Physics Units 3 & 4

Chapter 4 — Electric and magnetic fields

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

Significant effort has been made to ensure this content is legal. Please send any areas of concern to admin@vwx.au with appropriate document/legal references so errors can be verified and changes be made.

Contents

Theory

The field model. Electric charges exert forces on one another even when they are not touching, and even across an empty vacuum. Rather than treat this as mysterious "action at a distance", physicists picture a charge as altering the space around it: it sets up an **electric field** everywhere in the surrounding space. A second charge placed anywhere in that space then experiences a force *from the field at its own location*. The field is the intermediary — one charge makes the field, and the field pushes or pulls on the other. Gravitation, magnetism and electricity are all described in this same way, using a **field model** of a source that fills the space around it with a field to which other objects respond.

Electric field strength. The **electric field** $\vec{E}$ at a point is defined as the electric force per unit charge on a small, stationary **positive** test charge q placed there:

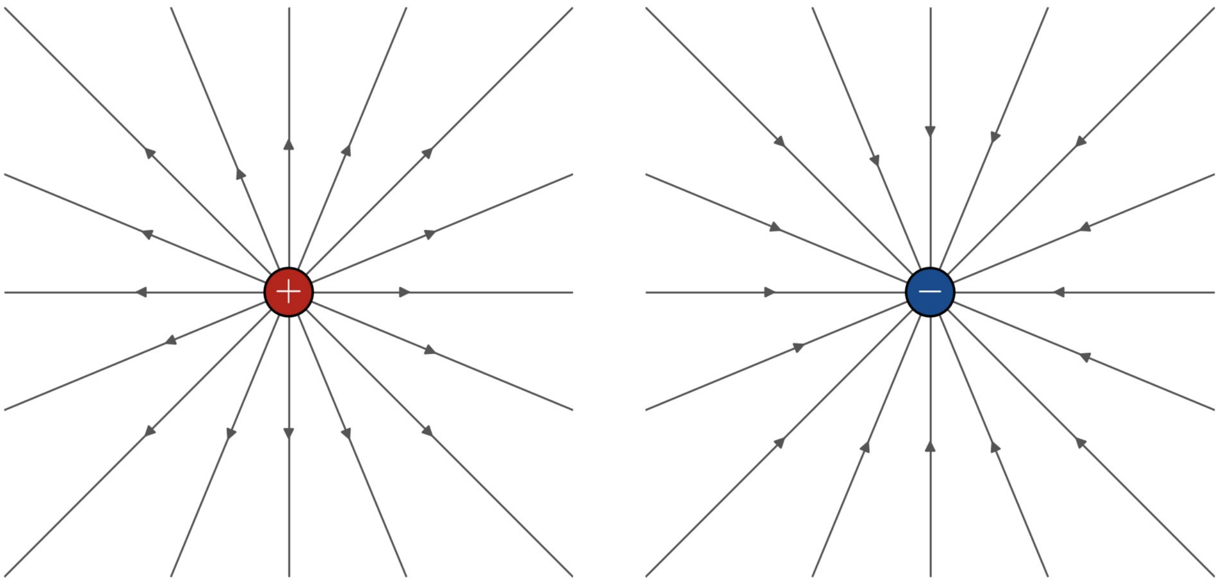
$$E = \frac{F}{q}.$$

It is a vector: its direction is the direction of the force on a *positive* charge, and its magnitude is measured in newtons per coulomb ($\mathrm{N\ C^{-1}}$). Rearranging gives the force on any charge q sitting in a known field, $F = qE$, developed below. A positive charge feels a force **along** the field; a negative charge feels a force **opposite** to the field.

The field of a point charge. A single point charge Q produces a field whose magnitude at a distance r from the charge is given by the **inverse-square law**

$$E = \frac{kQ}{r^2}, k = 8.99 \times 10^9\ \mathrm{N\ m^2\ C^{-2}}.$$

The field points **radially outward** from a positive charge and **radially inward** toward a negative charge. Because the field depends on $\frac{1}{r^2}$, moving twice as far from the charge reduces the field to one **quarter** of its value. For example, at $0.20\ \mathrm{m}$ from a $+2.0 \times 10^{-6}\ \mathrm{C}$ charge the field is about $4.5 \times 10^5\ \mathrm{N\ C^{-1}}$; at $0.40\ \mathrm{m}$ — twice as far — it has fallen to about $1.1 \times 10^5\ \mathrm{N\ C^{-1}}$, one-quarter as large.



isolated positive charge: field points radially outward

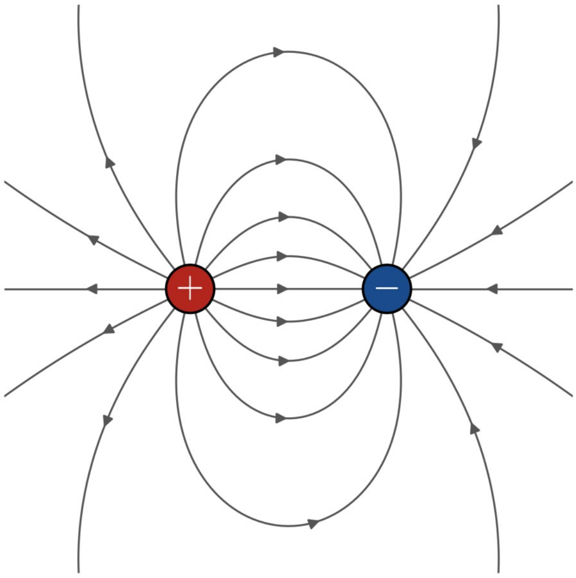
isolated negative charge: field points radially inward

Field lines, monopoles and dipoles. An electric field is mapped with **field lines** that follow these rules:

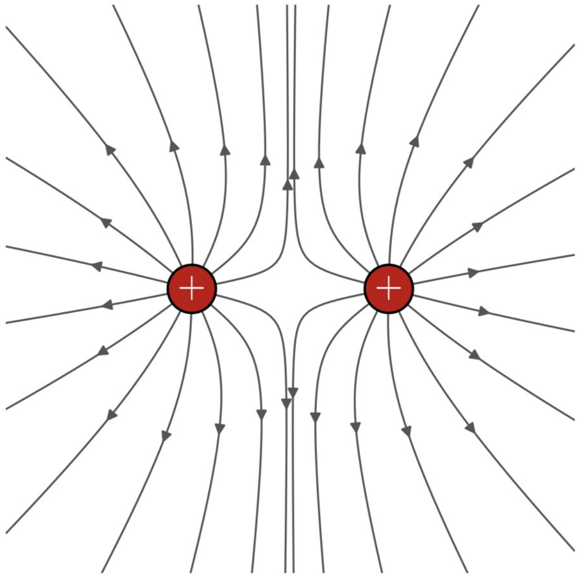
- a line points in the direction of the field — the direction of the force on a positive test charge;
- lines **begin on positive charges and end on negative charges** (or run to or from infinity);
- lines are drawn **closer together where the field is stronger** and further apart where it is weaker;

- field lines **never cross**, because the field has only one direction at each point.

A single isolated charge is an electric **monopole** — a positive charge is a source from which lines spread out, and a negative charge is a sink into which lines converge. Two equal and opposite charges placed near each other form an electric **dipole**, and the field lines run from the positive charge across to the negative charge. Two *like* charges repel, and midway between two equal like charges the two fields exactly cancel, giving a **null point** where $E = 0$.



dipole: lines run from the + charge to the - charge



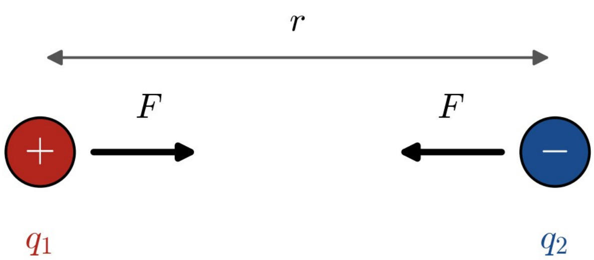
two like charges: field is zero at the midpoint (null point)

Electric charge, gravitational mass and magnetism differ in an important way here. Electric charge comes in **two signs**, so isolated electric monopoles of either sign exist, and charges can **attract or repel**. Gravitational **mass** also acts as a monopole source, but there is only one kind of mass and gravity is **only ever attractive** — masses only attract one another. Isolated **magnetic** poles (monopoles) have never been found: magnetic poles always occur as **dipoles** (a north–south pair), and cutting a magnet in two simply produces two smaller dipoles.

Coulomb's law. The force between two point charges q_1 and q_2 separated by a distance r is

$$F = \frac{k\,q_1 q_2}{r^2}.$$

The force is **repulsive** when the charges have the same sign and **attractive** when they have opposite signs. By Newton's third law the two charges push or pull on each other with **equal and opposite** forces — the force on q_1 by q_2 is equal in magnitude and opposite in direction to the force on q_2 by q_1. Coulomb's law is exactly $F = q\,E$ applied to the field of one charge: the field of q_1 at the location of q_2 is $E = \frac{k\,q_1}{r^2}$, and the force on q_2 is then $F = q_2 E = \frac{k\,q_1 q_2}{r^2}$.



$F = \frac{kq_1 q_2}{r^2}$ (equal and opposite)

Force on a charge in a field. Whenever a charge q sits in an external electric field of magnitude E, the field exerts a force on it of magnitude

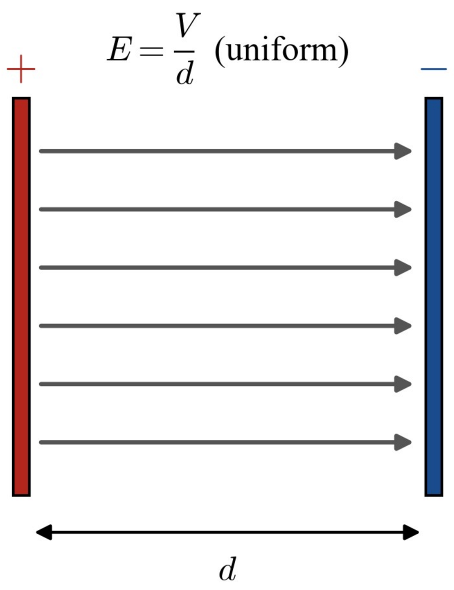
$$F = qE.$$

A positive charge is pushed **in the direction of the field**; a negative charge (such as an electron) is pushed **opposite to the field**.

Uniform fields between parallel plates. Two flat parallel conducting plates connected to a battery of potential difference V and separated by a distance d produce a field that is **uniform** in the region between them — the same magnitude and direction everywhere. Its field lines are straight, parallel and evenly spaced, running from the positive plate to the negative plate, and its magnitude is

$$E = \frac{V}{d}.$$

Here E may be measured in volts per metre ($\mathrm{V\,m^{-1}}$), which is identical to $\mathrm{N\,C^{-1}}$.



Fields are also classified as **uniform or non-uniform** and as **static or changing**. The field of a point charge is **non-uniform** (it weakens with distance and changes direction from place to place), while the field between parallel plates is **uniform**. A field is **static** if it does not change with time — for example the field of a fixed charge, or the field between plates held at a steady voltage — and **changing** if the source charges move or the voltage varies.

Work and energy in a uniform field. Moving a charge q through a potential difference V involves a work done by the electric force of

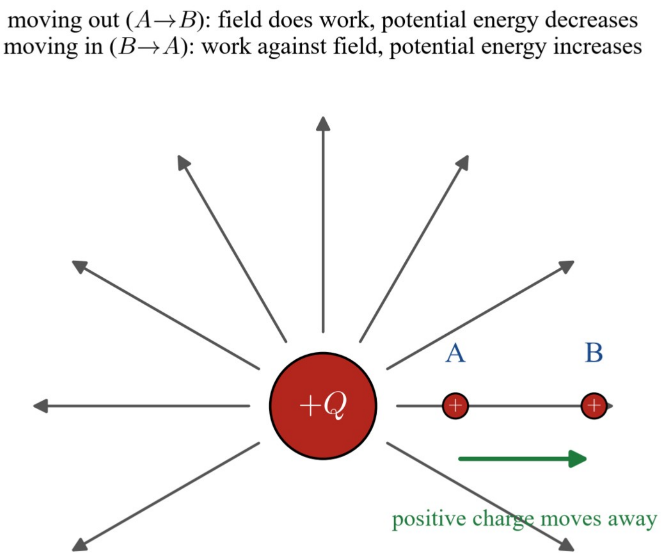
$$W = qV.$$

As a charge moves through the field, energy is transferred between **electric potential energy** and **kinetic energy**, just as a mass rising or falling exchanges gravitational potential energy and kinetic energy. When a **positive** charge moves in the direction of the field (from the positive plate toward the negative plate), the field does positive work on it: its electric potential energy **decreases** and its kinetic energy **increases**. To push the same positive charge the other way — toward the positive plate, against the field — an external agent must do work on it, and its electric potential energy **increases**. A negative charge behaves in the opposite sense, gaining kinetic energy as it moves toward the positive plate.

Qualitative changes for a charge in a point field. The same energy reasoning applies to a charge in the field of a point charge, with one important difference: electric forces can attract or repel, so the outcome depends on the signs. For two **like** charges — say a positive charge q near a positive source Q — the force on q points away from Q. As q moves **away** from Q, that force is in the direction of the motion, so the field does positive work on q and its electric potential energy **decreases**; pushing q **closer** to Q, against the repulsion, requires work by an external agent, so its electric potential energy **increases**. For two **unlike** charges the directions are reversed: the electric potential energy

decreases as the charges come together (the attractive force is in the direction of the motion) and **increases** as they are pulled apart. Two features hold in every case:

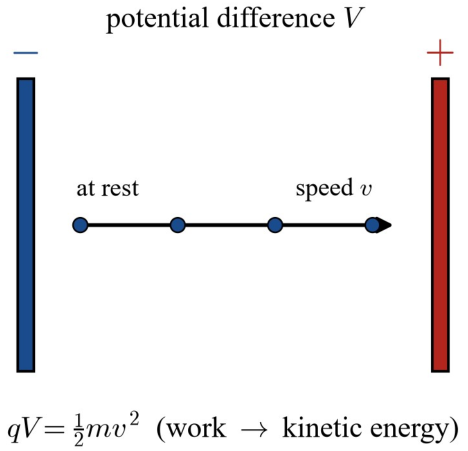
- whenever the field does positive work on a charge its electric potential energy decreases, and whenever work is done against the field its electric potential energy increases — the same rule as between the plates;
- because the field is stronger close to the source, the force on the charge is larger there, so a given step of distance changes the electric potential energy by **more** close to the source than far away — equal steps of distance do **not** produce equal steps of energy.



Accelerating a charge. If a charge q starts from rest and is accelerated through a potential difference V, all of the work done becomes kinetic energy:

$$qV = 1/2\, m v^2 \Rightarrow v = \sqrt{\frac{2qV}{m}}.$$

This is how an "electron gun" and a linear accelerator speed up charged particles.



The electron-volt. For single charged particles the joule is an inconveniently large unit of energy, so a smaller unit is used: the **electron-volt** (eV). One electron-volt is the energy gained by a charge of magnitude $q_e = 1.60 \times 10^{-19}$ C (the size of the charge on an electron or a proton) as it moves through a potential difference of 1 V:

$$1\,\text{eV} = q_e \times 1\,\text{V} = 1.60 \times 10^{-19}\,\text{J}.$$

To convert an energy from electron-volts to joules, **multiply** by 1.60×10^{-19}; to go from joules to electron-volts, **divide** by 1.60×10^{-19}. A particle carrying a charge of magnitude q_e accelerated through V volts gains exactly V electron-volts of kinetic energy — an electron accelerated through $250\,\mathrm{V}$ gains $250\,\mathrm{eV}$.

Throughout this subtopic $k=8.99\times10^{9}\,\mathrm{N\,m^{2}\,C^{-2}}$, $q_e=1.60\times10^{-19}\,\mathrm{C}$ and $m_e=9.11\times10^{-31}\,\mathrm{kg}$ are used. The charge of a proton is $+q_e$; where a proton or other particle mass is needed it is given in the question.

Worked examples

Example 1 — scaffolded

A point charge $Q=+4.0\times10^{-6}\,\mathrm{C}$ is fixed in place. Point P is $0.30\,\mathrm{m}$ from the charge. Calculate the magnitude and direction of the electric field at P, and then the magnitude and direction of the force on a charge $q=-2.0\times10^{-9}\,\mathrm{C}$ placed at P.

Working	Comment
$Q=4.0\times10^{-6}\,\mathrm{C}$, $r=0.30\,\mathrm{m}$, $k=8.99\times10^{9}\,\mathrm{N\,m^{2}\,C^{-2}}$.	List the data and the constant.
$E=\dfrac{kQ}{r^2}=\dfrac{8.99\times10^{9}\times4.0\times10^{-6}}{0.30^2}=4.00\times10^{5}\,\mathrm{N\,C^{-1}}$.	Point-charge field; the field points radially away from the positive Q.
Force on q: $F=\lvert q\rvert E=2.0\times10^{-9}\times3.996\times10^{5}=7.99\times10^{-4}\,\mathrm{N}$.	Use $F=qE$ with the magnitude of the charge, keeping the unrounded value of E during the working and rounding only at the end.
q is **negative**, so the force is **opposite** to the field — directed back **toward** Q (attraction).	A negative charge is pushed against the field direction.

Example 2 — scaffolded

Two horizontal parallel plates are $5.0\,\mathrm{cm}$ apart and connected to a $250\,\mathrm{V}$ supply. An electron ($q_e=1.60\times10^{-19}\,\mathrm{C}$, $m_e=9.11\times10^{-31}\,\mathrm{kg}$) is released from rest at the negative plate. Calculate the field between the plates, the force on the electron, the work done on it as it crosses to the positive plate (in joules and in electron-volts), and its speed on arrival.

Working	Comment
$V=250\,\mathrm{V}$, $d=5.0\,\mathrm{cm}=0.050\,\mathrm{m}$.	Convert the separation to metres.
$E=\dfrac{V}{d}=\dfrac{250}{0.050}=5.00\times10^{3}\,\mathrm{N\,C^{-1}}$.	Uniform field between the plates, from + plate to − plate.
$F=q_eE=1.60\times10^{-19}\times5.00\times10^{3}=8.00\times10^{-16}\,\mathrm{N}$.	Force on the electron; directed toward the positive plate.
$W=q_eV=1.60\times10^{-19}\times250=4.00\times10^{-17}\,\mathrm{J}=250\,\mathrm{eV}$.	Work done by the field; in eV it is just the $250\,\mathrm{V}$ crossed.
$q_eV=1/2\,m_ev^2\Rightarrow v=\sqrt{\dfrac{2q_eV}{m_e}}=\sqrt{\dfrac{2\times4.00\times10^{-17}}{9.11\times10^{-31}}}=9.3$.	All the work becomes kinetic energy (started from rest).

Example 3 — unscaffolded

Two positive point charges lie on a straight line: $Q_1=+6.0\times10^{-6}\,\mathrm{C}$ at the origin and $Q_2=+2.0\times10^{-6}\,\mathrm{C}$ at $x=0.40\,\mathrm{m}$. Calculate the magnitude and direction of the net electric field at the midpoint P ($x=0.20\,\mathrm{m}$).

The midpoint is $0.20\,\mathrm{m}$ from each charge. Each field points **away** from its (positive) source, so at P the field of Q_1 points in the $+x$ direction and the field of Q_2 points in the $-x$ direction; the net field is their difference. Taking $+x$ as positive,

$$E_1 = \frac{kQ_1}{r^2} = \frac{8.99\times10^9\times6.0\times10^{-6}}{0.20^2} = 1.348\times10^6\,\mathrm{N\,C^{-1}}(+x),$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}}{0.20^2} = 4.495\times10^5\,\mathrm{N\,C^{-1}}(-x).$$

The net field is

$$E_{\text{net}} = E_1 - E_2 = 1.348\times10^6 - 0.4495\times10^6 = 8.99\times10^5\,\mathrm{N\,C^{-1}}.$$

∴ the net field at P is $8.99\times10^5\,\mathrm{N\,C^{-1}}$, directed in the $+x$ direction (from Q_1 toward Q_2).

Example 4 — unscaffolded

An electron ($m_e = 9.11\times10^{-31}\,\mathrm{kg}$) is accelerated from rest through a potential difference of $1.2\,\mathrm{kV}$. Calculate its kinetic energy on emerging, expressed both in joules and in electron-volts, and its final speed.

The work done by the field equals the kinetic energy gained. Since $V = 1.2\,\mathrm{kV} = 1200\,\mathrm{V}$,

$$E_k = q_e V = 1.60\times10^{-19}\times1200 = 1.92\times10^{-16}\,\mathrm{J}.$$

Because the electron carries a charge of magnitude q_e and crosses $1200\,\mathrm{V}$, in electron-volts this is simply

$$E_k = 1200\,\mathrm{eV} = 1.2\,\mathrm{keV}.$$

The final speed follows from $q_e V = 1/2\, m_e v^2$:

$$v = \sqrt{\frac{2q_e V}{m_e}} = \sqrt{\frac{2\times1.92\times10^{-16}}{9.11\times10^{-31}}} = 2.05\times10^7\,\mathrm{m\,s^{-1}}.$$

∴ the electron gains $1.92\times10^{-16}\,\mathrm{J} = 1200\,\mathrm{eV}$ of kinetic energy and leaves at $2.05\times10^7\,\mathrm{m\,s^{-1}}$.

Practice questions

Question 1 A point charge $Q = +5.0\times10^{-6}\,\mathrm{C}$ is fixed in place. (a) Calculate the magnitude of the electric field at a point $0.20\,\mathrm{m}$ from the charge, and state its direction. (b) Calculate the magnitude of the field at a point $0.40\,\mathrm{m}$ from the charge, and comment on how it compares with your answer to part (a). (c) Calculate the magnitude and direction of the force on a charge $q = +3.0\times10^{-9}\,\mathrm{C}$ placed $0.20\,\mathrm{m}$ from Q.

Question 2 Two parallel plates $8.0\,\mathrm{mm}$ apart are connected to a $600\,\mathrm{V}$ supply. (a) Show that the electric field between the plates is $7.50\times10^4\,\mathrm{N\,C^{-1}}$. (b) Calculate the magnitude of the force on a charge $q = +2.5\times10^{-9}\,\mathrm{C}$ placed between the plates. (c) Calculate the work done by the field on this charge as it moves from one plate to the other.

Question 3 Two small charged spheres, $Q_1 = +8.0\times10^{-6}\,\mathrm{C}$ and $Q_2 = -3.0\times10^{-6}\,\mathrm{C}$, are held $0.25\,\mathrm{m}$ apart. (a) Calculate the magnitude of the electric force between them. (b) State whether the force is attractive or repulsive, and give the direction of the force on Q_2. (c) The separation is then doubled to $0.50\,\mathrm{m}$. Calculate the new magnitude of the force.

Question 4 An electron is accelerated from rest through a potential difference of $400\,\mathrm{V}$. (a) Calculate the work done on the electron by the field, in joules. (b) State the kinetic energy gained by the electron in electron-volts. (c) Calculate the final speed of the electron.

Question 5 (a) Sketch the electric field around an isolated negative point charge, showing at least six field lines with arrows, and describe how the spacing of the lines changes with distance from the charge. (b) Explain the difference between an electric monopole and an electric dipole, giving one example of each. (c) The region between two parallel plates connected to a steady battery is described as a uniform, static field. Justify both descriptions.

Question 6 A small positive charge is released from rest next to the positive plate of a pair of parallel plates. The field between the plates points from the positive plate toward the negative plate. (a) (Circle your answer.) As the charge moves across to the negative plate, its electric potential energy: *increases* / *decreases* / *stays the same*. Justify your choice. No calculations are required. (b) State the direction of the electric force on the charge and the direction of the electric field. (c) Describe the energy transformation that takes place as the charge crosses the gap.

Question 7 Calculate the magnitude of the electric field at a point $0.15\,\mathrm{m}$ from a point charge of $-6.0\times10^{-6}\,\mathrm{C}$, and state the direction of the field at that point.

Question 8 At a point $0.20\,\mathrm{m}$ from a point charge, the electric field has magnitude $3.0\times10^{5}\,\mathrm{N\ C^{-1}}$ and points **toward** the charge. Determine the size of the charge, and explain how the direction of the field tells you the sign of the charge.

Question 9 Two small spheres carry charges of $+7.5\times10^{-9}\,\mathrm{C}$ and $+4.0\times10^{-9}\,\mathrm{C}$ and are $3.0\,\mathrm{cm}$ apart. Calculate the magnitude of the electric force between them. Give your answer to three significant figures.

Question 10 A point charge $Q=+2.0\times10^{-6}\,\mathrm{C}$ is fixed in place. Show that the electric field at a distance of $0.12\,\mathrm{m}$ from the charge is $1.25\times10^{6}\,\mathrm{N\ C^{-1}}$, and hence calculate the magnitude of the force on a charge of $-5.0\times10^{-9}\,\mathrm{C}$ placed at that point.

Question 11 Two charges of equal magnitude, $+4.0\times10^{-6}\,\mathrm{C}$ and $-4.0\times10^{-6}\,\mathrm{C}$, are fixed $0.60\,\mathrm{m}$ apart. Calculate the magnitude and direction of the net electric field at the midpoint between them, and explain why the two contributions add rather than cancel there.

Question 12 A pair of parallel plates connected to a $90\,\mathrm{V}$ supply produces a uniform field of $3.0\times10^{4}\,\mathrm{N\ C^{-1}}$ between them. Calculate the separation of the plates.

Question 13 Two parallel plates $4.0\,\mathrm{cm}$ apart are connected to a $120\,\mathrm{V}$ supply. An electron is placed in the field between them. Calculate the field strength, the magnitude of the force on the electron, and the magnitude of the electron's acceleration.

Question 14 An electron is accelerated from rest through a potential difference of $2.5\,\mathrm{kV}$. Calculate its kinetic energy in electron-volts and in joules, and its final speed.

Question 15 A proton (charge $+q_e$, mass $1.67\times10^{-27}\,\mathrm{kg}$) is accelerated from rest through a potential difference of $5.0\,\mathrm{kV}$. Calculate its kinetic energy in joules and its final speed. Comment on why the proton's final speed is so much lower than that of an electron accelerated through the same voltage (Question 14 used a comparable voltage).

Question 16 An electron is moving at $1.5\times10^{7}\,\mathrm{m\ s^{-1}}$. (a) Calculate its kinetic energy in joules. (b) Express this kinetic energy in electron-volts.

Question 17 A charged dust particle of mass $5.0\times10^{-8}\,\mathrm{kg}$ carries a charge of $+2.0\times10^{-9}\,\mathrm{C}$. Starting from rest, it is accelerated through a potential difference of $3000\,\mathrm{V}$. Calculate its final speed.

Question 18 Electric field lines never cross one another. Explain why, referring to the definition of the electric field at a point. Set out your reasoning as a clear chain of steps.

Question 19 Compare gravitational, electric and magnetic fields with respect to (i) whether isolated monopoles exist, and (ii) whether the sources can attract, repel, or both. Refer to dipoles where relevant.

Question 20 A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$) and an electron (mass $9.11\times10^{-31}\,\mathrm{kg}$) are each placed in the same uniform field between two plates $5.0\,\mathrm{cm}$ apart connected to a $200\,\mathrm{V}$ supply. (a) Compare the directions of the electric force on the proton and on the electron. (b) Calculate the magnitude of the force on each particle, and account for the result. (c) Calculate the acceleration of each particle, and explain which one gains speed more rapidly and why.

Question 21 A pair of parallel plates is connected to a fixed $150\,\mathrm{V}$ supply, with the plates initially $2.0\,\mathrm{cm}$ apart. The separation is then increased to $4.0\,\mathrm{cm}$ while the supply voltage is held constant. (a) Calculate the field strength before and after the plates are moved. (b) State what happens to the magnitude of the force on an electron between the plates.

(c) An electron released from rest at one plate crosses to the other. Determine, with reasoning, whether its kinetic energy on arrival changes when the plates are moved apart.

Question 22 In an electron gun, electrons are accelerated from rest through a potential difference of $3.0\,\mathrm{kV}$. (Ignore relativistic effects.) (a) Calculate the kinetic energy of an electron in electron-volts and in joules. (b) Calculate the speed of an electron as it leaves the gun. (c) Through what potential difference would an electron need to be accelerated from rest to leave with **twice** this speed? Justify your answer.

Question 23 A positive point charge Q is fixed in place. A small positive charge q is moved directly away from Q, from a distance r to a distance $2r$. (a) (Circle your answer.) As q moves away from Q, its electric potential energy: *increases* / *decreases* / *stays the same*. Justify your choice in terms of the work done by or against the electric force. No calculations are required. (b) State how the answer to part (a) would differ if q were a negative charge moved through the same displacement, and explain why. (c) Compare the change in electric potential energy as q moves from r to $2r$ with the change as it moves from $2r$ to $3r$. Explain your reasoning. No calculations are required.

Answers

Question 1 (a) $1.12\times10^{6}\,\mathrm{N\ C^{-1}}$, directed away from Q; (b) $2.81\times10^{5}\,\mathrm{N\ C^{-1}}$, one-quarter of (a); (c) $3.37\times10^{-3}\,\mathrm{N}$, away from Q. **Question 2** (a) $7.50\times10^{4}\,\mathrm{N\ C^{-1}}$; (b) $1.88\times10^{-4}\,\mathrm{N}$; (c) $1.50\times10^{-6}\,\mathrm{J}$. **Question 3** (a) $3.45\,\mathrm{N}$; (b) attractive, force on Q_2 directed toward Q_1; (c) $0.863\,\mathrm{N}$. **Question 4** (a) $6.4\times10^{-17}\,\mathrm{J}$; (b) $400\,\mathrm{eV}$; (c) $1.19\times10^{7}\,\mathrm{m\ s^{-1}}$. **Question 5** (a) six radial lines pointing inward, more widely spaced further out; (b) monopole = single isolated charge, dipole = equal + and − pair; (c) uniform (same E everywhere between the plates) and static (steady voltage, so unchanging in time). **Question 6** (a) decreases; (b) force and field both directed from the positive plate toward the negative plate; (c) electric potential energy → kinetic energy. **Question 7** $2.40\times10^{6}\,\mathrm{N\ C^{-1}}$, directed toward the charge. **Question 8** $1.33\times10^{-6}\,\mathrm{C}$ (that is, 1.33 μC), negative. **Question 9** $3.00\times10^{-4}\,\mathrm{N}$. **Question 10** $E=1.25\times10^{6}\,\mathrm{N\ C^{-1}}$; $F=6.24\times10^{-3}\,\mathrm{N}$. **Question 11** $7.99\times10^{5}\,\mathrm{N\ C^{-1}}$, directed from the positive charge toward the negative charge. **Question 12** $3.0\times10^{-3}\,\mathrm{m}$ ($3.0\,\mathrm{mm}$). **Question 13** $3.00\times10^{3}\,\mathrm{N\ C^{-1}}$; $4.8\times10^{-16}\,\mathrm{N}$; $5.27\times10^{14}\,\mathrm{m\ s^{-2}}$. **Question 14** $2500\,\mathrm{eV}=4.0\times10^{-16}\,\mathrm{J}$; $2.96\times10^{7}\,\mathrm{m\ s^{-1}}$. **Question 15** $8.0\times10^{-16}\,\mathrm{J}$; $9.79\times10^{5}\,\mathrm{m\ s^{-1}}$. **Question 16** (a) $1.02\times10^{-16}\,\mathrm{J}$; (b) $641\,\mathrm{eV}$. **Question 17** $15.5\,\mathrm{m\ s^{-1}}$. **Question 18** A field line's direction is the unique field direction at each point; two crossing lines would give two directions there, which is impossible. **Question 19** Gravity: mass monopoles, attract only; electric: ± monopoles, attract and repel, dipoles form; magnetic: no monopoles, dipoles only, attract and repel. **Question 20** (a) opposite directions; (b) $6.4\times10^{-16}\,\mathrm{N}$ on each; (c) electron $7.03\times10^{14}\,\mathrm{m\ s^{-2}}$, proton $3.83\times10^{11}\,\mathrm{m\ s^{-2}}$; the electron accelerates far more (smaller mass). **Question 21** (a) $7500\,\mathrm{N\ C^{-1}}$ then $3750\,\mathrm{N\ C^{-1}}$; (b) it halves; (c) unchanged at $2.4\times10^{-17}\,\mathrm{J}$ ($150\,\mathrm{eV}$). **Question 22** (a) $3000\,\mathrm{eV}=4.8\times10^{-16}\,\mathrm{J}$; (b) $3.25\times10^{7}\,\mathrm{m\ s^{-1}}$; (c) $12\,\mathrm{kV}$. **Question 23** (a) decreases; (b) it increases instead — the force on a negative q points toward Q, so moving away is against the field; (c) larger for $r\to2r$ — the field, and the force on q, is stronger closer to Q.

Fully-worked solutions

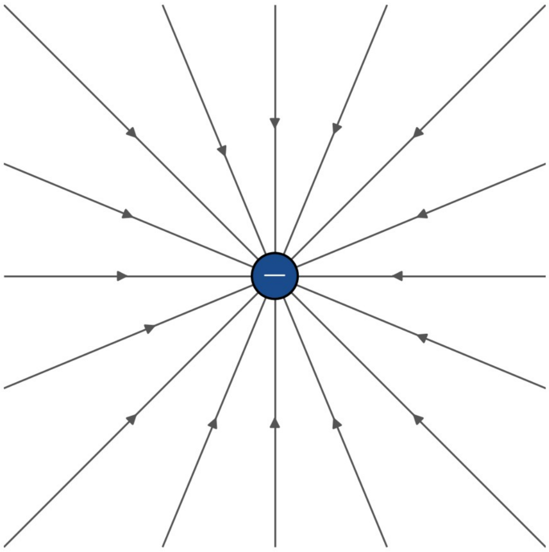
Question 1 $Q=5.0\times10^{-6}\,\mathrm{C}$. (a) $E=\dfrac{kQ}{r^2}=\dfrac{8.99\times10^{9}\times5.0\times10^{-6}}{0.20^2}=1.12\times10^{6}\,\mathrm{N\ C^{-1}}$, directed radially away from the positive charge. (b) At $r=0.40\,\mathrm{m}$: $E=\dfrac{8.99\times10^{9}\times5.0\times10^{-6}}{0.40^2}=2.81\times10^{5}\,\mathrm{N\ C^{-1}}$. The distance has doubled, so by the inverse-square law the field is one-quarter of its value in part (a). (c) Using the unrounded value of E from part (a), $F=qE=3.0\times10^{-9}\times1.124\times10^{6}=3.37\times10^{-3}\,\mathrm{N}$. The charge is positive, so the force is directed **away** from Q (the like charges repel).

Question 2 $V=600\,\mathrm{V}$, $d=8.0\,\mathrm{mm}=8.0\times10^{-3}\,\mathrm{m}$. (a) $E=\dfrac{V}{d}=\dfrac{600}{8.0\times10^{-3}}=7.50\times10^{4}\,\mathrm{N\ C^{-1}}$, as required. (b) $F=qE=2.5\times10^{-9}\times7.50\times10^{4}=1.88\times10^{-4}\,\mathrm{N}$. (c) $W=qV=2.5\times10^{-9}\times600=1.50\times10^{-6}\,\mathrm{J}$. (As a check, $W=Fd=1.88\times10^{-4}\times8.0\times10^{-3}=1.50\times10^{-6}\,\mathrm{J}$.)

Question 3 $Q_1=8.0\times10^{-6}\,\mathrm{C}$, $Q_2=3.0\times10^{-6}\,\mathrm{C}$ (magnitudes), $r=0.25\,\mathrm{m}$. (a) $F=\dfrac{kQ_1Q_2}{r^2}=\dfrac{8.99\times10^{9}\times8.0\times10^{-6}\times3.0\times10^{-6}}{0.25^2}=3.45\,\mathrm{N}$. (b) The charges have opposite signs, so the force is **attractive**; the force on Q_2 points toward Q_1 (and the force on Q_1 points toward Q_2 — equal and opposite, by Newton's third law). (c) Doubling r to $0.50\,\mathrm{m}$ quarters the force: $F=\dfrac{3.45}{4}=0.863\,\mathrm{N}$ (equivalently $\dfrac{8.99\times10^{9}\times8.0\times10^{-6}\times3.0\times10^{-6}}{0.50^2}$).

Question 4 $V = 400\,\text{V}$. (a) $W = q_e V = 1.60 \times 10^{-19} \times 400 = 6.4 \times 10^{-17}\,\text{J}$. (b) Crossing 400 V with a charge of magnitude q_e gives $E_k = 400\,\text{eV}$. (c) $q_e V = 1/2\, m_e v^2$, so $v = \sqrt{\dfrac{2 q_e V}{m_e}} = \sqrt{\dfrac{2 \times 6.4 \times 10^{-17}}{9.11 \times 10^{-31}}} = 1.19 \times 10^7\,\text{m s}^{-1}$.

Question 5 (a) The field lines are **radial and point inward**, toward the negative charge, since the field is the direction of the force on a positive test charge. The lines are drawn closer together near the charge (where the field is strong) and spread further apart with distance (where the field is weaker).



isolated negative charge: field points radially inward

(b) An electric **monopole** is a single isolated charge — for example one positive point charge, from which field lines radiate outward. An electric **dipole** is a pair of equal and opposite charges close together — for example a $+Q$ and a $-Q$ side by side — whose field lines run from the positive to the negative charge.

(c) It is **uniform** because between the plates the field has the same magnitude and direction at every point (straight, evenly spaced parallel lines). It is **static** because the battery holds the voltage steady, so the field does not change with time.

Question 6 (a) **Decreases.** The field points from the positive plate toward the negative plate, so the electric force on the positive charge is in the same direction as its motion. The field does positive work on the charge, converting electric potential energy into kinetic energy, so the electric potential energy falls. (b) Both the electric force on the positive charge and the electric field are directed from the positive plate toward the negative plate. (c) Electric potential energy is transformed into kinetic energy: the charge loses potential energy and speeds up as it crosses the gap.

Question 7 $E = \dfrac{kQ}{r^2} = \dfrac{8.99 \times 10^9 \times 6.0 \times 10^{-6}}{0.15^2} = 2.40 \times 10^6\,\text{N C}^{-1}$. Because the charge is negative, the field points **toward** the charge at that point.

Question 8 Rearranging $E = \dfrac{kQ}{r^2}$ for the charge:

$$Q = \frac{E r^2}{k} = \frac{3.0 \times 10^5 \times 0.20^2}{8.99 \times 10^9} = 1.33 \times 10^{-6}\,\text{C}.$$

The field points **toward** the charge, and the field direction is the direction of the force on a *positive* test charge — a positive test charge is attracted toward the source, so the source must be **negative**. The charge is $-1.33 \times 10^{-6}\,\text{C}$ (−1.33 μC).

Question 9 $F = \dfrac{k Q_1 Q_2}{r^2} = \dfrac{8.99 \times 10^9 \times 7.5 \times 10^{-9} \times 4.0 \times 10^{-9}}{(3.0 \times 10^{-2})^2} = 3.00 \times 10^{-4}\,\text{N}$ (to three significant figures). The charges are both positive, so the force is repulsive.

Question 10 $E=\dfrac{kQ}{r^2}=\dfrac{8.99\times10^{9}\times2.0\times10^{-6}}{0.12^2}=1.25\times10^{6}\,\mathrm{N\,C^{-1}}$, as required. Using the unrounded value of E, the force on the placed charge is

$$F=|q|E=5.0\times10^{-9}\times1.2486\times10^{6}=6.24\times10^{-3}\,\mathrm{N},$$

directed toward Q (the negative charge is attracted to the positive source).

Question 11 The midpoint is $0.30\,\mathrm{m}$ from each charge. The field of the positive charge points **away** from it (toward the negative charge), and the field of the negative charge points **toward** it (also toward the negative charge) — both contributions point the same way, so they **add**. Each has magnitude

$$E=\frac{kQ}{r^2}=\frac{8.99\times10^{9}\times4.0\times10^{-6}}{0.30^2}=4.00\times10^{5}\,\mathrm{N\,C^{-1}},$$

so, keeping the unrounded value of each contribution ($3.996\times10^{5}\,\mathrm{N\,C^{-1}}$), the net field is $E_{\mathrm{net}}=2\times3.996\times10^{5}=7.99\times10^{5}\,\mathrm{N\,C^{-1}}$, directed from the positive charge toward the negative charge.

Question 12 $E=\dfrac{V}{d}$, so $d=\dfrac{V}{E}=\dfrac{90}{3.0\times10^{4}}=3.0\times10^{-3}\,\mathrm{m}=3.0\,\mathrm{mm}$.

Question 13 $V=120\,\mathrm{V}$, $d=4.0\,\mathrm{cm}=0.040\,\mathrm{m}$. Field: $E=\dfrac{V}{d}=\dfrac{120}{0.040}=3.00\times10^{3}\,\mathrm{N\,C^{-1}}$. Force on the electron: $F=q_eE=1.60\times10^{-19}\times3.00\times10^{3}=4.8\times10^{-16}\,\mathrm{N}$. Acceleration (Newton's second law): $a=\dfrac{F}{m_e}=\dfrac{4.8\times10^{-16}}{9.11\times10^{-31}}=5.27\times10^{14}\,\mathrm{m\,s^{-2}}$.

Question 14 $V=2.5\,\mathrm{kV}=2500\,\mathrm{V}$. In electron-volts $E_k=2500\,\mathrm{eV}$, and in joules

$$E_k=q_eV=1.60\times10^{-19}\times2500=4.0\times10^{-16}\,\mathrm{J}.$$

Speed: $v=\sqrt{\dfrac{2q_eV}{m_e}}=\sqrt{\dfrac{2\times4.0\times10^{-16}}{9.11\times10^{-31}}}=2.96\times10^{7}\,\mathrm{m\,s^{-1}}$.

Question 15 $V=5.0\,\mathrm{kV}=5000\,\mathrm{V}$, proton charge $+q_e$, mass $m_p=1.67\times10^{-27}\,\mathrm{kg}$.

$$E_k=q_eV=1.60\times10^{-19}\times5000=8.0\times10^{-16}\,\mathrm{J},$$

$$v=\sqrt{\frac{2q_eV}{m_p}}=\sqrt{\frac{2\times8.0\times10^{-16}}{1.67\times10^{-27}}}=9.79\times10^{5}\,\mathrm{m\,s^{-1}}.$$

Both particles carry a charge of magnitude q_e, so a proton and an electron accelerated through the same voltage gain the **same** kinetic energy. But $v=\sqrt{2q_eV/m}$ depends on the mass: the proton is about 1830 times more massive than the electron, so for the same kinetic energy its speed is smaller by a factor of $\sqrt{1830}\approx43$, giving a much lower final speed.

Question 16 $v=1.5\times10^{7}\,\mathrm{m\,s^{-1}}$, $m_e=9.11\times10^{-31}\,\mathrm{kg}$. (a) $E_k=1/2\,m_ev^2=1/2\times9.11\times10^{-31}\times(1.5\times10^{7})^2=1.02\times10^{-16}\,\mathrm{J}$. (b) Using the unrounded kinetic energy from part (a), $E_k=\dfrac{1.025\times10^{-16}}{1.60\times10^{-19}}=641\,\mathrm{eV}$.

Question 17 All the work done becomes kinetic energy: $qV=1/2\,mv^2$, with $q=2.0\times10^{-9}\,\mathrm{C}$, $V=3000\,\mathrm{V}$, $m=5.0\times10^{-8}\,\mathrm{kg}$.

$$v=\sqrt{\frac{2qV}{m}}=\sqrt{\frac{2\times 2.0\times 10^{-9}\times 3000}{5.0\times 10^{-8}}}=\sqrt{240}=15.5\,\mathrm{m\,s^{-1}}.$$

Question 18 The reasoning runs as a chain:

1. At any point, a field line is drawn in the direction of the electric field there — that is, the direction of the electric force on a small positive test charge placed at that point.
2. The electric field at a point has a single, definite value: the contributions of all the source charges add as vectors to give **one** resultant vector, with one direction.
3. If two field lines crossed, then at the crossing point the field would have to point in two different directions at once.
4. That is impossible, so field lines never cross.

Question 19 - Gravitational field: the source is mass, which acts as a **monopole** (field lines point toward the mass). There is only one kind of mass, so gravity is **only ever attractive** — masses only attract one another, and there is no gravitational analogue of a repulsive charge or of a +/− dipole. - **Electric field:** charge comes in **two signs**, so isolated **monopoles** of either sign exist (a + charge sources lines, a − charge sinks them). Like charges **repel** and unlike charges **attract**, and a +/− pair forms an electric **dipole**. - **Magnetic field:** isolated magnetic **monopoles have never been found** — magnetic poles always come as **dipoles** (a north–south pair), and cutting a magnet yields two smaller dipoles. Like poles **repel** and unlike poles **attract**.

So all three are described by a field model, but only gravity is restricted to attraction; electric charges and magnetic poles can do both, electric charge has isolated monopoles of two signs, and magnetism has no monopoles at all.

Question 20 $V=200\,\mathrm{V}$, $d=5.0\,\mathrm{cm}=0.050\,\mathrm{m}$, so $E=\frac{V}{d}=\frac{200}{0.050}=4.00\times 10^{3}\,\mathrm{N\,C^{-1}}$. (a) The proton is positive, so the force on it is **along** the field; the electron is negative, so the force on it is **opposite** to the field. The two forces are in **opposite directions**. (b) $F=|q|E$, and both particles carry the same magnitude of charge q_e, so each feels the same force:

$$F=1.60\times 10^{-19}\times 4.00\times 10^{3}=6.4\times 10^{-16}\,\mathrm{N}.$$

The force depends only on the size of the charge and the field, not on the mass, so the forces are equal. (c) $a=\frac{F}{m}$: for the electron $a_e=\frac{6.4\times 10^{-16}}{9.11\times 10^{-31}}=7.03\times 10^{14}\,\mathrm{m\,s^{-2}}$; for the proton $a_p=\frac{6.4\times 10^{-16}}{1.67\times 10^{-27}}=3.83\times 10^{11}\,\mathrm{m\,s^{-2}}$. Although the forces are equal, the electron has far less mass, so it accelerates about 1830 times more rapidly and gains speed much faster than the proton.

Question 21 $V=150\,\mathrm{V}$ throughout. (a) At $d_1=2.0\,\mathrm{cm}=0.020\,\mathrm{m}$: $E_1=\frac{V}{d_1}=\frac{150}{0.020}=7500\,\mathrm{N\,C^{-1}}$. At $d_2=4.0\,\mathrm{cm}=0.040\,\mathrm{m}$: $E_2=\frac{150}{0.040}=3750\,\mathrm{N\,C^{-1}}$. (b) Since $F=q_eE$ and the field has halved, the force on the electron **halves** (from $1.2\times 10^{-15}\,\mathrm{N}$ to $6.0\times 10^{-16}\,\mathrm{N}$). (c) The kinetic energy gained in crossing the gap is $E_k=q_eV$, which depends only on the potential difference, not on the separation. The voltage is unchanged at $150\,\mathrm{V}$, so the electron arrives with the **same** kinetic energy either way: $E_k=1.60\times 10^{-19}\times 150=2.4\times 10^{-17}\,\mathrm{J}=150\,\mathrm{eV}$. (The field and force are weaker over the larger gap, but they act over a proportionally longer distance, so the work done — and hence the final kinetic energy and speed — is unchanged.)

Question 22 $V=3.0\,\mathrm{kV}=3000\,\mathrm{V}$. (a) $E_k=3000\,\mathrm{eV}$, and in joules $E_k=q_eV=1.60\times 10^{-19}\times 3000=4.8\times 10^{-16}\,\mathrm{J}$. (b) $v=\sqrt{\frac{2q_eV}{m_e}}=\sqrt{\frac{2\times 4.8\times 10^{-16}}{9.11\times 10^{-31}}}=3.25\times 10^{7}\,\mathrm{m\,s^{-1}}$. (c) From $q_eV=1/2\,m_ev^2$, the speed satisfies $v\propto\sqrt{V}$, so doubling the speed requires **four times** the kinetic energy and therefore four times the accelerating voltage:

$V^{'} = 4 \times 3000 = 12\,000\ \mathrm{V} = 12\ \mathrm{kV}$. (Check: using the unrounded speed from part (b), at $12\ \mathrm{kV}$, $v^{'} = 2 \times 3.246 \times 10^{7} = 6.49 \times 10^{7}\ \mathrm{m\ s^{-1}}$.)

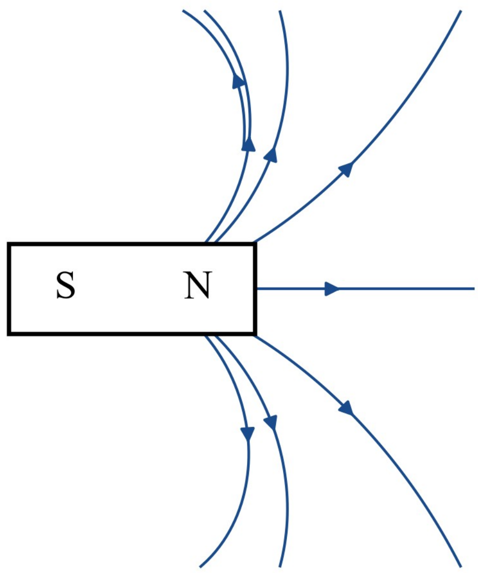
Question 23 (a) **Decreases.** Both charges are positive, so the electric force on q is repulsive — directed **away** from Q, the same direction as the motion. The field therefore does positive work on q, converting electric potential energy into kinetic energy, so the electric potential energy falls as q moves away. (b) If q were negative, the force on it would be attractive — directed **toward** Q, opposite to the displacement away from Q. Moving away would then require work to be done **against** the field by an external agent, so the electric potential energy would **increase** instead. (c) The change is **larger** from r to $2r$. The field of a point charge is stronger closer to the source (inverse-square law), so the force on q is larger throughout the interval from r to $2r$ than throughout the interval from $2r$ to $3r$. Over equal steps of distance, a larger force does more work, so more electric potential energy is lost in the first step than in the second.

Theory

The magnetic field model. A **magnetic field** is the region of influence around a magnet or a moving charge, in which another magnet or moving charge experiences a force *without contact*. Like the gravitational and electric fields, it is drawn with **field lines**: the line at each point gives the direction of the field $\vec{B}$, and the lines are drawn closer together where the field is stronger. By convention the field around a magnet points **out of a north (N) pole and into a south (S) pole**, and every field line forms a closed loop, continuing through the body of the magnet from S back to N. The magnitude of $\vec{B}$ is the **magnetic field strength**, measured in **tesla** (T).

Poles always come in pairs — no magnetic monopoles. A gravitational field has a single kind of source (mass), and an electric field can arise from an isolated positive *or* negative charge (a **monopole**). Magnetism is different: an isolated north or south pole has never been found. Every magnet is a **dipole** with both an N and an S pole, and cutting a magnet in two simply produces two smaller dipoles, each with its own N and S. This is why magnetic field lines always form continuous closed loops.

The field of a bar magnet. The field of a bar magnet is strongest and most concentrated near the poles and spreads out in between, so it is a **non-uniform** field. Two magnets brought together obey the rule that **like poles repel and unlike poles attract**.

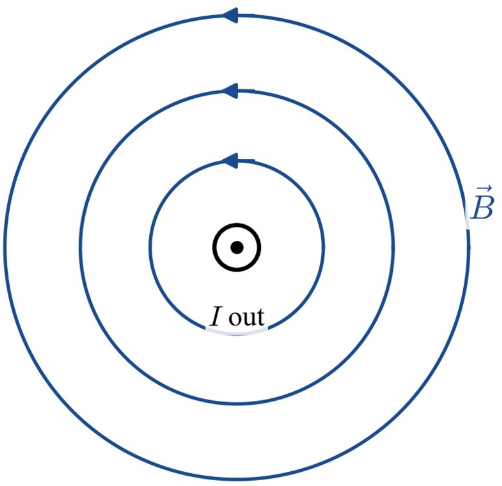


field lines leave N, enter S, and are closer together near the poles (the field is non-uniform)

The field of a straight current-carrying wire. A moving charge is also a source of a magnetic field, so an electric current produces one. Around a long straight wire the field lines are **concentric circles** centred on the wire, lying in planes perpendicular to it; the field is stronger close to the wire, so it too is non-uniform. Its direction is found with the **right-hand grip rule**, stated as a directional convention:

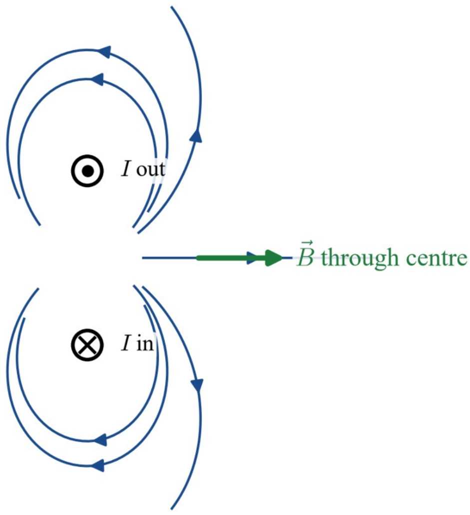
> Grip the wire with the right hand so the **thumb points along the conventional current** I; the **curled fingers** then point in the direction the field $\vec{B}$ circles the wire.

Fields directed out of the page are drawn as a dot $\odot$ (an arrow-tip coming toward you) and fields into the page as a cross $\otimes$ (an arrow-tail going away).



cross-section of a straight wire, current out of the page (⊙):
field lines are concentric circles (right-hand grip rule)

The field of a current loop. Bending the wire into a single loop concentrates the circular field of each part of the wire through the centre of the loop, producing a field like that of a very short bar magnet: the field passes straight through the middle, so one face of the loop behaves as a north pole and the other as a south pole.

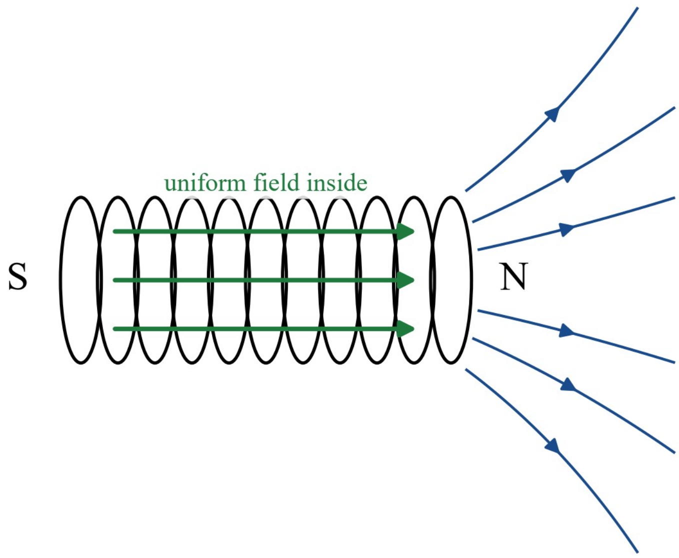


a current loop (edge-on) makes a field like a short bar magnet:
one face acts as N, the other as S

The field of a solenoid. A **solenoid** is a coil of many loops wound in a cylinder. The fields of the individual loops add, giving a field that outside the coil looks just like a bar magnet's, and **inside the coil is very nearly uniform** — the field lines there are straight, parallel and evenly spaced. The right-hand grip rule fixes the poles:

> Curl the right-hand fingers in the direction of the **conventional current** around the loops; the **thumb points along the field inside the solenoid**, toward the end that acts as the **north pole**.

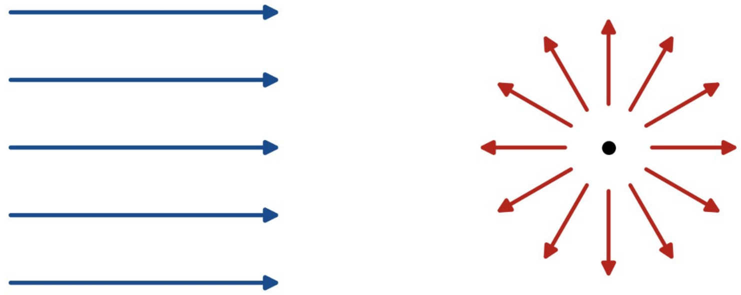
The field inside is made stronger by increasing the current, or by increasing the number of turns of wire; reversing the current reverses the field and swaps the poles.



a solenoid carrying a steady current: uniform field inside, bar-magnet-like field outside (right-hand grip rule sets N)

Classifying fields: static or changing, uniform or non-uniform. Any field can be described in two independent ways.

- A field is **static** if it does not change with time (a permanent magnet, or a coil carrying a steady direct current) and **changing** if it varies with time (a coil carrying an alternating current, or the field near a moving magnet).
- A field is **uniform** if it has the same magnitude *and* direction at every point (the field inside a long solenoid, or between the poles of a horseshoe magnet) and **non-uniform** if its magnitude or direction varies from place to place (the field of a bar magnet or of a single wire).



same magnitude and direction everywhere

magnitude and/or direction vary in space

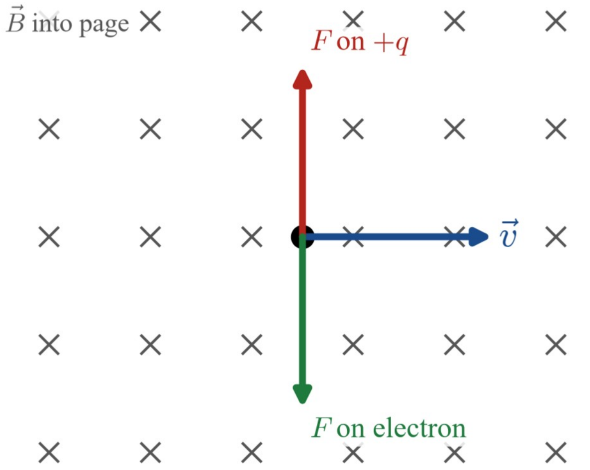
The force on a moving charge: $F = q\,v\,B$. A charge q moving with speed v through a magnetic field B experiences a magnetic force. When the velocity is **perpendicular** to the field, the magnitude is

$$F = q\,v\,B.$$

The force is always perpendicular to *both* $\vec{v}$ and $\vec{B}$. Its direction (for a positive charge) follows a right-hand rule, again a directional convention:

> Hold the right hand flat with the **thumb along the velocity** $\vec{v}$ of a positive charge and the **fingers along the field** $\vec{B}$; the **force comes out of the palm**.

For a negative charge, such as an electron, the force is in the **opposite** direction. When the velocity is **parallel** (or anti-parallel) to the field, the charge cuts across no field lines and the force is **zero**: $F = q\,v\,B$ is used only in these perpendicular or parallel cases.

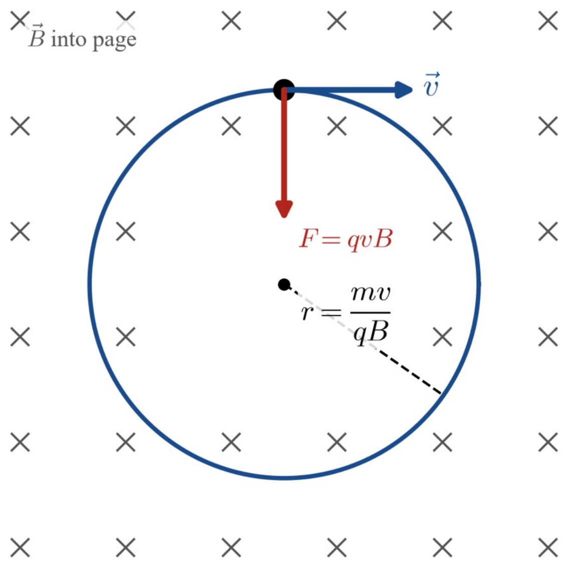


$F = qvB$ when $\vec{v} \perp \vec{B}$; F is perpendicular to both.
Right-hand rule gives F on a positive charge; reverse it for an electron.

Circular motion of a charged particle: $r = \dfrac{mv}{qB}$**.** Consider a charged particle entering a uniform field at right angles. The magnetic force is always perpendicular to the velocity, so it does no work on the particle: its **kinetic energy and speed stay constant**. A constant-magnitude force that is always perpendicular to the velocity is exactly a centripetal force, so the particle moves in a **circle**. Equating the magnetic force to the centripetal force,

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}.$$

This is applied only for particles with $v \ll c$, so that relativistic effects can be ignored. The radius grows with the momentum mv and shrinks in a stronger field. Because the motion is uniform circular motion, the period follows from $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{qB}$.

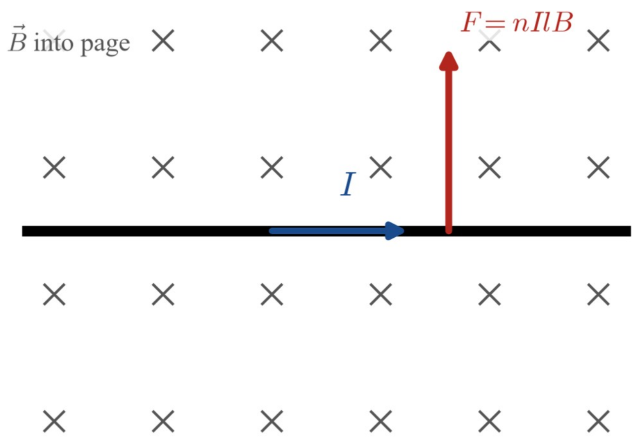


the magnetic force stays perpendicular to $\vec{v}$, so it does no work: the speed is constant and the path is a circle

The force on a current-carrying conductor: $F = nIlB$**.** A current is a stream of moving charges, so a current-carrying wire in an external magnetic field feels a force. For n conductors (for example the n turns of a coil), each of length l carrying current I, lying **perpendicular** to a field B, the magnitude is

$$F = n I l B.$$

The direction is found with the same right-hand rule as for a moving positive charge, using the **conventional current** direction in place of $\vec{v}$. As with a single charge, when the current is **parallel** to the field the force is **zero**.



a conductor of length l carrying current I, perpendicular to $\vec{B}$: force $F = nIlB$, perpendicular to both (right-hand rule)

Throughout this subtopic the data-sheet constants $q_e = 1.60 \times 10^{-19}$ C and $m_e = 9.11 \times 10^{-31}$ kg are used; where another particle is involved, its mass is given in the question.

Worked examples

Example 1 — scaffolded

An electron travels at 3.0×10^6 m s^{-1} to the right, at right angles to a uniform magnetic field of magnitude 0.25 T directed into the page. Calculate the magnitude of the magnetic force on the electron, and state its direction.

Working	Comment
The velocity is perpendicular to the field, so $F = q v B$ applies. Data: $v = 3.0 \times 10^6$ m s^{-1}, $B = 0.25$ T, $q = 1.60 \times 10^{-19}$ C.	Identify the perpendicular case; use the electron's charge magnitude.
$F = q v B = 1.60 \times 10^{-19} \times 3.0 \times 10^6 \times 0.25$.	Substitute into $F = q v B$.
$F = 1.2 \times 10^{-13}$ N.	Evaluate.
For a positive charge moving right in a field into the page the right-hand rule gives an upward force; the electron is negative, so its force is reversed — **downward**.	The force on an electron is opposite to the right-hand-rule direction.

Example 2 — scaffolded

An electron enters a uniform magnetic field of magnitude 0.012 T at right angles, moving at 2.5×10^6 m s^{-1}. Calculate the radius of its circular path, giving your answer to three significant figures, and state what provides the centripetal force.

Working	Comment
The magnetic force is perpendicular to $\vec{v}$, so it acts as the centripetal force: $q v B = \dfrac{m v^2}{r}$.	Set the magnetic force equal to the centripetal force.

Working	Comment
Rearranging, $r=\dfrac{m\,v}{q\,B}$.	Cancel one factor of v.
$r=\dfrac{9.11\times10^{-31}\times2.5\times10^{6}}{1.60\times10^{-19}\times0.012}$.	Substitute m_e, v, q_e, B.
$r=1.19\times10^{-3}\,\mathrm{m}$ (three significant figures).	Evaluate.
The magnetic force $q\,v\,B$ provides the centripetal force.	Name the force that curves the path.

Example 3 — unscaffolded

A rectangular coil of 12 turns carries a current of $2.5\,\mathrm{A}$. One side of the coil, of length $8.0\,\mathrm{cm}$, lies at right angles to a uniform magnetic field of magnitude $0.40\,\mathrm{T}$. Calculate the force on this side of the coil.

The side is perpendicular to the field, and $n=12$ conductors carry the current across it. Converting the length to metres, $l=8.0\,\mathrm{cm}=0.080\,\mathrm{m}$, so

$$F=n\,I\,l\,B=12\times2.5\times0.080\times0.40=0.96\,\mathrm{N}.$$

The direction, from the right-hand rule, is perpendicular to both the current and the field.

∴ the force on this side of the coil is $0.96\,\mathrm{N}$.

Example 4 — unscaffolded

A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) moves at $4.0\times10^{5}\,\mathrm{m\,s^{-1}}$ at right angles to a uniform magnetic field of magnitude $0.15\,\mathrm{T}$. Calculate the radius of its circular path and the period of its motion.

The magnetic force provides the centripetal force, $q\,v\,B=\dfrac{m\,v^2}{r}$, so

$$r=\frac{m\,v}{q\,B}=\frac{1.67\times10^{-27}\times4.0\times10^{5}}{1.60\times10^{-19}\times0.15}=2.78\times10^{-2}\,\mathrm{m}.$$

The path is uniform circular motion, so the period is

$$T=\frac{2\pi r}{v}=\frac{2\pi\left(2.78\times10^{-2}\right)}{4.0\times10^{5}}=4.37\times10^{-7}\,\mathrm{s}.$$

∴ the radius is $2.78\times10^{-2}\,\mathrm{m}$ and the period is $4.37\times10^{-7}\,\mathrm{s}$.

Practice questions

Question 1 An electron moves at $5.0\times10^{6}\,\mathrm{m\,s^{-1}}$ perpendicular to a uniform magnetic field of magnitude $0.30\,\mathrm{T}$. (a) Calculate the magnitude of the magnetic force on the electron. (b) State the direction of this force relative to the electron's velocity and to the field. (c) The electron's speed is then doubled. State the new magnitude of the magnetic force.

Question 2 An electron travels at $6.0\times10^{6}\,\mathrm{m\,s^{-1}}$ perpendicular to a uniform magnetic field of magnitude $0.045\,\mathrm{T}$. (a) Show that the magnitude of the magnetic force on the electron is $4.32\times10^{-14}\,\mathrm{N}$. (b) Taking this force as the centripetal force, calculate the radius of the electron's circular path. Give your answer to three significant figures. (c) State what provides the centripetal force on the electron.

Question 3 A straight wire of length $0.25\,\mathrm{m}$ carries a current of $6.0\,\mathrm{A}$ at right angles to a uniform magnetic field of magnitude $0.80\,\mathrm{T}$. (a) Calculate the magnitude of the force on the wire. (b) The wire is then rotated so that the current runs parallel to the field. State the magnitude of the force on the wire now. (c) State the rule used to find the direction of the force on the wire.

Question 4 A solenoid is connected to a battery so that it carries a steady direct current. (a) Draw the magnetic field pattern both inside and outside the solenoid, and mark the north pole. (b) Classify the field inside the solenoid as uniform or non-uniform, and as static or changing. Justify each choice. (c) State two changes that would increase the strength of the field inside the solenoid.

Question 5 A proton enters a uniform magnetic field that is directed into the page. At the instant it enters, the proton is moving to the right (east). (a) State the direction of the magnetic force on the proton at that instant. (b) Describe the shape of the path the proton then follows in the field. (c) Explain why the magnetic force does not change the proton's speed.

Question 6 A coil of 20 turns has one side of length $5.0\,\mathrm{cm}$ in a uniform magnetic field. When a current of $3.0\,\mathrm{A}$ flows and this side is perpendicular to the field, the force on the side is $0.30\,\mathrm{N}$. (a) Calculate the magnetic field strength. (b) The current is then reversed. State the effect on the force on this side of the coil. (c) The coil is rotated until this side is parallel to the field. State the magnitude of the force on the side then.

Question 7 An electron follows a circular path of radius $4.0\,\mathrm{mm}$ in a uniform magnetic field while moving at $3.5 \times 10^{6}\,\mathrm{m\ s^{-1}}$. Calculate the magnetic field strength.

Question 8 A straight conductor of length $0.40\,\mathrm{m}$ lies perpendicular to a uniform magnetic field of magnitude $0.60\,\mathrm{T}$. The force on the conductor is $0.72\,\mathrm{N}$. Calculate the current in the conductor.

Question 9 A particle carrying a charge of $3.2 \times 10^{-19}\,\mathrm{C}$ moves at $1.5 \times 10^{6}\,\mathrm{m\ s^{-1}}$ at right angles to a uniform magnetic field of magnitude $0.20\,\mathrm{T}$. Calculate the magnitude of the magnetic force on the particle.

Question 10 A proton (mass $1.67 \times 10^{-27}\,\mathrm{kg}$) moves at $8.0 \times 10^{5}\,\mathrm{m\ s^{-1}}$ perpendicular to a uniform magnetic field of magnitude $0.50\,\mathrm{T}$. Calculate the radius of the proton's circular path.

Question 11 An electron moves in a circle of radius $1.5\,\mathrm{mm}$ in a uniform magnetic field of magnitude $0.020\,\mathrm{T}$. Calculate the speed of the electron.

Question 12 A coil of 50 turns carries a current of $0.80\,\mathrm{A}$. One side of length $0.12\,\mathrm{m}$ lies perpendicular to a uniform magnetic field of magnitude $0.35\,\mathrm{T}$. Calculate the force on this side of the coil.

Question 13 An electron moves in a circle of radius $2.0\,\mathrm{mm}$ in a uniform magnetic field of magnitude $0.030\,\mathrm{T}$. (a) Show that the speed of the electron is $1.05 \times 10^{7}\,\mathrm{m\ s^{-1}}$. (b) Hence calculate the period of the electron's circular motion.

Question 14 An electron moving at constant speed enters a uniform magnetic field at right angles to the field. Explain, with a clear chain of reasoning, why the electron travels in a circle at constant speed rather than speeding up, slowing down, or continuing in a straight line.

Question 15 A student cuts a bar magnet in half, expecting to obtain a separate north pole and a separate south pole. Explain what actually happens, and state what this shows about magnetism, with reference to monopoles and dipoles.

Question 16 A long straight vertical wire carries a steady conventional current directed upward. (a) Describe the shape and direction of the magnetic field around the wire, referring to the right-hand grip rule. (b) Classify this field as static or changing, and as uniform or non-uniform, justifying each choice.

Question 17 A proton moves at $2.0 \times 10^{6}\,\mathrm{m\ s^{-1}}$ in a direction parallel to a uniform magnetic field of magnitude $0.30\,\mathrm{T}$. Calculate the magnetic force on the proton, and describe the subsequent motion of the proton, explaining your reasoning.

Question 18 An electron and a proton (mass $1.67 \times 10^{-27}\,\mathrm{kg}$) each enter the same uniform magnetic field at the same speed, both perpendicular to the field. Determine the ratio of the radius of the proton's path to the radius of the electron's path, and state how the two paths differ in direction of curvature.

Question 19 A straight wire carrying a current of $4.5\,\mathrm{A}$ is placed perpendicular to a uniform magnetic field of magnitude $0.25\,\mathrm{T}$. The force on the length of wire lying in the field is $0.45\,\mathrm{N}$. Calculate the length of wire that lies in the field.

Question 20 A horizontal wire runs east–west in a uniform magnetic field that is directed vertically downward. When a current flows in the wire, the wire experiences a force directed to the north. (a) Determine the direction of the conventional current in the wire. (b) The current is steady and the external field is produced by fixed magnets. Classify the field acting on the wire as static or changing, and as uniform or non-uniform, justifying each choice.

Question 21 A solenoid connected to a DC supply produces a magnetic field like that of a bar magnet. (a) Explain how the right-hand grip rule is used to identify which end of the solenoid is its north pole. (b) State the effect on the field inside the solenoid of (i) reversing the direction of the current, and (ii) doubling the current. (c) The DC supply is replaced by an AC supply. Classify the field inside the solenoid as static or changing, and explain your answer.

Answers

Question 1 (a) 2.4×10^{-13} N; (b) perpendicular to both the velocity and the field; (c) 4.8×10^{-13} N. **Question 2** (a) 4.32×10^{-14} N; (b) 7.59×10^{-4} m; (c) the magnetic force $q\,v\,B$. **Question 3** (a) 1.2 N; (b) 0 N; (c) the right-hand rule. **Question 4** (b) uniform and static; (c) e.g. increase the current; increase the number of turns. **Question 5** (a) toward the top of the page (north); (b) a circle; (c) the force is perpendicular to the velocity, so it does no work. **Question 6** (a) 0.10 T; (b) the force reverses direction, same magnitude; (c) 0 N. **Question 7** 4.98×10^{-3} T. **Question 8** 3.0 A. **Question 9** 9.6×10^{-14} N. **Question 10** 1.67×10^{-2} m. **Question 11** 5.27×10^{6} m s^{-1}. **Question 12** 1.68 N. **Question 13** (a) 1.05×10^{7} m s^{-1}; (b) 1.19×10^{-9} s. **Question 14** Magnetic force perpendicular to $\vec{v}$ ⇒ no work ⇒ constant speed; constant perpendicular force ⇒ centripetal ⇒ circular path. **Question 15** Two smaller magnets, each with N and S; no magnetic monopole exists — magnetism comes in dipoles. **Question 16** (a) concentric circles in horizontal planes, anticlockwise viewed from above; (b) static and non-uniform. **Question 17** 0 N ; the proton continues in a straight line at constant velocity. **Question 18** $r_p : r_e = 1.83 \times 10^{3}$; the two particles curve in opposite directions. **Question 19** 0.40 m. **Question 20** (a) toward the east; (b) static and uniform. **Question 21** (b) (i) the field reverses (poles swap); (ii) the field strength doubles; (c) changing.

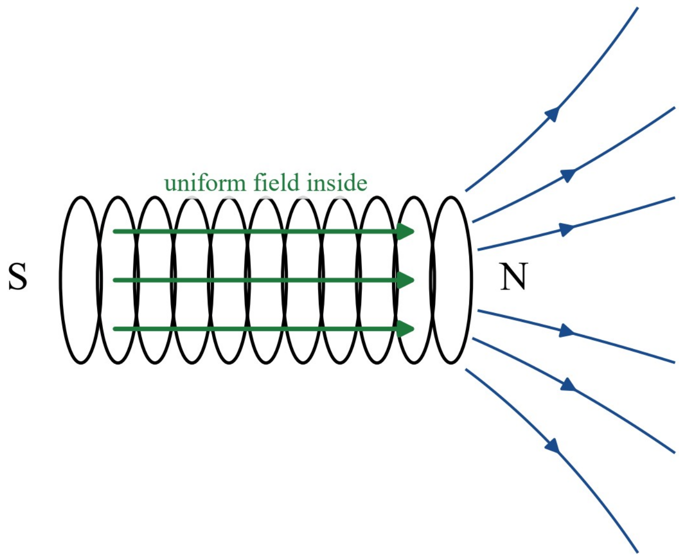
Fully-worked solutions

Question 1 $v = 5.0 \times 10^{6}$ m s^{-1}, $B = 0.30$ T, $q = 1.60 \times 10^{-19}$ C. (a) Perpendicular case: $F = q\,v\,B = 1.60 \times 10^{-19} \times 5.0 \times 10^{6} \times 0.30 = 2.4 \times 10^{-13}$ N. (b) The magnetic force is always perpendicular to both the velocity and the magnetic field. (c) $F = q\,v\,B$ is directly proportional to v, so doubling the speed doubles the force: $F = 4.8 \times 10^{-13}$ N.

Question 2 $v = 6.0 \times 10^{6}$ m s^{-1}, $B = 0.045$ T. (a) $F = q\,v\,B = 1.60 \times 10^{-19} \times 6.0 \times 10^{6} \times 0.045 = 4.32 \times 10^{-14}$ N, as required. (b) The magnetic force is the centripetal force, $F = \frac{m\,v^2}{r}$, so $r = \frac{m\,v^2}{F} = \frac{9.11 \times 10^{-31} \times \left(6.0 \times 10^{6}\right)^2}{4.32 \times 10^{-14}} = 7.59 \times 10^{-4}$ m. (Equivalently $r = \frac{m\,v}{q\,B}$.) (c) The magnetic force $q\,v\,B$ acting perpendicular to the velocity provides the centripetal force.

Question 3 $l = 0.25$ m, $I = 6.0$ A, $B = 0.80$ T, $n = 1$. (a) Perpendicular case: $F = n\,I\,l\,B = 1 \times 6.0 \times 0.25 \times 0.80 = 1.2$ N . (b) When the current is parallel to the field the force is zero: $F = 0$ N. (c) The direction is found using the right-hand rule (fingers along $\vec{B}$, thumb along the conventional current, force out of the palm).

Question 4 (a) Inside the solenoid the field lines are straight, parallel and evenly spaced; outside they spread out and return from the north end to the south end, exactly like a bar magnet. Curling the right hand's fingers along the current identifies the north end.



a solenoid carrying a steady current: uniform field inside, bar-magnet-like field outside (right-hand grip rule sets N)

(b) The field inside is **uniform** — the lines are parallel, equally spaced and in the same direction, so $\vec{B}$ has the same magnitude and direction throughout. It is **static** — the current is a steady direct current, so the field does not change with time.

(c) Any two of: increase the current in the coil; increase the number of turns of wire (turns per unit length). (Reversing the current would swap the poles but not increase the strength.)

Question 5 (a) For a positive charge moving to the right (east) in a field into the page, the right-hand rule gives a force toward the top of the page (**north**). (b) The force stays perpendicular to the velocity and has constant magnitude, so the proton follows a **circular** path. (c) The magnetic force is always perpendicular to the velocity, so it does no work on the proton. With no work done, the proton's kinetic energy — and therefore its speed — cannot change.

Question 6 $n = 20$, $I = 3.0\,\mathrm{A}$, $l = 5.0\,\mathrm{cm} = 0.050\,\mathrm{m}$, $F = 0.30\,\mathrm{N}$. (a) From $F = nIlB$, $B = \frac{F}{nIl} = \frac{0.30}{20 \times 3.0 \times 0.050} = 0.10\,\mathrm{T}$. (b) Reversing the current reverses the direction of the force; its magnitude is unchanged ($0.30\,\mathrm{N}$). (c) With the side parallel to the field the force is zero: $F = 0\,\mathrm{N}$.

Question 7 The magnetic force provides the centripetal force, $qvB = \frac{mv^2}{r}$, so $r = \frac{mv}{qB}$ and

$$B = \frac{mv}{qr} = \frac{9.11 \times 10^{-31} \times 3.5 \times 10^{6}}{1.60 \times 10^{-19} \times 4.0 \times 10^{-3}} = 4.98 \times 10^{-3}\,\mathrm{T}.$$

(Note $4.0\,\mathrm{mm} = 4.0 \times 10^{-3}\,\mathrm{m}$.)

Question 8 For a straight conductor perpendicular to the field, $F = nIlB$ with $n = 1$, so

$$I = \frac{F}{lB} = \frac{0.72}{0.40 \times 0.60} = 3.0\,\mathrm{A}.$$

Question 9 The velocity is perpendicular to the field, so

$$F = qvB = 3.2 \times 10^{-19} \times 1.5 \times 10^{6} \times 0.20 = 9.6 \times 10^{-14}\,\mathrm{N}.$$

Question 10 $qvB = \frac{mv^2}{r}$, so

$$r = \frac{mv}{qB} = \frac{1.67 \times 10^{-27} \times 8.0 \times 10^{5}}{1.60 \times 10^{-19} \times 0.50} = 1.67 \times 10^{-2}\,\mathrm{m}.$$

Question 11 From $r = \frac{m\,v}{q\,B}$, the speed is

$$v = \frac{q\,B\,r}{m} = \frac{1.60 \times 10^{-19} \times 0.020 \times 1.5 \times 10^{-3}}{9.11 \times 10^{-31}} = 5.27 \times 10^{6}\,\mathrm{m\ s^{-1}}.$$

(Note $1.5\,\mathrm{mm} = 1.5 \times 10^{-3}\,\mathrm{m}$.)

Question 12 Perpendicular case with $n = 50$:

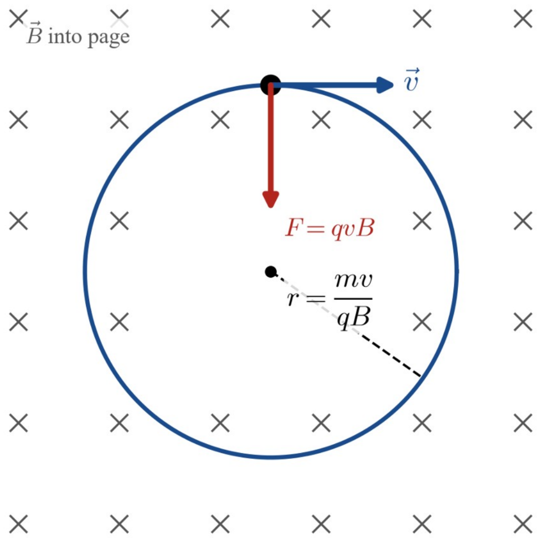
$$F = n\,I\,l\,B = 50 \times 0.80 \times 0.12 \times 0.35 = 1.68\,\mathrm{N}.$$

Question 13 $r = 2.0\,\mathrm{mm} = 2.0 \times 10^{-3}\,\mathrm{m}$, $B = 0.030\,\mathrm{T}$. (a) From $r = \frac{m\,v}{q\,B}$, $v = \frac{q\,B\,r}{m} = \frac{1.60 \times 10^{-19} \times 0.030 \times 2.0 \times 10^{-3}}{9.11 \times 10^{-31}} = 1.05 \times 10^{7}\,\mathrm{m\ s^{-1}}$, as required. (b) The motion is uniform circular motion, so $T = \frac{2\pi r}{v}$. Using the unrounded speed from part (a), $v = 1.0538 \times 10^{7}\,\mathrm{m\ s^{-1}}$,

$$T = \frac{2\pi\left(2.0 \times 10^{-3}\right)}{1.0538 \times 10^{7}} = 1.19 \times 10^{-9}\,\mathrm{s}.$$

(Equivalently $T = \frac{2\pi m}{q\,B}$, which is independent of v.)

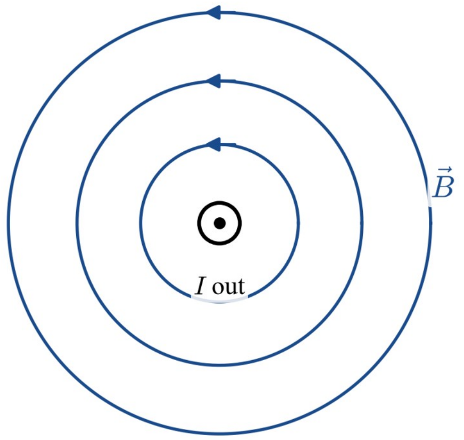
Question 14 The magnetic force on the electron is $F = q\,v\,B$, always directed **perpendicular to the velocity**. A force perpendicular to the motion does **no work**, so the electron's kinetic energy and therefore its **speed stay constant**. A force of constant magnitude that always points at right angles to the velocity is precisely a **centripetal** force, directed toward a fixed centre, so the electron moves in **uniform circular motion** of radius $r = \frac{m\,v}{q\,B}$. It cannot speed up or slow down (no work is done) and it cannot travel in a straight line (a net force always acts).



the magnetic force stays perpendicular to $\vec{v}$, so it does no work: the speed is constant and the path is a circle

Question 15 Cutting the magnet does **not** separate the poles. Each half becomes a complete, smaller bar magnet with **both** a north and a south pole, and the halves could be cut again with the same result. This shows that an isolated magnetic pole — a **monopole** — does not exist: magnetism always occurs as **dipoles**, which is why magnetic field lines always form closed loops rather than starting or ending on a single pole.

Question 16 (a) The field lines are **concentric circles** centred on the wire, lying in horizontal planes perpendicular to the wire. Gripping the wire with the right hand so the thumb points **up** along the conventional current, the fingers curl so that, viewed from above, the field circulates **anticlockwise** around the wire.



cross-section of a straight wire, current out of the page (⊙):
field lines are concentric circles (right-hand grip rule)

(b) The current is steady, so the field is **static** (it does not change with time). The field lines are circles that change direction from point to point and are more closely spaced near the wire, so the field is **non-uniform**.

Question 17 The velocity is **parallel** to the magnetic field, so the magnetic force is

$$F = 0\,\mathrm{N}.$$

With no magnetic force (and no other force stated) acting on it, the proton continues in a **straight line at constant velocity**, by Newton's first law. A magnetic force acts on a charged particle only when it has a component of velocity across the field lines.

Question 18 For a charged particle, $r = \dfrac{mv}{qB}$. The electron and proton have the same speed v, the same magnitude of charge q, and move in the same field B, so $r \propto m$. Hence

$$\frac{r_p}{r_e} = \frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}}{9.11 \times 10^{-31}} = 1.83 \times 10^{3}.$$

The proton's path has about 1830 times the radius of the electron's. Because their charges have opposite sign, the magnetic force is in opposite directions for the two particles, so they **curve in opposite senses** (one clockwise, the other anticlockwise).

Question 19 For the length of wire in the field, $F = nIlB$ with $n = 1$, so

$$l = \frac{F}{IB} = \frac{0.45}{4.5 \times 0.25} = 0.40\,\mathrm{m}.$$

Question 20 (a) Using the right-hand rule with the fingers pointing along $\vec{B}$ (vertically **down**) and the force (F, out of the palm) pointing **north**, the thumb — the conventional current — points to the **east**. So the conventional current flows from west to east. (b) The current is steady and the magnets are fixed, so the field does not change with time: it is **static**. The field is described as uniform (equal magnitude and direction across the wire), so it is **uniform**.

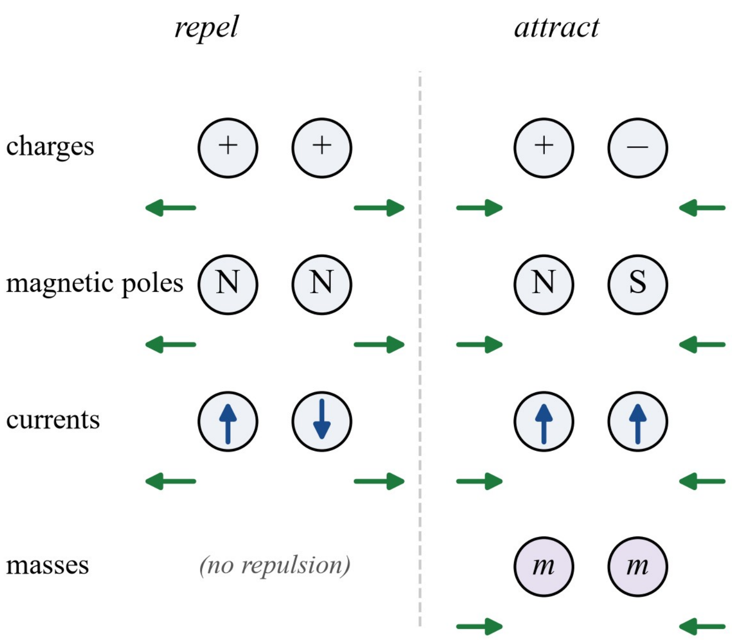
Question 21 (a) Curl the fingers of the right hand around the solenoid in the direction of the **conventional current** in the windings; the **thumb** then points along the internal field toward the end that is the **north pole**. (b) (i) Reversing the current reverses the internal field, so the north and south ends **swap**. (ii) The field strength is proportional to the current, so **doubling the current doubles** the field strength inside the solenoid. (c) With an alternating current the current continually changes size and reverses direction, so the magnetic field inside also continually changes magnitude and reverses direction: the field is **changing** (no longer static).

Theory

This subtopic applies the field ideas of 4A and 4B to two important devices: the **simple DC motor**, which uses the force on a current-carrying coil in a magnetic field to produce continuous rotation, and the **particle accelerator**, which uses electric and magnetic fields to speed up charged particles and steer them around a curved path. It also draws together how two fields interact — when the sources **attract** and when they **repel**.

Interaction of two fields: attraction and repulsion. A field lets one object exert a force on another without contact. The three fields studied in this course differ in an important way:

- **Electric charges** exert forces through electric fields. Like charges (both positive or both negative) **repel**; unlike charges **attract**.
- **Magnetic poles** exert forces through magnetic fields. Like poles (N–N or S–S) **repel**; unlike poles (N–S) **attract**.
- **Current-carrying conductors** set up magnetic fields and so exert magnetic forces on one another. Two parallel wires **attract** when their currents are in the same direction and **repel** when the currents are opposed.
- **Masses** exert forces through gravitational fields, but a gravitational field produces only **attraction**. There is no such thing as a repulsive gravitational force, and there is no negative mass — so masses can **only ever attract** one another.



So electric charges, magnetic poles and current-carrying conductors can each either attract or repel, depending on the arrangement of the sources, whereas masses attract and never repel. This single difference — that charge and magnetism come in two “signs” but mass comes in only one — is what allows electric and magnetic forces to both push and pull, and it underlies the operation of a motor.

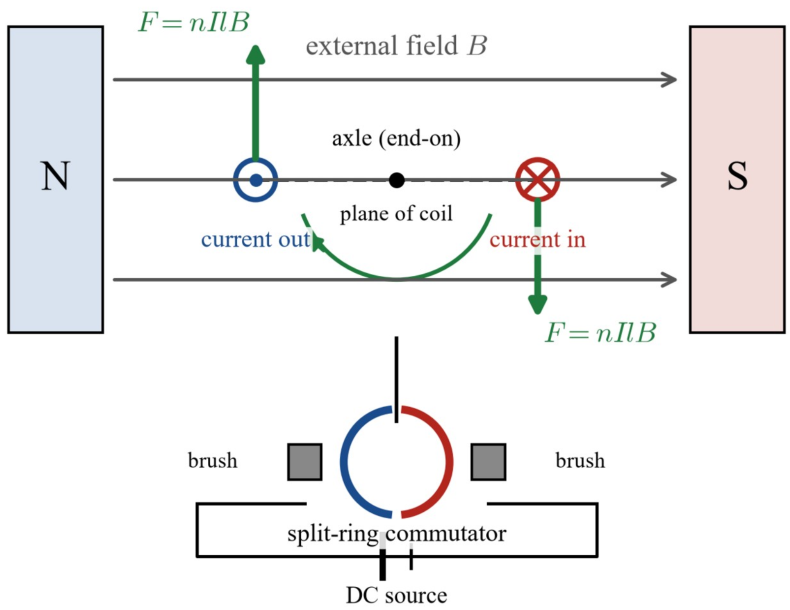
The force on a coil side. From 4B, the magnitude of the force on a straight conductor carrying a current I at right angles to a uniform external field B is

$$F = nIlB,$$

where l is the length of the conductor in the field and n is the number of conducting wires (the number of loops in a coil). The **direction** of this force is given by a hand rule used as a directional convention. Using the **right-hand rule** for conventional current: point the fingers of the right hand along the external field B and the thumb along the current

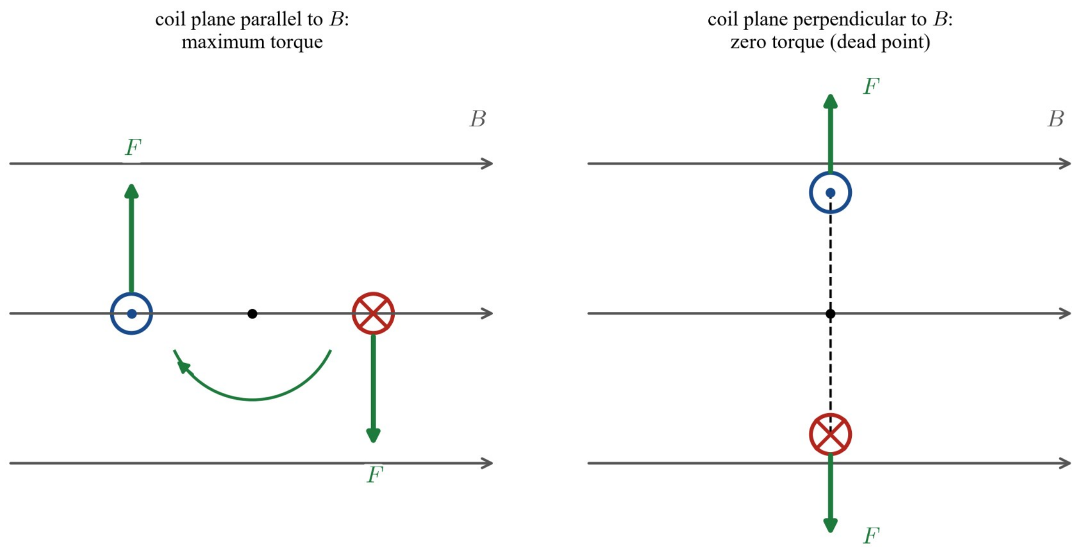
I; the palm then pushes in the direction of the force F. The force is always perpendicular to both the current and the field.

The simple DC motor. A simple DC motor consists of a single **coil** of n loops of wire, free to rotate about an axle (axis) that lies in a **uniform magnetic field** produced by fixed magnets. Direct current is fed to the coil through a **split-ring commutator** and two **brushes**.

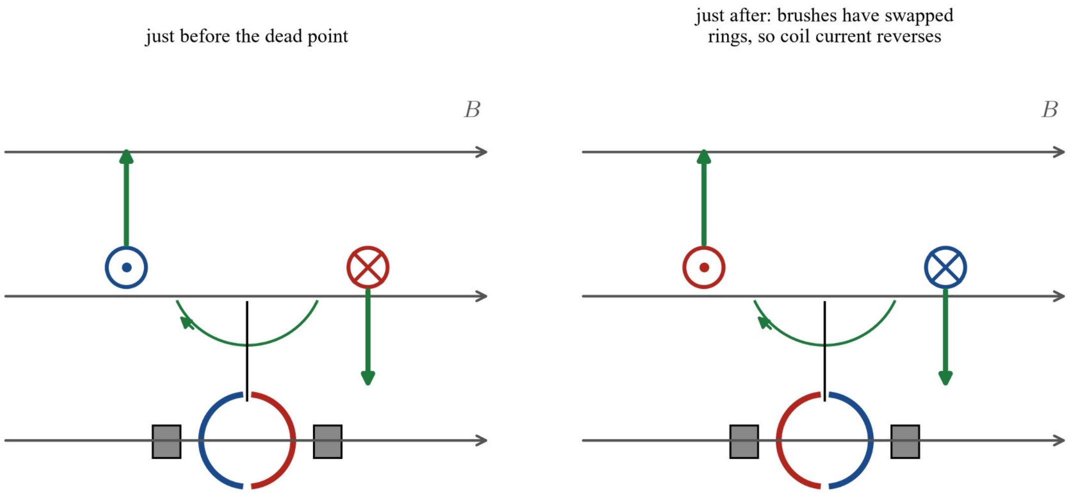


Consider the coil in the position where its plane is *parallel* to the field. The two sides that lie parallel to the axle carry current in **opposite** directions, so the external field pushes one side one way and the opposite side the other way. Each side feels a force of magnitude $F = n\,I\,l\,B$. The two forces are equal in magnitude, opposite in direction and act on opposite sides of the axle, so they do not cancel — instead they form a **couple** that produces a **torque** (turning effect) about the axle, and the coil begins to rotate.

Torque and the "dead point". As the coil turns toward the position where its plane is *perpendicular* to the field, the two side forces come into line with each other (both act along the same line through the axle). At that instant the forces produce **no torque** — this is the **dead point**. The turning effect is greatest when the plane of the coil is **parallel** to the field and falls to zero when the plane is **perpendicular** to the field.



The role of the split-ring commutator. If nothing changed the current, the coil would simply rotate to the dead point, be pushed back, and oscillate to rest. The **split-ring commutator** prevents this. It is a conducting ring divided into two halves, one connected to each end of the coil, turning with the coil against the two stationary brushes. Just as the coil passes the dead point, each half-ring moves from one brush to the other, so the **current through the coil reverses**. Reversing the current reverses the force on each side, so the torque continues to act in the **same rotational sense** rather than opposing the motion. The current is switched every half-turn, and the coil rotates **continuously in one direction**.



Qualitative factors affecting the torque. Because the force on each side is $F = n I l B$, the torque of a simple motor is increased by:

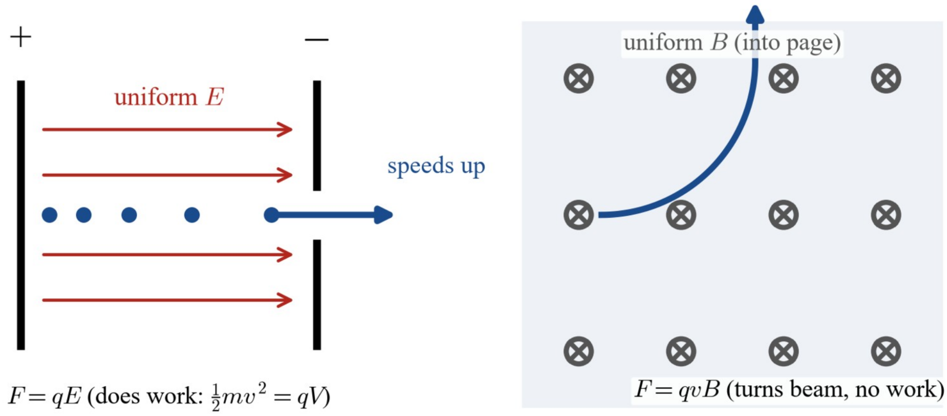
- increasing the **current** I in the coil;
- increasing the strength of the **external magnetic field** B;
- increasing the **number of loops** n in the coil.

(Increasing the area of the coil also increases the torque.) In this course these effects are treated **qualitatively** — you are expected to say *how* the torque changes, not to calculate a numerical torque. For example, doubling the current doubles the force on each side and so doubles the torque; tripling the number of loops triples it.

Particle accelerators. A particle accelerator uses fields to do two separate jobs on a charged particle: an **electric field speeds it up**, and a **magnetic field changes its direction**.

Linear acceleration by a uniform electric field. Between two charged plates a distance d apart with a potential difference V, there is a uniform field of magnitude $E = \frac{V}{d}$. A particle of charge q in this field feels a constant force $F = q E$ parallel to the field, so it accelerates in a **straight line**. The field does work $W = q V$ on the particle, which (starting from rest) appears as kinetic energy:

$$1/2\, m v^2 = q V \Rightarrow v = \sqrt{\frac{2 q V}{m}}.$$

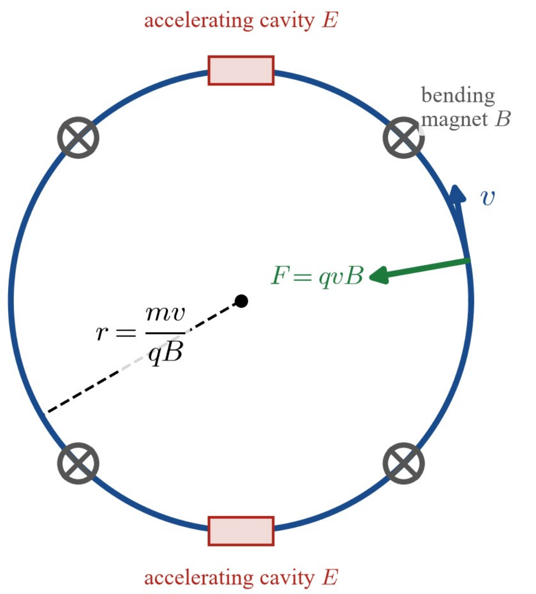


Direction change by a uniform magnetic field. When the particle enters a uniform magnetic field B at right angles to its velocity, it feels a force $F = q\,v\,B$. This force is **always perpendicular to the velocity**, so it does **no work** and cannot change the particle's speed — it changes only the **direction** of motion. The magnetic force acts as the centripetal force, holding the particle on a circular arc:

$$q\,v\,B = \frac{m\,v^2}{r} \Rightarrow r = \frac{m\,v}{q\,B}.$$

Synchrotrons. In a **synchrotron** the particles travel around a ring: bending magnets provide the uniform magnetic field that curves the path, and accelerating cavities apply electric fields that increase the speed. Within this course the motion is **modelled as uniform circular motion**, with the magnetic force providing the centripetal force. The radius of the path is $r = \frac{m\,v}{q\,B}$, and the period of one orbit is

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{q\,B}.$$



model as uniform circular motion; e.g. proton: $r = 4.18 \times 10^{-2}$ m

Throughout this subtopic the electron mass $m_e = 9.11 \times 10^{-31}$ kg and the elementary charge $q_e = 1.60 \times 10^{-19}$ C are used; the mass of a proton, 1.67×10^{-27} kg, is given where it is needed. The circular-motion relation $r = \frac{m v}{q B}$ applies while $v \ll c$, so all speeds here are kept well below the speed of light $c = 3.00 \times 10^{8}$ m s^{-1}.

Worked examples

Example 1 — scaffolded

A simple DC motor has a rectangular coil of 50 loops of wire. Each side of the coil that lies parallel to the axle has length 0.080 m and carries a current of 2.0 A. The coil is in a uniform magnetic field of 0.15 T directed from the N pole to the S pole, and at the instant shown the plane of the coil is parallel to the field. Calculate the magnitude of the force on one side of the coil, and describe the effect of the two side forces on the coil.

Working	Comment
$n = 50$, $l = 0.080$ m, $I = 2.0$ A, $B = 0.15$ T; the sides are perpendicular to B.	List the data; the force law $F = n I l B$ needs $I \perp B$.
$F = n I l B = 50 \times 2.0 \times 0.080 \times 0.15 = 1.2$ N.	Force on one side (the n loops act as n wires each of length l).
The opposite side carries current in the reverse direction, so its force is 1.2 N in the opposite direction.	The two sides carry opposite currents in the same field.
The two equal, opposite forces act on opposite sides of the axle, so they form a couple producing a torque that turns the coil.	Effect: a turning effect (torque), not a net linear force.

Example 2 — scaffolded

In one stage of a particle accelerator an electron, starting from rest, is accelerated through a potential difference of 1200 V and then enters a uniform magnetic field of 0.015 T directed perpendicular to its velocity. Take $q_e = 1.60 \times 10^{-19}$ C and $m_e = 9.11 \times 10^{-31}$ kg. Calculate the speed the electron reaches, and the radius of its circular path in the magnetic field.

Working	Comment
The electric field does work $q V$, which becomes kinetic energy: $1/2 m v^2 = q V$.	A uniform electric field accelerates the charge in a straight line.
$v = \sqrt{\frac{2 q V}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 1200}{9.11 \times 10^{-31}}} = 2.053 \times 10^{7}$ m s .	Speed gained; $v \approx 0.07\, c$, so the model $v \ll c$ holds.
In the field the force $q v B$ is perpendicular to v, so it changes direction only and acts as the centripetal force: $q v B = \frac{m v^2}{r}$.	The magnetic field does no work; it curves the path.
$r = \frac{m v}{q B} = \frac{9.11 \times 10^{-31} \times 2.053 \times 10^{7}}{1.60 \times 10^{-19} \times 0.015} = 7.79 \times 10^{-3}$ m.	Radius of the circular path.

Example 3 — unscaffolded

Explain how the split-ring commutator allows the coil of a simple DC motor to rotate continuously in one direction, rather than oscillating back and forth.

When current flows in the coil, the external magnetic field exerts a force $F = n I l B$ on each of the two sides. The two forces are equal and opposite and act on opposite sides of the axle, so they form a couple: the coil experiences a torque and begins to rotate.

As the coil turns, it approaches the position where its plane is perpendicular to the field. At this dead point the two side forces act along the same line, so the torque is momentarily zero; the coil's inertia carries it through. Just as the coil passes the dead point, each half of the split ring moves from one brush to the other, so the direction of the current in the coil is reversed. Reversing the current reverses the direction of the force on each side, so the torque continues to act in the same rotational sense instead of opposing the motion.

∴ because the commutator reverses the current every half-turn, the torque always drives the coil the same way, and the coil rotates continuously in one direction.

Example 4 — unscaffolded

A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) travels at $4.0\times10^{6}\,\mathrm{m\ s^{-1}}$ around a synchrotron in which a uniform magnetic field of $1.0\,\mathrm{T}$ holds it on a circular path. Modelling the motion as uniform circular motion, calculate the radius of the path, the period of one orbit, and the magnitude of the proton's acceleration.

The magnetic force provides the centripetal force, $q\,v\,B=\frac{m\,v^2}{r}$, so

$$r=\frac{m\,v}{q\,B}=\frac{1.67\times10^{-27}\times4.0\times10^{6}}{1.60\times10^{-19}\times1.0}=4.18\times10^{-2}\,\mathrm{m}.$$

The period is

$$T=\frac{2\pi r}{v}=\frac{2\pi m}{q\,B}=\frac{2\pi\times1.67\times10^{-27}}{1.60\times10^{-19}\times1.0}=6.56\times10^{-8}\,\mathrm{s}.$$

The centripetal acceleration is

$$a=\frac{v^2}{r}=\frac{(4.0\times10^{6})^2}{4.18\times10^{-2}}=3.83\times10^{14}\,\mathrm{m\ s^{-2}},\text{directed toward the centre.}$$

∴ the radius is $4.18\times10^{-2}\,\mathrm{m}$, the period is $6.56\times10^{-8}\,\mathrm{s}$, and the acceleration is $3.83\times10^{14}\,\mathrm{m\ s^{-2}}$ toward the centre of the ring.

Practice questions

Question 1 A simple DC motor has a coil of 60 loops. Each side parallel to the axle is $0.050\,\mathrm{m}$ long and carries a current of $3.0\,\mathrm{A}$ in a uniform field of $0.25\,\mathrm{T}$. The plane of the coil is parallel to the field. (a) Calculate the magnitude of the force on one side of the coil. (b) State the direction of the force on the opposite side relative to the first, and describe the combined effect of the two forces on the coil. (c) The current is then doubled. State the new force on one side of the coil.

Question 2 Two parallel plates $0.040\,\mathrm{m}$ apart have a potential difference of $800\,\mathrm{V}$ across them. An electron ($q_e=1.60\times10^{-19}\,\mathrm{C}$) is between the plates. (a) Calculate the magnitude of the uniform electric field between the plates. (b) Calculate the magnitude of the electric force on the electron. (c) Calculate the work done on the electron as it moves from the negative plate to the positive plate.

Question 3 A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) enters a uniform magnetic field of $0.50\,\mathrm{T}$ at right angles to its velocity, moving at $3.0\times10^{6}\,\mathrm{m\ s^{-1}}$. (a) Calculate the magnitude of the magnetic force on the proton. (b) Calculate the radius of its circular path. (c) Calculate the period of its circular motion.

Question 4 A simple DC motor has a coil that is free to rotate in a uniform magnetic field, fed by a split-ring commutator. (a) State the function of the split-ring commutator. (b) State the position of the coil, relative to the field, at which the torque on the coil is greatest. (c) Explain why the coil experiences no torque at the instant its plane is perpendicular to the field, even though the forces on its sides are not zero.

Question 5 A student is investigating how to increase the torque produced by a simple motor. (a) State three changes to the motor that would each increase the torque. (b) The student increases the number of loops in the coil from 20 to 60, leaving everything else unchanged. State the effect on the torque. (c) Explain why increasing the current increases the torque, referring to the force on the sides of the coil.

Question 6 This question is about the interaction of two fields. (a) (Circle your answer.) Two small spheres carry charges of the same sign. The electric force between them is: *attractive* / *repulsive*. Justify your choice. (b) State whether two long parallel wires carrying current in the same direction attract or repel. (c) State the only kind of interaction that two masses can have through their gravitational fields, and explain how this differs from the interaction between two magnetic poles.

Question 7 A simple DC motor has a coil of 80 loops. Each side parallel to the axle is $0.040\,\mathrm{m}$ long and carries a current of $1.5\,\mathrm{A}$ in a uniform field of $0.30\,\mathrm{T}$. Calculate the force on one side of the coil when the plane of the coil is parallel to the field.

Question 8 Show that an electron accelerated from rest through a potential difference of $1500\,\mathrm{V}$ reaches a speed of $2.30\times10^{7}\,\mathrm{m\ s^{-1}}$. Take $q_e = 1.60\times10^{-19}\,\mathrm{C}$ and $m_e = 9.11\times10^{-31}\,\mathrm{kg}$.

Question 9 The electron in Question 8 then enters a uniform magnetic field of $0.020\,\mathrm{T}$ at right angles to its velocity. Calculate the radius of the circular path it follows.

Question 10 A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$) moves in a circle in the uniform $1.2\,\mathrm{T}$ magnetic field of a synchrotron. Calculate the period of its orbit. Give your answer to three significant figures.

Question 11 A student claims that the coil of a simple DC motor experiences its greatest turning effect at the moment its plane is perpendicular to the magnetic field. Explain why this claim is incorrect, and state the position of the coil at which the torque is actually greatest.

Question 12 In a particle accelerator, a uniform electric field is used to increase a charged particle's speed and a uniform magnetic field is used to change its direction. Explain, with reference to the direction of the force in each case, why the electric field changes the particle's kinetic energy but the magnetic field does not.

Question 13 An electron (mass $9.11\times10^{-31}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) is released in a uniform electric field of magnitude $5.0\times10^{4}\,\mathrm{N\ C^{-1}}$. Calculate the magnitude of the electric force on the electron and its resulting acceleration.

Question 14 A student claims: "Because gravitational, electric and magnetic fields are all described by the same field model, each of them can produce both attraction and repulsion." Explain what is wrong with this claim, referring to charges, magnetic poles and masses.

Question 15 The coil of a simple motor has 50 loops, each side of length $0.10\,\mathrm{m}$, in a uniform field of $0.040\,\mathrm{T}$. Calculate the current required to produce a force of $2.0\,\mathrm{N}$ on each side of the coil.

Question 16 A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) moves at $6.0\times10^{6}\,\mathrm{m\ s^{-1}}$ in a uniform magnetic field of $0.75\,\mathrm{T}$, following a circular path. Calculate the radius of the path and the magnitude of the proton's centripetal acceleration.

Question 17 Describe and explain the motion of the coil of a simple motor if its split-ring commutator is replaced by two continuous slip rings connected to a DC source.

Question 18 Two parallel plates $0.030\,\mathrm{m}$ apart have a potential difference of $900\,\mathrm{V}$ across them. An electron (mass $9.11\times10^{-31}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) starts from rest at the negative plate. Calculate the magnitude of the uniform electric field between the plates, and the speed of the electron when it reaches the positive plate.

Question 19 Two simple motors, X and Y, sit in the same magnetic field, and the same current passes through each coil. The coils are identical in size, but coil X has 40 loops and coil Y has 80 loops. Explain which motor produces the greater torque, referring to the force on the sides of the coil.

Question 20 A proton (mass $1.67\times10^{-27}\,\mathrm{kg}$, charge $1.60\times10^{-19}\,\mathrm{C}$) is accelerated from rest through a potential difference of $2.0\times10^{5}\,\mathrm{V}$. Calculate the kinetic energy gained by the proton and its final speed.

Question 21 Two long, straight, parallel wires carry currents in the same direction and are observed to attract each other. By considering one wire sitting in the magnetic field produced by the other, explain why two current-carrying conductors can either attract or repel one another, whereas two masses can only ever attract.

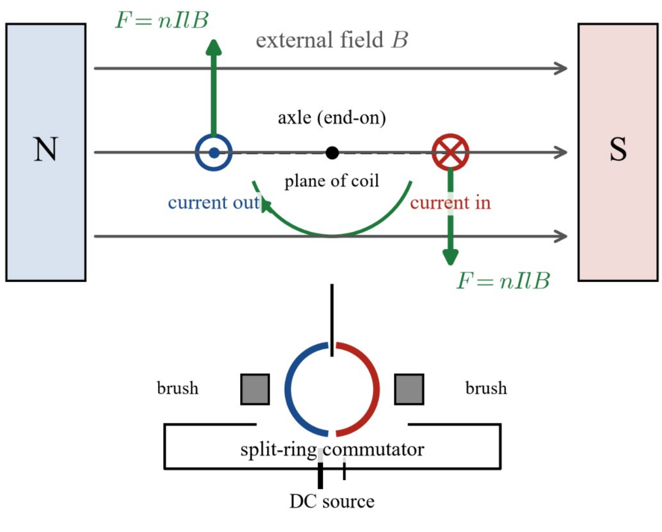
Question 22 A charged particle enters a region of uniform magnetic field and moves along a curved path at constant speed. Explain how the particle can be accelerating even though its speed does not change, and state the direction of its acceleration.

Answers

Question 1 (a) $2.25\,\text{N}$; (b) opposite direction, forming a couple that produces a torque turning the coil; (c) $4.5\,\text{N}$. **Question 2** (a) $2.0\times10^{4}\,\text{N C}^{-1}$; (b) $3.2\times10^{-15}\,\text{N}$; (c) $1.28\times10^{-16}\,\text{J}$. **Question 3** (a) $2.4\times10^{-13}\,\text{N}$; (b) $6.26\times10^{-2}\,\text{m}$; (c) $1.31\times10^{-7}\,\text{s}$. **Question 4** (a) it reverses the current in the coil every half-turn; (b) when the plane of the coil is parallel to the field; (c) the two side forces act along the same line, so they exert no torque. **Question 5** (a) increase the current, the field strength, or the number of loops (or the coil area); (b) the torque is tripled; (c) $F=nIlB\propto I$, so more current means a larger force on each side and hence a larger torque. **Question 6** (a) repulsive; (b) attract; (c) masses only attract, whereas two magnetic poles can either attract (unlike poles) or repel (like poles). **Question 7** $1.44\,\text{N}$. **Question 8** $v=2.30\times10^{7}\,\text{m s}^{-1}$ (shown). **Question 9** $6.55\times10^{-3}\,\text{m}$. **Question 10** $5.47\times10^{-8}\,\text{s}$. **Question 11** Incorrect: the torque is zero when the plane is perpendicular to the field (the dead point); it is greatest when the plane is parallel to the field. **Question 12** The electric force is parallel to the motion and does work (changes E_k); the magnetic force is perpendicular to the velocity and does no work (changes direction only). **Question 13** $8.0\times10^{-15}\,\text{N}$; $8.78\times10^{15}\,\text{m s}^{-2}$. **Question 14** Masses can only attract; charges and magnetic poles can attract or repel, so the claim is wrong for gravity. **Question 15** $10\,\text{A}$. **Question 16** $8.35\times10^{-2}\,\text{m}$; $4.31\times10^{14}\,\text{m s}^{-2}$. **Question 17** The coil turns to the perpendicular position, overshoots and is pushed back, so it oscillates about that position and stops — it does not rotate continuously. **Question 18** $3.0\times10^{4}\,\text{N C}^{-1}$; $1.78\times10^{7}\,\text{m s}^{-1}$. **Question 19** Motor Y: with the same current and twice as many loops, the force $nIlB$ on each side is doubled, so its torque is greater. **Question 20** $3.2\times10^{-14}\,\text{J}$; $6.19\times10^{6}\,\text{m s}^{-1}$. **Question 21** Each wire lies in the other's magnetic field and feels a force $F=nIlB$; reversing one current reverses the force, so the wires can attract or repel. Masses have only one sign, so gravity only attracts. **Question 22** Velocity is a vector; its direction is changing, so the particle accelerates even at constant speed. The acceleration is directed toward the centre of the circle.

Fully-worked solutions

Question 1 $n=60$, $l=0.050\,\text{m}$, $I=3.0\,\text{A}$, $B=0.25\,\text{T}$.



(a) $F=nIlB=60\times3.0\times0.050\times0.25=2.25\,\text{N}$.

(b) The opposite side carries current in the reverse direction, so the field pushes it with an equal force of $2.25\,\text{N}$ in the **opposite** direction. The two forces are equal, opposite and act on opposite sides of the axle, so they form a couple that produces a torque, turning the coil.

(c) $F\propto I$, so doubling the current doubles the force: $F=2\times2.25=4.5\,\text{N}$.

Question 2 $d=0.040\,\text{m}$, $V=800\,\text{V}$, $q=1.60\times10^{-19}\,\text{C}$. (a) $E=\dfrac{V}{d}=\dfrac{800}{0.040}=2.0\times10^{4}\,\text{N C}^{-1}$. (b) $F=qE=1.60\times10^{-19}\times2.0\times10^{4}=3.2\times10^{-15}\,\text{N}$. (c) $W=qV=1.60\times10^{-19}\times800=1.28\times10^{-16}\,\text{J}$.

Question 3 $B = 0.50\,\text{T}$, $v = 3.0 \times 10^6\,\text{m s}^{-1}$, $m = 1.67 \times 10^{-27}\,\text{kg}$, $q = 1.60 \times 10^{-19}\,\text{C}$. (a) $F = q\,v\,B = 1.60 \times 10^{-19} \times 3.0 \times 10^6 \times 0.50 = 2.4 \times 10^{-13}\,\text{N}$. (b) The magnetic force is the centripetal force: $q\,v\,B = \dfrac{m\,v^2}{r}$, so $r = \dfrac{m\,v}{q\,B} = \dfrac{1.67 \times 10^{-27} \times 3.0 \times 10^6}{1.60 \times 10^{-19} \times 0.50} = 6.26 \times 10^{-2}\,\text{m}$. (c) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{q\,B} = \dfrac{2\pi \times 1.67 \times 10^{-27}}{1.60 \times 10^{-19} \times 0.50} = 1.31 \times 10^{-7}\,\text{s}$.

Question 4 (a) The split-ring commutator **reverses the direction of the current in the coil every half-turn**, just as the coil passes through the position where its plane is perpendicular to the field. (b) The torque is greatest when the **plane of the coil is parallel to the field**. (c) When the plane of the coil is perpendicular to the field, the forces on the two sides still act (each is $F = n\,I\,l\,B$), but they now act along the **same line** through the axle. Two equal and opposite forces acting along the same line produce no turning effect, so the torque is zero — this is the dead point.

Question 5 (a) Increase the current I in the coil; increase the strength B of the external field; increase the number of loops n in the coil. (Increasing the coil area would also work.) (b) The force on each side is $F = n\,I\,l\,B \propto n$, so tripling the loops (from 20 to 60) **triples the torque**. (c) The force on each side of the coil is $F = n\,I\,l\,B$, which is proportional to the current I. A larger current therefore produces a larger force on each side, giving a larger couple and hence a larger torque.

Question 6 (a) **Repulsive.** Like charges (charges of the same sign) repel one another. (b) They **attract**. (c) Two masses can **only attract** each other through gravity — a gravitational field never produces repulsion, because there is only one sign of mass. Two magnetic poles, by contrast, can either attract (unlike poles, N–S) or repel (like poles, N–N or S–S), because magnetism has two kinds of pole.

Question 7 $F = n\,I\,l\,B = 80 \times 1.5 \times 0.040 \times 0.30 = 1.44\,\text{N}$.

Question 8 The electric field does work $q\,V$ on the electron, which starts from rest, so $1/2\,m\,v^2 = q\,V$ and

$$v = \sqrt{\frac{2\,q\,V}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 1500}{9.11 \times 10^{-31}}} = \sqrt{5.269 \times 10^{14}} = 2.30 \times 10^7\,\text{m s}^{-1},$$

as required. (This is about $0.08\,c$, so the non-relativistic model is valid.)

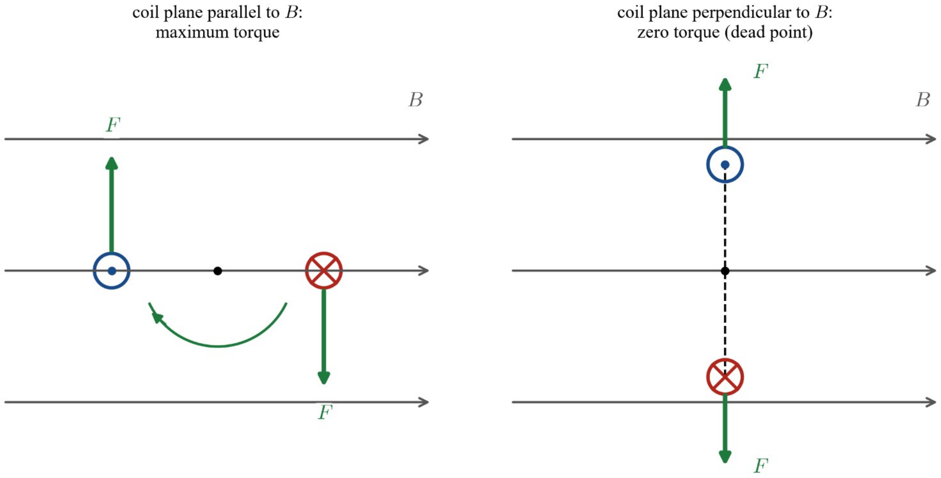
Question 9 Using $v = 2.30 \times 10^7\,\text{m s}^{-1}$ from Question 8, the magnetic force is the centripetal force, so

$$r = \frac{m\,v}{q\,B} = \frac{9.11 \times 10^{-31} \times 2.30 \times 10^7}{1.60 \times 10^{-19} \times 0.020} = 6.55 \times 10^{-3}\,\text{m}.$$

Question 10 The period does not depend on the speed:

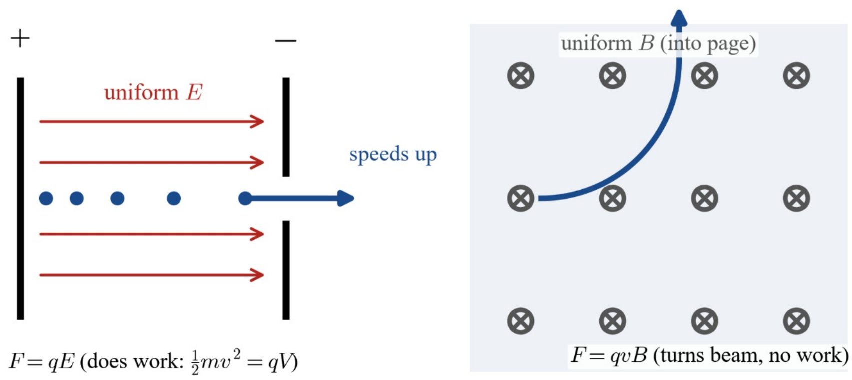
$$T = \frac{2\pi m}{q\,B} = \frac{2\pi \times 1.67 \times 10^{-27}}{1.60 \times 10^{-19} \times 1.2} = 5.47 \times 10^{-8}\,\text{s}\,(\text{to three significant figures}).$$

Question 11



The claim is incorrect. When the plane of the coil is perpendicular to the field, the two side forces act along the same line through the axle, so they produce **no** turning effect — this is the dead point, where the torque is **zero**, not greatest. The torque is greatest when the plane of the coil is **parallel** to the field, because then the two forces are perpendicular to the line joining them and act on opposite sides of the axle, giving the largest couple.

Question 12

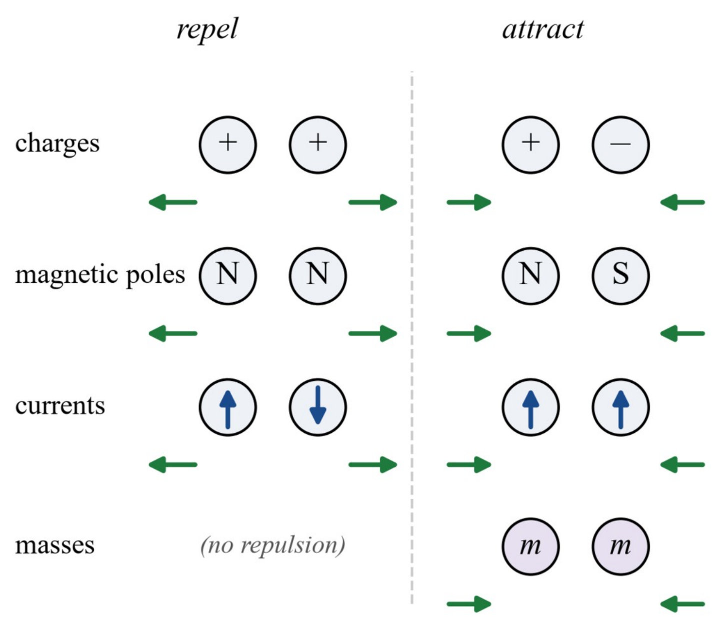


The electric force on the particle, $F = qE$, acts **parallel to the particle's motion** (along the field). A force with a component along the displacement does work, $W = qV$, which changes the particle's kinetic energy — so the electric field speeds the particle up. The magnetic force, $F = qvB$, always acts **perpendicular to the velocity**. A force perpendicular to the motion does **no work**, so it cannot change the particle's speed or kinetic energy; it changes only the **direction** of the velocity. That is why the electric field is used to accelerate the particle and the magnetic field is used to steer it.

Question 13 $F = qE = 1.60 \times 10^{-19} \times 5.0 \times 10^{4} = 8.0 \times 10^{-15}$ N. The acceleration follows from Newton's second law:

$$a = \frac{F}{m} = \frac{8.0 \times 10^{-15}}{9.11 \times 10^{-31}} = 8.78 \times 10^{15}\ \mathrm{m\ s^{-2}}.$$

Question 14

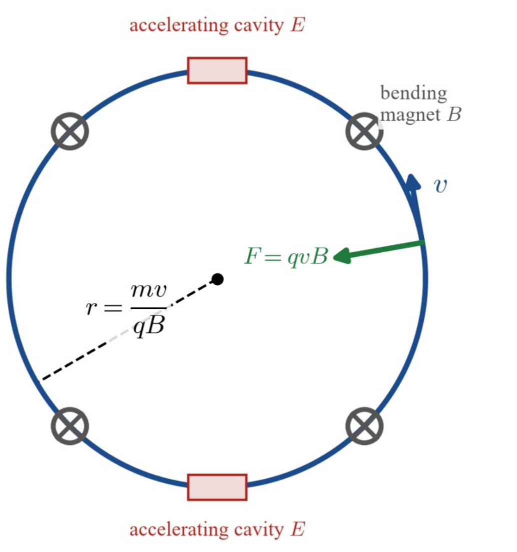


The claim is wrong for gravity. Electric charges can attract (unlike charges) or repel (like charges), and magnetic poles can attract (unlike poles) or repel (like poles), because charge and magnetism each come in two kinds. **Mass comes in only one kind**, and a gravitational field produces only attraction — there is no repulsive gravitational force. So masses can **only attract** one another, and it is incorrect to say that every field can produce both attraction and repulsion.

Question 15 Rearranging $F = n I l B$ for the current, with $F = 2.0\,\text{N}$, $n = 50$, $l = 0.10\,\text{m}$, $B = 0.040\,\text{T}$:

$$I = \frac{F}{n l B} = \frac{2.0}{50 \times 0.10 \times 0.040} = \frac{2.0}{0.20} = 10\,\text{A}.$$

Question 16



model as uniform circular motion; e.g. proton: $r = 4.18 \times 10^{-2}$ m

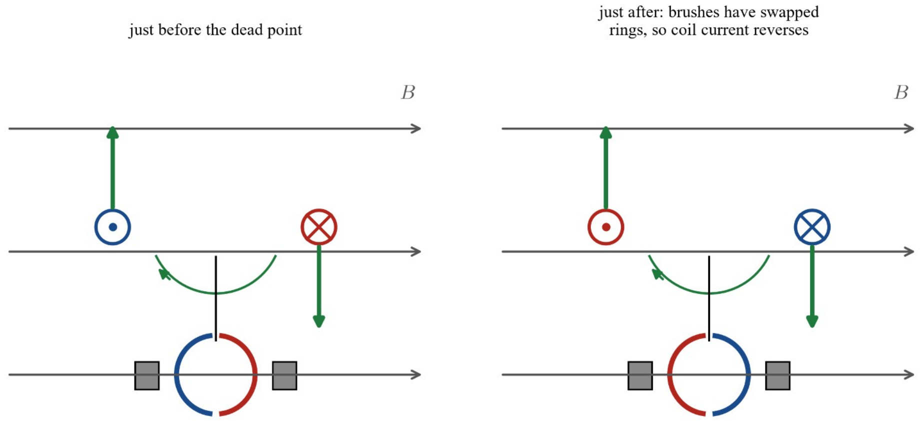
$m = 1.67 \times 10^{-27}\,\text{kg}$, $v = 6.0 \times 10^{6}\,\text{m s}^{-1}$, $B = 0.75\,\text{T}$. The magnetic force provides the centripetal force, $q v B = \dfrac{m v^2}{r}$, so

$$r=\frac{mv}{qB}=\frac{1.67\times10^{-27}\times6.0\times10^{6}}{1.60\times10^{-19}\times0.75}=8.35\times10^{-2}\,\mathrm{m}.$$

The centripetal acceleration is

$$a=\frac{v^2}{r}=\frac{(6.0\times10^{6})^2}{8.35\times10^{-2}}=4.31\times10^{14}\,\mathrm{m\ s^{-2}},\text{ toward the centre.}$$

Question 17



With slip rings the direction of the current in the coil is **never reversed**. Starting with its plane parallel to the field, the coil feels a torque and rotates toward the position where its plane is perpendicular to the field. Its inertia carries it past this dead point, but now — because the current has not reversed — the force on each side pushes the coil **back** the way it came. The coil therefore **oscillates** back and forth about the perpendicular position and, losing energy to friction, comes to rest there. It does not rotate continuously. A split-ring commutator is needed to reverse the current each half-turn and keep the torque acting in the one direction.

Question 18 $d=0.030\,\mathrm{m}$, $V=900\,\mathrm{V}$, $m=9.11\times10^{-31}\,\mathrm{kg}$, $q=1.60\times10^{-19}\,\mathrm{C}$. The field between the plates is

$$E=\frac{V}{d}=\frac{900}{0.030}=3.0\times10^{4}\,\mathrm{N\ C^{-1}}.$$

The field does work qV on the electron, which starts from rest, so $1/2\,mv^2=qV$ and

$$v=\sqrt{\frac{2qV}{m}}=\sqrt{\frac{2\times1.60\times10^{-19}\times900}{9.11\times10^{-31}}}=1.78\times10^{7}\,\mathrm{m\ s^{-1}}.$$

Question 19 The force on each side of a motor coil is $F=nIlB$. Motors X and Y carry the same current I, in the same field B, with coils of the same side length l, so the only difference is the number of loops n. Coil Y has twice as many loops as coil X (80 versus 40), so the force on each side of Y's coil is **twice** that on X's coil. A larger side force gives a larger couple, so ∴ **motor Y produces the greater torque** (twice that of motor X).

Question 20 $m=1.67\times10^{-27}\,\mathrm{kg}$, $q=1.60\times10^{-19}\,\mathrm{C}$, $V=2.0\times10^{5}\,\mathrm{V}$. The kinetic energy gained equals the work done by the field:

$$E_k=qV=1.60\times10^{-19}\times2.0\times10^{5}=3.2\times10^{-14}\,\mathrm{J}.$$

Then $1/2\,mv^2=E_k$, so

$$v=\sqrt{\frac{2E_k}{m}}=\sqrt{\frac{2\times3.2\times10^{-14}}{1.67\times10^{-27}}}=6.19\times10^{6}\,\mathrm{m\ s^{-1}}.$$

(This is about $0.02\,c$, so the non-relativistic model is valid.)

Question 21 Each wire produces a magnetic field in the region around it. The second wire, carrying its own current, sits in that field and so feels a force $F = n I l B$, and by Newton's third law it pushes back on the first wire with an equal and opposite force. Whether the force is attractive or repulsive depends on the **directions of the two currents**: parallel currents (same direction) attract, and antiparallel currents (opposite directions) repel — reversing one current changes an attraction into a repulsion. Two masses, by contrast, have only one kind of "charge" (mass is always positive) and a gravitational field only ever attracts, so two masses can **only attract** — there is no arrangement that makes them repel.

Question 22 Velocity is a vector, so it changes whenever its **direction** changes, even if its magnitude (the speed) stays the same. As the particle follows the curved path its direction is continually changing, so its velocity is changing and it is therefore **accelerating**. The magnetic force $F = q v B$ is always perpendicular to the velocity: it does no work (so the speed is constant) but it continually turns the velocity vector. This force, and hence the acceleration, is directed **toward the centre** of the circular path (it is centripetal).

Topic 2 Test A — Fields and interactions (multiple choice)

Time allowed: 25 minutes · 20 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used.

Choose the response that is correct or that best answers the question. A correct answer scores 1 mark; an incorrect answer scores 0. Marks are not deducted for incorrect answers. No marks are given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this test are not drawn to scale. The constants $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$*,* $M_E = 5.97 \times 10^{24}\,\mathrm{kg}$*,* $R_E = 6.37 \times 10^{6}\,\mathrm{m}$*,* $k = 8.99 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$*,* $q_e = 1.60 \times 10^{-19}\,\mathrm{C}$ *and* $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$ *are provided on the formula sheet.*

Question 1

A space probe moves from a point $2R$ from the centre of a planet to a point $5R$ from the centre, where R is the planet's radius. The gravitational field strength at $5R$, as a fraction of its value at $2R$, is closest to

A. 0.40

B. 2.50

C. 0.16

D. 6.25

Question 2

A satellite is in orbit at an altitude of $1.60 \times 10^{6}\,\mathrm{m}$ above Earth's surface. Which one of the following is closest to the gravitational field strength at the satellite's position?

A. $6.27\,\mathrm{N\,kg^{-1}}$

B. $156\,\mathrm{N\,kg^{-1}}$

C. $9.81\,\mathrm{N\,kg^{-1}}$

D. $4.91\,\mathrm{N\,kg^{-1}}$

Question 3

Which one of the following statements comparing gravitational, electric and magnetic fields is correct?

A. A gravitational field can be made repulsive by using a negative mass.

B. Isolated magnetic monopoles are the usual sources of magnetic fields.

C. Electric charges can only ever attract one another, never repel.

D. Electric charges exist as isolated monopoles of two signs, whereas magnetic poles occur only as dipoles.

Question 4

A $3.0\,\mathrm{kg}$ rock is lifted at constant speed through a vertical height of $4.0\,\mathrm{m}$ near Earth's surface. Which one of the following is closest to the increase in the rock's gravitational potential energy?

A. $12.0\,\mathrm{J}$

B. $118\,\mathrm{J}$

C. $58.9\,\mathrm{J}$

D. $29.4\,\mathrm{J}$

Question 5

A satellite is raised from a circular orbit of radius R to one of radius $3R$ around a planet. The change in its gravitational potential energy is best found from

A. the area under the force–distance graph between R and $3R$.

B. $mg\Delta h$, using the surface value of g.

C. the area under the field–distance graph between R and $3R$, without multiplying by the mass.

D. the gradient of the force–distance graph at radius $3R$.

Question 6

The area under the gravitational field–distance graph for a planet, between two radii, is 3.0×10^{7} J kg^{-1}. A 250 kg satellite is moved outward between these two radii. Which one of the following is closest to the change in the satellite's gravitational potential energy?

A. 3.0×10^{7} J

B. 3.75×10^{9} J

C. 7.5×10^{9} J

D. 1.2×10^{5} J

Question 7

A satellite moves in a circular orbit of radius 9.20×10^{6} m around Earth. Which one of the following is closest to the satellite's orbital speed?

A. 9.30×10^{3} m s^{-1}

B. 6.58×10^{3} m s^{-1}

C. 4.65×10^{3} m s^{-1}

D. 4.33×10^{7} m s^{-1}

Question 8

Two satellites orbit the same planet in circular orbits. Satellite Q has an orbital radius four times that of satellite P. The ratio of the period of Q to the period of P ($T_Q : T_P$) is closest to

A. $2:1$

B. $4:1$

C. $16:1$

D. $8:1$

Question 9

A satellite is in a circular orbit around Earth at a radius where the gravitational field strength is $2.0\,\mathrm{N\ kg^{-1}}$. Which one of the following is closest to the magnitude of the satellite's centripetal acceleration?

A. $2.0\,\mathrm{m\ s^{-2}}$

B. $9.81\,\mathrm{m\ s^{-2}}$

C. $0\,\mathrm{m\ s^{-2}}$

D. It cannot be found without knowing the satellite's mass.

Question 10

A satellite moves at constant speed in a circular orbit around Earth. Which one of the following best explains why the satellite is accelerating?

A. The forces on the satellite are balanced, which keeps its speed constant.

B. The satellite's speed is continually changing as it orbits.

C. The unbalanced gravitational force acts as the net force and, by Newton's second law, produces a centripetal acceleration.

D. By Newton's third law, the satellite pulls on Earth as hard as Earth pulls on it.

Question 11

Which one of the following is closest to the magnitude of the electric field at a point $0.25\,\mathrm{m}$ from an isolated point charge of $+7.0\times10^{-6}\,\mathrm{C}$?

A. $2.52\times10^{5}\,\mathrm{N\ C^{-1}}$

B. $6.29\times10^{4}\,\mathrm{N\ C^{-1}}$

C. $4.03\times10^{6}\,\mathrm{N\ C^{-1}}$

D. $1.01\times10^{6}\,\mathrm{N\ C^{-1}}$

Question 12

Two parallel plates $2.0\,\mathrm{cm}$ apart are connected to a $160\,\mathrm{V}$ supply. Which one of the following is closest to the magnitude of the uniform electric field between the plates?

A. $80\,\mathrm{N\ C^{-1}}$

B. $8.0\times10^{3}\,\mathrm{N\ C^{-1}}$

C. $3.2\,\mathrm{N\ C^{-1}}$

D. $4.0\times10^{3}\,\mathrm{N\ C^{-1}}$

Question 13

A small positive charge is released from rest next to the positive plate of two charged parallel plates and moves across to the negative plate. Which one of the following best describes the changes in the charge's electric potential energy and kinetic energy?

A. Its electric potential energy decreases and its kinetic energy increases.

B. Its electric potential energy increases and its kinetic energy decreases.

C. Both its electric potential energy and its kinetic energy increase.

D. Both remain constant because the field between the plates is uniform.

Question 14

An electron, starting from rest, is accelerated through a potential difference of $750\,\text{V}$. Which one of the following is closest to the electron's final speed?

A. $1.15 \times 10^{7}\,\text{m s}^{-1}$

B. $3.79 \times 10^{5}\,\text{m s}^{-1}$

C. $1.62 \times 10^{7}\,\text{m s}^{-1}$

D. $2.30 \times 10^{7}\,\text{m s}^{-1}$

Question 15

An electron moving at $4.0 \times 10^{6}\,\text{m s}^{-1}$ enters a uniform magnetic field of magnitude $0.050\,\text{T}$ at right angles to the field. Which one of the following is closest to the radius of the electron's circular path?

A. $0.835\,\text{m}$

B. $4.56 \times 10^{-4}\,\text{m}$

C. $1.82 \times 10^{3}\,\text{m}$

D. $2.28 \times 10^{-4}\,\text{m}$

Question 16

An electron travels in a circular path at constant speed in a uniform magnetic field. Which one of the following best explains why the electron's speed does not change?

A. The magnetic force becomes zero once the electron moves at constant speed.

B. The magnetic force does positive and negative work in equal amounts each half-circle.

C. The electric force on the electron balances the magnetic force.

D. The magnetic force is always perpendicular to the velocity, so it does no work on the electron.

Question 17

A straight wire of length $15\,\mathrm{cm}$ carries a current of $4.0\,\mathrm{A}$ at right angles to a uniform magnetic field of magnitude $0.60\,\mathrm{T}$. Which one of the following is closest to the magnitude of the force on the wire?

A. $0.36\,\mathrm{N}$

B. $2.4\,\mathrm{N}$

C. $36\,\mathrm{N}$

D. $0\,\mathrm{N}$

Question 18

In a simple DC motor, the essential function of the split-ring commutator is that

A. it supplies a larger current to the coil.

B. it provides the uniform magnetic field in which the coil rotates.

C. it reverses the current in the coil every half-turn, so the torque continues to act in the same rotational sense.

D. it holds the direction of the current in the coil fixed throughout each rotation.

Question 19

The torque on the coil of a simple DC motor is greatest

A. when the plane of the coil is perpendicular to the magnetic field.

B. when the plane of the coil is parallel to the magnetic field.

C. at the dead point, where the coil is momentarily stationary.

D. when the current in the coil is momentarily zero.

Question 20

The coil of a simple DC motor has 30 loops. Each side that lies parallel to the axle has length $0.080\,\mathrm{m}$ and carries a current of $4.0\,\mathrm{A}$ in a uniform magnetic field of magnitude $0.25\,\mathrm{T}$. At the instant the plane of the coil is parallel to the field, which one of the following is closest to the force on one side of the coil?

A. $0.080\,\mathrm{N}$

B. $30\,\mathrm{N}$

C. $4.8\,\mathrm{N}$

D. $2.4\,\mathrm{N}$

End of Topic 2 Test A

Topic 2 Test A — solutions

Question	Answer	Question	Answer
1	C	11	D
2	A	12	B
3	D	13	A
4	B	14	C
5	A	15	B
6	C	16	D
7	B	17	A
8	D	18	C
9	A	19	B
10	C	20	D

Question 1 — C. $g \propto \frac{1}{r^2}$, so $\frac{g(5R)}{g(2R)} = \left(\frac{2R}{5R}\right)^2 = \frac{4}{25} = 0.16$. **A** ($0.40 = 2/5$) drops the square; **B** (2.50) and **D** (6.25) use the inverted ratio $5/2$ (right ratio, wrong way and/or unsquared).

Question 2 — A. $r = R_E + h = 6.37 \times 10^6 + 1.60 \times 10^6 = 7.97 \times 10^6\,\text{m}$; $g = \frac{GM_E}{r^2} = 6.27\,\text{N kg}^{-1}$. **B** ($156$) is the classic altitude trap — using h alone as r; **C** (9.81) assumes the field is unchanged; **D** (4.91) just halves the surface value.

Question 3 — D. Charge comes in two signs, so isolated ± monopoles exist; magnetic poles have never been found in isolation (dipoles only). **A** is false (no negative mass); **B** is false (no magnetic monopoles); **C** is false (like charges repel).

Question 4 — B. Near the surface the field is uniform, so $E_g = mg\Delta h = 3.0 \times 9.81 \times 4.0 = 118\,\text{J}$. **A** ($12.0$) omits g; **C** (58.9) inserts a spurious $1/2$; **D** (29.4) omits Δh.

Question 5 — A. Over $R \to 3R$ the field is far from uniform, so $mg\Delta h$ (**B**) fails; the change in E_g is the area under the force–distance graph. **C** gives the change *per kilogram* (must be × mass); **D** confuses a gradient with an area.

Question 6 — C. $\Delta E_g = (\text{area}) \times m = 3.0 \times 10^7 \times 250 = 7.5 \times 10^9\,\text{J}$. **A** forgets the mass factor; **D** divides by the mass; **B** inserts a spurious $1/2$.

Question 7 — B. $v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{3.98 \times 10^{14}}{9.20 \times 10^6}} = 6.58 \times 10^3\,\text{m s}^{-1}$. **A** ($9.30 \times 10^3$) uses $\sqrt{2GM/r}$ (an energy-relation conflation); **C** (4.65×10^3) takes the radius as a diameter ($2r$); **D** omits the square root.

Question 8 — D. $T \propto r^{3/2}$, so $\frac{T_Q}{T_P} = 4^{3/2} = 8$. **B** ($4$) assumes $T \propto r$; **C** (16) assumes $T \propto r^2$; **A** (2) assumes $T \propto \sqrt{r}$.

Question 9 — A. The gravitational force is the only (net) force, so $a = \frac{v^2}{r} = \frac{GM}{r^2} = g = 2.0\,\text{m s}^{-2}$ at that radius. **B** uses the surface value; **C** wrongly assumes no acceleration ("no gravity"); **D** is wrong — the satellite's mass cancels.

Question 10 — C. The velocity vector changes direction, so the satellite accelerates; by Newton's second law an unbalanced net force (gravity) produces the centripetal acceleration. **A** and **B** are the classic misconceptions (forces are *not* balanced; the *speed* is constant); **D** invokes the wrong law (Newton's third).

Question 11 — D. $E=\frac{kQ}{r^2}=\frac{8.99\times10^9\times7.0\times10^{-6}}{0.25^2}=1.01\times10^6\,\mathrm{N\,C^{-1}}$. **A** ($2.52\times10^5$) does not square r; **B** (6.29×10^4) drops r^2 entirely; **C** (4.03×10^6) halves r.

Question 12 — B. $E=\frac{V}{d}=\frac{160}{0.020}=8.0\times10^3\,\mathrm{N\,C^{-1}}$. **A** (80) leaves d in centimetres; **C** (3.2) multiplies $V\times d$; **D** (4.0×10^3) uses twice the separation.

Question 13 — A. The field pushes the positive charge from the positive plate toward the negative plate — the direction of motion — so it does positive work: electric potential energy falls and kinetic energy rises. **B**/**C** get the energy directions wrong; **D** ignores that the charge still moves through a potential difference.

Question 14 — C. $v=\sqrt{\frac{2q_eV}{m_e}}=\sqrt{\frac{2\times1.60\times10^{-19}\times750}{9.11\times10^{-31}}}=1.62\times10^7\,\mathrm{m\,s^{-1}}$. **A** ($1.15\times10^7$) omits the factor 2; **D** (2.30×10^7) uses $2V$; **B** (3.79×10^5) uses the proton mass.

Question 15 — B. The magnetic force is centripetal: $r=\frac{m_ev}{q_eB}=\frac{9.11\times10^{-31}\times4.0\times10^6}{1.60\times10^{-19}\times0.050}=4.56\times10^{-4}\,\mathrm{m}$. **A** (0.835) uses the proton mass; **C** (1.82×10^3) uses $\frac{mv^2}{qB}$ (an extra v); **D** (2.28×10^{-4}) uses twice the field.

Question 16 — D. A force perpendicular to the velocity does no work, so kinetic energy — and speed — cannot change (only the direction does). **A** is false (the force is always $qvB\neq0$); **B** misdescribes the work (it is always zero, not cancelling); **C** invents a non-existent electric force.

Question 17 — A. $F=nIlB=1\times4.0\times0.15\times0.60=0.36\,\mathrm{N}$. **B** (2.4) omits the length; **C** (36) leaves the length in centimetres; **D** (0) is the parallel case, but here the current is perpendicular to the field.

Question 18 — C. The split ring reverses the coil current each half-turn (as the coil passes the dead point), so the torque keeps driving the coil the same way — giving continuous rotation. **A**/**B** describe other parts (supply, magnets); **D** describes slip rings, which would let the coil only oscillate.

Question 19 — B. The couple is largest when the two side forces are perpendicular to the line joining them — i.e. when the plane of the coil is parallel to the field. **A**/**C** describe the dead point (zero torque); **D** describes zero current (zero force).

Question 20 — D. $F=nIlB=30\times4.0\times0.080\times0.25=2.4\,\mathrm{N}$. **A** (0.080) omits the number of loops n; **B** (30) omits the length l; **C** (4.8) doubles the single-side force (both sides combined).

Topic 2 Test B — Fields and interactions (written responses)

Time allowed: 50 minutes · 40 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used. In questions where more than one mark is available, appropriate working must be shown.

Answer all questions in the spaces provided. Where a final answer has a required unit, give your answer in that unit. Unless otherwise indicated, the diagrams in this test are not drawn to scale.

Question 1 (5 marks)

The planet Theros has mass 6.4×10^{23} kg and radius 3.4×10^{6} m.

a. Show that the gravitational field strength at the surface of Theros is $3.69\ \text{N kg}^{-1}$. 1 mark

b. Calculate the force due to gravity on a 540 kg lander resting on the surface of Theros. Give your answer in N. 2 marks

c. Determine the distance from the centre of Theros at which the gravitational field strength is $1.00\ \text{N kg}^{-1}$. Give your answer in m. 2 marks

Question 2 (5 marks)

A 500 kg satellite is raised from Earth's surface to a high circular orbit. The area under the gravitational field–distance graph between the two radii is $3.2 \times 10^{7}\ \text{J kg}^{-1}$.

a. Calculate the change in the satellite's gravitational potential energy. Give your answer in J. 2 marks

b. (Circle your answer.) Compared with your answer to part **a.**, the change in gravitational potential energy for a 250 kg satellite raised between the same two radii would be: *larger* / *smaller* / *the same*. Justify your answer. No calculations are required. 2 marks

c. Explain why the change in gravitational potential energy in part **a.** cannot be found using $E_g = m g \Delta h$. 1 mark

Question 3 (6 marks)

A satellite orbits Earth in a circular orbit at an altitude of 1.20×10^{3} km.

a. Show that the orbital radius of the satellite is 7.57×10^{6} m. 1 mark

b. Calculate the orbital speed of the satellite. Give your answer in m s^{-1}. 2 marks

c. Calculate the period of the orbit. Give your answer in s. 2 marks

d. The satellite moves at constant speed, yet it is accelerating. With reference to Newton's second law, explain why, and state the direction of the acceleration. 1 mark

Question 4 (5 marks)

Two horizontal parallel plates are 4.0 cm apart and connected to a 500 V supply.

a. Show that the electric field between the plates is $1.25 \times 10^{4}\ \text{N C}^{-1}$. 1 mark

b. A small droplet carrying a charge of 3.2×10^{-19} C is placed between the plates. Calculate the magnitude of the electric force on the droplet. Give your answer in N. 2 marks

c. An electron is released from rest at the negative plate. Calculate the kinetic energy it gains, in joules, on reaching the positive plate. 2 marks

Question 5 (5 marks)

A proton (mass 1.67×10^{-27} kg, charge 1.60×10^{-19} C) enters a uniform magnetic field of magnitude $0.35\,\text{T}$ at right angles to the field, moving at $3.5\times10^{6}\,\text{m s}^{-1}$.

a. Show that the magnitude of the magnetic force on the proton is $1.96\times10^{-13}\,\text{N}$. 1 mark

b. Calculate the radius of the proton's circular path. Give your answer in m. 2 marks

c. Explain why the magnetic force changes the proton's direction of motion but not its speed. 2 marks

Question 6 (5 marks)

a. Explain the function of the split-ring commutator in a simple DC motor, and state what would happen to the motion of the coil if the commutator were replaced by two continuous slip rings connected to the DC source. 3 marks

b. A student doubles the number of loops in the coil and, in a separate trial, doubles the current in the coil. With reference to $F = nIlB$, state the effect of each change on the torque produced by the motor. 2 marks

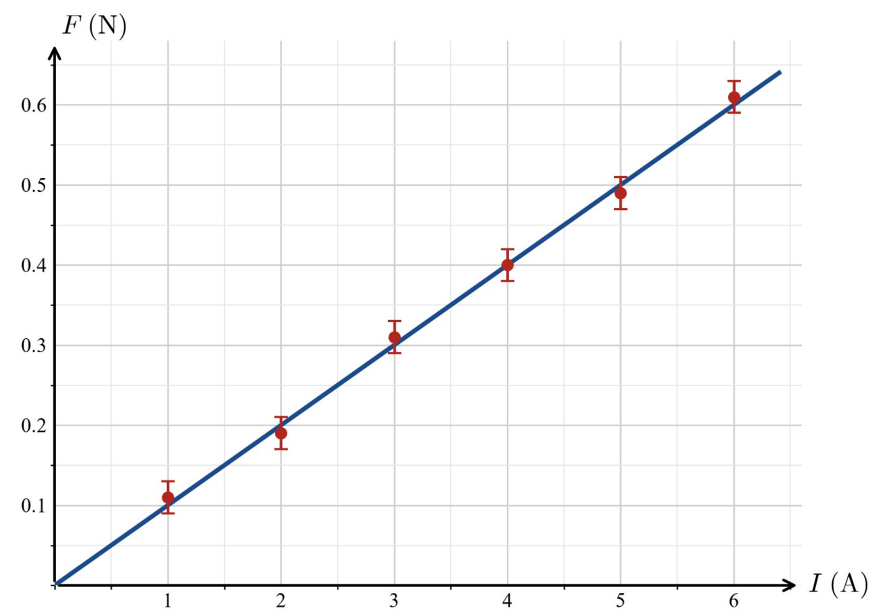
Question 7 (4 marks)

a. Compare gravitational, electric and magnetic fields in terms of (i) whether isolated monopoles exist, and (ii) whether their sources can attract, repel, or both. 3 marks

b. State the only kind of interaction two masses can have through their gravitational fields, and explain why two current-carrying conductors, by contrast, can either attract or repel one another. 1 mark

Question 8 (5 marks)

A student investigates the force F on a straight, horizontal conductor of length $0.25\,\text{m}$ that lies at right angles to a uniform magnetic field, by varying the current I in the conductor and measuring the force on it. The results are plotted below, with uncertainty bars, and a line of best fit is drawn through the origin.



a. Identify the independent variable and the dependent variable in this investigation. 1 mark

b. Using two well-separated points on the line of best fit, determine the gradient of the line. Give your answer in N A^{-1}. 2 marks

c. The conductor is a single wire ($n = 1$). Using the gradient from part **b.**, determine the magnitude of the magnetic field strength B. Give your answer in T. 2 marks

End of Topic 2 Test B

Topic 2 Test B — solutions

Question 1 $M=6.4\times10^{23}\,\text{kg},\ R=3.4\times10^{6}\,\text{m}$.

(a) $g=\dfrac{GM}{R^2}=\dfrac{(6.67\times10^{-11})(6.4\times10^{23})}{(3.4\times10^{6})^2}=\dfrac{4.269\times10^{13}}{1.156\times10^{13}}=3.69\,\text{N kg}^{-1}$, as required. **(1 mark: correct substitution and result.)**

(b) $F_g=mg=540\times3.6927=1.99\times10^{3}\,\text{N}$, directed toward the centre of Theros. **(1 mark** $F_g=mg$ used; **1 mark** correct value $1.99\times10^{3}\,\text{N}$.)

(c) Rearranging $g=\dfrac{GM}{r^2}$ for r:

$$r=\sqrt{\frac{GM}{g}}=\sqrt{\frac{(6.67\times10^{-11})(6.4\times10^{23})}{1.00}}=\sqrt{4.269\times10^{13}}=6.53\times10^{6}\,\text{m}.$$

∴ the field strength is $1.00\,\text{N kg}^{-1}$ at $6.53\times10^{6}\,\text{m}$ from the centre (about twice the planet's radius). **(1 mark** rearrangement; **1 mark** correct value.)

Question 2

(a) The area under the field–distance graph is the change in gravitational potential energy **per kilogram**, so

$$\Delta E_g=(\text{area})\times m=3.2\times10^{7}\times500=1.6\times10^{10}\,\text{J}.$$

(1 mark $\Delta E_g=\text{area}\times m$; **1 mark** correct value.)

(b) **Smaller.** The change in gravitational potential energy is proportional to the mass moved ($\Delta E_g\propto m$, because the force at every radius is proportional to m). The $250\,\text{kg}$ satellite has half the mass, so between the same two radii it gains half the gravitational potential energy — a smaller change. **(1 mark** circles *smaller*; **1 mark** justification via $\Delta E_g\propto m$. The circled option alone scores 0 without the reasoning.)

(c) $E_g=mg\Delta h$ assumes the gravitational field strength g is **constant**, which is only true very close to the surface. Over the large rise to a high orbit, $g=\dfrac{GM}{r^2}$ falls appreciably, so g is not constant and $mg\Delta h$ (which would use the surface value throughout) does not apply — the area-under-the-graph method must be used instead. **(1 mark: field not uniform / g varies over the range.)**

Question 3

(a) $h=1.20\times10^{3}\,\text{km}=1.20\times10^{6}\,\text{m}$, so

$$r=R_E+h=6.37\times10^{6}+1.20\times10^{6}=7.57\times10^{6}\,\text{m},$$

as required. (The altitude must be converted to metres and added to R_E.) **(1 mark: km → m and $r=R_E+h$.)**

(b) The gravitational force provides the net (centripetal) force, so $\dfrac{GM_E m}{r^2}=\dfrac{mv^2}{r}$ and

$$v=\sqrt{\frac{GM_E}{r}}=\sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{7.57\times10^{6}}}=7.25\times10^{3}\,\text{m s}^{-1}.$$

(1 mark $v=\sqrt{GM_E/r}$; **1 mark** correct value.)

(c) $T = \frac{2\pi r}{v} = \frac{2\pi(7.57\times10^6)}{7.25\times10^3} = 6.56\times10^3\,\text{s}$ (about 109 minutes). **(1 mark** method; **1 mark** correct value.
Equivalently $T = 2\pi\sqrt{r^3/GM_E} = 6.56\times10^3\,\text{s}$.)

(d) The satellite's speed is constant, but its velocity — a vector — continually changes **direction**, so the satellite is accelerating. By Newton's second law, $\Sigma F = ma$, this acceleration is produced by the unbalanced gravitational force acting as the net force. The acceleration is directed **toward Earth's centre** (centripetal). **(1 mark: changing-direction velocity + unbalanced gravitational force via N2, directed toward Earth's centre.)**

Question 4

(a) $E = \frac{V}{d} = \frac{500}{0.040} = 1.25\times10^4\,\text{N C}^{-1}$, as required (with $d = 4.0\,\text{cm} = 0.040\,\text{m}$). **(1 mark: correct conversion and result.)**

(b) $F = qE = 3.2\times10^{-19}\times1.25\times10^4 = 4.0\times10^{-15}\,\text{N}$. **(1 mark** $F = qE$; **1 mark** correct value.)

(c) All the work done by the field becomes kinetic energy:

$$E_k = W = q_e V = 1.60\times10^{-19}\times500 = 8.0\times10^{-17}\,\text{J}.$$

∴ the electron gains $8.0\times10^{-17}\,\text{J}$ ($=500\,\text{eV}$) of kinetic energy. **(1 mark** $E_k = q_e V$; **1 mark** correct value.)

Question 5

(a) The velocity is perpendicular to the field, so

$$F = qvB = 1.60\times10^{-19}\times3.5\times10^6\times0.35 = 1.96\times10^{-13}\,\text{N},$$

as required. **(1 mark: correct substitution and result.)**

(b) The magnetic force provides the centripetal force, $qvB = \frac{mv^2}{r}$, so

$$r = \frac{mv}{qB} = \frac{1.67\times10^{-27}\times3.5\times10^6}{1.60\times10^{-19}\times0.35} = 1.04\times10^{-1}\,\text{m}.$$

(1 mark $r = \frac{mv}{qB}$; **1 mark** correct value.)

(c) The magnetic force $F = qvB$ is always **perpendicular to the velocity**. A force perpendicular to the motion does **no work**, so the proton's kinetic energy — and therefore its speed — cannot change. The force does, however, continually change the **direction** of the velocity, acting as a centripetal force that curves the path into a circle. **(1 mark** force perpendicular to velocity ⇒ no work ⇒ constant speed; **1 mark** force continually changes direction ⇒ curved/circular path.)

Question 6

(a) When current flows, the external field exerts a force $F = nIlB$ on each side of the coil; the two side forces are equal, opposite and act on opposite sides of the axle, forming a couple that turns the coil. The **split-ring commutator reverses the current in the coil every half-turn**, just as the coil passes the dead point (plane perpendicular to the field), so the force on each side reverses and the torque keeps acting in the **same rotational sense** — the coil rotates continuously in one direction. With two **slip rings**, the current in the coil is never reversed, so past the dead point the side forces push the coil **back**: the coil merely **oscillates** about the perpendicular position and comes to rest — it does not rotate continuously. **(1 mark** commutator reverses current each half-turn; **1 mark** so torque stays in the same sense ⇒ continuous rotation; **1 mark** slip rings ⇒ oscillates/stops, no continuous rotation.)

(b) The force on each side is $F = nIlB$, and the torque is proportional to this force. Doubling the number of loops n **doubles** the force on each side and hence **doubles the torque**; doubling the current I likewise **doubles** the force and so **doubles the torque**. **(1 mark** doubling loops doubles torque; **1 mark** doubling current doubles torque — both via $F = nIlB$.)

Question 7

- **Gravitational field:** mass acts as a **monopole** source, but there is only one kind of mass, so gravity can **only attract** — masses never repel.
- **Electric field:** charge comes in **two signs**, so isolated **monopoles** of either sign exist; like charges **repel** and unlike charges **attract** (both attraction and repulsion).
- **Magnetic field:** isolated magnetic **monopoles do not exist** — poles occur only as **dipoles** (N–S pairs); like poles **repel** and unlike poles **attract** (both).

(1 mark gravity: monopole source, attract only; **1 mark** electric: two-sign monopoles, attract and repel; **1 mark** magnetic: no monopoles/dipoles only, attract and repel.)

(b) Two masses can **only attract** each other. Two current-carrying conductors can attract or repel because each sits in the magnetic field of the other and feels a force $F = nIlB$ whose direction depends on the current directions: parallel currents attract, antiparallel currents repel. Reversing one current reverses the force, which is impossible for gravity because mass comes in only one sign. **(1 mark: masses attract only; currents attract or repel depending on current directions.)**

Question 8

(a) The **independent variable** is the current I in the conductor (the quantity the student deliberately varies); the **dependent variable** is the force F on the conductor (the quantity measured in response). **(1 mark: both variables correctly identified.)**

(b) Using two well-separated points **on the line of best fit** — for example $(1.0\,\mathrm{A}, 0.10\,\mathrm{N})$ and $(5.0\,\mathrm{A}, 0.50\,\mathrm{N})$:

$$\text{gradient} = \frac{\Delta F}{\Delta I} = \frac{0.50 - 0.10}{5.0 - 1.0} = \frac{0.40}{4.0} = 0.10\,\mathrm{N\,A^{-1}}.$$

(1 mark two on-line, well-separated points used (not raw data points); **1 mark** correct gradient $0.10\,\mathrm{N\,A^{-1}}$.)

(c) For a single conductor perpendicular to the field, $F = nIlB = IlB$ (with $n = 1$), so a graph of F against I has gradient lB. Hence

$$B = \frac{\text{gradient}}{nl} = \frac{0.10}{1 \times 0.25} = 0.40\,\mathrm{T}.$$

$\therefore$ the magnetic field strength is $0.40\,\mathrm{T}$. **(1 mark** gradient $= lB$ identified; **1 mark** correct value $0.40\,\mathrm{T}$.)