Physics Units 3 & 4

Chapter 1 — Newton's laws of motion

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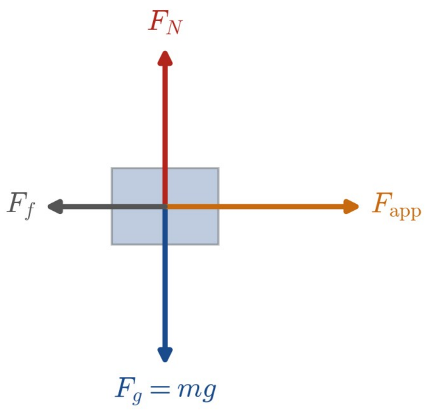
Theory

A **force** is a push or a pull exerted on one object by another. It is a vector: it has a magnitude, measured in **newtons** (N), and a direction. Every force acts *on* an object and is exerted *by* some other object, so a force is always named for the object it acts on and the object that provides it. When several forces act on an object, their vector sum is the **net force** (or resultant), F_{net}. A **free-body diagram** shows every force acting on one chosen object, drawn as an arrow from the object in the direction the force acts.



free-body diagram: every force acts ON the block

Newton's first law. An object remains at rest, or continues to move with constant velocity in a straight line, unless a non-zero net force acts on it. The tendency of an object to resist a change in its motion is its **inertia**, and mass is the measure of inertia. When the net force is zero the object is in **equilibrium**: it is either stationary or moving at constant velocity. A crucial consequence is that a moving object needs *no* force to keep moving at constant velocity — a force is required only to *change* an object's velocity (to speed it up, slow it down, or change its direction).

The force due to gravity and the normal force. Near the Earth's surface every object of mass m experiences a **force due to gravity** $F_g = mg$ directed vertically downward, where $g = 9.81\,\text{m s}^{-2}$ (equivalently $9.81\,\text{N kg}^{-1}$). A surface in contact with the object pushes on it with the **normal force** F_N, directed perpendicular to the surface, on the object by the surface. On a horizontal surface with no vertical acceleration the first law gives $F_N = mg$; but this equality is a consequence of balance, not a general rule — on an incline, or in a lift that is accelerating, $F_N \neq mg$.

Newton's second law. A non-zero net force gives an object an acceleration in the same direction as the net force, with magnitude

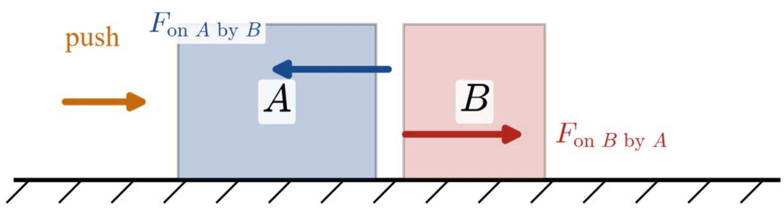
$$F_{\text{net}} = ma.$$

A net force of $1\,\text{N}$ gives a mass of $1\,\text{kg}$ an acceleration of $1\,\text{m s}^{-2}$. When the forces all act along one straight line, choose a positive direction and add them with signs; the sign of F_{net} then gives the direction of the acceleration.

Newton's third law. If object A exerts a force on object B, then B exerts an equal and opposite force on A:

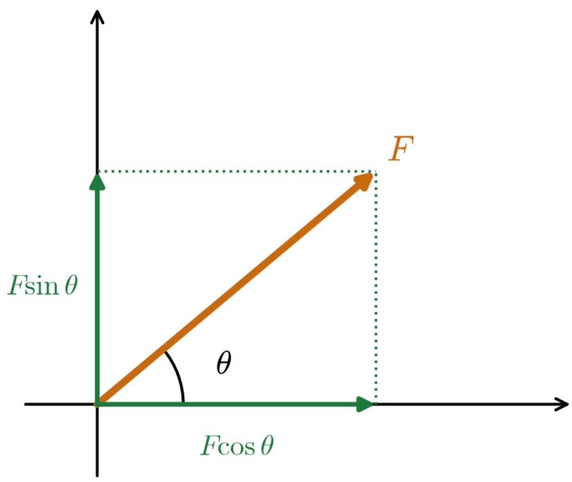
$$F_{\text{on B by A}} = -F_{\text{on A by B}}.$$

The two forces of such a pair are equal in magnitude, opposite in direction, of the *same type*, and — most importantly — they act on **different** objects, so they can never cancel each other. For a book resting on a table, the force due to gravity on the book by the Earth pairs with the gravitational force on the Earth by the book; the normal force on the book by the table pairs with the force on the table by the book. The force due to gravity F_g and the normal force F_N both act *on the book* and are therefore **not** a third-law pair — they are equal here only because the book is in equilibrium.



the contact pair is equal and opposite and acts on DIFFERENT bodies

Resolving forces in two dimensions. A single force can be replaced by two perpendicular **components** that together have the same effect. A force F making an angle θ with a chosen axis has a component $F\cos\theta$ along that axis and $F\sin\theta$ perpendicular to it.

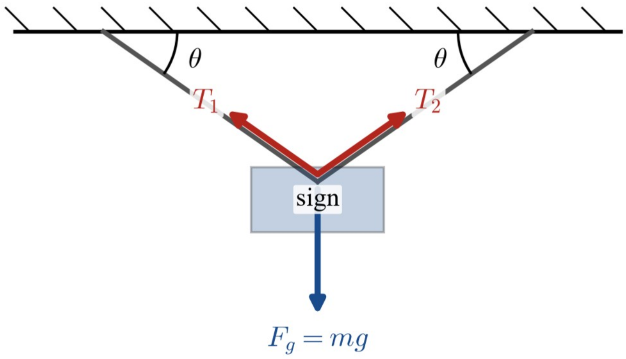


Once every force is resolved onto two perpendicular axes, the second law is applied to each axis separately: $\Sigma F_x = m a_x$ and $\Sigma F_y = m a_y$.

Equilibrium in two dimensions. A body acted on by several forces is in equilibrium when the components of the forces sum to zero in each of two perpendicular directions:

$$\Sigma F_x = 0, \Sigma F_y = 0.$$

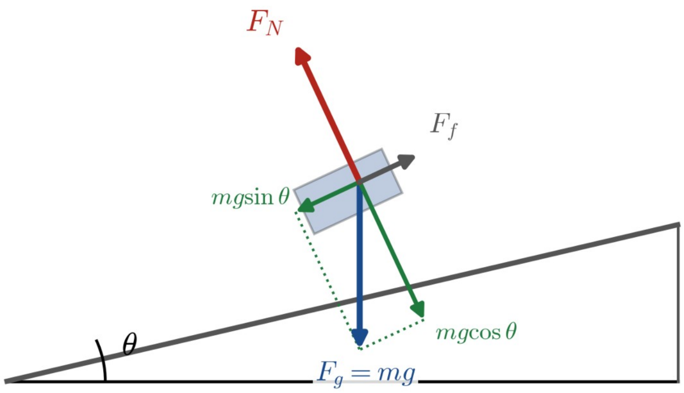
For example, a sign of mass m hung from two cables that each make an angle θ with the horizontal is held by two tensions and the force due to gravity. Resolving vertically, $2T\sin\theta = mg$, so $T = \frac{mg}{2\sin\theta}$; the horizontal components cancel. Notice that as θ becomes small the tension becomes very large — a nearly horizontal cable must pull very hard to support a modest load.



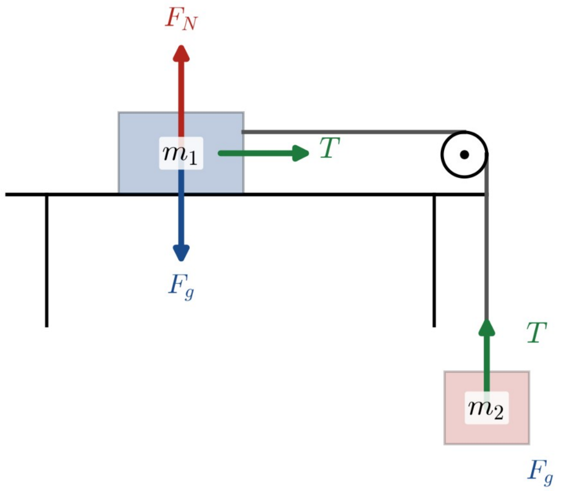
Inclined planes. For an object on a plane inclined at angle θ to the horizontal, it is convenient to resolve along and perpendicular to the slope. The force due to gravity has a component $m\,g\sin\theta$ directed *down the slope* and a component $m\,g\cos\theta$ directed *into the surface*. The normal force balances the perpendicular component, so

$$F_N = m\,g\cos\theta\,,$$

which is *less* than $m\,g$. Along the slope the second law applies: if the object moves at constant velocity the along-slope forces balance (equilibrium), while if it accelerates, F_{net} along the slope equals $m\,a$. On a frictionless incline the only along-slope force is $m\,g\sin\theta$, giving $a = g\sin\theta$ down the slope. A friction force F_f, when present, acts along the surface, opposing the motion or the tendency to move.



Connected bodies. Two objects joined by a light inextensible string, or in contact, or linked by a rigid rod, are constrained to move together and so share the same magnitude of acceleration. In an ideal (massless) string running over a frictionless pulley, the tension is the same at every point. A reliable method has two steps: **(1)** treat the whole system as a single body and find the common acceleration from the net *external* force; **(2)** then isolate one object and apply the second law to it to find the *internal* force (the string tension, or the contact force between touching bodies).



The tension in a string supporting a hanging mass equals mg only when the system is in equilibrium. When the system accelerates, the tension is greater than mg (if the mass accelerates upward) or less than mg (if it accelerates downward).

Throughout this subtopic the constant $g = 9.81\ \mathrm{m\ s^{-2}}$ (equivalently $9.81\ \mathrm{N\ kg^{-1}}$) is used.

Worked examples

Example 1 — scaffolded

A crate of mass $25\ \mathrm{kg}$ is pulled along a rough horizontal floor by a horizontal force of $90\ \mathrm{N}$. A constant friction force of $40\ \mathrm{N}$ opposes the motion. Calculate the normal force on the crate by the floor, the net force on the crate, and its acceleration.

Working	Comment
$m = 25\ \mathrm{kg}$, applied force $90\ \mathrm{N}$ horizontal, friction $F_f = 40\ \mathrm{N}$.	List the data; the motion and the net force are horizontal.
Vertical: no vertical acceleration, so $F_N = F_g = mg = 25 \times 9.81 = 245\ \mathrm{N}$.	The floor is horizontal, so the normal force balances the force due to gravity.
Horizontal: $F_{\text{net}} = 90 - 40 = 50\ \mathrm{N}$, in the direction of the pull.	Add the horizontal forces with signs (pull positive, friction negative).
$a = \dfrac{F_{\text{net}}}{m} = \dfrac{50}{25} = 2.0\ \mathrm{m\ s^{-2}}$, in the direction of the pull.	Newton's second law, $F_{\text{net}} = ma$.

Example 2 — scaffolded

A crate of mass $15\ \mathrm{kg}$ is pulled *up* a rough ramp inclined at 20° to the horizontal at **constant velocity** by a force F directed up the slope, parallel to the ramp. The friction force on the crate is $30\ \mathrm{N}$. Calculate the component of the force due to gravity along the ramp, the normal force on the crate, and the applied force F.

Working	Comment
Constant velocity $\Rightarrow$ equilibrium. Forces: F_g (down), F_N (perpendicular), F (up slope), friction $30\ \mathrm{N}$ (down slope, opposing motion).	The crate moves up, so friction acts down the slope.
$F_g = mg = 15 \times 9.81 = 147\ \mathrm{N}$.	Force due to gravity, vertically down.
Along the slope: $mg\sin\theta = 147 \times \sin 20^\circ = 50.3\ \mathrm{N}$ (down the slope).	Resolve F_g parallel to the ramp.
Perpendicular: $F_N = mg\cos\theta = 147 \times \cos 20^\circ = 138\ \mathrm{N}$.	The normal force balances the perpendicular component

Working	Comment
	of F_g.
Along the slope (equilibrium): $F = m\,g\sin\theta + F_f = 50.3 + 30 = 80.3\,\mathrm{N}$.	At constant velocity the up-slope and down-slope forces balance.

Example 3 — unscaffolded

A block of mass $3.0\,\mathrm{kg}$ rests on a smooth horizontal table and is connected by a light inextensible string, passing over a frictionless pulley at the edge of the table, to a mass of $2.0\,\mathrm{kg}$ that hangs vertically. Calculate the acceleration of the system and the tension in the string.

The two masses are connected, so they share the same acceleration a, and the string tension T is the same throughout. Treating the whole system as one body, the only external force driving the motion is the force due to gravity on the hanging mass, $F_g = m_2 g = 2.0 \times 9.81 = 19.62\,\mathrm{N}$, and the total mass moved is $m_1 + m_2 = 5.0\,\mathrm{kg}$:

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{19.62}{5.0} = 3.924\,\mathrm{m\ s^{-2}}.$$

To find the tension, isolate the block on the table. The only horizontal force acting on it is the tension, so

$$T = m_1 a = 3.0 \times 3.924 = 11.77\,\mathrm{N}.$$

(Check, using the hanging mass: $m_2 g - T = 19.62 - 11.77 = 7.85\,\mathrm{N} = 2.0 \times 3.924$. ✓)

∴ the acceleration is $3.92\,\mathrm{m\ s^{-2}}$ and the tension is $11.8\,\mathrm{N}$.

Example 4 — unscaffolded

A sign of mass $5.0\,\mathrm{kg}$ hangs in equilibrium from two cables attached to a horizontal ceiling. Each cable makes an angle of 35° with the ceiling. By resolving forces, calculate the tension in each cable.

Three forces act on the sign: the two cable tensions and the force due to gravity $F_g = m\,g = 5.0 \times 9.81 = 49.05\,\mathrm{N}$ downward. By the symmetry of the arrangement the two tensions are equal, each of magnitude T. Resolving vertically for equilibrium, the upward components of the two tensions support the force due to gravity:

$$2T\sin 35^\circ = m\,g \Rightarrow T = \frac{m\,g}{2\sin 35^\circ} = \frac{49.05}{2 \times 0.5736} = 42.8\,\mathrm{N}.$$

(The horizontal components, $T\cos 35^\circ$ on each side, are equal and opposite, so they cancel.)

∴ the tension in each cable is $42.8\,\mathrm{N}$.

Practice questions

Question 1 A crate of mass $30\,\mathrm{kg}$ is pulled along a rough horizontal floor by a horizontal force of $120\,\mathrm{N}$. A constant friction force of $45\,\mathrm{N}$ opposes the motion. (a) Calculate the normal force on the crate by the floor. (b) Calculate the net horizontal force on the crate. (c) Calculate the acceleration of the crate.

Question 2 A block of mass $6.0\,\mathrm{kg}$ is released from rest on a smooth (frictionless) ramp inclined at 25° to the horizontal. (a) Show that the component of the force due to gravity acting down the ramp is $24.9\,\mathrm{N}$. (b) Calculate the normal force on the block by the ramp. (c) Calculate the acceleration of the block down the ramp.

Question 3 Two blocks, A of mass $4.0\,\mathrm{kg}$ and B of mass $6.0\,\mathrm{kg}$, sit in contact with each other on a smooth horizontal floor. A horizontal force of $30\,\mathrm{N}$ is applied to block A, pushing both blocks along the floor. (a) Calculate the acceleration of the two blocks. (b) Calculate the net force on block B. (c) Hence state the magnitude of the contact force on B by A, and explain your reasoning.

Question 4 A ball of mass $2.0\,\mathrm{kg}$ hangs from a light string and is held aside by a horizontal force until the string makes an angle of 20° with the vertical. (a) Draw a free-body diagram of the ball, naming each force. (b) Calculate the tension in the string. (c) Calculate the horizontal force holding the ball aside.

Question 5 A book of mass $1.5\,\mathrm{kg}$ rests in equilibrium on a horizontal table. (a) Name the two forces acting on the book and state the magnitude of each. (b) For the force due to gravity on the book, state its Newton's third-law partner: name the force, the object it acts on, and its magnitude. (c) Explain why the force due to gravity on the book and the normal force on the book are *not* a Newton's third-law pair.

Question 6 A block of mass $5.0\,\mathrm{kg}$ rests on a smooth horizontal table and is connected by a light string over a frictionless pulley to a mass of $3.0\,\mathrm{kg}$ hanging vertically. (a) Draw a diagram showing the forces acting on each mass. (b) Calculate the acceleration of the system. (c) Calculate the tension in the string.

Question 7 A car of mass $1500\,\mathrm{kg}$ experiences a forward driving force of $4200\,\mathrm{N}$ and a total resistance force of $900\,\mathrm{N}$. Calculate the acceleration of the car.

Question 8 A crate of mass $12\,\mathrm{kg}$ slides *down* a rough incline angled at 30° to the horizontal. The friction force on the crate is $22\,\mathrm{N}$. Calculate the acceleration of the crate.

Question 9 A lamp of mass $8.0\,\mathrm{kg}$ hangs from two cables attached to a horizontal ceiling, each cable making an angle of 50° with the ceiling. Calculate the tension in each cable. Give your answer to three significant figures.

Question 10 A car tows a trailer of mass $400\,\mathrm{kg}$ by a rigid tow bar. The car and trailer accelerate together at $1.5\,\mathrm{m\,s^{-2}}$ along a straight, level road. Ignoring resistance forces on the trailer, calculate the force in the tow bar.

Question 11 A car travels along a straight, level highway at a constant velocity of $25\,\mathrm{m\,s^{-1}}$. With reference to Newton's first and second laws, state what can be said about the net force on the car, and account for the fact that the engine must still provide a forward driving force. No calculations are required.

Question 12 A swimmer propels herself forward through the water by pushing water backwards with her hands. With reference to Newton's third law, and using "force on … by …" language, explain how this pushes her forward.

Question 13 A block of mass $3.0\,\mathrm{kg}$ slides *down* a rough incline at 35° to the horizontal at **constant velocity**. (a) Show that the component of the force due to gravity acting down the incline is $16.9\,\mathrm{N}$. (b) Hence state the magnitude and direction of the friction force on the block, justifying your answer with reference to Newton's first law.

Question 14 A person pushes a lawnmower of mass $18\,\mathrm{kg}$ across level ground with a force of $150\,\mathrm{N}$ directed at 40° below the horizontal. (a) Calculate the horizontal and vertical components of the applied force. (b) Calculate the normal force on the mower by the ground. (c) Explain why the normal force is greater than the force due to gravity on the mower.

Question 15 Two masses, $4.0\,\mathrm{kg}$ and $6.0\,\mathrm{kg}$, are connected by a light inextensible string that passes over a frictionless pulley, and are released from rest. Calculate the acceleration of the masses and the tension in the string.

Question 16 A student of mass $60\,\mathrm{kg}$ stands at rest on the ground. The Earth exerts a force due to gravity on the student. (a) State the magnitude of this force. (b) State the Newton's third-law partner of this force: name the force, the object it acts on, and its direction. (c) Explain why the student and the Earth do not experience the same acceleration, even though the two forces of the pair are equal in magnitude.

Question 17 A traffic light of mass $12\,\mathrm{kg}$ is suspended from the midpoint of a cable strung between two poles. Each half of the cable makes an angle of 10° with the horizontal. Calculate the tension in the cable, and comment on its size compared with the force due to gravity on the traffic light.

Question 18 A crate of mass $20\,\mathrm{kg}$ rests on a ramp inclined at 18° to the horizontal. The maximum friction force the ramp surface can provide is $70\,\mathrm{N}$. Determine, with a calculation, whether the crate remains at rest on the ramp.

Question 19 A person of mass $70\,\mathrm{kg}$ stands on the floor of a lift. (a) Calculate the normal force on the person by the floor when the lift accelerates upward at $2.0\,\mathrm{m\,s^{-2}}$. (b) Calculate the normal force on the person by the floor when the lift accelerates downward at $2.0\,\mathrm{m\,s^{-2}}$.

Question 20 A block of mass $4.0\,\mathrm{kg}$ rests on a rough horizontal table and is connected by a light string over a frictionless pulley to a mass of $3.0\,\mathrm{kg}$ hanging vertically. As the system moves, the friction force on the block by the table is $6.0\,\mathrm{N}$. Calculate the acceleration of the system and the tension in the string.

Question 21 A car is travelling forward when the driver brakes sharply, and an unbelted passenger continues to move forward relative to the car. With reference to Newton's first law, explain why the passenger moves forward relative to the car, and identify the real force that a fastened seatbelt provides and its direction.

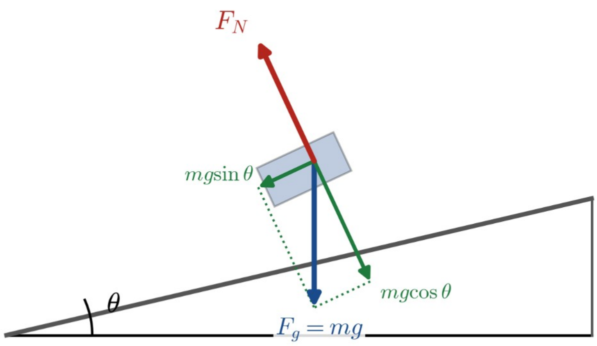
Answers

Question 1 (a) $294\,\mathrm{N}$; (b) $75\,\mathrm{N}$ in the direction of the pull; (c) $2.5\,\mathrm{m\,s^{-2}}$. **Question 2** (a) $24.9\,\mathrm{N}$; (b) $53.3\,\mathrm{N}$; (c) $4.15\,\mathrm{m\,s^{-2}}$ down the ramp. **Question 3** (a) $3.0\,\mathrm{m\,s^{-2}}$; (b) $18\,\mathrm{N}$; (c) $18\,\mathrm{N}$. **Question 4** (b) $20.9\,\mathrm{N}$; (c) $7.14\,\mathrm{N}$. **Question 5** (a) $F_g = 14.7\,\mathrm{N}$ down, $F_N = 14.7\,\mathrm{N}$ up; (b) the gravitational force on the Earth by the book, $14.7\,\mathrm{N}$, directed upward; (c) both forces act on the same object (the book). **Question 6** (b) $3.68\,\mathrm{m\,s^{-2}}$; (c) $18.4\,\mathrm{N}$. **Question 7** $2.2\,\mathrm{m\,s^{-2}}$. **Question 8** $3.07\,\mathrm{m\,s^{-2}}$ down the incline. **Question 9** $51.2\,\mathrm{N}$. **Question 10** $600\,\mathrm{N}$. **Question 11** The net force is zero (constant velocity); the driving force is balanced by the resistance forces. **Question 12** By Newton's third law the water pushes forward on the swimmer with a force equal and opposite to the backward force she exerts on the water. **Question 13** (a) $16.9\,\mathrm{N}$; (b) $16.9\,\mathrm{N}$ up the incline. **Question 14** (a) horizontal $115\,\mathrm{N}$, vertical $96.4\,\mathrm{N}$ (down); (b) $273\,\mathrm{N}$. **Question 15** $1.96\,\mathrm{m\,s^{-2}}$; $47.1\,\mathrm{N}$. **Question 16** (a) $589\,\mathrm{N}$; (b) the gravitational force on the Earth by the student, directed upward; (c) the Earth's mass is enormous, so its acceleration is negligible. **Question 17** $339\,\mathrm{N}$; far greater than the force due to gravity ($117.72\,\mathrm{N}$). **Question 18** Required along-slope force $60.6\,\mathrm{N} < 70\,\mathrm{N}$, so the crate remains at rest. **Question 19** (a) $827\,\mathrm{N}$; (b) $547\,\mathrm{N}$. **Question 20** $3.35\,\mathrm{m\,s^{-2}}$; $19.4\,\mathrm{N}$. **Question 21** The passenger continues forward by inertia (Newton's first law); the seatbelt provides a backward force on the passenger.

Fully-worked solutions

Question 1 $m = 30\,\mathrm{kg}$, applied force $120\,\mathrm{N}$, friction $F_f = 45\,\mathrm{N}$. (a) The floor is horizontal with no vertical acceleration, so $F_N = mg = 30 \times 9.81 = 294\,\mathrm{N}$. (b) $F_{\text{net}} = 120 - 45 = 75\,\mathrm{N}$, in the direction of the pull. (c) $a = \dfrac{F_{\text{net}}}{m} = \dfrac{75}{30} = 2.5\,\mathrm{m\,s^{-2}}$.

Question 2 $m = 6.0\,\mathrm{kg}$, $\theta = 25^\circ$, smooth ramp. $F_g = mg = 6.0 \times 9.81 = 58.86\,\mathrm{N}$.

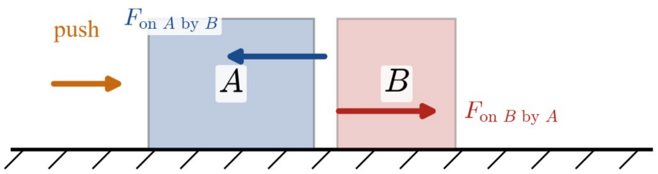


(a) The component of the force due to gravity along the ramp is $mg\sin\theta = 58.86 \times \sin 25^\circ = 24.9\,\mathrm{N}$, directed down the ramp, as required.

(b) Perpendicular to the ramp there is no acceleration, so $F_N = mg\cos\theta = 58.86 \times \cos 25^\circ = 53.3\,\mathrm{N}$.

(c) The ramp is smooth, so the only force along the slope is $mg\sin\theta$. By the second law, $a = \dfrac{mg\sin\theta}{m} = g\sin\theta = 9.81 \times \sin 25^\circ = 4.15\,\mathrm{m\,s^{-2}}$, directed down the ramp.

Question 3 $m_A = 4.0\,\mathrm{kg}$, $m_B = 6.0\,\mathrm{kg}$, applied force $30\,\mathrm{N}$, smooth floor.



the contact pair is equal and opposite and acts on DIFFERENT bodies

(a) Treating the two blocks as one body of mass 10 kg: $a = \dfrac{30}{4.0+6.0} = 3.0\,\text{m s}^{-2}$.

(b) Isolating block B, the net force on it is $F_{\text{net}} = m_B a = 6.0 \times 3.0 = 18\,\text{N}$.

(c) The only horizontal force on block B is the contact force pushing it forward, exerted on B by A. Since this is the net force on B, the contact force on B by A is 18 N. (By Newton's third law, block B pushes back on A with 18 N in the opposite direction.)

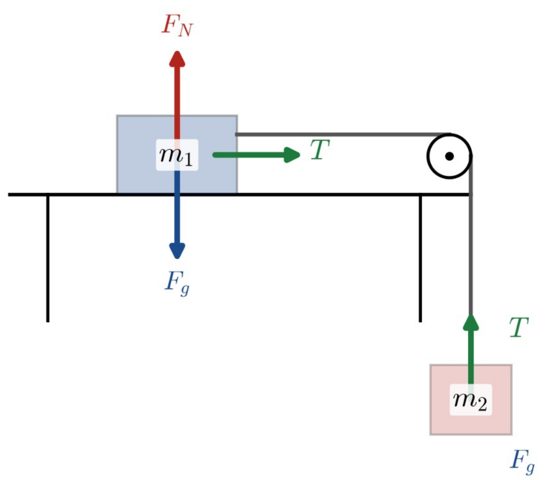
Question 4 $m = 2.0\,\text{kg}$, string 20° from the vertical. (a) Three forces act on the ball: the force due to gravity $F_g = mg$ vertically down, the tension F_T directed up along the string (that is, 20° from the vertical), and the applied horizontal force F pushing the ball aside. (b) The ball is in equilibrium. Resolving vertically, the vertical component of the tension supports the force due to gravity: $F_T \cos 20^\circ = mg$, so

$$F_T = \frac{mg}{\cos 20^\circ} = \frac{2.0 \times 9.81}{\cos 20^\circ} = \frac{19.62}{0.9397} = 20.9\,\text{N}.$$

(c) Resolving horizontally, $F = F_T \sin 20^\circ = mg \tan 20^\circ = 19.62 \times \tan 20^\circ = 7.14\,\text{N}$.

Question 5 $m = 1.5\,\text{kg}$. (a) The two forces acting on the book are the force due to gravity $F_g = mg = 1.5 \times 9.81 = 14.7\,\text{N}$ (down) and the normal force F_N on the book by the table (up). Since the book is in equilibrium, $F_N = F_g = 14.7\,\text{N}$. (b) The Newton's third-law partner of the force due to gravity on the book is the **gravitational force on the Earth by the book**: it acts on the Earth, is directed upward (toward the book), and has the same magnitude, 14.7 N. (c) A third-law pair consists of two forces that act on *different* objects. The force due to gravity F_g and the normal force F_N both act *on the book*, so they cannot be a third-law pair. They happen to be equal in magnitude here only because the book is in equilibrium (Newton's first law); they are also different types of force (one gravitational, one contact), and the partner of each acts on a different body (the Earth, and the table, respectively).

Question 6 $m_1 = 5.0\,\text{kg}$ (on the table), $m_2 = 3.0\,\text{kg}$ (hanging), smooth table.



(a) On the block on the table: the normal force F_N (up), the force due to gravity F_g (down) and the tension T (horizontal, toward the pulley). On the hanging mass: the tension T (up) and the force due to gravity $m_2 g$ (down).
(b) Treating the system as one body, the driving force is $m_2 g = 3.0 \times 9.81 = 29.43\,\mathrm{N}$ and the total mass is $8.0\,\mathrm{kg}$: $a = \dfrac{m_2 g}{m_1 + m_2} = \dfrac{29.43}{8.0} = 3.68\,\mathrm{m\ s^{-2}}$.
(c) Isolating the block on the table, the tension is the only horizontal force: $T = m_1 a = 5.0 \times 3.68 = 18.4\,\mathrm{N}$. (Check: $m_2 g - T = 29.43 - 18.4 = 11.0\,\mathrm{N} = 3.0 \times 3.68$. ✓)

Question 7 The net force is the driving force minus the resistance: $F_{\text{net}} = 4200 - 900 = 3300\,\mathrm{N}$. By the second law,

$$a = \frac{F_{\text{net}}}{m} = \frac{3300}{1500} = 2.2\,\mathrm{m\ s^{-2}}.$$

Question 8 $m = 12\,\mathrm{kg}$, $\theta = 30^\circ$, friction $F_f = 22\,\mathrm{N}$. The crate slides down, so friction acts up the slope. Along the slope, taking down-slope as positive,

$$F_{\text{net}} = m g \sin\theta - F_f = 12 \times 9.81 \times \sin 30^\circ - 22 = 58.86 - 22 = 36.86\,\mathrm{N}.$$

$$a = \frac{F_{\text{net}}}{m} = \frac{36.86}{12} = 3.07\,\mathrm{m\ s^{-2}}, \text{directed down the incline.}$$

Question 9 $m = 8.0\,\mathrm{kg}$, each cable 50° to the horizontal. By symmetry both tensions are equal. Resolving vertically for equilibrium, $2T\sin 50^\circ = m g$, so

$$T = \frac{m g}{2\sin 50^\circ} = \frac{8.0 \times 9.81}{2 \times 0.7660} = \frac{78.48}{1.532} = 51.2\,\mathrm{N}\ \text{(to three significant figures)}.$$

Question 10 The tow bar is the only horizontal force accelerating the trailer, so applying the second law to the trailer alone,

$$T = m_{\text{trailer}} a = 400 \times 1.5 = 600\,\mathrm{N}.$$

Question 11 At constant velocity the acceleration is zero. By Newton's second law, $F_{\text{net}} = m a = 0$, so the net force on the car is **zero**. This does not mean there are no forces: the forward driving force from the road on the tyres is exactly balanced by the resistance forces (air resistance and friction) acting backward, so the forces cancel and the car continues at constant velocity, in agreement with Newton's first law. The engine must keep providing the driving force precisely to balance these resistance forces; if it stopped, the resistance would produce a net backward force and the car would slow down.

Question 12 The swimmer's hands exert a backward force on the water — a force on the water by the swimmer. By Newton's third law the water exerts an equal and opposite force on the swimmer — a force on the swimmer by the water — directed forward. This forward force on the swimmer by the water is a real external force acting on her, and it is what accelerates her forward through the pool. The two forces act on different objects (the water and the swimmer), so they do not cancel.

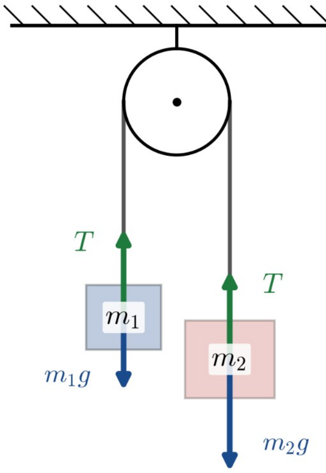
Question 13 $m = 3.0\,\mathrm{kg}$, $\theta = 35^\circ$, constant velocity down the incline. (a) The component of the force due to gravity down the incline is $m g \sin\theta = 3.0 \times 9.81 \times \sin 35^\circ = 16.9\,\mathrm{N}$, as required. (b) The block moves at constant velocity, so by Newton's first law it is in equilibrium and the net force along the incline is zero. The only along-slope forces are the $16.9\,\mathrm{N}$ component of the force due to gravity (down the slope) and friction. Friction must therefore be $16.9\,\mathrm{N}$ directed **up the incline**, exactly balancing the down-slope component.

Question 14 $F = 150\,\mathrm{N}$ at 40° below the horizontal, $m = 18\,\mathrm{kg}$. (a) Horizontal component: $F\cos 40^\circ = 150 \times \cos 40^\circ = 115\,\mathrm{N}$. Vertical component: $F\sin 40^\circ = 150 \times \sin 40^\circ = 96.4\,\mathrm{N}$, directed downward. (b) Vertically there is no acceleration. The normal force must balance both the force due to gravity and the downward component of the applied force:

$$F_N = mg + F\sin 40^\circ = 18\times 9.81 + 96.4 = 176.6 + 96.4 = 273\,\text{N}.$$

(c) Because the push is directed partly downward, it presses the mower into the ground. The surface must push back harder to prevent any vertical acceleration, so the normal force exceeds the force due to gravity by the downward component of the applied force (96.4 N).

Question 15 $m_1 = 4.0\,\text{kg}$, $m_2 = 6.0\,\text{kg}$ over a frictionless pulley; the heavier mass descends.



the heavier mass m_2 descends; both masses share the same $|a|$

Treating the system as one body, the net driving force is the difference in the forces due to gravity, $(m_2 - m_1)g$, and the total mass is $m_1 + m_2$:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = \frac{(6.0 - 4.0)\times 9.81}{10} = \frac{19.62}{10} = 1.96\,\text{m s}^{-2}.$$

For the tension, isolate the lighter mass, which accelerates upward: $T - m_1 g = m_1 a$, so

$$T = m_1(g + a) = 4.0(9.81 + 1.962) = 4.0\times 11.77 = 47.1\,\text{N}.$$

(Check, using the heavier mass: $m_2 g - T = 58.86 - 47.1 = 11.8\,\text{N} = 6.0\times 1.962$. ✓)

Question 16 $m = 60\,\text{kg}$. (a) The force due to gravity on the student is $F_g = mg = 60\times 9.81 = 589\,\text{N}$, directed downward. (b) Its Newton's third-law partner is the **gravitational force on the Earth by the student**: it acts on the Earth, is directed upward (toward the student), and has the same magnitude, $589\,\text{N}$. (c) The two forces are equal in magnitude but act on different objects. By Newton's second law each object's acceleration is $a = \frac{F}{m}$. The student's mass is $60\,\text{kg}$, but the Earth's mass is about $5.97\times 10^{24}\,\text{kg}$, so the same $589\,\text{N}$ force gives the Earth an utterly negligible acceleration while the student would accelerate noticeably. This is why we see objects fall toward the Earth and never notice the Earth moving toward them.

Question 17 $m = 12\,\text{kg}$, each half of the cable 10° to the horizontal. The force due to gravity on the traffic light is $mg = 12\times 9.81 = 117.72\,\text{N}$. By symmetry both cable tensions are equal; resolving vertically for equilibrium, $2T\sin 10^\circ = mg$, so

$$T = \frac{mg}{2\sin 10^\circ} = \frac{117.72}{2\times 0.1736} = 339\,\text{N}.$$

The tension (339 N) is far larger than the force due to gravity on the traffic light (117.72 N) — almost three times as large — because the cable is nearly horizontal, so only a small vertical component of each tension is available to support the load. A nearly horizontal cable must be pulled very hard, which is why such cables are prone to snapping.

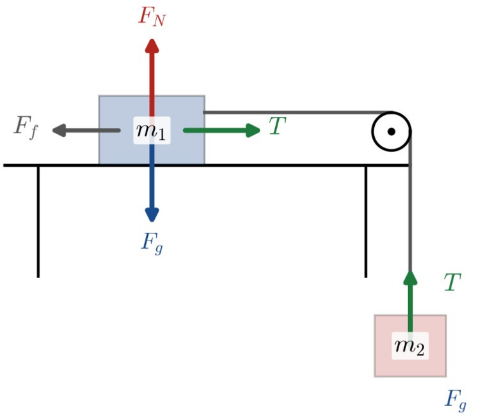
Question 18 The component of the force due to gravity acting down the ramp is

$$mg\sin\theta = 20 \times 9.81 \times \sin 18^{\circ} = 60.6\,\text{N}.$$

For the crate to stay at rest, friction must balance this along-slope force. The required friction is $60.6\,\text{N}$, which is *less* than the maximum the surface can provide ($70\,\text{N}$). The surface can therefore supply enough static friction to hold the crate in equilibrium, so the crate **remains at rest**.

Question 19 $m = 70\,\text{kg}$. Taking up as positive and applying Newton's second law in the vertical direction, $F_N - mg = ma$. (a) Accelerating upward, $a = +2.0\,\text{m s}^{-2}$: $F_N = m(g+a) = 70(9.81+2.0) = 70 \times 11.81 = 827\,\text{N}$. (b) Accelerating downward, $a = -2.0\,\text{m s}^{-2}$: $F_N = m(g+a) = 70(9.81-2.0) = 70 \times 7.81 = 547\,\text{N}$. The normal force is larger than mg ($= 687\,\text{N}$) when the lift accelerates upward and smaller when it accelerates downward.

Question 20 $m_1 = 4.0\,\text{kg}$ (on the table), $m_2 = 3.0\,\text{kg}$ (hanging), friction $F_f = 6.0\,\text{N}$.



Treating the system as one body, the driving force is $m_2 g$ and the opposing force is friction:

$$a = \frac{m_2 g - F_f}{m_1 + m_2} = \frac{3.0 \times 9.81 - 6.0}{7.0} = \frac{29.43 - 6.0}{7.0} = \frac{23.43}{7.0} = 3.35\,\text{m s}^{-2}.$$

For the tension, isolate the hanging mass, which accelerates downward: $m_2 g - T = m_2 a$, so

$$T = m_2(g-a) = 3.0(9.81 - 3.35) = 3.0 \times 6.46 = 19.4\,\text{N}.$$

(Check, using the block on the table: $T - F_f = 19.4 - 6.0 = 13.4\,\text{N} = 4.0 \times 3.35$. ✓)

Question 21 While the car is moving forward the passenger is moving forward with it. When the car brakes, a backward force (from the road on the tyres) acts on the *car*, slowing it. No such backward force acts on the unbelted passenger, so by Newton's first law the passenger continues to move forward in a straight line at constant velocity (inertia) — that is, the passenger moves forward *relative to the decelerating car*. A fastened seatbelt supplies the missing force: it exerts a **backward force on the passenger**, providing the net force needed to decelerate the passenger with the car. The passenger is not "thrown forward" by any forward force; the forward motion is simply their inertia carrying them on while the car slows beneath them.

Theory

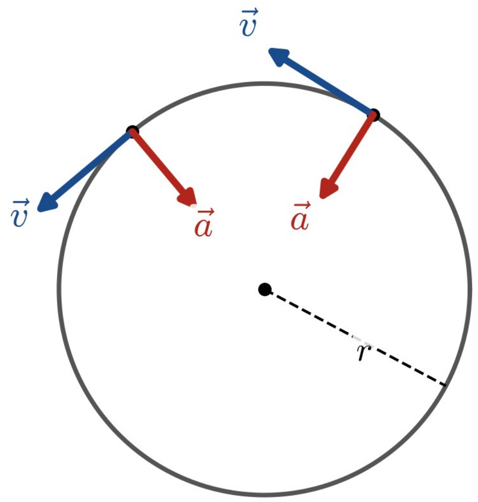
An object moves in **uniform circular motion** when it travels around a circle of fixed radius r at *constant speed*. Although the speed is constant, the **velocity** is not: the direction of motion changes continuously, always tangent to the circle. A changing velocity is an acceleration, so by Newton's second law a non-zero net force must act on the object at every instant.

Period, frequency and speed. The **period** T is the time for one complete revolution, and the **frequency** $f=\frac{1}{T}$ is the number of revolutions each second (in Hz). In one period the object travels the circumference $2\pi r$, so its constant speed is

$$v=\frac{2\pi r}{T}=2\pi r f\,.$$

Centripetal acceleration. Because the velocity continually turns toward the centre, the acceleration also points **toward the centre** of the circle — it is the **centripetal** (centre-seeking) acceleration. Its magnitude is

$$a=\frac{v^2}{r}=\frac{4\pi^2 r}{T^2}\,.$$



constant speed v, $a=\dfrac{v^2}{r}$ toward the centre

Newton's second law for circular motion. Applying $\Sigma F=ma$ with $a=\frac{v^2}{r}$ directed toward the centre gives the **net** force required to hold an object of mass m on the circle:

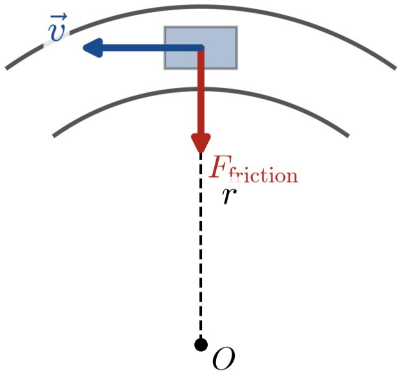
$$F_{\text{net}}=\frac{mv^2}{r}\text{, directed toward the centre.}$$

This is not a new kind of force. It is the **resultant** of the ordinary forces that act on the object — friction, the normal force, tension, the force due to gravity F_g. Whichever real forces are present must add, as vectors, to a resultant of magnitude $\frac{mv^2}{r}$ pointing at the centre. There is no outward ("centrifugal") force acting on the object.

A vehicle on a flat (level) circular road. On a level road the only horizontal force that can turn the car is the sideways friction on the tyres by the road, directed toward the centre of the bend. It alone provides the net force:

$$F_{\text{friction}} = \frac{m v^2}{r}.$$

The road can supply friction only up to some maximum value F_{max}. Setting $F_{\text{max}} = \frac{m v_{\text{max}}^2}{r}$ gives the greatest speed at which the bend can be taken; above it the required force exceeds what the road can provide and the car skids outward.



top view: friction on the car by the road provides the net force toward the centre

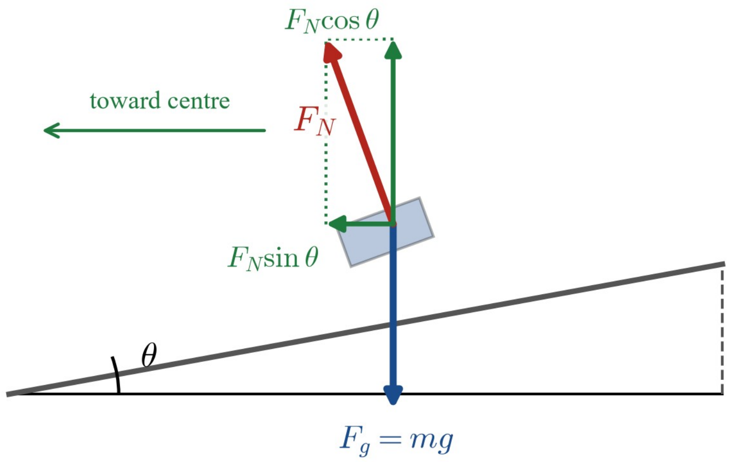
A vehicle on a banked track. A track banked at angle θ to the horizontal lets a vehicle turn *without relying on friction*. Two forces act on the car: the force due to gravity $F_g = m g$ (vertically down) and the normal force F_N on the car by the track (perpendicular to the surface). Resolving F_N into horizontal and vertical components:

- vertical (no vertical acceleration): $F_N \cos\theta = m g$;
- horizontal (this component is the net force, toward the centre): $F_N \sin\theta = \frac{m v^2}{r}$.

Dividing the horizontal equation by the vertical eliminates F_N and m:

$$\tan\theta = \frac{v^2}{r g} \Rightarrow v = \sqrt{r g \tan\theta}.$$

This single speed is the **design speed** — the speed for which the horizontal component of the normal force is exactly the required net force and **no friction is needed**. It is the horizontal component of the normal force (not gravity, and not friction) that provides the net force here. A car slower than the design speed tends to slide down the bank, so friction acts up the slope; a car faster than the design speed tends to slide up, so friction acts down the slope.



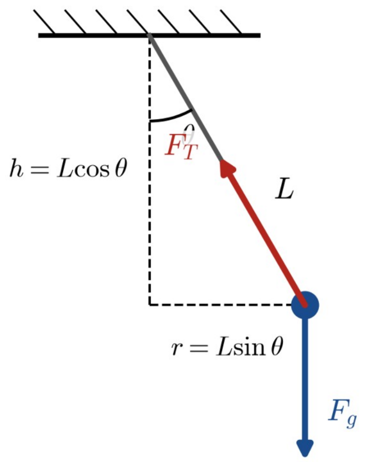
An object on the end of a string (horizontal circle). If a small object is whirled in a horizontal circle on a string — for example, a puck sliding on a smooth horizontal table, tethered to a central pin — the tension F_T on the object by the string is the only horizontal force and provides the net force:

$$F_T = \frac{mv^2}{r}.$$

The conical pendulum. When a mass hangs from a string of length L and swings in a horizontal circle, the string sweeps out a cone at angle θ to the vertical, and the circle has radius $r = L\sin\theta$. Two forces act on the mass: the tension F_T along the string and the force due to gravity $F_g = mg$ down. Resolving,

$$F_T\cos\theta = mg,\ F_T\sin\theta = \frac{mv^2}{r}.$$

Dividing gives $\tan\theta = \frac{v^2}{rg}$ — the same relation as the banked track — and the period is $T = \frac{2\pi r}{v}$.



Motion in a vertical circle (highest and lowest points only). When an object moves in a vertical circle — a ball on a string, a car through a dip or over a hump — the analysis here is restricted to the top and bottom, where the net force is vertical and points along the radius toward the centre.

At the **lowest point** the centre is directly above, so the net force points up. Taking up as positive, the support force F_N (tension, or a track pressing up) acts up and F_g acts down:

$$F_N - mg = \frac{mv^2}{r} \Rightarrow F_N = m\left(g + \frac{v^2}{r}\right).$$

The support force is *greater* than F_g.

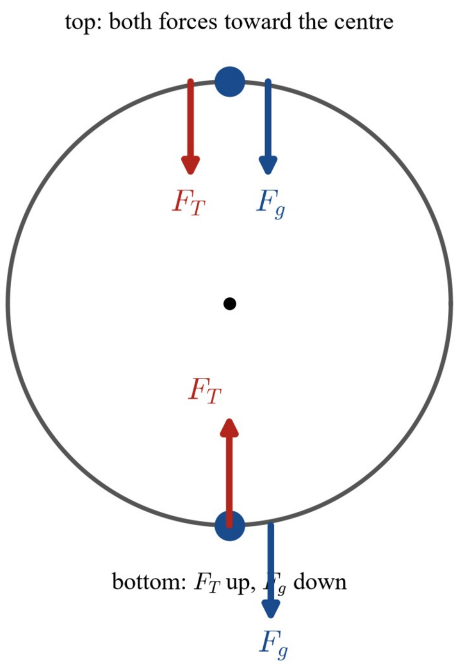
At the **highest point** of a circle traced on the inside (a ball on a string, or the inside of a loop) the centre is directly below, so the net force points down. Both the support force F_N **and** F_g point down, toward the centre:

$$F_N + mg = \frac{mv^2}{r} \Rightarrow F_N = m\left(\frac{v^2}{r} - g\right).$$

As the speed is reduced, F_N falls; the **minimum** speed for the object to stay on the path is reached when $F_N = 0$ and the force due to gravity alone provides the net force:

$$mg = \frac{mv_{\min}^2}{r} \Rightarrow v_{\min} = \sqrt{gr}.$$

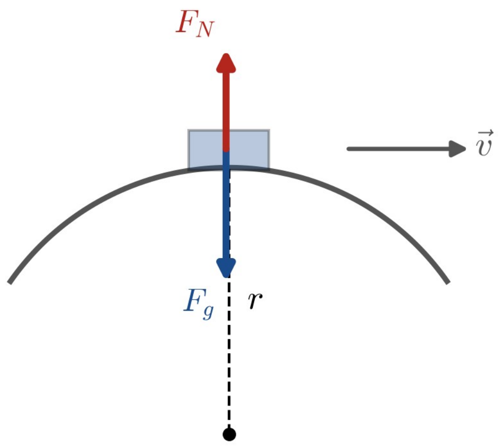
The force due to gravity does **not** vanish at the top; it still acts on the object there.



For a car going *over* a hump (a convex bridge) the geometry is reversed: the centre of the circle is below the car, so the normal force F_N on the car by the road acts up while F_g acts down, and

$$F_N = m\left(g - \frac{v^2}{r}\right).$$

The car leaves the road when $F_N = 0$, that is at $v = \sqrt{gr}$.



car at the top of a hump: F_N up, F_g down; net force down toward O

Satellites. Natural and artificial satellites move in (approximately) uniform circular motion, with the force due to gravity providing the net force. That case is developed with gravitational fields in Chapter 3 and is not treated further here.

Throughout this subtopic the constant $g = 9.81\ \text{m s}^{-2}$ (equivalently $9.81\ \text{N kg}^{-1}$) is used.

Worked examples

Example 1 — scaffolded

A car of mass $1200\ \text{kg}$ travels around a flat (level) circular bend of radius $45\ \text{m}$ at a constant speed of $15\ \text{m s}^{-1}$. Calculate the period of the motion, the centripetal acceleration, and the magnitude and direction of the net force on the car, naming the force that provides it.

Working	Comment
Uniform circular motion: $v = 15\ \text{m s}^{-1}$, $r = 45\ \text{m}$, $m = 1200\ \text{kg}$.	List the data and identify the model.
$v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(45)}{15} = 18.8\ \text{s}$.	Rearrange $v = \frac{2\pi r}{T}$ for the period.
$a = \frac{v^2}{r} = \frac{15^2}{45} = 5.00\ \text{m s}^{-2}$, toward the centre.	Centripetal acceleration; always directed at the centre.
$F_{\text{net}} = \frac{m v^2}{r} = 1200 \times 5.00 = 6.00 \times 10^3\ \text{N}$, toward the centre.	Newton's second law, $F_{\text{net}} = m a$, along the radius.
This net force is the sideways friction on the tyres by the road.	On a level road only friction can point toward the centre.

Example 2 — scaffolded

A racing track of radius $60\ \text{m}$ is banked at 20° to the horizontal. A car of mass $900\ \text{kg}$ rounds the track at the design speed (the speed for which no sideways friction is required). Calculate the design speed and the magnitude of the normal force on the car by the track.

Working	Comment
At the design speed only $F_g = m g$ (down) and F_N (perpendicular to the track) act on the car.	Frictionless design case: two forces only.

Working	Comment
Vertical: $F_N \cos\theta = m g$. Horizontal: $F_N \sin\theta = \frac{m v^2}{r}$ (toward the centre).	Resolve F_N; the horizontal component is the net force.
Dividing: $\tan\theta = \frac{v^2}{r g}$, so $v = \sqrt{r g \tan\theta}$.	Eliminates F_N and m.
$v = \sqrt{60 \times 9.81 \times \tan 20^\circ} = 14.6\,\text{m s}^{-1}$.	Substitute r, $g = 9.81\,\text{m s}^{-2}$, $\theta = 20^\circ$.
$F_N = \frac{m g}{\cos\theta} = \frac{900 \times 9.81}{\cos 20^\circ} = 9.40 \times 10^3\,\text{N}$.	From the vertical equation.

Example 3 — unscaffolded

A ball of mass $0.40\,\text{kg}$ is attached to a string and swung in a vertical circle of radius $0.80\,\text{m}$. At the highest point of the circle its speed is $4.0\,\text{m s}^{-1}$. Calculate the tension in the string at the highest point.

At the highest point the centre of the circle is directly below the ball, so the net force points down. Both the tension F_T and the force due to gravity $F_g = m g$ act downward, toward the centre:

$$F_T + m g = \frac{m v^2}{r}.$$

The centripetal acceleration is $\frac{v^2}{r} = \frac{4.0^2}{0.80} = 20\,\text{m s}^{-2}$, so

$$F_T = m\left(\frac{v^2}{r} - g\right) = 0.40(20 - 9.81) = 0.40 \times 10.19 = 4.076\,\text{N}.$$

$\therefore$ the tension on the ball by the string at the highest point is $4.08\,\text{N}$.

Example 4 — unscaffolded

A ball of mass $0.50\,\text{kg}$ hangs from a string of length $1.2\,\text{m}$ and swings as a conical pendulum, the string making an angle of 30° with the vertical. Calculate the tension in the string, the speed of the ball, and the period of the motion.

The ball moves in a horizontal circle of radius $r = L\sin\theta = 1.2\sin 30^\circ = 0.600\,\text{m}$. Two forces act on the ball: the tension F_T along the string and $F_g = m g$ down. Resolving vertically and horizontally,

$$F_T \cos\theta = m g\,,\ F_T \sin\theta = \frac{m v^2}{r}.$$

From the vertical equation,

$$F_T = \frac{m g}{\cos\theta} = \frac{0.50 \times 9.81}{\cos 30^\circ} = 5.66\,\text{N}.$$

Dividing the horizontal equation by the vertical gives $\tan\theta = \frac{v^2}{r g}$, so

$$v = \sqrt{r g \tan\theta} = \sqrt{0.600 \times 9.81 \times \tan 30^\circ} = 1.84\,\text{m s}^{-1}.$$

The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(0.600)}{1.84} = 2.05\,\text{s}.$$

$\therefore$ the tension is $5.66\,\text{N}$, the speed is $1.84\,\text{m s}^{-1}$, and the period is $2.05\,\text{s}$.

Practice questions

Question 1 A car of mass $1500\,\mathrm{kg}$ rounds a flat circular bend of radius $50\,\mathrm{m}$ at a constant speed of $20\,\mathrm{m\ s^{-1}}$. (a) Calculate the period of the car's circular motion. (b) Calculate the magnitude and direction of the centripetal acceleration. (c) Calculate the magnitude and direction of the net force on the car, and name the force that provides it.

Question 2 A section of road of radius $80\,\mathrm{m}$ is to be banked at 15° so that vehicles can round it without relying on friction. (a) Show that the design speed of a banked track is given by $v=\sqrt{r\,g\tan\theta}$. (b) Calculate the design speed of this bend. Give your answer to three significant figures. (c) A car travels around the bend more slowly than the design speed. State the direction along the track surface of the friction force that then acts on the car, and explain why it is needed.

Question 3 A puck of mass $0.25\,\mathrm{kg}$ slides on a smooth horizontal table, tethered to a central pin by a string of length $0.60\,\mathrm{m}$, and completes one revolution every $0.50\,\mathrm{s}$. (a) Calculate the speed of the puck. (b) Calculate its centripetal acceleration. (c) Calculate the tension in the string.

Question 4 A ball of mass $0.30\,\mathrm{kg}$ is swung on a string in a vertical circle of radius $0.90\,\mathrm{m}$. At the lowest point of the circle its speed is $5.0\,\mathrm{m\ s^{-1}}$. (a) Draw a diagram showing the forces acting on the ball at the lowest point. (b) Calculate the centripetal acceleration at the lowest point. (c) Calculate the tension in the string at the lowest point.

Question 5 A car of mass $1000\,\mathrm{kg}$ drives over the top of a circular hump (a convex bridge) of radius $20\,\mathrm{m}$ at a speed of $12\,\mathrm{m\ s^{-1}}$. (a) Draw a diagram showing the forces acting on the car at the top of the hump. (b) Calculate the normal force on the car by the road at the top of the hump. (c) Determine the maximum speed at which the car can cross the top of the hump while remaining in contact with the road.

Question 6 A car rounds a flat circular bend at a constant speed of $15\,\mathrm{m\ s^{-1}}$. (a) (Circle your answer.) As the car rounds the bend, its velocity is: *constant* / *changing*. Justify your choice. No calculations are required. (b) State the direction of the car's acceleration. (c) With reference to Newton's second law, explain why a net force must act on the car, and name the force that provides it.

Question 7 A child sits $2.5\,\mathrm{m}$ from the centre of a merry-go-round that rotates uniformly at 6.0 revolutions per minute. Calculate the child's speed and centripetal acceleration.

Question 8 A car of mass $1400\,\mathrm{kg}$ rounds a flat bend of radius $70\,\mathrm{m}$. The greatest sideways friction force the road can exert on the car is $8200\,\mathrm{N}$. Calculate the maximum speed at which the car can take the bend.

Question 9 A cycling track of radius $40\,\mathrm{m}$ is to be banked so that its design speed is $11\,\mathrm{m\ s^{-1}}$. Calculate the banking angle required.

Question 10 A ball of mass $0.60\,\mathrm{kg}$ on a string of length $1.5\,\mathrm{m}$ swings as a conical pendulum with the string 25° from the vertical. Calculate the tension in the string, the speed of the ball, and the period of the motion.

Question 11 A ball of mass $0.50\,\mathrm{kg}$ is swung on a string in a vertical circle of radius $0.70\,\mathrm{m}$. At the highest point its speed is $3.6\,\mathrm{m\ s^{-1}}$. Calculate the tension in the string at the highest point.

Question 12 A ball is swung on a string in a vertical circle of radius $1.2\,\mathrm{m}$. Calculate the minimum speed the ball can have at the highest point while keeping the string taut.

Question 13 A ball of mass $0.25\,\mathrm{kg}$ on a string of length $0.50\,\mathrm{m}$ is swung in a vertical circle. At the lowest point its speed is $4.0\,\mathrm{m\ s^{-1}}$. (a) Show that the centripetal acceleration at the lowest point is $32\,\mathrm{m\ s^{-2}}$. (b) Hence calculate the tension in the string at the lowest point.

Question 14 A car rounds a banked track at exactly the design speed. A student claims: "The car stays on the track because friction provides the force that pushes it toward the centre." Explain what is wrong with this statement, and identify correctly which force provides the net force and how.

Question 15 At the highest point of a vertical circle, a student says that the force due to gravity on a ball on a string briefly falls to zero. Explain why this is incorrect, with reference to the forces acting on the ball at that point.

Question 16 A ball is whirled in a horizontal circle on the end of a string. The string suddenly breaks. Describe the path the ball follows immediately afterwards, and explain your answer with reference to Newton's first law. (Ignore the effect of gravity for the instant just after the break.)

Question 17 A car of mass $1200\,\mathrm{kg}$ rounds a track of radius $90\,\mathrm{m}$ banked at 18°, travelling at the design speed. Calculate the design speed, the normal force on the car by the track, and the net force on the car.

Question 18 A car of mass $1600\,\mathrm{kg}$ rounds a flat bend of radius $55\,\mathrm{m}$ at $18\,\mathrm{m\ s^{-1}}$. The road can provide a maximum sideways friction force of $9000\,\mathrm{N}$. Determine, with a calculation, whether the car can round the bend at this speed.

Question 19 A person of mass $65\,\mathrm{kg}$ rides a Ferris wheel of radius $9.0\,\mathrm{m}$ that rotates uniformly, completing one revolution every $24\,\mathrm{s}$. Calculate the normal force on the person by the seat (a) at the lowest point and (b) at the highest point of the wheel.

Question 20 At a fairground swing ride, chairs hang from chains of length $4.0\,\mathrm{m}$ attached to a rotating disc at a distance of $5.0\,\mathrm{m}$ from the central axis. When the ride is running steadily, the chains make an angle of 35° with the vertical. Calculate the speed of a chair and the period of the ride.

Question 21 A passenger sitting in a car feels pushed toward the outside of the car as it rounds a sharp bend. With reference to Newton's laws, explain why there is no outward force acting on the passenger, and identify the real force that acts on the passenger and its direction.

Answers

Question 1 (a) $15.7\,\text{s}$; (b) $8.0\,\text{m s}^{-2}$ toward the centre; (c) $1.2\times10^{4}\,\text{N}$ toward the centre, provided by friction on the tyres by the road. **Question 2** (b) $14.5\,\text{m s}^{-1}$; (c) friction acts up the slope. **Question 3** (a) $7.54\,\text{m s}^{-1}$; (b) $94.7\,\text{m s}^{-2}$; (c) $23.7\,\text{N}$. **Question 4** (b) $27.8\,\text{m s}^{-2}$; (c) $11.3\,\text{N}$. **Question 5** (b) $2.61\times10^{3}\,\text{N}$; (c) $14.0\,\text{m s}^{-1}$. **Question 6** (a) changing; (b) toward the centre of the bend; (c) a net force acts, provided by friction on the tyres by the road. **Question 7** $1.57\,\text{m s}^{-1}$; $0.987\,\text{m s}^{-2}$. **Question 8** $20.2\,\text{m s}^{-1}$. **Question 9** 17.1°. **Question 10** $6.49\,\text{N}$; $1.70\,\text{m s}^{-1}$; $2.34\,\text{s}$. **Question 11** $4.35\,\text{N}$. **Question 12** $3.43\,\text{m s}^{-1}$. **Question 13** (a) $32\,\text{m s}^{-2}$; (b) $10.5\,\text{N}$. **Question 14** Friction is not needed at the design speed; the horizontal component of the normal force provides the net force. **Question 15** Incorrect: F_g still acts on the ball at the top; it is never zero. **Question 16** A straight line, tangent to the circle at the point of release. **Question 17** $16.9\,\text{m s}^{-1}$; $1.24\times10^{4}\,\text{N}$; $3.82\times10^{3}\,\text{N}$. **Question 18** Required net force $9.43\times10^{3}\,\text{N}>9000\,\text{N}$, so the car cannot round the bend (it skids). **Question 19** (a) $678\,\text{N}$; (b) $598\,\text{N}$. **Question 20** $7.08\,\text{m s}^{-1}$; $6.47\,\text{s}$. **Question 21** No outward force; the passenger's inertia carries them straight ahead while the inward force from the car provides the net force.

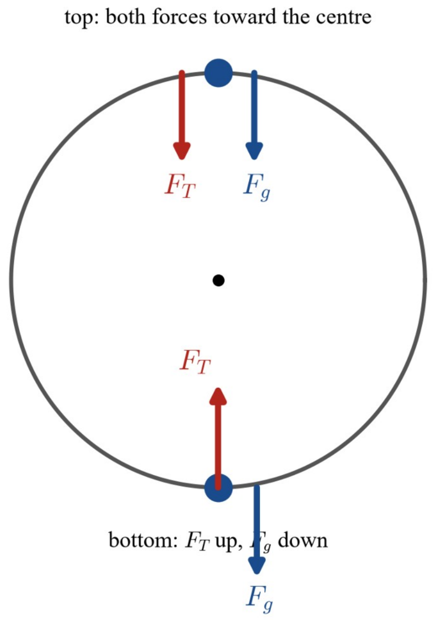
Fully-worked solutions

Question 1 $r=50\,\text{m}$, $v=20\,\text{m s}^{-1}$, $m=1500\,\text{kg}$. (a) $T=\dfrac{2\pi r}{v}=\dfrac{2\pi(50)}{20}=15.7\,\text{s}$. (b) $a=\dfrac{v^2}{r}=\dfrac{20^2}{50}=8.0\,\text{m s}^{-2}$, directed toward the centre of the bend. (c) $F_{\text{net}}=\dfrac{mv^2}{r}=1500\times8.0=1.2\times10^{4}\,\text{N}$, toward the centre. On a level road this is the sideways friction on the tyres by the road.

Question 2 (a) At the design speed the only forces on the car are $F_g=mg$ (down) and the normal force F_N (perpendicular to the surface). Vertically $F_N\cos\theta=mg$; horizontally (net force toward the centre) $F_N\sin\theta=\dfrac{mv^2}{r}$. Dividing the horizontal equation by the vertical equation gives $\tan\theta=\dfrac{v^2}{rg}$, hence $v=\sqrt{rg\tan\theta}$, as required. (b) $v=\sqrt{80\times9.81\times\tan15^{\circ}}=\sqrt{210.3}=14.5\,\text{m s}^{-1}$ (to three significant figures). (c) At less than the design speed the net force required is smaller, so the inward (horizontal) component of the normal force would be more than enough and the car tends to slide down the bank toward the centre. Friction therefore acts **up the slope** to hold the car on its circular path.

Question 3 $r=0.60\,\text{m}$, $T=0.50\,\text{s}$, $m=0.25\,\text{kg}$. (a) $v=\dfrac{2\pi r}{T}=\dfrac{2\pi(0.60)}{0.50}=7.54\,\text{m s}^{-1}$. (b) $a=\dfrac{4\pi^2r}{T^2}=\dfrac{4\pi^2(0.60)}{0.50^2}=94.7\,\text{m s}^{-2}$, toward the centre. (c) The tension on the puck by the string provides the net force: $F_T=ma=0.25\times94.7=23.7\,\text{N}$.

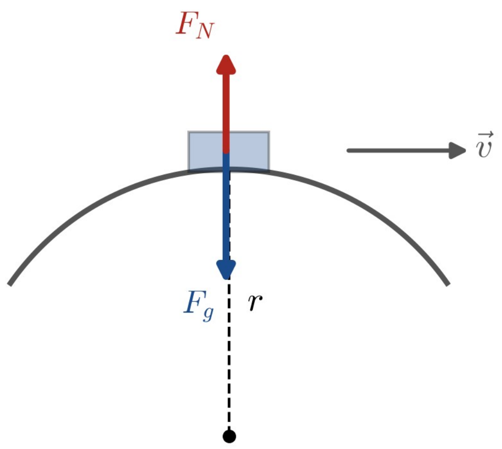
Question 4 $r=0.90\,\text{m}$, $v=5.0\,\text{m s}^{-1}$, $m=0.30\,\text{kg}$. (a) At the lowest point the centre of the circle is above the ball, so the tension F_T acts up (toward the centre) and $F_g=mg$ acts down.



(b) $a = \dfrac{v^2}{r} = \dfrac{5.0^2}{0.90} = 27.8\,\text{m s}^{-2}$, directed toward the centre (upward).

(c) Taking up as positive, $F_T - mg = \dfrac{mv^2}{r}$, so $F_T = m\left(g + \dfrac{v^2}{r}\right) = 0.30(9.81 + 27.78) = 0.30 \times 37.59 = 11.3\,\text{N}$.

Question 5 $r = 20\,\text{m}$, $v = 12\,\text{m s}^{-1}$, $m = 1000\,\text{kg}$. (a) At the top of the hump the centre of the circle is below the car; the normal force F_N on the car by the road acts up and $F_g = mg$ acts down. The net force is directed down, toward the centre.



car at the top of a hump: F_N up, F_g down; net force down toward O

(b) $mg - F_N = \dfrac{mv^2}{r}$, so $F_N = m\left(g - \dfrac{v^2}{r}\right) = 1000\left(9.81 - \dfrac{12^2}{20}\right) = 1000(9.81 - 7.20) = 2.61 \times 10^3\,\text{N}$.

(c) The car stays on the road while $F_N \geq 0$. Setting $F_N = 0$: $mg = \dfrac{mv_{\max}^2}{r}$, so

$v_{\max} = \sqrt{gr} = \sqrt{9.81 \times 20} = 14.0\,\text{m s}^{-1}$. (Since $12\,\text{m s}^{-1} < 14.0\,\text{m s}^{-1}$, the car does remain in contact.)

Question 6 (a) **Changing.** The speed is constant, but the direction of motion changes continuously as the car follows the bend, so the velocity — a vector — is changing. (b) The acceleration is directed toward the centre of the bend. (c) A changing velocity means a non-zero acceleration. By Newton's second law, $F_{\text{net}} = ma$, a non-zero acceleration requires a non-zero net force in the same direction — toward the centre. On a level road this net force is the sideways friction on the tyres by the road.

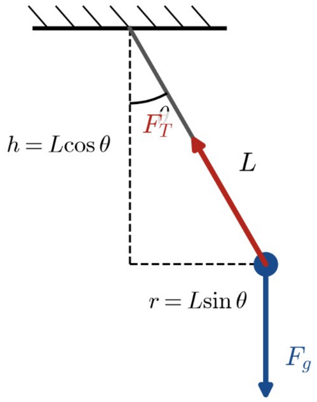
Question 7 $r = 2.5\,\text{m}$; $f = 6.0\,\text{rev min}^{-1} = \dfrac{6.0}{60}\,\text{Hz} = 0.10\,\text{Hz}$, so $T = \dfrac{1}{f} = 10\,\text{s}$. $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(2.5)}{10} = 1.57\,\text{m s}^{-1}$. $a = \dfrac{4\pi^2 r}{T^2} = \dfrac{4\pi^2(2.5)}{10^2} = 0.987\,\text{m s}^{-2}$, toward the centre.

Question 8 The friction force provides the net force, so $F_{\text{max}} = \dfrac{m v_{\text{max}}^2}{r}$, giving

$$v_{\text{max}} = \sqrt{\frac{F_{\text{max}} r}{m}} = \sqrt{\frac{8200 \times 70}{1400}} = \sqrt{410} = 20.2\,\text{m s}^{-1}.$$

Question 9 $\tan\theta = \dfrac{v^2}{rg} = \dfrac{11^2}{40 \times 9.81} = \dfrac{121}{392.4} = 0.3084$, so $\theta = \tan^{-1}(0.3084) = 17.1^\circ$.

Question 10 $r = L\sin\theta = 1.5\sin 25^\circ = 0.634\,\text{m}$.



$F_T = \dfrac{mg}{\cos\theta} = \dfrac{0.60 \times 9.81}{\cos 25^\circ} = 6.49\,\text{N}$. $v = \sqrt{rg\tan\theta} = \sqrt{0.634 \times 9.81 \times \tan 25^\circ} = 1.70\,\text{m s}^{-1}$.
$T = \dfrac{2\pi r}{v} = \dfrac{2\pi(0.634)}{1.70} = 2.34\,\text{s}$.

Question 11 At the highest point both F_T and F_g act toward the centre (down): $F_T + mg = \dfrac{mv^2}{r}$, so

$$F_T = m\left(\frac{v^2}{r} - g\right) = 0.50\left(\frac{3.6^2}{0.70} - 9.81\right) = 0.50(18.51 - 9.81) = 4.35\,\text{N}.$$

Question 12 At the minimum speed the string tension is zero and the force due to gravity alone provides the net force at the top: $mg = \dfrac{m v_{\text{min}}^2}{r}$, so $v_{\text{min}} = \sqrt{gr} = \sqrt{9.81 \times 1.2} = 3.43\,\text{m s}^{-1}$.

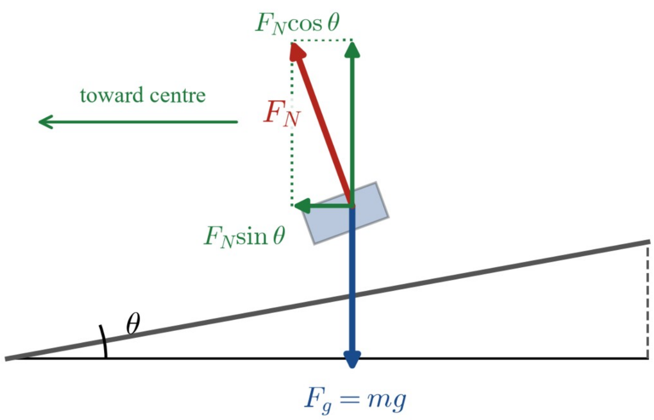
Question 13 $r = 0.50\,\text{m}$, $v = 4.0\,\text{m s}^{-1}$, $m = 0.25\,\text{kg}$. (a) $a = \dfrac{v^2}{r} = \dfrac{4.0^2}{0.50} = \dfrac{16}{0.50} = 32\,\text{m s}^{-2}$, as required. (b) At the lowest point $F_T - mg = ma$, so $F_T = m(g + a) = 0.25(9.81 + 32) = 0.25 \times 41.81 = 10.5\,\text{N}$.

Question 14 The statement is wrong on two counts. First, at exactly the design speed **no friction is needed at all**: only two forces act on the car — the force due to gravity F_g (down) and the normal force F_N on the car by the track (perpendicular to the surface). Second, it is the **horizontal component of the normal force**, $F_N \sin\theta$, that points toward the centre and provides the entire net force $\frac{mv^2}{r}$, while the vertical component $F_N \cos\theta$ balances F_g. Gravity is vertical and contributes nothing to the horizontal net force, and friction plays no part at the design speed.

Question 15 The claim is incorrect because the force due to gravity $F_g = mg$ does not disappear at the top of the circle — it acts on the ball at every point of the motion, always directed downward. At the highest point F_g points toward the centre and, together with the tension, provides the net force $\frac{mv^2}{r}$. Even at the minimum speed, where the string tension has fallen to zero, F_g is unchanged and is precisely what holds the ball on its circular path. The force due to gravity is never zero.

Question 16 Immediately after the string breaks there is no longer any net force on the ball (ignoring gravity for that instant). By Newton's first law, an object with no net force acting on it continues in a straight line at constant velocity. The ball therefore travels in a **straight line along the tangent** to the circle at the point where the string broke — in the direction of its velocity at that instant. It does not fly radially outward.

Question 17 $m = 1200\,\text{kg}$, $r = 90\,\text{m}$, $\theta = 18^\circ$.



Design speed: $v = \sqrt{rg\tan\theta} = \sqrt{90 \times 9.81 \times \tan 18^\circ} = 16.9\,\text{m s}^{-1}$. Normal force (from $F_N \cos\theta = mg$): $F_N = \frac{mg}{\cos\theta} = \frac{1200 \times 9.81}{\cos 18^\circ} = 1.24 \times 10^4\,\text{N}$. Net force:
$F_{\text{net}} = F_N \sin\theta = mg\tan\theta = 1200 \times 9.81 \times \tan 18^\circ = 3.82 \times 10^3\,\text{N}$, directed horizontally toward the centre.

Question 18 The net force required to round the bend at this speed is

$$F_{\text{req}} = \frac{mv^2}{r} = \frac{1600 \times 18^2}{55} = \frac{518400}{55} = 9.43 \times 10^3\,\text{N}.$$

This exceeds the maximum friction force the road can provide (9000 N). Because the road cannot supply the required net force, the car cannot maintain the circular path — it skids outward and does not round the bend at this speed.

Question 19 $r = 9.0\,\text{m}$, $T = 24\,\text{s}$, $m = 65\,\text{kg}$. $v = \frac{2\pi r}{T} = \frac{2\pi(9.0)}{24} = 2.36\,\text{m s}^{-1}$; $a = \frac{4\pi^2 r}{T^2} = 0.617\,\text{m s}^{-2}$. (a) Lowest point (centre above, net force up): $F_N - mg = ma$, so $F_N = m(g+a) = 65(9.81 + 0.617) = 678\,\text{N}$. (b) Highest point (centre below, net force down): $mg - F_N = ma$, so $F_N = m(g-a) = 65(9.81 - 0.617) = 598\,\text{N}$.

Question 20 The radius of the circle is $r = 5.0 + L\sin\theta = 5.0 + 4.0\sin 35^\circ = 7.29\,\text{m}$. As for any conical pendulum, $\tan\theta = \frac{v^2}{rg}$, so

$$v = \sqrt{rg\tan\theta} = \sqrt{7.29 \times 9.81 \times \tan 35^\circ} = 7.08\,\text{m s}^{-1}.$$

The period is $T=\dfrac{2\pi r}{v}=\dfrac{2\pi(7.29)}{7.08}=6.47\text{ s}$.

Question 21 There is no outward force on the passenger. By Newton's first law, the passenger's body tends to continue moving in a straight line (inertia). As the car turns, the side of the car — the seat and the door — pushes **inward** on the passenger, providing the net force directed toward the centre of the bend that makes the passenger follow the circular path. The sensation of being "thrown outward" is really the passenger's inertia carrying them straight ahead while the car turns inward beneath them; the only horizontal force actually acting on the passenger is this inward push from the car. There is no real outward ("centrifugal") force.

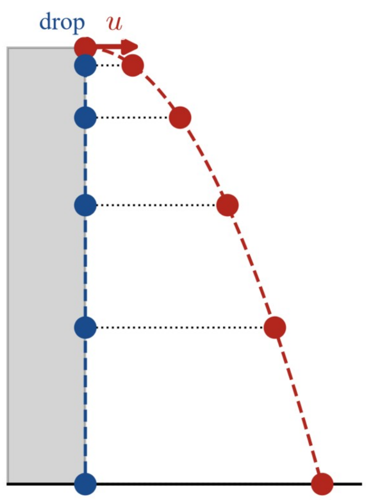
Theory

A **projectile** is an object moving near Earth's surface that, once launched, is acted on by only one force — the force due to gravity F_g. If air resistance is ignored, there is no horizontal force at all. The object therefore has a constant downward acceleration of magnitude $g = 9.81\ \mathrm{m\ s^{-2}}$ and no horizontal acceleration, whatever direction it was launched in.

The key idea: horizontal and vertical motions are independent. The single acceleration g is purely vertical, so it changes only the *vertical* part of the motion. The horizontal and vertical components of a projectile's motion can be analysed as two separate problems that share the **same clock** (the time t):

- **horizontally** there is no force, so the horizontal velocity is *constant*;
- **vertically** the motion is uniformly accelerated at g, exactly like an object thrown straight up or dropped.

The two motions are linked only through the time. This independence is why a ball dropped from a bench and a ball rolled horizontally off the same bench at the same instant strike the floor **together**: their vertical motions are identical, and the horizontal velocity of the second ball has no effect on how fast it falls.



equal heights at equal times: the two balls land together

Resolving the launch velocity. A projectile launched with speed u at an angle θ above the horizontal has initial velocity components

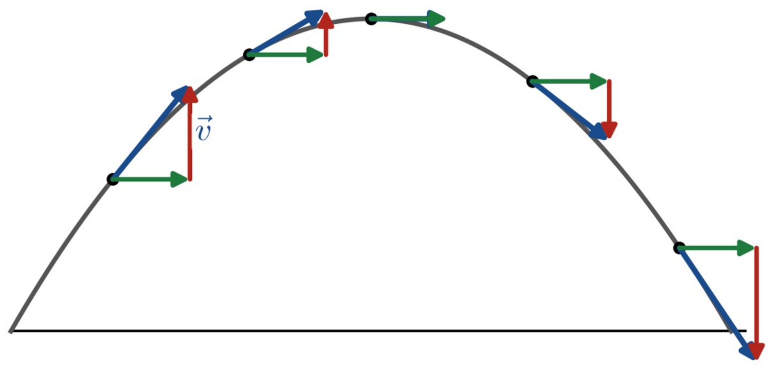
$$u_x = u\cos\theta, u_y = u\sin\theta.$$

Throughout this subtopic we take **up as positive**, so the vertical acceleration is $a_y = -g$. The horizontal velocity never changes; the vertical velocity does.

The equations of motion for each axis. Using the constant-acceleration equations separately:

- horizontal (constant velocity): $x = u_x t$;
- vertical (acceleration $-g$): $v_y = u_y - gt, y = u_y t - \frac{1}{2}gt^2, v_y^2 = u_y^2 - 2gy$.

Here x and y are the horizontal and vertical displacements from the launch point. At any instant the horizontal velocity component is $v_x = u_x = u\cos\theta$ (unchanged), while the vertical component v_y decreases, passes through zero, and then grows in the downward direction.

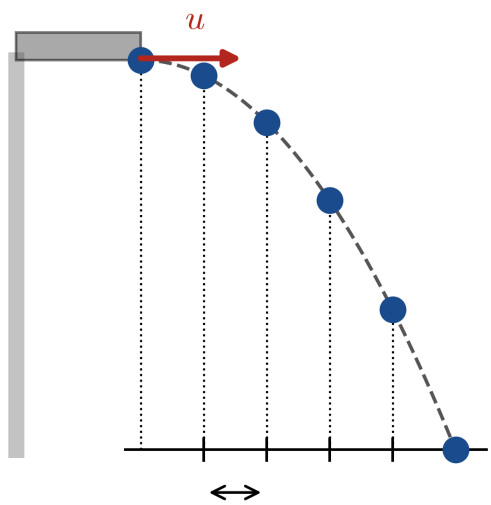


$v_x = u\cos\theta$ constant (green); v_y changes (red)

Horizontal launch. When an object is launched horizontally (for example rolling off a bench or fired horizontally), $\theta = 0$, so $u_y = 0$ and $u_x = u$. The object begins to fall the instant it leaves the edge. If it starts at a height h above the ground, the vertical motion is a fall from rest through h:

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}.$$

The time to reach the ground depends only on the height, **not** on the horizontal speed. The horizontal distance travelled (the range) is then $x = ut$. In equal time intervals the object covers equal horizontal steps but falls through ever-greater vertical distances.



equal horizontal steps in equal times;
drop grows as $\frac{1}{2}gt^2$

Launch at an angle from ground level. For a projectile launched from level ground that returns to the same height, the path is a symmetric parabola. At the **apex** (highest point) the vertical velocity is momentarily zero, $v_y = 0$, while the horizontal velocity is still u_x. Setting $v_y = u_y - gt = 0$ gives the time to the apex, $t_{\text{apex}} = \frac{u_y}{g}$, and the **maximum height** above the launch level:

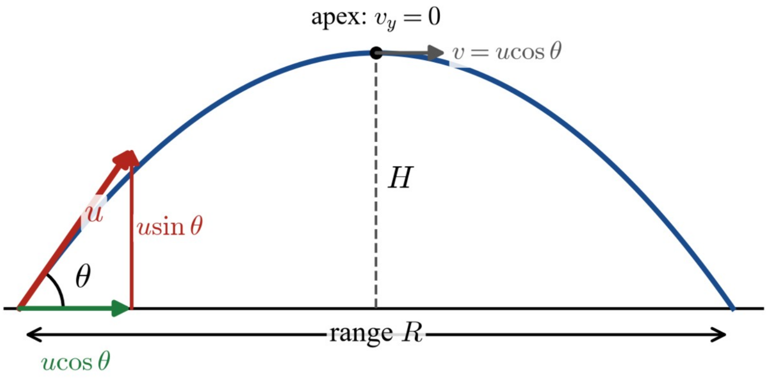
$$H = \frac{u_y^2}{2g} = \frac{(u\sin\theta)^2}{2g}.$$

By symmetry the object takes the same time to come back down, so the **time of flight** is

$$T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g},$$

and the **horizontal range** on level ground is

$$R = u_x T = \frac{u^2 \sin 2\theta}{g}.$$



Velocity at any point. Because the horizontal and vertical components are perpendicular, the speed and direction at any instant are found by combining them:

$$v = \sqrt{v_x^2 + v_y^2}, \tan\phi = \frac{v_y}{v_x},$$

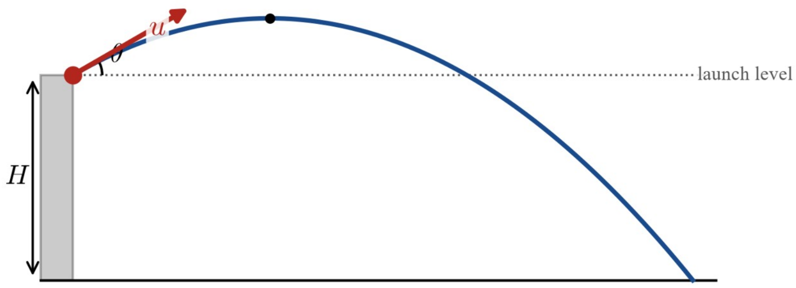
where ϕ is the angle of the velocity to the horizontal. On the way up v_y is positive (velocity points above the horizontal); on the way down v_y is negative (below the horizontal). The speed is a **minimum at the apex**, where it equals $u_x = u\cos\theta$, and is directed horizontally there.

Launch from a height at an angle. If a projectile is launched at an angle but lands at a *different* level (for example thrown from the top of a cliff), the path is **not** symmetric. The time of flight must be found from the full vertical equation. Taking up as positive with the landing point a height H *below* the launch point, the vertical displacement on landing is $-H$:

$$u_y t - \frac{1}{2} g t^2 = -H \Rightarrow \frac{1}{2} g t^2 - u_y t - H = 0,$$

a quadratic in t whose positive root is the time of flight. The horizontal distance is then $x = u_x t$. The speed on impact is quickest found from energy (equivalently $v_y^2 = u_y^2 + 2gH$): the landing speed is

$$v = \sqrt{u^2 + 2gH}.$$

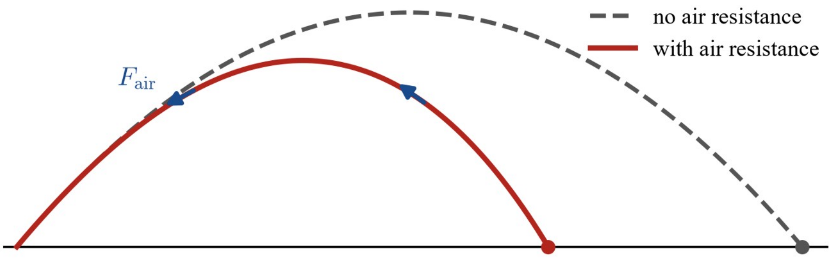


launched at an angle from a height: lands below the launch level

The effect of air resistance (qualitative). Real projectiles move through air, which exerts a resistive force on the object **opposite to its velocity** at every instant, growing larger as the speed increases. This changes the ideal picture above in several ways:

- the horizontal velocity is **no longer constant** — air resistance has a backward horizontal component that continually reduces it, so the object covers less ground and the **range is reduced**;

- the **maximum height is reduced**, because on the way up both the force due to gravity *and* air resistance act to slow the rise;
- the trajectory becomes **asymmetric**: the descent is steeper than the ascent and the object spends longer coming down than it did going up, so the path is no longer a symmetric parabola;
- energy is transferred to the air, so the object returns to its launch level with a **speed less than its launch speed** (in the ideal case the two are equal).



air resistance: lower apex, shorter range, steeper descent

Throughout this subtopic air resistance is ignored unless a question states otherwise, and the constant $g = 9.81\ \text{m s}^{-2}$ (equivalently $9.81\ \text{N kg}^{-1}$) is used.

Worked examples

Example 1 — scaffolded

A ball rolls off the edge of a bench $1.25\ \text{m}$ high with a horizontal speed of $3.5\ \text{m s}^{-1}$. Calculate the time of flight, the horizontal distance it travels before landing, and the speed and direction of its velocity as it strikes the floor. Ignore air resistance.

Working	Comment
Horizontal launch: $u_x = 3.5\ \text{m s}^{-1}$, $u_y = 0$, height $h = 1.25\ \text{m}$, $a = g = 9.81\ \text{m s}^{-2}$ down.	List the data; the launch is horizontal so $u_y = 0$.
Vertical (fall from rest through h): $h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.25)}{9.81}} = 0.505\ \text{s}$.	The vertical motion alone fixes the time of flight.
Horizontal: $x = u_x t = 3.5 \times 0.505 = 1.77\ \text{m}$.	Horizontal velocity is constant.
Vertical velocity on landing: $v_y = g t = 9.81 \times 0.505 = 4.95\ \text{m s}^{-1}$ (down).	Found from $v_y = u_y + g t$ with $u_y = 0$.
Speed: $v = \sqrt{u_x^2 + v_y^2} = \sqrt{3.5^2 + 4.95^2} = 6.06\ \text{m s}^{-1}$.	Combine the perpendicular components.
Direction: $\phi = \tan^{-1} \frac{v_y}{u_x} = \tan^{-1} \frac{4.95}{3.5} = 54.7^\circ$ below the horizontal.	Direction of the impact velocity.

Example 2 — scaffolded

A ball is kicked from level ground with a speed of $20\ \text{m s}^{-1}$ at 35° above the horizontal. Calculate the maximum height reached, the time of flight, and the horizontal range. Ignore air resistance.

Working	Comment
Resolve the launch velocity (up positive): $u_x = 20 \cos 35^\circ = 16.38\ \text{m s}^{-1}$, $u_y = 20 \sin 35^\circ = 11.47\ \text{m s}^{-1}$.	The horizontal component stays constant; the vertical component decreases at g.

Working	Comment
At the apex $v_y = 0$, so $H = \frac{u_y^2}{2g} = \frac{11.47^2}{2(9.81)} = 6.71\,\text{m}$.	Maximum height above the launch level.
Time of flight $T = \frac{2u_y}{g} = \frac{2(11.47)}{9.81} = 2.34\,\text{s}$.	Up and down take equal times (same launch and landing level).
Range $R = u_x T = 16.38 \times 2.34 = 38.3\,\text{m}$.	Constant horizontal velocity over the whole flight.

Example 3 — unscaffolded

A projectile is launched from level ground with a speed of $25\,\text{m s}^{-1}$ at 50° above the horizontal. Calculate the magnitude and direction of its velocity $1.5\,\text{s}$ after launch. Ignore air resistance.

Resolve the launch velocity, taking up as positive:

$$u_x = 25\cos 50^\circ = 16.07\,\text{m s}^{-1}(\text{constant}), u_y = 25\sin 50^\circ = 19.15\,\text{m s}^{-1}.$$

The horizontal velocity is unchanged, $v_x = 16.07\,\text{m s}^{-1}$. After $1.5\,\text{s}$ the vertical velocity is

$$v_y = u_y - g t = 19.15 - 9.81 \times 1.5 = 4.44\,\text{m s}^{-1}(\text{still upward}).$$

Combining the components,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16.07^2 + 4.44^2} = 16.7\,\text{m s}^{-1},$$

$$\phi = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{4.44}{16.07} = 15.4^\circ \text{ above the horizontal}.$$

$\therefore$ $1.5\,\text{s}$ after launch the velocity is $16.7\,\text{m s}^{-1}$ directed 15.4° above the horizontal.

Example 4 — unscaffolded

A stone is thrown from the top of a building $18\,\text{m}$ high with a speed of $15\,\text{m s}^{-1}$ at 25° above the horizontal. Calculate the time it takes to reach the ground, the horizontal distance from the base of the building where it lands, and its speed on impact. Ignore air resistance.

Resolve the launch velocity (up positive):

$$u_x = 15\cos 25^\circ = 13.59\,\text{m s}^{-1}, u_y = 15\sin 25^\circ = 6.339\,\text{m s}^{-1}.$$

The ground is $18\,\text{m}$ below the launch point, so the vertical displacement on landing is $-18\,\text{m}$:

$$u_y t - \frac{1}{2} g t^2 = -18 \Rightarrow 4.905t^2 - 6.339t - 18 = 0.$$

Taking the positive root,

$$t = \frac{6.339 + \sqrt{6.339^2 + 4(4.905)(18)}}{2(4.905)} = \frac{6.339 + \sqrt{393.3}}{9.81} = 2.67\,\text{s}.$$

The horizontal distance is $x = u_x t = 13.59 \times 2.67 = 36.3\,\text{m}$. The impact speed follows from energy (equivalently $v^2 = u^2 + 2gH$ with a drop of $H = 18\,\text{m}$):

$$v = \sqrt{u^2 + 2gH} = \sqrt{15^2 + 2(9.81)(18)} = \sqrt{578.2} = 24.0\,\text{m s}^{-1}.$$

$\therefore$ the stone lands after $2.67\,\text{s}$, a horizontal distance $36.3\,\text{m}$ from the base of the building, at a speed of $24.0\,\text{m s}^{-1}$.

Practice questions

Question 1 A marble rolls off the edge of a table $0.90\,\mathrm{m}$ high at a horizontal speed of $2.4\,\mathrm{m\,s^{-1}}$. Ignore air resistance. (a) Calculate the time taken for the marble to reach the floor. (b) Calculate the horizontal distance from the edge of the table to where it lands. (c) Calculate the speed and direction of the marble's velocity as it strikes the floor.

Question 2 A ball is launched from ground level with a speed of $18\,\mathrm{m\,s^{-1}}$ at an angle of 40° above the horizontal. Ignore air resistance. (a) Show that the time of flight is $2.36\,\mathrm{s}$. (b) Calculate the maximum height reached by the ball. (c) Calculate the horizontal range of the ball.

Question 3 A projectile is launched from ground level at $30\,\mathrm{m\,s^{-1}}$ at an angle of 60° above the horizontal. Ignore air resistance. (a) Calculate the horizontal and vertical components of the initial velocity. (b) Calculate the velocity (magnitude and direction) of the projectile $2.0\,\mathrm{s}$ after launch. (c) Calculate the speed of the projectile at the highest point of its path.

Question 4 From the top of a vertical cliff $40\,\mathrm{m}$ high, one stone is dropped from rest while, at the same instant, a second stone is thrown horizontally with a speed of $12\,\mathrm{m\,s^{-1}}$. Ignore air resistance. (a) State, with a justification referring to the vertical motion of each stone, which stone reaches the base of the cliff first. No calculations are required. (b) Calculate the time taken for the stones to reach the base of the cliff. (c) Calculate the horizontal distance from the base of the cliff at which the thrown stone lands.

Question 5 A ball is projected at an angle above the horizontal. In the absence of air resistance it would follow a symmetric parabola, but in reality air resistance acts on it throughout the flight. (a) State the direction of the air-resistance force on the ball at any instant during its flight. (b) Compare the maximum height and the horizontal range of the real trajectory with those of the ideal (no-air-resistance) path. (c) Explain why, with air resistance, the ball's impact speed is less than its launch speed and why its descent is steeper than its ascent.

Question 6 A stone is thrown from the top of a building $25\,\mathrm{m}$ high with a speed of $16\,\mathrm{m\,s^{-1}}$ at an angle of 30° above the horizontal. Ignore air resistance. (a) Calculate the vertical component of the initial velocity. (b) Calculate the time taken for the stone to reach the ground. (c) Calculate the horizontal distance from the base of the building to where the stone lands.

Question 7 A dart is thrown horizontally at $8.0\,\mathrm{m\,s^{-1}}$ from a height of $1.6\,\mathrm{m}$. Calculate how far horizontally it travels before it hits the ground. Ignore air resistance.

Question 8 A golf ball is struck from level ground at $32\,\mathrm{m\,s^{-1}}$ at 22° above the horizontal. Calculate the maximum height reached and the horizontal range. Ignore air resistance.

Question 9 A ball is projected from ground level at $24\,\mathrm{m\,s^{-1}}$ at an angle of 55° above the horizontal. Calculate the time of flight and the horizontal range. Ignore air resistance.

Question 10 A projectile is launched from level ground at $28\,\mathrm{m\,s^{-1}}$ at 48° above the horizontal. Calculate the magnitude and direction of its velocity $3.0\,\mathrm{s}$ after launch. Ignore air resistance.

Question 11 A ball is kicked from level ground at $27\,\mathrm{m\,s^{-1}}$ at 42° above the horizontal. Ignore air resistance. (a) Show that the maximum height reached is $16.6\,\mathrm{m}$. (b) State the speed of the ball when it returns to ground level, giving a reason.

Question 12 A ball is thrown from a platform $12\,\mathrm{m}$ above the ground with a speed of $16\,\mathrm{m\,s^{-1}}$ at 35° above the horizontal. Calculate the speed at which it strikes the ground. Ignore air resistance.

Question 13 A projectile is to be launched from level ground at an angle of 45° so that it lands $50\,\mathrm{m}$ away on the same level. Calculate the required launch speed. Give your answer to three significant figures. Ignore air resistance.

Question 14 A ball thrown horizontally from a height of $2.0\,\mathrm{m}$ lands a horizontal distance of $5.0\,\mathrm{m}$ away. Calculate the speed at which it was thrown. Ignore air resistance.

Question 15 A stone is thrown from level ground at $20\,\mathrm{m\,s^{-1}}$ at 65° above the horizontal. (a) A student claims that at the highest point of its path the stone's velocity and its acceleration are both zero. State what is wrong with this claim. (b) Calculate the stone's speed and its acceleration at the highest point.

Question 16 A ball is projected at a fixed speed and angle. Compared with its flight in a vacuum, when air resistance is taken into account the horizontal range of the ball is: (Circle your answer.) *greater* / *the same* / *less*. Justify your answer with reference to the horizontal force on the ball. No calculations are required.

Question 17 Two balls are projected from level ground with the same speed, one at 30° and the other at 60° above the horizontal. Ignore air resistance. (a) State, with a reason, whether the two balls have the same horizontal range, or whether one range is greater. (b) State, with a reason, which ball has the longer time of flight.

Question 18 Ignoring air resistance, explain why the horizontal component of a projectile's velocity remains constant throughout its flight. Refer to Newton's first law in your answer.

Question 19 A ball is kicked from level ground at $19\,\mathrm{m\,s^{-1}}$ at an angle of 52° above the horizontal, directly towards a wall $22\,\mathrm{m}$ away. The wall is $6.0\,\mathrm{m}$ high. Determine, with a calculation, whether the ball passes over the top of the wall. Ignore air resistance.

Question 20 A projectile is launched from the edge of a cliff $35\,\mathrm{m}$ above the sea, with a speed of $20\,\mathrm{m\,s^{-1}}$ at 40° above the horizontal. Ignore air resistance. (a) Calculate the total time of flight. (b) Calculate the horizontal distance travelled. (c) Calculate the speed and direction of the velocity when the projectile hits the sea.

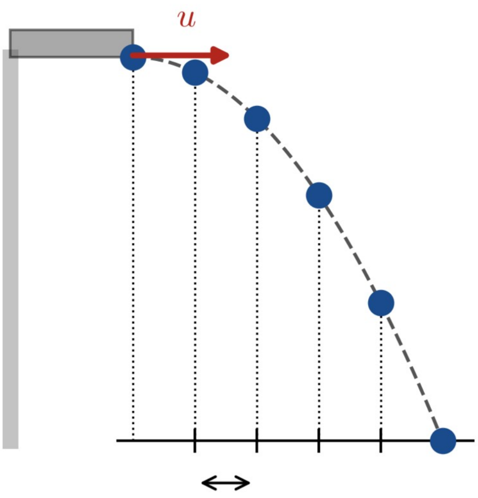
Question 21 A ball is projected at an angle above the horizontal and experiences air resistance throughout its flight. Compare the time taken to rise from the launch level to the maximum height with the time taken to fall from the maximum height back to the launch level. Explain your answer by referring to the direction of the air-resistance force during the ascent and during the descent.

Answers

Question 1 (a) $0.428\,\mathrm{s}$; (b) $1.03\,\mathrm{m}$; (c) $4.84\,\mathrm{m\,s^{-1}}$ at 60.3° below the horizontal. **Question 2** (a) $2.36\,\mathrm{s}$ (shown); (b) $6.82\,\mathrm{m}$; (c) $32.5\,\mathrm{m}$. **Question 3** (a) $u_x = 15.0\,\mathrm{m\,s^{-1}}$, $u_y = 26.0\,\mathrm{m\,s^{-1}}$; (b) $16.3\,\mathrm{m\,s^{-1}}$ at 23.0° above the horizontal; (c) $15.0\,\mathrm{m\,s^{-1}}$. **Question 4** (a) They reach the base at the same time; (b) $2.86\,\mathrm{s}$; (c) $34.3\,\mathrm{m}$. **Question 5** (a) opposite to the ball's velocity; (b) both are reduced; (c) qualitative — see solution. **Question 6** (a) $8.00\,\mathrm{m\,s^{-1}}$; (b) $3.22\,\mathrm{s}$; (c) $44.6\,\mathrm{m}$. **Question 7** $4.57\,\mathrm{m}$. **Question 8** $7.32\,\mathrm{m}$; $72.5\,\mathrm{m}$. **Question 9** $4.01\,\mathrm{s}$; $55.2\,\mathrm{m}$. **Question 10** $20.6\,\mathrm{m\,s^{-1}}$ at 24.7° below the horizontal. **Question 11** (a) $16.6\,\mathrm{m}$ (shown); (b) $27\,\mathrm{m\,s^{-1}}$. **Question 12** $22.2\,\mathrm{m\,s^{-1}}$. **Question 13** $22.1\,\mathrm{m\,s^{-1}}$. **Question 14** $7.83\,\mathrm{m\,s^{-1}}$. **Question 15** (a) the velocity is not zero (the horizontal component remains) and the acceleration is g throughout; (b) $8.45\,\mathrm{m\,s^{-1}}$; $9.81\,\mathrm{m\,s^{-2}}$ downward. **Question 16** less; the horizontal velocity is continually reduced by air resistance. **Question 17** (a) the same range — $\sin(2\times 30^\circ) = \sin(2\times 60^\circ)$ in $R = u^2 \sin 2\theta / g$; (b) the ball projected at 60° — its vertical launch component is larger. **Question 18** there is no horizontal force, so by Newton's first law the horizontal velocity is constant. **Question 19** the ball is $10.8\,\mathrm{m}$ high at the wall, which exceeds $6.0\,\mathrm{m}$, so it clears the wall. **Question 20** (a) $4.29\,\mathrm{s}$; (b) $65.7\,\mathrm{m}$; (c) $33.0\,\mathrm{m\,s^{-1}}$ at 62.3° below the horizontal. **Question 21** the fall takes longer than the rise — on the ascent air resistance adds to gravity, on the descent it opposes gravity; see solution.

Fully-worked solutions

Question 1 Horizontal launch: $u_x = 2.4\,\mathrm{m\,s^{-1}}$, $u_y = 0$, $h = 0.90\,\mathrm{m}$.



equal horizontal steps in equal times;
drop grows as $\frac{1}{2}gt^2$

(a) Fall from rest through h: $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(0.90)}{9.81}} = 0.428\,\mathrm{s}$.

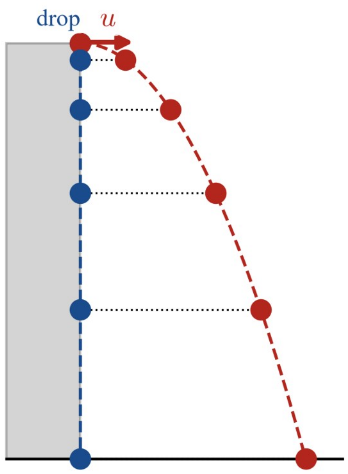
(b) $x = u_x t = 2.4 \times 0.428 = 1.03\,\mathrm{m}$.

(c) Vertical velocity on landing $v_y = gt = 9.81 \times 0.428 = 4.20\,\mathrm{m\,s^{-1}}$ (down). Speed $v = \sqrt{2.4^2 + 4.20^2} = 4.84\,\mathrm{m\,s^{-1}}$, at $\phi = \tan^{-1}\dfrac{4.20}{2.4} = 60.3^\circ$ below the horizontal.

Question 2 Resolve (up positive): $u_x = 18\cos 40^\circ = 13.79\,\mathrm{m\,s^{-1}}$, $u_y = 18\sin 40^\circ = 11.57\,\mathrm{m\,s^{-1}}$. (a) The ball returns to its launch level, so $T = \dfrac{2u_y}{g} = \dfrac{2(11.57)}{9.81} = 2.36\,\mathrm{s}$, as required. (b) $H = \dfrac{u_y^2}{2g} = \dfrac{11.57^2}{2(9.81)} = 6.82\,\mathrm{m}$. (c) $R = u_x T = 13.79 \times 2.36 = 32.5\,\mathrm{m}$.

Question 3 $u = 30\,\text{m s}^{-1}$, $\theta = 60^\circ$. (a) $u_x = 30\cos 60^\circ = 15.0\,\text{m s}^{-1}$; $u_y = 30\sin 60^\circ = 26.0\,\text{m s}^{-1}$. (b) After $2.0\,\text{s}$: $v_x = 15.0\,\text{m s}^{-1}$ (unchanged) and $v_y = u_y - gt = 25.98 - 9.81 \times 2.0 = 6.36\,\text{m s}^{-1}$ (up). Speed $v = \sqrt{15.0^2 + 6.36^2} = 16.3\,\text{m s}^{-1}$, at $\tan^{-1}\frac{6.36}{15.0} = 23.0^\circ$ above the horizontal. (c) At the highest point $v_y = 0$, so the speed equals the horizontal component: $v = u_x = 15.0\,\text{m s}^{-1}$.

Question 4 Both stones start with zero vertical velocity and fall the same height $h = 40\,\text{m}$.



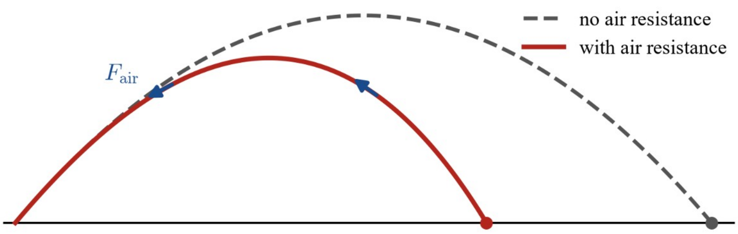
equal heights at equal times: the two balls land together

(a) They reach the base **at the same time**. The vertical motion of each stone is a fall from rest under gravity alone; the horizontal velocity of the thrown stone has no vertical component and does not affect its vertical motion (the two motions are independent). Both therefore fall for the same time.

(b) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(40)}{9.81}} = 2.86\,\text{s}$.

(c) The thrown stone travels $x = u_x t = 12 \times 2.86 = 34.3\,\text{m}$ horizontally.

Question 5



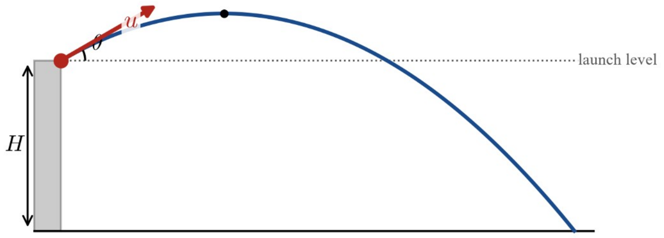
air resistance: lower apex, shorter range, steeper descent

(a) The air-resistance force acts **opposite to the ball's velocity** at every instant (directly against its instantaneous direction of motion).

(b) Both the maximum height and the horizontal range of the real trajectory are **smaller** than those of the ideal parabola.

(c) Air resistance opposes the motion and so does negative work on the ball, removing kinetic energy; the ball therefore returns to its launch level with a **smaller speed** than it was launched with. Because the horizontal velocity is continually reduced during the flight, the ball makes less horizontal progress as it descends while its downward vertical velocity keeps building, so the path steepens — the **descent is steeper than the ascent** and the trajectory is no longer symmetric.

Question 6 Resolve (up positive): $u_x = 16\cos 30^\circ = 13.86\,\text{m s}^{-1}$, $u_y = 16\sin 30^\circ = 8.00\,\text{m s}^{-1}$.



launched at an angle from a height: lands below the launch level

(a) $u_y = 16\sin 30^\circ = 8.00\ \text{m s}^{-1}$.

(b) The ground is $25\ \text{m}$ below the launch point, so $u_y t - \frac{1}{2} g t^2 = -25$, i.e. $4.905t^2 - 8.00t - 25 = 0$. Positive root: $t = \dfrac{8.00 + \sqrt{8.00^2 + 4(4.905)(25)}}{2(4.905)} = \dfrac{8.00 + \sqrt{554.5}}{9.81} = 3.22\ \text{s}$.

(c) $x = u_x t = 13.86 \times 3.22 = 44.6\ \text{m}$.

Question 7 Fall from rest through $h = 1.6\ \text{m}$: $t = \sqrt{\dfrac{2(1.6)}{9.81}} = 0.571\ \text{s}$. Horizontal distance $x = ut = 8.0 \times 0.571 = 4.57\ \text{m}$.

Question 8 $u = 32\ \text{m s}^{-1}$, $\theta = 22^\circ$; $u_y = 32\sin 22^\circ = 11.99\ \text{m s}^{-1}$. Maximum height $H = \dfrac{u_y^2}{2g} = \dfrac{11.99^2}{2(9.81)} = 7.32\ \text{m}$.

Range $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{32^2 \sin 44^\circ}{9.81} = 72.5\ \text{m}$.

Question 9 $u = 24\ \text{m s}^{-1}$, $\theta = 55^\circ$; $u_y = 24\sin 55^\circ = 19.66\ \text{m s}^{-1}$. Time of flight $T = \dfrac{2u_y}{g} = \dfrac{2(19.66)}{9.81} = 4.01\ \text{s}$. Range $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{24^2 \sin 110^\circ}{9.81} = 55.2\ \text{m}$.

Question 10 $u_x = 28\cos 48^\circ = 18.74\ \text{m s}^{-1}$, $u_y = 28\sin 48^\circ = 20.81\ \text{m s}^{-1}$. After $3.0\ \text{s}$: $v_y = 20.81 - 9.81 \times 3.0 = -8.62\ \text{m s}^{-1}$ (i.e. $8.62\ \text{m s}^{-1}$ downward). Speed $v = \sqrt{18.74^2 + 8.62^2} = 20.6\ \text{m s}^{-1}$, at $\tan^{-1} \dfrac{8.62}{18.74} = 24.7^\circ$ below the horizontal.

Question 11 $u = 27\ \text{m s}^{-1}$, $\theta = 42^\circ$; $u_y = 27\sin 42^\circ = 18.07\ \text{m s}^{-1}$. (a) $H = \dfrac{u_y^2}{2g} = \dfrac{18.07^2}{2(9.81)} = 16.6\ \text{m}$, as required. (b) The ball returns to the same level from which it was launched. Ignoring air resistance no energy is lost, so by symmetry (or conservation of energy) it returns with the **same speed** as at launch, $27\ \text{m s}^{-1}$ (directed 42° below the horizontal).

Question 12 Launch speed $u = 16\ \text{m s}^{-1}$; it lands a height $H = 12\ \text{m}$ below the launch point. Using energy, the impact speed is $v = \sqrt{u^2 + 2gH} = \sqrt{16^2 + 2(9.81)(12)} = \sqrt{491.4} = 22.2\ \text{m s}^{-1}$.

Question 13 On level ground $R = \dfrac{u^2 \sin 2\theta}{g}$. With $\theta = 45^\circ$, $\sin 2\theta = \sin 90^\circ = 1$, so $u = \sqrt{Rg} = \sqrt{50 \times 9.81} = \sqrt{490.5} = 22.1\ \text{m s}^{-1}$ (to three significant figures).

Question 14 Fall from rest through $h = 2.0\ \text{m}$: $t = \sqrt{\dfrac{2(2.0)}{9.81}} = 0.6386\ \text{s}$. The horizontal speed is $u = \dfrac{x}{t} = \dfrac{5.0}{0.6386} = 7.83\ \text{m s}^{-1}$.

Question 15 $u = 20\ \text{m s}^{-1}$, $\theta = 65^\circ$. (a) The claim is wrong on both counts. At the highest point the **vertical** velocity is zero, but the **horizontal** velocity is still $u_x = u\cos\theta \neq 0$, so the velocity is not zero — it is horizontal. The acceleration is the acceleration due to gravity, g downward, at *every* point of the flight, including the apex; it is never zero. (b) Speed at the apex $= u_x = 20\cos 65^\circ = 8.45\ \text{m s}^{-1}$ (horizontal). Acceleration $= 9.81\ \text{m s}^{-2}$ directed vertically downward.

Question 16 Less. When air resistance acts, it exerts a force on the ball opposite to the ball's velocity; this force has a horizontal component pointing backwards along the motion. That backward horizontal force gives the ball a horizontal deceleration, so the horizontal velocity — which would be constant in a vacuum — steadily decreases during the flight. The ball therefore covers less horizontal distance in the same descent, and the range is less than in a vacuum.

Question 17 Both balls share the same launch speed u, and on level ground $R = \dfrac{u^2\sin 2\theta}{g}$ and $T = \dfrac{2u\sin\theta}{g}$. (a) For the 30° projection $\sin 2\theta = \sin 60^\circ$; for the 60° projection $\sin 2\theta = \sin 120^\circ$. Since $\sin 120^\circ = \sin 60^\circ$, the two horizontal ranges are **equal** — complementary launch angles give the same range on level ground. (b) Since $\sin 60^\circ > \sin 30^\circ$, the ball projected at 60° has the larger vertical component of the launch velocity, and therefore the **longer** time of flight.

Question 18 In projectile motion (ignoring air resistance) the only force acting on the object is the force due to gravity F_g, which acts vertically downward. There is therefore **no horizontal force** on the projectile, so there is no horizontal acceleration. By Newton's first law, an object with no net force acting on it in a given direction moves at constant velocity in that direction; with no horizontal force, the horizontal component of the projectile's velocity stays constant throughout the flight.

Question 19 Resolve: $u_x = 19\cos 52^\circ = 11.70\ \text{m s}^{-1}$, $u_y = 19\sin 52^\circ = 14.97\ \text{m s}^{-1}$. Time to reach the wall (horizontal): $t = \dfrac{22}{11.70} = 1.88\ \text{s}$. Height of the ball at that instant:
$y = u_y t - \dfrac{1}{2}gt^2 = 14.97 \times 1.88 - 4.905 \times 1.88^2 = 28.14 - 17.34 = 10.8\ \text{m}$. Since $10.8\ \text{m} > 6.0\ \text{m}$, the ball is well above the top of the wall when it arrives, so it **passes over the wall**.

Question 20 Resolve (up positive): $u_x = 20\cos 40^\circ = 15.32\ \text{m s}^{-1}$, $u_y = 20\sin 40^\circ = 12.86\ \text{m s}^{-1}$. (a) The sea is $35\ \text{m}$ below the launch point, so $u_y t - \dfrac{1}{2}gt^2 = -35$, i.e. $4.905t^2 - 12.86t - 35 = 0$. Positive root:
$t = \dfrac{12.86 + \sqrt{12.86^2 + 4(4.905)(35)}}{2(4.905)} = \dfrac{12.86 + \sqrt{852.1}}{9.81} = 4.29\ \text{s}$. (b) $x = u_x t = 15.32 \times 4.29 = 65.7\ \text{m}$. (c) Impact speed from energy: $v = \sqrt{u^2 + 2gH} = \sqrt{20^2 + 2(9.81)(35)} = \sqrt{1087} = 33.0\ \text{m s}^{-1}$. The vertical component on impact is $v_y = \sqrt{u_y^2 + 2gH} = \sqrt{12.86^2 + 2(9.81)(35)} = 29.19\ \text{m s}^{-1}$ (down), so the direction is $\tan^{-1}\dfrac{29.19}{15.32} = 62.3^\circ$ below the horizontal.

Question 21 The descent takes **longer** than the ascent. The air-resistance force always acts opposite to the velocity, so its vertical component points downward on the way up and upward on the way down:

- On the **ascent** the ball moves upward and forward, so air resistance points downward and backward. Its downward vertical component is in the same direction as the force due to gravity F_g, so the downward acceleration is greater than g and the upward vertical velocity is lost quickly.
- On the **descent** the ball moves downward and forward, so air resistance points upward and backward. Its upward vertical component opposes F_g, so the downward acceleration is less than g and the downward vertical velocity builds more slowly.

Each half of the flight covers the same vertical distance (the maximum height), but with a smaller downward acceleration throughout the descent than throughout the ascent, the fall is completed more slowly — equivalently, air resistance continually removes kinetic energy, so the ball crosses the launch level moving downward with a smaller vertical speed than it had moving upward. In a vacuum the two times would be equal.