Physics Units 1 & 2

Chapter 8 — Forces and motion

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Chapter 8 Forces and motion — 8A Momentum and change in momentum

Theory

A supermarket trolley rolling towards you at walking pace is easy to stop; a car rolling towards you at the same pace is not. A tennis ball dropping into your hand from a shelf is easy to catch; the same ball arriving from a hard serve is not. In each pair the speed is the same and the mass is different, or the mass is the same and the speed is different. The quantity that combines the two — and that tells you how much an object's motion resists being changed — is called **momentum**.

The study design states the two tasks of this section: *"apply concepts of momentum to linear motion"* using $p = mv$, and *"explain changes in momentum as being caused by a net force"* using $\Delta p = F_{\text{net}} \Delta t$ (VCE Physics Study Design, Unit 2, Area of Study 1). Everything in this section is those two relationships applied to motion along one straight line.

Momentum. The **momentum** p of an object is the product of its mass and its velocity:

$$p = mv.$$

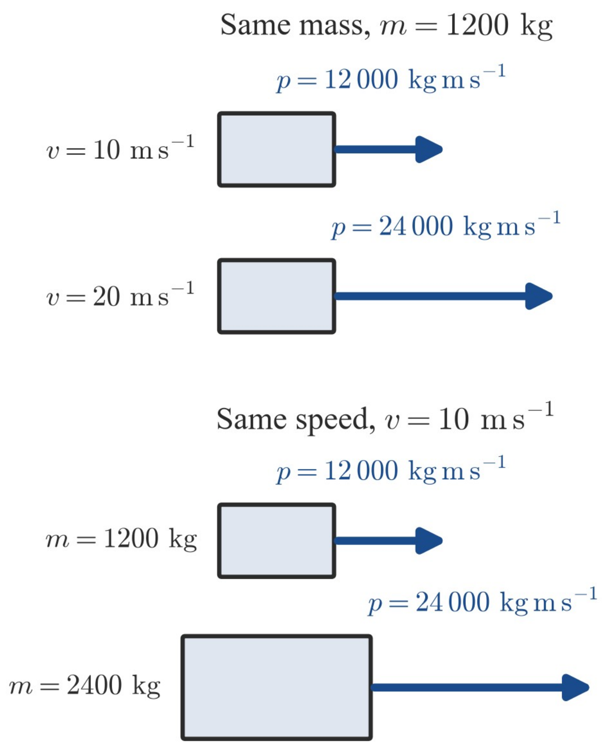
Momentum is measured in **kilogram metres per second** (kg m s^{-1}): a $2.0\,\text{kg}$ object moving at $3.0\,\text{m s}^{-1}$ has momentum $p = 2.0 \times 3.0 = 6.0\,\text{kg m s}^{-1}$. An object at rest has $v = 0$ and therefore zero momentum, whatever its mass.

Velocity is a vector, so momentum is a vector too: the momentum of an object points in the same direction as its velocity. A ball moving east has its momentum directed east. In this section every object moves along one straight line, and the direction of the motion is handled with a sign.

The sign convention. For motion along a straight line, choose one direction along the line and call it **positive**. A velocity in that direction is positive, a velocity in the opposite direction is negative, and the momentum carries the same sign as the velocity. Any answer may be stated as a magnitude with a direction ("$10\,\text{kg m s}^{-1}$ towards the bowler") or as a signed value with the convention stated ("$-10\,\text{kg m s}^{-1}$, taking towards the bowler as negative"). Both statements are the same answer.

Quantity	Symbol	Unit
momentum	p	kilogram metre per second (kg m s^{-1})
mass	m	kilogram (kg)
velocity	v	metre per second (m s^{-1})

Both mass and velocity count. In $p = mv$, doubling either factor doubles the momentum. The figure below shows the same car travelling at $10\,\text{m s}^{-1}$ and at $20\,\text{m s}^{-1}$ — doubling the speed doubles the momentum from $12\,000\,\text{kg m s}^{-1}$ to $24\,000\,\text{kg m s}^{-1}$ — and then the same speed with the mass doubled by loading the vehicle, which again doubles the momentum. The momentum arrows are drawn in proportion, so the comparisons can be read by eye.



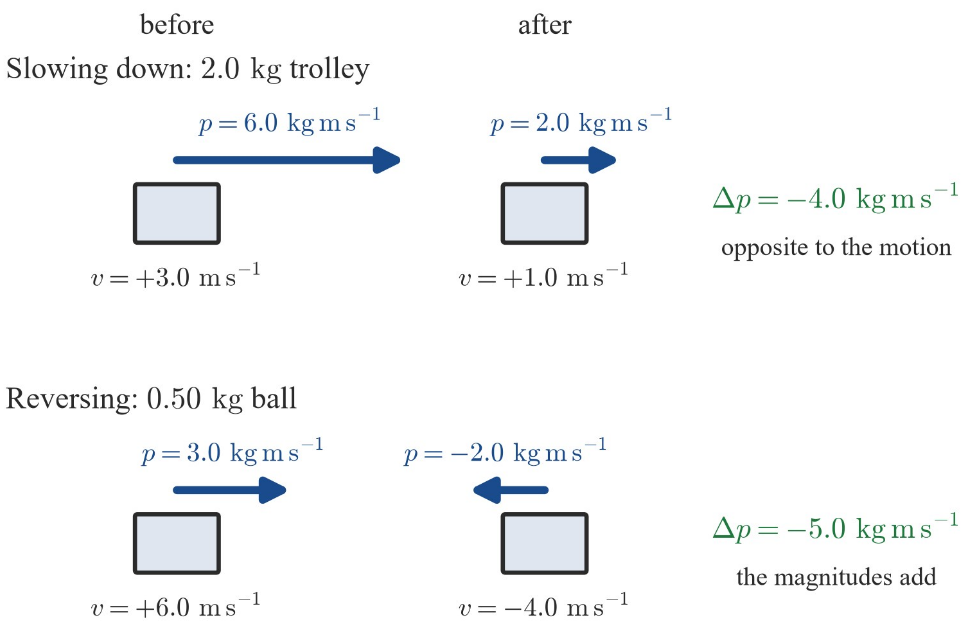
This is why a heavily loaded truck at $60\,\text{km h}^{-1}$ is a far more serious thing to stop than a car at $60\,\text{km h}^{-1}$: the truck carries several times the momentum, so several times the change of momentum is needed to bring it to rest.

Change in momentum. Momentum is interesting because it can *change*, and the change is what forces produce. The **change in momentum** Δp of an object is its final momentum minus its initial momentum. For an object of constant mass m whose velocity changes from u to v:

$$\Delta p = mv - mu = m(v - u).$$

Because momentum is a vector, the signs matter. Take motion to the right as positive.

- A $2.0\,\text{kg}$ trolley slowing from $+3.0\,\text{m s}^{-1}$ to $+1.0\,\text{m s}^{-1}$ changes its momentum by $\Delta p = 2.0 \times (1.0 - 3.0) = -4.0\,\text{kg m s}^{-1}$: the change is opposite to the motion, which is what "slowing down" means.
- A $0.50\,\text{kg}$ ball moving at $+6.0\,\text{m s}^{-1}$ that is struck straight back at $-4.0\,\text{m s}^{-1}$ changes its momentum by $\Delta p = 0.50 \times (-4.0 - 6.0) = -5.0\,\text{kg m s}^{-1}$.



Direction chosen as positive: to the right

The second case is the one that catches people out. The ball's speed only changed by $2.0\,\mathrm{m\ s^{-1}}$, yet its change of momentum ($5.0\,\mathrm{kg\ m\ s^{-1}}$) is *larger* than the trolley's, because reversing direction turns the two momenta against each other and their magnitudes add. Subtracting the magnitudes ($3.0-2.0=1.0$) would only be correct if the ball kept moving the same way. Whenever an object bounces or is struck back, expect a large change of momentum.

What changes momentum: the net force. A **force** is a push or a pull on an object. Usually several forces act on an object at once — the pushes and pulls of everything touching it, and of the Earth — and the **net force** F_{net} is the single force that has the same effect as all of them together. (How to find and combine forces properly is the subject of Sections 8B–8D; this section only needs the idea.) The key fact of this section is:

$$\Delta p = F_{\mathrm{net}}\,\Delta t\,.$$

A change in momentum is *caused* by a net force acting over a time interval Δt. If the net force on an object is zero, its momentum does not change at all: an object at rest stays at rest, and an object moving in a straight line keeps moving at the same speed, because there is nothing acting to change its momentum. If the net force is not zero, the momentum changes, and the change is in the direction of the net force. A larger net force, or the same net force acting for a longer time, produces a larger change of momentum. The relationship as written applies to a net force that is constant over the interval; when the force varies, F_{net} is read as the **average** net force over Δt.

For an object whose mass does not change, a change of momentum is a change of velocity — an acceleration — so $\Delta p = F_{\mathrm{net}}\,\Delta t$ is really a statement about how forces change motion. Section 8B develops that link in full, as Newton's three laws of motion.

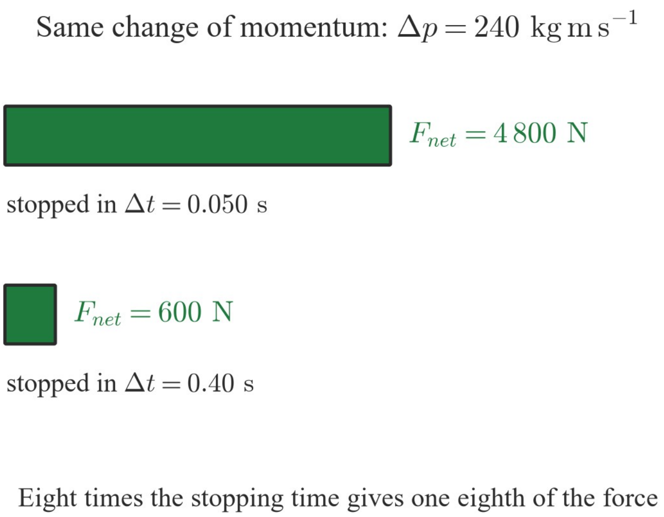
The relation rearranges two useful ways:

$$F_{\mathrm{net}} = \frac{\Delta p}{\Delta t} \text{ and } \Delta t = \frac{\Delta p}{F_{\mathrm{net}}}.$$

The first form says that the net force on an object equals the rate at which its momentum changes: to change an object's momentum quickly takes a large force, and a small force can only change it slowly.

The same change, different times. The rearranged forms make the practical point of this section. Suppose a $60\,\mathrm{kg}$ person lands with a momentum of $240\,\mathrm{kg\ m\ s^{-1}}$ and must be brought to rest, so that the magnitude of the change of momentum is fixed at $240\,\mathrm{kg\ m\ s^{-1}}$ either way. What the person does on landing then decides the force:

stopping time Δt	average net force $F_{\text{net}} = \Delta p / \Delta t$
$0.050\,\text{s}$ (legs held stiff)	$240 / 0.050 = 4800\,\text{N}$
$0.40\,\text{s}$ (knees bending)	$240 / 0.40 = 600\,\text{N}$



Eight times the stopping time gives one eighth of the force, for exactly the same change of momentum. This single idea explains a great many things you have met without the physics: landing with bent knees, catching a fast ball with hands that give and draw back, the crumple zones and airbags of cars, the mats under gymnasts and high-jumpers. All of them spread a fixed change of momentum over a longer time so that the force is smaller. A batter's or golfer's follow-through uses the same relationship in reverse: keeping the force applied to the ball for a longer contact time gives a larger change of momentum and so a faster ball.

Units 3 & 4 preview. The product $F\,\Delta t$ returns in Section 8E of this book, where it is given the name **impulse** and applied, with the conservation of momentum, to collisions between objects moving in a straight line. At Units 3 & 4 the same idea is extended to changing forces by reading the change of momentum from the area under a force–time graph, and momentum ideas are applied to motion in two dimensions. This section supplies the quantities all of those topics are built from — $p = m v$ and $\Delta p = F_{\text{net}}\,\Delta t$ are assumed known there and never re-taught. Content under this banner is not assessed in this book's topic tests or practice examinations.

Worked examples

Example 1 — scaffolded

An Australian-rules football has a mass of $0.45\,\text{kg}$. A player kicks it so that it leaves the boot at $30\,\text{m s}^{-1}$. (a) Calculate the momentum of the football as it leaves the boot. (b) A second kick sends the same ball off at $15\,\text{m s}^{-1}$. Calculate its momentum and compare it with part (a). (c) A cricket ball has a mass of $0.16\,\text{kg}$. Calculate the speed at which the cricket ball would have the same momentum as the football in part (a), and express your answer in km h^{-1} as well.

Working	Comment
$p = m v = 0.45 \times 30 = 13.5 \approx 14\,\text{kg m s}^{-1}$	Momentum is the product of mass and velocity; round to 2 significant figures at the end.
$p = 0.45 \times 15 = 6.75 \approx 6.8\,\text{kg m s}^{-1}$	Same mass, half the speed.
The momentum has halved.	In $p = m v$ with m fixed, p is proportional to v.
$v = \dfrac{p}{m} = \dfrac{13.5}{0.16} = 84.4 \approx 84\,\text{m s}^{-1}$	Rearrange $p = m v$; use the unrounded 13.5 so no rounding error creeps in.
$84.4 \times 3.6 \approx 300\,\text{km h}^{-1}$	Multiply m s^{-1} by 3.6 to get km h^{-1}.

The last part is the point: to carry the momentum of a kicked football, a cricket ball would have to travel at about $300\,\text{km h}^{-1}$, roughly twice the speed of the fastest recorded bowled deliveries. A light object needs a very large velocity to match the momentum of a heavier one.

Example 2 — scaffolded

A tennis ball of mass $0.058\,\text{kg}$ is served. While the ball is in contact with the strings, the racquet exerts an average net force of $320\,\text{N}$ on it for $0.010\,\text{s}$. The ball starts from rest. (a) Calculate the change of momentum of the ball during the contact. (b) Calculate the speed of the ball as it leaves the racquet. (c) Express this speed in km h^{-1} and comment.

Working	Comment
$\Delta p = F_{\text{net}}\,\Delta t = 320 \times 0.010 = 3.2\,\text{kg m s}^{-1}$	The change of momentum is the net force multiplied by the time it acts.
The ball starts from rest, so $p_{\text{initial}} = 0$ and $p_{\text{final}} = \Delta p = 3.2\,\text{kg m s}^{-1}$.	A change of momentum from rest *is* the final momentum.
$v = \dfrac{p}{m} = \dfrac{3.2}{0.058} = 55.2 \approx 55\,\text{m s}^{-1}$	Rearrange $p = m v$.
$55.2 \times 3.6 \approx 2.0 \times 10^2\,\text{km h}^{-1}$	About $200\,\text{km h}^{-1}$, the speed of a fast serve.

Example 3 — unscaffolded

A baseball of mass $0.15\,\text{kg}$ is pitched at $40\,\text{m s}^{-1}$ towards a batter, who hits it straight back towards the pitcher at $45\,\text{m s}^{-1}$. The ball is in contact with the bat for $0.010\,\text{s}$. Take the pitcher-to-batter direction as positive. Calculate the change of momentum of the ball and the average net force the bat exerts on it, and compare the change of momentum with what it would be if the ball were simply caught.

Take the pitcher-to-batter direction as positive, so $u = +40\,\text{m s}^{-1}$ and, because the ball returns the way it came, $v = -45\,\text{m s}^{-1}$. The change of momentum is

$$\Delta p = m(v - u) = 0.15 \times (-45 - 40) = 0.15 \times (-85) = -12.75 \approx -13\,\text{kg m s}^{-1}$$

to two significant figures: a change of $13\,\text{kg m s}^{-1}$ directed back towards the pitcher. The average net force follows from the rate of change of momentum:

$$F_{\text{net}} = \frac{\Delta p}{\Delta t} = \frac{-12.75}{0.010} = -1275 \approx -1.3 \times 10^3\,\text{N},$$

that is, an average net force of about $1300\,\text{N}$ directed towards the pitcher. If the ball had been caught instead, the change would have been $\Delta p = 0.15 \times (0 - 40) = -6.0\,\text{kg m s}^{-1}$. Reversing the ball more than doubles the change of momentum compared with stopping it — which is exactly why hitting a ball back takes (and delivers) a much larger force than catching it.

$\therefore\ \Delta p = -13\,\text{kg m s}^{-1}$ (towards the pitcher) and $F_{\text{net}} \approx -1.3 \times 10^3\,\text{N}$ on average during the contact.

Example 4 — unscaffolded

A $60\,\text{kg}$ basketballer lands after a rebound, meeting the floor at $4.0\,\text{m s}^{-1}$ and coming to rest. Compare the average net force on the player if the landing is stiff-legged and the player stops in $0.050\,\text{s}$ with a bent-knee landing that stops in $0.40\,\text{s}$, and explain what the comparison means for landing technique.

Taking downward as positive, the momentum at landing is $p = m v = 60 \times 4.0 = 240\,\text{kg m s}^{-1}$, and the player must end at rest, so the magnitude of the change of momentum is $240\,\text{kg m s}^{-1}$ for either landing. The two stopping times give

$$F_{\text{stiff}} = \frac{\Delta p}{\Delta t} = \frac{240}{0.050} = 4.8 \times 10^3\,\text{N},\ F_{\text{bent}} = \frac{240}{0.40} = 6.0 \times 10^2\,\text{N}.$$

Both forces act upward, opposite to the motion, as they must to remove a downward momentum. The bent-knee landing spreads the same change of momentum over eight times the time, so the average net force is eight times smaller: $4800\,\mathrm{N}$ against $600\,\mathrm{N}$. The stiff-legged landing delivers eight times the force to the player's joints for the very same landing, which is why every landing technique in sport — and every safety device in a car — works by lengthening the stopping time.

∴ the average net force is $4.8 \times 10^{3}\,\mathrm{N}$ stiff-legged and $6.0 \times 10^{2}\,\mathrm{N}$ with knees bent, a factor of eight smaller.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working, and state the direction of every vector quantity. Unless a question says otherwise, take the direction of the initial motion as positive.

Question 1 *(scaffolded)* (a) Write the relationship between momentum p, mass m and velocity v, and state the SI unit of momentum. (b) State whether momentum is a vector or a scalar, and what fixes its direction. (c) Calculate the momentum of a $0.60\,\mathrm{kg}$ basketball moving at $5.0\,\mathrm{m\ s^{-1}}$.

Question 2 *(scaffolded)* A car of mass $1200\,\mathrm{kg}$ travels in a straight line at $72\,\mathrm{km\ h^{-1}}$. (a) Express this speed in $\mathrm{m\ s^{-1}}$. (b) Calculate the momentum of the car. (c) At what speed, in $\mathrm{km\ h^{-1}}$, does the same car carry half of this momentum?

Question 3 *(scaffolded)* A truck of mass $3000\,\mathrm{kg}$ moves at $10\,\mathrm{m\ s^{-1}}$, and a car of mass $1000\,\mathrm{kg}$ moves at $30\,\mathrm{m\ s^{-1}}$ along the same straight road. (a) Calculate the momentum of each vehicle. (b) Both are brought to rest. State which requires the larger change of momentum, and justify your answer. (c) Calculate the momentum of the car if its speed doubles, and state the factor by which its momentum changes.

Question 4 *(scaffolded)* A $900\,\mathrm{kg}$ car slows in a straight line from $25\,\mathrm{m\ s^{-1}}$ to $10\,\mathrm{m\ s^{-1}}$. (a) Calculate the initial and final momentum of the car. (b) Calculate the change of momentum of the car, with its direction. (c) State the direction of the net force on the car while it slows, with a reason.

Question 5 *(scaffolded)* A student pushes a $2.0\,\mathrm{kg}$ dynamics trolley from rest along a straight bench. The push gives a constant net force of $5.0\,\mathrm{N}$ for $3.0\,\mathrm{s}$. (a) Calculate the change of momentum of the trolley. (b) Calculate the momentum of the trolley after $3.0\,\mathrm{s}$, and hence its speed. (c) The same net force acts on a trolley at rest for $6.0\,\mathrm{s}$ instead. Calculate the final speed.

Question 6 *(scaffolded)* At impact with a club, the momentum of a $0.045\,\mathrm{kg}$ golf ball changes from 0 to $3.0\,\mathrm{kg\ m\ s^{-1}}$. (a) State the change of momentum of the ball. (b) The contact lasts $0.0010\,\mathrm{s}$. Calculate the average net force on the ball. (c) A softer strike gives the same change of momentum over $0.0030\,\mathrm{s}$. Calculate the average net force in that case. (d) Describe the pattern your answers to (b) and (c) show, in words.

Question 7 A $1600\,\mathrm{kg}$ car travels along a straight country road at $100\,\mathrm{km\ h^{-1}}$. (a) Calculate the speed of the car in $\mathrm{m\ s^{-1}}$. (b) Calculate the momentum of the car. (c) The car enters a $50\,\mathrm{km\ h^{-1}}$ zone. Calculate its momentum at that speed and state the factor by which the momentum has changed.

Question 8 A cricket ball of mass $0.16\,\mathrm{kg}$ has a momentum of $4.0\,\mathrm{kg\ m\ s^{-1}}$. (a) Calculate the speed of the ball. (b) Express this speed in $\mathrm{km\ h^{-1}}$ and comment on whether it is a realistic bowling speed.

Question 9 A basketball of mass $0.60\,\mathrm{kg}$ falls onto a court floor at $6.0\,\mathrm{m\ s^{-1}}$ and bounces straight back up at $4.5\,\mathrm{m\ s^{-1}}$. Take upward as positive. (a) Calculate the momentum of the ball just before and just after the bounce. (b) Calculate the change of momentum of the ball during the bounce, with its direction. (c) Calculate the change of momentum the ball would undergo if it stopped dead on the floor instead, and state which change is the larger.

Question 10 A fully loaded road train and a small car travel at the same speed along a straight road. Use the momentum model to explain why the road train is far harder to bring to rest, and what this implies about the change of momentum and the stopping time for a given braking force.

Question 11 A netballer passes a stationary $0.45\,\text{kg}$ netball so that it leaves her hands at $12\,\text{m s}^{-1}$. Her hands are in contact with the ball for $0.050\,\text{s}$. (a) Calculate the momentum of the ball as it leaves her hands. (b) State the change of momentum of the ball during the pass. (c) Calculate the average net force on the ball during the contact.

Question 12 In a crash, the front of a modern car crumples progressively rather than staying rigid, and the car takes a longer time to come to rest than an identical rigid car would. Use $\Delta p = F_{\text{net}} \Delta t$ to explain why the crumpling reduces the risk of injury to the car's occupants.

Question 13 A $1000\,\text{kg}$ car travels in a straight line at a steady $15\,\text{m s}^{-1}$. (a) Calculate the momentum of the car. (b) State the change of momentum of the car over the next $10\,\text{s}$, and deduce the net force on the car over that interval. Explain your reasoning.

Question 14 A student watching a long drive says: "the ball keeps moving after it leaves the bat because it carries a force with it; when that force runs out, the ball stops." Evaluate this statement using the distinction between momentum and force developed in this section.

Question 15 A cyclist and bicycle of combined mass $85\,\text{kg}$ travel in a straight line at $8.0\,\text{m s}^{-1}$ and brake to rest in $4.0\,\text{s}$. (a) Calculate the initial momentum of cyclist and bicycle. (b) Calculate the change of momentum during the braking. (c) Calculate the average net force on cyclist and bicycle during the braking, with its direction.

Question 16 Explain, using $\Delta p = F_{\text{net}} \Delta t$, why catching a fast-moving ball stings the hands more than catching the same ball moving slowly, if the hands stop the ball in about the same time in each case.

Question 17 A $4000\,\text{kg}$ truck moving at $12\,\text{m s}^{-1}$ in a straight line brakes with a constant net force of $6000\,\text{N}$. (a) Calculate the initial momentum of the truck. (b) Calculate the time taken for the truck to stop. (c) A second identical truck brakes from the same speed with twice the net force. State the stopping time, with a reason.

Question 18 A high-jumper comes down onto a thick foam mat and is brought to rest. Use $\Delta p = F_{\text{net}} \Delta t$ to justify the use of the mat, and state what changes if the same jumper lands on bare ground instead.

Answers

Question 1 (a) $p = m v$; $\mathrm{kg\ m\ s^{-1}}$; (b) vector; the direction of the velocity; (c) $3.0\,\mathrm{kg\ m\ s^{-1}}$. **Question 2** (a) $20\,\mathrm{m\ s^{-1}}$; (b) $2.4 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$; (c) $36\,\mathrm{km\ h^{-1}}$. **Question 3** (a) truck 3.0×10^4, car $3.0 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$; (b) neither — see solutions; (c) $6.0 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$, a factor of 2. **Question 4** (a) $22\,500$ and $9000\,\mathrm{kg\ m\ s^{-1}}$; (b) $-13\,500\,\mathrm{kg\ m\ s^{-1}}$ (1.4×10^4 opposite to the motion, 2 s.f.); (c) opposite to the motion — see solutions. **Question 5** (a) $15\,\mathrm{kg\ m\ s^{-1}}$; (b) $15\,\mathrm{kg\ m\ s^{-1}}$, $7.5\,\mathrm{m\ s^{-1}}$; (c) $15\,\mathrm{m\ s^{-1}}$. **Question 6** (a) $3.0\,\mathrm{kg\ m\ s^{-1}}$; (b) $3000\,\mathrm{N}$; (c) $1000\,\mathrm{N}$; (d) see solutions. **Question 7** (a) $28\,\mathrm{m\ s^{-1}}$; (b) $4.4 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$; (c) $2.2 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$ — halved. **Question 8** (a) $25\,\mathrm{m\ s^{-1}}$; (b) $90\,\mathrm{km\ h^{-1}}$ — see solutions. **Question 9** (a) $-3.6\,\mathrm{kg\ m\ s^{-1}}$ before, $+2.7\,\mathrm{kg\ m\ s^{-1}}$ after; (b) $+6.3\,\mathrm{kg\ m\ s^{-1}}$ (upward); (c) $+3.6\,\mathrm{kg\ m\ s^{-1}}$ — the bounce is the larger change. **Question 10** See solutions (model answer). **Question 11** (a) $5.4\,\mathrm{kg\ m\ s^{-1}}$; (b) $5.4\,\mathrm{kg\ m\ s^{-1}}$; (c) $1.1 \times 10^2\,\mathrm{N}$. **Question 12** See solutions (model answer). **Question 13** (a) $1.5 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$; (b) $\Delta p = 0$, so $F_{\mathrm{net}} = 0$ — see solutions. **Question 14** See solutions (model answer). **Question 15** (a) $680\,\mathrm{kg\ m\ s^{-1}}$; (b) $-680\,\mathrm{kg\ m\ s^{-1}}$; (c) $170\,\mathrm{N}$ opposite to the motion. **Question 16** See solutions (model answer). **Question 17** (a) $4.8 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$; (b) $8.0\,\mathrm{s}$; (c) $4.0\,\mathrm{s}$. **Question 18** See solutions (model answer).

Fully-worked solutions

Question 1 (a) $p = m v$; momentum is measured in $\mathrm{kg\ m\ s^{-1}}$. (b) Momentum is a vector. Its direction is the direction of the velocity: mass is a positive scalar, so multiplying the velocity by the mass changes the size of the arrow but not its direction. (c) $p = m v = 0.60 \times 5.0 = 3.0\,\mathrm{kg\ m\ s^{-1}}$, in the direction of the ball's motion.

Question 2 (a) $72\,\mathrm{km\ h^{-1}} = 72 / 3.6 = 20\,\mathrm{m\ s^{-1}}$. (b) $p = m v = 1200 \times 20 = 24\,000\,\mathrm{kg\ m\ s^{-1}} = 2.4 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$. (c) With the mass fixed, p is proportional to v, so half the momentum means half the speed: $10\,\mathrm{m\ s^{-1}} = 36\,\mathrm{km\ h^{-1}}$.

Question 3 (a) Truck: $p = 3000 \times 10 = 30\,000\,\mathrm{kg\ m\ s^{-1}}$. Car: $p = 1000 \times 30 = 30\,000\,\mathrm{kg\ m\ s^{-1}}$. (b) Neither. Bringing each vehicle to rest removes the same momentum, $30\,000\,\mathrm{kg\ m\ s^{-1}}$, so the change of momentum is the same for both even though the car is lighter — its greater speed exactly makes up for its smaller mass. (With the same braking force, the two would take the same time to stop: $\Delta t = \Delta p / F_{\mathrm{net}}$.) (c) $p = 1000 \times 60 = 60\,000\,\mathrm{kg\ m\ s^{-1}} = 6.0 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$: doubling the speed doubles the momentum, a factor of 2.

Question 4 (a) $p_{\mathrm{initial}} = 900 \times 25 = 22\,500\,\mathrm{kg\ m\ s^{-1}}$; $p_{\mathrm{final}} = 900 \times 10 = 9000\,\mathrm{kg\ m\ s^{-1}}$. (b) $\Delta p = p_{\mathrm{final}} - p_{\mathrm{initial}} = 9000 - 22\,500 = -13\,500\,\mathrm{kg\ m\ s^{-1}}$: a change of $13\,500\,\mathrm{kg\ m\ s^{-1}}$ (1.4×10^4 to 2 significant figures) directed opposite to the motion. (c) The change of momentum is opposite to the motion, and Δp is always in the direction of the net force, so the net force on the car acts opposite to its motion. A net force opposing the motion is exactly what slowing down requires.

Question 5 (a) $\Delta p = F_{\mathrm{net}} \Delta t = 5.0 \times 3.0 = 15\,\mathrm{kg\ m\ s^{-1}}$. (b) The trolley starts from rest, so its momentum after $3.0\,\mathrm{s}$ is $15\,\mathrm{kg\ m\ s^{-1}}$, and $v = p / m = 15 / 2.0 = 7.5\,\mathrm{m\ s^{-1}}$. (c) $\Delta p = 5.0 \times 6.0 = 30\,\mathrm{kg\ m\ s^{-1}}$, so $v = 30 / 2.0 = 15\,\mathrm{m\ s^{-1}}$. Twice the time at the same force gives twice the change of momentum and twice the final speed.

Question 6 (a) $\Delta p = 3.0 - 0 = 3.0\,\mathrm{kg\ m\ s^{-1}}$. (b) $F_{\mathrm{net}} = \Delta p / \Delta t = 3.0 / 0.0010 = 3000\,\mathrm{N}$. (c) $F_{\mathrm{net}} = 3.0 / 0.0030 = 1000\,\mathrm{N}$. (d) The same change of momentum delivered over three times the time requires one third of the force. For a fixed Δp, the average net force is smaller the longer the time available to make the change.

Question 7 (a) $100\,\mathrm{km\ h^{-1}} = 100 / 3.6 = 27.8\,\mathrm{m\ s^{-1}}$. (b) $p = 1600 \times 27.8 = 44\,480\,\mathrm{kg\ m\ s^{-1}} \approx 4.4 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$. (c) $50\,\mathrm{km\ h^{-1}} = 13.9\,\mathrm{m\ s^{-1}}$; $p = 1600 \times 13.9 = 22\,240\,\mathrm{kg\ m\ s^{-1}} \approx 2.2 \times 10^4\,\mathrm{kg\ m\ s^{-1}}$. Halving the speed halves the momentum, because p is proportional to v at fixed mass.

Question 8 (a) $v = p / m = 4.0 / 0.16 = 25\,\mathrm{m\ s^{-1}}$. (b) $25 \times 3.6 = 90\,\mathrm{km\ h^{-1}}$. This is a realistic fast-bowling speed — pace bowlers deliver at roughly $80\text{–}150\,\mathrm{km\ h^{-1}}$ — so a momentum of $4.0\,\mathrm{kg\ m\ s^{-1}}$ is a sensible size for a cricket ball.

Question 9 (a) Just before the bounce: $p=0.60\times(-6.0)=-3.6\,\text{kg m s}^{-1}$ (downward). Just after: $p=0.60\times 4.5=+2.7\,\text{kg m s}^{-1}$ (upward). (b) $\Delta p=p_{\text{final}}-p_{\text{initial}}=+2.7-(-3.6)=+6.3\,\text{kg m s}^{-1}$: a change of $6.3\,\text{kg m s}^{-1}$ directed upward. (c) Stopping dead: $\Delta p=0-(-3.6)=+3.6\,\text{kg m s}^{-1}$. The bounce ($6.3\,\text{kg m s}^{-1}$) is the larger change, because reversing direction turns the before and after momenta against each other so their magnitudes add.

Question 10 The road train has many times the mass of the car and the same speed, so from $p=mv$ it carries many times the momentum. Bringing either vehicle to rest means removing all of its momentum, so the road train must undergo a much larger change of momentum than the car. Since $\Delta p=F_{\text{net}}\Delta t$, for a given (similar) braking force the road train needs a much longer time to stop — and if it must stop in the same time as the car, it needs a much larger braking force. Either way the road train is harder to stop, precisely because it carries more momentum.

Question 11 (a) $p=mv=0.45\times 12=5.4\,\text{kg m s}^{-1}$. (b) The ball starts at rest, so the change of momentum equals the final momentum: $\Delta p=5.4\,\text{kg m s}^{-1}$, in the direction of the pass. (c) $F_{\text{net}}=\Delta p/\Delta t=5.4/0.050=108\,\text{N}\approx 1.1\times 10^2\,\text{N}$.

Question 12 The occupants' change of momentum is fixed by the crash: whatever stops the car removes the same momentum, from the car's speed before impact down to rest. A rigid car would stop in a very short time, and from $F_{\text{net}}=\Delta p/\Delta t$ a short stopping time means a very large average net force on the occupants. Crumple zones collapse progressively during the crash, so the car takes a longer time Δt to come to rest. The same change of momentum is spread over that longer time, so the average net force on the occupants is smaller and the risk of injury is reduced. (Airbags and stretching seatbelts do the same thing for the occupants themselves: longer stopping time, smaller force.)

Question 13 (a) $p=mv=1000\times 15=15\,000\,\text{kg m s}^{-1}=1.5\times 10^4\,\text{kg m s}^{-1}$. (b) The car keeps the same speed and direction, so its momentum is unchanged: $\Delta p=0$. Since $\Delta p=F_{\text{net}}\Delta t$ and $\Delta t=10\,\text{s}$ is not zero, the net force must be zero: $F_{\text{net}}=\Delta p/\Delta t=0$. Momentum only changes when a net force acts, so a body whose momentum is constant has zero net force on it.

Question 14 The statement is incorrect in both halves. A moving object carries **momentum**, not force: force is something that acts *on* an object, from another object, while momentum is something an object *has* because of its mass and velocity. After the ball leaves the bat, nothing pushes it along, and with no net force in the direction of motion its momentum in that direction does not change — the ball keeps moving because nothing acts to change its momentum, not because a stored force propels it. The second half fails for the same reason: when the ball does stop, it is because a force (friction, air, a fielder) has acted to change its momentum, not because a force inside it has "run out". Forces change momentum; they are not carried by it.

Question 15 (a) $p=85\times 8.0=680\,\text{kg m s}^{-1}$, in the direction of travel. (b) The rider ends at rest, so $\Delta p=0-680=-680\,\text{kg m s}^{-1}$: a change of $680\,\text{kg m s}^{-1}$ opposite to the motion. (c) $F_{\text{net}}=\Delta p/\Delta t=-680/4.0=-170\,\text{N}$: an average net force of $170\,\text{N}$ directed opposite to the motion, supplied by friction between the brakes, the tyres and the road.

Question 16 The faster ball has the larger velocity and the same mass, so from $p=mv$ it carries more momentum. Catching either ball means removing all of its momentum, so the faster ball undergoes the larger change of momentum Δp. Since $F_{\text{net}}=\Delta p/\Delta t$, and the hands stop the ball in about the same time Δt in each case, the larger change of momentum requires a larger average net force on the hands — and it is the larger force that stings more. (Catching the fast ball "softly", by letting the hands give, works precisely by increasing Δt to bring the force back down.)

Question 17 (a) $p=4000\times 12=48\,000\,\text{kg m s}^{-1}=4.8\times 10^4\,\text{kg m s}^{-1}$. (b) Stopping removes all of the momentum, so $|\Delta p|=48\,000\,\text{kg m s}^{-1}$, and $\Delta t=\Delta p/F_{\text{net}}=48\,000/6000=8.0\,\text{s}$. (c) $4.0\,\text{s}$. The change of momentum is the same, and $\Delta t=\Delta p/F_{\text{net}}$, so doubling the net force halves the stopping time.

Question 18 The jumper arrives with a fixed momentum and must be brought to rest, so the change of momentum to be delivered is the same whatever the surface. The foam mat compresses as the jumper lands, lengthening the stopping time Δt. Since $F_{\text{net}}=\Delta p/\Delta t$, the same change of momentum spread over a longer time means a smaller average net force on the jumper's body, and the landing is safe. On bare ground the jumper stops in a much shorter time, so the same change of momentum is delivered more quickly and the average net force is correspondingly larger — large enough to cause injury. The mat is the landing version of bent knees on a jump: spread a fixed change of momentum over a longer time to reduce the force.

Theory

Push a supermarket trolley and let go, and it rolls on for a while before friction brings it to a stop. Kick a football and it keeps moving after your foot has lost contact with it. Everyday experience seems to say that a force is needed to keep something moving. In the seventeenth century Isaac Newton showed that this reading is wrong: forces do not *keep* things moving — forces *change* motion. His three laws of motion, published in 1687, are the subject of this section, and they are the foundation for almost everything that follows in this chapter.

The study design states the task of this section in one line: *"apply Newton's three laws of motion to a body on which forces act"*, using the two relationships $a = \frac{F_{\text{net}}}{m}$ (Newton's second law) and $F_{\text{on A by B}} = -F_{\text{on B by A}}$ (Newton's third law) (VCE Physics Study Design, Unit 2, Area of Study 1).

Forces and net force. A **force** is a push or a pull exerted on one object by another. A force is a vector: it has a magnitude (its size) and a direction, and it is measured in **newtons** (N). Every force acts *on* an object and is exerted *by* some other object, and this section writes forces using that pair of names — for example, the force on a tram by the rails, or the force on the Earth by a falling ball. Section 7A introduced vectors and scalars, and Section 8C develops this naming convention into the full force-vector model.

When several forces act on a body, their combined effect is the **net force**, F_{net}, the sum of all the forces. In this section every force on the body acts along one straight line, so forces in one direction are taken as positive, forces in the opposite direction as negative, and the net force is found by adding with signs. (Forces at angles to each other are added by components in Section 8D.) The sign of F_{net} gives the direction of the net force.

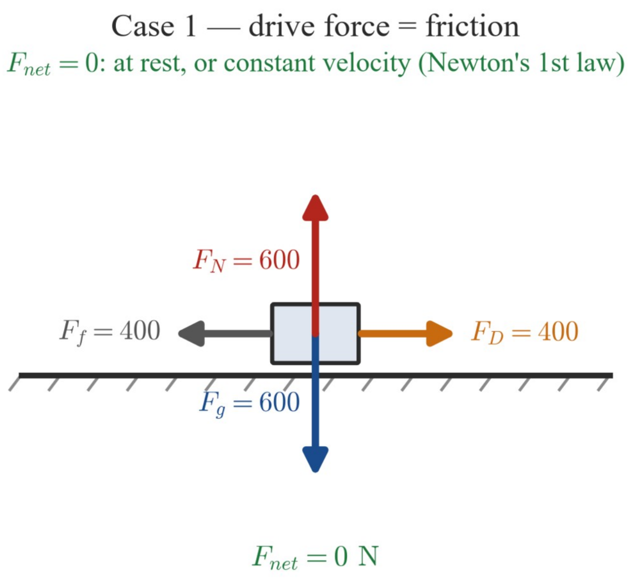
Throughout this section bodies are treated as sliding points — the forces may change a body's speed or direction, but they do not spin it. Turning effects (torques) are built in Section 8F.

Newton's first law. Newton's first law states the behaviour of a body when the forces on it balance:

> A body remains at rest, or continues to move with constant velocity in a straight line, unless a non-zero net force acts on it.

The tendency of a body to resist any change in its motion is called **inertia**, and the mass of a body is the measure of its inertia: the larger the mass, the harder it is to change the body's velocity.

A body with zero net force is in **equilibrium**. Equilibrium does not mean "not moving" — it means "not *changing* motion": the body is either at rest or moving at constant velocity in a straight line. The crucial consequence of the first law is that constant velocity needs *no* net force at all. A net force is needed only to *change* a velocity — to speed a body up, slow it down, or change its direction.



A tram travelling at constant velocity along a straight, level track is an example. Its motors provide a forward driving force, and friction and air resistance push backward. When the driving force equals the total resistance, the forces balance, $F_{\text{net}} = 0$, and the tram keeps its velocity. The trolley in the opening paragraph slows down precisely because *no one keeps pushing it*: friction gives it a backward net force, and that net force changes its motion.

Newton's second law. A non-zero net force produces an acceleration. Newton's second law gives the size and direction of that acceleration:

> The acceleration of a body is in the direction of the net force on it, with magnitude $a = \frac{F_{\text{net}}}{m}$, where m is the mass of the body.

The same law is usually written as

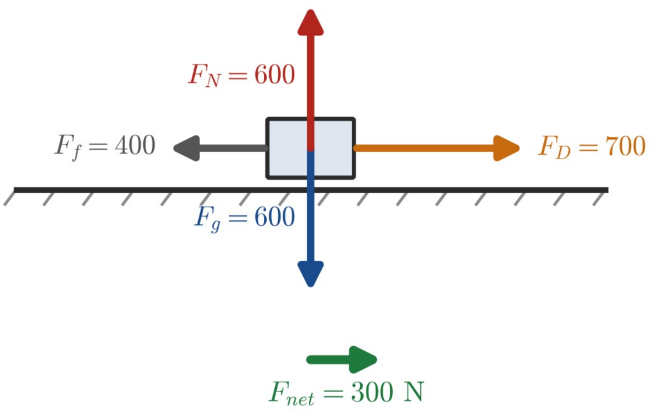
$$F_{\text{net}} = m a.$$

This is the equation that does most of the work in this section. It says two things at once:

- For a given mass, a larger net force gives a larger acceleration — doubling F_{net} doubles a.
- For a given net force, a larger mass gives a smaller acceleration — doubling m halves a. This is inertia in numbers: mass measures how strongly a body resists being accelerated.

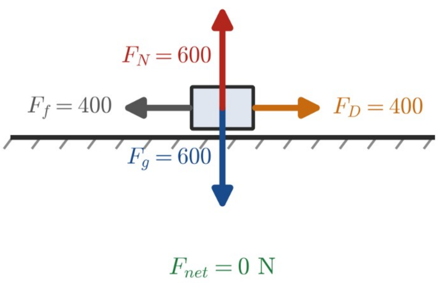
The unit of force is fixed by this equation: one **newton** is the net force that gives a mass of $1\,\text{kg}$ an acceleration of $1\,\text{m s}^{-2}$.

Case 2 — drive force > friction
$F_{net} > 0$: accelerating, $F_{net} = ma$ (Newton's 2nd law)

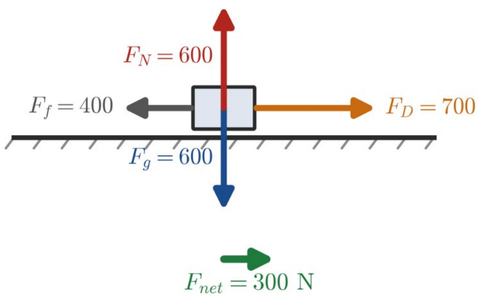


One point needs care: the acceleration is in the direction of the **net force**, which is not always the direction the body is moving. A braking car has a net force *opposite* to its velocity, and so it slows down. A body can move forward while accelerating backward — the velocity tells you which way the body is going; the net force tells you which way the velocity is *changing*.

Balanced: $F_D = F_f$
$a = 0$ — constant velocity



Unbalanced: $F_D > F_f$
$a = F_{net}/m$ — speeding up

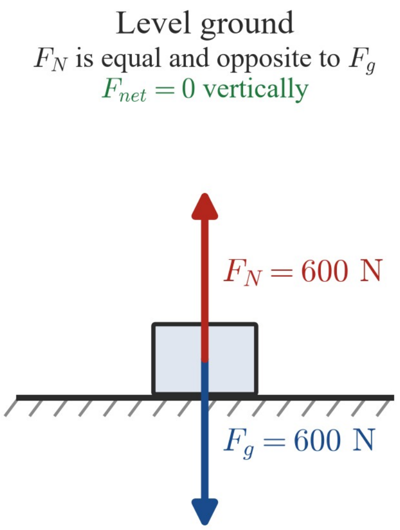


Two forces that appear everywhere. Two forces occur in nearly every situation in this section. Section 8C builds the full model of each; here they are introduced as far as Newton's laws need them.

- The **force due to gravity**, F_g, is the pull that the Earth exerts on every object near its surface. It acts vertically downward, and its magnitude is

$$F_g = mg,$$

  where $g = 9.8\ \text{N kg}^{-1}$ is the gravitational field strength near the surface of the Earth. (The same value written as $9.8\ \text{m s}^{-2}$ is the acceleration of a freely falling body.)
- The **normal force**, F_N, is the contact force a surface exerts on a body resting on it. It acts perpendicular to the surface. On level ground with no vertical acceleration, the vertical forces balance, so the first law gives $F_N = F_g$. That equality is a *consequence of balance*, not a rule — in an accelerating lift, as the worked examples show, $F_N \neq F_g$.



Two more forces appear in the questions. A **tension** is the pulling force carried by a rope, cable or string; it acts along the cable, on the body the cable is attached to. A **friction** force acts along a surface, opposing the motion (or attempted motion) of a body across it, and "resistance forces" (air resistance, rolling friction) likewise oppose motion. Driving forces — the forward push of the road on a car's tyres, or of the rails on a tram's wheels — are contact forces exerted by the surface on the vehicle.

Newton's third law. Forces never exist alone: they are always one half of an interaction between two bodies. Newton's third law states the symmetry of that interaction:

> If body A exerts a force on body B, then body B exerts a force of equal magnitude and opposite direction on body A.

Using the on/by naming convention, the two forces of a third-law pair are written

$$F_{\text{on A by B}} = -F_{\text{on B by A}}.$$

Read $F_{\text{on A by B}}$ as "the force on A by B": the first name is the body the force acts on, and the second is the body that provides it. The minus sign says the two forces point in opposite directions.

Every third-law pair has four properties:

1. the two forces are **equal in magnitude**;
2. they act in **opposite directions**;
3. they are the **same type** of force (both gravitational, or both contact forces);
4. they act on **different bodies** — one on A, one on B.

That last property is the one that is most often missed, and it is the whole point. Because the two forces act on *different* bodies, they can never cancel each other — cancellation only happens between forces that act on the *same* body. A swimmer moves forward because her hands exert a backward force on the water (a force on the water by the swimmer), and the water exerts an equal and opposite forward force on her (a force on the swimmer by the water). The forward force acts *on the swimmer*, so it is a real external force that accelerates her; the backward force acts on the water, and accelerates the water backward.

A crate resting on a floor shows two separate pairs at once. The force due to gravity on the crate by the Earth pairs with the gravitational force on the Earth by the crate. The normal force on the crate by the floor pairs with the contact force on the floor by the crate. Notice what is *not* a pair: the force due to gravity and the normal force both act *on the crate*. They are equal only because the crate is in equilibrium; they are different types of force, they act on the same body, and so they cannot be a third-law pair.

Equal forces do not mean equal effects. The second law applies to each body separately: $a = F / m$. The gravitational force the Earth exerts on a falling ball and the force the ball exerts on the Earth are equal in magnitude, but the ball's mass is small so its acceleration is obvious, while the Earth's mass is so enormous that its acceleration is far too small to notice.

Common wrong ideas. Three misconceptions are worth correcting now, because each one reappears in the questions below.

Wrong idea	What is wrong with it	The correction
"A moving body must have a forward net force on it."	Confuses *maintaining* a velocity with *changing* one.	Constant velocity means zero net force (first law). A forward driving force may be present, but it is balanced by resistance forces.
"In a collision, the bigger vehicle exerts the bigger force."	Breaks the third law.	The two forces are always equal in magnitude and opposite in direction. The different masses give different *accelerations*, not different forces.
"The two forces of a third-law pair cancel."	Cancelling requires forces on the *same* body.	The two forces act on different bodies, so each one affects the motion of the body it acts on. They never cancel.

A method for Newton's laws problems. Most problems in this section can be worked the same way:

1. **Choose the body** the question asks about.
2. **List the forces acting on that body** — and only those. Forces the body exerts on other things do not belong in the list. Sketch arrows if it helps; Section 8C turns this into the full force-diagram method.
3. **Choose a positive direction** along the line of the forces.
4. **Add the forces with signs** to find F_{net}.
5. If $F_{\text{net}} = 0$, the body is in equilibrium: use the first law (it is at rest or moving at constant velocity). If $F_{\text{net}} \neq 0$, use the second law: $a = \dfrac{F_{\text{net}}}{m}$, with the acceleration in the direction of the net force.

For two bodies that move together — a car towing a caravan, a locomotive pulling a wagon, two crates pushed in contact — the reliable two-step method is: **(1)** treat the pair as a single body and find the common acceleration from the net external force; **(2)** isolate one body and apply the second law to it to find the internal force between them (the tow-bar force, the coupling force, or the contact force). The third law then gives the matching force on the other body for free: same magnitude, opposite direction.

Where this leads. Section 8A showed that a net force changes a body's momentum; Newton's second law is the same idea written as a force–mass–acceleration relationship, and Section 8E returns to it as impulse. Sections 8C and 8D build the full vector model of forces and extend the method to forces at angles. The constant-acceleration equations of Section 7C combine with $F_{\text{net}} = m a$ whenever a question asks for times, distances or speeds as well as forces.

Throughout this section $g = 9.8\,\text{m s}^{-2}$ (equivalently $9.8\,\text{N kg}^{-1}$).

Worked examples

Example 1 — scaffolded

A tram of mass $30\,000\,\text{kg}$ travels along a straight, level stretch of track in Melbourne. Its motors provide a forward driving force of $9000\,\text{N}$, and resistance forces (friction and air resistance) total $3000\,\text{N}$. Calculate the normal force on the tram by the tracks, the net force on the tram, and its acceleration.

Working	Comment
$m = 30\,000\ \text{kg}$; driving force $9000\ \text{N}$ forward; resistance $3000\ \text{N}$ backward.	List the data. The motion and the net force are horizontal.
Vertical: no vertical acceleration, so $F_N = F_g = m\,g = 30\,000 \times 9.8 = 294\,000\ \text{N}$.	On level ground the normal force balances the force due to gravity (first law).
Horizontal: $F_{\text{net}} = 9000 - 3000 = 6000\ \text{N}$, forward.	Add the horizontal forces with signs: driving force positive, resistance negative.
$a = \dfrac{F_{\text{net}}}{m} = \dfrac{6000}{30\,000} = 0.20\ \text{m s}^{-2}$, forward.	Newton's second law: the acceleration is in the direction of the net force.

Example 2 — scaffolded

A crate of mass $25\ \text{kg}$ sits at rest on the horizontal floor of a warehouse. Identify all the forces acting on the crate, calculate their magnitudes, and state the Newton's third-law partner of each force using the on/by convention. Explain why the two forces on the crate are *not* a third-law pair.

Working	Comment
Forces on the crate: F_g downward (the force on the crate by the Earth) and F_N upward (the force on the crate by the floor).	List only the forces acting *on* the chosen body.
The crate is at rest, so $F_{\text{net}} = 0$ and $F_N = F_g = m\,g = 25 \times 9.8 = 245\ \text{N}$.	The first law (equilibrium) gives the size of the normal force.
Partner of F_g: the gravitational force on the Earth by the crate, $245\ \text{N}$, upward.	Third law: swap "on" and "by"; same type of force, same magnitude, opposite direction.
Partner of F_N: the contact force on the floor by the crate, $245\ \text{N}$, downward.	The same rule applied to the contact force.
F_g and F_N are not a pair: both act on the crate, and they are different types of force.	Third-law pairs always act on two *different* bodies; these are equal only because the crate is in equilibrium.

Example 3 — unscaffolded

A lift in a Melbourne office building has a total mass of $800\ \text{kg}$, including its passengers. It is supported by a single vertical cable. (a) Calculate the tension in the cable while the lift accelerates upward at $1.5\ \text{m s}^{-2}$. (b) Later, while moving upward, the lift slows down at $1.5\ \text{m s}^{-2}$. Calculate the tension in the cable during this slowing. Compare each tension with the total force due to gravity on the lift.

The total force due to gravity on the lift is

$$F_g = m\,g = 800 \times 9.8 = 7840\ \text{N},$$

directed downward. The two forces on the lift are the tension F_T (up) and F_g (down).

(a) Taking upward as positive, an upward acceleration of $1.5\ \text{m s}^{-2}$ requires an upward net force. By Newton's second law,

$$F_{\text{net}} = m\,a = 800 \times 1.5 = 1200\ \text{N}\ (\text{upward}).$$

Since $F_{\text{net}} = F_T - F_g$, the tension is

$$F_T = F_g + 1200 = 7840 + 1200 = 9040\ \text{N}.$$

(b) Moving upward but *slowing* means the acceleration is directed *downward*. With upward still positive, $a = -1.5\ \text{m s}^{-2}$, so

$$F_{\text{net}} = m\,a = 800 \times (-1.5) = -1200\,\text{N}\,,$$

a net force of $1200\,\text{N}$ downward. The tension is now

$$F_T = F_g - 1200 = 7840 - 1200 = 6640\,\text{N}\,.$$

∴ the tension is $9040\,\text{N}$ while the lift accelerates upward and $6640\,\text{N}$ while it slows. The tension is *greater* than the total force due to gravity ($7840\,\text{N}$) when the lift accelerates upward and *less* when the lift accelerates downward; the two are equal only when the lift is in equilibrium (at rest or moving at constant velocity).

Example 4 — unscaffolded

A freight train consists of a locomotive of mass $80\,000\,\text{kg}$ pulling a single wagon of mass $40\,000\,\text{kg}$ along a straight, level track. The driving force of the rails on the locomotive is $36\,000\,\text{N}$ forward, and resistance forces on the locomotive total $6000\,\text{N}$. Resistance forces on the wagon are negligible. Calculate the acceleration of the train and the force in the coupling between the locomotive and the wagon, and use Newton's third law to explain why the coupling force does not stop the train accelerating.

The two vehicles move together, so they share one acceleration. Treating the locomotive and wagon as a single body, the only external forces along the line of motion are the driving force forward and the locomotive's resistance backward:

$$F_{\text{net}} = 36\,000 - 6000 = 30\,000\,\text{N}\,.$$

The total mass is $80\,000 + 40\,000 = 120\,000\,\text{kg}$, so by Newton's second law

$$a = \frac{F_{\text{net}}}{m} = \frac{30\,000}{120\,000} = 0.25\,\text{m s}^{-2}\,(\text{forward})\,.$$

To find the coupling force, isolate the wagon. The only horizontal force on the wagon is the force of the coupling, directed forward — the force on the wagon by the locomotive. Applying the second law to the wagon alone,

$$F_{\text{on wagon by locomotive}} = m\,a = 40\,000 \times 0.25 = 10\,000\,\text{N}\,.$$

By Newton's third law, the wagon pulls back on the locomotive with the paired force — the force on the locomotive by the wagon — equal in magnitude ($10\,000\,\text{N}$) and opposite in direction (backward). These two forces do *not* cancel, because they act on different bodies. The locomotive still accelerates forward because its own forward driving force is larger than the two backward forces on it:

$$F_{\text{net on locomotive}} = 36\,000 - 6000 - 10\,000 = 20\,000\,\text{N} = 80\,000 \times 0.25\,.\ \checkmark$$

∴ the acceleration of the train is $0.25\,\text{m s}^{-2}$ forward, the coupling exerts $10\,000\,\text{N}$ on the wagon, and the wagon exerts $10\,000\,\text{N}$ back on the locomotive — a third-law pair acting on two different bodies, so it cannot cancel itself.

Practice questions

Question 1 *(scaffolded)* A car of mass $1200\,\text{kg}$ travels along a straight, level stretch of the Hume Freeway. The engine provides a forward driving force of $3000\,\text{N}$ and resistance forces total $600\,\text{N}$. (a) Calculate the net force on the car. (b) Calculate the acceleration of the car. (c) Later the car travels at constant velocity along the same road. State the net force on the car now, and justify your answer by name of the law you are using.

Question 2 *(scaffolded)* A shopper pushes a supermarket trolley along a straight, level aisle at constant velocity. A friction force of $25\,\text{N}$ opposes the motion. (a) State the acceleration of the trolley. (b) State the net force on the trolley. (c) Calculate the forward force the shopper applies to the trolley.

Question 3 *(scaffolded)* A cyclist and bicycle of total mass $90\,\text{kg}$ accelerate at $1.5\,\text{m s}^{-2}$ along a straight, level path. (a) Calculate the net force on the cyclist and bicycle. (b) The pedalling force is $200\,\text{N}$ forward. Calculate the total resistance force. (c) The cyclist then stops pedalling. State what happens to the speed of the bicycle and justify your answer in terms of the forces and Newton's second law.

Question 4 *(scaffolded)* A crane at the Port of Melbourne lifts a shipping container of mass $2000\,\mathrm{kg}$ vertically upward with an acceleration of $1.2\,\mathrm{m\ s^{-2}}$. (a) Calculate the force due to gravity on the container. (b) Taking upward as positive, write the net-force equation for the container in terms of the cable tension F_T and the force due to gravity F_g . (c) Calculate the tension in the cable.

Question 5 *(scaffolded)* In an investigation, two supermarket trolleys are pushed along a level aisle with the same net force of $30\,\mathrm{N}$. Trolley A has a mass of $15\,\mathrm{kg}$; trolley B is loaded with shopping and has a mass of $60\,\mathrm{kg}$. (a) Calculate the acceleration of trolley A. (b) Calculate the acceleration of trolley B. (c) State the relationship between mass and acceleration for a constant net force, and link it to the idea of inertia.

Question 6 *(scaffolded)* A football rests motionless in a player's hands. (a) Name the two forces acting on the ball, using the on/by convention, and state the direction of each. (b) State the Newton's third-law partner of the force due to gravity on the ball: name the force, state the body it acts on, and give its direction. (c) Explain why the two forces named in (a) are not a Newton's third-law pair.

Question 7 A banner of mass $4.0\,\mathrm{kg}$ hangs at rest from a single vertical cable strung above a street in Melbourne. (a) Calculate the force due to gravity on the banner. (b) Calculate the tension in the cable — the force on the banner by the cable — justifying your answer with reference to Newton's first law. (c) State the Newton's third-law partner of the tension force on the banner by the cable: name the force, the body it acts on, and its direction.

Question 8 A car of mass $1200\,\mathrm{kg}$ tows a caravan of mass $800\,\mathrm{kg}$ along a straight, level road. The car's engine provides a forward driving force of $3000\,\mathrm{N}$. Resistance forces are $700\,\mathrm{N}$ on the car and $300\,\mathrm{N}$ on the caravan. (a) Calculate the acceleration of the car and caravan. (b) Calculate the force in the tow bar — the force on the caravan by the car. (c) State the Newton's third-law partner of the force found in (b), giving its magnitude and direction.

Question 9 A car of mass $1000\,\mathrm{kg}$ travels at $20\,\mathrm{m\ s^{-1}}$ along a straight, level country road when the driver brakes hard. The braking force of the road on the tyres is constant at $5000\,\mathrm{N}$, directed backward. (a) Calculate the acceleration of the car, stating its direction relative to the motion. (b) Using the constant-acceleration equation $v=u+at$ from Section 7C, calculate the time taken for the car to stop. (c) State the direction of the net force relative to the velocity, and say what this tells you about the motion.

Question 10 A passenger of mass $60\,\mathrm{kg}$ stands on the floor of a lift in a Melbourne hotel. (a) Calculate the normal force on the passenger by the floor when the lift accelerates upward at $2.0\,\mathrm{m\ s^{-2}}$. (b) Calculate the normal force on the passenger by the floor when the lift accelerates downward at $2.0\,\mathrm{m\ s^{-2}}$. (c) State when the normal force is equal in magnitude to the force due to gravity on the passenger, justifying your answer.

Question 11 Two crates sit in contact on a smooth, level warehouse floor: crate A, of mass $40\,\mathrm{kg}$, and crate B, of mass $60\,\mathrm{kg}$. A worker pushes crate A horizontally with a force of $200\,\mathrm{N}$, pushing both crates across the floor. (a) Calculate the acceleration of the crates. (b) Calculate the net force on crate B. (c) Hence state the magnitude of the contact force on crate B by crate A, explaining your reasoning. (d) State the Newton's third-law partner of the force in (c), giving its magnitude and direction.

Question 12 A football player kicks a football of mass $0.45\,\mathrm{kg}$. During the brief contact, the foot exerts an average forward force of $90\,\mathrm{N}$ on the ball. (a) Calculate the acceleration of the ball during the contact. (b) State the force on the foot by the ball during the contact, giving its magnitude and direction. (c) The force in (b) acts on the player's foot and leg, which have a much larger mass than the ball. Use $a=\frac{F_{\text{net}}}{m}$ to explain why the acceleration of the foot is much smaller than the acceleration of the ball.

Question 13 A suburban passenger train in Melbourne has a total mass of $240\,000\,\mathrm{kg}$. Starting from rest at a station, the driving force of the rails on the train is $150\,000\,\mathrm{N}$ forward and resistance forces total $30\,000\,\mathrm{N}$. (a) Calculate the acceleration of the train. (b) Calculate the time the train takes to reach $20\,\mathrm{m\ s^{-1}}$. (c) Once the train travels at constant velocity, state what must be true about the driving force, and name the law that tells you.

Question 14 A car travelling along a straight road brakes sharply. An unbelted passenger continues to move forward relative to the car. With reference to Newton's first law, explain why the passenger moves forward relative to the car, and identify the real force a fastened seatbelt provides, stating the body that exerts it and its direction.

Question 15 A student says: *"if a tram is moving forward, there must be a forward net force on it."* Evaluate this claim with reference to Newton's first and second laws. In your answer, say what is true instead, and state what a genuinely forward net force would do to the tram.

Question 16 A swimmer propels herself forward through the water by pushing water backwards with her hands. Use Newton's third law, and "force on … by …" language, to explain how this pushes her forward, and explain why the two forces involved do not cancel each other.

Question 17 A large truck and a small car collide head-on at low speed on a country road. (a) Compare the magnitudes of the force on the truck by the car and the force on the car by the truck, justifying your answer. (b) Compare the magnitudes of the accelerations of the two vehicles during the collision, justifying your answer with $a = \frac{F_{\text{net}}}{m}$.

Question 18 A rocket accelerates upward from a launch pad by expelling hot gas downward from its engines. Use Newton's third law to explain why the rocket accelerates upward, and state whether the rocket can still accelerate in the vacuum of space, far from any atmosphere, justifying your answer.

Question 19 A horse pulls a cart along a level road. A student argues: *"By Newton's third law, the cart pulls backward on the horse with a force equal to the horse's pull, so the two forces cancel and the cart can never move."* Identify the error in this argument and explain how the cart can accelerate, making clear which forces act on which body.

Question 20 Car seats are fitted with headrests behind the occupant's head. With reference to Newton's first law, explain why the headrest matters when a stationary car is hit from behind and suddenly accelerates forward, naming the force the headrest provides and its direction.

Answers

Question 1 (a) $2400\,\mathrm{N}$ forward; (b) $2.0\,\mathrm{m\ s^{-2}}$ forward; (c) $0\,\mathrm{N}$ — constant velocity means equilibrium (Newton's first law). **Question 2** (a) $0\,\mathrm{m\ s^{-2}}$; (b) $0\,\mathrm{N}$; (c) $25\,\mathrm{N}$ forward. **Question 3** (a) $135\,\mathrm{N}$ forward; (b) $65\,\mathrm{N}$; (c) the speed decreases: with no pedalling force, resistance gives a backward net force, so the acceleration is backward. **Question 4** (a) $19\,600\,\mathrm{N}$ down; (b) $F_T - F_g = ma$; (c) $22\,000\,\mathrm{N}$. **Question 5** (a) $2.0\,\mathrm{m\ s^{-2}}$; (b) $0.50\,\mathrm{m\ s^{-2}}$; (c) for a constant net force, acceleration is inversely proportional to mass — a larger mass has more inertia and accelerates less. **Question 6** (a) the force due to gravity on the ball by the Earth (down) and the contact force on the ball by the hands (up), equal in magnitude; (b) the gravitational force on the Earth by the ball, directed upward; (c) both forces act on the same body (the ball), so they cannot be a third-law pair. **Question 7** (a) $39\,\mathrm{N}$; (b) $39\,\mathrm{N}$; (c) the force on the cable by the banner, $39\,\mathrm{N}$, downward. **Question 8** (a) $1.0\,\mathrm{m\ s^{-2}}$ forward; (b) $1100\,\mathrm{N}$ forward; (c) the force on the car by the caravan, $1100\,\mathrm{N}$ backward. **Question 9** (a) $5.0\,\mathrm{m\ s^{-2}}$ opposite to the motion; (b) $4.0\,\mathrm{s}$; (c) the net force is opposite to the velocity, so the car slows down. **Question 10** (a) $708\,\mathrm{N}$; (b) $468\,\mathrm{N}$; (c) when the lift is at rest or moving at constant velocity ($a=0$, equilibrium). **Question 11** (a) $2.0\,\mathrm{m\ s^{-2}}$; (b) $120\,\mathrm{N}$; (c) $120\,\mathrm{N}$ — the contact force is the only horizontal force on crate B; (d) the contact force on crate A by crate B, $120\,\mathrm{N}$, directed backward (opposite to the motion). **Question 12** (a) $200\,\mathrm{m\ s^{-2}}$ forward; (b) $90\,\mathrm{N}$ backward; (c) same magnitude of force on a much larger mass gives a much smaller acceleration. **Question 13** (a) $0.50\,\mathrm{m\ s^{-2}}$ forward; (b) $40\,\mathrm{s}$; (c) the driving force equals the total resistance force, so that $F_{\text{net}}=0$ (Newton's first law). **Question 14** The passenger continues forward by inertia while the car slows; the seatbelt provides a backward force on the passenger by the belt; see solutions for the full model answer. **Question 15** The claim is incorrect: constant velocity requires zero net force; see solutions for the full model answer. **Question 16** The water pushes forward on the swimmer with a force equal and opposite to the backward force of the swimmer on the water; the two forces act on different bodies, so they do not cancel; see solutions for the full model answer. **Question 17** (a) the two forces are equal in magnitude (Newton's third law); (b) the car has the larger acceleration, because the same force acts on the smaller mass. **Question 18** The expelled gas pushes upward on the rocket (Newton's third law pair of the rocket's downward push on the gas); yes — no air is needed; see solutions for the full model answer. **Question 19** The two third-law forces act on different bodies, so they cannot cancel; see solutions for the full model answer. **Question 20** The head tends to stay at rest by inertia while the car and seat accelerate forward; the headrest provides a forward force on the head; see solutions for the full model answer.

Fully-worked solutions

Question 1 $m=1200\,\mathrm{kg}$; driving force $3000\,\mathrm{N}$; resistance $600\,\mathrm{N}$. (a) Taking forward as positive, $F_{\text{net}}=3000-600=2400\,\mathrm{N}$, forward. (b) $a=\frac{F_{\text{net}}}{m}=\frac{2400}{1200}=2.0\,\mathrm{m\ s^{-2}}$, forward. (c) At constant velocity the car is in equilibrium, so by Newton's first law the net force is $0\,\mathrm{N}$: the driving force has dropped until it exactly balances the resistance forces.

Question 2 Constant velocity; friction $25\,\mathrm{N}$. (a) Constant velocity means no change of motion, so the acceleration is $0\,\mathrm{m\ s^{-2}}$. (b) By Newton's first law, $F_{\text{net}}=0\,\mathrm{N}$. (c) With zero net force, the forward push balances the friction: the applied force is $25\,\mathrm{N}$ forward.

Question 3 $m=90\,\mathrm{kg}$; $a=1.5\,\mathrm{m\ s^{-2}}$. (a) $F_{\text{net}}=ma=90\times1.5=135\,\mathrm{N}$, forward. (b) $F_{\text{net}}=F_{\text{drive}}-F_{\text{resistance}}$, so $F_{\text{resistance}}=200-135=65\,\mathrm{N}$. (c) With no pedalling force, the only horizontal force is the resistance force, directed backward. The net force is now $65\,\mathrm{N}$ backward, so by Newton's second law the acceleration is backward — opposite to the velocity — and the bicycle slows down.

Question 4 $m=2000\,\mathrm{kg}$; $a=1.2\,\mathrm{m\ s^{-2}}$ upward. (a) $F_g=mg=2000\times9.8=19\,600\,\mathrm{N}$, downward. (b) Taking upward as positive: $F_T-F_g=ma$. (c) $F_T=F_g+ma=19\,600+2000\times1.2=19\,600+2400=22\,000\,\mathrm{N}$.

Question 5 $F_{\text{net}}=30\,\mathrm{N}$ in both cases. (a) $a_A=\frac{30}{15}=2.0\,\mathrm{m\ s^{-2}}$. (b) $a_B=\frac{30}{60}=0.50\,\mathrm{m\ s^{-2}}$. (c) For a constant net force, the acceleration is inversely proportional to the mass: $a=\frac{F_{\text{net}}}{m}$. The loaded trolley has four times the mass and one quarter of the acceleration. This is inertia: the larger the mass, the more the body resists a change in its motion, so the same force produces a smaller change.

Question 6 The ball is motionless, so it is in equilibrium. (a) The two forces on the ball are the **force due to gravity on the ball by the Earth**, directed downward, and the **contact force on the ball by the hands**, directed upward. Because the ball is at rest, the two forces are equal in magnitude. (b) The third-law partner of the force due to gravity on the ball by the Earth is the **gravitational force on the Earth by the ball**: it acts on the Earth, is directed upward (toward the ball), and has the same magnitude. (c) A third-law pair always consists of forces acting on two *different* bodies. Both forces in (a) act on the *same* body — the ball — so they cannot be a third-law pair. They are equal only because the ball is in equilibrium, and they are different types of force (gravitational and contact).

Question 7 $m=4.0\,\mathrm{kg}$, at rest. (a) $F_g=mg=4.0\times 9.8=39.2\,\mathrm{N}$, which rounds to $39\,\mathrm{N}$ to two significant figures, directed downward. (b) The banner is at rest, so by Newton's first law it is in equilibrium and the net force on it is zero. The only forces on the banner are the tension (up) and the force due to gravity (down), so the tension in the cable — the force on the banner by the cable — equals $F_g=39\,\mathrm{N}$. (c) The third-law partner of the force on the banner by the cable is the **force on the cable by the banner**: same magnitude ($39\,\mathrm{N}$), opposite direction — downward.

Question 8 Car $1200\,\mathrm{kg}$; caravan $800\,\mathrm{kg}$; driving force $3000\,\mathrm{N}$; resistance $700\,\mathrm{N}$ (car) and $300\,\mathrm{N}$ (caravan). (a) Treat the car and caravan as one body. The net external force is $F_{\text{net}}=3000-700-300=2000\,\mathrm{N}$ and the total mass is $1200+800=2000\,\mathrm{kg}$, so

$$a=\frac{2000}{2000}=1.0\,\mathrm{m\,s^{-2}}\,(\text{forward}).$$

(b) Isolate the caravan. The horizontal forces on it are the tow-bar force F (forward, the force on the caravan by the car) and its resistance of $300\,\mathrm{N}$ (backward):

$$F-300=800\times 1.0\Rightarrow F=800+300=1100\,\mathrm{N}.$$

(Check with the car: $3000-700-1100=1200\,\mathrm{N}=1200\times 1.0$. ✓) (c) By Newton's third law, the partner is the **force on the car by the caravan**: $1100\,\mathrm{N}$, directed backward.

Question 9 $m=1000\,\mathrm{kg}$; $u=20\,\mathrm{m\,s^{-1}}$; braking force $5000\,\mathrm{N}$ backward. (a) Taking the direction of motion as positive, the braking force is negative:

$$a=\frac{F_{\text{net}}}{m}=\frac{-5000}{1000}=-5.0\,\mathrm{m\,s^{-2}}.$$

The acceleration is $5.0\,\mathrm{m\,s^{-2}}$ directed opposite to the motion. (b) With $v=0$ at rest, $v=u+at$ rearranges to

$$t=\frac{v-u}{a}=\frac{0-20}{-5.0}=4.0\,\mathrm{s}.$$

(c) The net force is directed opposite to the velocity. A net force opposite to the motion produces an acceleration opposite to the motion, so the car slows down — its velocity is forward while its acceleration is backward.

Question 10 $m=60\,\mathrm{kg}$. The vertical forces on the passenger are the normal force F_N (up) and the force due to gravity $F_g=mg=60\times 9.8=588\,\mathrm{N}$ (down). Take upward as positive and apply Newton's second law: $F_N-F_g=ma$, so $F_N=m(g+a)$. (a) Accelerating upward, $a=+2.0\,\mathrm{m\,s^{-2}}$:

$$F_N=60(9.8+2.0)=60\times 11.8=708\,\mathrm{N}.$$

(b) Accelerating downward, $a=-2.0\,\mathrm{m\,s^{-2}}$:

$$F_N=60(9.8-2.0)=60\times 7.8=468\,\mathrm{N}.$$

(c) $F_N = F_g$ only when $a = 0$ — that is, when the lift is at rest or moving at constant velocity (up or down). In equilibrium the vertical forces balance (Newton's first law). When the lift accelerates upward the normal force exceeds the force due to gravity; when it accelerates downward, the normal force is less.

Question 11 $m_A = 40\,\text{kg}$; $m_B = 60\,\text{kg}$; applied force $200\,\text{N}$ on crate A; smooth floor. (a) Treating the two crates as one body of mass $100\,\text{kg}$:

$$a = \frac{200}{40 + 60} = 2.0\,\text{m s}^{-2}.$$

(b) The net force on crate B is $F_{\text{net}} = m_B a = 60 \times 2.0 = 120\,\text{N}$, in the direction of the push. (c) The only horizontal force on crate B is the contact force from crate A (the floor is smooth), so the contact force on crate B by crate A equals the net force on crate B: $120\,\text{N}$. (d) By Newton's third law the partner is the **contact force on crate A by crate B**: $120\,\text{N}$, directed backward — opposite to the motion. (Check: the net force on crate A is then $200 - 120 = 80\,\text{N} = 40 \times 2.0$. ✓)

Question 12 $m = 0.45\,\text{kg}$; average force $90\,\text{N}$ forward. (a) $a = \frac{F_{\text{net}}}{m} = \frac{90}{0.45} = 200\,\text{m s}^{-2}$, forward, during the contact. (b) By Newton's third law the ball exerts an equal and opposite force on the foot: the force on the foot by the ball is $90\,\text{N}$, directed backward. (c) The two forces of the pair have equal magnitudes, but the acceleration each body receives is $a = \frac{F}{m}$ using its own mass. The player's foot and leg together have a mass far greater than the $0.45\,\text{kg}$ ball, so the same $90\,\text{N}$ gives the foot an acceleration far smaller than the ball's $200\,\text{m s}^{-2}$. Equal forces do not mean equal accelerations.

Question 13 $m = 240\,000\,\text{kg}$; driving force $150\,000\,\text{N}$; resistance $30\,000\,\text{N}$. (a) $F_{\text{net}} = 150\,000 - 30\,000 = 120\,000\,\text{N}$ forward, so

$$a = \frac{120\,000}{240\,000} = 0.50\,\text{m s}^{-2}\,(\text{forward}).$$

(b) From rest, $v = u + at$ gives

$$t = \frac{v - u}{a} = \frac{20 - 0}{0.50} = 40\,\text{s}.$$

(c) At constant velocity the train is in equilibrium, so by Newton's first law $F_{\text{net}} = 0$: the driving force must equal the total resistance force.

Question 14 A full-mark answer would include:

- before braking, the passenger is moving forward with the car;
- when the car brakes, a backward force acts on the *car* (from the road on the tyres), slowing the car — but no such backward force acts directly on the unbelted passenger;
- by Newton's first law the passenger continues to move forward in a straight line at constant velocity (inertia), while the car slows underneath them — so the passenger moves forward *relative to the car*;
- the passenger is not "thrown forward" by any forward force: the forward motion is simply inertia continuing while the car decelerates;
- a fastened seatbelt supplies the missing force: a **backward force on the passenger by the seatbelt**, providing the net force needed to slow the passenger down with the car.

Question 15 The claim is incorrect. A full-mark answer would include:

- Newton's first law: a body moving at constant velocity has zero net force on it — motion does not require a net force, only *changes* in motion do;

- Newton's second law: $a = \frac{F_{net}}{m}$ — a forward net force would produce a forward acceleration, meaning the tram would be *speeding up*, not merely moving;
- what is true instead: a tram moving forward at constant velocity has balanced forces — a forward driving force matched by backward resistance forces (friction and air resistance), so $F_{net} = 0$;
- a genuinely forward net force would make the tram accelerate — its velocity would increase.

Question 16 A full-mark answer would include:

- the swimmer's hands exert a backward force on the water — the force **on the water by the swimmer**;
- by Newton's third law, the water exerts an equal and opposite force on the swimmer — the force **on the swimmer by the water** — directed forward;
- this forward force on the swimmer is a real external force, and it is what accelerates her forward (Newton's second law);
- the two forces do not cancel because they act on different bodies — one on the water, one on the swimmer. Cancellation is only possible between forces acting on the same body.

Question 17 (a) By Newton's third law, the force on the truck by the car and the force on the car by the truck are a force pair: they are **equal in magnitude** and opposite in direction at every instant during the collision. The size and mass of the vehicles make no difference to this equality. (b) By Newton's second law, $a = \frac{F_{net}}{m}$. The two vehicles experience forces of equal magnitude, but the car has the smaller mass, so the car has the **larger acceleration** (a larger change in its velocity during the collision). Equal forces produce different accelerations because the masses are different.

Question 18 A full-mark answer would include:

- the rocket engines exert a downward force on the hot gas, expelling it — the force **on the gas by the rocket**;
- by Newton's third law, the gas exerts an equal and opposite upward force on the rocket — the force **on the rocket by the gas**;
- this upward force is a real external force on the rocket; when it exceeds the force due to gravity on the rocket, the net force is upward and the rocket accelerates upward;
- the rocket does **not** push on the ground or on the air — so it can accelerate in the vacuum of space, where there is no air at all. The interaction is entirely between the rocket and its own expelled gas, and that pair of forces exists in a vacuum just as it does in the atmosphere.

Question 19 The error is treating the two third-law forces as though they acted on the same body. A full-mark answer would include:

- the horse's pull acts **on the cart**, and the cart's pull acts **on the horse** — the two forces act on different bodies, so they can never cancel each other;
- forces cancel only when they act on the *same* body: whether the cart accelerates depends on the net force *on the cart*;
- the cart accelerates forward because the forward pull of the horse on the cart is larger than the backward resistance forces (friction) on the cart — so the net force on the cart is forward;
- the horse can pull because the ground pushes it forward: the horse's hooves push backward on the ground, and by Newton's third law the ground pushes forward on the horse (a friction force). When this forward force exceeds the backward pull of the cart on the horse, the horse also accelerates forward;
- for the horse-and-cart system as a whole, the external forward force of the ground on the horse exceeds the external resistance forces, so the whole system accelerates.

Question 20 A full-mark answer would include:

- before the impact, the occupant (including their head) is at rest;
- when the car is hit from behind, the car and the seat are suddenly accelerated forward;

- by Newton's first law, the head tends to remain at rest (inertia) while the seat pushes the torso forward — without support, the head lags behind the torso and the neck is forced into a damaging bend;
- the headrest supplies the missing force: a **forward force on the head by the headrest**, which accelerates the head forward together with the rest of the body, keeping the head and torso moving together.

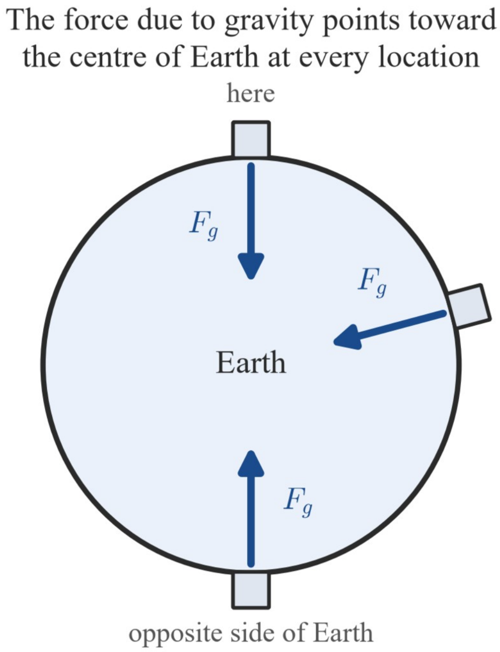
Theory

Forces are vectors. In Section 7A you sorted the quantities of motion into scalars and vectors, and a force is a vector: it has a **magnitude**, measured in newtons (N), and a **direction**. Section 8B showed how forces explain changes in motion through Newton's three laws of motion. To use a force in a calculation or a diagram you need three things: its magnitude, its direction, and the point at which it acts. This section sets up two tools the rest of the book — and Units 3 & 4 — rely on constantly: the force due to gravity, F_g, and a labelling convention that tells you exactly which object any force acts on and which object causes it.

The force due to gravity, F_g. The Earth attracts every object around it: a ball released from your hand accelerates downward, and the cause of that acceleration is the Earth's pull. The VCE Physics study design fixes the language this book uses for that pull: it is called the **force due to gravity** and is written F_g. The study design's own key knowledge models it as

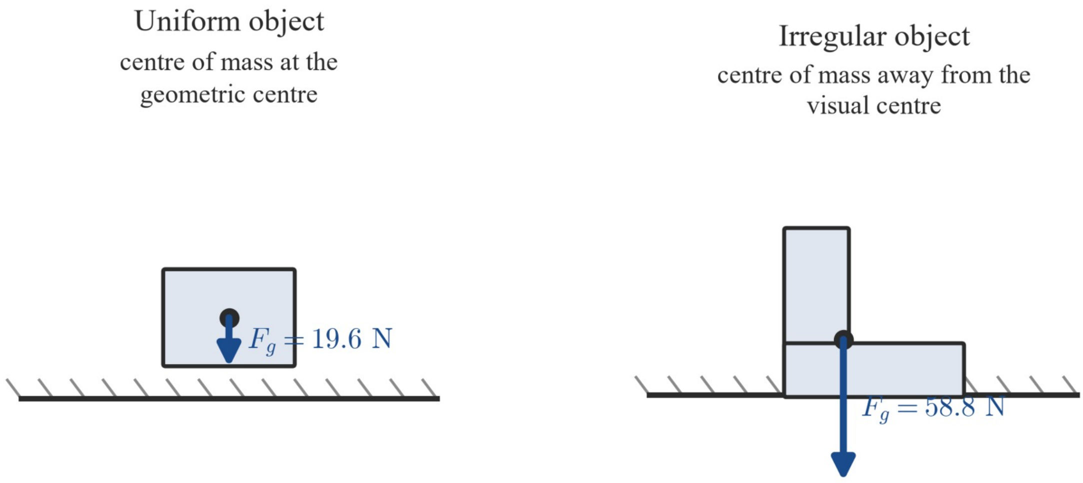
$$F_{\text{on body by Earth}} = mg,$$

where m is the object's mass in kilograms and g is the **gravitational field strength**, $9.8\ \text{N kg}^{-1}$ near the surface of Earth. The direction of F_g is toward the centre of the Earth, and the point at which it acts is the object's **centre of mass**.



'Down' is a direction toward Earth's centre. The word *down* behaves like the frame- dependent words discussed throughout this series: by itself it is incomplete, and once the location is stated it is exact. *Down* means *toward the centre of the Earth from where the object is*. On the figure above, the two F_g arrows at opposite sides of the Earth point in opposite directions in space, and both are correctly called *down*. From here on, every time this book says a force is downward, it means toward the centre of the Earth.

The centre of mass. The **centre of mass** of a body is the single point at which the whole mass of the body can be treated as gathered, for the purpose of the force due to gravity. For a uniform object with a symmetric shape the centre of mass is at the geometric centre; for an irregular object it lies elsewhere, as the right-hand panel shows. (For some shapes — a ring, a boomerang — the centre of mass is not even inside the material.) Wherever the centre of mass is, F_g is drawn as one arrow starting at that point and pointing toward the centre of the Earth.



F_g acts at the centre of mass; arrow length $\propto$ magnitude

What g tells you. $g = 9.8\,\mathrm{N\ kg^{-1}}$ says that near the surface of Earth, every kilogram of mass is attracted with a force of $9.8\,\mathrm{N}$. The force due to gravity is therefore proportional to mass: double the mass and F_g doubles. Note the unit: when g is doing the work of a **field strength** — force per unit mass — it is written in $\mathrm{N\ kg^{-1}}$. When the same quantity appears as the acceleration of a body in free fall near Earth's surface (Section 7C), it is written $9.8\,\mathrm{m\ s^{-2}}$. One quantity, two roles: $\mathrm{N\ kg^{-1}}$ for the field strength, $\mathrm{m\ s^{-2}}$ for the acceleration.

The value of g in this book. This book uses $g = 9.8\,\mathrm{N\ kg^{-1}}$ $(= 9.8\,\mathrm{m\ s^{-2}})$, the value printed in the Unit 2 key knowledge of the VCE Physics study design. The VCAA Formula Sheet, which is a Units 3 & 4 document, gives $g = 9.81\,\mathrm{m\ s^{-2}}$. The two values differ by about 0.1%, far less than the effect of rounding to two or three significant figures, so in practice they give the same recorded answer. At Units 3 & 4 you will use 9.81; the change is taught here, in Example 4, so it is never a surprise later.

Units 3 & 4 preview. At Units 3 & 4, g is no longer treated as a constant near Earth's surface: the gravitational field weakens with distance from Earth, and the force due to gravity is calculated from Newton's law of universal gravitation. The force-vector convention taught in this section carries across unchanged, and using it correctly is a marked requirement in VCAA assessment. Content under this banner is not assessed in this book's topic tests or practice examinations.

Mass and the force due to gravity are different quantities. It is easy to blur mass and F_g because they are proportional, but they are different kinds of quantity.

	Mass m	Force due to gravity F_g
What it is	the amount of matter in the body (a scalar)	the Earth's pull on the body (a vector)
Unit	kilogram (kg)	newton (N)
Relationship	—	$F_g = mg$
If the body were at the Moon's surface, where $g \approx 1.6\,\mathrm{N\ kg^{-1}}$	unchanged	smaller, because g is smaller

A person of mass $60\,\mathrm{kg}$ has $F_g = 60 \times 9.8 = 588\,\mathrm{N}$, $5.9 \times 10^2\,\mathrm{N}$ to two significant figures, on Earth, and $F_g = 60 \times 1.6 = 96\,\mathrm{N}$ at the Moon's surface, while the mass stays $60\,\mathrm{kg}$ in both places. The force changes with location; the mass does not.

Drawing a force: magnitude, direction, point of application. Because a force is a vector, a correct diagram shows all three of its properties: the length of the arrow is drawn proportional to the magnitude (and the value is labelled), the arrowhead shows the direction, and the arrow starts at the **point of application** — the place where the force acts. For a contact force this is where the contact happens: pushing a door near the handle and pushing it near the hinges have different effects, even for the same push. For the force due to gravity the point of application is the centre of mass. You will add forces as vectors, and separate them into components, in Section 8D; every force diagram from here on uses this drawing convention.

The 'force on A by B' convention. Every force is an interaction between two objects: one object acts on another. The house convention — the study design's own — names both objects every time a force is labelled:

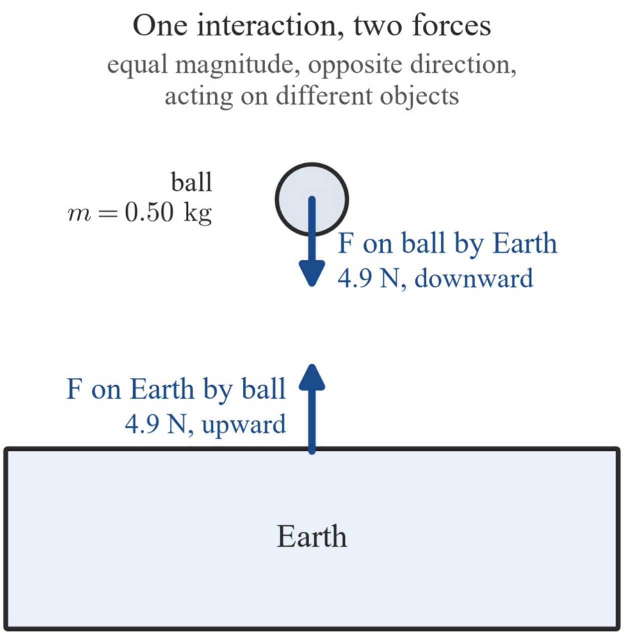
$$F_{\text{on A by B}} \text{ means the force on object A exerted by object B.}$$

The object that *feels* the force is named first; the object that *causes* it is named second. So the Earth's pull on a book is $F_{\text{on book by Earth}}$, and the upward push of a table on the same book is $F_{\text{on book by table}}$. (The force in the normal direction between two surfaces in contact is always called the **normal force** in this book, and Section 8D uses it fully.) The discipline pays every time forces are combined: only forces acting *on the same object* may be added together, and a label written in the convention makes it impossible to forget which object each force acts on.

Interaction pairs. If object A exerts a force on object B, then B at the same instant exerts a force on A of the same magnitude and opposite direction:

$$F_{\text{on A by B}} = -F_{\text{on B by A}}.$$

The minus sign means *opposite direction*: both sides are vectors, and the two forces have equal magnitude. Section 8B applies this statement as Newton's third law of motion; here it is pinned down as a labelling rule. The two halves of an interaction pair always share four features: equal magnitude, opposite direction, the same kind of interaction (both gravitational, or both contact pushes), and different objects acted upon. That last feature is the one most often lost: the two forces of a pair **never act on the same object**, so they never cancel each other out.



The figure shows the gravitational pair between a $0.50\,\text{kg}$ ball and the Earth. The force on the ball by the Earth is $0.50 \times 9.8 = 4.9\,\text{N}$, downward; the force on the Earth by the ball is the same $4.9\,\text{N}$, upward. The force on the ball accelerates the ball noticeably; the equal force on the Earth accelerates the Earth by far too little to observe, because the Earth's mass is enormous ($a = F_{\text{net}} / m$, Section 8B). Equal forces, very unequal effects — which is exactly why the labels must say which object each force acts on.

Worked examples

Example 1 — scaffolded

A cricket ball of the kind used in Australian professional cricket has a mass of $0.16\,\text{kg}$. (a) Write the relationship between the magnitude of the force due to gravity and the mass of the ball. (b) Calculate the magnitude of the force due to gravity on the ball. (c) State the direction of this force and the point at which it acts.

Working	Comment
$F_{\text{on ball by Earth}} = m\,g$	The study-design relationship; $g = 9.8\,\text{N kg}^{-1}$ near Earth's surface.
$F_g = 0.16 \times 9.8 = 1.568\,\text{N}$	Substitute with the mass in kilograms.
$F_g = 1.6\,\text{N}$ (2 significant figures)	0.16 and 9.8 each have 2 significant figures, so the answer is limited to 2.
Direction: toward the centre of the Earth; point of application: the centre of mass of the ball.	Direction and point of application are part of the model, not extras.

Example 2 — scaffolded

A mountain biker and bike together experience a force due to gravity of $780\,\text{N}$ while climbing in the You Yangs, west of Melbourne. (a) Rearrange $F_g = m\,g$ to make m the subject. (b) Calculate the total mass of rider and bike. (c) Check your answer by substituting it back.

Working	Comment
$m = \dfrac{F_g}{g}$	Divide both sides of $F_g = m\,g$ by g.
$m = \dfrac{780}{9.8} = 79.59\,\text{kg}$	Substitute; F_g in newtons, g in N kg^{-1}.
$m = 80\,\text{kg}$ (2 significant figures)	9.8 has 2 significant figures, so the answer is limited to 2.
Check: $80 \times 9.8 = 784\,\text{N} \approx 780\,\text{N}$ ✓	Agrees with the given force once both are rounded to 2 significant figures.

Example 3 — unscaffolded

A textbook of mass $0.85\,\text{kg}$ rests on a horizontal table. Name every force acting on the book using the *on A by B* convention, give the magnitude of the force due to gravity, and state the interaction-pair partner of each force.

The book touches two objects only: the table, and — without touching — the Earth. The Earth attracts the book: $F_{\text{on book by Earth}}$, acting at the book's centre of mass, directed toward the centre of the Earth. Its magnitude is

$$F_g = m\,g = 0.85 \times 9.8 = 8.33\,\text{N} = 8.3\,\text{N}$$

to two significant figures. The table supports the book with a contact force perpendicular to the surface — the normal force — written $F_{\text{on book by table}}$, acting upward at the contact surface.

The partner of each force swaps the two objects. The partner of $F_{\text{on book by Earth}}$ is $F_{\text{on Earth by book}}$: the book pulls the Earth upward with the same $8.3\,\text{N}$. The partner of $F_{\text{on book by table}}$ is $F_{\text{on table by book}}$: the book presses down on the table.

∴ the forces on the book are $F_{\text{on book by Earth}} = 8.3\,\text{N}$ (downward, at the centre of mass) and $F_{\text{on book by table}}$ (upward, at the contact surface); their interaction-pair partners are $F_{\text{on Earth by book}} = 8.3\,\text{N}$ upward on the Earth, and $F_{\text{on table by book}}$ downward on the table.

Example 4 — unscaffolded

A touring motorcycle with its rider has a mass of $255\,\text{kg}$. (a) Calculate the magnitude of the force due to gravity on the motorcycle and rider, using this book's value of g. (b) A Units 3 & 4 student repeats the calculation with the

VCAA Formula Sheet value $g = 9.81\ \text{m s}^{-2}$. Show both calculator values, and decide whether the two answers differ once recorded to the correct number of significant figures. (c) State which value of g you must use in this book, and why.

(a) Using $g = 9.8\ \text{N kg}^{-1}$:

$$F_g = mg = 255 \times 9.8 = 2499\ \text{N} = 2.5 \times 10^3\ \text{N}$$

to two significant figures (9.8 limits the answer to 2).

(b) With $g = 9.81\ \text{m s}^{-2}$:

$$F_g = 255 \times 9.81 = 2501.55\ \text{N} = 2.5 \times 10^3\ \text{N},$$

again $2.5 \times 10^3\ \text{N}$ to two significant figures. The raw calculator values differ by only $2.55\ \text{N}$ in about $2500\ \text{N}$ — roughly 0.1%, the same as the difference between 9.8 and 9.81 — which is far smaller than the effect of rounding, so the recorded answers are identical.

(c) This book uses $g = 9.8\ \text{N kg}^{-1}$, because that is the value the Unit 2 key knowledge of the VCE Physics study design gives for the gravitational field strength near Earth's surface. The value $9.81\ \text{m s}^{-2}$ comes from the VCAA Formula Sheet, a Units 3 & 4 document, and belongs to Units 3 & 4 work.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working. Take $g = 9.8\ \text{N kg}^{-1}$ *throughout.*

Question 1 *(scaffolded)* A koala in a wildlife sanctuary has a mass of $12\ \text{kg}$. (a) Write the relationship for the magnitude of the force due to gravity on the koala. (b) Calculate the magnitude of this force. (c) State its direction and its point of application.

Question 2 *(scaffolded)* The force due to gravity on a box of textbooks being carried upstairs is $74\ \text{N}$. (a) Rearrange $F_g = mg$ to make m the subject. (b) Calculate the mass of the box. (c) State the unit of your answer.

Question 3 *(scaffolded)* A mug rests on a bench. (a) Name the two forces acting on the mug, using the *on A by B* convention. (b) For each force, name its interaction-pair partner. (c) State which object each partner force acts on.

Question 4 *(scaffolded)* A $5.0\ \text{kg}$ bag of potting mix is taken from Melbourne to Perth. (a) State what happens to the mass of the bag. (b) Calculate the magnitude of the force due to gravity on the bag in each city. (c) Explain why the two values are the same.

Question 5 *(scaffolded)* A netball is thrown straight up into the air. Draw the force due to gravity on the ball — an arrow starting at the ball — when (a) the ball is moving up; (b) the ball is at the top of its flight; (c) the ball is moving down. (d) Explain why your arrow is the same in all three cases.

Question 6 *(scaffolded)* (a) Copy and complete the table by calculating the force due to gravity on each mass.

m (kg)	0.50	1.0	2.0	3.0
F_g (N)				

(b) Describe the pattern between mass and the force due to gravity shown by your table.

Question 7 An Australian-rules football has a mass of $0.46\ \text{kg}$. Calculate the magnitude of the force due to gravity on it.

Question 8 The force due to gravity on a loaded suitcase is $190\ \text{N}$. Calculate the mass of the suitcase.

Question 9 A small car has a mass of $950\,\mathrm{kg}$. (a) Calculate the magnitude of the force due to gravity on the car. (b) Name the force that forms an interaction pair with it, and state the magnitude and direction of that partner force.

Question 10 An adult African elephant can have a mass of about $5000\,\mathrm{kg}$. Calculate the magnitude of the force due to gravity on it, giving your answer in standard form.

Question 11 Two students stand on opposite sides of the Earth. Student A says that the two students' *down* directions point the same way in space, because down is down. Student B says they point in opposite directions in space. State which student is correct, and explain your reasoning.

Question 12 A netball of mass $0.45\,\mathrm{kg}$ is dropped from rest. (a) Calculate the magnitude of the force on the netball by the Earth. (b) Using the *on A by B* convention, name the partner force, and state its magnitude and direction. (c) A student argues: "The Earth does not move when the ball falls, so the ball cannot be pulling on the Earth." Evaluate this claim.

Question 13 A student draws the force due to gravity on a book resting on a table, and labels the downward force acting on the book as $F_{\text{on Earth by book}}$. Identify the error in the label, and write the label correctly.

Question 14 Explain why the force due to gravity on a body is drawn as a single arrow starting at the centre of mass, even when the body has an irregular shape such as a hammer or a boomerang.

Question 15 A student calculates the force due to gravity on a $3.0\,\mathrm{kg}$ object and records the answer as $F_g = 29.43\,\mathrm{N}$. Explain what is wrong with the way the answer is recorded, and write the answer correctly.

Question 16 Classify each of the following forces as a contact force or a non-contact force, justifying each answer: (a) the force due to gravity on a football while it is in flight; (b) the force on the ball by a boot as it kicks the ball; (c) the force on a bucket by a rope lifting the bucket.

Question 17 "Because $F_{\text{on A by B}} = -F_{\text{on B by A}}$, the two forces must always cancel, and nothing can ever accelerate." Identify the flaw in this argument.

Question 18 A physics textbook of mass $0.60\,\mathrm{kg}$ rests on a student's open palm. (a) Calculate the magnitude of the force due to gravity on the book. (b) The book is at rest. Use this fact to state the magnitude and direction of the force on the book by the palm. (c) Draw a diagram showing all four forces of the two interactions — the two forces on the book and their two interaction-pair partners — each arrow drawn on the object it acts on, and each labelled with the *on A by B* convention.

Answers

Question 1 (a) $F_g = m\,g$; (b) $1.2 \times 10^2\,\mathrm{N}$; (c) toward the centre of the Earth, acting at the koala's centre of mass. **Question 2** (a) $m = F_g / g$; (b) $7.6\,\mathrm{kg}$; (c) kilograms. **Question 3** (a) $F_{\text{on mug by Earth}}$ and $F_{\text{on mug by bench}}$; (b) $F_{\text{on Earth by mug}}$ and $F_{\text{on bench by mug}}$; (c) the Earth, and the bench, respectively. **Question 4** (a) it stays $5.0\,\mathrm{kg}$; (b) $49\,\mathrm{N}$ in both cities; (c) $g = 9.8\,\mathrm{N\ kg^{-1}}$ everywhere near Earth's surface, and F_g depends only on m and g — see solution. **Question 5** (a)–(c) the same downward arrow of magnitude $m\,g$ starting at the ball in every case; (d) F_g depends only on m and g, not on the motion of the ball. **Question 6** (a) $4.9\,\mathrm{N}$, $9.8\,\mathrm{N}$, $20\,\mathrm{N}$, $29\,\mathrm{N}$; (b) F_g is proportional to m — doubling the mass doubles the force. **Question 7** $4.5\,\mathrm{N}$. **Question 8** $19\,\mathrm{kg}$. **Question 9** (a) $9.3 \times 10^3\,\mathrm{N}$; (b) $F_{\text{on Earth by car}}$, $9.3 \times 10^3\,\mathrm{N}$, directed upward (toward the car). **Question 10** $4.9 \times 10^4\,\mathrm{N}$. **Question 11** Student B; *down* means toward the centre of the Earth, so opposite sides give opposite directions in space — see solution. **Question 12** (a) $4.4\,\mathrm{N}$; (b) $F_{\text{on Earth by netball}}$, $4.4\,\mathrm{N}$, upward on the Earth; (c) the claim is wrong — the pair always exists; the Earth's huge mass makes its response unobservable. **Question 13** The on- and by-objects are swapped; the force on the book is $F_{\text{on book by Earth}}$. **Question 14** F_g is modelled as acting at the centre of mass, where the whole mass may be treated as gathered — see solution. **Question 15** Too many significant figures; the answer should be $29\,\mathrm{N}$. **Question 16** (a) non-contact; (b) contact; (c) contact — see solution for the justifications. **Question 17** The two forces act on *different* objects, so they cannot cancel each other — see solution. **Question 18** (a) $5.9\,\mathrm{N}$; (b) $5.9\,\mathrm{N}$ upward; (c) see solution.

Fully-worked solutions

Question 1 $m = 12\,\mathrm{kg}$, $g = 9.8\,\mathrm{N\ kg^{-1}}$. (a) $F_{\text{on koala by Earth}} = m\,g$. (b) $F_g = m\,g = 12 \times 9.8 = 117.6\,\mathrm{N} = 1.2 \times 10^2\,\mathrm{N}$ to two significant figures (written in standard form so the two figures are unambiguous). (c) The force is directed toward the centre of the Earth, and it acts at the centre of mass of the koala.

Question 2 $F_g = 74\,\mathrm{N}$. (a) $F_g = m\,g \Rightarrow m = \dfrac{F_g}{g}$. (b) $m = \dfrac{74}{9.8} = 7.551\,\mathrm{kg} = 7.6\,\mathrm{kg}$ to two significant figures. (c) Mass is measured in kilograms (kg).

Question 3 (a) The forces on the mug are $F_{\text{on mug by Earth}}$ (the force due to gravity, downward, at the mug's centre of mass) and $F_{\text{on mug by bench}}$ (the normal force from the bench, upward, at the contact surface). (b) The partner of $F_{\text{on mug by Earth}}$ is $F_{\text{on Earth by mug}}$; the partner of $F_{\text{on mug by bench}}$ is $F_{\text{on bench by mug}}$. (c) $F_{\text{on Earth by mug}}$ acts on the Earth; $F_{\text{on bench by mug}}$ acts on the bench. In the convention, the first-named object is always the one the force acts on.

Question 4 (a) The mass is unchanged: $5.0\,\mathrm{kg}$ in Melbourne and in Perth. Mass is a property of the body, not of its location. (b) $F_g = m\,g = 5.0 \times 9.8 = 49\,\mathrm{N}$ in both cities. (c) Both cities are on the surface of the Earth, where g takes the same value $9.8\,\mathrm{N\ kg^{-1}}$; since F_g depends only on m and g, and neither changes, the force due to gravity is the same.

Question 5 (a)–(c) In all three positions the arrow starts at the ball, points toward the centre of the Earth, and has the same length (the same magnitude $m\,g$). (d) The force due to gravity depends only on the mass of the ball and on g; it does not depend on whether the ball is moving up, down, or is momentarily at rest at the top. (The *motion* is different in the three cases — Chapter 7 describes it — but the force is not.)

Question 6 (a) $F_g = m\,g$ for each row: $0.50 \times 9.8 = 4.9\,\mathrm{N}$; $1.0 \times 9.8 = 9.8\,\mathrm{N}$; $2.0 \times 9.8 = 19.6\,\mathrm{N} = 20\,\mathrm{N}$; $3.0 \times 9.8 = 29.4\,\mathrm{N} = 29\,\mathrm{N}$ (two significant figures where rounding is needed). (b) The unrounded values 4.9, 9.8, 19.6, 29.4 show that each time the mass doubles, the force doubles: F_g is **proportional** to m. This is exactly what $F_g = m\,g$ says for a constant g.

Question 7 $F_g = m\,g = 0.46 \times 9.8 = 4.508\,\mathrm{N} = 4.5\,\mathrm{N}$ to two significant figures.

Question 8 $m = \dfrac{F_g}{g} = \dfrac{190}{9.8} = 19.39\,\mathrm{kg} = 19\,\mathrm{kg}$ to two significant figures (limited by g).

Question 9 $m = 950\,\text{kg}$. (a) $F_g = m\,g = 950 \times 9.8 = 9310\,\text{N} = 9.3 \times 10^3\,\text{N}$ to two significant figures. (b) The partner force is $F_{\text{on Earth by car}}$: the car attracts the Earth with the same magnitude, $9.3 \times 10^3\,\text{N}$, directed upward (from the Earth's surface toward the car). Equal magnitude, opposite direction, different objects.

Question 10 $F_g = m\,g = 5000 \times 9.8 = 49\,000\,\text{N} = 4.9 \times 10^4\,\text{N}$ to two significant figures.

Question 11 Student B is correct. *Down* is a frame-dependent direction: it means toward the centre of the Earth from where the object is. For students on opposite sides of the Earth, "toward the centre" is the opposite direction in space for each of them, so their *down* directions are opposite. Neither student is wrong about their own local *down* — the two frames simply differ.

Question 12 $m = 0.45\,\text{kg}$. (a) $F_{\text{on netball by Earth}} = m\,g = 0.45 \times 9.8 = 4.41\,\text{N} = 4.4\,\text{N}$ to two significant figures. (b) The partner is $F_{\text{on Earth by netball}}$, of the same magnitude $4.4\,\text{N}$, directed upward on the Earth (toward the ball). (c) The claim confuses force with observable motion. The interaction pair exists whenever the two objects attract each other, whether or not anything appears to move. The Earth does experience the $4.4\,\text{N}$ pull; its resulting acceleration is $a = F / m$ with the Earth's enormous mass in the denominator, so it is far too small to observe. An unobservable effect is not an absent force.

Question 13 The label names the wrong object as the one feeling the force. $F_{\text{on Earth by book}}$ is the force the book exerts *on the Earth* — the partner force, which acts on the Earth, not on the book. The downward force acting on the book is $F_{\text{on book by Earth}}$. (The drawn arrow's direction was right; its label was not.)

Question 14 The model treats the whole mass of the body as gathered at one point — the centre of mass — for the purpose of the force due to gravity. Every small part of the body is attracted toward the Earth, and the combined effect of all those small pulls is exactly the same as one pull of size $m\,g$ acting at the centre of mass. That is true for any shape, regular or not, so one arrow at the centre of mass always represents F_g correctly; the only thing that changes for an irregular body is *where* the centre of mass lies.

Question 15 The inputs $3.0\,\text{kg}$ and $g = 9.8\,\text{N kg}^{-1}$ each have two significant figures, so the answer may not be recorded with more than two. Writing $29.43\,\text{N}$ pretends to four significant figures of precision that the data do not support. Correctly: $F_g = 3.0 \times 9.8 = 29.4\,\text{N}$, recorded as $29\,\text{N}$.

Question 16 (a) Non-contact: the Earth attracts the ball without touching it — the force acts even while the ball is in the air with nothing touching it. (b) Contact: the boot must touch the ball for the force to act; it ends the instant they separate. (c) Contact: the rope transmits the pull only while it is attached to and touching the bucket.

Question 17 Cancellation of two forces only makes sense for forces acting *on the same object*: $F_{\text{on A by B}}$ acts on A while $F_{\text{on B by A}}$ acts on B. When you ask whether object A accelerates, you add only the forces on A — and the force A exerts on B is not among them. So each object feels a single, uncancelled force from the other, and each can accelerate. (In Example 3 the book's two forces do not form a pair either — they act on the same object but come from two different causes — which is a separate point the argument misses.)

Question 18 $m = 0.60\,\text{kg}$. (a) $F_g = m\,g = 0.60 \times 9.8 = 5.88\,\text{N} = 5.9\,\text{N}$ to two significant figures. (b) The book is at rest, so by Newton's first law (Section 8B) the forces on it are balanced: the upward force on the book by the palm must equal the downward F_g in magnitude, $5.9\,\text{N}$, directed upward. (c) The four forces, each drawn on the object it acts on: on the book, $F_{\text{on book by Earth}}$ ($5.9\,\text{N}$, downward, from the book's centre of mass) and $F_{\text{on book by palm}}$ ($5.9\,\text{N}$, upward, at the contact with the palm); on the Earth, $F_{\text{on Earth by book}}$ ($5.9\,\text{N}$, upward); on the palm, $F_{\text{on palm by book}}$ ($5.9\,\text{N}$, downward). Note that the two forces on the book balance each other (same object) while each also has its partner on a *different* object (the two interactions) — the balancing of the forces on the book and the interaction pairs are two separate facts.

Scope — *VCE Physics Study Design, Unit 2, Area of Study 1*: "apply the vector model of forces, including vector addition and components of forces, to readily observable forces including the force due to gravity, friction and normal forces".

Theory

Section 8B applied Newton's three laws to bodies with forces acting on them, and Section 8C modelled forces as **vectors**: arrows with a magnitude (a size, measured in newtons, N) and a direction, drawn on a free-body diagram and labelled with the "force on A by B" convention. In those sections the forces mostly acted along one straight line, so adding them meant choosing a positive direction and adding with signs. Everyday forces rarely behave that neatly. A suitcase is pulled by a handle held at an angle, a toboggan runs down a slope, and two people push a box from different sides at once. This section develops the two pieces of machinery those situations need — **vector addition** and **components of forces** — and applies them to three readily observable forces: the force due to gravity F_g, the normal force F_N and the friction force F_f.

Adding forces that act along a line. Recall the one-dimensional rule from Section 8B: forces along a single straight line add as signed numbers. Forces pointing the same way add together; forces pointing opposite ways subtract. The sum is the **net force** F_{net}. If the net force is zero the forces are **balanced** and the object is in **equilibrium** — at rest, or moving at constant velocity in a straight line (Newton's first law). Everything in this section extends that same idea to forces that point in different directions.

Adding two perpendicular forces. Suppose two forces act at right angles on the one point of an object. Draw both arrows from that point: together they form two sides of a rectangle. Their combined effect is a single force called the **resultant** — here the same thing as the net force — drawn along the diagonal of that rectangle. Because the forces are at right angles, the magnitude of the resultant follows from Pythagoras' theorem:

$$F_{\text{net}} = \sqrt{F_1^2 + F_2^2}.$$

The direction of the resultant is found with trigonometry: if θ is the angle between the resultant and the force F_1, then

$$\tan\theta = \frac{F_2}{F_1}.$$

Notice that the resultant of two perpendicular forces is larger than either force alone but *smaller* than the two forces simply added — the forces work partly together, not fully together.

Adding forces head-to-tail. When the forces are not at right angles, the addition can be done as a scale diagram. Draw the first force as an arrow, to scale, pointing the right way. Draw the second arrow starting from the **head** (the arrow tip) of the first, with its own length and direction. Continue until every force has been drawn. The resultant is the arrow from the tail of the first force to the head of the last. The same resultant is obtained whatever order the forces are drawn in.

One case matters above all the others: if the chain of arrows closes — the head of the last force lands back on the tail of the first — the resultant is zero. The forces are balanced, $F_{\text{net}} = 0$, and the object is in equilibrium. A closed shape in a vector addition diagram is the signature of equilibrium, and it is Newton's first law (Section 8B) drawn as a picture.

Resolving a force into components. The reverse problem is just as useful: replacing one force by two perpendicular forces that together have exactly the same effect. Those two replacement forces are called the **components** of the original force. A force F that makes an angle θ with a chosen axis has

- a component $F\cos\theta$ along that axis, and
- a component $F\sin\theta$ perpendicular to it.

Because $\cos\theta$ and $\sin\theta$ are never greater than 1, a component can never be larger than the force it comes from. Components are signed: positive in the chosen positive direction of the axis, negative in the opposite direction. Once a force has been replaced by its components, work with the components *instead of* the original force — keeping both would count the force twice.

The net force from components. Component method turns any two-dimensional force problem into two one-dimensional ones. The routine is:

1. Choose two perpendicular axes — usually horizontal and vertical, or, for a body on a slope, along and perpendicular to the slope.
2. Resolve every force that does not already lie along one of the axes.
3. Add the components along each axis, keeping their signs: the sums are ΣF_x and ΣF_y.
4. If both sums are zero, the net force is zero and the object is in equilibrium.
5. Otherwise, recombine the two sums: the net force has magnitude $F_{\text{net}}=\sqrt{(\Sigma F_x)^2+(\Sigma F_y)^2}$ and makes an angle θ with the x-axis where $\tan\theta=\dfrac{\Sigma F_y}{\Sigma F_x}$.

When the net force is not zero, it points in the direction of the object's acceleration, and its magnitude and the acceleration are related by Newton's second law (Section 8B): $a=F_{\text{net}}/m$.

The normal force when a force acts at an angle. In Section 8C, an object resting on level ground with no vertical acceleration had a normal force equal in magnitude to the force due to gravity: $F_N=F_g$. That equality is a consequence of the vertical forces balancing — it is not a general rule, and component method shows when it fails.

Pull a suitcase along level ground with a handle held at an angle θ above the horizontal. The pull F has an upward vertical component $F\sin\theta$. The vertical forces are the upward normal force F_N, the upward component $F\sin\theta$, and the downward force due to gravity F_g. With no vertical acceleration they balance:

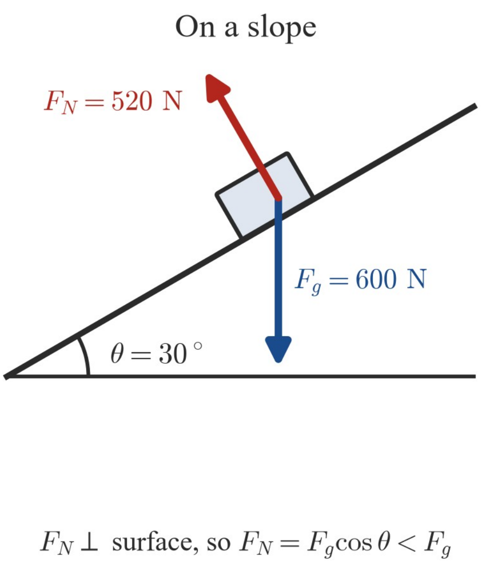
$$F_N+F\sin\theta=F_g \Rightarrow F_N=F_g-F\sin\theta.$$

The normal force is *less* than the force due to gravity: the angled pull carries part of the load the floor would otherwise support alone. Pushing *down* at an angle below the horizontal has the opposite effect — the downward component of the push presses the object harder into the surface, and the surface pushes back harder:

$$F_N=F_g+F\sin\theta.$$

If the upward component of a pull ever equals F_g, the normal force falls to zero and the object leaves the surface. The normal force has no fixed value of its own: it adjusts to whatever the other vertical forces require.

Forces on an inclined plane. For a body on a ramp inclined at an angle θ to the horizontal, the convenient axes are *along* the slope and *perpendicular* to the slope, because any motion is along the slope and the normal force is perpendicular to it.



The force due to gravity points vertically down, so it has components along both axes:

- down the slope: $mg\sin\theta$;
- into the slope: $mg\cos\theta$.

Perpendicular to the slope there is no acceleration, so the normal force balances the into-the-slope component:

$$F_N = mg\cos\theta.$$

This is less than mg, and it shrinks as the slope gets steeper because $\cos\theta$ decreases as θ increases. Along the slope, the down-slope component $mg\sin\theta$ tends to pull the body downhill; a friction force F_f, when present, acts along the surface opposing the motion or the tendency to move. If the body moves at constant velocity (or stays at rest), the along-slope forces balance; if it accelerates, $F_{\text{net}} = ma$ applies along the slope.

A routine for two-dimensional force problems. Every problem in this section follows the same five steps:

1. Choose the object of interest and draw its free-body diagram (Section 8C), labelling each force with the $F_{\text{on A by B}}$ convention — for example, the normal force on a suitcase by the floor is $F_{\text{on suitcase by floor}}$.
2. Choose two perpendicular axes — horizontal and vertical, or along and perpendicular to a slope.
3. Resolve every force that does not lie along one of the axes.
4. Add the components along each axis, keeping signs.
5. Interpret the sums: both zero means equilibrium (Newton's first law); otherwise find the net force and, where the mass is known, the acceleration (Newton's second law).

Units 3 & 4 preview. At Units 3 & 4 the vector model built here is used from the opening weeks of the course: Newton's laws applied in two dimensions (such as banked tracks and vertical circles), projectile motion split into two perpendicular axes, and the directions of field forces. Those sections assume that head-to-tail addition, resolution into components, and the $F_{\text{on A by B}}$ labelling convention are automatic, and they never re-teach them. Content under this banner is not assessed in this book's topic tests or practice examinations.

Throughout this section $g = 9.8\ \text{m s}^{-2}$ (equivalently $9.8\ \text{N kg}^{-1}$), as in Section 8C. Friction forces are treated as stated values or as values deduced from equilibrium.

Worked examples

Example 1 — scaffolded

Two friends are moving house. A box slides across a garage floor while one friend pushes on it with a force of $80\ \text{N}$ due east and the other pushes with a force of $60\ \text{N}$ due south. The box slides in the direction it is pushed. The floor exerts a friction force of $100\,\mathrm{N}$ opposing the motion of the box. (a) Calculate the resultant of the two pushes, giving both magnitude and direction. (b) Calculate the net force on the box. (c) Describe the motion of the box while the forces stay as stated, with reference to Newton's first law.

Working	Comment
(a) $F=\sqrt{80^2+60^2}=\sqrt{10\,000}=100\,\mathrm{N}$.	The two pushes are perpendicular, so they add by Pythagoras' theorem.
$\tan\theta=\dfrac{60}{80}=0.75$, so $\theta=37^\circ$ south of east.	The angle is measured from the eastward push toward the southward push.
(b) Friction opposes the motion, so it acts directly opposite the $100\,\mathrm{N}$ resultant: $F_{\mathrm{net}}=100-100=0\,\mathrm{N}$.	The friction force lies along the line of the motion.
(c) The net force is zero, so by Newton's first law (Section 8B) the box continues to slide at constant velocity.	Balanced forces produce no change of motion.

Example 2 — scaffolded

A $12\,\mathrm{kg}$ suitcase is pulled along level airport floor by a handle that exerts a force of $40\,\mathrm{N}$ at 30° above the horizontal. The suitcase stays on the floor throughout. (a) Calculate the horizontal and vertical components of the force exerted by the handle. (b) Calculate the force due to gravity on the suitcase. (c) Calculate the normal force on the suitcase by the floor.

Working	Comment
(a) $F_x=40\cos 30^\circ=34.6\,\mathrm{N}$ forward; $F_y=40\sin 30^\circ=20\,\mathrm{N}$ upward.	Resolve the handle force into horizontal and vertical components.
(b) $F_g=mg=12\times 9.8=117.6\,\mathrm{N}$, directed downward.	The force due to gravity acts vertically downward (Section 8C).
Vertically, there is no acceleration, so $F_N+20=117.6$.	Upward forces: F_N and the vertical component of the pull. Downward force: F_g.
(c) $F_N=97.6\,\mathrm{N}\approx 98\,\mathrm{N}$.	Round to two significant figures, matching the data.

The normal force ($98\,\mathrm{N}$) is less than the force due to gravity ($117.6\,\mathrm{N}$) because the upward component of the pull carries part of the load the floor would otherwise support alone.

Example 3 — unscaffolded

A toboggan carrying a rider has a total mass of $60\,\mathrm{kg}$. It slides down a straight snow slope at Mount Buller, inclined at 25° to the horizontal. The friction force on the toboggan is $150\,\mathrm{N}$. (a) Calculate the components of the force due to gravity on the toboggan along and perpendicular to the slope. (b) Calculate the normal force on the toboggan by the slope. (c) Calculate the acceleration of the toboggan.

Choose axes along the slope and perpendicular to the slope. The force due to gravity is $F_g=mg=60\times 9.8=588\,\mathrm{N}$, directed vertically downward.

(a) The component of F_g down the slope is

$$mg\sin\theta=588\times\sin 25^\circ=248.5\,\mathrm{N}\approx 250\,\mathrm{N},$$

and the component into the slope is

$$mg\cos\theta=588\times\cos 25^\circ=532.9\,\mathrm{N}\approx 530\,\mathrm{N}.$$

(b) Perpendicular to the slope there is no acceleration, so the normal force balances the into-the-slope component:

$$F_N=mg\cos\theta=530\,\mathrm{N}.$$

Notice that F_N is less than F_g; on a steeper slope it would be smaller still.

(c) Along the slope, taking downhill as positive, the net force is the down-slope component of F_g minus the friction:

$$F_{\text{net}} = 248.5 - 150 = 98.5\,\text{N}.$$

By Newton's second law (Section 8B),

$$a = \frac{F_{\text{net}}}{m} = \frac{98.5}{60} = 1.6\,\text{m s}^{-2},$$

directed down the slope.

∴ the components of F_g are $250\,\text{N}$ down the slope and $530\,\text{N}$ into the slope, the normal force is $530\,\text{N}$, and the toboggan accelerates at $1.6\,\text{m s}^{-2}$ down the slope.

Example 4 — unscaffolded

A $4.0\,\text{kg}$ banner hangs in equilibrium in a school gymnasium, supported by two ropes tied to the ceiling. The rope on the left makes an angle of 30° with the horizontal and the rope on the right makes an angle of 60° with the horizontal. Calculate the tension in each rope.

Three forces act on the banner: the force due to gravity $F_g = m g = 4.0 \times 9.8 = 39.2\,\text{N}$ downward, and the two rope tensions, T_L along the left rope and T_R along the right rope. Because the banner is in equilibrium, the components of these forces sum to zero along each axis.

Horizontally, the leftward component of T_L balances the rightward component of T_R:

$$T_L \cos 30^\circ = T_R \cos 60^\circ.$$

Vertically, the upward components of the two tensions together balance the force due to gravity:

$$T_L \sin 30^\circ + T_R \sin 60^\circ = m g.$$

From the horizontal equation, $T_R = T_L \dfrac{\cos 30^\circ}{\cos 60^\circ} = T_L \times \dfrac{0.8660}{0.5000} = 1.732\,T_L$. Substituting into the vertical equation:

$$T_L \sin 30^\circ + 1.732\,T_L \sin 60^\circ = 39.2$$

$$T_L (0.5000 + 1.732 \times 0.8660) = 39.2$$

$$2.0\,T_L = 39.2 \Rightarrow T_L = 19.6\,\text{N}.$$

Then $T_R = 1.732 \times 19.6 = 33.9\,\text{N}$, and to two significant figures

$$T_L = 20\,\text{N}, T_R = 34\,\text{N}.$$

(Check: the vertical components add to $19.6 \times 0.5000 + 33.9 \times 0.8660 = 9.8 + 29.4 = 39.2\,\text{N}$, which equals F_g. ✓)

∴ the tension is $20\,\text{N}$ in the left rope and $34\,\text{N}$ in the right rope. The steeper rope carries the larger tension: each newton of its tension contributes a larger upward component, so it does most of the supporting.

Practice questions

Unless stated otherwise, use $g = 9.8\,\text{m s}^{-2}$.

Question 1 *(scaffolded)* Two workers push a carton across a warehouse floor. One pushes due north with a force of $90\,\mathrm{N}$ and the other pushes due west with a force of $120\,\mathrm{N}$. The carton is sliding in the direction it is pushed, and the floor exerts a friction force of $70\,\mathrm{N}$ opposing the motion. The mass of the carton is $40\,\mathrm{kg}$. (a) Calculate the resultant of the two pushes, giving both magnitude and direction. (b) Calculate the net force on the carton. (c) Calculate the acceleration of the carton.

Question 2 *(scaffolded)* A rope from a winch pulls a small ferry along a straight section of the Murray River. The rope makes an angle of 35° with the direction of the river, and the tension in the rope is $200\,\mathrm{N}$. (a) Calculate the component of the tension acting along the river. (b) Calculate the component of the tension pulling the ferry toward the river bank. (c) The ferry does not drift sideways; it stays in line with the river. State what must be true of the sideways forces on the ferry.

Question 3 *(scaffolded)* A $12\,\mathrm{kg}$ crate is pulled across level ground by a rope at 37° above the horizontal. The tension in the rope is $50\,\mathrm{N}$. The crate stays on the ground. (a) Calculate the force due to gravity on the crate. (b) Calculate the vertical component of the tension. (c) Calculate the normal force on the crate by the ground.

Question 4 *(scaffolded)* A $25\,\mathrm{kg}$ box of tools rests on a ramp inclined at 20° to the horizontal. (a) Show that the component of the force due to gravity acting down the ramp is about $84\,\mathrm{N}$. (b) Calculate the normal force on the box by the ramp. (c) State the magnitude and the direction of the friction force on the box, justifying your answer.

Question 5 *(scaffolded)* A small metal ring is held at rest by three horizontal strings. String A pulls due east with a force of $30\,\mathrm{N}$ and string B pulls due north with a force of $40\,\mathrm{N}$. (a) Calculate the resultant of the forces exerted by strings A and B. (b) State the force that string C must exert to hold the ring at rest. (c) Describe the shape formed when the three forces are drawn head-to-tail.

Question 6 *(scaffolded)* Three horizontal forces act on a sled on a frozen pond: $100\,\mathrm{N}$ due east, $60\,\mathrm{N}$ due west and $50\,\mathrm{N}$ due north. (a) Find the sum of the east–west components and the sum of the north–south components. (b) Calculate the magnitude of the net force on the sled. (c) Calculate the direction of the net force, measured from east.

Question 7 Two rescuers pull a dinghy across a sandbank toward the water. One pulls due north with a force of $60\,\mathrm{N}$ and the other pulls due west with a force of $80\,\mathrm{N}$. Calculate the resultant of the two forces on the dinghy, giving both magnitude and direction.

Question 8 A parent pushes a pram with a force of $60\,\mathrm{N}$ directed 25° below the horizontal. The mass of the pram is $8.0\,\mathrm{kg}$, and it stays on level ground. Calculate the normal force on the pram by the ground, and explain why it is different from the force due to gravity on the pram.

Question 9 A $3.5\,\mathrm{kg}$ loudspeaker hangs in equilibrium from a horizontal stage truss, supported by two identical chains. Each chain makes an angle of 55° with the truss. Calculate the tension in each chain.

Question 10 A sled of mass $8.0\,\mathrm{kg}$ slides down a slope inclined at 30° to the horizontal. The friction force on the sled is $20\,\mathrm{N}$. Calculate the acceleration of the sled.

Question 11 A $15\,\mathrm{kg}$ box is placed on a ramp inclined at 22° to the horizontal. The maximum friction force the ramp surface can provide is $60\,\mathrm{N}$. Determine, with a calculation, whether the box remains at rest on the ramp.

Question 12 A trolley of mass $6.0\,\mathrm{kg}$ is held at rest on a smooth ramp inclined at 15° to the horizontal by a force directed up the ramp, parallel to the ramp. Calculate the magnitude of this holding force.

Question 13 A student writes: “The horizontal component of a force of $50\,\mathrm{N}$ directed 30° above the horizontal is $60\,\mathrm{N}$.” Identify and explain the error in this claim.

Question 14 A suitcase is pulled across level airport floor by a handle held at an angle above the horizontal. Explain why the normal force on the suitcase by the floor is less than the force due to gravity on the suitcase.

Question 15 A heavy wet towel is hung at the middle of a clothesline, so that each half of the line is almost horizontal. Explain why the tension in each half of the line must be much larger than the force due to gravity on the towel.

Question 16 A box rests on a ramp, and the angle of the ramp is slowly increased. With reference to components of forces, explain what happens to the normal force on the box by the ramp as the ramp becomes steeper.

Question 17 Two forces, $6.0\,\mathrm{N}$ and $8.0\,\mathrm{N}$, act on an object. One student says the resultant must be $10\,\mathrm{N}$. Another says the resultant could be any value between $2.0\,\mathrm{N}$ and $14\,\mathrm{N}$, depending on the angle between the two forces. Justify which student is correct.

Question 18 An object is at rest under the action of three forces. When the three forces are drawn head-to-tail, they form a closed triangle. Explain why the triangle closes, and state what it would mean if it did not.

Question 19 In an investigation, a small ring is held at rest by three horizontal strings pulled in different directions. Two of the strings exert forces of $3.0\,\mathrm{N}$ and $4.0\,\mathrm{N}$ at right angles to each other. The third string is attached to a spring balance. (a) Predict the reading of the spring balance. (b) Describe the direction of the force exerted by the third string relative to the resultant of the other two forces. (c) State what you would record in your logbook to test your prediction.

Question 20 A $20\,\mathrm{kg}$ crate is pulled across level ground at constant velocity by a rope at 20° above the horizontal. The tension in the rope is $100\,\mathrm{N}$. Calculate the friction force and the normal force on the crate by the ground, giving the reasoning behind each value.

Answers

Question 1 (a) $150\,\text{N}$, 37° north of west; (b) $80\,\text{N}$ in the direction of the resultant; (c) $2.0\,\text{m s}^{-2}$ in the same direction. **Question 2** (a) $160\,\text{N}$; (b) $110\,\text{N}$; (c) see solutions. **Question 3** (a) $120\,\text{N}$; (b) $30\,\text{N}$ upward; (c) $88\,\text{N}$. **Question 4** (a) $84\,\text{N}$; (b) $230\,\text{N}$; (c) $84\,\text{N}$ up the ramp. **Question 5** (a) $50\,\text{N}$, 53° north of east; (b) $50\,\text{N}$, 53° south of west; (c) a closed triangle. **Question 6** (a) $40\,\text{N}$ east and $50\,\text{N}$ north; (b) $64\,\text{N}$; (c) 51° north of east. **Question 7** $100\,\text{N}$, 37° north of west. **Question 8** $100\,\text{N}$; see solutions. **Question 9** $21\,\text{N}$. **Question 10** $2.4\,\text{m s}^{-2}$ down the slope. **Question 11** The box remains at rest: friction required is $55\,\text{N}$, less than the $60\,\text{N}$ available. **Question 12** $15\,\text{N}$ up the ramp. **Question 13** A component can never exceed the force it comes from: the horizontal component is $50\cos 30^{\circ} = 43\,\text{N}$, not $60\,\text{N}$. **Question 14** The upward component of the angled pull carries part of the load, so the floor supports less: $F_N = F_g - F\sin\theta < F_g$. **Question 15** Each half of the line is nearly horizontal, so only a small fraction of each tension acts vertically; the tensions must therefore be much larger than the force due to gravity on the towel. **Question 16** $F_N = mg\cos\theta$, and $\cos\theta$ gets smaller as the ramp gets steeper, so the normal force decreases. **Question 17** The second student: the resultant ranges from $2.0\,\text{N}$ (forces opposite) to $14\,\text{N}$ (forces in the same direction), with $10\,\text{N}$ only the right-angle case. **Question 18** The net force is zero, so the three vectors sum to zero and the triangle closes; an open triangle would mean a non-zero net force. **Question 19** (a) $5.0\,\text{N}$; (b), (c) see solutions. **Question 20** Friction $94\,\text{N}$ opposing the motion; normal force $160\,\text{N}$.

Fully-worked solutions

Question 1 Pushes of $90\,\text{N}$ north and $120\,\text{N}$ west; friction $70\,\text{N}$; $m = 40\,\text{kg}$. (a) The two pushes are perpendicular, so their resultant has magnitude

$$F = \sqrt{90^2 + 120^2} = \sqrt{22\,500} = 150\,\text{N}.$$

Its direction is given by $\tan\theta = \dfrac{90}{120} = 0.75$, so $\theta = 37^{\circ}$, measured from the westward direction toward north: the resultant is $150\,\text{N}$ at 37° north of west. (b) The carton moves in the direction of the resultant, so the friction force acts directly opposite it: $F_{\text{net}} = 150 - 70 = 80\,\text{N}$, in the direction of the resultant. (c) By Newton's second law (Section 8B), $a = \dfrac{F_{\text{net}}}{m} = \dfrac{80}{40} = 2.0\,\text{m s}^{-2}$, in the same direction.

Question 2 $T = 200\,\text{N}$ at 35° to the direction of the river. (a) Component along the river: $T\cos 35^{\circ} = 200 \times \cos 35^{\circ} = 163.8\,\text{N} \approx 160\,\text{N}$. (b) Component toward the bank: $T\sin 35^{\circ} = 200 \times \sin 35^{\circ} = 114.7\,\text{N} \approx 110\,\text{N}$. (c) Model answer. The ferry has no sideways acceleration, so by Newton's first law the net force sideways is zero: the sideways forces are balanced. The water must exert a sideways force on the hull (through the keel and rudder) of about $110\,\text{N}$, directed away from the bank, cancelling the sideways component of the rope's tension.

Question 3 $m = 12\,\text{kg}$, $T = 50\,\text{N}$ at 37° above the horizontal. (a) $F_g = mg = 12 \times 9.8 = 117.6\,\text{N}$, which rounds to $120\,\text{N}$ (two significant figures). (b) $T_y = T\sin 37^{\circ} = 50 \times \sin 37^{\circ} = 30.1\,\text{N}$, directed upward. (c) Vertically there is no acceleration, so the upward forces balance the downward force due to gravity: $F_N + 30.1 = 117.6$, giving $F_N = 87.5\,\text{N} \approx 88\,\text{N}$. The normal force is smaller than F_g because the upward component of the tension carries part of the load.

Question 4 $m = 25\,\text{kg}$, $\theta = 20^{\circ}$, box at rest. (a) $F_g = mg = 25 \times 9.8 = 245\,\text{N}$. The component down the ramp is

$$mg\sin\theta = 245 \times \sin 20^{\circ} = 83.8\,\text{N} \approx 84\,\text{N},$$

as required. (b) Perpendicular to the ramp there is no acceleration, so $F_N = mg\cos\theta = 245 \times \cos 20^{\circ} = 230\,\text{N}$. (c) The box is at rest, so it is in equilibrium and the forces along the ramp balance (Newton's first law). The friction force must therefore be $84\,\text{N}$, directed up the ramp, opposing the box's tendency to slide down.

Question 5 String A: $30\,\mathrm{N}$ east; string B: $40\,\mathrm{N}$ north. (a) The two forces are perpendicular, so their resultant is $\sqrt{30^2+40^2}=\sqrt{2500}=50\,\mathrm{N}$, at an angle θ given by $\tan\theta=\frac{40}{30}$, so $\theta=53^\circ$ north of east. (b) For the ring to stay at rest the net force on it must be zero, so string C must pull with $50\,\mathrm{N}$ in the direction exactly opposite the resultant: $50\,\mathrm{N}$ at 53° south of west. (c) Drawn head-to-tail, the three arrows return to their starting point: they form a closed triangle. A closed shape is the signature of equilibrium — the vector sum of the forces is zero.

Question 6 Forces: $100\,\mathrm{N}$ east, $60\,\mathrm{N}$ west, $50\,\mathrm{N}$ north. (a) East–west sum: $100-60=40\,\mathrm{N}$ east. North–south sum: $50\,\mathrm{N}$ north. (b) The two sums are perpendicular, so

$$F_{\text{net}}=\sqrt{40^2+50^2}=\sqrt{4100}=64\,\mathrm{N}.$$

(c) $\tan\theta=\frac{50}{40}=1.25$, so $\theta=51^\circ$ north of east.

Question 7 Forces: $60\,\mathrm{N}$ north and $80\,\mathrm{N}$ west. The two forces are perpendicular, so the resultant has magnitude

$$F=\sqrt{60^2+80^2}=\sqrt{10\,000}=100\,\mathrm{N}.$$

Its direction is given by $\tan\theta=\frac{60}{80}=0.75$, so $\theta=37^\circ$ measured from the westward direction toward north: the resultant is $100\,\mathrm{N}$ at 37° north of west.

Question 8 $F=60\,\mathrm{N}$ at 25° below the horizontal; $m=8.0\,\mathrm{kg}$. The vertical component of the push is $F\sin 25^\circ=60\times\sin 25^\circ=25.4\,\mathrm{N}$, directed downward. Vertically there is no acceleration, so the upward normal force balances both the force due to gravity and this downward component:

$$F_N=mg+F\sin 25^\circ=8.0\times 9.8+25.4=78.4+25.4=103.8\,\mathrm{N}\approx 100\,\mathrm{N}.$$

The normal force is greater than the force due to gravity ($78\,\mathrm{N}$): because the push is directed partly downward, the ground must push up hard enough to balance the force due to gravity *and* the downward component of the push.

Question 9 $m=3.5\,\mathrm{kg}$, each chain at 55° to the truss. By symmetry the two tensions are equal, each of magnitude T. Resolving vertically for equilibrium, the upward components of the two tensions balance the force due to gravity $F_g=mg=3.5\times 9.8=34.3\,\mathrm{N}$:

$$2T\sin 55^\circ=mg\Rightarrow T=\frac{mg}{2\sin 55^\circ}=\frac{34.3}{2\times 0.8192}=20.9\,\mathrm{N}\approx 21\,\mathrm{N}.$$

(The horizontal components of the two tensions are equal and opposite, so they cancel.)

Question 10 $m=8.0\,\mathrm{kg}$, $\theta=30^\circ$, friction $F_f=20\,\mathrm{N}$. The component of the force due to gravity down the slope is

$$mg\sin\theta=8.0\times 9.8\times\sin 30^\circ=78.4\times 0.5000=39.2\,\mathrm{N}.$$

The sled slides down, so friction acts up the slope. Taking down the slope as positive,

$$F_{\text{net}}=39.2-20=19.2\,\mathrm{N},$$

and by Newton's second law,

$$a=\frac{F_{\text{net}}}{m}=\frac{19.2}{8.0}=2.4\,\mathrm{m\,s^{-2}},$$

directed down the slope.

Question 11 $m=15\,\mathrm{kg}$, $\theta=22^\circ$, maximum available friction $60\,\mathrm{N}$. The component of the force due to gravity acting down the ramp is

$$m\,g\sin\theta = 15\times 9.8\times \sin 22^\circ = 147\times 0.3746 = 55.1\,\mathrm{N}.$$

For the box to remain at rest, friction must balance this with $55\,\mathrm{N}$ directed up the ramp. The surface can provide up to $60\,\mathrm{N}$, and $55\,\mathrm{N} < 60\,\mathrm{N}$, so the surface supplies enough friction and the box **remains at rest**.

Question 12 $m = 6.0\,\mathrm{kg}$, $\theta = 15^\circ$, smooth (frictionless) ramp. The trolley is held at rest, so the forces along the ramp balance: the holding force up the ramp equals the down-slope component of the force due to gravity,

$$F = m\,g\sin\theta = 6.0\times 9.8\times \sin 15^\circ = 58.8\times 0.2588 = 15.2\,\mathrm{N} \approx 15\,\mathrm{N}.$$

Question 13 Model answer. A component of a force can never be larger than the force itself: the horizontal component is $F\cos\theta$, and $\cos\theta$ is never greater than 1, so the horizontal component of a $50\,\mathrm{N}$ force is at most $50\,\mathrm{N}$ — the claimed $60\,\mathrm{N}$ is impossible. The correct value is $50\times\cos 30^\circ = 43\,\mathrm{N}$. The student has most likely divided by $\cos 30^\circ$ instead of multiplying, which inflates the value.

Question 14 Model answer. The suitcase has no vertical acceleration — it stays on the floor — so the vertical forces are balanced. The upward forces are the normal force F_N and the vertical component of the pull, $F\sin\theta$; the only downward force is the force due to gravity F_g. Hence $F_N + F\sin\theta = F_g$, and so $F_N = F_g - F\sin\theta$, which is less than F_g: the angled pull carries part of the load that the floor would otherwise support alone.

Question 15 Model answer. The towel hangs in equilibrium, so the vertical components of the two tensions must together balance the force due to gravity on the towel. When each half of the line is almost horizontal, each tension makes only a small angle with the horizontal, and only a small fraction of each tension ($T\sin\theta$) points upward. For those small vertical components to add up to F_g, the tensions themselves must be much larger than F_g. (If the line were perfectly horizontal, the tensions would have no vertical component at all and could not support the towel — a real clothesline always sags.)

Question 16 Model answer. Resolving the force due to gravity relative to the ramp, the component pressing the box into the ramp is $m\,g\cos\theta$. The box does not accelerate perpendicular to the ramp, so the normal force balances this component: $F_N = m\,g\cos\theta$. As the ramp becomes steeper, θ increases and $\cos\theta$ decreases, so the normal force decreases. At the limit of a vertical surface ($\theta = 90^\circ$), $\cos\theta = 0$ and the normal force would fall to zero.

Question 17 Model answer. The second student is correct. The resultant of two forces depends on the angle between them. When the two forces act in the same direction the resultant is $6.0 + 8.0 = 14\,\mathrm{N}$, its maximum; when they act in opposite directions it is $8.0 - 6.0 = 2.0\,\mathrm{N}$, its minimum; and for angles in between it takes every value between those limits. The first student's $10\,\mathrm{N}$ is only one special case — the value obtained when the two forces act at right angles, $\sqrt{6.0^2 + 8.0^2} = 10\,\mathrm{N}$.

Question 18 Model answer. The object is at rest, so by Newton's first law the net force on it is zero: the three forces add as vectors to zero. Drawn head-to-tail, the head of the last arrow lands exactly back on the tail of the first, and the triangle closes. If the triangle did not close, the vector sum would not be zero: the arrow needed to close the shape, drawn in reverse, would be the net force, and the object would accelerate in that direction instead of remaining at rest.

Question 19 (a) The ring is at rest, so the force from the third string balances the resultant of the other two forces. The two perpendicular forces of $3.0\,\mathrm{N}$ and $4.0\,\mathrm{N}$ have a resultant of $\sqrt{3.0^2 + 4.0^2} = 5.0\,\mathrm{N}$, so the predicted reading is $5.0\,\mathrm{N}$. (b) The third force points in the direction exactly opposite the resultant of the other two forces (180° from it). For example, if the $3.0\,\mathrm{N}$ force acts east and the $4.0\,\mathrm{N}$ force acts north, their resultant is $5.0\,\mathrm{N}$ at 53° north of east, and the third string pulls at 53° south of west. (c) Record the reading of the spring balance, the directions of all three strings (for example as a scale sketch of the three force arrows), and a comparison of the measured reading with the predicted $5.0\,\mathrm{N}$. Agreement within the precision of the equipment supports the vector model of forces.

Question 20 $m = 20\,\mathrm{kg}$, $T = 100\,\mathrm{N}$ at 20° above the horizontal, constant velocity. The horizontal component of the tension is $T\cos 20^\circ = 100\times\cos 20^\circ = 94.0\,\mathrm{N}$. Constant velocity means equilibrium (Newton's first law): the horizontal forces are balanced. The only horizontal forces are the horizontal component of the tension and the friction force, so the friction force is $94\,\mathrm{N}$, directed opposite the motion. Vertically, the upward forces balance the downward forces:

$$F_N + T\sin 20^\circ = mg \Rightarrow F_N = 20\times 9.8 - 100\times\sin 20^\circ = 196 - 34.2 = 161.8\,\mathrm{N} \approx 160\,\mathrm{N}.$$

Chapter 8 Forces and motion — 8E Impulse and momentum conservation in collisions

Scope — *VCE Physics Study Design, Unit 2, Area of Study 1*: "analyse impulse in an isolated system (for collisions between objects moving in a straight line): FΔt = mΔv"; and "investigate and analyse theoretically and practically momentum conservation in one dimension".

Theory

What happens in a collision. When a tennis racquet strikes a ball, when one rail wagon rolls into another, or when two ice-hockey players meet at the boards, the objects push on each other only while they are in contact. The contact lasts a very short time — often only a few thousandths of a second — but during that time each object exerts a force on the other, and those forces change the motion of both objects. This section applies the momentum ideas of Section 8A and Newton's laws of Section 8B to these interactions, with all motion along one straight line.

Momentum so far. In Section 8A the momentum of an object moving in a straight line was defined as $p = mv$: its mass times its velocity, measured in $\mathrm{kg\ m\ s^{-1}}$. Momentum is a vector — it has a direction, the direction of the velocity. Section 8A also established that a change in momentum is caused by a net force acting for a time:

$$\Delta p = F_{\text{net}} \Delta t .$$

The quantity on the right, force multiplied by the time for which it acts, is important enough to have its own name.

Impulse. The **impulse** delivered to an object is the product of the force on it and the time for which the force acts. For an object of constant mass m moving in a straight line, $\Delta p = m\Delta v$, so the key knowledge of the study design states the relationship as

$$F\Delta t = m\Delta v ,$$

where F is the force on the object, Δt is the time for which the force acts, and $\Delta v = v - u$ is the change in velocity (final velocity v minus initial velocity u). The impulse is $F\Delta t$. Its unit is the **newton-second** (N s); since $1\,\mathrm{N} = 1\,\mathrm{kg\ m\ s^{-2}}$, one N s is the same as one $\mathrm{kg\ m\ s^{-1}}$ — impulse and momentum change are measured in the same units, because impulse *is* the change in momentum.

Like force and momentum, impulse is a vector: it acts in the direction of the force, which is the direction of the change in momentum.

The force during a collision is an average. During a collision the contact force is not constant. When a ball hits a racquet the force starts at zero, rises to a maximum while the ball and strings are squashed together, then falls back to zero as they separate. In $F\Delta t = m\Delta v$, F is the **average force**: the constant force that, acting for the same time Δt, would produce the same change in momentum. Everything in this section can be answered with that average force.

Signs for straight-line motion. All the motion in this section is along one straight line, so directions are handled with signs. Choose one direction along the line as positive. Then:

- velocities in the positive direction are substituted as positive numbers, and velocities in the opposite direction as negative numbers;
- a positive answer for a velocity, a change in momentum or a force means the positive direction; a negative answer means the opposite direction.

Be careful with $\Delta v = v - u$ when an object reverses direction. A ball that comes in at $20\,\mathrm{m\ s^{-1}}$ and goes back out at $30\,\mathrm{m\ s^{-1}}$ (taking the outgoing direction as positive) has $u = -20\,\mathrm{m\ s^{-1}}$ and $v = +30\,\mathrm{m\ s^{-1}}$, so

$$\Delta v = v - u = 30 - (-20) = 50\,\mathrm{m\ s^{-1}},$$

not $10\,\mathrm{m\ s^{-1}}$. Reversing direction is a larger change in velocity than merely slowing down.

Controlling the force by controlling the time. Rearrange the impulse relationship as

$$F = \frac{m \Delta v}{\Delta t}.$$

For a given object and a given change in velocity, the product $m \Delta v$ is fixed, so the average force is *inversely proportional* to the contact time: a longer contact time means a smaller force, and a shorter contact time means a larger force. This one idea explains a great deal of everyday design:

- An **airbag** in a car brings the driver to rest over a longer time than a hard steering column would, so the average force on the driver is smaller.
- A person jumping down from a chair **bends their knees** on landing, extending the time taken to stop and so reducing the force on their legs.
- A fielder catching a cricket ball lets their **hands move back** with the ball, again lengthening the stopping time and reducing the force on their hands.

The same relationship works in reverse. To give a ball the largest possible change in momentum — to hit it as hard as possible — a large force must act for as long as possible. This is why a tennis player, a golfer or a baseball batter **follows through**: keeping the racquet, club or bat in contact with the ball for longer delivers a larger impulse $F \Delta t$, and so a larger change in the ball's momentum.

Systems and isolated systems. A **system** is the object or set of objects being studied. An **isolated system** is one on which no net external force acts: the forces between the objects inside the system (internal forces) can be large, but the forces on the system from outside (external forces) add to zero.

Two colliding objects form, to a good approximation, an isolated system during the collision itself. The contact time is so short that external forces such as friction and air resistance have almost no time to change the total momentum. On a horizontal surface the vertical external forces already balance — the normal force on each object from the surface balances the force due to gravity on it — so the motion along the surface is all that changes.

Conservation of momentum from Newton's laws. Momentum conservation can be derived from Newton's second and third laws (Section 8B). Consider two objects A and B that collide in a straight line.

1. During the collision, A exerts a force on B and B exerts a force on A. By Newton's third law these forces are equal in magnitude and opposite in direction:
$$F_{\text{on A by B}} = -F_{\text{on B by A}}.$$
2. Both forces act for the same contact time Δt, so the impulse delivered to A is equal in magnitude and opposite in direction to the impulse delivered to B:
$$F_{\text{on A by B}} \Delta t = -F_{\text{on B by A}} \Delta t.$$
3. By $F \Delta t = m \Delta v$, each impulse equals a change in momentum, so
$$\Delta p_A = -\Delta p_B.$$

Whatever momentum object A gains, object B loses, and vice versa. The **total momentum** of the two objects, $p_A + p_B$, is therefore unchanged by the collision. This is the **law of conservation of momentum**:

> In an isolated system, the total momentum before an interaction equals the total momentum after the interaction.

For two objects of masses m_1 and m_2 moving in a straight line with initial velocities u_1 and u_2 and final velocities v_1 and v_2, the law is written

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2,$$

with every velocity substituted with its sign. The total momentum before (the left side) equals the total momentum after (the right side).

Collisions where the objects stick together. The simplest collision to analyse is one in which the two objects join and move on together with one common final velocity v — a rail wagon coupling to another, a ball landing in a catcher's mitt, two skaters colliding and holding on. The conservation law becomes

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \text{, so } v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}.$$

In these sticking collisions the total momentum is conserved, but the motion is visibly different afterwards: usually the combined object moves more slowly than the moving object did before. Some of the **kinetic energy** — the energy an object has because it is moving, which you will calculate from Section 9C onwards as $E_k = 1/2 m v^2$ — has been transformed into thermal energy and sound, and used to deform the objects. Momentum conservation holds regardless: momentum is always conserved in an isolated system, even when kinetic energy is not.

Interactions where the objects push apart. Conservation of momentum applies to separations as well as collisions. Two ice-skaters initially at rest who push away from each other, or two trolleys released from a compressed spring between them, form an isolated system whose total momentum starts at zero and must remain zero:

$$0 = m_1 v_1 + m_2 v_2 \text{, so } m_1 v_1 = -m_2 v_2.$$

The two objects move in opposite directions, and the lighter one moves faster: the momenta are equal in magnitude and opposite in direction.

Practical investigation — momentum conservation in one dimension. The conservation law can be investigated with colliding trolleys on a low-friction track. A suitable framework:

- **Aim.** Test whether the total momentum of two trolleys before a straight-line collision equals the total momentum after it, for a sticking collision and for a collision in which the trolleys bounce apart.
- **Equipment.** Two dynamics trolleys (carts) on a smooth, level track; added masses; a way of making the trolleys stick (Velcro pads or a pin and cork) and a way of making them bounce apart (magnets or spring plungers); two light gates with interrupter cards of known width, or a smartphone camera for video analysis; a balance to measure the trolley masses.
- **Method.** Level the track: a trolley given a gentle push should move along it at constant speed, showing no net force along the track. Measure each trolley's mass. Record the speed of each trolley just before and just after the collision — with light gates, the speed is the card width divided by the interruption time. Take the direction of the first trolley's motion as positive, substitute velocities with signs, calculate the total momentum before ($m_1 u_1 + m_2 u_2$) and after ($m_1 v_1 + m_2 v_2$), and compare them. Repeat each arrangement several times, and repeat with different masses.
- **Expected results.** The total momentum after the collision is close to the total momentum before, in both directions of motion and for both collision types. The value after is usually slightly smaller, because friction and air resistance are external forces that slowly remove momentum from the system — the trolleys are only approximately isolated.
- **Safety and practice.** Keep feet clear of masses and trolleys that could fall from the bench, and secure the track so it cannot slide during the investigation. Record every reading — mass, card width, interruption time — with its unit in your logbook: the logbook is submitted as a requirement for satisfactory completion of the unit.

Where this leads. The one-dimensional collisions of this section are the foundation for the case study of Section 9E (for example, vehicle safety or motion in sport), where impulse arguments are applied in full. Students studying Option 2.11 (Section 11D) extend momentum transfer to collisions in two dimensions.

Units 3 & 4 preview. At Units 3 & 4 the impulse idea is extended to changing forces by treating the impulse as the area under a force–time graph, and collisions are classified as elastic or inelastic by comparing kinetic energy before and after. Momentum conservation itself is also applied in two dimensions. Content under this banner is not assessed in this book's topic tests or practice examinations.

Worked examples

Example 1 — scaffolded

In a cricket match, a ball of mass $160\,\mathrm{g}$ travels horizontally toward the wicket-keeper at $30\,\mathrm{m\ s^{-1}}$. The keeper catches it, letting her hands move back with the ball so that it comes to rest in $0.15\,\mathrm{s}$. (a) Calculate the change in momentum of the ball. Take the ball's original direction of travel as positive. (b) Calculate the average force on the ball by her hands during the catch. (c) Explain why this force is smaller than it would be if she held her hands still.

Working	Comment
$m=160\,\mathrm{g}=0.160\,\mathrm{kg}$; $u=+30\,\mathrm{m\ s^{-1}}$; $v=0$; $\Delta t=0.15\,\mathrm{s}$.	List the data, converting mass to kilograms.
(a) $\Delta p=m\Delta v=m(v-u)=0.160\times(0-30)=-4.8\,\mathrm{kg\ m\ s}$.	The change in momentum is negative: it acts opposite to the ball's original motion.
(b) $F\Delta t=m\Delta v$, so $F=\dfrac{m\Delta v}{\Delta t}=\dfrac{-4.8}{0.15}=-32\,\mathrm{N}$.	The average force is $32\,\mathrm{N}$, directed opposite to the ball's original motion. Both given values have 2 significant figures, so the answer is given to 2.
(c) The ball's change in momentum $m\Delta v$ is the same either way. Moving the hands back increases the stopping time Δt, and since $F=\dfrac{m\Delta v}{\Delta t}$, a larger Δt gives a smaller F.	Same impulse spread over a longer time means a smaller force.

Example 2 — scaffolded

In a grain receival yard in the Victorian Wimmera, a loaded grain wagon of mass $3.0\times10^4\,\mathrm{kg}$ rolls at $2.0\,\mathrm{m\ s^{-1}}$ toward a stationary wagon of mass $5.0\times10^4\,\mathrm{kg}$. The wagons couple together and move off as one. (a) Calculate the total momentum of the two wagons before the coupling. (b) State the total mass after the coupling. (c) Calculate the speed of the coupled wagons.

Working	Comment
(a) $p_{\text{before}}=m_1u_1+m_2u_2=(3.0\times10^4\times2.0)+(5.0\times10^4\times0)$.	The stationary wagon has zero momentum.
(b) $m_1+m_2=3.0\times10^4+5.0\times10^4=8.0\times10^4\,\mathrm{kg}$.	The coupled wagons move together as one object.
(c) $v=\dfrac{m_1u_1+m_2u_2}{m_1+m_2}=\dfrac{6.0\times10^4}{8.0\times10^4}=0.75\,\mathrm{m\ s^{-1}}$	Conservation of momentum for a sticking collision.

The coupled wagons move at $0.75\,\mathrm{m\ s^{-1}}$ in the original direction of the moving wagon. The much larger total mass moves more slowly than the single wagon did, even though the total momentum is unchanged.

Example 3 — unscaffolded

During a rally at the Australian Open at Melbourne Park, a tennis ball of mass $58\,\mathrm{g}$ arrives horizontally at $20\,\mathrm{m\ s^{-1}}$ and is returned horizontally at $30\,\mathrm{m\ s^{-1}}$ in the opposite direction. The ball is in contact with the strings for $5.0\,\mathrm{ms}$. Taking the direction of the returned ball as positive, calculate the change in momentum of the ball and the average force on the ball by the racquet.

The initial velocity is $u=-20\,\mathrm{m\ s^{-1}}$ (the ball arrives against the positive direction) and the final velocity is $v=+30\,\mathrm{m\ s^{-1}}$, so

$$\Delta v=v-u=30-(-20)=50\,\mathrm{m\ s^{-1}}.$$

With $m=58\,\mathrm{g}=0.058\,\mathrm{kg}$, the change in momentum is

$$\Delta p=m\,\Delta v=0.058\times 50=2.9\,\mathrm{N\ s},$$

in the direction of the return. The average force follows from $F\,\Delta t=m\,\Delta v$ with $\Delta t=5.0\,\mathrm{ms}=0.0050\,\mathrm{s}$:

$$F=\frac{m\,\Delta v}{\Delta t}=\frac{2.9}{0.0050}=580\,\mathrm{N}.$$

∴ the change in momentum of the ball is $2.9\,\mathrm{N\ s}$ and the average force on the ball by the racquet is $580\,\mathrm{N}$, both in the direction of the returned ball. Notice that Δv is $50\,\mathrm{m\ s^{-1}}$, not $10\,\mathrm{m\ s^{-1}}$: reversing the ball's direction is part of the change the racquet must produce.

Example 4 — unscaffolded

In an experiment like the practical investigation in this section, trolley A of mass $2.0\,\mathrm{kg}$ moves at $1.5\,\mathrm{m\ s^{-1}}$ toward trolley B of mass $1.0\,\mathrm{kg}$, which is at rest. After the collision the trolleys bounce apart, and trolley A continues in its original direction at $0.60\,\mathrm{m\ s^{-1}}$. (a) Calculate the velocity of trolley B after the collision. (b) The students find that the total momentum after the collision is slightly less than the total momentum before it. Explain why this does not contradict the conservation of momentum.

The total momentum before the collision is

$$m_A u_A+m_B u_B=2.0\times 1.5+1.0\times 0=3.0\,\mathrm{kg\ m\ s^{-1}}.$$

After the collision, trolley A carries $m_A v_A=2.0\times 0.60=1.2\,\mathrm{kg\ m\ s^{-1}}$, so conservation of momentum gives the momentum of trolley B:

$$m_B v_B=3.0-1.2=1.8\,\mathrm{kg\ m\ s^{-1}},\ v_B=\frac{1.8}{1.0}=1.8\,\mathrm{m\ s^{-1}}.$$

∴ trolley B moves at $1.8\,\mathrm{m\ s^{-1}}$ in the original direction of trolley A.

For part (b): the law of conservation of momentum applies to an isolated system, and the two trolleys are only approximately isolated. Friction in the axles and air resistance are external forces that act on the trolleys, slowly removing momentum from the system, and the speed measurements themselves carry some uncertainty. A small difference between the totals before and after is therefore expected; it is the closeness of the two totals, not their exact equality, that supports the conservation law.

Practice questions

Question 1 *(scaffolded)* A soccer ball of mass $0.43\,\mathrm{kg}$ is kicked from rest and leaves the boot at $25\,\mathrm{m\ s^{-1}}$. The boot is in contact with the ball for $0.010\,\mathrm{s}$. (a) Calculate the change in momentum of the ball. (b) Calculate the average force on the ball by the boot. (c) State the direction of this force relative to the direction the ball leaves.

Question 2 *(scaffolded)* In a rail yard, a truck of mass $4.0\times 10^3\,\mathrm{kg}$ rolls at $3.0\,\mathrm{m\ s^{-1}}$ toward a stationary truck of mass $2.0\times 10^3\,\mathrm{kg}$. The two trucks couple together and move off as one. (a) Calculate the total momentum of the trucks before the coupling. (b) State the total mass of the coupled trucks. (c) Calculate the speed of the coupled trucks. (d) Some of the kinetic energy of the moving truck is transformed during the coupling. Name two forms of energy into which it is transformed.

Question 3 *(scaffolded)* A trolley of mass $0.60\,\mathrm{kg}$ is initially at rest on a smooth track. A constant force of $12\,\mathrm{N}$ pushes it along the track for $0.25\,\mathrm{s}$. (a) Calculate the impulse delivered to the trolley. (b) Calculate the change in velocity of the trolley. (c) State the trolley's final velocity, including its direction.

Question 4 *(scaffolded)* A cricket ball of mass $0.16\,\mathrm{kg}$ arrives at the bat at $25\,\mathrm{m\ s^{-1}}$ and is hit straight back toward the bowler at $35\,\mathrm{m\ s^{-1}}$. The ball is in contact with the bat for $0.010\,\mathrm{s}$. Take the direction of the returned ball as positive. (a) State u and v for the ball, and calculate Δv. (b) Calculate the change in momentum of the ball. (c) Calculate the average force on the ball by the bat.

Question 5 *(scaffolded)* On an ice rink, player A of mass $80\,\mathrm{kg}$ skates at $3.0\,\mathrm{m\ s^{-1}}$ toward player B of mass $60\,\mathrm{kg}$, who skates at $4.0\,\mathrm{m\ s^{-1}}$ directly toward player A. The players collide and hold on to each other. Take player A's original direction as positive. (a) Calculate the momentum of player A before the collision. (b) Calculate the momentum of player B before the collision. (c) Calculate the total momentum of the two players before the collision. (d) Deduce the velocity of the two players immediately after the collision, and explain your reasoning.

Question 6 *(scaffolded)* In the practical investigation of this section, a trolley carries an interrupter card $5.0\,\mathrm{cm}$ wide. As the trolley passes through a light gate, the card blocks the beam for $0.050\,\mathrm{s}$. (a) Calculate the speed of the trolley as it passes the light gate. (b) Explain why the track must be level before measurements begin, referring to the forces on a trolley. (c) Give one reason for repeating each measurement several times.

Question 7 A tennis ball of mass $57\,\mathrm{g}$ is fired horizontally at a wall at $8.0\,\mathrm{m\ s^{-1}}$ and bounces straight back at $6.0\,\mathrm{m\ s^{-1}}$. The ball is in contact with the wall for $0.050\,\mathrm{s}$. Taking the direction away from the wall as positive, calculate the average net force on the ball during the contact.

Question 8 In a head-on crash, a car's driver is brought to rest whether they hit the hard steering wheel or hit an inflated airbag. Use $F\,\Delta t = m\,\Delta v$ to explain why the airbag greatly reduces the risk of injury.

Question 9 Two ice-skaters stand still on the ice facing each other, then push apart. Skater A has mass $70\,\mathrm{kg}$ and moves off at $2.4\,\mathrm{m\ s^{-1}}$ to the right. Skater B has mass $56\,\mathrm{kg}$. Calculate the velocity of skater B, taking right as positive.

Question 10 A water balloon thrown to you is caught without breaking by letting your hands move back toward your body as you catch it. Use the impulse relationship to explain why this works.

Question 11 On a smooth track, trolley A of mass $1.2\,\mathrm{kg}$ moves at $2.0\,\mathrm{m\ s^{-1}}$ toward trolley B, which is at rest. After the collision trolley A is stationary and trolley B moves in the original direction of A at $1.5\,\mathrm{m\ s^{-1}}$. Calculate the mass of trolley B.

Question 12 Two rail trucks couple together, and the coupled pair moves more slowly than the single moving truck did before the coupling. A student says: "The trucks have lost momentum, because they are moving more slowly." Identify the error in this statement and correct it.

Question 13 A trolley of mass $0.40\,\mathrm{kg}$ moves to the right at $3.0\,\mathrm{m\ s^{-1}}$ toward a trolley of mass $0.20\,\mathrm{kg}$ moving to the left at $3.0\,\mathrm{m\ s^{-1}}$. The trolleys collide and stick together. Taking right as positive, calculate the velocity of the joined trolleys.

Question 14 A golfer is told to "follow through" when driving the ball. Use the impulse relationship to justify this advice.

Question 15 In a momentum experiment, trolley A of mass $0.50\,\mathrm{kg}$ moves at $1.2\,\mathrm{m\ s^{-1}}$ toward trolley B of mass $0.30\,\mathrm{kg}$, which is at rest. The trolleys stick together on impact. (a) Calculate the speed of the joined trolleys predicted by conservation of momentum. (b) The speed measured after the collision is $0.72\,\mathrm{m\ s^{-1}}$. Give one reason why the measured speed is less than the predicted value.

Question 16 A large truck and a small car collide head-on. A student claims that the larger truck exerts a larger force on the car than the car exerts on the truck, because the car is more badly damaged. Using Newton's laws, explain why this claim is wrong, and why the car's motion changes more than the truck's.

Question 17 A car of mass $1400\,\mathrm{kg}$ travels at $15\,\mathrm{m\ s^{-1}}$ toward a school crossing. The driver brakes, and a constant braking force of 7000 N brings the car to rest. Use $F\,\Delta t = m\,\Delta v$ to calculate the time taken to stop.

Question 18 A person jumps down from a bench and lands on the floor. Use the impulse relationship to explain why bending the knees on landing reduces the risk of injury compared with landing stiff-legged.

Answers

Question 1 (a) $11\,\text{kg m s}^{-1}$; (b) $1.1\times10^{3}\,\text{N}$; (c) see solution. **Question 2** (a) $1.2\times10^{4}\,\text{kg m s}^{-1}$; (b) $6.0\times10^{3}\,\text{kg}$; (c) $2.0\,\text{m s}^{-1}$; (d) see solution. **Question 3** (a) $3.0\,\text{N s}$; (b) $5.0\,\text{m s}^{-1}$; (c) $5.0\,\text{m s}^{-1}$ in the direction of the force. **Question 4** (a) $\Delta v=60\,\text{m s}^{-1}$; (b) $9.6\,\text{kg m s}^{-1}$; (c) $9.6\times10^{2}\,\text{N}$. **Question 5** (a) $240\,\text{kg m s}^{-1}$; (b) $-240\,\text{kg m s}^{-1}$; (c) 0; (d) see solution. **Question 6** (a) $1.0\,\text{m s}^{-1}$; (b) see solution; (c) see solution. **Question 7** $16\,\text{N}$, away from the wall. **Question 8** See solution. **Question 9** $-3.0\,\text{m s}^{-1}$ (that is, $3.0\,\text{m s}^{-1}$ to the left). **Question 10** See solution. **Question 11** $1.6\,\text{kg}$. **Question 12** See solution. **Question 13** $1.0\,\text{m s}^{-1}$ to the right. **Question 14** See solution. **Question 15** (a) $0.75\,\text{m s}^{-1}$; (b) see solution. **Question 16** See solution. **Question 17** $3.0\,\text{s}$. **Question 18** See solution.

Fully-worked solutions

Question 1 $m=0.43\,\text{kg}$, $u=0$, $v=25\,\text{m s}^{-1}$, $\Delta t=0.010\,\text{s}$. (a) $\Delta p=m\,\Delta v=m(v-u)=0.43\times(25-0)=10.75\,\text{kg m s}^{-1}$, which rounds to $11\,\text{kg m s}^{-1}$ (2 significant figures). (b) $F=\dfrac{m\,\Delta v}{\Delta t}=\dfrac{10.75}{0.010}=1075\,\text{N}$, which rounds to $1.1\times10^{3}\,\text{N}$ (2 significant figures). (c) The force acts in the same direction as the change in momentum, which is the direction the ball leaves the boot: impulse and change in momentum always have the direction of the force that caused them.

Question 2 $m_1=4.0\times10^{3}\,\text{kg}$, $u_1=3.0\,\text{m s}^{-1}$; $m_2=2.0\times10^{3}\,\text{kg}$, $u_2=0$. (a) $p_{\text{before}}=m_1u_1+m_2u_2=(4.0\times10^{3}\times3.0)+0=1.2\times10^{4}\,\text{kg m s}^{-1}$. (b) Total mass $=4.0\times10^{3}+2.0\times10^{3}=6.0\times10^{3}\,\text{kg}$. (c) $v=\dfrac{m_1u_1+m_2u_2}{m_1+m_2}=\dfrac{1.2\times10^{4}}{6.0\times10^{3}}=2.0\,\text{m s}^{-1}$, in the original direction of the moving truck. (d) Thermal energy and sound; energy is also used to deform the coupling as the trucks jerk together. (Any two of thermal energy, sound, deformation energy.)

Question 3 $m=0.60\,\text{kg}$, $F=12\,\text{N}$, $\Delta t=0.25\,\text{s}$, $u=0$. (a) Impulse $=F\,\Delta t=12\times0.25=3.0\,\text{N s}$. (b) $F\,\Delta t=m\,\Delta v$, so $\Delta v=\dfrac{F\,\Delta t}{m}=\dfrac{3.0}{0.60}=5.0\,\text{m s}^{-1}$. (c) The trolley started from rest, so its final velocity is $5.0\,\text{m s}^{-1}$ in the direction of the applied force.

Question 4 $m=0.16\,\text{kg}$; taking the direction of the returned ball as positive: $u=-25\,\text{m s}^{-1}$, $v=+35\,\text{m s}^{-1}$; $\Delta t=0.010\,\text{s}$. (a) $\Delta v=v-u=35-(-25)=60\,\text{m s}^{-1}$. The change in velocity is $60\,\text{m s}^{-1}$, not $10\,\text{m s}^{-1}$, because the ball reverses direction. (b) $\Delta p=m\,\Delta v=0.16\times60=9.6\,\text{kg m s}^{-1}$, toward the bowler. (c) $F=\dfrac{m\,\Delta v}{\Delta t}=\dfrac{9.6}{0.010}=960\,\text{N}=9.6\times10^{2}\,\text{N}$ (2 significant figures), toward the bowler.

Question 5 $m_A=80\,\text{kg}$, $u_A=+3.0\,\text{m s}^{-1}$; $m_B=60\,\text{kg}$, $u_B=-4.0\,\text{m s}^{-1}$. (a) $p_A=m_Au_A=80\times3.0=240\,\text{kg m s}^{-1}$. (b) $p_B=m_Bu_B=60\times(-4.0)=-240\,\text{kg m s}^{-1}$. (c) $p_{\text{total}}=240+(-240)=0$. (d) The two players form an approximately isolated system during the brief collision, so the total momentum after the collision must also be zero. Their combined mass is not zero, so their velocity must be: they are stationary immediately after the collision.

Question 6 Card width $=5.0\,\text{cm}=0.050\,\text{m}$; interruption time $=0.050\,\text{s}$. (a) $v=\dfrac{\text{card width}}{\text{interruption time}}=\dfrac{0.050}{0.050}=1.0\,\text{m s}^{-1}$. (b) On a level track the normal force on a trolley by the track balances the force due to gravity on it vertically, so there is no component of either force along the track. A trolley not in a collision then moves at constant velocity, and the system is approximately isolated during the collision. If the track slopes, the component of the force due to gravity along the track is an external force that accelerates the trolleys and changes the total momentum. (c) Repeating each measurement and averaging reduces the effect of random errors — small, unpredictable variations from one run to the next — and makes any mistake in a single reading easier to spot.

Question 7 $m=57\,\text{g}=0.057\,\text{kg}$; taking away from the wall as positive: $u=-8.0\,\text{m s}^{-1}$, $v=+6.0\,\text{m s}^{-1}$; $\Delta t=0.050\,\text{s}$.

$$\Delta v=v-u=6.0-(-8.0)=14\,\text{m s}^{-1},$$

$$\Delta p=m\,\Delta v=0.057\times 14=0.798\,\text{N s},$$

$$F=\frac{m\,\Delta v}{\Delta t}=\frac{0.798}{0.050}=15.96\,\text{N}\approx 16\,\text{N}$$

to two significant figures, directed away from the wall.

Question 8 The driver's change in momentum $m\,\Delta v$ is the same in either case: the same mass goes from the same initial speed to rest. Since $F=\frac{m\,\Delta v}{\Delta t}$, the average force depends on the stopping time. The airbag compresses as the driver moves into it, bringing the driver to rest over a much longer time than the hard steering wheel would. The larger Δt gives a smaller average force, and a smaller force means less injury. (The airbag also spreads the force over a larger area of the body, which reduces damage further.)

Question 9 The skaters start at rest, so the total momentum before the push is zero. Taking right as positive and applying conservation of momentum:

$$0=m_A v_A+m_B v_B,\, v_B=-\frac{m_A v_A}{m_B}=-\frac{70\times 2.4}{56}=-3.0\,\text{m s}^{-1}.$$

Skater B moves at $3.0\,\text{m s}^{-1}$ to the left — opposite to skater A, and faster, because skater B is lighter.

Question 10 The balloon's change in momentum — from its incoming speed to rest — is fixed once it reaches you. Moving the hands back with the balloon lengthens the time Δt over which the balloon is brought to rest, and since $F=\frac{m\,\Delta v}{\Delta t}$, the average force on the balloon is smaller. A smaller force is not enough to burst the balloon; stopping it suddenly (small Δt) would need a large force that breaks the skin.

Question 11 $m_A=1.2\,\text{kg}$, $u_A=2.0\,\text{m s}^{-1}$, $u_B=0$; after: $v_A=0$, $v_B=1.5\,\text{m s}^{-1}$.

$$m_A u_A+m_B u_B=m_A v_A+m_B v_B,$$

$$1.2\times 2.0+0=0+m_B\times 1.5,\, m_B=\frac{2.4}{1.5}=1.6\,\text{kg}.$$

Question 12 Momentum depends on mass as well as velocity: $p=m\,v$. After the coupling the mass is larger than before, so a lower speed can still correspond to the same momentum. The correct statement is: the coupled trucks move more slowly *because* their combined mass is larger, and (if external forces are negligible) their total momentum is unchanged, not lost. The student has treated momentum as though it were speed.

Question 13 $m_1=0.40\,\text{kg}$, $u_1=+3.0\,\text{m s}^{-1}$; $m_2=0.20\,\text{kg}$, $u_2=-3.0\,\text{m s}^{-1}$.

$$p_{\text{before}}=0.40\times 3.0+0.20\times(-3.0)=1.2-0.6=0.6\,\text{kg m s}^{-1}.$$

The trolleys stick, so the combined mass is $0.60\,\text{kg}$:

$$v=\frac{0.6}{0.60}=1.0\,\text{m s}^{-1},$$

to the right, since the result is positive.

Question 14 The change in the ball's momentum equals the impulse delivered to it: $m\,\Delta v=F\,\Delta t$. Following through keeps the club in contact with the ball for a longer time Δt, so for the same force a larger impulse is delivered. A larger impulse gives the ball a larger change in momentum, and so a larger speed off the club face.

Question 15 $m_A = 0.50\,\text{kg}$, $u_A = 1.2\,\text{m s}^{-1}$; $m_B = 0.30\,\text{kg}$, $u_B = 0$. (a) $v = \frac{m_A u_A + m_B u_B}{m_A + m_B} = \frac{0.50 \times 1.2}{0.80} = \frac{0.60}{0.80} = 0.75\,\text{m s}^{-1}$. (b) The prediction assumes the trolleys form an isolated system, but friction in the axles and air resistance are external forces that remove momentum from the trolleys between the release and the measurement, so the measured speed comes out slightly low. (Small differences also arise from the limited precision of the timing measurements.)

Question 16 By Newton's third law, $F_{\text{on car by truck}} = -F_{\text{on truck by car}}$: the two forces are equal in magnitude and opposite in direction, whatever the masses of the vehicles. The larger damage to the car does not mean a larger force. What differs is the effect of the force: by Newton's second law, $a = F_{\text{net}} / m$, the same magnitude of force gives the smaller mass of the car a larger acceleration (a larger change in velocity), so the car's motion changes more than the truck's. By $F\,\Delta t = m\,\Delta v$, acting for the same contact time, the equal and opposite forces also give the two vehicles equal and opposite changes in momentum — which is why the total momentum of the pair is conserved.

Question 17 $m = 1400\,\text{kg}$, $u = 15\,\text{m s}^{-1}$, $v = 0$; the braking force acts against the motion, so take it as $F = -7000\,\text{N}$.

$$F\,\Delta t = m\,\Delta v,\ \Delta t = \frac{m(v-u)}{F} = \frac{1400 \times (0-15)}{-7000} = \frac{-21\,000}{-7000} = 3.0\,\text{s}.$$

The car takes $3.0\,\text{s}$ to stop.

Question 18 The person's change in momentum — from their speed just before landing to rest — is the same whichever way they land. Since $F = \frac{m\,\Delta v}{\Delta t}$, the average force is smaller when the stopping time Δt is larger. Bending the knees lets the body come to rest over a longer time than a stiff-legged landing does, so the average force on the legs and body is smaller, reducing the risk of injury.

Scope — *VCE Physics Study Design, Unit 2, Area of Study 1*: “calculate torque, τ = r⊥F”; and “analyse translational and rotational forces (torques) in simple structures in translational and rotational equilibrium”.

Theory

Forces can turn things. In the earlier sections of this chapter, forces changed the straight-line motion of objects: they changed momentum (Section 8A), they produced acceleration according to Newton’s laws (Section 8B), and they balanced one another when an object was at rest (Sections 8C and 8D). Many forces do something else as well: they make objects rotate. A push on a door turns it about its hinges. A pull on a spanner turns a nut about its axis. A child sitting on one side of a seesaw turns the plank about its pivot. This section builds the model that describes the turning effect of a force — the **torque** — and applies it to objects and simple structures that are in equilibrium.

Torque — the turning effect of a force. Consider a body that is free to rotate about a fixed point, called the **pivot** (a real hinge or axle is a line, but in a flat diagram it is drawn as one point). A force F applied to the body tends to rotate it about that pivot. The size of the turning effect depends on two things: how large the force is, and how far from the pivot it acts — measured in a particular way.

Extend the force’s **line of action**: the straight line through the point of application along which the force acts. Then drop a perpendicular from the pivot to that line. This perpendicular distance $r_\perp$ from the pivot to the line of action is the lever arm of the force. The **torque** of the force about the pivot is

$$\tau = r_\perp F,$$

measured in **newton-metres** ($\mathrm{N\ m}$): a force of $1\,\mathrm{N}$ acting with a lever arm of $1\,\mathrm{m}$ produces a torque of $1\,\mathrm{N\ m}$. The study design also calls torques *rotational forces* and ordinary forces *translational forces*, because a net force can produce a change in translational motion while a net torque produces a change in rotational motion. (Some texts call this quantity the *moment* of a force; this book uses *torque* throughout.)

A larger force produces a larger torque, and the same force produces a larger torque when it acts further from the pivot. A force of $20\,\mathrm{N}$ applied perpendicular to a spanner $0.30\,\mathrm{m}$ from a bolt gives a torque of $6.0\,\mathrm{N\ m}$; the same force applied $0.60\,\mathrm{m}$ from the bolt gives $12\,\mathrm{N\ m}$. This is why a longer spanner loosens a stubborn bolt more easily, and why door handles are placed at the edge of the door rather than near the hinges. A newton-metre has the same dimensions as a joule, but the joule is reserved for energy and work: torques are always quoted in $\mathrm{N\ m}$, never in joules.

When the force is not perpendicular. The definition $\tau = r_\perp F$ always uses the perpendicular distance from the pivot to the line of action. When the force makes an angle θ with the straight line joining the pivot to the point of application, the lever arm is shorter than that line:

$$r_\perp = r\sin\theta, \text{so}\ \tau = rF\sin\theta.$$

The same result follows by resolving the force into components, as in Section 8D: only the component of the force perpendicular to the arm, $F\sin\theta$, turns the body; the component along the arm points toward or away from the pivot and produces no torque at all. The torque is largest when the force is perpendicular to the arm ($\theta = 90^\circ$, so $\sin\theta = 1$), and it is zero when the force points along the arm ($\theta = 0^\circ$).

Torques have a direction of turning. A torque always tries to rotate the body one way or the other about the pivot: clockwise or anticlockwise. To combine torques, choose one direction of turning as positive and add torques with signs, exactly as straight-line forces and velocities were added with signs in Section 8E. The **net torque** is this sum: if the clockwise and anticlockwise torques cancel, the net torque is zero.

A force through the pivot produces no torque. If a force’s line of action passes through the pivot, then $r_\perp = 0$ and the torque of that force is zero, no matter how large the force is. Pushing on a door in a direction along the door, toward the hinges, produces no turning at all. One consequence is used constantly in this section: the force exerted *at* a pivot has zero torque *about* that pivot.

Translational equilibrium — a reminder. From Section 8B, an object is in **translational equilibrium** when the net force on it is zero:

$$F_{\text{net}} = 0 .$$

By Newton's first law, such an object remains at rest or continues to move in a straight line at constant velocity. The objects in this section are all at rest, so in every case the upward forces balance the downward forces, and the leftward forces balance the rightward forces.

Rotational equilibrium. A torque changes an object's rotational motion in the same sense that a force changes its straight-line motion. An object is in **rotational equilibrium** when the net torque on it is zero: its rotational motion does not change, so an object that is not rotating remains not rotating.

Zero net force alone is not enough for equilibrium. Two forces that are equal in magnitude and opposite in direction, but whose lines of action do not coincide, have a vector sum of zero yet produce torques in the same direction of turning — think of two hands turning a steering wheel, or the two hands on a tap handle. The net force is zero but the net torque is not, and the object rotates. Such a pair of forces is called a **couple**. Equilibrium therefore requires two conditions, not one:

> **Conditions for equilibrium.** A body is in equilibrium when both hold: - **translational equilibrium** — the net force on the body is zero, so it does not accelerate in any direction; - **rotational equilibrium** — the net torque on the body about any point is zero, so it does not begin to rotate.

The principle of moments. For calculations, the rotational condition is usually written as the **principle of moments**:

> When a body is in rotational equilibrium, the sum of the clockwise torques about any chosen point equals the sum of the anticlockwise torques about that same point.

The point may be chosen freely: a body at rest has zero net torque about *every* point, so the principle holds about whichever point is convenient. A useful trick follows from the zero-torque rule above: if the pivot is chosen at the point where an unknown force acts, that force's torque is zero and the force drops out of the torque equation entirely.

The force due to gravity acts at the centre of mass. Every part of an extended body is pulled downward by Earth, but as established in Section 8C the whole force due to gravity, $F_g = mg$, can be taken to act at one point: the **centre of mass** of the body. When calculating the torque produced by F_g, use the lever arm measured from the pivot to the centre of mass. For a uniform beam, plank or ruler, the centre of mass is at the geometric centre.

A method for equilibrium problems. Every problem in this section follows the same four steps.

1. **Draw and label.** Sketch the body and every force acting on it, with each force at its point of application: F_g at the centre of mass; the normal force at each support, perpendicular to the contact surface; tensions along cables, directed toward the cable.
2. **Choose a pivot.** Choose it at the point where an unknown force acts, if possible, so that force drops out of the torque equation.
3. **Balance the torques.** Find the lever arm $r_{\perp}$ of each force about the pivot, decide each force's direction of turning, and write the principle of moments: clockwise torques = anticlockwise torques.
4. **Balance the forces if needed.** If a second unknown remains, apply translational equilibrium: upward forces = downward forces, and leftward forces = rightward forces.

Simple structures in equilibrium. The study design asks for the analysis of *simple structures in translational and rotational equilibrium*: planks resting on supports, shelves on brackets, beams held by a pivot and a cable, and the jibs of cranes. The supports exert upward normal forces on the structure, cables exert tensions, and the structure's own F_g acts at its centre of mass. Every such analysis is the method above: the torque balance finds one unknown force, and the force balance finds the rest. The structure itself is treated as a rigid body that does not bend, stretch or break; what happens *inside* loaded members is left to Option 2.4 (Section 11B).

Practical investigation — the principle of moments. The principle of moments can be tested with a ruler balanced on a pivot. A suitable framework:

- **Aim.** Test whether, for a horizontal ruler balanced on a pivot, the sum of the clockwise torques about the pivot equals the sum of the anticlockwise torques.
- **Equipment.** A metre ruler; a pivot on which the ruler can balance (a knife-edge or a triangular prism); a set of slotted masses with hangers; a balance to measure the masses.
- **Method.** First find the balance point of the ruler alone: when the ruler rests horizontal, the pivot is directly below its centre of mass, near the $50.0\,\text{cm}$ mark for a uniform ruler. With the pivot at that mark, the ruler's own F_g acts through the pivot and produces no torque. Hang known masses on each side of the pivot and adjust their positions until the ruler balances horizontally. Record each mass and its distance from the pivot. Calculate the force due to gravity on each mass ($F = m g$) and its torque ($\tau = r_\perp F$), and compare the total clockwise torque with the total anticlockwise torque. Repeat for at least three different arrangements, including one with two masses on the same side of the pivot.
- **Expected results.** The clockwise and anticlockwise totals agree within the precision of the measurements. Small differences are expected: the ruler is not perfectly uniform, the pivot has a finite width rather than being a true point, and distances are read to the nearest millimetre.
- **Safety and practice.** Keep slotted masses clear of feet and secure the pivot so the ruler cannot slide off the bench. Record every reading — each mass and each distance, with its unit — in your logbook: the logbook is submitted as a requirement for satisfactory completion of the unit.

The equations of this section are summarised below.

Quantity	Symbol	Relationship	Unit
torque	τ	$\tau = r_\perp F$	N m
lever arm of an angled force	$r_\perp$	$r_\perp = r \sin\theta$	m
translational equilibrium	—	$F_{\text{net}} = 0$	—
rotational equilibrium	—	$\Sigma\tau = 0$ (clockwise = anticlockwise)	—

Where this leads. The torque model of this section is reused wherever a force acts at a distance from a pivot. Option 2.4 (Section 11B) applies it to the stability of structures — why a tower crane carries a counterweight, and when a loaded beam tips. Option 2.5 (Section 11C) applies the same model to the forces and torques in the human body. The case study of Section 9E can also involve a structure, where torques matter.

Units 3 & 4 preview. At Units 3 & 4 the torque idea reappears in a new setting: a coil carrying a current in a magnetic field experiences a torque, and that torque is what makes a DC motor turn. The argument there is the argument of this section — forces acting at a distance from an axis of rotation — applied to the magnetic forces on the sides of the coil. Content under this banner is not assessed in this book's topic tests or practice examinations.

Throughout this section $g = 9.8\,\text{N kg}^{-1}$ (equivalently $9.8\,\text{m s}^{-2}$), as established in Section 8C.

Worked examples

Example 1 — scaffolded

At a service centre in Geelong, a mechanic loosens a wheel nut using a spanner. She pulls with a force of $250\,\text{N}$ applied perpendicular to the spanner, at a point $0.40\,\text{m}$ from the centre of the nut.

(a) Calculate the torque about the nut.

(b) She slips a pipe over the handle so that her hands are $0.60\,\text{m}$ from the nut, and pulls again with $250\,\text{N}$, perpendicular to the spanner. Calculate the new torque.

(c) Explain why the pipe makes the nut easier to loosen.

Working	Comment
The force is perpendicular to the spanner, so $r_{\perp} = 0.40\,\text{m}$.	The lever arm is the perpendicular distance from the pivot (the nut) to the line of action of the force.
(a) $\tau = r_{\perp} F = 0.40 \times 250 = 100\,\text{N m}$	Torque = lever arm × force.
(b) $\tau = r_{\perp} F = 0.60 \times 250 = 150\,\text{N m}$	The same force now acts with a larger lever arm.
(c) Loosening the nut requires a particular torque. Since $\tau = r_{\perp} F$, a larger lever arm delivers that torque with a smaller force.	The pipe extends the distance from the pivot at which the mechanic's force acts.

Example 2 — scaffolded

In a Melbourne playground, two children balance on a uniform plank of mass $20\,\text{kg}$ and length $4.0\,\text{m}$ that rests on a pivot at its centre. Child A, of mass $30\,\text{kg}$, sits $1.5\,\text{m}$ to the left of the pivot. Child B has a mass of $25\,\text{kg}$. Take $g = 9.8\,\text{N kg}^{-1}$.

(a) Calculate the torque about the pivot produced by the force due to gravity on child A, and state its direction of turning.

(b) Calculate how far from the pivot child B must sit for the plank to balance horizontally.

(c) Calculate the force on the plank by the pivot when the plank is balanced.

Working	Comment
F_g on child A $= 30 \times 9.8 = 294\,\text{N}$, acting $1.5\,\text{m}$ left of the pivot.	The force due to gravity acts downward, at the child's position.
(a) $\tau_A = r_{\perp} F = 1.5 \times 294 = 441\,\text{N m}$, anticlockwise.	A force pulling down on the left side of the pivot turns the plank anticlockwise about it.
The plank's own F_g acts at its centre — at the pivot — so it produces no torque about the pivot.	The plank is uniform and the pivot is at its centre, so the lever arm of the plank's F_g is zero.
(b) F_g on child B $= 25 \times 9.8 = 245\,\text{N}$. By the principle of moments, $245 \times d = 441$, so $d = 1.8\,\text{m}$ to the right of the pivot.	Clockwise torque = anticlockwise torque; child B sits on the right so that her torque opposes child A's.
(c) Upward force on plank by pivot $= (20 + 30 + 25) \times 9.8 = 75 \times 9.8 = 735\,\text{N}$.	Translational equilibrium: the pivot supports the forces due to gravity on the plank and both children. The plank's F_g produced no torque, but it still counts in the force balance.

Example 3 — unscaffolded

On a building site in Ballarat, a painter stands on a uniform plank of mass $20\,\text{kg}$ and length $4.0\,\text{m}$ that rests on two trestles, one at each end of the plank. The painter has a mass of $70\,\text{kg}$ and stands $1.0\,\text{m}$ from the left-hand trestle. Calculate the force on the plank by each trestle. Take $g = 9.8\,\text{N kg}^{-1}$.

Four forces act on the plank: the force due to gravity on the plank, $F_g = 20 \times 9.8 = 196\,\text{N}$, acting at the centre of the plank ($2.0\,\text{m}$ from either end); the force due to gravity on the painter, $686\,\text{N}$, acting $1.0\,\text{m}$ from the left end; and the upward forces F_L and F_R exerted on the plank by the left and right trestles. The plank is at rest, so both equilibrium conditions apply.

Take torques about the left trestle; this choice makes F_L drop out, since it acts at the pivot. The two forces due to gravity act to the right of the pivot and produce clockwise torques, while F_R, acting upward $4.0\,\text{m}$ from the pivot, produces the anticlockwise torque:

$$F_R \times 4.0 = 196 \times 2.0 + 686 \times 1.0 = 392 + 686 = 1078\,\text{N m},$$

$$F_R = \frac{1078}{4.0} = 269.5\,\text{N}.$$

The vertical forces must also balance:

$$F_L + F_R = 196 + 686 = 882\,\text{N}, F_L = 882 - 269.5 = 612.5\,\text{N}.$$

(Check, taking torques about the right trestle instead: $F_L \times 4.0 = 196 \times 2.0 + 686 \times 3.0 = 392 + 2058 = 2450\,\text{N m}$, giving $F_L = 612.5\,\text{N}$. ✓)

∴ the force on the plank by the left trestle is $610\,\text{N}$ and the force on the plank by the right trestle is $270\,\text{N}$, both to two significant figures (the small difference between their sum, $880\,\text{N}$, and the total downward force, $882\,\text{N}$, is due to rounding). The painter stands near the left end, so the left trestle carries most of the load.

Example 4 — unscaffolded

At a marina on Port Phillip Bay, a small crane lifts supplies onto a boat. Its uniform boom (arm) is $4.0\,\text{m}$ long, has a mass of $40\,\text{kg}$, and is pivoted at its base; the boom is held horizontal by a cable attached to the far end of the boom, the cable making an angle of 30° with the boom. A crate of mass $60\,\text{kg}$ hangs from the far end of the boom. Take $g = 9.8\,\text{N kg}^{-1}$.

(a) Calculate the total torque about the pivot produced by the two forces due to gravity.

(b) Calculate the tension in the cable.

(c) Explain why the force on the boom by the pivot must have a horizontal component, and state its direction.

The force due to gravity on the boom is $40 \times 9.8 = 392\,\text{N}$, acting at the centre of the boom, $2.0\,\text{m}$ from the pivot; the force due to gravity on the crate is $60 \times 9.8 = 588\,\text{N}$, acting at the far end, $4.0\,\text{m}$ from the pivot. Both act downward to the right of the pivot, so both produce clockwise torques.

(a) The total clockwise torque about the pivot is

$$\tau = 392 \times 2.0 + 588 \times 4.0 = 784 + 2352 = 3136\,\text{N m},$$

that is, $3.1 \times 10^3\,\text{N m}$ clockwise, to two significant figures.

(b) The tension T in the cable acts at the far end of the boom, $4.0\,\text{m}$ from the pivot, at 30° to the boom. Only the component of the tension perpendicular to the boom produces a torque: $T \sin 30^\circ$, directed upward, giving an anticlockwise torque. The component of the tension along the boom points toward the pivot and produces no torque. The boom is in rotational equilibrium, so

$$T \sin 30^\circ \times 4.0 = 3136, T \times 0.50 \times 4.0 = 3136,$$

$$T = \frac{3136}{2.0} = 1568\,\text{N}.$$

∴ the tension in the cable is $1.6 \times 10^3\,\text{N}$, to two significant figures.

(c) The tension pulls the end of the boom along the cable, toward the point above the pivot. As well as its upward component, it therefore has a horizontal component, $T \cos 30^\circ = 1568 \times 0.866 = 1358\,\text{N}$, directed along the boom toward the pivot. The boom is in translational equilibrium, so the horizontal forces on it must balance: the pivot must exert a horizontal force on the boom of equal magnitude, about $1.4 \times 10^3\,\text{N}$, directed horizontally away from the pivot side — pushing the boom outward, to balance the inward pull of the cable.

Practice questions

Question 1 *(scaffolded)* A student pushes on a classroom door with a force of $40\,\text{N}$, perpendicular to the door, at a point $0.75\,\text{m}$ from the hinges. (a) Calculate the torque about the hinges. (b) The student pushes with the same force at a point $0.30\,\mathrm{m}$ from the hinges. Calculate the torque. (c) Use your answers to explain why a door is hard to open when it is pushed near the hinges.

Question 2 *(scaffolded)* A student loosens a bolt on a farm water tank using a spanner of length $0.60\,\mathrm{m}$. She pulls on the end of the spanner with a force of $50\,\mathrm{N}$, at an angle of 60° to the spanner. (a) Calculate the perpendicular distance from the bolt to the line of action of the force. (b) Calculate the torque about the bolt. (c) State the angle at which the same force would give the largest torque, and calculate that torque.

Question 3 *(scaffolded)* A uniform plank of length $3.0\,\mathrm{m}$ rests on a pivot at its centre. A mass of $4.0\,\mathrm{kg}$ rests on the plank $1.2\,\mathrm{m}$ to the left of the pivot. Take $g = 9.8\,\mathrm{N\ kg^{-1}}$. (a) Calculate the torque about the pivot produced by the force due to gravity on the $4.0\,\mathrm{kg}$ mass, and state its direction of turning. (b) State the condition for the plank to remain balanced horizontally. (c) Calculate where a $6.0\,\mathrm{kg}$ mass must be placed on the plank to balance it.

Question 4 *(scaffolded)* A uniform shelf of length $2.0\,\mathrm{m}$ and mass $5.0\,\mathrm{kg}$ rests on two brackets, one at each end of the shelf. An ornament of mass $3.0\,\mathrm{kg}$ rests on the shelf $0.50\,\mathrm{m}$ from the left-hand bracket. Take $g = 9.8\,\mathrm{N\ kg^{-1}}$. (a) Calculate the force due to gravity on the shelf and on the ornament. (b) By taking torques about the left-hand bracket, calculate the force on the shelf by the right-hand bracket. (c) Calculate the force on the shelf by the left-hand bracket.

Question 5 *(scaffolded)* A student turns a tap handle by pushing on opposite ends of the handle with two hands. Each hand pushes with a force of $10\,\mathrm{N}$, perpendicular to the handle, at a distance of $0.15\,\mathrm{m}$ from the centre of the tap, and the two forces produce rotation in the same direction. (a) Explain why the net force on the handle is zero even though the handle tends to rotate. (b) Calculate the total torque about the centre of the tap. (c) The handle is not turning: friction in the valve holds the tap in rotational equilibrium. State the torque that the valve exerts on the handle, including its direction of turning.

Question 6 *(scaffolded)* In a laboratory, a student balances a uniform metre ruler on a pivot at the $50.0\,\mathrm{cm}$ mark. She hangs a $0.20\,\mathrm{kg}$ mass at the $20.0\,\mathrm{cm}$ mark and a $0.50\,\mathrm{kg}$ mass at the mark that brings the ruler back into balance. Take $g = 9.8\,\mathrm{N\ kg^{-1}}$. (a) Calculate the torque about the pivot produced by the $0.20\,\mathrm{kg}$ mass, including its direction of turning. (b) Calculate the distance from the pivot at which the $0.50\,\mathrm{kg}$ mass must hang. (c) State the mark on the ruler at which the $0.50\,\mathrm{kg}$ mass hangs.

Question 7 During a roadside wheel change near Bendigo, a driver must produce a torque of at least $120\,\mathrm{N\ m}$ to loosen a wheel nut. The driver uses a spanner of length $0.30\,\mathrm{m}$ and applies the force perpendicular to the spanner, at its end. Calculate the smallest force needed.

Question 8 Two students are trying to loosen the same stubborn nut. Student A pulls at the very end of a spanner of length L with a force of magnitude F, at 30° to the handle. Student B pulls with the same force F at the midpoint of the handle, perpendicular to the handle. Show that the two students produce the same torque about the nut, and explain what this shows about producing a large torque.

Question 9 Two students carry a uniform plank of mass $20\,\mathrm{kg}$ and length $3.0\,\mathrm{m}$ horizontally, each holding one end. A toolbox of mass $10\,\mathrm{kg}$ rests on the plank $1.0\,\mathrm{m}$ from student A. Take $g = 9.8\,\mathrm{N\ kg^{-1}}$. Calculate the upward force on the plank exerted by each student.

Question 10 A gardener lifts a loaded wheelbarrow by its handles. The force due to gravity on the load acts about $0.50\,\mathrm{m}$ from the axle of the wheel, and her hands act about $1.5\,\mathrm{m}$ from the axle. Use the torque model to explain why lifting the wheelbarrow by its handles is much easier than lifting the load directly.

Question 11 A student pulls on a rope wrapped around the rim of a wheel of radius $0.25\,\mathrm{m}$. The pull has magnitude $80\,\mathrm{N}$ and makes an angle of 70° with the radius at the point where the rope leaves the wheel. Calculate the torque about the axle of the wheel.

Question 12 When analysing a boom held by a pivot and a cable, the torque equation is usually written about the pivot. Explain why this choice makes the calculation easier, referring to the torque produced by the force at the pivot.

Question 13 A student balances a uniform metre ruler on a knife-edge at the $35.0\,\mathrm{cm}$ mark by hanging a $0.12\,\mathrm{kg}$ mass at the $5.0\,\mathrm{cm}$ mark. Take $g = 9.8\,\mathrm{N\ kg^{-1}}$. Show that the mass of the ruler is $0.24\,\mathrm{kg}$.

Question 14 A uniform beam rests horizontally on a single pivot at its centre. A student says: "The upward force of the pivot must be larger than the force due to gravity on the beam, because the pivot has to support the beam and also balance the torques." Identify the error in this statement and correct it.

Question 15 On a building site in Southbank, a uniform jib of mass $120\,\mathrm{kg}$ and length $6.0\,\mathrm{m}$ is held horizontal by a pivot at one end and a vertical cable attached to the jib $4.0\,\mathrm{m}$ from the pivot. A load of mass $300\,\mathrm{kg}$ hangs from the far end of the jib. Take $g = 9.8\,\mathrm{N\,kg^{-1}}$. (a) Show that the total clockwise torque about the pivot produced by the two forces due to gravity is $2.1 \times 10^4\,\mathrm{N\,m}$. (b) Calculate the tension in the cable.

Question 16 Two students analyse the same balanced beam. One takes torques about the left-hand support; the other takes torques about the right-hand support. They calculate different individual torques. Explain why both students correctly conclude that the beam is in rotational equilibrium.

Question 17 A cyclist climbing a hill in the Dandenong Ranges pushes straight down on a pedal with a force of $350\,\mathrm{N}$. At that instant, the crank makes an angle of 60° with the vertical. The crank is $0.17\,\mathrm{m}$ long. (a) Calculate the torque about the axle. (b) State at what angle of the crank the torque is largest, and calculate its value.

Question 18 In the practical investigation of this section, the masses hang vertically from a horizontal ruler, and distances are read off the ruler. Explain why the distance read off the ruler is the lever arm $r_\perp$ in this arrangement, and state what extra step the calculation would need if a force were applied to the ruler at an angle instead.

Answers

Question 1 (a) $30\,\mathrm{N\,m}$; (b) $12\,\mathrm{N\,m}$; (c) see solution. **Question 2** (a) $0.52\,\mathrm{m}$; (b) $26\,\mathrm{N\,m}$; (c) 90° (perpendicular to the spanner), $30\,\mathrm{N\,m}$. **Question 3** (a) $47\,\mathrm{N\,m}$, anticlockwise; (b) see solution; (c) $0.80\,\mathrm{m}$ to the right of the pivot. **Question 4** (a) $49\,\mathrm{N}$, $29\,\mathrm{N}$; (b) $32\,\mathrm{N}$; (c) $47\,\mathrm{N}$. **Question 5** (a) see solution; (b) $3.0\,\mathrm{N\,m}$; (c) $3.0\,\mathrm{N\,m}$, opposite to the direction of turning of the applied forces. **Question 6** (a) $0.59\,\mathrm{N\,m}$, anticlockwise; (b) $0.12\,\mathrm{m}$; (c) the $62\,\mathrm{cm}$ mark. **Question 7** $400\,\mathrm{N}$. **Question 8** See solution — both torques equal $1/2\,F\,L$. **Question 9** Student A: $160\,\mathrm{N}$; student B: $130\,\mathrm{N}$ (to two significant figures). **Question 10** See solution. **Question 11** $19\,\mathrm{N\,m}$. **Question 12** See solution. **Question 13** See solution. **Question 14** See solution. **Question 15** (a) $2.1\times10^4\,\mathrm{N\,m}$; (b) $5.3\times10^3\,\mathrm{N}$. **Question 16** See solution. **Question 17** (a) $52\,\mathrm{N\,m}$; (b) crank horizontal (90° to the force): $60\,\mathrm{N\,m}$. **Question 18** See solution.

Fully-worked solutions

Question 1 $F=40\,\mathrm{N}$, applied perpendicular to the door. (a) The force is perpendicular to the door, so the lever arm is the distance from the hinges: $\tau=r_\perp F=0.75\times40=30\,\mathrm{N\,m}$. (b) $\tau=r_\perp F=0.30\times40=12\,\mathrm{N\,m}$. (c) The torque needed to swing the door is the same whichever point is pushed. Since $\tau=r_\perp F$, pushing near the hinges gives a small lever arm, so a much larger force is needed to produce the same torque.

Question 2 $F=50\,\mathrm{N}$ at 60° to the spanner; distance along the spanner $r=0.60\,\mathrm{m}$. (a) $r_\perp=r\sin\theta=0.60\times\sin60^\circ=0.60\times0.866=0.52\,\mathrm{m}$ (two significant figures). (b) $\tau=r_\perp F=0.5196\times50=26\,\mathrm{N\,m}$. (c) The torque is largest when the force is perpendicular to the spanner, $\theta=90^\circ$, so that the full distance $0.60\,\mathrm{m}$ is the lever arm: $\tau=0.60\times50=30\,\mathrm{N\,m}$.

Question 3 $m_1=4.0\,\mathrm{kg}$ at $1.2\,\mathrm{m}$ left of the pivot; uniform plank pivoted at its centre. (a) $F_g=4.0\times9.8=39.2\,\mathrm{N}$; $\tau=r_\perp F=1.2\times39.2=47.04\,\mathrm{N\,m}$, which rounds to $47\,\mathrm{N\,m}$. The mass is to the left of the pivot and pulls downward, so the turning is anticlockwise. (b) The plank balances when the net torque about the pivot is zero: the sum of the clockwise torques equals the sum of the anticlockwise torques (the principle of moments). The plank's own F_g acts through the pivot and produces no torque. (c) F_g on the $6.0\,\mathrm{kg}$ mass $=6.0\times9.8=58.8\,\mathrm{N}$. Balancing torques, $58.8\times d=47.04$, so $d=0.80\,\mathrm{m}$, to the right of the pivot so that its torque is clockwise.

Question 4 Shelf: $m=5.0\,\mathrm{kg}$, length $2.0\,\mathrm{m}$; ornament: $m=3.0\,\mathrm{kg}$ at $0.50\,\mathrm{m}$ from the left bracket. (a) Shelf: $F_g=5.0\times9.8=49\,\mathrm{N}$, acting at the centre, $1.0\,\mathrm{m}$ from either bracket. Ornament: $F_g=3.0\times9.8=29.4\,\mathrm{N}$, which rounds to $29\,\mathrm{N}$. (b) About the left bracket, the shelf and the ornament produce clockwise torques and the right bracket produces the anticlockwise torque. The force at the left bracket acts at the pivot and drops out:

$$F_R\times2.0=49\times1.0+29.4\times0.50=49+14.7=63.7\,\mathrm{N\,m},$$

so $F_R=31.85\,\mathrm{N}$, which rounds to $32\,\mathrm{N}$ (two significant figures). (c) Vertical force balance: $F_L+F_R=49+29.4=78.4\,\mathrm{N}$, so $F_L=78.4-31.85=46.55\,\mathrm{N}$, which rounds to $47\,\mathrm{N}$.

Question 5 Each hand: $F=10\,\mathrm{N}$ at $0.15\,\mathrm{m}$ from the centre. (a) The two forces are equal in magnitude and opposite in direction, so their vector sum is zero and the net force on the handle is zero. Their lines of action do not coincide, however: both forces produce torques in the same direction of turning about the centre. This pair is a couple — zero net force, non-zero net torque — so the handle tends to rotate. (b) Each force gives $\tau=r_\perp F=0.15\times10=1.5\,\mathrm{N\,m}$, and both act in the same sense, so the total torque is $2\times1.5=3.0\,\mathrm{N\,m}$. (c) For rotational equilibrium the net torque must be zero, so the valve exerts a torque of $3.0\,\mathrm{N\,m}$ on the handle, in the direction of turning opposite to that of the applied forces.

Question 6 Pivot at the $50.0\,\mathrm{cm}$ mark, the balance point of the uniform ruler, so the ruler's own F_g acts through the pivot and produces no torque. (a) $F_g=0.20\times9.8=1.96\,\mathrm{N}$, acting $0.30\,\mathrm{m}$ left of the pivot: $\tau=1.96\times0.30=0.588\,\mathrm{N\,m}$, which rounds to $0.59\,\mathrm{N\,m}$, anticlockwise (a downward force to the left of the pivot). (b) Balancing torques: $(0.50\times9.8)\times d=0.588$, so $4.9\times d=0.588$ and $d=0.12\,\mathrm{m}$. (c) The balancing mass hangs to the right of the pivot, at the $50.0+12.0=62\,\mathrm{cm}$ mark.

Question 7 The required torque is $\tau = 120\,\text{N m}$ and the largest lever arm available is the full length of the spanner, $r_{\perp} = 0.30\,\text{m}$, used with the force perpendicular to it:

$$F = \frac{\tau}{r_{\perp}} = \frac{120}{0.30} = 400\,\text{N}.$$

Any other direction of push would give a smaller lever arm and would need a larger force, so $400\,\text{N}$ is the smallest force that will do the job.

Question 8 For student A, the force acts at distance L from the nut but at 30° to the handle, so the lever arm is $L \sin 30^{\circ}$:

$$\tau_A = F \times L \sin 30^{\circ} = F \times 0.50\,L = 1/2\,F\,L.$$

For student B, the force acts at distance $L/2$ from the nut but perpendicular to the handle, so the lever arm is $L/2$:

$$\tau_B = F \times \frac{L}{2} \times \sin 90^{\circ} = 1/2\,F\,L.$$

The two torques are equal. Student A pulls twice as far from the pivot but at an angle that halves the effective lever arm ($\sin 30^{\circ} = 0.50$); student B pulls perpendicular to the handle but half as far from the pivot. The two effects exactly cancel. This shows that producing a large torque requires both a large distance from the pivot *and* a force applied as close to perpendicular as possible — gaining one at the expense of the other can leave the torque unchanged.

Question 9 Plank: $m = 20\,\text{kg}$, length $3.0\,\text{m}$; toolbox: $m = 10\,\text{kg}$ at $1.0\,\text{m}$ from student A. The forces on the plank are its own $F_g = 20 \times 9.8 = 196\,\text{N}$ at the centre ($1.5\,\text{m}$ from either end), the toolbox's $F_g = 10 \times 9.8 = 98\,\text{N}$ at $1.0\,\text{m}$ from student A, and the upward forces F_A and F_B at the two ends. Take torques about student A's hands, so that F_A drops out:

$$F_B \times 3.0 = 196 \times 1.5 + 98 \times 1.0 = 294 + 98 = 392\,\text{N m},$$

$$F_B = \frac{392}{3.0} = 130.7\,\text{N}.$$

Vertical force balance: $F_A + F_B = 196 + 98 = 294\,\text{N}$, so $F_A = 294 - 130.7 = 163.3\,\text{N}$. $\therefore$ to two significant figures, student A exerts $160\,\text{N}$ and student B exerts $130\,\text{N}$. The toolbox is nearer student A, so student A carries the larger share.

Question 10 The axle of the wheel is the pivot. The force due to gravity on the load, F_g, acts downward at $0.50\,\text{m}$ from the axle and produces a torque $F_g \times 0.50$ that would rotate the wheelbarrow one way about the axle. To hold the wheelbarrow up, the gardener's upward force at the handles must produce an equal torque the other way, acting at $1.5\,\text{m}$ from the axle:

$$F \times 1.5 = F_g \times 0.50\text{, so } F = \frac{F_g}{3}.$$

Because her force acts three times further from the pivot than the force due to gravity does, only one third of the force is needed to hold the load up. Lifting the load directly, without the pivot, would require the full F_g. (In the language of Section 11B, the wheelbarrow is a lever; the same torque model applies to the forces in the human body in Section 11C.)

Question 11 $F = 80\,\text{N}$, $r = 0.25\,\text{m}$, $\theta = 70^{\circ}$:

$$\tau = r\,F \sin\theta = 0.25 \times 80 \times \sin 70^{\circ} = 20 \times 0.9397 = 18.8\,\text{N m},$$

which rounds to $19\,\text{N m}$ (two significant figures).

Question 12 The force on the boom by the pivot acts *at* the pivot, so its line of action passes through the pivot: its lever arm about the pivot is zero, and its torque about the pivot is zero whatever its magnitude. Taking torques about the pivot therefore removes the unknown pivot force from the torque equation, leaving the cable tension as the only unknown. Once the tension is known, the pivot force can be found from the force balance ($F_{\text{net}}=0$). Choosing any other pivot would leave both unknowns in the torque equation at once.

Question 13 The knife-edge is at the $35.0\,\text{cm}$ mark. The $0.12\,\text{kg}$ mass hangs at the $5.0\,\text{cm}$ mark, $0.30\,\text{m}$ from the pivot, and produces an anticlockwise torque. The ruler is uniform, so its F_g acts at the $50.0\,\text{cm}$ mark, $0.15\,\text{m}$ from the pivot, producing a clockwise torque. For balance:

$$m_{\text{ruler}}\times 9.8\times 0.15=0.12\times 9.8\times 0.30\,.$$

The factor 9.8 cancels from both sides:

$$m_{\text{ruler}}=\frac{0.12\times 0.30}{0.15}=0.24\,\text{kg}\,,$$

as required.

Question 14 Torques are not forces and cannot be added to them: the error is treating “balancing the torques” as something that increases the upward force. For this beam, the torque balance settles nothing, because both relevant forces act through the pivot — the beam is uniform and the pivot is at its centre, so the line of action of F_g passes through the pivot, and both torques are zero. The force balance alone decides the answer: the beam is at rest, so the upward force of the pivot equals the force due to gravity on the beam exactly — no larger and no smaller.

Question 15 Jib: $m=120\,\text{kg}$, length $6.0\,\text{m}$; load: $m=300\,\text{kg}$ at the far end. (a) F_g on the jib $=120\times 9.8=1176\,\text{N}$, acting at the centre, $3.0\,\text{m}$ from the pivot; F_g on the load $=300\times 9.8=2940\,\text{N}$, acting $6.0\,\text{m}$ from the pivot. Both produce clockwise torques about the pivot:

$$\tau=1176\times 3.0+2940\times 6.0=3528+17\,640=21\,168\,\text{N m}\,,$$

which rounds to $2.1\times 10^4\,\text{N m}$ (two significant figures), as required. (b) The cable is vertical and acts $4.0\,\text{m}$ from the pivot, producing the anticlockwise torque. Rotational equilibrium about the pivot gives

$$T\times 4.0=21\,168\,,T=\frac{21\,168}{4.0}=5292\,\text{N}\,,$$

which rounds to $5.3\times 10^3\,\text{N}$ (two significant figures). The tension is much larger than the force due to gravity on the load alone, because the cable acts closer to the pivot than the load does.

Question 16 For a body at rest, rotational equilibrium holds about *any* point: if the net torque is zero, it is zero about every pivot. Choosing a different support changes the lever arm of each force, so the individual torques the two students calculate are different — but in each case the sum of the clockwise torques equals the sum of the anticlockwise torques, and both students correctly conclude that the beam is in rotational equilibrium. The choice of pivot changes the arithmetic, not the physics; a convenient pivot simply makes the calculation shorter.

Question 17 $F=350\,\text{N}$ (vertically downward), $r=0.17\,\text{m}$, $\theta=60^\circ$ between the crank and the force. (a) $\tau=r\,F\sin\theta=0.17\times 350\times\sin 60^\circ=59.5\times 0.866=51.5\,\text{N m}$, which rounds to $52\,\text{N m}$. (b) The torque is largest when the crank is horizontal, so that the downward force is perpendicular to it ($\theta=90^\circ$, $\sin\theta=1$):

$$\tau_{\max}=r\,F=0.17\times 350=59.5\,\text{N m}\,,$$

which rounds to $60\,\text{N m}$ (two significant figures). This is why cyclists push hardest when the cranks are near horizontal.

Question 18 The ruler is horizontal and the forces due to gravity on the hanging masses act vertically downward, so each line of action is perpendicular to the ruler. The perpendicular distance from the pivot to each line of action is therefore exactly the distance read off the ruler along it — the reading along the ruler *is* the lever arm $r_\perp$. If a force were applied to the ruler at an angle instead, the distance along the ruler would no longer be the lever arm: the calculation would need the perpendicular distance from the pivot to the line of action of that force, found from $r_{\perp} = r \sin\theta$, where θ is the angle between the ruler and the force.