Physics Units 3 & 4

Chapter 5 — Generation of electricity

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

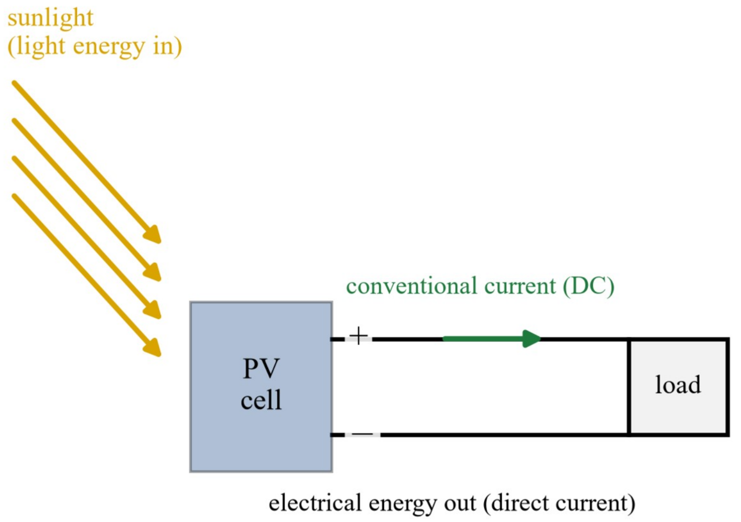
Contents

Theory

A **photovoltaic (PV) cell** — also called a solar cell — converts the energy of incident sunlight directly into electrical energy. When light falls on the cell it produces an electrical output straight away, silently and with no moving parts and no fuel. The internal process by which a cell does this is not required in this course; here a cell is treated simply as an **energy converter**: light energy in, electrical energy out.

Two features of that output matter for the rest of this subtopic:

- the cell produces a **direct current (DC)** — a current in one direction only, driven by a steady output voltage — for as long as the cell is illuminated;
- the cell only generates electricity **while light falls on it**. It does not store energy for later use; storing energy for the night requires a separate device such as a battery.



Electrical power and energy of a PV output. The electrical power delivered by an illuminated cell, module or array is

$$P = VI,$$

where V is the output voltage and I the output current. Over a time t the electrical energy delivered is

$$E = Pt.$$

In joules, t must be in seconds; for household quantities it is often more convenient to measure energy in **kilowatt-hours** ($\mathrm{kW\,h}$) — a power in kilowatts multiplied by a time in hours, with $1\,\mathrm{kW\,h} = 3.6 \times 10^6\,\mathrm{J}$.

Efficiency and the incident solar power. The **irradiance** G is the solar power falling on each square metre of a surface, measured in $\mathrm{W\,m^{-2}}$. A panel of area A facing the Sun therefore receives an incident solar power

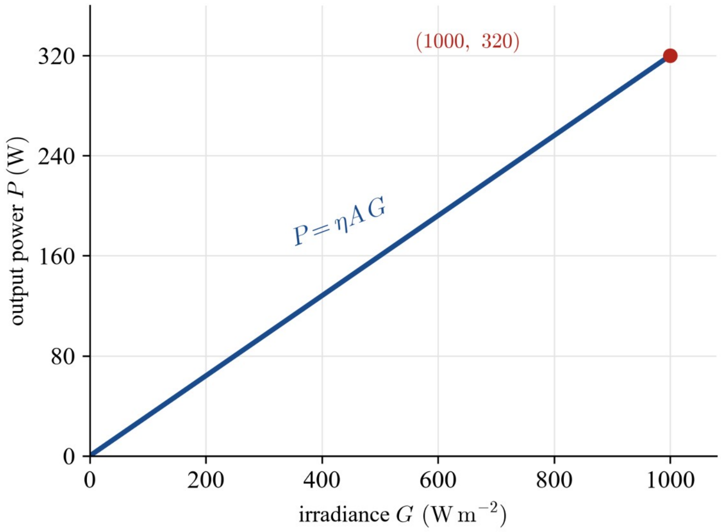
$$P_{\mathrm{in}} = GA.$$

The panel converts only a fraction of this to electricity. The **efficiency** is the ratio of the electrical power output to the incident solar power,

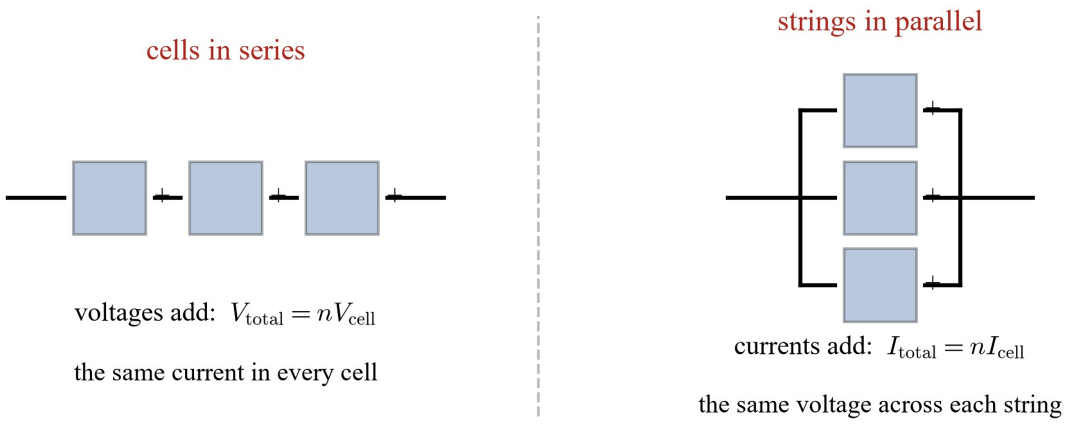
$$\eta = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}, \text{so } P_{\mathrm{out}} = \eta G A.$$

For a fixed panel (η and A constant) the electrical output is **proportional to the irradiance**: the graph of P_{out} against G is a straight line through the origin with gradient ηA. Typical commercial panels have efficiencies of about 15% to 22%.

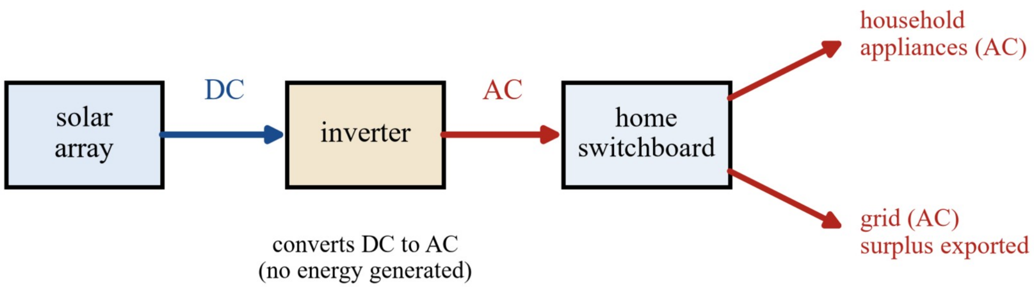
> **A note on scope.** The study design treats photovoltaic cells at a *describe* level (the production of electricity and the need for an inverter), and neither irradiance nor panel efficiency is named in the study design or on the formula sheet. The quantitative relation $P_{out} = \eta G A$ used here — and in a few assessment items in this series — therefore **may go beyond what the study design strictly requires**: it is a calculation rather than a "describe" task. It is included because it uses only the general efficiency concept with stem-supplied data, and supports the Area of Study 3 outcome of *analysing and evaluating an electricity generation and distribution system*. Treat it as useful extension if your teacher confirms it is not assessed.



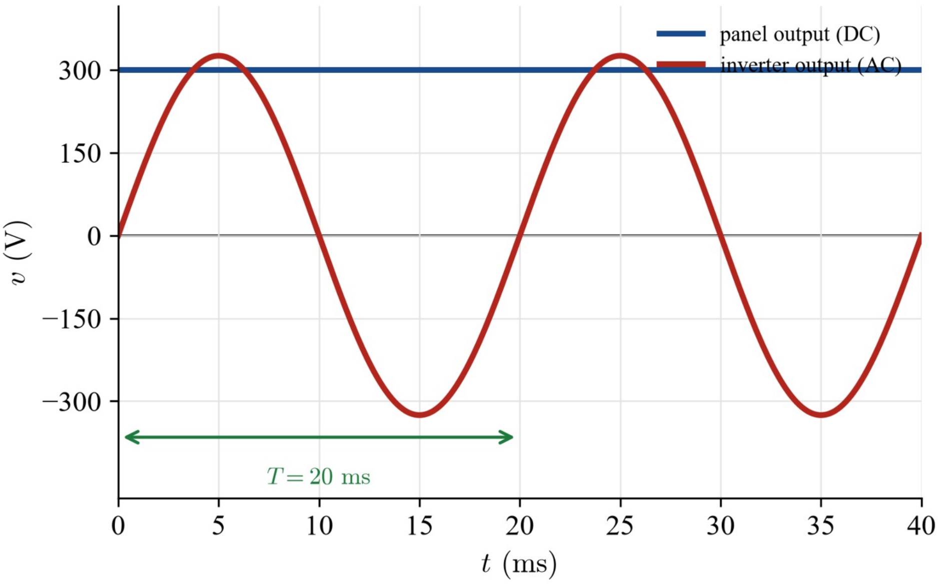
From cells to modules to arrays. A single cell produces an output voltage of only about 0.5 V, which is far too small to be useful on its own. Cells are therefore connected in **series** inside a module (a *panel*): in a series connection the cell voltages add while the same current passes through every cell, so a module of many cells produces a useful voltage of tens of volts. Modules (or strings of modules) may also be connected in **parallel**: in a parallel connection the currents add while the voltage stays the same, increasing the current the system can deliver. A rooftop **array** is many panels connected together in this way.



The household solar system and the need for an inverter. The array produces DC, but household appliances and the electricity grid run on **alternating current (AC)** — in Australia a supply that alternates at 50 Hz with a nominal voltage of 230 V. The DC output of the panels cannot be used directly by AC appliances, nor fed into the AC grid. An **inverter** is the component that converts the DC electricity from the panels into AC at the mains voltage and frequency, so that it can run the home's appliances and any surplus can be exported to the grid.



The inverter changes only the **form** of the electricity (DC to AC); it does not generate energy. For an ideal inverter the electrical energy leaving each second equals the electrical energy entering, by conservation of energy. The two traces below show the difference in form: the panel's output is a steady DC voltage, while the inverter's output is a sinusoidal AC voltage of period $20\,\text{ms}$ (a frequency of $50\,\text{Hz}$).



(AC is used by the grid partly because it can be readily stepped up and down by transformers for efficient transmission, as developed in Chapter 6.)

Throughout this subtopic, "full sun" refers to the standard test irradiance of $1000\,\text{W m}^{-2}$, and the household mains supply is taken as AC at $230\,\text{V}$ and $50\,\text{Hz}$.

Worked examples

Example 1 — scaffolded

A rooftop solar panel supplies a steady current of $7.5\,\text{A}$ at an output voltage of $32\,\text{V}$ while illuminated. Calculate the electrical power output of the panel, and the electrical energy it delivers in 4.0 hours, expressing this energy in both joules and kilowatt-hours.

Working	Comment
Illuminated panel: $V = 32\,\text{V}$, $I = 7.5\,\text{A}$, $t = 4.0\,\text{h}$.	List the data. The panel output is DC.
$P = V\,I = 32 \times 7.5 = 240\,\text{W}$.	Electrical power delivered to the system.
$E = P\,t = 240 \times (4.0 \times 3600) = 3.46 \times 10^{6}\,\text{J}$.	Energy in joules: convert the time to seconds ($4.0\,\text{h} = 1.44 \times 10^{4}\,\text{s}$).
Or $E = 0.240\,\text{kW} \times 4.0\,\text{h} = 0.96\,\text{kW h}$.	Power in kW × time in h gives kW h directly.

Example 2 — scaffolded

A solar panel of area $1.6\,\mathrm{m}^2$ receives sunlight of irradiance $1000\,\mathrm{W\,m}^{-2}$ and converts it to electrical energy with an efficiency of 20%. Calculate the incident solar power on the panel, the electrical power output, and the number of such panels needed to supply $3.2\,\mathrm{kW}$.

Working	Comment
$A=1.6\,\mathrm{m}^2$, $G=1000\,\mathrm{W\,m}^{-2}$, efficiency 20%, target $3.2\,\mathrm{kW}$.	G is the incident solar power per square metre.
$P_{\mathrm{in}}=G\,A=1000\times1.6=1600\,\mathrm{W}$.	Incident solar power on one panel.
$P_{\mathrm{out}}=0.20\times P_{\mathrm{in}}=0.20\times1600=320\,\mathrm{W}$.	Efficiency is the ratio of electrical output to solar input.
$N=\dfrac{3200}{320}=10$ panels.	Panels needed to reach $3.2\,\mathrm{kW}=3200\,\mathrm{W}$.

Example 3 — unscaffolded

A solar module is built from 36 identical cells connected in series. In full sun each cell produces an output of $0.55\,\mathrm{V}$ and a current of $8.0\,\mathrm{A}$. Calculate the module's output voltage, current and electrical power.

In a series connection the cell voltages add while the same current passes through every cell:

$$V=n\,V_{\mathrm{cell}}=36\times0.55=19.8\,\mathrm{V}\,,\,I=8.0\,\mathrm{A}\,.$$

The electrical power output of the module is

$$P=V\,I=19.8\times8.0=158\,\mathrm{W}\,.$$

∴ the module delivers $19.8\,\mathrm{V}$, $8.0\,\mathrm{A}$ and $158\,\mathrm{W}$.

Example 4 — unscaffolded

Explain why a rooftop photovoltaic system that supplies a home and exports surplus power to the grid must include an inverter. Refer to the form of the electricity produced by the panels and the form required by household appliances and the grid.

The solar panels produce a **direct current (DC)** output: the output voltage and current are steady and in one direction only. Household appliances and the electricity grid, however, operate on **alternating current (AC)** — in Australia a supply that alternates at $50\,\mathrm{Hz}$ with a nominal voltage of $230\,\mathrm{V}$. The panels' DC output cannot be used directly by AC appliances, nor connected to the AC grid. An **inverter** converts the DC electricity from the panels into AC at the mains voltage and frequency, so that it can run the home's appliances and be exported to the grid. The inverter changes only the *form* of the electricity (DC to AC); it does not generate additional energy.

∴ an inverter is required because the panels produce DC whereas the home and the grid require AC.

Practice questions

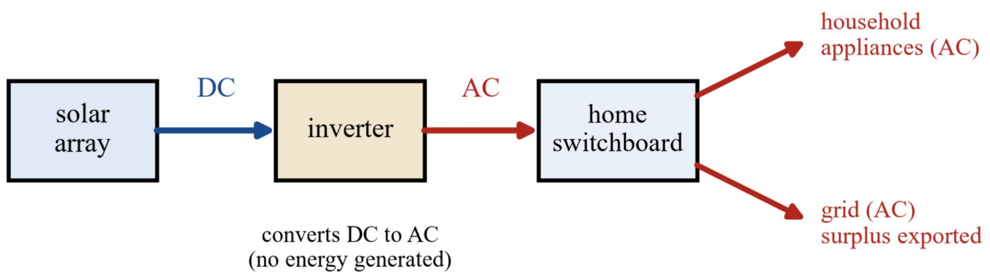
Question 1 A rooftop solar panel supplies a steady current of $6.0\,\mathrm{A}$ at an output voltage of $34\,\mathrm{V}$ while illuminated. (a) Calculate the electrical power output of the panel. (b) The panel delivers this output for $5.0\,\mathrm{hours}$ on a sunny day. Calculate the electrical energy it produces, in kilowatt-hours. (c) The household uses $8.5\,\mathrm{kW\,h}$ of electrical energy that day. Calculate the percentage of the household's daily energy use supplied by this panel.

Question 2 A solar panel of area $1.8\,\mathrm{m}^2$ receives sunlight of irradiance $950\,\mathrm{W\,m}^{-2}$ and converts it to electrical energy with an efficiency of 18%. (a) Calculate the incident solar power on the panel. (b) Calculate the electrical power output of the panel. (c) A newer panel of the same area in the same sunlight has an efficiency of 22%. Calculate its electrical power output. Give your answer to three significant figures.

Question 3 A household plans a 5.0 kW rooftop array using identical panels each rated at 250 W (the output of one panel in full sun). Each panel has an area of 1.6 m^2. (a) Calculate the number of panels required. (b) Calculate the total roof area the array will occupy. (c) On an average day the completed array delivers 3.0 kW for 5.0 hours. Calculate the electrical energy generated that day, in kilowatt-hours.

Question 4 A solar module is made of 40 identical cells connected in series. In full sun each cell produces 0.52 V and 9.0 A. (a) Calculate the output voltage of the module. (b) Calculate the electrical power output of the module. (c) Two identical modules are now connected in parallel. State the combined output current and calculate the combined electrical power output.

Question 5 The diagram shows the main components of a household rooftop solar system.



(a) State whether the electricity produced by the solar array is AC or DC.
(b) Name the component connected between the array and the switchboard, and state what conversion of the electricity it performs.
(c) Explain why this conversion is necessary before the electricity can be used by the household's appliances or exported to the grid.

Question 6 A student says: "The inverter in a solar system boosts the amount of energy that the solar panels produce." (a) (Circle your answer.) The main function of an inverter in a household solar system is to: *generate electricity / convert DC to AC / store energy for night-time / increase the number of panels*. (b) State the form of electricity produced by photovoltaic cells and the form required by the electricity grid. (c) Explain what is wrong with the student's statement, with reference to what the inverter actually does to the electricity and to the conservation of energy.

Question 7 A single photovoltaic cell in full sun delivers a current of 3.5 A at an output voltage of 0.58 V. Calculate its electrical power output.

Question 8 In terms of energy transformations, describe the sequence of energy changes that occurs in a rooftop solar system from the sunlight arriving at the panels to a household appliance running on the electricity supplied, naming the component responsible for each change of form.

Question 9 A rooftop panel produces more electrical power at midday than in the early morning, even on a cloudless day. Explain why, with reference to the quantity that determines the panel's electrical output.

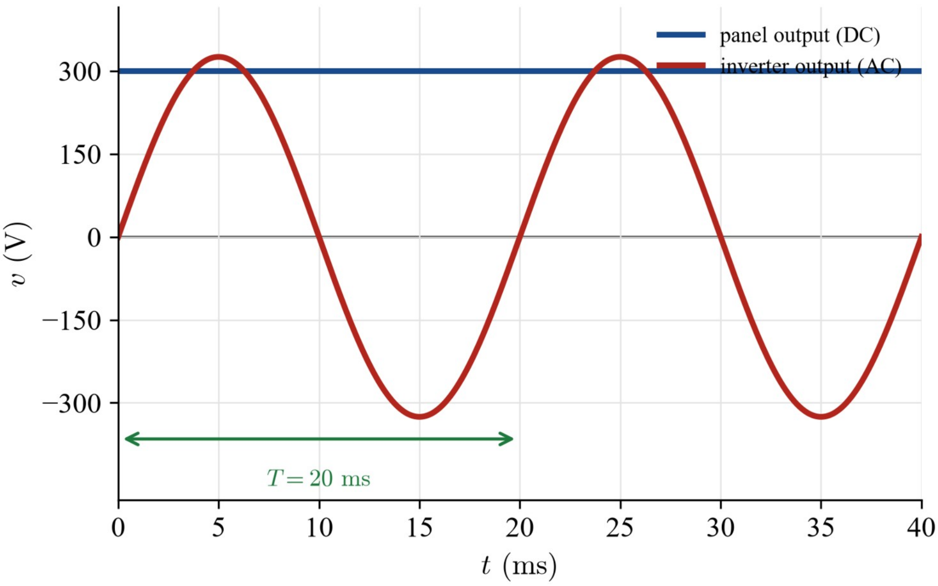
Question 10 A solar-farm panel of area 2.0 m^2 and efficiency 21% receives sunlight of irradiance 1000 W m^{-2}. (a) Show that the electrical power output of one panel is 420 W. (b) Hence determine the number of these panels needed for a solar farm with a total output of 1.05 MW.

Question 11 A single photovoltaic cell produces an output voltage of only about 0.5 V. Explain why the cells in a solar module are connected in series, referring to the voltage of one cell and the voltage a household solar system needs in order to operate.

Question 12 Three identical solar panels, each delivering 250 W at an output voltage of 30 V, are connected in parallel. Calculate the combined output current and the combined electrical power output of the three panels.

Question 13 A homeowner is told that although the solar panels on the roof generate electricity, the household's appliances cannot be run directly from the panels — an inverter must be connected in between. Explain, as a clear chain of reasoning, why this is so.

Question 14 The graph shows two voltage–time traces measured in a rooftop solar system: one at the output terminals of the solar array, and one at the output of the inverter.



(a) State which trace corresponds to the output of the inverter, and justify your answer.
(b) Determine the frequency of that trace.

Question 15 A household uses $20\,\mathrm{kW\,h}$ of electrical energy per day. A proposed rooftop array would deliver an average power of $4.2\,\mathrm{kW}$ for $5.5\,\mathrm{hours}$ of useful sunlight each day. Determine, with a calculation, whether the array would generate enough energy to meet the household's daily use.

Question 16 A student claims that a photovoltaic cell "stores energy from the Sun during the day and releases it at night." Explain what is wrong with this statement, and state correctly what a photovoltaic cell does.

Question 17 A solar panel of area $1.5\,\mathrm{m}^2$ in sunlight of irradiance $1000\,\mathrm{W\,m}^{-2}$ produces an electrical output of $285\,\mathrm{W}$. Calculate the efficiency of the panel, expressed as a percentage.

Question 18 A rooftop solar system uses an inverter that can be treated as ideal (no energy losses). Compare the electricity in the system before the inverter with the electricity after it, referring to (i) whether it is AC or DC, and (ii) whether the total electrical energy is changed by the inverter. Justify your answer.

Answers

Question 1 (a) $204\ \mathrm{W}$; (b) $1.02\ \mathrm{kW\ h}$; (c) 12.0%. **Question 2** (a) $1710\ \mathrm{W}$; (b) $308\ \mathrm{W}$; (c) $376\ \mathrm{W}$. **Question 3** (a) 20 panels; (b) $32\ \mathrm{m}^2$; (c) $15\ \mathrm{kW\ h}$. **Question 4** (a) $20.8\ \mathrm{V}$; (b) $187\ \mathrm{W}$; (c) $18.0\ \mathrm{A}$, $374\ \mathrm{W}$. **Question 5** (a) DC; (b) an inverter — it converts DC to AC; (c) household appliances and the grid run on AC, but the panels produce DC. **Question 6** (a) convert DC to AC; (b) the cells produce DC, the grid requires AC; (c) the inverter only converts the form (DC to AC) and, by conservation of energy, cannot increase the energy generated. **Question 7** $2.03\ \mathrm{W}$. **Question 8** light (radiant) energy → (PV cell) → electrical energy as DC → (inverter) → electrical energy as AC → (appliance) → useful output (light / heat / motion) plus some heat. **Question 9** the irradiance is greater at midday, so $P_{\mathrm{out}} = \eta G A$ is greater. **Question 10** (a) $420\ \mathrm{W}$; (b) 2500 panels. **Question 11** one cell's voltage (about $0.5\ \mathrm{V}$) is too small; connecting cells in series makes the voltages add to a useful value. **Question 12** $25.0\ \mathrm{A}$; $750\ \mathrm{W}$. **Question 13** the panels produce DC, but appliances and the grid need AC; the inverter converts DC to AC, so it must be connected between the panels and the appliances. **Question 14** (a) the sinusoidal (alternating) trace is the inverter output; (b) $50\ \mathrm{Hz}$. **Question 15** $23.1\ \mathrm{kW\ h} > 20\ \mathrm{kW\ h}$, so yes — with a surplus of $3.1\ \mathrm{kW\ h}$. **Question 16** a PV cell does not store energy; it converts light directly into electricity only while it is illuminated. **Question 17** 19.0%. **Question 18** before: DC; after: AC; an ideal inverter conserves energy, so the total electrical energy is unchanged.

Fully-worked solutions

Question 1 $V = 34\ \mathrm{V}$, $I = 6.0\ \mathrm{A}$. (a) $P = V I = 34 \times 6.0 = 204\ \mathrm{W}$. (b) $E = P t = 0.204\ \mathrm{kW} \times 5.0\ \mathrm{h} = 1.02\ \mathrm{kW\ h}$. (c) Fraction of the daily use $= \dfrac{1.02}{8.5} = 0.120$, that is 12.0% of the household's $8.5\ \mathrm{kW\ h}$.

Question 2 $A = 1.8\ \mathrm{m}^2$, $G = 950\ \mathrm{W\ m}^{-2}$. (a) $P_{\mathrm{in}} = G A = 950 \times 1.8 = 1710\ \mathrm{W}$. (b) $P_{\mathrm{out}} = 0.18 \times 1710 = 308\ \mathrm{W}$. (c) With an efficiency of 22%, $P_{\mathrm{out}} = 0.22 \times 1710 = 376\ \mathrm{W}$ (to three significant figures).

Question 3 Array target $5.0\ \mathrm{kW} = 5000\ \mathrm{W}$; panels rated $250\ \mathrm{W}$, each $1.6\ \mathrm{m}^2$. (a) $N = \dfrac{5000}{250} = 20$ panels. (b) Roof area $= 20 \times 1.6 = 32\ \mathrm{m}^2$. (c) $E = P t = 3.0\ \mathrm{kW} \times 5.0\ \mathrm{h} = 15\ \mathrm{kW\ h}$.

Question 4 $n = 40$ cells in series, each $0.52\ \mathrm{V}$ and $9.0\ \mathrm{A}$. (a) In series the voltages add: $V = 40 \times 0.52 = 20.8\ \mathrm{V}$. (b) $P = V I = 20.8 \times 9.0 = 187\ \mathrm{W}$. (c) In parallel the currents add and the voltage is unchanged: combined current $= 2 \times 9.0 = 18.0\ \mathrm{A}$, and combined power $= 2 \times 187 = 374\ \mathrm{W}$ (equivalently $20.8 \times 18.0 = 374\ \mathrm{W}$).

Question 5 (a) The array produces **DC**. (b) The component is an **inverter**; it converts the DC electricity from the array into AC. (c) Household appliances and the electricity grid are designed to operate on AC (in Australia, $230\ \mathrm{V}$, $50\ \mathrm{Hz}$). The array's DC output cannot run AC appliances or be fed into the AC grid, so it must first be converted to AC by the inverter.

Question 6 (a) **convert DC to AC**. (b) Photovoltaic cells produce DC; the grid requires AC. (c) The statement is wrong: the inverter does not generate or boost energy. It converts the *form* of the electricity from DC to AC. By conservation of energy, an ideal inverter delivers the same electrical energy as it receives (a real one delivers slightly less, losing a little as heat), so it can never increase the energy the panels produce. The energy generated is set by the panels and the sunlight, not by the inverter.

Question 7 $P = V I = 0.58 \times 3.5 = 2.03\ \mathrm{W}$.

Question 8 The energy transformations, in order, are:

- sunlight delivers **light (radiant) energy** to the panels;
- each **PV cell** converts this light energy into **electrical energy** in the form of DC;
- the **inverter** converts the electrical energy from DC to AC (the energy stays electrical; only its form changes);
- the **appliance** converts the AC electrical energy into its useful output — light, heat or motion, for example — together with some energy dissipated as heat.

Question 9 The electrical output of a panel is $P_{out} = \eta G A$, so with the efficiency η and area A fixed it depends on the irradiance G (the incident solar power per square metre). At midday the Sun is high in the sky, so its light strikes the panel more directly and passes through less atmosphere; the irradiance G is then greater than in the early morning, when the Sun is low. A larger G gives a larger P_{out}, so the panel produces more power at midday.

Question 10 $A = 2.0\,\mathrm{m}^2$, $\eta = 0.21$, $G = 1000\,\mathrm{W\,m^{-2}}$. (a) $P_{out} = \eta G A = 0.21 \times 1000 \times 2.0 = 420\,\mathrm{W}$, as required. (b) $N = \dfrac{1.05 \times 10^6}{420} = 2500$ panels.

Question 11 A single cell provides only about $0.5\,\mathrm{V}$, which is far too small to drive a household system or its inverter. When cells are connected in **series** their voltages add ($V_{total} = n V_{cell}$) while the same current passes through each cell, so a module of many cells produces a much larger, useful voltage (tens of volts). Connecting the cells in series is therefore how the small voltage of each individual cell is built up to the voltage the system needs.

Question 12 Each panel carries a current $I = \dfrac{P}{V} = \dfrac{250}{30} = 8.33\,\mathrm{A}$. In parallel the currents add while the voltage stays at $30\,\mathrm{V}$, so the combined current is $3 \times 8.33 = 25.0\,\mathrm{A}$. The combined power is $P = 3 \times 250 = 750\,\mathrm{W}$ (equivalently $30 \times 25.0 = 750\,\mathrm{W}$).

Question 13 The reasoning runs as a chain:

- the solar panels produce a **DC** output;
- household appliances (and the electricity grid) are built to run on **AC** — a supply of $230\,\mathrm{V}$, $50\,\mathrm{Hz}$ in Australia;
- DC and AC are different forms of electricity, so appliances designed for AC will not operate correctly on the panels' DC, and DC cannot be connected to the AC grid;
- an **inverter** converts the panels' DC into AC at the mains voltage and frequency;
- therefore the appliances must be supplied *through* the inverter, not directly from the panels.

Question 14 (a) The **sinusoidal (alternating) trace** is the inverter output. The inverter converts the panels' DC into AC, so its output voltage varies sinusoidally with time; the flat, constant trace is the array's steady DC output. (b) The AC trace completes one full cycle in a period $T = 20\,\mathrm{ms} = 0.020\,\mathrm{s}$, so its frequency is $f = \dfrac{1}{T} = \dfrac{1}{0.020} = 50\,\mathrm{Hz}$ — the mains frequency. (Its amplitude is about $325\,\mathrm{V}$.)

Question 15 The daily energy the array would generate is

$$E = P t = 4.2\,\mathrm{kW} \times 5.5\,\mathrm{h} = 23.1\,\mathrm{kW\,h}\,.$$

Since $23.1\,\mathrm{kW\,h} > 20\,\mathrm{kW\,h}$, the array generates more than the household uses in a day, with a surplus of $23.1 - 20 = 3.1\,\mathrm{kW\,h}$ that could be exported to the grid or stored. So yes, it would meet the daily use.

Question 16 A photovoltaic cell does **not** store energy. It converts the energy of light directly into electrical energy, and only while light is falling on it; the moment the Sun sets the cell stops producing electricity. To have electricity available at night the system must include a separate storage device — a **battery** — which is charged by the panels during the day and supplies energy after dark. The cell itself is a converter of light to electricity, not a store of energy.

Question 17 The incident solar power is $P_{in} = G A = 1000 \times 1.5 = 1500\,\mathrm{W}$. The efficiency is

$$\eta = \frac{P_{out}}{P_{in}} = \frac{285}{1500} = 0.19\,,$$

that is 19.0%.

Question 18 (i) Before the inverter the electricity is **DC** (the output of the panels); after the inverter it is **AC** (supplied to the home and the grid). (ii) An ideal inverter changes only the form of the electricity, not the amount of energy. By conservation of energy, the electrical energy leaving an ideal inverter each second equals the electrical energy entering it, so the power — and hence the total electrical energy — is **unchanged**. (A real inverter dissipates a small fraction as heat, so its AC output energy is slightly less than the DC input; an ideal inverter conserves it exactly.)

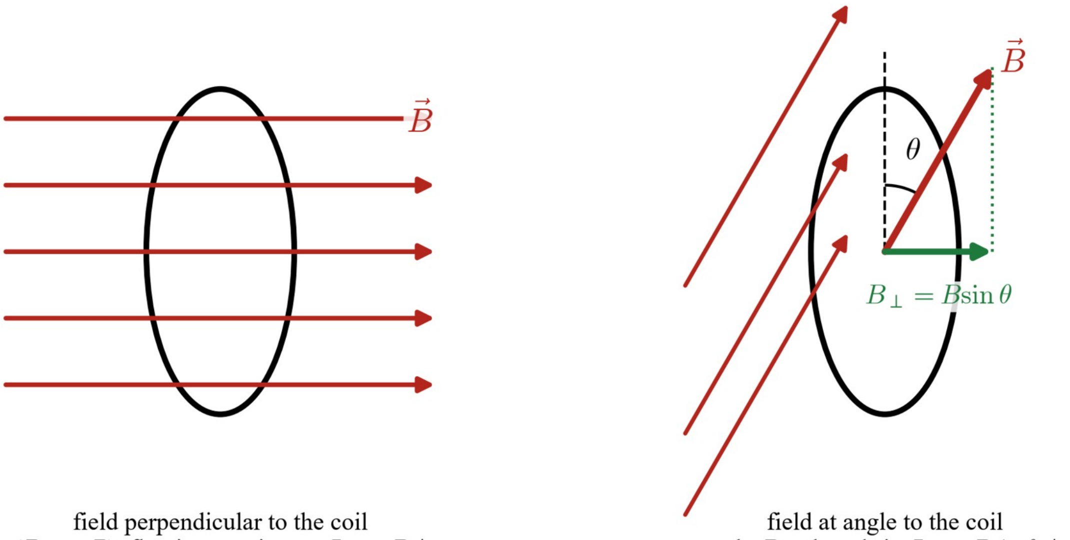
Theory

Magnetic flux. The **magnetic flux** Φ_B through a flat area measures how much magnetic field passes through that area. When the field is *perpendicular to the area* — that is, the field lines pass straight through the face of the loop — the flux is a maximum and

$$\Phi_B = B_\perp A\,,$$

where A is the area and $B_\perp$ is the component of the field perpendicular to that area. Flux is measured in **webers** (Wb), and $1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2}$.

The qualitative effect of angle. Only the part of the field that is perpendicular to the area threads the loop. If the field makes an angle θ to the *plane* of the coil, the perpendicular component is $B_\perp = B\sin\theta$, so $\Phi_B = B\sin\theta\, A$. The flux is therefore **greatest when the field is perpendicular to the coil** ($\theta = 90^\circ$, field straight through, $\Phi_B = B\,A$) and **falls to zero when the field lies in the plane of the coil** ($\theta = 0$, no field lines pass through). In this subtopic the quantitative calculations use a field perpendicular to the area; the angle dependence is treated qualitatively as above.



field perpendicular to the coil ($B_\perp = B$): flux is a maximum, $\Phi_B = BA$

field at angle to the coil only $B_\perp$ threads it: $\Phi_B = B\sin\theta\, A$

Electromagnetic induction. Whenever the magnetic flux through a coil **changes**, an electromotive force (**emf**) is induced across the coil. If the coil forms part of a closed circuit, this emf drives an induced current. It is the **change** in flux that matters: a steady flux, however large, induces nothing. The size of the induced emf is given by **Faraday's law**,

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}\,,$$

where N is the number of loops (turns) through which the flux passes, $\frac{\Delta\Phi_B}{\Delta t}$ is the rate of change of flux, and the minus sign gives the **direction** of the induced emf (discussed below). Faraday's law identifies three ways to increase the induced emf:

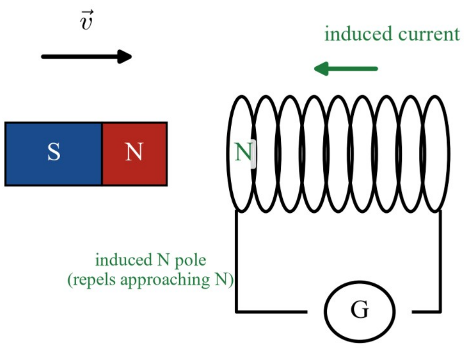
- a **faster rate of change of flux** $\frac{\Delta\Phi_B}{\Delta t}$ (a bigger change, or the same change in less time);
- a **greater number of loops** N through which the flux passes;
- and, since $\Phi_B = B_\perp A$, a larger field, a larger area, or a bigger swing in the angle.

Ways to change the flux. Because $\Phi_B = B_\perp A$, the flux through a coil changes if the **field strength** B changes, if the **area** A enclosed by the circuit changes, or if the **angle** between the field and the coil changes. Each produces an induced emf $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$; if the circuit has resistance R, the induced current is $I = \frac{\varepsilon}{R}$.

The direction of the induced emf (the reasoning chain). The minus sign in Faraday's law is **Lenz's law**: the induced current flows in the direction whose own magnetic field *opposes the change* in flux that produced it. Simply writing "the induced current opposes the change" earns nothing on its own — the direction must be justified by the following chain, which is exactly what an assessor rewards:

1. **Relative motion / change** — state what is moving or changing (for example, a magnet's north pole approaching the coil).
2. **Changing flux through the coil** — state how the flux Φ_B *through the coil* is changing as a result (increasing, or decreasing, and in which direction).
3. **Faraday's law** — a changing flux induces an emf, $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$, which drives an induced current.
4. **Opposition** — the induced current flows so that its own magnetic field *opposes the change*: it opposes an increase in flux, and reinforces a decrease. State the resulting current direction (or the pole the coil presents).

approaching N pole -> flux through coil increasing -> $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$ -> current opposes

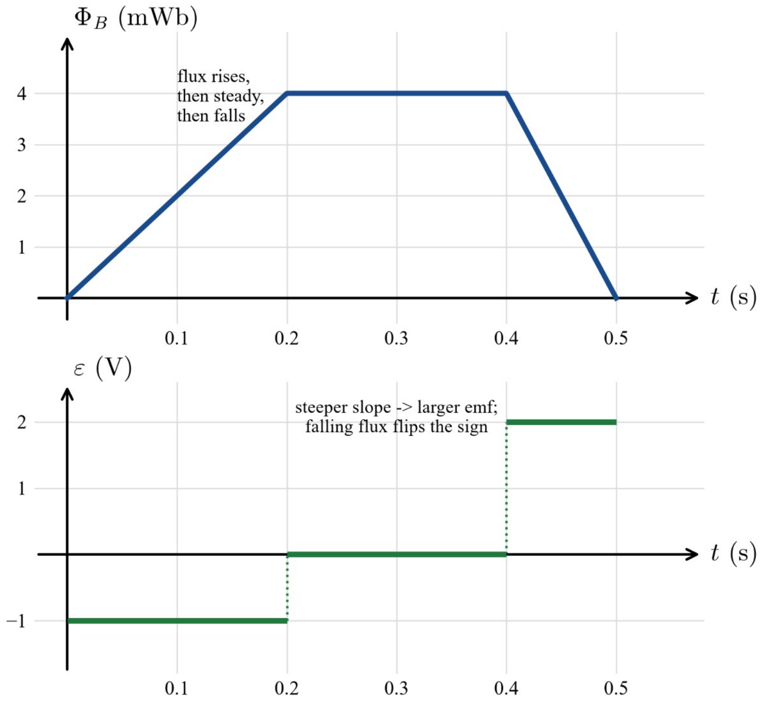


For a magnet's **north pole approaching** a coil, the flux through the coil increases, so the induced current makes the near face of the coil a **north pole** to repel the magnet (opposing the approach). When the north pole is **withdrawn**, the flux decreases, so the induced current reverses to make the near face a **south pole**, attracting the magnet (opposing the departure). In both cases the induced current opposes the *change*, never the motion for its own sake.

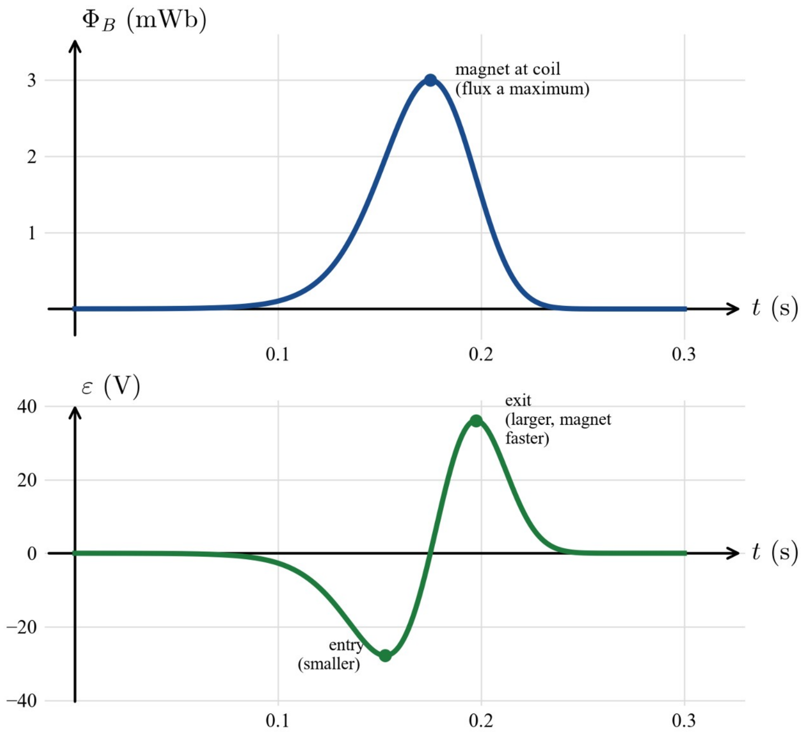
Energy and the minus sign. Because the induced current opposes the change, an external agent must do **work** to keep the flux changing — for example, to push the magnet in against the repulsion. That work is the source of the electrical energy dissipated in the circuit. If the induced current instead *aided* the change, the magnet would accelerate on its own and electrical energy would appear from nowhere, violating conservation of energy. The minus sign in Faraday's law is this bookkeeping.

Reading a flux–time graph. Since $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$, the induced emf is $-N$ times the **gradient** of a flux–time graph.

Where the flux rises steadily the emf is a constant of one sign; where the flux is constant the emf is zero; where the flux falls the emf takes the opposite sign. A steeper section of the graph means a larger emf.

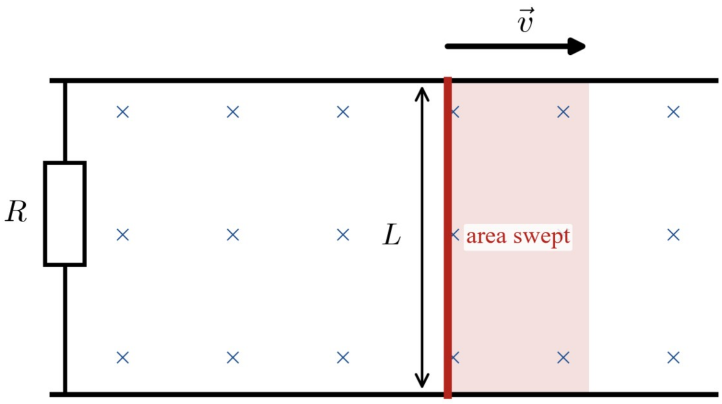


A magnet falling through a solenoid. As a magnet falls through a coil, the flux through the coil rises from zero to a maximum (magnet at the coil) and then falls back to zero. The emf, being $-N$ times the gradient, is therefore a **pulse of one sign as the magnet enters, then a pulse of the opposite sign as it leaves**. Because the magnet has accelerated under the force due to gravity, it is moving faster on the way out, so the flux changes more rapidly then: the **second pulse is larger and briefer** than the first.



Changing the area: a rod on rails. If a conducting rod of length L slides along two rails in a field B perpendicular to the plane of the circuit, the area enclosed grows, so the flux changes even though B is constant. In a time Δt the rod moves a distance d, sweeping an extra area $\Delta A = L d$, so

$$\varepsilon = N \frac{\Delta \Phi_B}{\Delta t} = N B \frac{\Delta A}{\Delta t}.$$



field B into the page; rod moves at v so the enclosed area grows: $\varepsilon = B\dfrac{\Delta A}{\Delta t}$

Throughout this subtopic, emf magnitudes are quoted as positive values and the induced-current direction is stated separately using the reasoning chain above.

Worked examples

Example 1 — scaffolded

A circular coil of 200 turns has radius 0.040 m. It lies in a uniform magnetic field directed perpendicular to the plane of the coil. The field strength increases steadily from 0.15 T to 0.65 T in 0.20 s. Calculate the initial and final flux through the coil, and the magnitude of the average induced emf.

Working	Comment
Field perpendicular to the coil, so $\Phi_B = BA$ with $A = \pi r^2 = \pi(0.040)^2 = 5.03 \times 10^{-3}\,\mathrm{m}^2$.	Maximum-flux case; find the area first.
$\Phi_i = B_i A = 0.15 \times 5.03 \times 10^{-3} = 7.54 \times 10^{-4}\,\mathrm{Wb}$.	Initial flux through one turn.
$\Phi_f = B_f A = 0.65 \times 5.03 \times 10^{-3} = 3.27 \times 10^{-3}\,\mathrm{Wb}$.	Final flux through one turn.
$\Delta\Phi_B = \Phi_f - \Phi_i = (0.65 - 0.15) \times 5.03 \times 10^{-3} = 2.51 \times 10$.	Only the change drives the emf.
$\varepsilon = N\dfrac{\Delta\Phi_B}{\Delta t} = 200 \times \dfrac{2.51 \times 10^{-3}}{0.20} = 2.51\,\mathrm{V}$.	Faraday's law; N multiplies the per-turn change.

Example 2 — scaffolded

The north pole of a bar magnet is pushed toward one end of a coil that is connected to a galvanometer. State, as a reasoning chain, the direction of the induced current (in terms of the magnetic pole presented by the near face of the coil), and describe how the reading changes if the magnet is instead pushed in more quickly.

Working	Comment
Relative motion: the magnet's north pole moves toward the coil — they approach each other.	Step 1: identify the change (not "the magnet has a field").
Changing flux: the field lines of the north pole pass through the coil, and as it approaches, the flux Φ_B *through the coil* increases.	Step 2: state the flux change through the coil.
Faraday's law: a changing flux induces an emf $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, driving an induced current.	Step 3: quote Faraday's law.
Opposition: the induced current flows so its magnetic	Step 4: direction from Lenz/energy, stated as a pole.

Working	Comment
field opposes the *increase* — the near face of the coil becomes a **north pole**, repelling the approaching magnet.	
Pushing the magnet in faster increases $\frac{\Delta \Phi_B}{\Delta t}$, so the emf and the galvanometer deflection are **larger** (same direction).	Rate of change of flux controls the size.

Example 3 — unscaffolded

A single conducting rod of length $0.30\,\mathrm{m}$ slides at a steady speed along two horizontal rails in a uniform magnetic field of $0.25\,\mathrm{T}$ directed vertically (perpendicular to the plane of the rails). The rod moves $0.20\,\mathrm{m}$ in $0.50\,\mathrm{s}$. The rails and rod form a circuit of total resistance $0.50\,\Omega$. Calculate the induced emf and the induced current.

The field is constant, but the area of the circuit grows as the rod moves, so the flux changes. In $0.50\,\mathrm{s}$ the rod sweeps an extra area

$$\Delta A = L d = 0.30 \times 0.20 = 0.060\,\mathrm{m}^2,$$

so the change in flux is $\Delta \Phi_B = B \Delta A = 0.25 \times 0.060 = 0.015\,\mathrm{Wb}$. With $N = 1$,

$$\varepsilon = \frac{\Delta \Phi_B}{\Delta t} = \frac{0.015}{0.50} = 0.030\,\mathrm{V}.$$

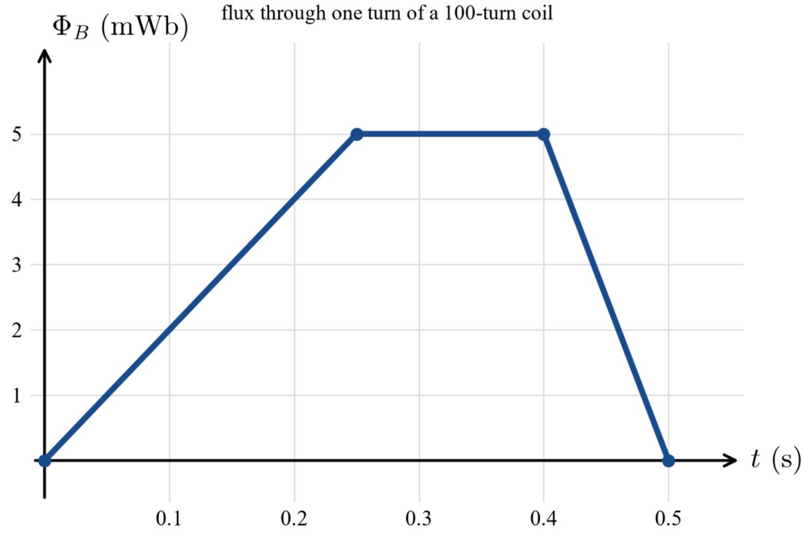
The induced current is

$$I = \frac{\varepsilon}{R} = \frac{0.030}{0.50} = 0.060\,\mathrm{A}.$$

∴ the induced emf is $0.030\,\mathrm{V}$ and the induced current is $0.060\,\mathrm{A}$.

Example 4 — unscaffolded

The graph shows the magnetic flux through **one turn** of a 100-turn coil as a function of time. Determine the magnitude of the induced emf in each of the three intervals.



The induced emf is N times the gradient of the flux–time graph.

Interval 0–0.25 s: the flux rises steadily, gradient $= \frac{5.0 \times 10^{-3}}{0.25} = 0.020\,\mathrm{Wb\,s^{-1}}$, so $\varepsilon = N \times 0.020 = 100 \times 0.020 = 2.0\,\mathrm{V}$.

Interval 0.25–$0.40\,\text{s}$: the flux is constant, gradient $=0$, so $\varepsilon=0$.

Interval 0.40–$0.50\,\text{s}$: the flux falls, gradient $=\dfrac{-5.0\times10^{-3}}{0.10}=-0.050\ \text{Wb s}^{-1}$, so $\varepsilon=100\times0.050=5.0\,\text{V}$, of the **opposite polarity** to the first interval.

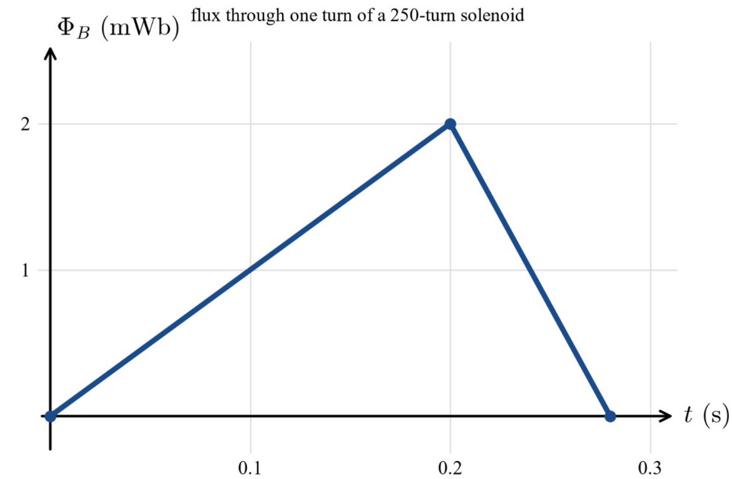
∴ the emf magnitudes are $2.0\,\text{V}$, 0 and $5.0\,\text{V}$; the steepest section gives the largest emf.

Practice questions

Question 1 A square coil of side $0.050\,\text{m}$ has 150 turns and lies in a uniform field directed perpendicular to its plane. The field increases steadily from $0.20\,\text{T}$ to $0.80\,\text{T}$ in $0.30\,\text{s}$. (a) Calculate the area of the coil. (b) Calculate the initial magnetic flux through one turn of the coil. (c) Calculate the magnitude of the average induced emf.

Question 2 A circular coil of 80 turns and radius $0.060\,\text{m}$ lies with its plane perpendicular to a uniform field. The field is reduced steadily from $0.40\,\text{T}$ to $0.10\,\text{T}$ in $0.10\,\text{s}$. The coil has resistance $2.0\,\Omega$. (a) Calculate the initial flux through one turn of the coil. (b) Calculate the magnitude of the change in flux through one turn. (c) Calculate the magnitude of the average induced emf and of the induced current.

Question 3 A magnet is dropped through a 250-turn solenoid. The graph shows the flux through one turn of the solenoid as a function of time.



(a) Calculate the magnitude of the induced emf while the magnet is entering (0–$0.20\,\text{s}$).
(b) Calculate the magnitude of the induced emf while the magnet is leaving (0.20–$0.28\,\text{s}$).
(c) With reference to the rate of change of flux, explain why the emf while the magnet leaves is larger than while it enters.

Question 4 A circular coil of radius $0.050\,\text{m}$ sits in a uniform field of $0.30\,\text{T}$. (a) Calculate the flux through the coil when the field is perpendicular to the plane of the coil. (b) The coil is slowly rotated until the field lies in the plane of the coil. State what happens to the flux, and explain why, with reference to $\Phi_B=B_\perp A$. (c) The field strength is increased to $0.50\,\text{T}$ while it remains perpendicular to the plane of the coil. Calculate the new flux through the coil.

Question 5 A conducting rod of length $0.40\,\text{m}$ slides at $0.50\,\text{m s}^{-1}$ along two rails in a uniform field of $0.35\,\text{T}$ perpendicular to the plane of the circuit. The circuit has resistance $1.4\,\Omega$. (a) Calculate the induced emf. (b) Calculate the induced current. (c) Using the reasoning chain, explain why an induced current flows even though the field is constant.

Question 6 The north pole of a bar magnet is held stationary just inside a coil connected to a galvanometer; the galvanometer reads zero. The magnet is then pulled straight out of the coil. (a) (Circle your answer.) While the magnet is being pulled out, the galvanometer needle: *stays at zero / deflects*. Justify your choice. No calculations are required. (b) State the magnetic pole presented by the near face of the coil as the magnet is withdrawn. (c) Set out the full reasoning chain (relative motion → changing flux → Faraday's law → opposition) that leads to your answer in part (b).

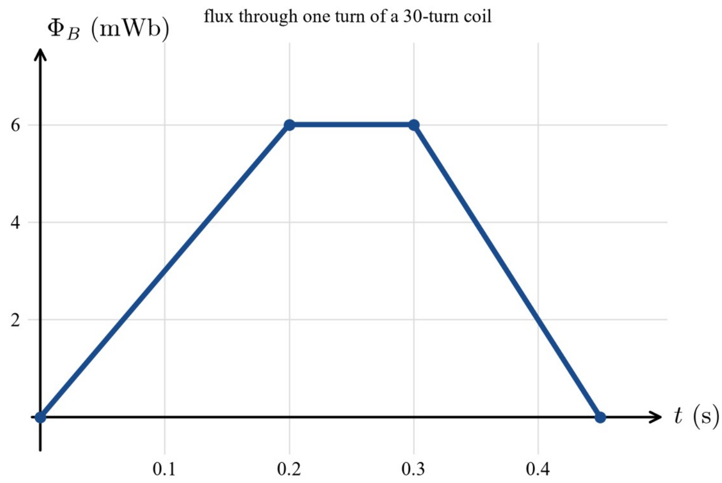
Question 7 A rectangular coil measuring $0.080\,\mathrm{m} \times 0.120\,\mathrm{m}$ has a single turn and lies with its plane perpendicular to a uniform field of $0.55\,\mathrm{T}$. Calculate the magnetic flux through the coil.

Question 8 A rectangular coil measuring $0.10\,\mathrm{m} \times 0.15\,\mathrm{m}$ has 120 turns and lies perpendicular to a uniform field. The field increases steadily from $0.10\,\mathrm{T}$ to $0.90\,\mathrm{T}$ in $0.50\,\mathrm{s}$. The coil has resistance $6.0\,\Omega$. Calculate the induced emf and the induced current.

Question 9 A circular coil of 250 turns and radius $0.045\,\mathrm{m}$ lies with its plane perpendicular to a uniform $0.20\,\mathrm{T}$ field, so that the flux through it is a maximum. The coil is rotated a quarter turn — until its plane is parallel to the field and the flux is zero — in $0.15\,\mathrm{s}$. (a) Show that the average induced emf is $2.12\,\mathrm{V}$. (b) Hence calculate the average induced current if the coil has resistance $4.0\,\Omega$.

Question 10 A flat coil of 60 turns and area $0.020\,\mathrm{m}^2$ lies perpendicular to a uniform field whose strength increases at a steady $2.0\,\mathrm{T\,s^{-1}}$. (a) Calculate the induced emf. (b) Determine the number of turns a coil of the same area would need, in the same changing field, to produce an induced emf of $5.0\,\mathrm{V}$.

Question 11 The graph shows the flux through one turn of a 30-turn coil as a function of time.



(a) Calculate the magnitude of the induced emf during the interval 0–$0.20\,\mathrm{s}$.
(b) Calculate the magnitude of the induced emf during the interval 0.30–$0.45\,\mathrm{s}$.
(c) State the interval in which the induced emf is greatest, and the interval in which it is zero.

Question 12 A bar magnet is released from rest just above a vertical solenoid connected to a data logger, and falls straight through it. Sketch the shape of the induced emf as a function of time, and explain, using the rate of change of flux, why the trace consists of two pulses of opposite sign with the second pulse larger than the first.

Question 13 The south pole of a strong magnet is pushed rapidly toward one end of a coil. Using the full reasoning chain, determine the magnetic pole presented by the near face of the coil, and state the effect this has on the moving magnet.

Question 14 Two coils are wound close together on the same soft-iron core. Coil 1 is connected to a battery through a switch; coil 2 is connected to a galvanometer. The switch, which was open, is now closed. (a) Explain why the galvanometer deflects briefly at the instant the switch is closed. (b) Explain why the galvanometer then returns to zero while the switch remains closed, even though a steady current flows in coil 1. (c) Explain what the galvanometer does at the instant the switch is later opened.

Question 15 A flat coil of 90 turns and radius $0.037\,\mathrm{m}$ lies perpendicular to a uniform field. The field increases steadily from 0 to $0.42\,\mathrm{T}$ in $0.25\,\mathrm{s}$. Calculate the average induced emf. Give your answer to three significant figures.

Question 16 A conducting rod of length $0.20\,\mathrm{m}$ slides along rails in a uniform field of $0.50\,\mathrm{T}$ perpendicular to the plane of the circuit. (a) Determine the speed at which the rod must move to induce an emf of $0.15\,\mathrm{V}$. (b) The circuit has resistance $0.30\,\Omega$. Calculate the induced current at this speed.

Question 17 A small coil is carried at constant velocity through a large region of uniform magnetic field, remaining entirely inside the field region throughout. A student predicts that a current is induced in the coil "because it is moving in a magnetic field". Explain, with reference to the flux through the coil, why no current is in fact induced while the coil is inside the uniform field.

Question 18 A magnet is pushed into a coil that is part of a closed circuit. Explain, using the reasoning chain and the principle of conservation of energy, why the person must do work to push the magnet in, and identify the source of the electrical energy dissipated in the circuit. State what the minus sign in $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$ represents.

Question 19 The same magnet is dropped through a fixed coil twice: first from a small height, and then from a much greater height so that it is moving faster as it passes through. Compare, with reasons, (a) the peak induced emf and (b) the duration of the emf pulse in the two drops.

Question 20 A circular coil of 300 turns and radius $0.030\,\mathrm{m}$ lies with its plane perpendicular to a uniform field directed into the page. The field decreases steadily from $0.80\,\mathrm{T}$ to $0.20\,\mathrm{T}$ in $0.30\,\mathrm{s}$. The coil has resistance $15\,\Omega$. (a) Calculate the induced emf. (b) Calculate the induced current. (c) Using the reasoning chain, state the direction of the induced current around the coil (as viewed from the front, with the field into the page).

Question 21 A square coil of side $0.10\,\mathrm{m}$ has 40 turns. A uniform field of $0.60\,\mathrm{T}$ is initially perpendicular to the plane of the coil. The coil is then rotated in $0.25\,\mathrm{s}$ until the field lies in the plane of the coil. (a) Calculate the initial flux through one turn of the coil. (b) Calculate the final flux through one turn of the coil. (c) Calculate the magnitude of the average induced emf.

Question 22 A student investigates the peak emf produced when a magnet falls through a coil. They wind coils with different numbers of turns and drop the same magnet through each from the same height, recording the peak emf from a data logger. (a) Identify the independent variable, the dependent variable, and one variable that must be controlled. (b) Predict, with reference to Faraday's law, how the peak emf should depend on the number of turns. (c) The student then repeats the experiment dropping the magnet from a greater height for a fixed coil. Predict and justify the effect on the peak emf.

Answers

Question 1 (a) $2.5\times10^{-3}\,\mathrm{m}^2$; (b) $5.0\times10^{-4}\,\mathrm{Wb}$; (c) $0.75\,\mathrm{V}$. **Question 2** (a) $4.52\times10^{-3}\,\mathrm{Wb}$; (b) $3.39\times10^{-3}\,\mathrm{Wb}$; (c) $2.71\,\mathrm{V}$, $1.36\,\mathrm{A}$. **Question 3** (a) $2.5\,\mathrm{V}$; (b) $6.25\,\mathrm{V}$; (c) the flux falls faster on exit (steeper gradient), so the emf is larger. **Question 4** (a) $2.36\times10^{-3}\,\mathrm{Wb}$; (b) the flux falls to zero; (c) $3.93\times10^{-3}\,\mathrm{Wb}$. **Question 5** (a) $0.070\,\mathrm{V}$; (b) $0.050\,\mathrm{A}$; (c) the area of the circuit changes, so the flux changes. **Question 6** (a) deflects; (b) south pole; (c) chain as set out in the solution. **Question 7** $5.28\times10^{-3}\,\mathrm{Wb}$. **Question 8** $2.88\,\mathrm{V}$; $0.48\,\mathrm{A}$. **Question 9** (a) $2.12\,\mathrm{V}$; (b) $0.530\,\mathrm{A}$. **Question 10** (a) $2.4\,\mathrm{V}$; (b) 125 turns. **Question 11** (a) $0.90\,\mathrm{V}$; (b) $1.20\,\mathrm{V}$; (c) greatest 0.30–$0.45\,\mathrm{s}$; zero 0.20–$0.30\,\mathrm{s}$. **Question 12** Two opposite pulses; the exit pulse is larger (magnet faster) — see solution. **Question 13** South pole presented (repels the approaching south pole), opposing its approach. **Question 14** Deflects on closing, returns to zero while steady, deflects the opposite way on opening. **Question 15** $0.650\,\mathrm{V}$. **Question 16** (a) $1.5\,\mathrm{m\,s^{-1}}$; (b) $0.50\,\mathrm{A}$. **Question 17** No change in flux while wholly inside a uniform field, so no induced emf. **Question 18** Work done pushing against the opposing force is the source of the electrical energy; the minus sign is Lenz's law (opposition). **Question 19** (a) larger peak emf from the greater height; (b) shorter pulse. **Question 20** (a) $1.70\,\mathrm{V}$; (b) $0.113\,\mathrm{A}$; (c) clockwise (as viewed from the front). **Question 21** (a) $6.0\times10^{-3}\,\mathrm{Wb}$; (b) $0\,\mathrm{Wb}$; (c) $0.96\,\mathrm{V}$. **Question 22** (a) IV: number of turns; DV: peak emf; controlled: drop height (or the magnet/coil area); (b) peak emf $\propto N$; (c) greater height → faster magnet → larger peak emf.

Fully-worked solutions

Question 1 $a=0.050\,\mathrm{m}$, $N=150$, field perpendicular so $\Phi_B=BA$. (a) $A=a^2=0.050^2=2.5\times10^{-3}\,\mathrm{m}^2$. (b) $\Phi_i=B_iA=0.20\times2.5\times10^{-3}=5.0\times10^{-4}\,\mathrm{Wb}$. (c) $\varepsilon=N\dfrac{\Delta\Phi_B}{\Delta t}=N\dfrac{(B_f-B_i)A}{\Delta t}=150\times\dfrac{(0.80-0.20)\times2.5\times10^{-3}}{0.30}=0.75\,\mathrm{V}$.

Question 2 $N=80$, $r=0.060\,\mathrm{m}$, $A=\pi r^2=1.13\times10^{-2}\,\mathrm{m}^2$, $R=2.0\,\Omega$. (a) $\Phi_i=B_iA=0.40\times1.13\times10^{-2}=4.52\times10^{-3}\,\mathrm{Wb}$. (b) $\Delta\Phi_B=(B_i-B_f)A=(0.40-0.10)\times1.13\times10^{-2}=3.39\times10^{-3}\,\mathrm{Wb}$. (c) $\varepsilon=N\dfrac{\Delta\Phi_B}{\Delta t}=80\times\dfrac{3.39\times10^{-3}}{0.10}=2.71\,\mathrm{V}$; $I=\dfrac{\varepsilon}{R}=\dfrac{2.71}{2.0}=1.36\,\mathrm{A}$.

Question 3 $N=250$; read the gradients from the graph. (a) Entry (0–$0.20\,\mathrm{s}$): gradient $=\dfrac{2.0\times10^{-3}}{0.20}=0.010\,\mathrm{Wb\,s^{-1}}$, so $\varepsilon=250\times0.010=2.5\,\mathrm{V}$. (b) Exit ($0.20$–$0.28\,\mathrm{s}$): gradient $=\dfrac{2.0\times10^{-3}}{0.08}=0.025\,\mathrm{Wb\,s^{-1}}$, so $\varepsilon=250\times0.025=6.25\,\mathrm{V}$. (c) The magnet accelerates under the force due to gravity, so it moves faster on the way out. The same flux change ($2.0\times10^{-3}\,\mathrm{Wb}$) then occurs in a shorter time, giving a larger rate of change of flux $\dfrac{\Delta\Phi_B}{\Delta t}$ and hence, by Faraday's law, a larger emf.

Question 4 $r=0.050\,\mathrm{m}$, $A=\pi r^2=7.85\times10^{-3}\,\mathrm{m}^2$, $B=0.30\,\mathrm{T}$. (a) Field perpendicular to the plane: $\Phi_B=BA=0.30\times7.85\times10^{-3}=2.36\times10^{-3}\,\mathrm{Wb}$. (b) The flux **falls to zero**. Only the component of the field perpendicular to the coil contributes to $\Phi_B=B_\perp A$; when the field lies in the plane of the coil, no field lines pass through the area, so $B_\perp=0$ and $\Phi_B=0$. (c) With the field still perpendicular but increased to $0.50\,\mathrm{T}$, $\Phi_B=BA=0.50\times7.85\times10^{-3}=3.93\times10^{-3}\,\mathrm{Wb}$.

Question 5 $L=0.40\,\mathrm{m}$, $v=0.50\,\mathrm{m\,s^{-1}}$, $B=0.35\,\mathrm{T}$, $R=1.4\,\Omega$. (a) In a time Δt the rod moves $d=v\Delta t$, sweeping area $\Delta A=Ld=Lv\Delta t$, so $\varepsilon=B\dfrac{\Delta A}{\Delta t}=BLv=0.35\times0.40\times0.50=0.070\,\mathrm{V}$. (b) $I=\dfrac{\varepsilon}{R}=\dfrac{0.070}{1.4}=0.050\,\mathrm{A}$. (c) Although B is constant, the rod's motion increases the area enclosed by the circuit, so the flux $\Phi_B=BA$ through the circuit increases. By Faraday's law a changing flux induces an emf, $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, which drives an induced current opposing the increase in flux. It is the change in flux (through changing area), not motion by itself, that induces the current.

Question 6 (a) **Deflects.** Pulling the magnet out changes the flux through the coil, and a changing flux induces an emf that drives a current through the galvanometer. (b) The near face of the coil presents a **south pole**. (c) **Relative motion:** the magnet's north pole moves away from the coil. **Changing flux:** the flux through the coil (due to the north pole) is decreasing. **Faraday's law:** a changing flux induces an emf $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, driving an induced current. **Opposition:** the induced current flows so as to oppose the *decrease* in flux — it acts to maintain the flux, making the near face a south pole, which attracts the departing north pole and so opposes its removal.

Question 7 Field perpendicular to the coil, so
$\Phi_B = BA = 0.55\times(0.080\times0.120) = 0.55\times9.6\times10^{-3} = 5.28\times10^{-3}\,\text{Wb}$.

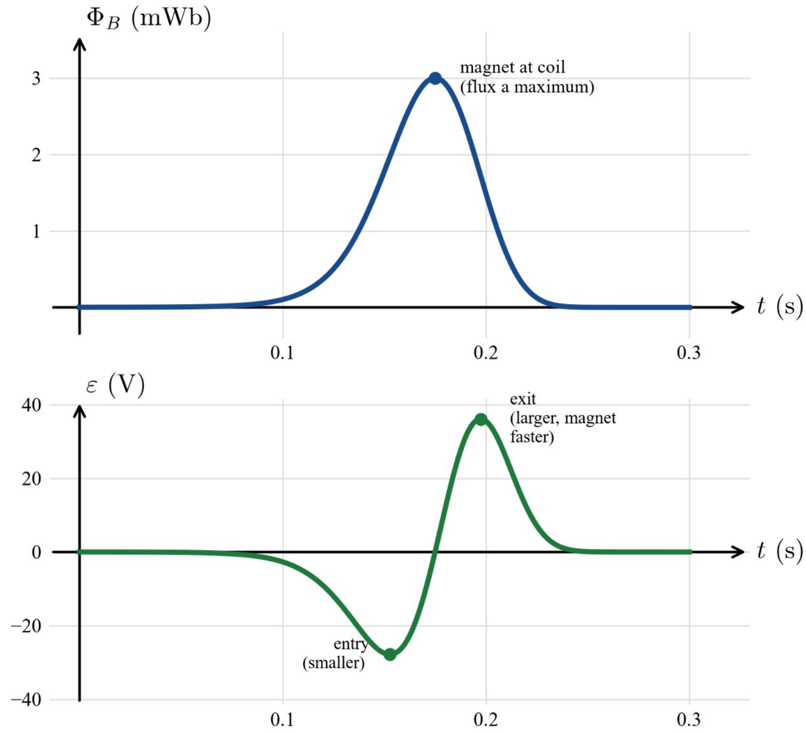
Question 8 $A = 0.10\times0.15 = 0.015\,\text{m}^2$, $N = 120$, $R = 6.0\,\Omega$.
$\varepsilon = N\dfrac{(B_f - B_i)A}{\Delta t} = 120\times\dfrac{(0.90-0.10)\times0.015}{0.50} = 120\times\dfrac{0.012}{0.50} = 2.88\,\text{V}$. $I = \dfrac{\varepsilon}{R} = \dfrac{2.88}{6.0} = 0.48\,\text{A}$.

Question 9 $N = 250$, $r = 0.045\,\text{m}$, $A = \pi r^2 = 6.36\times10^{-3}\,\text{m}^2$, $B = 0.20\,\text{T}$. (a) Face-on flux $\Phi = BA = 0.20\times6.36\times10^{-3} = 1.27\times10^{-3}\,\text{Wb}$; a quarter turn takes this to zero, so $\Delta\Phi_B = 1.27\times10^{-3}\,\text{Wb}$.
$\varepsilon = N\dfrac{\Delta\Phi_B}{\Delta t} = 250\times\dfrac{1.27\times10^{-3}}{0.15} = 2.12\,\text{V}$, as required. (b) $I = \dfrac{\varepsilon}{R} = \dfrac{2.12}{4.0} = 0.530\,\text{A}$.

Question 10 $A = 0.020\,\text{m}^2$; the field changes at $\dfrac{\Delta B}{\Delta t} = 2.0\,\text{T s}^{-1}$, so $\dfrac{\Delta\Phi_B}{\Delta t} = A\dfrac{\Delta B}{\Delta t} = 0.020\times2.0 = 0.040\,\text{Wb s}^{-1}$. (a) $\varepsilon = N\dfrac{\Delta\Phi_B}{\Delta t} = 60\times0.040 = 2.4\,\text{V}$. (b) For $\varepsilon = 5.0\,\text{V}$: $N = \dfrac{\varepsilon}{A\,\Delta B/\Delta t} = \dfrac{5.0}{0.040} = 125$ turns.

Question 11 $N = 30$; the induced emf is N times the gradient of the flux–time graph. (a) 0–0.20 s: gradient $= \dfrac{6.0\times10^{-3}}{0.20} = 0.030\,\text{Wb s}^{-1}$, so $\varepsilon = 30\times0.030 = 0.90\,\text{V}$. (b) 0.30–0.45 s: gradient $= \dfrac{-6.0\times10^{-3}}{0.15} = -0.040\,\text{Wb s}^{-1}$, so $\varepsilon = 30\times0.040 = 1.20\,\text{V}$. (c) The emf is greatest during 0.30–0.45 s (steepest gradient) and zero during 0.20–0.30 s (flux constant).

Question 12



As the magnet enters, the flux through the coil rises from zero to a maximum; as it leaves, the flux falls back to zero. Since $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, the emf is $-N$ times the gradient of the flux–time curve: it is a pulse of one sign while the flux is rising (entry) and a pulse of the **opposite sign** while the flux is falling (exit), passing through zero at the instant the flux is a maximum (magnet level with the coil). The magnet accelerates under the force due to gravity, so it is moving faster on the way out; the flux then changes over a shorter time, giving a larger rate of change of flux. Hence the **second pulse is larger in magnitude and briefer** than the first.

Question 13 Relative motion: the magnet's south pole moves toward the coil — they approach each other. **Changing flux:** magnetic field lines run *toward* a south pole, so as the south pole approaches, the flux through the coil (directed toward the magnet) increases. **Faraday's law:** a changing flux induces an emf $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, driving an induced current. **Opposition:** the induced current flows to oppose the *increase* — the near face of the coil becomes a **south pole**, which repels the approaching south pole of the magnet, opposing its approach. The induced current therefore exerts a retarding force on the moving magnet.

Question 14 (a) Closing the switch makes the current in coil 1 rise rapidly from zero, so the magnetic field it produces — and therefore the flux through coil 2 — rises rapidly. By Faraday's law this changing flux induces an emf in coil 2, driving a brief current that deflects the galvanometer. (b) Once the current in coil 1 is steady, its field and the flux through coil 2 are constant. With $\dfrac{\Delta\Phi_B}{\Delta t} = 0$, no emf is induced, so the galvanometer returns to zero even though a current still flows in coil 1. (c) Opening the switch makes the current in coil 1 fall rapidly to zero, so the flux through coil 2 falls rapidly. This changing flux induces an emf again, but of the opposite sign, so the galvanometer deflects briefly in the **opposite direction**.

Question 15 $N = 90$, $r = 0.037\,\text{m}$, $A = \pi r^2 = 4.30\times10^{-3}\,\text{m}^2$.

$$\varepsilon = N\frac{(B_f - B_i)A}{\Delta t} = 90\times\frac{(0.42-0)\times4.30\times10^{-3}}{0.25} = 0.650\,\text{V (to three significant figures)}.$$

Question 16 $L = 0.20\,\text{m}$, $B = 0.50\,\text{T}$. (a) $\varepsilon = BLv \Rightarrow v = \dfrac{\varepsilon}{BL} = \dfrac{0.15}{0.50\times0.20} = \dfrac{0.15}{0.10} = 1.5\,\text{m s}^{-1}$. (b) $I = \dfrac{\varepsilon}{R} = \dfrac{0.15}{0.30} = 0.50\,\text{A}$.

Question 17 While the coil is entirely inside a *uniform* field and moving with constant velocity, the field strength and the area of the coil do not change, and the field stays perpendicular to the same area, so the flux $\Phi_B = B_\perp A$ through the coil is **constant**. By Faraday's law, $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = 0$ when the flux does not change, so no emf and no current are induced. It is a *change* in flux — not motion through a field by itself — that induces a current. (A current would be induced only while the coil is entering or leaving the field region, where the flux does change.)

Question 18 Pushing the magnet in increases the flux through the coil, which by Faraday's law induces an emf and drives an induced current. By Lenz's law that current opposes the change: it makes the near face of the coil a north pole (for an approaching north pole), which **repels** the magnet. To keep pushing the magnet in, the person must therefore exert a force against this repulsion and do **work**. That mechanical work is the source of the electrical energy dissipated in the circuit's resistance — energy is transformed from the person's work into electrical energy and then heat, and is conserved. If the induced current instead *aided* the motion, the magnet would speed up on its own and electrical energy would be created from nothing, which is impossible. The minus sign in $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$ is exactly this: it states that the induced emf acts in the sense that **opposes** the change in flux (Lenz's law), as required by conservation of energy.

Question 19 In the second drop the magnet passes through the coil faster. (a) The flux through the coil changes by the same total amount (the same magnet, same coil), but in a shorter time, so the rate of change of flux $\frac{\Delta\Phi_B}{\Delta t}$ is greater. By Faraday's law the **peak emf is larger** for the drop from the greater height. (b) Because the magnet spends less time near the coil, the flux rises and falls more quickly, so each emf pulse is **shorter in duration**. (The magnet also accelerates as it falls, so within each drop the exit pulse is briefer and taller than the entry pulse.)

Question 20 $N = 300$, $r = 0.030\,\text{m}$, $A = \pi r^2 = 2.83\times10^{-3}\,\text{m}^2$, $R = 15\,\Omega$. (a) $\varepsilon = N\frac{(B_i - B_f)A}{\Delta t} = 300\times\frac{(0.80-0.20)\times2.83\times10^{-3}}{0.30} = 1.70\,\text{V}$. (b) $I = \frac{\varepsilon}{R} = \frac{1.70}{15} = 0.113\,\text{A}$. (c) **Relative motion / change:** the field into the page is decreasing. **Changing flux:** the flux into the page through the coil is decreasing. **Faraday's law:** a changing flux induces an emf and current. **Opposition:** the induced current flows to oppose the decrease — it acts to maintain the flux into the page, so by the right-hand rule it flows **clockwise** as viewed from the front (with the field into the page).

Question 21 $N = 40$, $A = 0.10^2 = 0.010\,\text{m}^2$, $B = 0.60\,\text{T}$. (a) Field perpendicular to the plane: $\Phi_i = BA = 0.60\times0.010 = 6.0\times10^{-3}\,\text{Wb}$. (b) Field in the plane of the coil: $B_\perp = 0$, so $\Phi_f = 0$. (c) $\varepsilon = N\frac{\Delta\Phi_B}{\Delta t} = 40\times\frac{(6.0-0)\times10^{-3}}{0.25} = 40\times\frac{6.0\times10^{-3}}{0.25} = 0.96\,\text{V}$.

Question 22 (a) Independent variable: the number of turns on the coil. Dependent variable: the peak emf. Controlled variables (any one): the height from which the magnet is dropped (so the speed through the coil is the same), the magnet used, and the cross-sectional area of the coil. (b) By Faraday's law, $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$. For the same magnet dropped from the same height, the flux change through one turn and its timing are unchanged, so the peak emf is directly proportional to the number of turns: doubling N doubles the peak emf. (c) Dropping the magnet from a greater height makes it faster as it passes through the coil. The same flux change then occurs in a shorter time, so $\frac{\Delta\Phi_B}{\Delta t}$ is larger and, by Faraday's law, the **peak emf increases**.

Theory

A **generator** turns mechanical energy into electrical energy: a coil is forced to rotate in a magnetic field, and the changing magnetic flux through the coil induces an emf. It is, in effect, a DC motor (4C) run in reverse — instead of a current in the coil producing rotation, a forced rotation produces a current. This subtopic explains how the same rotating coil can be made to deliver either **alternating (AC)** or **direct (DC)** voltage to an external circuit, depending only on how the ends of the coil are connected to that circuit: through **slip rings** (giving AC, in an **alternator**) or through a **split-ring commutator** (giving DC, in a **DC generator**).

The induced emf comes from electromagnetic induction (5B). The magnetic flux through a coil of area A in a field B is $\Phi_B = B_\perp A$, the product of the area and the component of the field perpendicular to it. Faraday's law gives the emf induced in a coil of N loops as

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t},$$

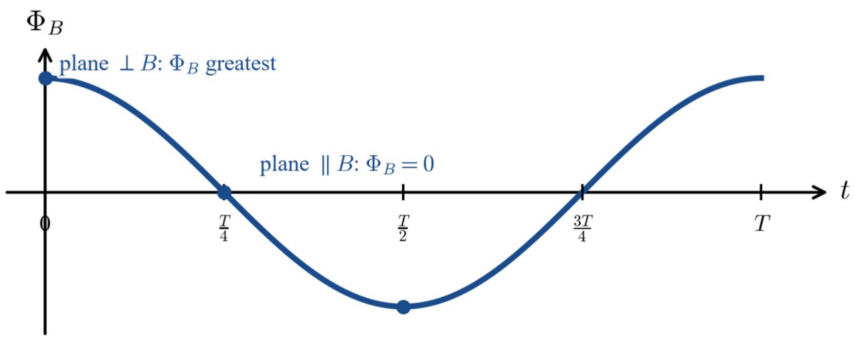
that is, N times the **rate of change** of flux. The minus sign encodes the direction of the emf (Lenz's law, 5B); for the size of the output voltage we use the magnitude $|\varepsilon| = N\frac{\Delta\Phi_B}{\Delta t}$.

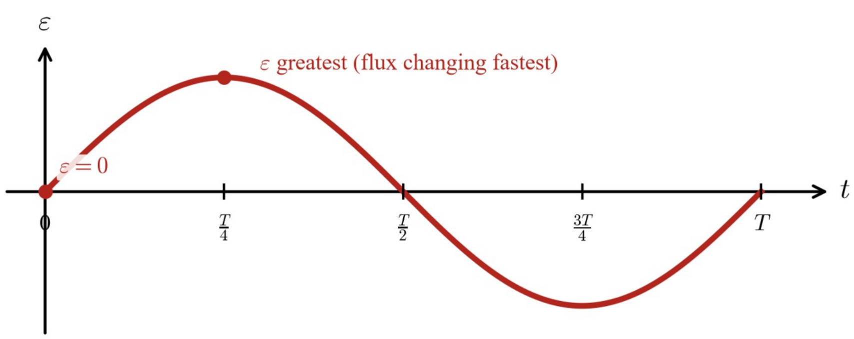
Why a rotating coil generates an emf. As the coil turns, the angle between its plane and the field changes continuously, so the flux threading the coil changes continuously — and a changing flux induces an emf. The chain is:

coil rotates → flux Φ_B through the coil changes → Faraday: $\varepsilon = -N\,\Delta\Phi_B/\Delta t$ → induced emf.

The emf depends on how *fast* the flux is changing, not on how *large* the flux is at that instant.

Where the emf is largest and where it is zero. Consider one revolution, and track both the flux and the emf.



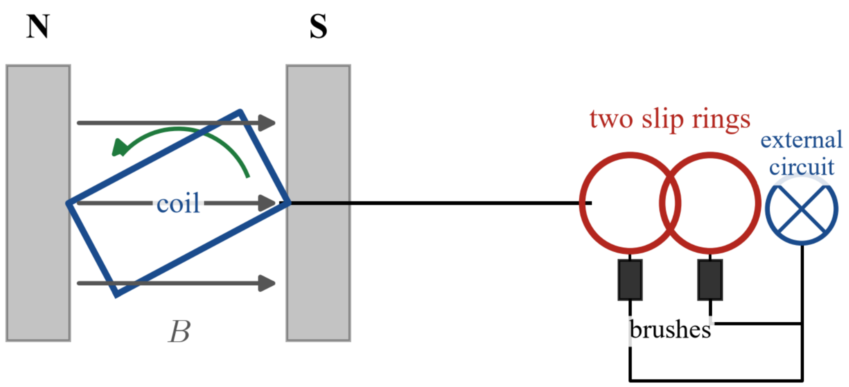


- When the **plane of the coil is perpendicular to the field**, the flux through it is at its greatest value ($\Phi_B = BA$). But at that instant the flux is momentarily *not changing* (it is turning over from increasing to decreasing), so the **emf is zero**. The coil sides are moving parallel to the field lines and are not cutting them.

- A quarter-turn later the **plane of the coil is parallel to the field**, so the flux through it is **zero** — yet this is where the flux is changing *fastest*, so the **emf is a maximum**. The coil sides are now moving perpendicular to the field lines and cut through them at the greatest rate.

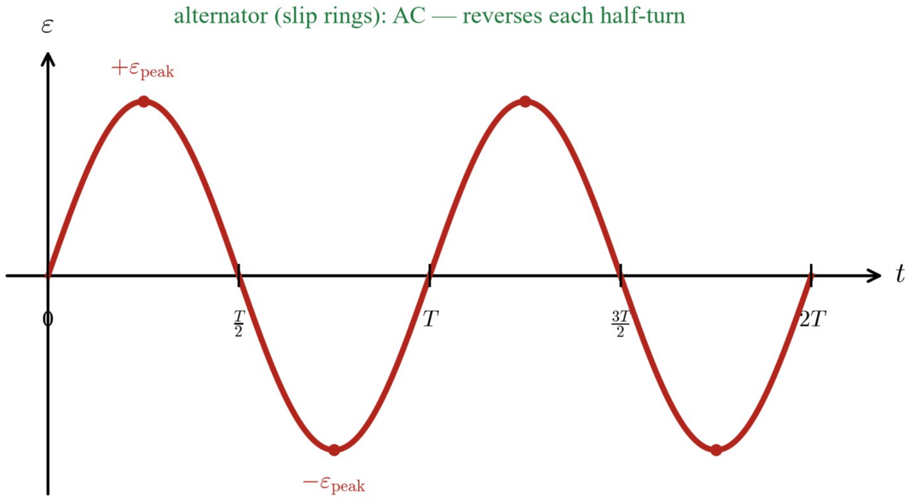
So the induced emf is a maximum exactly where the flux is zero, and zero where the flux is greatest; the emf curve is a quarter-cycle out of step with the flux curve. As the coil keeps turning, the emf rises and falls and — crucially — **reverses direction every half-turn**, because the coil sides swap which way they move through the field. Left to itself, a rotating coil produces an **alternating** emf.

The alternator: slip rings give AC. In an **alternator**, each end of the rotating coil is joined to its own **slip ring** — a complete conducting ring fixed to the axle. Two fixed **brushes** press against the two rings and carry the current out to the external circuit. Because each ring stays connected to the *same* end of the coil throughout the rotation, the external circuit "sees" the coil's own alternating emf unchanged.



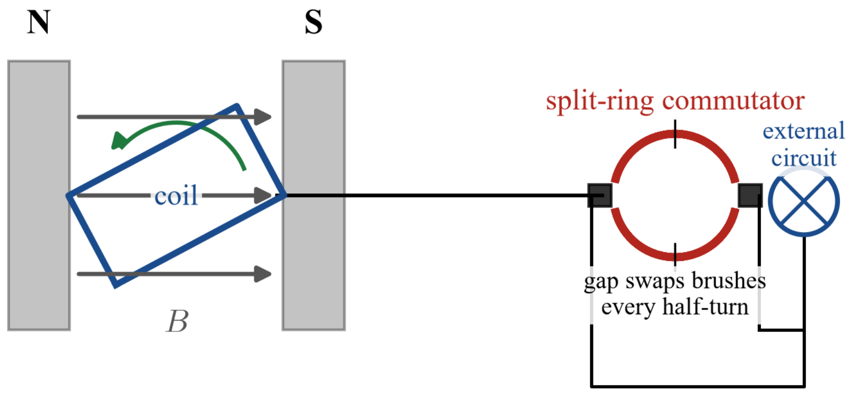
each end of the coil keeps its own ring for the whole rotation → AC output

The output is therefore an **alternating voltage** whose emf–time graph is a **sinusoid**: it rises to a positive peak $+\varepsilon_{\text{peak}}$, falls back through zero to a negative peak $-\varepsilon_{\text{peak}}$, and repeats.



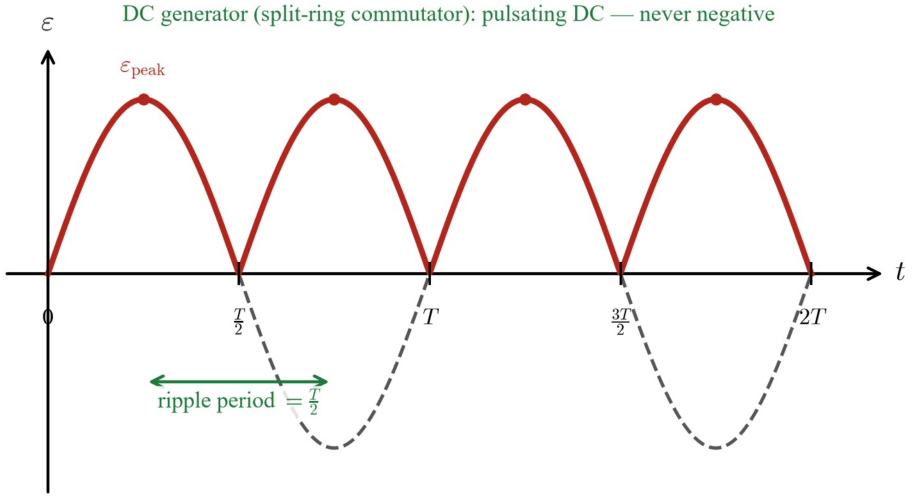
One complete cycle of the output corresponds to one revolution of the coil, so the **frequency of the AC output equals the frequency of rotation** of the coil, and the **period** of the output is $T = \dfrac{1}{f}$.

The DC generator: a split-ring commutator gives DC. A **DC generator** is built from the same rotating coil in the same field, but the ends of the coil are connected to the external circuit through a **split-ring commutator** — a single conducting ring split into two halves by insulating gaps, with each half joined to one end of the coil. Two fixed brushes press on opposite sides of the ring.



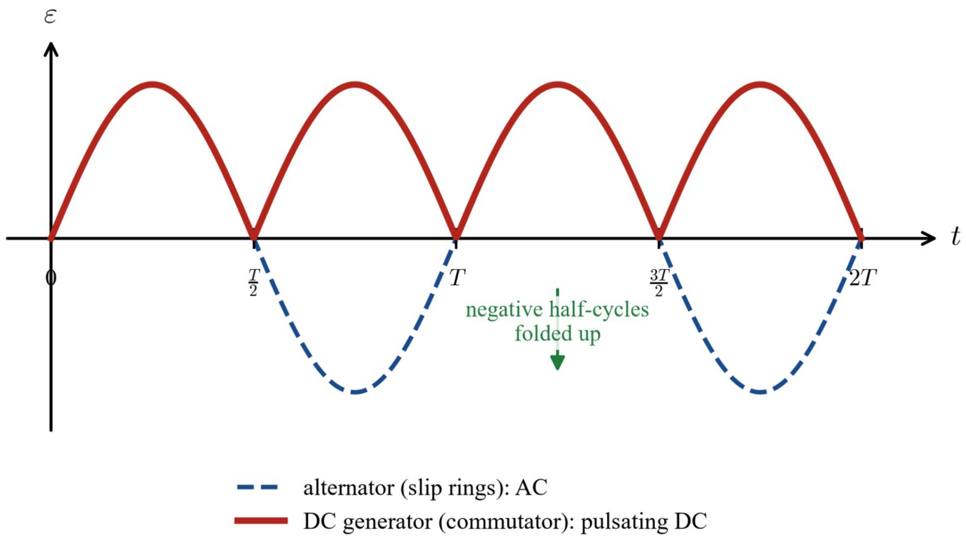
the split reverses the connection each half-turn, just as the coil's emf reverses → DC output

The split ring is arranged so that the **gaps pass the brushes at exactly the moments the coil's emf passes through zero** (when the plane of the coil is perpendicular to the field). At each of these instants each brush swaps from one half of the ring to the other, so the connection between the coil and the external circuit is **reversed every half-turn** — at precisely the moment the coil's own emf is about to reverse. The two reversals cancel, so the emf delivered to the external circuit **keeps the same polarity**: the negative half-cycles of the coil's emf are "folded up" to the positive side.



The result is a **direct voltage** (it never goes negative), but it is not steady — it rises and falls between zero and $\varepsilon_{\text{peak}}$. This is called **pulsating DC**. Because a sinusoid has two humps (one positive, one "folded-up") per revolution, the DC output has **two peaks per revolution**: the **ripple frequency is twice the rotation frequency** $(2f)$ and successive peaks are $\dfrac{T}{2}$ apart.

The single difference, side by side. The alternator and the DC generator are identical machines apart from the connection to the external circuit. Slip rings preserve the coil's alternating emf (AC); the split-ring commutator reverses the connection each half-turn and so rectifies it to same-polarity pulsating DC. The commutator does to the *output connection* of a generator exactly what it does to the *input current* of a motor — it reverses it every half-turn.



Calculating an average emf. The peak of the sinusoid depends on the details of the rotation, but a value that can be found directly from Faraday's law is the **average emf over a quarter turn**. As the coil turns from the plane-perpendicular position (flux $\Phi_B = BA$) to the plane-parallel position (flux $\Phi_B = 0$), the flux changes by $\Delta\Phi_B = BA$ in a time equal to a quarter of the period, $\frac{T}{4}$. Hence

$$|\varepsilon_{\text{avg}}| = N\frac{\Delta\Phi_B}{\Delta t} = \frac{NBA}{T/4}.$$

(The instantaneous peak emf is larger than this quarter-turn average, since the average includes the low-emf instants near the zero-crossing.)

What makes the peak emf larger. From the induction chain, the peak emf of a generator is larger when the flux changes more rapidly, so it increases if the coil has **more loops** N, a **larger area** A, sits in a **stronger field** B, or **rotates faster**. Turning the coil faster is special: it raises the **peak emf** *and* raises the **frequency** (shortens the period), because each revolution is completed in less time.

Throughout this subtopic the magnetic field is uniform and constant, and the coil rotates steadily; flux is measured in webers (Wb, where $1\,\text{Wb} = 1\,\text{T m}^2$) and emf in volts (V).

Worked examples

Example 1 — scaffolded

A rectangular coil of 100 loops measures $0.080\,\text{m}$ by $0.050\,\text{m}$ and rotates steadily at 25 revolutions per second in a uniform magnetic field of $0.20\,\text{T}$. Its ends are connected to the external circuit by two slip rings. Calculate the maximum flux through the coil and the average emf induced as the coil turns through a quarter revolution, and state the frequency of the output voltage.

Working	Comment
$N = 100,\ A = 0.080 \times 0.050 = 4.0 \times 10^{-3}\,\text{m}^2$, $B = 0.20\,\text{T},\ f = 25\,\text{Hz}$.	List the data; the area is length × width.
Maximum flux (plane perpendicular to B): $\Phi_B = B_\perp A = 0.20 \times 4.0 \times 10^{-3} = 8.0 \times 10^{-4}\,\text{Wb}$.	$\Phi_B = B_\perp A$ is greatest when the field is perpendicular to the coil's plane.
Over a quarter turn the flux changes from BA to 0: $\Delta\Phi_B = 8.0 \times 10^{-4}\,\text{Wb}$, in a time $\frac{T}{4} = \frac{1}{4 \times 25} = 0.010\,\text{s}$.	The coil goes from plane-perpendicular to plane-parallel; $T = \frac{1}{f}$.

Working	Comment
$\lvert \varepsilon_{avg} \rvert = N\dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{100 \times 8.0 \times 10^{-4}}{0.010} = 8.0\,\text{V}$.	Faraday's law with the average rate of change of flux.
With slip rings the output is AC at the rotation frequency: $f = 25\,\text{Hz}$.	One cycle of output per revolution.

Example 2 — scaffolded

The emf–time graph of an alternator is a sinusoid with a peak emf of $12\,\text{V}$ and a period of $0.025\,\text{s}$. Calculate the frequency of the output. Then state, with reasons, how the peak emf and the period would change if the coil were made to rotate twice as fast.

Working	Comment
The output is AC; its frequency is $f = \dfrac{1}{T} = \dfrac{1}{0.025} = 40\,\text{Hz}$.	Frequency of the output equals the rotation frequency.
Rotating twice as fast doubles the rotation frequency, so the output frequency becomes $2 \times 40 = 80\,\text{Hz}$ and the period halves to $\dfrac{0.025}{2} = 0.0125\,\text{s}$.	Each revolution now takes half as long.
The flux changes twice as fast, so the peak emf doubles: $2 \times 12 = 24\,\text{V}$.	Peak emf increases with the rate of change of flux (faster rotation).
The graph keeps its sinusoidal shape but is taller ($\pm 24\,\text{V}$) and more tightly spaced.	Still AC — slip rings do not change the shape.

Example 3 — unscaffolded

Explain how a split-ring commutator makes a simple generator produce a DC output, and describe how the emf–time graph of that output differs from the graph produced by the same coil connected to slip rings.

As the coil rotates in the magnetic field, the flux through it changes, inducing an emf $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$. This emf reverses direction every half-turn, because the coil sides swap which way they move through the field; on its own the coil produces an alternating emf.

The split-ring commutator is a single ring divided into two halves, each joined to one end of the coil, with the insulating gaps positioned so that they pass the brushes just as the coil's emf falls to zero. At each of these instants every brush moves from one half of the ring to the other, so the connection between the coil and the external circuit is reversed. This reversal happens at the same moment as the reversal of the coil's own emf, so the two cancel and the voltage supplied to the external circuit always has the **same polarity**.

∴ the output is direct (it never goes negative), but it rises and falls between zero and a peak value twice per revolution — a series of positive humps, or **pulsating DC**. With slip rings instead, each end of the coil stays connected to the same terminal, so the alternating emf is passed on unchanged and the graph is a full **sinusoid** that swings equally positive and negative.

Example 4 — unscaffolded

A generator has a coil of 250 loops, each of area $6.0 \times 10^{-3}\,\text{m}^2$, rotating at 19 revolutions per second in a uniform field of $0.15\,\text{T}$. Show that the average emf induced over a quarter turn is about $17.1\,\text{V}$, and state the frequency of the output if the coil is connected to slip rings.

The maximum flux through the coil (plane perpendicular to the field) is

$$\Phi_B = B_{\perp}A = 0.15 \times 6.0 \times 10^{-3} = 9.0 \times 10^{-4}\,\text{Wb}.$$

Over a quarter turn the flux falls from this value to zero, so $\Delta\Phi_B = 9.0\times10^{-4}\,\text{Wb}$, in a time $\frac{T}{4} = \frac{1}{4\times19} = 1.316\times10^{-2}\,\text{s}$. Therefore

$$|\varepsilon_{\text{avg}}| = N\frac{\Delta\Phi_B}{\Delta t} = \frac{250\times9.0\times10^{-4}}{1.316\times10^{-2}} = 17.1\,\text{V},$$

as required. With slip rings the output is AC at the rotation frequency.

∴ the average emf over a quarter turn is $17.1\,\text{V}$ and the output frequency is $19\,\text{Hz}$.

Practice questions

Question 1 A coil of 80 loops measures $0.060\,\text{m}$ by $0.10\,\text{m}$ and rotates at 30 revolutions per second in a uniform field of $0.25\,\text{T}$. (a) Calculate the maximum flux through the coil. (b) Calculate the change in flux, and the time taken, as the coil turns from the position where its plane is perpendicular to the field to the position where its plane is parallel to the field. (c) Hence calculate the average emf induced over this quarter turn.

Question 2 A single coil rotates steadily in a uniform magnetic field, with its ends connected to the external circuit by two slip rings. (a) Name the type of voltage (AC or DC) produced at the output, and name the device. (b) (Circle your answer.) At the instant the plane of the coil is perpendicular to the field, the flux through the coil is *maximum / zero* and the induced emf is *maximum / zero*. (c) Using the rotation → flux → emf chain, explain why the induced emf is zero at that instant even though the flux through the coil is greatest.

Question 3 The coil of a generator rotates at 50 revolutions per second. (a) The coil is connected to the external circuit by slip rings. State the frequency and the period of the output voltage. (b) The slip rings are then replaced by a split-ring commutator. State how many peaks the output shows per revolution, and state the ripple frequency. (c) Calculate the time between successive peaks of the DC output.

Question 4 The emf–time graph of a DC generator shows a series of positive humps that reach a peak of $9.0\,\text{V}$, with successive peaks $0.008\,\text{s}$ apart. (a) State whether the output is AC or DC, and justify your answer from the shape of the graph. (b) Determine the frequency at which the coil is rotating. (c) Describe how the graph would change if the split-ring commutator were replaced by two slip rings, including the peak value and the period of the new output.

Question 5 A simple DC generator uses a split-ring commutator. (a) State the function of the split-ring commutator in the generator. (b) Explain what happens to the coil's own induced emf every half-turn, and how the commutator responds at that instant. (c) Describe the shape of the output emf–time graph, and explain why it never goes below zero.

Question 6 Two alternators, X and Y, use identical coils rotating in identical magnetic fields. Coil X rotates at 20 revolutions per second and coil Y at 40 revolutions per second. (a) State and justify the ratio of the output frequencies of X and Y. (b) Alternator X produces a peak emf of $6.0\,\text{V}$. State the peak emf of alternator Y, and justify your answer. (c) Calculate the period of the output of alternator Y.

Question 7 A coil of 200 loops, each of area $2.5\times10^{-3}\,\text{m}^2$, rotates at 14 revolutions per second in a uniform field of $0.40\,\text{T}$. Calculate the average emf induced as the coil turns through a quarter revolution.

Question 8 A coil of 120 loops measures $0.075\,\text{m}$ by $0.045\,\text{m}$ and rotates at 18 revolutions per second in a uniform field of $0.22\,\text{T}$. (a) Show that the average emf induced over a quarter turn is about $6.42\,\text{V}$. (b) Hence state whether the peak emf of the sinusoidal output is larger or smaller than $6.42\,\text{V}$, and briefly explain why.

Question 9 The emf–time graph of a DC generator shows peaks that are $4.00\,\text{ms}$ apart. Determine the frequency at which the coil is rotating, and the frequency of the AC output that the same coil would produce through slip rings. Give your answers to three significant figures.

Question 10 The emf–time graph of an alternator is a sinusoid of peak emf $15\,\mathrm{V}$ and period $0.036\,\mathrm{s}$. Calculate the frequency of the output, and state how the graph of the same coil would differ if it were connected instead to a split-ring commutator (include the peak value and the spacing of the humps).

Question 11 A student says: "The slip rings of an alternator turn the coil's alternating emf into a direct voltage." Explain what is wrong with this statement. State what the slip rings actually do, and name the component that would be needed to obtain a DC output instead.

Question 12 A coil is rotated at a steady rate in a uniform magnetic field. Starting from the rotation of the coil, explain why an emf is induced, and explain why this emf is greatest at the instant the plane of the coil is parallel to the field.

Question 13 A DC generator and an alternator are built from the same rotating coil in the same magnetic field. Explain the single difference in their construction, and explain how that difference produces the different output voltages. Refer to the emf–time graph of each in your answer.

Question 14 A DC generator must produce a larger peak emf, but the rotation rate of its coil cannot be changed. State two changes to the generator that would increase the peak emf, and explain why each change has this effect.

Question 15 A simple DC motor and a simple DC generator can be built from identical components — a coil that rotates in a magnetic field, connected through a split-ring commutator. (a) State the energy transformation that takes place in each device. (b) Explain the role of the split-ring commutator in each device.

Question 16 A coil of 100 loops has an area of $4.0 \times 10^{-3}\,\mathrm{m}^2$ and rotates in a uniform field of $0.30\,\mathrm{T}$. Calculate the flux through the coil when its plane is perpendicular to the field and when its plane is parallel to the field, and state at which of these two positions the induced emf is a maximum, explaining your answer.

Question 17 On a single set of axes, sketch the emf–time graph produced over two revolutions by a coil connected to (i) two slip rings and (ii) a split-ring commutator, for the same steady rotation. Label the peak emf $\varepsilon_{\text{peak}}$ and the period T, and explain the difference in shape between the two graphs.

Question 18 An alternator produces a sinusoidal output with a peak emf of $20\,\mathrm{V}$ when its coil rotates at 30 revolutions per second. (a) Calculate the period of the output. Give your answer to three significant figures. (b) The slip rings are replaced by a split-ring commutator, the rotation unchanged. State the peak value of the new output and calculate the time between successive peaks.

Question 19 During one full rotation of a generator coil, a student claims that the emf must be constant because the strength of the magnetic field and the area of the coil do not change. Explain what is wrong with this reasoning, referring to the flux through the coil and its rate of change.

Question 20 A single coil is rotated steadily in a constant, uniform magnetic field. Explain how this one coil can be made to deliver either an alternating voltage or a direct voltage to an external circuit. In your answer, refer to the rotation → flux → emf chain, the connection to the external circuit in each case, and the emf–time graph produced in each case.

Answers

Question 1 (a) 1.5×10^{-3} Wb; (b) $\Delta\Phi_B=1.5\times10^{-3}$ Wb in 8.33×10^{-3} s; (c) 14.4 V. **Question 2** (a) AC, from an alternator; (b) flux maximum, emf zero; (c) the flux is momentarily not changing, so the rate of change (and hence the emf) is zero. **Question 3** (a) 50 Hz, 0.020 s; (b) two peaks per revolution, ripple frequency 100 Hz; (c) 0.010 s. **Question 4** (a) DC — the graph never goes below zero (always one polarity); (b) 62.5 Hz; (c) a full sinusoid, peak 9.0 V, period 0.016 s (negative humps no longer folded up). **Question 5** (a) it reverses the connection between the coil and the external circuit every half-turn; (b) the coil's emf reverses every half-turn, and the commutator reverses the connection at the same instant, so the two cancel; (c) a series of positive humps (pulsating DC); it never goes negative because the two reversals keep the output at one polarity. **Question 6** (a) $1:2$ (frequency equals rotation frequency); (b) 12 V (peak emf is proportional to the rotation rate); (c) 0.025 s. **Question 7** 11.2 V. **Question 8** (a) 6.42 V (shown); (b) larger, because the quarter-turn average includes the low-emf instants near the zero-crossing, whereas the peak is the single greatest value. **Question 9** 125 Hz (rotation and AC output are the same frequency). **Question 10** 27.8 Hz; the commutator output is the rectified graph — same peak 15 V, humps 0.018 s apart, never negative. **Question 11** Incorrect: slip rings keep the output AC; a split-ring commutator is needed for DC. **Question 12** An emf is induced because rotation changes the flux; it is greatest when the plane is parallel to the field, where the flux is changing fastest. **Question 13** The only difference is the connection to the external circuit: slip rings (AC, sinusoid) versus a split-ring commutator (DC, rectified humps). **Question 14** Any two of: more loops N (the emf is N times the rate of change of flux); a larger coil area A; a stronger magnetic field B. A larger area or a stronger field increases the flux $\Phi_B=B_\perp A$ that changes each turn, so the rate of change of flux is greater; in each case the peak emf is larger. **Question 15** (a) motor: electrical → mechanical; generator: mechanical → electrical; (b) motor: reverses the coil current each half-turn to keep the torque one way; generator: reverses the output connection each half-turn to keep the output one polarity. **Question 16** Plane perpendicular: 1.2×10^{-3} Wb; plane parallel: 0 Wb; the emf is a maximum when the plane is parallel (flux zero but changing fastest). **Question 17** Slip rings: full sinusoid ($\pm\varepsilon_{\text{peak}}$); commutator: rectified positive humps (twice as many, never negative), same peak and period. **Question 18** (a) 0.0333 s; (b) peak still 20 V, successive peaks 0.0167 s apart. **Question 19** Incorrect: the flux changes because the angle between the coil and the field changes as it rotates; the emf depends on the rate of change of flux, which varies through each revolution. **Question 20** Slip rings pass the coil's alternating emf on unchanged (AC, sinusoid); a split-ring commutator reverses the connection each half-turn (DC, pulsating humps).

Fully-worked solutions

Question 1 $N=80$, $A=0.060\times0.10=6.0\times10^{-3}\,\text{m}^2$, $B=0.25$ T, $f=30$ Hz. (a) Maximum flux (plane perpendicular to the field): $\Phi_B=B_\perp A=0.25\times6.0\times10^{-3}=1.5\times10^{-3}$ Wb. (b) Turning from plane-perpendicular ($\Phi_B=1.5\times10^{-3}$ Wb) to plane-parallel ($\Phi_B=0$), the change is $\Delta\Phi_B=1.5\times10^{-3}$ Wb. The time is a quarter period: $\frac{T}{4}=\frac{1}{4\times30}=8.33\times10^{-3}$ s. (c) $|\varepsilon_{\text{avg}}|=N\frac{\Delta\Phi_B}{\Delta t}=\frac{80\times1.5\times10^{-3}}{8.33\times10^{-3}}=14.4$ V.

Question 2 (a) The output is **AC** (alternating), produced by an **alternator**. (b) The flux is **maximum** and the induced emf is **zero**. (c) As the coil rotates, the flux through it changes and induces an emf equal to N times the rate of change of flux, $\varepsilon=-N\frac{\Delta\Phi_B}{\Delta t}$. At the instant the plane of the coil is perpendicular to the field the flux is at its greatest value, but it is momentarily **not changing** — it is turning over from increasing to decreasing. Because the rate of change of flux is zero at that instant, the induced emf is zero, even though the flux itself is largest. (The coil sides are moving parallel to the field lines and are not cutting them.)

Question 3 The coil rotates at $f=50$ Hz. (a) With slip rings the output is AC at the rotation frequency: $f=50$ Hz, period $T=\frac{1}{f}=0.020$ s. (b) A split-ring commutator folds the negative half-cycle up, giving **two peaks per revolution**; the ripple frequency is $2f=100$ Hz. (c) Successive peaks are half a period apart: $\frac{T}{2}=\frac{0.020}{2}=0.010$ s.

Question 4 (a) **DC.** The graph is a series of humps that stay on one side of the axis (never negative), so the output always has the same polarity — this is pulsating DC, as produced by a split-ring commutator. (b) The humps are $\frac{T}{2}$ apart, so $T = 2 \times 0.008 = 0.016\,\text{s}$ and the rotation frequency is $f = \frac{1}{T} = \frac{1}{0.016} = 62.5\,\text{Hz}$. (c) With slip rings the output would be a full **sinusoid**: the negative half-cycles are no longer folded up, so the graph swings between $+9.0\,\text{V}$ and $-9.0\,\text{V}$ with period $T = 0.016\,\text{s}$ (one cycle per revolution).

Question 5 (a) The split-ring commutator **reverses the connection between the coil and the external circuit every half-turn**, so that the output keeps the same polarity (DC). (b) As the coil rotates, its own induced emf **reverses direction every half-turn** (the coil sides swap which way they move through the field). The commutator's insulating gaps are placed so that each brush switches from one half of the ring to the other at exactly that instant, reversing the connection at the same moment the coil emf reverses. The two reversals cancel. (c) The output is a series of **positive humps** (pulsating DC), rising and falling between zero and the peak twice per revolution. It never goes below zero because the commutator reverses the connection each half-turn, so the "negative" half-cycles of the coil are delivered to the external circuit with the same polarity as the positive ones.

Question 6 Coils and fields are identical; only the rotation rates differ ($f_X = 20\,\text{Hz}$, $f_Y = 40\,\text{Hz}$). (a) The output frequency equals the rotation frequency, so $f_X : f_Y = 20:40 = 1:2$ — alternator Y's output has twice the frequency of X's. (b) The peak emf is proportional to how fast the coil turns (the flux changes twice as fast in Y). Since Y rotates twice as fast, its peak emf is twice that of X: $2 \times 6.0 = 12\,\text{V}$. (c) $T_Y = \frac{1}{f_Y} = \frac{1}{40} = 0.025\,\text{s}$.

Question 7 $N = 200$, $A = 2.5 \times 10^{-3}\,\text{m}^2$, $B = 0.40\,\text{T}$, $f = 14\,\text{Hz}$. The flux falls from $\Phi_B = BA = 0.40 \times 2.5 \times 10^{-3} = 1.0 \times 10^{-3}\,\text{Wb}$ to zero over a quarter turn, in a time $\frac{T}{4} = \frac{1}{4 \times 14} = 1.786 \times 10^{-2}\,\text{s}$. Hence

$$|\varepsilon_{\text{avg}}| = N\frac{\Delta\Phi_B}{\Delta t} = \frac{200 \times 1.0 \times 10^{-3}}{1.786 \times 10^{-2}} = 11.2\,\text{V}.$$

Question 8 $N = 120$, $A = 0.075 \times 0.045 = 3.375 \times 10^{-3}\,\text{m}^2$, $B = 0.22\,\text{T}$, $f = 18\,\text{Hz}$. (a) The flux changes by $\Delta\Phi_B = BA = 0.22 \times 3.375 \times 10^{-3} = 7.425 \times 10^{-4}\,\text{Wb}$ over a quarter turn, in a time $\frac{T}{4} = \frac{1}{4 \times 18} = 1.389 \times 10^{-2}\,\text{s}$. So

$$|\varepsilon_{\text{avg}}| = N\frac{\Delta\Phi_B}{\Delta t} = \frac{120 \times 7.425 \times 10^{-4}}{1.389 \times 10^{-2}} = 6.42\,\text{V},$$

as required. (b) The peak emf is **larger** than $6.42\,\text{V}$. The quarter-turn value is an *average* rate of change of flux, and it includes the instants near the zero-crossing where the emf is small; the peak is the single greatest instantaneous value and so exceeds the average.

Question 9 The humps of a DC generator are $\frac{T}{2}$ apart, so $\frac{T}{2} = 4.00\,\text{ms} = 4.00 \times 10^{-3}\,\text{s}$, giving $T = 8.00 \times 10^{-3}\,\text{s}$ and a rotation frequency

$$f = \frac{1}{T} = \frac{1}{8.00 \times 10^{-3}} = 125\,\text{Hz}.$$

Through slip rings the same coil produces AC at the rotation frequency, so the AC output frequency is also $125\,\text{Hz}$ (to three significant figures). (The DC ripple frequency itself is $2f = 250\,\text{Hz}$.)

Question 10 The output is AC, so its frequency equals the rotation frequency: $f = \frac{1}{T} = \frac{1}{0.036} = 27.8\,\text{Hz}$. Connected instead to a split-ring commutator, the same coil gives the **rectified** graph: the negative half-cycles are folded up, so the output is a series of positive humps of the **same peak**, $15\,\mathrm{V}$, that never go negative. There are now two humps per revolution, so successive peaks are $\frac{T}{2}=\frac{0.036}{2}=0.018\,\mathrm{s}$ apart.

Question 11 The statement is incorrect. **Slip rings do not change AC into DC.** Each slip ring is a complete ring that stays connected to the same end of the coil throughout the rotation, so the brushes simply carry the coil's own alternating emf out to the external circuit unchanged — the output remains **AC**. To obtain a DC output the coil must instead be connected through a **split-ring commutator**, whose gaps reverse the connection every half-turn and so keep the output at one polarity.

Question 12 As the coil is rotated, the angle between its plane and the magnetic field changes continuously, so the flux $\Phi_B=B_{\perp}A$ threading the coil changes continuously. By Faraday's law a changing flux induces an emf equal to N times its rate of change, $\varepsilon=-N\frac{\Delta\Phi_B}{\Delta t}$ — this is why a rotating coil generates an emf. The emf is greatest when the flux is changing **fastest**. When the plane of the coil is parallel to the field, the flux through the coil is momentarily zero, but this is exactly the position where it is changing most rapidly (the coil sides are moving perpendicular to the field and cut the field lines at the greatest rate). Therefore the induced emf reaches its maximum at that instant.

Question 13 The single difference is the **connection between the coil and the external circuit**. In an **alternator** each end of the coil is joined to its own **slip ring**, so the coil's alternating emf is passed to the circuit unchanged: the emf–time graph is a full **sinusoid** that swings equally positive and negative (AC). In a **DC generator** the ends are joined to a **split-ring commutator**, a single ring split in two whose gaps reverse the connection every half-turn, at the instant the coil emf reverses. This folds the negative half-cycles up to the positive side, so the emf–time graph is a series of **positive humps that never go negative** (pulsating DC). The rotating coil, field and induced emf are identical in both machines — only the output connection differs.

Question 14 The peak emf is larger when the flux through the coil changes more rapidly. With the rotation rate unchanged, the time taken for each quarter turn is fixed, so the peak emf is raised by increasing the amount of flux that changes, or by multiplying the induced emf. Any two of the following: - **Increase the number of loops N:** the total emf is N times the emf induced in a single loop, $\varepsilon=-N\frac{\Delta\Phi_B}{\Delta t}$, so more loops give a larger emf in proportion. - **Increase the coil area A:** the maximum flux $\Phi_B=B_{\perp}A$ is larger, so the change in flux $\Delta\Phi_B=BA$ over each quarter turn is larger; since this change occurs in the same time, the rate of change of flux — and hence the emf — is larger. - **Increase the field strength B:** a stronger field also increases $\Phi_B=B_{\perp}A$ and therefore $\Delta\Phi_B$, again raising the rate of change of flux and the peak emf.

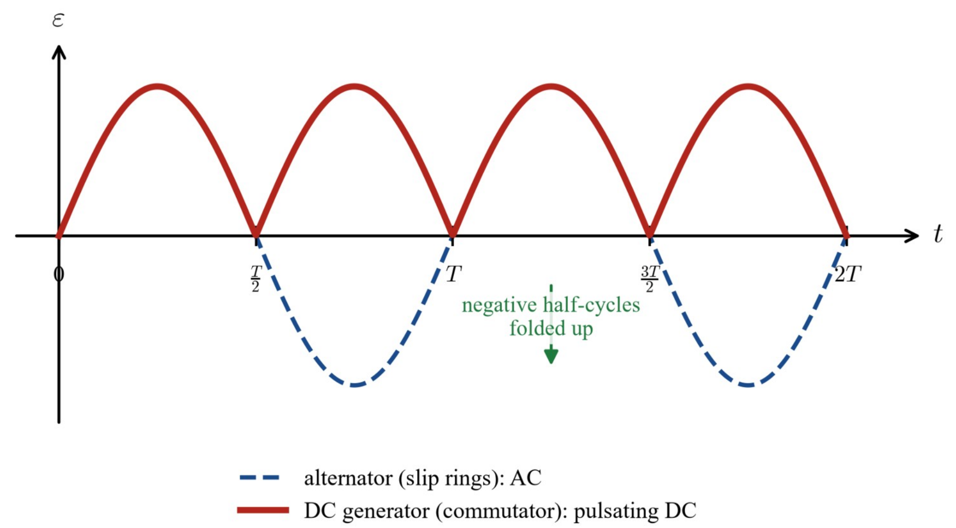
Question 15 (a) In a **motor** the energy transformation is **electrical → mechanical** (a current in the coil produces rotation). In a **generator** it is **mechanical → electrical** (a forced rotation produces a current). The two are the same device run in opposite directions. (b) In a **motor**, the split-ring commutator **reverses the current in the coil every half-turn**, so that the force on each side keeps turning the coil the same way (the torque never reverses). In a **generator**, the split-ring commutator **reverses the connection to the external circuit every half-turn**, so that the alternating emf induced in the coil is delivered to the circuit as a single-polarity (DC) output. In both cases the commutator reverses a connection each half-turn; the difference is whether it is switching the current *into* the coil or the emf *out of* it.

Question 16 $N=100$, $A=4.0\times10^{-3}\,\mathrm{m}^2$, $B=0.30\,\mathrm{T}$. When the plane of the coil is **perpendicular** to the field, the field is perpendicular to the coil's area, so the flux is greatest:

$$\Phi_B=B_{\perp}A=0.30\times4.0\times10^{-3}=1.2\times10^{-3}\,\mathrm{Wb}.$$

When the plane of the coil is **parallel** to the field, no field passes through the coil's area, so $\Phi_B=0\,\mathrm{Wb}$. The induced emf is a maximum at the **plane-parallel** position. Although the flux there is zero, it is changing at its greatest rate as the coil turns through that position, and the emf depends on the **rate of change** of flux, not on its value — so the emf peaks where the flux is momentarily zero.

Question 17



For the same steady rotation:

- **Slip rings (AC):** a full **sinusoid**, rising to $+\varepsilon_{\text{peak}}$, falling through zero to $-\varepsilon_{\text{peak}}$, one complete cycle per revolution (period T).
- **Split-ring commutator (DC):** the **rectified** version of that sinusoid — the negative half-cycles are folded up to the positive side, giving positive humps of the same peak $\varepsilon_{\text{peak}}$, two per revolution, that touch zero but never go negative.

The difference in shape arises because the slip rings keep each end of the coil connected to the same terminal (so the alternating emf is passed on unchanged), whereas the split-ring commutator reverses the connection every half-turn (so the negative half-cycles are flipped up and the output stays one polarity).

Question 18 The coil rotates at $f=30\,\text{Hz}$. (a) The output is AC at the rotation frequency, so its period is $T=\frac{1}{f}=\frac{1}{30}=0.0333\,\text{s}$ (to three significant figures). (b) A split-ring commutator does not change the peak value, so the peak of the new output is still $20\,\text{V}$. It does fold the negative half-cycles up, giving two humps per revolution, so successive peaks are $\frac{T}{2}=\frac{0.0333}{2}=0.0167\,\text{s}$ apart.

Question 19 The reasoning is wrong because the flux does **not** stay constant. The flux through the coil is $\Phi_B=B_{\perp}A$, the field component **perpendicular to the coil's area**; as the coil rotates, the angle between the coil and the field changes, so the perpendicular component — and hence the flux — changes continuously, even though B and A themselves are fixed. The induced emf depends on the **rate of change** of this flux, $\varepsilon=-N\frac{\Delta\Phi_B}{\Delta t}$, which varies through each revolution: it is zero when the plane is perpendicular to the field (flux greatest, momentarily unchanging) and greatest when the plane is parallel to the field (flux zero, changing fastest). The emf is therefore continually varying, not constant.

Question 20 In both cases the coil is rotated steadily in the constant field, so the flux through it changes continuously and, by Faraday's law, induces an emf: **rotation → the flux Φ_B through the coil changes →** $\varepsilon=-N\frac{\Delta\Phi_B}{\Delta t}$ **→ an induced emf** that reverses every half-turn. What determines the output is how the ends of the coil are connected to the external circuit.

- Connected through **two slip rings**, each end of the coil stays joined to the same terminal for the whole rotation, so the coil's alternating emf reaches the circuit unchanged. The output is **AC**, and its emf–time graph is a **sinusoid** swinging between $+\varepsilon_{\text{peak}}$ and $-\varepsilon_{\text{peak}}$, one cycle per revolution.
- Connected through a **split-ring commutator**, the gaps reverse the connection every half-turn, at the instant the coil emf reverses. The two reversals cancel, so the output keeps one polarity: the emf–time graph is a series of **positive humps** (pulsating DC), two per revolution, that never go below zero.

∴ the one rotating coil delivers AC when connected through slip rings and DC when connected through a split-ring commutator; the induction that generates the emf is identical, and only the output connection differs.

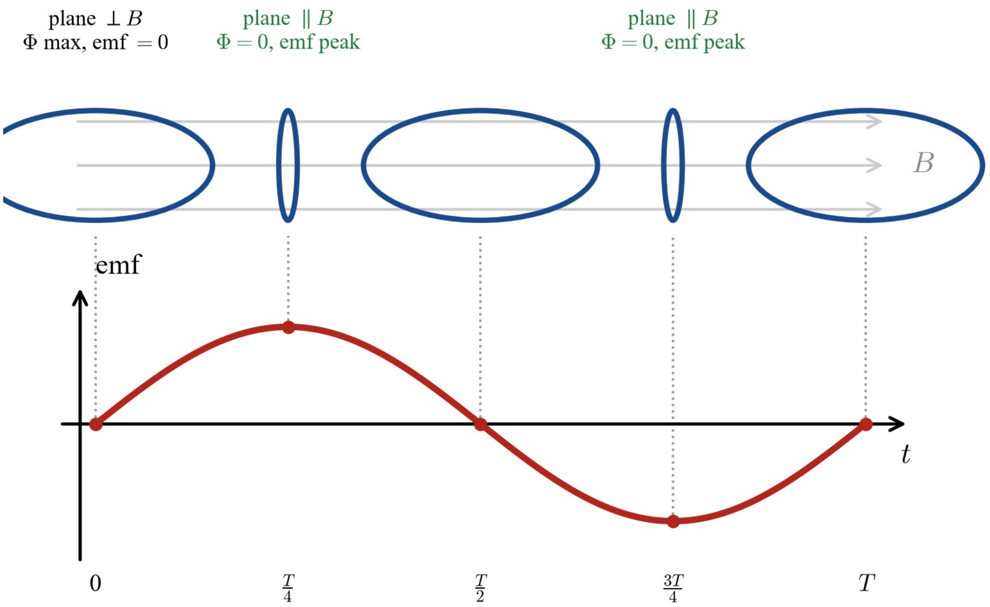
Theory

When a loop (coil) of wire is rotated at a steady rate in a **constant magnetic field**, the magnetic flux threading the loop changes continuously, and by electromagnetic induction (5B) an emf is induced across its ends. Because the loop turns uniformly, the flux — and therefore the induced emf — varies smoothly and repeatedly, first one way and then the other: the output is an **alternating voltage** (AC) with the shape of a **sine curve**. This is the voltage produced by an alternator (5C); here we study the *shape* of that output, and how to describe and compare such waveforms.

Why the output is sinusoidal. The size of the induced emf follows Faraday's law $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$ — it depends on how fast the flux is changing. As the loop turns:

- when the **plane of the loop is perpendicular** to the field, the flux through it is at its maximum, but for that instant it is momentarily *not changing*, so the induced emf is **zero**;
- a quarter-turn later the **plane of the loop is parallel** to the field; the flux is momentarily zero but is changing at its greatest rate, so the induced emf is at its **peak**.

Over one complete revolution the emf rises to a positive peak, falls back through zero to a negative peak and returns to zero — exactly one cycle of a sine curve. A single loop therefore produces **one full AC cycle per revolution**.

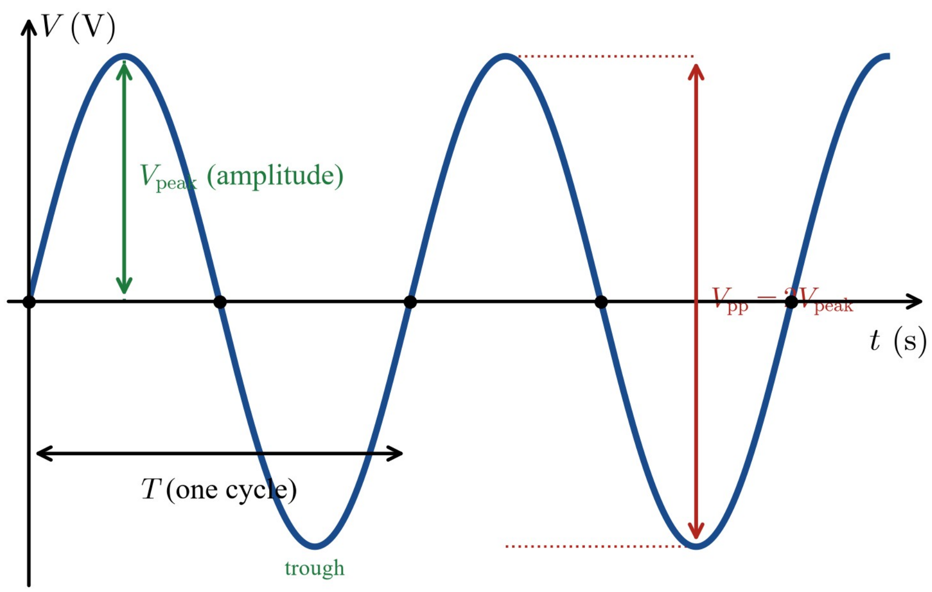


Period and frequency. The **period** T is the time for one complete cycle of the waveform, in seconds. The **frequency** $f = \dfrac{1}{T}$ is the number of complete cycles each second, measured in hertz (Hz). The two are reciprocals, so $T = \dfrac{1}{f}$.

Australian mains supply alternates at $50\,\mathrm{Hz}$, so its period is $T = \dfrac{1}{50} = 0.020\,\mathrm{s} = 20\,\mathrm{ms}$.

Amplitude, peak and peak-to-peak. The **amplitude** of the waveform is its greatest displacement from zero, the **peak voltage** V_{peak}. Because the sine curve swings symmetrically up to $+V_{\text{peak}}$ and down to $-V_{\text{peak}}$, the total vertical distance from the bottom of a trough to the top of a crest is the **peak-to-peak voltage**:

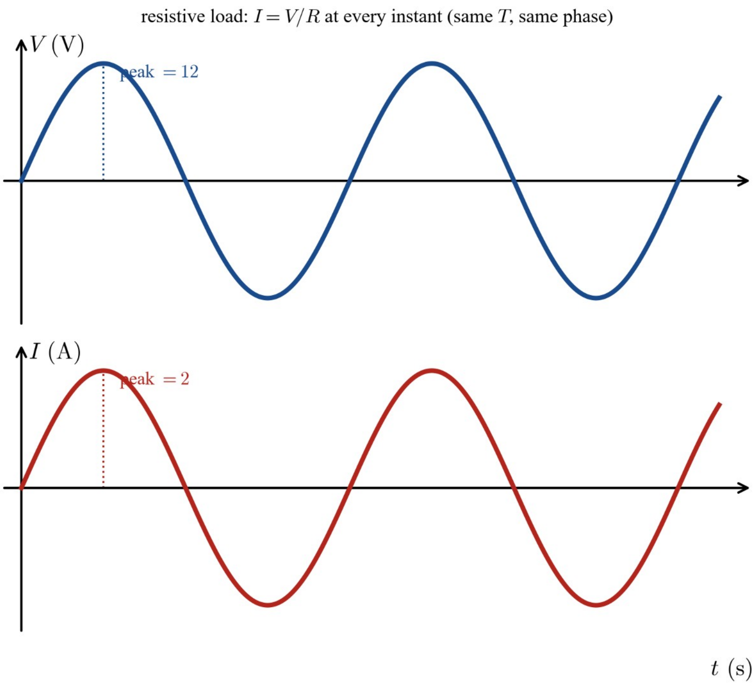
$$V_{\text{p-p}} = 2V_{\text{peak}}.$$



The current driven through a load behaves the same way: it has a **peak current** I_{peak} (its amplitude) and a **peak-to-peak current** $I_{\text{p-p}} = 2I_{\text{peak}}$. Written as a function of time the voltage is $V = V_{\text{peak}} \sin\left(\frac{2\pi t}{T}\right)$, which shows the amplitude V_{peak} and the period T directly; but for this subtopic the waveform is read from its graph rather than evaluated at particular instants.

Voltage and current in a resistive load. If the alternating voltage is connected across a resistor of resistance R, Ohm's law applies at every instant, so $I = \frac{V}{R}$ moment by moment. The current is therefore a sine curve of the *same period and frequency*, reaching its peak at the same time as the voltage (the two are **in phase**), with

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R}, I_{\text{p-p}} = \frac{V_{\text{p-p}}}{R} = 2I_{\text{peak}}.$$



Reading an AC graph. To describe a sinusoidal voltage or current from its graph:

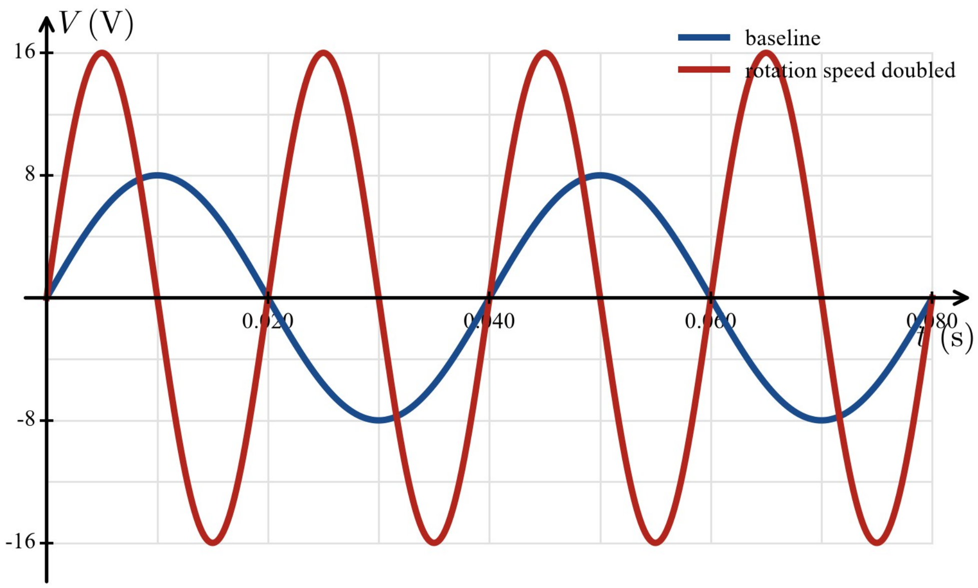
- read the **period** as the time for *one complete cycle* — for example between two successive crests, or between every second crossing of the zero line (a common error is to read only a half-cycle);

- find the **frequency** from $f = \dfrac{1}{T}$;
- read the **peak** value as the height of a crest above the zero line;
- the **peak-to-peak** value is the full height from a trough up to a crest, equal to twice the peak.

The effect of rotation speed. How fast the loop spins controls *two* features of the output at once:

- **Frequency and period.** A single loop gives one cycle per revolution, so spinning it faster produces more cycles each second: the **frequency increases**, and the **period shortens**, in direct proportion to the rotation rate. Double the rotation rate ⇒ double the frequency, half the period.
- **Amplitude.** Spinning faster also makes the flux change *more rapidly*, and by $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$ a faster rate of change of flux gives a larger induced emf. So the **peak voltage (amplitude) also increases**, in direct proportion to the rotation rate. Double the rotation rate ⇒ double the peak voltage (and double the peak-to-peak voltage).

Increasing the rotation speed therefore raises **both the frequency and the amplitude together** — a point often missed.



doubling the rotation speed doubles the frequency AND the peak voltage

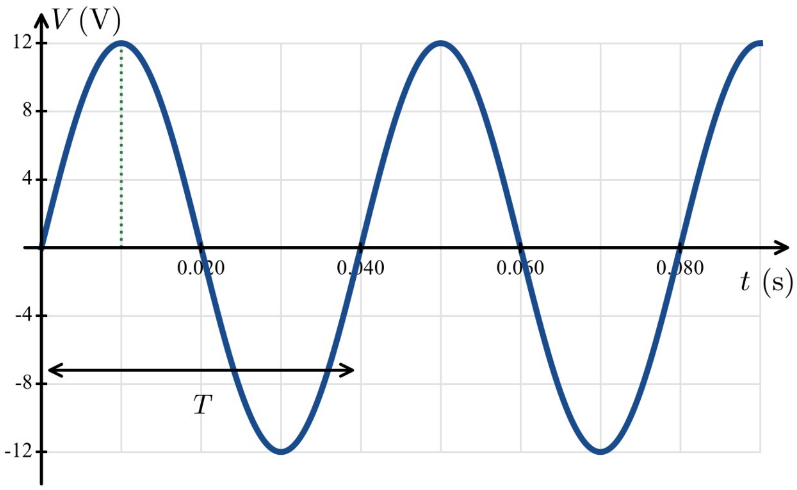
By contrast, the amplitude can be increased *without* changing the frequency: using **more turns** N on the loop, a **stronger field** B, or a **larger loop area** each raise the peak emf but leave the rotation rate — and hence the frequency — unchanged. The frequency of the output is set by the rotation rate alone.

A note on rms values. A single number is often quoted for an AC supply (for example, the "230 V" of household mains). That figure is neither the peak nor the peak-to-peak value: it is the **root-mean-square (rms)** value, the steady DC voltage that would deliver the same average power in a resistor. The rms value, and how it relates to the peak, is developed in the next chapter (6A); throughout this subtopic AC voltages and currents are described only by their **peak** and **peak-to-peak** values, read directly from the waveform.

Worked examples

Example 1 — scaffolded

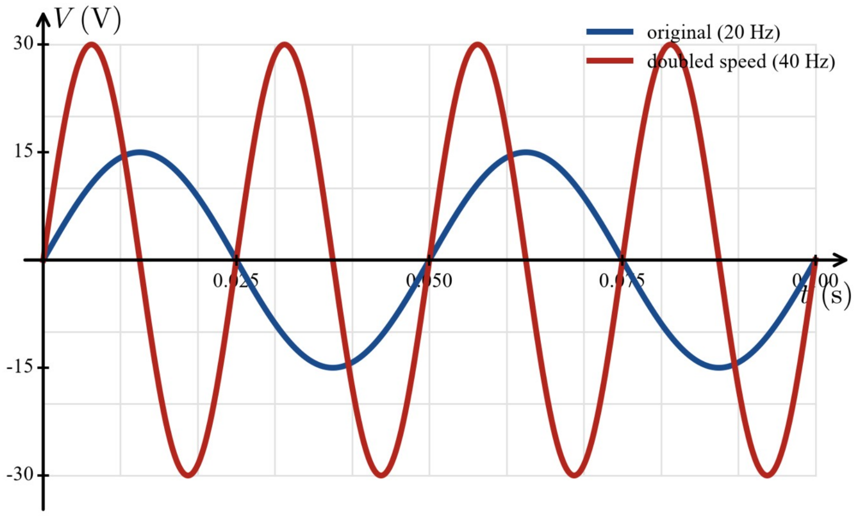
The graph shows the sinusoidal voltage produced by a rotating loop, connected across a resistor of resistance $6.0\,\Omega$. Determine the frequency and the peak-to-peak voltage of the supply, and the peak and peak-to-peak current in the resistor.



Working	Comment
From the graph, one complete cycle takes $T = 0.040\,\text{s}$, and a crest reaches $V_{\text{peak}} = 12\,\text{V}$.	Read the period (one full cycle) and the peak from the graph.
$f = \dfrac{1}{T} = \dfrac{1}{0.040} = 25\,\text{Hz}$.	Frequency is the number of cycles each second.
$V_{\text{p-p}} = 2V_{\text{peak}} = 2 \times 12 = 24\,\text{V}$.	Peak-to-peak is crest to trough = twice the peak.
$I_{\text{peak}} = \dfrac{V_{\text{peak}}}{R} = \dfrac{12}{6.0} = 2.0\,\text{A}$.	Ohm's law at the instant of peak voltage (resistive load).
$I_{\text{p-p}} = 2I_{\text{peak}} = 2 \times 2.0 = 4.0\,\text{A}$.	Current peak-to-peak.

Example 2 — scaffolded

A single loop rotating in a constant magnetic field produces a sinusoidal output of frequency $20\,\text{Hz}$ and peak voltage $15\,\text{V}$. The rotation rate of the loop is then **doubled**. Determine the new frequency, the new period, and the new peak and peak-to-peak voltage, and state what has happened to the frequency and the amplitude.



Working	Comment
Original output: $f_0 = 20\,\text{Hz}$, $V_{\text{peak},0} = 15\,\text{V}$, so $T_0 = \dfrac{1}{20} = 0.050\,\text{s}$.	Start from the given output.
Doubling the rotation rate doubles the cycles produced each second: $f = 2 \times 20 = 40\,\text{Hz}$, so $T = \dfrac{1}{40} = 0.025\,\text{s}$.	Frequency is set by how fast the loop turns; one cycle per revolution.

Working	Comment
The flux now changes twice as fast, so by $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$ the peak emf doubles: $V_{\text{peak}} = 2 \times 15 = 30\ \text{V}$.	Faster rotation ⇒ larger rate of change of flux ⇒ larger peak.
$V_{\text{p-p}} = 2 \times 30 = 60\ \text{V}$.	Peak-to-peak follows the new peak.
Both the frequency and the amplitude have doubled.	The signature result: rotation speed changes **both**.

Example 3 — unscaffolded

A single loop completes one rotation every $0.080\ \text{s}$ in a constant magnetic field and produces a peak voltage of $9.0\ \text{V}$ across a $3.0\ \Omega$ resistor. Determine the frequency of the output and the peak and peak-to-peak current. The loop is then rewound with **three times as many turns** and rotated at the same rate; determine the new peak voltage and the frequency.

One rotation gives one cycle, so the period is $T = 0.080\ \text{s}$ and

$$f = \frac{1}{T} = \frac{1}{0.080} = 12.5\ \text{Hz}.$$

Across the resistor, Ohm's law at the peak gives

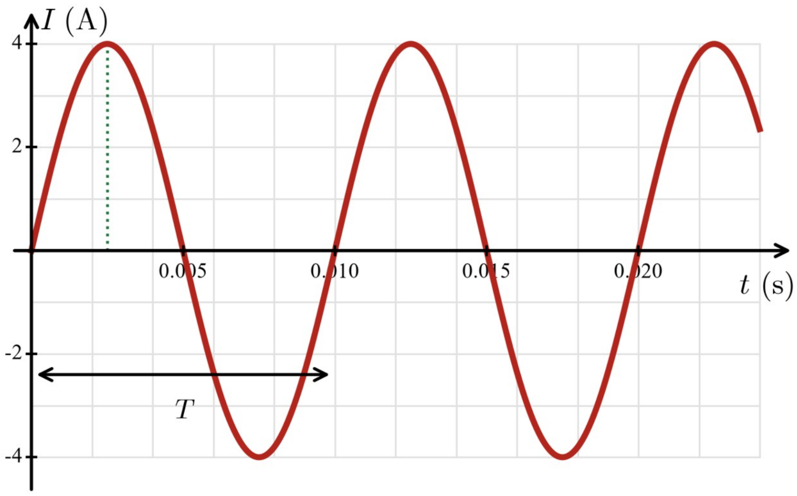
$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R} = \frac{9.0}{3.0} = 3.0\ \text{A}\,,\ I_{\text{p-p}} = 2\,I_{\text{peak}} = 6.0\ \text{A}.$$

Tripling the number of turns triples the peak emf (the amplitude is proportional to N), while the rotation rate — and therefore the frequency — is unchanged: $V_{\text{peak}} = 3 \times 9.0 = 27\ \text{V}$ at the same $12.5\ \text{Hz}$.

∴ the output is $12.5\ \text{Hz}$ with $I_{\text{peak}} = 3.0\ \text{A}$ and $I_{\text{p-p}} = 6.0\ \text{A}$; with three times the turns the peak voltage rises to $27\ \text{V}$ while the frequency stays at $12.5\ \text{Hz}$.

Example 4 — unscaffolded

The graph shows the alternating current supplied to a load by an alternator. Determine the frequency and the peak-to-peak current. The rotation rate of the loop is then **halved**; determine the new frequency, period, and peak and peak-to-peak current.



From the graph one complete cycle takes $T = 0.010\ \text{s}$, and a crest reaches $I_{\text{peak}} = 4.0\ \text{A}$, so

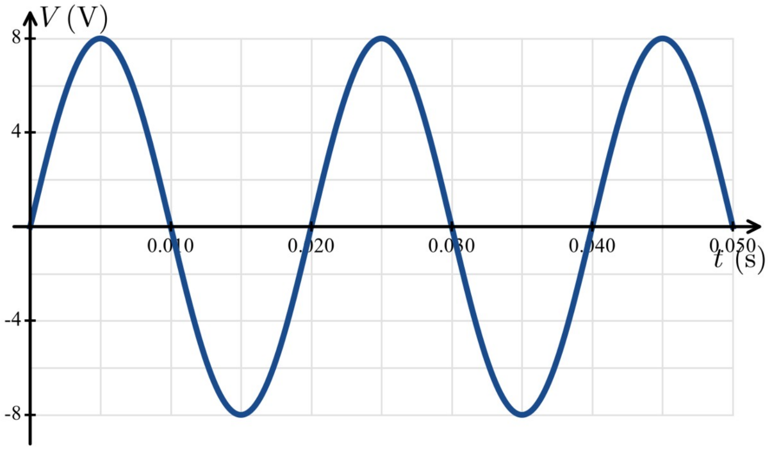
$$f = \frac{1}{T} = \frac{1}{0.010} = 100\ \text{Hz}\,,\ I_{\text{p-p}} = 2 \times 4.0 = 8.0\ \text{A}.$$

Halving the rotation rate halves both the frequency and the peak: $f' = 50\,\text{Hz}$, $T' = \frac{1}{50} = 0.020\,\text{s}$, $I'_{\text{peak}} = 1/2 \times 4.0 = 2.0\,\text{A}$ and $I'_{\text{p-p}} = 2 \times 2.0 = 4.0\,\text{A}$.

$\therefore$ the original output is $100\,\text{Hz}$ with $I_{\text{p-p}} = 8.0\,\text{A}$; at half the rotation rate it becomes $50\,\text{Hz}$, period $0.020\,\text{s}$, with $I_{\text{peak}} = 2.0\,\text{A}$ and $I_{\text{p-p}} = 4.0\,\text{A}$.

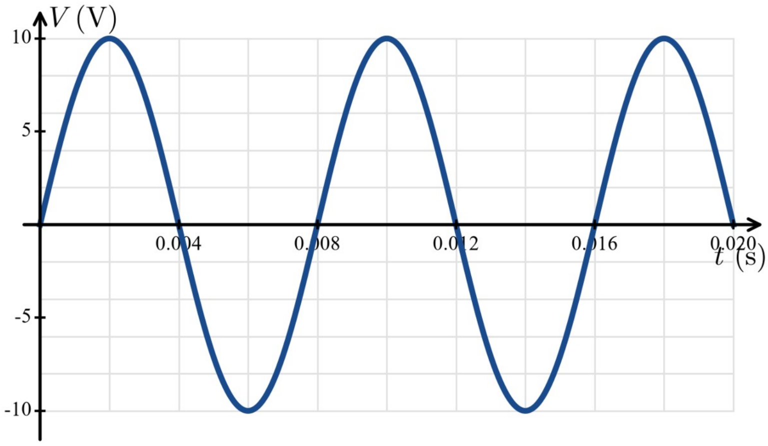
Practice questions

Question 1 The graph shows the voltage output of a loop rotating uniformly in a constant magnetic field.



(a) State the period of the voltage.
(b) Calculate the frequency.
(c) State the peak voltage and calculate the peak-to-peak voltage.

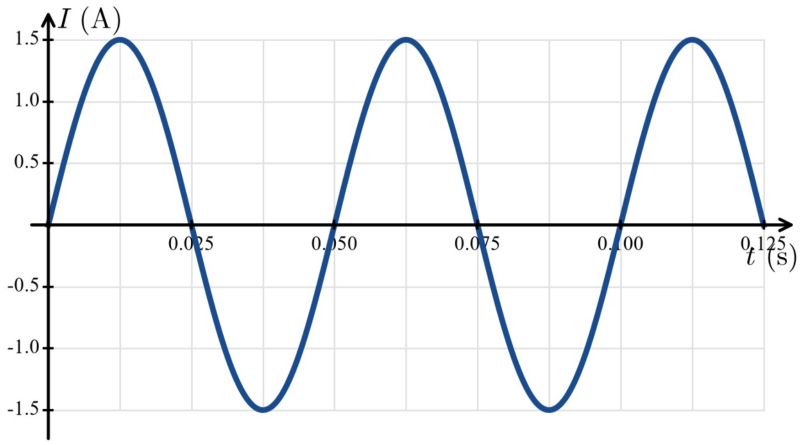
Question 2 The graph shows a sinusoidal voltage that is applied across a resistor of resistance $5.0\,\Omega$.



(a) Calculate the frequency of the voltage.
(b) Calculate the peak-to-peak voltage.
(c) Calculate the peak current and the peak-to-peak current in the resistor.

Question 3 A single loop rotates 30 times each second in a constant magnetic field, producing a peak voltage of $6.0\,\text{V}$. (a) Calculate the period of the output voltage. (b) The rotation rate is doubled. Calculate the new frequency and the new period. (c) Calculate the new peak voltage and the new peak-to-peak voltage, and state what has happened to both the frequency and the amplitude.

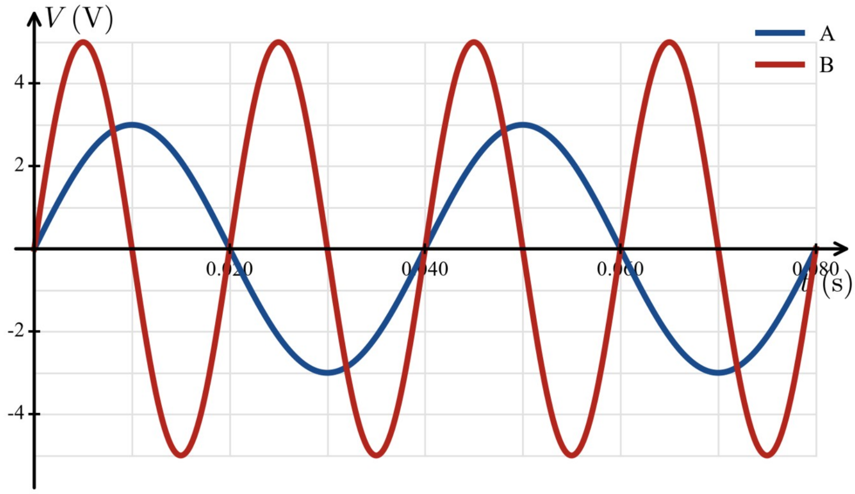
Question 4 The graph shows the alternating current delivered to a load.



(a) State the period of the current.
(b) Calculate the frequency.
(c) State the peak current and calculate the peak-to-peak current.

Question 5 A loop rotating in a constant magnetic field produces a sinusoidal output with a peak voltage of $7.0\,\mathrm{V}$ at a frequency of $40\,\mathrm{Hz}$. (a) The number of turns on the loop is doubled, with the rotation rate unchanged. State the new peak voltage and the new frequency. (b) Instead, starting again from the original output, the strength of the magnetic field is halved. State the new peak voltage. (c) Explain why changing the number of turns or the field strength alters the amplitude of the output but not its frequency.

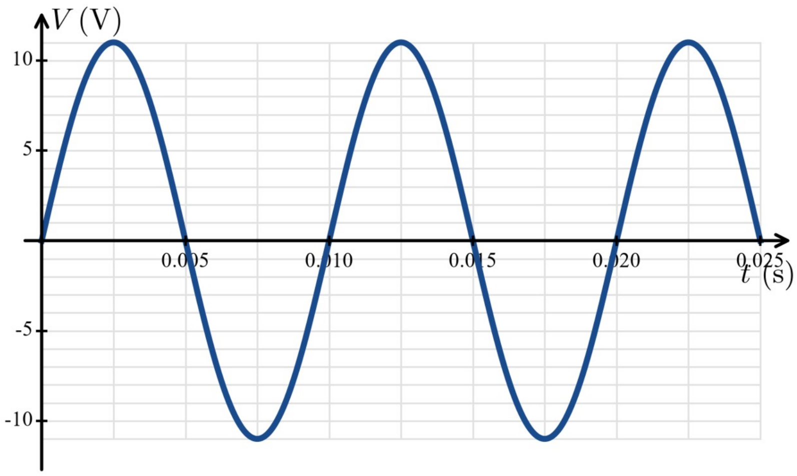
Question 6 The graph shows two AC voltages, **A** and **B**, produced by two identical loops rotating in the same constant magnetic field.



(a) (Circle your answer.) The voltage with the higher frequency is: **A** / **B**. Justify your choice. No calculations are required.
(b) State the peak-to-peak voltage of each waveform.
(c) State which loop is rotating faster, and explain how the graph shows this in *two* ways.

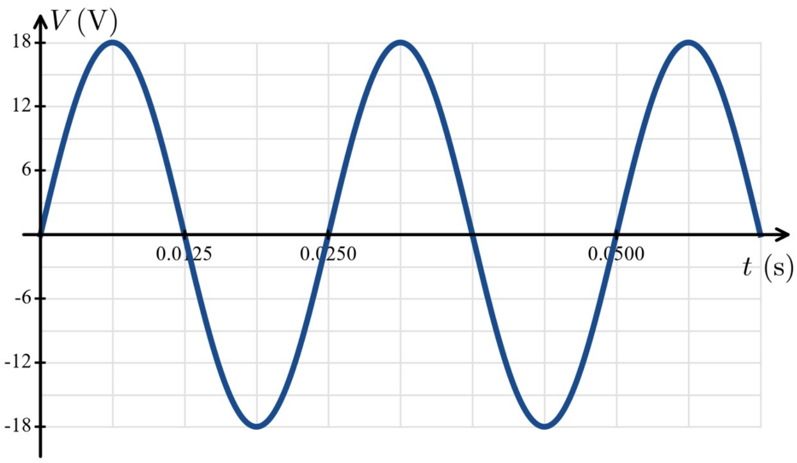
Question 7 An alternator produces a sinusoidal voltage of frequency $80\,\mathrm{Hz}$ and peak voltage $13\,\mathrm{V}$. Calculate the period of the output and its peak-to-peak voltage.

Question 8 The graph shows the voltage produced by a rotating loop.



Determine the period, the frequency, the peak voltage and the peak-to-peak voltage of the output.

Question 9 The sinusoidal voltage in the graph is applied across a resistor of resistance $4.0\,\Omega$.



(a) Calculate the peak current and the peak-to-peak current in the resistor.

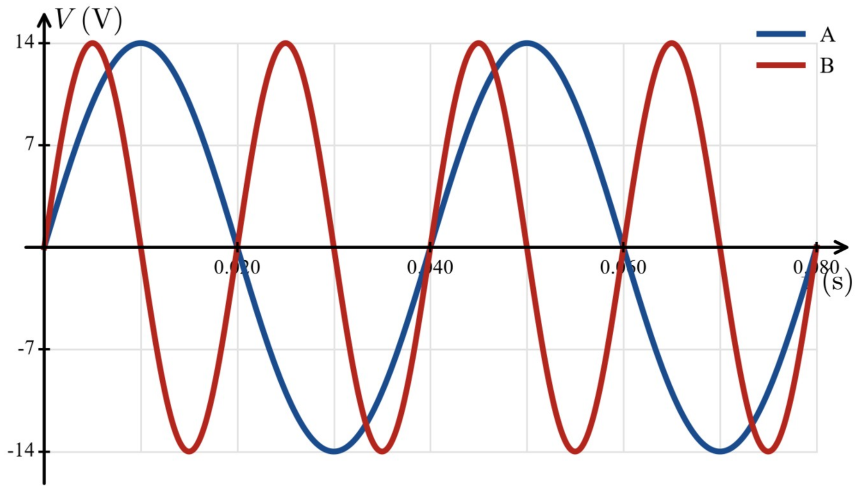
(b) On the same time axis, sketch the current–time graph for one complete cycle, marking the peak current and the period.

Question 10 A loop rotating 15 times each second produces a peak voltage of $5.0\,\mathrm{V}$. The rotation rate is tripled. Calculate the new frequency, the new period, and the new peak and peak-to-peak voltage.

Question 11 A sinusoidal voltage of peak value $17\,\mathrm{V}$ is applied across a resistor of resistance $6.0\,\Omega$. (a) Show that the peak current in the resistor is $2.83\,\mathrm{A}$. (b) Hence state the peak-to-peak current. (c) The rotation rate of the source is increased by $50\,\%$. Calculate the new peak current.

Question 12 Explain, with reference to the rate of change of magnetic flux through the loop, why increasing the rotation speed of a loop in a constant magnetic field increases **both** the frequency and the peak voltage of the AC output.

Question 13 The graph shows two AC voltages, **A** and **B**.



Compare the two waveforms with reference to period, frequency, amplitude and peak-to-peak voltage. State clearly which quantities are the same and which differ, giving values.

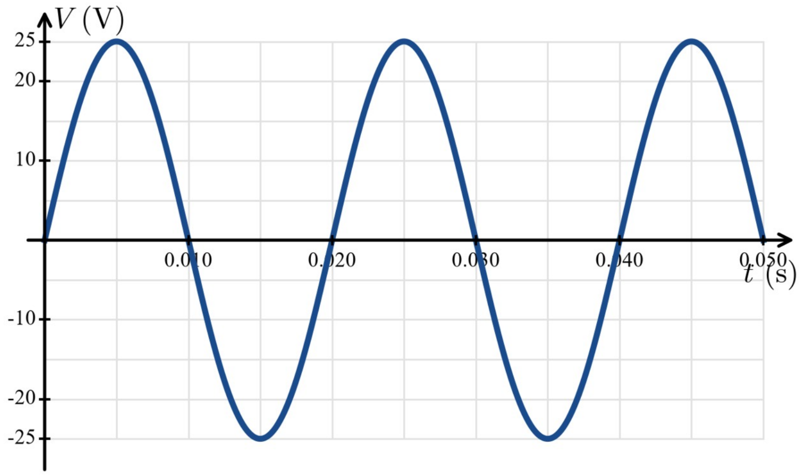
Question 14 A generator's loop completes one rotation in $0.0135\,\mathrm{s}$, producing a peak voltage of $9.0\,\mathrm{V}$. (a) Calculate the frequency of the output. Give your answer to three significant figures. (b) State the peak-to-peak voltage.

Question 15 A household power outlet is labelled "$230\,\mathrm{V}$". A student assumes that this is the peak voltage of the mains supply. (a) State whether $230\,\mathrm{V}$ is the peak, the peak-to-peak, or the rms value of the mains voltage. (b) Hence state whether the peak voltage of the mains is greater than, equal to, or less than $230\,\mathrm{V}$. (c) Explain why the peak and peak-to-peak values, rather than this single quoted value, are used to describe the shape of the AC waveform in this chapter.

Question 16 Sketch a voltage–time graph of a sinusoidal AC supply that has a peak voltage of $16\,\mathrm{V}$ and a frequency of $25\,\mathrm{Hz}$, showing two complete cycles. On your sketch mark the period T, the peak voltage and the peak-to-peak voltage, and state the peak-to-peak value.

Question 17 A loop rotating 30 times each second produces a peak voltage of $1.5\,\mathrm{V}$. The loop is rewound with three times as many turns and then rotated twice as fast. Calculate the new frequency and the new peak and peak-to-peak voltage.

Question 18 The graph shows an alternating voltage applied across a resistor of resistance $10\,\Omega$.



(a) State the peak-to-peak voltage and hence the peak voltage.
(b) Calculate the peak current and the peak-to-peak current in the resistor.
(c) Calculate the frequency of the supply.

Question 19 A student says: "Speeding up the loop in a generator makes the output voltage change more often, but the size of the peak voltage stays the same." Explain what is wrong with this statement, and describe correctly what happens to the peak voltage when the loop is rotated faster.

Question 20 Two alternators use identical magnetic fields. Alternator X has a loop that rotates 40 times each second and produces a peak voltage of 16 V. Alternator Y has a loop with **half as many turns** as X, rotated **twice as fast** as X. (a) Calculate the frequency of Y's output. (b) Determine the peak voltage of Y's output, and compare it with that of X. (c) Explain, in terms of frequency and amplitude, how the two outputs differ and how they are the same.

Answers

Question 1 (a) $0.020\,\mathrm{s}$; (b) $50\,\mathrm{Hz}$; (c) peak $8.0\,\mathrm{V}$, peak-to-peak $16\,\mathrm{V}$. **Question 2** (a) $125\,\mathrm{Hz}$; (b) $20\,\mathrm{V}$; (c) $I_{\text{peak}}=2.0\,\mathrm{A}$, $I_{\text{p-p}}=4.0\,\mathrm{A}$. **Question 3** (a) $0.0333\,\mathrm{s}$; (b) $60\,\mathrm{Hz}$, $0.0167\,\mathrm{s}$; (c) peak $12\,\mathrm{V}$, peak-to-peak $24\,\mathrm{V}$ — both the frequency and the amplitude have doubled. **Question 4** (a) $0.050\,\mathrm{s}$; (b) $20\,\mathrm{Hz}$; (c) peak $1.5\,\mathrm{A}$, peak-to-peak $3.0\,\mathrm{A}$. **Question 5** (a) $14\,\mathrm{V}$, still $40\,\mathrm{Hz}$; (b) $3.5\,\mathrm{V}$; (c) the amplitude depends on N and B; the frequency is set by the rotation rate alone. **Question 6** (a) **B** (its period is shorter, so it fits more cycles into the same time); (b) A: $6.0\,\mathrm{V}$, B: $10\,\mathrm{V}$; (c) B's loop rotates faster. **Question 7** $0.0125\,\mathrm{s}$ ($12.5\,\mathrm{ms}$); $26\,\mathrm{V}$. **Question 8** $T=0.010\,\mathrm{s}$; $f=100\,\mathrm{Hz}$; peak $11\,\mathrm{V}$; peak-to-peak $22\,\mathrm{V}$. **Question 9** (a) $I_{\text{peak}}=4.5\,\mathrm{A}$, $I_{\text{p-p}}=9.0\,\mathrm{A}$; (b) a sine curve in phase with the voltage, peak $4.5\,\mathrm{A}$, period $0.025\,\mathrm{s}$. **Question 10** $45\,\mathrm{Hz}$; $0.0222\,\mathrm{s}$; peak $15\,\mathrm{V}$, peak-to-peak $30\,\mathrm{V}$. **Question 11** (a) $2.83\,\mathrm{A}$; (b) $5.67\,\mathrm{A}$; (c) $4.25\,\mathrm{A}$. **Question 12** Faster rotation gives more cycles per second (higher f) **and** a faster rate of change of flux, so a larger peak emf (larger amplitude). **Question 13** Same amplitude ($V_{\text{peak}}=14\,\mathrm{V}$) and peak-to-peak ($28\,\mathrm{V}$); B has half the period ($0.020\,\mathrm{s}$ vs $0.040\,\mathrm{s}$) and double the frequency ($50\,\mathrm{Hz}$ vs $25\,\mathrm{Hz}$). **Question 14** (a) $74.1\,\mathrm{Hz}$; (b) $18\,\mathrm{V}$. **Question 15** (a) the rms value; (b) greater than $230\,\mathrm{V}$; (c) peak and peak-to-peak are read directly from the waveform and fix its actual maximum swing. **Question 16** $T=0.040\,\mathrm{s}$; peak-to-peak $32\,\mathrm{V}$. **Question 17** $60\,\mathrm{Hz}$; peak $9.0\,\mathrm{V}$, peak-to-peak $18\,\mathrm{V}$. **Question 18** (a) peak-to-peak $50\,\mathrm{V}$, peak $25\,\mathrm{V}$; (b) $I_{\text{peak}}=2.5\,\mathrm{A}$, $I_{\text{p-p}}=5.0\,\mathrm{A}$; (c) $50\,\mathrm{Hz}$. **Question 19** Incorrect: faster rotation also increases the peak voltage, because the flux changes more rapidly; both frequency and amplitude rise. **Question 20** (a) $80\,\mathrm{Hz}$; (b) $16\,\mathrm{V}$ — the same peak as X; (c) Y has double the frequency but the same amplitude.

Fully-worked solutions

Question 1 From the graph, one complete cycle takes $T=0.020\,\mathrm{s}$ and a crest reaches $8.0\,\mathrm{V}$. (a) $T=0.020\,\mathrm{s}$. (b) $f=\dfrac{1}{T}=\dfrac{1}{0.020}=50\,\mathrm{Hz}$. (c) The peak voltage is $V_{\text{peak}}=8.0\,\mathrm{V}$, so $V_{\text{p-p}}=2V_{\text{peak}}=2\times 8.0=16\,\mathrm{V}$.

Question 2 From the graph, $T=0.008\,\mathrm{s}$ and $V_{\text{peak}}=10\,\mathrm{V}$, across $R=5.0\,\Omega$. (a) $f=\dfrac{1}{0.008}=125\,\mathrm{Hz}$. (b) $V_{\text{p-p}}=2\times 10=20\,\mathrm{V}$. (c) $I_{\text{peak}}=\dfrac{V_{\text{peak}}}{R}=\dfrac{10}{5.0}=2.0\,\mathrm{A}$, and $I_{\text{p-p}}=2\times 2.0=4.0\,\mathrm{A}$.

Question 3 The loop turns 30 times per second, so $f_0=30\,\mathrm{Hz}$ and $V_{\text{peak},0}=6.0\,\mathrm{V}$. (a) $T_0=\dfrac{1}{f_0}=\dfrac{1}{30}=0.0333\,\mathrm{s}$. (b) Doubling the rotation rate doubles the frequency: $f=2\times 30=60\,\mathrm{Hz}$, so $T=\dfrac{1}{60}=0.0167\,\mathrm{s}$. (c) The flux changes twice as fast, so the peak doubles: $V_{\text{peak}}=2\times 6.0=12\,\mathrm{V}$ and $V_{\text{p-p}}=2\times 12=24\,\mathrm{V}$. Both the frequency and the amplitude have doubled.

Question 4 From the graph, one complete cycle takes $T=0.050\,\mathrm{s}$ and a crest reaches $1.5\,\mathrm{A}$. (a) $T=0.050\,\mathrm{s}$. (b) $f=\dfrac{1}{0.050}=20\,\mathrm{Hz}$. (c) The peak current is $I_{\text{peak}}=1.5\,\mathrm{A}$, so $I_{\text{p-p}}=2\times 1.5=3.0\,\mathrm{A}$.

Question 5 Original output: $V_{\text{peak}}=7.0\,\mathrm{V}$ at $40\,\mathrm{Hz}$. (a) The peak emf is proportional to the number of turns N, so doubling N doubles the peak: $V_{\text{peak}}=2\times 7.0=14\,\mathrm{V}$. The rotation rate is unchanged, so the frequency stays at $40\,\mathrm{Hz}$. (b) The peak emf is proportional to the field strength B, so halving B halves the peak: $V_{\text{peak}}=1/2\times 7.0=3.5\,\mathrm{V}$. (c) The size of the peak emf is set by $\varepsilon=-N\dfrac{\Delta\Phi_B}{\Delta t}$: increasing N, B (or the loop area) increases the flux and its rate of change, and hence the amplitude. None of these changes how fast the loop turns, and it is the rotation rate alone — one cycle per revolution — that fixes the frequency. So the amplitude changes but the frequency does not.

Question 6 (a) **B.** Its period is shorter (each cycle occupies less time on the graph), so it completes more cycles each second and therefore has the higher frequency. (b) Curve A has $V_{\text{peak}}=3.0\,\mathrm{V}$, so $V_{\text{p-p}}=6.0\,\mathrm{V}$; curve B has $V_{\text{peak}}=5.0\,\mathrm{V}$, so $V_{\text{p-p}}=10\,\mathrm{V}$. (c) The loops are identical and share the same field, so the only difference is how fast they turn: **B's loop is rotating faster.** The graph shows this in two ways — B has the shorter period (higher frequency) *and* the larger amplitude, and faster rotation increases both of these together.

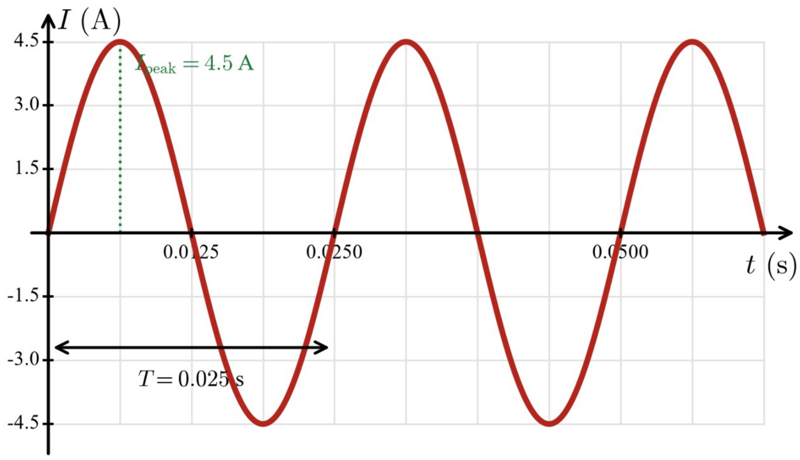
Question 7 $f=80\,\text{Hz}$, $V_{\text{peak}}=13\,\text{V}$.

$$T=\frac{1}{f}=\frac{1}{80}=0.0125\,\text{s}=12.5\,\text{ms}\,,V_{\text{p-p}}=2\times13=26\,\text{V}\,.$$

Question 8 From the graph, one complete cycle takes $T=0.010\,\text{s}$ and a crest reaches $11\,\text{V}$.

$$f=\frac{1}{T}=\frac{1}{0.010}=100\,\text{Hz}\,,V_{\text{peak}}=11\,\text{V}\,,V_{\text{p-p}}=2\times11=22\,\text{V}\,.$$

Question 9 From the graph, $V_{\text{peak}}=18\,\text{V}$ and $T=0.025\,\text{s}$, across $R=4.0\,\Omega$. (a) $I_{\text{peak}}=\frac{V_{\text{peak}}}{R}=\frac{18}{4.0}=4.5\,\text{A}$, and $I_{\text{p-p}}=2\times4.5=9.0\,\text{A}$. (b) In a resistor the current is in phase with the voltage, so the current–time graph is a sine curve of the same period $0.025\,\text{s}$, peaking at $4.5\,\text{A}$ at the same instants as the voltage:



Question 10 $f_0=15\,\text{Hz}$, $V_{\text{peak},0}=5.0\,\text{V}$; rotation rate tripled.

$$f=3\times15=45\,\text{Hz}\,,T=\frac{1}{45}=0.0222\,\text{s}\,.$$

The peak is proportional to the rotation rate, so it triples: $V_{\text{peak}}=3\times5.0=15\,\text{V}$ and $V_{\text{p-p}}=2\times15=30\,\text{V}$.

Question 11 $V_{\text{peak}}=17\,\text{V}$, $R=6.0\,\Omega$. (a) $I_{\text{peak}}=\frac{V_{\text{peak}}}{R}=\frac{17}{6.0}=2.833\,\text{A}=2.83\,\text{A}$ (to three significant figures), as required. (b) $I_{\text{p-p}}=2I_{\text{peak}}=2\times2.833=5.67\,\text{A}$. (c) Increasing the rotation rate by $50\,\%$ multiplies the peak current by 1.5 (the load is unchanged, so $I_{\text{peak}}\propto V_{\text{peak}}\propto$ rotation rate): $I'_{\text{peak}}=1.5\times2.833=4.25\,\text{A}$.

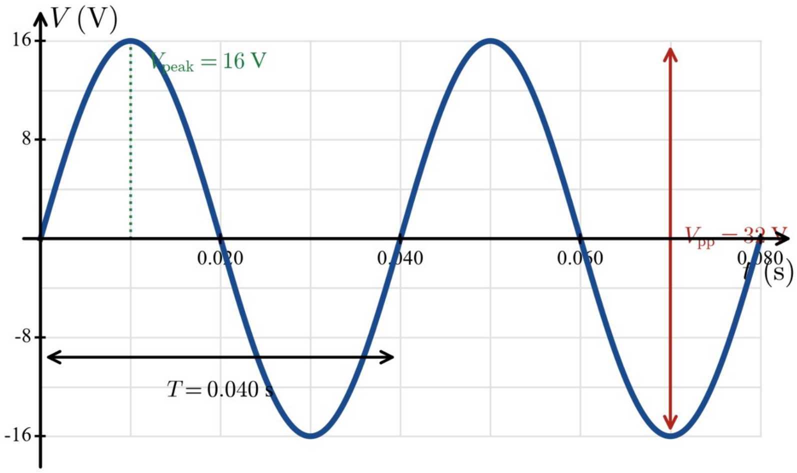
Question 12 As the loop rotates, the flux through it changes, and by Faraday's law $\varepsilon=-N\frac{\Delta\Phi_B}{\Delta t}$ the induced emf at any instant equals the rate of change of flux (times the number of turns). Rotating the loop faster has two consequences. First, the loop completes a revolution — and hence one full cycle of the output — in less time, so more cycles occur each second: the **frequency increases** (and the period shortens). Second, the same change in flux now happens in a shorter time, so the flux is changing more rapidly at every stage of the rotation; a greater $\frac{\Delta\Phi_B}{\Delta t}$ means a greater induced emf, so the **peak voltage (amplitude) increases** as well. Because both the number of cycles per second and the rate of change of flux scale with the rotation rate, increasing the rotation speed raises both the frequency and the peak voltage together.

Question 13 Reading each curve from the graph: both reach the same peak, $V_{\text{peak}}=14\,\text{V}$, so both have the same amplitude and the same peak-to-peak voltage, $V_{\text{p-p}}=2\times14=28\,\text{V}$. They differ in timing: A has period $T_A=0.040\,\text{s}$, so $f_A = \dfrac{1}{0.040} = 25\,\text{Hz}$, while B has period $T_B = 0.020\,\text{s}$, so $f_B = \dfrac{1}{0.020} = 50\,\text{Hz}$. B therefore has **half the period and double the frequency** of A, but the **same amplitude and peak-to-peak voltage**.

Question 14 One rotation gives one cycle, so $T = 0.0135\,\text{s}$. (a) $f = \dfrac{1}{T} = \dfrac{1}{0.0135} = 74.07\ldots = 74.1\,\text{Hz}$ (to three significant figures). (b) $V_{\text{p-p}} = 2V_{\text{peak}} = 2 \times 9.0 = 18\,\text{V}$.

Question 15 (a) The quoted $230\,\text{V}$ is the **rms value** of the mains voltage — the steady DC voltage that would deliver the same average power. (b) The peak voltage is **greater than** $230\,\text{V}$: the waveform rises above its DC-equivalent (rms) value to reach its crest at the top of each cycle. (The exact peak-to-rms relationship is developed in 6A.) (c) The peak and peak-to-peak values are read straight off the waveform and give its actual maximum swing above and below zero, so they describe the *shape* of the sinusoid directly. The rms value is a single averaged number that hides the shape; it is introduced in the next chapter to compare AC with DC on the basis of power.

Question 16 With $f = 25\,\text{Hz}$ the period is $T = \dfrac{1}{25} = 0.040\,\text{s}$, so two complete cycles span $0.080\,\text{s}$. The curve is a sine wave reaching $+16\,\text{V}$ at each crest and $-16\,\text{V}$ at each trough, giving $V_{\text{p-p}} = 2 \times 16 = 32\,\text{V}$:



Question 17 $f_0 = 30\,\text{Hz}$, $V_{\text{peak},0} = 1.5\,\text{V}$. Rotating twice as fast doubles the frequency: $f = 2 \times 30 = 60\,\text{Hz}$. The peak voltage is increased by *both* changes — three times the turns ($\times 3$) and twice the rotation rate ($\times 2$) — so

$$V_{\text{peak}} = 1.5 \times 3 \times 2 = 9.0\,\text{V},\ V_{\text{p-p}} = 2 \times 9.0 = 18\,\text{V}.$$

Question 18 From the graph the waveform runs from $-25\,\text{V}$ to $+25\,\text{V}$ with period $T = 0.020\,\text{s}$, across $R = 10\,\Omega$. (a) The peak-to-peak voltage is $V_{\text{p-p}} = 50\,\text{V}$, so the peak voltage is $V_{\text{peak}} = \dfrac{V_{\text{p-p}}}{2} = 25\,\text{V}$. (b) $I_{\text{peak}} = \dfrac{V_{\text{peak}}}{R} = \dfrac{25}{10} = 2.5\,\text{A}$, and $I_{\text{p-p}} = 2 \times 2.5 = 5.0\,\text{A}$. (c) $f = \dfrac{1}{T} = \dfrac{1}{0.020} = 50\,\text{Hz}$.

Question 19 The statement is only half right. Speeding up the loop does make the voltage alternate more often — the frequency increases and the period shortens — but it is wrong to say the peak voltage is unchanged. By $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$, rotating faster makes the flux through the loop change more rapidly, so the induced emf reaches a **larger** peak. The peak voltage (the amplitude) increases in proportion to the rotation rate, just as the frequency does: faster rotation raises *both* the frequency and the peak voltage.

Question 20 Alternator X: rotates 40 times per second, $V_{\text{peak,X}} = 16\,\text{V}$. Alternator Y: half the turns of X, rotated twice as fast, same field. (a) Rotating twice as fast doubles the frequency: $f_Y = 2 \times 40 = 80\,\text{Hz}$ (X itself outputs $40\,\text{Hz}$). (b) The peak voltage of Y is scaled from X's by two competing factors — twice the rotation rate ($\times 2$) but half the number of turns ($\times 1/2$):

$$V_{\text{peak,Y}} = 16 \times 2 \times 1/2 = 16\,\mathrm{V}\,.$$

So Y has the **same peak voltage as X**, $16\,\mathrm{V}$. (c) The two outputs **differ in frequency** — Y alternates at $80\,\mathrm{Hz}$, twice as often as X's $40\,\mathrm{Hz}$ — but have the **same amplitude**, because Y's faster rotation (which alone would double the peak) is exactly cancelled by its halved number of turns (which alone would halve the peak). This shows that the frequency is fixed by the rotation rate, while the amplitude depends on the rotation rate *and* the number of turns together.