Physics Units 1 & 2

Chapter 7 — Modelling motion

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Theory

Two questions about any journey. A walker sets out from her front gate, walks 300 m east to the shops, remembers she has forgotten her shopping bag, and walks 100 m back west toward home. If a friend later asks *how far did you go?*, the honest answer is 400 m — that is the length of the path she covered. If the friend asks *where did you end up?*, the answer is different: she finished 200 m east of where she started. Physics keeps these two questions separate, and the difference between them is the starting point of this chapter.

The quantities used to describe the motion of an object — time, distance, displacement, speed, velocity and acceleration — are called the **parameters of motion**. They fall into two families, and telling the families apart is the first skill this chapter builds.

Scalars and vectors. A **scalar** is a quantity that has a size (a **magnitude**) and a unit, and nothing else. Time is a scalar: “$40\,\mathrm{s}$” is a complete description — there is no direction attached to it. Distance and speed are scalars too, and so are mass, temperature and energy. A **vector** is a quantity that has both a magnitude *and* a direction. Displacement is a vector: “$200\,\mathrm{m}$ east” says where the walker ended up; “$200\,\mathrm{m}$” alone does not. Velocity and acceleration are also vectors, and later chapters add more — force (Chapter 8) and momentum (Section 8A) are vectors as well.

A quick test helps with classification: ask *does it make sense to ask “which way?”* Time does not have a direction; asking “which way is the time?” means nothing, so time is a scalar. A velocity does have a direction, so it is a vector.

Distance and displacement. The **distance** travelled, d, is the total length of the path taken, whatever shape that path has. Distance is a scalar, measured in metres (m). It only ever grows as an object moves: every step adds to it.

To say where an object is, you compare it with a chosen reference point called the **origin**, and you choose which way along the line counts as the **positive direction**. The object’s **position** is then a signed distance from the origin: a position of $+120\,\mathrm{m}$ is $120\,\mathrm{m}$ from the origin in the positive direction, and $-40\,\mathrm{m}$ is $40\,\mathrm{m}$ in the negative direction.

The **displacement**, s, of an object is the change in its position — how far, *and in which direction*, it is from its starting point. Displacement is a vector, measured in metres (m). Unlike distance, displacement depends only on the starting point and the finishing point, not on the route taken between them. The walker above covered a distance of $300\,\mathrm{m}+100\,\mathrm{m}=400\,\mathrm{m}$, but her displacement was $+200\,\mathrm{m}$ if east is chosen as positive. An object that returns to its start has a displacement of zero however far it has travelled: a swimmer who swims a lap of a pool ends where she began, so her displacement for the lap is $0\,\mathrm{m}$ while her distance is twice the pool length.

Speed and velocity. The **speed** of an object is how fast it is moving — the rate at which it covers distance. Speed is a scalar, measured in metres per second ($\mathrm{m\ s^{-1}}$). The **average speed** over a trip is

$$\text{average speed}=\frac{\text{total distance}}{\text{time taken}}.$$

The **velocity** of an object is the rate at which its displacement changes. Velocity is a vector, measured in $\mathrm{m\ s^{-1}}$ in a stated direction. The **average velocity** over a trip is

$$\text{average velocity}=\frac{\text{displacement}}{\text{time taken}}.$$

Because distance and displacement can be very different, average speed and average velocity can be very different too. Walk $250\,\mathrm{m}$ to the shops and $250\,\mathrm{m}$ back in a total of $600\,\mathrm{s}$: the average speed is $500/600=0.83\,\mathrm{m\ s^{-1}}$, but the displacement for the whole trip is $0\,\mathrm{m}$, so the average velocity is $0\,\mathrm{m\ s^{-1}}$. You moved the whole time, yet your average velocity was zero.

The **instantaneous velocity** is the velocity at one particular instant — the value a speedometer reads at that moment, together with the direction of travel at that moment. The speedometer reading alone is the **instantaneous speed**, the magnitude of the instantaneous velocity. Keeping speed and velocity separate matters: a car moving around a roundabout at a steady $30\,\text{km h}^{-1}$ has a constant *speed*, but its *velocity* is changing all the time because its direction keeps changing.

Representing vectors. A vector is drawn as an **arrow**. The length of the arrow represents the magnitude of the vector (drawn to a chosen scale), and the arrowhead points in the vector's direction. A longer arrow means a larger magnitude; an arrow pointing the other way means the opposite direction.

In writing, vectors are marked with an arrow above the symbol: the velocity is $\vec{v}$, the acceleration is $\vec{a}$. The same letter *without* the arrow stands for the magnitude only: $v=6.0\,\text{m s}^{-1}$ is a size with no direction, and a magnitude is never negative.

Along a single straight line there are only two possible directions, so there is a useful shortcut. Choose one direction as positive; then every vector along the line can be written as a signed number. With east chosen as positive, a velocity of $+6.0\,\text{m s}^{-1}$ means $6.0\,\text{m s}^{-1}$ east, and a velocity of $-6.0\,\text{m s}^{-1}$ means $6.0\,\text{m s}^{-1}$ west. The sign is not about size — it records direction. A velocity of $-8.0\,\text{m s}^{-1}$ is *faster* than $+3.0\,\text{m s}^{-1}$; it simply points the other way. This sign method is used throughout this chapter, because Chapter 7 deals with motion in a straight line. Combining vectors that point at angles to one another needs more than signs; that is developed in Section 8D when forces are combined.

Acceleration. A velocity can change in three ways: the object can speed up, slow down, or change direction. Any of these is a change in velocity, because velocity includes direction. The **acceleration** of an object is the rate at which its velocity changes. Acceleration is a vector, measured in metres per second per second (m s^{-2}). If an object's velocity changes from an initial value u to a final value v over a time t, its average acceleration is

$$a=\frac{v-u}{t}=\frac{\Delta v}{t},$$

where Δv (read "delta v") means the change in velocity. The symbols u, v, a, s and t are used throughout this chapter; they reappear in the constant-acceleration equations of Section 7C. If the velocity changes steadily, this average value is the acceleration at every instant.

Because acceleration is a vector, it has a direction, and the direction of the acceleration is the direction of the *change* in velocity — not necessarily the direction of motion.

- An object that is **speeding up** has its acceleration in the *same* direction as its velocity (the two carry the same sign).
- An object that is **slowing down** has its acceleration in the *opposite* direction to its velocity (the two carry opposite signs).

A negative acceleration does not automatically mean slowing down. A cyclist moving in the positive direction at $+6.0\,\text{m s}^{-1}$ who brakes to $+2.0\,\text{m s}^{-1}$ over $4.0\,\text{s}$ has

$$a=\frac{2.0-6.0}{4.0}=-1.0\,\text{m s}^{-2}:$$

the acceleration is negative, opposite to the motion, and the cyclist slows. But an object moving in the negative direction and speeding up also has a negative acceleration — here the acceleration points the same way as the motion. The sign of the acceleration alone never tells you whether an object is speeding up or slowing down; you must compare the signs of the velocity and the acceleration.

Combining vectors along a straight line. Scalars add like ordinary numbers: $2\,\text{s}+3\,\text{s}=5\,\text{s}$, whatever the direction of anything. Vectors along the same straight line also add — but the signs must be included, because each vector can point either way along the line. Stand on a moving walkway at an airport that carries you at $+0.80\,\text{m s}^{-1}$ relative to the ground, and walk along it at $+1.30\,\text{m s}^{-1}$ relative to the walkway: your velocity relative to the ground is $0.80+1.30=+2.10\,\text{m s}^{-1}$. Turn around and walk back along the walkway at the same pace and your velocity relative to the walkway is $-1.30\,\text{m s}^{-1}$, so your velocity relative to the ground is $0.80+(-1.30)=-0.50\,\text{m s}^{-1}$ — you slide backwards relative to the ground even though you are walking. Worked Example 3 below works through this situation in full.

The same signed subtraction gives a **change in velocity**: $\Delta v = v - u$. When an object reverses direction, the signs matter a great deal — Example 4 shows a ball that bounces off a wall.

The parameters of motion, summarised.

Parameter of motion	Symbol	Vector or scalar?	Unit
Time	t	scalar	second (s)
Distance	d	scalar	metre (m)
Displacement	s	vector	metre (m)
Speed	magnitude of $\vec{v}$	scalar	metres per second ($\mathrm{m\ s^{-1}}$)
Velocity	$\vec{v}$ (signed v in a straight line)	vector	metres per second ($\mathrm{m\ s^{-1}}$)
Acceleration	$\vec{a}$ (signed a in a straight line)	vector	metres per second per second ($\mathrm{m\ s^{-2}}$)

Later chapters of this book extend the list: momentum (Section 8A) and force (Chapter 8) are both vectors.

A note on units. Speeds and velocities in this section are in SI units, $\mathrm{m\ s^{-1}}$. Real-world speeds are often quoted in kilometres per hour ($\mathrm{km\ h^{-1}}$). Since $1\,\mathrm{km} = 1000\,\mathrm{m}$ and $1\,\mathrm{h} = 3600\,\mathrm{s}$, converting $\mathrm{km\ h^{-1}}$ to $\mathrm{m\ s^{-1}}$ means dividing by 3.6: for example, $72\,\mathrm{km\ h^{-1}} = 72 \div 3.6 = 20\,\mathrm{m\ s^{-1}}$. Convert to SI units before substituting into any calculation, as in Section 1A.

Units 3 & 4 preview. Everything in this section keeps vectors on a single straight line, where a sign is enough to record the direction. At Units 3 & 4 the same vector ideas are applied to vectors that point at angles to one another — projectile motion, circular motion, and the directions of field forces — and the vector skills built here are assumed, never re-taught. Content under this banner is not assessed in this book's topic tests or practice examinations.

Worked examples

Example 1 — scaffolded

At a Victorian school athletics carnival, a coach records the following quantities about a $100\,\mathrm{m}$ race: the time of the race, the length of the straight track, the winner's displacement from start to finish, the winner's average speed, the winner's average velocity, and the winner's acceleration near the finish line. Classify each quantity as a vector or a scalar.

Working	Comment
Time of the race — scalar.	A time has a size and a unit only; "which way?" has no meaning for time.
Length of the track — scalar.	A length (here, the distance of $100\,\mathrm{m}$) is a size only.
Winner's displacement — vector.	A displacement says how far *and in which direction* the winner is from the start.
Winner's average speed — scalar.	Speed is the rate of covering distance; it has magnitude only.
Winner's average velocity — vector.	Velocity is displacement divided by time, and displacement has a direction.
Winner's acceleration — vector.	Acceleration is the rate of change of velocity, and the change has a direction.

Notice the pattern: the three vectors (displacement, velocity, acceleration) all carry a direction that must be stated; the three scalars (time, distance, speed) are complete with a number and a unit alone.

Example 2 — scaffolded

At the Melbourne Sports and Aquatic Centre, a swimmer trains in the $50\,\mathrm{m}$ pool. She swims $50\,\mathrm{m}$ from one end of the pool to the other in $40\,\mathrm{s}$, touches the wall, and swims the $50\,\mathrm{m}$ back to her starting end in a further $60\,\mathrm{s}$. (a) What is the total distance she swims? (b) What is her displacement for the whole $100\,\mathrm{s}$? (c) Calculate her average speed. (d) Calculate her average velocity.

Working	Comment
(a) $d=50+50=100\,\mathrm{m}$	Distance is the total length of the path covered.
(b) $s=0\,\mathrm{m}$	She finishes where she started, so her position has not changed overall.
(c) $\text{average speed}=\dfrac{100}{100}=1.0\,\mathrm{m\,s^{-1}}$	Average speed uses the total distance and the total time.
(d) $\text{average velocity}=\dfrac{0}{100}=0\,\mathrm{m\,s^{-1}}$	Average velocity uses the displacement, which is zero.

The swimmer was moving for the full $100\,\mathrm{s}$, yet her average velocity is zero. A zero average velocity never says “nothing happened” — it says only that the trip ended where it began.

Example 3 — unscaffolded

At Melbourne Airport, a moving walkway carries passengers along a straight corridor at $0.80\,\mathrm{m\,s^{-1}}$ relative to the terminal floor. Take the direction the walkway moves as positive. (a) A passenger stands still on the walkway. What is her velocity relative to the terminal? (b) A passenger walks along the walkway in its direction at $1.30\,\mathrm{m\,s^{-1}}$ relative to the walkway. Calculate his velocity relative to the terminal. (c) A passenger at the wrong gate turns and walks back along the walkway at $1.30\,\mathrm{m\,s^{-1}}$ relative to the walkway. Calculate her velocity relative to the terminal, and describe what a person standing in the terminal sees.

(a) The standing passenger has no velocity relative to the walkway, so she simply moves with it: her velocity relative to the terminal is $+0.80\,\mathrm{m\,s^{-1}}$.

(b) Velocities along the same straight line combine by adding their signed values. Taking the walkway’s direction as positive:

$$v=+0.80+(+1.30)=+2.10\,\mathrm{m\,s^{-1}}.$$

Relative to the terminal he moves at $2.10\,\mathrm{m\,s^{-1}}$ in the walkway’s direction — faster than either motion alone.

(c) Walking back along the walkway means her velocity relative to the walkway points in the negative direction: $-1.30\,\mathrm{m\,s^{-1}}$. Adding the walkway’s velocity:

$$v=+0.80+(-1.30)=-0.50\,\mathrm{m\,s^{-1}}.$$

Relative to the terminal she moves at $0.50\,\mathrm{m\,s^{-1}}$ in the direction *opposite* the walkway’s motion: a person standing in the terminal sees her sliding backwards, even though she is walking. The two vectors point in opposite directions along the line, so the smaller one only partly cancels the larger one.

$\therefore$ the three velocities relative to the terminal are $+0.80\,\mathrm{m\,s^{-1}}$, $+2.10\,\mathrm{m\,s^{-1}}$ and $-0.50\,\mathrm{m\,s^{-1}}$.

Example 4 — unscaffolded

Before a practice session at Melbourne Park, a tennis ball is struck toward a practice wall. Take the direction toward the wall as positive. Just before impact the ball’s velocity is $+14\,\mathrm{m\,s^{-1}}$; it rebounds from the wall with a velocity of $-10\,\mathrm{m\,s^{-1}}$. The ball is in contact with the wall for $0.050\,\mathrm{s}$. (a) Calculate the change in velocity of the ball during the contact. (b) Calculate the average acceleration of the ball during the contact. (c) Explain what the sign of your answer to (b) tells you about the acceleration.

The change in velocity is the final velocity minus the initial velocity, with the signs kept:

$$\Delta v = v - u = (-10) - (+14) = -24\,\text{m s}^{-1}.$$

The change in velocity is $24\,\text{m s}^{-1}$ directed *away from the wall* (the negative direction). Note that the change is not $14 - 10 = 4\,\text{m s}^{-1}$: because the ball reverses direction, the two velocities point opposite ways and their effects add rather than cancel.

The average acceleration during the contact is

$$a = \frac{\Delta v}{t} = \frac{-24}{0.050} = -4.8 \times 10^{2}\,\text{m s}^{-2}$$

to two significant figures.

(c) The negative sign means the acceleration points in the negative direction — away from the wall. The ball arrived moving toward the wall (positive velocity) and left moving away from it (negative velocity), so during the contact its velocity changed in the negative direction; the acceleration points the same way as that change. This is the pattern of an object slowing down, stopping for an instant, and speeding up again in the opposite direction.

∴ $\Delta v = -24\,\text{m s}^{-1}$ and $a = -4.8 \times 10^{2}\,\text{m s}^{-2}$, both directed away from the wall.

Practice questions

Question 1 *(scaffolded)* The table lists quantities measured during a school athletics carnival. Copy and complete the table, classifying each quantity as a vector or a scalar and giving its SI unit. The first row has been done for you.

Quantity	Vector or scalar?	Unit
time of the race	scalar	s
distance run		
displacement of the runner from the start		
average speed of the runner		
average velocity of the runner		
acceleration of the runner		

Question 2 *(scaffolded)* A Route 96 tram travels $640\,\text{m}$ along a straight stretch of Bourke Street in Melbourne, away from its depot, and stops. It then reverses direction and travels $240\,\text{m}$ back along the same stretch. The whole movement takes $160\,\text{s}$. Take the direction away from the depot as positive. (a) What is the total distance travelled by the tram? (b) What is the displacement of the tram? (c) Calculate the average speed of the tram. (d) Calculate the average velocity of the tram.

Question 3 *(scaffolded)* Take east as the positive direction. Write each of the following as a signed velocity with its unit. (a) a car moving east at $12\,\text{m s}^{-1}$ (b) a car moving west at $12\,\text{m s}^{-1}$ (c) a cyclist moving west at $4.0\,\text{m s}^{-1}$ (d) a runner moving east at $4.0\,\text{m s}^{-1}$ (e) Which of your four velocities have the same speed? Do any two have the same velocity? Explain your answer.

Question 4 *(scaffolded)* A cyclist rides $3.0\,\text{km}$ from home along a straight bay road to a jetty in $15\,\text{min}$, then turns around and rides the same $3.0\,\text{km}$ back home in $25\,\text{min}$. (a) Convert the total distance to metres and the total time to seconds. (b) What is the cyclist's displacement for the whole ride? (c) Calculate the average speed of the cyclist for the whole ride. (d) Calculate the average velocity of the cyclist for the whole ride. (e) Explain why the answers to (c) and (d) are different, even though they describe the same ride.

Question 5 *(scaffolded)* A punt travels along a straight reach of the Yarra River. The river current is $1.5\,\mathrm{m\ s^{-1}}$ downstream, and the punt's motor drives it at $4.0\,\mathrm{m\ s^{-1}}$ relative to the water. Take downstream as the positive direction. (a) Calculate the punt's velocity relative to the river bank when it travels downstream. (b) Calculate the punt's velocity relative to the river bank when it travels upstream. (c) The punt travels upstream for $3.0\,\mathrm{min}$. Calculate how far along the bank it actually moves in that time.

Question 6 *(scaffolded)* A car travels along the West Gate Freeway in Melbourne at $25\,\mathrm{m\ s^{-1}}$. The direction of travel is taken as positive. The driver brakes, and $4.0\,\mathrm{s}$ later the car's velocity is $+5.0\,\mathrm{m\ s^{-1}}$. (a) Calculate the change in velocity of the car. (b) Calculate the average acceleration of the car during the braking. (c) Is the acceleration in the same direction as the velocity, or in the opposite direction? (d) Is the car speeding up or slowing down? Use the signs of the velocity and the acceleration to justify your answer.

Question 7 A car moves around a roundabout without its speedometer reading changing. (a) Is the speed of the car constant? (b) Is the velocity of the car constant? (c) Explain your answers, using the definitions of speed and velocity.

Question 8 A bushwalker walks $4.0\,\mathrm{km}$ north along a straight fire trail, then turns and walks $1.5\,\mathrm{km}$ back south. The whole walk takes $2.0\,\mathrm{h}$. Take north as the positive direction. (a) What is the total distance walked? (b) What is the bushwalker's displacement? (c) Calculate the average speed in $\mathrm{km\ h^{-1}}$. (d) Calculate the average velocity in $\mathrm{km\ h^{-1}}$. (e) Convert the average speed to $\mathrm{m\ s^{-1}}$.

Question 9 A student says: *"Average speed and average velocity are really the same thing — they even have the same units."* Evaluate this claim. In your answer, use one example from this section to support your evaluation, and state the one situation in which average speed and the magnitude of average velocity *are* equal.

Question 10 During a warm-up before a Melbourne United basketball game, a player drops a ball. Just before it hits the floor the ball's velocity is $-4.0\,\mathrm{m\ s^{-1}}$, taking upward as the positive direction. The ball rebounds upward with a velocity of $+3.0\,\mathrm{m\ s^{-1}}$. Calculate the change in velocity of the ball during the bounce, and state the direction of that change.

Question 11 A small delivery drone flies at $8.0\,\mathrm{m\ s^{-1}}$ relative to still air, along a straight delivery route. On the day of the flight a steady wind of $3.0\,\mathrm{m\ s^{-1}}$ blows along the route. (a) Calculate the greatest possible speed of the drone relative to the ground. (b) Calculate the smallest possible speed of the drone relative to the ground. (c) Explain why the greatest and smallest values occur when the wind blows exactly along, or exactly against, the route. Why can a wind blowing from the side not be combined with the drone's velocity by simple addition?

Question 12 The speedometer of a car reads $60\,\mathrm{km\ h^{-1}}$. (a) Is this reading the car's speed or its velocity? Justify your answer. (b) State what extra information you would need in order to state the car's velocity.

Question 13 A Metro Trains service starts from rest at a station and moves along a straight track, taking the direction of travel as positive. Over $20\,\mathrm{s}$ its velocity increases steadily from rest to $+15\,\mathrm{m\ s^{-1}}$. Later, approaching the next station, the train brakes steadily from $+15\,\mathrm{m\ s^{-1}}$ to rest in $10\,\mathrm{s}$. (a) Calculate the acceleration of the train as it leaves the station. (b) Calculate the acceleration of the train as it brakes. (c) Explain why the two accelerations have different signs, and say what each sign tells you about the direction of the acceleration.

Question 14 An object moves along a straight line. In each of the four cases below, state whether the object is speeding up or slowing down.

Case	Sign of velocity	Sign of acceleration
(i)	positive	positive
(ii)	positive	negative
(iii)	negative	positive
(iv)	negative	negative

Then state a single rule, in words, that decides the answer from the two signs.

Question 15 At a Victorian school championships meeting, a sprinter runs $100\,\mathrm{m}$ along a straight track in $12.5\,\mathrm{s}$, then walks slowly back along the track to the start, taking $80\,\mathrm{s}$ for the walk. (a) Calculate the sprinter's average speed during the running phase only. (b) Calculate the average speed for the whole trip (run plus walk). (c) What is the average velocity for the whole trip? (d) Explain why the answer to (c) is what it is, even though the sprinter was moving for the entire time.

Question 16 A news report describes a new suburban train service as *"travelling at a constant velocity of* $70\,\mathrm{km\ h^{-1}}$ *between stations"*. Use the ideas of this section to evaluate whether the train can really have kept a constant velocity for the whole journey between two stations.

Question 17 A surveyor sets a straight measuring line beside a track, with a marker as the origin and east as the positive direction. A runner training for the Melbourne Marathon is at position $+120\,\mathrm{m}$ at one instant, and $4.0\,\mathrm{s}$ later is at position $+40\,\mathrm{m}$. (a) Calculate the runner's displacement during the $4.0\,\mathrm{s}$. (b) Calculate the runner's average velocity during the $4.0\,\mathrm{s}$. (c) Describe the runner's motion in words.

Question 18 Two trams pass each other on parallel straight tracks. Take east as the positive direction. Tram A has a velocity of $+18\,\mathrm{m\ s^{-1}}$ and tram B has a velocity of $-18\,\mathrm{m\ s^{-1}}$. (a) Do the two trams have the same speed? Justify your answer. (b) Do the two trams have the same velocity? Justify your answer. (c) The velocity of tram B relative to tram A is found by subtracting tram A's velocity from tram B's velocity: $v_{\mathrm{rel}} = v_B - v_A$. Calculate v_{rel}, and state how fast a passenger in tram A sees tram B moving away.

Answers

Question 1 distance: scalar, m; displacement: vector, m; average speed: scalar, m s^{-1}; average velocity: vector, m s^{-1}; acceleration: vector, m s^{-2}. **Question 2** (a) $880\,\text{m}$; (b) $+400\,\text{m}$; (c) $5.5\,\text{m s}^{-1}$; (d) $+2.5\,\text{m s}^{-1}$. **Question 3** (a) $+12\,\text{m s}^{-1}$; (b) $-12\,\text{m s}^{-1}$; (c) $-4.0\,\text{m s}^{-1}$; (d) $+4.0\,\text{m s}^{-1}$; (e) see solution — no two are the same velocity. **Question 4** (a) $6000\,\text{m}$, $2400\,\text{s}$; (b) $0\,\text{m}$; (c) $2.5\,\text{m s}^{-1}$; (d) $0\,\text{m s}^{-1}$; (e) see solution. **Question 5** (a) $+5.5\,\text{m s}^{-1}$; (b) $-2.5\,\text{m s}^{-1}$; (c) $4.5\times10^{2}\,\text{m}$ upstream. **Question 6** (a) $-20\,\text{m s}^{-1}$; (b) $-5.0\,\text{m s}^{-2}$; (c) opposite; (d) slowing down — see solution. **Question 7** (a) yes; (b) no; (c) see solution. **Question 8** (a) $5.5\,\text{km}$; (b) $+2.5\,\text{km}$; (c) $2.8\,\text{km h}^{-1}$; (d) $+1.3\,\text{km h}^{-1}$; (e) $0.76\,\text{m s}^{-1}$. **Question 9** See solution — the claim is wrong; the two are equal only for straight-line motion without reversing direction. **Question 10** $+7.0\,\text{m s}^{-1}$, directed upward. **Question 11** (a) $11\,\text{m s}^{-1}$; (b) $5.0\,\text{m s}^{-1}$; (c) see solution. **Question 12** (a) speed — the reading has a magnitude but states no direction; (b) the direction of travel. **Question 13** (a) $+0.75\,\text{m s}^{-2}$; (b) $-1.5\,\text{m s}^{-2}$; (c) see solution. **Question 14** (i) speeding up; (ii) slowing down; (iii) slowing down; (iv) speeding up; rule: see solution. **Question 15** (a) $8.0\,\text{m s}^{-1}$; (b) $2.2\,\text{m s}^{-1}$; (c) $0\,\text{m s}^{-1}$; (d) see solution. **Question 16** See solution — it cannot; starting, stopping and any change of direction all change the velocity. **Question 17** (a) $-80\,\text{m}$; (b) $-20\,\text{m s}^{-1}$; (c) running west at $20\,\text{m s}^{-1}$. **Question 18** (a) yes — both $18\,\text{m s}^{-1}$; (b) no — opposite directions; (c) $-36\,\text{m s}^{-1}$; $36\,\text{m s}^{-1}$.

Fully-worked solutions

Question 1 Scalars have magnitude only; vectors have magnitude and direction. The units are the standard SI units for each parameter of motion.

Quantity	Vector or scalar?	Unit
time of the race	scalar	s
distance run	scalar	m
displacement of the runner from the start	vector	m
average speed of the runner	scalar	m s^{-1}
average velocity of the runner	vector	m s^{-1}
acceleration of the runner	vector	m s^{-2}

Displacement, velocity and acceleration each need a direction to be complete; time, distance and speed are complete with a number and a unit alone.

Question 2 (a) Distance is the total path length: $d = 640 + 240 = 880\,\text{m}$. (b) Displacement is the overall change in position: $s = +640 - 240 = +400\,\text{m}$ — $400\,\text{m}$ away from the depot. (c) $\text{average speed} = \frac{880}{160} = 5.5\,\text{m s}^{-1}$. (d) $\text{average velocity} = \frac{+400}{160} = +2.5\,\text{m s}^{-1}$, in the direction away from the depot. The average velocity is smaller than the average speed because part of the trip undid earlier displacement.

Question 3 (a) East is positive: $+12\,\text{m s}^{-1}$. (b) West is the negative direction: $-12\,\text{m s}^{-1}$. (c) $-4.0\,\text{m s}^{-1}$. (d) $+4.0\,\text{m s}^{-1}$. (e) Same speed: (a) and (b) both have a speed of $12\,\text{m s}^{-1}$, and (c) and (d) both have a speed of $4.0\,\text{m s}^{-1}$ — speed ignores direction. No two of the four have the same velocity: a velocity is only the same when both the magnitude and the direction match, and no pair matches on both — the two pairs going the same way, (a) with (d) and (b) with (c), differ in magnitude ($12\,\text{m s}^{-1}$ vs $4.0\,\text{m s}^{-1}$), while the two pairs with the same magnitude, (a) with (b) and (c) with (d), point in opposite directions.

Question 4 (a) Total distance $= 3.0\ \text{km} + 3.0\ \text{km} = 6.0\ \text{km} = 6000\ \text{m}$. Total time $= 15\ \text{min} + 25\ \text{min} = 40\ \text{min} = 40 \times 60 = 2400\ \text{s}$. (b) The cyclist finishes at home, where she started, so her displacement is $0\ \text{m}$. (c) $\text{average speed} = \dfrac{6000}{2400} = 2.5\ \text{m s}^{-1}$. (d) $\text{average velocity} = \dfrac{0}{2400} = 0\ \text{m s}^{-1}$. (e) Average speed is built from the distance travelled, which is $6000\ \text{m}$ and never shrinks as the ride goes on. Average velocity is built from the displacement, and the outward half of the ride is exactly undone by the return half, leaving zero. The two answers describe the same ride but answer different questions: how much ground was covered, and where the ride ended relative to where it began.

Question 5 (a) Downstream, the motor and the current point the same way, so their signed values add: $v = +4.0 + (+1.5) = +5.5\ \text{m s}^{-1}$. (b) Upstream, the motor points against the current: $v = -4.0 + (+1.5) = -2.5\ \text{m s}^{-1}$ — the punt moves upstream at $2.5\ \text{m s}^{-1}$ relative to the bank, slower than its speed through the water because the current pushes it back. (c) $t = 3.0\ \text{min} = 180\ \text{s}$. Distance along the bank $= vt = 2.5 \times 180 = 450\ \text{m}$, i.e. $4.5 \times 10^2\ \text{m}$ upstream (2 s.f.). Without allowing for the current, the punt would have covered $4.0 \times 180 = 720\ \text{m}$ through the water; the bank only sees $450\ \text{m}$ of progress.

Question 6 (a) $\Delta v = v - u = +5.0 - (+25) = -20\ \text{m s}^{-1}$. (b) $a = \dfrac{\Delta v}{t} = \dfrac{-20}{4.0} = -5.0\ \text{m s}^{-2}$. (c) The velocity is positive but the acceleration is negative: they point in opposite directions. (d) The car is slowing down. When the velocity and the acceleration carry opposite signs, the acceleration acts against the motion and the object slows.

Question 7 (a) Yes — the speedometer reading is unchanged, so the speed is constant. (b) No. (c) Speed is a scalar: it is the magnitude of the velocity, and it does not depend on direction, so a steady speedometer reading means a constant speed. Velocity is a vector: it is constant only if both its magnitude and its direction stay the same. Around the roundabout the car's direction of travel changes continuously, so its velocity changes continuously even though its speed never changes.

Question 8 (a) $d = 4.0 + 1.5 = 5.5\ \text{km}$. (b) $s = +4.0 - 1.5 = +2.5\ \text{km}$ — $2.5\ \text{km}$ north of the starting point. (c) $\text{average speed} = \dfrac{5.5}{2.0} = 2.75$, which rounds to $2.8\ \text{km h}^{-1}$ (2 s.f.). (d) $\text{average velocity} = \dfrac{+2.5}{2.0} = +1.25$, which rounds to $+1.3\ \text{km h}^{-1}$ (2 s.f.), north. (e) Converting km h^{-1} to m s^{-1} means dividing by 3.6: $2.75 \div 3.6 = 0.76\ \text{m s}^{-1}$ (2 s.f.). (Use the unrounded 2.75 for the conversion, not the rounded 2.8.)

Question 9 Model answer: The claim is wrong. Average speed and average velocity do share the same unit (m s^{-1}), but they are different quantities: average speed is total distance divided by time, and distance is a scalar; average velocity is displacement divided by time, and displacement is a vector. Whenever a trip doubles back on itself, the two differ. For example, the swimmer of Worked Example 2 swims $100\ \text{m}$ in $100\ \text{s}$, so her average speed is $1.0\ \text{m s}^{-1}$, but she finishes where she started, so her displacement is $0\ \text{m}$ and her average velocity is $0\ \text{m s}^{-1}$. The two are equal in magnitude only for motion in a single direction along a straight line, where the distance covered and the magnitude of the displacement are the same.

Question 10 The change in velocity is final minus initial, keeping the signs:

$$\Delta v = v - u = (+3.0) - (-4.0) = +7.0\ \text{m s}^{-1}.$$

The change in velocity is $7.0\ \text{m s}^{-1}$ directed upward (the positive direction). Subtracting a negative is the same as adding: the ball's velocity has swung from $4.0\ \text{m s}^{-1}$ downward to $3.0\ \text{m s}^{-1}$ upward, so the total change is the sum of the two magnitudes, not their difference.

Question 11 (a) The greatest ground speed occurs with the wind exactly behind the route (a tailwind): the two velocities point the same way and add: $8.0 + 3.0 = 11\ \text{m s}^{-1}$. (b) The smallest ground speed occurs with the wind exactly against the route (a headwind): $8.0 - 3.0 = 5.0\ \text{m s}^{-1}$. (c) When the wind is exactly along or exactly against the route, the wind velocity and the drone velocity lie on the same straight line, so they combine by signed addition — fully adding or fully opposing. A wind from the side points at an angle to the route, and vectors at angles cannot be combined by simple signed addition; the combination must take the angle into account. Combining vectors at angles is developed in Section 8D.

Question 12 (a) The reading is the car's speed. A speedometer shows how fast the car is moving — the magnitude of its velocity — but displays no direction, and a vector cannot be fully described by a magnitude alone. (b) To state the car's velocity you would also need the direction in which the car is travelling at that instant — for example, "$60\,\text{km h}^{-1}$ north".

Question 13 (a) Leaving the station: $u = 0\,\text{m s}^{-1}$, $v = +15\,\text{m s}^{-1}$, $t = 20\,\text{s}$.

$$a = \frac{v-u}{t} = \frac{15-0}{20} = +0.75\,\text{m s}^{-2}.$$

(b) Braking: $u = +15\,\text{m s}^{-1}$, $v = 0\,\text{m s}^{-1}$, $t = 10\,\text{s}$.

$$a = \frac{0-15}{10} = -1.5\,\text{m s}^{-2}.$$

(c) The signs differ because the acceleration points in different directions in the two cases. Leaving the station, the train speeds up in the positive direction, so the change in velocity is positive and the acceleration points forward, in the same direction as the motion. Braking, the train's velocity decreases: the change in velocity points opposite to the motion, so the acceleration is negative — backward. A negative acceleration here means slowing down only because the velocity is positive; the sign records direction, not "slowing".

Question 14 (i) Speeding up — velocity and acceleration have the same sign. (ii) Slowing down — the signs are opposite. (iii) Slowing down — the signs are opposite. (iv) Speeding up — both are negative, so the signs are the same.

Rule: an object speeds up when its velocity and acceleration carry the same sign (the same direction), and slows down when they carry opposite signs (opposite directions). The sign of the acceleration alone never decides it.

Question 15 (a) During the running phase: $\text{average speed} = \frac{100}{12.5} = 8.0\,\text{m s}^{-1}$. (b) Whole trip: total distance $= 100 + 100 = 200\,\text{m}$; total time $= 12.5 + 80 = 92.5\,\text{s}$.

$$\text{average speed} = \frac{200}{92.5} = 2.2\,\text{m s}^{-1}\,(2\text{ s.f.}).$$

(c) The sprinter finishes back at the start, so the displacement for the whole trip is $0\,\text{m}$ and the average velocity is $0\,\text{m s}^{-1}$.

(d) Average velocity is displacement divided by time. The outward run and the walk back undo one another, so the overall change of position is zero; dividing zero by any time gives zero. The sprinter was certainly moving — the average speed of $2.2\,\text{m s}^{-1}$ says so — but average velocity measures only where the trip ended relative to where it began.

Question 16 Model answer: The train cannot have kept a constant velocity for the whole journey between the stations. A constant velocity requires both a constant speed and an unchanging direction of travel. At the first station the train starts from rest, so its velocity increases from zero; at the second it brakes to rest, so its velocity decreases to zero — the magnitude of the velocity changes at both ends of the journey. If the track includes any curve or bend, the direction of travel changes as well, which is a further change in velocity even at steady speed. At best, the train keeps a roughly constant *speed* for part of the journey. The report's own wording gives the clue: "$70\,\text{km h}^{-1}$" without a direction is a speed, a scalar, not a velocity.

Question 17 (a) Displacement is the change in position: $s = 40 - 120 = -80\,\text{m}$. (b) $\text{average velocity} = \frac{s}{t} = \frac{-80}{4.0} = -20\,\text{m s}^{-1}$. (c) The negative sign means the runner moves in the negative direction — west — and the magnitude is $20\,\text{m s}^{-1}$: the runner is running west at $20\,\text{m s}^{-1}$.

Question 18 (a) Yes. Speed is the magnitude of velocity, and both trams have velocities of magnitude $18\,\mathrm{m\ s^{-1}}$. (b) No. Velocities are the same only when magnitude and direction both match. Tram A moves east ($+18\,\mathrm{m\ s^{-1}}$) and tram B moves west ($-18\,\mathrm{m\ s^{-1}}$): same magnitude, opposite directions. (c)

$$v_{\mathrm{rel}} = v_B - v_A = (-18) - (+18) = -36\,\mathrm{m\ s^{-1}}.$$

The magnitude is $36\,\mathrm{m\ s^{-1}}$: a passenger in tram A sees tram B pulling away at $36\,\mathrm{m\ s^{-1}}$. Each tram's own speed is only $18\,\mathrm{m\ s^{-1}}$, but because they move in opposite directions, the velocities combine by signed subtraction and the separation grows at the sum of the two speeds.

Theory

In Section 7A you met the parameters used to describe motion — distance, displacement, speed, velocity and acceleration — and identified each one as a scalar or a vector. This section puts those quantities onto graphs. A graph shows the whole history of a motion in one picture, and two simple readings — the **gradient** of the graph and the **area under** the graph — extract almost all of the information the graph holds. The motion analysed here is straight-line motion, and the emphasis is on motion whose velocity changes.

Uniform and non-uniform motion. An object moving in a straight line with constant velocity is in **uniform motion**: it covers equal displacements in equal intervals of time. A tram gliding along a straight track at a steady $2.5\,\mathrm{m\ s^{-1}}$ is close to this ideal case. Real motions usually change with time — a tram pulls away from a stop, a cyclist brakes for a light, a sprinter accelerates out of the blocks. The key knowledge for this section uses the term *non-uniform motion*, which the study design does not define. In this subtopic, **non-uniform motion** is treated as straight-line motion in which the velocity is changing — the object is speeding up, slowing down, or changing direction along the line — and every question here is answerable on that definition alone. (The *cause* of a change in velocity — a net force — is the work of Chapter 8. Here the task is to describe and analyse the motion from its graphs.)

The sign convention. All motion in this section is along one straight line. Choose one direction along the line as positive and stick to it. A positive velocity means motion in the positive direction; a negative velocity means motion in the opposite direction. The same applies to displacement and acceleration: the sign carries the direction, and the number carries the size.

Displacement–time graphs. A **displacement–time graph** plots the displacement s of the object from a chosen reference point (vertical axis, in metres) against time t (horizontal axis, in seconds). Its gradient is the velocity:

$$\text{gradient} = \frac{\Delta s}{\Delta t} = \frac{\text{change in displacement}}{\text{time taken}} = v\,.$$

The gradient carries its own units: metres divided by seconds, $\mathrm{m\ s^{-1}}$ — the unit of velocity. Read the gradient of a straight section by choosing two points on the line that are far apart, and dividing the rise (change in s) by the run (change in t). The shape of the graph describes the motion directly:

- a **straight line** means constant velocity — uniform motion. A steeper line means a larger speed;
- a **horizontal line** means the displacement is not changing: the object is at rest ($v=0$);
- a line with a **negative gradient** means the object is moving in the negative direction (and, while it is on the positive side of the reference point, moving back towards it);
- a **curve** means the velocity is changing — non-uniform motion. A curve that gets steeper as time goes on shows an object speeding up; a curve that flattens out shows an object slowing down.

The gradient of the straight line joining two points on a curve (a **chord**) gives the **average velocity** over the interval between those two points. It says nothing about the velocity at any single instant inside the interval — that needs a tangent, below.

Velocity–time graphs. A **velocity–time graph** plots the velocity v (vertical axis, in $\mathrm{m\ s^{-1}}$) against time t (horizontal axis, in seconds). Two readings are available from it. First, the gradient is the acceleration:

$$\text{gradient} = \frac{\Delta v}{\Delta t} = a\,,$$

in units of $\mathrm{m\ s^{-2}}$. Second, the **area under the graph** is the displacement. For constant velocity this is simply the area of a rectangle: displacement $= v \times t$. When the velocity changes, split the time into thin strips: within each strip the velocity is nearly constant, so the strip's area is nearly $v \times \Delta t$, and adding the strips — that is, finding the total area under the graph — gives the total displacement.

The shapes to know:

- a **horizontal line** means constant velocity and zero acceleration;
- a **straight sloping line** means the velocity changes by equal amounts in equal times — constant acceleration. Straight-line motion under constant acceleration is analysed numerically and algebraically in Section 7C; here it is read from the graph;
- a **curve** means the acceleration itself is changing;
- a line **below the time axis** means negative velocity — motion in the negative direction. Area below the time axis counts as *negative* displacement. The net displacement is the sum of the areas, taking their signs into account.

Acceleration–time graphs. An **acceleration–time graph** plots the acceleration a (vertical axis, in $\mathrm{m\ s^{-2}}$) against time. The value of the graph at any instant is the acceleration at that instant, and the area under the graph is the **change in velocity** over that time: $\Delta v = a \times \Delta t$ when the acceleration is constant, or the sum of strips when it is not. An acceleration–time graph that is a horizontal line at some value corresponds to a velocity–time graph that is a straight line with that value as its gradient.

The three graphs are summarised below.

Graph	Vertical axis	Gradient gives	Area under graph gives
displacement–time	s (m)	velocity ($\mathrm{m\ s^{-1}}$)	—
velocity–time	v ($\mathrm{m\ s^{-1}}$)	acceleration ($\mathrm{m\ s^{-2}}$)	displacement (m)
acceleration–time	a ($\mathrm{m\ s^{-2}}$)	not needed in this course	change in velocity ($\mathrm{m\ s^{-1}}$)

Instantaneous values: drawing a tangent. The **instantaneous velocity** is the velocity at one particular instant, as distinct from an average over an interval. On a curved displacement–time graph the gradient changes from point to point, so the instantaneous velocity at a point is the gradient of the **tangent** to the curve at that point. A tangent is a straight line that touches the curve at the one point and has the same slope as the curve there. To use one:

1. Mark the point on the curve where the instantaneous value is wanted.
2. Using a ruler, draw a straight line that touches the curve at that point only, with the same slope as the curve — the curve should sit symmetrically either side of the line near the point of contact.
3. Choose two points on the **tangent line** (not on the curve), far apart for accuracy, read their coordinates, and calculate the gradient of the tangent.

The same method gives the **instantaneous acceleration** from the tangent to a curved velocity–time graph.

Area under a curved graph: counting squares. When the line on a velocity–time graph is curved, the area underneath is estimated by counting squares on the grid:

1. Count the whole squares under the curve.
2. Estimate the partial squares along the curve — a common routine is to count each roughly half-filled square as half a square, or to pair partial squares that together make a whole.
3. Multiply the total number of squares by the displacement one square represents: (value per square on the vertical axis) × (value per square on the horizontal axis), with units $\mathrm{m\ s^{-1}} \times \mathrm{s} = \mathrm{m}$.

The result is an estimate, and its precision is limited by the judgment of the partial squares. A finer grid gives a better estimate.

Constructing a motion graph. Constructing a clear graph from a table of readings is a key science skill in its own right, whether the readings come from a motion sensor and data logger, video analysis, or a stopwatch and measured positions. The study design's guidance for presenting data in graphs (VCE Physics Study Design, © VCAA) states that graphs should include *"a title"*, *"axes labels with units"*, *"the independent variable on the horizontal axis (unless other conventions or usefulness override this)"*, *"uncertainty bars on data points (if provided with a numerical representation of the uncertainty)"* and *"a smooth trend line (straight or curved) through data points"*, and that *"extrapolation should be represented by dashed lines"*. In practice, for motion graphs:

1. Put time — the independent variable — on the horizontal axis, and the quantity being studied (s, v or a) on the vertical axis.
2. Label each axis with the quantity, its symbol and its unit: for example "Time t (s)" and "Velocity v ($\mathrm{m\ s^{-1}}$)".
3. Choose scales that spread the plotted points over most of the grid. The axes do not have to start at zero.
4. Plot every reading from the table as a clear point.
5. Draw the line of best fit: a single smooth trend line (straight or curved) that passes as close as possible to all the points. Do not join dot to dot.
6. Give the graph a title.

Two rules from the study design's treatment of data (VCE Physics Study Design, © VCAA) bind on every graph in this course, and they are worth quoting in full: *"that not all graphs will necessarily pass through the origin (0,0)"*, and *"that the origin may only be included as a data point if authentically measured."* A graph whose line of best fit meets the vertical axis away from zero is telling you something real — an initial displacement, or an initial velocity. Do not force a line through the origin, and do not add the origin as a data point unless a reading at $(0,0)$ was actually taken. Where two quantities are directly proportional, the study design notes that the graphed result is expected to be a linear relationship passing through the origin — but only the data can show whether that is so.

Extrapolation and intercepts. To **extrapolate** means to extend the trend line beyond the measured data. On a graph, extrapolated sections are drawn as dashed lines. Where an extrapolated (or measured) line crosses an axis is an **intercept**, and intercepts often carry direct physical meaning:

- the **vertical-axis intercept** of a velocity–time graph is the velocity at $t=0$ — the initial velocity;
- the **time-axis intercept** of a velocity–time graph is the time at which $v=0$ — the instant the object is momentarily at rest, for example the instant a braking tram stops or a thrown ball changes direction;
- the vertical-axis intercept of a displacement–time graph is the displacement at the moment timing began.

Extrapolation is only as good as the reason to believe the trend continues. Extending a straight trend a short distance to find an intercept is usually sound; projecting far beyond the last data point is not, because the motion may change — a braking vehicle stops, and its velocity does not keep falling below zero.

Units 3 & 4 preview. The two readings of this section — gradient as a rate of change, and area under a graph — reappear throughout Units 3 & 4: the impulse of a force is read as the area under a force–time graph, and the gradient of a maximum kinetic energy against frequency graph gives Planck's constant. Units 3 & 4 assumes both readings are fluent and does not re-teach them. Content under this banner is not assessed in this book's topic tests or practice examinations.

Worked examples

Example 1 — scaffolded

A tram moves along a straight section of track past a tram stop. Its displacement from the stop is measured every $2.0\,\mathrm{s}$, with the direction of travel taken as positive. The results are:

Time t (s)	Displacement s (m)
0.0	4.0
2.0	9.0
4.0	14.0
6.0	19.0
8.0	24.0

(a) Calculate the gradient of the displacement–time graph, using the readings at $t=2.0\,\mathrm{s}$ and $t=6.0\,\mathrm{s}$.
(b) State what the gradient represents, and describe the tram's motion.
(c) State the displacement at $t=0$ and explain why the graph does not pass through the origin.

Working	Comment
(a) $\text{gradient} = \frac{\Delta s}{\Delta t} = \frac{19.0 - 9.0}{6.0 - 2.0} = \frac{10.0}{4.0} = 2.5\,\text{m s}^{-1}$	Gradient is rise over run: change in displacement divided by change in time, with units.
(b) The gradient of a displacement–time graph is the velocity: $v = 2.5\,\text{m s}^{-1}$.	The same gradient is obtained from any pair of readings, so the velocity is constant.
The tram is in uniform motion at $2.5\,\text{m s}^{-1}$ in the positive direction.	Constant velocity: equal displacements in equal times.
(c) At $t = 0$, $s = 4.0\,\text{m}$.	The intercept on the displacement axis.
The graph does not pass through the origin because the tram was already $4.0\,\text{m}$ past the stop when timing began; a reading of $s = 0$ was never taken.	A graph need not pass through the origin, and the origin is a data point only if authentically measured.

Example 2 — scaffolded

A V/Line train travels in a straight line towards a station. Its velocity–time graph is a straight line falling from $18.0\,\text{m s}^{-1}$ at $t = 0$ to $6.0\,\text{m s}^{-1}$ at $t = 8.0\,\text{s}$. (a) Calculate the acceleration of the train. (b) Calculate the displacement of the train over the $8.0\,\text{s}$. (c) State what the sign of the acceleration tells you about the train's motion. (d) State what the vertical-axis intercept of the graph represents.

Working	Comment
(a) $a = \frac{\Delta v}{\Delta t} = \frac{6.0 - 18.0}{8.0 - 0.0} = \frac{-12.0}{8.0} = -1.5\,\text{m s}^{-2}$	The gradient of a velocity–time graph is the acceleration.
(b) Area $= \frac{1}{2}(18.0 + 6.0) \times 8.0 = 96\,\text{m}$	The area under a velocity–time graph is the displacement; the shape is a trapezium.
Check: rectangle $6.0 \times 8.0 = 48\,\text{m}$ plus triangle $\frac{1}{2} \times 8.0 \times 12.0 = 48\,\text{m}$; $48 + 48 = 96\,\text{m}$. ✓	The same area split two ways.
(c) The acceleration is negative, opposite in direction to the velocity, so the train is slowing down.	Velocity stays positive throughout; the negative gradient reduces it.
(d) The vertical-axis intercept, $18.0\,\text{m s}^{-1}$, is the velocity at $t = 0$: the train's initial velocity.	Reading the intercept.

Example 3 — unscaffolded

A sprinter accelerates from the blocks in a school athletics carnival $100\,\text{m}$ race, moving in a straight line. The displacement–time graph of the sprinter is a curve. At $t = 2.0\,\text{s}$ the sprinter's displacement is $7.5\,\text{m}$, and the tangent to the curve at that point passes through $(0.5\,\text{s}, 0.0\,\text{m})$ and $(3.5\,\text{s}, 15.0\,\text{m})$. (a) Calculate the sprinter's instantaneous velocity at $t = 2.0\,\text{s}$. (b) Calculate the sprinter's average velocity over the first $2.0\,\text{s}$. (c) Explain why the two answers differ.

The instantaneous velocity at $t = 2.0\,\text{s}$ is the gradient of the tangent at that point. Using the two given points on the tangent line:

$$v = \frac{\Delta s}{\Delta t} = \frac{15.0 - 0.0}{3.5 - 0.5} = \frac{15.0}{3.0} = 5.0\,\text{m s}^{-1}.$$

The average velocity over the first $2.0\,\text{s}$ uses the displacement read from the curve itself, $s = 7.5\,\text{m}$ at $t = 2.0\,\text{s}$:

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{7.5 - 0.0}{2.0 - 0.0} = 3.75\,\text{m s}^{-1} \approx 3.8\,\text{m s}^{-1}.$$

∴ the instantaneous velocity at $2.0\,\text{s}$ is $5.0\,\text{m s}^{-1}$ and the average velocity over the first $2.0\,\text{s}$ is $3.8\,\text{m s}^{-1}$. (c) The two values differ because the sprinter is speeding up: the velocity at the end of the interval is larger than the velocity at the start, so the average over the whole interval must sit below the final value. The average velocity is the gradient of the chord from the origin to the point at $2.0\,\mathrm{s}$; the instantaneous velocity is the gradient of the tangent at that point, and the curve steepens between them.

Example 4 — unscaffolded

A cyclist riding in a straight line brakes gently for a traffic light. Her velocity is measured every $2.0\,\mathrm{s}$:

Time t (s)	Velocity v ($\mathrm{m\ s^{-1}}$)
0.0	8.0
2.0	6.4
4.0	4.8
6.0	3.2
8.0	1.6

(a) Construct the velocity–time graph for these data, describing the axes, the plotted points and the trend line.
(b) Determine the acceleration of the cyclist.
(c) Use extrapolation to find the time at which the cyclist stops, and state the meaning of each intercept of the graph.
(d) Estimate the total distance the cyclist travels from the moment braking begins to the moment she stops.

The velocity falls by the same amount, $1.6\,\mathrm{m\ s^{-1}}$, in every $2.0\,\mathrm{s}$ interval, so the plotted points lie on a straight line. (a) The graph has time t (s) on the horizontal axis — the independent variable — and velocity v ($\mathrm{m\ s^{-1}}$) on the vertical axis, each axis labelled with its quantity and unit, and a scale chosen so the points spread over most of the grid. The five points are plotted and a single straight line of best fit is drawn through them.

(b) The acceleration is the gradient of the line, using the first and last readings:

$$a=\frac{\Delta v}{\Delta t}=\frac{1.6-8.0}{8.0-0.0}=\frac{-6.4}{8.0}=-0.80\,\mathrm{m\ s^{-2}}.$$

(c) The line continues with the same gradient below the last measured point, so it is extended as a dashed line down to $v=0$. The velocity falls by $0.80\,\mathrm{m\ s^{-1}}$ each second, so from $1.6\,\mathrm{m\ s^{-1}}$ at $8.0\,\mathrm{s}$ it reaches zero a further $1.6\,/\,0.80=2.0\,\mathrm{s}$ later, at $t=10.0\,\mathrm{s}$. The vertical-axis intercept, $8.0\,\mathrm{m\ s^{-1}}$, is the velocity at $t=0$ — the initial velocity when braking began. The time-axis intercept, $10.0\,\mathrm{s}$, is the instant at which $v=0$: the moment the cyclist stops.

(d) The distance travelled is the area under the velocity–time graph from $t=0$ to $t=10.0\,\mathrm{s}$ — a triangle of base $10.0\,\mathrm{s}$ and height $8.0\,\mathrm{m\ s^{-1}}$, including the extrapolated part:

$$\text{distance}=\frac{1}{2}\times 10.0\times 8.0=40\,\mathrm{m}.$$

∴ the cyclist's acceleration is $-0.80\,\mathrm{m\ s^{-2}}$, she stops at $t=10.0\,\mathrm{s}$, and she travels $40\,\mathrm{m}$ from the moment braking begins. The velocity stays positive throughout, so the distance and the displacement are the same.

Practice questions

Question 1 *(scaffolded)* A pedestrian walks in a straight line along a footpath. Her displacement from a tram stop, measured at three instants, is:

Time t (s)	Displacement s (m)
0.0	2.0
4.0	8.0
10.0	17.0

(a) Calculate the gradient of the displacement–time graph using the first two readings, and check it using the last two.
(b) State the pedestrian's velocity and describe her motion.
(c) State her displacement at $t=0$, and explain whether the origin should be plotted as a data point on her graph.

Question 2 *(scaffolded)* A car moves in a straight line. Its velocity–time graph is a straight line rising from $4.0\,\mathrm{m\,s^{-1}}$ at $t=0$ to $16.0\,\mathrm{m\,s^{-1}}$ at $t=6.0\,\mathrm{s}$. (a) Calculate the acceleration of the car. (b) Calculate the displacement of the car over the $6.0\,\mathrm{s}$. (c) State what the vertical-axis intercept of the graph represents.

Question 3 *(scaffolded)* In an investigation, a student measures the displacement of a toy car rolling along a straight track, using a metre ruler and a stopwatch:

Time t (s)	Displacement s (m)
0.0	0.00
1.0	0.30
2.0	0.60
3.0	0.90
4.0	1.20

(a) State the label that belongs on each axis of the displacement–time graph of these data.
(b) Describe the line of best fit for these points, and justify whether it is appropriate for the line to pass through the origin.
(c) Calculate the velocity of the car.

Question 4 *(scaffolded)* A cyclist braking gently in a straight line records the following velocities:

Time t (s)	Velocity v ($\mathrm{m\,s^{-1}}$)
0.0	6.0
2.0	4.5
4.0	3.0
6.0	1.5

(a) Describe the velocity–time graph of these data.
(b) Calculate the acceleration of the cyclist.
(c) Extrapolate the trend line to find the time at which the cyclist stops. State how the extrapolated part of the line should be drawn.
(d) State the meaning of the vertical-axis intercept of this graph.

Question 5 *(scaffolded)* A car travelling in a straight line at $3.0\,\mathrm{m\,s^{-1}}$ accelerates uniformly at $2.5\,\mathrm{m\,s^{-2}}$ for $4.0\,\mathrm{s}$. Its acceleration–time graph is a horizontal line at $2.5\,\mathrm{m\,s^{-2}}$ from $t=0$ to $t=4.0\,\mathrm{s}$. (a) Calculate the change in velocity of the car, using the area under the acceleration–time graph. (b) Calculate the final velocity of the car. (c) Describe the velocity–time graph that corresponds to this acceleration–time graph.

Question 6 *(scaffolded)* A skateboarder rolls from rest down a straight ramp. Her displacement is measured each second:

Time t (s)	Displacement s (m)
0.0	0.0
1.0	0.5
2.0	2.0
3.0	4.5

(a) Calculate her average velocity over the first $3.0\,\mathrm{s}$.
(b) Calculate her average velocity between $t=2.0\,\mathrm{s}$ and $t=3.0\,\mathrm{s}$.

(c) Compare the two answers and state what they show about her motion.

Question 7 A tram travels along a straight route. Its displacement–time graph consists of three straight sections: from $(0\,\text{s}, 0\,\text{m})$ to $(40\,\text{s}, 120\,\text{m})$; a horizontal section from $(40\,\text{s}, 120\,\text{m})$ to $(60\,\text{s}, 120\,\text{m})$; then from $(60\,\text{s}, 120\,\text{m})$ to $(100\,\text{s}, 300\,\text{m})$. (a) Calculate the velocity of the tram in each of the three sections. (b) Calculate the average speed of the tram for the whole journey. (c) State what the horizontal section of the graph represents.

Question 8 Two students look at a velocity–time graph that is a straight line with a positive gradient, starting from the origin. Student A says: "The object moves with constant velocity, because the line is straight." Student B says: "The object has constant acceleration, because the gradient is constant and not zero." State which student is correct, and explain the error in the other student's reasoning.

Question 9 The velocity–time graph of a car moving in a straight line is a straight line from $(0\,\text{s}, 0\,\text{m s}^{-1})$ to $(4.0\,\text{s}, 8.0\,\text{m s}^{-1})$, then a straight line from $(4.0\,\text{s}, 8.0\,\text{m s}^{-1})$ to $(10.0\,\text{s}, -4.0\,\text{m s}^{-1})$. (a) Calculate the acceleration of the car in each section. (b) Calculate the time at which the graph crosses the time axis, and state what this instant means. (c) Calculate the net displacement of the car over the $10.0\,\text{s}$. (d) Calculate the total distance travelled by the car over the $10.0\,\text{s}$.

Question 10 A student is asked to find the instantaneous velocity of an object at one point on a curved displacement–time graph. Explain why the student must draw the tangent at that point and use its gradient, rather than joining the point to the origin and calculating s/t.

Question 11 A car accelerates from rest along a straight test track. Its velocity–time graph is a curve that becomes less steep as time increases. At $t = 6.0\,\text{s}$, the tangent to the curve passes through $(2.0\,\text{s}, 5.0\,\text{m s}^{-1})$ and $(10.0\,\text{s}, 29.0\,\text{m s}^{-1})$. (a) Calculate the instantaneous acceleration of the car at $t = 6.0\,\text{s}$. (b) State what the changing steepness of the curve shows about the car's acceleration.

Question 12 A student measures the velocity of a trolley at $t = 2.0$, 4.0, 6.0 and $8.0\,\text{s}$, plots the four points on a velocity–time graph and draws a line of best fit that meets the vertical axis at $0.6\,\text{m s}^{-1}$. The student then adds the point $(0\,\text{s}, 0\,\text{m s}^{-1})$ to the graph, saying "graphs should start at the origin", and redraws the line through it. Evaluate the student's action, referring to the study design's rules for graphs.

Question 13 The velocity–time graph of a braking car is a curve drawn on a grid where each square is $1.0\,\text{s}$ wide and $2.0\,\text{m s}^{-1}$ tall. A student counts 21 whole squares under the curve and estimates the partial squares along the curve as equivalent to 3 more whole squares. (a) Calculate the displacement represented by one square. (b) Estimate the displacement of the car while braking. (c) Explain why the answer is an estimate rather than an exact value.

Question 14 A student records the velocity of a car braking to a stop over $8.0\,\text{s}$ and draws a velocity–time graph. To predict the velocity of the car at $t = 60\,\text{s}$, the student extends the line of the graph out that far. Evaluate the reliability of this prediction.

Question 15 A lift in a city office tower travels upwards in a straight line, starting from rest. Its acceleration–time graph has three sections: $+1.2\,\text{m s}^{-2}$ from $t = 0$ to $t = 2.0\,\text{s}$; $0\,\text{m s}^{-2}$ from $t = 2.0\,\text{s}$ to $t = 7.0\,\text{s}$; and $-1.2\,\text{m s}^{-2}$ from $t = 7.0\,\text{s}$ to $t = 9.0\,\text{s}$. Upwards is positive. (a) Calculate the change in velocity of the lift in each section. (b) State the maximum velocity of the lift and the time at which it is first reached. (c) Describe the motion of the lift over the $9.0\,\text{s}$, and state its velocity at $t = 9.0\,\text{s}$. (d) Calculate the total distance the lift travels in the $9.0\,\text{s}$.

Question 16 A tram starts from rest at a tram stop, speeds up uniformly, travels for a while at constant velocity, then brakes uniformly to rest at the next stop, moving in a straight line throughout. Describe, without calculations, the shape of (a) the tram's velocity–time graph and (b) the tram's displacement–time graph for the whole journey.

Question 17 A student walks in a straight corridor during a motion-sensor investigation. She walks $6.0\,\text{m}$ away from the sensor in $4.0\,\text{s}$ at constant velocity, then turns and walks straight back to the sensor in $3.0\,\text{s}$ at constant velocity. The direction away from the sensor is positive. (a) Calculate her average velocity for the whole $7.0\,\text{s}$. (b) Calculate her average speed for the whole $7.0\,\text{s}$. (c) Explain why the two answers differ.

Question 18 The velocity–time graph of an object moving in a straight line is a straight line from $(0\,\text{s}, 2.0\,\text{m s}^{-1})$ to $(5.0\,\text{s}, 12.0\,\text{m s}^{-1})$. A student calculates the acceleration and writes: “acceleration $= 5.0/10.0 = 0.50$”. Identify the error in the student’s working, and calculate the correct acceleration, including units.

Answers

Question 1 (a) $1.5\,\mathrm{m\ s^{-1}}$ both ways; (b) $1.5\,\mathrm{m\ s^{-1}}$, uniform motion away from the stop; (c) $2.0\,\mathrm{m}$; the origin was not measured, so it is not a data point. **Question 2** (a) $2.0\,\mathrm{m\ s^{-2}}$; (b) $60\,\mathrm{m}$; (c) the velocity at $t=0$ (initial velocity, $4.0\,\mathrm{m\ s^{-1}}$). **Question 3** (a) horizontal: "Time t (s)"; vertical: "Displacement s (m)"; (b) a straight line through the points — see solution; (c) $0.30\,\mathrm{m\ s^{-1}}$. **Question 4** (a) a straight line with negative gradient; (b) $-0.75\,\mathrm{m\ s^{-2}}$; (c) $8.0\,\mathrm{s}$, drawn as a dashed line; (d) the initial velocity, $6.0\,\mathrm{m\ s^{-1}}$. **Question 5** (a) $10.0\,\mathrm{m\ s^{-1}}$; (b) $13.0\,\mathrm{m\ s^{-1}}$; (c) a straight line with gradient $2.5\,\mathrm{m\ s^{-2}}$ starting at $3.0\,\mathrm{m\ s^{-1}}$. **Question 6** (a) $1.5\,\mathrm{m\ s^{-1}}$; (b) $2.5\,\mathrm{m\ s^{-1}}$; (c) the velocity is increasing — see solution. **Question 7** (a) $3.0\,\mathrm{m\ s^{-1}}$; $0\,\mathrm{m\ s^{-1}}$; $4.5\,\mathrm{m\ s^{-1}}$; (b) $3.0\,\mathrm{m\ s^{-1}}$; (c) the tram is stationary. **Question 8** Student B; see solution. **Question 9** (a) $2.0\,\mathrm{m\ s^{-2}}$ then $-2.0\,\mathrm{m\ s^{-2}}$; (b) $t=8.0\,\mathrm{s}$ — the car is momentarily at rest, then reverses; (c) $28\,\mathrm{m}$; (d) $36\,\mathrm{m}$. **Question 10** See solution. **Question 11** (a) $3.0\,\mathrm{m\ s^{-2}}$; (b) the acceleration is decreasing with time. **Question 12** See solution — the action is not justified. **Question 13** (a) $2.0\,\mathrm{m}$; (b) $\approx 48\,\mathrm{m}$; (c) see solution. **Question 14** See solution — not reliable. **Question 15** (a) $+2.4$, 0, $-2.4\,\mathrm{m\ s^{-1}}$; (b) $2.4\,\mathrm{m\ s^{-1}}$, first reached at $t=2.0\,\mathrm{s}$; (c) speeds up, moves uniformly, slows to rest; $0\,\mathrm{m\ s^{-1}}$; (d) $17\,\mathrm{m}$. **Question 16** See solution. **Question 17** (a) $0\,\mathrm{m\ s^{-1}}$; (b) $1.7\,\mathrm{m\ s^{-1}}$; (c) see solution. **Question 18** The student divided run by rise; $a=2.0\,\mathrm{m\ s^{-2}}$.

Fully-worked solutions

Question 1 Readings: $(0.0\,\mathrm{s}, 2.0\,\mathrm{m})$, $(4.0\,\mathrm{s}, 8.0\,\mathrm{m})$, $(10.0\,\mathrm{s}, 17.0\,\mathrm{m})$. (a) Using the first two readings: $\text{gradient}=\dfrac{8.0-2.0}{4.0-0.0}=\dfrac{6.0}{4.0}=1.5\,\mathrm{m\ s^{-1}}$. Check with the last two: $\dfrac{17.0-8.0}{10.0-4.0}=\dfrac{9.0}{6.0}=1.5\,\mathrm{m\ s^{-1}}$. ✓ (b) The gradient of a displacement–time graph is the velocity: $v=1.5\,\mathrm{m\ s^{-1}}$. The gradient is the same everywhere, so the pedestrian is in uniform motion, moving at a constant velocity in the positive direction, away from the tram stop. (c) At $t=0$ her displacement is $2.0\,\mathrm{m}$: she was already $2.0\,\mathrm{m}$ from the stop when timing began. The graph does not pass through the origin, and that is correct — the study design states that not all graphs will necessarily pass through the origin, and the origin may only be included as a data point if authentically measured. No reading at $(0\,\mathrm{s}, 0\,\mathrm{m})$ was taken here, so the origin is not a data point.

Question 2 Straight line from $(0\,\mathrm{s}, 4.0\,\mathrm{m\ s^{-1}})$ to $(6.0\,\mathrm{s}, 16.0\,\mathrm{m\ s^{-1}})$. (a) $a=\dfrac{\Delta v}{\Delta t}=\dfrac{16.0-4.0}{6.0-0.0}=\dfrac{12.0}{6.0}=2.0\,\mathrm{m\ s^{-2}}$. (b) The area under the graph is a trapezium: $\text{displacement}=\dfrac{1}{2}(4.0+16.0)\times 6.0=\dfrac{1}{2}\times 20.0\times 6.0=60\,\mathrm{m}$. (c) The vertical-axis intercept is the value of the velocity at $t=0$ — the car's initial velocity, $4.0\,\mathrm{m\ s^{-1}}$.

Question 3 (a) The horizontal axis is labelled "Time t (s)" and the vertical axis "Displacement s (m)" — each axis label carries the quantity, its symbol and its unit. (b) Model answer. The displacement increases by $0.30\,\mathrm{m}$ in every $1.0\,\mathrm{s}$ interval, so the points lie on a straight line, and the line of best fit is a single straight line passing as close as possible to all five points. It is appropriate for the line to pass through the origin here because the origin was authentically measured: the table records a reading of $s=0.00\,\mathrm{m}$ at $t=0.0\,\mathrm{s}$. The study design states that the origin may only be included as a data point if authentically measured — here it may. (c) $v=\dfrac{\Delta s}{\Delta t}=\dfrac{1.20-0.00}{4.0-0.0}=0.30\,\mathrm{m\ s^{-1}}$.

Question 4 Readings: $(0.0\,\mathrm{s}, 6.0)$, $(2.0\,\mathrm{s}, 4.5)$, $(4.0\,\mathrm{s}, 3.0)$, $(6.0\,\mathrm{s}, 1.5)$, in $\mathrm{m\ s^{-1}}$. (a) The velocity falls by $1.5\,\mathrm{m\ s^{-1}}$ every $2.0\,\mathrm{s}$, so the velocity–time graph is a straight line with a negative gradient. (b) $a=\dfrac{\Delta v}{\Delta t}=\dfrac{1.5-6.0}{6.0-0.0}=\dfrac{-4.5}{6.0}=-0.75\,\mathrm{m\ s^{-2}}$. (c) The velocity falls by $0.75\,\mathrm{m\ s^{-1}}$ each second. From $1.5\,\mathrm{m\ s^{-1}}$ at $6.0\,\mathrm{s}$, it reaches zero after $1.5/0.75=2.0\,\mathrm{s}$ more, at $t=8.0\,\mathrm{s}$. The line beyond the last measured point is an extrapolation and is drawn as a dashed line. (d) The vertical-axis intercept is the velocity at $t=0$ — the cyclist's initial velocity, $6.0\,\mathrm{m\ s^{-1}}$.

Question 5 $a=2.5\,\mathrm{m\ s^{-2}}$ for $4.0\,\mathrm{s}$; initial velocity $u=3.0\,\mathrm{m\ s^{-1}}$. (a) The area under an acceleration–time graph is the change in velocity: $\Delta v=2.5\times 4.0=10.0\,\mathrm{m\ s^{-1}}$. (b) final velocity $=3.0+10.0=13.0\,\mathrm{m\ s^{-1}}$. (c) Constant acceleration means velocity increasing by equal amounts in equal times: the velocity–time graph is a straight line with gradient $2.5\,\mathrm{m\,s^{-2}}$, starting at $3.0\,\mathrm{m\,s^{-1}}$ on the vertical axis.

Question 6 Readings: $(0.0\,\mathrm{s}, 0.0\,\mathrm{m})$, $(1.0\,\mathrm{s}, 0.5\,\mathrm{m})$, $(2.0\,\mathrm{s}, 2.0\,\mathrm{m})$, $(3.0\,\mathrm{s}, 4.5\,\mathrm{m})$. (a) $v_{\mathrm{avg}} = \dfrac{4.5-0.0}{3.0-0.0} = 1.5\,\mathrm{m\,s^{-1}}$. (b) $v_{\mathrm{avg}} = \dfrac{4.5-2.0}{3.0-2.0} = 2.5\,\mathrm{m\,s^{-1}}$. (c) Model answer. The average velocity for the last second, $2.5\,\mathrm{m\,s^{-1}}$, is larger than the average over the whole $3.0\,\mathrm{s}$, $1.5\,\mathrm{m\,s^{-1}}$. The skateboarder covers a larger displacement in each successive second, so her velocity is increasing: the displacement–time graph is a curve that gets steeper, and the motion is non-uniform.

Question 7 Three straight sections of the displacement–time graph. (a) Section 1: $v = \dfrac{120-0}{40-0} = 3.0\,\mathrm{m\,s^{-1}}$. Section 2: the line is horizontal, so $v = 0\,\mathrm{m\,s^{-1}}$. Section 3: $v = \dfrac{300-120}{100-60} = \dfrac{180}{40} = 4.5\,\mathrm{m\,s^{-1}}$. (b) Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{300}{100} = 3.0\,\mathrm{m\,s^{-1}}$. The motion is in one direction throughout, so the total distance is the final displacement, $300\,\mathrm{m}$. (c) The horizontal section means the displacement is not changing: the tram is stationary for $20\,\mathrm{s}$ — held at a tram stop or at signals.

Question 8 Model answer. Student B is correct. On a velocity–time graph the gradient is the acceleration. A straight line has a constant gradient, so the acceleration is constant, and a gradient that is not zero means the velocity is changing — that is exactly constant acceleration. Student A's error is to read the straightness of the line as constant velocity. Constant velocity on a velocity–time graph is a *horizontal* line (zero gradient, zero acceleration), whatever its height. A straight *sloping* line shows a velocity that changes uniformly.

Question 9 Section 1: $(0\,\mathrm{s}, 0\,\mathrm{m\,s^{-1}})$ to $(4.0\,\mathrm{s}, 8.0\,\mathrm{m\,s^{-1}})$. Section 2: $(4.0\,\mathrm{s}, 8.0\,\mathrm{m\,s^{-1}})$ to $(10.0\,\mathrm{s}, -4.0\,\mathrm{m\,s^{-1}})$. (a) Section 1: $a = \dfrac{8.0-0.0}{4.0-0.0} = 2.0\,\mathrm{m\,s^{-2}}$. Section 2: $a = \dfrac{-4.0-8.0}{10.0-4.0} = \dfrac{-12.0}{6.0} = -2.0\,\mathrm{m\,s^{-2}}$. (b) In section 2 the velocity falls from $8.0\,\mathrm{m\,s^{-1}}$ at $2.0\,\mathrm{m\,s^{-1}}$ each second, so it reaches zero after $8.0/2.0 = 4.0\,\mathrm{s}$: $t = 4.0+4.0 = 8.0\,\mathrm{s}$. At that instant the car is momentarily at rest; after it, the velocity is negative, so the car moves in the negative direction — it has reversed. (c) The net displacement is the sum of the signed areas: 0–4.0 s: $\dfrac{1}{2} \times 4.0 \times 8.0 = 16\,\mathrm{m}$; 4.0–8.0 s: $\dfrac{1}{2} \times 4.0 \times 8.0 = 16\,\mathrm{m}$; 8.0–10.0 s: $\dfrac{1}{2} \times 2.0 \times (-4.0) = -4.0\,\mathrm{m}$. Net displacement $= 16+16-4.0 = 28\,\mathrm{m}$. (d) The total distance counts all areas as positive: $16+16+4.0 = 36\,\mathrm{m}$.

Question 10 Model answer. Joining the point to the origin and calculating s/t gives the *average* velocity over the whole interval from the start of timing to that point: it uses the total displacement and the total time, and says nothing about how fast the object is moving at the one instant. On a curved graph the velocity is different at different instants, so the average over a long interval generally differs from the value at any particular point within it. The tangent touches the curve at the one point and has the same slope as the curve there, so the gradient of the tangent is the rate of change of displacement at that instant alone — the instantaneous velocity.

Question 11 Tangent at $t = 6.0\,\mathrm{s}$ through $(2.0\,\mathrm{s}, 5.0\,\mathrm{m\,s^{-1}})$ and $(10.0\,\mathrm{s}, 29.0\,\mathrm{m\,s^{-1}})$. (a) The instantaneous acceleration is the gradient of the tangent to the velocity–time graph: $a = \dfrac{\Delta v}{\Delta t} = \dfrac{29.0-5.0}{10.0-2.0} = \dfrac{24.0}{8.0} = 3.0\,\mathrm{m\,s^{-2}}$. (b) The curve becomes less steep as time increases. The gradient of a velocity–time graph is the acceleration, so the car's acceleration is decreasing with time: the velocity is still increasing, but by smaller amounts each second.

Question 12 Model answer. The student's action is not justified. The study design's treatment of data states that not all graphs will necessarily pass through the origin, and that the origin may only be included as a data point if authentically measured. Here no reading was taken at $t = 0$, so $(0\,\mathrm{s}, 0\,\mathrm{m\,s^{-1}})$ is not a data point and must not be added as one. The line of best fit is drawn from the four measured points, and its vertical-axis intercept of $0.6\,\mathrm{m\,s^{-1}}$ is a genuine result: it is the velocity the trolley had at $t = 0$, found by extrapolating the measured trend. Forcing the line through the origin ignores that measurement and distorts every gradient and every value read from the line. If the student wanted to know the velocity at $t=0$ directly, the correct step is to take a reading there — not to invent one.

Question 13 (a) One square represents $1.0\,\text{s}\times2.0\,\text{m s}^{-1}=2.0\,\text{m}$. (b) Total squares $\approx21+3=24$, so the displacement $\approx24\times2.0=48\,\text{m}$. (c) Model answer. The boundary of the region under the curve cuts through squares, and the student has to judge how much of each partial square is covered. Those judgments introduce uncertainty, so the total — and the displacement found from it — is an estimate. The finer the grid, the smaller each partial square and the better the estimate.

Question 14 Model answer. The prediction is not reliable. Extrapolation assumes the trend recorded in the data continues unchanged, and here there is good reason to believe it does not: the car is braking to a stop, so its velocity falls to zero and stays there — it does not keep falling. Extending the line to $t=60\,\text{s}$, far beyond the last measurement at $8.0\,\text{s}$, would even give negative velocities, which are meaningless for a car that has already stopped. A short extrapolation to find the stopping time is sound; an extension this far has no physical basis.

Question 15 Three sections of the acceleration–time graph; the lift starts from rest. (a) The area under an acceleration–time graph is the change in velocity. Section 1: $\Delta v=1.2\times2.0=+2.4\,\text{m s}^{-1}$. Section 2: $\Delta v=0\times5.0=0\,\text{m s}^{-1}$. Section 3: $\Delta v=-1.2\times2.0=-2.4\,\text{m s}^{-1}$. (b) The velocity rises from 0 to $2.4\,\text{m s}^{-1}$ in the first section, stays there in the second, then falls. The maximum velocity is $2.4\,\text{m s}^{-1}$, first reached at $t=2.0\,\text{s}$. (c) The lift speeds up uniformly for $2.0\,\text{s}$, moves with constant velocity $2.4\,\text{m s}^{-1}$ for $5.0\,\text{s}$, then slows down uniformly for $2.0\,\text{s}$, coming to rest: its velocity at $t=9.0\,\text{s}$ is $0\,\text{m s}^{-1}$ $(0+2.4+0-2.4=0)$. (d) The distance is the area under the corresponding velocity–time graph: section 1: $\frac{1}{2}\times2.0\times2.4=2.4\,\text{m}$; section 2: $2.4\times5.0=12.0\,\text{m}$; section 3: $\frac{1}{2}\times2.0\times2.4=2.4\,\text{m}$. Total $=2.4+12.0+2.4=16.8\,\text{m}\approx17\,\text{m}$ (2 significant figures).

Question 16 Model answer. (a) The velocity–time graph has three sections: a straight line rising from zero (velocity increasing uniformly as the tram speeds up), a horizontal line (constant velocity), and a straight line falling back to zero (velocity decreasing uniformly as the tram brakes). All of the graph sits on or above the time axis because the tram never reverses. (b) The displacement–time graph has three matching sections: a curve that gets steeper as the tram speeds up (gradient increasing), a straight line while the velocity is constant (constant gradient), and a curve that flattens out as the tram brakes (gradient decreasing), finishing horizontal when the tram is at rest. The graph never goes downhill, because the displacement keeps increasing.

Question 17 Outward: $+6.0\,\text{m}$ in $4.0\,\text{s}$. Return: $-6.0\,\text{m}$ in $3.0\,\text{s}$. (a) Average velocity $=\dfrac{\text{net displacement}}{\text{total time}}=\dfrac{6.0+(-6.0)}{7.0}=\dfrac{0}{7.0}=0\,\text{m s}^{-1}$. She finishes where she started, so her net displacement is zero. (b) Average speed $=\dfrac{\text{total distance}}{\text{total time}}=\dfrac{6.0+6.0}{7.0}=\dfrac{12.0}{7.0}=1.7\,\text{m s}^{-1}$ (2 significant figures). (c) Model answer. Average velocity uses the net displacement, which is zero because the outward and return displacements are equal and opposite, while average speed uses the total distance travelled, which is $12.0\,\text{m}$. Velocity is a vector, so the two halves cancel; speed is a scalar, so they add. The two averages answer different questions: “how far from the start, per second?” against “how much ground covered, per second?”.

Question 18 Model answer. The student has divided the run by the rise, calculating $\Delta t/\Delta v=5.0/10.0=0.50$, which is the reciprocal of the gradient; the units would be $\text{s}^2\,\text{m}^{-1}$, not the unit of acceleration. The gradient of a velocity–time graph is the change in velocity divided by the change in time:

$$a=\frac{\Delta v}{\Delta t}=\frac{12.0-2.0}{5.0-0.0}=\frac{10.0}{5.0}=2.0\,\text{m s}^{-2}.$$

The correct acceleration is $2.0\,\text{m s}^{-2}$.

Theory

Section 7A sorted the parameters of motion into vectors and scalars, and 7B showed how any straight-line motion can be described with graphs, even when the motion changes in a complicated way. This section combines those two ideas in the most important special case: **straight-line motion under constant acceleration**, motion in which the velocity changes by equal amounts in equal intervals of time. A train pulling away from a platform, a car braking in a straight lane and a ball travelling straight up and back down are all, to a very good approximation, motions of this kind. When the acceleration is constant, the three ways of describing the motion — numerically, with a table of values; graphically, with a velocity–time graph; and algebraically, with equations — must all agree with one another. Checking one description against another is part of the analysis.

The quantities and the sign convention. Five quantities describe the motion between a chosen start and a chosen end of the movement.

Quantity	Symbol	Unit	Vector or scalar?	Meaning
displacement	s	metre (m)	vector	change of position along the line, with a sign
initial velocity	u	$\mathrm{m\ s^{-1}}$	vector	velocity at the start of the timing
final velocity	v	$\mathrm{m\ s^{-1}}$	vector	velocity at the end of the timing
acceleration	a	$\mathrm{m\ s^{-2}}$	vector	rate of change of velocity
time interval	t	second (s)	scalar	length of time between start and end

(Displacement, velocity and acceleration are vectors and time is a scalar, per 7A.) In a straight-line problem, words such as *forward*, *up* and *to the right* are only directions once a frame has been stated: saying which way is positive is what makes every sign exact. So the first step in every problem here is to **choose the positive direction** and hold to it. Quantities directed along it are positive; quantities directed against it are negative. A negative acceleration does not automatically mean slowing down: if v is positive and a is negative the object slows, but if v and a are both negative the object speeds up in the negative direction. When an object slows down, the size of its acceleration is often called a **deceleration**.

Numerical analysis — the definition of acceleration. The acceleration is the change in velocity divided by the time taken:

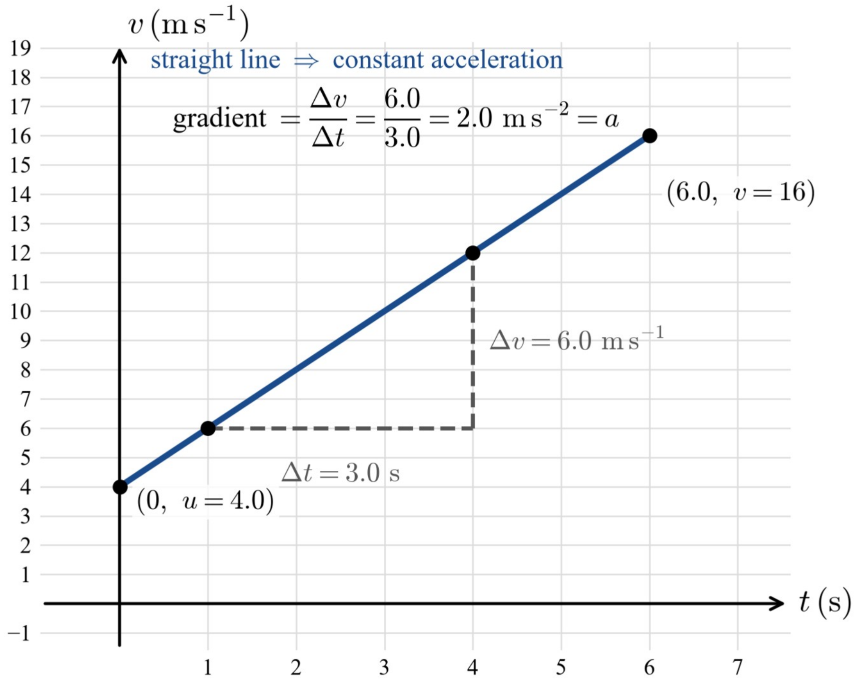
$$a = \frac{v - u}{t}.$$

A car moving along a straight road is timed each second, and its velocity is recorded:

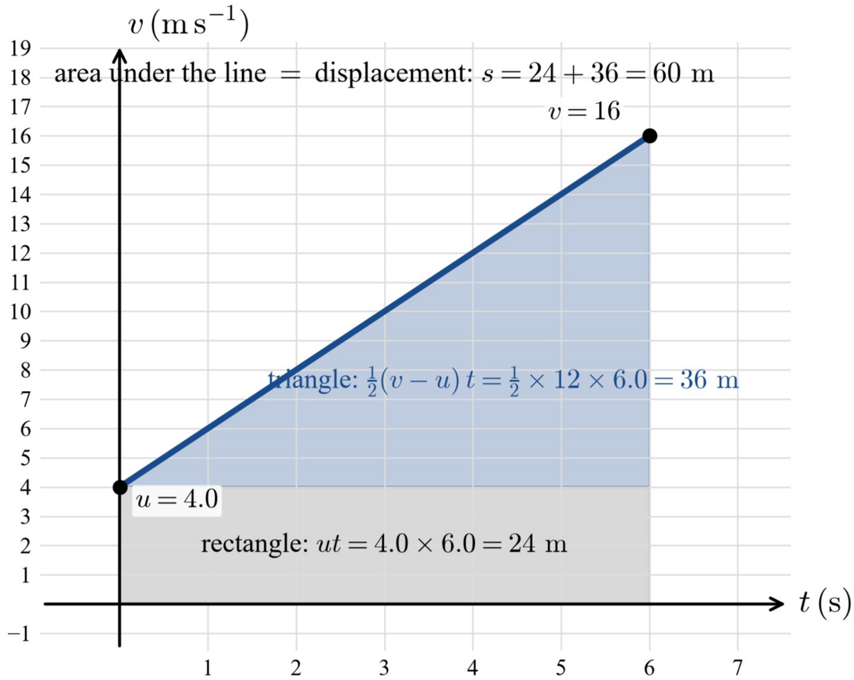
t (s)	0	1.0	2.0	3.0	4.0
v ($\mathrm{m\ s^{-1}}$)	2.0	3.5	5.0	6.5	8.0

Each second the velocity increases by the same amount, $1.5\,\mathrm{m\ s^{-1}}$, so the acceleration is constant with $a = \frac{8.0 - 2.0}{4.0} = 1.5\,\mathrm{m\ s^{-2}}$. Equal velocity steps in equal times is exactly what constant acceleration means, read off a table of values. A table like this one is the sort of data a motion sensor and data logger, a video analysis or a ticker-timer produces in a practical investigation; the analysis of it is the same whichever instrument made it.

Graphical analysis — gradient and area. Because the velocity changes by equal amounts in equal times, the velocity–time graph of a constant-acceleration motion is a **straight line**, and the line's gradient is the acceleration (7B develops the idea that a gradient is a rate). Consider an object already moving at $u = 4.0\,\mathrm{m\ s^{-1}}$ that accelerates at $2.0\,\mathrm{m\ s^{-2}}$; after $t = 6.0\,\mathrm{s}$ its velocity is $v = u + at = 4.0 + 2.0 \times 6.0 = 16\,\mathrm{m\ s^{-1}}$. Reading the gradient between $t = 1.0\,\mathrm{s}$ and $t = 4.0\,\mathrm{s}$ gives $a = \dfrac{\Delta v}{\Delta t} = \dfrac{6.0}{3.0} = 2.0\,\mathrm{m\ s^{-2}}$, as set.



7B also shows that the **area under a velocity–time line is the displacement**. For constant acceleration the area is exact, because the line is straight: the region under the line splits into a rectangle of area $ut = 4.0 \times 6.0 = 24\,\mathrm{m}$ and a triangle of area $\dfrac{1}{2}(v - u)t = \dfrac{1}{2} \times 12 \times 6.0 = 36\,\mathrm{m}$, so the displacement over the $6.0\,\mathrm{s}$ is $s = 24 + 36 = 60\,\mathrm{m}$.



Algebraic analysis — the five equations. The numerical and graphical readings translate into algebra. From the definition of acceleration,

$$v = u + at.$$

Adding the rectangle and the triangle, the displacement is the area under the line,

$$s = 1/2(u+v)t,$$

which says that the displacement equals the average of the initial and final velocities multiplied by the time — true because the velocity grows uniformly. Substituting $v = u + at$ into this gives

$$s = ut + 1/2at^2,$$

and substituting $u = v - at$ instead gives

$$s = vt - 1/2at^2.$$

Finally, eliminating t between the first two equations ($t = \frac{v-u}{a}$, then $s = \frac{1}{2}(u+v) \cdot \frac{v-u}{a} = \frac{v^2-u^2}{2a}$) gives

$$v^2 = u^2 + 2as.$$

These are the five constant-acceleration equations of the study design. Each one involves exactly four of the five quantities s, u, v, a, t, which is precisely what makes them easy to choose between:

If your data does not include	use
s	$v = u + at$
a	$s = 1/2(u+v)t$
v	$s = ut + 1/2at^2$
u	$s = vt - 1/2at^2$
t	$v^2 = u^2 + 2as$

A reliable method for every problem is: **(1)** choose the positive direction and state it; **(2)** write s, u, v, a, t in a column, filling in what is known (with signs) and marking what is wanted; **(3)** pick the equation that contains all of the knowns and the unknown; **(4)** substitute, solve, and report the answer with its unit, its sign or direction, and an appropriate number of significant figures (per 1A).

Free fall — the most familiar constant acceleration. Near the surface of the Earth, an object moving vertically with air resistance neglected keeps a constant downward acceleration, the **acceleration due to gravity**. Throughout Units 1 & 2 its value is taken as $g = 9.8\ \mathrm{m\ s^{-2}}$. (Chapter 8 explains *why* freely falling objects share this acceleration, by modelling the force due to gravity; there g also appears as the gravitational field strength $9.8\ \mathrm{N\ kg^{-1}}$. Here only its value as an acceleration is needed.) If the positive direction is chosen **down**, then $a = +9.8\ \mathrm{m\ s^{-2}}$ for every vertical motion; if **up** is chosen positive, then $a = -9.8\ \mathrm{m\ s^{-2}}$. An object dropped from rest has $u = 0$. An object thrown straight up has $v = 0$ at the very top of its flight — yet its acceleration is still $-9.8\ \mathrm{m\ s^{-2}}$ there, because the velocity is in the act of changing from up to down; if the acceleration were ever zero, the object would hang at the top forever.

When the equations apply. All five equations assume the acceleration is constant, and they must not be applied where it is not. A real car or train does not keep one acceleration for a whole journey; such a motion is handled in stages, each with its (approximately) constant acceleration, or with the graphical and numerical methods of 7B, which work for any straight-line motion, uniform or not.

Units 3 & 4 preview. At Units 3 & 4 the same five equations are applied to projectile motion by treating the horizontal and vertical directions as two separate straight-line motions, each with a constant acceleration. The fluency built in this section is exactly what that topic assumes. Content under this banner is not assessed in this book's topic tests or practice examinations.

Throughout this section the acceleration due to gravity is taken as $g = 9.8\ \mathrm{m\ s^{-2}}$ and air resistance is neglected unless stated.

Worked examples

Example 1 — scaffolded

A Metro Trains service leaves a platform on a straight section of track and accelerates at a constant $0.80\,\mathrm{m\ s^{-2}}$ for $30\,\mathrm{s}$. (a) Calculate the velocity it reaches. (b) Calculate the distance it has travelled from the platform.

Working	Comment
$u=0$ (from rest); $a=0.80\,\mathrm{m\ s^{-2}}$; $t=30\,\mathrm{s}$. Forward taken as positive.	List the data; the train starts from rest.
$v=u+at=0+0.80\times30=24\,\mathrm{m\ s^{-1}}$.	The unknown is v and s is not known: the equation without s.
$s=ut+1/2\,at^2=0+1/2\times0.80\times30^2=360\,\mathrm{m}$.	For the distance, the equation without v.

(Check: $s=1/2(u+v)t=1/2\times24\times30=360\,\mathrm{m}$. ✓ The velocity reached, $24\,\mathrm{m\ s^{-1}}$, is about $86\,\mathrm{km\ h^{-1}}$ — reasonable for a suburban service.)

Example 2 — scaffolded

A car is travelling at $72\,\mathrm{km\ h^{-1}}$ along a straight suburban road when the driver brakes for a red light, decelerating at a constant $5.0\,\mathrm{m\ s^{-2}}$ until the car stops. (a) Calculate the time taken to stop. (b) Calculate the braking distance.

Working	Comment
$72\,\mathrm{km\ h^{-1}}=72\times\dfrac{1000}{3600}=20\,\mathrm{m\ s^{-1}}$. With the direction of travel positive: $u=+20\,\mathrm{m\ s^{-1}}$, $v=0$, $a=-5.0\,\mathrm{m\ s^{-2}}$.	Convert to SI units first; the acceleration opposes the motion, so it is negative.
$v=u+at\Rightarrow t=\dfrac{v-u}{a}=\dfrac{0-20}{-5.0}=4.0\,\mathrm{s}$.	The equation without s, rearranged for t.
$v^2=u^2+2as\Rightarrow s=\dfrac{v^2-u^2}{2a}=\dfrac{0-400}{-10}=40\,\mathrm{m}$.	The equation without t, rearranged for s.

Example 3 — unscaffolded

A ball is thrown vertically upward from level ground with an initial velocity of $15\,\mathrm{m\ s^{-1}}$. Take up as positive and neglect air resistance. Calculate (i) the maximum height reached, (ii) the total time until the ball returns to its starting level, and (iii) the velocity of the ball as it returns.

With up positive, the acceleration is $a=-9.8\,\mathrm{m\ s^{-2}}$ for the whole flight, including at the top. (i) At the maximum height the velocity is momentarily zero, so with $u=15$, $v=0$:

$$v^2=u^2+2as\Rightarrow0=15^2-19.6\,s\Rightarrow s=\frac{225}{19.6}=11.5\,\mathrm{m}.$$

(ii) On returning to the starting level the displacement is $s=0$:

$$s=ut+1/2\,at^2\Rightarrow0=15t-4.9t^2=t(15-4.9t),$$

so $t=0$ (the start) or $t=\dfrac{15}{4.9}=3.1\,\mathrm{s}$. (iii) $v=u+at=15-9.8\times3.06=-15\,\mathrm{m\ s^{-1}}$: the ball returns at $15\,\mathrm{m\ s^{-1}}$ directed downward — the same speed it left with, in the opposite direction.

∴ the ball rises $11\,\mathrm{m}$ (to 2 significant figures), is in the air for $3.1\,\mathrm{s}$, and returns at $15\,\mathrm{m\ s^{-1}}$ downward.

Example 4 — unscaffolded

In a practical investigation, a dynamics trolley is released on a straight ramp and a motion sensor with data logger records its velocity:

t (s)	0.0	0.5	1.0	1.5	2.0
v (m s^{-1})	0.20	0.55	0.90	1.25	1.60

Show numerically that the acceleration is constant and find its value, then find the displacement over the $2.0\,\mathrm{s}$ by two different methods and compare the results.

Every $0.5\,\mathrm{s}$ the velocity increases by the same amount, $0.35\,\mathrm{m\,s^{-1}}$, in every interval, so the acceleration is constant, with

$$a=\frac{\Delta v}{\Delta t}=\frac{0.35}{0.5}=0.70\,\mathrm{m\,s^{-2}}.$$

Method 1, the average-velocity equation:

$$s=1/2(u+v)t=1/2(0.20+1.60)\times 2.0=1.8\,\mathrm{m}.$$

Method 2, the u, a, t equation:

$$s=ut+1/2at^2=0.20\times 2.0+1/2\times 0.70\times 2.0^2=0.40+1.40=1.8\,\mathrm{m}.$$

The two methods agree, as they must when the acceleration is constant. ✓ Notice that the velocity–time line for this data would not pass through the origin — the trolley was already rolling at $0.20\,\mathrm{m\,s^{-1}}$ when the timing began. That is expected: a graph need not pass through the origin, and here $t=0$ is an authentically measured point (per the 7B graphing conventions).

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the positive direction, the data column, the relationship, the substitution and the unit in your own working. Take $g=9.8\,\mathrm{m\,s^{-2}}$ where needed and neglect air resistance unless told otherwise.

Question 1 An object moves in a straight line with a constant acceleration of $2.0\,\mathrm{m\,s^{-2}}$. (a) Explain, in words, what the constant acceleration means for the velocity each second. (b) Copy and complete the table of velocities, given that $v=3.0\,\mathrm{m\,s^{-1}}$ at $t=0$ and $v=7.0\,\mathrm{m\,s^{-1}}$ at $t=2.0\,\mathrm{s}$.

t (s)	0	1.0	2.0	3.0
v (m s^{-1})	3.0	?	7.0	?

(c) Use two readings from your completed table and $a=\dfrac{v-u}{t}$ to confirm that the acceleration is $2.0\,\mathrm{m\,s^{-2}}$.

Question 2 A train starts from rest and accelerates at a constant $0.60\,\mathrm{m\,s^{-2}}$ along a straight track for $40\,\mathrm{s}$. (a) Calculate the velocity reached. (b) Calculate the distance travelled in the $40\,\mathrm{s}$. (c) Confirm your distance using $s=1/2(u+v)t$.

Question 3 A car moving at $20\,\mathrm{m\,s^{-1}}$ along a straight road brakes with a constant deceleration of $4.0\,\mathrm{m\,s^{-2}}$ until it stops. (a) Calculate the time taken to stop. (b) Calculate the braking distance using $v^2=u^2+2as$. (c) Confirm your distance using $s=1/2(u+v)t$.

Question 4 A stone is dropped from rest and falls $4.9\,\mathrm{m}$ to the ground. Take the positive direction as downward. (a) Explain why, with this choice of positive direction, $a=+9.8\,\mathrm{m\,s^{-2}}$. (b) Calculate the speed of the stone as it reaches the ground. (c) Calculate the time of the fall.

Question 5 A ball is thrown vertically upward with an initial velocity of $12\,\mathrm{m\,s^{-1}}$. Take up as positive. (a) State the value of the acceleration, with its sign. (b) Calculate the maximum height reached. (c) Calculate the time taken to reach the maximum height.

Question 6 A trolley runs down a straight ramp and its velocity is recorded each second:

t (s)	0	1.0	2.0	3.0	4.0
v ($\mathrm{m\ s^{-1}}$)	3.0	5.5	8.0	10.5	13.0

(a) Show, using the table, that the acceleration is constant.
(b) Calculate the acceleration.
(c) Calculate the displacement over the $4.0\,\mathrm{s}$ using $s = 1/2(u+v)t$.

Question 7 A light aircraft begins its take-off run from rest at Avalon Airport and accelerates along the runway at a constant $2.0\,\mathrm{m\ s^{-2}}$ until it reaches its take-off speed of $30\,\mathrm{m\ s^{-1}}$. (a) Calculate the time the take-off run takes. (b) Calculate the length of runway the aircraft uses.

Question 8 A tram moving at $6.0\,\mathrm{m\ s^{-1}}$ along a straight reserve accelerates at a constant $2.0\,\mathrm{m\ s^{-2}}$ over a distance of $40\,\mathrm{m}$. Calculate the tram's final velocity and the time taken to cover the $40\,\mathrm{m}$.

Question 9 A stone is dropped from rest into a deep well. It strikes the water $2.4\,\mathrm{s}$ later. Neglect air resistance. (a) Calculate the depth of the well. (b) Calculate the speed of the stone as it strikes the water.

Question 10 A driver travels at $20\,\mathrm{m\ s^{-1}}$ along a straight road. When a hazard appears, the driver takes $0.50\,\mathrm{s}$ to react, during which the car keeps its speed; the brakes then produce a constant deceleration of $5.0\,\mathrm{m\ s^{-2}}$ until the car stops. Calculate the total distance travelled from the moment the hazard appears to the moment the car stops.

Question 11 The velocity–time graph of an object moving in a straight line is a straight line passing through the points $(0, 2.0\,\mathrm{m\ s^{-1}})$ and $(5.0\,\mathrm{s}, 12\,\mathrm{m\ s^{-1}})$. (a) Calculate the acceleration of the object. (b) Calculate the displacement over the $5.0\,\mathrm{s}$. (c) A student says the graph must be wrong because it does not pass through the origin. Evaluate this claim.

Question 12 A ball is thrown vertically **downward** from the top of a cliff with an initial velocity of $5.0\,\mathrm{m\ s^{-1}}$ and falls $30\,\mathrm{m}$ to the base. Calculate the velocity of the ball at the base and the time of the fall.

Question 13 Explain why $s = 1/2(u+v)t$ gives the displacement of a constant- acceleration motion but cannot be used for a motion whose acceleration changes. Refer to velocity–time graphs in your explanation.

Question 14 A student claims: "an object moving with constant acceleration covers equal distances in equal times." Evaluate this claim with a calculation, using an object that starts from rest with $a = 2.0\,\mathrm{m\ s^{-2}}$.

Question 15 Explain what a negative acceleration means about an object's motion. Give one straight-line example in which a negative acceleration corresponds to slowing down and one in which it corresponds to speeding up.

Question 16 A ball is thrown straight up. At the very top of its flight a student claims that, because the velocity is zero, the acceleration must also be zero. Evaluate the claim.

Question 17 Describe the method you use to choose the correct constant-acceleration equation, and demonstrate it on this problem: a cyclist accelerates uniformly from $4.0\,\mathrm{m\ s^{-1}}$ to $10\,\mathrm{m\ s^{-1}}$ over $20\,\mathrm{s}$; calculate the distance covered.

Question 18 Speed limits outside Victorian schools are reduced to $40\,\mathrm{km\ h^{-1}}$, against $60\,\mathrm{km\ h^{-1}}$ on the surrounding roads. For a car braking with a constant deceleration of $5.0\,\mathrm{m\ s^{-2}}$, calculate the braking distance from each speed and use the results, together with $v^2 = u^2 + 2as$, to justify the lower limit.

Answers

Question 1 (a) the velocity increases by the same amount, $2.0\,\mathrm{m\,s^{-1}}$, in every second; (b) 5.0 and $9.0\,\mathrm{m\,s^{-1}}$; (c) see solutions. **Question 2** (a) $24\,\mathrm{m\,s^{-1}}$; (b) $480\,\mathrm{m}$; (c) $480\,\mathrm{m}$, confirmed. **Question 3** (a) $5.0\,\mathrm{s}$; (b) $50\,\mathrm{m}$; (c) $50\,\mathrm{m}$, confirmed. **Question 4** (a) see solutions (model answer); (b) $9.8\,\mathrm{m\,s^{-1}}$; (c) $1.0\,\mathrm{s}$. **Question 5** (a) $-9.8\,\mathrm{m\,s^{-2}}$; (b) $7.3\,\mathrm{m}$; (c) $1.2\,\mathrm{s}$. **Question 6** (a) see solutions; (b) $2.5\,\mathrm{m\,s^{-2}}$; (c) $32\,\mathrm{m}$. **Question 7** (a) $15\,\mathrm{s}$; (b) $225\,\mathrm{m}$. **Question 8** $14\,\mathrm{m\,s^{-1}}$; $4.0\,\mathrm{s}$. **Question 9** (a) $28\,\mathrm{m}$; (b) $24\,\mathrm{m\,s^{-1}}$. **Question 10** $50\,\mathrm{m}$. **Question 11** (a) $2.0\,\mathrm{m\,s^{-2}}$; (b) $35\,\mathrm{m}$; (c) see solutions (model answer). **Question 12** $25\,\mathrm{m\,s^{-1}}$ downward; $2.0\,\mathrm{s}$. **Question 13** See solutions (model answer). **Question 14** See solutions (model answer). **Question 15** See solutions (model answer). **Question 16** See solutions (model answer). **Question 17** See solutions (method + calculation). **Question 18** $12\,\mathrm{m}$ and $28\,\mathrm{m}$; see solutions for the justification.

Fully-worked solutions

Question 1 (a) A constant acceleration of $2.0\,\mathrm{m\,s^{-2}}$ means the velocity increases by $2.0\,\mathrm{m\,s^{-1}}$ in every second of the motion — the same change in every equal interval of time. (b) $v(1.0)=3.0+2.0=5.0\,\mathrm{m\,s^{-1}}$ and $v(3.0)=7.0+2.0=9.0\,\mathrm{m\,s^{-1}}$. (c) Using the readings at $t=0$ and $t=3.0\,\mathrm{s}$: $a=\dfrac{9.0-3.0}{3.0}=2.0\,\mathrm{m\,s^{-2}}$, as stated. ✓

Question 2 $u=0$, $a=0.60\,\mathrm{m\,s^{-2}}$, $t=40\,\mathrm{s}$. (a) $v=u+at=0+0.60\times40=24\,\mathrm{m\,s^{-1}}$. (b) $s=ut+1/2\,at^2=0+1/2\times0.60\times40^2=480\,\mathrm{m}$. (c) $s=1/2(u+v)t=1/2\times24\times40=480\,\mathrm{m}$. ✓ The two agree.

Question 3 Direction of travel positive: $u=+20\,\mathrm{m\,s^{-1}}$, $v=0$, $a=-4.0\,\mathrm{m\,s^{-2}}$. (a) $t=\dfrac{v-u}{a}=\dfrac{0-20}{-4.0}=5.0\,\mathrm{s}$. (b) $s=\dfrac{v^2-u^2}{2a}=\dfrac{0-400}{-8.0}=50\,\mathrm{m}$. (c) $s=1/2(u+v)t=1/2\times20\times5.0=50\,\mathrm{m}$. ✓

Question 4 (a) The stone falls, and the positive direction was chosen downward. The acceleration due to gravity is directed downward, along the chosen positive direction, so $a=+9.8\,\mathrm{m\,s^{-2}}$. (The sign of an acceleration says nothing by itself about speeding up or slowing down; it says how the velocity changes relative to the chosen positive direction.) (b) $v^2=u^2+2as=0+2\times9.8\times4.9=96.04$, so $v=9.8\,\mathrm{m\,s^{-1}}$. (c) $t=\dfrac{v-u}{a}=\dfrac{9.8}{9.8}=1.0\,\mathrm{s}$.

Question 5 Up positive: $u=+12\,\mathrm{m\,s^{-1}}$. (a) $a=-9.8\,\mathrm{m\,s^{-2}}$, directed downward against the chosen positive direction. (b) At the top $v=0$: $s=\dfrac{v^2-u^2}{2a}=\dfrac{0-144}{-19.6}=7.3\,\mathrm{m}$. (c) $t=\dfrac{v-u}{a}=\dfrac{0-12}{-9.8}=1.2\,\mathrm{s}$.

Question 6 (a) Each $1.0\,\mathrm{s}$ interval shows the same increase of $2.5\,\mathrm{m\,s^{-1}}$ (3.0 → 5.5 → 8.0 → 10.5 → 13.0), so the velocity changes by equal amounts in equal times: the acceleration is constant. (b) $a=\dfrac{13.0-3.0}{4.0}=2.5\,\mathrm{m\,s^{-2}}$. (c) $s=1/2(u+v)t=1/2(3.0+13.0)\times4.0=32\,\mathrm{m}$. (Check: $s=ut+1/2\,at^2=3.0\times4.0+1/2\times2.5\times16=12+20=32\,\mathrm{m}$. ✓)

Question 7 $u=0$, $a=2.0\,\mathrm{m\,s^{-2}}$, $v=30\,\mathrm{m\,s^{-1}}$. (a) $t=\dfrac{v-u}{a}=\dfrac{30}{2.0}=15\,\mathrm{s}$. (b) $s=1/2(u+v)t=1/2\times30\times15=225\,\mathrm{m}$.

Question 8 $u=6.0\,\mathrm{m\,s^{-1}}$, $a=2.0\,\mathrm{m\,s^{-2}}$, $s=40\,\mathrm{m}$. $v^2=u^2+2as=36+2\times2.0\times40=196$, so $v=14\,\mathrm{m\,s^{-1}}$. Then $t=\dfrac{v-u}{a}=\dfrac{14-6.0}{2.0}=4.0\,\mathrm{s}$.

Question 9 Down positive: $u=0$, $a=+9.8\,\mathrm{m\,s^{-2}}$, $t=2.4\,\mathrm{s}$. (a) $s=ut+1/2\,at^2=0+1/2\times9.8\times2.4^2=28.2\,\mathrm{m}\approx28\,\mathrm{m}$. (b) $v=u+at=9.8\times2.4=23.5\,\mathrm{m\,s^{-1}}\approx24\,\mathrm{m\,s^{-1}}$.

Question 10 The motion has two stages. Stage 1 (reaction): constant velocity, $s_1 = ut = 20 \times 0.50 = 10\,\mathrm{m}$. Stage 2 (braking): $u = 20\,\mathrm{m\ s^{-1}}$, $v = 0$, $a = -5.0\,\mathrm{m\ s^{-2}}$, so $s_2 = \dfrac{v^2 - u^2}{2a} = \dfrac{0 - 400}{-10} = 40\,\mathrm{m}$. Total distance $= 10 + 40 = 50\,\mathrm{m}$.

Question 11 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{12 - 2.0}{5.0} = 2.0\,\mathrm{m\ s^{-2}}$. (b) $s = 1/2(u+v)t = 1/2(2.0 + 12) \times 5.0 = 35\,\mathrm{m}$. (c) The claim is incorrect. The intercept of $2.0\,\mathrm{m\ s^{-1}}$ simply says the object was already moving at $2.0\,\mathrm{m\ s^{-1}}$ when the timing began — exactly as in Example 4, where the trolley was already rolling when the data logger started. A velocity–time graph need not pass through the origin; the origin is one possible data point only if the object was genuinely at rest at $t = 0$.

Question 12 Down positive: $u = +5.0\,\mathrm{m\ s^{-1}}$, $a = +9.8\,\mathrm{m\ s^{-2}}$, $s = +30\,\mathrm{m}$. $v^2 = u^2 + 2as = 25 + 588 = 613$, so $v = 24.8\,\mathrm{m\ s^{-1}} \approx 25\,\mathrm{m\ s^{-1}}$, directed downward. $t = \dfrac{v - u}{a} = \dfrac{24.8 - 5.0}{9.8} = 2.0\,\mathrm{s}$.

Question 13 The quantity $1/2(u+v)$ is the average velocity only when the velocity grows uniformly, so that the object spends as much time at each speed between u and v as at any other. On a velocity–time graph, constant acceleration is a straight line and the area under it — the displacement — is exactly the trapezium $1/2(u+v)t$. If the acceleration changes, the line curves. A curve that rises slowly at first and steeply at the end (small acceleration, then large) lies below the straight chord joining $(0, u)$ to (t, v), so the true area — the true displacement — is less than $1/2(u+v)t$; the object spends most of the time at low speed. The reverse curvature gives the opposite result. So the equation is exact only for the straight-line (constant-acceleration) case; for a changing acceleration the displacement must be read from the area under the actual curve (7B).

Question 14 The claim is false. Equal times bring equal *changes of velocity*, not equal distances. With $u = 0$ and $a = 2.0\,\mathrm{m\ s^{-2}}$: in the first second $s_1 = 1/2\,at^2 = 1/2 \times 2.0 \times 1^2 = 1.0\,\mathrm{m}$; by $t = 2.0\,\mathrm{s}$ the total is $s = 1/2 \times 2.0 \times 2.0^2 = 4.0\,\mathrm{m}$, so the second second covers $4.0 - 1.0 = 3.0\,\mathrm{m}$, three times the first. As the object moves faster, each equal time interval covers a greater distance — which is precisely why the velocity–time area grows more quickly as time passes.

Question 15 A negative acceleration is one directed against the chosen positive direction; it tells how the velocity changes relative to that frame, and its effect depends on the sign of the velocity. Slowing down: a car moving in the positive direction (v positive) brakes, so a is negative and the speed falls. Speeding up: the same car now reverses and moves in the negative direction (v negative) while a remains negative, and its speed grows in the negative direction — as when a dropped stone is analysed with up taken positive, $u = 0$ and $a = -9.8\,\mathrm{m\ s^{-2}}$: the (negative) velocity grows larger in magnitude every second. The object slows only when v and a have opposite signs, and speeds up when they have the same sign.

Question 16 The claim is incorrect. At the top the velocity is zero for an instant, but the velocity is changing there — from upward before the top to downward after it — and acceleration is the rate of that change. The acceleration is $-9.8\,\mathrm{m\ s^{-2}}$ (up positive) for the entire flight, including the top. If the acceleration were zero at the top, the velocity would stay at zero and the ball would remain suspended there; instead it turns and falls, which only a non-zero downward acceleration can produce. Zero velocity does not imply zero acceleration; it is the velocity, not the acceleration, that is momentarily zero.

Question 17 The method: choose the positive direction; list s, u, v, a, t with signs, known and wanted; then pick the one equation containing all of them — equivalently, the equation that does not contain the quantity that is neither given nor wanted. For the cyclist: $u = 4.0\,\mathrm{m\ s^{-1}}$, $v = 10\,\mathrm{m\ s^{-1}}$, $t = 20\,\mathrm{s}$, and s is wanted while a is neither given nor wanted, so use the equation without a: $s = 1/2(u+v)t = 1/2(4.0 + 10) \times 20 = 140\,\mathrm{m}$.

Question 18 $40\,\mathrm{km\ h^{-1}} = 40 \times \dfrac{1000}{3600} = 11.1\,\mathrm{m\ s^{-1}}$ and $60\,\mathrm{km\ h^{-1}} = 16.7\,\mathrm{m\ s^{-1}}$. With $v = 0$, $s = \dfrac{u^2}{2 \times 5.0}$: from $40\,\mathrm{km\ h^{-1}}$, $s = \dfrac{11.1^2}{10} = 12.3\,\mathrm{m} \approx 12\,\mathrm{m}$; from $60\,\mathrm{km\ h^{-1}}$, $s = \dfrac{16.7^2}{10} = 27.8\,\mathrm{m} \approx 28\,\mathrm{m}$. Because $v^2 = u^2 + 2as$ with $v = 0$

gives $s=\frac{u^2}{2a}$, the braking distance is proportional to the *square* of the speed: raising the speed by a factor of 1.5 raises the braking distance by $1.5^2=2.25$, which the numbers confirm ($27.8\div 12.3=2.26\approx 2.25$, the small difference due to rounding the speeds). At $60\,\text{km h}^{-1}$ a car needs over twice the road to stop, before any reaction distance is added; where children cross, that extra $16\,\text{m}$ is the difference between stopping and striking. The square-law makes the lower limit a strong protection, not a marginal one.