Physics Units 1 & 2

Chapter 11 — Options: forces, materials and motion

Aboriginal and Torres Strait Islander content has been excluded as accuracy cannot be guaranteed, and it is better to be respectful than cover the entire curriculum inaccurately.

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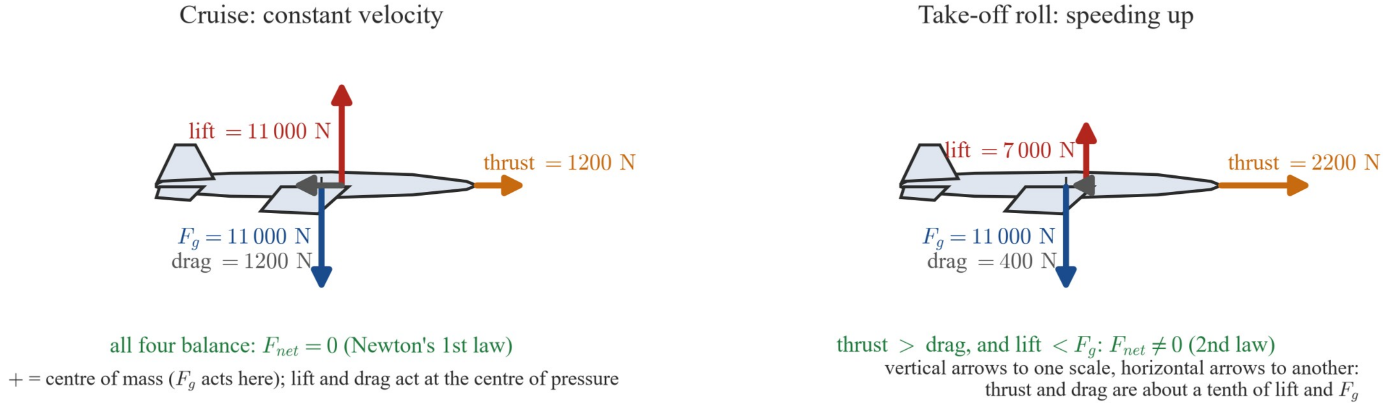
Theory

This section is **Option 2.3 — The physics of flight**, one of the eighteen options of Unit 2, Area of Study 2. A student studies exactly one option; this section is self-contained for a student who has chosen Option 2.3. It applies the force model of Chapter 8 — force vectors, Newton's laws, the force due to gravity F_g, and torques — and the motion ideas of Chapter 7 to a single question: how does an aircraft of many tonnes stay up, move forward, and do what the pilot asks of it? The section closes with the assessed Option task.

> **Scope — VCE Physics Study Design, Unit 2, Area of Study 2, Option 2.3 (key knowledge U2.2.O3.1–.5), quoted verbatim with the two formula images restored as printed:**
>
> - `U2.2.O3.1` "model the forces acting on an aircraft in flight as:" — "the force due to gravity, acting at the centre of mass" · "thrust" · "lift: $F_L = \frac{1}{2} C_L \rho v^2 A$, acting at the centre of pressure" · "drag: $F_D = \frac{1}{2} C_D \rho v^2 A$, acting at the centre of pressure"
> - `U2.2.O3.2` "explain the production of aerodynamic lift with reference to:" — "Bernoulli's principle and pressure differences" · "conservation of momentum and downwash"
> - `U2.2.O3.3` "compare contributions to aerodynamic drag, including skin friction, form and lift-induced"
> - `U2.2.O3.4` "explain the production of thrust with reference to Newton's laws of motion"
> - `U2.2.O3.5` "apply balance of forces and torques with reference to Newton's laws of motion to the different stages of flight and the control of the aircraft"
>
> In the two formulas, C_L is the **lift coefficient** and C_D the **drag coefficient** (numbers that collect the effects of shape and angle), ρ is the density of the air, v is the speed of the aircraft through the air, and A is the wing area.

The four-force model. An aircraft in flight is modelled with exactly four forces.

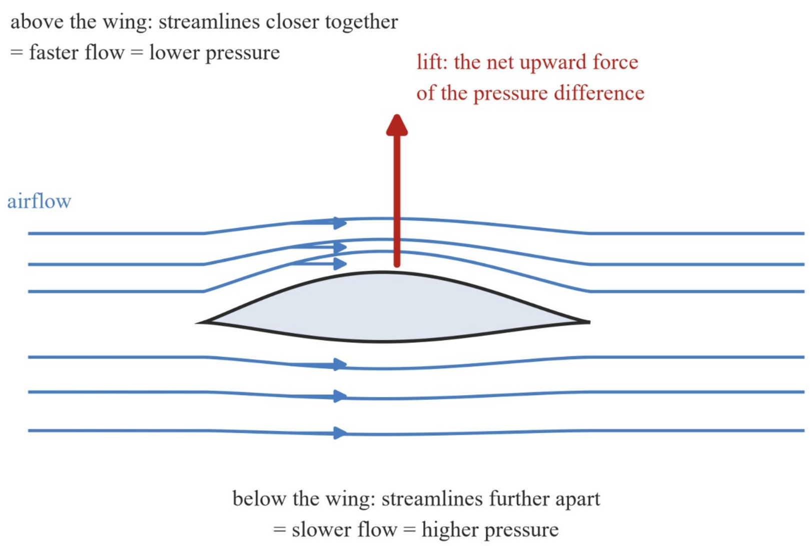
- The **force due to gravity** $F_g = mg$, where $g = 9.8\ \mathrm{N\ kg^{-1}}$ near the surface of Earth (Section 8C). It acts at the **centre of mass** of the aircraft, the single point at which the whole of F_g can be taken to act.
- **Thrust**, the forward force produced by the engines, along the direction the engines push.
- **Lift**, the net force the air exerts on the aircraft perpendicular (at right angles) to the direction of the airflow. It acts at the **centre of pressure**, the single point at which the whole of the air's lift can be taken to act.
- **Drag**, the net force the air exerts on the aircraft opposite to its motion through the air. It too is taken to act at the centre of pressure.



The two panels show the model at work. In steady cruise at constant velocity, all four forces balance in pairs — lift balances F_g, and thrust balances drag — so the net force is zero and, by Newton's first law, the velocity stays constant. During the take-off roll the aircraft is speeding up, so by Newton's second law the forces cannot balance:

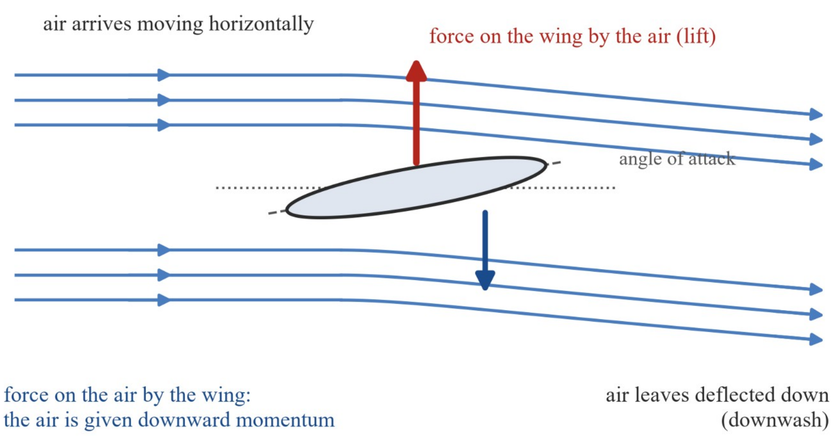
thrust is greater than drag, and lift has not yet grown to F_g (the runway still supports part of F_g through a normal force). Note the honest detail in the figure: thrust and drag are about a tenth of lift and F_g, so the vertical and horizontal pairs are drawn to different scales — what matters is the comparison of arrow lengths *within* each pair.

Lift — Bernoulli's principle and pressure differences. A wing is an **aerofoil**: a shaped body, curved more on top than below (that built-in curve is its **camber**). As the aircraft moves, the air that passes over the top surface is squeezed through a narrower gap — the streamlines above the wing run closer together — and so moves faster than the air below. **Bernoulli's principle** says that where a fluid flows faster, its pressure is lower. The faster flow above the wing therefore has a lower pressure than the slower flow below, and the pressure difference — higher below, lower above — pushes the wing up. Lift is the net upward force of that pressure difference acting over the whole wing area A.



The lift formula of the study design collects this physics: for a given wing, the lift grows with the coefficient C_L (which depends on the wing's shape and on its angle to the airflow), with the air density ρ, with the square of the airspeed v, and with the wing area A. The v^2 is the signature of Bernoulli's principle: double the speed and the pressure difference — and the lift — is four times larger.

Lift — conservation of momentum and downwash. The same lift can be explained without pressures, using momentum. Watch the air: it arrives at the wing moving horizontally, and it leaves the wing deflected downward. Its momentum has therefore changed — it has gained downward momentum — and a change of momentum requires a force (Newton's second law): the wing must have pushed the air down. By Newton's third law, the air pushes the wing up with an equal and opposite force. That upward force is the lift. The downward deflection of the air behind the wing is called the **downwash**, and the **angle of attack** — the angle between the wing and the oncoming airflow — sets how strongly the air is deflected.



The two explanations are not rivals; they are two views of the same event. The pressure difference over the wing *is* the mechanism by which the wing pushes the air down, and both views give the same lift force. In this section you should be able to use either, and to say how they agree.

Drag and its three contributions. Drag is not one effect but several, and the key knowledge asks you to compare three contributions.

- **Skin friction** is the rubbing of the air over the surface of the aircraft — the same idea as friction between solids, applied to a gas sliding over a skin. It grows with the area of the surface the air touches and with the speed, and it is why aircraft surfaces are smooth and clean.
- **Form drag** comes from the shape of the body. Air pushed aside at the front is compressed (higher pressure), and air closing in behind leaves a partial low-pressure wake; the pressure difference between front and back pushes the aircraft backwards. Blunt shapes make large wakes and large form drag; **streamlined** shapes let the air close in smoothly and cut it down. Landing gear, antennas and engines hung on pylons are all sources of form drag, which is why long-range aircraft retract their gear after take-off.
- **Lift-induced drag** is the price paid for lift. Deflecting the air down (the momentum view above) means the air pushes back on the wing up *and slightly backwards*; that backwards component is induced drag. It is largest when a large lift coefficient is needed — at low speed, or in a climb — and small in fast cruise, where the same lift is obtained mostly from speed rather than from strong deflection of the air.

At cruise, skin friction and form drag make up most of the total; at low speed and in the climb, the induced share grows. All three contributions grow with speed squared for a fixed shape and angle, which is why the drag formula has the same $\frac{1}{2}\rho v^2 A$ structure as the lift formula, with C_D collecting the shape-and-angle effects. That growth with v^2 holds at a fixed angle of attack; in level flight at fixed lift the wing instead works at a smaller angle of attack as speed rises, and the induced share then falls as speed rises.

Thrust and Newton's laws. Every engine that moves an aircraft — piston engine and propeller, turbofan, rocket — works the same way: it accelerates a mass of air or gas backwards, and the air or gas pushes the engine forwards. That is Newton's third law as the source of thrust. Newton's second law supplies the size: to accelerate a mass m of air by Δv in a time t requires a force $F = \frac{m \Delta v}{t}$ on the air, and the thrust on the aircraft is the equal and opposite force. A turbofan moves a large mass of air each second by a modest Δv; a rocket moves a small mass by a large Δv; the product — the rate of change of momentum — is the thrust in each case. In steady cruise the thrust exactly balances the drag (first law); on the take-off roll it exceeds the drag, and the difference accelerates the aircraft (second law).

The stages of flight. Newton's first and second laws sort out every stage of a flight. Any stage at constant velocity — including a steady climb or a steady descent — has zero net force; any stage that is speeding up, slowing down or turning has a non-zero net force.

Stage	Motion	Force balance
Take-off roll	speeding up along the runway	thrust > drag; lift $< F_g$, the runway supplies the rest through a normal force
Lift-off	lift reaches F_g	at rotation speed, lift $= F_g$ and the aircraft leaves the ground
Steady climb	constant velocity, path tilted up	all forces balance, but thrust must balance drag *plus* the component of F_g along the flight path, so thrust > drag
Steady cruise	constant velocity, level	lift $= F_g$ and thrust = drag
Steady descent	constant velocity, path tilted down	all forces balance, with thrust reduced below the drag
Landing roll	slowing down	drag plus braking forces > thrust

The steady climb is the stage that most often trips people up: because the velocity is constant, the forces balance even though the aircraft is rising. The climbing comes from the direction of the velocity, not from an unbalanced upward force. What is true is that the engine must work harder than in level cruise, because part of F_g now acts backwards along the flight path and the thrust must balance it as well as the drag.

Control of the aircraft: forces and torques. An aircraft must also rotate in a controlled way — nose up or down (**pitch**), wings up or down (**roll**), nose left or right (**yaw**) — and rotation is governed by torques about the centre of mass, exactly the model taught in Section 8F ($\tau = r_{\perp} F$; clockwise and anticlockwise torques balance in rotational equilibrium). That model is applied here, not re-taught.

The pilot's controls work by moving small forces far from the centre of mass, where they make large torques. Pulling back on the control column tilts the **elevator**, the movable part of the tailplane: the tail lift changes, and that force, acting several metres behind the centre of mass, produces a torque that raises or lowers the nose. This is how the pilot rotates the aircraft at lift-off — raising the nose increases the wing's angle of attack, which increases C_L and so the lift, until lift equals F_g. **Ailerons** at the wingtips change the lift on one wing while reducing it on the other, giving a rolling torque; the **rudder** on the fin makes a sideways force at the tail, giving a yawing torque.

In trimmed, steady flight the torques about the centre of mass also balance. The lift acting at the centre of pressure usually does not pass exactly through the centre of mass, so it makes a torque about it; the tailplane supplies a small force — often a downward one — several metres away, whose torque balances the wing's torque. Because the tail force acts far from the centre of mass, only a small force is needed (you will calculate one in Question 6). The same reasoning — a small force, a long lever arm, a balancing torque — is the one 8F developed for beams and cranes, here applied to an aircraft.

The communication skills assessed by the Option task at the end of this section — evaluating sources, using conventions and representations, discussing factors beyond the physics and taking a justified position — are taught once, in Sections 10A and 10B. This section applies them.

Worked examples

Example 1 — scaffolded

A Cessna 172S Skyhawk, a common four-seat training aircraft, has a maximum take-off mass of $1157\,\text{kg}$ and a wing area of $16.2\,\text{m}^2$. Take $g = 9.8\,\text{N kg}^{-1}$.

Working	Comment
$F_g = m\,g = 1157 \times 9.8 = 1.1 \times 10^4\,\text{N}$.	The force due to gravity at maximum take-off mass, acting at the centre of mass.
In steady level cruise the velocity is constant, so the net force is zero (Newton's first law) and the lift balances F_g: lift $= 1.1 \times 10^4\,\text{N}$.	Lift balances F_g in steady level flight — not "beats" it.
F_g acts at the centre of mass; the lift acts at the centre of pressure.	The two forces act at different points — which is why torques matter (Question 6).

∴ at maximum take-off mass the Cessna's F_g is about $1.1 \times 10^4\,\text{N}$, and in steady level cruise the lift equals it, acting at the centre of pressure.

Source: Textron Aviation, Cessna Skyhawk 172S specifications, retrieved 2026-08-18.

Example 2 — scaffolded

The same Cessna cruises at about $63\,\text{m s}^{-1}$ near sea level, where the air density is about $1.2\,\text{kg m}^{-3}$. Use the lift formula $F_L = \frac{1}{2} C_L \rho v^2 A$ to find the lift coefficient C_L in this cruise.

Working	Comment
Rearrange: $C_L = \frac{2F_L}{\rho v^2 A}$.	Make C_L the subject before substituting.
$F_L = 1.1339 \times 10^4\,\mathrm{N}$ from Example 1 (keep extra figures until the end).	In steady level cruise the lift equals F_g.
$C_L = \frac{2 \times 1.1339 \times 10^4}{1.2 \times 63^2 \times 16.2} = \frac{2.2677 \times 10^4}{7.7157 \times 10^4} = 0.29$.	$v^2 = 3969\,\mathrm{m^2\,s^{-2}}$; the units cancel, so C_L has no unit.

∴ the Cessna cruises at a lift coefficient of about 0.29 — a small number, because at cruise speed the wing needs only a gentle angle of attack to make enough lift.

Example 3 — unscaffolded

For the Cessna in the same cruise, take the drag coefficient as $C_D = 0.030$. Calculate the drag, and hence the thrust the engine must produce to hold the cruise.

The drag is

$$F_D = \frac{1}{2} C_D \rho v^2 A = \frac{1}{2} \times 0.030 \times 1.2 \times 63^2 \times 16.2 = 1.2 \times 10^3\,\mathrm{N}.$$

In steady cruise the velocity is constant, so by Newton’s first law the thrust balances the drag:

$$\text{thrust} = F_D = 1.2 \times 10^3\,\mathrm{N}.$$

∴ the drag is about $1.2 \times 10^3\,\mathrm{N}$ — roughly a tenth of the lift, as the four-force figure promised — and the engine holds the cruise with an equal thrust. Note how efficient steady flight is: the engine does not carry the aircraft up, it only pays for the drag; the lift is supplied free by the airflow over the wing.

Example 4 — unscaffolded

Aerobatic aircraft fly upside down. Using both models of lift, explain how a wing that is shaped to make lift the right way up can still make lift when inverted.

The shape of a wing helps, but it does not decide everything: the angle of attack — the angle between the wing and the oncoming air — decides how the air is deflected and how the streamlines crowd together. In the momentum view this is the clearer story. Lift is the reaction to the wing pushing air down. An inverted wing held at a sufficiently large angle of attack — the pilot holds the nose a little higher relative to the flight path than when the right way up — still deflects the air down, so the air still pushes the wing up, and the aircraft flies inverted. In the pressure view, the same tilt makes the flow over the (now upper) surface faster than the flow below, so the pressure is again lower above and higher below, and the pressure difference again pushes up. Both views give the same conclusion: lift follows the angle of attack as well as the shape. This is also why aerobatic aircraft such as the Extra 330 are built with wings of nearly symmetric cross-section: with the camber removed, the wing behaves the same way up and inverted, and the pilot simply sets the angle of attack.

Practice questions

Question 1 *(scaffolded)* An Airbus A350-1000 has a maximum take-off mass of $316\,\mathrm{t}$ ($3.16 \times 10^5\,\mathrm{kg}$) and a wing area of $442\,\mathrm{m^2}$. Take $g = 9.8\,\mathrm{N\,kg^{-1}}$. (a) Calculate the force due to gravity on the A350 at maximum take-off mass. (b) State the lift the wings must produce for the aircraft to leave the ground, and the physical law that connects the two at lift-off. (c) State where each of the two forces is taken to act.

Source: Airbus, A350 factsheet, retrieved 2026-08-18.

Question 2 *(scaffolded)* Early in a flight an A350-1000 of mass $3.16 \times 10^5\,\mathrm{kg}$ flies level at $80\,\mathrm{m\,s^{-1}}$ through air of density $1.2\,\mathrm{kg\,m^{-3}}$. (a) Write down the lift formula and rearrange it to make C_L the subject. (b) Calculate C_L at this speed. (Wing area $442\,\mathrm{m^2}$.) (c) Compare your value with the Cessna's cruise value of 0.29 (Example 2), and suggest why a large, heavily loaded aircraft flying slowly needs the larger coefficient.

Question 3 *(scaffolded)* In its cruise, the Cessna of Examples 1–3 has a total drag of $1.16 \times 10^3\,\mathrm{N}$, made up of about $45\,\%$ skin friction, $15\,\%$ form drag and $40\,\%$ lift-induced drag. (a) Calculate the size of each of the three contributions. (b) State which contribution is largest in this cruise. (c) In a steady climb at lower speed, the induced share rises to about $60\,\%$ of the total. Explain why the induced contribution grows when the aircraft flies more slowly.

Question 4 *(scaffolded)* A high-bypass turbofan engine at take-off power takes in and accelerates about $340\,\mathrm{kg}$ of air each second, increasing its backward speed by about $600\,\mathrm{m\,s^{-1}}$ (values assumed for this exercise). (a) Calculate the change of momentum given to the air each second. (b) Use Newton's second law to find the force the engine exerts on the air. (c) Use Newton's third law to state the thrust on the aircraft, and name the force pair.

Question 5 *(scaffolded)* Early in its take-off roll, a Cessna 172S with pilot and fuel — mass $1000\,\mathrm{kg}$ — has a thrust of $2200\,\mathrm{N}$ and a drag of $400\,\mathrm{N}$. (a) Calculate the net horizontal force on the aircraft. (b) Calculate its acceleration along the runway. (c) At this low speed the lift is small. Explain why the vertical forces still balance, naming the force the runway supplies.

Question 6 *(scaffolded)* In steady cruise, the Cessna's lift acts at the centre of pressure, $0.10\,\mathrm{m}$ behind the centre of mass, while $F_g = 1.1339 \times 10^4\,\mathrm{N}$ acts at the centre of mass. The tailplane is $3.0\,\mathrm{m}$ behind the centre of mass. (a) Calculate the torque about the centre of mass produced by the lift. (b) State the direction (up or down) of the tail force that balances this torque, with a one-line reason. (c) Calculate the size of that tail force. (Treat the lift as balancing F_g; the small correction to the lift is not required.)

Question 7 The Cessna 172S wing can reach a maximum lift coefficient of about 1.6 with flaps down. Calculate the minimum airspeed at which the wings can support the aircraft at maximum take-off mass ($1157\,\mathrm{kg}$) in air of density $1.2\,\mathrm{kg\,m^{-3}}$.

Question 8 Aircraft take off into the wind whenever they can. Using the lift formula, explain why a headwind shortens the take-off roll, and why the quantity that matters for lift is the speed through the air, not the speed over the ground.

Question 9 The drag formula has the same v^2 dependence as the lift formula. The Cessna's cruise drag is $1.2 \times 10^3\,\mathrm{N}$ at $63\,\mathrm{m\,s^{-1}}$. Assuming C_D unchanged, calculate the drag at $126\,\mathrm{m\,s^{-1}}$, and state the factor by which the required thrust has increased.

Question 10 In a steady climb at constant velocity the forces on an aircraft balance, yet the thrust is greater than the drag. Explain both facts at once, with reference to the direction of F_g relative to the flight path.

Question 11 A lightly loaded Cessna 172S of mass $950\,\mathrm{kg}$ is on its take-off roll with a thrust of $2.3 \times 10^3\,\mathrm{N}$ and a drag of $1.9 \times 10^3\,\mathrm{N}$. Calculate its acceleration.

Question 12 Extending the flaps increases the camber of the wing and raises the maximum lift coefficient. Explain why flaps allow a slower and shorter take-off and a slower, steeper approach to landing.

Question 13 In a trimmed cruise, the tailplane of a light aircraft exerts a downward force of $250\,\mathrm{N}$ at $3.2\,\mathrm{m}$ behind the centre of mass. The lift in this cruise is $1.0 \times 10^4\,\mathrm{N}$. Calculate how far behind the centre of mass the centre of pressure lies.

Question 14 To rotate at lift-off, the pilot pulls back on the control column and the elevator at the tail tilts. Explain, in terms of forces and torques about the centre of mass, how a small force at the tail raises the nose, and why raising the nose increases the lift.

Question 15 The Cessna cruises at $63\,\mathrm{m\,s^{-1}}$ in air of density $1.2\,\mathrm{kg\,m^{-3}}$ at a given C_L. At a higher altitude the density is $1.1\,\mathrm{kg\,m^{-3}}$. Calculate the airspeed needed at the higher altitude to produce the same lift at the same C_L, and comment on the result.

Question 16 The Cessna 172S has fixed landing gear; the A350-1000 retracts its gear after take-off. Identify which contribution to drag the gear is mainly responsible for, and explain why retracting it matters far more in cruise than on the take-off roll.

Question 17 Using the momentum view of lift, explain why producing lift always costs something — the lift-induced drag — and why that cost is proportionally larger at low speed than in fast cruise.

Question 18 An A350-1000 in high-altitude cruise flies through air of density $0.38\,\mathrm{kg\,m^{-3}}$. Its drag coefficient in this cruise is about 0.020, and the thrust holding the cruise is about $1.0 \times 10^{5}\,\mathrm{N}$. Use the drag formula to calculate the cruise speed. (Wing area $442\,\mathrm{m^2}$.) Comment on why aircraft cruise high and fast.

Answers

Question 1 (a) $3.1 \times 10^6\,\mathrm{N}$; (b) $3.1 \times 10^6\,\mathrm{N}$ — at lift-off the lift reaches F_g (Newton's first/second law: the vertical forces balance as the aircraft leaves the ground); (c) F_g at the centre of mass; lift at the centre of pressure. **Question 2** (a) $C_L = 2F_L/(\rho v^2 A)$; (b) 1.8; (c) see solution — more mass to carry at less speed needs a larger coefficient, i.e. a larger angle of attack. **Question 3** (a) skin friction $\approx 520\,\mathrm{N}$, form $\approx 170\,\mathrm{N}$, induced $\approx 460\,\mathrm{N}$; (b) skin friction; (c) see solution — at low speed the same lift needs a larger C_L, i.e. stronger downward deflection of the air. **Question 4** (a) $2.0 \times 10^5\,\mathrm{kg\ m\ s^{-1}}$ per second; (b) $2.0 \times 10^5\,\mathrm{N}$; (c) $2.0 \times 10^5\,\mathrm{N}$ forwards on the aircraft — the pair is engine-on-air (backwards) and air-on-engine (forwards). **Question 5** (a) $1800\,\mathrm{N}$; (b) $1.8\,\mathrm{m\ s^{-2}}$; (c) the runway supplies a normal force that, with the small lift, balances F_g. **Question 6** (a) $1.1 \times 10^3\,\mathrm{N\ m}$; (b) down — an upward lift behind the centre of mass pitches the nose down, so the tail must push down to balance; (c) $380\,\mathrm{N}$. **Question 7** $27\,\mathrm{m\ s^{-1}}$. **Question 8** See solution — lift depends on v^2 with v the speed of the air over the wing; a headwind adds to that speed at any groundspeed. **Question 9** $4.8 \times 10^3\,\mathrm{N}$; a factor of 4. **Question 10** See solution — constant velocity means zero net force, but part of F_g acts backwards along the climb path, so thrust balances drag plus that component. **Question 11** $0.42\,\mathrm{m\ s^{-2}}$. **Question 12** See solution — a larger C_L gives the required lift at a lower v, since $F_L \propto C_L v^2$. **Question 13** $0.080\,\mathrm{m}$. **Question 14** See solution — the tail force acts several metres from the centre of mass, so its torque is large; raising the nose raises the angle of attack and hence C_L and the lift. **Question 15** $66\,\mathrm{m\ s^{-1}}$; thinner air needs more speed for the same lift. **Question 16** Form drag; see solution — in cruise the gear's wake costs thrust for hours, and drag grows with v^2. **Question 17** See solution — the reaction to pushing air down is tilted slightly backwards; at low speed the same lift needs stronger deflection. **Question 18** $240\,\mathrm{m\ s^{-1}}$; high, thin air cuts the drag, and the v^2 in the lift formula makes the lost density cheap to buy back with speed.

Fully-worked solutions

Question 1 (a) $F_g = mg = 3.16 \times 10^5 \times 9.8 = 3.1 \times 10^6\,\mathrm{N}$. (b) At lift-off the lift must reach $3.1 \times 10^6\,\mathrm{N}$. As the lift grows to equal F_g, the vertical net force becomes zero and the runway's normal force falls to zero — the aircraft leaves the ground (Newton's first and second laws). (c) F_g is taken to act at the centre of mass; the lift at the centre of pressure.

Question 2 (a) $F_L = \frac{1}{2} C_L \rho v^2 A \Rightarrow C_L = \frac{2F_L}{\rho v^2 A}$. (b) $F_L = F_g = 3.0968 \times 10^6\,\mathrm{N}$, so

$$C_L = \frac{2 \times 3.0968 \times 10^6}{1.2 \times 80^2 \times 442} = \frac{6.1936 \times 10^6}{3.3946 \times 10^6} = 1.8.$$

(c) The A350 carries far more force due to gravity per square metre of wing than the Cessna, and at $80\,\mathrm{m\ s^{-1}}$ the v^2 term is still small; to make enough lift it must therefore use a much larger coefficient — a larger angle of attack and the help of flaps and slats. This is why take-off and climb are flown at high C_L, and cruise at low C_L and high speed.

Question 3 (a) Skin friction: $0.45 \times 1.16 \times 10^3 = 520\,\mathrm{N}$. Form: $0.15 \times 1.16 \times 10^3 = 170\,\mathrm{N}$. Induced: $0.40 \times 1.16 \times 10^3 = 460\,\mathrm{N}$. (b) Skin friction is the largest single contribution in this cruise. (c) Lift must still balance F_g in the climb, but at lower speed the $\rho v^2 A$ part of the lift formula is smaller, so C_L must be larger — the wing must deflect the air down more strongly. Stronger deflection means a larger backward component of the air's push on the wing, so the induced drag, and its share of the total, grows.

Question 4 (a) Each second, $\Delta p = m\Delta v = 340 \times 600 = 2.04 \times 10^5\,\mathrm{kg\ m\ s^{-1}}$. (b) Newton's second law: the force on the air equals its rate of change of momentum, $F = \frac{\Delta p}{t} = \frac{2.04 \times 10^5}{1} = 2.0 \times 10^5\,\mathrm{N}$, directed backwards on the air. (c) By Newton's third law the air exerts an equal and opposite force on the engine: a thrust of $2.0 \times 10^5\,\mathrm{N}$ forwards. The pair is the force on the air by the engine and the force on the engine by the air.

Question 5 (a) $F_{\text{net}} = 2200 - 400 = 1800\,\text{N}$ forwards. (b) $a = \dfrac{F_{\text{net}}}{m} = \dfrac{1800}{1000} = 1.8\,\text{m s}^{-2}$. (c) Vertically, the aircraft is not accelerating — it stays on the runway — so the vertical forces balance there too: the small lift plus the **normal force** from the runway balance F_g. As the speed grows, the lift grows (v^2) and the normal force shrinks, until at lift-off it reaches zero.

Question 6 (a) $\tau = r_{\perp} F = 0.10 \times 1.1339 \times 10^4 = 1.1 \times 10^3\,\text{N m}$. An upward lift acting behind the centre of mass tends to pitch the nose down. (b) The tail force must produce a nose-up torque, so it must act **downwards** on the tail, which lies behind the centre of mass. (c) Balancing torques about the centre of mass (rotational equilibrium, Section 8F):

$$F_{\text{tail}} \times 3.0 = 1133.9 \Rightarrow F_{\text{tail}} = \frac{1133.9}{3.0} = 380\,\text{N}.$$

A few hundred newtons at the tail balances over eleven thousand newtons at the wing — the lever arm does the work.

Question 7 At the minimum speed the wing is at maximum C_L and the lift equals F_g:

$$F_g = 1157 \times 9.8 = 1.1339 \times 10^4\,\text{N},$$

$$v = \sqrt{\frac{2F_L}{C_L \rho A}} = \sqrt{\frac{2 \times 1.1339 \times 10^4}{1.6 \times 1.2 \times 16.2}} = \sqrt{729} = 27\,\text{m s}^{-1}.$$

About $27\,\text{m s}^{-1}$ — near $100\,\text{km h}^{-1}$ — is the slowest the Cessna can fly at this mass; below it the wing stalls.

Question 8 Model answer. The v in the lift formula is the speed of the aircraft *through the air*, because lift is made by the airflow over the wing. With a headwind of, say, $10\,\text{m s}^{-1}$, an aircraft rolling at $50\,\text{m s}^{-1}$ over the ground already has an airspeed of $60\,\text{m s}^{-1}$, and since lift grows with v^2, it reaches the lift-off lift sooner — a shorter ground roll and a lower groundspeed at lift-off. With a tailwind the opposite happens, which is why runways are used into the wind wherever the wind allows.

Question 9 With C_D, ρ and A unchanged, $F_D \propto v^2$. Doubling v multiplies the drag by $2^2 = 4$:

$$F_D = 4 \times 1.2 \times 10^3 = 4.8 \times 10^3\,\text{N}.$$

The thrust needed to hold that speed is four times larger — one reason aircraft have a best-speed range rather than "as fast as possible".

Question 10 Model answer. A steady climb at constant velocity is an equilibrium: by Newton's first law the net force is zero, so the four forces (plus any small tail force) balance. But the flight path is tilted, and F_g still acts straight down; resolved along the path, part of F_g acts backwards along it. The thrust must balance the drag *and* that backwards component, so thrust > drag even though everything balances. Perpendicular to the path, the lift balances the remaining component of F_g — which is slightly less than F_g, so in a steady climb the lift is actually a little smaller than the force due to gravity.

Question 11 $F_{\text{net}} = 2.3 \times 10^3 - 1.9 \times 10^3 = 400\,\text{N}$, so $a = \dfrac{400}{950} = 0.42\,\text{m s}^{-2}$.

Question 12 Model answer. With flaps extended the wing has a larger C_L, and since $F_L = \frac{1}{2} C_L \rho v^2 A$, the lift needed to balance F_g is reached at a lower speed. A lower lift-off speed means a shorter take-off roll; a lower approach speed means a shorter landing roll and a steeper, slower approach over obstacles. The price is extra drag — flaps raise C_D too — which is why flaps are retracted once the aircraft is climbing or cruising.

Question 13 Rotational equilibrium about the centre of mass: the tail's nose-up torque must equal the lift's nose-down torque,

$$250 \times 3.2 = 1.0 \times 10^4 \times d \Rightarrow d = \frac{800}{1.0 \times 10^4} = 0.080\,\mathrm{m}.$$

The centre of pressure lies $0.080\,\mathrm{m}$ behind the centre of mass.

Question 14 Model answer. Tilting the elevator changes the tail's lift by a few hundred newtons, but that force acts several metres behind the centre of mass, so its torque $\tau = r_{\perp} F$ is large enough to rotate the whole aircraft (Section 8F). Pulling back makes the tail force push the tail down and the nose up. Raising the nose increases the wing's angle of attack, which increases C_L; since $F_L = \frac{1}{2} C_L \rho v^2 A$, the lift grows at the same speed, and at rotation it reaches F_g and the aircraft lifts off.

Question 15 Same lift at the same C_L and A means ρv^2 is unchanged, so

$$v_2 = v_1 \sqrt{\frac{\rho_1}{\rho_2}} = 63 \times \sqrt{\frac{1.2}{1.1}} = 66\,\mathrm{m\,s^{-1}}.$$

In thinner air the aircraft must fly faster to make the same lift — the beginning of the reason aircraft cruise high *and* fast (Question 18).

Question 16 Model answer. Landing gear is blunt and unstreamlined, so its contribution is mainly **form drag**: the gear and its struts push the air aside and leave a low-pressure wake behind them. On the take-off roll and in the climb the speed is low and the gear's drag is a small part of the total; in cruise, drag grows with v^2, and the gear's wake would demand thrust — fuel — for hours. Retracting the gear removes the wake, which is why every long-range aircraft, from the A350 upwards, retracts its gear as soon as it is safely airborne. The Cessna keeps fixed gear because at its low cruise speed the gear's drag is small and the simplicity and reliability are worth more.

Question 17 Model answer. In the momentum view, lift is the air's equal and opposite push on the wing, in answer to the wing pushing the air down. That reaction is perpendicular to the deflected flow only in the ideal case; in reality the force the air exerts on the wing is tilted slightly backwards from the vertical, and the backward component is the lift-induced drag. At low speed the $\rho v^2 A$ part of the lift formula is small, so the same lift needs a larger C_L — a stronger downward deflection of a given mass of air — and the stronger the deflection, the larger the backward tilt and the induced drag. In fast cruise the lift comes mostly from speed, the deflection is gentle, and the induced share is small.

Question 18 In steady cruise thrust = drag $= 1.0 \times 10^5\,\mathrm{N}$, so from $F_D = \frac{1}{2} C_D \rho v^2 A$:

$$v = \sqrt{\frac{2F_D}{C_D \rho A}} = \sqrt{\frac{2 \times 1.0 \times 10^5}{0.020 \times 0.38 \times 442}} = \sqrt{5.95 \times 10^4} = 240\,\mathrm{m\,s^{-1}}.$$

About $240\,\mathrm{m\,s^{-1}}$ — roughly $870\,\mathrm{km\,h^{-1}}$. High air is thin air: with ρ small, the drag is small for a given speed, and the lift lost to the thin air is bought back cheaply because lift also grows with v^2. Cruising high and fast is how a heavy aircraft makes the four forces balance cheaply.

Option task — Project Sunrise: how does a 300-tonne aircraft fly 17 000 km?

This is the assessed Option task that closes Option 2.3. It uses the study design's own Outcome 2 task types *"a physics-referenced response to an issue"* and *"an analysis, including calculations, of physics concepts applied to real-world contexts"*. The study design specifies no mark allocations at Units 1 & 2 — levels of achievement are a matter for school decision, and only satisfactory completion is reported to the VCAA — so the **30 marks below are house convention**. The task is assessed against the shared *Communicating physics* key knowledge ($U2.2.CP$.1–.5), which is taught in Sections 10A and 10B, applied to the physics of this section ($U2.2.O3$.1–.5).

Qantas's **Project Sunrise** plans nonstop scheduled flights from Sydney to London — about $17\,000\,\mathrm{km}$, roughly 20 hours — flown with the Airbus A350-1000, the same aircraft type used throughout this section. No scheduled flight of this length has ever been operated before; the route makes a good issue, because the physics that allows it is exactly the physics of this section, and the arguments for and against it reach beyond the physics.

Source: Qantas, Project Sunrise, retrieved 2026-08-18. Source: Airbus, A350 factsheet, retrieved 2026-08-18.

Data for this task (A350-1000): maximum take-off mass 3.16×10^5 kg; mass in cruise 2.4×10^5 kg; wing area $442\,\mathrm{m}^2$; cruise speed $244\,\mathrm{m\,s}^{-1}$ at an altitude where the air density is $0.38\,\mathrm{kg\,m}^{-3}$; cruise drag coefficient $C_D \approx 0.020$; $g = 9.8\,\mathrm{N\,kg}^{-1}$.

The task. Produce a response of 800–1200 words, with labelled diagrams and calculations, that does all of the following:

1. **Model the forces across the flight.** For the take-off roll, the steady climb, the steady cruise and the landing roll, state how the four forces compare (which balance, which do not, and why), using Newton's first and second laws. List the sources you used and evaluate the validity of each ($U2.2.O3.1$, $U2.2.O3.5$; $U2.2.CP.1$).
2. **Explain the lift twice.** Explain how the wings support the aircraft in cruise using both required models — Bernoulli's principle and pressure differences, and conservation of momentum and downwash — and state how the two explanations agree ($U2.2.O3.2$; $U2.2.CP.2$, $U2.2.CP.3$).
3. **Calculate.** Using the data above: (a) the force due to gravity at maximum take-off mass, and the lift at lift-off; (b) the lift coefficient C_L in the cruise; (c) the thrust in the cruise, and a comment comparing it with the force due to gravity. Show your working in full, with units and correct significant figures ($U2.2.O3.1$, $U2.2.O3.4$; $U2.2.CP.2$).
4. **Draw and discuss the model.** Include a labelled force diagram for the cruise, showing the four forces, their directions and their points of application (centre of mass and centre of pressure), and state at least one limitation of the four-force model — for example, that lift and drag are really spread over the whole surface, or that "steady" cruise is an average over turbulence and small climbs and descents ($U2.2.O3.1$; $U2.2.CP.3$).
5. **Judge the issue and take a position.** Ultra-long-haul flying has costs and benefits that the physics alone does not settle: fuel burn and greenhouse emissions over 20 hours against the time saved, the connections a nonstop Sydney–London service creates, and the engineering cost of an aircraft built for the route. Discuss at least one of these sociocultural, economic, legal or political factors genuinely, and finish with a justified position on the statement: *"Physics makes Project Sunrise possible; that does not, by itself, make it worthwhile."* ($U2.2.O3.3$; $U2.2.CP.4$, $U2.2.CP.5$).

Your teacher will set the due date and any in-class conditions. Work that uses data from a source must name the source and its retrieval date.

Mark allocation (30 marks — house convention).

Criterion	Study-design dot point	Marks
Validity of sources evaluated	$U2.2.CP.1$	4
Physics concepts, terms and conventions applied correctly, with full calculations	$U2.2.CP.2$	8
Force diagrams, models and representations used and organised clearly, with limitations discussed	$U2.2.CP.3$	6
Sociocultural, economic, legal or political factors discussed	$U2.2.CP.4$	4
Physics-based stance, recommendation or solution justified	$U2.2.CP.5$	8
Total		**30**

Marking rubric.

Band	$CP.1$ sources (4)	$CP.2$ physics (8)	$CP.3$ representations (6)	$CP.4$ factors (4)	$CP.5$ stance (8)
High	4: every source named and its validity evaluated	7–8: the four-force model, both lift models and Newton's laws used correctly throughout; calculations complete with units and correct significant figures	5–6: force diagrams complete and clearly labelled, points of application correct; limitations of the model discussed	4: at least one relevant factor discussed in genuine depth	7–8: position fully justified by the physics of the response
Medium	3: sources named and validity considered	4–6: model mostly correct; working mostly complete	3–4: diagrams present with minor omissions; limitations noted briefly	3: a relevant factor identified with some discussion	4–6: position given with partial justification
Low	1–2: sources listed with little or no evaluation	1–3: model partly correct; working incomplete	1–2: diagrams missing key labels, directions or points of application	1–2: a factor named with little discussion	1–3: position stated with little support
None	0	0	0	0	0

Marks are awarded for the quality of the reasoning, not for the conclusion reached: two students who take opposite positions on Project Sunrise can both receive full marks in $CP.5$ if both justify their position from correct physics.

Chapter 11 Options: forces, materials and motion — 11B Option 2.4: How do forces act on structures and materials?

Theory

This section is **Option 2.4 — The physics of structures and materials**, one of the eighteen options of Unit 2, Area of Study 2. A student studies exactly one option; this section is self-contained for a student who has chosen Option 2.4. It applies the force model of Chapter 8 — force vectors, Newton's laws, the force due to gravity F_g, and torques — and the energy ideas of Chapter 9 — work done and Hooke's Law — to things that are built to hold loads up: bridges, towers, cranes, roofs, balconies and shelves. The section closes with the assessed Option task.

> **Scope — VCE Physics Study Design, Unit 2, Area of Study 2, Option 2.4 (key knowledge U2.2.O4.1 –.5), quoted verbatim:**
>
> - `U2.2.O4.1` "identify different types of external forces due to loads such as compression, tension and shear, that can act on a body, including gravitational forces"
> - `U2.2.O4.2` "analyse the stability of structures using translational and rotational forces (torques) and identify that gravitational forces act at the centre of mass"
> - `U2.2.O4.3` "calculate the stress and strain resulting from the application of compressive and tensile forces and loads to materials in structures, σ = F/A, ε = Δl/l"
> - `U2.2.O4.4` "analyse the behaviour of materials under load in terms of extension and compression, including Young's modulus, E = σ/ε, area under stress versus strain graphs and the elastic or plastic behaviour of materials under load"
> - `U2.2.O4.5` "evaluate the suitability of different materials for use in structures, comparing tensile and compressive strength and stiffness or flexibility under load"
>
> In the formulas, σ is the stress, ε the strain and E the Young's modulus; each is defined in the theory below.

Structures and loads. A **structure** is anything built to support itself and to support other things. Every structure has to carry forces without breaking, without deforming too much, and without toppling over. The key knowledge for this option uses the term *load*, which the study design does not define. In this subtopic, a **load** is treated as any external force acting on a structure — the gravitational forces on the structure's own parts and on whatever the structure carries (vehicles, people, stored goods), together with pushes and pulls from wind, from moving loads and from the supports that hold the structure up — and every question here is answerable on that definition alone.

Gravitational forces and the centre of mass. Every part of a structure is pulled downwards by Earth. The force due to gravity on a body is $F_g = mg$, where $g = 9.8\ \text{N kg}^{-1}$ near the surface of Earth (Section 8C), and the total gravitational force on the body acts at its **centre of mass** — the single point at which the whole of F_g can be taken to act. A structure must carry its own gravitational forces as well as those of its load: a bridge supports its own deck as well as the traffic on it. Identifying the line of action of F_g through the centre of mass is the first step in any analysis of stability.

Compression, tension and shear. The external forces due to loads act on the members of a structure in three characteristic ways, and a member can experience more than one at a time.

- **Compression** squeezes a member: forces push inwards from opposite ends and the member shortens. Vertical columns, bridge piers, arches and the legs of a chair carry compressive forces.
- **Tension** stretches a member: forces pull outwards from opposite ends and the member gets longer and thinner. Cables, ropes, chains and tie rods carry tensile forces. A cable can only pull — if you push on a rope it simply goes slack — so cable members carry tension only.
- **Shear** acts when two neighbouring parts of a body are pushed sideways in opposite directions, so that they tend to slide across each other. The bolts that join two plates, the rivets in a steel frame, and the layers of a loaded beam experience shear. In this option shear is identified, not calculated; the calculations are limited to compressive and tensile forces, as the study design specifies.

When a horizontal beam sags under a load it **bends**, and bending combines the first two: the top of the beam is squeezed (compression) while the bottom is stretched (tension). This one fact explains a great deal about how real structures are built, as you will see below.

Stability of structures. The key knowledge for this option asks you to analyse the *stability* of structures, a term the study design does not define. In this subtopic, a structure is treated as **stable** when it remains in equilibrium under the forces acting on it — it does not translate and it does not rotate — and every question here is answerable on that definition alone.

The torque model used in this analysis is taught in Section 8F ($\tau = r_{\perp} F$, clockwise and anticlockwise torques, the conditions for equilibrium); it is applied here to structures, not re-taught. The two conditions a stable structure satisfies are:

- **translational equilibrium** — the net force on the structure is zero, so it does not accelerate in any direction. Upward support forces balance downward gravitational forces;
- **rotational equilibrium** — the net torque about any point is zero, so it does not begin to rotate. Clockwise torques balance anticlockwise torques.

Because the gravitational forces act at the centre of mass, stability often comes down to a single question: does the line of action of F_g fall inside the base of the structure? If it does, the torques can balance and the structure stands. If a structure tilts far enough that the line of action of F_g falls outside its base, the gravitational force produces a net torque that tips the structure further — it topples. Two design features follow directly: a **wider base** and a **lower centre of mass** both allow a structure to tilt further before the line of action of F_g leaves the base, and both make it harder to topple. A tower crane carries a large concrete balance block on its short arm so that the torques about the tower balance when a heavy load hangs from the long arm; a bridge deck stays level because the upward support forces from its piers produce torques that balance the torques of every load on the deck.

Stress. Whether a member breaks under a force depends on more than the size of the force — it also depends on how much material the force is spread across. A person standing on a wooden floor does not break it; the same person standing on the same floor in stiletto heels might dent it, because the same force acts over a far smaller area. The quantity that allows this comparison is the **stress**:

$$\sigma = \frac{F}{A},$$

where F is the magnitude of the compressive or tensile force applied to the member and A is the cross-sectional area over which the force acts. The SI unit of stress is the pascal, $\mathrm{Pa} = \mathrm{N\ m^{-2}}$; engineering values are commonly given in megapascals ($1\,\mathrm{MPa} = 10^6\,\mathrm{Pa}$) or gigapascals ($1\,\mathrm{GPa} = 10^9\,\mathrm{Pa}$). A **tensile stress** stretches a member; a **compressive stress** squeezes it. For the same force, a larger cross-sectional area gives a smaller stress — which is why columns that carry heavy loads are made thick, and why foundations spread a building's load over a wide area of ground.

Strain. The corresponding measure of how much a member deforms is the **strain**:

$$\varepsilon = \frac{\Delta l}{l},$$

where l is the original length of the member and Δl is its change in length — the extension under tension, or the shortening under compression. Strain is a ratio of two lengths, so it has no unit. It is sometimes quoted as a percentage: a strain of 1.0×10^{-3} is $0.10\,\%$. A strain of zero means the member has not deformed at all.

Young's modulus and stiffness. For many materials, at small strains, stress is directly proportional to strain: double the stress and the strain doubles. The constant of proportionality is **Young's modulus**:

$$E = \frac{\sigma}{\varepsilon}.$$

Young's modulus has the same unit as stress, the pascal, and is usually quoted in gigapascals. It is a measure of the **stiffness** of a material — its resistance to being deformed elastically. A material with a large Young's modulus (steel, about 200 GPa) is stiff: it takes a large stress to produce even a small strain. A material with a small Young's modulus (rubber, or timber loaded across its grain) is flexible: the same stress produces a larger strain. Stiffness is not the same as strength — a material can be stiff yet break at a modest stress, or flexible yet very hard to break. The distinction matters when materials are chosen for a structure, and it returns below.

This linear behaviour is the material-level version of Hooke's Law for a spring ($F = k\,x$, Section 9B). The spring constant k depends on the object — its material *and* its size and shape — while Young's modulus depends on the material alone. Two steel cables, one thick and one thin, have different spring constants but the same Young's modulus.

Stress–strain graphs. In a materials test, a sample of the material is pulled (or squeezed) in a machine while the force and the change in length are measured, and the stress and strain are calculated and graphed. A stress–strain graph for a ductile metal such as mild steel has the following features:

- a **straight-line region** starting at the origin, where stress is proportional to strain. The gradient of this line is Young's modulus. Deformation in this region is elastic;
- a **curved region** beyond the straight line, where the material **yields**: a small extra stress produces a large extra strain, and the deformation is no longer fully elastic;
- a **maximum stress** — the greatest stress the material sustains. The maximum tensile stress a material can withstand is its **tensile strength**;
- **fracture**, where the sample finally breaks, at a strain larger than the strain at the maximum stress for ductile materials.

Materials differ in how much curved region they have before fracture. A **brittle** material — glass, cast iron, concrete — shows little or no plastic deformation: it snaps soon after the straight-line region ends. A **ductile** material — mild steel, aluminium, copper — stretches a long way in the plastic region before it finally breaks, giving visible warning first.

Area under a stress–strain graph. The area between a stress–strain graph and the strain axis has a direct physical meaning: it is the energy absorbed per unit volume of the material up to that strain, in $\mathrm{J\,m^{-3}}$. To see why, multiply a stress by a strain:

$$\sigma \times \varepsilon = \frac{F}{A} \times \frac{\Delta l}{l} = \frac{F\,\Delta l}{A\,l} = \frac{\text{work done on the member}}{\text{volume of the member}},$$

since $A \times l$ is the volume of the member and $F\,\Delta l$ is the work done by the force (Section 9A). Within the straight-line region the area is a triangle, $1/2\,\sigma\,\varepsilon$; beyond it, the area is found by counting squares, exactly as for a curved velocity–time graph (Section 7B). A material whose graph encloses a large area up to fracture absorbs a large amount of energy before it breaks — a desirable property anywhere a structure might face sudden or impact loads.

Elastic and plastic behaviour. A material behaves **elastically** when it returns to its original shape and size after the load is removed. Elastic behaviour holds up to the **elastic limit** of the material. Beyond the elastic limit the behaviour is **plastic**: when the load is removed, some of the deformation remains, and the member is permanently stretched, bent or squeezed. A structure overloaded past its elastic limit is damaged even if it does not break — a scaffold plank that stays bent after a load is removed is no longer safe to use.

Choosing materials. Evaluating whether a material suits a structure means comparing what the loads demand against what the material offers:

- **tensile strength** — the maximum tensile stress the material can withstand before fracture, and **compressive strength** — the maximum compressive stress. The two strengths of one material can be very different;
- **stiffness** — a large Young's modulus, so the member deforms only a little under load — against **flexibility**, a small Young's modulus, where some give is wanted.

Some common structural materials, compared qualitatively:

Material	Tensile strength	Compressive strength	Stiffness	Typical structural use
steel	high	high	high (stiff)	beams, cables, tie rods, reinforcement
concrete	low	high	high (stiff)	columns, piers, foundations
timber	moderate	moderate	lower (more flexible)	frames, beams, small bridges
rope, cable	high (tension only)	none — goes slack	low	suspended loads, ties

Two consequences follow from the table. First, **concrete is strong where it is squeezed and weak where it is stretched**. A plain concrete beam under load bends, and its underside is in tension — exactly where concrete is weakest — so a plain concrete beam cracks and fails at modest loads, while a concrete column, which carries compression only, is an excellent use of the material. Second, the two materials can be combined: in **reinforced concrete**, steel bars are cast into the concrete near the surfaces that will be stretched, so that the steel carries the tensile stress and the concrete carries the compressive stress. Each material is placed where it is strongest.

A practical investigation connects directly to this section: hang a wire or a strip of material from a fixed support, add known loads, measure the extension, and graph stress against strain. From the graph you can read Young's modulus (the gradient of the straight region) and estimate the energy absorbed per unit volume (the area under the graph). The study design requires a minimum of ten hours of practical work across Areas of Study 1 and 2 of Unit 2; your teacher will organise how those hours are spent.

The equations of this section are summarised below.

Quantity	Symbol	Equation	Unit
stress	σ	$\sigma = F / A$	Pa (N m^{-2})
strain	ε	$\varepsilon = \Delta l / l$	none
Young's modulus	E	$E = \sigma / \varepsilon$	Pa
energy absorbed per unit volume	—	area under the stress–strain graph	J m^{-3}

Worked examples

Example 1 — scaffolded

A tower crane on a Melbourne building site lifts a steel beam of mass $1200\,\text{kg}$. The beam hangs at rest from a steel cable of original length $10\,\text{m}$ and cross-sectional area $6.0 \times 10^{-5}\,\text{m}^2$. The Young's modulus of the cable steel is $2.0 \times 10^{11}\,\text{Pa}$. Use $g = 9.8\,\text{N kg}^{-1}$.

(a) Calculate the magnitude of the tensile force in the cable.
(b) Calculate the tensile stress in the cable.
(c) Calculate the tensile strain in the cable.
(d) Calculate the extension of the cable under this load.

Working	Comment
(a) The beam hangs at rest, so the net force on it is zero: the upward tension in the cable equals the force due to gravity on the beam (Section 8B).	Equilibrium: forces balance when $a = 0$.
$F = F_g = mg = 1200 \times 9.8 = 1.2 \times 10^4\,\text{N}$	$F_g = mg$ with $g = 9.8\,\text{N kg}^{-1}$.
(b) $\sigma = \dfrac{F}{A} = \dfrac{11760}{6.0 \times 10^{-5}} = 2.0 \times 10^8\,\text{Pa}$	Stress = force ÷ cross-sectional area; final answer to 2 significant figures.

Working	Comment
(c) $E = \dfrac{\sigma}{\varepsilon}$, so $\varepsilon = \dfrac{\sigma}{E} = \dfrac{1.96 \times 10^{8}}{2.0 \times 10^{11}} = 9.8 \times 10^{-4}$	Strain has no unit. The unrounded stress is carried through.
(d) $\varepsilon = \dfrac{\Delta l}{l}$, so $\Delta l = \varepsilon\, l = 9.8 \times 10^{-4} \times 10 = 9.8 \times 10^{-3}\,\text{m} = 9.8\,\text{mm}$	Extension = strain × original length.

The cable stretches about $9.8\,\text{mm}$ when the beam is lifted — invisible in a photograph, but real, and exactly what the stress and strain calculations predict.

Example 2 — scaffolded

A footbridge crosses Merri Creek in Melbourne. Its walkway is uniform, of length $8.0\,\text{m}$ and mass $400\,\text{kg}$, and rests on a pier at each end. A maintenance vehicle of mass $600\,\text{kg}$ parks on the walkway, $2.0\,\text{m}$ from the left end. The bridge is at rest. Use $g = 9.8\,\text{N kg}^{-1}$.

(a) Identify the vertical forces acting on the walkway and state the two conditions for the walkway to be stable.
(b) Taking the left end of the walkway as the pivot, calculate the upward support force exerted by the right pier.
(c) Use translational equilibrium to calculate the upward support force exerted by the left pier.
(d) Check your answer to (c) by taking torques about the right end.

Working	Comment
(a) Downward: F_g on the walkway, $400 \times 9.8 = 3.9 \times 10^{3}\,\text{N}$, acting at the centre of the walkway ($4.0\,\text{m}$ from each end); F_g on the vehicle, $600 \times 9.8 = 5.9 \times 10^{3}\,\text{N}$, acting $2.0\,\text{m}$ from the left end. Upward: the support forces N_L and N_R at the two piers.	Gravitational forces act at the centre of mass (Section 8C).
Conditions: net force $= 0$ (translational equilibrium) and net torque $= 0$ about any point (rotational equilibrium).	The two stability conditions, applied in Section 8F.
(b) About the left end, the two gravitational forces give clockwise torques and N_R gives an anticlockwise torque:	The pivot force N_L has zero torque about the pivot — choosing the pivot at an unknown force removes it from the torque equation.
$N_R \times 8.0 = 3920 \times 4.0 + 5880 \times 2.0 = 15680 + 11760 = 2$	$\tau = r_{\perp} F$ for each force.
$N_R = \dfrac{27440}{8.0} = 3430 \approx 3.4 \times 10^{3}\,\text{N}$	Clockwise and anticlockwise torques balance.
(c) Upward forces = downward forces: $N_L + N_R = 3920 + 5880 = 9800$	Net force is zero.
$N_L = 9800 - 3430 = 6370 \approx 6.4 \times 10^{3}\,\text{N}$	The left pier carries the larger share, because the vehicle is closer to it.
(d) About the right end: $N_L \times 8.0 = 3920 \times 4.0 + 5880 \times 6.0 = 15680 + 35280 = 5$, giving $N_L = 6370\,\text{N}$. ✓	The same result from a different pivot — a check on the whole calculation.

Example 3 — unscaffolded

A sample of structural steel is tested in a materials-testing machine, and its stress–strain graph is drawn. The graph is a straight line from the origin to a strain of 2.0×10^{-3}, where the stress is $4.0 \times 10^{8}\,\text{Pa}$. The graph then curves, reaches a maximum stress of $5.0 \times 10^{8}\,\text{Pa}$ at a strain of 1.5×10^{-2}, and the sample finally fractures at a strain of 2.2×10^{-2}.

(a) Calculate Young's modulus for this steel.
(b) Calculate the energy absorbed per unit volume of the sample up to the end of the straight-line region.

(c) The sample is loaded to a strain of 1.0×10^{-2} and then unloaded. Describe what happens and name the behaviour shown.

(d) State whether this steel is ductile or brittle, justifying your answer from the graph.

The straight-line region ends at strain 2.0×10^{-3}, stress $4.0 \times 10^{8}\,\text{Pa}$.

(a) Young's modulus is the gradient of the straight-line region of the stress–strain graph:

$$E = \frac{\sigma}{\varepsilon} = \frac{4.0 \times 10^{8}}{2.0 \times 10^{-3}} = 2.0 \times 10^{11}\,\text{Pa}.$$

(b) Up to the end of the straight-line region the area under the graph is a triangle:

$$\text{energy per unit volume} = \frac{1}{2} \times 2.0 \times 10^{-3} \times 4.0 \times 10^{8} = 4.0 \times 10^{5}\,\text{J m}^{-3}.$$

(c) A strain of 1.0×10^{-2} lies well beyond the end of the straight-line region (2.0×10^{-3}), so the sample has passed its elastic limit. When the load is removed the sample does not return to its original length: it keeps a permanent extension. The behaviour is **plastic** deformation — the steel has yielded and been permanently stretched.

(d) ∴ the steel is **ductile**: the graph shows a long plastic region, with the strain increasing from 2.0×10^{-3} at the end of the elastic region to 2.2×10^{-2} at fracture — more than ten times the elastic strain. A brittle material, by contrast, would fracture almost immediately after the straight-line region ended, with little or no plastic deformation. The large plastic region also means the steel gives visible warning — it stretches — before it breaks.

Example 4 — unscaffolded

A small footbridge is to be built over a creek in rural Victoria. Under the design load, the maximum tensile stress in the underside of a beam is estimated at $8.0 \times 10^{6}\,\text{Pa}$ and the maximum compressive stress in the top of the beam at $6.0 \times 10^{6}\,\text{Pa}$. Two materials are proposed for the beam. Their approximate properties, supplied for this problem, are:

Material	Tensile strength	Compressive strength	Young's modulus
Material C (concrete)	$3.0 \times 10^{6}\,\text{Pa}$	$3.0 \times 10^{7}\,\text{Pa}$	$3.0 \times 10^{10}\,\text{Pa}$
Material S (steel)	$2.5 \times 10^{8}\,\text{Pa}$	$2.5 \times 10^{8}\,\text{Pa}$	$2.0 \times 10^{11}\,\text{Pa}$

(a) Decide, for each material, whether a plain (unreinforced) beam can carry the design load. Justify your decisions.

(b) Explain why material C is still an excellent choice for the piers (the vertical columns) of the bridge.

(c) Evaluate the two materials for the beam and recommend a solution, giving reasons based on the physics of this section.

(d) Material C: the underside of the beam is in tension, and the tensile stress there, $8.0 \times 10^{6}\,\text{Pa}$, exceeds the tensile strength of concrete, $3.0 \times 10^{6}\,\text{Pa}$. A plain concrete beam would crack on its underside and fail. Material C alone is **not suitable** for the beam.

Material S: both design stresses sit far below both strengths of steel ($8.0 \times 10^{6}\,\text{Pa}$ and $6.0 \times 10^{6}\,\text{Pa}$ against $2.5 \times 10^{8}\,\text{Pa}$), so a plain steel beam carries the design load with a large margin. Material S is **suitable**.

(b) A pier carries compression only — the loads above squeeze it. The compressive strength of concrete, $3.0 \times 10^{7}\,\text{Pa}$, comfortably exceeds the compressive stresses a pier experiences, and concrete's weakness in tension is never called upon. Concrete is also cheap and does not rust, so it suits a pier even though it would fail as a plain beam.

(c) Model answer. A plain concrete beam fails because bending puts its underside in tension, and concrete's tensile strength is far below the tensile stress the design load produces. Steel carries both the tensile and the compressive stresses with a wide margin, so a steel beam works on its own. The strongest overall solution is to combine the materials: use **reinforced concrete**, casting steel bars into the underside of the beam where the tensile stress acts, so that the steel carries the tension and the concrete carries the compression. Each material is placed where it is strongest. Any one of these positions — a steel beam, or a reinforced-concrete beam — is acceptable provided the justification compares the design stresses against the tensile and compressive strengths as above, and a full evaluation also notes a practical factor: steel is costly and must be protected from rust, while concrete is cheap but needs the steel bars where tension occurs.

Practice questions

Question 1 *(scaffolded)* A steel storage rack in a Melbourne warehouse has vertical uprights supporting horizontal shelf boards, and thin diagonal steel rods bracing the frame. (a) Boxes placed on a shelf board cause it to sag slightly, squeezing the top of the board and stretching the bottom. Identify the type of force at the top and at the bottom of the board. (b) The diagonal rods are stretched when the frame sways. Identify the type of force in a rod. (c) The uprights are squeezed by everything resting on them. Identify the type of force in an upright. (d) Name the force exerted on the rack by Earth, and state the point at which the total of this force acts.

Question 2 *(scaffolded)* A concrete column in an office building has a cross-sectional area of $0.25\,\mathrm{m}^2$ and supports a section of the building of mass $5000\,\mathrm{kg}$. Use $g = 9.8\,\mathrm{N\ kg}^{-1}$. (a) Calculate the force due to gravity on the section of the building. (b) Calculate the compressive stress in the column. (c) A second column, of twice the cross-sectional area, supports the same load. Calculate the stress in the second column and state the effect of the larger area.

Question 3 *(scaffolded)* A timber post of original length $3.0\,\mathrm{m}$ under a house shortens by $1.2\,\mathrm{mm}$ when the load above it is applied. (a) Convert $1.2\,\mathrm{mm}$ to metres. (b) Calculate the strain in the post. (c) Express the strain as a percentage.

Question 4 *(scaffolded)* In a laboratory test, a strip of a polymer is stretched. While the stress–strain graph is still a straight line, one reading gives a stress of $3.0 \times 10^{7}\,\mathrm{Pa}$ and a strain of 1.5×10^{-3}. (a) Calculate Young's modulus of the polymer. (b) A second polymer has a Young's modulus of $4.0 \times 10^{10}\,\mathrm{Pa}$. State which polymer is stiffer, and why. (c) Strips of the two polymers have identical dimensions and carry the same tensile force. State which strip stretches more, and justify your answer.

Question 5 *(scaffolded)* A tower crane at Docklands has a horizontal jib (arm). A large concrete balance block of mass $2000\,\mathrm{kg}$ sits on the jib $4.0\,\mathrm{m}$ from the tower on one side. A load hangs from the jib on the opposite side, $8.0\,\mathrm{m}$ from the tower. The mass of the jib itself may be ignored. Use $g = 9.8\,\mathrm{N\ kg}^{-1}$. (a) Calculate the torque about the tower produced by the balance block, and state the direction of this torque, given that the hanging load tends to rotate the jib clockwise. (b) Calculate the mass of the largest load the crane can hold at $8.0\,\mathrm{m}$ with the jib balanced. (c) Explain, in terms of torques, what happens if a heavier load is hung at the same point.

Question 6 *(scaffolded)* The stress–strain graph for a sample of a material is a straight line from the origin to a strain of 1.0×10^{-3}, where the stress is $6.0 \times 10^{7}\,\mathrm{Pa}$. The graph then curves, and the sample fractures at a strain of 1.6×10^{-3}. (a) Calculate Young's modulus of the material. (b) Calculate the energy absorbed per unit volume up to the end of the straight-line region. (c) State what happens if the sample is unloaded after reaching a strain of 1.3×10^{-3}, and name the behaviour.

Question 7 A steel tie rod in the roof structure of a Melbourne sports stadium carries a tensile force of $6.0 \times 10^{4}\,\mathrm{N}$. The design requirement is that the stress in the rod must not exceed $2.0 \times 10^{8}\,\mathrm{Pa}$. (a) Calculate the minimum cross-sectional area of the rod. (b) Explain why engineers design structures so that the stress in normal use sits well below the strength of the material.

Question 8 A student says: "Concrete is strong, so a plain concrete beam is a good choice for any bridge span." Evaluate this claim, referring to tensile and compressive strength and to the stresses in a loaded beam.

Question 9 A bookcase of height $2.0\,\mathrm{m}$ and width $0.80\,\mathrm{m}$ stands on a level floor. It is uniform, of mass $60\,\mathrm{kg}$, and a horizontal force is applied at its top, pushing perpendicular to its side. (a) Taking torques about the bottom edge on the side being pushed towards, calculate the force that just begins to tip the bookcase. Assume it tips before it slides. (b)

Explain why packing the heaviest books onto the bottom shelf makes the bookcase harder to topple, referring to the centre of mass and the line of action of the force due to gravity.

Question 10 A horizontal timber beam rests on a support at each end and sags when a heavy load is placed at its centre. State which surface of the beam — top or bottom — is in compression and which is in tension, and hence state where a crack would first appear in a plain concrete beam loaded this way. Justify your answer.

Question 11 Two wires, each of original length $4.0\,\mathrm{m}$ and cross-sectional area $2.0\times10^{-6}\,\mathrm{m}^2$, hang vertically from a roof beam. One is steel ($E = 2.0\times10^{11}\,\mathrm{Pa}$) and one is aluminium ($E = 7.0\times10^{10}\,\mathrm{Pa}$). Each wire supports a load of mass $50\,\mathrm{kg}$ at rest. Use $g = 9.8\,\mathrm{N\ kg^{-1}}$. (a) Calculate the extension of each wire. (b) Compare the two extensions and explain the difference in terms of Young's modulus.

Question 12 A farming family in rural Victoria is replacing a footbridge over a creek on their property. Two designs are quoted: steel beams, or timber beams. Compare the two materials for this use, referring to stiffness, strength and flexibility, and justify a recommendation. (Reasoning is assessed; either recommendation can receive full marks.)

Question 13 The stress–strain graph for a sample of a metal is a straight line from the origin to a strain of 2.5×10^{-3}, where the stress is $1.0\times10^{8}\,\mathrm{Pa}$. Beyond this point the graph is horizontal at $1.0\times10^{8}\,\mathrm{Pa}$ (the material stretches at constant stress) until it fractures at a strain of 1.0×10^{-2}. (a) Calculate Young's modulus of the metal. (b) Estimate the total energy absorbed per unit volume up to fracture. (c) Explain what a large area under a stress–strain graph means for a structure that might face sudden loads.

Question 14 A salesperson advertises a material: "It is very strong — it will barely stretch at all." Explain why this statement confuses two different properties of materials, and give an example of a material that is strong but stretches a lot, and one that is stiff but breaks easily.

Question 15 A column in a city office tower transmits a downward force of $2.0\times10^{6}\,\mathrm{N}$ to the concrete footing beneath it. The ground under the footing can safely support a maximum stress of $4.0\times10^{5}\,\mathrm{Pa}$. (a) Calculate the minimum area of the footing in contact with the ground. (b) Explain, in terms of stress, why the footing is made much wider than the column above it.

Question 16 A steel scaffold plank is overloaded on a building site. It does not break, but it remains bent after the load is removed. Explain what has happened to the plank in terms of elastic and plastic behaviour, and state whether the plank is safe to reuse. Justify your answer.

Question 17 Workers use a uniform plank of mass $200\,\mathrm{kg}$ and length $4.0\,\mathrm{m}$ as a temporary platform. The plank rests on the flat roof of a building, with $1.5\,\mathrm{m}$ of it extending horizontally over the edge, and the rest lying on the roof. Use $g = 9.8\,\mathrm{N\ kg^{-1}}$. (a) State where the force due to gravity on the plank acts, and how far this point is from the edge of the roof. (b) Calculate the torque about the edge of the roof produced by the force due to gravity on the plank. (c) Calculate the mass of the heaviest person who can stand at the outer end of the plank before it begins to tip. (d) Explain how sliding the plank further onto the roof increases the safe load.

Question 18 A small country town must replace a damaged single-span bridge over a creek. Two proposals are received: proposal A is a plain (unreinforced) concrete beam; proposal B is a steel-reinforced concrete beam. Evaluate the two proposals and recommend one, referring to tensile and compressive strength, the behaviour of materials under load, and one non-physics factor such as cost or appearance.

Answers

Question 1 (a) top: compression; bottom: tension; (b) tension; (c) compression; (d) the force due to gravity, F_g; at the centre of mass. **Question 2** (a) 4.9×10^4 N; (b) 2.0×10^5 Pa; (c) 9.8×10^4 Pa — doubled area halves the stress. **Question 3** (a) 1.2×10^{-3} m; (b) 4.0×10^{-4}; (c) 0.040%. **Question 4** (a) 2.0×10^{10} Pa; (b) the second polymer — larger E; (c) the first polymer strip — see solution. **Question 5** (a) 7.8×10^4 N m, anticlockwise; (b) 1.0×10^3 kg; (c) see solution — the net torque is no longer zero. **Question 6** (a) 6.0×10^{10} Pa; (b) 3.0×10^4 J m^{-3}; (c) a permanent deformation remains — plastic behaviour. **Question 7** (a) 3.0×10^{-4} m^2; (b) see solution. **Question 8** See solution — the claim is only partly correct. **Question 9** (a) 1.2×10^2 N; (b) see solution. **Question 10** Top in compression, bottom in tension; a crack appears in the underside (the tension side) — see solution. **Question 11** (a) steel 4.9 mm, aluminium 14 mm; (b) see solution. **Question 12** See solution — reasoning assessed; either recommendation acceptable if justified. **Question 13** (a) 4.0×10^{10} Pa; (b) $\approx 8.8 \times 10^5$ J m^{-3}; (c) see solution. **Question 14** See solution — strength ≠ stiffness. **Question 15** (a) 5.0 m^2; (b) see solution. **Question 16** See solution — plastic deformation; not safe to reuse. **Question 17** (a) at the centre of the plank, 0.50 m inside the edge; (b) 980 N m; (c) 67 kg; (d) see solution. **Question 18** See solution — proposal B recommended.

Fully-worked solutions

Question 1 (a) Bending combines compression and tension: the sagging board is squeezed along its top (compression) and stretched along its bottom (tension). (b) A stretched rod carries **tension**. (c) A squeezed upright carries **compression**. (d) The force exerted on the rack by Earth is the **force due to gravity**, F_g. The total gravitational force acts at the rack's **centre of mass**.

Question 2 (a) $F_g = mg = 5000 \times 9.8 = 4.9 \times 10^4\ \text{N}$. (b) $\sigma = \frac{F}{A} = \frac{4.9 \times 10^4}{0.25} = 1.96 \times 10^5 \approx 2.0 \times 10^5\ \text{Pa}$. (c) $\sigma = \frac{4.9 \times 10^4}{0.50} = 9.8 \times 10^4\ \text{Pa}$. Doubling the cross-sectional area halves the stress: the same force is spread over twice the material.

Question 3 (a) $1.2\ \text{mm} = 1.2 \times 10^{-3}\ \text{m}$. (b) $\varepsilon = \frac{\Delta l}{l} = \frac{1.2 \times 10^{-3}}{3.0} = 4.0 \times 10^{-4}$. The post is shortening, so this is a compressive strain. (c) $4.0 \times 10^{-4} = 0.040\%$.

Question 4 (a) $E = \frac{\sigma}{\varepsilon} = \frac{3.0 \times 10^7}{1.5 \times 10^{-3}} = 2.0 \times 10^{10}\ \text{Pa}$. (b) The second polymer, with $E = 4.0 \times 10^{10}\ \text{Pa}$, is stiffer. A larger Young's modulus means a larger stress is needed for the same strain. (c) Identical dimensions and the same force mean the same stress in both strips. Since $\varepsilon = \sigma / E$, the strip with the smaller Young's modulus — the first polymer — has the larger strain and stretches more.

Question 5 (a) $\tau = r_\perp F = 4.0 \times (2000 \times 9.8) = 4.0 \times 19600 = 7.8 \times 10^4\ \text{N m}$. The hanging load tends to rotate the jib clockwise, so the balance block's torque is **anticlockwise** — the two torques oppose each other. (b) For the jib to balance, the clockwise torque of the load equals the anticlockwise torque of the balance block: $F_{g,\text{load}} \times 8.0 = 78400$, so $F_{g,\text{load}} = 9800\ \text{N}$ and $m = \frac{9800}{9.8} = 1.0 \times 10^3\ \text{kg}$. (c) A heavier load produces a clockwise torque greater than the anticlockwise torque of the balance block. The net torque about the tower is no longer zero, so the jib is no longer in rotational equilibrium and begins to rotate towards the load side — the crane tips.

Question 6 (a) $E = \frac{\sigma}{\varepsilon} = \frac{6.0 \times 10^7}{1.0 \times 10^{-3}} = 6.0 \times 10^{10}\ \text{Pa}$. (b) Area $= \frac{1}{2} \times 1.0 \times 10^{-3} \times 6.0 \times 10^7 = 3.0 \times 10^4\ \text{J m}^{-3}$. (c) A strain of 1.3×10^{-3} lies beyond the straight-line region, which ended at 1.0×10^{-3}, so the sample has passed its elastic limit. When unloaded it does not return to its original length; a permanent deformation remains. The behaviour is **plastic**.

Question 7 (a) $\sigma=\frac{F}{A}$, so the minimum area is $A=\frac{F}{\sigma}=\frac{6.0\times10^{4}}{2.0\times10^{8}}=3.0\times10^{-4}\,\mathrm{m}^{2}$. (b) Model answer. The design stress assumes normal, expected loads. In real use a structure can face loads larger than expected — extra traffic, gusts of wind, a crowd moving together — and materials can weaken with time and rust. If the normal stress sits close to the strength of the material, any unexpected load takes the stress past the strength and the member breaks, possibly without warning. Keeping the normal stress well below the strength leaves a margin that covers the unexpected.

Question 8 Model answer. The claim is only partly correct. Concrete does have a high compressive strength, so it is an excellent material wherever loads squeeze it — columns, piers and foundations. But a beam carrying a load bends, and bending puts the underside of the beam in tension and the top in compression. Concrete's tensile strength is far smaller than its compressive strength, so the underside of a plain concrete beam cracks at modest loads and the beam fails. Concrete is therefore not a good choice for a bridge span unless steel reinforcement is added to carry the tensile stress — and it is never a good choice for a *plain* beam.

Question 9 (a) The bookcase is uniform, so its centre of mass is at its geometric centre: $0.40\,\mathrm{m}$ from either side. About the bottom edge on the side it tips towards, the force due to gravity acts at a perpendicular distance of $0.40\,\mathrm{m}$, and the applied force acts at a height of $2.0\,\mathrm{m}$. Tipping just begins when the two torques balance: $F\times2.0=F_g\times0.40=(60\times9.8)\times0.40=235$, so $F=\frac{235}{2.0}=1.2\times10^{2}\,\mathrm{N}$. (b) Model answer. Moving the heaviest books to the bottom shelf lowers the bookcase's centre of mass (the books are rearranged, not added). When the bookcase tilts, it topples only once the line of action of the force due to gravity falls outside its base. With a lower centre of mass, the bookcase must tilt through a larger angle before that happens, so it is harder to topple.

Question 10 Model answer. The sagging beam is compressed along its top surface and stretched — in tension — along its bottom surface. A crack in a plain concrete beam would therefore appear first in the underside, at mid-span, because concrete's tensile strength is far below its compressive strength: the tensile stress in the underside reaches the tensile strength long before the compressive stress in the top reaches the compressive strength. This is exactly why steel bars are placed near the underside of concrete beams.

Question 11 Each wire supports a load at rest, so the tensile force in each wire equals the force due to gravity on its load: $F=mg=50\times9.8=490\,\mathrm{N}$. The stress is the same in both wires: $\sigma=\frac{F}{A}=\frac{490}{2.0\times10^{-6}}=2.45\times10^{8}\,\mathrm{Pa}$. (a) Steel: $\varepsilon=\frac{\sigma}{E}=\frac{2.45\times10^{8}}{2.0\times10^{11}}=1.225\times10^{-3}$, so $\Delta l=\varepsilon l=1.225\times10^{-3}\times4.0=4.9\times10^{-3}\,\mathrm{m}=4.9\,\mathrm{mm}$. Aluminium: $\varepsilon=\frac{2.45\times10^{8}}{7.0\times10^{10}}=3.5\times10^{-3}$, so $\Delta l=3.5\times10^{-3}\times4.0=1.4\times10^{-2}\,\mathrm{m}=14\,\mathrm{mm}$. (b) The aluminium wire stretches about 2.9 times as much as the steel wire under the same stress. Both wires carry the same force and have the same dimensions, so both have the same stress; since $\varepsilon=\sigma/E$, the wire with the smaller Young's modulus has the larger strain. Aluminium is less stiff than steel, so it deforms more.

Question 12 Model answer. Steel has a larger Young's modulus than timber — a steel beam is stiffer and sags less under the same load — and steel's tensile strength is high, which matters on the tension side of a bending beam. Timber is more flexible and weaker, but it is lighter, cheaper, easier to work with on a farm, and does not rust; some flexibility is acceptable in a short footbridge. Both recommendations are defensible: steel if stiffness and strength are the priorities (longer span, heavier loads); timber if cost, ease of repair and lightness matter more, provided the span is short and the timber is checked and replaced as it ages. Marks are awarded for the comparison of stiffness, strength and flexibility and for a justified recommendation.

Question 13 (a) $E=\frac{\sigma}{\varepsilon}=\frac{1.0\times10^{8}}{2.5\times10^{-3}}=4.0\times10^{10}\,\mathrm{Pa}$. (b) The area has two parts: the elastic triangle, $\frac{1}{2}\times2.5\times10^{-3}\times1.0\times10^{8}=1.25\times10^{5}\,\mathrm{J\,m^{-3}}$, and the plastic rectangle, $1.0\times10^{8}\times(1.0\times10^{-2}-2.5\times10^{-3})=1.0\times10^{8}\times7.5\times10^{-3}=7.5\times10^{5}\,\mathrm{J\,m^{-3}}$. Total $\approx1.25\times10^{5}+7.5\times10^{5}=8.75\times10^{5}\approx8.8\times10^{5}\,\mathrm{J\,m^{-3}}$. (c) Model answer. The area under the graph is the energy absorbed per unit volume before fracture. A large area means each cubic metre of the material absorbs a large amount of energy before it breaks. In a structure that might face sudden loads — a falling object, a vehicle impact, a gust — the material can take in that energy by deforming rather than fracturing, so failure is less sudden and gives warning.

Question 14 Model answer. The statement confuses strength with stiffness. **Strength** is the maximum stress a material can withstand before it breaks; **stiffness** — measured by Young's modulus — is how little it deforms under a given stress. The two are independent: a material can be strong and flexible, or stiff and weak. A nylon rope is strong — it supports large tensile stresses before breaking — yet it stretches noticeably under load, so it is not stiff. Chalk, or ordinary glass, is stiff — it barely stretches at all under small stresses — yet it is brittle and breaks at a modest stress, so it is not strong. Barely stretching tells you the material is stiff; it tells you nothing about how much stress it can take.

Question 15 (a) $\sigma=\frac{F}{A}$, so the minimum area is $A=\frac{F}{\sigma}=\frac{2.0\times10^6}{4.0\times10^5}=5.0\,\mathrm{m}^2$. (b) Model answer. The footing transmits the same downward force to the ground, but over a much larger area than the column above it. Since stress is force divided by area, the wider footing reduces the stress on the ground below the ground's safe limit. Without it, the same force through the narrow column would produce a stress the ground could not support, and the building would sink or crack.

Question 16 Model answer. While overloaded, the stress in the plank passed its elastic limit, so the plank deformed plastically: when the load was removed, it did not return to its original shape and a permanent bend remains. The plank is **not safe to reuse**. The plastic deformation shows it has already been stressed beyond the region in which it behaves elastically; its shape is changed, it may carry hidden damage, and the load it can take before breaking is reduced. A bent member also carries loads differently from a straight one, so it should be withdrawn from use and replaced.

Question 17 The plank is uniform, so its centre of mass is at its midpoint, $2.0\,\mathrm{m}$ from either end. With $1.5\,\mathrm{m}$ over the edge, the midpoint is $2.0-1.5=0.50\,\mathrm{m}$ inside the edge of the roof. (a) The force due to gravity acts at the centre of mass, $0.50\,\mathrm{m}$ inside the edge. (b) $F_g=mg=200\times9.8=1960\,\mathrm{N}$, so the torque about the edge is $\tau=1960\times0.50=980\,\mathrm{N\ m}$ (tending to rotate the plank back onto the roof). (c) A person at the outer end produces a torque about the edge of $F_g\times1.5$ in the tipping direction. Tipping just begins when the two torques balance: $m\times9.8\times1.5=980$, so $m=\frac{980}{9.8\times1.5}=67\,\mathrm{kg}$. (d) Model answer. Sliding the plank further onto the roof moves its centre of mass further from the edge, increasing the torque of the plank's force due to gravity that holds it down. With a larger restoring torque available, a larger load — a heavier person, or the same person standing further out — can be supported before the torques become unbalanced and the plank tips.

Question 18 Model answer. A loaded bridge beam bends: its underside is in tension and its top in compression. Proposal A, a plain concrete beam, is not suitable: concrete's tensile strength is far below its compressive strength, so the tensile stress in the underside cracks the beam at modest loads, and an unreinforced concrete beam fails without warning once cracked. Proposal B, a steel-reinforced concrete beam, is suitable: the steel bars cast into the underside carry the tensile stress while the concrete carries the compressive stress, so each material is used where it is strongest. Recommendation: **proposal B**. The evaluation should also note one non-physics factor — for example, reinforced concrete costs more than plain concrete but far less than a full steel span, and it suits a small country bridge that must carry occasional heavy loads (farm machinery, delivery trucks) with little maintenance; appearance and heritage considerations may also favour a concrete finish. Marks are awarded for the comparison of tensile and compressive strength, the reference to elastic and plastic behaviour or cracking under load, and a justified recommendation.

Option task — Structures and materials report

This is the assessed Option task that closes Option 2.4. It uses the study design's own Outcome 2 task type *"an analysis, including calculations, of physics concepts applied to real-world contexts"*. The study design specifies no mark allocations at Units 1 & 2 — levels of achievement are a matter for school decision, and only satisfactory completion is reported to the VCAA — so the **30 marks below are house convention**. The task is assessed against the shared *Communicating physics* key knowledge ($U2.2.CP$.1–.5), which is taught in Sections 10A and 10B, applied to the physics of this section ($U2.2.O4$.1–.5).

The task. Choose a real structure in Australia — for example the West Gate Bridge or Bolte Bridge in Melbourne, the roof structure of the Melbourne Cricket Ground, Eureka Tower, a tower crane on a building site, a water tower, a grain silo, or a heritage stone arch bridge — and produce a report of 800–1200 words, with labelled diagrams and calculations, that does all of the following:

1. **Describe the structure and its loads.** State what the structure is for, identify its main loads, and identify the compressive, tensile and shear forces acting on at least three of its members. List the sources you used, and evaluate the validity of each source ($U2.2.O4.1$; $U2.2.CP.1$).
2. **Analyse the stability of one component.** Choose one component of the structure — a beam, a jib, a whole span — and draw a labelled force diagram showing each force, its direction and its point of application, with the gravitational forces acting at the centre of mass. Apply the conditions for translational and rotational equilibrium to the component ($U2.2.O4.2$; $U2.2.CP.2$, $U2.2.CP.3$).
3. **Calculate.** Using stated data with sources, calculate stress, strain, Young's modulus or an extension for one member of the structure. Show your working in full, with units and correct significant figures ($U2.2.O4.3$; $U2.2.CP.2$).
4. **Analyse the behaviour under load.** Explain how the material of your chosen member behaves under load, referring to a stress–strain graph, elastic and plastic behaviour, and what the area under the graph represents ($U2.2.O4.4$; $U2.2.CP.3$).
5. **Evaluate and take a position.** Evaluate the suitability of the material for its use, comparing tensile and compressive strength and stiffness or flexibility under load, and discuss at least one relevant sociocultural, economic, legal or political factor (cost, safety regulation, appearance, effect on the community). Finish with a justified recommendation or stance: is the material well chosen, or what would you change? ($U2.2.O4.5$; $U2.2.CP.4$, $U2.2.CP.5$).

Your teacher will set the due date and any in-class conditions. Work that uses data from a source must name the source and its retrieval date.

Mark allocation (30 marks — house convention).

Criterion	Study-design dot point	Marks
Validity of sources evaluated	$U2.2.CP.1$	4
Physics concepts, terms and conventions applied correctly, with full calculations	$U2.2.CP.2$	8
Force diagrams, models and representations used and organised clearly, with limitations discussed	$U2.2.CP.3$	6
Sociocultural, economic, legal or political factors discussed	$U2.2.CP.4$	4
Physics-based stance, recommendation or solution justified	$U2.2.CP.5$	8
Total		**30**

Marking rubric.

Band	$CP.1$ sources (4)	$CP.2$ physics (8)	$CP.3$ representations (6)	$CP.4$ factors (4)	$CP.5$ stance (8)
High	4: every source named and its validity evaluated	7–8: concepts and terms correct throughout; calculations complete with units and correct	5–6: diagrams and models complete, clearly labelled; limitations of the model discussed	4: at least one relevant factor discussed in genuine depth	7–8: recommendation fully justified by the physics of the report

Band	$CP.1$ sources (4)	$CP.2$ physics (8)	$CP.3$ representations (6)	$CP.4$ factors (4)	$CP.5$ stance (8)
		significant figures			
Medium	3: sources named and validity considered	4–6: concepts mostly correct; working mostly complete	3–4: diagrams present with minor omissions; limitations noted briefly	3: a relevant factor identified with some discussion	4–6: recommendation given with partial justification
Low	1–2: sources listed with little or no evaluation	1–3: concepts partly correct; working incomplete	1–2: diagrams missing key labels or forces	1–2: a factor named with little discussion	1–3: recommendation stated with little support
None	0	0	0	0	0

Marks are awarded for the quality of the reasoning, not for the conclusion reached: two students who reach opposite recommendations can both receive full marks in $CP.5$ if both justify their position from correct physics.

Chapter 11 Options: forces, materials and motion — 11C Option 2.5: How do forces act on the human body?

Theory

This section is **Option 2.5 — The physics of forces on the human body**, one of the eighteen options of Unit 2, Area of Study 2. A student studies exactly one option; this section is self-contained for a student who has chosen Option 2.5. It applies the force model of Chapter 8 — force vectors, Newton's laws, the force due to gravity F_g, the centre of mass and torques — to the human body: bones as load-bearing columns, muscles and tendons as cables, and joints as pivots. It then asks what happens when a damaged part is replaced by an artificial one, and closes with the assessed Option task. The torque model ($\tau = r_{\perp} F$ and the conditions for equilibrium) is owned by Section 8F; F_g, the centre of mass and the $F_{\text{on A by B}}$ convention are owned by Section 8C. They are applied here, not re-taught.

Scope — VCE Physics Study Design, Unit 2, Area of Study 2, Option 2.5 (key knowledge U2.2.O5.1 –.6), quoted verbatim with the three formula images restored as printed:

- `U2.2.O5.1` "apply centre of mass calculations to a body or system: $x_m = (x_1m_1 + x_2m_2 + \dots + x_n m_n)/(m_1 + m_2 + \dots + m_n)$"
- `U2.2.O5.2` "investigate and apply theoretically and practically translational forces (gravitational, compressive, tensile and shear) and torques"
- `U2.2.O5.3` "calculate the stress and strain resulting from the application of compressive and tensile forces and loads to materials in organic structures including bone and muscle using: $\sigma = F/A$ and $\varepsilon = \Delta l/l$"
- `U2.2.O5.4` "analyse the behaviour of living tissue under load with reference to extension and compression, including Young's modulus, $E = \sigma/\varepsilon$, area under stress versus strain graphs and the elastic or plastic behaviour of materials under load"
- `U2.2.O5.5` "investigate the suitability of different materials for use in the human body, including bone, tendons and muscle, by comparing tensile and compressive strength and stiffness, toughness, and flexibility under load"
- `U2.2.O5.6` "evaluate the development, performance and challenges of using artificial materials and structures in prosthetics, including external prostheses for the replacement of lost limbs, and internal prostheses such as hip or valve replacements"

In the formulas, σ is the stress, ε the strain and E the Young's modulus; each is defined in the theory below.

Forces on and in the body. The external forces on a standing person are familiar from Chapter 8: the **force due to gravity** $F_g = mg$, acting at the person's centre of mass, and the **normal forces** from the ground on each foot, directed perpendicular to the contact surface. Inside the body, forces are carried from one part to another, and they come in three types:

- **Compression** — forces that squeeze a part along its length. The bones of the leg and the discs between the vertebrae carry compression whenever you stand, walk or land from a jump.
- **Tension** — forces that stretch a part along its length. Muscles can only *pull*, never push, so muscles and the tendons that attach them to bone carry tension.
- **Shear** — forces that slide one layer of a material across another. Shear appears where one part slides past another, for example between vertebrae when the spine is bent sideways, or across a bone when it is twisted.

Every one of these forces is still a force *on* one object *by* another: the force on the femur by the hip, the force on the tendon by the muscle, the force on the foot by the ground. Name them in that $F_{\text{on A by B}}$ form and the third law, $F_{\text{on A by B}} = -F_{\text{on B by A}}$, applies exactly as it does anywhere else.

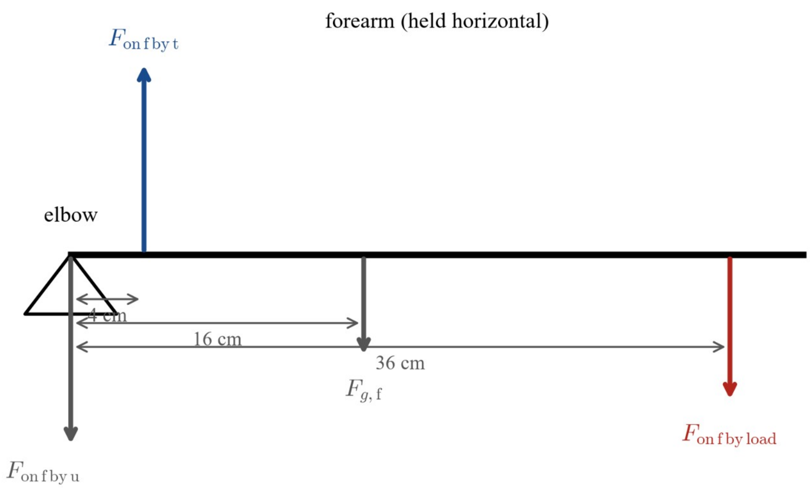
Centre of mass calculations. The **centre of mass** of a body is the single point at which the whole mass of the body can be treated as concentrated for the purpose of finding the force due to gravity. For a system made of several parts whose individual centres of mass are known, the position of the combined centre of mass is the mass-weighted average of the part positions. The study design writes this, for positions measured along one straight line from a chosen origin, as

$$x_m = \frac{x_1 m_1 + x_2 m_2 + \ldots + x_n m_n}{m_1 + m_2 + \ldots + m_n}.$$

Positions are measured from any origin you choose, and positions on the opposite side of the origin are negative. The formula says exactly what you would expect: the centre of mass sits closer to the more massive parts, and it moves whenever mass is moved — raise your arms, pick up a bag, lean forward, and your centre of mass shifts.

Centre of mass is what keeps you upright. A standing body is in equilibrium only while the line of the force due to gravity through the centre of mass falls **inside the base of support** — the area bounded by the feet. Lean until the centre of mass passes beyond the feet and the force due to gravity exerts an unbalanced torque about the edge of the base, and you begin to topple. This is why you automatically lean forward when you carry a load on your back and backward when you carry one in your arms: the body shifts itself so that the *combined* centre of mass stays over the feet.

Torques in the body: muscles work at short lever arms. A muscle pulls on a bone through its tendon, and the line of that pull usually passes very close to the joint the muscle acts on. The load the muscle must move — a held object, or the force due to gravity on a limb — acts much farther from the joint. Because a torque is $\tau = r_\perp F$, balancing the torque of a modest load at a long lever arm requires a *large* muscle force at a short lever arm.



Hold a heavy object in your hand with the forearm horizontal and the elbow is the pivot: the biceps tendon pulls upward a few centimetres from the elbow while the load pulls downward at arm's length. The muscle force is typically **five to ten times** the force due to gravity on the load, and the joint must then carry a large compressive force to balance the vertical forces. The same geometry explains lifting injuries: the upper body pivots about the lumbar spine, and the back muscles attach only about $5\,\text{cm}$ from that pivot. When you stoop, the centre of mass of the upper body — and of any load in the hands — moves to a much larger distance from the pivot, so the torques grow and the back muscles must pull with forces of thousands of newtons. Keeping a load close to the body keeps its lever arm small and the muscle force manageable.

Stress and strain in bone and muscle. A force alone does not tell you whether a part will break. A thick bone carries a larger force than a thin one made of the same tissue, because the force is shared over a larger area. The quantity that removes the size of the part from the problem is the **stress**:

$$\sigma = \frac{F}{A},$$

the force F carried perpendicular to a cross-section divided by the cross-sectional area A. Stress is measured in pascals ($\text{Pa} = \text{N m}^{-2}$); the stresses in bone and muscle are conveniently given in megapascals ($1\,\text{MPa} = 10^6\,\text{Pa}$). A part is in **compressive stress** when the forces squeeze it and in **tensile stress** when they stretch it.

Equally, an extension alone tells you little: a $1\,\text{mm}$ stretch in a $1\,\text{mm}$ sample is catastrophic, while in a $1\,\text{m}$ sample it is barely visible. The size-independent measure of deformation is the **strain**:

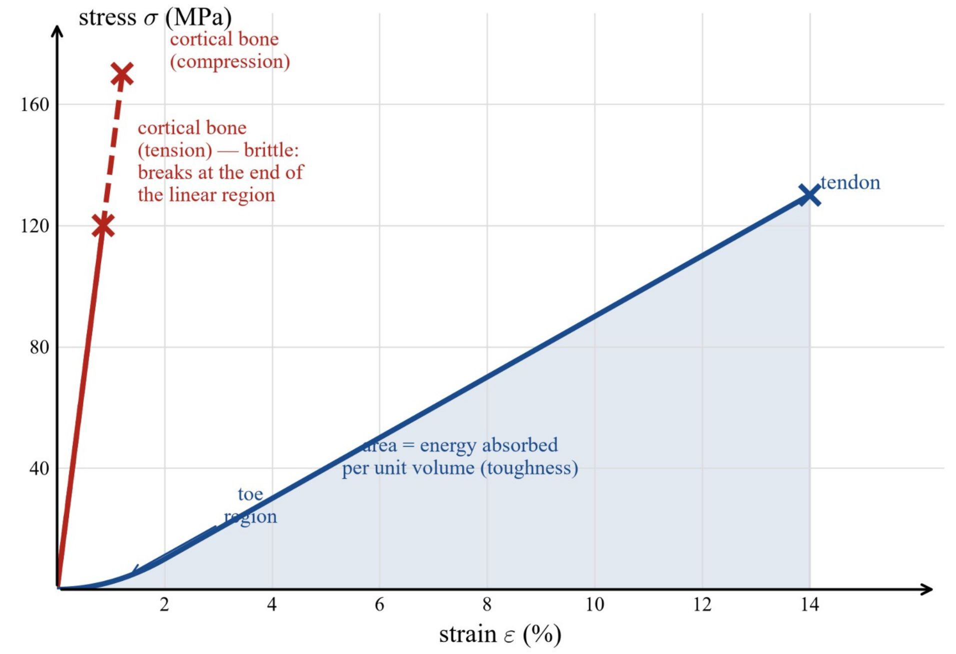
$$\varepsilon = \frac{\Delta l}{l},$$

the change in length Δl divided by the original length l. Strain is a ratio of two lengths, so it has **no unit**; it is often written as a percentage ($\varepsilon = 0.02$ is a $2\,\%$ strain).

The behaviour of living tissue under load: Young's modulus. In many materials, living tissue included, stress and strain are proportional while the load is light: double the stress and the strain doubles. This straight-line part of the behaviour is the **linear** or **elastic** region, and its gradient — stress divided by strain — is the **Young's modulus**:

$$E = \frac{\sigma}{\varepsilon}.$$

E has the units of stress (pascals), because strain has no unit. A large Young's modulus means a material is **stiff**: a large stress is needed to produce a given strain.



Real tissue data shows what these graphs look like for the body:

- **Cortical bone** (the dense outer shell of bone) is stiff and strong but **brittle**: its stress–strain graph is essentially a straight line that ends in sudden break, at a strain of only about $1\,\%$. It is strongest in compression (breaking stress about $170\,\mathrm{MPa}$), weaker in tension (104–$121\,\mathrm{MPa}$) and weakest in shear (about $51.6\,\mathrm{MPa}$).

  Source: Wikipedia, Bone, retrieved 2026-08-18.

- **Tendon** is much less stiff and far more stretchable. Its graph begins with a **toe region** of low stiffness — the crimped collagen fibres straightening out — and then becomes linear until the tendon fails at a strain of around 12–$15\,\%$ and a stress of about 100–$150\,\mathrm{MPa}$. Positional tendons have moduli of about 700–$1000\,\mathrm{MPa}$.

  Source: Wikipedia, Tendon, retrieved 2026-08-18.

For comparison, steel has $E \approx 200\,\mathrm{GPa}$, titanium alloy about $114\,\mathrm{GPa}$, and human cortical bone about $14\,\mathrm{GPa}$.

Source: Wikipedia, Young's modulus, retrieved 2026-08-18.

Behaviour under load is described as **elastic** if the part returns to its original length when the stress is removed, and **plastic** if it is permanently deformed. Bone shows almost no plastic region — it either returns or it breaks — while many engineering materials deform plastically long before they break.

Reading energy from the graph. In Section 9A you met the rule that the area under a force–extension graph is the work done on the sample. Dividing the force axis by area and the extension axis by length turns that into the stress–strain graph, so the **area under a stress–strain graph is the energy absorbed per unit volume** of the material as it is loaded. The key knowledge for this option compares materials by their *stiffness*, *toughness* and *flexibility* under load, terms the study design does not define. In this subtopic they are treated as follows, and every question here is answerable on these definitions alone:

- **Stiffness** — resistance to being deformed, measured by the Young's modulus E (the gradient of the linear region). Stiff means large E.
- **Toughness** — the energy per unit volume a material can absorb before it breaks, measured by the total area under the stress–strain graph up to breaking.
- **Flexibility** — the ability to undergo large strain without breaking or permanent damage; a flexible part reaches large ε on the graph.

On these definitions, the graph above shows that tendon is far *tougher* and far *more flexible* than bone, while bone is far *stiffer*. Tendon's toe region makes it extra flexible at low stress and stiff at high stress — compliant when the load is light, firm when the load is heavy.

Which materials suit which job in the body? Bone's strength pattern matches the way it is loaded: the leg bones and spine mostly carry compression, and compression is exactly what bone is strongest in ($170\,\text{MPa}$ against 104–$121\,\text{MPa}$ in tension and only about $51.6\,\text{MPa}$ in shear). Breaks therefore tend to come from bending — which puts one side of the bone in tension — or from twisting, which loads the bone in shear. Tendon suits its job in the opposite way: it needs to be strong in tension, which it is (failing at 100–$150\,\text{MPa}$), and stretchy enough to smooth out the sudden loads of running and jumping, which its toe region provides. Muscle is soft and operates at far lower stresses; its role is to *generate* tension, not to carry structural loads.

Artificial materials in the body: prosthetics. When a part fails beyond repair, physics sets the requirements for its replacement: the substitute must carry the same stresses without breaking, deform within acceptable strain limits, and not wear out or loosen over decades of loading cycles.

External prostheses replace lost limbs. Modern running blades are made of carbon-fibre composite: stiff enough to transfer the runner's push, light, and elastic in the sense that the blade bends on landing, storing elastic potential energy ($E_s = 1/2\,k\,x^2$, Section 9C), and returns most of that energy as it straightens during the push-off. The blade's curved shape also spreads the large ground force of landing over a longer contact time, softening the peak force on the stump.

Internal prostheses replace parts inside the body. In a total **hip replacement** the worn ball and socket are replaced by a metal stem and head running in a cup. The modern low-friction design is due to Sir John Charnley (1962), and the long-lived Exeter stem was first implanted at the Princess Elizabeth Orthopaedic Hospital in 1970. Current stems are made of titanium alloys or cobalt–chromium alloys, with cups of ultra-high-molecular-weight polyethylene or ceramic; about 58% of total hip replacements are estimated to last 25 years. The physics problems are wear and stiffness: abrasive wear of the liner can loosen the implant, and a metal stem ($E \approx 114\,\text{GPa}$ for titanium alloy) is still about eight times stiffer than bone ($14\,\text{GPa}$), so the stem carries most of the load and shields the bone around it from stress, which over years can allow that bone to weaken.

Source: Wikipedia, Hip replacement, retrieved 2026-08-18.

Heart valve replacements show the same trade-off between durability and compatibility with the body. Mechanical valves, made of titanium and carbon, can last 20–30 years, but they promote blood clots, so their recipients take anticoagulant (blood-thinning) medication for life. Tissue valves, usually made from bovine or porcine tissue, are less likely to cause clots and need no lifelong anticoagulants, but are less durable, typically lasting 10–20 years. Choosing between them is an evaluation of exactly the kind this option practises: the same physics requirements (strength, flexibility, fatigue over millions of cycles) balanced against the person the part is for.

Source: Wikipedia, Artificial heart valve, retrieved 2026-08-18.

Throughout this section $g = 9.8\,\text{m s}^{-2}$ (equivalently $9.8\,\text{N kg}^{-1}$).

Worked examples

Example 1 — scaffolded

A student of mass $62\,\mathrm{kg}$ stands upright. Take the origin at the point on the ground directly below the student's own centre of mass, so that the student's centre of mass is at $x=0$. The student then holds a school bag of mass $12\,\mathrm{kg}$ so that the bag's centre of mass is $0.45\,\mathrm{m}$ in front of the origin.

(a) Calculate the position of the combined centre of mass of student and bag.
(b) Explain what the student must do to stay balanced.

Working	Comment
$m_1=62\,\mathrm{kg}$ at $x_1=0$; $m_2=12\,\mathrm{kg}$ at $x_2=+0.45\,\mathrm{m}$.	List the parts and their positions from the chosen origin.
$x_m=\dfrac{x_1 m_1+x_2 m_2}{m_1+m_2}=\dfrac{62\times 0+12\times 0.45}{62+12}=\dfrac{5.4}{74}=0.073\,\mathrm{m}$.	The centre-of-mass formula: a mass-weighted average of the positions.
$x_m=0.073\,\mathrm{m}=7.3\,\mathrm{cm}$ in front of the student's original centre of mass.	The combined centre of mass has shifted toward the bag.

The combined centre of mass now lies $7.3\,\mathrm{cm}$ in front of where the student's centre of mass was. If the student stood perfectly upright, the force due to gravity on student and bag would act in front of the base of support and exert an unbalanced toppling torque about the feet. The student therefore leans back slightly, shifting the body's own mass backward until the combined centre of mass is once again above the feet.

Example 2 — scaffolded

When a person stands on one leg, the femur (thigh bone) carries a compressive force of about $700\,\mathrm{N}$. The smallest cross-sectional area of the femur is about $5.0\,\mathrm{cm}^2=5.0\times 10^{-4}\,\mathrm{m}^2$, and cortical bone has $E\approx 14\,\mathrm{GPa}$.

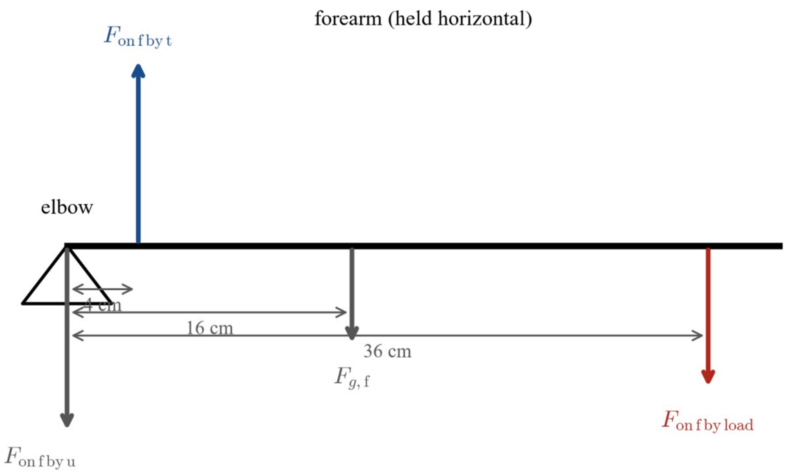
(a) Calculate the compressive stress in the femur.
(b) Calculate the resulting strain.
(c) Calculate the shortening of a $0.45\,\mathrm{m}$ length of the bone.
(d) Compare the stress with bone's compressive breaking stress of about $170\,\mathrm{MPa}$.

Working	Comment
$A=5.0\,\mathrm{cm}^2=5.0\times 10^{-4}\,\mathrm{m}^2$; $F=700\,\mathrm{N}$.	Convert the area to m^2 first ($1\,\mathrm{cm}^2=10^{-4}\,\mathrm{m}^2$).
$\sigma=\dfrac{F}{A}=\dfrac{700}{5.0\times 10^{-4}}=1.4\times 10^{6}\,\mathrm{Pa}=1.4\,\mathrm{MPa}$.	Stress is force per unit cross-sectional area.
$\varepsilon=\dfrac{\sigma}{E}=\dfrac{1.4\times 10^{6}}{14\times 10^{9}}=1.0\times 10^{-4}$.	Rearrange $E=\sigma/\varepsilon$; strain has no unit ($0.010\,\%$).
$\Delta l=\varepsilon l=1.0\times 10^{-4}\times 0.45=4.5\times 10^{-5}\,\mathrm{m}=0.045\,\mathrm{mm}$.	Strain is change in length over original length.

The stress of $1.4\,\mathrm{MPa}$ is only about $1\,\%$ of bone's compressive breaking stress of $170\,\mathrm{MPa}$, so standing loads the femur far below its limit — bone is built with a large safety margin for everyday loads.

Example 3 — unscaffolded

A person holds a $6.0\,\mathrm{kg}$ object in the hand with the forearm horizontal. Model the forearm as a horizontal rod pivoted at the elbow, as in the figure in the theory. The force due to gravity on the forearm is $20\,\mathrm{N}$, acting at its centre of mass $16\,\mathrm{cm}$ from the elbow; the object exerts a downward force of $60\,\mathrm{N}$ on the hand $36\,\mathrm{cm}$ from the elbow; and the biceps tendon pulls vertically upward on the forearm $4.0\,\mathrm{cm}$ from the elbow. Calculate the force in the tendon and the vertical force the upper arm exerts on the forearm at the elbow joint.



The forearm is in rotational equilibrium, so the torques about the elbow sum to zero. The joint force acts *at* the elbow, so its lever arm about the elbow is zero and it contributes no torque. Taking the tendon's torque as balancing the two downward torques, with $\tau = r_{\perp} F$:

$$F_t \times 0.040 = 20 \times 0.16 + 60 \times 0.36 = 3.2 + 21.6 = 24.8 \text{ N m}$$

$$F_t = \frac{24.8}{0.040} = 620 \text{ N}.$$

The forearm is also in translational equilibrium, so the vertical forces sum to zero. The tendon pulls up with 620 N while the downward forces total $20 + 60 = 80 \text{ N}$, so the upper arm must push *down* on the forearm at the elbow with

$$F_j = 620 - 80 = 540 \text{ N}.$$

∴ the tendon carries 620 N — more than ten times the force due to gravity on the held object — and the elbow joint carries a compressive force of 540 N. This is the short-lever-arm effect described in the theory: muscles act close to the joint, so their forces are always much larger than the loads they support.

Example 4 — unscaffolded

Model the upper body as a rigid rod pivoted at the lumbar spine, held horizontal by the back muscles, which pull with a lever arm of 5.0 cm about the pivot. The force due to gravity on the upper body is 350 N.

(i) Standing upright while holding a 90 N load, the centre of mass of the upper body is 5.0 cm in front of the pivot and the load is 10 cm in front of it. Calculate the force the back muscles must supply.

(ii) Stooping to pick the same load up off the ground, the upper body's centre of mass moves to 25 cm in front of the pivot and the load to 55 cm. Calculate the muscle force now, and compare.

(i) Rotational equilibrium about the pivot, $\Sigma \tau = 0$:

$$F_m \times 0.050 = 350 \times 0.050 + 90 \times 0.10 = 17.5 + 9.0 = 26.5 \text{ N m}$$

$$F_m = \frac{26.5}{0.050} = 530 \text{ N}.$$

(ii) Stooping:

$$F_m \times 0.050 = 350 \times 0.25 + 90 \times 0.55 = 87.5 + 49.5 = 137 \text{ N m}$$

$$F_m = \frac{137}{0.050} = 2740 \text{ N}.$$

∴ stooping raises the back-muscle force from about 530 N to about 2700 N — more than five times larger — even though the load has not changed. The load on the spine is set by the *torques*, and the torques grow with the lever arms. This is the physics behind the advice to lift with the knees and keep the load close to the body.

Practice questions

Question 1 *(scaffolded)* A straight arm hangs at the side of the body. Take the origin at the shoulder. The upper arm has mass $2.4\,\mathrm{kg}$ with its centre of mass $0.10\,\mathrm{m}$ from the shoulder, and the forearm and hand together have mass $1.6\,\mathrm{kg}$ with their centre of mass $0.32\,\mathrm{m}$ from the shoulder. (a) Write down the centre-of-mass formula for this two-part system. (b) Substitute the values and calculate x_m. (c) In one sentence, say whether the centre of mass of the whole arm is closer to the shoulder or to the forearm's centre of mass, and why.

Question 2 *(scaffolded)* For each situation, state whether the named part is mainly in compression, tension or shear. (a) The tibia (shin bone) of a person standing still. (b) The Achilles tendon while a person stands on tiptoe. (c) The disc between two vertebrae when a sideways force slides one vertebra across the other. (d) A person of mass $70\,\mathrm{kg}$ stands on the ground. Name, in $F_{\text{on A by B}}$ form, the force the ground exerts on each foot, and state its Newton's third-law partner.

Question 3 *(scaffolded)* The tibia of a person of mass $72\,\mathrm{kg}$ has a minimum cross-sectional area of $3.2\,\mathrm{cm}^2$. The person stands still with their mass shared equally between the two legs. (a) Show that the compressive force on each tibia is about $350\,\mathrm{N}$. (b) Calculate the compressive stress in each tibia in MPa. (Remember $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$.) (c) Using $E = 14\,\mathrm{GPa}$ for cortical bone, calculate the strain and the shortening of a $0.36\,\mathrm{m}$ length of the bone.

Question 4 *(scaffolded)* A tendon sample of cross-sectional area $1.5\,\mathrm{cm}^2$ and original length $0.20\,\mathrm{m}$ is stretched by $4.0\,\mathrm{mm}$ when a tensile force of $1200\,\mathrm{N}$ is applied. (a) Calculate the tensile stress in the sample. (b) Calculate the strain. (c) Hence calculate the Young's modulus of the sample over this stretch, and explain why your value is smaller than the 700–$1000\,\mathrm{MPa}$ linear modulus quoted for tendon in the theory.

Question 5 *(scaffolded)* Use the stress–strain graph in the theory and the values quoted there. (a) Approximating the bone (tension) curve as a triangle ending at $\sigma = 120\,\mathrm{MPa}$, $\varepsilon = 0.86\,\%$, calculate the area under it and state what this area represents. (b) Approximating the tendon curve as a triangle ending at $\sigma = 130\,\mathrm{MPa}$, $\varepsilon = 14\,\%$, calculate its area. (c) Hence estimate how many times more energy per unit volume tendon can absorb before breaking than bone, and name the property this compares.

Question 6 *(scaffolded)* A person of mass $70\,\mathrm{kg}$ stands on tiptoe on one foot. Model the foot as a lever pivoted at the ankle joint. The normal force from the ground on the foot is $686\,\mathrm{N}$, acting $12\,\mathrm{cm}$ in front of the ankle, and the Achilles tendon pulls upward on the heel with a lever arm of $5.0\,\mathrm{cm}$ behind the ankle. (a) Explain why the torque of the normal force about the ankle must be balanced by the torque of the Achilles tendon. (b) Calculate the tension in the Achilles tendon. (c) Compare your answer with the force due to gravity on the person, and comment.

Question 7 An outstretched arm is modelled as three segments measured from the shoulder: upper arm $2.2\,\mathrm{kg}$ with centre of mass at $0.10\,\mathrm{m}$; forearm $1.5\,\mathrm{kg}$ with centre of mass at $0.32\,\mathrm{m}$; hand $0.6\,\mathrm{kg}$ with centre of mass at $0.50\,\mathrm{m}$. Calculate the position of the centre of mass of the whole arm from the shoulder.

Question 8 A person holds an $8.0\,\mathrm{kg}$ object with the forearm horizontal. The force due to gravity on the forearm is $25\,\mathrm{N}$ acting $15\,\mathrm{cm}$ from the elbow; the object exerts $80\,\mathrm{N}$ downward $40\,\mathrm{cm}$ from the elbow; the biceps tendon pulls vertically upward $5.0\,\mathrm{cm}$ from the elbow. Calculate the tendon force and the vertical force on the forearm by the upper arm at the elbow.

Question 9 The disc between two lumbar vertebrae has a cross-sectional area of $1.1 \times 10^{-3}\,\mathrm{m}^2$ and behaves, for small loads, with a Young's modulus of about $2.0\,\mathrm{MPa}$. The disc carries a compressive force of $550\,\mathrm{N}$. (a) Calculate the compressive stress in the disc. (b) Calculate the strain. (c) Hence find the compression of a disc $9.0\,\mathrm{mm}$ thick, and comment on why the spine is able to compress noticeably under load while bone barely does.

Question 10 The Achilles tendon has a cross-sectional area of about $8.0 \times 10^{-5}\,\mathrm{m}^2$ and a length of about $0.25\,\mathrm{m}$. During running it can carry a tensile force of $2000\,\mathrm{N}$. Using $E = 1.0\,\mathrm{GPa}$ for the linear region of tendon: (a) calculate the tensile stress; (b) calculate the strain; (c) calculate the extension of the tendon, and compare the strain with the 12–15 % failure strain quoted in the theory.

Question 11 A femur has minimum cross-sectional area $5.0 \times 10^{-4}\,\mathrm{m}^2$ and cortical bone breaks in compression at about $170\,\mathrm{MPa}$. (a) Calculate the compressive force that would break the femur. (b) Standing on one leg loads the femur with about $700\,\mathrm{N}$; sprinting can load it with about $2100\,\mathrm{N}$. Calculate the ratio of the breaking force to each of these loads.

Question 12 Using the spine model of Example 4 (back muscles with a $5.0\,\mathrm{cm}$ lever arm about the lumbar pivot; upper body $380\,\mathrm{N}$): (a) calculate the back-muscle force when standing upright holding a $120\,\mathrm{N}$ load, with the upper body's centre of mass $6.0\,\mathrm{cm}$ and the load $10\,\mathrm{cm}$ in front of the pivot; (b) calculate the muscle force when stooped, with the upper body's centre of mass $30\,\mathrm{cm}$ and the load $60\,\mathrm{cm}$ in front of the pivot; (c) find the factor by which stooping multiplies the muscle force.

Question 13 A person of mass $60\,\mathrm{kg}$ stands with their centre of mass above the middle of their feet. They hold a $20\,\mathrm{kg}$ load whose centre of mass is $0.40\,\mathrm{m}$ in front of the person's centre of mass. (a) Taking the origin at the person's centre of mass, calculate the position of the combined centre of mass. (b) The feet of a standing person span about $0.25\,\mathrm{m}$ from heel to toe. Explain, using torques, whether the person can remain balanced standing rigidly upright, and what they will do in practice.

Question 14 Cortical bone breaks at about $170\,\mathrm{MPa}$ in compression, $104–121\,\mathrm{MPa}$ in tension and about $51.6\,\mathrm{MPa}$ in shear. Explain why this pattern of strengths is well matched to the way leg bones are loaded in everyday activity, and deduce the kinds of loading most likely to break a bone.

Question 15 With reference to torques about the lumbar spine, explain why carrying a $15\,\mathrm{kg}$ bag with arms fully extended in front of the body is far more tiring for the back muscles than carrying the same bag held against the chest. No calculation is required, but your answer should refer to lever arms.

Question 16 The tendon stress–strain graph begins with a toe region of low stiffness before the linear region. Explain one advantage this gives the body when a tendon first takes up a load, and one advantage of the stiffer linear region when the load is large.

Question 17 A hip-replacement stem of titanium alloy ($E \approx 114\,\mathrm{GPa}$) is about eight times stiffer than the bone ($E \approx 14\,\mathrm{GPa}$) it is fixed into, and a steel stem ($E \approx 200\,\mathrm{GPa}$) is stiffer still. Explain, using the idea that stiffer material carries a larger share of the load, why a very stiff stem can cause the surrounding bone to weaken over time, and why this is one reason titanium alloy is preferred to steel.

Question 18 Mechanical heart valves last 20–30 years but require lifelong anticoagulant medication; tissue valves need no anticoagulants but typically last 10–20 years. Evaluate which type of valve is the more suitable choice for (a) a 30-year-old and (b) a 78-year-old, giving reasons in each case.

Question 19 Running blades are made of carbon-fibre composite. With reference to elastic potential energy and the shape of a stress–strain graph, explain how a blade stores and returns energy during a stride, and why the material must be both stiff and highly elastic (returning to its shape without a plastic region).

Question 20 Return to the forearm of Example 3. (a) Name the Newton's third-law partner of the force on the forearm by the biceps tendon, stating the object it acts on and its direction. (b) A student claims that the upward tendon force and the downward joint force at the elbow form a Newton's third-law pair because the forearm is in equilibrium. Explain why this is wrong, and what the correct relationship between the two forces is.

Answers

Question 1 (a) $x_m = (x_1 m_1 + x_2 m_2)/(m_1 + m_2)$; (b) $0.19\,\mathrm{m}$ from the shoulder; (c) closer to the shoulder — see solution. **Question 2** (a) compression; (b) tension; (c) shear; (d) $F_{\text{on foot by ground}}$, paired with $F_{\text{on ground by foot}}$. **Question 3** (a) $350\,\mathrm{N}$; (b) $1.1\,\mathrm{MPa}$; (c) $\varepsilon = 7.9\times10^{-5}$, $\Delta l = 2.8\times10^{-5}\,\mathrm{m}$. **Question 4** (a) $8.0\,\mathrm{MPa}$; (b) 0.020; (c) $0.40\,\mathrm{GPa}$ — see solution for the comparison. **Question 5** (a) $5.2\times10^{5}\,\mathrm{J\,m^{-3}}$, the energy absorbed per unit volume; (b) $9.1\times10^{6}\,\mathrm{J\,m^{-3}}$; (c) about 18 times; toughness. **Question 6** (a) see solution; (b) $1.6\times10^{3}\,\mathrm{N}$; (c) about 2.4 times F_g — see solution. **Question 7** $0.23\,\mathrm{m}$ from the shoulder. **Question 8** tendon $715\,\mathrm{N}$; joint force $610\,\mathrm{N}$ downward. **Question 9** (a) $0.50\,\mathrm{MPa}$; (b) 0.25; (c) $2.3\,\mathrm{mm}$ — see solution. **Question 10** (a) $25\,\mathrm{MPa}$; (b) 0.025; (c) $6.3\,\mathrm{mm}$; well below the failure strain. **Question 11** (a) $85\,\mathrm{kN}$; (b) about 120 and about 40. **Question 12** (a) $700\,\mathrm{N}$; (b) $3700\,\mathrm{N}$; (c) about 5.3. **Question 13** (a) $0.10\,\mathrm{m}$ in front; (b) see solution. **Question 14** See solution — legs are loaded mainly in compression, bone's strongest mode. **Question 15** See solution — extended arms give the load a larger lever arm. **Question 16** See solution — toe region compliant at low load, linear region stiff at high load. **Question 17** See solution — a stiff stem shields the bone from stress. **Question 18** See solution — (a) mechanical; (b) tissue valve. **Question 19** See solution — elastic storage and return of E_s. **Question 20** (a) $F_{\text{on tendon by forearm}}$, downward on the tendon; (b) see solution.

Fully-worked solutions

Question 1 (a) $x_m = \dfrac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$. (b) $x_m = \dfrac{2.4\times0.10 + 1.6\times0.32}{2.4+1.6} = \dfrac{0.24+0.512}{4.0} = \dfrac{0.752}{4.0} = 0.188\,\mathrm{m} \approx 0.19\,\mathrm{m}$ from the shoulder. (c) $0.19\,\mathrm{m}$ is closer to the shoulder ($0.10\,\mathrm{m}$) than to the forearm's centre of mass ($0.32\,\mathrm{m}$), because the upper arm is the more massive part and the centre of mass is a mass-weighted average.

Question 2 (a) The tibia is squeezed along its length by the load above it: **compression**. (b) The Achilles tendon is stretched by the pull of the calf muscle: **tension**. (c) Sliding one vertebra across the other loads the disc in **shear**. (d) The ground exerts $F_{\text{on foot by ground}}$, a normal force directed upward, of magnitude $1/2\,m g = 1/2\times70\times9.8 = 343\,\mathrm{N}$ per foot. Its third-law partner is $F_{\text{on ground by foot}}$, equal in magnitude, directed downward, acting on the ground.

Question 3 (a) $F = 1/2\,m g = 1/2\times72\times9.8 = 352.8 \approx 350\,\mathrm{N}$. (b) $\sigma = \dfrac{F}{A} = \dfrac{352.8}{3.2\times10^{-4}} = 1.10\times10^{6}\,\mathrm{Pa} = 1.1\,\mathrm{MPa}$. (c) $\varepsilon = \dfrac{\sigma}{E} = \dfrac{1.10\times10^{6}}{14\times10^{9}} = 7.9\times10^{-5}$; $\Delta l = \varepsilon l = 7.9\times10^{-5}\times0.36 = 2.8\times10^{-5}\,\mathrm{m}$, i.e. $0.028\,\mathrm{mm}$ — bone barely compresses because E is large.

Question 4 (a) $\sigma = \dfrac{1200}{1.5\times10^{-4}} = 8.0\times10^{6}\,\mathrm{Pa} = 8.0\,\mathrm{MPa}$. (b) $\varepsilon = \dfrac{0.0040}{0.20} = 0.020$ (2%). (c) $E = \dfrac{\sigma}{\varepsilon} = \dfrac{8.0\times10^{6}}{0.020} = 4.0\times10^{8}\,\mathrm{Pa} = 0.40\,\mathrm{GPa}$. This is below the quoted linear modulus because the first part of the stretch lies in the low-stiffness toe region, so the average gradient over 0–2% strain is smaller than the gradient of the linear region alone.

Question 5 (a) Area $= 1/2\times120\times10^{6}\,\mathrm{Pa}\times0.0086 = 5.2\times10^{5}\,\mathrm{J\,m^{-3}}$. The area under a stress–strain graph is the energy absorbed per unit volume of the material. (b) Area $= 1/2\times130\times10^{6}\times0.14 = 9.1\times10^{6}\,\mathrm{J\,m^{-3}}$. (c) $\dfrac{9.1\times10^{6}}{5.2\times10^{5}} \approx 18$: tendon absorbs about 18 times more energy per unit volume before breaking. This compares **toughness**.

Question 6 (a) The foot is not rotating, so it is in rotational equilibrium and the net torque about any point — in particular the ankle — is zero. The normal force and the tendon force are the two forces with lever arms about the ankle, so their torques must balance. (b) $F_A\times0.050 = 686\times0.12$, so $F_A = \dfrac{686\times0.12}{0.050} = 1646 \approx 1.6\times10^{3}\,\mathrm{N}$. (c)

$F_g = mg = 70 \times 9.8 = 686\,\mathrm{N}$, so the tendon carries about 2.4 times the force due to gravity on the whole person — again because the muscle works at a much shorter lever arm than the load it balances.

Question 7 $x_m = \dfrac{2.2 \times 0.10 + 1.5 \times 0.32 + 0.6 \times 0.50}{2.2 + 1.5 + 0.6} = \dfrac{0.22 + 0.48 + 0.30}{4.3} = \dfrac{1.00}{4.3} = 0.23\,\mathrm{m}$ from the shoulder.

Question 8 Torques about the elbow: $F_t \times 0.050 = 25 \times 0.15 + 80 \times 0.40 = 3.75 + 32.0 = 35.75\,\mathrm{N\ m}$, so $F_t = \dfrac{35.75}{0.050} = 715\,\mathrm{N}$. Vertical equilibrium: $F_j = 715 - (25 + 80) = 610\,\mathrm{N}$, directed downward on the forearm at the elbow.

Question 9 (a) $\sigma = \dfrac{550}{1.1 \times 10^{-3}} = 5.0 \times 10^5\,\mathrm{Pa} = 0.50\,\mathrm{MPa}$. (b) $\varepsilon = \dfrac{\sigma}{E} = \dfrac{5.0 \times 10^5}{2.0 \times 10^6} = 0.25$. (c) $\Delta l = 0.25 \times 9.0\,\mathrm{mm} = 2.25 \approx 2.3\,\mathrm{mm}$. The disc's Young's modulus is thousands of times smaller than bone's, so the same stress produces a strain of $25\,\%$ instead of a tiny fraction of a percent: the spine compresses visibly and acts as a shock absorber, while bone stays rigid.

Question 10 (a) $\sigma = \dfrac{2000}{8.0 \times 10^{-5}} = 2.5 \times 10^7\,\mathrm{Pa} = 25\,\mathrm{MPa}$. (b) $\varepsilon = \dfrac{2.5 \times 10^7}{1.0 \times 10^9} = 0.025$ ($2.5\,\%$). (c) $\Delta l = 0.025 \times 0.25 = 6.3 \times 10^{-3}\,\mathrm{m} = 6.3\,\mathrm{mm}$. The $2.5\,\%$ strain is well inside the 12–$15\,\%$ failure strain, so the tendon carries running loads with a comfortable margin.

Question 11 (a) $F_{\text{break}} = \sigma_{\text{break}} A = 170 \times 10^6 \times 5.0 \times 10^{-4} = 8.5 \times 10^4\,\mathrm{N} = 85\,\mathrm{kN}$. (b) Standing: $\dfrac{85000}{700} \approx 120$; sprinting: $\dfrac{85000}{2100} \approx 40$. Bone carries everyday loads far below its breaking stress.

Question 12 (a) $F_m \times 0.050 = 380 \times 0.060 + 120 \times 0.10 = 22.8 + 12.0 = 34.8\,\mathrm{N\ m}$; $F_m = \dfrac{34.8}{0.050} = 696 \approx 700\,\mathrm{N}$. (b) $F_m \times 0.050 = 380 \times 0.30 + 120 \times 0.60 = 114 + 72 = 186\,\mathrm{N\ m}$; $F_m = \dfrac{186}{0.050} = 3720 \approx 3700\,\mathrm{N}$. (c) $\dfrac{3720}{696} \approx 5.3$: stooping multiplies the muscle force by about five.

Question 13 (a) $x_m = \dfrac{60 \times 0 + 20 \times 0.40}{60 + 20} = \dfrac{8.0}{80} = 0.10\,\mathrm{m}$ in front of the person's own centre of mass. (b) A shift of $0.10\,\mathrm{m}$ is within a $0.25\,\mathrm{m}$ foot span, so balance is possible — but only if the centre of mass lies above the feet. Standing rigidly upright with the feet where they were, the force due to gravity would act $0.10\,\mathrm{m}$ in front of the old line, outside the ankle pivot, giving an unbalanced toppling torque forward. In practice the person leans back, moving the body's own mass behind the original line, until the combined centre of mass is again above the feet.

Question 14 In standing, walking and jumping the long bones of the leg are loaded mainly along their length by the forces above them — that is, in compression — and $170\,\mathrm{MPa}$ in compression is bone's largest strength. The pattern therefore matches the everyday loading. The weaker modes are tension (104–$121\,\mathrm{MPa}$) and especially shear ($51.6\,\mathrm{MPa}$), so the loadings most likely to break a bone are those that put it in tension or shear: bending, which stretches the far side of the bone, and twisting, which shears it. This is consistent with the way most fractures occur — in falls and impacts that bend or twist the limb rather than simply compress it.

Question 15 The back muscles act at a fixed, small lever arm (about $5\,\mathrm{cm}$) about the lumbar pivot, so the force they must supply is set by the torque of the load about that pivot. With arms extended, the bag's centre of mass may be half a metre or more in front of the pivot, giving the bag's force due to gravity a large lever arm and therefore a large torque; the muscles must then pull with a force many times the bag's force due to gravity. Held against the chest, the same bag has a lever arm of a few centimetres, its torque is small, and the muscle force needed to balance it is small. The bag has not changed; its lever arm has, and torque is what the muscles must balance.

Question 16 At low load the toe region's small stiffness means the tendon stretches easily: it takes up slack gently and lets the joint move freely without resisting small motions — useful for smooth, low-force movement. Once the load is large the fibres are straight and the tendon is in its stiff linear region: the muscle's pull is then transferred to the bone efficiently, with little wasted stretch, and the rising stiffness also limits further extension, helping protect the tendon and the joint from over-stretch as the load grows toward the failure region.

Question 17 Where the stem and the bone share a load, they must stretch together, so they experience similar strain; at equal strain the stress $\sigma = E\,\varepsilon$ is larger in the material with the larger E, so the stiffer metal carries the larger share of the force. With a steel stem ($200\,\text{GPa}$ against the bone's $14\,\text{GPa}$) the bone carries very little of the load at all. Bone remodels in response to the stress it carries: bone that is shielded from stress is gradually resorbed and weakens, which can loosen the implant over the years (one of the long-term problems recorded for hip replacements). Titanium alloy, at $114\,\text{GPa}$, is still much stiffer than bone but the mismatch is nearly halved, so more of the load stays in the bone — one reason it replaced steel in modern stems.

Question 18 (a) For a 30-year-old, the mechanical valve is the stronger choice: they can expect decades of life, beyond the 10–20 year service of a tissue valve, so a tissue valve would probably need replacing; the lifelong anticoagulant medication is a real burden but a manageable one over a working life. (b) For a 78-year-old, the tissue valve is the stronger choice: its 10–20 year service is likely to cover the remainder of their life, avoiding both a re-operation and lifelong anticoagulants, which carry bleeding risks that rise with age. In both cases the evaluation balances durability against compatibility with the body — the same trade-off as any prosthetic material choice.

Question 19 As the runner lands, the ground force bends the blade; the material is loaded up its (linear) elastic region and the work done on it — the area under the force–extension graph — is stored as elastic potential energy, $E_s = 1/2\,k\,x^2$. Because the carbon composite has no plastic region, almost all of that energy is returned as the blade springs straight during push-off, adding to the push the runner's muscles provide. The material must be stiff enough that the blade does not collapse under the runner's force, yet highly elastic so that the large strains of landing are fully recovered each stride; a plastic region would absorb energy permanently, the blade would sag and heat instead of springing, and the runner would lose the return that makes the blade effective.

Question 20 (a) The partner of the force on the forearm by the tendon is the force **on the tendon by the forearm**: it acts on the tendon, is directed downward (opposite to the tendon's upward pull on the forearm), and has the same magnitude, $620\,\text{N}$ in Example 3. (b) A third-law pair always acts on two *different* objects. The tendon force and the joint force both act on the *same* object — the forearm — so they cannot be a third-law pair. Their relationship is an equilibrium one: with the forearm in translational equilibrium (Newton's first law), the vertical forces on it sum to zero, which is why the upward tendon force equals the sum of the downward joint force and the forces due to gravity on the forearm and load. Equal and opposite forces on one body are a first-law balance; equal and opposite forces on two bodies are a third-law pair.

Option task — Forces on the human body report

This is the assessed Option task that closes Option 2.5. It uses the study design's own Outcome 2 task type *"an analysis, including calculations, of physics concepts applied to real-world contexts"*. The study design specifies no mark allocations at Units 1 & 2 — levels of achievement are a matter for school decision, and only satisfactory completion is reported to the VCAA — so the **30 marks below are house convention**. The task is assessed against the shared *Communicating physics* key knowledge ($U2.2.CP.1$–.5), which is taught in Sections 10A and 10B, applied to the physics of this section ($U2.2.O5.1$–.6).

The task. Choose a real situation in Australia in which forces act on the human body — for example a sprinter driving out of the blocks at an athletics meet, an AFL player taking a mark, a gymnast holding a position on the rings, a worker lifting cartons in a warehouse, a cyclist riding to work, or a person living with a prosthetic such as a running blade, a hip replacement or an artificial heart valve — and produce a report of 800–1200 words, with labelled diagrams and calculations, that does all of the following:

1. **Describe the situation and its forces.** Describe the activity or the treatment, and identify the gravitational, compressive, tensile and shear forces acting on at least three parts of the body (or of the body and the prosthetic together). List the sources you used, and evaluate the validity of each source ($U2.2.O5.2$; $U2.2.CP.1$).

2. **Analyse the forces and torques on one part.** Choose one part — a limb acting as a lever, the spine during lifting, the foot on tiptoe — and draw a labelled force diagram showing each force, its direction and its point of application, with the gravitational forces acting at the centre of mass. Apply the conditions for translational and rotational equilibrium to the part, and/or locate the combined centre of mass of two or more parts using the centre-of-mass formula ($U2.2.O5.1$, $U2.2.O5.2$; $U2.2.CP.2$, $U2.2.CP.3$).
3. **Calculate.** Using stated data with sources, calculate stress, strain, Young's modulus or an extension for one structure of the body — a bone, a tendon or a disc. Show your working in full, with units and correct significant figures ($U2.2.O5.3$, $U2.2.O5.4$; $U2.2.CP.2$).
4. **Analyse the behaviour under load.** Explain how the material of your chosen structure behaves under load, referring to a stress–strain graph, elastic and plastic behaviour, and what the area under the graph represents ($U2.2.O5.4$; $U2.2.CP.3$).
5. **Evaluate and take a position.** Either evaluate the suitability of the living tissue for its job, comparing tensile and compressive strength and stiffness, toughness and flexibility under load ($U2.2.O5.5$), or evaluate the development, performance and challenges of the artificial material or structure in the prosthetic ($U2.2.O5.6$). In either case, discuss at least one relevant sociocultural, economic, legal or political factor (cost, access to treatment, safety regulation, the effect on the person's life), and finish with a justified recommendation or stance ($U2.2.CP.4$, $U2.2.CP.5$).

Your teacher will set the due date and any in-class conditions. Work that uses data from a source must name the source and its retrieval date.

Mark allocation (30 marks — house convention).

Criterion	Study-design dot point	Marks
Validity of sources evaluated	$U2.2.CP.1$	4
Physics concepts, terms and conventions applied correctly, with full calculations	$U2.2.CP.2$	8
Force diagrams, models and representations used and organised clearly, with limitations discussed	$U2.2.CP.3$	6
Sociocultural, economic, legal or political factors discussed	$U2.2.CP.4$	4
Physics-based stance, recommendation or solution justified	$U2.2.CP.5$	8
Total		**30**

Marking rubric.

Band	$CP.1$ sources (4)	$CP.2$ physics (8)	$CP.3$ representations (6)	$CP.4$ factors (4)	$CP.5$ stance (8)
High	4: every source named and its validity evaluated	7–8: concepts and terms correct throughout; calculations complete with units and correct significant figures	5–6: diagrams and models complete, clearly labelled; limitations of the model discussed	4: at least one relevant factor discussed in genuine depth	7–8: recommendation fully justified by the physics of the report
Medium	3: sources named and validity considered	4–6: concepts mostly correct; working mostly complete	3–4: diagrams present with minor omissions; limitations noted	3: a relevant factor identified with some discussion	4–6: recommendation given with partial

Band	$CP.1$ sources (4)	$CP.2$ physics (8)	$CP.3$ representations (6)	$CP.4$ factors (4)	$CP.5$ stance (8)
			briefly		justification
Low	1–2: sources listed with little or no evaluation	1–3: concepts partly correct; working incomplete	1–2: diagrams missing key labels or forces	1–2: a factor named with little discussion	1–3: recommendation stated with little support
None	0	0	0	0	0

Marks are awarded for the quality of the reasoning, not for the conclusion reached: two students who reach opposite recommendations can both receive full marks in $CP.5$ if both justify their position from correct physics.

Theory

This section is one of the eighteen options of Unit 2, Area of Study 2. A student studies exactly one option, and this one applies the physics of motion to ball sports. Everything in it stands on the core of Unit 2 Area of Study 1: momentum (8A), Newton's three laws (8B), the force due to gravity and force vectors (8C), components of forces including friction (8D), impulse and collisions in one dimension (8E), and the constant-acceleration equations (7C). The option extends those ideas into two dimensions, into collisions between balls and sporting equipment, and into the motion of balls through air.

The study design introduces the option as follows (VCE Physics Study Design, Unit 2, Area of Study 2, Option 2.11, © VCAA): *"In this option students focus on communicating physics understanding in the context of ball sports."* It suggests that students *"examine mechanics concepts including Newton's laws of motion"*, *"observe and analyse motion in one and two dimensions, study associated collisions and explore the factors that maximise the projection of the ball in various sports"*, and that contexts for exploration *"may include: ideas applied in a selected sport of interest or the comparison of a range of ball sports"*, for example *"the swing of a racquet, club, stick or ball; the throw, pitch or hurl of a ball; or the kick of a ball"*, and *"the relative influence of dynamics factors that affect the performance of the equipment used in ball sports, the effect of the use of different materials, or the aerodynamics of ball shapes (round, ovoid) and surface textures (smooth or dimpled)"*.

The **Communicating physics** key knowledge that prints under every option (U2.2.CP.1–.5) is taught once, in 10A and 10B. This section applies those skills, and the Option task at the end of the section assesses them.

The key knowledge for this option, quoted in full (VCE Physics Study Design, Unit 2, Area of Study 2, Option 2.11, © VCAA):

- *"investigate and calculate theoretically and practically the transfer of momentum in elastic and inelastic collisions (limited to two dimensions) including the use of the coefficient of restitution, e"*;
- *"investigate and apply theoretically and practically the coefficients of static and kinetic friction to sliding and rolling balls to calculate speeds using Newton's laws of motion and the equations of constant acceleration"*;
- *"explain rolling of spherical objects using angular and linear speeds: v = rω."*;
- *"calculate air resistance (drag) and terminal velocity: v_terminal = √(2mg / C_D ρA), where C_D is the drag coefficient, ρ is the gas density, A is the frontal area"*;
- *"investigate and apply theoretically and practically the equations of constant acceleration to calculate the flight of objects through the air (neglecting air resistance) in two dimensions"*;
- *"model and describe qualitatively the flight of:"* — *"a ball through the air when air resistance is not neglected"* and *"spinning sports balls with reference to the Magnus effect"*.

What "improving performance" means here. The option's question — *how can performance in ball sports be improved?* — uses the term *performance*, which the VCE Physics Study Design (© VCAA) does not define. This section therefore treats *improving performance* as making the ball's motion work better for the player: leaving the boot, bat or hand at a greater speed, travelling farther, bouncing more predictably, or curving in a way that is hard for an opponent to read. Each of those is a measurable feature of the ball's motion, and every question in this section is answerable on that definition alone. Other senses of "performance" — tactics, fitness, the rules of the competition — exist, but they are not interchangeable with this one and are not the subject of this section. If your Option task uses the phrase in a different sense, state that sense first.

The balls themselves. The official rules of each sport fix the mass and size of the ball, so the calculations in this section can be built on real, published values.

Ball	Mass	Size	Governing rules
tennis	56.0–59.4 g	diameter 6.54–6.86 cm	ITF, *Rules of Tennis*
cricket (men's)	155.9–163.0 g	circumference 22.4–	MCC, *Laws of Cricket*

Ball	Mass	Size	Governing rules
		22.9 cm (diameter about 7.1–7.3 cm)	
football (soccer, size 5)	410–450 g	circumference 68–70 cm (diameter about 22 cm)	FIFA, *Laws of the Game*
golf	not more than 45.93 g	diameter at least 42.67 mm	The R&A and USGA, *Rules of Golf*
table tennis	2.7 g	diameter 40 mm	ITTF, *Handbook*

Source: International Tennis Federation, Rules of Tennis, retrieved 2026-08-18. Source: Marylebone Cricket Club, Laws of Cricket, retrieved 2026-08-18. Source: FIFA, Laws of the Game, retrieved 2026-08-18. Source: The R&A and USGA, Rules of Golf, retrieved 2026-08-18. Source: International Table Tennis Federation, ITTF Handbook, retrieved 2026-08-18.

Momentum transfer in collisions. When a bat, racquet, club, boot or hand strikes a ball, the contact lasts only a few milliseconds, but during that time the implement exerts a large force on the ball and the ball's momentum changes. The impulse relationship of 8E, $\Delta p = F_{\text{net}} \Delta t$, says that the change in momentum of the ball equals the net force on it multiplied by the contact time. Two consequences follow for technique:

- for a given average force during contact, a **longer contact time** gives the ball a larger change in momentum — one reason coaches teach players to "hit through" the ball rather than tap at it;
- for a given change in momentum, a longer contact time **reduces the average force** on the hands and joints — one reason catching a fast ball "with soft hands", letting them move back as the ball arrives, is easier on the fingers than holding them rigid.

When two free objects collide — two balls on a green, for example — and no external net force acts during the brief contact, total momentum is conserved (8E). Momentum is a vector, so in two dimensions conservation applies to each of two perpendicular directions separately:

$$\Sigma p_x \text{ before} = \Sigma p_x \text{ after}, \Sigma p_y \text{ before} = \Sigma p_y \text{ after}.$$

A collision that is not head-on sends the objects away at angles; the component equations above are the tool for finding the missing velocity.

Elastic and inelastic collisions. Collisions are classified by what happens to the total kinetic energy. Both terms are defined here in their measurable sense, and every question in this section is answerable on these definitions alone:

- an **elastic collision** is one in which the total kinetic energy of the colliding objects is the same after the collision as before it;
- an **inelastic collision** is one in which some of the total kinetic energy is transformed — into thermal energy, sound, and energy of deformation of the ball or the surface. In a **perfectly inelastic** collision the objects move together afterwards as one combined object, which is the case where the most kinetic energy is lost from the motion.

Real sports collisions are almost always inelastic to some degree: a ball squashes on impact, the strings of a racquet stretch, and some of the energy of the motion ends up as thermal energy in the materials. When a ball hits a heavy, rigid surface such as a court or the turf, the surface barely moves, and the collision is usefully characterised by a single number.

The coefficient of restitution, e. The **coefficient of restitution** e of a collision is the ratio of the relative speed of separation after the collision to the relative speed of approach before it, measured along the line of the impact:

$$e = \frac{\text{relative speed of separation}}{\text{relative speed of approach}}.$$

For a ball dropped onto a massive, rigid surface, the surface's speed is zero before and after, so e is simply the ball's speed just after the bounce divided by its speed just before it. The value of e always lies between 0 and 1: $e=1$ describes a perfectly elastic bounce (no kinetic energy lost), and $e=0$ describes a ball that does not rebound at all.

The coefficient of restitution gives the standard drop test used to compare balls and surfaces. A ball dropped from rest through a height h reaches the surface at speed $u=\sqrt{2gh}$, and if it rebounds to a height h', it left the surface at speed $v=\sqrt{2gh'}$. Therefore

$$e=\frac{v}{u}=\sqrt{\frac{h'}{h}}.$$

Measuring two heights with a metre ruler is all the experiment needs. The fraction of the ball's kinetic energy retained in the bounce is e^2; the rest is transformed. A tennis ball on a hard court, for example, rebounds to a little over half its drop height, giving e of about 0.75: roughly $56\,\%$ of the kinetic energy survives the bounce.

The shape of the ball matters too. A round ball rebounds predictably because the normal force at the contact point acts along the line through the ball's centre. An ovoid ball — an Australian rules football — lands at an unknown orientation, so the direction of the normal force on the ball by the ground changes from bounce to bounce, which is why its bounces are famously hard to read.

Friction: sliding balls and rolling balls. A ball in contact with a surface experiences friction at the contact point (8D). Two coefficients describe the two regimes:

- the **coefficient of static friction** μ_s applies while the contact point is not sliding. Static friction adjusts up to a maximum value $\mu_s F_N$, where F_N is the normal force;
- the **coefficient of kinetic friction** μ_k applies while the ball is skidding across the surface. The kinetic friction force has magnitude $F_k=\mu_k F_N$ and acts opposite the sliding. In practice μ_k is smaller than μ_s for the same pair of surfaces.

For a ball **sliding** on a horizontal surface, the vertical forces balance ($F_N=F_g=mg$), so the kinetic friction force is $\mu_k mg$ and Newton's second law gives the deceleration:

$$a=-\mu_k g.$$

The deceleration does not depend on the mass of the ball. With a known, any of the constant-acceleration equations of 7C applies; the most useful here is $v^2=u^2+2as$, which gives the speed after a given distance, and the stopping distance $s=\dfrac{u^2}{2\mu_k g}$.

For a ball **rolling without slipping**, the point of contact is momentarily at rest relative to the surface, so the friction there is static friction, not kinetic friction. A ball rolling at constant speed on a level surface still slows gradually, because the ball and the surface each deform slightly at the contact; the resulting **rolling resistance** is well modelled at this level as a small constant force opposing the motion, giving a constant deceleration that the equations of constant acceleration can handle. If you measure how far a ball rolls before stopping, you can work backwards to the deceleration and to an effective coefficient $\mu=|a|/g$.

Rolling without slipping: $v=r\omega$. A rolling ball has two descriptions of its motion at once. Its centre moves in a straight line with **linear speed** v, and the ball spins about its centre with **angular speed** ω. Angles are measured in **radians**: one full turn is 2π radians, so an angle in radians equals the arc length it cuts off divided by the radius. The unit of angular speed is rad s^{-1}.

For a ball of radius r rolling without slipping, one full turn of the ball moves its centre forward by exactly one circumference, $2\pi r$. Matching the two descriptions of the motion gives the rolling condition:

$$v=r\omega.$$

Know both directions of this result: $\omega = v/r$ gives the spin rate of a ball with a known speed, and the number of revolutions a ball makes over a distance s is $s/(2\pi r)$.

Air resistance and terminal velocity. Every ball moving through air pushes air out of its way, and the air exerts a **drag force** on the ball, opposite the ball's velocity. At sports speeds the drag on a ball is well modelled by

$$F_D = 1/2\, C_D \rho A v^2,$$

where C_D is the **drag coefficient** (a number that depends on the shape and the surface texture of the ball), ρ is the density of the air (about $1.2\,\mathrm{kg\ m^{-3}}$ near sea level), A is the **frontal area** — the cross-sectional area the ball presents to the flow, $A = \pi r^2$ for a sphere — and v is the speed of the ball. The important feature is the v^2: double the speed and the drag is four times as large.

Typical measured drag coefficients at sports speeds: a smooth sphere has C_D of about 0.5; the felt of a tennis ball raises this to about 0.6; the dimples of a golf ball lower it to about 0.25.

Source: International Organization for Standardization, ISO 2533:1975 Standard Atmosphere (air density near sea level), retrieved 2026-08-18. Source: NASA Glenn Research Center, Beginner's Guide to Aeronautics (typical drag coefficients), retrieved 2026-08-18.

For a ball falling vertically, the force due to gravity $F_g = mg$ acts downward and the drag acts upward. As the ball speeds up, the drag grows until it equals F_g; the net force is then zero, the acceleration is zero, and the ball continues downward at constant speed. That speed is the **terminal velocity**. Setting $F_D = mg$ in the drag equation and rearranging gives the study design's formula:

$$v_{\text{terminal}} = \sqrt{\frac{2mg}{C_D \rho A}}.$$

Notice what the formula says: for the same size and shape (same C_D and A), a more massive ball has a higher terminal velocity, because the drag force at any given speed is the same but F_g is larger. The formula also gives the check on units: $\mathrm{kg\,m\,s^{-2}}$ divided by $(\mathrm{kg\,m^{-3}} \times \mathrm{m^2})$ leaves $\mathrm{m^2 s^{-2}}$, whose square root is $\mathrm{m\ s^{-1}}$.

The same drag acts on the horizontal part of a ball's flight, continually reducing its speed. That is why a ball struck or kicked at a speed above its terminal velocity slows noticeably before it arrives.

Flight without air resistance: two-dimensional projectile motion. A ball kicked, thrown or hit into the air — a **projectile** — moves in two dimensions. With air resistance neglected, only one force acts on the ball after release: the force due to gravity $F_g = mg$, vertically downward. The method is to resolve the launch velocity u into components and treat the two directions separately, because each direction obeys its own independent motion:

- **horizontal**: no force, so no acceleration — the horizontal velocity $u_x = u\cos\theta$ stays constant;
- **vertical**: constant acceleration $g = 9.8\,\mathrm{m\ s^{-2}}$ downward, so the constant-acceleration equations of 7C apply with $a = -g$ (taking upward as positive): $u_y = u\sin\theta$, $v_y = u_y - gt$, and $\Delta s_y = u_y t - 1/2\, g t^2$.

For a ball launched from level ground at angle θ and landing at the same height:

- the **time of flight** comes from setting $\Delta s_y = 0$: $t = \dfrac{2u_y}{g}$;
- the **maximum height** comes from setting $v_y = 0$ at the top: $h_{\max} = \dfrac{u_y^2}{2g}$;
- the **range** is horizontal speed multiplied by time of flight: $R = u_x \times \dfrac{2u_y}{g} = \dfrac{u^2 \sin 2\theta}{g}$.

The range expression is largest when $\sin 2\theta = 1$, that is, when $\theta = 45^\circ$: without air resistance, 45° is the launch angle that sends the ball farthest for a given launch speed. Two complementary angles (angles adding to 90°, such as 30° and 60°) give the same range.

Real flight: drag, spin and the Magnus effect. The no-drag model above is the starting model; real balls always feel air resistance, and the study design asks for the corrections to be described qualitatively.

With drag. Drag always opposes the velocity, so it reduces both the horizontal and the vertical components of the motion. Compared with the no-drag parabola, the real flight of a ball has a lower peak, a shorter range, and a descent that is steeper than the ascent: on the way up, drag and F_g act together against the upward motion, while on the way down, drag acts upward against F_g; meanwhile the horizontal speed, constant in the ideal model, is continually reduced. The path is no longer symmetric.

Spin: the Magnus effect. A spinning ball drags a thin layer of air around with it. On the side where the surface is moving in the same direction as the airflow past the ball, the air moves faster and the pressure is lower; on the opposite side, the air moves slower and the pressure is higher. The pressure difference produces a net force on the ball toward the lower-pressure side — the **Magnus effect**, described by Heinrich Magnus in 1852. The direction of the force depends on the spin:

- **topspin** (the top of the ball rotating forward) produces a downward Magnus force: the ball dips sharply, which is how a hard topspin tennis drive lands inside the court;
- **backspin** produces an upward Magnus force: the ball is partly supported against F_g, stays in the air longer and carries farther — the reason golfers want backspin on a drive;
- **sidespin** produces a sideways Magnus force: the ball curves away from a straight line, as in a bending free kick in football.

Spin also changes the bounce: a ball with topspin grabs the surface and kicks forward low and fast, while backspin tends to check the bounce. The surface texture matters here too: the dimples on a golf ball trip the airflow, shrinking the turbulent wake behind the ball and nearly halving C_D compared with a smooth sphere, which is why a dimpled ball carries so much farther.

Investigating ball sports practically. The key knowledge of this option uses the verbs *investigate* and *practically* throughout, and the practical work connects directly to the investigation skills of Chapter 14. Experiments that work well with school equipment:

- the **drop test** for e: drop a ball from a measured height onto a chosen surface, film the rebound against a ruler or read the peak height, calculate $e = \sqrt{h' / h}$, and compare balls or surfaces;
- the **rolling run-down**: roll a ball along a flat surface, measure speed at two marked positions (by timing gates, video analysis or measured distances), and find the deceleration and the effective friction coefficient;
- the **projectile check**: film a kicked or thrown ball against a marked background, extract positions frame by frame, and compare the measured time of flight and range with the no-drag model's predictions — the shortfall is a measure of the drag's effect.

Record readings, repeats and any changes to the method in your logbook: the logbook is required for satisfactory completion of the unit. Apply the safety habits of 14C — keep the flight path of a struck ball clear of people, and choose a space suited to the speeds involved.

Units 3 & 4 preview. The two-dimensional projectile model built here reappears in Units 3 & 4 as part of the core motion content, where it is applied more widely, and the momentum and collision ideas are extended to more complex systems. Units 3 & 4 assumes the component method and the conservation reasoning are already fluent. Content under this banner is not assessed in this book's topic tests or practice examinations.

Throughout this section, $g = 9.8\,\mathrm{m\ s^{-2}}$ ($9.8\,\mathrm{N\ kg^{-1}}$) is used, per the house convention for Units 1 & 2.

Worked examples

Example 1 — scaffolded

In a school investigation, a tennis ball of mass $57\ \mathrm{g}$ is dropped from rest through $1.00\ \mathrm{m}$ onto a hard court surface and rebounds to a height of $0.56\ \mathrm{m}$. (a) Calculate the speed of the ball just before it hits the surface. (b) Calculate the coefficient of restitution e for the bounce. (c) Calculate the speed of the ball just after it leaves the surface, and check it against the rebound height. (d) Determine the fraction of the kinetic energy retained in the bounce, and classify the collision.

Working	Comment
(a) $u=\sqrt{2gh}=\sqrt{2\times 9.8\times 1.00}=\sqrt{19.6}=4.43\ \mathrm{m\ s^{-1}}$	Energy transfer $E_g \rightarrow E_k$ during the fall; the speed from a drop height h is $\sqrt{2gh}$.
(b) $e=\sqrt{\dfrac{h'}{h}}=\sqrt{\dfrac{0.56}{1.00}}=0.748\approx 0.75$	The drop-test formula: e from the two heights.
(c) $v=eu=0.748\times 4.43=3.31\ \mathrm{m\ s^{-1}}$. Check: $\sqrt{2gh'}=\sqrt{2\times 9.8\times 0.56}=3.31\ \mathrm{m\ s^{-1}}$. ✓	The rebound speed is e times the approach speed; the check runs the fall formula on the rebound height.
(d) $\dfrac{E_k\ \text{after}}{E_k\ \text{before}}=\dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2}=\left(\dfrac{v}{u}\right)^2=e^2=0.748^2=0.56$	The mass cancels, so the energy ratio is e^2.
About $56\,\%$ of the kinetic energy is retained; the other $44\,\%$ is transformed into thermal energy, sound and deformation. The bounce is **inelastic**.	Any bounce with $e<1$ is inelastic.

Example 2 — scaffolded

A field hockey ball of mass $0.16\ \mathrm{kg}$ slides across a watered synthetic pitch with an initial speed of $8.0\ \mathrm{m\ s^{-1}}$. The coefficient of kinetic friction between the ball and the wet surface is 0.20. (a) Calculate the normal force on the ball by the pitch. (b) Calculate the kinetic friction force on the ball. (c) Calculate the acceleration of the ball. (d) Calculate the speed of the ball after it has travelled $10.0\ \mathrm{m}$.

Working	Comment
(a) Vertical equilibrium on a horizontal surface: $F_N=F_g=mg=0.16\times 9.8=1.57\ \mathrm{N}$.	The normal force balances the force due to gravity.
(b) $F_k=\mu_k F_N=0.20\times 1.57=0.31\ \mathrm{N}$, opposite the motion.	Kinetic friction applies because the ball is sliding.
(c) $a=\dfrac{F_{\text{net}}}{m}=\dfrac{-\mu_k mg}{m}=-\mu_k g=-0.20\times 9.8=-1.96\ \mathrm{m\ s^{-2}}$.	Newton’s second law; friction is the only horizontal force, and the mass cancels.
(d) $v^2=u^2+2as=8.0^2+2(-1.96)(10.0)=64-39.2=24.8$, so $v=\sqrt{24.8}=5.0\ \mathrm{m\ s^{-1}}$.	The constant-acceleration equation $v^2=u^2+2as$ from 7C.

Example 3 — unscaffolded

An Australian rules footballer place-kicks the ball from level ground at $25\ \mathrm{m\ s^{-1}}$, at 37° to the horizontal. Teammates are standing $60\ \mathrm{m}$ away down the field. Neglecting air resistance and taking $g=9.8\ \mathrm{m\ s^{-2}}$, (a) resolve the launch velocity into components; (b) calculate the time of flight; (c) calculate the range and state whether the ball reaches the teammates; (d) calculate the maximum height of the ball; (e) state one reason the real kick will not travel as far as your answer to (c).

Resolve the launch velocity:

$$u_x = u\cos\theta = 25\cos 37^\circ = 20.0\ \mathrm{m\ s^{-1}}, u_y = u\sin\theta = 25\sin 37^\circ = 15.0\ \mathrm{m\ s^{-1}}.$$

The horizontal motion has zero acceleration; the vertical motion has constant acceleration $-g$ (upward positive). The ball lands at its launch height, so set $\Delta s_y = 0$:

$$0 = u_y t - 1/2\, g t^2 \Rightarrow t = \frac{2u_y}{g} = \frac{2\times 15.0}{9.8} = 3.07\ \mathrm{s}.$$

The range is the constant horizontal speed multiplied by the time of flight:

$$R = u_x t = 20.0\times 3.07 = 61\ \mathrm{m}.$$

The ball reaches $61\ \mathrm{m} > 60\ \mathrm{m}$, so in this model it **reaches the teammates**. The maximum height occurs at half the time of flight, where $v_y = 0$:

$$h_{\max} = \frac{u_y^2}{2g} = \frac{15.0^2}{2\times 9.8} = \frac{225}{19.6} = 11.5\ \mathrm{m}.$$

(e) The calculation neglects air resistance. A real football experiences drag throughout the flight, which continually reduces both components of its velocity, so the real range is shorter than $61\ \mathrm{m}$ — and the ball's spin can add a Magnus force as well.

$\therefore$ $u_x = 20.0\ \mathrm{m\ s^{-1}}$, $u_y = 15.0\ \mathrm{m\ s^{-1}}$; time of flight $3.07\ \mathrm{s}$; range $61\ \mathrm{m}$ (the kick reaches the teammates in the model); maximum height $11.5\ \mathrm{m}$.

Example 4 — unscaffolded

Use the terminal velocity formula to compare the flight of a cricket ball (mass $0.160\ \mathrm{kg}$, diameter $7.1\ \mathrm{cm}$) with that of a table tennis ball (mass $2.7\ \mathrm{g}$, diameter $40\ \mathrm{mm}$). Take $C_D = 0.47$ for each ball, air density $\rho = 1.2\ \mathrm{kg\ m^{-3}}$ and $g = 9.8\ \mathrm{m\ s^{-2}}$. (a) Calculate the terminal velocity of each ball. (b) Explain, using your answers, why a fast cricket bowl slows much less in flight than a table tennis smash.

For each ball the frontal area is $A = \pi r^2$, and the terminal velocity is $v_{\text{terminal}} = \sqrt{2mg/(C_D \rho A)}$.

Cricket ball: $r = 0.0355\ \mathrm{m}$, so $A = \pi(0.0355)^2 = 3.96\times 10^{-3}\ \mathrm{m}^2$.

$$v_{\text{terminal}} = \sqrt{\frac{2\times 0.160\times 9.8}{0.47\times 1.2\times 3.96\times 10^{-3}}} = \sqrt{\frac{3.136}{2.23\times 10^{-3}}} = \sqrt{1400} = 37\ \mathrm{m\ s^{-1}} (\approx 135\ \mathrm{km\ h^{-1}}).$$

Table tennis ball: $r = 0.020\ \mathrm{m}$, so $A = \pi(0.020)^2 = 1.26\times 10^{-3}\ \mathrm{m}^2$.

$$v_{\text{terminal}} = \sqrt{\frac{2\times 0.0027\times 9.8}{0.47\times 1.2\times 1.26\times 10^{-3}}} = \sqrt{\frac{0.0529}{7.09\times 10^{-4}}} = \sqrt{74.6} = 8.6\ \mathrm{m\ s^{-1}}.$$

(b) The drag force at any given speed depends on C_D, A and v, not on the mass — but the force due to gravity does depend on the mass. The cricket ball has 59 times the mass of the table tennis ball but only about 3 times the frontal area, so its terminal velocity is much higher: $37\ \mathrm{m\ s^{-1}}$ against $8.6\ \mathrm{m\ s^{-1}}$. A fast cricket bowl at about $40\ \mathrm{m\ s^{-1}}$ is travelling not far above its terminal velocity, so drag has a moderate effect over the length of the pitch. A table tennis smash at $20\ \mathrm{m\ s^{-1}}$ is travelling at more than twice the ball's terminal velocity, so the drag force is several times F_g and the ball slows very quickly — which is why hard table tennis strokes are played close to the table.

$\therefore$ $v_{\text{terminal}} \approx 37\ \mathrm{m\ s^{-1}}$ for the cricket ball and $8.6\ \mathrm{m\ s^{-1}}$ for the table tennis ball.

Practice questions

Question 1 *(scaffolded)* In a rebound investigation, a basketball is dropped from rest through $1.80\,\mathrm{m}$ onto a sports hall floor and rebounds to $1.05\,\mathrm{m}$. (a) Write the expression for the coefficient of restitution e in terms of the drop height h and the rebound height h'. (b) Calculate e for this bounce. (c) Calculate the speed of the ball just before impact. (d) State the fraction of the kinetic energy retained in the bounce.

Question 2 *(scaffolded)* On a bowling green, bowl A of mass $1.5\,\mathrm{kg}$ travels at $2.4\,\mathrm{m\,s^{-1}}$ and strikes an identical bowl B at rest, head-on. After the collision, bowl A is stationary. (a) Calculate the total momentum of the two bowls before the collision. (b) Use conservation of momentum to find the velocity of bowl B after the collision. (c) Calculate the total kinetic energy before and after the collision, and classify the collision.

Question 3 *(scaffolded)* A field hockey ball slides along a dry pitch at $10.0\,\mathrm{m\,s^{-1}}$. The coefficient of kinetic friction between the ball and the pitch is 0.20. (a) Calculate the acceleration of the ball. (b) Write the constant-acceleration equation that gives the speed after a given distance, and substitute the values for a distance of $15\,\mathrm{m}$. (c) Calculate the speed of the ball after $15\,\mathrm{m}$.

Question 4 *(scaffolded)* A basketball of radius $0.12\,\mathrm{m}$ rolls without slipping along a court at $2.4\,\mathrm{m\,s^{-1}}$. (a) Calculate the angular speed of the ball. (b) Calculate the number of revolutions the ball makes each second. (c) Explain why the friction at the contact point of a rolling ball is static friction rather than kinetic friction.

Question 5 *(scaffolded)* A golfer chips a ball from level ground at $12\,\mathrm{m\,s^{-1}}$, at 30° to the horizontal. Neglect air resistance. (a) Calculate the horizontal and vertical components of the launch velocity. (b) Calculate the time of flight. (c) Calculate the range of the shot.

Question 6 *(scaffolded)* A tennis ball of mass $57\,\mathrm{g}$ and diameter $6.7\,\mathrm{cm}$ is served at $40\,\mathrm{m\,s^{-1}}$. Take $C_D = 0.60$ for the felt-covered ball and air density $\rho = 1.2\,\mathrm{kg\,m^{-3}}$. (a) Calculate the frontal area of the ball. (b) Calculate the drag force on the ball at the instant of the serve. (c) Calculate the force due to gravity on the ball and compare it with the drag force. Comment on what the comparison means for the serve.

Question 7 Two identical bowls, each of mass $1.4\,\mathrm{kg}$, collide on a green. Bowl A is travelling at $2.5\,\mathrm{m\,s^{-1}}$ along the x direction and strikes bowl B, which is at rest. After the collision, bowl A moves at $1.2\,\mathrm{m\,s^{-1}}$ at 40° to its original direction. (a) Use conservation of momentum in the x and y directions to find the velocity of bowl B after the collision, giving both magnitude and direction. (b) Calculate the total kinetic energy before and after the collision, and state whether the collision is elastic or inelastic.

Question 8 A student drops three balls from $1.00\,\mathrm{m}$ onto a concrete surface and measures the rebound heights: golf ball $0.78\,\mathrm{m}$; football (soccer) $0.72\,\mathrm{m}$; cricket ball $0.61\,\mathrm{m}$. (a) Calculate the coefficient of restitution for each ball. (b) Identify which ball retains the largest fraction of its kinetic energy in the bounce. (c) Describe the energy transfers and transformations that occur during one of these bounces, from release to the top of the rebound.

Question 9 A football (soccer) of mass $0.43\,\mathrm{kg}$ rolls across level grass at $8.0\,\mathrm{m\,s^{-1}}$ and comes to rest after $16\,\mathrm{m}$. Model the slowing as a constant deceleration caused by a constant resistive force. (a) Calculate the acceleration of the ball. (b) Calculate the net force on the ball during the slowing, and state its direction. (c) Assuming the resistive force can be written $\mu m g$, calculate the effective coefficient μ for the ball on this grass. (d) Identify the object that exerts the resistive force on the ball, using "force on … by …" language.

Question 10 A football (soccer) of radius $0.11\,\mathrm{m}$ rolls without slipping at $15\,\mathrm{m\,s^{-1}}$ across the pitch. (a) Calculate the angular speed of the ball. (b) Calculate the number of revolutions the ball makes while travelling $30\,\mathrm{m}$. (c) State the condition that must hold for the relationship $v = r\omega$ to apply.

Question 11 A football (soccer) of mass $0.43\,\mathrm{kg}$ and diameter $0.22\,\mathrm{m}$ moves through still air. Take $C_D = 0.47$ and $\rho = 1.2\,\mathrm{kg\,m^{-3}}$. (a) Calculate the terminal velocity of the ball. (b) A long pass is struck at $25\,\mathrm{m\,s^{-1}}$. Compare this speed with your answer to (a), and explain what must happen to the ball's speed during its flight. (c) Calculate the drag force on the ball at $25\,\mathrm{m\,s^{-1}}$ and compare it with the force due to gravity on the ball.

Question 12 Smooth golf balls were replaced long ago by dimpled ones. Explain, in terms of the airflow around the ball and the drag coefficient, why dimples improve the performance of a golf ball. No calculation is required.

Question 13 A player kicks a ball from level ground at $20\,\mathrm{m\,s^{-1}}$, neglecting air resistance. (a) Calculate the range for launch angles of 30°, 45° and 60°, using the component method for each. (b) Explain why the 30° and 60° kicks have the same range. (c) Explain why a real player aiming for maximum distance does not usually kick at 45°, even though the no-drag model says this angle is best.

Question 14 A tennis player hits a hard drive with heavy topspin, and the ball dips down into the court rather than sailing long. A golfer strikes a drive with backspin, and the ball stays in the air longer than a spinless shot would. (a) Explain the dipping of the topspin tennis ball, referring to the airflow on each side of the ball and the direction of the resulting force. (b) Explain the effect of the backspin on the golf ball's flight.

Question 15 A student claims: "Two balls with the same diameter but different masses, dropped from a great height, fall differently — the more massive ball reaches the higher terminal velocity." Evaluate this claim using the terminal velocity formula, and state what would happen to the two balls if the same experiment were performed in a vacuum.

Question 16 Coaches in striking and kicking sports constantly tell players to "follow through" — to keep the bat, racquet or leg moving well past the point of contact. Use the impulse–momentum relationship $\Delta p = F_{\mathrm{net}}\,\Delta t$ to explain why follow-through improves the speed of the struck ball, and why "giving" with the hands makes catching a fast ball more comfortable.

Question 17 A netball shooter releases the ball from a height of $1.6\,\mathrm{m}$ at $7.5\,\mathrm{m\,s^{-1}}$, at 55° above the horizontal. The ring is at a height of $3.05\,\mathrm{m}$. Neglect air resistance. (a) Calculate the horizontal and vertical components of the launch velocity. (b) Calculate the time at which the ball is at the height of the ring on its way down. (c) Calculate the horizontal distance from the shooter to the ring for the shot to arrive. (d) State one limitation of this model as a description of a real shot.

Question 18 Describe how the flight of a ball kicked high into the air with air resistance differs from the parabolic path predicted when air resistance is neglected. Address the height, the range, and the shapes of the ascent and the descent, and give a reason for each difference.

Question 19 A cricket ball and a tennis ball have similar diameters but the cricket ball is about three times as massive, and the tennis ball's felt gives it a larger drag coefficient. Both are delivered at the same speed through still air. Explain, referring to the drag force and Newton's second law, which ball slows more quickly, and predict which travels farther. Your answer should be qualitative; no calculation is required.

Question 20 You are asked to investigate how the surface affects the coefficient of restitution of a tennis ball, using the drop-test method of this section. (a) State the aim and a hypothesis for the investigation. (b) Identify the independent, dependent and two controlled variables. (c) Outline a procedure that would generate reliable primary data, including how you would handle repeats. (d) Identify one source of uncertainty in the measurements and suggest one improvement to the method. (e) State where the records of your readings and method changes must be kept, and why.

Answers

Question 1 (a) $e=\sqrt{h'/h}$; (b) 0.76; (c) $5.9\,\mathrm{m\,s^{-1}}$; (d) $58\,\%$. **Question 2** (a) $3.6\,\mathrm{kg\,m\,s^{-1}}$; (b) $2.4\,\mathrm{m\,s^{-1}}$ in A's original direction; (c) $4.32\,\mathrm{J}$ before and $4.32\,\mathrm{J}$ after — elastic. **Question 3** (a) $-2.0\,\mathrm{m\,s^{-2}}$; (b) $v^2=u^2+2as$: $v^2=100+2(-1.96)(15)=41$; (c) $6.4\,\mathrm{m\,s^{-1}}$. **Question 4** (a) $20\,\mathrm{rad\,s^{-1}}$; (b) 3.2 revolutions per second; (c) see solution — the contact point is momentarily at rest relative to the surface. **Question 5** (a) $u_x=10.4\,\mathrm{m\,s^{-1}}$, $u_y=6.0\,\mathrm{m\,s^{-1}}$; (b) $1.22\,\mathrm{s}$; (c) $12.7\,\mathrm{m}$. **Question 6** (a) $3.53\times10^{-3}\,\mathrm{m^2}$; (b) $2.0\,\mathrm{N}$; (c) $F_g=0.56\,\mathrm{N}$ — the drag is about 3.6 times F_g, so the serve slows quickly. **Question 7** (a) $1.76\,\mathrm{m\,s^{-1}}$ at 26° to the original line, on the opposite side from bowl A; (b) $4.4\,\mathrm{J}$ before, $3.2\,\mathrm{J}$ after — inelastic. **Question 8** (a) golf 0.88, football 0.85, cricket 0.78; (b) the golf ball; (c) see solution. **Question 9** (a) $-2.0\,\mathrm{m\,s^{-2}}$; (b) $0.86\,\mathrm{N}$ opposite the motion; (c) 0.20; (d) the force on the ball by the ground (grass). **Question 10** (a) $136\,\mathrm{rad\,s^{-1}}$; (b) 43 revolutions; (c) rolling without slipping. **Question 11** (a) $20\,\mathrm{m\,s^{-1}}$; (b) $25>20\,\mathrm{m\,s^{-1}}$, so the ball must slow; (c) $F_D=6.7\,\mathrm{N}$ against $F_g=4.2\,\mathrm{N}$. **Question 12** See solution — dimples lower C_D, so the drag at any speed is smaller. **Question 13** (a) $35\,\mathrm{m}$, $41\,\mathrm{m}$, $35\,\mathrm{m}$; (b) complementary angles give equal ranges; (c) air resistance, and the need for height and spin. **Question 14** See solution — the Magnus force is downward for topspin, upward for backspin. **Question 15** The claim is correct: $v_{\mathrm{terminal}}\propto\sqrt{m}$ for equal size; in a vacuum both fall identically with $a=g$. **Question 16** See solution — longer contact time means a larger Δp for the same force, and a smaller force for the same Δp. **Question 17** (a) $u_x=4.3\,\mathrm{m\,s^{-1}}$, $u_y=6.1\,\mathrm{m\,s^{-1}}$; (b) $0.94\,\mathrm{s}$; (c) $4.0\,\mathrm{m}$; (d) see solution — air resistance is neglected. **Question 18** See solution — lower peak, shorter range, steeper descent. **Question 19** See solution — the tennis ball slows more quickly; the cricket ball travels farther. **Question 20** See solution.

Fully-worked solutions

Question 1 $h=1.80\,\mathrm{m}$, $h'=1.05\,\mathrm{m}$. (a) From the drop-test derivation, $e=\sqrt{h'/h}$. (b) $e=\sqrt{1.05/1.80}=\sqrt{0.583}=0.76$. (c) $u=\sqrt{2gh}=\sqrt{2\times9.8\times1.80}=\sqrt{35.28}=5.9\,\mathrm{m\,s^{-1}}$. (d) The fraction of kinetic energy retained is $e^2=0.583$, so about $58\,\%$.

Question 2 $m_A=m_B=1.5\,\mathrm{kg}$; $u_A=2.4\,\mathrm{m\,s^{-1}}$; $u_B=0$; bowl A stationary after the collision. (a) Total momentum before: $p=m_Au_A+m_Bu_B=1.5\times2.4+0=3.6\,\mathrm{kg\,m\,s^{-1}}$, in bowl A's original direction. (b) With no external net force during the collision, total momentum is conserved, and all of the momentum after the collision is carried by bowl B: $m_Bv_B=3.6$, so $v_B=3.6/1.5=2.4\,\mathrm{m\,s^{-1}}$ in bowl A's original direction. (c) Before: $E_k=1/2\times1.5\times2.4^2=4.32\,\mathrm{J}$. After: $E_k=1/2\times1.5\times2.4^2=4.32\,\mathrm{J}$. The total kinetic energy is unchanged, so the collision is **elastic** (as the idealised data implies — real bowls collisions lose some kinetic energy).

Question 3 $u=10.0\,\mathrm{m\,s^{-1}}$, $\mu_k=0.20$, $s=15\,\mathrm{m}$. (a) For a sliding ball on a horizontal surface, $a=-\mu_kg=-0.20\times9.8=-1.96\,\mathrm{m\,s^{-2}}\approx-2.0\,\mathrm{m\,s^{-2}}$. (b) $v^2=u^2+2as$: $v^2=10.0^2+2(-1.96)(15)=100-58.8=41.2$. (c) $v=\sqrt{41.2}=6.4\,\mathrm{m\,s^{-1}}$.

Question 4 $r=0.12\,\mathrm{m}$, $v=2.4\,\mathrm{m\,s^{-1}}$. (a) $\omega=v/r=2.4/0.12=20\,\mathrm{rad\,s^{-1}}$. (b) One revolution is 2π radians, so revolutions per second $=\omega/(2\pi)=20/(2\pi)=3.2$. (c) Model answer. A ball rolling without slipping has its point of contact momentarily at rest relative to the surface — there is no sliding at the contact point. Kinetic friction applies only while surfaces slide across each other, so the friction at the contact point of a rolling ball is static friction. Kinetic friction would act only if the ball were skidding.

Question 5 $u=12\,\mathrm{m\,s^{-1}}$, $\theta=30^\circ$, level ground. (a) $u_x=u\cos30^\circ=12\times0.866=10.4\,\mathrm{m\,s^{-1}}$; $u_y=u\sin30^\circ=12\times0.500=6.0\,\mathrm{m\,s^{-1}}$. (b) Time of flight: $t=2u_y/g=2\times6.0/9.8=1.22\,\mathrm{s}$. (c) Range: $R=u_xt=10.4\times1.22=12.7\,\mathrm{m}$.

Question 6 $m=0.057\,\mathrm{kg}$, $d=0.067\,\mathrm{m}$ (so $r=0.0335\,\mathrm{m}$), $v=40\,\mathrm{m\,s^{-1}}$, $C_D=0.60$, $\rho=1.2\,\mathrm{kg\,m^{-3}}$. (a) $A=\pi r^2=\pi\times0.0335^2=3.53\times10^{-3}\,\mathrm{m^2}$. (b)

$F_D = 1/2\, C_D \rho A v^2 = 0.5 \times 0.60 \times 1.2 \times 3.53 \times 10^{-3} \times 40^2 = 0.36 \times 3.53 \times 10^{-3} \times 1600 = 2.0\,\text{N}$. (c) $F_g = m g = 0.057 \times 9.8 = 0.56\,\text{N}$. The drag at the instant of the serve is about 3.6 times the force due to gravity on the ball. The net force slowing the ball is large compared with F_g, so the ball decelerates quickly: a serve is never travelling at its launch speed by the time it reaches the receiver.

Question 7 $m_A = m_B = 1.4\,\text{kg}$; $u_A = 2.5\,\text{m s}^{-1}$ along $+x$; bowl B at rest; after the collision bowl A moves at $1.2\,\text{m s}^{-1}$ at 40° to $+x$ (take its deflection as $+y$). (a) Momentum before: $p_x = 1.4 \times 2.5 = 3.5\,\text{kg m s}^{-1}$; $p_y = 0$. x direction: $3.5 = m_A v_A \cos 40^\circ + m_B v_{Bx}$, so

$$v_{Bx} = \frac{3.5 - 1.4 \times 1.2 \times \cos 40^\circ}{1.4} = \frac{3.5 - 1.287}{1.4} = 1.58\,\text{m s}^{-1}.$$

y direction: $0 = m_A v_A \sin 40^\circ + m_B v_{By}$, so

$$v_{By} = -\frac{1.4 \times 1.2 \times \sin 40^\circ}{1.4} = -1.2 \sin 40^\circ = -0.77\,\text{m s}^{-1}.$$

Magnitude: $v_B = \sqrt{1.58^2 + 0.77^2} = \sqrt{3.09} = 1.76\,\text{m s}^{-1}$. Direction: $\theta = \tan^{-1}(0.77/1.58) = 26^\circ$ below the original line — that is, on the opposite side of the original direction from bowl A. (b) Before: $E_k = 1/2 \times 1.4 \times 2.5^2 = 4.375\,\text{J}$. After: $E_k = 1/2 \times 1.4 \times 1.2^2 + 1/2 \times 1.4 \times 1.76^2 = 1.01 + 2.16 = 3.17\,\text{J}$. The kinetic energy after ($3.2\,\text{J}$) is less than before ($4.4\,\text{J}$), so the collision is **inelastic**: about $28\,\%$ of the kinetic energy is transformed into thermal energy, sound and deformation.

Question 8 Drop height $h = 1.00\,\text{m}$ for all three balls. (a) $e = \sqrt{h'/h}$: golf ball: $e = \sqrt{0.78} = 0.88$; football: $e = \sqrt{0.72} = 0.85$; cricket ball: $e = \sqrt{0.61} = 0.78$. (b) The fraction of kinetic energy retained is e^2, which equals h'/h here. The golf ball ($e = 0.88$, retaining $78\,\%$) retains the largest fraction. (c) Model answer. On the way down, gravitational potential energy is transformed into kinetic energy of the ball. During the impact, the ball (and the surface slightly) deforms: kinetic energy is stored briefly as elastic energy in the deformed materials, and some is transformed into thermal energy and sound. As the ball rebounds, the stored elastic energy is transformed back into kinetic energy — less than before, because of the losses. On the way up, that kinetic energy is transformed back into gravitational potential energy, reaching the (lower) rebound height.

Question 9 $m = 0.43\,\text{kg}$, $u = 8.0\,\text{m s}^{-1}$, $v = 0$, $s = 16\,\text{m}$. (a) $v^2 = u^2 + 2as$ gives $0 = 64 + 2a(16)$, so $a = -64/32 = -2.0\,\text{m s}^{-2}$. (b) $F_{\text{net}} = ma = 0.43 \times 2.0 = 0.86\,\text{N}$, directed opposite the motion (the negative sign of a shows the force opposes the velocity). (c) If the resistive force is $\mu m g$, then $ma = \mu m g$, so $\mu = |a|/g = 2.0/9.8 = 0.20$. (d) Model answer. The resistive force is the force on the ball by the ground — the grass and the surface acting at the contact point (rolling resistance and friction), directed backward against the ball's motion.

Question 10 $r = 0.11\,\text{m}$, $v = 15\,\text{m s}^{-1}$. (a) $\omega = v/r = 15/0.11 = 136\,\text{rad s}^{-1}$. (b) One revolution advances the ball by one circumference, $2\pi r = 2\pi \times 0.11 = 0.69\,\text{m}$. Over $30\,\text{m}$: $30/0.69 = 43$ revolutions. (c) Model answer. The relationship $v = r\omega$ holds when the ball rolls without slipping — the point of contact is momentarily at rest relative to the surface, so the distance the centre travels equals the arc length the ball turns through. If the ball skids as well as rolls, $v \neq r\omega$.

Question 11 $m = 0.43\,\text{kg}$, $d = 0.22\,\text{m}$ (so $r = 0.11\,\text{m}$), $C_D = 0.47$, $\rho = 1.2\,\text{kg m}^{-3}$. (a) $A = \pi r^2 = \pi \times 0.11^2 = 0.0380\,\text{m}^2$.

$$v_{\text{terminal}} = \sqrt{\frac{2mg}{C_D \rho A}} = \sqrt{\frac{2 \times 0.43 \times 9.8}{0.47 \times 1.2 \times 0.0380}} = \sqrt{\frac{8.43}{0.0214}} = \sqrt{393} = 20\,\text{m s}^{-1}.$$

(b) Model answer. The pass is struck at $25\,\text{m s}^{-1}$, which is above the ball's terminal velocity of $20\,\text{m s}^{-1}$. At any speed above the terminal velocity the drag force is larger than F_g, so the ball experiences a large net force opposing its motion and must slow. The ball's speed falls steadily through the flight; it can only approach, never exceed, the terminal velocity as it travels. (c) $F_D = 1/2\, C_D \rho A v^2 = 0.5 \times 0.47 \times 1.2 \times 0.0380 \times 25^2 = 6.7\,\mathrm{N}$. $F_g = mg = 0.43 \times 9.8 = 4.2\,\mathrm{N}$. The drag at $25\,\mathrm{m\ s^{-1}}$ is about 1.6 times the force due to gravity on the ball — consistent with (b), since at the terminal velocity of $20\,\mathrm{m\ s^{-1}}$ the two forces would be equal.

Question 12 Model answer. As a ball moves through air, the flow separates from the surface and leaves a low-pressure turbulent wake behind the ball; the pressure difference between the front and the back of the ball is a major source of drag. The dimples on a golf ball trip the airflow near the surface into turbulence earlier, which keeps the flow attached to the ball for longer and narrows the wake. The smaller wake means a lower drag coefficient — about 0.25 for a dimpled ball against about 0.5 for a smooth sphere of the same size. With a smaller C_D, the drag force $F_D = 1/2\, C_D \rho A v^2$ at any given speed is about half as large, so the ball loses speed more slowly and carries much farther for the same strike. This is the study design's "surface texture (smooth or dimpled)" factor at work.

Question 13 $u = 20\,\mathrm{m\ s^{-1}}$, level ground, no air resistance; $t = 2u_y / g$ and $R = u_x t$. (a) 30°: $u_x = 20\cos 30^\circ = 17.3\,\mathrm{m\ s^{-1}}$, $u_y = 20\sin 30^\circ = 10.0\,\mathrm{m\ s^{-1}}$; $t = 2 \times 10.0 / 9.8 = 2.04\,\mathrm{s}$; $R = 17.3 \times 2.04 = 35\,\mathrm{m}$. 45°: $u_x = u_y = 20\cos 45^\circ = 14.1\,\mathrm{m\ s^{-1}}$; $t = 2 \times 14.1 / 9.8 = 2.89\,\mathrm{s}$; $R = 14.1 \times 2.89 = 41\,\mathrm{m}$. 60°: $u_x = 20\cos 60^\circ = 10.0\,\mathrm{m\ s^{-1}}$, $u_y = 20\sin 60^\circ = 17.3\,\mathrm{m\ s^{-1}}$; $t = 2 \times 17.3 / 9.8 = 3.53\,\mathrm{s}$; $R = 10.0 \times 3.53 = 35\,\mathrm{m}$. (b) Model answer. The 30° and 60° kicks exchange the roles of the two components: what one has as horizontal speed the other has as vertical speed. The low kick has more horizontal speed but less time of flight; the high kick has less horizontal speed but more time of flight. Since the range is the product $u_x \times t = u_x \times 2u_y / g$, swapping the components leaves the product unchanged. (This is the identity $\sin 2\theta = \sin(180^\circ - 2\theta)$: complementary angles give equal ranges.) (c) Model answer. The 45° result assumes no air resistance. A real ball experiences drag throughout the flight, which reduces both components of the velocity; drag also grows with speed, so the faster, flatter parts of the flight are penalised most, and the optimum angle drops below 45°. In many sports the ball also needs to clear opponents or land at a useful height, and spin (the Magnus effect) changes the flight as well — all reasons a player chooses the angle for the situation rather than always 45°.

Question 14 (a) Model answer. With topspin, the top of the ball rotates forward, so the top surface moves against the airflow past the ball and the air on that side moves slower and its pressure is higher; on the underside the surface moves in the same direction as the airflow, so the air there moves faster and its pressure is lower. The pressure difference produces a net force on the ball toward the lower-pressure side, downward — the Magnus force. The downward Magnus force adds to the force due to gravity, giving the ball a larger downward acceleration than g alone, so the ball dips and lands in the court instead of sailing long. (b) Model answer. With backspin the directions reverse: the underside of the ball rotates forward, so the underside moves against the airflow past the ball and the air there moves slower and its pressure is higher, while the top surface moves in the same direction as the airflow, so the air over the top moves faster and its pressure is lower. The pressure difference therefore produces a net force on the ball toward the lower-pressure side, upward — the Magnus force — partly opposing the force due to gravity. The net downward force is smaller than F_g, the ball takes longer to fall a given height, stays in the air longer, and carries farther than a spinless shot at the same launch speed. The upward Magnus force on a backspinning ball is often called lift.

Question 15 Model answer. The claim is correct. For two balls with the same diameter and the same drag coefficient, C_D, ρ and A are identical in the terminal velocity formula, so $v_{\text{terminal}} = \sqrt{2mg/(C_D \rho A)}$ is proportional to $\sqrt{m}$: the more massive ball has the higher terminal velocity. Physically, at any given speed both balls experience the same drag force, but the more massive ball has the larger F_g, so drag cancels F_g only at a higher speed. In a vacuum there is no air and therefore no drag: both balls experience only the force due to gravity, and by Newton's second law both accelerate at $a = F_g / m = g$, regardless of mass, so they fall together.

Question 16 Model answer. The impulse–momentum relationship $\Delta p = F_{\text{net}} \Delta t$ says the change in the ball's momentum equals the net force on it multiplied by the contact time. When a player follows through, the implement stays in contact with the ball for a longer time Δt. For the same average force during contact, the longer Δt gives the ball a larger change in momentum, so it leaves the bat, racquet or boot at a greater speed. The same relationship explains catching: the ball arriving at the hands must lose a fixed amount of momentum Δp to stop. If the hands are held rigid, the stopping time is short and the force on the hands is large; "giving" with the hands extends Δt, and the same Δp is then achieved with a smaller average force, making the catch more comfortable and more secure.

Question 17 $u = 7.5\,\mathrm{m\ s^{-1}}$, $\theta = 55^\circ$; release height $1.6\,\mathrm{m}$, ring height $3.05\,\mathrm{m}$, so the ball must rise by $\Delta s_y = 3.05 - 1.6 = 1.45\,\mathrm{m}$. Upward is positive; $a = -g = -9.8\,\mathrm{m\ s^{-2}}$. (a) $u_x = 7.5\cos 55^\circ = 4.3\,\mathrm{m\ s^{-1}}$; $u_y = 7.5\sin 55^\circ = 6.1\,\mathrm{m\ s^{-1}}$. (b) The vertical motion obeys $\Delta s_y = u_y t - 1/2\,g t^2$:

$$1.45 = 6.14t - 4.9t^2 \Rightarrow 4.9t^2 - 6.14t + 1.45 = 0.$$

The quadratic gives $t = \left(6.14 \pm \sqrt{6.14^2 - 4 \times 4.9 \times 1.45}\right) / \left(2 \times 4.9\right) = \left(6.14 \pm 3.05\right)/9.8$, so $t = 0.32\,\mathrm{s}$ (on the way up) or $t = 0.94\,\mathrm{s}$ (on the way down). A shot arriving at the ring must be descending, so $t = 0.94\,\mathrm{s}$. (c) The horizontal speed is constant: $x = u_x t = 4.30 \times 0.94 = 4.0\,\mathrm{m}$. (d) Model answer. The model neglects air resistance, which slows the ball in flight; a real shot released at these values would fall short of $4.0\,\mathrm{m}$, so the shooter would need a slightly greater launch speed, a slightly higher angle, or to stand closer. The model also treats the ball as a point and ignores any spin, which in a real shot can add a Magnus force.

Question 18 Model answer. With air resistance, the path is no longer a symmetric parabola. The peak is **lower** than the no-drag peak, because drag acts downward alongside the force due to gravity during the ascent, giving the ball a downward acceleration greater than g and removing vertical speed faster. The range is **shorter**, because drag always opposes the velocity and continually reduces the horizontal component, which stays constant in the no-drag model. The descent is **steeper** than the ascent: on the way down, drag acts upward against F_g, so the downward acceleration is less than g and the ball gains vertical speed more slowly than it lost it on the way up, while the horizontal speed keeps falling throughout — so the ball covers less horizontal distance per metre of fall on the descent. The result is a path whose second half is shorter and more vertical than its first half.

Question 19 Model answer. At the same speed, through the same air, the drag force $F_D = 1/2\,C_D \rho A v^2$ is larger on the tennis ball, because the felt raises its drag coefficient and the two balls have similar frontal areas. But the deceleration caused by the drag is what matters for the flight, and by Newton's second law $a = F_{\mathrm{net}}/m$: the tennis ball has about one third of the cricket ball's mass, so the same order of drag force produces a much larger deceleration. The tennis ball therefore slows more quickly, and the cricket ball travels farther. This is the same physics as the terminal velocity comparison of Worked example 4: the small mass and large C_D of the tennis ball give it a much lower terminal velocity, so any delivery well above that speed is rapidly dragged back down toward it.

Question 20 Model answer. (a) Aim: to determine how the rebound surface affects the coefficient of restitution of a tennis ball. Hypothesis (example): the harder the surface, the larger the coefficient of restitution, because less energy is lost to deformation of the surface. (b) Independent variable: the rebound surface (for example concrete, timber sports hall floor, carpet, grass). Dependent variable: the rebound height h', and the coefficient of restitution $e = \sqrt{h'/h}$ calculated from it. Controlled variables (any two): the drop height h; the same ball (same mass, pressure and temperature); the same release method (dropped from rest, no spin); the same measuring position and lighting. (c) Procedure (outline). Mark a drop height of, say, $1.00\,\mathrm{m}$ on a vertical ruler or against a wall. Release the ball from rest at the mark, without spin. Film the rebound from directly beside the ruler (or read the peak height) and record the rebound height. Repeat at least three times for each surface and use the mean rebound height. Calculate $e = \sqrt{h'/h}$ for each surface and compare. (d) Source of uncertainty (any one): judging the exact top of the rebound, which lasts only an instant; parallax when reading the height; small variations in the release. Improvement (any one, matching the source): use slow-motion video frame by frame to locate the rebound peak; keep the camera at the height of the ruler to reduce parallax; use a release clamp or guide so the ball drops without spin. (e) Model answer. All readings, repeats, calculations and changes to the method must be recorded in your logbook. The logbook authenticates your primary data — it is the record that the data are genuinely yours and genuinely taken — and submitting a logbook is a requirement for satisfactory completion of the unit.

Option task

This Option task is the assessed component of this option. It is one of the house Option tasks required for each of the seventeen authored options: a 30-mark assessed piece in one of the study design's own Outcome 2 formats, marked against the Communicating physics key knowledge (U2.2.CP.1–.5), which is taught in 10A and 10B. Options are excluded from the topic tests and practice examinations, so this task is the only assessed work for the option.

Marks in this task are house convention. The study design fixes no mark allocations for Units 1 & 2 assessment — levels of achievement are a matter for school decision — and the 30-mark total and the rubric below are house practice, not a VCAA requirement.

The task

Task type, from the study design's list of suitable assessment tasks for Unit 2 Outcome 2 (VCE Physics Study Design, © VCAA): *"an analysis, including calculations, of physics concepts applied to real-world contexts"*.

Task statement. Choose one ball sport — for example Australian rules football, cricket, tennis, netball, football (soccer), hockey, golf, basketball or lawn bowls — and write an analysis answering the question:

How can performance in your chosen ball sport be improved?

Your analysis must:

1. select **two** of the physics factors of this option — momentum transfer and the coefficient of restitution in collisions; friction and rolling; air resistance and terminal velocity; two-dimensional flight (projectile motion); spin and the Magnus effect — and explain each one correctly in the context of your sport;
2. include at least **two calculations of your own**, one for each factor, using real, sourced data for your sport's ball (mass, size, speed or measured rebound heights); every real datum carries a `Source:` line naming the publishing organisation, the document and the retrieval date;
3. include at least one **data representation** — a table or graph of data you have measured (for example drop-test rebound heights on different surfaces) or collated from a source;
4. discuss at least one **limitation** of the physics model you used — for example, neglecting air resistance in the flight calculation, or treating a collision as elastic when it is not;
5. evaluate the **validity of your sources**, stating why each is reliable or where it falls short;
6. discuss at least one **societal factor** bearing on your recommendation — the rules of the sport (for example equipment regulations), cost of equipment, access to facilities, safety, or fairness between players;
7. finish with a **justified recommendation** — a specific, physics-based way to improve performance in the sport, stating clearly the evidence your recommendation rests on.

Conditions. Length 800–1200 words (diagrams and the data representation not counted). A scientific calculator may be used. Primary data, if collected, must be recorded in your logbook. Where the task uses measurements, record units and significant figures throughout, and label every graph with quantities, symbols and units.

Marking rubric (30 marks)

Each criterion is worth 6 marks and is keyed to one Communicating physics key knowledge dot point. Award within a band for the quality of the response; the bands are house convention.

Criterion	Key knowledge (verbatim, VCE Physics Study Design, Unit 2, AoS 2, © VCAA)	Marks
1. Sources	*"evaluate validity of sources of information"*	6
2. Physics concepts	*"apply physics concepts specific to the investigation: definitions of key terms; and use of appropriate scientific terminology, conventions and representations"*	6
3. Models and data	*"apply the use of data representations, models and theories in organising and explaining observed phenomena and physics*	6

Criterion	Key knowledge (verbatim, VCE Physics Study Design, Unit 2, AoS 2, © VCAA)	Marks
	concepts, and discuss the limitations of the explanations”	
4. Societal context	*“discuss the influence of sociocultural, economic, legal and political factors relevant to the selected issue or application”*	6
5. Stance	*“apply physics understanding to justify a stance, opinion or solution to the selected issue or application”*	6

Criterion 1 — validity of sources (6 marks). 5–6: every real datum has a `Source:` line with organisation, document and retrieval date; the reliability of each source is explicitly evaluated (authority, currency, independence); at least one weakness in a source is identified. 3–4: sources are named and dated; validity is discussed in general terms without identifying weaknesses. 1–2: sources are listed but not evaluated, or evaluation is asserted without reasoning. 0: no sources given, or sources that cannot be identified.

Criterion 2 — physics concepts (6 marks). 5–6: both selected factors are explained correctly and completely, with key terms (for example *coefficient of restitution*, *terminal velocity*, *angular speed*, *Magnus effect*) defined at first use; the $F_{\text{on A by B}}$ convention and correct units are used throughout; no physics errors. 3–4: both factors are explained with minor errors or omissions; terminology mostly correct. 1–2: one factor explained; several physics errors or undefined terms. 0: physics content absent or incorrect throughout.

Criterion 3 — models and data (6 marks). 5–6: at least one clear data representation (table or correctly constructed graph) is included and interpreted; both required calculations are correct, with units and appropriate significant figures; at least one limitation of the model used is discussed and its effect on the result stated. 3–4: calculations correct; a data representation included; a limitation is named but its effect is not discussed. 1–2: one calculation attempted, or the data representation missing, or no limitation discussed. 0: no calculations and no data representation.

Criterion 4 — societal context (6 marks). 5–6: at least one societal factor (rules and regulations, cost, access, safety or fairness) is identified and discussed with a clear link to the chosen sport and to the recommendation. 3–4: a societal factor is identified and described, with a loose link to the recommendation. 1–2: a societal factor is named only. 0: no societal factor addressed.

Criterion 5 — justified stance (6 marks). 5–6: the recommendation is specific (a named change to equipment, technique or preparation), follows directly from the physics and the data presented, and its limitations are acknowledged. 3–4: a recommendation is given and supported by the physics, but is general or its limitations are not acknowledged. 1–2: a recommendation is stated with little or no support from the analysis. 0: no recommendation, or one contradicted by the analysis.

Total: /30