Physics Units 3 & 4

Chapter 3 — Gravitational fields and orbits

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Theory

The field model. Two masses attract each other even though nothing touches and nothing passes between them. Physics explains this action-at-a-distance with a **field model**: any mass alters the space around it, filling that space with a **gravitational field**, and a second mass placed in the field feels a **force due to gravity** at its own location. The field is the messenger — the first mass sets up the field everywhere, and the second mass responds only to the field where it sits.

The strength of the field at a point is the **gravitational field strength** g, defined as the force due to gravity per unit mass on a small test mass placed there:

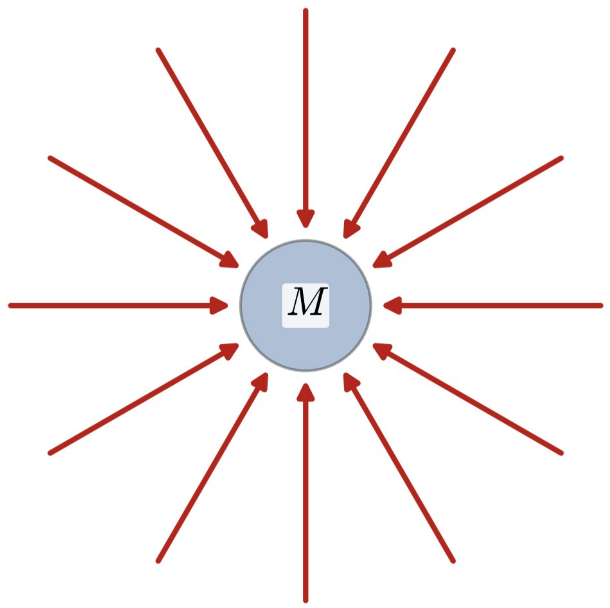
$$g = \frac{F_g}{m}, \text{so } F_g = m g.$$

g is a **vector**: it has a magnitude and a direction (the direction of the force due to gravity on a mass placed at that point). Its unit is the newton per kilogram, and because $F_g = m g$ has the form of $\Sigma F = m a$, that unit is equivalent to an acceleration: $1\,\mathrm{N\,kg^{-1}} = 1\,\mathrm{m\,s^{-2}}$.

Field of a point (or spherical) mass. A single mass M produces a field that, at a distance r from its centre, has magnitude

$$g = \frac{G M}{r^2},$$

where $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ is the universal gravitational constant. For a uniform sphere the field outside is exactly the same as if all the mass were concentrated at the centre, so r is always measured **from the centre** of the body, not from its surface. The field is directed **radially inward**, toward the centre of M — a mass placed anywhere in the field is pulled straight toward M.



field lines point toward the mass;
spacing widens as r increases (field weakens)

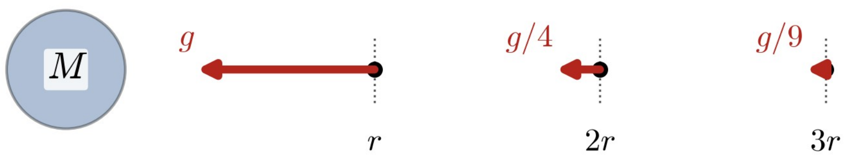
The force between two masses. Applying $F_g = m g$ with $g = \frac{G M}{r^2}$ to a second mass m gives Newton's law of universal gravitation:

$$F_g = \frac{G m_1 m_2}{r^2},$$

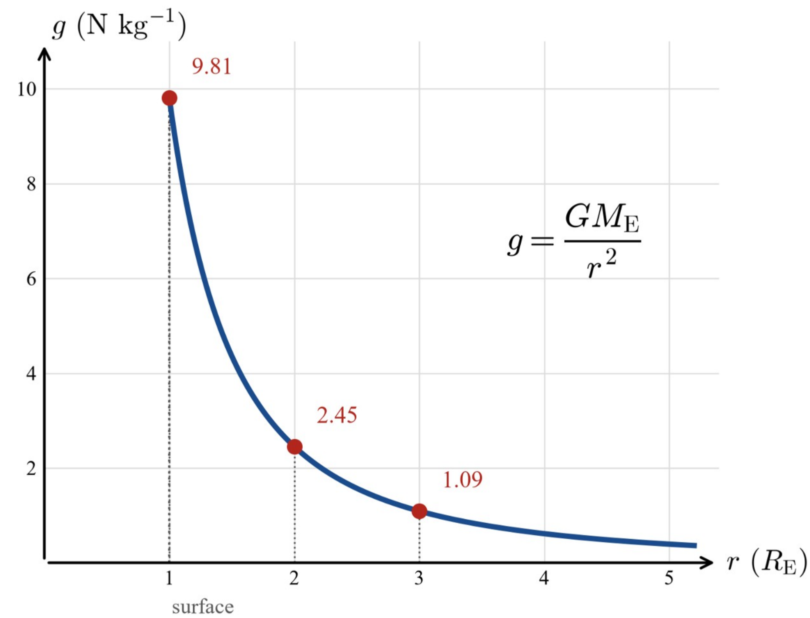
the force each of two masses m_1 and m_2 exerts on the other when their centres are a distance r apart. The two forces are equal in magnitude and opposite in direction (an F on A by B / F on B by A pair), and they are always **attractive**: masses only ever pull, never push. This is a key difference from electric charges and magnetic poles, which can attract *or* repel. Because a single mass produces only an inward field, gravity has **monopoles** but no repulsive counterpart — there is no gravitational equivalent of two like charges pushing apart.

Inverse-square behaviour. Both g and F_g depend on $\frac{1}{r^2}$, so the field weakens rapidly with distance. Doubling the distance from the centre reduces the field to $(1/2)^2 = 1/4$ of its value; tripling it reduces the field to $1/9$. The field never reaches zero at any finite distance, but it falls away steeply.

$$g \propto \frac{1}{r^2}$$



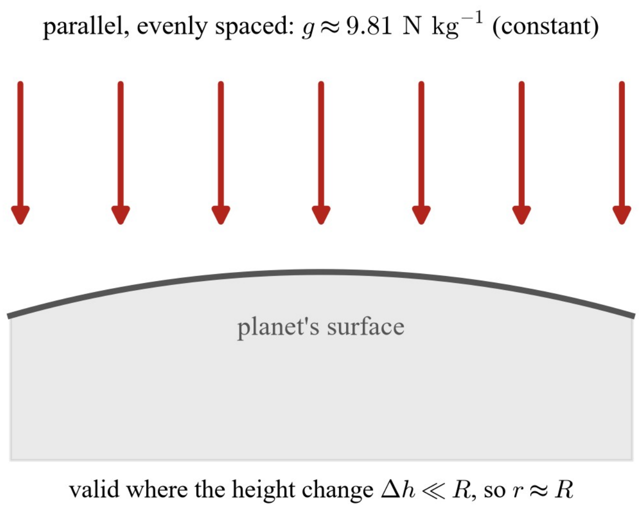
The graph of g against r for Earth shows this shape directly. Using $M_E = 5.97 \times 10^{24}$ kg and $R_E = 6.37 \times 10^6$ m, the field at the surface ($r = R_E$) is $9.81\ \text{N kg}^{-1}$; at $r = 2R_E$ it has dropped to $2.45\ \text{N kg}^{-1}$ (a quarter), and at $r = 3R_E$ to $1.09\ \text{N kg}^{-1}$ (a ninth).



Shape and direction of the field (field lines). A field is mapped with **field lines** drawn in the direction of the force due to gravity on a test mass. For an isolated mass the lines are straight, radial, and directed inward. Two rules make the lines quantitative:

- the **direction** of the field at any point is the direction of the field line there;
- the **spacing** of the lines shows the magnitude — lines are crowded together where the field is strong (close to the mass) and spread apart where it is weak (far away).

The uniform-field approximation near a surface. Close to the surface of a large body, over a region whose size is tiny compared with the body's radius, the radial field lines are so nearly parallel and so nearly equally spaced that the field can be treated as **uniform** — constant in both magnitude and direction. Near Earth's surface this constant value is $g = 9.81\,\mathrm{N\ kg^{-1}}$, directed vertically downward.

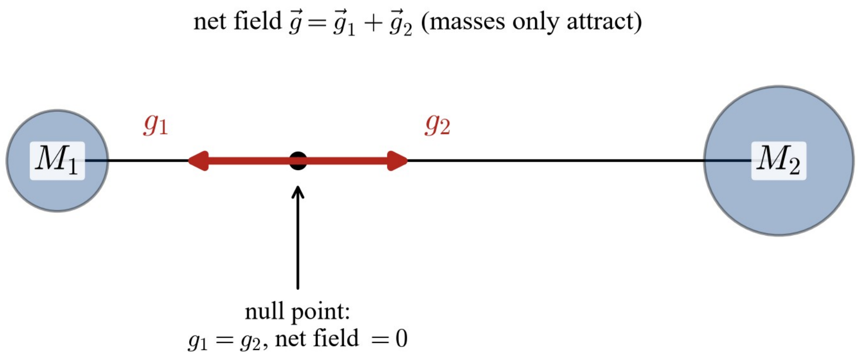


The approximation is good because $g = \frac{GM}{r^2}$ changes only when r changes appreciably. For a height change Δh that is very small compared with the radius R ($\Delta h \ll R$), the distance from the centre stays $r \approx R$, so g barely changes. Climbing a $100\,\mathrm{m}$ tower changes r by about one part in $64\,000$ of R_E, an utterly negligible change in g. The field is only *approximately* uniform, however — over distances comparable with R the inverse-square weakening cannot be ignored.

Classifying fields. Any field can be described in two independent ways:

- **static** (unchanging in time) or **changing** — the field of a fixed mass is static;
- **uniform** (same magnitude and direction everywhere) or **non-uniform** — the radial field of a point mass is non-uniform, while the near-surface approximation above is uniform.

Adding fields. When more than one mass is present, the gravitational field at a point is the **vector sum** of the fields each mass would produce there on its own, $\vec{g} = \vec{g}_1 + \vec{g}_2 + \cdots$. Somewhere along the line joining two masses the two fields point in opposite directions and can exactly cancel: this is a **null point**, where the net gravitational field is zero. Setting the magnitudes equal, $\frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2}$ locates it. The null point always lies **closer to the smaller mass**, because a weaker source must be approached more closely for its field to match the stronger one.



Throughout this subtopic the constants $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, $M_E = 5.97 \times 10^{24}\,\mathrm{kg}$, $R_E = 6.37 \times 10^{6}\,\mathrm{m}$ and (near Earth's surface) $g = 9.81\,\mathrm{N\,kg^{-1}}$ are used.

Worked examples

Example 1 — scaffolded

The planet Veros has mass $4.80 \times 10^{24}\,\mathrm{kg}$ and radius $5.20 \times 10^{6}\,\mathrm{m}$. Calculate the gravitational field strength at its surface, the force due to gravity on a $75\,\mathrm{kg}$ lander resting on the surface, and state the direction of the field.

Working	Comment
$M = 4.80 \times 10^{24}\,\mathrm{kg}$, $r = R = 5.20 \times 10^{6}\,\mathrm{m}$, $m = 75\,\mathrm{kg}$.	List the data; at the surface r equals the planet's radius.
$g = \dfrac{GM}{r^2} = \dfrac{(6.67 \times 10^{-11})(4.80 \times 10^{24})}{(5.20 \times 10^{6})^2}$.	Field strength of a spherical mass, r from the centre.
$= 11.8\,\mathrm{N\,kg^{-1}}$.	Evaluate.
$F_g = mg = 75 \times 11.84 = 888\,\mathrm{N}$.	Force due to gravity on a mass in the field.
The field points radially inward, toward the centre of Veros.	Gravitational fields are always directed toward the source mass.

Example 2 — scaffolded

Two spherical asteroids of masses $2.0 \times 10^{12}\,\mathrm{kg}$ and $3.5 \times 10^{12}\,\mathrm{kg}$ drift with their centres $8.0 \times 10^{3}\,\mathrm{m}$ apart. Calculate the gravitational force between them. The asteroids then drift apart until their separation is three times as large; calculate the new force.

Working	Comment
$m_1 = 2.0 \times 10^{12}\,\mathrm{kg}$, $m_2 = 3.5 \times 10^{12}\,\mathrm{kg}$, $r = 8.0 \times 10^{3}\,\mathrm{m}$.	List the data.
$F_g = \dfrac{G m_1 m_2}{r^2} = \dfrac{(6.67 \times 10^{-11})(2.0 \times 10^{12})(3.5 \times 10^{12})}{(8.0 \times 10^{3})^2}$.	Newton's law of universal gravitation.
$= 7.3 \times 10^{6}\,\mathrm{N}$ (attractive, on each asteroid toward the other).	Evaluate; the force is attractive.
Tripling r multiplies F_g by $\dfrac{1}{3^2} = \dfrac{1}{9}$.	Inverse-square dependence on separation.
$F_g' = \dfrac{7.30 \times 10^{6}}{9} = 8.11 \times 10^{5}\,\mathrm{N}$.	New force at three times the separation.

Example 3 — unscaffolded

A satellite-tracking station is being planned for a platform at an altitude of $3.60 \times 10^{6}\,\mathrm{m}$ above Earth's surface. Calculate the gravitational field strength at that altitude.

The field strength depends on the distance from Earth's **centre**, so the altitude must be added to Earth's radius:

$$r = R_E + h = 6.37 \times 10^{6} + 3.60 \times 10^{6} = 9.97 \times 10^{6}\,\mathrm{m}.$$

Then

$$g = \frac{G M_E}{r^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(9.97 \times 10^{6})^2} = 4.01\,\mathrm{N\,kg^{-1}}.$$

∴ the gravitational field strength at that altitude is $4.01\,\text{N kg}^{-1}$, directed toward Earth's centre. (Adding the altitude to R_E is essential — using h alone would be badly wrong.)

Example 4 — unscaffolded

Two stars have masses $M_1 = 2.0\times10^{30}\,\text{kg}$ and $M_2 = 8.0\times10^{30}\,\text{kg}$ and their centres are $3.0\times10^{11}\,\text{m}$ apart. Find the point on the line between them at which the net gravitational field is zero.

At the null point, a distance x from M_1 (and $d-x$ from M_2), the two field magnitudes are equal and the fields point in opposite directions:

$$\frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2} \Rightarrow \frac{d-x}{x} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{8.0\times10^{30}}{2.0\times10^{30}}} = 2.$$

So $d-x = 2x$, giving $x = \frac{d}{3} = \frac{3.0\times10^{11}}{3} = 1.0\times10^{11}\,\text{m}$.

∴ the net field is zero $1.0\times10^{11}\,\text{m}$ from the lighter star M_1 — one third of the way across, closer to the smaller mass, as expected.

Practice questions

Question 1 The moon Talos has mass $7.2\times10^{22}\,\text{kg}$ and radius $1.6\times10^{6}\,\text{m}$. (a) Calculate the gravitational field strength at the surface of Talos. (b) Calculate the force due to gravity on an $850\,\text{kg}$ rover resting on the surface. (c) State the direction of the gravitational field at the surface.

Question 2 A spherical planet of radius $4.0\times10^{6}\,\text{m}$ has a surface gravitational field strength of $6.4\,\text{N kg}^{-1}$. (a) Calculate the mass of the planet. (b) Calculate the gravitational field strength at an altitude equal to the planet's radius (that is, at a distance of $2R$ from the centre). (c) With reference to $g = \frac{GM}{r^2}$, explain why the field strength there is one quarter of its surface value.

Question 3 Two spherical asteroids of masses $4.0\times10^{12}\,\text{kg}$ and $6.0\times10^{12}\,\text{kg}$ have their centres $5.0\times10^{3}\,\text{m}$ apart. (a) Calculate the gravitational force between the two asteroids. (b) State the magnitude and direction of the force on the smaller asteroid by the larger one, and name the type of interaction. (c) The asteroids move until their separation is doubled. Calculate the new gravitational force between them.

Question 4 A spacecraft coasts through empty space near an isolated spherical planet. (a) Draw the gravitational field around the planet, showing at least six field lines with their directions. (b) Classify the planet's field as uniform or non-uniform, and as static or changing. Justify each choice. (c) Explain what the spacing of the field lines represents, and why the lines are drawn closer together nearer the planet.

Question 5 Use $M_E = 5.97\times10^{24}\,\text{kg}$ and $R_E = 6.37\times10^{6}\,\text{m}$. (a) Show that the gravitational field strength at Earth's surface is $9.81\,\text{N kg}^{-1}$. (b) Hence calculate the force due to gravity on a $64\,\text{kg}$ astronaut standing on Earth's surface. (c) The astronaut later travels to a location where the gravitational field strength is $2.45\,\text{N kg}^{-1}$. Calculate the force due to gravity on the astronaut there.

Question 6 Answer the following about the gravitational field model. (a) Explain what is meant by describing gravitation using a *field model*, referring to how one mass affects another without contact. (b) A student claims the gravitational field just above the ground is perfectly uniform. Explain why this is only an approximation, and state the condition under which it is a good one. (c) State one way in which the gravitational field around a mass differs from the electric field around a charge.

Question 7 A rocky planet has mass $3.3\times10^{24}\,\text{kg}$ and radius $4.5\times10^{6}\,\text{m}$. Calculate the gravitational field strength at its surface.

Question 8 A probe of mass $1200\,\mathrm{kg}$ is positioned at a distance of $2\,R_E$ from Earth's centre. Calculate the gravitational field strength at the probe's location and the force due to gravity on the probe. Give your answers to three significant figures.

Question 9 The surface gravitational field strength of a gas-giant planet is $24.8\,\mathrm{N\ kg^{-1}}$ and its radius is $7.0\times10^{7}\,\mathrm{m}$. Calculate the mass of the planet.

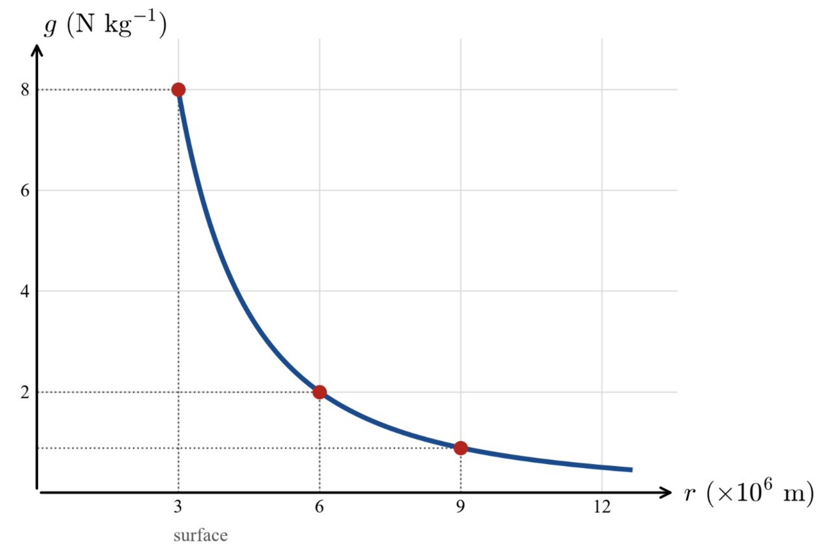
Question 10 A star has mass $2.5\times10^{30}\,\mathrm{kg}$. Determine the distance from the star's centre at which the gravitational field strength is $150\,\mathrm{N\ kg^{-1}}$.

Question 11 A planet has twice the mass of Earth and one-and-a-half times Earth's radius. Determine the gravitational field strength at its surface, expressing your answer both as a multiple of Earth's surface value and in $\mathrm{N\ kg^{-1}}$.

Question 12 The gravitational field strength at the surface of a planet of radius R is $12.0\,\mathrm{N\ kg^{-1}}$. Calculate the field strength at a distance of $2.5\,R$ from the planet's centre.

Question 13 A planet has mass $3.0\times10^{24}\,\mathrm{kg}$ and radius $4.0\times10^{6}\,\mathrm{m}$. Show that its surface gravitational field strength is $12.5\,\mathrm{N\ kg^{-1}}$, then determine the force due to gravity on a $95\,\mathrm{kg}$ astronaut standing on the surface and the field strength at an altitude equal to the planet's radius.

Question 14 The graph shows the gravitational field strength g against distance r from the centre of a spherical planet whose surface is at $r=3.0\times10^{6}\,\mathrm{m}$.



Use the graph to state the field strength at the surface and at $r=6.0\times10^{6}\,\mathrm{m}$, explain how these two values illustrate the inverse-square law, and determine the field strength at $r=9.0\times10^{6}\,\mathrm{m}$.

Question 15 A spacecraft travels from deep space directly toward a distant planet, finally landing on its surface. Describe how the gravitational field strength the spacecraft experiences changes during the journey, and classify the planet's gravitational field as uniform or non-uniform and as static or changing, justifying each classification.

Question 16 A planet of mass $6.0\times10^{24}\,\mathrm{kg}$ has a moon of mass $7.0\times10^{22}\,\mathrm{kg}$ orbiting with their centres $3.8\times10^{8}\,\mathrm{m}$ apart. Calculate the gravitational force the planet exerts on the moon and the gravitational field strength of the planet at the moon's location, and show that these two results are consistent with $F_g=mg$.

Question 17 A probe of mass $340\,\mathrm{kg}$ is at an altitude of $2.0\times10^{6}\,\mathrm{m}$ above the surface of a planet of radius $4.0\times10^{6}\,\mathrm{m}$ and mass $5.50\times10^{24}\,\mathrm{kg}$. Calculate the gravitational field strength at the probe's location and the force due to gravity on the probe.

Question 18 A student states: "An object twice as far from the centre of Earth experiences half the force due to gravity." Explain what is wrong with this statement, and state the correct factor by which the force due to gravity changes.

Question 19 Using the field model, explain why an isolated mass produces a field with the shape and direction shown for a gravitational monopole (field lines everywhere directed inward), and explain why there is no gravitational analogue of two like electric charges repelling each other.

Question 20 Earth (mass 5.97×10^{24} kg) and the Moon (mass 7.35×10^{22} kg) have their centres 3.84×10^{8} m apart. Determine the distance from Earth's centre to the point on the line joining them at which the net gravitational field is zero.

Question 21 Explain, with reference to $g = \frac{GM}{r^2}$, why the gravitational field within a small laboratory near Earth's surface can be treated as uniform, whereas the field mapped over a region extending several Earth radii into space cannot. In your answer, state what a set of parallel, evenly spaced field lines represents.

Answers

Question 1 (a) $1.88\,\mathrm{N\,kg^{-1}}$; (b) $1.59\times10^{3}\,\mathrm{N}$; (c) radially inward, toward the centre of Talos. **Question 2** (a) $1.54\times10^{24}\,\mathrm{kg}$; (b) $1.60\,\mathrm{N\,kg^{-1}}$; (c) at $2R$ the denominator r^2 is four times larger, so g is one quarter. **Question 3** (a) $6.40\times10^{7}\,\mathrm{N}$; (b) $6.40\times10^{7}\,\mathrm{N}$ toward the larger asteroid, an attractive (gravitational) interaction; (c) $1.60\times10^{7}\,\mathrm{N}$. **Question 4** (a) six or more straight lines directed radially inward; (b) non-uniform and static; (c) closer spacing means a stronger field. **Question 5** (a) $g=9.81\,\mathrm{N\,kg^{-1}}$; (b) $628\,\mathrm{N}$; (c) $157\,\mathrm{N}$. **Question 6** (a) a mass sets up a field that a second mass responds to; (b) only approximate — good when $\Delta h \ll R_E$; (c) gravity only attracts (no repulsion). **Question 7** $10.9\,\mathrm{N\,kg^{-1}}$. **Question 8** $2.45\,\mathrm{N\,kg^{-1}}$; $2.94\times10^{3}\,\mathrm{N}$. **Question 9** $1.82\times10^{27}\,\mathrm{kg}$. **Question 10** $1.05\times10^{9}\,\mathrm{m}$. **Question 11** 0.889 times Earth's value, i.e. $8.72\,\mathrm{N\,kg^{-1}}$. **Question 12** $1.92\,\mathrm{N\,kg^{-1}}$. **Question 13** $12.5\,\mathrm{N\,kg^{-1}}$; force due to gravity $1.19\times10^{3}\,\mathrm{N}$; field at altitude $3.13\,\mathrm{N\,kg^{-1}}$. **Question 14** $8.0\,\mathrm{N\,kg^{-1}}$ at the surface, $2.0\,\mathrm{N\,kg^{-1}}$ at $r=6.0\times10^{6}\,\mathrm{m}$; doubling r quarters g; $0.889\,\mathrm{N\,kg^{-1}}$ at $r=9.0\times10^{6}\,\mathrm{m}$. **Question 15** Field strength increases (as $1/r^2$) on approach; non-uniform (varies with position) and static (does not change in time). **Question 16** $1.94\times10^{20}\,\mathrm{N}$; $2.77\times10^{-3}\,\mathrm{N\,kg^{-1}}$; $F_g=mg$ reproduces the force. **Question 17** $10.2\,\mathrm{N\,kg^{-1}}$; $3.46\times10^{3}\,\mathrm{N}$. **Question 18** Wrong: the force follows an inverse-square law, so it falls to one quarter, not one half. **Question 19** A single mass makes only an inward (attractive) field; there is no negative mass to reverse it. **Question 20** $3.46\times10^{8}\,\mathrm{m}$ from Earth's centre. **Question 21** Uniform in the laboratory because $r\approx R_E$ (so g is nearly constant); non-uniform over several R_E because r changes greatly; parallel, evenly spaced lines represent a field constant in magnitude and direction.

Fully-worked solutions

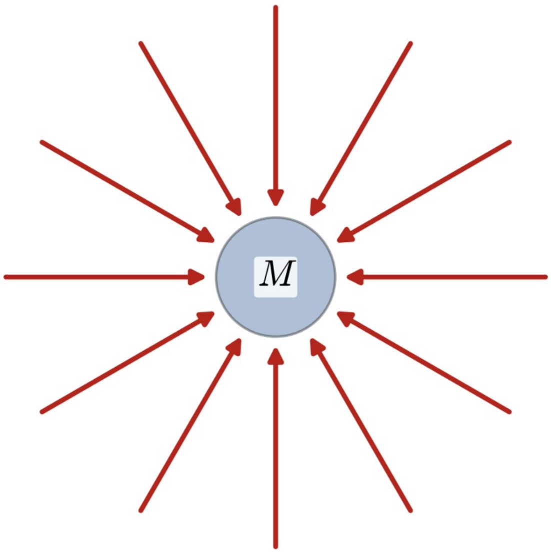
Question 1 $M=7.2\times10^{22}\,\mathrm{kg}$, $R=1.6\times10^{6}\,\mathrm{m}$, $m=850\,\mathrm{kg}$. (a) $g=\frac{GM}{R^2}=\frac{(6.67\times10^{-11})(7.2\times10^{22})}{(1.6\times10^{6})^2}=1.88\,\mathrm{N\,kg^{-1}}$. (b) $F_g=mg=850\times1.876=1.59\times10^{3}\,\mathrm{N}$. (c) The field points radially inward, toward the centre of Talos (vertically downward at the rover).

Question 2 $R=4.0\times10^{6}\,\mathrm{m}$, surface $g=6.4\,\mathrm{N\,kg^{-1}}$. (a) Rearranging $g=\frac{GM}{R^2}$ gives $M=\frac{gR^2}{G}=\frac{6.4\times(4.0\times10^{6})^2}{6.67\times10^{-11}}=1.54\times10^{24}\,\mathrm{kg}$. (b) At $r=2R$, $g=\frac{GM}{(2R)^2}=\frac{6.4}{4}=1.60\,\mathrm{N\,kg^{-1}}$. (c) In $g=\frac{GM}{r^2}$ the numerator GM is fixed for the planet, so $g\propto\frac{1}{r^2}$. Doubling the distance from the centre makes r^2 four times larger, so g becomes $\frac{1}{4}$ of its surface value — from 6.4 to $1.60\,\mathrm{N\,kg^{-1}}$.

Question 3 $m_1=4.0\times10^{12}\,\mathrm{kg}$, $m_2=6.0\times10^{12}\,\mathrm{kg}$, $r=5.0\times10^{3}\,\mathrm{m}$. (a) $F_g=\frac{Gm_1m_2}{r^2}=\frac{(6.67\times10^{-11})(4.0\times10^{12})(6.0\times10^{12})}{(5.0\times10^{3})^2}=6.40\times10^{7}\,\mathrm{N}$. (b) By Newton's third law the two forces are equal in magnitude, so the force on the smaller asteroid by the larger is $6.40\times10^{7}\,\mathrm{N}$, directed toward the larger asteroid. The interaction is attractive — masses only attract. (c) Doubling the separation multiplies F_g by $\frac{1}{2^2}=\frac{1}{4}$:

$$F_g'=\frac{6.40\times10^{7}}{4}=1.60\times10^{7}\,\mathrm{N}.$$

Question 4 (a) The field is a set of straight lines pointing radially inward, toward the planet's centre, evenly spaced around it.



field lines point toward the mass;
spacing widens as r increases (field weakens)

(b) The field is **non-uniform**: its magnitude decreases with distance ($g = \frac{GM}{r^2}$) and its direction changes from point to point (always toward the centre), so it is not the same everywhere. It is **static**: the planet is not changing, so the field does not change in time.
(c) The spacing of the field lines represents the magnitude of the field: where lines are close together the field is strong, and where they are far apart it is weak. The lines are drawn closer together nearer the planet because the field is stronger there (a fixed number of radial lines is squeezed into a smaller area as they converge on the centre), matching the inverse-square increase in g.

Question 5 $M_E = 5.97 \times 10^{24}\,\text{kg}$, $R_E = 6.37 \times 10^6\,\text{m}$. (a) $g = \frac{GM_E}{R_E^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)^2} = \frac{3.982 \times 10^{14}}{4.058 \times 10^{13}} = 9.81\,\text{N kg}^{-1}$, as required. (b) $F_g = mg = 64 \times 9.81 = 628\,\text{N}$, directed toward Earth's centre. (c) $F_g = mg = 64 \times 2.45 = 157\,\text{N}$.

Question 6 (a) In the field model, a mass modifies the space around it, setting up a gravitational field that exists at every point whether or not a second mass is present. A second mass placed at some point responds to the field **there**, feeling a force due to gravity, so the two masses interact without touching and without anything travelling between them — the field is the intermediary that carries the interaction across the gap. (b) The field near the ground is genuinely non-uniform, because $g = \frac{GM_E}{r^2}$ decreases with height and its direction points toward Earth's centre (so it is not exactly parallel everywhere). Over a small region, however, the change in r is negligible and the lines are nearly parallel and equally spaced, so the field is *approximately* uniform. This approximation is good only when the vertical extent of the region is very small compared with Earth's radius, $\Delta h \ll R_E$. (c) A gravitational field only ever attracts — it points inward toward the mass and there is no repulsive version — whereas an electric field can point toward *or* away from a charge and can attract or repel, because charge comes in two signs while mass comes in only one.

Question 7 $M = 3.3 \times 10^{24}\,\text{kg}$, $R = 4.5 \times 10^6\,\text{m}$. $g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(3.3 \times 10^{24})}{(4.5 \times 10^6)^2} = 10.9\,\text{N kg}^{-1}$, directed toward the planet's centre.

Question 8 $r=2R_E=2\times 6.37\times 10^6=1.274\times 10^7\,\mathrm{m}$, $m=1200\,\mathrm{kg}$.
$g=\frac{GM_E}{r^2}=\frac{(6.67\times 10^{-11})(5.97\times 10^{24})}{(1.274\times 10^7)^2}=2.45\,\mathrm{N\ kg^{-1}}$. $F_g=mg=1200\times 2.453=2.94\times 10^3\,\mathrm{N}$. (Equivalently, at $2R_E$ the field is $\frac{9.81}{4}=2.45\,\mathrm{N\ kg^{-1}}$.)

Question 9 Rearranging $g=\frac{GM}{R^2}$: $M=\frac{gR^2}{G}=\frac{24.8\times(7.0\times 10^7)^2}{6.67\times 10^{-11}}=1.82\times 10^{27}\,\mathrm{kg}$.

Question 10 Rearranging $g=\frac{GM}{r^2}$ for r: $r=\sqrt{\frac{GM}{g}}=\sqrt{\frac{(6.67\times 10^{-11})(2.5\times 10^{30})}{150}}=\sqrt{1.112\times 10^{18}}=1.05\times 10^9\,\mathrm{m}$.

Question 11 Writing the field as $g=\frac{GM}{R^2}$, form the ratio to Earth's surface field:

$$\frac{g_p}{g_E}=\frac{M_p/R_p^2}{M_E/R_E^2}=\frac{M_p}{M_E}\times\left(\frac{R_E}{R_p}\right)^2=2\times\left(\frac{1}{1.5}\right)^2=\frac{2}{2.25}=0.889.$$

So $g_p=0.889\times 9.81=8.72\,\mathrm{N\ kg^{-1}}$. The extra mass raises the field, but the larger radius (squared) more than offsets it, so the surface field is slightly *weaker* than Earth's.

Question 12 With GM fixed, $g\propto\frac{1}{r^2}$, so $\frac{g(2.5R)}{g(R)}=\left(\frac{R}{2.5R}\right)^2=\frac{1}{6.25}$. Hence $g(2.5R)=\frac{12.0}{6.25}=1.92\,\mathrm{N\ kg^{-1}}$.

Question 13 $M=3.0\times 10^{24}\,\mathrm{kg}$, $R=4.0\times 10^6\,\mathrm{m}$. Show that:
$g=\frac{GM}{R^2}=\frac{(6.67\times 10^{-11})(3.0\times 10^{24})}{(4.0\times 10^6)^2}=\frac{2.001\times 10^{14}}{1.6\times 10^{13}}=12.5\,\mathrm{N\ kg^{-1}}$, as required. Force due to gravity:
$F_g=mg=95\times 12.5=1.19\times 10^3\,\mathrm{N}$. Field at altitude $=R$ (so $r=2R$): $g=\frac{GM}{(2R)^2}=\frac{12.5}{4}=3.13\,\mathrm{N\ kg^{-1}}$.

Question 14 At the surface ($r=3.0\times 10^6\,\mathrm{m}$) the graph reads $g=8.0\,\mathrm{N\ kg^{-1}}$; at $r=6.0\times 10^6\,\mathrm{m}$ it reads $g=2.0\,\mathrm{N\ kg^{-1}}$. The distance from the centre has doubled (6.0×10^6 is $2R$) and the field has fallen to a quarter of its surface value ($\frac{8.0}{4}=2.0$), exactly the $g\propto\frac{1}{r^2}$ behaviour of the inverse-square law. At $r=9.0\times 10^6\,\mathrm{m}$ (three times the surface distance) the field is $\frac{8.0}{3^2}=\frac{8.0}{9}=0.889\,\mathrm{N\ kg^{-1}}$.

Question 15 As the spacecraft moves in from deep space, its distance r from the planet's centre decreases, so by $g=\frac{GM}{r^2}$ the gravitational field strength it experiences steadily **increases**, growing most rapidly in the final approach and reaching its greatest value at the surface. The field is **non-uniform**: its magnitude depends on position (it is larger closer to the planet) and its direction always points toward the planet's centre, so it is not the same at every point. The field is **static**: the planet is a fixed source, so the field at any given point does not change with time — it is only the spacecraft's *position within* the unchanging field that changes.

Question 16 $M=6.0\times 10^{24}\,\mathrm{kg}$, $m=7.0\times 10^{22}\,\mathrm{kg}$, $r=3.8\times 10^8\,\mathrm{m}$. Force:
$F_g=\frac{GMm}{r^2}=\frac{(6.67\times 10^{-11})(6.0\times 10^{24})(7.0\times 10^{22})}{(3.8\times 10^8)^2}=1.94\times 10^{20}\,\mathrm{N}$. Field of the planet at the moon:
$g=\frac{GM}{r^2}=\frac{(6.67\times 10^{-11})(6.0\times 10^{24})}{(3.8\times 10^8)^2}=2.77\times 10^{-3}\,\mathrm{N\ kg^{-1}}$. Consistency:
$F_g=mg=(7.0\times 10^{22})(2.77\times 10^{-3})=1.94\times 10^{20}\,\mathrm{N}$, the same as the direct calculation. The field strength is just the force per unit mass, so multiplying it by the moon's mass returns the force.

Question 17 $M = 5.50 \times 10^{24}$ kg, $R = 4.0 \times 10^6$ m, $h = 2.0 \times 10^6$ m, $m = 340$ kg. Distance from centre: $r = R + h = 4.0 \times 10^6 + 2.0 \times 10^6 = 6.0 \times 10^6$ m. $g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(5.50 \times 10^{24})}{(6.0 \times 10^6)^2} = 10.2\ \text{N kg}^{-1}$.

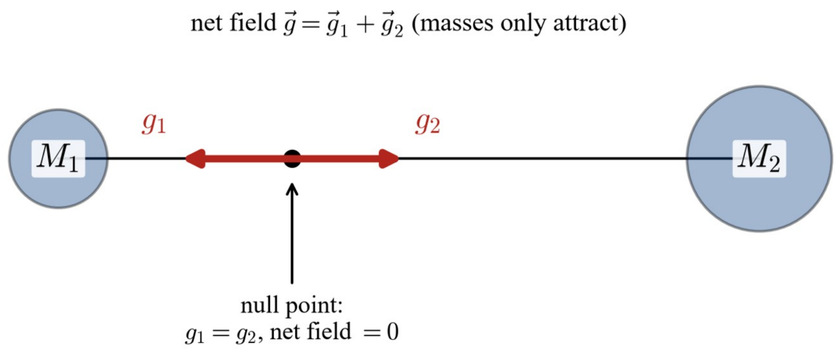
$F_g = mg = 340 \times 10.19 = 3.46 \times 10^3$ N, directed toward the planet's centre.

Question 18 The statement is wrong because the force due to gravity obeys an **inverse-square** law, $F_g = \frac{GMm}{r^2}$, not an inverse (first-power) law. Doubling the distance from the centre makes r^2 four times larger, so the force falls to $\frac{1}{2^2} = \frac{1}{4}$ of its original value, not $\frac{1}{2}$. The correct factor is **one quarter**.

Question 19 In the field model, a single mass sets up a field whose direction at every point is the direction of the force due to gravity on a small test mass placed there. Because gravity always **pulls** the test mass toward the source, the field lines everywhere point inward, toward the mass — the shape of a monopole field with no beginning of its own, only an ending on the mass. There is no gravitational analogue of two like charges repelling because mass comes in only **one sign**: there is no "negative mass" that a field could push away. Electric charge, by contrast, comes in two signs, so an electric field can point away from a positive charge and two positive charges repel. Since every mass attracts every other, gravitational fields can only converge inward and can never produce repulsion.

Question 20 Let x be the distance from Earth's centre to the null point, so the distance from the Moon's centre is $d - x$. At the null point the two field magnitudes are equal:

$$\frac{GM_E}{x^2} = \frac{GM_{\text{Moon}}}{(d-x)^2} \Rightarrow \frac{x}{d-x} = \sqrt{\frac{M_E}{M_{\text{Moon}}}} = \sqrt{\frac{5.97 \times 10^{24}}{7.35 \times 10^{22}}} = \sqrt{81.2} = 9.01.$$



So $x = 9.01(d - x)$, giving $x = \frac{9.01}{1 + 9.01} d = 0.900 \times 3.84 \times 10^8 = 3.46 \times 10^8$ m.

∴ the net gravitational field is zero 3.46×10^8 m from Earth's centre — about 90 % of the way to the Moon, much closer to the far smaller Moon, as expected.

Question 21 In $g = \frac{GM}{r^2}$ the field strength depends only on the distance r from Earth's centre. Inside a small laboratory the greatest possible change in r (a few metres of height) is utterly negligible beside $R_E = 6.37 \times 10^6$ m, so $r \approx R_E$ throughout and g stays essentially constant in both magnitude and direction; the field can therefore be treated as **uniform**. Over a region extending several Earth radii into space, r changes by factors of two or three, so g falls to a quarter, a ninth, and so on, and its direction fans inward toward the centre — the field varies strongly with position and is decidedly **non-uniform**. A set of parallel, evenly spaced field lines represents a field that is constant in magnitude (equal spacing) and constant in direction (parallel) — that is, a uniform field.

Theory

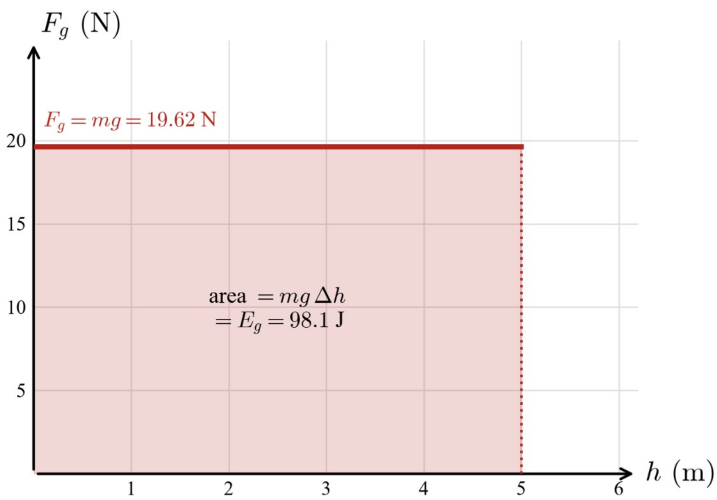
A mass placed in a gravitational field has **gravitational potential energy** (E_g) by virtue of its position in the field. When the mass moves, energy is transferred between E_g and other forms (such as kinetic energy). Only **changes** in gravitational potential energy have a definite physical meaning; a value of E_g is always quoted relative to a chosen **reference level** at which E_g is taken to be zero. Near Earth's surface the ground is the usual reference; for a mass in the field of a point mass, the natural reference is at an infinite distance away.

The force due to gravity does the work. As a mass moves in a gravitational field, the force due to gravity F_g acts on it. When the mass moves in the direction of F_g (toward the source) the force does **positive** work: E_g falls and the energy released can appear as kinetic energy. When the mass moves **against** F_g (away from the source) work must be done on the mass by some other agent: E_g rises. In every case the size of the change in gravitational potential energy equals the work done by (or against) the force due to gravity.

Uniform field near a surface: $E_g = mg\Delta h$. Close to the surface of a planet the field is very nearly **uniform**, so the force due to gravity on a mass m is constant, $F_g = mg$. Lifting the mass through a small vertical height Δh requires work $F_g \Delta h$, which is stored as gravitational potential energy:

$$E_g = mg\Delta h.$$

Because $F_g = mg$ is constant here, E_g is also the **area under the force–height graph** — a rectangle of height mg and width Δh:

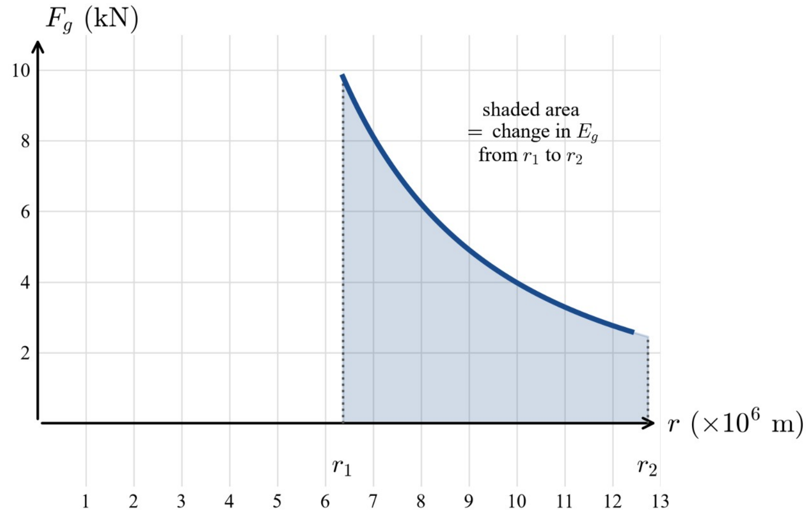


Throughout, $g = 9.81\ \text{N kg}^{-1}$ (equivalently $9.81\ \text{m s}^{-2}$) at Earth's surface.

Varying field: change in E_g from the area under a force–distance graph. Far from the surface the field is **not** uniform. For a point (or spherical) mass M, the force due to gravity on a mass m a distance r from the centre obeys the inverse-square law

$$F_g = \frac{GMm}{r^2},$$

with $G = 6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2}$. Because F_g now changes with distance, $E_g = mg\Delta h$ cannot be used. Instead the change in gravitational potential energy as the mass moves between two radii r_1 and r_2 equals the **area under the force–distance graph** between those radii:



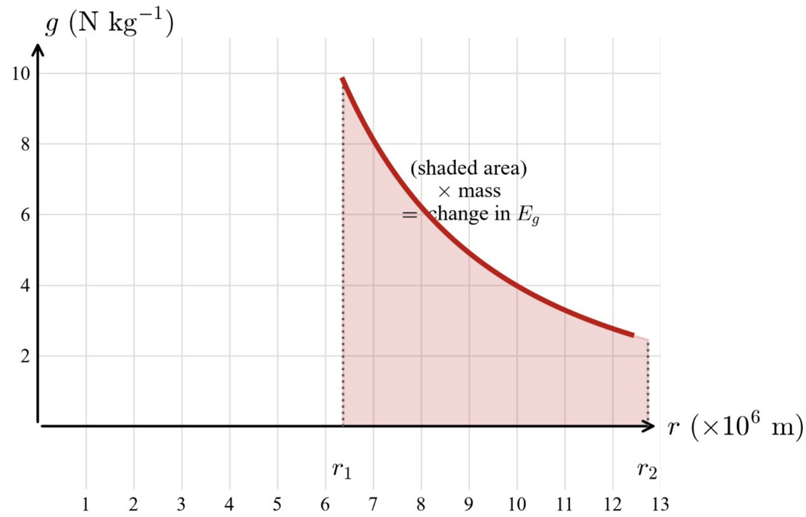
Moving **outward** (from r_1 to a larger r_2), this area is the work that must be done against gravity, so E_g **increases** by that amount. Moving **inward** (from r_2 to a smaller r_1), the same area is the work done *by* gravity, so E_g **decreases** by that amount and the energy is released (typically as kinetic energy). The area is the *magnitude* of the change; its sign is fixed by the direction of travel.

Varying field: change in E_g from the area under a field–distance graph. The gravitational field strength of a point mass is

$$g = \frac{GM}{r^2},$$

which is the force per unit mass ($F_g = mg$). The **area under the field–distance graph** between r_1 and r_2 therefore has units of $\mathrm{N\ kg^{-1}} \times \mathrm{m} = \mathrm{J\ kg^{-1}}$: it is the change in gravitational potential energy **per kilogram**. Multiplying that area by the mass gives the change in gravitational potential energy of the object:

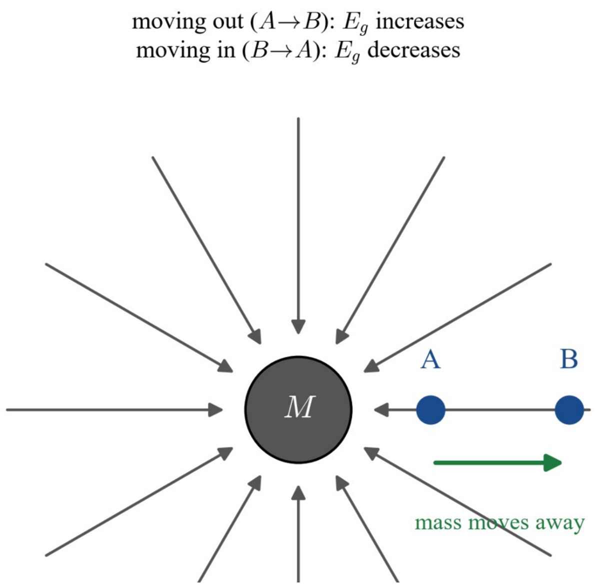
$$\Delta E_g = (\text{area under the } g - r \text{ graph}) \times m.$$



Estimating the area by counting squares. The area under a curved graph is found by counting the grid squares between the curve and the horizontal axis, between the two radii, and multiplying by the energy each square represents. Part-squares are estimated and added. This gives the change in gravitational potential energy (force–distance graph) or the change per kilogram (field–distance graph), and is the method used whenever the field varies with distance.

Qualitative changes for a mass in a point field. In the field of a point mass the field lines point radially **inward**, and the field weakens with distance as $1/r^2$. It follows that:

- As a mass moves **away** from the source, its gravitational potential energy **increases**; as it moves **toward** the source, E_g **decreases**.
- Because the field is stronger close in, the force due to gravity is larger there, so a given step of distance close to the source changes E_g by **more** than the same step far away. Equal steps of distance do **not** produce equal steps of energy.
- E_g never quite levels off until the mass is infinitely far away, where the field (and F_g) approach zero.



When each method applies. Over a small height near the surface the field barely changes, so $E_g = mg\Delta h$ and the area under the (almost flat) force–distance graph agree. Over a large change in distance the field falls appreciably, so $mg\Delta h$ — which assumes a constant g — **overestimates** the true change, and the area-under-the-graph method must be used instead.

Worked examples

Example 1 — scaffolded

A box of mass $8.0\,\text{kg}$ is lifted at constant speed through a vertical height of $2.5\,\text{m}$ near Earth's surface. Calculate the gain in gravitational potential energy, and the speed the box would have if it were then released and allowed to fall back through the same height (ignore air resistance).

Working	Comment
Near the surface the field is uniform: $m = 8.0\,\text{kg}$, $\Delta h = 2.5\,\text{m}$, $g = 9.81\,\text{N kg}^{-1}$.	Small height $\Rightarrow$ use $E_g = mg\Delta h$.
$E_g = mg\Delta h = 8.0 \times 9.81 \times 2.5 = 196\,\text{J}$.	The area under the constant F_g–h graph.
On release, all of this E_g converts to kinetic energy: $1/2\,mv^2 = mg\Delta h$.	Energy conservation; m cancels.
$v = \sqrt{2g\Delta h} = \sqrt{2 \times 9.81 \times 2.5} = 7.00\,\text{m s}^{-1}$.	Independent of mass.

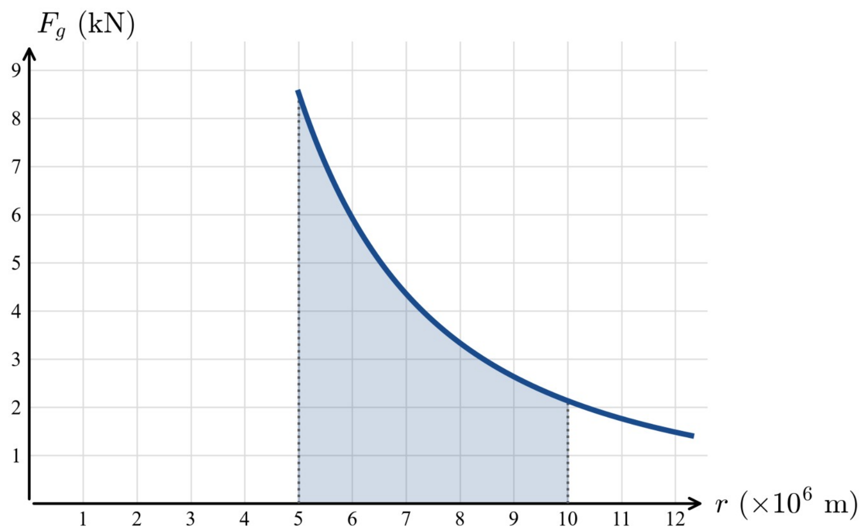
Example 2 — scaffolded

The field–distance graph below shows the gravitational field strength g of Earth against distance r from Earth's centre. Each grid square represents $1 \times 10^6\,\text{J kg}^{-1}$. A satellite of mass $200\,\text{kg}$ is raised from Earth's surface ($r = R_E = 6.37 \times 10^6\,\text{m}$) to $r = 3R_E$. Use the graph to estimate the change in its gravitational potential energy.

Working	Comment
The area under the g–r graph between R_E and $3R_E$ is the change in E_g *per kilogram*.	Area units: $\text{N kg}^{-1} \times \text{m} = \text{J kg}^{-1}$.
Counting squares between the curve and the axis gives about 42 squares, each $1 \times 10^6\ \text{J kg}^{-1}$: area $\approx 4.2 \times 10^7\ \text{J kg}^{-1}$.	Estimate part-squares and add.
$\Delta E_g = (\text{area}) \times m = 4.2 \times 10^7 \times 200$.	Multiply the per-kilogram change by the mass.
$\Delta E_g \approx 8.4 \times 10^9\ \text{J}$ (an increase, since the satellite moves outward).	Work is done against gravity.

Example 3 — unscaffolded

The force–distance graph below shows the force due to gravity F_g on an $800\ \text{kg}$ probe against its distance r from the centre of a planet. Each grid square represents $1 \times 10^9\ \text{J}$. Use the graph to estimate the change in the probe's gravitational potential energy as it is raised from the surface ($r = R$) to $r = 2R$.



The change in gravitational potential energy equals the area under the force–distance graph between R and $2R$. Counting the grid squares between the curve and the r-axis, from R to $2R$, gives approximately 21 squares. Each square represents $1 \times 10^9\ \text{J}$, so

$$\Delta E_g \approx 21 \times (1 \times 10^9) = 2.1 \times 10^{10}\ \text{J}.$$

The probe moves outward, against the force due to gravity, so this is an **increase** in gravitational potential energy; the energy is supplied by the rocket.

∴ the gravitational potential energy increases by about 2.1×10^{10} J.

Example 4 — unscaffolded

A probe of mass $400\,\mathrm{kg}$ falls freely from rest toward a planet, moving inward from r_2 to r_1. The area under the force–distance graph between r_1 and r_2 is 6.0×10^9 J. Ignoring any atmosphere, calculate the speed of the probe when it reaches r_1.

Moving inward, the force due to gravity does positive work on the probe, so its gravitational potential energy **decreases** by the area under the graph, and that energy appears as kinetic energy:

$$1/2\,m v^2=\Delta E_g=6.0\times10^9\,\mathrm{J}.$$

Solving for the speed,

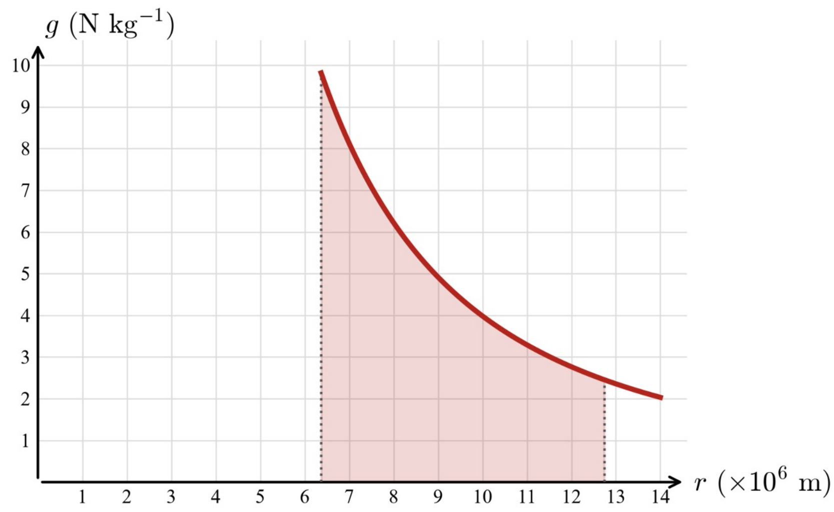
$$v=\sqrt{\frac{2\Delta E_g}{m}}=\sqrt{\frac{2\times6.0\times10^9}{400}}=\sqrt{3.0\times10^7}=5477\,\mathrm{m\,s^{-1}}.$$

∴ the probe reaches r_1 with a speed of $5.48\times10^3\,\mathrm{m\,s^{-1}}$.

Practice questions

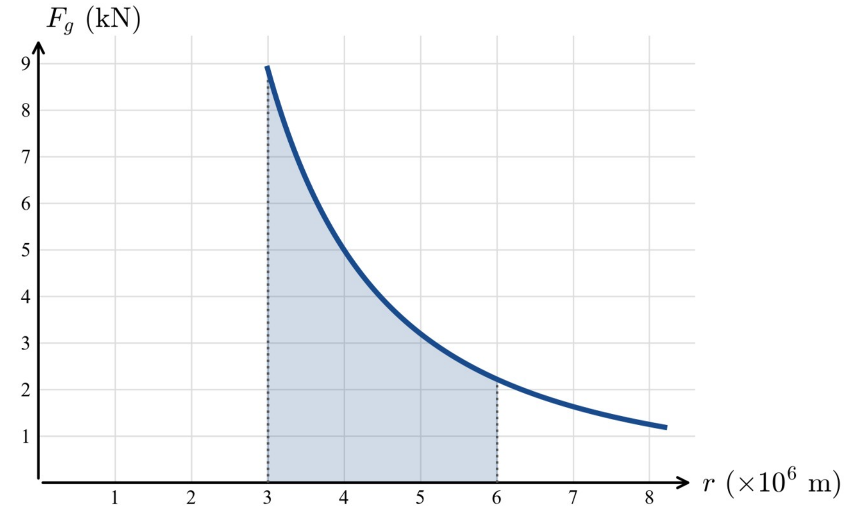
Question 1 A rock of mass $3.0\,\mathrm{kg}$ is lifted through a vertical height of $12\,\mathrm{m}$ near Earth's surface. (a) Calculate the gain in the rock's gravitational potential energy. (b) The rock is released from rest and falls back through the $12\,\mathrm{m}$. Calculate its speed just before it lands (ignore air resistance). (c) Explain why the relationship $E_g=mg\Delta h$ may be used here, referring to the gravitational field near Earth's surface.

Question 2 The field–distance graph below shows the gravitational field strength g of Earth against distance r from Earth's centre. Each grid square represents $1\times10^6\,\mathrm{J\,kg^{-1}}$. A satellite of mass $150\,\mathrm{kg}$ is raised from the surface ($r=R_E=6.37\times10^6\,\mathrm{m}$) to $r=2R_E$.



(a) State what the area under the g–r graph between two radii represents, including its units.
(b) Use the graph to estimate the area under the curve between R_E and $2R_E$.
(c) Hence calculate the change in the satellite's gravitational potential energy.

Question 3 The force–distance graph below shows the force due to gravity F_g on a $1500\,\mathrm{kg}$ spacecraft against its distance r from the centre of a planet. Each grid square represents 1×10^9 J.



(a) State what the area under the force–distance graph represents.
(b) The spacecraft is raised from the surface ($r = R$) to $r = 2R$. Use the graph to estimate the change in its gravitational potential energy.
(c) State whether the gravitational potential energy increases or decreases, and identify the source of the energy.

Question 4 A probe moves in the gravitational field of a distant planet, treated as a point mass. (a) State how the probe's gravitational potential energy changes as it moves from a point far away directly toward the planet. (b) Explain, in terms of the field and the work done by the force due to gravity, why the gravitational potential energy changes in this way. (c) State whether a $1\,\text{km}$ step taken close to the planet changes the probe's gravitational potential energy by more than, the same as, or less than a $1\,\text{km}$ step taken far away, and justify your answer.

Question 5 Earth has mass $M_E = 5.97 \times 10^{24}\,\text{kg}$ and radius $R_E = 6.37 \times 10^6\,\text{m}$. (a) Using $g = \frac{GM}{r^2}$, show that the gravitational field strength at Earth's surface is $9.81\,\text{N kg}^{-1}$. (b) A mass of $50\,\text{kg}$ is raised $120\,\text{m}$ near Earth's surface. Calculate the change in its gravitational potential energy. (c) Explain why $E_g = mg\Delta h$ gives an accurate result in part (b), but would **overestimate** the change in gravitational potential energy if the same mass were raised to an altitude of $3000\,\text{km}$.

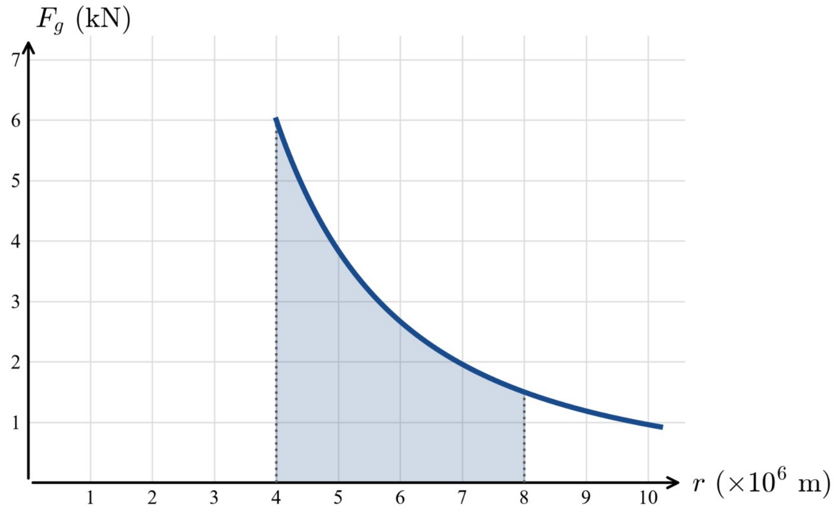
Question 6 A satellite is moved from a low circular orbit to a higher circular orbit around Earth. (a) (Circle your answer.) As it moves to the higher orbit, the satellite's gravitational potential energy: *increases* / *decreases* / *stays the same*. Justify your choice. (b) State whether the area under the force–distance graph between the two orbital radii represents the change in gravitational potential energy or the gravitational field strength. (c) Explain what happens to the size of the force due to gravity on the satellite as it moves to the higher orbit, with reference to the inverse-square law.

Question 7 A textbook of mass $1.5\,\text{kg}$ is raised from the floor to a shelf $2.2\,\text{m}$ above it. Calculate the gain in the book's gravitational potential energy.

Question 8 A climber of mass $60\,\text{kg}$ gains $1.6 \times 10^4\,\text{J}$ of gravitational potential energy while ascending a cliff. Assuming the field is uniform, calculate the vertical height climbed.

Question 9 A ball of mass $0.25\,\text{kg}$ is dropped from rest from a height of $8.0\,\text{m}$ above the ground. Ignoring air resistance, calculate the speed of the ball just before it lands. Give your answer to three significant figures.

Question 10 The force–distance graph below shows the force due to gravity F_g on a $600\,\text{kg}$ satellite against its distance r from the centre of a planet. Each grid square represents $1 \times 10^9\,\text{J}$. The satellite is raised from the surface ($r = R$) to $r = 2R$. Use the graph to determine the change in its gravitational potential energy.



Question 11 For a certain planet, the area under the field–distance graph between two radii r_1 and r_2 is 2.5×10^7 J kg^{-1}. (a) State the physical meaning of this area. (b) A spacecraft of mass 350 kg travels from r_1 to r_2. Calculate the change in its gravitational potential energy.

Question 12 A meteoroid of mass 50 kg falls from rest toward a planet, moving inward from r_2 to r_1. The area under the force–distance graph between r_1 and r_2 is 1.8×10^9 J. (a) State the size and direction (increase or decrease) of the change in the meteoroid's gravitational potential energy. (b) Assuming all of this energy becomes kinetic energy, calculate the speed of the meteoroid at r_1.

Question 13 A planet has mass 3.0×10^{24} kg and radius 6.0×10^6 m. (a) Show that the gravitational field strength at the planet's surface is 5.56 N kg^{-1}. (b) Hence calculate the change in gravitational potential energy when a 250 kg lander is raised 500 m above the surface, treating the field over this height as uniform.

Question 14 A spacecraft travels directly away from a planet, from the planet's surface out into deep space. With reference to the field model, describe and explain how the force due to gravity on the spacecraft and its gravitational potential energy each change during the journey.

Question 15 Explain why the change in gravitational potential energy of a rocket that rises from Earth's surface to an altitude equal to R_E **cannot** be found using $E_g = mg\Delta h$, and state what method should be used instead.

Question 16 Two satellites, of mass 400 kg and 1000 kg, are each raised from radius R to radius $2R$ around the same planet. The gravitational potential energy of the 400 kg satellite increases by 9.4×10^9 J. (a) Explain why the two satellites do not gain the same amount of gravitational potential energy. (b) Calculate the increase in gravitational potential energy of the 1000 kg satellite, and state the factor by which it exceeds that of the 400 kg satellite.

Question 17 A satellite of mass 800 kg is raised from Earth's surface to a high orbit. The area under the field–distance graph between the two radii is 5.26×10^7 J kg^{-1}. Calculate the change in the satellite's gravitational potential energy. Give your answer to three significant figures.

Question 18 A probe of mass 200 kg falls to the surface of a planet, starting from rest at a very great distance away. The area under the force–distance graph, from the surface outward to that distance, is 1.11×10^{10} J. Ignoring any atmosphere, calculate the speed at which the probe reaches the surface.

Question 19 A student says: "An object resting on the ground has zero gravitational potential energy, and an object in orbit has a large positive gravitational potential energy." Comment on this statement, explaining what can and cannot be determined about an object's gravitational potential energy.

Question 20 Explain why raising a satellite through each additional kilometre of altitude requires **less** energy the higher the satellite already is. Support your answer by referring to the shape of the force–distance graph for the planet.

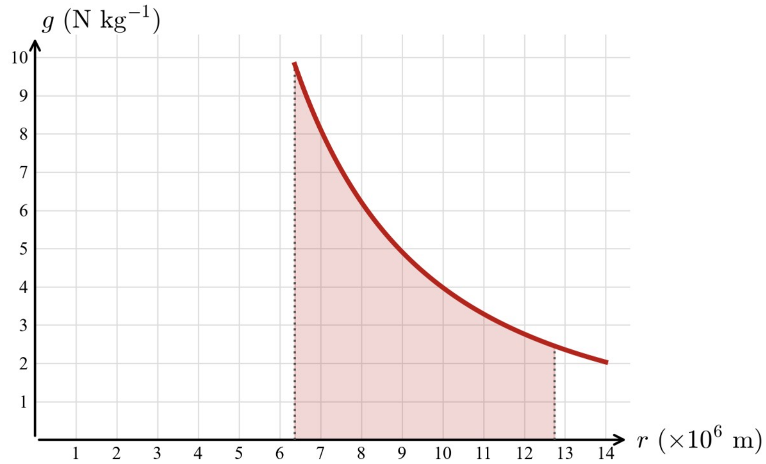
Answers

Question 1 (a) $353\,\mathrm{J}$; (b) $15.3\,\mathrm{m\ s^{-1}}$; (c) the field is uniform near the surface, so $F_g = mg$ is constant. **Question 2** (a) the change in gravitational potential energy per kilogram (J kg^{-1}); (b) $\approx 3.1\times10^{7}$ J kg^{-1}; (c) $\approx 4.7\times10^{9}$ J (increase). **Question 3** (a) the change in gravitational potential energy; (b) $\approx 1.3\times10^{10}$ J; (c) increases; energy supplied by the rocket/spacecraft engines. **Question 4** (a) decreases; (b) gravity does positive work on the inward-moving probe; (c) more, because the field (and F_g) is stronger close in. **Question 5** (a) $g = 9.81\,\mathrm{N\ kg^{-1}}$ (shown); (b) 5.89×10^{4} J; (c) over $120\,\mathrm{m}$, g is essentially constant, but over $3000\,\mathrm{km}$ it falls, so $m g \Delta h$ overestimates. **Question 6** (a) increases; (b) the change in gravitational potential energy; (c) it decreases as $1/r^2$. **Question 7** $32.4\,\mathrm{J}$. **Question 8** $27.2\,\mathrm{m}$. **Question 9** $12.5\,\mathrm{m\ s^{-1}}$. **Question 10** $\approx 1.2\times10^{10}$ J. **Question 11** (a) the change in gravitational potential energy per kilogram; (b) 8.75×10^{9} J. **Question 12** (a) a decrease of 1.8×10^{9} J; (b) $8.49\times10^{3}\,\mathrm{m\ s^{-1}}$. **Question 13** (a) $5.56\,\mathrm{N\ kg^{-1}}$ (shown); (b) 6.95×10^{5} J. **Question 14** F_g decreases as $1/r^2$ (never reaching zero except at infinity); E_g increases, by the area under the F_g–r graph, but at a decreasing rate per metre. **Question 15** g is not constant over that distance (it falls to $1/4$ its surface value by $2R_E$); use the area under the force–distance graph (or the field–distance graph × mass). **Question 16** (a) $\Delta E_g \propto m$; (b) 2.35×10^{10} J, a factor of 2.5. **Question 17** 4.21×10^{10} J. **Question 18** $1.05\times10^{4}\,\mathrm{m\ s^{-1}}$. **Question 19** Only *changes* in gravitational potential energy are meaningful; the zero is a chosen reference, so no absolute value exists. **Question 20** The force due to gravity weakens with distance, so the area under the F_g–r graph for each further kilometre is smaller.

Fully-worked solutions

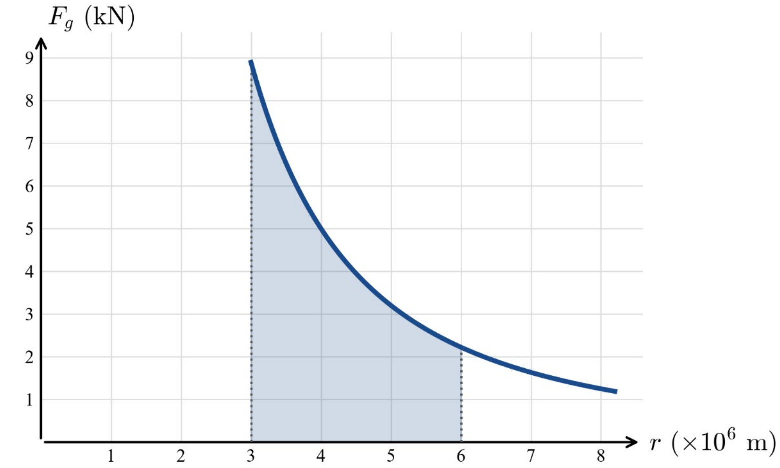
Question 1 $m = 3.0\,\mathrm{kg}$, $\Delta h = 12\,\mathrm{m}$, $g = 9.81\,\mathrm{N\ kg^{-1}}$. (a) Near the surface the field is uniform, so $E_g = m g \Delta h = 3.0\times9.81\times12 = 353\,\mathrm{J}$. (b) On release, all of this gravitational potential energy converts to kinetic energy: $1/2\, m v^2 = m g \Delta h$, so $v = \sqrt{2 g \Delta h} = \sqrt{2\times9.81\times12} = \sqrt{235.4} = 15.3\,\mathrm{m\ s^{-1}}$. (c) Over a height of $12\,\mathrm{m}$ — tiny compared with Earth's radius — the gravitational field is essentially uniform, so the force due to gravity $F_g = mg$ is constant. A constant force through a distance Δh does work $m g \Delta h$, which is exactly the area under the (flat) force–height graph, so $E_g = m g \Delta h$ applies.

Question 2



(a) The area under the field–distance graph between two radii is the change in gravitational potential energy **per kilogram** of the object, measured in J kg^{-1} (since $\mathrm{N\ kg^{-1}} \times \mathrm{m} = \mathrm{J\ kg^{-1}}$).
(b) Counting the grid squares between the curve and the r-axis, from R_E to $2R_E$, gives about 31 squares, each worth 1×10^{6} J kg^{-1}, so the area $\approx 3.1\times10^{7}$ J kg^{-1}.
(c) $\Delta E_g = (\text{area}) \times m = 3.1\times10^{7}\times150 \approx 4.7\times10^{9}$ J. The satellite moves outward, so this is an increase in gravitational potential energy.

Question 3



(a) The area under the force–distance graph between two radii is the change in the object's gravitational potential energy as it moves between them (the work done by, or against, the force due to gravity).

(b) Counting the grid squares under the curve from R to $2R$ gives approximately 13 squares, each worth 1×10^9 J, so $\Delta E_g \approx 13\times(1\times10^9)=1.3\times10^{10}$ J.

(c) The spacecraft moves outward, against the force due to gravity, so its gravitational potential energy **increases**. The energy is supplied by the spacecraft's engines (chemical energy of the fuel).

Question 4 (a) The probe's gravitational potential energy **decreases** as it moves toward the planet. (b) The field lines of a point mass point radially inward, so the force due to gravity on the probe points toward the planet — the same direction as the probe's motion. The force therefore does **positive** work on the probe, transferring energy out of the gravitational potential energy store (into kinetic energy), so E_g decreases. (c) A $1\,\text{km}$ step close to the planet changes E_g by **more** than a $1\,\text{km}$ step far away. The field strength, and hence the force due to gravity $F_g = GMm/r^2$, is larger close in, so more work is done per metre travelled; on the force–distance graph the curve is higher near the planet, so a $1\,\text{km}$ strip encloses a greater area there.

Question 5 (a) $g=\frac{GM_E}{R_E^2}=\frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6.37\times10^6)^2}=\frac{3.982\times10^{14}}{4.058\times10^{13}}=9.81\,\text{N kg}^{-1}$, as required. (b) Near the surface the field is uniform, so $E_g=mg\Delta h=50\times9.81\times120=5.89\times10^4$ J. (c) Over $120\,\text{m}$ the distance from Earth's centre changes by a negligible fraction of R_E, so g is effectively constant and $mg\Delta h$ is accurate. Over an altitude of $3000\,\text{km}$, however, r increases from $6.37\times10^6\,\text{m}$ to about $9.37\times10^6\,\text{m}$, and $g=GM/r^2$ falls to roughly half its surface value. Using the *surface* value of g throughout (as $mg\Delta h$ does) therefore assumes a stronger field than is actually present over most of the climb, and so **overestimates** the true change in gravitational potential energy. The correct value is the (smaller) area under the force–distance graph.

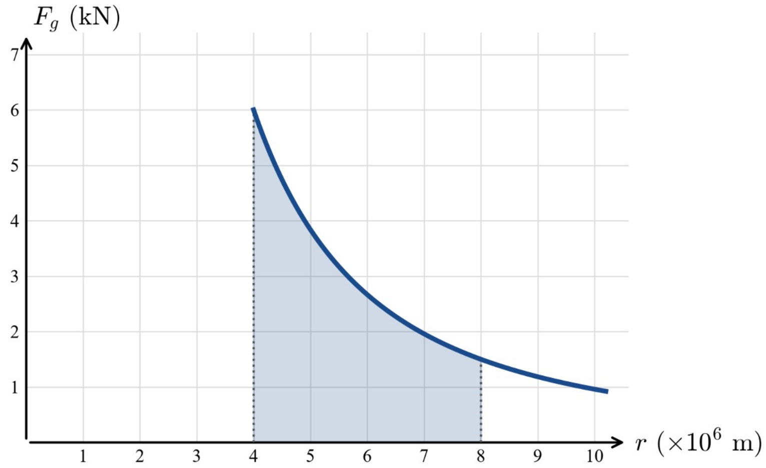
Question 6 (a) **Increases.** Moving to a higher orbit takes the satellite further from Earth, against the force due to gravity, so work is done on it and its gravitational potential energy rises. (b) It represents the **change in gravitational potential energy** between the two orbital radii. (c) The force due to gravity is $F_g=GMm/r^2$. As r increases to the higher orbit, F_g **decreases** in proportion to $1/r^2$ — for example, doubling r reduces the force to one quarter.

Question 7 Near the surface the field is uniform, so $E_g=mg\Delta h=1.5\times9.81\times2.2=32.4\,\text{J}$.

Question 8 Treating the field as uniform, $E_g=mg\Delta h$, so $\Delta h=\frac{E_g}{mg}=\frac{1.6\times10^4}{60\times9.81}=\frac{1.6\times10^4}{588.6}=27.2\,\text{m}$.

Question 9 Ignoring air resistance, the loss in gravitational potential energy equals the gain in kinetic energy: $1/2\,mv^2=mg\Delta h$, so $v=\sqrt{2g\Delta h}=\sqrt{2\times9.81\times8.0}=\sqrt{157.0}=12.5\,\text{m s}^{-1}$ (to three significant figures). (The released gravitational potential energy is $mg\Delta h=0.25\times9.81\times8.0=19.6$ J.)

Question 10



The change in gravitational potential energy is the area under the force–distance graph between R and $2R$. Counting the grid squares between the curve and the r-axis over this interval gives approximately 12 squares, each worth 1×10^9 J, so

$$\Delta E_g \approx 12 \times (1 \times 10^9) = 1.2 \times 10^{10}\,\text{J}.$$

Question 11 (a) It is the change in gravitational potential energy **per kilogram** of an object moving between r_1 and r_2 (units J kg^{-1}). (b) $\Delta E_g = (\text{area}) \times m = 2.5 \times 10^7 \times 350 = 8.75 \times 10^9\,\text{J}$.

Question 12 (a) The meteoroid moves **inward**, so its gravitational potential energy **decreases**; the size of the decrease equals the area under the graph, 1.8×10^9 J. (b) All of this becomes kinetic energy: $1/2\, m v^2 = 1.8 \times 10^9$, so

$$v = \sqrt{\frac{2 \times 1.8 \times 10^9}{50}} = \sqrt{7.2 \times 10^7} = 8.49 \times 10^3\,\text{m s}^{-1}.$$

Question 13 (a) $g = \dfrac{GM}{R^2} = \dfrac{6.67 \times 10^{-11} \times 3.0 \times 10^{24}}{(6.0 \times 10^6)^2} = \dfrac{2.001 \times 10^{14}}{3.6 \times 10^{13}} = 5.56\,\text{N kg}^{-1}$, as required. (b) Over 500 m — negligible compared with the planet's radius — the field is effectively uniform, so $E_g = m g \Delta h = 250 \times 5.56 \times 500 = 6.95 \times 10^5\,\text{J}$.

Question 14 As the spacecraft moves outward, its distance r from the planet's centre increases, so the force due to gravity $F_g = GMm/r^2$ **decreases** in proportion to $1/r^2$; it becomes very small far away but only reaches zero at an infinite distance. Because the spacecraft moves against this inward force, work is done against gravity, so its gravitational potential energy **increases**. The total increase is the area under the force–distance graph between the surface and the final distance. Since F_g shrinks with distance, each successive kilometre adds less area, so E_g rises **rapidly at first and then more and more slowly** as the field weakens.

Question 15 $E_g = m g \Delta h$ assumes the gravitational field strength g is **constant** over the rise, which is only true very close to the surface. Rising to an altitude equal to R_E takes the rocket from $r = R_E$ to $r = 2R_E$, over which $g = GM/r^2$ falls to one quarter of its surface value — far from constant. Using $m g \Delta h$ with the surface value of g would therefore be badly wrong (it overestimates). The change in gravitational potential energy must instead be found as the **area under the force–distance graph** between R_E and $2R_E$ (equivalently, the area under the field–distance graph over the same interval, multiplied by the rocket's mass).

Question 16 (a) The change in gravitational potential energy is proportional to the mass moved ($\Delta E_g \propto m$, since $F_g \propto m$ at every distance and the change is the area under the force–distance graph). The two satellites travel between the same radii in the same field, so the more massive one experiences a larger force at every point and gains more gravitational potential energy. (b) The masses are in the ratio $\dfrac{1000}{400} = 2.5$, so ΔE_g is 2.5 times larger:

$\Delta E_g = 2.5 \times 9.4 \times 10^9 = 2.35 \times 10^{10}$ J. The 1000 kg satellite gains a factor of 2.5 more gravitational potential energy than the 400 kg satellite.

Question 17 The area under the field–distance graph is the change in gravitational potential energy per kilogram, so multiplying by the mass gives the total change:

$$\Delta E_g = (\text{area}) \times m = 5.26 \times 10^7 \times 800 = 4.21 \times 10^{10}\ \text{J}.$$

Question 18 Starting from rest a great distance away, the probe's gravitational potential energy decreases by the area under the force–distance graph as it falls, and all of it becomes kinetic energy on arrival: $1/2\, m v^2 = 1.11 \times 10^{10}$ J. Hence

$$v = \sqrt{\frac{2 \times 1.11 \times 10^{10}}{200}} = \sqrt{1.11 \times 10^8} = 1.05 \times 10^4\ \text{m s}^{-1}.$$

Question 19 The statement is only partly correct. **Only changes in gravitational potential energy have physical meaning** — the amount by which E_g rises or falls as an object moves between two positions. There is no absolute value of E_g; any value quoted depends on where the zero (reference level) is chosen. Taking the ground as the zero is a perfectly valid choice, and *relative to that reference* an object in orbit does have a larger (positive) gravitational potential energy, because reaching orbit requires work against gravity. But the ground is not "really" zero: with the more usual point-mass reference at infinity, every object nearer the planet has a **negative** gravitational potential energy. So the student's numbers are meaningful only as *differences* from a stated reference, not as absolute quantities.

Question 20 The energy needed to raise a satellite through a small extra height is the area of the thin strip under the force–distance graph at that height — approximately (force due to gravity) × (extra height). Because the force due to gravity $F_g = G M m / r^2$ **decreases** as the satellite gets higher, the graph is lower there, so each additional kilometre of altitude encloses a **smaller** area and therefore requires **less** energy than the same kilometre lower down. In short, the falling height of the force–distance curve with increasing r means the change in gravitational potential energy per kilometre steadily diminishes.

Theory

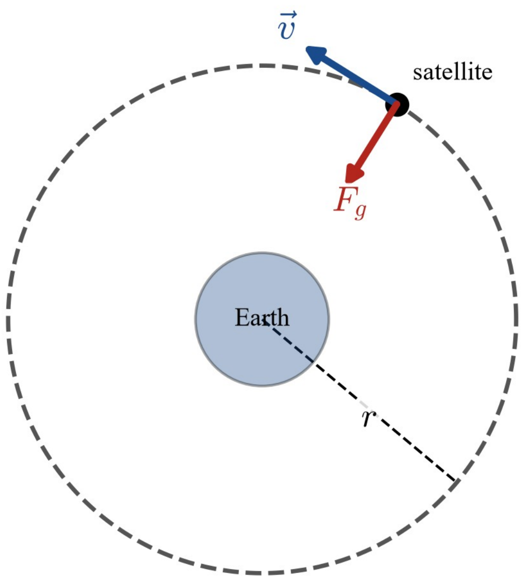
A **satellite** is any object that moves in orbit around a much more massive body under the action of gravity alone. **Natural satellites** include the Moon (orbiting Earth) and the planets (orbiting the Sun); **artificial satellites** are the spacecraft placed in orbit by rockets — communications satellites, weather satellites, navigation satellites and crewed stations. In this subtopic every orbit is modelled as **uniform circular motion**: the satellite moves in a circle of fixed radius r at *constant speed*, so although the speed does not change, the direction of the velocity does, and the motion is therefore accelerated.

The gravitational force provides the net force. The only force acting on an orbiting satellite is the gravitational force F_g exerted on it by the central body, directed toward the centre of that body. From the gravitational field (Subtopic 3A), a body of mass M produces a field of magnitude $g=\frac{GM}{r^2}$ at distance r, so the gravitational force on a satellite of mass m is

$$F_g=\frac{GMm}{r^2}\text{, directed toward the centre.}$$

Because the motion is circular, this single force is the **net** force, and it points toward the centre of the circle — it is the centripetal force. Setting the gravitational force equal to the centripetal requirement $\frac{mv^2}{r}$,

$$\frac{GMm}{r^2}=\frac{mv^2}{r}.$$

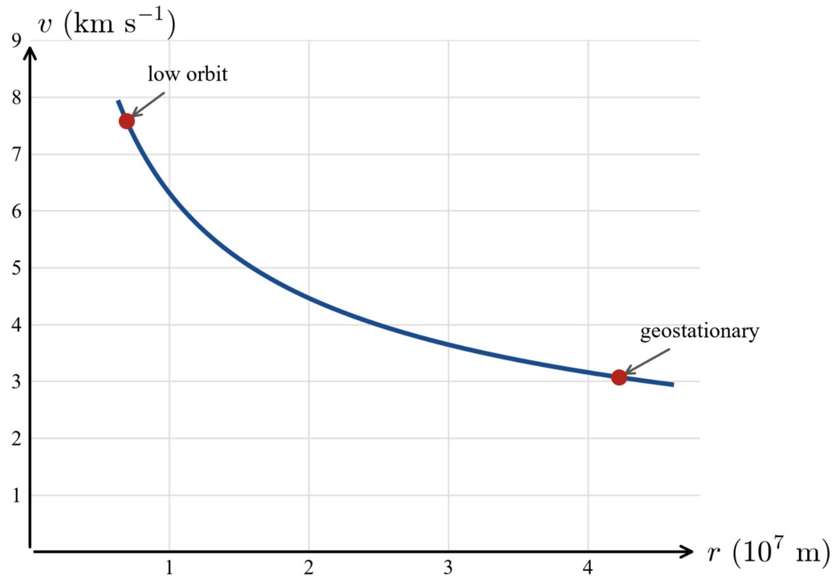


F_g is the net force; $a=\frac{v^2}{r}=\frac{GM}{r^2}$ toward the centre

Orbital speed. The satellite mass m cancels from both sides of the equation above, leaving

$$v=\sqrt{\frac{GM}{r}}.$$

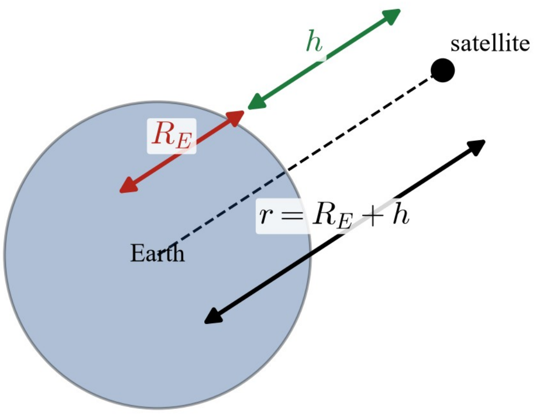
The orbital speed depends only on the mass M of the central body and the orbital radius r — **not** on the mass of the satellite. Two satellites sharing the same orbit travel at the same speed regardless of their sizes. Because $v \propto \frac{1}{\sqrt{r}}$, a satellite in a larger orbit moves more slowly.



Orbital radius versus altitude (the most common error). The radius r in every orbital equation is measured **from the centre of the central body**, not from its surface. For an Earth satellite at **altitude** h above the surface,

$$r = R_E + h,$$

where $R_E = 6.37 \times 10^6$ m is Earth's radius. An altitude quoted in kilometres must first be converted to metres and then added to R_E. Forgetting to add R_E — or leaving the altitude in kilometres — is the single most common mistake in orbital calculations.



orbital radius is measured from Earth's centre:
$r = R_E + h$ (never the altitude h alone)

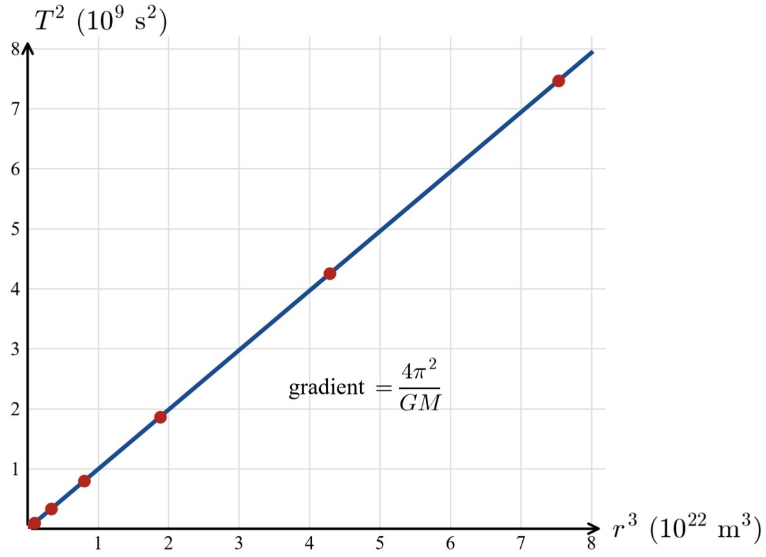
Period and the T^2–r^3 relationship. In one period T the satellite travels the circumference $2\pi r$, so its speed is $v = \frac{2\pi r}{T}$, and the centripetal acceleration can be written

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}.$$

Equating the gravitational force to the centripetal requirement in the form $\frac{GM}{r^2}=\frac{4\pi^2 r}{T^2}$ and rearranging gives

$$T^2=\frac{4\pi^2 r^3}{GM}\text{, so } T=2\pi\sqrt{\frac{r^3}{GM}}.$$

For every satellite of a given central body the ratio $\frac{T^2}{r^3}=\frac{4\pi^2}{GM}$ is the **same constant**. A graph of T^2 against r^3 is therefore a straight line through the origin whose gradient is $\frac{4\pi^2}{GM}$ — a useful way to find the mass of the central body from orbital data.

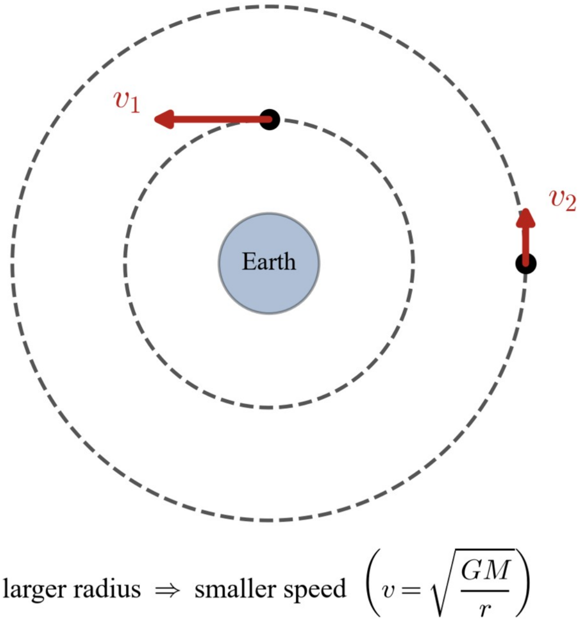


The acceleration equals the local field strength. Since the gravitational force is the net force, the satellite's centripetal acceleration is

$$a=\frac{v^2}{r}=\frac{GM}{r^2}=g \text{ at that radius.}$$

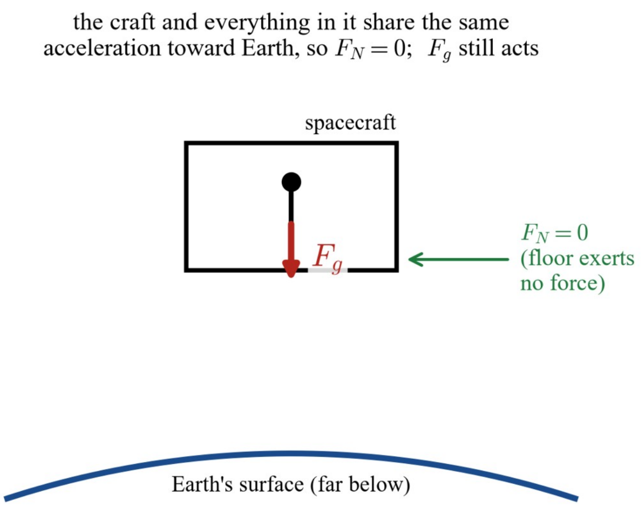
An orbiting satellite accelerates toward Earth at exactly the local gravitational field strength. At Earth's surface $g=\frac{GM_E}{R_E^2}=9.81\,\text{N kg}^{-1}$; at greater radius g is smaller, but it is never zero.

Higher orbits are slower. Combining $v=\sqrt{\frac{GM}{r}}$ with $T=2\pi\sqrt{\frac{r^3}{GM}}$ shows that a satellite in a higher orbit has both a smaller orbital speed and a longer period. A low-Earth-orbit satellite circles in about 90 minutes; a distant satellite may take many hours or, like the Moon, weeks.



Why a satellite accelerates (Newton's second law). A satellite in a circular orbit moves at constant speed, yet it is **accelerating**, because its velocity — a vector — is continually changing direction. The acceleration points toward the centre of the orbit. By **Newton's second law**, $\Sigma F = m a$, a non-zero acceleration requires a non-zero *net* force in the same direction. That net force is the **unbalanced gravitational force** F_g acting on the satellite. It is a common error to say that the forces on an orbiting satellite are balanced, or to appeal to Newton's first or third law; the correct statement is that a single unbalanced force — gravity — acts as the net force and, by Newton's second law, produces the centripetal acceleration $\frac{v^2}{r}$.

The normal force in orbit. Consider an astronaut inside an orbiting spacecraft. Both the astronaut and the craft are in the same circular orbit, so both have the same centripetal acceleration toward Earth, produced entirely by the gravitational force acting on each of them. Because the floor does not need to push the astronaut to give them this acceleration, the **normal force** F_N on the astronaut by the floor is **zero**. The gravitational force on the astronaut has **not** vanished — it is still $F_g = m g$ at that altitude, and it is precisely this force that provides the centripetal acceleration. A released object floats beside the astronaut for the same reason: object, astronaut and craft all accelerate toward Earth together at the same rate.



Throughout this subtopic the VCAA constants are used verbatim: $G=6.67\times10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$, $M_E=5.97\times10^{24}\,\text{kg}$ and $R_E=6.37\times10^6\,\text{m}$, giving $GM_E=3.98\times10^{14}\,\text{m}^3\,\text{s}^{-2}$. Masses of other central bodies are supplied where needed.

Worked examples

Example 1 — scaffolded

A satellite orbits Earth in a circular orbit at an altitude of $550\,\text{km}$ above the surface. Calculate the orbital radius, the orbital speed, the period of the orbit, and the centripetal acceleration of the satellite.

Working	Comment
Model: uniform circular motion; the gravitational force provides the net force.	The satellite's own mass will cancel.
Altitude in metres: $h=550\,\text{km}=5.50\times10^5\,\text{m}$.	Convert km to m before doing anything else.
$r=R_E+h=6.37\times10^6+5.50\times10^5=6.92\times10^6\,\text{m}$.	Radius is measured from Earth's centre — add R_E.
$v=\sqrt{\dfrac{GM_E}{r}}=\sqrt{\dfrac{(6.67\times10^{-11})(5.97\times10^{24})}{6.92\times10^6}}=7.59\times10^3$.	From $\dfrac{GM_E m}{r^2}=\dfrac{mv^2}{r}$.
$T=\dfrac{2\pi r}{v}=\dfrac{2\pi(6.92\times10^6)}{7.59\times10^3}=5.73\times10^3\,\text{s}$.	About 95 minutes — a typical low orbit.
$a=\dfrac{v^2}{r}=\dfrac{GM_E}{r^2}=8.32\,\text{m s}^{-2}$, toward Earth's centre.	The centripetal acceleration equals the local field strength g.

Example 2 — scaffolded

A geostationary satellite stays permanently above the same point on Earth's equator, so it must complete one orbit in exactly one day. Show that the orbital radius of a satellite of period T is $r=\left(\dfrac{GM_ET^2}{4\pi^2}\right)^{1/3}$, and hence find the altitude of a geostationary satellite above Earth's surface.

Working	Comment
Required period: $T=24\,\text{h}=8.64\times10^4\,\text{s}$.	One revolution per day keeps it above a fixed point.
Gravitational force = net force with $a=\dfrac{4\pi^2 r}{T^2}$: $\dfrac{GM_E m}{r^2}=\dfrac{4\pi^2 m r}{T^2}$.	Equate F_g to the centripetal requirement.
Cancel m and rearrange: $r^3=\dfrac{GM_ET^2}{4\pi^2}$, so $r=\left(\dfrac{GM_ET^2}{4\pi^2}\right)^{1/3}$.	Show-that: the satellite mass cancels.
$r=\left(\dfrac{(6.67\times10^{-11})(5.97\times10^{24})(8.64\times10^4)^2}{4\pi^2}\right)^{1/3}=4.2$.	Substitute the constants.
$h=r-R_E=4.223\times10^7-6.37\times10^6=3.59\times10^7\,\text{m}$.	Subtract R_E to get the altitude — about $35\,900\,\text{km}$.

Example 3 — unscaffolded

A satellite moves around Earth in a circular orbit at constant speed. Explain why the satellite is accelerating, identify the force responsible, and state the direction of the acceleration. Refer to Newton's second law in your answer.

Although the satellite's *speed* is constant, its *velocity* is a vector whose direction changes continuously as it moves around the circle — the velocity is always tangent to the orbit. A continuously changing velocity is, by definition, an acceleration, and for circular motion this acceleration is directed toward the centre of the orbit (centripetal). By Newton's second law, $\Sigma F = m a$, a non-zero acceleration must be caused by a non-zero net force acting in the same direction. The only force acting on the satellite is the gravitational force F_g exerted on it by Earth, directed toward Earth's centre; this single **unbalanced** force is the net force.

∴ the satellite accelerates because the gravitational force on it is unbalanced, and by Newton's second law this net force produces a centripetal acceleration $\frac{v^2}{r}$ directed toward Earth's centre.

Example 4 — unscaffolded

The Moon is a natural satellite of Earth, moving in an approximately circular orbit of radius $3.84 \times 10^8\,\text{m}$. Modelling its motion as uniform circular motion, calculate the Moon's orbital speed and its period, giving the period in both seconds and days.

The gravitational force on the Moon by Earth provides the net force, so the orbital speed is

$$v = \sqrt{\frac{G M_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{3.84 \times 10^8}} = 1.02 \times 10^3\,\text{m s}^{-1}.$$

The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(3.84 \times 10^8)}{1.02 \times 10^3} = 2.37 \times 10^6\,\text{s}.$$

Converting to days, $T = \frac{2.37 \times 10^6}{8.64 \times 10^4} = 27.4\,\text{days}$.

∴ the Moon's orbital speed is $1.02 \times 10^3\,\text{m s}^{-1}$ and its period is $2.37 \times 10^6\,\text{s}$, or about 27.4 days.

Practice questions

Question 1 A satellite orbits Earth in a circular orbit at an altitude of $800\,\text{km}$. (a) Calculate the orbital radius of the satellite. (b) Calculate the orbital speed of the satellite. (c) Calculate the period of the orbit.

Question 2 A satellite of mass m orbits Earth in a circular orbit of radius r. (a) Starting from the fact that the gravitational force provides the net force, show that the orbital speed is $v = \sqrt{\frac{G M_E}{r}}$, and name the force that provides the net force. (b) Hence explain why two satellites of different mass in the same orbit travel at the same speed. (c) Calculate the orbital speed of a satellite at a radius of $1.20 \times 10^7\,\text{m}$ from Earth's centre.

Question 3 A satellite orbits Earth in a circular orbit of radius $8.00 \times 10^6\,\text{m}$. (a) Show that the period of a satellite is given by $T = 2\pi\sqrt{\frac{r^3}{G M_E}}$. (b) Show that the period of this satellite is $7.12 \times 10^3\,\text{s}$. (c) Hence calculate the orbital speed of the satellite.

Question 4 A communications satellite moves around Earth in a circular orbit at constant speed. (a) State whether the satellite is accelerating, and give the direction of any acceleration. (b) With reference to Newton's second law, explain why the satellite accelerates, and name the force responsible. (c) A student claims that the forces on the satellite must be balanced because its speed does not change. Explain what is wrong with this claim.

Question 5 An astronaut of mass $72\,\mathrm{kg}$ is inside a spacecraft that orbits Earth in a circular orbit at an altitude of $350\,\mathrm{km}$. (a) Calculate the gravitational field strength at this altitude. (b) Calculate the gravitational force on the astronaut. (c) Explain why the normal force on the astronaut by the floor of the spacecraft is zero, even though the gravitational force on the astronaut is not zero.

Question 6 Mars is a natural satellite of the Sun, moving in an approximately circular orbit of radius $2.28 \times 10^{11}\,\mathrm{m}$. The mass of the Sun is $1.99 \times 10^{30}\,\mathrm{kg}$. (a) Calculate the orbital speed of Mars. (b) Calculate the orbital period of Mars, in seconds. (c) Express this period in days.

Question 7 A satellite orbits Earth in a circular orbit at an altitude of $2000\,\mathrm{km}$. Calculate its orbital speed and its period.

Question 8 A satellite is to orbit Earth in a circular orbit with a period of $12\,\mathrm{hours}$. Calculate its orbital radius and its altitude above Earth's surface.

Question 9 A planet moves around the Sun (mass $1.99 \times 10^{30}\,\mathrm{kg}$) in a circular orbit of radius $1.08 \times 10^{11}\,\mathrm{m}$. Calculate the planet's orbital speed and its period in Earth days.

Question 10 A moon orbits Jupiter (mass $1.90 \times 10^{27}\,\mathrm{kg}$) in a circular orbit of radius $4.22 \times 10^{8}\,\mathrm{m}$. Calculate the moon's orbital period, in days.

Question 11 Two satellites orbit the same planet: satellite A at radius r and satellite B at radius $2r$. Show that the ratio of their orbital speeds is $\frac{v_A}{v_B} = \sqrt{2}$, and state, with justification, which satellite has the longer period.

Question 12 A satellite orbits a planet in a circular orbit of radius $9.60 \times 10^{6}\,\mathrm{m}$ with a period of $7.20 \times 10^{3}\,\mathrm{s}$. Calculate the mass of the planet.

Question 13 A satellite orbits Earth in a circular orbit of radius $1.50 \times 10^{7}\,\mathrm{m}$. Calculate the centripetal acceleration of the satellite, compare it with the gravitational field strength at that radius, and explain the result.

Question 14 An artificial satellite travels around Earth at constant speed in a circular orbit. A student argues: *"Because the satellite's speed is constant, its velocity is constant, so it is not accelerating and no net force acts on it."* Identify the two errors in this reasoning and give the correct physics, with reference to Newton's second law.

Question 15 Inside an orbiting space station, a released object floats beside an astronaut instead of falling to the floor, and the floor exerts no normal force on the astronaut. Explain both observations in terms of the gravitational force acting and the acceleration of the station and its contents.

Question 16 It is sometimes said that there is "no gravity" acting on a spacecraft in low Earth orbit at an altitude of about $400\,\mathrm{km}$. By calculating the gravitational field strength at this altitude and comparing it with the value at Earth's surface, explain why this statement is incorrect.

Question 17 Two satellites orbit Earth, one at an altitude of $300\,\mathrm{km}$ and the other at an altitude of $35\,800\,\mathrm{km}$. Calculate the orbital speed of each, and state which satellite moves faster.

Question 18 An Earth satellite has an orbital speed of $6.50 \times 10^{3}\,\mathrm{m\,s^{-1}}$. Calculate its orbital radius and its altitude above Earth's surface. Give your answer to three significant figures.

Question 19 For several moons of a distant planet, a graph of T^2 against r^3 is a straight line passing through the origin with a gradient of $9.30 \times 10^{-14}\,\mathrm{s^2\,m^{-3}}$. (a) Explain why this graph is a straight line through the origin. (b) Use the gradient to calculate the mass of the planet.

Question 20 Explain, with reference to the gravitational force and the orbital-speed relationship, why a satellite in a higher circular orbit moves more slowly than a satellite in a lower circular orbit.

Question 21 The International Space Station orbits Earth in a circular orbit at an altitude of $420\,\mathrm{km}$. Calculate (i) its orbital speed, (ii) its orbital period, and (iii) the number of complete orbits it makes in one day.

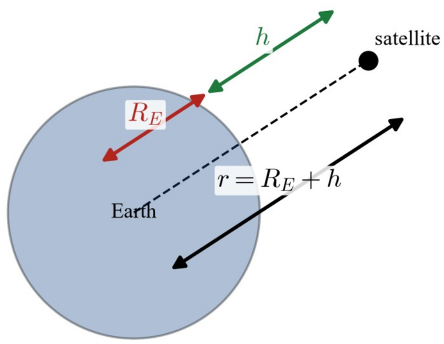
Question 22 A geostationary satellite must remain above a fixed point on Earth's equator. (a) State the period such a satellite must have. (b) A proposed satellite is to orbit at an altitude of $25\,000\,\mathrm{km}$. By calculating its period, determine whether it could be geostationary.

Answers

Question 1 (a) 7.17×10^{6} m; (b) 7.45×10^{3} m s^{-1}; (c) 6.05×10^{3} s. **Question 2** (c) 5.76×10^{3} m s^{-1}. **Question 3** (b) 7.12×10^{3} s; (c) 7.06×10^{3} m s^{-1}. **Question 4** (a) accelerating, toward Earth's centre; (b) unbalanced gravitational force is the net force (Newton's second law); (c) constant speed does not mean balanced forces — the velocity changes direction. **Question 5** (a) 8.82 N kg^{-1}; (b) 635 N; (c) $F_N=0$ because astronaut and craft share the same acceleration, provided entirely by gravity. **Question 6** (a) 2.41×10^{4} m s^{-1}; (b) 5.94×10^{7} s; (c) 687 days. **Question 7** 6.90×10^{3} m s^{-1}; 7.62×10^{3} s. **Question 8** $r=2.66\times10^{7}$ m; altitude $=2.02\times10^{7}$ m. **Question 9** 3.51×10^{4} m s^{-1}; 1.94×10^{7} s $=224$ days. **Question 10** 1.53×10^{5} s $=1.77$ days. **Question 11** $\frac{v_A}{v_B}=\sqrt{2}\approx1.41$; satellite B (larger radius) has the longer period. **Question 12** 1.01×10^{25} kg. **Question 13** $a=1.77$ m s^{-2}, equal to $g=1.77$ N kg^{-1} at that radius. **Question 14** Speed constant does not mean velocity constant (direction changes); the satellite accelerates, so a net force (gravity) does act. **Question 15** Both float/experience $F_N=0$ because object, astronaut and station share the same acceleration toward Earth; F_g still acts. **Question 16** $g=8.69$ N kg^{-1}, about 89 % of the surface value 9.81 N kg^{-1} — far from zero. **Question 17** 300 km: 7.73×10^{3} m s^{-1}; 35 800 km: 3.07×10^{3} m s^{-1}; the lower satellite moves faster. **Question 18** $r=9.42\times10^{6}$ m; altitude $=3.05\times10^{6}$ m. **Question 19** (b) 6.36×10^{24} kg. **Question 20** Larger r gives a weaker field and, from $v=\sqrt{GM/r}$, a smaller orbital speed. **Question 21** (i) 7.66×10^{3} m s^{-1}; (ii) 5.57×10^{3} s; (iii) about 15.5 orbits. **Question 22** (a) 24 h $=8.64\times10^{4}$ s; (b) period $=5.53\times10^{4}$ s (about 15.4 h), not 24 h, so it could not be geostationary.

Fully-worked solutions

Question 1 Altitude $h=800\text{ km}=8.00\times10^{5}\text{ m}$.



orbital radius is measured from Earth's centre:
$r = R_E + h$ (never the altitude h alone)

(a) $r=R_E+h=6.37\times10^{6}+8.00\times10^{5}=7.17\times10^{6}$ m.

(b) $v=\sqrt{\frac{GM_E}{r}}=\sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{7.17\times10^{6}}}=7.45\times10^{3}\text{ m s}^{-1}$.

(c) $T=\frac{2\pi r}{v}=\frac{2\pi(7.17\times10^{6})}{7.45\times10^{3}}=6.05\times10^{3}\text{ s}$.

Question 2 (a) The only force on the satellite is the gravitational force $F_g=\frac{GM_Em}{r^2}$ on it by Earth, directed toward Earth's centre. As the motion is circular this is the net (centripetal) force, so $\frac{GM_Em}{r^2}=\frac{mv^2}{r}$. The mass m cancels, leaving $v^2=\frac{GM_E}{r}$, hence $v=\sqrt{\frac{GM_E}{r}}$, as required. The net force is provided by the gravitational force. (b) The satellite mass m cancels out of the equation, so v depends only on M_E and r. Two satellites in the same orbit have the same r and therefore the same speed, whatever their masses. (c) $v=\sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{1.20\times10^{7}}}=5.76\times10^{3}\,\mathrm{m\ s^{-1}}$.

Question 3 (a) The gravitational force is the net force, and with $a=\frac{4\pi^2 r}{T^2}$, $\frac{GM_E m}{r^2}=\frac{4\pi^2 m r}{T^2}$. Cancelling m and rearranging gives $T^2=\frac{4\pi^2 r^3}{GM_E}$, so $T=2\pi\sqrt{\frac{r^3}{GM_E}}$, as required. (b) $T=2\pi\sqrt{\frac{(8.00\times10^{6})^3}{(6.67\times10^{-11})(5.97\times10^{24})}}=2\pi\sqrt{1.29\times10^{6}}=7.12\times10^{3}\,\mathrm{s}$. (c) $v=\frac{2\pi r}{T}=\frac{2\pi(8.00\times10^{6})}{7.12\times10^{3}}=7.06\times10^{3}\,\mathrm{m\ s^{-1}}$. (Equivalently $v=\sqrt{GM_E/r}=7.06\times10^{3}\,\mathrm{m\ s^{-1}}$.)

Question 4 (a) The satellite **is** accelerating, and the acceleration is directed toward Earth’s centre (centripetal). (b) The speed is constant, but the direction of the velocity changes continuously, so the velocity — a vector — is changing; a changing velocity is an acceleration. By Newton’s second law, $\Sigma F=ma$, this acceleration must be produced by a non-zero net force in the same direction. That net force is the unbalanced gravitational force F_g on the satellite by Earth, directed toward Earth’s centre. (c) A constant *speed* does not imply balanced forces. The velocity is still changing direction, so the satellite is accelerating, and an acceleration requires an unbalanced net force. If the forces really were balanced, Newton’s first law would have the satellite move in a straight line at constant velocity — it would not follow a circular orbit at all. The single gravitational force is unbalanced and acts as the net force.

Question 5 $r=R_E+h=6.37\times10^{6}+3.50\times10^{5}=6.72\times10^{6}\,\mathrm{m}$. (a) $g=\frac{GM_E}{r^2}=\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.72\times10^{6})^2}=8.82\,\mathrm{N\ kg^{-1}}$. (b) $F_g=mg=72\times8.82=635\,\mathrm{N}$, directed toward Earth’s centre. (c) The astronaut and the spacecraft are in the same orbit, so both have the same centripetal acceleration toward Earth. This acceleration is produced entirely by the gravitational force acting on each of them, so the floor does not need to push on the astronaut — the normal force F_N on the astronaut by the floor is therefore zero. The gravitational force on the astronaut is unchanged (635 N) and is exactly what supplies the centripetal acceleration.

Question 6 $r=2.28\times10^{11}\,\mathrm{m}$, central mass $M=1.99\times10^{30}\,\mathrm{kg}$, so $GM=1.33\times10^{20}\,\mathrm{m^3\,s^{-2}}$. (a) $v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{1.33\times10^{20}}{2.28\times10^{11}}}=2.41\times10^{4}\,\mathrm{m\ s^{-1}}$. (b) $T=\frac{2\pi r}{v}=\frac{2\pi(2.28\times10^{11})}{2.41\times10^{4}}=5.94\times10^{7}\,\mathrm{s}$. (c) $T=\frac{5.94\times10^{7}}{8.64\times10^{4}}=687\,\mathrm{days}$.

Question 7 $r=R_E+h=6.37\times10^{6}+2.00\times10^{6}=8.37\times10^{6}\,\mathrm{m}$. $v=\sqrt{\frac{GM_E}{r}}=\sqrt{\frac{3.98\times10^{14}}{8.37\times10^{6}}}=6.90\times10^{3}\,\mathrm{m\ s^{-1}}$. $T=\frac{2\pi r}{v}=\frac{2\pi(8.37\times10^{6})}{6.90\times10^{3}}=7.62\times10^{3}\,\mathrm{s}$.

Question 8 $T=12\,\mathrm{h}=4.32\times10^{4}\,\mathrm{s}$. From $T^2=\frac{4\pi^2 r^3}{GM_E}$,

$$r=\left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}=\left(\frac{(3.98\times10^{14})(4.32\times10^{4})^2}{4\pi^2}\right)^{1/3}=2.66\times10^{7}\,\mathrm{m}.$$

Altitude $h = r - R_E = 2.660\times10^7 - 6.37\times10^6 = 2.02\times10^7$ m (about 20 200 km).

Question 9 $GM = (6.67\times10^{-11})(1.99\times10^{30}) = 1.33\times10^{20}\,\text{m}^3\,\text{s}^{-2}$. $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{1.33\times10^{20}}{1.08\times10^{11}}} = 3.51\times10^4\,\text{m s}^{-1}$.

$T = \frac{2\pi r}{v} = \frac{2\pi(1.08\times10^{11})}{3.51\times10^4} = 1.94\times10^7\,\text{s} = \frac{1.94\times10^7}{8.64\times10^4} = 224\,\text{days}$.

Question 10 $GM = (6.67\times10^{-11})(1.90\times10^{27}) = 1.27\times10^{17}\,\text{m}^3\,\text{s}^{-2}$.
$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(4.22\times10^8)^3}{1.27\times10^{17}}} = 1.53\times10^5\,\text{s}$. In days, $T = \frac{1.53\times10^5}{8.64\times10^4} = 1.77\,\text{days}$.

Question 11 For any satellite, $v = \sqrt{\frac{GM}{r}}$. Therefore

$$\frac{v_A}{v_B} = \frac{\sqrt{GM/r}}{\sqrt{GM/2r}} = \sqrt{\frac{2r}{r}} = \sqrt{2} \approx 1.41\,.$$

Satellite A (smaller radius) is the faster, so satellite **B**, with the larger radius, has the longer period. This also follows from $T = 2\pi\sqrt{r^3/GM}$: a larger r gives both a larger circumference to travel and a smaller speed, so T increases with r.

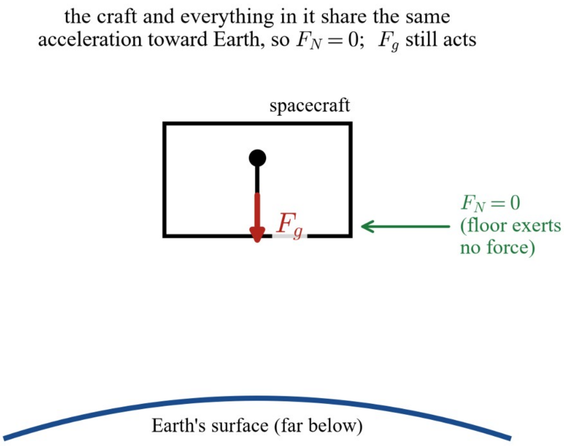
Question 12 From $T^2 = \frac{4\pi^2 r^3}{GM}$, the mass of the planet is

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(9.60\times10^6)^3}{(6.67\times10^{-11})(7.20\times10^3)^2} = 1.01\times10^{25}\,\text{kg}\,.$$

Question 13 $a = \frac{GM_E}{r^2} = \frac{3.98\times10^{14}}{(1.50\times10^7)^2} = 1.77\,\text{m s}^{-2}$, directed toward Earth's centre. The gravitational field strength at that radius is $g = \frac{GM_E}{r^2} = 1.77\,\text{N kg}^{-1}$ — numerically equal to the acceleration. They are equal because the gravitational force is the only force acting and therefore provides the entire centripetal force: the satellite's acceleration is exactly the local field strength, $a = g$.

Question 14 *First error:* a constant speed does **not** mean a constant velocity. Velocity is a vector; as the satellite moves around the circle its direction changes continuously, so the velocity is changing even though the speed is not. *Second error:* because the velocity is changing, the satellite **is** accelerating (toward the centre of the orbit), and by Newton's second law, $\Sigma F = ma$, a net force must therefore act on it. That net force is the unbalanced gravitational force on the satellite by Earth, directed toward Earth's centre. Correct statement: the satellite moves at constant speed but changing velocity, so it is accelerating, and the gravitational force is the unbalanced net force that produces this centripetal acceleration.

Question 15



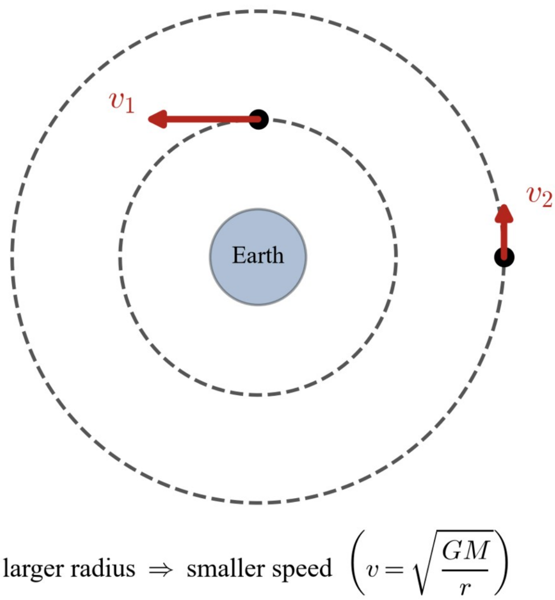
The space station, the astronaut and the released object are all in the same orbit and therefore all have the **same acceleration** toward Earth, produced entirely by the gravitational force acting on each of them. Because the released object accelerates toward Earth at exactly the same rate as the floor of the station beside it, it does not approach the floor — it appears to float alongside the astronaut. For the same reason the floor does not have to push on the astronaut to give them their acceleration, so the normal force F_N on the astronaut by the floor is zero. The gravitational force has not disappeared: it still acts on the astronaut and on the object, and it is precisely this force that provides their centripetal acceleration.

Question 16 At altitude $400\,\mathrm{km}$, $r = R_E + h = 6.37\times10^6 + 4.00\times10^5 = 6.77\times10^6\,\mathrm{m}$, so

$$g = \frac{GM_E}{r^2} = \frac{3.98\times10^{14}}{(6.77\times10^6)^2} = 8.69\,\mathrm{N\ kg^{-1}}.$$

At Earth's surface $g = \dfrac{GM_E}{R_E^2} = 9.81\,\mathrm{N\ kg^{-1}}$. The ratio is $\dfrac{8.69}{9.81} = 0.885$, so the field at this altitude is about $89\,\%$ of its surface value — nowhere near zero. Gravity is still strong at $400\,\mathrm{km}$; indeed it is the gravitational force that keeps the spacecraft in orbit. The apparent floating of astronauts is because they and their craft share the same acceleration, not because gravity is absent.

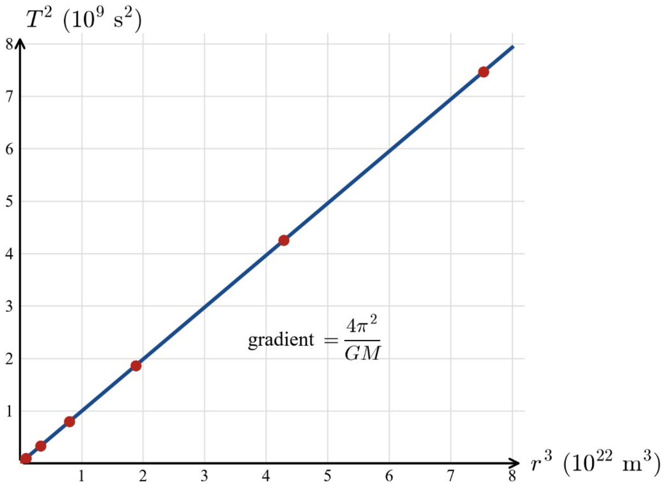
Question 17



Lower satellite: $r = 6.37 \times 10^6 + 3.00 \times 10^5 = 6.67 \times 10^6$ m, so $v = \sqrt{\frac{3.98 \times 10^{14}}{6.67 \times 10^6}} = 7.73 \times 10^3$ m s^{-1}. Higher satellite: $r = 6.37 \times 10^6 + 3.58 \times 10^7 = 4.22 \times 10^7$ m, so $v = \sqrt{\frac{3.98 \times 10^{14}}{4.22 \times 10^7}} = 3.07 \times 10^3$ m s^{-1}. The satellite in the lower orbit moves faster, consistent with $v \propto \frac{1}{\sqrt{r}}$.

Question 18 From $v = \sqrt{\frac{G M_E}{r}}$, $r = \frac{G M_E}{v^2} = \frac{3.98 \times 10^{14}}{(6.50 \times 10^3)^2} = 9.42 \times 10^6$ m. Altitude $h = r - R_E = 9.42 \times 10^6 - 6.37 \times 10^6 = 3.05 \times 10^6$ m (to three significant figures).

Question 19



(a) For satellites of one central body, $T^2 = \frac{4\pi^2}{GM} r^3$. This has the form $T^2 = (\text{constant}) \times r^3$, a direct proportion between T^2 and r^3, so the graph is a straight line; and because $T^2 = 0$ when $r^3 = 0$, the line passes through the origin.

(b) The gradient is $\frac{4\pi^2}{GM}$, so

$$M = \frac{4\pi^2}{G \times \text{gradient}} = \frac{4\pi^2}{(6.67 \times 10^{-11})(9.30 \times 10^{-14})} = 6.36 \times 10^{24}\,\text{kg}.$$

Question 20 The gravitational force provides the net (centripetal) force on a satellite, and this force weakens with distance as $F_g = \frac{G M_E m}{r^2}$. Equating it to $\frac{m v^2}{r}$ and cancelling m gives $v = \sqrt{\frac{G M_E}{r}}$. Since $v \propto \frac{1}{\sqrt{r}}$, a larger orbital radius corresponds to a smaller orbital speed: a satellite in a higher orbit needs (and has) a smaller centripetal acceleration, matched by the weaker gravitational field there, so it moves more slowly than a satellite in a lower orbit.

Question 21 $r = R_E + h = 6.37 \times 10^6 + 4.20 \times 10^5 = 6.79 \times 10^6$ m. (i) $v = \sqrt{\frac{3.98 \times 10^{14}}{6.79 \times 10^6}} = 7.66 \times 10^3$ m s^{-1}. (ii) $T = \frac{2\pi r}{v} = \frac{2\pi(6.79 \times 10^6)}{7.66 \times 10^3} = 5.57 \times 10^3$ s. (iii) Orbits per day $= \frac{8.64 \times 10^4}{5.57 \times 10^3} = 15.5$, so the station makes about 15 or 16 complete orbits (about 15.5) each day.

Question 22 (a) To stay above a fixed point on the equator the satellite must turn with Earth, completing one orbit per day: $T = 24\,\text{h} = 8.64 \times 10^4$ s. (b) At altitude 25 000 km, $r = R_E + h = 6.37 \times 10^6 + 2.50 \times 10^7 = 3.14 \times 10^7$ m, so

$$T = 2\pi\sqrt{\frac{r^3}{G M_E}} = 2\pi\sqrt{\frac{(3.14\times10^7)^3}{3.98\times10^{14}}} = 5.53\times10^4\,\mathrm{s}\,,$$

which is about $15.4\,\mathrm{h}$. This is not equal to the required $24\,\mathrm{h}$, so a satellite at this altitude could **not** be geostationary — it would drift relative to the ground.