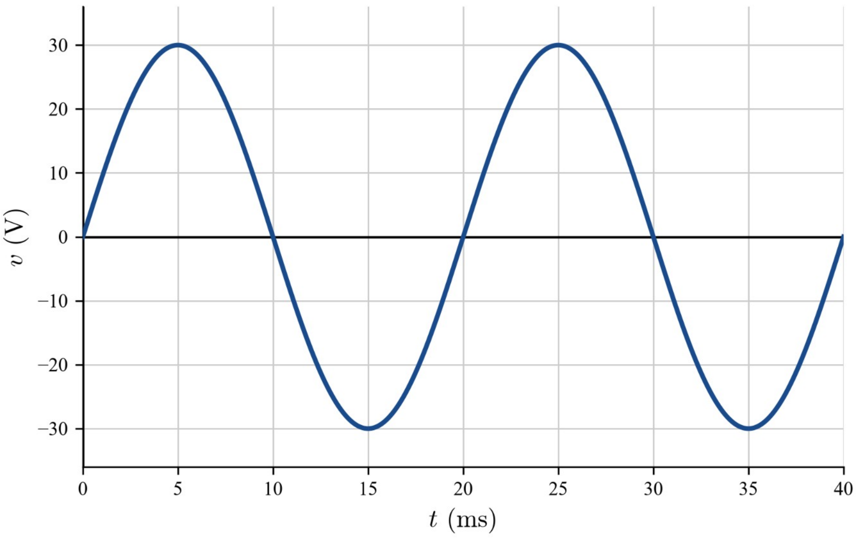
Question 1 (a) 340 V; (b) 120 V; (c) 2.00 A. **Question 2** (a) 18 V rms is the steady DC voltage that would dissipate the same average power in the resistor; (b) 36 W; (c) 25.5 V. **Question 3** (a) 30 V; (b) 21.2 V; (c) 22.5 W. **Question 4** (a) 5.22 A; (b) 7.38 A; (c) $44.1\,\Omega$. **Question 5** (a) 2.83 A; (b) 80 W; (c) 160 W, twice the average power. **Question 6** (a) *less*; (b) 14.1 V; (c) 14.1 V. **Question 7** 4.38 W. **Question 8** 10.0 A; 14.1 A. **Question 9** rms current 4.24 A; average power 216 W. **Question 10** The mains is rated at its rms (DC-equivalent) voltage $325/\sqrt{2}=230\text{ V}$, the voltage that determines the average power delivered. **Question 11** peak instantaneous power 24 W; average power 12 W. **Question 12** 40 V; 56.6 V; 40 V. **Question 13** Because v and i vary through the cycle; the factor is $1/2$. **Question 14** rms current 3.54 A; average power 75 W. **Question 15** Y dissipates half the average power of X. **Question 16** 0.261 A; $882\,\Omega$; 325 V. **Question 17** (a) $V_{\text{peak}} I_{\text{peak}}$ is the peak instantaneous power, twice the average power; (b) 50 W; (c) $V_{\text{RMS}}=28.3\text{ V}$, $I_{\text{RMS}}=1.77\text{ A}$, $P=V_{\text{RMS}} I_{\text{RMS}}=50\text{ W}$. **Question 18** 5.0 A; $1.15\times10^3\text{ W}$; $2.30\times10^3\text{ W}$.

Fully-worked solutions

Question 1 $V_{\text{peak}}=170\text{ V}$, $R=60\,\Omega$. (a) $V_{\text{p-p}}=2V_{\text{peak}}=2\times170=340\text{ V}$. (b) $V_{\text{RMS}}=\dfrac{V_{\text{peak}}}{\sqrt{2}}=\dfrac{170}{\sqrt{2}}=120\text{ V}$. (c) Using rms values, $I_{\text{RMS}}=\dfrac{V_{\text{RMS}}}{R}=\dfrac{120}{60}=2.00\text{ A}$.

Question 2 (a) An rms voltage of 18 V means that the alternating supply delivers, on average, the same power to the resistor as a **constant DC voltage of** 18 V would deliver to the same resistor. The rms value is the DC-equivalent voltage for power. (b) $P=\dfrac{V_{\text{RMS}}^2}{R}=\dfrac{18^2}{9.0}=\dfrac{324}{9.0}=36\text{ W}$. (c) $V_{\text{peak}}=V_{\text{RMS}}\sqrt{2}=18\sqrt{2}=25.5\text{ V}$.

Question 3 $R=20\,\Omega$.



(a) The curve reaches $+30\text{ V}$ and -30 V, so the peak voltage is $V_{\text{peak}}=30\text{ V}$ (the peak-to-peak value is 60 V).

(b) $V_{\text{RMS}}=\dfrac{V_{\text{peak}}}{\sqrt{2}}=\dfrac{30}{\sqrt{2}}=21.2\text{ V}$.

(c) $P=\dfrac{V_{\text{RMS}}^2}{R}=\dfrac{(30/\sqrt{2})^2}{20}=\dfrac{450}{20}=22.5\text{ W}$, using the unrounded rms voltage.

Question 4 $P = 1200\ \text{W}$, $V_{\text{RMS}} = 230\ \text{V}$. (a) $I_{\text{RMS}} = \dfrac{P}{V_{\text{RMS}}} = \dfrac{1200}{230} = 5.22\ \text{A}$. (b) $I_{\text{peak}} = I_{\text{RMS}}\sqrt{2} = 5.22 \times \sqrt{2} = 7.38\ \text{A}$. (c) $R = \dfrac{V_{\text{RMS}}^2}{P} = \dfrac{230^2}{1200} = \dfrac{52900}{1200} = 44.1\ \Omega$ (equivalently $R = \dfrac{V_{\text{RMS}}}{I_{\text{RMS}}} = \dfrac{230}{5.22}$).

Question 5 $R = 10\ \Omega$, $I_{\text{peak}} = 4.0\ \text{A}$. (a) $I_{\text{RMS}} = \dfrac{I_{\text{peak}}}{\sqrt{2}} = \dfrac{4.0}{\sqrt{2}} = 2.83\ \text{A}$. (b) $P = I_{\text{RMS}}^2 R = \left(\dfrac{4.0}{\sqrt{2}}\right)^2 \times 10 = 8.0 \times 10 = 80\ \text{W}$, using the unrounded rms current. (c) The peak instantaneous power is $P_{\text{peak}} = I_{\text{peak}}^2 R = 4.0^2 \times 10 = 160\ \text{W}$. This is **twice** the average power ($160 = 2 \times 80$), as expected for a sinusoidal supply.

Question 6 Identical resistors, resistance R. (a) **Less.** The average power dissipated depends on the rms (DC-equivalent) voltage. The AC supply has an rms voltage of $\dfrac{20}{\sqrt{2}} = 14.1\ \text{V}$, which is smaller than the steady $20\ \text{V}$ across resistor P. Since $P = \dfrac{V_{\text{RMS}}^2}{R}$, the smaller rms voltage means resistor Q dissipates less average power. (b) $V_{\text{RMS}} = \dfrac{V_{\text{peak}}}{\sqrt{2}} = \dfrac{20}{\sqrt{2}} = 14.1\ \text{V}$. (c) To dissipate the same average power in the identical resistor, the DC voltage must equal the AC rms voltage — that is what "DC-equivalent" means. The new DC voltage is $14.1\ \text{V}$.

Question 7 $R = 33\ \Omega$, $V_{\text{p-p}} = 34\ \text{V}$, so $V_{\text{peak}} = \dfrac{34}{2} = 17\ \text{V}$ and $V_{\text{RMS}} = \dfrac{17}{\sqrt{2}}\ \text{V}$.

$$P = \frac{V_{\text{RMS}}^2}{R} = \frac{(17/\sqrt{2})^2}{33} = \frac{144.5}{33} = 4.38\ \text{W}\ \text{(to three significant figures)}.$$

Question 8 $P = 2400\ \text{W}$, $V_{\text{RMS}} = 240\ \text{V}$. $I_{\text{RMS}} = \dfrac{P}{V_{\text{RMS}}} = \dfrac{2400}{240} = 10.0\ \text{A}$; $I_{\text{peak}} = I_{\text{RMS}}\sqrt{2} = 10.0 \times \sqrt{2} = 14.1\ \text{A}$.

Question 9 For a sinusoid, $I_{\text{RMS}} = \dfrac{I_{\text{peak}}}{\sqrt{2}} = \dfrac{6.0}{\sqrt{2}} = 4.24\ \text{A}$, as required. The average power in a $12\ \Omega$ resistor is

$$P = I_{\text{RMS}}^2 R = \left(\frac{6.0}{\sqrt{2}}\right)^2 \times 12 = 18.0 \times 12 = 216\ \text{W},$$

using the unrounded rms current.

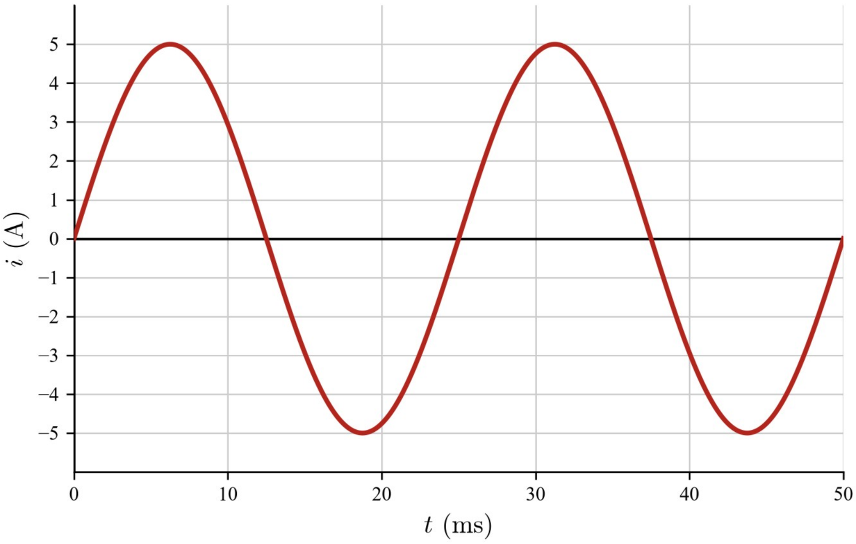
Question 10 The two numbers describe the same waveform. The peak voltage of $325\ \text{V}$ is the greatest instantaneous voltage, reached only momentarily at the top of each cycle; the voltage spends most of the cycle well below this and reverses direction each half cycle. What matters for the power delivered to a resistive component is the **rms** value, defined as the constant DC voltage that would dissipate the **same average power**. For this waveform $V_{\text{RMS}} = \dfrac{325}{\sqrt{2}} = 230\ \text{V}$. Rating the supply at $230\ \text{V}$ therefore tells the user the DC-equivalent voltage — the value that, used in $P = \dfrac{V_{\text{RMS}}^2}{R}$, correctly gives the average power a device will draw. Rating it at the peak $325\ \text{V}$ would overstate the effective voltage and the power delivered.

Question 11 $V_{\text{peak}} = 12\ \text{V}$, $R = 6.0\ \Omega$. Peak instantaneous power: $P_{\text{peak}} = \dfrac{V_{\text{peak}}^2}{R} = \dfrac{12^2}{6.0} = \dfrac{144}{6.0} = 24\ \text{W}$. Average power: $P = \dfrac{V_{\text{RMS}}^2}{R} = \dfrac{(12/\sqrt{2})^2}{6.0} = \dfrac{72}{6.0} = 12\ \text{W}$ — half the peak instantaneous power, as expected.

Question 12 $R = 20\,\Omega$, average power $P = 80\text{ W}$. From $P = \frac{V_{\text{RMS}}^2}{R}$, the rms voltage is $V_{\text{RMS}} = \sqrt{PR} = \sqrt{80 \times 20} = \sqrt{1600} = 40\text{ V}$. The peak voltage is $V_{\text{peak}} = V_{\text{RMS}}\sqrt{2} = 40\sqrt{2} = 56.6\text{ V}$. The constant DC voltage that would dissipate the same 80 W in the resistor is the DC-equivalent, equal to the rms voltage: 40 V.

Question 13 The instantaneous power delivered to the resistor is $p = v\,i$, and both v and i vary continuously through the cycle. The product $v\,i$ reaches its greatest value $V_{\text{peak}} I_{\text{peak}}$ only at the instants of peak voltage and current, and falls to zero each time the voltage passes through zero. So $V_{\text{peak}} I_{\text{peak}}$ is the **peak instantaneous** power, not the average. Because the average of the squared sinusoid over a cycle is one half, the average power is $P = V_{\text{RMS}} I_{\text{RMS}} = \frac{V_{\text{peak}} I_{\text{peak}}}{2}$. The factor is $1/2$.

Question 14 $R = 6.0\,\Omega$.



The current reaches $+5.0\text{ A}$ and -5.0 A, so $I_{\text{peak}} = 5.0\text{ A}$.

$$I_{\text{RMS}} = \frac{I_{\text{peak}}}{\sqrt{2}} = \frac{5.0}{\sqrt{2}} = 3.54\text{ A}\,,\, P = I_{\text{RMS}}^2 R = \left(\frac{5.0}{\sqrt{2}}\right)^2 \times 6.0 = 12.5 \times 6.0 = 75\text{ W}\,,$$

using the unrounded rms current.

Question 15 The elements are identical (same resistance R). Element X, on a steady DC voltage of 50 V, dissipates $P_X = \frac{50^2}{R}$. Element Y, on an AC supply of peak voltage 50 V, has an rms (DC-equivalent) voltage of $V_{\text{RMS}} = \frac{50}{\sqrt{2}} = 35.4\text{ V}$ and dissipates $P_Y = \frac{V_{\text{RMS}}^2}{R} = \frac{(50/\sqrt{2})^2}{R} = \frac{50^2/2}{R}$. Taking the ratio,

$$\frac{P_Y}{P_X} = \frac{50^2/2}{50^2} = \frac{1}{2}.$$

Element Y dissipates **half** the average power of element X, because it is the rms voltage (35.4 V), not the peak voltage, that is the DC-equivalent for power, and the rms voltage of Y is smaller than the DC voltage across X by the factor $\sqrt{2}$ (so the power, which depends on voltage squared, is halved).

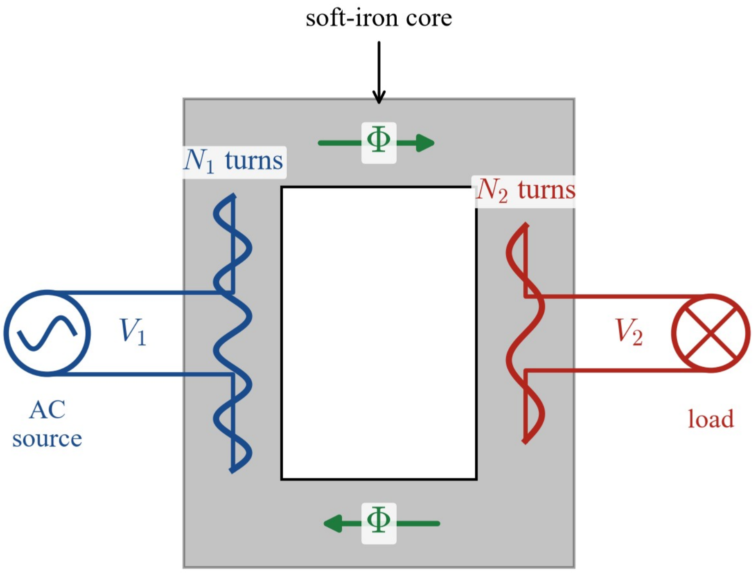
Question 16 $P = 60\text{ W}$, $V_{\text{RMS}} = 230\text{ V}$. $I_{\text{RMS}} = \frac{P}{V_{\text{RMS}}} = \frac{60}{230} = 0.261\text{ A}$. $R = \frac{V_{\text{RMS}}^2}{P} = \frac{230^2}{60} = \frac{52900}{60} = 882\,\Omega$. $V_{\text{peak}} = V_{\text{RMS}}\sqrt{2} = 230\sqrt{2} = 325\text{ V}$.

Question 17 $V_{\text{peak}} = 40\ \text{V}$, $I_{\text{peak}} = 2.5\ \text{A}$. (a) The product $V_{\text{peak}} I_{\text{peak}} = 40 \times 2.5 = 100\ \text{W}$ is the **peak instantaneous power** — the greatest power delivered, reached only momentarily at the peaks of the cycle. Because the voltage and current both vary through the cycle, the average power is smaller: for a sinusoidal supply it is **half** the peak instantaneous power. The student's method therefore overstates the power by a factor of two. (b) $P = \dfrac{V_{\text{peak}} I_{\text{peak}}}{2} = \dfrac{100}{2} = 50\ \text{W}$. (c) $V_{\text{RMS}} = \dfrac{V_{\text{peak}}}{\sqrt{2}} = \dfrac{40}{\sqrt{2}} = 28.3\ \text{V}$ and $I_{\text{RMS}} = \dfrac{I_{\text{peak}}}{\sqrt{2}} = \dfrac{2.5}{\sqrt{2}} = 1.77\ \text{A}$. Using these rms (DC-equivalent) values, $P = V_{\text{RMS}} I_{\text{RMS}} = \dfrac{40}{\sqrt{2}} \times \dfrac{2.5}{\sqrt{2}} = \dfrac{100}{2} = 50\ \text{W}$, which agrees with (b).

Question 18 $R = 46\ \Omega$, $V_{\text{RMS}} = 230\ \text{V}$. $I_{\text{RMS}} = \dfrac{V_{\text{RMS}}}{R} = \dfrac{230}{46} = 5.0\ \text{A}$. Average power: $P = V_{\text{RMS}} I_{\text{RMS}} = 230 \times 5.0 = 1.15 \times 10^3\ \text{W}$. Peak instantaneous power: $P_{\text{peak}} = 2P = 2 \times 1150 = 2.30 \times 10^3\ \text{W}$ (equivalently $V_{\text{peak}} I_{\text{peak}} = (230\sqrt{2})(5.0\sqrt{2}) = 230 \times 5.0 \times 2$).

Theory

A **transformer** is a device that changes the voltage of an alternating supply. In its simplest form it has two coils of insulated wire — the **primary** (input) coil of N_1 turns and the **secondary** (output) coil of N_2 turns — wound on a common **soft-iron core**. The two coils are not electrically connected: energy passes from one to the other entirely through the magnetic field in the core, by **electromagnetic induction**.



primary and secondary share the same changing core flux

Transformer action explained by electromagnetic induction. The operation is a chain of cause and effect, and it is worth stating each link:

- The primary coil is connected to an **alternating** supply, so it carries an **alternating current**.
- This continually changing current sets up a continually **changing magnetic flux** Φ in the iron core. (Flux is $\Phi_B = B_{\perp} A$; a changing current means a changing field B, so a changing flux.)
- The core guides almost all of this flux around to the secondary coil, so the **same changing flux threads the secondary**.
- A changing flux through the secondary coil **induces an emf** in it, by Faraday's law $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$.

The result is an alternating output voltage across the secondary. The soft-iron core matters because iron is easily magnetised and **concentrates and channels the flux**, so that nearly all the flux made by the primary passes through the secondary (maximising the flux linkage between the coils).

The ideal transformer equation. In an **ideal** transformer the windings have no resistance and all the flux is linked, so the applied primary voltage equals the emf induced in the primary and the secondary voltage equals the emf induced in the secondary. Each turn of wire encloses the *same* changing flux, so each turn has the same induced emf $\dfrac{\Delta\Phi_B}{\Delta t}$.

For the two coils,

$$V_1 = N_1\frac{\Delta\Phi_B}{\Delta t}, V_2 = N_2\frac{\Delta\Phi_B}{\Delta t}.$$

Dividing one equation by the other eliminates the common rate of change of flux:

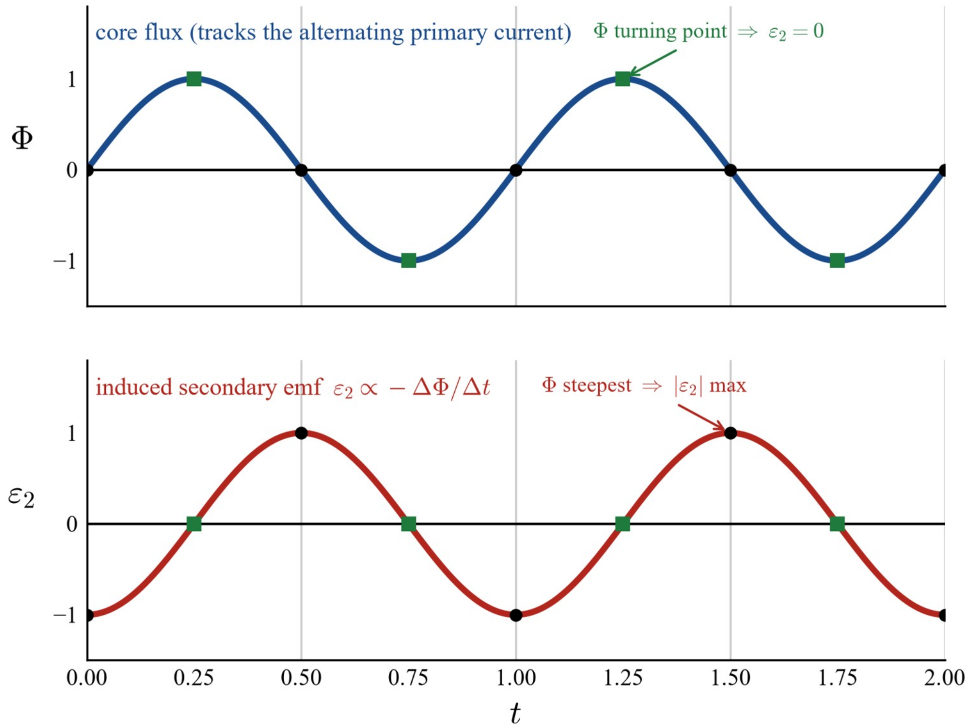
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}.$$

The output voltage is therefore set by the **turns ratio**: $V_2 = V_1 \frac{N_2}{N_1}$.

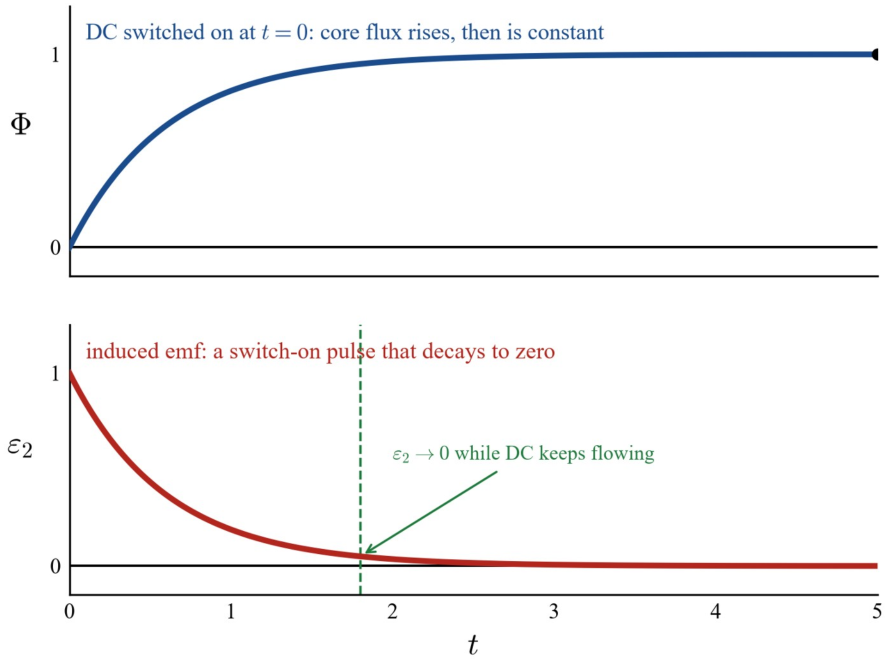
Why a transformer requires AC. A transformer produces an output only while the flux in the core is *changing*, because only a changing flux induces an emf. An alternating primary current makes the flux change all the time, so an emf is induced continuously.

It is **not** an explanation to say that "a transformer needs AC because it only works with alternating current" — that just restates the fact and earns no marks. The physical reason is the flux-change chain: **alternating current → continuously changing core flux → changing flux through the secondary → an emf is induced** ($\varepsilon = -N\,\Delta\Phi_B/\Delta t$). If the primary instead carried a steady **direct current**, the flux would be constant, $\Delta\Phi_B/\Delta t = 0$, and **no emf would be induced** in the secondary — apart from a brief pulse at the instant the DC is switched on or off, while the current (and flux) is momentarily changing.

The graphs below make the rate-of-change idea explicit. With AC, the induced secondary emf is **greatest when the flux passes through zero** (where the flux is changing fastest) and **zero when the flux is at a maximum or minimum** (where, for an instant, it is not changing).



With DC switched on at $t = 0$, the flux rises and then holds steady; the induced emf is only a decaying switch-on pulse and falls to zero while the direct current keeps flowing.



Ideal transformers conserve power. An **ideal** transformer is one that transfers **100%** of the input power to the output — it has no resistive heating and no other losses. Energy conservation then requires the output power to equal the input power:

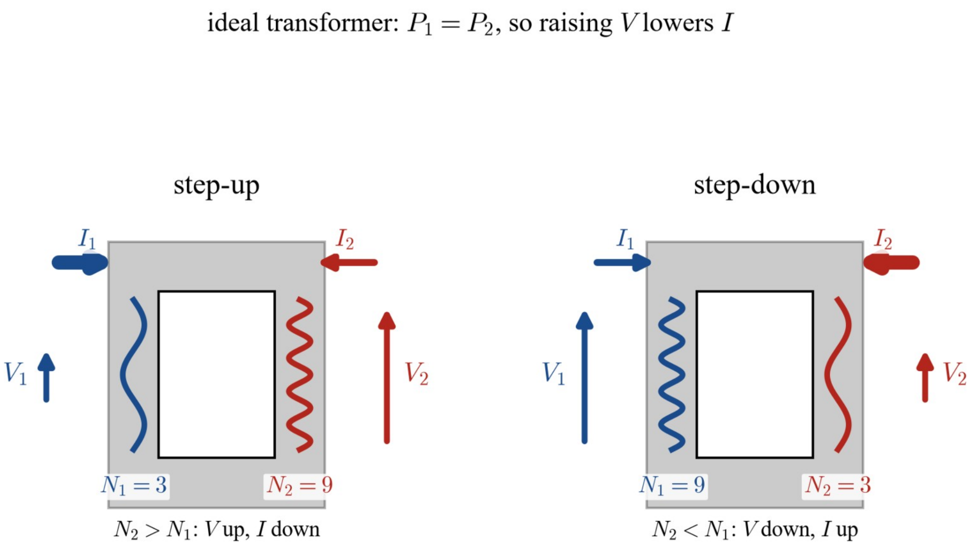
$$P_1 = P_2 \Rightarrow V_1 I_1 = V_2 I_2 .$$

Rearranging gives the current relationship, and combining it with the turns ratio gives the single result quoted on the formula sheet:

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

Notice that the current ratio is **inverted**: the coil with more turns has the higher voltage but the **smaller** current. So a transformer that **steps voltage up must step current down**, and one that steps voltage down steps current up. (Real transformers are slightly less than 100% efficient, dissipating a small amount of energy as heat; every question in this subtopic treats the transformer as ideal unless told otherwise.)

Step-up and step-down. If $N_2 > N_1$ the secondary voltage is higher than the primary — a **step-up** transformer. If $N_2 < N_1$ the secondary voltage is lower — a **step-down** transformer. The same device run "backwards" swaps the two roles, because the turns ratio is simply reversed.



Step-up and step-down transformers are the reason electricity is transmitted efficiently. Power is generated at a moderate voltage, **stepped up** to a very high voltage for transmission (which, for a given power, means a smaller line current and therefore less energy wasted as heat in the lines), and then **stepped down** in stages to the safe voltages used in homes and factories. The quantitative treatment of transmission-line losses is developed in the next subtopic; here the focus is the transformer itself.

A note on values. The alternating voltages and currents quoted for transformers are **rms** values (the DC-equivalent values of the previous subtopic), and the power relationship $P = VI$ uses those rms values. The turns/voltage/current ratios above hold equally for rms and for peak values.

Worked examples

Example 1 — scaffolded

A transformer has a primary of 800 turns connected to a $240\ \text{V}$ AC supply, and a secondary of 60 turns. The secondary delivers a current of $3.0\ \text{A}$ to a device. Assuming the transformer is ideal, find the secondary voltage, the primary current, and confirm that the input and output powers are equal.

Working	Comment
$N_1 = 800$, $N_2 = 60$, $V_1 = 240\ \text{V}$, $I_2 = 3.0\ \text{A}$.	List the data; the transformer is ideal.
$N_2 < N_1$, so this is a **step-down** transformer.	Fewer secondary turns ⇒ lower output voltage.
$V_2 = V_1 \dfrac{N_2}{N_1} = 240 \times \dfrac{60}{800} = 18\ \text{V}$.	From $\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2}$.
$I_1 = I_2 \dfrac{N_2}{N_1} = 3.0 \times \dfrac{60}{800} = 0.225\ \text{A}$.	From $\dfrac{I_2}{I_1} = \dfrac{N_1}{N_2}$ (current ratio is inverted).
$P_2 = V_2 I_2 = 18 \times 3.0 = 54\ \text{W}$; $P_1 = V_1 I_1 = 240 \times 0.225 = 54\ \text{W}$.	$P_1 = P_2$: power is conserved in an ideal transformer.

Example 2 — scaffolded

A step-up transformer raises $240\ \text{V}$ to $6.0\ \text{kV}$. Its primary has 500 turns and draws a current of $30\ \text{A}$ from the supply. Assuming the transformer is ideal, find the number of turns on the secondary and the secondary (output) current.

Working	Comment
$V_1 = 240\ \text{V}$, $V_2 = 6.0\ \text{kV} = 6000\ \text{V}$, $N_1 = 500$, $I_1 = 30\ \text{A}$.	List the data; convert kV to V.
$N_2 = N_1 \dfrac{V_2}{V_1} = 500 \times \dfrac{6000}{240} = 12500$ turns.	From $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$; $N_2 > N_1$ confirms step-up.
$I_2 = I_1 \dfrac{N_1}{N_2} = 30 \times \dfrac{500}{12500} = 1.2\ \text{A}$.	From $\dfrac{I_2}{I_1} = \dfrac{N_1}{N_2}$.
Check: $P_1 = 240 \times 30 = 7200\ \text{W}$; $P_2 = 6000 \times 1.2 = 7200\ \text{W}$.	Stepping voltage up steps current down; power is unchanged.

Example 3 — unscaffolded

An ideal transformer runs a $12\ \text{V}$, $36\ \text{W}$ lamp from a $240\ \text{V}$ AC supply. Calculate the current in the lamp, the current drawn from the supply, and the ratio of primary to secondary turns.

The lamp is the load on the secondary, so $V_2 = 12\ \text{V}$ and $P_2 = 36\ \text{W}$:

$$I_2 = \frac{P_2}{V_2} = \frac{36}{12} = 3.0\ \text{A}.$$

The transformer is ideal, so the input power equals the output power, $P_1 = P_2 = 36\ \text{W}$, and the current drawn from the supply is

$$I_1 = \frac{P_1}{V_1} = \frac{36}{240} = 0.15\ \text{A}.$$

The turns ratio equals the voltage ratio:

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{240}{12} = 20.$$

∴ the lamp current is $3.0\ \text{A}$, the supply current is $0.15\ \text{A}$, and the primary and secondary turns are in the ratio $20:1$.

Example 4 — unscaffolded

Explain, with reference to electromagnetic induction, why an ideal transformer produces an output voltage when its primary is connected to an AC supply, but produces no steady output when the primary is connected to a battery (DC).

An emf is induced in the secondary coil only when the magnetic flux threading it is **changing** ($\varepsilon = -N\,\Delta\Phi_B/\Delta t$). With an **AC** supply the primary current reverses continually, so the flux it produces in the core is continually changing; this changing flux passes through the secondary and induces an alternating emf in it at every instant. With a **battery**, the primary carries a steady direct current, so once the current has settled the core flux is constant. Then $\Delta\Phi_B/\Delta t = 0$, and no emf is induced in the secondary — there is only a brief pulse at the moment the battery is connected or disconnected, while the current (and hence the flux) is momentarily changing.

∴ a transformer needs a *changing* flux, which a continually changing (alternating) current supplies but a steady direct current does not.

Practice questions

Question 1 A transformer has a primary of 1500 turns connected to a $240\ \text{V}$ AC supply and a secondary of 100 turns. The secondary supplies a current of $4.5\ \text{A}$. Assume the transformer is ideal. (a) State whether this is a step-up or step-down transformer, and justify your answer. (b) Calculate the secondary voltage. (c) Calculate the current drawn by the primary.

Question 2 A transformer connected to a $240\ \text{V}$ AC supply has a primary of 300 turns and a secondary of 720 turns. The secondary delivers $2.5\ \text{A}$ to a load. Assume the transformer is ideal. (a) Calculate the secondary voltage. (b) State whether the transformer steps the voltage up or down. (c) Calculate the current drawn from the supply.

Question 3 An ideal transformer connected to a $240\ \text{V}$ AC supply produces a secondary voltage of $40\ \text{V}$, from which a heater draws $6.0\ \text{A}$. (a) Calculate the power delivered to the heater. (b) State the power drawn from the supply, and the assumption you have used. (c) Calculate the current drawn from the supply.

Question 4 A transformer is used to step $22\ \text{kV}$ down to $240\ \text{V}$ for local distribution. The secondary has 120 turns. Assume the transformer is ideal. (a) Calculate the ratio $N_1:N_2$. (b) Calculate the number of primary turns. (c) A load draws $22\ \text{A}$ from the secondary. Calculate the primary current.

Question 5 A transformer has its primary coil connected to an AC supply and its secondary coil connected to a lamp; both coils are wound on the same iron core. (a) Describe how the alternating current in the primary produces a changing magnetic flux in the core. (b) Explain, with reference to Faraday's law, how an emf is induced in the secondary coil. (c) Explain the purpose of the soft-iron core in the operation of the transformer.

Question 6 A transformer nameplate reads "$240\ \text{V}$ / $16\ \text{V}$". (a) (Circle your answer.) Used as marked, this is a *step-up* / *step-down* transformer. Justify your choice. (b) Calculate the ratio of primary to secondary turns, $N_1:N_2$. (c) A student mistakenly connects the $16\ \text{V}$ coil to a $240\ \text{V}$ AC supply. Calculate the voltage that then appears across the other coil, and state what type of transformer it now acts as.

Question 7 An ideal transformer connected to a $240\ \text{V}$ AC supply has a primary of 960 turns and a secondary of 40 turns. The secondary supplies a current of $5.0\ \text{A}$. Calculate the secondary voltage and the primary current.

Question 8 A transformer steps $415\ \text{V}$ down to run a $24\ \text{V}$ device. Its primary has 2500 turns. Assume the transformer is ideal. (a) Show that the secondary has 145 turns. Give your answer to three significant figures. (b) The device draws $3.2\ \text{A}$. Calculate the current drawn from the $415\ \text{V}$ supply.

Question 9 An ideal transformer supplies $13.2\ \text{kW}$ to a workshop at $220\ \text{V}$, drawing its input from a $6.6\ \text{kV}$ line. Calculate: (a) the current supplied to the workshop; (b) the current drawn from the $6.6\ \text{kV}$ line; (c) the ratio of primary to secondary turns.

Question 10 A student writes: "A transformer must be run on AC rather than DC because a transformer only works with alternating current." Explain why this statement would earn no marks, and give the correct physical reason a transformer requires an alternating (changing) current in its primary coil.

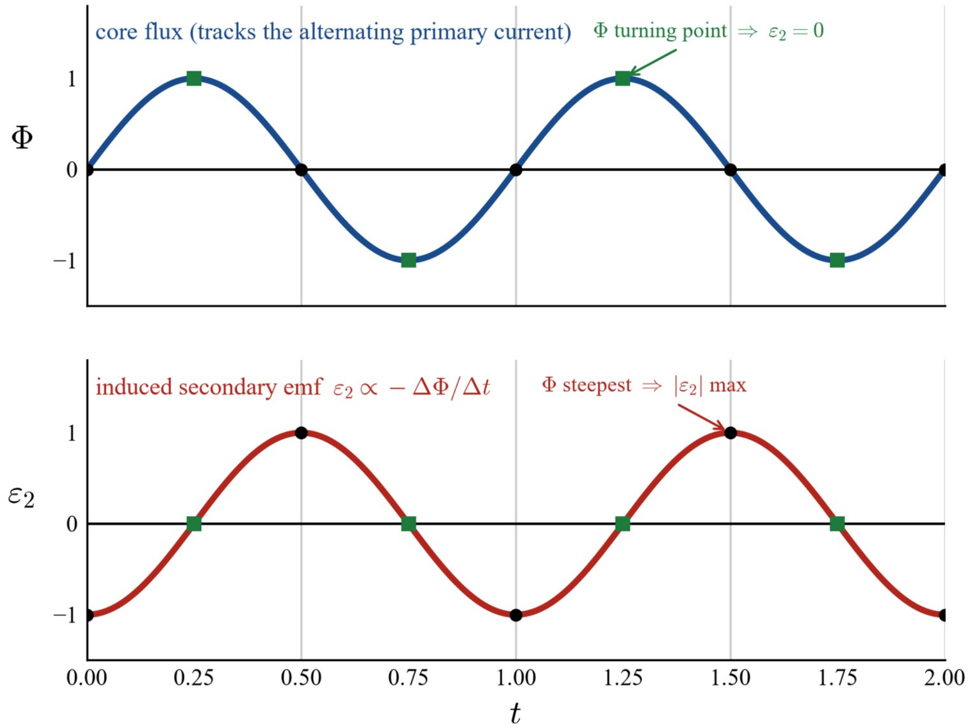
Question 11 Describe the sequence of physical events by which an alternating voltage applied to the primary coil produces an alternating voltage across the secondary coil of an ideal transformer. Your answer should refer to the current, the magnetic flux and the induced emf.

Question 12 A step-up transformer raises $240\ \text{V}$ to $3.0\ \text{kV}$ for a transmission line. The primary draws $50\ \text{A}$ from the supply. Assuming the transformer is ideal, calculate the current delivered to the line, and show that the power delivered to the line equals the power drawn from the supply.

Question 13 State what is meant by an **ideal** transformer. Then, using the relationship $P_1 = P_2$, explain why a transformer that steps the voltage up must step the current down.

Question 14 An ideal transformer has 1200 turns on its primary, connected to a $240\ \text{V}$ AC supply. The secondary is required to deliver $5.0\ \text{V}$. (a) Calculate the number of secondary turns. (b) The secondary supplies $1.2\ \text{A}$ to a device. Calculate the current in the primary.

Question 15 The graphs show the magnetic flux Φ in the core of an ideal transformer, and the resulting induced secondary emf ε_2, as functions of time when the primary is driven by an AC supply.



(a) Explain why the induced secondary emf is zero at the instants when the flux is at a maximum or a minimum.

(b) Explain why a steady direct current in the primary would induce no lasting emf in the secondary, with reference to the rate of change of flux.

Question 16 An ideal transformer runs a $24\,\mathrm{V}$, $120\,\mathrm{W}$ appliance from a $240\,\mathrm{V}$ AC supply. Calculate the current in the appliance, the current drawn from the supply, and the ratio of primary to secondary turns.

Question 17 When a battery is first connected to the primary coil of a transformer, a brief pulse of voltage appears across the secondary, but it quickly falls to zero even though the battery stays connected. Explain this observation with reference to the magnetic flux in the core.

Question 18 A power station generates $20\,\mathrm{MW}$ at $23\,\mathrm{kV}$. An ideal step-up transformer with a turns ratio $N_1 : N_2 = 1 : 22$ raises the voltage for transmission. (a) Calculate the transmission voltage. (b) Calculate the current in the primary of the transformer. Give your answer to three significant figures. (c) Calculate the current in the secondary (the transmission line).

Question 19 An **ideal** transformer is assumed to transfer $100\,\%$ of the input power to the output; a real transformer is slightly less efficient because some energy is dissipated as heat. (a) Write the equation relating input power and output power for an ideal transformer. (b) A real transformer delivers $95\,\%$ of its input power. If the input power is $500\,\mathrm{W}$, calculate the output power and the power dissipated as heat in the transformer.

Question 20 Explain why step-up and step-down transformers are essential to an efficient electricity transmission system. Your answer should refer to how a transformer changes both the voltage and the current, and to the ideal power relationship $P_1 = P_2$.

Answers

Question 1 (a) step-down ($N_2 < N_1$); (b) $16\,\mathrm{V}$; (c) $0.30\,\mathrm{A}$. **Question 2** (a) $576\,\mathrm{V}$; (b) step-up; (c) $6.0\,\mathrm{A}$. **Question 3** (a) $240\,\mathrm{W}$; (b) $240\,\mathrm{W}$ (ideal: $P_1 = P_2$); (c) $1.0\,\mathrm{A}$. **Question 4** (a) $N_1 : N_2 \approx 91.7 : 1$; (b) 11000 turns; (c) $0.24\,\mathrm{A}$. **Question 5** (a)–(c) qualitative — see solutions. **Question 6** (a) step-down; (b) $15 : 1$; (c) $3600\,\mathrm{V}$, acting as a step-up transformer. **Question 7** $V_2 = 10\,\mathrm{V}$; $I_1 = 0.208\,\mathrm{A}$. **Question 8** (a) 145 turns; (b) $0.185\,\mathrm{A}$. **Question 9** (a) $60\,\mathrm{A}$; (b) $2.0\,\mathrm{A}$; (c) $30 : 1$. **Question 10** qualitative — the statement is circular; the correct reason is the flux-change chain. **Question 11** qualitative — see solutions. **Question 12** $I_2 = 4.0\,\mathrm{A}$; $P_1 = P_2 = 12\,\mathrm{kW}$. **Question 13** qualitative — ideal = 100% power transfer; $V_1 I_1 = V_2 I_2$. **Question 14** (a) 25 turns; (b) $0.025\,\mathrm{A}$. **Question 15** qualitative — see solutions. **Question 16** $I_2 = 5.0\,\mathrm{A}$; $I_1 = 0.50\,\mathrm{A}$; $N_1 : N_2 = 10 : 1$. **Question 17** qualitative — see solutions. **Question 18** (a) $506\,\mathrm{kV}$; (b) $870\,\mathrm{A}$; (c) $39.5\,\mathrm{A}$. **Question 19** (a) $P_1 = P_2$; (b) output $475\,\mathrm{W}$, dissipated $25\,\mathrm{W}$. **Question 20** qualitative — see solutions.

Fully-worked solutions

Question 1 $N_1 = 1500$, $N_2 = 100$, $V_1 = 240\,\mathrm{V}$, $I_2 = 4.5\,\mathrm{A}$; ideal. (a) The secondary has fewer turns than the primary ($N_2 < N_1$), so the output voltage is lower than the input: this is a **step-down** transformer. (b) $V_2 = V_1 \dfrac{N_2}{N_1} = 240 \times \dfrac{100}{1500} = 16\,\mathrm{V}$. (c) $I_1 = I_2 \dfrac{N_2}{N_1} = 4.5 \times \dfrac{100}{1500} = 0.30\,\mathrm{A}$. (Check: $P_2 = 16 \times 4.5 = 72\,\mathrm{W} = 240 \times 0.30 = P_1$.)

Question 2 $V_1 = 240\,\mathrm{V}$, $N_1 = 300$, $N_2 = 720$, $I_2 = 2.5\,\mathrm{A}$; ideal. (a) $V_2 = V_1 \dfrac{N_2}{N_1} = 240 \times \dfrac{720}{300} = 576\,\mathrm{V}$. (b) $N_2 > N_1$ and $V_2 > V_1$, so the transformer **steps the voltage up**. (c) $I_1 = I_2 \dfrac{N_2}{N_1} = 2.5 \times \dfrac{720}{300} = 6.0\,\mathrm{A}$. (Check: $P_1 = 240 \times 6.0 = 1440\,\mathrm{W} = 576 \times 2.5 = P_2$.)

Question 3 $V_1 = 240\,\mathrm{V}$, $V_2 = 40\,\mathrm{V}$, $I_2 = 6.0\,\mathrm{A}$. (a) $P_2 = V_2 I_2 = 40 \times 6.0 = 240\,\mathrm{W}$. (b) The transformer is assumed **ideal**, so all the input power is transferred to the output: $P_1 = P_2 = 240\,\mathrm{W}$. (c) $I_1 = \dfrac{P_1}{V_1} = \dfrac{240}{240} = 1.0\,\mathrm{A}$.

Question 4 $V_1 = 22\,\mathrm{kV} = 22000\,\mathrm{V}$, $V_2 = 240\,\mathrm{V}$, $N_2 = 120$; ideal. (a) $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{22000}{240} = 91.7$, so $N_1 : N_2 \approx 91.7 : 1$. (b) $N_1 = N_2 \dfrac{V_1}{V_2} = 120 \times \dfrac{22000}{240} = 11000$ turns. (c) $I_1 = I_2 \dfrac{V_2}{V_1} = 22 \times \dfrac{240}{22000} = 0.24\,\mathrm{A}$. (Equivalently $I_1 = P / V_1 = (240 \times 22) / 22000 = 0.24\,\mathrm{A}$.)

Question 5 (a) The primary is connected to an AC supply, so it carries an alternating current — a current that grows, falls, reverses and grows again continually. A current in the primary magnetises the core ($\Phi_B = B_{\perp} A$), and because the current is always changing, the magnetic flux it produces in the core is always changing too. (b) The soft-iron core carries this changing flux around to the secondary coil, so the same changing flux threads every turn of the secondary. By Faraday's law, a coil experiencing a changing flux has an emf induced in it, $\varepsilon = -N_2 \dfrac{\Delta \Phi_B}{\Delta t}$. Because the flux is changing at every instant, an alternating emf is induced in the secondary continuously. (c) Iron is easily magnetised, so the core **concentrates and channels the magnetic flux**, guiding almost all of the flux produced by the primary through the secondary. This maximises the flux linkage between the coils, so nearly all the changing flux made by the primary contributes to the emf induced in the secondary, making the energy transfer efficient.

Question 6 Nameplate $240\,\mathrm{V}$ / $16\,\mathrm{V}$. (a) **Step-down.** Used as marked, the $240\,\mathrm{V}$ side is the primary (input) and the $16\,\mathrm{V}$ side is the secondary (output); the output voltage is lower than the input, so the transformer steps the voltage down. (b) $\frac{N_1}{N_2}=\frac{V_1}{V_2}=\frac{240}{16}=15$, so $N_1:N_2=15:1$. (c) The turns ratio is fixed at $15:1$. Connecting the $16\,\mathrm{V}$ (fewer-turns) coil to the $240\,\mathrm{V}$ supply makes it the primary, so

$$V_{\text{out}}=240\times\frac{15}{1}=3600\,\mathrm{V}.$$

With the low-turns coil as the input, the device now acts as a **step-up** transformer (and $3600\,\mathrm{V}$ is a dangerous output — hence the warning against this mistake).

Question 7 $V_1=240\,\mathrm{V}$, $N_1=960$, $N_2=40$, $I_2=5.0\,\mathrm{A}$; ideal.

$$V_2=V_1\frac{N_2}{N_1}=240\times\frac{40}{960}=10\,\mathrm{V}\,,\,I_1=I_2\frac{N_2}{N_1}=5.0\times\frac{40}{960}=0.208\,\mathrm{A}.$$

Question 8 $V_1=415\,\mathrm{V}$, $V_2=24\,\mathrm{V}$, $N_1=2500$; ideal. (a) $N_2=N_1\frac{V_2}{V_1}=2500\times\frac{24}{415}=144.6=145$ turns (to three significant figures). (b) $I_1=I_2\frac{V_2}{V_1}=3.2\times\frac{24}{415}=0.185\,\mathrm{A}$.

Question 9 $P=13.2\,\mathrm{kW}=13200\,\mathrm{W}$, workshop (secondary) at $V_2=220\,\mathrm{V}$, line (primary) at $V_1=6.6\,\mathrm{kV}=6600\,\mathrm{V}$; ideal, so $P_1=P_2=13200\,\mathrm{W}$. (a) $I_2=\frac{P}{V_2}=\frac{13200}{220}=60\,\mathrm{A}$. (b) $I_1=\frac{P}{V_1}=\frac{13200}{6600}=2.0\,\mathrm{A}$. (c) $\frac{N_1}{N_2}=\frac{V_1}{V_2}=\frac{6600}{220}=30$, so $N_1:N_2=30:1$.

Question 10 The statement is **circular**: "it only works with alternating current" simply repeats the claim that it needs AC, without giving a physical reason, so it earns no marks. The correct reason is the electromagnetic-induction chain. An emf is induced in the secondary only when the magnetic flux through it is **changing** ($\varepsilon=-N\,\Delta\Phi_B/\Delta t$). An **alternating** current in the primary is continually changing, so it produces a continually **changing flux** in the core; this changing flux threads the secondary and **induces an emf** in it. A steady direct current would give a constant flux ($\Delta\Phi_B/\Delta t=0$) and hence no induced emf. It is the need for a *changing flux*, supplied by a *changing current*, that makes AC necessary — not the bare fact that "it needs AC".

Question 11 The chain of events is: 1. The alternating voltage drives an **alternating current** in the primary coil. 2. This continually changing current produces a continually **changing magnetic flux** Φ in the soft-iron core. 3. The core channels this changing flux so that it **threads the secondary coil**; the same changing flux passes through all N_2 turns. 4. By Faraday's law, the changing flux **induces an emf** in the secondary, $\varepsilon=-N_2\frac{\Delta\Phi_B}{\Delta t}$. Because the primary current alternates, the flux alternates, so the induced secondary emf — and hence the output voltage — is also **alternating**, at the same frequency as the supply.

Question 12 $V_1=240\,\mathrm{V}$, $V_2=3.0\,\mathrm{kV}=3000\,\mathrm{V}$, $I_1=50\,\mathrm{A}$; ideal.

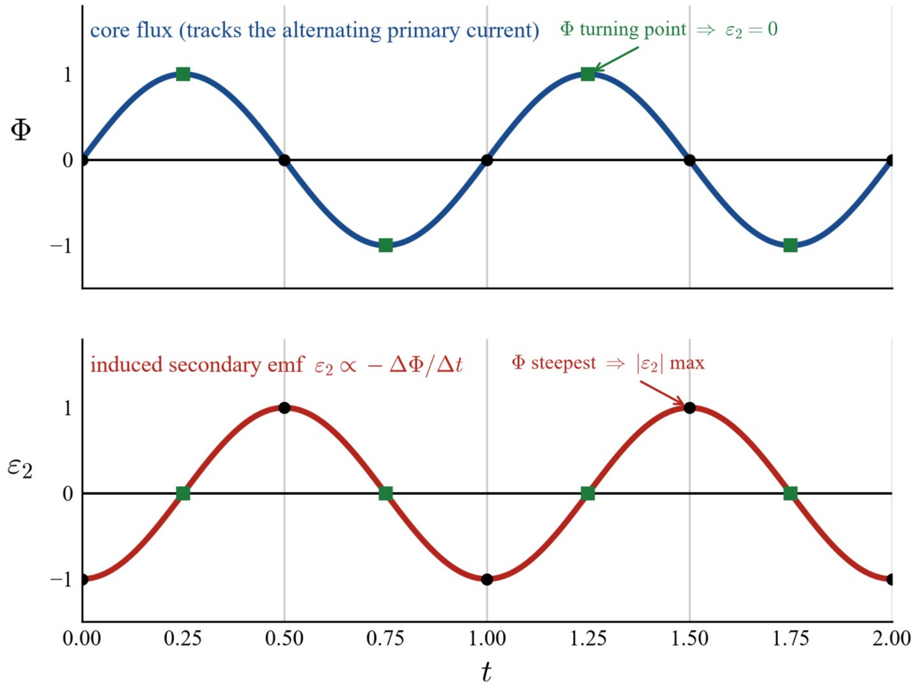
$$I_2=I_1\frac{V_1}{V_2}=50\times\frac{240}{3000}=4.0\,\mathrm{A}.$$

Power drawn from the supply: $P_1=V_1I_1=240\times50=12000\,\mathrm{W}=12\,\mathrm{kW}$. Power delivered to the line: $P_2=V_2I_2=3000\times4.0=12000\,\mathrm{W}=12\,\mathrm{kW}$. The two are equal, as expected for an ideal transformer ($P_1=P_2$).

Question 13 An **ideal transformer** is one that transfers **100%** of the input electrical power to the output — it is assumed to have no resistive heating in its windings and no other energy losses, so the output power equals the input power, $P_1=P_2$. Writing this as $V_1I_1=V_2I_2$: if the transformer steps the voltage **up**, so that $V_2>V_1$, then for the product V_2I_2 to stay equal to V_1I_1 the secondary current must be correspondingly **smaller**, $I_2<I_1$. Conserving power therefore forces a step-up in voltage to be accompanied by a step-down in current (and vice versa).

Question 14 $N_1 = 1200$, $V_1 = 240\ \text{V}$, $V_2 = 5.0\ \text{V}$; ideal. (a) $N_2 = N_1 \dfrac{V_2}{V_1} = 1200 \times \dfrac{5.0}{240} = 25$ turns. (b) $I_1 = I_2 \dfrac{V_2}{V_1} = 1.2 \times \dfrac{5.0}{240} = 0.025\ \text{A}$.

Question 15



(a) The induced secondary emf depends on the **rate of change** of flux, $\varepsilon_2 = -N_2 \Delta\Phi_B / \Delta t$. At the instant the flux reaches a maximum or a minimum it is, for that instant, momentarily **not changing** — the flux–time graph is flat there (its gradient is zero). With $\Delta\Phi_B / \Delta t = 0$, the induced emf is zero. (Correspondingly, the emf is largest where the flux passes through zero, because that is where the flux is changing fastest.)

(b) A steady direct current would produce a **constant** flux in the core. A constant flux has $\Delta\Phi_B / \Delta t = 0$ at all times — the flux–time graph would be a flat horizontal line — so no emf is induced. (Only at the moment the DC is switched on or off, while the flux is rising or falling, is there a brief non-zero rate of change and hence a short-lived emf.)

Question 16 $V_1 = 240\ \text{V}$, appliance $V_2 = 24\ \text{V}$, $P_2 = 120\ \text{W}$; ideal.

$$I_2 = \frac{P_2}{V_2} = \frac{120}{24} = 5.0\ \text{A}\,,\ I_1 = \frac{P_1}{V_1} = \frac{120}{240} = 0.50\ \text{A}\,,\ \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{240}{24} = 10\,.$$

So the appliance draws $5.0\ \text{A}$, the supply delivers $0.50\ \text{A}$, and $N_1 : N_2 = 10 : 1$.

Question 17 At the instant the battery is connected, the primary current rises rapidly from zero towards its steady value, so the flux in the core is **increasing** — it is changing. This changing flux threads the secondary and induces an emf there, giving the brief pulse. Once the current reaches its steady DC value, the flux becomes **constant**; then $\Delta\Phi_B / \Delta t = 0$, so no further emf is induced even though the battery is still connected and current still flows. The output falls to zero because the flux, though large, is no longer changing.

Question 18 $P = 20\ \text{MW} = 20 \times 10^6\ \text{W}$, $V_1 = 23\ \text{kV} = 23000\ \text{V}$, turns ratio $N_1 : N_2 = 1 : 22$; ideal. (a) $V_2 = V_1 \dfrac{N_2}{N_1} = 23000 \times 22 = 506000\ \text{V} = 506\ \text{kV}$. (b) $I_1 = \dfrac{P}{V_1} = \dfrac{20 \times 10^6}{23000} = 870\ \text{A}$ (to three significant figures). (c) $I_2 = \dfrac{P}{V_2} = \dfrac{20 \times 10^6}{506000} = 39.5\ \text{A}$ (equivalently $I_2 = I_1 / 22 = 39.5\ \text{A}$). Stepping the voltage up by a factor of 22 steps the current down by the same factor.

Question 19 (a) For an ideal transformer, output power equals input power: $P_1 = P_2$ (equivalently $V_1 I_1 = V_2 I_2$). (b) Output power $= 0.95 \times 500 = 475\ \text{W}$. The remaining power is dissipated as heat: $P_{\text{dissipated}} = 500 - 475 = 25\ \text{W}$.

Question 20 Transmitting electrical power over long distances wastes energy as heat in the resistance of the lines, and for a given power this loss is smaller when the line current is smaller. A **step-up** transformer at the generator raises the voltage to a very high value; because the transformer is (nearly) ideal, $P_1 = P_2$ means $V_1 I_1 = V_2 I_2$, so raising the voltage necessarily **lowers the current** carried by the transmission line — reducing the energy wasted as heat. At the far end, **step-down** transformers reduce the voltage again, in stages, to the lower values that are safe to use in homes and factories. Without transformers the voltage could not be raised for efficient transmission and then lowered for safe use, so the ability to change voltage (and, through $P_1 = P_2$, the current) is what makes large-scale transmission practical. Transformers change only voltage and current in this way, not the total power, and they require the alternating supply that the grid provides.

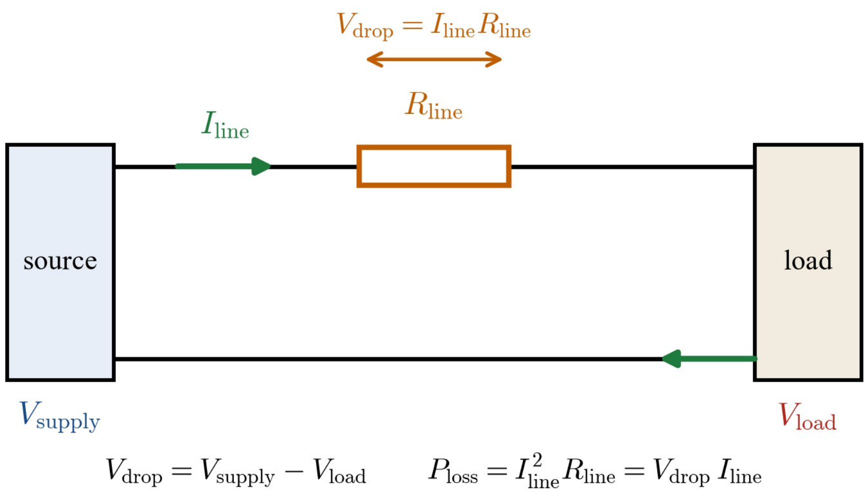
Theory

Electricity is generated at power stations, solar farms and wind farms that are often far from the homes, towns and factories that use it. The energy travels along **transmission lines** — long conductors that, however good, have a small but non-zero **resistance** R_{line}. Whenever a current flows through this resistance the line warms up, and the energy that heats the line is energy that never reaches the consumer. Analysing a supply system means tracking how much power is delivered and how much is lost along the way.

The transmission line as a resistance. Model the whole line (both conductors of the supply circuit) as a single resistance R_{line} in series between the supply and the load. The same current I_{line} flows through the line and the load. By Ohm's law, this current through R_{line} produces a **voltage drop** along the line:

$$V_{\text{drop}} = I_{\text{line}} R_{\text{line}}.$$

The voltage arriving at the load end is therefore lower than the voltage sent from the supply end.



Power lost in the line. The power dissipated as heat in the line resistance is

$$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = V_{\text{drop}} I_{\text{line}}.$$

This power is transformed from electrical energy into heat (thermal energy) in the conductors and radiated to the surroundings; it is genuinely lost from the supply. Note that the current is **squared**: the power lost is very sensitive to the size of the line current.

The two end voltages (the safe method). In many problems you are told the voltage at each end of the line — the **supply-end** voltage V_{supply} and the **load-end** voltage V_{load} — together with the line resistance. Because the drop along the line is exactly the difference between these two voltages,

$$V_{\text{drop}} = V_{\text{supply}} - V_{\text{load}},$$

the line current follows directly from Ohm's law applied to the line:

$$I_{\text{line}} = \frac{V_{\text{drop}}}{R_{\text{line}}} = \frac{V_{\text{supply}} - V_{\text{load}}}{R_{\text{line}}}.$$

Once I_{line} is known, everything else follows:

- power delivered to the load: $P_{\text{load}} = V_{\text{load}} I_{\text{line}}$;

- power lost in the line: $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$;
- power supplied at the sending end: $P_{\text{in}} = V_{\text{supply}} I_{\text{line}} = P_{\text{load}} + P_{\text{loss}}$.

A warning about "rated power". A common and costly mistake is to read a device's **rated power** from the question — "a 5 kW heater", "the town draws 50 kW" — and use $I = P/V$ with that number to find the line current. A rating is the power the device is designed to use at its own rated voltage; it is not, in general, the actual power delivered, and it is not the quantity that fixes the line current. **When you are given the two end voltages and the line resistance, the current is $I_{\text{line}} = (V_{\text{supply}} - V_{\text{load}})/R_{\text{line}}$ — use it.** Reach for a rated power only when the problem actually states the power fed into the line together with the voltage there.

Why electricity is transmitted at high voltage. Suppose a fixed amount of power P must be sent along a given line. The current needed to carry it depends on the voltage at which it is transmitted:

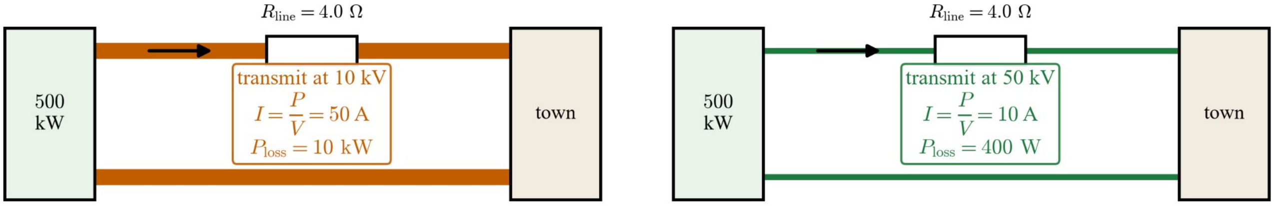
$$I_{\text{line}} = \frac{P}{V}.$$

Raising the transmission voltage lowers the current in the same proportion. Because $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$ depends on the **square** of the current,

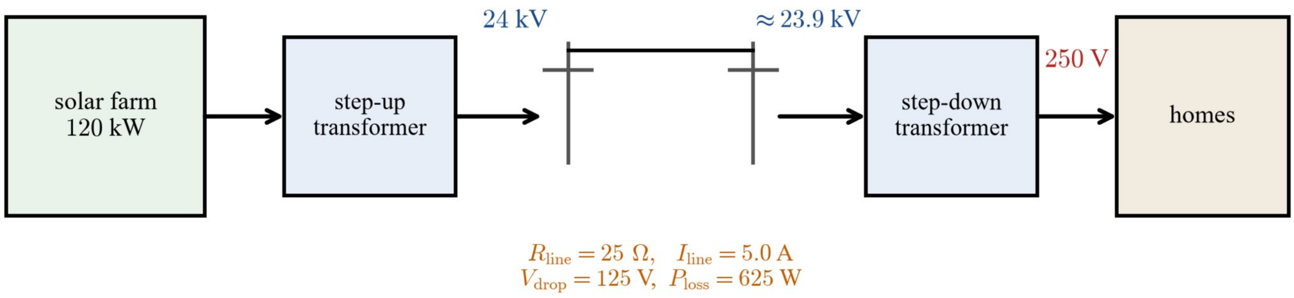
$$P_{\text{loss}} = \left(\frac{P}{V}\right)^2 R_{\text{line}},$$

the power lost is inversely proportional to the square of the transmission voltage. Transmitting at a higher voltage — hence a smaller current — cuts the line loss dramatically. Raising the voltage by a factor of 5, for instance, lowers the current by a factor of 5 and the power lost by a factor of $5^2 = 25$.

same power, same line: $\times 5$ voltage $\Rightarrow$ $\div 5$ current $\Rightarrow$ $\div 25$ power lost



The full supply chain. Real networks use **transformers** to change the voltage. A step-up transformer raises the generator's output to a high voltage for transmission (keeping the current, and therefore the loss, low); at the far end a step-down transformer lowers the voltage again for safe use in homes and factories. For an ideal transformer the power out equals the power in, and $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$. Because a transformer relies on a *changing* magnetic flux, the whole system must run on alternating current (AC).



The figure follows a worked chain: a solar farm feeds 120 kW into the line at 24 kV, so $I_{line} = 120000 / 24000 = 5.0$ A. With $R_{line} = 25\ \Omega$ the drop is $V_{drop} = 5.0 \times 25 = 125$ V and the loss is only $P_{loss} = 5.0^2 \times 25 = 625$ W — just 0.52% of the transmitted power. The **efficiency** of transmission is the fraction of the fed-in power that reaches the load,

$$\text{efficiency} = \frac{P_{load}}{P_{in}} = \frac{P_{in} - P_{loss}}{P_{in}},$$

usually quoted as a percentage.

Throughout this subtopic, final answers are given to two or three significant figures matching the data, and the same current flows through the line and the load in each single-line model.

Worked examples

Example 1 — scaffolded

A transmission line of total resistance $8.0\ \Omega$ carries a current of 25 A from a wind turbine to a substation. Calculate the voltage drop across the line and the power dissipated as heat in it.

Working	Comment
$R_{line} = 8.0\ \Omega$, $I_{line} = 25$ A.	List the data; the line is modelled as a series resistance.
$V_{drop} = I_{line} R_{line} = 25 \times 8.0 = 200$ V.	Ohm's law along the line.
$P_{loss} = I_{line}^2 R_{line} = 25^2 \times 8.0 = 5000\ \text{W} = 5.0$ kW.	Power dissipated in the line resistance.

Example 2 — scaffolded

A solar farm sends power to a town along a transmission line. The voltage at the solar-farm (supply) end of the line is 6.6 kV and the voltage at the town (load) end is 6.3 kV. The line has a total resistance of $15\ \Omega$. Calculate the voltage drop across the line, the current in the line, the power delivered to the town and the power lost in the line.

Working	Comment
$V_{supply} = 6.6\ \text{kV} = 6600$ V, $V_{load} = 6.3\ \text{kV} = 6300$ V, $R_{line} = 15\ \Omega$.	Convert kV to V before substituting.
$V_{drop} = V_{supply} - V_{load} = 6600 - 6300 = 300$ V.	The drop is the difference of the two end voltages.
$I_{line} = \frac{V_{drop}}{R_{line}} = \frac{300}{15} = 20$ A.	Ohm's law on the line gives the current — not the town's rated power.
$P_{load} = V_{load} I_{line} = 6300 \times 20 = 1.26 \times 10^5$ W.	Power delivered to the town.
$P_{loss} = I_{line}^2 R_{line} = 20^2 \times 15 = 6000\ \text{W} = 6.0$ kW.	Power wasted heating the line.

Example 3 — unscaffolded

A wind turbine generates $60\,\text{kW}$ of electrical power, which is fed into a transmission line at $5.0\,\text{kV}$. The line has a total resistance of $6.0\,\Omega$. Calculate the current in the line, the power lost in the line, and the percentage of the generated power that is lost.

Here the power fed into the line and the voltage there are given, so the current follows from $P = V\,I$:

$$I_{\text{line}} = \frac{P}{V} = \frac{60000}{5000} = 12\,\text{A}\,.$$

The power lost heating the line is

$$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 12^2 \times 6.0 = 864\,\text{W}\,.$$

As a fraction of the generated power, $\dfrac{864}{60000} = 0.0144$, i.e. 1.44%.

∴ the line current is $12\,\text{A}$, the power lost is $864\,\text{W}$, and 1.44% of the generated power is lost in the line.

Example 4 — unscaffolded

A power station delivers $500\,\text{kW}$ to a distant town along a line of total resistance $4.0\,\Omega$. Compare the power lost in the line when the power is transmitted at $10\,\text{kV}$ with the loss when it is transmitted at $50\,\text{kV}$.

For a fixed transmitted power the line current is $I_{\text{line}} = P/V$, and the loss is $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$.

At $10\,\text{kV}$:

$$I_{\text{line}} = \frac{500000}{10000} = 50\,\text{A}\,, P_{\text{loss}} = 50^2 \times 4.0 = 10000\,\text{W} = 10\,\text{kW}\,.$$

At $50\,\text{kV}$:

$$I_{\text{line}} = \frac{500000}{50000} = 10\,\text{A}\,, P_{\text{loss}} = 10^2 \times 4.0 = 400\,\text{W}\,.$$

∴ raising the transmission voltage by a factor of 5 reduces the current by a factor of 5 and the power lost by a factor of $5^2 = 25$ — from $10\,\text{kW}$ down to $400\,\text{W}$.

Practice questions

Question 1 A transmission cable of total resistance $5.0\,\Omega$ carries a current of $30\,\text{A}$. (a) Calculate the voltage drop across the cable. (b) Calculate the power dissipated as heat in the cable. (c) The cable delivers power to a load at $400\,\text{V}$. Calculate the power delivered to the load.

Question 2 A solar farm transmits electricity to a nearby town. The voltage measured at the solar-farm end of the transmission line is $3.30\,\text{kV}$ and the voltage measured at the town end is $3.12\,\text{kV}$. The line has a total resistance of $3.0\,\Omega$. (a) Calculate the voltage drop across the transmission line. (b) Calculate the current in the transmission line. (c) Calculate the power delivered to the town and the power lost in the line.

Question 3 A generator supplies $72\,\text{kW}$ of power to a transmission line of total resistance $10\,\Omega$. (a) Calculate the line current and the power lost in the line when the power is transmitted at $3.0\,\text{kV}$. (b) Calculate the line current and the power lost when the same power is transmitted at $9.0\,\text{kV}$. (c) With reference to your answers, explain why electricity is transmitted at high voltage.

Question 4 A solar farm generates $120\,\text{kW}$, which is raised by a step-up transformer to $24\,\text{kV}$ and sent along a transmission line of total resistance $25\,\Omega$ to a step-down transformer near a town. (a) Calculate the current in the transmission line. (b) Calculate the voltage drop across the line, and hence the voltage at the far (step-down) end of the line. (c) Calculate the power lost in the line and the percentage of the generated power that reaches the step-down transformer.

Question 5 A factory is supplied through a transmission line of total resistance $5.0\,\Omega$. The voltage at the supply end of the line is $2.40\,\text{kV}$ and the voltage at the factory end is $2.32\,\text{kV}$. The factory's machinery carries a total power rating of $50\,\text{kW}$. (a) Calculate the voltage drop across the line. (b) Calculate the current in the line. (c) Calculate the actual power delivered to the factory, and explain why the $50\,\text{kW}$ rating should not be used to find the line current.

Question 6 A transmission line carries a fixed amount of power from a wind farm to a town. (a) (Circle your answer.) To reduce the power lost in the line, the voltage at which the power is transmitted should be: *increased* / *decreased*. Justify your choice with reference to $P = V\,I$ and $P_{\text{loss}} = I_{\text{line}}^2\,R_{\text{line}}$. No calculations are required. (b) State the energy transformation that accounts for the power lost in the line. (c) Explain why replacing the line with thicker cables of the same length also reduces the power lost, for a given line current.

Question 7 A transmission line carries a current of $45\,\text{A}$ and has a total resistance of $3.2\,\Omega$. Calculate the voltage drop across the line and the power dissipated in it.

Question 8 The voltage at the sending end of a transmission line is $11.0\,\text{kV}$ and the voltage at the receiving end is $10.72\,\text{kV}$. The line has a total resistance of $8.0\,\Omega$. Calculate the current in the line, the power lost in the line, and the power delivered to the load.

Question 9 A wind farm generates $200\,\text{kW}$ and transmits it at $16\,\text{kV}$ along a line of total resistance $6.0\,\Omega$. Calculate the line current and the power lost in the line, and determine the percentage of the generated power that is lost. Give your answer for the percentage to three significant figures.

Question 10 A transmission line carries a fixed amount of power. Explain, as a chain of reasoning, what happens to the power lost in the line if the transmission voltage is doubled while the transmitted power is unchanged. No numerical data are given; reason from the relevant relationships.

Question 11 The voltage drop across a transmission line is $210\,\text{V}$ when it carries a current of $15\,\text{A}$. Calculate the resistance of the line and the power dissipated in it.

Question 12 A solar farm feeds $110\,\text{kW}$ into a transmission line at $10\,\text{kV}$. The line has a total resistance of $10\,\Omega$. Calculate the power delivered to the far end of the line and the efficiency of the transmission (the percentage of the fed-in power that is delivered).

Question 13 Explain, as a step-by-step chain of reasoning, why electrical power is transmitted over long distances at very high voltage rather than at the low voltage used inside homes. Refer to $P = V\,I$ and $P_{\text{loss}} = I_{\text{line}}^2\,R_{\text{line}}$ in your answer.

Question 14 A student is asked to find the current in a transmission line. The question gives the voltage at each end of the line and the line resistance, and also mentions that the house at the end of the line contains a $9.0\,\text{kW}$ heater. The student writes $I = P / V = 9000 / 230 = 39\,\text{A}$. Explain what is wrong with this approach and describe the correct method for finding the line current.

Question 15 A transmission line continuously dissipates $8.0\,\text{kW}$ of power as heat, and electrical energy is sold at $\$0.22$ per kilowatt-hour. Calculate the energy lost in the line over one week (7 days), expressing your answer in both kilowatt-hours and joules, and hence the cost of that wasted energy.

Question 16 The two ends of a transmission line are at $6.60\,\text{kV}$ and $6.48\,\text{kV}$, and the line has a total resistance of $4.0\,\Omega$. Calculate the current in the line, the power lost in the line, and the power delivered to the load.

Question 17 A power station transmits $400\,\text{kW}$ along a line of total resistance $4.0\,\Omega$ at a voltage of $16\,\text{kV}$. (a) Show that the current in the transmission line is $25.0\,\text{A}$. (b) Hence calculate the power lost in the line and the percentage of the transmitted power that is lost.

Question 18 A wind turbine produces $350\,\text{kW}$ of electrical power. A step-up transformer raises the voltage to $25\,\text{kV}$ before the power is transmitted along a line of total resistance $15\,\Omega$ to a town, where a step-down transformer supplies homes at $250\,\text{V}$. Assume both transformers are ideal. (a) Calculate the current in the transmission line. (b) Calculate the power lost in the line and the voltage drop across it. (c) Calculate the voltage at the town end of the line, before the step-down transformer. (d) Calculate the power delivered to the town and the efficiency of the transmission.

Question 19 Two identical towns are each supplied with $100\,\mathrm{kW}$ along identical transmission lines. Town A is supplied at $4.0\,\mathrm{kV}$ and Town B at $20\,\mathrm{kV}$. Without calculating the actual power losses, state which town's line wastes more power, determine by what factor, and explain your reasoning.

Question 20 A generator maintains $540\,\mathrm{V}$ across the start of a two-wire transmission line. The line has a total resistance of $5.0\,\Omega$ and supplies a heater of resistance $25\,\Omega$ at its far end. Calculate the current in the line, the power lost in the line, and the power delivered to the heater.

Question 21 Explain, in terms of the line's resistance, why the voltage at the load end of a long transmission line is lower than the voltage at the supply end, and state what becomes of the electrical energy that does not reach the load.

Question 22 To reduce the energy wasted in a transmission line, one engineer proposes halving the resistance of the line (by using thicker cables), while another proposes doubling the transmission voltage. Assuming the transmitted power is unchanged, evaluate the two proposals by comparing the factor by which each reduces the power lost in the line.

Answers

Question 1 (a) $150\,\text{V}$; (b) $4500\,\text{W} = 4.5\,\text{kW}$; (c) $1.20 \times 10^4\,\text{W} = 12\,\text{kW}$. **Question 2** (a) $180\,\text{V}$; (b) $60\,\text{A}$; (c) $P_{\text{load}} = 1.87 \times 10^5\,\text{W}$, $P_{\text{loss}} = 1.08 \times 10^4\,\text{W} = 10.8\,\text{kW}$. **Question 3** (a) $24\,\text{A}$, $5760\,\text{W}$; (b) $8.0\,\text{A}$, $640\,\text{W}$; (c) high voltage gives low current, so the $I^2 R$ loss is much smaller. **Question 4** (a) $5.0\,\text{A}$; (b) $125\,\text{V}$, $2.39 \times 10^4\,\text{V}$; (c) $625\,\text{W}$, 99.5%. **Question 5** (a) $80\,\text{V}$; (b) $16\,\text{A}$; (c) $3.71 \times 10^4\,\text{W}$; the rating is not the actual delivered power. **Question 6** (a) increased; (b) electrical energy → heat (thermal energy) in the line; (c) thicker cable → lower resistance → smaller $I^2 R$ loss. **Question 7** $144\,\text{V}$; $6480\,\text{W}$. **Question 8** $35\,\text{A}$; $9800\,\text{W}$; $3.75 \times 10^5\,\text{W}$. **Question 9** $12.5\,\text{A}$; $938\,\text{W}$; 0.469%. **Question 10** Falls to one quarter. **Question 11** $14\,\Omega$; $3150\,\text{W}$. **Question 12** $1.09 \times 10^5\,\text{W}$ delivered; efficiency 98.9%. **Question 13** High voltage → small current → small $I^2 R$ line loss (full chain in the solution). **Question 14** The rated power must not be used; the current is $(V_{\text{supply}} - V_{\text{load}})/R_{\text{line}}$. **Question 15** $1.34 \times 10^3\,\text{kW h} = 4.84 \times 10^9\,\text{J}$; about \$296. **Question 16** $30\,\text{A}$; $3600\,\text{W}$; $1.94 \times 10^5\,\text{W}$. **Question 17** (a) $25.0\,\text{A}$; (b) $2500\,\text{W}$, 0.625%. **Question 18** (a) $14\,\text{A}$; (b) $2940\,\text{W}$, $210\,\text{V}$; (c) $2.48 \times 10^4\,\text{V}$; (d) $3.47 \times 10^5\,\text{W}$, 99.2%. **Question 19** Town A; by a factor of 25; because $I_{\text{line}} \propto 1/V$ and $P_{\text{loss}} \propto I_{\text{line}}^2$. **Question 20** $18\,\text{A}$; $1620\,\text{W}$; $8100\,\text{W}$. **Question 21** $V_{\text{load}} = V_{\text{supply}} - I_{\text{line}} R_{\text{line}} < V_{\text{supply}}$; the missing energy becomes heat in the line. **Question 22** Halving R_{line} halves the loss; doubling the voltage quarters it; doubling the voltage is more effective.

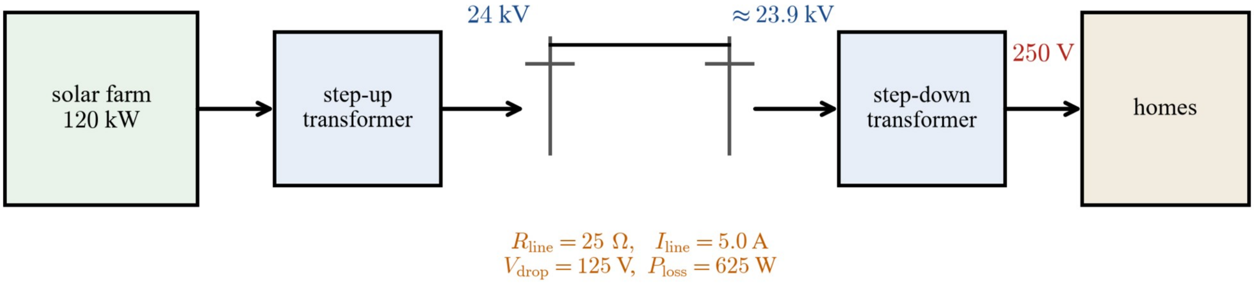
Fully-worked solutions

Question 1 $R_{\text{line}} = 5.0\,\Omega$, $I_{\text{line}} = 30\,\text{A}$, load at $V_{\text{load}} = 400\,\text{V}$. (a) $V_{\text{drop}} = I_{\text{line}} R_{\text{line}} = 30 \times 5.0 = 150\,\text{V}$. (b) $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 30^2 \times 5.0 = 4500\,\text{W} = 4.5\,\text{kW}$. (c) $P_{\text{load}} = V_{\text{load}} I_{\text{line}} = 400 \times 30 = 1.20 \times 10^4\,\text{W} = 12\,\text{kW}$.

Question 2 $V_{\text{supply}} = 3300\,\text{V}$, $V_{\text{load}} = 3120\,\text{V}$, $R_{\text{line}} = 3.0\,\Omega$. (a) $V_{\text{drop}} = 3300 - 3120 = 180\,\text{V}$. (b) $I_{\text{line}} = \frac{V_{\text{drop}}}{R_{\text{line}}} = \frac{180}{3.0} = 60\,\text{A}$. (c) $P_{\text{load}} = V_{\text{load}} I_{\text{line}} = 3120 \times 60 = 1.87 \times 10^5\,\text{W}$; $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 60^2 \times 3.0 = 1.08 \times 10^4\,\text{W} = 10.8\,\text{kW}$.

Question 3 $P = 72000\,\text{W}$, $R_{\text{line}} = 10\,\Omega$. (a) At $3.0\,\text{kV}$: $I_{\text{line}} = \frac{P}{V} = \frac{72000}{3000} = 24\,\text{A}$; $P_{\text{loss}} = 24^2 \times 10 = 5760\,\text{W}$. (b) At $9.0\,\text{kV}$: $I_{\text{line}} = \frac{72000}{9000} = 8.0\,\text{A}$; $P_{\text{loss}} = 8.0^2 \times 10 = 640\,\text{W}$. (c) Tripling the transmission voltage (from $3.0\,\text{kV}$ to $9.0\,\text{kV}$) cuts the current to one-third, since $I_{\text{line}} = P/V$. Because $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$ depends on the square of the current, the loss falls to one-ninth — from $5760\,\text{W}$ to $640\,\text{W}$. Transmitting at high voltage therefore keeps the line current, and hence the $I^2 R$ heating loss, small.

Question 4 $P = 120000\,\text{W}$, transmission voltage $V = 24000\,\text{V}$, $R_{\text{line}} = 25\,\Omega$.

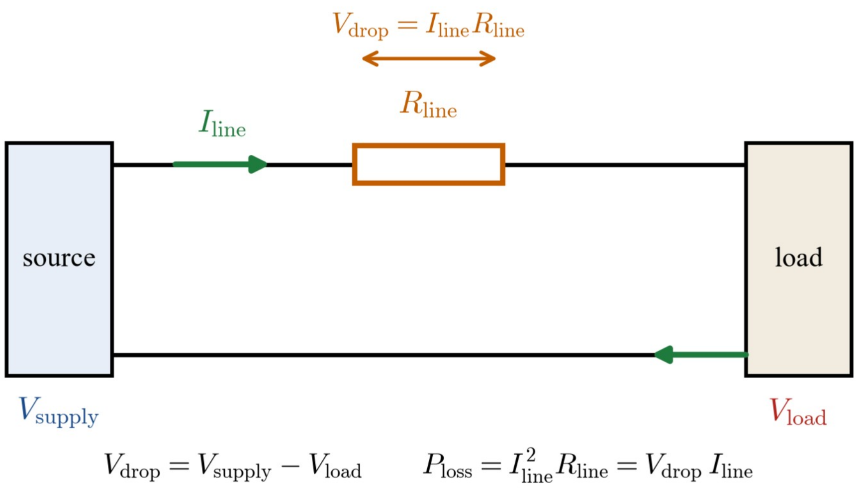


(a) $I_{\text{line}} = \frac{P}{V} = \frac{120000}{24000} = 5.0\,\text{A}$.

(b) $V_{\text{drop}} = I_{\text{line}} R_{\text{line}} = 5.0 \times 25 = 125\,\text{V}$; the voltage at the far end is $24000 - 125 = 23875\,\text{V} = 2.39 \times 10^4\,\text{V}$.

(c) $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 5.0^2 \times 25 = 625\,\text{W}$. The power reaching the step-down transformer is $120000 - 625 = 119375\,\text{W}$, a fraction $\frac{119375}{120000} = 0.995$ of the generated power, i.e. 99.5%.

Question 5 $V_{\text{supply}} = 2400\,\text{V}$, $V_{\text{load}} = 2320\,\text{V}$, $R_{\text{line}} = 5.0\,\Omega$.



(a) $V_{\text{drop}} = 2400 - 2320 = 80\,\text{V}$.

(b) $I_{\text{line}} = \frac{V_{\text{drop}}}{R_{\text{line}}} = \frac{80}{5.0} = 16\,\text{A}$.

(c) $P_{\text{load}} = V_{\text{load}} I_{\text{line}} = 2320 \times 16 = 3.71 \times 10^4\,\text{W}$. The 50 kW figure is the machinery's rated power — the power it would draw at its rated voltage — not the actual power delivered here, and it plays no part in fixing the line current. The current is set by the drop across the line and the line resistance, $I_{\text{line}} = (V_{\text{supply}} - V_{\text{load}}) / R_{\text{line}}$; using 50 kW in $I = P / V$ would give a different, incorrect current.

Question 6 (a) **Increased.** For a fixed transmitted power, $P = VI$ means a higher transmission voltage requires a smaller line current. Since $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$, a smaller current gives a smaller power loss, so the voltage should be increased. (b) Electrical energy is transformed into heat (thermal energy) in the resistance of the line. (c) Thicker cables of the same length have a lower resistance R_{line}. For a given line current, $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$ is then smaller, so less power is wasted heating the line.

Question 7 $I_{\text{line}} = 45\,\text{A}$, $R_{\text{line}} = 3.2\,\Omega$. $V_{\text{drop}} = I_{\text{line}} R_{\text{line}} = 45 \times 3.2 = 144\,\text{V}$. $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 45^2 \times 3.2 = 6480\,\text{W}$.

Question 8 $V_{\text{supply}} = 11000\,\text{V}$, $V_{\text{load}} = 10720\,\text{V}$, $R_{\text{line}} = 8.0\,\Omega$. $V_{\text{drop}} = 11000 - 10720 = 280\,\text{V}$, so $I_{\text{line}} = \frac{280}{8.0} = 35\,\text{A}$. $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 35^2 \times 8.0 = 9800\,\text{W}$. $P_{\text{load}} = V_{\text{load}} I_{\text{line}} = 10720 \times 35 = 3.75 \times 10^5\,\text{W}$.

Question 9 $P = 200000\,\text{W}$, $V = 16000\,\text{V}$, $R_{\text{line}} = 6.0\,\Omega$. $I_{\text{line}} = \frac{P}{V} = \frac{200000}{16000} = 12.5\,\text{A}$.

$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 12.5^2 \times 6.0 = 938\,\text{W}$ (to 3 s.f.). Fraction lost: $\frac{937.5}{200000} = 4.69 \times 10^{-3}$, i.e. 0.469% (to three significant figures).

Question 10 For a fixed transmitted power the line current is $I_{\text{line}} = P / V$, so doubling the transmission voltage halves the line current. The power lost is $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$, which depends on the square of the current; halving the current multiplies the loss by $(1/2)^2 = 1/4$. ∴ the power lost in the line falls to one quarter of its previous value.

Question 11 $V_{\text{drop}} = 210\text{ V}$, $I_{\text{line}} = 15\text{ A}$. $R_{\text{line}} = \dfrac{V_{\text{drop}}}{I_{\text{line}}} = \dfrac{210}{15} = 14\,\Omega$. $P_{\text{loss}} = V_{\text{drop}} I_{\text{line}} = 210 \times 15 = 3150\text{ W}$.

Question 12 $P = 110000\text{ W}$, $V = 10000\text{ V}$, $R_{\text{line}} = 10\,\Omega$. $I_{\text{line}} = \dfrac{P}{V} = \dfrac{110000}{10000} = 11\text{ A}$.
$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 11^2 \times 10 = 1210\text{ W}$. Power delivered $= 110000 - 1210 = 108790\text{ W} = 1.09 \times 10^5\text{ W}$. Efficiency: $\dfrac{108790}{110000} = 0.989$, i.e. 98.9%.

Question 13 A power station must deliver a certain power P. The power carried is $P = V\,I$, so for a chosen transmission voltage V the line current is $I_{\text{line}} = P / V$ — the higher the voltage, the smaller the current needed to carry the same power. The line has resistance R_{line}, and the power wasted heating it is $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$, which grows with the **square** of the current. Transmitting at a very high voltage makes I_{line} small, so $I_{\text{line}}^2 R_{\text{line}}$ — and therefore the energy lost in the lines — is small. The low voltage used inside homes would need a very large current to carry the same power, giving a large $I^2 R$ loss (and requiring unsafe, very thick conductors). ∴ power is transmitted at high voltage and stepped down to a safe voltage only where it is used.

Question 14 The student has used the heater's **rated power** to find the current, which is wrong. First, the 9.0 kW rating is the power the heater is designed to draw at its rated operating voltage, not necessarily the power actually delivered through this line, and 230 V is not stated to be the voltage at the load. Second, and more importantly, the line current is already fully determined by the information given — the two end voltages and the line resistance — through Ohm's law applied to the line. The correct method is

$$I_{\text{line}} = \frac{V_{\text{drop}}}{R_{\text{line}}} = \frac{V_{\text{supply}} - V_{\text{load}}}{R_{\text{line}}},$$

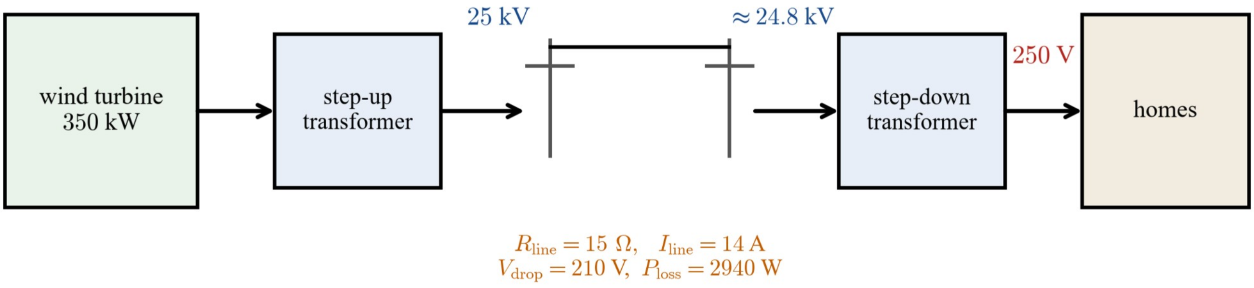
using the difference between the supply-end and load-end voltages. A rated power should be used only if the question states the power fed into the line together with the voltage there.

Question 15 $P_{\text{loss}} = 8.0\text{ kW}$, running for $t = 7\text{ days} = 7 \times 24 = 168\text{ h}$. Energy lost $= P_{\text{loss}} \times t = 8.0 \times 168 = 1344\text{ kW h} = 1.34 \times 10^3\text{ kW h}$. Using $1\text{ kW h} = 3.6 \times 10^6\text{ J}$, this is $1344 \times 3.6 \times 10^6 = 4.84 \times 10^9\text{ J}$. At \$0.22 per kW h, the cost is $1344 \times 0.22 = 295.68$, i.e. about \$296.

Question 16 $V_{\text{supply}} = 6600\text{ V}$, $V_{\text{load}} = 6480\text{ V}$, $R_{\text{line}} = 4.0\,\Omega$. $V_{\text{drop}} = 6600 - 6480 = 120\text{ V}$, so $I_{\text{line}} = \dfrac{120}{4.0} = 30\text{ A}$.
$P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 30^2 \times 4.0 = 3600\text{ W}$. $P_{\text{load}} = V_{\text{load}} I_{\text{line}} = 6480 \times 30 = 1.94 \times 10^5\text{ W}$.

Question 17 $P = 400000\text{ W}$, $V = 16000\text{ V}$, $R_{\text{line}} = 4.0\,\Omega$. (a) $I_{\text{line}} = \dfrac{P}{V} = \dfrac{400000}{16000} = 25.0\text{ A}$, as required. (b) $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 25.0^2 \times 4.0 = 2500\text{ W}$. As a fraction of the transmitted power, $\dfrac{2500}{400000} = 0.00625$, i.e. 0.625%.

Question 18 $P = 350000\text{ W}$, transmission voltage $V = 25000\text{ V}$, $R_{\text{line}} = 15\,\Omega$.



(a) $I_{line} = \frac{P}{V} = \frac{350000}{25000} = 14\text{ A}$.

(b) $P_{loss} = I_{line}^2 R_{line} = 14^2 \times 15 = 2940\text{ W}$; $V_{drop} = I_{line} R_{line} = 14 \times 15 = 210\text{ V}$.

(c) Voltage at the town end $= 25000 - 210 = 24790\text{ V} = 2.48 \times 10^4\text{ V}$.

(d) Power delivered $= 350000 - 2940 = 347060\text{ W} = 3.47 \times 10^5\text{ W}$; efficiency $\frac{347060}{350000} = 0.992$, i.e. 99.2%.

Question 19 Town A's line wastes more power. Each line carries the same power $P = 100\text{ kW}$, so the line current is $I_{line} = P/V$: Town A (at 4.0 kV) carries 5 times the current of Town B (at 20 kV), because its voltage is 5 times lower. The power wasted is $P_{loss} = I_{line}^2 R_{line}$, and the lines are identical (same R_{line}), so the loss scales with the square of the current. ∴ Town A's line wastes $5^2 = 25$ times as much power as Town B's.

Question 20 $V_{supply} = 540\text{ V}$, $R_{line} = 5.0\ \Omega$, heater resistance $R_{load} = 25\ \Omega$. The line and heater are in series, so the total resistance is $R_{line} + R_{load} = 5.0 + 25 = 30\ \Omega$ and the current is

$$I_{line} = \frac{V_{supply}}{R_{line} + R_{load}} = \frac{540}{30} = 18\text{ A}.$$

Power lost in the line: $P_{loss} = I_{line}^2 R_{line} = 18^2 \times 5.0 = 1620\text{ W}$. Power delivered to the heater: $P_{load} = I_{line}^2 R_{load} = 18^2 \times 25 = 8100\text{ W}$. ∴ the current is 18 A, the line wastes 1620 W, and 8100 W is delivered to the heater.

Question 21 A transmission line has resistance R_{line}, and the load current I_{line} flows through it. By Ohm's law this produces a voltage drop $V_{drop} = I_{line} R_{line}$ along the line, so the voltage arriving at the load is $V_{load} = V_{supply} - I_{line} R_{line}$, which is less than the supply-end voltage. The electrical energy that does not reach the load is dissipated as heat in the resistance of the conductors — the line warms up and radiates the energy to the surroundings — at a rate $P_{loss} = I_{line}^2 R_{line}$. Energy is conserved: the power supplied equals the power delivered to the load plus the power turned into heat in the line.

Question 22 Both proposals reduce the loss $P_{loss} = I_{line}^2 R_{line}$, but by different factors. Halving the line resistance halves the loss directly, since $P_{loss} \propto R_{line}$ at a fixed current — a factor of $1/2$. Doubling the transmission voltage, at fixed transmitted power, halves the line current ($I_{line} = P/V$); because $P_{loss} \propto I_{line}^2$, the loss falls by a factor of $(1/2)^2 = 1/4$. ∴ doubling the voltage is the more effective measure — it cuts the loss to a quarter, whereas halving the resistance only cuts it to a half.

Topic 3 Test A — Electricity generation and transmission (multiple choice)

Time allowed: 25 minutes · 20 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used.

Choose the response that is correct or that best answers the question. A correct answer scores 1 mark; an incorrect answer scores 0. Marks are not deducted for incorrect answers. No marks are given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this test are not drawn to scale. The relations $\Phi_B = B_\perp A$, $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, $V_{\text{rms}} = \dfrac{1}{\sqrt{2}}V_{\text{peak}}$, $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{I_2}{I_1}$, $P = VI$, $I = \dfrac{V}{R}$, $V_{\text{drop}} = I_{\text{line}}R_{\text{line}}$ *and* $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$ *are provided on the formula sheet.*

Question 1

A household rooftop solar (photovoltaic) system is connected to the home's wiring and to the electricity grid through an inverter. Which one of the following best describes the function of the inverter?

A. It increases the power and the voltage generated by the solar panels.

B. It converts the direct current (DC) produced by the panels into alternating current (AC) for household appliances and the grid.

C. It converts the alternating current (AC) produced by the panels into direct current (DC).

D. It stores the electrical energy generated during the day for use at night.

Question 2

A photovoltaic (solar) cell is exposed to sunlight. Which one of the following best describes the production of electricity by the cell?

A. Energy from the sunlight is converted directly into electrical energy in the cell, producing an alternating current (AC).

B. Sunlight heats the cell, and the thermal energy is then converted into electrical energy.

C. Energy from the sunlight is converted directly into electrical energy in the cell, producing a direct current (DC).

D. Sunlight is stored in the cell as chemical energy during the day and released as electrical energy at night.

Question 3

A single circular loop of wire of radius $8.0\,\text{cm}$ is placed in a uniform magnetic field of magnitude $0.35\,\text{T}$ directed perpendicular to the plane of the loop. Which one of the following is closest to the magnetic flux through the loop?

A. $7.0\times10^{-3}\,\text{Wb}$

B. $0.18\,\text{Wb}$

C. $2.8\times10^{-2}\,\text{Wb}$

D. $2.2\times10^{-3}\,\text{Wb}$

Question 4

A coil of 200 turns is linked by a magnetic flux that falls steadily from 4.0×10^{-3} Wb to 1.0×10^{-3} Wb in a time of 0.015 s. Which one of the following is closest to the magnitude of the average emf induced in the coil?

A. 0.20 V

B. 53.3 V

C. 0.60 V

D. 40 V

Question 5

The north pole of a bar magnet is pushed towards a coil that is connected to a galvanometer. Which one of the following best explains the direction of the current induced in the coil?

A. As the magnet approaches, the flux through the coil increases, so the induced current flows in the direction that makes the near face of the coil a north pole, opposing the increase in flux.

B. The induced current flows in the direction that makes the near face of the coil a south pole, attracting the magnet and increasing the flux through the coil.

C. The moving magnetic field of the magnet exerts a force directly on the electrons in the wire, pushing them around the coil.

D. A current is induced because the magnet's field is strong, but its direction cannot be predicted without knowing the resistance of the coil.

Question 6

A single coil is rotated at a steady rate in a uniform magnetic field. To make this device deliver direct current (DC) to an external circuit, the ends of the coil should be connected to

A. two slip rings, which keep the connection to the external circuit the same throughout each rotation.

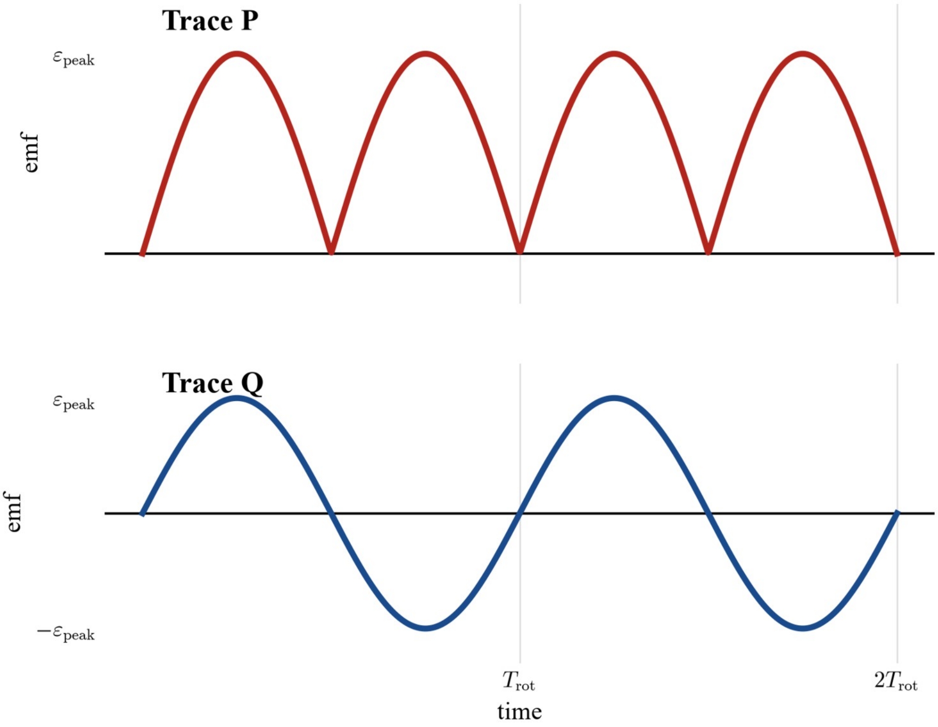
B. a split-ring commutator, which reverses the connection to the external circuit every half-turn.

C. a step-up transformer placed on the output.

D. an inverter, which changes the output to alternating current.

Question 7

The graph below shows two output traces, P and Q, obtained from the same single coil rotating at the same steady rate in a uniform magnetic field, using two different connections to the external circuit.



Which one of the following is correct?

A. Trace P (the all-positive humps) is produced by slip rings, and trace Q (the sinusoid) by a split-ring commutator.

B. Trace P (the all-positive humps) is produced by a split-ring commutator, and trace Q (the sinusoid) by slip rings.

C. Both traces are produced by slip rings; a split-ring commutator cannot produce an induced emf.

D. Trace P has half the frequency of trace Q.

Question 8

The coil of a simple alternator rotates at 1500 revolutions per minute in a constant magnetic field. Which one of the following is closest to the frequency of the sinusoidal AC voltage it produces?

A. 1500 Hz

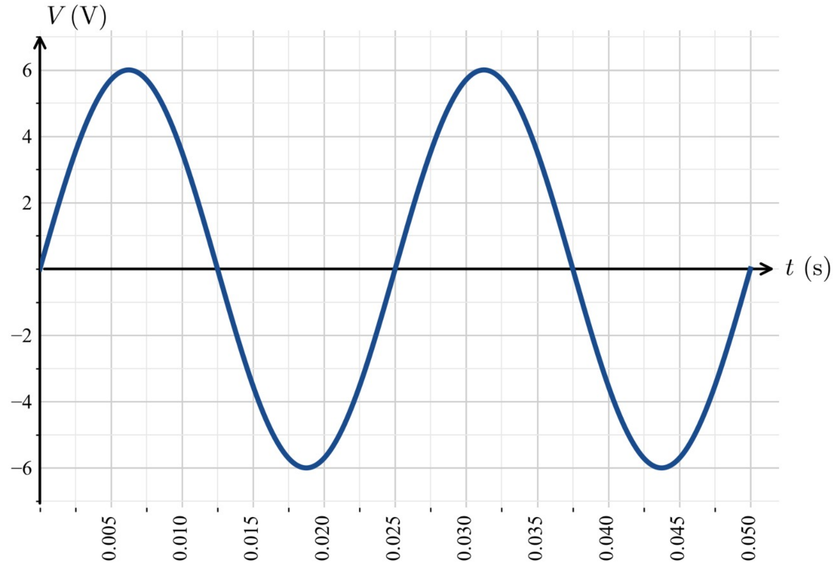
B. 50 Hz

C. 25 Hz

D. 12.5 Hz

Question 9

The graph below shows the alternating voltage across a resistor as a function of time.



Which one of the following is closest to the frequency of this AC voltage?

A. 0.025 Hz

B. 20 Hz

C. 80 Hz

D. 40 Hz

Question 10

An alternating supply with a peak voltage of 12 V is connected across a 4.0 Ω resistor. Which one of the following is closest to the peak-to-peak current in the resistor?

A. 6.0 A

B. 3.0 A

C. 1.5 A

D. 4.24 A

Question 11

The single coil of an AC generator is made to rotate twice as fast in the same magnetic field. Which one of the following best describes the change in the output voltage?

A. The frequency doubles, but the peak voltage is unchanged.

B. The peak voltage doubles, but the frequency is unchanged.

C. Both the frequency and the peak voltage double.

D. Both the frequency and the peak voltage are halved.

Question 12

An alternating voltage has a peak-to-peak value of 68 V. Which one of the following is closest to its rms voltage?

A. 48 V

B. 24 V

C. 34 V

D. 17 V

Question 13

Which one of the following best describes the meaning of the rms (root-mean-square) value of an alternating voltage?

A. It is the value of a steady DC voltage that would dissipate the same average power in a given resistor.

B. It is the maximum (peak) value that the voltage reaches during a cycle.

C. It is the average of the voltage over one complete cycle.

D. It is the peak-to-peak voltage divided by two.

Question 14

An AC supply with an rms voltage of 25 V is connected across a 10 Ω resistor. Which one of the following is closest to the average power dissipated in the resistor?

A. 125 W

B. 31.3 W

C. 2.5 W

D. 62.5 W

Question 15

An ideal transformer has 800 turns on its primary coil and 200 turns on its secondary coil. The primary is connected to a 240 V AC supply. Which one of the following is closest to the secondary (output) voltage?

A. 960 V

B. 240 V

C. 60 V

D. 480 V

Question 16

An ideal transformer steps a 250 V AC supply up to 2500 V. The current in the primary coil is 12 A. Which one of the following is closest to the current in the secondary coil?

A. 120 A

B. 1.2 A

C. 12 A

D. 2.4 A

Question 17

Which one of the following best explains why a transformer operates with alternating current (AC) but not with steady direct current (DC)?

A. A transformer works by electromagnetic induction: only a changing current in the primary produces a changing flux through the secondary, which is needed to induce an emf. A steady DC gives a constant flux and no induced emf.

B. A transformer is designed to change AC voltages, and a DC voltage simply cannot be changed.

C. A steady direct current would melt the primary coil, whereas alternating current does not.

D. The iron core conducts alternating current but does not conduct direct current.

Question 18

A transmission line of total resistance $8.0\,\Omega$ carries a current of $15\,\text{A}$. Which one of the following is closest to the power dissipated in the line?

A. $120\,\text{W}$

B. $960\,\text{W}$

C. $900\,\text{W}$

D. $1800\,\text{W}$

Question 19

Electricity is transmitted over long distances at very high voltage. Which one of the following best explains why?

A. A higher voltage forces the current through the line resistance more quickly, so less energy is lost.

B. A higher voltage reduces the resistance of the transmission line.

C. For a given power, a higher transmission voltage means a lower line current ($P=V\,I$); since the line loss is $P_{\text{loss}}=I^2R$, the lower current greatly reduces the power dissipated in the line.

D. A higher voltage increases the line current, and a larger current is transmitted more efficiently.

Question 20

A power station supplies a factory through transmission lines of total resistance $3.0\,\Omega$. The voltage at the sending (station) end is $5000\,\text{V}$ and the voltage measured at the factory end is $4880\,\text{V}$. Which one of the following is closest to the power dissipated in the transmission lines?

A. $2.00\times10^5\,\text{W}$

B. $533\,\text{W}$

C. $120\,\text{W}$

D. $4800\,\text{W}$

End of Topic 3 Test A

Topic 3 Test A — solutions

Question	Answer	Question	Answer
1	B	11	C
2	C	12	B
3	A	13	A
4	D	14	D
5	A	15	C
6	B	16	B
7	B	17	A
8	C	18	D
9	D	19	C
10	A	20	D

Question 1 — B. Photovoltaic cells generate direct current, but household appliances and the grid run on alternating current; the inverter converts DC to AC. **A** treats the inverter as an amplifier (it cannot create energy); **C** reverses the conversion; **D** describes a battery, not an inverter.

Question 2 — C. Photovoltaic cells convert energy from sunlight directly into electrical energy, and their output is a direct current (DC). **A** names the right conversion but the wrong current type; **B** describes a thermal process rather than the photovoltaic effect; **D** describes storage in a battery, but a cell generates electricity only while it is illuminated.

Question 3 — A. With the field perpendicular to the plane the flux is a maximum: $\Phi_B = BA = 0.35 \times \pi(0.080)^2 = 7.0 \times 10^{-3}\,\text{Wb}$. **B** (0.18) uses the circumference $2\pi r$ instead of the area; **C** (2.8×10^{-2}) uses the diameter as the radius; **D** (2.2×10^{-3}) omits the factor π.

Question 4 — D. $\varepsilon = N\dfrac{\Delta\Phi_B}{\Delta t} = 200 \times \dfrac{(4.0 - 1.0) \times 10^{-3}}{0.015} = 40\,\text{V}$. **A** (0.20) omits the number of turns; **B** (53.3) uses the initial flux 4.0×10^{-3} instead of the change; **C** (0.60) omits the time interval.

Question 5 — A. This is the full induction chain: motion of the magnet increases the flux through the coil, so by Faraday's law an emf is induced, and the induced current flows so as to oppose the increase — making the near face a north pole (repelling the approaching north pole). **B** has the current opposing the wrong way (it would reinforce the change); **C** is the misconception that the field acts directly on the conduction electrons; **D** is false — the direction is fully determined, independent of resistance.

Question 6 — B. A split-ring commutator reverses the connection between the coil and the external circuit every half-turn, so the output never reverses polarity — direct current. **A** describes slip rings, which give AC; **C** and **D** describe devices that do not rectify a rotating coil's output (and an inverter would produce AC).

Question 7 — B. For one coil turning at one rate, the slip-ring output is a full sinusoid (AC), whereas the split-ring commutator folds the negative half-cycles up into all-positive humps (DC). So trace P (humps) is the commutator and trace Q (sinusoid) is the slip rings. **A** swaps them; **C** is false (a rotating coil always induces an emf); **D** is false — the commutator humps come twice per revolution, so trace P has *twice* the frequency of trace Q.

Question 8 — C. A single coil produces one complete AC cycle per revolution, so the output frequency equals the rotation frequency: $f = \dfrac{1500}{60} = 25\,\text{Hz}$. **A** (1500) fails to convert revolutions per minute to per second; **B** (50) assumes two cycles per revolution; **D** (12.5) halves the correct value.

Question 9 — D. One complete cycle spans $T = 0.025\,\text{s}$, so $f = \dfrac{1}{T} = 40\,\text{Hz}$. **A** (0.025) quotes the period as if it were the frequency; **B** (20) reads the whole $0.050\,\text{s}$ window (two complete cycles) as one period; **C** (80) reads only half a cycle ($0.0125\,\text{s}$) as the period.

Question 10 — A. $I_{\text{peak}} = \dfrac{V_{\text{peak}}}{R} = \dfrac{12}{4.0} = 3.0\ \text{A}$, so the peak-to-peak current is $I_{\text{p-p}} = 2 \times 3.0 = 6.0\ \text{A}$. **B** (3.0) gives the peak, not the peak-to-peak; **C** (1.5) inserts a spurious factor of $1/2$; **D** (4.24) takes the rms of the peak-to-peak value.

Question 11 — C. Faster rotation gives more cycles per second (higher frequency) **and** a faster rate of change of flux (larger peak emf), so both double. **A** is the common frequency-only misconception; **B** keeps the frequency fixed; **D** has the changes the wrong way.

Question 12 — B. $V_{\text{peak}} = \dfrac{V_{\text{p-p}}}{2} = 34\ \text{V}$, so $V_{\text{rms}} = \dfrac{V_{\text{peak}}}{\sqrt{2}} = \dfrac{34}{\sqrt{2}} = 24\ \text{V}$. **A** (48) divides the peak-to-peak value by $\sqrt{2}$ without first halving it; **C** (34) is the peak; **D** (17) halves the peak.

Question 13 — A. The rms value is the equivalent steady DC voltage that would deliver the same average power to a resistor — the reason rms is used to state AC quantities. **B** is the peak; **C** is the cycle-average, which is zero for a symmetric AC waveform; **D** describes the peak (half of peak-to-peak).

Question 14 — D. $P_{\text{avg}} = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{25^2}{10} = 62.5\ \text{W}$. **A** (125) uses the peak voltage, giving the peak instantaneous power (twice the average); **B** (31.3) inserts a spurious $1/2$; **C** (2.5) is the current V / R, not a power.

Question 15 — C. For an ideal transformer $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$, so $V_2 = 240 \times \dfrac{200}{800} = 60\ \text{V}$. **A** (960) inverts the turns ratio (steps up instead of down); **B** (240) leaves the voltage unchanged; **D** (480) uses a factor of 2 the wrong way.

Question 16 — B. For an ideal transformer power is conserved, $V_1 I_1 = V_2 I_2$, so $I_2 = \dfrac{V_1 I_1}{V_2} = \dfrac{250 \times 12}{2500} = 1.2\ \text{A}$ (step-up in voltage means step-down in current). **A** (120) inverts the ratio; **C** (12) leaves the current unchanged; **D** (2.4) is a factor-of-two slip.

Question 17 — A. A transformer relies on electromagnetic induction: an alternating primary current produces a continually changing flux in the core, which induces an emf in the secondary. A steady DC gives a constant flux, so no emf is induced. **B** is a circular restatement (a tautology) with no mechanism; **C** and **D** are physically false.

Question 18 — D. $P_{\text{loss}} = I^2 R = 15^2 \times 8.0 = 1800\ \text{W}$. **A** (120) computes $I R$, the voltage drop, not a power; **B** (960) squares R instead of I; **C** (900) inserts a spurious $1/2$.

Question 19 — C. For a fixed power, $P = V I$ means a higher transmission voltage draws a smaller line current; because the line loss is $I^2 R$, halving the current quarters the loss. **A** is meaningless hand-waving; **B** is false (voltage does not change the line's resistance); **D** has the current going the wrong way.

Question 20 — D. Use the two end voltages: $V_{\text{drop}} = 5000 - 4880 = 120\ \text{V}$, so $I_{\text{line}} = \dfrac{V_{\text{drop}}}{R} = \dfrac{120}{3.0} = 40\ \text{A}$ and $P_{\text{loss}} = I^2 R = 40^2 \times 3.0 = 4800\ \text{W}$ (equivalently $\dfrac{V_{\text{drop}}^2}{R}$). **A** ($2.00 \times 10^5$) multiplies the *station* voltage by the line current; **B** (533) divides by R instead of multiplying; **C** (120) quotes the voltage drop as if it were the power.

Topic 3 Test B — Electricity generation and transmission (written responses)

Time allowed: 50 minutes · 40 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used. In questions where more than one mark is available, appropriate working must be shown.

Answer all questions in the spaces provided. Where a final answer has a required unit, give your answer in that unit. Unless otherwise indicated, the diagrams in this test are not drawn to scale.

Question 1 (4 marks)

A rooftop photovoltaic array is connected to the household wiring and to the electricity grid through an inverter.

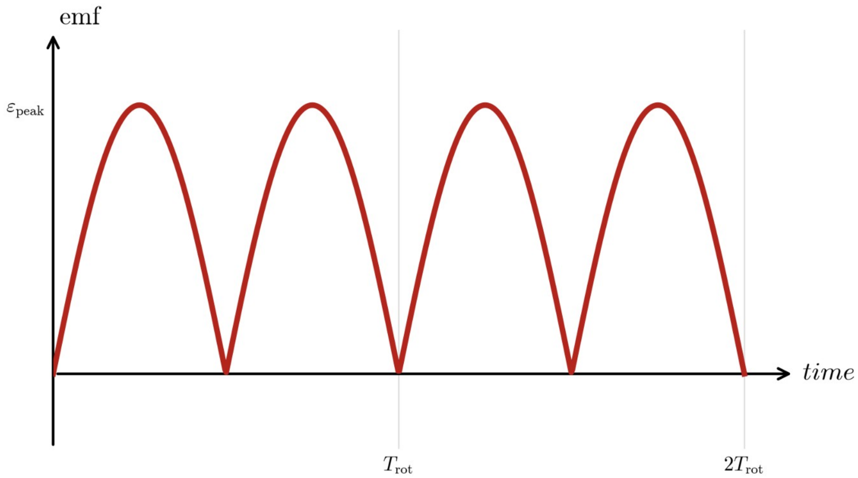
a. Describe how the photovoltaic array produces electricity when sunlight falls on it. 2 marks

b. During the day the array delivers an average output of $2.4\,\text{kW}$ for 5.0 hours. Calculate the electrical energy it generates in that time. Give your answer in kW h. 1 mark

c. Explain why the inverter is necessary. 1 mark

Question 2 (5 marks)

A simple generator has a single coil that rotates at a constant rate in a uniform magnetic field. Its output is shown in the graph below.



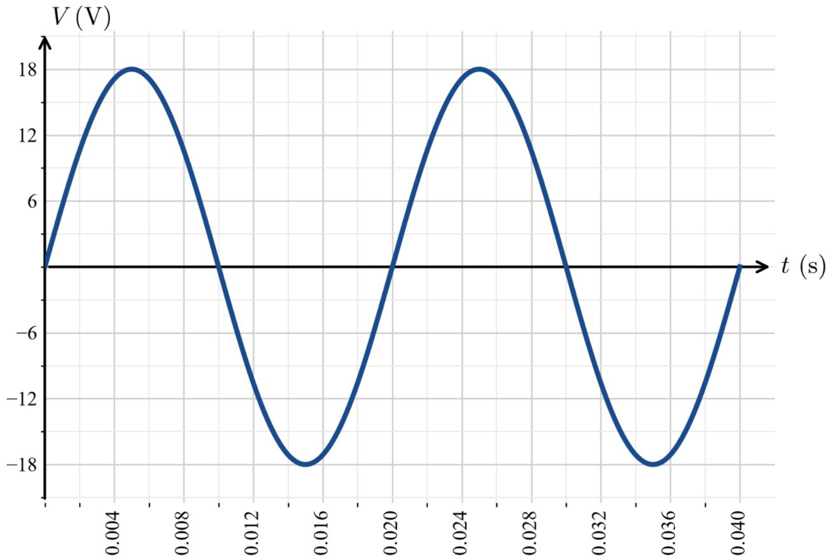
a. Explain why the emf induced in the coil is a maximum at the instant the plane of the coil is parallel to the magnetic field. 2 marks

b. (Circle your answer.) The output shown never falls below zero. This output is produced by connecting the coil to: *a split-ring commutator* / *two slip rings*. Justify your choice. 2 marks

c. State how the output trace would differ if the other connection had been used instead. 1 mark

Question 3 (5 marks)

The graph below shows the alternating voltage produced by a generator across a resistor of resistance $12\,\Omega$.



a. Determine the frequency of the AC voltage. Give your answer in Hz. 1 mark

b. State the peak-to-peak voltage of the AC. Give your answer in V. 1 mark

c. Calculate the peak current and the peak-to-peak current in the resistor. Give your answers in A. 2 marks

d. The generator's coil is then rotated at half its original rate. State the new frequency, and describe the effect on the peak voltage of the output. 1 mark

Question 4 (5 marks)

An alternating voltage with a peak value of $60\ \text{V}$ is applied across a resistor of resistance $18\ \Omega$.

a. Show that the rms voltage of the supply is $42.4\ \text{V}$. 1 mark

b. Calculate the average power dissipated in the resistor. Give your answer in W. 2 marks

c. Explain what the rms voltage of $42.4\ \text{V}$ means in terms of an equivalent steady DC voltage. 2 marks

Question 5 (6 marks)

An ideal transformer has 1000 turns on its primary coil and 50 turns on its secondary coil. The primary coil is connected to a $300\ \text{V}$ AC supply, and the secondary coil delivers a current of $4.0\ \text{A}$ to a device.

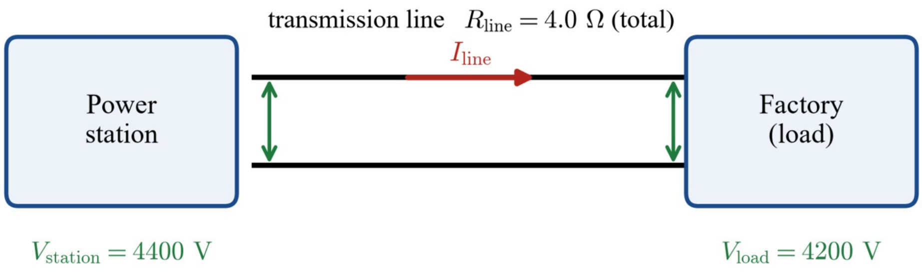
a. Show that the secondary (output) voltage is $15\ \text{V}$. 1 mark

b. Calculate the current in the primary coil. Give your answer in A. 2 marks

c. The $300\ \text{V}$ AC supply is replaced by a $300\ \text{V}$ steady DC supply. Explain, with reference to electromagnetic induction, why the transformer then produces no output voltage. 3 marks

Question 6 (7 marks)

A power station supplies a factory through transmission lines of total resistance $4.0\ \Omega$, as shown below. The voltage at the sending (station) end is $4400\ \text{V}$ and the voltage measured across the factory is $4200\ \text{V}$.



a. Show that the current in the transmission line is 50 A. 2 marks

b. Calculate the power dissipated in the transmission lines. Give your answer in W. 2 marks

c. Calculate the power delivered to the factory. Give your answer in W. 1 mark

d. Explain why transmitting the power at a much higher voltage (with a step-up transformer at the station) would reduce the power lost in the lines. 2 marks

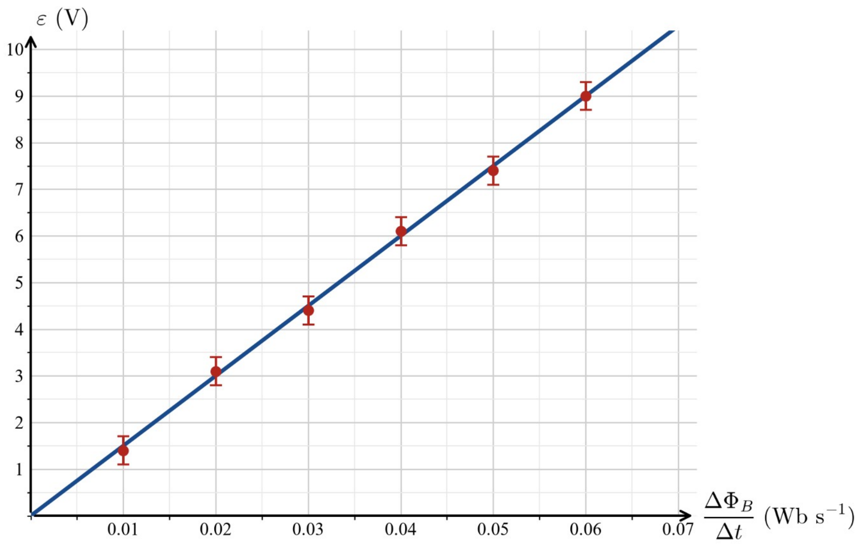
Question 7 (8 marks)

A student investigates electromagnetic induction. In the first part, she pushes the north pole of a bar magnet into a coil that is connected to a sensitive galvanometer. In the second part, she uses apparatus that produces a known, steady rate of change of magnetic flux through a fixed coil, and measures the average emf induced for several different rates of change of flux.

a. As the magnet is pushed into the coil, describe, as a sequence of steps, how an emf is induced in the coil and how the direction of the induced current is determined. 3 marks

b. For the second part of the investigation, identify the independent variable, the dependent variable, and one variable that must be held constant. 2 marks

The student's results are plotted below, with uncertainty bars, and a line of best fit is drawn through the origin.



c. Using two well-separated points on the line of best fit, determine the gradient of the line. 2 marks

d. State the physical quantity that the gradient of this graph represents. 1 mark

End of Topic 3 Test B

Topic 3 Test B — solutions

Question 1

(a) Energy from the sunlight is converted directly into electrical energy in the photovoltaic cells, and the output of the cells is a direct current (DC). **(1 mark** sunlight energy converted directly to electrical energy; **1 mark** the output is direct current (DC).)

(b) $E = P\,t = 2.4\,\text{kW} \times 5.0\,\text{h} = 12\,\text{kW h}$. **(1 mark: correct value** $12\,\text{kW h}$**.)**

(c) The photovoltaic cells generate direct current (DC), but household appliances and the grid operate on alternating current (AC). The inverter converts the DC output of the panels into AC so it can be used in the home and fed to the grid. **(1 mark: PV output is DC, appliances/grid need AC, so the inverter converts DC to AC.)**

Question 2

(a) As the coil rotates, the flux through it changes, and the induced emf is proportional to the *rate* of change of flux, $\varepsilon = N\dfrac{\Delta\Phi_B}{\Delta t}$. When the plane of the coil is parallel to the field, the flux through the coil is instantaneously zero but is changing at its fastest rate (the coil sides are cutting field lines most rapidly), so the emf is a maximum. **(1 mark** emf depends on the rate of change of flux; **1 mark** when the plane is parallel to the field the flux is changing fastest, so the emf is maximum.)

(b) **A split-ring commutator.** The output never reverses polarity (it stays positive), which is direct current. A split-ring commutator reverses the connection between the coil and the external circuit every half-turn — at the instant the coil's own emf reverses — so the two reversals cancel and the output keeps one polarity. **(1 mark** circles *a split-ring commutator*; **1 mark** justification: it reverses the connection each half-turn so the output stays one polarity (DC). The circled option alone scores 0 without the reasoning.)

(c) With two slip rings the output would be a full sinusoid — the negative half-cycles would appear as equal humps below the axis (alternating current), with the same peak value but twice the period of the trace shown: the sinusoid completes one cycle per revolution, whereas the rectified trace shown repeats every half-revolution. **(1 mark: slip rings give a full sinusoid / AC with equal negative humps.)**

Question 3 $R = 12\,\Omega$; from the graph the peak voltage is $18\,\text{V}$ and one period is $T = 0.020\,\text{s}$.

(a) $f = \dfrac{1}{T} = \dfrac{1}{0.020} = 50\,\text{Hz}$. **(1 mark: correct value** $50\,\text{Hz}$**.)**

(b) The trace swings from $+18\,\text{V}$ to $-18\,\text{V}$, so the peak-to-peak voltage is $V_{\text{p-p}} = 2 \times 18 = 36\,\text{V}$. **(1 mark: correct value** $36\,\text{V}$**.)**

(c) $I_{\text{peak}} = \dfrac{V_{\text{peak}}}{R} = \dfrac{18}{12} = 1.5\,\text{A}$, so $I_{\text{p-p}} = 2 \times 1.5 = 3.0\,\text{A}$. **(1 mark** $I_{\text{peak}} = 1.5\,\text{A}$; **1 mark** $I_{\text{p-p}} = 3.0\,\text{A}$.)

(d) The frequency is set by the rotation rate alone, so it halves to $25\,\text{Hz}$. The peak voltage also depends on the rate of change of flux, so at half the rotation rate the peak voltage halves — from $18\,\text{V}$ to $9.0\,\text{V}$. **(1 mark: new frequency** $25\,\text{Hz}$ **and peak voltage halves to** $9.0\,\text{V}$**.)**

Question 4 $V_{\text{peak}} = 60\,\text{V}$, $R = 18\,\Omega$.

(a) $V_{\text{rms}} = \dfrac{V_{\text{peak}}}{\sqrt{2}} = \dfrac{60}{\sqrt{2}} = 42.4\,\text{V}$, as required. **(1 mark: correct substitution and result.)**

(b) $P_{\text{avg}} = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{42.43^2}{18} = \dfrac{1800}{18} = 100\,\text{W}$ (equivalently $\dfrac{V_{\text{peak}}^2}{2R} = \dfrac{3600}{36} = 100\,\text{W}$). **(1 mark** uses V_{rms} (not the peak) in $P = V^2/R$; **1 mark** correct value $100\,\text{W}$.)

(c) The rms voltage of $42.4\,\text{V}$ is the value of a *steady DC voltage* that would dissipate the same average power in the same resistor. In other words, this AC supply delivers, on average, the same power to the $18\,\Omega$ resistor as a $42.4\ \mathrm{V}$ DC supply would. **(1 mark** rms = equivalent steady DC voltage; **1 mark** that delivers the same average power in the resistor.)

Question 5 Ideal transformer: $N_1 = 1000$, $N_2 = 50$, $V_1 = 300\ \mathrm{V}$, $I_2 = 4.0\ \mathrm{A}$.

(a) $V_2 = V_1 \dfrac{N_2}{N_1} = 300 \times \dfrac{50}{1000} = 15\ \mathrm{V}$, as required. **(1 mark: correct substitution and result.)**

(b) For an ideal transformer power is conserved, $V_1 I_1 = V_2 I_2$, so

$$I_1 = \frac{V_2 I_2}{V_1} = \frac{15 \times 4.0}{300} = 0.20\ \mathrm{A}.$$

∴ the primary current is $0.20\ \mathrm{A}$ (equivalently $I_1 = I_2 \dfrac{N_2}{N_1} = 4.0 \times \dfrac{50}{1000} = 0.20\ \mathrm{A}$). **(1 mark** $V_1 I_1 = V_2 I_2$ (or the turns ratio for current); **1 mark** correct value $0.20\ \mathrm{A}$.)

(c) A transformer works by electromagnetic induction. The primary current sets up a magnetic flux in the core; an emf is induced in the secondary only when that flux is *changing*, since $\varepsilon = N \dfrac{\Delta \Phi_B}{\Delta t}$. A steady DC supply drives a constant current, which produces a *constant* flux in the core. With no change in flux there is no rate of change of flux through the secondary, so no emf is induced and the output voltage is zero. **(1 mark** transformer action is electromagnetic induction; **1 mark** steady DC gives a constant current and hence constant flux; **1 mark** no changing flux ⇒ no induced emf ⇒ zero output. A bare "transformers only work on AC" without the induction reasoning does not earn full marks.)

Question 6 $R_{\text{line}} = 4.0\ \Omega$, $V_{\text{station}} = 4400\ \mathrm{V}$, $V_{\text{load}} = 4200\ \mathrm{V}$.

(a) The voltage drop along the line is $V_{\text{drop}} = V_{\text{station}} - V_{\text{load}} = 4400 - 4200 = 200\ \mathrm{V}$. The line current is

$$I_{\text{line}} = \frac{V_{\text{drop}}}{R_{\text{line}}} = \frac{200}{4.0} = 50\ \mathrm{A},$$

as required. **(1 mark** $V_{\text{drop}} = 200\ \mathrm{V}$ from the two end voltages; **1 mark** $I_{\text{line}} = 50\ \mathrm{A}$.)

(b) $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}} = 50^2 \times 4.0 = 1.0 \times 10^4\ \mathrm{W}$ (equivalently $V_{\text{drop}} I_{\text{line}} = 200 \times 50$). **(1 mark** $P_{\text{loss}} = I^2 R$; **1 mark** correct value $1.0 \times 10^4\ \mathrm{W}$.)

(c) $P_{\text{load}} = V_{\text{load}} I_{\text{line}} = 4200 \times 50 = 2.1 \times 10^5\ \mathrm{W}$. **(1 mark: correct value** $2.1 \times 10^5\ \mathrm{W}$**.)** (The power supplied is $V_{\text{station}} I_{\text{line}} = 2.2 \times 10^5\ \mathrm{W}$, so about $95\,\%$ reaches the factory.)

(d) The power delivered is fixed, so from $P = VI$ a higher transmission voltage means a smaller line current. Because the line loss is $P_{\text{loss}} = I_{\text{line}}^2 R_{\text{line}}$, it depends on the *square* of the current: reducing the current by transmitting at high voltage cuts the loss steeply (halving the current quarters the loss). The line resistance is unchanged, but far less power is wasted as heat in it. **(1 mark** higher voltage ⇒ lower line current ($P = VI$); **1 mark** $P_{\text{loss}} = I^2 R$ falls steeply as the current falls.)

Question 7

(a) The sequence is:

- Pushing the magnet in moves it relative to the coil, so the magnetic flux through the coil **increases** (more field lines pass through it as the magnet nears).
- Because the flux through the coil is **changing**, by Faraday's law an emf is induced: $\varepsilon = N \dfrac{\Delta \Phi_B}{\Delta t}$.
- This emf drives an induced current whose direction **opposes the change** that produced it (Lenz's law): the current flows so that the near face of the coil becomes a **north pole**, repelling the approaching magnet.

(1 mark relative motion ⇒ changing/increasing flux through the coil; **1 mark** changing flux ⇒ induced emf by Faraday's law; **1 mark** induced current opposes the change (near face becomes north / repels the magnet). A bare recitation of "Lenz's law" without the flux-change chain does not earn full marks.)

(b) **Independent variable:** the rate of change of magnetic flux, $\frac{\Delta\Phi_B}{\Delta t}$ (the quantity deliberately varied).

Dependent variable: the average induced emf ε (the quantity measured). **Controlled variable:** the number of turns N on the coil (also acceptable: the coil's area, or the same coil throughout). **(1 mark** IV and DV both correct; **1 mark** a valid controlled variable.)

(c) Using two well-separated points **on the line of best fit** — for example $(0.010\ \mathrm{Wb\ s^{-1}}, 1.5\ \mathrm{V})$ and $(0.060\ \mathrm{Wb\ s^{-1}}, 9.0\ \mathrm{V})$:

$$\text{gradient} = \frac{\Delta\varepsilon}{\Delta(\Delta\Phi_B/\Delta t)} = \frac{9.0-1.5}{0.060-0.010} = \frac{7.5}{0.050} = 150.$$

(1 mark two well-separated points *on the line of best fit* (not raw data points); **1 mark** correct gradient 150.)

(d) From $\varepsilon = N\frac{\Delta\Phi_B}{\Delta t}$, a graph of ε against $\frac{\Delta\Phi_B}{\Delta t}$ is a straight line through the origin of gradient N. So the gradient represents the **number of turns on the coil** ($N = 150$). **(1 mark: the gradient is the number of turns N on the coil.)**