Physics Units 3 & 4

Chapter 2 — Force, energy and mass

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Theory

When a force acts on an object and the object moves, the force transfers energy to or from the object. That energy transfer is called **work**. For a constant force acting along the line of motion, the work done is the product of the force and the displacement through which it acts:

$$W = F\,s\,.$$

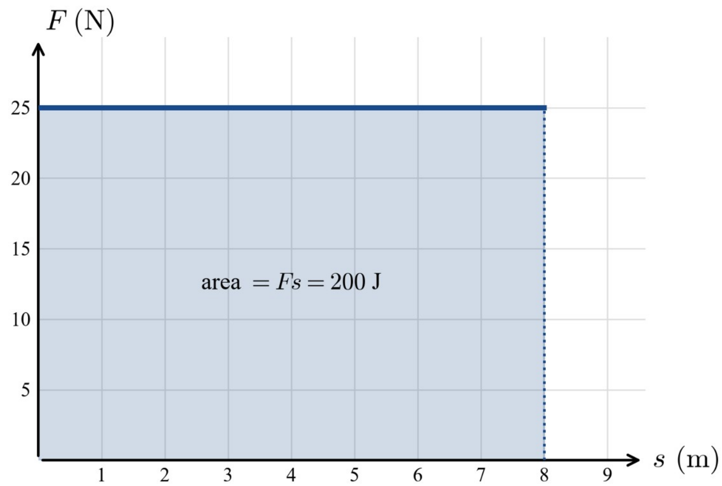
Work is a scalar and is measured in **joules** (J); one joule is the work done by a force of $1\,\mathrm{N}$ acting through $1\,\mathrm{m}$ ($1\,\mathrm{J} = 1\,\mathrm{N\,m}$).

Only the force component along the motion does work. If a constant force F acts at an angle θ to the displacement s, only the component $F\cos\theta$ lies along the motion, so

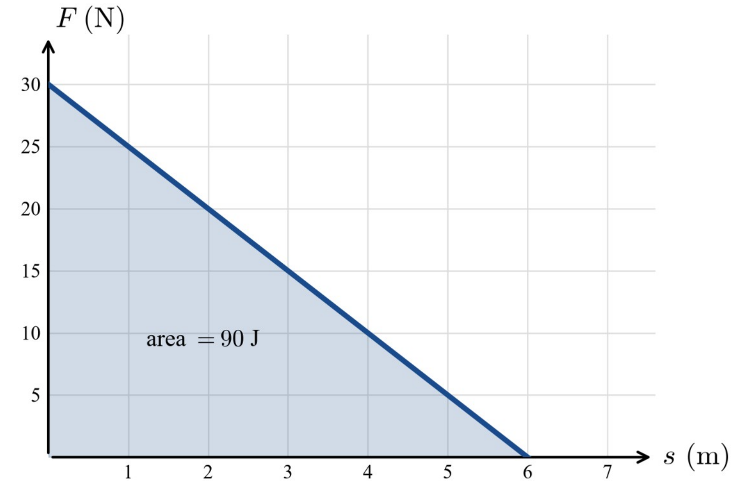
$$W = F\,s\cos\theta\,.$$

A force **perpendicular** to the motion ($\theta = 90^\circ$) does **no work at all**: for example the normal force on an object sliding along a level floor, and the net (centripetal) force on an object in uniform circular motion, are each perpendicular to the motion and transfer no energy. Work is **positive** when the force has a component in the direction of motion (the force adds energy) and **negative** when it opposes the motion (the force removes energy) — friction acting on a sliding object does negative work.

Work from the area under a force–distance graph (one dimension). When the force varies with position, $W = F\,s$ cannot be used directly. Instead, the work done as the object moves along a straight line equals the **area between the force–distance graph and the distance axis**. For a constant force this area is a rectangle, $W = F\,s$:



For a force that changes with position the work is still the area under the graph — a triangle, a trapezium, or a combination of shapes added together:



Where the graph dips **below** the axis (the force opposes the motion), that area counts as **negative** work, and the **net** work is the signed sum of the areas above and below the axis.

Kinetic energy. A moving object carries **kinetic energy**, the energy associated with its motion. At the low speeds treated here,

$$E_k = 1/2\, m v^2,$$

measured in joules. The kinetic energy grows with the *square* of the speed, so doubling the speed quadruples the kinetic energy.

The work–energy theorem. The **net** work done on an object equals the change in its kinetic energy:

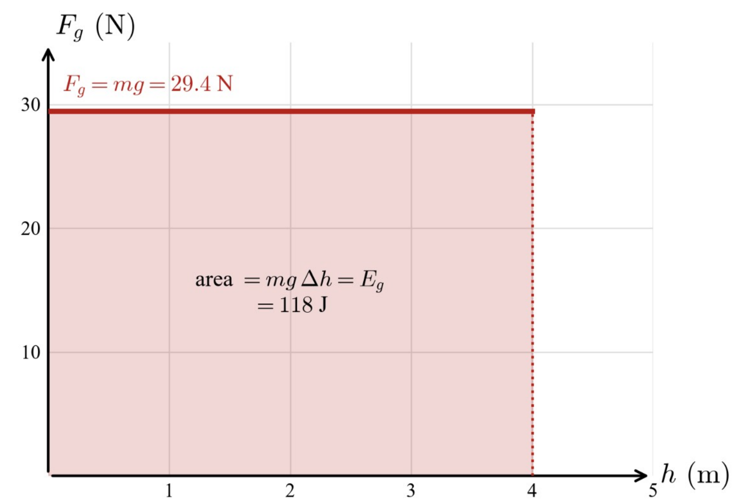
$$W_{\text{net}} = \Delta E_k = 1/2\, m v^2 - 1/2\, m u^2,$$

where u and v are the initial and final speeds. This links the two ideas above: the area under the net force–distance graph is the change in kinetic energy. (This area is **work**, not impulse — impulse is the area under a force–*time* graph, treated in the next subtopic.)

Gravitational potential energy. Near Earth's surface the force due to gravity on an object of mass m is $F_g = m g$, and it is very nearly constant. Lifting the object through a vertical height Δh stores **gravitational potential energy**:

$$E_g = m g \Delta h.$$

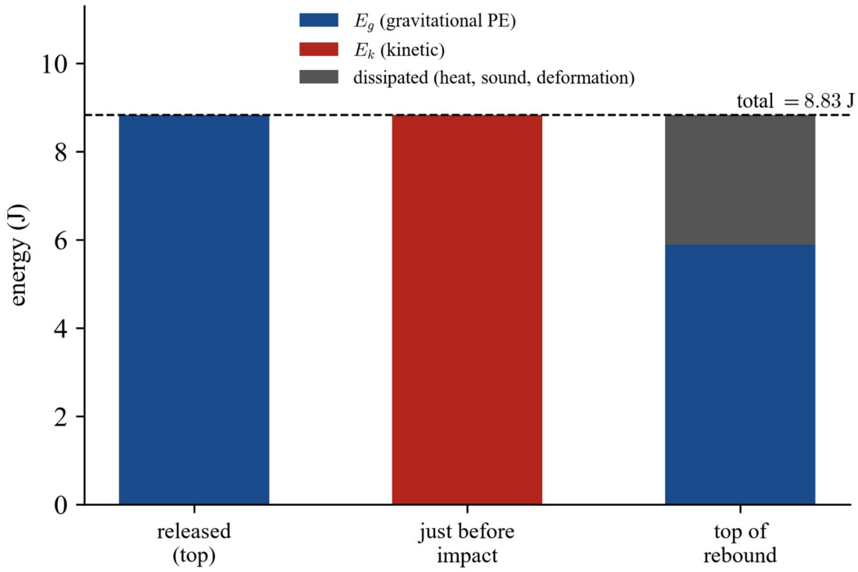
Because $F_g = m g$ is constant near the surface, E_g is also the area under the (horizontal) force–height graph — a rectangle of height $m g$ and width Δh:



Only the **vertical** part of any movement changes E_g; moving horizontally does not change an object's gravitational potential energy. Throughout this subtopic $g = 9.81\ \text{m s}^{-2}$ (equivalently $9.81\ \text{N kg}^{-1}$).

Energy transformations and conservation. Energy is never created or destroyed, only transformed from one form to another. In mechanical systems it moves between kinetic energy, gravitational potential energy and elastic potential energy (the last is stored in stretched or compressed objects such as springs, developed in 2C). In an **ideal** case — no friction and no air resistance — the total mechanical energy $E_k + E_g$ stays constant, so an object speeds up exactly as it loses height and slows exactly as it gains height.

In real situations some mechanical energy is always **dissipated to the environment**, considered as a combination of **heat, sound and deformation of material**. A car braking to rest, a box skidding across a floor, and a ball that rebounds to a lower height than it was dropped from all lose mechanical energy this way. The total energy is still conserved — the “missing” mechanical energy now exists as thermal energy, sound and permanent deformation in the surroundings — but the object’s kinetic and potential energies no longer add up to their starting value. The bar chart below tracks a bouncing ball: the total stays fixed while, at the top of the rebound, part of the original energy has been dissipated:



Whether kinetic energy is conserved when two objects collide distinguishes elastic from inelastic collisions; that test is examined alongside momentum in 2B.

Worked examples

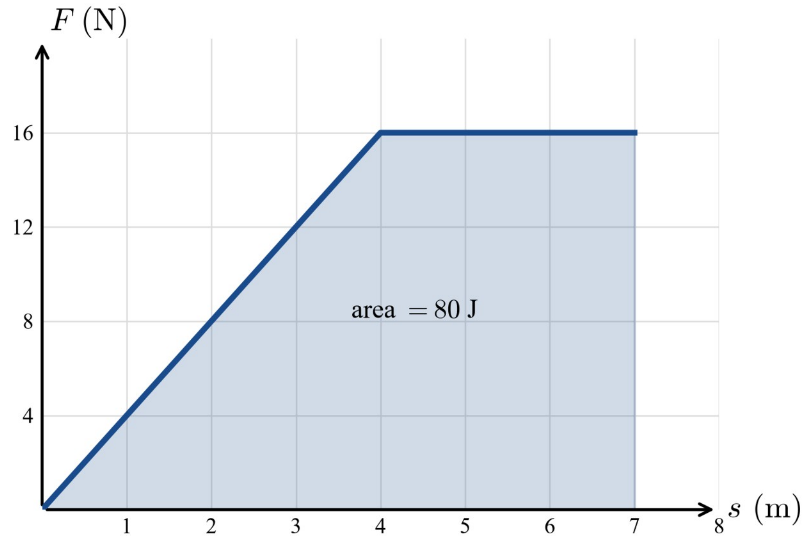
Example 1 — scaffolded

A $3.0\,\mathrm{kg}$ trolley on a frictionless horizontal track is pushed from rest by a constant horizontal force of $12\,\mathrm{N}$ through a distance of $4.0\,\mathrm{m}$. Calculate the work done on the trolley, its kinetic energy after $4.0\,\mathrm{m}$, and its speed at that point.

Working	Comment
$m = 3.0\,\mathrm{kg}$, $F = 12\,\mathrm{N}$ (along the motion), $s = 4.0\,\mathrm{m}$, from rest.	List the data; the force is parallel to the displacement.
$W = F\,s = 12 \times 4.0 = 48\,\mathrm{J}$.	Work done by a constant force along the line of motion.
Track is frictionless, so all the work becomes kinetic energy: $E_k = W = 48\,\mathrm{J}$.	Work–energy theorem: $W_{\mathrm{net}} = \Delta E_k$ (starts from rest).
$E_k = 1/2\,m\,v^2 \Rightarrow v = \sqrt{\dfrac{2\,E_k}{m}} = \sqrt{\dfrac{2 \times 48}{3.0}} = 5.66\,\mathrm{m\,s^{-1}}$.	Rearrange for speed.

Example 2 — scaffolded

A $2.0\,\mathrm{kg}$ trolley starts from rest on a frictionless track. The horizontal force pushing it varies with position as shown. Calculate the work done on the trolley over the first $7.0\,\mathrm{m}$ and its speed at that point.



Working	Comment
Work = area under the force–distance graph.	The force varies, so use the area, not $W = F\,s$.
Ramp, $0 \rightarrow 4.0\,\mathrm{m}$: area $= 1/2 \times 4.0 \times 16 = 32\,\mathrm{J}$.	Triangle: $1/2$ × base × height.
Level, $4.0 \rightarrow 7.0\,\mathrm{m}$: area $= 16 \times 3.0 = 48\,\mathrm{J}$.	Rectangle: constant force × distance.
Total $W = 32 + 48 = 80\,\mathrm{J}$.	Add the areas (both above the axis ⇒ positive work).
From rest, $W = 1/2\,m\,v^2 \Rightarrow v = \sqrt{\dfrac{2 \times 80}{2.0}} = 8.94\,\mathrm{m\ s^{-1}}$.	Net work equals the gain in kinetic energy.

Example 3 — unscaffolded

A $3.0\,\mathrm{kg}$ box is released from rest and slides down a ramp, descending a vertical height of $2.5\,\mathrm{m}$. It reaches the bottom moving at $5.0\,\mathrm{m\ s^{-1}}$. Calculate the energy dissipated to the environment during the descent, and state the forms it takes.

As the box descends, its gravitational potential energy decreases by

$$E_g = m\,g\,\Delta h = 3.0 \times 9.81 \times 2.5 = 73.575\,\mathrm{J}\,.$$

Its kinetic energy at the bottom is

$$E_k = 1/2\,m\,v^2 = 1/2 \times 3.0 \times 5.0^2 = 37.5\,\mathrm{J}\,.$$

Total energy is conserved, so the gravitational potential energy that has **not** become kinetic energy must have been dissipated to the environment:

$$E_{\text{dissipated}} = E_g - E_k = 73.575 - 37.5 = 36.075\,\mathrm{J}\,.$$

∴ about $36.1\,\mathrm{J}$ is dissipated to the environment, mainly as heat (with a little sound) produced by friction between the box and the ramp.

Example 4 — unscaffolded

A child pulls a $20\,\mathrm{kg}$ sled $12\,\mathrm{m}$ across smooth (frictionless) ice using a rope held at 25° above the horizontal, with a constant force of $140\,\mathrm{N}$. The sled starts from rest. Calculate the work done by the rope on the sled and the sled's final speed.

Only the horizontal component of the rope's force, $F\cos\theta$, lies along the displacement, so

$$W = F\,s\cos\theta = 140 \times 12 \times \cos 25^\circ = 1522.6\,\mathrm{J} \approx 1.52 \times 10^3\,\mathrm{J}\,.$$

The ice is frictionless, so this work all becomes kinetic energy:

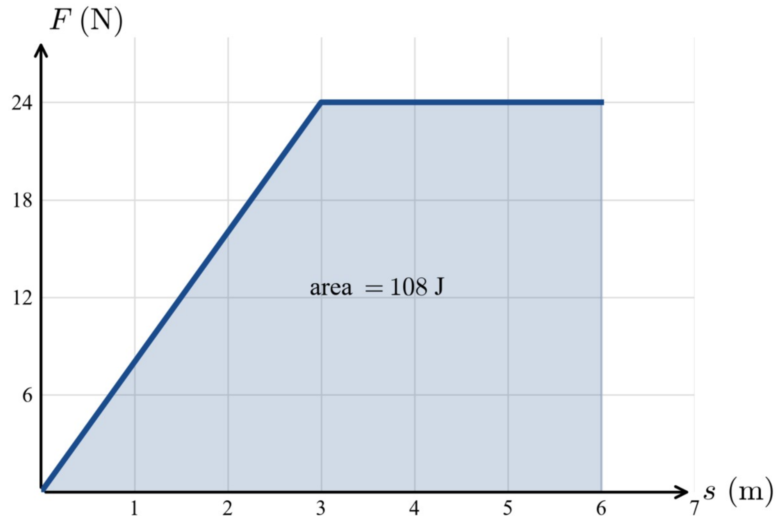
$$1/2 m v^2 = W \Rightarrow v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 1522.6}{20}} = 12.3\,\mathrm{m\,s^{-1}}.$$

∴ the rope does 1.52×10^3 J of work and the sled reaches $12.3\,\mathrm{m\,s^{-1}}$. (The vertical component $F \sin\theta$ is perpendicular to the motion and does no work.)

Practice questions

Question 1 A $2.0\,\mathrm{kg}$ box is pushed from rest across a frictionless floor by a constant horizontal force of $9.0\,\mathrm{N}$ through $4.0\,\mathrm{m}$. (a) Calculate the work done on the box. (b) Calculate the kinetic energy of the box after $4.0\,\mathrm{m}$. (c) Calculate the speed of the box after $4.0\,\mathrm{m}$.

Question 2 A $4.0\,\mathrm{kg}$ trolley starts from rest on a frictionless track. The horizontal force acting on it varies with position as shown.



(a) Show that the work done on the trolley over the first $6.0\,\mathrm{m}$ is $108\,\mathrm{J}$.
(b) Calculate the kinetic energy of the trolley after $6.0\,\mathrm{m}$.
(c) Calculate the speed of the trolley after $6.0\,\mathrm{m}$.

Question 3 A $0.30\,\mathrm{kg}$ ball is dropped from rest from a height of $2.5\,\mathrm{m}$. Air resistance is negligible. (a) Calculate the gravitational potential energy lost by the ball as it falls to the ground. (b) Calculate the kinetic energy of the ball just before it reaches the ground. (c) Calculate the speed of the ball just before it reaches the ground.

Question 4 A $1200\,\mathrm{kg}$ car travelling at $15\,\mathrm{m\,s^{-1}}$ brakes to rest. (a) Calculate the initial kinetic energy of the car. (b) The car stops in a braking distance of $22\,\mathrm{m}$. Calculate the average braking force. (c) State the form(s) into which the car's kinetic energy is transformed as it brakes.

Question 5 A $2.0\,\mathrm{kg}$ block is released from rest at the top of a ramp and descends a vertical height of $1.5\,\mathrm{m}$, reaching the bottom at $4.0\,\mathrm{m\,s^{-1}}$. (a) Calculate the gravitational potential energy converted as the block descends. (b) Calculate the kinetic energy of the block at the bottom of the ramp. (c) Calculate the energy dissipated to the environment, and state the form(s) it takes.

Question 6 A ball is dropped onto a hard floor and bounces. (a) (Circle your answer.) The height the ball rebounds to is *less than* / *equal to* / *greater than* the height from which it was dropped. Justify your choice with reference to energy. (b) Name the forms into which the "missing" mechanical energy is transformed. (c) Explain, with reference to conservation of energy, why the total energy of the ball and its surroundings is still conserved even though the ball does not rebound to its original height.

Question 7 A worker pushes a $40\,\mathrm{kg}$ crate $6.0\,\mathrm{m}$ across a level floor with a constant horizontal force of $85\,\mathrm{N}$. Calculate the work done by the worker on the crate.

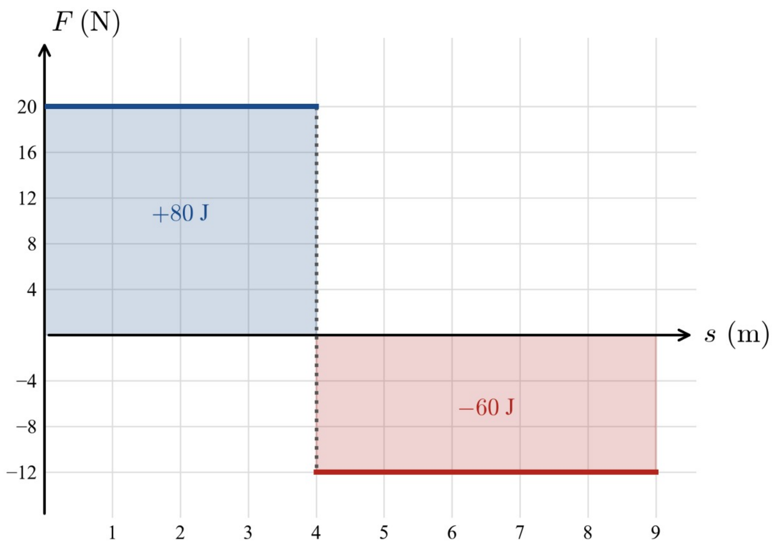
Question 8 A person pulls a sled $15\,\mathrm{m}$ across level ground using a rope held at 30° above the horizontal, applying a constant force of $120\,\mathrm{N}$. Calculate the work done by the rope on the sled. Give your answer to three significant figures.

Question 9 A $1500\,\mathrm{kg}$ car increases its speed from $12\,\mathrm{m\,s^{-1}}$ to $24\,\mathrm{m\,s^{-1}}$. (a) Calculate the kinetic energy of the car at each speed. (b) State the factor by which the kinetic energy increases when the speed doubles, and explain why.

Question 10 A $0.20\,\mathrm{kg}$ ice-hockey puck moving at $8.0\,\mathrm{m\,s^{-1}}$ slides across rough ice and comes to rest after $12\,\mathrm{m}$. Calculate the average friction force on the puck and the energy dissipated to the environment.

Question 11 A $65\,\mathrm{kg}$ hiker walks up a straight path $200\,\mathrm{m}$ long inclined at 12° to the horizontal. Calculate the gain in the hiker's gravitational potential energy.

Question 12 An object of mass $5.0\,\mathrm{kg}$ moves in a straight line. The force acting on it along the line of motion varies with position as shown; a positive force drives the object forward and a negative force opposes its motion.



The object passes $s = 0$ moving at $4.0\,\mathrm{m\,s^{-1}}$. Calculate the total (net) work done on the object between $s = 0$ and $s = 9.0\,\mathrm{m}$, and hence its speed at $s = 9.0\,\mathrm{m}$.

Question 13 A $0.40\,\mathrm{kg}$ pendulum bob is pulled aside until it has risen $0.25\,\mathrm{m}$ above its lowest point, then released. (a) Assuming no energy is dissipated, show that the bob's speed at the lowest point of its swing is $2.21\,\mathrm{m\,s^{-1}}$. (b) In practice the bob reaches the lowest point moving at only $2.0\,\mathrm{m\,s^{-1}}$. Calculate the energy dissipated to the environment during the swing.

Question 14 A student walks $10\,\mathrm{m}$ across a level room at constant speed while carrying a $5.0\,\mathrm{kg}$ box at constant height. With reference to the definition of work, explain why the upward force the student exerts on the box does no work on it.

Question 15 Looking at a force–distance graph for an object moving in a straight line, a student says: "The area under this graph gives the impulse on the object." Explain what is wrong with this statement, and state correctly what the area under a force–distance graph represents.

Question 16 A $20\,\mathrm{kg}$ crate slides off the end of a moving conveyor belt onto a level floor and skids to rest. Describe the energy transformations from the moment the crate leaves the belt until it stops, in terms of kinetic energy and energy dissipated to the environment.

Question 17 A $1.5\,\mathrm{kg}$ ball is thrown vertically upward at $6.0\,\mathrm{m\,s^{-1}}$. Air resistance is negligible. (a) Calculate the kinetic energy of the ball as it leaves the thrower's hand. (b) Using energy conservation, calculate the maximum height the ball rises above the launch point.

Question 18 A $3.0\,\mathrm{kg}$ block is pulled from rest $5.0\,\mathrm{m}$ along a rough horizontal surface by a constant horizontal force of $25\,\mathrm{N}$. A constant friction force of $10\,\mathrm{N}$ opposes the motion. Calculate the block's speed after $5.0\,\mathrm{m}$, and the energy dissipated to the environment over that distance.

Question 19 Object A has mass $4.0\,\mathrm{kg}$ and moves at $3.0\,\mathrm{m\,s^{-1}}$; object B has mass $1.0\,\mathrm{kg}$ and moves at $6.0\,\mathrm{m\,s^{-1}}$. Compare the kinetic energies of the two objects, supporting your answer with calculations.

Question 20 A cyclist and bicycle of combined mass $78\,\mathrm{kg}$ freewheel from rest down a hill, descending a vertical height of $20\,\mathrm{m}$, and reach the bottom travelling at $15\,\mathrm{m\,s^{-1}}$. Determine the energy dissipated to the environment during the descent.

Question 21 A book is lifted at constant speed from the floor to a shelf, and is then held stationary on the shelf. With reference to the definition of work, justify whether work is done on the book (a) while it is being lifted, and (b) while it is held stationary on the shelf.

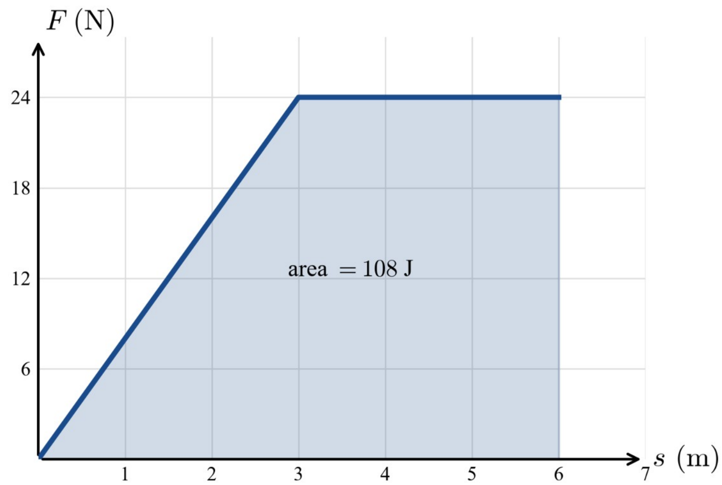
Answers

Question 1 (a) $36\,\mathrm{J}$; (b) $36\,\mathrm{J}$; (c) $6.00\,\mathrm{m\,s^{-1}}$. **Question 2** (a) $108\,\mathrm{J}$ (shown); (b) $108\,\mathrm{J}$; (c) $7.35\,\mathrm{m\,s^{-1}}$. **Question 3** (a) $7.36\,\mathrm{J}$; (b) $7.36\,\mathrm{J}$; (c) $7.00\,\mathrm{m\,s^{-1}}$. **Question 4** (a) $1.35\times10^5\,\mathrm{J}$; (b) $6.14\times10^3\,\mathrm{N}$; (c) mostly heat (in the brakes, tyres and road), with some sound — dissipated to the environment. **Question 5** (a) $29.4\,\mathrm{J}$; (b) $16.0\,\mathrm{J}$; (c) $13.4\,\mathrm{J}$, as heat (and a little sound) from friction. **Question 6** (a) less than; (b) heat, sound and deformation of material; (c) total energy is conserved — the dissipated energy is still present in the surroundings. **Question 7** $510\,\mathrm{J}$. **Question 8** $1.56\times10^3\,\mathrm{J}$. **Question 9** (a) $1.08\times10^5\,\mathrm{J}$ and $4.32\times10^5\,\mathrm{J}$; (b) factor of 4 (kinetic energy $\propto v^2$). **Question 10** friction $0.533\,\mathrm{N}$; energy dissipated $6.4\,\mathrm{J}$. **Question 11** $2.65\times10^4\,\mathrm{J}$ (height gain $41.6\,\mathrm{m}$). **Question 12** net work $20\,\mathrm{J}$; speed $4.90\,\mathrm{m\,s^{-1}}$. **Question 13** (a) $2.21\,\mathrm{m\,s^{-1}}$ (shown); (b) $0.181\,\mathrm{J}$. **Question 14** The supporting force is vertical while the displacement is horizontal; the force is perpendicular to the motion, so it does no work. **Question 15** The area under a force–*distance* graph is the work done ($=\Delta E_k$), not impulse; impulse is the area under a force–*time* graph. **Question 16** Kinetic energy of the crate → energy dissipated to the environment (mainly heat, with some sound and slight deformation) by friction, until the crate stops. **Question 17** (a) $27\,\mathrm{J}$; (b) $1.83\,\mathrm{m}$. **Question 18** $7.07\,\mathrm{m\,s^{-1}}$; energy dissipated $50\,\mathrm{J}$. **Question 19** Equal: both have $18\,\mathrm{J}$. **Question 20** $6.53\times10^3\,\mathrm{J}$. **Question 21** (a) yes — work $mg\Delta h$ is done against gravity, stored as gravitational potential energy; (b) no — there is no displacement, so no work is done.

Fully-worked solutions

Question 1 $m=2.0\,\mathrm{kg}$, $F=9.0\,\mathrm{N}$, $s=4.0\,\mathrm{m}$, from rest, frictionless. (a) $W=Fs=9.0\times4.0=36\,\mathrm{J}$. (b) The track is frictionless, so all the work becomes kinetic energy: $E_k=W=36\,\mathrm{J}$. (c) $E_k=1/2\,mv^2\Rightarrow v=\sqrt{\dfrac{2E_k}{m}}=\sqrt{\dfrac{2\times36}{2.0}}=\sqrt{36}=6.00\,\mathrm{m\,s^{-1}}$.

Question 2 $m=4.0\,\mathrm{kg}$, from rest.



(a) Work = area under the force–distance graph. Triangle ($0\to3.0\,\mathrm{m}$): $1/2\times3.0\times24=36\,\mathrm{J}$. Rectangle ($3.0\to6.0\,\mathrm{m}$): $24\times3.0=72\,\mathrm{J}$. Total $W=36+72=108\,\mathrm{J}$, as required.
(b) From rest all the work becomes kinetic energy: $E_k=W=108\,\mathrm{J}$.
(c) $v=\sqrt{\dfrac{2E_k}{m}}=\sqrt{\dfrac{2\times108}{4.0}}=\sqrt{54}=7.35\,\mathrm{m\,s^{-1}}$.

Question 3 $m=0.30\,\mathrm{kg}$, $h=2.5\,\mathrm{m}$. (a) $E_g=mg\Delta h=0.30\times9.81\times2.5=7.36\,\mathrm{J}$. (b) With negligible air resistance, all the gravitational potential energy becomes kinetic energy: $E_k=7.36\,\mathrm{J}$. (c) $E_k=1/2\,mv^2\Rightarrow v=\sqrt{2gh}=\sqrt{2\times9.81\times2.5}=\sqrt{49.05}=7.00\,\mathrm{m\,s^{-1}}$.

Question 4 $m=1200\,\mathrm{kg}$, $v=15\,\mathrm{m\,s^{-1}}$, braking distance $d=22\,\mathrm{m}$. (a) $E_k=1/2\,mv^2=1/2\times1200\times15^2=1.35\times10^5\,\mathrm{J}$. (b) The braking force removes all this kinetic energy over $22\,\mathrm{m}$, so

$F \times d = E_k$, giving $F = \dfrac{E_k}{d} = \dfrac{135000}{22} = 6.14 \times 10^3\,\mathrm{N}$. (c) The kinetic energy is dissipated to the environment, mostly as **heat** in the brakes, tyres and road, together with some **sound**.

Question 5 $m = 2.0\,\mathrm{kg}$, $\Delta h = 1.5\,\mathrm{m}$, bottom speed $v = 4.0\,\mathrm{m\ s^{-1}}$. (a) $E_g = m g \Delta h = 2.0 \times 9.81 \times 1.5 = 29.4\,\mathrm{J}$. (b) $E_k = 1/2\, m v^2 = 1/2 \times 2.0 \times 4.0^2 = 16.0\,\mathrm{J}$. (c) Total energy is conserved, so the energy dissipated is $E_g - E_k = 29.43 - 16.0 = 13.4\,\mathrm{J}$, released as **heat** (and a little **sound**) by friction between the block and the ramp.

Question 6 (a) **Less than.** The ball's mechanical energy at the top of its bounce equals its gravitational potential energy there, $m g \Delta h$. Some mechanical energy is dissipated during the bounce, so the ball has less energy afterwards and cannot rise as high. (b) The lost mechanical energy is transformed into **heat, sound and deformation of material** (of the ball and the floor). (c) Energy is never created or destroyed. During the bounce, part of the ball's kinetic energy is transformed into heat, sound and deformation, which spread into the ball, the floor and the air. That energy still exists — it has simply changed form and moved into the surroundings — so the **total** energy of the ball and its surroundings is unchanged. Only the ball's **mechanical** energy has decreased, which is why it rebounds to a lower height.

Question 7 $W = F s = 85 \times 6.0 = 510\,\mathrm{J}$.

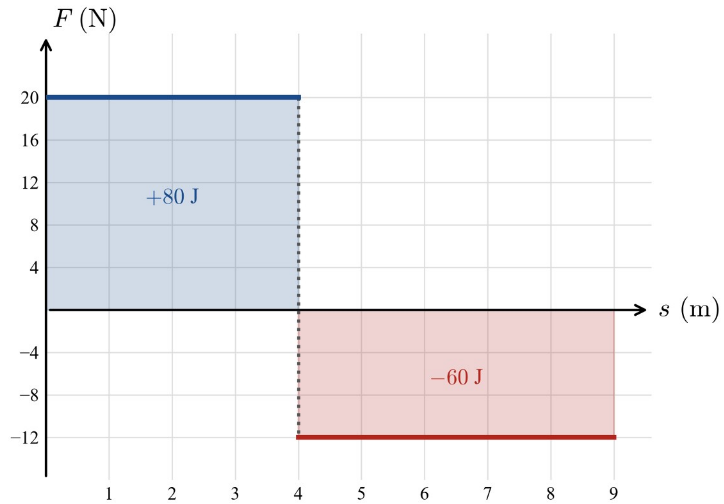
Question 8 Only the horizontal component of the force does work: $W = F s \cos\theta = 120 \times 15 \times \cos 30^\circ = 1558.8\,\mathrm{J} = 1.56 \times 10^3\,\mathrm{J}$ (to three significant figures).

Question 9 $m = 1500\,\mathrm{kg}$. (a) At $12\,\mathrm{m\ s^{-1}}$: $E_k = 1/2 \times 1500 \times 12^2 = 1.08 \times 10^5\,\mathrm{J}$. At $24\,\mathrm{m\ s^{-1}}$: $E_k = 1/2 \times 1500 \times 24^2 = 4.32 \times 10^5\,\mathrm{J}$. (b) The kinetic energy increases by a factor of 4. Kinetic energy is proportional to the square of the speed ($E_k = 1/2\, m v^2$), so doubling the speed multiplies E_k by $2^2 = 4$.

Question 10 $m = 0.20\,\mathrm{kg}$, $v = 8.0\,\mathrm{m\ s^{-1}}$, $d = 12\,\mathrm{m}$. The puck's kinetic energy is $E_k = 1/2\, m v^2 = 1/2 \times 0.20 \times 8.0^2 = 6.4\,\mathrm{J}$. Friction removes all of it over $12\,\mathrm{m}$, so the friction force is $F = \dfrac{E_k}{d} = \dfrac{6.4}{12} = 0.533\,\mathrm{N}$, and the energy dissipated to the environment (as heat and sound) is $6.4\,\mathrm{J}$.

Question 11 The vertical height gained is $\Delta h = 200 \sin 12^\circ = 41.6\,\mathrm{m}$. Hence $E_g = m g \Delta h = 65 \times 9.81 \times 41.58 = 2.65 \times 10^4\,\mathrm{J}$.

Question 12 $m = 5.0\,\mathrm{kg}$; enters at $u = 4.0\,\mathrm{m\ s^{-1}}$.



Net work = signed area under the graph. Above the axis ($0 \to 4.0\,\mathrm{m}$): $+20 \times 4.0 = +80\,\mathrm{J}$. Below the axis ($4.0 \to 9.0\,\mathrm{m}$): $-12 \times 5.0 = -60\,\mathrm{J}$. Net work $W = 80 - 60 = 20\,\mathrm{J}$. By the work–energy theorem $W = E_k - 1/2\, m u^2$.

The initial kinetic energy is $1/2 \times 5.0 \times 4.0^2 = 40\,\text{J}$, so the final kinetic energy is $40 + 20 = 60\,\text{J}$. Then $v = \sqrt{\dfrac{2 \times 60}{5.0}} = \sqrt{24} = 4.90\,\text{m s}^{-1}$.

Question 13 $m = 0.40\,\text{kg}$, $\Delta h = 0.25\,\text{m}$. (a) With no energy dissipated, all the gravitational potential energy becomes kinetic energy: $1/2\,m v^2 = m g \Delta h$, so $v = \sqrt{2 g \Delta h} = \sqrt{2 \times 9.81 \times 0.25} = \sqrt{4.905} = 2.21\,\text{m s}^{-1}$, as required. (b) The gravitational potential energy released is $E_g = m g \Delta h = 0.40 \times 9.81 \times 0.25 = 0.981\,\text{J}$. The actual kinetic energy at the lowest point is $E_k = 1/2 \times 0.40 \times 2.0^2 = 0.80\,\text{J}$. The energy dissipated to the environment (as heat and sound, chiefly from air resistance) is $E_g - E_k = 0.981 - 0.80 = 0.181\,\text{J}$.

Question 14 Work done by a force equals the force multiplied by the displacement **in the direction of the force**. The force the student exerts to support the box is directed vertically upward, but the box's displacement is horizontal. The force is therefore perpendicular to the displacement, so it has no component along the motion and transfers no energy to the box — the work it does is zero. (The box's kinetic energy and height are both unchanged, consistent with no work being done on it.)

Question 15 The statement is wrong: the area under a force–**distance** graph is the **work done** on the object, which equals its change in kinetic energy ($W = \Delta E_k$), and is measured in joules. Impulse is a different quantity — it is the area under a force–**time** graph ($F \Delta t$), equals the change in momentum, and is measured in N s. The two areas come from graphs with different horizontal axes and must not be confused.

Question 16 As the crate slides on the floor, friction acts backward on it, opposite to its motion, and does negative work on it. The crate's **kinetic energy** is therefore steadily transformed into energy **dissipated to the environment** — mainly **heat** at the crate–floor contact, with some **sound** and slight **deformation** of the crate and floor surfaces. This continues until all the crate's kinetic energy has been dissipated and it comes to rest. The total energy is conserved throughout; it has simply changed from ordered kinetic energy into thermal and sound energy spread through the surroundings.

Question 17 $m = 1.5\,\text{kg}$, $v = 6.0\,\text{m s}^{-1}$. (a) $E_k = 1/2\,m v^2 = 1/2 \times 1.5 \times 6.0^2 = 27\,\text{J}$. (b) With negligible air resistance, at the highest point all the kinetic energy has become gravitational potential energy: $1/2\,m v^2 = m g \Delta h$, so $\Delta h = \dfrac{v^2}{2g} = \dfrac{6.0^2}{2 \times 9.81} = 1.83\,\text{m}$.

Question 18 $m = 3.0\,\text{kg}$, applied force $25\,\text{N}$, friction $10\,\text{N}$, $s = 5.0\,\text{m}$, from rest. The net force along the motion is $25 - 10 = 15\,\text{N}$, so the net work is $W_{\text{net}} = 15 \times 5.0 = 75\,\text{J}$. By the work–energy theorem this equals the kinetic energy gained (from rest): $1/2\,m v^2 = 75\,\text{J}$, giving $v = \sqrt{\dfrac{2 \times 75}{3.0}} = \sqrt{50} = 7.07\,\text{m s}^{-1}$. The friction force does $10 \times 5.0 = 50\,\text{J}$ of negative work, so $50\,\text{J}$ is dissipated to the environment as heat and sound.

Question 19 $E_{k,\text{A}} = 1/2 \times 4.0 \times 3.0^2 = 18\,\text{J}$; $E_{k,\text{B}} = 1/2 \times 1.0 \times 6.0^2 = 18\,\text{J}$. The two objects have **equal** kinetic energies ($18\,\text{J}$ each). Although B has one-quarter the mass of A, it moves twice as fast, and because kinetic energy depends on the square of the speed, B's fourfold advantage in v^2 exactly offsets its quarter share of the mass.

Question 20 $m = 78\,\text{kg}$, $\Delta h = 20\,\text{m}$, bottom speed $v = 15\,\text{m s}^{-1}$. Gravitational potential energy released: $E_g = m g \Delta h = 78 \times 9.81 \times 20 = 1.53 \times 10^4\,\text{J}$. Kinetic energy gained: $E_k = 1/2\,m v^2 = 1/2 \times 78 \times 15^2 = 8775\,\text{J} = 8.78 \times 10^3\,\text{J}$. Total energy is conserved, so the energy dissipated to the environment (as heat and sound in the tyres, bearings and air) is $E_g - E_k = 15303.6 - 8775 = 6.53 \times 10^3\,\text{J}$.

Question 21 (a) **Yes, work is done.** As the book is lifted, the person applies an upward force through an upward displacement, so the force acts along the direction of motion. Work equal to $m g \Delta h$ is done against the force due to gravity and is stored as the book's gravitational potential energy. (b) **No work is done.** While the book is held stationary on the shelf there is no displacement. Work is force multiplied by the displacement in the direction of the force, and with zero displacement the work is zero — even though a supporting force is still present.

Theory

Momentum. The **momentum** of an object of mass m moving with velocity v is

$$p = mv.$$

Momentum is a **vector**: it has the same direction as the velocity. In this subtopic every collision happens along a single straight line, so direction is handled with a **sign**. Choose one direction as positive; an object moving the other way then has a negative velocity and a negative momentum. The unit of momentum is $\mathrm{kg\ m\ s^{-1}}$, which is identical to the newton second (N s).

Impulse and Newton's second law. Newton's second law can be written in terms of momentum. If a constant net force F acts on an object for a time Δt, the object's momentum changes by

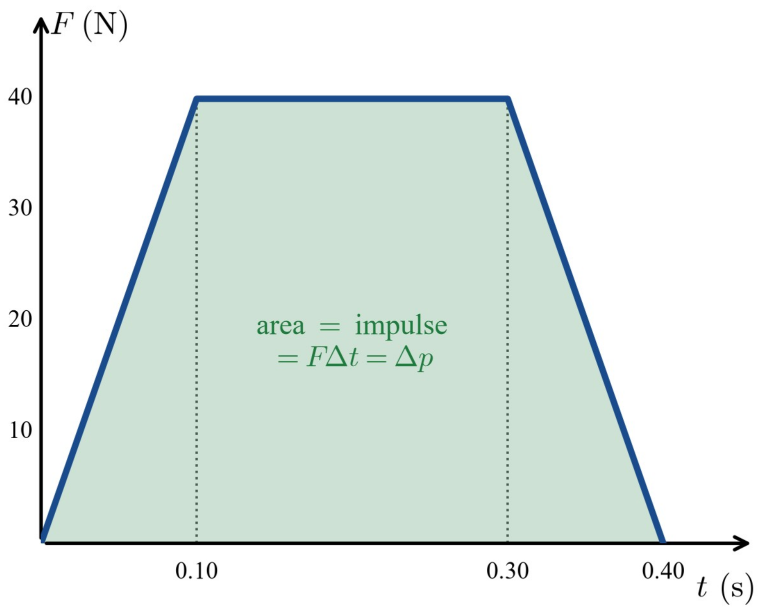
$$F\Delta t = \Delta p = m\Delta v = mv - mu,$$

where u is the initial velocity and v the final velocity. The quantity $F\Delta t$ is the **impulse** of the force. Impulse is a vector with the same direction as the force, and its unit is the newton second ($\mathrm{N\ s}$). Rearranged, $F = \dfrac{\Delta p}{\Delta t}$ says that the net force equals the **rate of change of momentum** — the most general form of the second law. Because $m\Delta v = \Delta p$, delivering a given change in momentum with a **smaller force** requires a **longer time**, and vice versa.

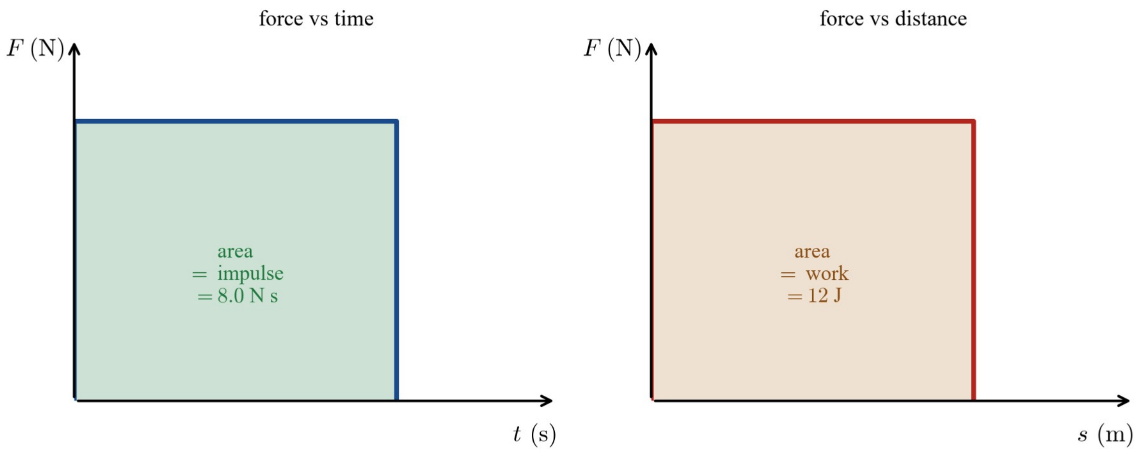
Impulse as the area under a force–time graph. When the force is not constant, the impulse is the **area under the force–time (F–t) graph**:

$$\text{impulse} = \text{area under the } F-t \text{ graph} = \Delta p.$$

The average force is then $F_{\text{avg}} = \dfrac{\text{area}}{\Delta t}$ — the height of a rectangle of the same width and the same area. For the trapezoidal pulse below the area is $1/2\,(0.40 + 0.20) \times 40 = 12\,\mathrm{N\ s}$, so the impulse is $12\,\mathrm{N\ s}$.



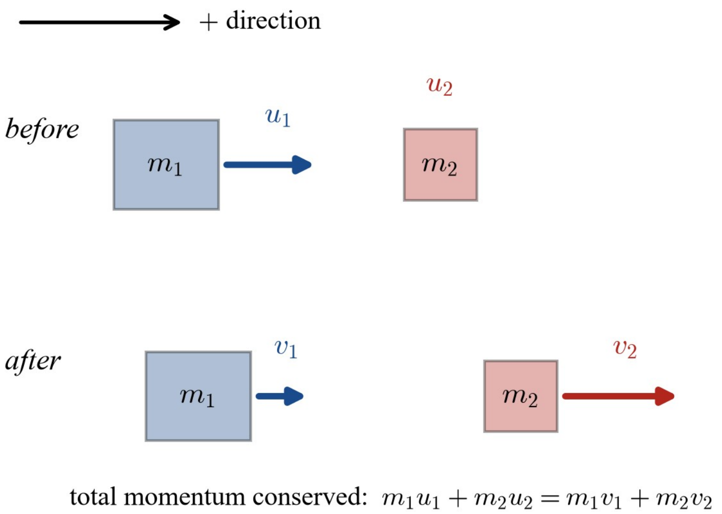
Impulse is not work — the area trap. The area under a graph means different things depending on what is plotted. The area under a **force–time** graph is the **impulse** (unit $\mathrm{N\ s}$), and it equals the change in *momentum*. The area under a **force–distance** graph is the **work done** (unit J), and it equals the change in *kinetic* energy (subtopic 2A). These are different physical quantities with different units; the two areas below are numerically unrelated. A force–distance area is **never** an impulse.



Conservation of momentum in an isolated system. An **isolated system** is one on which the net external force is zero, so the only forces that change the objects' momenta are the internal forces they exert on one another. When two objects interact, Newton's third law says the force on object 1 by object 2 is equal in magnitude and opposite in direction to the force on object 2 by object 1. They act for the **same** contact time Δt, so the two impulses are equal and opposite, and therefore the momentum *gained* by one object is exactly the momentum *lost* by the other. The **total momentum of the system is unchanged**:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 .$$

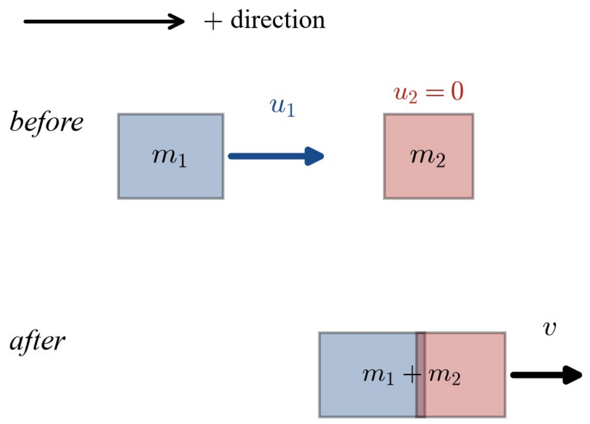
This holds for any interaction in an isolated system — collisions, explosions, pushes — provided the signs of the velocities are kept consistent with the chosen positive direction.



Elastic and inelastic collisions — the kinetic-energy test. Momentum is conserved in **every** collision in an isolated system. Kinetic energy, however, is not always conserved. Compare the total kinetic energy $E_k = 1/2\, m v^2$ before and after:

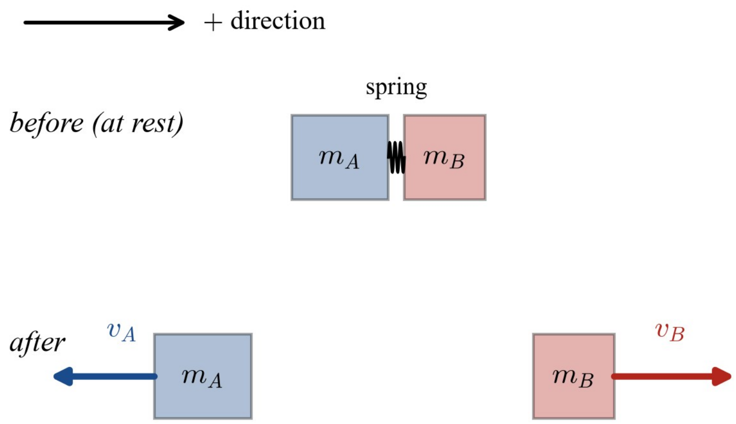
- a collision is **elastic** if the total kinetic energy is **unchanged**;
- a collision is **inelastic** if the total kinetic energy **decreases** — some kinetic energy is transformed into heat, sound and permanent deformation of the colliding objects.

A **perfectly inelastic** collision is one in which the objects **stick together** and move off with a common velocity; this loses the most kinetic energy consistent with conserving momentum. It is a common error to say that momentum is "not conserved" in an inelastic collision — it *is* conserved; it is the **kinetic energy** that decreases.



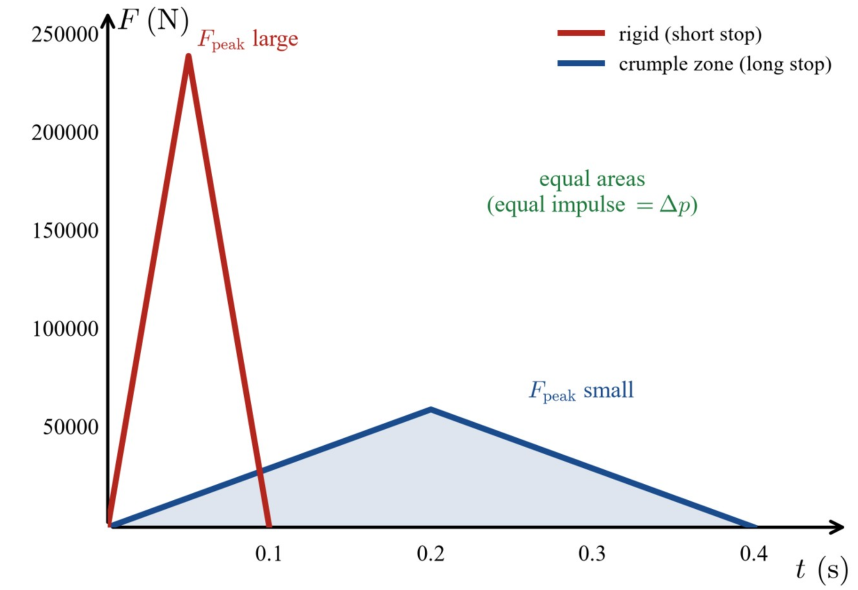
objects stick: $m_1 u_1 = (m_1 + m_2)\, v$; E_k decreases (inelastic)

Explosions and recoil. An explosion is the reverse of a perfectly inelastic collision: objects initially at rest (or moving together) push apart. If the system starts at rest, its total momentum is zero and must stay zero, so the fragments carry **equal and opposite** momenta. A cannon and its shell, a rifle and its bullet, or two spring-loaded carts all recoil this way. The lighter object moves off with the greater speed, and — because $E_k = \frac{p^2}{2m}$ for a given momentum magnitude p — it also carries the larger share of the kinetic energy.



total momentum stays zero: $m_A v_A + m_B v_B = 0$

Impulse and vehicle safety. In a crash an occupant undergoes a fixed change in momentum $\Delta p = m \Delta v$ as they are brought to rest, so the impulse delivered is fixed. Since $F_{\text{avg}} = \frac{\Delta p}{\Delta t}$, **increasing the time** over which the momentum change occurs **reduces the average force**. Crumple zones, airbags, seatbelts that stretch, and bending one's knees on landing all work this way: the same impulse (the same area under the F–t graph) is spread over a longer time, so the peak force falls. The two pulses below have equal areas — equal impulse and equal Δp — but the longer collision has a much smaller peak force.



Where an impact speed must first be found from a fall or a run-up, the kinematics of Chapter 1 are used with the constant $g=9.81\,\text{m s}^{-2}$.

Worked examples

Example 1 — scaffolded

A tennis ball of mass $0.058\,\text{kg}$ approaches a racquet horizontally at $25\,\text{m s}^{-1}$ and is returned along the same line in the opposite direction at $32\,\text{m s}^{-1}$. The racquet and ball are in contact for $5.0\,\text{ms}$. Calculate the change in momentum of the ball, the impulse on the ball, and the average force the racquet exerts on the ball.

Working	Comment
$m=0.058\,\text{kg}$; take the outgoing direction as positive, so $u=-25\,\text{m s}^{-1}$ and $v=+32\,\text{m s}^{-1}$; $\Delta t=5.0\times10^{-3}\,\text{s}$.	Set a sign convention; the incoming velocity is negative because it is opposite to the chosen positive direction.
$\Delta p=m(v-u)=0.058(32-(-25))=0.058\times57=3.3$.	Change in momentum: subtract the initial from the final momentum.
Impulse $=\Delta p=3.31\,\text{N s}$, directed away from the racquet.	The impulse equals the change in momentum ($1\,\text{N s}=1\,\text{kg m s}^{-1}$).
$F=\dfrac{\Delta p}{\Delta t}=\dfrac{3.306}{5.0\times10^{-3}}=661\,\text{N}$.	From $F\,\Delta t=\Delta p$; same direction as the impulse.

Example 2 — scaffolded

A railway truck of mass $3.0\times10^{3}\,\text{kg}$ moving at $2.0\,\text{m s}^{-1}$ collides with a stationary truck of mass $1.0\times10^{3}\,\text{kg}$ and the two couple together. Calculate their common velocity immediately after the collision, and determine whether the collision is elastic or inelastic.

Working	Comment
$m_1=3.0\times10^{3}\,\text{kg}$, $u_1=2.0\,\text{m s}^{-1}$; $m_2=1.0\times10^{3}\,\text{kg}$, $u_2=0$. Take the direction of motion as positive.	List the data; the system (both trucks) is isolated horizontally.
Momentum: $m_1u_1+m_2u_2=(m_1+m_2)v$.	Conservation of momentum; the trucks share one velocity v after coupling.
$v=\dfrac{m_1u_1}{m_1+m_2}=\dfrac{3000\times2.0}{4000}=1.5\,\text{m s}^{-1}$.	Solve for the common velocity.
$E_k(\text{before})=1/2\,m_1u_1^2=1/2\times3000\times2.0^2=6.00\times10^3$	Total kinetic energy before (truck 2 contributes none).

Working	Comment
.	
$E_k(\text{after}) = 1/2(m_1+m_2)v^2 = 1/2 \times 4000 \times 1.5^2 = 4.50$ ›	Total kinetic energy after.
.	
E_k falls by 1.50×10^3 J, so the collision is **inelastic**; momentum is still conserved.	The kinetic-energy test: a decrease in E_k means inelastic.

Example 3 — unscaffolded

A cannon of mass $8.0 \times 10^2\,\text{kg}$ fires a shell of mass $4.0\,\text{kg}$ horizontally at $1.8 \times 10^2\,\text{m s}^{-1}$. Assuming the ground exerts negligible horizontal force during the firing, calculate the recoil velocity of the cannon.

Before firing, the cannon and shell are at rest, so the total momentum of the isolated system is zero. Taking the firing direction as positive, conservation of momentum gives

$$0 = m_\text{s} v_\text{s} + m_\text{c} v_\text{c},$$

so

$$v_\text{c} = -\frac{m_\text{s} v_\text{s}}{m_\text{c}} = -\frac{4.0 \times 180}{800} = -\frac{720}{800} = -0.90\,\text{m s}^{-1}.$$

The negative sign shows the cannon moves opposite to the shell.

∴ the cannon recoils at $0.90\,\text{m s}^{-1}$ in the direction opposite to the shell.

Example 4 — unscaffolded

A $2.0\,\text{kg}$ trolley moving to the right at $4.0\,\text{m s}^{-1}$ collides head-on with a $1.0\,\text{kg}$ trolley moving to the left at $2.0\,\text{m s}^{-1}$. After the collision the $2.0\,\text{kg}$ trolley continues to the right at $1.5\,\text{m s}^{-1}$. Find the velocity of the $1.0\,\text{kg}$ trolley after the collision, and determine whether the collision is elastic.

Take **right** as positive: $u_A = +4.0\,\text{m s}^{-1}$, $u_B = -2.0\,\text{m s}^{-1}$. The total momentum before is

$$p = m_A u_A + m_B u_B = 2.0(4.0) + 1.0(-2.0) = 8.0 - 2.0 = 6.0\,\text{kg m s}^{-1}.$$

Conserving momentum, with $v_A = +1.5\,\text{m s}^{-1}$,

$$m_A v_A + m_B v_B = 6.0 \Rightarrow 2.0(1.5) + 1.0\,v_B = 6.0 \Rightarrow v_B = +3.0\,\text{m s}^{-1}.$$

Testing kinetic energy,

$$E_k(\text{before}) = 1/2(2.0)(4.0)^2 + 1/2(1.0)(2.0)^2 = 16 + 2.0 = 18\,\text{J},$$

$$E_k(\text{after}) = 1/2(2.0)(1.5)^2 + 1/2(1.0)(3.0)^2 = 2.25 + 4.5 = 6.75\,\text{J}.$$

The kinetic energy decreases, so the collision is inelastic.

∴ the $1.0\,\text{kg}$ trolley moves at $3.0\,\text{m s}^{-1}$ to the right, and the collision is inelastic (E_k falls from 18 J to 6.75 J).

Practice questions

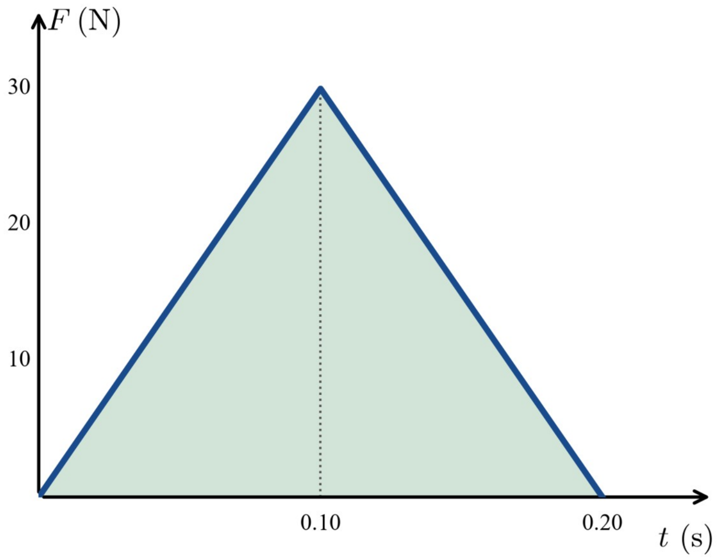
Question 1 A delivery van of mass $2500\,\text{kg}$ accelerates from rest to $12\,\text{m s}^{-1}$ along a straight road. (a) Calculate the change in momentum of the van. (b) The change takes $8.0\,\text{s}$. Calculate the impulse on the van and the average net force. (c) State the direction of the impulse.

Question 2 A golf club strikes a stationary $0.045\,\text{kg}$ golf ball, exerting an average force of $9.0\times10^{2}\,\text{N}$ for $1.2\,\text{ms}$. (a) Calculate the impulse delivered to the ball. (b) Calculate the speed with which the ball leaves the tee. (c) With reference to Newton's third law, explain why the impulse on the club head has the same magnitude as the impulse on the ball.

Question 3 A $0.80\,\text{kg}$ trolley moving at $3.0\,\text{m s}^{-1}$ collides with, and sticks to, a stationary $1.2\,\text{kg}$ trolley on a smooth track. (a) Calculate the common velocity of the trolleys after the collision. (b) Calculate the total kinetic energy before and after the collision. (c) State whether the collision is elastic or inelastic, justifying your answer with your kinetic energy values.

Question 4 A $60\,\text{kg}$ student stands at rest on a frictionless icy surface and throws a $2.0\,\text{kg}$ medicine ball horizontally at $6.0\,\text{m s}^{-1}$. (a) Calculate the momentum of the ball after the throw. (b) Using conservation of momentum, calculate the recoil speed of the student. (c) Compare the magnitudes of the momenta of the student and the ball after the throw, and explain your comparison.

Question 5 A $0.20\,\text{kg}$ ball, initially at rest on a smooth horizontal surface, experiences the horizontal force shown in the graph.



(a) Calculate the impulse delivered to the ball.
(b) Calculate the speed of the ball just after the force stops acting.
(c) A student says the area under the graph gives the work done on the ball. Explain why this is incorrect, and state what the area actually represents.

Question 6 A $1200\,\text{kg}$ car travelling at $15\,\text{m s}^{-1}$ is brought to rest in a collision. (a) Calculate the magnitude of the impulse on the car. (b) A rigid car stops in $0.10\,\text{s}$; a car with a crumple zone stops in $0.40\,\text{s}$. Calculate the average force on the car in each case. (c) With reference to $F\,\Delta t = m\,\Delta v$, explain how a crumple zone reduces the force on the occupants.

Question 7 A $7.5\,\text{kg}$ bowling ball rolls at $4.0\,\text{m s}^{-1}$. Calculate the speed a $0.058\,\text{kg}$ tennis ball would need in order to have the same momentum as the bowling ball. Give your answer to three significant figures.

Question 8 A $0.30\,\text{kg}$ ball travelling horizontally at $8.0\,\text{m s}^{-1}$ strikes a wall and rebounds along the same line at $6.0\,\text{m s}^{-1}$. Calculate the impulse on the ball by the wall.

Question 9 A $0.145\,\text{kg}$ baseball moving at $40\,\text{m s}^{-1}$ is caught, the glove bringing it to rest in $0.025\,\text{s}$. Determine the average force the glove exerts on the ball.

Question 10 A $1.5\,\text{kg}$ trolley moving at $2.0\,\text{m s}^{-1}$ collides with a stationary $1.0\,\text{kg}$ trolley on a smooth track. After the collision the $1.5\,\text{kg}$ trolley continues in the same direction at $0.80\,\text{m s}^{-1}$. Calculate the velocity of the $1.0\,\text{kg}$ trolley after the collision.

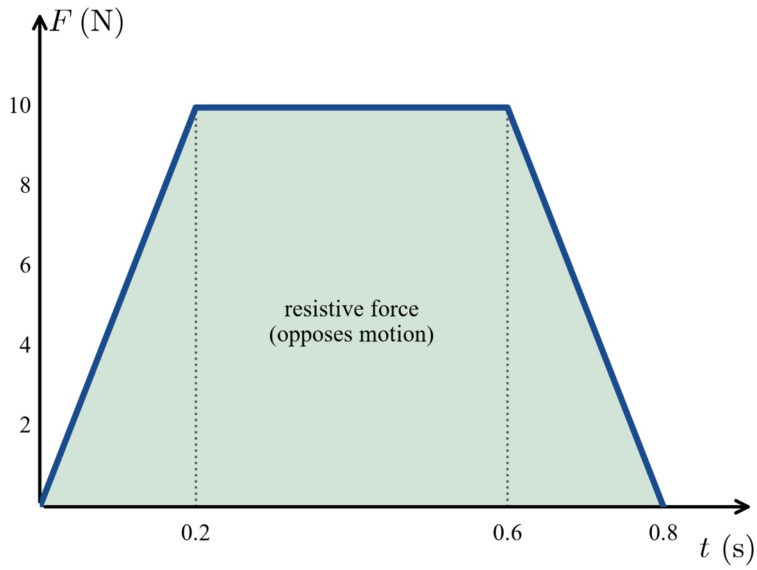
Question 11 A $0.50\,\mathrm{kg}$ cart moving at $6.0\,\mathrm{m\ s^{-1}}$ collides with an identical $0.50\,\mathrm{kg}$ cart at rest. After the collision the first cart is stationary and the second moves off at $6.0\,\mathrm{m\ s^{-1}}$. Show, using kinetic energy, that this collision is elastic.

Question 12 Two carts initially at rest on a smooth track are pushed apart by a compressed spring released between them. Cart A ($0.40\,\mathrm{kg}$) moves off to the left at $1.5\,\mathrm{m\ s^{-1}}$; cart B has mass $0.60\,\mathrm{kg}$. Calculate the velocity of cart B, and state the total momentum of the two-cart system after release.

Question 13 A $1000\,\mathrm{kg}$ car travelling at $20\,\mathrm{m\ s^{-1}}$ runs into the back of a stationary $1500\,\mathrm{kg}$ car, and the two lock together. (a) Show that the common velocity immediately after the collision is $8.00\,\mathrm{m\ s^{-1}}$. (b) Hence determine the fraction of the original kinetic energy that is dissipated in the collision.

Question 14 Explain, with reference to impulse and Newton's laws, how an airbag reduces the risk of injury to a driver during a collision. Your answer should form a reasoning chain, not merely a definition.

Question 15 A $2.0\,\mathrm{kg}$ object moving at $5.0\,\mathrm{m\ s^{-1}}$ across a rough surface experiences the resistive force shown, which opposes its motion.



Calculate the impulse of the resistive force and the speed of the object after the force has finished acting.

Question 16 A $0.50\,\mathrm{kg}$ ball is dropped from a height of $1.8\,\mathrm{m}$ onto a hard floor and rebounds to a height of $0.80\,\mathrm{m}$. It is in contact with the floor for $0.020\,\mathrm{s}$. Calculate the average net force on the ball while it is in contact with the floor. (Ignore the change in momentum due to gravity during the brief contact.)

Question 17 In one experiment the area under a force–time graph for a moving object is found to be $8.0\,\mathrm{N\ s}$; in a separate experiment the area under a force–distance graph for a push on the object is found to be $12\,\mathrm{J}$. (a) State the physical quantity given by the $8.0\,\mathrm{N\ s}$ area, and the quantity it is equal to. (b) State the physical quantity given by the $12\,\mathrm{J}$ area, and the quantity it is equal to. (c) A student claims that both areas give the impulse on the object. Explain why this is wrong.

Question 18 A $0.020\,\mathrm{kg}$ bullet moving at $4.0\times10^{2}\,\mathrm{m\ s^{-1}}$ strikes and embeds itself in a $2.0\,\mathrm{kg}$ wooden block resting on a frictionless surface. (a) Calculate the velocity of the block-and-bullet immediately after impact. (b) Determine the percentage of the original kinetic energy that is converted to other forms.

Question 19 A $0.15\,\mathrm{kg}$ ball strikes a wall horizontally at $12\,\mathrm{m\ s^{-1}}$. In case A the ball sticks to the wall; in case B it rebounds along the same line at $9.0\,\mathrm{m\ s^{-1}}$. Calculate the impulse on the ball in each case, and explain why the rebound involves the larger impulse.

Question 20 A $3.6\,\mathrm{kg}$ rifle fires a $0.012\,\mathrm{kg}$ bullet at $350\,\mathrm{m\ s^{-1}}$. Calculate the recoil speed of the rifle, and the ratio of the bullet's kinetic energy to the rifle's kinetic energy. Comment on what your ratio shows about how the kinetic energy is shared.

Question 21 After a head-on collision between a heavy truck and a light car, a student argues: “The truck pushes on the car with a larger force than the car pushes on the truck, so momentum is not conserved.” Explain what is wrong with this argument, using Newton’s third law and the idea of an isolated system, and state what is true about the changes in momentum of the two vehicles.

Question 22 A $0.50\,\mathrm{kg}$ trolley A moving to the right at $8.0\,\mathrm{m\ s^{-1}}$ collides with a stationary $0.30\,\mathrm{kg}$ trolley B on a smooth track. After the collision A continues to the right at $2.0\,\mathrm{m\ s^{-1}}$. (a) Calculate the velocity of trolley B after the collision. (b) Determine whether the collision is elastic. (c) If instead the trolleys had stuck together, calculate their common velocity and the kinetic energy lost in that case.

Answers

Question 1 (a) $3.0\times10^{4}\,\mathrm{kg\ m\ s^{-1}}$; (b) impulse $3.0\times10^{4}\,\mathrm{N\ s}$, force $3.75\times10^{3}\,\mathrm{N}$; (c) in the direction of motion (forward). **Question 2** (a) $1.08\,\mathrm{N\ s}$; (b) $24\,\mathrm{m\ s^{-1}}$; (c) equal and opposite forces (Newton's third law) act for the same time, so equal-magnitude impulses. **Question 3** (a) $1.2\,\mathrm{m\ s^{-1}}$; (b) $3.6\,\mathrm{J}$ before, $1.44\,\mathrm{J}$ after; (c) inelastic (E_k decreases). **Question 4** (a) $12\,\mathrm{kg\ m\ s^{-1}}$; (b) $0.20\,\mathrm{m\ s^{-1}}$; (c) equal magnitudes ($12\,\mathrm{kg\ m\ s^{-1}}$ each), opposite directions. **Question 5** (a) $3.0\,\mathrm{N\ s}$; (b) $15\,\mathrm{m\ s^{-1}}$; (c) the area is the impulse, not the work. **Question 6** (a) $1.8\times10^{4}\,\mathrm{N\ s}$; (b) rigid $1.8\times10^{5}\,\mathrm{N}$, crumple $4.5\times10^{4}\,\mathrm{N}$; (c) same impulse over a longer time gives a smaller force. **Question 7** $517\,\mathrm{m\ s^{-1}}$. **Question 8** $4.2\,\mathrm{N\ s}$ (directed away from the wall). **Question 9** $232\,\mathrm{N}$. **Question 10** $1.8\,\mathrm{m\ s^{-1}}$ in the original direction. **Question 11** $E_k=9.0\,\mathrm{J}$ before and after ⇒ elastic. **Question 12** $1.0\,\mathrm{m\ s^{-1}}$ to the right; total momentum $=0$. **Question 13** (a) $8.00\,\mathrm{m\ s^{-1}}$; (b) 0.60 (60%). **Question 14** Longer contact time ⇒ smaller average force for the same Δp. **Question 15** $6.0\,\mathrm{N\ s}$, directed opposite to the motion; $2.0\,\mathrm{m\ s^{-1}}$. **Question 16** $248\,\mathrm{N}$, directed upward. **Question 17** (a) impulse, equal to the change in momentum; (b) work, equal to the change in kinetic energy; (c) only the F–t area is impulse. **Question 18** (a) $3.96\,\mathrm{m\ s^{-1}}$; (b) $99.0\,\%$. **Question 19** case A $1.8\,\mathrm{N\ s}$, case B $3.15\,\mathrm{N\ s}$; the rebound reverses the momentum, a larger change. **Question 20** $1.17\,\mathrm{m\ s^{-1}}$; ratio 300. **Question 21** Forces are equal and opposite (Newton's third law); momentum changes are equal and opposite; total momentum is conserved. **Question 22** (a) $10.0\,\mathrm{m\ s^{-1}}$; (b) elastic ($E_k=16\,\mathrm{J}$ before and after); (c) $5.0\,\mathrm{m\ s^{-1}}$, $6.0\,\mathrm{J}$ lost.

Fully-worked solutions

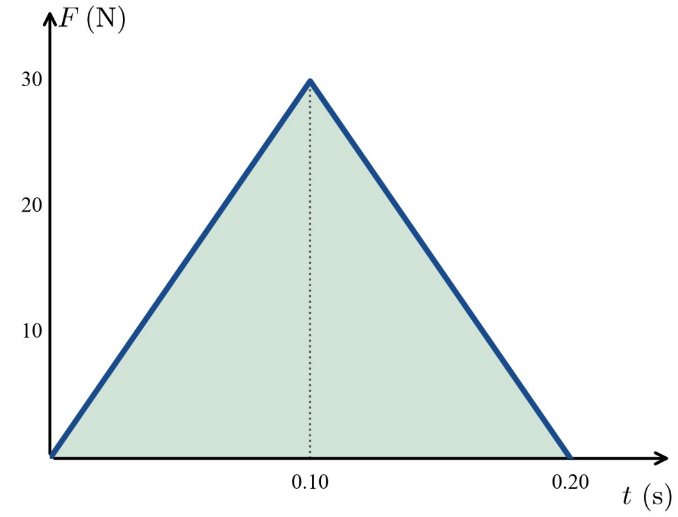
Question 1 $m=2500\,\mathrm{kg}$; from rest ($u=0$) to $v=12\,\mathrm{m\ s^{-1}}$. (a) $\Delta p=m\,\Delta v=2500\times12=3.0\times10^{4}\,\mathrm{kg\ m\ s^{-1}}$. (b) Impulse $=\Delta p=3.0\times10^{4}\,\mathrm{N\ s}$. Average net force $F=\frac{\Delta p}{\Delta t}=\frac{30000}{8.0}=3.75\times10^{3}\,\mathrm{N}$. (c) The impulse (and the net force) point in the direction of motion, i.e. forward.

Question 2 $m=0.045\,\mathrm{kg}$, $F=9.0\times10^{2}\,\mathrm{N}$, $\Delta t=1.2\times10^{-3}\,\mathrm{s}$. (a) Impulse $=F\,\Delta t=900\times1.2\times10^{-3}=1.08\,\mathrm{N\ s}$. (b) The ball starts from rest, so $m\,v=\text{impulse}$, giving $v=\frac{1.08}{0.045}=24\,\mathrm{m\ s^{-1}}$. (c) By Newton's third law the force on the ball by the club and the force on the club by the ball are equal in magnitude and opposite in direction. They act for the same contact time Δt, so the impulses $F\,\Delta t$ on the two bodies are equal in magnitude (and opposite in direction).

Question 3 $m_1=0.80\,\mathrm{kg}$, $u_1=3.0\,\mathrm{m\ s^{-1}}$, $m_2=1.2\,\mathrm{kg}$, $u_2=0$; the trolleys stick. (a) $m_1u_1=(m_1+m_2)v\Rightarrow v=\frac{0.80\times3.0}{2.0}=1.2\,\mathrm{m\ s^{-1}}$. (b) $E_k(\text{before})=1/2(0.80)(3.0)^2=3.6\,\mathrm{J}$; $E_k(\text{after})=1/2(2.0)(1.2)^2=1.44\,\mathrm{J}$. (c) The kinetic energy falls from $3.6\,\mathrm{J}$ to $1.44\,\mathrm{J}$, so the collision is **inelastic**. (Momentum is still conserved.)

Question 4 $m_\mathrm{s}=60\,\mathrm{kg}$, $m_\mathrm{b}=2.0\,\mathrm{kg}$, $v_\mathrm{b}=6.0\,\mathrm{m\ s^{-1}}$; the system starts at rest. (a) $p_\mathrm{b}=m_\mathrm{b}v_\mathrm{b}=2.0\times6.0=12\,\mathrm{kg\ m\ s^{-1}}$. (b) Total momentum stays zero: $m_\mathrm{s}v_\mathrm{s}+p_\mathrm{b}=0$, so $v_\mathrm{s}=-\frac{12}{60}=-0.20\,\mathrm{m\ s^{-1}}$; the student recoils at $0.20\,\mathrm{m\ s^{-1}}$. (c) The momenta are equal in magnitude ($12\,\mathrm{kg\ m\ s^{-1}}$ each) and opposite in direction, because the total momentum began at zero and must remain zero — the forward momentum of the ball is balanced by the backward momentum of the student.

Question 5 $m=0.20\,\mathrm{kg}$, starting from rest. The graph is a triangle of base $0.20\,\mathrm{s}$ and height $30\,\mathrm{N}$.



(a) Impulse = area $= 1/2 \times 0.20 \times 30 = 3.0\,\mathrm{N\ s}$.

(b) $m\,v = \text{impulse} \Rightarrow v = \dfrac{3.0}{0.20} = 15\,\mathrm{m\ s^{-1}}$.

(c) The area under a force–**time** graph is the **impulse** (unit N s), which equals the change in momentum — not the work done. Work is the area under a force–**distance** graph (unit J). The student has confused the two graphs.

Question 6 $m = 1200\,\mathrm{kg}$, $\Delta v = 15\,\mathrm{m\ s^{-1}}$ (brought to rest). (a) $|\text{impulse}| = m\,\Delta v = 1200 \times 15 = 1.8 \times 10^4\,\mathrm{N\ s}$. (b) Rigid: $F = \dfrac{1.8 \times 10^4}{0.10} = 1.8 \times 10^5\,\mathrm{N}$. Crumple zone: $F = \dfrac{1.8 \times 10^4}{0.40} = 4.5 \times 10^4\,\mathrm{N}$. (c) The car (and each occupant) must undergo the same change in momentum $\Delta p = m\,\Delta v$ whichever way it stops, so the impulse $F\,\Delta t$ is fixed. A crumple zone increases the stopping time Δt; since $F = \dfrac{\Delta p}{\Delta t}$, a larger Δt gives a smaller average force F — here it is four times smaller — reducing the force transmitted to the occupants.

Question 7 Momentum of the bowling ball: $p = 7.5 \times 4.0 = 30\,\mathrm{kg\ m\ s^{-1}}$. For the tennis ball to have the same momentum, $v = \dfrac{p}{m} = \dfrac{30}{0.058} = 517\,\mathrm{m\ s^{-1}}$ (to three significant figures).

Question 8 Take the rebound direction as positive: $u = -8.0\,\mathrm{m\ s^{-1}}$, $v = +6.0\,\mathrm{m\ s^{-1}}$, $m = 0.30\,\mathrm{kg}$.

$$\text{impulse} = m(v-u) = 0.30(6.0-(-8.0)) = 0.30 \times 14 = 4.2\,\mathrm{N\ s},$$

directed away from the wall.

Question 9 $\Delta p = m\,\Delta v = 0.145 \times 40 = 5.8\,\mathrm{kg\ m\ s^{-1}}$. $F = \dfrac{\Delta p}{\Delta t} = \dfrac{5.8}{0.025} = 232\,\mathrm{N}$.

Question 10 Take the initial direction as positive: $m_1 = 1.5\,\mathrm{kg}$, $u_1 = 2.0\,\mathrm{m\ s^{-1}}$, $m_2 = 1.0\,\mathrm{kg}$, $u_2 = 0$, $v_1 = 0.80\,\mathrm{m\ s^{-1}}$.

$$m_1 u_1 = m_1 v_1 + m_2 v_2 \Rightarrow 1.5(2.0) = 1.5(0.80) + 1.0\,v_2 \Rightarrow v_2 = \frac{3.0-1.2}{1.0} = 1.8\,\mathrm{m\ s^{-1}},$$

in the original direction.

Question 11 Take the initial direction as positive: $m_1 = m_2 = 0.50\,\mathrm{kg}$, $u_1 = 6.0\,\mathrm{m\ s^{-1}}$, $u_2 = 0$; after, $v_1 = 0$, $v_2 = 6.0\,\mathrm{m\ s^{-1}}$. (Momentum checks: $0.50 \times 6.0 = 0.50 \times 6.0$.)

$$E_k(\text{before}) = 1/2(0.50)(6.0)^2 = 9.0\,\mathrm{J},\ E_k(\text{after}) = 1/2(0.50)(6.0)^2 = 9.0\,\mathrm{J}.$$

The total kinetic energy is unchanged, so the collision is **elastic**.

Question 12 Take the right as positive: before release the system is at rest, so total momentum $= 0$. Cart A: $m_A = 0.40\,\text{kg}$, $v_A = -1.5\,\text{m s}^{-1}$; cart B: $m_B = 0.60\,\text{kg}$.

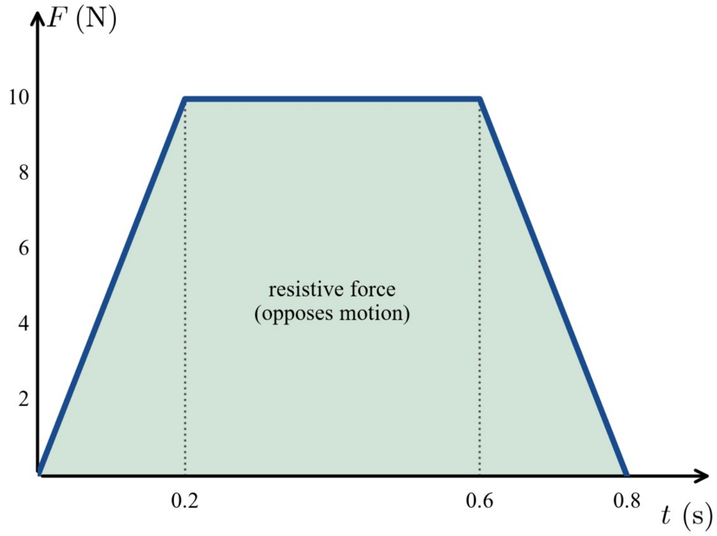
$$m_A v_A + m_B v_B = 0 \Rightarrow 0.40(-1.5) + 0.60\,v_B = 0 \Rightarrow v_B = \frac{0.60}{0.60} = 1.0\,\text{m s}^{-1}(\text{to the right}).$$

The total momentum after release is $0.40(-1.5) + 0.60(1.0) = -0.60 + 0.60 = 0$, unchanged from before.

Question 13 $m_1 = 1000\,\text{kg}$, $u_1 = 20\,\text{m s}^{-1}$, $m_2 = 1500\,\text{kg}$, $u_2 = 0$; the cars lock together. (a) $v = \dfrac{m_1 u_1}{m_1 + m_2} = \dfrac{1000 \times 20}{2500} = \dfrac{20000}{2500} = 8.00\,\text{m s}^{-1}$, as required. (b) $E_k(\text{before}) = 1/2(1000)(20)^2 = 2.00 \times 10^5\,\text{J}$; $E_k(\text{after}) = 1/2(2500)(8.00)^2 = 8.00 \times 10^4\,\text{J}$. Energy dissipated $= 2.00 \times 10^5 - 8.00 \times 10^4 = 1.20 \times 10^5\,\text{J}$. Fraction $= \dfrac{1.20 \times 10^5}{2.00 \times 10^5} = 0.60$, i.e. 60% is dissipated.

Question 14 In a collision the driver's momentum must change from $m v$ to zero, a fixed change $\Delta p = m\,\Delta v$, so a fixed impulse must be delivered. Because impulse $= F\,\Delta t = \Delta p$, the average force depends on the time over which that momentum change happens: $F = \dfrac{\Delta p}{\Delta t}$. An airbag inflates and then compresses as the driver moves into it, so the driver decelerates over a longer time Δt than they would against the rigid steering wheel or dashboard. With Δp fixed, a larger Δt gives a smaller average force F on the driver. The airbag also spreads that force over a larger area of the body, lowering the pressure on any one part. Both effects reduce the risk of injury.

Question 15 $m = 2.0\,\text{kg}$, $u = 5.0\,\text{m s}^{-1}$. The force opposes the motion, so its impulse is negative. Its magnitude is the area of the trapezium: area $= 1/2(0.80 + 0.40) \times 10 = 6.0\,\text{N s}$.



The change in velocity is $\Delta v = \dfrac{\text{impulse}}{m} = \dfrac{-6.0}{2.0} = -3.0\,\text{m s}^{-1}$, so the final speed is $v = 5.0 - 3.0 = 2.0\,\text{m s}^{-1}$ (still in the original direction).

Question 16 Take **up** as positive; $m = 0.50\,\text{kg}$, $\Delta t = 0.020\,\text{s}$. Impact speed (falling $1.8\,\text{m}$): $v_1 = \sqrt{2 g h_1} = \sqrt{2 \times 9.81 \times 1.8} = 5.94\,\text{m s}^{-1}$ downward, so $u = -5.94\,\text{m s}^{-1}$. Rebound speed (rising $0.80\,\text{m}$): $v_2 = \sqrt{2 g h_2} = \sqrt{2 \times 9.81 \times 0.80} = 3.96\,\text{m s}^{-1}$ upward, so $v = +3.96\,\text{m s}^{-1}$.

$$F = \frac{m(v-u)}{\Delta t} = \frac{0.50(3.96 - (-5.94))}{0.020} = \frac{0.50 \times 9.90}{0.020} = 248\,\text{N},$$

directed upward.

Question 17 (a) The area under a force–time graph is the **impulse**, which is equal to the object's **change in momentum** Δp. Here that impulse is $8.0\,\mathrm{N\,s}$. (b) The area under a force–distance graph is the **work done**, which is equal to the object's **change in kinetic energy** ΔE_k. Here that work is $12\,\mathrm{J}$. (c) Only the force–time area is the impulse. The force–distance area is work — a different physical quantity, with a different unit (J, not N s) and equal to the change in kinetic energy, not momentum. The two areas are computed from different graphs and are not interchangeable, so it is wrong to call the force–distance area an impulse.

Question 18 $m_{\mathrm{b}}=0.020\,\mathrm{kg}$, $u_{\mathrm{b}}=4.0\times10^2\,\mathrm{m\,s^{-1}}$, $M=2.0\,\mathrm{kg}$ (block, at rest); the bullet embeds. (a) $m_{\mathrm{b}}u_{\mathrm{b}}=(m_{\mathrm{b}}+M)v\Rightarrow v=\dfrac{0.020\times400}{2.02}=\dfrac{8.0}{2.02}=3.96\,\mathrm{m\,s^{-1}}$. (b) $E_k(\text{before})=1/2(0.020)(400)^2=1600\,\mathrm{J}$; $E_k(\text{after})=1/2(2.02)(3.96)^2=15.8\,\mathrm{J}$. Percentage converted $=\dfrac{1600-15.8}{1600}\times100=99.0\,\%$.

Question 19 Take the rebound direction as positive; $m=0.15\,\mathrm{kg}$, approach $u=-12\,\mathrm{m\,s^{-1}}$. Case A (sticks, $v=0$): $\text{impulse}=m(v-u)=0.15(0-(-12))=1.8\,\mathrm{N\,s}$. Case B (rebounds, $v=+9.0\,\mathrm{m\,s^{-1}}$): $\text{impulse}=m(v-u)=0.15(9.0-(-12))=0.15\times21=3.15\,\mathrm{N\,s}$. The rebound involves the larger impulse because the ball's velocity not only drops to zero but reverses, giving a larger change in momentum (a change of $21\,\mathrm{m\,s^{-1}}$ in velocity rather than $12\,\mathrm{m\,s^{-1}}$).

Question 20 $M=3.6\,\mathrm{kg}$, $m=0.012\,\mathrm{kg}$, $v=350\,\mathrm{m\,s^{-1}}$; the system starts at rest. Recoil: $MV=mv\Rightarrow V=\dfrac{0.012\times350}{3.6}=\dfrac{4.2}{3.6}=1.17\,\mathrm{m\,s^{-1}}$. Kinetic energies: $E_k(\text{bullet})=1/2(0.012)(350)^2=735\,\mathrm{J}$; $E_k(\text{rifle})=1/2(3.6)(1.17)^2=2.45\,\mathrm{J}$. Ratio $=\dfrac{735}{2.45}=300$. Although the bullet and rifle carry equal and opposite momenta, the bullet carries 300 times the kinetic energy: for a given momentum $E_k=\dfrac{p^2}{2m}$, so the lighter object receives far more of the kinetic energy.

Question 21 The argument is wrong. By Newton's third law the force on the car by the truck and the force on the truck by the car are equal in magnitude and opposite in direction — the truck does **not** push harder on the car than the car pushes on the truck. Because these forces act for the same contact time, the impulses on the two vehicles are equal and opposite, so their changes in momentum are equal and opposite: the momentum gained by one is exactly the momentum lost by the other. In this isolated system the **total** momentum is therefore conserved. What differs is the acceleration: the lighter car, having less mass, undergoes a much greater acceleration (and velocity change) than the truck for the same force, which is why the car is more severely affected even though the forces are equal.

Question 22 Take the right as positive: $m_A=0.50\,\mathrm{kg}$, $u_A=8.0\,\mathrm{m\,s^{-1}}$, $m_B=0.30\,\mathrm{kg}$, $u_B=0$, $v_A=2.0\,\mathrm{m\,s^{-1}}$. (a) $m_Au_A=m_Av_A+m_Bv_B\Rightarrow0.50(8.0)=0.50(2.0)+0.30v_B\Rightarrow v_B=\dfrac{4.0-1.0}{0.30}=10.0\,\mathrm{m\,s^{-1}}$. (b) $E_k(\text{before})=1/2(0.50)(8.0)^2=16\,\mathrm{J}$; $E_k(\text{after})=1/2(0.50)(2.0)^2+1/2(0.30)(10.0)^2=1.0+15=16\,\mathrm{J}$. The kinetic energy is unchanged, so the collision is **elastic**. (c) If they stuck together: $v=\dfrac{m_Au_A}{m_A+m_B}=\dfrac{4.0}{0.80}=5.0\,\mathrm{m\,s^{-1}}$. Then $E_k(\text{after})=1/2(0.80)(5.0)^2=10\,\mathrm{J}$, so the kinetic energy lost would be $16-10=6.0\,\mathrm{J}$.

Theory

Hooke's law. When a spring is stretched or compressed, it pushes or pulls back with a **restoring force** that tries to return it to its natural (unstretched) length. For an **ideal spring** the size of this force is proportional to the distance x the spring has been stretched or compressed from its natural length:

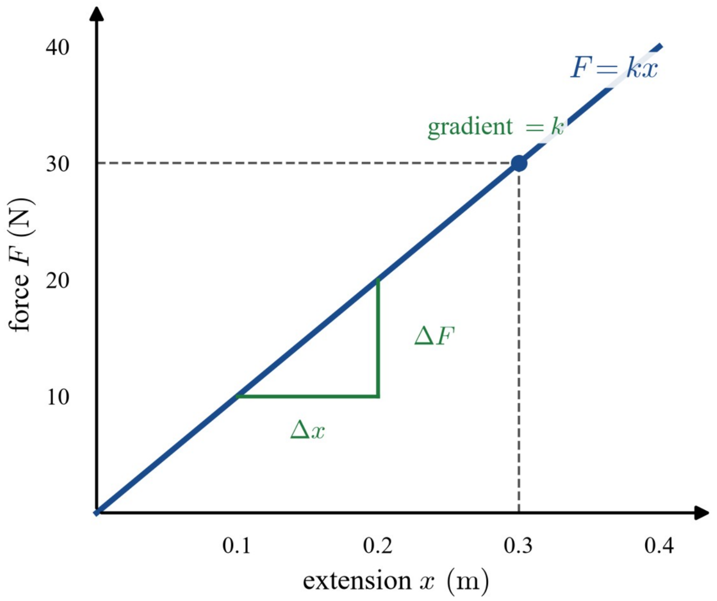
$$F = -k x.$$

The constant k is the **spring constant** (or stiffness), measured in $\mathrm{N\ m^{-1}}$. The negative sign shows that the force is a *restoring* force — it always acts **opposite** to the displacement x. In this subtopic we work with **magnitudes**, so the size of the applied force (or of the spring force) at extension x is

$$F = k x.$$

A stiff spring (large k) needs a large force to stretch it a given distance; a soft spring (small k) needs only a small force.

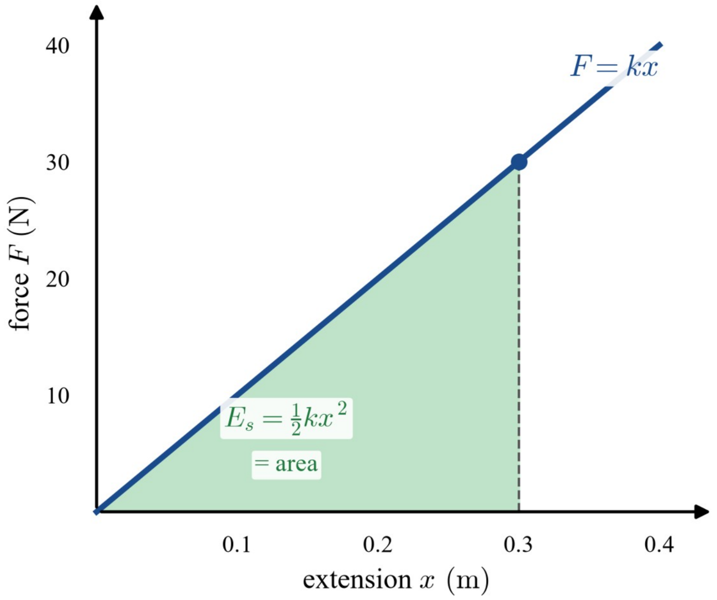
The force–extension graph. A graph of applied force F against extension x for an ideal spring is a **straight line through the origin**, and its **gradient is the spring constant** k. Reading k from the gradient of measured data is the standard way to find the stiffness of a real spring.



Elastic potential energy. Stretching or compressing a spring stores energy in it — **elastic potential energy** E_s. The work done by the applied force equals the **area under the force–extension graph**. For an ideal spring this area is a triangle of base x and height $F = k x$, so

$$E_s = \text{area} = 1/2\, x (k x) = 1/2\, k x^2.$$

The two forms are the same result: $E_s = 1/2\, k x^2 = 1/2\, F x$, measured in joules.



The area rule is more general than $1/2\,k\,x^2$. The energy stored is *always* the area under the force–extension graph, whatever the shape of the graph. The formula $E_s = 1/2\,k\,x^2$ applies **only** to an ideal (Hookean) spring, whose graph is a straight line. If a spring does not obey Hooke's law — for example one that stiffens as it stretches — the graph is not a straight line and the stored energy must be found as the **area** (by counting squares, or by splitting the region into triangles and trapezia), not from $1/2\,k\,x^2$.

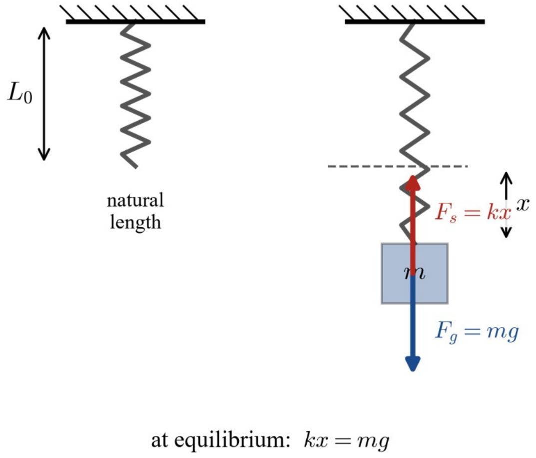
Work to change the extension. The work needed to stretch an ideal spring **from** x_1 **to** x_2 is the area under the graph between those extensions:

$$W = 1/2\,k\,x_2^2 - 1/2\,k\,x_1^2 = 1/2\,k\left(x_2^2 - x_1^2\right).$$

Because the graph is a straight line, this region is a trapezium and the work can equally be found as $W = 1/2\left(F_1 + F_2\right)\left(x_2 - x_1\right)$.

Spring–gravity systems. When a mass is supported by a spring, both the spring force and the force due to gravity $F_g = m\,g$ act on it. If a mass m hangs at rest from a vertical spring, it sits at the **equilibrium extension** x_{eq}, where the upward spring force balances the downward force due to gravity:

$$k\,x_{\text{eq}} = m\,g \Rightarrow x_{\text{eq}} = \frac{m\,g}{k}.$$



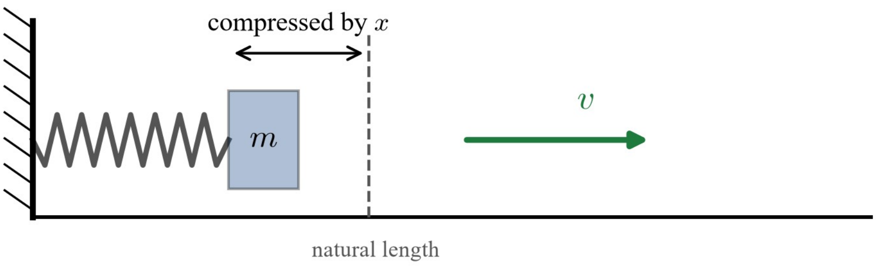
As a mass moves in a spring–gravity system, energy is continually exchanged between three stores: **kinetic energy** $E_k = 1/2\, m\, v^2$, **gravitational potential energy** $E_g = m\, g\, \Delta h$, and **elastic potential energy** $E_s = 1/2\, k\, x^2$. If no energy is dissipated, the **total** mechanical energy is conserved:

$$E_k + E_g + E_s = \text{constant}.$$

A useful check point is where the moving mass is **momentarily at rest** (for example at the highest or lowest point of its motion): there $E_k = 0$, so the energy is shared between E_g and E_s only.

Note that the equilibrium point is **not** where all of the gravitational potential energy has become elastic potential energy. Lowering a mass *slowly* to $x_{\text{eq}} = m\, g / k$ stores only $E_s = 1/2\, k\, x_{\text{eq}}^2 = 1/2\, m\, g\, x_{\text{eq}}$, which is exactly **half** of the gravitational potential energy lost, $m\, g\, x_{\text{eq}}$; the other half is transferred to whatever lowers the mass. Only if the mass is *released* from the natural length and allowed to fall freely does it descend to $x = 2\, m\, g / k$ (twice the equilibrium extension) before coming momentarily to rest, where all the lost gravitational potential energy has become elastic potential energy.

Launching and energy transformations. A compressed spring is a convenient energy store. When it is released against an object, the stored elastic potential energy is transformed into other forms: on a smooth horizontal surface it becomes kinetic energy, $E_s \rightarrow E_k$, giving a launch speed from $1/2\, k\, x^2 = 1/2\, m\, v^2$; fired vertically it becomes gravitational potential energy at the top of the flight, $E_s \rightarrow E_g$, giving a height from $1/2\, k\, x^2 = m\, g\, \Delta h$.



on release: $E_s = \frac{1}{2}kx^2 \;\rightarrow\; E_k = \frac{1}{2}mv^2$

Dissipation and the ideal-spring assumption. Real systems never conserve mechanical energy perfectly. Whenever a spring is worked repeatedly, or an object slides on a rough surface, some mechanical energy is **dissipated to the environment** — a combination of **heat**, **sound**, and permanent **deformation** of the material — and air resistance transfers still more. Treating a spring as an **ideal spring** means assuming that:

- it is *massless* (its own kinetic and potential energy can be ignored);
- it *obeys Hooke's law* ($F = k\, x$) over the whole range of extensions used; and
- it stores and returns energy with *no dissipation*, so all the elastic potential energy is recovered on release.

These assumptions make the energy accounting exact. Because a real spring falls short of them, a real launcher always sends an object a little slower, or a little lower, than the ideal calculation predicts.

Throughout this subtopic the constant $g = 9.81\ \text{m s}^{-2}$ (equivalently $9.81\ \text{N kg}^{-1}$) is used, and springs are assumed ideal unless stated otherwise.

Worked examples

Example 1 — scaffolded

A spring that obeys Hooke's law is stretched by a steady pulling force. A force of $12\ \text{N}$ produces an extension of $0.080\ \text{m}$. Calculate the spring constant and the elastic potential energy stored at this extension.

Working	Comment
Hooke's law (magnitudes): $F = k\, x$, with $F = 12\ \text{N}$,	List the data and choose the magnitude form of Hooke's

Working	Comment
$x = 0.080\,\text{m}$.	law.
$k = \dfrac{F}{x} = \dfrac{12}{0.080} = 150\,\text{N m}^{-1}$.	The spring constant is the gradient of the force–extension graph.
$E_s = 1/2\,k\,x^2 = 1/2(150)(0.080)^2 = 0.48\,\text{J}$.	Elastic PE = area under the force–extension graph.
Check: $E_s = 1/2\,F\,x = 1/2(12)(0.080) = 0.48\,\text{J}$.	The same triangular area, $1/2 \times \text{base} \times \text{height}$.

Example 2 — scaffolded

A spring of constant $200\,\text{N m}^{-1}$ is compressed $0.15\,\text{m}$ and used to launch a $0.25\,\text{kg}$ trolley along a frictionless horizontal track. Calculate the elastic potential energy stored, and the speed of the trolley as it leaves the spring.

Working	Comment
$E_s = 1/2\,k\,x^2 = 1/2(200)(0.15)^2 = 2.25\,\text{J}$.	Elastic PE stored in the compressed spring.
Track frictionless: all elastic PE becomes kinetic energy, $E_s = 1/2\,m\,v^2$.	Energy transformation $E_s \rightarrow E_k$; mechanical energy conserved.
$v = \sqrt{\dfrac{2\,E_s}{m}} = \sqrt{\dfrac{2(2.25)}{0.25}} = \sqrt{18} = 4.24\,\text{m s}^{-1}$.	Rearrange for v and substitute.

Example 3 — unscaffolded

A toy launcher has a spring of constant $320\,\text{N m}^{-1}$. The spring is compressed $0.10\,\text{m}$, a $0.050\,\text{kg}$ disc is placed on top, and the spring is released, firing the disc straight up. Ignoring air resistance and the mass of the spring, calculate the maximum height the disc rises above its release point.

The elastic potential energy stored in the compressed spring is

$$E_s = 1/2\,k\,x^2 = 1/2(320)(0.10)^2 = 1.6\,\text{J}.$$

At the highest point the disc is momentarily at rest, so all of this energy has been transformed into gravitational potential energy, $E_s = m\,g\,\Delta h$. Hence

$$\Delta h = \frac{E_s}{m\,g} = \frac{1.6}{0.050 \times 9.81} = \frac{1.6}{0.4905} = 3.262\,\text{m}.$$

$\therefore$ the disc rises to a maximum height of $3.26\,\text{m}$ above its release point.

Example 4 — unscaffolded

A spring of constant $80\,\text{N m}^{-1}$ is already stretched by $0.10\,\text{m}$. Calculate the work required to stretch it further, to a total extension of $0.25\,\text{m}$.

The work done equals the area under the force–extension graph between $x_1 = 0.10\,\text{m}$ and $x_2 = 0.25\,\text{m}$. Because $F = k\,x$, this is the difference of two triangular areas:

$$W = 1/2\,k\left(x_2^2 - x_1^2\right) = 1/2(80)\left(0.25^2 - 0.10^2\right) = 40(0.0625 - 0.0100) = 2.1\,\text{J}.$$

As a check, the region is a trapezium with parallel sides $F_1 = k\,x_1 = 8.0\,\text{N}$ and $F_2 = k\,x_2 = 20\,\text{N}$ and width $x_2 - x_1 = 0.15\,\text{m}$:

$$W = 1/2(F_1 + F_2)(x_2 - x_1) = 1/2(8.0 + 20)(0.15) = 2.1\,\text{J}.$$

$\therefore$ the work required is $2.1\,\text{J}$.

Practice questions

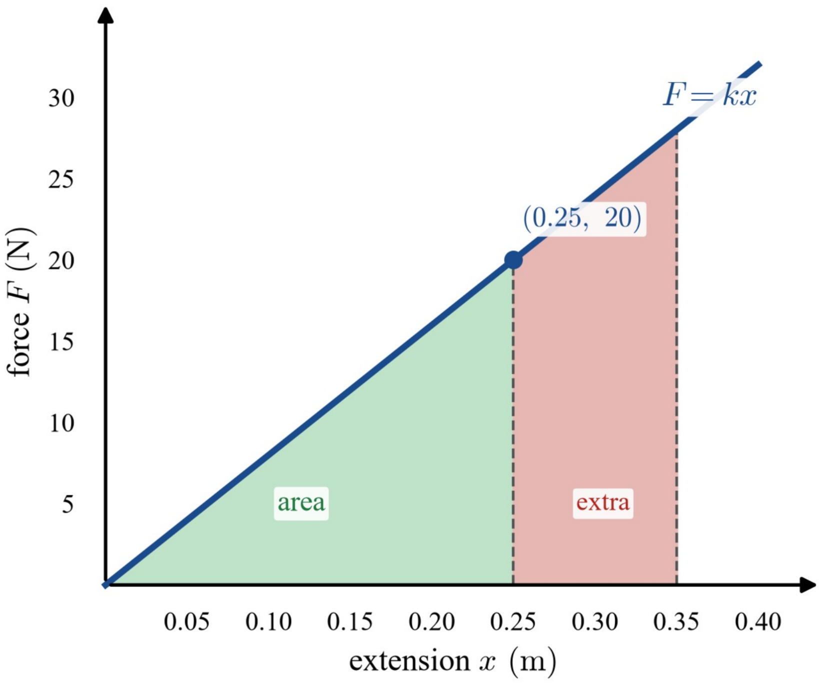
Question 1 A spring that obeys Hooke's law extends by $0.060\,\mathrm{m}$ when a force of $18\,\mathrm{N}$ is applied. (a) Calculate the spring constant. (b) Calculate the extension produced by a force of $30\,\mathrm{N}$ (assume the spring is still within its elastic limit). (c) Calculate the elastic potential energy stored at an extension of $0.060\,\mathrm{m}$.

Question 2 A mass of $0.60\,\mathrm{kg}$ hangs at rest from a vertical spring, extending it by $0.15\,\mathrm{m}$. (a) Calculate the spring constant. (b) Calculate the elastic potential energy stored in the spring. (c) Calculate the gravitational potential energy lost by the mass as it was lowered slowly from the spring's natural length to this position, and comment on how your answer compares with the elastic potential energy found in part (b).

Question 3 A spring of constant $250\,\mathrm{N\ m^{-1}}$ is compressed $0.12\,\mathrm{m}$ against a $0.30\,\mathrm{kg}$ trolley on a frictionless horizontal track, then released. (a) Calculate the elastic potential energy stored in the compressed spring. (b) Calculate the speed of the trolley as it leaves the spring. (c) State the energy transformation that occurs, and explain why the trolley would leave the spring more slowly if the track were rough.

Question 4 A toy popper uses a spring of constant $400\,\mathrm{N\ m^{-1}}$ compressed by $0.050\,\mathrm{m}$ to fire a $0.020\,\mathrm{kg}$ disc straight up. (a) Show that the elastic potential energy stored is $0.500\,\mathrm{J}$. (b) Calculate the maximum height the disc rises above its release point, ignoring air resistance. (c) In practice the disc rises to a lower height than your answer to part (b). Give one reason.

Question 5 The force–extension graph of a spring is shown below.



(a) Determine the spring constant from the graph.
(b) Use the area under the graph to find the elastic potential energy stored at an extension of $0.25\,\mathrm{m}$.
(c) Calculate the extra energy needed to stretch the spring from an extension of $0.25\,\mathrm{m}$ to an extension of $0.35\,\mathrm{m}$.

Question 6 A ball is launched straight up by a compressed spring and later falls back. (a) Describe the sequence of energy transformations from the instant the spring is released to the moment the ball reaches its highest point. (b) State two assumptions involved in treating the spring as an ideal spring. (c) A real spring launches the ball to a lower height than an ideal spring would. Identify the forms into which the "missing" energy has been transformed.

Question 7 A spring stretches by $0.045\,\mathrm{m}$ when a force of $6.0\,\mathrm{N}$ is applied to it. Calculate the force needed to stretch the same spring by $0.070\,\mathrm{m}$.

Question 8 A spring of constant $120\,\mathrm{N\ m^{-1}}$ is compressed by $0.15\,\mathrm{m}$. Calculate the elastic potential energy stored.

Question 9 A compressed spring of constant $500\,\mathrm{N\,m^{-1}}$ stores $4.5\,\mathrm{J}$ of elastic potential energy. Calculate the compression of the spring. Give your answer to three significant figures.

Question 10 A spring launches a $0.15\,\mathrm{kg}$ ball to a speed of $6.0\,\mathrm{m\,s^{-1}}$ from a compression of $0.080\,\mathrm{m}$ on a frictionless horizontal surface. Calculate the spring constant.

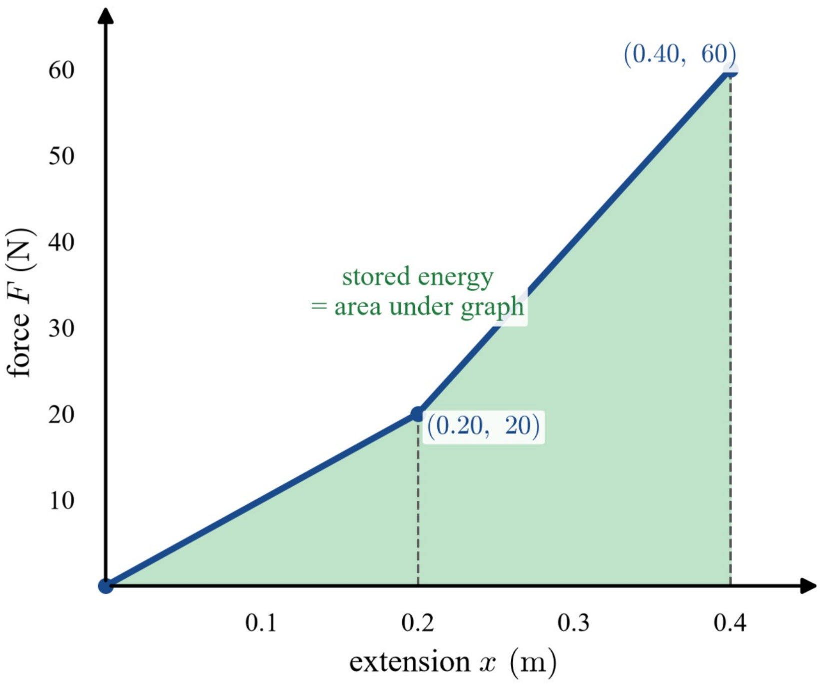
Question 11 A vertical spring of constant $60\,\mathrm{N\,m^{-1}}$ extends by $0.12\,\mathrm{m}$ when a mass hangs from it at rest. Calculate the mass.

Question 12 A $0.20\,\mathrm{kg}$ block is placed on top of a vertical spring of constant $180\,\mathrm{N\,m^{-1}}$ at the spring's natural length, and released from rest. Calculate the maximum compression of the spring.

Question 13 A spring of constant $800\,\mathrm{N\,m^{-1}}$ is compressed by $0.060\,\mathrm{m}$ and launches a $0.040\,\mathrm{kg}$ puck horizontally on a frictionless surface. (a) Show that the elastic potential energy stored is $1.44\,\mathrm{J}$. (b) Hence calculate the launch speed of the puck.

Question 14 Spring A has a larger spring constant than spring B. (a) If both springs are stretched by the **same extension**, state which one stores more elastic potential energy, and justify your answer. (b) If instead both springs are stretched by the **same applied force**, state which one stores more elastic potential energy, and justify your answer.

Question 15 The force–extension graph of a non-ideal (stiffening) spring is shown below. It is made up of two straight sections.



Use the area under the graph to determine the elastic potential energy stored when the spring is stretched by $0.40\,\mathrm{m}$. Explain why the formula $E_s = 1/2\,k\,x^2$ cannot be used here.

Question 16 A child bounces up and down on a pogo stick. Consider one bounce, from the instant the spring first begins to compress, through maximum compression, to the moment the child returns to the same height. (a) Describe the energy transformations that occur during this bounce. (b) Explain, in terms of energy, why a real pogo stick gradually bounces lower unless the child does work to keep it going.

Question 17 A spring of constant $600\,\mathrm{N\,m^{-1}}$ is compressed by $0.10\,\mathrm{m}$ and pushes a $0.40\,\mathrm{kg}$ block across a rough horizontal floor. The block leaves the spring moving at $2.5\,\mathrm{m\,s^{-1}}$. Calculate the energy dissipated as heat and sound by friction while the spring was expanding.

Question 18 A student lowers a $1.2\,\mathrm{kg}$ mass slowly onto a hanging spring until it rests at equilibrium, extending the spring by $0.20\,\mathrm{m}$. The student claims that, because the mass descended $0.20\,\mathrm{m}$, the gravitational potential energy lost must all have been stored as elastic potential energy in the spring. By calculating both energies, show that the claim is wrong, and explain where the difference has gone.

Question 19 A $0.30\,\text{kg}$ ball is dropped from rest from a height of $0.50\,\text{m}$ above the top of a vertical spring of constant $240\,\text{N m}^{-1}$. Calculate the maximum compression of the spring. (Ignore air resistance and the mass of the spring.)

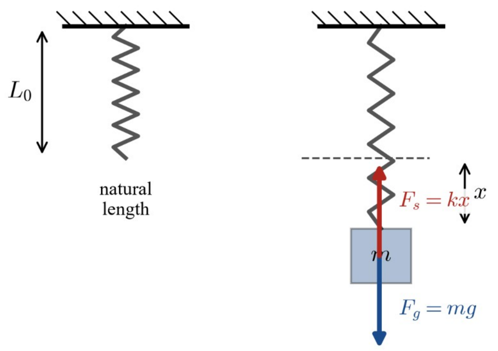
Answers

Question 1 (a) $300\ \mathrm{N\ m^{-1}}$; (b) $0.10\ \mathrm{m}$; (c) $0.54\ \mathrm{J}$. **Question 2** (a) $39.2\ \mathrm{N\ m^{-1}}$; (b) $0.441\ \mathrm{J}$; (c) $0.883\ \mathrm{J}$ lost — exactly twice the elastic PE stored. **Question 3** (a) $1.8\ \mathrm{J}$; (b) $3.46\ \mathrm{m\ s^{-1}}$; (c) elastic PE → kinetic energy; on a rough track some energy is dissipated as heat and sound, so less becomes kinetic energy. **Question 4** (a) $0.500\ \mathrm{J}$; (b) $2.55\ \mathrm{m}$; (c) air resistance / a real spring is not ideal (dissipation). **Question 5** (a) $80\ \mathrm{N\ m^{-1}}$; (b) $2.5\ \mathrm{J}$; (c) $2.4\ \mathrm{J}$. **Question 6** (a) elastic PE → kinetic energy → gravitational PE; (b) any two of: massless spring, obeys Hooke's law, no energy dissipated; (c) heat, sound and deformation (plus energy to air resistance). **Question 7** $9.33\ \mathrm{N}$. **Question 8** $1.35\ \mathrm{J}$. **Question 9** $0.134\ \mathrm{m}$. **Question 10** $844\ \mathrm{N\ m^{-1}}$. **Question 11** $0.734\ \mathrm{kg}$. **Question 12** $0.0218\ \mathrm{m}$. **Question 13** (a) $1.44\ \mathrm{J}$; (b) $8.49\ \mathrm{m\ s^{-1}}$. **Question 14** (a) spring A; (b) spring B. **Question 15** $10\ \mathrm{J}$; k is not constant, so the graph is not a straight line and $E_s = 1/2\,k\,x^2$ does not apply. **Question 16** (a) kinetic + gravitational PE → elastic PE → kinetic + gravitational PE; (b) each cycle dissipates energy as heat, sound and deformation, so mechanical energy falls unless replaced. **Question 17** $1.75\ \mathrm{J}$. **Question 18** E_g lost $= 2.35\ \mathrm{J}$, E_s stored $= 1.18\ \mathrm{J}$; the claim is wrong — only half is stored, the rest is transferred to the student's hand. **Question 19** $0.124\ \mathrm{m}$.

Fully-worked solutions

Question 1 $F = 18\ \mathrm{N}$, $x = 0.060\ \mathrm{m}$. (a) $k = \dfrac{F}{x} = \dfrac{18}{0.060} = 300\ \mathrm{N\ m^{-1}}$. (b) $x = \dfrac{F}{k} = \dfrac{30}{300} = 0.10\ \mathrm{m}$. (c) $E_s = 1/2\,k\,x^2 = 1/2(300)(0.060)^2 = 0.54\ \mathrm{J}$.

Question 2 $m = 0.60\ \mathrm{kg}$, $x = 0.15\ \mathrm{m}$.



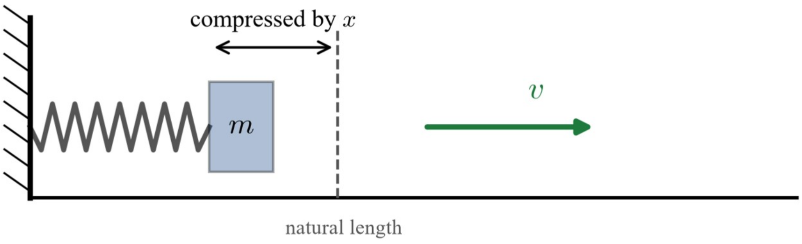
at equilibrium: $kx = mg$

(a) At equilibrium the spring force balances the force due to gravity: $k\,x = m\,g$, so $k = \dfrac{m\,g}{x} = \dfrac{0.60 \times 9.81}{0.15} = 39.2\ \mathrm{N\ m^{-1}}$.

(b) $E_s = 1/2\,k\,x^2 = 1/2(39.24)(0.15)^2 = 0.441\ \mathrm{J}$.

(c) Gravitational PE lost $= m\,g\,\Delta h = m\,g\,x = 0.60 \times 9.81 \times 0.15 = 0.883\ \mathrm{J}$. This is **twice** the elastic potential energy stored ($0.441\ \mathrm{J}$). The mass was lowered slowly, so its kinetic energy stayed zero throughout; the "missing" half of the gravitational potential energy was transferred to whatever lowered it (the student's hand), not stored in the spring. The equilibrium extension is therefore *not* the point at which all the lost gravitational potential energy has become elastic potential energy.

Question 3 $k = 250\ \mathrm{N\ m^{-1}}$, $x = 0.12\ \mathrm{m}$, $m = 0.30\ \mathrm{kg}$.



on release: $E_s = \frac{1}{2}kx^2 \rightarrow E_k = \frac{1}{2}mv^2$

(a) $E_s = 1/2\,k\,x^2 = 1/2(250)(0.12)^2 = 1.8\,\text{J}$.

(b) The track is frictionless, so all the elastic potential energy becomes kinetic energy: $1/2\,m\,v^2 = E_s$, giving $v = \sqrt{\frac{2E_s}{m}} = \sqrt{\frac{2(1.8)}{0.30}} = \sqrt{12} = 3.46\,\text{m s}^{-1}$.

(c) The transformation is elastic potential energy → kinetic energy. On a rough track the block also does work against friction, so some of the stored energy is dissipated to the environment as heat and sound. Less energy is left as kinetic energy, so the trolley leaves the spring at a lower speed.

Question 4 $k = 400\,\text{N m}^{-1}$, $x = 0.050\,\text{m}$, $m = 0.020\,\text{kg}$. (a) $E_s = 1/2\,k\,x^2 = 1/2(400)(0.050)^2 = 1/2(400)(0.0025) = 0.500\,\text{J}$, as required. (b) At the highest point the disc is momentarily at rest, so $E_s = m\,g\,\Delta h$ and $\Delta h = \frac{E_s}{m\,g} = \frac{0.500}{0.020 \times 9.81} = 2.55\,\text{m}$. (c) In practice some energy is transformed to heat, sound and deformation in the real (non-ideal) spring, and the disc does work against air resistance as it rises; less energy reaches the gravitational-PE store, so the disc rises less high.

Question 5 (a) The graph is a straight line through the origin passing through $(0.25\,\text{m}, 20\,\text{N})$, so the spring constant is the gradient: $k = \frac{20}{0.25} = 80\,\text{N m}^{-1}$. (b) The elastic potential energy is the area of the triangle under the graph up to $x = 0.25\,\text{m}$: $E_s = 1/2(0.25)(20) = 2.5\,\text{J}$. (c) The extra energy is the area under the graph between $0.25\,\text{m}$ and $0.35\,\text{m}$: $W = 1/2\,k\left(0.35^2 - 0.25^2\right) = 1/2(80)(0.1225 - 0.0625) = 40 \times 0.0600 = 2.4\,\text{J}$.

Question 6 (a) At release the spring's **elastic potential energy** is transformed into **kinetic energy** as the ball is pushed up and speeds up. Once the ball leaves the spring, its kinetic energy is progressively transformed into **gravitational potential energy** as it rises and slows, until at the highest point (momentarily at rest) essentially all of it is gravitational potential energy. The chain is elastic PE → kinetic energy → gravitational PE. (b) Any two of: the spring is **massless**; the spring **obeys Hooke's law** ($F = k\,x$) over the range used; the spring **dissipates no energy**, returning all its stored energy on release. (c) The "missing" energy is transformed into **heat** and **sound**, and into permanent **deformation** of the real spring; a further part is transferred to the surrounding air as the ball does work against air resistance. These are energy dissipated to the environment, so less energy reaches the gravitational-PE store and the ball rises less high.

Question 7 The spring constant is $k = \frac{F}{x} = \frac{6.0}{0.045} = 133.3\,\text{N m}^{-1}$. The force to produce an extension of $0.070\,\text{m}$ is

$$F = k\,x = 133.3 \times 0.070 = 9.33\,\text{N}.$$

(Equivalently, since $F \propto x$: $F = 6.0 \times \frac{0.070}{0.045} = 9.33\,\text{N}$.)

Question 8 $E_s = 1/2\,k\,x^2 = 1/2(120)(0.15)^2 = 1/2(120)(0.0225) = 1.35\,\text{J}$.

Question 9 From $E_s = 1/2\,k\,x^2$, $x = \sqrt{\frac{2E_s}{k}} = \sqrt{\frac{2(4.5)}{500}} = \sqrt{0.018} = 0.134\,\text{m}$ (to three significant figures).

Question 10 On a frictionless surface all the elastic potential energy becomes kinetic energy: $1/2\,k\,x^2 = 1/2\,m\,v^2$, so

$$k = \frac{m v^2}{x^2} = \frac{0.15 \times 6.0^2}{0.080^2} = \frac{5.4}{0.0064} = 844\,\mathrm{N\,m^{-1}}.$$

Question 11 At equilibrium $k x = m g$, so $m = \frac{k x}{g} = \frac{60 \times 0.12}{9.81} = \frac{7.2}{9.81} = 0.734\,\mathrm{kg}$.

Question 12 The block is released from rest at the natural length and is momentarily at rest again at maximum compression x, so its kinetic energy is zero at both points. All the gravitational potential energy lost has become elastic potential energy:

$$m g x = 1/2 k x^2 \Rightarrow x = \frac{2 m g}{k} = \frac{2(0.20)(9.81)}{180} = \frac{3.924}{180} = 0.0218\,\mathrm{m}.$$

Question 13 $k = 800\,\mathrm{N\,m^{-1}}$, $x = 0.060\,\mathrm{m}$, $m = 0.040\,\mathrm{kg}$. (a) $E_s = 1/2 k x^2 = 1/2(800)(0.060)^2 = 1/2(800)(0.0036) = 1.44\,\mathrm{J}$, as required. (b) The surface is frictionless, so $1/2 m v^2 = E_s$ and $v = \sqrt{\frac{2E_s}{m}} = \sqrt{\frac{2(1.44)}{0.040}} = \sqrt{72} = 8.49\,\mathrm{m\,s^{-1}}$.

Question 14 (a) For the **same extension** x, the stored energy is $E_s = 1/2 k x^2$. With x fixed, a larger k gives a larger E_s, so **spring A** (the larger spring constant) stores more elastic potential energy. (b) For the **same applied force** F, each spring stretches to $x = \frac{F}{k}$, so the stored energy is $E_s = 1/2 k x^2 = 1/2 k \left(\frac{F}{k}\right)^2 = \frac{F^2}{2k}$. With F fixed, a larger k now gives a *smaller* E_s, because the stiffer spring stretches less far. So **spring B** (the smaller spring constant) stores more. The key is that "same extension" and "same force" are different conditions and give opposite answers.

Question 15 The spring is non-ideal, so the stored energy is the total area under the graph, found as a triangle plus a trapezium:

$$E_s = 1/2(0.20)(20) + 1/2(20+60)(0.20) = 2.0 + 8.0 = 10\,\mathrm{J}$$

(the first term is the triangle from $x = 0$ to $x = 0.20\,\mathrm{m}$; the second is the trapezium from $x = 0.20\,\mathrm{m}$ to $x = 0.40\,\mathrm{m}$). The formula $E_s = 1/2 k x^2$ cannot be used because the graph is not a single straight line through the origin: the spring stiffens (its gradient increases), so k is not constant and there is no single value to substitute. The area rule, however, still applies.

Question 16 (a) As the spring compresses, the child's **kinetic energy** and **gravitational potential energy** are transformed into **elastic potential energy**, which is greatest at maximum compression (where the child is momentarily at rest). As the spring then extends, this elastic potential energy is transformed back into kinetic energy and gravitational potential energy, pushing the child up again. Over the bounce: kinetic + gravitational PE → elastic PE → kinetic + gravitational PE. (b) In an ideal spring no energy is lost and the child would return to exactly the same height each time. In reality, every compression–extension cycle dissipates some mechanical energy to the environment as **heat**, **sound** and **deformation** (and to air resistance). The total mechanical energy therefore decreases with each bounce, so the child rises a little less each time — unless the child does work (pushing with their legs) to replace the dissipated energy.

Question 17 $k = 600\,\mathrm{N\,m^{-1}}$, $x = 0.10\,\mathrm{m}$, $m = 0.40\,\mathrm{kg}$, $v = 2.5\,\mathrm{m\,s^{-1}}$. The elastic potential energy released by the spring is

$$E_s = 1/2 k x^2 = 1/2(600)(0.10)^2 = 3.0\,\mathrm{J},$$

while the kinetic energy the block actually gains is

$$E_k = 1/2 m v^2 = 1/2(0.40)(2.5)^2 = 1.25\,\mathrm{J}.$$

By conservation of energy, the difference has been dissipated by friction as heat and sound:

$$E_{\text{dissipated}} = E_s - E_k = 3.0 - 1.25 = 1.75\,\text{J}.$$

Question 18 $m = 1.2\,\text{kg}$, $x = 0.20\,\text{m}$. The gravitational potential energy lost as the mass descends is

$$E_g = m\,g\,\Delta h = m\,g\,x = 1.2 \times 9.81 \times 0.20 = 2.35\,\text{J}.$$

At equilibrium $k\,x = m\,g$, so $k = \frac{m\,g}{x} = \frac{1.2 \times 9.81}{0.20} = 58.9\,\text{N m}^{-1}$, and the elastic potential energy stored is

$$E_s = 1/2\,k\,x^2 = 1/2(58.86)(0.20)^2 = 1.18\,\text{J}.$$

Only $1.18\,\text{J}$ is stored in the spring, which is exactly **half** of the $2.35\,\text{J}$ of gravitational potential energy lost — so the claim is wrong. Because the mass was lowered slowly, its kinetic energy remained zero throughout; the remaining $1.18\,\text{J}$ was transferred out of the system, doing negative work on the student's hand (the hand held the mass back all the way down). The gravitational potential energy is shared equally between the spring and the hand, not stored entirely in the spring.

Question 19 Take the ball momentarily at rest at maximum compression x. It started at rest a height $0.50\,\text{m}$ above the spring and has then descended a further distance x, so the total drop is $(0.50 + x)$ and all of the lost gravitational potential energy has become elastic potential energy:

$$m\,g(0.50 + x) = 1/2\,k\,x^2.$$

Substituting $m = 0.30\,\text{kg}$, $g = 9.81\,\text{m s}^{-2}$, $k = 240\,\text{N m}^{-1}$:

$$2.943(0.50 + x) = 120\,x^2 \Rightarrow 120\,x^2 - 2.943\,x - 1.4715 = 0.$$

Solving the quadratic for the positive root,

$$x = \frac{2.943 + \sqrt{(-2.943)^2 + 4(120)(1.4715)}}{2(120)} = \frac{2.943 + \sqrt{714.98}}{240} = \frac{29.68}{240} = 0.124\,\text{m}.$$

$\therefore$ the maximum compression of the spring is $0.124\,\text{m}$.

Topic 1 Test A — Motion and energy (multiple choice)

Time allowed: 25 minutes · 20 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used.

Choose the response that is correct or that best answers the question. A correct answer scores 1 mark; an incorrect answer scores 0. Marks are not deducted for incorrect answers. No marks are given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this test are not drawn to scale. Where required, take the gravitational field strength to be $g = 9.81\,\mathrm{N\ kg^{-1}}$.

Question 1

A hammer drives a nail into a plank. At the instant of impact the hammer exerts a force on the nail. According to Newton's third law, the reaction to this force is

A. the force on the hammer by the nail, equal in magnitude and opposite in direction.

B. the force on the nail by the plank, equal in magnitude and opposite in direction.

C. the force due to gravity on the nail, acting downward.

D. the normal force on the hammer by the nail, in the same direction as the hammer's motion.

Question 2

A light string connects a $2.0\,\mathrm{kg}$ trolley behind a $3.0\,\mathrm{kg}$ trolley on a frictionless horizontal floor. A horizontal force of $20\,\mathrm{N}$ pulls the $3.0\,\mathrm{kg}$ trolley, so that both trolleys accelerate together. Which one of the following is closest to the tension in the connecting string?

A. $20\,\mathrm{N}$

B. $12\,\mathrm{N}$

C. $8.0\,\mathrm{N}$

D. $4.0\,\mathrm{N}$

Question 3

A $5.0\,\mathrm{kg}$ block is pushed up a rough incline at 30° to the horizontal at constant velocity by a force acting parallel to the incline. The friction force on the block is $8.0\,\mathrm{N}$. Which one of the following is closest to the magnitude of the applied force?

A. $50.5\,\mathrm{N}$

B. $24.5\,\mathrm{N}$

C. $16.5\,\mathrm{N}$

D. $32.5\,\mathrm{N}$

Question 4

A $0.20\,\mathrm{kg}$ ball on a string moves in a horizontal circle of radius $0.80\,\mathrm{m}$ at a constant speed of $3.0\,\mathrm{m\ s^{-1}}$. Which one of the following is closest to the net force on the ball?

A. $0.75\,\mathrm{N}$

B. $2.25\,\mathrm{N}$

C. $11.25\,\mathrm{N}$

D. $1.8\,\mathrm{N}$

Question 5

A $1500\,\mathrm{kg}$ car rounds a flat, level curve of radius $40\,\mathrm{m}$. The maximum sideways friction the road can provide on the tyres is $6.0 \times 10^{3}\,\mathrm{N}$. Which one of the following is closest to the maximum speed at which the car can round the curve without sliding?

A. $12.6\,\mathrm{m\ s^{-1}}$

B. $8.94\,\mathrm{m\ s^{-1}}$

C. $17.9\,\mathrm{m\ s^{-1}}$

D. $160\,\mathrm{m\ s^{-1}}$

Question 6

An object moves in uniform circular motion at constant speed. Which one of the following is correct?

A. Its velocity is constant, so its acceleration is zero.

B. Its speed is constant, but its velocity and acceleration continually change; the acceleration is directed toward the centre.

C. Its acceleration is directed along its velocity, in the direction of motion.

D. The net force on it is directed outward, away from the centre.

Question 7

A ball rolls off the edge of a bench $1.25\,\mathrm{m}$ high, leaving the edge horizontally at $3.0\,\mathrm{m\ s^{-1}}$. Ignoring air resistance, which one of the following is closest to the time the ball takes to reach the floor?

A. $0.357\,\mathrm{s}$

B. $0.255\,\mathrm{s}$

C. $0.714\,\mathrm{s}$

D. $0.505\,\mathrm{s}$

Question 8

A ball is launched from ground level at $25\,\mathrm{m\ s^{-1}}$ at 40° above the horizontal. Ignoring air resistance, which one of the following is closest to the maximum height it reaches?

A. $26.3\,\mathrm{m}$

B. $18.7\,\mathrm{m}$

C. $13.2\,\mathrm{m}$

D. $31.9\,\mathrm{m}$

Question 9

Two projectiles are launched from ground level with the same speed, one at 30° and the other at 60° above the horizontal. Ignoring air resistance, which one of the following is correct?

A. Both projectiles have the same horizontal range.

B. The 60° projectile has the greater horizontal range.

C. Both projectiles reach the same maximum height.

D. The 30° projectile spends longer in the air.

Question 10

A stone is thrown from ground level at $20\,\text{m s}^{-1}$ at 50° above the horizontal. Ignoring air resistance, which one of the following is closest to the stone's speed $1.0\,\text{s}$ after launch?

A. $20.0\,\text{m s}^{-1}$

B. $12.9\,\text{m s}^{-1}$

C. $16.2\,\text{m s}^{-1}$

D. $14.0\,\text{m s}^{-1}$

Question 11

A constant horizontal force of $15\,\text{N}$ acts through a distance of $6.0\,\text{m}$ on a $3.0\,\text{kg}$ box, initially at rest on a frictionless surface. Which one of the following is closest to the box's final speed?

A. $5.48\,\text{m s}^{-1}$

B. $7.75\,\text{m s}^{-1}$

C. $30.0\,\text{m s}^{-1}$

D. $3.87\,\text{m s}^{-1}$

Question 12

A child slides down a straight playground slide at constant speed. Which one of the following best describes the energy transformation as she descends?

A. Her gravitational potential energy is converted entirely to kinetic energy.

B. Her kinetic energy is converted to gravitational potential energy.

C. Her gravitational potential energy is converted to elastic potential energy stored in the slide.

D. Her gravitational potential energy is transferred to heat and sound by friction; her kinetic energy is unchanged because her speed is constant.

Question 13

A car travelling on a level road brakes and slows down. Which one of the following is correct about the work done by the net force on the car during braking?

A. The net work is positive and equals the car's loss of kinetic energy.

B. The net work is zero because the road is level.

C. The net work is negative and equals the change in the car's kinetic energy.

D. The net work is positive and equals the car's original kinetic energy.

Question 14

A $1200\,\mathrm{kg}$ car travelling at $15\,\mathrm{m\ s^{-1}}$ is brought to rest in a collision lasting $0.12\,\mathrm{s}$. Which one of the following is closest to the magnitude of the average net force on the car?

A. $1.5\times10^{5}\,\mathrm{N}$

B. $1.8\times10^{4}\,\mathrm{N}$

C. $2.16\times10^{3}\,\mathrm{N}$

D. $1.13\times10^{6}\,\mathrm{N}$

Question 15

On a frictionless track a $0.30\,\mathrm{kg}$ trolley moving right at $5.0\,\mathrm{m\ s^{-1}}$ overtakes and collides with a $0.20\,\mathrm{kg}$ trolley moving right at $1.0\,\mathrm{m\ s^{-1}}$. After the collision the $0.30\,\mathrm{kg}$ trolley continues right at $2.0\,\mathrm{m\ s^{-1}}$. Which one of the following is closest to the velocity of the $0.20\,\mathrm{kg}$ trolley after the collision?

A. $8.5\,\mathrm{m\ s^{-1}}$ right

B. $4.5\,\mathrm{m\ s^{-1}}$ right

C. $5.5\,\mathrm{m\ s^{-1}}$ right

D. $3.0\,\mathrm{m\ s^{-1}}$ right

Question 16

A single constant force acts on a trolley. One graph plots the force against time; a second plots the same force against the trolley's displacement. Which one of the following is correct?

A. The area under both graphs gives the impulse on the trolley.

B. The area under the force–time graph gives the impulse; the area under the force–displacement graph gives the work done.

C. The area under the force–time graph gives the work done; the area under the force–displacement graph gives the impulse.

D. The area under both graphs gives the change in kinetic energy.

Question 17

Two carts collide on a frictionless track that forms an isolated system. Which one of the following is always true, whether the collision is elastic or inelastic?

A. Both total momentum and total kinetic energy are conserved.

B. Total kinetic energy is conserved, but total momentum may change.

C. Neither quantity is conserved if the carts stick together.

D. Total momentum is conserved; total kinetic energy is conserved only if the collision is elastic.

Question 18

A spring that obeys Hooke's law and has a stiffness of $320\,\mathrm{N\,m^{-1}}$ is compressed by $0.10\,\mathrm{m}$. Which one of the following is closest to the elastic potential energy stored in the spring?

A. $1.6\,\mathrm{J}$

B. $3.2\,\mathrm{J}$

C. $16\,\mathrm{J}$

D. $32\,\mathrm{J}$

Question 19

A spring of stiffness $160\,\mathrm{N\,m^{-1}}$ is compressed by $0.10\,\mathrm{m}$ and used to launch a $0.20\,\mathrm{kg}$ cart along a frictionless horizontal track. Which one of the following is closest to the launch speed of the cart?

A. $8.94\,\mathrm{m\,s^{-1}}$

B. $2.0\,\mathrm{m\,s^{-1}}$

C. $2.83\,\mathrm{m\,s^{-1}}$

D. $4.0\,\mathrm{m\,s^{-1}}$

Question 20

For a spring that obeys Hooke's law, the elastic potential energy stored when it is stretched by an amount x is represented on a force–extension graph by

A. the gradient of the graph, equal to k.

B. the area under the graph, equal to $1/2\,k\,x^2$.

C. the product of the maximum force and the extension, $k\,x^2$.

D. the area under the corresponding force–time graph.

End of Topic 1 Test A

Topic 1 Test A — solutions

Question	Answer	Question	Answer
1	A	11	B
2	C	12	D
3	D	13	C
4	B	14	A
5	A	15	C
6	B	16	B
7	D	17	D
8	C	18	A
9	A	19	C
10	D	20	B

Question 1 — A. The reaction to "the force on the nail by the hammer" is the equal and opposite "force on the hammer by the nail"; a third-law pair acts on the two *different* interacting bodies. **B** names a different interaction (nail–plank); **C** is the force due to gravity, not the contact-pair partner; **D** misstates both the type of force and its direction.

Question 2 — C. $a=\frac{20}{2.0+3.0}=4.0\,\mathrm{m\,s^{-2}}$; the string accelerates only the trailing $2.0\,\mathrm{kg}$ trolley, so $F_T=2.0\times4.0=8.0\,\mathrm{N}$. **A** quotes the whole applied force; **B** uses the leading trolley's mass ($3.0\,\mathrm{kg}$); **D** quotes the acceleration ($4.0\,\mathrm{m\,s^{-2}}$) as a force.

Question 3 — D. Constant velocity up the incline: $F=mg\sin\theta+f=5.0\times9.81\times\sin30^\circ+8.0=24.5+8.0=32.5\,\mathrm{N}$. **A** uses $\cos$ instead of $\sin$; **B** omits friction; **C** subtracts friction (treating it as if it aided the motion).

Question 4 — B. $F=\frac{mv^2}{r}=\frac{0.20\times3.0^2}{0.80}=2.25\,\mathrm{N}$, directed toward the centre. **A** does not square v; **C** omits the mass; **D** omits dividing by r.

Question 5 — A. The maximum friction supplies the centripetal force: $v=\sqrt{\frac{Fr}{m}}=\sqrt{\frac{6.0\times10^3\times40}{1500}}=\sqrt{160}=12.6\,\mathrm{m\,s^{-1}}$. **B** halves the radius (diameter/radius slip); **C** doubles it; **D** omits the square root.

Question 6 — B. In uniform circular motion the speed is constant, but the velocity (a vector) and the acceleration continually change direction; the acceleration is centripetal (toward the centre). **A** confuses constant speed with constant velocity; **C** points the acceleration along the velocity; **D** invents an outward force.

Question 7 — D. The vertical motion is a fall from rest: $t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times1.25}{9.81}}=0.505\,\mathrm{s}$ (the horizontal launch speed does not affect the time). **A** drops the factor 2; **B** omits the square root; **C** uses $4h$ instead of $2h$.

Question 8 — C. $H=\frac{u_y^2}{2g}$ with $u_y=25\sin40^\circ=16.1\,\mathrm{m\,s^{-1}}$, giving $H=13.2\,\mathrm{m}$. **A** omits the factor 2; **B** uses the horizontal component ($\cos$); **D** uses the full launch speed instead of its vertical component.

Question 9 — A. Complementary launch angles (30° and 60°) give equal horizontal ranges. **B** is the "steeper travels further" misconception; **C** is false (the 60° projectile reaches the greater height); **D** is false (the 60° projectile has the longer flight time).

Question 10 — D. $u_x = 20\cos 50^{\circ} = 12.9\,\text{m s}^{-1}$, $u_y = 20\sin 50^{\circ} = 15.3\,\text{m s}^{-1}$; after $1.0\,\text{s}$, $v_y = 15.3 - 9.81 = 5.51\,\text{m s}^{-1}$, so $v = \sqrt{12.9^2 + 5.51^2} = 14.0\,\text{m s}^{-1}$. **A** assumes the speed is unchanged; **B** keeps only the horizontal component; **C** takes $v_y = g\,t$ (as if dropped from rest) instead of $u_y - g\,t$.

Question 11 — B. $W = F\,s = 15 \times 6.0 = 90\,\text{J} = 1/2\,m\,v^2$, so $v = \sqrt{\dfrac{2 \times 90}{3.0}} = 7.75\,\text{m s}^{-1}$. **A** drops the factor 2; **C** omits the square root; **D** inserts a spurious extra $1/2$.

Question 12 — D. At constant speed the kinetic energy does not change, so all the gravitational potential energy lost is transferred by friction to heat and sound. **A** ignores the constant speed (there is no gain in kinetic energy); **B** reverses the transformation; **C** invents elastic potential energy in the slide.

Question 13 — C. The work–energy theorem gives $W_{\text{net}} = \Delta E_k$; braking lowers the kinetic energy, so the net work is negative and equals that (negative) change. **A** has the wrong sign; **B** ignores the work done by friction along the road; **D** confuses ΔE_k with the initial kinetic energy.

Question 14 — A. $F = \dfrac{\Delta p}{\Delta t} = \dfrac{1200 \times 15}{0.12} = 1.5 \times 10^5\,\text{N}$. **B** stops at the impulse ($1.8 \times 10^4\,\text{N s}$); **C** multiplies by Δt instead of dividing; **D** uses $1/2\,m\,v^2$ (energy) in place of momentum.

Question 15 — C. Conservation of momentum: $0.30 \times 5.0 + 0.20 \times 1.0 = 0.30 \times 2.0 + 0.20 \times v$, so $v = \dfrac{1.7 - 0.6}{0.20} = 5.5\,\text{m s}^{-1}$. **A** drops the $0.30\,\text{kg}$ trolley's remaining momentum; **B** drops the $0.20\,\text{kg}$ trolley's initial momentum; **D** uses only the $0.30\,\text{kg}$ trolley's final momentum.

Question 16 — B. The area under a force–*time* graph is the impulse ($= \Delta p$); the area under a force–*displacement* graph is the work done ($= \Delta E_k$). **A** and **D** collapse the two; **C** swaps them — the classic impulse-versus-work trap.

Question 17 — D. In an isolated collision the total momentum is always conserved; kinetic energy is conserved only when the collision is elastic. **A** holds only for elastic collisions; **B** has it backwards (momentum is the conserved quantity); **C** is false — momentum is still conserved when the carts stick together.

Question 18 — A. $E_s = 1/2\,k\,x^2 = 1/2 \times 320 \times 0.10^2 = 1.6\,\text{J}$. **B** omits the $1/2$; **C** fails to square x; **D** quotes $k\,x$ (the force at that compression), not the energy.

Question 19 — C. $1/2\,k\,x^2 = 1/2\,m\,v^2$, so $v = x\sqrt{\dfrac{k}{m}} = 0.10\sqrt{\dfrac{160}{0.20}} = 2.83\,\text{m s}^{-1}$. **A** does not square x; **B** carries an extra $1/2$; **D** omits the $1/2$ in the spring energy.

Question 20 — B. On a force–extension graph the stored energy is the area under the straight line, $1/2\,k\,x^2$. **A** gives the gradient ($= k$), not the area; **C** is the full rectangle $k\,x^2$ (twice the true area); **D** refers to the wrong graph (a force–time area is impulse).

Topic 1 Test B — Motion and energy (written responses)

Time allowed: 50 minutes · 40 marks · One scientific calculator permitted · The formula sheet (back of the Practice Examinations booklet) may be used. In questions where more than one mark is available, appropriate working must be shown.

Answer all questions in the spaces provided. Where a final answer has a required unit, give your answer in that unit. Unless otherwise indicated, the diagrams in this test are not drawn to scale. Take the gravitational field strength to be $g = 9.81\,\mathrm{N\ kg^{-1}}$.

Question 1 (5 marks)

A $1500\,\mathrm{kg}$ car tows a $500\,\mathrm{kg}$ caravan along a straight, level road. The car and caravan accelerate forward together at $0.80\,\mathrm{m\ s^{-2}}$. The total resistance force (friction and air resistance) is $900\,\mathrm{N}$ on the car and $300\,\mathrm{N}$ on the caravan.

a. Show that the total driving force on the car is $2.80 \times 10^{3}\,\mathrm{N}$. 1 mark

b. Calculate the tension in the towbar connecting the car and the caravan. Give your answer in N. 2 marks

c. The driver then brakes so that the car and caravan decelerate. Describe how the force in the towbar differs from your answer to part **b.**, and explain why, with reference to Newton's laws of motion. 2 marks

Question 2 (5 marks)

A $0.40\,\mathrm{kg}$ puck on a light string moves in a horizontal circle of radius $0.75\,\mathrm{m}$ on a frictionless table, completing one revolution every $1.2\,\mathrm{s}$.

a. Show that the speed of the puck is $3.93\,\mathrm{m\ s^{-1}}$. 1 mark

b. Calculate the tension in the string. Give your answer in N. 2 marks

c. The string will snap if its tension exceeds $20\,\mathrm{N}$. Determine the maximum speed at which the puck can move in this circle. Give your answer in $\mathrm{m\ s^{-1}}$. 2 marks

Question 3 (6 marks)

A ball is kicked from ground level on a flat field at $22\,\mathrm{m\ s^{-1}}$, at 30° above the horizontal. Ignore air resistance except in part **d.**

a. Show that the vertical component of the launch velocity is $11.0\,\mathrm{m\ s^{-1}}$. 1 mark

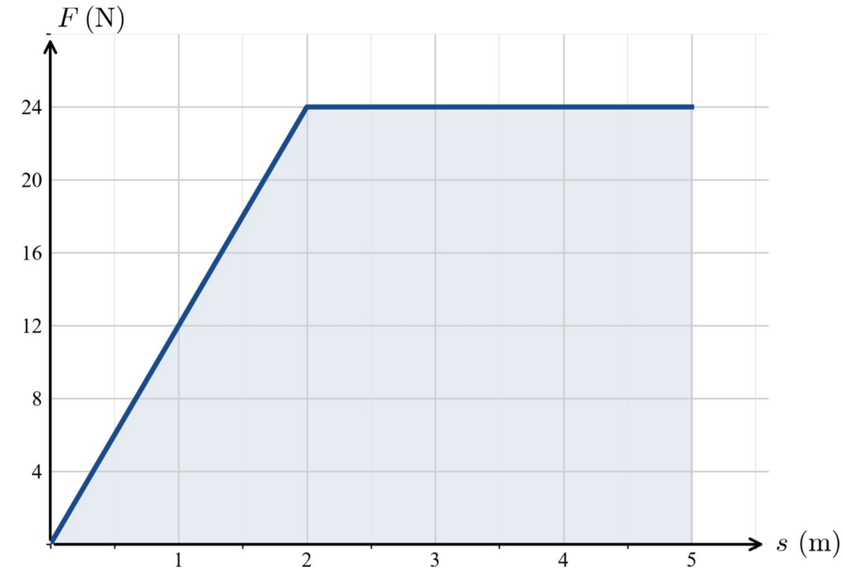
b. Calculate the time the ball is in the air before it returns to ground level. Give your answer in s. 2 marks

c. Calculate the horizontal range of the ball. Give your answer in m. 2 marks

d. (Circle your answer.) If air resistance were significant, the ball's horizontal range would be: *greater* / *smaller* / *the same*. Justify your answer. No calculations are required. 1 mark

Question 4 (5 marks)

A $2.5\,\mathrm{kg}$ trolley starts from rest on a rough horizontal track. The horizontal force applied to it varies with distance as shown: the force increases uniformly from 0 to $24\,\mathrm{N}$ over the first $2.0\,\mathrm{m}$, then stays constant at $24\,\mathrm{N}$ up to $5.0\,\mathrm{m}$.



a. Show that the work done by the applied force over the $5.0\,\mathrm{m}$ is $96\,\mathrm{J}$. 1 mark

b. A constant friction force of $6.0\,\mathrm{N}$ also acts on the trolley over the $5.0\,\mathrm{m}$. Calculate the trolley's kinetic energy at the $5.0\,\mathrm{m}$ mark. Give your answer in J. 2 marks

c. Calculate the trolley's speed at the $5.0\,\mathrm{m}$ mark. Give your answer in $\mathrm{m\,s^{-1}}$. 1 mark

d. State what has become of the energy removed from the trolley by friction, in terms of energy transformation. 1 mark

Question 5 (6 marks)

A $1100\,\mathrm{kg}$ car (X) moving east at $16\,\mathrm{m\,s^{-1}}$ collides with a stationary $1400\,\mathrm{kg}$ car (Y). The two cars lock together on impact.

a. Show that the common velocity of the wreckage immediately after the collision is $7.04\,\mathrm{m\,s^{-1}}$ east. 1 mark

b. By calculating the total kinetic energy before and after the collision, determine whether the collision is elastic or inelastic. 2 marks

c. The cars are in contact for $0.15\,\mathrm{s}$. Calculate the magnitude of the average force exerted on car Y by car X during the collision. Give your answer in N. 2 marks

d. Explain, in terms of impulse, why fitting car X with a crumple zone that increases the contact time would reduce the force on its occupants. 1 mark

Question 6 (5 marks)

A spring of stiffness $800\,\mathrm{N\,m^{-1}}$ that obeys Hooke's law is compressed by $0.090\,\mathrm{m}$. A $0.30\,\mathrm{kg}$ ball is placed against the compressed spring and then released so that it is launched vertically upward.

a. Show that the elastic potential energy stored in the compressed spring is $3.24\,\mathrm{J}$. 1 mark

b. Assuming no energy is dissipated, calculate the maximum height the ball rises above its launch point. Give your answer in m. 2 marks

c. State two assumptions of the ideal-spring model that you relied on in part **b.** 2 marks

Question 7 (3 marks)

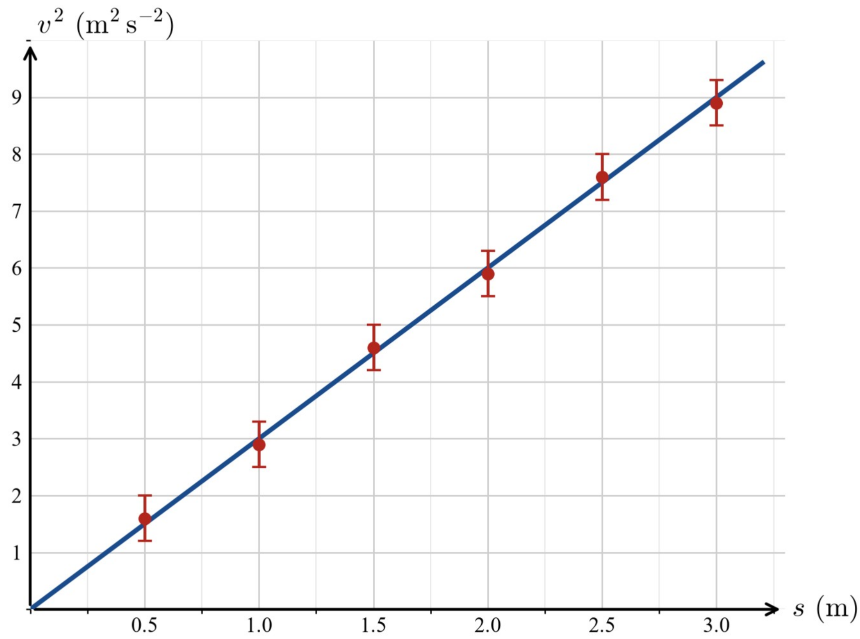
Two trucks, P and Q, have the same mass. Truck Q is moving at twice the speed of truck P.

a. Compare the kinetic energies of the two trucks. 1 mark

b. Each truck is brought to rest by the same constant braking force. Compare the distances the two trucks travel while stopping, explaining your reasoning in terms of work and energy. 2 marks

Question 8 (5 marks)

A student releases a $0.80\,\mathrm{kg}$ dynamics trolley from rest on a straight track and uses a motion sensor to measure its speed v after it has travelled various distances s. She plots v^2 against s, with uncertainty bars, and draws a line of best fit through the origin.



a. Identify the independent variable and the dependent variable in this investigation. 1 mark

b. Using two well-separated points on the line of best fit, determine the gradient of the line. Give your answer in $\mathrm{m\,s^{-2}}$. 2 marks

c. The motion obeys $v^2 = u^2 + 2as$ with $u = 0$. Use the gradient to determine (i) the trolley's acceleration and (ii) the magnitude of the net force on the trolley. Give the force in N. 2 marks

End of Topic 1 Test B

Topic 1 Test B — solutions

Question 1 Total mass $=1500+500=2000\,\mathrm{kg}$; $a=0.80\,\mathrm{m\ s^{-2}}$; total resistance $=900+300=1200\,\mathrm{N}$.

(a) For the whole system, $\Sigma F=ma=2000\times0.80=1600\,\mathrm{N}$. This net force is the driving force minus the total resistance, so

$$F_{\text{drive}}=ma+F_{\text{resist}}=1600+1200=2.80\times10^{3}\,\mathrm{N},$$

as required. **(1 mark: system $\Sigma F=ma$ balanced against the total resistance to give the driving force.)**

(b) Isolating the caravan (mass $500\,\mathrm{kg}$): the forward force on it is the towbar tension F_T, opposed by its $300\,\mathrm{N}$ resistance, and its net force is $500\times0.80=400\,\mathrm{N}$. So

$$F_T-300=500\times0.80=400\Rightarrow F_T=700\,\mathrm{N}.$$

∴ the towbar tension is $700\,\mathrm{N}$. (**1 mark** isolates the caravan with $\Sigma F=ma$; **1 mark** correct value $700\,\mathrm{N}$.)

(c) In part **b.** the towbar is in **tension** — it pulls the caravan **forward**. During braking the car decelerates; by inertia (Newton's first law) the caravan tends to keep moving forward, so the towbar force **reverses**: the towbar is now in **compression** and pushes **backward** on the caravan. By Newton's second law this backward force provides the caravan's deceleration; by Newton's third law the caravan pushes forward on the car with an equal and opposite force. (**1 mark** the towbar force reverses direction / becomes a backward (compression) force on the caravan; **1 mark** justified via inertia (N1) and $\Sigma F=ma$ (N2), i.e. a net backward force is now needed to decelerate the caravan.)

Question 2 $m=0.40\,\mathrm{kg}$, $r=0.75\,\mathrm{m}$, $T=1.2\,\mathrm{s}$.

(a) $v=\dfrac{2\pi r}{T}=\dfrac{2\pi(0.75)}{1.2}=3.93\,\mathrm{m\ s^{-1}}$, as required. **(1 mark: correct substitution and result.)**

(b) The string tension provides the net (centripetal) force:

$$F_T=\frac{mv^2}{r}=\frac{0.40\times3.927^2}{0.75}=\frac{6.168}{0.75}=8.22\,\mathrm{N}.$$

(**1 mark** $F_T=\dfrac{mv^2}{r}$; **1 mark** correct value $8.22\,\mathrm{N}$.)

(c) At the maximum tension the string provides the maximum centripetal force, so

$$v_{max}=\sqrt{\frac{F_{max}r}{m}}=\sqrt{\frac{20\times0.75}{0.40}}=\sqrt{37.5}=6.12\,\mathrm{m\ s^{-1}}.$$

∴ the puck can move at up to $6.12\,\mathrm{m\ s^{-1}}$ before the string snaps. (**1 mark** rearranges $F_{max}=\dfrac{mv_{max}^2}{r}$; **1 mark** correct value $6.12\,\mathrm{m\ s^{-1}}$.)

Question 3 $u=22\,\mathrm{m\ s^{-1}}$, $\theta=30^\circ$.

(a) $u_y=u\sin\theta=22\sin30^\circ=22\times0.5=11.0\,\mathrm{m\ s^{-1}}$, as required. **(1 mark: correct vertical component.)**

(b) The ball returns to its launch level, so (taking up as positive, and using the symmetry of the flight)

$$t=\frac{2u_y}{g}=\frac{2\times11.0}{9.81}=2.24\,\mathrm{s}.$$

(**1 mark** $t=\dfrac{2u_y}{g}$ (or equivalent); **1 mark** correct value $2.24\,\mathrm{s}$.)

(c) The horizontal component is $u_x = 22\cos 30^\circ = 19.05\,\mathrm{m\ s^{-1}}$, and it is constant, so

$$R = u_x t = 19.05 \times 2.243 = 42.7\,\mathrm{m}.$$

(1 mark $R = u_x t$ with the correct u_x; **1 mark** correct value $42.7\,\mathrm{m}$.)

(d) **Smaller.** Air resistance acts opposite to the ball's velocity throughout the flight, continually reducing both the horizontal and vertical components of the velocity. The ball therefore travels a shorter horizontal distance before landing (the flight is also cut short), so the range is less than the ideal value. **(1 mark: circles *smaller* AND justifies via air resistance opposing the motion / reducing the horizontal velocity. The circled option alone scores 0.)**

Question 4 $m = 2.5\,\mathrm{kg}$, from rest.

(a) Work = area under the force–distance graph. Triangle ($0 \to 2.0\,\mathrm{m}$): $1/2 \times 2.0 \times 24 = 24\,\mathrm{J}$. Rectangle ($2.0 \to 5.0\,\mathrm{m}$): $24 \times 3.0 = 72\,\mathrm{J}$. Total $W = 24 + 72 = 96\,\mathrm{J}$, as required. **(1 mark: correct area giving** $96\,\mathrm{J}$**.)**

(b) Friction does negative work $W_f = -F_f s = -6.0 \times 5.0 = -30\,\mathrm{J}$. The kinetic energy is the net work done from rest:

$$E_k = W - F_f s = 96 - 30 = 66\,\mathrm{J}.$$

(1 mark subtracts the friction work $30\,\mathrm{J}$; **1 mark** correct value $66\,\mathrm{J}$.)

(c) $E_k = 1/2\, m v^2 \Rightarrow v = \sqrt{\dfrac{2E_k}{m}} = \sqrt{\dfrac{2 \times 66}{2.5}} = \sqrt{52.8} = 7.27\,\mathrm{m\ s^{-1}}$. **(1 mark: correct value** $7.27\,\mathrm{m\ s^{-1}}$**.)**

(d) The $30\,\mathrm{J}$ removed by friction is transferred to the surroundings, mostly as **heat** in the track and the trolley's wheels, with a little **sound**; total energy is conserved. **(1 mark: dissipated as heat (and sound) to the surroundings / total energy conserved.)**

Question 5 $m_X = 1100\,\mathrm{kg}$, $u_X = 16\,\mathrm{m\ s^{-1}}$ east, $m_Y = 1400\,\mathrm{kg}$, $u_Y = 0$; the cars lock together.

(a) Conservation of momentum: $m_X u_X = (m_X + m_Y)v$, so

$$v = \frac{1100 \times 16}{1100 + 1400} = \frac{17600}{2500} = 7.04\,\mathrm{m\ s^{-1}}\ \text{east},$$

as required. **(1 mark: momentum conservation giving** $7.04\,\mathrm{m\ s^{-1}}$**.)**

(b) $E_k(\text{before}) = 1/2\, m_X u_X^2 = 1/2 \times 1100 \times 16^2 = 1.41 \times 10^5\,\mathrm{J}$.
$E_k(\text{after}) = 1/2(m_X + m_Y)v^2 = 1/2 \times 2500 \times 7.04^2 = 6.20 \times 10^4\,\mathrm{J}$. The kinetic energy falls from $1.41 \times 10^5\,\mathrm{J}$ to $6.20 \times 10^4\,\mathrm{J}$, so the collision is **inelastic**. (Momentum is still conserved.) **(1 mark** both kinetic energies correct; **1 mark** concludes inelastic because E_k decreases.)

(c) The impulse on car Y equals its change in momentum: $\Delta p_Y = m_Y v = 1400 \times 7.04 = 9856\,\mathrm{N\ s}$. The average force is

$$F = \frac{\Delta p_Y}{\Delta t} = \frac{9856}{0.15} = 6.57 \times 10^4\,\mathrm{N}.$$

∴ car X exerts an average force of $6.57 \times 10^4\,\mathrm{N}$ on car Y. **(1 mark** $F = \dfrac{\Delta p_Y}{\Delta t}$ with $\Delta p_Y = m_Y v$; **1 mark** correct value $6.57 \times 10^4\,\mathrm{N}$.)

(d) The occupants undergo a fixed change in momentum Δp (from the collision speed to rest). Their impulse $F\Delta t = \Delta p$ is therefore fixed, so **increasing the contact time** Δt **decreases the average force** F ($F = \Delta p / \Delta t$) — the crumple zone spreads the same momentum change over a longer time, reducing the force on the occupants. **(1 mark: same** Δp**, so a longer** Δt **gives a smaller** F **via** $F\Delta t = \Delta p$**.)**

Question 6 $k = 800\,\mathrm{N\,m^{-1}}$, $x = 0.090\,\mathrm{m}$, $m = 0.30\,\mathrm{kg}$.

(a) $E_s = 1/2\,k\,x^2 = 1/2 \times 800 \times 0.090^2 = 1/2 \times 800 \times 0.0081 = 3.24\,\mathrm{J}$, as required. **(1 mark: correct substitution and result.)**

(b) With no energy dissipated, the stored elastic potential energy becomes gravitational potential energy at the top: $E_s = m\,g\,\Delta h$, so

$$\Delta h = \frac{E_s}{m\,g} = \frac{3.24}{0.30 \times 9.81} = \frac{3.24}{2.943} = 1.10\,\mathrm{m}.$$

∴ the ball rises $1.10\,\mathrm{m}$ above its launch point. **(1 mark** $E_s = m\,g\,\Delta h$; **1 mark** correct value $1.10\,\mathrm{m}$.)

(c) Any **two** of:

- the spring is **massless**, so all the stored energy is transferred to the ball (none remains as kinetic energy of the spring);
- the spring **obeys Hooke's law**, so the stored energy is $E_s = 1/2\,k\,x^2$;
- **no energy is dissipated** (the spring is perfectly elastic — no heat or sound).

(1 mark each for any two valid ideal-spring assumptions.)

Question 7 Let truck P have mass m and speed v; truck Q has mass m and speed $2\,v$.

(a) $E_k \propto v^2$, so $\dfrac{E_{k,Q}}{E_{k,P}} = \dfrac{1/2\,m(2\,v)^2}{1/2\,m\,v^2} = 4$. Truck Q has **four times** the kinetic energy of truck P. **(1 mark: Q has 4 × the kinetic energy, via $E_k \propto v^2$.)**

(b) The braking force F does work $F\,d$ to remove all the kinetic energy, so $F\,d = E_k$ and the stopping distance is $d = \dfrac{E_k}{F}$. With the same force F, the stopping distance is proportional to the kinetic energy. Truck Q has $4\,\times$ the kinetic energy, so it travels **four times** the stopping distance of truck P. **(1 mark** $F\,d = E_k \Rightarrow d \propto E_k$ for a fixed force; **1 mark** so $d_Q = 4\,d_P$.)

Question 8 $m = 0.80\,\mathrm{kg}$; the trolley starts from rest, so $u = 0$ and $v^2 = 2\,a\,s$.

(a) The **independent variable** is the distance travelled s (deliberately varied); the **dependent variable** is the trolley's speed v (measured by the sensor), from which v^2 is calculated and plotted. **(1 mark: both variables correctly identified.)**

(b) Using two well-separated points **on the line of best fit** — for example $(1.0\,\mathrm{m}, 3.0\,\mathrm{m^2\,s^{-2}})$ and $(3.0\,\mathrm{m}, 9.0\,\mathrm{m^2\,s^{-2}})$:

$$\text{gradient} = \frac{\Delta(v^2)}{\Delta s} = \frac{9.0 - 3.0}{3.0 - 1.0} = \frac{6.0}{2.0} = 3.0\,\mathrm{m\,s^{-2}}.$$

(1 mark two well-separated on-line points used (not raw data points); **1 mark** correct gradient $3.0\,\mathrm{m\,s^{-2}}$.)

(c) Comparing $v^2 = 2\,a\,s$ with a straight line through the origin, the gradient equals $2\,a$. Hence

$$a = \frac{\text{gradient}}{2} = \frac{3.0}{2} = 1.5\,\mathrm{m\,s^{-2}},\ F = m\,a = 0.80 \times 1.5 = 1.2\,\mathrm{N}.$$

∴ the acceleration is $1.5\,\mathrm{m\,s^{-2}}$ and the net force on the trolley is $1.2\,\mathrm{N}$. **(1 mark** gradient $= 2\,a \Rightarrow a = 1.5\,\mathrm{m\,s^{-2}}$; **1 mark** $F = m\,a = 1.2\,\mathrm{N}$.)