# Mathematical Methods Units 1 & 2

## Chapter 2 — Coordinate geometry and straight-line graphs



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### Contents

## Chapter 2 Coordinate geometry and straight-line graphs — 2A The distance and midpoint formulas

### Theory

Every point in the plane has coordinates $(x, y)$. This section finds the **distance** between two points and the **midpoint** of the interval joining them.

**Distance between two points.** For $A(x_1, y_1)$ and $B(x_2, y_2)$, dropping a horizontal and a vertical line creates a right-angled triangle with legs $x_2 - x_1$ and $y_2 - y_1$. By Pythagoras' theorem the length of the interval $AB$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Because each difference is **squared**, it does not matter which point is called $A$ and which is $B$. Give the answer as an **exact value** (a simplified surd) unless a decimal is asked for.

**Midpoint of an interval.** The midpoint $M$ of the interval joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is the average of the coordinates:

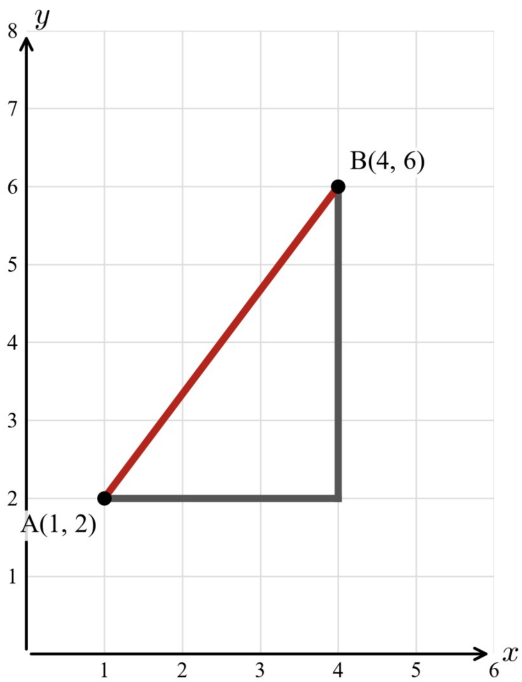
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

### Worked examples

#### *Example 1 — scaffolded*

Find the distance between $A(1, 2)$ and $B(4, 6)$.

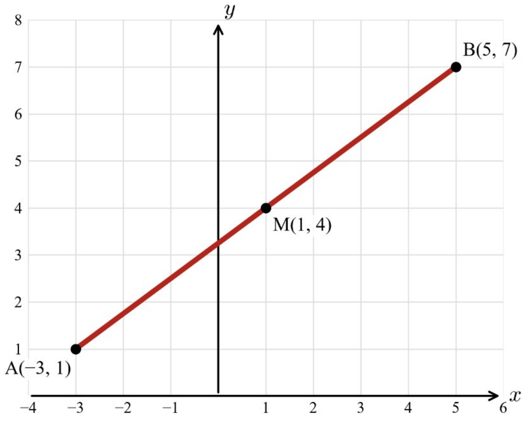
| Working | Comment |
|---|---|
| $x_1 = 1, y_1 = 2, x_2 = 4, y_2 = 6.$ | Label the coordinates. |
| $d = \sqrt{(4-1)^2 + (6-2)^2}$ | Substitute into the distance formula. |
| $= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$ | Evaluate each difference, then square. |
| $= 5$ | Simplify the surd (here it is exact). |



***Example 2 — scaffolded***

Find the midpoint of the interval joining $A(-3,1)$ and $B(5,7)$.

| Working | Comment |
|---|---|
| $M=\left(\frac{-3+5}{2},\frac{1+7}{2}\right)$ | Substitute into the midpoint formula. |
| $=\left(\frac{2}{2},\frac{8}{2}\right)$ | Add the coordinates. |
| $=(1,4)$ | Simplify. |

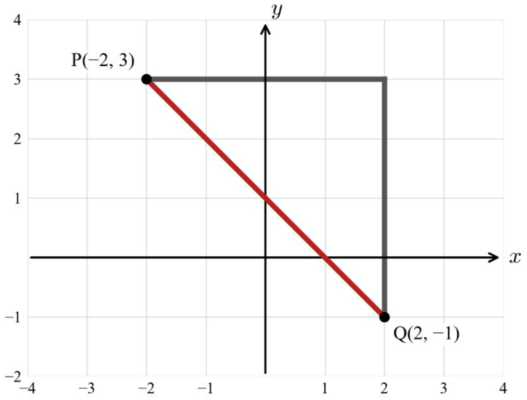



***Example 3 — unscaffolded***

Find the exact distance between $P(-2,3)$ and $Q(2,-1)$.

$$d=\sqrt{(2-(-2))^2+(-1-3)^2}=\sqrt{4^2+(-4)^2}=\sqrt{16+16}=\sqrt{32}.$$

Since $32=16\times 2$, $\sqrt{32}=4\sqrt{2}$. $\therefore$ the distance is $4\sqrt{2}$.



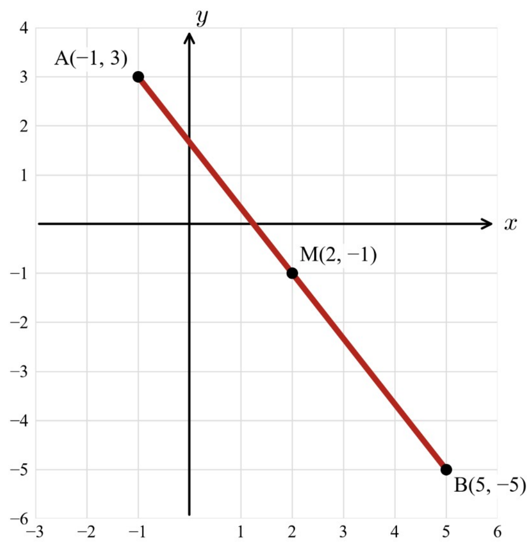

***Example 4 — unscaffolded***

The midpoint of the interval $AB$ is $M(2,-1)$. Given $A(-1,3)$, find the coordinates of $B$.

Let $B=(x,y)$. Then $\dfrac{-1+x}{2}=2$ and $\dfrac{3+y}{2}=-1$.

So $-1+x=4\Rightarrow x=5$, and $3+y=-2\Rightarrow y=-5$.

$\therefore B=(5,-5)$.




## Practice questions

### *Scaffolded*

**Q1.** Find the distance between $A(0,0)$ and $B(8,15)$.

**Q2.** Find the distance between $A(1,1)$ and $B(7,9)$.

**Q3.** Find the midpoint of the interval joining $A(2,4)$ and $B(8,10)$.

**Q4.** Find the midpoint of the interval joining $A(-4,2)$ and $B(2,-6)$.

**Q5.** Find the exact distance between the origin and $B(3,3)$.

**Q6.** The midpoint of $AB$ is the origin $(0,0)$ and $A=(-3,4)$. Find $B$.

### *Unscaffolded*

**Q7.** Find the exact distance between $(2,3)$ and $(4,8)$.

**Q8.** Find the distance between $(-1,-2)$ and $(4,10)$.

**Q9.** Find the exact distance between $(1,2)$ and $(4,8)$.

**Q10.** Find the exact distance between $(-2,3)$ and $(3,-2)$.

**Q11.** Find the midpoint of the interval joining $(3,-5)$ and $(9,1)$.

**Q12.** Find the midpoint of the interval joining $(-7,4)$ and $(3,-2)$.

**Q13.** The midpoint of $AB$ is $(4,1)$ and $A=(1,-2)$. Find $B$.

**Q14.** The midpoint of $PQ$ is $(-1,2)$ and $P=(3,-4)$. Find $Q$.

**Q15.** A triangle has vertices $A(1,1)$, $B(5,1)$ and $C(3,5)$. By finding the side lengths, show that the triangle is isosceles.

**Q16.** Show that the point $P(1,4)$ is equidistant from $A(0,1)$ and $B(4,3)$.

**Q17.** Two towns are at $A(2,5)$ and $B(7,11)$ on a map, where each unit represents one kilometre. Find the straight-line distance between the towns.

**Q18.** For $A(-4,-2)$ and $B(6,4)$, find the midpoint of $AB$ and the exact length of $AB$.

## Answers

**Q1.** $17$. **Q2.** $10$. **Q3.** $(5,7)$. **Q4.** $(-1,-2)$. **Q5.** $3\sqrt{2}$. **Q6.** $(3,-4)$. **Q7.** $\sqrt{29}$. **Q8.** $13$. **Q9.** $3\sqrt{5}$. **Q10.** $5\sqrt{2}$. **Q11.** $(6,-2)$. **Q12.** $(-2,1)$. **Q13.** $(7,4)$. **Q14.** $(-5,8)$. **Q15.** $AC=BC=2\sqrt{5}$, so isosceles. **Q16.** $PA=PB=\sqrt{10}$. **Q17.** $\sqrt{61}$ km. **Q18.** midpoint $(1,1)$; length $2\sqrt{34}$.

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## Fully-worked solutions

**Q1.** $d=\sqrt{(8-0)^2+(15-0)^2}=\sqrt{64+225}=\sqrt{289}=17$.

**Q2.** $d=\sqrt{(7-1)^2+(9-1)^2}=\sqrt{36+64}=\sqrt{100}=10$.

**Q3.** $M=\left(\dfrac{2+8}{2},\dfrac{4+10}{2}\right)=(5,7)$.

**Q4.** $M=\left(\dfrac{-4+2}{2},\dfrac{2+(-6)}{2}\right)=(-1,-2)$.

**Q5.** $d=\sqrt{3^2+3^2}=\sqrt{18}=3\sqrt{2}$.

**Q6.** Let $B=(x,y)$: $\dfrac{-3+x}{2}=0\Rightarrow x=3$; $\dfrac{4+y}{2}=0\Rightarrow y=-4$. $\therefore B=(3,-4)$.

**Q7.** $d=\sqrt{(4-2)^2+(8-3)^2}=\sqrt{4+25}=\sqrt{29}$.

**Q8.** $d=\sqrt{(4-(-1))^2+(10-(-2))^2}=\sqrt{25+144}=\sqrt{169}=13$.

**Q9.** $d=\sqrt{(4-1)^2+(8-2)^2}=\sqrt{9+36}=\sqrt{45}=3\sqrt{5}$.

**Q10.** $d=\sqrt{(3-(-2))^2+(-2-3)^2}=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$.

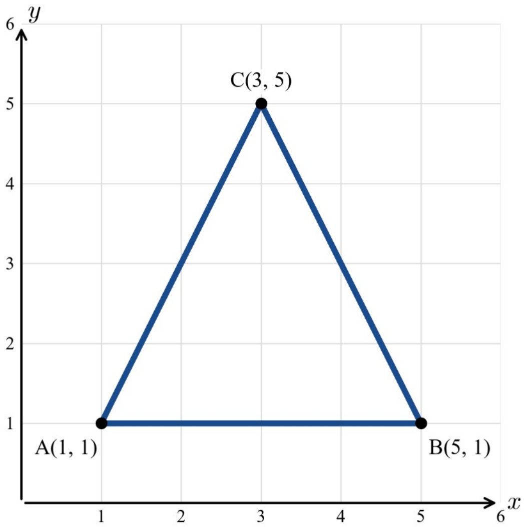
**Q11.** $M=\left(\dfrac{3+9}{2},\dfrac{-5+1}{2}\right)=(6,-2)$.

**Q12.** $M=\left(\dfrac{-7+3}{2},\dfrac{4+(-2)}{2}\right)=(-2,1)$.

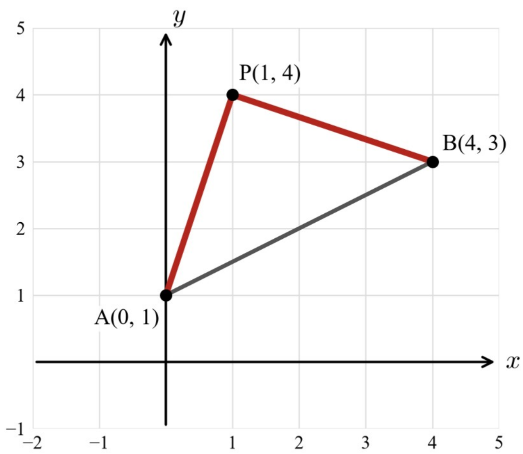
**Q13.** Let $B=(x,y)$: $\dfrac{1+x}{2}=4\Rightarrow x=7$; $\dfrac{-2+y}{2}=1\Rightarrow y=4$. $\therefore B=(7,4)$.

**Q14.** Let $Q=(x,y)$: $\dfrac{3+x}{2}=-1\Rightarrow x=-5$; $\dfrac{-4+y}{2}=2\Rightarrow y=8$. $\therefore Q=(-5,8)$.

**Q15.** $AB=\sqrt{(5-1)^2+0^2}=4$; $AC=\sqrt{(3-1)^2+(5-1)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$;
$BC=\sqrt{(3-5)^2+(5-1)^2}=\sqrt{4+16}=2\sqrt{5}$. Since $AC=BC$, the triangle is isosceles.




**Q16.** $PA = \sqrt{(1-0)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$; $PB = \sqrt{(4-1)^2 + (3-4)^2} = \sqrt{9+1} = \sqrt{10}$. Since $PA = PB$, $P$ is equidistant from $A$ and $B$.




**Q17.** $d = \sqrt{(7-2)^2 + (11-5)^2} = \sqrt{25+36} = \sqrt{61}$. $\therefore$ the towns are $\sqrt{61}$ km apart.

**Q18.** Midpoint $= \left(\frac{-4+6}{2}, \frac{-2+4}{2}\right) = (1, 1)$; $AB = \sqrt{(6-(-4))^2 + (4-(-2))^2} = \sqrt{100+36} = \sqrt{136} = 2\sqrt{34}$.

### Theory

The **gradient** of a straight line measures its steepness. For two points $(x_1, y_1)$ and $(x_2, y_2)$ on the line, the gradient is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}.$$

- A **positive** gradient rises to the right; a **negative** gradient falls to the right.
- A **horizontal** line has gradient $m = 0$ (no rise).
- A **vertical** line has an **undefined** gradient (the run is $0$, so the fraction is undefined).

The gradient is also related to the line's **angle of inclination** $\theta$ (the angle the line makes with the positive direction of the $x$-axis, measured anticlockwise, $0° \le \theta < 180°$):

$$m = \tan\theta.$$

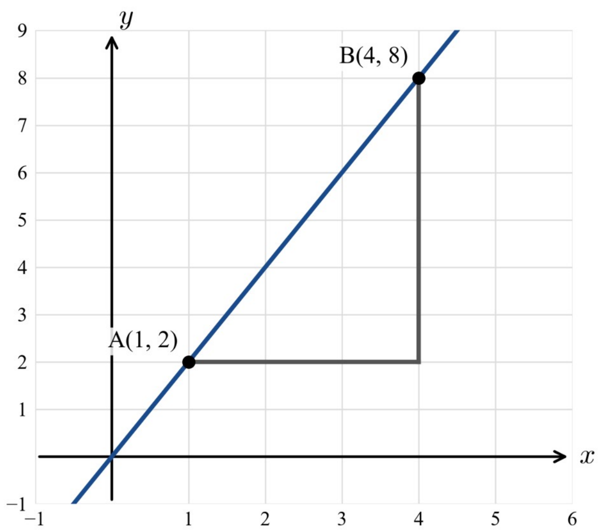
It does not matter which point is labelled $(x_1, y_1)$, provided the coordinates are subtracted in the same order in the numerator and denominator.

### Worked examples

#### *Example 1 — scaffolded*

Find the gradient of the line through $A(1, 2)$ and $B(4, 8)$.

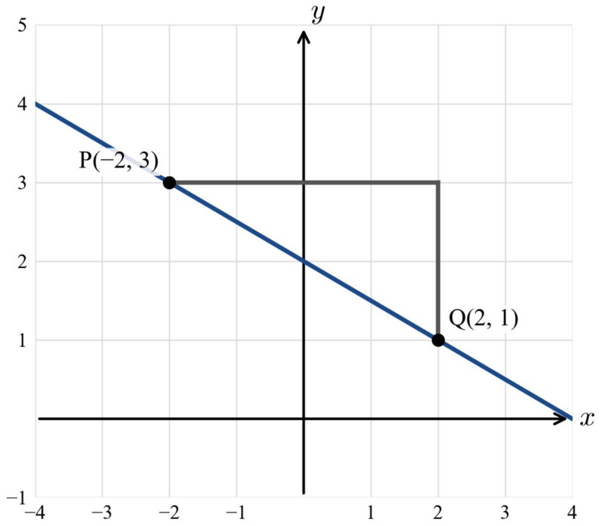
| Working | Comment |
|---|---|
| Let $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (4, 8)$. | Label the two points. |
| $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{4-1}$ | Substitute into the gradient formula. |
| $= \frac{6}{3} = 2$ | rise $= 6$, run $= 3$. |




#### *Example 2 — scaffolded*

Find the gradient of the line through $P(-2, 3)$ and $Q(2, 1)$.

| Working | Comment |
| --- | --- |
| Let $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (2, 1)$. | Label the points. |
| $m = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4}$ | Take care with the double negative in the denominator. |
| $= -\dfrac{1}{2}$ | The negative gradient means the line falls to the right. |

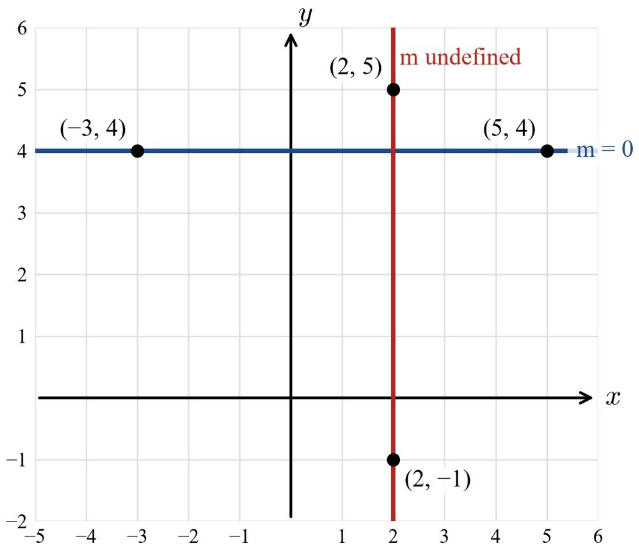



### *Example 3 — unscaffolded*

Find the gradient of the line through **(a)** $(-3, 4)$ and $(5, 4)$; **(b)** $(2, -1)$ and $(2, 5)$.

**(a)** $m = \dfrac{4-4}{5-(-3)} = \dfrac{0}{8} = 0$. The line is **horizontal**.

**(b)** $m = \dfrac{5-(-1)}{2-2} = \dfrac{6}{0}$, which is **undefined**. The line is **vertical**.




### *Example 4 — unscaffolded*

The line through $(1, 3)$ and $(a, 9)$ has gradient 2. Find the value of $a$.

$$m = \frac{9-3}{a-1} = 2 \Rightarrow \frac{6}{a-1} = 2 \Rightarrow 6 = 2(a-1) \Rightarrow a-1 = 3.$$

$\therefore a = 4.$

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## Practice questions

### *Scaffolded*

**Q1.** Find the gradient of the line through $(2,1)$ and $(5,4)$.

**Q2.** Find the gradient of the line through $(-1,4)$ and $(3,-4)$.

**Q3.** Find the gradient of the line through the origin and $(4,3)$.

**Q4.** Find the gradient of the line through $(-2,-5)$ and $(4,-5)$, and describe the line.

**Q5.** Find the gradient of the line through $(3,1)$ and $(3,8)$, and describe the line.

**Q6.** A line has gradient 3 and passes through $(1,2)$. Find the $y$-coordinate of the point on the line with $x = 4$.

### *Unscaffolded*

Find the gradient of the line through each pair of points (Q7–Q12), then complete Q13–Q18 as directed.

**Q7.** $(1,3)$ and $(3,-3)$

**Q8.** $(-1,5)$ and $(1,-3)$

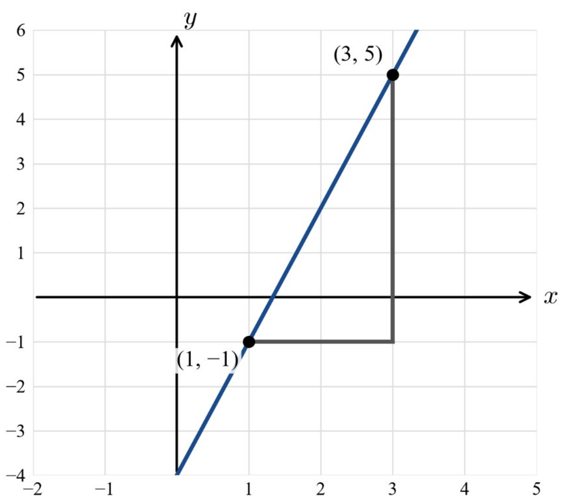
**Q9.** $(-3,-2)$ and $(3,2)$

**Q10.** $(5,2)$ and $(-1,-1)$

**Q11.** $(-4,3)$ and $(6,3)$

**Q12.** $(-2,1)$ and $(-2,9)$

**Q13.** Find the gradient of the line shown in the graph below.




**Q14.** The line through $(2,5)$ and $(a,11)$ has gradient 2. Find $a$.

**Q15.** The line through $(1,b)$ and $(4,10)$ has gradient 1. Find $b$.

**Q16.** Find the angle of inclination of a line with gradient 1.

**Q17.** Find the angle of inclination of a line with gradient $-1$.

**Q18.** Find the gradient of the line through $(-1,-4)$ and $(3,2)$.

## Answers

**Q1.** 1. **Q2.** $-2$. **Q3.** $3/4$. **Q4.** 0; horizontal line. **Q5.** undefined; vertical line. **Q6.** 11. **Q7.** $-3$. **Q8.** $-4$. **Q9.** $2/3$. **Q10.** $1/2$. **Q11.** 0. **Q12.** undefined. **Q13.** 3. **Q14.** $a=5$. **Q15.** $b=7$. **Q16.** $45°$. **Q17.** $135°$. **Q18.** $3/2$.

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## Fully-worked solutions

**Q1.** $m=\dfrac{4-1}{5-2}=\dfrac{3}{3}=1$.

**Q2.** $m=\dfrac{-4-4}{3-(-1)}=\dfrac{-8}{4}=-2$.

**Q3.** $m=\dfrac{3-0}{4-0}=\dfrac{3}{4}$.

**Q4.** $m=\dfrac{-5-(-5)}{4-(-2)}=\dfrac{0}{6}=0$. The line is **horizontal**.

**Q5.** $m=\dfrac{8-1}{3-3}=\dfrac{7}{0}$, which is **undefined**. The line is **vertical**.

**Q6.** $m=\dfrac{y-2}{4-1}=3 \Rightarrow y-2=9 \Rightarrow y=11$.

**Q7.** $m=\dfrac{-3-3}{3-1}=\dfrac{-6}{2}=-3$.

**Q8.** $m=\dfrac{-3-5}{1-(-1)}=\dfrac{-8}{2}=-4$.

**Q9.** $m=\dfrac{2-(-2)}{3-(-3)}=\dfrac{4}{6}=\dfrac{2}{3}$.

**Q10.** $m=\dfrac{-1-2}{-1-5}=\dfrac{-3}{-6}=\dfrac{1}{2}$.

**Q11.** $m=\dfrac{3-3}{6-(-4)}=\dfrac{0}{10}=0$.

**Q12.** $m=\dfrac{9-1}{-2-(-2)}=\dfrac{8}{0}$, which is **undefined**.

**Q13.** The line passes through $(1,-1)$ and $(3,5)$, so $m=\dfrac{5-(-1)}{3-1}=\dfrac{6}{2}=3$.

**Q14.** $\dfrac{11-5}{a-2}=2 \Rightarrow \dfrac{6}{a-2}=2 \Rightarrow a-2=3 \Rightarrow a=5$.

**Q15.** $\dfrac{10-b}{4-1}=1 \Rightarrow 10-b=3 \Rightarrow b=7$.

**Q16.** $\tan\theta=1 \Rightarrow \theta=45°$.

**Q17.** $\tan\theta=-1$. Since the angle of inclination satisfies $0° \le \theta < 180°$, $\theta=180°-45°=135°$.

**Q18.** $m=\dfrac{2-(-4)}{3-(-1)}=\dfrac{6}{4}=\dfrac{3}{2}$.

## Chapter 2 Coordinate geometry and straight-line graphs — 2C Equations of straight lines

### Theory

Every non-vertical straight line can be written as $y = mx + c$.

**Gradient–intercept form.** In $y = mx + c$, the number $m$ is the **gradient** and $c$ is the $y$-intercept (the line cuts the $y$-axis at $(0, c)$). If you know $m$ and $c$, write the equation directly.

**Point–gradient form.** If you know the gradient $m$ and one point $(x_1, y_1)$ on the line, use

$$y - y_1 = m(x - x_1),$$

then rearrange into gradient–intercept form.

**From two points.** If the line passes through $(x_1, y_1)$ and $(x_2, y_2)$, first find the gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

then substitute $m$ and either point into the point–gradient form.

**Horizontal and vertical lines.** A **horizontal** line has gradient $0$ and equation $y = c$. A **vertical** line has an undefined gradient and equation $x = a$.

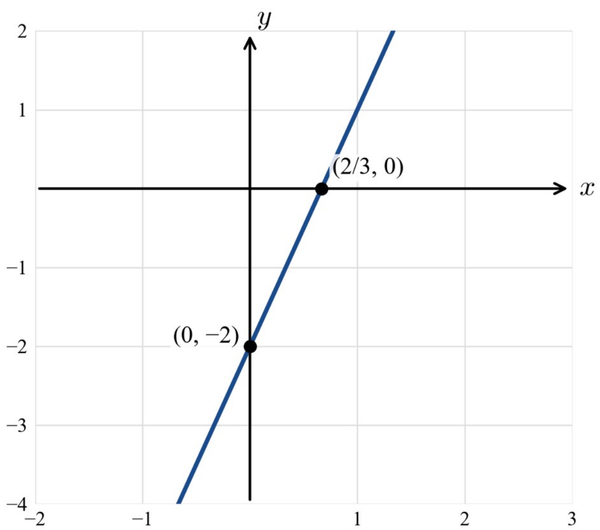
**General form.** A line may also be written in the general form $ax + by + c = 0$ with integer coefficients. To convert, clear fractions and move all terms to one side.

### Worked examples

#### *Example 1 — scaffolded*

Find the equation of the line with gradient $3$ and $y$-intercept $-2$, and sketch it.

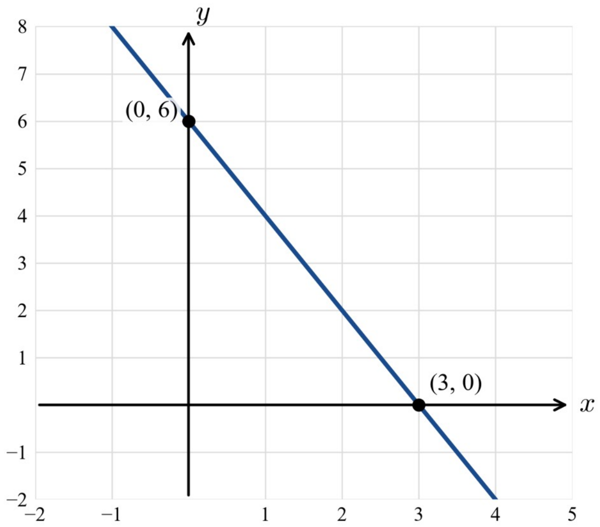
| Working | Comment |
|---|---|
| Gradient $m = 3$; $y$-intercept $c = -2$. | Read the two values from the question. |
| $y = mx + c = 3x - 2$. | Substitute into gradient–intercept form. |



***Example 2 — scaffolded***

Find the equation of the line with gradient $-2$ that passes through $(1,4)$.

| Working | Comment |
|---|---|
| $m=-2$, and $(x_1,y_1)=(1,4)$. | Identify the gradient and the point. |
| $y-4=-2(x-1)$. | Substitute into the point–gradient form $y-y_1=m(x-x_1)$. |
| $y-4=-2x+2$, so $y=-2x+6$. | Expand and make $y$ the subject. |



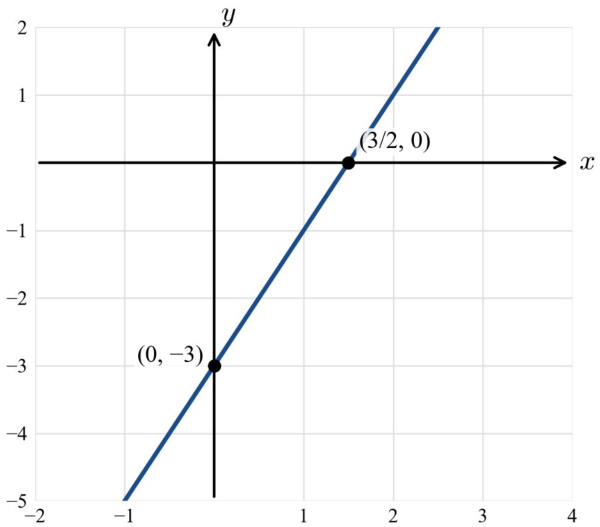

***Example 3 — unscaffolded***

Find the equation of the line passing through $(2,1)$ and $(6,9)$.

$m=\frac{9-1}{6-2}=\frac{8}{4}=2$.

Using $(2,1)$: $y-1=2(x-2)\Rightarrow y-1=2x-4$.

$\therefore y=2x-3$.



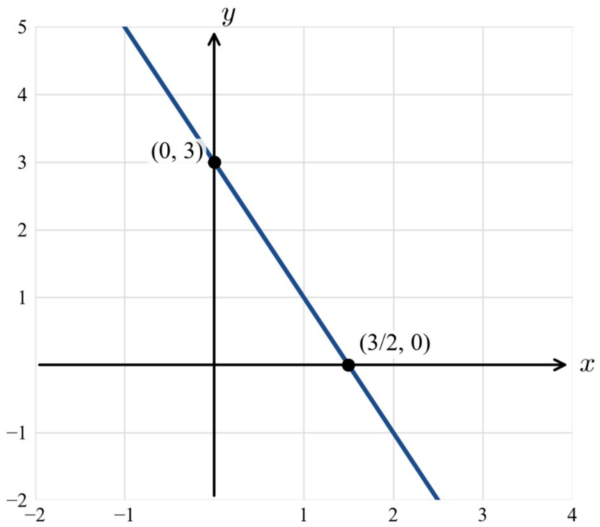

***Example 4 — unscaffolded***

Find the equation of the line through $(-1,5)$ and $(3,-3)$, giving your answer in general form.

$m = \frac{-3-5}{3-(-1)} = \frac{-8}{4} = -2$.

Using $(-1, 5)$: $y - 5 = -2(x+1) \Rightarrow y = -2x + 3$.

$\therefore 2x + y - 3 = 0$.

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## Practice questions

### *Scaffolded*

**Q1.** For the line with gradient $4$ and $y$-intercept $-1$: (a) state $m$ and $c$; (b) write the equation; (c) sketch it.

**Q2.** For the line with gradient $-3$ passing through $(0, 5)$: (a) state $m$ and $c$; (b) write the equation; (c) sketch it.

**Q3.** For the line with gradient $2$ passing through $(1, 3)$: (a) write the point–gradient form; (b) simplify to gradient–intercept form; (c) sketch it.

**Q4.** For the line through $(0, 4)$ and $(2, 10)$: (a) find the gradient; (b) find the equation; (c) sketch it.

**Q5.** For the line through $(-2, 1)$ and $(4, 4)$: (a) find the gradient; (b) find the equation; (c) sketch it.

**Q6.** Write the equations of the horizontal and vertical lines through $(3, -2)$, and sketch both on the one set of axes.

### *Unscaffolded*

Find the equation of each line (in gradient–intercept form unless otherwise stated), and sketch it.

**Q7.** Gradient $5$ and $y$-intercept $2$.

**Q8.** Gradient $-1$, passing through $(0, -4)$.

**Q9.** Gradient $4$, passing through $(2, 5)$.

**Q10.** Gradient $-2$, passing through $(-1, 3)$.

**Q11.** Passing through $(1, 2)$ and $(4, 11)$.

**Q12.** Passing through $(-2, 6)$ and $(2, -2)$.

**Q13.** Passing through $(-1, -1)$ and $(3, -5)$.

**Q14.** Passing through $(-3, -1)$ and $(3, 3)$.

**Q15.** Passing through $(0, -2)$ and $(2, 1)$.

**Q16.** The horizontal line through $(-4, 6)$.

**Q17.** The vertical line through $(-4, 6)$.

**Q18.** Passing through $(1, -2)$ with gradient $3/4$; give your answer in general form.

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### Answers

**Q1.** $y=4x-1$. **Q2.** $y=-3x+5$. **Q3.** $y=2x+1$. **Q4.** $y=3x+4$. **Q5.** $y=1/2x+2$. **Q6.** $y=-2$ and $x=3$. **Q7.** $y=5x+2$. **Q8.** $y=-x-4$. **Q9.** $y=4x-3$. **Q10.** $y=-2x+1$. **Q11.** $y=3x-1$. **Q12.** $y=-2x+2$. **Q13.** $y=-x-2$. **Q14.** $y=2/3x+1$. **Q15.** $y=3/2x-2$. **Q16.** $y=6$. **Q17.** $x=-4$. **Q18.** $3x-4y-11=0$.

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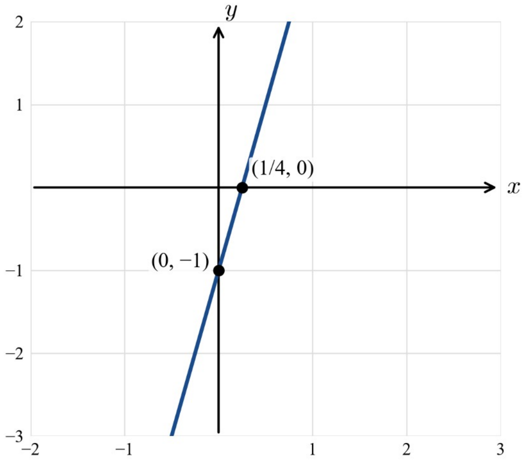
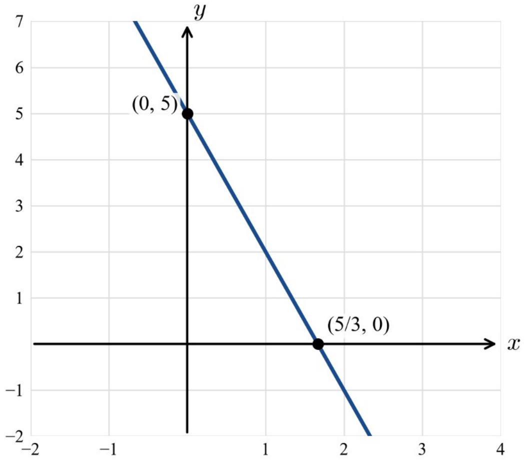
### Fully-worked solutions

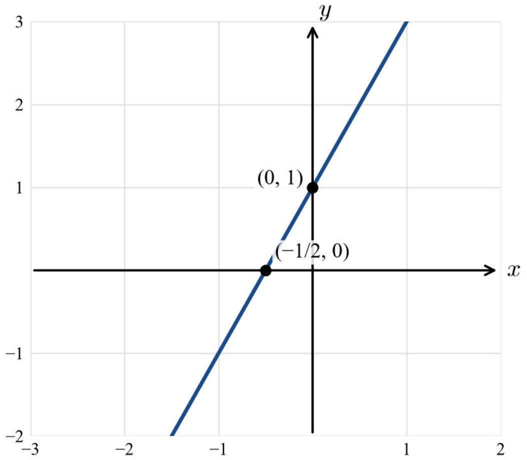
**Q1.** $m=4$, $c=-1$, so $y=4x-1$.



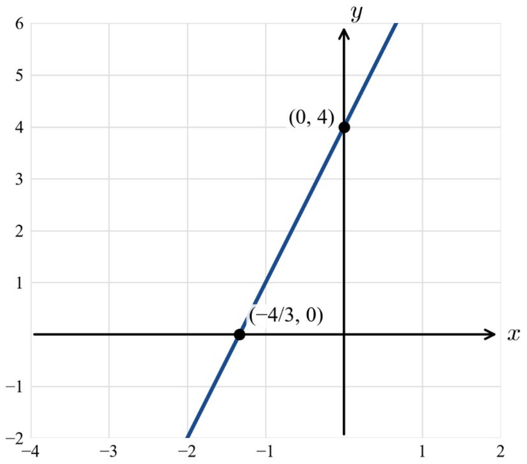

**Q2.** $m=-3$, $c=5$, so $y=-3x+5$.



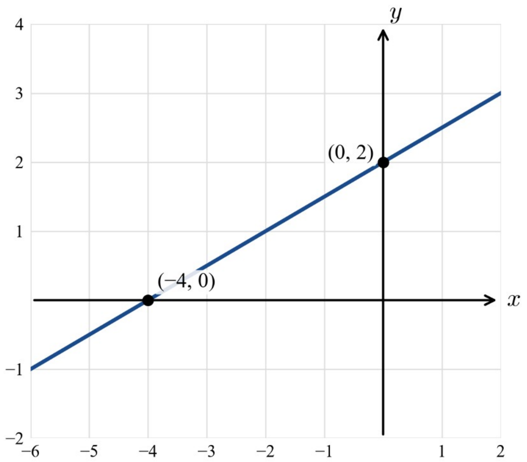

**Q3.** $y-3=2(x-1) \Rightarrow y-3=2x-2 \Rightarrow y=2x+1$.



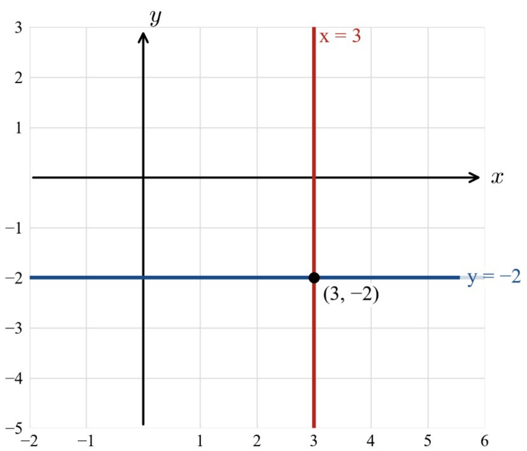
**Q4.** $m = \dfrac{10-4}{2-0} = 3$; the $y$-intercept is 4, so $y = 3x + 4$.



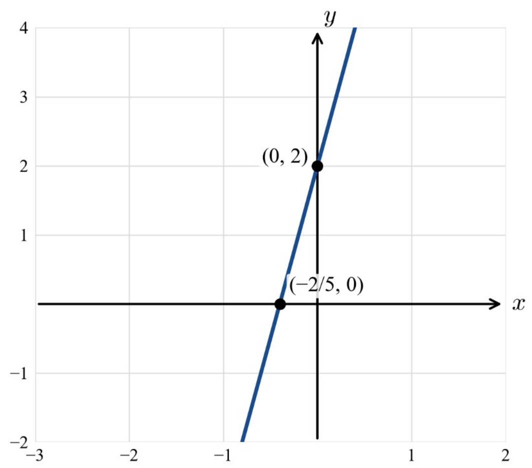

**Q5.** $m = \dfrac{4-1}{4-(-2)} = \dfrac{3}{6} = 1/2$. Using $(-2, 1)$: $y - 1 = 1/2(x+2) \Rightarrow y = 1/2x + 2$.



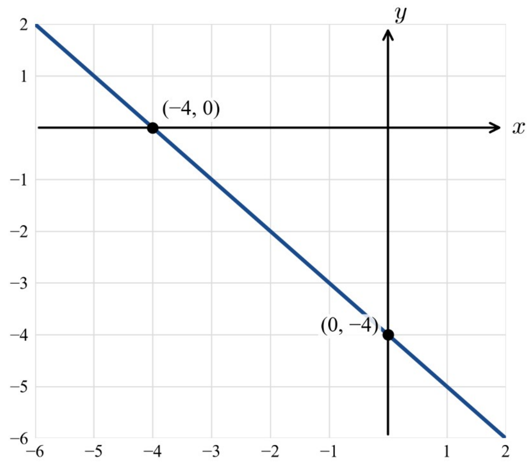

**Q6.** A horizontal line through $(3, -2)$ has equation $y = -2$; a vertical line through $(3, -2)$ has equation $x = 3$.



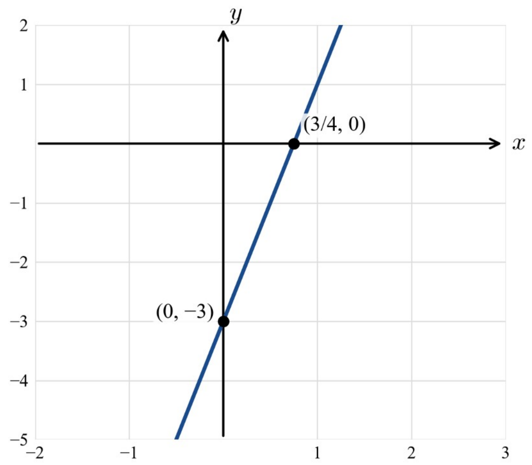

**Q7.** $y = 5x + 2$.



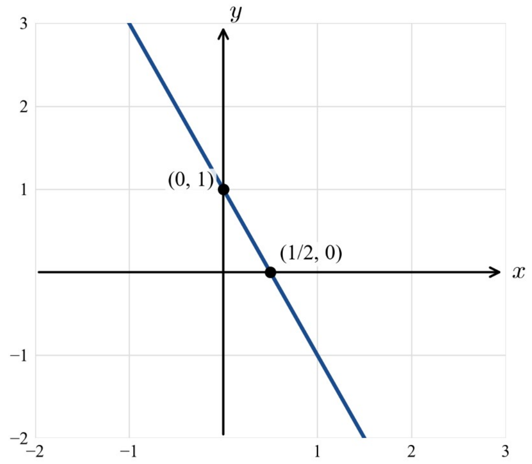

**Q8.** $c=-4$, so $y=-x-4$.



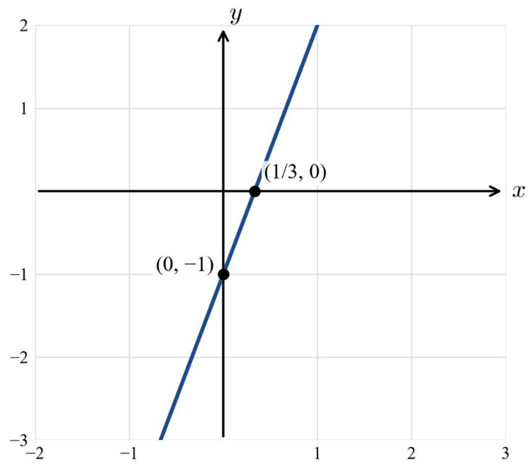

**Q9.** $y-5=4(x-2)\Rightarrow y=4x-3$.



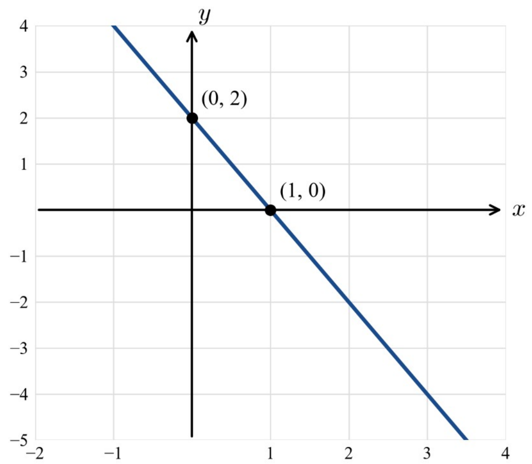

**Q10.** $y-3=-2(x+1)\Rightarrow y=-2x+1$.



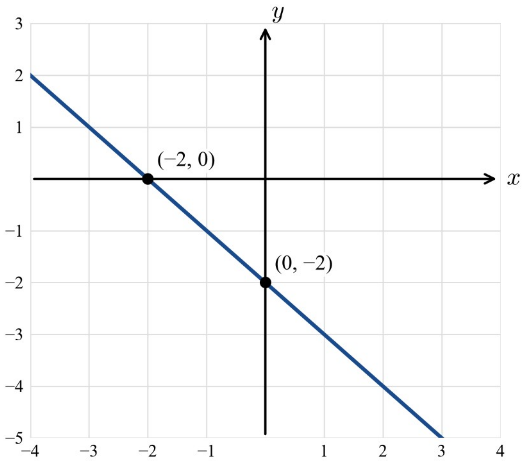

**Q11.** $m=\frac{11-2}{4-1}=3$. Using $(1,2)$: $y-2=3(x-1)\Rightarrow y=3x-1$.



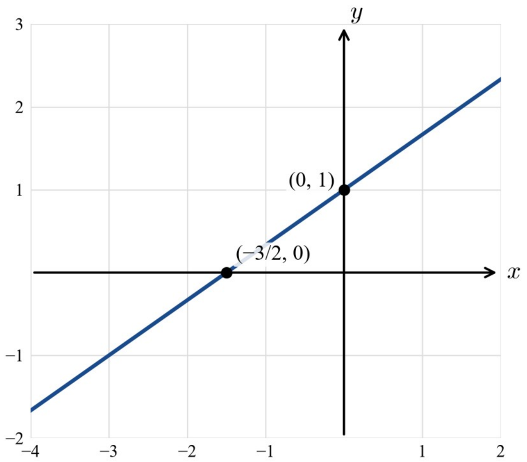

**Q12.** $m=\frac{-2-6}{2-(-2)}=\frac{-8}{4}=-2$. Using $(2,-2)$: $y+2=-2(x-2)\Rightarrow y=-2x+2$.



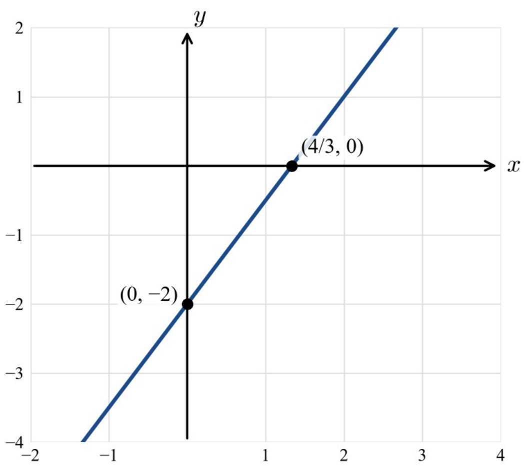

**Q13.** $m=\frac{-5-(-1)}{3-(-1)}=\frac{-4}{4}=-1$. Using $(-1,-1)$: $y+1=-(x+1)\Rightarrow y=-x-2$.



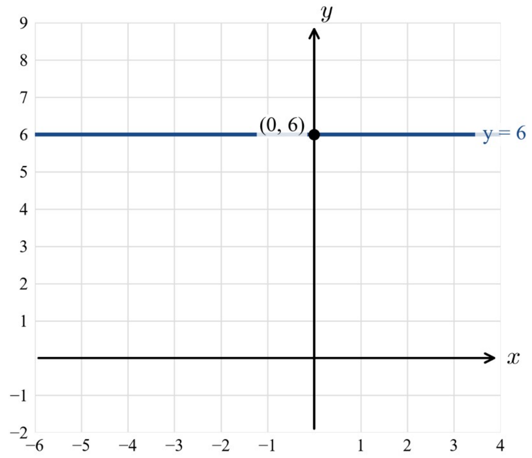

**Q14.** $m=\dfrac{3-(-1)}{3-(-3)}=\dfrac{4}{6}=2/3$. Using $(3,3)$: $y-3=2/3(x-3)\Rightarrow y=2/3x+1$.



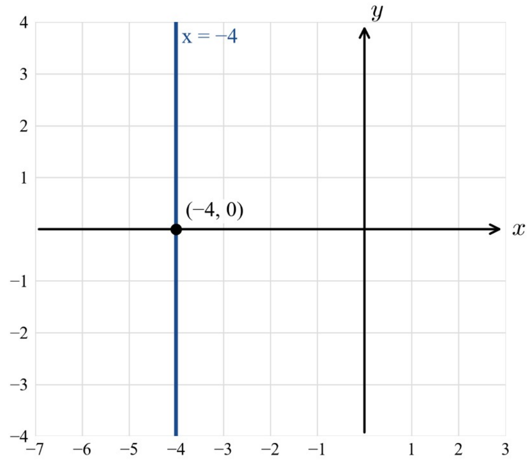

**Q15.** $m=\dfrac{1-(-2)}{2-0}=3/2$; the $y$-intercept is $-2$, so $y=3/2x-2$.



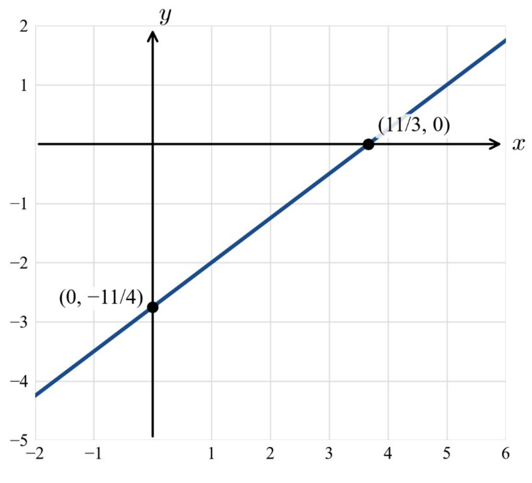

**Q16.** A horizontal line has equation $y=c$; through $(-4,6)$ this is $y=6$.




**Q17.** A vertical line has equation $x=a$; through $(-4,6)$ this is $x=-4$.




**Q18.** $y-(-2)=3/4(x-1) \Rightarrow y+2=3/4x-3/4 \Rightarrow y=3/4x-11/4$. Multiplying by 4: $4y=3x-11$, so in general form $3x-4y-11=0$.



# Chapter 2 Coordinate geometry and straight-line graphs — 2D Sketching straight-line graphs

## Theory

A non-vertical straight line has the equation $y=mx+c$, where $m$ is the **gradient** and $c$ is the $y$**-intercept**. To draw a line you need two points, or one point and the gradient. Two efficient methods:

**Gradient–intercept method.** Plot the $y$-intercept $(0,c)$, then use the gradient $m=\frac{\text{rise}}{\text{run}}$ to step to a second point, and rule a line through them.

**Intercept method.** Find both axis intercepts and join them:

- $y$**-intercept:** put $x=0$, giving $(0,c)$.
- $x$**-intercept:** put $y=0$ and solve for $x$.

**Special lines.**

- A **horizontal line** $y=k$ has gradient $0$ and passes through $(0,k)$.
- A **vertical line** $x=a$ has an undefined gradient and passes through $(a,0)$.
- A line **through the origin** has $c=0$, i.e. $y=mx$, passing through $(0,0)$.

**General form.** A line may be written as $ax+by+c=0$. Rearrange it into $y=mx+c$, or find the two intercepts directly.

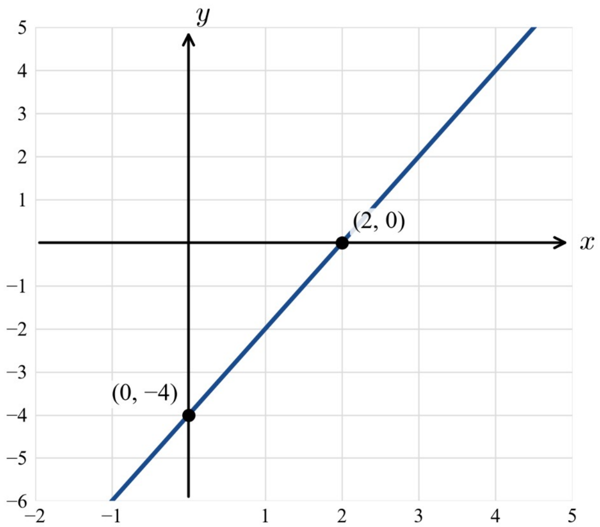
To **sketch** a line: find two points (usually the two intercepts), plot them, and rule a straight line through them, extending it in both directions.

## Worked examples

### *Example 1 — scaffolded*

Sketch the graph of $y=2x-4$, labelling the axis intercepts.

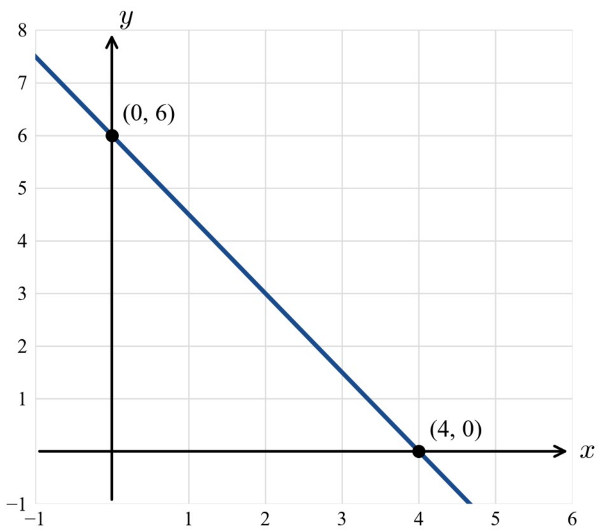
| Working | Comment |
|---|---|
| Gradient $m=2$; $y$-intercept $c=-4$. | Read directly from $y=mx+c$. |
| $y$-intercept: $(0,-4)$. | Put $x=0$. |
| $x$-intercept: put $y=0$: $2x-4=0\Rightarrow x=2$, so $(2,0)$. | Put $y=0$ and solve. |
| Plot $(0,-4)$ and $(2,0)$ and rule a line through them. | Two points determine the line. |




### *Example 2 — scaffolded*

Sketch the graph of $3x+2y=12$ using its axis intercepts.

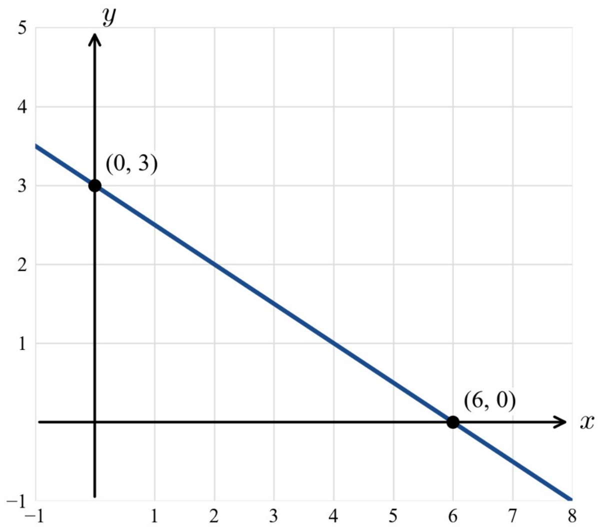
| Working | Comment |
|---|---|
| $x$-intercept: put $y=0$: $3x=12 \Rightarrow x=4$, so $(4,0)$. | Put $y=0$. |
| $y$-intercept: put $x=0$: $2y=12 \Rightarrow y=6$, so $(0,6)$. | Put $x=0$. |
| Plot $(4,0)$ and $(0,6)$ and rule a line. | The intercept method suits general form. |




### *Example 3 — unscaffolded*

Sketch the graph of $y=-1/2x+3$.

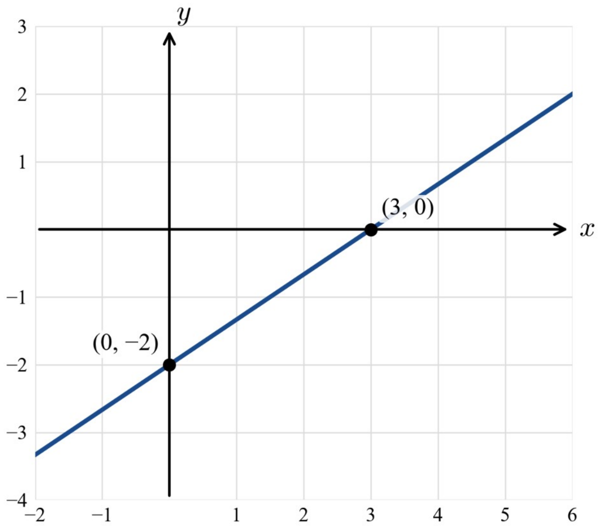
Gradient $-1/2$; $y$-intercept $(0,3)$. $x$-intercept: $-1/2x+3=0 \Rightarrow x=6$, so $(6,0)$.




***Example 4 — unscaffolded***

Sketch the graph of $2x-3y-6=0$.

Rearrange: $3y=2x-6 \Rightarrow y=2/3x-2$. $y$-intercept $(0,-2)$; $x$-intercept $y=0 \Rightarrow 2/3x=2 \Rightarrow x=3$, so $(3,0)$.

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## Practice questions

***Scaffolded***

**Q1.** For $y=x+2$: (a) state the gradient and the $y$-intercept; (b) find the $x$-intercept; (c) sketch the graph.

**Q2.** For $y=-3x+6$: (a) state the gradient and the $y$-intercept; (b) find the $x$-intercept; (c) sketch the graph.

**Q3.** Sketch the graph of $2x+y=4$ using its axis intercepts.

**Q4.** Sketch the graph of $y=1/2x-1$.

**Q5.** Sketch the line $y=3$.

**Q6.** Sketch the line $x=-2$.

***Unscaffolded***

Sketch each of the following, labelling the axis intercepts.

**Q7.** $y=3x-6$

**Q8.** $y = -x + 4$

**Q9.** $y = 1/2x + 1$

**Q10.** $y = -3/2x + 3$

**Q11.** $x + y = 5$

**Q12.** $3x - 4y = 12$

**Q13.** $y = 2x$

**Q14.** $y = -3x$

**Q15.** $y = -2$

**Q16.** $x = 4$

**Q17.** $2x + 5y - 10 = 0$

**Q18.** $4x - 3y + 12 = 0$

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### Answers

**Q1.** Gradient 1, $y$-int $(0,2)$; $x$-int $(-2,0)$. **Q2.** Gradient $-3$, $y$-int $(0,6)$; $x$-int $(2,0)$. **Q3.** $x$-int $(2,0)$, $y$-int $(0,4)$. **Q4.** $y$-int $(0,-1)$, $x$-int $(2,0)$. **Q5.** Horizontal line through $(0,3)$. **Q6.** Vertical line through $(-2,0)$. **Q7.** $(2,0)$, $(0,-6)$. **Q8.** $(4,0)$, $(0,4)$. **Q9.** $(-2,0)$, $(0,1)$. **Q10.** $(2,0)$, $(0,3)$. **Q11.** $(5,0)$, $(0,5)$. **Q12.** $(4,0)$, $(0,-3)$. **Q13.** Through $(0,0)$ and $(1,2)$. **Q14.** Through $(0,0)$ and $(1,-3)$. **Q15.** Horizontal line through $(0,-2)$. **Q16.** Vertical line through $(4,0)$. **Q17.** $(5,0)$, $(0,2)$. **Q18.** $(-3,0)$, $(0,4)$.

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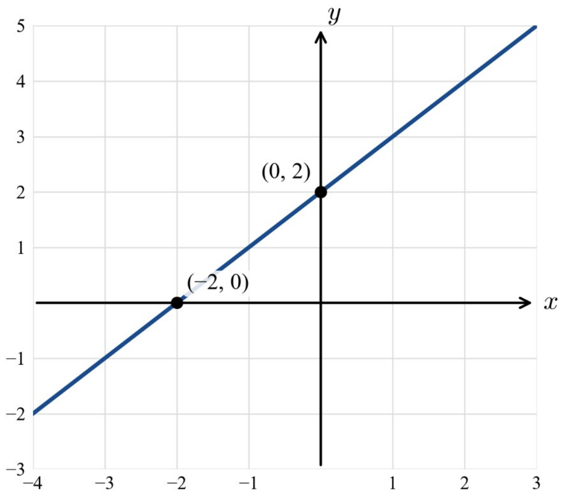
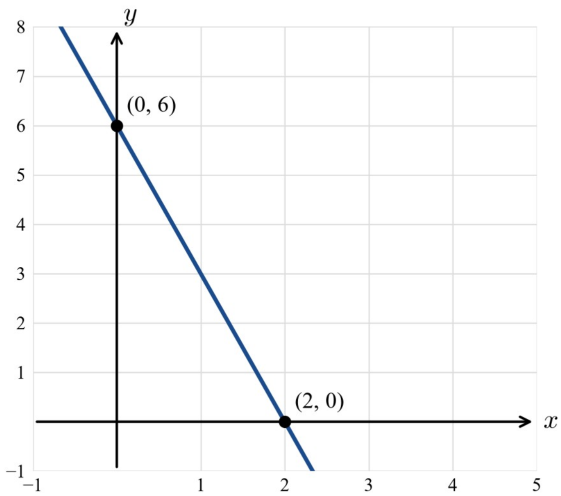
### Fully-worked solutions

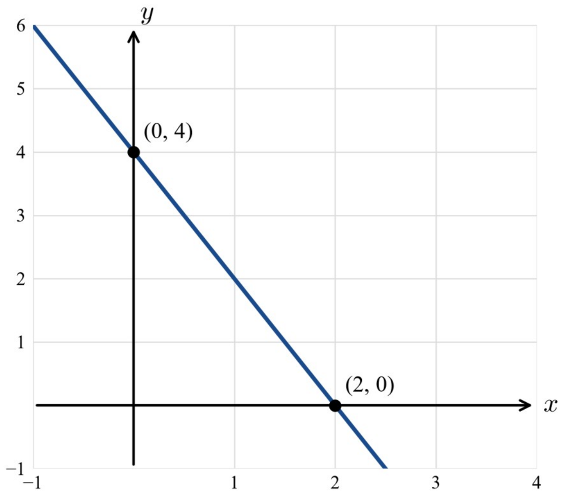
**Q1.** $y=x+2$: gradient 1, $y$-intercept $(0,2)$; $x$-intercept $y=0 \Rightarrow x=-2$, so $(-2,0)$.



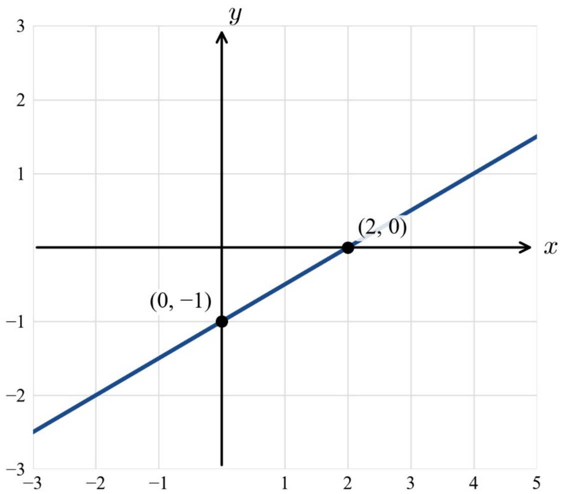

**Q2.** $y=-3x+6$: gradient $-3$, $y$-intercept $(0,6)$; $x$-intercept $-3x+6=0 \Rightarrow x=2$, so $(2,0)$.



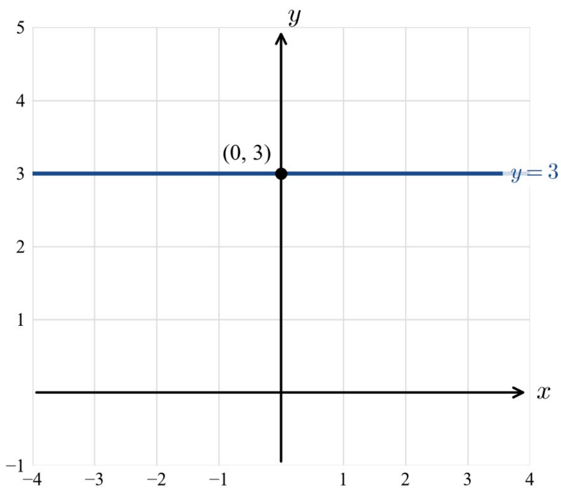

**Q3.** $2x+y=4$: $x$-intercept $y=0 \Rightarrow x=2$, so $(2,0)$; $y$-intercept $x=0 \Rightarrow y=4$, so $(0,4)$.



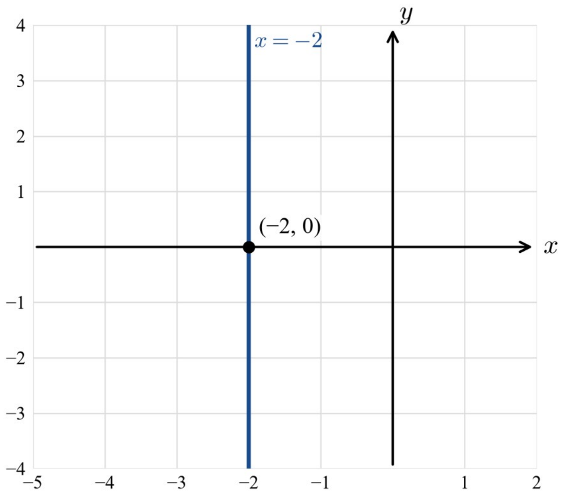

**Q4.** $y = 1/2\,x - 1$: $y$-intercept $(0, -1)$; $x$-intercept $1/2\,x - 1 = 0 \Rightarrow x = 2$, so $(2, 0)$.



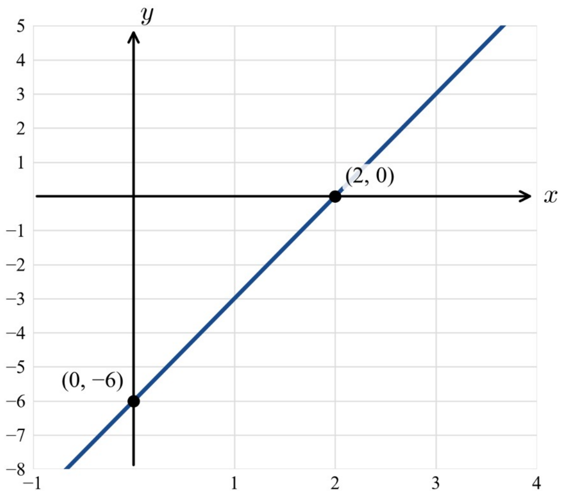

**Q5.** $y = 3$ is a horizontal line through $(0, 3)$, parallel to the $x$-axis.



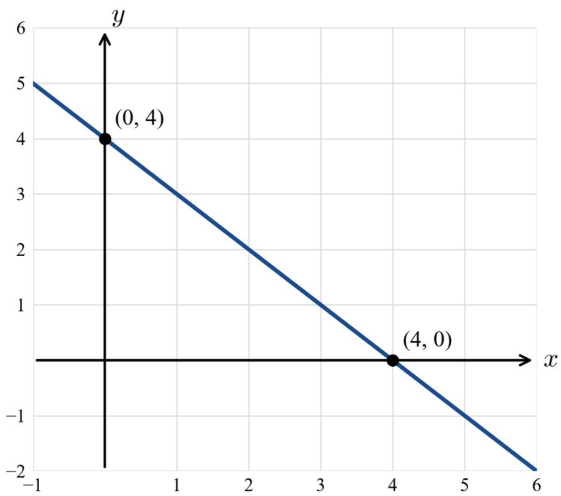

**Q6.** $x = -2$ is a vertical line through $(-2, 0)$, parallel to the $y$-axis.



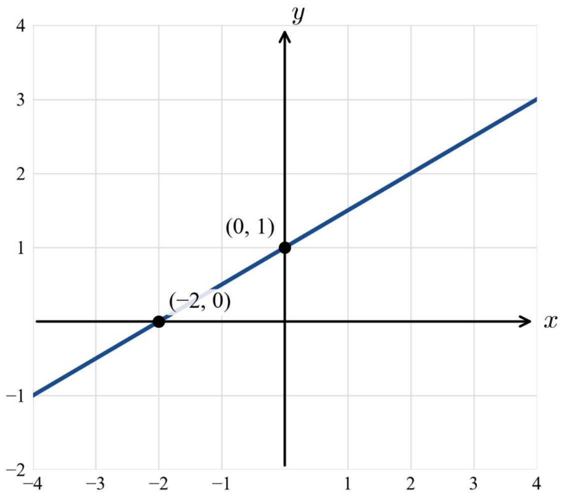

**Q7.** $y = 3x - 6$: $y$-intercept $(0, -6)$; $x$-intercept $3x - 6 = 0 \Rightarrow x = 2$, so $(2, 0)$.



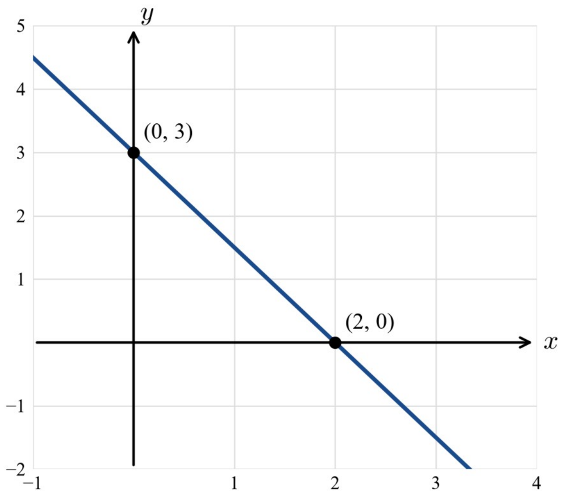

**Q8.** $y = -x + 4$: $y$-intercept $(0, 4)$; $x$-intercept $-x + 4 = 0 \Rightarrow x = 4$, so $(4, 0)$.



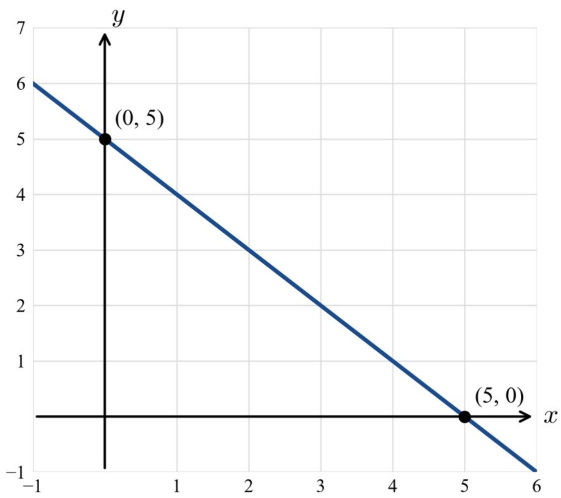

**Q9.** $y = 1/2x + 1$: $y$-intercept $(0, 1)$; $x$-intercept $1/2x + 1 = 0 \Rightarrow x = -2$, so $(-2, 0)$.



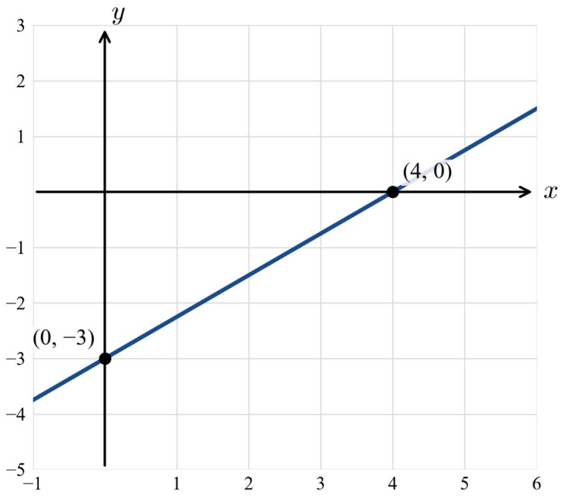

**Q10.** $y = -3/2\,x + 3$: $y$-intercept $(0, 3)$; $x$-intercept $-3/2\,x + 3 = 0 \Rightarrow x = 2$, so $(2, 0)$.



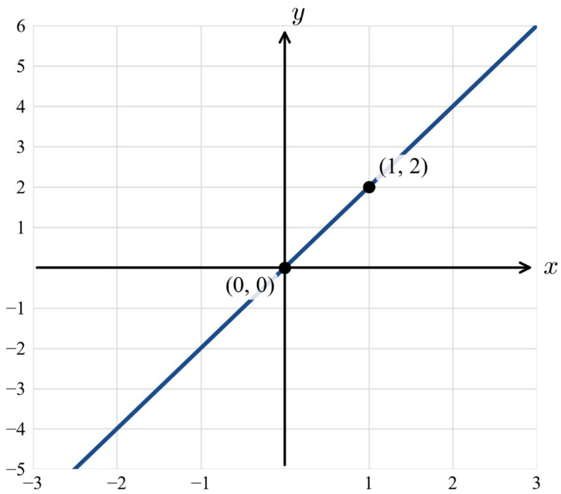

**Q11.** $x + y = 5$: $x$-intercept $(5, 0)$; $y$-intercept $(0, 5)$.



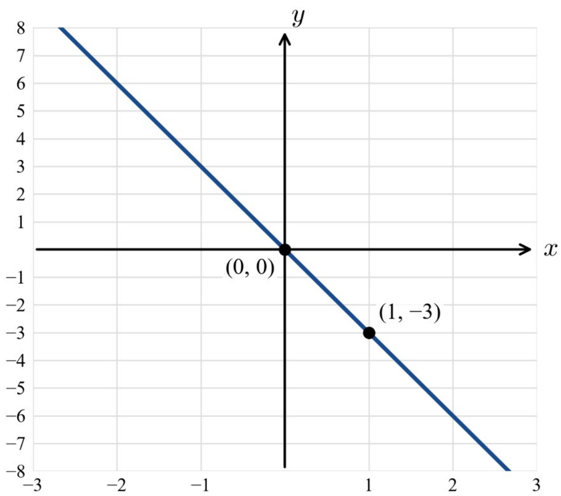

**Q12.** $3x - 4y = 12$: $x$-intercept $y = 0 \Rightarrow x = 4$, so $(4, 0)$; $y$-intercept $x = 0 \Rightarrow -4y = 12 \Rightarrow y = -3$, so $(0, -3)$.



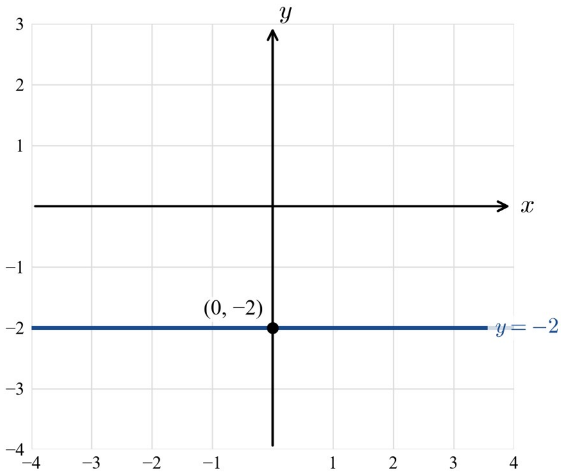

**Q13.** $y=2x$ passes through the origin $(0,0)$; using the gradient, a second point is $(1,2)$.



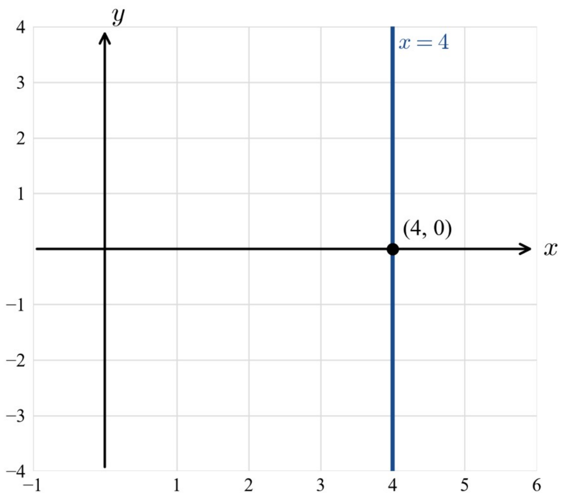

**Q14.** $y=-3x$ passes through the origin $(0,0)$; a second point is $(1,-3)$.



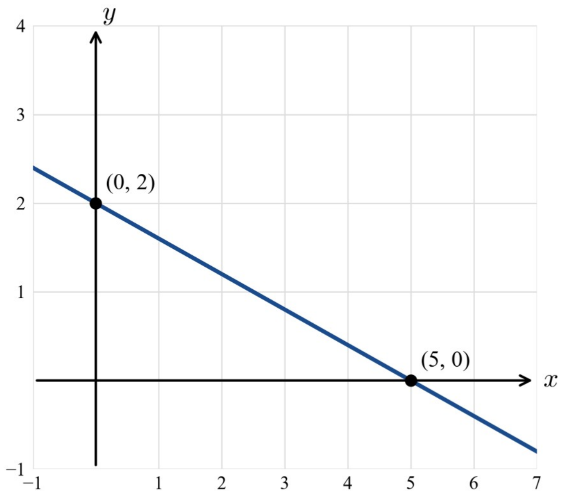

**Q15.** $y=-2$ is a horizontal line through $(0,-2)$.



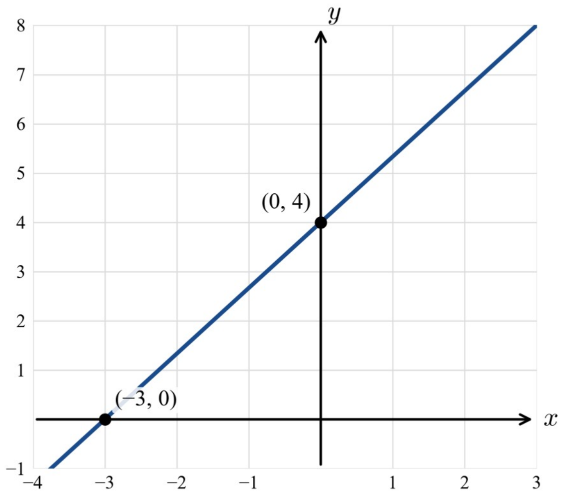

**Q16.** $x=4$ is a vertical line through $(4,0)$.




**Q17.** $2x+5y-10=0 \Rightarrow y=-2/5x+2$: $y$-intercept $(0,2)$; $x$-intercept $y=0 \Rightarrow x=5$, so $(5,0)$.




**Q18.** $4x-3y+12=0 \Rightarrow y=4/3x+4$: $y$-intercept $(0,4)$; $x$-intercept $y=0 \Rightarrow x=-3$, so $(-3,0)$.


y
x
(0, 4)
(−3, 0)
8
7
6
5
4
3
2
1
−1
−4
−3
−2
−1
1
2
3

## Chapter 2 Coordinate geometry and straight-line graphs — 2E Parallel and perpendicular lines

### Theory

The gradient of a straight line tells us how the line is tilted. Comparing gradients tells us when two lines are **parallel** or **perpendicular**.

**Parallel lines.** Two non-vertical lines are **parallel** when they have **equal gradients**:

$$m_1 = m_2 .$$

They have the same steepness and never meet (unless they are the same line).

**Perpendicular lines.** Two lines are **perpendicular** (they meet at a right angle) when the **product of their gradients is** $-1$:

$$m_1 m_2 = -1 \text{, equivalently } m_2 = -\frac{1}{m_1} .$$

That is, the gradient of a perpendicular line is the **negative reciprocal** of the original gradient. (A horizontal line and a vertical line are also perpendicular — a special case not covered by the formula.)

**Finding the equation of a parallel or perpendicular line.** To find the line through a given point $(x_1, y_1)$ that is parallel or perpendicular to a known line:

1. find the gradient $m$ of the known line;
2. take $m$ (for parallel) or $-\frac{1}{m}$ (for perpendicular) as the new gradient;
3. substitute into the **point–gradient form** $y - y_1 = m(x - x_1)$ and simplify.

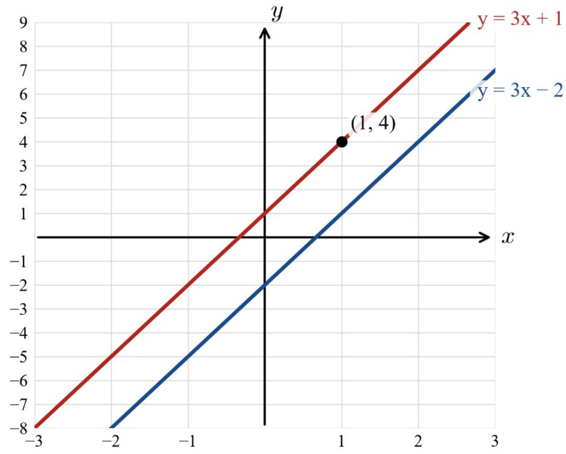
**Perpendicular bisector.** The perpendicular bisector of the segment joining two points passes through the **midpoint** of the segment and is **perpendicular** to it.

### Worked examples

#### *Example 1 — scaffolded*

Find the equation of the line parallel to $y = 3x - 2$ that passes through $(1, 4)$.

| Working | Comment |
| --- | --- |
| The line $y = 3x - 2$ has gradient $3$, so the parallel line also has gradient $m = 3$. | Parallel lines have **equal** gradients. |
| $y - 4 = 3(x - 1)$ | Point–gradient form with $m = 3$, $(x_1, y_1) = (1, 4)$. |
| $y - 4 = 3x - 3$, so $y = 3x + 1$. | Expand and make $y$ the subject. |

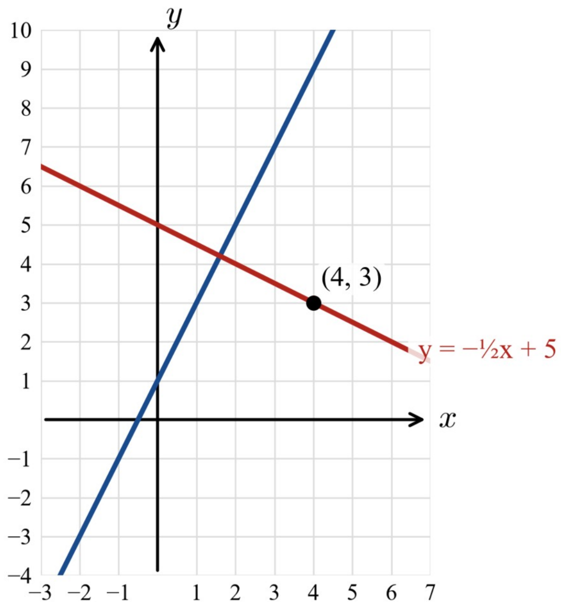



### *Example 2 — scaffolded*

Find the equation of the line perpendicular to $y=2x+1$ that passes through $(4,3)$.

| Working | Comment |
|---|---|
| The line $y=2x+1$ has gradient 2. The perpendicular gradient is $m=-\frac{1}{2}$. | Perpendicular gradient = negative reciprocal ( $m_1 m_2=-1$). |
| $y-3=-\frac{1}{2}(x-4)$ | Point–gradient form. |
| $y-3=-\frac{1}{2}x+2$, so $y=-\frac{1}{2}x+5$. | Expand and simplify. |

y = 2x + 1



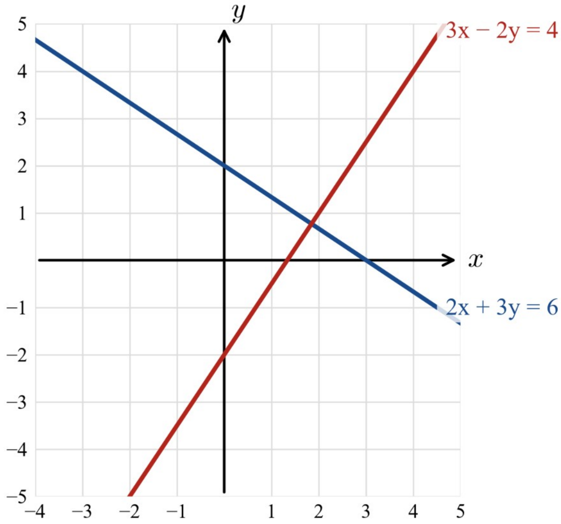

### *Example 3 — unscaffolded*

Determine whether the lines $2x+3y=6$ and $3x-2y=4$ are parallel, perpendicular, or neither.

Rearranging each into the form $y=mx+c$:

$$2x+3y=6 \Rightarrow y=-2/3x+2, m_1=-2/3; 3x-2y=4 \Rightarrow y=3/2x-2, m_2=3/2.$$

$m_1 m_2 = -2/3 \times 3/2 = -1$. $\therefore$ the lines are **perpendicular**.




***Example 4 — unscaffolded***

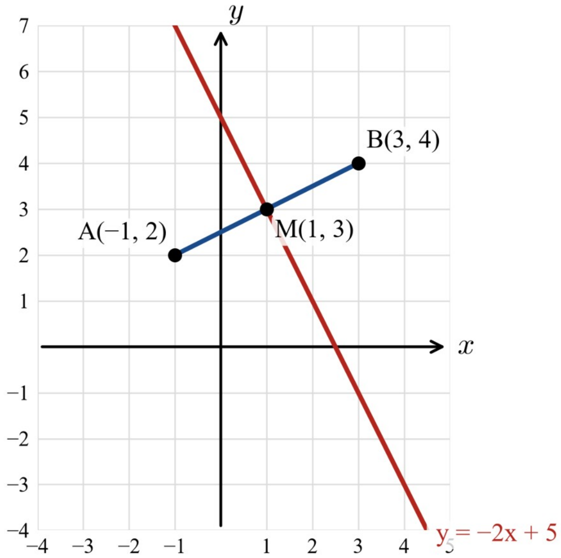
Find the equation of the perpendicular bisector of the segment joining $A(-1,2)$ and $B(3,4)$.

Midpoint: $M = \left(\dfrac{-1+3}{2}, \dfrac{2+4}{2}\right) = (1,3)$.

Gradient of $AB$: $m_{AB} = \dfrac{4-2}{3-(-1)} = \dfrac{2}{4} = \dfrac{1}{2}$, so the perpendicular gradient is $-2$.

Through $M(1,3)$: $y-3=-2(x-1)$, so $y=-2x+5$.

$\therefore$ the perpendicular bisector is $y=-2x+5$.

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## Practice questions

### *Scaffolded*

**Q1.** Find the equation of the line parallel to $y=4x-1$ that passes through $(2,3)$.

**Q2.** Find the equation of the line perpendicular to $y=1/2x+2$ that passes through $(1,-2)$.

**Q3.** Show that the lines $y=3x+2$ and $y=3x-5$ are parallel.

**Q4.** Show that the lines $y=2x+1$ and $y=-1/2x+4$ are perpendicular.

**Q5.** Find the equation of the line parallel to $2x+y=5$ that passes through the origin.

**Q6.** Find the equation of the line perpendicular to $y=-3x+1$ that passes through $(3,0)$.

***Unscaffolded***

**Q7.** Find the equation of the line parallel to $y=5x+2$ through $(-1,3)$.

**Q8.** Find the equation of the line parallel to $y=-2x+7$ through $(2,-1)$.

**Q9.** Find the equation of the line perpendicular to $y=3x-4$ through $(6,1)$.

**Q10.** Find the equation of the line perpendicular to $y=-1/4x+2$ through $(-2,5)$.

**Q11.** Determine whether $3x-y=2$ and $x+3y=6$ are parallel, perpendicular, or neither.

**Q12.** Determine whether $y=2x+1$ and $4x-2y=3$ are parallel, perpendicular, or neither.

**Q13.** Determine whether $y=1/2x$ and $y=2x$ are parallel, perpendicular, or neither.

**Q14.** Find the perpendicular bisector of the segment joining $(2,1)$ and $(6,5)$.

**Q15.** Find the perpendicular bisector of the segment joining $(-3,0)$ and $(1,4)$.

**Q16.** Find the equation of the line through $(0,4)$ parallel to the line through $(1,1)$ and $(3,7)$.

**Q17.** Find the equation of the line through $(2,-3)$ perpendicular to the line joining $(-1,2)$ and $(5,4)$.

**Q18.** Show that the lines $5x+2y=10$ and $2x-5y=3$ are perpendicular.

---

### Answers

**Q1.** $y=4x-5$. **Q2.** $y=-2x$. **Q3.** Both have gradient $3$ → parallel. **Q4.** $2\times(-1/2)=-1$ → perpendicular. **Q5.** $y=-2x$. **Q6.** $y=1/3x-1$. **Q7.** $y=5x+8$. **Q8.** $y=-2x+3$. **Q9.** $y=-1/3x+3$. **Q10.** $y=4x+13$. **Q11.** Perpendicular ($m_1=3, m_2=-1/3$). **Q12.** Parallel (both $m=2$). **Q13.** Neither. **Q14.** $y=-x+7$. **Q15.** $y=-x+1$. **Q16.** $y=3x+4$. **Q17.** $y=-3x+3$. **Q18.** $m_1=-5/2, m_2=2/5$, product $-1$ → perpendicular.

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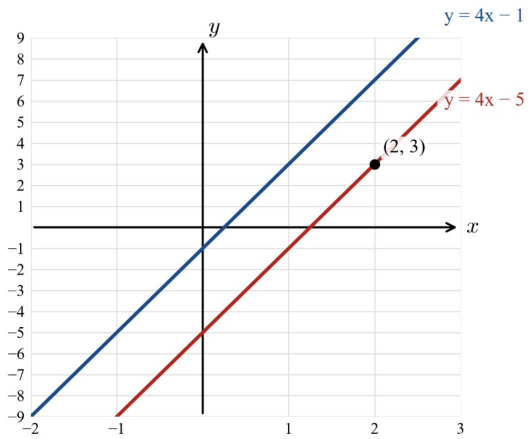
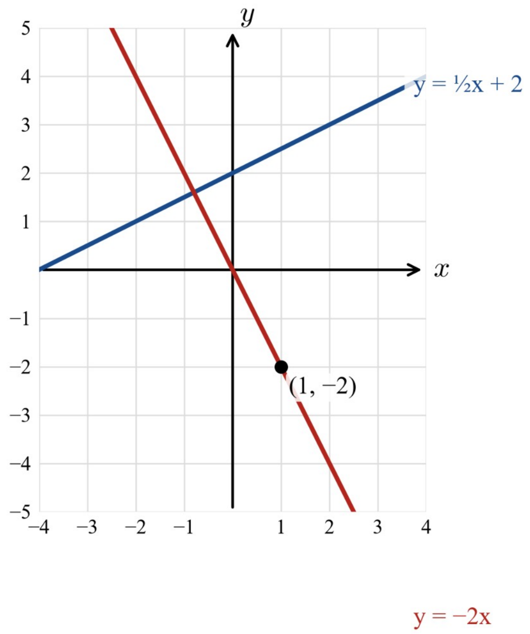
### Fully-worked solutions

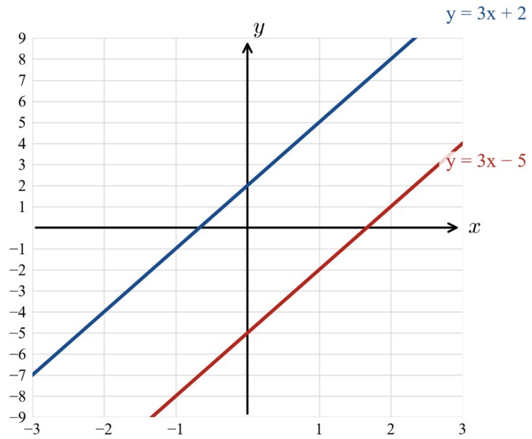
**Q1.** Gradient $=4$ (parallel). $y-3=4(x-2)\Rightarrow y=4x-5$.



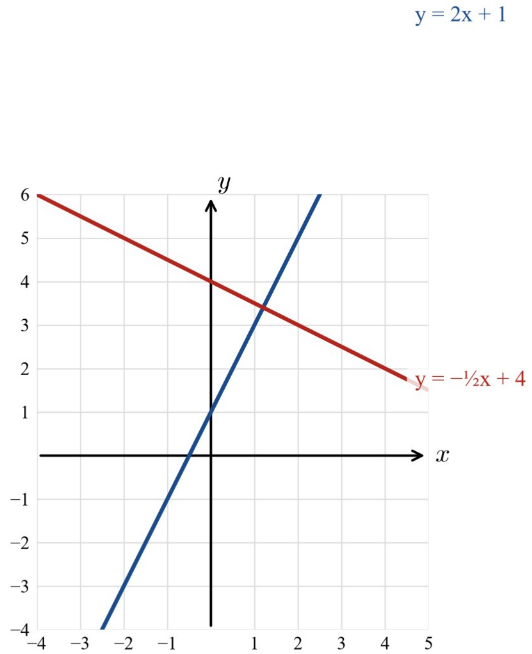

**Q2.** Perpendicular gradient $=-2$. $y-(-2)=-2(x-1)\Rightarrow y=-2x$.



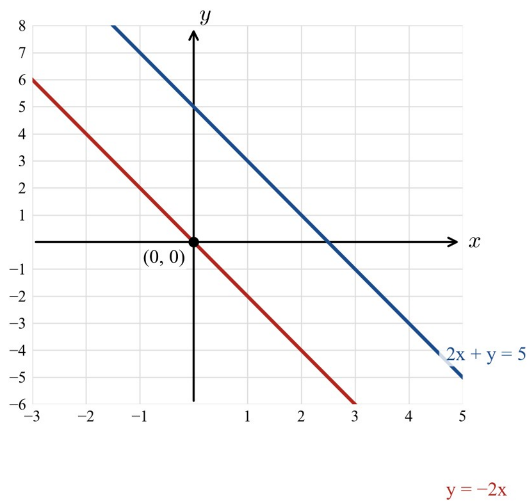

**Q3.** $y=3x+2$ has gradient $3$ and $y=3x-5$ has gradient $3$. Equal gradients (and different $y$-intercepts), so the lines are **parallel**.



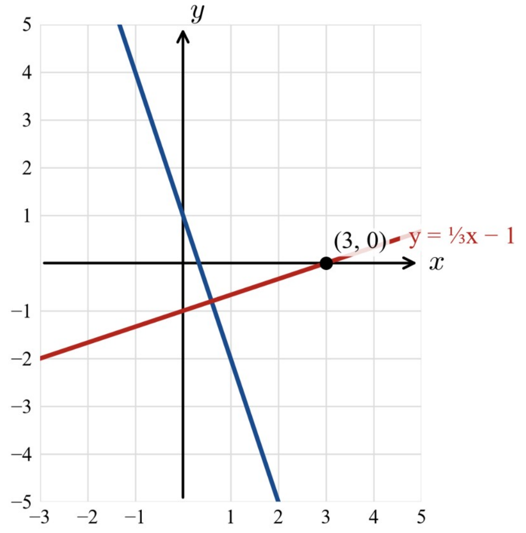

**Q4.** Gradients are $2$ and $-1/2$. Product $= 2 \times -1/2 = -1$, so the lines are **perpendicular**.




**Q5.** $2x + y = 5 \Rightarrow y = -2x + 5$, gradient $-2$. Parallel line through $(0,0)$: $y = -2x$.



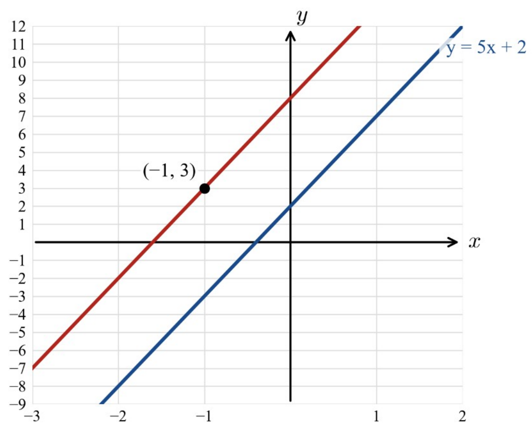

**Q6.** Gradient of $y = -3x + 1$ is $-3$; perpendicular gradient $= 1/3$. Through $(3,0)$:
$y - 0 = 1/3(x - 3) \Rightarrow y = 1/3x - 1$.



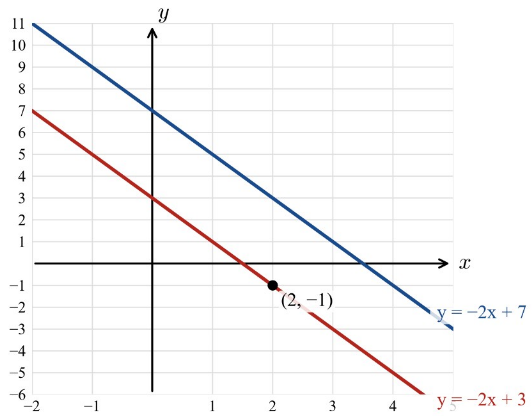

y = −3x + 1

**Q7.** Parallel gradient $=5$. $y-3=5(x+1)\Rightarrow y=5x+8$.

y = 5x + 8

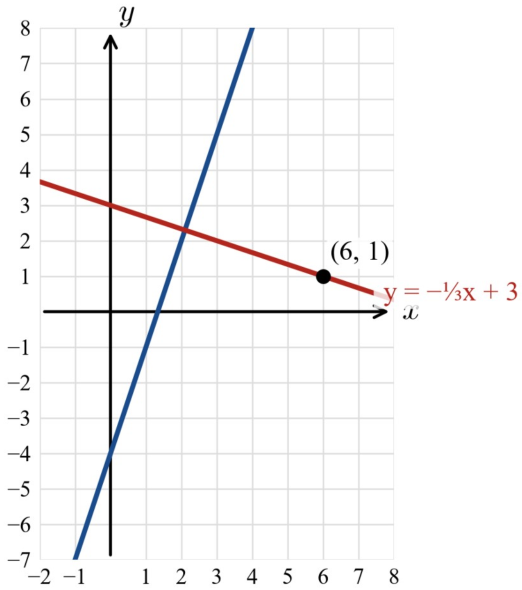



**Q8.** Parallel gradient $=-2$. $y+1=-2(x-2)\Rightarrow y=-2x+3$.



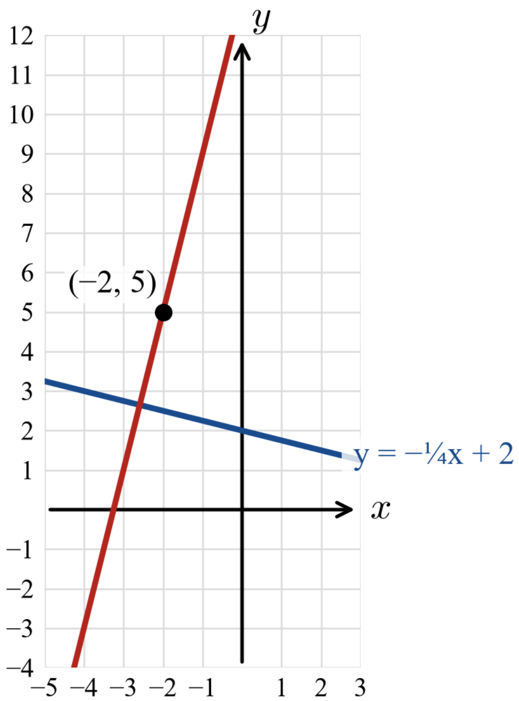
**Q9.** Perpendicular gradient $= -1/3$. $y-1=-1/3(x-6) \Rightarrow y=-1/3x+3$.

y = 3x − 4



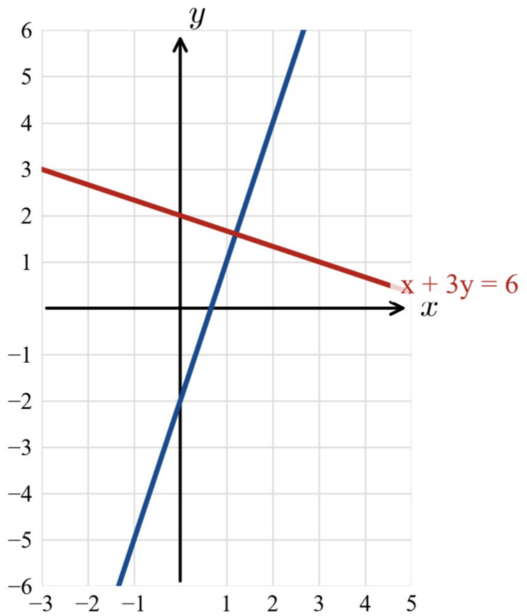

**Q10.** Gradient of the given line is $-1/4$; perpendicular gradient $=4$. Through $(-2,5)$:
$y-5=4(x+2) \Rightarrow y=4x+13$.

y = 4x + 13

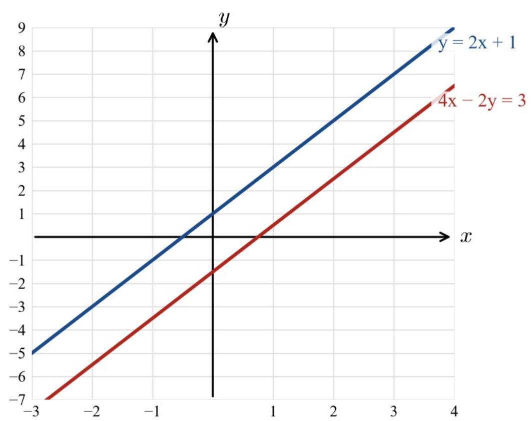



**Q11.** $3x-y=2 \Rightarrow y=3x-2$ ($m_1=3$); $x+3y=6 \Rightarrow y=-1/3x+2$ ($m_2=-1/3$). $m_1m_2=-1$, so **perpendicular**.

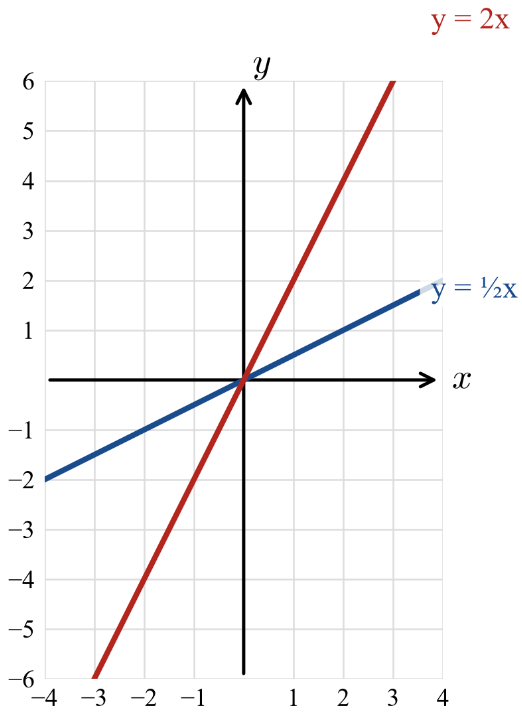
3x − y = 2



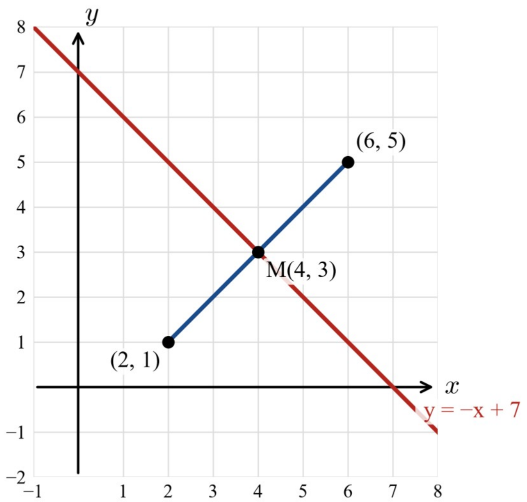

**Q12.** $y=2x+1$ ($m_1=2$); $4x-2y=3 \Rightarrow y=2x-3/2$ ($m_2=2$). Equal gradients, so **parallel**.



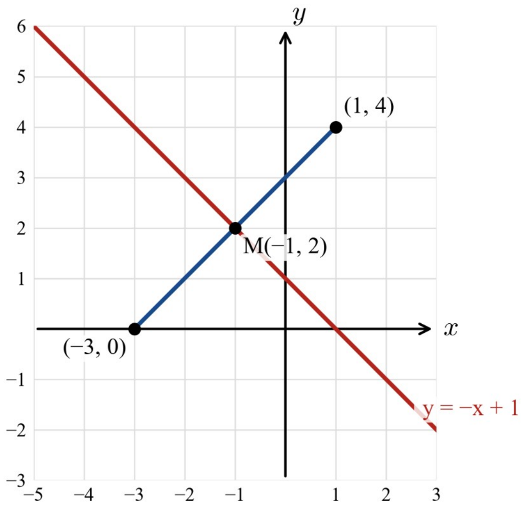

**Q13.** Gradients $1/2$ and $2$. They are not equal, and $1/2\times 2=1\neq -1$, so the lines are **neither** parallel nor perpendicular.



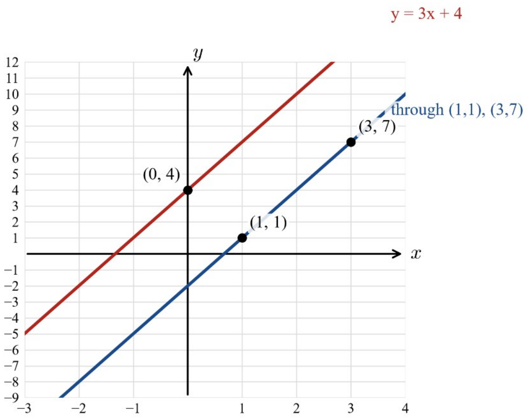

**Q14.** Midpoint $(4, 3)$; gradient of the segment $= \dfrac{5-1}{6-2} = 1$; perpendicular gradient $= -1$. Through $(4, 3)$:
$y - 3 = -1(x - 4) \Rightarrow y = -x + 7$.



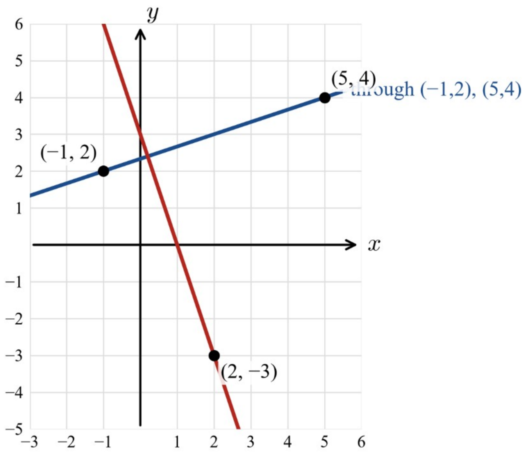

**Q15.** Midpoint $(-1, 2)$; gradient of the segment $= \dfrac{4-0}{1-(-3)} = 1$; perpendicular gradient $= -1$. Through $(-1, 2)$:
$y - 2 = -1(x + 1) \Rightarrow y = -x + 1$.



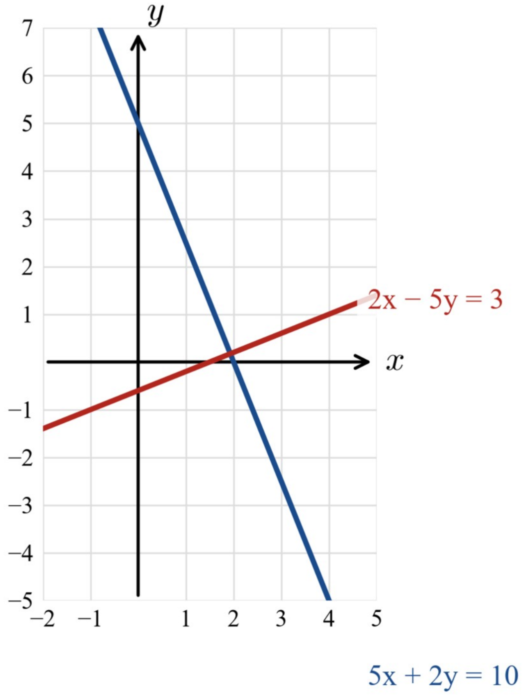

**Q16.** Gradient of the line through $(1,1)$ and $(3,7)$ is $\dfrac{7-1}{3-1}=3$. A parallel line through $(0,4)$: $y=3x+4$.




**Q17.** Gradient of the line joining $(-1,2)$ and $(5,4)$ is $\dfrac{4-2}{5-(-1)}=\dfrac{2}{6}=\dfrac{1}{3}$; perpendicular gradient $=-3$. Through $(2,-3)$: $y+3=-3(x-2)\Rightarrow y=-3x+3$.



**Q18.** $5x+2y=10 \Rightarrow y=-5/2x+5$ ($m_1=-5/2$); $2x-5y=3 \Rightarrow y=2/5x-3/5$ ($m_2=2/5$).
$m_1 m_2=-5/2\times 2/5=-1$, so the lines are **perpendicular**.



## Chapter 2 Coordinate geometry and straight-line graphs — 2F Families of linear functions

### Theory

A **family of straight lines** is a set of lines that share a common feature, described by a **parameter** (a letter that can take many values). Changing the parameter gives a different **member** of the family.

**Parallel family — fixed gradient.** The family $y = mx + c$ with a *fixed* gradient $m$ and $c$ varying consists of all lines **parallel** to one another. For example, every line $y = 2x + c$ has gradient $2$.

**Concurrent family — fixed point.** A family whose members all pass through one fixed point. Two common forms:

- $y = mx + c$ with a *fixed* $c$ and $m$ varying — every member has the same $y$-intercept, so all pass through $(0, c)$. For example, every line $y = mx + 2$ passes through $(0, 2)$.
- Using the **point–gradient form**, all lines through a fixed point $(x_1, y_1)$ are $y - y_1 = m(x - x_1)$, with $m$ varying.

**Finding a particular member.** To pick out the one member satisfying an extra condition, substitute that condition and solve for the parameter. Useful facts:

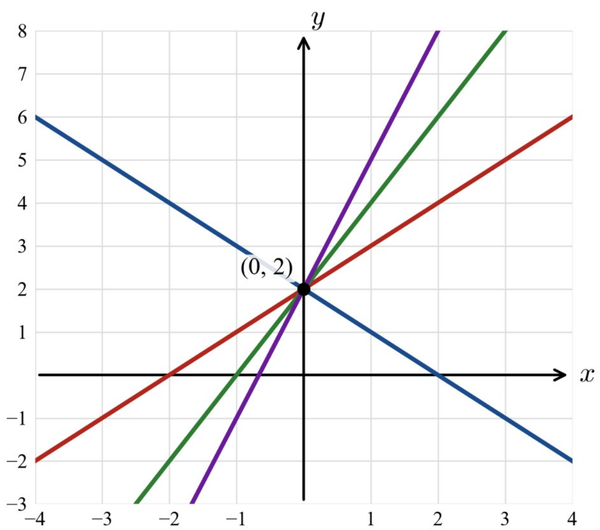
- **Parallel** lines have **equal gradients**.
- **Perpendicular** lines have gradients whose product is $-1$ (so the gradient is the negative reciprocal).

### Worked examples

#### *Example 1 — scaffolded*

Consider the family of lines $y = mx + 2$.

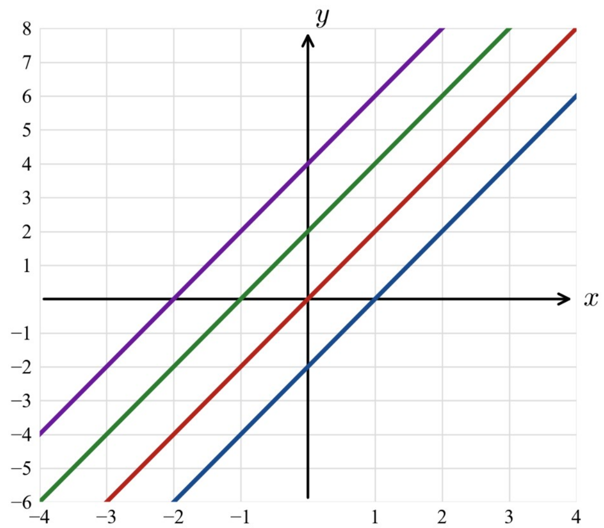
| Working | Comment |
|---|---|
| **(a)** Every member has $y$-intercept $2$, so all lines pass through $(0, 2)$. | $c = 2$ is fixed; only the gradient $m$ changes. |
| **(b)** Member with gradient $3$: $y = 3x + 2$. | Substitute $m = 3$. |
| **(c)** Member through $(2, -4)$: $-4 = m(2) + 2 \Rightarrow 2m = -6 \Rightarrow m = -3$. So $y = -3x + 2$. | Substitute the point and solve for $m$. |




#### *Example 2 — scaffolded*

Consider the family of lines $y = 2x + c$.

| Working | Comment |
|---|---|
| **(a)** Every member has gradient 2, so all lines are **parallel**. | $m=2$ is fixed; only the intercept $c$ changes. |
| **(b)** Member through $(1,5)$: $5=2(1)+c \Rightarrow c=3$. So $y=2x+3$. | Substitute the point and solve for $c$. |
| **(c)** Member with $y$-intercept $-3$: $y=2x-3$. | The $y$-intercept is $c$. |

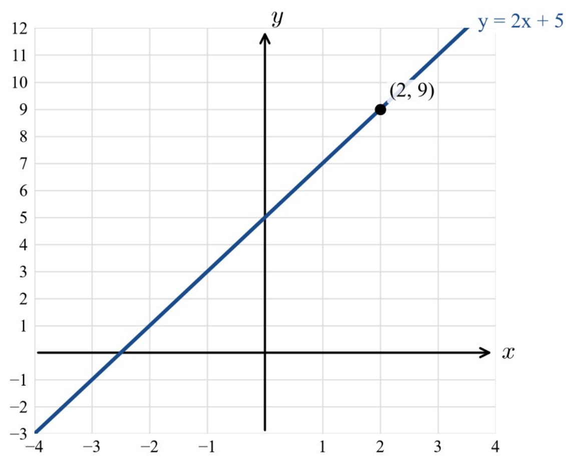



### *Example 3 — unscaffolded*

The line $y=(k-1)x+(k+2)$ passes through the point $(2,9)$. Find $k$ and the equation of the line.

Substitute $(2,9)$: $9=(k-1)(2)+(k+2)=2k-2+k+2=3k$, so $k=3$.

$\therefore y=(3-1)x+(3+2)=2x+5$.

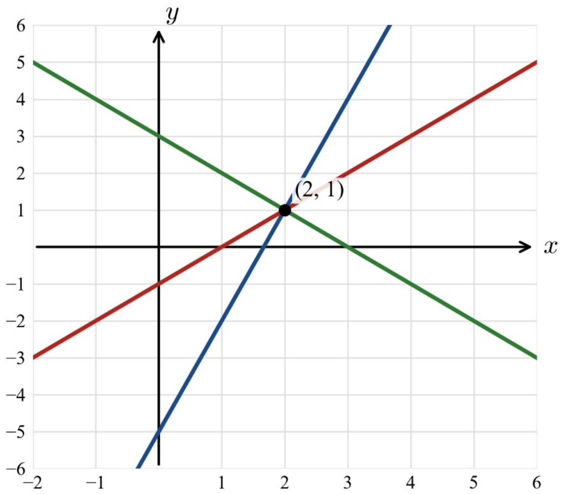



### *Example 4 — unscaffolded*

All members of a family pass through $(2,1)$, so the family is $y-1=m(x-2)$.

Member parallel to $y=3x+1$: equal gradients, so $m=3$, giving $y-1=3(x-2)$, that is $y=3x-5$.

Member through $(5,4)$: $4-1=m(5-2) \Rightarrow 3=3m \Rightarrow m=1$, giving $y=x-1$.




## Practice questions

### *Scaffolded*

**Q1.** For the family $y = mx - 1$: (a) state the point all members pass through; (b) find the member with gradient $2$; (c) find the member passing through $(1, 4)$.

**Q2.** For the family $y = -3x + c$: (a) describe the family; (b) find the member through $(2, 1)$; (c) find the member with $y$-intercept $4$.

**Q3.** Every member of the family $y = mx + 4$ passes through $(0, 4)$. Find the member that is **perpendicular** to $y = 2x + 1$.

**Q4.** Find the member of the family $y = 2x + c$ that passes through $(-1, 5)$.

**Q5.** The line $y = (k - 1)x + 2$ passes through $(1, 5)$. Find $k$ and the equation of the line.

**Q6.** Every member of the family $y = m(x - 3)$ passes through $(3, 0)$. Find the member with $y$-intercept $-6$.

### *Unscaffolded*

**Q7.** Describe the family $y = 4x + c$, then find the member passing through $(1, 1)$.

**Q8.** Describe the family $y = mx + 5$, then find the member with gradient $-2$.

**Q9.** Find the member of the family $y = mx - 2$ that passes through $(3, 7)$.

**Q10.** Find the member of the family $y = 3x + c$ that passes through $(-2, 1)$.

**Q11.** Find the member of the family $y = mx + 1$ that is perpendicular to $y = 1/2x + 4$.

**Q12.** Find the member of the family of lines through $(0, -3)$ that is parallel to $y = -x + 2$.

**Q13.** The line $y = (k - 2)x + 3$ passes through $(2, 11)$. Find $k$ and the equation.

**Q14.** The line $y = (2k + 1)x - k$ passes through $(1, 4)$. Find $k$ and the equation.

**Q15.** All members of a family pass through $(2, 5)$. Write the family in point–gradient form, then find the member with gradient $-1$.

**Q16.** All members of a family pass through $(-1, 4)$. Find the member parallel to $y = 2x - 3$.

**Q17.** Find the member of the family $y = mx + 6$ whose $x$-intercept is 3.

**Q18.** The line $y=(k+1)x+(2k-3)$ passes through $(1,2)$. Find $k$ and the equation.

## Answers

**Q1.** (a) $(0,-1)$; (b) $y=2x-1$; (c) $y=5x-1$. **Q2.** (a) parallel, gradient $-3$; (b) $y=-3x+7$; (c) $y=-3x+4$. **Q3.** $y=-1/2x+4$. **Q4.** $y=2x+7$. **Q5.** $k=4$, $y=3x+2$. **Q6.** $y=2x-6$. **Q7.** parallel, gradient $4$; $y=4x-3$. **Q8.** all through $(0,5)$; $y=-2x+5$. **Q9.** $y=3x-2$. **Q10.** $y=3x+7$. **Q11.** $y=-2x+1$. **Q12.** $y=-x-3$. **Q13.** $k=6$, $y=4x+3$. **Q14.** $k=3$, $y=7x-3$. **Q15.** $y-5=m(x-2)$; $y=-x+7$. **Q16.** $y=2x+6$. **Q17.** $y=-2x+6$. **Q18.** $k=4/3$, $y=7/3x-1/3$.

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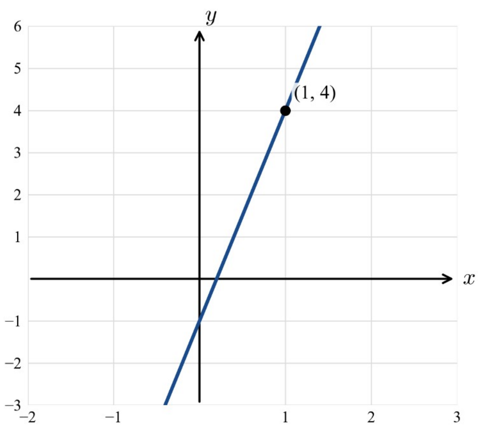
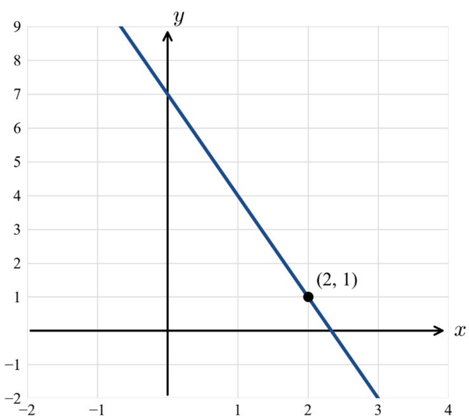
## Fully-worked solutions

**Q1.** $y=mx-1$. (a) Every member has $y$-intercept $-1$, so all pass through $(0,-1)$. (b) Gradient 2: $y=2x-1$. (c) Through $(1,4)$: $4=m(1)-1 \Rightarrow m=5$, so $y=5x-1$.

y = 5x − 1

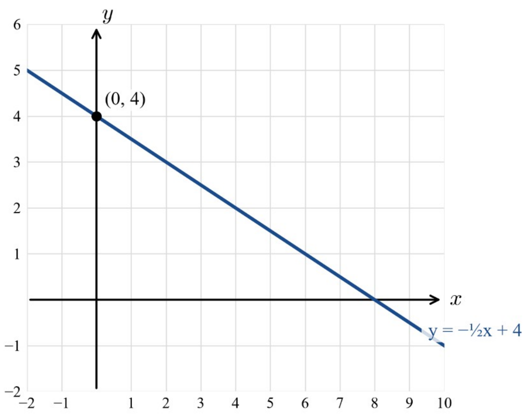



**Q2.** $y=-3x+c$. (a) Every member has gradient $-3$, so all are parallel. (b) Through $(2,1)$: $1=-3(2)+c \Rightarrow c=7$, so $y=-3x+7$. (c) $y$-intercept $4$: $y=-3x+4$.

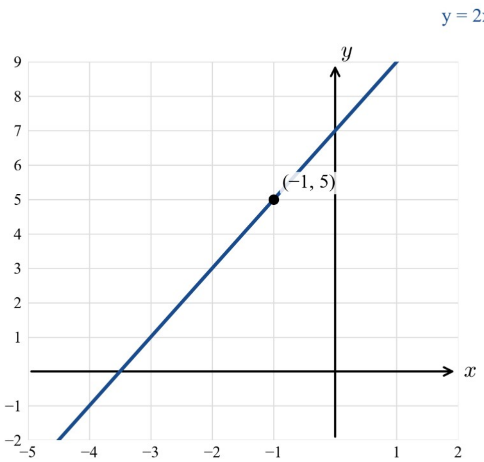



y = −3x + 7

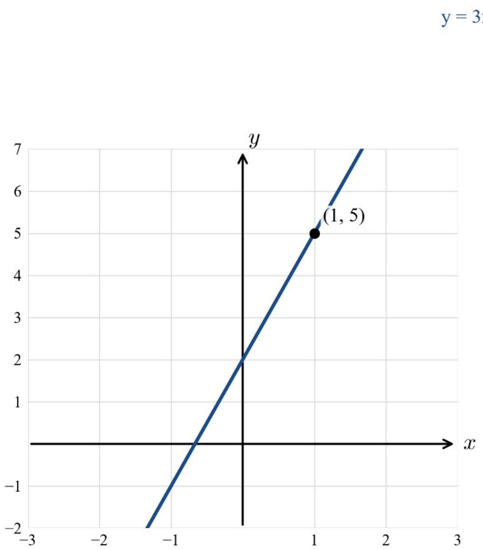
**Q3.** The gradient of $y=2x+1$ is 2, so a perpendicular line has gradient $-1/2$ (the negative reciprocal). Since the member has $y$-intercept $4$, $y=-1/2x+4$.




**Q4.** Through $(-1,5)$: $5=2(-1)+c \Rightarrow c=7$. $\therefore\ y=2x+7$.

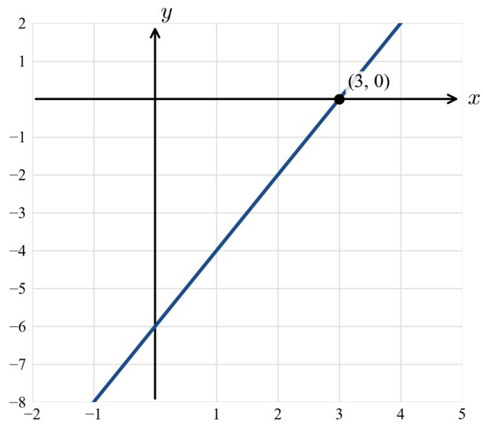



**Q5.** Substitute $(1,5)$: $5=(k-1)(1)+2=k+1 \Rightarrow k=4$. $\therefore\ y=(4-1)x+2=3x+2$.



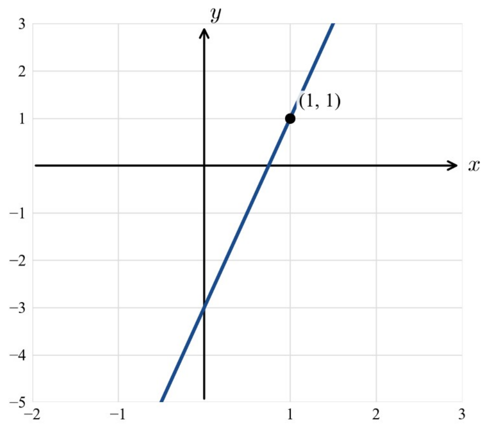

**Q6.** $y=m(x-3)$. The $y$-intercept is at $x=0$: $y=m(0-3)=-3m$. Setting $-3m=-6$ gives $m=2$.
$\therefore\ y=2(x-3)=2x-6$.

y = 2x − 6

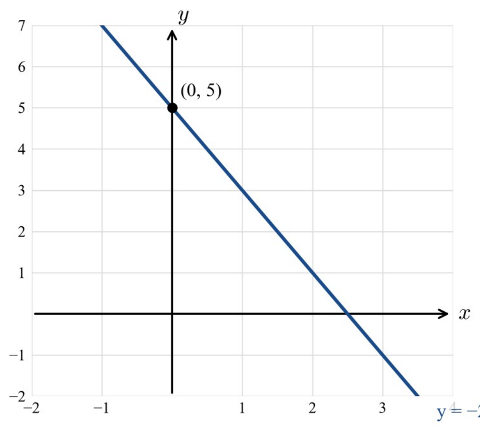



**Q7.** Every member of $y=4x+c$ has gradient 4, so the family is the set of lines parallel to $y=4x$. Through $(1,1)$: $1=4(1)+c \Rightarrow c=-3$, so $y=4x-3$.

y = 4x − 3



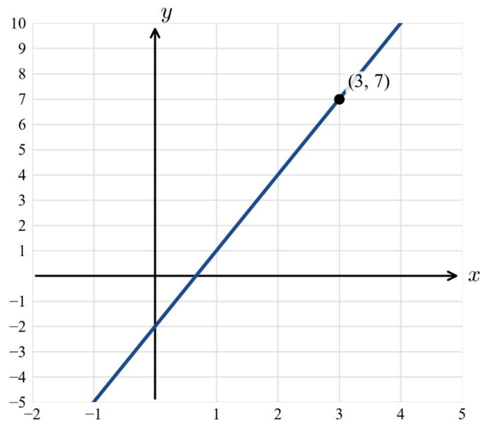

**Q8.** Every member of $y=mx+5$ passes through $(0,5)$. With gradient $-2$: $y=-2x+5$.




y = −2x + 5

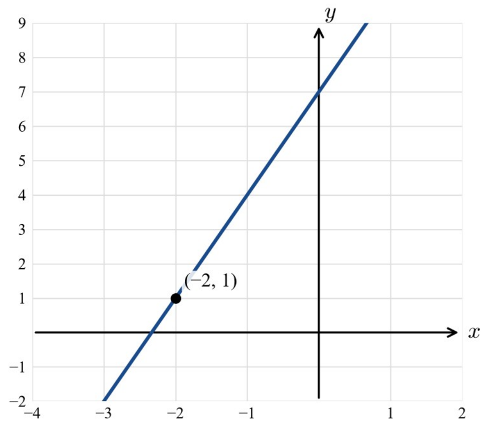
**Q9.** Through $(3,7)$: $7=m(3)-2 \Rightarrow 3m=9 \Rightarrow m=3$. $\therefore$ $y=3x-2$.

y = 3x − 2

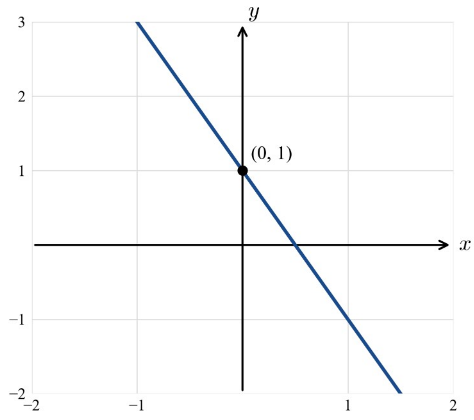



**Q10.** Through $(-2,1)$: $1=3(-2)+c \Rightarrow c=7$. $\therefore\ y=3x+7$.

y = 3x + 7

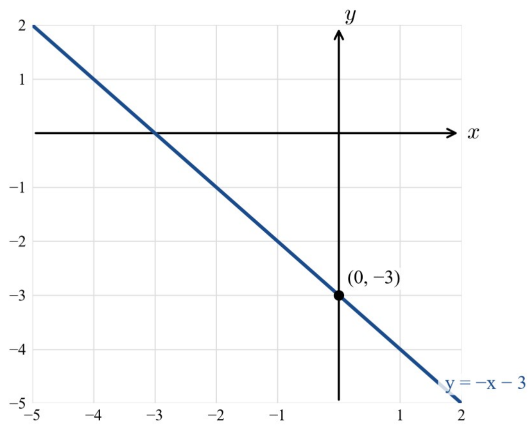



**Q11.** The gradient of $y=1/2x+4$ is $1/2$, so a perpendicular line has gradient $-2$. With $y$-intercept 1: $y=-2x+1$.

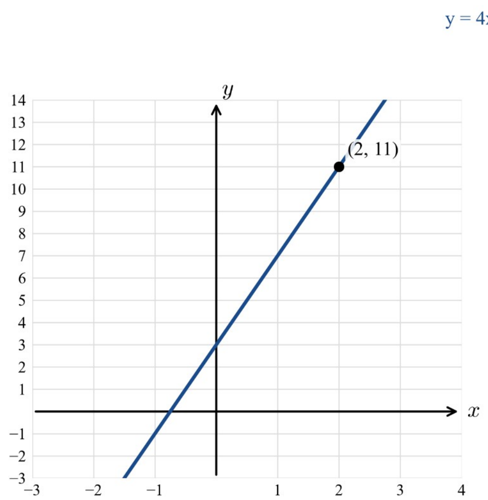



y = −2x + 1

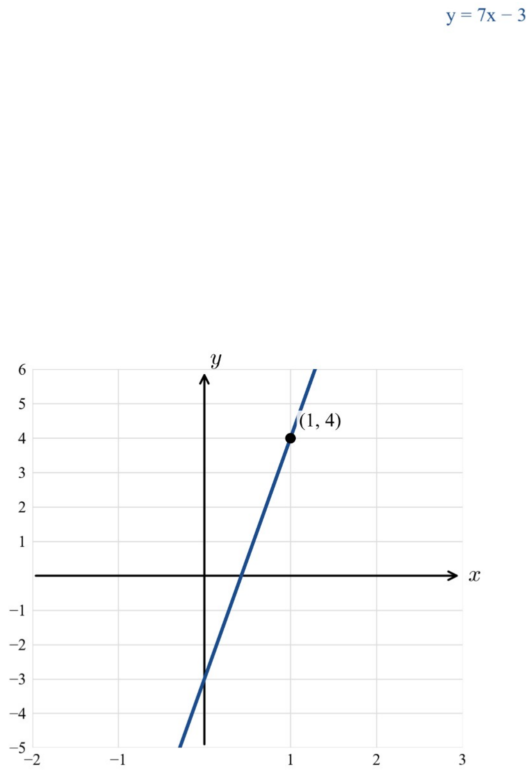
**Q12.** Parallel to $y=-x+2$ means gradient $-1$; with $y$-intercept $-3$: $y=-x-3$.



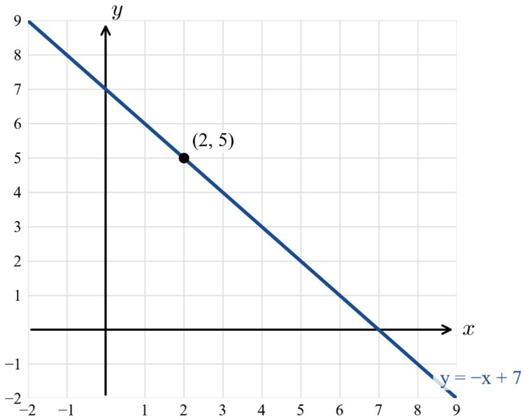

**Q13.** Substitute $(2, 11)$: $11 = (k-2)(2)+3 = 2k-1 \Rightarrow 2k = 12 \Rightarrow k = 6$. $\therefore\ y = 4x+3$.



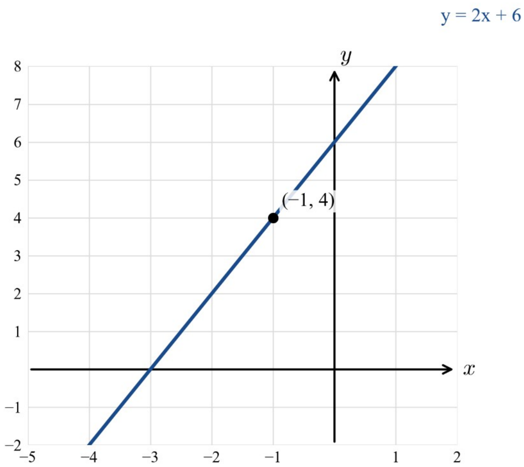

**Q14.** Substitute $(1, 4)$: $4 = (2k+1)(1) - k = k+1 \Rightarrow k = 3$. $\therefore\ y = 7x-3$.



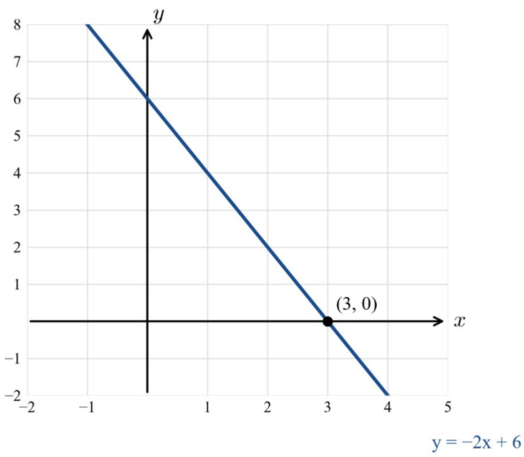

**Q15.** All lines through $(2, 5)$ are $y-5 = m(x-2)$. With gradient $-1$: $y-5 = -(x-2) \Rightarrow y = -x+7$.




**Q16.** All lines through $(-1, 4)$ are $y-4=m(x+1)$. Parallel to $y=2x-3$ means $m=2$: $y-4=2(x+1) \Rightarrow y=2x+6$.

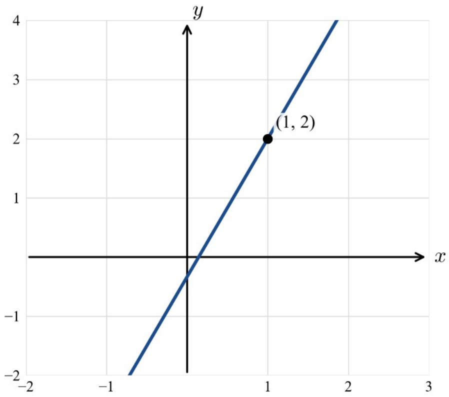



**Q17.** The $x$-intercept is 3, so the line passes through $(3, 0)$: $0=m(3)+6 \Rightarrow m=-2$. $\therefore y=-2x+6$.




**Q18.** Substitute $(1, 2)$: $2=(k+1)(1)+(2k-3)=3k-2 \Rightarrow 3k=4 \Rightarrow k=4/3$. $\therefore y=7/3\,x-1/3$.

y = 7/3 x − 1/3



### Theory

Many real situations involve a quantity that changes at a **constant rate**. These can be modelled by a straight line.

**The model.** A linear model has the form

$$y = mx + c,$$

where $x$ and $y$ are the quantities being related (often with their own letters and units).

- The **gradient** $m$ is the **rate of change** — how much $y$ changes for each 1-unit increase in $x$. Its units are (units of $y$) per (unit of $x$).
- The $y$**-intercept** $c$ is the **initial or fixed value** — the value of $y$ when $x = 0$.

**Finding the rule from two data points** $(x_1, y_1)$ and $(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

then substitute one point to find $c$.

**Using a model.** Substitute a value to **predict** the other quantity. Always keep a **sensible domain** — only use values of $x$ for which the model makes sense (for example $t \geq 0$ for time).

**Comparing two models.** Two linear models cost the same (or meet) where they are **equal** — solve the two rules simultaneously. This is the “break-even” point.

Always **interpret** the gradient and intercept in the context of the problem, and include units in the final answer.

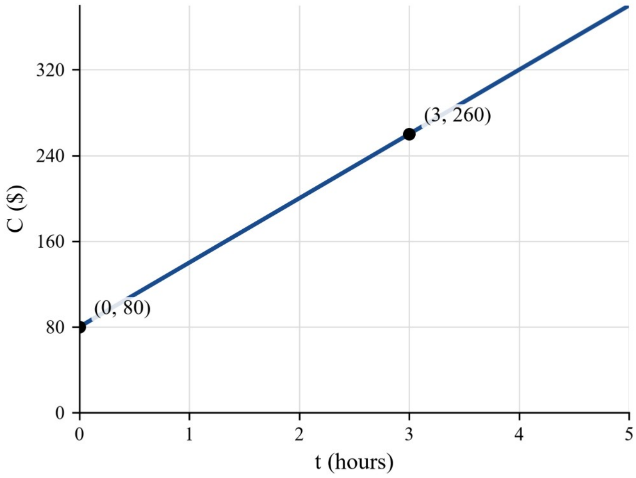
### Worked examples

#### *Example 1 — scaffolded*

A plumber charges an \$80 call-out fee plus \$60 per hour. Let $C$ be the total cost, in dollars, of a job lasting $t$ hours.

| Working | Comment |
|---|---|
| Cost = fixed fee + rate × time: $C = 60t + 80$. | The \$60 per hour is the rate (gradient); the \$80 fee is the fixed value (intercept). |
| For a 3-hour job: $C = 60(3) + 80 = 180 + 80 = 260$. | Substitute $t = 3$. |
| The gradient 60 means the cost rises by \$60 for each extra hour; the intercept 80 is the call-out fee (the cost when $t = 0$). | Interpret both numbers in context. |

∴ a 3-hour job costs \$260.

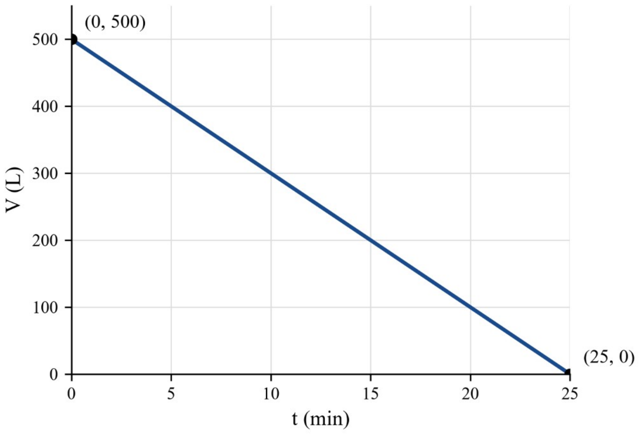



***Example 2 — scaffolded***

A tank holds 500 L of water and drains at a constant 20 L per minute. Let $V$ be the volume, in litres, after $t$ minutes.

| Working | Comment |
|---|---|
| $V = 500 - 20t$. | Starts at 500 L and falls by 20 L each minute, so the gradient is $-20$. |
| The tank is empty when $V = 0$:<br>$500 - 20t = 0 \Rightarrow 20t = 500 \Rightarrow t = 25$. | Solve for $t$. |
| Sensible domain: $0 \le t \le 25$. | The model only applies while there is water in the tank. |

$\therefore$ the tank is empty after 25 minutes.

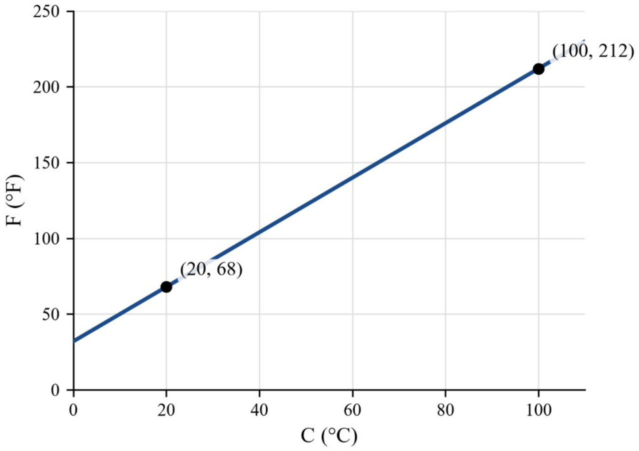



***Example 3 — unscaffolded***

Degrees Fahrenheit $F$ and degrees Celsius $C$ are related by $F = 1.8C + 32$. (a) Convert $20^\circ$C to °F. (b) Convert $212^\circ$F to °C.

(a) $F = 1.8(20) + 32 = 36 + 32 = 68$. $\therefore 20^\circ\mathrm{C} = 68^\circ\mathrm{F}$.

(b) $212 = 1.8C + 32 \Rightarrow 1.8C = 180 \Rightarrow C = 100$. $\therefore 212^\circ\mathrm{F} = 100^\circ\mathrm{C}$.



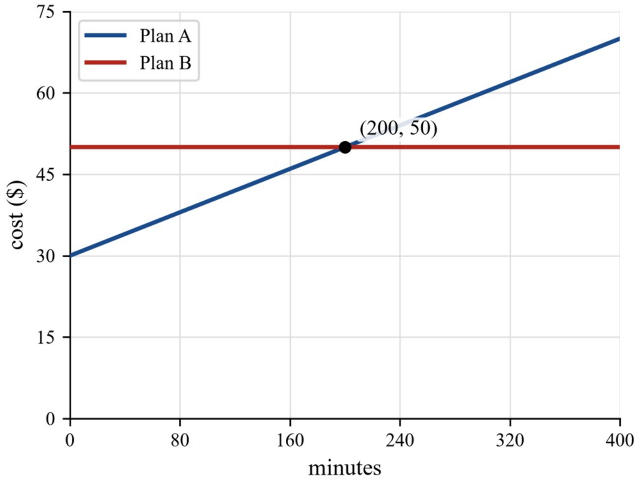

### *Example 4 — unscaffolded*

Plan A costs \$30 per month plus \$0.10 per minute of calls; Plan B costs a flat \$50 per month. For how many minutes of calls do the two plans cost the same?

Let $x$ be the number of minutes. Plan A: $C = 0.1x + 30$; Plan B: $C = 50$.

Equal cost: $0.1x + 30 = 50 \Rightarrow 0.1x = 20 \Rightarrow x = 200$.

∴ the plans cost the same for 200 minutes of calls (Plan A is cheaper below 200 minutes, Plan B above).

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## Practice questions

### *Scaffolded*

**Q1.** A gym charges a \$50 joining fee plus \$20 per month. Let $C$ be the total cost after $n$ months. (a) Write a rule for $C$. (b) Find the cost after 6 months. (c) State what the gradient and the intercept represent.

**Q2.** A car is 100 km from home and travels towards home at 80 km/h. Let $d$ be the distance (km) from home after $t$ hours. (a) Write a rule for $d$. (b) Find $d$ after 1 hour. (c) When does the car reach home?

**Q3.** A candle is 25 cm tall and burns down at 2 cm per hour. Let $h$ be its height after $t$ hours. (a) Write a rule for $h$. (b) Find the height after 5 hours. (c) How long until the candle burns out?

**Q4.** A straight line passes through $(2, 7)$ and $(4, 13)$. Find its rule $y = mx + c$.

**Q5.** A printing service charges a \$15 setup fee plus \$0.20 per page. Let $C$ be the cost of $n$ pages. (a) Write a rule. (b) Find the cost of 100 pages. (c) Interpret the gradient.

**Q6.** Company A charges \$40 plus \$0.50 per item; Company B charges \$25 plus \$1.00 per item. For how many items do they charge the same, and what is that cost?

### *Unscaffolded*

For each of the following, form or use a linear model and answer the question. State units.

**Q7.** A taxi charges a \$4.20 flagfall plus \$2.10 per kilometre. Find the fare for an 8 km trip.

**Q8.** A phone plan costs \$15 per month plus \$0.05 per text message. Find the monthly cost when 300 texts are sent.

**Q9.** A cyclist is 180 km from the finish and rides at a constant 90 km/h. (a) Write a rule for the distance remaining, $r$, after $t$ hours. (b) Find $r$ after 1.5 hours. (c) When does the cyclist finish?

**Q10.** A pool contains 2000 L and is filled at 150 L per minute. (a) Write a rule for the volume $V$ after $t$ minutes. (b) Find $V$ after 10 minutes. (c) The pool is full at 5000 L — how long does filling take?

**Q11.** Fahrenheit and Celsius are related by $F=1.8C+32$. Find the Celsius temperature when $F=50$.

**Q12.** A straight line passes through $(1,5)$ and $(3,11)$. (a) Find its rule. (b) Hence find $y$ when $x=10$.

**Q13.** A salesperson's weekly salary is $S=0.05x+800$, where $x$ is the value of sales in dollars. (a) State what the gradient and intercept represent. (b) Find the salary when sales are \$10,000.

**Q14.** A seedling is 12 cm tall in week 2 and 20 cm tall in week 6, growing linearly. (a) Find a rule for the height $h$ in week $t$. (b) Predict the height in week 10.

**Q15.** Gym A charges a \$60 joining fee plus \$15 per month; Gym B charges \$30 per month with no joining fee. After how many months is the total cost the same?

**Q16.** On a certain day, \$1 AUD converts to \$0.65 USD. (a) Convert \$200 AUD to USD. (b) How many AUD equal \$91 USD?

**Q17.** A new car worth \$30,000 depreciates by \$2500 each year. (a) Write a rule for the value $V$ after $t$ years. (b) Find the value after 4 years. (c) When is the car worth nothing according to the model?

**Q18.** A worker is paid a \$200 base plus \$18 per hour. Find the pay for 38 hours of work.

### Answers

**Q1.** $C = 20n + 50$; \$170; gradient \$20/month, intercept \$50 joining fee. **Q2.** $d = 100 - 80t$; 20 km; after 1.25 hours (1 h 15 min). **Q3.** $h = 25 - 2t$; 15 cm; after 12.5 hours. **Q4.** $y = 3x + 1$. **Q5.** $C = 0.2n + 15$; \$35; gradient \$0.20 per page. **Q6.** 30 items; \$55. **Q7.** \$21.00. **Q8.** \$30.00. **Q9.** $r = 180 - 90t$; 45 km; after 2 hours. **Q10.** $V = 150t + 2000$; 3500 L; 20 minutes. **Q11.** $10^{\circ}$C. **Q12.** $y = 3x + 2$; $y = 32$. **Q13.** gradient = 5% commission on sales, intercept = \$800 base; \$1300. **Q14.** $h = 2t + 8$; 28 cm. **Q15.** 4 months. **Q16.** \$130 USD; \$140 AUD. **Q17.** $V = 30\,000 - 2500t$; \$20,000; after 12 years. **Q18.** \$884.

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### Fully-worked solutions

**Q1.** (a) $C = 20n + 50$. (b) $C = 20(6) + 50 = 120 + 50 = 170$, so \$170. (c) The gradient \$20 is the monthly rate; the intercept \$50 is the fixed joining fee (cost when $n = 0$).

**Q2.** (a) $d = 100 - 80t$. (b) $d = 100 - 80(1) = 20$ km. (c) $d = 0$: $100 - 80t = 0 \Rightarrow t = \frac{100}{80} = 1.25$ hours (1 h 15 min).

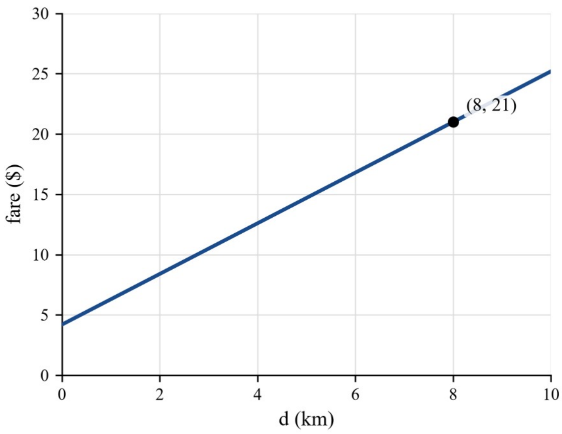
**Q3.** (a) $h = 25 - 2t$. (b) $h = 25 - 2(5) = 15$ cm. (c) $h = 0$: $25 - 2t = 0 \Rightarrow t = 12.5$ hours.

**Q4.** $m = \frac{13 - 7}{4 - 2} = \frac{6}{2} = 3$. Using $(2, 7)$: $7 = 3(2) + c \Rightarrow c = 1$. $\therefore y = 3x + 1$.

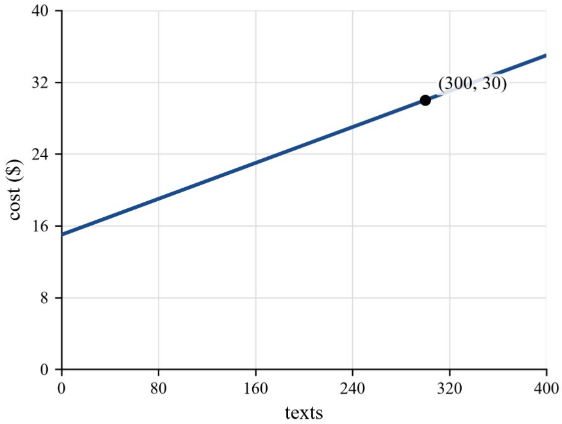
**Q5.** (a) $C = 0.2n + 15$. (b) $C = 0.2(100) + 15 = 20 + 15 = 35$, so \$35. (c) The gradient \$0.20 is the cost of each additional page.

**Q6.** Let $x$ be the number of items. A: $C = 0.5x + 40$; B: $C = x + 25$. Equal: $0.5x + 40 = x + 25 \Rightarrow 15 = 0.5x \Rightarrow x = 30$. Cost $= 30 + 25 = 55$. $\therefore$ 30 items, each costing \$55.

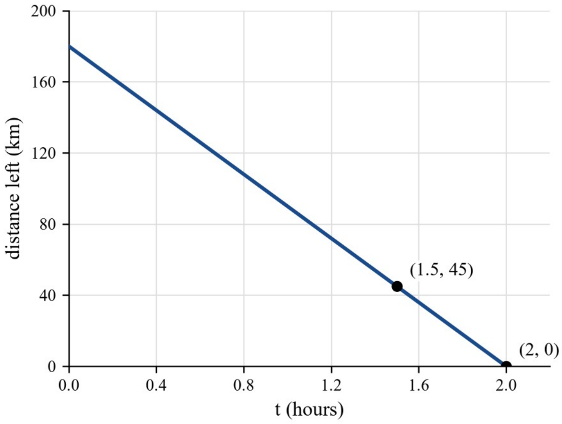
**Q7.** Fare $F = 2.1d + 4.2$. For $d = 8$: $F = 2.1(8) + 4.2 = 16.8 + 4.2 = 21$. $\therefore$ the fare is \$21.00.



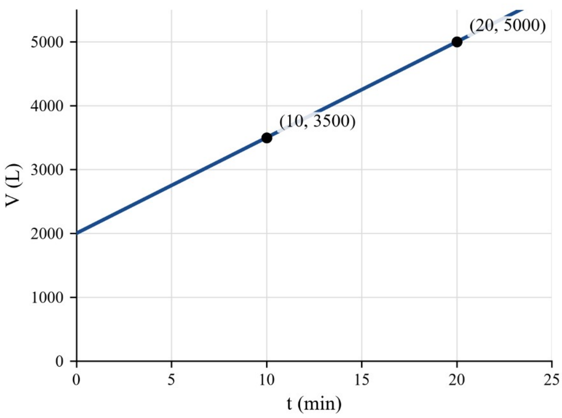

**Q8.** Cost $C = 0.05t + 15$ (where $t$ = number of texts). For $t = 300$: $C = 0.05(300) + 15 = 15 + 15 = 30$. $\therefore$ the monthly cost is \$30.00.



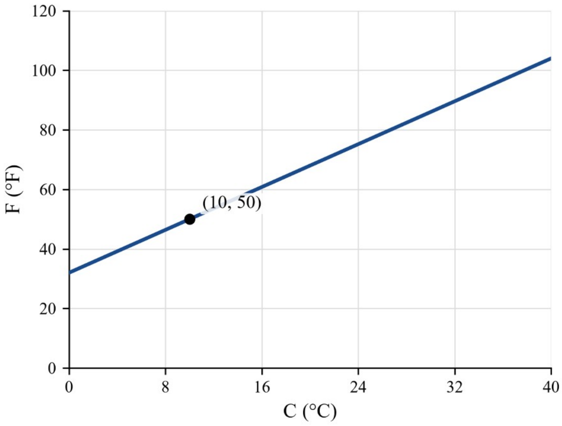

**Q9.** (a) $r = 180 - 90t$. (b) $r = 180 - 90(1.5) = 180 - 135 = 45$ km. (c) $r = 0$: $180 - 90t = 0 \Rightarrow t = 2$ hours.



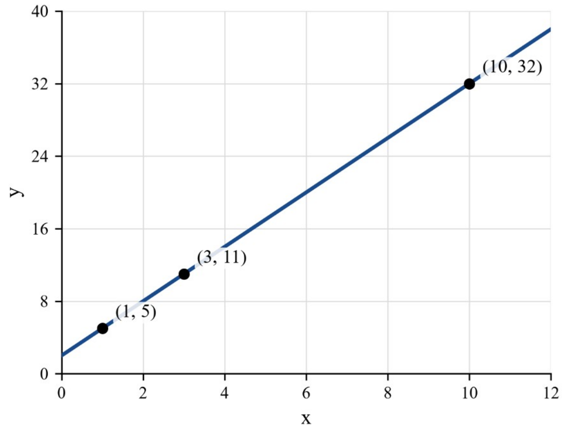

**Q10.** (a) $V = 150t + 2000$. (b) $V = 150(10) + 2000 = 3500$ L. (c) $V = 5000$:
$150t + 2000 = 5000 \Rightarrow 150t = 3000 \Rightarrow t = 20$ minutes.



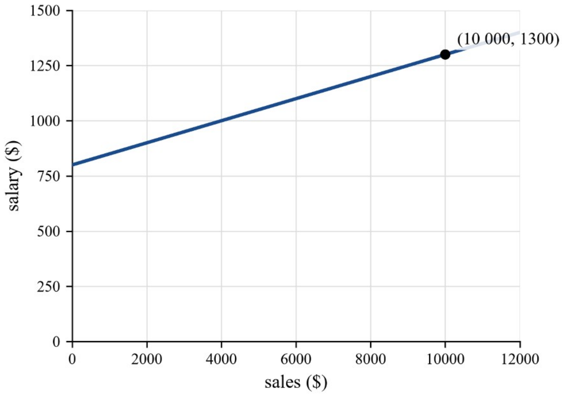

**Q11.** $50 = 1.8C + 32 \Rightarrow 1.8C = 18 \Rightarrow C = 10$. $\therefore$ the temperature is $10^{\circ}$C.



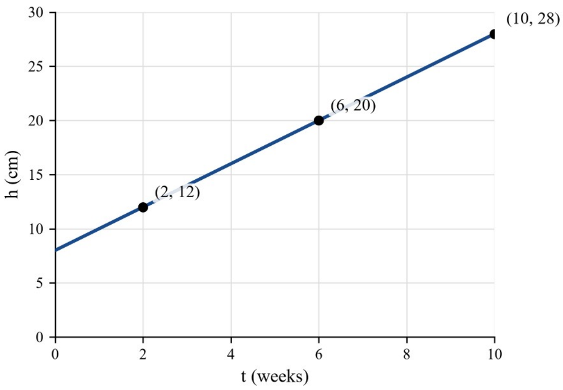
**Q12.** (a) $m = \frac{11-5}{3-1} = 3$; using $(1, 5)$: $5 = 3(1) + c \Rightarrow c = 2$, so $y = 3x + 2$. (b) $y = 3(10) + 2 = 32$.



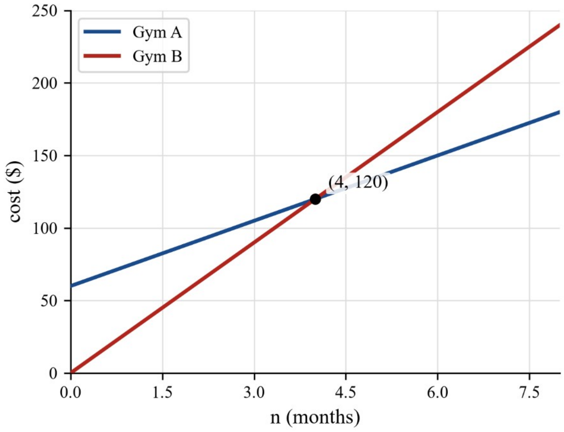

**Q13.** (a) The gradient $0.05$ means the salary rises by $0.05 for every $1 of sales (a 5% commission); the intercept $800 is the fixed base salary. (b) $S = 0.05(10\,000) + 800 = 500 + 800 = 1300$, so $1300.



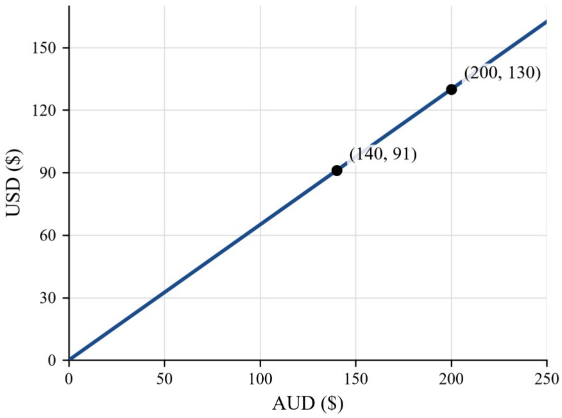

**Q14.** (a) $m = \frac{20-12}{6-2} = \frac{8}{4} = 2$; using $(2, 12)$: $12 = 2(2) + c \Rightarrow c = 8$, so $h = 2t + 8$. (b) $h = 2(10) + 8 = 28$ cm.



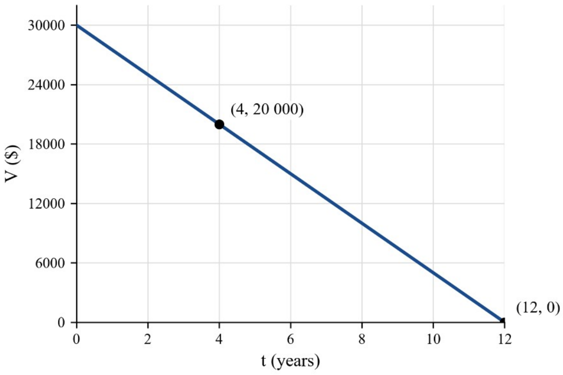

**Q15.** Gym A: $C = 15n + 60$; Gym B: $C = 30n$. Equal: $15n + 60 = 30n \Rightarrow 60 = 15n \Rightarrow n = 4$. ∴ the costs are equal after 4 months (each $120).



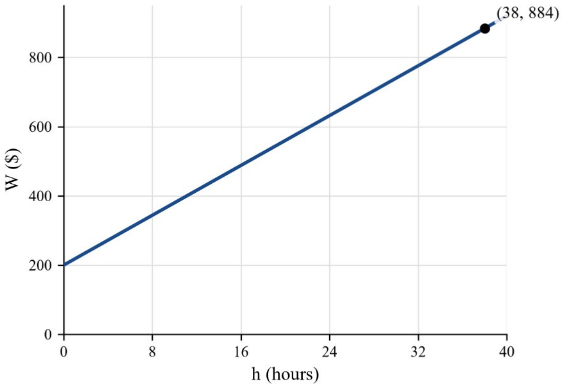

**Q16.** Model: $U = 0.65A$ (USD from AUD). (a) $U = 0.65(200) = 130$, so \$130 USD. (b) $91 = 0.65A \Rightarrow A = \dfrac{91}{0.65} = 140$, so \$140 AUD.




**Q17.** (a) $V = 30\,000 - 2500t$. (b) $V = 30\,000 - 2500(4) = 30\,000 - 10\,000 = 20\,000$, so \$20,000. (c) $V = 0$: $30\,000 - 2500t = 0 \Rightarrow 2500t = 30\,000 \Rightarrow t = 12$ years.




**Q18.** Pay $W = 18h + 200$. For $h = 38$: $W = 18(38) + 200 = 684 + 200 = 884$. $\therefore$ the pay is \$884.



## Chapter 2 Coordinate geometry and straight-line graphs — 2H Systems of simultaneous linear equations

### Theory

Two linear equations considered together are called **simultaneous equations**. Because the graph of each is a straight line, the **solution** of the pair is the **point of intersection** of the two lines — the single point $(x, y)$ that lies on both.

**Solving graphically.** Draw both lines accurately (using the gradient and $y$-intercept, or a table of values), then read off the coordinates of the point where they cross. Always **check** the result by substituting it back into both equations.

**How many solutions?** Compare the two lines:

- **Different gradients** — the lines cross exactly once, so there is **one solution**.
- **Equal gradients, different $y$-intercepts** — the lines are **parallel** and never meet, so there is **no solution**.
- **Equal gradients and equal $y$-intercepts** — the equations describe the **same line**, so every point on it is a solution: **infinitely many solutions**.

This means you can predict the number of solutions **without solving**, just by comparing gradients (and, if the gradients are equal, the $y$-intercepts).

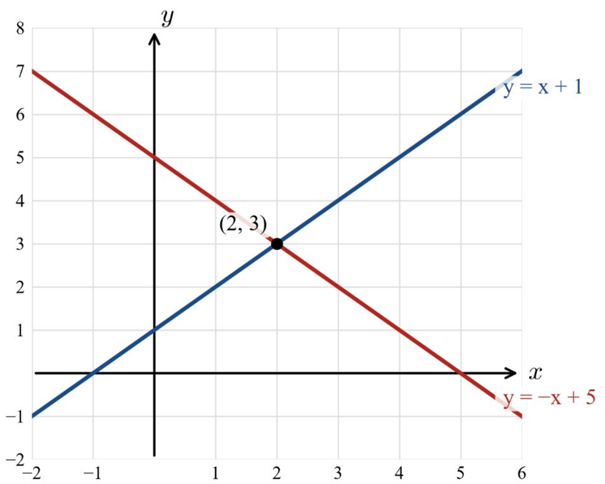
### Worked examples

#### *Example 1 — scaffolded*

Solve the simultaneous equations $y = x + 1$ and $y = -x + 5$ graphically, and check your answer.

| Working | Comment |
|---|---|
| $y = x + 1$ has gradient 1 and $y$-intercept $(0, 1)$; $y = -x + 5$ has gradient $-1$ and $y$-intercept $(0, 5)$. | Read the gradient and $y$-intercept from each rule. |
| The lines cross at $(2, 3)$. | Draw both lines and read the point of intersection. |
| Check: $x + 1 = -x + 5 \Rightarrow 2x = 4 \Rightarrow x = 2$, then $y = 2 + 1 = 3$. | Confirm algebraically by equating the two expressions for $y$. |

$\therefore$ the solution is $(2, 3)$.

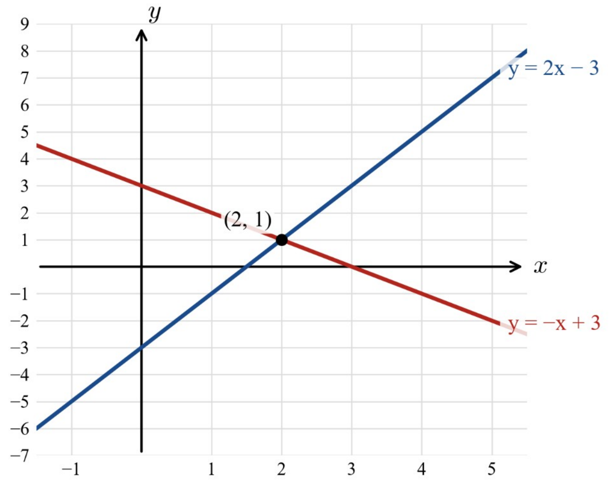



#### *Example 2 — scaffolded*

Solve $y = 2x - 3$ and $y = -x + 3$ graphically, and check your answer.

| Working | Comment |
|---|---|
| $y=2x-3$: gradient 2, $y$-intercept $(0,-3)$; $y=-x+3$: gradient $-1$, $y$-intercept $(0,3)$. | Different gradients, so expect exactly one solution. |
| The lines cross at $(2,1)$. | Read the intersection from the graph. |
| Check: $2x-3=-x+3 \Rightarrow 3x=6 \Rightarrow x=2$, then $y=2(2)-3=1$. | Confirm algebraically. |

∴ the solution is $(2,1)$.



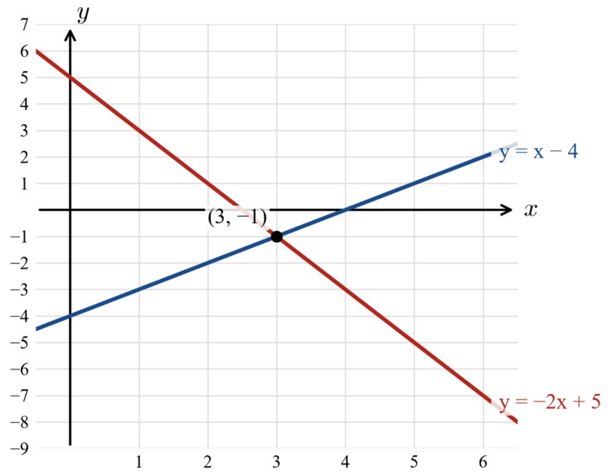

### *Example 3 — unscaffolded*

Solve $y=x-4$ and $y=-2x+5$ graphically.

The lines have different gradients (1 and $-2$), so they meet once. From the graph they cross at $(3,-1)$.

Check: $x-4=-2x+5 \Rightarrow 3x=9 \Rightarrow x=3$, then $y=3-4=-1$.

∴ the solution is $(3,-1)$.

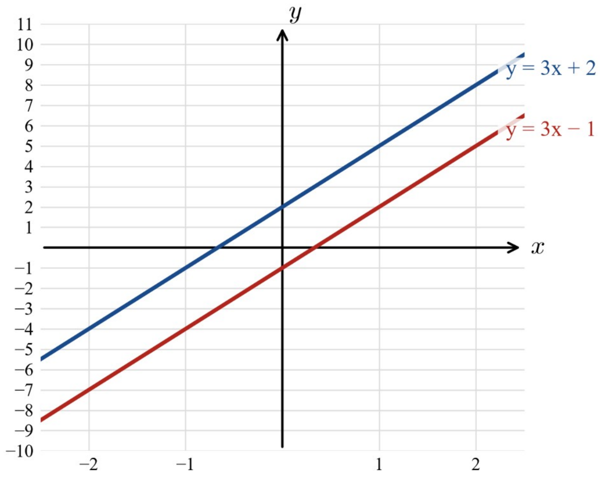



### *Example 4 — unscaffolded*

How many solutions does the pair $y=3x+2$ and $y=3x-1$ have?

Both lines have gradient 3, but their $y$-intercepts differ (2 and $-1$). Equal gradients with different $y$-intercepts means the lines are **parallel**.

∴ there is **no solution** — the lines never meet.




## Practice questions

### *Scaffolded*

**Q1.** For $y=x+2$ and $y=-x+4$: (a) state the gradient and $y$-intercept of each line; (b) draw both lines and read the point of intersection; (c) check your answer algebraically.

**Q2.** For $y=2x$ and $y=x+2$: (a) draw both lines; (b) read the intersection; (c) check.

**Q3.** For $y=x+1$ and $y=-2x+4$: (a) draw both lines; (b) read the intersection; (c) check.

**Q4.** For $y=2x+1$ and $y=2x-4$: (a) compare the gradients; (b) hence state how many solutions the pair has and why.

**Q5.** For $y=2x+3$ and $2y=4x+6$: (a) write the second equation in the form $y=mx+c$; (b) hence state how many solutions the pair has and why.

**Q6.** For $y=x-1$ and $y=-x+5$: (a) draw both lines; (b) read the intersection; (c) check.

### *Unscaffolded*

Solve each pair graphically, then check algebraically. For any pair without a single solution, state whether there are **no** solutions or **infinitely many**, and why.

**Q7.** $y=x+3$ and $y=-2x+6$

**Q8.** $y=3x-5$ and $y=x+1$

**Q9.** $y=-x+1$ and $y=x-3$

**Q10.** $y=-2x-1$ and $y=x-4$

**Q11.** $y=1/2x+3$ and $y=-x$

**Q12.** $x+y=6$ and $x-y=2$

**Q13.** $y=4x$ and $y=2x+4$

**Q14.** $y=-3x+2$ and $y=-3x-5$

**Q15.** $y=1/2x-2$ and $2x-4y=8$

**Q16.** $y=x-1$ and $y=-3x+11$

**Q17.** $y=-x+2$ and $y=2x-7$

**Q18.** $y=2x-1$ and $y=-x+5$

### Answers

**Q1.** $(1,3)$. **Q2.** $(2,4)$. **Q3.** $(1,2)$. **Q4.** No solution (parallel — equal gradients, different $y$-intercepts). **Q5.** Infinitely many (same line). **Q6.** $(3,2)$. **Q7.** $(1,4)$. **Q8.** $(3,4)$. **Q9.** $(2,-1)$. **Q10.** $(1,-3)$. **Q11.** $(-2,2)$. **Q12.** $(4,2)$. **Q13.** $(2,8)$. **Q14.** No solution (parallel). **Q15.** Infinitely many (same line). **Q16.** $(3,2)$. **Q17.** $(3,-1)$. **Q18.** $(2,3)$.

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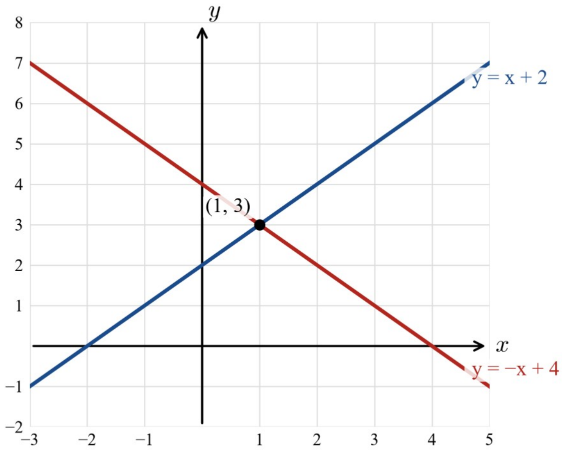
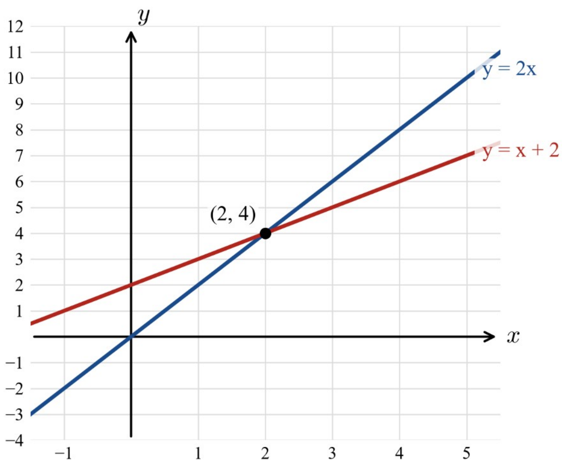
### Fully-worked solutions

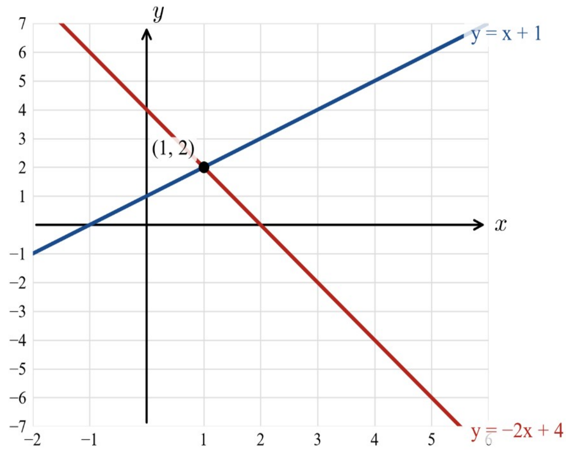
**Q1.** $y=x+2$ (gradient 1, $y$-int $(0,2)$) and $y=-x+4$ (gradient $-1$, $y$-int $(0,4)$). Equate: $x+2=-x+4 \Rightarrow 2x=2 \Rightarrow x=1$, then $y=3$. $\therefore (1,3)$.



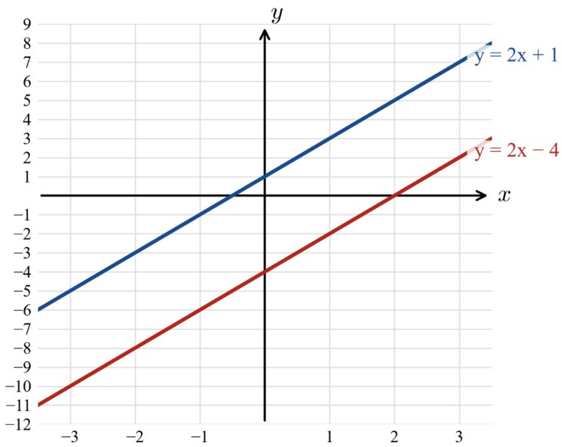

**Q2.** $y=2x$ and $y=x+2$. Equate: $2x=x+2 \Rightarrow x=2$, then $y=4$. $\therefore (2,4)$.



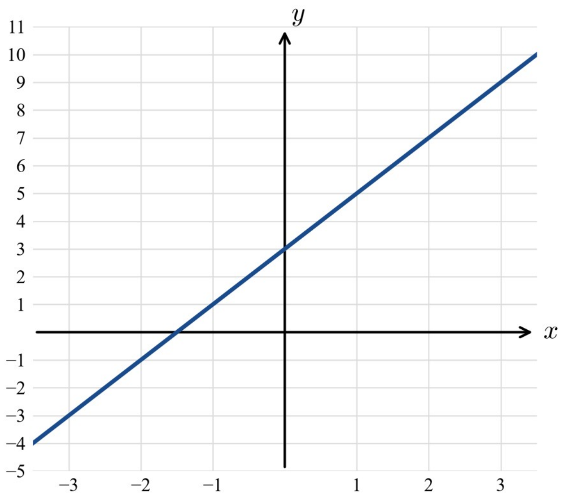

**Q3.** $y=x+1$ and $y=-2x+4$. Equate: $x+1=-2x+4 \Rightarrow 3x=3 \Rightarrow x=1$, then $y=2$. $\therefore (1,2)$.



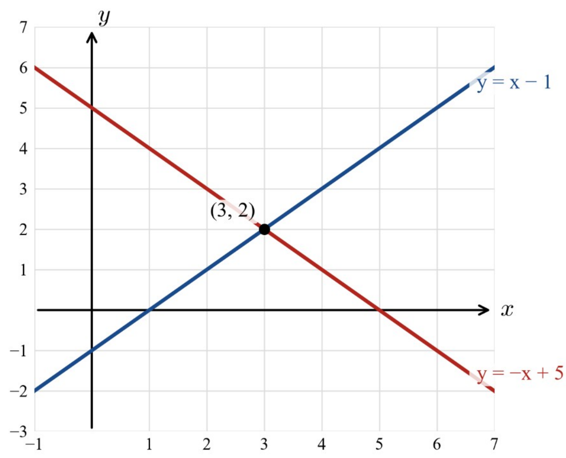

**Q4.** Both lines have gradient $2$ but different $y$-intercepts ($1$ and $-4$), so they are parallel. ∴ **no solution**.



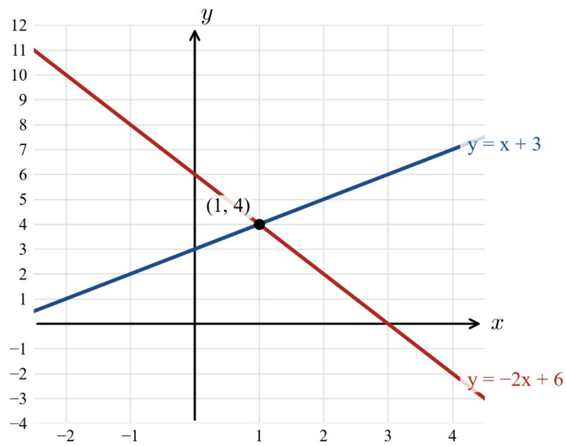

**Q5.** $2y = 4x + 6 \Rightarrow y = 2x + 3$, which is identical to the first equation. The two equations describe the **same line**. ∴ **infinitely many solutions**.



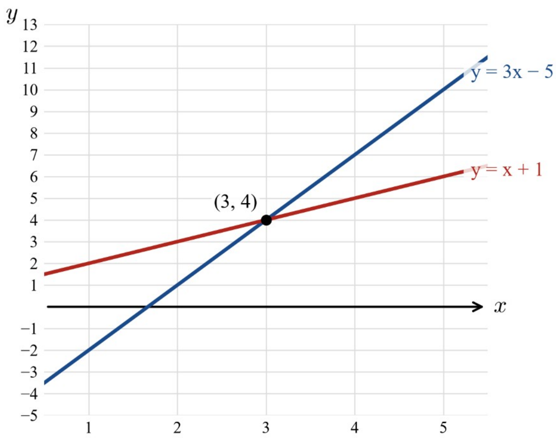

**Q6.** $y = x - 1$ and $y = -x + 5$. Equate: $x - 1 = -x + 5 \Rightarrow 2x = 6 \Rightarrow x = 3$, then $y = 2$. $\therefore (3, 2)$.



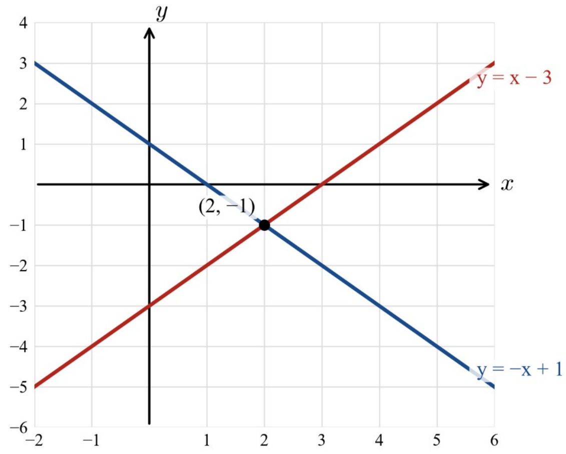

**Q7.** $x+3=-2x+6 \Rightarrow 3x=3 \Rightarrow x=1$, then $y=4$. $\therefore (1,4)$.



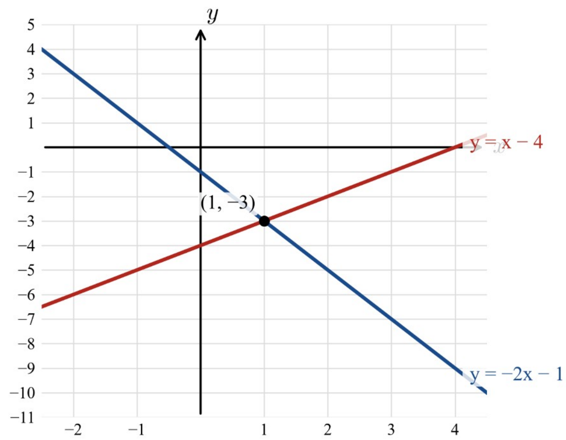

**Q8.** $3x-5=x+1 \Rightarrow 2x=6 \Rightarrow x=3$, then $y=4$. $\therefore (3,4)$.



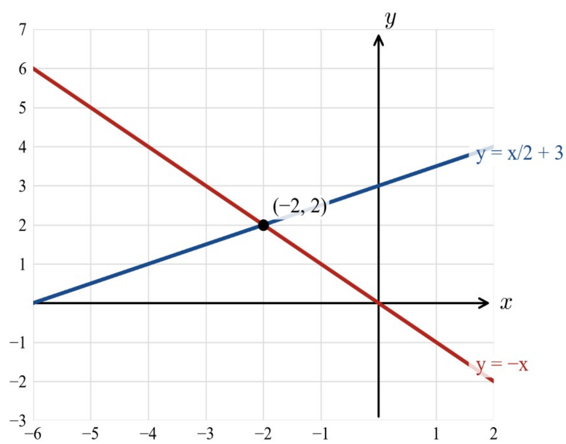

**Q9.** $-x+1=x-3 \Rightarrow 4=2x \Rightarrow x=2$, then $y=-1$. $\therefore (2,-1)$.



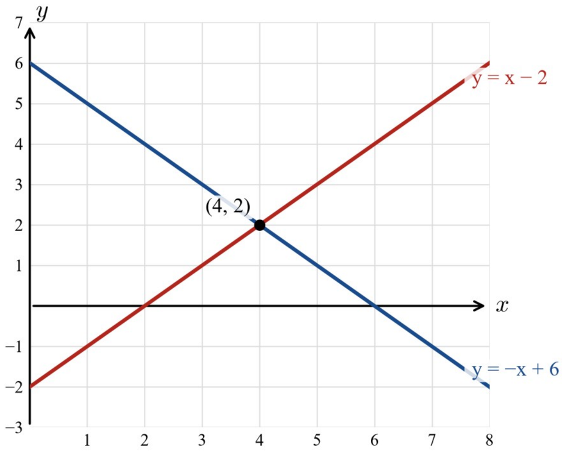

**Q10.** $-2x-1=x-4 \Rightarrow 3=3x \Rightarrow x=1$, then $y=-3$. $\therefore (1,-3)$.



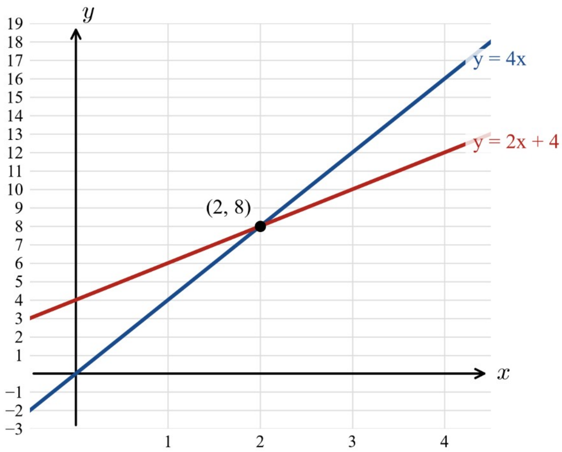

**Q11.** $1/2x+3=-x \Rightarrow 3/2x=-3 \Rightarrow x=-2$, then $y=2$. $\therefore (-2,2)$.



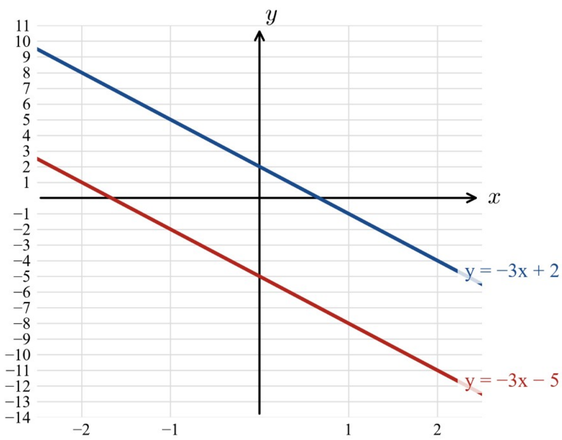

**Q12.** From the graphs (or by elimination): adding $x+y=6$ and $x-y=2$ gives $2x=8$, so $x=4$ and $y=2$. $\therefore (4,2)$.



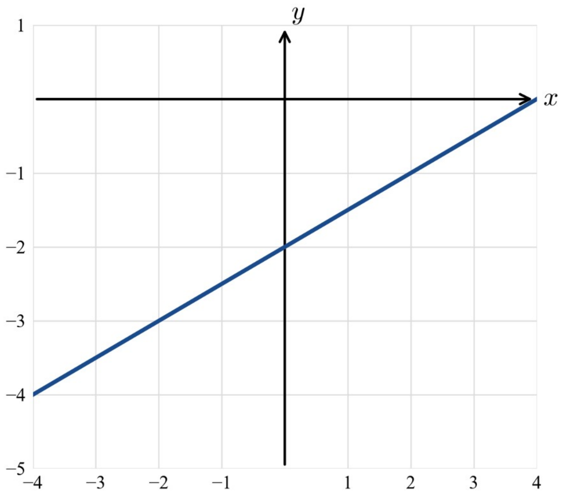

**Q13.** $4x = 2x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$, then $y = 8$. $\therefore (2, 8)$.



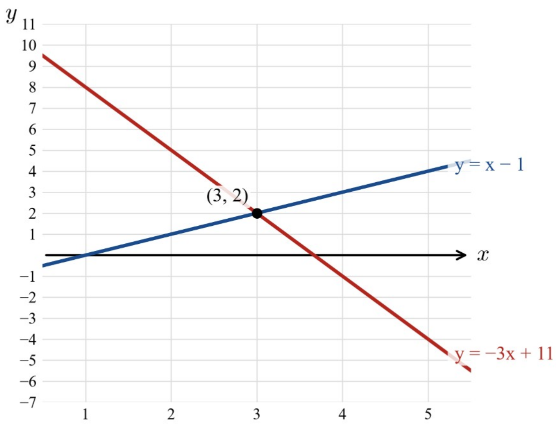

**Q14.** Both lines have gradient $-3$ with different $y$-intercepts (2 and $-5$), so they are parallel. ∴ **no solution**.



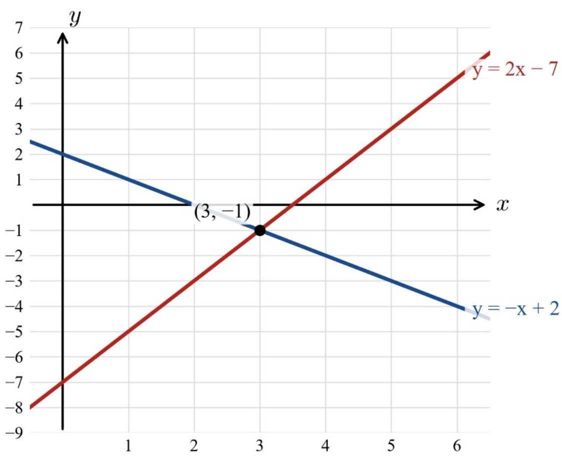

**Q15.** $2x - 4y = 8 \Rightarrow -4y = 8 - 2x \Rightarrow y = 1/2\,x - 2$, identical to the first equation — the **same line**. ∴ **infinitely many solutions**.



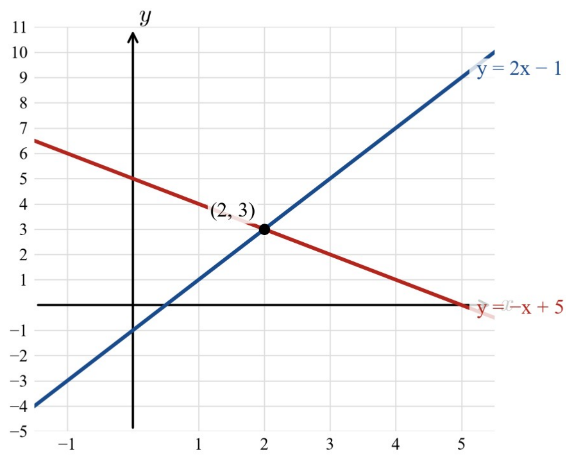

**Q16.** $x-1=-3x+11 \Rightarrow 4x=12 \Rightarrow x=3$, then $y=2$. $\therefore (3,2)$.




**Q17.** $-x+2=2x-7 \Rightarrow 9=3x \Rightarrow x=3$, then $y=-1$. $\therefore (3,-1)$.




**Q18.** $2x-1=-x+5 \Rightarrow 3x=6 \Rightarrow x=2$, then $y=3$. $\therefore (2,3)$.


y
y = 2x − 1
(2, 3)
y = −x + 5
11
10
9
8
7
6
5
4
3
2
1
−1
−2
−3
−4
−5
−1
1
2
3
4
5