# Physics Units 1 & 2

## Chapter 9 — Energy and motion



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### Contents

**Scope** — *VCE Physics Study Design, Unit 2, Area of Study 1*: "apply the concept of work done by a force using:" "work done = force × displacement: W = Fs cos θ, where force is constant"; and "work done = area under force vs distance graph".

### Theory

**Forces that move things.** When a locomotive hauls carriages along a track, when an AFL player drives a tackling sled across the ground, or when a crane raises a container at the Port of Melbourne, a force acts on an object and the object moves while the force acts. In physics, the force is said to do **work** on the object. Work is not a kind of force and not a kind of motion: it is the energy transferred by a force. This section builds the two ways of calculating the work done by a force that the study design requires — from the force and the displacement, and from the area under a force–distance graph.

**Work done by a force acting along the motion.** When a constant force $F$ acts on an object in the same direction as the object's displacement $s$, the work done by the force is

$$W = F\,s\,.$$

In words: work done = force × displacement. Work done is measured in **joules** (J). One joule is the work done when a force of $1\,\mathrm{N}$ moves an object $1\,\mathrm{m}$ in the direction of the force:

$$1\,\mathrm{J} = 1\,\mathrm{N\,m}\,.$$

The force due to gravity on an apple of mass about $100\,\mathrm{g}$ is close to $1\,\mathrm{N}$, so lifting the apple $1\,\mathrm{m}$ does about $1\,\mathrm{J}$ of work — a useful feel for the size of a joule.

Force and displacement are both vectors (Section 7A), but work done itself is a **scalar**: it has a size, and (as you will see below) a sign, but no direction.

**Work done by a force at an angle: $W = F\,s\cos\theta$.** Forces often act at an angle to the direction of motion — the handle of a wheeled bag, the tow rope of a water-skier, a rope pulling a sledge. Only the component of the force in the direction of the displacement does work. Resolving a force into components was introduced in Section 8D: the component of $F$ along the displacement is $F\cos\theta$, where $\theta$ is the angle between the force and the displacement. The study design states the result for a constant force as

$$W = F\,s\cos\theta\,.$$

The component of the force perpendicular to the displacement, $F\sin\theta$, does no work at all: it acts at $90^\circ$ to the motion and transfers no energy along it.

Note the condition attached to the formula: $W = F\,s\cos\theta$ applies **only when the force is constant**. When the force changes while the object moves, use the graph method later in this section.

**Three angles worth remembering.**

- $\theta = 0^\circ$: $\cos 0^\circ = 1$, so $W = F\,s$. The force acts in the direction of the motion and does its maximum work — a locomotive pulling along the track, a push straight along the direction a trolley rolls.
- $\theta = 90^\circ$: $\cos 90^\circ = 0$, so $W = 0$. A force perpendicular to the motion does **no work**. The normal force on a block sliding along a level bench, the force due to gravity on a car travelling along a level road, and the upward supporting force on a tray carried horizontally across a room are each perpendicular to the displacement and each does zero work.
- $\theta = 180^\circ$: $\cos 180^\circ = -1$, so $W = -F\,s$. A force acting directly against the motion does **negative work** — friction on a sliding football player, or the braking force on a tram.

**The sign of work done: adding or removing energy.** Work done is an energy transfer, and the sign tells you which way the energy moves.

- **Positive work** transfers energy **to** the object: the force has a component in the direction of the motion.
- **Negative work** takes energy **from** the object: the force has a component against the motion. The energy removed is most often transformed into thermal energy (and some sound) by friction or air resistance — this is why the brakes of a tram or a bicycle warm up when they slow the vehicle.
- **Zero work** transfers no energy, whatever the size of the force.

**Work done by several forces.** When more than one force acts on an object, each force has its own work done, and the overall energy transfer is the sum of them. Consider a box pulled along a level floor at constant speed. The pulling force does positive work; the friction force, equal in size and opposite in direction, does the same amount of negative work; the normal force on the box by the floor and the force due to gravity on the box both act perpendicular to the motion and do zero work. The total work done on the box is zero — consistent with Newton's first law (Section 8B), since an object with zero net force keeps a constant velocity and its kinetic energy does not change.

From Section 9C you will calculate these energy changes directly — kinetic energy, gravitational potential energy and elastic potential energy — and use energy conservation. Work done is the mechanism that moves the energy between stores.

**Work done = area under a force–distance graph.** Plot the force on the vertical axis against the distance moved on the horizontal axis. For a constant force the graph is a horizontal line, and the work done is the area between the line and the distance axis — a rectangle of height $F$ and width $s$:

$$\text{area} = F \times s = W\,.$$

The area under a force–distance graph is the work done even when the force is not constant and $W = F\,s$ cannot be used. Break the area into rectangles and triangles and add them, or count squares on the grid. Two shapes appear over and over:

- A **rectangle** — a constant force $F$ acting over a distance $s$: work done $= F\,s$.
- A **triangle** — a force that increases steadily from zero to $F$ over a distance $s$: work done $= 1/2 \times s \times F = 1/2\,F\,s$. This is the same as using the average force, $1/2\,F$, over the distance.

In Section 9B you will meet the force in a stretched spring, which increases in exactly this triangular way with extension; the triangular area under its force–distance graph becomes the elastic potential energy stored in the spring.

One sign rule carries over to graphs: an area where the force points **against** the motion (counted below the axis on a signed graph) is **negative** work — energy taken from the object.

**Measuring work done.** Work done is straightforward to measure in the laboratory. Pull a block along the bench with a spring balance or a force sensor and read the force; measure the distance the block moves while the force acts. If the force is roughly constant, the work done is the force times the distance. If the force changes, record force against distance and find the area under the graph. Section 9B uses exactly this method for a spring.

**Where this leads.** The triangular area of this section becomes the elastic potential energy of a spring in Section 9B; Section 9C uses energy conservation to track the kinetic, gravitational potential and elastic potential energy stores that work done moves energy between; Section 9D turns work done into a rate (power) and an efficiency; and Section 9E applies all of it in a case study. Chapters 7–9 are assessed together in Topic Test T4.

**Units 3 & 4 preview.** At Units 3 & 4 the connection between work and energy becomes a calculation tool: the total work done on an object is set equal to its change in kinetic energy and used to calculate speeds, and the area-under-the-graph method of this section is extended to graphs of field strength against distance. Content under this banner is not assessed in this book's topic tests or practice examinations.

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## Worked examples

### *Example 1 — scaffolded*

At Melbourne Airport, a traveller pulls a suitcase $60\,\text{m}$ along the level floor of the terminal. The force on the suitcase by the strap is a constant $80\,\text{N}$, and the strap makes an angle of $40^\circ$ above the horizontal. (a) Calculate the component of this force in the direction of the suitcase's motion. (b) Calculate the work done on the suitcase by the force from the strap. (c) The vertical component of the force does no work on the suitcase. Explain why.

| Working | Comment |
|---|---|
| $F = 80\,\mathrm{N}$, $\theta = 40^{\circ}$, $s = 60\,\mathrm{m}$. | List the data. |
| (a) $F\cos\theta = 80 \times \cos 40^{\circ} = 61\,\mathrm{N}$. | The component of the force along the displacement is $F\cos\theta$. |
| (b) $W = F\,s\cos\theta = 80 \times 60 \times \cos 40^{\circ} = 3.7 \times 10^{3}\,\mathrm{J}$. | The force is constant, so $W = F\,s\cos\theta$ applies. The given values have 2 significant figures, so the answer is given to 2. |
| (c) The vertical component acts at $90^{\circ}$ to the displacement, and $\cos 90^{\circ} = 0$, so the work it does is zero. | Only the component of the force in the direction of the motion does work. |

### *Example 2 — scaffolded*

The Australian bobsleigh team practises the push-start of a run. During the push, the force on the sled by the athletes is a constant $350\,\mathrm{N}$ for the first $10\,\mathrm{m}$ of the track, and then decreases steadily to zero over the next $10\,\mathrm{m}$. (a) State what is equal to the area under a force–distance graph. (b) Calculate the work done on the sled over the first $10\,\mathrm{m}$. (c) Calculate the total work done on the sled over the full $20\,\mathrm{m}$.

| Working | Comment |
|---|---|
| (a) The work done by the force. | Work done = area under a force–distance graph. |
| (b) The graph is a rectangle: area $= 350 \times 10 = 3500\,\mathrm{J}$. | Constant force: rectangle of height $F$, width $s$. |
| (c) The second half is a triangle: area $= 1/2 \times 10 \times 350 = 1750\,\mathrm{J}$. Total $= 3500 + 1750 = 5250\,\mathrm{J} \approx 5.3 \times 10^{3}\,\mathrm{J}$. | Force decreasing steadily to zero: triangle, $1/2 \times$ base × height. Both areas are above the axis, so both are positive work. The answer is given to 2 significant figures. |

### *Example 3 — unscaffolded*

A tram travelling along Flinders Street in Melbourne brakes to a stop. The braking force on the tram is a constant $2.5 \times 10^{4}\,\mathrm{N}$, directed against the motion, and the tram moves $40\,\mathrm{m}$ while it slows to rest. Calculate the work done on the tram by the braking force, and explain the meaning of the sign of your answer.

The braking force acts at $\theta = 180^{\circ}$ to the displacement, and $\cos 180^{\circ} = -1$, so

$$W = F\,s\cos\theta = 2.5 \times 10^{4} \times 40 \times (-1) = -1.0 \times 10^{6}\,\mathrm{J}.$$

$\therefore$ the work done on the tram by the braking force is $-1.0 \times 10^{6}\,\mathrm{J}$, or $-1.0\,\mathrm{MJ}$. The negative sign means the force takes energy from the tram rather than giving energy to it: the braking force acts against the motion, so the tram's kinetic energy decreases. The energy removed is transformed mainly into thermal energy in the brakes and the wheels, with a little sound.

### *Example 4 — unscaffolded*

At the Port of Melbourne, a crane lifts a cargo crate of mass $1.2 \times 10^{3}\,\mathrm{kg}$ vertically through $12\,\mathrm{m}$ at constant speed. (a) Explain why the force on the crate by the crane equals the force due to gravity on the crate, and calculate this force. (b) Calculate the work done on the crate by the force from the crane. (c) Calculate the work done on the crate by the force due to gravity. (d) Use your answers to explain why the crate's kinetic energy does not change during the lift.

For part (a): the crate moves at constant speed, so its acceleration is zero and the forces on it are balanced (Newton's first law, Section 8B). The upward force on the crate by the crane therefore equals the force due to gravity on the crate:

$$F = F_g = m\,g = 1.2 \times 10^{3} \times 9.8 = 11\,760\,\mathrm{N}.$$

For part (b): the force from the crane acts in the direction of the displacement ($\theta = 0^\circ$), so

$$W = F\,s\cos\theta = 11\,760 \times 12 \times \cos 0^\circ = 1.4 \times 10^5\ \mathrm{J}$$

to 2 significant figures.

For part (c): the force due to gravity acts downward while the displacement is upward, so $\theta = 180^\circ$ and $\cos 180^\circ = -1$:

$$W = F_g\,s\cos 180^\circ = 11\,760 \times 12 \times (-1) = -1.4 \times 10^5\ \mathrm{J}.$$

For part (d): the total work done on the crate is the sum of the two works:

$$W_{\text{total}} = 1.4 \times 10^5 + \left(-1.4 \times 10^5\right) = 0.$$

∴ the crane does $1.4 \times 10^5$ J of work and the force due to gravity does $-1.4 \times 10^5$ J of work. The two cancel, so the total work done on the crate is zero and its kinetic energy is unchanged — which is exactly what "constant speed" means. The energy transferred by the crane is not lost: it is stored in the crate–Earth system, and Section 9C calculates this stored energy as gravitational potential energy.

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## Practice questions

**Question 1** At training, an AFL player pushes a tackling sled $8.0\ \mathrm{m}$ along the ground with a constant horizontal force of $250\ \mathrm{N}$. (a) State the expression for the work done by a constant force acting in the direction of the displacement. (b) Calculate the work done on the sled by the player. (c) Name the main form of energy the sled gains because of this work.

**Question 2** A student pushes a mop $10\ \mathrm{m}$ across a floor. The force along the mop handle is a constant $60\ \mathrm{N}$, and the handle makes an angle of $35^\circ$ with the horizontal. (a) Calculate the component of the force in the direction of the mop's motion. (b) Calculate the work done on the mop by the force from the handle. (c) State the component of the force that does no work on the mop.

**Question 3** A student lifts a school bag of mass $5.0\ \mathrm{kg}$ from the floor to a bench $0.90\ \mathrm{m}$ higher, moving it at constant speed. Use $g = 9.8\ \mathrm{N\ kg^{-1}}$. (a) Calculate the force due to gravity on the bag. (b) State the force on the bag by the student's hand, with a reason. (c) Calculate the work done on the bag by the student's hand.

**Question 4** In a netball match, a player dives and slides $2.5\ \mathrm{m}$ along the court. During the slide, a constant friction force of $180\ \mathrm{N}$ on the player by the court acts against her motion. (a) State the angle between the friction force and the player's displacement. (b) Calculate the work done on the player by the friction force. (c) State the form of energy into which this energy is mainly transformed.

**Question 5** A constant force of $40\ \mathrm{N}$ pushes a trolley along a straight, level track, and the force–distance graph for the motion is a horizontal line from $0$ to $12\ \mathrm{m}$. (a) State what is equal to the area between the graph and the distance axis. (b) Calculate that area, and hence the work done by the force. (c) The trolley then rolls a further $3.0\ \mathrm{m}$ with no force applied. State the work done by the pushing force over this $3.0\ \mathrm{m}$, with a reason.

**Question 6** In a practical activity, a student uses a spring balance to pull a wooden block along the bench at constant speed. The balance reads $2.4\ \mathrm{N}$ and the block moves $0.80\ \mathrm{m}$. (a) State the two measurements needed to calculate the work done by the spring balance. (b) Calculate the work done on the block by the spring balance. (c) Give one reason for repeating the measurement several times and averaging.

**Question 7** At St Kilda Beach, a kite-surfer is pulled along the water by the kite. The pull on the rider by the lines is a constant $240\ \mathrm{N}$ at an angle of $30^\circ$ to the direction of travel. Calculate the work done on the rider by the lines over $50\ \mathrm{m}$ of travel.

**Question 8** A waiter carries a tray of drinks horizontally across a room at constant speed, supporting the tray with an upward force. A customer says the waiter does work on the tray the whole time. Explain whether the upward force on the tray by the waiter's hand does work on the tray.

**Question 9** A trolley is pushed along a straight track. For the first $5.0\,\mathrm{m}$ the force on the trolley is a constant $20\,\mathrm{N}$; over the next $5.0\,\mathrm{m}$ the force decreases steadily to zero. Calculate the total work done on the trolley over the full $10\,\mathrm{m}$ .

**Question 10** A student holds a bag of groceries still at their side while waiting, and says it feels tiring, so they must be doing work on the bag. Identify the error in this statement and correct it, using the physics definition of work done.

**Question 11** A mango of mass $0.20\,\mathrm{kg}$ falls $4.0\,\mathrm{m}$ from a tree to the ground. Calculate the work done on the mango by the force due to gravity during the fall, and explain the sign of your answer. Use $g = 9.8\,\mathrm{N\ kg^{-1}}$.

**Question 12** A skydiver descends at constant speed with an open parachute. Explain why the work done on the skydiver by the air resistance is negative, and state where the energy removed from the skydiver goes.

**Question 13** A commuter of mass $62\,\mathrm{kg}$ climbs the stairs from the concourse to a platform at Flinders Street Station in Melbourne, gaining a vertical height of $3.6\,\mathrm{m}$. Calculate the work done against the force due to gravity during the climb. Use $g = 9.8\,\mathrm{N\ kg^{-1}}$.

**Question 14** The commuter in Question 13 could take the lift instead of the stairs to reach the same platform. Compare the work done against the force due to gravity on the two routes, and justify your answer.

**Question 15** A force of $40\,\mathrm{N}$ pulls a box $6.0\,\mathrm{m}$ along a floor while a constant friction force of $15\,\mathrm{N}$ acts in the opposite direction. (a) Calculate the work done on the box by each force. (b) Calculate the total work done on the box.

**Question 16** A force on an object increases steadily from zero to $F$ while the object moves a distance $s$. A student says the work done is $F s$. Explain why this is not correct, and state the correct expression for the work done.

**Question 17** Section 8E analysed a cricket catch using impulse. The same catch can be analysed using work: as the wicket-keeper stops the ball, her hands move back $0.25\,\mathrm{m}$ while exerting an average force of $48\,\mathrm{N}$ against the ball's motion. Calculate the work done on the ball by her hands.

**Question 18** Two cranes at a building site lift identical containers of bricks through the same height. Crane A takes $40\,\mathrm{s}$; crane B takes $20\,\mathrm{s}$. A student says crane B does more work on its container because it is faster. Identify the error in this statement and correct it.

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### Answers

**Question 1** (a) $W = F\,s$; (b) $2.0 \times 10^3$ J; (c) see solution. **Question 2** (a) 49 N; (b) $4.9 \times 10^2$ J; (c) see solution. **Question 3** (a) 49 N; (b) 49 N upward; (c) 44 J. **Question 4** (a) $180^\circ$; (b) $-450$ J; (c) see solution. **Question 5** (a) the work done by the force; (b) 480 J; (c) see solution. **Question 6** (a) see solution; (b) 1.9 J; (c) see solution. **Question 7** $1.0 \times 10^4$ J. **Question 8** See solution. **Question 9** 150 J. **Question 10** See solution. **Question 11** 7.8 J, positive; see solution. **Question 12** See solution. **Question 13** $2.2 \times 10^3$ J. **Question 14** See solution. **Question 15** (a) $+240$ J by the pulling force, $-90$ J by friction; (b) 150 J. **Question 16** See solution. **Question 17** $-12$ J. **Question 18** See solution.

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### Fully-worked solutions

**Question 1** $F = 250$ N, $s = 8.0$ m, force along the displacement. (a) $W = F\,s$ for a constant force acting in the direction of the displacement. (b) $W = F\,s = 250 \times 8.0 = 2000\text{ J} = 2.0 \times 10^3$ J (2 significant figures). (c) Kinetic energy — the sled speeds up as the player does work on it. (Some of the energy is also transformed into thermal energy by friction between the sled and the ground.)

**Question 2** $F = 60$ N, $\theta = 35^\circ$, $s = 10$ m. (a) Component along the motion: $F\cos\theta = 60 \times \cos 35^\circ = 49$ N (2 significant figures). (b) $W = F\,s\cos\theta = 60 \times 10 \times \cos 35^\circ = 490\text{ J} = 4.9 \times 10^2$ J. (c) The component perpendicular to the floor, $F\sin\theta$, acts at $90^\circ$ to the displacement and does no work on the mop.

**Question 3** $m = 5.0$ kg, $s = 0.90$ m, constant speed, $g = 9.8\text{ N kg}^{-1}$. (a) $F_g = m\,g = 5.0 \times 9.8 = 49$ N. (b) The bag moves at constant speed, so the forces on it are balanced (Newton's first law): the force on the bag by the student's hand equals the force due to gravity, 49 N upward. (c) The hand's force acts along the displacement, so $W = F\,s = 49 \times 0.90 = 44$ J (2 significant figures). This energy is stored and will be calculated as gravitational potential energy in Section 9C.

**Question 4** $F = 180$ N, $s = 2.5$ m, force against the motion. (a) The friction force acts directly against the displacement, so $\theta = 180^\circ$. (b) $W = F\,s\cos\theta = 180 \times 2.5 \times \cos 180^\circ = -450$ J. (c) Thermal energy (in the player's clothing and the court surface), with a little sound.

**Question 5** $F = 40$ N for $s = 12$ m, then $F = 0$ for 3.0 m. (a) The work done by the force. (b) Area $= 40 \times 12 = 480$ J, so the work done by the force is 480 J. (c) Zero. Work done is force times displacement, and over the extra 3.0 m the pushing force is zero, so it does no work even though the trolley keeps moving.

**Question 6** Balance reading $= 2.4$ N; distance moved $= 0.80$ m. (a) The force applied, and the distance the block moves in the direction of the force. (b) $W = F\,s = 2.4 \times 0.80 = 1.9$ J (2 significant figures). (c) Repeating and averaging reduces the effect of random errors — small, unpredictable variations from one run to the next — and makes a mistake in a single reading easier to spot. Record each reading, with its unit, in your logbook.

**Question 7** $F = 240$ N, $\theta = 30^\circ$, $s = 50$ m.

$$W = F\,s\cos\theta = 240 \times 50 \times \cos 30^\circ = 10\,392\text{ J} \approx 1.0 \times 10^4\text{ J}$$

to 2 significant figures. Only the component of the pull along the direction of travel, $F\cos 30^\circ$, does work.

**Question 8** No work is done on the tray by the upward force. The force on the tray by the waiter's hand is vertical, while the tray's displacement is horizontal: the force is perpendicular to the motion, so $\theta = 90^\circ$ and $W = F\,s\cos 90^\circ = 0$. In physics, work done requires a displacement in the direction of the force; the customer is using "work" in its everyday sense, which is not the physics definition. (The waiter's muscles do transform chemical energy internally while holding the tray, but that energy transfer is not work done on the tray.)

**Question 9** First 5.0 m: rectangle of height 20 N, width 5.0 m: area $= 20 \times 5.0 = 100$ J. Second 5.0 m: force decreases steadily to zero, so the area is a triangle: $1/2 \times 5.0 \times 20 = 50$ J. Total work done $= 100 + 50 = 150$ J.

**Question 10** The error is treating effort as work. Work done by a force requires the object to move: work done = force × displacement. The bag is held still, so its displacement is zero and the work done on the bag by the student's hand is zero, however tiring the effort feels. The tiredness comes from chemical energy transformed inside the student's muscles to keep them tensed; that energy is not transferred to the bag as work.

**Question 11** $m = 0.20\,\text{kg}$, $s = 4.0\,\text{m}$, $g = 9.8\,\text{N kg}^{-1}$. The force due to gravity on the mango is

$$F_g = m\,g = 0.20 \times 9.8 = 1.96\,\text{N},$$

and it acts downward, in the same direction as the fall ($\theta = 0^\circ$):

$$W = F\,s = 1.96 \times 4.0 = 7.8\,\text{J}$$

to 2 significant figures. The work done is positive because the force due to gravity acts in the direction of the displacement, transferring energy to the mango as it falls.

**Question 12** The air resistance on the skydiver acts upward, against the downward motion, so $\theta = 180^\circ$ and the work it does is negative: it takes energy from the skydiver. The energy removed is transformed into thermal energy of the surrounding air and the parachute, with a little sound. (Since the skydiver descends at constant speed, the negative work done by the air resistance exactly cancels the positive work done by the force due to gravity, so the skydiver's kinetic energy does not change.)

**Question 13** $m = 62\,\text{kg}$, vertical height $= 3.6\,\text{m}$, $g = 9.8\,\text{N kg}^{-1}$. The force due to gravity on the commuter is

$$F_g = m\,g = 62 \times 9.8 = 607.6\,\text{N}.$$

Work done against the force due to gravity is the force due to gravity multiplied by the vertical height gained:

$$W = 607.6 \times 3.6 = 2187\,\text{J} \approx 2.2 \times 10^3\,\text{J}$$

to 2 significant figures.

**Question 14** The work done against the force due to gravity is the same on both routes. The force due to gravity is vertical, so only the vertical part of the movement matters: on both the stairs and the lift the commuter gains the same vertical height of $3.6\,\text{m}$, and the work done against the force due to gravity is $m\,g \times$ vertical height in each case. The length or shape of the path does not change the work done against the force due to gravity.

**Question 15** $s = 6.0\,\text{m}$; pulling force $= 40\,\text{N}$ along the motion; friction $= 15\,\text{N}$ against the motion. (a) Work done by the pulling force: $W = 40 \times 6.0 = +240\,\text{J}$. Work done by friction: $W = 15 \times 6.0 \times \cos 180^\circ = -90\,\text{J}$. (b) Total work done $= 240 + (-90) = 150\,\text{J}$. The positive total means energy is transferred to the box overall, so the box speeds up.

**Question 16** $W = F\,s$ applies only when the force is constant, and here the force changes — it starts at zero and ends at $F$, so there is no single value of $F$ to use over the whole distance. On a force–distance graph the area is a triangle of base $s$ and height $F$, so the work done is $1/2\,F\,s$. Equivalently, the average force during the motion is $1/2\,F$, and work done = average force × distance.

**Question 17** $F = 48\,\text{N}$, $s = 0.25\,\text{m}$, force against the ball's motion ($\theta = 180^\circ$).

$$W = F\,s\cos\theta = 48 \times 0.25 \times (-1) = -12\,\text{J}.$$

The work done on the ball by her hands is $-12\,\text{J}$: the force removes energy from the ball as it is stopped. (Section 8E analysed the same event with impulse — force × time; here the analysis uses force × distance.)

**Question 18** Both cranes do the same work on their containers. Work done against the force due to gravity depends on the force needed (equal to the force due to gravity on the container) and the height lifted, and both containers are identical and lifted through the same height. What differs is how quickly the energy is transferred: crane B does the same work in half the time, so it transfers energy at a greater rate. Rate of energy transfer is power, the topic of Section 9D.

## Theory

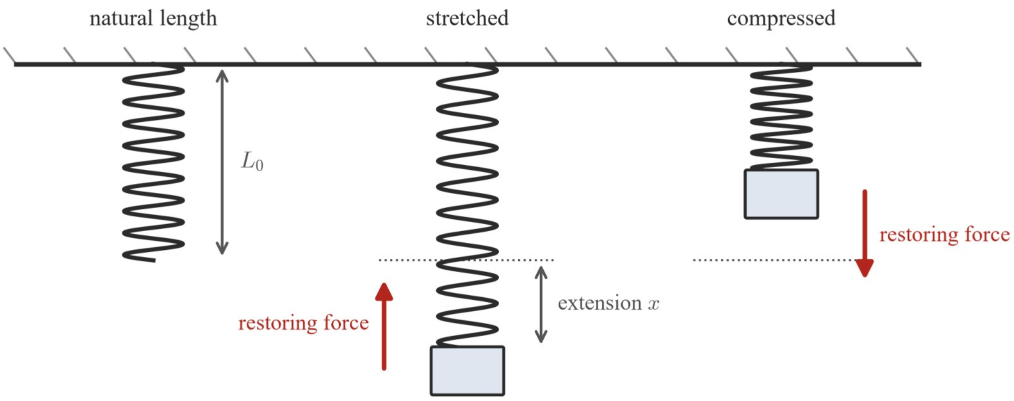
A retracting pen, a flyscreen door closer, a trampoline, a pogo stick and the suspension of every car and ute all rely on the same behaviour: a coiled spring changes its length when forces act on it, and springs back when those forces are removed. Pull on such a spring gently and it stretches a little; pull harder and it stretches further. The link between the force and the stretch is simple enough to be captured in a single equation. That equation is **Hooke’s Law**, and it is one of the most widely used models in physics and engineering.

The study design states the task of this section: *“investigate and analyse theoretically and practically Hooke’s Law for an ideal spring: F = −kx, where x is extension”* (VCE Physics Study Design, Unit 2, Area of Study 1). Everything in this section is that one relationship — what the symbols mean, how the law is applied, and where real springs stop obeying it.

**Springs, natural length and extension.** A spring that is not being pushed or pulled has a length called its **natural length**, $L_0$. When forces stretch the spring, its length increases; the increase is called the **extension**, $x$:

$$x = L - L_0,$$

where $L$ is the stretched length. Extension is a distance and is measured in metres (m). When forces squash the spring instead, its length becomes less than $L_0$ and the spring is **compressed**; the compression is measured in the same way, as a distance from the natural length. The figure below shows a spring at its natural length, extended and compressed.




the restoring force always pushes or pulls the spring back
toward its natural length

**The restoring force.** A stretched or compressed spring does not simply sit in its new shape — it pushes or pulls back. Hold one end of a spring, stretch it, and you feel it pull your hand back towards its other end. The force a spring exerts because it has been extended is called the **restoring force**, because it always acts to restore the spring to its natural length: a stretched spring pulls inwards, a compressed spring pushes outwards, and in both cases the force points back towards the natural length. This direction rule is one half of Hooke’s Law.

**Hooke’s Law.** For many springs, provided they are not stretched too far, the restoring force is directly proportional to the extension: double the extension and the restoring force doubles, triple the extension and the force triples. An **ideal spring** is a model spring for which this proportionality is exact, in both extension and compression. For an ideal spring, the restoring force $F$ and the extension $x$ are related by

$$F = -k\,x \text{ (Hooke’s Law)}$$

where $k$ is the **spring constant** of the spring. Hooke’s Law can be read two ways: as a statement about size (a larger extension needs a larger force, in fixed ratio) and as a statement about direction (the minus sign — see below).

The **spring constant** $k$ measures how stiff a spring is:

$$k = \frac{F}{x}.$$

A stiff spring has a large $k$: it takes a large force to produce even a small extension. A soft spring has a small $k$: a modest force stretches it a long way. The unit of the spring constant is the **newton per metre** ($\mathrm{N\ m^{-1}}$): a spring with $k = 200\ \mathrm{N\ m^{-1}}$ needs $200\ \mathrm{N}$ of force for each metre of extension, or $4.0\ \mathrm{N}$ for each $2.0\ \mathrm{cm}$. To get a feel for the sizes: the small spring inside a retracting pen has $k$ of roughly $100\ \mathrm{N\ m^{-1}}$, while each suspension spring of a car has $k$ of tens of thousands of $\mathrm{N\ m^{-1}}$.

| Quantity | Symbol | Unit |
|---|---|---|
| restoring force | $F$ | newton ($\mathrm{N}$) |
| spring constant | $k$ | newton per metre ($\mathrm{N\ m^{-1}}$) |
| extension | $x$ | metre ($\mathrm{m}$) |

**The minus sign: force towards home.** In $F = -k\,x$, the direction is handled with a sign, as for motion in a straight line (Section 7A). Take the direction of the extension as positive. Then:

- For an **extension**, $x$ is positive, so $F = -k\,x$ is negative: the restoring force points opposite to the extension, back towards the natural length. With $k = 200\ \mathrm{N\ m^{-1}}$ and $x = +0.050\ \mathrm{m}$, $F = -200 \times 0.050 = -10\ \mathrm{N}$.
- For a **compression**, $x$ is negative, so $F = -k\,x$ is positive: the restoring force points opposite to the compression — again back towards the natural length. With the same spring compressed by $0.050\ \mathrm{m}$, $x = -0.050\ \mathrm{m}$ and $F = -200 \times (-0.050) = +10\ \mathrm{N}$.

Either way, the spring's force always points back towards the natural length, which is why it is called a restoring force. For calculations where only the size of the force matters, the magnitude form

$$F = k\,x$$

is used, with $x$ read as the size of the extension or compression and the direction stated in words.

**Springs at rest: a hanging mass.** The most common way to apply a known force to a spring is to hang a known mass from it. Consider a mass $m$ hanging at rest from a vertical spring. Two forces act on the mass: the force due to gravity, $F_{\text{on mass by Earth}} = m\,g$, directed downward (Section 8C), and the upward restoring force of the spring, $F_{\text{on mass by spring}}$. Because the mass is at rest, the net force on it is zero (Newton's first law, Section 8B), so the two forces have equal magnitudes:

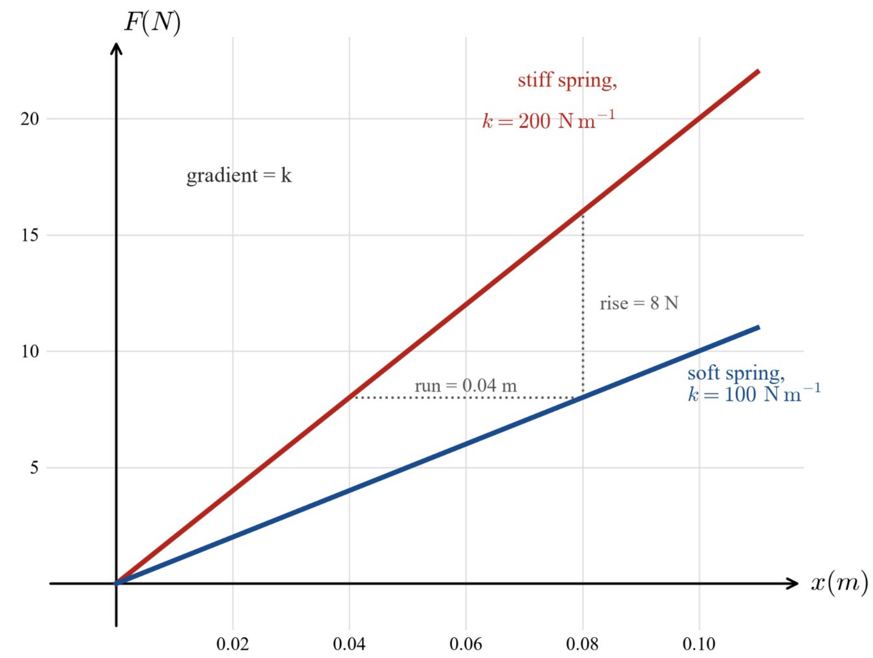
$$k\,x = m\,g.$$

The spring stretches until its restoring force balances the force due to gravity on the mass, and then stops stretching. In this book, $g = 9.8\ \mathrm{N\ kg^{-1}}$ near the surface of Earth (Section 8C). Notice also the third-law pair (Section 8B): the spring pulls up on the mass, and the mass pulls down on the spring with an equal and opposite force, $F_{\text{on mass by spring}} = -F_{\text{on spring by mass}}$. It is this downward force on the spring that stretches it.

**The force–extension graph.** The behaviour of an ideal spring is summarised by its **force–extension graph**: the restoring force $F$ on the vertical axis against the extension $x$ on the horizontal axis. Because $F$ is proportional to $x$, the graph is a straight line through the origin, and its gradient is the spring constant:

$$\text{gradient} = \frac{\Delta F}{\Delta x} = k.$$

A steep gradient means a stiff spring (large $k$); a shallow gradient means a soft spring (small $k$). Two springs drawn on the same axes can be compared at a glance: for the same force, the spring with the smaller gradient stretches further.




**Investigating Hooke's Law practically.** Hooke's Law is the kind of relationship that should be tested rather than taken on trust, and the test is one of the classic investigations of this course.

*The method.* Suspend a spring from a clamp stand beside a metre ruler. Measure and record the length of the unloaded spring — this is the natural length $L_0$. Hang a slotted mass from the spring, wait until the mass is at rest, and record the new length $L$. The extension is $x = L - L_0$, and the force stretching the spring is the force due to gravity on the hanging mass, $F = m g$, with $g = 9.8\,\mathrm{N\ kg}^{-1}$. Repeat with increasing masses. The independent variable is the applied force (set by the hanging mass); the dependent variable is the extension; the same spring, the same ruler and readings taken at rest are among the controlled variables.

*The results table.* A typical set of results, with the masses converted to forces using $F = m g$, looks like this:

| Hanging mass (g) | 0 | 100 | 200 | 300 | 400 |
|---|---|---|---|---|---|
| Applied force $F = m g$ (N) | 0 | 0.98 | 1.96 | 2.94 | 3.92 |
| Extension $x$ (cm) | 0 | 2.0 | 4.0 | 6.0 | 8.0 |

Every row gives the same ratio $F / x = 49\,\mathrm{N\ m}^{-1}$, so this spring obeys Hooke's Law over the range measured, with $k = 49\,\mathrm{N\ m}^{-1}$.

*The graph.* Plot $F$ on the vertical axis against $x$ on the horizontal axis, label each axis with its quantity and unit, and draw the line of best fit — a single straight line drawn as close as possible to all the points, not a dot-to-dot join. Here the unloaded reading ($F = 0$, $x = 0$) was actually measured, because $L_0$ was recorded before any mass was hung, so that point may be included as a data point. If the unloaded length had not been measured, the origin would not be a data point and could not be plotted. The gradient of the line is the spring constant:

$$k = \frac{\Delta F}{\Delta x} = \frac{3.92}{0.080} = 49\,\mathrm{N\ m}^{-1}.$$

Record the table, the graph and your evaluation of the method in your logbook. Two safety points: slotted masses dropped on feet hurt, so lift them carefully and keep feet clear; and do not keep adding masses past the point where the spring stops behaving itself — a spring stretched beyond its limit can be permanently damaged or snap.

**Where the model breaks: the elastic limit.** Real springs are only approximately ideal. For any real spring there is a greatest extension beyond which the proportionality between $F$ and $x$ fails: on a force–extension graph the points stop following the straight line and begin to curve. If the spring is stretched past this point and the loads are then removed, it does not return to its natural length but stays permanently longer. The greatest extension from which a spring still returns to its natural length when unloaded is called its **elastic limit**. The ideal-spring model — and every calculation in this section — applies only within the elastic limit. This is why springs in real devices are chosen so that the forces they experience keep them well inside their elastic range, and why the hanging-mass investigation above must stop adding masses once the readings stop growing in equal steps.

**Springs at work.** The devices named at the start of this section all put $F=-kx$ to work. A spring balance (a spring with a pointer and a scale) measures force because the extension is proportional to the force — which is also why its scale marks are equally spaced. Vehicle suspension uses springs chosen to be soft enough to absorb bumps but stiff enough to carry the vehicle's load, a choice made through $k$. Trampolines, pogo sticks and spring-loaded clips all store energy while they are deformed and give it back when they return to shape. Stretching or compressing a spring requires a force acting through a distance — work is done on the spring (Section 9A) — and that work is stored, ready to be returned. Section 9C names this stored energy **elastic potential energy** and quantifies it; this section supplies the force model that the energy treatment is built on.

**Units 3 & 4 preview.** The energy stored in ideal springs returns in Section 9C of this book, where it is quantified as $E_s=\frac{1}{2}kx^2$ and used in mechanical energy-conservation problems. At Units 3 & 4, springs reappear in energy-transformation problems involving transfers between elastic potential energy, kinetic energy and gravitational potential energy, and Hooke's Law is assumed knowledge there — it is never re-taught. Content under this banner is not assessed in this book's topic tests or practice examinations.

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## Worked examples

### *Example 1 — scaffolded*

A school spring balance reads forces up to $10\,\text{N}$. When a force of $4.0\,\text{N}$ is applied to its hook, the spring inside extends by $2.0\,\text{cm}$. (a) Calculate the spring constant of the spring. (b) Calculate the extension when the balance reads $7.0\,\text{N}$. (c) Explain why the scale marks on the balance are equally spaced.

| Working | Comment |
|---|---|
| $x=2.0\,\text{cm}=0.020\,\text{m}$ | Convert the extension to metres before using it in $F=kx$. |
| $k=\frac{F}{x}=\frac{4.0}{0.020}=200\,\text{N m}^{-1}$ | Hooke's Law rearranged for the spring constant. |
| $x=\frac{F}{k}=\frac{7.0}{200}=0.035\,\text{m}=3.5\,\text{cm}$ | Use the magnitude form $F=kx$ rearranged for $x$. |
| The marks are equally spaced because $F=kx$: equal steps in force produce equal steps in extension. | Extension is proportional to force for an ideal spring. |

### *Example 2 — scaffolded*

A student investigates a spring and records the following data.

| Applied force $F$ (N) | 0 | 2.0 | 4.0 | 6.0 | 8.0 |
|---|---|---|---|---|---|
| Extension $x$ (cm) | 0 | 1.6 | 3.2 | 4.8 | 6.4 |

(a) Show that the data are consistent with Hooke's Law and determine the spring constant.
(b) Predict the force needed to extend the spring by $9.0\,\text{cm}$.
(c) Predict the extension produced by a force of $5.0\,\text{N}$.

| Working | Comment |
| --- | --- |
| $\frac{2.0}{0.016}=\frac{4.0}{0.032}=\frac{6.0}{0.048}=\frac{8.0}{0.064}=125$ | The ratio $F/x$ is the same for every reading. |
| The force is proportional to the extension, so the spring obeys Hooke's Law over this range; $k=125\,\mathrm{N\,m^{-1}}$ ( $1.3\times10^{2}\,\mathrm{N\,m^{-1}}$ to 2 significant figures). | A constant $F/x$ ratio *is* Hooke's Law. Keep the unrounded value for the next parts. |
| $F=k\,x=125\times0.090=11.25\approx11\,\mathrm{N}$ | Magnitude form of Hooke's Law; round to 2 significant figures at the end. |
| $x=\frac{F}{k}=\frac{5.0}{125}=0.040\,\mathrm{m}=4.0\,\mathrm{cm}$ | Rearrange $F=k\,x$ for $x$. |

### *Example 3 — unscaffolded*

A $0.80\,\mathrm{kg}$ mass hangs at rest from a vertical spring of spring constant $k=160\,\mathrm{N\,m^{-1}}$. Calculate the extension of the spring, and describe the forces on the mass and on the spring using the *on A by B* convention.

Two forces act on the mass: the force due to gravity, $F_{\text{on mass by Earth}}=m\,g=0.80\times9.8=7.84\,\mathrm{N}$, directed downward, and the upward restoring force of the spring, $F_{\text{on mass by spring}}$. The mass is at rest, so by Newton's first law the net force on it is zero and the two forces have equal magnitudes:

$$F_{\text{on mass by spring}}=7.84\,\mathrm{N}\ \text{upward}\,.$$

The extension follows from the magnitude form of Hooke's Law:

$$x=\frac{F}{k}=\frac{7.84}{160}=0.049\,\mathrm{m}=4.9\,\mathrm{cm}\,.$$

The interaction pair (Newton's third law) is $F_{\text{on mass by spring}}=-F_{\text{on spring by mass}}$: the spring pulls up on the mass with $7.84\,\mathrm{N}$, and the mass pulls down on the spring with $7.84\,\mathrm{N}$. It is this downward force on the spring that stretches it — the forces on the mass explain why the mass is at rest, and the force on the spring explains why the spring is extended.

∴ the extension is $4.9\,\mathrm{cm}$; $F_{\text{on mass by spring}}=7.8\,\mathrm{N}$ upward balances $F_{\text{on mass by Earth}}=7.8\,\mathrm{N}$ downward, and the third-law partner of the spring's pull is $F_{\text{on spring by mass}}=7.8\,\mathrm{N}$ downward.

### *Example 4 — unscaffolded*

The spring inside a luggage scale compresses when a bag is placed on the platform. A $12\,\mathrm{kg}$ bag compresses the spring by $3.0\,\mathrm{cm}$. (a) Calculate the spring constant of the spring. (b) A second bag compresses the same spring by $4.5\,\mathrm{cm}$. Calculate the mass of the second bag.

The force compressing the spring is the force due to gravity on the first bag:

$$F=m\,g=12\times9.8=117.6\,\mathrm{N}\,.$$

The spring constant follows from $k=F/x$ with $x=3.0\,\mathrm{cm}=0.030\,\mathrm{m}$:

$$k=\frac{117.6}{0.030}=3920\approx3.9\times10^{3}\,\mathrm{N\,m^{-1}}.$$

For the second bag, the compression is $x=4.5\,\mathrm{cm}=0.045\,\mathrm{m}$, so the force due to gravity on it is

$$F=k\,x=3920\times0.045=176.4\,\mathrm{N}\,,$$

and its mass is

$$m=\frac{F}{g}=\frac{176.4}{9.8}=18\,\mathrm{kg}\,.$$

A check by proportion: $4.5\,\text{cm}$ is 1.5 times $3.0\,\text{cm}$, and $18\,\text{kg}$ is 1.5 times $12\,\text{kg}$ — force and compression grow together in fixed ratio, as Hooke's Law requires.

$\therefore\ k \approx 3.9 \times 10^3\,\text{N m}^{-1}$ and the second bag has a mass of $18\,\text{kg}$.

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## Practice questions

*Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working, and state the direction of every force you report. Take $g = 9.8\,\text{N kg}^{-1}$ near the surface of Earth, and treat every spring as ideal unless the question says otherwise.*

**Question 1** (a) Write Hooke's Law for an ideal spring and state what each symbol represents. (b) State the SI unit of the spring constant. (c) Explain what the minus sign in Hooke's Law says about the direction of the spring's force.

**Question 2** A spring has a spring constant of $200\,\text{N m}^{-1}$. (a) Calculate the force needed to extend it by $5.0\,\text{cm}$. (b) State the extension if the force is doubled, with a reason. (c) Calculate the force needed to extend it by $8.0\,\text{cm}$.

**Question 3** The spring in a door handle compresses by $2.0\,\text{cm}$ when the handle is pushed with a force of $6.0\,\text{N}$. (a) Calculate the spring constant of the spring. (b) Calculate the force needed to compress it by $5.0\,\text{cm}$. (c) State the assumption your calculations make about the behaviour of the spring.

**Question 4** The spring in a clothes peg compresses by $1.0\,\text{cm}$ when squeezed with a force of $8.0\,\text{N}$. (a) Calculate the spring constant of the spring. (b) Calculate the force needed to compress it by $2.5\,\text{cm}$. (c) State the direction of the force the compressed spring exerts on the fingers squeezing it.

**Question 5** A spring extends by $2.0\,\text{cm}$ for each newton of force applied to it. (a) Calculate the spring constant of the spring. (b) Calculate its extension when the applied force is $7.0\,\text{N}$. (c) Calculate the applied force when the extension is $9.0\,\text{cm}$.

**Question 6** The force–extension graph of a spring is a straight line through the origin. At an extension of $0.080\,\text{m}$, the force is $24\,\text{N}$. (a) State what the gradient of a force–extension graph represents. (b) Calculate the spring constant of the spring. (c) Calculate the force at an extension of $0.050\,\text{m}$.

**Question 7** A spring balance with a spring constant of $500\,\text{N m}^{-1}$ is used to find the mass of a bag of potatoes. The bag hangs at rest from the hook and has a mass of $7.5\,\text{kg}$. Calculate the extension of the spring, and comment on whether this extension is a sensible size for a hand-held balance.

**Question 8** Explain why the force in Hooke's Law is called a **restoring force**. Your explanation should refer to the direction of the force when the spring is extended and when it is compressed.

**Question 9** One of the springs of a trampoline has a spring constant of $3.0 \times 10^3\,\text{N m}^{-1}$. (a) Calculate the force in the spring when it is stretched by $8.0\,\text{cm}$. (b) When a second person lands on the trampoline, the same spring stretches momentarily to $12\,\text{cm}$. Calculate the force in the spring then, and compare it with part (a).

**Question 10** A student investigating a spring finds that the force–extension graph is a straight line up to a force of $8.0\,\text{N}$ (extension $4.0\,\text{cm}$), after which the points curve away from the line. When all the masses are removed, the spring is permanently $2.0\,\text{cm}$ longer than when the investigation started. (a) Calculate the spring constant of the spring from the linear part of the graph. (b) Explain why Hooke's Law is applied only up to $8.0\,\text{N}$ in this investigation. (c) Explain why the spring did not return to its original length.

**Question 11** A child of mass $40\,\text{kg}$ stands at rest on a pogo stick, compressing its spring by $6.0\,\text{cm}$. Treat all of the child's force due to gravity as acting through the single spring, and neglect the mass of the pogo stick itself. (a) Calculate the force compressing the spring. (b) Calculate the spring constant of the spring.

**Question 12** A $2.0\,\mathrm{kg}$ mass hangs at rest from a vertical spring. A student says: "The spring must pull up on the mass harder than gravity pulls down, otherwise the mass would fall." Evaluate this statement using Newton's laws and the *on A by B* convention, including the magnitude of each force on the mass.

**Question 13** The force–extension graph of a spring is a straight line through the origin, and the force is $25\,\mathrm{N}$ at an extension of $0.10\,\mathrm{m}$. (a) Calculate the spring constant of the spring. (b) Calculate the extension when the force is $60\,\mathrm{N}$. (c) A student claims the same spring could be used, unchanged, to measure forces up to $500\,\mathrm{N}$. Evaluate this claim.

**Question 14** The suspension spring of a car has a far larger spring constant than the spring in a retracting pen. Explain what this means for the force each spring produces at a given extension, and for the extension each spring produces at a given force, referring to the gradient of a force–extension graph.

**Question 15** Two springs, A and B, have spring constants $150\,\mathrm{N\,m^{-1}}$ and $400\,\mathrm{N\,m^{-1}}$. Each is pulled with a force of $12\,\mathrm{N}$. (a) Calculate the extension of each spring. (b) State which spring extends more, and explain why in terms of the spring constant.

**Question 16** A student carrying out the hanging-masses investigation forgets to record the length of the unloaded spring. They measure the total length $L$ of the spring for each hanging mass and plot force $F$ against $L$. The graph is a straight line, but it does not pass through the origin. (a) Explain why the line does not pass through the origin. (b) Explain why the gradient of this graph still equals the spring constant $k$. (c) State whether the student may include the origin $(0,0)$ as a data point on this graph, referring to the rule for when the origin may be plotted.

**Question 17** A trailer carries a $300\,\mathrm{kg}$ load, shared equally between two identical suspension springs, each with a spring constant of $2.5\times10^{4}\,\mathrm{N\,m^{-1}}$. (a) Calculate the compression of each spring. (b) A second, identical load is added. State the new compression of each spring and the condition under which your answer holds.

**Question 18** A student wants to build a simple force meter that can measure forces up to $20\,\mathrm{N}$, with the extension kept to $10\,\mathrm{cm}$ or less. (a) Calculate the spring constant a spring would need to extend by exactly $10\,\mathrm{cm}$ at $20\,\mathrm{N}$. (b) The student is offered two springs: spring P with $k=100\,\mathrm{N\,m^{-1}}$ and spring Q with $k=400\,\mathrm{N\,m^{-1}}$. Determine the extension of each spring at $20\,\mathrm{N}$ and state which spring meets the requirement, with a reason.

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## Answers

**Question 1** (a) $F = -kx$: $F$ restoring force, $k$ spring constant, $x$ extension; (b) $\mathrm{N\ m^{-1}}$; (c) see solutions — the force acts opposite to the extension. **Question 2** (a) $10\,\mathrm{N}$; (b) $10\,\mathrm{cm}$; (c) $16\,\mathrm{N}$. **Question 3** (a) $300\,\mathrm{N\ m^{-1}}$; (b) $15\,\mathrm{N}$; (c) see solutions — Hooke's Law holds over this range. **Question 4** (a) $800\,\mathrm{N\ m^{-1}}$; (b) $20\,\mathrm{N}$; (c) opposite to the compression, pushing back against the fingers. **Question 5** (a) $50\,\mathrm{N\ m^{-1}}$; (b) $14\,\mathrm{cm}$; (c) $4.5\,\mathrm{N}$. **Question 6** (a) the spring constant $k$; (b) $300\,\mathrm{N\ m^{-1}}$; (c) $15\,\mathrm{N}$. **Question 7** $0.15\,\mathrm{m}$ $(15\,\mathrm{cm})$ — see solutions for the comment. **Question 8** See solutions (model answer). **Question 9** (a) $240\,\mathrm{N}$; (b) $360\,\mathrm{N}$, a factor of 1.5 larger. **Question 10** (a) $200\,\mathrm{N\ m^{-1}}$; (b), (c) see solutions. **Question 11** (a) $3.9 \times 10^2\,\mathrm{N}$; (b) $6.5 \times 10^3\,\mathrm{N\ m^{-1}}$. **Question 12** See solutions (model answer) — the statement is incorrect; both forces are $20\,\mathrm{N}$. **Question 13** (a) $250\,\mathrm{N\ m^{-1}}$; (b) $0.24\,\mathrm{m}$; (c) see solutions — the claim is not justified. **Question 14** See solutions (model answer). **Question 15** (a) A: $8.0\,\mathrm{cm}$, B: $3.0\,\mathrm{cm}$; (b) spring A — see solutions. **Question 16** See solutions (model answer). **Question 17** (a) $5.9\,\mathrm{cm}$; (b) doubles to about $12\,\mathrm{cm}$ — see solutions for the condition. **Question 18** (a) $200\,\mathrm{N\ m^{-1}}$; (b) spring Q — see solutions.

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## Fully-worked solutions

**Question 1** (a) Hooke's Law is $F = -kx$, where $F$ is the restoring force exerted by the spring, $k$ is the spring constant and $x$ is the extension of the spring from its natural length. (b) The SI unit of the spring constant is the newton per metre, $\mathrm{N\ m^{-1}}$. (c) The minus sign means the spring's force acts in the opposite direction to the extension: when the spring is stretched one way, the force pulls back the other way, towards the natural length. That is why $F$ is a restoring force.

**Question 2** (a) $F = kx = 200 \times 0.050 = 10\,\mathrm{N}$. (b) $10\,\mathrm{cm}$. In $F = kx$ with $k$ fixed, $F$ is proportional to $x$, so doubling the force doubles the extension. (c) $F = kx = 200 \times 0.080 = 16\,\mathrm{N}$.

**Question 3** (a) $k = F / x = 6.0 / 0.020 = 300\,\mathrm{N\ m^{-1}}$. (b) $F = kx = 300 \times 0.050 = 15\,\mathrm{N}$. (c) The calculations assume the spring obeys Hooke's Law over the whole range used — that is, the spring behaves ideally (within its elastic limit) up to at least $5.0\,\mathrm{cm}$ of compression.

**Question 4** (a) $k = F / x = 8.0 / 0.010 = 800\,\mathrm{N\ m^{-1}}$. (b) $F = kx = 800 \times 0.025 = 20\,\mathrm{N}$. (c) The force acts opposite to the compression: the squeezed spring pushes back against the fingers, trying to return to its natural length.

**Question 5** (a) $1\,\mathrm{N}$ produces $0.020\,\mathrm{m}$ of extension, so $k = F / x = 1 / 0.020 = 50\,\mathrm{N\ m^{-1}}$. (b) $x = F / k = 7.0 / 50 = 0.14\,\mathrm{m} = 14\,\mathrm{cm}$. (c) $F = kx = 50 \times 0.090 = 4.5\,\mathrm{N}$.

**Question 6** (a) The gradient of a force–extension graph is the spring constant $k$. (b) $k = \Delta F / \Delta x = 24 / 0.080 = 300\,\mathrm{N\ m^{-1}}$. (c) $F = kx = 300 \times 0.050 = 15\,\mathrm{N}$.

**Question 7** The force stretching the spring is the force due to gravity on the bag: $F = mg = 7.5 \times 9.8 = 73.5\,\mathrm{N}$. The extension is $x = F / k = 73.5 / 500 = 0.147 \approx 0.15\,\mathrm{m}$, or $15\,\mathrm{cm}$ to 2 significant figures. This is a sensible size for a hand-held balance: large enough to read clearly on a scale, small enough to fit inside a device you can hold.

**Question 8** The force is called a restoring force because it always acts to restore the spring to its natural length. When the spring is extended, $x$ is positive and $F = -kx$ is negative: the force acts opposite to the extension, pulling the spring back in. When the spring is compressed, $x$ is negative and $F = -kx$ is positive: the force acts opposite to the compression, pushing the spring back out. In both cases the force points back towards the natural length — it restores rather than stretches further.

**Question 9** (a) $F = kx = 3.0 \times 10^3 \times 0.080 = 240\,\mathrm{N}$. (b) $F = kx = 3.0 \times 10^3 \times 0.12 = 360\,\mathrm{N}$. The force is $360 / 240 = 1.5$ times the force in part (a): the extension increased by a factor of 1.5, and because $F$ is proportional to $x$, the force increases by the same factor.

**Question 10** (a) From the linear part, $k = F / x = 8.0 / 0.040 = 200\,\mathrm{N\ m^{-1}}$. (b) Hooke's Law applies only while force is proportional to extension — that is, while the graph is a straight line. Beyond $8.0\,\mathrm{N}$ the points curve, so $F / x$ is no longer constant and the spring no longer obeys Hooke's Law there. The straight-line region ends because the spring has been stretched beyond the limit of its ideal behaviour. (c) The spring was stretched beyond its elastic limit — the greatest extension from which it can still return to its natural length when unloaded. Past that limit the spring is permanently deformed, so when the masses are removed it remains $2.0\,\mathrm{cm}$ longer than its original natural length.

**Question 11** (a) The force compressing the spring is the force due to gravity on the child: $F=mg=40\times9.8=392\approx3.9\times10^{2}\,\mathrm{N}$, directed downward. (b) $k=F/x=392/0.060=6533\approx6.5\times10^{3}\,\mathrm{N\ m^{-1}}$, using the unrounded $392\,\mathrm{N}$.

**Question 12** The statement is incorrect. The mass hangs at rest, so its acceleration is zero and by Newton's first law (equivalently, $a=F_{\text{net}}/m$ with $a=0$) the net force on it is zero. The two forces on the mass — $F_{\text{on mass by Earth}}=mg=2.0\times9.8=19.6\approx20\,\mathrm{N}$ downward, and $F_{\text{on mass by spring}}$ upward — must therefore be equal in magnitude: $F_{\text{on mass by spring}}=20\,\mathrm{N}$ upward. The spring pulls up with exactly the same force that gravity pulls down, not a larger one; if it pulled harder the mass would accelerate upward, not hang at rest. (The third-law partner of the spring's upward pull is $F_{\text{on spring by mass}}=20\,\mathrm{N}$ downward — the force that stretches the spring. It acts on the spring, not on the mass, so it is not part of the balance of forces on the mass.)

**Question 13** (a) $k=F/x=25/0.10=250\,\mathrm{N\ m^{-1}}$. (b) $x=F/k=60/250=0.24\,\mathrm{m}$. (c) The claim is not justified. At $500\,\mathrm{N}$ the straight-line model predicts $x=F/k=500/250=2.0\,\mathrm{m}$ — a two-metre stretch of a laboratory spring, which is not physically reasonable. More importantly, the linear model has only been tested up to the measured range; a real spring would reach its elastic limit long before $2.0\,\mathrm{m}$, become permanently deformed or break, and stop obeying Hooke's Law. Extrapolating the straight line this far is not reliable.

**Question 14** A larger spring constant means a stiffer spring. On a force–extension graph the car's suspension spring has the steeper gradient, because gradient $=k$. At a given extension, the suspension spring therefore exerts a much larger restoring force than the pen spring — it takes a large force to stretch it even a little, which is what supporting a car requires. At a given force, the extension $x=F/k$ is smaller for the suspension spring: the same force that stretches the pen spring several centimetres compresses the suspension spring only a few millimetres.

**Question 15** (a) Spring A: $x=F/k=12/150=0.080\,\mathrm{m}=8.0\,\mathrm{cm}$. Spring B: $x=F/k=12/400=0.030\,\mathrm{m}=3.0\,\mathrm{cm}$. (b) Spring A extends more. For the same applied force, the extension is $x=F/k$: the smaller the spring constant, the larger the extension. Spring A is the softer spring.

**Question 16** (a) The measured length $L$ is the natural length plus the extension, $L=L_0+x$. At zero force the spring still has length $L_0$, not zero, so the line crosses the length axis at $L_0$ and does not pass through the origin. (b) Since $x=L-L_0$, Hooke's Law gives $F=kx=k(L-L_0)$. When the force changes, the change in length equals the change in extension ($\Delta L=\Delta x$), so the gradient is $\Delta F/\Delta L=\Delta F/\Delta x=k$: the gradient still gives the spring constant, even though the intercept does not. (c) No. The study design's graph rule is that the origin may only be included as a data point if it was authentically measured. The spring was never measured at zero length — at zero force its length was $L_0$ — so $(0,0)$ is not a measurement and must not be plotted. (Had the student plotted $F$ against the extension $x$ instead, the unloaded state would have been an authentically measured point and could have been included.)

**Question 17** (a) The force due to gravity on the load is $F=mg=300\times9.8=2940\,\mathrm{N}$. Shared equally, each spring supports $2940/2=1470\,\mathrm{N}$. The compression of each spring is $x=F/k=1470/(2.5\times10^{4})=0.0588\approx0.059\,\mathrm{m}$, or $5.9\,\mathrm{cm}$ to 2 significant figures. (b) Doubling the load doubles the force on each spring, so the compression doubles to about $12\,\mathrm{cm}$. This holds only while the springs remain within their elastic limits — Hooke's Law applies only in the linear range.

**Question 18** (a) $k=F/x=20/0.10=200\,\mathrm{N\ m^{-1}}$. (b) Spring P: $x=F/k=20/100=0.20\,\mathrm{m}=20\,\mathrm{cm}$ — more than the $10\,\mathrm{cm}$ limit, so spring P fails the requirement. Spring Q: $x=20/400=0.050\,\mathrm{m}=5.0\,\mathrm{cm}$ — within the limit, so spring Q is the suitable choice. Spring Q's larger spring constant keeps the extension small enough at the full $20\,\mathrm{N}$.

**Scope** — *VCE Physics Study Design, Unit 2, Area of Study 1*: "analyse and model mechanical energy transfers and transformations using energy conservation:", with "changes in gravitational potential energy near Earth's surface: E_g = mgΔh"; "elastic potential energy in ideal springs: E_s = ½kx²"; and "kinetic energy: E_k = ½mv²".

## Theory

**Energy so far.** Energy is the quantity we keep track of when anything changes: a ball speeds up, water heats up, a spring stretches, a lamp glows. Energy is measured in **joules** (J), and Unit 1 followed it through many forms — thermal energy, electromagnetic radiation, nuclear energy and electrical energy — as it was transferred between objects and transformed from one form to another. Chapter 9 now connects energy to motion. Section 9A treated the **work** done by a force as one way of transferring energy: when a force acts on an object while the object moves, energy is transferred to or from that object. Section 9B introduced the **ideal spring** and Hooke's Law. This section defines the three energy stores involved in motion and stretching, and shows how the conservation of energy links them.

**Kinetic energy — the energy of motion.** An object has **kinetic energy** because it is moving. For an object of mass $m$ moving with speed $v$, the kinetic energy is

$$E_k = 1/2\, m v^2.$$

Mass is measured in kilograms (kg) and speed in metres per second ($\mathrm{m\ s^{-1}}$), giving kinetic energy in joules: $1\,\mathrm{J} = 1\,\mathrm{kg\ m^2 s^{-2}}$. Kinetic energy is a **scalar** — it has a size but no direction. A car travelling north at $15\,\mathrm{m\ s^{-1}}$ and a car travelling south at $15\,\mathrm{m\ s^{-1}}$ have the same kinetic energy.

Because speed is squared, kinetic energy grows quickly with speed. An object moving at $2v$ has four times the kinetic energy of the same object moving at $v$, and an object moving at $3v$ has nine times as much. This is why high-speed crashes are so much more damaging than low-speed ones: a car at $20\,\mathrm{m\ s^{-1}}$ carries four times the kinetic energy of the same car at $10\,\mathrm{m\ s^{-1}}$, and all of that energy must be transferred elsewhere (into deforming the vehicles, into thermal energy and into sound) during the crash.

**Changes in gravitational potential energy.** An object raised above the ground has energy stored because of its position in Earth's gravitational field. This stored energy is called **gravitational potential energy**. Near Earth's surface, raising an object of mass $m$ through a height $\Delta h$ increases its gravitational potential energy by

$$\Delta E_g = m g \Delta h,$$

where $g = 9.8\,\mathrm{N\ kg^{-1}}$ is the gravitational field strength (Section 8C). The relationship comes from the work done in lifting: to raise the object at constant speed, an upward force equal to the force due to gravity, $F_g = m g$, must act through the height $\Delta h$, and the work done by that force, $m g \Delta h$, is stored as gravitational potential energy.

Only **changes** in gravitational potential energy are physically meaningful, because the zero level is a choice. A textbook $1.0\,\mathrm{m}$ above the floor has one value of height measured from the floor and a different value measured from the benchtop, but if the book is lifted a further $0.5\,\mathrm{m}$, the increase $\Delta E_g = m g \Delta h$ is the same whichever level the heights are measured from. In any problem, choose one reference level for zero height and measure all heights from it; only the difference in height between two positions enters the calculation. The same is true when an object falls: the gravitational potential energy decreases, and $\Delta h$ is the vertical distance fallen.

**Elastic potential energy.** Section 9B showed that the force needed to stretch or compress an ideal spring by an extension $x$ obeys Hooke's Law: the force is proportional to the extension, with spring constant $k$ measured in $\mathrm{N\ m^{-1}}$. Because the force grows steadily from zero (at the natural length) to its largest value, the energy stored in the stretched spring is

$$E_s = 1/2\, k x^2.$$

The factor $1/2$ appears because the force is not constant during the stretching. As Section 9A showed, work done can be found from the area under a force–distance graph; for an ideal spring that graph is a straight line from the origin, and the area under it is a triangle — half the base times the height, $1/2\,x \times k\,x = 1/2\,k\,x^2$. The stored energy is the same whether the spring is stretched or compressed: $x$ is the extension or compression measured from the spring's natural length.

**Mechanical energy.** The kinetic energy of moving objects, the gravitational potential energy of raised objects and the elastic potential energy of stretched or compressed springs are together called **mechanical energy**:

$$E_{\text{mech}} = E_k + E_g + E_s.$$

In any particular situation one or more of these terms may be zero, and it is common to write only the terms that are present.

**The law of conservation of energy.** Energy cannot be created or destroyed. It can only be transferred from one object to another, or transformed from one form to another. The total amount of energy stays the same.

> The total energy of a system is conserved: energy may change form or move from one object to another, but it is never created and never destroyed.

**Conservation of mechanical energy.** When only the force due to gravity and ideal spring forces transfer energy — no friction, no air resistance, no driving force from an engine or a person — the mechanical energy of the system stays constant. Energy simply moves between the kinetic, gravitational and elastic stores:

$$E_{k,\text{initial}} + E_{g,\text{initial}} + E_{s,\text{initial}} = E_{k,\text{final}} + E_{g,\text{final}} + E_{s,\text{final}}.$$

A ball dropped from rest through a height $\Delta h$ loses gravitational potential energy $m\,g\,\Delta h$ and gains the same amount of kinetic energy. If air resistance is neglected,

$$m\,g\,\Delta h = 1/2\,m\,v^2, \text{so } v = \sqrt{2\,g\,\Delta h}.$$

The mass cancels: heavy and light objects dropped through the same height reach the same speed. A pendulum bob swings the same energy back and forth — gravitational potential energy at the top of each swing becomes kinetic energy at the lowest point, where the speed is greatest, and then becomes gravitational potential energy again on the other side. A child on a trampoline transforms kinetic energy into elastic potential energy as the mat stretches at the bottom of each bounce, and back into kinetic energy and gravitational potential energy on the way up.

Notice that the conservation method needs only the starting and ending states — the heights, the speeds and the spring extensions. The path taken between them does not matter. A rider on a curved slide and a rider on a straight slide, both starting from rest at the same height and both neglecting friction, reach the bottom with the same speed, because both have the same decrease in height.

**When mechanical energy is not conserved.** Real motion usually involves **dissipative forces** — forces such as friction and air resistance that transform mechanical energy into thermal energy of the objects and their surroundings, and into sound. When a sliding object comes to rest, or a bouncing ball returns to a lower height after each bounce, mechanical energy has not been destroyed: it has been transformed into forms that are harder to collect and use again. The total energy is still conserved, but the mechanical energy of the objects is smaller afterwards.

Mechanical energy can also increase. An engine, a motor or a person pushing does work on a system and transfers energy into it: a cyclist climbing a hill gains gravitational potential energy because the cyclist's muscles are doing work, transferring chemical energy from food into mechanical energy (and thermal energy).

**A method for energy-conservation problems.**

1. Choose the system — the object or objects whose energy you will follow — and check whether mechanical energy is conserved: no work done against friction or air resistance, and no driving force adding energy.
2. Choose a reference level for zero height, and measure all heights from it.
3. Identify the initial state and the final state. For each, note the height $h$, the speed $v$ and the extension or compression $x$ of any spring.

4. Write conservation of mechanical energy: $E_{k,\mathrm{i}}+E_{g,\mathrm{i}}+E_{s,\mathrm{i}}=E_{k,\mathrm{f}}+E_{g,\mathrm{f}}+E_{s,\mathrm{f}}$, leaving out any terms that are zero.
5. Substitute with units, solve, and check that the answer is reasonable.

**Practical investigation — testing energy conservation with a pendulum.** The conservation of mechanical energy can be investigated with a pendulum bob swinging from a fixed support. A suitable framework:

- **Aim.** Test whether the decrease in gravitational potential energy of a pendulum bob as it swings to the lowest point equals the kinetic energy of the bob at that point.
- **Equipment.** A pendulum bob hung from a length of string attached to a clamped stand; a balance to measure the mass of the bob; a ruler to measure heights; a light gate and data logger placed at the lowest point of the swing (or a smartphone camera for video analysis).
- **Method.** Measure the mass of the bob. Pull the bob aside so that its centre is a measured vertical height $\Delta h$ above the lowest point of the swing, and release it from rest. Record the speed $v$ of the bob at the lowest point — with a light gate, the speed is the diameter of the bob divided by the interruption time. Calculate the decrease in gravitational potential energy, $\Delta E_g = m g \Delta h$, and the kinetic energy at the lowest point, $E_k = 1/2\, m v^2$, and compare them. Repeat for several release heights.
- **Expected results.** The kinetic energy at the lowest point is close to, but slightly less than, the decrease in gravitational potential energy. The small difference is the mechanical energy transformed into thermal energy by air resistance and by friction at the support.
- **Safety and practice.** Clamp the stand securely so it cannot topple while the bob swings, and keep faces and hands clear of the arc of the swinging bob. Record every reading — mass, height, interruption time — with its unit in your logbook: the logbook is submitted as a requirement for satisfactory completion of the unit.

**Where this leads.** Section 9D uses energy to define power (the rate of energy transfer) and efficiency. The case study of Section 9E brings the energy ideas of this chapter together with the momentum ideas of Chapter 8, for example in vehicle safety or motion in sport. Students studying Option 2.11 (Section 11D) apply kinetic and potential energy to balls in flight and in collisions.

**Units 3 & 4 preview.** At Units 3 & 4 the link between work and kinetic energy is made precise: the net work done on an object equals its change in kinetic energy. Energy conservation is then applied to situations in which friction or other forces do work, by accounting for that work explicitly, and gravitational potential energy is generalised beyond the near-Earth form $\Delta E_g = m g \Delta h$ used here. Content under this banner is not assessed in this book's topic tests or practice examinations.

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## Worked examples

### *Example 1 — scaffolded*

A skier of mass $70\,\mathrm{kg}$ starts down a run at Mount Buller at $4.0\,\mathrm{m\ s^{-1}}$. Over the run, the vertical drop is $25\,\mathrm{m}$. Model the run as frictionless. (a) Calculate the skier's kinetic energy at the start. (b) Calculate the decrease in the skier's gravitational potential energy during the descent. (c) Calculate the skier's speed at the bottom of the run.

| Working | Comment |
| --- | --- |
| $m=70\,\mathrm{kg}$; $u=4.0\,\mathrm{m\ s^{-1}}$; $\Delta h=25\,\mathrm{m}$; $g=9.8\,\mathrm{N\ kg^{-1}}$. | List the data. Heights are measured from the bottom of the run. |
| (a) $E_k=1/2\,m u^2=1/2\times 70\times 4.0^2=560\,\mathrm{J}$. | Kinetic energy at the start. |
| (b) $\Delta E_g=m g \Delta h=70\times 9.8\times 25=17\,150\,\mathrm{J}$. | The gravitational potential energy decreases by this amount during the descent. |
| (c) With no friction, mechanical energy is conserved: $E_k$ at the bottom $=560+17\,150=17\,710\,\mathrm{J}$. Then $1/2\,m v^2=17\,710$, so | The initial kinetic energy plus the gravitational potential energy released becomes kinetic energy at the bottom. Data are given to 2 significant figures, so the final answer is given to 2. |

| Working | Comment |
|---|---|
| $v=\sqrt{\dfrac{2\times 17\,710}{70}}=\sqrt{506}=22.5\,\mathrm{m\ s^{-1}}\approx 22\,\mathrm{m\ s^{-1}}$. | |

A real skier reaches a lower speed than this: work done against friction between the skis and the snow, and against air resistance, transforms some of the mechanical energy into thermal energy.

### *Example 2 — scaffolded*

In a school experiment, a trolley of mass $0.80\,\mathrm{kg}$ is placed against a compressed spring on a smooth bench. The spring has spring constant $200\,\mathrm{N\ m^{-1}}$ and is compressed by $5.0\,\mathrm{cm}$. The trolley is released. (a) Calculate the elastic potential energy stored in the compressed spring. (b) State the main energy transformation as the trolley is launched. (c) Calculate the maximum speed reached by the trolley.

| Working | Comment |
|---|---|
| $m=0.80\,\mathrm{kg}$; $k=200\,\mathrm{N\ m^{-1}}$; $x=5.0\,\mathrm{cm}=0.050\,\mathrm{m}$. | List the data, converting the compression to metres. |
| (a) $E_s=1/2\,k\,x^2=1/2\times 200\times 0.050^2=0.25\,\mathrm{J}$. | Elastic potential energy of the compressed spring. |
| (b) Elastic potential energy of the spring is transformed into kinetic energy of the trolley (with a small amount transformed into thermal energy and sound). | The spring does work on the trolley as it returns to its natural length. |
| (c) With all $0.25\,\mathrm{J}$ becoming kinetic energy: $1/2\,m\,v^2=0.25$, so $v=\sqrt{\dfrac{2\times 0.25}{0.80}}=\sqrt{0.625}=0.79\,\mathrm{m\ s^{-1}}$. | The speed is greatest when the spring reaches its natural length and all the stored energy has been transferred. |

### *Example 3 — unscaffolded*

At Luna Park Melbourne, a roller-coaster train of total mass $2000\,\mathrm{kg}$ passes point $A$, at the top of a drop, at $4.0\,\mathrm{m\ s^{-1}}$. Point $A$ is $20\,\mathrm{m}$ above point $B$, the lowest point of the drop. (a) Calculate the total mechanical energy of the ride at point $A$, taking point $B$ as the reference level for height. (b) Calculate the speed of the ride at point $B$ if mechanical energy is conserved between $A$ and $B$. (c) The measured speed at point $B$ is $16\,\mathrm{m\ s^{-1}}$. Calculate the mechanical energy transformed into thermal energy and sound between $A$ and $B$, and explain why it is transformed.

The kinetic energy at point $A$ is

$$E_k=1/2\,m\,v^2=1/2\times 2000\times 4.0^2=16\,000\,\mathrm{J},$$

and the gravitational potential energy at point $A$, measured from point $B$, is

$$E_g=m\,g\,\Delta h=2000\times 9.8\times 20=392\,000\,\mathrm{J}.$$

$\therefore$ the total mechanical energy at point $A$ is

$$E_{\mathrm{mech}}=16\,000+392\,000=408\,000\,\mathrm{J}=4.08\times 10^5\,\mathrm{J}.$$

For part (b), if mechanical energy is conserved, all $408\,000\,\mathrm{J}$ is kinetic energy at point $B$:

$$1/2\,m\,v^2=408\,000,\ v=\sqrt{\frac{2\times 408\,000}{2000}}=\sqrt{408}=20.2\,\mathrm{m\ s^{-1}}\approx 20\,\mathrm{m\ s^{-1}}.$$

For part (c), the kinetic energy actually measured at point $B$ is

$$E_k=1/2\,m\,v^2=1/2\times 2000\times 16^2=256\,000\,\mathrm{J},$$

so the mechanical energy transformed into other forms between $A$ and $B$ is

$$408\,000 - 256\,000 = 152\,000\ \mathrm{J} \approx 1.5 \times 10^5\ \mathrm{J}.$$

This energy is transformed into thermal energy and sound by friction in the wheels and track and by air resistance. Energy overall is still conserved — it is the mechanical energy of the ride that has decreased, not the total energy.

### *Example 4 — unscaffolded*

A spring-loaded launcher fires a ball of mass $0.050\ \mathrm{kg}$ vertically upwards. The spring has spring constant $120\ \mathrm{N\ m^{-1}}$ and is compressed by $8.0\ \mathrm{cm}$. (a) Calculate the elastic potential energy stored in the compressed spring. (b) Calculate the maximum height the ball reaches above its release position, assuming all the stored energy becomes gravitational potential energy. (c) Give two reasons why the actual height gain is less than the value in part (b).

The elastic potential energy stored in the spring is

$$E_s = 1/2\, k\, x^2 = 1/2 \times 120 \times 0.080^2 = 0.384\ \mathrm{J} \approx 0.38\ \mathrm{J}.$$

For part (b), setting this equal to the gain in gravitational potential energy,

$$m\, g\, \Delta h = 0.384,\ \Delta h = \frac{0.384}{0.050 \times 9.8} = \frac{0.384}{0.49} = 0.78\ \mathrm{m}.$$

∴ the ball rises about $0.78\ \mathrm{m}$ above its release position in this ideal model.

For part (c): first, air resistance does work against the ball as it rises, transforming some of the mechanical energy into thermal energy of the ball and the air. Second, not all of the stored energy reaches the ball — some is transformed into thermal energy and sound by friction in the launcher mechanism as the spring expands. The actual height gain is therefore smaller than $0.78\ \mathrm{m}$.

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## Practice questions

*Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: set out your own working, including the data, the relationship used, the substitution and the unit. Take $g = 9.8\ \mathrm{N\ kg^{-1}}$ where needed, and neglect air resistance unless told otherwise.*

**Question 1** *(scaffolded)* A little penguin of mass $1.2\ \mathrm{kg}$ toboggans across the snow on its belly at $3.0\ \mathrm{m\ s^{-1}}$. (a) Write the expression for the kinetic energy of the penguin. (b) Calculate the penguin's kinetic energy. (c) The penguin speeds up to $6.0\ \mathrm{m\ s^{-1}}$. By what factor does its kinetic energy increase? Explain your answer using the kinetic energy expression.

**Question 2** *(scaffolded)* A footballer of mass $80\ \mathrm{kg}$ climbs the stairs at the MCG, rising $12\ \mathrm{m}$ from the concourse to the stand. (a) Write the expression for the change in gravitational potential energy. (b) Calculate the footballer's gain in gravitational potential energy. (c) State the main energy transformation that provides this increase.

**Question 3** *(scaffolded)* A spring of spring constant $50\ \mathrm{N\ m^{-1}}$ is stretched $0.10\ \mathrm{m}$ from its natural length. (a) Write the expression for the elastic potential energy stored in the spring. (b) Calculate the energy stored. (c) The spring is then stretched to twice the extension, $0.20\ \mathrm{m}$. By what factor does the stored energy increase? Explain your answer using the elastic potential energy expression.

**Question 4** *(scaffolded)* A cricket ball of mass $0.16\ \mathrm{kg}$ is dropped from rest from a height of $2.0\ \mathrm{m}$ above the ground. (a) Calculate the decrease in the ball's gravitational potential energy during the fall. (b) State the ball's kinetic energy just before it hits the ground, assuming mechanical energy is conserved. (c) Calculate the ball's speed just before it hits the ground.

**Question 5** *(scaffolded)* In a hydroelectric scheme, $1000\ \mathrm{kg}$ of water falls through a vertical drop of $400\ \mathrm{m}$. (a) Calculate the decrease in gravitational potential energy of the water. (b) If all of this energy became kinetic energy, calculate the speed the water would reach. (c) In a real scheme the water does not reach this speed. Explain where the energy goes instead.

**Question 6** *(scaffolded)* A gymnast of mass $50\,\mathrm{kg}$ lands on a trampoline and depresses the mat by $0.80\,\mathrm{m}$ to the lowest point. Model the trampoline as an ideal spring of spring constant $5000\,\mathrm{N\ m^{-1}}$. (a) Calculate the elastic potential energy stored in the trampoline at the lowest point. (b) State the main energy transformations as the gymnast rebounds upwards. (c) Assuming all of the stored elastic potential energy becomes gravitational potential energy, calculate the height gained by the gymnast from the lowest point.

**Question 7** A car of mass $1200\,\mathrm{kg}$ travels along a country road at $20\,\mathrm{m\ s^{-1}}$. Calculate the car's kinetic energy.

**Question 8** Object A has mass $m$ and speed $v$. Object B has mass $2m$ and speed $v/2$. Which object has the greater kinetic energy, and by what factor? Justify your answer with calculations.

**Question 9** A pole vaulter of mass $75\,\mathrm{kg}$ plants the pole while running at $9.2\,\mathrm{m\ s^{-1}}$. (a) Calculate the vaulter's kinetic energy at the instant the pole is planted. (b) If all of this kinetic energy became gravitational potential energy, calculate the height the vaulter's centre of mass would gain. (c) In real vaults the height gained is less than your answer to part (b). Give two reasons why.

**Question 10** A student says: "When a ball is thrown upwards it slows down, so it is losing energy. This shows that energy is not conserved." Identify the error in this statement and correct it.

**Question 11** At a skate park, a skateboarder of mass $55\,\mathrm{kg}$ drops into a halfpipe from rest at a point $2.5\,\mathrm{m}$ above the lowest point. (a) Calculate the skateboarder's speed at the lowest point, neglecting friction. (b) The measured speed at the lowest point is $6.0\,\mathrm{m\ s^{-1}}$. Calculate the mechanical energy transformed into thermal energy and sound during the drop.

**Question 12** Two water slides at an aquatic park start at the same height and finish at the same low point. One slide is short and steep; the other is long and curved. Use conservation of energy to explain whether riders reach the bottom at the same speed on both slides, neglecting friction.

**Question 13** A spring-loaded launcher fires a ball of mass $0.060\,\mathrm{kg}$ horizontally along a smooth bench. The spring has spring constant $400\,\mathrm{N\ m^{-1}}$ and is compressed by $5.0\,\mathrm{cm}$. (a) Calculate the elastic potential energy stored in the spring. (b) Calculate the launch speed of the ball.

**Question 14** A basketball is dropped from a height of $1.5\,\mathrm{m}$ onto a timber floor and rebounds to a height of $1.0\,\mathrm{m}$. Explain, using energy ideas, why the rebound height is less than the drop height.

**Question 15** At a building site, a crane lifts a concrete panel of mass $1200\,\mathrm{kg}$ at constant speed from the ground to a height of $8.0\,\mathrm{m}$. (a) Calculate the gain in gravitational potential energy of the panel. (b) The panel later slips and falls. Neglecting air resistance, calculate its speed just before it reaches the ground.

**Question 16** A student claims: "A heavier skier reaches a greater speed at the bottom of a slope, because a heavier skier starts with more gravitational potential energy." Evaluate this claim using conservation of energy for a frictionless slope.

**Question 17** A bungee jumper of mass $60\,\mathrm{kg}$ jumps from a platform. The jumper falls freely for $20\,\mathrm{m}$ before the cord starts to stretch, and comes momentarily to rest a further $25\,\mathrm{m}$ below. (a) Calculate the total decrease in the jumper's gravitational potential energy between the platform and the lowest point. (b) Assuming all of this energy becomes elastic potential energy of the cord, model the cord as an ideal spring and calculate its spring constant. (c) Give one reason why the ideal-spring model is only an approximation for a real bungee cord.

**Question 18** In the pendulum investigation of this section, a bob of mass $0.50\,\mathrm{kg}$ is released from rest with its centre $0.20\,\mathrm{m}$ above the lowest point of the swing. (a) Calculate the decrease in the bob's gravitational potential energy between release and the lowest point. (b) Calculate the predicted speed of the bob at the lowest point. (c) A light gate measures a speed of $1.8\,\mathrm{m\ s^{-1}}$ at the lowest point. Give one reason why the measured speed is less than the predicted value.

### Answers

**Question 1** (a) $E_k = 1/2\, m v^2$; (b) 5.4 J; (c) factor of 4, see solution. **Question 2** (a) $\Delta E_g = m g \Delta h$; (b) $9.4 \times 10^3$ J; (c) see solution. **Question 3** (a) $E_s = 1/2\, k x^2$; (b) 0.25 J; (c) factor of 4, see solution. **Question 4** (a) 3.1 J; (b) 3.1 J; (c) $6.3\ \mathrm{m\ s^{-1}}$. **Question 5** (a) $3.92 \times 10^6$ J; (b) $89\ \mathrm{m\ s^{-1}}$; (c) see solution. **Question 6** (a) $1.6 \times 10^3$ J; (b) see solution; (c) 3.3 m. **Question 7** $2.4 \times 10^5$ J. **Question 8** Object A, by a factor of 2. **Question 9** (a) $3.2 \times 10^3$ J; (b) 4.3 m; (c) see solution. **Question 10** See solution. **Question 11** (a) $7.0\ \mathrm{m\ s^{-1}}$; (b) $3.6 \times 10^2$ J. **Question 12** See solution. **Question 13** (a) 0.50 J; (b) $4.1\ \mathrm{m\ s^{-1}}$. **Question 14** See solution. **Question 15** (a) $9.4 \times 10^4$ J; (b) $13\ \mathrm{m\ s^{-1}}$. **Question 16** See solution. **Question 17** (a) $2.6 \times 10^4$ J; (b) $85\ \mathrm{N\ m^{-1}}$; (c) see solution. **Question 18** (a) 0.98 J; (b) $2.0\ \mathrm{m\ s^{-1}}$; (c) see solution.

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### Fully-worked solutions

**Question 1** $m = 1.2$ kg. (a) $E_k = 1/2\, m v^2$. (b) $E_k = 1/2 \times 1.2 \times 3.0^2 = 5.4$ J. (c) Since $E_k$ is proportional to $v^2$, doubling the speed multiplies the kinetic energy by $2^2 = 4$. Checking: $1/2 \times 1.2 \times 6.0^2 = 21.6$ J, which is $5.4\ \mathrm{J} \times 4$.

**Question 2** $m = 80$ kg, $\Delta h = 12$ m. (a) $\Delta E_g = m g \Delta h$. (b) $\Delta E_g = 80 \times 9.8 \times 12 = 9408\ \mathrm{J} \approx 9.4 \times 10^3$ J. (c) The footballer's muscles do the work of climbing, transforming chemical energy from food into gravitational potential energy (and thermal energy of the body and surroundings).

**Question 3** $k = 50\ \mathrm{N\ m^{-1}}$. (a) $E_s = 1/2\, k x^2$. (b) $E_s = 1/2 \times 50 \times 0.10^2 = 0.25$ J. (c) Since $E_s$ is proportional to $x^2$, doubling the extension multiplies the stored energy by $2^2 = 4$. Checking: $1/2 \times 50 \times 0.20^2 = 1.0$ J, which is $0.25\ \mathrm{J} \times 4$.

**Question 4** $m = 0.16$ kg, $\Delta h = 2.0$ m, released from rest. (a) $\Delta E_g = m g \Delta h = 0.16 \times 9.8 \times 2.0 = 3.136\ \mathrm{J} \approx 3.1$ J. (b) If mechanical energy is conserved, the decrease in gravitational potential energy equals the increase in kinetic energy: $E_k = 3.1$ J just before impact. (c) $1/2\, m v^2 = 3.136$, so $v = \sqrt{\dfrac{2 \times 3.136}{0.16}} = \sqrt{39.2} = 6.26\ \mathrm{m\ s^{-1}} \approx 6.3\ \mathrm{m\ s^{-1}}$.

**Question 5** $m = 1000$ kg, $\Delta h = 400$ m. (a) $\Delta E_g = m g \Delta h = 1000 \times 9.8 \times 400 = 3.92 \times 10^6$ J. (b) Setting $1/2\, m v^2 = 3.92 \times 10^6$: $v = \sqrt{\dfrac{2 \times 3.92 \times 10^6}{1000}} = \sqrt{7840} = 88.5\ \mathrm{m\ s^{-1}} \approx 89\ \mathrm{m\ s^{-1}}$. (Equivalently, $v = \sqrt{2 g \Delta h}$, which does not depend on the mass.) (c) The water does work on the turbine blades, transferring energy to the turbine and then to the generator, and some energy is transformed into thermal energy and sound by friction as the water moves through the pipes and past the blades. The water's kinetic energy at the bottom is much smaller than the ideal value.

**Question 6** $m = 50$ kg, $x = 0.80$ m, $k = 5000\ \mathrm{N\ m^{-1}}$. (a) $E_s = 1/2\, k x^2 = 1/2 \times 5000 \times 0.80^2 = 1600\ \mathrm{J} = 1.6 \times 10^3$ J. (b) As the mat returns to its flat shape, elastic potential energy of the trampoline is transformed into kinetic energy of the gymnast; as the gymnast rises, that kinetic energy is transformed into gravitational potential energy. (c) $m g \Delta h = 1600$, so $\Delta h = \dfrac{1600}{50 \times 9.8} = \dfrac{1600}{490} = 3.27\ \mathrm{m} \approx 3.3$ m above the lowest point.

**Question 7** $m = 1200$ kg, $v = 20\ \mathrm{m\ s^{-1}}$.

$$E_k = 1/2\, m v^2 = 1/2 \times 1200 \times 20^2 = 240\,000\ \mathrm{J} = 2.4 \times 10^5\ \mathrm{J}.$$

**Question 8** Object A: $E_{k,A} = 1/2\, m v^2$. Object B:

$$E_{k,B} = 1/2(2m)\left(\frac{v}{2}\right)^2 = 1/2 \times 2m \times \frac{v^2}{4} = 1/4\, m v^2.$$

Object A has the greater kinetic energy: $E_{k,A} = 2 \times E_{k,B}$. Doubling the mass doubles the kinetic energy, but halving the speed divides it by four, so the speed change has the larger effect.

**Question 9** $m = 75\,\text{kg}$, $v = 9.2\,\text{m s}^{-1}$. (a) $E_k = 1/2\,m\,v^2 = 1/2 \times 75 \times 9.2^2 = 3174\,\text{J} \approx 3.2 \times 10^3\,\text{J}$. (b) $m\,g\,\Delta h = 3174$, so $\Delta h = \dfrac{3174}{75 \times 9.8} = \dfrac{3174}{735} = 4.3\,\text{m}$. (c) First, at the top of the vault the vaulter is still moving horizontally, so some kinetic energy remains and has not become gravitational potential energy. Second, some energy is transformed into thermal energy and sound as the pole bends and straightens, and work is done against air resistance. (A model answer should include any two of these reasons.)

**Question 10** The ball is not losing energy — its kinetic energy is being transformed into gravitational potential energy as it rises. If air resistance is neglected, the mechanical energy ($E_k + E_g$) stays constant: as $E_k$ falls, $E_g$ rises by the same amount. The law of conservation of energy is about the total energy, not about any single form of it. The student has mistaken a transformation for a loss.

**Question 11** $m = 55\,\text{kg}$, $\Delta h = 2.5\,\text{m}$, released from rest. (a) Conservation of mechanical energy: $1/2\,m\,v^2 = m\,g\,\Delta h$, so $v = \sqrt{2\,g\,\Delta h} = \sqrt{2 \times 9.8 \times 2.5} = \sqrt{49} = 7.0\,\text{m s}^{-1}$. (b) The decrease in gravitational potential energy is $m\,g\,\Delta h = 55 \times 9.8 \times 2.5 = 1347.5\,\text{J}$. The kinetic energy actually measured at the bottom is $1/2\,m\,v^2 = 1/2 \times 55 \times 6.0^2 = 990\,\text{J}$. The difference, $1347.5 - 990 = 357.5\,\text{J} \approx 3.6 \times 10^2\,\text{J}$, is the mechanical energy transformed into thermal energy and sound by friction in the wheels and bearings and by air resistance.

**Question 12** The riders reach the bottom at the same speed. Neglecting friction, mechanical energy is conserved on each slide, and both riders start from rest at the same height. Each rider's decrease in gravitational potential energy, $m\,g\,\Delta h$, is therefore the same, and all of it becomes kinetic energy: $1/2\,m\,v^2 = m\,g\,\Delta h$, giving $v = \sqrt{2\,g\,\Delta h}$. The speed at the bottom depends only on the vertical drop, not on the path taken. (The riders do not take the same time: the steep slide is quicker.)

**Question 13** $m = 0.060\,\text{kg}$, $k = 400\,\text{N m}^{-1}$, $x = 5.0\,\text{cm} = 0.050\,\text{m}$. (a) $E_s = 1/2\,k\,x^2 = 1/2 \times 400 \times 0.050^2 = 0.50\,\text{J}$. (b) With all $0.50\,\text{J}$ becoming kinetic energy: $1/2\,m\,v^2 = 0.50$, so $v = \sqrt{\dfrac{2 \times 0.50}{0.060}} = \sqrt{16.7} = 4.1\,\text{m s}^{-1}$.

**Question 14** As the ball falls, gravitational potential energy is transformed into kinetic energy. During the impact with the floor, the ball and the floor deform, and some of the kinetic energy is transformed into thermal energy and sound; only the remainder is transformed back into kinetic energy as the ball rebounds. Since the ball leaves the floor with less kinetic energy than it had just before impact, it rises to a lower height: the rebound's kinetic energy becomes gravitational potential energy $m\,g\,\Delta h$ with a smaller $\Delta h$. Energy overall is conserved — the "missing" energy is in the thermal energy of the ball, the floor and the air, and in sound.

**Question 15** $m = 1200\,\text{kg}$, $\Delta h = 8.0\,\text{m}$. (a) $\Delta E_g = m\,g\,\Delta h = 1200 \times 9.8 \times 8.0 = 94\,080\,\text{J} \approx 9.4 \times 10^4\,\text{J}$. (b) Neglecting air resistance, the gain in kinetic energy equals the loss of gravitational potential energy during the fall: $1/2\,m\,v^2 = m\,g\,\Delta h$, so $v = \sqrt{2\,g\,\Delta h} = \sqrt{2 \times 9.8 \times 8.0} = \sqrt{156.8} = 12.5\,\text{m s}^{-1} \approx 13\,\text{m s}^{-1}$ to two significant figures.

**Question 16** The claim is incorrect. On a frictionless slope, conservation of mechanical energy gives $m\,g\,\Delta h = 1/2\,m\,v^2$. The mass $m$ appears on both sides and cancels, giving $v = \sqrt{2\,g\,\Delta h}$: the speed at the bottom depends only on the vertical drop and $g$, not on the mass. A heavier skier does start with more gravitational potential energy, but reaching a given speed also requires proportionally more kinetic energy ($E_k = 1/2\,m\,v^2$), so the two effects exactly balance. Both skiers reach the same speed.

**Question 17** $m = 60\,\text{kg}$; total fall $= 20\,\text{m} + 25\,\text{m} = 45\,\text{m}$. (a) $\Delta E_g = m\,g\,\Delta h = 60 \times 9.8 \times 45 = 26\,460\,\text{J} \approx 2.6 \times 10^4\,\text{J}$. (b) Setting $E_s = 1/2\,k\,x^2 = 26\,460\,\text{J}$ with $x = 25\,\text{m}$:

$$k = \frac{2 \times 26\,460}{25^2} = \frac{52\,920}{625} = 84.7\,\text{N m}^{-1} \approx 85\,\text{N m}^{-1}.$$

(c) A real bungee cord does not obey Hooke's Law exactly — its force is not proportional to extension throughout the stretch — and some mechanical energy is transformed into thermal energy within the cord as it stretches. Either reason makes the ideal-spring calculation approximate.

**Question 18** $m = 0.50\,\text{kg}$, $\Delta h = 0.20\,\text{m}$, released from rest. (a) $\Delta E_g = m g \Delta h = 0.50 \times 9.8 \times 0.20 = 0.98\,\text{J}$. (b) Conservation of mechanical energy: $1/2 m v^2 = 0.98$, so $v = \sqrt{\frac{2 \times 0.98}{0.50}} = \sqrt{3.92} = 2.0\,\text{m s}^{-1}$. (c) Air resistance on the moving bob, and friction at the support, do work during the swing and transform a small amount of mechanical energy into thermal energy of the surroundings, so the measured kinetic energy — and speed — at the lowest point is slightly less than the ideal prediction.

**Scope** — *VCE Physics Study Design, Unit 2, Area of Study 1*: "analyse rate of energy transfer using power: P = E/t"; and "calculate the efficiency of an energy transfer system: η = useful energy out / total energy in".

## Theory

**Two questions about every energy transfer.** Sections 9A and 9C established that energy can be transferred from one place to another and transformed from one form to another, and that the total amount of energy is always conserved. Conservation alone, however, does not tell the whole story. A crane lifting steel beams to the top of a building, a student climbing the stairs to the science lab, and a solar panel on a house roof are all transferring energy — but in each case two further questions matter. *How quickly is the energy being transferred?* And *how much of the energy supplied actually ends up where it is wanted?* This section answers those two questions with two quantities: **power** and **efficiency**.

**Power — the rate of energy transfer.** The **power** of a device or a person is the rate at which it transfers energy: the amount of energy transferred divided by the time taken. The relationship is

$$P = \frac{E}{t},$$

where $P$ is the power, $E$ is the energy transferred, and $t$ is the time taken. The SI unit of power is the **watt** (W):

$$1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}.$$

A device with a power of $1\,\mathrm{W}$ transfers $1\,\mathrm{J}$ of energy every second. Larger powers are written in kilowatts and megawatts: $1\,\mathrm{kW} = 1000\,\mathrm{W}$ and $1\,\mathrm{MW} = 10^6\,\mathrm{W}$.

Power is a scalar: it has a size but no direction. It tells you how *quickly* energy is moved or transformed, not how much is moved in total. A $2400\,\mathrm{W}$ kettle transfers $2400\,\mathrm{J}$ of energy every second it is switched on; a $60\,\mathrm{W}$ lamp transfers $60\,\mathrm{J}$ every second. The kettle transfers far more energy *per second*, but if the lamp is left on all night it can transfer far more energy in total. Power and energy are related but different quantities, and keeping them separate is the key skill of this section.

Rearranging $P = \frac{E}{t}$ gives the other two forms used throughout this section:

$$E = P\,t \text{ and } t = \frac{E}{P}.$$

The first of these, energy = power × time, is the reason the kilowatt-hour of Section 5C is a unit of energy: $1\,\mathrm{kW\,h}$ is the energy transferred by a $1\,\mathrm{kW}$ device running for $1\,\mathrm{h}$, and $1\,\mathrm{kW\,h} = 3.6 \times 10^6\,\mathrm{J}$.

Work, from Section 9A, is one way of transferring energy, so power can equally be described as the rate of doing work. The form $P = \frac{E}{t}$ is the one used here, because it applies to *every* energy transfer — mechanical, electrical, thermal or otherwise. In electric circuits the same rate idea appears as $P = V\,I$ (Section 5A); that is the electrical version of the same quantity, and the two forms give the same watts for the same device.

**The same energy, different powers.** Two students of the same mass who climb the same flight of stairs gain the same gravitational potential energy $E_g = m\,g\,\Delta h$ (Section 9C). The one who sprints up does it in a shorter time, so she transfers that energy at a greater rate — she has the greater power. The energy transferred is the same; only the rate differs. The same reasoning explains why two cranes lifting identical loads through the same height can have very different powers: the faster crane delivers the same energy in less time.

**Efficiency — how much of the energy arrives usefully.** No real device transfers all of its input energy to the intended place or form. In an electric motor raising a load, most of the electrical energy becomes gravitational potential energy of the load — but some warms the motor and its cables, and a little becomes sound. In a car engine, only part of the chemical energy of the fuel becomes kinetic energy of the car; the rest warms the engine, the coolant and the exhaust gases, and some becomes sound. Energy is never destroyed in these processes — conservation of energy still holds — but some of it is **dissipated**, meaning spread out into the surroundings in forms that are no longer useful for the task.

The **efficiency** of an energy transfer system measures the fraction of the input energy that leaves the system in the useful form:

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}},$$

where $\eta$ (the Greek letter *eta*) is the efficiency. Efficiency is a ratio of two energies, so it has **no unit**. It may be given as a decimal ($\eta = 0.74$) or as a percentage ($74\,\%$, found by multiplying the decimal by $100$). Since a real system can never deliver more useful energy than the total energy supplied to it, $\eta \leq 1$ (or $100\,\%$); a claim of an efficiency greater than $100\,\%$ would mean creating energy, which conservation of energy forbids.

Because $E = P\,t$, a system that runs steadily for a time $t$ satisfies

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}} = \frac{P_{\text{useful}}\,t}{P_{\text{total}}\,t} = \frac{P_{\text{useful}}}{P_{\text{total}}},$$

so the same ratio holds for the *powers* of a steadily operating system: efficiency is also the useful power output divided by the total power input. This is not a new relationship — it is $P = \frac{E}{t}$ applied to both the input and the output, with the common time cancelling.

**Following the energy.** For every device, the total energy in equals the useful energy out plus the energy dissipated:

$$\text{total energy in} = \text{useful energy out} + \text{energy dissipated}.$$

A motor that is supplied with $100\text{ J}$ of electrical energy and delivers $74\text{ J}$ of gravitational potential energy has an efficiency of $74\,\%$; the remaining $26\text{ J}$ has not vanished — it has warmed the motor and its surroundings. This bookkeeping is the pattern behind every efficiency calculation in this section.

Efficiency can be improved by reducing the pathways that dissipate energy: lubricating moving parts to reduce friction, insulating hot systems, streamlining objects that move through air, and choosing devices (such as LED lamps) that waste a smaller fraction of their input energy. Improving efficiency does not change the physics of conservation — it changes how much input energy is needed for a given useful output.

**Practical investigation — measuring a person's power output.** The power of a student can be measured directly by timing them up a flight of stairs. A suitable framework:

- **Aim.** Measure the average power output of a student climbing a flight of stairs, and compare the power when walking with the power when climbing at a steady jog.
- **Equipment.** A flight of stairs; a metre ruler or tape measure (to measure the height of one step and count the steps, giving the total vertical height $\Delta h$); a stopwatch; a balance to measure the student's mass.
- **Method.** Measure the total vertical height $\Delta h$ climbed. The student climbs the stairs at a steady walking pace while the time $t$ is measured. Calculate the gain in gravitational potential energy $E_g = m\,g\,\Delta h$ and the average power $P = \frac{E_g}{t}$. Repeat several times, then repeat the whole procedure climbing at a steady jog.
- **Expected results.** For the same student and the same stairs, $E_g$ is the same every climb; the jogged climb has a shorter time, so a greater power. Typical values for a student are a few hundred watts. The power found this way is the *useful mechanical power output* only — the chemical energy the student's body uses is several times larger, because the body itself is an inefficient engine.
- **Safety and practice.** Use a handrail, climb one at a time, wear sensible footwear, and do not rush to the point of losing footing. Record every measurement — mass, step height, number of steps, time — with its unit in your logbook: the logbook is submitted as a requirement for satisfactory completion of the unit.

Throughout this section, where the gravitational field strength is needed, $g = 9.8\ \text{N kg}^{-1}$, the value near the surface of Earth (Section 8C).

**Where this leads.** Power and efficiency are the language of the case study in Section 9E, where motion concepts are applied to contexts such as sport, vehicle safety and devices. Efficiency also recurs in the options of Chapter 10–13: in Option 2.1 (Section 10C) and Option 2.2 (Section 10D) when energy sources for society are compared, and in Option 2.12 (Section 12E) when AC electricity is used to charge a DC device. The electrical power $P = V\,I$ and the kilowatt-hour of Sections 5A and 5C are the same ideas applied to circuits.

**Units 3 & 4 preview.** At Units 3 & 4 the relationships of this section are applied at a larger scale: $P = \frac{E}{t}$ and efficiency are used to analyse electrical power generation and the energy dissipated during long-distance transmission, and the energy-conservation modelling of Section 9C is extended to fields. Content under this banner is not assessed in this book's topic tests or practice examinations.

## Worked examples

### *Example 1 — scaffolded*

A spectator of mass $70\ \text{kg}$ climbs the stairs to the top tier at the Melbourne Cricket Ground, gaining a vertical height of $30\ \text{m}$ in $75\ \text{s}$. (a) Calculate the increase in her gravitational potential energy. (b) Calculate her average power during the climb. (c) A second spectator of the same mass climbs the same stairs more slowly, taking $150\ \text{s}$. Calculate her average power, and explain why it differs from the first value.

| Working | Comment |
|---|---|
| $m = 70\ \text{kg}$, $\Delta h = 30\ \text{m}$, $t = 75\ \text{s}$, $g = 9.8\ \text{N kg}^{-1}$. | List the data; the useful energy transferred is the gain in gravitational potential energy. |
| (a) $E_g = m\,g\,\Delta h = 70 \times 9.8 \times 30 = 20\,580\ \text{J} \approx 2.1 \times 10^4\ \text{J}$. | From Section 9C, to 2 significant figures. |
| (b) $P = \frac{E}{t} = \frac{20\,580}{75} = 274.4\ \text{W} \approx 270\ \text{W}$. | Power is the rate of energy transfer, to 2 significant figures. |
| (c) The energy transferred is the same ($20\,580\ \text{J}$), since the mass and height are the same. $P = \frac{20\,580}{150} = 137.2\ \text{W} \approx 140\ \text{W}$. | The same energy in twice the time is transferred at half the rate. |

### *Example 2 — scaffolded*

On a construction site at Fishermans Bend in Melbourne, an electric winch is supplied with $60\,000\ \text{J}$ of electrical energy while it raises a load of mass $150\ \text{kg}$ through a vertical height of $30\ \text{m}$. (a) Calculate the useful energy output of the winch. (b) Calculate the efficiency of the winch, as a decimal and as a percentage. (c) Name the main form into which the remaining energy is transformed.

| Working | Comment |
|---|---|
| $E_{\text{in}} = 60\,000\ \text{J}$, $m = 150\ \text{kg}$, $\Delta h = 30\ \text{m}$. | The useful output is the gain in gravitational potential energy of the load. |
| (a) $E_{\text{useful}} = m\,g\,\Delta h = 150 \times 9.8 \times 30 = 44\,100\ \text{J} \approx 4.4 \times 10^4\ \text{J}$. | To 2 significant figures. |
| (b) $\eta = \frac{\text{useful energy out}}{\text{total energy in}} = \frac{44\,100}{60\,000} = 0.735 \approx 0.74$, that is $74\,\%$. | Efficiency is a ratio and has no unit. |
| (c) The remaining $60\,000 - 44\,100 = 15\,900\ \text{J}$ warms the motor, the cables and the surroundings — it becomes | The energy is dissipated, not destroyed. |

thermal energy, with a small amount of sound.

### *Example 3 — unscaffolded*

In the Snowy Mountains Scheme, water released from a high reservoir falls through tunnels to turbines far below. Consider one turbine through which $12\,000\,\mathrm{kg}$ of water falls vertically through $200\,\mathrm{m}$ every second. Calculate the power available from the falling water, and hence calculate the electrical power produced if the turbine and generator together operate with an efficiency of $85\,\%$.

Each second, the water loses gravitational potential energy

$$E_g = m\,g\,\Delta h = 12\,000 \times 9.8 \times 200 = 2.352 \times 10^7\,\mathrm{J} \approx 2.4 \times 10^7\,\mathrm{J}.$$

Since this energy is transferred every second, the power available from the falling water is

$$P = \frac{E}{t} = \frac{2.352 \times 10^7}{1} = 2.352 \times 10^7\,\mathrm{W} \approx 24\,\mathrm{MW}.$$

The turbine and generator deliver $85\,\%$ of this as electrical energy each second, so the electrical power produced is

$$P_{\mathrm{electrical}} = 0.85 \times 2.352 \times 10^7 = 1.999 \times 10^7\,\mathrm{W} \approx 2.0 \times 10^7\,\mathrm{W} = 20\,\mathrm{MW}.$$

∴ the power available from the falling water is about $24\,\mathrm{MW}$, and the electrical power produced is about $20\,\mathrm{MW}$. The remaining $15\,\%$ is dissipated — it warms the turbine, the generator and the water itself, and some becomes sound and turbulence.

### *Example 4 — unscaffolded*

A cyclist (combined mass of rider and bicycle $80\,\mathrm{kg}$) rides from the base of Mount Dandenong to the summit, a vertical rise of $500\,\mathrm{m}$, in $50\,\mathrm{min}$. During the climb her body transforms $2.1 \times 10^6\,\mathrm{J}$ of chemical energy from food. Calculate the gain in gravitational potential energy, the efficiency of her body during the climb, and her useful power output, and explain what becomes of the remaining energy.

The gain in gravitational potential energy is

$$E_g = m\,g\,\Delta h = 80 \times 9.8 \times 500 = 3.92 \times 10^5\,\mathrm{J}.$$

This is the useful energy output of the climb. With a total energy input from food of $2.1 \times 10^6\,\mathrm{J}$, the efficiency of her body is

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}} = \frac{3.92 \times 10^5}{2.1 \times 10^6} = 0.187 \approx 0.19,$$

or about $19\,\%$. Her useful power output is the rate at which gravitational potential energy is gained, with $t = 50\,\mathrm{min} = 3000\,\mathrm{s}$:

$$P = \frac{E_g}{t} = \frac{3.92 \times 10^5}{3000} = 130.7\,\mathrm{W} \approx 130\,\mathrm{W}.$$

∴ the gain in gravitational potential energy is $3.9 \times 10^5\,\mathrm{J}$, the efficiency of her body is about $19\,\%$, and her useful power output is about $130\,\mathrm{W}$. The remaining $81\,\%$ of the chemical energy — roughly $1.7 \times 10^6\,\mathrm{J}$ — is not destroyed: most of it becomes thermal energy in her muscles and her surroundings, which is why she warms up, sweats and breathes harder during the climb.

## Practice questions

**Question 1** An electric kettle in the school staff room is rated at $2400\,\mathrm{W}$. (a) State the energy the kettle transfers each second while it is switched on. (b) Calculate the energy transferred by the kettle in $2.0\,\mathrm{min}$ of operation. (c) A travel kettle rated at $1200\,\mathrm{W}$ is used to transfer the same energy as in part (b). Calculate the time it takes.

**Question 2** A student of mass $52\,\mathrm{kg}$ runs up a flight of stairs to the first-floor science lab, gaining a vertical height of $4.0\,\mathrm{m}$ in $5.0\,\mathrm{s}$. (a) Calculate the increase in the student's gravitational potential energy. (b) Calculate the student's average power during the climb. (c) The student later walks up the same stairs in $10\,\mathrm{s}$. State the energy transferred and calculate the average power, and explain the difference from part (b).

**Question 3** An electric motor is supplied with $50\,000\,\mathrm{J}$ of electrical energy while it raises a crate of mass $80\,\mathrm{kg}$ through a vertical height of $45\,\mathrm{m}$. (a) Calculate the useful energy output of the motor. (b) Calculate the efficiency of the motor, as a decimal and as a percentage. (c) Name the main form into which the remaining energy is transformed.

**Question 4** Two cranes at the Port of Melbourne each lift an identical container of mass $1200\,\mathrm{kg}$ from a ship through a vertical height of $15\,\mathrm{m}$. Crane A takes $30\,\mathrm{s}$; crane B takes $20\,\mathrm{s}$. (a) Calculate the increase in gravitational potential energy of each container. (b) Calculate the average power of each crane during its lift. (c) State which crane is the more powerful, and explain your reasoning.

**Question 5** A solar panel on the roof of a house in Bendigo receives $8.0\,\mathrm{MJ}$ of energy from sunlight during one hour of a clear day, and transforms $1.6\,\mathrm{MJ}$ of this into electrical energy. (a) Identify the total energy input and the useful energy output of the panel. (b) Calculate the efficiency of the panel. (c) State the main form into which the remaining energy is transformed.

**Question 6** A $2.0\,\mathrm{kW}$ electric heater operates for $3.0\,\mathrm{h}$. (a) Calculate the energy transferred by the heater, in $\mathrm{kW\,h}$. (b) Calculate the same energy in joules. (c) A $0.60\,\mathrm{kW}$ heater is operated for $10\,\mathrm{h}$. Calculate the energy it transfers and explain how a lower-power device can transfer the same total energy.

**Question 7** At the school athletics carnival, a sprinter of mass $64\,\mathrm{kg}$ accelerates from rest to $9.0\,\mathrm{m\,s^{-1}}$ in $1.5\,\mathrm{s}$. Calculate the average rate at which her kinetic energy increases during this time. (This is the power going into her kinetic energy only.)

**Question 8** A student says: "My microwave oven has a power rating of $1100\,\mathrm{W}$ and the toaster has a power rating of $900\,\mathrm{W}$, so the microwave must transfer more energy than the toaster." Explain why this conclusion does not necessarily follow, using the relationship between energy, power and time.

**Question 9** A $2400\,\mathrm{W}$ kettle takes $45\,\mathrm{s}$ to bring its water to the boil. During this time the water gains $96\,000\,\mathrm{J}$ of thermal energy (a quantity treated in Section 2B). Calculate the efficiency of the kettle.

**Question 10** Electric motors, lamps and phone chargers all become warm while they are operating. Explain why this happens, and why it means no real energy transfer system can be $100\,\%$ efficient.

**Question 11** The Snowy 2.0 project pumps water uphill into a reservoir when cheap electricity is available. While pumping, the motor and pump together receive an input power of $2.0\,\mathrm{MW}$ and operate with an efficiency of $75\,\%$. (a) Calculate the rate at which the pumped water gains gravitational potential energy. (b) The water is pumped through a vertical height of $100\,\mathrm{m}$. Calculate the mass of water lifted each second.

**Question 12** A student says: "An inefficient device destroys some of the energy supplied to it." Identify the error in this statement and correct it, referring to the conservation of energy.

**Question 13** A hiker of total mass $75\,\mathrm{kg}$ (including pack) climbs the You Yangs, gaining a vertical height of $400\,\mathrm{m}$ in $2.0\,\mathrm{h}$. The human body operates with an efficiency of about $20\,\%$ during such a climb. (a) Calculate the increase in the hiker's gravitational potential energy. (b) Calculate the hiker's useful power output during the climb. (c) Calculate the total chemical energy the hiker's body uses during the climb. (d) Calculate the total rate at which the hiker's body uses energy.

**Question 14** A more efficient appliance can deliver the same useful energy output as a less efficient one while drawing less total energy from the supply. Use the definition of efficiency to explain this, and state what happens to the energy that is not delivered usefully.

**Question 15** An electric car has a battery that stores $60\,\mathrm{kW\,h}$ of energy. The electric motor and drivetrain together operate with an efficiency of $90\,\%$. (a) Calculate the useful mechanical energy available from the battery, in joules. (b) At a constant speed on a country highway the car requires a useful mechanical power of $20\,\mathrm{kW}$. Calculate how long the battery can maintain this speed. (c) Name the main forms into which this mechanical energy is eventually transformed as the car cruises at constant speed.

**Question 16** Pumped-storage hydroelectricity uses surplus electrical energy to pump water uphill, then allows the water to flow back down through turbines when electricity is needed. Explain why the electrical energy generated when the water flows back down is always less than the electrical energy that was used to pump the water up.

**Question 17** A water pump used on a farm must deliver a useful power output of $450\,\mathrm{W}$. The pump operates with an efficiency of $75\,\%$. (a) Calculate the input power the pump requires. (b) Calculate the energy dissipated by the pump in $10\,\mathrm{min}$ of operation.

**Question 18** When a person climbs stairs, only about one-fifth of the chemical energy used by the body becomes gravitational potential energy. Identify the main form into which the rest of the energy is transformed, and use this to explain why the person becomes warmer and breathes faster during the climb.

### Answers

**Question 1** (a) $2400\,\text{J}$; (b) $2.9 \times 10^5\,\text{J}$; (c) $240\,\text{s}$ ($4.0\,\text{min}$). **Question 2** (a) $2.0 \times 10^3\,\text{J}$; (b) $410\,\text{W}$; (c) same energy, $200\,\text{W}$; see solution. **Question 3** (a) $3.5 \times 10^4\,\text{J}$; (b) $0.71$ ($71\,\%$); (c) see solution. **Question 4** (a) $1.8 \times 10^5\,\text{J}$ each; (b) crane A $5900\,\text{W}$, crane B $8800\,\text{W}$; (c) see solution. **Question 5** (a) see solution; (b) $20\,\%$; (c) see solution. **Question 6** (a) $6.0\,\text{kW h}$; (b) $2.2 \times 10^7\,\text{J}$; (c) $6.0\,\text{kW h}$; see solution. **Question 7** $1.7 \times 10^3\,\text{W}$. **Question 8** See solution. **Question 9** $89\,\%$. **Question 10** See solution. **Question 11** (a) $1.5 \times 10^6\,\text{W}$; (b) $1.5 \times 10^3\,\text{kg}$ each second. **Question 12** See solution. **Question 13** (a) $2.9 \times 10^5\,\text{J}$; (b) $41\,\text{W}$; (c) $1.5 \times 10^6\,\text{J}$; (d) $200\,\text{W}$. **Question 14** See solution. **Question 15** (a) $1.9 \times 10^8\,\text{J}$; (b) $2.7\,\text{h}$; (c) see solution. **Question 16** See solution. **Question 17** (a) $600\,\text{W}$; (b) $9.0 \times 10^4\,\text{J}$. **Question 18** See solution.

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### Fully-worked solutions

**Question 1** $P = 2400\,\text{W}$. (a) Power is the energy transferred each second, so the kettle transfers $2400\,\text{J}$ every second. (b) $t = 2.0\,\text{min} = 120\,\text{s}$. $E = P\,t = 2400 \times 120 = 288\,000\,\text{J} \approx 2.9 \times 10^5\,\text{J}$ (2 significant figures). (c) $t = \dfrac{E}{P} = \dfrac{288\,000}{1200} = 240\,\text{s}$, which is $4.0\,\text{min}$. The lower-power kettle transfers the same energy at a lower rate, so it takes longer.

**Question 2** $m = 52\,\text{kg}$, $\Delta h = 4.0\,\text{m}$, $t = 5.0\,\text{s}$. (a) $E_g = m\,g\,\Delta h = 52 \times 9.8 \times 4.0 = 2038\,\text{J} \approx 2.0 \times 10^3\,\text{J}$ (2 significant figures). (b) $P = \dfrac{E}{t} = \dfrac{2038}{5.0} = 407.7\,\text{W} \approx 410\,\text{W}$ (2 significant figures). (c) The energy transferred is the same, $2.0 \times 10^3\,\text{J}$, because the same mass is raised through the same height. The time is longer ($10\,\text{s}$), so $P = \dfrac{2038}{10} = 203.8\,\text{W} \approx 200\,\text{W}$. Walking transfers the same energy at a lower rate, so the power is lower.

**Question 3** $E_{\text{in}} = 50\,000\,\text{J}$, $m = 80\,\text{kg}$, $\Delta h = 45\,\text{m}$. (a) The useful output is the gain in gravitational potential energy of the crate: $E_{\text{useful}} = m\,g\,\Delta h = 80 \times 9.8 \times 45 = 35\,280\,\text{J} \approx 3.5 \times 10^4\,\text{J}$. (b) $\eta = \dfrac{\text{useful energy out}}{\text{total energy in}} = \dfrac{35\,280}{50\,000} = 0.7056 \approx 0.71$, that is $71\,\%$ (2 significant figures). (c) The remaining $50\,000 - 35\,280 = 14\,720\,\text{J}$ becomes mostly thermal energy — the motor warms as current passes through it and friction acts in its moving parts — with a small amount of sound.

**Question 4** $m = 1200\,\text{kg}$, $\Delta h = 15\,\text{m}$; $t_A = 30\,\text{s}$, $t_B = 20\,\text{s}$. (a) Each container gains $E_g = m\,g\,\Delta h = 1200 \times 9.8 \times 15 = 176\,400\,\text{J} \approx 1.8 \times 10^5\,\text{J}$. (b) Crane A: $P = \dfrac{176\,400}{30} = 5880\,\text{W} \approx 5900\,\text{W}$. Crane B: $P = \dfrac{176\,400}{20} = 8820\,\text{W} \approx 8800\,\text{W}$ (both to 2 significant figures). (c) Crane B is the more powerful. Both cranes transfer the same energy, but crane B transfers it in a shorter time, and power is the rate of energy transfer: the same energy in less time is a greater power.

**Question 5** Total energy in $= 8.0\,\text{MJ}$ (from sunlight); useful energy out $= 1.6\,\text{MJ}$ (electrical). (a) The total energy input is the $8.0\,\text{MJ}$ of energy received from sunlight; the useful energy output is the $1.6\,\text{MJ}$ transformed into electrical energy. (b) $\eta = \dfrac{1.6}{8.0} = 0.20$, that is $20\,\%$. (c) Most of the remaining $6.4\,\text{MJ}$ becomes thermal energy — the panel and the roof warm up — and some is reflected back into the surroundings.

**Question 6** $P = 2.0\,\text{kW}$, $t = 3.0\,\text{h}$. (a) $E = P\,t = 2.0 \times 3.0 = 6.0\,\text{kW h}$. (b) $6.0\,\text{kW h} = 6.0 \times 3.6 \times 10^6\,\text{J} = 2.16 \times 10^7\,\text{J} \approx 2.2 \times 10^7\,\text{J}$. (c) $E = 0.60\,\text{kW} \times 10\,\text{h} = 6.0\,\text{kW h}$ — the same energy. Since $E = P\,t$, a device of lower power transfers the same total energy if it operates for a proportionally longer time. Energy depends on power *and* time together, not on power alone.

**Question 7** $m = 64\,\text{kg}$, $u = 0$, $v = 9.0\,\text{m s}^{-1}$, $t = 1.5\,\text{s}$. The kinetic energy gained is

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 64 \times 9.0^2 = 2592\,\text{J}.$$

The average rate at which kinetic energy increases is therefore

$$P = \frac{E}{t} = \frac{2592}{1.5} = 1728\,\text{W} \approx 1.7 \times 10^3\,\text{W}$$

to 2 significant figures. This is only the power going into the sprinter's kinetic energy; the total rate at which her body uses energy is larger, because her body is not $100\,\%$ efficient.

**Question 8** Energy transferred depends on power *and* time: $E = P\,t$. A higher power rating means more energy transferred *each second*, but it says nothing about how long the device runs. For example, the microwave operating for $2.0\,\text{min}$ transfers $E = 1100 \times 120 = 132\,000\,\text{J}$, while the toaster operating for $10\,\text{min}$ transfers $E = 900 \times 600 = 540\,000\,\text{J}$ — more than four times as much, despite its lower power rating. The conclusion follows only if both devices operate for the same time; the student has confused power (a rate) with energy (a total amount).

**Question 9** $P = 2400\,\text{W}$, $t = 45\,\text{s}$; useful energy out $= 96\,000\,\text{J}$. Total energy input: $E_{\text{in}} = P\,t = 2400 \times 45 = 108\,000\,\text{J}$.

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}} = \frac{96\,000}{108\,000} = 0.888\ldots \approx 0.89,$$

that is $89\,\%$ (2 significant figures). The remaining $11\,\%$ warms the kettle, the bench and the surrounding air.

**Question 10** In every real device, some of the input energy is transferred to unintended places: motor windings and phone-charger components warm as current passes through them, moving parts warm through friction, and some energy is carried away as sound. All of these pathways end by spreading energy into the surroundings as thermal energy. Because some input energy is always dissipated in this way, the useful energy output is always less than the total energy input, and the efficiency is always less than $100\,\%$. This does not contradict conservation of energy — the energy is spread out, not destroyed.

**Question 11** $P_{\text{in}} = 2.0 \times 10^6\,\text{W}$, $\eta = 0.75$, $\Delta h = 100\,\text{m}$. (a) The useful power output — the rate at which the water gains gravitational potential energy — is

$$P_{\text{useful}} = \eta \times P_{\text{in}} = 0.75 \times 2.0 \times 10^6 = 1.5 \times 10^6\,\text{W}.$$

(b) Each second the water gains $1.5 \times 10^6\,\text{J}$ of gravitational potential energy, so with $E_g = m\,g\,\Delta h$:

$$m = \frac{E_g}{g\,\Delta h} = \frac{1.5 \times 10^6}{9.8 \times 100} = 1530\,\text{kg} \approx 1.5 \times 10^3\,\text{kg}$$

to 2 significant figures. About $1.5 \times 10^3\,\text{kg}$ of water is lifted each second.

**Question 12** The statement treats inefficiency as though energy were destroyed, which conservation of energy forbids: energy cannot be created or destroyed, only transferred or transformed. In an inefficient device, all of the input energy still leaves the device — the total energy out equals the total energy in. The difference from an efficient device is that a larger fraction leaves in forms that are not useful for the task, mostly thermal energy spread into the surroundings. The correct statement is: an inefficient device transfers *some of the energy supplied to it into unwanted forms*, not that it destroys energy.

**Question 13** $m = 75\,\text{kg}$, $\Delta h = 400\,\text{m}$, $t = 2.0\,\text{h} = 7200\,\text{s}$, $\eta = 0.20$. (a) $E_g = m\,g\,\Delta h = 75 \times 9.8 \times 400 = 294\,000\,\text{J} \approx 2.9 \times 10^5\,\text{J}$. (b) $P = \frac{E_g}{t} = \frac{294\,000}{7200} = 40.8\,\text{W} \approx 41\,\text{W}$ (2 significant figures). (c) Rearranging $\eta = \frac{\text{useful energy out}}{\text{total energy in}}$: total energy in $= \frac{\text{useful energy out}}{\eta} = \frac{294\,000}{0.20} = 1.47 \times 10^6\,\text{J} \approx 1.5 \times 10^6\,\text{J}$. (d) $\frac{1.47 \times 10^6}{7200} = 204\,\text{W} \approx 200\,\text{W}$. The body uses energy at about five times the useful power output, because only about one-fifth of the chemical energy becomes gravitational potential energy.

**Question 14** Efficiency is $\eta = \frac{\text{useful energy out}}{\text{total energy in}}$, which rearranges to total energy in $= \frac{\text{useful energy out}}{\eta}$. For a fixed useful energy output — the same amount of light, the same lift of a load, the same heated water — the total energy input is the useful output divided by the efficiency, so a *larger* efficiency gives a *smaller* total energy input. That is what makes the appliance efficient: the same task from less supplied energy. The energy that is not delivered usefully is not destroyed; it is dissipated, mostly as thermal energy to the surroundings (with small amounts as sound), warming the device and its environment.

**Question 15** Battery energy $= 60\ \text{kW h}$; $\eta = 0.90$; useful power required $= 20\ \text{kW} = 20\,000\ \text{W}$. (a) $60\ \text{kW h} = 60 \times 3.6 \times 10^6\ \text{J} = 2.16 \times 10^8\ \text{J}$. Useful mechanical energy $= 0.90 \times 2.16 \times 10^8 = 1.944 \times 10^8\ \text{J} \approx 1.9 \times 10^8\ \text{J}$ (2 significant figures). (b) $t = \frac{E}{P} = \frac{1.944 \times 10^8}{20\,000} = 9720\ \text{s} = 2.7\ \text{h}$. (c) At constant speed the car's kinetic energy is unchanged, so the useful mechanical energy is used to overcome resistance forces — air resistance and friction in the tyres and moving parts — and is transformed into thermal energy of the surrounding air, the road and the car's components.

**Question 16** Energy is lost at every stage of the round trip, so the overall efficiency is less than $100\,\%$. On the way up, the electric motor and the pump dissipate some of the input electrical energy as thermal energy and sound, so the gravitational potential energy gained by the water is already less than the electrical energy used. On the way down, the falling water loses some of that gravitational potential energy to friction and turbulence in the pipes and turbine, and the turbine and generator transform the remainder into electrical energy with their own thermal losses. Each stage has an efficiency below $100\,\%$, and the overall efficiency is the product of the stage efficiencies — so the electrical energy generated is always less than the electrical energy originally used to pump the water. (This is why pumped storage is described as storing energy, not creating it.)

**Question 17** $P_{\text{useful}} = 450\ \text{W}$, $\eta = 0.75$. (a) From $\eta = \frac{P_{\text{useful}}}{P_{\text{total}}}$ (valid because the pump runs steadily, so the time cancels):

$$P_{\text{total}} = \frac{P_{\text{useful}}}{\eta} = \frac{450}{0.75} = 600\ \text{W}.$$

(b) In $t = 10\ \text{min} = 600\ \text{s}$: total energy in $= 600 \times 600 = 360\,000\ \text{J}$, and useful energy out $= 450 \times 600 = 270\,000\ \text{J}$. The energy dissipated is the difference:

$$360\,000 - 270\,000 = 90\,000\ \text{J} = 9.0 \times 10^4\ \text{J}.$$

(Equivalently, the wasted power is $600 - 450 = 150\ \text{W}$, and $150 \times 600 = 90\,000\ \text{J}$.)

**Question 18** The remaining four-fifths of the chemical energy is transformed mainly into thermal energy in the climber's muscles and body. Because this energy is being produced continuously during the climb, the body's temperature rises; the body responds by increasing blood flow to the skin, sweating and breathing faster to transfer the excess thermal energy to the surroundings. The total energy is still conserved — the chemical energy from food has simply been shared between gravitational potential energy (the useful one-fifth) and thermal energy (the dissipated four-fifths).

**Scope** — *VCE Physics Study Design, Unit 2, Area of Study 1*: "investigate the application of motion concepts through a case study, for example, through motion in sport, vehicle safety, a device or a structure."

### Theory

**What this section is for.** Each section of Chapters 7–9 introduced one idea at a time: vectors and scalars, motion graphs, constant acceleration, momentum, Newton's laws, forces, impulse, work, springs, energy conservation, power and efficiency. Real situations do not arrive sorted by chapter. A car braking to a stop involves constant acceleration, work done by a force, energy transformation and momentum all at once; a kicked football involves impulse, kinetic energy, power and efficiency in a single action. This section is the capstone of Unit 2 Area of Study 1: it chooses real situations and analyses each of them with every idea that applies. That is what a case study is for.

**A note about the terms.** The study design uses the term *case study* but does not define it. This section therefore treats a case study as follows: one chosen real-world situation — a movement in sport, a vehicle-safety feature, a device or a structure — analysed in depth using the physics taught so far, by identifying the concepts that apply, building a simplified model of the situation, calculating with that model, and then evaluating the model's assumptions and limits. Every question in this section is answerable on that definition alone. (Case study is also one of the investigation methodologies listed in Section 14A for Unit 2 Area of Study 3.)

**The motion toolkit.** The whole of Unit 2 Area of Study 1 so far is summarised below. In a case study, the first step is always to ask which rows of this table the situation involves.

| Concept | Core relationships | Section |
|---|---|---|
| Scalars and vectors | displacement, velocity, acceleration are vectors; distance, speed, mass are scalars | 7A |
| Motion graphs | gradient gives a rate; area under a velocity–time graph gives displacement | 7B |
| Constant acceleration | $v=u+at$; $v^2=u^2+2as$; $s=1/2(u+v)t$ | 7C |
| Momentum | $p=mv$; $\Delta p=F_{\text{net}}\Delta t$ | 8A |
| Newton's laws | $a=F_{\text{net}}/m$; $F_{\text{on A by B}}=-F_{\text{on B by A}}$ | 8B |
| Force due to gravity | $F_{\text{on body by Earth}}=mg$, acting at the centre of mass | 8C |
| Adding forces | vector addition; components along two perpendicular axes | 8D |
| Impulse and momentum conservation | $F\Delta t=m\Delta v$; $m_1u_1+m_2u_2=m_1v_1+m_2v_2$ | 8E |
| Torque and equilibrium | $\tau=r_{\perp}F$; net force zero and net torque zero | 8F |
| Work | $W=Fs\cos\theta$; area under a force–distance graph | 9A |
| Springs | $F=-kx$; $E_s=1/2kx^2$ | 9C |
| Mechanical energy | $E_g=mg\Delta h$; $E_k=1/2mv^2$; conservation of mechanical energy | 9C |

| Concept | Core relationships | Section |
|---|---|---|
| Power and efficiency | $P=E/t$; $\eta=\dfrac{\text{useful energy out}}{\text{total energy in}}$ | 9D |

Throughout this section, $g=9.8\,\mathrm{m\ s^{-2}}$ (equivalently $9.8\,\mathrm{N\ kg^{-1}}$), as everywhere in this book (Section 8C).

**Case study 1: vehicle safety.** A car approaching a hazard goes through two distinct stages, and each stage is different physics.

- *The reaction stage.* Between seeing the hazard and the brakes taking hold, the car continues at constant velocity. With zero net force along the direction of travel, Newton's first law (Section 8B) applies and the distance covered is simply speed multiplied by time — a car at $50\,\mathrm{km\ h^{-1}}$, which is $13.9\,\mathrm{m\ s^{-1}}$, travels about $14\,\mathrm{m}$ in one second. Any factor that lengthens the reaction time lengthens this distance.
- *The braking stage.* Once the brakes act, a roughly constant braking force opposes the motion. Two routes give the same stopping distance. By constant acceleration (Section 7C), $a=F_{\mathrm{net}}/m$ and $v^2=u^2+2as$. By work and energy (Sections 9A and 9C), the braking force does negative work ($\theta=180^\circ$ in $W=Fs\cos\theta$), removing the car's kinetic energy: $Fd=1/2\,mv^2$, where $d$ is the braking distance. Both routes are used in the worked examples below, as a check on each other. The kinetic energy removed by the brakes is transformed mainly into thermal energy in the brakes, the tyres and the road.

Because $E_k=1/2\,mv^2$, kinetic energy depends on the *square* of the speed. A car at $60\,\mathrm{km\ h^{-1}}$ has $\left(\dfrac{60}{50}\right)^2=1.44$ times the kinetic energy of the same car at $50\,\mathrm{km\ h^{-1}}$ — $44\,\%$ more energy for only $20\,\%$ more speed. For the same braking force, the stopping distance is proportional to the kinetic energy, so it grows by the same factor. This is the physics behind speed limits: in Victoria the default speed limit in a built-up area is $50\,\mathrm{km\ h^{-1}}$, and $40\,\mathrm{km\ h^{-1}}$ zones apply around schools at the times children are arriving and leaving.

*Source: VicRoads, Speed limits, retrieved 2026-08-18.*

The crash itself is an impulse problem (Section 8E). The occupants must lose all of their momentum, and $m\Delta v$ is fixed by the speed before the crash, so the only thing design can control is the stopping time and distance. Since

$$F=\frac{m\Delta v}{\Delta t},$$

a longer stopping time means a smaller average force. Three features work on exactly this principle:

- **Crumple zones** are structures at the front and rear of the car designed to deform during a crash. Crushing the metal does the work of absorbing the car's kinetic energy — work that would otherwise be done on the occupants — and the deformation lengthens the collision time. The passenger cabin, by contrast, is built to stay rigid.
- **Airbags** inflate in the first instants of a crash and bring the driver or passenger to rest over a much longer time than a hard steering wheel or dashboard would, and they spread the force over a larger area of the body.
- **Seatbelts** supply the backward force that decelerates the occupant with the car — without one, Newton's first law carries the occupant forward at the car's original speed until something else stops them. Modern seatbelts stretch slightly, adding to the stopping time.

Notice that the same change in momentum happens in every one of these cases; the safety features do not reduce the change in momentum, they spread it over more time and more distance, and that reduces the force.

**Case study 2: motion in sport.** Almost every action in sport is an application of the same toolkit.

- *Giving a ball speed.* When a boot, bat, racket or club strikes a ball, the ball's change in momentum equals the impulse $F\Delta t$ delivered to it (Section 8E). To maximise the speed of the ball, a player applies a large force and keeps contact going for as long as possible — which is what a follow-through does.

- *The power of an impact.* Because the contact time of a strike is only a few thousandths of a second, even a modest transfer of energy happens at a very high rate: $P = E / t$. Impact powers of several kilowatts are common in sport, far above what the same muscles could sustain for a whole game.
- *The efficiency of the body.* The energy delivered to a ball comes from chemical energy in the player's muscles, and only part of it ends up as kinetic energy of the ball. The efficiency of the transfer (Section 9D) is well below $100\,\%$: the rest is transformed into thermal energy in the muscles and the surroundings, and into sound and deformation at impact.
- *Landing safely.* Long-jump pits, gymnastics mats, the bending of knees on landing and the padded floors beneath a climbing wall all use the impulse idea in the same direction as airbags: a longer stopping time and distance mean a smaller force on the body.

**Case study 3: a device or a structure.** The study design also allows the case study to be a device or a structure.

- *Springs as energy stores and absorbers.* A spring buffer at the end of a rail siding, a trampoline, a pogo stick and the suspension of a car all rely on the spring as an energy store: kinetic energy or gravitational potential energy is transformed into elastic potential energy $E_s = 1/2\,k\,x^2$ (Section 9C) and then back again, or — where damping is wanted — gradually into thermal energy. Because the spring force grows with extension ( $F = k\,x$), a spring starts softly and pushes harder the more it is compressed, which makes it a gentle way to stop something.
- *Structures in equilibrium.* A shelf bracket, a ladder or a footbridge stays still because it is in both translational and rotational equilibrium (Section 8F): the forces on it sum to zero, and the torques about any point sum to zero. A case study of a structure identifies every force using the $F_{\text{on A by B}}$ convention of Section 8C and checks both conditions.

**Your own case study: an investigation framework.** The study design asks you to *investigate* the application of motion concepts through a case study, and this book's Topic Test 4 draws on this section as well. A suitable framework:

- **Aim and question.** Choose one specific, bounded situation — for example, *How does the speed of a car affect its stopping distance?* or *What average force acts on a football when it is kicked?* State the aim as a question about that one situation.
- **Identify the physics.** List which rows of the toolkit table above are involved, and which sections of this book you will draw on.
- **Build the model and state its assumptions.** Decide what to treat as constant, what to neglect and which system is (approximately) isolated — constant braking force, air resistance negligible, an ideal spring. Every assumption should be written down, because the evaluation will return to them.
- **Data.** Use values given to you or measurements you generate yourself. If you use published values, take them from a reputable organisation and record the organisation, the document and the retrieval date, as the *Source:* lines in this section do. Record everything in your logbook: the logbook is submitted as a requirement for satisfactory completion of the unit.
- **Calculate.** Apply the relevant relationships, keeping track of units, signs and significant figures as in every section since 1A.
- **Evaluate.** Return to each assumption and ask how the answer would change if the assumption were relaxed. State the limits of your model honestly.

The study design's list of suitable assessment tasks for Unit 2 Outcomes 1 and 2 includes, among others, *"an analysis, including calculations, of physics concepts applied to real-world contexts"* and *"a report of an application of physics concepts to a real-world context"* (*VCE Physics Study Design, Unit 2*). A case study of the kind built in this section suits both.

**Key ideas**

- A case study is one real situation analysed in depth: identify the concepts that apply, model the situation, calculate, then evaluate the model's assumptions.

- Braking is constant-acceleration motion and work–energy transformation at the same time; the two routes give the same stopping distance and check each other.
- Kinetic energy grows with the square of the speed, so a small increase in speed brings a larger increase in stopping distance and in the energy a crash must absorb.
- Safety features (crumple zones, airbags, seatbelts, landing mats) do not reduce the change of momentum; they spread it over more time and distance, reducing the average force ($F = m\,\Delta v / \Delta t$).
- Impacts deliver large power because the contact time is very short; the body's transfer of energy to a ball is far from $100\,\%$ efficient.
- Springs are energy stores ($E_s = 1/2\,k\,x^2$) that can absorb motion gently; structures in equilibrium satisfy both force balance and torque balance.

**Units 3 & 4 preview.** At Units 3 & 4 the same ideas are extended to motion in two dimensions (projectiles and circular motion) and to more complex momentum and energy systems. Content under this banner is not assessed in this book's topic tests or practice examinations. Within Unit 2 itself, students who choose Option 2.11 (Section 11D, ball sports) or Options 2.4 and 2.5 (Sections 11B and 11C, structures and forces on the body) continue the case-study approach with new concepts.

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## Worked examples

### *Example 1 — scaffolded*

A car of mass $1200\,\text{kg}$ travels at $50\,\text{km h}^{-1}$ along a straight suburban street. The driver brakes hard, and a constant braking force of $6000\,\text{N}$ brings the car to rest. (a) Convert the car's speed to $\text{m s}^{-1}$. (b) Calculate the kinetic energy of the car before braking. (c) Use work and energy to calculate the braking distance. (d) If the driver's reaction time is $0.80\,\text{s}$, calculate the distance travelled during the reaction stage, and hence the total stopping distance.

| Working | Comment |
|---|---|
| (a) $v = \dfrac{50}{3.6} = 13.9\,\text{m s}^{-1}$. | Divide by $3.6$ to convert $\text{km h}^{-1}$ to $\text{m s}^{-1}$. |
| (b) $E_k = 1/2\,m\,v^2 = 1/2 \times 1200 \times 13.89^2 = 1.16 \times 10^5\,\text{J}$. | Keep an extra figure before squaring; $13.89^2 = 192.9$. |
| (c) $F\,d = E_k$, so $d = \dfrac{E_k}{F} = \dfrac{1.16 \times 10^5}{6000} = 19\,\text{m}$. | The braking force does negative work that removes the car's kinetic energy (Section 9A, 9C). Check with Section 7C: $a = F/m = 5.0\,\text{m s}^{-2}$ and $s = u^2/(2a) = 192.9/10 = 19\,\text{m}$. ✓ |
| (d) Reaction distance $= v\,t = 13.9 \times 0.80 = 11\,\text{m}$; total $= 11 + 19 = 30\,\text{m}$. | During the reaction stage the velocity is constant (Newton's first law). The reaction stage adds more than half again to the braking distance. |

### *Example 2 — scaffolded*

In a crash-test laboratory, a crash-test dummy of mass $70\,\text{kg}$ is brought to rest from the car's original speed of $13.9\,\text{m s}^{-1}$ (the $50\,\text{km h}^{-1}$ of Example 1). Take the car's original direction of travel as positive. (a) Calculate the change in momentum of the dummy. (b) Calculate the average force on the dummy if it is stopped in $0.020\,\text{s}$ by a hard surface such as an unpadded dashboard. (c) Calculate the average force on the dummy if it is stopped in $0.50\,\text{s}$ by an airbag. (d) Compare the two forces and explain the difference.

| Working | Comment |
|---|---|
| (a) $\Delta p = m(v - u) = 70 \times (0 - 13.9) = -973\,\text{kg m s}^{-1}$. | The change in momentum is directed opposite to the original motion; its magnitude is $973\,\text{kg m s}^{-1}$. |
| (b) $F\,\Delta t = m\,\Delta v$, so $F = \dfrac{m\,\Delta v}{\Delta t} = \dfrac{-973}{0.020} = -4.9 \times 10^4\,\text{N}$ | Magnitude $4.9 \times 10^4\,\text{N}$, opposite to the original motion (2 significant figures). |

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| Working | Comment |
|---|---|
| (c) $F=\dfrac{-973}{0.50}=-1.9\times10^{3}\,\mathrm{N}$. | The same change of momentum, spread over a longer time. |
| (d) $\dfrac{4.9\times10^{4}}{1.9\times10^{3}}=25$: the force with the airbag is about 25 times smaller. | $m\,\Delta v$ is fixed, so $F=m\,\Delta v/\Delta t$ is inversely proportional to the stopping time. |

***Example 3 — unscaffolded***

At the MCG, a player kicks an Australian rules football of mass $0.46\,\mathrm{kg}$. The ball starts from rest and leaves the player's boot at $28\,\mathrm{m\ s^{-1}}$. The boot is in contact with the ball for $0.012\,\mathrm{s}$, and the energy supplied to the kick by the player's leg muscles is $600\,\mathrm{J}$. Calculate the average force on the ball by the boot, the kinetic energy the ball leaves with, the power of the impact, and the efficiency of the energy transfer from the leg to the ball.

The change in momentum of the ball is

$$\Delta p=m(v-u)=0.46\times(28-0)=12.88\,\mathrm{kg\ m\ s^{-1}},$$

so by $F\,\Delta t=m\,\Delta v$ the average force is

$$F=\frac{m\,\Delta v}{\Delta t}=\frac{12.88}{0.012}=1073\,\mathrm{N}\approx1.1\times10^{3}\,\mathrm{N}.$$

The kinetic energy of the ball as it leaves the boot is

$$E_k=1/2\,m\,v^2=1/2\times0.46\times28^2=180\,\mathrm{J}.$$

That energy is delivered in only $0.012\,\mathrm{s}$, so the power of the impact is

$$P=\frac{E}{t}=\frac{180}{0.012}=1.5\times10^{4}\,\mathrm{W},$$

fifteen kilowatts — very much larger than the power the same muscles could sustain for a whole match, because the contact time is so short. Finally, the efficiency of the transfer from the leg's $600\,\mathrm{J}$ to the ball's kinetic energy is

$$\eta=\frac{\text{useful energy out}}{\text{total energy in}}=\frac{180}{600}=0.30,$$

or $30\,\%$. The remaining $70\,\%$ is transformed mainly into thermal energy in the muscles and into sound and deformation at impact.

∴ the average force is $1.1\times10^{3}\,\mathrm{N}$, the ball leaves with $180\,\mathrm{J}$ of kinetic energy, the impact power is $1.5\times10^{4}\,\mathrm{W}$, and the kick is $30\,\%$ efficient.

***Example 4 — unscaffolded***

At the end of a rail siding, a wagon of mass $3.0\times10^{3}\,\mathrm{kg}$ rolls at $2.0\,\mathrm{m\ s^{-1}}$ into a spring buffer and is brought to rest after the buffer compresses $0.20\,\mathrm{m}$. Treat the buffer as an ideal spring and neglect friction. Calculate the spring constant of the buffer and the maximum force the buffer exerts on the wagon, and compare the maximum force with the average force during the compression.

The wagon's kinetic energy just before contact is

$$E_k=1/2\,m\,v^2=1/2\times3.0\times10^{3}\times2.0^2=6000\,\mathrm{J}.$$

All of it is transformed into elastic potential energy in the buffer (Section 9C), so with $E_s=1/2\,k\,x^2$ and $x=0.20\,\mathrm{m}$,

$$k=\frac{2E_s}{x^2}=\frac{2\times6000}{0.20^2}=\frac{12\,000}{0.040}=3.0\times10^{5}\,\mathrm{N\ m^{-1}}.$$

The maximum force occurs at maximum compression, when by Hooke's law (Section 9B)

$$F_{max} = k\,x = 3.0 \times 10^5 \times 0.20 = 6.0 \times 10^4\,\mathrm{N}.$$

At that instant the wagon's deceleration is $a = F_{max}/m = 20\,\mathrm{m\ s^{-2}}$, about twice $g$. The *average* force during the compression is smaller, because the spring force grows from zero as the buffer compresses. Using work, $F_{avg}\,x = E_k$, so

$$F_{avg} = \frac{E_k}{x} = \frac{6000}{0.20} = 3.0 \times 10^4\,\mathrm{N},$$

exactly half the maximum force. For an ideal spring, the average stopping force is always half the maximum, which is one reason a spring stops things more gently than a rigid barrier does.

$\therefore$ $k = 3.0 \times 10^5\,\mathrm{N\ m^{-1}}$, $F_{max} = 6.0 \times 10^4\,\mathrm{N}$, and the average force is $3.0 \times 10^4\,\mathrm{N}$, half the maximum.

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### Practice questions

**Question 1** A car of mass $1000\,\mathrm{kg}$ travels at $50\,\mathrm{km\ h^{-1}}$ along a straight road. The driver brakes hard, and a constant braking force of $4000\,\mathrm{N}$ brings the car to rest. (a) Convert the car's speed to $\mathrm{m\ s^{-1}}$. (b) Calculate the kinetic energy of the car before braking. (c) Calculate the braking distance. (d) The driver's reaction time is $1.0\,\mathrm{s}$. Calculate the total stopping distance from the moment the driver sees the hazard.

**Question 2** A car of mass $1200\,\mathrm{kg}$ travelling at $15\,\mathrm{m\ s^{-1}}$ crashes head-on into a rigid barrier and is brought to rest. (a) Calculate the kinetic energy of the car just before impact. (b) State the total work done on the car by the barrier in bringing it to rest. (c) The car's crumple zone deforms by $0.50\,\mathrm{m}$ during the crash. Calculate the average force on the car by the barrier. (d) An older car with a rigid front end would have stopped over only $0.05\,\mathrm{m}$. Calculate the average force in that case, and comment on the comparison.

**Question 3** A golfer drives a ball of mass $0.046\,\mathrm{kg}$ from rest. The ball leaves the club at $70\,\mathrm{m\ s^{-1}}$, and the club is in contact with the ball for $0.0005\,\mathrm{s}$. The kinetic energy of the club head just before impact is $250\,\mathrm{J}$. (a) Calculate the change in momentum of the ball. (b) Calculate the average force on the ball by the club. (c) Calculate the kinetic energy of the ball as it leaves the club. (d) Calculate the efficiency of the energy transfer from the club head to the ball.

**Question 4** A gymnast of mass $50\,\mathrm{kg}$ falls from rest from a height of $1.6\,\mathrm{m}$ onto a thick foam mat, landing upright on her feet. The mat compresses $0.80\,\mathrm{m}$ as it brings her to rest. (a) Calculate the kinetic energy of the gymnast at the instant she first touches the mat. (b) Calculate her speed at that instant. (c) Calculate the average upward force on the gymnast by the mat during the compression. (d) If the same fall ended on a hard floor that stopped her in $0.10\,\mathrm{m}$, the average force would be about $8.3 \times 10^3\,\mathrm{N}$. Use your answer to (c) to explain why mats reduce the risk of injury.

**Question 5** A car of mass $900\,\mathrm{kg}$ brakes with the same constant braking force of $4500\,\mathrm{N}$ from each of two speeds, $50\,\mathrm{km\ h^{-1}}$ and $60\,\mathrm{km\ h^{-1}}$. (a) Calculate the kinetic energy of the car at $50\,\mathrm{km\ h^{-1}}$. (b) Calculate the kinetic energy of the car at $60\,\mathrm{km\ h^{-1}}$. (c) Calculate the two braking distances and their ratio. (d) In Victoria, the speed limit around schools is $40\,\mathrm{km\ h^{-1}}$ when children are arriving and leaving. Using your results and the kinetic energy at $40\,\mathrm{km\ h^{-1}}$, justify this lower limit.

*Source: VicRoads, Speed limits, retrieved 2026-08-18.*

**Question 6** A shelf of mass $1.5\,\mathrm{kg}$ is held horizontally by a bracket fixed to a wall. Books of total mass $4.5\,\mathrm{kg}$ rest on the shelf. (a) Calculate the total force due to gravity on the shelf and books. (b) State the total upward support force the bracket must provide for the shelf to be in translational equilibrium. (c) With reference to $\tau = r_{\perp} F$ (Section 8F), explain why the bracket is less likely to fail if the books are placed close to the wall rather than at the outer edge of the shelf.

**Question 7** A cyclist can sustain a power output of $250\,\mathrm{W}$. Calculate the energy she transfers in $1.0\,\mathrm{hour}$ of riding, and hence the height through which that energy could raise the cyclist and her bicycle, of total mass $85\,\mathrm{kg}$, if it were all converted into gravitational potential energy. Comment on your result.

**Question 8** A raw egg dropped onto a concrete floor breaks, but the same egg dropped from the same height onto a pillow survives. Use the impulse relationship $F\,\Delta t = m\,\Delta v$ to explain why.

**Question 9** A V/Line commuter train of mass $1.2 \times 10^{5}\,\mathrm{kg}$ approaches Southern Cross Station at $25\,\mathrm{m\ s^{-1}}$. The brakes apply a constant force of $8.0 \times 10^{4}\,\mathrm{N}$. (a) Calculate the kinetic energy of the train. (b) Calculate the distance the train needs to stop. (c) Calculate the time taken to stop.

**Question 10** A student reads that a car's crumple zone must absorb the car's kinetic energy whatever its design, and concludes: "Crumple zones make no difference to the occupants — the same energy has to go somewhere either way." Identify what is correct in this statement, and use physics to explain why the conclusion is wrong.

**Question 11** A squash ball is dropped from a height of $1.8\,\mathrm{m}$ and rebounds to a height of $1.2\,\mathrm{m}$. Treat the rebound height as the useful energy output of the bounce, and calculate the efficiency of the bounce. State where the remaining energy goes.

**Question 12** Two roller-coaster cars, one carrying a single rider and one carrying four, are released from rest at the top of the same first hill. A student predicts that the heavily loaded car will be moving faster at the bottom, because it has more gravitational potential energy. Use energy conservation to evaluate this prediction, stating any assumption you make.

**Question 13** A stunt performer of mass $60\,\mathrm{kg}$ falls from rest through $10\,\mathrm{m}$ into a safety net. The net stretches $2.0\,\mathrm{m}$ as it brings the performer to rest. Calculate the average upward force on the performer by the net, and compare it with the force due to gravity on the performer.

**Question 14** A proposal suggests raising the default built-up area speed limit in Victoria from $50\,\mathrm{km\ h^{-1}}$ to $60\,\mathrm{km\ h^{-1}}$ so that trips are shorter. Using the physics of kinetic energy and stopping distance, evaluate this proposal. (You do not need factors other than physics.)

**Question 15** A crane lifts a load of mass $400\,\mathrm{kg}$ vertically through $12\,\mathrm{m}$ in $60\,\mathrm{s}$ at constant velocity. Calculate the energy transferred to the load's gravitational potential energy, and the power of the crane during the lift.

**Question 16** Bicycle helmets contain a foam liner designed to crush during a crash, and manufacturers state that a helmet must be replaced after any significant impact. Use the language of work and energy to explain both how the liner protects the head and why it works only once.

**Question 17** A jumper of mass $45\,\mathrm{kg}$ falls from rest through $1.2\,\mathrm{m}$ onto a trampoline. The trampoline surface stretches $0.60\,\mathrm{m}$ as it brings the jumper to rest at the lowest point. Modelling the trampoline as a single ideal spring, calculate its effective spring constant.

**Question 18** In a case study of car braking, a student makes three modelling assumptions: the braking force is constant, air resistance is negligible, and during any collision the vehicles form an isolated system. For each assumption, explain why it is made and how relaxing it would affect the student's results.

## Answers

**Question 1** (a) $13.9\,\mathrm{m\,s^{-1}}$; (b) $9.6\times10^4\,\mathrm{J}$; (c) $24\,\mathrm{m}$; (d) $38\,\mathrm{m}$. **Question 2** (a) $1.35\times10^5\,\mathrm{J}$; (b) $-1.35\times10^5\,\mathrm{J}$; (c) $2.7\times10^5\,\mathrm{N}$; (d) $2.7\times10^6\,\mathrm{N}$, ten times larger; see solution. **Question 3** (a) $3.2\,\mathrm{kg\,m\,s^{-1}}$; (b) $6.4\times10^3\,\mathrm{N}$; (c) $1.1\times10^2\,\mathrm{J}$; (d) $0.45$ ($45\,\%$). **Question 4** (a) $784\,\mathrm{J}$; (b) $5.6\,\mathrm{m\,s^{-1}}$; (c) $1.5\times10^3\,\mathrm{N}$; (d) see solution. **Question 5** (a) $8.7\times10^4\,\mathrm{J}$; (b) $1.3\times10^5\,\mathrm{J}$; (c) $19\,\mathrm{m}$ and $28\,\mathrm{m}$, ratio $1.4$; (d) see solution. **Question 6** (a) $59\,\mathrm{N}$; (b) $59\,\mathrm{N}$ upward; (c) see solution. **Question 7** $9.0\times10^5\,\mathrm{J}$; $1.1\times10^3\,\mathrm{m}$; see solution. **Question 8** See solution. **Question 9** (a) $3.75\times10^7\,\mathrm{J}$; (b) $470\,\mathrm{m}$; (c) $38\,\mathrm{s}$. **Question 10** See solution. **Question 11** $0.67$ ($67\,\%$); the rest is transformed mainly into thermal energy in the ball. **Question 12** See solution. **Question 13** $3.5\times10^3\,\mathrm{N}$; six times the force due to gravity ($588\,\mathrm{N}$). **Question 14** See solution. **Question 15** $4.7\times10^4\,\mathrm{J}$; $7.8\times10^2\,\mathrm{W}$. **Question 16** See solution. **Question 17** $4.4\times10^3\,\mathrm{N\,m^{-1}}$. **Question 18** See solution.

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## Fully-worked solutions

**Question 1** $m=1000\,\mathrm{kg}$, braking force $F=4000\,\mathrm{N}$. (a) $v=\dfrac{50}{3.6}=13.9\,\mathrm{m\,s^{-1}}$. (b) Keeping an extra figure, $v=13.89\,\mathrm{m\,s^{-1}}$: $E_k=1/2\,m v^2=1/2\times1000\times13.89^2=1/2\times1000\times192.9=96\,450\,\mathrm{J}\approx9.6\times10^4\,\mathrm{J}$. (c) The braking force does the work of removing this kinetic energy: $F d=E_k$, so $d=\dfrac{96\,450}{4000}=24.1\,\mathrm{m}\approx24\,\mathrm{m}$ (2 significant figures). (d) During the reaction stage the car moves at constant velocity: distance $=v t=13.9\times1.0=13.9\,\mathrm{m}\approx14\,\mathrm{m}$. Total stopping distance $=14+24=38\,\mathrm{m}$.

**Question 2** $m=1200\,\mathrm{kg}$, $u=15\,\mathrm{m\,s^{-1}}$, $v=0$. (a) $E_k=1/2\,m u^2=1/2\times1200\times15^2=1/2\times1200\times225=135\,000\,\mathrm{J}=1.35\times10^5\,\mathrm{J}$. (b) The barrier's force acts opposite to the car's displacement ($\theta=180^\circ$), so the work done on the car by the barrier is negative: $W=-1.35\times10^5\,\mathrm{J}$. Its magnitude equals the kinetic energy removed. (c) With the energy removed over $0.50\,\mathrm{m}$: $F=\dfrac{E_k}{d}=\dfrac{1.35\times10^5}{0.50}=2.7\times10^5\,\mathrm{N}$. (d) Stopping over $0.05\,\mathrm{m}$ instead: $F=\dfrac{1.35\times10^5}{0.05}=2.7\times10^6\,\mathrm{N}$, ten times larger. The same kinetic energy removed over one-tenth of the distance requires ten times the average force. A crumple zone lengthens the stopping distance (and time), which is exactly what reduces the force on the car — and on its occupants.

**Question 3** $m=0.046\,\mathrm{kg}$, $u=0$, $v=70\,\mathrm{m\,s^{-1}}$, $\Delta t=0.0005\,\mathrm{s}$, club-head energy $250\,\mathrm{J}$. (a) $\Delta p=m(v-u)=0.046\times70=3.22\,\mathrm{kg\,m\,s^{-1}}\approx3.2\,\mathrm{kg\,m\,s^{-1}}$ in the direction the ball leaves. (b) $F=\dfrac{m\,\Delta v}{\Delta t}=\dfrac{3.22}{0.0005}=6440\,\mathrm{N}\approx6.4\times10^3\,\mathrm{N}$. (c) $E_k=1/2\,m v^2=1/2\times0.046\times70^2=1/2\times0.046\times4900=112.7\,\mathrm{J}\approx1.1\times10^2\,\mathrm{J}$. (d) $\eta=\dfrac{\text{useful energy out}}{\text{total energy in}}=\dfrac{112.7}{250}=0.45$, so $45\,\%$ of the club head's kinetic energy becomes kinetic energy of the ball; the rest is transformed into thermal energy, sound and deformation of ball and club face.

**Question 4** $m=50\,\mathrm{kg}$, fall $h=1.6\,\mathrm{m}$ before contact, mat compression $0.80\,\mathrm{m}$. (a) By conservation of energy (Section 9C), the gravitational potential energy lost in the $1.6\,\mathrm{m}$ fall becomes kinetic energy: $E_k=m g\,\Delta h=50\times9.8\times1.6=784\,\mathrm{J}$. (b) From $E_k=1/2\,m v^2$: $v=\sqrt{\dfrac{2E_k}{m}}=\sqrt{\dfrac{2\times784}{50}}=\sqrt{31.36}=5.6\,\mathrm{m\,s^{-1}}$, downward. (Equivalently, $v^2=u^2+2as$ with $a=g$ gives $v^2=2\times9.8\times1.6=31.36$.) (c) Measure heights from the lowest point. The gymnast loses gravitational potential energy over the full $1.6+0.80=2.4\,\mathrm{m}$, and starts and finishes at rest, so all of it is removed by the mat's upward force over the $0.80\,\mathrm{m}$ compression:

$$F\times0.80=m g\times2.4=50\times9.8\times2.4=1176\,\mathrm{J},\ F=\frac{1176}{0.80}=1470\,\mathrm{N}\approx1.5\times10^3\,\mathrm{N}.$$

(d) The mat's average force ($1470\,\mathrm{N}$) is about six times smaller than the hard floor's ($8.3\times10^3\,\mathrm{N}$). Both surfaces must remove the same energy, but the mat removes it over a longer distance and time, so the average force is smaller. A smaller force on the body means less risk of injury.

**Question 5** $m=900\,\mathrm{kg}$, braking force $F=4500\,\mathrm{N}$. (a) $50\,\mathrm{km\ h^{-1}}=13.89\,\mathrm{m\ s^{-1}}$: $E_k=1/2\times900\times13.89^2=450\times192.9=86\,805\,\mathrm{J}\approx8.7\times10^4\,\mathrm{J}$. (b) $60\,\mathrm{km\ h^{-1}}=16.67\,\mathrm{m\ s^{-1}}$: $E_k=1/2\times900\times16.67^2=450\times277.9=125\,055\,\mathrm{J}\approx1.3\times10^5\,\mathrm{J}$. (c) $d=\frac{E_k}{F}$: at $50\,\mathrm{km\ h^{-1}}$, $d=\frac{86\,805}{4500}=19.3\,\mathrm{m}$; at $60\,\mathrm{km\ h^{-1}}$, $d=\frac{125\,055}{4500}=27.8\,\mathrm{m}$. The ratio is $\frac{27.8}{19.3}=1.44$, which equals $\left(\frac{60}{50}\right)^2$: for the same braking force, stopping distance grows with the square of the speed. (d) At $40\,\mathrm{km\ h^{-1}}=11.11\,\mathrm{m\ s^{-1}}$: $E_k=450\times11.11^2=450\times123.4=55\,530\,\mathrm{J}\approx5.6\times10^4\,\mathrm{J}$ and $d=\frac{55\,530}{4500}=12.3\,\mathrm{m}$. Compared with $50\,\mathrm{km\ h^{-1}}$, the braking distance is about $7\,\mathrm{m}$ shorter — roughly one and a half car lengths — and a crash at this speed has only about two-thirds of the kinetic energy to absorb. Around schools, where children may step onto the road with little warning, both the shorter stopping distance and the lower impact energy justify the $40\,\mathrm{km\ h^{-1}}$ limit.

**Question 6** Shelf $1.5\,\mathrm{kg}$, books $4.5\,\mathrm{kg}$; total mass $6.0\,\mathrm{kg}$. (a) $F_g=mg=6.0\times9.8=58.8\,\mathrm{N}\approx59\,\mathrm{N}$, directed downward, acting at the centre of mass of the shelf-and-books system. (b) For translational equilibrium the forces sum to zero, so the total upward support force on the shelf by the bracket equals the total force due to gravity: $59\,\mathrm{N}$ upward. (c) About the bracket's fixing point, the forces due to gravity on the shelf and books exert a torque $\tau=r_\perp F$, where $r_\perp$ is the perpendicular distance from the fixing to the line of action of the force. Books placed at the outer edge of the shelf have a larger $r_\perp$ and so exert a larger torque; the bracket must then supply a larger counter-torque to keep the shelf in rotational equilibrium (Section 8F). Placing the books close to the wall makes $r_\perp$ small, the torque is smaller, and the bracket is less likely to fail.

**Question 7** $P=250\,\mathrm{W}$, $t=1.0\,\mathrm{h}=3600\,\mathrm{s}$, $m=85\,\mathrm{kg}$. The energy transferred is $E=Pt=250\times3600=900\,000\,\mathrm{J}=9.0\times10^5\,\mathrm{J}$. Setting this equal to $E_g=mg\Delta h$:

$$\Delta h=\frac{E}{mg}=\frac{9.0\times10^5}{85\times9.8}=\frac{9.0\times10^5}{833}=1080\,\mathrm{m}\approx1.1\times10^3\,\mathrm{m}.$$

This is the height of a large mountain: a sustained human power output is modest ($250\,\mathrm{W}$ is comparable with a few bright incandescent light bulbs), but applied for an hour it transfers a great deal of energy. In practice most of the cyclist's energy is used against resistance forces, not climbing.

**Question 8** The egg's change in momentum — from its speed just before impact to rest — is the same whichever surface stops it, since both falls start from the same height. Since $F=\frac{m\Delta v}{\Delta t}$, the average force depends on the stopping time. The pillow compresses under the egg, bringing it to rest over a much longer time (and distance) than the unyielding concrete. The larger $\Delta t$ gives a much smaller average force, and a force below what the shell can withstand does not break it. The concrete stops the egg in a very short time, requiring a force larger than the shell can support.

**Question 9** $m=1.2\times10^5\,\mathrm{kg}$, $u=25\,\mathrm{m\ s^{-1}}$, braking force $F=8.0\times10^4\,\mathrm{N}$. (a) $E_k=1/2\,mu^2=1/2\times1.2\times10^5\times25^2=1/2\times1.2\times10^5\times625=3.75\times10^7\,\mathrm{J}$. (b) The braking force removes this energy over the stopping distance: $Fd=E_k$, so $d=\frac{3.75\times10^7}{8.0\times10^4}=468.75\,\mathrm{m}\approx470\,\mathrm{m}$ (2 significant figures). (c) By Newton's second law the deceleration is $a=\frac{F}{m}=\frac{8.0\times10^4}{1.2\times10^5}=0.667\,\mathrm{m\ s^{-2}}$. With constant acceleration, $v=u+at$ and $v=0$: $t=\frac{u}{a}=\frac{25}{0.667}=37.5\,\mathrm{s}\approx38\,\mathrm{s}$. (Check: at the average speed of $12.5\,\mathrm{m\ s^{-1}}$ for $37.5\,\mathrm{s}$, the train covers $469\,\mathrm{m}$. ✓)

**Question 10** What is correct: the car's initial kinetic energy must be removed whatever the car's design, and in both cases it is transformed mainly into thermal energy and the energy of deformation. Energy accounting alone really is the same. The conclusion is wrong because injury depends on *force*, not on the total energy. By $W = F s$, removing the same energy over a longer distance requires a smaller average force, and by $F \Delta t = m \Delta v$, spreading the same momentum change over a longer time also reduces the force. A crumple zone is designed to deform, so it lengthens the stopping distance and time, and the occupants experience a much smaller force than they would against a rigid front end that stops almost instantly. The crumple zone also decides *where* the deformation energy goes: into crushing cheap metal, rather than into the rigid passenger cabin and its occupants. Same energy — very different force, and very different outcome.

**Question 11** Taking the ground as the zero of height, the ball starts with gravitational potential energy $E_g = m g h_1$ at $h_1 = 1.8\,\mathrm{m}$ and rebounds to $E_g = m g h_2$ at $h_2 = 1.2\,\mathrm{m}$, so

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}} = \frac{m g h_2}{m g h_1} = \frac{h_2}{h_1} = \frac{1.2}{1.8} = 0.67\,,$$

or $67\,\%$. The mass cancels: the efficiency of the bounce depends only on the two heights. The remaining $33\,\%$ of the energy is transformed mainly into thermal energy in the ball (squash balls are well known for warming up during play), with small amounts into sound and into deforming the ball and the floor.

**Question 12** The prediction is wrong, provided friction and air resistance are neglected. At the top, each car has $E_g = m g \Delta h$; at the bottom this has become $E_k = 1/2\, m v^2$. Setting them equal:

$$m g \Delta h = 1/2\, m v^2 \Rightarrow v = \sqrt{2 g \Delta h}\,.$$

The mass cancels, so both cars reach the bottom with the same speed. The loaded car does indeed have more gravitational potential energy — but it also has proportionally more mass to accelerate, and the two effects exactly cancel. (With friction present, the result is only approximate; real roller-coaster designers must account for friction and air resistance, which is why the model's assumptions matter.)

**Question 13** $m = 60\,\mathrm{kg}$; the performer falls $10\,\mathrm{m}$ and the net stretches a further $2.0\,\mathrm{m}$, so the total fall to the lowest point is $12\,\mathrm{m}$. The performer starts and finishes at rest, so the gravitational potential energy lost over the full $12\,\mathrm{m}$ is removed by the net's upward force over the $2.0\,\mathrm{m}$ stretch:

$$F \times 2.0 = m g \times 12 = 60 \times 9.8 \times 12 = 7056\,\mathrm{J}\,, F = \frac{7056}{2.0} = 3528\,\mathrm{N} \approx 3.5 \times 10^3\,\mathrm{N}\,.$$

The force due to gravity on the performer is $F_g = m g = 60 \times 9.8 = 588\,\mathrm{N}$, so the net's average force is six times the force due to gravity. Large as it is, this force is spread over the whole body and applied over $2.0\,\mathrm{m}$; a hard floor would supply the same change of momentum over a few centimetres, at a force large enough to cause injury.

**Question 14** The evaluation should weigh the kinetic-energy and stopping-distance consequences of the higher speed. Kinetic energy grows with the square of the speed: at $60\,\mathrm{km\,h^{-1}}$ a car has $\left(\frac{60}{50}\right)^2 = 1.44$ times the kinetic energy it has at $50\,\mathrm{km\,h^{-1}}$ — $44\,\%$ more. For the same braking force the stopping distance grows by the same factor (Question 5), so a driver who could stop in $19\,\mathrm{m}$ from $50\,\mathrm{km\,h^{-1}}$ needs about $28\,\mathrm{m}$ from $60\,\mathrm{km\,h^{-1}}$: roughly two car lengths more road in which to avoid a hazard. In a crash, $44\,\%$ more kinetic energy must be absorbed by crumple zones, seatbelts, airbags and bodies that are no stronger than before, so the average forces and the risk of injury rise. The trip-time saving from the extra $10\,\mathrm{km\,h^{-1}}$ is small over short built-up distances, while the physics costs — longer stopping distances and more energetic crashes — apply to every vehicle on every trip. On physics grounds alone, the proposal is not supported.

**Question 15** $m = 400\,\mathrm{kg}$, $\Delta h = 12\,\mathrm{m}$, $t = 60\,\mathrm{s}$, constant velocity. The energy transferred to gravitational potential energy is

$$E_g = m g \Delta h = 400 \times 9.8 \times 12 = 47\,040\,\mathrm{J} \approx 4.7 \times 10^4\,\mathrm{J}\,.$$

The power is the rate of energy transfer:

$$P=\frac{E}{t}=\frac{47\,040}{60}=784\text{ W}\approx 7.8\times 10^{2}\text{ W}.$$

(Constant velocity means the crane's upward force equals the force due to gravity on the load; no energy goes into kinetic energy.)

**Question 16** In a crash the head has kinetic energy that must be removed before the head stops. The foam liner removes it by doing work over a distance: as the liner crushes, the force on the head acts through the crushing distance, $W=F\,s$, transforming the head's kinetic energy into the energy of deformation of the foam and into thermal energy. Because the crushing distance is comparatively long, the average force on the skull is much smaller than it would be against a hard surface that stops the head in a few millimetres; and since $F\,\Delta t=m\,\Delta v$, the longer stopping time likewise reduces the force. The liner works only once because the foam is crushed permanently: the energy has gone into rearranging the foam's structure, and a crushed liner has no crushing distance left to give. A second impact would stop the head over a much shorter distance, at a much larger force — which is why the helmet must be replaced.

**Question 17** $m=45\text{ kg}$; fall $1.2\text{ m}$ before contact, then a further stretch $x=0.60\text{ m}$ to the lowest point, where the jumper is momentarily at rest. Measure heights from the lowest point. The gravitational potential energy lost over the full $1.2+0.60=1.8\text{ m}$ has all become elastic potential energy in the trampoline:

$$m\,g\left(h+x\right)=1/2\,k\,x^{2},$$

$$k=\frac{2\,m\,g\left(h+x\right)}{x^{2}}=\frac{2\times 45\times 9.8\times 1.8}{0.60^{2}}=\frac{1587.6}{0.36}=4410\text{ N m}^{-1}\approx 4.4\times 10^{3}\text{ N m}^{-1}.$$

(Check of the model's scale: at maximum stretch the trampoline's force is $k\,x=4410\times 0.60=2646\text{ N}$, about six times the jumper's force due to gravity of $441\text{ N}$ — consistent with the force ratios found for other soft landings in this section.)

**Question 18** *Constant braking force.* Made so that a single force can be used in $W=F\,d$ and so the constant-acceleration equations of Section 7C apply. In reality the braking force builds up as the brakes are applied and changes as the tyres and road warm. Relaxing the assumption would make the true stopping distance slightly different from the calculated one — typically a little shorter at first, while the brakes build to full force — and the motion would no longer have a single constant acceleration. *Air resistance negligible.* Made so that the net force opposing the motion is just the braking force, keeping $F_{\text{net}}$ constant. At urban speeds air resistance is small compared with a hard braking force, so the assumption is a good one. Relaxing it would add a speed-dependent opposing force, making the car stop slightly sooner than the model predicts — the difference is small at $50\text{ km h}^{-1}$ and larger at highway speeds. *Isolated system during the collision.* Made so that conservation of momentum can be applied: external forces such as friction and air resistance have almost no time to act during the brief contact, as in Section 8E. Relaxing it means the total momentum of the vehicles is only approximately conserved; the approximation is excellent for the collision itself but would matter over the longer sliding phase that follows. In each case the assumption keeps the model solvable with the tools of this chapter, and the evaluation's job is to say how the answer would shift when it is relaxed.

## Topic 4 Test A — Motion, forces and energy (multiple choice)

*Time allowed: 25 minutes · 20 marks · One scientific calculator permitted · The formula sheet may be used. The mark total and time allowed follow ClassBasics house convention; the VCE Physics Units 1 & 2 Study Design fixes no mark allocations for school-based assessment.*

*Choose the response that is correct or that best answers the question. A correct answer scores 1 mark; an incorrect answer scores 0. Marks are not deducted for incorrect answers. No marks are given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this test are not drawn to scale. Where required, take the gravitational field strength to be* $g = 9.8\,\mathrm{N\ kg^{-1}}$.

**Question 1**

Which one of the following rows contains only vector quantities?

**A.** distance, speed, energy

**B.** displacement, velocity, force

**C.** momentum, time, mass

**D.** acceleration, temperature, velocity

**Question 2**

An object moves in a straight line. Its velocity decreases uniformly from $+8.0\,\mathrm{m\ s^{-1}}$ at time $t = 0$ to zero at $t = 4.0\,\mathrm{s}$. The displacement of the object over this interval is

**A.** $2.0\,\mathrm{m}$

**B.** $32\,\mathrm{m}$

**C.** $16\,\mathrm{m}$

**D.** $0\,\mathrm{m}$

**Question 3**

A motorcycle travelling at $4.0\,\mathrm{m\ s^{-1}}$ accelerates uniformly at $2.5\,\mathrm{m\ s^{-2}}$ for $6.0\,\mathrm{s}$. Its final speed is

**A.** $15.0\,\mathrm{m\ s^{-1}}$

**B.** $19.0\,\mathrm{m\ s^{-1}}$

**C.** $23.0\,\mathrm{m\ s^{-1}}$

**D.** $11.5\,\mathrm{m\ s^{-1}}$

**Question 4**

A cyclist travelling at $5.0\,\mathrm{m\ s^{-1}}$ accelerates uniformly at $1.5\,\mathrm{m\ s^{-2}}$ over a distance of $48\,\mathrm{m}$. The cyclist's final speed is

**A.** $13.0\,\mathrm{m\ s^{-1}}$

**B.** $12.0\,\mathrm{m\ s^{-1}}$

**C.** $169\,\mathrm{m\ s^{-1}}$

**D.** $9.8\,\mathrm{m\ s^{-1}}$

**Question 5**

A roof tile slips from rest off a roof and falls freely for $3.0\,\text{s}$. Ignoring air resistance, the distance the tile falls is closest to

**A.** $29\,\text{m}$

**B.** $44\,\text{m}$

**C.** $15\,\text{m}$

**D.** $88\,\text{m}$

**Question 6**

Which one of the following objects has the greatest momentum?

**A.** an object of mass $0.50\,\text{kg}$ moving at $8.0\,\text{m s}^{-1}$

**B.** an object of mass $2.0\,\text{kg}$ moving at $3.0\,\text{m s}^{-1}$

**C.** an object of mass $5.0\,\text{kg}$ moving at $1.5\,\text{m s}^{-1}$

**D.** an object of mass $10\,\text{kg}$ moving at $0.60\,\text{m s}^{-1}$

**Question 7**

A constant net force of $12\,\text{N}$ acts on a $3.0\,\text{kg}$ object for $2.0\,\text{s}$. The change in momentum of the object is

**A.** $12\,\text{N s}$

**B.** $6.0\,\text{N s}$

**C.** $36\,\text{N s}$

**D.** $24\,\text{N s}$

**Question 8**

A laptop rests on a desk. One of the forces acting on the laptop is the normal force on the laptop by the desk. The Newton's-third-law partner of this force is

**A.** the force due to gravity on the laptop by the Earth.

**B.** the force due to gravity on the Earth by the laptop.

**C.** the force on the desk by the laptop.

**D.** the normal force on the Earth by the desk.

**Question 9**

A $5.0\,\text{kg}$ box is pushed along a horizontal floor. The push has magnitude $30\,\text{N}$, and the friction force opposing the motion is $10\,\text{N}$. The acceleration of the box is

**A.** $6.0\,\text{m s}^{-2}$

**B.** $8.0\,\text{m s}^{-2}$

**C.** $4.0\,\text{m s}^{-2}$

**D.** $2.0\,\text{m s}^{-2}$

**Question 10**

The force due to gravity acting on a student of mass $60\,\mathrm{kg}$ at Earth's surface is closest to

**A.** $600\,\mathrm{N}$

**B.** $588\,\mathrm{N}$

**C.** $6.1\,\mathrm{N}$

**D.** $5880\,\mathrm{N}$

**Question 11**

A $5.0\,\mathrm{kg}$ box rests on a horizontal floor. The magnitude of the normal force acting on the box is

**A.** $0\,\mathrm{N}$

**B.** $5.0\,\mathrm{N}$

**C.** $98\,\mathrm{N}$

**D.** $49\,\mathrm{N}$

**Question 12**

Two forces act on a ring at right angles to each other: $3.0\,\mathrm{N}$ toward the north and $4.0\,\mathrm{N}$ toward the east. The magnitude of the resultant force is

**A.** $7.0\,\mathrm{N}$

**B.** $5.0\,\mathrm{N}$

**C.** $1.0\,\mathrm{N}$

**D.** $3.5\,\mathrm{N}$

**Question 13**

A $4.0\,\mathrm{kg}$ parcel slides along a smooth bench at $3.0\,\mathrm{m\,s^{-1}}$ and lands in a stationary $8.0\,\mathrm{kg}$ box. The parcel and the box move together. Their common speed is closest to

**A.** $1.5\,\mathrm{m\,s^{-1}}$

**B.** $3.0\,\mathrm{m\,s^{-1}}$

**C.** $1.0\,\mathrm{m\,s^{-1}}$

**D.** $0.50\,\mathrm{m\,s^{-1}}$

**Question 14**

A hockey ball of mass $0.16\,\mathrm{kg}$ travelling at $15\,\mathrm{m\,s^{-1}}$ is brought to rest by a goalkeeper's stick in $0.050\,\mathrm{s}$. The magnitude of the average force on the ball by the stick is

**A.** $2.4\,\mathrm{N}$

**B.** $48\,\mathrm{N}$

**C.** $0.48\,\mathrm{N}$

**D.** $480\,\mathrm{N}$

**Question 15**

A farmer pushes on a farm gate with a force of $20\,\mathrm{N}$, perpendicular to the gate, at a point $1.5\,\mathrm{m}$ from the hinges. The torque about the hinges is

**A.** $30\,\mathrm{N\ m}$

**B.** $13\,\mathrm{N\ m}$

**C.** $20\,\mathrm{N\ m}$

**D.** $15\,\mathrm{N\ m}$

**Question 16**

An object is in static equilibrium. Which one of the following must be true?

**A.** The net force on the object is zero and the net torque on the object is zero.

**B.** The net force on the object is zero, but the net torque may be non-zero.

**C.** No forces act on the object.

**D.** All the forces acting on the object have the same magnitude.

**Question 17**

A child pulls a garden wagon $20\,\mathrm{m}$ along level ground. The force in the handle is $50\,\mathrm{N}$, directed at $30^{\circ}$ above the horizontal. The work done on the wagon by this force is closest to

**A.** $1000\,\mathrm{J}$

**B.** $500\,\mathrm{J}$

**C.** $577\,\mathrm{J}$

**D.** $866\,\mathrm{J}$

**Question 18**

A spring of spring constant $400\,\mathrm{N\ m^{-1}}$ is compressed by $0.050\,\mathrm{m}$. The elastic potential energy stored in the spring is

**A.** $0.50\,\mathrm{J}$

**B.** $1.0\,\mathrm{J}$

**C.** $10\,\mathrm{J}$

**D.** $20\,\mathrm{J}$

**Question 19**

A marble is released from rest at a height of $0.80\,\mathrm{m}$ above the bottom of a smooth curved track. Ignoring friction, the speed of the marble at the bottom of the track is closest to

**A.** $2.8\,\mathrm{m\ s^{-1}}$

**B.** $15.7\,\mathrm{m\ s^{-1}}$

**C.** $19.6\,\mathrm{m\ s^{-1}}$

**D.** $4.0\,\mathrm{m\ s^{-1}}$

**Question 20**

A worker pushes a load up a ramp. The worker does $4000\,\mathrm{J}$ of work on the load, and the load gains $3200\,\mathrm{J}$ of gravitational potential energy. The efficiency of the ramp as a machine is

**A.** $20\,\%$

**B.** $25\,\%$

**C.** $80\,\%$

**D.** $125\,\%$

*End of Topic 4 Test A*

## Topic 4 Test A — solutions

| Question | Answer | Question | Answer |
|---|---|---|---|
| 1 | B | 11 | D |
| 2 | C | 12 | B |
| 3 | B | 13 | C |
| 4 | A | 14 | B |
| 5 | B | 15 | A |
| 6 | C | 16 | A |
| 7 | D | 17 | D |
| 8 | C | 18 | A |
| 9 | C | 19 | D |
| 10 | B | 20 | C |

**Question 1 — B.** Displacement, velocity and force each have both magnitude and direction, so all three are vectors. **A** lists only scalars; **C** mixes the vector momentum with the scalars time and mass; **D** mixes the vectors acceleration and velocity with the scalar temperature.

**Question 2 — C.** On a velocity–time graph the displacement is the area between the line and the time axis: a triangle of base $4.0\,\mathrm{s}$ and height $8.0\,\mathrm{m\ s^{-1}}$, so $1/2\times 4.0\times 8.0=16\,\mathrm{m}$. **A** is the magnitude of the gradient (the acceleration, $2.0\,\mathrm{m\ s^{-2}}$); **B** uses the full rectangle $8.0\times 4.0$; **D** wrongly treats slowing down as meaning no displacement.

**Question 3 — B.** $v=u+at=4.0+2.5\times 6.0=19.0\,\mathrm{m\ s^{-1}}$. **A** quotes $at$ alone, dropping the initial velocity; **C** adds the initial velocity twice ($2u+at$); **D** halves the time interval ($u+a\times 3.0=11.5$).

**Question 4 — A.** $v^2=u^2+2as=5.0^2+2\times 1.5\times 48=25+144=169$, so $v=13.0\,\mathrm{m\ s^{-1}}$. **B** ignores the initial velocity ($\sqrt{144}=12$); **C** quotes $v^2$ as if it were $v$; **D** omits the factor 2 ($\sqrt{25+72}=9.8$).

**Question 5 — B.** $s=1/2\,g t^2=1/2\times 9.8\times 3.0^2=44.1\,\mathrm{m}$, closest to $44\,\mathrm{m}$. **A** uses $gt$ (the final velocity) as if it were a distance; **C** uses $1/2\,gt$; **D** omits the factor $1/2$ ($gt^2=88\,\mathrm{m}$).

**Question 6 — C.** $p=mv$: **A** gives $0.50\times 8.0=4.0\,\mathrm{kg\ m\ s^{-1}}$; **B** gives $2.0\times 3.0=6.0\,\mathrm{kg\ m\ s^{-1}}$; **C** gives $5.0\times 1.5=7.5\,\mathrm{kg\ m\ s^{-1}}$; **D** gives $10\times 0.60=6.0\,\mathrm{kg\ m\ s^{-1}}$. Object **C** has the greatest momentum.

**Question 7 — D.** $\Delta p=F_{\mathrm{net}}\Delta t=12\times 2.0=24\,\mathrm{N\ s}$; the mass is not needed. **A** quotes the force; **B** divides the force by the time; **C** also multiplies by the mass ($12\times 3.0=36$).

**Question 8 — C.** A third-law pair acts on the two *different* bodies of the one interaction: the partner of "the force on the laptop by the desk" is "the force on the desk by the laptop". **A** is the force that *balances* the normal force on the laptop (same body — an equilibrium pair, not a third-law pair); **B** is the partner of the force due to gravity on the laptop; **D** is not the interaction pair of this contact force.

**Question 9 — C.** $F_{\mathrm{net}}=30-10=20\,\mathrm{N}$; $a=\dfrac{F_{\mathrm{net}}}{m}=\dfrac{20}{5.0}=4.0\,\mathrm{m\ s^{-2}}$. **A** ignores friction ($30/5.0$); **B** adds friction instead of subtracting it ($40/5.0$); **D** divides the net force by the friction force.

**Question 10 — B.** $F_g=mg=60\times 9.8=588\,\mathrm{N}$. **A** rounds $g$ to 10; **C** divides by $g$ instead of multiplying; **D** misplaces the decimal point ($g=98$).

**Question 11 — D.** The box has no vertical acceleration, so the vertical forces balance: the normal force equals the force due to gravity, $N=mg=5.0\times 9.8=49\,\mathrm{N}$. **A** assumes the floor supplies no support; **B** quotes the mass; **C** doubles the value.

**Question 12 — B.** The forces are perpendicular, so $R=\sqrt{3.0^2+4.0^2}=5.0\,\mathrm{N}$. **A** adds them arithmetically (only valid for parallel forces); **C** subtracts them (only valid for antiparallel forces); **D** averages them.

**Question 13 — C.** Momentum is conserved: $4.0 \times 3.0 = (4.0 + 8.0)v$, so $v = \frac{12}{12} = 1.0\ \text{m s}^{-1}$. **A** divides the momentum by the box's mass only ($12/8.0 = 1.5$); **B** keeps the parcel's original speed; **D** divides the parcel's mass by the box's mass ($4.0/8.0 = 0.50$) instead of conserving momentum.

**Question 14 — B.** $F = \frac{\Delta p}{\Delta t} = \frac{0.16 \times 15}{0.050} = \frac{2.4}{0.050} = 48\ \text{N}$. **A** stops at the impulse ($2.4\ \text{N s}$); **C** and **D** misplace the decimal point by a factor of $100$ in opposite directions.

**Question 15 — A.** The push is perpendicular to the gate, so $\tau = r_{\perp} F = 1.5 \times 20 = 30\ \text{N m}$. **B** divides force by distance ($20/1.5$); **C** quotes the force alone; **D** halves the lever arm.

**Question 16 — A.** Static equilibrium requires both conditions: zero net force (so no translational acceleration) and zero net torque (so no rotational acceleration). **B** forgets the torque condition; **C** confuses equilibrium with having no forces at all (forces may act and still cancel); **D** is not a requirement of equilibrium.

**Question 17 — D.** $W = F s \cos\theta = 50 \times 20 \times \cos 30^{\circ} = 1000 \times 0.866 = 866\ \text{J}$. **A** ignores the angle; **B** uses $\sin 30^{\circ}$ (the vertical component of the force does no work along the ground); **C** uses $\tan 30^{\circ}$.

**Question 18 — A.** $E_s = 1/2\, k x^2 = 1/2 \times 400 \times 0.050^2 = 0.50\ \text{J}$. **B** omits the $1/2$; **C** fails to square $x$ ($1/2\, k x = 10\ \text{J}$); **D** quotes $k x = 20\ \text{N}$, which is the force at that compression, not an energy.

**Question 19 — D.** With no friction, $E_g$ converts entirely to $E_k$: $v = \sqrt{2 g h} = \sqrt{2 \times 9.8 \times 0.80} = \sqrt{15.68} = 3.96\ \text{m s}^{-1}$, closest to $4.0\ \text{m s}^{-1}$. **A** uses $\sqrt{g h}$; **B** quotes $2 g h$ as if it were $v$; **C** quotes $2 g$, ignoring the height.

**Question 20 — C.** $\eta = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{3200}{4000} = 0.80 = 80\,\%$. **A** is the wasted fraction ($800/4000 = 20\,\%$); **B** is wasted over useful ($800/3200 = 25\,\%$); **D** inverts the ratio ($4000/3200 = 125\,\%$, impossible for an efficiency).

## Topic 4 Test B — Motion, forces and energy (written responses)

*Time allowed: 50 minutes · 40 marks · One scientific calculator permitted · The formula sheet may be used. In questions where more than one mark is available, appropriate working must be shown. The mark total and time allowed follow ClassBasics house convention; the VCE Physics Units 1 & 2 Study Design fixes no mark allocations for school-based assessment.*

*Answer all questions in the spaces provided. Where a final answer has a required unit, give your answer in that unit. Unless otherwise indicated, the diagrams in this test are not drawn to scale. Take the gravitational field strength to be* $g = 9.8\ \text{N kg}^{-1}$.

**Question 1** (4 marks)

A delivery van travelling at $10\ \text{m s}^{-1}$ along a straight road accelerates uniformly at $2.0\ \text{m s}^{-2}$ for $8.0\ \text{s}$.

**a.** Calculate the final velocity of the van. Give your answer in $\text{m s}^{-1}$. 1 mark

**b.** Calculate the distance the van travels during the $8.0\ \text{s}$. Give your answer in m. 2 marks

**c.** State what the area between the line and the time axis on a velocity–time graph of this motion represents. 1 mark

**Question 2** (5 marks)

A loaded wheelie bin of mass $25\ \text{kg}$ is at rest on level ground. A student pushes the bin horizontally with a force of $90\ \text{N}$, and a friction force of $40\ \text{N}$ opposes the motion.

**a.** Identify the two vertical forces acting on the bin, giving each in the form "the force on the bin by …". 1 mark

**b.** Calculate the net horizontal force on the bin and use it to find the bin's acceleration. Give your answers in N and $\text{m s}^{-2}$. 2 marks

**c.** Calculate the magnitude of the normal force acting on the bin. Give your answer in N. 1 mark

**d.** State the Newton's-third-law partner of the force on the bin by the student, including its magnitude and direction. 1 mark

**Question 3** (6 marks)

A student of mass $40\ \text{kg}$ runs at $3.0\ \text{m s}^{-1}$ along a smooth level path and jumps onto a stationary trolley of mass $10\ \text{kg}$. The student and the trolley move off together.

**a.** Show that the speed of the student and trolley immediately after landing is $2.4\ \text{m s}^{-1}$. 1 mark

**b.** Calculate the total kinetic energy of the student–trolley system immediately before and immediately after the landing, and use your answers to name the type of collision. 2 marks

**c.** During the landing, the student's feet are in contact with the trolley for $0.40\ \text{s}$. Calculate the average force on the trolley by the student during this time. Give your answer in N. 2 marks

**d.** Account for the kinetic energy "lost" in this interaction, in terms of energy transformation. 1 mark

**Question 4** (5 marks)

A tractor tows a hay cart at constant speed along a level farm track. The tension in the towbar is $600\ \text{N}$, the cart travels $150\ \text{m}$, and the journey takes $50\ \text{s}$.

**a.** Calculate the work done on the cart by the towbar over the $150\ \text{m}$. Give your answer in J. 1 mark

**b.** Calculate the work done on the cart by the friction force over the $150\ \text{m}$. Give your answer in J. 1 mark

**c.** Use your answers to parts **a.** and **b.** to explain why the cart's kinetic energy does not change. 1 mark

**d.** Calculate the power delivered to the cart by the towbar. Give your answer in W. 1 mark

**e.** On a soft patch of track the friction force becomes greater than the towbar tension. State, with a reason, what happens to the speed of the cart. 1 mark

**Question 5** (5 marks)

A toy jumping frog of mass $0.050\,\text{kg}$ contains a spring of spring constant $500\,\text{N m}^{-1}$. The frog is pushed down until the spring is compressed by $0.040\,\text{m}$, then released, and it leaves the ground moving vertically.

**a.** Show that the elastic potential energy stored in the compressed spring is $0.40\,\text{J}$. 1 mark

**b.** Calculate the magnitude of the force required to hold the spring at this compression. Give your answer in N. 1 mark

**c.** Assuming all the stored elastic potential energy becomes kinetic energy as the frog leaves the ground, calculate the take-off speed of the frog. Give your answer in $\text{m s}^{-1}$. 2 marks

**d.** Calculate the maximum height reached by the frog above the point where it leaves the ground. Give your answer in m. 1 mark

**Question 6** (6 marks)

A uniform rod of mass $1.0\,\text{kg}$ and length $1.00\,\text{m}$ is held horizontally by a hinge on a wall at its left-hand end. A vertical spring balance supports the rod at its right-hand end. A shop sign of mass $2.5\,\text{kg}$ hangs from the rod at a point $0.40\,\text{m}$ from the hinge.

**a.** Calculate the torque about the hinge produced by the force due to gravity on the rod, and state its direction of turning. Give your answer in N m. 1 mark

**b.** By taking torques about the hinge, show that the spring balance reads $14.7\,\text{N}$. 2 marks

**c.** Calculate the force on the rod by the hinge. Give your answer in N. 2 marks

**d.** The sign is moved to a point closer to the hinge. State, with a reason, what happens to the spring balance reading. 1 mark

**Question 7** (3 marks)

Two forklifts each lift an identical pallet of mass $200\,\text{kg}$ through the same vertical height of $1.5\,\text{m}$. Forklift A takes $4.0\,\text{s}$; forklift B takes $6.0\,\text{s}$.

**a.** Calculate the gain in gravitational potential energy of each pallet. Give your answer in J. 1 mark

**b.** Calculate the average useful power output of each forklift during its lift, and state which forklift delivers the greater useful power. Give your answers in W. 1 mark

**c.** Each forklift's engine consumes more energy than the pallet gains. Explain this observation in terms of efficiency. 1 mark

**Question 8** (6 marks)

A group of students investigates how the extension of a spring depends on the force applied to it. They hang known masses from the spring, measure the extension from the spring's natural length each time, and plot force (on the vertical axis) against extension (on the horizontal axis). The points lie close to a straight line that passes through the origin. The students' line of best fit passes through the points $(0.020\,\text{m}, 1.0\,\text{N})$ and $(0.10\,\text{m}, 5.0\,\text{N})$.

**a.** Identify the independent variable and the dependent variable in this investigation. 1 mark

**b.** Calculate the gradient of the line of best fit. Give your answer in $\text{N m}^{-1}$. 2 marks

**c.** State the physical quantity represented by this gradient. 1 mark

**d.** Use the line of best fit to predict the extension of the spring when a mass of $0.80\,\text{kg}$ is hung from it. Give your answer in m. 1 mark

**e.** When several groups repeat the measurement with a $0.30\,\mathrm{kg}$ mass, they obtain slightly different values of the extension each time. Name this feature of data quality, and give one way the students could improve it. 1 mark

*End of Topic 4 Test B*

## Topic 4 Test B — solutions

**Question 1** $u = 10\ \mathrm{m\ s^{-1}}$, $a = 2.0\ \mathrm{m\ s^{-2}}$, $t = 8.0\ \mathrm{s}$.

(a) $v = u + at = 10 + 2.0 \times 8.0 = 26\ \mathrm{m\ s^{-1}}$. **(1 mark: correct value $26\ \mathrm{m\ s^{-1}}$.)**

(b) $s = ut + 1/2\,at^2 = 10 \times 8.0 + 1/2 \times 2.0 \times 8.0^2 = 80 + 64 = 144\ \mathrm{m}$. (**1 mark** correct substitution into $s = ut + 1/2\,at^2$; **1 mark** correct value $144\ \mathrm{m}$.)

(c) The area between the line and the time axis on a velocity–time graph is the displacement of the object (here, $144\ \mathrm{m}$). **(1 mark: area = displacement.)**

**Question 2** $m = 25\ \mathrm{kg}$; push $90\ \mathrm{N}$; friction $40\ \mathrm{N}$.

(a) The two vertical forces are the force on the bin by the Earth (the force due to gravity, downward) and the force on the bin by the ground (the normal force, upward). **(1 mark: both forces identified, in the required form.)**

(b) $F_{\text{net}} = 90 - 40 = 50\ \mathrm{N}$ in the direction of the push. By Newton's second law,

$$a = \frac{F_{\text{net}}}{m} = \frac{50}{25} = 2.0\ \mathrm{m\ s^{-2}}.$$

(**1 mark** net force $50\ \mathrm{N}$; **1 mark** correct acceleration $2.0\ \mathrm{m\ s^{-2}}$.)

(c) The bin has no vertical acceleration, so the vertical forces balance: the normal force equals the force due to gravity,

$$N = mg = 25 \times 9.8 = 245\ \mathrm{N}.$$

**(1 mark: correct value $245\ \mathrm{N}$.)**

(d) The partner is the force on the student by the bin, of magnitude $90\ \mathrm{N}$, directed opposite to the push (backward on the student). By Newton's third law, $F_{\text{on student by bin}} = -F_{\text{on bin by student}}$. **(1 mark: correct partner force with magnitude and direction.)**

**Question 3** $m_1 = 40\ \mathrm{kg}$ at $3.0\ \mathrm{m\ s^{-1}}$; $m_2 = 10\ \mathrm{kg}$ stationary; they move off together.

(a) Conservation of momentum: $m_1 u_1 = (m_1 + m_2)v$, so

$$v = \frac{40 \times 3.0}{40 + 10} = \frac{120}{50} = 2.4\ \mathrm{m\ s^{-1}},$$

as required. **(1 mark: momentum conservation giving $2.4\ \mathrm{m\ s^{-1}}$.)**

(b) Before: $E_k = 1/2\,m_1 u_1^2 = 1/2 \times 40 \times 3.0^2 = 180\ \mathrm{J}$. After: $E_k = 1/2(m_1 + m_2)v^2 = 1/2 \times 50 \times 2.4^2 = 144\ \mathrm{J}$. The total kinetic energy falls from $180\ \mathrm{J}$ to $144\ \mathrm{J}$, so the collision is **inelastic**. (Momentum is still conserved.) (**1 mark** both kinetic energies correct; **1 mark** names the collision inelastic because $E_k$ decreases.)

(c) The impulse on the trolley equals its change in momentum: $\Delta p = 10 \times 2.4 = 24\ \mathrm{N\ s}$. The average force is

$$F = \frac{\Delta p}{\Delta t} = \frac{24}{0.40} = 60\ \mathrm{N},$$

in the direction of motion. (**1 mark** $F = \Delta p / \Delta t$ with $\Delta p = 24\ \mathrm{N\ s}$; **1 mark** correct value $60\ \mathrm{N}$ in the direction of motion.)

(d) The "lost" $36\ \mathrm{J}$ of kinetic energy is transformed mainly into heat and sound at the contact between the student's shoes and the trolley's deck, with a little internal vibration of the student and trolley. Total energy is conserved. **(1 mark: transformed to heat and sound (internal energy) / total energy conserved.)**

**Question 4** Tension $600\ \mathrm{N}$; $s = 150\ \mathrm{m}$; $t = 50\ \mathrm{s}$; constant speed.

(a) $W = F s = 600 \times 150 = 9.0 \times 10^4$ J (positive — the tension acts along the motion). **(1 mark: correct value $9.0 \times 10^4$ J.)**

(b) Constant speed means zero net force (Newton's first law), so the friction force has magnitude $600$ N and opposes the motion:

$$W = -F s = -600 \times 150 = -9.0 \times 10^4 \text{ J}.$$

**(1 mark: correct value $-9.0 \times 10^4$ J, with the negative sign.)**

(c) The net work done on the cart is $9.0 \times 10^4 \text{ J} - 9.0 \times 10^4 \text{ J} = 0$. The cart moves at constant speed, so its kinetic energy does not change: parts (a) and (b) show that the towbar transfers exactly as much energy to the cart as friction transfers out. **(1 mark: zero net work → no change in kinetic energy.)**

(d) $P = \dfrac{W}{t} = \dfrac{9.0 \times 10^4}{50} = 1800$ W. **(1 mark: correct value** $1800$ W.**)**

(e) The cart slows down. The net force is now backward, so the net work is negative: kinetic energy is removed from the cart and its speed decreases (Newton's second law). **(1 mark: slows, because a backward net force / negative net work removes kinetic energy.)**

**Question 5** $m = 0.050$ kg, $k = 500 \text{ N m}^{-1}$, $x = 0.040$ m.

(a) $E_s = 1/2\, k x^2 = 1/2 \times 500 \times 0.040^2 = 1/2 \times 500 \times 0.0016 = 0.40$ J, as required. **(1 mark: correct substitution and result.)**

(b) By Hooke's law, $F = k x = 500 \times 0.040 = 20$ N. **(1 mark: correct value** $20$ N.**)**

(c) Setting the stored energy equal to the kinetic energy at take-off,

$$1/2\, m v^2 = 0.40 \Rightarrow v^2 = \frac{2 \times 0.40}{0.050} = 16,$$

so $v = 4.0 \text{ m s}^{-1}$. **(1 mark** $1/2\, m v^2 = E_s$ set up; **1 mark** correct value $4.0 \text{ m s}^{-1}$.)

(d) The kinetic energy converts to gravitational potential energy at the top of the flight: $m g h = 0.40$, so

$$h = \frac{0.40}{m g} = \frac{0.40}{0.050 \times 9.8} = 0.816 \text{ m},$$

which rounds to $0.82$ m (equivalently, $h = v^2 / 2g = 16 / 19.6$). **(1 mark: correct value** $0.82$ m.**)**

**Question 6** Rod: $m = 1.0$ kg, length $1.00$ m; sign: $m = 2.5$ kg at $0.40$ m from the hinge.

(a) The rod is uniform, so its force due to gravity $F_g = 1.0 \times 9.8 = 9.8$ N acts at its centre, $0.50$ m from the hinge:

$$\tau = r_{\perp} F = 0.50 \times 9.8 = 4.9 \text{ N m},$$

turning the rod clockwise about the hinge. **(1 mark: correct torque** $4.9$ N m **with clockwise direction.)**

(b) The sign's force due to gravity is $2.5 \times 9.8 = 24.5$ N. Taking torques about the hinge, the two forces due to gravity give clockwise torques and the spring balance gives the anticlockwise torque. The hinge force acts at the pivot and produces no torque. Rotational equilibrium gives

$$F \times 1.00 = 9.8 \times 0.50 + 24.5 \times 0.40 = 4.9 + 9.8 = 14.7 \text{ N m},$$

so the balance reads $F = 14.7$ N, as required. **(1 mark** both clockwise torques correct (4.9 and 9.8 N m); **1 mark** correct balance reading 14.7 N.)

(c) The rod has no vertical acceleration, so the upward forces balance the downward forces:

$$F_{\text{hinge}} + 14.7 = 9.8 + 24.5 = 34.3 \text{ N},$$

giving $F_{\text{hinge}} = 19.6\,\text{N}$, upward. **(1 mark** translational-equilibrium equation set up; **1 mark** correct value $19.6\,\text{N}$ upward.)

(d) The reading decreases. Moving the sign closer to the hinge reduces its lever arm, and therefore its torque, about the hinge; the spring balance then needs to supply a smaller force to keep the net torque at zero. **(1 mark: reading decreases, justified via the smaller lever arm/torque.)**

**Question 7** $m = 200\,\text{kg}$, $h = 1.5\,\text{m}$; times $4.0\,\text{s}$ and $6.0\,\text{s}$.

(a) $\Delta E_g = m g h = 200 \times 9.8 \times 1.5 = 2940\,\text{J}$ (about $2.9 \times 10^3\,\text{J}$) for each pallet. **(1 mark: correct value $2940\,\text{J}$.)**

(b) $P_A = \frac{2940}{4.0} = 735\,\text{W}$; $P_B = \frac{2940}{6.0} = 490\,\text{W}$. Forklift A delivers the greater useful power (same energy transferred in less time). **(1 mark: both powers correct and forklift A identified.)**

(c) No machine is $100\,\%$ efficient: some of the energy consumed by each engine is dissipated to the surroundings — as heat and sound in the engine and hydraulics — rather than becoming gravitational potential energy of the pallet. Efficiency is the ratio of useful energy output to total energy input, and here that ratio is less than 1. **(1 mark: energy dissipated to surroundings / efficiency less than 1.)**

**Question 8** Force–extension investigation; line of best fit through $(0.020\,\text{m}, 1.0\,\text{N})$ and $(0.10\,\text{m}, 5.0\,\text{N})$.

(a) The independent variable is the force applied to the spring (set by the mass hung on it); the dependent variable is the extension of the spring. **(1 mark: both variables correctly identified.)**

(b) Using the two given points on the line of best fit:

$$\text{gradient} = \frac{\Delta F}{\Delta x} = \frac{5.0 - 1.0}{0.10 - 0.020} = \frac{4.0}{0.080} = 50\,\text{N m}^{-1}.$$

**(1 mark** two well-separated on-line points used; **1 mark** correct gradient $50\,\text{N m}^{-1}$.)

(c) Since $F = k x$, the gradient of the force–extension line is the spring constant $k$ (here, $50\,\text{N m}^{-1}$). **(1 mark: gradient = spring constant $k$.)**

(d) The force due to gravity on a $0.80\,\text{kg}$ mass is $F = m g = 0.80 \times 9.8 = 7.84\,\text{N}$. Using the line, $x = \frac{F}{k} = \frac{7.84}{50} = 0.1568\,\text{m}$, which rounds to $0.16\,\text{m}$. This is a prediction beyond the measured range, valid only if the spring remains within its elastic limit. **(1 mark: correct predicted extension $0.16\,\text{m}$.)**

(e) This spread of repeated readings is a limit of **precision**. It could be improved by repeating each reading and taking an average, by using a ruler with finer scale divisions, or by fixing a pointer to the end of the spring so the extension is read consistently (avoiding parallax). **(1 mark: names precision AND gives one valid improvement.)**