Physics Units 1 & 2

Chapter 5 — Modelling electricity

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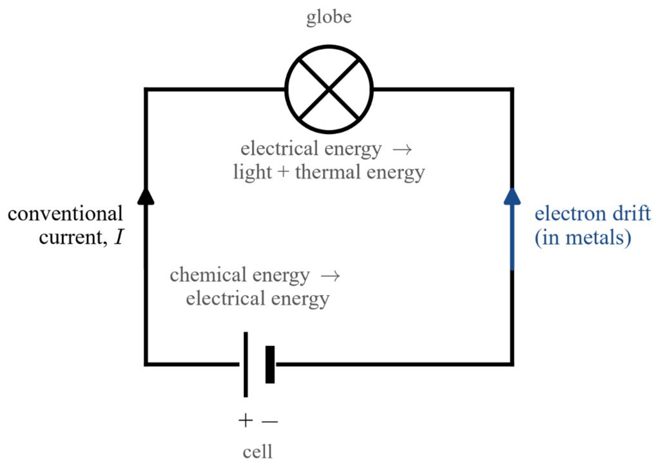
Chapter 5 Modelling electricity — 5A Charge, current, potential difference, energy and power

Theory

An electric **circuit** is a closed loop of conducting material through which charge can flow. A torch, a bicycle light and a phone charger are all circuits, and the model built here — five quantities and three relationships — describes all of them. The five quantities are **electric charge** Q, **electric current** I, **potential difference** V, **energy** E and **power** P.



Electric charge. All matter contains electric charge. There are two kinds: positive charge, carried by the protons in the nucleus of an atom, and negative charge, carried by the electrons around it. In an ordinary atom the two are present in equal numbers, so the atom is electrically neutral (uncharged). Charge can move from one object to another — rubbing a plastic comb with a cloth moves electrons across, leaving both charged — but charge is never created or destroyed: it is **conserved**. The SI unit of charge is the **coulomb** (C). Charge also comes in packets that cannot be split: the proton carries $+e$ and the electron carries $-e$, where $e = 1.60 \times 10^{-19}$ C, so every charge is a whole-number multiple of e. One coulomb is a large amount of charge: it is the charge of 6.25×10^{18} electrons. In a metal, some electrons are free to drift from atom to atom, and these drifting electrons carry the charge through a wire. In some solutions, moving ions (charged atoms or molecules) can carry charge as well.

Electric current. An electric **current** is the flow of charge, and its size is the rate of that flow:

$$I = \frac{Q}{t},$$

where Q is the charge that passes any point in the circuit in time t. The unit of current is the **ampere** (A), and $1\,\mathrm{A} = 1\,\mathrm{C\,s^{-1}}$: a current of $0.30\,\mathrm{A}$ means $0.30\,\mathrm{C}$ of charge passes every point of the circuit each second. The globe of a torch carries about $0.3\,\mathrm{A}$; a kettle on the household supply about $9\,\mathrm{A}$; the starter motor of a car briefly carries hundreds of amperes.

The direction of the current. By **convention**, the direction of a current is the direction in which positive charge would flow: outside the source, from the positive terminal, through the circuit, to the negative terminal. This convention was fixed before the electron was discovered. In a metal wire the charge that actually drifts is the negative electrons, which drift the opposite way. Every circuit diagram in this book uses conventional current, and every calculation of I, V, E or P gives the same result whichever picture you use.

Direct and alternating current. A battery produces **direct current (DC)**: the charge drifts in one direction the whole time, because the battery's terminals keep a fixed polarity (one terminal is always + and the other always −). The household supply is **alternating current (AC)**, in which the direction of the drift reverses periodically; the Australian supply is $230\,\mathrm{V}$ at a frequency of $50\,\mathrm{Hz}$ (50 cycles each second). Circuits in the home are modelled later in this book (Chapter 6, Section 6B); in this section we work with DC circuits.

A key modelling fact. In a single loop the current is the same at every point. Charge is conserved — it cannot be created, destroyed or stored up inside a globe — so the rate at which charge flows into any part of the loop equals the rate at which it flows out. **The current is not used up.** What a component takes from the charge is energy, not charge.

Potential difference. As charge flows through a component, the electrical energy it carries is transformed into other forms — in a globe, into light and thermal energy. The **potential difference** (voltage) across a component is the energy transferred per unit charge that passes through it:

$$V = \frac{E}{Q}.$$

The unit of potential difference is the **volt** (V), and $1\ \mathrm{V} = 1\ \mathrm{J\ C^{-1}}$. If the potential difference across a globe is $6.0\ \mathrm{V}$, every coulomb of charge passing through it hands over $6.0\ \mathrm{J}$ of electrical energy to the globe.

A source does the reverse. In a cell, chemical energy is transformed into electrical energy, and the potential difference across the terminals of the source tells how much energy each coulomb receives: a $1.5\ \mathrm{V}$ cell gives $1.5\ \mathrm{J}$ to every coulomb of charge that passes through it.

Energy in circuits. Rearranging $V = E/Q$ gives the energy transferred when charge Q passes through a potential difference V:

$$E = V\,Q.$$

Since $Q = I\,t$, this can also be written $E = V\,I\,t$. In the simple model the connecting wires transfer no energy, so all of the energy the source gives to the charge is handed over in the components: follow one coulomb around a $6.0\ \mathrm{V}$ loop and it carries $6.0\ \mathrm{J}$ out of the cell and delivers $6.0\ \mathrm{J}$ in the globe.

Power. Power is the rate at which energy is transferred:

$$P = \frac{E}{t},$$

with unit the **watt** (W), where $1\ \mathrm{W} = 1\ \mathrm{J\ s^{-1}}$. For any part of a circuit, combining $P = E/t$ with $E = V\,Q$ and $Q = I\,t$ gives

$$P = V\,I.$$

A device rated at $10\ \mathrm{W}$ transforms $10\ \mathrm{J}$ of electrical energy into other forms every second. Larger powers are measured in kilowatts ($1\ \mathrm{kW} = 1000\ \mathrm{W}$).

Measuring I and V. Current is measured with an **ammeter**, connected into the loop so the current flows through it; potential difference is measured with a **voltmeter**, connected across the component of interest (a multimeter can do both). Which meter to choose, and why it is connected that way, is taken up in 5C; the way each component restricts the current — its resistance — is the subject of 5D.

Quantity	Symbol	Unit	Meaning
charge	Q	coulomb (C)	amount of electricity carried by the moving charge
current	I	ampere (A)	rate of flow of charge: $I = Q/t$
potential difference	V	volt (V)	energy transferred per unit charge: $V = E/Q$
energy	E	joule (J)	amount transferred: $E = V\,Q = V\,I\,t$
power	P	watt (W)	rate of energy transfer: $P = E/t = V\,I$

Throughout this section the charge of one electron has magnitude $e = 1.60 \times 10^{-19}$ C.

Worked examples

Example 1 — scaffolded

The globe of a hand torch carries a current of 0.30 A when switched on. (a) Calculate the charge that passes through the globe in 2.0 minutes. (b) Calculate how many electrons make up this charge.

Working	Comment
$I = 0.30\,\text{A}$; $t = 2.0\,\text{min} = 2.0 \times 60 = 120\,\text{s}$.	List the data; convert the time to seconds first.
$Q = I\,t = 0.30 \times 120 = 36\,\text{C}$.	Rearrange $I = Q/t$ to $Q = I\,t$.
$n = \dfrac{Q}{e} = \dfrac{36}{1.60 \times 10^{-19}} = 2.25 \times 10^{20} \approx 2.3 \times 10^{20}$ electrons.	Each electron carries charge e, so divide the total charge by e.

Example 2 — scaffolded

When a car engine is started, its 12 V battery drives the starter motor; 150 C of charge passes through the motor while the engine turns over. (a) Calculate the energy transferred to the motor. (b) If turning over takes 2.0 s, calculate the average power delivered to the motor.

Working	Comment
$E = V\,Q = 12 \times 150 = 1800\,\text{J}$.	Rearrange $V = E/Q$ to $E = V\,Q$: each coulomb hands over 12 J.
$P = \dfrac{E}{t} = \dfrac{1800}{2.0} = 900\,\text{W}$.	Power is energy per second — a large power, which is why a starter runs only briefly.

Example 3 — unscaffolded

A student charges a phone from a 5.0 V USB power supply that delivers a current of 2.0 A. The charge takes 90 minutes. Calculate the power delivered to the phone and the total energy transferred in that time, then check the energy by a second method.

The power delivered to the phone is

$$P = V\,I = 5.0 \times 2.0 = 10\,\text{W}.$$

The time in seconds is $t = 90 \times 60 = 5400$ s, so the energy transferred is

$$E = P\,t = 10 \times 5400 = 54\,000\,\text{J} = 54\,\text{kJ}.$$

Check, using the charge: $Q = I\,t = 2.0 \times 5400 = 10\,800$ C, and therefore $E = V\,Q = 5.0 \times 10\,800 = 54\,000$ J, which agrees. ✓

∴ the power is 10 W and the energy transferred is 54 kJ.

Example 4 — unscaffolded

A bicycle light's battery pack is 6.0 V, and its globe carries a current of 0.50 A. The light is left on for 4.0 hours. Calculate the power of the globe, the charge through it and the energy transferred, and describe the energy transformations involved.

The power of the globe is

$$P = V\,I = 6.0 \times 0.50 = 3.0\,\text{W}.$$

The time is $t = 4.0 \times 3600 = 14\,400$ s, so the charge through the globe is

$$Q = I\,t = 0.50 \times 14\,400 = 7200\,\mathrm{C},$$

and the energy transferred is

$$E = V\,Q = 6.0 \times 7200 = 43\,200\,\mathrm{J} \approx 43\,\mathrm{kJ}.$$

(Equally, $E = P\,t = 3.0 \times 14\,400 = 43\,200\,\mathrm{J}$.) In the battery, chemical energy is transformed into electrical energy; in the globe, electrical energy is transformed into light and thermal energy.

∴ the globe's power is $3.0\,\mathrm{W}$, the charge is $7200\,\mathrm{C}$ and the energy transferred is $43\,200\,\mathrm{J}$.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working. Take $e = 1.60 \times 10^{-19}\,\mathrm{C}$ *where needed.*

Question 1 (a) Write the SI units of charge and of current, and state what a current of one ampere means in terms of coulombs and seconds. (b) Express a current of $250\,\mathrm{mA}$ in amperes. (c) A current of $0.80\,\mathrm{A}$ flows for $10\,\mathrm{s}$. Calculate the charge.

Question 2 A calculator operates on a current of $0.020\,\mathrm{A}$. It is switched on for 5.0 minutes. (a) Express this time in seconds. (b) Calculate the charge through the calculator. (c) Calculate the number of electrons that make up this charge.

Question 3 When $20\,\mathrm{C}$ of charge passes through a small electric motor, $60\,\mathrm{J}$ of energy is transferred. (a) Calculate the potential difference across the motor. (b) Calculate the energy transferred when $45\,\mathrm{C}$ of charge passes through it. (c) If the $20\,\mathrm{C}$ passes in $4.0\,\mathrm{s}$, calculate the power of the motor.

Question 4 An electric kettle in a Victorian home is connected to the $230\,\mathrm{V}$ supply and is rated at $2000\,\mathrm{W}$. (a) Calculate the energy transferred in 3.0 minutes. (b) Calculate the current in the kettle's element. (c) Calculate the charge through the element in that time.

Question 5 A car headlight globe operates from the car's $12\,\mathrm{V}$ battery and carries a current of $4.0\,\mathrm{A}$. The light is left on for 30 minutes with the engine off. (a) Calculate the power of the globe. (b) Calculate the energy transferred. (c) Calculate the charge through the globe.

Question 6 An LED lamp is rated "$9.2\,\mathrm{W}$, $230\,\mathrm{V}$". (a) Calculate the current in the lamp when it operates normally. (b) Calculate the charge through the lamp in 1.0 hour. (c) Calculate the energy transferred in that hour.

Question 7 A torch globe carries a current of $0.25\,\mathrm{A}$ for $40\,\mathrm{s}$. Calculate the charge through the globe and the number of electrons that make up this charge.

Question 8 A bar radiator in a Victorian home draws a current of $4.3\,\mathrm{A}$ from the $230\,\mathrm{V}$ supply. Calculate the power of the radiator and the energy it transfers in 2.0 hours.

Question 9 A mobile phone's lithium-ion cell has a potential difference of $3.7\,\mathrm{V}$ and delivers an average current of $1.5\,\mathrm{A}$ for 2.0 hours while playing music. Calculate the charge supplied and the energy transferred.

Question 10 Starting from $V = E / Q$ and $I = Q / t$, show that the power delivered to any part of a circuit is $P = V\,I$. State the definition of power you use.

Question 11 A $12\,\mathrm{V}$ cooling fan passes $24\,\mathrm{C}$ of charge in 2.0 minutes. Calculate the current in the fan, its power, and the energy transferred in that time.

Question 12 A $9.0\,\mathrm{V}$ battery drives a small radio with a current of $0.20\,\mathrm{A}$ for 10 minutes. Calculate the charge through the radio, the energy transferred, and the number of electrons that make up the charge.

Question 13 A student measures the current in the wire on each side of a globe in a single-loop circuit and obtains the same value at both points. Explain, with reference to what happens to the charge, why this must be so.

Question 14 Explain what is meant by the conventional current. In a metal wire, how does the direction of the conventional current compare with the direction of the electron drift, and why do the two differ?

Question 15 A single loop consists of a $6.0\,\mathrm{V}$ battery and one globe. Follow one coulomb of charge once around the loop and state the energy transfers that involve that coulomb at the battery, in the connecting wires and at the globe.

Question 16 A student claims: "the current is used up in the globe, so the current in the wire after the globe is smaller than before it". Evaluate this claim, making clear what is actually transferred from the charge to the globe.

Question 17 Distinguish between direct current and alternating current, giving one source of each. Explain why a battery can only deliver direct current.

Question 18 Two globes connected to the $230\,\mathrm{V}$ supply are rated at $60\,\mathrm{W}$ and $100\,\mathrm{W}$. (a) Which globe transfers energy at the greater rate? Justify your answer. (b) Both operate for the same time. Which transfers the greater energy? Justify your answer. (c) A student argues that the $100\,\mathrm{W}$ globe must carry the larger current. Use $P = V\,I$ to justify or refute the argument.

Answers

Question 1 (a) charge: coulomb, C; current: ampere, A; 1 A means 1 C of charge passes a point each second; (b) 0.25 A; (c) 8.0 C. **Question 2** (a) 300 s; (b) 6.0 C; (c) 3.8×10^{19}. **Question 3** (a) 3.0 V; (b) 135 J; (c) 15 W. **Question 4** (a) 360 kJ; (b) 8.7 A; (c) 1.6×10^{3} C. **Question 5** (a) 48 W; (b) 86 400 J (≈ 86 kJ); (c) 7200 C. **Question 6** (a) 0.040 A; (b) 144 C; (c) 33 120 J (≈ 33 kJ). **Question 7** 10 C; 6.3×10^{19} electrons. **Question 8** 989 W (≈ 0.99 kW); 7.1×10^{6} J. **Question 9** 1.1×10^{4} C (10 800 C); 4.0×10^{4} J (40 kJ). **Question 10** See solutions. **Question 11** 0.20 A; 2.4 W; 288 J. **Question 12** 120 C; 1080 J; 7.5×10^{20} electrons. **Question 13** See solutions (model answer). **Question 14** See solutions (model answer). **Question 15** See solutions (model answer). **Question 16** See solutions (model answer). **Question 17** See solutions (model answer). **Question 18** (a) the 100 W globe; (b) the 100 W globe; (c) the argument is correct — see solutions.

Fully-worked solutions

Question 1 (a) The unit of charge is the coulomb, C; the unit of current is the ampere, A. A current of 1 A means 1 C of charge passes any point in the circuit each second ($1\text{ A} = 1\text{ C s}^{-1}$). (b) $250\text{ mA} = 250 \times 10^{-3}\text{ A} = 0.25\text{ A}$. (c) $Q = I\,t = 0.80 \times 10 = 8.0\text{ C}$.

Question 2 (a) $t = 5.0 \times 60 = 300\text{ s}$. (b) $Q = I\,t = 0.020 \times 300 = 6.0\text{ C}$. (c) $n = \dfrac{Q}{e} = \dfrac{6.0}{1.60 \times 10^{-19}} = 3.75 \times 10^{19} \approx 3.8 \times 10^{19}$ electrons.

Question 3 (a) $V = \dfrac{E}{Q} = \dfrac{60}{20} = 3.0\text{ V}$. (b) $E = V\,Q = 3.0 \times 45 = 135\text{ J}$. (c) $P = \dfrac{E}{t} = \dfrac{60}{4.0} = 15\text{ W}$.

Question 4 (a) $t = 3.0 \times 60 = 180\text{ s}$; $E = P\,t = 2000 \times 180 = 360\,000\text{ J} = 360\text{ kJ}$. (b) $I = \dfrac{P}{V} = \dfrac{2000}{230} = 8.7\text{ A}$. (c) $Q = I\,t = 8.696 \times 180 = 1565\text{ C} \approx 1.6 \times 10^{3}\text{ C}$. (Equally, $Q = E / V = 360\,000 / 230$.)

Question 5 (a) $P = V\,I = 12 \times 4.0 = 48\text{ W}$. (b) $t = 30 \times 60 = 1800\text{ s}$; $E = P\,t = 48 \times 1800 = 86\,400\text{ J}$. (c) $Q = I\,t = 4.0 \times 1800 = 7200\text{ C}$.

Question 6 (a) $I = \dfrac{P}{V} = \dfrac{9.2}{230} = 0.040\text{ A}$. (b) $Q = I\,t = 0.040 \times 3600 = 144\text{ C}$. (c) $E = P\,t = 9.2 \times 3600 = 33\,120\text{ J}$.

Question 7 $Q = I\,t = 0.25 \times 40 = 10\text{ C}$; $n = \dfrac{Q}{e} = \dfrac{10}{1.60 \times 10^{-19}} = 6.25 \times 10^{19} \approx 6.3 \times 10^{19}$ electrons.

Question 8 $P = V\,I = 230 \times 4.3 = 989\text{ W} \approx 0.99\text{ kW}$. $t = 2.0 \times 3600 = 7200\text{ s}$; $E = P\,t = 989 \times 7200 = 7.12 \times 10^{6}\text{ J} \approx 7.1 \times 10^{6}\text{ J}$.

Question 9 $t = 2.0 \times 3600 = 7200\text{ s}$; $Q = I\,t = 1.5 \times 7200 = 10\,800\text{ C} \approx 1.1 \times 10^{4}\text{ C}$. $E = V\,Q = 3.7 \times 10\,800 = 39\,960\text{ J} \approx 4.0 \times 10^{4}\text{ J}$.

Question 10 Power is, by definition, the rate of energy transfer: $P = E / t$. From $V = E / Q$, the energy transferred is $E = V\,Q$. Substituting, $P = \dfrac{V\,Q}{t} = V\dfrac{Q}{t}$, and since $I = Q / t$, $P = V\,I$. ■

Question 11 $t = 2.0 \times 60 = 120\text{ s}$. $I = \dfrac{Q}{t} = \dfrac{24}{120} = 0.20\text{ A}$. $P = V\,I = 12 \times 0.20 = 2.4\text{ W}$. $E = P\,t = 2.4 \times 120 = 288\text{ J}$. (Check: $E = V\,Q = 12 \times 24 = 288\text{ J}$. ✓)

Question 12 $t = 10 \times 60 = 600\text{ s}$; $Q = I\,t = 0.20 \times 600 = 120\text{ C}$. $E = V\,Q = 9.0 \times 120 = 1080\text{ J}$. $n = \dfrac{120}{1.60 \times 10^{-19}} = 7.5 \times 10^{20}$ electrons.

Question 13 The loop is a closed path: charge cannot escape from it and cannot pile up inside the globe or the wires. Charge is conserved — it is neither created nor destroyed — so the rate at which charge enters the globe equals the rate at which charge leaves it. Current is exactly this rate of flow of charge, so the current is the same at every point of a single loop, which is what the two equal readings show. What the globe takes from the charge is energy (transformed to light and thermal energy), not charge: the charge arrives carrying energy and leaves with less of it, but the amount of charge flowing per second is unchanged.

Question 14 The conventional current is defined as the direction in which positive charge would flow: outside the source, from the positive terminal, through the circuit, to the negative terminal. In a metal wire the charge that actually drifts is the negative electrons, and they drift the opposite way — through the external circuit, from the negative terminal toward the positive one. The two differ because the convention was fixed long before the electron was discovered; it is kept because every calculation of current, potential difference, energy and power gives the same result under either picture.

Question 15 At the battery the coulomb receives $6.0\,\text{J}$ of energy, transformed from the battery's chemical store ($V = E / Q$: $6.0\,\text{V}$ means $6.0\,\text{J}$ per coulomb). In the connecting wires no energy is transferred in this simple model, so the coulomb carries its $6.0\,\text{J}$ unchanged. At the globe, across which the potential difference is $6.0\,\text{V}$, the coulomb hands over its $6.0\,\text{J}$ as light and thermal energy. It returns to the battery unchanged itself: the coulomb is the carrier of the energy, not the fuel.

Question 16 The claim is incorrect. Current is the rate of flow of charge, and charge is conserved: it is not used up, created or stored, so the current is the same in the wire before the globe, in the globe, and in the wire after it. What is transferred in the globe is *energy*: the potential difference across the globe is the energy handed over per coulomb of charge. The correct statement is that the charge enters the globe carrying energy and leaves with less of it, while the rate of flow of charge is unchanged. The claim confuses energy, which is transferred, with charge, which is merely the carrier.

Question 17 In a direct current the charge drifts in one direction the whole time; an example source is any battery, such as the cell in a torch. In an alternating current the direction of the drift reverses periodically; an example source is the household supply ($230\,\text{V}$, $50\,\text{Hz}$ in Australia). A battery can only deliver direct current because its terminals keep a fixed polarity — the chemical reactions inside the battery always push charge the same way through the external circuit, so the drift never reverses.

Question 18 (a) The $100\,\text{W}$ globe. Power is the rate of energy transfer, so it transforms $100\,\text{J}$ each second against $60\,\text{J}$ each second. (b) The $100\,\text{W}$ globe: $E = P\,t$, so for the same time t the greater power transfers the greater energy. (c) The argument is correct. Both globes have the same potential difference, $V = 230\,\text{V}$, and from $P = V\,I$, $I = P / V$: the $100\,\text{W}$ globe carries $I = 100 / 230 = 0.43\,\text{A}$ while the $60\,\text{W}$ globe carries $I = 60 / 230 = 0.26\,\text{A}$. With V the same, the greater power must go with the greater current, exactly as the student argued.

Theory

Electric current is invisible. The charge carriers inside a wire are far too small to see, and nobody can watch them move around a circuit. To reason about something that cannot be observed directly, physicists build **models**: simplified pictures that stand in for the real thing. An **analogy** is a model that explains something unfamiliar by comparing it with something familiar — water moving through pipes, a rope running in a loop, or delivery vans driving a round. This subtopic looks at several of those comparisons: what each one gets right, where each one breaks down, and how to tell the difference.

The study design itself introduces this area of study with that exact idea: *"Modelling is a useful tool in developing concepts that explain physical phenomena that cannot be directly observed"* (VCE Physics Study Design, Unit 1, Area of Study 3).

What every analogy must match. Section 5A established the quantities that any analogy has to reproduce:

- the **electric current** I is the rate of flow of charge, $I = \frac{Q}{t}$, measured in **amperes** (A), where $1\text{ A} = 1\text{ C s}^{-1}$;
- the **potential difference** V between two points is the energy transferred per unit charge, $V = \frac{E}{Q}$, measured in **volts** (V), where $1\text{ V} = 1\text{ J C}^{-1}$;
- a cell (or battery) transfers energy *to* the charges; components such as lamps, buzzers and heaters transfer energy *from* the charges to their surroundings.

Five facts about circuits are the test that every analogy must pass:

1. **The moving charges are already in the wires.** A cell does not create charge, and it does not store charge waiting to pour out. The wires are full of charge carriers before the cell is ever connected.
2. **Current is not used up.** Charge is conserved — it is neither created nor destroyed. In a single series loop the current has the same value at every point: the current into any component equals the current out of it.
3. **The cell does work on the charges.** The potential difference across a cell is the energy it gives to each coulomb of charge that passes through it.
4. **Components take energy from the charges.** The potential difference across a component is the energy each coulomb of charge gives up there.
5. **The effect starts everywhere at once.** When the loop is completed, charges begin drifting everywhere in the loop together, so a lamp lights immediately — even though any individual charge drifts only slowly.

One more distinction matters for every analogy: current is a flow *through* something (through a wire, through a component), while potential difference is measured *across* two points (across a cell, across a component).

Which way does the current go? In metal wires the moving charge carriers are **electrons**, which carry negative charge. Electrons drift from the negative terminal of the cell, around the loop, toward the positive terminal. By convention, the direction of an electric current is defined as the direction that positive charge would move: from the positive terminal, through the circuit, to the negative terminal. The two directions are opposite, but nothing in what follows depends on which one is chosen — what matters is that the flow is around a closed loop.

Analogy 1 — The water circuit. A pump drives water around a closed loop of pipe. The water turns a water wheel as it passes, and a valve can open or close the loop.

Water circuit	Electric circuit	What matches
Water	Charge carriers (electrons in a metal wire)	The stuff that flows

Water circuit	Electric circuit	What matches
Pump	Cell or battery	Keeps the flow going; gives energy to what flows
Pipes	Connecting wires	The path the flow follows
Flow rate (litres per second)	Current (coulombs per second)	How much passes one point each second
Pressure difference between two points	Potential difference between two points	The push that drives the flow
Water wheel	Lamp, motor or heater	A device that takes energy from the flow
Valve	Switch	Opens or closes the loop

What the water circuit gets right:

- Flow needs a difference. Water flows only where there is a pressure difference; charge flows only where there is a potential difference. No difference, no flow.
- The pump gives energy to the water, and the wheel takes energy from the water — yet the same water keeps flowing. The flow rate is the same everywhere around one loop: whatever water enters the wheel leaves it again. This matches facts 2 and 4.
- Close the valve anywhere and the flow stops everywhere at once, matching fact 5.
- A pump can maintain a pressure difference even when the valve is closed and nothing flows. This matches a useful truth: an open switch can have a potential difference across it even though the current is zero.

Where the water circuit breaks down:

- Water can leak or spill out of a pipe. Charge does not leak out of a wire: the charge that enters a component leaves it again.
- Water runs downhill under gravity, so pipes must slope. A circuit works in any position — the charges are driven by the electric field set up by the cell, not pulled downward by gravity.
- Cut a pipe and water pours out. Cut a wire and nothing spills: the charge stays in the wire and the flow simply stops.
- Water is visible; current is not.
- A pump-and-tank picture suggests the cell is a container full of charge waiting to flow out. That contradicts fact 1: the charge is already in every wire.

Analogy 2 — The rope loop. Imagine a loop of rope threaded around a course. A hand crank keeps the rope moving, and the rope passes through a tight tube that it rubs against and warms.

Rope loop	Electric circuit	What matches
The rope	Charge carriers	Already present everywhere in the loop
The turning crank	Cell or battery	Does work to keep the flow going
Rate the rope passes a point	Current	How much passes one point each second
Tight tube the rope rubs through	Lamp or other component	Where the flow’s energy is transferred (the tube warms by friction)
A clamp holding the rope still	Open switch	The rope (charge) is still there, but it is not moving

What the rope loop gets right:

- The rope already fills the whole loop before the crank turns. This matches fact 1 exactly: the charge is already in every wire; the crank does not create rope, and the cell does not create charge.
- When the crank starts turning, every part of the rope begins to move at once. This matches fact 5: close the switch and drifting begins everywhere together, so the lamp lights immediately even though any one charge drifts slowly.
- The same rope passes every point each second — the current is the same everywhere in a single series loop (fact 2).
- The crank does work on the rope, and the rope warms the tight tube: energy in, energy out, while the rope itself keeps circulating (facts 3 and 4).

Where the rope loop breaks down:

- A rope is one continuous object, but a current is a flow of many separate charge carriers.
- There is no clean way to say how much energy is given to each *unit* of rope, so the rope loop does not represent $V = \dfrac{E}{Q}$ — energy per unit charge — very well.
- One rope cannot branch, so parallel circuits cannot be shown at all.
- Friction warming a tube suits a heater, but it is a poor picture of a lamp giving out light.

Analogy 3 — The delivery vans. Vans leave a bakery loaded with loaves of bread, deliver them to shops around a route, and return empty to be loaded again.

Delivery round	Electric circuit	What matches
Bakery	Cell or battery	Where the carriers pick up energy
Shops	Lamp or other component	Where the carriers drop off energy
One van	A fixed parcel of charge, say one coulomb	The carrier that circulates
Loaves of bread	Parcels of energy (joules)	What is picked up and delivered
Vans passing a point each hour	Current	The rate of flow

What the delivery round gets right:

- It represents $V = \dfrac{E}{Q}$ directly. The number of loaves each van picks up at the bakery is the energy each coulomb receives at the cell: a $12\,\mathrm{V}$ battery loads each coulomb with $12\,\mathrm{J}$, just as a bakery loads twelve loaves per van.
- The vans are not used up. They keep circulating, which matches fact 2: charge is not consumed.
- Every shop receives loaves: every component receives energy from the charges passing through it (fact 4), while the vans themselves come back to be reloaded (fact 3).
- More vans per hour means a bigger current; more loaves per van means a bigger potential difference. The two quantities stay separate, as they must.

Where the delivery round breaks down:

- Vans take time to drive from the bakery to the shops. This wrongly suggests a delay between closing a switch and a lamp lighting. In a real circuit the energy transfer starts everywhere at once — there is no waiting for the first van to arrive (fact 5).
- Loaves ride *inside* the vans, which suggests energy is carried inside the charge like luggage. In a circuit the energy is transferred by the electric field, not carried within the charges.
- Roads branch and vans choose routes; a series loop offers no choices.
- The analogy has no good picture of the push — potential difference as a *difference* that drives the flow. The vans simply drive.

Other analogies. Two more are worth knowing, if only to practise the same evaluation. A **hydronic heating system** — a boiler warms water, a pump circulates it through pipes to radiators, and cooler water returns to be warmed again — is good at showing that the carrier circulates and is never used up while energy is delivered from one source to many places; but it wrongly suggests that charges are "hot" leaving the cell and "cool" returning to it, and it splits the pump and the boiler into two devices where a single cell does both jobs. A **ski-lift** picture — the lift does work raising skiers, who then ski downhill through a village giving up their energy — captures the idea of height difference standing in for potential difference, but it depends on gravity, which is exactly what a circuit does not depend on.

How to evaluate an analogy. Every analogy is a model, and models are tools, not the real thing. To evaluate one, ask three questions:

1. **What does it get right?** Which parts of the circuit does it match, and which of the five facts above does it reproduce?
2. **Where does it break down?** What does the analogy predict that a real circuit does not do? A prediction that fails is where the analogy stops being useful.
3. **What can it not show at all?** Every analogy leaves something out — parallel branches, the direction of electron drift, the speed of the effect.

Knowing where a model fails is part of understanding it. A student who can name an analogy's limitations understands the circuit better than one who can only repeat the analogy.

Three common wrong ideas, and their corrections:

Wrong idea	What is wrong with it	The correction
"Current is used up as it passes through a component."	Treats current as a substance that can be spent.	Charge is conserved: the current into a component equals the current out of it. It is *energy* that is transferred, not charge.
"A battery is a store of charge."	Contradicts fact 1.	A battery stores chemical energy and transfers it to charges that are already in the wires. A flat battery has not run out of charge.
"The current reaches the first lamp first, so it lights first."	Assumes charges must travel from the cell to the lamp before anything happens.	Drifting begins everywhere in the loop at once when the switch closes (fact 5), so lamps in series light together.

From the next subtopic onward, the book works with circuit diagrams and measured values directly — meters in 5C, and resistance and I–V graphs from 5D. The analogies remain available as a check on your own thinking: whenever a circuit result feels surprising, ask which analogy predicts it, and where that analogy breaks down.

Worked examples

Example 1 — scaffolded

On a market garden, a pump pushes irrigation water through a closed loop of pipe. A flow meter records $240\,\mathrm{L}$ of water passing one point in $2.0\,\mathrm{min}$. In the electric circuit that is analogous to this water loop, $30\,\mathrm{C}$ of charge pass one point in the same time. (a) Calculate the water flow rate in litres per second. (b) Calculate the current in the corresponding circuit. (c) Copy and complete the mapping: litres correspond to …; the flow meter corresponds to …; the pump corresponds to …; the pressure difference corresponds to …

Working	Comment
Data: $240\,\mathrm{L}$ in $2.0\,\mathrm{min} = 120\,\mathrm{s}$; $30\,\mathrm{C}$ in the same $120\,\mathrm{s}$.	Convert minutes to seconds; compare the two systems over the same time interval.
(a) Flow rate $= \dfrac{240}{120} = 2.0\,\mathrm{L\,s^{-1}}$.	Litres per second is the rate of flow of water.

Working	Comment
(b) $I=\frac{Q}{t}=\frac{30}{120}=0.25\,\mathrm{A}$.	Current is the rate of flow of charge; coulombs per second are amperes.
(c) litres ↔ coulombs; flow meter ↔ ammeter; pump ↔ cell; pressure difference ↔ potential difference.	Litres measure water as coulombs measure charge; a flow meter measures flow as an ammeter measures current; the pump drives the water as the cell drives the charge.

Example 2 — scaffolded

Use the delivery-van analogy. A torch cell has a potential difference of $3.0\,\mathrm{V}$, one van represents $1\,\mathrm{C}$ of charge, and one loaf represents $1\,\mathrm{J}$ of energy. (a) State the energy given to each coulomb that passes through the cell, and translate your answer into the analogy. (b) Calculate the energy transferred to $12\,\mathrm{C}$ of charge passing through the cell. (c) If each van instead represents $2.0\,\mathrm{C}$ of charge, calculate the energy each van carries away from the bakery.

Working	Comment
(a) From $V=\frac{E}{Q}$, each coulomb receives $3.0\,\mathrm{J}$. In the analogy: each van picks up 3 loaves.	Potential difference is the energy transferred per unit charge.
(b) $E=V\,Q=3.0\times 12=36\,\mathrm{J}$.	Rearrange $V=\frac{E}{Q}$ to $E=V\,Q$.
(c) $E=3.0\times 2.0=6.0\,\mathrm{J}$, so each van carries 6 loaves.	Energy is proportional to charge: doubling the charge per van doubles the loaves per van.

Example 3 — unscaffolded

A series circuit contains a cell, a switch and two identical lamps. An ammeter A_1 placed in the loop before the first lamp reads $0.40\,\mathrm{A}$. A student claims: *"the current is used up by the first lamp, so less current reaches the second one."* (a) State the reading of a second ammeter A_2 placed after the second lamp, and justify your answer. (b) Calculate the charge that passes through the first lamp in $30\,\mathrm{s}$, and the charge that leaves it in the same time. (c) Identify the error in the student's claim and give the correct explanation, including why two identical lamps in series are dimmer than a single lamp would be.

(a) A_2 reads $0.40\,\mathrm{A}$. Charge is conserved — it is not created, destroyed or used up — so in a single series loop the current has the same value at every point. Whatever charge flows into the lamps flows out of them.

(b) From $I=\frac{Q}{t}$ rearranged to $Q=I\,t$:

$$Q=0.40\times 30=12\,\mathrm{C}$$

passing into the first lamp, and $12\,\mathrm{C}$ leaving it in the same $30\,\mathrm{s}$.

(c) The error is treating current as a substance that can be spent. The lamps transfer *energy* from the charges, not charge itself: each coulomb gives some of its energy to the first lamp and the remainder to the second. With two lamps in series, the energy supplied by the cell is shared between them, so each lamp transfers less energy per second than a single lamp would — that is why each is dimmer.

∴ A_2 reads $0.40\,\mathrm{A}$; $12\,\mathrm{C}$ pass in and $12\,\mathrm{C}$ pass out; the claim confuses energy (which is transferred) with charge (which is conserved).

Example 4 — unscaffolded

A $9.0\,\mathrm{V}$ battery powers a small motor. The current in the motor is $0.30\,\mathrm{A}$ and the motor runs for $20\,\mathrm{s}$. (a) Calculate the charge that passes through the motor. (b) Calculate the energy the battery transfers to that charge. (c) In the rope-loop analogy, state what corresponds to (i) the charge found in (a), (ii) the energy found in (b), and (iii) an open switch, and give one limitation of the rope-loop analogy that your answer to (iii) suggests.

The charge follows from $I = \frac{Q}{t}$ rearranged to $Q = I\,t$:

$$Q = 0.30 \times 20 = 6.0\,\mathrm{C}.$$

The energy follows from $V = \frac{E}{Q}$ rearranged to $E = V\,Q$:

$$E = 9.0 \times 6.0 = 54\,\mathrm{J}.$$

In the rope-loop analogy: (i) the $6.0\,\mathrm{C}$ of charge corresponds to the length of rope that passes a point in the $20\,\mathrm{s}$; (ii) the $54\,\mathrm{J}$ corresponds to the work done by the crank on the rope; (iii) an open switch corresponds to a clamp holding the rope still — the rope is present, but it is not moving, just as the charges remain in the wires when the switch is open. One limitation this reveals: a clamp is a mechanical block at one point, but opening a real switch stops drifting everywhere in the loop at once, not just at the switch.

$\therefore$ $Q = 6.0\,\mathrm{C}$, $E = 54\,\mathrm{J}$; mapping and limitation as above.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working.

Question 1 An irrigation channel carries water to a farm. In $5.0\,\mathrm{min}$, $1800\,\mathrm{L}$ of water pass one point in the channel. (a) Calculate the flow rate of the water in litres per second. (b) In a circuit analogous to this channel, $90\,\mathrm{C}$ of charge pass one point in the same $5.0\,\mathrm{min}$. Calculate the current in the circuit. (c) State which electric-circuit quantity each of the following corresponds to in this analogy: the pump; the pressure difference between two points in the channel; the litre.

Question 2 A car battery has a potential difference of $12\,\mathrm{V}$. In the delivery-van analogy, one van represents $1\,\mathrm{C}$ of charge and one loaf represents $1\,\mathrm{J}$ of energy. (a) State the number of loaves each van picks up at the bakery. (b) Calculate the energy transferred to $5.0\,\mathrm{C}$ of charge passing through the battery. (c) While the engine is being started, $200\,\mathrm{C}$ of charge pass through the battery. Calculate the energy transferred to that charge.

Question 3 A cell, an ammeter A_1, a lamp and an ammeter A_2 are connected in series. A_1 reads $0.50\,\mathrm{A}$. (a) State the reading of A_2, justifying your answer. (b) Calculate the charge that passes through the lamp in $2.0\,\mathrm{min}$. (c) In the water-circuit analogy the lamp corresponds to a water wheel. State whether the flow rate of the water changes as it passes through the wheel, and use your answer to explain the reading of A_2.

Question 4 In a rope-loop model of a circuit, $2.0\,\mathrm{m}$ of rope pass one point in $5.0\,\mathrm{s}$ while the crank turns. (a) Calculate the rate at which the rope passes that point. (b) State what corresponds to each of the following in the rope model: the cell; a lamp; an open switch. (c) Give one limitation of the rope-loop analogy.

Question 5 A $4.5\,\mathrm{V}$ battery powers a buzzer, and the full potential difference of the battery is across the buzzer. A charge of $8.0\,\mathrm{C}$ passes through the buzzer in $10\,\mathrm{s}$. (a) Calculate the current in the buzzer. (b) State the energy transferred from each coulomb of charge as it passes through the buzzer. (c) Calculate the total energy transferred to the buzzer in the $10\,\mathrm{s}$.

Question 6 A student writes about the water-circuit analogy: *"the pump makes the water flow, just as a battery makes the charge. The battery is full of charge, waiting to flow out."* (a) Identify the incorrect idea in the student's statement. (b) Rewrite the second sentence correctly. (c) State one thing the water-circuit analogy gets right about potential difference.

Question 7 A battery has a potential difference of $6.0\,\mathrm{V}$. (a) State the energy the battery gives to each coulomb of charge that passes through it. (b) A charge of $15\,\mathrm{C}$ passes through the battery. Calculate the energy transferred to that charge.

Question 8 The lamp in a torch carries a current of $0.25\,\mathrm{A}$ when the torch is on. (a) Calculate the charge that passes through the lamp in $10\,\mathrm{min}$. (b) The cells provide a combined potential difference of $3.0\,\mathrm{V}$. Calculate the energy transferred to that charge by the cells.

Question 9 In a water model of a circuit, a pump keeps water flowing around a loop of pipe, and $500\,\mathrm{L}$ of water pass one point in the loop in $4.0\,\mathrm{min}$. (a) Calculate the flow rate of the water in litres per second. (b) The analogous electric circuit carries a current of $0.50\,\mathrm{A}$. Calculate the charge that passes one point in the circuit in $4.0\,\mathrm{min}$. (c) Name the circuit device that corresponds to the water flow meter.

Question 10 A $12\,\mathrm{V}$ car battery transfers $600\,\mathrm{J}$ of energy to the charges as an engine is started. (a) Calculate the charge that passes through the battery during starting. (b) Starting takes $5.0\,\mathrm{s}$. Calculate the average current during starting.

Question 11 Two lamps are connected in series with a cell and a switch. With the switch closed, the current at a point between the cell and the first lamp is $0.35\,\mathrm{A}$. (a) State the current between the two lamps and the current after the second lamp, justifying your answers. (b) Calculate the time taken for $20\,\mathrm{C}$ of charge to pass through the first lamp.

Question 12 In a circuit, $48\,\mathrm{C}$ of charge pass one point in $4.0\,\mathrm{min}$. (a) Calculate the current. (b) The potential difference across the component is $12\,\mathrm{V}$. Calculate the energy transferred to the component as the $48\,\mathrm{C}$ pass through it.

Question 13 A student says: *"the current into a lamp is greater than the current out of the lamp, because some of the current is converted into light and thermal energy."* Evaluate this claim. In your answer, identify what is wrong, state the correct relationship between the two currents, and explain what actually changes as charge passes through the lamp.

Question 14 The rope-loop analogy pictures a loop of rope already threaded around a course before the crank turns. (a) Explain how this analogy accounts for a lamp lighting as soon as the switch is closed, even though each individual charge drifts only slowly. (b) State one limitation of the rope-loop analogy.

Question 15 Compare the water-circuit analogy with the delivery-van analogy. Justify which of the two better represents the definition of potential difference, $V = \frac{E}{Q}$.

Question 16 A hydronic heating system warms a house: a boiler heats water, a pump circulates it through pipes to radiators in each room, and cooler water returns to the boiler. (a) Match each of the following to its electric-circuit counterpart: the boiler and pump together; the pipes; the circulating water; a radiator; the cooler returning water. (b) Give one strength and one limitation of this analogy.

Question 17 Two students discuss a flat battery. Student A says: *"the battery has run out of charge."* Student B says: *"the battery has run out of energy."* Decide which student is correct, justifying your answer in terms of charges and energy.

Question 18 A teacher needs an analogy for each of two lessons. Lesson 1 teaches that the current is the same everywhere in a series loop; lesson 2 teaches that potential difference is the energy transferred per unit charge. (a) Recommend one analogy from this subtopic for each lesson, justifying each choice. (b) For each analogy you chose, state one limitation the teacher should mention.

Answers

Question 1 (a) $6.0\,\mathrm{L\ s^{-1}}$; (b) $0.30\,\mathrm{A}$; (c) pump ↔ cell; pressure difference ↔ potential difference; litre ↔ coulomb. **Question 2** (a) 12 loaves; (b) $60\,\mathrm{J}$; (c) $2400\,\mathrm{J}$. **Question 3** (a) $0.50\,\mathrm{A}$; (b) $60\,\mathrm{C}$; (c) no — the wheel takes energy from the water, not water, so the flow rate is unchanged before and after it. **Question 4** (a) $0.40\,\mathrm{m\ s^{-1}}$; (b) crank ↔ cell; tight tube the rope rubs through ↔ lamp; clamp holding the rope still ↔ open switch; (c) e.g. a rope is one object, but a current is a flow of many separate charges. **Question 5** (a) $0.80\,\mathrm{A}$; (b) $4.5\,\mathrm{J}$; (c) $36\,\mathrm{J}$. **Question 6** (a) the battery does not make or store charge as its supply; (b) the charges are already in every wire — the battery transfers energy to them; (c) e.g. a flow needs a difference: no pressure difference, no water flow, just as no potential difference means no current. **Question 7** (a) $6.0\,\mathrm{J}$; (b) $90\,\mathrm{J}$. **Question 8** (a) $150\,\mathrm{C}$; (b) $450\,\mathrm{J}$. **Question 9** (a) $2.1\,\mathrm{L\ s^{-1}}$; (b) $120\,\mathrm{C}$; (c) ammeter. **Question 10** (a) $50\,\mathrm{C}$; (b) $10\,\mathrm{A}$. **Question 11** (a) $0.35\,\mathrm{A}$ at both points; (b) $57\,\mathrm{s}$. **Question 12** (a) $0.20\,\mathrm{A}$; (b) $576\,\mathrm{J}$. **Question 13** The claim is incorrect: current in equals current out; it is energy, not charge, that is transferred in the lamp. **Question 14** (a) Charge is already in every wire, and drifting begins everywhere in the loop at once when the switch closes; (b) e.g. a single rope cannot show parallel branches. **Question 15** The delivery-van analogy: loaves per van maps directly onto joules per coulomb. **Question 16** Boiler and pump ↔ cell; pipes ↔ wires; circulating water ↔ charge; radiator ↔ lamp; cooler returning water ↔ charges after giving up energy. Strength: the carrier is never used up. Limitation: it suggests charges are hotter after the cell. **Question 17** Student B: a flat battery can no longer transfer energy to the charges; the charges themselves are still present. **Question 18** Lesson 1: the rope loop; lesson 2: the delivery vans.

Fully-worked solutions

Question 1 $1800\,\mathrm{L}$ in $5.0\,\mathrm{min} = 300\,\mathrm{s}$; $90\,\mathrm{C}$ in the same $300\,\mathrm{s}$. (a) Flow rate $= \frac{1800}{300} = 6.0\,\mathrm{L\ s^{-1}}$. (b) $I = \frac{Q}{t} = \frac{90}{300} = 0.30\,\mathrm{A}$. (c) The pump corresponds to the **cell** (it keeps the flow going and gives energy to what flows); the pressure difference between two points corresponds to the **potential difference** (the push that drives the flow); the litre corresponds to the **coulomb** (the unit used to measure how much of the flowing stuff there is).

Question 2 $V = 12\,\mathrm{V}$; one van $= 1\,\mathrm{C}$; one loaf $= 1\,\mathrm{J}$. (a) From $V = \frac{E}{Q}$, each coulomb receives $12\,\mathrm{J}$, so each van picks up **12 loaves**. (b) $E = VQ = 12 \times 5.0 = 60\,\mathrm{J}$. (c) $E = VQ = 12 \times 200 = 2400\,\mathrm{J}$.

Question 3 Series circuit; A_1 reads $0.50\,\mathrm{A}$; $t = 2.0\,\mathrm{min} = 120\,\mathrm{s}$. (a) A_2 reads $0.50\,\mathrm{A}$. Charge is conserved — it is not created, destroyed or used up — so in a single series loop the current has the same value at every point: what flows into the lamp flows out of it. (b) $Q = It = 0.50 \times 120 = 60\,\mathrm{C}$. (c) The flow rate of the water does **not** change as it passes through the wheel. The wheel takes *energy* from the water, not water: the same water that enters the wheel leaves it, still flowing at the same rate. The lamp behaves the same way — it transfers energy from the charges, but the charge itself is conserved, so the current after the lamp (A_2) equals the current before it (A_1).

Question 4 $2.0\,\mathrm{m}$ of rope pass one point in $5.0\,\mathrm{s}$. (a) Rate $= \frac{2.0}{5.0} = 0.40\,\mathrm{m\ s^{-1}}$. This rope speed corresponds to the current in the circuit analogy. (b) The cell corresponds to the **crank** (it does the work that keeps the flow going); a lamp corresponds to the **tight tube the rope rubs through** (where the flow's energy is transferred — the tube warms); an open switch corresponds to a **clamp holding the rope still** (the rope, like the charge, is still present, but it is not moving). (c) Any one of: a rope is one continuous object, while a current is a flow of many separate charge carriers; the rope model gives no clean way to represent energy per unit charge ($V = \frac{E}{Q}$); one rope cannot branch, so parallel circuits cannot be shown; friction warming a tube is a poor picture of a lamp giving out light.

Question 5 $V = 4.5\,\mathrm{V}$ (all across the buzzer); $Q = 8.0\,\mathrm{C}$; $t = 10\,\mathrm{s}$. (a) $I = \frac{Q}{t} = \frac{8.0}{10} = 0.80\,\mathrm{A}$. (b) From $V = \frac{E}{Q}$, the energy transferred from each coulomb is $4.5\,\mathrm{J}$. (In the delivery analogy: each van drops off 4.5 loaves at the shop.) (c) $E = VQ = 4.5 \times 8.0 = 36\,\mathrm{J}$.

Question 6 (a) The incorrect idea is that a battery *makes* charge or holds a store of it waiting to flow out. This contradicts the fact that the moving charges are already present in every wire before the battery is connected. (b) A correct version: *"The battery transfers energy to the charges that are already in the wires, and keeps them moving around the loop."* (c) Any one of: a flow happens only while there is a difference — a pressure difference for water, a potential difference for charge; the pump maintains the pressure difference, as the cell maintains the potential difference; with the valve closed (switch open) the pressure difference can still exist even though nothing flows.

Question 7 $V = 6.0\,\text{V}$. (a) From $V = \dfrac{E}{Q}$, the battery gives $6.0\,\text{J}$ to each coulomb. (b) $E = V\,Q = 6.0 \times 15 = 90\,\text{J}$.

Question 8 $I = 0.25\,\text{A}$; $t = 10\,\text{min} = 600\,\text{s}$; $V = 3.0\,\text{V}$. (a) $Q = I\,t = 0.25 \times 600 = 150\,\text{C}$. (b) $E = V\,Q = 3.0 \times 150 = 450\,\text{J}$.

Question 9 $500\,\text{L}$ in $4.0\,\text{min} = 240\,\text{s}$; circuit current $0.50\,\text{A}$. (a) Flow rate $= \dfrac{500}{240} = 2.08\ldots = 2.1\,\text{L s}^{-1}$. (b) $Q = I\,t = 0.50 \times 240 = 120\,\text{C}$. (c) The flow meter corresponds to an **ammeter** — each measures how much of the flow passes one point each second.

Question 10 $V = 12\,\text{V}$; $E = 600\,\text{J}$; $t = 5.0\,\text{s}$. (a) From $V = \dfrac{E}{Q}$, $Q = \dfrac{E}{V} = \dfrac{600}{12} = 50\,\text{C}$. (b) $I = \dfrac{Q}{t} = \dfrac{50}{5.0} = 10\,\text{A}$.

Question 11 Series circuit; current $0.35\,\text{A}$ between the cell and the first lamp. (a) The current between the two lamps is $0.35\,\text{A}$ and the current after the second lamp is $0.35\,\text{A}$. Charge is conserved, so in a single series loop the current has the same value at every point — no charge is lost or used up at either lamp. (b) $t = \dfrac{Q}{I} = \dfrac{20}{0.35} = 57\,\text{s}$.

Question 12 $Q = 48\,\text{C}$; $t = 4.0\,\text{min} = 240\,\text{s}$; $V = 12\,\text{V}$. (a) $I = \dfrac{Q}{t} = \dfrac{48}{240} = 0.20\,\text{A}$. (b) $E = V\,Q = 12 \times 48 = 576\,\text{J}$.

Question 13 The claim is incorrect. A full-mark answer would include:

- the error: the student treats current as a substance that can be converted into other forms, but current is a rate of flow of charge, and charge is conserved;
- the correct relationship: the current into the lamp equals the current out of it;
- what actually changes: the *energy* of the charges. Each coulomb gives up energy as it passes through the lamp ($E = V\,Q$), and that energy is transferred to the surroundings as light and thermal energy;
- analogy support (for example, water circuit): the flow rate of water through a water wheel is the same on both sides — the wheel takes energy from the water, not water.

Question 14 (a) A full-mark answer would include:

- the rope is already threaded everywhere around the loop before the crank turns, as charge is already present in every wire before the switch closes;
- when the crank starts turning, every part of the rope begins moving at once;
- likewise, when the switch closes, drifting begins everywhere in the loop at once, so the lamp starts lighting immediately;
- the slow drift of any individual charge (like the slow motion of one marked point on the rope) does not delay the start of the flow.

(b) Any one of: a rope is one continuous object rather than many separate charge carriers; the rope loop cannot represent parallel branches; the rope model gives no clean picture of energy per unit charge; friction warming a tube is a poor picture of light from a lamp.

Question 15 The delivery-van analogy is the better representation of $V = \dfrac{E}{Q}$. A full-mark answer would include:

- the delivery analogy counts carriers and energy separately: each van stands for a fixed parcel of charge (a coulomb) and each loaf for a fixed parcel of energy (a joule), so "loaves per van" maps directly onto "joules per coulomb" — which is exactly what potential difference is;
- a $12\,\text{V}$ battery is simply "12 loaves per van", so the analogy makes the definition countable;
- the water circuit's "stuff" is continuous, not divided into equal carriers, so "energy per unit charge" cannot be counted in it; its pressure difference is better read as the *push* driving the flow than as an amount of energy per coulomb;
- conclusion: for teaching $V = \frac{E}{Q}$, use the delivery vans; the water circuit is better for the idea that a difference is needed to drive a flow.

Question 16 (a) The boiler and pump together $\leftrightarrow$ the **cell** (source of energy and the driver of the flow — in a circuit one device does both jobs); the pipes $\leftrightarrow$ the **connecting wires**; the circulating water $\leftrightarrow$ the **charge carriers**; a radiator $\leftrightarrow$ a **lamp or other component** (where energy is transferred to the surroundings); the cooler returning water $\leftrightarrow$ the **charges after they have given up their energy**, returning to the cell to receive energy again. (b) Strength (any one): the carrier circulates and is never used up — energy is delivered from one source to several places while the water keeps returning, which matches charge conservation; the picture of energy being carried around a loop to where it is needed is a good one. Limitation (any one): it suggests the charges are "hot" leaving the cell and "cool" returning — charges are not hotter on one side of the cell than the other; the system splits the pump (driver of flow) and the boiler (energy source) into two devices, where a single cell does both jobs.

Question 17 Student B is correct. A full-mark answer would include:

- the moving charges are already in the wires and terminals; a battery does not supply charge, it transfers energy to charge;
- a cell transfers chemical energy to the charges passing through it — its potential difference is the energy it gives to each coulomb;
- a flat battery can no longer transfer enough energy to each coulomb (its potential difference falls), but the charges are still present in the circuit;
- analogy: in the delivery round, a closed bakery means the vans are still on the roads but no loaves are being loaded — it has run out of loaves (energy), not vans (charge).

Question 18 (a) Lesson 1 (current the same everywhere in a series loop): the **rope-loop analogy** — the same rope passes every point each second, and the rope already fills the loop, so it makes both conservation of charge and the equal current vivid. The water circuit also works (the same water passes every point of one loop), but the rope makes the "already everywhere" point more strongly. Lesson 2 (potential difference as energy per unit charge): the **delivery-van analogy** — loaves per van maps directly onto joules per coulomb, making $V = \frac{E}{Q}$ countable. (b) For the rope loop (any one): one rope cannot branch, so parallel circuits cannot be shown; a rope is one object rather than many separate carriers; there is no clean picture of energy per unit charge. For the delivery vans (any one): vans take time to travel from bakery to shop, wrongly suggesting a delay between closing the switch and the lamp lighting; loaves riding inside the vans suggest energy is carried inside the charges, whereas in a circuit the energy is transferred by the electric field; the analogy has no good picture of the push that drives the flow.

Theory

In 5A you met the quantities used to describe electric circuits: charge (Q), electric current (I), potential difference (V), energy (E) and power (P). These quantities cannot be seen, so they are measured with instruments called **meters**. This subtopic looks at three meters — the ammeter, the voltmeter and the multimeter — and at how to *justify* the choice and connection of each one. It then introduces the **kilowatt-hour**, the unit of energy in which household electricity is measured and billed.

What every meter must do. A meter is connected to a circuit, takes a reading, and displays it. But a meter can only work by taking a little energy from the circuit it is measuring. A well designed meter takes so little energy that the circuit behaves almost exactly as it would with the meter absent. The better the meter, the less it disturbs the circuit — and the more the reading can be trusted. Each meter below is described two ways: as an **ideal** meter (zero disturbance, the model used in this course) and as a real meter that gets close to that ideal.

The ammeter — measuring current. Electric current is the rate of flow of charge ($I = Q / t$, from 5A), measured in **amperes** (A). An **ammeter** measures the current *through* a component. Since current is charge passing each second, the ammeter must sit in the path that the charge takes: one connection of the circuit is broken and the ammeter is inserted into the gap, so that every coulomb passing through the component also passes through the meter. Components connected in a single path so that the same current passes through each of them are said to be connected **in series**. An ammeter is always connected **in series** with the component whose current is to be measured. In circuit diagrams an ammeter is drawn as a circle containing the letter A.

Resistance is a measure of how strongly a component opposes the current through it; it is treated in full in 5D, and here it is needed only to compare meters. An ideal ammeter has **zero resistance**: it does not oppose the current at all, adding nothing to the circuit and leaving the current exactly as it was. Real ammeters are built with very low resistance for the same reason. This is the justification for the design: an ammeter with a noticeable resistance would reduce the very current it is trying to measure, and the reading would be lower than the true value.

Connecting an ammeter *across* a supply (in parallel, the way a voltmeter goes) is a mistake. Because the ammeter's resistance is so low, almost nothing limits the current around that loop, so the current becomes very large. The meter overheats or its internal fuse melts (blows). School meters are protected by fuses for exactly this reason — but the habit to build is: check the connection before closing the switch.

The voltmeter — measuring potential difference. Potential difference is the energy transferred per unit charge as charge passes through a component ($V = E / Q$, from 5A), measured in **volts** (V). A **voltmeter** measures the potential difference *across* a component, so it is connected across the component, touching the circuit at two points — one on each side of the component. Components connected across the same two points, giving the charge separate paths to follow, are said to be connected **in parallel**. A voltmeter is always connected **in parallel** with the component whose potential difference is to be measured. In circuit diagrams a voltmeter is drawn as a circle containing the letter V. (Series and parallel circuits are treated in full in 5D; here you only need the connection idea for the meters.)

An ideal voltmeter has **infinite resistance**: it draws no current at all from the circuit. Real voltmeters have very high resistance — often millions of ohms. The justification is the mirror image of the ammeter's: a voltmeter with low resistance would provide an extra path and draw current through itself, changing the current in the component and so changing the potential difference it is trying to measure. A high-resistance voltmeter leaves the circuit almost undisturbed, so its reading can be trusted.

The multimeter. A **multimeter** is a single instrument that can be switched to work as an ammeter, a voltmeter or an **ohmmeter** (a meter that measures resistance — the quantity covered in 5D). The user selects the function on a dial, and the leads plug into the terminals marked for that function. The same physical meter therefore has a very low resistance when it is set to measure current, and a very high resistance when it is set to measure potential difference.

Two practices follow from this and are worth justifying. First, select the function *before* connecting the meter: a multimeter left set to measure current, but connected across a supply like a voltmeter, behaves like an ammeter across a supply — a very large current flows and the fuse blows. Second, on a meter with manual ranges, start on the highest range and step down until the reading is clear: this protects the meter from a current or voltage larger than expected, and the lowest range that still works gives the most precise reading. (An auto-ranging meter chooses the range itself.) When measuring the resistance of a component with the ohmmeter function, the component must be disconnected from the circuit, because the ohmmeter pushes its own small current through the component to take the reading.

For a DC circuit, connect the red lead (marked +) toward the positive side of the supply and the black lead (COM) toward the negative side. If the leads are reversed, an analogue needle tries to move backwards, and a digital meter displays a negative value, such as $-0.30\,\mathrm{A}$.

Justifying a meter choice. To justify the use of a meter in a circuit, state three things: the quantity being measured (which fixes which meter), the connection (series or parallel, which follows from what the quantity means), and the resistance property that keeps the reading trustworthy. The pattern is summarised below.

Meter	Quantity measured	Unit	Connection	Ideal resistance
Ammeter	current through a component	ampere (A)	in series with the component	zero
Voltmeter	potential difference across a component	volt (V)	in parallel across the component	infinite
Multimeter	current, potential difference or resistance, as selected	A, V or ohm	as above, once the function is selected	very low (current mode) or very high (voltage mode)

Practical note. In class, meter work is done on low-voltage DC circuits — batteries or a lab power pack. Choose the function and range before connecting, have the teacher check an unfamiliar circuit before the switch is closed, and record each reading, with its unit and the meter setting used, in your logbook. The household supply is $230\,\mathrm{V}$ AC at $50\,\mathrm{Hz}$ and is dangerous: meters are never connected to household wiring or to the inside of appliances in class. This matches the study design's rule that students are not permitted to carry out electrical work on the internal components of equipment operating above $50\,\mathrm{V}$ AC or $120\,\mathrm{V}$ ripple-free DC. A household electricity meter is read from the outside; it is never opened.

The kilowatt-hour. Power is the rate of energy transfer ($P = E/t$, from 5A), measured in watts: $1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$. The joule is a small unit for household purposes. A $1\,\mathrm{kW}$ appliance running for one hour transfers $3.6 \times 10^6\,\mathrm{J}$, so household energy use is measured in **kilowatt-hours** instead. One **kilowatt-hour** ($1\,\mathrm{kW\,h}$) is the energy transferred by a device of power $1\,\mathrm{kW}$ operating for $1\,\mathrm{hour}$.

Using $E = Pt$ with the power in kilowatts and the time in hours gives the energy directly in kilowatt-hours:

$$E = Pt\,(E \text{ in kW h}, P \text{ in kW}, t \text{ in h}).$$

The kilowatt-hour is a perfectly ordinary unit of energy, and it converts to joules through the definition of the watt and the hour:

$$1\,\mathrm{kW\,h} = 1000\,\mathrm{W} \times 3600\,\mathrm{s} = 3.6 \times 10^6\,\mathrm{J}.$$

This equivalence is printed on the formula sheet. To convert kW h to J, multiply by 3.6×10^6; to convert J to kW h, divide by 3.6×10^6.

Do not be misled by the name: the **kilowatt** is the unit of power in the name, and the **hour** is a time — the kilowatt-hour is power multiplied by time, which is energy. A kilowatt-hour is never a unit of power.

Household appliances carry a label stating their power rating, for example $2.0\,\mathrm{kW}$ for a kettle or $10\,\mathrm{W}$ for an LED lamp. The meter on a house (the **electricity meter**, in the meter box) keeps a running total of the energy delivered to the home, in kW h. The electricity retailer charges a price for each kW h used, so

$$\text{cost} = \text{energy in kW h} \times \text{price per kW h}.$$

You may see "kWh" printed on bills and appliance websites; in this course the study design's form, kW h, is used.

Units 3 & 4 preview. The kW h ↔ J conversion returns in Units 3 & 4, where electricity generation, transmission and household consumption are analysed quantitatively. Fluency with $1\,\mathrm{kW\ h} = 3.6 \times 10^6\,\mathrm{J}$, built here, is assumed at that point. This preview is not assessed in Units 1 & 2.

Worked examples

Example 1 — scaffolded

An air conditioner in a bedroom is labelled $2.5\,\mathrm{kW}$. It runs for $3.0\,\mathrm{h}$ on a hot afternoon. (a) Calculate the energy it transfers, in kW h. (b) Convert this energy to joules.

Working	Comment
$P = 2.5\,\mathrm{kW}$, $t = 3.0\,\mathrm{h}$, find E.	Power is already in kW and time in h, so $E = P\,t$ gives kW h directly.
(a) $E = P\,t = 2.5 \times 3.0 = 7.5\,\mathrm{kW\ h}$.	Energy in kW h = power in kW × time in h.
(b) $1\,\mathrm{kW\ h} = 3.6 \times 10^6\,\mathrm{J}$, so $E = 7.5 \times 3.6 \times 10^6 = 2.7 \times 10^7\,\mathrm{J}$.	Multiply by 3.6×10^6 to convert kW h to J.

Check by the joule route: $t = 3.0 \times 3600 = 10\,800\,\mathrm{s}$ and $P = 2500\,\mathrm{W}$, so $E = P\,t = 2500 \times 10\,800 = 2.7 \times 10^7\,\mathrm{J}$. ✓ The two routes agree.

Example 2 — scaffolded

A student builds a circuit with a $6.0\,\mathrm{V}$ battery, a switch and a small lamp. They want to measure (a) the current through the lamp and (b) the potential difference across the lamp when it is lit. For each measurement, state the meter used, how it is connected, and justify the connection by referring to the meter's resistance.

Working	Comment
(a) Current through the lamp: use an ammeter.	Current is the rate of flow of charge, so the meter must sit in the path of the charge.
Break the circuit at one point and insert the ammeter so it is in series with the lamp.	In series, the same current passes through the lamp and the meter, so the meter reads the lamp's current.
The ammeter has very low resistance (ideally zero).	Justification: a low resistance means inserting the meter does not reduce the current being measured.
(b) Potential difference across the lamp: use a voltmeter.	Potential difference is the energy transferred per unit charge across the lamp, measured between its two ends.
Connect the voltmeter across the lamp, in parallel with it.	The two leads touch the circuit at the points just before and just after the lamp.
The voltmeter has very high resistance (ideally infinite).	Justification: a high resistance means the meter draws almost no current, so the lamp's potential difference is unchanged by the measurement.

Example 3 — unscaffolded

A student leaves a $10\,\mathrm{W}$ LED desk lamp on for $6.0\,\mathrm{h}$ each day for 30 days. The electricity price is $30\,\mathrm{c}$ per kW h. (a) Calculate the energy used, in kW h. (b) Convert this energy to joules. (c) Calculate the cost.

Convert the power to kilowatts and find the total time first:

$$P = 10\,\mathrm{W} = 0.010\,\mathrm{kW}\,,\, t = 6.0 \times 30 = 180\,\mathrm{h}\,.$$

Then

$$E = P\,t = 0.010 \times 180 = 1.8\,\mathrm{kW\ h}\,.$$

Converting to joules,

$$E = 1.8 \times 3.6 \times 10^6 = 6.5 \times 10^6\ \text{J}.$$

The cost is the energy multiplied by the price per kW h:

$$\text{cost} = 1.8 \times 30 = 54\ \text{c}.$$

∴ the lamp uses $1.8\ \text{kW h}$ (6.5×10^6 J), costing $54\ \text{c}$.

Example 4 — unscaffolded

A student sets a multimeter to measure current and connects its leads directly across the terminals of a $6.0\ \text{V}$ battery. The meter's fuse blows immediately. Explain, with reference to the meter's resistance in this setting, why the current was so large, and describe how the multimeter should instead have been connected to measure the current through a lamp.

In its current-measuring setting the multimeter works as an ammeter, so its resistance is very low, close to zero. Connected directly across the battery, the loop contains almost no resistance to limit the current, so by the circuit model the current becomes very large. The large current heats the meter's internal fuse until the fuse melts and breaks the circuit, protecting the meter and the wiring. (A very low-resistance path like this across a supply is called a **short circuit**.)

To measure the current through a lamp correctly, the student should keep the meter in its current setting but break the circuit at one point and connect the meter in series with the lamp, so that the same current passes through the lamp and the meter. With a manual-range meter, the highest current range should be selected first, then stepped down for a clearer reading.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working.

Question 1 For each of the ammeter and the voltmeter: (a) state the electrical quantity it measures and the unit of that quantity; (b) state how it is connected relative to the component being measured (in series or in parallel).

Question 2 A student uses a multimeter in a circuit containing a battery, a switch and a lamp. (a) State the function the student should select, and the connection needed, to measure the potential difference across the lamp. (b) State the function the student should select, and the connection needed, to measure the current through the lamp. (c) Explain why connecting the multimeter directly across the battery while it is set to measure current is likely to blow its fuse.

Question 3 (a) Copy and complete: "One kilowatt-hour is the energy transferred by a device of power … operating for …". (b) Show that $1\ \text{kW h} = 3.6 \times 10^6$ J. (c) Convert $2.5\ \text{kW h}$ to joules.

Question 4 A reverse-cycle air conditioner labelled $3.5\ \text{kW}$ runs for $2.0\ \text{h}$ on a cold morning. (a) Calculate the energy it transfers, in kW h. (b) Convert this energy to joules. (c) Calculate the cost of this running time if the electricity price is $28\ \text{c}$ per kW h.

Question 5 (a) State the ideal resistance of an ammeter and explain why a real ammeter is designed to have a resistance close to this value. (b) State the ideal resistance of a voltmeter and explain why a real voltmeter is designed to have a resistance close to this value. (c) A digital voltmeter has a resistance of several million ohms. An older style of voltmeter can have a resistance of only a few thousand ohms. Justify why the digital meter gives the more trustworthy reading when connected across a lamp.

Question 6 A toaster labelled $2000\ \text{W}$ transfers $7.2\ \text{MJ}$ of energy while toasting a batch of bread. (a) Convert $7.2\ \text{MJ}$ to kW h. (b) Calculate the toasting time, in hours. (c) State the energy the toaster would transfer in half this time, in kW h.

Question 7 A pond pump labelled $0.40\ \text{kW}$ runs for $45\ \text{min}$. Calculate the energy it transfers (a) in kW h and (b) in joules.

Question 8 A refrigerator labelled $0.15\,\mathrm{kW}$ runs for an average of $8.0\,\mathrm{h}$ each day. Calculate the energy it uses over 7 days, in kW h, and the cost at $30\,\mathrm{c}$ per kW h.

Question 9 The battery of an electric scooter stores $1.8\,\mathrm{MJ}$ of energy. (a) Convert this energy to kW h. (b) Calculate how long the battery can run a $250\,\mathrm{W}$ motor.

Question 10 One evening, a $2.0\,\mathrm{kW}$ heater runs for $1.5\,\mathrm{h}$ and a television labelled $0.12\,\mathrm{kW}$ runs for $5.0\,\mathrm{h}$. Calculate the energy used by each appliance and state which uses more energy, and by how much.

Question 11 Each evening, a household replaces a $60\,\mathrm{W}$ light bulb with a $10\,\mathrm{W}$ LED lamp that gives the same light; either lamp is used for $5.0\,\mathrm{h}$ per evening. The electricity price is $32\,\mathrm{c}$ per kW h. Calculate the difference in energy used, and the difference in cost, per evening.

Question 12 An appliance uses $1.5\,\mathrm{kW\,h}$ of energy in $45\,\mathrm{min}$. Calculate the power of the appliance, in kW.

Question 13 Explain why the resistance of an ideal ammeter is zero, and describe how the reading would be affected if a real ammeter with a noticeable resistance were inserted into a circuit.

Question 14 A voltmeter is connected across a lamp in a working circuit. Explain why the voltmeter must have a very high resistance, and describe what would happen to the circuit if the voltmeter's resistance were low instead.

Question 15 A student writes: "The kilowatt-hour must be a unit of power, because the word *kilowatt* is in its name." Identify the error in this statement and correct it, using the relationship between energy, power and time.

Question 16 A lab group has one multimeter and needs to measure both the current through, and the potential difference across, a resistor in a working circuit. Describe the steps the group should follow, and justify why the meter's function and connection must change between the two measurements.

Question 17 Household electricity bills state the energy used in kW h rather than in joules. Give two reasons why the kW h is the more practical unit for this purpose.

Question 18 A student accidentally connects a voltmeter in series with a lamp and a battery, instead of across the lamp. Predict what is observed — the behaviour of the lamp and the voltmeter reading — and explain the observation with reference to the voltmeter's resistance.

Answers

Question 1 Ammeter: current, ampere (A), in series with the component. Voltmeter: potential difference, volt (V), in parallel across the component. **Question 2** (a) voltmeter function, in parallel across the lamp; (b) ammeter function, in series with the lamp; (c) in current mode the meter's resistance is very low, so the current is very large and blows the fuse. **Question 3** (a) $1\,\text{kW}$, $1\,\text{hour}$; (b) $1000\,\text{W} \times 3600\,\text{s} = 3.6 \times 10^6\,\text{J}$; (c) $9.0 \times 10^6\,\text{J}$. **Question 4** (a) $7.0\,\text{kW h}$; (b) $2.5 \times 10^7\,\text{J}$; (c) $196\,\text{c}$, that is, \$1.96. **Question 5** (a) zero; so inserting the meter does not reduce the current being measured; (b) infinite; so the meter draws no current and does not change the potential difference being measured; (c) the higher resistance draws less current, disturbing the circuit less. **Question 6** (a) $2.0\,\text{kW h}$; (b) $1.0\,\text{h}$; (c) $1.0\,\text{kW h}$. **Question 7** (a) $0.30\,\text{kW h}$; (b) $1.1 \times 10^6\,\text{J}$. **Question 8** $8.4\,\text{kW h}$; $252\,\text{c}$, that is, \$2.52. **Question 9** (a) $0.50\,\text{kW h}$; (b) $2.0\,\text{h}$. **Question 10** heater $3.0\,\text{kW h}$, television $0.60\,\text{kW h}$; the heater uses $2.4\,\text{kW h}$ more (five times as much). **Question 11** $0.25\,\text{kW h}$ less per evening; $8.0\,\text{c}$ less per evening. **Question 12** $2.0\,\text{kW}$. **Question 13** Zero resistance means the meter does not change the current it measures; a noticeable resistance would make the reading lower than the true current. **Question 14** A high resistance means the meter draws almost no current and leaves the potential difference unchanged; a low resistance would change the circuit and the reading. **Question 15** The kW h is a power multiplied by a time, so it is a unit of energy ($3.6 \times 10^6\,\text{J}$), not a unit of power. **Question 16** Ammeter function in series first, then voltmeter function in parallel; each quantity needs its own connection and its own meter resistance. **Question 17** Numbers stay small and manageable, and power in kW times time in h gives the billed unit directly. **Question 18** The lamp goes out; the voltmeter reads approximately the supply potential difference.

Fully-worked solutions

Question 1 An ammeter measures the electric current through a component, in amperes (A); it is connected in series with the component, so that the same current passes through both. A voltmeter measures the potential difference across a component, in volts (V); it is connected in parallel, across the component.

Question 2 (a) The student selects the voltmeter function and connects the meter in parallel, across the lamp: potential difference is measured between two points, one on each side of the lamp. (b) The student selects the ammeter function and connects the meter in series with the lamp: the circuit is broken at one point and the meter inserted, so the lamp's current passes through it. (c) In its current-measuring setting the multimeter has a very low resistance. Connected across the battery it provides a path with almost nothing to limit the current, so the current becomes very large and melts (blows) the meter's internal fuse.

Question 3 (a) One kilowatt-hour is the energy transferred by a device of power $1\,\text{kW}$ operating for $1\,\text{hour}$. (b) $1\,\text{kW h} = 1\,\text{kW} \times 1\,\text{h} = 1000\,\text{W} \times 3600\,\text{s} = 3.6 \times 10^6\,\text{J}$, since $1\,\text{W} = 1\,\text{J s}^{-1}$. (c) $E = 2.5 \times 3.6 \times 10^6 = 9.0 \times 10^6\,\text{J}$.

Question 4 (a) $E = Pt = 3.5 \times 2.0 = 7.0\,\text{kW h}$. (b) $E = 7.0 \times 3.6 \times 10^6 = 2.52 \times 10^7\,\text{J} = 2.5 \times 10^7\,\text{J}$ (to two significant figures). (c) $\text{cost} = 7.0 \times 28 = 196\,\text{c}$, that is, \$1.96.

Question 5 (a) The ideal ammeter has zero resistance. It is connected in series, in the path of the current, so any resistance it adds reduces the current. A real ammeter keeps its resistance as close to zero as possible so that inserting it does not change the current being measured. (b) The ideal voltmeter has infinite resistance. It is connected in parallel, so any current it draws is current removed from the component. A real voltmeter keeps its resistance as high as possible so that it draws almost no current and leaves the potential difference being measured unchanged. (c) Connected across the lamp, the meter draws some current through itself. The digital meter's resistance is several million ohms, far higher than the older meter's few thousand ohms, so it draws far less current, disturbs the circuit far less, and its reading is closer to the potential difference that existed before the meter was connected.

Question 6 (a) $7.2\,\text{MJ} = 7.2 \times 10^6\,\text{J}$. Dividing by $3.6 \times 10^6\,\text{J}$ per kW h: $E = \dfrac{7.2 \times 10^6}{3.6 \times 10^6} = 2.0\,\text{kW h}$. (b) $P = 2000\,\text{W} = 2.0\,\text{kW}$. From $E = Pt$, $t = \dfrac{E}{P} = \dfrac{2.0}{2.0} = 1.0\,\text{h}$. (c) At constant power the energy is proportional to the time, so in half the time the toaster transfers $1.0\,\text{kW h}$.

Question 7 $t = 45\,\text{min} = 0.75\,\text{h}$. (a) $E = Pt = 0.40 \times 0.75 = 0.30\,\text{kW h}$. (b) $E = 0.30 \times 3.6 \times 10^6 = 1.08 \times 10^6\,\text{J} = 1.1 \times 10^6\,\text{J}$ (to two significant figures).

Question 8 Energy per day: $E = Pt = 0.15 \times 8.0 = 1.2\,\text{kW h}$. Over 7 days, $E = 1.2 \times 7 = 8.4\,\text{kW h}$. Cost: $8.4 \times 30 = 252\,\text{c}$, that is, \$2.52.

Question 9 (a) $E = \dfrac{1.8 \times 10^6}{3.6 \times 10^6} = 0.50\,\text{kW h}$. (b) $P = 250\,\text{W} = 0.25\,\text{kW}$. From $E = Pt$, $t = \dfrac{E}{P} = \dfrac{0.50}{0.25} = 2.0\,\text{h}$.

Question 10 Heater: $E = Pt = 2.0 \times 1.5 = 3.0\,\text{kW h}$. Television: $E = Pt = 0.12 \times 5.0 = 0.60\,\text{kW h}$. The heater uses more energy, by $3.0 - 0.60 = 2.4\,\text{kW h}$. It uses five times as much as the television ($3.0 / 0.60 = 5.0$), even though it ran for less time, because its power is so much greater.

Question 11 Energy per evening, light bulb: $E = Pt = 0.060 \times 5.0 = 0.30\,\text{kW h}$. Energy per evening, LED lamp: $E = Pt = 0.010 \times 5.0 = 0.050\,\text{kW h}$. The LED lamp uses $0.30 - 0.050 = 0.25\,\text{kW h}$ less per evening. Cost per evening: light bulb $0.30 \times 32 = 9.6\,\text{c}$; LED lamp $0.050 \times 32 = 1.6\,\text{c}$. The saving is $9.6 - 1.6 = 8.0\,\text{c}$ per evening.

Question 12 $t = 45\,\text{min} = 0.75\,\text{h}$. From $E = Pt$,

$$P = \frac{E}{t} = \frac{1.5}{0.75} = 2.0\,\text{kW}.$$

Question 13 Model answer. An ammeter is connected in series, so the current being measured passes through it. An ideal ammeter has zero resistance because then it adds nothing to the circuit and the current is exactly what it was before the meter was inserted. If a real ammeter had a noticeable resistance, the total resistance of the circuit would increase when the meter is added, the current would fall, and the meter would read a value lower than the current that existed before the meter was connected — the meter would be changing the very quantity it is measuring.

Question 14 Model answer. A voltmeter is connected in parallel across the lamp, so it provides an extra path for the charge. If its resistance is very high, it draws almost no current, the circuit behaves as if the meter were absent, and the reading is the lamp's true potential difference. If the voltmeter's resistance were low, it would draw a significant current through itself, changing the current in the rest of the circuit and therefore changing the potential difference across the lamp. The reading would then differ from the potential difference that existed before the meter was connected.

Question 15 Model answer. The error is treating a product of power and time as if it were a power. The kilowatt (kW) is indeed a unit of power, but the kilowatt-hour is a power multiplied by a time: from $E = Pt$, an energy equals a power multiplied by the time for which that power is delivered. So $1\,\text{kW h} = 1\,\text{kW} \times 1\,\text{h} = 3.6 \times 10^6\,\text{J}$, which is a quantity of energy. The kilowatt-hour is a unit of energy, not of power.

Question 16 Model answer. Steps: (1) Set the multimeter to its ammeter (current) function, break the circuit at one point, connect the meter in series with the resistor, and read the current. (2) Disconnect the meter, set it to its voltmeter function, connect it in parallel across the resistor, and read the potential difference. Justification: the two quantities need different connections because current is measured in the path of the charge (series) while potential difference is measured between two points (parallel). The meter's internal resistance is also different in the two functions — very low for current, very high for potential difference — so leaving it in the wrong function would disturb the circuit wrongly and could blow its fuse if the low-resistance current setting were connected across the supply.

Question 17 Model answer. Any two of: (1) appliance powers are naturally quoted in kilowatts and usage times in hours, so $E = Pt$ gives the energy directly without any unit conversion; (2) household energies become convenient numbers — a home uses tens of kW h per day, which would be hundreds of millions of joules — so bills, meters and comparisons stay compact and readable; (3) the electricity meter can keep its running total directly in the unit that the retailer charges for, so no conversion is needed between the meter and the bill.

Question 18 Model answer. The voltmeter's resistance is very high, so with the voltmeter in series the total resistance of the circuit is very large and the current becomes extremely small. The lamp receives too little current to glow, so it goes out (or stays unlit). Almost all of the supply's potential difference now appears across the high-resistance voltmeter rather than the lamp, so the voltmeter reads approximately the supply potential difference.

Theory

In 5C, resistance appeared while justifying the meters: it is a measure of how strongly a component opposes the current through it. This subtopic makes resistance quantitative. It defines resistance as a ratio that can be measured, shows how the behaviour of any device is summarised by a current–potential difference graph, and builds the models for the total resistance of components connected in series and in parallel.

Resistance as a ratio. In 5A, the electric current I through a component was defined as the rate of flow of charge, in amperes (A), and the potential difference V across it as the energy transferred per unit charge, in volts (V). When the potential difference across a component is increased, the current through it generally increases too. The link between the two is the component's **resistance**. The resistance R of a component is defined as the ratio of the potential difference across it to the current through it:

$$R=\frac{V}{I}.$$

The unit of resistance is the **ohm** (Ω): a component has a resistance of $1\,\Omega$ when a potential difference of $1\,\mathrm{V}$ across it produces a current of $1\,\mathrm{A}$, so

$$1\,\Omega=1\,\mathrm{V\,A^{-1}}.$$

Rearranging the definition gives the two other forms you will use constantly, including the formula-sheet form $I=\frac{V}{R}$:

$$V=I\,R \text{ and } I=\frac{V}{R}.$$

Resistance is a property of the component itself. A component with a larger resistance needs a larger potential difference to push the same current through it. The three forms of the relationship answer three different questions: $R=V\,/\,I$ finds the resistance from measurements, $I=V\,/\,R$ predicts the current a given supply will drive, and $V=I\,R$ finds the potential difference needed for a chosen current, or the potential difference across one component in a circuit.

Ohmic and non-ohmic devices. For some components, the current is directly proportional to the potential difference: doubling V doubles I, so the ratio $R=V\,/\,I$ has the same value at every setting. A device whose resistance is constant (independent of the potential difference across it) is called **ohmic**. A metal-film resistor, while its temperature stays constant, is ohmic. The constant-resistance behaviour is so common that it has a name of its own: the linear relationship between current and potential difference is often called Ohm's law.

Many devices are **non-ohmic**: their resistance changes as the potential difference, and therefore the current, changes. A device is non-ohmic when $R=V\,/\,I$ does not come out the same at every measurement. The components where this matters most in electronics — lamps, diodes, thermistors, light dependent resistors (LDRs) and light-emitting diodes (LEDs) — are studied in 6C; here the non-ohmic idea is needed only as the contrast that makes the ohmic case precise.

The I–V graph. The behaviour of a device is summarised in one picture by its **I–V graph**: a graph of current (on the vertical axis) against potential difference (on the horizontal axis). To obtain one, connect the device in a circuit with a battery or low-voltage lab power pack and a way to vary the potential difference across it, measure the current with an ammeter in series and the potential difference with a voltmeter in parallel — the connections justified in 5C — and take paired readings of V and I at several settings. Organise the readings in a table and plot them, as described below.

- For an **ohmic** device the plotted points lie on a **straight line through the origin**. The line is straight because the ratio $I\,/\,V$ is the same at every point, and it passes through the origin because with zero potential difference there is genuinely zero current.
- For a **non-ohmic** device the points do not lie on a straight line.

Two readings from an I–V graph are important. The **resistance at any point** is always $R = V / I$: divide the potential difference at that point by the current there. For an ohmic device this gives the same resistance everywhere on the line. The **gradient** of the I–V graph of an ohmic device is

$$\text{gradient} = \frac{\Delta I}{\Delta V} = \frac{1}{R},$$

so a steep gradient means a small resistance, and a shallow gradient means a large resistance. Be careful not to call the gradient the resistance: the gradient of an I–V graph is the *reciprocal* of the resistance, $1 / R$ (the reciprocal of R is the quantity $1 / R$). The resistance itself is found from $R = V / I$ at a point on the line.

A common non-ohmic example is a lamp with a metal filament — the thin wire inside the lamp that glows when it becomes hot. As the current grows, the filament gets hotter, and the resistance of the hot metal is larger than that of the cold metal. The I–V graph of the lamp bends away from the straight ohmic line: the current still increases with potential difference, but by less than proportion, because the resistance is rising as the device heats. Only the shape of the graph is needed here; the physics of lamps and other non-ohmic components belongs to 6C.

Organising and presenting data. An I–V investigation produces pairs of readings, and the way those readings are organised and presented is a key science skill in its own right.

The results table. Record the readings in a table with one column per quantity. Every column heading carries the quantity and its unit, for example "Potential difference V (V)" and "Current I (A)". Order the rows by increasing potential difference, and write each reading to the number of decimal places the meter actually displays. If you repeat a reading, record each attempt and show the mean. A completed table for an ohmic resistor looks like this:

Potential difference V (V)	Current I (A)
0.0	0.00
1.5	0.15
3.0	0.30
4.5	0.45
6.0	0.60

Every row of this table gives the same ratio $V / I = 10\,\Omega$, so the resistor is ohmic.

The graph. Label each axis with the quantity and its unit — current I (A) on the vertical axis, potential difference V (V) on the horizontal axis. Choose a scale that spreads the plotted points over most of the grid, plot every point from the table, then draw the best-fit line: a single straight line drawn as close as possible to all the points, not a dot-to-dot join. For an ohmic device the line of best fit is straight and passes through the origin. Record the table and the graph in your logbook, along with the meter ranges used. As always in this chapter, work only with low-voltage DC supplies — batteries or a school power pack — and have the teacher check the circuit before closing the switch.

Series circuits and equivalent resistance. Components connected end to end in a single path so that the same current passes through each of them are connected **in series** (the connection used for an ammeter, from 5C). In a series circuit:

- the current is the same through every component — there is only one path for the charge;
- the potential differences across the components add up to the potential difference of the supply: $V = V_1 + V_2 + \ldots$. This follows from energy: the energy per unit charge given to the charge by the battery ($V = E / Q$, from 5A) equals the total energy per unit charge the charge transfers to the components as it goes around.

Apply $V = I R$ to each resistor. With the same current I through each one, $V_1 = I R_1$, $V_2 = I R_2$, and so on, so

$$V = I R_1 + I R_2 + \ldots = I \left(R_1 + R_2 + \ldots \right).$$

A single resistor whose resistance is $R_1 + R_2 + \ldots$ would draw exactly the same current from the same supply. This single resistance is called the **equivalent resistance**, R_{eq}, of the combination. For resistors in series,

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n.$$

Adding a resistor in series always **increases** the equivalent resistance: the charge must now pass through more opposition on its single path around the circuit.

Parallel circuits and equivalent resistance. Components connected across the same two points, giving the charge separate paths to follow, are connected **in parallel** (the connection used for a voltmeter, from 5C). Each separate path is called a **branch**. In a parallel circuit:

- the potential difference is the same across every branch — each branch connects the same two points;
- the currents in the branches add up to the total current from the supply: $I = I_1 + I_2 + \ldots$. Charge is conserved, so the charge arriving at a branch point each second ($I = Q/t$, from 5A) splits between the paths.

Apply $I = V/R$ to each branch. With the same potential difference V across each one, $I_1 = V/R_1$, $I_2 = V/R_2$, and so on, so

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \ldots = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right).$$

A single resistor that draws this same total current from the same supply has an equivalent resistance R_{eq} given by

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}.$$

Adding a resistor in parallel always **decreases** the equivalent resistance. A new branch is an extra path for the charge, so the total current drawn from the supply increases at the same potential difference, and $R = V/I$ for the whole combination becomes smaller. The equivalent resistance of a parallel combination is always **smaller than the smallest branch resistance**.

Two special cases are worth knowing. For **two** resistors in parallel, the formula above rearranges to

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2},$$

and for n **identical** resistors of resistance R in parallel,

$$R_{\text{eq}} = \frac{R}{n}.$$

Using the equivalent resistance. Once R_{eq} is known, the whole combination can be replaced in the mind by one resistor of that value: the current drawn from the supply is then simply $I = V/R_{\text{eq}}$. If the components are ohmic, the I–V graph of the combination is still a straight line through the origin — with a shallower gradient than any of its parts for a series combination (larger resistance) and a steeper gradient for a parallel combination (smaller resistance). Circuits that mix series and parallel parts are analysed in 5E, and the deliberate sharing of a supply's potential difference across series components — the voltage divider — is studied in 5F.

The two circuit models are summarised below.

Property	Series	Parallel
Connection	one path, end to end	branches across the same two points
Current	same through every component	shared: $I = I_1 + I_2 + \ldots$
Potential difference	shared: $V = V_1 + V_2 + \ldots$	same across every branch
Equivalent resistance	$R_{\text{eq}} = R_1 + R_2 + \ldots$	$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
Adding a resistor	R_{eq} increases	R_{eq} decreases

Units 3 & 4 preview. The resistance model built here — $R = V / I$, series and parallel equivalence, and reading behaviour from I–V graphs — is the arithmetic base of Units 3 & 4 circuit work, where power generation and transmission are analysed and transmission lines are modelled as resistances in series with the load. These skills are assumed there, not re-taught. Content under this banner is not assessed in this book's topic tests or practice examinations.

Worked examples

Example 1 — scaffolded

A student measures the current through a resistor for several potential differences across it and records the results in the table.

Potential difference V (V)	Current I (A)
0.0	0.00
1.5	0.10
3.0	0.20
4.5	0.30
6.0	0.40

(a) Calculate the resistance of the resistor when the potential difference across it is $4.5\,\mathrm{V}$.
(b) Show by calculation whether the resistance has the same value at $1.5\,\mathrm{V}$ and at $6.0\,\mathrm{V}$.
(c) State whether the resistor is ohmic, and describe the shape of its I–V graph.

Working	Comment
At $V = 4.5\,\mathrm{V}$, the table gives $I = 0.30\,\mathrm{A}$.	Read the matching pair of values from the table.
(a) $R = \dfrac{V}{I} = \dfrac{4.5}{0.30} = 15\,\Omega$.	Resistance is the ratio of potential difference to current.
(b) At $1.5\,\mathrm{V}$: $R = \dfrac{1.5}{0.10} = 15\,\Omega$. At $6.0\,\mathrm{V}$: $R = \dfrac{6.0}{0.40} = 15\,\Omega$.	Test the same ratio at two other readings.
The ratio V / I is $15\,\Omega$ at every reading.	The resistance is constant.
(c) The resistor is ohmic, and its I–V graph is a straight line through the origin.	Constant resistance is the defining property of an ohmic device.

Example 2 — scaffolded

Resistors of $3.0\,\Omega$, $5.0\,\Omega$ and $2.0\,\Omega$ are connected in series with a $6.0\,\mathrm{V}$ battery and a switch. (a) Calculate the equivalent resistance of the three resistors. (b) Calculate the current in the circuit when the switch is closed. (c) Calculate the potential difference across the $5.0\,\Omega$ resistor.

Working	Comment
(a) $R_{\mathrm{eq}} = R_1 + R_2 + R_3 = 3.0 + 5.0 + 2.0 = 10\,\Omega$.	Resistances in series add.
(b) $I = \dfrac{V}{R_{\mathrm{eq}}} = \dfrac{6.0}{10} = 0.60\,\mathrm{A}$.	The same current passes through every component in a series circuit.
(c) $V_2 = I\,R_2 = 0.60 \times 5.0 = 3.0\,\mathrm{V}$.	Apply $V = I\,R$ to the $5.0\,\Omega$ resistor, using the series current.

Check: the other two potential differences are $V_1 = 0.60 \times 3.0 = 1.8\,\mathrm{V}$ and $V_3 = 0.60 \times 2.0 = 1.2\,\mathrm{V}$, and $1.8 + 3.0 + 1.2 = 6.0\,\mathrm{V}$ — the shared potential differences add back up to the battery's potential difference. ✓

Example 3 — unscaffolded

A $12\,\Omega$ resistor and a $6.0\,\Omega$ resistor are connected in parallel across a $12\,\text{V}$ DC supply. Calculate the equivalent resistance of the two resistors, the current in each branch, and the total current drawn from the supply, and check that the branch currents add up to the total current.

The equivalent resistance follows from the parallel formula:

$$\frac{1}{R_{\text{eq}}}=\frac{1}{12}+\frac{1}{6.0}=\frac{1}{12}+\frac{2}{12}=\frac{3}{12}=\frac{1}{4.0},$$

so $R_{\text{eq}}=4.0\,\Omega$. Notice that this is smaller than the smallest branch resistance, as it must be for a parallel combination. Each branch has the full supply potential difference across it, so

$$I_1=\frac{V}{R_1}=\frac{12}{12}=1.0\,\text{A}\,,\,I_2=\frac{V}{R_2}=\frac{12}{6.0}=2.0\,\text{A}\,.$$

The total current drawn from the supply is

$$I=\frac{V}{R_{\text{eq}}}=\frac{12}{4.0}=3.0\,\text{A}\,.$$

(Check: $I_1+I_2=1.0+2.0=3.0\,\text{A}$, which equals the total current. ✓)

∴ the equivalent resistance is $4.0\,\Omega$, the branch currents are $1.0\,\text{A}$ and $2.0\,\text{A}$, and the total current is $3.0\,\text{A}$. The smaller branch resistance carries the larger branch current, because both branches share the same potential difference.

Example 4 — unscaffolded

Two devices are tested with an ammeter and a voltmeter, with the following results.

Device A (a resistor):

Potential difference V (V)	Current I (A)
2.0	0.10
4.0	0.20
6.0	0.30

Device B (a small lamp):

Potential difference V (V)	Current I (A)
2.0	1.00
4.0	1.60
6.0	2.00
8.0	2.30

(a) Calculate the resistance of each device at each measurement.
(b) Identify which device is ohmic, giving a reason.
(c) Explain the trend in the resistance of device B.

For device A, $R=V\,/\,I$ at each reading gives $2.0\,/\,0.10=20\,\Omega$, $4.0\,/\,0.20=20\,\Omega$ and $6.0\,/\,0.30=20\,\Omega$. For device B, the same calculation gives $2.0\,/\,1.00=2.0\,\Omega$, $4.0\,/\,1.60=2.5\,\Omega$, $6.0\,/\,2.00=3.0\,\Omega$ and $8.0\,/\,2.30=3.5\,\Omega$ (to two significant figures).

∴ (a) device A has a resistance of $20\,\Omega$ at every reading, while device B's resistance rises from $2.0\,\Omega$ to $3.5\,\Omega$ as the potential difference increases. (b) Device A is ohmic, because its resistance has the same value at every measurement; device B is non-ohmic, because its resistance changes. (c) Device B is a lamp: as the current through its metal filament increases, the filament becomes hotter, and the resistance of the hot metal is larger than when the filament is cooler. The rising resistance is a heating effect, and it makes the I–V graph of the lamp bend away from a straight line.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working.

Unless stated otherwise, treat each battery's potential difference as unchanged and ignore the resistance of the connecting wires.

Question 1 A student measures the current through a device for several potential differences across it and records the results in the table.

Potential difference V (V)	Current I (A)
0.0	0.00
2.0	0.25
4.0	0.50
6.0	0.75
8.0	1.00

(a) Calculate the resistance of the device when the potential difference across it is $6.0\,\mathrm{V}$.
(b) State what the values of $V\,/\,I$ in the table show about the device.
(c) Describe the shape of the I–V graph of this device.

Question 2 A $4.0\,\Omega$ resistor and an $8.0\,\Omega$ resistor are connected in series with a $6.0\,\mathrm{V}$ battery. (a) Write the relationship for the equivalent resistance of resistors in series. (b) Calculate the equivalent resistance of the two resistors. (c) Calculate the current in the circuit.

Question 3 A $24\,\Omega$ resistor and an $8.0\,\Omega$ resistor are connected in parallel. (a) Write the relationship for the equivalent resistance of resistors in parallel. (b) Calculate the equivalent resistance of the combination. (c) Compare your answer in (b) with the resistance of the smaller resistor, and state the general rule this illustrates.

Question 4 During an I–V investigation, a student writes the following readings in her logbook in the order she took them: "$3.0\,\mathrm{V}$, $0.30\,\mathrm{A}$; $6.0\,\mathrm{V}$, $0.60\,\mathrm{A}$; $1.5\,\mathrm{V}$, $0.15\,\mathrm{A}$; $4.5\,\mathrm{V}$, $0.45\,\mathrm{A}$". (a) Organise the readings into a results table, in order of increasing potential difference, with column headings that include the units. (b) Calculate the resistance of the device. (c) State the label that belongs on each axis of the I–V graph of these data.

Question 5 A $2.0\,\Omega$ resistor and a $6.0\,\Omega$ resistor are connected in series. The current in the circuit is $0.50\,\mathrm{A}$. (a) Calculate the potential difference across each resistor. (b) Calculate the potential difference of the battery. (c) Calculate the equivalent resistance of the circuit from the battery's potential difference and the current, and check that it equals the sum of the two resistances.

Question 6 Three identical $30\,\Omega$ resistors are connected in parallel. (a) Calculate the equivalent resistance of the combination. (b) A fourth $30\,\Omega$ resistor is added in parallel. Calculate the new equivalent resistance. (c) Compare each equivalent resistance with $30\,\Omega$.

Question 7 A resistor is connected across a $9.0\,\mathrm{V}$ battery and carries a current of $0.30\,\mathrm{A}$. Calculate the resistance of the resistor.

Question 8 Resistors of $2.0\,\Omega$, $3.0\,\Omega$, $4.0\,\Omega$ and $6.0\,\Omega$ are connected in series with a $6.0\,\mathrm{V}$ battery. Calculate the current in the circuit and the potential difference across the $6.0\,\Omega$ resistor.

Question 9 Resistors of $20\,\Omega$, $30\,\Omega$ and $60\,\Omega$ are connected in parallel across a $6.0\,\mathrm{V}$ battery. (a) Calculate the equivalent resistance of the combination. (b) Calculate the total current drawn from the battery. (c) Calculate the current in each branch and check that the branch currents add to give the total current.

Question 10 Explain why adding a resistor in series with a circuit increases the equivalent resistance of the circuit. Refer to the path of the charge and the way potential differences are shared in a series circuit.

Question 11 Explain why the equivalent resistance of a parallel combination is always smaller than the smallest branch resistance. Refer to the current drawn from the supply.

Question 12 A student measures the current through a small lamp for several potential differences across it, with the following results.

Potential difference V (V)	Current I (A)
0.50	0.40
1.0	0.60
2.0	0.80
3.0	0.90

(a) Calculate the resistance of the lamp at each measurement.
(b) State whether the lamp is ohmic, giving a reason.
(c) Explain the trend in your values of resistance.

Question 13 The I–V graph of an ohmic device is a straight line through the origin. When the potential difference across the device is $9.0\,\mathrm{V}$, the current through it is $0.45\,\mathrm{A}$. (a) Calculate the resistance of the device. (b) Calculate the gradient of the I–V graph and state the relationship between this gradient and the resistance. (c) Calculate the current when the potential difference is $3.0\,\mathrm{V}$.

Question 14 A student is asked for the equivalent resistance of a $6.0\,\Omega$ resistor and a $3.0\,\Omega$ resistor connected in series. The student writes: "$1/R_{\mathrm{eq}} = 1/6.0 + 1/3.0 = 3/6.0$, so $R_{\mathrm{eq}} = 2.0\,\Omega$". Identify the error in this working and calculate the correct equivalent resistance.

Question 15 Two resistors are connected in parallel across a $12\,\mathrm{V}$ battery. The total current drawn from the battery is $3.0\,\mathrm{A}$, and one of the resistors has a resistance of $6.0\,\Omega$. Calculate the resistance of the other resistor.

Question 16 The I–V graph of an ohmic device is a straight line through the origin, and the point $(4.0\,\mathrm{V}, 0.20\,\mathrm{A})$ lies on the line. (a) Calculate the resistance of the device. (b) A student writes: "The gradient of the line is $0.20/4.0 = 0.050$, so the resistance of the device is $0.050\,\Omega$." Evaluate this claim.

Question 17 The headlamp bulb of a car is designed for the car's $12\,\mathrm{V}$ battery. When the bulb is lit at full brightness, the current through it is $4.0\,\mathrm{A}$. Calculate the resistance of the bulb at this operating point. (A lamp filament is non-ohmic; the value you calculate is the resistance at this operating point.)

Question 18 Two identical resistors, each of resistance R, are connected first in series and then in parallel, each time across the same battery of potential difference V. Compare the current drawn from the battery in the two cases, and explain the difference with reference to the equivalent resistance of each arrangement.

Answers

Question 1 (a) $8.0\,\Omega$; (b) the resistance is constant, so the device is ohmic; (c) a straight line through the origin. **Question 2** (a) $R_{\text{eq}} = R_1 + R_2 + \ldots$; (b) $12\,\Omega$; (c) $0.50\,\text{A}$. **Question 3** (a) $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \ldots$; (b) $6.0\,\Omega$; (c) smaller than $8.0\,\Omega$ — see solutions. **Question 4** (a) see solutions; (b) $10\,\Omega$; (c) vertical: current I (A); horizontal: potential difference V (V). **Question 5** (a) $1.0\,\text{V}$ and $3.0\,\text{V}$; (b) $4.0\,\text{V}$; (c) $8.0\,\Omega$, which equals $2.0 + 6.0$. **Question 6** (a) $10\,\Omega$; (b) $7.5\,\Omega$; (c) both are smaller than $30\,\Omega$. **Question 7** $30\,\Omega$. **Question 8** $0.40\,\text{A}$; $2.4\,\text{V}$. **Question 9** (a) $10\,\Omega$; (b) $0.60\,\text{A}$; (c) $0.30\,\text{A}$, $0.20\,\text{A}$, $0.10\,\text{A}$ — they add to $0.60\,\text{A}$. **Question 10** See solution. **Question 11** See solution. **Question 12** (a) $1.3\,\Omega$, $1.7\,\Omega$, $2.5\,\Omega$, $3.3\,\Omega$; (b) non-ohmic — the resistance is not constant; (c) see solution. **Question 13** (a) $20\,\Omega$; (b) $0.050\,\text{A V}^{-1}$, the gradient equals $1/R$; (c) $0.15\,\text{A}$. **Question 14** The student used the parallel formula for a series circuit; $R_{\text{eq}} = 9.0\,\Omega$. **Question 15** $12\,\Omega$. **Question 16** (a) $20\,\Omega$; (b) see solution. **Question 17** $3.0\,\Omega$. **Question 18** The parallel current is four times the series current; see solution.

Fully-worked solutions

Question 1 The table pairs each potential difference with a current. (a) At $V = 6.0\,\text{V}$, $I = 0.75\,\text{A}$, so $R = \frac{V}{I} = \frac{6.0}{0.75} = 8.0\,\Omega$. (b) The ratio V/I is $8.0\,\Omega$ at every reading ($2.0/0.25 = 4.0/0.50 = 6.0/0.75 = 8.0/1.00 = 8.0\,\Omega$). The resistance is constant, so the device is ohmic. (c) An ohmic device has an I–V graph that is a straight line through the origin.

Question 2 $R_1 = 4.0\,\Omega$, $R_2 = 8.0\,\Omega$, $V = 6.0\,\text{V}$. (a) For resistors in series, $R_{\text{eq}} = R_1 + R_2 + \ldots$ (b) $R_{\text{eq}} = 4.0 + 8.0 = 12\,\Omega$. (c) $I = \frac{V}{R_{\text{eq}}} = \frac{6.0}{12} = 0.50\,\text{A}$.

Question 3 $R_1 = 24\,\Omega$, $R_2 = 8.0\,\Omega$. (a) For resistors in parallel, $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$ (b) $\frac{1}{R_{\text{eq}}} = \frac{1}{24} + \frac{1}{8.0} = \frac{1}{24} + \frac{3}{24} = \frac{4}{24} = \frac{1}{6.0}$, so $R_{\text{eq}} = 6.0\,\Omega$. (c) The equivalent resistance, $6.0\,\Omega$, is smaller than the smaller resistor, $8.0\,\Omega$. This illustrates the general rule that the equivalent resistance of a parallel combination is always smaller than the smallest branch resistance: adding a branch gives the charge an extra path, so more current is drawn at the same potential difference.

Question 4 (a) The organised table, in order of increasing potential difference:

Potential difference V (V)	Current I (A)
1.5	0.15
3.0	0.30
4.5	0.45
6.0	0.60

(b) Every reading gives $R = \frac{V}{I} = 10\,\Omega$ (for example $1.5/0.15 = 10$), so the resistance of the device is $10\,\Omega$.

(c) The vertical axis is labelled "current I (A)" and the horizontal axis "potential difference V (V)" — each axis label carries the quantity and its unit.

Question 5 $R_1 = 2.0\,\Omega$, $R_2 = 6.0\,\Omega$, $I = 0.50\,\text{A}$. (a) The same current passes through both resistors, so $V_1 = I R_1 = 0.50 \times 2.0 = 1.0\,\text{V}$ and $V_2 = I R_2 = 0.50 \times 6.0 = 3.0\,\text{V}$. (b) The potential differences across series components add up to the battery's potential difference: $V = V_1 + V_2 = 1.0 + 3.0 = 4.0\,\text{V}$. (c) $R_{\text{eq}} = \frac{V}{I} = \frac{4.0}{0.50} = 8.0\,\Omega$, which equals $R_1 + R_2 = 2.0 + 6.0 = 8.0\,\Omega$. ✓

Question 6 Three identical resistors of $R=30\,\Omega$ in parallel. (a) For n identical resistors in parallel, $R_{\text{eq}}=\frac{R}{n}=\frac{30}{3}=10\,\Omega$. (Equivalently, $\frac{1}{R_{\text{eq}}}=\frac{3}{30}=\frac{1}{10}$.) (b) With four resistors, $R_{\text{eq}}=\frac{30}{4}=7.5\,\Omega$. (c) Both equivalent resistances are smaller than the $30\,\Omega$ of a single resistor, and adding the fourth branch makes the equivalent resistance smaller still — each extra parallel path draws more current from the supply at the same potential difference.

Question 7 $V=9.0\,\text{V}$, $I=0.30\,\text{A}$.

$$R=\frac{V}{I}=\frac{9.0}{0.30}=30\,\Omega.$$

Question 8 $R_{\text{eq}}=2.0+3.0+4.0+6.0=15\,\Omega$, and $V=6.0\,\text{V}$. The current is

$$I=\frac{V}{R_{\text{eq}}}=\frac{6.0}{15}=0.40\,\text{A}.$$

The same current passes through every component in series, so the potential difference across the $6.0\,\Omega$ resistor is

$$V=I\,R=0.40\times 6.0=2.4\,\text{V}.$$

Question 9 $R_1=20\,\Omega$, $R_2=30\,\Omega$, $R_3=60\,\Omega$, $V=6.0\,\text{V}$. (a) $\frac{1}{R_{\text{eq}}}=\frac{1}{20}+\frac{1}{30}+\frac{1}{60}=\frac{3}{60}+\frac{2}{60}+\frac{1}{60}=\frac{6}{60}=\frac{1}{10}$, so $R_{\text{eq}}=10\,\Omega$. (b) $I=\frac{V}{R_{\text{eq}}}=\frac{6.0}{10}=0.60\,\text{A}$. (c) Each branch has $6.0\,\text{V}$ across it: $I_1=\frac{6.0}{20}=0.30\,\text{A}$, $I_2=\frac{6.0}{30}=0.20\,\text{A}$, $I_3=\frac{6.0}{60}=0.10\,\text{A}$. Check: $0.30+0.20+0.10=0.60\,\text{A}$, which equals the total current. ✓

Question 10 Model answer. In a series circuit there is a single path, so the same current passes through every component, and the potential differences across the components add up to the supply's potential difference. With $V=I\,R$ for each component, the supply's potential difference is $V=I\,R_1+I\,R_2+\ldots=I\left(R_1+R_2+\ldots\right)$, which is the potential difference needed to drive the same current through a single resistance of $R_1+R_2+\ldots$ The charge must pass through all of the opposition in turn, one piece after another on its one path, so the total opposition — the equivalent resistance — is the sum of the individual resistances and is larger than any one of them.

Question 11 Model answer. Every branch of a parallel combination has the full supply potential difference across it. Adding a branch adds another path for the charge, so the total current drawn from the supply increases while the potential difference stays the same. The equivalent resistance of the whole combination is $R_{\text{eq}}=V/I$; with the same V and a larger I, this ratio becomes smaller. Since each branch alone already draws some current, the combined current is always larger than the current in any single branch, so the equivalent resistance is always smaller than the resistance of any one branch — including the smallest.

Question 12 (a) $R=V/I$ at each reading: $0.50/0.40=1.25\approx 1.3\,\Omega$; $1.0/0.60=1.67\approx 1.7\,\Omega$; $2.0/0.80=2.5\,\Omega$; $3.0/0.90=3.33\approx 3.3\,\Omega$ (to two significant figures). (b) The lamp is non-ohmic: its resistance does not have the same value at every measurement, so the current is not proportional to the potential difference. (c) Model answer. The resistance increases as the potential difference and current increase. The lamp's metal filament becomes hotter as the current through it grows, and the resistance of the hot metal is larger than when the filament is cooler. The rising resistance is a heating effect of the increasing current.

Question 13 $V=9.0\,\text{V}$, $I=0.45\,\text{A}$, ohmic device. (a) $R=\frac{V}{I}=\frac{9.0}{0.45}=20\,\Omega$. (b) The line passes through the origin, so its gradient is $\frac{\Delta I}{\Delta V}=\frac{0.45}{9.0}=0.050\,\text{A}\,\text{V}^{-1}$. The gradient of the I–V graph of an ohmic device equals the reciprocal of the resistance: $\text{gradient}=1/R=1/20=0.050\,\text{A}\,\text{V}^{-1}$. (c) With the resistance constant, $I=\frac{V}{R}=\frac{3.0}{20}=0.15\,\text{A}$.

Question 14 Model answer. The resistors are in series, so their resistances add: $R_{\mathrm{eq}}=6.0+3.0=9.0\,\Omega$. The error is that the student used the formula for resistors in *parallel* ($1/R_{\mathrm{eq}}=1/R_1+1/R_2+\ldots$) for a series circuit. Applying the parallel formula to $6.0\,\Omega$ and $3.0\,\Omega$ gives $R_{\mathrm{eq}}=2.0\,\Omega$, which is the equivalent resistance the two resistors would have if they were connected in parallel, not in series. The correct equivalent resistance of the series circuit is $9.0\,\Omega$.

Question 15 $V=12\,\mathrm{V}$, total current $I=3.0\,\mathrm{A}$, one branch $R_1=6.0\,\Omega$. The equivalent resistance of the parallel pair is

$$R_{\mathrm{eq}}=\frac{V}{I}=\frac{12}{3.0}=4.0\,\Omega.$$

Then

$$\frac{1}{R_2}=\frac{1}{R_{\mathrm{eq}}}-\frac{1}{R_1}=\frac{1}{4.0}-\frac{1}{6.0}=\frac{3}{12}-\frac{2}{12}=\frac{1}{12},$$

so $R_2=12\,\Omega$. (Check: $12\,\Omega$ in parallel with $6.0\,\Omega$ gives $R_{\mathrm{eq}}=\dfrac{12\times 6.0}{12+6.0}=\dfrac{72}{18}=4.0\,\Omega$. ✓)

Question 16 (a) The point ($4.0\,\mathrm{V}$, $0.20\,\mathrm{A}$) lies on the line, so $R=\dfrac{V}{I}=\dfrac{4.0}{0.20}=20\,\Omega$. (b) Model answer. The claim is incorrect. The quantity $0.20/4.0=0.050\,\mathrm{A\,V^{-1}}$ is the gradient of the I–V graph, and the gradient of an I–V graph is the reciprocal of the resistance, $1/R$ — not the resistance itself. The resistance is $R=V/I=4.0/0.20=20\,\Omega$. The student has found $1/R=0.050\,\mathrm{A\,V^{-1}}$ and treated it as R; the units also show the mistake, since a resistance is measured in ohms, not in $\mathrm{A\,V^{-1}}$.

Question 17 $V=12\,\mathrm{V}$, $I=4.0\,\mathrm{A}$.

$$R=\frac{V}{I}=\frac{12}{4.0}=3.0\,\Omega.$$

Because the filament is non-ohmic, this is the resistance at the full-brightness operating point; the resistance of the cold filament before the lamp is switched on is smaller.

Question 18 Model answer. In series, the equivalent resistance is $R_{\mathrm{eq}}=R+R=2R$, so the current drawn from the battery is $I_{\mathrm{series}}=\dfrac{V}{2R}$. In parallel, $\dfrac{1}{R_{\mathrm{eq}}}=\dfrac{1}{R}+\dfrac{1}{R}=\dfrac{2}{R}$, so $R_{\mathrm{eq}}=\dfrac{R}{2}$ and the current is $I_{\mathrm{parallel}}=\dfrac{V}{R/2}=\dfrac{2V}{R}$. Comparing the two:

$$\frac{I_{\mathrm{parallel}}}{I_{\mathrm{series}}}=\frac{2V/R}{V/2R}=4,$$

so the parallel arrangement draws four times the current of the series arrangement from the same battery. The difference comes from the equivalent resistances: the series combination has four times the resistance of the parallel combination ($2R$ compared with $R/2$), because series connection forces the charge through both resistors on one path while parallel connection gives the charge two paths, each of resistance R.

Chapter 5 Modelling electricity — 5E Equivalent resistance of combined series and parallel circuits

Theory

Open up almost any battery-operated device — a camping lantern, a model railway controller, a portable speaker — and you will not find a single resistor connected to a battery. You will find several components, some connected end-to-end so that the same current flows through them, some connected side-by-side across the same two points so that the same potential difference acts across them. Real circuits are **combinations** of series and parallel connections. This section shows how to calculate and analyse the **equivalent resistance** of such combined circuits — the study-design skill *“calculate and analyse the equivalent resistance of circuits comprising parallel and series resistance”* (Unit 1, Area of Study 3).

The idea of an equivalent resistance. Suppose a combination of resistors is connected to a battery of potential difference V and the battery delivers a current I. The **equivalent resistance** of the combination is the single resistance that, connected to the same battery, would draw exactly the same current:

$$R_{\text{equivalent}} = \frac{V}{I}.$$

If you replaced the whole combination with this one resistor, the battery would not notice the difference. In this section the symbol R_{eq} is used as a short form of $R_{\text{equivalent}}$ once the meaning is established.

The two tools carried over from Section 5D. Section 5D derives the equivalent resistance of a purely series arrangement and of a purely parallel arrangement. Those two results are the only machinery this section adds to:

- **Series:** resistors carrying the same current, one after another,
$$R_{\text{equivalent}} = R_1 + R_2 + \cdots + R_n.$$
- **Parallel:** resistors connected across the same two points, sharing the same potential difference,
$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}.$$

For the common case of exactly **two** resistors in parallel, the parallel rule can be rearranged into a convenient shortcut:

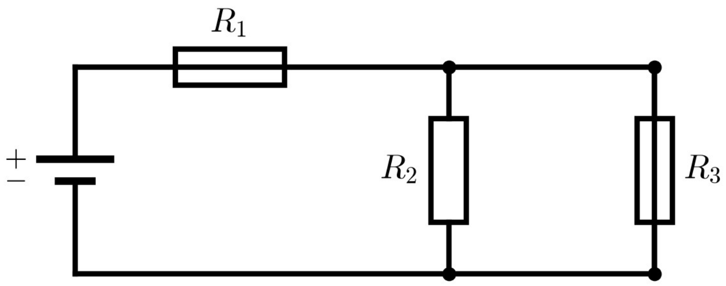
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2},$$

the product of the two resistances divided by their sum. The shortcut is only the parallel rule in disguise; with three or more branches the full reciprocal form must be used.

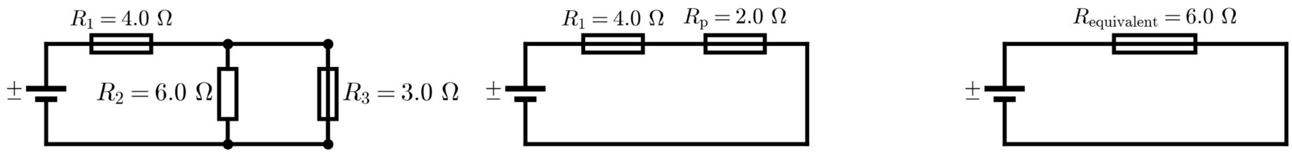
Throughout this section the resistors are treated as **ohmic** (their resistance is constant, so $V = I R$ applies with the same R at every current — Section 5D), the battery is ideal (it maintains its stated potential difference whatever current it delivers), and the connecting wires have negligible resistance.

The method: simplify step by step. A combined circuit is analysed by repeatedly finding a group of resistors that is *purely* in series or *purely* in parallel, replacing that group with its equivalent resistance, and redrawing the circuit. Each redraw is a simpler circuit that behaves exactly like the original as far as the battery is concerned. The steps are:

1. Look for a group of resistors connected in series with one another, or a group connected in parallel with one another. Start with the most tightly nested group, usually the group farthest from the battery.
2. Replace that group with its equivalent resistance using the series or the parallel rule.
3. Redraw the circuit. Repeat until a single resistance remains between the battery terminals — that is $R_{\text{equivalent}}$ for the whole circuit.



The figure above shows the simplest combined circuit: R_1 in series with the parallel pair R_2, R_3. The junction dots mark the points between which R_2 and R_3 are connected — the same two points, so the pair is in parallel. R_1 is *not* in series with R_2 alone, because the current through R_1 splits at the junction: some goes through R_2, some through R_3.



(a) Original circuit

(b) Parallel pair replaced with its equivalent resistance

(c) Single equivalent resistance

In the figure above, the parallel pair (b) is replaced first, and the two series resistors that remain are then added (c). Two habits prevent nearly every error in this work: **check before combining** that the group really is in series (same current through each member) or really in parallel (same potential difference across each member), and **redraw after every step** so the next group is easy to see.

Working backwards: currents and potential differences. The equivalent resistance tells you the battery current, $I = V / R_\mathrm{eq}$. To find the current in, and potential difference across, each individual resistor, work backwards through the redrawn circuits, using two facts:

- **A series group** carries one current. The potential difference across the group is shared between its members, each taking $V = I\,R$ for its own resistance.
- **A parallel group** has one potential difference across it. The current entering the group splits between the branches, each branch carrying $I = V / R$ for its own resistance; the branch currents add up to the current entering the group.

Every completed analysis can therefore be checked in two ways: the potential differences across the series pieces must add up to the battery potential difference, and the branch currents at every junction must add up to the current arriving at that junction.

Two rules worth knowing before you calculate.

- Adding a resistor **in series** always *increases* the equivalent resistance: the same current then has to be driven through more resistance, so R_eq is larger than the largest member of the series group.
- Adding a resistor **in parallel** always *decreases* the equivalent resistance: an extra path is opened for the current, so at the same potential difference the battery delivers more current, and $R = V / I$ is therefore smaller. The equivalent resistance of a parallel group is smaller than the smallest branch in it.

A pair of resistors of $6.0\,\Omega$ and $3.0\,\Omega$ makes the point: in series they give $9.0\,\Omega$ (more than either); in parallel they give $2.0\,\Omega$ (less than either).

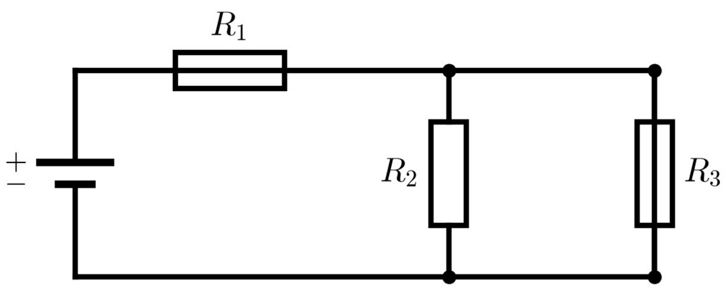
Why engineers care. The equivalent resistance fixes the current the battery must supply, $I = V / R_\mathrm{eq}$, and a battery holds a fixed amount of charge, so by $I = Q / t$ a larger current empties it in a shorter time (Section 5A). Choosing series or parallel connections is how a designer sets the currents a device draws and the life of its battery. Comparing how power is shared in series and parallel arrangements is taken up further in Section 6A; a series pair used deliberately to tap off a chosen potential difference is treated as a *voltage divider* in Section 5F. In the laboratory the reduction can be checked directly: with the battery disconnected, an ohmmeter (Section 5C) placed across the terminals of the combination should read the calculated $R_{\text{equivalent}}$.

The method works for any circuit that can be broken into series and parallel groups, which is every circuit in this section. A few arrangements cannot be reduced this way at all; they lie beyond this study design and are not needed here.

Worked examples

Example 1 — scaffolded

A $12\,\text{V}$ battery is connected to the circuit shown: a resistor $R_1 = 4.0\,\Omega$ in series with a parallel pair, $R_2 = 6.0\,\Omega$ and $R_3 = 3.0\,\Omega$. (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current delivered by the battery. (c) Calculate the potential difference across the parallel pair and check that the potential differences around the circuit are consistent.



Working	Comment
R_2, R_3 in parallel: $\frac{1}{R_p} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{6.0} + \frac{2}{6.0} = \frac{3}{6.0} = \frac{1}{2.0}$, so $R_p = 2.0\,\Omega$.	Same two points on either side of R_2 and R_3, so the pair is in parallel. Two-resistor shortcut gives the same result: $(6.0 \times 3.0)/(6.0 + 3.0) = 2.0\,\Omega$.
$R_{eq} = R_1 + R_p = 4.0 + 2.0 = 6.0\,\Omega$.	R_1 carries the whole current, so it is in series with the pair. (a) answer.
$I = \frac{V}{R_{eq}} = \frac{12}{6.0} = 2.0\,\text{A}$.	(b) answer: the battery current.
$V_p = I\,R_p = 2.0 \times 2.0 = 4.0\,\text{V}$; and $V_1 = I\,R_1 = 2.0 \times 4.0 = 8.0\,\text{V}$.	One current flows through the series pieces, so each takes $V = I\,R$.
Check: $V_1 + V_p = 8.0 + 4.0 = 12\,\text{V}$ = battery potential difference. ✓	The potential differences across the series pieces must add up to the supply.

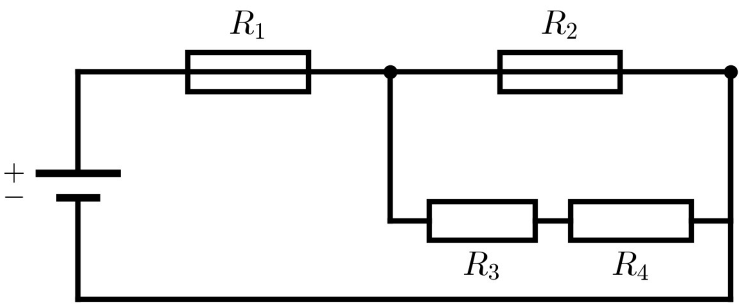
Example 2 — scaffolded

In a school practical, a $12.0\,\text{V}$ battery pack is connected to a $10.0\,\Omega$ resistor in series with a parallel pair of $15.0\,\Omega$ and $30.0\,\Omega$ resistors (the same shape as the circuit of Example 1). (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current delivered by the battery pack. (c) Calculate the potential difference across the parallel pair, the current in each of its resistors, and check the currents at the junction.

Working	Comment
$\frac{1}{R_p}=\frac{1}{15.0}+\frac{1}{30.0}=\frac{2}{30.0}+\frac{1}{30.0}=\frac{3}{30.0}=\frac{1}{10.0}$, so $R_p=10.0\,\Omega$.	Parallel pair first.
$R_{eq}=10.0+10.0=20.0\,\Omega$.	Series addition. (a) answer.
$I=\frac{12.0}{20.0}=0.60\text{ A}$.	(b) answer.
$V_p=I\,R_p=0.60\times10.0=6.0\text{ V}$.	One potential difference across the whole parallel pair.
$I_{15}=\frac{6.0}{15.0}=0.40\text{ A}$; $I_{30}=\frac{6.0}{30.0}=0.20\text{ A}$.	Each branch takes $I=V/R$ at the shared 6.0 V.
Check: $0.40+0.20=0.60\text{ A}$ = current entering the junction. ✓	The smaller branch resistance carries the larger share of the current.

Example 3 — unscaffolded

An 18.0 V supply is connected to the circuit shown. $R_1=6.0\,\Omega$ is in series with a parallel combination whose upper branch is $R_2=8.0\,\Omega$ and whose lower branch is $R_3=2.0\,\Omega$ in series with $R_4=6.0\,\Omega$. Calculate the equivalent resistance of the circuit, the current drawn from the supply, and the current in, and potential difference across, each resistor.



The most tightly nested group is the series pair in the lower branch:

$$R_{34}=R_3+R_4=2.0+6.0=8.0\,\Omega.$$

That $8.0\,\Omega$ branch is in parallel with the $8.0\,\Omega$ upper branch:

$$R_p=\frac{8.0\times8.0}{8.0+8.0}=4.0\,\Omega,$$

and the parallel combination is in series with R_1:

$$R_{eq}=R_1+R_p=6.0+4.0=10.0\,\Omega.$$

The supply current is therefore

$$I=\frac{V}{R_{eq}}=\frac{18.0}{10.0}=1.8\text{ A}.$$

Working backwards, R_1 carries the whole current, so

$$V_1=I\,R_1=1.8\times6.0=10.8\text{ V},\;V_p=I\,R_p=1.8\times4.0=7.2\text{ V},$$

and indeed $10.8+7.2=18.0\,\mathrm{V}$, the supply potential difference. The $7.2\,\mathrm{V}$ acts across each branch, so the branch currents are

$$I_2=\frac{7.2}{8.0}=0.90\,\mathrm{A}\,,\,I_{34}=\frac{7.2}{8.0}=0.90\,\mathrm{A}\,,$$

which add to the $1.8\,\mathrm{A}$ entering the junction, as they must. R_3 and R_4 carry the lower branch current, so

$$V_3=0.90\times 2.0=1.8\,\mathrm{V}\,,\,V_4=0.90\times 6.0=5.4\,\mathrm{V}\,,$$

and $1.8+5.4=7.2\,\mathrm{V}$, the potential difference across the branch.

$\therefore$ $R_{eq}=10.0\,\Omega$; supply current 1.8 A; R_1: 1.8 A, 10.8 V; R_2: 0.90 A, 7.2 V; R_3: 0.90 A, 1.8 V; R_4: 0.90 A, 5.4 V.

Example 4 — unscaffolded

A camping lantern is modelled as a $12\,\mathrm{V}$ battery connected to a $4.0\,\Omega$ resistor in series with a bank of identical lamps, each modelled as a $24\,\Omega$ resistor, connected in parallel. Initially the bank has two lamps. (a) Calculate the equivalent resistance of the circuit and the current in each lamp. A third identical lamp is now connected in parallel with the first two. (b) Calculate the new equivalent resistance and the new current in each lamp. (c) Explain the effect of adding the third lamp on the battery current and on the current in each of the original lamps.

Before the third lamp. Two $24\,\Omega$ lamps in parallel give

$$R_p=\frac{24}{2}=12\,\Omega\,,\,R_{eq}=4.0+12=16\,\Omega\,,$$

so the battery current is $I=12/16=0.75\,\mathrm{A}$. The potential difference across the lamp bank is $V_p=0.75\times 12=9.0\,\mathrm{V}$, and each lamp carries

$$I_{lamp}=\frac{9.0}{24}=0.375\approx 0.38\,\mathrm{A}\,.$$

After the third lamp. Three $24\,\Omega$ lamps in parallel give

$$R_p=\frac{24}{3}=8.0\,\Omega\,,\,R_{eq}=4.0+8.0=12\,\Omega\,,$$

so the battery current rises to $I=12/12=1.0\,\mathrm{A}$. The potential difference across the bank is now $V_p=1.0\times 8.0=8.0\,\mathrm{V}$, and each lamp carries

$$I_{lamp}=\frac{8.0}{24}=0.333\approx 0.33\,\mathrm{A}\,.$$

(c) Explanation. Adding a branch in parallel lowers the resistance of the bank ($12\,\Omega \rightarrow 8.0\,\Omega$) and therefore lowers R_{eq} ($16\,\Omega \rightarrow 12\,\Omega$), so the battery current *rises* ($0.75\,\mathrm{A} \rightarrow 1.0\,\mathrm{A}$) — an extra path lets the battery deliver more current. But the larger battery current also increases the potential difference taken by the series $4.0\,\Omega$ resistor ($3.0\,\mathrm{V} \rightarrow 4.0\,\mathrm{V}$), leaving a *smaller* potential difference across the bank ($9.0\,\mathrm{V} \rightarrow 8.0\,\mathrm{V}$). Each original lamp therefore carries slightly *less* current than before ($0.38\,\mathrm{A} \rightarrow 0.33\,\mathrm{A}$). Adding a parallel branch always raises the battery current; because of the series resistor, it slightly dims the lamps already there.

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working.

Question 1 Two $6.0\,\Omega$ resistors are connected in parallel, and this combination is connected in series with a $3.0\,\Omega$ resistor. (a) Calculate the equivalent resistance of the parallel pair. (b) Calculate the equivalent resistance of the whole circuit. (c) Compare your answer to (b) with the resistance of the larger single resistor in the circuit, and comment.

Question 2 In a school practical, a $9.0\,\text{V}$ battery is connected to a $2.0\,\Omega$ resistor in series with a parallel pair of $3.0\,\Omega$ and $6.0\,\Omega$ resistors. (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current delivered by the battery. (c) Calculate the potential difference across the parallel pair and the current in each of its resistors, and check that the two branch currents add to your answer in (b).

Question 3 A $20\,\text{V}$ supply is connected to a $7.0\,\Omega$ resistor in series with a parallel pair of $12\,\Omega$ and $4.0\,\Omega$ resistors. (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current drawn from the supply. (c) Calculate the current in the $12\,\Omega$ resistor.

Question 4 (a) State the effect on the equivalent resistance of a group of adding one more resistor in series with it, and of adding one more resistor in parallel with it. (b) Explain, in terms of the paths available to the current, why a parallel combination has a smaller equivalent resistance than its smallest branch. (c) Two resistors each of $8.0\,\Omega$ are connected (i) in series, (ii) in parallel. Calculate the equivalent resistance in each case and compare both with $8.0\,\Omega$.

Question 5 A $12.0\,\text{V}$ battery is connected to the following circuit: a $4.0\,\Omega$ resistor and a second $4.0\,\Omega$ resistor connected in series; this series combination connected in parallel with an $8.0\,\Omega$ resistor; and the whole combination connected in series with a $2.0\,\Omega$ resistor. (a) Calculate the resistance of the series combination of the two $4.0\,\Omega$ resistors. (b) Calculate the equivalent resistance of the whole circuit. (c) Calculate the current delivered by the battery. (d) Calculate the potential difference across the $8.0\,\Omega$ resistor, and hence the current in it and in each $4.0\,\Omega$ resistor.

Question 6 A $12\,\text{V}$ battery is connected to a $2.0\,\Omega$ resistor in series with a parallel pair of $6.0\,\Omega$ and $3.0\,\Omega$ resistors. (a) Calculate the equivalent resistance of the circuit. (b) Calculate the current delivered by the battery. (c) Calculate the power delivered by the battery. (d) Calculate the energy transferred by the battery in 5.0 minutes.

Question 7 Calculate the equivalent resistance of a $5.0\,\Omega$ resistor in series with a parallel pair of $4.0\,\Omega$ and $12\,\Omega$ resistors.

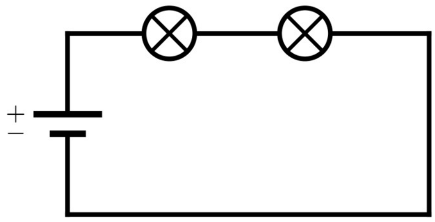
Question 8 A $15\,\text{V}$ supply is connected to a $5.0\,\Omega$ resistor in series with a parallel pair of $20\,\Omega$ resistors. Calculate the equivalent resistance of the circuit, the current drawn from the supply, and the current in each $20\,\Omega$ resistor.

Question 9 A $24\,\text{V}$ supply is connected to a circuit with the arrangement shown in the figure for Example 3: $R_1 = 4.0\,\Omega$ in series with a parallel combination whose upper branch is $R_2 = 12\,\Omega$ and whose lower branch is $R_3 = 3.0\,\Omega$ in series with $R_4 = 9.0\,\Omega$. Calculate the current in the $9.0\,\Omega$ resistor and the potential difference across it.

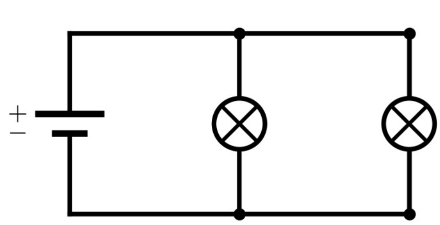
Question 10 An unknown resistor is connected in parallel with a $6.0\,\Omega$ resistor, and the combination is connected in series with a $2.0\,\Omega$ resistor. The equivalent resistance of the whole circuit is measured to be $5.0\,\Omega$. Calculate the resistance of the unknown resistor.

Question 11 A student writes: *"Adding another resistor in parallel must increase the equivalent resistance, because the circuit then contains more resistance."* Identify the error in this claim and give the correct reasoning, supporting it with a numerical example.

Question 12 Two identical $12\,\Omega$ resistors are connected to a $12\,\text{V}$ battery, first in series and then in parallel, as in the figure below. Calculate the battery current in each arrangement, and explain the difference in terms of the paths available to the current and the potential difference each resistor receives.



(a) Two lamps in series

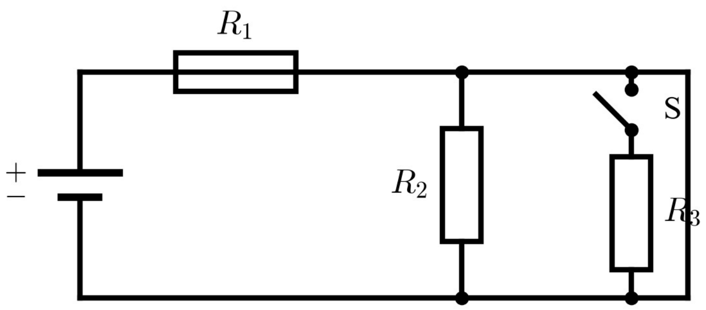


(b) Two lamps in parallel

Question 13 An 18 V supply is connected to a 2.4 Ω resistor in series with a parallel pair of 6.0 Ω and 9.0 Ω resistors. Show that the equivalent resistance of the circuit is 6.0 Ω, and hence calculate the supply current and the current in each of the two parallel resistors.

Question 14 Two identical lamps, each modelled as a 24 Ω resistor, are connected to a 12 V battery, first in series and then in parallel (figure of Question 12). Determine the battery current in each arrangement, state which arrangement draws the larger current, and explain which arrangement keeps the battery going longer. Justify your answer with reference to the current drawn and the power delivered by the battery.

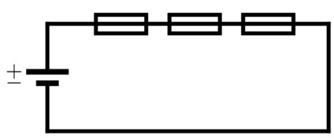
Question 15 The circuit shown contains a 12 V battery, a 4.0 Ω resistor, and a 12 Ω resistor; a switch S connects a second 12 Ω resistor in parallel with the first. Calculate the current drawn from the battery with the switch open and with it closed, and explain the change.



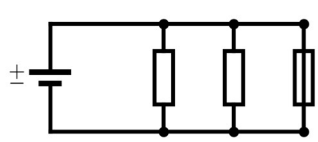
Question 16 Explain why the equivalent resistance of a parallel combination is always smaller than the resistance of its smallest branch. Illustrate your explanation with a 3.0 Ω and a 6.0 Ω resistor.

Question 17 A student has a 6.0 Ω resistor and a 3.0 Ω resistor and wants (i) the largest possible equivalent resistance from them, and (ii) the smallest possible equivalent resistance. State how the resistors should be connected in each case, calculate the equivalent resistance obtained, and justify why no other connection can do better.

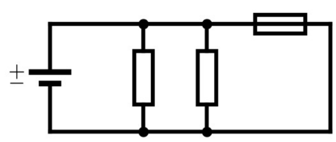
Question 18 Three identical 6.0 Ω resistors are connected to a 6.0 V battery in each of the arrangements shown. For each arrangement calculate the equivalent resistance and the current drawn from the battery, and state which arrangement draws the largest current.



(i) All three in series



(ii) All three in parallel



(iii) Two in parallel, then in series with the third

Answers

Question 1 (a) $3.0\,\Omega$; (b) $6.0\,\Omega$; (c) $6.0\,\Omega$ equals one $6.0\,\Omega$ resistor and is far less than the $12\,\Omega$ the two would give in series — the parallel pair more than halves the resistance it contributes. **Question 2** (a) $4.0\,\Omega$; (b) 2.25 A ($\approx 2.3\text{ A}$); (c) 4.5 V; 1.5 A and 0.75 A, which add to 2.25 A. ✓ **Question 3** (a) $10\,\Omega$; (b) 2.0 A; (c) 0.50 A. **Question 4** (a) series: increases; parallel: decreases; (b) an extra branch opens an extra path, so at the same potential difference more current flows and $R = V / I$ is smaller; (c) (i) $16\,\Omega$, (ii) $4.0\,\Omega$; the series value is above $8.0\,\Omega$ and the parallel value below it. **Question 5** (a) $8.0\,\Omega$; (b) $6.0\,\Omega$; (c) 2.0 A; (d) 8.0 V; 1.0 A in the $8.0\,\Omega$ resistor and 1.0 A in the series string (1.0 A through each $4.0\,\Omega$ resistor). **Question 6** (a) $4.0\,\Omega$; (b) 3.0 A; (c) 36 W; (d) $1.1\times10^4\text{ J}$ (11 kJ). **Question 7** $8.0\,\Omega$. **Question 8** $15\,\Omega$; 1.0 A; 0.50 A in each $20\,\Omega$ resistor. **Question 9** 1.2 A; 11 V (10.8 V before rounding). **Question 10** $6.0\,\Omega$. **Question 11** The error is treating resistance as a quantity that simply accumulates; a parallel branch adds a *path*, so at the same potential difference more current flows and the equivalent resistance falls — e.g. $6.0\,\Omega$ alone becomes $2.0\,\Omega$ when $3.0\,\Omega$ is added in parallel. **Question 12** series: 0.50 A; parallel: 2.0 A; in parallel each resistor receives the full 12 V and there are two independent paths, so each carries 1.0 A. **Question 13** $R_\text{p} = 3.6\,\Omega$, $R_\text{eq} = 6.0\,\Omega$; 3.0 A; 1.8 A in the $6.0\,\Omega$ and 1.2 A in the $9.0\,\Omega$ resistor. **Question 14** series: 0.25 A; parallel: 1.0 A; parallel draws the larger current; series keeps the battery going longer (smaller current, smaller power: 3.0 W vs 12 W). **Question 15** open: 0.75 A; closed: 1.2 A; closing the switch adds a parallel path, lowering the resistance of the pair from $12\,\Omega$ to $6.0\,\Omega$ and R_eq from $16\,\Omega$ to $10\,\Omega$. **Question 16** The extra path carries current of its own at the same potential difference, so the total current exceeds the current in the smallest branch alone, making $R = V / I$ smaller than that branch; here $6.0\,\Omega$ in parallel with $3.0\,\Omega$ gives $2.0\,\Omega$, which is less than $3.0\,\Omega$. **Question 17** (i) series, $9.0\,\Omega$; (ii) parallel, $2.0\,\Omega$; series adds resistances so it is the largest combination, parallel always reduces so it is the smallest. **Question 18** (i) $18\,\Omega$, 0.33 A; (ii) $2.0\,\Omega$, 3.0 A; (iii) $9.0\,\Omega$, 0.67 A; arrangement (ii) draws the largest current.

Fully-worked solutions

Question 1 $R_\text{p} = (6.0\times6.0)/(6.0+6.0) = 3.0\,\Omega$. (b) $R_\text{eq} = 3.0+3.0 = 6.0\,\Omega$. (c) The whole circuit ($6.0\,\Omega$) has the same resistance as a single $6.0\,\Omega$ resistor, and much less than the $12\,\Omega$ the pair would contribute in series. Putting resistors in parallel is a way of *reducing* the resistance a group contributes to a circuit.

Question 2 $R_\text{p} = (3.0\times6.0)/(3.0+6.0) = 2.0\,\Omega$. (a) $R_\text{eq} = 2.0+2.0 = 4.0\,\Omega$. (b) $I = 9.0/4.0 = 2.25\text{ A}$ ($\approx 2.3\text{ A}$ to two significant figures). (c) $V_\text{p} = 2.25\times2.0 = 4.5\text{ V}$; $I_3 = 4.5/3.0 = 1.5\text{ A}$; $I_6 = 4.5/6.0 = 0.75\text{ A}$. Check: $1.5+0.75 = 2.25\text{ A}$, the battery current. ✓

Question 3 $R_\text{p} = (12\times4.0)/(12+4.0) = 48/16 = 3.0\,\Omega$. (a) $R_\text{eq} = 7.0+3.0 = 10\,\Omega$. (b) $I = 20/10 = 2.0\text{ A}$. (c) $V_\text{p} = 2.0\times3.0 = 6.0\text{ V}$, so $I_{12} = 6.0/12 = 0.50\text{ A}$. (The $4.0\,\Omega$ resistor carries $6.0/4.0 = 1.5\text{ A}$; $0.50+1.5 = 2.0\text{ A}$. ✓)

Question 4 (a) Adding a resistor in series increases the equivalent resistance; adding one in parallel decreases it. (b) In a parallel group every branch is connected across the same potential difference. Adding a branch opens an extra path, so the battery must deliver extra current through the new branch while the existing branches carry what they carried before. With more current at the same potential difference, $R = V / I$ is smaller — the equivalent resistance falls below the smallest branch. (c) (i) $8.0+8.0 = 16\,\Omega$; (ii) $(8.0\times8.0)/(8.0+8.0) = 4.0\,\Omega$. The series value ($16\,\Omega$) is greater than either resistor; the parallel value ($4.0\,\Omega$) is less than either — as the two rules require.

Question 5 (a) $4.0+4.0 = 8.0\,\Omega$. (b) The $8.0\,\Omega$ string in parallel with $8.0\,\Omega$: $R_\text{p} = 4.0\,\Omega$; then $R_\text{eq} = 4.0+2.0 = 6.0\,\Omega$. (c) $I = 12.0/6.0 = 2.0\text{ A}$. (d) $V_2 = 2.0\times2.0 = 4.0\text{ V}$ across the $2.0\,\Omega$ resistor, leaving $V_\text{p} = 12.0-4.0 = 8.0\text{ V}$ across the parallel combination. Hence $I_8 = 8.0/8.0 = 1.0\text{ A}$ through the $8.0\,\Omega$ resistor, and the series string carries $8.0/8.0 = 1.0\text{ A}$ — so each $4.0\,\Omega$ resistor carries 1.0 A (taking 4.0 V each). Check: $1.0+1.0 = 2.0\text{ A}$. ✓

Question 6 $R_\text{p} = (6.0\times3.0)/(6.0+3.0) = 2.0\,\Omega$. (a) $R_\text{eq} = 2.0+2.0 = 4.0\,\Omega$. (b) $I = 12/4.0 = 3.0\text{ A}$. (c) $P = VI = 12\times3.0 = 36\text{ W}$. (d) $t = 5.0\times60 = 300\text{ s}$; $E = Pt = 36\times300 = 1.08\times10^4\text{ J} \approx 1.1\times10^4\text{ J}$ (11 kJ).

Question 7 $R_\text{p} = (4.0\times12)/(4.0+12) = 48/16 = 3.0\,\Omega$; $R_\text{eq} = 5.0+3.0 = 8.0\,\Omega$.

Question 8 $R_p = 20/2 = 10\,\Omega$; $R_{eq} = 5.0 + 10 = 15\,\Omega$. $I = 15/15 = 1.0\text{ A}$. $V_p = 1.0 \times 10 = 10\text{ V}$, so each $20\,\Omega$ resistor carries $10/20 = 0.50$ A (check: $0.50 + 0.50 = 1.0$ A ✓).

Question 9 Lower branch: $R_{34} = 3.0 + 9.0 = 12\,\Omega$, equal to the upper branch. $R_p = 12/2 = 6.0\,\Omega$; $R_{eq} = 4.0 + 6.0 = 10\,\Omega$; $I = 24/10 = 2.4\text{ A}$. $V_p = 2.4 \times 6.0 = 14.4\text{ V}$. The two equal branches share this equally: $I_{34} = 14.4/12 = 1.2\text{ A}$. The $9.0\,\Omega$ resistor carries this branch current, 1.2 A, and takes $V_9 = 1.2 \times 9.0 = 10.8 \approx 11\text{ V}$. (Check: $V_3 = 1.2 \times 3.0 = 3.6\text{ V}$; $3.6 + 10.8 = 14.4\text{ V}$. ✓)

Question 10 The parallel pair must contribute $R_p = 5.0 - 2.0 = 3.0\,\Omega$. Then

$$\frac{1}{R_x} = \frac{1}{R_p} - \frac{1}{6.0} = \frac{1}{3.0} - \frac{1}{6.0} = \frac{2}{6.0} - \frac{1}{6.0} = \frac{1}{6.0},$$

so $R_x = 6.0\,\Omega$. (Two equal resistors in parallel give half of one — consistent.)

Question 11 The claim confuses *amount of resistive material* with *equivalent resistance*. Resistance in a circuit is defined by $R = V/I$: connecting a branch in parallel keeps the same potential difference but opens an extra path, so the combination draws **more** current, and $R = V/I$ is therefore **smaller**. The reciprocal rule shows it algebraically: adding a branch adds a positive term $1/R$ to $1/R_{eq}$, so R_{eq} must fall. Example: a $6.0\,\Omega$ resistor alone has equivalent resistance $6.0\,\Omega$; adding $3.0\,\Omega$ in parallel gives $(6.0 \times 3.0)/(6.0 + 3.0) = 2.0\,\Omega$ — less than either branch, not more.

Question 12 Series: $R_{eq} = 12 + 12 = 24\,\Omega$, so $I = 12/24 = 0.50\text{ A}$. Parallel: $R_{eq} = 12/2 = 6.0\,\Omega$, so $I = 12/6.0 = 2.0\text{ A}$. In series there is one path, and the 12 V is shared (6.0 V each), so the same 0.50 A flows through both. In parallel there are two independent paths, each resistor receives the full 12 V and carries $12/12 = 1.0\text{ A}$, giving 2.0 A in total — four times the series current.

Question 13 $1/R_p = 1/6.0 + 1/9.0 = 3/18 + 2/18 = 5/18$, so $R_p = 3.6\,\Omega$ and $R_{eq} = 2.4 + 3.6 = 6.0\,\Omega$, as required. $I = 18/6.0 = 3.0\text{ A}$. $V_p = 3.0 \times 3.6 = 10.8\text{ V}$; $I_6 = 10.8/6.0 = 1.8\text{ A}$; $I_9 = 10.8/9.0 = 1.2\text{ A}$. Check: $1.8 + 1.2 = 3.0\text{ A}$. ✓

Question 14 Series: $R_{eq} = 48\,\Omega$, $I = 12/48 = 0.25\text{ A}$; the battery delivers $P = VI = 12 \times 0.25 = 3.0\text{ W}$. Parallel: $R_{eq} = 12\,\Omega$, $I = 12/12 = 1.0\text{ A}$; $P = 12 \times 1.0 = 12\text{ W}$. The parallel arrangement draws the larger current (four times larger). A battery stores a fixed amount of energy, so the arrangement that draws less power empties it more slowly: the **series** arrangement keeps the battery going longer (at the price of dimmer lamps, since each lamp then receives far less power).

Question 15 Switch open: only the first $12\,\Omega$ resistor is connected; $R_{eq} = 4.0 + 12 = 16\,\Omega$, $I = 12/16 = 0.75\text{ A}$. Switch closed: the pair gives $12/2 = 6.0\,\Omega$; $R_{eq} = 4.0 + 6.0 = 10\,\Omega$, $I = 12/10 = 1.2\text{ A}$. Closing the switch adds a second path in parallel, halving the resistance of the pair and so lowering R_{eq}; at the same battery potential difference the current rises from 0.75 A to 1.2 A.

Question 16 The potential difference V across a parallel group acts on every branch. The smallest branch alone would carry V/R_{small}; the other branches each carry additional current of their own, so the total current I is greater than V/R_{small}. Since $R_{eq} = V/I$, a larger current at the same potential difference means $R_{eq} < R_{small}$. Example: $3.0\,\Omega$ in parallel with $6.0\,\Omega$ gives $(3.0 \times 6.0)/(3.0 + 6.0) = 2.0\,\Omega$, which is indeed less than the smallest branch ($3.0\,\Omega$).

Question 17 (i) Largest: connect them in series, $6.0 + 3.0 = 9.0\,\Omega$. Every series connection adds resistances, and any parallel connection would reduce the total, so series is the largest value obtainable. (ii) Smallest: connect them in parallel, $(6.0 \times 3.0)/(6.0 + 3.0) = 2.0\,\Omega$. Every parallel connection reduces the equivalent resistance below the smallest member, and any series element would add to it, so parallel is the smallest value obtainable.

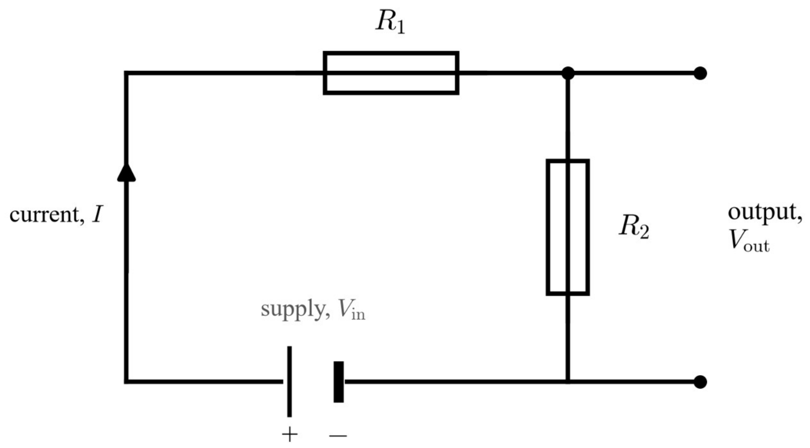
Question 18 (i) All in series: $R_{eq} = 6.0 + 6.0 + 6.0 = 18\,\Omega$; $I = 6.0/18 = 0.33\text{ A}$. (ii) All in parallel: $1/R_{eq} = 1/6.0 + 1/6.0 + 1/6.0 = 3/6.0 = 1/2.0$, so $R_{eq} = 2.0\,\Omega$; $I = 6.0/2.0 = 3.0\text{ A}$. (iii) Two in parallel then in series with the third: $R_p = 6.0/2 = 3.0\,\Omega$; $R_{eq} = 3.0 + 6.0 = 9.0\,\Omega$; $I = 6.0/9.0 = 0.67\text{ A}$. Arrangement (ii) draws the largest current, because parallel connections lower the equivalent resistance while series connections raise it: $2.0\,\Omega < 9.0\,\Omega < 18\,\Omega$, and $I = V / R_{\text{eq}}$.

Chapter 5 Modelling electricity — 5F Voltage dividers

Theory

A torch cell provides $1.5\,\mathrm{V}$, a rectangular transistor battery provides $9\,\mathrm{V}$, the battery of a car provides $12\,\mathrm{V}$, and a USB charger provides $5.0\,\mathrm{V}$. Each of these supplies delivers a **fixed** potential difference — and yet the component we want to connect often needs a smaller potential difference than the supply gives. A **voltage divider** is a simple circuit that takes the potential difference of a supply and delivers a chosen part of it as an output. Voltage dividers sit inside many familiar devices: the volume control of a radio, the fuel gauge of a car, light- and temperature-sensing circuits, and the multimeter you met in 5C. This section builds the model the study design asks you to apply — *analyse circuits comprising voltage dividers* (*VCE Physics Study Design*, Unit 1, Area of Study 3) — using only the series-circuit ideas already built in 5D.

The circuit. A voltage divider is two resistors, R_1 and R_2, connected **in series** across a supply of potential difference V_{in}. The **output**, V_{out}, is taken across R_2: one output connection is made at the point where R_1 and R_2 meet — the **junction** of the divider — and the other at the negative terminal of the supply.



Because R_1 and R_2 are in series, the results of 5D apply:

- the **same current** I passes through R_1 and R_2;
- the potential differences across R_1 and across R_2 **add up to the supply potential difference**, $V_1+V_2=V_{\text{in}}$;
- the equivalent resistance of the pair is R_1+R_2.

Throughout this section the connecting wires are treated as having zero resistance and the resistors as ohmic, and — unless a load is drawn — **nothing takes current from the output**: the divider is said to be **unloaded**. (Connecting something to the output is treated later in this section.)

Deriving the divider equation. Since the whole loop is one series path, the current through it is set by the supply potential difference and the equivalent resistance:

$$I=\frac{V_{\text{in}}}{R_1+R_2}.$$

The output is the potential difference across R_2, and for a resistor $V=I\,R$ (5D), so

$$V_{\text{out}}=I\,R_2=V_{\text{in}}\times\frac{R_2}{R_1+R_2}.$$

This result is the **voltage divider equation**: the output is the supply potential difference multiplied by the fraction $\frac{R_2}{R_1+R_2}$. The same reasoning gives the potential difference across R_1 as $V_1 = V_{\text{in}} \times \frac{R_1}{R_1+R_2}$, and the two shares add to V_{in}, as they must.

Quantity	Relationship	Meaning
current in the divider	$I = \frac{V_{\text{in}}}{R_1+R_2}$	one series path, so one current
output across R_2	$V_{\text{out}} = V_{\text{in}} \times \frac{R_2}{R_1+R_2}$	the output is the supply's fraction $\frac{R_2}{R_1+R_2}$
drop across R_1	$V_1 = V_{\text{in}} \times \frac{R_1}{R_1+R_2}$	the rest of the supply potential difference
shares add	$V_1 + V_{\text{out}} = V_{\text{in}}$	the two resistors split the whole supply

What the equation tells us.

1. **The output is always smaller than the supply.** The fraction $\frac{R_2}{R_1+R_2}$ is always less than 1, so $V_{\text{out}} < V_{\text{in}}$. A divider can only reduce a potential difference, never increase it. The energy picture agrees: $V = E/Q$ (5A) says each coulomb of charge leaves the supply carrying V_{in} joules, and gives all of it up across the two resistors. The share given up across R_2 is part of that total, so it can never exceed it.
2. **The larger resistor takes the larger share.** The same current passes through both resistors and $V = I\,R$, so the potential differences are in the same ratio as the resistances: $\frac{V_1}{V_{\text{out}}} = \frac{R_1}{R_2}$. Double R_1 and it takes double the potential difference of R_2.
3. **Equal resistors halve the supply.** If $R_1 = R_2$ the fraction is $\frac{1}{2}$, so $V_{\text{out}} = \frac{V_{\text{in}}}{2}$.
4. **More resistors divide further.** Three or more resistors in series each take $V_{\text{in}} \times \frac{R_i}{R_{\text{total}}}$, and n *equal* resistors share the supply equally, each taking $\frac{V_{\text{in}}}{n}$.

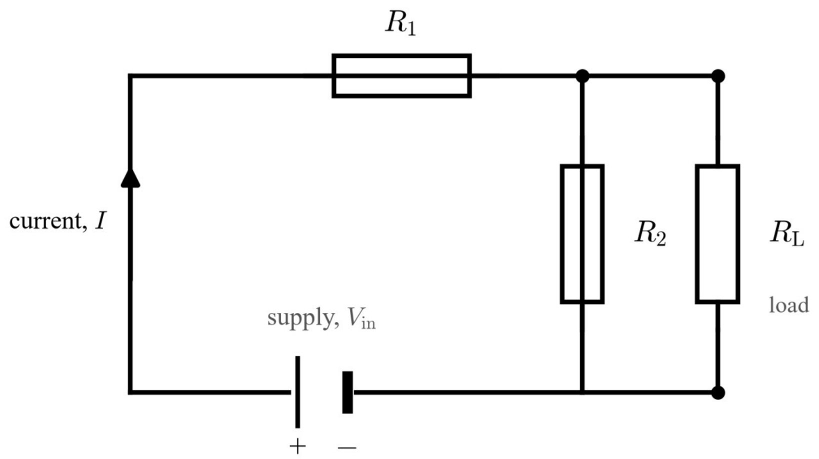
Designing a divider. To obtain a chosen output, choose resistances whose ratio gives the required fraction. For example, to take $3.0\,\text{V}$ from a $9.0\,\text{V}$ supply the fraction must be $\frac{1}{3}$, which needs R_2 to be one-third of the total, that is, $R_1 = 2\,R_2$ — such as $R_1 = 2.0\,\text{k}\,\Omega$ with $R_2 = 1.0\,\text{k}\,\Omega$. The ratio fixes the output; the *actual* resistance values then fix the current the divider draws ($I = V_{\text{in}}/(R_1+R_2)$) and therefore the power it takes from the supply ($P = V\,I$, from 5A). Higher values draw less current and waste less of the battery's energy, but — as we see below — the output of a high-resistance divider is more easily pulled down by anything connected to it. Whatever the values, the resistors must be able to carry the current and release the power as heat without overheating.

Adjustable dividers. Replace one arm of the divider with a **variable resistor** and the ratio becomes adjustable: turning the control moves the output continuously from $0\,\text{V}$ (the variable resistor set to zero) toward the full supply potential difference. A radio's volume control and a lamp dimmer work in this way. One common form of adjustable divider is the **potentiometer**, a resistor with a sliding contact; variable resistors and potentiometers as components are treated in 6C.

Sensing with a divider — a look ahead. Suppose one arm of a divider is a component whose resistance depends on a physical condition — a **thermistor**, whose resistance changes with temperature, or a **light-dependent resistor**

(LDR), whose resistance changes with the light falling on it. As the condition changes, the resistance of that arm changes, the fraction changes, and V_{out} changes with it. The divider has turned a change in the physical world into a change in potential difference that an electronic circuit can respond to — switching a heater on when it is cold, or a lamp on when it is dark. The components themselves, and how their resistance behaves, are covered in 6C, and a complete trigger circuit is built in Option 2.12 (12E). The divider idea is the same in every case: *a changing resistance in one arm gives a changing output.*

Connecting a load to the output. A divider exists to feed its output to something — a gauge, a sensor circuit, a voltmeter. Whatever is connected joins the two output connections, and is therefore **in parallel with** R_2 (the arrangement of 5D). The resistance of the lower arm becomes the parallel equivalent of R_2 and the load (5E), which is always *smaller* than R_2 alone, so the fraction taken by the lower arm falls and **the output potential difference drops below its unloaded value**. Worked Example 4 puts numbers to this.



The voltmeter of 5C is a special case of a load. An **ideal** voltmeter has infinite resistance and draws no current, so it reads the unloaded output exactly; that is the model assumed throughout this section unless a question states otherwise. A real voltmeter has a large but finite resistance and reads slightly low, especially across a high-resistance divider — one more reason voltmeters are built with the highest resistance possible. In class, divider investigations use low-voltage DC supplies only — batteries or a laboratory power pack — under the study design's rule quoted in 5C that students do not work on the internal components of equipment above $50\,\text{V}$ AC or $120\,\text{V}$ ripple-free DC. Record each reading, with its unit, in your logbook.

Units 3 & 4 preview. The series reasoning built here underpins the treatment of electricity transmission in Units 3 & 4, where a long transmission line is modelled as a resistance in series with the load: line and load divide the supply potential difference between them, and the potential difference and power lost along the line are found with exactly the divider reasoning of this section. This preview is not assessed in Units 1 & 2.

Worked examples

Example 1 — scaffolded

A school electronics club is building a sensor circuit. The sensor module needs $3.0\,\text{V}$, but the club's supply is a $9.0\,\text{V}$ battery. They build a divider with $R_1 = 2.0\,\text{k}\Omega$ and $R_2 = 1.0\,\text{k}\Omega$ and take the output across R_2. (a) Calculate the equivalent resistance of the divider and the current in it. (b) Calculate the output potential difference and comment on whether it suits the sensor. (c) Calculate the potential difference across R_1 and check that the two shares add to the supply.

Working	Comment
$R_1 = 2000\,\Omega$, $R_2 = 1000\,\Omega$, $V_{\text{in}} = 9.0\,\text{V}$.	List the data; convert $\text{k}\Omega$ to Ω.

Working	Comment
(a) $R_1 + R_2 = 3000\,\Omega$; $I = \dfrac{9.0}{3000} = 3.0 \times 10^{-3}\,\text{A}$.	Series resistances add (5D). One series path, so one current.
(b) $V_{\text{out}} = I\,R_2 = 3.0 \times 10^{-3} \times 1000 = 3.0\,\text{V}$.	Or by the divider equation: $9.0 \times \dfrac{1000}{3000} = 3.0\,\text{V}$.
The output is exactly the $3.0\,\text{V}$ the sensor needs.	The divider does its job while unloaded.
(c) $V_1 = I\,R_1 = 3.0 \times 10^{-3} \times 2000 = 6.0\,\text{V}$.	The larger resistor takes the larger share.
$6.0 + 3.0 = 9.0\,\text{V} = V_{\text{in}}$. ✓	The shares must add to the supply.

Example 2 — scaffolded

A camper trailer's electrical system runs from a $12\,\text{V}$ battery. A small lamp rated $4.0\,\text{V}$, $0.80\,\text{W}$ is to run at its rated values from the $12\,\text{V}$ battery by placing a fixed resistor in series with it — the resistor and the lamp together dividing the supply between them. (a) State the potential difference the series resistor must take. (b) Calculate the current in the lamp at its rated values, and hence the resistance the series resistor must have. (c) Calculate the power released by the series resistor and comment on this way of running the lamp.

Working	Comment
(a) $12 - 4.0 = 8.0\,\text{V}$ across the series resistor.	The two components in series divide the $12\,\text{V}$ between them.
(b) At its rated values, $I = \dfrac{P}{V} = \dfrac{0.80}{4.0} = 0.20\,\text{A}$.	$P = V\,I$ rearranged (5A). The same current passes through lamp and resistor, since they are in series.
$R = \dfrac{V}{I} = \dfrac{8.0}{0.20} = 40\,\Omega$.	$V = I\,R$ rearranged (5D).
(c) $P = V\,I = 8.0 \times 0.20 = 1.6\,\text{W}$.	The power the resistor releases as heat.
$1.6\,\text{W}$ is twice the lamp's $0.80\,\text{W}$.	Two-thirds of the energy drawn from the battery heats the resistor instead of lighting the lamp. A $4.0\,\text{V}$ supply would waste none.

A lamp's filament is non-ohmic — its resistance changes as it warms (non-ohmic behaviour is investigated in 6C) — so the calculation is done at the rated operating point, where $4.0\,\text{V}$ drives $0.20\,\text{A}$, rather than by assuming a fixed lamp resistance.

Example 3 — unscaffolded

The fuel gauge of a car is driven by a **sender unit**: a float on the fuel surface moves a contact along a resistance track, so the sender's resistance rises as the tank fills. Model the sender as a resistance R_s that varies from $0\,\Omega$ (tank empty) to $120\,\Omega$ (tank full), in series with a fixed $60\,\Omega$ resistor across the car's $12\,\text{V}$ battery, the output being taken across the sender by a voltmeter that draws negligible current. Calculate the voltmeter reading when the tank is empty, half full and full, and explain how the arrangement works as a fuel gauge.

The sender and the fixed resistor form a voltage divider across the $12\,\text{V}$ supply, with the output across the sender:

$$V_{\text{out}} = 12 \times \frac{R_s}{60 + R_s}.$$

Empty ($R_s = 0\,\Omega$): the sender takes no share of the supply potential difference, so

$$V_{\text{out}} = 12 \times \frac{0}{60} = 0\,\text{V}.$$

Half full ($R_s = 60\,\Omega$): the two arms are equal, so each takes half,

$$V_{out} = 12 \times \frac{60}{120} = 6.0\,\text{V}\,.$$

Full ($R_s = 120\,\Omega$): the sender is now the larger arm and takes the larger share,

$$V_{out} = 12 \times \frac{120}{180} = 8.0\,\text{V}\,.$$

As fuel is added the float raises the contact, R_s increases, and the sender — the arm across which the gauge reads — takes a growing share of the $12\,\text{V}$, so the voltmeter reading rises and can be marked as a fuel level. The reading never reaches $12\,\text{V}$ because the fixed $60\,\Omega$ resistor always takes some share of the supply potential difference. Notice too that the scale is not even: half a tank gives $6.0\,\text{V}$, three-quarters of the full reading of $8.0\,\text{V}$, so equal changes of fuel level give unequal changes of reading — a divider's output is not proportional to one arm's resistance.

Example 4 — unscaffolded

A student builds a divider from two $1.0\,\text{k}\,\Omega$ resistors across a $9.0\,\text{V}$ battery, expecting an output of $4.5\,\text{V}$. A device of resistance $1.0\,\text{k}\,\Omega$ is then connected across the output as a load. Calculate the new output potential difference, compare it with the unloaded value, and calculate the current the battery now supplies.

With no load, the equal resistors halve the supply: $V_{out} = 4.5\,\text{V}$, and the battery current is $I = \frac{9.0}{2000} = 4.5\,\text{mA}$.

Once the load is connected it is in parallel with R_2 (5D), so the lower arm of the divider becomes (5E)

$$R_{lower} = \frac{R_2 R_L}{R_2 + R_L} = \frac{1000 \times 1000}{2000} = 500\,\Omega\,.$$

The divider is now $R_1 = 1000\,\Omega$ above a $500\,\Omega$ lower arm, giving an equivalent resistance of $1500\,\Omega$ and a battery current of

$$I = \frac{9.0}{1500} = 6.0 \times 10^{-3}\,\text{A} = 6.0\,\text{mA}\,.$$

The output is the potential difference across the lower arm:

$$V_{out} = I\,R_{lower} = 6.0 \times 10^{-3} \times 500 = 3.0\,\text{V}\,.$$

∴ connecting the load pulls the output down from $4.5\,\text{V}$ to $3.0\,\text{V}$, while the battery current *rises* from $4.5\,\text{mA}$ to $6.0\,\text{mA}$. The load has reduced the lower arm's share of the supply potential difference — exactly the behaviour described in the Theory section. (An ideal voltmeter, drawing no current, would still read $4.5\,\text{V}$; a real voltmeter would read slightly below it.)

Practice questions

Questions 1–6 are scaffolded: the parts lead you through the working. Questions 7–18 are unscaffolded: for every calculation, set out the quantity, the relationship, the substitution and the unit in your own working.

Unless stated otherwise, treat all resistors as ohmic, all connecting wires as having zero resistance, and all meters as ideal.

Question 1 A divider across an $8.0\,\text{V}$ battery has $R_1 = 20\,\Omega$ and $R_2 = 60\,\Omega$, with the output taken across R_2. (a) Write the voltage divider equation for V_{out} in terms of V_{in}, R_1 and R_2. (b) Calculate the equivalent resistance of the divider and the current in it. (c) Calculate V_{out} and the potential difference across R_1, and check that the two shares add to the supply.

Question 2 A solar-powered weather station in a school garden has a $12\,\mathrm{V}$ battery, but its soil-moisture sensor needs $4.0\,\mathrm{V}$. The students build a divider with the sensor across R_2. (a) State the fraction of the supply potential difference that must appear at the output, and hence the required ratio of R_2 to R_1+R_2. (b) With $R_2=100\,\Omega$, calculate R_1. (c) Calculate the current in the divider and the power it draws from the battery.

Question 3 Two $470\,\Omega$ resistors in series are connected across a $9.0\,\mathrm{V}$ supply. (a) Show that the output taken across one of the resistors is $4.5\,\mathrm{V}$. (b) A third $470\,\Omega$ resistor is added in series. Calculate the potential difference across each resistor and the output taken across any two of them. (c) State, in general, the potential difference across each of n equal resistors in series across a supply V_{in}.

Question 4 A divider with $R_1=1.0\,\mathrm{k}\,\Omega$ and $R_2=3.0\,\mathrm{k}\,\Omega$ is connected across a $12\,\mathrm{V}$ battery, the output taken across R_2. (a) Calculate the current in the divider and the output potential difference. (b) Calculate the power released by each resistor. (c) State where this energy goes.

Question 5 A divider is built from a $5.0\,\mathrm{V}$ supply, a fixed $200\,\Omega$ resistor as R_1, and a variable resistor (range 0–$1.0\,\mathrm{k}\,\Omega$) as R_2, the output taken across the variable resistor. (a) Calculate the output when the variable resistor is set to $300\,\Omega$. (b) Calculate the setting that gives an output of $2.0\,\mathrm{V}$. (c) State the smallest and largest outputs available as the setting is turned through its range.

Question 6 (a) State what fixes the fraction of the supply potential difference that appears at the output of an unloaded divider. (b) Explain, with reference to the energy carried by each coulomb of charge, why the output of a simple divider is always less than the supply potential difference. (c) A divider of $330\,\Omega$ and $680\,\Omega$ is connected across a $6.0\,\mathrm{V}$ supply. State which resistor takes the larger share, and justify your answer by calculating the potential difference across each resistor.

Question 7 A school robotics club's microcontroller module needs about $3.3\,\mathrm{V}$, and the club's supply is a $5.0\,\mathrm{V}$ USB power pack. They build a divider with $R_1=1.0\,\mathrm{k}\,\Omega$ and $R_2=2.0\,\mathrm{k}\,\Omega$, the output across R_2. Calculate the output and comment on whether the divider suits the module.

Question 8 A sensor is to receive $2.0\,\mathrm{V}$ from a $9.0\,\mathrm{V}$ battery through a divider whose lower arm is $R_2=500\,\Omega$. Calculate R_1, and state the nearest common resistor value to your answer.

Question 9 Three resistors of $100\,\Omega$, $200\,\Omega$ and $300\,\Omega$ are connected in series across a $12\,\mathrm{V}$ battery. Calculate the potential difference across each resistor, and state across which single resistor an output of $4.0\,\mathrm{V}$ could be taken.

Question 10 A divider of two $100\,\Omega$ resistors is connected across a $12\,\mathrm{V}$ battery, the output taken across one of them. A load of $100\,\Omega$ is then connected across the output. Calculate (a) the unloaded output, (b) the loaded output, and (c) the current the battery supplies once the load is connected.

Question 11 In the fuel-gauge divider of Worked Example 3 ($12\,\mathrm{V}$ supply, fixed $60\,\Omega$ resistor, sender varying from $0\,\Omega$ to $120\,\Omega$), calculate the sender resistance at which the voltmeter reads $4.0\,\mathrm{V}$, and state what fraction of full that resistance represents.

Question 12 A divider across a $9.0\,\mathrm{V}$ battery has $R_1=4.0\,\mathrm{k}\,\Omega$ and gives an output of $6.0\,\mathrm{V}$ across R_2. Calculate R_2 and the current in the divider.

Question 13 With reference to $V=E/Q$, explain why the potential differences across R_1 and R_2 of a divider must add to the supply potential difference, and use this to justify the claim that the output of a simple divider can never exceed the supply.

Question 14 An unloaded divider draws current from its battery the whole time it is connected, even though nothing is being powered at the output. State where the energy goes, and justify why a divider is a poor way of running a device that draws a large current, such as an electric motor, compared with using a supply that provides the required potential difference directly.

Question 15 Explain, in your own words, how the float, the sender unit and the fixed resistor of the fuel gauge of Worked Example 3 turn a fuel level into a voltmeter reading. State what happens to the reading as fuel is used, and suggest why equal changes of fuel level do not produce equal changes of reading.

Question 16 Explain why connecting a device across the output of a divider reduces the output potential difference, referring to the resistance of the lower arm. Explain why the reduction is small when the device's resistance is much larger than R_2, and connect this to the way voltmeters are designed (5C).

Question 17 To obtain $3.0\,\mathrm{V}$ from a $9.0\,\mathrm{V}$ battery, a student may use the pair $10\,\Omega$ and $5\,\Omega$ or the pair $10\,\mathrm{k}\,\Omega$ and $5\,\mathrm{k}\,\Omega$. Both give the same unloaded output. Calculate the current drawn from the battery and the power taken from it in each case, and justify which pair is the better choice for a battery-powered device. State one disadvantage of the higher-resistance pair.

Question 18 Explain why the volume control of a radio uses a variable resistor rather than two fixed resistors, and state the range of outputs such an adjustable divider can provide from a fixed supply. Name two other devices that rely on an adjustable divider of this kind.

Answers

Question 1 (a) $V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$; (b) $80\,\Omega$, $0.10\,\text{A}$; (c) $6.0\,\text{V}$ and $2.0\,\text{V}$; $6.0 + 2.0 = 8.0\,\text{V}$ ✓. **Question 2** (a) $\frac{1}{3}$; (b) $200\,\Omega$; (c) $0.040\,\text{A}$, $0.48\,\text{W}$. **Question 3** (a) $4.5\,\text{V}$; (b) $3.0\,\text{V}$ each, $6.0\,\text{V}$ across any two; (c) $\frac{V_{in}}{n}$. **Question 4** (a) $3.0\,\text{mA}$, $9.0\,\text{V}$; (b) $9.0\,\text{mW}$ and $27\,\text{mW}$; (c) see solutions. **Question 5** (a) $3.0\,\text{V}$; (b) $130\,\Omega$ (to two significant figures); (c) $0\,\text{V}$ to $4.2\,\text{V}$. **Question 6** (a) the ratio $\frac{R_2}{R_1 + R_2}$; (b) see solutions; (c) the $680\,\Omega$ resistor; $4.0\,\text{V}$ against $2.0\,\text{V}$. **Question 7** $3.3\,\text{V}$; yes — close to the $3.3\,\text{V}$ the module needs. **Question 8** $1750\,\Omega$; the nearest common value is $1.8\,\text{k}\,\Omega$. **Question 9** $2.0\,\text{V}$, $4.0\,\text{V}$, $6.0\,\text{V}$; across the $200\,\Omega$ resistor. **Question 10** (a) $6.0\,\text{V}$; (b) $4.0\,\text{V}$; (c) $0.080\,\text{A}$. **Question 11** $30\,\Omega$; one quarter of full. **Question 12** $8.0\,\text{k}\,\Omega$; $0.75\,\text{mA}$. **Question 13** See solutions. **Question 14** See solutions. **Question 15** See solutions. **Question 16** See solutions. **Question 17** $0.60\,\text{A}$ and $5.4\,\text{W}$; $0.60\,\text{mA}$ and $5.4\,\text{mW}$; the higher-resistance pair; see solutions. **Question 18** See solutions.

Fully-worked solutions

Question 1 $V_{in} = 8.0\,\text{V}$, $R_1 = 20\,\Omega$, $R_2 = 60\,\Omega$. (a) $V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$. (b) $R_1 + R_2 = 80\,\Omega$; $I = \frac{8.0}{80} = 0.10\,\text{A}$. (c) $V_{out} = I\,R_2 = 0.10 \times 60 = 6.0\,\text{V}$; $V_1 = I\,R_1 = 0.10 \times 20 = 2.0\,\text{V}$; $6.0 + 2.0 = 8.0\,\text{V}$, the supply potential difference. ✓

Question 2 $V_{in} = 12\,\text{V}$, required output $4.0\,\text{V}$. (a) The fraction is $\frac{4.0}{12} = \frac{1}{3}$, so $\frac{R_2}{R_1 + R_2} = \frac{1}{3}$. (b) With $R_2 = 100\,\Omega$: $R_1 + R_2 = 3\,R_2 = 300\,\Omega$, so $R_1 = 200\,\Omega$. (c) $I = \frac{12}{300} = 0.040\,\text{A}$; $P = V\,I = 12 \times 0.040 = 0.48\,\text{W}$.

Question 3 (a) Equal resistors take equal shares, so each takes half: $V_{out} = \frac{9.0}{2} = 4.5\,\text{V}$. (Check: $I = \frac{9.0}{940} = 9.6 \times 10^{-3}\,\text{A}$; $V = I\,R = 9.6 \times 10^{-3} \times 470 = 4.5\,\text{V}$.) (b) $R_{total} = 3 \times 470 = 1410\,\Omega$; $I = \frac{9.0}{1410} = 6.4 \times 10^{-3}\,\text{A}$; each resistor takes $V = I\,R = 6.4 \times 10^{-3} \times 470 = 3.0\,\text{V}$, and any two in series take $3.0 + 3.0 = 6.0\,\text{V}$. (c) n equal resistors share the supply equally: each takes $\frac{V_{in}}{n}$.

Question 4 $V_{in} = 12\,\text{V}$, $R_1 = 1000\,\Omega$, $R_2 = 3000\,\Omega$. (a) $I = \frac{12}{4000} = 3.0 \times 10^{-3}\,\text{A}$; $V_{out} = I\,R_2 = 3.0 \times 10^{-3} \times 3000 = 9.0\,\text{V}$. (b) $V_1 = I\,R_1 = 3.0\,\text{V}$, so $P_1 = V_1\,I = 3.0 \times 3.0 \times 10^{-3} = 9.0\,\text{mW}$; $P_2 = V_{out}\,I = 9.0 \times 3.0 \times 10^{-3} = 27\,\text{mW}$. (Check: $P_1 + P_2 = 36\,\text{mW} = V_{in}\,I = 12 \times 3.0 \times 10^{-3}$.) (c) The energy is released as heat in the two resistors.

Question 5 $V_{in} = 5.0\,\text{V}$, $R_1 = 200\,\Omega$, variable R_2. (a) $V_{out} = 5.0 \times \frac{300}{200 + 300} = 3.0\,\text{V}$. (b) $2.0 = 5.0 \times \frac{R_2}{200 + R_2}$ gives $2.0(200 + R_2) = 5.0\,R_2$, so $400 = 3.0\,R_2$ and $R_2 = 133\,\Omega = 130\,\Omega$ (to two significant figures). (c) At the $0\,\Omega$ setting the output is $0\,\text{V}$; at the $1.0\,\text{k}\,\Omega$ setting, $V_{out} = 5.0 \times \frac{1000}{1200} = 4.2\,\text{V}$. The range is $0\,\text{V}$ to $4.2\,\text{V}$.

Question 6 (a) The fraction $\frac{R_2}{R_1 + R_2}$ — that is, the ratio of the two resistances. (b) $V = E / Q$ says that each coulomb leaves the supply carrying V_{in} joules and gives all of it up across the two resistors in series. The share given up across R_2 is part of that total, so the output potential difference is necessarily less than V_{in}. (c) The same current passes through both, and $V = I\,R$, so the larger resistance takes the larger share — the $680\,\Omega$ resistor.

$I=\frac{6.0}{1010}=5.9\times10^{-3}$ A; $V_{330}=IR=1.96\text{ V}=2.0\text{ V}$ (to two significant figures); $V_{680}=IR=4.04\text{ V}=4.0\text{ V}$ (to two significant figures).

Question 7 $V_{\text{out}}=5.0\times\frac{2000}{1000+2000}=3.3\text{ V}$ (to two significant figures). This is close to the 3.3 V the module needs, so the divider suits it while the module draws negligible current.

Question 8 The fraction must be $\frac{2.0}{9.0}$, so $\frac{500}{R_1+500}=\frac{2.0}{9.0}$, giving $R_1+500=\frac{9.0\times500}{2.0}=2250\,\Omega$ and $R_1=1750\,\Omega$. The nearest common resistor value is $1.8\text{ k}\Omega$, which would give $V_{\text{out}}=9.0\times\frac{500}{2300}=2.0\text{ V}$ (to two significant figures) — still suitable.

Question 9 $R_{\text{total}}=600\,\Omega$; $I=\frac{12}{600}=0.020\text{ A}$. Across the $100\,\Omega$ resistor: 2.0 V; across the $200\,\Omega$ resistor: 4.0 V; across the $300\,\Omega$ resistor: 6.0 V. An output of 4.0 V is taken across the $200\,\Omega$ resistor.

Question 10 (a) Equal resistors halve the supply: $V_{\text{out}}=6.0\text{ V}$. (b) Loaded, the lower arm is $\frac{100\times100}{200}=50\,\Omega$; the divider is $100\,\Omega$ above $50\,\Omega$, so $V_{\text{out}}=12\times\frac{50}{150}=4.0\text{ V}$. (c) $I=\frac{12}{150}=0.080\text{ A}$.

Question 11 $4.0=12\times\frac{R_s}{60+R_s}$ gives $4.0(60+R_s)=12R_s$, so $240=8.0R_s$ and $R_s=30\,\Omega$. With resistance proportional to fuel level, $\frac{30}{120}=\frac{1}{4}$: the gauge reads 4.0 V at a quarter tank.

Question 12 The fraction is $\frac{6.0}{9.0}=\frac{2}{3}$, so R_2 must be two-thirds of the total: $\frac{R_2}{4000+R_2}=\frac{2}{3}$ gives $3R_2=8000+2R_2$, so $R_2=8000\,\Omega=8.0\text{ k}\Omega$. $I=\frac{9.0}{12\,000}=7.5\times10^{-4}\text{ A}=0.75\text{ mA}$.

Question 13 Model answer. $V=E/Q$ is the energy transferred per coulomb. A coulomb that goes once around the loop leaves the supply carrying V_{in} joules and returns to the supply having given all of that energy up — part across R_1, the rest across R_2. Energy is conserved, so the two transfers must add to the total: $V_1+V_2=V_{\text{in}}$. Since the share given up across R_2 is a *part* of the total V_{in} joules per coulomb, the output potential difference can never exceed the supply potential difference; a divider cannot hand a coulomb more energy across one resistor than the coulomb received from the supply.

Question 14 Model answer. The current the divider draws passes through R_1 and R_2 continuously, and in a resistor the electrical energy is transferred to the surroundings as heat, at the rate $P=VI$. Even with nothing connected at the output, this heating continues for as long as the divider is switched on. For a device that draws a large current, the divider would have to carry that same current, and the power released as heat in its resistors would be large — most of the supply's energy would warm the divider rather than drive the device, the battery would run down quickly, and the resistors could overheat. A supply providing the required potential difference directly passes no current through a divider at all, so none of this energy is wasted.

Question 15 Model answer. The float rides on the fuel surface, so as the level changes the float moves the contact and changes the sender's resistance. The sender and the fixed resistor form a voltage divider across the 12 V battery, with the voltmeter reading the sender's share of the supply potential difference. As fuel is used the level falls, the sender resistance decreases, the sender takes a smaller share of the 12 V, and the reading falls — so the voltmeter can be marked as a fuel gauge. Equal changes of fuel level give equal changes of sender resistance, but the divider's output is

$V_{\text{in}} \times \dfrac{R_s}{60+R_s}$, which is not proportional to R_s, so equal resistance changes give unequal voltage changes and the marked scale is not evenly spaced.

Question 16 Model answer. The device joins the two output connections, so it is in parallel with R_2, and the resistance of the lower arm becomes the parallel equivalent of R_2 and the device, which is smaller than R_2 alone (5E). A smaller lower arm takes a smaller share of the supply potential difference, so the output falls. If the device's resistance is much larger than R_2, the parallel equivalent is only slightly below R_2, the lower arm's share barely changes, and the output barely falls. This is exactly why voltmeters are built with a very high resistance (5C): a high-resistance voltmeter loads the circuit it measures only very slightly, so its reading is close to the potential difference that existed before it was connected.

Question 17 Both pairs give $\dfrac{R_2}{R_1+R_2}=\dfrac{1}{3}$, so $V_{\text{out}}=3.0\,\text{V}$. Low-resistance pair: $I=\dfrac{9.0}{15}=0.60\,\text{A}$; $P=V_{\text{in}}I=9.0\times0.60=5.4\,\text{W}$. High-resistance pair: $I=\dfrac{9.0}{15\,000}=6.0\times10^{-4}\,\text{A}=0.60\,\text{mA}$; $P=9.0\times6.0\times10^{-4}=5.4\times10^{-3}\,\text{W}=5.4\,\text{mW}$. Model answer for the justification: the high-resistance pair is the better choice for a battery-powered device. It takes one thousandth of the current and one thousandth of the power, so it wastes one thousandth of the battery's energy and the battery lasts far longer; a small battery could not sustain $0.60\,\text{A}$ for long in any case. The disadvantage of the high-resistance pair is that its output is more easily pulled down by a load connected across it (Question 16): the same load resistance disturbs a high-resistance divider far more than a low-resistance one.

Question 18 Model answer. A fixed pair of resistors gives one fixed output; a volume control must be *adjustable*, so the listener can choose any loudness. A variable resistor makes the divider's ratio adjustable, so the output can be set anywhere in a continuous range rather than at one value. Turned through its range, an adjustable divider can provide outputs from $0\,\text{V}$ (the variable arm set to zero) up to nearly the full supply potential difference (exactly V_{in} if the other arm can be set to zero, or if the variable arm is made much larger than it). Other devices using an adjustable divider of this kind include a lamp dimmer and an adjustable bench (laboratory) power supply; a joystick or a throttle position sensor works the same way. (The variable-resistor component itself — the potentiometer — is treated in 6C.)